Computer Algebra Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

4-Trig-functions/4.7-Miscellaneous/141-4.7.7-Trig-functions

Nasser M. Abbasi

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [950]. This is test number [141].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	99.58 (946)	0.42 (4)
Rubi	99.37 (944)	0.63 (6)
Fricas	96.00 (912)	4.00 (38)
Maple	94.95 (902)	5.05 (48)
Giac	76.32 (725)	23.68 (225)
Mupad	73.68 (700)	26.32 (250)
Maxima	69.05 (656)	30.95 (294)
Sympy	45.68 (434)	54.32 (516)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

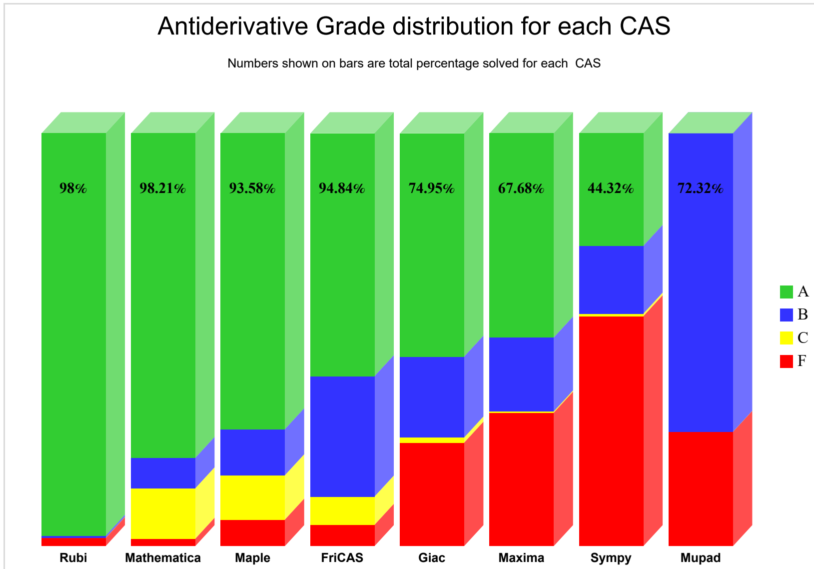
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

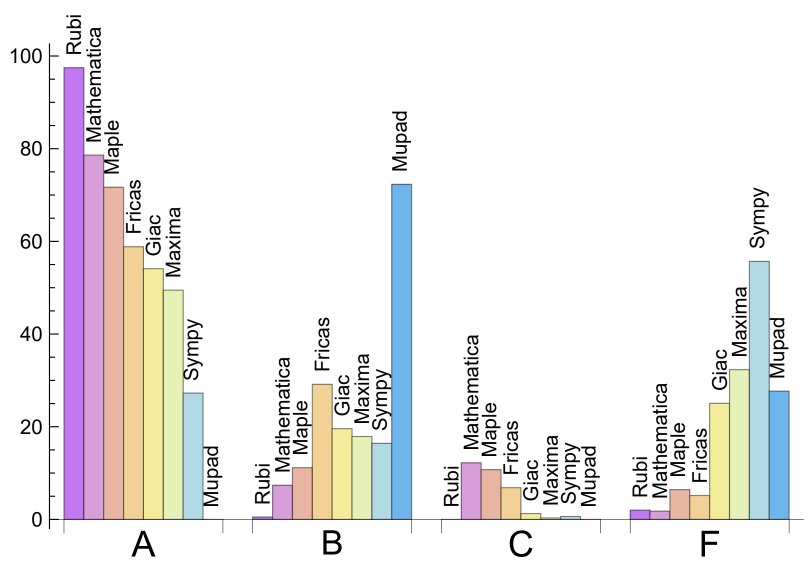
System	% A grade	% B grade	% C grade	% F grade
Rubi	96.947	0.947	0.105	2.000
Mathematica	78.632	7.368	12.211	1.789
Maple	71.684	11.158	10.737	6.421
Fricas	58.842	29.158	6.842	5.158
Giac	54.105	19.579	1.263	25.053
Maxima	49.474	17.895	0.316	32.316
Sympy	27.263	16.421	0.632	55.684
Mupad	0.000	72.316	0.000	27.684

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Mathematica	4	0.00	100.00	0.00
Rubi	6	100.00	0.00	0.00
Fricas	38	47.37	0.00	52.63
Maple	48	87.50	12.50	0.00
Giac	225	79.11	16.44	4.44
Mupad	250	0.00	100.00	0.00
Maxima	294	65.65	1.36	32.99
Sympy	516	67.25	31.40	1.36

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.29
Rubi	0.38
Maxima	0.54
Giac	1.22
Mathematica	1.48
Maple	4.08
Sympy	4.89
Mupad	24.73

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Rubi	73.39	1.03	40.50	1.00
Mupad	175.74	2.27	30.50	1.06
Maxima	207.53	3.51	29.00	1.00
Fricas	251.55	2.25	51.50	1.39
Sympy	470.73	25.49	36.00	1.40
Mathematica	630.50	4.62	40.00	1.00
Giac	833.20	13.52	39.00	1.12
Maple	957.10	4.31	36.00	1.05

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

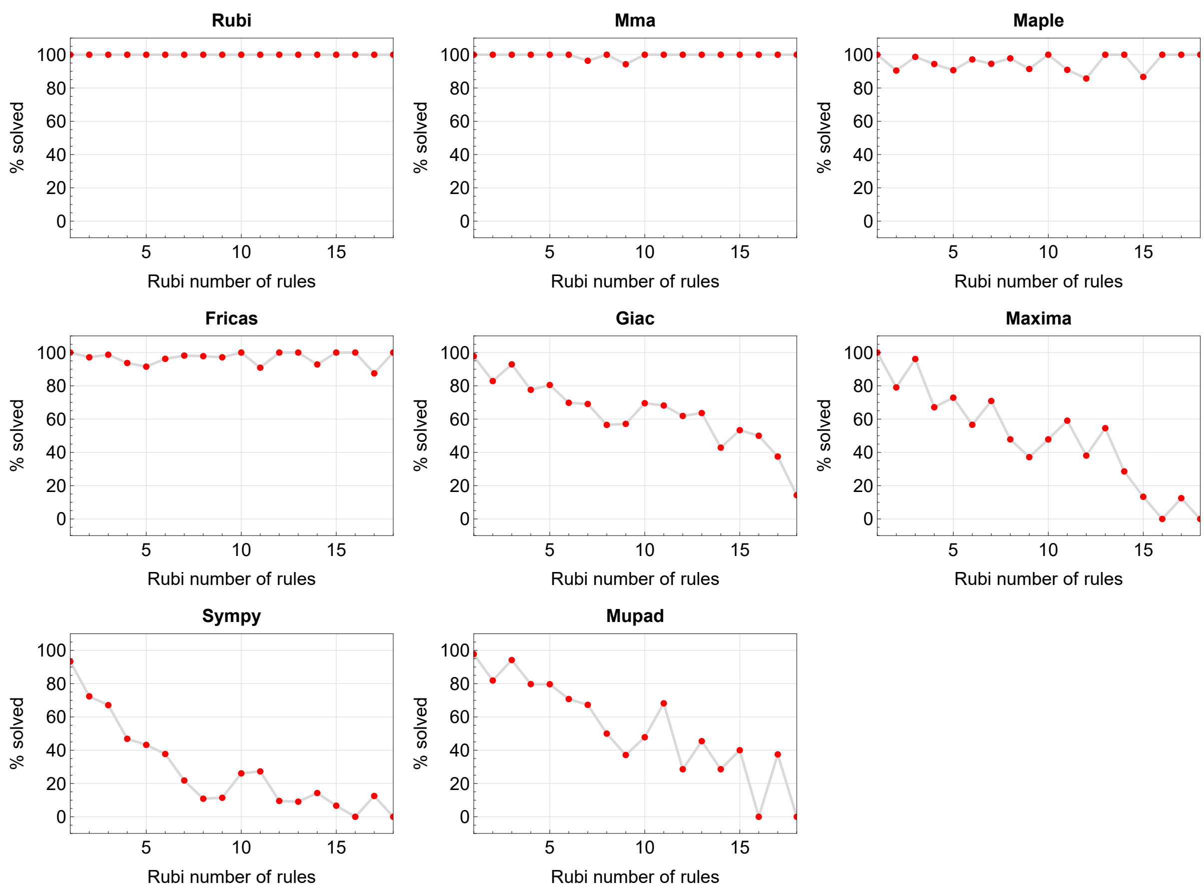


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

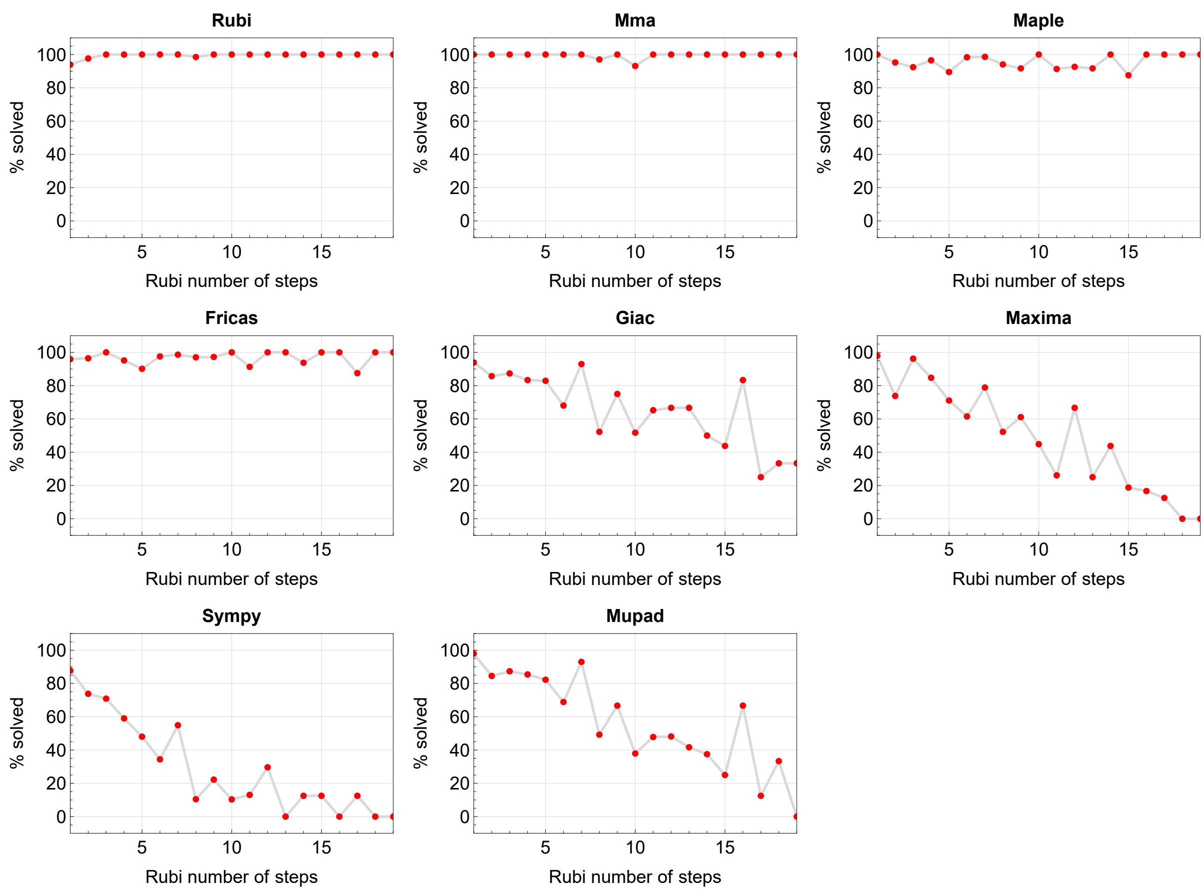


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

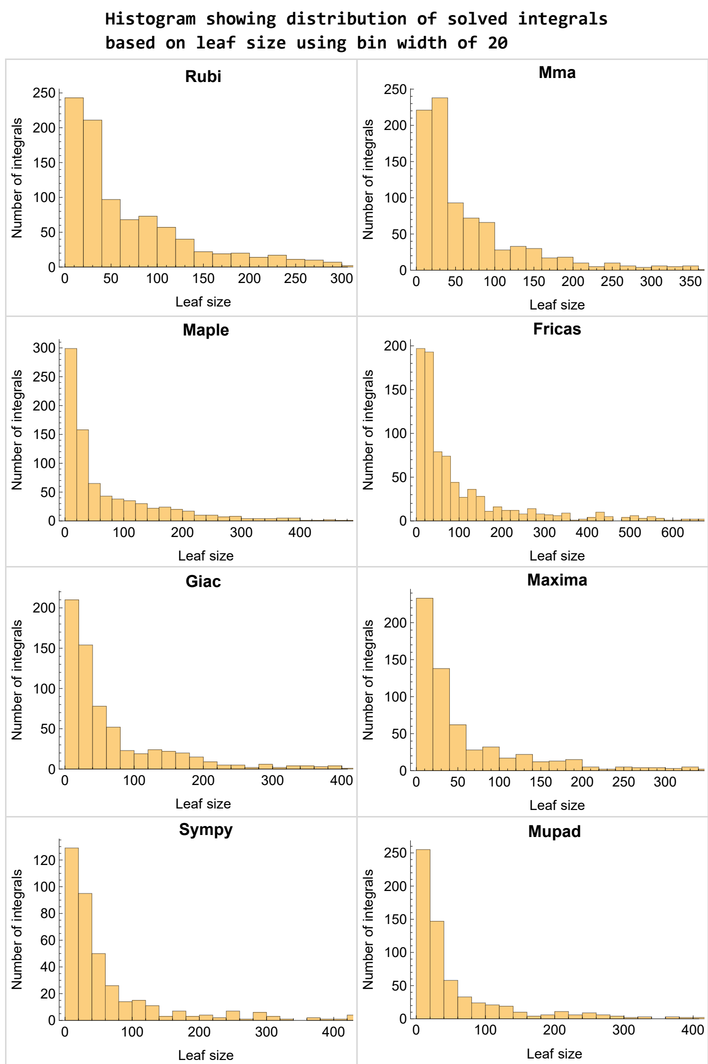


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

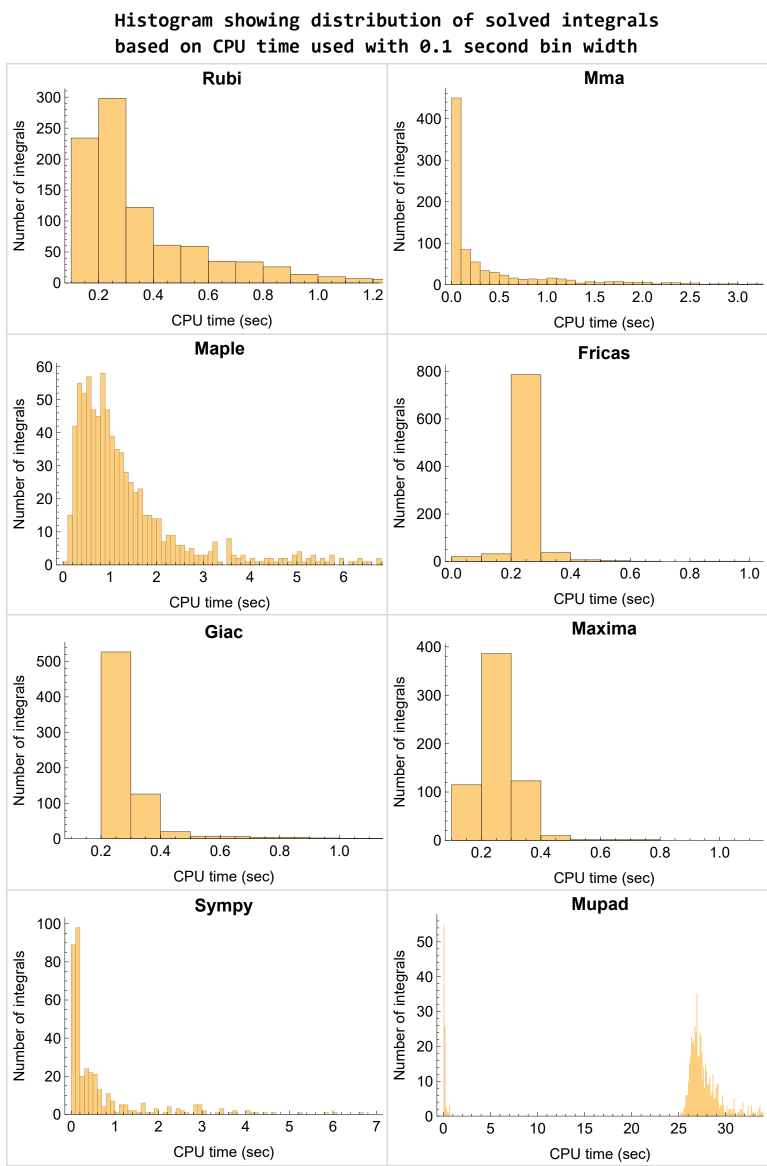


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

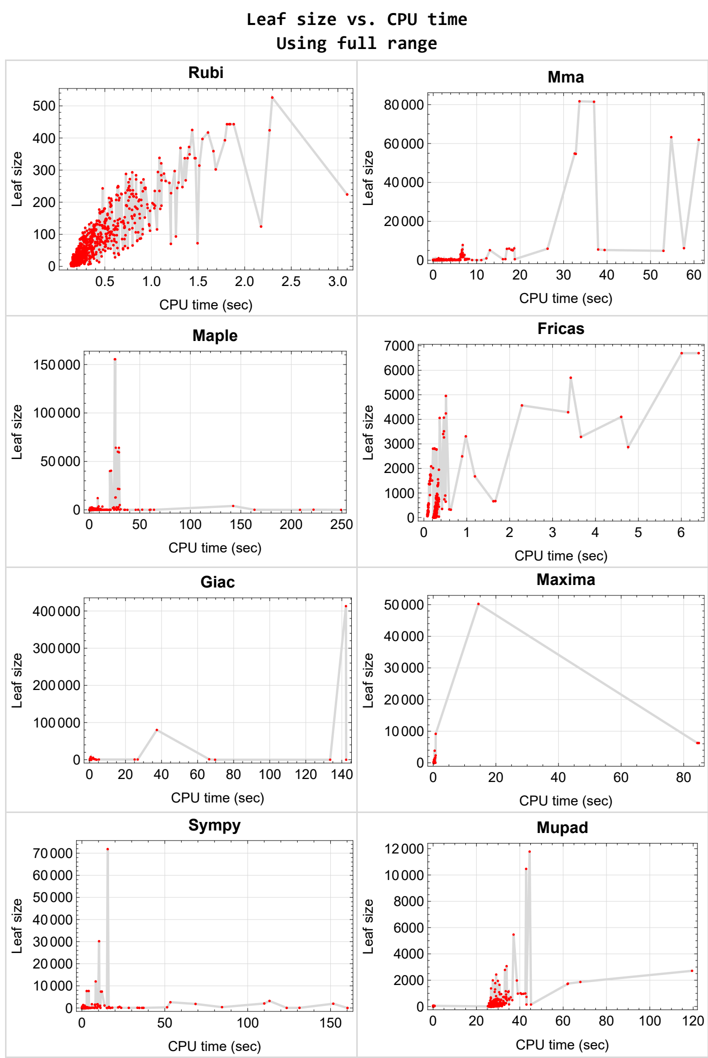


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{42, 43, 56, 57, 180, 582, 583, 584, 644, 645, 646, 647, 932}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {48, 49, 50, 318, 319, 320, 321, 634, 635, 636, 799, 801, 866, 888, 897, 899, 902}

Mathematica {87, 89, 107, 109, 160, 163, 240, 242, 244, 246, 404, 409, 410, 411, 412, 414, 415, 416, 430, 431, 432, 434, 437, 438, 439, 440, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 556, 557, 558, 559, 560, 561, 859}

Maple {34, 410, 411, 414, 415, 416, 448, 450, 451, 452, 453, 455, 456, 457, 462, 463, 464, 465, 466, 467, 469, 470, 471, 504, 505, 506, 507, 515, 556, 557, 558, 559, 560, 561}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

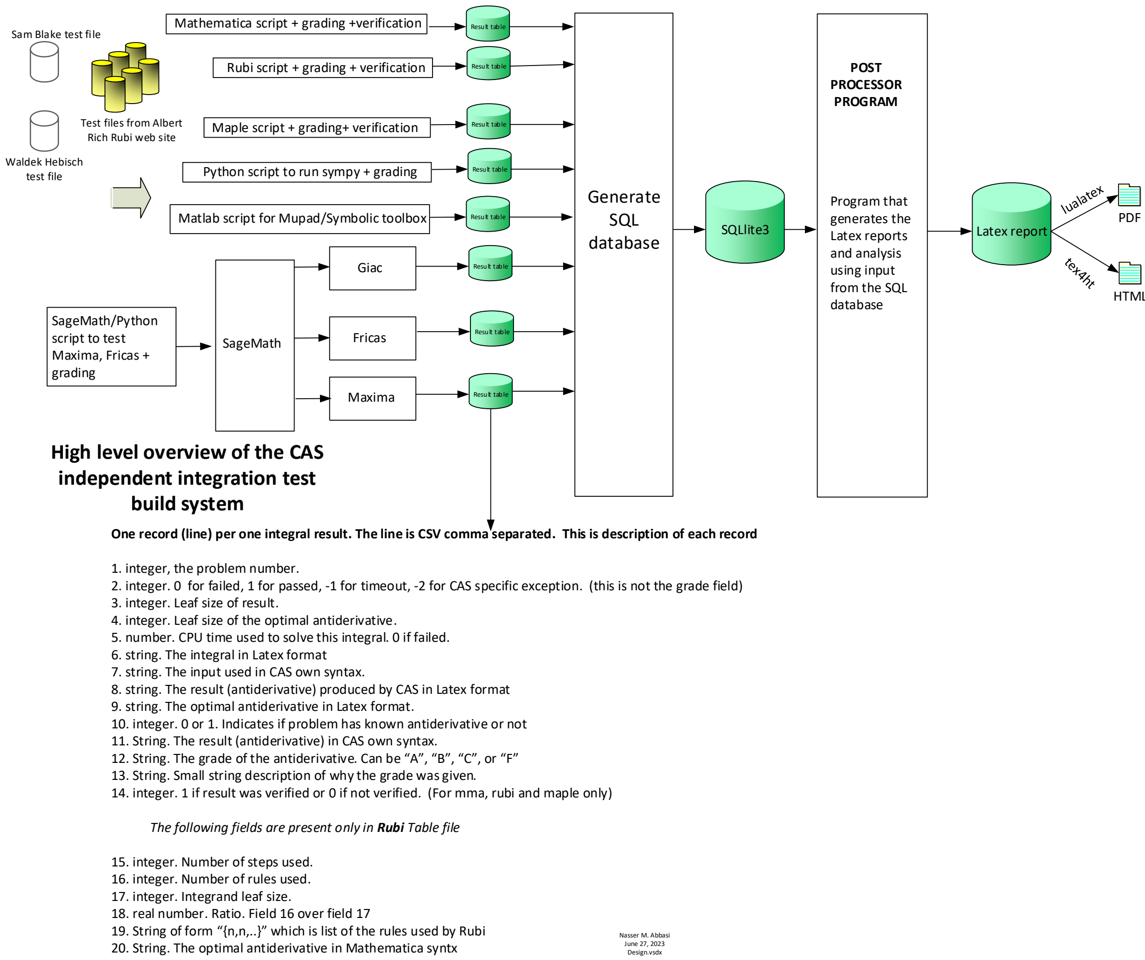
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 44, 45, 46, 47, 49, 50, 51, 52, 53, 54, 55, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 752, 753, 754, 755, 756, 757, 758, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 797, 798, 799, 800, 801, 802, 803, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 828, 829, 830, 831, 832, 833, 834, 835, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 913, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950 }

B grade { 48, 555, 759, 804, 827, 836, 858, 860, 912 }

C grade { 751 }

F normal fail { 796, 859, 914, 915, 931, 933 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 33, 39, 40, 41, 44, 45, 48, 49, 50, 53, 54, 55, 58, 59, 60, 61, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 86, 88, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 111, 113, 115, 116, 117, 118, 119, 120, 121, 122, 124, 126, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 161, 162, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 177, 178, 179, 181, 183, 184, 185, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 229, 230, 231, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 264, 265, 266, 267, 268, 270, 272, 273, 275, 276, 277, 279, 282, 283, 284, 285, 286, 288, 289, 290, 291, 292, 293, 294, 295, 296, 298, 299, 300, 301, 302, 303, 304, 305, 307, 308, 309, 311, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 330, 332, 333, 334, 335, 336, 337, 338, 339, 340, 342, 344, 345, 346, 347, 348, 349, 350, 352, 353, 357, 358, 359, 360, 363, 364, 365, 367, 368, 369, 371, 372, 373, 374, 375, 376, 377, 381, 382, 383, 385, 386, 388, 389, 390, 392, 393, 395, 396, 397, 398, 399, 400, 401, 417, 418, 419, 423, 424, 425, 426, 443, 444, 445, 446, 447, 458, 459, 460, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 498, 499, 500, 501, 502, 504, 505, 506, 507, 508, 509, 515, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 550, 551, 552, 553, 554, 555, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 585, 586, 587, 589, 590, 591, 592, 593, 594, 595, 596, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 639, 641, 642, 643, 648, 649, 651, 652, 653, 655, 656, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 674, 675, 676, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 696, 697, 698, 699, 700, 701, 702, 703, 704, 706, 707, 708, 714, 715, 716, 717, 718, 720, 721, 722, 723, 724, 725, 726, 727, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 782, 783, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 804, 805, 806, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 827, 828, 829, 830, 831, 832, 833, 835, 836, 837, 838, 839, 840, 841, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 862, 863, 864, 865, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 905, 906, 907, 908, 909, 911, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 928, 929, 930, 931, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 946, 947, 948, 949, 950 }

B grade { 35, 85, 105, 106, 108, 110, 114, 123, 127, 160, 163, 189, 263, 269, 271, 274, 278, 280, 281, 287, 297, 306, 310, 312, 329, 331, 341, 343, 361, 362, 366, 370, 378, 380, 384, 387, 391, 394, 402, 461, 497, 548, 549, 581, 638, 640, 650, 654, 673, 677, 694, 695, 705, 709, 710, 711, 712, 713, 728, 759, 781, 784, 803, 807, 826, 834, 861, 904, 927, 945 }

C grade { 31, 32, 34, 36, 37, 38, 46, 47, 51, 52, 62, 87, 89, 107, 109, 112, 125, 174, 175, 176, 182, 203, 228, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 351, 354, 355, 356, 379, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 420, 421, 422, 427, 428, 429, 430, 431, 432, 433, 434, 437, 438, 439, 440, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 503, 510, 511, 512, 513, 514, 516, 517, 556, 557, 558, 559, 560, 561, 588, 597, 657, 719, 842, 866, 867, 910, 912 }

F normal fail { }

F(-1) timedout fail { 435, 436, 441, 442 }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 35, 36, 37, 38, 44, 45, 46, 47, 48, 49, 50, 51, 52, 58, 59, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 80, 81, 85, 86, 88, 89, 90, 91, 92, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 111, 116, 117, 120, 121, 122, 123, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 143, 144, 150, 151, 152, 159, 162, 184, 185, 186, 187, 188, 189, 190, 191, 192, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 272, 276, 277, 278, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 319, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 387, 388, 389, 390, 391, 392, 395, 396, 397, 398, 399, 400, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 436, 437, 438, 442, 443, 445, 446, 447, 458, 459, 460, 461, 472, 473, 474, 475, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 499, 500, 501, 503, 508, 509, 510, 511, 512, 513, 514, 516, 518, 519, 520, 521, 522, 523, 524, 526, 527, 528, 529, 530, 531, 533, 534, 535, 536, 538, 539, 540, 541, 543, 544, 545, 546, 548, 549, 550, 551, 553, 554, 555, 562, 563, 567, 568, 569, 570, 571, 572, 576, 589, 590, 593, 598, 599, 602, 603, 604, 605, 606, 611, 612, 613, 614, 637, 641, 642, 643, 648, 649, 650, 651, 652, 653, 654, 655, 656, 658, 659, 660, 661, 665, 666, 667, 668, 669, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 690, 691, 692, 693, 694, 695, 696, 697, 698, 700, 701, 702, 703, 704, 705, 706, 707, 708, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 794, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 861, 862, 863, 864, 866, 867, 868, 869, 870, 871, 872, 873, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 886, 887, 889, 890, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 905, 906, 907, 908, 909, 910, 911, 913, 914, 915, 916, 917, 919, 920, 921, 922, 923, 924, 925, 926, 928, 929, 930, 931, 934, 935, 936, 937, 938, 939, 940, 941, 942, 944, 945, 946, 947, 948, 949 }

B grade { 110, 145, 146, 148, 149, 160, 161, 163, 164, 193, 194, 195, 196, 197, 198, 199, 200, 219, 271, 320, 321, 333, 334, 335, 337, 338, 339, 340, 386, 393, 394, 401, 410, 411, 412, 413, 414, 415, 416, 433, 434, 435, 439, 440, 441, 497, 498, 517, 532, 537, 542, 547, 552, 556, 557, 558, 559, 560, 561, 564, 565, 573, 574, 575, 577, 578, 579, 580, 581, 607, 608, 609, 610, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 638, 639, 640, 670, 709, 729, 759, 793, 795, 859, 874, 885, 888, 904, 912 }

C grade { 34, 76, 77, 78, 82, 83, 84, 87, 93, 107, 108, 109, 112, 113, 114, 118, 119, 124, 139, 140, 141, 142, 147, 153, 154, 155, 156, 157, 158, 230, 273, 274, 275, 279, 293, 294, 295, 336, 360, 361, 362, 402, 403, 404, 405, 406, 407, 408, 409, 444, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 476, 502, 504, 505, 506, 507, 515, 525, 588, 591, 597, 600, 633, 634, 635, 636, 699, 741, 742, 743, 744, 745, 746, 747, 748, 836, 860, 891, 927, 933, 943, 950 }

F normal fail { 39, 40, 41, 53, 54, 55, 60, 79, 115, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 217, 218, 318, 566, 592, 601, 657, 662, 663, 664, 687, 688, 689, 865, 918 }

F(-1) timedout fail { 585, 586, 587, 594, 595, 596 }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 33, 35, 36, 37, 38, 44, 45, 48, 50, 51, 52, 58, 59, 61, 66, 67, 68, 69, 70, 71, 72, 73, 75, 76, 77, 78, 86, 88, 91, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 107, 109, 111, 113, 117, 124, 126, 129, 130, 132, 134, 136, 138, 147, 148, 149, 150, 151, 152, 153, 155, 157, 158, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 220, 221, 222, 223, 224, 225, 227, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 268, 270, 273, 275, 276, 277, 278, 279, 280, 282, 284, 285, 286, 288, 290, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 305, 307, 309, 310, 311, 313, 314, 315, 316, 317, 321, 322, 323, 324, 326, 333, 334, 335, 336, 337, 341, 342, 344, 345, 346, 348, 355, 356, 357, 358, 359, 360, 363, 364, 365, 370, 371, 372, 373, 374, 375, 376, 381, 382, 383, 388, 389, 390, 395, 396, 397, 398, 417, 418, 419, 423, 424, 425, 426, 430, 431, 432, 433, 437, 438, 439, 440, 444, 447, 461, 472, 473, 474, 476, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 499, 500, 501, 504, 505, 506, 507, 508, 510, 511, 512, 514, 515, 516, 517, 518, 519, 520, 522, 523, 524, 525, 526, 527, 528, 529, 532, 538, 539, 543, 544, 548, 549, 553, 554, 555, 562, 563, 567, 568, 569, 570, 585, 586, 587, 588, 589, 590, 591, 594, 595, 596, 597, 598, 599, 600, 603, 604, 605, 606, 607, 609, 610, 611, 612, 613, 614, 615, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 635, 636, 637, 641, 642, 648, 649, 654, 656, 658, 659, 660, 661, 662, 663, 664, 665, 666, 672, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 692, 693, 696, 700, 705, 706, 708, 715, 717, 723, 724, 725, 726, 727, 729, 730, 731, 734, 735, 736, 737, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 753, 754, 755, 756, 757, 758, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 792, 794, 796, 799, 800, 802, 803, 804, 806, 807, 808, 809, 811, 812, 813, 814, 815, 817, 819, 820, 821, 822, 824, 827, 828, 830, 831, 832, 833, 834, 835, 837, 838, 839, 840, 841, 842, 843, 844, 845, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 860, 861, 866, 867, 874, 880, 881, 882, 883, 884, 886, 887, 889, 890, 891, 892, 893, 894, 895, 896, 898, 900, 901, 903, 905, 907, 908, 909, 910, 911, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 925, 926, 927, 928, 929, 930, 933, 934, 935, 936, 937, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950 }

B grade { 49, 62, 64, 65, 74, 80, 81, 82, 83, 84, 85, 87, 89, 90, 92, 93, 94, 105, 106, 108, 110, 112, 114, 116, 118, 119, 120, 121, 122, 123, 125, 127, 128, 131, 133, 135, 137, 139, 140, 141, 142, 143, 144, 145, 146, 154, 156, 159, 160, 161, 162, 163, 164, 195, 196, 197, 198, 214, 215, 216, 219, 226, 228, 229, 230, 231, 267, 269, 271, 272, 274, 281, 283, 287, 289, 291, 304, 306, 308, 312, 318, 319, 320, 325, 327, 328, 329, 330, 331, 332, 338, 339, 340, 343, 347, 349, 350, 351, 352, 353, 354, 361, 362, 366, 367, 368, 369, 377, 378, 379, 380, 384, 385, 386, 387, 391, 392, 393, 394, 399, 400, 401, 402, 420, 421, 422, 427, 428, 429, 434, 435, 436, 441, 442, 443, 445, 446, 458, 459, 460, 475, 477, 478, 479, 480, 496, 497, 498, 502, 503, 509, 513, 521, 530, 531, 533, 534, 535, 536, 537, 540, 541, 542, 545, 546, 547, 550, 551, 552, 564, 565, 571, 572, 579, 580, 581, 592, 593, 601, 602, 608, 616, 633, 634, 638, 639, 640, 643, 650, 651, 652, 653, 655, 657, 667, 668, 669, 670, 671, 673, 674, 675, 676, 677, 690, 691, 694, 695, 697, 698, 699, 701, 702, 703, 704, 707, 709, 710, 711, 712, 713, 714, 716, 718, 719, 720, 721, 722, 728, 732, 733, 738, 752, 759, 775, 791, 793, 795, 797, 798, 801, 805, 810, 816, 818, 823, 825, 826, 829, 836, 846, 847, 859, 862, 863, 864, 868, 869, 870, 871, 872, 873, 875, 876, 877, 878, 879, 885, 888, 897, 899, 902, 904, 906, 912, 913, 924 }

C grade { 31, 32, 46, 47, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 556, 557, 558, 559, 560, 561, 576, 577, 578, 938, 939 }

F normal fail { 34, 39, 40, 41, 53, 54, 55, 60, 63, 79, 115, 181, 217, 218, 566, 573, 574, 575 }

F(-1) timedout fail { }

F(-2) exception fail { 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 182, 865, 931, 932 }

2.1.5 Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 33, 35, 44, 45, 48, 49, 50, 58, 59, 60, 62, 66, 67, 68, 69, 70, 71, 72, 73, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 121, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 159, 162, 184, 185, 186, 187, 188, 191, 192, 193, 194, 195, 196, 201, 202, 203, 204, 207, 208, 209, 212, 213, 214, 220, 221, 222, 223, 224, 225, 226, 227, 229, 231, 252, 253, 254, 255, 256, 263, 264, 265, 266, 267, 274, 276, 277, 279, 283, 284, 285, 286, 287, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 308, 310, 312, 322, 323, 324, 325, 327, 329, 331, 333, 341, 342, 343, 344, 345, 346, 347, 348, 350, 355, 356, 357, 358, 359, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 378, 380, 381, 382, 383, 388, 389, 390, 395, 396, 397, 398, 461, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 499, 500, 504, 505, 510, 511, 512, 514, 515, 516, 517, 518, 519, 522, 523, 528, 567, 568, 569, 590, 599, 637, 638, 639, 640, 641, 642, 643, 648, 649, 650, 651, 652, 653, 655, 656, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 716, 717, 718, 719, 720, 721, 723, 724, 725, 726, 727, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 745, 746, 749, 751, 753, 754, 755, 756, 758, 759, 760, 763, 764, 765, 766, 767, 768, 769, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 798, 799, 800, 801, 802, 803, 806, 807, 808, 809, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 828, 830, 831, 832, 833, 834, 835, 837, 838, 839, 840, 843, 844, 845, 852, 854, 857, 862, 863, 864, 867, 868, 869, 870, 871, 872, 873, 880, 881, 882, 883, 884, 885, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 905, 908, 909, 910, 911, 913, 914, 916, 917, 920, 923, 924, 925, 926, 929, 934, 935, 936, 937, 940, 941, 943, 945, 946, 948, 950 }

B grade { 61, 64, 65, 74, 80, 81, 82, 85, 86, 90, 91, 92, 105, 110, 111, 112, 116, 117, 122, 123, 124, 125, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 157, 158, 189, 190, 197, 198, 219, 228, 230, 248, 249, 250, 251, 257, 258, 259, 260, 261, 262, 268, 270, 272, 273, 275, 278, 280, 281, 282, 288, 290, 292, 293, 294, 295, 307, 309, 311, 313, 314, 315, 316, 317, 326, 328, 330, 332, 334, 335, 336, 349, 351, 352, 353, 354, 377, 379, 384, 385, 386, 387, 391, 392, 393, 394, 444, 513, 529, 530, 531, 532, 533, 534, 589, 591, 593, 598, 600, 602, 607, 608, 609, 610, 615, 616, 654, 657, 670, 715, 722, 728, 750, 752, 757, 761, 770, 771, 796, 797, 804, 805, 810, 827, 829, 836, 841, 842, 850, 851, 853, 855, 856, 858, 859, 860, 866, 874, 876, 877, 878, 879, 886, 887, 888, 904, 915, 918, 919, 927, 928, 930, 933, 944, 947, 949 }

C grade { 623, 907, 921 }

F normal fail { 31, 32, 34, 36, 37, 38, 39, 40, 41, 46, 47, 51, 52, 53, 54, 55, 63, 75, 76, 77, 78, 79, 83, 84, 87, 88, 89, 93, 94, 106, 107, 108, 109, 113, 114, 115, 118, 119, 120, 126, 127, 160, 161, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 217, 218, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 318, 319, 320, 321, 337, 338, 339, 340, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 497, 498, 503, 509, 556, 557, 558, 559, 560, 561, 566, 573, 574, 575, 576, 577, 578, 579, 580, 581, 603, 604, 605, 606, 611, 612, 613, 614, 619, 620, 621, 622, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 743, 744, 747, 748, 762, 861, 865, 875, 906, 912, 922, 931, 938, 939, 942 }

F(-1) timedout fail { 617, 618, 635, 636 }

F(-2) exception fail { 153, 154, 155, 156, 199, 200, 205, 206, 210, 211, 215, 216, 269, 271, 289, 291, 360, 361, 362, 399, 400, 401, 402, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 445, 446, 447, 458, 459, 460, 501, 502, 506, 507, 508, 520, 521, 524, 525, 526, 527, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 562, 563, 564, 565, 570, 571, 572, 585, 586, 587, 588, 592, 594, 595, 596, 597, 601, 634, 846, 847, 848, 849 }

2.1.6 Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 33, 35, 44, 45, 48, 49, 50, 58, 59, 61, 62, 66, 67, 68, 69, 70, 71, 72, 73, 86, 88, 91, 93, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 111, 113, 117, 119, 120, 124, 126, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 150, 151, 152, 159, 165, 166, 167, 174, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 198, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 220, 222, 223, 224, 225, 226, 227, 229, 231, 248, 252, 253, 254, 255, 256, 257, 258, 259, 260, 263, 264, 265, 266, 268, 269, 270, 272, 274, 276, 279, 281, 283, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 305, 306, 308, 310, 312, 322, 323, 324, 326, 327, 328, 329, 330, 331, 332, 337, 338, 344, 345, 346, 348, 349, 350, 351, 352, 353, 355, 356, 357, 358, 359, 360, 361, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 388, 389, 390, 395, 396, 397, 398, 399, 400, 443, 444, 445, 446, 447, 458, 459, 460, 461, 472, 473, 474, 476, 477, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 499, 500, 504, 505, 506, 507, 512, 516, 517, 520, 522, 523, 525, 526, 527, 529, 530, 531, 532, 533, 535, 536, 540, 541, 545, 546, 550, 551, 562, 563, 567, 568, 569, 570, 571, 572, 589, 591, 598, 600, 640, 641, 643, 648, 649, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 665, 666, 668, 671, 672, 673, 674, 675, 677, 678, 679, 680, 681, 682, 684, 685, 686, 687, 690, 691, 692, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 708, 710, 711, 713, 714, 715, 716, 724, 725, 726, 727, 729, 730, 731, 735, 736, 737, 738, 739, 749, 751, 753, 754, 755, 756, 758, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 794, 795, 798, 799, 800, 801, 802, 803, 804, 806, 807, 808, 810, 811, 812, 814, 817, 818, 819, 820, 821, 822, 823, 824, 828, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 843, 844, 845, 846, 847, 851, 852, 853, 854, 855, 856, 857, 860, 866, 867, 880, 881, 882, 883, 884, 887, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 905, 906, 907, 908, 909, 911, 913, 916, 919, 920, 921, 922, 923, 924, 925, 926, 929, 934, 935, 936, 937, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950 }

B grade { 36, 37, 38, 51, 52, 64, 65, 75, 80, 81, 82, 83, 84, 85, 87, 89, 90, 92, 94, 110, 112, 114, 116, 118, 121, 122, 123, 125, 127, 139, 140, 141, 142, 143, 144, 145, 146, 157, 158, 162, 171, 172, 173, 175, 176, 183, 185, 196, 197, 199, 200, 219, 221, 228, 230, 249, 250, 251, 261, 262, 267, 271, 273, 275, 277, 278, 280, 282, 284, 304, 307, 309, 311, 313, 325, 336, 339, 340, 341, 342, 343, 347, 354, 362, 384, 385, 386, 387, 391, 392, 393, 394, 401, 402, 475, 478, 496, 501, 502, 508, 509, 510, 511, 513, 514, 515, 518, 519, 521, 524, 528, 534, 537, 538, 539, 542, 543, 544, 547, 548, 549, 552, 553, 554, 555, 564, 565, 590, 599, 605, 606, 637, 638, 639, 642, 650, 667, 670, 676, 683, 707, 717, 718, 719, 720, 721, 722, 728, 732, 743, 744, 747, 748, 752, 757, 759, 775, 793, 796, 797, 805, 813, 815, 816, 825, 826, 827, 829, 842, 848, 849, 850, 861, 862, 863, 864, 885, 886, 888, 904, 910, 915, 927, 928, 930, 933 }

C grade { 168, 169, 170, 585, 586, 587, 588, 594, 595, 596, 597, 712 }

F normal fail { 31, 32, 34, 39, 40, 41, 46, 47, 53, 54, 55, 60, 63, 74, 76, 77, 78, 79, 105, 106, 107, 108, 109, 115, 147, 148, 149, 153, 154, 155, 156, 160, 161, 163, 164, 177, 178, 179, 181, 182, 184, 217, 218, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 314, 315, 316, 317, 318, 319, 320, 321, 333, 334, 335, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 423, 424, 425, 426, 427, 428, 448, 449, 450, 453, 454, 455, 462, 463, 464, 465, 466, 467, 468, 469, 470, 497, 498, 556, 557, 558, 559, 560, 561, 566, 575, 576, 577, 578, 579, 580, 581, 592, 593, 601, 602, 607, 608, 609, 610, 622, 623, 633, 634, 635, 636, 662, 663, 664, 669, 688, 689, 709, 723, 733, 734, 740, 741, 742, 745, 746, 750, 809, 858, 859, 865, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 912, 917, 918, 931, 938, 939 }

F(-1) timedout fail { 422, 429, 435, 436, 441, 442, 451, 452, 456, 457, 471, 503, 573, 574, 603, 604, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 624, 625, 626, 627, 628, 629, 630, 631, 632, 914 }

F(-2) exception fail { 430, 431, 432, 433, 434, 437, 438, 439, 440, 693 }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 33, 35, 44, 45, 48, 49, 50, 58, 59, 61, 62, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 150, 155, 159, 162, 165, 166, 167, 171, 172, 173, 183, 185, 186, 187, 188, 189, 190, 191, 192, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 317, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 336, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 419, 426, 443, 444, 445, 446, 447, 458, 459, 460, 461, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 499, 500, 501, 502, 503, 508, 509, 510, 511, 512, 513, 518, 519, 520, 521, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 562, 563, 564, 565, 567, 568, 569, 570, 571, 572, 589, 590, 598, 599, 603, 604, 605, 606, 611, 612, 613, 614, 633, 637, 638, 639, 640, 641, 642, 643, 648, 649, 650, 651, 652, 653, 654, 655, 656, 659, 660, 661, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 680, 681, 682, 683, 684, 685, 686, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 712, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 852, 854, 855, 856, 857, 858, 861, 862, 863, 864, 865, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 923, 924, 925, 926, 927, 928, 929, 930, 931, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950 }

C grade { }

F normal fail { }

F(-1) timedout fail { 31, 32, 34, 36, 37, 38, 39, 40, 41, 46, 47, 51, 52, 53, 54, 55, 60, 63, 79, 115, 148, 149, 151, 152, 153, 154, 156, 157, 158, 160, 161, 163, 164, 168, 169, 170, 174, 175, 176, 177, 178, 179, 181, 182, 184, 193, 194, 195, 196, 197, 198, 199, 217, 218, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 259, 260, 261, 262, 314, 315, 316, 318, 319, 320, 321, 333, 334, 335, 337, 338, 339, 340, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 420, 421, 422, 423, 424, 425, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 497, 498, 504, 505, 506, 507, 514, 515, 516, 517, 522, 523, 524, 525, 556, 557, 558, 559, 560, 561, 566, 573, 574, 575, 576, 577, 578, 579, 580, 581, 585, 586, 587, 588, 591, 592, 593, 594, 595, 596, 597, 600, 601, 602, 607, 608, 609, 610, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 634, 635, 636, 657, 658, 662, 663, 664, 679, 687, 688, 689, 709, 710, 711, 713, 850, 851, 853, 859, 860, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 921, 922 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 1, 8, 15, 17, 18, 19, 22, 29, 30, 33, 35, 44, 45, 48, 49, 50, 58, 59, 62, 66, 67, 68, 70, 71, 72, 73, 91, 95, 96, 97, 98, 99, 100, 101, 102, 103, 124, 136, 185, 186, 187, 188, 203, 204, 225, 248, 249, 250, 251, 252, 253, 254, 255, 256, 264, 265, 266, 267, 274, 276, 277, 284, 285, 286, 287, 294, 296, 303, 304, 305, 322, 323, 324, 341, 342, 343, 344, 357, 358, 363, 365, 374, 376, 381, 383, 388, 390, 396, 397, 398, 444, 481, 510, 511, 526, 527, 528, 538, 539, 543, 544, 548, 549, 553, 554, 567, 569, 589, 590, 598, 599, 648, 651, 652, 654, 655, 656, 665, 674, 675, 676, 677, 678, 679, 680, 681, 701, 702, 703, 704, 705, 706, 707, 714, 715, 718, 720, 721, 722, 723, 724, 725, 726, 728, 730, 731, 732, 735, 736, 737, 752, 754, 755, 756, 758, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 794, 795, 797, 799, 800, 801, 806, 807, 808, 809, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 823, 824, 825, 826, 827, 830, 831, 832, 833, 834, 835, 837, 838, 839, 840, 842, 843, 844, 845, 846, 847, 852, 855, 856, 884, 886, 887, 889, 890, 891, 892, 893, 894, 895, 900, 901, 902, 904, 905, 907, 908, 911, 913, 916, 920, 921, 923, 924, 925, 926, 929, 930, 933, 937, 943, 944 }

B grade { 3, 4, 5, 10, 11, 12, 24, 25, 26, 69, 80, 90, 92, 104, 110, 121, 122, 123, 125, 128, 129, 130, 131, 132, 133, 134, 135, 137, 138, 139, 140, 141, 142, 146, 155, 201, 202, 205, 206, 207, 208, 210, 211, 212, 213, 219, 220, 221, 222, 223, 224, 263, 268, 270, 272, 273, 275, 278, 280, 282, 283, 293, 295, 297, 306, 325, 355, 356, 364, 366, 370, 371, 372, 373, 375, 377, 382, 384, 389, 391, 395, 399, 472, 473, 474, 475, 476, 477, 478, 479, 480, 482, 483, 499, 500, 562, 568, 591, 600, 637, 638, 639, 640, 641, 642, 643, 649, 653, 659, 660, 661, 666, 672, 684, 685, 686, 694, 695, 719, 727, 738, 751, 753, 759, 760, 775, 793, 798, 802, 803, 804, 805, 821, 822, 828, 836, 841, 848, 849, 854, 862, 880, 881, 882, 883, 885, 888, 909, 934, 935, 936, 940, 941, 942, 945, 946 }

C grade { 226, 349, 352, 512, 529, 532 }

F normal fail { 2, 6, 7, 9, 13, 14, 16, 20, 21, 23, 27, 28, 31, 32, 34, 36, 37, 39, 40, 41, 46, 47, 51, 52, 53, 54, 55, 60, 61, 63, 64, 65, 74, 75, 76, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 93, 94, 105, 106, 107, 108, 109, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 126, 127, 143, 144, 145, 147, 148, 150, 151, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 189, 190, 191, 192, 194, 195, 196, 197, 198, 215, 216, 217, 218, 227, 234, 235, 236, 237, 243, 244, 245, 258, 259, 260, 261, 262, 269, 271, 279, 281, 288, 289, 290, 298, 299, 300, 307, 308, 309, 310, 311, 312, 316, 317, 318, 319, 320, 326, 327, 328, 329, 335, 336, 337, 338, 339, 345, 346, 347, 348, 404, 405, 406, 407, 408, 411, 412, 413, 414, 415, 418, 419, 420, 421, 422, 425, 426, 427, 428, 429, 432, 433, 434, 435, 436, 438, 439, 440, 441, 442, 445, 446, 447, 448, 449, 450, 451, 454, 455, 458, 459, 460, 461, 462, 463, 464, 465, 468, 469, 484, 485, 486, 487, 488, 489, 490, 493, 496, 497, 498, 505, 506, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 557, 558, 559, 574, 575, 576, 577, 578, 579, 580, 581, 585, 587, 588, 592, 593, 594, 596, 597, 601, 602, 633, 634, 650, 662, 663, 664, 667, 668, 669, 670, 671, 673, 683, 687, 688, 689, 690, 691, 692, 693, 696, 697, 698, 699, 700, 708, 709, 710, 711, 712, 713, 716, 717, 729, 733, 734, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 757, 761, 774, 810, 829, 850, 851, 853, 857, 858, 859, 860, 861, 863, 864, 865, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 903, 910, 912, 914, 915, 917, 918, 919, 922, 927, 928, 931, 938, 939, 947, 948, 949, 950 }

F(-1) timedout fail { 38, 149, 152, 153, 154, 156, 171, 193, 199, 200, 209, 214, 228, 229, 230, 231, 232, 233, 238, 239, 240, 241, 242, 246, 247, 257, 291, 292, 301, 302, 313, 314, 315, 321, 330, 331, 332, 333, 334, 340, 351, 354, 359, 360, 361, 362, 367, 368, 369, 378, 379, 380, 385, 386, 387, 392, 393, 394, 400, 401, 402, 403, 409, 410, 416, 417, 423, 424, 430, 431, 437, 443, 452, 453, 456, 457, 466, 467, 470, 471, 491, 492, 494, 495, 501, 502, 503, 504, 507, 508, 509, 531, 534, 535, 536, 537, 540, 541, 542, 545, 546, 547, 550, 551, 552, 555, 556, 560, 561, 563, 564, 565, 566, 570, 571, 572, 573, 586, 595, 603, 604, 605, 606, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 624, 625, 626, 627, 628, 629, 630, 631, 632, 635, 636, 657, 658, 682, 750, 762, 796, 866, 867, 896, 897, 898, 899, 906 }

F(-2) exception fail { 350, 353, 513, 530, 533, 607, 623 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	49	22	17	26	33	32	57	35
N.S.	1	1.11	0.50	0.39	0.59	0.75	0.73	1.30	0.80
time (sec)	N/A	0.188	0.089	0.367	0.283	0.250	0.144	0.263	27.402

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	49	22	17	26	33	0	57	16
N.S.	1	1.11	0.50	0.39	0.59	0.75	0.00	1.30	0.36
time (sec)	N/A	0.270	0.016	0.612	0.294	0.238	0.000	0.304	27.129

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	22	22	17	16	43	246	57	35
N.S.	1	0.46	0.46	0.35	0.33	0.90	5.12	1.19	0.73
time (sec)	N/A	0.164	0.230	0.868	0.300	0.246	2.894	0.269	27.296

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	22	22	17	16	43	246	57	35
N.S.	1	0.46	0.46	0.35	0.33	0.90	5.12	1.19	0.73
time (sec)	N/A	0.179	0.838	0.781	0.284	0.251	3.026	0.277	27.497

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	22	22	17	16	43	246	57	35
N.S.	1	0.46	0.46	0.35	0.33	0.90	5.12	1.19	0.73
time (sec)	N/A	0.195	0.024	0.872	0.300	0.245	3.042	0.290	27.211

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	22	22	17	16	43	0	16	16
N.S.	1	0.46	0.46	0.35	0.33	0.90	0.00	0.33	0.33
time (sec)	N/A	0.207	0.009	1.368	0.300	0.247	0.000	0.909	26.598

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	22	22	17	16	43	0	57	16
N.S.	1	0.46	0.46	0.35	0.33	0.90	0.00	1.19	0.33
time (sec)	N/A	0.204	0.009	1.234	0.331	0.245	0.000	0.358	25.686

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	22	22	17	54	74	42	39	16
N.S.	1	0.37	0.37	0.28	0.90	1.23	0.70	0.65	0.27
time (sec)	N/A	0.176	0.105	0.559	0.293	0.237	0.151	0.304	25.657

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	22	22	17	54	74	0	39	16
N.S.	1	0.37	0.37	0.28	0.90	1.23	0.00	0.65	0.27
time (sec)	N/A	0.248	0.081	0.937	0.287	0.246	0.000	0.338	26.303

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	22	22	17	34	86	1644	39	16
N.S.	1	0.37	0.37	0.28	0.57	1.43	27.40	0.65	0.27
time (sec)	N/A	0.181	0.438	0.674	0.295	0.238	9.466	0.296	26.408

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	22	22	17	34	86	1644	39	16
N.S.	1	0.37	0.37	0.28	0.57	1.43	27.40	0.65	0.27
time (sec)	N/A	0.177	1.096	0.612	0.290	0.247	9.499	0.288	26.027

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	22	22	17	34	86	1644	39	16
N.S.	1	0.37	0.37	0.28	0.57	1.43	27.40	0.65	0.27
time (sec)	N/A	0.194	0.019	0.890	0.303	0.247	9.243	0.293	26.700

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	22	22	17	34	86	0	39	16
N.S.	1	0.37	0.37	0.28	0.57	1.43	0.00	0.65	0.27
time (sec)	N/A	0.204	0.009	1.248	0.294	0.244	0.000	0.936	26.664

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	22	22	17	34	86	0	39	16
N.S.	1	0.37	0.37	0.28	0.57	1.43	0.00	0.65	0.27
time (sec)	N/A	0.206	0.025	1.012	0.293	0.260	0.000	0.399	26.282

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	47	22	18	27	31	34	57	36
N.S.	1	1.12	0.52	0.43	0.64	0.74	0.81	1.36	0.86
time (sec)	N/A	0.195	0.176	0.520	0.316	0.241	0.129	0.266	26.692

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	47	22	18	27	31	0	57	17
N.S.	1	1.12	0.52	0.43	0.64	0.74	0.00	1.36	0.40
time (sec)	N/A	0.239	0.027	0.932	0.306	0.239	0.000	0.326	26.817

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	22	22	18	17	43	76	57	36
N.S.	1	0.46	0.46	0.38	0.35	0.90	1.58	1.19	0.75
time (sec)	N/A	0.167	0.011	1.273	0.292	0.250	0.282	0.267	26.949

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	22	22	18	17	43	76	57	36
N.S.	1	0.46	0.46	0.38	0.35	0.90	1.58	1.19	0.75
time (sec)	N/A	0.177	0.488	1.238	0.319	0.244	0.263	0.273	27.455

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	22	22	18	17	43	76	57	36
N.S.	1	0.46	0.46	0.38	0.35	0.90	1.58	1.19	0.75
time (sec)	N/A	0.193	0.014	1.428	0.309	0.253	0.310	0.278	26.477

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	22	22	18	17	43	0	17	17
N.S.	1	0.46	0.46	0.38	0.35	0.90	0.00	0.35	0.35
time (sec)	N/A	0.198	0.025	1.153	0.301	0.261	0.000	0.772	26.893

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	22	22	18	17	43	0	57	17
N.S.	1	0.46	0.46	0.38	0.35	0.90	0.00	1.19	0.35
time (sec)	N/A	0.206	0.017	1.010	0.297	0.246	0.000	0.390	27.368

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	22	22	18	53	74	39	39	17
N.S.	1	0.36	0.36	0.30	0.87	1.21	0.64	0.64	0.28
time (sec)	N/A	0.175	0.114	0.478	0.292	0.248	0.153	0.280	28.127

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	22	22	18	53	74	0	39	17
N.S.	1	0.36	0.36	0.30	0.87	1.21	0.00	0.64	0.28
time (sec)	N/A	0.239	0.017	0.912	0.285	0.249	0.000	0.327	28.620

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	22	22	18	32	85	1481	39	17
N.S.	1	0.36	0.36	0.30	0.52	1.39	24.28	0.64	0.28
time (sec)	N/A	0.177	1.214	0.607	0.297	0.245	7.832	0.295	27.985

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	22	22	18	32	85	1481	39	17
N.S.	1	0.36	0.36	0.30	0.52	1.39	24.28	0.64	0.28
time (sec)	N/A	0.177	0.563	0.597	0.294	0.253	7.951	0.284	27.786

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	22	22	18	32	85	1481	39	17
N.S.	1	0.36	0.36	0.30	0.52	1.39	24.28	0.64	0.28
time (sec)	N/A	0.196	0.024	0.865	0.303	0.243	7.917	0.297	27.873

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	22	22	18	32	85	0	37	17
N.S.	1	0.36	0.36	0.30	0.52	1.39	0.00	0.61	0.28
time (sec)	N/A	0.201	0.009	1.207	0.294	0.271	0.000	0.791	28.687

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	22	22	18	32	85	0	39	17
N.S.	1	0.36	0.36	0.30	0.52	1.39	0.00	0.64	0.28
time (sec)	N/A	0.205	0.020	1.079	0.294	0.239	0.000	0.419	28.103

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	22	41	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.73	1.37	0.80	0.80
time (sec)	N/A	0.198	0.050	0.464	0.208	0.253	0.068	0.253	0.108

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	48	47	48	46	85	46	46
N.S.	1	1.00	0.86	0.84	0.86	0.82	1.52	0.82	0.82
time (sec)	N/A	0.239	0.063	1.517	0.211	0.249	0.095	0.259	28.848

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	163	233	0	187	0	0	0
N.S.	1	1.00	0.77	1.09	0.00	0.88	0.00	0.00	0.00
time (sec)	N/A	0.634	0.819	1.136	0.000	0.251	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	245	328	0	434	0	0	0
N.S.	1	1.00	0.90	1.21	0.00	1.60	0.00	0.00	0.00
time (sec)	N/A	0.905	1.650	1.671	0.000	0.257	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	10	8	8
N.S.	1	1.00	1.00	0.90	0.80	0.80	1.00	0.80	0.80
time (sec)	N/A	0.194	0.033	0.562	0.203	0.249	0.107	0.258	26.975

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	28	28	46	72	0	0	0	0	0
N.S.	1	1.00	1.64	2.57	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.593	0.692	0.685	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	26	12	17	22	10	11	11
N.S.	1	1.00	2.17	1.00	1.42	1.83	0.83	0.92	0.92
time (sec)	N/A	0.189	0.045	0.439	0.209	0.233	0.642	0.268	28.698

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	108	324	142	0	104	0	630	0
N.S.	1	1.08	3.24	1.42	0.00	1.04	0.00	6.30	0.00
time (sec)	N/A	0.545	0.801	1.467	0.000	0.249	0.000	3.572	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	115	495	195	0	113	0	681	0
N.S.	1	1.07	4.63	1.82	0.00	1.06	0.00	6.36	0.00
time (sec)	N/A	0.581	3.916	1.494	0.000	0.265	0.000	25.070	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	180	692	295	0	212	0	1239	0
N.S.	1	0.93	3.57	1.52	0.00	1.09	0.00	6.39	0.00
time (sec)	N/A	0.482	7.776	1.733	0.000	0.285	0.000	66.416	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	57	53	0	0	0	0	0	0
N.S.	1	0.98	0.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.330	0.356	0.000	0.000	0.000	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	57	57	0	0	0	0	0	0
N.S.	1	0.98	0.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.307	0.221	0.000	0.000	0.000	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	26	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.256	0.131	0.000	0.000	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	36	32	35	34	48	34	35
N.S.	1	1.00	1.06	0.94	1.03	1.00	1.41	1.00	1.03
time (sec)	N/A	0.264	3.594	0.678	0.378	0.261	10.182	0.349	28.516

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	36	38	32	696	45	51	34	35
N.S.	1	1.00	1.06	0.89	19.33	1.25	1.42	0.94	0.97
time (sec)	N/A	0.295	20.517	0.678	0.456	0.244	36.783	0.362	28.717

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	26	25	24	23	41	24	24
N.S.	1	1.00	0.87	0.83	0.80	0.77	1.37	0.80	0.80
time (sec)	N/A	0.202	0.051	0.480	0.220	0.240	0.065	0.273	28.912

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	51	48	46	44	85	46	48
N.S.	1	1.00	0.91	0.86	0.82	0.79	1.52	0.82	0.86
time (sec)	N/A	0.244	0.075	1.471	0.209	0.248	0.092	0.272	0.110

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	164	231	0	189	0	0	0
N.S.	1	1.00	0.77	1.08	0.00	0.89	0.00	0.00	0.00
time (sec)	N/A	0.530	0.538	1.093	0.000	0.259	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	243	326	0	436	0	0	0
N.S.	1	1.00	0.90	1.20	0.00	1.61	0.00	0.00	0.00
time (sec)	N/A	0.831	0.669	1.740	0.000	0.273	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	A	A	A	A	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	27	10	9	8	8	8	8	8
N.S.	1	2.70	1.00	0.90	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.326	0.034	0.522	0.207	0.245	0.142	0.264	25.944

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	38	22	18	17	37	20	17	15
N.S.	1	1.73	1.00	0.82	0.77	1.68	0.91	0.77	0.68
time (sec)	N/A	0.380	0.054	0.572	0.298	0.235	0.203	0.262	26.192

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	36	20	16	16	15	15	19	16
N.S.	1	1.50	0.83	0.67	0.67	0.62	0.62	0.79	0.67
time (sec)	N/A	0.635	0.721	4.674	0.203	0.237	1.979	0.267	26.749

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	108	319	142	0	105	0	633	0
N.S.	1	1.07	3.16	1.41	0.00	1.04	0.00	6.27	0.00
time (sec)	N/A	0.564	0.789	1.540	0.000	0.242	0.000	5.257	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	114	497	195	0	109	0	683	0
N.S.	1	1.07	4.64	1.82	0.00	1.02	0.00	6.38	0.00
time (sec)	N/A	0.582	3.567	1.655	0.000	0.247	0.000	26.794	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	57	53	0	0	0	0	0	0
N.S.	1	0.98	0.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.332	0.026	0.000	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	57	51	0	0	0	0	0	0
N.S.	1	0.98	0.88	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.302	0.032	0.000	0.000	0.000	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	26	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.255	0.008	0.000	0.000	0.000	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	36	32	35	34	48	34	35
N.S.	1	1.00	1.06	0.94	1.03	1.00	1.41	1.00	1.03
time (sec)	N/A	0.264	3.261	0.562	0.475	0.249	6.662	0.369	26.471

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	36	38	32	420	34	51	34	35
N.S.	1	1.00	1.06	0.89	11.67	0.94	1.42	0.94	0.97
time (sec)	N/A	0.296	13.936	0.688	0.430	0.252	21.289	0.347	25.939

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	7	13	10	8	19
N.S.	1	1.00	1.00	0.89	0.78	1.44	1.11	0.89	2.11
time (sec)	N/A	0.179	0.015	0.090	0.223	0.252	0.077	0.269	27.295

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	18	15	12	12	14	12	20
N.S.	1	1.00	1.12	0.94	0.75	0.75	0.88	0.75	1.25
time (sec)	N/A	0.190	0.029	0.364	0.290	0.246	0.079	0.276	26.939

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	70	90	70	0	80	0	0	0	0
N.S.	1	1.29	1.00	0.00	1.14	0.00	0.00	0.00	0.00
time (sec)	N/A	0.415	0.020	0.000	0.302	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	18	18	83	23	0	18	21
N.S.	1	1.00	0.95	0.95	4.37	1.21	0.00	0.95	1.11
time (sec)	N/A	0.156	1.152	0.923	0.293	0.243	0.000	4.002	28.418

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	18	26	16	14	28	15	22	20
N.S.	1	1.12	1.62	1.00	0.88	1.75	0.94	1.38	1.25
time (sec)	N/A	0.198	0.089	0.359	0.294	0.248	0.077	0.294	28.060

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	85	120	0	0	0	0	0
N.S.	1	1.00	0.92	1.30	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.393	2.759	8.656	0.000	0.000	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	33	35	43	6257	72	0	66	29
N.S.	1	0.94	1.00	1.23	178.77	2.06	0.00	1.89	0.83
time (sec)	N/A	0.208	0.034	1.219	84.817	0.265	0.000	0.277	0.115

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	33	35	43	6257	72	0	66	29
N.S.	1	0.94	1.00	1.23	178.77	2.06	0.00	1.89	0.83
time (sec)	N/A	0.216	0.003	0.872	84.357	0.256	0.000	0.280	0.003

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	10	20	6	6
N.S.	1	1.00	1.00	0.80	0.73	0.67	1.33	0.40	0.40
time (sec)	N/A	0.164	0.025	0.576	0.213	0.226	0.124	0.259	0.041

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	20	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	1.18	0.76	0.76
time (sec)	N/A	0.165	0.009	0.757	0.186	0.235	0.128	0.265	0.037

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	18	20	13	13
N.S.	1	1.00	1.00	0.82	0.76	1.06	1.18	0.76	0.76
time (sec)	N/A	0.164	0.024	0.852	0.192	0.242	0.123	0.264	0.041

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	25	28	28	26	78	29	64
N.S.	1	1.00	0.71	0.80	0.80	0.74	2.23	0.83	1.83
time (sec)	N/A	0.186	0.041	0.697	0.195	0.249	0.252	0.256	26.429

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	9	20	11	9
N.S.	1	1.00	1.00	0.80	0.73	0.60	1.33	0.73	0.60
time (sec)	N/A	0.162	0.024	0.603	0.190	0.236	0.126	0.260	0.027

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	20	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	1.18	0.76	0.76
time (sec)	N/A	0.167	0.007	0.773	0.184	0.246	0.127	0.265	0.028

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	17	20	13	17
N.S.	1	1.00	1.00	0.82	0.76	1.00	1.18	0.76	1.00
time (sec)	N/A	0.165	0.025	0.783	0.204	0.246	0.131	0.267	0.037

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	24	28	28	24	37	29	55
N.S.	1	1.00	0.69	0.80	0.80	0.69	1.06	0.83	1.57
time (sec)	N/A	0.187	0.036	0.677	0.197	0.242	0.261	0.264	26.462

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	27	20	18	141	38	0	0	17
N.S.	1	1.35	1.00	0.90	7.05	1.90	0.00	0.00	0.85
time (sec)	N/A	0.186	0.016	0.715	0.287	0.250	0.000	0.000	25.638

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	57	21	38	0	39	0	364	26
N.S.	1	1.21	0.45	0.81	0.00	0.83	0.00	7.74	0.55
time (sec)	N/A	0.270	0.029	0.999	0.000	0.243	0.000	0.349	26.009

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	80	69	55	0	101	0	0	103
N.S.	1	1.13	0.97	0.77	0.00	1.42	0.00	0.00	1.45
time (sec)	N/A	0.274	0.062	1.206	0.000	0.251	0.000	0.000	25.838

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	100	236	0	136	0	0	107
N.S.	1	1.00	0.89	2.11	0.00	1.21	0.00	0.00	0.96
time (sec)	N/A	0.389	0.125	0.842	0.000	0.268	0.000	0.000	26.722

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	93	84	103	0	134	0	0	131
N.S.	1	1.04	0.94	1.16	0.00	1.51	0.00	0.00	1.47
time (sec)	N/A	0.355	0.108	0.793	0.000	0.275	0.000	0.000	27.666

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	105	105	200	0	0	0	0	0	0
N.S.	1	1.00	1.90	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.259	0.121	0.000	0.000	0.000	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	14	10	12	37	19	19	19	10
N.S.	1	1.40	1.00	1.20	3.70	1.90	1.90	1.90	1.00
time (sec)	N/A	0.188	0.009	0.269	0.298	0.251	0.362	0.256	25.275

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	127	36	0	34	16
N.S.	1	1.00	1.00	0.85	6.35	1.80	0.00	1.70	0.80
time (sec)	N/A	0.186	0.015	0.358	0.303	0.244	0.000	0.273	26.052

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	30	28	88	173	52	0	50	29
N.S.	1	1.07	1.00	3.14	6.18	1.86	0.00	1.79	1.04
time (sec)	N/A	0.249	0.026	0.384	0.292	0.253	0.000	0.263	25.320

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	76	55	0	127	0	111	119
N.S.	1	1.00	0.93	0.67	0.00	1.55	0.00	1.35	1.45
time (sec)	N/A	0.356	0.203	0.448	0.000	0.265	0.000	0.361	26.918

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	42	38	124	0	70	0	70	37
N.S.	1	1.11	1.00	3.26	0.00	1.84	0.00	1.84	0.97
time (sec)	N/A	0.279	0.046	0.420	0.000	0.261	0.000	0.273	28.128

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	35	13	129	33	0	49	12
N.S.	1	1.00	2.33	0.87	8.60	2.20	0.00	3.27	0.80
time (sec)	N/A	0.180	0.063	1.574	0.281	0.242	0.000	0.281	0.232

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	27	17	18	81	19	0	24	15
N.S.	1	1.29	0.81	0.86	3.86	0.90	0.00	1.14	0.71
time (sec)	N/A	0.186	0.009	1.549	0.279	0.250	0.000	0.263	27.299

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	4814	47	0	121	0	133	112
N.S.	1	1.00	67.80	0.66	0.00	1.70	0.00	1.87	1.58
time (sec)	N/A	0.231	52.974	1.747	0.000	0.270	0.000	0.268	27.294

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	80	57	43	0	72	0	67	47
N.S.	1	1.29	0.92	0.69	0.00	1.16	0.00	1.08	0.76
time (sec)	N/A	0.294	0.581	2.086	0.000	0.288	0.000	0.276	0.668

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	627	80	0	153	0	182	118
N.S.	1	1.00	7.38	0.94	0.00	1.80	0.00	2.14	1.39
time (sec)	N/A	0.276	7.333	1.830	0.000	0.277	0.000	0.280	27.361

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	9	35	17	15	17	5
N.S.	1	1.00	1.00	1.29	5.00	2.43	2.14	2.43	0.71
time (sec)	N/A	0.193	0.002	0.293	0.300	0.238	0.404	0.264	26.990

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	15	15	14	125	58	76	31	17
N.S.	1	0.33	0.33	0.31	2.78	1.29	1.69	0.69	0.38
time (sec)	N/A	0.186	0.048	0.618	0.327	0.262	0.803	0.281	28.315

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	28	171	50	294	48	27
N.S.	1	1.00	1.00	1.08	6.58	1.92	11.31	1.85	1.04
time (sec)	N/A	0.200	0.040	0.683	0.300	0.257	3.423	0.271	27.584

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	87	84	42	0	231	0	105	217
N.S.	1	0.53	0.51	0.25	0.00	1.40	0.00	0.64	1.32
time (sec)	N/A	0.263	0.166	0.852	0.000	0.284	0.000	0.345	27.981

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	38	30	47	0	68	0	68	35
N.S.	1	1.06	0.83	1.31	0.00	1.89	0.00	1.89	0.97
time (sec)	N/A	0.239	0.040	0.721	0.000	0.273	0.000	0.275	28.125

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	16	6	7	6	8	5	6	6
N.S.	1	2.00	0.75	0.88	0.75	1.00	0.62	0.75	0.75
time (sec)	N/A	0.191	0.025	0.387	0.197	0.245	0.380	0.269	28.014

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.192	0.005	0.290	0.198	0.255	1.272	0.277	0.030

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	6	22	6	6
N.S.	1	1.00	1.00	0.80	0.73	0.40	1.47	0.40	0.40
time (sec)	N/A	0.166	0.007	0.390	0.200	0.241	0.127	0.266	0.029

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	22	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	1.29	0.76	0.76
time (sec)	N/A	0.164	0.007	0.487	0.210	0.251	0.128	0.272	0.034

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	22	13	14
N.S.	1	1.00	1.00	0.82	0.76	0.76	1.29	0.76	0.82
time (sec)	N/A	0.165	0.007	0.471	0.199	0.240	0.128	0.257	0.034

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	26	28	27	25	44	29	57
N.S.	1	1.00	0.74	0.80	0.77	0.71	1.26	0.83	1.63
time (sec)	N/A	0.187	0.038	0.410	0.199	0.254	0.274	0.261	27.347

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	11	12	20	11	9
N.S.	1	1.00	1.00	0.80	0.73	0.80	1.33	0.73	0.60
time (sec)	N/A	0.162	0.008	0.371	0.200	0.233	0.128	0.260	0.030

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	7	20	13	7
N.S.	1	1.00	1.00	0.82	0.76	0.41	1.18	0.76	0.41
time (sec)	N/A	0.168	0.007	0.490	0.201	0.244	0.126	0.267	0.026

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	18	20	13	13
N.S.	1	1.00	1.00	0.82	0.76	1.06	1.18	0.76	0.76
time (sec)	N/A	0.165	0.006	0.460	0.192	0.252	0.130	0.265	0.032

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	25	28	28	25	56	29	58
N.S.	1	1.00	0.71	0.80	0.80	0.71	1.60	0.83	1.66
time (sec)	N/A	0.185	0.030	0.379	0.197	0.239	0.266	0.260	0.144

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	27	45	18	133	38	0	0	42
N.S.	1	1.35	2.25	0.90	6.65	1.90	0.00	0.00	2.10
time (sec)	N/A	0.187	0.085	1.010	0.282	0.250	0.000	0.000	28.274

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	48	19	0	38	0	0	42
N.S.	1	1.00	2.29	0.90	0.00	1.81	0.00	0.00	2.00
time (sec)	N/A	0.183	0.056	0.987	0.000	0.251	0.000	0.000	27.938

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	71	80	6161	54	0	101	0	0	295
N.S.	1	1.13	86.77	0.76	0.00	1.42	0.00	0.00	4.15
time (sec)	N/A	0.260	57.693	1.392	0.000	0.266	0.000	0.000	27.721

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	215	54	0	129	0	0	407
N.S.	1	1.00	2.56	0.64	0.00	1.54	0.00	0.00	4.85
time (sec)	N/A	0.305	1.043	1.137	0.000	0.276	0.000	0.000	27.898

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	A	F	F	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	89	93	628	99	0	134	0	0	787
N.S.	1	1.04	7.06	1.11	0.00	1.51	0.00	0.00	8.84
time (sec)	N/A	0.324	8.210	1.170	0.000	0.261	0.000	0.000	29.059

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	14	25	20	37	21	19	19	20
N.S.	1	1.40	2.50	2.00	3.70	2.10	1.90	1.90	2.00
time (sec)	N/A	0.182	0.102	1.118	0.207	0.251	0.358	0.276	26.277

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	55	47	36	131	39	0	39	39
N.S.	1	1.22	1.04	0.80	2.91	0.87	0.00	0.87	0.87
time (sec)	N/A	0.270	0.061	1.437	0.289	0.255	0.000	0.269	26.613

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	30	73	81	165	53	0	50	67
N.S.	1	1.07	2.61	2.89	5.89	1.89	0.00	1.79	2.39
time (sec)	N/A	0.241	0.108	1.111	0.299	0.266	0.000	0.271	26.447

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	133	224	0	137	0	117	611
N.S.	1	1.00	1.21	2.04	0.00	1.25	0.00	1.06	5.55
time (sec)	N/A	0.372	0.176	1.246	0.000	0.261	0.000	0.279	26.667

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	42	87	113	0	71	0	70	86
N.S.	1	1.11	2.29	2.97	0.00	1.87	0.00	1.84	2.26
time (sec)	N/A	0.260	0.097	1.143	0.000	0.251	0.000	0.275	27.399

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	92	92	179	0	0	0	0	0	0
N.S.	1	1.00	1.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.255	0.209	0.000	0.000	0.000	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	13	137	33	0	31	12
N.S.	1	1.00	1.00	0.87	9.13	2.20	0.00	2.07	0.80
time (sec)	N/A	0.176	0.019	1.563	0.294	0.249	0.000	0.260	0.172

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	15	15	13	76	53	0	31	16
N.S.	1	0.34	0.34	0.30	1.73	1.20	0.00	0.70	0.36
time (sec)	N/A	0.188	0.031	1.696	0.308	0.257	0.000	0.302	26.795

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	67	46	0	121	0	99	95
N.S.	1	1.00	0.94	0.65	0.00	1.70	0.00	1.39	1.34
time (sec)	N/A	0.224	0.129	1.790	0.000	0.255	0.000	0.307	26.856

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	89	84	44	0	231	0	105	217
N.S.	1	0.55	0.52	0.27	0.00	1.42	0.00	0.64	1.33
time (sec)	N/A	0.280	0.111	1.948	0.000	0.301	0.000	0.349	26.767

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	81	80	0	154	0	132	118
N.S.	1	1.00	0.95	0.94	0.00	1.81	0.00	1.55	1.39
time (sec)	N/A	0.264	0.107	1.810	0.000	0.274	0.000	0.308	26.135

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	14	19	21	20	21	10
N.S.	1	1.00	1.00	1.40	1.90	2.10	2.00	2.10	1.00
time (sec)	N/A	0.183	0.010	1.611	0.205	0.267	0.491	0.258	26.923

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	18	14	18	129	25	427	25	12
N.S.	1	1.29	1.00	1.29	9.21	1.79	30.50	1.79	0.86
time (sec)	N/A	0.188	0.010	1.667	0.294	0.246	1.951	0.255	0.032

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	21	9	35	19	15	17	5
N.S.	1	1.00	3.00	1.29	5.00	2.71	2.14	2.43	0.71
time (sec)	N/A	0.187	0.005	1.408	0.215	0.246	0.427	0.259	0.037

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	27	21	27	129	19	17	24	17
N.S.	1	1.29	1.00	1.29	6.14	0.90	0.81	1.14	0.81
time (sec)	N/A	0.184	0.013	1.678	0.289	0.247	0.561	0.261	26.320

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	66	28	163	52	248	48	55
N.S.	1	1.00	2.54	1.08	6.27	2.00	9.54	1.85	2.12
time (sec)	N/A	0.194	0.054	1.786	0.316	0.257	2.844	0.277	26.562

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	80	57	80	0	72	0	67	51
N.S.	1	1.29	0.92	1.29	0.00	1.16	0.00	1.08	0.82
time (sec)	N/A	0.273	0.079	1.812	0.000	0.245	0.000	0.265	27.454

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	37	83	47	0	70	0	68	74
N.S.	1	1.03	2.31	1.31	0.00	1.94	0.00	1.89	2.06
time (sec)	N/A	0.236	0.066	1.795	0.000	0.271	0.000	0.273	26.828

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	26	25	57	63	25	57
N.S.	1	1.00	1.00	0.79	0.76	1.73	1.91	0.76	1.73
time (sec)	N/A	0.203	0.079	0.766	0.211	0.282	0.596	0.280	0.111

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	26	25	39	71	25	78
N.S.	1	1.00	1.00	0.79	0.76	1.18	2.15	0.76	2.36
time (sec)	N/A	0.213	0.072	0.847	0.192	0.254	0.602	0.266	27.047

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	32	48	19	25
N.S.	1	1.00	1.00	0.80	0.76	1.28	1.92	0.76	1.00
time (sec)	N/A	0.203	0.047	0.548	0.189	0.240	0.262	0.263	27.607

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	17	42	42	17	17
N.S.	1	1.00	1.00	0.78	0.74	1.83	1.83	0.74	0.74
time (sec)	N/A	0.194	0.046	0.592	0.190	0.252	0.274	0.255	26.957

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	26	25	49	65	49	150
N.S.	1	1.00	1.00	0.79	0.76	1.48	1.97	1.48	4.55
time (sec)	N/A	0.204	0.014	0.771	0.196	0.298	0.613	0.272	27.025

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	24	23	67	70	23	198
N.S.	1	1.00	1.00	0.77	0.74	2.16	2.26	0.74	6.39
time (sec)	N/A	0.202	0.070	0.847	0.190	0.305	0.610	0.284	28.183

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	32	31	25	226	31	25
N.S.	1	1.00	1.00	0.78	0.76	0.61	5.51	0.76	0.61
time (sec)	N/A	0.216	0.073	1.214	0.211	0.252	1.510	0.262	0.089

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	26	24	23	50	58	23	36
N.S.	1	1.00	0.96	0.89	0.85	1.85	2.15	0.85	1.33
time (sec)	N/A	0.184	0.044	0.448	0.200	0.252	0.182	0.265	27.005

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	26	24	23	44	61	23	36
N.S.	1	1.00	0.96	0.89	0.85	1.63	2.26	0.85	1.33
time (sec)	N/A	0.194	0.032	0.467	0.191	0.249	0.167	0.261	27.344

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	26	24	23	50	58	23	36
N.S.	1	1.00	0.96	0.89	0.85	1.85	2.15	0.85	1.33
time (sec)	N/A	0.186	0.019	0.402	0.189	0.250	0.163	0.258	27.377

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	26	24	23	44	58	23	36
N.S.	1	1.00	0.96	0.89	0.85	1.63	2.15	0.85	1.33
time (sec)	N/A	0.184	0.017	0.464	0.210	0.270	0.161	0.267	27.899

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	47	31	173	371	145	7672	242	207
N.S.	1	1.21	0.79	4.44	9.51	3.72	196.72	6.21	5.31
time (sec)	N/A	0.289	0.411	0.362	0.220	0.249	2.877	0.293	31.894

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	41	28	145	290	145	7679	245	196
N.S.	1	1.21	0.82	4.26	8.53	4.26	225.85	7.21	5.76
time (sec)	N/A	0.284	0.413	0.394	0.220	0.268	4.071	0.286	31.883

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	51	31	177	549	118	7404	348	207
N.S.	1	1.31	0.79	4.54	14.08	3.03	189.85	8.92	5.31
time (sec)	N/A	0.288	0.393	0.373	0.248	0.263	11.460	0.296	31.687

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	44	30	149	432	118	7417	345	200
N.S.	1	1.29	0.88	4.38	12.71	3.47	218.15	10.15	5.88
time (sec)	N/A	0.285	0.383	0.388	0.240	0.258	12.039	0.292	31.828

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	28	54	349	107	0	171	249
N.S.	1	1.00	0.78	1.50	9.69	2.97	0.00	4.75	6.92
time (sec)	N/A	0.223	0.175	1.419	0.221	0.262	0.000	0.303	32.329

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	26	53	322	93	0	169	249
N.S.	1	1.00	0.79	1.61	9.76	2.82	0.00	5.12	7.55
time (sec)	N/A	0.219	0.169	1.381	0.216	0.273	0.000	0.313	33.694

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	40	28	79	564	110	0	396	249
N.S.	1	1.11	0.78	2.19	15.67	3.06	0.00	11.00	6.92
time (sec)	N/A	0.230	0.189	1.490	0.225	0.271	0.000	0.294	33.332

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	37	29	80	536	96	1824	397	249
N.S.	1	1.12	0.88	2.42	16.24	2.91	55.27	12.03	7.55
time (sec)	N/A	0.232	0.170	1.484	0.218	0.262	68.548	0.312	32.906

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	50	57	22	0	0	20
N.S.	1	1.00	1.00	3.85	4.38	1.69	0.00	0.00	1.54
time (sec)	N/A	0.233	0.074	2.470	0.300	0.254	0.000	0.000	27.875

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	55	23	522	57	26	0	0	0
N.S.	1	1.77	0.74	16.84	1.84	0.84	0.00	0.00	0.00
time (sec)	N/A	0.318	0.075	2.098	0.293	0.251	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	76	29	283	82	38	0	0	0
N.S.	1	1.52	0.58	5.66	1.64	0.76	0.00	0.00	0.00
time (sec)	N/A	0.391	0.186	2.685	0.296	0.254	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	188	19	0	12	18
N.S.	1	1.00	1.00	0.92	14.46	1.46	0.00	0.92	1.38
time (sec)	N/A	0.236	0.096	1.915	0.350	0.267	0.000	0.267	25.581

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	55	21	17	314	23	0	19	0
N.S.	1	1.77	0.68	0.55	10.13	0.74	0.00	0.61	0.00
time (sec)	N/A	0.328	0.071	1.664	0.357	0.251	0.000	0.263	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	76	29	29	427	35	0	27	0
N.S.	1	1.52	0.58	0.58	8.54	0.70	0.00	0.54	0.00
time (sec)	N/A	0.415	0.118	4.340	0.367	0.259	0.000	0.275	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	64	56	159	0	236	0	0	0
N.S.	1	1.10	0.97	2.74	0.00	4.07	0.00	0.00	0.00
time (sec)	N/A	0.271	0.129	0.487	0.000	0.253	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	95	84	257	0	459	0	0	0
N.S.	1	1.12	0.99	3.02	0.00	5.40	0.00	0.00	0.00
time (sec)	N/A	0.373	0.230	0.721	0.000	0.267	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	58	154	0	227	1935	0	132
N.S.	1	1.00	0.98	2.61	0.00	3.85	32.80	0.00	2.24
time (sec)	N/A	0.265	0.088	0.402	0.000	0.266	151.660	0.000	29.711

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	103	85	250	0	417	0	0	0
N.S.	1	1.17	0.97	2.84	0.00	4.74	0.00	0.00	0.00
time (sec)	N/A	0.355	0.250	0.586	0.000	0.269	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	53	48	86	250	80	0	322	0
N.S.	1	1.06	0.96	1.72	5.00	1.60	0.00	6.44	0.00
time (sec)	N/A	0.359	0.166	2.013	0.208	0.266	0.000	0.376	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	54	48	87	250	81	0	322	0
N.S.	1	1.08	0.96	1.74	5.00	1.62	0.00	6.44	0.00
time (sec)	N/A	0.367	0.149	1.062	0.225	0.256	0.000	0.375	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	24	23	205	0	40	24
N.S.	1	1.00	1.00	0.75	0.72	6.41	0.00	1.25	0.75
time (sec)	N/A	0.227	0.063	1.645	0.282	0.259	0.000	0.477	26.638

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	211	241	512	1003	0	3284	0	0	0
N.S.	1	1.14	2.43	4.75	0.00	15.56	0.00	0.00	0.00
time (sec)	N/A	0.907	2.204	2.148	0.000	3.662	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	337	349	294	1251	0	4568	0	0	0
N.S.	1	1.04	0.87	3.71	0.00	13.55	0.00	0.00	0.00
time (sec)	N/A	1.369	1.017	2.101	0.000	2.286	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	34	43	300	0	76	45
N.S.	1	1.00	1.00	0.85	1.08	7.50	0.00	1.90	1.12
time (sec)	N/A	0.621	0.967	2.046	0.283	0.303	0.000	0.672	26.761

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	267	289	751	1670	0	4100	0	0	0
N.S.	1	1.08	2.81	6.25	0.00	15.36	0.00	0.00	0.00
time (sec)	N/A	1.103	4.097	2.657	0.000	4.599	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	407	417	314	2061	0	5696	0	0	0
N.S.	1	1.02	0.77	5.06	0.00	14.00	0.00	0.00	0.00
time (sec)	N/A	1.568	1.724	2.414	0.000	3.423	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	155	97	61	0	0	0	0	156	111
N.S.	1	0.63	0.39	0.00	0.00	0.00	0.00	1.01	0.72
time (sec)	N/A	0.599	3.128	0.000	0.000	0.000	0.000	0.301	28.107

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	118	78	54	0	0	0	0	119	86
N.S.	1	0.66	0.46	0.00	0.00	0.00	0.00	1.01	0.73
time (sec)	N/A	0.498	1.513	0.000	0.000	0.000	0.000	0.316	27.416

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	A	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	74	58	44	0	0	0	0	82	61
N.S.	1	0.78	0.59	0.00	0.00	0.00	0.00	1.11	0.82
time (sec)	N/A	0.358	0.782	0.000	0.000	0.000	0.000	0.303	27.021

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	C	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	86	52	52	0	0	0	0	274	0
N.S.	1	0.60	0.60	0.00	0.00	0.00	0.00	3.19	0.00
time (sec)	N/A	0.488	0.756	0.000	0.000	0.000	0.000	0.323	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	C	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	123	66	65	0	0	0	0	886	0
N.S.	1	0.54	0.53	0.00	0.00	0.00	0.00	7.20	0.00
time (sec)	N/A	0.564	0.922	0.000	0.000	0.000	0.000	0.411	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	C	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	176	85	87	0	0	0	0	1022	0
N.S.	1	0.48	0.49	0.00	0.00	0.00	0.00	5.81	0.00
time (sec)	N/A	0.668	0.977	0.000	0.000	0.000	0.000	0.424	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	393	206	113	0	0	0	0	1487	216
N.S.	1	0.52	0.29	0.00	0.00	0.00	0.00	3.78	0.55
time (sec)	N/A	0.624	1.380	0.000	0.000	0.000	0.000	0.565	29.196

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	265	151	95	0	0	0	0	739	159
N.S.	1	0.57	0.36	0.00	0.00	0.00	0.00	2.79	0.60
time (sec)	N/A	0.533	1.045	0.000	0.000	0.000	0.000	0.427	28.362

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	168	115	73	0	0	0	0	311	123
N.S.	1	0.68	0.43	0.00	0.00	0.00	0.00	1.85	0.73
time (sec)	N/A	0.402	0.889	0.000	0.000	0.000	0.000	0.339	29.200

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-2)	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	186	79	150	0	0	0	0	160	0
N.S.	1	0.42	0.81	0.00	0.00	0.00	0.00	0.86	0.00
time (sec)	N/A	0.811	2.567	0.000	0.000	0.000	0.000	0.320	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-2)	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	273	106	231	0	0	0	0	608	0
N.S.	1	0.39	0.85	0.00	0.00	0.00	0.00	2.23	0.00
time (sec)	N/A	0.821	2.706	0.000	0.000	0.000	0.000	0.383	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-2)	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	385	152	317	0	0	0	0	1502	0
N.S.	1	0.39	0.82	0.00	0.00	0.00	0.00	3.90	0.00
time (sec)	N/A	0.890	2.999	0.000	0.000	0.000	0.000	0.511	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	767	372	247	0	0	0	0	0	0
N.S.	1	0.49	0.32	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.369	2.849	0.000	0.000	0.000	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	555	268	194	0	0	0	0	0	0
N.S.	1	0.48	0.35	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.091	2.499	0.000	0.000	0.000	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	355	176	154	0	0	0	0	0	0
N.S.	1	0.50	0.43	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.761	2.030	0.000	0.000	0.000	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	37	37	39	33	35	0	34	35	35
N.S.	1	1.00	1.05	0.89	0.95	0.00	0.92	0.95	0.95
time (sec)	N/A	0.837	4.391	0.270	1.047	0.000	0.906	0.386	26.835

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	536	224	192	0	0	0	0	0	0
N.S.	1	0.42	0.36	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.950	2.298	0.000	0.000	0.000	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	280	124	154	0	0	0	0	0	0
N.S.	1	0.44	0.55	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.048	1.903	0.000	0.000	0.000	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	171	93	150	0	0	72	0	446	88
N.S.	1	0.54	0.88	0.00	0.00	0.42	0.00	2.61	0.51
time (sec)	N/A	1.196	1.023	0.000	0.000	0.253	0.000	0.380	28.316

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	141	85	154	56	75	0	0	0
N.S.	1	0.47	0.28	0.51	0.19	0.25	0.00	0.00	0.00
time (sec)	N/A	0.574	0.128	0.572	0.298	0.261	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	20	24	23	32	27	51	54
N.S.	1	1.00	1.11	1.33	1.28	1.78	1.50	2.83	3.00
time (sec)	N/A	0.404	0.027	0.611	0.195	0.259	0.842	0.282	26.885

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	67	53	54	60	61	100	403
N.S.	1	1.00	1.18	0.93	0.95	1.05	1.07	1.75	7.07
time (sec)	N/A	0.621	0.420	0.727	0.209	0.255	1.241	0.269	27.834

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	81	80	67	84	77	92	125	431
N.S.	1	1.08	1.07	0.89	1.12	1.03	1.23	1.67	5.75
time (sec)	N/A	0.820	0.410	1.500	0.196	0.266	2.879	0.285	28.291

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	115	97	80	118	89	116	149	460
N.S.	1	1.11	0.93	0.77	1.13	0.86	1.12	1.43	4.42
time (sec)	N/A	1.037	0.484	1.857	0.203	0.265	6.662	0.270	26.380

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	71	35	63	47	0	46	25
N.S.	1	1.00	2.84	1.40	2.52	1.88	0.00	1.84	1.00
time (sec)	N/A	0.342	0.219	0.340	0.201	0.253	0.000	0.274	26.663

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	76	51	93	85	0	77	50
N.S.	1	1.00	1.58	1.06	1.94	1.77	0.00	1.60	1.04
time (sec)	N/A	0.486	0.307	0.333	0.205	0.267	0.000	0.270	27.591

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	85	88	64	119	122	0	102	92
N.S.	1	1.13	1.17	0.85	1.59	1.63	0.00	1.36	1.23
time (sec)	N/A	0.693	0.427	0.381	0.199	0.255	0.000	0.276	26.179

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	116	104	79	143	158	0	126	140
N.S.	1	1.21	1.08	0.82	1.49	1.65	0.00	1.31	1.46
time (sec)	N/A	0.899	0.721	0.389	0.211	0.273	0.000	0.270	26.962

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	106	78	225	43	123	0	134	0
N.S.	1	1.08	0.80	2.30	0.44	1.26	0.00	1.37	0.00
time (sec)	N/A	0.822	0.127	3.619	0.335	0.249	0.000	0.279	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	75	62	188	26	99	0	92	0
N.S.	1	1.04	0.86	2.61	0.36	1.38	0.00	1.28	0.00
time (sec)	N/A	0.594	0.080	2.111	0.319	0.261	0.000	0.279	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	47	154	13	81	0	61	0
N.S.	1	1.00	1.07	3.50	0.30	1.84	0.00	1.39	0.00
time (sec)	N/A	0.395	0.028	2.096	0.324	0.250	0.000	0.272	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	52	194	58	116	0	120	0
N.S.	1	1.00	0.76	2.85	0.85	1.71	0.00	1.76	0.00
time (sec)	N/A	0.497	0.037	2.046	0.334	0.259	0.000	0.293	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	97	73	270	9153	187	0	158	0
N.S.	1	1.05	0.79	2.93	99.49	2.03	0.00	1.72	0.00
time (sec)	N/A	0.705	0.291	2.567	0.788	0.255	0.000	0.281	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	131	95	322	50224	234	0	183	0
N.S.	1	1.09	0.79	2.68	418.53	1.95	0.00	1.52	0.00
time (sec)	N/A	0.941	0.389	2.788	14.433	0.258	0.000	0.293	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	58	0	35	0	283	0
N.S.	1	1.00	1.00	2.32	0.00	1.40	0.00	11.32	0.00
time (sec)	N/A	0.246	0.219	0.707	0.000	0.261	0.000	0.367	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	57	0	36	0	397	68
N.S.	1	1.00	1.00	2.38	0.00	1.50	0.00	16.54	2.83
time (sec)	N/A	0.253	0.123	0.618	0.000	0.241	0.000	0.358	26.224

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	10	9	8	15	41	8	8
N.S.	1	1.00	1.25	1.12	1.00	1.88	5.12	1.00	1.00
time (sec)	N/A	0.271	0.009	0.291	0.278	0.252	0.441	0.253	27.277

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	19	24	18	15	35	248	49	26
N.S.	1	0.53	0.67	0.50	0.42	0.97	6.89	1.36	0.72
time (sec)	N/A	0.232	0.046	0.684	0.280	0.264	21.850	0.254	26.930

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	23	9	10	15	12	16	8
N.S.	1	1.00	2.88	1.12	1.25	1.88	1.50	2.00	1.00
time (sec)	N/A	0.269	0.015	0.286	0.277	0.250	0.351	0.278	25.786

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	20	23	19	16	35	61	49	27
N.S.	1	0.54	0.62	0.51	0.43	0.95	1.65	1.32	0.73
time (sec)	N/A	0.237	0.046	0.603	0.314	0.253	0.835	0.260	26.541

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	14	0	13	61	26	13
N.S.	1	1.00	1.00	1.00	0.00	0.93	4.36	1.86	0.93
time (sec)	N/A	0.333	0.024	0.394	0.000	0.237	28.273	0.262	27.946

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	73	88	0	254	2608	110	2429
N.S.	1	1.00	0.70	0.84	0.00	2.42	24.84	1.05	23.13
time (sec)	N/A	0.442	0.150	0.529	0.000	0.277	53.367	0.269	29.122

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	31	34	30	29	45	143	62	242
N.S.	1	0.54	0.60	0.53	0.51	0.79	2.51	1.09	4.25
time (sec)	N/A	0.282	0.195	0.504	0.294	0.259	4.136	0.266	26.732

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	17	19	24	29	13
N.S.	1	1.00	1.00	0.93	1.13	1.27	1.60	1.93	0.87
time (sec)	N/A	0.245	0.045	0.444	0.289	0.241	0.417	0.265	26.059

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	51	47	44	40	228	0	58	1987
N.S.	1	1.04	0.96	0.90	0.82	4.65	0.00	1.18	40.55
time (sec)	N/A	0.285	0.562	0.576	0.293	0.290	0.000	0.273	27.565

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	15	13	14	0	12	56	22	13
N.S.	1	1.15	1.00	1.08	0.00	0.92	4.31	1.69	1.00
time (sec)	N/A	0.325	0.030	0.642	0.000	0.235	37.137	0.271	27.427

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	72	86	0	262	2035	93	1646
N.S.	1	1.00	0.72	0.86	0.00	2.62	20.35	0.93	16.46
time (sec)	N/A	0.417	0.142	0.576	0.000	0.257	110.014	0.268	30.621

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	33	31	29	28	45	520	62	249
N.S.	1	0.59	0.55	0.52	0.50	0.80	9.29	1.11	4.45
time (sec)	N/A	0.326	0.231	0.829	0.298	0.266	22.714	0.269	25.776

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	12	14	13	14	18	51	23	12
N.S.	1	0.86	1.00	0.93	1.00	1.29	3.64	1.64	0.86
time (sec)	N/A	0.254	0.013	0.767	0.302	0.250	0.436	0.257	26.724

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	51	47	46	42	255	0	70	1774
N.S.	1	1.04	0.96	0.94	0.86	5.20	0.00	1.43	36.20
time (sec)	N/A	0.306	0.568	0.876	0.298	0.275	0.000	0.277	28.666

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	79	98	106	0	318	0	125	1302
N.S.	1	1.07	1.32	1.43	0.00	4.30	0.00	1.69	17.59
time (sec)	N/A	0.599	0.604	0.562	0.000	0.628	0.000	0.278	28.612

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	85	102	90	0	332	0	110	463
N.S.	1	1.18	1.42	1.25	0.00	4.61	0.00	1.53	6.43
time (sec)	N/A	0.578	1.000	0.875	0.000	0.592	0.000	0.273	27.903

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	136	136	94	0	0	0	0	0	0
N.S.	1	1.00	0.69	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.349	0.218	0.000	0.000	0.000	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	95	95	88	0	0	0	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.271	0.156	0.000	0.000	0.000	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	120	246	271	257	257	461	316	422
N.S.	1	0.94	1.94	2.13	2.02	2.02	3.63	2.49	3.32
time (sec)	N/A	0.287	2.175	3.194	0.216	0.273	0.514	0.499	32.374

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	167	192	235	238	219	770	235	519
N.S.	1	1.04	1.19	1.46	1.48	1.36	4.78	1.46	3.22
time (sec)	N/A	0.431	2.218	2.365	0.222	0.259	0.414	0.385	28.246

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	89	156	167	172	155	267	187	248
N.S.	1	0.95	1.66	1.78	1.83	1.65	2.84	1.99	2.64
time (sec)	N/A	0.240	0.252	2.438	0.222	0.249	0.269	0.342	27.392

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	111	107	135	136	121	381	122	320
N.S.	1	1.03	0.99	1.25	1.26	1.12	3.53	1.13	2.96
time (sec)	N/A	0.304	0.213	1.698	0.233	0.249	0.197	0.305	27.992

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	56	81	75	84	77	117	91	104
N.S.	1	0.97	1.40	1.29	1.45	1.33	2.02	1.57	1.79
time (sec)	N/A	0.207	0.201	1.776	0.207	0.256	0.126	0.288	27.643

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	52	57	68	52	128	50	63
N.S.	1	1.00	0.95	1.04	1.24	0.95	2.33	0.91	1.15
time (sec)	N/A	0.198	0.052	0.809	0.217	0.244	0.094	0.271	25.830

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	46	23	24	23	31	24	38
N.S.	1	1.00	1.92	0.96	1.00	0.96	1.29	1.00	1.58
time (sec)	N/A	0.152	0.007	0.422	0.196	0.242	0.069	0.259	25.834

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	45	43	80	131	163	74	39
N.S.	1	1.00	0.96	0.91	1.70	2.79	3.47	1.57	0.83
time (sec)	N/A	0.206	0.040	0.638	0.301	0.250	2.923	0.301	28.704

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	21	21	57	0	20	47
N.S.	1	1.00	1.00	0.66	0.66	1.78	0.00	0.62	1.47
time (sec)	N/A	0.188	0.022	0.767	0.223	0.239	0.000	0.266	28.647

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	132	191	326	294	0	221	260
N.S.	1	1.00	1.28	1.85	3.17	2.85	0.00	2.15	2.52
time (sec)	N/A	0.327	0.225	0.973	0.323	0.270	0.000	0.311	31.504

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	85	64	85	217	0	50	222
N.S.	1	1.00	0.87	0.65	0.87	2.21	0.00	0.51	2.27
time (sec)	N/A	0.314	0.196	1.066	0.216	0.258	0.000	0.290	27.892

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	170	157	454	822	544	0	588	719
N.S.	1	1.09	1.01	2.91	5.27	3.49	0.00	3.77	4.61
time (sec)	N/A	0.466	1.032	2.222	0.325	0.288	0.000	0.346	32.328

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	165	182	125	174	441	0	118	470
N.S.	1	1.09	1.21	0.83	1.15	2.92	0.00	0.78	3.11
time (sec)	N/A	0.449	0.505	2.306	0.236	0.282	0.000	0.316	31.225

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	189	205	185	0	261	0	0	0
N.S.	1	1.02	1.10	0.99	0.00	1.40	0.00	0.00	0.00
time (sec)	N/A	0.562	1.433	1.365	0.000	0.108	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	256	250	0	247	0	0	0
N.S.	1	1.00	1.95	1.91	0.00	1.89	0.00	0.00	0.00
time (sec)	N/A	0.414	1.290	1.144	0.000	0.109	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	143	165	0	159	0	0	0
N.S.	1	1.00	1.09	1.26	0.00	1.21	0.00	0.00	0.00
time (sec)	N/A	0.422	1.119	1.084	0.000	0.089	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	268	163	0	163	0	0	0
N.S.	1	1.00	3.57	2.17	0.00	2.17	0.00	0.00	0.00
time (sec)	N/A	0.267	0.883	3.509	0.000	0.088	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	92	124	0	128	0	0	0
N.S.	1	1.00	1.23	1.65	0.00	1.71	0.00	0.00	0.00
time (sec)	N/A	0.278	0.211	0.997	0.000	0.080	0.000	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	219	232	0	278	0	0	0
N.S.	1	1.00	1.59	1.68	0.00	2.01	0.00	0.00	0.00
time (sec)	N/A	0.440	2.381	1.084	0.000	0.091	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	145	182	0	406	0	0	0
N.S.	1	1.00	1.02	1.28	0.00	2.86	0.00	0.00	0.00
time (sec)	N/A	0.412	1.424	1.046	0.000	0.105	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	207	277	297	0	552	0	0	0
N.S.	1	1.05	1.41	1.51	0.00	2.80	0.00	0.00	0.00
time (sec)	N/A	0.600	1.930	1.092	0.000	0.116	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	120	125	153	128	0	118	0	0	0
N.S.	1	1.04	1.28	1.07	0.00	0.98	0.00	0.00	0.00
time (sec)	N/A	0.489	0.455	1.134	0.000	0.089	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	199	174	0	111	0	0	0
N.S.	1	1.00	2.65	2.32	0.00	1.48	0.00	0.00	0.00
time (sec)	N/A	0.344	0.929	0.871	0.000	0.086	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	133	108	0	96	0	0	0
N.S.	1	1.00	1.77	1.44	0.00	1.28	0.00	0.00	0.00
time (sec)	N/A	0.335	0.501	1.046	0.000	0.085	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	184	112	0	63	0	0	0
N.S.	1	1.00	6.81	4.15	0.00	2.33	0.00	0.00	0.00
time (sec)	N/A	0.220	0.817	1.637	0.000	0.087	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	88	85	0	58	0	0	0
N.S.	1	1.00	3.26	3.15	0.00	2.15	0.00	0.00	0.00
time (sec)	N/A	0.219	0.193	1.336	0.000	0.087	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	190	162	0	158	0	0	0
N.S.	1	1.00	2.60	2.22	0.00	2.16	0.00	0.00	0.00
time (sec)	N/A	0.339	1.015	0.861	0.000	0.087	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	157	118	0	189	0	0	0
N.S.	1	1.00	2.09	1.57	0.00	2.52	0.00	0.00	0.00
time (sec)	N/A	0.337	0.627	0.865	0.000	0.089	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	123	224	205	0	264	0	0	0
N.S.	1	1.02	1.87	1.71	0.00	2.20	0.00	0.00	0.00
time (sec)	N/A	0.480	1.512	0.908	0.000	0.090	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	31	31	59	23	42	23	0
N.S.	1	1.00	0.97	0.97	1.84	0.72	1.31	0.72	0.00
time (sec)	N/A	0.186	0.102	0.642	0.329	0.240	0.131	0.321	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	19	132	17	36	52	84
N.S.	1	1.00	1.00	0.61	4.26	0.55	1.16	1.68	2.71
time (sec)	N/A	0.186	0.516	1.952	0.225	0.258	0.092	0.312	27.281

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	19	83	17	36	52	66
N.S.	1	1.00	1.00	0.61	2.68	0.55	1.16	1.68	2.13
time (sec)	N/A	0.184	0.268	1.927	0.227	0.257	0.085	0.280	26.299

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	19	69	17	36	52	44
N.S.	1	1.00	1.00	0.61	2.23	0.55	1.16	1.68	1.42
time (sec)	N/A	0.181	0.159	1.011	0.207	0.228	0.085	0.274	26.862

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	51	17	24	15	26	24	20
N.S.	1	1.00	1.96	0.65	0.92	0.58	1.00	0.92	0.77
time (sec)	N/A	0.150	0.007	0.434	0.232	0.238	0.072	0.266	26.904

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	19	29	17	31	21	25
N.S.	1	1.00	1.00	0.66	1.00	0.59	1.07	0.72	0.86
time (sec)	N/A	0.184	0.015	0.823	0.225	0.239	0.085	0.275	26.923

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	19	22	17	44	30	31
N.S.	1	1.00	1.00	0.61	0.71	0.55	1.42	0.97	1.00
time (sec)	N/A	0.183	0.020	0.820	0.211	0.239	0.079	0.268	27.303

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	19	29	17	44	36	68
N.S.	1	1.00	1.00	0.61	0.94	0.55	1.42	1.16	2.19
time (sec)	N/A	0.180	0.020	1.006	0.231	0.227	0.084	0.273	28.248

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	19	29	17	44	44	91
N.S.	1	1.00	1.00	0.61	0.94	0.55	1.42	1.42	2.94
time (sec)	N/A	0.180	0.414	1.007	0.227	0.238	0.086	0.291	27.705

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	32	28	51	17	0	17	35
N.S.	1	1.00	0.97	0.85	1.55	0.52	0.00	0.52	1.06
time (sec)	N/A	0.184	0.102	1.805	0.321	0.238	0.000	0.389	0.616

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	32	28	51	17	0	25	33
N.S.	1	1.00	0.97	0.85	1.55	0.52	0.00	0.76	1.00
time (sec)	N/A	0.182	0.069	0.787	0.323	0.240	0.000	0.276	28.239

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	30	20	51	17	0	25	0
N.S.	1	1.00	0.97	0.65	1.65	0.55	0.00	0.81	0.00
time (sec)	N/A	0.181	0.040	0.716	0.321	0.242	0.000	0.267	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	30	20	51	17	0	25	0
N.S.	1	1.00	0.97	0.65	1.65	0.55	0.00	0.81	0.00
time (sec)	N/A	0.183	0.048	0.730	0.331	0.237	0.000	0.260	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	32	28	51	17	0	65	0
N.S.	1	1.00	0.97	0.85	1.55	0.52	0.00	1.97	0.00
time (sec)	N/A	0.183	0.048	0.710	0.313	0.233	0.000	0.801	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	32	28	51	17	0	67	0
N.S.	1	1.00	0.97	0.85	1.55	0.52	0.00	2.03	0.00
time (sec)	N/A	0.185	0.047	0.824	0.326	0.251	0.000	0.666	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	172	303	164	204	166	308	178	272
N.S.	1	1.15	2.03	1.10	1.37	1.11	2.07	1.19	1.83
time (sec)	N/A	0.455	1.015	60.632	0.225	0.270	2.123	0.268	29.643

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	107	96	74	72	80	97	131	115
N.S.	1	1.07	0.96	0.74	0.72	0.80	0.97	1.31	1.15
time (sec)	N/A	0.542	0.273	17.590	0.327	0.265	1.189	0.286	28.716

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	95	123	74	95	85	122	86	126
N.S.	1	1.27	1.64	0.99	1.27	1.13	1.63	1.15	1.68
time (sec)	N/A	0.353	0.451	4.855	0.233	0.249	1.273	0.272	28.614

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	25	26	26	29	22	40	40
N.S.	1	1.00	0.93	0.96	0.96	1.07	0.81	1.48	1.48
time (sec)	N/A	0.262	0.044	1.513	0.314	0.243	0.353	0.260	29.288

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	16	14	25	24	34	37
N.S.	1	1.00	1.00	1.33	1.17	2.08	2.00	2.83	3.08
time (sec)	N/A	0.144	0.010	0.384	0.208	0.262	0.058	0.265	29.042

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	50	11	32	12	55
N.S.	1	1.00	1.00	1.09	4.55	1.00	2.91	1.09	5.00
time (sec)	N/A	0.219	0.007	0.551	0.303	0.283	0.134	0.260	31.001

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	77	344	92	0	308	0	94	604
N.S.	1	1.17	5.21	1.39	0.00	4.67	0.00	1.42	9.15
time (sec)	N/A	0.421	1.611	1.680	0.000	0.292	0.000	0.270	29.684

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	46	40	50	201	83	503	43	106
N.S.	1	0.90	0.78	0.98	3.94	1.63	9.86	0.84	2.08
time (sec)	N/A	0.291	0.210	6.999	0.314	0.254	1.061	0.267	29.630

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	182	2661	358	0	931	0	369	2782
N.S.	1	1.17	17.06	2.29	0.00	5.97	0.00	2.37	17.83
time (sec)	N/A	0.828	6.850	28.200	0.000	0.317	0.000	0.284	33.156

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	89	86	102	483	217	1719	91	541
N.S.	1	0.88	0.85	1.01	4.78	2.15	17.02	0.90	5.36
time (sec)	N/A	0.332	0.282	163.533	0.341	0.269	6.003	0.262	30.436

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	54	51	141	38	68	62	59
N.S.	1	1.00	1.80	1.70	4.70	1.27	2.27	2.07	1.97
time (sec)	N/A	0.257	0.044	8.766	0.224	0.259	1.978	0.267	29.073

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	64	32	28	61	44	20	20
N.S.	1	1.00	2.13	1.07	0.93	2.03	1.47	0.67	0.67
time (sec)	N/A	0.359	0.127	3.327	0.304	0.243	1.128	0.267	28.806

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	31	35	52	21	44	48	43
N.S.	1	1.00	1.72	1.94	2.89	1.17	2.44	2.67	2.39
time (sec)	N/A	0.243	0.024	1.643	0.220	0.254	1.200	0.275	28.301

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	14	15	14	28	10	14	14
N.S.	1	1.00	0.88	0.94	0.88	1.75	0.62	0.88	0.88
time (sec)	N/A	0.278	0.015	1.176	0.290	0.241	0.319	0.286	28.581

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	9	9	13	10	9	20	31	19
N.S.	1	0.69	0.69	1.00	0.77	0.69	1.54	2.38	1.46
time (sec)	N/A	0.137	0.001	0.317	0.225	0.241	0.061	0.272	30.081

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	16	6	31	5	17	22	21
N.S.	1	1.00	3.20	1.20	6.20	1.00	3.40	4.40	4.20
time (sec)	N/A	0.200	0.007	0.349	0.220	0.251	0.063	0.277	30.209

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	27	17	28	25	0	14	14
N.S.	1	1.00	1.93	1.21	2.00	1.79	0.00	1.00	1.00
time (sec)	N/A	0.221	0.025	0.592	0.305	0.233	0.000	0.276	30.143

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	34	17	64	20	301	45	41
N.S.	1	1.00	2.12	1.06	4.00	1.25	18.81	2.81	2.56
time (sec)	N/A	0.243	0.021	1.145	0.321	0.251	0.455	0.275	30.533

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	62	29	64	63	0	20	19
N.S.	1	1.00	2.38	1.12	2.46	2.42	0.00	0.77	0.73
time (sec)	N/A	0.288	0.066	2.801	0.305	0.246	0.000	0.257	31.757

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	39	23	92	35	1059	64	61
N.S.	1	1.00	1.77	1.05	4.18	1.59	48.14	2.91	2.77
time (sec)	N/A	0.248	0.044	6.852	0.307	0.246	1.660	0.267	30.561

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	173	143	167	188	292	308	169	174
N.S.	1	1.14	0.94	1.10	1.24	1.92	2.03	1.11	1.14
time (sec)	N/A	0.455	0.602	30.267	0.232	0.269	51.393	0.268	28.978

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	107	95	72	80	95	97	215	127
N.S.	1	1.06	0.94	0.71	0.79	0.94	0.96	2.13	1.26
time (sec)	N/A	0.558	0.293	10.311	0.298	0.242	16.150	0.268	29.139

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	98	79	80	87	128	124	86	82
N.S.	1	1.27	1.03	1.04	1.13	1.66	1.61	1.12	1.06
time (sec)	N/A	0.360	0.281	3.150	0.224	0.256	5.851	0.272	29.609

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	24	29	29	28	31	52	30
N.S.	1	1.00	0.83	1.00	1.00	0.97	1.07	1.79	1.03
time (sec)	N/A	0.272	0.165	1.115	0.317	0.244	1.135	0.278	30.308

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	30	16	15	27	24	15	27
N.S.	1	1.00	2.50	1.33	1.25	2.25	2.00	1.25	2.25
time (sec)	N/A	0.143	0.005	0.298	0.217	0.249	0.052	0.271	30.147

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	45	12	0	13	36
N.S.	1	1.00	1.00	1.08	3.75	1.00	0.00	1.08	3.00
time (sec)	N/A	0.223	0.032	0.535	0.308	0.250	0.000	0.264	30.744

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	72	71	86	0	307	0	107	440
N.S.	1	1.07	1.06	1.28	0.00	4.58	0.00	1.60	6.57
time (sec)	N/A	0.411	0.239	1.435	0.000	0.281	0.000	0.264	29.270

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	47	77	49	177	70	0	45	311
N.S.	1	0.94	1.54	0.98	3.54	1.40	0.00	0.90	6.22
time (sec)	N/A	0.294	0.194	5.556	0.313	0.282	0.000	0.249	28.501

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	179	150	197	0	878	0	282	3068
N.S.	1	1.13	0.94	1.24	0.00	5.52	0.00	1.77	19.30
time (sec)	N/A	0.832	0.414	20.358	0.000	0.308	0.000	0.270	33.917

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	87	138	107	497	166	0	93	538
N.S.	1	0.87	1.38	1.07	4.97	1.66	0.00	0.93	5.38
time (sec)	N/A	0.332	0.376	64.037	0.337	0.279	0.000	0.260	29.437

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	32	45	125	37	68	22	34
N.S.	1	1.00	1.14	1.61	4.46	1.32	2.43	0.79	1.21
time (sec)	N/A	0.255	0.072	1.439	0.218	0.241	131.279	0.264	27.529

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	31	56	36	44	20	16
N.S.	1	1.00	1.00	1.03	1.87	1.20	1.47	0.67	0.53
time (sec)	N/A	0.366	0.055	1.664	0.295	0.241	34.608	0.255	27.681

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	32	46	22	46	18	25
N.S.	1	1.00	1.00	1.60	2.30	1.10	2.30	0.90	1.25
time (sec)	N/A	0.247	0.044	0.986	0.215	0.247	8.997	0.252	28.133

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	12	15	16	16	17	12	10
N.S.	1	1.00	0.75	0.94	1.00	1.00	1.06	0.75	0.62
time (sec)	N/A	0.288	0.027	0.924	0.299	0.249	1.487	0.271	27.546

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	23	13	12	7	20	11	19
N.S.	1	1.00	2.56	1.44	1.33	0.78	2.22	1.22	2.11
time (sec)	N/A	0.136	0.003	0.287	0.231	0.252	0.049	0.268	27.419

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	9	8	14	9	0	7	9
N.S.	1	1.00	1.29	1.14	2.00	1.29	0.00	1.00	1.29
time (sec)	N/A	0.208	0.016	0.423	0.334	0.245	0.000	0.262	29.578

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	12	15	23	18	0	10	10
N.S.	1	1.00	0.86	1.07	1.64	1.29	0.00	0.71	0.71
time (sec)	N/A	0.229	0.015	0.464	0.322	0.254	0.000	0.259	28.429

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	18	15	28	21	0	14	18
N.S.	1	1.00	1.29	1.07	2.00	1.50	0.00	1.00	1.29
time (sec)	N/A	0.251	0.016	0.806	0.326	0.250	0.000	0.253	29.133

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	30	23	35	40	0	16	16
N.S.	1	1.00	1.15	0.88	1.35	1.54	0.00	0.62	0.62
time (sec)	N/A	0.297	0.019	1.598	0.327	0.257	0.000	0.269	28.945

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	32	2	39	38	0	22	26
N.S.	1	1.00	1.33	0.08	1.62	1.58	0.00	0.92	1.08
time (sec)	N/A	0.251	0.017	3.732	0.320	0.266	0.000	0.264	28.201

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	56	38	39	36	51	44	39	59
N.S.	1	1.27	0.86	0.89	0.82	1.16	1.00	0.89	1.34
time (sec)	N/A	0.211	0.038	2.079	0.221	0.242	3.454	0.270	29.116

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	39	61	29	37	57	42	99	75
N.S.	1	1.15	1.79	0.85	1.09	1.68	1.24	2.91	2.21
time (sec)	N/A	0.269	0.028	1.566	0.231	0.268	1.413	0.262	28.558

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	29	18	15	16	20	15	23	21
N.S.	1	1.32	0.82	0.68	0.73	0.91	0.68	1.05	0.95
time (sec)	N/A	0.193	0.021	0.967	0.243	0.241	0.508	0.264	28.878

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	19	12	11	21	19	9	8
N.S.	1	1.00	2.38	1.50	1.38	2.62	2.38	1.12	1.00
time (sec)	N/A	0.137	0.003	0.200	0.231	0.255	0.050	0.264	0.028

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	5	17	4	0	17	12
N.S.	1	1.00	1.00	2.50	8.50	2.00	0.00	8.50	6.00
time (sec)	N/A	0.191	0.005	0.399	0.233	0.229	0.000	0.269	29.315

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	14	0	6	6
N.S.	1	1.00	1.00	0.88	0.75	1.75	0.00	0.75	0.75
time (sec)	N/A	0.168	0.003	0.464	0.218	0.255	0.000	0.262	29.133

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	103	14	0	59	13
N.S.	1	1.00	1.00	0.82	6.06	0.82	0.00	3.47	0.76
time (sec)	N/A	0.234	0.024	0.613	0.228	0.234	0.000	0.267	28.035

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	37	14	13	26	0	13	23
N.S.	1	1.00	2.18	0.82	0.76	1.53	0.00	0.76	1.35
time (sec)	N/A	0.185	0.024	0.783	0.237	0.230	0.000	0.260	28.665

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	187	20	0	101	19
N.S.	1	1.00	1.00	0.80	7.48	0.80	0.00	4.04	0.76
time (sec)	N/A	0.239	0.024	0.944	0.228	0.235	0.000	0.291	29.588

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	57	20	19	40	0	19	33
N.S.	1	1.00	2.28	0.80	0.76	1.60	0.00	0.76	1.32
time (sec)	N/A	0.198	0.022	1.381	0.220	0.243	0.000	0.269	28.922

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	26	271	26	0	143	25
N.S.	1	1.00	1.00	0.79	8.21	0.79	0.00	4.33	0.76
time (sec)	N/A	0.249	0.024	1.866	0.233	0.244	0.000	0.262	28.922

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	97	37	36	578	44	0	0	0
N.S.	1	1.33	0.51	0.49	7.92	0.60	0.00	0.00	0.00
time (sec)	N/A	0.588	0.096	2.602	0.426	0.253	0.000	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	76	29	29	427	35	0	0	0
N.S.	1	1.52	0.58	0.58	8.54	0.70	0.00	0.00	0.00
time (sec)	N/A	0.462	0.052	2.589	0.416	0.258	0.000	0.000	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	55	21	17	314	23	0	0	0
N.S.	1	1.77	0.68	0.55	10.13	0.74	0.00	0.00	0.00
time (sec)	N/A	0.362	0.019	1.783	0.393	0.248	0.000	0.000	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	188	19	0	0	15
N.S.	1	1.00	1.00	0.92	14.46	1.46	0.00	0.00	1.15
time (sec)	N/A	0.282	0.011	1.784	0.409	0.252	0.000	0.000	27.750

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	60	36	44	0	0	124	0	0	0
N.S.	1	0.60	0.73	0.00	0.00	2.07	0.00	0.00	0.00
time (sec)	N/A	0.376	0.193	0.000	0.000	0.274	0.000	0.000	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	78	60	99	0	152	0	0	0
N.S.	1	0.98	0.75	1.24	0.00	1.90	0.00	0.00	0.00
time (sec)	N/A	0.490	0.159	15.632	0.000	0.288	0.000	0.000	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	69	158	0	165	0	0	0
N.S.	1	1.00	0.70	1.60	0.00	1.67	0.00	0.00	0.00
time (sec)	N/A	0.583	0.434	16.576	0.000	0.293	0.000	0.000	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	120	74	186	0	167	0	0	0
N.S.	1	1.02	0.63	1.58	0.00	1.42	0.00	0.00	0.00
time (sec)	N/A	0.684	0.256	16.421	0.000	0.300	0.000	0.000	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	58	38	29	26	37	44	35	80
N.S.	1	1.32	0.86	0.66	0.59	0.84	1.00	0.80	1.82
time (sec)	N/A	0.226	0.041	2.319	0.232	0.249	4.650	0.269	29.230

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	39	38	30	37	49	42	39	68
N.S.	1	1.15	1.12	0.88	1.09	1.44	1.24	1.15	2.00
time (sec)	N/A	0.263	0.012	1.664	0.224	0.245	1.627	0.268	26.102

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	29	16	13	12	22	14	18	49
N.S.	1	1.32	0.73	0.59	0.55	1.00	0.64	0.82	2.23
time (sec)	N/A	0.192	0.022	0.980	0.216	0.243	0.585	0.280	26.339

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	12	11	21	19	29	14
N.S.	1	1.00	1.00	1.50	1.38	2.62	2.38	3.62	1.75
time (sec)	N/A	0.135	0.009	0.218	0.217	0.244	0.047	0.265	27.760

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	21	6	0	6	6
N.S.	1	1.00	1.00	1.25	5.25	1.50	0.00	1.50	1.50
time (sec)	N/A	0.204	0.006	0.397	0.215	0.248	0.000	0.263	25.782

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	18	0	6	6
N.S.	1	1.00	1.00	0.88	0.75	2.25	0.00	0.75	0.75
time (sec)	N/A	0.168	0.003	0.448	0.207	0.228	0.000	0.269	27.900

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	73	28	0	14	14
N.S.	1	1.00	1.00	0.82	4.29	1.65	0.00	0.82	0.82
time (sec)	N/A	0.243	0.011	0.474	0.216	0.237	0.000	0.261	28.270

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	37	14	14	39	0	14	16
N.S.	1	1.00	2.18	0.82	0.82	2.29	0.00	0.82	0.94
time (sec)	N/A	0.185	0.029	0.487	0.215	0.248	0.000	0.283	28.086

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	121	46	0	20	20
N.S.	1	1.00	1.00	0.80	4.84	1.84	0.00	0.80	0.80
time (sec)	N/A	0.250	0.013	0.489	0.220	0.251	0.000	0.273	28.139

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	57	20	20	57	0	20	46
N.S.	1	1.00	2.28	0.80	0.80	2.28	0.00	0.80	1.84
time (sec)	N/A	0.197	0.033	0.537	0.206	0.233	0.000	0.291	28.088

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	26	169	64	0	26	109
N.S.	1	1.00	1.00	0.79	5.12	1.94	0.00	0.79	3.30
time (sec)	N/A	0.253	0.021	0.593	0.220	0.234	0.000	0.283	27.341

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	97	37	286	82	44	0	0	0
N.S.	1	1.33	0.51	3.92	1.12	0.60	0.00	0.00	0.00
time (sec)	N/A	0.545	0.197	11.325	0.323	0.267	0.000	0.000	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	76	29	280	82	38	0	0	0
N.S.	1	1.52	0.58	5.60	1.64	0.76	0.00	0.00	0.00
time (sec)	N/A	0.430	0.049	9.228	0.317	0.263	0.000	0.000	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	55	23	519	57	26	0	0	0
N.S.	1	1.77	0.74	16.74	1.84	0.84	0.00	0.00	0.00
time (sec)	N/A	0.344	0.021	2.212	0.310	0.240	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	50	57	22	0	46	20
N.S.	1	1.00	1.00	3.85	4.38	1.69	0.00	3.54	1.54
time (sec)	N/A	0.261	0.014	1.770	0.314	0.237	0.000	0.321	28.278

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	42	43	90	0	72	0	35	0
N.S.	1	0.81	0.83	1.73	0.00	1.38	0.00	0.67	0.00
time (sec)	N/A	0.365	0.111	1.135	0.000	0.267	0.000	0.337	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	84	56	166	0	119	0	95	0
N.S.	1	1.17	0.78	2.31	0.00	1.65	0.00	1.32	0.00
time (sec)	N/A	0.438	0.152	1.319	0.000	0.267	0.000	0.342	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	105	73	195	0	147	0	151	0
N.S.	1	1.15	0.80	2.14	0.00	1.62	0.00	1.66	0.00
time (sec)	N/A	0.538	0.733	1.229	0.000	0.274	0.000	0.384	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	126	74	217	0	171	0	210	0
N.S.	1	1.15	0.67	1.97	0.00	1.55	0.00	1.91	0.00
time (sec)	N/A	0.632	0.333	1.220	0.000	0.276	0.000	0.460	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	129	66	68	78	90	1375	88
N.S.	1	1.00	2.35	1.20	1.24	1.42	1.64	25.00	1.60
time (sec)	N/A	0.310	0.148	0.568	0.302	0.254	1.630	3.096	29.097

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	40	39	42	47	46	173	65
N.S.	1	1.00	1.05	1.03	1.11	1.24	1.21	4.55	1.71
time (sec)	N/A	0.257	0.027	10.224	0.220	0.267	1.650	0.457	28.718

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	60	25	28	44	31	177	61
N.S.	1	1.00	2.40	1.00	1.12	1.76	1.24	7.08	2.44
time (sec)	N/A	0.260	0.077	2.950	0.311	0.253	0.503	0.335	28.601

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	8	12	8	11	22
N.S.	1	1.00	1.00	1.10	0.80	1.20	0.80	1.10	2.20
time (sec)	N/A	0.138	0.002	0.256	0.213	0.256	0.022	0.276	27.306

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	35	24	25	35	0	28	16
N.S.	1	1.00	1.46	1.00	1.04	1.46	0.00	1.17	0.67
time (sec)	N/A	0.362	0.017	0.474	0.217	0.250	0.000	0.283	27.929

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	57	32	45	34	0	31	40
N.S.	1	1.00	1.73	0.97	1.36	1.03	0.00	0.94	1.21
time (sec)	N/A	0.318	0.019	0.589	0.223	0.257	0.000	0.287	29.090

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	83	56	73	130	0	95	48
N.S.	1	1.00	1.38	0.93	1.22	2.17	0.00	1.58	0.80
time (sec)	N/A	0.289	0.018	0.886	0.223	0.255	0.000	0.276	29.385

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	129	64	97	78	0	65	87
N.S.	1	1.00	1.98	0.98	1.49	1.20	0.00	1.00	1.34
time (sec)	N/A	0.410	0.018	1.655	0.235	0.241	0.000	0.284	29.113

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	68	117	153	144	634	131	695
N.S.	1	1.00	0.92	1.58	2.07	1.95	8.57	1.77	9.39
time (sec)	N/A	0.302	0.243	0.800	0.323	0.263	10.676	0.318	35.824

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-2)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	82	108	146	200	0	130	105
N.S.	1	1.00	1.09	1.44	1.95	2.67	0.00	1.73	1.40
time (sec)	N/A	0.308	0.296	0.730	0.308	0.252	0.000	0.315	29.622

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	128	132	177	338	279	0	199	227
N.S.	1	1.10	1.14	1.53	2.91	2.41	0.00	1.72	1.96
time (sec)	N/A	0.467	0.389	0.955	0.326	0.266	0.000	0.346	30.332

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	67	117	153	143	673	131	692
N.S.	1	1.00	0.92	1.60	2.10	1.96	9.22	1.79	9.48
time (sec)	N/A	0.293	0.135	0.696	0.306	0.268	10.602	0.340	33.801

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-2)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	82	109	156	201	0	132	126
N.S.	1	1.00	1.08	1.43	2.05	2.64	0.00	1.74	1.66
time (sec)	N/A	0.306	0.211	0.668	0.325	0.254	0.000	0.307	27.855

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	128	118	204	366	279	0	245	251
N.S.	1	1.10	1.02	1.76	3.16	2.41	0.00	2.11	2.16
time (sec)	N/A	0.462	0.276	0.845	0.336	0.257	0.000	0.339	27.079

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	248	238	331	354	221	857	287	522
N.S.	1	1.01	0.97	1.35	1.44	0.90	3.48	1.17	2.12
time (sec)	N/A	0.624	2.080	3.266	0.247	0.266	0.556	0.402	31.392

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	177	163	219	207	145	415	199	261
N.S.	1	0.99	0.92	1.23	1.16	0.81	2.33	1.12	1.47
time (sec)	N/A	0.431	1.508	2.348	0.248	0.251	0.345	0.323	32.702

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	106	111	106	113	81	192	92	100
N.S.	1	0.91	0.96	0.91	0.97	0.70	1.66	0.79	0.86
time (sec)	N/A	0.279	1.211	1.022	0.236	0.254	0.163	0.287	28.228

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	36	36	35	34	42	35	48
N.S.	1	1.00	0.97	0.97	0.95	0.92	1.14	0.95	1.30
time (sec)	N/A	0.160	0.060	0.542	0.214	0.250	0.069	0.274	28.288

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	50	40	75	0	43	38
N.S.	1	1.00	1.00	1.02	0.82	1.53	0.00	0.88	0.78
time (sec)	N/A	0.224	0.208	1.035	0.209	0.251	0.000	0.309	27.230

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	98	129	0	192	0	158	274
N.S.	1	1.00	0.76	1.00	0.00	1.49	0.00	1.22	2.12
time (sec)	N/A	0.381	0.269	1.375	0.000	0.256	0.000	0.309	29.110

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	211	420	246	0	490	0	342	592
N.S.	1	1.10	2.20	1.29	0.00	2.57	0.00	1.79	3.10
time (sec)	N/A	0.559	1.845	1.976	0.000	0.302	0.000	0.417	33.506

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	259	293	533	511	0	739	0	593	1004
N.S.	1	1.13	2.06	1.97	0.00	2.85	0.00	2.29	3.88
time (sec)	N/A	0.774	1.739	2.785	0.000	0.478	0.000	0.682	40.356

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	170	135	152	191	134	291	151	239
N.S.	1	1.08	0.86	0.97	1.22	0.85	1.85	0.96	1.52
time (sec)	N/A	0.465	1.455	2.038	0.234	0.260	0.156	0.313	27.661

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	86	92	77	99	71	170	78	96
N.S.	1	1.06	1.14	0.95	1.22	0.88	2.10	0.96	1.19
time (sec)	N/A	0.266	0.509	0.852	0.239	0.246	0.104	0.285	27.297

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	53	30	29	27	39	29	29
N.S.	1	1.00	1.83	1.03	1.00	0.93	1.34	1.00	1.00
time (sec)	N/A	0.159	0.061	0.477	0.209	0.237	0.068	0.263	27.047

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	57	23	29	60	63	23	22
N.S.	1	1.00	2.28	0.92	1.16	2.40	2.52	0.92	0.88
time (sec)	N/A	0.194	0.124	0.721	0.216	0.272	0.633	0.281	26.760

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	115	68	90	154	0	83	79
N.S.	1	1.00	1.53	0.91	1.20	2.05	0.00	1.11	1.05
time (sec)	N/A	0.307	0.479	0.985	0.227	0.261	0.000	0.281	26.773

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	139	186	131	190	433	0	164	162
N.S.	1	1.04	1.39	0.98	1.42	3.23	0.00	1.22	1.21
time (sec)	N/A	0.496	2.233	1.135	0.234	0.258	0.000	0.289	26.349

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	218	397	215	307	791	0	291	260
N.S.	1	1.05	1.92	1.04	1.48	3.82	0.00	1.41	1.26
time (sec)	N/A	0.793	4.437	1.413	0.252	0.303	0.000	0.340	26.991

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	50	21	28	31	36	21	20
N.S.	1	1.00	2.17	0.91	1.22	1.35	1.57	0.91	0.87
time (sec)	N/A	0.188	0.131	0.712	0.212	0.254	0.317	0.295	27.015

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	93	48	80	100	168	65	59
N.S.	1	1.00	1.24	0.64	1.07	1.33	2.24	0.87	0.79
time (sec)	N/A	0.299	0.255	0.917	0.218	0.251	0.827	0.301	27.880

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	134	135	78	146	143	423	102	90
N.S.	1	1.09	1.10	0.63	1.19	1.16	3.44	0.83	0.73
time (sec)	N/A	0.488	0.564	0.938	0.225	0.246	2.611	0.310	27.893

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	188	247	106	208	237	792	132	161
N.S.	1	1.12	1.47	0.63	1.24	1.41	4.71	0.79	0.96
time (sec)	N/A	0.714	1.163	0.952	0.245	0.248	9.733	0.313	27.120

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	171	136	152	188	134	291	151	258
N.S.	1	1.09	0.87	0.97	1.20	0.85	1.85	0.96	1.64
time (sec)	N/A	0.455	1.730	2.088	0.222	0.253	0.158	0.316	28.034

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	86	92	77	98	71	170	78	84
N.S.	1	1.06	1.14	0.95	1.21	0.88	2.10	0.96	1.04
time (sec)	N/A	0.255	0.765	0.852	0.211	0.241	0.104	0.284	26.002

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	53	30	29	28	39	29	29
N.S.	1	1.00	1.83	1.03	1.00	0.97	1.34	1.00	1.00
time (sec)	N/A	0.155	0.040	0.490	0.214	0.239	0.067	0.269	27.286

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	50	36	54	60	95	41	26
N.S.	1	1.00	2.00	1.44	2.16	2.40	3.80	1.64	1.04
time (sec)	N/A	0.187	1.131	0.786	0.212	0.253	0.709	0.285	27.442

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	229	88	137	162	0	110	91
N.S.	1	1.00	3.05	1.17	1.83	2.16	0.00	1.47	1.21
time (sec)	N/A	0.311	5.410	0.986	0.223	0.263	0.000	0.290	27.320

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	138	350	176	265	438	0	229	186
N.S.	1	1.03	2.61	1.31	1.98	3.27	0.00	1.71	1.39
time (sec)	N/A	0.507	7.658	1.182	0.235	0.274	0.000	0.304	28.772

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	218	494	253	382	796	0	347	301
N.S.	1	1.05	2.39	1.22	1.85	3.85	0.00	1.68	1.45
time (sec)	N/A	0.800	7.967	1.535	0.261	0.281	0.000	0.324	30.810

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	170	135	151	191	127	291	151	292
N.S.	1	1.08	0.86	0.96	1.22	0.81	1.85	0.96	1.86
time (sec)	N/A	0.458	1.446	2.191	0.219	0.252	0.163	0.293	28.531

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	86	92	80	99	72	170	79	127
N.S.	1	1.06	1.14	0.99	1.22	0.89	2.10	0.98	1.57
time (sec)	N/A	0.263	0.482	0.819	0.209	0.243	0.112	0.276	27.882

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	53	30	29	27	39	29	29
N.S.	1	1.00	1.83	1.03	1.00	0.93	1.34	1.00	1.00
time (sec)	N/A	0.158	0.054	0.490	0.219	0.247	0.077	0.268	27.689

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	93	43	66	54	107	79	33
N.S.	1	1.00	2.82	1.30	2.00	1.64	3.24	2.39	1.00
time (sec)	N/A	0.195	0.118	0.904	0.220	0.266	0.998	0.286	27.366

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	162	122	185	148	0	187	126
N.S.	1	1.00	1.95	1.47	2.23	1.78	0.00	2.25	1.52
time (sec)	N/A	0.306	0.469	1.161	0.229	0.266	0.000	0.294	26.784

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	146	255	264	493	420	0	458	360
N.S.	1	1.03	1.80	1.86	3.47	2.96	0.00	3.23	2.54
time (sec)	N/A	0.506	1.916	1.954	0.234	0.265	0.000	0.302	30.880

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	226	632	398	963	729	0	957	730
N.S.	1	1.05	2.94	1.85	4.48	3.39	0.00	4.45	3.40
time (sec)	N/A	0.790	1.748	3.181	0.266	0.302	0.000	0.334	32.740

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	169	136	152	188	126	291	151	292
N.S.	1	1.08	0.87	0.97	1.20	0.80	1.85	0.96	1.86
time (sec)	N/A	0.452	1.671	2.199	0.243	0.288	0.161	0.290	27.160

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	86	92	80	98	70	170	79	128
N.S.	1	1.06	1.14	0.99	1.21	0.86	2.10	0.98	1.58
time (sec)	N/A	0.257	0.658	0.796	0.211	0.245	0.109	0.268	27.045

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	53	30	29	26	39	29	29
N.S.	1	1.00	1.83	1.03	1.00	0.90	1.34	1.00	1.00
time (sec)	N/A	0.153	0.039	0.428	0.237	0.243	0.073	0.270	26.380

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	96	47	62	57	109	79	32
N.S.	1	1.00	2.91	1.42	1.88	1.73	3.30	2.39	0.97
time (sec)	N/A	0.190	0.414	0.879	0.232	0.270	0.979	0.282	27.432

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	166	130	182	154	0	189	126
N.S.	1	1.00	2.00	1.57	2.19	1.86	0.00	2.28	1.52
time (sec)	N/A	0.307	0.891	1.166	0.238	0.258	0.000	0.295	27.148

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	147	261	275	491	423	0	458	361
N.S.	1	1.04	1.84	1.94	3.46	2.98	0.00	3.23	2.54
time (sec)	N/A	0.516	2.330	1.976	0.252	0.264	0.000	0.321	32.503

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	225	636	410	959	735	0	957	731
N.S.	1	1.05	2.96	1.91	4.46	3.42	0.00	4.45	3.40
time (sec)	N/A	0.784	1.872	3.044	0.266	0.289	0.000	0.357	33.674

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	284	237	293	330	255	682	286	376
N.S.	1	1.09	0.91	1.13	1.27	0.98	2.62	1.10	1.45
time (sec)	N/A	0.822	1.988	3.200	0.231	0.268	0.290	0.339	28.617

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	180	144	168	189	147	294	167	333
N.S.	1	1.06	0.85	0.99	1.11	0.86	1.73	0.98	1.96
time (sec)	N/A	0.477	1.368	1.984	0.226	0.241	0.159	0.299	29.068

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	89	77	88	100	73	162	81	125
N.S.	1	0.98	0.85	0.97	1.10	0.80	1.78	0.89	1.37
time (sec)	N/A	0.257	0.505	0.814	0.208	0.241	0.108	0.284	29.423

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	49	28	27	26	34	27	40
N.S.	1	1.00	1.81	1.04	1.00	0.96	1.26	1.00	1.48
time (sec)	N/A	0.157	0.006	0.512	0.227	0.249	0.083	0.272	26.436

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	67	57	61	0	434	3179	89	75
N.S.	1	1.10	0.93	1.00	0.00	7.11	52.11	1.46	1.23
time (sec)	N/A	0.237	0.177	0.806	0.000	0.297	113.176	0.269	28.842

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	127	116	203	0	819	0	214	195
N.S.	1	1.05	0.96	1.68	0.00	6.77	0.00	1.77	1.61
time (sec)	N/A	0.388	0.334	1.108	0.000	0.301	0.000	0.272	27.542

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	223	200	611	0	1947	0	856	700
N.S.	1	1.13	1.02	3.10	0.00	9.88	0.00	4.35	3.55
time (sec)	N/A	0.626	0.734	1.723	0.000	0.351	0.000	0.320	32.720

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	338	606	1655	0	4069	0	2558	1946
N.S.	1	1.16	2.08	5.67	0.00	13.93	0.00	8.76	6.66
time (sec)	N/A	1.044	1.713	4.406	0.000	0.466	0.000	0.425	29.885

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	193	399	821	0	180	0	0	0
N.S.	1	1.04	2.16	4.44	0.00	0.97	0.00	0.00	0.00
time (sec)	N/A	1.003	4.940	2.045	0.000	0.099	0.000	0.000	0.000

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	139	142	349	806	0	159	0	0	0
N.S.	1	1.02	2.51	5.80	0.00	1.14	0.00	0.00	0.00
time (sec)	N/A	0.699	2.856	1.339	0.000	0.095	0.000	0.000	0.000

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	326	461	0	120	0	0	0
N.S.	1	1.00	7.24	10.24	0.00	2.67	0.00	0.00	0.00
time (sec)	N/A	0.238	1.937	2.762	0.000	0.085	0.000	0.000	0.000

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	128	152	0	60	0	0	0
N.S.	1	1.00	2.84	3.38	0.00	1.33	0.00	0.00	0.00
time (sec)	N/A	0.248	0.334	1.592	0.000	0.080	0.000	0.000	0.000

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	390	425	0	278	0	0	0
N.S.	1	1.00	4.15	4.52	0.00	2.96	0.00	0.00	0.00
time (sec)	N/A	0.377	4.788	1.348	0.000	0.097	0.000	0.000	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	193	430	586	0	431	0	0	0
N.S.	1	1.03	2.30	3.13	0.00	2.30	0.00	0.00	0.00
time (sec)	N/A	0.944	2.466	1.621	0.000	0.099	0.000	0.000	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	233	244	436	649	0	577	0	0	0
N.S.	1	1.05	1.87	2.79	0.00	2.48	0.00	0.00	0.00
time (sec)	N/A	1.233	2.998	1.697	0.000	0.109	0.000	0.000	0.000

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	347	359	3767	2270	0	1595	0	0	0
N.S.	1	1.03	10.86	6.54	0.00	4.60	0.00	0.00	0.00
time (sec)	N/A	1.624	6.765	3.022	0.000	0.152	0.000	0.000	0.000

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	283	284	2190	1494	0	1483	0	0	0
N.S.	1	1.00	7.74	5.28	0.00	5.24	0.00	0.00	0.00
time (sec)	N/A	1.066	6.344	1.985	0.000	0.135	0.000	0.000	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	1408	691	0	1371	0	0	0
N.S.	1	1.00	13.04	6.40	0.00	12.69	0.00	0.00	0.00
time (sec)	N/A	0.361	6.308	5.309	0.000	0.117	0.000	0.000	0.000

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	285	295	0	506	0	0	0
N.S.	1	1.00	2.64	2.73	0.00	4.69	0.00	0.00	0.00
time (sec)	N/A	0.366	0.677	1.348	0.000	0.106	0.000	0.000	0.000

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	186	186	1540	2645	0	1737	0	0	0
N.S.	1	1.00	8.28	14.22	0.00	9.34	0.00	0.00	0.00
time (sec)	N/A	0.547	6.436	2.437	0.000	0.147	0.000	0.000	0.000

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	382	397	2408	3642	0	2805	0	0	0
N.S.	1	1.04	6.30	9.53	0.00	7.34	0.00	0.00	0.00
time (sec)	N/A	1.532	6.694	9.276	0.000	0.213	0.000	0.000	0.000

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	490	526	4116	5028	0	4955	0	0	0
N.S.	1	1.07	8.40	10.26	0.00	10.11	0.00	0.00	0.00
time (sec)	N/A	2.255	6.565	30.194	0.000	0.513	0.000	0.000	0.000

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	142	130	74	0	101	0	0	0
N.S.	1	1.02	0.94	0.53	0.00	0.73	0.00	0.00	0.00
time (sec)	N/A	0.407	6.248	1.062	0.000	0.236	0.000	0.000	0.000

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	104	60	0	81	0	0	0
N.S.	1	1.00	1.12	0.65	0.00	0.87	0.00	0.00	0.00
time (sec)	N/A	0.292	1.850	0.668	0.000	0.237	0.000	0.000	0.000

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	75	50	0	61	0	0	39
N.S.	1	1.00	1.70	1.14	0.00	1.39	0.00	0.00	0.89
time (sec)	N/A	0.188	0.037	0.764	0.000	0.239	0.000	0.000	0.416

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	101	77	0	147	0	0	0
N.S.	1	1.00	2.10	1.60	0.00	3.06	0.00	0.00	0.00
time (sec)	N/A	0.260	0.204	2.901	0.000	0.251	0.000	0.000	0.000

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	154	117	0	268	0	0	0
N.S.	1	1.00	1.60	1.22	0.00	2.79	0.00	0.00	0.00
time (sec)	N/A	0.364	0.312	0.769	0.000	0.262	0.000	0.000	0.000

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	147	180	190	0	341	0	0	0
N.S.	1	1.04	1.27	1.34	0.00	2.40	0.00	0.00	0.00
time (sec)	N/A	0.502	0.413	0.907	0.000	0.259	0.000	0.000	0.000

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	193	151	86	0	121	0	0	0
N.S.	1	1.04	0.82	0.46	0.00	0.65	0.00	0.00	0.00
time (sec)	N/A	0.547	7.295	1.194	0.000	0.245	0.000	0.000	0.000

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	142	127	74	0	101	0	0	0
N.S.	1	1.02	0.91	0.53	0.00	0.73	0.00	0.00	0.00
time (sec)	N/A	0.407	5.842	0.840	0.000	0.254	0.000	0.000	0.000

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	103	60	0	80	0	0	0
N.S.	1	1.00	1.11	0.65	0.00	0.86	0.00	0.00	0.00
time (sec)	N/A	0.294	4.045	0.684	0.000	0.238	0.000	0.000	0.000

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	75	50	0	59	0	0	39
N.S.	1	1.00	1.70	1.14	0.00	1.34	0.00	0.00	0.89
time (sec)	N/A	0.190	0.032	0.648	0.000	0.240	0.000	0.000	0.551

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	99	77	0	88	0	0	0
N.S.	1	1.00	2.02	1.57	0.00	1.80	0.00	0.00	0.00
time (sec)	N/A	0.256	0.163	2.324	0.000	0.248	0.000	0.000	0.000

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	152	118	0	210	0	0	0
N.S.	1	1.00	1.58	1.23	0.00	2.19	0.00	0.00	0.00
time (sec)	N/A	0.364	0.310	0.773	0.000	0.257	0.000	0.000	0.000

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	147	178	190	0	280	0	0	0
N.S.	1	1.04	1.25	1.34	0.00	1.97	0.00	0.00	0.00
time (sec)	N/A	0.499	0.441	0.881	0.000	0.251	0.000	0.000	0.000

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	258	272	5490	306	0	268	0	0	0
N.S.	1	1.05	21.28	1.19	0.00	1.04	0.00	0.00	0.00
time (sec)	N/A	0.757	37.941	1.635	0.000	0.261	0.000	0.000	0.000

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	190	199	5377	200	0	189	0	0	0
N.S.	1	1.05	28.30	1.05	0.00	0.99	0.00	0.00	0.00
time (sec)	N/A	0.535	18.056	1.350	0.000	0.256	0.000	0.000	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	A	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	5279	126	0	125	0	0	0
N.S.	1	1.00	41.90	1.00	0.00	0.99	0.00	0.00	0.00
time (sec)	N/A	0.370	17.981	1.361	0.000	0.256	0.000	0.000	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	565	113	0	80	0	0	0
N.S.	1	1.00	10.27	2.05	0.00	1.45	0.00	0.00	0.00
time (sec)	N/A	0.221	16.601	1.279	0.000	0.247	0.000	0.000	0.000

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	63264	172	0	349	0	0	0
N.S.	1	1.00	718.91	1.95	0.00	3.97	0.00	0.00	0.00
time (sec)	N/A	0.352	54.766	5.571	0.000	0.425	0.000	0.000	0.000

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	B	F(-2)	B	F	F(-1)	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	0	350	0	646	0	0	0
N.S.	1	1.00	0.00	2.19	0.00	4.04	0.00	0.00	0.00
time (sec)	N/A	0.516	0.000	1.540	0.000	0.509	0.000	0.000	0.000

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	F(-2)	B	F	F(-1)	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	226	244	0	350	0	895	0	0	0
N.S.	1	1.08	0.00	1.55	0.00	3.96	0.00	0.00	0.00
time (sec)	N/A	0.739	0.000	1.597	0.000	0.499	0.000	0.000	0.000

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	196	205	5238	204	0	192	0	0	0
N.S.	1	1.05	26.72	1.04	0.00	0.98	0.00	0.00	0.00
time (sec)	N/A	0.560	39.428	1.451	0.000	0.261	0.000	0.000	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	A	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	5142	130	0	127	0	0	0
N.S.	1	1.00	39.55	1.00	0.00	0.98	0.00	0.00	0.00
time (sec)	N/A	0.372	13.034	1.381	0.000	0.257	0.000	0.000	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	A	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	5053	117	0	80	0	0	0
N.S.	1	1.00	88.65	2.05	0.00	1.40	0.00	0.00	0.00
time (sec)	N/A	0.226	18.300	1.338	0.000	0.252	0.000	0.000	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	A	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	61904	175	0	107	0	0	0
N.S.	1	1.00	680.26	1.92	0.00	1.18	0.00	0.00	0.00
time (sec)	N/A	0.345	61.111	4.727	0.000	0.257	0.000	0.000	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	B	F(-2)	B	F	F(-1)	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	0	364	0	442	0	0	0
N.S.	1	1.00	0.00	2.22	0.00	2.70	0.00	0.00	0.00
time (sec)	N/A	0.528	0.000	1.582	0.000	0.305	0.000	0.000	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	F(-2)	B	F	F(-1)	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	232	250	0	364	0	655	0	0	0
N.S.	1	1.08	0.00	1.57	0.00	2.82	0.00	0.00	0.00
time (sec)	N/A	0.758	0.000	1.629	0.000	0.349	0.000	0.000	0.000

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	107	80	176	0	579	0	160	950
N.S.	1	1.06	0.79	1.74	0.00	5.73	0.00	1.58	9.41
time (sec)	N/A	0.333	0.278	0.799	0.000	0.315	0.000	0.285	40.867

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	30	22	20	41	11	22	25	34
N.S.	1	1.36	1.00	0.91	1.86	0.50	1.00	1.14	1.55
time (sec)	N/A	0.227	0.080	0.544	0.296	0.246	0.142	0.292	26.115

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	102	79	171	0	553	0	158	988
N.S.	1	1.05	0.81	1.76	0.00	5.70	0.00	1.63	10.19
time (sec)	N/A	0.386	0.229	0.766	0.000	0.315	0.000	0.272	42.593

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	56	50	53	0	349	0	73	47
N.S.	1	1.10	0.98	1.04	0.00	6.84	0.00	1.43	0.92
time (sec)	N/A	0.274	0.044	0.654	0.000	0.316	0.000	0.289	27.996

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	148	120	180	0	663	0	161	977
N.S.	1	1.04	0.85	1.27	0.00	4.67	0.00	1.13	6.88
time (sec)	N/A	0.653	0.341	1.510	0.000	1.620	0.000	0.310	39.084

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	371	337	2490	64069	0	1504	0	0	0
N.S.	1	0.91	6.71	172.69	0.00	4.05	0.00	0.00	0.00
time (sec)	N/A	1.448	6.819	29.328	0.000	0.159	0.000	0.000	0.000

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	1580	2660	0	1367	0	0	0
N.S.	1	1.00	13.39	22.54	0.00	11.58	0.00	0.00	0.00
time (sec)	N/A	0.506	6.803	26.931	0.000	0.138	0.000	0.000	0.000

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	118	118	339	1018	0	504	0	0	0
N.S.	1	1.00	2.87	8.63	0.00	4.27	0.00	0.00	0.00
time (sec)	N/A	0.498	1.262	25.329	0.000	0.101	0.000	0.000	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F(-1)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	240	237	1732	40064	0	1724	0	0	0
N.S.	1	0.99	7.22	166.93	0.00	7.18	0.00	0.00	0.00
time (sec)	N/A	0.738	6.943	20.450	0.000	0.158	0.000	0.000	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	492	443	2708	155460	0	2769	0	0	0
N.S.	1	0.90	5.50	315.98	0.00	5.63	0.00	0.00	0.00
time (sec)	N/A	1.849	7.014	25.615	0.000	0.295	0.000	0.000	0.000

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	371	337	81485	64099	0	1500	0	0	0
N.S.	1	0.91	219.64	172.77	0.00	4.04	0.00	0.00	0.00
time (sec)	N/A	1.315	36.993	26.216	0.000	0.150	0.000	0.000	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	54676	2660	0	1371	0	0	0
N.S.	1	1.00	463.36	22.54	0.00	11.62	0.00	0.00	0.00
time (sec)	N/A	0.513	32.799	24.267	0.000	0.136	0.000	0.000	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	118	118	506	1012	0	506	0	0	0
N.S.	1	1.00	4.29	8.58	0.00	4.29	0.00	0.00	0.00
time (sec)	N/A	0.511	16.024	25.260	0.000	0.101	0.000	0.000	0.000

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	240	237	54829	40403	0	1752	0	0	0
N.S.	1	0.99	228.45	168.35	0.00	7.30	0.00	0.00	0.00
time (sec)	N/A	0.739	32.543	21.589	0.000	0.154	0.000	0.000	0.000

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	492	443	81703	155452	0	2816	0	0	0
N.S.	1	0.90	166.06	315.96	0.00	5.72	0.00	0.00	0.00
time (sec)	N/A	1.790	33.649	25.230	0.000	0.248	0.000	0.000	0.000

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	103	80	176	0	555	0	158	965
N.S.	1	1.05	0.82	1.80	0.00	5.66	0.00	1.61	9.85
time (sec)	N/A	0.377	0.332	0.758	0.000	0.305	0.000	0.283	41.768

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	56	50	53	0	349	0	73	47
N.S.	1	1.10	0.98	1.04	0.00	6.84	0.00	1.43	0.92
time (sec)	N/A	0.270	0.054	0.682	0.000	0.305	0.000	0.276	27.288

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	104	123	0	669	0	142	531
N.S.	1	1.00	0.87	1.02	0.00	5.58	0.00	1.18	4.42
time (sec)	N/A	0.599	0.982	1.052	0.000	1.667	0.000	0.286	35.161

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	17	51	10	14	24	0	22	9
N.S.	1	0.81	2.43	0.48	0.67	1.14	0.00	1.05	0.43
time (sec)	N/A	0.240	0.037	0.437	0.317	0.248	0.000	0.271	27.333

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	371	337	2490	21493	0	1511	0	0	0
N.S.	1	0.91	6.71	57.93	0.00	4.07	0.00	0.00	0.00
time (sec)	N/A	1.438	6.813	29.776	0.000	0.149	0.000	0.000	0.000

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	118	118	1580	1862	0	1361	0	0	0
N.S.	1	1.00	13.39	15.78	0.00	11.53	0.00	0.00	0.00
time (sec)	N/A	0.521	6.678	28.197	0.000	0.142	0.000	0.000	0.000

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	118	118	339	644	0	508	0	0	0
N.S.	1	1.00	2.87	5.46	0.00	4.31	0.00	0.00	0.00
time (sec)	N/A	0.508	1.082	31.073	0.000	0.102	0.000	0.000	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	240	237	1732	12702	0	1711	0	0	0
N.S.	1	0.99	7.22	52.92	0.00	7.13	0.00	0.00	0.00
time (sec)	N/A	0.770	6.876	25.719	0.000	0.161	0.000	0.000	0.000

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	492	443	2708	59351	0	2777	0	0	0
N.S.	1	0.90	5.50	120.63	0.00	5.64	0.00	0.00	0.00
time (sec)	N/A	1.827	7.193	29.434	0.000	0.296	0.000	0.000	0.000

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	371	337	5904	21719	0	1500	0	0	0
N.S.	1	0.91	15.91	58.54	0.00	4.04	0.00	0.00	0.00
time (sec)	N/A	1.358	26.310	28.387	0.000	0.147	0.000	0.000	0.000

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	5835	1851	0	1371	0	0	0
N.S.	1	1.00	49.45	15.69	0.00	11.62	0.00	0.00	0.00
time (sec)	N/A	0.517	16.941	23.301	0.000	0.136	0.000	0.000	0.000

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	118	118	519	716	0	506	0	0	0
N.S.	1	1.00	4.40	6.07	0.00	4.29	0.00	0.00	0.00
time (sec)	N/A	0.541	18.765	28.566	0.000	0.098	0.000	0.000	0.000

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	240	237	5959	12838	0	1746	0	0	0
N.S.	1	0.99	24.83	53.49	0.00	7.28	0.00	0.00	0.00
time (sec)	N/A	0.786	17.465	26.129	0.000	0.148	0.000	0.000	0.000

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	492	443	6066	60004	0	2798	0	0	0
N.S.	1	0.90	12.33	121.96	0.00	5.69	0.00	0.00	0.00
time (sec)	N/A	1.829	18.635	28.204	0.000	0.253	0.000	0.000	0.000

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	10	1	1
N.S.	1	1.00	1.00	2.00	1.00	1.00	10.00	1.00	1.00
time (sec)	N/A	0.150	0.019	0.300	0.305	0.221	0.195	0.285	28.216

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	22	1	1
N.S.	1	1.00	1.00	2.00	1.00	1.00	22.00	1.00	1.00
time (sec)	N/A	0.153	0.004	0.241	0.288	0.225	0.326	0.281	27.814

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	34	1	1
N.S.	1	1.00	1.00	2.00	1.00	1.00	34.00	1.00	1.00
time (sec)	N/A	0.156	0.001	0.230	0.304	0.217	0.604	0.281	27.169

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	3	9	4	15	23	36	33	3
N.S.	1	0.27	0.82	0.36	1.36	2.09	3.27	3.00	0.27
time (sec)	N/A	0.168	0.049	0.595	0.205	0.243	0.174	0.284	27.891

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	8	13	12	15	48	6	6
N.S.	1	1.00	0.62	1.00	0.92	1.15	3.69	0.46	0.46
time (sec)	N/A	0.177	0.027	0.524	0.205	0.229	0.634	0.279	27.177

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	30	22	48	38	74	765	37	32
N.S.	1	0.94	0.69	1.50	1.19	2.31	23.91	1.16	1.00
time (sec)	N/A	0.193	0.034	0.766	0.225	0.246	1.614	0.276	27.572

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	9	35	12007	20	9
N.S.	1	1.00	1.00	1.11	1.00	3.89	1334.11	2.22	1.00
time (sec)	N/A	0.181	0.124	1.798	0.284	0.253	8.295	0.278	29.704

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	31	30189	22	11
N.S.	1	1.00	1.00	1.09	1.00	2.82	2744.45	2.00	1.00
time (sec)	N/A	0.183	0.085	3.571	0.300	0.254	10.340	0.270	27.904

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	15	43	71839	26	15
N.S.	1	1.00	1.00	1.07	1.00	2.87	4789.27	1.73	1.00
time (sec)	N/A	0.193	0.074	3.513	0.315	0.255	15.531	0.286	28.071

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	25	25	18	17	43	87	61	36
N.S.	1	0.47	0.47	0.34	0.32	0.81	1.64	1.15	0.68
time (sec)	N/A	0.187	0.156	0.677	0.315	0.245	0.321	0.294	28.914

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	45	36	38	35	182	216	48	51
N.S.	1	1.05	0.84	0.88	0.81	4.23	5.02	1.12	1.19
time (sec)	N/A	0.291	0.305	0.530	0.302	0.276	0.583	0.280	27.917

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	45	36	38	35	181	211	48	48
N.S.	1	1.05	0.84	0.88	0.81	4.21	4.91	1.12	1.12
time (sec)	N/A	0.266	0.089	0.516	0.323	0.260	0.594	0.272	27.645

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	21	19	18	15	35	0	15	15
N.S.	1	0.58	0.53	0.50	0.42	0.97	0.00	0.42	0.42
time (sec)	N/A	0.191	0.141	3.813	0.282	0.254	0.000	0.264	27.151

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	33	42	27	27	68	0	27	27
N.S.	1	0.67	0.86	0.55	0.55	1.39	0.00	0.55	0.55
time (sec)	N/A	0.210	0.255	0.280	0.297	0.252	0.000	0.268	27.224

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	64	79	2	45	100	0	39	40
N.S.	1	0.86	1.07	0.03	0.61	1.35	0.00	0.53	0.54
time (sec)	N/A	0.222	0.254	14.445	0.291	0.259	0.000	0.271	28.557

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	0	1	1
N.S.	1	1.00	1.00	2.00	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.157	0.001	0.518	0.297	0.229	0.000	0.263	27.835

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	0	1	1
N.S.	1	1.00	1.00	2.00	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.160	0.003	5.348	0.296	0.224	0.000	0.257	27.109

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	0	1	1
N.S.	1	1.00	1.00	2.00	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.159	0.001	59.677	0.289	0.220	0.000	0.266	28.384

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	21	19	19	16	35	0	49	16
N.S.	1	0.57	0.51	0.51	0.43	0.95	0.00	1.32	0.43
time (sec)	N/A	0.187	0.123	3.243	0.288	0.247	0.000	0.278	27.624

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	32	64	28	27	66	0	60	27
N.S.	1	0.68	1.36	0.60	0.57	1.40	0.00	1.28	0.57
time (sec)	N/A	0.200	0.174	0.320	0.311	0.267	0.000	0.263	27.913

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	62	66	2	42	98	0	69	43
N.S.	1	0.86	0.92	0.03	0.58	1.36	0.00	0.96	0.60
time (sec)	N/A	0.238	0.237	11.542	0.312	0.244	0.000	0.269	29.586

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	3	0	3	3
N.S.	1	1.00	1.00	1.33	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.163	0.001	0.463	0.306	0.230	0.000	0.273	26.306

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	0	1	1
N.S.	1	1.00	1.00	2.00	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.162	0.001	3.673	0.310	0.233	0.000	0.279	27.007

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	3	0	3	3
N.S.	1	1.00	1.00	1.33	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.168	0.001	37.707	0.292	0.226	0.000	0.267	26.513

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	27	26	259	0	61	43
N.S.	1	1.00	1.00	0.82	0.79	7.85	0.00	1.85	1.30
time (sec)	N/A	0.209	0.116	0.598	0.290	0.282	0.000	0.264	27.186

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	261	1022	820	0	2869	0	0	0
N.S.	1	1.09	4.28	3.43	0.00	12.00	0.00	0.00	0.00
time (sec)	N/A	0.880	1.229	0.909	0.000	4.762	0.000	0.000	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	365	369	279	1161	0	4289	0	0	0
N.S.	1	1.01	0.76	3.18	0.00	11.75	0.00	0.00	0.00
time (sec)	N/A	1.292	5.661	0.914	0.000	3.362	0.000	0.000	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	234	149	172	246	150	566	158	456
N.S.	1	1.20	0.76	0.88	1.26	0.77	2.90	0.81	2.34
time (sec)	N/A	0.882	2.400	2.348	0.218	0.258	0.326	0.310	29.601

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	113	77	90	112	76	204	79	88
N.S.	1	1.04	0.71	0.83	1.03	0.70	1.87	0.72	0.81
time (sec)	N/A	0.360	1.125	1.093	0.223	0.258	0.138	0.269	27.241

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	32	23	24	0	23	0	50	39
N.S.	1	1.39	1.00	1.04	0.00	1.00	0.00	2.17	1.70
time (sec)	N/A	0.327	0.342	0.805	0.000	0.243	0.000	0.289	27.389

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	215	140	386	0	795	0	435	497
N.S.	1	1.37	0.89	2.46	0.00	5.06	0.00	2.77	3.17
time (sec)	N/A	0.821	1.105	1.938	0.000	0.293	0.000	0.326	30.188

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	250	286	263	0	6695	0	0	10465
N.S.	1	1.03	1.18	1.09	0.00	27.67	0.00	0.00	43.24
time (sec)	N/A	0.644	1.389	5.269	0.000	6.008	0.000	0.000	42.929

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	331	216	140	191	556	112	0	229	0
N.S.	1	0.65	0.42	0.58	1.68	0.34	0.00	0.69	0.00
time (sec)	N/A	0.753	2.549	3.697	0.347	0.248	0.000	0.300	0.000

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	185	99	70	72	187	43	0	94	0
N.S.	1	0.54	0.38	0.39	1.01	0.23	0.00	0.51	0.00
time (sec)	N/A	0.350	0.387	1.812	0.365	0.275	0.000	0.273	0.000

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	137	114	85	88	0	204	0	110	0
N.S.	1	0.83	0.62	0.64	0.00	1.49	0.00	0.80	0.00
time (sec)	N/A	0.520	0.452	2.051	0.000	0.262	0.000	0.296	0.000

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	239	190	144	250	0	527	0	292	0
N.S.	1	0.79	0.60	1.05	0.00	2.21	0.00	1.22	0.00
time (sec)	N/A	0.713	0.848	2.089	0.000	0.286	0.000	0.325	0.000

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	20	11	12	0	11	0	32	24
N.S.	1	1.82	1.00	1.09	0.00	1.00	0.00	2.91	2.18
time (sec)	N/A	0.283	0.109	0.427	0.000	0.243	0.000	0.287	26.940

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	241	254	0	6697	0	5300	11781
N.S.	1	1.00	0.98	1.03	0.00	27.22	0.00	21.54	47.89
time (sec)	N/A	0.621	1.142	5.289	0.000	6.404	0.000	2.039	44.542

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	150	153	155	150	149	248	2007	205
N.S.	1	1.04	1.06	1.08	1.04	1.03	1.72	13.94	1.42
time (sec)	N/A	0.852	1.723	0.257	0.309	0.254	0.159	2.431	27.076

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	88	77	74	74	122	592	105
N.S.	1	1.00	1.22	1.07	1.03	1.03	1.69	8.22	1.46
time (sec)	N/A	0.386	0.279	0.211	0.298	0.240	0.099	0.638	26.975

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	125	187	125	161	191	1360	199	152
N.S.	1	1.24	1.85	1.24	1.59	1.89	13.47	1.97	1.50
time (sec)	N/A	0.697	1.908	0.383	0.313	0.254	0.749	0.581	28.050

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	261	308	218	419	580	0	437	388
N.S.	1	1.32	1.56	1.11	2.13	2.94	0.00	2.22	1.97
time (sec)	N/A	1.271	4.045	0.542	0.308	0.283	0.000	0.875	28.773

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	149	147	158	166	102	0	1535	0
N.S.	1	0.52	0.52	0.56	0.58	0.36	0.00	5.40	0.00
time (sec)	N/A	0.800	1.150	1.342	0.310	0.253	0.000	1.165	0.000

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	122	82	58	75	65	38	0	323	0
N.S.	1	0.67	0.48	0.61	0.53	0.31	0.00	2.65	0.00
time (sec)	N/A	0.447	0.240	1.171	0.308	0.265	0.000	0.402	0.000

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	102	111	114	137	71	0	161	0
N.S.	1	0.74	0.80	0.83	0.99	0.51	0.00	1.17	0.00
time (sec)	N/A	0.549	0.837	0.842	0.313	0.248	0.000	0.369	0.000

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	235	268	622	498	355	0	481	0
N.S.	1	0.74	0.85	1.97	1.58	1.12	0.00	1.52	0.00
time (sec)	N/A	1.061	3.596	0.696	0.334	0.270	0.000	0.551	0.000

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	207	212	200	299	198	0	447	323
N.S.	1	1.12	1.15	1.09	1.62	1.08	0.00	2.43	1.76
time (sec)	N/A	0.872	0.123	2.881	0.220	0.288	0.000	0.388	26.598

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	80	64	95	126	125	0	182	160
N.S.	1	1.05	0.84	1.25	1.66	1.64	0.00	2.39	2.11
time (sec)	N/A	0.285	0.219	1.496	0.222	0.274	0.000	0.298	26.342

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	135	97	120	0	279	0	139	444
N.S.	1	1.47	1.05	1.30	0.00	3.03	0.00	1.51	4.83
time (sec)	N/A	0.808	0.956	0.742	0.000	0.279	0.000	0.308	26.382

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	302	276	307	0	1335	0	469	5469
N.S.	1	1.31	1.20	1.33	0.00	5.80	0.00	2.04	23.78
time (sec)	N/A	1.645	2.391	1.524	0.000	0.331	0.000	0.387	37.127

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	359	190	128	300	440	162	0	605	0
N.S.	1	0.53	0.36	0.84	1.23	0.45	0.00	1.69	0.00
time (sec)	N/A	0.739	0.619	5.181	0.335	0.266	0.000	0.378	0.000

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	87	67	177	164	85	0	208	0
N.S.	1	0.50	0.39	1.02	0.95	0.49	0.00	1.20	0.00
time (sec)	N/A	0.514	0.254	4.932	0.326	0.257	0.000	0.311	0.000

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	115	92	147	0	184	0	383	0
N.S.	1	0.81	0.65	1.04	0.00	1.30	0.00	2.70	0.00
time (sec)	N/A	0.622	0.620	5.085	0.000	0.269	0.000	0.548	0.000

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	330	268	216	678	0	798	0	405	0
N.S.	1	0.81	0.65	2.05	0.00	2.42	0.00	1.23	0.00
time (sec)	N/A	1.336	2.505	4.599	0.000	0.343	0.000	0.385	0.000

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	19	8	0	6	8	14	16
N.S.	1	1.00	1.12	0.47	0.00	0.35	0.47	0.82	0.94
time (sec)	N/A	0.193	0.034	0.855	0.000	0.234	0.045	0.260	26.936

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	19	8	0	6	10	14	13
N.S.	1	1.00	1.12	0.47	0.00	0.35	0.59	0.82	0.76
time (sec)	N/A	0.194	0.025	0.830	0.000	0.253	0.038	0.261	26.711

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	6	11	7	16	32
N.S.	1	1.00	1.00	1.17	1.00	1.83	1.17	2.67	5.33
time (sec)	N/A	0.171	0.058	0.621	0.219	0.255	0.056	0.283	26.926

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	39	66	181	59	360	77	1976
N.S.	1	1.00	0.83	1.40	3.85	1.26	7.66	1.64	42.04
time (sec)	N/A	0.213	0.212	0.891	0.321	0.238	0.351	0.277	38.642

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-2)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	75	113	271	194	0	132	129
N.S.	1	1.00	1.01	1.53	3.66	2.62	0.00	1.78	1.74
time (sec)	N/A	0.312	0.296	0.863	0.323	0.254	0.000	0.300	26.139

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	64	37	199	152	0	26	95
N.S.	1	1.00	0.97	0.56	3.02	2.30	0.00	0.39	1.44
time (sec)	N/A	0.297	0.240	0.801	0.268	0.247	0.000	0.302	26.245

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	78	167	243	155	1030	148	1099
N.S.	1	1.00	0.93	1.99	2.89	1.85	12.26	1.76	13.08
time (sec)	N/A	0.309	0.387	0.890	0.353	0.267	10.948	0.312	34.612

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-2)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	92	124	286	226	0	150	141
N.S.	1	1.00	1.08	1.46	3.36	2.66	0.00	1.76	1.66
time (sec)	N/A	0.316	0.407	0.756	0.324	0.268	0.000	0.295	27.597

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	141	122	218	451	311	0	270	264
N.S.	1	1.09	0.95	1.69	3.50	2.41	0.00	2.09	2.05
time (sec)	N/A	0.497	0.686	0.895	0.376	0.259	0.000	0.321	27.373

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	113	95	185	0	625	0	178	1709
N.S.	1	0.98	0.83	1.61	0.00	5.43	0.00	1.55	14.86
time (sec)	N/A	0.361	0.274	0.920	0.000	0.322	0.000	0.279	62.049

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	119	118	206	0	1277	0	209	205
N.S.	1	1.05	1.04	1.82	0.00	11.30	0.00	1.85	1.81
time (sec)	N/A	0.378	0.325	0.933	0.000	0.321	0.000	0.303	26.221

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	225	326	853	0	3402	0	1162	946
N.S.	1	1.12	1.63	4.26	0.00	17.01	0.00	5.81	4.73
time (sec)	N/A	0.674	0.684	1.727	0.000	0.450	0.000	0.368	31.451

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	147	86	0	72	94	168	99
N.S.	1	1.00	1.75	1.02	0.00	0.86	1.12	2.00	1.18
time (sec)	N/A	0.231	0.210	1.207	0.000	0.236	0.324	0.271	28.308

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	147	79	0	56	73	168	584
N.S.	1	1.00	1.75	0.94	0.00	0.67	0.87	2.00	6.95
time (sec)	N/A	0.228	0.195	1.188	0.000	0.246	0.311	0.281	33.712

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	114	96	187	0	625	0	177	1741
N.S.	1	0.98	0.83	1.61	0.00	5.39	0.00	1.53	15.01
time (sec)	N/A	0.347	0.363	0.978	0.000	0.319	0.000	0.278	62.212

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	120	123	207	0	1301	0	206	204
N.S.	1	1.05	1.08	1.82	0.00	11.41	0.00	1.81	1.79
time (sec)	N/A	0.385	0.480	0.908	0.000	0.319	0.000	0.294	28.451

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	226	361	832	0	3513	0	1054	912
N.S.	1	1.13	1.80	4.16	0.00	17.56	0.00	5.27	4.56
time (sec)	N/A	0.662	0.894	1.738	0.000	0.468	0.000	0.378	33.695

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	152	84	0	71	100	169	96
N.S.	1	1.00	1.79	0.99	0.00	0.84	1.18	1.99	1.13
time (sec)	N/A	0.232	0.400	1.212	0.000	0.250	0.400	0.270	29.903

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	152	77	0	57	78	169	96
N.S.	1	1.00	1.79	0.91	0.00	0.67	0.92	1.99	1.13
time (sec)	N/A	0.230	0.359	1.303	0.000	0.247	0.403	0.273	28.448

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	125	98	217	0	687	0	187	1864
N.S.	1	1.05	0.82	1.82	0.00	5.77	0.00	1.57	15.66
time (sec)	N/A	0.372	0.491	1.050	0.000	0.329	0.000	0.271	67.959

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	116	116	206	0	1316	0	205	202
N.S.	1	1.05	1.05	1.87	0.00	11.96	0.00	1.86	1.84
time (sec)	N/A	0.375	0.513	0.919	0.000	0.323	0.000	0.291	27.406

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	221	311	795	0	3264	0	1034	923
N.S.	1	1.12	1.58	4.04	0.00	16.57	0.00	5.25	4.69
time (sec)	N/A	0.640	0.930	1.609	0.000	0.468	0.000	0.361	30.834

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	87	195	118	0	78	110	178	118
N.S.	1	0.95	2.12	1.28	0.00	0.85	1.20	1.93	1.28
time (sec)	N/A	0.246	0.549	1.286	0.000	0.251	0.431	0.271	29.100

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	85	195	116	0	68	99	178	118
N.S.	1	0.94	2.17	1.29	0.00	0.76	1.10	1.98	1.31
time (sec)	N/A	0.245	0.564	1.254	0.000	0.256	0.424	0.270	27.996

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	129	110	227	0	711	0	199	2711
N.S.	1	0.98	0.84	1.73	0.00	5.43	0.00	1.52	20.69
time (sec)	N/A	0.386	0.523	1.418	0.000	0.322	0.000	0.283	119.488

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	133	137	231	0	1556	0	241	227
N.S.	1	1.05	1.08	1.82	0.00	12.25	0.00	1.90	1.79
time (sec)	N/A	0.401	0.685	0.947	0.000	0.329	0.000	0.309	26.585

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	262	452	1080	0	4240	0	1506	1160
N.S.	1	1.11	1.91	4.56	0.00	17.89	0.00	6.35	4.89
time (sec)	N/A	0.748	1.261	1.868	0.000	0.513	0.000	0.368	33.519

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	165	136	0	89	129	203	132
N.S.	1	1.00	1.57	1.30	0.00	0.85	1.23	1.93	1.26
time (sec)	N/A	0.259	0.563	1.325	0.000	0.243	0.693	0.267	31.678

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	167	135	0	73	105	203	133
N.S.	1	1.00	1.62	1.31	0.00	0.71	1.02	1.97	1.29
time (sec)	N/A	0.260	0.583	1.316	0.000	0.240	0.685	0.268	31.921

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	A	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	68	32	36	0	24	0	68	62
N.S.	1	2.83	1.33	1.50	0.00	1.00	0.00	2.83	2.58
time (sec)	N/A	0.256	0.160	0.762	0.000	0.238	0.000	0.301	26.049

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	390	424	7823	3452	0	2014	0	0	0
N.S.	1	1.09	20.06	8.85	0.00	5.16	0.00	0.00	0.00
time (sec)	N/A	2.232	6.784	12.958	0.000	0.214	0.000	0.000	0.000

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	294	314	5218	2205	0	1692	0	0	0
N.S.	1	1.07	17.75	7.50	0.00	5.76	0.00	0.00	0.00
time (sec)	N/A	1.485	6.594	8.221	0.000	0.157	0.000	0.000	0.000

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	229	235	3006	1438	0	1498	0	0	0
N.S.	1	1.03	13.13	6.28	0.00	6.54	0.00	0.00	0.00
time (sec)	N/A	1.034	6.267	5.767	0.000	0.139	0.000	0.000	0.000

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	180	180	570	766	0	1352	0	0	0
N.S.	1	1.00	3.17	4.26	0.00	7.51	0.00	0.00	0.00
time (sec)	N/A	0.727	5.966	8.368	0.000	0.128	0.000	0.000	0.000

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	250	266	3176	2911	0	2086	0	0	0
N.S.	1	1.06	12.70	11.64	0.00	8.34	0.00	0.00	0.00
time (sec)	N/A	1.122	6.449	28.981	0.000	0.169	0.000	0.000	0.000

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	378	393	5554	4020	0	4057	0	0	0
N.S.	1	1.04	14.69	10.63	0.00	10.73	0.00	0.00	0.00
time (sec)	N/A	1.742	6.727	142.432	0.000	0.368	0.000	0.000	0.000

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	90	80	96	0	346	1110	136	1143
N.S.	1	1.07	0.95	1.14	0.00	4.12	13.21	1.62	13.61
time (sec)	N/A	0.462	3.666	1.493	0.000	0.277	13.611	0.282	34.846

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	124	114	155	0	458	0	180	227
N.S.	1	1.05	0.97	1.31	0.00	3.88	0.00	1.53	1.92
time (sec)	N/A	0.530	2.585	1.599	0.000	0.263	0.000	0.287	26.954

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	205	174	373	0	880	0	571	557
N.S.	1	1.11	0.94	2.02	0.00	4.76	0.00	3.09	3.01
time (sec)	N/A	0.743	1.770	1.906	0.000	0.287	0.000	0.325	28.880

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	294	244	715	0	1411	0	1281	1085
N.S.	1	1.14	0.95	2.77	0.00	5.47	0.00	4.97	4.21
time (sec)	N/A	1.047	3.142	2.568	0.000	0.336	0.000	0.343	30.499

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	131	131	145	0	0	0	0	0	0
N.S.	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.331	0.684	0.000	0.000	0.000	0.000	0.000	0.000

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	112	75	84	80	97	190	75	125
N.S.	1	1.05	0.70	0.79	0.75	0.91	1.78	0.70	1.17
time (sec)	N/A	0.367	0.866	1.053	0.226	0.270	0.360	0.307	27.188

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	48	46	48	63	129	46	78
N.S.	1	1.00	0.79	0.75	0.79	1.03	2.11	0.75	1.28
time (sec)	N/A	0.241	0.501	0.816	0.209	0.262	0.176	0.273	27.068

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	38	19	18	22	26	18	22
N.S.	1	1.00	1.90	0.95	0.90	1.10	1.30	0.90	1.10
time (sec)	N/A	0.151	0.004	0.585	0.207	0.250	0.082	0.261	26.767

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	45	0	290	0	61	44
N.S.	1	1.00	1.00	0.94	0.00	6.04	0.00	1.27	0.92
time (sec)	N/A	0.263	0.300	0.829	0.000	0.282	0.000	0.264	27.348

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	100	94	114	0	493	0	116	181
N.S.	1	1.05	0.99	1.20	0.00	5.19	0.00	1.22	1.91
time (sec)	N/A	0.379	0.554	1.355	0.000	0.280	0.000	0.283	27.402

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	169	120	271	0	969	0	252	396
N.S.	1	1.13	0.81	1.82	0.00	6.50	0.00	1.69	2.66
time (sec)	N/A	0.574	1.064	2.629	0.000	0.306	0.000	0.290	27.388

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	265	260	202	1138	0	0	0	0	0
N.S.	1	0.98	0.76	4.29	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.168	2.051	2.582	0.000	0.000	0.000	0.000	0.000

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	208	167	844	0	0	0	0	0
N.S.	1	0.98	0.79	3.98	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.879	2.029	1.567	0.000	0.000	0.000	0.000	0.000

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	75	311	0	0	0	0	0
N.S.	1	1.00	0.99	4.09	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.359	0.401	1.229	0.000	0.000	0.000	0.000	0.000

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	70	165	0	304	0	0	0
N.S.	1	1.00	0.92	2.17	0.00	4.00	0.00	0.00	0.00
time (sec)	N/A	0.355	0.394	0.848	0.000	0.097	0.000	0.000	0.000

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	101	570	0	914	0	0	0
N.S.	1	1.00	0.71	3.99	0.00	6.39	0.00	0.00	0.00
time (sec)	N/A	0.502	0.817	1.201	0.000	0.141	0.000	0.000	0.000

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	297	201	1554	0	1506	0	0	0
N.S.	1	1.01	0.68	5.27	0.00	5.11	0.00	0.00	0.00
time (sec)	N/A	1.214	1.616	1.429	0.000	0.194	0.000	0.000	0.000

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	461	425	340	2282	0	3308	0	0	0
N.S.	1	0.92	0.74	4.95	0.00	7.18	0.00	0.00	0.00
time (sec)	N/A	1.408	0.932	1.019	0.000	0.978	0.000	0.000	0.000

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	340	321	256	1782	0	2492	0	0	0
N.S.	1	0.94	0.75	5.24	0.00	7.33	0.00	0.00	0.00
time (sec)	N/A	1.071	0.903	0.955	0.000	0.893	0.000	0.000	0.000

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	229	788	1284	0	1676	0	0	0
N.S.	1	1.02	3.50	5.71	0.00	7.45	0.00	0.00	0.00
time (sec)	N/A	0.700	1.252	0.907	0.000	1.190	0.000	0.000	0.000

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	15	14	16	16
N.S.	1	1.00	1.14	1.00	1.14	1.07	1.00	1.14	1.14
time (sec)	N/A	0.255	2.524	0.638	0.362	0.241	24.994	0.261	26.466

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	36	34	36	66	53	36	36
N.S.	1	1.00	1.06	1.00	1.06	1.94	1.56	1.06	1.06
time (sec)	N/A	0.386	8.961	0.957	2.805	0.256	82.028	0.323	28.265

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	33	33	35	33	35	70	53	35	35
N.S.	1	1.00	1.06	1.00	1.06	2.12	1.61	1.06	1.06
time (sec)	N/A	0.372	7.174	0.986	2.242	0.261	82.926	0.305	27.857

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	F(-2)	A	F	C	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	175	244	198	0	0	186	0	7347	0
N.S.	1	1.39	1.13	0.00	0.00	1.06	0.00	41.98	0.00
time (sec)	N/A	0.904	1.632	0.000	0.000	0.292	0.000	0.724	0.000

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	F(-2)	A	F(-1)	C	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	131	181	142	0	0	142	0	4175	0
N.S.	1	1.38	1.08	0.00	0.00	1.08	0.00	31.87	0.00
time (sec)	N/A	1.001	1.451	0.000	0.000	0.277	0.000	0.551	0.000

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	F(-2)	A	F	C	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	80	101	77	0	0	77	0	1033	0
N.S.	1	1.26	0.96	0.00	0.00	0.96	0.00	12.91	0.00
time (sec)	N/A	0.506	1.359	0.000	0.000	0.263	0.000	0.394	0.000

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-2)	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	69	160	120	0	45	0	496	0
N.S.	1	1.23	2.86	2.14	0.00	0.80	0.00	8.86	0.00
time (sec)	N/A	0.431	2.053	1.609	0.000	0.248	0.000	0.359	0.000

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	24	38	114	24	20	39	24
N.S.	1	1.00	0.69	1.09	3.26	0.69	0.57	1.11	0.69
time (sec)	N/A	0.187	0.330	1.807	0.200	0.241	2.200	0.291	27.147

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	20	21	19	42	23
N.S.	1	1.00	1.00	1.05	1.00	1.05	0.95	2.10	1.15
time (sec)	N/A	0.202	0.024	0.382	0.214	0.252	2.236	0.288	0.173

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	32	39	100	34	112	53	0
N.S.	1	1.00	0.91	1.11	2.86	0.97	3.20	1.51	0.00
time (sec)	N/A	0.246	1.474	1.303	0.217	0.237	2.918	0.284	0.000

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	104	119	157	0	0	295	0	0	0
N.S.	1	1.14	1.51	0.00	0.00	2.84	0.00	0.00	0.00
time (sec)	N/A	0.465	0.901	0.000	0.000	0.284	0.000	0.000	0.000

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	158	102	172	594	410	0	0	0
N.S.	1	1.24	0.80	1.35	4.68	3.23	0.00	0.00	0.00
time (sec)	N/A	0.794	1.297	1.815	0.264	0.286	0.000	0.000	0.000

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	F(-2)	A	F	C	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	176	245	194	0	0	162	0	7279	0
N.S.	1	1.39	1.10	0.00	0.00	0.92	0.00	41.36	0.00
time (sec)	N/A	0.914	1.188	0.000	0.000	0.279	0.000	0.714	0.000

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	F(-2)	A	F(-1)	C	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	132	179	136	0	0	135	0	3130	0
N.S.	1	1.36	1.03	0.00	0.00	1.02	0.00	23.71	0.00
time (sec)	N/A	1.031	0.756	0.000	0.000	0.263	0.000	0.482	0.000

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	F(-2)	A	F	C	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	80	101	71	0	0	73	0	997	0
N.S.	1	1.26	0.89	0.00	0.00	0.91	0.00	12.46	0.00
time (sec)	N/A	0.504	1.014	0.000	0.000	0.260	0.000	0.398	0.000

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F(-2)	A	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	67	181	119	0	43	0	366	0
N.S.	1	1.20	3.23	2.12	0.00	0.77	0.00	6.54	0.00
time (sec)	N/A	0.437	1.859	2.379	0.000	0.248	0.000	0.343	0.000

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	22	31	114	23	20	32	22
N.S.	1	1.00	0.65	0.91	3.35	0.68	0.59	0.94	0.65
time (sec)	N/A	0.182	0.224	2.402	0.211	0.231	2.241	0.285	0.268

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	19	22	20	40	22
N.S.	1	1.00	1.00	1.05	1.00	1.16	1.05	2.11	1.16
time (sec)	N/A	0.212	0.019	0.573	0.200	0.247	2.242	0.285	0.125

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	31	40	100	36	109	52	0
N.S.	1	1.00	0.94	1.21	3.03	1.09	3.30	1.58	0.00
time (sec)	N/A	0.247	1.005	1.891	0.216	0.258	2.414	0.272	0.000

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	110	126	176	0	0	290	0	0	0
N.S.	1	1.15	1.60	0.00	0.00	2.64	0.00	0.00	0.00
time (sec)	N/A	0.475	1.159	0.000	0.000	0.270	0.000	0.000	0.000

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	156	130	141	372	382	0	0	0
N.S.	1	1.26	1.05	1.14	3.00	3.08	0.00	0.00	0.00
time (sec)	N/A	0.785	1.832	3.539	0.324	0.277	0.000	0.000	0.000

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	173	64	90	0	106	0	0	463
N.S.	1	1.10	0.41	0.57	0.00	0.68	0.00	0.00	2.95
time (sec)	N/A	0.980	1.091	7.235	0.000	0.266	0.000	0.000	34.127

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	121	62	80	0	84	0	0	148
N.S.	1	1.10	0.56	0.73	0.00	0.76	0.00	0.00	1.35
time (sec)	N/A	0.676	0.729	6.547	0.000	0.259	0.000	0.000	43.075

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	44	70	0	64	0	413066	129
N.S.	1	1.00	0.61	0.97	0.00	0.89	0.00	5737.03	1.79
time (sec)	N/A	0.488	0.527	6.748	0.000	0.258	0.000	142.288	32.891

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	30	44	0	40	0	80097	87
N.S.	1	1.00	0.91	1.33	0.00	1.21	0.00	2427.18	2.64
time (sec)	N/A	0.293	0.295	6.319	0.000	0.248	0.000	37.399	27.416

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	73	122	430	201	0	0	0
N.S.	1	1.00	1.62	2.71	9.56	4.47	0.00	0.00	0.00
time (sec)	N/A	0.257	0.215	7.244	0.352	0.285	0.000	0.000	0.000

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	92	341	1049	351	0	0	0
N.S.	1	1.00	1.10	4.06	12.49	4.18	0.00	0.00	0.00
time (sec)	N/A	0.428	0.455	6.326	0.454	0.285	0.000	0.000	0.000

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	134	105	571	1421	419	0	0	0
N.S.	1	1.04	0.81	4.43	11.02	3.25	0.00	0.00	0.00
time (sec)	N/A	0.611	0.800	6.141	0.511	0.296	0.000	0.000	0.000

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	186	116	811	2333	481	0	0	0
N.S.	1	1.06	0.66	4.61	13.26	2.73	0.00	0.00	0.00
time (sec)	N/A	0.805	1.078	7.424	0.762	0.281	0.000	0.000	0.000

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	228	85	101	0	132	0	0	594
N.S.	1	1.10	0.41	0.49	0.00	0.63	0.00	0.00	2.86
time (sec)	N/A	1.186	1.221	6.246	0.000	0.242	0.000	0.000	34.921

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	163	73	91	0	111	0	0	479
N.S.	1	1.10	0.49	0.61	0.00	0.75	0.00	0.00	3.24
time (sec)	N/A	0.862	0.756	5.799	0.000	0.245	0.000	0.000	36.544

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	115	59	81	0	88	0	0	149
N.S.	1	1.05	0.54	0.74	0.00	0.80	0.00	0.00	1.35
time (sec)	N/A	0.624	0.631	5.671	0.000	0.241	0.000	0.000	45.174

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	51	71	0	67	0	0	158
N.S.	1	1.00	0.68	0.95	0.00	0.89	0.00	0.00	2.11
time (sec)	N/A	0.401	0.484	5.744	0.000	0.250	0.000	0.000	35.041

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	86	224	1317	296	0	0	0
N.S.	1	1.00	1.08	2.80	16.46	3.70	0.00	0.00	0.00
time (sec)	N/A	0.358	0.316	17.211	0.470	0.291	0.000	0.000	0.000

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	124	93	457	1058	369	0	0	0
N.S.	1	1.44	1.08	5.31	12.30	4.29	0.00	0.00	0.00
time (sec)	N/A	0.604	0.601	4.950	0.459	0.286	0.000	0.000	0.000

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	137	105	702	0	437	0	0	0
N.S.	1	1.03	0.79	5.28	0.00	3.29	0.00	0.00	0.00
time (sec)	N/A	0.684	1.031	5.323	0.000	0.285	0.000	0.000	0.000

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-1)	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	189	117	958	0	503	0	0	0
N.S.	1	1.04	0.64	5.26	0.00	2.76	0.00	0.00	0.00
time (sec)	N/A	0.889	1.176	6.480	0.000	0.277	0.000	0.000	0.000

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	189	112	892	0	380	0	0	0
N.S.	1	1.08	0.64	5.10	0.00	2.17	0.00	0.00	0.00
time (sec)	N/A	1.088	1.791	11.228	0.000	0.283	0.000	0.000	0.000

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	136	89	607	0	294	0	0	0
N.S.	1	1.05	0.69	4.71	0.00	2.28	0.00	0.00	0.00
time (sec)	N/A	0.740	1.157	9.807	0.000	0.292	0.000	0.000	0.000

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	67	370	0	245	0	0	0
N.S.	1	1.00	0.76	4.20	0.00	2.78	0.00	0.00	0.00
time (sec)	N/A	0.546	0.736	4.093	0.000	0.291	0.000	0.000	0.000

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	64	209	0	146	0	0	0
N.S.	1	1.00	1.16	3.80	0.00	2.65	0.00	0.00	0.00
time (sec)	N/A	0.298	0.390	3.510	0.000	0.275	0.000	0.000	0.000

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	C	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	96	260	1685	309	0	0	0
N.S.	1	1.00	0.96	2.60	16.85	3.09	0.00	0.00	0.00
time (sec)	N/A	0.447	0.507	4.072	0.646	0.272	0.000	0.000	0.000

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	163	590	0	481	0	0	0
N.S.	1	1.00	1.18	4.28	0.00	3.49	0.00	0.00	0.00
time (sec)	N/A	0.658	2.426	3.283	0.000	0.282	0.000	0.000	0.000

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	197	186	989	0	569	0	0	0
N.S.	1	1.08	1.02	5.43	0.00	3.13	0.00	0.00	0.00
time (sec)	N/A	1.003	3.983	3.566	0.000	0.281	0.000	0.000	0.000

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	189	100	1070	0	350	0	0	0
N.S.	1	1.05	0.56	5.94	0.00	1.94	0.00	0.00	0.00
time (sec)	N/A	1.027	2.974	9.491	0.000	0.287	0.000	0.000	0.000

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	133	94	495	0	276	0	0	0
N.S.	1	1.04	0.73	3.87	0.00	2.16	0.00	0.00	0.00
time (sec)	N/A	0.697	1.664	3.909	0.000	0.285	0.000	0.000	0.000

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	84	380	0	272	0	0	0
N.S.	1	1.00	0.90	4.09	0.00	2.92	0.00	0.00	0.00
time (sec)	N/A	0.518	1.433	3.891	0.000	0.283	0.000	0.000	0.000

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	83	377	0	269	0	0	0
N.S.	1	1.00	0.89	4.05	0.00	2.89	0.00	0.00	0.00
time (sec)	N/A	0.433	1.035	4.179	0.000	0.275	0.000	0.000	0.000

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	144	198	552	0	438	0	0	0
N.S.	1	1.04	1.43	4.00	0.00	3.17	0.00	0.00	0.00
time (sec)	N/A	0.626	3.205	4.367	0.000	0.269	0.000	0.000	0.000

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	182	217	1075	0	528	0	0	0
N.S.	1	1.02	1.22	6.04	0.00	2.97	0.00	0.00	0.00
time (sec)	N/A	0.947	5.859	2.957	0.000	0.286	0.000	0.000	0.000

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	247	357	1632	0	616	0	0	0
N.S.	1	1.06	1.53	6.97	0.00	2.63	0.00	0.00	0.00
time (sec)	N/A	1.339	7.595	3.258	0.000	0.278	0.000	0.000	0.000

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	119	0	29	0	0	14
N.S.	1	1.00	1.00	7.44	0.00	1.81	0.00	0.00	0.88
time (sec)	N/A	0.270	0.068	1.266	0.000	0.251	0.000	0.000	26.371

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	B	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	55	80	396	0	120	0	0	0
N.S.	1	0.80	1.16	5.74	0.00	1.74	0.00	0.00	0.00
time (sec)	N/A	0.566	1.807	1.615	0.000	0.265	0.000	0.000	0.000

Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-1)	A	F(-1)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	57	122	396	0	120	0	0	0
N.S.	1	0.72	1.54	5.01	0.00	1.52	0.00	0.00	0.00
time (sec)	N/A	0.721	2.225	1.898	0.000	0.259	0.000	0.000	0.000

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-1)	A	F(-1)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	65	92	761	0	136	0	0	0
N.S.	1	0.68	0.97	8.01	0.00	1.43	0.00	0.00	0.00
time (sec)	N/A	0.771	4.515	2.834	0.000	0.263	0.000	0.000	0.000

Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	31	30	59	138	85	63
N.S.	1	1.00	1.00	1.03	1.00	1.97	4.60	2.83	2.10
time (sec)	N/A	0.245	5.450	0.673	0.224	0.286	18.292	1.047	30.763

Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	938	187	24	122	129	142	185
N.S.	1	1.00	36.08	7.19	0.92	4.69	4.96	5.46	7.12
time (sec)	N/A	0.227	12.232	222.002	0.204	0.270	2.514	0.806	27.192

Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	152	24	92	100	321	100
N.S.	1	1.00	1.00	5.85	0.92	3.54	3.85	12.35	3.85
time (sec)	N/A	0.222	5.946	48.376	0.208	0.256	0.998	0.700	26.844

Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	67	57	24	61	73	45	61
N.S.	1	1.00	2.58	2.19	0.92	2.35	2.81	1.73	2.35
time (sec)	N/A	0.206	0.058	8.280	0.205	0.245	0.414	0.432	26.372

Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	29	23	22	33	61	42	133
N.S.	1	1.00	1.32	1.05	1.00	1.50	2.77	1.91	6.05
time (sec)	N/A	0.226	3.226	2.250	0.201	0.267	3.778	133.527	29.034

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	27	25	24	29	49	108	47
N.S.	1	1.00	1.12	1.04	1.00	1.21	2.04	4.50	1.96
time (sec)	N/A	0.222	1.574	4.659	0.193	0.250	11.183	0.624	26.693

Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	29	25	24	63	80	37	291
N.S.	1	1.00	1.12	0.96	0.92	2.42	3.08	1.42	11.19
time (sec)	N/A	0.225	3.993	18.498	0.200	0.246	23.681	142.414	30.843

Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	15	20	20
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.83	1.11	1.11
time (sec)	N/A	0.203	0.083	0.116	0.306	0.232	0.443	0.276	26.712

Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	15	20	20
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.83	1.11	1.11
time (sec)	N/A	0.206	0.070	0.115	0.301	0.229	0.488	0.262	27.582

Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	17	22	22
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.85	1.10	1.10
time (sec)	N/A	0.207	0.118	0.102	0.287	0.234	0.376	0.360	27.419

Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	17	22	22
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.85	1.10	1.10
time (sec)	N/A	0.204	0.107	0.113	0.283	0.240	3.257	0.315	27.162

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	12	15	17	13	12
N.S.	1	1.00	1.00	1.08	1.00	1.25	1.42	1.08	1.00
time (sec)	N/A	0.173	0.009	0.353	0.198	0.242	0.084	0.276	0.074

Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	20	23	63	20	20
N.S.	1	1.00	1.00	1.05	1.00	1.15	3.15	1.00	1.00
time (sec)	N/A	0.181	0.029	2.420	0.199	0.255	0.518	0.282	27.363

Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	16	6	5	36	0	14	5
N.S.	1	1.00	3.20	1.20	1.00	7.20	0.00	2.80	1.00
time (sec)	N/A	0.173	0.053	0.478	0.275	0.244	0.000	0.272	26.473

Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	5	20	5	5	5
N.S.	1	1.00	1.00	1.20	1.00	4.00	1.00	1.00	1.00
time (sec)	N/A	0.181	4.392	0.257	0.204	0.259	0.134	0.271	0.117

Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	33	21	23	17	73	34	17	22
N.S.	1	1.18	0.75	0.82	0.61	2.61	1.21	0.61	0.79
time (sec)	N/A	0.234	2.048	0.577	0.217	0.245	0.599	0.280	26.841

Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	21	20	168	54	20	20
N.S.	1	1.00	1.00	0.81	0.77	6.46	2.08	0.77	0.77
time (sec)	N/A	0.244	4.550	4.431	0.197	0.246	3.393	0.269	26.752

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	40	137	32	103	39	46	39	39
N.S.	1	1.11	3.81	0.89	2.86	1.08	1.28	1.08	1.08
time (sec)	N/A	0.256	0.236	0.822	0.204	0.256	0.924	0.279	0.107

Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	7	22	7	7	7
N.S.	1	1.00	1.00	0.89	0.78	2.44	0.78	0.78	0.78
time (sec)	N/A	0.186	2.314	0.308	0.203	0.240	0.148	0.294	26.802

Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	29	24	18	17	17	26	17	17
N.S.	1	0.94	0.77	0.58	0.55	0.55	0.84	0.55	0.55
time (sec)	N/A	0.208	0.095	0.531	0.205	0.241	0.147	0.269	0.142

Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	B	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	9	9	162	0	18	29	0	7	0
N.S.	1	1.00	18.00	0.00	2.00	3.22	0.00	0.78	0.00
time (sec)	N/A	0.233	2.643	0.000	0.303	0.284	0.000	0.280	0.000

Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	59	52	51	34	0	75	0
N.S.	1	1.00	0.83	0.73	0.72	0.48	0.00	1.06	0.00
time (sec)	N/A	0.234	0.148	0.852	0.195	0.257	0.000	0.282	0.000

Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	18	17	17	39	17	17
N.S.	1	1.00	1.00	1.00	0.94	0.94	2.17	0.94	0.94
time (sec)	N/A	0.187	0.059	0.547	0.203	0.231	0.138	0.270	0.134

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	23	23	23	53	23	23
N.S.	1	1.00	1.00	1.00	1.00	1.00	2.30	1.00	1.00
time (sec)	N/A	0.188	0.110	0.622	0.193	0.248	4.242	0.273	26.557

Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	23	23	23	23	51	23	23
N.S.	1	1.00	0.96	0.96	0.96	0.96	2.12	0.96	0.96
time (sec)	N/A	0.183	0.023	0.365	0.198	0.248	0.303	0.295	25.867

Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	14	0	14	14	0	0	0
N.S.	1	1.00	1.00	0.00	1.00	1.00	0.00	0.00	0.00
time (sec)	N/A	0.185	0.036	0.000	0.251	0.258	0.000	0.000	0.000

Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	19	19	19	0	20	20	0	0	0
N.S.	1	1.00	1.00	0.00	1.05	1.05	0.00	0.00	0.00
time (sec)	N/A	0.189	0.051	0.000	0.244	0.242	0.000	0.000	0.000

Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	19	0	20	20	0	0	0
N.S.	1	1.00	0.95	0.00	1.00	1.00	0.00	0.00	0.00
time (sec)	N/A	0.190	0.042	0.000	0.249	0.238	0.000	0.000	0.000

Problem 665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	11	14	12	11
N.S.	1	1.00	1.00	1.09	1.00	1.00	1.27	1.09	1.00
time (sec)	N/A	0.172	0.004	0.407	0.203	0.248	0.080	0.267	0.039

Problem 666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	19	22	56	19	19
N.S.	1	1.00	1.00	1.05	1.00	1.16	2.95	1.00	1.00
time (sec)	N/A	0.177	0.022	0.957	0.204	0.250	0.502	0.283	26.717

Problem 667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	39	0	16	9
N.S.	1	1.00	1.00	1.33	1.00	13.00	0.00	5.33	3.00
time (sec)	N/A	0.170	0.009	0.520	0.296	0.261	0.000	0.265	0.028

Problem 668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	6	5	53	0	5	5
N.S.	1	1.00	1.00	0.86	0.71	7.57	0.00	0.71	0.71
time (sec)	N/A	0.180	0.009	0.582	0.275	0.249	0.000	0.273	26.855

Problem 669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	9	71	0	0	9
N.S.	1	1.00	1.00	0.77	0.69	5.46	0.00	0.00	0.69
time (sec)	N/A	0.183	0.027	0.591	0.275	0.248	0.000	0.000	26.913

Problem 670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	48	38	79	0	38	47
N.S.	1	1.00	1.00	2.29	1.81	3.76	0.00	1.81	2.24
time (sec)	N/A	0.189	0.016	0.642	0.193	0.238	0.000	0.282	26.725

Problem 671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	22	61	0	22	20
N.S.	1	1.00	1.00	0.82	0.79	2.18	0.00	0.79	0.71
time (sec)	N/A	0.186	0.020	0.372	0.309	0.267	0.000	0.274	27.247

Problem 672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	12	27	10	10
N.S.	1	1.00	1.00	0.79	0.71	0.86	1.93	0.71	0.71
time (sec)	N/A	0.183	0.006	0.218	0.193	0.244	0.159	0.276	0.130

Problem 673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	40	17	20	35	0	20	14
N.S.	1	1.00	2.11	0.89	1.05	1.84	0.00	1.05	0.74
time (sec)	N/A	0.197	0.020	1.217	0.200	0.351	0.000	0.279	26.437

Problem 674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	3	17	3	3	3
N.S.	1	1.00	1.00	1.33	1.00	5.67	1.00	1.00	1.00
time (sec)	N/A	0.164	1.241	0.256	0.199	0.241	0.135	0.263	26.960

Problem 675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	4	41	5	4	4
N.S.	1	1.00	1.00	1.25	1.00	10.25	1.25	1.00	1.00
time (sec)	N/A	0.214	3.763	10.411	0.208	0.251	0.877	0.264	27.626

Problem 676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	9	8	47	10	29	21
N.S.	1	1.00	1.00	2.25	2.00	11.75	2.50	7.25	5.25
time (sec)	N/A	0.176	0.007	0.374	0.201	0.247	0.617	0.285	27.370

Problem 677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	40	128	32	39	103	44	39	73
N.S.	1	1.11	3.56	0.89	1.08	2.86	1.22	1.08	2.03
time (sec)	N/A	0.235	0.256	0.590	0.213	0.257	0.919	0.269	0.105

Problem 678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	9	9	8	8	12	8	8
N.S.	1	1.00	0.64	0.64	0.57	0.57	0.86	0.57	0.57
time (sec)	N/A	0.186	0.024	0.345	0.201	0.245	0.149	0.273	26.694

Problem 679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	34	20	17	19	28	36	13	0
N.S.	1	1.36	0.80	0.68	0.76	1.12	1.44	0.52	0.00
time (sec)	N/A	0.253	0.023	0.668	0.198	0.256	0.430	0.271	0.000

Problem 680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	8	7	7	8	7	7
N.S.	1	1.00	1.00	0.80	0.70	0.70	0.80	0.70	0.70
time (sec)	N/A	0.177	0.014	0.266	0.231	0.247	0.147	0.281	26.993

Problem 681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	6	5	5	7	5	6
N.S.	1	1.00	1.00	1.00	0.83	0.83	1.17	0.83	1.00
time (sec)	N/A	0.160	0.011	0.477	0.230	0.236	0.136	0.265	26.310

Problem 682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	7	7	7	6	0	12	7
N.S.	1	1.00	0.70	0.70	0.70	0.60	0.00	1.20	0.70
time (sec)	N/A	0.168	0.018	208.582	0.690	0.255	0.000	0.275	26.283

Problem 683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	13	7	12	0	18	7
N.S.	1	1.00	1.00	1.30	0.70	1.20	0.00	1.80	0.70
time (sec)	N/A	0.165	0.011	0.770	0.212	0.264	0.000	0.291	26.000

Problem 684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	17	16	16	36	16	16
N.S.	1	1.00	1.00	1.00	0.94	0.94	2.12	0.94	0.94
time (sec)	N/A	0.178	0.016	0.982	0.195	0.244	0.143	0.271	0.123

Problem 685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	22	22	22	49	22	22
N.S.	1	1.00	1.00	1.00	1.00	1.00	2.23	1.00	1.00
time (sec)	N/A	0.185	0.102	1.060	0.205	0.235	4.017	0.282	25.774

Problem 686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	22	22	22	22	48	22	22
N.S.	1	1.00	0.96	0.96	0.96	0.96	2.09	0.96	0.96
time (sec)	N/A	0.185	0.024	0.641	0.201	0.255	0.273	0.278	26.207

Problem 687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	13	13	13	0	13	13	0	13	0
N.S.	1	1.00	1.00	0.00	1.00	1.00	0.00	1.00	0.00
time (sec)	N/A	0.182	0.082	0.000	0.247	0.251	0.000	0.280	0.000

Problem 688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	18	0	19	19	0	0	0
N.S.	1	1.00	1.00	0.00	1.06	1.06	0.00	0.00	0.00
time (sec)	N/A	0.188	0.094	0.000	0.263	0.232	0.000	0.000	0.000

Problem 689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	19	19	18	0	19	19	0	0	0
N.S.	1	1.00	0.95	0.00	1.00	1.00	0.00	0.00	0.00
time (sec)	N/A	0.185	0.037	0.000	0.286	0.249	0.000	0.000	0.000

Problem 690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	40	0	12	11
N.S.	1	1.00	1.00	1.09	1.00	3.64	0.00	1.09	1.00
time (sec)	N/A	0.184	0.071	1.529	0.194	0.254	0.000	0.272	26.485

Problem 691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	3	9	4	15	23	0	17	3
N.S.	1	0.27	0.82	0.36	1.36	2.09	0.00	1.55	0.27
time (sec)	N/A	0.176	0.003	0.954	0.214	0.245	0.000	0.281	26.550

Problem 692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	11	9	8	7	21	0	7	7
N.S.	1	0.41	0.33	0.30	0.26	0.78	0.00	0.26	0.26
time (sec)	N/A	0.176	0.062	6.790	0.293	0.248	0.000	0.275	26.377

Problem 693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	19	37	0	0	37
N.S.	1	1.00	1.00	1.05	1.00	1.95	0.00	0.00	1.95
time (sec)	N/A	0.191	0.138	12.981	0.231	0.253	0.000	0.000	27.661

Problem 694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	6	11	5	4	12	27	4	4
N.S.	1	1.50	2.75	1.25	1.00	3.00	6.75	1.00	1.00
time (sec)	N/A	0.204	0.006	5.987	0.291	0.257	0.269	0.275	26.025

Problem 695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	11	5	4	12	27	4	4
N.S.	1	1.00	2.75	1.25	1.00	3.00	6.75	1.00	1.00
time (sec)	N/A	0.214	0.002	5.948	0.295	0.262	0.277	0.291	25.599

Problem 696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	5	31	6	5	35	0	5	5
N.S.	1	0.15	0.94	0.18	0.15	1.06	0.00	0.15	0.15
time (sec)	N/A	0.207	0.082	8.091	0.306	0.246	0.000	0.300	26.792

Problem 697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	15	16	18	17	36	0	19	16
N.S.	1	1.50	1.60	1.80	1.70	3.60	0.00	1.90	1.60
time (sec)	N/A	0.218	0.151	1.348	0.204	0.243	0.000	0.308	27.752

Problem 698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	15	16	16	15	36	0	17	14
N.S.	1	1.50	1.60	1.60	1.50	3.60	0.00	1.70	1.40
time (sec)	N/A	0.227	0.121	1.408	0.209	0.260	0.000	0.279	26.108

Problem 699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	115	74	38	89	441	0	61	75
N.S.	1	0.65	0.42	0.22	0.51	2.51	0.00	0.35	0.43
time (sec)	N/A	0.329	0.196	1.424	0.301	0.351	0.000	0.288	27.291

Problem 700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	20	22	18	17	48	0	17	17
N.S.	1	0.38	0.42	0.34	0.32	0.91	0.00	0.32	0.32
time (sec)	N/A	0.226	0.105	0.839	0.274	0.260	0.000	0.268	26.821

Problem 701	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	26	28	28	71	29	29	27
N.S.	1	1.00	0.93	1.00	1.00	2.54	1.04	1.04	0.96
time (sec)	N/A	0.271	0.168	1.128	0.206	0.280	1.348	0.286	28.344

Problem 702	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	52	60	63	122	56	64	65
N.S.	1	1.00	0.98	1.13	1.19	2.30	1.06	1.21	1.23
time (sec)	N/A	0.323	0.206	1.935	0.218	0.261	1.890	0.278	28.520

Problem 703	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	122	116	118	201	95	123	122
N.S.	1	1.00	1.56	1.49	1.51	2.58	1.22	1.58	1.56
time (sec)	N/A	0.341	0.464	3.895	0.212	0.300	2.843	0.281	28.903

Problem 704	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	36	10	10	12
N.S.	1	1.00	1.00	0.92	0.83	3.00	0.83	0.83	1.00
time (sec)	N/A	0.233	0.050	1.794	0.221	0.253	160.229	0.271	28.609

Problem 705	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	67	26	25	46	27	25	25
N.S.	1	1.00	2.03	0.79	0.76	1.39	0.82	0.76	0.76
time (sec)	N/A	0.288	0.038	5.069	0.207	0.259	15.055	0.279	27.056

Problem 706	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	26	32	24	23	52	41	24	30
N.S.	1	0.57	0.70	0.52	0.50	1.13	0.89	0.52	0.65
time (sec)	N/A	0.264	6.054	0.997	0.314	0.265	2.498	0.287	26.259

Problem 707	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	4	12	3	15	4
N.S.	1	1.00	1.00	1.25	1.00	3.00	0.75	3.75	1.00
time (sec)	N/A	0.165	0.002	1.162	0.294	0.237	3.766	0.282	26.470

Problem 708	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	17	29	14	13	29	0	15	9
N.S.	1	0.81	1.38	0.67	0.62	1.38	0.00	0.71	0.43
time (sec)	N/A	0.277	0.095	0.624	0.228	0.240	0.000	0.276	27.385

Problem 709	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	43	109	8	25	0	0	0
N.S.	1	1.00	4.78	12.11	0.89	2.78	0.00	0.00	0.00
time (sec)	N/A	0.197	0.038	16.838	0.310	0.258	0.000	0.000	0.000

Problem 710	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	47	8	7	45	0	7	0
N.S.	1	1.00	5.22	0.89	0.78	5.00	0.00	0.78	0.00
time (sec)	N/A	0.197	0.056	0.856	0.331	0.261	0.000	0.294	0.000

Problem 711	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	46	13	16	67	0	17	0
N.S.	1	1.00	3.29	0.93	1.14	4.79	0.00	1.21	0.00
time (sec)	N/A	0.192	0.045	0.884	0.241	0.257	0.000	0.281	0.000

Problem 712	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	52	33	30	78	0	142	19
N.S.	1	1.00	2.74	1.74	1.58	4.11	0.00	7.47	1.00
time (sec)	N/A	0.211	0.443	0.885	0.303	0.260	0.000	0.305	26.145

Problem 713	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	63	21	20	72	0	20	0
N.S.	1	1.00	2.42	0.81	0.77	2.77	0.00	0.77	0.00
time (sec)	N/A	0.198	0.110	0.965	0.304	0.256	0.000	0.277	0.000

Problem 714	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	4	3	8	3	3	3
N.S.	1	1.00	1.00	1.00	0.75	2.00	0.75	0.75	0.75
time (sec)	N/A	0.155	0.056	0.596	0.198	0.243	0.461	0.266	26.558

Problem 715	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	25	14	42	20	19	20	14
N.S.	1	1.00	1.47	0.82	2.47	1.18	1.12	1.18	0.82
time (sec)	N/A	0.258	0.024	19.937	0.224	0.247	0.911	0.286	26.295

Problem 716	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	20	13	12	45	0	22	16
N.S.	1	1.00	1.67	1.08	1.00	3.75	0.00	1.83	1.33
time (sec)	N/A	0.189	0.046	0.530	0.197	0.259	0.000	0.278	26.573

Problem 717	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	20	38	0	41	43
N.S.	1	1.00	1.00	1.05	1.00	1.90	0.00	2.05	2.15
time (sec)	N/A	0.195	0.019	9.101	0.219	0.249	0.000	0.299	26.633

Problem 718	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	8	14	3	16	6
N.S.	1	1.00	1.00	1.17	1.33	2.33	0.50	2.67	1.00
time (sec)	N/A	0.160	0.003	1.086	0.298	0.251	2.466	0.283	26.316

Problem 719	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	8	22	7	8	14	27	16	6
N.S.	1	1.33	3.67	1.17	1.33	2.33	4.50	2.67	1.00
time (sec)	N/A	0.220	0.006	3.270	0.297	0.240	0.256	0.288	26.245

Problem 720	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	56	47	46	76	31	68	35
N.S.	1	1.00	2.00	1.68	1.64	2.71	1.11	2.43	1.25
time (sec)	N/A	0.258	1.248	0.852	0.214	0.271	9.240	0.280	26.269

Problem 721	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	62	62	92	182	58	139	92
N.S.	1	1.00	1.17	1.17	1.74	3.43	1.09	2.62	1.74
time (sec)	N/A	0.310	2.169	0.764	0.213	0.276	36.277	0.311	27.290

Problem 722	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	135	118	161	320	97	232	141
N.S.	1	1.00	1.73	1.51	2.06	4.10	1.24	2.97	1.81
time (sec)	N/A	0.326	3.079	1.167	0.223	0.293	33.514	0.290	26.514

Problem 723	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	6	5	9	5	0	7
N.S.	1	1.00	1.00	1.00	0.83	1.50	0.83	0.00	1.17
time (sec)	N/A	0.168	0.069	0.427	0.217	0.238	17.170	0.000	26.254

Problem 724	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	20	20	12	11	19	14	22	48
N.S.	1	1.82	1.82	1.09	1.00	1.73	1.27	2.00	4.36
time (sec)	N/A	0.207	0.018	0.351	0.203	0.260	0.122	0.273	26.063

Problem 725	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	4	5	5	3	5	7
N.S.	1	1.00	1.00	0.80	1.00	1.00	0.60	1.00	1.40
time (sec)	N/A	0.186	0.037	2.258	0.290	0.243	0.095	0.263	26.468

Problem 726	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	8	7	7	8	7	13
N.S.	1	1.00	1.00	0.73	0.64	0.64	0.73	0.64	1.18
time (sec)	N/A	0.195	0.046	2.358	0.286	0.256	0.110	0.260	26.192

Problem 727	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	9	12	7	9	15	7	9
N.S.	1	1.00	1.29	1.71	1.00	1.29	2.14	1.00	1.29
time (sec)	N/A	0.191	0.006	0.371	0.205	0.247	0.100	0.278	26.596

Problem 728	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	7	38	6	33	27	5	36	7
N.S.	1	1.40	7.60	1.20	6.60	5.40	1.00	7.20	1.40
time (sec)	N/A	0.215	0.028	0.591	0.212	0.262	0.336	0.285	26.997

Problem 729	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	25	13	16	0	21	13
N.S.	1	1.00	1.00	1.92	1.00	1.23	0.00	1.62	1.00
time (sec)	N/A	0.236	0.014	0.687	0.292	0.252	0.000	0.292	0.122

Problem 730	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	4	3	5	3	5	5
N.S.	1	1.00	1.00	1.00	0.75	1.25	0.75	1.25	1.25
time (sec)	N/A	0.180	0.008	2.780	0.199	0.254	0.173	0.269	27.733

Problem 731	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	9	11	7	11	11
N.S.	1	1.00	1.00	1.11	1.00	1.22	0.78	1.22	1.22
time (sec)	N/A	0.185	0.008	21.101	0.200	0.242	0.170	0.286	27.581

Problem 732	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	29	12	31	18
N.S.	1	1.00	1.00	0.92	0.83	2.42	1.00	2.58	1.50
time (sec)	N/A	0.229	0.096	0.451	0.197	0.253	0.359	0.292	25.970

Problem 733	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	42	42	65	35	122	0	0	36
N.S.	1	0.98	0.98	1.51	0.81	2.84	0.00	0.00	0.84
time (sec)	N/A	0.258	0.045	1.013	0.282	0.261	0.000	0.000	26.702

Problem 734	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	34	20	20	0	0	18
N.S.	1	1.00	1.00	1.55	0.91	0.91	0.00	0.00	0.82
time (sec)	N/A	0.254	0.021	0.642	0.280	0.235	0.000	0.000	26.300

Problem 735	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	20	20	13	12	20	17	22	31
N.S.	1	1.67	1.67	1.08	1.00	1.67	1.42	1.83	2.58
time (sec)	N/A	0.205	0.015	0.371	0.201	0.245	0.111	0.278	26.752

Problem 736	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	14	12	14	14
N.S.	1	1.00	1.00	0.93	0.86	1.00	0.86	1.00	1.00
time (sec)	N/A	0.189	0.020	1.423	0.206	0.252	0.155	0.311	26.654

Problem 737	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	6	3	3	5	3	26
N.S.	1	1.00	1.00	2.00	1.00	1.00	1.67	1.00	8.67
time (sec)	N/A	0.188	0.011	0.510	0.296	0.270	0.088	0.269	27.014

Problem 738	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	47	41	35	49	73	151	48	57
N.S.	1	1.09	0.95	0.81	1.14	1.70	3.51	1.12	1.33
time (sec)	N/A	0.220	0.373	0.875	0.286	0.248	0.523	0.356	27.435

Problem 739	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	21	0	21	34
N.S.	1	1.00	1.00	1.07	1.00	1.50	0.00	1.50	2.43
time (sec)	N/A	0.229	0.021	0.811	0.284	0.252	0.000	0.282	26.833

Problem 740	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	47	28	38	37	57	0	0	28
N.S.	1	1.09	0.65	0.88	0.86	1.33	0.00	0.00	0.65
time (sec)	N/A	0.261	0.046	1.201	0.274	0.257	0.000	0.000	26.342

Problem 741	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	41	28	104	37	27	0	0	27
N.S.	1	0.95	0.65	2.42	0.86	0.63	0.00	0.00	0.63
time (sec)	N/A	0.221	0.110	1.334	0.236	0.241	0.000	0.000	27.671

Problem 742	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	41	28	104	37	27	0	0	27
N.S.	1	0.95	0.65	2.42	0.86	0.63	0.00	0.00	0.63
time (sec)	N/A	0.219	0.013	1.107	0.208	0.246	0.000	0.000	0.002

Problem 743	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	62	36	122	0	33	0	138	33
N.S.	1	0.97	0.56	1.91	0.00	0.52	0.00	2.16	0.52
time (sec)	N/A	0.221	0.129	1.408	0.000	0.247	0.000	0.317	28.514

Problem 744	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	62	36	122	0	33	0	138	33
N.S.	1	0.97	0.56	1.91	0.00	0.52	0.00	2.16	0.52
time (sec)	N/A	0.222	0.028	1.086	0.000	0.251	0.000	0.330	0.004

Problem 745	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	41	28	105	37	27	0	0	27
N.S.	1	0.95	0.65	2.44	0.86	0.63	0.00	0.00	0.63
time (sec)	N/A	0.214	0.100	1.249	0.218	0.247	0.000	0.000	29.002

Problem 746	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	41	28	105	37	27	0	0	27
N.S.	1	0.95	0.65	2.44	0.86	0.63	0.00	0.00	0.63
time (sec)	N/A	0.214	0.012	1.020	0.224	0.248	0.000	0.000	0.003

Problem 747	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	62	36	123	0	33	0	195	33
N.S.	1	0.97	0.56	1.92	0.00	0.52	0.00	3.05	0.52
time (sec)	N/A	0.221	0.096	1.284	0.000	0.250	0.000	0.338	26.769

Problem 748	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	62	36	123	0	33	0	195	33
N.S.	1	0.97	0.56	1.92	0.00	0.52	0.00	3.05	0.52
time (sec)	N/A	0.219	0.013	1.034	0.000	0.242	0.000	0.336	0.003

Problem 749	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	7	12	0	7	27
N.S.	1	1.00	1.00	0.89	0.78	1.33	0.00	0.78	3.00
time (sec)	N/A	0.168	0.009	1.243	0.194	0.232	0.000	0.269	29.117

Problem 750	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	265	7	0	0	27
N.S.	1	1.00	1.00	0.89	29.44	0.78	0.00	0.00	3.00
time (sec)	N/A	0.164	0.010	0.596	0.307	0.233	0.000	0.000	27.264

Problem 751	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	5	3	5	4	4	14	4	4
N.S.	1	1.67	1.00	1.67	1.33	1.33	4.67	1.33	1.33
time (sec)	N/A	0.149	0.001	0.356	0.283	0.233	0.060	0.263	0.035

Problem 752	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	74	18	8	103	8
N.S.	1	1.00	1.00	1.12	9.25	2.25	1.00	12.88	1.00
time (sec)	N/A	0.196	0.008	0.262	0.287	0.258	0.318	0.286	26.838

Problem 753	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	43	28	23	22	27	76	22	22
N.S.	1	1.26	0.82	0.68	0.65	0.79	2.24	0.65	0.65
time (sec)	N/A	0.233	0.018	1.135	0.207	0.257	0.171	0.275	26.926

Problem 754	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	7	6	6	10	6	6
N.S.	1	1.00	1.00	0.70	0.60	0.60	1.00	0.60	0.60
time (sec)	N/A	0.155	0.003	0.332	0.191	0.240	0.127	0.274	0.090

Problem 755	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	9	8	14	15	9	12
N.S.	1	1.00	1.00	0.82	0.73	1.27	1.36	0.82	1.09
time (sec)	N/A	0.179	0.008	0.184	0.185	0.238	0.191	0.292	27.399

Problem 756	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	9	8	7	9	8	7	7
N.S.	1	1.00	1.29	1.14	1.00	1.29	1.14	1.00	1.00
time (sec)	N/A	0.199	0.005	2.500	0.199	0.245	1.340	0.280	0.065

Problem 757	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	74	28	0	103	15
N.S.	1	1.00	1.00	0.84	3.89	1.47	0.00	5.42	0.79
time (sec)	N/A	0.211	0.010	0.477	0.272	0.248	0.000	0.287	25.919

Problem 758	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	26	24	26	29	32	23	25
N.S.	1	1.00	0.70	0.65	0.70	0.78	0.86	0.62	0.68
time (sec)	N/A	0.168	0.029	0.600	0.187	0.235	0.188	0.284	26.391

Problem 759	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	B	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	129	49	38	8	39	236	37	49
N.S.	1	10.75	4.08	3.17	0.67	3.25	19.67	3.08	4.08
time (sec)	N/A	0.433	0.059	2.077	0.203	0.285	0.032	0.276	26.575

Problem 760	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	15	8	7	13	19	8	19
N.S.	1	1.00	1.67	0.89	0.78	1.44	2.11	0.89	2.11
time (sec)	N/A	0.172	0.010	0.297	0.192	0.244	0.079	0.263	26.402

Problem 761	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	35	12	0	6	14
N.S.	1	1.00	1.00	0.88	4.38	1.50	0.00	0.75	1.75
time (sec)	N/A	0.187	0.018	1.185	0.197	0.238	0.000	0.272	0.132

Problem 762	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	0	13	0	11	13
N.S.	1	1.00	1.00	0.80	0.00	0.87	0.00	0.73	0.87
time (sec)	N/A	0.195	0.015	26.895	0.000	0.259	0.000	0.270	26.975

Problem 763	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	20	16	15	15	19	15	15
N.S.	1	1.00	0.87	0.70	0.65	0.65	0.83	0.65	0.65
time (sec)	N/A	0.178	0.011	0.323	0.275	0.246	0.103	0.285	0.097

Problem 764	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	30	29	29	42	29	33
N.S.	1	1.00	1.00	0.81	0.78	0.78	1.14	0.78	0.89
time (sec)	N/A	0.186	0.046	2.114	0.199	0.256	0.131	0.272	26.433

Problem 765	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	12	9	8	10	8	10	8
N.S.	1	1.00	1.20	0.90	0.80	1.00	0.80	1.00	0.80
time (sec)	N/A	0.200	0.015	0.311	0.181	0.240	0.073	0.268	26.601

Problem 766	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.62	0.75	0.75
time (sec)	N/A	0.167	0.003	0.615	0.196	0.239	0.064	0.283	0.070

Problem 767	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	7	8	8
N.S.	1	1.00	1.00	0.90	0.80	0.80	0.70	0.80	0.80
time (sec)	N/A	0.175	0.005	0.144	0.199	0.257	0.087	0.270	0.067

Problem 768	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.62	0.75	0.75
time (sec)	N/A	0.174	0.003	0.139	0.192	0.253	0.117	0.272	25.877

Problem 769	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	7	8	8
N.S.	1	1.00	1.00	0.90	0.80	0.80	0.70	0.80	0.80
time (sec)	N/A	0.178	0.016	0.520	0.193	0.248	0.060	0.278	26.994

Problem 770	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	56	8	5	8	8
N.S.	1	1.00	1.00	0.88	7.00	1.00	0.62	1.00	1.00
time (sec)	N/A	0.236	0.007	0.188	0.202	0.247	0.106	0.272	0.099

Problem 771	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	12	13	67	13	7	10	10
N.S.	1	1.00	1.20	1.30	6.70	1.30	0.70	1.00	1.00
time (sec)	N/A	0.189	0.020	0.211	0.187	0.232	0.091	0.287	26.483

Problem 772	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	15	12	10	13
N.S.	1	1.00	1.00	0.91	0.82	1.36	1.09	0.91	1.18
time (sec)	N/A	0.180	0.019	0.304	0.200	0.250	0.056	0.290	0.364

Problem 773	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	10	12	12	10	13
N.S.	1	1.00	1.00	0.92	0.83	1.00	1.00	0.83	1.08
time (sec)	N/A	0.147	0.014	0.371	0.192	0.253	0.475	0.259	0.070

Problem 774	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	25	18	18	29	0	18	16
N.S.	1	1.00	1.19	0.86	0.86	1.38	0.00	0.86	0.76
time (sec)	N/A	0.236	0.015	0.809	0.191	0.238	0.000	0.279	26.576

Problem 775	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	15	40	32	39	15
N.S.	1	1.00	1.00	0.84	0.79	2.11	1.68	2.05	0.79
time (sec)	N/A	0.191	0.011	0.449	0.194	0.258	0.601	0.282	26.648

Problem 776	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	52	35	33	32	32	48	32	43
N.S.	1	1.11	0.74	0.70	0.68	0.68	1.02	0.68	0.91
time (sec)	N/A	0.428	0.034	0.174	0.188	0.257	4.439	0.272	26.935

Problem 777	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	39	28	26	25	29	32	25	25
N.S.	1	1.11	0.80	0.74	0.71	0.83	0.91	0.71	0.71
time (sec)	N/A	0.304	0.053	0.365	0.196	0.259	0.420	0.271	26.467

Problem 778	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	4	4	3	4	4
N.S.	1	1.00	1.00	1.25	1.00	1.00	0.75	1.00	1.00
time (sec)	N/A	0.167	0.001	0.230	0.185	0.254	0.067	0.272	26.524

Problem 779	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	6	6	5	6	6
N.S.	1	1.00	1.00	1.17	1.00	1.00	0.83	1.00	1.00
time (sec)	N/A	0.178	0.004	0.240	0.186	0.253	0.087	0.286	0.071

Problem 780	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	7	7	5	7	7
N.S.	1	1.00	1.00	1.14	1.00	1.00	0.71	1.00	1.00
time (sec)	N/A	0.131	0.011	1.213	0.269	0.286	0.045	0.277	0.068

Problem 781	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	21	9	8	8	8	8	8
N.S.	1	1.00	2.10	0.90	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.177	0.021	0.534	0.183	0.242	0.090	0.273	0.053

Problem 782	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	7	8	8
N.S.	1	1.00	1.00	0.90	0.80	0.80	0.70	0.80	0.80
time (sec)	N/A	0.176	0.004	0.214	0.189	0.251	0.072	0.268	0.055

Problem 783	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	7	8	8
N.S.	1	1.00	1.00	0.90	0.80	0.80	0.70	0.80	0.80
time (sec)	N/A	0.136	0.002	0.187	0.195	0.243	0.120	0.269	0.055

Problem 784	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	21	9	8	8	8	8	8
N.S.	1	1.00	2.10	0.90	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.179	0.016	0.217	0.182	0.252	0.123	0.264	27.293

Problem 785	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	6	6	5	6	6
N.S.	1	1.00	1.00	1.17	1.00	1.00	0.83	1.00	1.00
time (sec)	N/A	0.171	0.001	0.128	0.203	0.243	0.094	0.273	0.069

Problem 786	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	14	14	15	14	14
N.S.	1	1.00	1.00	1.07	1.00	1.00	1.07	1.00	1.00
time (sec)	N/A	0.193	0.029	0.381	0.198	0.237	0.067	0.263	0.079

Problem 787	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	17	20	17	16	16	15	16	16
N.S.	1	0.85	1.00	0.85	0.80	0.80	0.75	0.80	0.80
time (sec)	N/A	0.220	0.010	0.277	0.188	0.240	0.240	0.264	27.183

Problem 788	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	16	14	15	17	20	15	15
N.S.	1	1.00	0.70	0.61	0.65	0.74	0.87	0.65	0.65
time (sec)	N/A	0.153	0.013	0.295	0.188	0.246	0.180	0.276	0.032

Problem 789	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	18	20	17	16	16	15	16	16
N.S.	1	0.90	1.00	0.85	0.80	0.80	0.75	0.80	0.80
time (sec)	N/A	0.224	0.004	0.563	0.199	0.237	0.124	0.281	26.821

Problem 790	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	17	20	17	16	16	15	16	16
N.S.	1	0.85	1.00	0.85	0.80	0.80	0.75	0.80	0.80
time (sec)	N/A	0.224	0.008	0.648	0.191	0.234	0.129	0.263	0.064

Problem 791	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	7	19	7	7	7
N.S.	1	1.00	1.00	0.89	0.78	2.11	0.78	0.78	0.78
time (sec)	N/A	0.181	1.158	0.443	0.183	0.257	0.139	0.271	0.109

Problem 792	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	9	11	10	9	17
N.S.	1	1.00	1.00	0.91	0.82	1.00	0.91	0.82	1.55
time (sec)	N/A	0.196	0.083	0.362	0.191	0.253	0.082	0.263	26.785

Problem 793	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	51	8	45	36	61	8
N.S.	1	1.00	1.00	5.10	0.80	4.50	3.60	6.10	0.80
time (sec)	N/A	0.176	0.019	3.204	0.187	0.252	0.133	0.256	26.609

Problem 794	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	9	13	10	8	8	6
N.S.	1	1.00	1.00	1.00	1.44	1.11	0.89	0.89	0.67
time (sec)	N/A	0.210	0.003	1.395	0.190	0.237	0.924	0.266	26.360

Problem 795	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	20	7	11	7	7	7
N.S.	1	1.00	1.00	4.00	1.40	2.20	1.40	1.40	1.40
time (sec)	N/A	0.175	0.017	1.872	0.182	0.250	0.031	0.270	26.728

Problem 796	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	B	A	F(-1)	B	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	0	13	11	88	14	0	794	14
N.S.	1	0.00	1.00	0.85	6.77	1.08	0.00	61.08	1.08
time (sec)	N/A	0.000	0.594	2.227	0.523	0.235	0.000	0.282	26.894

Problem 797	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	104	20	8	52	9
N.S.	1	1.00	1.00	1.11	11.56	2.22	0.89	5.78	1.00
time (sec)	N/A	0.200	0.016	0.207	0.182	0.255	0.274	0.306	0.035

Problem 798	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	15	14	31	37	14	18
N.S.	1	1.00	1.00	0.75	0.70	1.55	1.85	0.70	0.90
time (sec)	N/A	0.173	0.013	0.757	0.204	0.252	0.125	0.279	0.033

Problem 799	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	18	19	15	15	15	22	15	14
N.S.	1	0.95	1.00	0.79	0.79	0.79	1.16	0.79	0.74
time (sec)	N/A	0.202	0.013	1.183	0.208	0.241	0.116	0.303	28.065

Problem 800	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	16	14	10	18	16
N.S.	1	1.00	1.00	0.93	1.14	1.00	0.71	1.29	1.14
time (sec)	N/A	0.173	0.008	1.596	0.200	0.258	0.032	0.285	26.748

Problem 801	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	18	20	29	20	37	20	28	32
N.S.	1	0.82	0.91	1.32	0.91	1.68	0.91	1.27	1.45
time (sec)	N/A	0.201	0.020	4.276	0.211	0.259	0.040	0.280	27.407

Problem 802	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	7	6	5	5	12	5	5
N.S.	1	1.00	1.40	1.20	1.00	1.00	2.40	1.00	1.00
time (sec)	N/A	0.168	0.006	0.336	0.207	0.240	0.829	0.277	26.920

Problem 803	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	23	13	5	7	12	7	5
N.S.	1	1.00	3.29	1.86	0.71	1.00	1.71	1.00	0.71
time (sec)	N/A	0.166	0.004	0.444	0.200	0.250	0.736	0.271	25.727

Problem 804	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	11	8	6	11	5	14	9	6
N.S.	1	2.20	1.60	1.20	2.20	1.00	2.80	1.80	1.20
time (sec)	N/A	0.179	0.002	3.151	0.214	0.239	1.642	0.282	25.924

Problem 805	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	30	18	69	35	32	29	24
N.S.	1	1.00	2.00	1.20	4.60	2.33	2.13	1.93	1.60
time (sec)	N/A	0.211	0.025	1.263	0.281	0.246	0.728	0.290	27.138

Problem 806	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	17	12	11	15	12	15	11
N.S.	1	1.00	1.55	1.09	1.00	1.36	1.09	1.36	1.00
time (sec)	N/A	0.217	0.154	0.523	0.211	0.254	0.080	0.280	0.112

Problem 807	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	82	18	17	17	19	17	17
N.S.	1	1.00	4.10	0.90	0.85	0.85	0.95	0.85	0.85
time (sec)	N/A	0.215	0.168	1.132	0.293	0.252	0.145	0.289	26.855

Problem 808	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	13	10	12	9
N.S.	1	1.00	1.00	1.09	1.00	1.18	0.91	1.09	0.82
time (sec)	N/A	0.191	0.015	0.375	0.194	0.247	0.071	0.270	0.192

Problem 809	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	35	26	31	34	27	36	0	29
N.S.	1	1.35	1.00	1.19	1.31	1.04	1.38	0.00	1.12
time (sec)	N/A	0.223	0.077	1.428	0.279	0.245	0.436	0.000	27.358

Problem 810	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	39	24	220	35	0	28	16
N.S.	1	1.00	1.62	1.00	9.17	1.46	0.00	1.17	0.67
time (sec)	N/A	0.192	0.035	0.867	0.209	0.238	0.000	0.275	27.486

Problem 811	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	29	26	38	26	39	26	34
N.S.	1	1.00	0.72	0.65	0.95	0.65	0.98	0.65	0.85
time (sec)	N/A	0.235	0.094	0.513	0.206	0.242	0.090	0.266	0.080

Problem 812	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	22	20	19	21	26	19	19
N.S.	1	1.00	0.81	0.74	0.70	0.78	0.96	0.70	0.70
time (sec)	N/A	0.160	0.040	0.342	0.197	0.239	0.165	0.268	0.034

Problem 813	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	31	18	17	12	26	42	12
N.S.	1	1.00	1.35	0.78	0.74	0.52	1.13	1.83	0.52
time (sec)	N/A	0.206	0.019	0.643	0.213	0.232	0.112	0.296	0.138

Problem 814	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	46	25	24	36	27	24	24
N.S.	1	1.00	1.53	0.83	0.80	1.20	0.90	0.80	0.80
time (sec)	N/A	0.166	0.052	0.592	0.198	0.255	0.027	0.275	26.174

Problem 815	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	6	6	3	14	6
N.S.	1	1.00	1.00	1.17	1.00	1.00	0.50	2.33	1.00
time (sec)	N/A	0.165	0.001	1.201	0.214	0.245	0.502	0.269	26.604

Problem 816	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	7	15	8	30	12
N.S.	1	1.00	1.00	1.14	1.00	2.14	1.14	4.29	1.71
time (sec)	N/A	0.194	0.002	5.481	0.197	0.241	2.904	0.269	25.428

Problem 817	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	11	10	10	13	10	14	13	6
N.S.	1	0.85	0.77	0.77	1.00	0.77	1.08	1.00	0.46
time (sec)	N/A	0.147	0.008	1.230	0.202	0.251	0.027	0.276	25.759

Problem 818	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	11	22	12	11	14
N.S.	1	1.00	1.00	0.91	1.00	2.00	1.09	1.00	1.27
time (sec)	N/A	0.140	0.024	1.255	0.204	0.241	0.023	0.302	26.630

Problem 819	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	15	17	10	18	10	11	24
N.S.	1	1.00	1.50	1.70	1.00	1.80	1.00	1.10	2.40
time (sec)	N/A	0.139	0.030	0.626	0.198	0.266	0.023	0.281	27.429

Problem 820	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	8	8	8
N.S.	1	1.00	1.00	0.90	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.175	0.011	0.496	0.194	0.232	0.061	0.264	0.067

Problem 821	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	13	12	14	32	12	12
N.S.	1	1.00	1.00	0.72	0.67	0.78	1.78	0.67	0.67
time (sec)	N/A	0.205	0.027	0.415	0.191	0.241	0.156	0.266	26.256

Problem 822	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	8	31	8	8
N.S.	1	1.00	1.00	0.90	0.80	0.80	3.10	0.80	0.80
time (sec)	N/A	0.147	0.005	0.128	0.220	0.238	0.501	0.271	26.625

Problem 823	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	25	11	10	21	12	10	14
N.S.	1	1.00	1.56	0.69	0.62	1.31	0.75	0.62	0.88
time (sec)	N/A	0.170	0.022	0.284	0.218	0.234	0.116	0.318	26.110

Problem 824	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	7	13	8	9	9
N.S.	1	1.00	1.00	1.14	1.00	1.86	1.14	1.29	1.29
time (sec)	N/A	0.176	0.006	0.264	0.201	0.265	0.052	0.270	0.092

Problem 825	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	17	16	25	15	41	15
N.S.	1	1.00	1.00	1.31	1.23	1.92	1.15	3.15	1.15
time (sec)	N/A	0.192	0.019	0.338	0.207	0.248	7.638	0.287	26.882

Problem 826	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	21	10	10	23	10	43	31
N.S.	1	1.00	4.20	2.00	2.00	4.60	2.00	8.60	6.20
time (sec)	N/A	0.188	0.015	0.123	0.209	0.237	0.533	0.270	27.840

Problem 827	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	19	9	8	15	7	8	16	26
N.S.	1	2.71	1.29	1.14	2.14	1.00	1.14	2.29	3.71
time (sec)	N/A	0.221	0.004	1.335	0.219	0.264	1.160	0.274	26.437

Problem 828	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	4	4	20	4	4
N.S.	1	1.00	1.00	1.25	1.00	1.00	5.00	1.00	1.00
time (sec)	N/A	0.143	0.002	1.509	0.192	0.241	0.152	0.262	27.409

Problem 829	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	21	21	14	74	27	0	81	13
N.S.	1	1.62	1.62	1.08	5.69	2.08	0.00	6.23	1.00
time (sec)	N/A	0.220	0.006	0.434	0.285	0.271	0.000	0.292	26.951

Problem 830	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	31	18	28	20	22	20	20	18
N.S.	1	1.94	1.12	1.75	1.25	1.38	1.25	1.25	1.12
time (sec)	N/A	0.193	0.010	2.332	0.287	0.243	0.020	0.264	26.207

Problem 831	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	50	26	34	35	28	36	31	26
N.S.	1	1.56	0.81	1.06	1.09	0.88	1.12	0.97	0.81
time (sec)	N/A	0.201	0.030	34.701	0.302	0.247	0.020	0.273	27.698

Problem 832	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	31	16	11	10	19	12	10	18
N.S.	1	1.72	0.89	0.61	0.56	1.06	0.67	0.56	1.00
time (sec)	N/A	0.213	0.031	0.159	0.212	0.252	0.023	0.279	0.051

Problem 833	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	48	32	23	18	25	32	22	24
N.S.	1	1.41	0.94	0.68	0.53	0.74	0.94	0.65	0.71
time (sec)	N/A	0.277	0.011	0.217	0.227	0.250	0.019	0.269	0.057

Problem 834	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	19	33	14	13	21	12	13	13
N.S.	1	1.46	2.54	1.08	1.00	1.62	0.92	1.00	1.00
time (sec)	N/A	0.184	0.009	0.275	0.242	0.240	0.019	0.276	27.233

Problem 835	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	63	24	17	16	31	31	16	33
N.S.	1	1.37	0.52	0.37	0.35	0.67	0.67	0.35	0.72
time (sec)	N/A	0.335	0.025	1.042	0.243	0.249	0.023	0.278	0.052

Problem 836	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	C	B	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	21	9	20	20	19	20	18	13
N.S.	1	2.33	1.00	2.22	2.22	2.11	2.22	2.00	1.44
time (sec)	N/A	0.194	0.010	0.376	0.225	0.243	0.113	0.271	28.609

Problem 837	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	17	13	13	12	12	13	11
N.S.	1	1.00	1.21	0.93	0.93	0.86	0.86	0.93	0.79
time (sec)	N/A	0.144	0.005	0.451	0.212	0.242	0.022	0.263	26.402

Problem 838	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	1	0	1	1
N.S.	1	1.00	1.00	2.00	1.00	1.00	0.00	1.00	1.00
time (sec)	N/A	0.128	0.001	0.313	0.246	0.243	0.024	0.274	26.419

Problem 839	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	8	7	6	6	7	6	6
N.S.	1	1.00	1.33	1.17	1.00	1.00	1.17	1.00	1.00
time (sec)	N/A	0.134	0.002	0.517	0.274	0.233	0.021	0.269	26.148

Problem 840	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	9	9	7	9	9
N.S.	1	1.00	1.00	1.11	1.00	1.00	0.78	1.00	1.00
time (sec)	N/A	0.161	0.009	0.556	0.234	0.243	0.112	0.275	0.101

Problem 841	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	35	6	22	6	6
N.S.	1	1.00	1.00	0.88	4.38	0.75	2.75	0.75	0.75
time (sec)	N/A	0.140	0.004	0.173	0.220	0.232	0.064	0.277	26.381

Problem 842	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	51	9	51	8	5	26	8
N.S.	1	1.00	8.50	1.50	8.50	1.33	0.83	4.33	1.33
time (sec)	N/A	0.308	0.041	0.212	0.402	0.236	0.140	0.286	0.131

Problem 843	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	8	8	8	8
N.S.	1	1.00	1.00	0.88	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.338	0.017	0.150	0.236	0.240	0.110	0.271	26.830

Problem 844	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	12	9	8	8	7	6	8
N.S.	1	1.00	1.50	1.12	1.00	1.00	0.88	0.75	1.00
time (sec)	N/A	0.144	0.014	0.112	0.237	0.237	0.105	0.276	26.623

Problem 845	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	8	7	8	8
N.S.	1	1.00	1.00	0.88	1.00	1.00	0.88	1.00	1.00
time (sec)	N/A	0.323	0.015	0.126	0.234	0.263	0.111	0.261	26.988

Problem 846	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	91	55	80	0	320	124	77	1108
N.S.	1	1.65	1.00	1.45	0.00	5.82	2.25	1.40	20.15
time (sec)	N/A	0.392	0.101	2.215	0.000	0.284	2.909	0.287	28.410

Problem 847	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	83	54	72	0	322	124	77	1374
N.S.	1	1.51	0.98	1.31	0.00	5.85	2.25	1.40	24.98
time (sec)	N/A	0.328	0.072	2.247	0.000	0.302	2.981	0.279	26.735

Problem 848	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	66	48	49	0	225	432	141	108
N.S.	1	1.27	0.92	0.94	0.00	4.33	8.31	2.71	2.08
time (sec)	N/A	0.294	0.121	2.228	0.000	0.286	16.323	0.287	26.847

Problem 849	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	54	50	51	0	224	432	159	684
N.S.	1	1.04	0.96	0.98	0.00	4.31	8.31	3.06	13.15
time (sec)	N/A	0.267	0.070	2.147	0.000	0.280	16.276	0.302	26.608

Problem 850	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	31	30	76	91	0	56	0
N.S.	1	1.00	1.03	1.00	2.53	3.03	0.00	1.87	0.00
time (sec)	N/A	0.207	0.030	0.663	0.337	0.261	0.000	0.412	0.000

Problem 851	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	31	51	84	0	27	0
N.S.	1	1.00	1.00	1.00	1.65	2.71	0.00	0.87	0.00
time (sec)	N/A	0.211	0.045	0.717	0.238	0.267	0.000	0.284	0.000

Problem 852	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.145	0.003	0.331	0.226	0.244	0.106	0.279	27.194

Problem 853	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	22	25	41	21	0	14	0
N.S.	1	1.00	1.16	1.32	2.16	1.11	0.00	0.74	0.00
time (sec)	N/A	0.192	0.022	0.901	0.296	0.259	0.000	0.271	0.000

Problem 854	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	34	16	13	12	23	476	12	12
N.S.	1	1.17	0.55	0.45	0.41	0.79	16.41	0.41	0.41
time (sec)	N/A	0.257	0.018	0.298	0.207	0.260	8.127	0.267	26.874

Problem 855	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	43	29	36	103	26	32	34	45
N.S.	1	1.48	1.00	1.24	3.55	0.90	1.10	1.17	1.55
time (sec)	N/A	0.308	5.020	0.872	0.296	0.255	0.109	0.278	26.568

Problem 856	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	43	29	36	103	26	32	34	45
N.S.	1	1.48	1.00	1.24	3.55	0.90	1.10	1.17	1.55
time (sec)	N/A	0.271	5.018	0.746	0.282	0.247	0.109	0.297	26.397

Problem 857	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	38	31	30	46	0	34	32
N.S.	1	1.00	0.86	0.70	0.68	1.05	0.00	0.77	0.73
time (sec)	N/A	0.238	0.058	0.905	0.210	0.244	0.000	0.278	0.082

Problem 858	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	100	19	16	145	15	0	0	15
N.S.	1	5.26	1.00	0.84	7.63	0.79	0.00	0.00	0.79
time (sec)	N/A	0.872	0.090	5.019	0.337	0.252	0.000	0.000	26.858

Problem 859	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	B	B	B	F	F	F(-1)
verified	N/A	N/A	No	Yes	TBD	TBD	TBD	TBD	TBD
size	44	0	68	124	518	96	0	0	0
N.S.	1	0.00	1.55	2.82	11.77	2.18	0.00	0.00	0.00
time (sec)	N/A	0.000	0.940	10.200	0.487	0.264	0.000	0.000	0.000

Problem 860	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	C	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	72	17	12132	329	3	0	11	0
N.S.	1	3.79	0.89	638.53	17.32	0.16	0.00	0.58	0.00
time (sec)	N/A	1.469	0.064	8.306	0.353	0.245	0.000	0.281	0.000

Problem 861	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	37	10	0	21	0	55	14
N.S.	1	1.00	2.85	0.77	0.00	1.62	0.00	4.23	1.08
time (sec)	N/A	0.421	0.058	1.049	0.000	0.252	0.000	0.372	0.369

Problem 862	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	25	29	68	29
N.S.	1	1.00	1.00	0.79	0.71	1.79	2.07	4.86	2.07
time (sec)	N/A	0.214	0.086	0.490	0.217	0.253	0.178	0.288	26.595

Problem 863	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	30	18	21	35	0	128	24
N.S.	1	1.00	1.20	0.72	0.84	1.40	0.00	5.12	0.96
time (sec)	N/A	0.287	0.256	0.692	0.236	0.253	0.000	0.271	27.378

Problem 864	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	18	18	21	44	0	128	24
N.S.	1	1.00	0.72	0.72	0.84	1.76	0.00	5.12	0.96
time (sec)	N/A	0.272	0.033	0.651	0.222	0.240	0.000	0.268	27.031

Problem 865	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	20	20	17	0	0	0	0	0	77
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	3.85
time (sec)	N/A	0.300	1.517	0.000	0.000	0.000	0.000	0.000	26.735

Problem 866	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	A	F(-1)	A	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	95	78	37	136	34	0	97	0
N.S.	1	1.25	1.03	0.49	1.79	0.45	0.00	1.28	0.00
time (sec)	N/A	0.395	0.338	1.193	0.303	0.243	0.000	0.287	0.000

Problem 867	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	44	40	48	86	41	0	44	0
N.S.	1	0.54	0.49	0.59	1.06	0.51	0.00	0.54	0.00
time (sec)	N/A	0.443	0.044	0.937	0.202	0.250	0.000	0.277	0.000

Problem 868	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	53	69	83	79	124	0	0	0
N.S.	1	0.70	0.91	1.09	1.04	1.63	0.00	0.00	0.00
time (sec)	N/A	0.643	0.097	1.039	0.310	0.264	0.000	0.000	0.000

Problem 869	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	87	99	106	107	227	0	0	0
N.S.	1	0.68	0.77	0.83	0.84	1.77	0.00	0.00	0.00
time (sec)	N/A	0.824	0.099	0.941	0.328	0.280	0.000	0.000	0.000

Problem 870	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	127	147	137	131	327	0	0	0
N.S.	1	0.68	0.79	0.74	0.70	1.76	0.00	0.00	0.00
time (sec)	N/A	0.910	0.127	0.935	0.339	0.298	0.000	0.000	0.000

Problem 871	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	63	50	127	83	138	0	0	0
N.S.	1	0.78	0.62	1.57	1.02	1.70	0.00	0.00	0.00
time (sec)	N/A	0.674	0.080	1.108	0.292	0.283	0.000	0.000	0.000

Problem 872	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	86	75	153	113	248	0	0	0
N.S.	1	0.79	0.69	1.40	1.04	2.28	0.00	0.00	0.00
time (sec)	N/A	0.826	0.089	0.999	0.307	0.275	0.000	0.000	0.000

Problem 873	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	111	87	181	137	356	0	0	0
N.S.	1	0.78	0.61	1.27	0.96	2.49	0.00	0.00	0.00
time (sec)	N/A	0.937	0.093	0.991	0.291	0.281	0.000	0.000	0.000

Problem 874	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	62	108	290	294	140	0	0	0
N.S.	1	0.59	1.03	2.76	2.80	1.33	0.00	0.00	0.00
time (sec)	N/A	0.493	0.090	1.125	0.315	0.260	0.000	0.000	0.000

Problem 875	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	153	174	200	0	337	0	0	0
N.S.	1	0.68	0.77	0.89	0.00	1.50	0.00	0.00	0.00
time (sec)	N/A	0.699	0.131	1.167	0.000	0.300	0.000	0.000	0.000

Problem 876	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	341	225	290	250	567	539	0	0	0
N.S.	1	0.66	0.85	0.73	1.66	1.58	0.00	0.00	0.00
time (sec)	N/A	0.806	0.324	1.344	0.326	0.332	0.000	0.000	0.000

Problem 877	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	79	85	140	423	270	0	0	0
N.S.	1	0.56	0.60	0.99	2.98	1.90	0.00	0.00	0.00
time (sec)	N/A	0.559	0.259	1.035	0.322	0.288	0.000	0.000	0.000

Problem 878	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	115	138	191	639	550	0	0	0
N.S.	1	0.52	0.63	0.87	2.90	2.50	0.00	0.00	0.00
time (sec)	N/A	0.716	0.514	1.045	0.361	0.317	0.000	0.000	0.000

Problem 879	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	203	191	254	852	740	0	0	0
N.S.	1	0.57	0.54	0.71	2.39	2.08	0.00	0.00	0.00
time (sec)	N/A	0.808	0.837	0.913	0.420	0.353	0.000	0.000	0.000

Problem 880	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	17	112	13	14
N.S.	1	1.00	1.00	0.80	0.76	0.68	4.48	0.52	0.56
time (sec)	N/A	0.202	0.010	1.361	0.195	0.236	0.870	0.266	26.887

Problem 881	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	23	22	25	114	22	22
N.S.	1	1.00	1.00	0.77	0.73	0.83	3.80	0.73	0.73
time (sec)	N/A	0.205	0.008	1.413	0.220	0.241	0.858	0.272	26.011

Problem 882	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	23	22	25	116	22	22
N.S.	1	1.00	1.00	0.77	0.73	0.83	3.87	0.73	0.73
time (sec)	N/A	0.203	0.007	1.194	0.208	0.248	0.895	0.272	26.087

Problem 883	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	19	19	114	19	19
N.S.	1	1.00	1.00	0.80	0.76	0.76	4.56	0.76	0.76
time (sec)	N/A	0.198	0.009	1.240	0.223	0.252	0.861	0.270	26.572

Problem 884	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	7	6	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75
time (sec)	N/A	0.171	0.010	0.521	0.236	0.257	0.065	0.295	26.712

Problem 885	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	20	9	34	46	19	20
N.S.	1	1.00	1.00	1.82	0.82	3.09	4.18	1.73	1.82
time (sec)	N/A	0.171	0.042	2.072	0.222	0.246	0.257	0.265	26.365

Problem 886	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	18	18	12	133	15	8	52	16
N.S.	1	1.64	1.64	1.09	12.09	1.36	0.73	4.73	1.45
time (sec)	N/A	0.207	0.016	1.059	0.211	0.247	0.383	0.263	26.483

Problem 887	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	10	12	9	18	13	7	8	8
N.S.	1	0.83	1.00	0.75	1.50	1.08	0.58	0.67	0.67
time (sec)	N/A	0.204	0.053	4.722	0.215	0.246	0.595	0.268	26.591

Problem 888	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	23	17	27	26	33	29	24	19
N.S.	1	1.92	1.42	2.25	2.17	2.75	2.42	2.00	1.58
time (sec)	N/A	0.223	0.015	0.490	0.215	0.243	0.332	0.277	0.099

Problem 889	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	20	20	19	20	19
N.S.	1	1.00	1.00	0.95	1.05	1.05	1.00	1.05	1.00
time (sec)	N/A	0.225	0.017	249.260	0.194	0.245	2.522	0.278	27.223

Problem 890	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	53	52	50	49	65	51	51	69
N.S.	1	1.20	1.18	1.14	1.11	1.48	1.16	1.16	1.57
time (sec)	N/A	0.210	0.025	3.522	0.192	0.257	0.054	0.367	26.253

Problem 891	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	35	57	33	32	34	33	30
N.S.	1	1.00	0.95	1.54	0.89	0.86	0.92	0.89	0.81
time (sec)	N/A	0.197	0.095	5.074	0.192	0.234	0.043	0.558	26.445

Problem 892	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	63	68	58	57	73	61	59	85
N.S.	1	1.17	1.26	1.07	1.06	1.35	1.13	1.09	1.57
time (sec)	N/A	0.214	0.023	35.374	0.190	0.246	0.062	0.409	31.821

Problem 893	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	33	37	30	31	34	42	33	33
N.S.	1	0.89	1.00	0.81	0.84	0.92	1.14	0.89	0.89
time (sec)	N/A	0.248	0.098	1.030	0.187	0.243	123.549	0.270	26.125

Problem 894	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	54	42	46	46	49	49	42	69
N.S.	1	1.59	1.24	1.35	1.35	1.44	1.44	1.24	2.03
time (sec)	N/A	0.191	0.008	3.036	0.190	0.253	0.070	0.263	26.814

Problem 895	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	47	43	68	41	52	42	41	42
N.S.	1	1.09	1.00	1.58	0.95	1.21	0.98	0.95	0.98
time (sec)	N/A	0.204	0.018	4.932	0.187	0.255	0.046	0.294	26.543

Problem 896	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	85	87	72	73	92	0	73	74
N.S.	1	0.98	1.00	0.83	0.84	1.06	0.00	0.84	0.85
time (sec)	N/A	0.399	0.056	1.155	0.194	0.277	0.000	0.416	25.897

Problem 897	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	48	42	37	44	79	0	52	71
N.S.	1	1.14	1.00	0.88	1.05	1.88	0.00	1.24	1.69
time (sec)	N/A	0.346	0.040	14.661	0.187	0.264	0.000	0.271	26.990

Problem 898	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	49	57	84	0	57	57
N.S.	1	1.00	1.00	0.78	0.90	1.33	0.00	0.90	0.90
time (sec)	N/A	0.386	0.030	0.950	0.196	0.261	0.000	0.282	26.374

Problem 899	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	58	60	49	52	103	0	60	84
N.S.	1	0.97	1.00	0.82	0.87	1.72	0.00	1.00	1.40
time (sec)	N/A	0.361	0.028	45.951	0.202	0.262	0.000	0.335	28.169

Problem 900	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	59	38	41	42	44	41	42
N.S.	1	1.00	1.26	0.81	0.87	0.89	0.94	0.87	0.89
time (sec)	N/A	0.343	0.122	1.595	0.196	0.241	3.632	69.700	29.497

Problem 901	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	37	38	37	41	44	39	37
N.S.	1	1.00	0.86	0.88	0.86	0.95	1.02	0.91	0.86
time (sec)	N/A	0.352	0.022	0.781	0.186	0.251	0.423	0.280	0.105

Problem 902	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	46	42	69	44	79	41	52	71
N.S.	1	1.10	1.00	1.64	1.05	1.88	0.98	1.24	1.69
time (sec)	N/A	0.222	0.004	0.292	0.201	0.270	0.051	0.276	27.178

Problem 903	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	41	63	55	91	82	0	72	41
N.S.	1	0.55	0.85	0.74	1.23	1.11	0.00	0.97	0.55
time (sec)	N/A	0.576	0.084	0.422	0.275	0.249	0.000	0.324	27.273

Problem 904	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	47	32	176	39	17	949	34
N.S.	1	1.00	3.36	2.29	12.57	2.79	1.21	67.79	2.43
time (sec)	N/A	0.192	0.042	0.294	0.279	0.243	0.851	0.449	26.920

Problem 905	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	12	10	12	12	10
N.S.	1	1.00	1.00	0.93	0.86	0.71	0.86	0.86	0.71
time (sec)	N/A	0.202	0.013	0.707	0.196	0.258	0.509	0.277	0.219

Problem 906	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	15	11	10	0	24	0	9	18
N.S.	1	1.36	1.00	0.91	0.00	2.18	0.00	0.82	1.64
time (sec)	N/A	0.231	0.017	0.792	0.000	0.293	0.000	0.288	27.114

Problem 907	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	37	32	33	19	32	32	34	33
N.S.	1	1.09	0.94	0.97	0.56	0.94	0.94	1.00	0.97
time (sec)	N/A	0.356	0.024	0.307	0.231	0.229	0.488	0.271	26.508

Problem 908	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	25	18	17	17	24	17	18
N.S.	1	1.00	1.19	0.86	0.81	0.81	1.14	0.81	0.86
time (sec)	N/A	0.194	0.035	0.365	0.197	0.259	0.249	0.288	0.089

Problem 909	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	103	55	60	51	66	189	58	69
N.S.	1	1.04	0.56	0.61	0.52	0.67	1.91	0.59	0.70
time (sec)	N/A	0.313	0.147	1.624	0.207	0.259	0.189	0.279	26.537

Problem 910	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	40	41	59	51	0	70	64
N.S.	1	1.00	1.08	1.11	1.59	1.38	0.00	1.89	1.73
time (sec)	N/A	0.267	0.229	0.857	0.280	0.258	0.000	0.313	26.130

Problem 911	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	49	42	41	73	90	41	45
N.S.	1	1.00	0.86	0.74	0.72	1.28	1.58	0.72	0.79
time (sec)	N/A	0.242	0.080	0.897	0.190	0.244	0.609	0.284	26.355

Problem 912	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	C	B	F	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	243	68	96	0	85	0	0	51
N.S.	1	4.26	1.19	1.68	0.00	1.49	0.00	0.00	0.89
time (sec)	N/A	0.458	0.045	24.511	0.000	0.263	0.000	0.000	28.437

Problem 913	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	8	7	17	7	10	10
N.S.	1	1.00	1.00	1.60	1.40	3.40	1.40	2.00	2.00
time (sec)	N/A	0.197	0.002	1.298	0.199	0.236	1.143	0.294	26.478

Problem 914	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	A	A	F	F(-1)	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	0	11	15	11	11	0	0	11
N.S.	1	0.00	1.00	1.36	1.00	1.00	0.00	0.00	1.00
time (sec)	N/A	0.000	0.344	1.819	0.260	0.259	0.000	0.000	27.236

Problem 915	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	B	A	F	B	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	0	24	22	74	27	0	93	38
N.S.	1	0.00	0.89	0.81	2.74	1.00	0.00	3.44	1.41
time (sec)	N/A	0.000	0.168	3.518	0.282	0.243	0.000	0.285	0.630

Problem 916	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	75	50	48	66	50	100	51	65
N.S.	1	1.15	0.77	0.74	1.02	0.77	1.54	0.78	1.00
time (sec)	N/A	0.472	0.055	2.255	0.205	0.236	0.182	0.270	26.115

Problem 917	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	17	10	14	0	0	10
N.S.	1	1.00	1.00	2.12	1.25	1.75	0.00	0.00	1.25
time (sec)	N/A	0.167	0.023	0.243	0.197	0.249	0.000	0.000	0.192

Problem 918	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	B	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	32	32	32	0	125	41	0	0	45
N.S.	1	1.00	1.00	0.00	3.91	1.28	0.00	0.00	1.41
time (sec)	N/A	0.204	0.063	0.000	0.297	0.261	0.000	0.000	26.673

Problem 919	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	45	3	0	3	26
N.S.	1	1.00	1.00	1.33	15.00	1.00	0.00	1.00	8.67
time (sec)	N/A	0.217	0.014	0.651	0.217	0.263	0.000	0.265	27.038

Problem 920	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	32	31	35	27	39	27	34
N.S.	1	1.00	0.91	0.89	1.00	0.77	1.11	0.77	0.97
time (sec)	N/A	0.240	0.075	0.255	0.186	0.249	0.094	0.265	0.081

Problem 921	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	17	6	5	6	0
N.S.	1	1.00	1.00	0.88	2.12	0.75	0.62	0.75	0.00
time (sec)	N/A	0.142	0.004	0.286	0.227	0.249	0.343	0.268	0.000

Problem 922	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	29	29	30	0	29	0	29	0
N.S.	1	0.78	0.78	0.81	0.00	0.78	0.00	0.78	0.00
time (sec)	N/A	0.339	0.054	0.451	0.000	0.248	0.000	0.280	0.000

Problem 923	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	10	10	10	10	41
N.S.	1	1.00	1.00	0.79	0.71	0.71	0.71	0.71	2.93
time (sec)	N/A	0.154	0.005	0.226	0.183	0.239	5.247	0.307	27.747

Problem 924	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	8	20	7	8	19
N.S.	1	1.00	1.00	0.90	0.80	2.00	0.70	0.80	1.90
time (sec)	N/A	0.185	0.003	0.268	0.192	0.264	0.503	0.268	26.966

Problem 925	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	27	15	56
N.S.	1	1.00	1.00	0.94	0.88	0.88	1.59	0.88	3.29
time (sec)	N/A	0.171	0.015	0.551	0.191	0.246	1.227	0.271	26.987

Problem 926	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	149	120	127	126	85	241	155	147
N.S.	1	1.16	0.93	0.98	0.98	0.66	1.87	1.20	1.14
time (sec)	N/A	0.533	0.386	1.987	0.221	0.271	3.411	0.281	27.283

Problem 927	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	128	352	160	3830	115	0	1363	730
N.S.	1	1.16	3.20	1.45	34.82	1.05	0.00	12.39	6.64
time (sec)	N/A	0.566	0.795	52.376	0.458	0.279	0.000	0.845	43.077

Problem 928	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	36	12	0	20	14
N.S.	1	1.00	1.00	1.17	6.00	2.00	0.00	3.33	2.33
time (sec)	N/A	0.198	0.021	0.937	0.196	0.233	0.000	0.262	26.919

Problem 929	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	4	4	3	4	4
N.S.	1	1.00	1.00	1.25	1.00	1.00	0.75	1.00	1.00
time (sec)	N/A	0.168	0.001	0.144	0.202	0.246	0.095	0.265	27.682

Problem 930	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	30	24	98	39	22	908	23
N.S.	1	1.00	1.11	0.89	3.63	1.44	0.81	33.63	0.85
time (sec)	N/A	0.231	0.018	0.392	0.277	0.254	0.676	0.404	0.128

Problem 931	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	A	F	F(-2)	F	F	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	0	26	28	0	0	0	0	22
N.S.	1	0.00	1.00	1.08	0.00	0.00	0.00	0.00	0.85
time (sec)	N/A	0.000	0.362	1.366	0.000	0.000	0.000	0.000	28.340

Problem 932	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	26	28	24	26	0	26	26	26
N.S.	1	1.00	1.08	0.92	1.00	0.00	1.00	1.00	1.00
time (sec)	N/A	0.281	5.237	0.031	1.409	0.000	1.883	0.393	27.610

Problem 933	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	C	B	A	A	B	B
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	0	9	24	82	10	10	83	10
N.S.	1	0.00	1.00	2.67	9.11	1.11	1.11	9.22	1.11
time (sec)	N/A	0.000	0.356	1.288	0.333	0.249	0.146	0.268	26.757

Problem 934	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	79	76	60	57	60	150	62	111
N.S.	1	1.03	0.99	0.78	0.74	0.78	1.95	0.81	1.44
time (sec)	N/A	0.473	0.109	1.987	0.211	0.244	0.168	0.272	29.709

Problem 935	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	134	125	131	123	326	143	210
N.S.	1	1.00	0.83	0.78	0.81	0.76	2.02	0.89	1.30
time (sec)	N/A	0.758	0.308	2.758	0.199	0.257	0.496	0.305	30.876

Problem 936	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	86	105	70	79	72	201	70	73
N.S.	1	0.97	1.18	0.79	0.89	0.81	2.26	0.79	0.82
time (sec)	N/A	0.494	0.144	1.893	0.192	0.250	0.175	0.260	27.213

Problem 937	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	167	182	218	162	541	186	456
N.S.	1	1.00	0.58	0.63	0.76	0.56	1.88	0.65	1.58
time (sec)	N/A	0.687	1.658	3.716	0.221	0.252	0.546	0.311	28.969

Problem 938	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	55	132	0	108	0	0	51
N.S.	1	1.00	0.90	2.16	0.00	1.77	0.00	0.00	0.84
time (sec)	N/A	0.470	0.606	1.421	0.000	0.096	0.000	0.000	27.982

Problem 939	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	C	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	137	256	0	210	0	0	129
N.S.	1	1.00	0.93	1.73	0.00	1.42	0.00	0.00	0.87
time (sec)	N/A	0.492	1.129	2.597	0.000	0.107	0.000	0.000	31.509

Problem 940	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	62	43	32	50	30	73	31	35
N.S.	1	1.82	1.26	0.94	1.47	0.88	2.15	0.91	1.03
time (sec)	N/A	0.316	0.075	0.736	0.208	0.257	0.484	0.345	27.673

Problem 941	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	66	43	34	62	30	71	31	35
N.S.	1	1.83	1.19	0.94	1.72	0.83	1.97	0.86	0.97
time (sec)	N/A	0.322	0.121	0.763	0.212	0.256	0.509	0.367	27.052

Problem 942	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	119	73	111	0	150	379	128	226
N.S.	1	0.99	0.61	0.92	0.00	1.25	3.16	1.07	1.88
time (sec)	N/A	0.766	8.946	1.424	0.000	0.277	84.523	0.481	28.342

Problem 943	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	70	25	72	58	74	122	48	23
N.S.	1	0.97	0.35	1.00	0.81	1.03	1.69	0.67	0.32
time (sec)	N/A	0.334	0.029	1.046	0.319	0.277	1.716	0.350	25.950

Problem 944	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	56	42	51	154	42	76	52	105
N.S.	1	1.02	0.76	0.93	2.80	0.76	1.38	0.95	1.91
time (sec)	N/A	0.514	10.011	1.220	0.310	0.254	0.423	0.354	27.042

Problem 945	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	33	15	22	16	32	14	14
N.S.	1	1.00	2.06	0.94	1.38	1.00	2.00	0.88	0.88
time (sec)	N/A	0.244	0.011	0.625	0.222	0.246	0.118	0.296	26.985

Problem 946	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	22	31	29	50
N.S.	1	1.00	1.00	1.06	1.00	1.22	1.72	1.61	2.78
time (sec)	N/A	0.194	0.186	0.623	0.187	0.249	0.208	0.287	27.438

Problem 947	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	31	19	30	70	22	0	29	50
N.S.	1	1.63	1.00	1.58	3.68	1.16	0.00	1.53	2.63
time (sec)	N/A	0.445	0.414	1.007	0.289	0.243	0.000	0.317	27.567

Problem 948	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	23	33	15	23	17	0	14	14
N.S.	1	1.35	1.94	0.88	1.35	1.00	0.00	0.82	0.82
time (sec)	N/A	0.323	0.010	0.737	0.204	0.236	0.000	0.313	26.226

Problem 949	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	56	42	51	154	42	0	52	106
N.S.	1	1.04	0.78	0.94	2.85	0.78	0.00	0.96	1.96
time (sec)	N/A	0.648	11.061	1.291	0.287	0.266	0.000	0.381	26.948

Problem 950	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	70	26	72	58	74	0	48	23
N.S.	1	0.97	0.36	1.00	0.81	1.03	0.00	0.67	0.32
time (sec)	N/A	1.133	0.022	1.020	0.283	0.261	0.000	0.389	26.874

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [345] had the largest ratio of [1.8571400000000001]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	3	1.11	14	0.214
2	A	6	6	1.11	27	0.222
3	A	4	3	0.46	12	0.250
4	A	4	3	0.46	14	0.214
5	A	4	3	0.46	21	0.143
6	A	4	3	0.46	23	0.130
7	A	4	3	0.46	21	0.143
8	A	5	4	0.37	14	0.286
9	A	8	7	0.37	25	0.280
10	A	4	3	0.37	14	0.214
11	A	4	3	0.37	14	0.214
12	A	4	3	0.37	23	0.130
13	A	4	3	0.37	23	0.130
14	A	4	3	0.37	23	0.130
15	A	3	3	1.12	12	0.250
16	A	5	5	1.12	25	0.200
17	A	4	3	0.46	14	0.214
18	A	4	3	0.46	12	0.250
19	A	4	3	0.46	21	0.143
20	A	4	3	0.46	21	0.143
21	A	4	3	0.46	23	0.130
22	A	5	4	0.36	14	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	7	6	0.36	25	0.240
24	A	4	3	0.36	14	0.214
25	A	4	3	0.36	14	0.214
26	A	4	3	0.36	21	0.143
27	A	4	3	0.36	21	0.143
28	A	4	3	0.36	23	0.130
29	A	2	2	1.00	6	0.333
30	A	2	2	1.00	6	0.333
31	A	2	2	1.00	16	0.125
32	A	2	2	1.00	19	0.105
33	A	5	4	1.00	16	0.250
34	A	4	4	1.00	27	0.148
35	A	4	3	1.00	11	0.273
36	A	10	9	1.08	14	0.643
37	A	11	10	1.07	16	0.625
38	A	5	4	0.93	16	0.250
39	A	5	4	0.98	36	0.111
40	A	5	4	0.98	36	0.111
41	A	4	3	1.00	34	0.088
42	N/A	4	0	1.00	34	0.000
43	N/A	4	0	1.00	36	0.000
44	A	2	2	1.00	6	0.333
45	A	2	2	1.00	6	0.333
46	A	2	2	1.00	16	0.125
47	A	2	2	1.00	19	0.105
48	B	6	5	2.70	21	0.238
49	A	6	5	1.73	27	0.185
50	A	9	8	1.50	37	0.216
51	A	11	10	1.07	14	0.714
52	A	11	10	1.07	16	0.625
53	A	5	4	0.98	36	0.111
54	A	5	4	0.98	36	0.111
55	A	4	3	1.00	34	0.088

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
56	N/A	4	0	1.00	34	0.000
57	N/A	4	0	1.00	36	0.000
58	A	4	3	1.00	12	0.250
59	A	5	4	1.00	14	0.286
60	A	8	7	1.29	12	0.583
61	A	1	1	1.00	33	0.030
62	A	5	4	1.12	14	0.286
63	A	4	4	1.00	25	0.160
64	A	5	4	0.94	16	0.250
65	A	5	4	0.94	15	0.267
66	A	2	2	1.00	7	0.286
67	A	2	2	1.00	7	0.286
68	A	2	2	1.00	7	0.286
69	A	2	2	1.00	7	0.286
70	A	2	2	1.00	7	0.286
71	A	2	2	1.00	7	0.286
72	A	2	2	1.00	7	0.286
73	A	2	2	1.00	7	0.286
74	A	6	5	1.35	7	0.714
75	A	8	7	1.21	7	1.000
76	A	8	7	1.13	7	1.000
77	A	5	4	1.00	7	0.571
78	A	6	5	1.04	7	0.714
79	A	2	2	1.00	7	0.286
80	A	6	5	1.40	7	0.714
81	A	5	4	1.00	7	0.571
82	A	6	5	1.07	7	0.714
83	A	5	4	1.00	7	0.571
84	A	6	5	1.11	7	0.714
85	A	4	3	1.00	7	0.429
86	A	8	7	1.29	7	1.000
87	A	5	4	1.00	7	0.571
88	A	6	5	1.29	7	0.714

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
89	A	5	4	1.00	7	0.571
90	A	4	4	1.00	7	0.571
91	A	4	3	0.33	7	0.429
92	A	5	4	1.00	7	0.571
93	A	5	4	0.53	7	0.571
94	A	6	5	1.06	7	0.714
95	A	5	4	2.00	7	0.571
96	A	4	4	1.00	9	0.444
97	A	2	2	1.00	7	0.286
98	A	2	2	1.00	7	0.286
99	A	2	2	1.00	7	0.286
100	A	2	2	1.00	7	0.286
101	A	2	2	1.00	7	0.286
102	A	2	2	1.00	7	0.286
103	A	2	2	1.00	7	0.286
104	A	2	2	1.00	7	0.286
105	A	6	5	1.35	7	0.714
106	A	5	4	1.00	7	0.571
107	A	8	7	1.13	7	1.000
108	A	5	4	1.00	7	0.571
109	A	6	5	1.04	7	0.714
110	A	6	5	1.40	7	0.714
111	A	8	7	1.22	7	1.000
112	A	6	5	1.07	7	0.714
113	A	5	4	1.00	7	0.571
114	A	6	5	1.11	7	0.714
115	A	2	2	1.00	7	0.286
116	A	4	3	1.00	7	0.429
117	A	4	3	0.34	7	0.429
118	A	5	4	1.00	7	0.571
119	A	5	4	0.55	7	0.571
120	A	5	4	1.00	7	0.571
121	A	5	4	1.00	7	0.571
122	A	5	4	1.29	9	0.444

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
123	A	4	4	1.00	7	0.571
124	A	7	6	1.29	7	0.857
125	A	5	4	1.00	7	0.571
126	A	6	5	1.29	7	0.714
127	A	6	5	1.03	7	0.714
128	A	3	3	1.00	9	0.333
129	A	3	3	1.00	11	0.273
130	A	3	3	1.00	11	0.273
131	A	3	3	1.00	9	0.333
132	A	3	3	1.00	9	0.333
133	A	3	3	1.00	11	0.273
134	A	3	3	1.00	13	0.231
135	A	2	2	1.00	13	0.154
136	A	2	2	1.00	14	0.143
137	A	2	2	1.00	13	0.154
138	A	2	2	1.00	14	0.143
139	A	4	4	1.21	13	0.308
140	A	4	4	1.21	14	0.286
141	A	5	5	1.31	13	0.385
142	A	5	5	1.29	14	0.357
143	A	3	3	1.00	13	0.231
144	A	3	3	1.00	14	0.214
145	A	4	4	1.11	13	0.308
146	A	4	4	1.12	14	0.286
147	A	4	4	1.00	9	0.444
148	A	6	6	1.77	9	0.667
149	A	8	8	1.52	9	0.889
150	A	4	4	1.00	9	0.444
151	A	6	6	1.77	9	0.667
152	A	8	8	1.52	9	0.889
153	A	6	5	1.10	12	0.417
154	A	10	9	1.12	12	0.750
155	A	5	4	1.00	12	0.333
156	A	9	8	1.17	12	0.667

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
157	A	5	5	1.06	14	0.357
158	A	6	6	1.08	14	0.429
159	A	4	3	1.00	23	0.130
160	A	9	8	1.14	24	0.333
161	A	10	9	1.04	26	0.346
162	A	4	3	1.00	33	0.091
163	A	9	8	1.08	34	0.235
164	A	10	9	1.02	36	0.250
165	A	11	11	0.63	33	0.333
166	A	8	8	0.66	33	0.242
167	A	6	6	0.78	31	0.194
168	A	6	6	0.60	33	0.182
169	A	9	9	0.54	33	0.273
170	A	11	11	0.48	33	0.333
171	A	5	5	0.52	33	0.152
172	A	5	5	0.57	33	0.152
173	A	4	4	0.68	31	0.129
174	A	5	5	0.42	33	0.152
175	A	5	5	0.39	33	0.152
176	A	5	5	0.39	33	0.152
177	A	5	5	0.49	37	0.135
178	A	5	5	0.48	37	0.135
179	A	5	5	0.50	35	0.143
180	N/A	5	0	1.00	37	0.000
181	A	5	5	0.42	33	0.152
182	A	5	5	0.44	33	0.152
183	A	5	5	0.54	31	0.161
184	A	3	3	0.47	22	0.136
185	A	10	10	1.00	13	0.769
186	A	12	12	1.00	15	0.800
187	A	15	15	1.08	15	1.000
188	A	17	17	1.11	15	1.133
189	A	7	7	1.00	15	0.467

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
190	A	9	9	1.00	15	0.600
191	A	11	11	1.13	15	0.733
192	A	13	13	1.21	15	0.867
193	A	14	13	1.08	17	0.765
194	A	11	10	1.04	17	0.588
195	A	8	7	1.00	17	0.412
196	A	10	9	1.00	17	0.529
197	A	13	12	1.05	17	0.706
198	A	16	15	1.09	17	0.882
199	A	5	4	1.00	16	0.250
200	A	5	4	1.00	16	0.250
201	A	8	7	1.00	17	0.412
202	A	6	5	0.53	17	0.294
203	A	8	7	1.00	17	0.412
204	A	6	5	0.54	17	0.294
205	A	5	5	1.00	22	0.227
206	A	3	3	1.00	17	0.176
207	A	6	5	0.54	19	0.263
208	A	6	5	1.00	20	0.250
209	A	6	5	1.04	19	0.263
210	A	5	5	1.15	22	0.227
211	A	3	3	1.00	17	0.176
212	A	5	4	0.59	19	0.211
213	A	5	5	0.86	20	0.250
214	A	6	5	1.04	19	0.263
215	A	12	11	1.07	17	0.647
216	A	13	12	1.18	17	0.706
217	A	4	4	1.00	19	0.211
218	A	4	4	1.00	19	0.211
219	A	5	4	0.94	19	0.211
220	A	7	7	1.04	19	0.368
221	A	5	4	0.95	19	0.211
222	A	5	5	1.03	19	0.263
223	A	4	3	0.97	19	0.158

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
224	A	3	3	1.00	19	0.158
225	A	1	1	1.00	17	0.059
226	A	4	3	1.00	19	0.158
227	A	2	2	1.00	19	0.105
228	A	6	5	1.00	19	0.263
229	A	4	4	1.00	19	0.211
230	A	8	7	1.09	19	0.368
231	A	6	6	1.09	19	0.316
232	A	8	8	1.02	21	0.381
233	A	6	6	1.00	21	0.286
234	A	6	6	1.00	21	0.286
235	A	4	4	1.00	21	0.190
236	A	4	4	1.00	21	0.190
237	A	6	6	1.00	21	0.286
238	A	6	6	1.00	21	0.286
239	A	8	8	1.05	21	0.381
240	A	8	8	1.04	21	0.381
241	A	6	6	1.00	21	0.286
242	A	6	6	1.00	21	0.286
243	A	4	4	1.00	21	0.190
244	A	4	4	1.00	21	0.190
245	A	6	6	1.00	21	0.286
246	A	6	6	1.00	21	0.286
247	A	8	8	1.02	21	0.381
248	A	2	2	1.00	22	0.091
249	A	2	2	1.00	22	0.091
250	A	2	2	1.00	22	0.091
251	A	2	2	1.00	22	0.091
252	A	1	1	1.00	20	0.050
253	A	2	2	1.00	22	0.091
254	A	2	2	1.00	22	0.091
255	A	2	2	1.00	22	0.091
256	A	2	2	1.00	22	0.091
257	A	2	2	1.00	24	0.083

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
258	A	2	2	1.00	24	0.083
259	A	2	2	1.00	24	0.083
260	A	2	2	1.00	24	0.083
261	A	2	2	1.00	24	0.083
262	A	2	2	1.00	24	0.083
263	A	7	6	1.15	11	0.545
264	A	10	10	1.07	11	0.909
265	A	7	6	1.27	11	0.545
266	A	5	5	1.00	11	0.455
267	A	1	1	1.00	9	0.111
268	A	6	5	1.00	11	0.455
269	A	11	10	1.17	11	0.909
270	A	7	6	0.90	11	0.545
271	A	16	15	1.17	11	1.364
272	A	7	6	0.88	11	0.545
273	A	7	6	1.00	7	0.857
274	A	9	9	1.00	7	1.286
275	A	7	6	1.00	7	0.857
276	A	7	7	1.00	7	1.000
277	A	1	1	0.69	5	0.200
278	A	6	5	1.00	7	0.714
279	A	5	5	1.00	7	0.714
280	A	7	6	1.00	7	0.857
281	A	7	7	1.00	7	1.000
282	A	7	6	1.00	7	0.857
283	A	7	6	1.14	11	0.545
284	A	9	9	1.06	11	0.818
285	A	7	6	1.27	11	0.545
286	A	5	5	1.00	11	0.455
287	A	1	1	1.00	9	0.111
288	A	6	5	1.00	11	0.455
289	A	11	10	1.07	11	0.909
290	A	7	6	0.94	11	0.545
291	A	16	15	1.13	11	1.364

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
292	A	7	6	0.87	11	0.545
293	A	7	6	1.00	7	0.857
294	A	9	9	1.00	7	1.286
295	A	7	6	1.00	7	0.857
296	A	7	7	1.00	7	1.000
297	A	1	1	1.00	5	0.200
298	A	6	5	1.00	7	0.714
299	A	5	5	1.00	7	0.714
300	A	7	6	1.00	7	0.857
301	A	7	7	1.00	7	1.000
302	A	7	6	1.00	7	0.857
303	A	8	7	1.27	9	0.778
304	A	9	8	1.15	9	0.889
305	A	6	5	1.32	9	0.556
306	A	1	1	1.00	7	0.143
307	A	6	5	1.00	9	0.556
308	A	4	3	1.00	9	0.333
309	A	8	7	1.00	9	0.778
310	A	4	3	1.00	9	0.333
311	A	7	6	1.00	9	0.667
312	A	5	4	1.00	9	0.444
313	A	8	7	1.00	9	0.778
314	A	12	12	1.33	11	1.091
315	A	10	10	1.52	11	0.909
316	A	8	8	1.77	11	0.727
317	A	6	6	1.00	11	0.545
318	A	13	12	0.60	11	1.091
319	A	15	14	0.98	11	1.273
320	A	17	16	1.00	11	1.455
321	A	19	18	1.02	11	1.636
322	A	7	6	1.32	9	0.667
323	A	8	7	1.15	9	0.778
324	A	6	5	1.32	9	0.556
325	A	1	1	1.00	7	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
326	A	7	6	1.00	9	0.667
327	A	4	3	1.00	9	0.333
328	A	9	8	1.00	9	0.889
329	A	4	3	1.00	9	0.333
330	A	8	7	1.00	9	0.778
331	A	5	4	1.00	9	0.444
332	A	9	8	1.00	9	0.889
333	A	12	12	1.33	11	1.091
334	A	10	10	1.52	11	0.909
335	A	8	8	1.77	11	0.727
336	A	6	6	1.00	11	0.545
337	A	13	12	0.81	11	1.091
338	A	15	14	1.17	11	1.273
339	A	17	16	1.15	11	1.455
340	A	19	18	1.15	11	1.636
341	A	5	5	1.00	7	0.714
342	A	8	7	1.00	7	1.000
343	A	5	5	1.00	7	0.714
344	A	1	1	1.00	5	0.200
345	A	14	13	1.00	7	1.857
346	A	5	5	1.00	7	0.714
347	A	8	7	1.00	7	1.000
348	A	5	5	1.00	7	0.714
349	A	6	5	1.00	18	0.278
350	A	6	5	1.00	18	0.278
351	A	8	7	1.10	18	0.389
352	A	6	5	1.00	18	0.278
353	A	6	5	1.00	18	0.278
354	A	8	7	1.10	18	0.389
355	A	7	7	1.01	30	0.233
356	A	5	5	0.99	30	0.167
357	A	3	3	0.91	30	0.100
358	A	1	1	1.00	28	0.036
359	A	2	2	1.00	30	0.067

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
360	A	4	4	1.00	30	0.133
361	A	6	6	1.10	30	0.200
362	A	8	8	1.13	30	0.267
363	A	6	6	1.08	24	0.250
364	A	3	3	1.06	24	0.125
365	A	1	1	1.00	22	0.045
366	A	4	3	1.00	24	0.125
367	A	8	7	1.00	24	0.292
368	A	9	8	1.04	24	0.333
369	A	12	11	1.05	24	0.458
370	A	4	3	1.00	24	0.125
371	A	7	6	1.00	24	0.250
372	A	9	8	1.09	24	0.333
373	A	12	11	1.12	24	0.458
374	A	6	6	1.09	24	0.250
375	A	3	3	1.06	24	0.125
376	A	1	1	1.00	22	0.045
377	A	4	3	1.00	24	0.125
378	A	8	7	1.00	24	0.292
379	A	9	8	1.03	24	0.333
380	A	12	11	1.05	24	0.458
381	A	6	6	1.08	24	0.250
382	A	3	3	1.06	24	0.125
383	A	1	1	1.00	22	0.045
384	A	4	3	1.00	24	0.125
385	A	8	7	1.00	24	0.292
386	A	9	8	1.03	24	0.333
387	A	12	11	1.05	24	0.458
388	A	6	6	1.08	24	0.250
389	A	3	3	1.06	24	0.125
390	A	1	1	1.00	22	0.045
391	A	4	3	1.00	24	0.125
392	A	8	7	1.00	24	0.292
393	A	9	8	1.04	24	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
394	A	12	11	1.05	24	0.458
395	A	7	7	1.09	20	0.350
396	A	5	5	1.06	20	0.250
397	A	3	3	0.98	20	0.150
398	A	1	1	1.00	18	0.056
399	A	5	4	1.10	20	0.200
400	A	9	8	1.05	20	0.400
401	A	10	9	1.13	20	0.450
402	A	13	12	1.16	20	0.600
403	A	13	13	1.04	22	0.591
404	A	11	11	1.02	22	0.500
405	A	4	4	1.00	22	0.182
406	A	4	4	1.00	22	0.182
407	A	6	6	1.00	22	0.273
408	A	15	15	1.03	22	0.682
409	A	18	18	1.05	22	0.818
410	A	15	15	1.03	22	0.682
411	A	12	12	1.00	22	0.545
412	A	4	4	1.00	22	0.182
413	A	4	4	1.00	22	0.182
414	A	6	6	1.00	22	0.273
415	A	15	15	1.04	22	0.682
416	A	18	18	1.07	22	0.818
417	A	6	6	1.02	22	0.273
418	A	4	4	1.00	22	0.182
419	A	2	2	1.00	22	0.091
420	A	6	5	1.00	22	0.227
421	A	8	7	1.00	22	0.318
422	A	10	9	1.04	22	0.409
423	A	8	8	1.04	22	0.364
424	A	6	6	1.02	22	0.273
425	A	4	4	1.00	22	0.182
426	A	2	2	1.00	22	0.091
427	A	6	5	1.00	22	0.227

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
428	A	8	7	1.00	22	0.318
429	A	10	9	1.04	22	0.409
430	A	8	8	1.05	32	0.250
431	A	6	6	1.05	32	0.188
432	A	4	4	1.00	32	0.125
433	A	2	2	1.00	32	0.062
434	A	6	5	1.00	32	0.156
435	A	8	7	1.00	32	0.219
436	A	10	9	1.08	32	0.281
437	A	6	6	1.05	34	0.176
438	A	4	4	1.00	34	0.118
439	A	2	2	1.00	34	0.059
440	A	6	5	1.00	34	0.147
441	A	8	7	1.00	34	0.206
442	A	10	9	1.08	34	0.265
443	A	7	6	1.06	15	0.400
444	A	6	5	1.36	11	0.455
445	A	9	8	1.05	12	0.667
446	A	7	6	1.10	15	0.400
447	A	15	14	1.04	17	0.824
448	A	14	14	0.91	33	0.424
449	A	6	6	1.00	33	0.182
450	A	6	6	1.00	33	0.182
451	A	8	8	0.99	33	0.242
452	A	17	17	0.90	33	0.515
453	A	14	14	0.91	33	0.424
454	A	6	6	1.00	33	0.182
455	A	6	6	1.00	33	0.182
456	A	8	8	0.99	33	0.242
457	A	17	17	0.90	33	0.515
458	A	9	8	1.05	12	0.667
459	A	7	6	1.10	15	0.400
460	A	8	7	1.00	17	0.412
461	A	7	6	0.81	15	0.400

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
462	A	14	14	0.91	33	0.424
463	A	6	6	1.00	33	0.182
464	A	6	6	1.00	33	0.182
465	A	8	8	0.99	33	0.242
466	A	17	17	0.90	33	0.515
467	A	14	14	0.91	33	0.424
468	A	6	6	1.00	33	0.182
469	A	6	6	1.00	33	0.182
470	A	8	8	0.99	33	0.242
471	A	17	17	0.90	33	0.515
472	A	3	3	1.00	11	0.273
473	A	3	3	1.00	11	0.273
474	A	3	3	1.00	11	0.273
475	A	4	3	0.27	13	0.231
476	A	4	3	1.00	13	0.231
477	A	6	5	0.94	13	0.385
478	A	4	3	1.00	15	0.200
479	A	4	3	1.00	15	0.200
480	A	4	3	1.00	19	0.158
481	A	4	3	0.47	23	0.130
482	A	6	5	1.05	20	0.250
483	A	6	5	1.05	20	0.250
484	A	5	4	0.58	11	0.364
485	A	7	6	0.67	11	0.545
486	A	9	8	0.86	11	0.727
487	A	3	3	1.00	13	0.231
488	A	3	3	1.00	13	0.231
489	A	3	3	1.00	13	0.231
490	A	5	4	0.57	11	0.364
491	A	7	6	0.68	11	0.545
492	A	8	7	0.86	11	0.636
493	A	3	3	1.00	13	0.231
494	A	3	3	1.00	13	0.231
495	A	3	3	1.00	13	0.231

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
496	A	4	3	1.00	16	0.188
497	A	9	8	1.09	18	0.444
498	A	10	9	1.01	20	0.450
499	A	12	12	1.20	39	0.308
500	A	4	4	1.04	37	0.108
501	A	6	6	1.39	39	0.154
502	A	16	15	1.37	39	0.385
503	A	7	6	1.03	21	0.286
504	A	9	9	0.65	41	0.220
505	A	5	5	0.54	41	0.122
506	A	10	9	0.83	41	0.220
507	A	14	13	0.79	41	0.317
508	A	6	6	1.82	27	0.222
509	A	6	5	1.00	21	0.238
510	A	14	14	1.04	39	0.359
511	A	6	6	1.00	37	0.162
512	A	9	9	1.24	39	0.231
513	A	14	14	1.32	39	0.359
514	A	12	12	0.52	41	0.293
515	A	7	7	0.67	41	0.171
516	A	7	7	0.74	41	0.171
517	A	12	12	0.74	41	0.293
518	A	10	10	1.12	39	0.256
519	A	3	3	1.05	37	0.081
520	A	13	12	1.47	39	0.308
521	A	18	17	1.31	39	0.436
522	A	8	8	0.53	41	0.195
523	A	10	9	0.50	41	0.220
524	A	11	10	0.81	41	0.244
525	A	15	14	0.81	41	0.341
526	A	2	2	1.00	21	0.095
527	A	2	2	1.00	21	0.095
528	A	2	2	1.00	15	0.133
529	A	2	2	1.00	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
530	A	6	5	1.00	21	0.238
531	A	5	5	1.00	21	0.238
532	A	6	5	1.00	22	0.227
533	A	6	5	1.00	22	0.227
534	A	8	7	1.09	22	0.318
535	A	7	6	0.98	19	0.316
536	A	7	6	1.05	19	0.316
537	A	10	9	1.12	19	0.474
538	A	2	2	1.00	22	0.091
539	A	2	2	1.00	22	0.091
540	A	7	6	0.98	19	0.316
541	A	7	6	1.05	19	0.316
542	A	10	9	1.13	19	0.474
543	A	2	2	1.00	22	0.091
544	A	2	2	1.00	22	0.091
545	A	7	6	1.05	22	0.273
546	A	7	6	1.05	22	0.273
547	A	9	8	1.12	22	0.364
548	A	2	2	0.95	25	0.080
549	A	2	2	0.94	25	0.080
550	A	7	6	0.98	23	0.261
551	A	7	6	1.05	23	0.261
552	A	10	9	1.11	23	0.391
553	A	2	2	1.00	26	0.077
554	A	2	2	1.00	26	0.077
555	B	2	2	2.83	30	0.067
556	A	18	18	1.09	27	0.667
557	A	15	15	1.07	27	0.556
558	A	12	12	1.03	27	0.444
559	A	9	9	1.00	27	0.333
560	A	12	12	1.06	27	0.444
561	A	15	15	1.04	27	0.556
562	A	11	10	1.07	31	0.323
563	A	13	12	1.05	31	0.387

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
564	A	16	15	1.11	31	0.484
565	A	18	17	1.14	31	0.548
566	A	7	6	1.00	18	0.333
567	A	7	7	1.05	18	0.389
568	A	4	4	1.00	18	0.222
569	A	1	1	1.00	16	0.062
570	A	7	6	1.00	18	0.333
571	A	10	9	1.05	18	0.500
572	A	14	13	1.13	18	0.722
573	A	17	17	0.98	20	0.850
574	A	14	14	0.98	20	0.700
575	A	6	6	1.00	20	0.300
576	A	6	6	1.00	20	0.300
577	A	9	9	1.00	20	0.450
578	A	18	18	1.01	20	0.900
579	A	12	11	0.92	14	0.786
580	A	11	10	0.94	14	0.714
581	A	10	9	1.02	12	0.750
582	N/A	3	0	1.00	14	0.000
583	N/A	3	0	1.00	34	0.000
584	N/A	3	0	1.00	33	0.000
585	A	12	12	1.39	26	0.462
586	A	15	15	1.38	26	0.577
587	A	9	9	1.26	26	0.346
588	A	8	8	1.23	26	0.308
589	A	1	1	1.00	23	0.043
590	A	1	1	1.00	22	0.045
591	A	5	4	1.00	20	0.200
592	A	8	7	1.14	24	0.292
593	A	15	14	1.24	26	0.538
594	A	12	12	1.39	24	0.500
595	A	15	15	1.36	24	0.625
596	A	9	9	1.26	24	0.375

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
597	A	8	8	1.20	24	0.333
598	A	1	1	1.00	21	0.048
599	A	1	1	1.00	20	0.050
600	A	5	4	1.00	18	0.222
601	A	8	7	1.15	22	0.318
602	A	14	13	1.26	24	0.542
603	A	11	11	1.10	31	0.355
604	A	9	9	1.10	31	0.290
605	A	6	6	1.00	31	0.194
606	A	4	4	1.00	29	0.138
607	A	6	5	1.00	20	0.250
608	A	8	7	1.00	29	0.241
609	A	10	9	1.04	31	0.290
610	A	12	11	1.06	31	0.355
611	A	15	15	1.10	31	0.484
612	A	11	11	1.10	31	0.355
613	A	8	8	1.05	31	0.258
614	A	6	6	1.00	29	0.207
615	A	9	8	1.00	20	0.400
616	A	11	10	1.44	29	0.345
617	A	12	11	1.03	31	0.355
618	A	14	13	1.04	31	0.419
619	A	13	12	1.08	31	0.387
620	A	11	10	1.05	31	0.323
621	A	8	7	1.00	31	0.226
622	A	6	5	1.00	29	0.172
623	A	10	9	1.00	20	0.450
624	A	12	11	1.00	29	0.379
625	A	16	15	1.08	31	0.484
626	A	14	13	1.05	31	0.419
627	A	11	10	1.04	31	0.323
628	A	8	7	1.00	31	0.226
629	A	8	7	1.00	29	0.241
630	A	13	12	1.04	20	0.600

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
631	A	15	14	1.02	29	0.483
632	A	19	18	1.06	31	0.581
633	A	5	4	1.00	13	0.308
634	A	8	7	0.80	21	0.333
635	A	8	7	0.72	28	0.250
636	A	8	7	0.68	30	0.233
637	A	2	2	1.00	43	0.047
638	A	2	2	1.00	43	0.047
639	A	2	2	1.00	43	0.047
640	A	2	2	1.00	41	0.049
641	A	2	2	1.00	43	0.047
642	A	2	2	1.00	43	0.047
643	A	2	2	1.00	43	0.047
644	N/A	3	0	1.00	18	0.000
645	N/A	3	0	1.00	18	0.000
646	N/A	3	0	1.00	20	0.000
647	N/A	3	0	1.00	20	0.000
648	A	4	3	1.00	11	0.273
649	A	4	3	1.00	11	0.273
650	A	5	4	1.00	13	0.308
651	A	4	3	1.00	6	0.500
652	A	6	5	1.18	11	0.455
653	A	5	4	1.00	13	0.308
654	A	6	5	1.11	17	0.294
655	A	4	3	1.00	10	0.300
656	A	4	3	0.94	21	0.143
657	A	5	4	1.00	19	0.211
658	A	5	4	1.00	15	0.267
659	A	3	2	1.00	17	0.118
660	A	3	2	1.00	22	0.091
661	A	3	2	1.00	22	0.091
662	A	3	2	1.00	17	0.118
663	A	3	2	1.00	22	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
664	A	3	2	1.00	22	0.091
665	A	4	3	1.00	11	0.273
666	A	4	3	1.00	11	0.273
667	A	4	3	1.00	13	0.231
668	A	4	3	1.00	15	0.200
669	A	4	3	1.00	19	0.158
670	A	6	5	1.00	11	0.455
671	A	5	4	1.00	15	0.267
672	A	4	3	1.00	15	0.200
673	A	5	4	1.00	16	0.250
674	A	4	3	1.00	6	0.500
675	A	5	4	1.00	10	0.400
676	A	4	3	1.00	6	0.500
677	A	6	5	1.11	17	0.294
678	A	4	3	1.00	9	0.333
679	A	7	6	1.36	13	0.462
680	A	3	2	1.00	17	0.118
681	A	3	2	1.00	9	0.222
682	A	3	2	1.00	12	0.167
683	A	3	2	1.00	18	0.111
684	A	3	2	1.00	17	0.118
685	A	3	2	1.00	22	0.091
686	A	3	2	1.00	22	0.091
687	A	3	2	1.00	17	0.118
688	A	3	2	1.00	22	0.091
689	A	3	2	1.00	22	0.091
690	A	4	3	1.00	13	0.231
691	A	4	3	0.27	15	0.200
692	A	4	3	0.41	13	0.231
693	A	4	3	1.00	13	0.231
694	A	4	3	1.50	15	0.200
695	A	3	3	1.00	19	0.158
696	A	5	4	0.15	17	0.235
697	A	5	4	1.50	16	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
698	A	6	5	1.50	18	0.278
699	A	11	10	0.65	15	0.667
700	A	5	4	0.38	19	0.211
701	A	5	4	1.00	19	0.211
702	A	5	4	1.00	21	0.190
703	A	5	4	1.00	21	0.190
704	A	4	3	1.00	17	0.176
705	A	7	6	1.00	17	0.353
706	A	7	6	0.57	19	0.316
707	A	3	3	1.00	11	0.273
708	A	5	4	0.81	17	0.235
709	A	4	3	1.00	17	0.176
710	A	4	3	1.00	17	0.176
711	A	5	4	1.00	15	0.267
712	A	5	4	1.00	17	0.235
713	A	5	4	1.00	17	0.235
714	A	3	2	1.00	9	0.222
715	A	7	6	1.00	15	0.400
716	A	4	3	1.00	13	0.231
717	A	4	3	1.00	13	0.231
718	A	3	3	1.00	11	0.273
719	A	5	4	1.33	15	0.267
720	A	5	4	1.00	19	0.211
721	A	5	4	1.00	21	0.190
722	A	5	4	1.00	21	0.190
723	A	3	2	1.00	11	0.182
724	A	6	5	1.82	13	0.385
725	A	4	3	1.00	13	0.231
726	A	4	3	1.00	15	0.200
727	A	4	3	1.00	14	0.214
728	A	5	4	1.40	15	0.267
729	A	4	3	1.00	15	0.200
730	A	3	2	1.00	9	0.222
731	A	3	2	1.00	9	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
732	A	4	3	1.00	19	0.158
733	A	5	4	0.98	23	0.174
734	A	4	3	1.00	23	0.130
735	A	6	5	1.67	13	0.385
736	A	3	2	1.00	15	0.133
737	A	4	3	1.00	13	0.231
738	A	5	4	1.09	21	0.190
739	A	4	3	1.00	15	0.200
740	A	5	4	1.09	23	0.174
741	A	5	4	0.95	20	0.200
742	A	5	4	0.95	19	0.211
743	A	5	4	0.97	24	0.167
744	A	5	4	0.97	21	0.190
745	A	5	4	0.95	20	0.200
746	A	5	4	0.95	19	0.211
747	A	5	4	0.97	24	0.167
748	A	5	4	0.97	21	0.190
749	A	1	1	1.00	8	0.125
750	A	1	1	1.00	8	0.125
751	C	4	3	1.67	11	0.273
752	A	5	5	1.00	6	0.833
753	A	7	6	1.26	8	0.750
754	A	4	3	1.00	9	0.333
755	A	4	3	1.00	12	0.250
756	A	4	3	1.00	13	0.231
757	A	5	5	1.00	8	0.625
758	A	1	1	1.00	12	0.083
759	B	1	1	10.75	20	0.050
760	A	5	4	1.00	6	0.667
761	A	5	4	1.00	8	0.500
762	A	6	5	1.00	15	0.333
763	A	4	3	1.00	15	0.200
764	A	2	2	1.00	17	0.118
765	A	7	6	1.00	13	0.462

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
766	A	4	3	1.00	6	0.500
767	A	4	3	1.00	10	0.300
768	A	4	3	1.00	8	0.375
769	A	4	3	1.00	10	0.300
770	A	5	4	1.00	10	0.400
771	A	5	4	1.00	10	0.400
772	A	4	3	1.00	8	0.375
773	A	3	3	1.00	8	0.375
774	A	5	4	1.00	18	0.222
775	A	2	2	1.00	16	0.125
776	A	14	13	1.11	8	1.625
777	A	3	2	1.11	17	0.118
778	A	5	4	1.00	7	0.571
779	A	5	4	1.00	11	0.364
780	A	1	1	1.00	10	0.100
781	A	4	3	1.00	8	0.375
782	A	4	3	1.00	8	0.375
783	A	1	1	1.00	10	0.100
784	A	4	3	1.00	10	0.300
785	A	5	4	1.00	9	0.444
786	A	4	4	1.00	8	0.500
787	A	7	6	0.85	8	0.750
788	A	1	1	1.00	8	0.125
789	A	6	5	0.90	8	0.625
790	A	7	6	0.85	8	0.750
791	A	4	3	1.00	8	0.375
792	A	4	3	1.00	13	0.231
793	A	1	1	1.00	11	0.091
794	A	6	5	1.00	9	0.556
795	A	5	4	1.00	7	0.571
796	F	0	0	N/A	0.000	N/A
797	A	5	5	1.00	6	0.833
798	A	2	2	1.00	11	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
799	A	5	4	0.95	8	0.500
800	A	5	4	1.00	7	0.571
801	A	7	6	0.82	9	0.667
802	A	4	3	1.00	9	0.333
803	A	4	3	1.00	7	0.429
804	B	4	3	2.20	13	0.231
805	A	3	3	1.00	10	0.300
806	A	5	4	1.00	21	0.190
807	A	5	4	1.00	15	0.267
808	A	5	4	1.00	12	0.333
809	A	7	6	1.35	16	0.375
810	A	6	5	1.00	11	0.455
811	A	2	2	1.00	13	0.154
812	A	1	1	1.00	10	0.100
813	A	5	4	1.00	13	0.308
814	A	1	1	1.00	12	0.083
815	A	3	3	1.00	8	0.375
816	A	3	3	1.00	10	0.300
817	A	2	2	0.85	15	0.133
818	A	1	1	1.00	11	0.091
819	A	1	1	1.00	13	0.077
820	A	4	3	1.00	8	0.375
821	A	5	4	1.00	28	0.143
822	A	1	1	1.00	12	0.083
823	A	4	3	1.00	23	0.130
824	A	5	4	1.00	7	0.571
825	A	4	3	1.00	10	0.300
826	A	4	3	1.00	8	0.375
827	B	6	5	2.71	13	0.385
828	A	1	1	1.00	20	0.050
829	A	6	6	1.62	9	0.667
830	A	7	6	1.94	10	0.600
831	A	7	6	1.56	9	0.667
832	A	6	6	1.72	10	0.600

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
833	A	8	8	1.41	10	0.800
834	A	6	5	1.46	10	0.500
835	A	10	10	1.37	10	1.000
836	B	8	7	2.33	14	0.500
837	A	1	1	1.00	13	0.077
838	A	1	1	1.00	9	0.111
839	A	1	1	1.00	11	0.091
840	A	3	2	1.00	7	0.286
841	A	1	1	1.00	9	0.111
842	A	2	2	1.00	10	0.200
843	A	6	5	1.00	18	0.278
844	A	1	1	1.00	18	0.056
845	A	5	4	1.00	18	0.222
846	A	11	10	1.65	15	0.667
847	A	11	10	1.51	15	0.667
848	A	5	4	1.27	15	0.267
849	A	5	4	1.04	15	0.267
850	A	5	4	1.00	21	0.190
851	A	6	5	1.00	21	0.238
852	A	1	1	1.00	14	0.071
853	A	6	5	1.00	15	0.333
854	A	7	6	1.17	14	0.429
855	A	5	4	1.48	16	0.250
856	A	5	4	1.48	16	0.250
857	A	6	5	1.00	15	0.333
858	B	7	6	5.26	18	0.333
859	F	0	0	N/A	0.000	N/A
860	B	3	3	3.79	16	0.188
861	A	8	8	1.00	12	0.667
862	A	4	3	1.00	15	0.200
863	A	9	8	1.00	15	0.533
864	A	9	8	1.00	15	0.533
865	A	2	2	1.00	18	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
866	A	13	12	1.25	23	0.522
867	A	12	11	0.54	23	0.478
868	A	6	5	0.70	16	0.312
869	A	7	6	0.68	18	0.333
870	A	8	7	0.68	18	0.389
871	A	9	8	0.78	16	0.500
872	A	10	9	0.79	18	0.500
873	A	11	10	0.78	18	0.556
874	A	3	3	0.59	16	0.188
875	A	5	5	0.68	18	0.278
876	A	5	5	0.66	18	0.278
877	A	3	3	0.56	16	0.188
878	A	5	5	0.52	18	0.278
879	A	5	5	0.57	18	0.278
880	A	3	3	1.00	11	0.273
881	A	3	3	1.00	11	0.273
882	A	3	3	1.00	11	0.273
883	A	3	3	1.00	11	0.273
884	A	4	3	1.00	6	0.500
885	A	2	2	1.00	15	0.133
886	A	6	5	1.64	9	0.556
887	A	6	5	0.83	15	0.333
888	A	6	5	1.92	16	0.312
889	A	5	4	1.00	15	0.267
890	A	6	5	1.20	13	0.385
891	A	5	4	1.00	13	0.308
892	A	6	5	1.17	13	0.385
893	A	7	6	0.89	15	0.400
894	A	7	6	1.59	7	0.857
895	A	5	4	1.09	13	0.308
896	A	10	9	0.98	27	0.333
897	A	8	7	1.14	27	0.259
898	A	9	8	1.00	27	0.296
899	A	10	9	0.97	27	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
900	A	9	8	1.00	21	0.381
901	A	7	6	1.00	23	0.261
902	A	7	6	1.10	13	0.462
903	A	14	13	0.55	15	0.867
904	A	3	3	1.00	10	0.300
905	A	5	4	1.00	17	0.235
906	A	7	6	1.36	17	0.353
907	A	12	11	1.09	8	1.375
908	A	5	4	1.00	13	0.308
909	A	6	6	1.04	16	0.375
910	A	3	3	1.00	13	0.231
911	A	2	2	1.00	12	0.167
912	B	3	3	4.26	18	0.167
913	A	6	5	1.00	9	0.556
914	F	0	0	N/A	0.000	N/A
915	F	0	0	N/A	0.000	N/A
916	A	12	11	1.15	12	0.917
917	A	4	3	1.00	9	0.333
918	A	2	2	1.00	21	0.095
919	A	6	5	1.00	10	0.500
920	A	2	2	1.00	13	0.154
921	A	1	1	1.00	8	0.125
922	A	4	3	0.78	12	0.250
923	A	1	1	1.00	18	0.056
924	A	1	1	1.00	14	0.071
925	A	1	1	1.00	22	0.045
926	A	12	11	1.16	22	0.500
927	A	11	10	1.16	22	0.455
928	A	5	4	1.00	10	0.400
929	A	5	4	1.00	9	0.444
930	A	5	5	1.00	14	0.357
931	F	0	0	N/A	0.000	N/A
932	N/A	1	0	1.00	26	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
933	F	0	0	N/A	0.000	N/A
934	A	12	11	1.03	28	0.393
935	A	15	14	1.00	30	0.467
936	A	12	11	0.97	36	0.306
937	A	5	5	1.00	38	0.132
938	A	5	5	1.00	29	0.172
939	A	5	5	1.00	31	0.161
940	A	4	4	1.82	27	0.148
941	A	4	4	1.83	27	0.148
942	A	5	4	0.99	39	0.103
943	A	7	6	0.97	39	0.154
944	A	5	4	1.02	39	0.103
945	A	3	3	1.00	39	0.077
946	A	2	2	1.00	31	0.065
947	A	6	5	1.63	31	0.161
948	A	5	4	1.35	39	0.103
949	A	6	5	1.04	39	0.128
950	A	8	7	0.97	39	0.179

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \frac{2}{3-\cos(4+6x)}\,dx$

3.1.1 Optimal result

Integrand size = 14, antiderivative size = 44

$$\int \frac{2}{3-\cos(4+6x)}\,dx=\frac{x}{\sqrt{2}}+\frac{\arctan\left(\frac{\sin(4+6x)}{3+2\sqrt{2}-\cos(4+6x)}\right)}{3\sqrt{2}}$$

output
```
1/2*x*2^(1/2)+1/6*arctan(sin(4+6*x)/(3-cos(4+6*x)+2*2^(1/2)))*2^(1/2)
```

3.1.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.50

$$\int \frac{2}{3-\cos(4+6x)}\,dx=\frac{\arctan\left(\sqrt{2}\tan(2+3x)\right)}{3\sqrt{2}}$$

input
```
Integrate[2/(3 - Cos[4 + 6*x]),x]
```

output
```
ArcTan[Sqrt[2]*Tan[2 + 3*x]]/(3*Sqrt[2])
```

3.1.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {27, 3042, 3136}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2}{3-\cos(6x+4)}dx$$
$$\Big\downarrow \text{27}$$
$$2\int \frac{1}{3-\cos(6x+4)}dx$$
$$\Big\downarrow \text{3042}$$
$$2\int \frac{1}{3-\sin\left(6x+\frac{\pi}{2}+4\right)}dx$$
$$\Big\downarrow \text{3136}$$
$$2\left(\frac{\arctan\left(\frac{\sin(6x+4)}{-\cos(6x+4)+2\sqrt{2}+3}\right)}{6\sqrt{2}}+\frac{x}{2\sqrt{2}}\right)$$

input
```
Int[2/(3 - Cos[4 + 6*x]),x]
```

output
```
2*(x/(2*Sqrt[2]) + ArcTan[Sin[4 + 6*x]/(3 + 2*Sqrt[2] - Cos[4 + 6*x])]/(6*
Sqrt[2]))
```

3.1.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3136
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

3.1.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.39

method	result	size
derivativedivides	$\frac{\sqrt{2}\,\arctan\left(\tan(2+3x)\sqrt{2}\right)}{6}$	17
default	$\frac{\sqrt{2}\,\arctan\left(\tan(2+3x)\sqrt{2}\right)}{6}$	17
risch	$\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{2i(2+3x)}-3-2\sqrt{2}\right)}{12}-\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{2i(2+3x)}-3+2\sqrt{2}\right)}{12}$	48

input
```
int(2/(3-cos(4+6*x)),x,method=_RETURNVERBOSE)
```

output
```
1/6*2^(1/2)*arctan(tan(2+3*x)*2^(1/2))
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.75

$$\int \frac{2}{3-\cos(4+6x)}\,dx = -\frac{1}{12}\,\sqrt{2}\arctan\left(\frac{3\,\sqrt{2}\cos\left(6\,x+4\right)-\sqrt{2}}{4\,\sin\left(6\,x+4\right)}\right)$$

input
```
integrate(2/(3-cos(4+6*x)),x, algorithm="fricas")
```

output
```
-1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(6*x + 4) - sqrt(2))/sin(6*x + 4))
```

3.1.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.73

$$\int \frac{2}{3-\cos(4+6x)}\,dx = \frac{\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(3x+2\right)\right)+\pi\left\lfloor\frac{3x-\frac{\pi}{2}+2}{\pi}\right\rfloor\right)}{6}$$

input
```
integrate(2/(3-cos(4+6*x)),x)
```

output
```
sqrt(2)*(atan(sqrt(2)*tan(3*x + 2)) + pi*floor((3*x - pi/2 + 2)/pi))/6
```

3.1.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.59

$$\int \frac{2}{3-\cos(4+6x)}\,dx = \frac{1}{6}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sin\left(6\,x+4\right)}{\cos\left(6\,x+4\right)+1}\right)$$

input
```
integrate(2/(3-cos(4+6*x)),x, algorithm="maxima")
```

output
```
1/6*sqrt(2)*arctan(sqrt(2)*sin(6*x + 4)/(cos(6*x + 4) + 1))
```

3.1.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.30

$$\int \frac{2}{3-\cos(4+6x)}\,dx$$
$$= \frac{1}{6}\sqrt{2}\left(3\,x+\arctan\left(-\frac{\sqrt{2}\sin\left(6\,x+4\right)-2\,\sin\left(6\,x+4\right)}{\sqrt{2}\cos\left(6\,x+4\right)+\sqrt{2}-2\,\cos\left(6\,x+4\right)+2}\right)+2\right)$$

input
```
integrate(2/(3-cos(4+6*x)),x, algorithm="giac")
```

output
```
1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - 2*sin(6*x + 4))/(sqrt(2
)*cos(6*x + 4) + sqrt(2) - 2*cos(6*x + 4) + 2)) + 2)
```

3.1.9 Mupad [B] (verification not implemented)

Time = 27.40 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

$$\int \frac{2}{3-\cos(4+6x)}\,dx = \frac{\sqrt{2}\,(3\,x-\operatorname{atan}(\tan(3\,x+2)))}{6}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\tan(3\,x+2)\right)}{6}$$

input
```
int(-2/(cos(6*x + 4) - 3),x)
```

output
```
(2^(1/2)*(3*x - atan(tan(3*x + 2))))/6 + (2^(1/2)*atan(2^(1/2)*tan(3*x + 2
)))/6
```

3.2 $\int \frac{2\csc(4+6x)}{-\cot(4+6x)+3\csc(4+6x)}\,dx$

3.2.1 Optimal result

Integrand size = 27, antiderivative size = 44

$$\int \frac{2\csc(4+6x)}{-\cot(4+6x)+3\csc(4+6x)}\,dx = \frac{x}{\sqrt{2}} + \frac{\arctan\left(\frac{\sin(4+6x)}{3+2\sqrt{2}-\cos(4+6x)}\right)}{3\sqrt{2}}$$

output
```
1/2*x*2^(1/2)+1/6*arctan(sin(4+6*x)/(3-cos(4+6*x)+2*2^(1/2)))*2^(1/2)
```

3.2.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.50

$$\int \frac{2\csc(4+6x)}{-\cot(4+6x)+3\csc(4+6x)}\,dx = \frac{\arctan\left(\sqrt{2}\tan(2+3x)\right)}{3\sqrt{2}}$$

input
```
Integrate[(2*Csc[4 + 6*x])/(-Cot[4 + 6*x] + 3*Csc[4 + 6*x]),x]
```

output
```
ArcTan[Sqrt[2]*Tan[2 + 3*x]]/(3*Sqrt[2])
```

3.2.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {27, 25, 3042, 3645, 3042, 3137}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2\csc(6x+4)}{3\csc(6x+4)-\cot(6x+4)}dx$$
$$\Big\downarrow \text{27}$$
$$2\int -\frac{\csc(6x+4)}{\cot(6x+4)-3\csc(6x+4)}dx$$
$$\Big\downarrow \text{25}$$
$$-2\int \frac{\csc(6x+4)}{\cot(6x+4)-3\csc(6x+4)}dx$$
$$\Big\downarrow \text{3042}$$
$$-2\int \frac{\csc(6x+4)}{\cot(6x+4)-3\csc(6x+4)}dx$$
$$\Big\downarrow \text{3645}$$
$$-2\int \frac{1}{\cos(6x+4)-3}dx$$
$$\Big\downarrow \text{3042}$$
$$-2\int \frac{1}{\sin\left(6x+\frac{\pi}{2}+4\right)-3}dx$$
$$\Big\downarrow \text{3137}$$
$$-2\left(-\frac{\arctan\left(\frac{\sin(6x+4)}{-\cos(6x+4)+2\sqrt{2}+3}\right)}{6\sqrt{2}}-\frac{x}{2\sqrt{2}}\right)$$

input
```
Int[(2*Csc[4 + 6*x])/(-Cot[4 + 6*x] + 3*Csc[4 + 6*x]),x]
```

output
```
-2*(-1/2*x/Sqrt[2] - ArcTan[Sin[4 + 6*x]/(3 + 2*Sqrt[2] - Cos[4 + 6*x])]/(
6*Sqrt[2]))
```

3.2.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3137
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[-x/q, x] - Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a -
q + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &
& NegQ[a]
```

rule 3645
```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Int[1/(b + a*Sin[d + e*x]
 + c*Cos[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && I
ntegerQ[n]
```

3.2.4 Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.39

method	result	size
derivativedivides	$\frac{\sqrt{2}\,\arctan\left(\tan(2+3x)\sqrt{2}\right)}{6}$	17
default	$\frac{\sqrt{2}\,\arctan\left(\tan(2+3x)\sqrt{2}\right)}{6}$	17
risch	$\frac{i\sqrt{2}\,\ln\left(e^{2i(2+3x)}-3-2\sqrt{2}\right)}{12}-\frac{i\sqrt{2}\,\ln\left(e^{2i(2+3x)}-3+2\sqrt{2}\right)}{12}$	48

input
```
int(2*csc(4+6*x)/(-cot(4+6*x)+3*csc(4+6*x)),x,method=_RETURNVERBOSE)
```

output
```
1/6*2^(1/2)*arctan(tan(2+3*x)*2^(1/2))
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.75

$$\int \frac{2\csc(4+6x)}{-\cot(4+6x)+3\csc(4+6x)}\,dx = -\frac{1}{12}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(6x+4)-\sqrt{2}}{4\sin(6x+4)}\right)$$

input
```
integrate(2*csc(4+6*x)/(-cot(4+6*x)+3*csc(4+6*x)),x, algorithm="fricas")
```

output
```
-1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(6*x + 4) - sqrt(2))/sin(6*x + 4))
```

3.2.6 Sympy [F]

$$\int \frac{2\csc(4+6x)}{-\cot(4+6x)+3\csc(4+6x)}\,dx = -2\int \frac{\csc(6x+4)}{\cot(6x+4)-3\csc(6x+4)}\,dx$$

input
```
integrate(2*csc(4+6*x)/(-cot(4+6*x)+3*csc(4+6*x)),x)
```

output
```
-2*Integral(csc(6*x + 4)/(cot(6*x + 4) - 3*csc(6*x + 4)), x)
```

3.2.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.59

$$\int \frac{2\csc(4+6x)}{-\cot(4+6x)+3\csc(4+6x)}\,dx = \frac{1}{6}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sin(6x+4)}{\cos(6x+4)+1}\right)$$

input
```
integrate(2*csc(4+6*x)/(-cot(4+6*x)+3*csc(4+6*x)),x, algorithm="maxima")
```

output
```
1/6*sqrt(2)*arctan(sqrt(2)*sin(6*x + 4)/(cos(6*x + 4) + 1))
```

3.2.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.30

$$\int \frac{2\csc(4+6x)}{-\cot(4+6x)+3\csc(4+6x)}\,dx = \frac{1}{6}\sqrt{2}\left(3\,x+\arctan\left(-\frac{\sqrt{2}\sin\left(6\,x+4\right)-2\,\sin\left(6\,x+4\right)}{\sqrt{2}\cos\left(6\,x+4\right)+\sqrt{2}-2\,\cos\left(6\,x+4\right)+2}\right)+2\right)$$

input
```
integrate(2*csc(4+6*x)/(-cot(4+6*x)+3*csc(4+6*x)),x, algorithm="giac")
```

output
```
1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - 2*sin(6*x + 4))/(sqrt(2
)*cos(6*x + 4) + sqrt(2) - 2*cos(6*x + 4) + 2)) + 2)
```

3.2.9 Mupad [B] (verification not implemented)

Time = 27.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.36

$$\int \frac{2\csc(4+6x)}{-\cot(4+6x)+3\csc(4+6x)}\,dx = \frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\tan(3\,x+2)\right)}{6}$$

input
```
int(-2/(sin(6*x + 4)*(cot(6*x + 4) - 3/sin(6*x + 4))),x)
```

output
```
(2^(1/2)*atan(2^(1/2)*tan(3*x + 2)))/6
```

3.3 $\int \frac{1}{1+\sin^2(2+3x)}\,dx$

3.3.1 Optimal result

Integrand size = 12, antiderivative size = 48

$$\int \frac{1}{1+\sin^2(2+3x)}\,dx=\frac{x}{\sqrt{2}}+\frac{\arctan\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\sin^2(2+3x)}\right)}{3\sqrt{2}}$$

output
```
1/2*x*2^(1/2)+1/6*arctan(cos(2+3*x)*sin(2+3*x)/(1+sin(2+3*x)^2+2^(1/2)))*2
^(1/2)
```

3.3.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.46

$$\int \frac{1}{1+\sin^2(2+3x)}\,dx=\frac{\arctan\left(\sqrt{2}\tan(2+3x)\right)}{3\sqrt{2}}$$

input
```
Integrate[(1 + Sin[2 + 3*x]^2)^(-1),x]
```

output
```
ArcTan[Sqrt[2]*Tan[2 + 3*x]]/(3*Sqrt[2])
```

3.3.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.46, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3660, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sin^2(3x+2)+1} \, dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\sin(3x+2)^2+1}dx$$

$$\Big\downarrow \ 3660$$

$$\frac{1}{3}\int \frac{1}{2\tan^2(3x+2)+1}d\tan(3x+2)$$

$$\Big\downarrow \ 216$$

$$\frac{\arctan\left(\sqrt{2}\tan(3x+2)\right)}{3\sqrt{2}}$$

input
```
Int[(1 + Sin[2 + 3*x]^2)^(-1),x]
```

output
```
ArcTan[Sqrt[2]*Tan[2 + 3*x]]/(3*Sqrt[2])
```

3.3.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

3.3.4 Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.35

method	result	size
derivativedivides	$\frac{\sqrt{2}\,\arctan\left(\tan(2+3x)\sqrt{2}\right)}{6}$	17
default	$\frac{\sqrt{2}\,\arctan\left(\tan(2+3x)\sqrt{2}\right)}{6}$	17
risch	$\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{2i(2+3x)}-3-2\sqrt{2}\right)}{12}-\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{2i(2+3x)}-3+2\sqrt{2}\right)}{12}$	48

input
```
int(1/(1+sin(2+3*x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/6*2^(1/2)*arctan(tan(2+3*x)*2^(1/2))
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

$$\int \frac{1}{1+\sin^2(2+3x)}\,dx=-\frac{1}{12}\,\sqrt{2}\arctan\left(\frac{3\,\sqrt{2}\cos\left(3\,x+2\right)^2-2\,\sqrt{2}}{4\,\cos\left(3\,x+2\right)\sin\left(3\,x+2\right)}\right)$$

input
```
integrate(1/(1+sin(2+3*x)^2),x, algorithm="fricas")
```

output
```
-1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(3*x + 2)^2 - 2*sqrt(2))/(cos(3*x +
 2)*sin(3*x + 2)))
```

3.3.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(44) = 88.

Time = 2.89 (sec) , antiderivative size = 246, normalized size of antiderivative = 5.12

$$\int \frac{1}{1+\sin^2(2+3x)}\,dx = \frac{47321\sqrt{2}\sqrt{3-2\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{3x}{2}+1\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{83160\sqrt{2}+117606} + \frac{66922\sqrt{3-2\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{3x}{2}+1\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{83160\sqrt{2}+117606} + \frac{8119\sqrt{2}\sqrt{2\sqrt{2}+3}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{3x}{2}+1\right)}{\sqrt{2\sqrt{2}+3}}\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{83160\sqrt{2}+117606} + \frac{11482\sqrt{2\sqrt{2}+3}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{3x}{2}+1\right)}{\sqrt{2\sqrt{2}+3}}\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{83160\sqrt{2}+117606}$$

input
```
integrate(1/(1+sin(2+3*x)**2),x)
```

output
```
47321*sqrt(2)*sqrt(3 - 2*sqrt(2))*(atan(tan(3*x/2 + 1)/sqrt(3 - 2*sqrt(2))
) + pi*floor((3*x/2 - pi/2 + 1)/pi))/(83160*sqrt(2) + 117606) + 66922*sqrt
(3 - 2*sqrt(2))*(atan(tan(3*x/2 + 1)/sqrt(3 - 2*sqrt(2))) + pi*floor((3*x/
2 - pi/2 + 1)/pi))/(83160*sqrt(2) + 117606) + 8119*sqrt(2)*sqrt(2*sqrt(2)
+ 3)*(atan(tan(3*x/2 + 1)/sqrt(2*sqrt(2) + 3)) + pi*floor((3*x/2 - pi/2 +
1)/pi))/(83160*sqrt(2) + 117606) + 11482*sqrt(2*sqrt(2) + 3)*(atan(tan(3*x
/2 + 1)/sqrt(2*sqrt(2) + 3)) + pi*floor((3*x/2 - pi/2 + 1)/pi))/(83160*sqr
t(2) + 117606)
```

3.3.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.33

$$\int \frac{1}{1+\sin^2(2+3x)}\,dx = \frac{1}{6}\sqrt{2}\arctan\left(\sqrt{2}\tan\left(3\,x+2\right)\right)$$

input
```
integrate(1/(1+sin(2+3*x)^2),x, algorithm="maxima")
```

output
```
1/6*sqrt(2)*arctan(sqrt(2)*tan(3*x + 2))
```

3.3.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int \frac{1}{1+\sin^2(2+3x)}\,dx$$
$$=\frac{1}{6}\sqrt{2}\left(3\,x+\arctan\left(-\frac{\sqrt{2}\sin\left(6\,x+4\right)-2\,\sin\left(6\,x+4\right)}{\sqrt{2}\cos\left(6\,x+4\right)+\sqrt{2}-2\,\cos\left(6\,x+4\right)+2}\right)+2\right)$$

input
```
integrate(1/(1+sin(2+3*x)^2),x, algorithm="giac")
```

output
```
1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - 2*sin(6*x + 4))/(sqrt(2
)*cos(6*x + 4) + sqrt(2) - 2*cos(6*x + 4) + 2)) + 2)
```

3.3.9 Mupad [B] (verification not implemented)

Time = 27.30 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.73

$$\int \frac{1}{1+\sin^2(2+3x)}\,dx=\frac{\sqrt{2}\,\left(3\,x-\mathrm{atan}(\tan(3\,x+2))\right)}{6}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\tan(3\,x+2)\right)}{6}$$

input
```
int(1/(sin(3*x + 2)^2 + 1),x)
```

output
```
(2^(1/2)*(3*x - atan(tan(3*x + 2))))/6 + (2^(1/2)*atan(2^(1/2)*tan(3*x + 2
)))/6
```

3.4 $\int \frac{1}{2-\cos^2(2+3x)}\,dx$

3.4.1 Optimal result

Integrand size = 14, antiderivative size = 48

$$\int \frac{1}{2-\cos^2(2+3x)}\,dx=\frac{x}{\sqrt{2}}+\frac{\arctan\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\sin^2(2+3x)}\right)}{3\sqrt{2}}$$

output
```
1/2*x*2^(1/2)+1/6*arctan(cos(2+3*x)*sin(2+3*x)/(1+sin(2+3*x)^2+2^(1/2)))*2
^(1/2)
```

3.4.2 Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.46

$$\int \frac{1}{2-\cos^2(2+3x)}\,dx=\frac{\arctan\left(\sqrt{2}\tan(2+3x)\right)}{3\sqrt{2}}$$

input
```
Integrate[(2 - Cos[2 + 3*x]^2)^(-1),x]
```

output
```
ArcTan[Sqrt[2]*Tan[2 + 3*x]]/(3*Sqrt[2])
```

3.4.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.46, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 3660, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{2-\cos^2(3x+2)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{2-\sin\left(3x+\frac{\pi}{2}+2\right)^2}dx$$

$$\Big\downarrow \text{ 3660}$$

$$-\frac{1}{3}\int \frac{1}{\cot^2(3x+2)+2}d\cot(3x+2)$$

$$\Big\downarrow \text{ 216}$$

$$-\frac{\arctan\left(\frac{\cot(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

input
```
Int[(2 - Cos[2 + 3*x]^2)^(-1),x]
```

output
```
-1/3*ArcTan[Cot[2 + 3*x]/Sqrt[2]]/Sqrt[2]
```

3.4.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

3.4.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.35

method	result	size
derivativedivides	$\frac{\sqrt{2}\,\arctan\left(\tan(2+3x)\sqrt{2}\right)}{6}$	17
default	$\frac{\sqrt{2}\,\arctan\left(\tan(2+3x)\sqrt{2}\right)}{6}$	17
risch	$\frac{i\sqrt{2}\,\ln\left(e^{2i(2+3x)}-3-2\sqrt{2}\right)}{12}-\frac{i\sqrt{2}\,\ln\left(e^{2i(2+3x)}-3+2\sqrt{2}\right)}{12}$	48

input
```
int(1/(2-cos(2+3*x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/6*2^(1/2)*arctan(tan(2+3*x)*2^(1/2))
```

3.4.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

$$\int \frac{1}{2-\cos^2(2+3x)}\,dx = -\frac{1}{12}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(3x+2)^2-2\sqrt{2}}{4\cos(3x+2)\sin(3x+2)}\right)$$

input
```
integrate(1/(2-cos(2+3*x)^2),x, algorithm="fricas")
```

output
```
-1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(3*x + 2)^2 - 2*sqrt(2))/(cos(3*x +
 2)*sin(3*x + 2)))
```

3.4.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(44) = 88.

Time = 3.03 (sec) , antiderivative size = 246, normalized size of antiderivative = 5.12

$$\int \frac{1}{2-\cos^2(2+3x)}\,dx = \frac{47321\sqrt{2}\sqrt{3-2\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{3x}{2}+1\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{83160\sqrt{2}+117606} + \frac{66922\sqrt{3-2\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{3x}{2}+1\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{83160\sqrt{2}+117606} + \frac{8119\sqrt{2}\sqrt{2\sqrt{2}+3}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{3x}{2}+1\right)}{\sqrt{2\sqrt{2}+3}}\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{83160\sqrt{2}+117606} + \frac{11482\sqrt{2\sqrt{2}+3}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{3x}{2}+1\right)}{\sqrt{2\sqrt{2}+3}}\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{83160\sqrt{2}+117606}$$

input
```
integrate(1/(2-cos(2+3*x)**2),x)
```

output
```
47321*sqrt(2)*sqrt(3 - 2*sqrt(2))*(atan(tan(3*x/2 + 1)/sqrt(3 - 2*sqrt(2))
) + pi*floor((3*x/2 - pi/2 + 1)/pi))/(83160*sqrt(2) + 117606) + 66922*sqrt
(3 - 2*sqrt(2))*(atan(tan(3*x/2 + 1)/sqrt(3 - 2*sqrt(2))) + pi*floor((3*x/
2 - pi/2 + 1)/pi))/(83160*sqrt(2) + 117606) + 8119*sqrt(2)*sqrt(2*sqrt(2)
+ 3)*(atan(tan(3*x/2 + 1)/sqrt(2*sqrt(2) + 3)) + pi*floor((3*x/2 - pi/2 +
1)/pi))/(83160*sqrt(2) + 117606) + 11482*sqrt(2*sqrt(2) + 3)*(atan(tan(3*x
/2 + 1)/sqrt(2*sqrt(2) + 3)) + pi*floor((3*x/2 - pi/2 + 1)/pi))/(83160*sqr
t(2) + 117606)
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.33

$$\int \frac{1}{2-\cos^2(2+3x)}\,dx = \frac{1}{6}\sqrt{2}\arctan\left(\sqrt{2}\tan\left(3\,x+2\right)\right)$$

input
```
integrate(1/(2-cos(2+3*x)^2),x, algorithm="maxima")
```

output
```
1/6*sqrt(2)*arctan(sqrt(2)*tan(3*x + 2))
```

3.4.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int \frac{1}{2-\cos^2(2+3x)}\,dx = \frac{1}{6}\sqrt{2}\left(3x+\arctan\left(-\frac{\sqrt{2}\sin(6x+4)-2\sin(6x+4)}{\sqrt{2}\cos(6x+4)+\sqrt{2}-2\cos(6x+4)+2}\right)+2\right)$$

input
```
integrate(1/(2-cos(2+3*x)^2),x, algorithm="giac")
```

output
```
1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - 2*sin(6*x + 4))/(sqrt(2
)*cos(6*x + 4) + sqrt(2) - 2*cos(6*x + 4) + 2)) + 2)
```

3.4.9 Mupad [B] (verification not implemented)

Time = 27.50 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.73

$$\int \frac{1}{2-\cos^2(2+3x)}\,dx = \frac{\sqrt{2}\,(3x-\mathrm{atan}(\tan(3x+2)))}{6}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\tan(3x+2)\right)}{6}$$

input
```
int(-1/(cos(3*x + 2)^2 - 2),x)
```

output
```
(2^(1/2)*(3*x - atan(tan(3*x + 2))))/6 + (2^(1/2)*atan(2^(1/2)*tan(3*x + 2
)))/6
```

3.5 $\int \frac{1}{\cos^2(2+3x)+2\sin^2(2+3x)}\,dx$

3.5.1 Optimal result

Integrand size = 21, antiderivative size = 48

$$\int \frac{1}{\cos^2(2+3x)+2\sin^2(2+3x)}\,dx=\frac{x}{\sqrt{2}}+\frac{\arctan\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\sin^2(2+3x)}\right)}{3\sqrt{2}}$$

output
```
1/2*x*2^(1/2)+1/6*arctan(cos(2+3*x)*sin(2+3*x)/(1+sin(2+3*x)^2+2^(1/2)))*2
^(1/2)
```

3.5.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.46

$$\int \frac{1}{\cos^2(2+3x)+2\sin^2(2+3x)}\,dx=\frac{\arctan\left(\sqrt{2}\tan(2+3x)\right)}{3\sqrt{2}}$$

input
```
Integrate[(Cos[2 + 3*x]^2 + 2*Sin[2 + 3*x]^2)^(-1),x]
```

output
```
ArcTan[Sqrt[2]*Tan[2 + 3*x]]/(3*Sqrt[2])
```

3.5.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.46, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 4889, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{2\sin^2(3x+2)+\cos^2(3x+2)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{2\sin(3x+2)^2+\cos(3x+2)^2}dx$$
$$\Big\downarrow \text{4889}$$
$$\frac{1}{3}\int \frac{1}{2\tan^2(3x+2)+1}d\tan(3x+2)$$
$$\Big\downarrow \text{216}$$
$$\frac{\arctan\left(\sqrt{2}\tan(3x+2)\right)}{3\sqrt{2}}$$

input
```
Int[(Cos[2 + 3*x]^2 + 2*Sin[2 + 3*x]^2)^(-1),x]
```

output
```
ArcTan[Sqrt[2]*Tan[2 + 3*x]]/(3*Sqrt[2])
```

3.5.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.5.4 Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.35

method	result	size
derivativedivides	$\frac{\sqrt{2}\,\arctan\left(\tan(2+3x)\sqrt{2}\right)}{6}$	17
default	$\frac{\sqrt{2}\,\arctan\left(\tan(2+3x)\sqrt{2}\right)}{6}$	17
risch	$\frac{i\sqrt{2}\,\ln\left(e^{2i(2+3x)}-3-2\sqrt{2}\right)}{12}-\frac{i\sqrt{2}\,\ln\left(e^{2i(2+3x)}-3+2\sqrt{2}\right)}{12}$	48

input
```
int(1/(cos(2+3*x)^2+2*sin(2+3*x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/6*2^(1/2)*arctan(tan(2+3*x)*2^(1/2))
```

3.5.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

$$\int \frac{1}{\cos^2(2+3x)+2\sin^2(2+3x)}\,dx = -\frac{1}{12}\,\sqrt{2}\arctan\left(\frac{3\,\sqrt{2}\cos\left(3\,x+2\right)^2-2\,\sqrt{2}}{4\,\cos\left(3\,x+2\right)\sin\left(3\,x+2\right)}\right)$$

input
```
integrate(1/(cos(2+3*x)^2+2*sin(2+3*x)^2),x, algorithm="fricas")
```

output
```
-1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(3*x + 2)^2 - 2*sqrt(2))/(cos(3*x +
 2)*sin(3*x + 2)))
```

3.5.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(44) = 88$.

Time = 3.04 (sec) , antiderivative size = 246, normalized size of antiderivative = 5.12

$$\int \frac{1}{\cos^2(2+3x)+2\sin^2(2+3x)}\,dx = \frac{47321\sqrt{2}\sqrt{3-2\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{3x}{2}+1\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{83160\sqrt{2}+117606} + \frac{66922\sqrt{3-2\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{3x}{2}+1\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{83160\sqrt{2}+117606} + \frac{8119\sqrt{2}\sqrt{2\sqrt{2}+3}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{3x}{2}+1\right)}{\sqrt{2\sqrt{2}+3}}\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{83160\sqrt{2}+117606} + \frac{11482\sqrt{2\sqrt{2}+3}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{3x}{2}+1\right)}{\sqrt{2\sqrt{2}+3}}\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{83160\sqrt{2}+117606}$$

input
```
integrate(1/(cos(2+3*x)**2+2*sin(2+3*x)**2),x)
```

output
```
47321*sqrt(2)*sqrt(3 - 2*sqrt(2))*(atan(tan(3*x/2 + 1)/sqrt(3 - 2*sqrt(2))
) + pi*floor((3*x/2 - pi/2 + 1)/pi))/(83160*sqrt(2) + 117606) + 66922*sqrt
(3 - 2*sqrt(2))*(atan(tan(3*x/2 + 1)/sqrt(3 - 2*sqrt(2))) + pi*floor((3*x/
2 - pi/2 + 1)/pi))/(83160*sqrt(2) + 117606) + 8119*sqrt(2)*sqrt(2*sqrt(2)
+ 3)*(atan(tan(3*x/2 + 1)/sqrt(2*sqrt(2) + 3)) + pi*floor((3*x/2 - pi/2 +
1)/pi))/(83160*sqrt(2) + 117606) + 11482*sqrt(2*sqrt(2) + 3)*(atan(tan(3*x
/2 + 1)/sqrt(2*sqrt(2) + 3)) + pi*floor((3*x/2 - pi/2 + 1)/pi))/(83160*sqr
t(2) + 117606)
```

3.5.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.33

$$\int \frac{1}{\cos^2(2+3x)+2\sin^2(2+3x)}\,dx = \frac{1}{6}\sqrt{2}\arctan\left(\sqrt{2}\tan\left(3x+2\right)\right)$$

input
```
integrate(1/(cos(2+3*x)^2+2*sin(2+3*x)^2),x, algorithm="maxima")
```

output
```
1/6*sqrt(2)*arctan(sqrt(2)*tan(3*x + 2))
```

3.5.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int \frac{1}{\cos^2(2+3x)+2\sin^2(2+3x)}\,dx = \frac{1}{6}\sqrt{2}\left(3\,x+\arctan\left(-\frac{\sqrt{2}\sin\left(6\,x+4\right)-2\,\sin\left(6\,x+4\right)}{\sqrt{2}\cos\left(6\,x+4\right)+\sqrt{2}-2\,\cos\left(6\,x+4\right)+2}\right)+2\right)$$

input
```
integrate(1/(cos(2+3*x)^2+2*sin(2+3*x)^2),x, algorithm="giac")
```

output
```
1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - 2*sin(6*x + 4))/(sqrt(2
)*cos(6*x + 4) + sqrt(2) - 2*cos(6*x + 4) + 2)) + 2)
```

3.5.9 Mupad [B] (verification not implemented)

Time = 27.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.73

$$\int \frac{1}{\cos^2(2+3x)+2\sin^2(2+3x)}\,dx = \frac{\sqrt{2}\,(3\,x-\operatorname{atan}(\tan(3\,x+2)))}{6} + \frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}\tan(3\,x+2)\right)}{6}$$

input
```
int(1/(2*sin(3*x + 2)^2 + cos(3*x + 2)^2),x)
```

output
```
(2^(1/2)*(3*x - atan(tan(3*x + 2))))/6 + (2^(1/2)*atan(2^(1/2)*tan(3*x + 2
)))/6
```

3.6 $\int \frac{\sec^2(2+3x)}{1+2\tan^2(2+3x)}\,dx$

3.6.1 Optimal result

Integrand size = 23, antiderivative size = 48

$$\int \frac{\sec^2(2+3x)}{1+2\tan^2(2+3x)}\,dx=\frac{x}{\sqrt{2}}+\frac{\arctan\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\sin^2(2+3x)}\right)}{3\sqrt{2}}$$

output
```
1/2*x*2^(1/2)+1/6*arctan(cos(2+3*x)*sin(2+3*x)/(1+sin(2+3*x)^2+2^(1/2)))*2
^(1/2)
```

3.6.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.46

$$\int \frac{\sec^2(2+3x)}{1+2\tan^2(2+3x)}\,dx=\frac{\arctan\left(\sqrt{2}\tan(2+3x)\right)}{3\sqrt{2}}$$

input
```
Integrate[Sec[2 + 3*x]^2/(1 + 2*Tan[2 + 3*x]^2),x]
```

output
```
ArcTan[Sqrt[2]*Tan[2 + 3*x]]/(3*Sqrt[2])
```

3.6.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.46, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3042, 4158, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(3x+2)}{2\tan^2(3x+2)+1}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sec(3x+2)^2}{2\tan(3x+2)^2+1}dx$$

$$\Big\downarrow \text{ 4158}$$

$$\frac{1}{3}\int \frac{1}{2\tan^2(3x+2)+1}d\tan(3x+2)$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\arctan\left(\sqrt{2}\tan(3x+2)\right)}{3\sqrt{2}}$$

input
```
Int[Sec[2 + 3*x]^2/(1 + 2*Tan[2 + 3*x]^2),x]
```

output
```
ArcTan[Sqrt[2]*Tan[2 + 3*x]]/(3*Sqrt[2])
```

3.6.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

3.6.4 Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.35

method	result	size
derivativedivides	$\frac{\sqrt{2}\,\arctan\left(\tan(2+3x)\sqrt{2}\right)}{6}$	17
default	$\frac{\sqrt{2}\,\arctan\left(\tan(2+3x)\sqrt{2}\right)}{6}$	17
risch	$\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{2i(2+3x)}-3-2\sqrt{2}\right)}{12}-\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{2i(2+3x)}-3+2\sqrt{2}\right)}{12}$	48

input
```
int(sec(2+3*x)^2/(1+2*tan(2+3*x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/6*2^(1/2)*arctan(tan(2+3*x)*2^(1/2))
```

3.6.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

$$\int \frac{\sec^2(2+3x)}{1+2\tan^2(2+3x)}\,dx = -\frac{1}{12}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos\left(3x+2\right)^2-2\sqrt{2}}{4\cos\left(3x+2\right)\sin\left(3x+2\right)}\right)$$

input
```
integrate(sec(2+3*x)^2/(1+2*tan(2+3*x)^2),x, algorithm="fricas")
```

output
```
-1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(3*x + 2)^2 - 2*sqrt(2))/(cos(3*x +
 2)*sin(3*x + 2)))
```

3.6.6 Sympy [F]

$$\int \frac{\sec^2(2+3x)}{1+2\tan^2(2+3x)}\,dx = \int \frac{\sec^2(3x+2)}{2\tan^2(3x+2)+1}\,dx$$

input
```
integrate(sec(2+3*x)**2/(1+2*tan(2+3*x)**2),x)
```

output
```
Integral(sec(3*x + 2)**2/(2*tan(3*x + 2)**2 + 1), x)
```

3.6.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.33

$$\int \frac{\sec^2(2+3x)}{1+2\tan^2(2+3x)}\,dx = \frac{1}{6}\sqrt{2}\arctan\left(\sqrt{2}\tan(3x+2)\right)$$

input
```
integrate(sec(2+3*x)^2/(1+2*tan(2+3*x)^2),x, algorithm="maxima")
```

output
```
1/6*sqrt(2)*arctan(sqrt(2)*tan(3*x + 2))
```

3.6.8 Giac [A] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.33

$$\int \frac{\sec^2(2+3x)}{1+2\tan^2(2+3x)}\,dx = \frac{1}{6}\sqrt{2}\arctan\left(\sqrt{2}\tan(3x+2)\right)$$

input
```
integrate(sec(2+3*x)^2/(1+2*tan(2+3*x)^2),x, algorithm="giac")
```

output
```
1/6*sqrt(2)*arctan(sqrt(2)*tan(3*x + 2))
```

3.6.9 Mupad [B] (verification not implemented)

Time = 26.60 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.33

$$\int \frac{\sec^2(2+3x)}{1+2\tan^2(2+3x)}\,dx = \frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\tan(3\,x+2)\right)}{6}$$

input
```
int(1/(cos(3*x + 2)^2*(2*tan(3*x + 2)^2 + 1)),x)
```

output
```
(2^(1/2)*atan(2^(1/2)*tan(3*x + 2)))/6
```

3.7 $\int \frac{\csc^2(2+3x)}{2+\cot^2(2+3x)}\,dx$

3.7.1 Optimal result

Integrand size = 21, antiderivative size = 48

$$\int \frac{\csc^2(2+3x)}{2+\cot^2(2+3x)}\,dx = \frac{x}{\sqrt{2}} + \frac{\arctan\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\sin^2(2+3x)}\right)}{3\sqrt{2}}$$

output
```
1/2*x*2^(1/2)+1/6*arctan(cos(2+3*x)*sin(2+3*x)/(1+sin(2+3*x)^2+2^(1/2)))*2
^(1/2)
```

3.7.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.46

$$\int \frac{\csc^2(2+3x)}{2+\cot^2(2+3x)}\,dx = \frac{\arctan\left(\sqrt{2}\tan(2+3x)\right)}{3\sqrt{2}}$$

input
```
Integrate[Csc[2 + 3*x]^2/(2 + Cot[2 + 3*x]^2),x]
```

output
```
ArcTan[Sqrt[2]*Tan[2 + 3*x]]/(3*Sqrt[2])
```

3.7.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.46, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4158, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(3x+2)}{\cot^2(3x+2)+2}dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\sec\left(3x+\frac{\pi}{2}+2\right)^2}{\tan\left(3x+\frac{\pi}{2}+2\right)^2+2}dx$$
$$\Big\downarrow \; 4158$$
$$-\frac{1}{3}\int \frac{1}{\cot^2(3x+2)+2}d\cot(3x+2)$$
$$\Big\downarrow \; 216$$
$$-\frac{\arctan\left(\frac{\cot(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

input
```
Int[Csc[2 + 3*x]^2/(2 + Cot[2 + 3*x]^2),x]
```

output
```
-1/3*ArcTan[Cot[2 + 3*x]/Sqrt[2]]/Sqrt[2]
```

3.7.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

3.7.4 Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.35

method	result	size
derivativedivides	$\frac{\sqrt{2}\,\arctan\left(\tan(2+3x)\sqrt{2}\right)}{6}$	17
default	$\frac{\sqrt{2}\,\arctan\left(\tan(2+3x)\sqrt{2}\right)}{6}$	17
risch	$\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{2i(2+3x)}-3-2\sqrt{2}\right)}{12}-\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{2i(2+3x)}-3+2\sqrt{2}\right)}{12}$	48

input
```
int(csc(2+3*x)^2/(2+cot(2+3*x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/6*2^(1/2)*arctan(tan(2+3*x)*2^(1/2))
```

3.7.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

$$\int \frac{\csc^2(2+3x)}{2+\cot^2(2+3x)}\,dx = -\frac{1}{12}\,\sqrt{2}\arctan\left(\frac{3\,\sqrt{2}\cos\left(3\,x+2\right)^2-2\,\sqrt{2}}{4\,\cos\left(3\,x+2\right)\sin\left(3\,x+2\right)}\right)$$

input
```
integrate(csc(2+3*x)^2/(2+cot(2+3*x)^2),x, algorithm="fricas")
```

output
```
-1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(3*x + 2)^2 - 2*sqrt(2))/(cos(3*x +
 2)*sin(3*x + 2)))
```

3.7.6 Sympy [F]

$$\int \frac{\csc^2(2+3x)}{2+\cot^2(2+3x)}\,dx = \int \frac{\csc^2(3x+2)}{\cot^2(3x+2)+2}\,dx$$

input
```
integrate(csc(2+3*x)**2/(2+cot(2+3*x)**2),x)
```

output
```
Integral(csc(3*x + 2)**2/(cot(3*x + 2)**2 + 2), x)
```

3.7.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.33

$$\int \frac{\csc^2(2+3x)}{2+\cot^2(2+3x)}\,dx = \frac{1}{6}\sqrt{2}\arctan\left(\sqrt{2}\tan(3x+2)\right)$$

input
```
integrate(csc(2+3*x)^2/(2+cot(2+3*x)^2),x, algorithm="maxima")
```

output
```
1/6*sqrt(2)*arctan(sqrt(2)*tan(3*x + 2))
```

3.7.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int \frac{\csc^2(2+3x)}{2+\cot^2(2+3x)}\,dx$$
$$= \frac{1}{6}\sqrt{2}\left(3x+\arctan\left(-\frac{\sqrt{2}\sin(6x+4)-2\sin(6x+4)}{\sqrt{2}\cos(6x+4)+\sqrt{2}-2\cos(6x+4)+2}\right)+2\right)$$

input
```
integrate(csc(2+3*x)^2/(2+cot(2+3*x)^2),x, algorithm="giac")
```

output
```
1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - 2*sin(6*x + 4))/(sqrt(2
)*cos(6*x + 4) + sqrt(2) - 2*cos(6*x + 4) + 2)) + 2)
```

3.7.9 Mupad [B] (verification not implemented)

Time = 25.69 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.33

$$\int \frac{\csc^2(2+3x)}{2+\cot^2(2+3x)}\,dx = \frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\tan(3\,x+2)\right)}{6}$$

input
```
int(1/(sin(3*x + 2)^2*(cot(3*x + 2)^2 + 2)),x)
```

output
```
(2^(1/2)*atan(2^(1/2)*tan(3*x + 2)))/6
```

3.8 $\int \frac{2}{1-3\cos(4+6x)}\,dx$

3.8.1 Optimal result

Integrand size = 14, antiderivative size = 60

$$\int \frac{2}{1-3\cos(4+6x)}\,dx = \frac{\log\left(\cos(2+3x)-\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}} - \frac{\log\left(\cos(2+3x)+\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}}$$

output
```
1/12*ln(cos(2+3*x)-sin(2+3*x)*2^(1/2))*2^(1/2)-1/12*ln(cos(2+3*x)+sin(2+3*
x)*2^(1/2))*2^(1/2)
```

3.8.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.37

$$\int \frac{2}{1-3\cos(4+6x)}\,dx = -\frac{\operatorname{arctanh}\left(\sqrt{2}\tan(2+3x)\right)}{3\sqrt{2}}$$

input
```
Integrate[2/(1 - 3*Cos[4 + 6*x]),x]
```

output
```
-1/3*ArcTanh[Sqrt[2]*Tan[2 + 3*x]]/Sqrt[2]
```

3.8.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.37, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {27, 3042, 3138, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2}{1-3\cos(6x+4)}dx$$

$$\Big\downarrow \text{27}$$

$$2\int \frac{1}{1-3\cos(6x+4)}dx$$

$$\Big\downarrow \text{3042}$$

$$2\int \frac{1}{1-3\sin\left(6x+\frac{\pi}{2}+4\right)}dx$$

$$\Big\downarrow \text{3138}$$

$$\frac{2}{3}\int \frac{1}{4\tan^2(3x+2)-2}d\tan(3x+2)$$

$$\Big\downarrow \text{220}$$

$$-\frac{\operatorname{arctanh}\left(\sqrt{2}\tan(3x+2)\right)}{3\sqrt{2}}$$

input
```
Int[2/(1 - 3*Cos[4 + 6*x]),x]
```

output
```
-1/3*ArcTanh[Sqrt[2]*Tan[2 + 3*x]]/Sqrt[2]
```

3.8.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

3.8.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.28

method	result	size
derivativedivides	$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\tan(2+3x)\sqrt{2}\right)}{6}$	17
default	$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\tan(2+3x)\sqrt{2}\right)}{6}$	17
risch	$\frac{\sqrt{2}\,\ln\left(e^{2i(2+3x)}-\frac{1}{3}-\frac{2i\sqrt{2}}{3}\right)}{12}-\frac{\sqrt{2}\,\ln\left(e^{2i(2+3x)}-\frac{1}{3}+\frac{2i\sqrt{2}}{3}\right)}{12}$	48

input
```
int(2/(1-3*cos(4+6*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/6*2^(1/2)*arctanh(tan(2+3*x)*2^(1/2))
```

3.8.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.23

$$\int \frac{2}{1-3\cos(4+6x)}\,dx$$
$$=\frac{1}{24}\sqrt{2}\log\left(-\frac{7\cos(6x+4)^2-4\left(\sqrt{2}\cos(6x+4)-3\sqrt{2}\right)\sin(6x+4)+6\cos(6x+4)-17}{9\cos(6x+4)^2-6\cos(6x+4)+1}\right)$$

input
```
integrate(2/(1-3*cos(4+6*x)),x, algorithm="fricas")
```

output
```
1/24*sqrt(2)*log(-(7*cos(6*x + 4)^2 - 4*(sqrt(2)*cos(6*x + 4) - 3*sqrt(2))
*sin(6*x + 4) + 6*cos(6*x + 4) - 17)/(9*cos(6*x + 4)^2 - 6*cos(6*x + 4) +
1))
```

3.8.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.70

$$\int \frac{2}{1-3\cos(4+6x)}\,dx = \frac{\sqrt{2}\log\left(\tan\left(3x+2\right)-\frac{\sqrt{2}}{2}\right)}{12} - \frac{\sqrt{2}\log\left(\tan\left(3x+2\right)+\frac{\sqrt{2}}{2}\right)}{12}$$

input
```
integrate(2/(1-3*cos(4+6*x)),x)
```

output
```
sqrt(2)*log(tan(3*x + 2) - sqrt(2)/2)/12 - sqrt(2)*log(tan(3*x + 2) + sqrt
(2)/2)/12
```

3.8.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.90

$$\int \frac{2}{1-3\cos(4+6x)}\,dx = \frac{1}{12}\,\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{2\,\sin(6\,x+4)}{\cos(6\,x+4)+1}}{\sqrt{2}+\frac{2\,\sin(6\,x+4)}{\cos(6\,x+4)+1}}\right)$$

input
```
integrate(2/(1-3*cos(4+6*x)),x, algorithm="maxima")
```

output
```
1/12*sqrt(2)*log(-(sqrt(2) - 2*sin(6*x + 4)/(cos(6*x + 4) + 1))/(sqrt(2) +
 2*sin(6*x + 4)/(cos(6*x + 4) + 1)))
```

3.8.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.65

$$\int \frac{2}{1-3\cos(4+6x)}\,dx = \frac{1}{12}\,\sqrt{2}\log\left(\frac{\left|-2\,\sqrt{2}+4\,\tan\left(3\,x+2\right)\right|}{\left|2\,\sqrt{2}+4\,\tan\left(3\,x+2\right)\right|}\right)$$

input
```
integrate(2/(1-3*cos(4+6*x)),x, algorithm="giac")
```

output
```
1/12*sqrt(2)*log(abs(-2*sqrt(2) + 4*tan(3*x + 2))/abs(2*sqrt(2) + 4*tan(3*
x + 2)))
```

3.8.9 Mupad [B] (verification not implemented)

Time = 25.66 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.27

$$\int \frac{2}{1-3\cos(4+6x)}\,dx = -\frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\tan(3\,x+2)\right)}{6}$$

input
```
int(-2/(3*cos(6*x + 4) - 1),x)
```

output
```
-(2^(1/2)*atanh(2^(1/2)*tan(3*x + 2)))/6
```

3.9 $\int \frac{2\csc(4+6x)}{-3\cot(4+6x)+\csc(4+6x)}\,dx$

3.9.1 Optimal result

Integrand size = 25, antiderivative size = 60

$$\int \frac{2\csc(4+6x)}{-3\cot(4+6x)+\csc(4+6x)}\,dx = \frac{\log\left(\cos(2+3x)-\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}} - \frac{\log\left(\cos(2+3x)+\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}}$$

output
```
1/12*ln(cos(2+3*x)-sin(2+3*x)*2^(1/2))*2^(1/2)-1/12*ln(cos(2+3*x)+sin(2+3*
x)*2^(1/2))*2^(1/2)
```

3.9.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.37

$$\int \frac{2\csc(4+6x)}{-3\cot(4+6x)+\csc(4+6x)}\,dx = -\frac{\operatorname{arctanh}\left(\sqrt{2}\tan(2+3x)\right)}{3\sqrt{2}}$$

input
```
Integrate[(2*Csc[4 + 6*x])/(-3*Cot[4 + 6*x] + Csc[4 + 6*x]),x]
```

output
```
-1/3*ArcTanh[Sqrt[2]*Tan[2 + 3*x]]/Sqrt[2]
```

3.9.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.37, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {27, 25, 3042, 3645, 3042, 3138, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2\csc(6x+4)}{\csc(6x+4)-3\cot(6x+4)}dx$$

$$\Big\downarrow \text{27}$$

$$2\int -\frac{\csc(6x+4)}{3\cot(6x+4)-\csc(6x+4)}dx$$

$$\Big\downarrow \text{25}$$

$$-2\int \frac{\csc(6x+4)}{3\cot(6x+4)-\csc(6x+4)}dx$$

$$\Big\downarrow \text{3042}$$

$$-2\int \frac{\csc(6x+4)}{3\cot(6x+4)-\csc(6x+4)}dx$$

$$\Big\downarrow \text{3645}$$

$$-2\int \frac{1}{3\cos(6x+4)-1}dx$$

$$\Big\downarrow \text{3042}$$

$$-2\int \frac{1}{3\sin\left(6x+\frac{\pi}{2}+4\right)-1}dx$$

$$\Big\downarrow \text{3138}$$

$$-\frac{2}{3}\int \frac{1}{2-4\tan^2(3x+2)}d\tan(3x+2)$$

$$\Big\downarrow \text{219}$$

$$-\frac{\operatorname{arctanh}\left(\sqrt{2}\tan(3x+2)\right)}{3\sqrt{2}}$$

input
```
Int[(2*Csc[4 + 6*x])/(-3*Cot[4 + 6*x] + Csc[4 + 6*x]),x]
```

output
```
-1/3*ArcTanh[Sqrt[2]*Tan[2 + 3*x]]/Sqrt[2]
```

3.9.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3645
```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Int[1/(b + a*Sin[d + e*x]
 + c*Cos[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && I
ntegerQ[n]
```

3.9.4 Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.28

method	result	size
derivativedivides	$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\tan(2+3x)\sqrt{2}\right)}{6}$	17
default	$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\tan(2+3x)\sqrt{2}\right)}{6}$	17
risch	$\frac{\sqrt{2}\,\ln\left(e^{2i(2+3x)}-\frac{1}{3}-\frac{2i\sqrt{2}}{3}\right)}{12}-\frac{\sqrt{2}\,\ln\left(e^{2i(2+3x)}-\frac{1}{3}+\frac{2i\sqrt{2}}{3}\right)}{12}$	48

input
```
int(2*csc(4+6*x)/(-3*cot(4+6*x)+csc(4+6*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/6*2^(1/2)*arctanh(tan(2+3*x)*2^(1/2))
```

3.9.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.23

$$\int \frac{2\csc(4+6x)}{-3\cot(4+6x)+\csc(4+6x)}\,dx$$
$$= \frac{1}{24}\sqrt{2}\log\left(-\frac{7\cos(6x+4)^2-4\left(\sqrt{2}\cos(6x+4)-3\sqrt{2}\right)\sin(6x+4)+6\cos(6x+4)-17}{9\cos(6x+4)^2-6\cos(6x+4)+1}\right)$$

input
```
integrate(2*csc(4+6*x)/(-3*cot(4+6*x)+csc(4+6*x)),x, algorithm="fricas")
```

output
```
1/24*sqrt(2)*log(-(7*cos(6*x + 4)^2 - 4*(sqrt(2)*cos(6*x + 4) - 3*sqrt(2))
*sin(6*x + 4) + 6*cos(6*x + 4) - 17)/(9*cos(6*x + 4)^2 - 6*cos(6*x + 4) +
1))
```

3.9.6 Sympy [F]

$$\int \frac{2\csc(4+6x)}{-3\cot(4+6x)+\csc(4+6x)}\,dx = -2\int \frac{\csc(6x+4)}{3\cot(6x+4)-\csc(6x+4)}\,dx$$

input
```
integrate(2*csc(4+6*x)/(-3*cot(4+6*x)+csc(4+6*x)),x)
```

output
```
-2*Integral(csc(6*x + 4)/(3*cot(6*x + 4) - csc(6*x + 4)), x)
```

3.9.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.90

$$\int \frac{2\csc(4+6x)}{-3\cot(4+6x)+\csc(4+6x)}\,dx = \frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{2\sin(6x+4)}{\cos(6x+4)+1}}{\sqrt{2}+\frac{2\sin(6x+4)}{\cos(6x+4)+1}}\right)$$

input
```
integrate(2*csc(4+6*x)/(-3*cot(4+6*x)+csc(4+6*x)),x, algorithm="maxima")
```

output
```
1/12*sqrt(2)*log(-(sqrt(2) - 2*sin(6*x + 4)/(cos(6*x + 4) + 1))/(sqrt(2) +
 2*sin(6*x + 4)/(cos(6*x + 4) + 1)))
```

3.9.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.65

$$\int \frac{2\csc(4+6x)}{-3\cot(4+6x)+\csc(4+6x)}\,dx = \frac{1}{12}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\tan(3x+2)\right|}{\left|2\sqrt{2}+4\tan(3x+2)\right|}\right)$$

input
```
integrate(2*csc(4+6*x)/(-3*cot(4+6*x)+csc(4+6*x)),x, algorithm="giac")
```

output
```
1/12*sqrt(2)*log(abs(-2*sqrt(2) + 4*tan(3*x + 2))/abs(2*sqrt(2) + 4*tan(3*
x + 2)))
```

3.9.9 Mupad [B] (verification not implemented)

Time = 26.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.27

$$\int \frac{2\csc(4+6x)}{-3\cot(4+6x)+\csc(4+6x)}\,dx = -\frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\tan(3x+2)\right)}{6}$$

input
```
int(-2/(sin(6*x + 4)*(3*cot(6*x + 4) - 1/sin(6*x + 4))),x)
```

output
```
-(2^(1/2)*atanh(2^(1/2)*tan(3*x + 2)))/6
```

3.10 $\int \frac{1}{-1+3\sin^2(2+3x)}\,dx$

3.10.1 Optimal result

Integrand size = 14, antiderivative size = 60

$$\int \frac{1}{-1+3\sin^2(2+3x)}\,dx = \frac{\log\left(\cos(2+3x)-\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}} - \frac{\log\left(\cos(2+3x)+\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}}$$

output
```
1/12*ln(cos(2+3*x)-sin(2+3*x)*2^(1/2))*2^(1/2)-1/12*ln(cos(2+3*x)+sin(2+3*
x)*2^(1/2))*2^(1/2)
```

3.10.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.37

$$\int \frac{1}{-1+3\sin^2(2+3x)}\,dx = -\frac{\operatorname{arctanh}\left(\sqrt{2}\tan(2+3x)\right)}{3\sqrt{2}}$$

input
```
Integrate[(-1 + 3*Sin[2 + 3*x]^2)^(-1),x]
```

output
```
-1/3*ArcTanh[Sqrt[2]*Tan[2 + 3*x]]/Sqrt[2]
```

3.10.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.37, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 3660, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{3\sin^2(3x+2)-1}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{3\sin(3x+2)^2-1}dx$$

$$\Big\downarrow \text{3660}$$

$$\frac{1}{3}\int \frac{1}{2\tan^2(3x+2)-1}d\tan(3x+2)$$

$$\Big\downarrow \text{220}$$

$$-\frac{\text{arctanh}\left(\sqrt{2}\tan(3x+2)\right)}{3\sqrt{2}}$$

input
```
Int[(-1 + 3*Sin[2 + 3*x]^2)^(-1),x]
```

output
```
-1/3*ArcTanh[Sqrt[2]*Tan[2 + 3*x]]/Sqrt[2]
```

3.10.3.1 Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

3.10.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.28

method	result	size
derivativedivides	$-\frac{\sqrt{2}\, \operatorname{arctanh}\left(\tan(2+3x)\sqrt{2}\right)}{6}$	17
default	$-\frac{\sqrt{2}\, \operatorname{arctanh}\left(\tan(2+3x)\sqrt{2}\right)}{6}$	17
risch	$\frac{\sqrt{2}\, \ln\left(e^{2i(2+3x)}-\frac{1}{3}-\frac{2i\sqrt{2}}{3}\right)}{12}-\frac{\sqrt{2}\, \ln\left(e^{2i(2+3x)}-\frac{1}{3}+\frac{2i\sqrt{2}}{3}\right)}{12}$	48

input
```
int(1/(-1+3*sin(2+3*x)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*2^(1/2)*arctanh(tan(2+3*x)*2^(1/2))
```

3.10.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.43

$$\int \frac{1}{-1+3\sin^2(2+3x)}\, dx$$
$$= \frac{1}{24}\sqrt{2}\log\left(-\frac{7\cos(3x+2)^4-4\cos(3x+2)^2-4\left(\sqrt{2}\cos(3x+2)^3-2\sqrt{2}\cos(3x+2)\right)\sin(3x+2)}{9\cos(3x+2)^4-12\cos(3x+2)^2+4}\right.$$

input
```
integrate(1/(-1+3*sin(2+3*x)^2),x, algorithm="fricas")
```

output
```
1/24*sqrt(2)*log(-(7*cos(3*x + 2)^4 - 4*cos(3*x + 2)^2 - 4*(sqrt(2)*cos(3*
x + 2)^3 - 2*sqrt(2)*cos(3*x + 2))*sin(3*x + 2) - 4)/(9*cos(3*x + 2)^4 - 1
2*cos(3*x + 2)^2 + 4))
```

3.10.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1644 vs. $2(53) = 106$.

Time = 9.47 (sec) , antiderivative size = 1644, normalized size of antiderivative = 27.40

$$\int \frac{1}{-1+3\sin^2(2+3x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(-1+3*sin(2+3*x)**2),x)
```

output
```
-1387702511766624*sqrt(5 - 2*sqrt(6))*log(tan(3*x/2 + 1) - sqrt(5 - 2*sqrt
(6)))/(-467972363532675 - 191048917396548*sqrt(6) + 13665597568857156*sqrt
(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 33473741073918339*sqrt(5 - 2
*sqrt(6))*sqrt(2*sqrt(6) + 5)) - 566527178101133*sqrt(6)*sqrt(5 - 2*sqrt(6
))*log(tan(3*x/2 + 1) - sqrt(5 - 2*sqrt(6)))/(-467972363532675 - 191048917
396548*sqrt(6) + 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt
(6) + 5) + 33473741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5)) + 13
76499295618884*sqrt(2*sqrt(6) + 5)*log(tan(3*x/2 + 1) - sqrt(5 - 2*sqrt(6)
))/(-467972363532675 - 191048917396548*sqrt(6) + 13665597568857156*sqrt(6)
*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 33473741073918339*sqrt(5 - 2*sq
rt(6))*sqrt(2*sqrt(6) + 5)) + 561953484261121*sqrt(6)*sqrt(2*sqrt(6) + 5)*
log(tan(3*x/2 + 1) - sqrt(5 - 2*sqrt(6)))/(-467972363532675 - 191048917396
548*sqrt(6) + 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6)
 + 5) + 33473741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5)) - 12479
44371758796*sqrt(2*sqrt(6) + 5)*log(tan(3*x/2 + 1) + sqrt(5 - 2*sqrt(6)))/
(-467972363532675 - 191048917396548*sqrt(6) + 13665597568857156*sqrt(6)*sq
rt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 33473741073918339*sqrt(5 - 2*sqrt(
6))*sqrt(2*sqrt(6) + 5)) - 509471156364528*sqrt(6)*sqrt(2*sqrt(6) + 5)*log
(tan(3*x/2 + 1) + sqrt(5 - 2*sqrt(6)))/(-467972363532675 - 191048917396548
*sqrt(6) + 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6)...
```

3.10.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.57

$$\int \frac{1}{-1+3\sin^2(2+3x)}\,dx = \frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-2\tan(3x+2)}{\sqrt{2}+2\tan(3x+2)}\right)$$

input
```
integrate(1/(-1+3*sin(2+3*x)^2),x, algorithm="maxima")
```

output
```
1/12*sqrt(2)*log(-(sqrt(2) - 2*tan(3*x + 2))/(sqrt(2) + 2*tan(3*x + 2)))
```

3.10.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.65

$$\int \frac{1}{-1+3\sin^2(2+3x)}\,dx = \frac{1}{12}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\tan\left(3x+2\right)\right|}{\left|2\sqrt{2}+4\tan\left(3x+2\right)\right|}\right)$$

input
```
integrate(1/(-1+3*sin(2+3*x)^2),x, algorithm="giac")
```

output
```
1/12*sqrt(2)*log(abs(-2*sqrt(2) + 4*tan(3*x + 2))/abs(2*sqrt(2) + 4*tan(3*
x + 2)))
```

3.10.9 Mupad [B] (verification not implemented)

Time = 26.41 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.27

$$\int \frac{1}{-1+3\sin^2(2+3x)}\,dx = -\frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\tan(3x+2)\right)}{6}$$

input
```
int(1/(3*sin(3*x + 2)^2 - 1),x)
```

output
```
-(2^(1/2)*atanh(2^(1/2)*tan(3*x + 2)))/6
```

3.11 $\int \frac{1}{2-3\cos^2(2+3x)}\,dx$

3.11.1 Optimal result

Integrand size = 14, antiderivative size = 60

$$\int \frac{1}{2-3\cos^2(2+3x)}\,dx = \frac{\log\left(\cos(2+3x)-\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}} - \frac{\log\left(\cos(2+3x)+\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}}$$

output
```
1/12*ln(cos(2+3*x)-sin(2+3*x)*2^(1/2))*2^(1/2)-1/12*ln(cos(2+3*x)+sin(2+3*
x)*2^(1/2))*2^(1/2)
```

3.11.2 Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.37

$$\int \frac{1}{2-3\cos^2(2+3x)}\,dx = -\frac{\operatorname{arctanh}\left(\sqrt{2}\tan(2+3x)\right)}{3\sqrt{2}}$$

input
```
Integrate[(2 - 3*Cos[2 + 3*x]^2)^(-1),x]
```

output
```
-1/3*ArcTanh[Sqrt[2]*Tan[2 + 3*x]]/Sqrt[2]
```

3.11.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.37, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 3660, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{2-3\cos^2(3x+2)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{2-3\sin\left(3x+\frac{\pi}{2}+2\right)^2}dx$$

$$\Big\downarrow \text{ 3660}$$

$$-\frac{1}{3}\int \frac{1}{2-\cot^2(3x+2)}d\cot(3x+2)$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{\text{arctanh}\left(\frac{\cot(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

input
```
Int[(2 - 3*Cos[2 + 3*x]^2)^(-1),x]
```

output
```
-1/3*ArcTanh[Cot[2 + 3*x]/Sqrt[2]]/Sqrt[2]
```

3.11.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

3.11.4 Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.28

method	result	size
derivativedivides	$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\tan(2+3x)\sqrt{2}\right)}{6}$	17
default	$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\tan(2+3x)\sqrt{2}\right)}{6}$	17
risch	$\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2i(2+3x)}-\frac{1}{3}-\frac{2i\sqrt{2}}{3}\right)}{12}-\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2i(2+3x)}-\frac{1}{3}+\frac{2i\sqrt{2}}{3}\right)}{12}$	48

input
```
int(1/(2-3*cos(2+3*x)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*2^(1/2)*arctanh(tan(2+3*x)*2^(1/2))
```

3.11.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.43

$$\int \frac{1}{2-3\cos^2(2+3x)}\,dx$$
$$=\frac{1}{24}\sqrt{2}\log\left(-\frac{7\cos(3x+2)^4-4\cos(3x+2)^2-4\left(\sqrt{2}\cos(3x+2)^3-2\sqrt{2}\cos(3x+2)\right)\sin(3x+2)}{9\cos(3x+2)^4-12\cos(3x+2)^2+4}\right.$$

input
```
integrate(1/(2-3*cos(2+3*x)^2),x, algorithm="fricas")
```

output
```
1/24*sqrt(2)*log(-(7*cos(3*x + 2)^4 - 4*cos(3*x + 2)^2 - 4*(sqrt(2)*cos(3*
x + 2)^3 - 2*sqrt(2)*cos(3*x + 2))*sin(3*x + 2) - 4)/(9*cos(3*x + 2)^4 - 1
2*cos(3*x + 2)^2 + 4))
```

3.11.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1644 vs. $2(53) = 106$.

Time = 9.50 (sec) , antiderivative size = 1644, normalized size of antiderivative = 27.40

$$\int \frac{1}{2 - 3\cos^2(2 + 3x)}\, dx = \text{Too large to display}$$

input
```
integrate(1/(2-3*cos(2+3*x)**2),x)
```

output
```
-1387702511766624*sqrt(5 - 2*sqrt(6))*log(tan(3*x/2 + 1) - sqrt(5 - 2*sqrt
(6)))/(-467972363532675 - 191048917396548*sqrt(6) + 13665597568857156*sqrt
(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 33473741073918339*sqrt(5 - 2
*sqrt(6))*sqrt(2*sqrt(6) + 5)) - 566527178101133*sqrt(6)*sqrt(5 - 2*sqrt(6
))*log(tan(3*x/2 + 1) - sqrt(5 - 2*sqrt(6)))/(-467972363532675 - 191048917
396548*sqrt(6) + 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt
(6) + 5) + 33473741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5)) + 13
76499295618884*sqrt(2*sqrt(6) + 5)*log(tan(3*x/2 + 1) - sqrt(5 - 2*sqrt(6)
))/(-467972363532675 - 191048917396548*sqrt(6) + 13665597568857156*sqrt(6)
*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 33473741073918339*sqrt(5 - 2*sq
rt(6))*sqrt(2*sqrt(6) + 5)) + 561953484261121*sqrt(6)*sqrt(2*sqrt(6) + 5)*
log(tan(3*x/2 + 1) - sqrt(5 - 2*sqrt(6)))/(-467972363532675 - 191048917396
548*sqrt(6) + 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6)
 + 5) + 33473741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5)) - 12479
44371758796*sqrt(2*sqrt(6) + 5)*log(tan(3*x/2 + 1) + sqrt(5 - 2*sqrt(6)))/
(-467972363532675 - 191048917396548*sqrt(6) + 13665597568857156*sqrt(6)*sq
rt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 33473741073918339*sqrt(5 - 2*sqrt(
6))*sqrt(2*sqrt(6) + 5)) - 509471156364528*sqrt(6)*sqrt(2*sqrt(6) + 5)*log
(tan(3*x/2 + 1) + sqrt(5 - 2*sqrt(6)))/(-467972363532675 - 191048917396548
*sqrt(6) + 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6)...
```

3.11.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.57

$$\int \frac{1}{2 - 3\cos^2(2 + 3x)}\, dx = \frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2} - 2\tan(3x + 2)}{\sqrt{2} + 2\tan(3x + 2)}\right)$$

input
```
integrate(1/(2-3*cos(2+3*x)^2),x, algorithm="maxima")
```

output
```
1/12*sqrt(2)*log(-(sqrt(2) - 2*tan(3*x + 2))/(sqrt(2) + 2*tan(3*x + 2)))
```

3.11.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.65

$$\int \frac{1}{2-3\cos^2(2+3x)}\,dx = \frac{1}{12}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\tan\left(3x+2\right)\right|}{\left|2\sqrt{2}+4\tan\left(3x+2\right)\right|}\right)$$

input
```
integrate(1/(2-3*cos(2+3*x)^2),x, algorithm="giac")
```

output
```
1/12*sqrt(2)*log(abs(-2*sqrt(2) + 4*tan(3*x + 2))/abs(2*sqrt(2) + 4*tan(3*
x + 2)))
```

3.11.9 Mupad [B] (verification not implemented)

Time = 26.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.27

$$\int \frac{1}{2-3\cos^2(2+3x)}\,dx = -\frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\tan(3x+2)\right)}{6}$$

input
```
int(-1/(3*cos(3*x + 2)^2 - 2),x)
```

output
```
-(2^(1/2)*atanh(2^(1/2)*tan(3*x + 2)))/6
```

3.12 $\int \frac{1}{-\cos^2(2+3x)+2\sin^2(2+3x)}\,dx$

3.12.1 Optimal result

Integrand size = 23, antiderivative size = 60

$$\int \frac{1}{-\cos^2(2+3x)+2\sin^2(2+3x)}\,dx = \frac{\log\left(\cos(2+3x)-\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}} - \frac{\log\left(\cos(2+3x)+\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}}$$

output
```
1/12*ln(cos(2+3*x)-sin(2+3*x)*2^(1/2))*2^(1/2)-1/12*ln(cos(2+3*x)+sin(2+3*
x)*2^(1/2))*2^(1/2)
```

3.12.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.37

$$\int \frac{1}{-\cos^2(2+3x)+2\sin^2(2+3x)}\,dx = -\frac{\operatorname{arctanh}\left(\sqrt{2}\tan(2+3x)\right)}{3\sqrt{2}}$$

input
```
Integrate[(-Cos[2 + 3*x]^2 + 2*Sin[2 + 3*x]^2)^(-1),x]
```

output
```
-1/3*ArcTanh[Sqrt[2]*Tan[2 + 3*x]]/Sqrt[2]
```

3.12.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.37, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3042, 4889, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{2\sin^2(3x+2)-\cos^2(3x+2)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{2\sin(3x+2)^2-\cos(3x+2)^2}dx$$
$$\Big\downarrow \text{4889}$$
$$\frac{1}{3}\int \frac{1}{2\tan^2(3x+2)-1}d\tan(3x+2)$$
$$\Big\downarrow \text{220}$$
$$-\frac{\text{arctanh}\left(\sqrt{2}\tan(3x+2)\right)}{3\sqrt{2}}$$

input
```
Int[(-Cos[2 + 3*x]^2 + 2*Sin[2 + 3*x]^2)^(-1),x]
```

output
```
-1/3*ArcTanh[Sqrt[2]*Tan[2 + 3*x]]/Sqrt[2]
```

3.12.3.1 Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.12.4 Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.28

method	result	size
derivativedivides	$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\tan(2+3x)\sqrt{2}\right)}{6}$	17
default	$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\tan(2+3x)\sqrt{2}\right)}{6}$	17
risch	$\frac{\sqrt{2}\,\ln\left(e^{2i(2+3x)}-\frac{1}{3}-\frac{2i\sqrt{2}}{3}\right)}{12}-\frac{\sqrt{2}\,\ln\left(e^{2i(2+3x)}-\frac{1}{3}+\frac{2i\sqrt{2}}{3}\right)}{12}$	48

input
```
int(1/(-cos(2+3*x)^2+2*sin(2+3*x)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*2^(1/2)*arctanh(tan(2+3*x)*2^(1/2))
```

3.12.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.43

$$\int \frac{1}{-\cos^2(2+3x)+2\sin^2(2+3x)}\,dx$$
$$=\frac{1}{24}\sqrt{2}\log\left(-\frac{7\cos(3x+2)^4-4\cos(3x+2)^2-4\left(\sqrt{2}\cos(3x+2)^3-2\sqrt{2}\cos(3x+2)\right)\sin(3x+2)}{9\cos(3x+2)^4-12\cos(3x+2)^2+4}\right.$$

input
```
integrate(1/(-cos(2+3*x)^2+2*sin(2+3*x)^2),x, algorithm="fricas")
```

output
```
1/24*sqrt(2)*log(-(7*cos(3*x + 2)^4 - 4*cos(3*x + 2)^2 - 4*(sqrt(2)*cos(3*
x + 2)^3 - 2*sqrt(2)*cos(3*x + 2))*sin(3*x + 2) - 4)/(9*cos(3*x + 2)^4 - 1
2*cos(3*x + 2)^2 + 4))
```

3.12.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1644 vs. $2(53) = 106$.

Time = 9.24 (sec) , antiderivative size = 1644, normalized size of antiderivative = 27.40

$$\int \frac{1}{-\cos^2(2+3x)+2\sin^2(2+3x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(-cos(2+3*x)**2+2*sin(2+3*x)**2),x)
```

output
```
-1387702511766624*sqrt(5 - 2*sqrt(6))*log(tan(3*x/2 + 1) - sqrt(5 - 2*sqrt
(6)))/(-467972363532675 - 191048917396548*sqrt(6) + 13665597568857156*sqrt
(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 33473741073918339*sqrt(5 - 2
*sqrt(6))*sqrt(2*sqrt(6) + 5)) - 566527178101133*sqrt(6)*sqrt(5 - 2*sqrt(6
))*log(tan(3*x/2 + 1) - sqrt(5 - 2*sqrt(6)))/(-467972363532675 - 191048917
396548*sqrt(6) + 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt
(6) + 5) + 33473741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5)) + 13
76499295618884*sqrt(2*sqrt(6) + 5)*log(tan(3*x/2 + 1) - sqrt(5 - 2*sqrt(6)
))/(-467972363532675 - 191048917396548*sqrt(6) + 13665597568857156*sqrt(6)
*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 33473741073918339*sqrt(5 - 2*sq
rt(6))*sqrt(2*sqrt(6) + 5)) + 561953484261121*sqrt(6)*sqrt(2*sqrt(6) + 5)*
log(tan(3*x/2 + 1) - sqrt(5 - 2*sqrt(6)))/(-467972363532675 - 191048917396
548*sqrt(6) + 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6)
 + 5) + 33473741073918339*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5)) - 12479
44371758796*sqrt(2*sqrt(6) + 5)*log(tan(3*x/2 + 1) + sqrt(5 - 2*sqrt(6)))/
(-467972363532675 - 191048917396548*sqrt(6) + 13665597568857156*sqrt(6)*sq
rt(5 - 2*sqrt(6))*sqrt(2*sqrt(6) + 5) + 33473741073918339*sqrt(5 - 2*sqrt(
6))*sqrt(2*sqrt(6) + 5)) - 509471156364528*sqrt(6)*sqrt(2*sqrt(6) + 5)*log
(tan(3*x/2 + 1) + sqrt(5 - 2*sqrt(6)))/(-467972363532675 - 191048917396548
*sqrt(6) + 13665597568857156*sqrt(6)*sqrt(5 - 2*sqrt(6))*sqrt(2*sqrt(6)...
```

3.12.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.57

$$\int \frac{1}{-\cos^2(2+3x)+2\sin^2(2+3x)}\,dx = \frac{1}{12}\,\sqrt{2}\log\left(-\frac{\sqrt{2}-2\,\tan\left(3\,x+2\right)}{\sqrt{2}+2\,\tan\left(3\,x+2\right)}\right)$$

input
```
integrate(1/(-cos(2+3*x)^2+2*sin(2+3*x)^2),x, algorithm="maxima")
```

output
```
1/12*sqrt(2)*log(-(sqrt(2) - 2*tan(3*x + 2))/(sqrt(2) + 2*tan(3*x + 2)))
```

3.12.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.65

$$\int \frac{1}{-\cos^2(2+3x)+2\sin^2(2+3x)}\,dx = \frac{1}{12}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\tan\left(3x+2\right)\right|}{\left|2\sqrt{2}+4\tan\left(3x+2\right)\right|}\right)$$

input
```
integrate(1/(-cos(2+3*x)^2+2*sin(2+3*x)^2),x, algorithm="giac")
```

output
```
1/12*sqrt(2)*log(abs(-2*sqrt(2) + 4*tan(3*x + 2))/abs(2*sqrt(2) + 4*tan(3*
x + 2)))
```

3.12.9 Mupad [B] (verification not implemented)

Time = 26.70 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.27

$$\int \frac{1}{-\cos^2(2+3x)+2\sin^2(2+3x)}\,dx = -\frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\tan(3x+2)\right)}{6}$$

input
```
int(1/(2*sin(3*x + 2)^2 - cos(3*x + 2)^2),x)
```

output
```
-(2^(1/2)*atanh(2^(1/2)*tan(3*x + 2)))/6
```

3.13 $\int \frac{\sec^2(2+3x)}{-1+2\tan^2(2+3x)}\,dx$

3.13.1 Optimal result

Integrand size = 23, antiderivative size = 60

$$\int \frac{\sec^2(2+3x)}{-1+2\tan^2(2+3x)}\,dx = \frac{\log\left(\cos(2+3x)-\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}} - \frac{\log\left(\cos(2+3x)+\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}}$$

output
```
1/12*ln(cos(2+3*x)-sin(2+3*x)*2^(1/2))*2^(1/2)-1/12*ln(cos(2+3*x)+sin(2+3*
x)*2^(1/2))*2^(1/2)
```

3.13.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.37

$$\int \frac{\sec^2(2+3x)}{-1+2\tan^2(2+3x)}\,dx = -\frac{\operatorname{arctanh}\left(\sqrt{2}\tan(2+3x)\right)}{3\sqrt{2}}$$

input
```
Integrate[Sec[2 + 3*x]^2/(-1 + 2*Tan[2 + 3*x]^2),x]
```

output
```
-1/3*ArcTanh[Sqrt[2]*Tan[2 + 3*x]]/Sqrt[2]
```

3.13.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.37, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3042, 4158, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(3x+2)}{2\tan^2(3x+2)-1}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sec(3x+2)^2}{2\tan(3x+2)^2-1}dx$$
$$\Big\downarrow \text{4158}$$
$$\frac{1}{3}\int \frac{1}{2\tan^2(3x+2)-1}d\tan(3x+2)$$
$$\Big\downarrow \text{220}$$
$$-\frac{\text{arctanh}\left(\sqrt{2}\tan(3x+2)\right)}{3\sqrt{2}}$$

input
```
Int[Sec[2 + 3*x]^2/(-1 + 2*Tan[2 + 3*x]^2),x]
```

output
```
-1/3*ArcTanh[Sqrt[2]*Tan[2 + 3*x]]/Sqrt[2]
```

3.13.3.1 Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

3.13.4 Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.28

method	result	size
derivativedivides	$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\tan(2+3x)\sqrt{2}\right)}{6}$	17
default	$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\tan(2+3x)\sqrt{2}\right)}{6}$	17
risch	$\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2i(2+3x)}-\frac{1}{3}-\frac{2i\sqrt{2}}{3}\right)}{12}-\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2i(2+3x)}-\frac{1}{3}+\frac{2i\sqrt{2}}{3}\right)}{12}$	48

input
```
int(sec(2+3*x)^2/(-1+2*tan(2+3*x)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*2^(1/2)*arctanh(tan(2+3*x)*2^(1/2))
```

3.13.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.43

$$\int \frac{\sec^2(2+3x)}{-1+2\tan^2(2+3x)}\,dx$$
$$=\frac{1}{24}\sqrt{2}\log\left(-\frac{7\cos(3x+2)^4-4\cos(3x+2)^2-4\left(\sqrt{2}\cos(3x+2)^3-2\sqrt{2}\cos(3x+2)\right)\sin(3x+2)}{9\cos(3x+2)^4-12\cos(3x+2)^2+4}\right.$$

input
```
integrate(sec(2+3*x)^2/(-1+2*tan(2+3*x)^2),x, algorithm="fricas")
```

output
```
1/24*sqrt(2)*log(-(7*cos(3*x + 2)^4 - 4*cos(3*x + 2)^2 - 4*(sqrt(2)*cos(3*
x + 2)^3 - 2*sqrt(2)*cos(3*x + 2))*sin(3*x + 2) - 4)/(9*cos(3*x + 2)^4 - 1
2*cos(3*x + 2)^2 + 4))
```

3.13.6 Sympy [F]

$$\int \frac{\sec^2(2+3x)}{-1+2\tan^2(2+3x)}\,dx = \int \frac{\sec^2(3x+2)}{2\tan^2(3x+2)-1}\,dx$$

input
```
integrate(sec(2+3*x)**2/(-1+2*tan(2+3*x)**2),x)
```

output
```
Integral(sec(3*x + 2)**2/(2*tan(3*x + 2)**2 - 1), x)
```

3.13.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.57

$$\int \frac{\sec^2(2+3x)}{-1+2\tan^2(2+3x)}\,dx = \frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-2\tan(3x+2)}{\sqrt{2}+2\tan(3x+2)}\right)$$

input
```
integrate(sec(2+3*x)^2/(-1+2*tan(2+3*x)^2),x, algorithm="maxima")
```

output
```
1/12*sqrt(2)*log(-(sqrt(2) - 2*tan(3*x + 2))/(sqrt(2) + 2*tan(3*x + 2)))
```

3.13.8 Giac [A] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.65

$$\int \frac{\sec^2(2+3x)}{-1+2\tan^2(2+3x)}\,dx = -\frac{1}{12}\sqrt{2}\log\left(\left|\frac{1}{2}\sqrt{2}+\tan(3x+2)\right|\right) + \frac{1}{12}\sqrt{2}\log\left(\left|-\frac{1}{2}\sqrt{2}+\tan(3x+2)\right|\right)$$

input
```
integrate(sec(2+3*x)^2/(-1+2*tan(2+3*x)^2),x, algorithm="giac")
```

output
```
-1/12*sqrt(2)*log(abs(1/2*sqrt(2) + tan(3*x + 2))) + 1/12*sqrt(2)*log(abs(
-1/2*sqrt(2) + tan(3*x + 2)))
```

3.13.9 Mupad [B] (verification not implemented)

Time = 26.66 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.27

$$\int \frac{\sec^2(2+3x)}{-1+2\tan^2(2+3x)}\,dx = -\frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\tan(3\,x+2)\right)}{6}$$

input
```
int(1/(cos(3*x + 2)^2*(2*tan(3*x + 2)^2 - 1)),x)
```

output
```
-(2^(1/2)*atanh(2^(1/2)*tan(3*x + 2)))/6
```

3.14 $\int \frac{\csc^2(2+3x)}{2-\cot^2(2+3x)}\,dx$

3.14.1 Optimal result

Integrand size = 23, antiderivative size = 60

$$\int \frac{\csc^2(2+3x)}{2-\cot^2(2+3x)}\,dx = \frac{\log\left(\cos(2+3x)-\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}} - \frac{\log\left(\cos(2+3x)+\sqrt{2}\sin(2+3x)\right)}{6\sqrt{2}}$$

output
```
1/12*ln(cos(2+3*x)-sin(2+3*x)*2^(1/2))*2^(1/2)-1/12*ln(cos(2+3*x)+sin(2+3*
x)*2^(1/2))*2^(1/2)
```

3.14.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.37

$$\int \frac{\csc^2(2+3x)}{2-\cot^2(2+3x)}\,dx = -\frac{\operatorname{arctanh}\left(\sqrt{2}\tan(2+3x)\right)}{3\sqrt{2}}$$

input
```
Integrate[Csc[2 + 3*x]^2/(2 - Cot[2 + 3*x]^2),x]
```

output
```
-1/3*ArcTanh[Sqrt[2]*Tan[2 + 3*x]]/Sqrt[2]
```

3.14.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.37, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3042, 4158, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(3x+2)}{2-\cot^2(3x+2)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sec\left(3x+\frac{\pi}{2}+2\right)^2}{2-\tan\left(3x+\frac{\pi}{2}+2\right)^2}dx$$

$$\Big\downarrow \text{ 4158}$$

$$-\frac{1}{3}\int \frac{1}{2-\cot^2(3x+2)}d\cot(3x+2)$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{\text{arctanh}\left(\frac{\cot(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

input
```
Int[Csc[2 + 3*x]^2/(2 - Cot[2 + 3*x]^2),x]
```

output
```
-1/3*ArcTanh[Cot[2 + 3*x]/Sqrt[2]]/Sqrt[2]
```

3.14.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

3.14.4 Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.28

method	result	size
derivativedivides	$-\frac{\sqrt{2}\, \operatorname{arctanh}\left(\tan (2+3x)\sqrt{2}\right)}{6}$	17
default	$-\frac{\sqrt{2}\, \operatorname{arctanh}\left(\tan (2+3x)\sqrt{2}\right)}{6}$	17
risch	$\frac{\sqrt{2}\, \ln\left(\mathrm{e}^{2i(2+3x)}-\frac{1}{3}-\frac{2i\sqrt{2}}{3}\right)}{12}-\frac{\sqrt{2}\, \ln\left(\mathrm{e}^{2i(2+3x)}-\frac{1}{3}+\frac{2i\sqrt{2}}{3}\right)}{12}$	48

input
```
int(csc(2+3*x)^2/(2-cot(2+3*x)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*2^(1/2)*arctanh(tan(2+3*x)*2^(1/2))
```

3.14.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.43

$$\int \frac{\csc^2(2+3x)}{2-\cot^2(2+3x)}\,dx$$
$$=\frac{1}{24}\sqrt{2}\log\left(-\frac{7\cos(3x+2)^4-4\cos(3x+2)^2-4\left(\sqrt{2}\cos(3x+2)^3-2\sqrt{2}\cos(3x+2)\right)\sin(3x+2)}{9\cos(3x+2)^4-12\cos(3x+2)^2+4}\right.$$

input
```
integrate(csc(2+3*x)^2/(2-cot(2+3*x)^2),x, algorithm="fricas")
```

output
```
1/24*sqrt(2)*log(-(7*cos(3*x + 2)^4 - 4*cos(3*x + 2)^2 - 4*(sqrt(2)*cos(3*
x + 2)^3 - 2*sqrt(2)*cos(3*x + 2))*sin(3*x + 2) - 4)/(9*cos(3*x + 2)^4 - 1
2*cos(3*x + 2)^2 + 4))
```

3.14.6 Sympy [F]

$$\int \frac{\csc^2(2+3x)}{2-\cot^2(2+3x)}\,dx = -\int \frac{\csc^2(3x+2)}{\cot^2(3x+2)-2}\,dx$$

input
```
integrate(csc(2+3*x)**2/(2-cot(2+3*x)**2),x)
```

output
```
-Integral(csc(3*x + 2)**2/(cot(3*x + 2)**2 - 2), x)
```

3.14.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.57

$$\int \frac{\csc^2(2+3x)}{2-\cot^2(2+3x)}\,dx = \frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-2\tan(3x+2)}{\sqrt{2}+2\tan(3x+2)}\right)$$

input
```
integrate(csc(2+3*x)^2/(2-cot(2+3*x)^2),x, algorithm="maxima")
```

output
```
1/12*sqrt(2)*log(-(sqrt(2) - 2*tan(3*x + 2))/(sqrt(2) + 2*tan(3*x + 2)))
```

3.14.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.65

$$\int \frac{\csc^2(2+3x)}{2-\cot^2(2+3x)}\,dx = \frac{1}{12}\sqrt{2}\log\left(\frac{|-2\sqrt{2}+4\tan(3x+2)|}{|2\sqrt{2}+4\tan(3x+2)|}\right)$$

input
```
integrate(csc(2+3*x)^2/(2-cot(2+3*x)^2),x, algorithm="giac")
```

output
```
1/12*sqrt(2)*log(abs(-2*sqrt(2) + 4*tan(3*x + 2))/abs(2*sqrt(2) + 4*tan(3*
x + 2)))
```

3.14.9 Mupad [B] (verification not implemented)

Time = 26.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.27

$$\int \frac{\csc^2(2+3x)}{2-\cot^2(2+3x)}\,dx = -\frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\tan(3\,x+2)\right)}{6}$$

input
```
int(-1/(sin(3*x + 2)^2*(cot(3*x + 2)^2 - 2)),x)
```

output
```
-(2^(1/2)*atanh(2^(1/2)*tan(3*x + 2)))/6
```

3.15 $\int \frac{2}{3+\cos(4+6x)}\,dx$

3.15.1 Optimal result

Integrand size = 12, antiderivative size = 42

$$\int \frac{2}{3+\cos(4+6x)}\,dx = \frac{x}{\sqrt{2}} - \frac{\arctan\left(\frac{\sin(4+6x)}{3+2\sqrt{2}+\cos(4+6x)}\right)}{3\sqrt{2}}$$

output
```
1/2*x*2^(1/2)-1/6*arctan(sin(4+6*x)/(3+cos(4+6*x)+2*2^(1/2)))*2^(1/2)
```

3.15.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.52

$$\int \frac{2}{3+\cos(4+6x)}\,dx = \frac{\arctan\left(\frac{\tan(2+3x)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

input
```
Integrate[2/(3 + Cos[4 + 6*x]),x]
```

output
```
ArcTan[Tan[2 + 3*x]/Sqrt[2]]/(3*Sqrt[2])
```

3.15.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {27, 3042, 3136}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2}{\cos(6x+4)+3}dx$$

$$\Big\downarrow \; 27$$

$$2\int \frac{1}{\cos(6x+4)+3}dx$$

$$\Big\downarrow \; 3042$$

$$2\int \frac{1}{\sin\left(6x+\frac{\pi}{2}+4\right)+3}dx$$

$$\Big\downarrow \; 3136$$

$$2\left(\frac{x}{2\sqrt{2}}-\frac{\arctan\left(\frac{\sin(6x+4)}{\cos(6x+4)+2\sqrt{2}+3}\right)}{6\sqrt{2}}\right)$$

input
```
Int[2/(3 + Cos[4 + 6*x]),x]
```

output
```
2*(x/(2*Sqrt[2]) - ArcTan[Sin[4 + 6*x]/(3 + 2*Sqrt[2] + Cos[4 + 6*x])]/(6*
Sqrt[2]))
```

3.15.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3136
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

3.15.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.43

method	result	size
derivativedivides	$\frac{\sqrt{2}\,\arctan\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)}{6}$	18
default	$\frac{\sqrt{2}\,\arctan\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)}{6}$	18
risch	$\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{2i(2+3x)}+3+2\sqrt{2}\right)}{12}-\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{2i(2+3x)}+3-2\sqrt{2}\right)}{12}$	48

input
```
int(2/(3+cos(4+6*x)),x,method=_RETURNVERBOSE)
```

output
```
1/6*2^(1/2)*arctan(1/2*tan(2+3*x)*2^(1/2))
```

3.15.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.74

$$\int \frac{2}{3+\cos(4+6x)}\,dx = -\frac{1}{12}\,\sqrt{2}\arctan\left(\frac{3\,\sqrt{2}\cos\left(6\,x+4\right)+\sqrt{2}}{4\,\sin\left(6\,x+4\right)}\right)$$

input
```
integrate(2/(3+cos(4+6*x)),x, algorithm="fricas")
```

output
```
-1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(6*x + 4) + sqrt(2))/sin(6*x + 4))
```

3.15.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81

$$\int \frac{2}{3+\cos(4+6x)}\,dx = \frac{\sqrt{2}\left(\operatorname{atan}\left(\frac{\sqrt{2}\tan{(3x+2)}}{2}\right)+\pi\left\lfloor\frac{3x-\frac{\pi}{2}+2}{\pi}\right\rfloor\right)}{6}$$

input
```
integrate(2/(3+cos(4+6*x)),x)
```

output
```
sqrt(2)*(atan(sqrt(2)*tan(3*x + 2)/2) + pi*floor((3*x - pi/2 + 2)/pi))/6
```

3.15.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.64

$$\int \frac{2}{3+\cos(4+6x)}\,dx = \frac{1}{6}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sin{(6x+4)}}{2(\cos{(6x+4)}+1)}\right)$$

input
```
integrate(2/(3+cos(4+6*x)),x, algorithm="maxima")
```

output
```
1/6*sqrt(2)*arctan(1/2*sqrt(2)*sin(6*x + 4)/(cos(6*x + 4) + 1))
```

3.15.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.36

$$\int \frac{2}{3+\cos(4+6x)}\,dx = \frac{1}{6}\sqrt{2}\left(3x+\arctan\left(-\frac{\sqrt{2}\sin{(6x+4)}-\sin{(6x+4)}}{\sqrt{2}\cos{(6x+4)}+\sqrt{2}-\cos{(6x+4)}+1}\right)+2\right)$$

input
```
integrate(2/(3+cos(4+6*x)),x, algorithm="giac")
```

output
```
1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - sin(6*x + 4))/(sqrt(2)*
cos(6*x + 4) + sqrt(2) - cos(6*x + 4) + 1)) + 2)
```

3.15.9 Mupad [B] (verification not implemented)

Time = 26.69 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

$$\int \frac{2}{3+\cos(4+6x)}\,dx = \frac{\sqrt{2}\,(3\,x-\mathrm{atan}(\tan(3\,x+2)))}{6}+\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\tan(3\,x+2)}{2}\right)}{6}$$

input
```
int(2/(cos(6*x + 4) + 3),x)
```

output
```
(2^(1/2)*(3*x - atan(tan(3*x + 2))))/6 + (2^(1/2)*atan((2^(1/2)*tan(3*x +
2))/2))/6
```

3.16 $\int \frac{2\csc(4+6x)}{\cot(4+6x)+3\csc(4+6x)}\,dx$

3.16.1 Optimal result

Integrand size = 25, antiderivative size = 42

$$\int \frac{2\csc(4+6x)}{\cot(4+6x)+3\csc(4+6x)}\,dx=\frac{x}{\sqrt{2}}-\frac{\arctan\left(\frac{\sin(4+6x)}{3+2\sqrt{2}+\cos(4+6x)}\right)}{3\sqrt{2}}$$

output
```
1/2*x*2^(1/2)-1/6*arctan(sin(4+6*x)/(3+cos(4+6*x)+2*2^(1/2)))*2^(1/2)
```

3.16.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.52

$$\int \frac{2\csc(4+6x)}{\cot(4+6x)+3\csc(4+6x)}\,dx=\frac{\arctan\left(\frac{\tan(2+3x)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

input
```
Integrate[(2*Csc[4 + 6*x])/(Cot[4 + 6*x] + 3*Csc[4 + 6*x]),x]
```

output
```
ArcTan[Tan[2 + 3*x]/Sqrt[2]]/(3*Sqrt[2])
```

3.16.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {27, 3042, 3645, 3042, 3136}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2\csc(6x+4)}{\cot(6x+4)+3\csc(6x+4)}dx$$

$$\Big\downarrow \text{27}$$

$$2\int \frac{\csc(6x+4)}{\cot(6x+4)+3\csc(6x+4)}dx$$

$$\Big\downarrow \text{3042}$$

$$2\int \frac{\csc(6x+4)}{\cot(6x+4)+3\csc(6x+4)}dx$$

$$\Big\downarrow \text{3645}$$

$$2\int \frac{1}{\cos(6x+4)+3}dx$$

$$\Big\downarrow \text{3042}$$

$$2\int \frac{1}{\sin\left(6x+\frac{\pi}{2}+4\right)+3}dx$$

$$\Big\downarrow \text{3136}$$

$$2\left(\frac{x}{2\sqrt{2}}-\frac{\arctan\left(\frac{\sin(6x+4)}{\cos(6x+4)+2\sqrt{2}+3}\right)}{6\sqrt{2}}\right)$$

input
```
Int[(2*Csc[4 + 6*x])/(Cot[4 + 6*x] + 3*Csc[4 + 6*x]),x]
```

output
```
2*(x/(2*Sqrt[2]) - ArcTan[Sin[4 + 6*x]/(3 + 2*Sqrt[2] + Cos[4 + 6*x])]/(6*
Sqrt[2]))
```

3.16.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3136
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

rule 3645
```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Int[1/(b + a*Sin[d + e*x]
 + c*Cos[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && I
ntegerQ[n]
```

3.16.4 Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.43

method	result	size
derivativedivides	$\frac{\sqrt{2}\, \arctan\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)}{6}$	18
default	$\frac{\sqrt{2}\, \arctan\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)}{6}$	18
risch	$\frac{i\sqrt{2}\, \ln\left(e^{2i(2+3x)}+3+2\sqrt{2}\right)}{12}-\frac{i\sqrt{2}\, \ln\left(e^{2i(2+3x)}+3-2\sqrt{2}\right)}{12}$	48

input
```
int(2*csc(4+6*x)/(cot(4+6*x)+3*csc(4+6*x)),x,method=_RETURNVERBOSE)
```

output
```
1/6*2^(1/2)*arctan(1/2*tan(2+3*x)*2^(1/2))
```

3.16.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.74

$$\int \frac{2\csc(4+6x)}{\cot(4+6x)+3\csc(4+6x)}\,dx = -\frac{1}{12}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(6x+4)+\sqrt{2}}{4\sin(6x+4)}\right)$$

input
```
integrate(2*csc(4+6*x)/(cot(4+6*x)+3*csc(4+6*x)),x, algorithm="fricas")
```

output
```
-1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(6*x + 4) + sqrt(2))/sin(6*x + 4))
```

3.16.6 Sympy [F]

$$\int \frac{2\csc(4+6x)}{\cot(4+6x)+3\csc(4+6x)}\,dx = 2\int \frac{\csc(6x+4)}{\cot(6x+4)+3\csc(6x+4)}\,dx$$

input
```
integrate(2*csc(4+6*x)/(cot(4+6*x)+3*csc(4+6*x)),x)
```

output
```
2*Integral(csc(6*x + 4)/(cot(6*x + 4) + 3*csc(6*x + 4)), x)
```

3.16.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.64

$$\int \frac{2\csc(4+6x)}{\cot(4+6x)+3\csc(4+6x)}\,dx = \frac{1}{6}\sqrt{2}\arctan\left(\frac{\sqrt{2}\sin(6x+4)}{2(\cos(6x+4)+1)}\right)$$

input
```
integrate(2*csc(4+6*x)/(cot(4+6*x)+3*csc(4+6*x)),x, algorithm="maxima")
```

output
```
1/6*sqrt(2)*arctan(1/2*sqrt(2)*sin(6*x + 4)/(cos(6*x + 4) + 1))
```

3.16.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.36

$$\int \frac{2\csc(4+6x)}{\cot(4+6x)+3\csc(4+6x)}\,dx = \frac{1}{6}\sqrt{2}\left(3\,x+\arctan\left(-\frac{\sqrt{2}\sin\left(6\,x+4\right)-\sin\left(6\,x+4\right)}{\sqrt{2}\cos\left(6\,x+4\right)+\sqrt{2}-\cos\left(6\,x+4\right)+1}\right)+2\right)$$

input
```
integrate(2*csc(4+6*x)/(cot(4+6*x)+3*csc(4+6*x)),x, algorithm="giac")
```

output
```
1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - sin(6*x + 4))/(sqrt(2)*
cos(6*x + 4) + sqrt(2) - cos(6*x + 4) + 1)) + 2)
```

3.16.9 Mupad [B] (verification not implemented)

Time = 26.82 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.40

$$\int \frac{2\csc(4+6x)}{\cot(4+6x)+3\csc(4+6x)}\,dx = \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\tan(3\,x+2)}{2}\right)}{6}$$

input
```
int(2/(sin(6*x + 4)*(cot(6*x + 4) + 3/sin(6*x + 4))),x)
```

output
```
(2^(1/2)*atan((2^(1/2)*tan(3*x + 2))/2))/6
```

3.17 $\int \frac{1}{2-\sin^2(2+3x)}\,dx$

3.17.1 Optimal result

Integrand size = 14, antiderivative size = 48

$$\int \frac{1}{2-\sin^2(2+3x)}\,dx = \frac{x}{\sqrt{2}} - \frac{\arctan\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\cos^2(2+3x)}\right)}{3\sqrt{2}}$$

output
```
1/2*x*2^(1/2)-1/6*arctan(cos(2+3*x)*sin(2+3*x)/(1+cos(2+3*x)^2+2^(1/2)))*2
^(1/2)
```

3.17.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.46

$$\int \frac{1}{2-\sin^2(2+3x)}\,dx = \frac{\arctan\left(\frac{\tan(2+3x)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

input
```
Integrate[(2 - Sin[2 + 3*x]^2)^(-1),x]
```

output
```
ArcTan[Tan[2 + 3*x]/Sqrt[2]]/(3*Sqrt[2])
```

3.17.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.46, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 3660, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{2-\sin^2(3x+2)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{2-\sin(3x+2)^2}dx$$
$$\Big\downarrow \text{3660}$$
$$\frac{1}{3}\int \frac{1}{\tan^2(3x+2)+2}d\tan(3x+2)$$
$$\Big\downarrow \text{216}$$
$$\frac{\arctan\left(\frac{\tan(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

input
```
Int[(2 - Sin[2 + 3*x]^2)^(-1),x]
```

output
```
ArcTan[Tan[2 + 3*x]/Sqrt[2]]/(3*Sqrt[2])
```

3.17.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

3.17.4 Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.38

method	result	size
derivativedivides	$\frac{\sqrt{2}\,\arctan\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)}{6}$	18
default	$\frac{\sqrt{2}\,\arctan\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)}{6}$	18
risch	$\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{2i(2+3x)}+3+2\sqrt{2}\right)}{12}-\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{2i(2+3x)}+3-2\sqrt{2}\right)}{12}$	48

input
```
int(1/(2-sin(2+3*x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/6*2^(1/2)*arctan(1/2*tan(2+3*x)*2^(1/2))
```

3.17.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

$$\int \frac{1}{2-\sin^2(2+3x)}\,dx = -\frac{1}{12}\,\sqrt{2}\arctan\left(\frac{3\,\sqrt{2}\cos\left(3\,x+2\right)^2-\sqrt{2}}{4\,\cos\left(3\,x+2\right)\sin\left(3\,x+2\right)}\right)$$

input
```
integrate(1/(2-sin(2+3*x)^2),x, algorithm="fricas")
```

output
```
-1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(3*x + 2)^2 - sqrt(2))/(cos(3*x + 2
)*sin(3*x + 2)))
```

3.17.6 Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.58

$$\int \frac{1}{2-\sin^2(2+3x)}\,dx = \frac{\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{3x}{2}+1\right)-1\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{6} + \frac{\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{3x}{2}+1\right)+1\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{6}$$

input
```
integrate(1/(2-sin(2+3*x)**2),x)
```

output
```
sqrt(2)*(atan(sqrt(2)*tan(3*x/2 + 1) - 1) + pi*floor((3*x/2 - pi/2 + 1)/pi
))/6 + sqrt(2)*(atan(sqrt(2)*tan(3*x/2 + 1) + 1) + pi*floor((3*x/2 - pi/2
+ 1)/pi))/6
```

3.17.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.35

$$\int \frac{1}{2-\sin^2(2+3x)}\,dx = \frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\tan\left(3x+2\right)\right)$$

input
```
integrate(1/(2-sin(2+3*x)^2),x, algorithm="maxima")
```

output
```
1/6*sqrt(2)*arctan(1/2*sqrt(2)*tan(3*x + 2))
```

3.17.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int \frac{1}{2-\sin^2(2+3x)}\,dx = \frac{1}{6}\sqrt{2}\left(3x+\arctan\left(-\frac{\sqrt{2}\sin\left(6x+4\right)-\sin\left(6x+4\right)}{\sqrt{2}\cos\left(6x+4\right)+\sqrt{2}-\cos\left(6x+4\right)+1}\right)+2\right)$$

input
```
integrate(1/(2-sin(2+3*x)^2),x, algorithm="giac")
```

output
```
1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - sin(6*x + 4))/(sqrt(2)*
cos(6*x + 4) + sqrt(2) - cos(6*x + 4) + 1)) + 2)
```

3.17.9 Mupad [B] (verification not implemented)

Time = 26.95 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.75

$$\int \frac{1}{2-\sin^2(2+3x)}\,dx = \frac{\sqrt{2}\,(3\,x-\operatorname{atan}(\tan(3\,x+2)))}{6} + \frac{\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}\tan(3\,x+2)}{2}\right)}{6}$$

input
```
int(-1/(sin(3*x + 2)^2 - 2),x)
```

output
```
(2^(1/2)*(3*x - atan(tan(3*x + 2))))/6 + (2^(1/2)*atan((2^(1/2)*tan(3*x +
2))/2))/6
```

3.18 $\int \frac{1}{1+\cos^2(2+3x)} dx$

3.18.1 Optimal result

Integrand size = 12, antiderivative size = 48

$$\int \frac{1}{1+\cos^2(2+3x)} dx = \frac{x}{\sqrt{2}} - \frac{\arctan\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\cos^2(2+3x)}\right)}{3\sqrt{2}}$$

output
```
1/2*x*2^(1/2)-1/6*arctan(cos(2+3*x)*sin(2+3*x)/(1+cos(2+3*x)^2+2^(1/2)))*2
^(1/2)
```

3.18.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.46

$$\int \frac{1}{1+\cos^2(2+3x)} dx = \frac{\arctan\left(\frac{\tan(2+3x)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

input
```
Integrate[(1 + Cos[2 + 3*x]^2)^(-1),x]
```

output
```
ArcTan[Tan[2 + 3*x]/Sqrt[2]]/(3*Sqrt[2])
```

3.18.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.46, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3660, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^2(3x+2)+1}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin\left(3x+\frac{\pi}{2}+2\right)^2+1}dx$$

$$\Big\downarrow \text{ 3660}$$

$$-\frac{1}{3}\int \frac{1}{2\cot^2(3x+2)+1}d\cot(3x+2)$$

$$\Big\downarrow \text{ 216}$$

$$-\frac{\arctan\left(\sqrt{2}\cot(3x+2)\right)}{3\sqrt{2}}$$

input
```
Int[(1 + Cos[2 + 3*x]^2)^(-1),x]
```

output
```
-1/3*ArcTan[Sqrt[2]*Cot[2 + 3*x]]/Sqrt[2]
```

3.18.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

3.18.4 Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.38

method	result	size
derivativedivides	$\frac{\sqrt{2}\,\arctan\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)}{6}$	18
default	$\frac{\sqrt{2}\,\arctan\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)}{6}$	18
risch	$\frac{i\sqrt{2}\,\ln\left(e^{2i(2+3x)}+3+2\sqrt{2}\right)}{12}-\frac{i\sqrt{2}\,\ln\left(e^{2i(2+3x)}+3-2\sqrt{2}\right)}{12}$	48

input
```
int(1/(1+cos(2+3*x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/6*2^(1/2)*arctan(1/2*tan(2+3*x)*2^(1/2))
```

3.18.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

$$\int \frac{1}{1+\cos^2(2+3x)}\,dx = -\frac{1}{12}\,\sqrt{2}\arctan\left(\frac{3\,\sqrt{2}\cos\left(3\,x+2\right)^2-\sqrt{2}}{4\,\cos\left(3\,x+2\right)\sin\left(3\,x+2\right)}\right)$$

input
```
integrate(1/(1+cos(2+3*x)^2),x, algorithm="fricas")
```

output
```
-1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(3*x + 2)^2 - sqrt(2))/(cos(3*x + 2
)*sin(3*x + 2)))
```

3.18.6 Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.58

$$\int \frac{1}{1+\cos^2(2+3x)}\,dx = \frac{\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{3x}{2}+1\right)-1\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{6} + \frac{\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{3x}{2}+1\right)+1\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{6}$$

input
```
integrate(1/(1+cos(2+3*x)**2),x)
```

output
```
sqrt(2)*(atan(sqrt(2)*tan(3*x/2 + 1) - 1) + pi*floor((3*x/2 - pi/2 + 1)/pi
))/6 + sqrt(2)*(atan(sqrt(2)*tan(3*x/2 + 1) + 1) + pi*floor((3*x/2 - pi/2
+ 1)/pi))/6
```

3.18.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.35

$$\int \frac{1}{1+\cos^2(2+3x)}\,dx = \frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\tan(3x+2)\right)$$

input
```
integrate(1/(1+cos(2+3*x)^2),x, algorithm="maxima")
```

output
```
1/6*sqrt(2)*arctan(1/2*sqrt(2)*tan(3*x + 2))
```

3.18.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int \frac{1}{1+\cos^2(2+3x)}\,dx = \frac{1}{6}\sqrt{2}\left(3x+\arctan\left(-\frac{\sqrt{2}\sin(6x+4)-\sin(6x+4)}{\sqrt{2}\cos(6x+4)+\sqrt{2}-\cos(6x+4)+1}\right)+2\right)$$

input
```
integrate(1/(1+cos(2+3*x)^2),x, algorithm="giac")
```

output
```
1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - sin(6*x + 4))/(sqrt(2)*
cos(6*x + 4) + sqrt(2) - cos(6*x + 4) + 1)) + 2)
```

3.18.9 Mupad [B] (verification not implemented)

Time = 27.45 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.75

$$\int \frac{1}{1+\cos^2(2+3x)}\,dx = \frac{\sqrt{2}\,(3\,x-\operatorname{atan}(\tan(3\,x+2)))}{6}+\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\tan(3\,x+2)}{2}\right)}{6}$$

input
```
int(1/(cos(3*x + 2)^2 + 1),x)
```

output
```
(2^(1/2)*(3*x - atan(tan(3*x + 2))))/6 + (2^(1/2)*atan((2^(1/2)*tan(3*x +
2))/2))/6
```

3.19 $\int \frac{1}{2\cos^2(2+3x)+\sin^2(2+3x)}\,dx$

3.19.1 Optimal result

Integrand size = 21, antiderivative size = 48

$$\int \frac{1}{2\cos^2(2+3x)+\sin^2(2+3x)}\,dx=\frac{x}{\sqrt{2}}-\frac{\arctan\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\cos^2(2+3x)}\right)}{3\sqrt{2}}$$

output
```
1/2*x*2^(1/2)-1/6*arctan(cos(2+3*x)*sin(2+3*x)/(1+cos(2+3*x)^2+2^(1/2)))*2
^(1/2)
```

3.19.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.46

$$\int \frac{1}{2\cos^2(2+3x)+\sin^2(2+3x)}\,dx=\frac{\arctan\left(\frac{\tan(2+3x)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

input
```
Integrate[(2*Cos[2 + 3*x]^2 + Sin[2 + 3*x]^2)^(-1),x]
```

output
```
ArcTan[Tan[2 + 3*x]/Sqrt[2]]/(3*Sqrt[2])
```

3.19.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.46, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 4889, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sin^2(3x+2)+2\cos^2(3x+2)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(3x+2)^2+2\cos(3x+2)^2}dx$$

$$\Big\downarrow \text{4889}$$

$$\frac{1}{3}\int \frac{1}{\tan^2(3x+2)+2}d\tan(3x+2)$$

$$\Big\downarrow \text{216}$$

$$\frac{\arctan\left(\frac{\tan(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

input
```
Int[(2*Cos[2 + 3*x]^2 + Sin[2 + 3*x]^2)^(-1),x]
```

output
```
ArcTan[Tan[2 + 3*x]/Sqrt[2]]/(3*Sqrt[2])
```

3.19.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.19.4 Maple [A] (verified)

Time = 1.43 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.38

method	result	size
derivativedivides	$\frac{\sqrt{2}\,\arctan\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)}{6}$	18
default	$\frac{\sqrt{2}\,\arctan\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)}{6}$	18
risch	$\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{2i(2+3x)}+3+2\sqrt{2}\right)}{12}-\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{2i(2+3x)}+3-2\sqrt{2}\right)}{12}$	48

input
```
int(1/(2*cos(2+3*x)^2+sin(2+3*x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/6*2^(1/2)*arctan(1/2*tan(2+3*x)*2^(1/2))
```

3.19.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

$$\int \frac{1}{2\cos^2(2+3x)+\sin^2(2+3x)}\,dx = -\frac{1}{12}\,\sqrt{2}\arctan\left(\frac{3\,\sqrt{2}\cos\left(3\,x+2\right)^2-\sqrt{2}}{4\,\cos\left(3\,x+2\right)\sin\left(3\,x+2\right)}\right)$$

input
```
integrate(1/(2*cos(2+3*x)^2+sin(2+3*x)^2),x, algorithm="fricas")
```

output
```
-1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(3*x + 2)^2 - sqrt(2))/(cos(3*x + 2
)*sin(3*x + 2)))
```

3.19.6 Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.58

$$\int \frac{1}{2\cos^2(2+3x)+\sin^2(2+3x)}\,dx = \frac{\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{3x}{2}+1\right)-1\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{6} + \frac{\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{3x}{2}+1\right)+1\right)+\pi\left\lfloor\frac{\frac{3x}{2}-\frac{\pi}{2}+1}{\pi}\right\rfloor\right)}{6}$$

input
```
integrate(1/(2*cos(2+3*x)**2+sin(2+3*x)**2),x)
```

output
```
sqrt(2)*(atan(sqrt(2)*tan(3*x/2 + 1) - 1) + pi*floor((3*x/2 - pi/2 + 1)/pi
))/6 + sqrt(2)*(atan(sqrt(2)*tan(3*x/2 + 1) + 1) + pi*floor((3*x/2 - pi/2
+ 1)/pi))/6
```

3.19.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.35

$$\int \frac{1}{2\cos^2(2+3x)+\sin^2(2+3x)}\,dx = \frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\tan(3x+2)\right)$$

input
```
integrate(1/(2*cos(2+3*x)^2+sin(2+3*x)^2),x, algorithm="maxima")
```

output
```
1/6*sqrt(2)*arctan(1/2*sqrt(2)*tan(3*x + 2))
```

3.19.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int \frac{1}{2\cos^2(2+3x)+\sin^2(2+3x)}\,dx = \frac{1}{6}\sqrt{2}\left(3x+\arctan\left(-\frac{\sqrt{2}\sin(6x+4)-\sin(6x+4)}{\sqrt{2}\cos(6x+4)+\sqrt{2}-\cos(6x+4)+1}\right)+2\right)$$

input
```
integrate(1/(2*cos(2+3*x)^2+sin(2+3*x)^2),x, algorithm="giac")
```

output
```
1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - sin(6*x + 4))/(sqrt(2)*
cos(6*x + 4) + sqrt(2) - cos(6*x + 4) + 1)) + 2)
```

3.19.9 Mupad [B] (verification not implemented)

Time = 26.48 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.75

$$\int \frac{1}{2\cos^2(2+3x)+\sin^2(2+3x)}\,dx = \frac{\sqrt{2}\,(3\,x-\operatorname{atan}(\tan(3\,x+2)))}{6}+\frac{\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}\tan(3\,x+2)}{2}\right)}{6}$$

input
```
int(1/(sin(3*x + 2)^2 + 2*cos(3*x + 2)^2),x)
```

output
```
(2^(1/2)*(3*x - atan(tan(3*x + 2))))/6 + (2^(1/2)*atan((2^(1/2)*tan(3*x +
2))/2))/6
```

3.20 $\int \frac{\sec^2(2+3x)}{2+\tan^2(2+3x)}\,dx$

3.20.1 Optimal result

Integrand size = 21, antiderivative size = 48

$$\int \frac{\sec^2(2+3x)}{2+\tan^2(2+3x)}\,dx = \frac{x}{\sqrt{2}} - \frac{\arctan\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\cos^2(2+3x)}\right)}{3\sqrt{2}}$$

output
```
1/2*x*2^(1/2)-1/6*arctan(cos(2+3*x)*sin(2+3*x)/(1+cos(2+3*x)^2+2^(1/2)))*2
^(1/2)
```

3.20.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.46

$$\int \frac{\sec^2(2+3x)}{2+\tan^2(2+3x)}\,dx = \frac{\arctan\left(\frac{\tan(2+3x)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

input
```
Integrate[Sec[2 + 3*x]^2/(2 + Tan[2 + 3*x]^2),x]
```

output
```
ArcTan[Tan[2 + 3*x]/Sqrt[2]]/(3*Sqrt[2])
```

3.20.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.46, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 4158, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(3x+2)}{\tan^2(3x+2)+2} \, dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sec(3x+2)^2}{\tan(3x+2)^2+2}dx$$

$$\Big\downarrow \; 4158$$

$$\frac{1}{3}\int \frac{1}{\tan^2(3x+2)+2}d\tan(3x+2)$$

$$\Big\downarrow \; 216$$

$$\frac{\arctan\left(\frac{\tan(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

input
```
Int[Sec[2 + 3*x]^2/(2 + Tan[2 + 3*x]^2),x]
```

output
```
ArcTan[Tan[2 + 3*x]/Sqrt[2]]/(3*Sqrt[2])
```

3.20.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

3.20.4 Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.38

method	result	size
derivativedivides	$\frac{\sqrt{2}\,\arctan\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)}{6}$	18
default	$\frac{\sqrt{2}\,\arctan\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)}{6}$	18
risch	$\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{2i(2+3x)}+3+2\sqrt{2}\right)}{12}-\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{2i(2+3x)}+3-2\sqrt{2}\right)}{12}$	48

input
```
int(sec(2+3*x)^2/(2+tan(2+3*x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/6*2^(1/2)*arctan(1/2*tan(2+3*x)*2^(1/2))
```

3.20.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

$$\int \frac{\sec^2(2+3x)}{2+\tan^2(2+3x)}\,dx = -\frac{1}{12}\,\sqrt{2}\arctan\left(\frac{3\,\sqrt{2}\cos\left(3\,x+2\right)^2-\sqrt{2}}{4\,\cos\left(3\,x+2\right)\sin\left(3\,x+2\right)}\right)$$

input
```
integrate(sec(2+3*x)^2/(2+tan(2+3*x)^2),x, algorithm="fricas")
```

output
```
-1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(3*x + 2)^2 - sqrt(2))/(cos(3*x + 2
)*sin(3*x + 2)))
```

3.20.6 Sympy [F]

$$\int \frac{\sec^2(2+3x)}{2+\tan^2(2+3x)}\,dx = \int \frac{\sec^2(3x+2)}{\tan^2(3x+2)+2}\,dx$$

input
```
integrate(sec(2+3*x)**2/(2+tan(2+3*x)**2),x)
```

output
```
Integral(sec(3*x + 2)**2/(tan(3*x + 2)**2 + 2), x)
```

3.20.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.35

$$\int \frac{\sec^2(2+3x)}{2+\tan^2(2+3x)}\,dx = \frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\tan(3x+2)\right)$$

input
```
integrate(sec(2+3*x)^2/(2+tan(2+3*x)^2),x, algorithm="maxima")
```

output
```
1/6*sqrt(2)*arctan(1/2*sqrt(2)*tan(3*x + 2))
```

3.20.8 Giac [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.35

$$\int \frac{\sec^2(2+3x)}{2+\tan^2(2+3x)}\,dx = \frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\tan(3x+2)\right)$$

input
```
integrate(sec(2+3*x)^2/(2+tan(2+3*x)^2),x, algorithm="giac")
```

output
```
1/6*sqrt(2)*arctan(1/2*sqrt(2)*tan(3*x + 2))
```

3.20.9 Mupad [B] (verification not implemented)

Time = 26.89 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.35

$$\int \frac{\sec^2(2+3x)}{2+\tan^2(2+3x)}\,dx = \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\tan(3\,x+2)}{2}\right)}{6}$$

input
```
int(1/(cos(3*x + 2)^2*(tan(3*x + 2)^2 + 2)),x)
```

output
```
(2^(1/2)*atan((2^(1/2)*tan(3*x + 2))/2))/6
```

3.21 $\int \frac{\csc^2(2+3x)}{1+2\cot^2(2+3x)}\,dx$

3.21.1 Optimal result

Integrand size = 23, antiderivative size = 48

$$\int \frac{\csc^2(2+3x)}{1+2\cot^2(2+3x)}\,dx = \frac{x}{\sqrt{2}} - \frac{\arctan\left(\frac{\cos(2+3x)\sin(2+3x)}{1+\sqrt{2}+\cos^2(2+3x)}\right)}{3\sqrt{2}}$$

output
```
1/2*x*2^(1/2)-1/6*arctan(cos(2+3*x)*sin(2+3*x)/(1+cos(2+3*x)^2+2^(1/2)))*2
^(1/2)
```

3.21.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.46

$$\int \frac{\csc^2(2+3x)}{1+2\cot^2(2+3x)}\,dx = \frac{\arctan\left(\frac{\tan(2+3x)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

input
```
Integrate[Csc[2 + 3*x]^2/(1 + 2*Cot[2 + 3*x]^2),x]
```

output
```
ArcTan[Tan[2 + 3*x]/Sqrt[2]]/(3*Sqrt[2])
```

3.21.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.46, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4158, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(3x+2)}{2\cot^2(3x+2)+1}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec\left(3x+\frac{\pi}{2}+2\right)^2}{2\tan\left(3x+\frac{\pi}{2}+2\right)^2+1}dx$$

$$\Big\downarrow \text{4158}$$

$$-\frac{1}{3}\int \frac{1}{2\cot^2(3x+2)+1}d\cot(3x+2)$$

$$\Big\downarrow \text{216}$$

$$-\frac{\arctan\left(\sqrt{2}\cot(3x+2)\right)}{3\sqrt{2}}$$

input
```
Int[Csc[2 + 3*x]^2/(1 + 2*Cot[2 + 3*x]^2),x]
```

output
```
-1/3*ArcTan[Sqrt[2]*Cot[2 + 3*x]]/Sqrt[2]
```

3.21.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

3.21.4 Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.38

method	result	size
derivativedivides	$\frac{\sqrt{2}\,\arctan\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)}{6}$	18
default	$\frac{\sqrt{2}\,\arctan\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)}{6}$	18
risch	$\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{2i(2+3x)}+3+2\sqrt{2}\right)}{12}-\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{2i(2+3x)}+3-2\sqrt{2}\right)}{12}$	48

input
```
int(csc(2+3*x)^2/(1+2*cot(2+3*x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/6*2^(1/2)*arctan(1/2*tan(2+3*x)*2^(1/2))
```

3.21.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

$$\int \frac{\csc^2(2+3x)}{1+2\cot^2(2+3x)}\,dx = -\frac{1}{12}\,\sqrt{2}\arctan\left(\frac{3\,\sqrt{2}\cos\left(3\,x+2\right)^2-\sqrt{2}}{4\,\cos\left(3\,x+2\right)\sin\left(3\,x+2\right)}\right)$$

input
```
integrate(csc(2+3*x)^2/(1+2*cot(2+3*x)^2),x, algorithm="fricas")
```

output
```
-1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(3*x + 2)^2 - sqrt(2))/(cos(3*x + 2
)*sin(3*x + 2)))
```

3.21.6 Sympy [F]

$$\int \frac{\csc^2(2+3x)}{1+2\cot^2(2+3x)}\,dx = \int \frac{\csc^2(3x+2)}{2\cot^2(3x+2)+1}\,dx$$

input
```
integrate(csc(2+3*x)**2/(1+2*cot(2+3*x)**2),x)
```

output
```
Integral(csc(3*x + 2)**2/(2*cot(3*x + 2)**2 + 1), x)
```

3.21.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.35

$$\int \frac{\csc^2(2+3x)}{1+2\cot^2(2+3x)}\,dx = \frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\tan(3x+2)\right)$$

input
```
integrate(csc(2+3*x)^2/(1+2*cot(2+3*x)^2),x, algorithm="maxima")
```

output
```
1/6*sqrt(2)*arctan(1/2*sqrt(2)*tan(3*x + 2))
```

3.21.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int \frac{\csc^2(2+3x)}{1+2\cot^2(2+3x)}\,dx$$
$$= \frac{1}{6}\sqrt{2}\left(3x+\arctan\left(-\frac{\sqrt{2}\sin(6x+4)-\sin(6x+4)}{\sqrt{2}\cos(6x+4)+\sqrt{2}-\cos(6x+4)+1}\right)+2\right)$$

input
```
integrate(csc(2+3*x)^2/(1+2*cot(2+3*x)^2),x, algorithm="giac")
```

output
```
1/6*sqrt(2)*(3*x + arctan(-(sqrt(2)*sin(6*x + 4) - sin(6*x + 4))/(sqrt(2)*
cos(6*x + 4) + sqrt(2) - cos(6*x + 4) + 1)) + 2)
```

3.21.9 Mupad [B] (verification not implemented)

Time = 27.37 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.35

$$\int \frac{\csc^2(2+3x)}{1+2\cot^2(2+3x)}\,dx = \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\tan(3\,x+2)}{2}\right)}{6}$$

input
```
int(1/(sin(3*x + 2)^2*(2*cot(3*x + 2)^2 + 1)),x)
```

output
```
(2^(1/2)*atan((2^(1/2)*tan(3*x + 2))/2))/6
```

3.22 $\int -\frac{2}{1+3\cos(4+6x)}\,dx$

3.22.1 Optimal result

Integrand size = 14, antiderivative size = 61

$$\int -\frac{2}{1+3\cos(4+6x)}\,dx = \frac{\log\left(\sqrt{2}\cos(2+3x)-\sin(2+3x)\right)}{6\sqrt{2}} - \frac{\log\left(\sqrt{2}\cos(2+3x)+\sin(2+3x)\right)}{6\sqrt{2}}$$

output
```
1/12*ln(-sin(2+3*x)+cos(2+3*x)*2^(1/2))*2^(1/2)-1/12*ln(sin(2+3*x)+cos(2+3
*x)*2^(1/2))*2^(1/2)
```

3.22.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.36

$$\int -\frac{2}{1+3\cos(4+6x)}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\tan(2+3x)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

input
```
Integrate[-2/(1 + 3*Cos[4 + 6*x]),x]
```

output
```
-1/3*ArcTanh[Tan[2 + 3*x]/Sqrt[2]]/Sqrt[2]
```

3.22.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.36, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {27, 3042, 3138, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int -\frac{2}{3\cos(6x+4)+1}dx$$

$$\Big\downarrow \; 27$$

$$-2\int \frac{1}{3\cos(6x+4)+1}dx$$

$$\Big\downarrow \; 3042$$

$$-2\int \frac{1}{3\sin\left(6x+\frac{\pi}{2}+4\right)+1}dx$$

$$\Big\downarrow \; 3138$$

$$-\frac{2}{3}\int \frac{1}{4-2\tan^2(3x+2)}d\tan(3x+2)$$

$$\Big\downarrow \; 219$$

$$-\frac{\text{arctanh}\left(\frac{\tan(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

input
```
Int[-2/(1 + 3*Cos[4 + 6*x]),x]
```

output
```
-1/3*ArcTanh[Tan[2 + 3*x]/Sqrt[2]]/Sqrt[2]
```

3.22.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

3.22.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.30

method	result	size
derivativedivides	$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)}{6}$	18
default	$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)}{6}$	18
risch	$-\frac{\sqrt{2}\,\ln\left(e^{2i(2+3x)}+\frac{1}{3}+\frac{2i\sqrt{2}}{3}\right)}{12}+\frac{\sqrt{2}\,\ln\left(e^{2i(2+3x)}+\frac{1}{3}-\frac{2i\sqrt{2}}{3}\right)}{12}$	48

input
```
int(-2/(1+3*cos(4+6*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/6*2^(1/2)*arctanh(1/2*tan(2+3*x)*2^(1/2))
```

3.22.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.21

$$\int -\frac{2}{1+3\cos(4+6x)}\,dx$$
$$=\frac{1}{24}\sqrt{2}\log\left(-\frac{7\cos(6x+4)^2+4\left(\sqrt{2}\cos(6x+4)+3\sqrt{2}\right)\sin(6x+4)-6\cos(6x+4)-17}{9\cos(6x+4)^2+6\cos(6x+4)+1}\right)$$

input
```
integrate(-2/(1+3*cos(4+6*x)),x, algorithm="fricas")
```

output
```
1/24*sqrt(2)*log(-(7*cos(6*x + 4)^2 + 4*(sqrt(2)*cos(6*x + 4) + 3*sqrt(2))
*sin(6*x + 4) - 6*cos(6*x + 4) - 17)/(9*cos(6*x + 4)^2 + 6*cos(6*x + 4) +
1))
```

3.22.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.64

$$\int -\frac{2}{1+3\cos(4+6x)}\,dx = \frac{\sqrt{2}\log\left(\tan\left(3x+2\right)-\sqrt{2}\right)}{12} - \frac{\sqrt{2}\log\left(\tan\left(3x+2\right)+\sqrt{2}\right)}{12}$$

input
```
integrate(-2/(1+3*cos(4+6*x)),x)
```

output
```
sqrt(2)*log(tan(3*x + 2) - sqrt(2))/12 - sqrt(2)*log(tan(3*x + 2) + sqrt(2
))/12
```

3.22.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int -\frac{2}{1+3\cos(4+6x)}\,dx = \frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{\sin(6x+4)}{\cos(6x+4)+1}}{\sqrt{2}+\frac{\sin(6x+4)}{\cos(6x+4)+1}}\right)$$

input
```
integrate(-2/(1+3*cos(4+6*x)),x, algorithm="maxima")
```

output
```
1/12*sqrt(2)*log(-(sqrt(2) - sin(6*x + 4)/(cos(6*x + 4) + 1))/(sqrt(2) + s
in(6*x + 4)/(cos(6*x + 4) + 1)))
```

3.22.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.64

$$\int -\frac{2}{1+3\cos(4+6x)}\,dx = \frac{1}{12}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\tan\left(3x+2\right)\right|}{\left|2\sqrt{2}+2\tan\left(3x+2\right)\right|}\right)$$

input
```
integrate(-2/(1+3*cos(4+6*x)),x, algorithm="giac")
```

output
```
1/12*sqrt(2)*log(abs(-2*sqrt(2) + 2*tan(3*x + 2))/abs(2*sqrt(2) + 2*tan(3*
x + 2)))
```

3.22.9 Mupad [B] (verification not implemented)

Time = 28.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.28

$$\int -\frac{2}{1+3\cos(4+6x)}\,dx = -\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\tan(3x+2)}{2}\right)}{6}$$

input
```
int(-2/(3*cos(6*x + 4) + 1),x)
```

output
```
-(2^(1/2)*atanh((2^(1/2)*tan(3*x + 2))/2))/6
```

3.23 $\int -\frac{2\csc(4+6x)}{3\cot(4+6x)+\csc(4+6x)}\,dx$

3.23.1 Optimal result

Integrand size = 25, antiderivative size = 61

$$\int -\frac{2\csc(4+6x)}{3\cot(4+6x)+\csc(4+6x)}\,dx = \frac{\log\left(\sqrt{2}\cos(2+3x)-\sin(2+3x)\right)}{6\sqrt{2}} - \frac{\log\left(\sqrt{2}\cos(2+3x)+\sin(2+3x)\right)}{6\sqrt{2}}$$

output
```
1/12*ln(-sin(2+3*x)+cos(2+3*x)*2^(1/2))*2^(1/2)-1/12*ln(sin(2+3*x)+cos(2+3
*x)*2^(1/2))*2^(1/2)
```

3.23.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.36

$$\int -\frac{2\csc(4+6x)}{3\cot(4+6x)+\csc(4+6x)}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\tan(2+3x)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

input
```
Integrate[(-2*Csc[4 + 6*x])/(3*Cot[4 + 6*x] + Csc[4 + 6*x]),x]
```

output
```
-1/3*ArcTanh[Tan[2 + 3*x]/Sqrt[2]]/Sqrt[2]
```

3.23.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.36, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {27, 3042, 3645, 3042, 3138, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int -\frac{2\csc(6x+4)}{3\cot(6x+4)+\csc(6x+4)}dx$$

$$\Big\downarrow \; 27$$

$$-2\int \frac{\csc(6x+4)}{3\cot(6x+4)+\csc(6x+4)}dx$$

$$\Big\downarrow \; 3042$$

$$-2\int \frac{\csc(6x+4)}{3\cot(6x+4)+\csc(6x+4)}dx$$

$$\Big\downarrow \; 3645$$

$$-2\int \frac{1}{3\cos(6x+4)+1}dx$$

$$\Big\downarrow \; 3042$$

$$-2\int \frac{1}{3\sin\left(6x+\frac{\pi}{2}+4\right)+1}dx$$

$$\Big\downarrow \; 3138$$

$$-\frac{2}{3}\int \frac{1}{4-2\tan^2(3x+2)}d\tan(3x+2)$$

$$\Big\downarrow \; 219$$

$$-\frac{\operatorname{arctanh}\left(\frac{\tan(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

input
```
Int[(-2*Csc[4 + 6*x])/(3*Cot[4 + 6*x] + Csc[4 + 6*x]),x]
```

output
```
-1/3*ArcTanh[Tan[2 + 3*x]/Sqrt[2]]/Sqrt[2]
```

3.23.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3645
```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Int[1/(b + a*Sin[d + e*x]
 + c*Cos[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && I
ntegerQ[n]
```

3.23.4 Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.30

method	result	size
derivativedivides	$-\frac{\sqrt{2}\, \operatorname{arctanh}\left(\frac{\tan (2+3 x) \sqrt{2}}{2}\right)}{6}$	18
default	$-\frac{\sqrt{2}\, \operatorname{arctanh}\left(\frac{\tan (2+3 x) \sqrt{2}}{2}\right)}{6}$	18
risch	$-\frac{\sqrt{2}\, \ln \left(\mathrm{e}^{2 i (2+3 x)}+\frac{1}{3}+\frac{2 i \sqrt{2}}{3}\right)}{12}+\frac{\sqrt{2}\, \ln \left(\mathrm{e}^{2 i (2+3 x)}+\frac{1}{3}-\frac{2 i \sqrt{2}}{3}\right)}{12}$	48

input
```
int(-2*csc(4+6*x)/(3*cot(4+6*x)+csc(4+6*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/6*2^(1/2)*arctanh(1/2*tan(2+3*x)*2^(1/2))
```

3.23.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.21

$$\int -\frac{2\csc(4+6x)}{3\cot(4+6x)+\csc(4+6x)}\,dx$$
$$=\frac{1}{24}\sqrt{2}\log\left(-\frac{7\cos\left(6\,x+4\right)^{2}+4\left(\sqrt{2}\cos\left(6\,x+4\right)+3\sqrt{2}\right)\sin\left(6\,x+4\right)-6\cos\left(6\,x+4\right)-17}{9\cos\left(6\,x+4\right)^{2}+6\cos\left(6\,x+4\right)+1}\right)$$

input
```
integrate(-2*csc(4+6*x)/(3*cot(4+6*x)+csc(4+6*x)),x, algorithm="fricas")
```

output
```
1/24*sqrt(2)*log(-(7*cos(6*x + 4)^2 + 4*(sqrt(2)*cos(6*x + 4) + 3*sqrt(2))
*sin(6*x + 4) - 6*cos(6*x + 4) - 17)/(9*cos(6*x + 4)^2 + 6*cos(6*x + 4) +
1))
```

3.23.6 Sympy [F]

$$\int -\frac{2\csc(4+6x)}{3\cot(4+6x)+\csc(4+6x)}\,dx=-2\int \frac{\csc\left(6x+4\right)}{3\cot\left(6x+4\right)+\csc\left(6x+4\right)}\,dx$$

input
```
integrate(-2*csc(4+6*x)/(3*cot(4+6*x)+csc(4+6*x)),x)
```

output
```
-2*Integral(csc(6*x + 4)/(3*cot(6*x + 4) + csc(6*x + 4)), x)
```

3.23.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int -\frac{2\csc(4+6x)}{3\cot(4+6x)+\csc(4+6x)}\,dx=\frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{\sin(6\,x+4)}{\cos(6\,x+4)+1}}{\sqrt{2}+\frac{\sin(6\,x+4)}{\cos(6\,x+4)+1}}\right)$$

input
```
integrate(-2*csc(4+6*x)/(3*cot(4+6*x)+csc(4+6*x)),x, algorithm="maxima")
```

output
```
1/12*sqrt(2)*log(-(sqrt(2) - sin(6*x + 4)/(cos(6*x + 4) + 1))/(sqrt(2) + s
in(6*x + 4)/(cos(6*x + 4) + 1)))
```

3.23.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.64

$$\int -\frac{2\csc(4+6x)}{3\cot(4+6x)+\csc(4+6x)}\,dx = \frac{1}{12}\,\sqrt{2}\log\left(\frac{\left|-2\,\sqrt{2}+2\,\tan\left(3\,x+2\right)\right|}{\left|2\,\sqrt{2}+2\,\tan\left(3\,x+2\right)\right|}\right)$$

input
```
integrate(-2*csc(4+6*x)/(3*cot(4+6*x)+csc(4+6*x)),x, algorithm="giac")
```

output
```
1/12*sqrt(2)*log(abs(-2*sqrt(2) + 2*tan(3*x + 2))/abs(2*sqrt(2) + 2*tan(3*
x + 2)))
```

3.23.9 Mupad [B] (verification not implemented)

Time = 28.62 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.28

$$\int -\frac{2\csc(4+6x)}{3\cot(4+6x)+\csc(4+6x)}\,dx = -\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\tan(3\,x+2)}{2}\right)}{6}$$

input
```
int(-2/(sin(6*x + 4)*(3*cot(6*x + 4) + 1/sin(6*x + 4))),x)
```

output
```
-(2^(1/2)*atanh((2^(1/2)*tan(3*x + 2))/2))/6
```

3.24 $\int \frac{1}{-2+3\sin^2(2+3x)}\,dx$

3.24.1 Optimal result

Integrand size = 14, antiderivative size = 61

$$\int \frac{1}{-2+3\sin^2(2+3x)}\,dx = \frac{\log\left(\sqrt{2}\cos(2+3x)-\sin(2+3x)\right)}{6\sqrt{2}} - \frac{\log\left(\sqrt{2}\cos(2+3x)+\sin(2+3x)\right)}{6\sqrt{2}}$$

output
```
1/12*ln(-sin(2+3*x)+cos(2+3*x)*2^(1/2))*2^(1/2)-1/12*ln(sin(2+3*x)+cos(2+3
*x)*2^(1/2))*2^(1/2)
```

3.24.2 Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.36

$$\int \frac{1}{-2+3\sin^2(2+3x)}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\tan(2+3x)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

input
```
Integrate[(-2 + 3*Sin[2 + 3*x]^2)^(-1),x]
```

output
```
-1/3*ArcTanh[Tan[2 + 3*x]/Sqrt[2]]/Sqrt[2]
```

3.24.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.36, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 3660, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{3\sin^2(3x+2)-2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{3\sin(3x+2)^2-2}dx$$
$$\Big\downarrow \text{3660}$$
$$\frac{1}{3}\int \frac{1}{\tan^2(3x+2)-2}d\tan(3x+2)$$
$$\Big\downarrow \text{220}$$
$$-\frac{\text{arctanh}\left(\frac{\tan(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

input
```
Int[(-2 + 3*Sin[2 + 3*x]^2)^(-1),x]
```

output
```
-1/3*ArcTanh[Tan[2 + 3*x]/Sqrt[2]]/Sqrt[2]
```

3.24.3.1 Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

3.24.4 Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.30

method	result	size
derivativedivides	$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)}{6}$	18
default	$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)}{6}$	18
risch	$-\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2i(2+3x)}+\frac{1}{3}+\frac{2i\sqrt{2}}{3}\right)}{12}+\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2i(2+3x)}+\frac{1}{3}-\frac{2i\sqrt{2}}{3}\right)}{12}$	48

input
```
int(1/(-2+3*sin(2+3*x)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*2^(1/2)*arctanh(1/2*tan(2+3*x)*2^(1/2))
```

3.24.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.39

$$\int \frac{1}{-2+3\sin^2(2+3x)}\,dx$$
$$=\frac{1}{24}\sqrt{2}\log\left(-\frac{7\cos(3x+2)^4-10\cos(3x+2)^2+4\left(\sqrt{2}\cos(3x+2)^3+\sqrt{2}\cos(3x+2)\right)\sin(3x+2)\cdots}{9\cos(3x+2)^4-6\cos(3x+2)^2+1}\right.$$

input
```
integrate(1/(-2+3*sin(2+3*x)^2),x, algorithm="fricas")
```

output
```
1/24*sqrt(2)*log(-(7*cos(3*x + 2)^4 - 10*cos(3*x + 2)^2 + 4*(sqrt(2)*cos(3
*x + 2)^3 + sqrt(2)*cos(3*x + 2))*sin(3*x + 2) - 1)/(9*cos(3*x + 2)^4 - 6*
cos(3*x + 2)^2 + 1))
```

3.24.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1481 vs. $2(53) = 106$.

Time = 7.83 (sec) , antiderivative size = 1481, normalized size of antiderivative = 24.28

$$\int \frac{1}{-2+3\sin^2(2+3x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(-2+3*sin(2+3*x)**2),x)
```

output
```
-2108299*sqrt(3)*sqrt(sqrt(3) + 2)*log(tan(3*x/2 + 1) - sqrt(2 - sqrt(3)))
/(-16557465*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 846881*sqrt(3) + 1466841
 + 9559457*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) - 57873*sqrt(2 - s
qrt(3))*log(tan(3*x/2 + 1) - sqrt(2 - sqrt(3)))/(-16557465*sqrt(2 - sqrt(3
))*sqrt(sqrt(3) + 2) - 846881*sqrt(3) + 1466841 + 9559457*sqrt(3)*sqrt(2 -
 sqrt(3))*sqrt(sqrt(3) + 2)) + 33413*sqrt(3)*sqrt(2 - sqrt(3))*log(tan(3*x
/2 + 1) - sqrt(2 - sqrt(3)))/(-16557465*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2
) - 846881*sqrt(3) + 1466841 + 9559457*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt
(3) + 2)) + 3651681*sqrt(sqrt(3) + 2)*log(tan(3*x/2 + 1) - sqrt(2 - sqrt(3
)))/(-16557465*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 846881*sqrt(3) + 1466
841 + 9559457*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) - 4768180*sqrt(
sqrt(3) + 2)*log(tan(3*x/2 + 1) + sqrt(2 - sqrt(3)))/(-16557465*sqrt(2 - s
qrt(3))*sqrt(sqrt(3) + 2) - 846881*sqrt(3) + 1466841 + 9559457*sqrt(3)*sqr
t(2 - sqrt(3))*sqrt(sqrt(3) + 2)) - 206136*sqrt(3)*sqrt(2 - sqrt(3))*log(t
an(3*x/2 + 1) + sqrt(2 - sqrt(3)))/(-16557465*sqrt(2 - sqrt(3))*sqrt(sqrt(
3) + 2) - 846881*sqrt(3) + 1466841 + 9559457*sqrt(3)*sqrt(2 - sqrt(3))*sqr
t(sqrt(3) + 2)) + 357038*sqrt(2 - sqrt(3))*log(tan(3*x/2 + 1) + sqrt(2 - s
qrt(3)))/(-16557465*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 846881*sqrt(3) +
 1466841 + 9559457*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) + 2752910*
sqrt(3)*sqrt(sqrt(3) + 2)*log(tan(3*x/2 + 1) + sqrt(2 - sqrt(3)))/(-165...
```

3.24.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.52

$$\int \frac{1}{-2+3\sin^2(2+3x)}\,dx = \frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-\tan(3x+2)}{\sqrt{2}+\tan(3x+2)}\right)$$

input
```
integrate(1/(-2+3*sin(2+3*x)^2),x, algorithm="maxima")
```

output
```
1/12*sqrt(2)*log(-(sqrt(2) - tan(3*x + 2))/(sqrt(2) + tan(3*x + 2)))
```

3.24.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.64

$$\int \frac{1}{-2+3\sin^2(2+3x)}\,dx = \frac{1}{12}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\,\tan\left(3\,x+2\right)\right|}{\left|2\sqrt{2}+2\,\tan\left(3\,x+2\right)\right|}\right)$$

input
```
integrate(1/(-2+3*sin(2+3*x)^2),x, algorithm="giac")
```

output
```
1/12*sqrt(2)*log(abs(-2*sqrt(2) + 2*tan(3*x + 2))/abs(2*sqrt(2) + 2*tan(3*
x + 2)))
```

3.24.9 Mupad [B] (verification not implemented)

Time = 27.99 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.28

$$\int \frac{1}{-2+3\sin^2(2+3x)}\,dx = -\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\tan(3\,x+2)}{2}\right)}{6}$$

input
```
int(1/(3*sin(3*x + 2)^2 - 2),x)
```

output
```
-(2^(1/2)*atanh((2^(1/2)*tan(3*x + 2))/2))/6
```

3.25 $\int \frac{1}{1-3\cos^2(2+3x)}\,dx$

3.25.1 Optimal result

Integrand size = 14, antiderivative size = 61

$$\int \frac{1}{1-3\cos^2(2+3x)}\,dx = \frac{\log\left(\sqrt{2}\cos(2+3x)-\sin(2+3x)\right)}{6\sqrt{2}} - \frac{\log\left(\sqrt{2}\cos(2+3x)+\sin(2+3x)\right)}{6\sqrt{2}}$$

output
```
1/12*ln(-sin(2+3*x)+cos(2+3*x)*2^(1/2))*2^(1/2)-1/12*ln(sin(2+3*x)+cos(2+3
*x)*2^(1/2))*2^(1/2)
```

3.25.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.36

$$\int \frac{1}{1-3\cos^2(2+3x)}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\tan(2+3x)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

input
```
Integrate[(1 - 3*Cos[2 + 3*x]^2)^(-1),x]
```

output
```
-1/3*ArcTanh[Tan[2 + 3*x]/Sqrt[2]]/Sqrt[2]
```

3.25.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.36, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 3660, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{1-3\cos^2(3x+2)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{1-3\sin\left(3x+\frac{\pi}{2}+2\right)^2}dx$$

$$\Big\downarrow \text{3660}$$

$$-\frac{1}{3}\int \frac{1}{1-2\cot^2(3x+2)}d\cot(3x+2)$$

$$\Big\downarrow \text{219}$$

$$-\frac{\text{arctanh}\left(\sqrt{2}\cot(3x+2)\right)}{3\sqrt{2}}$$

input
```
Int[(1 - 3*Cos[2 + 3*x]^2)^(-1),x]
```

output
```
-1/3*ArcTanh[Sqrt[2]*Cot[2 + 3*x]]/Sqrt[2]
```

3.25.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

3.25.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.30

method	result	size
derivativedivides	$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)}{6}$	18
default	$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)}{6}$	18
risch	$-\frac{\sqrt{2}\,\ln\left(e^{2i(2+3x)}+\frac{1}{3}+\frac{2i\sqrt{2}}{3}\right)}{12}+\frac{\sqrt{2}\,\ln\left(e^{2i(2+3x)}+\frac{1}{3}-\frac{2i\sqrt{2}}{3}\right)}{12}$	48

input
```
int(1/(1-3*cos(2+3*x)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*2^(1/2)*arctanh(1/2*tan(2+3*x)*2^(1/2))
```

3.25.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.39

$$\int \frac{1}{1-3\cos^2(2+3x)}\,dx$$
$$=\frac{1}{24}\sqrt{2}\log\left(-\frac{7\cos(3x+2)^4-10\cos(3x+2)^2+4\left(\sqrt{2}\cos(3x+2)^3+\sqrt{2}\cos(3x+2)\right)\sin(3x+2)}{9\cos(3x+2)^4-6\cos(3x+2)^2+1}\right.$$

input
```
integrate(1/(1-3*cos(2+3*x)^2),x, algorithm="fricas")
```

output
```
1/24*sqrt(2)*log(-(7*cos(3*x + 2)^4 - 10*cos(3*x + 2)^2 + 4*(sqrt(2)*cos(3
*x + 2)^3 + sqrt(2)*cos(3*x + 2))*sin(3*x + 2) - 1)/(9*cos(3*x + 2)^4 - 6*
cos(3*x + 2)^2 + 1))
```

3.25.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1481 vs. $2(53) = 106$.

Time = 7.95 (sec) , antiderivative size = 1481, normalized size of antiderivative = 24.28

$$\int \frac{1}{1-3\cos^2(2+3x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(1-3*cos(2+3*x)**2),x)
```

output
```
-2108299*sqrt(3)*sqrt(sqrt(3) + 2)*log(tan(3*x/2 + 1) - sqrt(2 - sqrt(3)))
/(-16557465*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 846881*sqrt(3) + 1466841
 + 9559457*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) - 57873*sqrt(2 - s
qrt(3))*log(tan(3*x/2 + 1) - sqrt(2 - sqrt(3)))/(-16557465*sqrt(2 - sqrt(3
))*sqrt(sqrt(3) + 2) - 846881*sqrt(3) + 1466841 + 9559457*sqrt(3)*sqrt(2 -
 sqrt(3))*sqrt(sqrt(3) + 2)) + 33413*sqrt(3)*sqrt(2 - sqrt(3))*log(tan(3*x
/2 + 1) - sqrt(2 - sqrt(3)))/(-16557465*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2
) - 846881*sqrt(3) + 1466841 + 9559457*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt
(3) + 2)) + 3651681*sqrt(sqrt(3) + 2)*log(tan(3*x/2 + 1) - sqrt(2 - sqrt(3
)))/(-16557465*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 846881*sqrt(3) + 1466
841 + 9559457*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) - 4768180*sqrt(
sqrt(3) + 2)*log(tan(3*x/2 + 1) + sqrt(2 - sqrt(3)))/(-16557465*sqrt(2 - s
qrt(3))*sqrt(sqrt(3) + 2) - 846881*sqrt(3) + 1466841 + 9559457*sqrt(3)*sqr
t(2 - sqrt(3))*sqrt(sqrt(3) + 2)) - 206136*sqrt(3)*sqrt(2 - sqrt(3))*log(t
an(3*x/2 + 1) + sqrt(2 - sqrt(3)))/(-16557465*sqrt(2 - sqrt(3))*sqrt(sqrt(
3) + 2) - 846881*sqrt(3) + 1466841 + 9559457*sqrt(3)*sqrt(2 - sqrt(3))*sqr
t(sqrt(3) + 2)) + 357038*sqrt(2 - sqrt(3))*log(tan(3*x/2 + 1) + sqrt(2 - s
qrt(3)))/(-16557465*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 846881*sqrt(3) +
 1466841 + 9559457*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) + 2752910*
sqrt(3)*sqrt(sqrt(3) + 2)*log(tan(3*x/2 + 1) + sqrt(2 - sqrt(3)))/(-165...
```

3.25.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.52

$$\int \frac{1}{1-3\cos^2(2+3x)}\,dx = \frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-\tan(3\,x+2)}{\sqrt{2}+\tan(3\,x+2)}\right)$$

input
```
integrate(1/(1-3*cos(2+3*x)^2),x, algorithm="maxima")
```

output
```
1/12*sqrt(2)*log(-(sqrt(2) - tan(3*x + 2))/(sqrt(2) + tan(3*x + 2)))
```

3.25.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.64

$$\int \frac{1}{1-3\cos^2(2+3x)}\,dx = \frac{1}{12}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\tan\left(3x+2\right)\right|}{\left|2\sqrt{2}+2\tan\left(3x+2\right)\right|}\right)$$

input
```
integrate(1/(1-3*cos(2+3*x)^2),x, algorithm="giac")
```

output
```
1/12*sqrt(2)*log(abs(-2*sqrt(2) + 2*tan(3*x + 2))/abs(2*sqrt(2) + 2*tan(3*
x + 2)))
```

3.25.9 Mupad [B] (verification not implemented)

Time = 27.79 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.28

$$\int \frac{1}{1-3\cos^2(2+3x)}\,dx = -\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\tan(3x+2)}{2}\right)}{6}$$

input
```
int(-1/(3*cos(3*x + 2)^2 - 1),x)
```

output
```
-(2^(1/2)*atanh((2^(1/2)*tan(3*x + 2))/2))/6
```

3.26 $\int \frac{1}{-2\cos^2(2+3x)+\sin^2(2+3x)}\,dx$

3.26.1 Optimal result

Integrand size = 21, antiderivative size = 61

$$\int \frac{1}{-2\cos^2(2+3x)+\sin^2(2+3x)}\,dx = \frac{\log\left(\sqrt{2}\cos(2+3x)-\sin(2+3x)\right)}{6\sqrt{2}} - \frac{\log\left(\sqrt{2}\cos(2+3x)+\sin(2+3x)\right)}{6\sqrt{2}}$$

output
```
1/12*ln(-sin(2+3*x)+cos(2+3*x)*2^(1/2))*2^(1/2)-1/12*ln(sin(2+3*x)+cos(2+3
*x)*2^(1/2))*2^(1/2)
```

3.26.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.36

$$\int \frac{1}{-2\cos^2(2+3x)+\sin^2(2+3x)}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\tan(2+3x)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

input
```
Integrate[(-2*Cos[2 + 3*x]^2 + Sin[2 + 3*x]^2)^(-1),x]
```

output
```
-1/3*ArcTanh[Tan[2 + 3*x]/Sqrt[2]]/Sqrt[2]
```

3.26.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.36, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 4889, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sin^2(3x+2)-2\cos^2(3x+2)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sin(3x+2)^2-2\cos(3x+2)^2}dx$$
$$\Big\downarrow \text{4889}$$
$$\frac{1}{3}\int \frac{1}{\tan^2(3x+2)-2}d\tan(3x+2)$$
$$\Big\downarrow \text{220}$$
$$-\frac{\text{arctanh}\left(\frac{\tan(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

input
```
Int[(-2*Cos[2 + 3*x]^2 + Sin[2 + 3*x]^2)^(-1),x]
```

output
```
-1/3*ArcTanh[Tan[2 + 3*x]/Sqrt[2]]/Sqrt[2]
```

3.26.3.1 Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.26.4 Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.30

method	result	size
derivativedivides	$-\frac{\sqrt{2}\, \operatorname{arctanh}\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)}{6}$	18
default	$-\frac{\sqrt{2}\, \operatorname{arctanh}\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)}{6}$	18
risch	$-\frac{\sqrt{2}\, \ln\left(\mathrm{e}^{2i(2+3x)}+\frac{1}{3}+\frac{2i\sqrt{2}}{3}\right)}{12}+\frac{\sqrt{2}\, \ln\left(\mathrm{e}^{2i(2+3x)}+\frac{1}{3}-\frac{2i\sqrt{2}}{3}\right)}{12}$	48

input
```
int(1/(-2*cos(2+3*x)^2+sin(2+3*x)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*2^(1/2)*arctanh(1/2*tan(2+3*x)*2^(1/2))
```

3.26.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.39

$$\int \frac{1}{-2\cos^2(2+3x)+\sin^2(2+3x)}\,dx$$
$$=\frac{1}{24}\sqrt{2}\log\left(-\frac{7\cos(3x+2)^4-10\cos(3x+2)^2+4\left(\sqrt{2}\cos(3x+2)^3+\sqrt{2}\cos(3x+2)\right)\sin(3x+2)-}{9\cos(3x+2)^4-6\cos(3x+2)^2+1}\right.$$

input
```
integrate(1/(-2*cos(2+3*x)^2+sin(2+3*x)^2),x, algorithm="fricas")
```

output
```
1/24*sqrt(2)*log(-(7*cos(3*x + 2)^4 - 10*cos(3*x + 2)^2 + 4*(sqrt(2)*cos(3
*x + 2)^3 + sqrt(2)*cos(3*x + 2))*sin(3*x + 2) - 1)/(9*cos(3*x + 2)^4 - 6*
cos(3*x + 2)^2 + 1))
```

3.26.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1481 vs. $2(53) = 106$.

Time = 7.92 (sec) , antiderivative size = 1481, normalized size of antiderivative = 24.28

$$\int \frac{1}{-2\cos^2(2+3x)+\sin^2(2+3x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(-2*cos(2+3*x)**2+sin(2+3*x)**2),x)
```

output
```
-2108299*sqrt(3)*sqrt(sqrt(3) + 2)*log(tan(3*x/2 + 1) - sqrt(2 - sqrt(3)))
/(-16557465*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 846881*sqrt(3) + 1466841
 + 9559457*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) - 57873*sqrt(2 - s
qrt(3))*log(tan(3*x/2 + 1) - sqrt(2 - sqrt(3)))/(-16557465*sqrt(2 - sqrt(3
))*sqrt(sqrt(3) + 2) - 846881*sqrt(3) + 1466841 + 9559457*sqrt(3)*sqrt(2 -
 sqrt(3))*sqrt(sqrt(3) + 2)) + 33413*sqrt(3)*sqrt(2 - sqrt(3))*log(tan(3*x
/2 + 1) - sqrt(2 - sqrt(3)))/(-16557465*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2
) - 846881*sqrt(3) + 1466841 + 9559457*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt
(3) + 2)) + 3651681*sqrt(sqrt(3) + 2)*log(tan(3*x/2 + 1) - sqrt(2 - sqrt(3
)))/(-16557465*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 846881*sqrt(3) + 1466
841 + 9559457*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) - 4768180*sqrt(
sqrt(3) + 2)*log(tan(3*x/2 + 1) + sqrt(2 - sqrt(3)))/(-16557465*sqrt(2 - s
qrt(3))*sqrt(sqrt(3) + 2) - 846881*sqrt(3) + 1466841 + 9559457*sqrt(3)*sqr
t(2 - sqrt(3))*sqrt(sqrt(3) + 2)) - 206136*sqrt(3)*sqrt(2 - sqrt(3))*log(t
an(3*x/2 + 1) + sqrt(2 - sqrt(3)))/(-16557465*sqrt(2 - sqrt(3))*sqrt(sqrt(
3) + 2) - 846881*sqrt(3) + 1466841 + 9559457*sqrt(3)*sqrt(2 - sqrt(3))*sqr
t(sqrt(3) + 2)) + 357038*sqrt(2 - sqrt(3))*log(tan(3*x/2 + 1) + sqrt(2 - s
qrt(3)))/(-16557465*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2) - 846881*sqrt(3) +
 1466841 + 9559457*sqrt(3)*sqrt(2 - sqrt(3))*sqrt(sqrt(3) + 2)) + 2752910*
sqrt(3)*sqrt(sqrt(3) + 2)*log(tan(3*x/2 + 1) + sqrt(2 - sqrt(3)))/(-165...
```

3.26.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.52

$$\int \frac{1}{-2\cos^2(2+3x)+\sin^2(2+3x)}\,dx = \frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-\tan(3x+2)}{\sqrt{2}+\tan(3x+2)}\right)$$

input
```
integrate(1/(-2*cos(2+3*x)^2+sin(2+3*x)^2),x, algorithm="maxima")
```

output
```
1/12*sqrt(2)*log(-(sqrt(2) - tan(3*x + 2))/(sqrt(2) + tan(3*x + 2)))
```

3.26.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.64

$$\int \frac{1}{-2\cos^2(2+3x)+\sin^2(2+3x)}\,dx = \frac{1}{12}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\tan(3x+2)\right|}{\left|2\sqrt{2}+2\tan(3x+2)\right|}\right)$$

input
```
integrate(1/(-2*cos(2+3*x)^2+sin(2+3*x)^2),x, algorithm="giac")
```

output
```
1/12*sqrt(2)*log(abs(-2*sqrt(2) + 2*tan(3*x + 2))/abs(2*sqrt(2) + 2*tan(3*
x + 2)))
```

3.26.9 Mupad [B] (verification not implemented)

Time = 27.87 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.28

$$\int \frac{1}{-2\cos^2(2+3x)+\sin^2(2+3x)}\,dx = -\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\tan(3x+2)}{2}\right)}{6}$$

input
```
int(1/(sin(3*x + 2)^2 - 2*cos(3*x + 2)^2),x)
```

output
```
-(2^(1/2)*atanh((2^(1/2)*tan(3*x + 2))/2))/6
```

3.27 $\int \frac{\sec^2(2+3x)}{-2+\tan^2(2+3x)}\,dx$

3.27.1 Optimal result

Integrand size = 21, antiderivative size = 61

$$\int \frac{\sec^2(2+3x)}{-2+\tan^2(2+3x)}\,dx = \frac{\log\left(\sqrt{2}\cos(2+3x)-\sin(2+3x)\right)}{6\sqrt{2}} - \frac{\log\left(\sqrt{2}\cos(2+3x)+\sin(2+3x)\right)}{6\sqrt{2}}$$

output
```
1/12*ln(-sin(2+3*x)+cos(2+3*x)*2^(1/2))*2^(1/2)-1/12*ln(sin(2+3*x)+cos(2+3
*x)*2^(1/2))*2^(1/2)
```

3.27.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.36

$$\int \frac{\sec^2(2+3x)}{-2+\tan^2(2+3x)}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\tan(2+3x)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

input
```
Integrate[Sec[2 + 3*x]^2/(-2 + Tan[2 + 3*x]^2),x]
```

output
```
-1/3*ArcTanh[Tan[2 + 3*x]/Sqrt[2]]/Sqrt[2]
```

3.27.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.36, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4158, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(3x+2)}{\tan^2(3x+2)-2}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\sec(3x+2)^2}{\tan(3x+2)^2-2}dx$$
$$\Big\downarrow \text{ 4158}$$
$$\frac{1}{3}\int \frac{1}{\tan^2(3x+2)-2}d\tan(3x+2)$$
$$\Big\downarrow \text{ 220}$$
$$-\frac{\text{arctanh}\left(\frac{\tan(3x+2)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

input
```
Int[Sec[2 + 3*x]^2/(-2 + Tan[2 + 3*x]^2),x]
```

output
```
-1/3*ArcTanh[Tan[2 + 3*x]/Sqrt[2]]/Sqrt[2]
```

3.27.3.1 Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

3.27.4 Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.30

method	result	size
derivativedivides	$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)}{6}$	18
default	$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)}{6}$	18
risch	$-\frac{\sqrt{2}\,\ln\left(e^{2i(2+3x)}+\frac{1}{3}+\frac{2i\sqrt{2}}{3}\right)}{12}+\frac{\sqrt{2}\,\ln\left(e^{2i(2+3x)}+\frac{1}{3}-\frac{2i\sqrt{2}}{3}\right)}{12}$	48

input
```
int(sec(2+3*x)^2/(-2+tan(2+3*x)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*2^(1/2)*arctanh(1/2*tan(2+3*x)*2^(1/2))
```

3.27.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.39

$$\int \frac{\sec^2(2+3x)}{-2+\tan^2(2+3x)}\,dx$$
$$=\frac{1}{24}\sqrt{2}\log\left(-\frac{7\cos\left(3x+2\right)^4-10\cos\left(3x+2\right)^2+4\left(\sqrt{2}\cos\left(3x+2\right)^3+\sqrt{2}\cos\left(3x+2\right)\right)\sin\left(3x+2\right)}{9\cos\left(3x+2\right)^4-6\cos\left(3x+2\right)^2+1}\right.$$

input
```
integrate(sec(2+3*x)^2/(-2+tan(2+3*x)^2),x, algorithm="fricas")
```

output
```
1/24*sqrt(2)*log(-(7*cos(3*x + 2)^4 - 10*cos(3*x + 2)^2 + 4*(sqrt(2)*cos(3
*x + 2)^3 + sqrt(2)*cos(3*x + 2))*sin(3*x + 2) - 1)/(9*cos(3*x + 2)^4 - 6*
cos(3*x + 2)^2 + 1))
```

3.27.6 Sympy [F]

$$\int \frac{\sec^2(2+3x)}{-2+\tan^2(2+3x)}\,dx = \int \frac{\sec^2(3x+2)}{\tan^2(3x+2)-2}\,dx$$

input
```
integrate(sec(2+3*x)**2/(-2+tan(2+3*x)**2),x)
```

output
```
Integral(sec(3*x + 2)**2/(tan(3*x + 2)**2 - 2), x)
```

3.27.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.52

$$\int \frac{\sec^2(2+3x)}{-2+\tan^2(2+3x)}\,dx = \frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-\tan(3x+2)}{\sqrt{2}+\tan(3x+2)}\right)$$

input
```
integrate(sec(2+3*x)^2/(-2+tan(2+3*x)^2),x, algorithm="maxima")
```

output
```
1/12*sqrt(2)*log(-(sqrt(2) - tan(3*x + 2))/(sqrt(2) + tan(3*x + 2)))
```

3.27.8 Giac [A] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.61

$$\int \frac{\sec^2(2+3x)}{-2+\tan^2(2+3x)}\,dx = -\frac{1}{12}\sqrt{2}\log\left(\left|\sqrt{2}+\tan(3x+2)\right|\right) + \frac{1}{12}\sqrt{2}\log\left(\left|-\sqrt{2}+\tan(3x+2)\right|\right)$$

input
```
integrate(sec(2+3*x)^2/(-2+tan(2+3*x)^2),x, algorithm="giac")
```

output
```
-1/12*sqrt(2)*log(abs(sqrt(2) + tan(3*x + 2))) + 1/12*sqrt(2)*log(abs(-sqr
t(2) + tan(3*x + 2)))
```

3.27.9 Mupad [B] (verification not implemented)

Time = 28.69 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.28

$$\int \frac{\sec^2(2+3x)}{-2+\tan^2(2+3x)}\,dx = -\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\tan(3\,x+2)}{2}\right)}{6}$$

input
```
int(1/(cos(3*x + 2)^2*(tan(3*x + 2)^2 - 2)),x)
```

output
```
-(2^(1/2)*atanh((2^(1/2)*tan(3*x + 2))/2))/6
```

3.28 $\int \frac{\csc^2(2+3x)}{1-2\cot^2(2+3x)}\,dx$

3.28.1 Optimal result

Integrand size = 23, antiderivative size = 61

$$\int \frac{\csc^2(2+3x)}{1-2\cot^2(2+3x)}\,dx = \frac{\log\left(\sqrt{2}\cos(2+3x)-\sin(2+3x)\right)}{6\sqrt{2}} - \frac{\log\left(\sqrt{2}\cos(2+3x)+\sin(2+3x)\right)}{6\sqrt{2}}$$

output
```
1/12*ln(-sin(2+3*x)+cos(2+3*x)*2^(1/2))*2^(1/2)-1/12*ln(sin(2+3*x)+cos(2+3
*x)*2^(1/2))*2^(1/2)
```

3.28.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.36

$$\int \frac{\csc^2(2+3x)}{1-2\cot^2(2+3x)}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\tan(2+3x)}{\sqrt{2}}\right)}{3\sqrt{2}}$$

input
```
Integrate[Csc[2 + 3*x]^2/(1 - 2*Cot[2 + 3*x]^2),x]
```

output
```
-1/3*ArcTanh[Tan[2 + 3*x]/Sqrt[2]]/Sqrt[2]
```

3.28.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.36, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3042, 4158, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(3x+2)}{1-2\cot^2(3x+2)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sec\left(3x+\frac{\pi}{2}+2\right)^2}{1-2\tan\left(3x+\frac{\pi}{2}+2\right)^2}dx$$

$$\Big\downarrow \text{ 4158}$$

$$-\frac{1}{3}\int \frac{1}{1-2\cot^2(3x+2)}d\cot(3x+2)$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{\operatorname{arctanh}\left(\sqrt{2}\cot(3x+2)\right)}{3\sqrt{2}}$$

input
```
Int[Csc[2 + 3*x]^2/(1 - 2*Cot[2 + 3*x]^2),x]
```

output
```
-1/3*ArcTanh[Sqrt[2]*Cot[2 + 3*x]]/Sqrt[2]
```

3.28.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

3.28.4 Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.30

method	result	size
derivativedivides	$-\frac{\sqrt{2}\, \operatorname{arctanh}\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)}{6}$	18
default	$-\frac{\sqrt{2}\, \operatorname{arctanh}\left(\frac{\tan(2+3x)\sqrt{2}}{2}\right)}{6}$	18
risch	$-\frac{\sqrt{2}\, \ln\left(e^{2i(2+3x)}+\frac{1}{3}+\frac{2i\sqrt{2}}{3}\right)}{12}+\frac{\sqrt{2}\, \ln\left(e^{2i(2+3x)}+\frac{1}{3}-\frac{2i\sqrt{2}}{3}\right)}{12}$	48

input
```
int(csc(2+3*x)^2/(1-2*cot(2+3*x)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*2^(1/2)*arctanh(1/2*tan(2+3*x)*2^(1/2))
```

3.28.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.39

$$\int \frac{\csc^2(2+3x)}{1-2\cot^2(2+3x)}\,dx$$
$$=\frac{1}{24}\sqrt{2}\log\left(-\frac{7\cos(3x+2)^4-10\cos(3x+2)^2+4\left(\sqrt{2}\cos(3x+2)^3+\sqrt{2}\cos(3x+2)\right)\sin(3x+2)-}{9\cos(3x+2)^4-6\cos(3x+2)^2+1}\right.$$

input
```
integrate(csc(2+3*x)^2/(1-2*cot(2+3*x)^2),x, algorithm="fricas")
```

output
```
1/24*sqrt(2)*log(-(7*cos(3*x + 2)^4 - 10*cos(3*x + 2)^2 + 4*(sqrt(2)*cos(3
*x + 2)^3 + sqrt(2)*cos(3*x + 2))*sin(3*x + 2) - 1)/(9*cos(3*x + 2)^4 - 6*
cos(3*x + 2)^2 + 1))
```

3.28.6 Sympy [F]

$$\int \frac{\csc^2(2+3x)}{1-2\cot^2(2+3x)}\,dx = -\int \frac{\csc^2(3x+2)}{2\cot^2(3x+2)-1}\,dx$$

input
```
integrate(csc(2+3*x)**2/(1-2*cot(2+3*x)**2),x)
```

output
```
-Integral(csc(3*x + 2)**2/(2*cot(3*x + 2)**2 - 1), x)
```

3.28.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.52

$$\int \frac{\csc^2(2+3x)}{1-2\cot^2(2+3x)}\,dx = \frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-\tan(3x+2)}{\sqrt{2}+\tan(3x+2)}\right)$$

input
```
integrate(csc(2+3*x)^2/(1-2*cot(2+3*x)^2),x, algorithm="maxima")
```

output
```
1/12*sqrt(2)*log(-(sqrt(2) - tan(3*x + 2))/(sqrt(2) + tan(3*x + 2)))
```

3.28.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.64

$$\int \frac{\csc^2(2+3x)}{1-2\cot^2(2+3x)}\,dx = \frac{1}{12}\sqrt{2}\log\left(\frac{|-2\sqrt{2}+2\tan(3x+2)|}{|2\sqrt{2}+2\tan(3x+2)|}\right)$$

input
```
integrate(csc(2+3*x)^2/(1-2*cot(2+3*x)^2),x, algorithm="giac")
```

output
```
1/12*sqrt(2)*log(abs(-2*sqrt(2) + 2*tan(3*x + 2))/abs(2*sqrt(2) + 2*tan(3*
x + 2)))
```

3.28.9 Mupad [B] (verification not implemented)

Time = 28.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.28

$$\int \frac{\csc^2(2+3x)}{1-2\cot^2(2+3x)}\,dx = -\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\tan(3\,x+2)}{2}\right)}{6}$$

input
```
int(-1/(sin(3*x + 2)^2*(2*cot(3*x + 2)^2 - 1)),x)
```

output
```
-(2^(1/2)*atanh((2^(1/2)*tan(3*x + 2))/2))/6
```

3.29 $\int (x+\sin(x))^2 \, dx$

3.29.1 Optimal result

Integrand size = 6, antiderivative size = 30

$$\int (x+\sin(x))^2 \, dx = \frac{x}{2}+\frac{x^3}{3}-2x\cos(x)+2\sin(x)-\frac{1}{2}\cos(x)\sin(x)$$

output
```
1/2*x+1/3*x^3-2*x*cos(x)+2*sin(x)-1/2*cos(x)*sin(x)
```

3.29.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (x+\sin(x))^2 \, dx = \frac{1}{6}x\left(3+2x^2\right)-2x\cos(x)+2\sin(x)-\frac{1}{4}\sin(2x)$$

input
```
Integrate[(x + Sin[x])^2,x]
```

output
```
(x*(3 + 2*x^2))/6 - 2*x*Cos[x] + 2*Sin[x] - Sin[2*x]/4
```

3.29.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x+\sin(x))^2\,dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(x^2+\sin^2(x)+2x\sin(x)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^3}{3}+\frac{x}{2}+2\sin(x)-2x\cos(x)-\frac{1}{2}\sin(x)\cos(x)$$

input
```
Int[(x + Sin[x])^2,x]
```

output
```
x/2 + x^3/3 - 2*x*Cos[x] + 2*Sin[x] - (Cos[x]*Sin[x])/2
```

3.29.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.29.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{x}{2}+\frac{x^{3}}{3}-2x\cos\left(x\right)+2\sin\left(x\right)-\frac{\cos\left(x\right)\sin\left(x\right)}{2}$	25
risch	$\frac{x^{3}}{3}+\frac{x}{2}-2x\cos\left(x\right)+2\sin\left(x\right)-\frac{\sin\left(2x\right)}{4}$	25
parallelrisch	$\frac{x^{3}}{3}+\frac{x}{2}-2x\cos\left(x\right)+2\sin\left(x\right)-\frac{\sin\left(2x\right)}{4}$	25
parts	$\frac{x}{2}+\frac{x^{3}}{3}-2x\cos\left(x\right)+2\sin\left(x\right)-\frac{\cos\left(x\right)\sin\left(x\right)}{2}$	25
norman	$\frac{x\tan\left(\frac{x}{2}\right)^{2}-\frac{3x}{2}+\frac{x^{3}}{3}+5\tan\left(\frac{x}{2}\right)^{3}+\frac{5x\tan\left(\frac{x}{2}\right)^{4}}{2}+\frac{2x^{3}\tan\left(\frac{x}{2}\right)^{2}}{3}+\frac{x^{3}\tan\left(\frac{x}{2}\right)^{4}}{3}+3\tan\left(\frac{x}{2}\right)}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)^{2}}$	74

input
```
int((x+sin(x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x+1/3*x^3-2*x*cos(x)+2*sin(x)-1/2*cos(x)*sin(x)
```

3.29.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int (x+\sin(x))^2\,dx = \frac{1}{3}\,x^{3}-2\,x\cos\left(x\right)-\frac{1}{2}\left(\cos\left(x\right)-4\right)\sin\left(x\right)+\frac{1}{2}\,x$$

input
```
integrate((x+sin(x))^2,x, algorithm="fricas")
```

output
```
1/3*x^3 - 2*x*cos(x) - 1/2*(cos(x) - 4)*sin(x) + 1/2*x
```

3.29.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.37

$$\int (x+\sin(x))^2\,dx = \frac{x^{3}}{3}+\frac{x\sin^{2}\left(x\right)}{2}+\frac{x\cos^{2}\left(x\right)}{2}-2x\cos\left(x\right)-\frac{\sin\left(x\right)\cos\left(x\right)}{2}+2\sin\left(x\right)$$

input
```
integrate((x+sin(x))**2,x)
```

output
```
x**3/3 + x*sin(x)**2/2 + x*cos(x)**2/2 - 2*x*cos(x) - sin(x)*cos(x)/2 + 2*
sin(x)
```

3.29.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int (x+\sin(x))^2\,dx = \frac{1}{3}\,x^3 - 2\,x\cos(x) + \frac{1}{2}\,x - \frac{1}{4}\,\sin(2\,x) + 2\,\sin(x)$$

input
```
integrate((x+sin(x))^2,x, algorithm="maxima")
```

output
```
1/3*x^3 - 2*x*cos(x) + 1/2*x - 1/4*sin(2*x) + 2*sin(x)
```

3.29.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int (x+\sin(x))^2\,dx = \frac{1}{3}\,x^3 - 2\,x\cos(x) + \frac{1}{2}\,x - \frac{1}{4}\,\sin(2\,x) + 2\,\sin(x)$$

input
```
integrate((x+sin(x))^2,x, algorithm="giac")
```

output
```
1/3*x^3 - 2*x*cos(x) + 1/2*x - 1/4*sin(2*x) + 2*sin(x)
```

3.29.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int (x+\sin(x))^2\,dx = \frac{x}{2} + 2\,\sin(x) - \frac{\cos(x)\,\sin(x)}{2} - 2\,x\,\cos(x) + \frac{x^3}{3}$$

input
```
int((x + sin(x))^2,x)
```

output
```
x/2 + 2*sin(x) - (cos(x)*sin(x))/2 - 2*x*cos(x) + x^3/3
```

3.30 $\int (x+\sin(x))^3 \, dx$

3.30.1 Optimal result

Integrand size = 6, antiderivative size = 56

$$\int (x+\sin(x))^3 \, dx = \frac{3x^2}{4} + \frac{x^4}{4} + 5\cos(x) - 3x^2\cos(x) + \frac{\cos^3(x)}{3} + 6x\sin(x) - \frac{3}{2}x\cos(x)\sin(x) + \frac{3\sin^2(x)}{4}$$

output
```
3/4*x^2+1/4*x^4+5*cos(x)-3*x^2*cos(x)+1/3*cos(x)^3+6*x*sin(x)-3/2*x*cos(x)
*sin(x)+3/4*sin(x)^2
```

3.30.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int (x+\sin(x))^3 \, dx = \frac{1}{24}\left(-18\left(-7+4x^2\right)\cos(x) - 9\cos(2x) + 2\cos(3x) + 6x\left(3x + x^3 + 24\sin(x) - 3\sin(2x)\right)\right)$$

input
```
Integrate[(x + Sin[x])^3,x]
```

output
```
(-18*(-7 + 4*x^2)*Cos[x] - 9*Cos[2*x] + 2*Cos[3*x] + 6*x*(3*x + x^3 + 24*S
in[x] - 3*Sin[2*x]))/24
```

3.30.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x+\sin(x))^3\,dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(x^3+3x^2\sin(x)+\sin^3(x)+3x\sin^2(x)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^4}{4}+\frac{3x^2}{4}-3x^2\cos(x)+\frac{3\sin^2(x)}{4}+6x\sin(x)+\frac{\cos^3(x)}{3}+5\cos(x)-\frac{3}{2}x\sin(x)\cos(x)$$

input
```
Int[(x + Sin[x])^3,x]
```

output
```
(3*x^2)/4 + x^4/4 + 5*Cos[x] - 3*x^2*Cos[x] + Cos[x]^3/3 + 6*x*Sin[x] - (3
*x*Cos[x]*Sin[x])/2 + (3*Sin[x]^2)/4
```

3.30.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.30.4 Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.84

method	result
risch	$\frac{x^4}{4}+\frac{3x^2}{4}+\frac{9}{16}+\left(\frac{21}{4}-3x^2\right)\cos\left(x\right)+6x\sin\left(x\right)+\frac{\cos(3x)}{12}-\frac{3\cos(2x)}{8}-\frac{3x\sin(2x)}{4}$
parallelrisch	$\frac{17}{24}+\frac{x^4}{4}+\frac{3x^2}{4}-3x^2\cos\left(x\right)+6x\sin\left(x\right)-\frac{3x\sin(2x)}{4}+\frac{\cos(3x)}{12}+\frac{21\cos(x)}{4}-\frac{3\cos(2x)}{8}$
default	$-\frac{\left(2+\sin(x)^2\right)\cos(x)}{3}+3x\left(-\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)-\frac{3x^2}{4}+\frac{3\sin(x)^2}{4}-3x^2\cos\left(x\right)+6\cos\left(x\right)+6x\sin\left(\right.$
parts	$-\frac{\left(2+\sin(x)^2\right)\cos(x)}{3}+3x\left(-\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)-\frac{3x^2}{4}+\frac{3\sin(x)^2}{4}-3x^2\cos\left(x\right)+6\cos\left(x\right)+6x\sin\left(\right.$
norman	$\frac{15\tan\left(\frac{x}{2}\right)^4+23\tan\left(\frac{x}{2}\right)^2-\frac{9x^2}{4}+\frac{x^4}{4}+9x\tan\left(\frac{x}{2}\right)+15x\tan\left(\frac{x}{2}\right)^5-\frac{3x^2\tan\left(\frac{x}{2}\right)^2}{4}+\frac{21x^2\tan\left(\frac{x}{2}\right)^4}{4}+\frac{15x^2\tan\left(\frac{x}{2}\right)^6}{4}+\frac{3x^4\tan\left(\frac{x}{2}\right)^2}{4}+\frac{3x}{}}{\left(1+\tan\left(\frac{x}{2}\right)^2\right)^3}$

input
```
int((x+sin(x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4+3/4*x^2+9/16+(21/4-3*x^2)*cos(x)+6*x*sin(x)+1/12*cos(3*x)-3/8*cos(
2*x)-3/4*x*sin(2*x)
```

3.30.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int (x+\sin(x))^3\,dx = \frac{1}{4}\,x^4+\frac{1}{3}\,\cos\left(x\right)^3+\frac{3}{4}\,x^2-\left(3\,x^2-5\right)\cos\left(x\right) - \frac{3}{4}\,\cos\left(x\right)^2-\frac{3}{2}\left(x\cos\left(x\right)-4\,x\right)\sin\left(x\right)$$

input
```
integrate((x+sin(x))^3,x, algorithm="fricas")
```

output
```
1/4*x^4 + 1/3*cos(x)^3 + 3/4*x^2 - (3*x^2 - 5)*cos(x) - 3/4*cos(x)^2 - 3/2
*(x*cos(x) - 4*x)*sin(x)
```

3.30.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.52

$$\int (x+\sin(x))^3\,dx = \frac{x^4}{4} + \frac{3x^2\sin^2(x)}{4} + \frac{3x^2\cos^2(x)}{4} - 3x^2\cos(x) - \frac{3x\sin(x)\cos(x)}{2} + 6x\sin(x) - \sin^2(x)\cos(x) + \frac{3\sin^2(x)}{4} - \frac{2\cos^3(x)}{3} + 6\cos(x)$$

input
```
integrate((x+sin(x))**3,x)
```

output
```
x**4/4 + 3*x**2*sin(x)**2/4 + 3*x**2*cos(x)**2/4 - 3*x**2*cos(x) - 3*x*sin
(x)*cos(x)/2 + 6*x*sin(x) - sin(x)**2*cos(x) + 3*sin(x)**2/4 - 2*cos(x)**3
/3 + 6*cos(x)
```

3.30.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int (x+\sin(x))^3\,dx = \frac{1}{4}x^4 + \frac{1}{3}\cos(x)^3 + \frac{3}{4}x^2 - 3(x^2-2)\cos(x) - \frac{3}{4}x\sin(2x) + 6x\sin(x) - \frac{3}{8}\cos(2x) - \cos(x)$$

input
```
integrate((x+sin(x))^3,x, algorithm="maxima")
```

output
```
1/4*x^4 + 1/3*cos(x)^3 + 3/4*x^2 - 3*(x^2 - 2)*cos(x) - 3/4*x*sin(2*x) + 6
*x*sin(x) - 3/8*cos(2*x) - cos(x)
```

3.30.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int (x+\sin(x))^3\,dx = \frac{1}{4}x^4 + \frac{3}{4}x^2 - \frac{3}{4}(4x^2-7)\cos(x) - \frac{3}{4}x\sin(2x) + 6x\sin(x) + \frac{1}{12}\cos(3x) - \frac{3}{8}\cos(2x)$$

input
```
integrate((x+sin(x))^3,x, algorithm="giac")
```

output
```
1/4*x^4 + 3/4*x^2 - 3/4*(4*x^2 - 7)*cos(x) - 3/4*x*sin(2*x) + 6*x*sin(x) +
 1/12*cos(3*x) - 3/8*cos(2*x)
```

3.30.9 Mupad [B] (verification not implemented)

Time = 28.85 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int (x+\sin(x))^3\,dx = 5\,\cos(x) - 3\,x^2\,\cos(x) - \frac{3\cos(x)^2}{4} + \frac{\cos(x)^3}{3} + 6\,x\,\sin(x) + \frac{3\,x^2}{4} + \frac{x^4}{4} - \frac{3\,x\,\cos(x)\,\sin(x)}{2}$$

input
```
int((x + sin(x))^3,x)
```

output
```
5*cos(x) - 3*x^2*cos(x) - (3*cos(x)^2)/4 + cos(x)^3/3 + 6*x*sin(x) + (3*x^
2)/4 + x^4/4 - (3*x*cos(x)*sin(x))/2
```

3.31 $\int \frac{\sin(a+bx)}{c+dx^2} dx$

3.31.1 Optimal result

Integrand size = 16, antiderivative size = 213

$$\int \frac{\sin(a+bx)}{c+dx^2}\,dx = -\frac{\text{CosIntegral}\left(\frac{b\sqrt{-c}}{\sqrt{d}}+bx\right)\sin\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}} + \frac{\text{CosIntegral}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)\sin\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}} - \frac{\cos\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Si}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\sqrt{d}} - \frac{\cos\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Si}\left(\frac{b\sqrt{-c}}{\sqrt{d}}+bx\right)}{2\sqrt{-c}\sqrt{d}}$$

output
```
1/2*cos(a+b*(-c)^(1/2)/d^(1/2))*Si(b*x-b*(-c)^(1/2)/d^(1/2))/(-c)^(1/2)/d^
(1/2)-1/2*cos(a-b*(-c)^(1/2)/d^(1/2))*Si(b*x+b*(-c)^(1/2)/d^(1/2))/(-c)^(1
/2)/d^(1/2)-1/2*Ci(b*x+b*(-c)^(1/2)/d^(1/2))*sin(a-b*(-c)^(1/2)/d^(1/2))/(
-c)^(1/2)/d^(1/2)+1/2*Ci(-b*x+b*(-c)^(1/2)/d^(1/2))*sin(a+b*(-c)^(1/2)/d^(
1/2))/(-c)^(1/2)/d^(1/2)
```

3.31.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.82 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.77

$$\int \frac{\sin(a+bx)}{c+dx^2}\,dx = \frac{e^{-ia-\frac{b\sqrt{c}}{\sqrt{d}}}\left(e^{\frac{2b\sqrt{c}}{\sqrt{d}}}\operatorname{ExpIntegralEi}\left(-\frac{b\sqrt{c}}{\sqrt{d}}-ibx\right)-\operatorname{ExpIntegralEi}\left(\frac{b\sqrt{c}}{\sqrt{d}}-ibx\right)+e^{2ia}\left(e^{\frac{2b\sqrt{c}}{\sqrt{d}}}\operatorname{ExpIntegralEi}\left(-\right.\right.\right.}{4\sqrt{c}\sqrt{d}}$$

input
```
Integrate[Sin[a + b*x]/(c + d*x^2),x]
```

output
```
(E^((-I)*a - (b*Sqrt[c])/Sqrt[d])*(E^((2*b*Sqrt[c])/Sqrt[d])*ExpIntegralEi
[-((b*Sqrt[c])/Sqrt[d]) - I*b*x] - ExpIntegralEi[(b*Sqrt[c])/Sqrt[d] - I*b
*x] + E^((2*I)*a)*(E^((2*b*Sqrt[c])/Sqrt[d])*ExpIntegralEi[-((b*Sqrt[c])/S
qrt[d]) + I*b*x] - ExpIntegralEi[(b*Sqrt[c])/Sqrt[d] + I*b*x])))/(4*Sqrt[c
]*Sqrt[d])
```

3.31.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3814, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)}{c+dx^2}\,dx$$

$$\Big\downarrow \ 3814$$

$$\int \left(\frac{\sqrt{-c}\sin(a+bx)}{2c\left(\sqrt{-c}-\sqrt{d}x\right)}+\frac{\sqrt{-c}\sin(a+bx)}{2c\left(\sqrt{-c}+\sqrt{d}x\right)}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{\sin\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{CosIntegral}\left(xb+\frac{\sqrt{-c}b}{\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}}+\frac{\sin\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{CosIntegral}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\sqrt{d}}-\frac{\cos\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Si}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\sqrt{d}}-\frac{\cos\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Si}\left(xb+\frac{\sqrt{-c}b}{\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}}$$

input
```
Int[Sin[a + b*x]/(c + d*x^2),x]
```

output
```
-1/2*(CosIntegral[(b*Sqrt[-c])/Sqrt[d] + b*x]*Sin[a - (b*Sqrt[-c])/Sqrt[d]
])/(Sqrt[-c]*Sqrt[d]) + (CosIntegral[(b*Sqrt[-c])/Sqrt[d] - b*x]*Sin[a + (
b*Sqrt[-c])/Sqrt[d]])/(2*Sqrt[-c]*Sqrt[d]) - (Cos[a + (b*Sqrt[-c])/Sqrt[d]
]*SinIntegral[(b*Sqrt[-c])/Sqrt[d] - b*x])/(2*Sqrt[-c]*Sqrt[d]) - (Cos[a -
 (b*Sqrt[-c])/Sqrt[d]]*SinIntegral[(b*Sqrt[-c])/Sqrt[d] + b*x])/(2*Sqrt[-c
]*Sqrt[d])
```

3.31.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3814
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Int
[ExpandIntegrand[Sin[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

3.31.4 Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.09

method	result
derivativedivides	$b\left(-\frac{-\operatorname{Si}\left(-xb-a+\frac{b\sqrt{-cd}+ad}{d}\right)\cos\left(\frac{b\sqrt{-cd}+ad}{d}\right)+\operatorname{Ci}\left(xb+a-\frac{b\sqrt{-cd}+ad}{d}\right)\sin\left(\frac{b\sqrt{-cd}+ad}{d}\right)}{2d\left(-\frac{b\sqrt{-cd}+ad}{d}+a\right)}-\frac{-\operatorname{Si}\left(-xb-a-\frac{b\sqrt{-cd}-}{d}\right.}{}\right.$
default	$b\left(-\frac{-\operatorname{Si}\left(-xb-a+\frac{b\sqrt{-cd}+ad}{d}\right)\cos\left(\frac{b\sqrt{-cd}+ad}{d}\right)+\operatorname{Ci}\left(xb+a-\frac{b\sqrt{-cd}+ad}{d}\right)\sin\left(\frac{b\sqrt{-cd}+ad}{d}\right)}{2d\left(-\frac{b\sqrt{-cd}+ad}{d}+a\right)}-\frac{-\operatorname{Si}\left(-xb-a-\frac{b\sqrt{-cd}-}{d}\right.}{}\right.$
risch	$-\frac{\sqrt{cd}\,e^{\frac{iad+b\sqrt{cd}}{d}}\operatorname{Ei}_1\left(\frac{iad+b\sqrt{cd}-(ibx+ia)d}{d}\right)}{4cd}+\frac{\sqrt{cd}\,e^{\frac{iad-b\sqrt{cd}}{d}}\operatorname{Ei}_1\left(\frac{iad-b\sqrt{cd}-(ibx+ia)d}{d}\right)}{4cd}+\frac{\sqrt{cd}\operatorname{Ei}_1\left(-\frac{iad+b\sqrt{cd}-}{d}\right.}{4}$

input
```
int(sin(b*x+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
b*(-1/2/d/(-(b*(-c*d)^(1/2)+a*d)/d+a)*(-Si(-x*b-a+(b*(-c*d)^(1/2)+a*d)/d)*
cos((b*(-c*d)^(1/2)+a*d)/d)+Ci(x*b+a-(b*(-c*d)^(1/2)+a*d)/d)*sin((b*(-c*d)
^(1/2)+a*d)/d))-1/2/d/((b*(-c*d)^(1/2)-a*d)/d+a)*(-Si(-x*b-a-(b*(-c*d)^(1/
2)-a*d)/d)*cos((b*(-c*d)^(1/2)-a*d)/d)-Ci(x*b+a+(b*(-c*d)^(1/2)-a*d)/d)*si
n((b*(-c*d)^(1/2)-a*d)/d)))
```

3.31.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.88

$$\int \frac{\sin(a+bx)}{c+dx^2}\,dx$$
$$= \frac{\sqrt{\frac{b^2c}{d}}\operatorname{Ei}\left(i\,bx-\sqrt{\frac{b^2c}{d}}\right)e^{\left(i\,a+\sqrt{\frac{b^2c}{d}}\right)}-\sqrt{\frac{b^2c}{d}}\operatorname{Ei}\left(i\,bx+\sqrt{\frac{b^2c}{d}}\right)e^{\left(i\,a-\sqrt{\frac{b^2c}{d}}\right)}+\sqrt{\frac{b^2c}{d}}\operatorname{Ei}\left(-i\,bx-\sqrt{\frac{b^2c}{d}}\right)e^{\left(-i\,a\right.}}{4\,bc}$$

input
```
integrate(sin(b*x+a)/(d*x^2+c),x, algorithm="fricas")
```

output
```
1/4*(sqrt(b^2*c/d)*Ei(I*b*x - sqrt(b^2*c/d))*e^(I*a + sqrt(b^2*c/d)) - sqr
t(b^2*c/d)*Ei(I*b*x + sqrt(b^2*c/d))*e^(I*a - sqrt(b^2*c/d)) + sqrt(b^2*c/
d)*Ei(-I*b*x - sqrt(b^2*c/d))*e^(-I*a + sqrt(b^2*c/d)) - sqrt(b^2*c/d)*Ei(
-I*b*x + sqrt(b^2*c/d))*e^(-I*a - sqrt(b^2*c/d)))/(b*c)
```

3.31.6 Sympy [F]

$$\int \frac{\sin(a+bx)}{c+dx^2}\,dx = \int \frac{\sin(a+bx)}{c+dx^2}\,dx$$

input
```
integrate(sin(b*x+a)/(d*x**2+c),x)
```

output
```
Integral(sin(a + b*x)/(c + d*x**2), x)
```

3.31.7 Maxima [F]

$$\int \frac{\sin(a+bx)}{c+dx^2}\,dx = \int \frac{\sin(bx+a)}{dx^2+c}\,dx$$

input
```
integrate(sin(b*x+a)/(d*x^2+c),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)/(d*x^2 + c), x)
```

3.31.8 Giac [F]

$$\int \frac{\sin(a+bx)}{c+dx^2}\,dx = \int \frac{\sin(bx+a)}{dx^2+c}\,dx$$

input
```
integrate(sin(b*x+a)/(d*x^2+c),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)/(d*x^2 + c), x)
```

3.31.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(a+bx)}{c+dx^2}\,dx = \int \frac{\sin(a+b\,x)}{d\,x^2+c}\,dx$$

input
```
int(sin(a + b*x)/(c + d*x^2),x)
```

output
```
int(sin(a + b*x)/(c + d*x^2), x)
```

3.32 $\int \frac{\sin(a+bx)}{c+dx+ex^2}\,dx$

3.32.1 Optimal result

Integrand size = 19, antiderivative size = 271

$$\int \frac{\sin(a+bx)}{c+dx+ex^2}\,dx=\frac{\text{CosIntegral}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)\sin\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)}{\sqrt{d^2-4ce}}-\frac{\text{CosIntegral}\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}+bx\right)\sin\left(a-\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}\right)}{\sqrt{d^2-4ce}}+\frac{\cos\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\text{Si}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}-\frac{\cos\left(a-\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}\right)\text{Si}\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}$$

output
```
cos(a-1/2*b*(d-(-4*c*e+d^2)^(1/2))/e)*Si(b*x+1/2*b*(d-(-4*c*e+d^2)^(1/2))/
e)/(-4*c*e+d^2)^(1/2)-cos(a-1/2*b*(d+(-4*c*e+d^2)^(1/2))/e)*Si(b*x+1/2*b*(
d+(-4*c*e+d^2)^(1/2))/e)/(-4*c*e+d^2)^(1/2)+Ci(b*x+1/2*b*(d-(-4*c*e+d^2)^(
1/2))/e)*sin(a-1/2*b*(d-(-4*c*e+d^2)^(1/2))/e)/(-4*c*e+d^2)^(1/2)-Ci(b*x+1
/2*b*(d+(-4*c*e+d^2)^(1/2))/e)*sin(a-1/2*b*(d+(-4*c*e+d^2)^(1/2))/e)/(-4*c
*e+d^2)^(1/2)
```

3.32.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.65 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.90

$$\int \frac{\sin(a+bx)}{c+dx+ex^2}\,dx$$
$$= \frac{ie^{-\frac{1}{2}i\left(2a+\frac{b\left(d+\sqrt{d^2-4ce}\right)}{e}\right)}\left(e^{\frac{ibd}{e}}\,\text{ExpIntegralEi}\left(-\frac{ib\left(d-\sqrt{d^2-4ce}+2ex\right)}{2e}\right)-e^{i\left(2a+\frac{b\sqrt{d^2-4ce}}{e}\right)}\,\text{ExpIntegralEi}\left(\frac{ib\left(d-\sqrt{\ }\right.}{}\right.\right.}{2\sqrt{\ }}$$

input
```
Integrate[Sin[a + b*x]/(c + d*x + e*x^2),x]
```

output
```
((I/2)*(E^((I*b*d)/e)*ExpIntegralEi[((-1/2*I)*b*(d - Sqrt[d^2 - 4*c*e] + 2
*e*x))/e] - E^(I*(2*a + (b*Sqrt[d^2 - 4*c*e])/e))*ExpIntegralEi[((I/2)*b*(
d - Sqrt[d^2 - 4*c*e] + 2*e*x))/e] - E^((I*b*(d + Sqrt[d^2 - 4*c*e]))/e)*E
xpIntegralEi[((-1/2*I)*b*(d + Sqrt[d^2 - 4*c*e] + 2*e*x))/e] + E^((2*I)*a)
*ExpIntegralEi[((I/2)*b*(d + Sqrt[d^2 - 4*c*e] + 2*e*x))/e]))/(Sqrt[d^2 -
4*c*e]*E^((I/2)*(2*a + (b*(d + Sqrt[d^2 - 4*c*e]))/e)))
```

3.32.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {7279, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)}{c+dx+ex^2}\,dx$$
$$\Big\downarrow \ 7279$$
$$\int \left(\frac{2e\sin(a+bx)}{\sqrt{d^2-4ce}\left(-\sqrt{d^2-4ce}+d+2ex\right)}-\frac{2e\sin(a+bx)}{\sqrt{d^2-4ce}\left(\sqrt{d^2-4ce}+d+2ex\right)}\right)dx$$
$$\Big\downarrow \ 2009$$

$$\frac{\sin\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\text{CosIntegral}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}-\frac{\sin\left(a-\frac{b\left(\sqrt{d^2-4ce}+d\right)}{2e}\right)\text{CosIntegral}\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}+\frac{\cos\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\text{Si}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}-\frac{\cos\left(a-\frac{b\left(\sqrt{d^2-4ce}+d\right)}{2e}\right)\text{Si}\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}$$

input
```
Int[Sin[a + b*x]/(c + d*x + e*x^2),x]
```

output
```
(CosIntegral[(b*(d - Sqrt[d^2 - 4*c*e]))/(2*e) + b*x]*Sin[a - (b*(d - Sqrt
[d^2 - 4*c*e]))/(2*e)])/Sqrt[d^2 - 4*c*e] - (CosIntegral[(b*(d + Sqrt[d^2
- 4*c*e]))/(2*e) + b*x]*Sin[a - (b*(d + Sqrt[d^2 - 4*c*e]))/(2*e)])/Sqrt[d
^2 - 4*c*e] + (Cos[a - (b*(d - Sqrt[d^2 - 4*c*e]))/(2*e)]*SinIntegral[(b*(
d - Sqrt[d^2 - 4*c*e]))/(2*e) + b*x])/Sqrt[d^2 - 4*c*e] - (Cos[a - (b*(d +
 Sqrt[d^2 - 4*c*e]))/(2*e)]*SinIntegral[(b*(d + Sqrt[d^2 - 4*c*e]))/(2*e)
+ b*x])/Sqrt[d^2 - 4*c*e]
```

3.32.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7279
```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

3.32.4 Maple [A] (verified)

Time = 1.67 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.21

method	result
derivativedivides	$b\left(\frac{-\operatorname{Si}\left(-xb-a+\frac{2ae-bd+\sqrt{-4b^2ce+b^2d^2}}{2e}\right)\cos\left(\frac{2ae-bd+\sqrt{-4b^2ce+b^2d^2}}{2e}\right)+\operatorname{Ci}\left(xb+a-\frac{2ae-bd+\sqrt{-4b^2ce+b^2d^2}}{2e}\right)\sin\left(\frac{2\ldots}{}\right.}{\sqrt{-4b^2ce+b^2d^2}}\right.$
default	$b\left(\frac{-\operatorname{Si}\left(-xb-a+\frac{2ae-bd+\sqrt{-4b^2ce+b^2d^2}}{2e}\right)\cos\left(\frac{2ae-bd+\sqrt{-4b^2ce+b^2d^2}}{2e}\right)+\operatorname{Ci}\left(xb+a-\frac{2ae-bd+\sqrt{-4b^2ce+b^2d^2}}{2e}\right)\sin\left(\frac{2\ldots}{}\right.}{\sqrt{-4b^2ce+b^2d^2}}\right.$
risch	$\frac{\sqrt{4b^2ce-b^2d^2}\,\operatorname{Ei}_1\left(\frac{2iae-ibd-2e(ibx+ia)-\sqrt{4b^2ce-b^2d^2}}{2e}\right)e^{\frac{2iae-ibd-\sqrt{4b^2ce-b^2d^2}}{2e}}}{2b(4ce-d^2)}-\frac{\sqrt{4b^2ce-b^2d^2}\,\operatorname{Ei}_1\left(\frac{2iae-ibd-2e(i\ldots}{}\right.}{}$

input
```
int(sin(b*x+a)/(e*x^2+d*x+c),x,method=_RETURNVERBOSE)
```

output
```
b*(1/(-4*b^2*c*e+b^2*d^2)^(1/2)*(-Si(-x*b-a+1/2/e*(2*a*e-b*d+(-4*b^2*c*e+b
^2*d^2)^(1/2)))*cos(1/2/e*(2*a*e-b*d+(-4*b^2*c*e+b^2*d^2)^(1/2)))+Ci(x*b+a
-1/2/e*(2*a*e-b*d+(-4*b^2*c*e+b^2*d^2)^(1/2)))*sin(1/2/e*(2*a*e-b*d+(-4*b^
2*c*e+b^2*d^2)^(1/2))))-1/(-4*b^2*c*e+b^2*d^2)^(1/2)*(-Si(-x*b-a-1/2*(-2*a
*e+b*d+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)*cos(1/2*(-2*a*e+b*d+(-4*b^2*c*e+b^2*
d^2)^(1/2))/e)-Ci(x*b+a+1/2*(-2*a*e+b*d+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)*sin
(1/2*(-2*a*e+b*d+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)))
```

3.32.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.60

$$\int \frac{\sin(a+bx)}{c+dx+ex^2}\,dx =$$

$$-\frac{e\sqrt{-\frac{b^2d^2-4\,b^2ce}{e^2}}\operatorname{Ei}\left(\frac{-2i\,bex-i\,bd-e\sqrt{-\frac{b^2d^2-4\,b^2ce}{e^2}}}{2\,e}\right)e^{\left(\frac{i\,bd-2i\,ae+e\sqrt{-\frac{b^2d^2-4\,b^2ce}{e^2}}}{2\,e}\right)}-e\sqrt{-\frac{b^2d^2-4\,b^2ce}{e^2}}\operatorname{Ei}\left(\frac{-2i\,bex-i\,bd+e\ldots}{2\,e\ldots}\right.}{}$$

input
```
integrate(sin(b*x+a)/(e*x^2+d*x+c),x, algorithm="fricas")
```

output
```
-1/2*(e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(1/2*(-2*I*b*e*x - I*b*d - e*sq
rt(-(b^2*d^2 - 4*b^2*c*e)/e^2))/e)*e^(1/2*(I*b*d - 2*I*a*e + e*sqrt(-(b^2*
d^2 - 4*b^2*c*e)/e^2))/e) - e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(1/2*(-2*
I*b*e*x - I*b*d + e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2))/e)*e^(1/2*(I*b*d - 2
*I*a*e - e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2))/e) + e*sqrt(-(b^2*d^2 - 4*b^2
*c*e)/e^2)*Ei(1/2*(2*I*b*e*x + I*b*d - e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2))
/e)*e^(1/2*(-I*b*d + 2*I*a*e + e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2))/e) - e*
sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(1/2*(2*I*b*e*x + I*b*d + e*sqrt(-(b^2*
d^2 - 4*b^2*c*e)/e^2))/e)*e^(1/2*(-I*b*d + 2*I*a*e - e*sqrt(-(b^2*d^2 - 4*
b^2*c*e)/e^2))/e))/(b*d^2 - 4*b*c*e)
```

3.32.6 Sympy [F]

$$\int \frac{\sin(a+bx)}{c+dx+ex^2}\,dx = \int \frac{\sin{(a+bx)}}{c+dx+ex^2}\,dx$$

input
```
integrate(sin(b*x+a)/(e*x**2+d*x+c),x)
```

output
```
Integral(sin(a + b*x)/(c + d*x + e*x**2), x)
```

3.32.7 Maxima [F]

$$\int \frac{\sin(a+bx)}{c+dx+ex^2}\,dx = \int \frac{\sin{(bx+a)}}{ex^2+dx+c}\,dx$$

input
```
integrate(sin(b*x+a)/(e*x^2+d*x+c),x, algorithm="maxima")
```

output
```
integrate(sin(b*x + a)/(e*x^2 + d*x + c), x)
```

3.32.8 Giac [F]

$$\int \frac{\sin(a+bx)}{c+dx+ex^2}\,dx = \int \frac{\sin(bx+a)}{ex^2+dx+c}\,dx$$

input
```
integrate(sin(b*x+a)/(e*x^2+d*x+c),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)/(e*x^2 + d*x + c), x)
```

3.32.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(a+bx)}{c+dx+ex^2}\,dx = \int \frac{\sin(a+b\,x)}{e\,x^2+d\,x+c}\,dx$$

input
```
int(sin(a + b*x)/(c + d*x + e*x^2),x)
```

output
```
int(sin(a + b*x)/(c + d*x + e*x^2), x)
```

3.33 $\int \frac{\sin(\sqrt{-7+x})}{\sqrt{-7+x}} \, dx$

3.33.1 Optimal result

Integrand size = 16, antiderivative size = 10

$$\int \frac{\sin\left(\sqrt{-7+x}\right)}{\sqrt{-7+x}} \, dx = -2\cos\left(\sqrt{-7+x}\right)$$

output
```
-2*cos((-7+x)^(1/2))
```

3.33.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(\sqrt{-7+x}\right)}{\sqrt{-7+x}} \, dx = -2\cos\left(\sqrt{-7+x}\right)$$

input
```
Integrate[Sin[Sqrt[-7 + x]]/Sqrt[-7 + x],x]
```

output
```
-2*Cos[Sqrt[-7 + x]]
```

3.33.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3912, 30, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin\left(\sqrt{x-7}\right)}{\sqrt{x-7}}dx$$

$$\Big\downarrow \text{ 3912}$$

$$2\int \sin\left(\sqrt{x-7}\right)d\sqrt{x-7}$$

$$\Big\downarrow \text{ 3042}$$

$$2\int \sin\left(\sqrt{x-7}\right)d\sqrt{x-7}$$

$$\Big\downarrow \text{ 3118}$$

$$-2\cos\left(\sqrt{x-7}\right)$$

input
```
Int[Sin[Sqrt[-7 + x]]/Sqrt[-7 + x],x]
```

output
```
-2*Cos[Sqrt[-7 + x]]
```

3.33.3.1 Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3912
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Simp[1/(n*f)   Subst[Int[ExpandIntegra
nd[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
 x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p
, 0] && IntegerQ[1/n]
```

3.33.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$-2\cos\left(\sqrt{-7+x}\right)$	9
default	$-2\cos\left(\sqrt{-7+x}\right)$	9

input
```
int(sin((-7+x)^(1/2))/(-7+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*cos((-7+x)^(1/2))
```

3.33.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\sin\left(\sqrt{-7+x}\right)}{\sqrt{-7+x}}\,dx = -2\,\cos\left(\sqrt{x-7}\right)$$

input
```
integrate(sin((-7+x)^(1/2))/(-7+x)^(1/2),x, algorithm="fricas")
```

output
```
-2*cos(sqrt(x - 7))
```

3.33.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\sin \left(\sqrt{-7+x}\right)}{\sqrt{-7+x}}\,dx = -2\cos \left(\sqrt{x-7}\right)$$

input
```
integrate(sin((-7+x)**(1/2))/(-7+x)**(1/2),x)
```

output
```
-2*cos(sqrt(x - 7))
```

3.33.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\sin \left(\sqrt{-7+x}\right)}{\sqrt{-7+x}}\,dx = -2\,\cos \left(\sqrt{x-7}\right)$$

input
```
integrate(sin((-7+x)^(1/2))/(-7+x)^(1/2),x, algorithm="maxima")
```

output
```
-2*cos(sqrt(x - 7))
```

3.33.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\sin \left(\sqrt{-7+x}\right)}{\sqrt{-7+x}}\,dx = -2\,\cos \left(\sqrt{x-7}\right)$$

input
```
integrate(sin((-7+x)^(1/2))/(-7+x)^(1/2),x, algorithm="giac")
```

output
```
-2*cos(sqrt(x - 7))
```

3.33.9 Mupad [B] (verification not implemented)

Time = 26.98 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\sin\left(\sqrt{-7+x}\right)}{\sqrt{-7+x}}\,dx = -2\,\cos\left(\sqrt{x-7}\right)$$

input
```
int(sin((x - 7)^(1/2))/(x - 7)^(1/2),x)
```

output
```
-2*cos((x - 7)^(1/2))
```

3.34 $\int \frac{\sqrt{b-\frac{a}{x^2}}\sin(x)}{\sqrt{a-bx^2}}\,dx$

3.34.1 Optimal result

Integrand size = 27, antiderivative size = 28

$$\int \frac{\sqrt{b-\frac{a}{x^2}}\sin(x)}{\sqrt{a-bx^2}}\,dx=\frac{\sqrt{b-\frac{a}{x^2}}x\text{Si}(x)}{\sqrt{a-bx^2}}$$

output
```
x*Si(x)*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2)
```

3.34.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.64

$$\int \frac{\sqrt{b-\frac{a}{x^2}}\sin(x)}{\sqrt{a-bx^2}}\,dx=\frac{i\sqrt{b-\frac{a}{x^2}}x(\text{ExpIntegralEi}(-ix)-\text{ExpIntegralEi}(ix))}{2\sqrt{a-bx^2}}$$

input
```
Integrate[(Sqrt[b - a/x^2]*Sin[x])/Sqrt[a - b*x^2],x]
```

output
```
((I/2)*Sqrt[b - a/x^2]*x*(ExpIntegralEi[(-I)*x] - ExpIntegralEi[I*x]))/Sqr
t[a - b*x^2]
```

3.34.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {7272, 283, 3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}dx$$

$$\Big\downarrow \text{7272}$$

$$\frac{x\sqrt{b-\frac{a}{x^2}}\int \frac{\sqrt{1-\frac{bx^2}{a}}\sin(x)}{x\sqrt{a-bx^2}}dx}{\sqrt{1-\frac{bx^2}{a}}}$$

$$\Big\downarrow \text{283}$$

$$\frac{x\sqrt{b-\frac{a}{x^2}}\int \frac{\sin(x)}{x}dx}{\sqrt{a-bx^2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x\sqrt{b-\frac{a}{x^2}}\int \frac{\sin(x)}{x}dx}{\sqrt{a-bx^2}}$$

$$\Big\downarrow \text{3780}$$

$$\frac{x\text{Si}(x)\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}}$$

input
```
Int[(Sqrt[b - a/x^2]*Sin[x])/Sqrt[a - b*x^2],x]
```

output
```
(Sqrt[b - a/x^2]*x*SinIntegral[x])/Sqrt[a - b*x^2]
```

3.34.3.1 Defintions of rubi rules used

rule 283
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Sy
mbol] :> Simp[(a + b*x^n)^p/(c + d*x^n)^p   Int[u*(c + d*x^n)^(p + q), x],
x] /; FreeQ[{a, b, c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] &&  !SimplerQ[a
+ b*x^n, c + d*x^n]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 7272
```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
   Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !
IntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

3.34.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.68 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.57

method	result	size
risch	$-\frac{\sqrt{-\frac{-b\,x^{2}+a}{x^{2}}}\,\left(b\,x^{2}-a\right)x\sqrt{\frac{-b\,x^{2}+a}{b\,x^{2}-a}}\,\left(-i\,\mathrm{Si}\left(x\right)+\frac{i\pi\,\operatorname{csgn}\left(x\right)}{2}\right)}{\left(-b\,x^{2}+a\right)^{\frac{3}{2}}}$	72

input
```
int(sin(x)*(b-1/x^2*a)^(1/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(-b*x^2+a)/x^2)^(1/2)*(b*x^2-a)/(-b*x^2+a)^(3/2)*x*(1/(b*x^2-a)*(-b*x^2
+a))^(1/2)*(-I*Si(x)+1/2*I*Pi*csgn(x))
```

3.34.5 Fricas [F]

$$\int \frac{\sqrt{b-\frac{a}{x^2}}\sin(x)}{\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{b-\frac{a}{x^2}}\sin\left(x\right)}{\sqrt{-bx^2+a}}\,dx$$

input
```
integrate(sin(x)*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-b*x^2 + a)*sqrt((b*x^2 - a)/x^2)*sin(x)/(b*x^2 - a), x)
```

3.34.6 Sympy [F]

$$\int \frac{\sqrt{b-\frac{a}{x^2}}\sin(x)}{\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{-\frac{a}{x^2}+b}\sin\left(x\right)}{\sqrt{a-bx^2}}\,dx$$

input
```
integrate(sin(x)*(b-a/x**2)**(1/2)/(-b*x**2+a)**(1/2),x)
```

output
```
Integral(sqrt(-a/x**2 + b)*sin(x)/sqrt(a - b*x**2), x)
```

3.34.7 Maxima [F]

$$\int \frac{\sqrt{b-\frac{a}{x^2}}\sin(x)}{\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{b-\frac{a}{x^2}}\sin\left(x\right)}{\sqrt{-bx^2+a}}\,dx$$

input
```
integrate(sin(x)*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b - a/x^2)*sin(x)/sqrt(-b*x^2 + a), x)
```

3.34.8 Giac [F]

$$\int \frac{\sqrt{b-\frac{a}{x^2}}\sin(x)}{\sqrt{a-bx^2}}\,dx = \int \frac{\sqrt{b-\frac{a}{x^2}}\sin\left(x\right)}{\sqrt{-bx^2+a}}\,dx$$

input
```
integrate(sin(x)*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b - a/x^2)*sin(x)/sqrt(-b*x^2 + a), x)
```

3.34.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{b-\frac{a}{x^2}}\sin(x)}{\sqrt{a-bx^2}}\,dx = \int \frac{\sin\left(x\right)\,\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-b\,x^2}}\,dx$$

input
```
int((sin(x)*(b - a/x^2)^(1/2))/(a - b*x^2)^(1/2),x)
```

output
```
int((sin(x)*(b - a/x^2)^(1/2))/(a - b*x^2)^(1/2), x)
```

3.35 $\int \frac{1}{x(1+\sin(\log(x)))} dx$

3.35.1 Optimal result

Integrand size = 11, antiderivative size = 12

$$\int \frac{1}{x(1+\sin(\log(x)))} dx = -\frac{\cos(\log(x))}{1+\sin(\log(x))}$$

output
```
-cos(ln(x))/(1+sin(ln(x)))
```

3.35.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 26 vs. 2(12) = 24.

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.17

$$\int \frac{1}{x(1+\sin(\log(x)))} dx = \frac{2\sin\left(\frac{\log(x)}{2}\right)}{\cos\left(\frac{\log(x)}{2}\right)+\sin\left(\frac{\log(x)}{2}\right)}$$

input
```
Integrate[1/(x*(1 + Sin[Log[x]])),x]
```

output
```
(2*Sin[Log[x]/2])/(Cos[Log[x]/2] + Sin[Log[x]/2])
```

3.35.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3039, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(\sin(\log(x))+1)}dx$$
$$\Big\downarrow \text{3039}$$
$$\int \frac{1}{\sin(\log(x))+1}d\log(x)$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sin(\log(x))+1}d\log(x)$$
$$\Big\downarrow \text{3127}$$
$$-\frac{\cos(\log(x))}{\sin(\log(x))+1}$$

input
```
Int[1/(x*(1 + Sin[Log[x]])),x]
```

output
```
-(Cos[Log[x]]/(1 + Sin[Log[x]]))
```

3.35.3.1 Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

3.35.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$-\frac{2}{\tan\left(\frac{\ln(x)}{2}\right)+1}$	12
default	$-\frac{2}{\tan\left(\frac{\ln(x)}{2}\right)+1}$	12
norman	$-\frac{2}{\tan\left(\frac{\ln(x)}{2}\right)+1}$	12
risch	$-\frac{2}{x^i+i}$	12
parallelrisch	$-\frac{2}{\tan\left(\ln\left(\sqrt{x}\right)\right)+1}$	12

input
```
int(1/x/(1+sin(ln(x))),x,method=_RETURNVERBOSE)
```

output
```
-2/(tan(1/2*ln(x))+1)
```

3.35.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{1}{x(1+\sin(\log(x)))}\,dx = -\frac{\cos(\log(x)) - \sin(\log(x)) + 1}{\cos(\log(x)) + \sin(\log(x)) + 1}$$

input
```
integrate(1/x/(1+sin(log(x))),x, algorithm="fricas")
```

output
```
-(cos(log(x)) - sin(log(x)) + 1)/(cos(log(x)) + sin(log(x)) + 1)
```

3.35.6 Sympy [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{x(1+\sin(\log(x)))}\,dx = -\frac{2}{\tan\left(\frac{\log(x)}{2}\right)+1}$$

input
```
integrate(1/x/(1+sin(ln(x))),x)
```

output
```
-2/(tan(log(x)/2) + 1)
```

3.35.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \frac{1}{x(1+\sin(\log(x)))}\,dx = -\frac{2}{\frac{\sin(\log(x))}{\cos(\log(x))+1}+1}$$

input
```
integrate(1/x/(1+sin(log(x))),x, algorithm="maxima")
```

output
```
-2/(sin(log(x))/(cos(log(x)) + 1) + 1)
```

3.35.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{1}{x(1+\sin(\log(x)))}\,dx = -\frac{2}{\tan\left(\frac{1}{2}\log(x)\right)+1}$$

input
```
integrate(1/x/(1+sin(log(x))),x, algorithm="giac")
```

output
```
-2/(tan(1/2*log(x)) + 1)
```

3.35.9 Mupad [B] (verification not implemented)

Time = 28.70 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{1}{x(1+\sin(\log(x)))}\,dx = -\frac{2}{\tan\left(\frac{\ln(x)}{2}\right)+1}$$

input
```
int(1/(x*(sin(log(x)) + 1)),x)
```

output
```
-2/(tan(log(x)/2) + 1)
```

3.36 $\int \sin\left(\frac{a+bx}{c+dx}\right)\,dx$

3.36.1 Optimal result

Integrand size = 14, antiderivative size = 100

$$\int \sin\left(\frac{a+bx}{c+dx}\right)\,dx=\frac{(bc-ad)\cos\left(\frac{b}{d}\right)\text{CosIntegral}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}+\frac{(c+dx)\sin\left(\frac{a+bx}{c+dx}\right)}{d}+\frac{(bc-ad)\sin\left(\frac{b}{d}\right)\text{Si}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}$$

output
```
(-a*d+b*c)*Ci((-a*d+b*c)/d/(d*x+c))*cos(b/d)/d^2+(-a*d+b*c)*Si((-a*d+b*c)/
d/(d*x+c))*sin(b/d)/d^2+(d*x+c)*sin((b*x+a)/(d*x+c))/d
```

3.36.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.80 (sec) , antiderivative size = 324, normalized size of antiderivative = 3.24

$$\int \sin\left(\frac{a+bx}{c+dx}\right)\,dx$$
$$=\frac{icde^{-\frac{i(a+bx)}{c+dx}}-icde^{\frac{i(a+bx)}{c+dx}}+2d^2x\cos\left(\frac{-bc+ad}{d(c+dx)}\right)\sin\left(\frac{b}{d}\right)+2d^2x\cos\left(\frac{b}{d}\right)\sin\left(\frac{-bc+ad}{d(c+dx)}\right)+(bc-ad)\left(\text{CosIntegra}\right.}{}$$

input
```
Integrate[Sin[(a + b*x)/(c + d*x)],x]
```

output
```
((I*c*d)/E^((I*(a + b*x))/(c + d*x)) - I*c*d*E^((I*(a + b*x))/(c + d*x)) +
 2*d^2*x*Cos[(-(b*c) + a*d)/(d*(c + d*x))]*Sin[b/d] + 2*d^2*x*Cos[b/d]*Sin
[(-(b*c) + a*d)/(d*(c + d*x))] + (b*c - a*d)*(CosIntegral[(-(b*c) + a*d)/(
d*(c + d*x))]*(Cos[b/d] - I*Sin[b/d]) + CosIntegral[(b*c - a*d)/(c*d + d^2
*x)]*(Cos[b/d] + I*Sin[b/d]) + I*Cos[b/d]*SinIntegral[(-(b*c) + a*d)/(d*(c
 + d*x))] - Sin[b/d]*SinIntegral[(-(b*c) + a*d)/(d*(c + d*x))] + I*Cos[b/d
]*SinIntegral[(b*c - a*d)/(c*d + d^2*x)] + Sin[b/d]*SinIntegral[(b*c - a*d
)/(c*d + d^2*x)]))/(2*d^2)
```

3.36.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {5074, 3042, 3778, 3042, 3784, 25, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin\left(\frac{a+bx}{c+dx}\right) dx$$

$$\Big\downarrow \text{5074}$$

$$-\frac{\int (c+dx)^2 \sin\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right) d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int (c+dx)^2 \sin\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right) d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \text{3778}$$

$$-\frac{-\frac{(bc-ad)\int (c+dx)\cos\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)d\frac{1}{c+dx}}{d}-\left((c+dx)\sin\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{(bc-ad)\int (c+dx)\sin\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}+\frac{\pi}{2}\right)d\frac{1}{c+dx}}{d}-\left((c+dx)\sin\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)\right)}{d}$$

$$\Big\downarrow \text{3784}$$

$$-\frac{-\frac{(bc-ad)\left(\cos\left(\frac{b}{d}\right)\int(c+dx)\cos\left(\frac{bc-ad}{d(c+dx)}\right)d\frac{1}{c+dx}-\sin\left(\frac{b}{d}\right)\int-\left((c+dx)\sin\left(\frac{bc-ad}{d(c+dx)}\right)\right)d\frac{1}{c+dx}\right)}{d}-\left((c+dx)\sin\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)\right)}{d}$$

$$\Big\downarrow \text{25}$$

$$-\frac{-\frac{(bc-ad)\left(\sin\left(\frac{b}{d}\right)\int(c+dx)\sin\left(\frac{bc-ad}{d(c+dx)}\right)d\frac{1}{c+dx}+\cos\left(\frac{b}{d}\right)\int(c+dx)\cos\left(\frac{bc-ad}{d(c+dx)}\right)d\frac{1}{c+dx}\right)}{d}-\left((c+dx)\sin\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{(bc-ad)\left(\sin\left(\frac{b}{d}\right)\int(c+dx)\sin\left(\frac{bc-ad}{d(c+dx)}\right)d\frac{1}{c+dx}+\cos\left(\frac{b}{d}\right)\int(c+dx)\sin\left(\frac{bc-ad}{d(c+dx)}+\frac{\pi}{2}\right)d\frac{1}{c+dx}\right)}{d}-\left((c+dx)\sin\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)\right)}{d}$$

$$\Big\downarrow \text{3780}$$

$$-\frac{-\frac{(bc-ad)\left(\cos\left(\frac{b}{d}\right)\int(c+dx)\sin\left(\frac{bc-ad}{d(c+dx)}+\frac{\pi}{2}\right)d\frac{1}{c+dx}+\sin\left(\frac{b}{d}\right)\text{Si}\left(\frac{bc-ad}{d(c+dx)}\right)\right)}{d}-\left((c+dx)\sin\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)\right)}{d}$$

$$\Big\downarrow \text{3783}$$

$$-\frac{-\frac{(bc-ad)\left(\cos\left(\frac{b}{d}\right)\text{CosIntegral}\left(\frac{bc-ad}{d(c+dx)}\right)+\sin\left(\frac{b}{d}\right)\text{Si}\left(\frac{bc-ad}{d(c+dx)}\right)\right)}{d}-\left((c+dx)\sin\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)\right)}{d}$$

input
```
Int[Sin[(a + b*x)/(c + d*x)],x]
```

output
```
-((-((c + d*x)*Sin[b/d - (b*c - a*d)/(d*(c + d*x))]) - ((b*c - a*d)*(Cos[b
/d]*CosIntegral[(b*c - a*d)/(d*(c + d*x))] + Sin[b/d]*SinIntegral[(b*c - a
*d)/(d*(c + d*x))]))/d)/d)
```

3.36.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 5074
```
Int[Sin[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol]
 :> Simp[-d^(-1)   Subst[Int[Sin[b*(e/d) - e*(b*c - a*d)*(x/d)]^n/x^2, x],
x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a*
d, 0]
```

3.36.4 Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.42

method	result
derivativedivides	$-(ad-cb)\left(-\frac{\sin\left(\frac{b}{d}+\frac{ad-cb}{d(dx+c)}\right)}{\left(\left(\frac{b}{d}+\frac{ad-cb}{d(dx+c)}\right)d-b\right)d}+\frac{-\frac{\mathrm{Si}\left(\frac{ad-cb}{d(dx+c)}\right)\sin\left(\frac{b}{d}\right)}{d}+\frac{\mathrm{Ci}\left(\frac{ad-cb}{d(dx+c)}\right)\cos\left(\frac{b}{d}\right)}{d}}{d}\right)$
default	$-(ad-cb)\left(-\frac{\sin\left(\frac{b}{d}+\frac{ad-cb}{d(dx+c)}\right)}{\left(\left(\frac{b}{d}+\frac{ad-cb}{d(dx+c)}\right)d-b\right)d}+\frac{-\frac{\mathrm{Si}\left(\frac{ad-cb}{d(dx+c)}\right)\sin\left(\frac{b}{d}\right)}{d}+\frac{\mathrm{Ci}\left(\frac{ad-cb}{d(dx+c)}\right)\cos\left(\frac{b}{d}\right)}{d}}{d}\right)$
risch	$\frac{\mathrm{Ei}_1\left(-\frac{i(ad-cb)}{d(dx+c)}\right)e^{\frac{ib}{d}}a}{2d}-\frac{\mathrm{Ei}_1\left(-\frac{i(ad-cb)}{d(dx+c)}\right)e^{\frac{ib}{d}}cb}{2d^2}-\frac{ie^{-\frac{ib}{d}}\pi\,\mathrm{csgn}\left(\frac{ad-cb}{d(dx+c)}\right)a}{2d}+\frac{ie^{-\frac{ib}{d}}\pi\,\mathrm{csgn}\left(\frac{ad-cb}{d(dx+c)}\right)bc}{2d^2}+ie^{-\frac{ib}{d}}$

input
```
int(sin((b*x+a)/(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-(a*d-b*c)*(-sin(b/d+(a*d-b*c)/d/(d*x+c))/((b/d+(a*d-b*c)/d/(d*x+c))*d-b)/
d+(-Si((a*d-b*c)/d/(d*x+c))*sin(b/d)/d+Ci((a*d-b*c)/d/(d*x+c))*cos(b/d)/d)
/d)
```

3.36.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.04

$$\int \sin\left(\frac{a+bx}{c+dx}\right) dx = \frac{(bc-ad)\cos\left(\frac{b}{d}\right)\operatorname{Ci}\left(-\frac{bc-ad}{d^2x+cd}\right)-(bc-ad)\sin\left(\frac{b}{d}\right)\operatorname{Si}\left(-\frac{bc-ad}{d^2x+cd}\right)+(d^2x+cd)\sin\left(\frac{bx+a}{dx+c}\right)}{d^2}$$

input
```
integrate(sin((b*x+a)/(d*x+c)),x, algorithm="fricas")
```

output
```
((b*c - a*d)*cos(b/d)*cos_integral(-(b*c - a*d)/(d^2*x + c*d)) - (b*c - a*
d)*sin(b/d)*sin_integral(-(b*c - a*d)/(d^2*x + c*d)) + (d^2*x + c*d)*sin((
b*x + a)/(d*x + c)))/d^2
```

3.36.6 Sympy [F]

$$\int \sin\left(\frac{a+bx}{c+dx}\right) dx = \int \sin\left(\frac{a+bx}{c+dx}\right) dx$$

input
```
integrate(sin((b*x+a)/(d*x+c)),x)
```

output
```
Integral(sin((a + b*x)/(c + d*x)), x)
```

3.36.7 Maxima [F]

$$\int \sin\left(\frac{a+bx}{c+dx}\right)\,dx = \int \sin\left(\frac{bx+a}{dx+c}\right)\,dx$$

input
```
integrate(sin((b*x+a)/(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(sin((b*x + a)/(d*x + c)), x)
```

3.36.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 630 vs. $2(100) = 200$.

Time = 3.57 (sec) , antiderivative size = 630, normalized size of antiderivative = 6.30

$$\int \sin\left(\frac{a+bx}{c+dx}\right)\,dx$$
$$= \frac{\left(b^3c^2\cos\left(\frac{b}{d}\right)\operatorname{Ci}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right) - 2\,ab^2cd\cos\left(\frac{b}{d}\right)\operatorname{Ci}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right) - \frac{(bx+a)b^2c^2d\cos\left(\frac{b}{d}\right)\operatorname{Ci}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right)}{dx+c} + a^2bd\right.}{}$$

input
```
integrate(sin((b*x+a)/(d*x+c)),x, algorithm="giac")
```

output
```
(b^3*c^2*cos(b/d)*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d) - 2*a*b^2*c
*d*cos(b/d)*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d) - (b*x + a)*b^2*c
^2*d*cos(b/d)*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) + a^2
*b*d^2*cos(b/d)*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d) + 2*(b*x + a)
*a*b*c*d^2*cos(b/d)*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d)/(d*x + c)
 - (b*x + a)*a^2*d^3*cos(b/d)*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d)
/(d*x + c) + b^3*c^2*sin(b/d)*sin_integral((b - (b*x + a)*d/(d*x + c))/d)
- 2*a*b^2*c*d*sin(b/d)*sin_integral((b - (b*x + a)*d/(d*x + c))/d) - (b*x
+ a)*b^2*c^2*d*sin(b/d)*sin_integral((b - (b*x + a)*d/(d*x + c))/d)/(d*x +
 c) + a^2*b*d^2*sin(b/d)*sin_integral((b - (b*x + a)*d/(d*x + c))/d) + 2*(
b*x + a)*a*b*c*d^2*sin(b/d)*sin_integral((b - (b*x + a)*d/(d*x + c))/d)/(d
*x + c) - (b*x + a)*a^2*d^3*sin(b/d)*sin_integral((b - (b*x + a)*d/(d*x +
c))/d)/(d*x + c) + b^2*c^2*d*sin((b*x + a)/(d*x + c)) - 2*a*b*c*d^2*sin((b
*x + a)/(d*x + c)) + a^2*d^3*sin((b*x + a)/(d*x + c)))*(b*c/(b*c - a*d)^2
- a*d/(b*c - a*d)^2)/(b*d^2 - (b*x + a)*d^3/(d*x + c))
```

3.36.9 Mupad [F(-1)]

Timed out.

$$\int \sin\left(\frac{a+bx}{c+dx}\right)\,dx = \int \sin\left(\frac{a+b\,x}{c+d\,x}\right)\,dx$$

input
```
int(sin((a + b*x)/(c + d*x)),x)
```

output
```
int(sin((a + b*x)/(c + d*x)), x)
```

3.37 $\int \sin^2\left(\frac{a+bx}{c+dx}\right)\,dx$

3.37.1 Optimal result

Integrand size = 16, antiderivative size = 107

$$\int \sin^2\left(\frac{a+bx}{c+dx}\right)\,dx = \frac{(bc-ad)\,\mathrm{CosIntegral}\left(\frac{2(bc-ad)}{d(c+dx)}\right)\sin\left(\frac{2b}{d}\right)}{d^2} + \frac{(c+dx)\sin^2\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{(bc-ad)\cos\left(\frac{2b}{d}\right)\mathrm{Si}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}$$

output
```
-(-a*d+b*c)*cos(2*b/d)*Si(2*(-a*d+b*c)/d/(d*x+c))/d^2+(-a*d+b*c)*Ci(2*(-a*
d+b*c)/d/(d*x+c))*sin(2*b/d)/d^2+(d*x+c)*sin((b*x+a)/(d*x+c))^2/d
```

3.37.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.92 (sec) , antiderivative size = 495, normalized size of antiderivative = 4.63

$$\int \sin^2\left(\frac{a+bx}{c+dx}\right)\,dx$$
$$= \frac{-cde^{-\frac{2i(a+bx)}{c+dx}} - cde^{\frac{2i(a+bx)}{c+dx}} + 2d^2x - 2d^2x\cos\left(\frac{2b}{d}\right)\cos\left(\frac{2(-bc+ad)}{d(c+dx)}\right) + 2(bc-ad)\,\mathrm{CosIntegral}\left(\frac{2bc-2ad}{cd+d^2x}\right)(-i c \cdots}{\cdots}$$

input
```
Integrate[Sin[(a + b*x)/(c + d*x)]^2,x]
```

output
```
(-((c*d)/E^(((2*I)*(a + b*x))/(c + d*x))) - c*d*E^(((2*I)*(a + b*x))/(c +
d*x)) + 2*d^2*x - 2*d^2*x*Cos[(2*b)/d]*Cos[(2*(-(b*c) + a*d))/(d*(c + d*x)
)] + 2*(b*c - a*d)*CosIntegral[(2*b*c - 2*a*d)/(c*d + d^2*x)]*((-I)*Cos[(2
*b)/d] + Sin[(2*b)/d]) + 2*(b*c - a*d)*CosIntegral[(2*(-(b*c) + a*d))/(d*(
c + d*x))]*(I*Cos[(2*b)/d] + Sin[(2*b)/d]) + 2*d^2*x*Sin[(2*b)/d]*Sin[(2*(
-(b*c) + a*d))/(d*(c + d*x))] + 2*b*c*Cos[(2*b)/d]*SinIntegral[(2*(-(b*c)
+ a*d))/(d*(c + d*x))] - 2*a*d*Cos[(2*b)/d]*SinIntegral[(2*(-(b*c) + a*d))
/(d*(c + d*x))] + (2*I)*b*c*Sin[(2*b)/d]*SinIntegral[(2*(-(b*c) + a*d))/(d
*(c + d*x))] - (2*I)*a*d*Sin[(2*b)/d]*SinIntegral[(2*(-(b*c) + a*d))/(d*(c
 + d*x))] - 2*b*c*Cos[(2*b)/d]*SinIntegral[(2*b*c - 2*a*d)/(c*d + d^2*x)]
+ 2*a*d*Cos[(2*b)/d]*SinIntegral[(2*b*c - 2*a*d)/(c*d + d^2*x)] + (2*I)*b*
c*Sin[(2*b)/d]*SinIntegral[(2*b*c - 2*a*d)/(c*d + d^2*x)] - (2*I)*a*d*Sin[
(2*b)/d]*SinIntegral[(2*b*c - 2*a*d)/(c*d + d^2*x)])/(4*d^2)
```

3.37.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5074, 3042, 3794, 27, 3042, 3784, 25, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2\left(\frac{a+bx}{c+dx}\right)dx$$

$$\Big\downarrow \; 5074$$

$$-\frac{\int (c+dx)^2\sin^2\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\int (c+dx)^2\sin\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)^2d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \; 3794$$

$$-\frac{-\frac{2(bc-ad)\int \frac{1}{2}(c+dx)\sin\left(\frac{2b}{d}-\frac{2(bc-ad)}{d(c+dx)}\right)d\frac{1}{c+dx}}{d}-\left((c+dx)\sin^2\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)\right)}{d}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{(bc-ad)\int (c+dx)\sin\left(\frac{2b}{d}-\frac{2(bc-ad)}{d(c+dx)}\right)d\frac{1}{c+dx}}{d}-\left((c+dx)\sin^2\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{(bc-ad)\int (c+dx)\sin\left(\frac{2b}{d}-\frac{2(bc-ad)}{d(c+dx)}\right)d\frac{1}{c+dx}}{d}-\left((c+dx)\sin^2\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)\right)}{d}$$

$$\Big\downarrow \text{3784}$$

$$-\frac{-\frac{(bc-ad)\left(\sin\left(\frac{2b}{d}\right)\int (c+dx)\cos\left(\frac{2(bc-ad)}{d(c+dx)}\right)d\frac{1}{c+dx}+\cos\left(\frac{2b}{d}\right)\int -\left((c+dx)\sin\left(\frac{2(bc-ad)}{d(c+dx)}\right)\right)d\frac{1}{c+dx}\right)}{d}-\left((c+dx)\sin^2\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)\right)}{d}$$

$$\Big\downarrow \text{25}$$

$$-\frac{-\frac{(bc-ad)\left(\sin\left(\frac{2b}{d}\right)\int (c+dx)\cos\left(\frac{2(bc-ad)}{d(c+dx)}\right)d\frac{1}{c+dx}-\cos\left(\frac{2b}{d}\right)\int (c+dx)\sin\left(\frac{2(bc-ad)}{d(c+dx)}\right)d\frac{1}{c+dx}\right)}{d}-\left((c+dx)\sin^2\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{(bc-ad)\left(\sin\left(\frac{2b}{d}\right)\int (c+dx)\sin\left(\frac{2(bc-ad)}{d(c+dx)}+\frac{\pi}{2}\right)d\frac{1}{c+dx}-\cos\left(\frac{2b}{d}\right)\int (c+dx)\sin\left(\frac{2(bc-ad)}{d(c+dx)}\right)d\frac{1}{c+dx}\right)}{d}-\left((c+dx)\sin^2\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)\right)}{d}$$

$$\Big\downarrow \text{3780}$$

$$-\frac{-\frac{(bc-ad)\left(\sin\left(\frac{2b}{d}\right)\int (c+dx)\sin\left(\frac{2(bc-ad)}{d(c+dx)}+\frac{\pi}{2}\right)d\frac{1}{c+dx}-\cos\left(\frac{2b}{d}\right)\text{Si}\left(\frac{2(bc-ad)}{d(c+dx)}\right)\right)}{d}-\left((c+dx)\sin^2\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)\right)}{d}$$

$$\Big\downarrow \text{3783}$$

$$-\frac{-\frac{(bc-ad)\left(\sin\left(\frac{2b}{d}\right)\text{CosIntegral}\left(\frac{2(bc-ad)}{d(c+dx)}\right)-\cos\left(\frac{2b}{d}\right)\text{Si}\left(\frac{2(bc-ad)}{d(c+dx)}\right)\right)}{d}-\left((c+dx)\sin^2\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)\right)}{d}$$

input
```
Int[Sin[(a + b*x)/(c + d*x)]^2,x]
```

output
```
-((-((c + d*x)*Sin[b/d - (b*c - a*d)/(d*(c + d*x))]^2) - ((b*c - a*d)*(Cos
Integral[(2*(b*c - a*d))/(d*(c + d*x))]*Sin[(2*b)/d] - Cos[(2*b)/d]*SinInt
egral[(2*(b*c - a*d))/(d*(c + d*x))]))/d)/d)
```

3.37. $\int \sin^2\left(\frac{a+bx}{c+dx}\right)\,dx$

3.37.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

rule 5074
```
Int[Sin[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol]
 :> Simp[-d^(-1)   Subst[Int[Sin[b*(e/d) - e*(b*c - a*d)*(x/d)]^n/x^2, x],
x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a*
d, 0]
```

3.37.4 Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.82

method	result
derivativedivides	$-\frac{(ad-cb)\left(-\frac{d}{2\left(\left(\frac{b}{d}+\frac{ad-cb}{d(dx+c)}\right)d-b\right)}-\frac{d^2\left(-\frac{2\cos\left(\frac{2ad-2cb}{d(dx+c)}+\frac{2b}{d}\right)}{\left(\left(\frac{b}{d}+\frac{ad-cb}{d(dx+c)}\right)d-b\right)d}-\frac{2\left(\frac{2\,\mathrm{Si}\left(\frac{2ad-2cb}{d(dx+c)}\right)\cos\left(\frac{2b}{d}\right)}{d}+\frac{2\,\mathrm{Ci}\left(\frac{2ad-2cb}{d(dx+c)}\right)\sin\left(\frac{2b}{d}\right)}{d}\right)}{d}\right)}{4}\right)}{d^2}$
default	$-\frac{(ad-cb)\left(-\frac{d}{2\left(\left(\frac{b}{d}+\frac{ad-cb}{d(dx+c)}\right)d-b\right)}-\frac{d^2\left(-\frac{2\cos\left(\frac{2ad-2cb}{d(dx+c)}+\frac{2b}{d}\right)}{\left(\left(\frac{b}{d}+\frac{ad-cb}{d(dx+c)}\right)d-b\right)d}-\frac{2\left(\frac{2\,\mathrm{Si}\left(\frac{2ad-2cb}{d(dx+c)}\right)\cos\left(\frac{2b}{d}\right)}{d}+\frac{2\,\mathrm{Ci}\left(\frac{2ad-2cb}{d(dx+c)}\right)\sin\left(\frac{2b}{d}\right)}{d}\right)}{d}\right)}{4}\right)}{d^2}$
risch	$\frac{\mathrm{e}^{-\frac{2ib}{d}}\,\mathrm{csgn}\left(\frac{ad-cb}{d(dx+c)}\right)\pi a}{2d}-\frac{\mathrm{e}^{-\frac{2ib}{d}}\,\mathrm{csgn}\left(\frac{ad-cb}{d(dx+c)}\right)\pi bc}{2d^2}-\frac{\mathrm{e}^{-\frac{2ib}{d}}\,\mathrm{Si}\left(\frac{2ad-2cb}{d(dx+c)}\right)a}{d}+\frac{\mathrm{e}^{-\frac{2ib}{d}}\,\mathrm{Si}\left(\frac{2ad-2cb}{d(dx+c)}\right)bc}{d^2}+\frac{i\mathrm{e}^{-\frac{2ib}{d}}\,\mathrm{E}}{}$

input
```
int(sin((b*x+a)/(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/d^2*(a*d-b*c)*(-1/2*d/((b/d+(a*d-b*c)/d/(d*x+c))*d-b)-1/4*d^2*(-2*cos(2
*(a*d-b*c)/d/(d*x+c)+2*b/d)/((b/d+(a*d-b*c)/d/(d*x+c))*d-b)/d-2*(2*Si(2*(a
*d-b*c)/d/(d*x+c))*cos(2*b/d)/d+2*Ci(2*(a*d-b*c)/d/(d*x+c))*sin(2*b/d)/d)/
d))
```

3.37.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.06

$$\int \sin^2\left(\frac{a+bx}{c+dx}\right)\,dx$$
$$=\frac{d^2x-(d^2x+cd)\cos\left(\frac{bx+a}{dx+c}\right)^2+(bc-ad)\,\mathrm{Ci}\left(-\frac{2\,(bc-ad)}{d^2x+cd}\right)\sin\left(\frac{2\,b}{d}\right)+(bc-ad)\cos\left(\frac{2\,b}{d}\right)\mathrm{Si}\left(-\frac{2\,(bc-ad)}{d^2x+cd}\right)}{d^2}$$

input
```
integrate(sin((b*x+a)/(d*x+c))^2,x, algorithm="fricas")
```

output
```
(d^2*x - (d^2*x + c*d)*cos((b*x + a)/(d*x + c))^2 + (b*c - a*d)*cos_integr
al(-2*(b*c - a*d)/(d^2*x + c*d))*sin(2*b/d) + (b*c - a*d)*cos(2*b/d)*sin_i
ntegral(-2*(b*c - a*d)/(d^2*x + c*d)))/d^2
```

3.37.6 Sympy [F]

$$\int \sin^2\left(\frac{a+bx}{c+dx}\right)\,dx = \int \sin^2\left(\frac{a+bx}{c+dx}\right)dx$$

input
```
integrate(sin((b*x+a)/(d*x+c))**2,x)
```

output
```
Integral(sin((a + b*x)/(c + d*x))**2, x)
```

3.37.7 Maxima [F]

$$\int \sin^2\left(\frac{a+bx}{c+dx}\right)\,dx = \int \sin\left(\frac{bx+a}{dx+c}\right)^2\,dx$$

input
```
integrate(sin((b*x+a)/(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/2*x - 1/2*integrate(cos(2*(b*x + a)/(d*x + c)), x)
```

3.37.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 681 vs. $2(107) = 214$.

Time = 25.07 (sec) , antiderivative size = 681, normalized size of antiderivative = 6.36

$$\int \sin^2\left(\frac{a+bx}{c+dx}\right)\,dx$$
$$= \frac{\left(2\,b^3c^2\,\mathrm{Ci}\left(-\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)\sin\left(\frac{2\,b}{d}\right) - 4\,ab^2cd\,\mathrm{Ci}\left(-\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)\sin\left(\frac{2\,b}{d}\right) - \frac{2\,(bx+a)b^2c^2d\,\mathrm{Ci}\left(-\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)}{dx+c}\right.}{}$$

input
```
integrate(sin((b*x+a)/(d*x+c))^2,x, algorithm="giac")
```

output
```
1/2*(2*b^3*c^2*cos_integral(-2*(b - (b*x + a)*d/(d*x + c))/d)*sin(2*b/d) -
 4*a*b^2*c*d*cos_integral(-2*(b - (b*x + a)*d/(d*x + c))/d)*sin(2*b/d) - 2
*(b*x + a)*b^2*c^2*d*cos_integral(-2*(b - (b*x + a)*d/(d*x + c))/d)*sin(2*
b/d)/(d*x + c) + 2*a^2*b*d^2*cos_integral(-2*(b - (b*x + a)*d/(d*x + c))/d
)*sin(2*b/d) + 4*(b*x + a)*a*b*c*d^2*cos_integral(-2*(b - (b*x + a)*d/(d*x
 + c))/d)*sin(2*b/d)/(d*x + c) - 2*(b*x + a)*a^2*d^3*cos_integral(-2*(b -
(b*x + a)*d/(d*x + c))/d)*sin(2*b/d)/(d*x + c) - 2*b^3*c^2*cos(2*b/d)*sin_
integral(2*(b - (b*x + a)*d/(d*x + c))/d) + 4*a*b^2*c*d*cos(2*b/d)*sin_int
egral(2*(b - (b*x + a)*d/(d*x + c))/d) + 2*(b*x + a)*b^2*c^2*d*cos(2*b/d)*
sin_integral(2*(b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) - 2*a^2*b*d^2*cos(
2*b/d)*sin_integral(2*(b - (b*x + a)*d/(d*x + c))/d) - 4*(b*x + a)*a*b*c*d
^2*cos(2*b/d)*sin_integral(2*(b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) + 2*
(b*x + a)*a^2*d^3*cos(2*b/d)*sin_integral(2*(b - (b*x + a)*d/(d*x + c))/d)
/(d*x + c) - b^2*c^2*d*cos(2*(b*x + a)/(d*x + c)) + 2*a*b*c*d^2*cos(2*(b*x
 + a)/(d*x + c)) - a^2*d^3*cos(2*(b*x + a)/(d*x + c)) + b^2*c^2*d - 2*a*b*
c*d^2 + a^2*d^3)*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)/(b*d^2 - (b*x + a
)*d^3/(d*x + c))
```

3.37.9 Mupad [F(-1)]

Timed out.

$$\int \sin^2\left(\frac{a+bx}{c+dx}\right)\,dx=\int \sin\left(\frac{a+b\,x}{c+d\,x}\right)^2\,dx$$

input
```
int(sin((a + b*x)/(c + d*x))^2,x)
```

output
```
int(sin((a + b*x)/(c + d*x))^2, x)
```

3.38 $\int \sin^3\left(\frac{a+bx}{c+dx}\right) dx$

3.38.1 Optimal result

Integrand size = 16, antiderivative size = 194

$$\int \sin^3\left(\frac{a+bx}{c+dx}\right) dx = \frac{3(bc-ad)\cos\left(\frac{b}{d}\right)\text{CosIntegral}\left(\frac{bc-ad}{d(c+dx)}\right)}{4d^2} - \frac{3(bc-ad)\cos\left(\frac{3b}{d}\right)\text{CosIntegral}\left(\frac{3(bc-ad)}{d(c+dx)}\right)}{4d^2} + \frac{(c+dx)\sin^3\left(\frac{a+bx}{c+dx}\right)}{d} + \frac{3(bc-ad)\sin\left(\frac{b}{d}\right)\text{Si}\left(\frac{bc-ad}{d(c+dx)}\right)}{4d^2} - \frac{3(bc-ad)\sin\left(\frac{3b}{d}\right)\text{Si}\left(\frac{3(bc-ad)}{d(c+dx)}\right)}{4d^2}$$

output
```
3/4*(-a*d+b*c)*Ci((-a*d+b*c)/d/(d*x+c))*cos(b/d)/d^2-3/4*(-a*d+b*c)*Ci(3*(
-a*d+b*c)/d/(d*x+c))*cos(3*b/d)/d^2+3/4*(-a*d+b*c)*Si((-a*d+b*c)/d/(d*x+c)
)*sin(b/d)/d^2-3/4*(-a*d+b*c)*Si(3*(-a*d+b*c)/d/(d*x+c))*sin(3*b/d)/d^2+(d
*x+c)*sin((b*x+a)/(d*x+c))^3/d
```

3.38.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.78 (sec) , antiderivative size = 692, normalized size of antiderivative = 3.57

$$\int \sin^3\left(\frac{a+bx}{c+dx}\right) dx = \frac{3icde^{-\frac{i(a+bx)}{c+dx}} - 3icde^{\frac{i(a+bx)}{c+dx}} - icde^{-\frac{3i(a+bx)}{c+dx}} + icde^{\frac{3i(a+bx)}{c+dx}} + 6d^2x\cos\left(\frac{-bc+ad}{d(c+dx)}\right)\sin\left(\frac{b}{d}\right) - 2d^2x\cos\left(\frac{3(-bc+ad)}{d(c+dx)}\right)}{}$$

input
```
Integrate[Sin[(a + b*x)/(c + d*x)]^3,x]
```

output
```
(((3*I)*c*d)/E^((I*(a + b*x))/(c + d*x)) - (3*I)*c*d*E^((I*(a + b*x))/(c +
 d*x)) - (I*c*d)/E^(((3*I)*(a + b*x))/(c + d*x)) + I*c*d*E^(((3*I)*(a + b*
x))/(c + d*x)) + 6*d^2*x*Cos[(-(b*c) + a*d)/(d*(c + d*x))]*Sin[b/d] - 2*d^
2*x*Cos[(3*(-(b*c) + a*d))/(d*(c + d*x))]*Sin[(3*b)/d] + 6*d^2*x*Cos[b/d]*
Sin[(-(b*c) + a*d)/(d*(c + d*x))] - 2*d^2*x*Cos[(3*b)/d]*Sin[(3*(-(b*c) +
a*d))/(d*(c + d*x))] + 3*(b*c - a*d)*(-(Cos[(3*b)/d]*CosIntegral[(3*b*c -
3*a*d)/(c*d + d^2*x)]) + Cos[b/d]*CosIntegral[(b*c - a*d)/(c*d + d^2*x)] +
 CosIntegral[(-(b*c) + a*d)/(d*(c + d*x))]*(Cos[b/d] - I*Sin[b/d]) + I*Cos
Integral[(b*c - a*d)/(c*d + d^2*x)]*Sin[b/d] - CosIntegral[(3*(-(b*c) + a*
d))/(d*(c + d*x))]*(Cos[(3*b)/d] - I*Sin[(3*b)/d]) - I*CosIntegral[(3*b*c
- 3*a*d)/(c*d + d^2*x)]*Sin[(3*b)/d] + I*Cos[b/d]*SinIntegral[(-(b*c) + a*
d)/(d*(c + d*x))] - Sin[b/d]*SinIntegral[(-(b*c) + a*d)/(d*(c + d*x))] - I
*Cos[(3*b)/d]*SinIntegral[(3*(-(b*c) + a*d))/(d*(c + d*x))] + Sin[(3*b)/d]
*SinIntegral[(3*(-(b*c) + a*d))/(d*(c + d*x))] - I*Cos[(3*b)/d]*SinIntegra
l[(3*b*c - 3*a*d)/(c*d + d^2*x)] - Sin[(3*b)/d]*SinIntegral[(3*b*c - 3*a*d
)/(c*d + d^2*x)] + I*Cos[b/d]*SinIntegral[(b*c - a*d)/(c*d + d^2*x)] + Sin
[b/d]*SinIntegral[(b*c - a*d)/(c*d + d^2*x)]))/(8*d^2)
```

3.38.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5074, 3042, 3794, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3\left(\frac{a+bx}{c+dx}\right)dx$$
$$\Big\downarrow \text{5074}$$
$$-\frac{\int (c+dx)^2\sin^3\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)d\frac{1}{c+dx}}{d}$$
$$\Big\downarrow \text{3042}$$
$$-\frac{\int (c+dx)^2\sin\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)^3d\frac{1}{c+dx}}{d}$$
$$\Big\downarrow \text{3794}$$

$$-\frac{-\frac{3(bc-ad)\int\left(\frac{1}{4}(c+dx)\cos\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)-\frac{1}{4}(c+dx)\cos\left(\frac{3b}{d}-\frac{3(bc-ad)}{d(c+dx)}\right)\right)d\frac{1}{c+dx}}{d}-\left((c+dx)\sin^3\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)\right)}{d}$$

$\big\downarrow$ 2009

$$-\frac{-\frac{3(bc-ad)\left(\frac{1}{4}\cos\left(\frac{b}{d}\right)\text{CosIntegral}\left(\frac{bc-ad}{d(c+dx)}\right)-\frac{1}{4}\cos\left(\frac{3b}{d}\right)\text{CosIntegral}\left(\frac{3(bc-ad)}{d(c+dx)}\right)+\frac{1}{4}\sin\left(\frac{b}{d}\right)\text{Si}\left(\frac{bc-ad}{d(c+dx)}\right)-\frac{1}{4}\sin\left(\frac{3b}{d}\right)\text{Si}\left(\frac{3(bc-ad)}{d(c+dx)}\right)\right)}{d}-\left((c\right.}{d}$$

input
```
Int[Sin[(a + b*x)/(c + d*x)]^3,x]
```

output
```
-((-((c + d*x)*Sin[b/d - (b*c - a*d)/(d*(c + d*x))]^3) - (3*(b*c - a*d)*((
Cos[b/d]*CosIntegral[(b*c - a*d)/(d*(c + d*x))])/4 - (Cos[(3*b)/d]*CosInte
gral[(3*(b*c - a*d))/(d*(c + d*x))])/4 + (Sin[b/d]*SinIntegral[(b*c - a*d)
/(d*(c + d*x))])/4 - (Sin[(3*b)/d]*SinIntegral[(3*(b*c - a*d))/(d*(c + d*x
))])/4))/d)/d)
```

3.38.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

rule 5074
```
Int[Sin[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol]
 :> Simp[-d^(-1)   Subst[Int[Sin[b*(e/d) - e*(b*c - a*d)*(x/d)]^n/x^2, x],
x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a*
d, 0]
```

3.38.4 Maple [A] (verified)

Time = 1.73 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.52

method	result
derivativedivides	$-\frac{(ad-cb)\left(-\frac{d^2\left(-\frac{3\sin\left(\frac{3ad-3cb}{d(dx+c)}+\frac{3b}{d}\right)}{\left(\left(\frac{b}{d}+\frac{ad-cb}{d(dx+c)}\right)d-b\right)d}+\frac{-\frac{9\,\mathrm{Si}\left(\frac{3ad-3cb}{d(dx+c)}\right)\sin\left(\frac{3b}{d}\right)}{d}+\frac{9\,\mathrm{Ci}\left(\frac{3ad-3cb}{d(dx+c)}\right)\cos\left(\frac{3b}{d}\right)}{d}}{d}\right)}{12}+\frac{3d^2\left(-\frac{\sin\left(\frac{b}{d}+\frac{ad}{d(d\ldots}\right)}{\left(\left(\frac{b}{d}+\frac{ad-cb}{d(dx+c\ldots}\right.\right.}\ldots\right)}{\ldots}\right)}{d^2}\ldots$
default	$-\frac{(ad-cb)\left(-\frac{d^2\left(-\frac{3\sin\left(\frac{3ad-3cb}{d(dx+c)}+\frac{3b}{d}\right)}{\left(\left(\frac{b}{d}+\frac{ad-cb}{d(dx+c)}\right)d-b\right)d}+\frac{-\frac{9\,\mathrm{Si}\left(\frac{3ad-3cb}{d(dx+c)}\right)\sin\left(\frac{3b}{d}\right)}{d}+\frac{9\,\mathrm{Ci}\left(\frac{3ad-3cb}{d(dx+c)}\right)\cos\left(\frac{3b}{d}\right)}{d}}{d}\right)}{12}+\frac{3d^2\left(-\frac{\sin\left(\frac{b}{d}+\frac{ad}{d(d\ldots}\right)}{\left(\left(\frac{b}{d}+\frac{ad-cb}{d(dx+c\ldots}\right.\right.}\ldots\right)}{\ldots}\right)}{d^2}\ldots$
risch	$-\frac{3ie^{-\frac{ib}{d}}\,\mathrm{Si}\left(\frac{ad-cb}{d(dx+c)}\right)bc}{4d^2}-\frac{3ie^{-\frac{ib}{d}}\pi\,\mathrm{csgn}\left(\frac{ad-cb}{d(dx+c)}\right)a}{8d}+\frac{3ie^{-\frac{ib}{d}}\,\mathrm{Si}\left(\frac{ad-cb}{d(dx+c)}\right)a}{4d}+\frac{3ie^{-\frac{ib}{d}}\pi\,\mathrm{csgn}\left(\frac{ad-cb}{d(dx+c)}\right)bc}{8d^2}-\ldots$

input
```
int(sin((b*x+a)/(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/d^2*(a*d-b*c)*(-1/12*d^2*(-3*sin(3*(a*d-b*c)/d/(d*x+c)+3*b/d)/((b/d+(a*
d-b*c)/d/(d*x+c))*d-b)/d+3*(-3*Si(3*(a*d-b*c)/d/(d*x+c))*sin(3*b/d)/d+3*Ci
(3*(a*d-b*c)/d/(d*x+c))*cos(3*b/d)/d)/d)+3/4*d^2*(-sin(b/d+(a*d-b*c)/d/(d*
x+c))/((b/d+(a*d-b*c)/d/(d*x+c))*d-b)/d+(-Si((a*d-b*c)/d/(d*x+c))*sin(b/d)
/d+Ci((a*d-b*c)/d/(d*x+c))*cos(b/d)/d)/d))
```

3.38.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.09

$$\int \sin^3\left(\frac{a+bx}{c+dx}\right)\,dx$$
$$=\frac{3\,(bc-ad)\cos\left(\frac{b}{d}\right)\mathrm{Ci}\left(-\frac{bc-ad}{d^2x+cd}\right)-3\,(bc-ad)\cos\left(\frac{3\,b}{d}\right)\mathrm{Ci}\left(-\frac{3\,(bc-ad)}{d^2x+cd}\right)-3\,(bc-ad)\sin\left(\frac{b}{d}\right)\mathrm{Si}\left(-\frac{bc-ad}{d^2x+cd}\right)+\ldots}{4\,d^2}$$

input
```
integrate(sin((b*x+a)/(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/4*(3*(b*c - a*d)*cos(b/d)*cos_integral(-(b*c - a*d)/(d^2*x + c*d)) - 3*(
b*c - a*d)*cos(3*b/d)*cos_integral(-3*(b*c - a*d)/(d^2*x + c*d)) - 3*(b*c
- a*d)*sin(b/d)*sin_integral(-(b*c - a*d)/(d^2*x + c*d)) + 3*(b*c - a*d)*s
in(3*b/d)*sin_integral(-3*(b*c - a*d)/(d^2*x + c*d)) + 4*(d^2*x - (d^2*x +
 c*d)*cos((b*x + a)/(d*x + c))^2 + c*d)*sin((b*x + a)/(d*x + c)))/d^2
```

3.38.6 Sympy [F(-1)]

Timed out.

$$\int \sin^3\left(\frac{a+bx}{c+dx}\right)\,dx = \text{Timed out}$$

input
```
integrate(sin((b*x+a)/(d*x+c))**3,x)
```

output
```
Timed out
```

3.38.7 Maxima [F]

$$\int \sin^3\left(\frac{a+bx}{c+dx}\right)\,dx = \int \sin\left(\frac{bx+a}{dx+c}\right)^3\,dx$$

input
```
integrate(sin((b*x+a)/(d*x+c))^3,x, algorithm="maxima")
```

output
```
integrate(sin((b*x + a)/(d*x + c))^3, x)
```

3.38.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1239 vs. $2(186) = 372$.

Time = 66.42 (sec) , antiderivative size = 1239, normalized size of antiderivative = 6.39

$$\int \sin^3\left(\frac{a+bx}{c+dx}\right)\,dx = \text{Too large to display}$$

input
```
integrate(sin((b*x+a)/(d*x+c))^3,x, algorithm="giac")
```

output
```
1/4*(3*b^3*c^2*cos(b/d)*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d) - 6*a
*b^2*c*d*cos(b/d)*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d) - 3*(b*x +
a)*b^2*c^2*d*cos(b/d)*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d)/(d*x +
c) + 3*a^2*b*d^2*cos(b/d)*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d) + 6
*(b*x + a)*a*b*c*d^2*cos(b/d)*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d)
/(d*x + c) - 3*(b*x + a)*a^2*d^3*cos(b/d)*cos_integral(-(b - (b*x + a)*d/(
d*x + c))/d)/(d*x + c) - 3*b^3*c^2*cos(3*b/d)*cos_integral(-3*(b - (b*x +
a)*d/(d*x + c))/d) + 6*a*b^2*c*d*cos(3*b/d)*cos_integral(-3*(b - (b*x + a)
*d/(d*x + c))/d) + 3*(b*x + a)*b^2*c^2*d*cos(3*b/d)*cos_integral(-3*(b - (
b*x + a)*d/(d*x + c))/d)/(d*x + c) - 3*a^2*b*d^2*cos(3*b/d)*cos_integral(-
3*(b - (b*x + a)*d/(d*x + c))/d) - 6*(b*x + a)*a*b*c*d^2*cos(3*b/d)*cos_in
tegral(-3*(b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) + 3*(b*x + a)*a^2*d^3*c
os(3*b/d)*cos_integral(-3*(b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) - 3*b^3
*c^2*sin(3*b/d)*sin_integral(3*(b - (b*x + a)*d/(d*x + c))/d) + 6*a*b^2*c*
d*sin(3*b/d)*sin_integral(3*(b - (b*x + a)*d/(d*x + c))/d) + 3*(b*x + a)*b
^2*c^2*d*sin(3*b/d)*sin_integral(3*(b - (b*x + a)*d/(d*x + c))/d)/(d*x + c
) - 3*a^2*b*d^2*sin(3*b/d)*sin_integral(3*(b - (b*x + a)*d/(d*x + c))/d) -
 6*(b*x + a)*a*b*c*d^2*sin(3*b/d)*sin_integral(3*(b - (b*x + a)*d/(d*x + c
))/d)/(d*x + c) + 3*(b*x + a)*a^2*d^3*sin(3*b/d)*sin_integral(3*(b - (b*x
+ a)*d/(d*x + c))/d)/(d*x + c) + 3*b^3*c^2*sin(b/d)*sin_integral((b - (...
```

3.38.9 Mupad [F(-1)]

Timed out.

$$\int \sin^3\left(\frac{a+bx}{c+dx}\right)\,dx = \int \sin\left(\frac{a+b\,x}{c+d\,x}\right)^3\,dx$$

input
```
int(sin((a + b*x)/(c + d*x))^3,x)
```

output
```
int(sin((a + b*x)/(c + d*x))^3, x)
```

3.39 $\int \frac{\sin^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

3.39.1 Optimal result

Integrand size = 36, antiderivative size = 58

$$\int \frac{\sin^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{3\operatorname{Si}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a} + \frac{\operatorname{Si}\left(\frac{3\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}$$

output
```
-3/4*Si((-a*x+1)^(1/2)/(a*x+1)^(1/2))/a+1/4*Si(3*(-a*x+1)^(1/2)/(a*x+1)^(1
/2))/a
```

3.39.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.91

$$\int \frac{\sin^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \frac{-3\operatorname{Si}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right) + \operatorname{Si}\left(\frac{3\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}$$

input
```
Integrate[Sin[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^3/(1 - a^2*x^2),x]
```

output
```
(-3*SinIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]] + SinIntegral[(3*Sqrt[1 - a*x
])/Sqrt[1 + a*x]])/(4*a)
```

3.39.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {7232, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{1-a^2x^2}dx$$

$$\Big\downarrow \text{7232}$$

$$-\frac{\int \frac{\sqrt{ax+1}\sin^3\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sqrt{ax+1}\sin\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)^3}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{\int \left(\frac{3\sqrt{ax+1}\sin\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4\sqrt{1-ax}}-\frac{\sqrt{ax+1}\sin\left(\frac{3\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4\sqrt{1-ax}}\right)d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{3}{4}\text{Si}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{1}{4}\text{Si}\left(\frac{3\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

input
```
Int[Sin[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^3/(1 - a^2*x^2),x]
```

output
```
-(((3*SinIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]])/4 - SinIntegral[(3*Sqrt[1
- a*x])/Sqrt[1 + a*x]]/4)/a)
```

3.39.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 7232
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Simp[2*e*(g/(C*(e*f - d
*g)))   Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]],
x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] &&
EqQ[e*f + d*g, 0] && IGtQ[n, 0]
```

3.39.4 Maple [F]

$$\int \frac{\sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^{3}}{-a^{2}x^{2}+1}dx$$

input
```
int(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x)
```

output
```
int(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x)
```

3.39.5 Fricas [F]

$$\int \frac{\sin^{3}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^{2}x^{2}}\,dx = \int -\frac{\sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^{3}}{a^{2}x^{2}-1}\,dx$$

input
```
integrate(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x, algorithm="f
ricas")
```

output
```
integral((cos(sqrt(-a*x + 1)/sqrt(a*x + 1))^2 - 1)*sin(sqrt(-a*x + 1)/sqrt
(a*x + 1))/(a^2*x^2 - 1), x)
```

3.39.6 Sympy [F]

$$\int \frac{\sin^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\int \frac{\sin^3\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

input
```
integrate(sin((-a*x+1)**(1/2)/(a*x+1)**(1/2))**3/(-a**2*x**2+1),x)
```

output
```
-Integral(sin(sqrt(-a*x + 1)/sqrt(a*x + 1))**3/(a**2*x**2 - 1), x)
```

3.39.7 Maxima [F]

$$\int \frac{\sin^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^3}{a^2x^2-1}\,dx$$

input
```
integrate(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x, algorithm="m
axima")
```

output
```
-integrate(sin(sqrt(-a*x + 1)/sqrt(a*x + 1))^3/(a^2*x^2 - 1), x)
```

3.39.8 Giac [F]

$$\int \frac{\sin^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^3}{a^2x^2-1}\,dx$$

input
```
integrate(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x, algorithm="g
iac")
```

output
```
integrate(-sin(sqrt(-a*x + 1)/sqrt(a*x + 1))^3/(a^2*x^2 - 1), x)
```

3.39.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\int \frac{\sin\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)^3}{a^2\,x^2-1}\,dx$$

input
```
int(-sin((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^3/(a^2*x^2 - 1),x)
```

output
```
-int(sin((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^3/(a^2*x^2 - 1), x)
```

3.40 $\int \frac{\sin^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

3.40.1 Optimal result

Integrand size = 36, antiderivative size = 58

$$\int \frac{\sin^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \frac{\text{CosIntegral}\left(\frac{2\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a} - \frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}$$

output
```
1/2*Ci(2*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a-1/2*ln((-a*x+1)^(1/2)/(a*x+1)^(1/
2))/a
```

3.40.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

$$\int \frac{\sin^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \frac{\text{CosIntegral}\left(\frac{2\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a} - \frac{\log(1-ax)}{4a} + \frac{\log(1+ax)}{4a}$$

input
```
Integrate[Sin[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

output
```
CosIntegral[(2*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/(2*a) - Log[1 - a*x]/(4*a) +
Log[1 + a*x]/(4*a)
```

3.40.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {7232, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{1-a^2x^2}dx$$

$$\Big\downarrow \text{7232}$$

$$-\frac{\int \frac{\sqrt{ax+1}\sin^2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sqrt{ax+1}\sin\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)^2}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{\int \left(\frac{\sqrt{ax+1}}{2\sqrt{1-ax}}-\frac{\sqrt{ax+1}\cos\left(\frac{2\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2\sqrt{1-ax}}\right)d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{1}{2}\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)-\frac{1}{2}\text{CosIntegral}\left(\frac{2\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

input
```
Int[Sin[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

output
```
-((-1/2*CosIntegral[(2*Sqrt[1 - a*x])/Sqrt[1 + a*x]] + Log[Sqrt[1 - a*x]/S
qrt[1 + a*x]]/2)/a)
```

3.40.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 7232
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Simp[2*e*(g/(C*(e*f - d
*g)))   Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]],
x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] &&
EqQ[e*f + d*g, 0] && IGtQ[n, 0]
```

3.40.4 Maple [F]

$$\int \frac{\sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^{2}}{-a^{2}x^{2}+1}dx$$

input
```
int(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x)
```

output
```
int(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x)
```

3.40.5 Fricas [F]

$$\int \frac{\sin^{2}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^{2}x^{2}}\,dx = \int -\frac{\sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^{2}}{a^{2}x^{2}-1}\,dx$$

input
```
integrate(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="f
ricas")
```

output
```
integral((cos(sqrt(-a*x + 1)/sqrt(a*x + 1))^2 - 1)/(a^2*x^2 - 1), x)
```

3.40.6 Sympy [F]

$$\int \frac{\sin^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\int \frac{\sin^2\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

input
```
integrate(sin((-a*x+1)**(1/2)/(a*x+1)**(1/2))**2/(-a**2*x**2+1),x)
```

output
```
-Integral(sin(sqrt(-a*x + 1)/sqrt(a*x + 1))**2/(a**2*x**2 - 1), x)
```

3.40.7 Maxima [F]

$$\int \frac{\sin^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}{a^2x^2-1}\,dx$$

input
```
integrate(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="m
axima")
```

output
```
1/4*(4*a*integrate(1/4*cos(2*sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1),
x) + 4*a*integrate(1/4*cos(2*sqrt(-a*x + 1)/sqrt(a*x + 1))/((a^2*x^2 - 1)*
cos(2*sqrt(-a*x + 1)/sqrt(a*x + 1))^2 + (a^2*x^2 - 1)*sin(2*sqrt(-a*x + 1)
/sqrt(a*x + 1))^2), x) + log(a*x + 1) - log(a*x - 1))/a
```

3.40.8 Giac [F]

$$\int \frac{\sin^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}{a^2x^2-1}\,dx$$

input
```
integrate(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="g
iac")
```

output
```
integrate(-sin(sqrt(-a*x + 1)/sqrt(a*x + 1))^2/(a^2*x^2 - 1), x)
```

3.40.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\int \frac{\sin\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)^2}{a^2\,x^2-1}\,dx$$

input
```
int(-sin((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2/(a^2*x^2 - 1),x)
```

output
```
-int(sin((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2/(a^2*x^2 - 1), x)
```

3.41 $\int \frac{\sin\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

3.41.1 Optimal result

Integrand size = 34, antiderivative size = 26

$$\int \frac{\sin\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx=-\frac{\operatorname{Si}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

output
```
-Si((-a*x+1)^(1/2)/(a*x+1)^(1/2))/a
```

3.41.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\sin\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx=-\frac{\operatorname{Si}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

input
```
Integrate[Sin[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

output
```
-(SinIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)
```

3.41.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {7232, 3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{1-a^2x^2}\,dx$$

$$\Big\downarrow \text{ 7232}$$

$$-\frac{\int \frac{\sqrt{ax+1}\sin\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\int \frac{\sqrt{ax+1}\sin\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \text{ 3780}$$

$$-\frac{\text{Si}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

input
```
Int[Sin[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

output
```
-(SinIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)
```

3.41.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 7232
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Simp[2*e*(g/(C*(e*f - d
*g)))   Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]],
x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] &&
EqQ[e*f + d*g, 0] && IGtQ[n, 0]
```

3.41.4 Maple [F]

$$\int \frac{\sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{-a^2x^2+1}dx$$

input
```
int(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

output
```
int(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

3.41.5 Fricas [F]

$$\int \frac{\sin\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

input
```
integrate(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="fri
cas")
```

output
```
integral(-sin(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

3.41.6 Sympy [F]

$$\int \frac{\sin\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\int \frac{\sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

input
```
integrate(sin((-a*x+1)**(1/2)/(a*x+1)**(1/2))/(-a**2*x**2+1),x)
```

output
```
-Integral(sin(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a**2*x**2 - 1), x)
```

3.41.7 Maxima [F]

$$\int \frac{\sin\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

input
```
integrate(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="max
ima")
```

output
```
-integrate(sin(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

3.41.8 Giac [F]

$$\int \frac{\sin\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

input
```
integrate(sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="gia
c")
```

output
```
integrate(-sin(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

3.41.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\int \frac{\sin\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)}{a^2\,x^2-1}\,dx$$

input
```
int(-sin((1 - a*x)^(1/2)/(a*x + 1)^(1/2))/(a^2*x^2 - 1),x)
```

output
```
-int(sin((1 - a*x)^(1/2)/(a*x + 1)^(1/2))/(a^2*x^2 - 1), x)
```

3.42 $\int \frac{\csc\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

3.42.1 Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{\csc\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \operatorname{Int}\left(\frac{\csc\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{(1-ax)(1+ax)},x\right)$$

output
```
Unintegrable(csc((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a*x+1)/(a*x+1),x)
```

3.42.2 Mathematica [N/A]

Not integrable

Time = 3.59 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\csc\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int \frac{\csc\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$$

input
```
Integrate[Csc[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

output
```
Integrate[Csc[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2), x]
```

3.42.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7232, 3042, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{1-a^2x^2}dx$$

$$\Big\downarrow \text{7232}$$

$$-\frac{\int \frac{\sqrt{ax+1}\csc\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sqrt{ax+1}\csc\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \text{4680}$$

$$-\frac{\int \frac{\sqrt{ax+1}\csc\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

input
```
Int[Csc[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

output
```
$Aborted
```

3.42.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 7232
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Simp[2*e*(g/(C*(e*f - d
*g)))   Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]],
x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] &&
EqQ[e*f + d*g, 0] && IGtQ[n, 0]
```

3.42.4 Maple [N/A] (verified)

Not integrable

Time = 0.68 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{1}{\left(-a^{2} x^{2}+1\right) \sin \left(\frac{\sqrt{-a x +1}}{\sqrt{a x +1}}\right)}d x$$

input
```
int(1/(-a^2*x^2+1)/sin((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x)
```

output
```
int(1/(-a^2*x^2+1)/sin((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x)
```

3.42.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\csc \left(\frac{\sqrt{1-a x}}{\sqrt{1+a x}}\right)}{1-a^2 x^2} \, dx=\int { -\frac{1}{{\left(a^{2} x^{2} - 1\right)} \sin \left(\frac{\sqrt{-a x + 1}}{\sqrt{a x + 1}}\right)} } \,d x$$

input
```
integrate(1/(-a^2*x^2+1)/sin((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x, algorithm="f
ricas")
```

output
```
integral(-1/((a^2*x^2 - 1)*sin(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)
```

3.42.6 Sympy [N/A]

Not integrable

Time = 10.18 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.41

$$\int \frac{\csc\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\int \frac{1}{a^2x^2\sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right) - \sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}\,dx$$

input
```
integrate(1/(-a**2*x**2+1)/sin((-a*x+1)**(1/2)/(a*x+1)**(1/2)),x)
```

output
```
-Integral(1/(a**2*x**2*sin(sqrt(-a*x + 1)/sqrt(a*x + 1)) - sin(sqrt(-a*x +
 1)/sqrt(a*x + 1))), x)
```

3.42.7 Maxima [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int \frac{\csc\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{1}{(a^2x^2-1)\sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)/sin((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x, algorithm="m
axima")
```

output
```
-integrate(1/((a^2*x^2 - 1)*sin(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)
```

3.42.8 Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\csc\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{1}{(a^2x^2-1)\sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)/sin((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x, algorithm="g
iac")
```

output
```
integrate(-1/((a^2*x^2 - 1)*sin(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)
```

3.42.9 Mupad [N/A]

Not integrable

Time = 28.52 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int \frac{\csc\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\int \frac{1}{\sin\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)\,(a^2\,x^2-1)}\,dx$$

input
```
int(-1/(sin((1 - a*x)^(1/2)/(a*x + 1)^(1/2))*(a^2*x^2 - 1)),x)
```

output
```
-int(1/(sin((1 - a*x)^(1/2)/(a*x + 1)^(1/2))*(a^2*x^2 - 1)), x)
```

3.43 $\int \frac{\csc^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} \, dx$

3.43.1 Optimal result

Integrand size = 36, antiderivative size = 36

$$\int \frac{\csc^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} \, dx = \text{Int}\left(\frac{\csc^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{(1-ax)(1+ax)}, x\right)$$

output
```
Unintegrable(csc((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a*x+1)/(a*x+1),x)
```

3.43.2 Mathematica [N/A]

Not integrable

Time = 20.52 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\csc^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} \, dx = \int \frac{\csc^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} \, dx$$

input
```
Integrate[Csc[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

output
```
Integrate[Csc[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2), x]
```

3.43.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7232, 3042, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{1-a^2x^2}dx$$

$$\Big\downarrow \text{7232}$$

$$-\frac{\int \frac{\sqrt{ax+1}\csc^2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sqrt{ax+1}\csc\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)^2}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \text{4680}$$

$$-\frac{\int \frac{\sqrt{ax+1}\csc^2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

input
```
Int[Csc[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

output
```
$Aborted
```

3.43.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 7232
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Simp[2*e*(g/(C*(e*f - d
*g)))   Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]],
x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] &&
EqQ[e*f + d*g, 0] && IGtQ[n, 0]
```

3.43.4 Maple [N/A] (verified)

Not integrable

Time = 0.68 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{1}{\left(-a^{2} x^{2}+1\right) \sin \left(\frac{\sqrt{-a x +1}}{\sqrt{a x +1}}\right)^{2}}d x$$

input
```
int(1/(-a^2*x^2+1)/sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x)
```

output
```
int(1/(-a^2*x^2+1)/sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x)
```

3.43.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.25

$$\int \frac{\csc^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx=\int -\frac{1}{\left(a^{2} x^{2}-1\right) \sin \left(\frac{\sqrt{-a x +1}}{\sqrt{a x +1}}\right)^{2}}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)/sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x, algorithm=
"fricas")
```

output
```
integral(-1/(a^2*x^2 - (a^2*x^2 - 1)*cos(sqrt(-a*x + 1)/sqrt(a*x + 1))^2 -
 1), x)
```

3.43.6 Sympy [N/A]

Not integrable

Time = 36.78 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.42

$$\int \frac{\csc^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\int \frac{1}{a^2x^2\sin^2\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right) - \sin^2\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}\,dx$$

input
```
integrate(1/(-a**2*x**2+1)/sin((-a*x+1)**(1/2)/(a*x+1)**(1/2))**2,x)
```

output
```
-Integral(1/(a**2*x**2*sin(sqrt(-a*x + 1)/sqrt(a*x + 1))**2 - sin(sqrt(-a*
x + 1)/sqrt(a*x + 1))**2), x)
```

3.43.7 Maxima [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 696, normalized size of antiderivative = 19.33

$$\int \frac{\csc^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{1}{(a^2x^2-1)\sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)/sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x, algorithm=
"maxima")
```

output
```
((a^2*x + (a^2*x - a)*cos(2*sqrt(-a*x + 1)/sqrt(a*x + 1))^2 + (a^2*x - a)*
sin(2*sqrt(-a*x + 1)/sqrt(a*x + 1))^2 - 2*(a^2*x - a)*cos(2*sqrt(-a*x + 1)
/sqrt(a*x + 1)) - a)*integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)*sin(sqrt(-a*x
+ 1)/sqrt(a*x + 1))/(a^3*x^3 - a^2*x^2 + (a^3*x^3 - a^2*x^2 - a*x + 1)*cos
(sqrt(-a*x + 1)/sqrt(a*x + 1))^2 + (a^3*x^3 - a^2*x^2 - a*x + 1)*sin(sqrt(
-a*x + 1)/sqrt(a*x + 1))^2 - a*x + 2*(a^3*x^3 - a^2*x^2 - a*x + 1)*cos(sqr
t(-a*x + 1)/sqrt(a*x + 1)) + 1), x) - (a^2*x + (a^2*x - a)*cos(2*sqrt(-a*x
 + 1)/sqrt(a*x + 1))^2 + (a^2*x - a)*sin(2*sqrt(-a*x + 1)/sqrt(a*x + 1))^2
 - 2*(a^2*x - a)*cos(2*sqrt(-a*x + 1)/sqrt(a*x + 1)) - a)*integrate(sqrt(a
*x + 1)*sqrt(-a*x + 1)*sin(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^3*x^3 - a^2*x^
2 + (a^3*x^3 - a^2*x^2 - a*x + 1)*cos(sqrt(-a*x + 1)/sqrt(a*x + 1))^2 + (a
^3*x^3 - a^2*x^2 - a*x + 1)*sin(sqrt(-a*x + 1)/sqrt(a*x + 1))^2 - a*x - 2*
(a^3*x^3 - a^2*x^2 - a*x + 1)*cos(sqrt(-a*x + 1)/sqrt(a*x + 1)) + 1), x) -
 2*sqrt(a*x + 1)*sqrt(-a*x + 1)*sin(2*sqrt(-a*x + 1)/sqrt(a*x + 1)))/(a^2*
x + (a^2*x - a)*cos(2*sqrt(-a*x + 1)/sqrt(a*x + 1))^2 + (a^2*x - a)*sin(2*
sqrt(-a*x + 1)/sqrt(a*x + 1))^2 - 2*(a^2*x - a)*cos(2*sqrt(-a*x + 1)/sqrt(
a*x + 1)) - a)
```

3.43.8 Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \frac{\csc^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{1}{(a^2x^2-1)\sin\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)/sin((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x, algorithm=
"giac")
```

output
```
integrate(-1/((a^2*x^2 - 1)*sin(sqrt(-a*x + 1)/sqrt(a*x + 1))^2), x)
```

3.43.9 Mupad [N/A]

Not integrable

Time = 28.72 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int \frac{\csc^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\int \frac{1}{\sin\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)^2\,\left(a^2\,x^2-1\right)}\,dx$$

input
```
int(-1/(sin((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2*(a^2*x^2 - 1)),x)
```

output
```
-int(1/(sin((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2*(a^2*x^2 - 1)), x)
```

3.44 $\int (x+\cos(x))^2 \, dx$

3.44.1 Optimal result

Integrand size = 6, antiderivative size = 30

$$\int (x+\cos(x))^2 \, dx = \frac{x}{2} + \frac{x^3}{3} + 2\cos(x) + 2x\sin(x) + \frac{1}{2}\cos(x)\sin(x)$$

output
```
1/2*x+1/3*x^3+2*cos(x)+2*x*sin(x)+1/2*cos(x)*sin(x)
```

3.44.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int (x+\cos(x))^2 \, dx = \frac{1}{6}\left(3\cos(x)(4+\sin(x)) + x\left(3+2x^2+12\sin(x)\right)\right)$$

input
```
Integrate[(x + Cos[x])^2,x]
```

output
```
(3*Cos[x]*(4 + Sin[x]) + x*(3 + 2*x^2 + 12*Sin[x]))/6
```

3.44.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x+\cos(x))^2 \, dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(x^2+\cos^2(x)+2x\cos(x)\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^3}{3}+\frac{x}{2}+2x\sin(x)+2\cos(x)+\frac{1}{2}\sin(x)\cos(x)$$

input
```
Int[(x + Cos[x])^2,x]
```

output
```
x/2 + x^3/3 + 2*Cos[x] + 2*x*Sin[x] + (Cos[x]*Sin[x])/2
```

3.44.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.44.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	siz
default	$\frac{x}{2}+\frac{x^3}{3}+2\cos(x)+2x\sin(x)+\frac{\cos(x)\sin(x)}{2}$	25
risch	$\frac{x^3}{3}+\frac{x}{2}+2\cos(x)+2x\sin(x)+\frac{\sin(2x)}{4}$	25
parts	$\frac{x}{2}+\frac{x^3}{3}+2\cos(x)+2x\sin(x)+\frac{\cos(x)\sin(x)}{2}$	25
parallelrisch	$\frac{x^3}{3}+\frac{x}{2}+2+2x\sin(x)+\frac{\sin(2x)}{4}+2\cos(x)$	26
norman	$\frac{x\tan\left(\frac{x}{2}\right)^2+4\tan\left(\frac{x}{2}\right)^2+\frac{x}{2}+\frac{x^3}{3}-\tan\left(\frac{x}{2}\right)^3+4x\tan\left(\frac{x}{2}\right)+\frac{x\tan\left(\frac{x}{2}\right)^4}{2}+\frac{2x^3\tan\left(\frac{x}{2}\right)^2}{3}+\frac{x^3\tan\left(\frac{x}{2}\right)^4}{3}+4\tan\left(\frac{x}{2}\right)^3x+4+\tan\left(\frac{x}{2}\right)}{\left(1+\tan\left(\frac{x}{2}\right)^2\right)^2}$	97

input
```
int((x+cos(x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x+1/3*x^3+2*cos(x)+2*x*sin(x)+1/2*cos(x)*sin(x)
```

3.44.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

$$\int (x+\cos(x))^2\,dx=\frac{1}{3}\,x^3+\frac{1}{2}\,(4\,x+\cos(x))\sin(x)+\frac{1}{2}\,x+2\,\cos(x)$$

input
```
integrate((x+cos(x))^2,x, algorithm="fricas")
```

output
```
1/3*x^3 + 1/2*(4*x + cos(x))*sin(x) + 1/2*x + 2*cos(x)
```

3.44.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.37

$$\int (x+\cos(x))^2\,dx=\frac{x^3}{3}+\frac{x\sin^2(x)}{2}+2x\sin(x)+\frac{x\cos^2(x)}{2}+\frac{\sin(x)\cos(x)}{2}+2\cos(x)$$

input
```
integrate((x+cos(x))**2,x)
```

output
```
x**3/3 + x*sin(x)**2/2 + 2*x*sin(x) + x*cos(x)**2/2 + sin(x)*cos(x)/2 + 2*
cos(x)
```

3.44.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int (x+\cos(x))^2\,dx = \frac{1}{3}\,x^3 + 2\,x\sin(x) + \frac{1}{2}\,x + 2\,\cos(x) + \frac{1}{4}\,\sin(2\,x)$$

input
```
integrate((x+cos(x))^2,x, algorithm="maxima")
```

output
```
1/3*x^3 + 2*x*sin(x) + 1/2*x + 2*cos(x) + 1/4*sin(2*x)
```

3.44.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int (x+\cos(x))^2\,dx = \frac{1}{3}\,x^3 + 2\,x\sin(x) + \frac{1}{2}\,x + 2\,\cos(x) + \frac{1}{4}\,\sin(2\,x)$$

input
```
integrate((x+cos(x))^2,x, algorithm="giac")
```

output
```
1/3*x^3 + 2*x*sin(x) + 1/2*x + 2*cos(x) + 1/4*sin(2*x)
```

3.44.9 Mupad [B] (verification not implemented)

Time = 28.91 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int (x+\cos(x))^2\,dx = \frac{x}{2} + 2\,\cos(x) + \frac{\cos(x)\,\sin(x)}{2} + 2\,x\,\sin(x) + \frac{x^3}{3}$$

input
```
int((x + cos(x))^2,x)
```

output
```
x/2 + 2*cos(x) + (cos(x)*sin(x))/2 + 2*x*sin(x) + x^3/3
```

3.45 $\int (x+\cos(x))^3\,dx$

3.45.1 Optimal result

Integrand size = 6, antiderivative size = 56

$$\int (x+\cos(x))^3\,dx=\frac{3x^2}{4}+\frac{x^4}{4}+6x\cos(x)+\frac{3\cos^2(x)}{4}-5\sin(x)+3x^2\sin(x)+\frac{3}{2}x\cos(x)\sin(x)-\frac{\sin^3(x)}{3}$$

output
```
3/4*x^2+1/4*x^4+6*x*cos(x)+3/4*cos(x)^2-5*sin(x)+3*x^2*sin(x)+3/2*x*cos(x)
*sin(x)-1/3*sin(x)^3
```

3.45.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.91

$$\int (x+\cos(x))^3\,dx=6x\cos(x)+\frac{3}{8}\cos(2x)+\frac{1}{12}\left(9x^2+3x^4+9\left(-7+4x^2\right)\sin(x)+9x\sin(2x)+\sin(3x)\right)$$

input
```
Integrate[(x + Cos[x])^3,x]
```

output
```
6*x*Cos[x] + (3*Cos[2*x])/8 + (9*x^2 + 3*x^4 + 9*(-7 + 4*x^2)*Sin[x] + 9*x
*Sin[2*x] + Sin[3*x])/12
```

3.45.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x+\cos(x))^3 \, dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(x^3+3 x^2 \cos(x)+\cos^3(x)+3 x \cos^2(x)\right) \, dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^4}{4}+\frac{3 x^2}{4}+3 x^2 \sin(x)-\frac{\sin^3(x)}{3}-5 \sin(x)+\frac{3 \cos^2(x)}{4}+6 x \cos(x)+\frac{3}{2} x \sin(x) \cos(x)$$

input
```
Int[(x + Cos[x])^3,x]
```

output
```
(3*x^2)/4 + x^4/4 + 6*x*Cos[x] + (3*Cos[x]^2)/4 - 5*Sin[x] + 3*x^2*Sin[x]
+ (3*x*Cos[x]*Sin[x])/2 - Sin[x]^3/3
```

3.45.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.45.4 Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

method	result
risch	$\frac{x^4}{4}+\frac{3x^2}{4}+\frac{9}{16}+6x\cos(x)+\frac{3(4x^2-7)\sin(x)}{4}+\frac{\sin(3x)}{12}+\frac{3\cos(2x)}{8}+\frac{3x\sin(2x)}{4}$
parallelrisch	$\frac{5}{8}+\frac{x^4}{4}+\frac{3x^2}{4}+3x^2\sin(x)+6x\cos(x)+\frac{3x\sin(2x)}{4}+\frac{\sin(3x)}{12}-\frac{21\sin(x)}{4}+\frac{3\cos(2x)}{8}$
default	$\frac{(2+\cos(x)^2)\sin(x)}{3}+3x\left(\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)-\frac{3x^2}{4}-\frac{3\sin(x)^2}{4}+3x^2\sin(x)-6\sin(x)+6x\cos(x)-$
parts	$\frac{(2+\cos(x)^2)\sin(x)}{3}+3x\left(\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)-\frac{3x^2}{4}-\frac{3\sin(x)^2}{4}+3x^2\sin(x)-6\sin(x)+6x\cos(x)-$
norman	$\frac{-3\tan(\frac{x}{2})^2-3\tan(\frac{x}{2})^4+6x+\frac{3x^2}{4}+\frac{x^4}{4}-\frac{68\tan(\frac{x}{2})^3}{3}-10\tan(\frac{x}{2})^5+3x\tan(\frac{x}{2})+6x\tan(\frac{x}{2})^2-6x\tan(\frac{x}{2})^4-3x\tan(\frac{x}{2})^5-6x\tan}{}$

input
```
int((x+cos(x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4+3/4*x^2+9/16+6*x*cos(x)+3/4*(4*x^2-7)*sin(x)+1/12*sin(3*x)+3/8*cos
(2*x)+3/4*x*sin(2*x)
```

3.45.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.79

$$\int (x+\cos(x))^3\,dx = \frac{1}{4}x^4+\frac{3}{4}x^2+6x\cos(x)+\frac{3}{4}\cos(x)^2 + \frac{1}{6}\left(18x^2+9x\cos(x)+2\cos(x)^2-32\right)\sin(x)$$

input
```
integrate((x+cos(x))^3,x, algorithm="fricas")
```

output
```
1/4*x^4 + 3/4*x^2 + 6*x*cos(x) + 3/4*cos(x)^2 + 1/6*(18*x^2 + 9*x*cos(x) +
 2*cos(x)^2 - 32)*sin(x)
```

3.45.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.52

$$\int (x+\cos(x))^3\,dx = \frac{x^4}{4}+\frac{3x^2\sin^2(x)}{4}+3x^2\sin(x)+\frac{3x^2\cos^2(x)}{4}+\frac{3x\sin(x)\cos(x)}{2}+6x\cos(x)+\frac{2\sin^3(x)}{3}-\frac{3\sin^2(x)}{4}+\sin(x)\cos^2(x)-6\sin(x)$$

input
```
integrate((x+cos(x))**3,x)
```

output
```
x**4/4 + 3*x**2*sin(x)**2/4 + 3*x**2*sin(x) + 3*x**2*cos(x)**2/4 + 3*x*sin
(x)*cos(x)/2 + 6*x*cos(x) + 2*sin(x)**3/3 - 3*sin(x)**2/4 + sin(x)*cos(x)*
*2 - 6*sin(x)
```

3.45.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int (x+\cos(x))^3\,dx = \frac{1}{4}x^4-\frac{1}{3}\sin(x)^3+\frac{3}{4}x^2+6x\cos(x)+\frac{3}{4}x\sin(2x)+3(x^2-2)\sin(x)+\frac{3}{8}\cos(2x)+\sin(x)$$

input
```
integrate((x+cos(x))^3,x, algorithm="maxima")
```

output
```
1/4*x^4 - 1/3*sin(x)^3 + 3/4*x^2 + 6*x*cos(x) + 3/4*x*sin(2*x) + 3*(x^2 -
2)*sin(x) + 3/8*cos(2*x) + sin(x)
```

3.45.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int (x+\cos(x))^3\,dx = \frac{1}{4}x^4+\frac{3}{4}x^2+6x\cos(x)+\frac{3}{4}x\sin(2x)+\frac{3}{4}(4x^2-7)\sin(x)+\frac{3}{8}\cos(2x)+\frac{1}{12}\sin(3x)$$

input
```
integrate((x+cos(x))^3,x, algorithm="giac")
```

output
```
1/4*x^4 + 3/4*x^2 + 6*x*cos(x) + 3/4*x*sin(2*x) + 3/4*(4*x^2 - 7)*sin(x) +
 3/8*cos(2*x) + 1/12*sin(3*x)
```

3.45.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int (x+\cos(x))^3\,dx = 3\,x^2\,\sin\left(x\right)-\frac{16\,\sin\left(x\right)}{3}+\frac{3\cos\left(x\right)^2}{4}+\frac{\cos\left(x\right)^2\,\sin\left(x\right)}{3}+6\,x\,\cos\left(x\right)+\frac{3\,x^2}{4}+\frac{x^4}{4}+\frac{3\,x\,\cos\left(x\right)\,\sin\left(x\right)}{2}$$

input
```
int((x + cos(x))^3,x)
```

output
```
3*x^2*sin(x) - (16*sin(x))/3 + (3*cos(x)^2)/4 + (cos(x)^2*sin(x))/3 + 6*x*
cos(x) + (3*x^2)/4 + x^4/4 + (3*x*cos(x)*sin(x))/2
```

3.46 $\int \frac{\cos(a+bx)}{c+dx^2} \, dx$

3.46.1 Optimal result

Integrand size = 16, antiderivative size = 213

$$\int \frac{\cos(a+bx)}{c+dx^2} \, dx = \frac{\cos\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right) \text{CosIntegral}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\sqrt{d}} - \frac{\cos\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right) \text{CosIntegral}\left(\frac{b\sqrt{-c}}{\sqrt{d}}+bx\right)}{2\sqrt{-c}\sqrt{d}} + \frac{\sin\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right) \text{Si}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\sqrt{d}} + \frac{\sin\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right) \text{Si}\left(\frac{b\sqrt{-c}}{\sqrt{d}}+bx\right)}{2\sqrt{-c}\sqrt{d}}$$

output
```
-1/2*Ci(b*x+b*(-c)^(1/2)/d^(1/2))*cos(a-b*(-c)^(1/2)/d^(1/2))/(-c)^(1/2)/d
^(1/2)+1/2*Ci(-b*x+b*(-c)^(1/2)/d^(1/2))*cos(a+b*(-c)^(1/2)/d^(1/2))/(-c)^
(1/2)/d^(1/2)+1/2*Si(b*x+b*(-c)^(1/2)/d^(1/2))*sin(a-b*(-c)^(1/2)/d^(1/2))
/(-c)^(1/2)/d^(1/2)-1/2*Si(b*x-b*(-c)^(1/2)/d^(1/2))*sin(a+b*(-c)^(1/2)/d^
(1/2))/(-c)^(1/2)/d^(1/2)
```

3.46.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.77

$$\int \frac{\cos(a+bx)}{c+dx^2}\,dx = \frac{ie^{-ia-\frac{b\sqrt{c}}{\sqrt{d}}}\left(-e^{\frac{2b\sqrt{c}}{\sqrt{d}}}\text{ExpIntegralEi}\left(-\frac{b\sqrt{c}}{\sqrt{d}}-ibx\right)+\text{ExpIntegralEi}\left(\frac{b\sqrt{c}}{\sqrt{d}}-ibx\right)+e^{2ia}\left(e^{\frac{2b\sqrt{c}}{\sqrt{d}}}\text{ExpIntegralEi}\right.\right.}{4\sqrt{c}\sqrt{d}}$$

input
```
Integrate[Cos[a + b*x]/(c + d*x^2),x]
```

output
```
((I/4)*E^((-I)*a - (b*Sqrt[c])/Sqrt[d])*(-(E^((2*b*Sqrt[c])/Sqrt[d])*ExpIn
tegralEi[-((b*Sqrt[c])/Sqrt[d]) - I*b*x]) + ExpIntegralEi[(b*Sqrt[c])/Sqrt
[d] - I*b*x] + E^((2*I)*a)*(E^((2*b*Sqrt[c])/Sqrt[d])*ExpIntegralEi[-((b*S
qrt[c])/Sqrt[d]) + I*b*x] - ExpIntegralEi[(b*Sqrt[c])/Sqrt[d] + I*b*x])))/
(Sqrt[c]*Sqrt[d])
```

3.46.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3815, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(a+bx)}{c+dx^2}\,dx$$

$$\Big\downarrow \; 3815$$

$$\int \left(\frac{\sqrt{-c}\cos(a+bx)}{2c\left(\sqrt{-c}-\sqrt{d}x\right)}+\frac{\sqrt{-c}\cos(a+bx)}{2c\left(\sqrt{-c}+\sqrt{d}x\right)}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{\cos\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{CosIntegral}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\sqrt{d}}-\frac{\cos\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{CosIntegral}\left(xb+\frac{\sqrt{-c}b}{\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}}+\frac{\sin\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Si}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\sqrt{d}}+\frac{\sin\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Si}\left(xb+\frac{\sqrt{-c}b}{\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}}$$

input
```
Int[Cos[a + b*x]/(c + d*x^2),x]
```

output
```
(Cos[a + (b*Sqrt[-c])/Sqrt[d]]*CosIntegral[(b*Sqrt[-c])/Sqrt[d] - b*x])/(2
*Sqrt[-c]*Sqrt[d]) - (Cos[a - (b*Sqrt[-c])/Sqrt[d]]*CosIntegral[(b*Sqrt[-c
])/Sqrt[d] + b*x])/(2*Sqrt[-c]*Sqrt[d]) + (Sin[a + (b*Sqrt[-c])/Sqrt[d]]*S
inIntegral[(b*Sqrt[-c])/Sqrt[d] - b*x])/(2*Sqrt[-c]*Sqrt[d]) + (Sin[a - (b
*Sqrt[-c])/Sqrt[d]]*SinIntegral[(b*Sqrt[-c])/Sqrt[d] + b*x])/(2*Sqrt[-c]*S
qrt[d])
```

3.46.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3815
```
Int[Cos[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int
[ExpandIntegrand[Cos[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

3.46.4 Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.08

method	result
derivativedivides	$b\left(-\frac{\text{Si}\left(-xb-a+\frac{b\sqrt{-cd}+ad}{d}\right)\sin\left(\frac{b\sqrt{-cd}+ad}{d}\right)+\text{Ci}\left(xb+a-\frac{b\sqrt{-cd}+ad}{d}\right)\cos\left(\frac{b\sqrt{-cd}+ad}{d}\right)}{2d\left(-\frac{b\sqrt{-cd}+ad}{d}+a\right)}-\frac{-\text{Si}\left(-xb-a-\frac{b\sqrt{-cd}-ad}{d}\right.}{}\right.$
default	$b\left(-\frac{\text{Si}\left(-xb-a+\frac{b\sqrt{-cd}+ad}{d}\right)\sin\left(\frac{b\sqrt{-cd}+ad}{d}\right)+\text{Ci}\left(xb+a-\frac{b\sqrt{-cd}+ad}{d}\right)\cos\left(\frac{b\sqrt{-cd}+ad}{d}\right)}{2d\left(-\frac{b\sqrt{-cd}+ad}{d}+a\right)}-\frac{-\text{Si}\left(-xb-a-\frac{b\sqrt{-cd}-ad}{d}\right.}{}\right.$
risch	$-\frac{i\sqrt{cd}\,\mathrm{e}^{-\frac{iad+b\sqrt{cd}}{d}}\,\text{Ei}_1\left(-\frac{iad+b\sqrt{cd}-(ibx+ia)d}{d}\right)}{4cd}-\frac{i\sqrt{cd}\,\mathrm{e}^{\frac{iad+b\sqrt{cd}}{d}}\,\text{Ei}_1\left(\frac{iad+b\sqrt{cd}-(ibx+ia)d}{d}\right)}{4cd}+\frac{i\sqrt{cd}\,\mathrm{e}^{\frac{iad-b\sqrt{cd}}{d}}}{}$

input
```
int(cos(b*x+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
b*(-1/2/d/(-(b*(-c*d)^(1/2)+a*d)/d+a)*(Si(-x*b-a+(b*(-c*d)^(1/2)+a*d)/d)*s
in((b*(-c*d)^(1/2)+a*d)/d)+Ci(x*b+a-(b*(-c*d)^(1/2)+a*d)/d)*cos((b*(-c*d)^
(1/2)+a*d)/d))-1/2/d/((b*(-c*d)^(1/2)-a*d)/d+a)*(-Si(-x*b-a-(b*(-c*d)^(1/2
)-a*d)/d)*sin((b*(-c*d)^(1/2)-a*d)/d)+Ci(x*b+a+(b*(-c*d)^(1/2)-a*d)/d)*cos
((b*(-c*d)^(1/2)-a*d)/d))
```

3.46.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.89

$$\int \frac{\cos(a+bx)}{c+dx^2}\,dx =$$
$$-\frac{-i\sqrt{\frac{b^2c}{d}}\,\mathrm{Ei}\left(i\,bx-\sqrt{\frac{b^2c}{d}}\right)e^{\left(i\,a+\sqrt{\frac{b^2c}{d}}\right)}+i\sqrt{\frac{b^2c}{d}}\,\mathrm{Ei}\left(i\,bx+\sqrt{\frac{b^2c}{d}}\right)e^{\left(i\,a-\sqrt{\frac{b^2c}{d}}\right)}+i\sqrt{\frac{b^2c}{d}}\,\mathrm{Ei}\left(-i\,bx-\sqrt{\frac{b^2c}{d}}\right.}{4\,bc}$$

input
```
integrate(cos(b*x+a)/(d*x^2+c),x, algorithm="fricas")
```

output
```
-1/4*(-I*sqrt(b^2*c/d)*Ei(I*b*x - sqrt(b^2*c/d))*e^(I*a + sqrt(b^2*c/d)) +
 I*sqrt(b^2*c/d)*Ei(I*b*x + sqrt(b^2*c/d))*e^(I*a - sqrt(b^2*c/d)) + I*sqr
t(b^2*c/d)*Ei(-I*b*x - sqrt(b^2*c/d))*e^(-I*a + sqrt(b^2*c/d)) - I*sqrt(b^
2*c/d)*Ei(-I*b*x + sqrt(b^2*c/d))*e^(-I*a - sqrt(b^2*c/d)))/(b*c)
```

3.46.6 Sympy [F]

$$\int \frac{\cos(a+bx)}{c+dx^2}\,dx = \int \frac{\cos{(a+bx)}}{c+dx^2}\,dx$$

input
```
integrate(cos(b*x+a)/(d*x**2+c),x)
```

output
```
Integral(cos(a + b*x)/(c + d*x**2), x)
```

3.46.7 Maxima [F]

$$\int \frac{\cos(a+bx)}{c+dx^2}\,dx=\int \frac{\cos(bx+a)}{dx^2+c}\,dx$$

input
```
integrate(cos(b*x+a)/(d*x^2+c),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)/(d*x^2 + c), x)
```

3.46.8 Giac [F]

$$\int \frac{\cos(a+bx)}{c+dx^2}\,dx=\int \frac{\cos(bx+a)}{dx^2+c}\,dx$$

input
```
integrate(cos(b*x+a)/(d*x^2+c),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)/(d*x^2 + c), x)
```

3.46.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)}{c+dx^2}\,dx=\int \frac{\cos(a+b\,x)}{d\,x^2+c}\,dx$$

input
```
int(cos(a + b*x)/(c + d*x^2),x)
```

output
```
int(cos(a + b*x)/(c + d*x^2), x)
```

3.47 $\int \frac{\cos(a+bx)}{c+dx+ex^2}\,dx$

3.47.1 Optimal result

Integrand size = 19, antiderivative size = 271

$$\int \frac{\cos(a+bx)}{c+dx+ex^2}\,dx = \frac{\cos\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\operatorname{CosIntegral}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}} - \frac{\cos\left(a-\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}\right)\operatorname{CosIntegral}\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}} - \frac{\sin\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\operatorname{Si}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}} + \frac{\sin\left(a-\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}\right)\operatorname{Si}\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}$$

output
```
Ci(b*x+1/2*b*(d-(-4*c*e+d^2)^(1/2))/e)*cos(a-1/2*b*(d-(-4*c*e+d^2)^(1/2))/
e)/(-4*c*e+d^2)^(1/2)-Ci(b*x+1/2*b*(d+(-4*c*e+d^2)^(1/2))/e)*cos(a-1/2*b*(
d+(-4*c*e+d^2)^(1/2))/e)/(-4*c*e+d^2)^(1/2)-Si(b*x+1/2*b*(d-(-4*c*e+d^2)^(
1/2))/e)*sin(a-1/2*b*(d-(-4*c*e+d^2)^(1/2))/e)/(-4*c*e+d^2)^(1/2)+Si(b*x+1
/2*b*(d+(-4*c*e+d^2)^(1/2))/e)*sin(a-1/2*b*(d+(-4*c*e+d^2)^(1/2))/e)/(-4*c
*e+d^2)^(1/2)
```

3.47.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.90

$$\int \frac{\cos(a+bx)}{c+dx+ex^2}\,dx$$
$$= \frac{e^{-\frac{1}{2}i\left(2a+\frac{b\left(d+\sqrt{d^2-4ce}\right)}{e}\right)}\left(e^{\frac{ibd}{e}}\text{ExpIntegralEi}\left(-\frac{ib\left(d-\sqrt{d^2-4ce}+2ex\right)}{2e}\right)+e^{i\left(2a+\frac{b\sqrt{d^2-4ce}}{e}\right)}\text{ExpIntegralEi}\left(\frac{ib\left(d-\sqrt{\ldots}\right.}{\ldots}\right.\right.}{2\sqrt{\ldots}}$$

input
```
Integrate[Cos[a + b*x]/(c + d*x + e*x^2),x]
```

output
```
(E^((I*b*d)/e)*ExpIntegralEi[((-1/2*I)*b*(d - Sqrt[d^2 - 4*c*e] + 2*e*x))/
e] + E^(I*(2*a + (b*Sqrt[d^2 - 4*c*e])/e))*ExpIntegralEi[((I/2)*b*(d - Sqr
t[d^2 - 4*c*e] + 2*e*x))/e] - E^((I*b*(d + Sqrt[d^2 - 4*c*e]))/e)*ExpInteg
ralEi[((-1/2*I)*b*(d + Sqrt[d^2 - 4*c*e] + 2*e*x))/e] - E^((2*I)*a)*ExpInt
egralEi[((I/2)*b*(d + Sqrt[d^2 - 4*c*e] + 2*e*x))/e])/(2*Sqrt[d^2 - 4*c*e]
*E^((I/2)*(2*a + (b*(d + Sqrt[d^2 - 4*c*e]))/e)))
```

3.47.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {7279, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(a+bx)}{c+dx+ex^2}\,dx$$
$$\Big\downarrow \text{ 7279}$$
$$\int \left(\frac{2e\cos(a+bx)}{\sqrt{d^2-4ce}\left(-\sqrt{d^2-4ce}+d+2ex\right)}-\frac{2e\cos(a+bx)}{\sqrt{d^2-4ce}\left(\sqrt{d^2-4ce}+d+2ex\right)}\right)dx$$
$$\Big\downarrow \text{ 2009}$$

$$\frac{\cos\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\text{CosIntegral}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}-\frac{\cos\left(a-\frac{b\left(\sqrt{d^2-4ce}+d\right)}{2e}\right)\text{CosIntegral}\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}-\frac{\sin\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\text{Si}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}+\frac{\sin\left(a-\frac{b\left(\sqrt{d^2-4ce}+d\right)}{2e}\right)\text{Si}\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}$$

input
```
Int[Cos[a + b*x]/(c + d*x + e*x^2),x]
```

output
```
(Cos[a - (b*(d - Sqrt[d^2 - 4*c*e]))/(2*e)]*CosIntegral[(b*(d - Sqrt[d^2 -
 4*c*e]))/(2*e) + b*x])/Sqrt[d^2 - 4*c*e] - (Cos[a - (b*(d + Sqrt[d^2 - 4*
c*e]))/(2*e)]*CosIntegral[(b*(d + Sqrt[d^2 - 4*c*e]))/(2*e) + b*x])/Sqrt[d
^2 - 4*c*e] - (Sin[a - (b*(d - Sqrt[d^2 - 4*c*e]))/(2*e)]*SinIntegral[(b*(
d - Sqrt[d^2 - 4*c*e]))/(2*e) + b*x])/Sqrt[d^2 - 4*c*e] + (Sin[a - (b*(d +
 Sqrt[d^2 - 4*c*e]))/(2*e)]*SinIntegral[(b*(d + Sqrt[d^2 - 4*c*e]))/(2*e)
+ b*x])/Sqrt[d^2 - 4*c*e]
```

3.47.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7279
```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

3.47.4 Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.20

method	result
derivativedivides	$b\left(\frac{\mathrm{Si}\left(-xb-a+\frac{2ae-bd+\sqrt{-4b^2ce+b^2d^2}}{2e}\right)\sin\left(\frac{2ae-bd+\sqrt{-4b^2ce+b^2d^2}}{2e}\right)+\mathrm{Ci}\left(xb+a-\frac{2ae-bd+\sqrt{-4b^2ce+b^2d^2}}{2e}\right)\cos\left(\frac{2ae-}{}\right.}{\sqrt{-4b^2ce+b^2d^2}}\right.$
default	$b\left(\frac{\mathrm{Si}\left(-xb-a+\frac{2ae-bd+\sqrt{-4b^2ce+b^2d^2}}{2e}\right)\sin\left(\frac{2ae-bd+\sqrt{-4b^2ce+b^2d^2}}{2e}\right)+\mathrm{Ci}\left(xb+a-\frac{2ae-bd+\sqrt{-4b^2ce+b^2d^2}}{2e}\right)\cos\left(\frac{2ae-}{}\right.}{\sqrt{-4b^2ce+b^2d^2}}\right.$
risch	$-\frac{i\sqrt{4b^2ce-b^2d^2}\ \mathrm{Ei}_1\left(-\frac{2iae-ibd-2e(ibx+ia)+\sqrt{4b^2ce-b^2d^2}}{2e}\right)\mathrm{e}^{-\frac{2iae-ibd+\sqrt{4b^2ce-b^2d^2}}{2e}}}{2b(4ce-d^2)}+\frac{i\sqrt{4b^2ce-b^2d^2}\ \mathrm{Ei}_1\left(-\frac{2iae-}{}\right.}{}$

input
```
int(cos(b*x+a)/(e*x^2+d*x+c),x,method=_RETURNVERBOSE)
```

output
```
b*(1/(-4*b^2*c*e+b^2*d^2)^(1/2)*(Si(-x*b-a+1/2/e*(2*a*e-b*d+(-4*b^2*c*e+b^
2*d^2)^(1/2)))*sin(1/2/e*(2*a*e-b*d+(-4*b^2*c*e+b^2*d^2)^(1/2)))+Ci(x*b+a-
1/2/e*(2*a*e-b*d+(-4*b^2*c*e+b^2*d^2)^(1/2)))*cos(1/2/e*(2*a*e-b*d+(-4*b^2
*c*e+b^2*d^2)^(1/2))))-1/(-4*b^2*c*e+b^2*d^2)^(1/2)*(-Si(-x*b-a-1/2*(-2*a*
e+b*d+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)*sin(1/2*(-2*a*e+b*d+(-4*b^2*c*e+b^2*d
^2)^(1/2))/e)+Ci(x*b+a+1/2*(-2*a*e+b*d+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)*cos(
1/2*(-2*a*e+b*d+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)))
```

3.47.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.61

$$\int \frac{\cos(a+bx)}{c+dx+ex^2}\,dx =$$

$$-\frac{-i\,e\sqrt{-\frac{b^2d^2-4\,b^2ce}{e^2}}\mathrm{Ei}\left(\frac{-2i\,bex-i\,bd-e\sqrt{-\frac{b^2d^2-4\,b^2ce}{e^2}}}{2\,e}\right)e^{\left(\frac{i\,bd-2i\,ae+e\sqrt{-\frac{b^2d^2-4\,b^2ce}{e^2}}}{2\,e}\right)}+i\,e\sqrt{-\frac{b^2d^2-4\,b^2ce}{e^2}}\mathrm{Ei}\left(\frac{-2i\,bex-i}{}\right.}{}$$

input
```
integrate(cos(b*x+a)/(e*x^2+d*x+c),x, algorithm="fricas")
```

output
```
-1/2*(-I*e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(1/2*(-2*I*b*e*x - I*b*d - e
*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2))/e)*e^(1/2*(I*b*d - 2*I*a*e + e*sqrt(-(b
^2*d^2 - 4*b^2*c*e)/e^2))/e) + I*e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(1/2
*(-2*I*b*e*x - I*b*d + e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2))/e)*e^(1/2*(I*b*
d - 2*I*a*e - e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2))/e) + I*e*sqrt(-(b^2*d^2
- 4*b^2*c*e)/e^2)*Ei(1/2*(2*I*b*e*x + I*b*d - e*sqrt(-(b^2*d^2 - 4*b^2*c*e
)/e^2))/e)*e^(1/2*(-I*b*d + 2*I*a*e + e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2))/
e) - I*e*sqrt(-(b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(1/2*(2*I*b*e*x + I*b*d + e*sq
rt(-(b^2*d^2 - 4*b^2*c*e)/e^2))/e)*e^(1/2*(-I*b*d + 2*I*a*e - e*sqrt(-(b^2
*d^2 - 4*b^2*c*e)/e^2))/e))/(b*d^2 - 4*b*c*e)
```

3.47.6 Sympy [F]

$$\int \frac{\cos(a+bx)}{c+dx+ex^2}\,dx = \int \frac{\cos{(a+bx)}}{c+dx+ex^2}\,dx$$

input
```
integrate(cos(b*x+a)/(e*x**2+d*x+c),x)
```

output
```
Integral(cos(a + b*x)/(c + d*x + e*x**2), x)
```

3.47.7 Maxima [F]

$$\int \frac{\cos(a+bx)}{c+dx+ex^2}\,dx = \int \frac{\cos{(bx+a)}}{ex^2+dx+c}\,dx$$

input
```
integrate(cos(b*x+a)/(e*x^2+d*x+c),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)/(e*x^2 + d*x + c), x)
```

3.47.8 Giac [F]

$$\int \frac{\cos(a+bx)}{c+dx+ex^2}\,dx=\int \frac{\cos(bx+a)}{ex^2+dx+c}\,dx$$

input
```
integrate(cos(b*x+a)/(e*x^2+d*x+c),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)/(e*x^2 + d*x + c), x)
```

3.47.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)}{c+dx+ex^2}\,dx=\int \frac{\cos(a+b\,x)}{e\,x^2+d\,x+c}\,dx$$

input
```
int(cos(a + b*x)/(c + d*x + e*x^2),x)
```

output
```
int(cos(a + b*x)/(c + d*x + e*x^2), x)
```

3.48 $\int \frac{x \cos\left(\sqrt{1+x^2}\right)}{\sqrt{1+x^2}}\,dx$

3.48.1 Optimal result

Integrand size = 21, antiderivative size = 10

$$\int \frac{x \cos\left(\sqrt{1+x^2}\right)}{\sqrt{1+x^2}}\,dx = \sin\left(\sqrt{1+x^2}\right)$$

output
```
sin((x^2+1)^(1/2))
```

3.48.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{x \cos\left(\sqrt{1+x^2}\right)}{\sqrt{1+x^2}}\,dx = \sin\left(\sqrt{1+x^2}\right)$$

input
```
Integrate[(x*Cos[Sqrt[1 + x^2]])/Sqrt[1 + x^2],x]
```

output
```
Sin[Sqrt[1 + x^2]]
```

3.48.3 Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 27 vs. $2(10) = 20$.

Time = 0.33 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.70, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {7266, 3913, 30, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\cos\left(\sqrt{x^2+1}\right)}{\sqrt{x^2+1}}dx$$

$$\Big\downarrow \text{7266}$$

$$\frac{1}{2}\int \frac{\cos\left(\sqrt{x^2+1}\right)}{\sqrt{x^2+1}}dx^2$$

$$\Big\downarrow \text{3913}$$

$$\int \frac{\sqrt{x^2+1}\cos\left(\sqrt{x^2+1}\right)}{\sqrt{x^4}}d\sqrt{x^2+1}$$

$$\Big\downarrow \text{30}$$

$$\frac{\sqrt{x^2+1}\int \cos\left(\sqrt{x^2+1}\right)d\sqrt{x^2+1}}{\sqrt{x^4}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{x^2+1}\int \sin\left(\sqrt{x^2+1}+\frac{\pi}{2}\right)d\sqrt{x^2+1}}{\sqrt{x^4}}$$

$$\Big\downarrow \text{3117}$$

$$\frac{\sqrt{x^2+1}\sin\left(\sqrt{x^2+1}\right)}{\sqrt{x^4}}$$

input
```
Int[(x*Cos[Sqrt[1 + x^2]])/Sqrt[1 + x^2],x]
```

output
```
(Sqrt[1 + x^2]*Sin[Sqrt[1 + x^2]])/Sqrt[x^4]
```

3.48.3.1 Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3913
```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[1/(n*f)   Subst[Int[ExpandIntegra
nd[(a + b*Cos[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
 x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p
, 0] && IntegerQ[1/n]
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

3.48.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$\sin\left(\sqrt{x^2+1}\right)$	9
default	$\sin\left(\sqrt{x^2+1}\right)$	9

input
```
int(x*cos((x^2+1)^(1/2))/(x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
sin((x^2+1)^(1/2))
```

3.48.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{x\cos\left(\sqrt{1+x^2}\right)}{\sqrt{1+x^2}}\,dx = \sin\left(\sqrt{x^2+1}\right)$$

input
```
integrate(x*cos((x^2+1)^(1/2))/(x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
sin(sqrt(x^2 + 1))
```

3.48.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{x\cos\left(\sqrt{1+x^2}\right)}{\sqrt{1+x^2}}\,dx = \sin\left(\sqrt{x^2+1}\right)$$

input
```
integrate(x*cos((x**2+1)**(1/2))/(x**2+1)**(1/2),x)
```

output
```
sin(sqrt(x**2 + 1))
```

3.48.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{x\cos\left(\sqrt{1+x^2}\right)}{\sqrt{1+x^2}}\,dx = \sin\left(\sqrt{x^2+1}\right)$$

input
```
integrate(x*cos((x^2+1)^(1/2))/(x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
sin(sqrt(x^2 + 1))
```

3.48.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{x \cos \left(\sqrt{1+x^2}\right)}{\sqrt{1+x^2}} \, dx = \sin \left(\sqrt{x^2+1}\right)$$

input
```
integrate(x*cos((x^2+1)^(1/2))/(x^2+1)^(1/2),x, algorithm="giac")
```

output
```
sin(sqrt(x^2 + 1))
```

3.48.9 Mupad [B] (verification not implemented)

Time = 25.94 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{x \cos \left(\sqrt{1+x^2}\right)}{\sqrt{1+x^2}} \, dx = \sin \left(\sqrt{x^2+1}\right)$$

input
```
int((x*cos((x^2 + 1)^(1/2)))/(x^2 + 1)^(1/2),x)
```

output
```
sin((x^2 + 1)^(1/2))
```

3.49 $\int \frac{x\cos\left(\sqrt{3}\sqrt{2+x^2}\right)}{\sqrt{2+x^2}}\,dx$

3.49.1 Optimal result

Integrand size = 27, antiderivative size = 22

$$\int \frac{x\cos\left(\sqrt{3}\sqrt{2+x^2}\right)}{\sqrt{2+x^2}}\,dx = \frac{\sin\left(\sqrt{3}\sqrt{2+x^2}\right)}{\sqrt{3}}$$

output
```
1/3*sin(3^(1/2)*(x^2+2)^(1/2))*3^(1/2)
```

3.49.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x\cos\left(\sqrt{3}\sqrt{2+x^2}\right)}{\sqrt{2+x^2}}\,dx = \frac{\sin\left(\sqrt{3}\sqrt{2+x^2}\right)}{\sqrt{3}}$$

input
```
Integrate[(x*Cos[Sqrt[3]*Sqrt[2 + x^2]])/Sqrt[2 + x^2],x]
```

output
```
Sin[Sqrt[3]*Sqrt[2 + x^2]]/Sqrt[3]
```

3.49.3 Rubi [A] (warning: unable to verify)

Time = 0.38 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.73, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {7266, 3913, 30, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\cos\left(\sqrt{3}\sqrt{x^2+2}\right)}{\sqrt{x^2+2}}dx$$

$$\Big\downarrow \text{7266}$$

$$\frac{1}{2}\int \frac{\cos\left(\sqrt{3}\sqrt{x^2+2}\right)}{\sqrt{x^2+2}}dx^2$$

$$\Big\downarrow \text{3913}$$

$$\int \frac{\sqrt{x^2+2}\cos\left(\sqrt{3}\sqrt{x^2+2}\right)}{\sqrt{x^4}}d\sqrt{x^2+2}$$

$$\Big\downarrow \text{30}$$

$$\frac{\sqrt{x^2+2}\int \cos\left(\sqrt{3}\sqrt{x^2+2}\right)d\sqrt{x^2+2}}{\sqrt{x^4}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{x^2+2}\int \sin\left(\sqrt{3}\sqrt{x^2+2}+\frac{\pi}{2}\right)d\sqrt{x^2+2}}{\sqrt{x^4}}$$

$$\Big\downarrow \text{3117}$$

$$\frac{\sqrt{x^2+2}\sin\left(\sqrt{3}\sqrt{x^2+2}\right)}{\sqrt{3}\sqrt{x^4}}$$

input
```
Int[(x*Cos[Sqrt[3]*Sqrt[2 + x^2]])/Sqrt[2 + x^2],x]
```

output
```
(Sqrt[2 + x^2]*Sin[Sqrt[3]*Sqrt[2 + x^2]])/(Sqrt[3]*Sqrt[x^4])
```

3.49.3.1 Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3913
```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[1/(n*f)   Subst[Int[ExpandIntegra
nd[(a + b*Cos[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
 x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p
, 0] && IntegerQ[1/n]
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

3.49.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{\sin\left(\sqrt{3}\,\sqrt{x^2+2}\right)\sqrt{3}}{3}$	18
default	$\frac{\sin\left(\sqrt{3}\,\sqrt{x^2+2}\right)\sqrt{3}}{3}$	18

input
```
int(x*cos(3^(1/2)*(x^2+2)^(1/2))/(x^2+2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*sin(3^(1/2)*(x^2+2)^(1/2))*3^(1/2)
```

3.49.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(17) = 34$.

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int \frac{x\cos\left(\sqrt{3}\sqrt{2+x^2}\right)}{\sqrt{2+x^2}}\,dx = \frac{2\sqrt{3}\tan\left(\frac{1}{2}\sqrt{3}\sqrt{x^2+2}\right)}{3\left(\tan\left(\frac{1}{2}\sqrt{3}\sqrt{x^2+2}\right)^2+1\right)}$$

input
```
integrate(x*cos(3^(1/2)*(x^2+2)^(1/2))/(x^2+2)^(1/2),x, algorithm="fricas"
)
```

output
```
2/3*sqrt(3)*tan(1/2*sqrt(3)*sqrt(x^2 + 2))/(tan(1/2*sqrt(3)*sqrt(x^2 + 2))
^2 + 1)
```

3.49.6 Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x\cos\left(\sqrt{3}\sqrt{2+x^2}\right)}{\sqrt{2+x^2}}\,dx = \frac{\sqrt{3}\sin\left(\sqrt{3}\sqrt{x^2+2}\right)}{3}$$

input
```
integrate(x*cos(3**(1/2)*(x**2+2)**(1/2))/(x**2+2)**(1/2),x)
```

output
```
sqrt(3)*sin(sqrt(3)*sqrt(x**2 + 2))/3
```

3.49.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \frac{x\cos\left(\sqrt{3}\sqrt{2+x^2}\right)}{\sqrt{2+x^2}}\,dx = \frac{1}{3}\sqrt{3}\sin\left(\sqrt{3}\sqrt{x^2+2}\right)$$

input
```
integrate(x*cos(3^(1/2)*(x^2+2)^(1/2))/(x^2+2)^(1/2),x, algorithm="maxima"
)
```

output
```
1/3*sqrt(3)*sin(sqrt(3)*sqrt(x^2 + 2))
```

3.49.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \frac{x\cos\left(\sqrt{3}\sqrt{2+x^2}\right)}{\sqrt{2+x^2}}\,dx = \frac{1}{3}\sqrt{3}\sin\left(\sqrt{3}\sqrt{x^2+2}\right)$$

input
```
integrate(x*cos(3^(1/2)*(x^2+2)^(1/2))/(x^2+2)^(1/2),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*sin(sqrt(3)*sqrt(x^2 + 2))
```

3.49.9 Mupad [B] (verification not implemented)

Time = 26.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int \frac{x\cos\left(\sqrt{3}\sqrt{2+x^2}\right)}{\sqrt{2+x^2}}\,dx = \frac{\sqrt{3}\,\sin\left(\sqrt{3\,x^2+6}\right)}{3}$$

input
```
int((x*cos(3^(1/2)*(x^2 + 2)^(1/2)))/(x^2 + 2)^(1/2),x)
```

output
```
(3^(1/2)*sin((3*x^2 + 6)^(1/2)))/3
```

3.50 $\int \frac{(-1+2x)\cos\left(\sqrt{6+3(-1+2x)^2}\right)}{\sqrt{6+3(-1+2x)^2}}\,dx$

3.50.1 Optimal result

Integrand size = 37, antiderivative size = 24

$$\int \frac{(-1+2x)\cos\left(\sqrt{6+3(-1+2x)^2}\right)}{\sqrt{6+3(-1+2x)^2}}\,dx = \frac{1}{6}\sin\left(\sqrt{3}\sqrt{2+(-1+2x)^2}\right)$$

output
```
1/6*sin(3^(1/2)*(2+(-1+2*x)^2)^(1/2))
```

3.50.2 Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{(-1+2x)\cos\left(\sqrt{6+3(-1+2x)^2}\right)}{\sqrt{6+3(-1+2x)^2}}\,dx = \frac{1}{6}\sin\left(\sqrt{6+3(1-2x)^2}\right)$$

input
```
Integrate[((-1 + 2*x)*Cos[Sqrt[6 + 3*(-1 + 2*x)^2]])/Sqrt[6 + 3*(-1 + 2*x)
^2],x]
```

output
```
Sin[Sqrt[6 + 3*(1 - 2*x)^2]]/6
```

3.50.3 Rubi [A] (warning: unable to verify)

Time = 0.64 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.50, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {7281, 27, 7266, 3913, 27, 30, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x-1)\cos\left(\sqrt{3(2x-1)^2+6}\right)}{\sqrt{3(2x-1)^2+6}}dx$$

$$\Big\downarrow \text{ 7281}$$

$$\frac{1}{2}\int \frac{(2x-1)\cos\left(\sqrt{3(2x-1)^2+6}\right)}{\sqrt{3}\sqrt{(2x-1)^2+2}}d(2x-1)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{(2x-1)\cos\left(\sqrt{3(2x-1)^2+6}\right)}{\sqrt{(2x-1)^2+2}}d(2x-1)}{2\sqrt{3}}$$

$$\Big\downarrow \text{ 7266}$$

$$\frac{\int \frac{\cos\left(\sqrt{3(2x-1)^2+6}\right)}{\sqrt{2x+1}}d(2x-1)^2}{4\sqrt{3}}$$

$$\Big\downarrow \text{ 3913}$$

$$\frac{\int \frac{\sqrt{3}\sqrt{3(2x-1)^2+6}\cos(1-2x)}{\sqrt{(2x-1)^4}}d\sqrt{3(2x-1)^2+6}}{6\sqrt{3}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{6}\int \frac{\sqrt{3(2x-1)^2+6}\cos(1-2x)}{\sqrt{(2x-1)^4}}d\sqrt{3(2x-1)^2+6}$$

$$\Big\downarrow \text{ 30}$$

$$\frac{\sqrt{3(2x-1)^2+6}\int \cos(1-2x)d\sqrt{3(2x-1)^2+6}}{6\sqrt{(2x-1)^4}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{3(2x-1)^2+6}\int \sin\left(\sqrt{3(2x-1)^2+6}+\frac{\pi}{2}\right)d\sqrt{3(2x-1)^2+6}}{6\sqrt{(2x-1)^4}}$$

$$\Big\downarrow \text{3117}$$

$$-\frac{\sqrt{3(2x-1)^2+6}\sin(1-2x)}{6\sqrt{(2x-1)^4}}$$

input
```
Int[((-1 + 2*x)*Cos[Sqrt[6 + 3*(-1 + 2*x)^2]])/Sqrt[6 + 3*(-1 + 2*x)^2],x]
```

output
```
-1/6*(Sqrt[6 + 3*(-1 + 2*x)^2]*Sin[1 - 2*x])/Sqrt[(-1 + 2*x)^4]
```

3.50.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3913
```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.)*((g_
.) + (h_.)*(x_))^(m_.), x_Symbol] :> Simp[1/(n*f)   Subst[Int[ExpandIntegra
nd[(a + b*Cos[c + d*x])^p, x^(1/n - 1)*(g - e*(h/f) + h*(x^(1/n)/f))^m, x],
 x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p
, 0] && IntegerQ[1/n]
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

3.50.4 Maple [A] (verified)

Time = 4.67 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

method	result	size
default	$\frac{\sin\left(\sqrt{12x^2-12x+9}\right)}{6}$	16

input
```
int((-1+2*x)*cos((6+3*(-1+2*x)^2)^(1/2))/(6+3*(-1+2*x)^2)^(1/2),x,method=_
RETURNVERBOSE)
```

output
```
1/6*sin((12*x^2-12*x+9)^(1/2))
```

3.50.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

$$\int \frac{(-1+2x)\cos\left(\sqrt{6+3(-1+2x)^2}\right)}{\sqrt{6+3(-1+2x)^2}}\,dx = \frac{1}{6}\sin\left(\sqrt{12\,x^2-12\,x+9}\right)$$

input
```
integrate((-1+2*x)*cos((6+3*(-1+2*x)^2)^(1/2))/(6+3*(-1+2*x)^2)^(1/2),x, a
lgorithm="fricas")
```

output
```
1/6*sin(sqrt(12*x^2 - 12*x + 9))
```

3.50.6 Sympy [A] (verification not implemented)

Time = 1.98 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.62

$$\int \frac{(-1+2x)\cos\left(\sqrt{6+3(-1+2x)^2}\right)}{\sqrt{6+3(-1+2x)^2}}\,dx = \frac{\sin\left(\sqrt{3\,(2x-1)^2+6}\right)}{6}$$

input
```
integrate((-1+2*x)*cos((6+3*(-1+2*x)**2)**(1/2))/(6+3*(-1+2*x)**2)**(1/2),
x)
```

output
```
sin(sqrt(3*(2*x - 1)**2 + 6))/6
```

3.50.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \frac{(-1+2x)\cos\left(\sqrt{6+3(-1+2x)^2}\right)}{\sqrt{6+3(-1+2x)^2}}\,dx = \frac{1}{6}\sin\left(\sqrt{3\,(2\,x-1)^2+6}\right)$$

input
```
integrate((-1+2*x)*cos((6+3*(-1+2*x)^2)^(1/2))/(6+3*(-1+2*x)^2)^(1/2),x, a
lgorithm="maxima")
```

output
```
1/6*sin(sqrt(3*(2*x - 1)^2 + 6))
```

3.50.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \frac{(-1+2x)\cos\left(\sqrt{6+3(-1+2x)^2}\right)}{\sqrt{6+3(-1+2x)^2}}\,dx = \frac{1}{6}\sin\left(\sqrt{3}\sqrt{4\,x^2-4\,x+3}\right)$$

input
```
integrate((-1+2*x)*cos((6+3*(-1+2*x)^2)^(1/2))/(6+3*(-1+2*x)^2)^(1/2),x, a
lgorithm="giac")
```

output
```
1/6*sin(sqrt(3)*sqrt(4*x^2 - 4*x + 3))
```

3.50.9 Mupad [B] (verification not implemented)

Time = 26.75 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \frac{(-1+2x)\cos\left(\sqrt{6+3(-1+2x)^2}\right)}{\sqrt{6+3(-1+2x)^2}}\,dx = \frac{\sin\left(\sqrt{3\,(2\,x-1)^2+6}\right)}{6}$$

input
```
int((cos((3*(2*x - 1)^2 + 6)^(1/2))*(2*x - 1))/(3*(2*x - 1)^2 + 6)^(1/2),x
)
```

output
```
sin((3*(2*x - 1)^2 + 6)^(1/2))/6
```

3.51 $\int \cos\left(\frac{a+bx}{c+dx}\right) dx$

3.51.1 Optimal result

Integrand size = 14, antiderivative size = 101

$$\int \cos\left(\frac{a+bx}{c+dx}\right) dx = \frac{(c+dx)\cos\left(\frac{a+bx}{c+dx}\right)}{d} - \frac{(bc-ad)\operatorname{CosIntegral}\left(\frac{bc-ad}{d(c+dx)}\right)\sin\left(\frac{b}{d}\right)}{d^2} + \frac{(bc-ad)\cos\left(\frac{b}{d}\right)\operatorname{Si}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}$$

output
```
(d*x+c)*cos((b*x+a)/(d*x+c))/d+(-a*d+b*c)*cos(b/d)*Si((-a*d+b*c)/d/(d*x+c)
)/d^2-(-a*d+b*c)*Ci((-a*d+b*c)/d/(d*x+c))*sin(b/d)/d^2
```

3.51.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.79 (sec) , antiderivative size = 319, normalized size of antiderivative = 3.16

$$\int \cos\left(\frac{a+bx}{c+dx}\right) dx = \frac{cde^{-\frac{i(a+bx)}{c+dx}} + cde^{\frac{i(a+bx)}{c+dx}} + 2d^2x\cos\left(\frac{b}{d}\right)\cos\left(\frac{-bc+ad}{d(c+dx)}\right) - 2d^2x\sin\left(\frac{b}{d}\right)\sin\left(\frac{-bc+ad}{d(c+dx)}\right) - (bc-ad)\left(\operatorname{CosIntegral}\right.}{}$$

input
```
Integrate[Cos[(a + b*x)/(c + d*x)],x]
```

output
```
((c*d)/E^((I*(a + b*x))/(c + d*x)) + c*d*E^((I*(a + b*x))/(c + d*x)) + 2*d
^2*x*Cos[b/d]*Cos[(-(b*c) + a*d)/(d*(c + d*x))] - 2*d^2*x*Sin[b/d]*Sin[(-(
b*c) + a*d)/(d*(c + d*x))] - (b*c - a*d)*(CosIntegral[(b*c - a*d)/(c*d + d
^2*x)]*((-I)*Cos[b/d] + Sin[b/d]) + CosIntegral[(-(b*c) + a*d)/(d*(c + d*x
))]*(I*Cos[b/d] + Sin[b/d]) + Cos[b/d]*SinIntegral[(-(b*c) + a*d)/(d*(c +
d*x))] + I*Sin[b/d]*SinIntegral[(-(b*c) + a*d)/(d*(c + d*x))] - Cos[b/d]*S
inIntegral[(b*c - a*d)/(c*d + d^2*x)] + I*Sin[b/d]*SinIntegral[(b*c - a*d)
/(c*d + d^2*x)]))/(2*d^2)
```

3.51.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {5075, 3042, 3778, 25, 3042, 3784, 25, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos\left(\frac{a+bx}{c+dx}\right)dx$$

$$\Big\downarrow \text{5075}$$

$$-\frac{\int (c+dx)^2\cos\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int (c+dx)^2\sin\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}+\frac{\pi}{2}\right)d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \text{3778}$$

$$-\frac{-\frac{(bc-ad)\int -\left((c+dx)\sin\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)\right)d\frac{1}{c+dx}}{d}-\left((c+dx)\cos\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)\right)}{d}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{(bc-ad)\int (c+dx)\sin\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)d\frac{1}{c+dx}}{d}-(c+dx)\cos\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{(bc-ad)\int (c+dx)\sin\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)d\frac{1}{c+dx}}{d}-(c+dx)\cos\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)}{d}$$

$$\Big\downarrow \text{3784}$$

$$-\frac{\frac{(bc-ad)\left(\sin\left(\frac{b}{d}\right)\int (c+dx)\cos\left(\frac{bc-ad}{d(c+dx)}\right)d\frac{1}{c+dx}+\cos\left(\frac{b}{d}\right)\int -\left((c+dx)\sin\left(\frac{bc-ad}{d(c+dx)}\right)\right)d\frac{1}{c+dx}\right)}{d}-(c+dx)\cos\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)}{d}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{(bc-ad)\left(\sin\left(\frac{b}{d}\right)\int (c+dx)\cos\left(\frac{bc-ad}{d(c+dx)}\right)d\frac{1}{c+dx}-\cos\left(\frac{b}{d}\right)\int (c+dx)\sin\left(\frac{bc-ad}{d(c+dx)}\right)d\frac{1}{c+dx}\right)}{d}-(c+dx)\cos\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{(bc-ad)\left(\sin\left(\frac{b}{d}\right)\int (c+dx)\sin\left(\frac{bc-ad}{d(c+dx)}+\frac{\pi}{2}\right)d\frac{1}{c+dx}-\cos\left(\frac{b}{d}\right)\int (c+dx)\sin\left(\frac{bc-ad}{d(c+dx)}\right)d\frac{1}{c+dx}\right)}{d}-(c+dx)\cos\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)}{d}$$

$$\Big\downarrow \text{3780}$$

$$-\frac{\frac{(bc-ad)\left(\sin\left(\frac{b}{d}\right)\int (c+dx)\sin\left(\frac{bc-ad}{d(c+dx)}+\frac{\pi}{2}\right)d\frac{1}{c+dx}-\cos\left(\frac{b}{d}\right)\text{Si}\left(\frac{bc-ad}{d(c+dx)}\right)\right)}{d}-(c+dx)\cos\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)}{d}$$

$$\Big\downarrow \text{3783}$$

$$-\frac{\frac{(bc-ad)\left(\sin\left(\frac{b}{d}\right)\text{CosIntegral}\left(\frac{bc-ad}{d(c+dx)}\right)-\cos\left(\frac{b}{d}\right)\text{Si}\left(\frac{bc-ad}{d(c+dx)}\right)\right)}{d}-(c+dx)\cos\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)}{d}$$

input
```
Int[Cos[(a + b*x)/(c + d*x)],x]
```

output
```
-((-((c + d*x)*Cos[b/d - (b*c - a*d)/(d*(c + d*x))]) + ((b*c - a*d)*(CosIn
tegral[(b*c - a*d)/(d*(c + d*x))]*Sin[b/d] - Cos[b/d]*SinIntegral[(b*c - a
*d)/(d*(c + d*x))]))/d)/d)
```

3.51.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 5075
```
Int[Cos[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol]
 :> Simp[-d^(-1)   Subst[Int[Cos[b*(e/d) - e*(b*c - a*d)*(x/d)]^n/x^2, x],
x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a*
d, 0]
```

3.51.4 Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.41

method	result
derivativedivides	$-(ad-cb)\left(-\frac{\cos\left(\frac{b}{d}+\frac{ad-cb}{d(dx+c)}\right)}{\left(\left(\frac{b}{d}+\frac{ad-cb}{d(dx+c)}\right)d-b\right)d}-\frac{\frac{\operatorname{Si}\left(\frac{ad-cb}{d(dx+c)}\right)\cos\left(\frac{b}{d}\right)}{d}+\frac{\operatorname{Ci}\left(\frac{ad-cb}{d(dx+c)}\right)\sin\left(\frac{b}{d}\right)}{d}}{d}\right)$
default	$-(ad-cb)\left(-\frac{\cos\left(\frac{b}{d}+\frac{ad-cb}{d(dx+c)}\right)}{\left(\left(\frac{b}{d}+\frac{ad-cb}{d(dx+c)}\right)d-b\right)d}-\frac{\frac{\operatorname{Si}\left(\frac{ad-cb}{d(dx+c)}\right)\cos\left(\frac{b}{d}\right)}{d}+\frac{\operatorname{Ci}\left(\frac{ad-cb}{d(dx+c)}\right)\sin\left(\frac{b}{d}\right)}{d}}{d}\right)$
risch	$\frac{i\operatorname{Ei}_1\left(-\frac{i(ad-cb)}{d(dx+c)}\right)e^{\frac{ib}{d}}a}{2d}-\frac{i\operatorname{Ei}_1\left(-\frac{i(ad-cb)}{d(dx+c)}\right)e^{\frac{ib}{d}}cb}{2d^2}-\frac{e^{-\frac{ib}{d}}\pi\operatorname{csgn}\left(\frac{ad-cb}{d(dx+c)}\right)a}{2d}+\frac{e^{-\frac{ib}{d}}\pi\operatorname{csgn}\left(\frac{ad-cb}{d(dx+c)}\right)bc}{2d^2}+\frac{e^{-\frac{ib}{d}}}{}$

input
```
int(cos((b*x+a)/(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-(a*d-b*c)*(-cos(b/d+(a*d-b*c)/d/(d*x+c))/((b/d+(a*d-b*c)/d/(d*x+c))*d-b)/
d-(Si((a*d-b*c)/d/(d*x+c))*cos(b/d)/d+Ci((a*d-b*c)/d/(d*x+c))*sin(b/d)/d)/
d)
```

3.51.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.04

$$\int \cos\left(\frac{a+bx}{c+dx}\right)\,dx = -\frac{(bc-ad)\operatorname{Ci}\left(-\frac{bc-ad}{d^2x+cd}\right)\sin\left(\frac{b}{d}\right)+(bc-ad)\cos\left(\frac{b}{d}\right)\operatorname{Si}\left(-\frac{bc-ad}{d^2x+cd}\right)-(d^2x+cd)\cos\left(\frac{bx+a}{dx+c}\right)}{d^2}$$

input
```
integrate(cos((b*x+a)/(d*x+c)),x, algorithm="fricas")
```

output
```
-((b*c - a*d)*cos_integral(-(b*c - a*d)/(d^2*x + c*d))*sin(b/d) + (b*c - a
*d)*cos(b/d)*sin_integral(-(b*c - a*d)/(d^2*x + c*d)) - (d^2*x + c*d)*cos(
(b*x + a)/(d*x + c)))/d^2
```

3.51.6 Sympy [F]

$$\int \cos\left(\frac{a+bx}{c+dx}\right)\,dx = \int \cos\left(\frac{a+bx}{c+dx}\right)\,dx$$

input
```
integrate(cos((b*x+a)/(d*x+c)),x)
```

output
```
Integral(cos((a + b*x)/(c + d*x)), x)
```

3.51.7 Maxima [F]

$$\int \cos \left(\frac{a+bx}{c+dx}\right) dx = \int \cos \left(\frac{bx+a}{dx+c}\right) dx$$

input
```
integrate(cos((b*x+a)/(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(cos((b*x + a)/(d*x + c)), x)
```

3.51.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 633 vs. $2(101) = 202$.

Time = 5.26 (sec) , antiderivative size = 633, normalized size of antiderivative = 6.27

$$\int \cos \left(\frac{a+bx}{c+dx}\right) dx =$$

$$-\frac{\left(b^3c^2 \operatorname{Ci}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right) \sin\left(\frac{b}{d}\right) - 2\,ab^2cd \operatorname{Ci}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right) \sin\left(\frac{b}{d}\right) - \frac{(bx+a)b^2c^2d \operatorname{Ci}\left(-\frac{b-\frac{(bx+a)d}{dx+c}}{d}\right) \sin\left(\frac{b}{d}\right)}{dx+c} + a^2 \cdots\right.}{\cdots}$$

input
```
integrate(cos((b*x+a)/(d*x+c)),x, algorithm="giac")
```

output
```
-(b^3*c^2*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d)*sin(b/d) - 2*a*b^2*
c*d*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d)*sin(b/d) - (b*x + a)*b^2*
c^2*d*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d)*sin(b/d)/(d*x + c) + a^
2*b*d^2*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d)*sin(b/d) + 2*(b*x + a
)*a*b*c*d^2*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d)*sin(b/d)/(d*x + c
) - (b*x + a)*a^2*d^3*cos_integral(-(b - (b*x + a)*d/(d*x + c))/d)*sin(b/d
)/(d*x + c) - b^3*c^2*cos(b/d)*sin_integral((b - (b*x + a)*d/(d*x + c))/d)
 + 2*a*b^2*c*d*cos(b/d)*sin_integral((b - (b*x + a)*d/(d*x + c))/d) + (b*x
 + a)*b^2*c^2*d*cos(b/d)*sin_integral((b - (b*x + a)*d/(d*x + c))/d)/(d*x
+ c) - a^2*b*d^2*cos(b/d)*sin_integral((b - (b*x + a)*d/(d*x + c))/d) - 2*
(b*x + a)*a*b*c*d^2*cos(b/d)*sin_integral((b - (b*x + a)*d/(d*x + c))/d)/(
d*x + c) + (b*x + a)*a^2*d^3*cos(b/d)*sin_integral((b - (b*x + a)*d/(d*x +
 c))/d)/(d*x + c) - b^2*c^2*d*cos((b*x + a)/(d*x + c)) + 2*a*b*c*d^2*cos((
b*x + a)/(d*x + c)) - a^2*d^3*cos((b*x + a)/(d*x + c)))*(b*c/(b*c - a*d)^2
 - a*d/(b*c - a*d)^2)/(b*d^2 - (b*x + a)*d^3/(d*x + c))
```

3.51.9 Mupad [F(-1)]

Timed out.

$$\int \cos\left(\frac{a+bx}{c+dx}\right)\,dx = \int \cos\left(\frac{a+b\,x}{c+d\,x}\right)\,dx$$

input
```
int(cos((a + b*x)/(c + d*x)),x)
```

output
```
int(cos((a + b*x)/(c + d*x)), x)
```

3.52 $\int \cos^2\left(\frac{a+bx}{c+dx}\right)\,dx$

3.52.1 Optimal result

Integrand size = 16, antiderivative size = 107

$$\int \cos^2\left(\frac{a+bx}{c+dx}\right)\,dx=\frac{(c+dx)\cos^2\left(\frac{a+bx}{c+dx}\right)}{d}-\frac{(bc-ad)\,\text{CosIntegral}\left(\frac{2(bc-ad)}{d(c+dx)}\right)\sin\left(\frac{2b}{d}\right)}{d^2}+\frac{(bc-ad)\cos\left(\frac{2b}{d}\right)\text{Si}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}$$

output
```
(d*x+c)*cos((b*x+a)/(d*x+c))^2/d+(-a*d+b*c)*cos(2*b/d)*Si(2*(-a*d+b*c)/d/(
d*x+c))/d^2-(-a*d+b*c)*Ci(2*(-a*d+b*c)/d/(d*x+c))*sin(2*b/d)/d^2
```

3.52.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.57 (sec) , antiderivative size = 497, normalized size of antiderivative = 4.64

$$\int \cos^2\left(\frac{a+bx}{c+dx}\right)\,dx$$
$$=\frac{cde^{-\frac{2i(a+bx)}{c+dx}}+cde^{\frac{2i(a+bx)}{c+dx}}+2d^2x+2d^2x\cos\left(\frac{2b}{d}\right)\cos\left(\frac{2(-bc+ad)}{d(c+dx)}\right)-2i(bc-ad)\,\text{CosIntegral}\left(\frac{2(-bc+ad)}{d(c+dx)}\right)(\cos}{}$$

input
```
Integrate[Cos[(a + b*x)/(c + d*x)]^2,x]
```

output
```
((c*d)/E^(((2*I)*(a + b*x))/(c + d*x)) + c*d*E^(((2*I)*(a + b*x))/(c + d*x
)) + 2*d^2*x + 2*d^2*x*Cos[(2*b)/d]*Cos[(2*(-(b*c) + a*d))/(d*(c + d*x))]
- (2*I)*(b*c - a*d)*CosIntegral[(2*(-(b*c) + a*d))/(d*(c + d*x))]*(Cos[(2*
b)/d] - I*Sin[(2*b)/d]) + (2*I)*(b*c - a*d)*CosIntegral[(2*b*c - 2*a*d)/(c
*d + d^2*x)]*(Cos[(2*b)/d] + I*Sin[(2*b)/d]) - 2*d^2*x*Sin[(2*b)/d]*Sin[(2
*(-(b*c) + a*d))/(d*(c + d*x))] - 2*b*c*Cos[(2*b)/d]*SinIntegral[(2*(-(b*c
) + a*d))/(d*(c + d*x))] + 2*a*d*Cos[(2*b)/d]*SinIntegral[(2*(-(b*c) + a*d
))/(d*(c + d*x))] - (2*I)*b*c*Sin[(2*b)/d]*SinIntegral[(2*(-(b*c) + a*d))/
(d*(c + d*x))] + (2*I)*a*d*Sin[(2*b)/d]*SinIntegral[(2*(-(b*c) + a*d))/(d*
(c + d*x))] + 2*b*c*Cos[(2*b)/d]*SinIntegral[(2*b*c - 2*a*d)/(c*d + d^2*x)
] - 2*a*d*Cos[(2*b)/d]*SinIntegral[(2*b*c - 2*a*d)/(c*d + d^2*x)] - (2*I)*
b*c*Sin[(2*b)/d]*SinIntegral[(2*b*c - 2*a*d)/(c*d + d^2*x)] + (2*I)*a*d*Si
n[(2*b)/d]*SinIntegral[(2*b*c - 2*a*d)/(c*d + d^2*x)])/(4*d^2)
```

3.52.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5075, 3042, 3794, 27, 3042, 3784, 25, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2\left(\frac{a+bx}{c+dx}\right)dx$$

$$\Big\downarrow \text{ 5075}$$

$$-\frac{\int (c+dx)^2\cos^2\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\int (c+dx)^2\sin\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}+\frac{\pi}{2}\right)^2d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \text{ 3794}$$

$$-\frac{-\frac{2(bc-ad)\int -\frac{1}{2}(c+dx)\sin\left(\frac{2b}{d}-\frac{2(bc-ad)}{d(c+dx)}\right)d\frac{1}{c+dx}}{d}-\left((c+dx)\cos^2\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)\right)}{d}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\frac{(bc-ad)\int (c+dx)\sin\left(\frac{2b}{d}-\frac{2(bc-ad)}{d(c+dx)}\right)d\frac{1}{c+dx}}{d}-(c+dx)\cos^2\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)}{d}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{(bc-ad)\int (c+dx)\sin\left(\frac{2b}{d}-\frac{2(bc-ad)}{d(c+dx)}\right)d\frac{1}{c+dx}}{d}-(c+dx)\cos^2\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)}{d}$$

$\Big\downarrow$ 3784

$$-\frac{\frac{(bc-ad)\left(\sin\left(\frac{2b}{d}\right)\int (c+dx)\cos\left(\frac{2(bc-ad)}{d(c+dx)}\right)d\frac{1}{c+dx}+\cos\left(\frac{2b}{d}\right)\int -\left((c+dx)\sin\left(\frac{2(bc-ad)}{d(c+dx)}\right)\right)d\frac{1}{c+dx}\right)}{d}-(c+dx)\cos^2\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)}{d}$$

$\Big\downarrow$ 25

$$-\frac{\frac{(bc-ad)\left(\sin\left(\frac{2b}{d}\right)\int (c+dx)\cos\left(\frac{2(bc-ad)}{d(c+dx)}\right)d\frac{1}{c+dx}-\cos\left(\frac{2b}{d}\right)\int (c+dx)\sin\left(\frac{2(bc-ad)}{d(c+dx)}\right)d\frac{1}{c+dx}\right)}{d}-(c+dx)\cos^2\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)}{d}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{(bc-ad)\left(\sin\left(\frac{2b}{d}\right)\int (c+dx)\sin\left(\frac{2(bc-ad)}{d(c+dx)}+\frac{\pi}{2}\right)d\frac{1}{c+dx}-\cos\left(\frac{2b}{d}\right)\int (c+dx)\sin\left(\frac{2(bc-ad)}{d(c+dx)}\right)d\frac{1}{c+dx}\right)}{d}-(c+dx)\cos^2\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)}{d}$$

$\Big\downarrow$ 3780

$$-\frac{\frac{(bc-ad)\left(\sin\left(\frac{2b}{d}\right)\int (c+dx)\sin\left(\frac{2(bc-ad)}{d(c+dx)}+\frac{\pi}{2}\right)d\frac{1}{c+dx}-\cos\left(\frac{2b}{d}\right)\text{Si}\left(\frac{2(bc-ad)}{d(c+dx)}\right)\right)}{d}-(c+dx)\cos^2\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)}{d}$$

$\Big\downarrow$ 3783

$$-\frac{\frac{(bc-ad)\left(\sin\left(\frac{2b}{d}\right)\text{CosIntegral}\left(\frac{2(bc-ad)}{d(c+dx)}\right)-\cos\left(\frac{2b}{d}\right)\text{Si}\left(\frac{2(bc-ad)}{d(c+dx)}\right)\right)}{d}-(c+dx)\cos^2\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)}{d}$$

input
```
Int[Cos[(a + b*x)/(c + d*x)]^2,x]
```

output
```
-((-((c + d*x)*Cos[b/d - (b*c - a*d)/(d*(c + d*x))]^2) + ((b*c - a*d)*(Cos
Integral[(2*(b*c - a*d))/(d*(c + d*x))]*Sin[(2*b)/d] - Cos[(2*b)/d]*SinInt
egral[(2*(b*c - a*d))/(d*(c + d*x))]))/d)/d)
```

3.52. $\int \cos^2\left(\frac{a+bx}{c+dx}\right)\,dx$

3.52.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

rule 5075
```
Int[Cos[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol]
 :> Simp[-d^(-1)   Subst[Int[Cos[b*(e/d) - e*(b*c - a*d)*(x/d)]^n/x^2, x],
x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c - a*
d, 0]
```

3.52.4 Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.82

method	result
derivativedivides	$-\frac{(ad-cb)\left(\frac{d^2\left(-\frac{2\cos\left(\frac{2ad-2cb}{d(dx+c)}+\frac{2b}{d}\right)}{\left(\left(\frac{b}{d}+\frac{ad-cb}{d(dx+c)}\right)d-b\right)d}-\frac{2\left(\frac{2\,\mathrm{Si}\left(\frac{2ad-2cb}{d(dx+c)}\right)\cos\left(\frac{2b}{d}\right)}{d}+\frac{2\,\mathrm{Ci}\left(\frac{2ad-2cb}{d(dx+c)}\right)\sin\left(\frac{2b}{d}\right)}{d}\right)}{d}\right)}{4}-\frac{d}{2\left(\left(\frac{b}{d}+\frac{ad-cb}{d(dx+c)}\right)d-\cdots}\right.}{d^2}$
default	$-\frac{(ad-cb)\left(\frac{d^2\left(-\frac{2\cos\left(\frac{2ad-2cb}{d(dx+c)}+\frac{2b}{d}\right)}{\left(\left(\frac{b}{d}+\frac{ad-cb}{d(dx+c)}\right)d-b\right)d}-\frac{2\left(\frac{2\,\mathrm{Si}\left(\frac{2ad-2cb}{d(dx+c)}\right)\cos\left(\frac{2b}{d}\right)}{d}+\frac{2\,\mathrm{Ci}\left(\frac{2ad-2cb}{d(dx+c)}\right)\sin\left(\frac{2b}{d}\right)}{d}\right)}{d}\right)}{4}-\frac{d}{2\left(\left(\frac{b}{d}+\frac{ad-cb}{d(dx+c)}\right)d-\cdots}\right.}{d^2}$
risch	$-\frac{\mathrm{e}^{-\frac{2ib}{d}}\,\mathrm{csgn}\left(\frac{ad-cb}{d(dx+c)}\right)\pi a}{2d}+\frac{\mathrm{e}^{-\frac{2ib}{d}}\,\mathrm{csgn}\left(\frac{ad-cb}{d(dx+c)}\right)\pi bc}{2d^2}+\frac{\mathrm{e}^{-\frac{2ib}{d}}\,\mathrm{Si}\left(\frac{2ad-2cb}{d(dx+c)}\right)a}{d}-\frac{\mathrm{e}^{-\frac{2ib}{d}}\,\mathrm{Si}\left(\frac{2ad-2cb}{d(dx+c)}\right)bc}{d^2}-i\mathrm{e}^{-\frac{2i\cdots}{\cdots}}$

input
```
int(cos((b*x+a)/(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/d^2*(a*d-b*c)*(1/4*d^2*(-2*cos(2*(a*d-b*c)/d/(d*x+c)+2*b/d)/((b/d+(a*d-
b*c)/d/(d*x+c))*d-b)/d-2*(2*Si(2*(a*d-b*c)/d/(d*x+c))*cos(2*b/d)/d+2*Ci(2*
(a*d-b*c)/d/(d*x+c))*sin(2*b/d)/d)/d)-1/2*d/((b/d+(a*d-b*c)/d/(d*x+c))*d-b
))
```

3.52.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.02

$$\int \cos^2\left(\frac{a+bx}{c+dx}\right)\,dx$$
$$=\frac{(d^2x+cd)\cos\left(\frac{bx+a}{dx+c}\right)^2-(bc-ad)\,\mathrm{Ci}\left(-\frac{2\,(bc-ad)}{d^2x+cd}\right)\sin\left(\frac{2\,b}{d}\right)-(bc-ad)\cos\left(\frac{2\,b}{d}\right)\mathrm{Si}\left(-\frac{2\,(bc-ad)}{d^2x+cd}\right)}{d^2}$$

input
```
integrate(cos((b*x+a)/(d*x+c))^2,x, algorithm="fricas")
```

output
```
((d^2*x + c*d)*cos((b*x + a)/(d*x + c))^2 - (b*c - a*d)*cos_integral(-2*(b
*c - a*d)/(d^2*x + c*d))*sin(2*b/d) - (b*c - a*d)*cos(2*b/d)*sin_integral(
-2*(b*c - a*d)/(d^2*x + c*d)))/d^2
```

3.52.6 Sympy [F]

$$\int \cos^2\left(\frac{a+bx}{c+dx}\right)\,dx = \int \cos^2\left(\frac{a+bx}{c+dx}\right)dx$$

input
```
integrate(cos((b*x+a)/(d*x+c))**2,x)
```

output
```
Integral(cos((a + b*x)/(c + d*x))**2, x)
```

3.52.7 Maxima [F]

$$\int \cos^2\left(\frac{a+bx}{c+dx}\right)\,dx = \int \cos\left(\frac{bx+a}{dx+c}\right)^2\,dx$$

input
```
integrate(cos((b*x+a)/(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/2*x + 1/2*integrate(cos(2*(b*x + a)/(d*x + c)), x)
```

3.52.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 683 vs. $2(107) = 214$.

Time = 26.79 (sec) , antiderivative size = 683, normalized size of antiderivative = 6.38

$$\int \cos^2\left(\frac{a+bx}{c+dx}\right)\,dx =$$

$$-\frac{\left(2\,b^3c^2\operatorname{Ci}\left(-\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)\sin\left(\frac{2\,b}{d}\right)-4\,ab^2cd\operatorname{Ci}\left(-\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right)\sin\left(\frac{2\,b}{d}\right)-\frac{2\,(bx+a)b^2c^2d\operatorname{Ci}\left(-\frac{2\left(b-\frac{(bx+a)d}{dx+c}\right)}{d}\right.}{dx+c}}{}$$

input
```
integrate(cos((b*x+a)/(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/2*(2*b^3*c^2*cos_integral(-2*(b - (b*x + a)*d/(d*x + c))/d)*sin(2*b/d)
- 4*a*b^2*c*d*cos_integral(-2*(b - (b*x + a)*d/(d*x + c))/d)*sin(2*b/d) -
2*(b*x + a)*b^2*c^2*d*cos_integral(-2*(b - (b*x + a)*d/(d*x + c))/d)*sin(2
*b/d)/(d*x + c) + 2*a^2*b*d^2*cos_integral(-2*(b - (b*x + a)*d/(d*x + c))/
d)*sin(2*b/d) + 4*(b*x + a)*a*b*c*d^2*cos_integral(-2*(b - (b*x + a)*d/(d*
x + c))/d)*sin(2*b/d)/(d*x + c) - 2*(b*x + a)*a^2*d^3*cos_integral(-2*(b -
 (b*x + a)*d/(d*x + c))/d)*sin(2*b/d)/(d*x + c) - 2*b^3*c^2*cos(2*b/d)*sin
_integral(2*(b - (b*x + a)*d/(d*x + c))/d) + 4*a*b^2*c*d*cos(2*b/d)*sin_in
tegral(2*(b - (b*x + a)*d/(d*x + c))/d) + 2*(b*x + a)*b^2*c^2*d*cos(2*b/d)
*sin_integral(2*(b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) - 2*a^2*b*d^2*cos
(2*b/d)*sin_integral(2*(b - (b*x + a)*d/(d*x + c))/d) - 4*(b*x + a)*a*b*c*
d^2*cos(2*b/d)*sin_integral(2*(b - (b*x + a)*d/(d*x + c))/d)/(d*x + c) + 2
*(b*x + a)*a^2*d^3*cos(2*b/d)*sin_integral(2*(b - (b*x + a)*d/(d*x + c))/d
)/(d*x + c) - b^2*c^2*d*cos(2*(b*x + a)/(d*x + c)) + 2*a*b*c*d^2*cos(2*(b*
x + a)/(d*x + c)) - a^2*d^3*cos(2*(b*x + a)/(d*x + c)) - b^2*c^2*d + 2*a*b
*c*d^2 - a^2*d^3)*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)/(b*d^2 - (b*x +
a)*d^3/(d*x + c))
```

3.52.9 Mupad [F(-1)]

Timed out.

$$\int \cos^2\left(\frac{a+bx}{c+dx}\right)\,dx=\int \cos\left(\frac{a+b\,x}{c+d\,x}\right)^2\,dx$$

input
```
int(cos((a + b*x)/(c + d*x))^2,x)
```

output
```
int(cos((a + b*x)/(c + d*x))^2, x)
```

3.53 $\int \frac{\cos^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

3.53.1 Optimal result

Integrand size = 36, antiderivative size = 58

$$\int \frac{\cos^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx=-\frac{3\,\text{CosIntegral}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}-\frac{\text{CosIntegral}\left(\frac{3\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}$$

output
```
-3/4*Ci((-a*x+1)^(1/2)/(a*x+1)^(1/2))/a-1/4*Ci(3*(-a*x+1)^(1/2)/(a*x+1)^(1
/2))/a
```

3.53.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.91

$$\int \frac{\cos^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx=-\frac{3\,\text{CosIntegral}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)+\text{CosIntegral}\left(\frac{3\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}$$

input
```
Integrate[Cos[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^3/(1 - a^2*x^2),x]
```

output
```
-1/4*(3*CosIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]] + CosIntegral[(3*Sqrt[1 -
 a*x])/Sqrt[1 + a*x]])/a
```

3.53.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {7232, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{1-a^2x^2}dx$$

$$\Big\downarrow \text{7232}$$

$$-\frac{\int \frac{\sqrt{ax+1}\cos^3\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sqrt{ax+1}\sin\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\pi}{2}\right)^3}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{\int \left(\frac{3\sqrt{ax+1}\cos\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4\sqrt{1-ax}}+\frac{\sqrt{ax+1}\cos\left(\frac{3\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4\sqrt{1-ax}}\right)d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{3}{4}\text{CosIntegral}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\frac{1}{4}\text{CosIntegral}\left(\frac{3\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

input
```
Int[Cos[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^3/(1 - a^2*x^2),x]
```

output
```
-(((3*CosIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]])/4 + CosIntegral[(3*Sqrt[1
- a*x])/Sqrt[1 + a*x]]/4)/a)
```

3.53.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 7232
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Simp[2*e*(g/(C*(e*f - d
*g)))   Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]],
x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] &&
EqQ[e*f + d*g, 0] && IGtQ[n, 0]
```

3.53.4 Maple [F]

$$\int \frac{\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^{3}}{-a^{2}x^{2}+1}dx$$

input
```
int(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x)
```

output
```
int(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x)
```

3.53.5 Fricas [F]

$$\int \frac{\cos^{3}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^{2}x^{2}}\,dx=\int -\frac{\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^{3}}{a^{2}x^{2}-1}\,dx$$

input
```
integrate(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x, algorithm="f
ricas")
```

output
```
integral(-cos(sqrt(-a*x + 1)/sqrt(a*x + 1))^3/(a^2*x^2 - 1), x)
```

3.53.6 Sympy [F]

$$\int \frac{\cos^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\int \frac{\cos^3\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

input
```
integrate(cos((-a*x+1)**(1/2)/(a*x+1)**(1/2))**3/(-a**2*x**2+1),x)
```

output
```
-Integral(cos(sqrt(-a*x + 1)/sqrt(a*x + 1))**3/(a**2*x**2 - 1), x)
```

3.53.7 Maxima [F]

$$\int \frac{\cos^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^3}{a^2x^2-1}\,dx$$

input
```
integrate(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x, algorithm="m
axima")
```

output
```
-integrate(cos(sqrt(-a*x + 1)/sqrt(a*x + 1))^3/(a^2*x^2 - 1), x)
```

3.53.8 Giac [F]

$$\int \frac{\cos^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^3}{a^2x^2-1}\,dx$$

input
```
integrate(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x, algorithm="g
iac")
```

output
```
integrate(-cos(sqrt(-a*x + 1)/sqrt(a*x + 1))^3/(a^2*x^2 - 1), x)
```

3.53.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\int \frac{\cos\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)^3}{a^2\,x^2-1}\,dx$$

input
```
int(-cos((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^3/(a^2*x^2 - 1),x)
```

output
```
-int(cos((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^3/(a^2*x^2 - 1), x)
```

3.54 $\int \frac{\cos^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

3.54.1 Optimal result

Integrand size = 36, antiderivative size = 58

$$\int \frac{\cos^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\text{CosIntegral}\left(\frac{2\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a} - \frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}$$

output
```
-1/2*Ci(2*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a-1/2*ln((-a*x+1)^(1/2)/(a*x+1)^(1
/2))/a
```

3.54.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.88

$$\int \frac{\cos^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\text{CosIntegral}\left(\frac{2\sqrt{1-ax}}{\sqrt{1+ax}}\right) + \log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}$$

input
```
Integrate[Cos[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

output
```
-1/2*(CosIntegral[(2*Sqrt[1 - a*x])/Sqrt[1 + a*x]] + Log[Sqrt[1 - a*x]/Sqr
t[1 + a*x]])/a
```

3.54.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {7232, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{1-a^2x^2}dx$$

$$\Big\downarrow \text{7232}$$

$$-\frac{\int \frac{\sqrt{ax+1}\cos^2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sqrt{ax+1}\sin\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\pi}{2}\right)^2}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{\int \left(\frac{\sqrt{ax+1}\cos\left(\frac{2\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2\sqrt{1-ax}}+\frac{\sqrt{ax+1}}{2\sqrt{1-ax}}\right)d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{1}{2}\text{CosIntegral}\left(\frac{2\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\frac{1}{2}\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

input
```
Int[Cos[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

output
```
-((CosIntegral[(2*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/2 + Log[Sqrt[1 - a*x]/Sqrt
[1 + a*x]]/2)/a)
```

3.54.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 7232
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Simp[2*e*(g/(C*(e*f - d
*g)))   Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]],
x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] &&
EqQ[e*f + d*g, 0] && IGtQ[n, 0]
```

3.54.4 Maple [F]

$$\int \frac{\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}{-a^2x^2+1}dx$$

input
```
int(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x)
```

output
```
int(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x)
```

3.54.5 Fricas [F]

$$\int \frac{\cos^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}{a^2x^2-1}\,dx$$

input
```
integrate(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="f
ricas")
```

output
```
integral(-cos(sqrt(-a*x + 1)/sqrt(a*x + 1))^2/(a^2*x^2 - 1), x)
```

3.54.6 Sympy [F]

$$\int \frac{\cos^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\int \frac{\cos^2\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

input
```
integrate(cos((-a*x+1)**(1/2)/(a*x+1)**(1/2))**2/(-a**2*x**2+1),x)
```

output
```
-Integral(cos(sqrt(-a*x + 1)/sqrt(a*x + 1))**2/(a**2*x**2 - 1), x)
```

3.54.7 Maxima [F]

$$\int \frac{\cos^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}{a^2x^2-1}\,dx$$

input
```
integrate(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="m
axima")
```

output
```
-1/4*(4*a*integrate(1/4*cos(2*sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1),
 x) + 4*a*integrate(1/4*cos(2*sqrt(-a*x + 1)/sqrt(a*x + 1))/((a^2*x^2 - 1)
*cos(2*sqrt(-a*x + 1)/sqrt(a*x + 1))^2 + (a^2*x^2 - 1)*sin(2*sqrt(-a*x + 1
)/sqrt(a*x + 1))^2), x) - log(a*x + 1) + log(a*x - 1))/a
```

3.54.8 Giac [F]

$$\int \frac{\cos^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}{a^2x^2-1}\,dx$$

input
```
integrate(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="g
iac")
```

output
```
integrate(-cos(sqrt(-a*x + 1)/sqrt(a*x + 1))^2/(a^2*x^2 - 1), x)
```

3.54.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\int \frac{\cos\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)^2}{a^2\,x^2-1}\,dx$$

input
```
int(-cos((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2/(a^2*x^2 - 1),x)
```

output
```
-int(cos((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2/(a^2*x^2 - 1), x)
```

3.55 $\int \frac{\cos\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

3.55.1 Optimal result

Integrand size = 34, antiderivative size = 26

$$\int \frac{\cos\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\text{CosIntegral}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

output
```
-Ci((-a*x+1)^(1/2)/(a*x+1)^(1/2))/a
```

3.55.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\text{CosIntegral}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

input
```
Integrate[Cos[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

output
```
-(CosIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)
```

3.55.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {7232, 3042, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{1-a^2x^2}dx$$

$$\Big\downarrow \ 7232$$

$$-\frac{\int \frac{\sqrt{ax+1}\cos\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\int \frac{\sqrt{ax+1}\sin\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\pi}{2}\right)}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \ 3783$$

$$-\frac{\text{CosIntegral}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

input
```
Int[Cos[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

output
```
-(CosIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)
```

3.55.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 7232
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Simp[2*e*(g/(C*(e*f - d
*g)))   Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]],
x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] &&
EqQ[e*f + d*g, 0] && IGtQ[n, 0]
```

3.55.4 Maple [F]

$$\int \frac{\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{-a^2x^2+1}dx$$

input
```
int(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

output
```
int(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

3.55.5 Fricas [F]

$$\int \frac{\cos\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

input
```
integrate(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="fri
cas")
```

output
```
integral(-cos(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

3.55.6 Sympy [F]

$$\int \frac{\cos\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\int \frac{\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

input
```
integrate(cos((-a*x+1)**(1/2)/(a*x+1)**(1/2))/(-a**2*x**2+1),x)
```

output
```
-Integral(cos(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a**2*x**2 - 1), x)
```

3.55.7 Maxima [F]

$$\int \frac{\cos\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

input
```
integrate(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="max
ima")
```

output
```
-integrate(cos(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

3.55.8 Giac [F]

$$\int \frac{\cos\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

input
```
integrate(cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="gia
c")
```

output
```
integrate(-cos(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

3.55.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\int \frac{\cos\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)}{a^2\,x^2-1}\,dx$$

input
```
int(-cos((1 - a*x)^(1/2)/(a*x + 1)^(1/2))/(a^2*x^2 - 1),x)
```

output
```
-int(cos((1 - a*x)^(1/2)/(a*x + 1)^(1/2))/(a^2*x^2 - 1), x)
```

3.56 $\int \frac{\sec\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

3.56.1 Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{\sec\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx=\text{Int}\left(\frac{\sec\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{(1-ax)(1+ax)},x\right)$$

output
```
Unintegrable(sec((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a*x+1)/(a*x+1),x)
```

3.56.2 Mathematica [N/A]

Not integrable

Time = 3.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\sec\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx=\int \frac{\sec\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$$

input
```
Integrate[Sec[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

output
```
Integrate[Sec[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2), x]
```

3.56.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7232, 3042, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{1-a^2x^2}\,dx$$

$$\Big\downarrow \; 7232$$

$$-\frac{\int \frac{\sqrt{ax+1}\sec\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\int \frac{\sqrt{ax+1}\csc\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\pi}{2}\right)}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \; 4680$$

$$-\frac{\int \frac{\sqrt{ax+1}\sec\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

input
```
Int[Sec[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

output
```
$Aborted
```

3.56.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 7232
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Simp[2*e*(g/(C*(e*f - d
*g)))   Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]],
x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] &&
EqQ[e*f + d*g, 0] && IGtQ[n, 0]
```

3.56.4 Maple [N/A] (verified)

Not integrable

Time = 0.56 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{1}{\left(-a^{2} x^{2}+1\right) \cos \left(\frac{\sqrt{-a x +1}}{\sqrt{a x +1}}\right)}d x$$

input
```
int(1/(-a^2*x^2+1)/cos((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x)
```

output
```
int(1/(-a^2*x^2+1)/cos((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x)
```

3.56.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\sec \left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} \, dx=\int -\frac{1}{(a^{2} x^{2} - 1) \cos \left(\frac{\sqrt{-a x + 1}}{\sqrt{a x + 1}}\right)} \,d x$$

input
```
integrate(1/(-a^2*x^2+1)/cos((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x, algorithm="f
ricas")
```

output
```
integral(-1/((a^2*x^2 - 1)*cos(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)
```

3.56.6 Sympy [N/A]

Not integrable

Time = 6.66 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.41

$$\int \frac{\sec\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\int \frac{1}{a^2x^2\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)-\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}\,dx$$

input
```
integrate(1/(-a**2*x**2+1)/cos((-a*x+1)**(1/2)/(a*x+1)**(1/2)),x)
```

output
```
-Integral(1/(a**2*x**2*cos(sqrt(-a*x + 1)/sqrt(a*x + 1)) - cos(sqrt(-a*x +
 1)/sqrt(a*x + 1))), x)
```

3.56.7 Maxima [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int \frac{\sec\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{1}{(a^2x^2-1)\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)/cos((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x, algorithm="m
axima")
```

output
```
-integrate(1/((a^2*x^2 - 1)*cos(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)
```

3.56.8 Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\sec\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{1}{(a^2x^2-1)\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)/cos((-a*x+1)^(1/2)/(a*x+1)^(1/2)),x, algorithm="g
iac")
```

output
```
integrate(-1/((a^2*x^2 - 1)*cos(sqrt(-a*x + 1)/sqrt(a*x + 1))), x)
```

3.56.9 Mupad [N/A]

Not integrable

Time = 26.47 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int \frac{\sec\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\int \frac{1}{\cos\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)\,(a^2\,x^2-1)}\,dx$$

input
```
int(-1/(cos((1 - a*x)^(1/2)/(a*x + 1)^(1/2))*(a^2*x^2 - 1)),x)
```

output
```
-int(1/(cos((1 - a*x)^(1/2)/(a*x + 1)^(1/2))*(a^2*x^2 - 1)), x)
```

3.57 $\int \frac{\sec^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

3.57.1 Optimal result

Integrand size = 36, antiderivative size = 36

$$\int \frac{\sec^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \text{Int}\left(\frac{\sec^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{(1-ax)(1+ax)},x\right)$$

output
```
Unintegrable(sec((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a*x+1)/(a*x+1),x)
```

3.57.2 Mathematica [N/A]

Not integrable

Time = 13.94 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\sec^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int \frac{\sec^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$$

input
```
Integrate[Sec[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

output
```
Integrate[Sec[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2), x]
```

3.57.3 Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7232, 3042, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{1-a^2x^2}dx$$

$$\Big\downarrow \text{7232}$$

$$-\frac{\int \frac{\sqrt{ax+1}\sec^2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sqrt{ax+1}\csc\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\pi}{2}\right)^2}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \text{4680}$$

$$-\frac{\int \frac{\sqrt{ax+1}\sec^2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

input
```
Int[Sec[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

output
```
$Aborted
```

3.57.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 7232
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Simp[2*e*(g/(C*(e*f - d
*g)))   Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]],
x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] &&
EqQ[e*f + d*g, 0] && IGtQ[n, 0]
```

3.57.4 Maple [N/A] (verified)

Not integrable

Time = 0.69 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{1}{\left(-a^{2} x^{2}+1\right) \cos \left(\frac{\sqrt{-a x +1}}{\sqrt{a x +1}}\right)^{2}}d x$$

input
```
int(1/(-a^2*x^2+1)/cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x)
```

output
```
int(1/(-a^2*x^2+1)/cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x)
```

3.57.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \frac{\sec^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx=\int -\frac{1}{\left(a^{2} x^{2}-1\right) \cos \left(\frac{\sqrt{-a x +1}}{\sqrt{a x +1}}\right)^{2}}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)/cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x, algorithm=
"fricas")
```

output
```
integral(-1/((a^2*x^2 - 1)*cos(sqrt(-a*x + 1)/sqrt(a*x + 1))^2), x)
```

3.57.6 Sympy [N/A]

Not integrable

Time = 21.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.42

$$\int \frac{\sec^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\int \frac{1}{a^2x^2\cos^2\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)-\cos^2\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}\,dx$$

input
```
integrate(1/(-a**2*x**2+1)/cos((-a*x+1)**(1/2)/(a*x+1)**(1/2))**2,x)
```

output
```
-Integral(1/(a**2*x**2*cos(sqrt(-a*x + 1)/sqrt(a*x + 1))**2 - cos(sqrt(-a*
x + 1)/sqrt(a*x + 1))**2), x)
```

3.57.7 Maxima [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 420, normalized size of antiderivative = 11.67

$$\int \frac{\sec^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{1}{(a^2x^2-1)\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)/cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x, algorithm=
"maxima")
```

output
```
2*((a^2*x + (a^2*x - a)*cos(2*sqrt(-a*x + 1)/sqrt(a*x + 1))^2 + (a^2*x - a
)*sin(2*sqrt(-a*x + 1)/sqrt(a*x + 1))^2 + 2*(a^2*x - a)*cos(2*sqrt(-a*x +
1)/sqrt(a*x + 1)) - a)*integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)*sin(2*sqrt(-
a*x + 1)/sqrt(a*x + 1))/(a^3*x^3 - a^2*x^2 + (a^3*x^3 - a^2*x^2 - a*x + 1)
*cos(2*sqrt(-a*x + 1)/sqrt(a*x + 1))^2 + (a^3*x^3 - a^2*x^2 - a*x + 1)*sin
(2*sqrt(-a*x + 1)/sqrt(a*x + 1))^2 - a*x + 2*(a^3*x^3 - a^2*x^2 - a*x + 1)
*cos(2*sqrt(-a*x + 1)/sqrt(a*x + 1)) + 1), x) + sqrt(a*x + 1)*sqrt(-a*x +
1)*sin(2*sqrt(-a*x + 1)/sqrt(a*x + 1)))/(a^2*x + (a^2*x - a)*cos(2*sqrt(-a
*x + 1)/sqrt(a*x + 1))^2 + (a^2*x - a)*sin(2*sqrt(-a*x + 1)/sqrt(a*x + 1))
^2 + 2*(a^2*x - a)*cos(2*sqrt(-a*x + 1)/sqrt(a*x + 1)) - a)
```

3.57.8 Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \frac{\sec^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx=\int -\frac{1}{(a^2x^2-1)\cos\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}\,dx$$

input
```
integrate(1/(-a^2*x^2+1)/cos((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2,x, algorithm=
"giac")
```

output
```
integrate(-1/((a^2*x^2 - 1)*cos(sqrt(-a*x + 1)/sqrt(a*x + 1))^2), x)
```

3.57.9 Mupad [N/A]

Not integrable

Time = 25.94 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int \frac{\sec^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx=-\int \frac{1}{\cos\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)^2\,(a^2\,x^2-1)}\,dx$$

input
```
int(-1/(cos((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2*(a^2*x^2 - 1)),x)
```

output
```
-int(1/(cos((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2*(a^2*x^2 - 1)), x)
```

3.58 $\int \frac{\tan(\sqrt{x})}{\sqrt{x}}\,dx$

3.58.1 Optimal result

Integrand size = 12, antiderivative size = 9

$$\int \frac{\tan\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -2\log\left(\cos\left(\sqrt{x}\right)\right)$$

output
```
-2*ln(cos(x^(1/2)))
```

3.58.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{\tan\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -2\log\left(\cos\left(\sqrt{x}\right)\right)$$

input
```
Integrate[Tan[Sqrt[x]]/Sqrt[x],x]
```

output
```
-2*Log[Cos[Sqrt[x]]]
```

3.58.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4234, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(\sqrt{x})}{\sqrt{x}}\,dx$$
$$\Big\downarrow \text{ 4234}$$
$$2\int \tan(\sqrt{x})\,d\sqrt{x}$$
$$\Big\downarrow \text{ 3042}$$
$$2\int \tan(\sqrt{x})\,d\sqrt{x}$$
$$\Big\downarrow \text{ 3956}$$
$$-2\log(\cos(\sqrt{x}))$$

input
```
Int[Tan[Sqrt[x]]/Sqrt[x],x]
```

output
```
-2*Log[Cos[Sqrt[x]]]
```

3.58.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4234
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Tan[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Tan[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
 1)/n], 0] && IntegerQ[p]
```

3.58.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$-2\ln\left(\cos\left(\sqrt{x}\right)\right)$	8
default	$-2\ln\left(\cos\left(\sqrt{x}\right)\right)$	8

input
```
int(tan(x^(1/2))/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*ln(cos(x^(1/2)))
```

3.58.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.44

$$\int \frac{\tan\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -\log\left(\frac{1}{\tan\left(\sqrt{x}\right)^2+1}\right)$$

input
```
integrate(tan(x^(1/2))/x^(1/2),x, algorithm="fricas")
```

output
```
-log(1/(tan(sqrt(x))^2 + 1))
```

3.58.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

$$\int \frac{\tan\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \log\left(\tan^2\left(\sqrt{x}\right)+1\right)$$

input
```
integrate(tan(x**(1/2))/x**(1/2),x)
```

output
```
log(tan(sqrt(x))**2 + 1)
```

3.58.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{\tan\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\,\log\left(\sec\left(\sqrt{x}\right)\right)$$

input
```
integrate(tan(x^(1/2))/x^(1/2),x, algorithm="maxima")
```

output
```
2*log(sec(sqrt(x)))
```

3.58.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int \frac{\tan\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -2\,\log\left(\left|\cos\left(\sqrt{x}\right)\right|\right)$$

input
```
integrate(tan(x^(1/2))/x^(1/2),x, algorithm="giac")
```

output
```
-2*log(abs(cos(sqrt(x))))
```

3.58.9 Mupad [B] (verification not implemented)

Time = 27.30 (sec) , antiderivative size = 19, normalized size of antiderivative = 2.11

$$\int \frac{\tan\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -2\,\ln\left(\mathrm{e}^{\sqrt{x}\,2\mathrm{i}}+1\right)+\sqrt{x}\,2\mathrm{i}$$

input
```
int(tan(x^(1/2))/x^(1/2),x)
```

output
```
x^(1/2)*2i - 2*log(exp(x^(1/2)*2i) + 1)
```

3.59 $\int \frac{\tan^2(\sqrt{x})}{\sqrt{x}}\,dx$

3.59.1 Optimal result

Integrand size = 14, antiderivative size = 16

$$\int \frac{\tan^2\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -2\sqrt{x} + 2\tan\left(\sqrt{x}\right)$$

output
```
-2*x^(1/2)+2*tan(x^(1/2))
```

3.59.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tan^2\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -2\arctan\left(\tan\left(\sqrt{x}\right)\right) + 2\tan\left(\sqrt{x}\right)$$

input
```
Integrate[Tan[Sqrt[x]]^2/Sqrt[x],x]
```

output
```
-2*ArcTan[Tan[Sqrt[x]]] + 2*Tan[Sqrt[x]]
```

3.59.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4234, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2\left(\sqrt{x}\right)}{\sqrt{x}}dx$$

$$\Big\downarrow \text{ 4234}$$

$$2\int \tan^2\left(\sqrt{x}\right)d\sqrt{x}$$

$$\Big\downarrow \text{ 3042}$$

$$2\int \tan\left(\sqrt{x}\right)^2d\sqrt{x}$$

$$\Big\downarrow \text{ 3954}$$

$$2\left(\tan\left(\sqrt{x}\right)-\int 1d\sqrt{x}\right)$$

$$\Big\downarrow \text{ 24}$$

$$2\left(\tan\left(\sqrt{x}\right)-\sqrt{x}\right)$$

input
```
Int[Tan[Sqrt[x]]^2/Sqrt[x],x]
```

output
```
2*(-Sqrt[x] + Tan[Sqrt[x]])
```

3.59.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4234
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Tan[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Tan[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
 1)/n], 0] && IntegerQ[p]
```

3.59.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$2\tan\left(\sqrt{x}\right)-2\arctan\left(\tan\left(\sqrt{x}\right)\right)$	15
default	$2\tan\left(\sqrt{x}\right)-2\arctan\left(\tan\left(\sqrt{x}\right)\right)$	15

input
```
int(tan(x^(1/2))^2/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*tan(x^(1/2))-2*arctan(tan(x^(1/2)))
```

3.59.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{\tan^2\left(\sqrt{x}\right)}{\sqrt{x}}\,dx=-2\sqrt{x}+2\tan\left(\sqrt{x}\right)$$

input
```
integrate(tan(x^(1/2))^2/x^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(x) + 2*tan(sqrt(x))
```

3.59.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\tan^2\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -2\sqrt{x} + 2\tan\left(\sqrt{x}\right)$$

input
```
integrate(tan(x**(1/2))**2/x**(1/2),x)
```

output
```
-2*sqrt(x) + 2*tan(sqrt(x))
```

3.59.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{\tan^2\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -2\sqrt{x} + 2\tan\left(\sqrt{x}\right)$$

input
```
integrate(tan(x^(1/2))^2/x^(1/2),x, algorithm="maxima")
```

output
```
-2*sqrt(x) + 2*tan(sqrt(x))
```

3.59.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{\tan^2\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -2\sqrt{x} + 2\tan\left(\sqrt{x}\right)$$

input
```
integrate(tan(x^(1/2))^2/x^(1/2),x, algorithm="giac")
```

output
```
-2*sqrt(x) + 2*tan(sqrt(x))
```

3.59.9 Mupad [B] (verification not implemented)

Time = 26.94 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{\tan^2\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -2\sqrt{x} + \frac{4\mathrm{i}}{\mathrm{e}^{\sqrt{x}\,2\mathrm{i}} + 1}$$

input
```
int(tan(x^(1/2))^2/x^(1/2),x)
```

output
```
4i/(exp(x^(1/2)*2i) + 1) - 2*x^(1/2)
```

3.60 $\int \sqrt{x} \tan \left(\sqrt{x}\right) \, dx$

3.60.1 Optimal result

Integrand size = 12, antiderivative size = 70

$$\int \sqrt{x} \tan \left(\sqrt{x}\right) \, dx = \frac{2}{3} i x^{3/2} - 2x \log \left(1 + e^{2i\sqrt{x}}\right) + 2i\sqrt{x} \operatorname{PolyLog}\left(2, -e^{2i\sqrt{x}}\right) - \operatorname{PolyLog}\left(3, -e^{2i\sqrt{x}}\right)$$

output
```
2/3*I*x^(3/2)-2*x*ln(1+exp(2*I*x^(1/2)))-polylog(3,-exp(2*I*x^(1/2)))+2*I*
polylog(2,-exp(2*I*x^(1/2)))*x^(1/2)
```

3.60.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \sqrt{x} \tan \left(\sqrt{x}\right) \, dx = \frac{2}{3} i x^{3/2} - 2x \log \left(1 + e^{2i\sqrt{x}}\right) + 2i\sqrt{x} \operatorname{PolyLog}\left(2, -e^{2i\sqrt{x}}\right) - \operatorname{PolyLog}\left(3, -e^{2i\sqrt{x}}\right)$$

input
```
Integrate[Sqrt[x]*Tan[Sqrt[x]],x]
```

output
```
((2*I)/3)*x^(3/2) - 2*x*Log[1 + E^((2*I)*Sqrt[x])] + (2*I)*Sqrt[x]*PolyLog
[2, -E^((2*I)*Sqrt[x])] - PolyLog[3, -E^((2*I)*Sqrt[x])]
```

3.60.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.29, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {4234, 3042, 4202, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\tan\left(\sqrt{x}\right)\,dx$$

$$\Big\downarrow \text{4234}$$

$$2\int x\tan\left(\sqrt{x}\right)d\sqrt{x}$$

$$\Big\downarrow \text{3042}$$

$$2\int x\tan\left(\sqrt{x}\right)d\sqrt{x}$$

$$\Big\downarrow \text{4202}$$

$$2\left(\frac{1}{3}ix^{3/2}-2i\int \frac{e^{2i\sqrt{x}}x}{1+e^{2i\sqrt{x}}}d\sqrt{x}\right)$$

$$\Big\downarrow \text{2620}$$

$$2\left(\frac{1}{3}ix^{3/2}-2i\left(i\int \sqrt{x}\log\left(1+e^{2i\sqrt{x}}\right)d\sqrt{x}-\frac{1}{2}ix\log\left(1+e^{2i\sqrt{x}}\right)\right)\right)$$

$$\Big\downarrow \text{3011}$$

$$2\left(\frac{1}{3}ix^{3/2}-2i\left(i\left(\frac{1}{2}i\sqrt{x}\,\text{PolyLog}\left(2,-e^{2i\sqrt{x}}\right)-\frac{1}{2}i\int \text{PolyLog}\left(2,-e^{2i\sqrt{x}}\right)d\sqrt{x}\right)-\frac{1}{2}ix\log\left(1+e^{2i\sqrt{x}}\right)\right)\right)$$

$$\Big\downarrow \text{2720}$$

$$2\left(\frac{1}{3}ix^{3/2}-2i\left(i\left(\frac{1}{2}i\sqrt{x}\,\text{PolyLog}\left(2,-e^{2i\sqrt{x}}\right)-\frac{1}{4}\int \frac{\text{PolyLog}\left(2,-e^{2i\sqrt{x}}\right)}{\sqrt{x}}de^{2i\sqrt{x}}\right)-\frac{1}{2}ix\log\left(1+e^{2i\sqrt{x}}\right)\right)\right)$$

$$\Big\downarrow \text{7143}$$

$$2\left(\frac{1}{3}ix^{3/2}-2i\left(i\left(\frac{1}{2}i\sqrt{x}\,\text{PolyLog}\left(2,-e^{2i\sqrt{x}}\right)-\frac{1}{4}\text{PolyLog}\left(3,-e^{2i\sqrt{x}}\right)\right)-\frac{1}{2}ix\log\left(1+e^{2i\sqrt{x}}\right)\right)\right)$$

input
```
Int[Sqrt[x]*Tan[Sqrt[x]],x]
```

output
```
2*((I/3)*x^(3/2) - (2*I)*((-1/2*I)*x*Log[1 + E^((2*I)*Sqrt[x])] + I*((I/2)
*Sqrt[x]*PolyLog[2, -E^((2*I)*Sqrt[x])] - PolyLog[3, -E^((2*I)*Sqrt[x])]/4
)))
```

3.60.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4234
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Tan[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Tan[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
 1)/n], 0] && IntegerQ[p]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.60.4 Maple [F]

$$\int \sqrt{x}\, \tan\left(\sqrt{x}\right) dx$$

input
```
int(x^(1/2)*tan(x^(1/2)),x)
```

output
```
int(x^(1/2)*tan(x^(1/2)),x)
```

3.60.5 Fricas [F]

$$\int \sqrt{x} \tan\left(\sqrt{x}\right)\, dx = \int \sqrt{x} \tan\left(\sqrt{x}\right)\, dx$$

input
```
integrate(x^(1/2)*tan(x^(1/2)),x, algorithm="fricas")
```

output
```
integral(sqrt(x)*tan(sqrt(x)), x)
```

3.60.6 Sympy [F]

$$\int \sqrt{x} \tan\left(\sqrt{x}\right)\, dx = \int \sqrt{x} \tan\left(\sqrt{x}\right) dx$$

input
```
integrate(x**(1/2)*tan(x**(1/2)),x)
```

output
```
Integral(sqrt(x)*tan(sqrt(x)), x)
```

3.60.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.14

$$\int \sqrt{x}\tan\left(\sqrt{x}\right)\,dx = -2i\,x\arctan\left(\sin\left(2\sqrt{x}\right),\cos\left(2\sqrt{x}\right)+1\right) - x\log\left(\cos\left(2\sqrt{x}\right)^2+\sin\left(2\sqrt{x}\right)^2+2\cos\left(2\sqrt{x}\right)+1\right) + \frac{2}{3}i\,x^{\frac{3}{2}} + 2i\sqrt{x}\mathrm{Li}_2\left(-e^{\left(2i\sqrt{x}\right)}\right) - \mathrm{Li}_3\left(-e^{\left(2i\sqrt{x}\right)}\right)$$

input
```
integrate(x^(1/2)*tan(x^(1/2)),x, algorithm="maxima")
```

output
```
-2*I*x*arctan2(sin(2*sqrt(x)), cos(2*sqrt(x)) + 1) - x*log(cos(2*sqrt(x))^
2 + sin(2*sqrt(x))^2 + 2*cos(2*sqrt(x)) + 1) + 2/3*I*x^(3/2) + 2*I*sqrt(x)
*dilog(-e^(2*I*sqrt(x))) - polylog(3, -e^(2*I*sqrt(x)))
```

3.60.8 Giac [F]

$$\int \sqrt{x}\tan\left(\sqrt{x}\right)\,dx = \int \sqrt{x}\tan\left(\sqrt{x}\right)\,dx$$

input
```
integrate(x^(1/2)*tan(x^(1/2)),x, algorithm="giac")
```

output
```
integrate(sqrt(x)*tan(sqrt(x)), x)
```

3.60.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}\tan\left(\sqrt{x}\right)\,dx = \int \sqrt{x}\tan\left(\sqrt{x}\right)\,dx$$

input
```
int(x^(1/2)*tan(x^(1/2)),x)
```

output
```
int(x^(1/2)*tan(x^(1/2)), x)
```

3.61 $\int \left(\frac{b \tan(a+bx+cx^2)}{2c} + x \tan\left(a+bx+cx^2\right)\right) dx$

3.61.1 Optimal result

Integrand size = 33, antiderivative size = 19

$$\int \left(\frac{b \tan\left(a+bx+cx^2\right)}{2c} + x \tan\left(a+bx+cx^2\right)\right) dx = -\frac{\log\left(\cos\left(a+bx+cx^2\right)\right)}{2c}$$

output
```
-1/2*ln(cos(c*x^2+b*x+a))/c
```

3.61.2 Mathematica [A] (verified)

Time = 1.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \left(\frac{b \tan\left(a+bx+cx^2\right)}{2c} + x \tan\left(a+bx+cx^2\right)\right) dx = -\frac{\log(\cos(a+x(b+cx)))}{2c}$$

input
```
Integrate[(b*Tan[a + b*x + c*x^2])/(2*c) + x*Tan[a + b*x + c*x^2],x]
```

output
```
-1/2*Log[Cos[a + x*(b + c*x)]]/c
```

3.61.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.030, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x\tan\left(a+bx+cx^2\right)+\frac{b\tan\left(a+bx+cx^2\right)}{2c}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\log\left(\cos\left(a+bx+cx^2\right)\right)}{2c}$$

input
```
Int[(b*Tan[a + b*x + c*x^2])/(2*c) + x*Tan[a + b*x + c*x^2],x]
```

output
```
-1/2*Log[Cos[a + b*x + c*x^2]]/c
```

3.61.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.61.4 Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result	size
default	$-\frac{\ln\left(\cos\left(c\,x^{2}+x b+a\right)\right)}{2c}$	18
norman	$\frac{\ln\left(1+\tan\left(c\,x^{2}+x b+a\right)^{2}\right)}{4c}$	22
parallelrisch	$\frac{\ln\left(1+\tan\left(c\,x^{2}+x b+a\right)^{2}\right)}{4c}$	22
risch	$\frac{i x^{2}}{2}+\frac{i x b}{2c}+\frac{i a}{c}-\frac{\ln\left({\mathrm e}^{2 i \left(c\,x^{2}+x b+a\right)}+1\right)}{2c}$	45

input
```
int(1/2*b*tan(c*x^2+b*x+a)/c+x*tan(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(cos(c*x^2+b*x+a))/c
```

3.61.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21

$$\int \left(\frac{b\tan\left(a+bx+cx^2\right)}{2c}+x\tan\left(a+bx+cx^2\right)\right)\,dx=-\frac{\log\left(\frac{1}{\tan\left(cx^2+bx+a\right)^2+1}\right)}{4\,c}$$

input
```
integrate(1/2*b*tan(c*x^2+b*x+a)/c+x*tan(c*x^2+b*x+a),x, algorithm="fricas
")
```

output
```
-1/4*log(1/(tan(c*x^2 + b*x + a)^2 + 1))/c
```

3.61.6 Sympy [F]

$$\int \left(\frac{b\tan\left(a+bx+cx^2\right)}{2c}+x\tan\left(a+bx+cx^2\right)\right)\,dx=\frac{\int b\tan\left(a+bx+cx^2\right)dx+\int 2cx\tan\left(a+bx+cx^2\right)dx}{2c}$$

input
```
integrate(1/2*b*tan(c*x**2+b*x+a)/c+x*tan(c*x**2+b*x+a),x)
```

output
```
(Integral(b*tan(a + b*x + c*x**2), x) + Integral(2*c*x*tan(a + b*x + c*x**
2), x))/(2*c)
```

3.61.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(17) = 34.

Time = 0.29 (sec) , antiderivative size = 83, normalized size of antiderivative = 4.37

$$\int \left(\frac{b\tan\left(a+bx+cx^2\right)}{2c}+x\tan\left(a+bx+cx^2\right)\right)\,dx=-\frac{\log\left(\cos\left(2\,cx^2\right)^2+2\,\cos\left(2\,cx^2\right)\cos\left(2\,bx+2\,a\right)+\cos\left(2\,bx+2\,a\right)^2+\sin\left(2\,cx^2\right)^2-2\,\sin\left(2\,cx^2\right)\sin\left(2\,bx\right.\right.}{4\,c}$$

input
```
integrate(1/2*b*tan(c*x^2+b*x+a)/c+x*tan(c*x^2+b*x+a),x, algorithm="maxima
")
```

output
```
-1/4*log(cos(2*c*x^2)^2 + 2*cos(2*c*x^2)*cos(2*b*x + 2*a) + cos(2*b*x + 2*
a)^2 + sin(2*c*x^2)^2 - 2*sin(2*c*x^2)*sin(2*b*x + 2*a) + sin(2*b*x + 2*a)
^2)/c
```

3.61.8 Giac [A] (verification not implemented)

Time = 4.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \left(\frac{b\tan\left(a+bx+cx^2\right)}{2c}+x\tan\left(a+bx+cx^2\right)\right)\,dx=-\frac{\log\left(\left|\cos\left(cx^2+bx+a\right)\right|\right)}{2\,c}$$

input
```
integrate(1/2*b*tan(c*x^2+b*x+a)/c+x*tan(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
-1/2*log(abs(cos(c*x^2 + b*x + a)))/c
```

3.61.9 Mupad [B] (verification not implemented)

Time = 28.42 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \left(\frac{b\tan\left(a+bx+cx^2\right)}{2c}+x\tan\left(a+bx+cx^2\right)\right)\,dx=\frac{\ln\left({\mathrm{tan}\left(c\,x^2+b\,x+a\right)}^2+1\right)}{4\,c}$$

input
```
int(x*tan(a + b*x + c*x^2) + (b*tan(a + b*x + c*x^2))/(2*c),x)
```

output
```
log(tan(a + b*x + c*x^2)^2 + 1)/(4*c)
```

3.62 $\int \frac{\cot^2(\sqrt{x})}{\sqrt{x}}\,dx$

3.62.1 Optimal result

Integrand size = 14, antiderivative size = 16

$$\int \frac{\cot^2\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -2\sqrt{x} - 2\cot\left(\sqrt{x}\right)$$

output
```
-2*cot(x^(1/2))-2*x^(1/2)
```

3.62.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int \frac{\cot^2\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -2\cot\left(\sqrt{x}\right)\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2\left(\sqrt{x}\right)\right)$$

input
```
Integrate[Cot[Sqrt[x]]^2/Sqrt[x],x]
```

output
```
-2*Cot[Sqrt[x]]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[Sqrt[x]]^2]
```

3.62.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4235, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2\left(\sqrt{x}\right)}{\sqrt{x}}dx$$

$$\Big\downarrow \text{4235}$$

$$2\int \cot^2\left(\sqrt{x}\right)d\sqrt{x}$$

$$\Big\downarrow \text{3042}$$

$$2\int \tan\left(\sqrt{x}+\frac{\pi}{2}\right)^2d\sqrt{x}$$

$$\Big\downarrow \text{3954}$$

$$2\left(-\int 1d\sqrt{x}-\cot\left(\sqrt{x}\right)\right)$$

$$\Big\downarrow \text{24}$$

$$2\left(-\sqrt{x}-\cot\left(\sqrt{x}\right)\right)$$

input
```
Int[Cot[Sqrt[x]]^2/Sqrt[x],x]
```

output
```
2*(-Sqrt[x] - Cot[Sqrt[x]])
```

3.62.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4235
```
Int[((a_.) + Cot[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cot[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
 1)/n], 0] && IntegerQ[p]
```

3.62.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$-2\cot\left(\sqrt{x}\right)+\pi-2\,\mathrm{arccot}\left(\cot\left(\sqrt{x}\right)\right)$	16
default	$-2\cot\left(\sqrt{x}\right)+\pi-2\,\mathrm{arccot}\left(\cot\left(\sqrt{x}\right)\right)$	16

input
```
int(cot(x^(1/2))^2/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*cot(x^(1/2))+Pi-2*arccot(cot(x^(1/2)))
```

3.62.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. 2(12) = 24.

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.75

$$\int \frac{\cot^2\left(\sqrt{x}\right)}{\sqrt{x}}\,dx=-\frac{2\left(\sqrt{x}\sin\left(2\sqrt{x}\right)+\cos\left(2\sqrt{x}\right)+1\right)}{\sin\left(2\sqrt{x}\right)}$$

input
```
integrate(cot(x^(1/2))^2/x^(1/2),x, algorithm="fricas")
```

output
```
-2*(sqrt(x)*sin(2*sqrt(x)) + cos(2*sqrt(x)) + 1)/sin(2*sqrt(x))
```

3.62.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\cot^2\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -2\sqrt{x} - 2\cot\left(\sqrt{x}\right)$$

input
```
integrate(cot(x**(1/2))**2/x**(1/2),x)
```

output
```
-2*sqrt(x) - 2*cot(sqrt(x))
```

3.62.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\cot^2\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -2\sqrt{x} - \frac{2}{\tan\left(\sqrt{x}\right)}$$

input
```
integrate(cot(x^(1/2))^2/x^(1/2),x, algorithm="maxima")
```

output
```
-2*sqrt(x) - 2/tan(sqrt(x))
```

3.62.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{\cot^2\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -2\sqrt{x} - \frac{1}{\tan\left(\frac{1}{2}\sqrt{x}\right)} + \tan\left(\frac{1}{2}\sqrt{x}\right)$$

input
```
integrate(cot(x^(1/2))^2/x^(1/2),x, algorithm="giac")
```

output
```
-2*sqrt(x) - 1/tan(1/2*sqrt(x)) + tan(1/2*sqrt(x))
```

3.62.9 Mupad [B] (verification not implemented)

Time = 28.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{\cot^2\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -2\sqrt{x} - \frac{4\mathrm{i}}{\mathrm{e}^{\sqrt{x}\,2\mathrm{i}} - 1}$$

input
```
int(cot(x^(1/2))^2/x^(1/2),x)
```

output
```
- 4i/(exp(x^(1/2)*2i) - 1) - 2*x^(1/2)
```

3.63 $\int \frac{\sqrt{a+b\sec(c+dx)}}{1+\cos(c+dx)}\,dx$

3.63.1 Optimal result

Integrand size = 25, antiderivative size = 92

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{1+\cos(c+dx)}\,dx=\frac{E\left(\arcsin\left(\frac{\tan(c+dx)}{1+\sec(c+dx)}\right)|\frac{a-b}{a+b}\right)\sqrt{\frac{1}{1+\sec(c+dx)}}\sqrt{a+b\sec(c+dx)}}{d\sqrt{\frac{a+b\sec(c+dx)}{(a+b)(1+\sec(c+dx))}}}$$

output
```
EllipticE(tan(d*x+c)/(1+sec(d*x+c)),((a-b)/(a+b))^(1/2))*(1/(1+sec(d*x+c))
)^(1/2)*(a+b*sec(d*x+c))^(1/2)/d/((a+b*sec(d*x+c))/(a+b)/(1+sec(d*x+c)))^(
1/2)
```

3.63.2 Mathematica [A] (verified)

Time = 2.76 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{1+\cos(c+dx)}\,dx=\frac{E\left(\arcsin\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)\sqrt{\frac{1}{1+\sec(c+dx)}}\sqrt{a+b\sec(c+dx)}}{d\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}}$$

input
```
Integrate[Sqrt[a + b*Sec[c + d*x]]/(1 + Cos[c + d*x]),x]
```

output
```
(EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[(1 + Sec[c + d*
x])^(-1)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/((a + b)*
(1 + Cos[c + d*x]))])
```

3.63.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3042, 3308, 3042, 4456}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{\cos(c+dx)+1}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\sin\left(c+dx+\frac{\pi}{2}\right)+1}dx$$

$$\Big\downarrow \ 3308$$

$$\int \frac{\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{\sec(c+dx)+1}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\csc\left(c+dx+\frac{\pi}{2}\right)\sqrt{a+b\csc\left(c+dx+\frac{\pi}{2}\right)}}{\csc\left(c+dx+\frac{\pi}{2}\right)+1}dx$$

$$\Big\downarrow \ 4456$$

$$\frac{\sqrt{\frac{1}{\sec(c+dx)+1}}\sqrt{a+b\sec(c+dx)}E\left(\arcsin\left(\frac{\tan(c+dx)}{\sec(c+dx)+1}\right)|\frac{a-b}{a+b}\right)}{d\sqrt{\frac{a+b\sec(c+dx)}{(a+b)(\sec(c+dx)+1)}}}$$

input
```
Int[Sqrt[a + b*Sec[c + d*x]]/(1 + Cos[c + d*x]),x]
```

output
```
(EllipticE[ArcSin[Tan[c + d*x]/(1 + Sec[c + d*x])], (a - b)/(a + b)]*Sqrt[
(1 + Sec[c + d*x])^(-1)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(a + b*Sec[c +
d*x])/((a + b)*(1 + Sec[c + d*x]))])
```

3.63.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3308
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_)*((a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)])^(m_.), x_Symbol] :> Int[(b + a*Csc[e + f*x])^m*((c + d*Csc[e +
f*x])^n/Csc[e + f*x]^m), x] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !Integer
Q[n] && IntegerQ[m]
```

rule 4456
```
Int[(csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)])/(c
sc[(e_.) + (f_.)*(x_)]*(d_.) + (c_)), x_Symbol] :> Simp[(-Sqrt[a + b*Csc[e
+ f*x]])*(Sqrt[c/(c + d*Csc[e + f*x])]/(d*f*Sqrt[c*d*((a + b*Csc[e + f*x])/
((b*c + a*d)*(c + d*Csc[e + f*x]))]]))*EllipticE[ArcSin[c*(Cot[e + f*x]/(c
+ d*Csc[e + f*x]))], -(b*c - a*d)/(b*c + a*d)], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && EqQ[c^2 - d^2, 0]
```

3.63.4 Maple [A] (verified)

Time = 8.66 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.30

method	result	size
default	$\frac{(\cos(dx+c)+1)\sqrt{\frac{b+\cos(dx+c)a}{(\cos(dx+c)+1)(a+b)}}\,\sqrt{\frac{\cos(dx+c)}{\cos(dx+c)+1}}\,\operatorname{EllipticE}\left(\cot(dx+c)-\csc(dx+c),\sqrt{\frac{a-b}{a+b}}\right)\sqrt{a+b\sec(dx+c)}\,(-a-b)}{d(b+\cos(dx+c)a)}$	120

input
```
int((a+b*sec(d*x+c))^(1/2)/(cos(d*x+c)+1),x,method=_RETURNVERBOSE)
```

output
```
1/d*(cos(d*x+c)+1)*((b+cos(d*x+c)*a)/(cos(d*x+c)+1)/(a+b))^(1/2)*(cos(d*x+
c)/(cos(d*x+c)+1))^(1/2)*EllipticE(cot(d*x+c)-csc(d*x+c),((a-b)/(a+b))^(1/
2))*(a+b*sec(d*x+c))^(1/2)/(b+cos(d*x+c)*a)*(-a-b)
```

3.63.5 Fricas [F]

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{1+\cos(c+dx)}\,dx = \int \frac{\sqrt{b\sec\left(dx+c\right)+a}}{\cos\left(dx+c\right)+1}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/(1+cos(d*x+c)),x, algorithm="fricas")
```

output
```
integral(sqrt(b*sec(d*x + c) + a)/(cos(d*x + c) + 1), x)
```

3.63.6 Sympy [F]

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{1+\cos(c+dx)}\,dx = \int \frac{\sqrt{a+b\sec\left(c+dx\right)}}{\cos\left(c+dx\right)+1}\,dx$$

input
```
integrate((a+b*sec(d*x+c))**(1/2)/(1+cos(d*x+c)),x)
```

output
```
Integral(sqrt(a + b*sec(c + d*x))/(cos(c + d*x) + 1), x)
```

3.63.7 Maxima [F]

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{1+\cos(c+dx)}\,dx = \int \frac{\sqrt{b\sec\left(dx+c\right)+a}}{\cos\left(dx+c\right)+1}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/(1+cos(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)/(cos(d*x + c) + 1), x)
```

3.63.8 Giac [F]

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{1+\cos(c+dx)}\,dx = \int \frac{\sqrt{b\sec(dx+c)+a}}{\cos(dx+c)+1}\,dx$$

input
```
integrate((a+b*sec(d*x+c))^(1/2)/(1+cos(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sqrt(b*sec(d*x + c) + a)/(cos(d*x + c) + 1), x)
```

3.63.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sec(c+dx)}}{1+\cos(c+dx)}\,dx = \int \frac{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}{\cos(c+d\,x)+1}\,dx$$

input
```
int((a + b/cos(c + d*x))^(1/2)/(cos(c + d*x) + 1),x)
```

output
```
int((a + b/cos(c + d*x))^(1/2)/(cos(c + d*x) + 1), x)
```

3.64 $\int \sec(a+bx)\sec(2a+2bx)\,dx$

3.64.1 Optimal result

Integrand size = 16, antiderivative size = 35

$$\int \sec(a+bx)\sec(2a+2bx)\,dx = -\frac{\operatorname{arctanh}(\sin(a+bx))}{b}+\frac{\sqrt{2}\operatorname{arctanh}\left(\sqrt{2}\sin(a+bx)\right)}{b}$$

output
```
-arctanh(sin(b*x+a))/b+arctanh(sin(b*x+a)*2^(1/2))*2^(1/2)/b
```

3.64.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \sec(a+bx)\sec(2a+2bx)\,dx = -\frac{\operatorname{arctanh}(\sin(a+bx))}{b}+\frac{\sqrt{2}\operatorname{arctanh}\left(\sqrt{2}\sin(a+bx)\right)}{b}$$

input
```
Integrate[Sec[a + b*x]*Sec[2*a + 2*b*x],x]
```

output
```
-(ArcTanh[Sin[a + b*x]]/b) + (Sqrt[2]*ArcTanh[Sqrt[2]*Sin[a + b*x]])/b
```

3.64.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 4864, 1406, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(a+bx)\sec(2a+2bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(a+bx)\cos(2a+2bx)}dx$$

$$\Big\downarrow \text{4864}$$

$$\frac{\int \frac{1}{2\sin^4(a+bx)-3\sin^2(a+bx)+1}d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{1406}$$

$$\frac{2\int \frac{1}{2\sin^2(a+bx)-2}d\sin(a+bx)-2\int \frac{1}{2\sin^2(a+bx)-1}d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{220}$$

$$\frac{\sqrt{2}\text{arctanh}\left(\sqrt{2}\sin(a+bx)\right)-\text{arctanh}(\sin(a+bx))}{b}$$

input
```
Int[Sec[a + b*x]*Sec[2*a + 2*b*x],x]
```

output
```
(-ArcTanh[Sin[a + b*x]] + Sqrt[2]*ArcTanh[Sqrt[2]*Sin[a + b*x]])/b
```

3.64.3.1 Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4864
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[(1 - d^2*x
^2)^((n - 1)/2), Sin[c*(a + b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x]
/; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

3.64.4 Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.23

method	result	size
default	$\frac{-\frac{\ln(\sin(xb+a)+1)}{2}+\frac{\ln(\sin(xb+a)-1)}{2}+\sqrt{2}\,\operatorname{arctanh}\left(\sin(xb+a)\sqrt{2}\right)}{b}$	43
risch	$\frac{\ln\left(\mathrm{e}^{i(xb+a)}-i\right)}{b}-\frac{\ln\left(i+\mathrm{e}^{i(xb+a)}\right)}{b}+\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2i(xb+a)}+i\sqrt{2}\,\mathrm{e}^{i(xb+a)}-1\right)}{2b}-\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2i(xb+a)}-i\sqrt{2}\,\mathrm{e}^{i(xb+a)}-1\right)}{2b}$	107

input
```
int(sec(b*x+a)*sec(2*b*x+2*a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/2*ln(sin(b*x+a)+1)+1/2*ln(sin(b*x+a)-1)+2^(1/2)*arctanh(sin(b*x+a)
*2^(1/2)))
```

3.64.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(31) = 62$.

Time = 0.26 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.06

$$\int \sec(a+bx)\sec(2a+2bx)\,dx = \frac{\sqrt{2}\log\left(-\frac{2\cos(bx+a)^2-2\sqrt{2}\sin(bx+a)-3}{2\cos(bx+a)^2-1}\right)-\log(\sin(bx+a)+1)+\log(-\sin(bx+a)+1)}{2b}$$

input
```
integrate(sec(b*x+a)*sec(2*b*x+2*a),x, algorithm="fricas")
```

output
```
1/2*(sqrt(2)*log(-(2*cos(b*x + a)^2 - 2*sqrt(2)*sin(b*x + a) - 3)/(2*cos(b
*x + a)^2 - 1)) - log(sin(b*x + a) + 1) + log(-sin(b*x + a) + 1))/b
```

3.64.6 Sympy [F]

$$\int \sec(a+bx)\sec(2a+2bx)\,dx = \int \sec(a+bx)\sec(2a+2bx)\,dx$$

input
```
integrate(sec(b*x+a)*sec(2*b*x+2*a),x)
```

output
```
Integral(sec(a + b*x)*sec(2*a + 2*b*x), x)
```

3.64.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6257 vs. $2(31) = 62$.

Time = 84.82 (sec) , antiderivative size = 6257, normalized size of antiderivative = 178.77

$$\int \sec(a+bx)\sec(2a+2bx)\,dx = \text{Too large to display}$$

input
```
integrate(sec(b*x+a)*sec(2*b*x+2*a),x, algorithm="maxima")
```

output
```
-1/8*(2*(sqrt(2)*cos(3*a)*cos(3/4*arctan2(sin(4*a), cos(4*a))) - sqrt(2)*c
os(a)*cos(1/4*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(3*a)*sin(3/4*arct
an2(sin(4*a), cos(4*a))) - sqrt(2)*sin(a)*sin(1/4*arctan2(sin(4*a), cos(4*
a))))*arctan2(sqrt(2)*cos(1/4*arctan2(sin(4*a), cos(4*a)))*sin(b*x) + sqrt
(2)*cos(b*x)*sin(1/4*arctan2(sin(4*a), cos(4*a))) + cos(1/2*arctan2(sin(4*
a), cos(4*a)))*sin(2*b*x) + cos(2*b*x)*sin(1/2*arctan2(sin(4*a), cos(4*a))
), sqrt(2)*cos(b*x)*cos(1/4*arctan2(sin(4*a), cos(4*a))) - sqrt(2)*sin(b*x
)*sin(1/4*arctan2(sin(4*a), cos(4*a))) + cos(2*b*x)*cos(1/2*arctan2(sin(4*
a), cos(4*a))) - sin(2*b*x)*sin(1/2*arctan2(sin(4*a), cos(4*a))) + 1) - 2*
(sqrt(2)*cos(3*a)*cos(3/4*arctan2(sin(4*a), cos(4*a))) - sqrt(2)*cos(a)*co
s(1/4*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(3*a)*sin(3/4*arctan2(sin(
4*a), cos(4*a))) - sqrt(2)*sin(a)*sin(1/4*arctan2(sin(4*a), cos(4*a))))*ar
ctan2(-sqrt(2)*cos(1/4*arctan2(sin(4*a), cos(4*a)))*sin(b*x) - sqrt(2)*cos
(b*x)*sin(1/4*arctan2(sin(4*a), cos(4*a))) + cos(1/2*arctan2(sin(4*a), cos
(4*a)))*sin(2*b*x) + cos(2*b*x)*sin(1/2*arctan2(sin(4*a), cos(4*a))), -sqr
t(2)*cos(b*x)*cos(1/4*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(b*x)*sin(
1/4*arctan2(sin(4*a), cos(4*a))) + cos(2*b*x)*cos(1/2*arctan2(sin(4*a), co
s(4*a))) - sin(2*b*x)*sin(1/2*arctan2(sin(4*a), cos(4*a))) + 1) - 2*((sqrt
(2)*cos(3*a)*cos(1/2*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(3*a)*sin(1
/2*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*cos(a))*cos(1/2*pi + 1/4*arct...
```

3.64.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(31) = 62$.

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.89

$$\int \sec(a+bx)\sec(2a+2bx)\,dx$$
$$= -\frac{\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin(bx+a)\right|}{\left|2\sqrt{2}+4\sin(bx+a)\right|}\right)+\log\left(\sin\left(bx+a\right)+1\right)-\log\left(-\sin\left(bx+a\right)+1\right)}{2b}$$

input
```
integrate(sec(b*x+a)*sec(2*b*x+2*a),x, algorithm="giac")
```

output
```
-1/2*(sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(b*x + a))/abs(2*sqrt(2) + 4*sin(b
*x + a))) + log(sin(b*x + a) + 1) - log(-sin(b*x + a) + 1))/b
```

3.64.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \sec(a+bx)\sec(2a+2bx)\,dx = -\frac{\operatorname{atanh}(\sin(a+b\,x)) - \sqrt{2}\operatorname{atanh}\left(\sqrt{2}\,\sin(a+b\,x)\right)}{b}$$

input
```
int(1/(cos(a + b*x)*cos(2*a + 2*b*x)),x)
```

output
```
-(atanh(sin(a + b*x)) - 2^(1/2)*atanh(2^(1/2)*sin(a + b*x)))/b
```

3.65 $\int \sec(a+bx)\sec(2(a+bx))\,dx$

3.65.1 Optimal result

Integrand size = 15, antiderivative size = 35

$$\int \sec(a+bx)\sec(2(a+bx))\,dx = -\frac{\operatorname{arctanh}(\sin(a+bx))}{b}+\frac{\sqrt{2}\operatorname{arctanh}\left(\sqrt{2}\sin(a+bx)\right)}{b}$$

output
```
-arctanh(sin(b*x+a))/b+arctanh(sin(b*x+a)*2^(1/2))*2^(1/2)/b
```

3.65.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \sec(a+bx)\sec(2(a+bx))\,dx = -\frac{\operatorname{arctanh}(\sin(a+bx))}{b}+\frac{\sqrt{2}\operatorname{arctanh}\left(\sqrt{2}\sin(a+bx)\right)}{b}$$

input
```
Integrate[Sec[a + b*x]*Sec[2*(a + b*x)],x]
```

output
```
-(ArcTanh[Sin[a + b*x]]/b) + (Sqrt[2]*ArcTanh[Sqrt[2]*Sin[a + b*x]])/b
```

3.65.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 4864, 1406, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(a+bx)\sec(2(a+bx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(a+bx)\cos(2a+2bx)}dx$$

$$\Big\downarrow \text{4864}$$

$$\frac{\int \frac{1}{2\sin^4(a+bx)-3\sin^2(a+bx)+1}d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{1406}$$

$$\frac{2\int \frac{1}{2\sin^2(a+bx)-2}d\sin(a+bx)-2\int \frac{1}{2\sin^2(a+bx)-1}d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{220}$$

$$\frac{\sqrt{2}\text{arctanh}\left(\sqrt{2}\sin(a+bx)\right)-\text{arctanh}(\sin(a+bx))}{b}$$

input
```
Int[Sec[a + b*x]*Sec[2*(a + b*x)],x]
```

output
```
(-ArcTanh[Sin[a + b*x]] + Sqrt[2]*ArcTanh[Sqrt[2]*Sin[a + b*x]])/b
```

3.65.3.1 Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4864
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[(1 - d^2*x
^2)^((n - 1)/2), Sin[c*(a + b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x]
/; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

3.65.4 Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.23

method	result	size
default	$\frac{-\frac{\ln(\sin(xb+a)+1)}{2}+\frac{\ln(\sin(xb+a)-1)}{2}+\sqrt{2}\,\operatorname{arctanh}\left(\sin(xb+a)\sqrt{2}\right)}{b}$	43
risch	$\frac{\ln\left(\mathrm{e}^{i(xb+a)}-i\right)}{b}-\frac{\ln\left(i+\mathrm{e}^{i(xb+a)}\right)}{b}+\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2i(xb+a)}+i\sqrt{2}\,\mathrm{e}^{i(xb+a)}-1\right)}{2b}-\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2i(xb+a)}-i\sqrt{2}\,\mathrm{e}^{i(xb+a)}-1\right)}{2b}$	107

input
```
int(sec(b*x+a)*sec(2*b*x+2*a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/2*ln(sin(b*x+a)+1)+1/2*ln(sin(b*x+a)-1)+2^(1/2)*arctanh(sin(b*x+a)
*2^(1/2)))
```

3.65.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(31) = 62$.

Time = 0.26 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.06

$$\int \sec(a+bx)\sec(2(a+bx))\,dx = \frac{\sqrt{2}\log\left(-\frac{2\cos(bx+a)^2-2\sqrt{2}\sin(bx+a)-3}{2\cos(bx+a)^2-1}\right)-\log(\sin(bx+a)+1)+\log(-\sin(bx+a)+1)}{2b}$$

input
```
integrate(sec(b*x+a)*sec(2*b*x+2*a),x, algorithm="fricas")
```

output
```
1/2*(sqrt(2)*log(-(2*cos(b*x + a)^2 - 2*sqrt(2)*sin(b*x + a) - 3)/(2*cos(b
*x + a)^2 - 1)) - log(sin(b*x + a) + 1) + log(-sin(b*x + a) + 1))/b
```

3.65.6 Sympy [F]

$$\int \sec(a+bx)\sec(2(a+bx))\,dx = \int \sec(a+bx)\sec(2a+2bx)\,dx$$

input
```
integrate(sec(b*x+a)*sec(2*b*x+2*a),x)
```

output
```
Integral(sec(a + b*x)*sec(2*a + 2*b*x), x)
```

3.65.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6257 vs. $2(31) = 62$.

Time = 84.36 (sec) , antiderivative size = 6257, normalized size of antiderivative = 178.77

$$\int \sec(a+bx)\sec(2(a+bx))\,dx = \text{Too large to display}$$

input
```
integrate(sec(b*x+a)*sec(2*b*x+2*a),x, algorithm="maxima")
```

output
```
-1/8*(2*(sqrt(2)*cos(3*a)*cos(3/4*arctan2(sin(4*a), cos(4*a))) - sqrt(2)*c
os(a)*cos(1/4*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(3*a)*sin(3/4*arct
an2(sin(4*a), cos(4*a))) - sqrt(2)*sin(a)*sin(1/4*arctan2(sin(4*a), cos(4*
a))))*arctan2(sqrt(2)*cos(1/4*arctan2(sin(4*a), cos(4*a)))*sin(b*x) + sqrt
(2)*cos(b*x)*sin(1/4*arctan2(sin(4*a), cos(4*a))) + cos(1/2*arctan2(sin(4*
a), cos(4*a)))*sin(2*b*x) + cos(2*b*x)*sin(1/2*arctan2(sin(4*a), cos(4*a))
), sqrt(2)*cos(b*x)*cos(1/4*arctan2(sin(4*a), cos(4*a))) - sqrt(2)*sin(b*x
)*sin(1/4*arctan2(sin(4*a), cos(4*a))) + cos(2*b*x)*cos(1/2*arctan2(sin(4*
a), cos(4*a))) - sin(2*b*x)*sin(1/2*arctan2(sin(4*a), cos(4*a))) + 1) - 2*
(sqrt(2)*cos(3*a)*cos(3/4*arctan2(sin(4*a), cos(4*a))) - sqrt(2)*cos(a)*co
s(1/4*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(3*a)*sin(3/4*arctan2(sin(
4*a), cos(4*a))) - sqrt(2)*sin(a)*sin(1/4*arctan2(sin(4*a), cos(4*a))))*ar
ctan2(-sqrt(2)*cos(1/4*arctan2(sin(4*a), cos(4*a)))*sin(b*x) - sqrt(2)*cos
(b*x)*sin(1/4*arctan2(sin(4*a), cos(4*a))) + cos(1/2*arctan2(sin(4*a), cos
(4*a)))*sin(2*b*x) + cos(2*b*x)*sin(1/2*arctan2(sin(4*a), cos(4*a))), -sqr
t(2)*cos(b*x)*cos(1/4*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(b*x)*sin(
1/4*arctan2(sin(4*a), cos(4*a))) + cos(2*b*x)*cos(1/2*arctan2(sin(4*a), co
s(4*a))) - sin(2*b*x)*sin(1/2*arctan2(sin(4*a), cos(4*a))) + 1) - 2*((sqrt
(2)*cos(3*a)*cos(1/2*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*sin(3*a)*sin(1
/2*arctan2(sin(4*a), cos(4*a))) + sqrt(2)*cos(a))*cos(1/2*pi + 1/4*arct...
```

3.65.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(31) = 62$.

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.89

$$\int \sec(a+bx)\sec(2(a+bx))\,dx$$
$$= -\frac{\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin(bx+a)\right|}{\left|2\sqrt{2}+4\sin(bx+a)\right|}\right)+\log\left(\sin\left(bx+a\right)+1\right)-\log\left(-\sin\left(bx+a\right)+1\right)}{2b}$$

input
```
integrate(sec(b*x+a)*sec(2*b*x+2*a),x, algorithm="giac")
```

output
```
-1/2*(sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(b*x + a))/abs(2*sqrt(2) + 4*sin(b
*x + a))) + log(sin(b*x + a) + 1) - log(-sin(b*x + a) + 1))/b
```

3.65.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \sec(a+bx)\sec(2(a+bx))\,dx = -\frac{\operatorname{atanh}(\sin(a+b\,x)) - \sqrt{2}\operatorname{atanh}\left(\sqrt{2}\,\sin(a+b\,x)\right)}{b}$$

input
```
int(1/(cos(a + b*x)*cos(2*a + 2*b*x)),x)
```

output
```
-(atanh(sin(a + b*x)) - 2^(1/2)*atanh(2^(1/2)*sin(a + b*x)))/b
```

3.66 $\int \sin(x)\sin(2x)\,dx$

3.66.1 Optimal result

Integrand size = 7, antiderivative size = 15

$$\int \sin(x)\sin(2x)\,dx = \frac{\sin(x)}{2} - \frac{1}{6}\sin(3x)$$

output
```
1/2*sin(x)-1/6*sin(3*x)
```

3.66.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \sin(x)\sin(2x)\,dx = \frac{\sin(x)}{2} - \frac{1}{6}\sin(3x)$$

input
```
Integrate[Sin[x]*Sin[2*x],x]
```

output
```
Sin[x]/2 - Sin[3*x]/6
```

3.66.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 4770}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\sin(2x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x)\sin(2x)dx$$

$$\Big\downarrow \text{4770}$$

$$\frac{\sin(x)}{2}-\frac{1}{6}\sin(3x)$$

input
```
Int[Sin[x]*Sin[2*x],x]
```

output
```
Sin[x]/2 - Sin[3*x]/6
```

3.66.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4770
```
Int[sin[(a_.) + (b_.)*(x_)]*sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

3.66.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{\sin(x)}{2}-\frac{\sin(3x)}{6}$	12
risch	$\frac{\sin(x)}{2}-\frac{\sin(3x)}{6}$	12
parallelrisch	$\frac{\sin(x)}{2}-\frac{\sin(3x)}{6}$	12
norman	$\frac{-\frac{2\tan(x)\tan\left(\frac{x}{2}\right)^{2}}{3}+\frac{4\tan(x)^{2}\tan\left(\frac{x}{2}\right)}{3}+\frac{2\tan(x)}{3}-\frac{4\tan\left(\frac{x}{2}\right)}{3}}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)\left(1+\tan(x)^{2}\right)}$	51

input
```
int(sin(x)*sin(2*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*sin(x)-1/6*sin(3*x)
```

3.66.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.67

$$\int \sin(x)\sin(2x)\,dx = -\frac{2}{3}\left(\cos\left(x\right)^{2}-1\right)\sin\left(x\right)$$

input
```
integrate(sin(x)*sin(2*x),x, algorithm="fricas")
```

output
```
-2/3*(cos(x)^2 - 1)*sin(x)
```

3.66.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \sin(x)\sin(2x)\,dx = -\frac{2\sin\left(x\right)\cos\left(2x\right)}{3}+\frac{\sin\left(2x\right)\cos\left(x\right)}{3}$$

input
```
integrate(sin(x)*sin(2*x),x)
```

output
```
-2*sin(x)*cos(2*x)/3 + sin(2*x)*cos(x)/3
```

3.66.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \sin(x)\sin(2x)\,dx = -\frac{1}{6}\sin(3x) + \frac{1}{2}\sin(x)$$

input
```
integrate(sin(x)*sin(2*x),x, algorithm="maxima")
```

output
```
-1/6*sin(3*x) + 1/2*sin(x)
```

3.66.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.40

$$\int \sin(x)\sin(2x)\,dx = \frac{2}{3}\sin(x)^3$$

input
```
integrate(sin(x)*sin(2*x),x, algorithm="giac")
```

output
```
2/3*sin(x)^3
```

3.66.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.40

$$\int \sin(x)\sin(2x)\,dx = \frac{2\sin(x)^3}{3}$$

input
```
int(sin(2*x)*sin(x),x)
```

output
```
(2*sin(x)^3)/3
```

3.67 $\int \sin(x)\sin(3x)\,dx$

3.67.1 Optimal result

Integrand size = 7, antiderivative size = 17

$$\int \sin(x)\sin(3x)\,dx = \frac{1}{4}\sin(2x) - \frac{1}{8}\sin(4x)$$

output
```
1/4*sin(2*x)-1/8*sin(4*x)
```

3.67.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \sin(x)\sin(3x)\,dx = \frac{1}{4}\sin(2x) - \frac{1}{8}\sin(4x)$$

input
```
Integrate[Sin[x]*Sin[3*x],x]
```

output
```
Sin[2*x]/4 - Sin[4*x]/8
```

3.67.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 4770}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\sin(3x)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sin(x)\sin(3x)dx$$

$$\Big\downarrow \ 4770$$

$$\frac{1}{4}\sin(2x)-\frac{1}{8}\sin(4x)$$

input
```
Int[Sin[x]*Sin[3*x],x]
```

output
```
Sin[2*x]/4 - Sin[4*x]/8
```

3.67.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4770
```
Int[sin[(a_.) + (b_.)*(x_)]*sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

3.67.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{\sin(2x)}{4}-\frac{\sin(4x)}{8}$	14
risch	$\frac{\sin(2x)}{4}-\frac{\sin(4x)}{8}$	14
parallelrisch	$\frac{\sin(2x)}{4}-\frac{\sin(4x)}{8}$	14
norman	$\frac{\frac{3\tan\left(\frac{x}{2}\right)\tan\left(\frac{3x}{2}\right)^{2}}{4}-\frac{\tan\left(\frac{x}{2}\right)^{2}\tan\left(\frac{3x}{2}\right)}{4}-\frac{3\tan\left(\frac{x}{2}\right)}{4}+\frac{\tan\left(\frac{3x}{2}\right)}{4}}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)\left(1+\tan\left(\frac{3x}{2}\right)^{2}\right)}$	59

input
```
int(sin(x)*sin(3*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*sin(2*x)-1/8*sin(4*x)
```

3.67.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \sin(x)\sin(3x)\,dx = -\left(\cos\left(x\right)^{3}-\cos\left(x\right)\right)\sin\left(x\right)$$

input
```
integrate(sin(x)*sin(3*x),x, algorithm="fricas")
```

output
```
-(cos(x)^3 - cos(x))*sin(x)
```

3.67.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \sin(x)\sin(3x)\,dx = -\frac{3\sin\left(x\right)\cos\left(3x\right)}{8}+\frac{\sin\left(3x\right)\cos\left(x\right)}{8}$$

input
```
integrate(sin(x)*sin(3*x),x)
```

output
```
-3*sin(x)*cos(3*x)/8 + sin(3*x)*cos(x)/8
```

3.67.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \sin(x)\sin(3x)\,dx = -\frac{1}{8}\sin(4x) + \frac{1}{4}\sin(2x)$$

input
```
integrate(sin(x)*sin(3*x),x, algorithm="maxima")
```

output
```
-1/8*sin(4*x) + 1/4*sin(2*x)
```

3.67.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \sin(x)\sin(3x)\,dx = -\frac{1}{8}\sin(4x) + \frac{1}{4}\sin(2x)$$

input
```
integrate(sin(x)*sin(3*x),x, algorithm="giac")
```

output
```
-1/8*sin(4*x) + 1/4*sin(2*x)
```

3.67.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \sin(x)\sin(3x)\,dx = \frac{\sin(2x)}{4} - \frac{\sin(4x)}{8}$$

input
```
int(sin(3*x)*sin(x),x)
```

output
```
sin(2*x)/4 - sin(4*x)/8
```

3.68 $\int \sin(x)\sin(4x)\,dx$

3.68.1 Optimal result

Integrand size = 7, antiderivative size = 17

$$\int \sin(x)\sin(4x)\,dx = \frac{1}{6}\sin(3x) - \frac{1}{10}\sin(5x)$$

output
```
1/6*sin(3*x)-1/10*sin(5*x)
```

3.68.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \sin(x)\sin(4x)\,dx = \frac{1}{6}\sin(3x) - \frac{1}{10}\sin(5x)$$

input
```
Integrate[Sin[x]*Sin[4*x],x]
```

output
```
Sin[3*x]/6 - Sin[5*x]/10
```

3.68.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 4770}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\sin(4x)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin(x)\sin(4x)dx$$

$$\Big\downarrow \; 4770$$

$$\frac{1}{6}\sin(3x)-\frac{1}{10}\sin(5x)$$

input
```
Int[Sin[x]*Sin[4*x],x]
```

output
```
Sin[3*x]/6 - Sin[5*x]/10
```

3.68.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4770
```
Int[sin[(a_.) + (b_.)*(x_)]*sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

3.68.4 Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{\sin(3x)}{6}-\frac{\sin(5x)}{10}$	14
risch	$\frac{\sin(3x)}{6}-\frac{\sin(5x)}{10}$	14
parallelrisch	$\frac{\sin(3x)}{6}-\frac{\sin(5x)}{10}$	14
norman	$\frac{-\frac{2\tan(2x)\tan\left(\frac{x}{2}\right)^2}{15}+\frac{8\tan(2x)^2\tan\left(\frac{x}{2}\right)}{15}+\frac{2\tan(2x)}{15}-\frac{8\tan\left(\frac{x}{2}\right)}{15}}{\left(1+\tan\left(\frac{x}{2}\right)^2\right)\left(1+\tan(2x)^2\right)}$	59

input
```
int(sin(x)*sin(4*x),x,method=_RETURNVERBOSE)
```

output
```
1/6*sin(3*x)-1/10*sin(5*x)
```

3.68.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int \sin(x)\sin(4x)\,dx = -\frac{4}{15}\left(6\cos(x)^4-7\cos(x)^2+1\right)\sin(x)$$

input
```
integrate(sin(x)*sin(4*x),x, algorithm="fricas")
```

output
```
-4/15*(6*cos(x)^4 - 7*cos(x)^2 + 1)*sin(x)
```

3.68.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \sin(x)\sin(4x)\,dx = -\frac{4\sin(x)\cos(4x)}{15}+\frac{\sin(4x)\cos(x)}{15}$$

input
```
integrate(sin(x)*sin(4*x),x)
```

output
```
-4*sin(x)*cos(4*x)/15 + sin(4*x)*cos(x)/15
```

3.68.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \sin(x)\sin(4x)\,dx = -\frac{1}{10}\sin(5x) + \frac{1}{6}\sin(3x)$$

input
```
integrate(sin(x)*sin(4*x),x, algorithm="maxima")
```

output
```
-1/10*sin(5*x) + 1/6*sin(3*x)
```

3.68.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \sin(x)\sin(4x)\,dx = -\frac{8}{5}\sin(x)^5 + \frac{4}{3}\sin(x)^3$$

input
```
integrate(sin(x)*sin(4*x),x, algorithm="giac")
```

output
```
-8/5*sin(x)^5 + 4/3*sin(x)^3
```

3.68.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \sin(x)\sin(4x)\,dx = \frac{\sin(3x)}{6} - \frac{\sin(5x)}{10}$$

input
```
int(sin(4*x)*sin(x),x)
```

output
```
sin(3*x)/6 - sin(5*x)/10
```

3.69 $\int \sin(x)\sin(mx)\,dx$

3.69.1 Optimal result

Integrand size = 7, antiderivative size = 35

$$\int \sin(x)\sin(mx)\,dx = \frac{\sin((1-m)x)}{2(1-m)} - \frac{\sin((1+m)x)}{2(1+m)}$$

output
```
1/2*sin((1-m)*x)/(1-m)-1/2*sin((1+m)*x)/(1+m)
```

3.69.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

$$\int \sin(x)\sin(mx)\,dx = \frac{-m\cos(mx)\sin(x)+\cos(x)\sin(mx)}{-1+m^2}$$

input
```
Integrate[Sin[x]*Sin[m*x],x]
```

output
```
(-(m*Cos[m*x]*Sin[x]) + Cos[x]*Sin[m*x])/(-1 + m^2)
```

3.69.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5080, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\sin(mx)\,dx$$

$$\Big\downarrow \text{5080}$$

$$\int \left(\frac{1}{2}\cos((1-m)x)-\frac{1}{2}\cos((m+1)x)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sin((1-m)x)}{2(1-m)}-\frac{\sin((m+1)x)}{2(m+1)}$$

input
```
Int[Sin[x]*Sin[m*x],x]
```

output
```
Sin[(1 - m)*x]/(2*(1 - m)) - Sin[(1 + m)*x]/(2*(1 + m))
```

3.69.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5080
```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

3.69.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{\sin(x(-1+m))}{-2+2m}-\frac{\sin((1+m)x)}{2(1+m)}$	28
parallelrisch	$\frac{(1-m)\sin((1+m)x)+(1+m)\sin(x(-1+m))}{2m^2-2}$	34
risch	$\frac{\sin(x(-1+m))}{-2+2m}+\frac{\sin((1+m)x)}{2(-1+m)(1+m)}-\frac{\sin((1+m)x)m}{2(-1+m)(1+m)}$	52
norman	$\frac{\frac{2\tan\left(\frac{mx}{2}\right)}{m^2-1}-\frac{2m\tan\left(\frac{x}{2}\right)}{m^2-1}-\frac{2\tan\left(\frac{x}{2}\right)^2\tan\left(\frac{mx}{2}\right)}{m^2-1}+\frac{2m\tan\left(\frac{x}{2}\right)\tan\left(\frac{mx}{2}\right)^2}{m^2-1}}{\left(1+\tan\left(\frac{x}{2}\right)^2\right)\left(1+\tan\left(\frac{mx}{2}\right)^2\right)}$	93

input
```
int(sin(x)*sin(m*x),x,method=_RETURNVERBOSE)
```

output
```
1/2/(-1+m)*sin(x*(-1+m))-1/2*sin((1+m)*x)/(1+m)
```

3.69.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

$$\int \sin(x)\sin(mx)\,dx = -\frac{m\cos(mx)\sin(x)-\cos(x)\sin(mx)}{m^2-1}$$

input
```
integrate(sin(x)*sin(m*x),x, algorithm="fricas")
```

output
```
-(m*cos(m*x)*sin(x) - cos(x)*sin(m*x))/(m^2 - 1)
```

3.69.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(22) = 44$.

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.23

$$\int \sin(x)\sin(mx)\,dx = \begin{cases} -\frac{x\sin^2(x)}{2}-\frac{x\cos^2(x)}{2}+\frac{\sin(x)\cos(x)}{2} & \text{for } m=-1 \\ \frac{x\sin^2(x)}{2}+\frac{x\cos^2(x)}{2}-\frac{\sin(x)\cos(x)}{2} & \text{for } m=1 \\ -\frac{m\sin(x)\cos(mx)}{m^2-1}+\frac{\sin(mx)\cos(x)}{m^2-1} & \text{otherwise} \end{cases}$$

input
```
integrate(sin(x)*sin(m*x),x)
```

output
```
Piecewise((-x*sin(x)**2/2 - x*cos(x)**2/2 + sin(x)*cos(x)/2, Eq(m, -1)), (
x*sin(x)**2/2 + x*cos(x)**2/2 - sin(x)*cos(x)/2, Eq(m, 1)), (-m*sin(x)*cos
(m*x)/(m**2 - 1) + sin(m*x)*cos(x)/(m**2 - 1), True))
```

3.69.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

$$\int \sin(x)\sin(mx)\,dx = -\frac{\sin\left((m+1)x\right)}{2\,(m+1)} - \frac{\sin\left(-(m-1)x\right)}{2\,(m-1)}$$

input
```
integrate(sin(x)*sin(m*x),x, algorithm="maxima")
```

output
```
-1/2*sin((m + 1)*x)/(m + 1) - 1/2*sin(-(m - 1)*x)/(m - 1)
```

3.69.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \sin(x)\sin(mx)\,dx = -\frac{\sin\left(mx+x\right)}{2\,(m+1)} + \frac{\sin\left(mx-x\right)}{2\,(m-1)}$$

input
```
integrate(sin(x)*sin(m*x),x, algorithm="giac")
```

output
```
-1/2*sin(m*x + x)/(m + 1) + 1/2*sin(m*x - x)/(m - 1)
```

3.69.9 Mupad [B] (verification not implemented)

Time = 26.43 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.83

$$\int \sin(x)\sin(mx)\,dx = \begin{cases} \frac{x}{2} - \frac{\sin(2\,x)}{4} & \text{if } m = 1 \\ \frac{\sin(2\,x)}{4} - \frac{x}{2} & \text{if } m = -1 \\ \frac{\sin(x\,(m-1))}{2\,m-2} - \frac{\sin(x\,(m+1))}{2\,m+2} & \text{if } m \neq -1 \land m \neq 1 \end{cases}$$

input
```
int(sin(m*x)*sin(x),x)
```

output
```
piecewise(m == 1, x/2 - sin(2*x)/4, m == -1, - x/2 + sin(2*x)/4, m ~= -1 &
 m ~= 1, sin(x*(m - 1))/(2*m - 2) - sin(x*(m + 1))/(2*m + 2))
```

3.69. $\int \sin(x)\sin(mx)\,dx$

3.70 $\int \cos(2x)\sin(x)\,dx$

3.70.1 Optimal result

Integrand size = 7, antiderivative size = 15

$$\int \cos(2x)\sin(x)\,dx = \frac{\cos(x)}{2} - \frac{1}{6}\cos(3x)$$

output
```
1/2*cos(x)-1/6*cos(3*x)
```

3.70.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cos(2x)\sin(x)\,dx = \frac{\cos(x)}{2} - \frac{1}{6}\cos(3x)$$

input
```
Integrate[Cos[2*x]*Sin[x],x]
```

output
```
Cos[x]/2 - Cos[3*x]/6
```

3.70.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 4772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\cos(2x)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin(x)\cos(2x)dx$$

$$\Big\downarrow \; 4772$$

$$\frac{\cos(x)}{2}-\frac{1}{6}\cos(3x)$$

input
```
Int[Cos[2*x]*Sin[x],x]
```

output
```
Cos[x]/2 - Cos[3*x]/6
```

3.70.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4772
```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[-Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

3.70.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{\cos(x)}{2}-\frac{\cos(3x)}{6}$	12
risch	$\frac{\cos(x)}{2}-\frac{\cos(3x)}{6}$	12
parallelrisch	$-\frac{1}{3}+\frac{\cos(x)}{2}-\frac{\cos(3x)}{6}$	13
norman	$\frac{-\frac{2\tan(x)^2}{3}-\frac{2\tan(\frac{x}{2})^2}{3}+\frac{8\tan(\frac{x}{2})\tan(x)}{3}}{\left(1+\tan(\frac{x}{2})^2\right)\left(1+\tan(x)^2\right)}$	43

input
```
int(cos(2*x)*sin(x),x,method=_RETURNVERBOSE)
```

output
```
1/2*cos(x)-1/6*cos(3*x)
```

3.70.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.60

$$\int \cos(2x)\sin(x)\,dx = -\frac{2}{3}\cos(x)^3 + \cos(x)$$

input
```
integrate(cos(2*x)*sin(x),x, algorithm="fricas")
```

output
```
-2/3*cos(x)^3 + cos(x)
```

3.70.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \cos(2x)\sin(x)\,dx = \frac{2\sin(x)\sin(2x)}{3} + \frac{\cos(x)\cos(2x)}{3}$$

input
```
integrate(cos(2*x)*sin(x),x)
```

output
```
2*sin(x)*sin(2*x)/3 + cos(x)*cos(2*x)/3
```

3.70.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \cos(2x)\sin(x)\,dx = -\frac{1}{6}\cos(3x) + \frac{1}{2}\cos(x)$$

input
```
integrate(cos(2*x)*sin(x),x, algorithm="maxima")
```

output
```
-1/6*cos(3*x) + 1/2*cos(x)
```

3.70.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \cos(2x)\sin(x)\,dx = -\frac{1}{6}\cos(3x) + \frac{1}{2}\cos(x)$$

input
```
integrate(cos(2*x)*sin(x),x, algorithm="giac")
```

output
```
-1/6*cos(3*x) + 1/2*cos(x)
```

3.70.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.60

$$\int \cos(2x)\sin(x)\,dx = \cos(x) - \frac{2\cos(x)^3}{3}$$

input
```
int(cos(2*x)*sin(x),x)
```

output
```
cos(x) - (2*cos(x)^3)/3
```

3.71 $\int \cos(3x)\sin(x)\,dx$

3.71.1 Optimal result

Integrand size = 7, antiderivative size = 17

$$\int \cos(3x)\sin(x)\,dx = \frac{1}{4}\cos(2x) - \frac{1}{8}\cos(4x)$$

output
```
1/4*cos(2*x)-1/8*cos(4*x)
```

3.71.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cos(3x)\sin(x)\,dx = \frac{\cos^2(x)}{2} - \frac{1}{8}\cos(4x)$$

input
```
Integrate[Cos[3*x]*Sin[x],x]
```

output
```
Cos[x]^2/2 - Cos[4*x]/8
```

3.71.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 4772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\cos(3x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x)\cos(3x)dx$$

$$\Big\downarrow \text{4772}$$

$$\frac{1}{4}\cos(2x)-\frac{1}{8}\cos(4x)$$

input
```
Int[Cos[3*x]*Sin[x],x]
```

output
```
Cos[2*x]/4 - Cos[4*x]/8
```

3.71.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4772
```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[-Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

3.71.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{\cos(2x)}{4}-\frac{\cos(4x)}{8}$	14
risch	$\frac{\cos(2x)}{4}-\frac{\cos(4x)}{8}$	14
parallelrisch	$-\frac{\cos(4x)}{8}-\frac{1}{8}+\frac{\cos(2x)}{4}$	15
norman	$\frac{-\frac{\tan\left(\frac{x}{2}\right)^{2}}{4}-\frac{\tan\left(\frac{3x}{2}\right)^{2}}{4}+\frac{3\tan\left(\frac{x}{2}\right)\tan\left(\frac{3x}{2}\right)}{2}}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)\left(1+\tan\left(\frac{3x}{2}\right)^{2}\right)}$	49

input
```
int(cos(3*x)*sin(x),x,method=_RETURNVERBOSE)
```

output
```
1/4*cos(2*x)-1/8*cos(4*x)
```

3.71.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(3x)\sin(x)\,dx = -\cos\left(x\right)^{4} + \frac{3}{2}\cos\left(x\right)^{2}$$

input
```
integrate(cos(3*x)*sin(x),x, algorithm="fricas")
```

output
```
-cos(x)^4 + 3/2*cos(x)^2
```

3.71.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \cos(3x)\sin(x)\,dx = \frac{3\sin\left(x\right)\sin\left(3x\right)}{8} + \frac{\cos\left(x\right)\cos\left(3x\right)}{8}$$

input
```
integrate(cos(3*x)*sin(x),x)
```

output
```
3*sin(x)*sin(3*x)/8 + cos(x)*cos(3*x)/8
```

3.71.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(3x)\sin(x)\,dx = -\frac{1}{8}\cos(4x) + \frac{1}{4}\cos(2x)$$

input
```
integrate(cos(3*x)*sin(x),x, algorithm="maxima")
```

output
```
-1/8*cos(4*x) + 1/4*cos(2*x)
```

3.71.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(3x)\sin(x)\,dx = -\sin(x)^4 + \frac{1}{2}\sin(x)^2$$

input
```
integrate(cos(3*x)*sin(x),x, algorithm="giac")
```

output
```
-sin(x)^4 + 1/2*sin(x)^2
```

3.71.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(3x)\sin(x)\,dx = \frac{3\cos(x)^2}{2} - \cos(x)^4$$

input
```
int(cos(3*x)*sin(x),x)
```

output
```
(3*cos(x)^2)/2 - cos(x)^4
```

3.72 $\int \cos(4x)\sin(x)\,dx$

3.72.1 Optimal result

Integrand size = 7, antiderivative size = 17

$$\int \cos(4x)\sin(x)\,dx = \frac{1}{6}\cos(3x) - \frac{1}{10}\cos(5x)$$

output
```
1/6*cos(3*x)-1/10*cos(5*x)
```

3.72.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cos(4x)\sin(x)\,dx = \frac{1}{6}\cos(3x) - \frac{1}{10}\cos(5x)$$

input
```
Integrate[Cos[4*x]*Sin[x],x]
```

output
```
Cos[3*x]/6 - Cos[5*x]/10
```

3.72.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\cos(4x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(x)\cos(4x)dx$$

$$\Big\downarrow \text{ 4772}$$

$$\frac{1}{6}\cos(3x) - \frac{1}{10}\cos(5x)$$

input
```
Int[Cos[4*x]*Sin[x],x]
```

output
```
Cos[3*x]/6 - Cos[5*x]/10
```

3.72.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4772
```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[-Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

3.72.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{\cos(3x)}{6}-\frac{\cos(5x)}{10}$	14
risch	$\frac{\cos(3x)}{6}-\frac{\cos(5x)}{10}$	14
parallelrisch	$-\frac{1}{15}+\frac{\cos(3x)}{6}-\frac{\cos(5x)}{10}$	15
norman	$\frac{-\frac{2\tan(2x)^2}{15}-\frac{2\tan\left(\frac{x}{2}\right)^2}{15}+\frac{16\tan\left(\frac{x}{2}\right)\tan(2x)}{15}}{\left(1+\tan\left(\frac{x}{2}\right)^2\right)\left(1+\tan(2x)^2\right)}$	49

input
```
int(cos(4*x)*sin(x),x,method=_RETURNVERBOSE)
```

output
```
1/6*cos(3*x)-1/10*cos(5*x)
```

3.72.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cos(4x)\sin(x)\,dx = -\frac{8}{5}\cos(x)^5+\frac{8}{3}\cos(x)^3-\cos(x)$$

input
```
integrate(cos(4*x)*sin(x),x, algorithm="fricas")
```

output
```
-8/5*cos(x)^5 + 8/3*cos(x)^3 - cos(x)
```

3.72.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \cos(4x)\sin(x)\,dx = \frac{4\sin(x)\sin(4x)}{15}+\frac{\cos(x)\cos(4x)}{15}$$

input
```
integrate(cos(4*x)*sin(x),x)
```

output
```
4*sin(x)*sin(4*x)/15 + cos(x)*cos(4*x)/15
```

3.72.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(4x)\sin(x)\,dx = -\frac{1}{10}\cos(5x) + \frac{1}{6}\cos(3x)$$

input
```
integrate(cos(4*x)*sin(x),x, algorithm="maxima")
```

output
```
-1/10*cos(5*x) + 1/6*cos(3*x)
```

3.72.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(4x)\sin(x)\,dx = -\frac{1}{10}\cos(5x) + \frac{1}{6}\cos(3x)$$

input
```
integrate(cos(4*x)*sin(x),x, algorithm="giac")
```

output
```
-1/10*cos(5*x) + 1/6*cos(3*x)
```

3.72.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cos(4x)\sin(x)\,dx = -\frac{8\cos(x)^5}{5} + \frac{8\cos(x)^3}{3} - \cos(x)$$

input
```
int(cos(4*x)*sin(x),x)
```

output
```
(8*cos(x)^3)/3 - cos(x) - (8*cos(x)^5)/5
```

3.73 $\int \cos(mx)\sin(x)\,dx$

3.73.1 Optimal result

Integrand size = 7, antiderivative size = 35

$$\int \cos(mx)\sin(x)\,dx = -\frac{\cos((1-m)x)}{2(1-m)} - \frac{\cos((1+m)x)}{2(1+m)}$$

output
```
-1/2*cos((1-m)*x)/(1-m)-1/2*cos((1+m)*x)/(1+m)
```

3.73.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.69

$$\int \cos(mx)\sin(x)\,dx = \frac{\cos(x)\cos(mx) + m\sin(x)\sin(mx)}{-1+m^2}$$

input
```
Integrate[Cos[m*x]*Sin[x],x]
```

output
```
(Cos[x]*Cos[m*x] + m*Sin[x]*Sin[m*x])/(-1 + m^2)
```

3.73.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5085, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\cos(mx)\,dx$$

$$\Big\downarrow \text{5085}$$

$$\int \left(\frac{1}{2}\sin((1-m)x)+\frac{1}{2}\sin((m+1)x)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\cos((1-m)x)}{2(1-m)}-\frac{\cos((m+1)x)}{2(m+1)}$$

input
```
Int[Cos[m*x]*Sin[x],x]
```

output
```
-1/2*Cos[(1 - m)*x]/(1 - m) - Cos[(1 + m)*x]/(2*(1 + m))
```

3.73.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5085
```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

3.73.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{\cos(x(-1+m))}{-2+2m}-\frac{\cos((1+m)x)}{2(1+m)}$	28
parallelrisch	$\frac{(1+m)\cos(x(-1+m))-2+(1-m)\cos((1+m)x)}{2m^2-2}$	35
risch	$\frac{\cos(x(-1+m))}{-2+2m}+\frac{\cos((1+m)x)}{2(1+m)(-1+m)}-\frac{\cos((1+m)x)m}{2(1+m)(-1+m)}$	52
norman	$\frac{-\frac{2\tan(\frac{x}{2})^2}{m^2-1}-\frac{2\tan(\frac{mx}{2})^2}{m^2-1}+\frac{4m\tan(\frac{x}{2})\tan(\frac{mx}{2})}{m^2-1}}{\left(1+\tan(\frac{x}{2})^2\right)\left(1+\tan(\frac{mx}{2})^2\right)}$	74

input
```
int(cos(m*x)*sin(x),x,method=_RETURNVERBOSE)
```

output
```
1/2/(-1+m)*cos(x*(-1+m))-1/2*cos((1+m)*x)/(1+m)
```

3.73.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.69

$$\int \cos(mx)\sin(x)\,dx = \frac{m\sin(mx)\sin(x)+\cos(mx)\cos(x)}{m^2-1}$$

input
```
integrate(cos(m*x)*sin(x),x, algorithm="fricas")
```

output
```
(m*sin(m*x)*sin(x) + cos(m*x)*cos(x))/(m^2 - 1)
```

3.73.6 Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \cos(mx)\sin(x)\,dx = \begin{cases} \frac{\sin^2(x)}{2} & \text{for } m=-1 \vee m=1 \\ \frac{m\sin(x)\sin(mx)}{m^2-1}+\frac{\cos(x)\cos(mx)}{m^2-1} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(m*x)*sin(x),x)
```

output
```
Piecewise((sin(x)**2/2, Eq(m, -1) | Eq(m, 1)), (m*sin(x)*sin(m*x)/(m**2 -
1) + cos(x)*cos(m*x)/(m**2 - 1), True))
```

3.73.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

$$\int \cos(mx)\sin(x)\,dx = -\frac{\cos\left((m+1)x\right)}{2\,(m+1)} + \frac{\cos\left(-(m-1)x\right)}{2\,(m-1)}$$

input
```
integrate(cos(m*x)*sin(x),x, algorithm="maxima")
```

output
```
-1/2*cos((m + 1)*x)/(m + 1) + 1/2*cos(-(m - 1)*x)/(m - 1)
```

3.73.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \cos(mx)\sin(x)\,dx = -\frac{\cos\left(mx+x\right)}{2\,(m+1)} + \frac{\cos\left(mx-x\right)}{2\,(m-1)}$$

input
```
integrate(cos(m*x)*sin(x),x, algorithm="giac")
```

output
```
-1/2*cos(m*x + x)/(m + 1) + 1/2*cos(m*x - x)/(m - 1)
```

3.73.9 Mupad [B] (verification not implemented)

Time = 26.46 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.57

$$\int \cos(mx)\sin(x)\,dx = \begin{cases} \frac{\sin(x)^2}{2} & \text{if}\ \ m=-1\vee m=1 \\ \frac{\cos(x\,(m-1))}{2\,m-2} - \frac{\cos(x\,(m+1))}{2\,m+2} & \text{if}\ \ m\neq -1\wedge m\neq 1 \end{cases}$$

input
```
int(cos(m*x)*sin(x),x)
```

output
```
piecewise(m == -1 | m == 1, sin(x)^2/2, m ~= -1 & m ~= 1, cos(x*(m - 1))/(
2*m - 2) - cos(x*(m + 1))/(2*m + 2))
```

3.74 $\int \sin(x)\tan(2x)\,dx$

3.74.1 Optimal result

Integrand size = 7, antiderivative size = 20

$$\int \sin(x)\tan(2x)\,dx = \frac{\operatorname{arctanh}\left(\sqrt{2}\sin(x)\right)}{\sqrt{2}} - \sin(x)$$

output
```
-sin(x)+1/2*arctanh(sin(x)*2^(1/2))*2^(1/2)
```

3.74.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \sin(x)\tan(2x)\,dx = \frac{\operatorname{arctanh}\left(\sqrt{2}\sin(x)\right)}{\sqrt{2}} - \sin(x)$$

input
```
Integrate[Sin[x]*Tan[2*x],x]
```

output
```
ArcTanh[Sqrt[2]*Sin[x]]/Sqrt[2] - Sin[x]
```

3.74.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3042, 4878, 27, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\tan(2x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x)\tan(2x)dx$$

$$\Big\downarrow \text{4878}$$

$$\int \frac{2\sin^2(x)}{1-2\sin^2(x)}d\sin(x)$$

$$\Big\downarrow \text{27}$$

$$2\int \frac{\sin^2(x)}{1-2\sin^2(x)}d\sin(x)$$

$$\Big\downarrow \text{262}$$

$$2\left(\frac{1}{2}\int \frac{1}{1-2\sin^2(x)}d\sin(x)-\frac{\sin(x)}{2}\right)$$

$$\Big\downarrow \text{219}$$

$$2\left(\frac{\text{arctanh}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}}-\frac{\sin(x)}{2}\right)$$

input
```
Int[Sin[x]*Tan[2*x],x]
```

output
```
2*(ArcTanh[Sqrt[2]*Sin[x]]/(2*Sqrt[2]) - Sin[x]/2)
```

3.74.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

3.74.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$-\sin\left(x\right)+\frac{\operatorname{arctanh}\left(\sin\left(x\right)\sqrt{2}\right)\sqrt{2}}{2}$	18
default	$-\sin\left(x\right)+\frac{\operatorname{arctanh}\left(\sin\left(x\right)\sqrt{2}\right)\sqrt{2}}{2}$	18
risch	$\frac{i\mathrm{e}^{ix}}{2}-\frac{i\mathrm{e}^{-ix}}{2}-\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}-i\sqrt{2}\,\mathrm{e}^{ix}-1\right)}{4}+\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}+i\sqrt{2}\,\mathrm{e}^{ix}-1\right)}{4}$	66

input
```
int(sin(x)*tan(2*x),x,method=_RETURNVERBOSE)
```

output
```
-sin(x)+1/2*arctanh(sin(x)*2^(1/2))*2^(1/2)
```

3.74.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(17) = 34.

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.90

$$\int \sin(x)\tan(2x)\,dx = \frac{1}{4}\sqrt{2}\log\left(-\frac{2\cos(x)^2 - 2\sqrt{2}\sin(x) - 3}{2\cos(x)^2 - 1}\right) - \sin(x)$$

input
```
integrate(sin(x)*tan(2*x),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*log(-(2*cos(x)^2 - 2*sqrt(2)*sin(x) - 3)/(2*cos(x)^2 - 1)) - s
in(x)
```

3.74.6 Sympy [F]

$$\int \sin(x)\tan(2x)\,dx = \int \sin(x)\tan(2x)\,dx$$

input
```
integrate(sin(x)*tan(2*x),x)
```

output
```
Integral(sin(x)*tan(2*x), x)
```

3.74.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(17) = 34.

Time = 0.29 (sec) , antiderivative size = 141, normalized size of antiderivative = 7.05

$$\begin{aligned}\int \sin(x)\tan(2x)\,dx = {} & \frac{1}{8}\sqrt{2}\log\left(2\cos(x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) + 2\sqrt{2}\sin(x) + 2\right) \\ & - \frac{1}{8}\sqrt{2}\log\left(2\cos(x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) - 2\sqrt{2}\sin(x) + 2\right) \\ & + \frac{1}{8}\sqrt{2}\log\left(2\cos(x)^2 + 2\sin(x)^2 - 2\sqrt{2}\cos(x) + 2\sqrt{2}\sin(x) + 2\right) \\ & - \frac{1}{8}\sqrt{2}\log\left(2\cos(x)^2 + 2\sin(x)^2 - 2\sqrt{2}\cos(x) - 2\sqrt{2}\sin(x) + 2\right) \\ & - \sin(x)\end{aligned}$$

input
```
integrate(sin(x)*tan(2*x),x, algorithm="maxima")
```

output
```
1/8*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*sqrt(2)*sin
(x) + 2) - 1/8*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) - 2*
sqrt(2)*sin(x) + 2) + 1/8*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)*
cos(x) + 2*sqrt(2)*sin(x) + 2) - 1/8*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 -
 2*sqrt(2)*cos(x) - 2*sqrt(2)*sin(x) + 2) - sin(x)
```

3.74.8 Giac [F]

$$\int \sin(x)\tan(2x)\,dx = \int \sin{(x)}\tan{(2\,x)}\;dx$$

input
```
integrate(sin(x)*tan(2*x),x, algorithm="giac")
```

output
```
integrate(sin(x)*tan(2*x), x)
```

3.74.9 Mupad [B] (verification not implemented)

Time = 25.64 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \sin(x)\tan(2x)\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\,\sin{(x)}\right)}{2} - \sin{(x)}$$

input
```
int(tan(2*x)*sin(x),x)
```

output
```
(2^(1/2)*atanh(2^(1/2)*sin(x)))/2 - sin(x)
```

3.75 $\int \sin(x)\tan(3x)\,dx$

3.75.1 Optimal result

Integrand size = 7, antiderivative size = 47

$$\int \sin(x)\tan(3x)\,dx = -\frac{1}{6}\log(1-2\sin(x)) - \frac{1}{6}\log(1-\sin(x)) + \frac{1}{6}\log(1+\sin(x)) + \frac{1}{6}\log(1+2\sin(x)) - \sin(x)$$

output
```
-1/6*ln(1-2*sin(x))-1/6*ln(1-sin(x))+1/6*ln(1+sin(x))+1/6*ln(1+2*sin(x))-s
in(x)
```

3.75.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.45

$$\int \sin(x)\tan(3x)\,dx = \frac{1}{3}\text{arctanh}(\sin(x)) + \frac{1}{3}\text{arctanh}(2\sin(x)) - \sin(x)$$

input
```
Integrate[Sin[x]*Tan[3*x],x]
```

output
```
ArcTanh[Sin[x]]/3 + ArcTanh[2*Sin[x]]/3 - Sin[x]
```

3.75.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.21, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 4878, 1602, 27, 1475, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\tan(3x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x)\tan(3x)dx$$

$$\Big\downarrow \text{4878}$$

$$\int \frac{\sin^2(x)\left(3-4\sin^2(x)\right)}{4\sin^4(x)-5\sin^2(x)+1}d\sin(x)$$

$$\Big\downarrow \text{1602}$$

$$-\frac{1}{4}\int -\frac{4\left(1-2\sin^2(x)\right)}{4\sin^4(x)-5\sin^2(x)+1}d\sin(x)-\sin(x)$$

$$\Big\downarrow \text{27}$$

$$\int \frac{1-2\sin^2(x)}{4\sin^4(x)-5\sin^2(x)+1}d\sin(x)-\sin(x)$$

$$\Big\downarrow \text{1475}$$

$$-\frac{1}{4}\int \frac{1}{\sin^2(x)-\frac{\sin(x)}{2}-\frac{1}{2}}d\sin(x)-\frac{1}{4}\int \frac{1}{\sin^2(x)+\frac{\sin(x)}{2}-\frac{1}{2}}d\sin(x)-\sin(x)$$

$$\Big\downarrow \text{1081}$$

$$-\frac{1}{4}\int \left(-\frac{2}{3(\sin(x)+1)}-\frac{4}{3(1-2\sin(x))}\right)d\sin(x)-\frac{1}{4}\int \left(-\frac{4}{3(2\sin(x)+1)}-\frac{2}{3(1-\sin(x))}\right)d\sin(x)-\sin(x)$$

$$\Big\downarrow \text{2009}$$

$$-\sin(x)+\frac{1}{4}\left(\frac{2}{3}\log(\sin(x)+1)-\frac{2}{3}\log(1-2\sin(x))\right)+\frac{1}{4}\left(\frac{2}{3}\log(2\sin(x)+1)-\frac{2}{3}\log(1-\sin(x))\right)$$

input
```
Int[Sin[x]*Tan[3*x],x]
```

output
```
((-2*Log[1 - 2*Sin[x]])/3 + (2*Log[1 + Sin[x]])/3)/4 + ((-2*Log[1 - Sin[x]
])/3 + (2*Log[1 + 2*Sin[x]])/3)/4 - Sin[x]
```

3.75.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

3.75.4 Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{\ln(1+2\sin(x))}{6}-\frac{\ln(\sin(x)-1)}{6}+\frac{\ln(1+\sin(x))}{6}-\frac{\ln(2\sin(x)-1)}{6}-\sin(x)$	38
risch	$\frac{i e^{ix}}{2}-\frac{i e^{-ix}}{2}-\frac{\ln(e^{ix}-i)}{3}+\frac{\ln(i+e^{ix})}{3}+\frac{\ln(i e^{ix}+e^{2ix}-1)}{6}-\frac{\ln(-i e^{ix}+e^{2ix}-1)}{6}$	76

input
```
int(sin(x)*tan(3*x),x,method=_RETURNVERBOSE)
```

output
```
1/6*ln(1+2*sin(x))-1/6*ln(sin(x)-1)+1/6*ln(1+sin(x))-1/6*ln(2*sin(x)-1)-si
n(x)
```

3.75.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \sin(x)\tan(3x)\,dx = \frac{1}{6}\log(2\sin(x)+1)+\frac{1}{6}\log(\sin(x)+1) - \frac{1}{6}\log(-\sin(x)+1)-\frac{1}{6}\log(-2\sin(x)+1)-\sin(x)$$

input
```
integrate(sin(x)*tan(3*x),x, algorithm="fricas")
```

output
```
1/6*log(2*sin(x) + 1) + 1/6*log(sin(x) + 1) - 1/6*log(-sin(x) + 1) - 1/6*l
og(-2*sin(x) + 1) - sin(x)
```

3.75.6 Sympy [F]

$$\int \sin(x)\tan(3x)\,dx = \int \sin{(x)}\tan{(3x)}\,dx$$

input
```
integrate(sin(x)*tan(3*x),x)
```

output
```
Integral(sin(x)*tan(3*x), x)
```

3.75.7 Maxima [F]

$$\int \sin(x)\tan(3x)\,dx = \int \sin{(x)}\tan{(3\,x)}\;dx$$

input
```
integrate(sin(x)*tan(3*x),x, algorithm="maxima")
```

output
```
integrate(-1/3*((cos(3*x) + cos(x))*cos(4*x) - (cos(2*x) - 1)*cos(3*x) - c
os(2*x)*cos(x) + (sin(3*x) + sin(x))*sin(4*x) - sin(3*x)*sin(2*x) - sin(2*
x)*sin(x) + cos(x))/(2*(cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - cos(2*x)^2 -
 sin(4*x)^2 + 2*sin(4*x)*sin(2*x) - sin(2*x)^2 + 2*cos(2*x) - 1), x) + 1/6
*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - 1/6*log(cos(x)^2 + sin(x)^2 - 2
*sin(x) + 1) - sin(x)
```

3.75.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(39) = 78$.

Time = 0.35 (sec) , antiderivative size = 364, normalized size of antiderivative = 7.74

$$\int \sin(x)\tan(3x)\,dx$$
$$= \frac{\log\left(\frac{\tan\left(\frac{1}{2}x\right)^4+8\tan\left(\frac{1}{2}x\right)^3+18\tan\left(\frac{1}{2}x\right)^2+8\tan\left(\frac{1}{2}x\right)+1}{\tan\left(\frac{1}{2}x\right)^4+2\tan\left(\frac{1}{2}x\right)^2+1}\right)\tan\left(\frac{1}{2}x\right)^2-\log\left(\frac{\tan\left(\frac{1}{2}x\right)^4-8\tan\left(\frac{1}{2}x\right)^3+18\tan\left(\frac{1}{2}x\right)^2-8\tan\left(\frac{1}{2}x\right)+}{\tan\left(\frac{1}{2}x\right)^4+2\tan\left(\frac{1}{2}x\right)^2+1}\right.}{}$$

input
```
integrate(sin(x)*tan(3*x),x, algorithm="giac")
```

output
```
1/12*(log((tan(1/2*x)^4 + 8*tan(1/2*x)^3 + 18*tan(1/2*x)^2 + 8*tan(1/2*x)
+ 1)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x)^2 - log((tan(1/2*x)^4
 - 8*tan(1/2*x)^3 + 18*tan(1/2*x)^2 - 8*tan(1/2*x) + 1)/(tan(1/2*x)^4 + 2*
tan(1/2*x)^2 + 1))*tan(1/2*x)^2 + 2*log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1
)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^2 - 2*log(2*(tan(1/2*x)^2 - 2*tan(1/2*x)
+ 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^2 + log((tan(1/2*x)^4 + 8*tan(1/2*x)^3
 + 18*tan(1/2*x)^2 + 8*tan(1/2*x) + 1)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1)
) - log((tan(1/2*x)^4 - 8*tan(1/2*x)^3 + 18*tan(1/2*x)^2 - 8*tan(1/2*x) +
1)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1)) + 2*log(2*(tan(1/2*x)^2 + 2*tan(1/
2*x) + 1)/(tan(1/2*x)^2 + 1)) - 2*log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/
(tan(1/2*x)^2 + 1)) - 24*tan(1/2*x))/(tan(1/2*x)^2 + 1)
```

3.75.9 Mupad [B] (verification not implemented)

Time = 26.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.55

$$\int \sin(x)\tan(3x)\,dx = \frac{2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{3} + \frac{\mathrm{atanh}(2\,\sin\left(x\right))}{3} - \sin\left(x\right)$$

input
```
int(tan(3*x)*sin(x),x)
```

output
```
(2*atanh(sin(x/2)/cos(x/2)))/3 + atanh(2*sin(x))/3 - sin(x)
```

3.76 $\int \sin(x)\tan(4x)\,dx$

3.76.1 Optimal result

Integrand size = 7, antiderivative size = 71

$$\int \sin(x)\tan(4x)\,dx = \frac{1}{4}\sqrt{2-\sqrt{2}}\text{arctanh}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right) + \frac{1}{4}\sqrt{2+\sqrt{2}}\text{arctanh}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{2}}}\right) - \sin(x)$$

output
```
-sin(x)+1/4*arctanh(2*sin(x)/(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)+1/4*arct
anh(2*sin(x)/(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)
```

3.76.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.97

$$\int \sin(x)\tan(4x)\,dx = \frac{1}{4}\left(\sqrt{2-\sqrt{2}}\text{arctanh}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right) + \sqrt{2+\sqrt{2}}\text{arctanh}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{2}}}\right) - 4\sin(x)\right)$$

input
```
Integrate[Sin[x]*Tan[4*x],x]
```

output
```
(Sqrt[2 - Sqrt[2]]*ArcTanh[(2*Sin[x])/Sqrt[2 - Sqrt[2]]] + Sqrt[2 + Sqrt[2
]]*ArcTanh[(2*Sin[x])/Sqrt[2 + Sqrt[2]]] - 4*Sin[x])/4
```

3.76.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 4878, 27, 1602, 27, 1480, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\tan(4x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x)\tan(4x)dx$$

$$\Big\downarrow \text{4878}$$

$$\int \frac{4\sin^2(x)\left(1-2\sin^2(x)\right)}{8\sin^4(x)-8\sin^2(x)+1}d\sin(x)$$

$$\Big\downarrow \text{27}$$

$$4\int \frac{\sin^2(x)\left(1-2\sin^2(x)\right)}{8\sin^4(x)-8\sin^2(x)+1}d\sin(x)$$

$$\Big\downarrow \text{1602}$$

$$4\left(-\frac{1}{8}\int -\frac{2(1-4\sin^2(x))}{8\sin^4(x)-8\sin^2(x)+1}d\sin(x)-\frac{\sin(x)}{4}\right)$$

$$\Big\downarrow \text{27}$$

$$4\left(\frac{1}{4}\int \frac{1-4\sin^2(x)}{8\sin^4(x)-8\sin^2(x)+1}d\sin(x)-\frac{\sin(x)}{4}\right)$$

$$\Big\downarrow \text{1480}$$

$$4\left(\frac{1}{4}\left(-\left(\left(2-\sqrt{2}\right)\int \frac{1}{8\sin^2(x)-2\left(2-\sqrt{2}\right)}d\sin(x)\right)-\left(2+\sqrt{2}\right)\int \frac{1}{8\sin^2(x)-2\left(2+\sqrt{2}\right)}d\sin(x)\right)-\frac{\sin(x)}{4}\right)$$

$$\Big\downarrow \text{220}$$

$$4\left(\frac{1}{4}\left(\frac{1}{4}\sqrt{2-\sqrt{2}}\text{arctanh}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right)+\frac{1}{4}\sqrt{2+\sqrt{2}}\text{arctanh}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{2}}}\right)\right)-\frac{\sin(x)}{4}\right)$$

input
```
Int[Sin[x]*Tan[4*x],x]
```

output
```
4*(((Sqrt[2 - Sqrt[2]]*ArcTanh[(2*Sin[x])/Sqrt[2 - Sqrt[2]]])/4 + (Sqrt[2
+ Sqrt[2]]*ArcTanh[(2*Sin[x])/Sqrt[2 + Sqrt[2]]])/4)/4 - Sin[x]/4)
```

3.76.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

3.76.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.21 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.77

method	result
risch	$\frac{i e^{ix}}{2}-\frac{i e^{-ix}}{2}-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(128\_Z^4-32\_Z^2+1\right)}\_R\ln\left(e^{2ix}-4i\_R\,e^{ix}-1\right)\right)}{2}$
default	$\frac{\sqrt{2}\,\sqrt{2+\sqrt{2}}\,\operatorname{arctanh}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{2}}}\right)}{4}+\frac{\left(\sqrt{2}-2\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2-\sqrt{2}}}-\sin(x)-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2+\sqrt{2}}}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{2\sin}{\sqrt{2-}}\right.}{4\sqrt{2-\sqrt{2}}}$

input
```
int(sin(x)*tan(4*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*I*exp(I*x)-1/2*I*exp(-I*x)-1/2*sum(_R*ln(exp(2*I*x)-4*I*_R*exp(I*x)-1)
,_R=RootOf(128*_Z^4-32*_Z^2+1))
```

3.76.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.42

$$\begin{aligned}\int \sin(x)\tan(4x)\,dx = {} & \frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\sqrt{\sqrt{2}+2}+2\sin(x)\right) \\ & -\frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\sqrt{\sqrt{2}+2}-2\sin(x)\right) \\ & +\frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(\sqrt{-\sqrt{2}+2}+2\sin(x)\right) \\ & -\frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(\sqrt{-\sqrt{2}+2}-2\sin(x)\right)-\sin(x)\end{aligned}$$

input
```
integrate(sin(x)*tan(4*x),x, algorithm="fricas")
```

output
```
1/8*sqrt(sqrt(2) + 2)*log(sqrt(sqrt(2) + 2) + 2*sin(x)) - 1/8*sqrt(sqrt(2)
 + 2)*log(sqrt(sqrt(2) + 2) - 2*sin(x)) + 1/8*sqrt(-sqrt(2) + 2)*log(sqrt(
-sqrt(2) + 2) + 2*sin(x)) - 1/8*sqrt(-sqrt(2) + 2)*log(sqrt(-sqrt(2) + 2)
- 2*sin(x)) - sin(x)
```

3.76.6 Sympy [F]

$$\int \sin(x)\tan(4x)\,dx = \int \sin{(x)}\tan{(4x)}\,dx$$

input
```
integrate(sin(x)*tan(4*x),x)
```

output
```
Integral(sin(x)*tan(4*x), x)
```

3.76.7 Maxima [F]

$$\int \sin(x)\tan(4x)\,dx = \int \sin{(x)}\tan{(4\,x)}\;dx$$

input
```
integrate(sin(x)*tan(4*x),x, algorithm="maxima")
```

output
```
integrate(((cos(7*x) + cos(x))*cos(8*x) + (sin(7*x) + sin(x))*sin(8*x) + c
os(7*x) + cos(x))/(cos(8*x)^2 + sin(8*x)^2 + 2*cos(8*x) + 1), x) - sin(x)
```

3.76.8 Giac [F]

$$\int \sin(x)\tan(4x)\,dx = \int \sin{(x)}\tan{(4\,x)}\;dx$$

input
```
integrate(sin(x)*tan(4*x),x, algorithm="giac")
```

output
```
integrate(sin(x)*tan(4*x), x)
```

3.76.9 Mupad [B] (verification not implemented)

Time = 25.84 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.45

$$\int \sin(x)\tan(4x)\,dx = \frac{\operatorname{atanh}\left(\frac{34\sin(x)\sqrt{\sqrt{2}+2}+24\sqrt{2}\sin(x)\sqrt{\sqrt{2}+2}}{41\sqrt{2}+58}\right)\sqrt{\sqrt{2}+2}}{4} - \sin(x) - \frac{\operatorname{atanh}\left(\frac{34\sin(x)\sqrt{2-\sqrt{2}}-24\sqrt{2}\sin(x)\sqrt{2-\sqrt{2}}}{41\sqrt{2}-58}\right)\sqrt{2-\sqrt{2}}}{4}$$

input
```
int(tan(4*x)*sin(x),x)
```

output
```
(atanh((34*sin(x)*(2^(1/2) + 2)^(1/2) + 24*2^(1/2)*sin(x)*(2^(1/2) + 2)^(1
/2))/(41*2^(1/2) + 58))*(2^(1/2) + 2)^(1/2))/4 - sin(x) - (atanh((34*sin(x
)*(2 - 2^(1/2))^(1/2) - 24*2^(1/2)*sin(x)*(2 - 2^(1/2))^(1/2))/(41*2^(1/2)
 - 58))*(2 - 2^(1/2))^(1/2))/4
```

3.77 $\int \sin(x)\tan(5x)\,dx$

3.77.1 Optimal result

Integrand size = 7, antiderivative size = 112

$$\begin{aligned}\int \sin(x)\tan(5x)\,dx =& \frac{1}{5}\text{arctanh}(\sin(x))-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(1-\sqrt{5}-4\sin(x)\right)\\&-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(1+\sqrt{5}-4\sin(x)\right)\\&+\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(1-\sqrt{5}+4\sin(x)\right)\\&+\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(1+\sqrt{5}+4\sin(x)\right)-\sin(x)\end{aligned}$$

output
```
1/5*arctanh(sin(x))-sin(x)-1/20*ln(1-4*sin(x)-5^(1/2))*(-5^(1/2)+1)+1/20*l
n(1+4*sin(x)-5^(1/2))*(-5^(1/2)+1)-1/20*ln(1-4*sin(x)+5^(1/2))*(5^(1/2)+1)
+1/20*ln(1+4*sin(x)+5^(1/2))*(5^(1/2)+1)
```

3.77.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.89

$$\begin{aligned}\int \sin(x)\tan(5x)\,dx =& \frac{1}{20}\Big(4\text{arctanh}(\sin(x))+\left(-1+\sqrt{5}\right)\log\left(1-\sqrt{5}-4\sin(x)\right)\\&-\left(1+\sqrt{5}\right)\log\left(1+\sqrt{5}-4\sin(x)\right)\\&-\left(-1+\sqrt{5}\right)\log\left(1-\sqrt{5}+4\sin(x)\right)\\&+\left(1+\sqrt{5}\right)\log\left(1+\sqrt{5}+4\sin(x)\right)-20\sin(x)\Big)\end{aligned}$$

input
```
Integrate[Sin[x]*Tan[5*x],x]
```

output
```
(4*ArcTanh[Sin[x]] + (-1 + Sqrt[5])*Log[1 - Sqrt[5] - 4*Sin[x]] - (1 + Sqr
t[5])*Log[1 + Sqrt[5] - 4*Sin[x]] - (-1 + Sqrt[5])*Log[1 - Sqrt[5] + 4*Sin
[x]] + (1 + Sqrt[5])*Log[1 + Sqrt[5] + 4*Sin[x]] - 20*Sin[x])/20
```

3.77.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4878, 2460, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\tan(5x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x)\tan(5x)dx$$

$$\Big\downarrow \text{4878}$$

$$\int \frac{\sin^2(x)\left(16\sin^4(x)-20\sin^2(x)+5\right)}{-16\sin^6(x)+28\sin^4(x)-13\sin^2(x)+1}d\sin(x)$$

$$\Big\downarrow \text{2460}$$

$$\int \left(\frac{2(\sin(x)-1)}{5\left(4\sin^2(x)+2\sin(x)-1\right)}-\frac{1}{5\left(\sin^2(x)-1\right)}-\frac{2(\sin(x)+1)}{5\left(4\sin^2(x)-2\sin(x)-1\right)}-1\right)d\sin(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}\text{arctanh}(\sin(x))-\sin(x)-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(-4\sin(x)-\sqrt{5}+1\right)-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(-4\sin(x)+\sqrt{5}+1\right)+\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(4\sin(x)-\sqrt{5}+1\right)+\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(4\sin(x)+\sqrt{5}+1\right)$$

input
```
Int[Sin[x]*Tan[5*x],x]
```

output
```
ArcTanh[Sin[x]]/5 - ((1 - Sqrt[5])*Log[1 - Sqrt[5] - 4*Sin[x]])/20 - ((1 +
 Sqrt[5])*Log[1 + Sqrt[5] - 4*Sin[x]])/20 + ((1 - Sqrt[5])*Log[1 - Sqrt[5]
 + 4*Sin[x]])/20 + ((1 + Sqrt[5])*Log[1 + Sqrt[5] + 4*Sin[x]])/20 - Sin[x]
```

3.77.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2460
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px /. x -> Sqrt[x]]},
Int[ExpandIntegrand[u*(Qx /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[Q
x, x]]] /; PolyQ[Px, x^2] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&
 !TrinomialQ[Px, x] && ILtQ[p, 0] && RationalFunctionQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

3.77.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.84 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.11

method	result
risch	$\frac{i\mathrm{e}^{ix}}{2}-\frac{i\mathrm{e}^{-ix}}{2}+\frac{\ln\left(i+\mathrm{e}^{ix}\right)}{5}-\frac{\ln\left(\mathrm{e}^{ix}-i\right)}{5}+\frac{\ln\left(\mathrm{e}^{2ix}-\frac{i\left(\sqrt{5}-1\right)\mathrm{e}^{ix}}{2}-1\right)}{20}-\frac{\ln\left(\mathrm{e}^{2ix}-\frac{i\left(\sqrt{5}-1\right)\mathrm{e}^{ix}}{2}-1\right)\sqrt{5}}{20}+\frac{\ln\left(\mathrm{e}^{2ix}+\frac{i\left(\sqrt{5}+1\right)}{2}\right.}{20}$

input
```
int(sin(x)*tan(5*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*I*exp(I*x)-1/2*I*exp(-I*x)+1/5*ln(I+exp(I*x))-1/5*ln(exp(I*x)-I)+1/20*
ln(exp(2*I*x)-1/2*I*(5^(1/2)-1)*exp(I*x)-1)-1/20*ln(exp(2*I*x)-1/2*I*(5^(1
/2)-1)*exp(I*x)-1)*5^(1/2)+1/20*ln(exp(2*I*x)+1/2*I*(5^(1/2)+1)*exp(I*x)-1
)+1/20*ln(exp(2*I*x)+1/2*I*(5^(1/2)+1)*exp(I*x)-1)*5^(1/2)-1/20*ln(exp(2*I
*x)-1/2*I*(5^(1/2)+1)*exp(I*x)-1)-1/20*ln(exp(2*I*x)-1/2*I*(5^(1/2)+1)*exp
(I*x)-1)*5^(1/2)-1/20*ln(exp(2*I*x)+1/2*I*(5^(1/2)-1)*exp(I*x)-1)+1/20*ln(
exp(2*I*x)+1/2*I*(5^(1/2)-1)*exp(I*x)-1)*5^(1/2)
```

3.77.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.21

$$\begin{aligned}\int \sin(x)\tan(5x)\,dx &= \frac{1}{20}\sqrt{5}\log\left(\frac{8\cos(x)^2-4(\sqrt{5}-1)\sin(x)+\sqrt{5}-11}{4\cos(x)^2+2\sin(x)-3}\right)\\ &\quad+\frac{1}{20}\sqrt{5}\log\left(-\frac{8\cos(x)^2-4(\sqrt{5}+1)\sin(x)-\sqrt{5}-11}{4\cos(x)^2-2\sin(x)-3}\right)\\ &\quad-\frac{1}{20}\log\left(4\cos(x)^2+2\sin(x)-3\right)\\ &\quad+\frac{1}{20}\log\left(4\cos(x)^2-2\sin(x)-3\right)\\ &\quad+\frac{1}{10}\log(\sin(x)+1)-\frac{1}{10}\log(-\sin(x)+1)-\sin(x)\end{aligned}$$

input
```
integrate(sin(x)*tan(5*x),x, algorithm="fricas")
```

output
```
1/20*sqrt(5)*log((8*cos(x)^2 - 4*(sqrt(5) - 1)*sin(x) + sqrt(5) - 11)/(4*c
os(x)^2 + 2*sin(x) - 3)) + 1/20*sqrt(5)*log(-(8*cos(x)^2 - 4*(sqrt(5) + 1)
*sin(x) - sqrt(5) - 11)/(4*cos(x)^2 - 2*sin(x) - 3)) - 1/20*log(4*cos(x)^2
 + 2*sin(x) - 3) + 1/20*log(4*cos(x)^2 - 2*sin(x) - 3) + 1/10*log(sin(x) +
 1) - 1/10*log(-sin(x) + 1) - sin(x)
```

3.77.6 Sympy [F]

$$\int \sin(x)\tan(5x)\,dx = \int \sin{(x)}\tan{(5x)}\,dx$$

input
```
integrate(sin(x)*tan(5*x),x)
```

output
```
Integral(sin(x)*tan(5*x), x)
```

3.77.7 Maxima [F]

$$\int \sin(x)\tan(5x)\,dx = \int \sin{(x)}\tan{(5\,x)}\ dx$$

input
```
integrate(sin(x)*tan(5*x),x, algorithm="maxima")
```

output
```
integrate(-1/5*((3*cos(7*x) - cos(5*x) - cos(3*x) + 3*cos(x))*cos(8*x) - 3
*(cos(6*x) - cos(4*x) + cos(2*x) - 1)*cos(7*x) + (cos(5*x) + cos(3*x) - 3*
cos(x))*cos(6*x) - (cos(4*x) - cos(2*x) + 1)*cos(5*x) - (cos(3*x) - 3*cos(
x))*cos(4*x) + (cos(2*x) - 1)*cos(3*x) - 3*cos(2*x)*cos(x) + (3*sin(7*x) -
 sin(5*x) - sin(3*x) + 3*sin(x))*sin(8*x) - 3*(sin(6*x) - sin(4*x) + sin(2
*x))*sin(7*x) + (sin(5*x) + sin(3*x) - 3*sin(x))*sin(6*x) - (sin(4*x) - si
n(2*x))*sin(5*x) - (sin(3*x) - 3*sin(x))*sin(4*x) + sin(3*x)*sin(2*x) - 3*
sin(2*x)*sin(x) + 3*cos(x))/(2*(cos(6*x) - cos(4*x) + cos(2*x) - 1)*cos(8*
x) - cos(8*x)^2 + 2*(cos(4*x) - cos(2*x) + 1)*cos(6*x) - cos(6*x)^2 + 2*(c
os(2*x) - 1)*cos(4*x) - cos(4*x)^2 - cos(2*x)^2 + 2*(sin(6*x) - sin(4*x) +
 sin(2*x))*sin(8*x) - sin(8*x)^2 + 2*(sin(4*x) - sin(2*x))*sin(6*x) - sin(
6*x)^2 - sin(4*x)^2 + 2*sin(4*x)*sin(2*x) - sin(2*x)^2 + 2*cos(2*x) - 1),
x) + 1/10*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - 1/10*log(cos(x)^2 + si
n(x)^2 - 2*sin(x) + 1) - sin(x)
```

3.77.8 Giac [F]

$$\int \sin(x)\tan(5x)\,dx = \int \sin(x)\tan(5x)\,dx$$

input
```
integrate(sin(x)*tan(5*x),x, algorithm="giac")
```

output
```
integrate(sin(x)*tan(5*x), x)
```

3.77.9 Mupad [B] (verification not implemented)

Time = 26.72 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.96

$$\int \sin(x)\tan(5x)\,dx = \frac{2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{5} + \frac{\mathrm{atan}\left(\frac{\sin(x)\,1042\mathrm{i}-\sqrt{5}\,\sin(x)\,466\mathrm{i}}{377\,\sqrt{5}-843}\right)\,1\mathrm{i}}{10} - \frac{\mathrm{atanh}\left(\sin(x)-\sqrt{5}\,\sin(x)\right)}{10} - \sin(x) - \frac{\sqrt{5}\,\mathrm{atanh}\left(\sin(x)-\sqrt{5}\,\sin(x)\right)}{10} - \frac{\sqrt{5}\,\mathrm{atan}\left(\frac{\sin(x)\,1042\mathrm{i}-\sqrt{5}\,\sin(x)\,466\mathrm{i}}{377\,\sqrt{5}-843}\right)\,1\mathrm{i}}{10}$$

input
```
int(tan(5*x)*sin(x),x)
```

output
```
(atan((sin(x)*1042i - 5^(1/2)*sin(x)*466i)/(377*5^(1/2) - 843))*1i)/10 - a
tanh(sin(x) - 5^(1/2)*sin(x))/10 + (2*atanh(sin(x/2)/cos(x/2)))/5 - sin(x)
 - (5^(1/2)*atanh(sin(x) - 5^(1/2)*sin(x)))/10 - (5^(1/2)*atan((sin(x)*104
2i - 5^(1/2)*sin(x)*466i)/(377*5^(1/2) - 843))*1i)/10
```

3.78 $\int \sin(x)\tan(6x)\,dx$

3.78.1 Optimal result

Integrand size = 7, antiderivative size = 89

$$\int \sin(x)\tan(6x)\,dx = \frac{\operatorname{arctanh}\left(\sqrt{2}\sin(x)\right)}{3\sqrt{2}} + \frac{1}{6}\sqrt{2-\sqrt{3}}\operatorname{arctanh}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{3}}}\right) + \frac{1}{6}\sqrt{2+\sqrt{3}}\operatorname{arctanh}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{3}}}\right) - \sin(x)$$

output
```
-sin(x)+1/6*arctanh(sin(x)*2^(1/2))*2^(1/2)+1/6*arctanh(2*sin(x)/(1/2*6^(1
/2)-1/2*2^(1/2)))*(1/2*6^(1/2)-1/2*2^(1/2))+1/6*arctanh(2*sin(x)/(1/2*6^(1
/2)+1/2*2^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))
```

3.78.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.94

$$\int \sin(x)\tan(6x)\,dx = \frac{1}{6}\left(\sqrt{2}\operatorname{arctanh}\left(\sqrt{2}\sin(x)\right) + \sqrt{2-\sqrt{3}}\operatorname{arctanh}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{3}}}\right) + \sqrt{2+\sqrt{3}}\operatorname{arctanh}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{3}}}\right) - 6\sin(x)\right)$$

input
```
Integrate[Sin[x]*Tan[6*x],x]
```

output
```
(Sqrt[2]*ArcTanh[Sqrt[2]*Sin[x]] + Sqrt[2 - Sqrt[3]]*ArcTanh[(2*Sin[x])/Sq
rt[2 - Sqrt[3]]] + Sqrt[2 + Sqrt[3]]*ArcTanh[(2*Sin[x])/Sqrt[2 + Sqrt[3]]]
 - 6*Sin[x])/6
```

3.78.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 4878, 27, 2460, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\tan(6x)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin(x)\tan(6x)dx$$
$$\Big\downarrow \text{4878}$$
$$\int \frac{2\sin^2(x)\left(16\sin^4(x)-16\sin^2(x)+3\right)}{-32\sin^6(x)+48\sin^4(x)-18\sin^2(x)+1}d\sin(x)$$
$$\Big\downarrow \text{27}$$
$$2\int \frac{\sin^2(x)\left(16\sin^4(x)-16\sin^2(x)+3\right)}{-32\sin^6(x)+48\sin^4(x)-18\sin^2(x)+1}d\sin(x)$$
$$\Big\downarrow \text{2460}$$
$$2\int \left(\frac{1-8\sin^2(x)}{3\left(16\sin^4(x)-16\sin^2(x)+1\right)}-\frac{1}{6\left(2\sin^2(x)-1\right)}-\frac{1}{2}\right)d\sin(x)$$
$$\Big\downarrow \text{2009}$$
$$2\left(\frac{\text{arctanh}\left(\sqrt{2}\sin(x)\right)}{6\sqrt{2}}+\frac{1}{12}\sqrt{2-\sqrt{3}}\text{arctanh}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{12}\sqrt{2+\sqrt{3}}\text{arctanh}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{3}}}\right)-\frac{\sin(x)}{2}\right)$$

input
```
Int[Sin[x]*Tan[6*x],x]
```

output
```
2*(ArcTanh[Sqrt[2]*Sin[x]]/(6*Sqrt[2]) + (Sqrt[2 - Sqrt[3]]*ArcTanh[(2*Sin
[x])/Sqrt[2 - Sqrt[3]]])/12 + (Sqrt[2 + Sqrt[3]]*ArcTanh[(2*Sin[x])/Sqrt[2
 + Sqrt[3]]])/12 - Sin[x]/2)
```

3.78.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2460
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px /. x -> Sqrt[x]]},
Int[ExpandIntegrand[u*(Qx /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[Q
x, x]]] /; PolyQ[Px, x^2] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&
 !TrinomialQ[Px, x] && ILtQ[p, 0] && RationalFunctionQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

3.78.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.79 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.16

method	result
risch	$\frac{i e^{ix}}{2}-\frac{i e^{-ix}}{2}-\frac{\sqrt{2}\,\ln\left(e^{2ix}-i\sqrt{2}\,e^{ix}-1\right)}{12}+\frac{\sqrt{2}\,\ln\left(e^{2ix}+i\sqrt{2}\,e^{ix}-1\right)}{12}-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(1296\_Z^4-144\_Z^2+1\right)}\_R\ln\left(e^{2ix}-6i\_\right.\right.}{2}$

input
```
int(sin(x)*tan(6*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*I*exp(I*x)-1/2*I*exp(-I*x)-1/12*2^(1/2)*ln(exp(2*I*x)-I*2^(1/2)*exp(I*
x)-1)+1/12*2^(1/2)*ln(exp(2*I*x)+I*2^(1/2)*exp(I*x)-1)-1/2*sum(_R*ln(exp(2
*I*x)-6*I*_R*exp(I*x)-1),_R=RootOf(1296*_Z^4-144*_Z^2+1))
```

3.78.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.51

$$\begin{aligned}\int \sin(x)\tan(6x)\,dx &= \frac{1}{12}\sqrt{\sqrt{3}+2}\log\left(\sqrt{\sqrt{3}+2}+2\,\sin(x)\right)\\ &\quad-\frac{1}{12}\sqrt{\sqrt{3}+2}\log\left(\sqrt{\sqrt{3}+2}-2\,\sin(x)\right)\\ &\quad+\frac{1}{12}\sqrt{-\sqrt{3}+2}\log\left(\sqrt{-\sqrt{3}+2}+2\,\sin(x)\right)\\ &\quad-\frac{1}{12}\sqrt{-\sqrt{3}+2}\log\left(\sqrt{-\sqrt{3}+2}-2\,\sin(x)\right)\\ &\quad+\frac{1}{12}\sqrt{2}\log\left(-\frac{2\,\cos(x)^2-2\,\sqrt{2}\sin(x)-3}{2\,\cos(x)^2-1}\right)-\sin(x)\end{aligned}$$

input
```
integrate(sin(x)*tan(6*x),x, algorithm="fricas")
```

output
```
1/12*sqrt(sqrt(3) + 2)*log(sqrt(sqrt(3) + 2) + 2*sin(x)) - 1/12*sqrt(sqrt(
3) + 2)*log(sqrt(sqrt(3) + 2) - 2*sin(x)) + 1/12*sqrt(-sqrt(3) + 2)*log(sq
rt(-sqrt(3) + 2) + 2*sin(x)) - 1/12*sqrt(-sqrt(3) + 2)*log(sqrt(-sqrt(3) +
 2) - 2*sin(x)) + 1/12*sqrt(2)*log(-(2*cos(x)^2 - 2*sqrt(2)*sin(x) - 3)/(2
*cos(x)^2 - 1)) - sin(x)
```

3.78.6 Sympy [F]

$$\int \sin(x)\tan(6x)\,dx = \int \sin{(x)}\tan{(6x)}\,dx$$

input
```
integrate(sin(x)*tan(6*x),x)
```

output
```
Integral(sin(x)*tan(6*x), x)
```

3.78.7 Maxima [F]

$$\int \sin(x)\tan(6x)\,dx = \int \sin{(x)}\tan{(6\,x)}\,\,dx$$

input
```
integrate(sin(x)*tan(6*x),x, algorithm="maxima")
```

output
```
1/24*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*sqrt(2)*si
n(x) + 2) - 1/24*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) -
2*sqrt(2)*sin(x) + 2) + 1/24*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(
2)*cos(x) + 2*sqrt(2)*sin(x) + 2) - 1/24*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)
^2 - 2*sqrt(2)*cos(x) - 2*sqrt(2)*sin(x) + 2) + integrate(-1/3*((2*cos(7*x
) - cos(5*x) - cos(3*x) + 2*cos(x))*cos(8*x) - 2*(cos(4*x) - 1)*cos(7*x) +
 (cos(4*x) - 1)*cos(5*x) + (cos(3*x) - 2*cos(x))*cos(4*x) + (2*sin(7*x) -
sin(5*x) - sin(3*x) + 2*sin(x))*sin(8*x) + (sin(3*x) - 2*sin(x))*sin(4*x)
- 2*sin(7*x)*sin(4*x) + sin(5*x)*sin(4*x) - cos(3*x) + 2*cos(x))/(2*(cos(4
*x) - 1)*cos(8*x) - cos(8*x)^2 - cos(4*x)^2 - sin(8*x)^2 + 2*sin(8*x)*sin(
4*x) - sin(4*x)^2 + 2*cos(4*x) - 1), x) - sin(x)
```

3.78.8 Giac [F]

$$\int \sin(x)\tan(6x)\,dx = \int \sin{(x)}\tan{(6\,x)}\,\,dx$$

input
```
integrate(sin(x)*tan(6*x),x, algorithm="giac")
```

output
```
integrate(sin(x)*tan(6*x), x)
```

3.78.9 Mupad [B] (verification not implemented)

Time = 27.67 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.47

$$\int \sin(x)\tan(6x)\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\,\sin(x)\right)}{6} - \sin(x) - \frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\,\sin(x) - \sqrt{6}\,\sin(x)\right)}{12} - \frac{\sqrt{6}\,\mathrm{atanh}\left(\sqrt{2}\,\sin(x) - \sqrt{6}\,\sin(x)\right)}{12} + \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sin(x)\,102818\mathrm{i} - \sqrt{6}\,\sin(x)\,59362\mathrm{i}}{40545\,\sqrt{2}\,\sqrt{6} - 140452}\right)\,1\mathrm{i}}{12} - \frac{\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sin(x)\,102818\mathrm{i} - \sqrt{6}\,\sin(x)\,59362\mathrm{i}}{40545\,\sqrt{2}\,\sqrt{6} - 140452}\right)\,1\mathrm{i}}{12}$$

input
```
int(tan(6*x)*sin(x),x)
```

output
```
(2^(1/2)*atan((2^(1/2)*sin(x)*102818i - 6^(1/2)*sin(x)*59362i)/(40545*2^(1
/2)*6^(1/2) - 140452))*1i)/12 - sin(x) - (6^(1/2)*atan((2^(1/2)*sin(x)*102
818i - 6^(1/2)*sin(x)*59362i)/(40545*2^(1/2)*6^(1/2) - 140452))*1i)/12 + (
2^(1/2)*atanh(2^(1/2)*sin(x)))/6 - (2^(1/2)*atanh(2^(1/2)*sin(x) - 6^(1/2)
*sin(x)))/12 - (6^(1/2)*atanh(2^(1/2)*sin(x) - 6^(1/2)*sin(x)))/12
```

3.79 $\int \sin(x)\tan(nx)\,dx$

3.79.1 Optimal result

Integrand size = 7, antiderivative size = 105

$$\int \sin(x)\tan(nx)\,dx = \frac{1}{2}ie^{-ix} + \frac{1}{2}ie^{ix} - ie^{-ix}\,\text{Hypergeometric2F1}\left(1,-\frac{1}{2n},1-\frac{1}{2n},-e^{2inx}\right) - ie^{ix}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-e^{2inx}\right)$$

output
```
1/2*I/exp(I*x)+1/2*I*exp(I*x)-I*hypergeom([1, -1/2/n],[1-1/2/n],-exp(2*I*n
*x))/exp(I*x)-I*exp(I*x)*hypergeom([1, 1/2/n],[1+1/2/n],-exp(2*I*n*x))
```

3.79.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.90

$$\int \sin(x)\tan(nx)\,dx = -\frac{ie^{-2ix}\left(e^{i(x+2nx)}(1+2n)\,\text{Hypergeometric2F1}\left(1,1-\frac{1}{2n},2-\frac{1}{2n},-e^{2inx}\right)+(-1+2n)\left(-e^{i(3+2n)x}\,\text{Hyperg}\right.\right.}{}$$

input
```
Integrate[Sin[x]*Tan[n*x],x]
```

output
```
((-1/2*I)*(E^(I*(x + 2*n*x))*(1 + 2*n)*Hypergeometric2F1[1, 1 - 1/(2*n), 2
 - 1/(2*n), -E^((2*I)*n*x)] + (-1 + 2*n)*(-(E^(I*(3 + 2*n)*x)*Hypergeometr
ic2F1[1, 1 + 1/(2*n), 2 + 1/(2*n), -E^((2*I)*n*x)]) + E^(I*x)*(1 + 2*n)*(H
ypergeometric2F1[1, -1/2*1/n, 1 - 1/(2*n), -E^((2*I)*n*x)] + E^((2*I)*x)*H
ypergeometric2F1[1, 1/(2*n), 1 + 1/(2*n), -E^((2*I)*n*x)]))))/(E^((2*I)*x)
*(-1 + 4*n^2))
```

3.79.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5068, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\tan(nx)\,dx$$

$$\Big\downarrow \text{5068}$$

$$\int \left(-\frac{e^{-ix}}{1+e^{2inx}}+\frac{e^{ix}}{1+e^{2inx}}+\frac{e^{-ix}}{2}-\frac{e^{ix}}{2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-ie^{-ix}\,\text{Hypergeometric2F1}\left(1,-\frac{1}{2n},1-\frac{1}{2n},-e^{2inx}\right)-ie^{ix}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),-e^{2inx}\right)+\frac{1}{2}ie^{-ix}+\frac{1}{2}ie^{ix}$$

input
```
Int[Sin[x]*Tan[n*x],x]
```

output
```
(I/2)/E^(I*x) + (I/2)*E^(I*x) - (I*Hypergeometric2F1[1, -1/2*1/n, 1 - 1/(2
*n), -E^((2*I)*n*x)])/E^(I*x) - I*E^(I*x)*Hypergeometric2F1[1, 1/(2*n), (2
 + n^(-1))/2, -E^((2*I)*n*x)]
```

3.79.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5068
```
Int[Sin[(a_.) + (b_.)*(x_)]*Tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Int[1/(E^
(I*(a + b*x))*2) - E^(I*(a + b*x))/2 - 1/(E^(I*(a + b*x))*(1 + E^(2*I*(c +
d*x)))) + E^(I*(a + b*x))/(1 + E^(2*I*(c + d*x))), x] /; FreeQ[{a, b, c, d}
, x] && NeQ[b^2 - d^2, 0]
```

3.79.4 Maple [F]

$$\int \sin(x)\tan(nx)\,dx$$

input
```
int(sin(x)*tan(n*x),x)
```

output
```
int(sin(x)*tan(n*x),x)
```

3.79.5 Fricas [F]

$$\int \sin(x)\tan(nx)\,dx = \int \sin(x)\tan(nx)\,dx$$

input
```
integrate(sin(x)*tan(n*x),x, algorithm="fricas")
```

output
```
integral(sin(x)*tan(n*x), x)
```

3.79.6 Sympy [F]

$$\int \sin(x)\tan(nx)\,dx = \int \sin(x)\tan(nx)\,dx$$

input
```
integrate(sin(x)*tan(n*x),x)
```

output
```
Integral(sin(x)*tan(n*x), x)
```

3.79.7 Maxima [F]

$$\int \sin(x)\tan(nx)\,dx = \int \sin{(x)}\tan{(nx)}\,dx$$

input
```
integrate(sin(x)*tan(n*x),x, algorithm="maxima")
```

output
```
integrate(sin(x)*tan(n*x), x)
```

3.79.8 Giac [F]

$$\int \sin(x)\tan(nx)\,dx = \int \sin{(x)}\tan{(nx)}\,dx$$

input
```
integrate(sin(x)*tan(n*x),x, algorithm="giac")
```

output
```
integrate(sin(x)*tan(n*x), x)
```

3.79.9 Mupad [F(-1)]

Timed out.

$$\int \sin(x)\tan(nx)\,dx = \int \tan(n\,x)\,\sin{(x)}\,dx$$

input
```
int(tan(n*x)*sin(x),x)
```

output
```
int(tan(n*x)*sin(x), x)
```

3.80 $\int \cot(2x)\sin(x)\,dx$

3.80.1 Optimal result

Integrand size = 7, antiderivative size = 10

$$\int \cot(2x)\sin(x)\,dx = -\frac{1}{2}\text{arctanh}(\sin(x)) + \sin(x)$$

output
```
-1/2*arctanh(sin(x))+sin(x)
```

3.80.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \cot(2x)\sin(x)\,dx = -\frac{1}{2}\text{arctanh}(\sin(x)) + \sin(x)$$

input
```
Integrate[Cot[2*x]*Sin[x],x]
```

output
```
-1/2*ArcTanh[Sin[x]] + Sin[x]
```

3.80.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.40, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3042, 4878, 27, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\cot(2x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cot(2x)}{\csc(x)}dx$$

$$\Big\downarrow \text{4878}$$

$$\int \frac{1-2\sin^2(x)}{2\left(1-\sin^2(x)\right)}d\sin(x)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{1-2\sin^2(x)}{1-\sin^2(x)}d\sin(x)$$

$$\Big\downarrow \text{299}$$

$$\frac{1}{2}\left(2\sin(x)-\int \frac{1}{1-\sin^2(x)}d\sin(x)\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}(2\sin(x)-\text{arctanh}(\sin(x)))$$

input
```
Int[Cot[2*x]*Sin[x],x]
```

output
```
(-ArcTanh[Sin[x]] + 2*Sin[x])/2
```

3.80.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

3.80.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

method	result	size
default	$-\frac{\ln(\sec(x)+\tan(x))}{2}+\sin(x)$	12
risch	$-\frac{i e^{ix}}{2}+\frac{i e^{-ix}}{2}+\frac{\ln(e^{ix}-i)}{2}-\frac{\ln(i+e^{ix})}{2}$	40

input
```
int(cot(2*x)*sin(x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(sec(x)+tan(x))+sin(x)
```

3.80.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(8) = 16.

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.90

$$\int \cot(2x)\sin(x)\,dx = -\frac{1}{4}\log\left(\sin\left(x\right)+1\right)+\frac{1}{4}\log\left(-\sin\left(x\right)+1\right)+\sin\left(x\right)$$

input
```
integrate(cot(2*x)*sin(x),x, algorithm="fricas")
```

output
```
-1/4*log(sin(x) + 1) + 1/4*log(-sin(x) + 1) + sin(x)
```

3.80.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(8) = 16.

Time = 0.36 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.90

$$\int \cot(2x)\sin(x)\,dx = \frac{\log\left(\sin\left(x\right)-1\right)}{4}-\frac{\log\left(\sin\left(x\right)+1\right)}{4}+\sin\left(x\right)$$

input
```
integrate(cot(2*x)*sin(x),x)
```

output
```
log(sin(x) - 1)/4 - log(sin(x) + 1)/4 + sin(x)
```

3.80.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(8) = 16.

Time = 0.30 (sec) , antiderivative size = 37, normalized size of antiderivative = 3.70

$$\int \cot(2x)\sin(x)\,dx = -\frac{1}{4}\log\left(\cos\left(x\right)^{2}+\sin\left(x\right)^{2}+2\,\sin\left(x\right)+1\right) + \frac{1}{4}\log\left(\cos\left(x\right)^{2}+\sin\left(x\right)^{2}-2\,\sin\left(x\right)+1\right)+\sin\left(x\right)$$

input
```
integrate(cot(2*x)*sin(x),x, algorithm="maxima")
```

output
```
-1/4*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) + 1/4*log(cos(x)^2 + sin(x)^2
 - 2*sin(x) + 1) + sin(x)
```

3.80.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(8) = 16.

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.90

$$\int \cot(2x)\sin(x)\,dx = -\frac{1}{4}\log\left(\sin\left(x\right)+1\right)+\frac{1}{4}\log\left(-\sin\left(x\right)+1\right)+\sin\left(x\right)$$

input
```
integrate(cot(2*x)*sin(x),x, algorithm="giac")
```

output
```
-1/4*log(sin(x) + 1) + 1/4*log(-sin(x) + 1) + sin(x)
```

3.80.9 Mupad [B] (verification not implemented)

Time = 25.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \cot(2x)\sin(x)\,dx = \sin\left(x\right)-\mathrm{atanh}\left(\tan\left(\frac{x}{2}\right)\right)$$

input
```
int(cot(2*x)*sin(x),x)
```

output
```
sin(x) - atanh(tan(x/2))
```

3.81 $\int \cot(3x)\sin(x)\,dx$

3.81.1 Optimal result

Integrand size = 7, antiderivative size = 20

$$\int \cot(3x)\sin(x)\,dx = -\frac{\operatorname{arctanh}\left(\frac{2\sin(x)}{\sqrt{3}}\right)}{\sqrt{3}} + \sin(x)$$

output
```
sin(x)-1/3*arctanh(2/3*sin(x)*3^(1/2))*3^(1/2)
```

3.81.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \cot(3x)\sin(x)\,dx = -\frac{\operatorname{arctanh}\left(\frac{2\sin(x)}{\sqrt{3}}\right)}{\sqrt{3}} + \sin(x)$$

input
```
Integrate[Cot[3*x]*Sin[x],x]
```

output
```
-(ArcTanh[(2*Sin[x])/Sqrt[3]]/Sqrt[3]) + Sin[x]
```

3.81.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3042, 4878, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\cot(3x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cot(3x)}{\csc(x)}dx$$

$$\Big\downarrow \text{ 4878}$$

$$\int \frac{1-4\sin^2(x)}{3-4\sin^2(x)}d\sin(x)$$

$$\Big\downarrow \text{ 299}$$

$$\sin(x)-2\int \frac{1}{3-4\sin^2(x)}d\sin(x)$$

$$\Big\downarrow \text{ 219}$$

$$\sin(x)-\frac{\text{arctanh}\left(\frac{2\sin(x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Int[Cot[3*x]*Sin[x],x]
```

output
```
-(ArcTanh[(2*Sin[x])/Sqrt[3]]/Sqrt[3]) + Sin[x]
```

3.81.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

3.81.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\sin\left(x\right)-\frac{\operatorname{arctanh}\left(\frac{2\sin\left(x\right)\sqrt{3}}{3}\right)\sqrt{3}}{3}$	17
default	$\sin\left(x\right)-\frac{\operatorname{arctanh}\left(\frac{2\sin\left(x\right)\sqrt{3}}{3}\right)\sqrt{3}}{3}$	17
risch	$-\frac{i{\mathrm e}^{i x}}{2}+\frac{i{\mathrm e}^{-i x}}{2}+\frac{\sqrt{3}\,\ln\left({\mathrm e}^{2 i x}-i\sqrt{3}\,{\mathrm e}^{i x}-1\right)}{6}-\frac{\sqrt{3}\,\ln\left({\mathrm e}^{2 i x}+i\sqrt{3}\,{\mathrm e}^{i x}-1\right)}{6}$	66

input
```
int(cot(3*x)*sin(x),x,method=_RETURNVERBOSE)
```

output
```
sin(x)-1/3*arctanh(2/3*sin(x)*3^(1/2))*3^(1/2)
```

3.81.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(16) = 32$.

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \cot(3x)\sin(x)\,dx = \frac{1}{6}\sqrt{3}\log\left(-\frac{4\cos(x)^2 + 4\sqrt{3}\sin(x) - 7}{4\cos(x)^2 - 1}\right) + \sin(x)$$

input
```
integrate(cot(3*x)*sin(x),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*log(-(4*cos(x)^2 + 4*sqrt(3)*sin(x) - 7)/(4*cos(x)^2 - 1)) + s
in(x)
```

3.81.6 Sympy [F]

$$\int \cot(3x)\sin(x)\,dx = \int \sin(x)\cot(3x)\,dx$$

input
```
integrate(cot(3*x)*sin(x),x)
```

output
```
Integral(sin(x)*cot(3*x), x)
```

3.81.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(16) = 32$.

Time = 0.30 (sec) , antiderivative size = 127, normalized size of antiderivative = 6.35

$$\begin{aligned}\int \cot(3x)\sin(x)\,dx = &-\frac{1}{12}\sqrt{3}\log\left(\frac{4}{3}\cos(x)^2 + \frac{4}{3}\sin(x)^2 + \frac{4}{3}\sqrt{3}\sin(x) + \frac{4}{3}\cos(x) + \frac{4}{3}\right)\\ &-\frac{1}{12}\sqrt{3}\log\left(\frac{4}{3}\cos(x)^2 + \frac{4}{3}\sin(x)^2 + \frac{4}{3}\sqrt{3}\sin(x) - \frac{4}{3}\cos(x) + \frac{4}{3}\right)\\ &+\frac{1}{12}\sqrt{3}\log\left(\frac{4}{3}\cos(x)^2 + \frac{4}{3}\sin(x)^2 - \frac{4}{3}\sqrt{3}\sin(x) + \frac{4}{3}\cos(x) + \frac{4}{3}\right)\\ &+\frac{1}{12}\sqrt{3}\log\left(\frac{4}{3}\cos(x)^2 + \frac{4}{3}\sin(x)^2 - \frac{4}{3}\sqrt{3}\sin(x) - \frac{4}{3}\cos(x) + \frac{4}{3}\right)\\ &+\sin(x)\end{aligned}$$

input
```
integrate(cot(3*x)*sin(x),x, algorithm="maxima")
```

output
```
-1/12*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 + 4/3*sqrt(3)*sin(x) + 4/3*c
os(x) + 4/3) - 1/12*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 + 4/3*sqrt(3)*
sin(x) - 4/3*cos(x) + 4/3) + 1/12*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2
- 4/3*sqrt(3)*sin(x) + 4/3*cos(x) + 4/3) + 1/12*sqrt(3)*log(4/3*cos(x)^2 +
 4/3*sin(x)^2 - 4/3*sqrt(3)*sin(x) - 4/3*cos(x) + 4/3) + sin(x)
```

3.81.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(16) = 32$.

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.70

$$\int \cot(3x)\sin(x)\,dx = \frac{1}{6}\sqrt{3}\log\left(\frac{|-4\sqrt{3}+8\sin(x)|}{|4\sqrt{3}+8\sin(x)|}\right)+\sin(x)$$

input
```
integrate(cot(3*x)*sin(x),x, algorithm="giac")
```

output
```
1/6*sqrt(3)*log(abs(-4*sqrt(3) + 8*sin(x))/abs(4*sqrt(3) + 8*sin(x))) + si
n(x)
```

3.81.9 Mupad [B] (verification not implemented)

Time = 26.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \cot(3x)\sin(x)\,dx = \sin(x) - \frac{\sqrt{3}\,\mathrm{atanh}\left(\frac{2\sqrt{3}\sin(x)}{3}\right)}{3}$$

input
```
int(cot(3*x)*sin(x),x)
```

output
```
sin(x) - (3^(1/2)*atanh((2*3^(1/2)*sin(x))/3))/3
```

3.82 $\int \cot(4x)\sin(x)\,dx$

3.82.1 Optimal result

Integrand size = 7, antiderivative size = 28

$$\int \cot(4x)\sin(x)\,dx = -\frac{1}{4}\text{arctanh}(\sin(x)) - \frac{\text{arctanh}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}} + \sin(x)$$

output
```
-1/4*arctanh(sin(x))+sin(x)-1/4*arctanh(sin(x)*2^(1/2))*2^(1/2)
```

3.82.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \cot(4x)\sin(x)\,dx = -\frac{1}{4}\text{arctanh}(\sin(x)) - \frac{\text{arctanh}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}} + \sin(x)$$

input
```
Integrate[Cot[4*x]*Sin[x],x]
```

output
```
-1/4*ArcTanh[Sin[x]] - ArcTanh[Sqrt[2]*Sin[x]]/(2*Sqrt[2]) + Sin[x]
```

3.82.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3042, 4878, 27, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\cot(4x)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\cot(4x)}{\csc(x)}dx$$

$$\Big\downarrow \; 4878$$

$$\int \frac{8\sin^4(x)-8\sin^2(x)+1}{4\left(2\sin^4(x)-3\sin^2(x)+1\right)}d\sin(x)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{4}\int \frac{8\sin^4(x)-8\sin^2(x)+1}{2\sin^4(x)-3\sin^2(x)+1}d\sin(x)$$

$$\Big\downarrow \; 2205$$

$$\frac{1}{4}\int \left(4-\frac{3-4\sin^2(x)}{2\sin^4(x)-3\sin^2(x)+1}\right)d\sin(x)$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{4}\left(-\text{arctanh}(\sin(x))-\sqrt{2}\text{arctanh}\left(\sqrt{2}\sin(x)\right)+4\sin(x)\right)$$

input
```
Int[Cot[4*x]*Sin[x],x]
```

output
```
(-ArcTanh[Sin[x]] - Sqrt[2]*ArcTanh[Sqrt[2]*Sin[x]] + 4*Sin[x])/4
```

3.82.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

3.82.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 88, normalized size of antiderivative = 3.14

method	result	size
risch	$-\frac{i e^{i x}}{2}+\frac{i e^{-i x}}{2}+\frac{\ln \left(e^{i x}-i\right)}{4}-\frac{\ln \left(i+e^{i x}\right)}{4}+\frac{\sqrt{2}\, \ln \left(e^{2 i x}-i \sqrt{2}\, e^{i x}-1\right)}{8}-\frac{\sqrt{2}\, \ln \left(e^{2 i x}+i \sqrt{2}\, e^{i x}-1\right)}{8}$	88

input
```
int(cot(4*x)*sin(x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*exp(I*x)+1/2*I*exp(-I*x)+1/4*ln(exp(I*x)-I)-1/4*ln(I+exp(I*x))+1/8*
2^(1/2)*ln(exp(2*I*x)-I*2^(1/2)*exp(I*x)-1)-1/8*2^(1/2)*ln(exp(2*I*x)+I*2^
(1/2)*exp(I*x)-1)
```

3.82.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(20) = 40$.

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.86

$$\int \cot(4x)\sin(x)\,dx = \frac{1}{8}\sqrt{2}\log\left(-\frac{2\cos(x)^2 + 2\sqrt{2}\sin(x) - 3}{2\cos(x)^2 - 1}\right) - \frac{1}{8}\log(\sin(x) + 1) + \frac{1}{8}\log(-\sin(x) + 1) + \sin(x)$$

input
```
integrate(cot(4*x)*sin(x),x, algorithm="fricas")
```

output
```
1/8*sqrt(2)*log(-(2*cos(x)^2 + 2*sqrt(2)*sin(x) - 3)/(2*cos(x)^2 - 1)) - 1
/8*log(sin(x) + 1) + 1/8*log(-sin(x) + 1) + sin(x)
```

3.82.6 Sympy [F]

$$\int \cot(4x)\sin(x)\,dx = \int \sin(x)\cot(4x)\,dx$$

input
```
integrate(cot(4*x)*sin(x),x)
```

output
```
Integral(sin(x)*cot(4*x), x)
```

3.82.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(20) = 40$.

Time = 0.29 (sec) , antiderivative size = 173, normalized size of antiderivative = 6.18

$$\begin{aligned}\int \cot(4x)\sin(x)\,dx = &-\frac{1}{16}\sqrt{2}\log\left(2\cos(x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) + 2\sqrt{2}\sin(x) + 2\right)\\ &+\frac{1}{16}\sqrt{2}\log\left(2\cos(x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) - 2\sqrt{2}\sin(x) + 2\right)\\ &-\frac{1}{16}\sqrt{2}\log\left(2\cos(x)^2 + 2\sin(x)^2 - 2\sqrt{2}\cos(x) + 2\sqrt{2}\sin(x) + 2\right)\\ &+\frac{1}{16}\sqrt{2}\log\left(2\cos(x)^2 + 2\sin(x)^2 - 2\sqrt{2}\cos(x) - 2\sqrt{2}\sin(x) + 2\right)\\ &-\frac{1}{8}\log\left(\cos(x)^2 + \sin(x)^2 + 2\sin(x) + 1\right)\\ &+\frac{1}{8}\log\left(\cos(x)^2 + \sin(x)^2 - 2\sin(x) + 1\right) + \sin(x)\end{aligned}$$

input
```
integrate(cot(4*x)*sin(x),x, algorithm="maxima")
```

output
```
-1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*sqrt(2)*s
in(x) + 2) + 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) -
 2*sqrt(2)*sin(x) + 2) - 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt
(2)*cos(x) + 2*sqrt(2)*sin(x) + 2) + 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x
)^2 - 2*sqrt(2)*cos(x) - 2*sqrt(2)*sin(x) + 2) - 1/8*log(cos(x)^2 + sin(x)
^2 + 2*sin(x) + 1) + 1/8*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1) + sin(x)
```

3.82.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(20) = 40$.

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.79

$$\int \cot(4x)\sin(x)\,dx = \frac{1}{8}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin(x)\right|}{\left|2\sqrt{2}+4\sin(x)\right|}\right) - \frac{1}{8}\log(\sin(x)+1) + \frac{1}{8}\log(-\sin(x)+1) + \sin(x)$$

input
```
integrate(cot(4*x)*sin(x),x, algorithm="giac")
```

output
```
1/8*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(x))/abs(2*sqrt(2) + 4*sin(x))) - 1/
8*log(sin(x) + 1) + 1/8*log(-sin(x) + 1) + sin(x)
```

3.82.9 Mupad [B] (verification not implemented)

Time = 25.32 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \cot(4x)\sin(x)\,dx = \sin(x) - \frac{\operatorname{atanh}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{2} - \frac{\sqrt{2}\operatorname{atanh}\left(\sqrt{2}\sin(x)\right)}{4}$$

input
```
int(cot(4*x)*sin(x),x)
```

output
```
sin(x) - atanh(sin(x/2)/cos(x/2))/2 - (2^(1/2)*atanh(2^(1/2)*sin(x)))/4
```

3.83 $\int \cot(5x)\sin(x)\,dx$

3.83.1 Optimal result

Integrand size = 7, antiderivative size = 82

$$\int \cot(5x)\sin(x)\,dx = -\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\operatorname{arctanh}\left(2\sqrt{\frac{2}{5+\sqrt{5}}}\sin(x)\right) - \frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}\sin(x)\right) + \sin(x)$$

output
```
sin(x)-1/10*arctanh(1/5*sin(x)*(50+10*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2)
-1/10*arctanh(2*sin(x)*2^(1/2)/(5+5^(1/2))^(1/2))*(10+2*5^(1/2))^(1/2)
```

3.83.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.93

$$\int \cot(5x)\sin(x)\,dx = \frac{1}{10}\left(-\sqrt{10-2\sqrt{5}}\operatorname{arctanh}\left(\sqrt{2+\frac{2}{\sqrt{5}}}\sin(x)\right) - \sqrt{2\left(5+\sqrt{5}\right)}\operatorname{arctanh}\left(2\sqrt{\frac{2}{5+\sqrt{5}}}\sin(x)\right) + 10\sin(x)\right)$$

input
```
Integrate[Cot[5*x]*Sin[x],x]
```

output
```
(-(Sqrt[10 - 2*Sqrt[5]]*ArcTanh[Sqrt[2 + 2/Sqrt[5]]*Sin[x]]) - Sqrt[2*(5 +
 Sqrt[5])]*ArcTanh[2*Sqrt[2/(5 + Sqrt[5])]*Sin[x]] + 10*Sin[x])/10
```

3.83.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4878, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\cot(5x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cot(5x)}{\csc(x)}dx$$

$$\Big\downarrow \text{4878}$$

$$\int \frac{16\sin^4(x)-12\sin^2(x)+1}{16\sin^4(x)-20\sin^2(x)+5}d\sin(x)$$

$$\Big\downarrow \text{2205}$$

$$\int \left(1-\frac{4(1-2\sin^2(x))}{16\sin^4(x)-20\sin^2(x)+5}\right)d\sin(x)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\text{arctanh}\left(2\sqrt{\frac{2}{5+\sqrt{5}}}\sin(x)\right)-\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\text{arctanh}\left(\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}\sin(x)\right)+\sin(x)$$

input
```
Int[Cot[5*x]*Sin[x],x]
```

output
```
-1/5*(Sqrt[(5 + Sqrt[5])/2]*ArcTanh[2*Sqrt[2/(5 + Sqrt[5])]*Sin[x]]) - (Sq
rt[(5 - Sqrt[5])/2]*ArcTanh[Sqrt[(2*(5 + Sqrt[5]))/5]*Sin[x]])/5 + Sin[x]
```

3.83.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

3.83.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.45 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.67

method	result	size
risch	$-\frac{i\mathrm{e}^{ix}}{2}+\frac{i\mathrm{e}^{-ix}}{2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(125\_Z^{4}-25\_Z^{2}+1\right)}\_R\ln\left(\mathrm{e}^{2ix}-5i\_R\,\mathrm{e}^{ix}-1\right)\right)}{2}$	55

input
```
int(cot(5*x)*sin(x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*exp(I*x)+1/2*I*exp(-I*x)+1/2*sum(_R*ln(exp(2*I*x)-5*I*_R*exp(I*x)-1
),_R=RootOf(125*_Z^4-25*_Z^2+1))
```

3.83.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. 2(54) = 108.

Time = 0.26 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.55

$$\int \cot(5x)\sin(x)\,dx = -\frac{1}{20}\sqrt{2}\sqrt{\sqrt{5}+5}\log\left(\sqrt{2}\sqrt{\sqrt{5}+5}+4\sin(x)\right) + \frac{1}{20}\sqrt{2}\sqrt{\sqrt{5}+5}\log\left(\sqrt{2}\sqrt{\sqrt{5}+5}-4\sin(x)\right) - \frac{1}{20}\sqrt{2}\sqrt{-\sqrt{5}+5}\log\left(\sqrt{2}\sqrt{-\sqrt{5}+5}+4\sin(x)\right) + \frac{1}{20}\sqrt{2}\sqrt{-\sqrt{5}+5}\log\left(\sqrt{2}\sqrt{-\sqrt{5}+5}-4\sin(x)\right) + \sin(x)$$

input
```
integrate(cot(5*x)*sin(x),x, algorithm="fricas")
```

output
```
-1/20*sqrt(2)*sqrt(sqrt(5) + 5)*log(sqrt(2)*sqrt(sqrt(5) + 5) + 4*sin(x))
+ 1/20*sqrt(2)*sqrt(sqrt(5) + 5)*log(sqrt(2)*sqrt(sqrt(5) + 5) - 4*sin(x))
 - 1/20*sqrt(2)*sqrt(-sqrt(5) + 5)*log(sqrt(2)*sqrt(-sqrt(5) + 5) + 4*sin(
x)) + 1/20*sqrt(2)*sqrt(-sqrt(5) + 5)*log(sqrt(2)*sqrt(-sqrt(5) + 5) - 4*s
in(x)) + sin(x)
```

3.83.6 Sympy [F]

$$\int \cot(5x)\sin(x)\,dx = \int \sin(x)\cot(5x)\,dx$$

input
```
integrate(cot(5*x)*sin(x),x)
```

output
```
Integral(sin(x)*cot(5*x), x)
```

3.83.7 Maxima [F]

$$\int \cot(5x)\sin(x)\,dx = \int \cot(5x)\sin(x)\,dx$$

input
```
integrate(cot(5*x)*sin(x),x, algorithm="maxima")
```

output
```
-integrate(1/2*((cos(3*x) + cos(2*x) + cos(x))*cos(4*x) + (2*cos(2*x) + 2*
cos(x) + 1)*cos(3*x) + cos(3*x)^2 + (2*cos(x) + 1)*cos(2*x) + cos(2*x)^2 +
 cos(x)^2 + (sin(3*x) + sin(2*x) + sin(x))*sin(4*x) + 2*(sin(2*x) + sin(x)
)*sin(3*x) + sin(3*x)^2 + sin(2*x)^2 + 2*sin(2*x)*sin(x) + sin(x)^2 + cos(
x))/(2*(cos(3*x) + cos(2*x) + cos(x) + 1)*cos(4*x) + cos(4*x)^2 + 2*(cos(2
*x) + cos(x) + 1)*cos(3*x) + cos(3*x)^2 + 2*(cos(x) + 1)*cos(2*x) + cos(2*
x)^2 + cos(x)^2 + 2*(sin(3*x) + sin(2*x) + sin(x))*sin(4*x) + sin(4*x)^2 +
 2*(sin(2*x) + sin(x))*sin(3*x) + sin(3*x)^2 + sin(2*x)^2 + 2*sin(2*x)*sin
(x) + sin(x)^2 + 2*cos(x) + 1), x) - integrate(-1/2*((cos(3*x) - cos(2*x)
+ cos(x))*cos(4*x) + (2*cos(2*x) - 2*cos(x) + 1)*cos(3*x) - cos(3*x)^2 + (
2*cos(x) - 1)*cos(2*x) - cos(2*x)^2 - cos(x)^2 + (sin(3*x) - sin(2*x) + si
n(x))*sin(4*x) + 2*(sin(2*x) - sin(x))*sin(3*x) - sin(3*x)^2 - sin(2*x)^2
+ 2*sin(2*x)*sin(x) - sin(x)^2 + cos(x))/(2*(cos(3*x) - cos(2*x) + cos(x)
- 1)*cos(4*x) - cos(4*x)^2 + 2*(cos(2*x) - cos(x) + 1)*cos(3*x) - cos(3*x)
^2 + 2*(cos(x) - 1)*cos(2*x) - cos(2*x)^2 - cos(x)^2 + 2*(sin(3*x) - sin(2
*x) + sin(x))*sin(4*x) - sin(4*x)^2 + 2*(sin(2*x) - sin(x))*sin(3*x) - sin
(3*x)^2 - sin(2*x)^2 + 2*sin(2*x)*sin(x) - sin(x)^2 + 2*cos(x) - 1), x) +
sin(x)
```

3.83.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(54) = 108$.

Time = 0.36 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.35

$$\begin{aligned}\int \cot(5x)\sin(x)\,dx = &-\frac{1}{20}\sqrt{2\sqrt{5}+10}\log\left(\left|\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}+5}+\sin(x)\right|\right)\\ &+\frac{1}{20}\sqrt{2\sqrt{5}+10}\log\left(\left|-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{\sqrt{5}+5}+\sin(x)\right|\right)\\ &-\frac{1}{20}\sqrt{-2\sqrt{5}+10}\log\left(\left|\sqrt{-\frac{1}{8}\sqrt{5}+\frac{5}{8}}+\sin(x)\right|\right)\\ &+\frac{1}{20}\sqrt{-2\sqrt{5}+10}\log\left(\left|-\sqrt{-\frac{1}{8}\sqrt{5}+\frac{5}{8}}+\sin(x)\right|\right)+\sin(x)\end{aligned}$$

input
```
integrate(cot(5*x)*sin(x),x, algorithm="giac")
```

output
```
-1/20*sqrt(2*sqrt(5) + 10)*log(abs(1/2*sqrt(1/2)*sqrt(sqrt(5) + 5) + sin(x
))) + 1/20*sqrt(2*sqrt(5) + 10)*log(abs(-1/2*sqrt(1/2)*sqrt(sqrt(5) + 5) +
 sin(x))) - 1/20*sqrt(-2*sqrt(5) + 10)*log(abs(sqrt(-1/8*sqrt(5) + 5/8) +
sin(x))) + 1/20*sqrt(-2*sqrt(5) + 10)*log(abs(-sqrt(-1/8*sqrt(5) + 5/8) +
sin(x))) + sin(x)
```

3.83.9 Mupad [B] (verification not implemented)

Time = 26.92 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.45

$$\int \cot(5x)\sin(x)\,dx = \sin(x) - \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\frac{25\sqrt{2}\sin(x)\sqrt{\sqrt{5}+5}}{2}+\frac{11\sqrt{2}\sqrt{5}\sin(x)\sqrt{\sqrt{5}+5}}{2}}{20\sqrt{5}+45}\right)\sqrt{\sqrt{5}+5}}{10} + \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\frac{25\sqrt{2}\sin(x)\sqrt{5-\sqrt{5}}}{2}-\frac{11\sqrt{2}\sqrt{5}\sin(x)\sqrt{5-\sqrt{5}}}{2}}{20\sqrt{5}-45}\right)\sqrt{5-\sqrt{5}}}{10}$$

input
```
int(cot(5*x)*sin(x),x)
```

output
```
sin(x) - (2^(1/2)*atanh(((25*2^(1/2)*sin(x)*(5^(1/2) + 5)^(1/2))/2 + (11*2
^(1/2)*5^(1/2)*sin(x)*(5^(1/2) + 5)^(1/2))/2)/(20*5^(1/2) + 45))*(5^(1/2)
+ 5)^(1/2))/10 + (2^(1/2)*atanh(((25*2^(1/2)*sin(x)*(5 - 5^(1/2))^(1/2))/2
 - (11*2^(1/2)*5^(1/2)*sin(x)*(5 - 5^(1/2))^(1/2))/2)/(20*5^(1/2) - 45))*(
5 - 5^(1/2))^(1/2))/10
```

3.84 $\int \cot(6x)\sin(x)\,dx$

3.84.1 Optimal result

Integrand size = 7, antiderivative size = 38

$$\int \cot(6x)\sin(x)\,dx = -\frac{1}{6}\text{arctanh}(\sin(x)) - \frac{1}{6}\text{arctanh}(2\sin(x)) - \frac{\text{arctanh}\left(\frac{2\sin(x)}{\sqrt{3}}\right)}{2\sqrt{3}} + \sin(x)$$

output
```
-1/6*arctanh(sin(x))-1/6*arctanh(2*sin(x))+sin(x)-1/6*arctanh(2/3*sin(x)*3
^(1/2))*3^(1/2)
```

3.84.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \cot(6x)\sin(x)\,dx = -\frac{1}{6}\text{arctanh}(\sin(x)) - \frac{1}{6}\text{arctanh}(2\sin(x)) - \frac{\text{arctanh}\left(\frac{2\sin(x)}{\sqrt{3}}\right)}{2\sqrt{3}} + \sin(x)$$

input
```
Integrate[Cot[6*x]*Sin[x],x]
```

output
```
-1/6*ArcTanh[Sin[x]] - ArcTanh[2*Sin[x]]/6 - ArcTanh[(2*Sin[x])/Sqrt[3]]/(
2*Sqrt[3]) + Sin[x]
```

3.84.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3042, 4878, 27, 2460, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\cot(6x)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\cot(6x)}{\csc(x)}dx$$

$$\Big\downarrow \ 4878$$

$$\int \frac{-32\sin^6(x)+48\sin^4(x)-18\sin^2(x)+1}{2\left(-16\sin^6(x)+32\sin^4(x)-19\sin^2(x)+3\right)}d\sin(x)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{2}\int \frac{-32\sin^6(x)+48\sin^4(x)-18\sin^2(x)+1}{-16\sin^6(x)+32\sin^4(x)-19\sin^2(x)+3}d\sin(x)$$

$$\Big\downarrow \ 2460$$

$$\frac{1}{2}\int \left(\frac{2}{4\sin^2(x)-3}+\frac{2}{3\left(4\sin^2(x)-1\right)}+2+\frac{1}{3\left(\sin^2(x)-1\right)}\right)d\sin(x)$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(-\frac{1}{3}\text{arctanh}(\sin(x))-\frac{1}{3}\text{arctanh}(2\sin(x))-\frac{\text{arctanh}\left(\frac{2\sin(x)}{\sqrt{3}}\right)}{\sqrt{3}}+2\sin(x)\right)$$

input
```
Int[Cot[6*x]*Sin[x],x]
```

output
```
(-1/3*ArcTanh[Sin[x]] - ArcTanh[2*Sin[x]]/3 - ArcTanh[(2*Sin[x])/Sqrt[3]]/
Sqrt[3] + 2*Sin[x])/2
```

3.84.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2460
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px /. x -> Sqrt[x]]},
Int[ExpandIntegrand[u*(Qx /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[Q
x, x]]] /; PolyQ[Px, x^2] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&
 !TrinomialQ[Px, x] && ILtQ[p, 0] && RationalFunctionQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

3.84.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 124, normalized size of antiderivative = 3.26

method	result
risch	$-\frac{i e^{i x}}{2}+\frac{i e^{-i x}}{2}-\frac{\ln \left(i+e^{i x}\right)}{6}+\frac{\ln \left(e^{i x}-i\right)}{6}+\frac{\sqrt{3}\, \ln \left(e^{2 i x}-i \sqrt{3}\, e^{i x}-1\right)}{12}-\frac{\sqrt{3}\, \ln \left(e^{2 i x}+i \sqrt{3}\, e^{i x}-1\right)}{12}-\frac{\ln \left(i e^{i x}+e^{2 i x}-1\right)}{12}+$

input
```
int(cot(6*x)*sin(x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*exp(I*x)+1/2*I*exp(-I*x)-1/6*ln(I+exp(I*x))+1/6*ln(exp(I*x)-I)+1/12
*3^(1/2)*ln(exp(2*I*x)-I*3^(1/2)*exp(I*x)-1)-1/12*3^(1/2)*ln(exp(2*I*x)+I*
3^(1/2)*exp(I*x)-1)-1/12*ln(I*exp(I*x)+exp(2*I*x)-1)+1/12*ln(-I*exp(I*x)+e
xp(2*I*x)-1)
```

3.84.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(28) = 56.

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.84

$$\int \cot(6x)\sin(x)\,dx = \frac{1}{12}\sqrt{3}\log\left(-\frac{4\cos(x)^2+4\sqrt{3}\sin(x)-7}{4\cos(x)^2-1}\right) - \frac{1}{12}\log(2\sin(x)+1) - \frac{1}{12}\log(\sin(x)+1) + \frac{1}{12}\log(-\sin(x)+1) + \frac{1}{12}\log(-2\sin(x)+1) + \sin(x)$$

input
```
integrate(cot(6*x)*sin(x),x, algorithm="fricas")
```

output
```
1/12*sqrt(3)*log(-(4*cos(x)^2 + 4*sqrt(3)*sin(x) - 7)/(4*cos(x)^2 - 1)) -
1/12*log(2*sin(x) + 1) - 1/12*log(sin(x) + 1) + 1/12*log(-sin(x) + 1) + 1/
12*log(-2*sin(x) + 1) + sin(x)
```

3.84.6 Sympy [F]

$$\int \cot(6x)\sin(x)\,dx = \int \sin(x)\cot(6x)\,dx$$

input
```
integrate(cot(6*x)*sin(x),x)
```

output
```
Integral(sin(x)*cot(6*x), x)
```

3.84.7 Maxima [F]

$$\int \cot(6x)\sin(x)\,dx = \int \cot(6x)\sin(x)\,dx$$

input
```
integrate(cot(6*x)*sin(x),x, algorithm="maxima")
```

output
```
-1/24*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 + 4/3*sqrt(3)*sin(x) + 4/3*c
os(x) + 4/3) - 1/24*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 + 4/3*sqrt(3)*
sin(x) - 4/3*cos(x) + 4/3) + 1/24*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2
- 4/3*sqrt(3)*sin(x) + 4/3*cos(x) + 4/3) + 1/24*sqrt(3)*log(4/3*cos(x)^2 +
 4/3*sin(x)^2 - 4/3*sqrt(3)*sin(x) - 4/3*cos(x) + 4/3) - integrate(-1/6*((
cos(3*x) + cos(x))*cos(4*x) - (cos(2*x) - 1)*cos(3*x) - cos(2*x)*cos(x) +
(sin(3*x) + sin(x))*sin(4*x) - sin(3*x)*sin(2*x) - sin(2*x)*sin(x) + cos(x
))/(2*(cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - cos(2*x)^2 - sin(4*x)^2 + 2*s
in(4*x)*sin(2*x) - sin(2*x)^2 + 2*cos(2*x) - 1), x) - 1/12*log(cos(x)^2 +
sin(x)^2 + 2*sin(x) + 1) + 1/12*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1) +
sin(x)
```

3.84.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(28) = 56$.

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.84

$$\int \cot(6x)\sin(x)\,dx = \frac{1}{12}\sqrt{3}\log\left(\frac{|-4\sqrt{3}+8\sin(x)|}{|4\sqrt{3}+8\sin(x)|}\right) - \frac{1}{12}\log(\sin(x)+1) + \frac{1}{12}\log(-\sin(x)+1) - \frac{1}{12}\log(|2\sin(x)+1|) + \frac{1}{12}\log(|2\sin(x)-1|) + \sin(x)$$

input
```
integrate(cot(6*x)*sin(x),x, algorithm="giac")
```

output
```
1/12*sqrt(3)*log(abs(-4*sqrt(3) + 8*sin(x))/abs(4*sqrt(3) + 8*sin(x))) - 1
/12*log(sin(x) + 1) + 1/12*log(-sin(x) + 1) - 1/12*log(abs(2*sin(x) + 1))
+ 1/12*log(abs(2*sin(x) - 1)) + sin(x)
```

3.84.9 Mupad [B] (verification not implemented)

Time = 28.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \cot(6x)\sin(x)\,dx = \sin(x) - \frac{\operatorname{atanh}(2\sin(x))}{6} - \frac{\operatorname{atanh}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{3} - \frac{\sqrt{3}\operatorname{atanh}\left(\frac{2\sqrt{3}\sin(x)}{3}\right)}{6}$$

input
```
int(cot(6*x)*sin(x),x)
```

output
```
sin(x) - atanh(2*sin(x))/6 - atanh(sin(x/2)/cos(x/2))/3 - (3^(1/2)*atanh((
2*3^(1/2)*sin(x))/3))/6
```

3.84. $\int \cot(6x)\sin(x)\,dx$

3.85 $\int \sec(2x)\sin(x)\,dx$

3.85.1 Optimal result

Integrand size = 7, antiderivative size = 15

$$\int \sec(2x)\sin(x)\,dx = \frac{\operatorname{arctanh}\left(\sqrt{2}\cos(x)\right)}{\sqrt{2}}$$

output
```
1/2*arctanh(cos(x)*2^(1/2))*2^(1/2)
```

3.85.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 35 vs. $2(15) = 30$.

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.33

$$\int \sec(2x)\sin(x)\,dx = \frac{\operatorname{arctanh}\left(\sqrt{2}-\tan\left(\frac{x}{2}\right)\right)+\operatorname{arctanh}\left(\sqrt{2}+\tan\left(\frac{x}{2}\right)\right)}{\sqrt{2}}$$

input
```
Integrate[Sec[2*x]*Sin[x],x]
```

output
```
(ArcTanh[Sqrt[2] - Tan[x/2]] + ArcTanh[Sqrt[2] + Tan[x/2]])/Sqrt[2]
```

3.85.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 4857, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\sec(2x)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin(x)}{\cos(2x)}dx$$

$$\Big\downarrow \; 4857$$

$$-\int \frac{1}{2\cos^2(x)-1}d\cos(x)$$

$$\Big\downarrow \; 220$$

$$\frac{\text{arctanh}\left(\sqrt{2}\cos(x)\right)}{\sqrt{2}}$$

input
```
Int[Sec[2*x]*Sin[x],x]
```

output
```
ArcTanh[Sqrt[2]*Cos[x]]/Sqrt[2]
```

3.85.3.1 Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4857
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[1, Cos[c*(a +
b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*
x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

3.85.4 Maple [A] (verified)

Time = 1.57 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{\operatorname{arctanh}\left(\cos(x)\sqrt{2}\right)\sqrt{2}}{2}$	13
risch	$\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}+\sqrt{2}\,\mathrm{e}^{ix}+1\right)}{4}-\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}-\sqrt{2}\,\mathrm{e}^{ix}+1\right)}{4}$	47

input
```
int(sec(2*x)*sin(x),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctanh(cos(x)*2^(1/2))*2^(1/2)
```

3.85.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(12) = 24.

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.20

$$\int \sec(2x)\sin(x)\,dx = \frac{1}{4}\sqrt{2}\log\left(-\frac{2\cos(x)^2+2\sqrt{2}\cos(x)+1}{2\cos(x)^2-1}\right)$$

input
```
integrate(sec(2*x)*sin(x),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*log(-(2*cos(x)^2 + 2*sqrt(2)*cos(x) + 1)/(2*cos(x)^2 - 1))
```

3.85.6 Sympy [F]

$$\int \sec(2x)\sin(x)\,dx = \int \sin(x)\sec(2x)\,dx$$

input
```
integrate(sec(2*x)*sin(x),x)
```

output
```
Integral(sin(x)*sec(2*x), x)
```

3.85.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(12) = 24.

Time = 0.28 (sec) , antiderivative size = 129, normalized size of antiderivative = 8.60

$$\int \sec(2x)\sin(x)\,dx = \frac{1}{8}\sqrt{2}\log\left(2\sqrt{2}\sin(2x)\sin(x) + 2\left(\sqrt{2}\cos(x) + 1\right)\cos(2x) + \cos(2x)^2 + 2\cos(x)^2 + \sin(2x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) + 1\right) - \frac{1}{8}\sqrt{2}\log\left(-2\sqrt{2}\sin(2x)\sin(x) - 2\left(\sqrt{2}\cos(x) - 1\right)\cos(2x) + \cos(2x)^2 + 2\cos(x)^2 + \sin(2x)^2 + 2\sin(x)^2 - 2\sqrt{2}\cos(x) + 1\right)$$

input
```
integrate(sec(2*x)*sin(x),x, algorithm="maxima")
```

output
```
1/8*sqrt(2)*log(2*sqrt(2)*sin(2*x)*sin(x) + 2*(sqrt(2)*cos(x) + 1)*cos(2*x
) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) +
 1) - 1/8*sqrt(2)*log(-2*sqrt(2)*sin(2*x)*sin(x) - 2*(sqrt(2)*cos(x) - 1)*
cos(2*x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 - 2*sqrt(2)*c
os(x) + 1)
```

3.85.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(12) = 24$.

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 3.27

$$\int \sec(2x)\sin(x)\,dx = \frac{1}{4}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2} - \frac{2(\cos(x)-1)}{\cos(x)+1} - 6\right|}{\left|4\sqrt{2} - \frac{2(\cos(x)-1)}{\cos(x)+1} - 6\right|}\right)$$

input
```
integrate(sec(2*x)*sin(x),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*log(abs(-4*sqrt(2) - 2*(cos(x) - 1)/(cos(x) + 1) - 6)/abs(4*sq
rt(2) - 2*(cos(x) - 1)/(cos(x) + 1) - 6))
```

3.85.9 Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \sec(2x)\sin(x)\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\,\cos(x)\right)}{2}$$

input
```
int(sin(x)/cos(2*x),x)
```

output
```
(2^(1/2)*atanh(2^(1/2)*cos(x)))/2
```

3.86 $\int \sec(3x)\sin(x)\,dx$

3.86.1 Optimal result

Integrand size = 7, antiderivative size = 21

$$\int \sec(3x)\sin(x)\,dx = \frac{1}{3}\log(\cos(x)) - \frac{1}{6}\log\left(3-4\cos^2(x)\right)$$

output
```
1/3*ln(cos(x))-1/6*ln(3-4*cos(x)^2)
```

3.86.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \sec(3x)\sin(x)\,dx = -\frac{1}{3}\text{arctanh}\left(\frac{1}{3}\left(-5+8\sin^2(x)\right)\right)$$

input
```
Integrate[Sec[3*x]*Sin[x],x]
```

output
```
-1/3*ArcTanh[(-5 + 8*Sin[x]^2)/3]
```

3.86.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 4857, 25, 243, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\sec(3x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)}{\cos(3x)}dx$$

$$\Big\downarrow \text{4857}$$

$$-\int -\frac{\sec(x)}{3-4\cos^2(x)}d\cos(x)$$

$$\Big\downarrow \text{25}$$

$$\int \frac{\sec(x)}{3-4\cos^2(x)}d\cos(x)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \frac{\sec(x)}{3-4\cos^2(x)}d\cos^2(x)$$

$$\Big\downarrow \text{47}$$

$$\frac{1}{2}\left(\frac{4}{3}\int \frac{1}{3-4\cos^2(x)}d\cos^2(x)+\frac{1}{3}\int \sec(x)d\cos^2(x)\right)$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{2}\left(\frac{4}{3}\int \frac{1}{3-4\cos^2(x)}d\cos^2(x)+\frac{1}{3}\log\left(\cos^2(x)\right)\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(\frac{1}{3}\log\left(\cos^2(x)\right)-\frac{1}{3}\log\left(3-4\cos^2(x)\right)\right)$$

input
```
Int[Sec[3*x]*Sin[x],x]
```

output
```
(Log[Cos[x]^2]/3 - Log[3 - 4*Cos[x]^2]/3)/2
```

3.86.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4857
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[1, Cos[c*(a +
b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*
x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

3.86.4 Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{\ln(\cos(x))}{3}-\frac{\ln\left(4\cos(x)^{2}-3\right)}{6}$	18
risch	$\frac{\ln(e^{2ix}+1)}{3}-\frac{\ln(e^{4ix}-e^{2ix}+1)}{6}$	29

input
```
int(sec(3*x)*sin(x),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(cos(x))-1/6*ln(4*cos(x)^2-3)
```

3.86.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \sec(3x)\sin(x)\,dx = -\frac{1}{6}\log\left(4\cos(x)^2-3\right)+\frac{1}{3}\log\left(-\cos(x)\right)$$

input
```
integrate(sec(3*x)*sin(x),x, algorithm="fricas")
```

output
```
-1/6*log(4*cos(x)^2 - 3) + 1/3*log(-cos(x))
```

3.86.6 Sympy [F]

$$\int \sec(3x)\sin(x)\,dx = \int \sin(x)\sec(3x)\,dx$$

input
```
integrate(sec(3*x)*sin(x),x)
```

output
```
Integral(sin(x)*sec(3*x), x)
```

3.86.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(17) = 34.

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 3.86

$$\int \sec(3x)\sin(x)\,dx = -\frac{1}{12}\log\left(-2\left(\cos(2x)-1\right)\cos(4x)+\cos(4x)^2+\cos(2x)^2+\sin(4x)^2-2\sin(4x)\sin(2x)+\sin(2x)^2-2\cos(2x)+1\right)+\frac{1}{6}\log\left(\cos(2x)^2+\sin(2x)^2+2\cos(2x)+1\right)$$

input
```
integrate(sec(3*x)*sin(x),x, algorithm="maxima")
```

output
```
-1/12*log(-2*(cos(2*x) - 1)*cos(4*x) + cos(4*x)^2 + cos(2*x)^2 + sin(4*x)^
2 - 2*sin(4*x)*sin(2*x) + sin(2*x)^2 - 2*cos(2*x) + 1) + 1/6*log(cos(2*x)^
2 + sin(2*x)^2 + 2*cos(2*x) + 1)
```

3.86.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int \sec(3x)\sin(x)\,dx = \frac{1}{6}\log\left(-\sin(x)^2+1\right) - \frac{1}{6}\log\left(\left|4\sin(x)^2-1\right|\right)$$

input
```
integrate(sec(3*x)*sin(x),x, algorithm="giac")
```

output
```
1/6*log(-sin(x)^2 + 1) - 1/6*log(abs(4*sin(x)^2 - 1))
```

3.86.9 Mupad [B] (verification not implemented)

Time = 27.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \sec(3x)\sin(x)\,dx = \frac{\ln(\cos(x))}{3} - \frac{\ln\left(\cos(x)^2-\frac{3}{4}\right)}{6}$$

input
```
int(sin(x)/cos(3*x),x)
```

output
```
log(cos(x))/3 - log(cos(x)^2 - 3/4)/6
```

3.87 $\int \sec(4x)\sin(x)\,dx$

3.87.1 Optimal result

Integrand size = 7, antiderivative size = 71

$$\int \sec(4x)\sin(x)\,dx = -\frac{\operatorname{arctanh}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}}+\frac{\operatorname{arctanh}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}$$

output
```
-1/2*arctanh(2*cos(x)/(2-2^(1/2))^(1/2))/(4-2*2^(1/2))^(1/2)+1/2*arctanh(2
*cos(x)/(2+2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2)
```

3.87.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 52.97 (sec) , antiderivative size = 4814, normalized size of antiderivative = 67.80

$$\int \sec(4x)\sin(x)\,dx = \text{Result too large to show}$$

input
```
Integrate[Sec[4*x]*Sin[x],x]
```

output
```
((-2*(-1)^(3/8)*(1 + Sqrt[2])*x - (2*(-1)^(1/4)*(-2 - (1 - I)*(-1)^(5/8) +
 (-1)^(5/8)*Sqrt[2])*ArcTan[(-Cos[x] + (1 + Sqrt[2])*Sin[x])/(2*(-1)^(3/8)
 + Cos[x] - Sqrt[2]*Cos[x] + Sin[x])]])/((-1 + I) + 2*(-1)^(3/8) + Sqrt[2])
 - (2*(1 - I)^(3/2)*2^(1/4)*((-3 - I) + 2*(-1)^(5/8) + (2 + I)*Sqrt[2] - (
2 + 2*I)*(-1)^(3/8)*Sqrt[2] + 2*(-1)^(5/8)*Sqrt[2])*ArcTan[((1 + I) + I*Sq
rt[2] + ((-1 + I) + 2*(-1)^(3/8) + Sqrt[2])*Tan[x/2])/(Sqrt[1 - I]*2^(3/4)
)])/((-1 + I) + 2*(-1)^(3/8) + Sqrt[2]) + 2*(-1)^(3/8)*Log[Sec[x/2]^2] + (
(-1)^(3/4)*(-2 - (1 - I)*(-1)^(5/8) + (-1)^(5/8)*Sqrt[2])*Log[-(Sec[x/2]^4
*(-2 + (1 - I)*Sqrt[2] + 2*(-1)^(3/8)*(-1 + Sqrt[2])*Cos[x] + Sqrt[2]*Cos[
2*x] - 2*(-1)^(3/8)*Sin[x] + Sqrt[2]*Sin[2*x]))])/((-1 + I) + 2*(-1)^(3/8)
 + Sqrt[2]))*((-1/2 - I/2)/(((-1 + I) + Sqrt[1 - I]*Sqrt[1 + I])*(-((-1 -
I)^(3/2)*(1 - I)^(1/4)*(1 + I)^(1/4)) - (1 + I)*Cos[x] + I*Sqrt[1 - I]*Sqr
t[1 + I]*Cos[x] + (1 - I)*Sin[x] + Sqrt[1 - I]*Sqrt[1 + I]*Sin[x])) - Sin[
x]/(Sqrt[-1 - I]*(1 - I)^(1/4)*(1 + I)^(1/4)*((-1 + I) + Sqrt[1 - I]*Sqrt[
1 + I])*(-((-1 - I)^(3/2)*(1 - I)^(1/4)*(1 + I)^(1/4)) - (1 + I)*Cos[x] +
I*Sqrt[1 - I]*Sqrt[1 + I]*Cos[x] + (1 - I)*Sin[x] + Sqrt[1 - I]*Sqrt[1 + I
]*Sin[x])) - ((I/2)*Sqrt[-1 - I]*(1 - I)^(1/4)*(1 + I)^(1/4)*Sin[x])/(((-1
 + I) + Sqrt[1 - I]*Sqrt[1 + I])*(-((-1 - I)^(3/2)*(1 - I)^(1/4)*(1 + I)^(
1/4)) - (1 + I)*Cos[x] + I*Sqrt[1 - I]*Sqrt[1 + I]*Cos[x] + (1 - I)*Sin[x]
 + Sqrt[1 - I]*Sqrt[1 + I]*Sin[x]))))/(-2*(-1)^(3/8)*(1 + Sqrt[2]) - (2...
```

3.87.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4857, 1406, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\sec(4x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)}{\cos(4x)}dx$$

$$\Big\downarrow \text{4857}$$

$$-\int \frac{1}{8\cos^4(x)-8\cos^2(x)+1}d\cos(x)$$

$$\Big\downarrow \text{1406}$$

$$\sqrt{2}\int \frac{1}{8\cos^2(x)-2\left(2-\sqrt{2}\right)}d\cos(x)-\sqrt{2}\int \frac{1}{8\cos^2(x)-2\left(2+\sqrt{2}\right)}d\cos(x)$$

$$\Big\downarrow \; 220$$

$$\frac{\operatorname{arctanh}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}-\frac{\operatorname{arctanh}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}}$$

input
```
Int[Sec[4*x]*Sin[x],x]
```

output
```
-1/2*ArcTanh[(2*Cos[x])/Sqrt[2 - Sqrt[2]]]/Sqrt[2*(2 - Sqrt[2])] + ArcTanh
[(2*Cos[x])/Sqrt[2 + Sqrt[2]]]/(2*Sqrt[2*(2 + Sqrt[2])])
```

3.87.3.1 Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4857
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[1, Cos[c*(a +
b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*
x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

3.87.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.75 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.66

method	result	size
risch	$-i\left(\sum_{\_R=\mathrm{RootOf}\left(2048\_Z^{4}+128\_Z^{2}+1\right)}\_R\ln\left(\mathrm{e}^{2ix}+\left(-512i\_R^{3}-24i\_R\right)\mathrm{e}^{ix}+1\right)\right)$	47
default	$\frac{\sqrt{2}\,\mathrm{arctanh}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2+\sqrt{2}}}-\frac{\sqrt{2}\,\mathrm{arctanh}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2-\sqrt{2}}}$	54

input
```
int(sec(4*x)*sin(x),x,method=_RETURNVERBOSE)
```

output
```
-I*sum(_R*ln(exp(2*I*x)+(-512*I*_R^3-24*I*_R)*exp(I*x)+1),_R=RootOf(2048*_
Z^4+128*_Z^2+1))
```

3.87.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(49) = 98.

Time = 0.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.70

$$\begin{aligned}\int \sec(4x)\sin(x)\,dx = &-\frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\sqrt{\sqrt{2}+2}\left(\sqrt{2}-1\right)+2\cos\left(x\right)\right)\\ &+\frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\sqrt{\sqrt{2}+2}\left(\sqrt{2}-1\right)-2\cos\left(x\right)\right)\\ &+\frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(\left(\sqrt{2}+1\right)\sqrt{-\sqrt{2}+2}+2\cos\left(x\right)\right)\\ &-\frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(\left(\sqrt{2}+1\right)\sqrt{-\sqrt{2}+2}-2\cos\left(x\right)\right)\end{aligned}$$

input
```
integrate(sec(4*x)*sin(x),x, algorithm="fricas")
```

output
```
-1/8*sqrt(sqrt(2) + 2)*log(sqrt(sqrt(2) + 2)*(sqrt(2) - 1) + 2*cos(x)) + 1
/8*sqrt(sqrt(2) + 2)*log(sqrt(sqrt(2) + 2)*(sqrt(2) - 1) - 2*cos(x)) + 1/8
*sqrt(-sqrt(2) + 2)*log((sqrt(2) + 1)*sqrt(-sqrt(2) + 2) + 2*cos(x)) - 1/8
*sqrt(-sqrt(2) + 2)*log((sqrt(2) + 1)*sqrt(-sqrt(2) + 2) - 2*cos(x))
```

3.87.6 Sympy [F]

$$\int \sec(4x)\sin(x)\,dx = \int \sin{\left(x \right)} \sec{\left(4x \right)}\,dx$$

input
```
integrate(sec(4*x)*sin(x),x)
```

output
```
Integral(sin(x)*sec(4*x), x)
```

3.87.7 Maxima [F]

$$\int \sec(4x)\sin(x)\,dx = \int \sec\left(4\,x\right)\sin\left(x\right)\,dx$$

input
```
integrate(sec(4*x)*sin(x),x, algorithm="maxima")
```

output
```
integrate(sec(4*x)*sin(x), x)
```

3.87.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. 2(49) = 98.

Time = 0.27 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.87

$$\begin{aligned}\int \sec(4x)\sin(x)\,dx =& -\frac{2.16139547686000\log\left(-\frac{\cos(x)-1}{\cos(x)+1}-0.0395661298966000\right)}{\frac{140\,(\cos(x)-1)}{\cos(x)+1}+28.1312524456150}\\&-\frac{4.18450863968000\log\left(-\frac{\cos(x)-1}{\cos(x)+1}-0.446462692172000\right)}{\frac{140\,(\cos(x)-1)}{\cos(x)+1}+44.3876588494000}\\&-\frac{20.9929814212000\log\left(-\frac{\cos(x)-1}{\cos(x)+1}-2.23982880884000\right)}{\frac{140\,(\cos(x)-1)}{\cos(x)+1}+404.466590643000}\\&-\frac{1380.66111446200\log\left(-\frac{\cos(x)-1}{\cos(x)+1}-25.2741423691000\right)}{\frac{140\,(\cos(x)-1)}{\cos(x)+1}-10892.9855019000}\end{aligned}$$

input
```
integrate(sec(4*x)*sin(x),x, algorithm="giac")
```

output
```
-2.16139547686000*log(-(cos(x) - 1)/(cos(x) + 1) - 0.0395661298966000)/(14
0*(cos(x) - 1)/(cos(x) + 1) + 28.1312524456150) - 4.18450863968000*log(-(c
os(x) - 1)/(cos(x) + 1) - 0.446462692172000)/(140*(cos(x) - 1)/(cos(x) + 1
) + 44.3876588494000) - 20.9929814212000*log(-(cos(x) - 1)/(cos(x) + 1) -
2.23982880884000)/(140*(cos(x) - 1)/(cos(x) + 1) + 404.466590643000) - 138
0.66111446200*log(-(cos(x) - 1)/(cos(x) + 1) - 25.2741423691000)/(140*(cos
(x) - 1)/(cos(x) + 1) - 10892.9855019000)
```

3.87.9 Mupad [B] (verification not implemented)

Time = 27.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.58

$$\int \sec(4x)\sin(x)\,dx = \frac{\operatorname{atanh}\left(\frac{\cos(x)\sqrt{2-\sqrt{2}}}{64\left(\frac{\sqrt{2}}{128}-\frac{1}{64}\right)} - \frac{\sqrt{2}\cos(x)\sqrt{2-\sqrt{2}}}{64\left(\frac{\sqrt{2}}{128}-\frac{1}{64}\right)}\right)\sqrt{2-\sqrt{2}}}{4} - \frac{\operatorname{atanh}\left(\frac{\cos(x)\sqrt{\sqrt{2}+2}}{64\left(\frac{\sqrt{2}}{128}+\frac{1}{64}\right)} + \frac{\sqrt{2}\cos(x)\sqrt{\sqrt{2}+2}}{64\left(\frac{\sqrt{2}}{128}+\frac{1}{64}\right)}\right)\sqrt{\sqrt{2}+2}}{4}$$

input
```
int(sin(x)/cos(4*x),x)
```

output
```
(atanh((cos(x)*(2 - 2^(1/2))^(1/2))/(64*(2^(1/2)/128 - 1/64)) - (2^(1/2)*c
os(x)*(2 - 2^(1/2))^(1/2))/(64*(2^(1/2)/128 - 1/64)))*(2 - 2^(1/2))^(1/2))
/4 - (atanh((cos(x)*(2^(1/2) + 2)^(1/2))/(64*(2^(1/2)/128 + 1/64)) + (2^(1
/2)*cos(x)*(2^(1/2) + 2)^(1/2))/(64*(2^(1/2)/128 + 1/64)))*(2^(1/2) + 2)^(
1/2))/4
```

3.88 $\int \sec(5x)\sin(x)\,dx$

3.88.1 Optimal result

Integrand size = 7, antiderivative size = 62

$$\int \sec(5x)\sin(x)\,dx = -\frac{1}{5}\log(\cos(x)) + \frac{1}{20}\left(1+\sqrt{5}\right)\log\left(5-\sqrt{5}-8\cos^2(x)\right) + \frac{1}{20}\left(1-\sqrt{5}\right)\log\left(5+\sqrt{5}-8\cos^2(x)\right)$$

output
```
-1/5*ln(cos(x))+1/20*ln(5-8*cos(x)^2+5^(1/2))*(-5^(1/2)+1)+1/20*ln(5-8*cos
(x)^2-5^(1/2))*(5^(1/2)+1)
```

3.88.2 Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

$$\int \sec(5x)\sin(x)\,dx = \frac{1}{20}\left(-4\log(\cos(x)) - \left(-1+\sqrt{5}\right)\log\left(-1-\sqrt{5}+4\cos(2x)\right) + \left(1+\sqrt{5}\right)\log\left(-1+\sqrt{5}+4\cos(2x)\right)\right)$$

input
```
Integrate[Sec[5*x]*Sin[x],x]
```

output
```
(-4*Log[Cos[x]] - (-1 + Sqrt[5])*Log[-1 - Sqrt[5] + 4*Cos[2*x]] + (1 + Sqr
t[5])*Log[-1 + Sqrt[5] + 4*Cos[2*x]])/20
```

3.88.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.29, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3042, 4857, 1434, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\sec(5x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)}{\cos(5x)}dx$$

$$\Big\downarrow \text{4857}$$

$$-\int \frac{\sec(x)}{16\cos^4(x)-20\cos^2(x)+5}d\cos(x)$$

$$\Big\downarrow \text{1434}$$

$$-\frac{1}{2}\int \frac{\sec(x)}{16\cos^4(x)-20\cos^2(x)+5}d\cos^2(x)$$

$$\Big\downarrow \text{1141}$$

$$-8\int \left(\frac{\sec(x)}{80}+\frac{1}{\sqrt{5}\left(5-\sqrt{5}\right)\left(-8\cos^2(x)-\sqrt{5}+5\right)}-\frac{1}{\sqrt{5}\left(5+\sqrt{5}\right)\left(-8\cos^2(x)+\sqrt{5}+5\right)}\right)d\cos^2(x)$$

$$\Big\downarrow \text{2009}$$

$$-8\left(\frac{1}{80}\log\left(\cos^2(x)\right)-\frac{\log\left(-8\cos^2(x)-\sqrt{5}+5\right)}{8\sqrt{5}\left(5-\sqrt{5}\right)}+\frac{\log\left(-8\cos^2(x)+\sqrt{5}+5\right)}{8\sqrt{5}\left(5+\sqrt{5}\right)}\right)$$

input
```
Int[Sec[5*x]*Sin[x],x]
```

output
```
-8*(Log[Cos[x]^2]/80 - Log[5 - Sqrt[5] - 8*Cos[x]^2]/(8*Sqrt[5]*(5 - Sqrt[
5])) + Log[5 + Sqrt[5] - 8*Cos[x]^2]/(8*Sqrt[5]*(5 + Sqrt[5])))
```

3.88.3.1 Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4857
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[1, Cos[c*(a +
b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*
x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

3.88.4 Maple [A] (verified)

Time = 2.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.69

method	result
default	$-\frac{\ln(\cos(x))}{5}+\frac{\ln\left(16\cos(x)^4-20\cos(x)^2+5\right)}{20}+\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{\left(32\cos(x)^2-20\right)\sqrt{5}}{20}\right)}{10}$
risch	$-\frac{\ln(e^{2ix}+1)}{5}+\frac{\ln\left(e^{4ix}+\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)e^{2ix}+1\right)}{20}+\frac{\ln\left(e^{4ix}+\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)e^{2ix}+1\right)\sqrt{5}}{20}+\frac{\ln\left(e^{4ix}+\left(-\frac{1}{2}-\frac{\sqrt{5}}{2}\right)e^{2ix}+1\right)}{20}-\frac{\ln\left(e^{4ix}+\left(-\right.\right.}{}$

input
```
int(sec(5*x)*sin(x),x,method=_RETURNVERBOSE)
```

output
```
-1/5*ln(cos(x))+1/20*ln(16*cos(x)^4-20*cos(x)^2+5)+1/10*5^(1/2)*arctanh(1/
20*(32*cos(x)^2-20)*5^(1/2))
```

3.88.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.16

$$\int \sec(5x)\sin(x)\,dx = \frac{1}{20}\sqrt{5}\log\left(\frac{32\cos(x)^4 + 8(\sqrt{5}-5)\cos(x)^2 - 5\sqrt{5} + 15}{16\cos(x)^4 - 20\cos(x)^2 + 5}\right) + \frac{1}{20}\log\left(16\cos(x)^4 - 20\cos(x)^2 + 5\right) - \frac{1}{5}\log(-\cos(x))$$

input
```
integrate(sec(5*x)*sin(x),x, algorithm="fricas")
```

output
```
1/20*sqrt(5)*log((32*cos(x)^4 + 8*(sqrt(5) - 5)*cos(x)^2 - 5*sqrt(5) + 15)
/(16*cos(x)^4 - 20*cos(x)^2 + 5)) + 1/20*log(16*cos(x)^4 - 20*cos(x)^2 + 5
) - 1/5*log(-cos(x))
```

3.88.6 Sympy [F]

$$\int \sec(5x)\sin(x)\,dx = \int \sin(x)\sec(5x)\,dx$$

input
```
integrate(sec(5*x)*sin(x),x)
```

output
```
Integral(sin(x)*sec(5*x), x)
```

3.88.7 Maxima [F]

$$\int \sec(5x)\sin(x)\,dx = \int \sec(5x)\sin(x)\,dx$$

input
```
integrate(sec(5*x)*sin(x),x, algorithm="maxima")
```

output
```
1/5*integrate(-(cos(4*x)*sin(8*x) - cos(8*x)*sin(4*x) + cos(3/2*arctan2(si
n(4*x), cos(4*x)))*sin(4*x) + cos(1/2*arctan2(sin(4*x), cos(4*x)))*sin(4*x
) - cos(4*x)*sin(3/2*arctan2(sin(4*x), cos(4*x))) - cos(4*x)*sin(1/2*arcta
n2(sin(4*x), cos(4*x))) - sin(4*x))/(2*(cos(4*x) + 1)*cos(8*x) + cos(8*x)^
2 + cos(4*x)^2 - 2*(cos(8*x) + cos(4*x) - cos(1/2*arctan2(sin(4*x), cos(4*
x))) + 1)*cos(3/2*arctan2(sin(4*x), cos(4*x))) + cos(3/2*arctan2(sin(4*x),
 cos(4*x)))^2 - 2*(cos(8*x) + cos(4*x) + 1)*cos(1/2*arctan2(sin(4*x), cos(
4*x))) + cos(1/2*arctan2(sin(4*x), cos(4*x)))^2 + sin(8*x)^2 + 2*sin(8*x)*
sin(4*x) + sin(4*x)^2 - 2*(sin(8*x) + sin(4*x) - sin(1/2*arctan2(sin(4*x),
 cos(4*x))))*sin(3/2*arctan2(sin(4*x), cos(4*x))) + sin(3/2*arctan2(sin(4*
x), cos(4*x)))^2 - 2*(sin(8*x) + sin(4*x))*sin(1/2*arctan2(sin(4*x), cos(4
*x))) + sin(1/2*arctan2(sin(4*x), cos(4*x)))^2 + 2*cos(4*x) + 1), x) + 4/5
*integrate(-(cos(2*x)*sin(8*x) - cos(2*x)*sin(6*x) + cos(2*x)*sin(4*x) - c
os(8*x)*sin(2*x) + cos(6*x)*sin(2*x) - cos(4*x)*sin(2*x) - sin(2*x))/(2*(c
os(6*x) - cos(4*x) + cos(2*x) - 1)*cos(8*x) - cos(8*x)^2 + 2*(cos(4*x) - c
os(2*x) + 1)*cos(6*x) - cos(6*x)^2 + 2*(cos(2*x) - 1)*cos(4*x) - cos(4*x)^
2 - cos(2*x)^2 + 2*(sin(6*x) - sin(4*x) + sin(2*x))*sin(8*x) - sin(8*x)^2
+ 2*(sin(4*x) - sin(2*x))*sin(6*x) - sin(6*x)^2 - sin(4*x)^2 + 2*sin(4*x)*
sin(2*x) - sin(2*x)^2 + 2*cos(2*x) - 1), x) - 2/5*integrate(-(cos(4/3*arct
an2(sin(6*x), cos(6*x)))*sin(6*x) + cos(2/3*arctan2(sin(6*x), cos(6*x))...
```

3.88.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.08

$$\int \sec(5x)\sin(x)\,dx = \frac{1}{20}\sqrt{5}\log\left(\frac{\left|32\,\sin\left(x\right)^2-4\,\sqrt{5}-12\right|}{\left|32\,\sin\left(x\right)^2+4\,\sqrt{5}-12\right|}\right)-\frac{1}{10}\,\log\left(-\sin\left(x\right)^2+1\right)+\frac{1}{20}\,\log\left(\left|16\,\sin\left(x\right)^4-12\,\sin\left(x\right)^2+1\right|\right)$$

input
```
integrate(sec(5*x)*sin(x),x, algorithm="giac")
```

output
```
1/20*sqrt(5)*log(abs(32*sin(x)^2 - 4*sqrt(5) - 12)/abs(32*sin(x)^2 + 4*sqr
t(5) - 12)) - 1/10*log(-sin(x)^2 + 1) + 1/20*log(abs(16*sin(x)^4 - 12*sin(
x)^2 + 1))
```

3.88.9 Mupad [B] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.76

$$\int \sec(5x)\sin(x)\,dx = \ln\left(\cos(x)^2 + \frac{\sqrt{5}}{8} - \frac{5}{8}\right)\left(\frac{\sqrt{5}}{20} + \frac{1}{20}\right) - \ln\left(\cos(x)^2 - \frac{\sqrt{5}}{8} - \frac{5}{8}\right)\left(\frac{\sqrt{5}}{20} - \frac{1}{20}\right) - \frac{\ln(\cos(x))}{5}$$

input
```
int(sin(x)/cos(5*x),x)
```

output
```
log(cos(x)^2 + 5^(1/2)/8 - 5/8)*(5^(1/2)/20 + 1/20) - log(cos(x)^2 - 5^(1/
2)/8 - 5/8)*(5^(1/2)/20 - 1/20) - log(cos(x))/5
```

3.89 $\int \sec(6x)\sin(x)\,dx$

3.89.1 Optimal result

Integrand size = 7, antiderivative size = 85

$$\int \sec(6x)\sin(x)\,dx = -\frac{\operatorname{arctanh}\left(\sqrt{2}\cos(x)\right)}{3\sqrt{2}} + \frac{\operatorname{arctanh}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{3}}}\right)}{6\sqrt{2-\sqrt{3}}} + \frac{\operatorname{arctanh}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{3}}}\right)}{6\sqrt{2+\sqrt{3}}}$$

output
```
-1/6*arctanh(cos(x)*2^(1/2))*2^(1/2)+1/6*arctanh(2*cos(x)/(1/2*6^(1/2)-1/2
*2^(1/2)))/(1/2*6^(1/2)-1/2*2^(1/2))+1/6*arctanh(2*cos(x)/(1/2*6^(1/2)+1/2
*2^(1/2)))/(1/2*6^(1/2)+1/2*2^(1/2))
```

3.89.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 7.33 (sec) , antiderivative size = 627, normalized size of antiderivative = 7.38

$$\int \sec(6x)\sin(x)\,dx$$

$$= \frac{1}{24}\left((-4-4i)(-1)^{3/4}\operatorname{arctanh}\left(\frac{-1+\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right) - (4-4i)\sqrt[4]{-1}\operatorname{arctanh}\left(\frac{1+\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right)\right.$$

$$+ \frac{2(1+\sqrt{2})\left(x+2\sqrt{3}\operatorname{arctanh}\left(\frac{2+\left(2+\sqrt{2}\right)\tan\left(\frac{x}{2}\right)}{\sqrt{6}}\right) - \log\left(\sec^2\left(\frac{x}{2}\right)\right) + \log\left(-\sec^2\left(\frac{x}{2}\right)\left(\sqrt{2}-2\cos(x)+2\right.\right.\right.}{2+\sqrt{2}}$$

$$-\sqrt{2}\left(x-2\sqrt{3}\operatorname{arctanh}\left(\frac{\sqrt{2}+(-1+\sqrt{2})\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right)-\log\left(\sec^2\left(\frac{x}{2}\right)\right)+\log\left(\sec^2\left(\frac{x}{2}\right)\left(1+\sqrt{2}\cos(x)-\right.\right.\right.$$

$$+ \frac{2\left(2(\sqrt{2}+\sqrt{3})\operatorname{arctanh}\left(\frac{2+\left(2+\sqrt{6}\right)\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right) + (3+\sqrt{6})\left(x-\log\left(\sec^2\left(\frac{x}{2}\right)\right)+\log\left(-\sec^2\left(\frac{x}{2}\right)\left(\sqrt{6}-2\,\mathrm{c}\right.\right.\right.\right.}{(12+5\sqrt{6})\cos(2x)+2\cos(x)\left(5+2\sqrt{6}+5\sqrt{6}\sin(x)\right)-2\,(}$$

$$+ \frac{\left(-2(-2+\sqrt{6})\operatorname{arctanh}\left(\sqrt{2}+(\sqrt{2}-\sqrt{3})\tan\left(\frac{x}{2}\right)\right)+(3\sqrt{2}-2\sqrt{3})\left(x-\log\left(\sec^2\left(\frac{x}{2}\right)\right)+\log\left(-\sec^2\right.\right.\right.}{(-12+5\sqrt{6})\cos(2x)+2\cos(x)\left(-5+2\sqrt{6}+5\sqrt{6}\right.}$$

input
```
Integrate[Sec[6*x]*Sin[x],x]
```

output
```
((-4 - 4*I)*(-1)^(3/4)*ArcTanh[(-1 + Tan[x/2])/Sqrt[2]] - (4 - 4*I)*(-1)^(
1/4)*ArcTanh[(1 + Tan[x/2])/Sqrt[2]] + (2*(1 + Sqrt[2])*(x + 2*Sqrt[3]*Arc
Tanh[(2 + (2 + Sqrt[2])*Tan[x/2])/Sqrt[6]] - Log[Sec[x/2]^2] + Log[-(Sec[x
/2]^2*(Sqrt[2] - 2*Cos[x] + 2*Sin[x]))]))/(2 + Sqrt[2]) - Sqrt[2]*(x - 2*S
qrt[3]*ArcTanh[(Sqrt[2] + (-1 + Sqrt[2])*Tan[x/2])/Sqrt[3]] - Log[Sec[x/2]
^2] + Log[Sec[x/2]^2*(1 + Sqrt[2]*Cos[x] - Sqrt[2]*Sin[x])]) + (2*(2*(Sqrt
[2] + Sqrt[3])*ArcTanh[(2 + (2 + Sqrt[6])*Tan[x/2])/Sqrt[2]] + (3 + Sqrt[6
])*(x - Log[Sec[x/2]^2] + Log[-(Sec[x/2]^2*(Sqrt[6] - 2*Cos[x] + 2*Sin[x])
)]))*(1 + Sqrt[6]*Sin[x])*(3 + Sqrt[6] - (2 + Sqrt[6])*Cos[x] + (2 + Sqrt[
6])*Sin[x]))/((12 + 5*Sqrt[6])*Cos[2*x] + 2*Cos[x]*(5 + 2*Sqrt[6] + 5*Sqrt
[6]*Sin[x]) - 2*(12 + 5*Sqrt[6] + 4*(5 + 2*Sqrt[6])*Sin[x] - 6*Sin[2*x]))
+ ((-2*(-2 + Sqrt[6])*ArcTanh[Sqrt[2] + (Sqrt[2] - Sqrt[3])*Tan[x/2]] + (3
*Sqrt[2] - 2*Sqrt[3])*(x - Log[Sec[x/2]^2] + Log[-(Sec[x/2]^2*(Sqrt[3] + S
qrt[2]*Cos[x] - Sqrt[2]*Sin[x]))]))*(Sqrt[2] - 2*Sqrt[3]*Sin[x])*(-3 + Sqr
t[6] - (-2 + Sqrt[6])*Cos[x] + (-2 + Sqrt[6])*Sin[x]))/((-12 + 5*Sqrt[6])*
Cos[2*x] + 2*Cos[x]*(-5 + 2*Sqrt[6] + 5*Sqrt[6]*Sin[x]) - 2*(-12 + 5*Sqrt[
6] + 4*(-5 + 2*Sqrt[6])*Sin[x] + 6*Sin[2*x])))/24
```

3.89. $\int \sec(6x)\sin(x)\,dx$

3.89.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4857, 2460, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\sec(6x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)}{\cos(6x)}dx$$

$$\Big\downarrow \text{4857}$$

$$-\int \frac{1}{32\cos^6(x)-48\cos^4(x)+18\cos^2(x)-1}d\cos(x)$$

$$\Big\downarrow \text{2460}$$

$$-\int \left(\frac{4(2\cos^2(x)-1)}{3\,(16\cos^4(x)-16\cos^2(x)+1)}-\frac{1}{3\,(2\cos^2(x)-1)}\right)d\cos(x)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\text{arctanh}\left(\sqrt{2}\cos(x)\right)}{3\sqrt{2}}+\frac{\text{arctanh}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{3}}}\right)}{6\sqrt{2-\sqrt{3}}}+\frac{\text{arctanh}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{3}}}\right)}{6\sqrt{2+\sqrt{3}}}$$

input
```
Int[Sec[6*x]*Sin[x],x]
```

output
```
-1/3*ArcTanh[Sqrt[2]*Cos[x]]/Sqrt[2] + ArcTanh[(2*Cos[x])/Sqrt[2 - Sqrt[3]
]]/(6*Sqrt[2 - Sqrt[3]]) + ArcTanh[(2*Cos[x])/Sqrt[2 + Sqrt[3]]]/(6*Sqrt[2
 + Sqrt[3]])
```

3.89.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2460
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px /. x -> Sqrt[x]]},
Int[ExpandIntegrand[u*(Qx /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[Q
x, x]]] /; PolyQ[Px, x^2] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&
 !TrinomialQ[Px, x] && ILtQ[p, 0] && RationalFunctionQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4857
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[1, Cos[c*(a +
b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*
x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

3.89.4 Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.94

method	result
default	$\frac{2\,\mathrm{arctanh}\left(\frac{8\cos(x)}{2\sqrt{6}+2\sqrt{2}}\right)}{3\left(2\sqrt{6}+2\sqrt{2}\right)}+\frac{2\,\mathrm{arctanh}\left(\frac{8\cos(x)}{2\sqrt{6}-2\sqrt{2}}\right)}{3\left(2\sqrt{6}-2\sqrt{2}\right)}-\frac{\mathrm{arctanh}\left(\cos(x)\sqrt{2}\right)\sqrt{2}}{6}$
risch	$\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}-\sqrt{2}\,\mathrm{e}^{ix}+1\right)}{12}-\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}+\sqrt{2}\,\mathrm{e}^{ix}+1\right)}{12}-i\left(\sum_{\_R=\mathrm{RootOf}(20736\_Z^4+576\_Z^2+1)}\_R\ln\left(\mathrm{e}^{2ix}+\left(-1728i\_\right.\right.\right.$

input
```
int(sec(6*x)*sin(x),x,method=_RETURNVERBOSE)
```

output
```
2/3/(2*6^(1/2)+2*2^(1/2))*arctanh(8*cos(x)/(2*6^(1/2)+2*2^(1/2)))+2/3/(2*6
^(1/2)-2*2^(1/2))*arctanh(8*cos(x)/(2*6^(1/2)-2*2^(1/2)))-1/6*arctanh(cos(
x)*2^(1/2))*2^(1/2)
```

3.89.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(67) = 134.

Time = 0.28 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.80

$$\int \sec(6x)\sin(x)\,dx = -\frac{1}{12}\sqrt{\sqrt{3}+2}\log\left(\sqrt{\sqrt{3}+2}\left(\sqrt{3}-2\right)+2\cos(x)\right) + \frac{1}{12}\sqrt{\sqrt{3}+2}\log\left(\sqrt{\sqrt{3}+2}\left(\sqrt{3}-2\right)-2\cos(x)\right) + \frac{1}{12}\sqrt{-\sqrt{3}+2}\log\left(\left(\sqrt{3}+2\right)\sqrt{-\sqrt{3}+2}+2\cos(x)\right) - \frac{1}{12}\sqrt{-\sqrt{3}+2}\log\left(\left(\sqrt{3}+2\right)\sqrt{-\sqrt{3}+2}-2\cos(x)\right) + \frac{1}{12}\sqrt{2}\log\left(\frac{2\cos(x)^2-2\sqrt{2}\cos(x)+1}{2\cos(x)^2-1}\right)$$

input
```
integrate(sec(6*x)*sin(x),x, algorithm="fricas")
```

output
```
-1/12*sqrt(sqrt(3) + 2)*log(sqrt(sqrt(3) + 2)*(sqrt(3) - 2) + 2*cos(x)) +
1/12*sqrt(sqrt(3) + 2)*log(sqrt(sqrt(3) + 2)*(sqrt(3) - 2) - 2*cos(x)) + 1
/12*sqrt(-sqrt(3) + 2)*log((sqrt(3) + 2)*sqrt(-sqrt(3) + 2) + 2*cos(x)) -
1/12*sqrt(-sqrt(3) + 2)*log((sqrt(3) + 2)*sqrt(-sqrt(3) + 2) - 2*cos(x)) +
 1/12*sqrt(2)*log((2*cos(x)^2 - 2*sqrt(2)*cos(x) + 1)/(2*cos(x)^2 - 1))
```

3.89.6 Sympy [F]

$$\int \sec(6x)\sin(x)\,dx = \int \sin(x)\sec(6x)\,dx$$

input
```
integrate(sec(6*x)*sin(x),x)
```

output
```
Integral(sin(x)*sec(6*x), x)
```

3.89.7 Maxima [F]

$$\int \sec(6x)\sin(x)\,dx = \int \sec(6x)\sin(x)\,dx$$

input
```
integrate(sec(6*x)*sin(x),x, algorithm="maxima")
```

output
```
-1/24*sqrt(2)*log(2*sqrt(2)*sin(2*x)*sin(x) + 2*(sqrt(2)*cos(x) + 1)*cos(2
*x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x)
 + 1) + 1/24*sqrt(2)*log(-2*sqrt(2)*sin(2*x)*sin(x) - 2*(sqrt(2)*cos(x) -
1)*cos(2*x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 - 2*sqrt(2
)*cos(x) + 1) - integrate(1/3*((sin(7*x) - sin(5*x) + sin(3*x) - sin(x))*c
os(8*x) - (sin(3*x) - sin(x))*cos(4*x) - (cos(7*x) - cos(5*x) + cos(3*x) -
 cos(x))*sin(8*x) - (cos(4*x) - 1)*sin(7*x) + (cos(4*x) - 1)*sin(5*x) + (c
os(3*x) - cos(x))*sin(4*x) + cos(7*x)*sin(4*x) - cos(5*x)*sin(4*x) + sin(3
*x) - sin(x))/(2*(cos(4*x) - 1)*cos(8*x) - cos(8*x)^2 - cos(4*x)^2 - sin(8
*x)^2 + 2*sin(8*x)*sin(4*x) - sin(4*x)^2 + 2*cos(4*x) - 1), x)
```

3.89.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(67) = 134$.

Time = 0.28 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.14

$$\begin{aligned}\int \sec(6x)\sin(x)\,dx = &-\frac{1}{12}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}-\frac{2(\cos(x)-1)}{\cos(x)+1}-6\right|}{\left|4\sqrt{2}-\frac{2(\cos(x)-1)}{\cos(x)+1}-6\right|}\right)\\
&-\frac{2.39014968180000\log\left(-\frac{\cos(x)-1}{\cos(x)+1}-0.0173323801210000\right)}{\frac{268(\cos(x)-1)}{\cos(x)+1}+60.0540532247402}\\
&+\frac{5.82951931426000\log\left(-\frac{\cos(x)-1}{\cos(x)+1}-0.588790706481000\right)}{\frac{268(\cos(x)-1)}{\cos(x)+1}+121.584934401100}\\
&+\frac{16.8155413244667\log\left(-\frac{\cos(x)-1}{\cos(x)+1}-1.69839637242000\right)}{\frac{268(\cos(x)-1)}{\cos(x)+1}+559.622604171000}\\
&-\frac{7956.25491093333\log\left(-\frac{\cos(x)-1}{\cos(x)+1}-57.6954805410000\right)}{\frac{268(\cos(x)-1)}{\cos(x)+1}-168981.261592000}\end{aligned}$$

input
```
integrate(sec(6*x)*sin(x),x, algorithm="giac")
```

output
```
-1/12*sqrt(2)*log(abs(-4*sqrt(2) - 2*(cos(x) - 1)/(cos(x) + 1) - 6)/abs(4*
sqrt(2) - 2*(cos(x) - 1)/(cos(x) + 1) - 6)) - 2.39014968180000*log(-(cos(x
) - 1)/(cos(x) + 1) - 0.0173323801210000)/(268*(cos(x) - 1)/(cos(x) + 1) +
 60.0540532247402) + 5.82951931426000*log(-(cos(x) - 1)/(cos(x) + 1) - 0.5
88790706481000)/(268*(cos(x) - 1)/(cos(x) + 1) + 121.584934401100) + 16.81
55413244667*log(-(cos(x) - 1)/(cos(x) + 1) - 1.69839637242000)/(268*(cos(x
) - 1)/(cos(x) + 1) + 559.622604171000) - 7956.25491093333*log(-(cos(x) -
1)/(cos(x) + 1) - 57.6954805410000)/(268*(cos(x) - 1)/(cos(x) + 1) - 16898
1.261592000)
```

3.89.9 Mupad [B] (verification not implemented)

Time = 27.36 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.39

$$\int \sec(6x)\sin(x)\,dx = \operatorname{atanh}\left(\frac{5\sqrt{2}\cos(x)}{2097152\left(\frac{\sqrt{2}\sqrt{6}}{4194304}+\frac{1}{1048576}\right)} + \frac{3\sqrt{6}\cos(x)}{2097152\left(\frac{\sqrt{2}\sqrt{6}}{4194304}+\frac{1}{1048576}\right)}\right)\left(\frac{\sqrt{2}}{12}+\frac{\sqrt{6}}{12}\right) - \operatorname{atanh}\left(\frac{5\sqrt{2}\cos(x)}{2097152\left(\frac{\sqrt{2}\sqrt{6}}{4194304}-\frac{1}{1048576}\right)} - \frac{3\sqrt{6}\cos(x)}{2097152\left(\frac{\sqrt{2}\sqrt{6}}{4194304}-\frac{1}{1048576}\right)}\right)\left(\frac{\sqrt{2}}{12}-\frac{\sqrt{6}}{12}\right) - \frac{\sqrt{2}\operatorname{atanh}\left(\sqrt{2}\cos(x)\right)}{6}$$

input
```
int(sin(x)/cos(6*x),x)
```

output
```
atanh((5*2^(1/2)*cos(x))/(2097152*((2^(1/2)*6^(1/2))/4194304 + 1/1048576))
 + (3*6^(1/2)*cos(x))/(2097152*((2^(1/2)*6^(1/2))/4194304 + 1/1048576)))*(
2^(1/2)/12 + 6^(1/2)/12) - atanh((5*2^(1/2)*cos(x))/(2097152*((2^(1/2)*6^(
1/2))/4194304 - 1/1048576)) - (3*6^(1/2)*cos(x))/(2097152*((2^(1/2)*6^(1/2
))/4194304 - 1/1048576)))*(2^(1/2)/12 - 6^(1/2)/12) - (2^(1/2)*atanh(2^(1/
2)*cos(x)))/6
```

3.90 $\int \csc(2x)\sin(x)\,dx$

3.90.1 Optimal result

Integrand size = 7, antiderivative size = 7

$$\int \csc(2x)\sin(x)\,dx = \frac{1}{2}\text{arctanh}(\sin(x))$$

output
```
1/2*arctanh(sin(x))
```

3.90.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \csc(2x)\sin(x)\,dx = \frac{1}{2}\text{arctanh}(\sin(x))$$

input
```
Integrate[Csc[2*x]*Sin[x],x]
```

output
```
ArcTanh[Sin[x]]/2
```

3.90.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3042, 4776, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\csc(2x)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin(x)}{\sin(2x)}dx$$

$$\Big\downarrow \; 4776$$

$$\frac{\int \sec(x)dx}{2}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{2}\int \csc\left(x+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \; 4257$$

$$\frac{1}{2}\text{arctanh}(\sin(x))$$

input
```
Int[Csc[2*x]*Sin[x],x]
```

output
```
ArcTanh[Sin[x]]/2
```

3.90.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.90.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

method	result	size
default	$\frac{\ln(\sec(x)+\tan(x))}{2}$	9
risch	$\frac{\ln(i+e^{ix})}{2}-\frac{\ln(e^{ix}-i)}{2}$	24

input
```
int(csc(2*x)*sin(x),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(sec(x)+tan(x))
```

3.90.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(5) = 10.

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.43

$$\int \csc(2x)\sin(x)\,dx = \frac{1}{4}\log\left(\sin\left(x\right)+1\right) - \frac{1}{4}\log\left(-\sin\left(x\right)+1\right)$$

input
```
integrate(csc(2*x)*sin(x),x, algorithm="fricas")
```

output
```
1/4*log(sin(x) + 1) - 1/4*log(-sin(x) + 1)
```

3.90.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(5) = 10$.

Time = 0.40 (sec) , antiderivative size = 15, normalized size of antiderivative = 2.14

$$\int \csc(2x)\sin(x)\,dx = -\frac{\log(\sin(x)-1)}{4} + \frac{\log(\sin(x)+1)}{4}$$

input
```
integrate(csc(2*x)*sin(x),x)
```

output
```
-log(sin(x) - 1)/4 + log(sin(x) + 1)/4
```

3.90.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(5) = 10$.

Time = 0.30 (sec) , antiderivative size = 35, normalized size of antiderivative = 5.00

$$\int \csc(2x)\sin(x)\,dx = \frac{1}{4}\log\left(\cos(x)^2 + \sin(x)^2 + 2\sin(x) + 1\right) - \frac{1}{4}\log\left(\cos(x)^2 + \sin(x)^2 - 2\sin(x) + 1\right)$$

input
```
integrate(csc(2*x)*sin(x),x, algorithm="maxima")
```

output
```
1/4*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - 1/4*log(cos(x)^2 + sin(x)^2
- 2*sin(x) + 1)
```

3.90.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(5) = 10$.

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.43

$$\int \csc(2x)\sin(x)\,dx = \frac{1}{4}\log(\sin(x)+1) - \frac{1}{4}\log(-\sin(x)+1)$$

input
```
integrate(csc(2*x)*sin(x),x, algorithm="giac")
```

output
```
1/4*log(sin(x) + 1) - 1/4*log(-sin(x) + 1)
```

3.90.9 Mupad [B] (verification not implemented)

Time = 26.99 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \csc(2x)\sin(x)\,dx = \frac{\operatorname{atanh}(\sin(x))}{2}$$

input
```
int(sin(x)/sin(2*x),x)
```

output
```
atanh(sin(x))/2
```

3.91 $\int \csc(3x)\sin(x)\,dx$

3.91.1 Optimal result

Integrand size = 7, antiderivative size = 45

$$\int \csc(3x)\sin(x)\,dx = -\frac{\log\left(\sqrt{3}\cos(x)-\sin(x)\right)}{2\sqrt{3}}+\frac{\log\left(\sqrt{3}\cos(x)+\sin(x)\right)}{2\sqrt{3}}$$

output
```
-1/6*ln(-sin(x)+cos(x)*3^(1/2))*3^(1/2)+1/6*ln(sin(x)+cos(x)*3^(1/2))*3^(1
/2)
```

3.91.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.33

$$\int \csc(3x)\sin(x)\,dx = \frac{\operatorname{arctanh}\left(\frac{\tan(x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Integrate[Csc[3*x]*Sin[x],x]
```

output
```
ArcTanh[Tan[x]/Sqrt[3]]/Sqrt[3]
```

3.91.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.33, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4889, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\csc(3x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)}{\sin(3x)}dx$$

$$\Big\downarrow \text{4889}$$

$$\int \frac{1}{3-\tan^2(x)}d\tan(x)$$

$$\Big\downarrow \text{219}$$

$$\frac{\operatorname{arctanh}\left(\frac{\tan(x)}{\sqrt{3}}\right)}{\sqrt{3}}$$

input
```
Int[Csc[3*x]*Sin[x],x]
```

output
```
ArcTanh[Tan[x]/Sqrt[3]]/Sqrt[3]
```

3.91.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.91.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.31

method	result	size
default	$\frac{\sqrt{3}\,\operatorname{arctanh}\left(\frac{\tan(x)\sqrt{3}}{3}\right)}{3}$	14
risch	$\frac{\sqrt{3}\,\ln\left(\mathrm{e}^{2ix}+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}{6}-\frac{\sqrt{3}\,\ln\left(\mathrm{e}^{2ix}+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{6}$	40

input
```
int(csc(3*x)*sin(x),x,method=_RETURNVERBOSE)
```

output
```
1/3*3^(1/2)*arctanh(1/3*tan(x)*3^(1/2))
```

3.91.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.29

$$\int \csc(3x)\sin(x)\,dx$$
$$= \frac{1}{12}\sqrt{3}\log\left(-\frac{8\cos(x)^4-16\cos(x)^2-4\left(2\sqrt{3}\cos(x)^3+\sqrt{3}\cos(x)\right)\sin(x)-1}{16\cos(x)^4-8\cos(x)^2+1}\right)$$

input
```
integrate(csc(3*x)*sin(x),x, algorithm="fricas")
```

output
```
1/12*sqrt(3)*log(-(8*cos(x)^4 - 16*cos(x)^2 - 4*(2*sqrt(3)*cos(x)^3 + sqrt
(3)*cos(x))*sin(x) - 1)/(16*cos(x)^4 - 8*cos(x)^2 + 1))
```

3.91.6 Sympy [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.69

$$\int \csc(3x)\sin(x)\,dx = \frac{\sqrt{3}\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{3}\right)}{6} - \frac{\sqrt{3}\log\left(\tan\left(\frac{x}{2}\right)-\frac{\sqrt{3}}{3}\right)}{6} + \frac{\sqrt{3}\log\left(\tan\left(\frac{x}{2}\right)+\frac{\sqrt{3}}{3}\right)}{6} - \frac{\sqrt{3}\log\left(\tan\left(\frac{x}{2}\right)+\sqrt{3}\right)}{6}$$

input
```
integrate(csc(3*x)*sin(x),x)
```

output
```
sqrt(3)*log(tan(x/2) - sqrt(3))/6 - sqrt(3)*log(tan(x/2) - sqrt(3)/3)/6 +
sqrt(3)*log(tan(x/2) + sqrt(3)/3)/6 - sqrt(3)*log(tan(x/2) + sqrt(3))/6
```

3.91.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(33) = 66$.

Time = 0.33 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.78

$$\begin{aligned}\int \csc(3x)\sin(x)\,dx = &-\frac{1}{12}\sqrt{3}\log\left(\frac{4}{3}\cos(x)^2+\frac{4}{3}\sin(x)^2+\frac{4}{3}\sqrt{3}\sin(x)+\frac{4}{3}\cos(x)+\frac{4}{3}\right)\\ &+\frac{1}{12}\sqrt{3}\log\left(\frac{4}{3}\cos(x)^2+\frac{4}{3}\sin(x)^2+\frac{4}{3}\sqrt{3}\sin(x)-\frac{4}{3}\cos(x)+\frac{4}{3}\right)\\ &+\frac{1}{12}\sqrt{3}\log\left(\frac{4}{3}\cos(x)^2+\frac{4}{3}\sin(x)^2-\frac{4}{3}\sqrt{3}\sin(x)+\frac{4}{3}\cos(x)+\frac{4}{3}\right)\\ &-\frac{1}{12}\sqrt{3}\log\left(\frac{4}{3}\cos(x)^2+\frac{4}{3}\sin(x)^2-\frac{4}{3}\sqrt{3}\sin(x)-\frac{4}{3}\cos(x)+\frac{4}{3}\right)\end{aligned}$$

input
```
integrate(csc(3*x)*sin(x),x, algorithm="maxima")
```

output
```
-1/12*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 + 4/3*sqrt(3)*sin(x) + 4/3*c
os(x) + 4/3) + 1/12*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 + 4/3*sqrt(3)*
sin(x) - 4/3*cos(x) + 4/3) + 1/12*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2
- 4/3*sqrt(3)*sin(x) + 4/3*cos(x) + 4/3) - 1/12*sqrt(3)*log(4/3*cos(x)^2 +
 4/3*sin(x)^2 - 4/3*sqrt(3)*sin(x) - 4/3*cos(x) + 4/3)
```

3.91.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.69

$$\int \csc(3x)\sin(x)\,dx = -\frac{1}{6}\sqrt{3}\log\left(\frac{\left|-2\sqrt{3}+2\tan\left(x\right)\right|}{\left|2\sqrt{3}+2\tan\left(x\right)\right|}\right)$$

input
```
integrate(csc(3*x)*sin(x),x, algorithm="giac")
```

output
```
-1/6*sqrt(3)*log(abs(-2*sqrt(3) + 2*tan(x))/abs(2*sqrt(3) + 2*tan(x)))
```

3.91.9 Mupad [B] (verification not implemented)

Time = 28.31 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.38

$$\int \csc(3x)\sin(x)\,dx = \frac{\sqrt{3}\,\mathrm{atanh}\left(\frac{\sqrt{3}\,\sin(x)}{3\,\cos(x)}\right)}{3}$$

input
```
int(sin(x)/sin(3*x),x)
```

output
```
(3^(1/2)*atanh((3^(1/2)*sin(x))/(3*cos(x))))/3
```

3.92 $\int \csc(4x)\sin(x)\,dx$

3.92.1 Optimal result

Integrand size = 7, antiderivative size = 26

$$\int \csc(4x)\sin(x)\,dx = -\frac{1}{4}\operatorname{arctanh}(\sin(x)) + \frac{\operatorname{arctanh}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}}$$

output
```
-1/4*arctanh(sin(x))+1/4*arctanh(sin(x)*2^(1/2))*2^(1/2)
```

3.92.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \csc(4x)\sin(x)\,dx = -\frac{1}{4}\operatorname{arctanh}(\sin(x)) + \frac{\operatorname{arctanh}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}}$$

input
```
Integrate[Csc[4*x]*Sin[x],x]
```

output
```
-1/4*ArcTanh[Sin[x]] + ArcTanh[Sqrt[2]*Sin[x]]/(2*Sqrt[2])
```

3.92.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3042, 4878, 1406, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\csc(4x)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin(x)}{\sin(4x)}dx$$

$$\Big\downarrow \; 4878$$

$$\int \frac{1}{8\sin^4(x)-12\sin^2(x)+4}d\sin(x)$$

$$\Big\downarrow \; 1406$$

$$2\int \frac{1}{8\sin^2(x)-8}d\sin(x)-2\int \frac{1}{8\sin^2(x)-4}d\sin(x)$$

$$\Big\downarrow \; 220$$

$$\frac{\operatorname{arctanh}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}}-\frac{1}{4}\operatorname{arctanh}(\sin(x))$$

input
```
Int[Csc[4*x]*Sin[x],x]
```

output
```
-1/4*ArcTanh[Sin[x]] + ArcTanh[Sqrt[2]*Sin[x]]/(2*Sqrt[2])
```

3.92.3.1 Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

3.92.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

method	result	size
default	$\frac{\ln(\sin(x)-1)}{8}+\frac{\operatorname{arctanh}\left(\sin(x)\sqrt{2}\right)\sqrt{2}}{4}-\frac{\ln(1+\sin(x))}{8}$	28
risch	$-\frac{\ln\left(i+\mathrm{e}^{ix}\right)}{4}+\frac{\ln\left(\mathrm{e}^{ix}-i\right)}{4}+\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}+i\sqrt{2}\,\mathrm{e}^{ix}-1\right)}{8}-\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}-i\sqrt{2}\,\mathrm{e}^{ix}-1\right)}{8}$	72

input
```
int(csc(4*x)*sin(x),x,method=_RETURNVERBOSE)
```

output
```
1/8*ln(sin(x)-1)+1/4*arctanh(sin(x)*2^(1/2))*2^(1/2)-1/8*ln(1+sin(x))
```

3.92.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. 2(18) = 36.

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.92

$$\int \csc(4x)\sin(x)\,dx = \frac{1}{8}\sqrt{2}\log\left(-\frac{2\cos(x)^2-2\sqrt{2}\sin(x)-3}{2\cos(x)^2-1}\right) - \frac{1}{8}\log(\sin(x)+1)+\frac{1}{8}\log(-\sin(x)+1)$$

input
```
integrate(csc(4*x)*sin(x),x, algorithm="fricas")
```

output
```
1/8*sqrt(2)*log(-(2*cos(x)^2 - 2*sqrt(2)*sin(x) - 3)/(2*cos(x)^2 - 1)) - 1
/8*log(sin(x) + 1) + 1/8*log(-sin(x) + 1)
```

3.92.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(22) = 44$.

Time = 3.42 (sec) , antiderivative size = 294, normalized size of antiderivative = 11.31

$$\begin{aligned}\int \csc(4x)\sin(x)\,dx &= \frac{27720\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)-1\right)}{110880\sqrt{2}+156808} + \frac{39202\log\left(\tan\left(\frac{x}{2}\right)-1\right)}{110880\sqrt{2}+156808} \\ &- \frac{39202\log\left(\tan\left(\frac{x}{2}\right)+1\right)}{110880\sqrt{2}+156808} - \frac{27720\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)+1\right)}{110880\sqrt{2}+156808} \\ &+ \frac{27720\log\left(\tan\left(\frac{x}{2}\right)-1+\sqrt{2}\right)}{110880\sqrt{2}+156808} + \frac{19601\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)-1+\sqrt{2}\right)}{110880\sqrt{2}+156808} \\ &+ \frac{27720\log\left(\tan\left(\frac{x}{2}\right)+1+\sqrt{2}\right)}{110880\sqrt{2}+156808} + \frac{19601\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)+1+\sqrt{2}\right)}{110880\sqrt{2}+156808} \\ &- \frac{19601\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{2}-1\right)}{110880\sqrt{2}+156808} - \frac{27720\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{2}-1\right)}{110880\sqrt{2}+156808} \\ &- \frac{19601\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{2}+1\right)}{110880\sqrt{2}+156808} - \frac{27720\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{2}+1\right)}{110880\sqrt{2}+156808}\end{aligned}$$

input
```
integrate(csc(4*x)*sin(x),x)
```

output
```
27720*sqrt(2)*log(tan(x/2) - 1)/(110880*sqrt(2) + 156808) + 39202*log(tan(
x/2) - 1)/(110880*sqrt(2) + 156808) - 39202*log(tan(x/2) + 1)/(110880*sqrt
(2) + 156808) - 27720*sqrt(2)*log(tan(x/2) + 1)/(110880*sqrt(2) + 156808)
+ 27720*log(tan(x/2) - 1 + sqrt(2))/(110880*sqrt(2) + 156808) + 19601*sqrt
(2)*log(tan(x/2) - 1 + sqrt(2))/(110880*sqrt(2) + 156808) + 27720*log(tan(
x/2) + 1 + sqrt(2))/(110880*sqrt(2) + 156808) + 19601*sqrt(2)*log(tan(x/2)
 + 1 + sqrt(2))/(110880*sqrt(2) + 156808) - 19601*sqrt(2)*log(tan(x/2) - s
qrt(2) - 1)/(110880*sqrt(2) + 156808) - 27720*log(tan(x/2) - sqrt(2) - 1)/
(110880*sqrt(2) + 156808) - 19601*sqrt(2)*log(tan(x/2) - sqrt(2) + 1)/(110
880*sqrt(2) + 156808) - 27720*log(tan(x/2) - sqrt(2) + 1)/(110880*sqrt(2)
+ 156808)
```

3.92.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(18) = 36$.

Time = 0.30 (sec) , antiderivative size = 171, normalized size of antiderivative = 6.58

$$\int \csc(4x)\sin(x)\,dx = \frac{1}{16}\sqrt{2}\log\left(2\cos(x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) + 2\sqrt{2}\sin(x) + 2\right) - \frac{1}{16}\sqrt{2}\log\left(2\cos(x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) - 2\sqrt{2}\sin(x) + 2\right) + \frac{1}{16}\sqrt{2}\log\left(2\cos(x)^2 + 2\sin(x)^2 - 2\sqrt{2}\cos(x) + 2\sqrt{2}\sin(x) + 2\right) - \frac{1}{16}\sqrt{2}\log\left(2\cos(x)^2 + 2\sin(x)^2 - 2\sqrt{2}\cos(x) - 2\sqrt{2}\sin(x) + 2\right) - \frac{1}{8}\log\left(\cos(x)^2 + \sin(x)^2 + 2\sin(x) + 1\right) + \frac{1}{8}\log\left(\cos(x)^2 + \sin(x)^2 - 2\sin(x) + 1\right)$$

input
```
integrate(csc(4*x)*sin(x),x, algorithm="maxima")
```

output
```
1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*sqrt(2)*si
n(x) + 2) - 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) -
2*sqrt(2)*sin(x) + 2) + 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(
2)*cos(x) + 2*sqrt(2)*sin(x) + 2) - 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)
^2 - 2*sqrt(2)*cos(x) - 2*sqrt(2)*sin(x) + 2) - 1/8*log(cos(x)^2 + sin(x)^
2 + 2*sin(x) + 1) + 1/8*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)
```

3.92.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(18) = 36$.

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.85

$$\int \csc(4x)\sin(x)\,dx = -\frac{1}{8}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2} + 4\sin(x)\right|}{\left|2\sqrt{2} + 4\sin(x)\right|}\right) - \frac{1}{8}\log(\sin(x) + 1) + \frac{1}{8}\log(-\sin(x) + 1)$$

input
```
integrate(csc(4*x)*sin(x),x, algorithm="giac")
```

output
```
-1/8*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(x))/abs(2*sqrt(2) + 4*sin(x))) - 1
/8*log(sin(x) + 1) + 1/8*log(-sin(x) + 1)
```

3.92.9 Mupad [B] (verification not implemented)

Time = 27.58 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

$$\int \csc(4x)\sin(x)\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\,\sin(x)\right)}{4} - \frac{\mathrm{atanh}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{2}$$

input
```
int(sin(x)/sin(4*x),x)
```

output
```
(2^(1/2)*atanh(2^(1/2)*sin(x)))/4 - atanh(sin(x/2)/cos(x/2))/2
```

3.93 $\int \csc(5x)\sin(x)\,dx$

3.93.1 Optimal result

Integrand size = 7, antiderivative size = 165

$$\begin{aligned}\int \csc(5x)\sin(x)\,dx = &-\frac{1}{10}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\log\left(\sqrt{5-2\sqrt{5}}\cos(x)-\sin(x)\right)\\ &+\frac{1}{10}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\log\left(\sqrt{5+2\sqrt{5}}\cos(x)-\sin(x)\right)\\ &+\frac{1}{10}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\log\left(\sqrt{5-2\sqrt{5}}\cos(x)+\sin(x)\right)\\ &-\frac{1}{10}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\log\left(\sqrt{5+2\sqrt{5}}\cos(x)+\sin(x)\right)\end{aligned}$$

output
```
-1/20*ln(-sin(x)+cos(x)*(5-2*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2)+1/20*ln(
sin(x)+cos(x)*(5-2*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2)+1/20*ln(-sin(x)+co
s(x)*(5+2*5^(1/2))^(1/2))*(10+2*5^(1/2))^(1/2)-1/20*ln(sin(x)+cos(x)*(5+2*
5^(1/2))^(1/2))*(10+2*5^(1/2))^(1/2)
```

3.93.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.51

$$\int \csc(5x)\sin(x)\,dx = \frac{\sqrt{5+\sqrt{5}}\text{arctanh}\left(\frac{\left(-3+\sqrt{5}\right)\tan(x)}{\sqrt{10-2\sqrt{5}}}\right)+\sqrt{5-\sqrt{5}}\text{arctanh}\left(\frac{\left(3+\sqrt{5}\right)\tan(x)}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)}{5\sqrt{2}}$$

input
```
Integrate[Csc[5*x]*Sin[x],x]
```

output
```
(Sqrt[5 + Sqrt[5]]*ArcTanh[((-3 + Sqrt[5])*Tan[x])/Sqrt[10 - 2*Sqrt[5]]] +
 Sqrt[5 - Sqrt[5]]*ArcTanh[((3 + Sqrt[5])*Tan[x])/Sqrt[2*(5 + Sqrt[5])]])/
(5*Sqrt[2])
```

3.93.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.53, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3042, 4889, 1480, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\csc(5x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)}{\sin(5x)}dx$$

$$\Big\downarrow \text{4889}$$

$$\int \frac{\tan^2(x)+1}{\tan^4(x)-10\tan^2(x)+5}d\tan(x)$$

$$\Big\downarrow \text{1480}$$

$$\frac{1}{10}\left(5+3\sqrt{5}\right)\int \frac{1}{\tan^2(x)-2\sqrt{5}-5}d\tan(x)+\frac{1}{10}\left(5-3\sqrt{5}\right)\int \frac{1}{\tan^2(x)+2\sqrt{5}-5}d\tan(x)$$

$$\Big\downarrow \text{220}$$

$$-\frac{(5-3\sqrt{5})\operatorname{arctanh}\left(\frac{\tan(x)}{\sqrt{5-2\sqrt{5}}}\right)}{10\sqrt{5-2\sqrt{5}}}-\frac{(5+3\sqrt{5})\operatorname{arctanh}\left(\frac{\tan(x)}{\sqrt{5+2\sqrt{5}}}\right)}{10\sqrt{5+2\sqrt{5}}}$$

input
```
Int[Csc[5*x]*Sin[x],x]
```

output
```
-1/10*((5 - 3*Sqrt[5])*ArcTanh[Tan[x]/Sqrt[5 - 2*Sqrt[5]]])/Sqrt[5 - 2*Sqr
t[5]] - ((5 + 3*Sqrt[5])*ArcTanh[Tan[x]/Sqrt[5 + 2*Sqrt[5]]])/(10*Sqrt[5 +
 2*Sqrt[5]])
```

3.93.3.1 Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.93.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.85 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.25

method	result	size
risch	$\sum_{\_R=\mathrm{RootOf}\left(2000\_Z^4-100\_Z^2+1\right)} \_R\ln\left(\mathrm{e}^{2ix}-500i\_R^3+50\_R^2+15i\_R-1\right)$	42
default	$-\frac{\left(\sqrt{5}-3\right)\sqrt{5}\,\operatorname{arctanh}\left(\frac{\tan(x)}{\sqrt{5-2\sqrt{5}}}\right)}{10\sqrt{5-2\sqrt{5}}}-\frac{\left(3+\sqrt{5}\right)\sqrt{5}\,\operatorname{arctanh}\left(\frac{\tan(x)}{\sqrt{5+2\sqrt{5}}}\right)}{10\sqrt{5+2\sqrt{5}}}$	66

input
```
int(csc(5*x)*sin(x),x,method=_RETURNVERBOSE)
```

output
```
sum(_R*ln(exp(2*I*x)-500*I*_R^3+50*_R^2+15*I*_R-1),_R=RootOf(2000*_Z^4-100
*_Z^2+1))
```

3.93.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. 2(113) = 226.

Time = 0.28 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.40

$$\begin{aligned}\int \csc(5x)\sin(x)\,dx = &-\frac{1}{40}\sqrt{2}\sqrt{\sqrt{5}+5}\log\left(\left(\sqrt{5}\sqrt{2}-\sqrt{2}\right)\sqrt{\sqrt{5}+5}\cos(x)\sin(x)\right.\\&\left.+2\left(\sqrt{5}+1\right)\cos(x)^2-\sqrt{5}+3\right)\\&+\frac{1}{40}\sqrt{2}\sqrt{\sqrt{5}+5}\log\left(-\left(\sqrt{5}\sqrt{2}-\sqrt{2}\right)\sqrt{\sqrt{5}+5}\cos(x)\sin(x)\right.\\&\left.+2\left(\sqrt{5}+1\right)\cos(x)^2-\sqrt{5}+3\right)\\&-\frac{1}{40}\sqrt{2}\sqrt{-\sqrt{5}+5}\log\left(\left(\sqrt{5}\sqrt{2}+\sqrt{2}\right)\sqrt{-\sqrt{5}+5}\cos(x)\sin(x)\right.\\&\left.+2\left(\sqrt{5}-1\right)\cos(x)^2-\sqrt{5}-3\right)\\&+\frac{1}{40}\sqrt{2}\sqrt{-\sqrt{5}+5}\log\left(-\left(\sqrt{5}\sqrt{2}+\sqrt{2}\right)\sqrt{-\sqrt{5}+5}\cos(x)\sin(x)\right.\\&\left.+2\left(\sqrt{5}-1\right)\cos(x)^2-\sqrt{5}-3\right)\end{aligned}$$

input
```
integrate(csc(5*x)*sin(x),x, algorithm="fricas")
```

output
```
-1/40*sqrt(2)*sqrt(sqrt(5) + 5)*log((sqrt(5)*sqrt(2) - sqrt(2))*sqrt(sqrt(
5) + 5)*cos(x)*sin(x) + 2*(sqrt(5) + 1)*cos(x)^2 - sqrt(5) + 3) + 1/40*sqr
t(2)*sqrt(sqrt(5) + 5)*log(-(sqrt(5)*sqrt(2) - sqrt(2))*sqrt(sqrt(5) + 5)*
cos(x)*sin(x) + 2*(sqrt(5) + 1)*cos(x)^2 - sqrt(5) + 3) - 1/40*sqrt(2)*sqr
t(-sqrt(5) + 5)*log((sqrt(5)*sqrt(2) + sqrt(2))*sqrt(-sqrt(5) + 5)*cos(x)*
sin(x) + 2*(sqrt(5) - 1)*cos(x)^2 - sqrt(5) - 3) + 1/40*sqrt(2)*sqrt(-sqrt
(5) + 5)*log(-(sqrt(5)*sqrt(2) + sqrt(2))*sqrt(-sqrt(5) + 5)*cos(x)*sin(x)
 + 2*(sqrt(5) - 1)*cos(x)^2 - sqrt(5) - 3)
```

3.93.6 Sympy [F]

$$\int \csc(5x)\sin(x)\,dx = \int \sin(x)\csc(5x)\,dx$$

input
```
integrate(csc(5*x)*sin(x),x)
```

output
```
Integral(sin(x)*csc(5*x), x)
```

3.93.7 Maxima [F]

$$\int \csc(5x)\sin(x)\,dx = \int \csc(5x)\sin(x)\,dx$$

input
```
integrate(csc(5*x)*sin(x),x, algorithm="maxima")
```

output
```
integrate(csc(5*x)*sin(x), x)
```

3.93.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.64

$$\begin{aligned}\int \csc(5x)\sin(x)\,dx = & -\frac{1}{20}\sqrt{2\sqrt{5}+10}\log\left(\left|\sqrt{2\sqrt{5}+5}+\tan(x)\right|\right) \\ & +\frac{1}{20}\sqrt{2\sqrt{5}+10}\log\left(\left|-\sqrt{2\sqrt{5}+5}+\tan(x)\right|\right) \\ & +\frac{1}{20}\sqrt{-2\sqrt{5}+10}\log\left(\left|\sqrt{-2\sqrt{5}+5}+\tan(x)\right|\right) \\ & -\frac{1}{20}\sqrt{-2\sqrt{5}+10}\log\left(\left|-\sqrt{-2\sqrt{5}+5}+\tan(x)\right|\right)\end{aligned}$$

input
```
integrate(csc(5*x)*sin(x),x, algorithm="giac")
```

output
```
-1/20*sqrt(2*sqrt(5) + 10)*log(abs(sqrt(2*sqrt(5) + 5) + tan(x))) + 1/20*s
qrt(2*sqrt(5) + 10)*log(abs(-sqrt(2*sqrt(5) + 5) + tan(x))) + 1/20*sqrt(-2
*sqrt(5) + 10)*log(abs(sqrt(-2*sqrt(5) + 5) + tan(x))) - 1/20*sqrt(-2*sqrt
(5) + 10)*log(abs(-sqrt(-2*sqrt(5) + 5) + tan(x)))
```

3.93.9 Mupad [B] (verification not implemented)

Time = 27.98 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.32

$$\int \csc(5x)\sin(x)\,dx$$

$$= \frac{\sqrt{2}\,\mathrm{atanh}\left(-\frac{34359738368\sqrt{2}\tan\left(\frac{x}{2}\right)\sqrt{\sqrt{5}+5}}{1953125\left(\frac{90194313216\sqrt{5}}{1953125}-\frac{90194313216\sqrt{5}\tan\left(\frac{x}{2}\right)^2}{1953125}-\frac{201863462912\tan\left(\frac{x}{2}\right)^2}{1953125}+\frac{201863462912}{1953125}\right)} - \frac{773}{9765625\left(\frac{90194313216\sqrt{5}}{1953125}-\frac{901}{}\right.}\right.}{10}$$

$$-\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{77309411328\sqrt{2}\sqrt{5}\tan\left(\frac{x}{2}\right)\sqrt{5-\sqrt{5}}}{9765625\left(\frac{90194313216\sqrt{5}}{1953125}-\frac{90194313216\sqrt{5}\tan\left(\frac{x}{2}\right)^2}{1953125}+\frac{201863462912\tan\left(\frac{x}{2}\right)^2}{1953125}-\frac{201863462912}{1953125}\right)} - \frac{3}{1953125\left(\frac{90194313216\sqrt{5}}{1953125}-\frac{901}{}\right.}\right.}{10}$$

input
```
int(sin(x)/sin(5*x),x)
```

output

```
(2^(1/2)*atanh(- (34359738368*2^(1/2)*tan(x/2)*(5^(1/2) + 5)^(1/2))/(19531
25*((90194313216*5^(1/2))/1953125 - (90194313216*5^(1/2)*tan(x/2)^2)/19531
25 - (201863462912*tan(x/2)^2)/1953125 + 201863462912/1953125)) - (7730941
1328*2^(1/2)*5^(1/2)*tan(x/2)*(5^(1/2) + 5)^(1/2))/(9765625*((90194313216*
5^(1/2))/1953125 - (90194313216*5^(1/2)*tan(x/2)^2)/1953125 - (20186346291
2*tan(x/2)^2)/1953125 + 201863462912/1953125)))*(5^(1/2) + 5)^(1/2))/10 -
(2^(1/2)*atanh((77309411328*2^(1/2)*5^(1/2)*tan(x/2)*(5 - 5^(1/2))^(1/2))/
(9765625*((90194313216*5^(1/2))/1953125 - (90194313216*5^(1/2)*tan(x/2)^2)
/1953125 + (201863462912*tan(x/2)^2)/1953125 - 201863462912/1953125)) - (3
4359738368*2^(1/2)*tan(x/2)*(5 - 5^(1/2))^(1/2))/(1953125*((90194313216*5^
(1/2))/1953125 - (90194313216*5^(1/2)*tan(x/2)^2)/1953125 + (201863462912*
tan(x/2)^2)/1953125 - 201863462912/1953125)))*(5 - 5^(1/2))^(1/2))/10
```

3.93. $\int \csc(5x)\sin(x)\,dx$

3.94 $\int \csc(6x)\sin(x)\,dx$

3.94.1 Optimal result

Integrand size = 7, antiderivative size = 36

$$\int \csc(6x)\sin(x)\,dx = \frac{1}{6}\text{arctanh}(\sin(x)) + \frac{1}{6}\text{arctanh}(2\sin(x)) - \frac{\text{arctanh}\left(\frac{2\sin(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

output
```
1/6*arctanh(sin(x))+1/6*arctanh(2*sin(x))-1/6*arctanh(2/3*sin(x)*3^(1/2))*
3^(1/2)
```

3.94.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int \csc(6x)\sin(x)\,dx = \frac{1}{6}\left(\text{arctanh}(\sin(x)) + \text{arctanh}(2\sin(x)) - \sqrt{3}\text{arctanh}\left(\frac{2\sin(x)}{\sqrt{3}}\right)\right)$$

input
```
Integrate[Csc[6*x]*Sin[x],x]
```

output
```
(ArcTanh[Sin[x]] + ArcTanh[2*Sin[x]] - Sqrt[3]*ArcTanh[(2*Sin[x])/Sqrt[3]]
)/6
```

3.94.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3042, 4878, 27, 2460, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\csc(6x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)}{\sin(6x)}dx$$

$$\Big\downarrow \text{4878}$$

$$\int \frac{1}{2\left(-16\sin^6(x)+32\sin^4(x)-19\sin^2(x)+3\right)}d\sin(x)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{1}{-16\sin^6(x)+32\sin^4(x)-19\sin^2(x)+3}d\sin(x)$$

$$\Big\downarrow \text{2460}$$

$$\frac{1}{2}\int \left(\frac{2}{4\sin^2(x)-3}-\frac{2}{3\left(4\sin^2(x)-1\right)}-\frac{1}{3\left(\sin^2(x)-1\right)}\right)d\sin(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{3}\text{arctanh}(\sin(x))+\frac{1}{3}\text{arctanh}(2\sin(x))-\frac{\text{arctanh}\left(\frac{2\sin(x)}{\sqrt{3}}\right)}{\sqrt{3}}\right)$$

input
```
Int[Csc[6*x]*Sin[x],x]
```

output
```
(ArcTanh[Sin[x]]/3 + ArcTanh[2*Sin[x]]/3 - ArcTanh[(2*Sin[x])/Sqrt[3]]/Sqr
t[3])/2
```

3.94.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2460
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px /. x -> Sqrt[x]]},
Int[ExpandIntegrand[u*(Qx /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[Q
x, x]]] /; PolyQ[Px, x^2] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&
 !TrinomialQ[Px, x] && ILtQ[p, 0] && RationalFunctionQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

3.94.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.31

method	result
default	$\frac{\ln(1+\sin(x))}{12}-\frac{\ln(\sin(x)-1)}{12}-\frac{\operatorname{arctanh}\left(\frac{2\sin(x)\sqrt{3}}{3}\right)\sqrt{3}}{6}-\frac{\ln(2\sin(x)-1)}{12}+\frac{\ln(1+2\sin(x))}{12}$
risch	$\frac{\ln(i+e^{ix})}{6}-\frac{\ln(e^{ix}-i)}{6}+\frac{\sqrt{3}\ln\left(e^{2ix}-i\sqrt{3}\,e^{ix}-1\right)}{12}-\frac{\sqrt{3}\ln\left(e^{2ix}+i\sqrt{3}\,e^{ix}-1\right)}{12}-\frac{\ln(-ie^{ix}+e^{2ix}-1)}{12}+\frac{\ln(ie^{ix}+e^{2ix}-1)}{12}$

input
```
int(csc(6*x)*sin(x),x,method=_RETURNVERBOSE)
```

output
```
1/12*ln(1+sin(x))-1/12*ln(sin(x)-1)-1/6*arctanh(2/3*sin(x)*3^(1/2))*3^(1/2
)-1/12*ln(2*sin(x)-1)+1/12*ln(1+2*sin(x))
```

3.94.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(26) = 52$.

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.89

$$\int \csc(6x)\sin(x)\,dx = \frac{1}{12}\sqrt{3}\log\left(-\frac{4\cos(x)^2 + 4\sqrt{3}\sin(x) - 7}{4\cos(x)^2 - 1}\right) + \frac{1}{12}\log(2\sin(x) + 1) + \frac{1}{12}\log(\sin(x) + 1) - \frac{1}{12}\log(-\sin(x) + 1) - \frac{1}{12}\log(-2\sin(x) + 1)$$

input
```
integrate(csc(6*x)*sin(x),x, algorithm="fricas")
```

output
```
1/12*sqrt(3)*log(-(4*cos(x)^2 + 4*sqrt(3)*sin(x) - 7)/(4*cos(x)^2 - 1)) +
1/12*log(2*sin(x) + 1) + 1/12*log(sin(x) + 1) - 1/12*log(-sin(x) + 1) - 1/
12*log(-2*sin(x) + 1)
```

3.94.6 Sympy [F]

$$\int \csc(6x)\sin(x)\,dx = \int \sin(x)\csc(6x)\,dx$$

input
```
integrate(csc(6*x)*sin(x),x)
```

output
```
Integral(sin(x)*csc(6*x), x)
```

3.94.7 Maxima [F]

$$\int \csc(6x)\sin(x)\,dx = \int \csc(6x)\sin(x)\,dx$$

input
```
integrate(csc(6*x)*sin(x),x, algorithm="maxima")
```

output
```
-1/24*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 + 4/3*sqrt(3)*sin(x) + 4/3*c
os(x) + 4/3) - 1/24*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2 + 4/3*sqrt(3)*
sin(x) - 4/3*cos(x) + 4/3) + 1/24*sqrt(3)*log(4/3*cos(x)^2 + 4/3*sin(x)^2
- 4/3*sqrt(3)*sin(x) + 4/3*cos(x) + 4/3) + 1/24*sqrt(3)*log(4/3*cos(x)^2 +
 4/3*sin(x)^2 - 4/3*sqrt(3)*sin(x) - 4/3*cos(x) + 4/3) + integrate(-1/6*((
cos(3*x) + cos(x))*cos(4*x) - (cos(2*x) - 1)*cos(3*x) - cos(2*x)*cos(x) +
(sin(3*x) + sin(x))*sin(4*x) - sin(3*x)*sin(2*x) - sin(2*x)*sin(x) + cos(x
))/(2*(cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - cos(2*x)^2 - sin(4*x)^2 + 2*s
in(4*x)*sin(2*x) - sin(2*x)^2 + 2*cos(2*x) - 1), x) + 1/12*log(cos(x)^2 +
sin(x)^2 + 2*sin(x) + 1) - 1/12*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)
```

3.94.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(26) = 52$.

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.89

$$\int \csc(6x)\sin(x)\,dx = \frac{1}{12}\sqrt{3}\log\left(\frac{\left|-4\sqrt{3}+8\sin(x)\right|}{\left|4\sqrt{3}+8\sin(x)\right|}\right) + \frac{1}{12}\log(\sin(x)+1) - \frac{1}{12}\log(-\sin(x)+1) + \frac{1}{12}\log(|2\sin(x)+1|) - \frac{1}{12}\log(|2\sin(x)-1|)$$

input
```
integrate(csc(6*x)*sin(x),x, algorithm="giac")
```

output
```
1/12*sqrt(3)*log(abs(-4*sqrt(3) + 8*sin(x))/abs(4*sqrt(3) + 8*sin(x))) + 1
/12*log(sin(x) + 1) - 1/12*log(-sin(x) + 1) + 1/12*log(abs(2*sin(x) + 1))
- 1/12*log(abs(2*sin(x) - 1))
```

3.94.9 Mupad [B] (verification not implemented)

Time = 28.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int \csc(6x)\sin(x)\,dx = \frac{\operatorname{atanh}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{3} + \frac{\operatorname{atanh}(2\sin(x))}{6} - \frac{\sqrt{3}\operatorname{atanh}\left(\frac{2\sqrt{3}\sin(x)}{3}\right)}{6}$$

input
```
int(sin(x)/sin(6*x),x)
```

output
```
atanh(sin(x/2)/cos(x/2))/3 + atanh(2*sin(x))/6 - (3^(1/2)*atanh((2*3^(1/2)
*sin(x))/3))/6
```

3.94. $\int \csc(6x)\sin(x)\,dx$

3.95 $\int \csc(x)\sin(3x)\,dx$

3.95.1 Optimal result

Integrand size = 7, antiderivative size = 8

$$\int \csc(x)\sin(3x)\,dx = x + 2\cos(x)\sin(x)$$

output
```
x+2*cos(x)*sin(x)
```

3.95.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \csc(x)\sin(3x)\,dx = x + \sin(2x)$$

input
```
Integrate[Csc[x]*Sin[3*x],x]
```

output
```
x + Sin[2*x]
```

3.95.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4889, 298, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(3x)\csc(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(3x)}{\sin(x)}dx$$

$$\Big\downarrow \text{4889}$$

$$\int \frac{3-\tan^2(x)}{\left(\tan^2(x)+1\right)^2}d\tan(x)$$

$$\Big\downarrow \text{298}$$

$$\int \frac{1}{\tan^2(x)+1}d\tan(x)+\frac{2\tan(x)}{\tan^2(x)+1}$$

$$\Big\downarrow \text{216}$$

$$\arctan(\tan(x))+\frac{2\tan(x)}{\tan^2(x)+1}$$

input
```
Int[Csc[x]*Sin[3*x],x]
```

output
```
ArcTan[Tan[x]] + (2*Tan[x])/(1 + Tan[x]^2)
```

3.95.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.95.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
risch	$x+\sin(2x)$	7
default	$x+2\cos(x)\sin(x)$	9

input
```
int(csc(x)*sin(3*x),x,method=_RETURNVERBOSE)
```

output
```
x+sin(2*x)
```

3.95.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \csc(x)\sin(3x)\,dx = 2\cos(x)\sin(x) + x$$

input
```
integrate(csc(x)*sin(3*x),x, algorithm="fricas")
```

output
```
2*cos(x)*sin(x) + x
```

3.95.6 Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \csc(x)\sin(3x)\,dx = x + \sin(2x)$$

input
```
integrate(csc(x)*sin(3*x),x)
```

output
```
x + sin(2*x)
```

3.95.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \csc(x)\sin(3x)\,dx = x + \sin(2\,x)$$

input
```
integrate(csc(x)*sin(3*x),x, algorithm="maxima")
```

output
```
x + sin(2*x)
```

3.95.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \csc(x)\sin(3x)\,dx = x + \sin(2\,x)$$

input
```
integrate(csc(x)*sin(3*x),x, algorithm="giac")
```

output
```
x + sin(2*x)
```

3.95.9 Mupad [B] (verification not implemented)

Time = 28.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \csc(x)\sin(3x)\,dx = x + \sin(2\,x)$$

input
```
int(sin(3*x)/sin(x),x)
```

output
```
x + sin(2*x)
```

3.96 $\int \csc(3x)\sin(6x)\,dx$

3.96.1 Optimal result

Integrand size = 9, antiderivative size = 8

$$\int \csc(3x)\sin(6x)\,dx = \frac{2}{3}\sin(3x)$$

output
```
2/3*sin(3*x)
```

3.96.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \csc(3x)\sin(6x)\,dx = \frac{2}{3}\sin(3x)$$

input
```
Integrate[Csc[3*x]*Sin[6*x],x]
```

output
```
(2*Sin[3*x])/3
```

3.96.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3042, 4776, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(6x)\csc(3x)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin(6x)}{\sin(3x)}dx$$

$$\Big\downarrow \; 4776$$

$$2\int \cos(3x)dx$$

$$\Big\downarrow \; 3042$$

$$2\int \sin\left(3x+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \; 3117$$

$$\frac{2}{3}\sin(3x)$$

input
```
Int[Csc[3*x]*Sin[6*x],x]
```

output
```
(2*Sin[3*x])/3
```

3.96.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4776
```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/f^p   Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x],
x] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.96.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
risch	$\frac{2\sin(3x)}{3}$	7
derivativedivides	$\frac{2}{3\csc(3x)}$	9
default	$\frac{2}{3\csc(3x)}$	9

input
```
int(csc(3*x)*sin(6*x),x,method=_RETURNVERBOSE)
```

output
```
2/3*sin(3*x)
```

3.96.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \csc(3x)\sin(6x)\,dx = \frac{2}{3}\,\sin\left(3\,x\right)$$

input
```
integrate(csc(3*x)*sin(6*x),x, algorithm="fricas")
```

output
```
2/3*sin(3*x)
```

3.96.6 Sympy [A] (verification not implemented)

Time = 1.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \csc(3x)\sin(6x)\,dx = \frac{2\sin{(3x)}}{3}$$

input
```
integrate(csc(3*x)*sin(6*x),x)
```

output
```
2*sin(3*x)/3
```

3.96.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \csc(3x)\sin(6x)\,dx = \frac{2}{3}\,\sin\left(3\,x\right)$$

input
```
integrate(csc(3*x)*sin(6*x),x, algorithm="maxima")
```

output
```
2/3*sin(3*x)
```

3.96.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \csc(3x)\sin(6x)\,dx = \frac{2}{3}\,\sin\left(3\,x\right)$$

input
```
integrate(csc(3*x)*sin(6*x),x, algorithm="giac")
```

output
```
2/3*sin(3*x)
```

3.96.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \csc(3x)\sin(6x)\,dx = \frac{2\,\sin\left(3\,x\right)}{3}$$

input
```
int(sin(6*x)/sin(3*x),x)
```

output
```
(2*sin(3*x))/3
```

3.97 $\int \cos(x)\sin(2x)\,dx$

3.97.1 Optimal result

Integrand size = 7, antiderivative size = 15

$$\int \cos(x)\sin(2x)\,dx = -\frac{\cos(x)}{2} - \frac{1}{6}\cos(3x)$$

output
```
-1/2*cos(x)-1/6*cos(3*x)
```

3.97.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cos(x)\sin(2x)\,dx = -\frac{\cos(x)}{2} - \frac{1}{6}\cos(3x)$$

input
```
Integrate[Cos[x]*Sin[2*x],x]
```

output
```
-1/2*Cos[x] - Cos[3*x]/6
```

3.97.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(2x)\cos(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(2x)\cos(x)dx$$

$$\Big\downarrow \text{4772}$$

$$-\frac{\cos(x)}{2} - \frac{1}{6}\cos(3x)$$

input
```
Int[Cos[x]*Sin[2*x],x]
```

output
```
-1/2*Cos[x] - Cos[3*x]/6
```

3.97.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4772
```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[-Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

3.97.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{\cos(x)}{2}-\frac{\cos(3x)}{6}$	12
risch	$-\frac{\cos(x)}{2}-\frac{\cos(3x)}{6}$	12
parallelrisch	$\frac{2}{3}-\frac{\cos(x)}{2}-\frac{\cos(3x)}{6}$	13
norman	$\frac{\frac{4\tan(x)^2}{3}+\frac{4\tan(\frac{x}{2})^2}{3}-\frac{4\tan(\frac{x}{2})\tan(x)}{3}}{\left(1+\tan(\frac{x}{2})^2\right)\left(1+\tan(x)^2\right)}$	43

input
```
int(cos(x)*sin(2*x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*cos(x)-1/6*cos(3*x)
```

3.97.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.40

$$\int \cos(x)\sin(2x)\,dx = -\frac{2}{3}\cos(x)^3$$

input
```
integrate(cos(x)*sin(2*x),x, algorithm="fricas")
```

output
```
-2/3*cos(x)^3
```

3.97.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int \cos(x)\sin(2x)\,dx = -\frac{\sin(x)\sin(2x)}{3}-\frac{2\cos(x)\cos(2x)}{3}$$

input
```
integrate(cos(x)*sin(2*x),x)
```

output
```
-sin(x)*sin(2*x)/3 - 2*cos(x)*cos(2*x)/3
```

3.97.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \cos(x)\sin(2x)\,dx = -\frac{1}{6}\cos(3x) - \frac{1}{2}\cos(x)$$

input
```
integrate(cos(x)*sin(2*x),x, algorithm="maxima")
```

output
```
-1/6*cos(3*x) - 1/2*cos(x)
```

3.97.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.40

$$\int \cos(x)\sin(2x)\,dx = -\frac{2}{3}\cos(x)^3$$

input
```
integrate(cos(x)*sin(2*x),x, algorithm="giac")
```

output
```
-2/3*cos(x)^3
```

3.97.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.40

$$\int \cos(x)\sin(2x)\,dx = -\frac{2\cos(x)^3}{3}$$

input
```
int(sin(2*x)*cos(x),x)
```

output
```
-(2*cos(x)^3)/3
```

3.98 $\int \cos(x)\sin(3x)\,dx$

3.98.1 Optimal result

Integrand size = 7, antiderivative size = 17

$$\int \cos(x)\sin(3x)\,dx = -\frac{1}{4}\cos(2x) - \frac{1}{8}\cos(4x)$$

output
```
-1/4*cos(2*x)-1/8*cos(4*x)
```

3.98.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cos(x)\sin(3x)\,dx = -\frac{1}{2}\cos^2(x) - \frac{1}{8}\cos(4x)$$

input
```
Integrate[Cos[x]*Sin[3*x],x]
```

output
```
-1/2*Cos[x]^2 - Cos[4*x]/8
```

3.98.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(3x)\cos(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(3x)\cos(x)dx$$

$$\Big\downarrow \text{4772}$$

$$-\frac{1}{4}\cos(2x)-\frac{1}{8}\cos(4x)$$

input
```
Int[Cos[x]*Sin[3*x],x]
```

output
```
-1/4*Cos[2*x] - Cos[4*x]/8
```

3.98.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4772
```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[-Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

3.98.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{\cos(2x)}{4}-\frac{\cos(4x)}{8}$	14
risch	$-\frac{\cos(2x)}{4}-\frac{\cos(4x)}{8}$	14
parallelrisch	$-\frac{\cos(4x)}{8}+\frac{3}{8}-\frac{\cos(2x)}{4}$	15
norman	$\frac{\frac{3\tan\left(\frac{x}{2}\right)^{2}}{4}+\frac{3\tan\left(\frac{3x}{2}\right)^{2}}{4}-\frac{\tan\left(\frac{x}{2}\right)\tan\left(\frac{3x}{2}\right)}{2}}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)\left(1+\tan\left(\frac{3x}{2}\right)^{2}\right)}$	49

input
```
int(cos(x)*sin(3*x),x,method=_RETURNVERBOSE)
```

output
```
-1/4*cos(2*x)-1/8*cos(4*x)
```

3.98.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(x)\sin(3x)\,dx = -\cos{(x)}^{4} + \frac{1}{2}\cos{(x)}^{2}$$

input
```
integrate(cos(x)*sin(3*x),x, algorithm="fricas")
```

output
```
-cos(x)^4 + 1/2*cos(x)^2
```

3.98.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.29

$$\int \cos(x)\sin(3x)\,dx = -\frac{\sin{(x)}\sin{(3x)}}{8} - \frac{3\cos{(x)}\cos{(3x)}}{8}$$

input
```
integrate(cos(x)*sin(3*x),x)
```

output
```
-sin(x)*sin(3*x)/8 - 3*cos(x)*cos(3*x)/8
```

3.98.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(x)\sin(3x)\,dx = -\frac{1}{8}\cos(4x) - \frac{1}{4}\cos(2x)$$

input
```
integrate(cos(x)*sin(3*x),x, algorithm="maxima")
```

output
```
-1/8*cos(4*x) - 1/4*cos(2*x)
```

3.98.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(x)\sin(3x)\,dx = -\cos(x)^4 + \frac{1}{2}\cos(x)^2$$

input
```
integrate(cos(x)*sin(3*x),x, algorithm="giac")
```

output
```
-cos(x)^4 + 1/2*cos(x)^2
```

3.98.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(x)\sin(3x)\,dx = \frac{\cos(x)^2}{2} - \cos(x)^4$$

input
```
int(sin(3*x)*cos(x),x)
```

output
```
cos(x)^2/2 - cos(x)^4
```

3.99 $\int \cos(x)\sin(4x)\,dx$

3.99.1 Optimal result

Integrand size = 7, antiderivative size = 17

$$\int \cos(x)\sin(4x)\,dx = -\frac{1}{6}\cos(3x) - \frac{1}{10}\cos(5x)$$

output
```
-1/6*cos(3*x)-1/10*cos(5*x)
```

3.99.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cos(x)\sin(4x)\,dx = -\frac{1}{6}\cos(3x) - \frac{1}{10}\cos(5x)$$

input
```
Integrate[Cos[x]*Sin[4*x],x]
```

output
```
-1/6*Cos[3*x] - Cos[5*x]/10
```

3.99.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 4772}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(4x)\cos(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(4x)\cos(x)dx$$

$$\Big\downarrow \text{4772}$$

$$-\frac{1}{6}\cos(3x)-\frac{1}{10}\cos(5x)$$

input
```
Int[Cos[x]*Sin[4*x],x]
```

output
```
-1/6*Cos[3*x] - Cos[5*x]/10
```

3.99.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4772
```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[-Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

3.99.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{\cos(3x)}{6}-\frac{\cos(5x)}{10}$	14
risch	$-\frac{\cos(3x)}{6}-\frac{\cos(5x)}{10}$	14
parallelrisch	$\frac{4}{15}-\frac{\cos(3x)}{6}-\frac{\cos(5x)}{10}$	15
norman	$\frac{\frac{8\tan(2x)^2}{15}+\frac{8\tan\left(\frac{x}{2}\right)^2}{15}-\frac{4\tan\left(\frac{x}{2}\right)\tan(2x)}{15}}{\left(1+\tan\left(\frac{x}{2}\right)^2\right)\left(1+\tan(2x)^2\right)}$	49

input
```
int(cos(x)*sin(4*x),x,method=_RETURNVERBOSE)
```

output
```
-1/6*cos(3*x)-1/10*cos(5*x)
```

3.99.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(x)\sin(4x)\,dx = -\frac{8}{5}\cos(x)^5+\frac{4}{3}\cos(x)^3$$

input
```
integrate(cos(x)*sin(4*x),x, algorithm="fricas")
```

output
```
-8/5*cos(x)^5 + 4/3*cos(x)^3
```

3.99.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.29

$$\int \cos(x)\sin(4x)\,dx = -\frac{\sin(x)\sin(4x)}{15}-\frac{4\cos(x)\cos(4x)}{15}$$

input
```
integrate(cos(x)*sin(4*x),x)
```

output
```
-sin(x)*sin(4*x)/15 - 4*cos(x)*cos(4*x)/15
```

3.99.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(x)\sin(4x)\,dx = -\frac{1}{10}\cos(5x) - \frac{1}{6}\cos(3x)$$

input
```
integrate(cos(x)*sin(4*x),x, algorithm="maxima")
```

output
```
-1/10*cos(5*x) - 1/6*cos(3*x)
```

3.99.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(x)\sin(4x)\,dx = -\frac{8}{5}\cos(x)^5 + \frac{4}{3}\cos(x)^3$$

input
```
integrate(cos(x)*sin(4*x),x, algorithm="giac")
```

output
```
-8/5*cos(x)^5 + 4/3*cos(x)^3
```

3.99.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \cos(x)\sin(4x)\,dx = -\frac{4\cos(x)^3\,(6\cos(x)^2 - 5)}{15}$$

input
```
int(sin(4*x)*cos(x),x)
```

output
```
-(4*cos(x)^3*(6*cos(x)^2 - 5))/15
```

3.100 $\int \cos(x)\sin(mx)\,dx$

3.100.1 Optimal result

Integrand size = 7, antiderivative size = 35

$$\int \cos(x)\sin(mx)\,dx = \frac{\cos((1-m)x)}{2(1-m)} - \frac{\cos((1+m)x)}{2(1+m)}$$

output
```
1/2*cos((1-m)*x)/(1-m)-1/2*cos((1+m)*x)/(1+m)
```

3.100.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

$$\int \cos(x)\sin(mx)\,dx = \frac{m\cos(x)\cos(mx)+\sin(x)\sin(mx)}{1-m^2}$$

input
```
Integrate[Cos[x]*Sin[m*x],x]
```

output
```
(m*Cos[x]*Cos[m*x] + Sin[x]*Sin[m*x])/(1 - m^2)
```

3.100.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5085, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\sin(mx)\,dx$$

$$\Big\downarrow \text{5085}$$

$$\int \left(\frac{1}{2}\sin((m+1)x)-\frac{1}{2}\sin((1-m)x)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\cos((1-m)x)}{2(1-m)}-\frac{\cos((m+1)x)}{2(m+1)}$$

input
```
Int[Cos[x]*Sin[m*x],x]
```

output
```
Cos[(1 - m)*x]/(2*(1 - m)) - Cos[(1 + m)*x]/(2*(1 + m))
```

3.100.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5085
```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

3.100.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{\cos(x(-1+m))}{2(-1+m)}-\frac{\cos((1+m)x)}{2(1+m)}$	28
risch	$-\frac{\cos(x(-1+m))}{2(-1+m)}-\frac{\cos((1+m)x)}{2(1+m)}$	28
parallelrisch	$\frac{(-1-m)\cos(x(-1+m))+(1-m)\cos((1+m)x)+2m}{2m^2-2}$	39
norman	$\frac{\frac{2m\tan\left(\frac{x}{2}\right)^2}{m^2-1}+\frac{2m\tan\left(\frac{mx}{2}\right)^2}{m^2-1}-\frac{4\tan\left(\frac{x}{2}\right)\tan\left(\frac{mx}{2}\right)}{m^2-1}}{\left(1+\tan\left(\frac{x}{2}\right)^2\right)\left(1+\tan\left(\frac{mx}{2}\right)^2\right)}$	75

input
```
int(cos(x)*sin(m*x),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(-1+m)*cos(x*(-1+m))-1/2*cos((1+m)*x)/(1+m)
```

3.100.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

$$\int \cos(x)\sin(mx)\,dx = -\frac{m\cos(mx)\cos(x)+\sin(mx)\sin(x)}{m^2-1}$$

input
```
integrate(cos(x)*sin(m*x),x, algorithm="fricas")
```

output
```
-(m*cos(m*x)*cos(x) + sin(m*x)*sin(x))/(m^2 - 1)
```

3.100.6 Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

$$\int \cos(x)\sin(mx)\,dx = \begin{cases} -\frac{\sin^2(x)}{2} & \text{for } m=-1 \\ \frac{\sin^2(x)}{2} & \text{for } m=1 \\ -\frac{m\cos(x)\cos(mx)}{m^2-1}-\frac{\sin(x)\sin(mx)}{m^2-1} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(x)*sin(m*x),x)
```

output
```
Piecewise((-sin(x)**2/2, Eq(m, -1)), (sin(x)**2/2, Eq(m, 1)), (-m*cos(x)*c
os(m*x)/(m**2 - 1) - sin(x)*sin(m*x)/(m**2 - 1), True))
```

3.100.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \cos(x)\sin(mx)\,dx = -\frac{\cos\left((m+1)x\right)}{2\left(m+1\right)} - \frac{\cos\left((m-1)x\right)}{2\left(m-1\right)}$$

input
```
integrate(cos(x)*sin(m*x),x, algorithm="maxima")
```

output
```
-1/2*cos((m + 1)*x)/(m + 1) - 1/2*cos((m - 1)*x)/(m - 1)
```

3.100.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \cos(x)\sin(mx)\,dx = -\frac{\cos\left(mx+x\right)}{2\left(m+1\right)} - \frac{\cos\left(mx-x\right)}{2\left(m-1\right)}$$

input
```
integrate(cos(x)*sin(m*x),x, algorithm="giac")
```

output
```
-1/2*cos(m*x + x)/(m + 1) - 1/2*cos(m*x - x)/(m - 1)
```

3.100.9 Mupad [B] (verification not implemented)

Time = 27.35 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.63

$$\int \cos(x)\sin(mx)\,dx = \begin{cases} \frac{\sin(x)^2}{2} & \text{if } m=1 \\ \frac{\cos(x)^2}{2} & \text{if } m=-1 \\ -\frac{\cos(x\,(m-1))}{2\,m-2} - \frac{\cos(x\,(m+1))}{2\,m+2} & \text{if } m\neq -1 \land m\neq 1 \end{cases}$$

input
```
int(sin(m*x)*cos(x),x)
```

output
```
piecewise(m == 1, sin(x)^2/2, m == -1, cos(x)^2/2, m ~= -1 & m ~= 1, - cos
(x*(m - 1))/(2*m - 2) - cos(x*(m + 1))/(2*m + 2))
```

3.100. $\int \cos(x) \sin(mx) \, dx$

3.101 $\int \cos(x)\cos(2x)\,dx$

3.101.1 Optimal result

Integrand size = 7, antiderivative size = 15

$$\int \cos(x)\cos(2x)\,dx = \frac{\sin(x)}{2} + \frac{1}{6}\sin(3x)$$

output
```
1/2*sin(x)+1/6*sin(3*x)
```

3.101.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cos(x)\cos(2x)\,dx = \frac{\sin(x)}{2} + \frac{1}{6}\sin(3x)$$

input
```
Integrate[Cos[x]*Cos[2*x],x]
```

output
```
Sin[x]/2 + Sin[3*x]/6
```

3.101.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 4771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\cos(2x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \cos(x)\cos(2x)dx$$

$$\Big\downarrow \text{ 4771}$$

$$\frac{\sin(x)}{2}+\frac{1}{6}\sin(3x)$$

input
```
Int[Cos[x]*Cos[2*x],x]
```

output
```
Sin[x]/2 + Sin[3*x]/6
```

3.101.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4771
```
Int[cos[(a_.) + (b_.)*(x_)]*cos[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] + Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

3.101.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{\sin(x)}{2}+\frac{\sin(3x)}{6}$	12
risch	$\frac{\sin(x)}{2}+\frac{\sin(3x)}{6}$	12
parallelrisch	$\frac{\sin(x)}{2}+\frac{\sin(3x)}{6}$	12
norman	$\frac{-\frac{4\tan(x)\tan\left(\frac{x}{2}\right)^{2}}{3}+\frac{2\tan(x)^{2}\tan\left(\frac{x}{2}\right)}{3}+\frac{4\tan(x)}{3}-\frac{2\tan\left(\frac{x}{2}\right)}{3}}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)\left(1+\tan(x)^{2}\right)}$	51

input
```
int(cos(x)*cos(2*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*sin(x)+1/6*sin(3*x)
```

3.101.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \cos(x)\cos(2x)\,dx = \frac{1}{3}\left(2\cos\left(x\right)^{2}+1\right)\sin\left(x\right)$$

input
```
integrate(cos(x)*cos(2*x),x, algorithm="fricas")
```

output
```
1/3*(2*cos(x)^2 + 1)*sin(x)
```

3.101.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.33

$$\int \cos(x)\cos(2x)\,dx = -\frac{\sin\left(x\right)\cos\left(2x\right)}{3}+\frac{2\sin\left(2x\right)\cos\left(x\right)}{3}$$

input
```
integrate(cos(x)*cos(2*x),x)
```

output
```
-sin(x)*cos(2*x)/3 + 2*sin(2*x)*cos(x)/3
```

3.101.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \cos(x)\cos(2x)\,dx = \frac{1}{6}\sin(3x) + \frac{1}{2}\sin(x)$$

input
```
integrate(cos(x)*cos(2*x),x, algorithm="maxima")
```

output
```
1/6*sin(3*x) + 1/2*sin(x)
```

3.101.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \cos(x)\cos(2x)\,dx = \frac{1}{6}\sin(3x) + \frac{1}{2}\sin(x)$$

input
```
integrate(cos(x)*cos(2*x),x, algorithm="giac")
```

output
```
1/6*sin(3*x) + 1/2*sin(x)
```

3.101.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.60

$$\int \cos(x)\cos(2x)\,dx = \sin(x) - \frac{2\sin(x)^3}{3}$$

input
```
int(cos(2*x)*cos(x),x)
```

output
```
sin(x) - (2*sin(x)^3)/3
```

3.102 $\int \cos(x)\cos(3x)\,dx$

3.102.1 Optimal result

Integrand size = 7, antiderivative size = 17

$$\int \cos(x)\cos(3x)\,dx = \frac{1}{4}\sin(2x) + \frac{1}{8}\sin(4x)$$

output
```
1/4*sin(2*x)+1/8*sin(4*x)
```

3.102.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cos(x)\cos(3x)\,dx = \frac{1}{4}\sin(2x) + \frac{1}{8}\sin(4x)$$

input
```
Integrate[Cos[x]*Cos[3*x],x]
```

output
```
Sin[2*x]/4 + Sin[4*x]/8
```

3.102.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 4771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\cos(3x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(x)\cos(3x)dx$$

$$\Big\downarrow \text{4771}$$

$$\frac{1}{4}\sin(2x)+\frac{1}{8}\sin(4x)$$

input
```
Int[Cos[x]*Cos[3*x],x]
```

output
```
Sin[2*x]/4 + Sin[4*x]/8
```

3.102.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4771
```
Int[cos[(a_.) + (b_.)*(x_)]*cos[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] + Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

3.102.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{\sin(2x)}{4}+\frac{\sin(4x)}{8}$	14
risch	$\frac{\sin(2x)}{4}+\frac{\sin(4x)}{8}$	14
parallelrisch	$\frac{\sin(2x)}{4}+\frac{\sin(4x)}{8}$	14
norman	$\frac{\frac{\tan\left(\frac{x}{2}\right)\tan\left(\frac{3x}{2}\right)^2}{4}-\frac{3\tan\left(\frac{x}{2}\right)^2\tan\left(\frac{3x}{2}\right)}{4}-\frac{\tan\left(\frac{x}{2}\right)}{4}+\frac{3\tan\left(\frac{3x}{2}\right)}{4}}{\left(1+\tan\left(\frac{x}{2}\right)^2\right)\left(1+\tan\left(\frac{3x}{2}\right)^2\right)}$	59

input
```
int(cos(x)*cos(3*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*sin(2*x)+1/8*sin(4*x)
```

3.102.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.41

$$\int \cos(x)\cos(3x)\,dx = \cos(x)^3\sin(x)$$

input
```
integrate(cos(x)*cos(3*x),x, algorithm="fricas")
```

output
```
cos(x)^3*sin(x)
```

3.102.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \cos(x)\cos(3x)\,dx = -\frac{\sin(x)\cos(3x)}{8}+\frac{3\sin(3x)\cos(x)}{8}$$

input
```
integrate(cos(x)*cos(3*x),x)
```

output
```
-sin(x)*cos(3*x)/8 + 3*sin(3*x)*cos(x)/8
```

3.102.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(x)\cos(3x)\,dx = \frac{1}{8}\sin(4x) + \frac{1}{4}\sin(2x)$$

input
```
integrate(cos(x)*cos(3*x),x, algorithm="maxima")
```

output
```
1/8*sin(4*x) + 1/4*sin(2*x)
```

3.102.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(x)\cos(3x)\,dx = \frac{1}{8}\sin(4x) + \frac{1}{4}\sin(2x)$$

input
```
integrate(cos(x)*cos(3*x),x, algorithm="giac")
```

output
```
1/8*sin(4*x) + 1/4*sin(2*x)
```

3.102.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.41

$$\int \cos(x)\cos(3x)\,dx = \cos(x)^3\sin(x)$$

input
```
int(cos(3*x)*cos(x),x)
```

output
```
cos(x)^3*sin(x)
```

3.103 $\int \cos(x)\cos(4x)\,dx$

3.103.1 Optimal result

Integrand size = 7, antiderivative size = 17

$$\int \cos(x)\cos(4x)\,dx = \frac{1}{6}\sin(3x) + \frac{1}{10}\sin(5x)$$

output
```
1/6*sin(3*x)+1/10*sin(5*x)
```

3.103.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \cos(x)\cos(4x)\,dx = \frac{1}{6}\sin(3x) + \frac{1}{10}\sin(5x)$$

input
```
Integrate[Cos[x]*Cos[4*x],x]
```

output
```
Sin[3*x]/6 + Sin[5*x]/10
```

3.103.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 4771}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\cos(4x)\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \cos(x)\cos(4x)dx$$
$$\Big\downarrow \; 4771$$
$$\frac{1}{6}\sin(3x)+\frac{1}{10}\sin(5x)$$

input
```
Int[Cos[x]*Cos[4*x],x]
```

output
```
Sin[3*x]/6 + Sin[5*x]/10
```

3.103.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4771
```
Int[cos[(a_.) + (b_.)*(x_)]*cos[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] + Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

3.103.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{\sin(3x)}{6}+\frac{\sin(5x)}{10}$	14
risch	$\frac{\sin(3x)}{6}+\frac{\sin(5x)}{10}$	14
parallelrisch	$\frac{\sin(3x)}{6}+\frac{\sin(5x)}{10}$	14
norman	$\frac{-\frac{8\tan(2x)\tan\left(\frac{x}{2}\right)^{2}}{15}+\frac{2\tan(2x)^{2}\tan\left(\frac{x}{2}\right)}{15}+\frac{8\tan(2x)}{15}-\frac{2\tan\left(\frac{x}{2}\right)}{15}}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)\left(1+\tan(2x)^{2}\right)}$	59

input
```
int(cos(x)*cos(4*x),x,method=_RETURNVERBOSE)
```

output
```
1/6*sin(3*x)+1/10*sin(5*x)
```

3.103.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

$$\int\cos(x)\cos(4x)\,dx=\frac{1}{15}\left(24\cos(x)^{4}-8\cos(x)^{2}-1\right)\sin(x)$$

input
```
integrate(cos(x)*cos(4*x),x, algorithm="fricas")
```

output
```
1/15*(24*cos(x)^4 - 8*cos(x)^2 - 1)*sin(x)
```

3.103.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int\cos(x)\cos(4x)\,dx=-\frac{\sin(x)\cos(4x)}{15}+\frac{4\sin(4x)\cos(x)}{15}$$

input
```
integrate(cos(x)*cos(4*x),x)
```

output
```
-sin(x)*cos(4*x)/15 + 4*sin(4*x)*cos(x)/15
```

3.103.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(x)\cos(4x)\,dx = \frac{1}{10}\sin(5x) + \frac{1}{6}\sin(3x)$$

input
```
integrate(cos(x)*cos(4*x),x, algorithm="maxima")
```

output
```
1/10*sin(5*x) + 1/6*sin(3*x)
```

3.103.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(x)\cos(4x)\,dx = \frac{1}{10}\sin(5x) + \frac{1}{6}\sin(3x)$$

input
```
integrate(cos(x)*cos(4*x),x, algorithm="giac")
```

output
```
1/10*sin(5*x) + 1/6*sin(3*x)
```

3.103.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \cos(x)\cos(4x)\,dx = \frac{\sin(3x)}{6} + \frac{\sin(5x)}{10}$$

input
```
int(cos(4*x)*cos(x),x)
```

output
```
sin(3*x)/6 + sin(5*x)/10
```

3.104 $\int \cos(x)\cos(mx)\,dx$

3.104.1 Optimal result

Integrand size = 7, antiderivative size = 35

$$\int \cos(x)\cos(mx)\,dx = \frac{\sin((1-m)x)}{2(1-m)} + \frac{\sin((1+m)x)}{2(1+m)}$$

output
```
1/2*sin((1-m)*x)/(1-m)+1/2*sin((1+m)*x)/(1+m)
```

3.104.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

$$\int \cos(x)\cos(mx)\,dx = \frac{-\cos(mx)\sin(x) + m\cos(x)\sin(mx)}{-1+m^2}$$

input
```
Integrate[Cos[x]*Cos[m*x],x]
```

output
```
(-(Cos[m*x]*Sin[x]) + m*Cos[x]*Sin[m*x])/(-1 + m^2)
```

3.104.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\cos(mx)\,dx$$

$$\Big\downarrow \text{5081}$$

$$\int \left(\frac{1}{2}\cos((1-m)x)+\frac{1}{2}\cos((m+1)x)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sin((1-m)x)}{2(1-m)}+\frac{\sin((m+1)x)}{2(m+1)}$$

input
```
Int[Cos[x]*Cos[m*x],x]
```

output
```
Sin[(1 - m)*x]/(2*(1 - m)) + Sin[(1 + m)*x]/(2*(1 + m))
```

3.104.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5081
```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

3.104.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{\sin(x(-1+m))}{-2+2m}+\frac{\sin((1+m)x)}{2+2m}$	28
risch	$\frac{\sin(x(-1+m))}{-2+2m}+\frac{\sin((1+m)x)}{2+2m}$	28
parallelrisch	$\frac{(1+m)\sin(x(-1+m))+\sin((1+m)x)(-1+m)}{2m^2-2}$	32
norman	$\frac{-\frac{2\tan\left(\frac{x}{2}\right)}{m^2-1}+\frac{2m\tan\left(\frac{mx}{2}\right)}{m^2-1}+\frac{2\tan\left(\frac{x}{2}\right)\tan\left(\frac{mx}{2}\right)^2}{m^2-1}-\frac{2m\tan\left(\frac{x}{2}\right)^2\tan\left(\frac{mx}{2}\right)}{m^2-1}}{\left(1+\tan\left(\frac{x}{2}\right)^2\right)\left(1+\tan\left(\frac{mx}{2}\right)^2\right)}$	93

input
```
int(cos(x)*cos(m*x),x,method=_RETURNVERBOSE)
```

output
```
1/2/(-1+m)*sin(x*(-1+m))+1/2*sin((1+m)*x)/(1+m)
```

3.104.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

$$\int \cos(x)\cos(mx)\,dx = \frac{m\cos{(x)}\sin{(mx)}-\cos{(mx)}\sin{(x)}}{m^2-1}$$

input
```
integrate(cos(x)*cos(m*x),x, algorithm="fricas")
```

output
```
(m*cos(x)*sin(m*x) - cos(m*x)*sin(x))/(m^2 - 1)
```

3.104.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(22) = 44.

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.60

$$\int \cos(x)\cos(mx)\,dx = \begin{cases} \frac{x\sin^2{(x)}}{2}+\frac{x\cos^2{(x)}}{2}+\frac{\sin{(x)}\cos{(x)}}{2} & \text{for } m=-1 \vee m=1 \\ \frac{m\sin{(mx)}\cos{(x)}}{m^2-1}-\frac{\sin{(x)}\cos{(mx)}}{m^2-1} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(x)*cos(m*x),x)
```

output
```
Piecewise((x*sin(x)**2/2 + x*cos(x)**2/2 + sin(x)*cos(x)/2, Eq(m, -1) | Eq
(m, 1)), (m*sin(m*x)*cos(x)/(m**2 - 1) - sin(x)*cos(m*x)/(m**2 - 1), True)
)
```

3.104.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

$$\int \cos(x)\cos(mx)\,dx = \frac{\sin\left((m+1)x\right)}{2\,(m+1)} - \frac{\sin\left(-(m-1)x\right)}{2\,(m-1)}$$

input
```
integrate(cos(x)*cos(m*x),x, algorithm="maxima")
```

output
```
1/2*sin((m + 1)*x)/(m + 1) - 1/2*sin(-(m - 1)*x)/(m - 1)
```

3.104.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \cos(x)\cos(mx)\,dx = \frac{\sin\left(mx+x\right)}{2\,(m+1)} + \frac{\sin\left(mx-x\right)}{2\,(m-1)}$$

input
```
integrate(cos(x)*cos(m*x),x, algorithm="giac")
```

output
```
1/2*sin(m*x + x)/(m + 1) + 1/2*sin(m*x - x)/(m - 1)
```

3.104.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.66

$$\int \cos(x)\cos(mx)\,dx = \begin{cases} \frac{x}{2} + \frac{\sin(2\,x)}{4} & \text{if } m = -1 \vee m = 1 \\ \frac{\sin(x\,(m-1))}{2\,m-2} + \frac{\sin(x\,(m+1))}{2\,m+2} & \text{if } m \neq -1 \wedge m \neq 1 \end{cases}$$

input
```
int(cos(m*x)*cos(x),x)
```

output
```
piecewise(m == -1 | m == 1, x/2 + sin(2*x)/4, m ~= -1 & m ~= 1, sin(x*(m -
 1))/(2*m - 2) + sin(x*(m + 1))/(2*m + 2))
```

3.104. $\int \cos(x) \cos(mx) \, dx$

3.105 $\int \cos(x)\tan(2x)\,dx$

3.105.1 Optimal result

Integrand size = 7, antiderivative size = 20

$$\int \cos(x)\tan(2x)\,dx = \frac{\operatorname{arctanh}\left(\sqrt{2}\cos(x)\right)}{\sqrt{2}} - \cos(x)$$

output
```
-cos(x)+1/2*arctanh(cos(x)*2^(1/2))*2^(1/2)
```

3.105.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 45 vs. $2(20) = 40$.

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.25

$$\int \cos(x)\tan(2x)\,dx = \frac{\operatorname{arctanh}\left(\sqrt{2}-\tan\left(\frac{x}{2}\right)\right)}{\sqrt{2}} + \frac{\operatorname{arctanh}\left(\sqrt{2}+\tan\left(\frac{x}{2}\right)\right)}{\sqrt{2}} - \cos(x)$$

input
```
Integrate[Cos[x]*Tan[2*x],x]
```

output
```
ArcTanh[Sqrt[2] - Tan[x/2]]/Sqrt[2] + ArcTanh[Sqrt[2] + Tan[x/2]]/Sqrt[2]
- Cos[x]
```

3.105.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3042, 4879, 27, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\tan(2x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(2x)}{\sec(x)}dx$$

$$\Big\downarrow \text{4879}$$

$$-\int -\frac{2\cos^2(x)}{1-2\cos^2(x)}d\cos(x)$$

$$\Big\downarrow \text{27}$$

$$2\int \frac{\cos^2(x)}{1-2\cos^2(x)}d\cos(x)$$

$$\Big\downarrow \text{262}$$

$$2\left(\frac{1}{2}\int \frac{1}{1-2\cos^2(x)}d\cos(x)-\frac{\cos(x)}{2}\right)$$

$$\Big\downarrow \text{219}$$

$$2\left(\frac{\text{arctanh}\left(\sqrt{2}\cos(x)\right)}{2\sqrt{2}}-\frac{\cos(x)}{2}\right)$$

input
```
Int[Cos[x]*Tan[2*x],x]
```

output
```
2*(ArcTanh[Sqrt[2]*Cos[x]]/(2*Sqrt[2]) - Cos[x]/2)
```

3.105.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4879
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cos[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Cos[v]/d
, u/Sin[v], x], x], x, Cos[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[Nonfree
Factors[Cos[v], x], u/Sin[v], x]]
```

3.105.4 Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$-\cos\left(x\right)+\frac{\operatorname{arctanh}\left(\cos\left(x\right)\sqrt{2}\right)\sqrt{2}}{2}$	18
default	$-\cos\left(x\right)+\frac{\operatorname{arctanh}\left(\cos\left(x\right)\sqrt{2}\right)\sqrt{2}}{2}$	18
risch	$-\frac{{\mathrm e}^{i x}}{2}-\frac{{\mathrm e}^{-i x}}{2}+\frac{\sqrt{2}\, \ln\left({\mathrm e}^{2 i x}+\sqrt{2}\, {\mathrm e}^{i x}+1\right)}{4}-\frac{\sqrt{2}\, \ln\left({\mathrm e}^{2 i x}-\sqrt{2}\, {\mathrm e}^{i x}+1\right)}{4}$	61

input
```
int(cos(x)*tan(2*x),x,method=_RETURNVERBOSE)
```

output
```
-cos(x)+1/2*arctanh(cos(x)*2^(1/2))*2^(1/2)
```

3.105.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(17) = 34.

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.90

$$\int \cos(x)\tan(2x)\,dx = \frac{1}{4}\sqrt{2}\log\left(-\frac{2\cos(x)^2 + 2\sqrt{2}\cos(x) + 1}{2\cos(x)^2 - 1}\right) - \cos(x)$$

input
```
integrate(cos(x)*tan(2*x),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*log(-(2*cos(x)^2 + 2*sqrt(2)*cos(x) + 1)/(2*cos(x)^2 - 1)) - c
os(x)
```

3.105.6 Sympy [F]

$$\int \cos(x)\tan(2x)\,dx = \int \cos(x)\tan(2x)\,dx$$

input
```
integrate(cos(x)*tan(2*x),x)
```

output
```
Integral(cos(x)*tan(2*x), x)
```

3.105.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. 2(17) = 34.

Time = 0.28 (sec) , antiderivative size = 133, normalized size of antiderivative = 6.65

$$\begin{aligned}\int \cos(x)\tan(2x)\,dx = {} & \frac{1}{8}\sqrt{2}\log\Big(2\sqrt{2}\sin(2x)\sin(x) + 2\left(\sqrt{2}\cos(x) + 1\right)\cos(2x) \\ & + \cos(2x)^2 + 2\cos(x)^2 + \sin(2x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) + 1\Big) \\ & - \frac{1}{8}\sqrt{2}\log\Big(-2\sqrt{2}\sin(2x)\sin(x) - 2\left(\sqrt{2}\cos(x) - 1\right)\cos(2x) \\ & + \cos(2x)^2 + 2\cos(x)^2 + \sin(2x)^2 + 2\sin(x)^2 - 2\sqrt{2}\cos(x) + 1\Big) \\ & - \cos(x)\end{aligned}$$

input
```
integrate(cos(x)*tan(2*x),x, algorithm="maxima")
```

output
```
1/8*sqrt(2)*log(2*sqrt(2)*sin(2*x)*sin(x) + 2*(sqrt(2)*cos(x) + 1)*cos(2*x
) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) +
 1) - 1/8*sqrt(2)*log(-2*sqrt(2)*sin(2*x)*sin(x) - 2*(sqrt(2)*cos(x) - 1)*
cos(2*x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 - 2*sqrt(2)*c
os(x) + 1) - cos(x)
```

3.105.8 Giac [F]

$$\int \cos(x)\tan(2x)\,dx = \int \cos\left(x\right)\tan\left(2\,x\right)\,dx$$

input
```
integrate(cos(x)*tan(2*x),x, algorithm="giac")
```

output
```
integrate(cos(x)*tan(2*x), x)
```

3.105.9 Mupad [B] (verification not implemented)

Time = 28.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.10

$$\int \cos(x)\tan(2x)\,dx = -\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{8\,\sqrt{2}\,\tan\left(\frac{x}{2}\right)^2}{12\,\tan\left(\frac{x}{2}\right)^2-4}\right)}{2} - \frac{2}{\tan\left(\frac{x}{2}\right)^2+1}$$

input
```
int(tan(2*x)*cos(x),x)
```

output
```
- (2^(1/2)*atanh((8*2^(1/2)*tan(x/2)^2)/(12*tan(x/2)^2 - 4)))/2 - 2/(tan(x
/2)^2 + 1)
```

3.106 $\int \cos(x)\tan(3x)\,dx$

3.106.1 Optimal result

Integrand size = 7, antiderivative size = 21

$$\int \cos(x)\tan(3x)\,dx = \frac{\operatorname{arctanh}\left(\frac{2\cos(x)}{\sqrt{3}}\right)}{\sqrt{3}} - \cos(x)$$

output
```
-cos(x)+1/3*arctanh(2/3*cos(x)*3^(1/2))*3^(1/2)
```

3.106.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 48 vs. 2(21) = 42.

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.29

$$\int \cos(x)\tan(3x)\,dx = -\frac{\operatorname{arctanh}\left(\frac{-2+\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{\operatorname{arctanh}\left(\frac{2+\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right)}{\sqrt{3}} - \cos(x)$$

input
```
Integrate[Cos[x]*Tan[3*x],x]
```

output
```
-(ArcTanh[(-2 + Tan[x/2])/Sqrt[3]]/Sqrt[3]) + ArcTanh[(2 + Tan[x/2])/Sqrt[
3]]/Sqrt[3] - Cos[x]
```

3.106.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3042, 4879, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\tan(3x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\tan(3x)}{\sec(x)}dx$$

$$\Big\downarrow \text{ 4879}$$

$$-\int \frac{1-4\cos^2(x)}{3-4\cos^2(x)}d\cos(x)$$

$$\Big\downarrow \text{ 299}$$

$$2\int \frac{1}{3-4\cos^2(x)}d\cos(x)-\cos(x)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\text{arctanh}\left(\frac{2\cos(x)}{\sqrt{3}}\right)}{\sqrt{3}}-\cos(x)$$

input
```
Int[Cos[x]*Tan[3*x],x]
```

output
```
ArcTanh[(2*Cos[x])/Sqrt[3]]/Sqrt[3] - Cos[x]
```

3.106.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4879
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cos[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Cos[v]/d
, u/Sin[v], x], x], x, Cos[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[Nonfree
Factors[Cos[v], x], u/Sin[v], x]]
```

3.106.4 Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$-\cos\left(x\right)+\frac{\operatorname{arctanh}\left(\frac{2\cos\left(x\right)\sqrt{3}}{3}\right)\sqrt{3}}{3}$	19
default	$-\cos\left(x\right)+\frac{\operatorname{arctanh}\left(\frac{2\cos\left(x\right)\sqrt{3}}{3}\right)\sqrt{3}}{3}$	19
risch	$-\frac{e^{ix}}{2}-\frac{e^{-ix}}{2}+\frac{\sqrt{3}\,\ln\left(e^{2ix}+\sqrt{3}\,e^{ix}+1\right)}{6}-\frac{\sqrt{3}\,\ln\left(e^{2ix}-\sqrt{3}\,e^{ix}+1\right)}{6}$	61

input
```
int(cos(x)*tan(3*x),x,method=_RETURNVERBOSE)
```

output
```
-cos(x)+1/3*arctanh(2/3*cos(x)*3^(1/2))*3^(1/2)
```

3.106.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(18) = 36$.

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.81

$$\int \cos(x)\tan(3x)\,dx = \frac{1}{6}\sqrt{3}\log\left(-\frac{4\,\cos(x)^2 + 4\sqrt{3}\cos(x) + 3}{4\,\cos(x)^2 - 3}\right) - \cos(x)$$

input
```
integrate(cos(x)*tan(3*x),x, algorithm="fricas")
```

output
```
1/6*sqrt(3)*log(-(4*cos(x)^2 + 4*sqrt(3)*cos(x) + 3)/(4*cos(x)^2 - 3)) - c
os(x)
```

3.106.6 Sympy [F]

$$\int \cos(x)\tan(3x)\,dx = \int \cos(x)\tan(3x)\,dx$$

input
```
integrate(cos(x)*tan(3*x),x)
```

output
```
Integral(cos(x)*tan(3*x), x)
```

3.106.7 Maxima [F]

$$\int \cos(x)\tan(3x)\,dx = \int \cos(x)\tan(3\,x)\,dx$$

input
```
integrate(cos(x)*tan(3*x),x, algorithm="maxima")
```

output
```
-cos(x) - integrate(((sin(3*x) - sin(x))*cos(4*x) - (cos(3*x) - cos(x))*si
n(4*x) - (cos(2*x) - 1)*sin(3*x) + cos(3*x)*sin(2*x) - cos(x)*sin(2*x) + c
os(2*x)*sin(x) - sin(x))/(2*(cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - cos(2*x
)^2 - sin(4*x)^2 + 2*sin(4*x)*sin(2*x) - sin(2*x)^2 + 2*cos(2*x) - 1), x)
```

3.106.8 Giac [F]

$$\int \cos(x)\tan(3x)\,dx = \int \cos\left(x\right)\tan\left(3\,x\right)\,dx$$

input
```
integrate(cos(x)*tan(3*x),x, algorithm="giac")
```

output
```
integrate(cos(x)*tan(3*x), x)
```

3.106.9 Mupad [B] (verification not implemented)

Time = 27.94 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.00

$$\int \cos(x)\tan(3x)\,dx = -\frac{\sqrt{3}\,\mathrm{atanh}\left(\frac{32\,\sqrt{3}\,\tan\left(\frac{x}{2}\right)^2}{3\left(\frac{56\,\tan\left(\frac{x}{2}\right)^2}{3}-\frac{8}{3}\right)}\right)}{3}-\frac{2}{\tan\left(\frac{x}{2}\right)^2+1}$$

input
```
int(tan(3*x)*cos(x),x)
```

output
```
- (3^(1/2)*atanh((32*3^(1/2)*tan(x/2)^2)/(3*((56*tan(x/2)^2)/3 - 8/3))))/3
 - 2/(tan(x/2)^2 + 1)
```

3.107 $\int \cos(x)\tan(4x)\,dx$

3.107.1 Optimal result

Integrand size = 7, antiderivative size = 71

$$\int \cos(x)\tan(4x)\,dx = \frac{1}{4}\sqrt{2-\sqrt{2}}\text{arctanh}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{2}}}\right) + \frac{1}{4}\sqrt{2+\sqrt{2}}\text{arctanh}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{2}}}\right) - \cos(x)$$

output
```
-cos(x)+1/4*arctanh(2*cos(x)/(2-2^(1/2))^(1/2))*(2-2^(1/2))^(1/2)+1/4*arct
anh(2*cos(x)/(2+2^(1/2))^(1/2))*(2+2^(1/2))^(1/2)
```

3.107.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 57.69 (sec) , antiderivative size = 6161, normalized size of antiderivative = 86.77

$$\int \cos(x)\tan(4x)\,dx = \text{Result too large to show}$$

input
```
Integrate[Cos[x]*Tan[4*x],x]
```

output
```
Result too large to show
```

3.107.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 4879, 27, 1602, 27, 1480, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\tan(4x)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\tan(4x)}{\sec(x)}dx$$

$$\Big\downarrow \; 4879$$

$$-\int -\frac{4\cos^2(x)\left(1-2\cos^2(x)\right)}{8\cos^4(x)-8\cos^2(x)+1}d\cos(x)$$

$$\Big\downarrow \; 27$$

$$4\int \frac{\cos^2(x)\left(1-2\cos^2(x)\right)}{8\cos^4(x)-8\cos^2(x)+1}d\cos(x)$$

$$\Big\downarrow \; 1602$$

$$4\left(-\frac{1}{8}\int -\frac{2\left(1-4\cos^2(x)\right)}{8\cos^4(x)-8\cos^2(x)+1}d\cos(x)-\frac{\cos(x)}{4}\right)$$

$$\Big\downarrow \; 27$$

$$4\left(\frac{1}{4}\int \frac{1-4\cos^2(x)}{8\cos^4(x)-8\cos^2(x)+1}d\cos(x)-\frac{\cos(x)}{4}\right)$$

$$\Big\downarrow \; 1480$$

$$4\left(\frac{1}{4}\left(-\left(\left(2-\sqrt{2}\right)\int \frac{1}{8\cos^2(x)-2\left(2-\sqrt{2}\right)}d\cos(x)\right)-\left(2+\sqrt{2}\right)\int \frac{1}{8\cos^2(x)-2\left(2+\sqrt{2}\right)}d\cos(x)\right)-\frac{\cos(x)}{4}\right)$$

$$\Big\downarrow \; 220$$

$$4\left(\frac{1}{4}\left(\frac{1}{4}\sqrt{2-\sqrt{2}}\text{arctanh}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{2}}}\right)+\frac{1}{4}\sqrt{2+\sqrt{2}}\text{arctanh}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{2}}}\right)\right)-\frac{\cos(x)}{4}\right)$$

input
```
Int[Cos[x]*Tan[4*x],x]
```

output
```
4*(((Sqrt[2 - Sqrt[2]]*ArcTanh[(2*Cos[x])/Sqrt[2 - Sqrt[2]]])/4 + (Sqrt[2
+ Sqrt[2]]*ArcTanh[(2*Cos[x])/Sqrt[2 + Sqrt[2]]])/4)/4 - Cos[x]/4)
```

3.107.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4879
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cos[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Cos[v]/d
, u/Sin[v], x], x], x, Cos[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[Nonfree
Factors[Cos[v], x], u/Sin[v], x]]
```

3.107.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.39 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{e^{ix}}{2}-\frac{e^{-ix}}{2}-i\left(\sum_{\_R=\operatorname{RootOf}(2048\_Z^4+128\_Z^2+1)}\_R\ln\left(e^{2ix}-8i\_R\,e^{ix}+1\right)\right)$
default	$-\frac{\sqrt{2}\,\sqrt{2+\sqrt{2}}\,\operatorname{arctanh}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{2}}}\right)}{4}-\frac{\left(\sqrt{2}-2\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2-\sqrt{2}}}-\cos(x)+\frac{\left(2\sqrt{2}+3\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2+\sqrt{2}}}+\frac{(-3}{}$

input
```
int(cos(x)*tan(4*x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*exp(I*x)-1/2*exp(-I*x)-I*sum(_R*ln(exp(2*I*x)-8*I*_R*exp(I*x)+1),_R=R
ootOf(2048*_Z^4+128*_Z^2+1))
```

3.107.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.42

$$\int \cos(x)\tan(4x)\,dx = \frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\sqrt{\sqrt{2}+2}+2\cos(x)\right) - \frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\sqrt{\sqrt{2}+2}-2\cos(x)\right) + \frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(\sqrt{-\sqrt{2}+2}+2\cos(x)\right) - \frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(\sqrt{-\sqrt{2}+2}-2\cos(x)\right) - \cos(x)$$

input
```
integrate(cos(x)*tan(4*x),x, algorithm="fricas")
```

output
```
1/8*sqrt(sqrt(2) + 2)*log(sqrt(sqrt(2) + 2) + 2*cos(x)) - 1/8*sqrt(sqrt(2)
 + 2)*log(sqrt(sqrt(2) + 2) - 2*cos(x)) + 1/8*sqrt(-sqrt(2) + 2)*log(sqrt(
-sqrt(2) + 2) + 2*cos(x)) - 1/8*sqrt(-sqrt(2) + 2)*log(sqrt(-sqrt(2) + 2)
- 2*cos(x)) - cos(x)
```

3.107.6 Sympy [F]

$$\int \cos(x)\tan(4x)\,dx = \int \cos(x)\tan(4x)\,dx$$

input
```
integrate(cos(x)*tan(4*x),x)
```

output
```
Integral(cos(x)*tan(4*x), x)
```

3.107.7 Maxima [F]

$$\int \cos(x)\tan(4x)\,dx = \int \cos(x)\tan(4x)\,dx$$

input
```
integrate(cos(x)*tan(4*x),x, algorithm="maxima")
```

output
```
-cos(x) - integrate(-((sin(7*x) - sin(x))*cos(8*x) - (cos(7*x) - cos(x))*s
in(8*x) + sin(7*x) - sin(x))/(cos(8*x)^2 + sin(8*x)^2 + 2*cos(8*x) + 1), x
)
```

3.107.8 Giac [F]

$$\int \cos(x)\tan(4x)\,dx = \int \cos(x)\tan(4x)\,dx$$

input
```
integrate(cos(x)*tan(4*x),x, algorithm="giac")
```

output
```
integrate(cos(x)*tan(4*x), x)
```

3.107.9 Mupad [B] (verification not implemented)

Time = 27.72 (sec) , antiderivative size = 295, normalized size of antiderivative = 4.15

$$\int \cos(x)\tan(4x)\,dx =$$

$$-\frac{\operatorname{atanh}\left(\frac{219747975168\tan\left(\frac{x}{2}\right)^2\sqrt{2-\sqrt{2}}}{6098518016\sqrt{2}-254015438848\sqrt{2}\tan\left(\frac{x}{2}\right)^2+386664497152\tan\left(\frac{x}{2}\right)^2-20887633920}-\frac{1597190963}{6098518016\sqrt{2}-254015438848\sqrt{2}\tan\left(\frac{x}{2}\right)}\right.}{}$$

$$-\frac{2}{\tan\left(\frac{x}{2}\right)^2+1}$$

$$-\frac{\operatorname{atanh}\left(\frac{15971909632\sqrt{\sqrt{2}+2}}{6098518016\sqrt{2}-254015438848\sqrt{2}\tan\left(\frac{x}{2}\right)^2-386664497152\tan\left(\frac{x}{2}\right)^2+20887633920}-\frac{219747975168\,\mathrm{ta}}{6098518016\sqrt{2}-254015438848\sqrt{2}\tan\left(\frac{x}{2}\right)}\right.}{}$$

input
```
int(tan(4*x)*cos(x),x)
```

output
```
- (atanh((219747975168*tan(x/2)^2*(2 - 2^(1/2))^(1/2))/(6098518016*2^(1/2)
 - 254015438848*2^(1/2)*tan(x/2)^2 + 386664497152*tan(x/2)^2 - 20887633920
) - (15971909632*(2 - 2^(1/2))^(1/2))/(6098518016*2^(1/2) - 254015438848*2
^(1/2)*tan(x/2)^2 + 386664497152*tan(x/2)^2 - 20887633920) - (130056978432
*2^(1/2)*tan(x/2)^2*(2 - 2^(1/2))^(1/2))/(6098518016*2^(1/2) - 25401543884
8*2^(1/2)*tan(x/2)^2 + 386664497152*tan(x/2)^2 - 20887633920))*(2 - 2^(1/2
))^(1/2))/4 - 2/(tan(x/2)^2 + 1) - (atanh((15971909632*(2^(1/2) + 2)^(1/2)
)/(6098518016*2^(1/2) - 254015438848*2^(1/2)*tan(x/2)^2 - 386664497152*tan
(x/2)^2 + 20887633920) - (219747975168*tan(x/2)^2*(2^(1/2) + 2)^(1/2))/(60
98518016*2^(1/2) - 254015438848*2^(1/2)*tan(x/2)^2 - 386664497152*tan(x/2)
^2 + 20887633920) - (130056978432*2^(1/2)*tan(x/2)^2*(2^(1/2) + 2)^(1/2))/
(6098518016*2^(1/2) - 254015438848*2^(1/2)*tan(x/2)^2 - 386664497152*tan(x
/2)^2 + 20887633920))*(2^(1/2) + 2)^(1/2))/4
```

3.108 $\int \cos(x)\tan(5x)\,dx$

3.108.1 Optimal result

Integrand size = 7, antiderivative size = 84

$$\int \cos(x)\tan(5x)\,dx = \frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\operatorname{arctanh}\left(2\sqrt{\frac{2}{5+\sqrt{5}}}\cos(x)\right) + \frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}\cos(x)\right) - \cos(x)$$

output
```
-cos(x)+1/10*arctanh(1/5*cos(x)*(50+10*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2
)+1/10*arctanh(2*cos(x)*2^(1/2)/(5+5^(1/2))^(1/2))*(10+2*5^(1/2))^(1/2)
```

3.108.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 215 vs. $2(84) = 168$.

Time = 1.04 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.56

$$\int \cos(x)\tan(5x)\,dx = \frac{(1+\sqrt{5})\,\text{arctanh}\left(\frac{4-\left(-1+\sqrt{5}\right)\tan\left(\frac{x}{2}\right)}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)}{\sqrt{10\,(5+\sqrt{5})}} + \frac{(1+\sqrt{5})\,\text{arctanh}\left(\frac{4+\left(-1+\sqrt{5}\right)\tan\left(\frac{x}{2}\right)}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)}{\sqrt{10\,(5+\sqrt{5})}} + \frac{(-1+\sqrt{5})\,\text{arctanh}\left(\frac{4-\left(1+\sqrt{5}\right)\tan\left(\frac{x}{2}\right)}{\sqrt{10-2\sqrt{5}}}\right)}{\sqrt{50-10\sqrt{5}}} + \frac{(-1+\sqrt{5})\,\text{arctanh}\left(\frac{4+\left(1+\sqrt{5}\right)\tan\left(\frac{x}{2}\right)}{\sqrt{10-2\sqrt{5}}}\right)}{\sqrt{50-10\sqrt{5}}} - \cos(x)$$

input
```
Integrate[Cos[x]*Tan[5*x],x]
```

output
```
((1 + Sqrt[5])*ArcTanh[(4 - (-1 + Sqrt[5])*Tan[x/2])/Sqrt[2*(5 + Sqrt[5])]
])/Sqrt[10*(5 + Sqrt[5])] + ((1 + Sqrt[5])*ArcTanh[(4 + (-1 + Sqrt[5])*Tan
[x/2])/Sqrt[2*(5 + Sqrt[5])]])/Sqrt[10*(5 + Sqrt[5])] + ((-1 + Sqrt[5])*Ar
cTanh[(4 - (1 + Sqrt[5])*Tan[x/2])/Sqrt[10 - 2*Sqrt[5]]])/Sqrt[50 - 10*Sqr
t[5]] + ((-1 + Sqrt[5])*ArcTanh[(4 + (1 + Sqrt[5])*Tan[x/2])/Sqrt[10 - 2*S
qrt[5]]])/Sqrt[50 - 10*Sqrt[5]] - Cos[x]
```

3.108.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4879, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\tan(5x)\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\tan(5x)}{\sec(x)}dx$$
$$\Big\downarrow \; 4879$$
$$-\int \frac{16\cos^4(x)-12\cos^2(x)+1}{16\cos^4(x)-20\cos^2(x)+5}d\cos(x)$$
$$\Big\downarrow \; 2205$$
$$-\int \left(1-\frac{4(1-2\cos^2(x))}{16\cos^4(x)-20\cos^2(x)+5}\right)d\cos(x)$$
$$\Big\downarrow \; 2009$$
$$\frac{1}{5}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\operatorname{arctanh}\left(2\sqrt{\frac{2}{5+\sqrt{5}}}\cos(x)\right)+\frac{1}{5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\operatorname{arctanh}\left(\sqrt{\frac{2}{5}\left(5+\sqrt{5}\right)}\cos(x)\right)-\cos(x)$$

input
```
Int[Cos[x]*Tan[5*x],x]
```

output
```
(Sqrt[(5 + Sqrt[5])/2]*ArcTanh[2*Sqrt[2/(5 + Sqrt[5])]*Cos[x]])/5 + (Sqrt[
(5 - Sqrt[5])/2]*ArcTanh[Sqrt[(2*(5 + Sqrt[5]))/5]*Cos[x]])/5 - Cos[x]
```

3.108.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4879
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cos[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Cos[v]/d
, u/Sin[v], x], x], x, Cos[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[Nonfree
Factors[Cos[v], x], u/Sin[v], x]]
```

3.108.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.14 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.64

method	result	size
risch	$-\frac{e^{ix}}{2}-\frac{e^{-ix}}{2}-i\left(\sum_{\_R=\mathrm{RootOf}\left(2000\_Z^{4}+100\_Z^{2}+1\right)}\_R\ln\left(e^{2ix}-10i\_R\,e^{ix}+1\right)\right)$	54

input
```
int(cos(x)*tan(5*x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*exp(I*x)-1/2*exp(-I*x)-I*sum(_R*ln(exp(2*I*x)-10*I*_R*exp(I*x)+1),_R=
RootOf(2000*_Z^4+100*_Z^2+1))
```

3.108.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(56) = 112$.

Time = 0.28 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.54

$$\begin{aligned}\int \cos(x)\tan(5x)\,dx = {} & \frac{1}{20}\sqrt{2}\sqrt{\sqrt{5}+5}\log\left(\sqrt{2}\sqrt{\sqrt{5}+5}+4\cos(x)\right) \\ & -\frac{1}{20}\sqrt{2}\sqrt{\sqrt{5}+5}\log\left(\sqrt{2}\sqrt{\sqrt{5}+5}-4\cos(x)\right) \\ & +\frac{1}{20}\sqrt{2}\sqrt{-\sqrt{5}+5}\log\left(\sqrt{2}\sqrt{-\sqrt{5}+5}+4\cos(x)\right) \\ & -\frac{1}{20}\sqrt{2}\sqrt{-\sqrt{5}+5}\log\left(\sqrt{2}\sqrt{-\sqrt{5}+5}-4\cos(x)\right)-\cos(x)\end{aligned}$$

input
```
integrate(cos(x)*tan(5*x),x, algorithm="fricas")
```

output
```
1/20*sqrt(2)*sqrt(sqrt(5) + 5)*log(sqrt(2)*sqrt(sqrt(5) + 5) + 4*cos(x)) -
 1/20*sqrt(2)*sqrt(sqrt(5) + 5)*log(sqrt(2)*sqrt(sqrt(5) + 5) - 4*cos(x))
+ 1/20*sqrt(2)*sqrt(-sqrt(5) + 5)*log(sqrt(2)*sqrt(-sqrt(5) + 5) + 4*cos(x
)) - 1/20*sqrt(2)*sqrt(-sqrt(5) + 5)*log(sqrt(2)*sqrt(-sqrt(5) + 5) - 4*co
s(x)) - cos(x)
```

3.108.6 Sympy [F]

$$\int \cos(x)\tan(5x)\,dx = \int \cos{(x)}\tan{(5x)}\,dx$$

input
```
integrate(cos(x)*tan(5*x),x)
```

output
```
Integral(cos(x)*tan(5*x), x)
```

3.108.7 Maxima [F]

$$\int \cos(x)\tan(5x)\,dx = \int \cos{(x)}\tan{(5\,x)}\,dx$$

input
```
integrate(cos(x)*tan(5*x),x, algorithm="maxima")
```

output
```
-cos(x) - integrate(((sin(7*x) - sin(5*x) + sin(3*x) - sin(x))*cos(8*x) +
(sin(6*x) - sin(4*x) + sin(2*x))*cos(7*x) + (sin(5*x) - sin(3*x) + sin(x))
*cos(6*x) + (sin(4*x) - sin(2*x))*cos(5*x) + (sin(3*x) - sin(x))*cos(4*x)
- (cos(7*x) - cos(5*x) + cos(3*x) - cos(x))*sin(8*x) - (cos(6*x) - cos(4*x
) + cos(2*x) - 1)*sin(7*x) - (cos(5*x) - cos(3*x) + cos(x))*sin(6*x) - (co
s(4*x) - cos(2*x) + 1)*sin(5*x) - (cos(3*x) - cos(x))*sin(4*x) - (cos(2*x)
 - 1)*sin(3*x) + cos(3*x)*sin(2*x) - cos(x)*sin(2*x) + cos(2*x)*sin(x) - s
in(x))/(2*(cos(6*x) - cos(4*x) + cos(2*x) - 1)*cos(8*x) - cos(8*x)^2 + 2*(
cos(4*x) - cos(2*x) + 1)*cos(6*x) - cos(6*x)^2 + 2*(cos(2*x) - 1)*cos(4*x)
 - cos(4*x)^2 - cos(2*x)^2 + 2*(sin(6*x) - sin(4*x) + sin(2*x))*sin(8*x) -
 sin(8*x)^2 + 2*(sin(4*x) - sin(2*x))*sin(6*x) - sin(6*x)^2 - sin(4*x)^2 +
 2*sin(4*x)*sin(2*x) - sin(2*x)^2 + 2*cos(2*x) - 1), x)
```

3.108.8 Giac [F]

$$\int \cos(x)\tan(5x)\,dx = \int \cos(x)\tan(5x)\,dx$$

input
```
integrate(cos(x)*tan(5*x),x, algorithm="giac")
```

output
```
integrate(cos(x)*tan(5*x), x)
```

3.108.9 Mupad [B] (verification not implemented)

Time = 27.90 (sec) , antiderivative size = 407, normalized size of antiderivative = 4.85

$$\int \cos(x)\tan(5x)\,dx$$

$$= \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{18032420192256\sqrt{2}\tan\left(\frac{x}{2}\right)^2\sqrt{\sqrt{5}+5}}{\frac{8851927597056\sqrt{5}}{25}-\frac{676375744741376\sqrt{5}\tan\left(\frac{x}{2}\right)^2}{25}-\frac{333433343574016\tan\left(\frac{x}{2}\right)^2}{5}+2398739234816}-\frac{8\cdots}{25\left(\frac{8851927597056\sqrt{5}}{25}-\frac{67637574474\cdots}{}\cdots\right.}\cdots\right.}{\cdots}$$

$$-\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{867583393792\sqrt{2}\sqrt{5}\sqrt{5-\sqrt{5}}}{25\left(\frac{8851927597056\sqrt{5}}{25}-\frac{676375744741376\sqrt{5}\tan\left(\frac{x}{2}\right)^2}{25}+\frac{333433343574016\tan\left(\frac{x}{2}\right)^2}{5}-2398739234816\right)}-\frac{\cdots}{5\left(\frac{8851927597056\sqrt{5}}{25}-\frac{67\cdots}{}\cdots\right.}\cdots\right.}{\cdots}$$

$$-\frac{2}{\tan\left(\frac{x}{2}\right)^2+1}$$

input
```
int(tan(5*x)*cos(x),x)
```

output
```
(2^(1/2)*atanh((18032420192256*2^(1/2)*tan(x/2)^2*(5^(1/2) + 5)^(1/2))/((8
851927597056*5^(1/2))/25 - (676375744741376*5^(1/2)*tan(x/2)^2)/25 - (3334
33343574016*tan(x/2)^2)/5 + 2398739234816) - (867583393792*2^(1/2)*5^(1/2)
*(5^(1/2) + 5)^(1/2))/(25*((8851927597056*5^(1/2))/25 - (676375744741376*5
^(1/2)*tan(x/2)^2)/25 - (333433343574016*tan(x/2)^2)/5 + 2398739234816)) -
 (3805341024256*2^(1/2)*(5^(1/2) + 5)^(1/2))/(5*((8851927597056*5^(1/2))/2
5 - (676375744741376*5^(1/2)*tan(x/2)^2)/25 - (333433343574016*tan(x/2)^2)
/5 + 2398739234816)) + (6886980059136*2^(1/2)*5^(1/2)*tan(x/2)^2*(5^(1/2)
+ 5)^(1/2))/((8851927597056*5^(1/2))/25 - (676375744741376*5^(1/2)*tan(x/2
)^2)/25 - (333433343574016*tan(x/2)^2)/5 + 2398739234816))*(5^(1/2) + 5)^(
1/2))/10 - (2^(1/2)*atanh((867583393792*2^(1/2)*5^(1/2)*(5 - 5^(1/2))^(1/2
))/(25*((8851927597056*5^(1/2))/25 - (676375744741376*5^(1/2)*tan(x/2)^2)/
25 + (333433343574016*tan(x/2)^2)/5 - 2398739234816)) - (3805341024256*2^(
1/2)*(5 - 5^(1/2))^(1/2))/(5*((8851927597056*5^(1/2))/25 - (67637574474137
6*5^(1/2)*tan(x/2)^2)/25 + (333433343574016*tan(x/2)^2)/5 - 2398739234816)
) + (18032420192256*2^(1/2)*tan(x/2)^2*(5 - 5^(1/2))^(1/2))/((885192759705
6*5^(1/2))/25 - (676375744741376*5^(1/2)*tan(x/2)^2)/25 + (333433343574016
*tan(x/2)^2)/5 - 2398739234816) - (6886980059136*2^(1/2)*5^(1/2)*tan(x/2)^
2*(5 - 5^(1/2))^(1/2))/((8851927597056*5^(1/2))/25 - (676375744741376*5^(1
/2)*tan(x/2)^2)/25 + (333433343574016*tan(x/2)^2)/5 - 2398739234816))*(...
```

3.108. $\int \cos(x)\tan(5x)\,dx$

3.109 $\int \cos(x)\tan(6x)\,dx$

3.109.1 Optimal result

Integrand size = 7, antiderivative size = 89

$$\int \cos(x)\tan(6x)\,dx = \frac{\operatorname{arctanh}\left(\sqrt{2}\cos(x)\right)}{3\sqrt{2}} + \frac{1}{6}\sqrt{2-\sqrt{3}}\operatorname{arctanh}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{3}}}\right) + \frac{1}{6}\sqrt{2+\sqrt{3}}\operatorname{arctanh}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{3}}}\right) - \cos(x)$$

output
```
-cos(x)+1/6*arctanh(cos(x)*2^(1/2))*2^(1/2)+1/6*arctanh(2*cos(x)/(1/2*6^(1
/2)-1/2*2^(1/2)))*(1/2*6^(1/2)-1/2*2^(1/2))+1/6*arctanh(2*cos(x)/(1/2*6^(1
/2)+1/2*2^(1/2)))*(1/2*6^(1/2)+1/2*2^(1/2))
```

3.109.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 8.21 (sec) , antiderivative size = 628, normalized size of antiderivative = 7.06

$$\int \cos(x)\tan(6x)\,dx = \frac{1}{24}\left((4+4i)(-1)^{3/4}\text{arctanh}\left(\frac{-1+\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right) + (4-4i)\sqrt[4]{-1}\text{arctanh}\left(\frac{1+\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right) - 24\cos(x) - \frac{2(1+\sqrt{2})\left(x-2\sqrt{3}\text{arctanh}\left(\frac{2+\left(2+\sqrt{2}\right)\tan\left(\frac{x}{2}\right)}{\sqrt{6}}\right) - \log\left(\sec^2\left(\frac{x}{2}\right)\right) + \log\left(-\sec^2\left(\frac{x}{2}\right)\left(\sqrt{2}-2\cos(x)+2\right.\right.\right.}{2+\sqrt{2}} + \sqrt{2}\left(x+2\sqrt{3}\text{arctanh}\left(\frac{\sqrt{2}+(-1+\sqrt{2})\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right) - \log\left(\sec^2\left(\frac{x}{2}\right)\right) + \log\left(\sec^2\left(\frac{x}{2}\right)\left(1+\sqrt{2}\cos(x)-\right.\right.\right. - \frac{2\left(2(-2+\sqrt{6})\text{arctanh}\left(\sqrt{2}+(\sqrt{2}-\sqrt{3})\tan\left(\frac{x}{2}\right)\right) + (3\sqrt{2}-2\sqrt{3})\left(x-\log\left(\sec^2\left(\frac{x}{2}\right)\right) + \log\left(-\sec^2\right.\right.\right.}{-36+15\sqrt{6}+\left(20-8\sqrt{6}\right)\cos(x)+\left(12-5\sqrt{6}\right)\cos} + \frac{2\left(-2\left(\sqrt{2}+\sqrt{3}\right)\text{arctanh}\left(\frac{2+\left(2+\sqrt{6}\right)\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right) + \left(3+\sqrt{6}\right)\left(x-\log\left(\sec^2\left(\frac{x}{2}\right)\right) + \log\left(-\sec^2\left(\frac{x}{2}\right)\left(\sqrt{6}-2\right.\right.\right.\right.}{-36-15\sqrt{6}+4\left(5+2\sqrt{6}\right)\cos(x)+\left(12+5\sqrt{6}\right)\cos(2x)-50\text{s}}$$

input
```
Integrate[Cos[x]*Tan[6*x],x]
```

output
```
((4 + 4*I)*(-1)^(3/4)*ArcTanh[(-1 + Tan[x/2])/Sqrt[2]] + (4 - 4*I)*(-1)^(1
/4)*ArcTanh[(1 + Tan[x/2])/Sqrt[2]] - 24*Cos[x] - (2*(1 + Sqrt[2])*(x - 2*
Sqrt[3]*ArcTanh[(2 + (2 + Sqrt[2])*Tan[x/2])/Sqrt[6]] - Log[Sec[x/2]^2] +
Log[-(Sec[x/2]^2*(Sqrt[2] - 2*Cos[x] + 2*Sin[x]))]))/(2 + Sqrt[2]) + Sqrt[
2]*(x + 2*Sqrt[3]*ArcTanh[(Sqrt[2] + (-1 + Sqrt[2])*Tan[x/2])/Sqrt[3]] - L
og[Sec[x/2]^2] + Log[Sec[x/2]^2*(1 + Sqrt[2]*Cos[x] - Sqrt[2]*Sin[x])]) -
(2*(2*(-2 + Sqrt[6])*ArcTanh[Sqrt[2] + (Sqrt[2] - Sqrt[3])*Tan[x/2]] + (3*
Sqrt[2] - 2*Sqrt[3])*(x - Log[Sec[x/2]^2] + Log[-(Sec[x/2]^2*(Sqrt[3] + Sq
rt[2]*Cos[x] - Sqrt[2]*Sin[x]))]))*(Sqrt[2] - Sqrt[3]*Sin[x])*(-3 + Sqrt[6
] - (-2 + Sqrt[6])*Cos[x] + (-2 + Sqrt[6])*Sin[x]))/(-36 + 15*Sqrt[6] + (2
0 - 8*Sqrt[6])*Cos[x] + (12 - 5*Sqrt[6])*Cos[2*x] - 50*Sin[x] + 20*Sqrt[6]
*Sin[x] + 12*Sin[2*x] - 5*Sqrt[6]*Sin[2*x]) + (2*(-2*(Sqrt[2] + Sqrt[3])*A
rcTanh[(2 + (2 + Sqrt[6])*Tan[x/2])/Sqrt[2]] + (3 + Sqrt[6])*(x - Log[Sec[
x/2]^2] + Log[-(Sec[x/2]^2*(Sqrt[6] - 2*Cos[x] + 2*Sin[x]))]))*(2 + Sqrt[6
]*Sin[x])*(3 + Sqrt[6] - (2 + Sqrt[6])*Cos[x] + (2 + Sqrt[6])*Sin[x]))/(-3
6 - 15*Sqrt[6] + 4*(5 + 2*Sqrt[6])*Cos[x] + (12 + 5*Sqrt[6])*Cos[2*x] - 50
*Sin[x] - 20*Sqrt[6]*Sin[x] + 12*Sin[2*x] + 5*Sqrt[6]*Sin[2*x]))/24
```

3.109.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 4879, 27, 2460, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\tan(6x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(6x)}{\sec(x)}dx$$

$$\Big\downarrow \text{4879}$$

$$-\int -\frac{2\cos^2(x)\left(16\cos^4(x)-16\cos^2(x)+3\right)}{-32\cos^6(x)+48\cos^4(x)-18\cos^2(x)+1}d\cos(x)$$

$$\Big\downarrow \text{27}$$

$$2\int \frac{\cos^2(x)\left(16\cos^4(x)-16\cos^2(x)+3\right)}{-32\cos^6(x)+48\cos^4(x)-18\cos^2(x)+1}d\cos(x)$$

$$\Big\downarrow 2460$$

$$2\int \left(\frac{1-8\cos^2(x)}{3\left(16\cos^4(x)-16\cos^2(x)+1\right)}-\frac{1}{6\left(2\cos^2(x)-1\right)}-\frac{1}{2}\right)d\cos(x)$$

$$\Big\downarrow 2009$$

$$2\left(\frac{\text{arctanh}\left(\sqrt{2}\cos(x)\right)}{6\sqrt{2}}+\frac{1}{12}\sqrt{2-\sqrt{3}}\text{arctanh}\left(\frac{2\cos(x)}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{12}\sqrt{2+\sqrt{3}}\text{arctanh}\left(\frac{2\cos(x)}{\sqrt{2+\sqrt{3}}}\right)-\frac{\cos(x)}{2}\right)$$

input
```
Int[Cos[x]*Tan[6*x],x]
```

output
```
2*(ArcTanh[Sqrt[2]*Cos[x]]/(6*Sqrt[2]) + (Sqrt[2 - Sqrt[3]]*ArcTanh[(2*Cos
[x])/Sqrt[2 - Sqrt[3]]])/12 + (Sqrt[2 + Sqrt[3]]*ArcTanh[(2*Cos[x])/Sqrt[2
 + Sqrt[3]]])/12 - Cos[x]/2)
```

3.109.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2460
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px /. x -> Sqrt[x]]},
Int[ExpandIntegrand[u*(Qx /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[Q
x, x]]] /; PolyQ[Px, x^2] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&
 !TrinomialQ[Px, x] && ILtQ[p, 0] && RationalFunctionQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4879
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cos[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Cos[v]/d
, u/Sin[v], x], x], x, Cos[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[Nonfree
Factors[Cos[v], x], u/Sin[v], x]]
```

3.109.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.17 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.11

method	result
risch	$-\frac{e^{ix}}{2}-\frac{e^{-ix}}{2}-i\left(\sum_{\_R=\operatorname{RootOf}(20736\_Z^4+576\_Z^2+1)}\_R\ln\left(e^{2ix}-12i\_R\,e^{ix}+1\right)\right)+\frac{\sqrt{2}\,\ln\left(e^{2ix}+\sqrt{2}\,e^{ix}+1\right)}{12}$

input
```
int(cos(x)*tan(6*x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*exp(I*x)-1/2*exp(-I*x)-I*sum(_R*ln(exp(2*I*x)-12*I*_R*exp(I*x)+1),_R=
RootOf(20736*_Z^4+576*_Z^2+1))+1/12*2^(1/2)*ln(exp(2*I*x)+2^(1/2)*exp(I*x)
+1)-1/12*2^(1/2)*ln(exp(2*I*x)-2^(1/2)*exp(I*x)+1)
```

3.109.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.51

$$\begin{aligned}\int \cos(x)\tan(6x)\,dx &= \frac{1}{12}\sqrt{\sqrt{3}+2}\log\left(\sqrt{\sqrt{3}+2}+2\cos(x)\right)\\&\quad-\frac{1}{12}\sqrt{\sqrt{3}+2}\log\left(\sqrt{\sqrt{3}+2}-2\cos(x)\right)\\&\quad+\frac{1}{12}\sqrt{-\sqrt{3}+2}\log\left(\sqrt{-\sqrt{3}+2}+2\cos(x)\right)\\&\quad-\frac{1}{12}\sqrt{-\sqrt{3}+2}\log\left(\sqrt{-\sqrt{3}+2}-2\cos(x)\right)\\&\quad+\frac{1}{12}\sqrt{2}\log\left(-\frac{2\cos(x)^2+2\sqrt{2}\cos(x)+1}{2\cos(x)^2-1}\right)-\cos(x)\end{aligned}$$

input
```
integrate(cos(x)*tan(6*x),x, algorithm="fricas")
```

output
```
1/12*sqrt(sqrt(3) + 2)*log(sqrt(sqrt(3) + 2) + 2*cos(x)) - 1/12*sqrt(sqrt(
3) + 2)*log(sqrt(sqrt(3) + 2) - 2*cos(x)) + 1/12*sqrt(-sqrt(3) + 2)*log(sq
rt(-sqrt(3) + 2) + 2*cos(x)) - 1/12*sqrt(-sqrt(3) + 2)*log(sqrt(-sqrt(3) +
 2) - 2*cos(x)) + 1/12*sqrt(2)*log(-(2*cos(x)^2 + 2*sqrt(2)*cos(x) + 1)/(2
*cos(x)^2 - 1)) - cos(x)
```

3.109.6 Sympy [F]

$$\int \cos(x)\tan(6x)\,dx = \int \cos{(x)}\tan{(6x)}\,dx$$

input
```
integrate(cos(x)*tan(6*x),x)
```

output
```
Integral(cos(x)*tan(6*x), x)
```

3.109.7 Maxima [F]

$$\int \cos(x)\tan(6x)\,dx = \int \cos{(x)}\tan{(6\,x)}\,dx$$

input
```
integrate(cos(x)*tan(6*x),x, algorithm="maxima")
```

output
```
1/24*sqrt(2)*log(2*sqrt(2)*sin(2*x)*sin(x) + 2*(sqrt(2)*cos(x) + 1)*cos(2*
x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x)
+ 1) - 1/24*sqrt(2)*log(-2*sqrt(2)*sin(2*x)*sin(x) - 2*(sqrt(2)*cos(x) - 1
)*cos(2*x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 - 2*sqrt(2)
*cos(x) + 1) - cos(x) - integrate(1/3*((2*sin(7*x) + sin(5*x) - sin(3*x) -
 2*sin(x))*cos(8*x) + (sin(3*x) + 2*sin(x))*cos(4*x) - (2*cos(7*x) + cos(5
*x) - cos(3*x) - 2*cos(x))*sin(8*x) - 2*(cos(4*x) - 1)*sin(7*x) - (cos(4*x
) - 1)*sin(5*x) - (cos(3*x) + 2*cos(x))*sin(4*x) + 2*cos(7*x)*sin(4*x) + c
os(5*x)*sin(4*x) - sin(3*x) - 2*sin(x))/(2*(cos(4*x) - 1)*cos(8*x) - cos(8
*x)^2 - cos(4*x)^2 - sin(8*x)^2 + 2*sin(8*x)*sin(4*x) - sin(4*x)^2 + 2*cos
(4*x) - 1), x)
```

3.109.8 Giac [F]

$$\int \cos(x)\tan(6x)\,dx = \int \cos{(x)}\tan{(6\,x)}\,dx$$

input
```
integrate(cos(x)*tan(6*x),x, algorithm="giac")
```

output
```
integrate(cos(x)*tan(6*x), x)
```

3.109.9 Mupad [B] (verification not implemented)

Time = 29.06 (sec) , antiderivative size = 787, normalized size of antiderivative = 8.84

$$\int \cos(x)\tan(6x)\,dx = \text{Too large to display}$$

input
```
int(tan(6*x)*cos(x),x)
```

output
```
(6^(1/2)*(atan((2^(1/2)*321030945816576i)/(2132548963043330304 00*tan(x/2)^
4 - 1292758292627954380 80*tan(x/2)^2 + 2176593611144037376) + (6^(1/2)*888
405273481134080i)/(213254896304333030400*tan(x/2)^4 - 12927582926279543808
0*tan(x/2)^2 + 2176593611144037376) - (2^(1/2)*tan(x/2)^2*1871105472480256
0i)/(213254896304333030400*tan(x/2)^4 - 129275829262795438080*tan(x/2)^2 +
 2176593611144037376) + (2^(1/2)*tan(x/2)^4*10905601889064960i)/(213254896
304333030400*tan(x/2)^4 - 129275829262795438080*tan(x/2)^2 + 2176593611144
037376) - (6^(1/2)*tan(x/2)^2*527658334623522877 44i)/(21325489630433303040
0*tan(x/2)^4 - 129275829262795438080*tan(x/2)^2 + 2176593611144037376) + (
6^(1/2)*tan(x/2)^4*870546504971060121 60i)/(213254896304333030400*tan(x/2)^
4 - 129275829262795438080*tan(x/2)^2 + 2176593611144037376)) + atan((2^(1/
2)*144332550458980178819048433254 4i)/(589232404262260650654553866240*2^(1/
2)*6^(1/2) + 1191297171699098884409493395865 60*tan(x/2)^2 - 34367271726987
9599464668620390 40*2^(1/2)*6^(1/2)*tan(x/2)^2 - 20870903094507989978345572
92544) - (6^(1/2)*85204713977120434661674188800 0i)/(58923240426226065065 45
53866240*2^(1/2)*6^(1/2) + 119129717169909888440949339586560*tan(x/2)^2 -
343672717269879599464668620390 40*2^(1/2)*6^(1/2)*tan(x/2)^2 - 208709030945
0798997834557292544) - (2^(1/2)*tan(x/2)^2*8418228357130530454356858241024
0i)/(589232404262260650654553866240*2^(1/2)*6^(1/2) + 1191297171699098 8844
0949339586560*tan(x/2)^2 - 343672717269879599464668620390 40*2^(1/2)*6^(...
```

3.110 $\int \cos(x)\cot(2x)\,dx$

3.110.1 Optimal result

Integrand size = 7, antiderivative size = 10

$$\int \cos(x)\cot(2x)\,dx = -\frac{1}{2}\text{arctanh}(\cos(x)) + \cos(x)$$

output
```
-1/2*arctanh(cos(x))+cos(x)
```

3.110.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 25 vs. 2(10) = 20.

Time = 0.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.50

$$\int \cos(x)\cot(2x)\,dx = \cos(x) - \frac{1}{2}\log\left(\cos\left(\frac{x}{2}\right)\right) + \frac{1}{2}\log\left(\sin\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[Cos[x]*Cot[2*x],x]
```

output
```
Cos[x] - Log[Cos[x/2]]/2 + Log[Sin[x/2]]/2
```

3.110.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.40, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 4879, 27, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\cot(2x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \cos(x)\cot(2x)dx$$

$$\Big\downarrow \text{ 4879}$$

$$-\int -\frac{1-2\cos^2(x)}{2\,(1-\cos^2(x))}d\cos(x)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\int \frac{1-2\cos^2(x)}{1-\cos^2(x)}d\cos(x)$$

$$\Big\downarrow \text{ 299}$$

$$\frac{1}{2}\left(2\cos(x)-\int \frac{1}{1-\cos^2(x)}d\cos(x)\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2}(2\cos(x)-\text{arctanh}(\cos(x)))$$

input
```
Int[Cos[x]*Cot[2*x],x]
```

output
```
(-ArcTanh[Cos[x]] + 2*Cos[x])/2
```

3.110.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4879
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cos[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Cos[v]/d
, u/Sin[v], x], x], x, Cos[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[Nonfree
Factors[Cos[v], x], u/Sin[v], x]]
```

3.110.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(8) = 16$.

Time = 1.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.00

method	result	size
default	$\cos(x)+\ln(\csc(x)-\cot(x))+\frac{\ln(\cot(x)+\csc(x))}{2}$	20
risch	$\frac{e^{ix}}{2}+\frac{e^{-ix}}{2}-\frac{\ln(e^{ix}+1)}{2}+\frac{\ln(e^{ix}-1)}{2}$	36

input
```
int(cos(x)*cot(2*x),x,method=_RETURNVERBOSE)
```

output
```
cos(x)+ln(csc(x)-cot(x))+1/2*ln(cot(x)+csc(x))
```

3.110.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(8) = 16$.

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 2.10

$$\int \cos(x)\cot(2x)\,dx = \cos(x) - \frac{1}{4}\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right) + \frac{1}{4}\log\left(-\frac{1}{2}\cos(x) + \frac{1}{2}\right)$$

input
```
integrate(cos(x)*cot(2*x),x, algorithm="fricas")
```

output
```
cos(x) - 1/4*log(1/2*cos(x) + 1/2) + 1/4*log(-1/2*cos(x) + 1/2)
```

3.110.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(8) = 16$.

Time = 0.36 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.90

$$\int \cos(x)\cot(2x)\,dx = \frac{\log(\cos(x) - 1)}{4} - \frac{\log(\cos(x) + 1)}{4} + \cos(x)$$

input
```
integrate(cos(x)*cot(2*x),x)
```

output
```
log(cos(x) - 1)/4 - log(cos(x) + 1)/4 + cos(x)
```

3.110.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(8) = 16$.

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 3.70

$$\int \cos(x)\cot(2x)\,dx = \cos(x) - \frac{1}{4}\log\left(\cos(x)^2 + \sin(x)^2 + 2\cos(x) + 1\right) + \frac{1}{4}\log\left(\cos(x)^2 + \sin(x)^2 - 2\cos(x) + 1\right)$$

input
```
integrate(cos(x)*cot(2*x),x, algorithm="maxima")
```

output
```
cos(x) - 1/4*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/4*log(cos(x)^2 +
sin(x)^2 - 2*cos(x) + 1)
```

3.110.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(8) = 16$.

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.90

$$\int \cos(x)\cot(2x)\,dx = \cos(x) - \frac{1}{4}\log(\cos(x)+1) + \frac{1}{4}\log(-\cos(x)+1)$$

input
```
integrate(cos(x)*cot(2*x),x, algorithm="giac")
```

output
```
cos(x) - 1/4*log(cos(x) + 1) + 1/4*log(-cos(x) + 1)
```

3.110.9 Mupad [B] (verification not implemented)

Time = 26.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.00

$$\int \cos(x)\cot(2x)\,dx = \frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2} + \frac{2}{\tan\left(\frac{x}{2}\right)^2 + 1}$$

input
```
int(cot(2*x)*cos(x),x)
```

output
```
log(tan(x/2))/2 + 2/(tan(x/2)^2 + 1)
```

3.111 $\int \cos(x)\cot(3x)\,dx$

3.111.1 Optimal result

Integrand size = 7, antiderivative size = 45

$$\int \cos(x)\cot(3x)\,dx = \cos(x)+\frac{1}{6}\log(1-2\cos(x))+\frac{1}{6}\log(1-\cos(x))-\frac{1}{6}\log(1+\cos(x))-\frac{1}{6}\log(1+2\cos(x))$$

output
```
cos(x)+1/6*ln(1-2*cos(x))+1/6*ln(1-cos(x))-1/6*ln(1+cos(x))-1/6*ln(1+2*cos
(x))
```

3.111.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.04

$$\int \cos(x)\cot(3x)\,dx = \cos(x)-\frac{1}{3}\log\left(\cos\left(\frac{x}{2}\right)\right)+\frac{1}{6}\log(1-2\cos(x))-\frac{1}{6}\log(1+2\cos(x))+\frac{1}{3}\log\left(\sin\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[Cos[x]*Cot[3*x],x]
```

output
```
Cos[x] - Log[Cos[x/2]]/3 + Log[1 - 2*Cos[x]]/6 - Log[1 + 2*Cos[x]]/6 + Log
[Sin[x/2]]/3
```

3.111.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.22, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 4879, 1602, 27, 1475, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\cot(3x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(x)\cot(3x)dx$$

$$\Big\downarrow \text{4879}$$

$$-\int \frac{\cos^2(x)\left(3-4\cos^2(x)\right)}{4\cos^4(x)-5\cos^2(x)+1}d\cos(x)$$

$$\Big\downarrow \text{1602}$$

$$\frac{1}{4}\int -\frac{4\left(1-2\cos^2(x)\right)}{4\cos^4(x)-5\cos^2(x)+1}d\cos(x)+\cos(x)$$

$$\Big\downarrow \text{27}$$

$$\cos(x)-\int \frac{1-2\cos^2(x)}{4\cos^4(x)-5\cos^2(x)+1}d\cos(x)$$

$$\Big\downarrow \text{1475}$$

$$\frac{1}{4}\int \frac{1}{\cos^2(x)-\frac{\cos(x)}{2}-\frac{1}{2}}d\cos(x)+\frac{1}{4}\int \frac{1}{\cos^2(x)+\frac{\cos(x)}{2}-\frac{1}{2}}d\cos(x)+\cos(x)$$

$$\Big\downarrow \text{1081}$$

$$\frac{1}{4}\int \left(-\frac{2}{3(\cos(x)+1)}-\frac{4}{3(1-2\cos(x))}\right)d\cos(x)+\frac{1}{4}\int \left(-\frac{4}{3(2\cos(x)+1)}-\frac{2}{3(1-\cos(x))}\right)d\cos(x)+\cos(x)$$

$$\Big\downarrow \text{2009}$$

$$\cos(x)+\frac{1}{4}\left(\frac{2}{3}\log(1-2\cos(x))-\frac{2}{3}\log(\cos(x)+1)\right)+\frac{1}{4}\left(\frac{2}{3}\log(1-\cos(x))-\frac{2}{3}\log(2\cos(x)+1)\right)$$

input
```
Int[Cos[x]*Cot[3*x],x]
```

output
```
Cos[x] + ((2*Log[1 - 2*Cos[x]])/3 - (2*Log[1 + Cos[x]])/3)/4 + ((2*Log[1 -
 Cos[x]])/3 - (2*Log[1 + 2*Cos[x]])/3)/4
```

3.111.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1602
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Simp[f^2/(c*(m + 4*p + 3))   Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p
+ 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c
, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] |
| IntegerQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4879
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cos[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Cos[v]/d
, u/Sin[v], x], x], x, Cos[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[Nonfree
Factors[Cos[v], x], u/Sin[v], x]]
```

3.111.4 Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{\ln(1+2\cos(x))}{6}+\frac{\ln(\cos(x)-1)}{6}-\frac{\ln(\cos(x)+1)}{6}+\frac{\ln(2\cos(x)-1)}{6}+\cos(x)$	36
risch	$\frac{e^{ix}}{2}+\frac{e^{-ix}}{2}+\frac{\ln(e^{ix}-1)}{3}-\frac{\ln(e^{ix}+1)}{3}+\frac{\ln(e^{2ix}-e^{ix}+1)}{6}-\frac{\ln(e^{2ix}+e^{ix}+1)}{6}$	68

input
```
int(cos(x)*cot(3*x),x,method=_RETURNVERBOSE)
```

output
```
-1/6*ln(1+2*cos(x))+1/6*ln(cos(x)-1)-1/6*ln(cos(x)+1)+1/6*ln(2*cos(x)-1)+c
os(x)
```

3.111.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int\cos(x)\cot(3x)\,dx=\cos(x)-\frac{1}{6}\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+\frac{1}{6}\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)+\frac{1}{6}\log(-2\cos(x)+1)-\frac{1}{6}\log(-2\cos(x)-1)$$

input
```
integrate(cos(x)*cot(3*x),x, algorithm="fricas")
```

output
```
cos(x) - 1/6*log(1/2*cos(x) + 1/2) + 1/6*log(-1/2*cos(x) + 1/2) + 1/6*log(
-2*cos(x) + 1) - 1/6*log(-2*cos(x) - 1)
```

3.111.6 Sympy [F]

$$\int \cos(x)\cot(3x)\,dx = \int \cos{(x)}\cot{(3x)}\,dx$$

input
```
integrate(cos(x)*cot(3*x),x)
```

output
```
Integral(cos(x)*cot(3*x), x)
```

3.111.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(37) = 74.

Time = 0.29 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.91

$$\begin{aligned}\int \cos(x)\cot(3x)\,dx = {} & \cos{(x)} - \frac{1}{12}\log\left(2\left(\cos{(x)}+1\right)\cos{(2x)} + \cos{(2x)}^2 + \cos{(x)}^2 \right. \\ & \left. + \sin{(2x)}^2 + 2\sin{(2x)}\sin{(x)} + \sin{(x)}^2 + 2\cos{(x)} + 1\right) \\ & + \frac{1}{12}\log\left(-2\left(\cos{(x)}-1\right)\cos{(2x)} + \cos{(2x)}^2 + \cos{(x)}^2 + \sin{(2x)}^2 \right. \\ & \left. - 2\sin{(2x)}\sin{(x)} + \sin{(x)}^2 - 2\cos{(x)} + 1\right) \\ & - \frac{1}{6}\log\left(\cos{(x)}^2 + \sin{(x)}^2 + 2\cos{(x)} + 1\right) \\ & + \frac{1}{6}\log\left(\cos{(x)}^2 + \sin{(x)}^2 - 2\cos{(x)} + 1\right)\end{aligned}$$

input
```
integrate(cos(x)*cot(3*x),x, algorithm="maxima")
```

output
```
cos(x) - 1/12*log(2*(cos(x) + 1)*cos(2*x) + cos(2*x)^2 + cos(x)^2 + sin(2*
x)^2 + 2*sin(2*x)*sin(x) + sin(x)^2 + 2*cos(x) + 1) + 1/12*log(-2*(cos(x)
- 1)*cos(2*x) + cos(2*x)^2 + cos(x)^2 + sin(2*x)^2 - 2*sin(2*x)*sin(x) + s
in(x)^2 - 2*cos(x) + 1) - 1/6*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/
6*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)
```

3.111.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int \cos(x)\cot(3x)\,dx = \cos(x) - \frac{1}{6}\log(\cos(x)+1) + \frac{1}{6}\log(-\cos(x)+1) - \frac{1}{6}\log(|2\cos(x)+1|) + \frac{1}{6}\log(|2\cos(x)-1|)$$

input
```
integrate(cos(x)*cot(3*x),x, algorithm="giac")
```

output
```
cos(x) - 1/6*log(cos(x) + 1) + 1/6*log(-cos(x) + 1) - 1/6*log(abs(2*cos(x)
 + 1)) + 1/6*log(abs(2*cos(x) - 1))
```

3.111.9 Mupad [B] (verification not implemented)

Time = 26.61 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int \cos(x)\cot(3x)\,dx = \frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{3} + \frac{\operatorname{atanh}\left(\frac{8}{183\left(\frac{488\tan\left(\frac{x}{2}\right)^2}{243}-\frac{56}{81}\right)}+\frac{121}{122}\right)}{3} + \frac{2}{\tan\left(\frac{x}{2}\right)^2+1}$$

input
```
int(cot(3*x)*cos(x),x)
```

output
```
log(tan(x/2))/3 + atanh(8/(183*((488*tan(x/2)^2)/243 - 56/81)) + 121/122)/
3 + 2/(tan(x/2)^2 + 1)
```

3.112 $\int \cos(x)\cot(4x)\,dx$

3.112.1 Optimal result

Integrand size = 7, antiderivative size = 28

$$\int \cos(x)\cot(4x)\,dx = -\frac{1}{4}\operatorname{arctanh}(\cos(x)) - \frac{\operatorname{arctanh}\left(\sqrt{2}\cos(x)\right)}{2\sqrt{2}} + \cos(x)$$

output
```
-1/4*arctanh(cos(x))+cos(x)-1/4*arctanh(cos(x)*2^(1/2))*2^(1/2)
```

3.112.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.61

$$\int \cos(x)\cot(4x)\,dx = \frac{1}{4}\left((-1-i)(-1)^{3/4}\operatorname{arctanh}\left(\frac{-1+\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right) - (1-i)\sqrt[4]{-1}\operatorname{arctanh}\left(\frac{1+\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right) + 4\cos(x) - \log\left(\cos\left(\frac{x}{2}\right)\right) + \log\left(\sin\left(\frac{x}{2}\right)\right)\right)$$

input
```
Integrate[Cos[x]*Cot[4*x],x]
```

output
```
((-1 - I)*(-1)^(3/4)*ArcTanh[(-1 + Tan[x/2])/Sqrt[2]] - (1 - I)*(-1)^(1/4)
*ArcTanh[(1 + Tan[x/2])/Sqrt[2]] + 4*Cos[x] - Log[Cos[x/2]] + Log[Sin[x/2]
])/4
```

3.112.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 4879, 27, 2205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\cot(4x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(x)\cot(4x)dx$$

$$\Big\downarrow \text{4879}$$

$$-\int -\frac{8\cos^4(x)-8\cos^2(x)+1}{4\left(2\cos^4(x)-3\cos^2(x)+1\right)}d\cos(x)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\int \frac{8\cos^4(x)-8\cos^2(x)+1}{2\cos^4(x)-3\cos^2(x)+1}d\cos(x)$$

$$\Big\downarrow \text{2205}$$

$$\frac{1}{4}\int \left(4-\frac{3-4\cos^2(x)}{2\cos^4(x)-3\cos^2(x)+1}\right)d\cos(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left(-\text{arctanh}(\cos(x))-\sqrt{2}\text{arctanh}\left(\sqrt{2}\cos(x)\right)+4\cos(x)\right)$$

input
```
Int[Cos[x]*Cot[4*x],x]
```

output
```
(-ArcTanh[Cos[x]] - Sqrt[2]*ArcTanh[Sqrt[2]*Cos[x]] + 4*Cos[x])/4
```

3.112.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2205
```
Int[(Px_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Px/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Px, x^
2] && Expon[Px, x^2] > 1
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4879
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cos[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Cos[v]/d
, u/Sin[v], x], x], x, Cos[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[Nonfree
Factors[Cos[v], x], u/Sin[v], x]]
```

3.112.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.11 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.89

method	result	size
risch	$\frac{e^{ix}}{2}+\frac{e^{-ix}}{2}+\frac{\ln(e^{ix}-1)}{4}-\frac{\ln(e^{ix}+1)}{4}-\frac{\sqrt{2}\,\ln\left(e^{2ix}+\sqrt{2}\,e^{ix}+1\right)}{8}+\frac{\sqrt{2}\,\ln\left(e^{2ix}-\sqrt{2}\,e^{ix}+1\right)}{8}$	81

input
```
int(cos(x)*cot(4*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(I*x)+1/2*exp(-I*x)+1/4*ln(exp(I*x)-1)-1/4*ln(exp(I*x)+1)-1/8*2^(1/
2)*ln(exp(2*I*x)+2^(1/2)*exp(I*x)+1)+1/8*2^(1/2)*ln(exp(2*I*x)-2^(1/2)*exp
(I*x)+1)
```

3.112.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(20) = 40$.

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.89

$$\int \cos(x)\cot(4x)\,dx = \frac{1}{8}\sqrt{2}\log\left(\frac{2\cos(x)^2 - 2\sqrt{2}\cos(x) + 1}{2\cos(x)^2 - 1}\right) + \cos(x) - \frac{1}{8}\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right) + \frac{1}{8}\log\left(-\frac{1}{2}\cos(x) + \frac{1}{2}\right)$$

input
```
integrate(cos(x)*cot(4*x),x, algorithm="fricas")
```

output
```
1/8*sqrt(2)*log((2*cos(x)^2 - 2*sqrt(2)*cos(x) + 1)/(2*cos(x)^2 - 1)) + co
s(x) - 1/8*log(1/2*cos(x) + 1/2) + 1/8*log(-1/2*cos(x) + 1/2)
```

3.112.6 Sympy [F]

$$\int \cos(x)\cot(4x)\,dx = \int \cos(x)\cot(4x)\,dx$$

input
```
integrate(cos(x)*cot(4*x),x)
```

output
```
Integral(cos(x)*cot(4*x), x)
```

3.112.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(20) = 40$.

Time = 0.30 (sec) , antiderivative size = 165, normalized size of antiderivative = 5.89

$$\int \cos(x)\cot(4x)\,dx = -\frac{1}{16}\sqrt{2}\log\left(2\sqrt{2}\sin(2x)\sin(x) + 2\left(\sqrt{2}\cos(x) + 1\right)\cos(2x) + \cos(2x)^2 + 2\cos(x)^2 + \sin(2x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) + 1\right) + \frac{1}{16}\sqrt{2}\log\left(-2\sqrt{2}\sin(2x)\sin(x) - 2\left(\sqrt{2}\cos(x) - 1\right)\cos(2x) + \cos(2x)^2 + 2\cos(x)^2 + \sin(2x)^2 + 2\sin(x)^2 - 2\sqrt{2}\cos(x) + 1\right) + \cos(x) - \frac{1}{8}\log\left(\cos(x)^2 + \sin(x)^2 + 2\cos(x) + 1\right) + \frac{1}{8}\log\left(\cos(x)^2 + \sin(x)^2 - 2\cos(x) + 1\right)$$

input
```
integrate(cos(x)*cot(4*x),x, algorithm="maxima")
```

output
```
-1/16*sqrt(2)*log(2*sqrt(2)*sin(2*x)*sin(x) + 2*(sqrt(2)*cos(x) + 1)*cos(2
*x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x)
 + 1) + 1/16*sqrt(2)*log(-2*sqrt(2)*sin(2*x)*sin(x) - 2*(sqrt(2)*cos(x) -
1)*cos(2*x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 - 2*sqrt(2
)*cos(x) + 1) + cos(x) - 1/8*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/8
*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)
```

3.112.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(20) = 40$.

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.79

$$\int \cos(x)\cot(4x)\,dx = \frac{1}{8}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\cos(x)\right|}{\left|2\sqrt{2}+4\cos(x)\right|}\right)+\cos(x) - \frac{1}{8}\log(\cos(x)+1)+\frac{1}{8}\log(-\cos(x)+1)$$

input
```
integrate(cos(x)*cot(4*x),x, algorithm="giac")
```

output
```
1/8*sqrt(2)*log(abs(-2*sqrt(2) + 4*cos(x))/abs(2*sqrt(2) + 4*cos(x))) + co
s(x) - 1/8*log(cos(x) + 1) + 1/8*log(-cos(x) + 1)
```

3.112.9 Mupad [B] (verification not implemented)

Time = 26.45 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.39

$$\int \cos(x)\cot(4x)\,dx = \frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{4} - \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{7\sqrt{2}}{8\left(\frac{29\tan\left(\frac{x}{2}\right)^2}{4}-\frac{5}{4}\right)}-\frac{41\sqrt{2}\tan\left(\frac{x}{2}\right)^2}{8\left(\frac{29\tan\left(\frac{x}{2}\right)^2}{4}-\frac{5}{4}\right)}\right)}{4} + \frac{2}{\tan\left(\frac{x}{2}\right)^2+1}$$

input
```
int(cot(4*x)*cos(x),x)
```

output
```
log(tan(x/2))/4 - (2^(1/2)*atanh((7*2^(1/2))/(8*((29*tan(x/2)^2)/4 - 5/4))
 - (41*2^(1/2)*tan(x/2)^2)/(8*((29*tan(x/2)^2)/4 - 5/4))))/4 + 2/(tan(x/2)
^2 + 1)
```

3.112. $\int \cos(x)\cot(4x)\,dx$

3.113 $\int \cos(x)\cot(5x)\,dx$

3.113.1 Optimal result

Integrand size = 7, antiderivative size = 110

$$\begin{aligned}\int \cos(x)\cot(5x)\,dx = &-\frac{1}{5}\text{arctanh}(\cos(x))+\cos(x)+\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(1-\sqrt{5}-4\cos(x)\right)\\ &+\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(1+\sqrt{5}-4\cos(x)\right)\\ &-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(1-\sqrt{5}+4\cos(x)\right)\\ &-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(1+\sqrt{5}+4\cos(x)\right)\end{aligned}$$

output
```
-1/5*arctanh(cos(x))+cos(x)+1/20*ln(1-4*cos(x)-5^(1/2))*(-5^(1/2)+1)-1/20*
ln(1+4*cos(x)-5^(1/2))*(-5^(1/2)+1)+1/20*ln(1-4*cos(x)+5^(1/2))*(5^(1/2)+1
)-1/20*ln(1+4*cos(x)+5^(1/2))*(5^(1/2)+1)
```

3.113.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.21

$$\begin{aligned}\int \cos(x)\cot(5x)\,dx = \frac{1}{100}\Bigg(&100\cos(x)-20\log\left(\cos\left(\frac{x}{2}\right)\right)\\ &+\sqrt{5}\left(-5+\sqrt{5}\right)\log\left(1-\sqrt{5}-4\cos(x)\right)\\ &+\sqrt{5}\left(5+\sqrt{5}\right)\log\left(1+\sqrt{5}-4\cos(x)\right)\\ &-\sqrt{5}\left(-5+\sqrt{5}\right)\log\left(1-\sqrt{5}+4\cos(x)\right)\\ &-\sqrt{5}\left(5+\sqrt{5}\right)\log\left(1+\sqrt{5}+4\cos(x)\right)+20\log\left(\sin\left(\frac{x}{2}\right)\right)\Bigg)\end{aligned}$$

input
```
Integrate[Cos[x]*Cot[5*x],x]
```

output
```
(100*Cos[x] - 20*Log[Cos[x/2]] + Sqrt[5]*(-5 + Sqrt[5])*Log[1 - Sqrt[5] -
4*Cos[x]] + Sqrt[5]*(5 + Sqrt[5])*Log[1 + Sqrt[5] - 4*Cos[x]] - Sqrt[5]*(-
5 + Sqrt[5])*Log[1 - Sqrt[5] + 4*Cos[x]] - Sqrt[5]*(5 + Sqrt[5])*Log[1 + S
qrt[5] + 4*Cos[x]] + 20*Log[Sin[x/2]])/100
```

3.113.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4879, 2460, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\cot(5x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(x)\cot(5x)dx$$

$$\Big\downarrow \text{4879}$$

$$-\int \frac{\cos^2(x)\left(16\cos^4(x)-20\cos^2(x)+5\right)}{-16\cos^6(x)+28\cos^4(x)-13\cos^2(x)+1}d\cos(x)$$

$$\Big\downarrow \text{2460}$$

$$-\int \left(\frac{2(\cos(x)-1)}{5\left(4\cos^2(x)+2\cos(x)-1\right)}-\frac{1}{5\left(\cos^2(x)-1\right)}-\frac{2(\cos(x)+1)}{5\left(4\cos^2(x)-2\cos(x)-1\right)}-1\right)d\cos(x)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{5}\text{arctanh}(\cos(x))+\cos(x)+\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(-4\cos(x)-\sqrt{5}+1\right)+\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(-4\cos(x)+\sqrt{5}+1\right)-\frac{1}{20}\left(1-\sqrt{5}\right)\log\left(4\cos(x)-\sqrt{5}+1\right)-\frac{1}{20}\left(1+\sqrt{5}\right)\log\left(4\cos(x)+\sqrt{5}+1\right)$$

input
```
Int[Cos[x]*Cot[5*x],x]
```

output
```
-1/5*ArcTanh[Cos[x]] + Cos[x] + ((1 - Sqrt[5])*Log[1 - Sqrt[5] - 4*Cos[x]]
)/20 + ((1 + Sqrt[5])*Log[1 + Sqrt[5] - 4*Cos[x]])/20 - ((1 - Sqrt[5])*Log
[1 - Sqrt[5] + 4*Cos[x]])/20 - ((1 + Sqrt[5])*Log[1 + Sqrt[5] + 4*Cos[x]])
/20
```

3.113.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2460
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px /. x -> Sqrt[x]]},
Int[ExpandIntegrand[u*(Qx /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[Q
x, x]]] /; PolyQ[Px, x^2] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&
 !TrinomialQ[Px, x] && ILtQ[p, 0] && RationalFunctionQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4879
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cos[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Cos[v]/d
, u/Sin[v], x], x], x, Cos[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[Nonfree
Factors[Cos[v], x], u/Sin[v], x]]
```

3.113.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.25 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.04

method	result
risch	$\frac{\mathrm{e}^{i x}}{2}+\frac{\mathrm{e}^{-i x}}{2}-\frac{\ln\left(\mathrm{e}^{i x}+1\right)}{5}+\frac{\ln\left(\mathrm{e}^{i x}-1\right)}{5}+\frac{\ln\left(\mathrm{e}^{2 i x}+\frac{\left(\sqrt{5}-1\right)\mathrm{e}^{i x}}{2}+1\right)}{20}-\frac{\ln\left(\mathrm{e}^{2 i x}+\frac{\left(\sqrt{5}-1\right)\mathrm{e}^{i x}}{2}+1\right)\sqrt{5}}{20}+\frac{\ln\left(\mathrm{e}^{2 i x}-\frac{\left(\sqrt{5}+1\right)\mathrm{e}^{i x}}{2}\right.}{20}$

input
```
int(cos(x)*cot(5*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(I*x)+1/2*exp(-I*x)-1/5*ln(exp(I*x)+1)+1/5*ln(exp(I*x)-1)+1/20*ln(e
xp(2*I*x)+1/2*(5^(1/2)-1)*exp(I*x)+1)-1/20*ln(exp(2*I*x)+1/2*(5^(1/2)-1)*e
xp(I*x)+1)*5^(1/2)+1/20*ln(exp(2*I*x)-1/2*(5^(1/2)+1)*exp(I*x)+1)+1/20*ln(
exp(2*I*x)-1/2*(5^(1/2)+1)*exp(I*x)+1)*5^(1/2)-1/20*ln(exp(2*I*x)+1/2*(5^(
1/2)+1)*exp(I*x)+1)-1/20*ln(exp(2*I*x)+1/2*(5^(1/2)+1)*exp(I*x)+1)*5^(1/2)
-1/20*ln(exp(2*I*x)-1/2*(5^(1/2)-1)*exp(I*x)+1)+1/20*ln(exp(2*I*x)-1/2*(5^
(1/2)-1)*exp(I*x)+1)*5^(1/2)
```

3.113.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.25

$$\begin{aligned}\int \cos(x)\cot(5x)\,dx &= \frac{1}{20}\sqrt{5}\log\left(-\frac{4\left(\sqrt{5}-1\right)\cos\left(x\right)-8\cos\left(x\right)^{2}+\sqrt{5}-3}{4\cos\left(x\right)^{2}+2\cos\left(x\right)-1}\right)\\&\quad+\frac{1}{20}\sqrt{5}\log\left(-\frac{4\left(\sqrt{5}+1\right)\cos\left(x\right)-8\cos\left(x\right)^{2}-\sqrt{5}-3}{4\cos\left(x\right)^{2}-2\cos\left(x\right)-1}\right)\\&\quad+\cos\left(x\right)-\frac{1}{20}\log\left(4\cos\left(x\right)^{2}+2\cos\left(x\right)-1\right)\\&\quad+\frac{1}{20}\log\left(4\cos\left(x\right)^{2}-2\cos\left(x\right)-1\right)\\&\quad-\frac{1}{10}\log\left(\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)+\frac{1}{10}\log\left(-\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)\end{aligned}$$

input
```
integrate(cos(x)*cot(5*x),x, algorithm="fricas")
```

output
```
1/20*sqrt(5)*log(-(4*(sqrt(5) - 1)*cos(x) - 8*cos(x)^2 + sqrt(5) - 3)/(4*c
os(x)^2 + 2*cos(x) - 1)) + 1/20*sqrt(5)*log(-(4*(sqrt(5) + 1)*cos(x) - 8*c
os(x)^2 - sqrt(5) - 3)/(4*cos(x)^2 - 2*cos(x) - 1)) + cos(x) - 1/20*log(4*
cos(x)^2 + 2*cos(x) - 1) + 1/20*log(4*cos(x)^2 - 2*cos(x) - 1) - 1/10*log(
1/2*cos(x) + 1/2) + 1/10*log(-1/2*cos(x) + 1/2)
```

3.113.6 Sympy [F]

$$\int \cos(x)\cot(5x)\,dx = \int \cos\left(x\right)\cot\left(5x\right)\,dx$$

input
```
integrate(cos(x)*cot(5*x),x)
```

output
```
Integral(cos(x)*cot(5*x), x)
```

3.113.7 Maxima [F]

$$\int \cos(x)\cot(5x)\,dx = \int \cos\left(x\right)\cot\left(5\,x\right)\,dx$$

input
```
integrate(cos(x)*cot(5*x),x, algorithm="maxima")
```

output
```
cos(x) + 1/10*integrate(-(cos(2*x)*sin(4*x) - cos(4*x)*sin(2*x) + cos(3/2*
arctan2(sin(2*x), cos(2*x)))*sin(2*x) + cos(1/2*arctan2(sin(2*x), cos(2*x)
))*sin(2*x) - cos(2*x)*sin(3/2*arctan2(sin(2*x), cos(2*x))) - cos(2*x)*sin
(1/2*arctan2(sin(2*x), cos(2*x))) - sin(2*x))/(2*(cos(2*x) + 1)*cos(4*x) +
 cos(4*x)^2 + cos(2*x)^2 - 2*(cos(4*x) + cos(2*x) - cos(1/2*arctan2(sin(2*
x), cos(2*x))) + 1)*cos(3/2*arctan2(sin(2*x), cos(2*x))) + cos(3/2*arctan2
(sin(2*x), cos(2*x)))^2 - 2*(cos(4*x) + cos(2*x) + 1)*cos(1/2*arctan2(sin(
2*x), cos(2*x))) + cos(1/2*arctan2(sin(2*x), cos(2*x)))^2 + sin(4*x)^2 + 2
*sin(4*x)*sin(2*x) + sin(2*x)^2 - 2*(sin(4*x) + sin(2*x) - sin(1/2*arctan2
(sin(2*x), cos(2*x))))*sin(3/2*arctan2(sin(2*x), cos(2*x))) + sin(3/2*arct
an2(sin(2*x), cos(2*x)))^2 - 2*(sin(4*x) + sin(2*x))*sin(1/2*arctan2(sin(2
*x), cos(2*x))) + sin(1/2*arctan2(sin(2*x), cos(2*x)))^2 + 2*cos(2*x) + 1)
, x) + 1/10*integrate((cos(2*x)*sin(4*x) - cos(4*x)*sin(2*x) - cos(3/2*arc
tan2(sin(2*x), cos(2*x)))*sin(2*x) - cos(1/2*arctan2(sin(2*x), cos(2*x)))*
sin(2*x) + cos(2*x)*sin(3/2*arctan2(sin(2*x), cos(2*x))) + cos(2*x)*sin(1/
2*arctan2(sin(2*x), cos(2*x))) - sin(2*x))/(2*(cos(2*x) + 1)*cos(4*x) + co
s(4*x)^2 + cos(2*x)^2 + 2*(cos(4*x) + cos(2*x) + cos(1/2*arctan2(sin(2*x),
 cos(2*x))) + 1)*cos(3/2*arctan2(sin(2*x), cos(2*x))) + cos(3/2*arctan2(si
n(2*x), cos(2*x)))^2 + 2*(cos(4*x) + cos(2*x) + 1)*cos(1/2*arctan2(sin(2*x
), cos(2*x))) + cos(1/2*arctan2(sin(2*x), cos(2*x)))^2 + sin(4*x)^2 + 2...
```

3.113.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.06

$$\int \cos(x)\cot(5x)\,dx = \frac{1}{20}\sqrt{5}\log\left(\frac{\left|-2\sqrt{5}+8\cos(x)+2\right|}{\left|2\sqrt{5}+8\cos(x)+2\right|}\right) + \frac{1}{20}\sqrt{5}\log\left(\frac{\left|-2\sqrt{5}+8\cos(x)-2\right|}{\left|2\sqrt{5}+8\cos(x)-2\right|}\right) + \cos(x) - \frac{1}{10}\log(\cos(x)+1) + \frac{1}{10}\log(-\cos(x)+1) - \frac{1}{20}\log\left(\left|4\cos(x)^2+2\cos(x)-1\right|\right) + \frac{1}{20}\log\left(\left|4\cos(x)^2-2\cos(x)-1\right|\right)$$

input
```
integrate(cos(x)*cot(5*x),x, algorithm="giac")
```

output
```
1/20*sqrt(5)*log(abs(-2*sqrt(5) + 8*cos(x) + 2)/abs(2*sqrt(5) + 8*cos(x) +
 2)) + 1/20*sqrt(5)*log(abs(-2*sqrt(5) + 8*cos(x) - 2)/abs(2*sqrt(5) + 8*c
os(x) - 2)) + cos(x) - 1/10*log(cos(x) + 1) + 1/10*log(-cos(x) + 1) - 1/20
*log(abs(4*cos(x)^2 + 2*cos(x) - 1)) + 1/20*log(abs(4*cos(x)^2 - 2*cos(x)
- 1))
```

3.113.9 Mupad [B] (verification not implemented)

Time = 26.67 (sec) , antiderivative size = 611, normalized size of antiderivative = 5.55

$$\int \cos(x)\cot(5x)\,dx = \text{Too large to display}$$

input
```
int(cot(5*x)*cos(x),x)
```

output
```
(atan((tan(x/2)^2*4813499234516992i)/(1220703125*((213485644414976*5^(1/2)
)/1220703125 - (2152646198689792*5^(1/2)*tan(x/2)^2)/1220703125 - (4959229
085483008*tan(x/2)^2)/1220703125 + 110872433262592/244140625)) - 954873235
37408i/(244140625*((213485644414976*5^(1/2))/1220703125 - (215264619868979
2*5^(1/2)*tan(x/2)^2)/1220703125 - (4959229085483008*tan(x/2)^2)/122070312
5 + 110872433262592/244140625)) - (5^(1/2)*247887795585024i)/(1220703125*(
(213485644414976*5^(1/2))/1220703125 - (2152646198689792*5^(1/2)*tan(x/2)^
2)/1220703125 - (4959229085483008*tan(x/2)^2)/1220703125 + 110872433262592
/244140625)) + (5^(1/2)*tan(x/2)^2*2217818569310208i)/(1220703125*((213485
644414976*5^(1/2))/1220703125 - (2152646198689792*5^(1/2)*tan(x/2)^2)/1220
703125 - (4959229085483008*tan(x/2)^2)/1220703125 + 110872433262592/244140
625)))*1i)/10 + (atan(95487323537408i/(244140625*((213485644414976*5^(1/2)
)/1220703125 - (2152646198689792*5^(1/2)*tan(x/2)^2)/1220703125 + (4959229
085483008*tan(x/2)^2)/1220703125 - 110872433262592/244140625)) - (5^(1/2)*
247887795585024i)/(1220703125*((213485644414976*5^(1/2))/1220703125 - (215
2646198689792*5^(1/2)*tan(x/2)^2)/1220703125 + (4959229085483008*tan(x/2)^
2)/1220703125 - 110872433262592/244140625)) - (tan(x/2)^2*4813499234516992
i)/(1220703125*((213485644414976*5^(1/2))/1220703125 - (2152646198689792*5
^(1/2)*tan(x/2)^2)/1220703125 + (4959229085483008*tan(x/2)^2)/1220703125 -
 110872433262592/244140625)) + (5^(1/2)*tan(x/2)^2*2217818569310208i)/(...
```

3.113. $\int \cos(x)\cot(5x)\,dx$

3.114 $\int \cos(x)\cot(6x)\,dx$

3.114.1 Optimal result

Integrand size = 7, antiderivative size = 38

$$\int \cos(x)\cot(6x)\,dx = -\frac{1}{6}\text{arctanh}(\cos(x)) - \frac{1}{6}\text{arctanh}(2\cos(x)) - \frac{\text{arctanh}\left(\frac{2\cos(x)}{\sqrt{3}}\right)}{2\sqrt{3}} + \cos(x)$$

output
```
-1/6*arctanh(cos(x))-1/6*arctanh(2*cos(x))+cos(x)-1/6*arctanh(2/3*cos(x)*3
^(1/2))*3^(1/2)
```

3.114.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 87 vs. 2(38) = 76.

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.29

$$\int \cos(x)\cot(6x)\,dx = \frac{1}{12}\left(2\sqrt{3}\text{arctanh}\left(\frac{-2+\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right) - 2\sqrt{3}\text{arctanh}\left(\frac{2+\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right) + 12\cos(x) - 2\log\left(\cos\left(\frac{x}{2}\right)\right) + \log(1-2\cos(x)) - \log(1+2\cos(x)) + 2\log\left(\sin\left(\frac{x}{2}\right)\right)\right)$$

input
```
Integrate[Cos[x]*Cot[6*x],x]
```

output
```
(2*Sqrt[3]*ArcTanh[(-2 + Tan[x/2])/Sqrt[3]] - 2*Sqrt[3]*ArcTanh[(2 + Tan[x
/2])/Sqrt[3]] + 12*Cos[x] - 2*Log[Cos[x/2]] + Log[1 - 2*Cos[x]] - Log[1 +
2*Cos[x]] + 2*Log[Sin[x/2]])/12
```

3.114.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 4879, 27, 2460, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\cot(6x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(x)\cot(6x)dx$$

$$\Big\downarrow \text{4879}$$

$$-\int -\frac{-32\cos^6(x)+48\cos^4(x)-18\cos^2(x)+1}{2\left(-16\cos^6(x)+32\cos^4(x)-19\cos^2(x)+3\right)}d\cos(x)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{-32\cos^6(x)+48\cos^4(x)-18\cos^2(x)+1}{-16\cos^6(x)+32\cos^4(x)-19\cos^2(x)+3}d\cos(x)$$

$$\Big\downarrow \text{2460}$$

$$\frac{1}{2}\int \left(\frac{2}{4\cos^2(x)-3}+\frac{2}{3\left(4\cos^2(x)-1\right)}+2+\frac{1}{3\left(\cos^2(x)-1\right)}\right)d\cos(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{1}{3}\text{arctanh}(\cos(x))-\frac{1}{3}\text{arctanh}(2\cos(x))-\frac{\text{arctanh}\left(\frac{2\cos(x)}{\sqrt{3}}\right)}{\sqrt{3}}+2\cos(x)\right)$$

input
```
Int[Cos[x]*Cot[6*x],x]
```

output
```
(-1/3*ArcTanh[Cos[x]] - ArcTanh[2*Cos[x]]/3 - ArcTanh[(2*Cos[x])/Sqrt[3]]/
Sqrt[3] + 2*Cos[x])/2
```

3.114.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2460
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px /. x -> Sqrt[x]]},
Int[ExpandIntegrand[u*(Qx /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[Q
x, x]]] /; PolyQ[Px, x^2] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&
 !TrinomialQ[Px, x] && ILtQ[p, 0] && RationalFunctionQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4879
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cos[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Cos[v]/d
, u/Sin[v], x], x], x, Cos[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[Nonfree
Factors[Cos[v], x], u/Sin[v], x]]
```

3.114.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.14 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.97

method	result
risch	$\frac{e^{ix}}{2}+\frac{e^{-ix}}{2}+\frac{\ln\left(e^{ix}-1\right)}{6}-\frac{\ln\left(e^{ix}+1\right)}{6}-\frac{\sqrt{3}\,\ln\left(e^{2ix}+\sqrt{3}\,e^{ix}+1\right)}{12}+\frac{\sqrt{3}\,\ln\left(e^{2ix}-\sqrt{3}\,e^{ix}+1\right)}{12}+\frac{\ln\left(e^{2ix}-e^{ix}+1\right)}{12}-\frac{\ln(e^{2i}$

input
```
int(cos(x)*cot(6*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(I*x)+1/2*exp(-I*x)+1/6*ln(exp(I*x)-1)-1/6*ln(exp(I*x)+1)-1/12*3^(1
/2)*ln(exp(2*I*x)+3^(1/2)*exp(I*x)+1)+1/12*3^(1/2)*ln(exp(2*I*x)-3^(1/2)*e
xp(I*x)+1)+1/12*ln(exp(2*I*x)-exp(I*x)+1)-1/12*ln(exp(2*I*x)+exp(I*x)+1)
```

3.114.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(28) = 56$.

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.87

$$\int \cos(x)\cot(6x)\,dx = \frac{1}{12}\sqrt{3}\log\left(\frac{4\cos(x)^2 - 4\sqrt{3}\cos(x) + 3}{4\cos(x)^2 - 3}\right) + \cos(x) - \frac{1}{12}\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right) + \frac{1}{12}\log\left(-\frac{1}{2}\cos(x) + \frac{1}{2}\right) + \frac{1}{12}\log(-2\cos(x) + 1) - \frac{1}{12}\log(-2\cos(x) - 1)$$

input
```
integrate(cos(x)*cot(6*x),x, algorithm="fricas")
```

output
```
1/12*sqrt(3)*log((4*cos(x)^2 - 4*sqrt(3)*cos(x) + 3)/(4*cos(x)^2 - 3)) + c
os(x) - 1/12*log(1/2*cos(x) + 1/2) + 1/12*log(-1/2*cos(x) + 1/2) + 1/12*lo
g(-2*cos(x) + 1) - 1/12*log(-2*cos(x) - 1)
```

3.114.6 Sympy [F]

$$\int \cos(x)\cot(6x)\,dx = \int \cos(x)\cot(6x)\,dx$$

input
```
integrate(cos(x)*cot(6*x),x)
```

output
```
Integral(cos(x)*cot(6*x), x)
```

3.114.7 Maxima [F]

$$\int \cos(x)\cot(6x)\,dx = \int \cos(x)\cot(6x)\,dx$$

input
```
integrate(cos(x)*cot(6*x),x, algorithm="maxima")
```

output
```
cos(x) + integrate(1/2*((sin(3*x) - sin(x))*cos(4*x) - (cos(3*x) - cos(x))
*sin(4*x) - (cos(2*x) - 1)*sin(3*x) + cos(3*x)*sin(2*x) - cos(x)*sin(2*x)
+ cos(2*x)*sin(x) - sin(x))/(2*(cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - cos(
2*x)^2 - sin(4*x)^2 + 2*sin(4*x)*sin(2*x) - sin(2*x)^2 + 2*cos(2*x) - 1),
x) - 1/24*log(2*(cos(x) + 1)*cos(2*x) + cos(2*x)^2 + cos(x)^2 + sin(2*x)^2
 + 2*sin(2*x)*sin(x) + sin(x)^2 + 2*cos(x) + 1) + 1/24*log(-2*(cos(x) - 1)
*cos(2*x) + cos(2*x)^2 + cos(x)^2 + sin(2*x)^2 - 2*sin(2*x)*sin(x) + sin(x
)^2 - 2*cos(x) + 1) - 1/12*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/12*
log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)
```

3.114.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(28) = 56$.

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.84

$$\int \cos(x)\cot(6x)\,dx = \frac{1}{12}\sqrt{3}\log\left(\frac{\left|-4\sqrt{3}+8\cos(x)\right|}{\left|4\sqrt{3}+8\cos(x)\right|}\right)+\cos(x) - \frac{1}{12}\log(\cos(x)+1)+\frac{1}{12}\log(-\cos(x)+1) - \frac{1}{12}\log(|2\cos(x)+1|)+\frac{1}{12}\log(|2\cos(x)-1|)$$

input
```
integrate(cos(x)*cot(6*x),x, algorithm="giac")
```

output
```
1/12*sqrt(3)*log(abs(-4*sqrt(3) + 8*cos(x))/abs(4*sqrt(3) + 8*cos(x))) + c
os(x) - 1/12*log(cos(x) + 1) + 1/12*log(-cos(x) + 1) - 1/12*log(abs(2*cos(
x) + 1)) + 1/12*log(abs(2*cos(x) - 1))
```

3.114.9 Mupad [B] (verification not implemented)

Time = 27.40 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.26

$$\int \cos(x)\cot(6x)\,dx$$
$$= \frac{\operatorname{atanh}\left(\frac{1073741824}{10761687\left(\frac{427973089951744\tan\left(\frac{x}{2}\right)^2}{14348907}-\frac{47552804159488}{4782969}\right)}+\frac{797161}{797162}\right)}{6}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{6} - \frac{\sqrt{3}\operatorname{atanh}\left(\frac{303181204553728\sqrt{3}}{4782969\left(\frac{7314051205955584\tan\left(\frac{x}{2}\right)^2}{4782969}-\frac{525125250187264}{4782969}\right)}-\frac{4222769432625152\sqrt{3}\tan\left(\frac{x}{2}\right)^2}{4782969\left(\frac{7314051205955584\tan\left(\frac{x}{2}\right)^2}{4782969}-\frac{525125250187264}{4782969}\right)}\right)}{6} + \frac{2}{\tan\left(\frac{x}{2}\right)^2+1}$$

input
```
int(cot(6*x)*cos(x),x)
```

output
```
atanh(1073741824/(10761687*((427973089951744*tan(x/2)^2)/14348907 - 475528
04159488/4782969)) + 797161/797162)/6 + log(tan(x/2))/6 - (3^(1/2)*atanh((
303181204553728*3^(1/2))/(4782969*((7314051205955584*tan(x/2)^2)/4782969 -
 525125250187264/4782969)) - (4222769432625152*3^(1/2)*tan(x/2)^2)/(478296
9*((7314051205955584*tan(x/2)^2)/4782969 - 525125250187264/4782969))))/6 +
 2/(tan(x/2)^2 + 1)
```

3.115 $\int \cos(x)\cot(nx)\,dx$

3.115.1 Optimal result

Integrand size = 7, antiderivative size = 92

$$\int \cos(x)\cot(nx)\,dx = -\frac{1}{2}e^{-ix}+\frac{e^{ix}}{2}+e^{-ix}\,\text{Hypergeometric2F1}\left(1,-\frac{1}{2n},1-\frac{1}{2n},e^{2inx}\right)-e^{ix}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),e^{2inx}\right)$$

output
```
-1/2/exp(I*x)+1/2*exp(I*x)+hypergeom([1, -1/2/n],[1-1/2/n],exp(2*I*n*x))/e
xp(I*x)-exp(I*x)*hypergeom([1, 1/2/n],[1+1/2/n],exp(2*I*n*x))
```

3.115.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.95

$$\int \cos(x)\cot(nx)\,dx = \frac{1}{2}e^{-2ix}\left(-\frac{e^{i(x+2nx)}\,\text{Hypergeometric2F1}\left(1,1-\frac{1}{2n},2-\frac{1}{2n},e^{2inx}\right)}{-1+2n}-\frac{e^{i(3+2n)x}\,\text{Hypergeometric2F1}\left(1,1+\frac{1}{2n},2+\frac{1}{2n},e^{2inx}\right)}{1+2n}+e^{ix}\,\text{Hypergeometric2F1}\left(1,-\frac{1}{2n},1-\frac{1}{2n},e^{2inx}\right)-e^{3ix}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},1+\frac{1}{2n},e^{2inx}\right)\right)$$

input
```
Integrate[Cos[x]*Cot[n*x],x]
```

output
```
(-((E^(I*(x + 2*n*x))*Hypergeometric2F1[1, 1 - 1/(2*n), 2 - 1/(2*n), E^((2
*I)*n*x)])/(-1 + 2*n)) - (E^(I*(3 + 2*n)*x)*Hypergeometric2F1[1, 1 + 1/(2*
n), 2 + 1/(2*n), E^((2*I)*n*x)])/(1 + 2*n) + E^(I*x)*Hypergeometric2F1[1,
-1/2*1/n, 1 - 1/(2*n), E^((2*I)*n*x)] - E^((3*I)*x)*Hypergeometric2F1[1, 1
/(2*n), 1 + 1/(2*n), E^((2*I)*n*x)])/(2*E^((2*I)*x))
```

3.115.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5069, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\cot(nx)\,dx$$

$$\Big\downarrow \text{5069}$$

$$\int \left(-\frac{ie^{-ix}}{1-e^{2inx}}-\frac{ie^{ix}}{1-e^{2inx}}+\frac{1}{2}ie^{-ix}+\frac{1}{2}ie^{ix}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$e^{-ix}\,\text{Hypergeometric2F1}\left(1,-\frac{1}{2n},1-\frac{1}{2n},e^{2inx}\right)-e^{ix}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2n},\frac{1}{2}\left(2+\frac{1}{n}\right),e^{2inx}\right)-\frac{e^{-ix}}{2}+\frac{e^{ix}}{2}$$

input
```
Int[Cos[x]*Cot[n*x],x]
```

output
```
-1/2*1/E^(I*x) + E^(I*x)/2 + Hypergeometric2F1[1, -1/2*1/n, 1 - 1/(2*n), E
^((2*I)*n*x)]/E^(I*x) - E^(I*x)*Hypergeometric2F1[1, 1/(2*n), (2 + n^(-1))
/2, E^((2*I)*n*x)]
```

3.115.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5069
```
Int[Cos[(a_.) + (b_.)*(x_)]*Cot[(c_.) + (d_.)*(x_)], x_Symbol] :> Int[I*(1/
(E^(I*(a + b*x))*2)) + I*(E^(I*(a + b*x))/2) - I*(1/(E^(I*(a + b*x))*(1 - E
^(2*I*(c + d*x))))) - I*(E^(I*(a + b*x))/(1 - E^(2*I*(c + d*x)))), x] /; Fr
eeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

3.115.4 Maple [F]

$$\int \cos (x) \cot (nx) \, dx$$

input
```
int(cos(x)*cot(n*x),x)
```

output
```
int(cos(x)*cot(n*x),x)
```

3.115.5 Fricas [F]

$$\int \cos(x)\cot(nx)\,dx = \int \cos (x) \cot (nx) \, dx$$

input
```
integrate(cos(x)*cot(n*x),x, algorithm="fricas")
```

output
```
integral(cos(x)*cot(n*x), x)
```

3.115.6 Sympy [F]

$$\int \cos(x)\cot(nx)\,dx = \int \cos (x) \cot (nx) \, dx$$

input
```
integrate(cos(x)*cot(n*x),x)
```

output
```
Integral(cos(x)*cot(n*x), x)
```

3.115.7 Maxima [F]

$$\int \cos(x)\cot(nx)\,dx = \int \cos\left(x\right)\cot\left(nx\right)\,dx$$

input
```
integrate(cos(x)*cot(n*x),x, algorithm="maxima")
```

output
```
integrate(cos(x)*cot(n*x), x)
```

3.115.8 Giac [F]

$$\int \cos(x)\cot(nx)\,dx = \int \cos\left(x\right)\cot\left(nx\right)\,dx$$

input
```
integrate(cos(x)*cot(n*x),x, algorithm="giac")
```

output
```
integrate(cos(x)*cot(n*x), x)
```

3.115.9 Mupad [F(-1)]

Timed out.

$$\int \cos(x)\cot(nx)\,dx = \int \cot(n\,x)\,\cos\left(x\right)\,dx$$

input
```
int(cot(n*x)*cos(x),x)
```

output
```
int(cot(n*x)*cos(x), x)
```

3.116 $\int \cos(x)\sec(2x)\,dx$

3.116.1 Optimal result

Integrand size = 7, antiderivative size = 15

$$\int \cos(x)\sec(2x)\,dx = \frac{\operatorname{arctanh}\left(\sqrt{2}\sin(x)\right)}{\sqrt{2}}$$

output
```
1/2*arctanh(sin(x)*2^(1/2))*2^(1/2)
```

3.116.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \cos(x)\sec(2x)\,dx = \frac{\operatorname{arctanh}\left(\sqrt{2}\sin(x)\right)}{\sqrt{2}}$$

input
```
Integrate[Cos[x]*Sec[2*x],x]
```

output
```
ArcTanh[Sqrt[2]*Sin[x]]/Sqrt[2]
```

3.116.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 4856, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\sec(2x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(x)}{\cos(2x)}dx$$

$$\Big\downarrow \text{ 4856}$$

$$\int \frac{1}{1-2\sin^2(x)}d\sin(x)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\text{arctanh}\left(\sqrt{2}\sin(x)\right)}{\sqrt{2}}$$

input
```
Int[Cos[x]*Sec[2*x],x]
```

output
```
ArcTanh[Sqrt[2]*Sin[x]]/Sqrt[2]
```

3.116.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4856
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

3.116.4 Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{\operatorname{arctanh}\left(\sin(x)\sqrt{2}\right)\sqrt{2}}{2}$	13
risch	$\frac{\sqrt{2}\,\ln\left(e^{2ix}+i\sqrt{2}\,e^{ix}-1\right)}{4}-\frac{\sqrt{2}\,\ln\left(e^{2ix}-i\sqrt{2}\,e^{ix}-1\right)}{4}$	50

input
```
int(cos(x)*sec(2*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctanh(sin(x)*2^(1/2))*2^(1/2)
```

3.116.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(12) = 24.

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.20

$$\int \cos(x)\sec(2x)\,dx = \frac{1}{4}\sqrt{2}\log\left(-\frac{2\cos(x)^2 - 2\sqrt{2}\sin(x) - 3}{2\cos(x)^2 - 1}\right)$$

input
```
integrate(cos(x)*sec(2*x),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*log(-(2*cos(x)^2 - 2*sqrt(2)*sin(x) - 3)/(2*cos(x)^2 - 1))
```

3.116.6 Sympy [F]

$$\int \cos(x)\sec(2x)\,dx = \int \cos(x)\sec(2x)\,dx$$

input
```
integrate(cos(x)*sec(2*x),x)
```

output
```
Integral(cos(x)*sec(2*x), x)
```

3.116.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(12) = 24.

Time = 0.29 (sec) , antiderivative size = 137, normalized size of antiderivative = 9.13

$$\begin{aligned}\int \cos(x)\sec(2x)\,dx =& \frac{1}{8}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2+2\sqrt{2}\cos(x)+2\sqrt{2}\sin(x)+2\right)\\&-\frac{1}{8}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2+2\sqrt{2}\cos(x)-2\sqrt{2}\sin(x)+2\right)\\&+\frac{1}{8}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2-2\sqrt{2}\cos(x)+2\sqrt{2}\sin(x)+2\right)\\&-\frac{1}{8}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2-2\sqrt{2}\cos(x)-2\sqrt{2}\sin(x)+2\right)\end{aligned}$$

input
```
integrate(cos(x)*sec(2*x),x, algorithm="maxima")
```

output
```
1/8*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*sqrt(2)*sin
(x) + 2) - 1/8*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) - 2*
sqrt(2)*sin(x) + 2) + 1/8*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)*
cos(x) + 2*sqrt(2)*sin(x) + 2) - 1/8*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 -
 2*sqrt(2)*cos(x) - 2*sqrt(2)*sin(x) + 2)
```

3.116.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(12) = 24.

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \cos(x)\sec(2x)\,dx = \frac{1}{4}\sqrt{2}\log\left(\left|\frac{1}{2}\sqrt{2}+\sin\left(x\right)\right|\right) - \frac{1}{4}\sqrt{2}\log\left(\left|-\frac{1}{2}\sqrt{2}+\sin\left(x\right)\right|\right)$$

input
```
integrate(cos(x)*sec(2*x),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*log(abs(1/2*sqrt(2) + sin(x))) - 1/4*sqrt(2)*log(abs(-1/2*sqrt
(2) + sin(x)))
```

3.116.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

$$\int \cos(x)\sec(2x)\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\sqrt{2}\,\sin\left(x\right)\right)}{2}$$

input
```
int(cos(x)/cos(2*x),x)
```

output
```
(2^(1/2)*atanh(2^(1/2)*sin(x)))/2
```

3.117 $\int \cos(x)\sec(3x)\,dx$

3.117.1 Optimal result

Integrand size = 7, antiderivative size = 44

$$\int \cos(x)\sec(3x)\,dx = -\frac{\log\left(\cos(x)-\sqrt{3}\sin(x)\right)}{2\sqrt{3}} + \frac{\log\left(\cos(x)+\sqrt{3}\sin(x)\right)}{2\sqrt{3}}$$

output
```
-1/6*ln(cos(x)-sin(x)*3^(1/2))*3^(1/2)+1/6*ln(cos(x)+sin(x)*3^(1/2))*3^(1/
2)
```

3.117.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.34

$$\int \cos(x)\sec(3x)\,dx = \frac{\operatorname{arctanh}\left(\sqrt{3}\tan(x)\right)}{\sqrt{3}}$$

input
```
Integrate[Cos[x]*Sec[3*x],x]
```

output
```
ArcTanh[Sqrt[3]*Tan[x]]/Sqrt[3]
```

3.117.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.34, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 4889, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\sec(3x)\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{\cos(x)}{\cos(3x)}dx$$
$$\Big\downarrow \ 4889$$
$$\int \frac{1}{1-3\tan^2(x)}d\tan(x)$$
$$\Big\downarrow \ 219$$
$$\frac{\text{arctanh}\left(\sqrt{3}\tan(x)\right)}{\sqrt{3}}$$

input
```
Int[Cos[x]*Sec[3*x],x]
```

output
```
ArcTanh[Sqrt[3]*Tan[x]]/Sqrt[3]
```

3.117.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.117.4 Maple [A] (verified)

Time = 1.70 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.30

method	result	size
default	$\frac{\sqrt{3}\,\operatorname{arctanh}\left(\tan(x)\sqrt{3}\right)}{3}$	13
risch	$\frac{\sqrt{3}\,\ln\left(e^{2ix}-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}{6}-\frac{\sqrt{3}\,\ln\left(e^{2ix}-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{6}$	40

input
```
int(cos(x)*sec(3*x),x,method=_RETURNVERBOSE)
```

output
```
1/3*3^(1/2)*arctanh(tan(x)*3^(1/2))
```

3.117.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.20

$$\int \cos(x)\sec(3x)\,dx = \frac{1}{12}\sqrt{3}\log\left(-\frac{8\cos(x)^4+4\left(2\sqrt{3}\cos(x)^3-3\sqrt{3}\cos(x)\right)\sin(x)-9}{16\cos(x)^4-24\cos(x)^2+9}\right)$$

input
```
integrate(cos(x)*sec(3*x),x, algorithm="fricas")
```

output
```
1/12*sqrt(3)*log(-(8*cos(x)^4 + 4*(2*sqrt(3)*cos(x)^3 - 3*sqrt(3)*cos(x))*
sin(x) - 9)/(16*cos(x)^4 - 24*cos(x)^2 + 9))
```

3.117.6 Sympy [F]

$$\int \cos(x)\sec(3x)\,dx = \int \cos(x)\sec(3x)\,dx$$

input
```
integrate(cos(x)*sec(3*x),x)
```

output
```
Integral(cos(x)*sec(3*x), x)
```

3.117.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(32) = 64$.

Time = 0.31 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.73

$$\int \cos(x)\sec(3x)\,dx$$
$$= \frac{1}{12}\sqrt{3}\left(\log\left(\frac{4}{3}\cos(2x)^2 + \frac{4}{3}\sin(2x)^2 + \frac{4}{3}\sqrt{3}\sin(2x) - \frac{4}{3}\cos(2x) + \frac{4}{3}\right) - \log\left(\frac{4}{3}\cos(2x)^2 + \frac{4}{3}\sin\right.\right.$$

input
```
integrate(cos(x)*sec(3*x),x, algorithm="maxima")
```

output
```
1/12*sqrt(3)*(log(4/3*cos(2*x)^2 + 4/3*sin(2*x)^2 + 4/3*sqrt(3)*sin(2*x) -
 4/3*cos(2*x) + 4/3) - log(4/3*cos(2*x)^2 + 4/3*sin(2*x)^2 - 4/3*sqrt(3)*s
in(2*x) - 4/3*cos(2*x) + 4/3))
```

3.117.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.70

$$\int \cos(x)\sec(3x)\,dx = -\frac{1}{6}\sqrt{3}\log\left(\frac{\left|-2\sqrt{3}+6\tan(x)\right|}{\left|2\sqrt{3}+6\tan(x)\right|}\right)$$

input
```
integrate(cos(x)*sec(3*x),x, algorithm="giac")
```

output
```
-1/6*sqrt(3)*log(abs(-2*sqrt(3) + 6*tan(x))/abs(2*sqrt(3) + 6*tan(x)))
```

3.117.9 Mupad [B] (verification not implemented)

Time = 26.79 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.36

$$\int \cos(x)\sec(3x)\,dx = \frac{\sqrt{3}\,\mathrm{atanh}\left(\frac{\sqrt{3}\,\sin(x)}{\cos(x)}\right)}{3}$$

input
```
int(cos(x)/cos(3*x),x)
```

output
```
(3^(1/2)*atanh((3^(1/2)*sin(x))/cos(x)))/3
```

3.118 $\int \cos(x)\sec(4x)\,dx$

3.118.1 Optimal result

Integrand size = 7, antiderivative size = 71

$$\int \cos(x)\sec(4x)\,dx = \frac{\operatorname{arctanh}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}} - \frac{\operatorname{arctanh}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}$$

output
```
1/2*arctanh(2*sin(x)/(2-2^(1/2))^(1/2))/(4-2*2^(1/2))^(1/2)-1/2*arctanh(2*
sin(x)/(2+2^(1/2))^(1/2))/(4+2*2^(1/2))^(1/2)
```

3.118.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.94

$$\int \cos(x)\sec(4x)\,dx = \frac{1}{4}\sqrt{2+\sqrt{2}}\operatorname{arctanh}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right) - \frac{\operatorname{arctanh}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}$$

input
```
Integrate[Cos[x]*Sec[4*x],x]
```

output
```
(Sqrt[2 + Sqrt[2]]*ArcTanh[(2*Sin[x])/Sqrt[2 - Sqrt[2]]])/4 - ArcTanh[(2*S
in[x])/Sqrt[2 + Sqrt[2]]]/(2*Sqrt[2*(2 + Sqrt[2])])
```

3.118.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3042, 4856, 1406, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\sec(4x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(x)}{\cos(4x)}dx$$

$$\Big\downarrow \text{4856}$$

$$\int \frac{1}{8\sin^4(x)-8\sin^2(x)+1}d\sin(x)$$

$$\Big\downarrow \text{1406}$$

$$\sqrt{2}\int \frac{1}{8\sin^2(x)-2\left(2+\sqrt{2}\right)}d\sin(x)-\sqrt{2}\int \frac{1}{8\sin^2(x)-2\left(2-\sqrt{2}\right)}d\sin(x)$$

$$\Big\downarrow \text{220}$$

$$\frac{\operatorname{arctanh}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}}-\frac{\operatorname{arctanh}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}$$

input
```
Int[Cos[x]*Sec[4*x],x]
```

output
```
ArcTanh[(2*Sin[x])/Sqrt[2 - Sqrt[2]]]/(2*Sqrt[2*(2 - Sqrt[2])]) - ArcTanh[
(2*Sin[x])/Sqrt[2 + Sqrt[2]]]/(2*Sqrt[2*(2 + Sqrt[2])])
```

3.118.3.1 Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4856
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

3.118.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.79 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.65

method	result	size
risch	$2\left(\sum_{\_R=\operatorname{RootOf}\left(32768\_Z^4-512\_Z^2+1\right)}\_R\ln\left(e^{2ix}+\left(4096i\_R^3-48i\_R\right)e^{ix}-1\right)\right)$	46
default	$-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{2}}}\right)}{4\sqrt{2+\sqrt{2}}}+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{2}}}\right)}{4\sqrt{2-\sqrt{2}}}$	54

input
```
int(cos(x)*sec(4*x),x,method=_RETURNVERBOSE)
```

output
```
2*sum(_R*ln(exp(2*I*x)+(4096*I*_R^3-48*I*_R)*exp(I*x)-1),_R=RootOf(32768*_
Z^4-512*_Z^2+1))
```

3.118.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(49) = 98.

Time = 0.26 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.70

$$\int \cos(x)\sec(4x)\,dx = \frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\sqrt{\sqrt{2}+2}\left(\sqrt{2}-1\right)+2\sin(x)\right) - \frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\sqrt{\sqrt{2}+2}\left(\sqrt{2}-1\right)-2\sin(x)\right) - \frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(\left(\sqrt{2}+1\right)\sqrt{-\sqrt{2}+2}+2\sin(x)\right) + \frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(\left(\sqrt{2}+1\right)\sqrt{-\sqrt{2}+2}-2\sin(x)\right)$$

input
```
integrate(cos(x)*sec(4*x),x, algorithm="fricas")
```

output
```
1/8*sqrt(sqrt(2) + 2)*log(sqrt(sqrt(2) + 2)*(sqrt(2) - 1) + 2*sin(x)) - 1/
8*sqrt(sqrt(2) + 2)*log(sqrt(sqrt(2) + 2)*(sqrt(2) - 1) - 2*sin(x)) - 1/8*
sqrt(-sqrt(2) + 2)*log((sqrt(2) + 1)*sqrt(-sqrt(2) + 2) + 2*sin(x)) + 1/8*
sqrt(-sqrt(2) + 2)*log((sqrt(2) + 1)*sqrt(-sqrt(2) + 2) - 2*sin(x))
```

3.118.6 Sympy [F]

$$\int \cos(x)\sec(4x)\,dx = \int \cos(x)\sec(4x)\,dx$$

input
```
integrate(cos(x)*sec(4*x),x)
```

output
```
Integral(cos(x)*sec(4*x), x)
```

3.118.7 Maxima [F]

$$\int \cos(x)\sec(4x)\,dx = \int \cos(x)\sec(4x)\,dx$$

input
```
integrate(cos(x)*sec(4*x),x, algorithm="maxima")
```

output
```
integrate(cos(x)*sec(4*x), x)
```

3.118.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(49) = 98.

Time = 0.31 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.39

$$\begin{aligned}\int \cos(x)\sec(4x)\,dx = &-\frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(\left|\frac{1}{2}\sqrt{\sqrt{2}+2}+\sin(x)\right|\right)\\&+\frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(\left|-\frac{1}{2}\sqrt{\sqrt{2}+2}+\sin(x)\right|\right)\\&+\frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\left|\sqrt{-\frac{1}{4}\sqrt{2}+\frac{1}{2}}+\sin(x)\right|\right)\\&-\frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\left|-\sqrt{-\frac{1}{4}\sqrt{2}+\frac{1}{2}}+\sin(x)\right|\right)\end{aligned}$$

input
```
integrate(cos(x)*sec(4*x),x, algorithm="giac")
```

output
```
-1/8*sqrt(-sqrt(2) + 2)*log(abs(1/2*sqrt(sqrt(2) + 2) + sin(x))) + 1/8*sqr
t(-sqrt(2) + 2)*log(abs(-1/2*sqrt(sqrt(2) + 2) + sin(x))) + 1/8*sqrt(sqrt(
2) + 2)*log(abs(sqrt(-1/4*sqrt(2) + 1/2) + sin(x))) - 1/8*sqrt(sqrt(2) + 2
)*log(abs(-sqrt(-1/4*sqrt(2) + 1/2) + sin(x)))
```

3.118.9 Mupad [B] (verification not implemented)

Time = 26.86 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.34

$$\int \cos(x)\sec(4x)\,dx = \frac{\operatorname{atanh}\left(\frac{2\sin(x)\sqrt{\sqrt{2}+2}+2\sqrt{2}\sin(x)\sqrt{\sqrt{2}+2}}{\sqrt{2}+2}\right)\sqrt{\sqrt{2}+2}}{4} - \frac{\operatorname{atanh}\left(\frac{2\sin(x)\sqrt{2-\sqrt{2}}-2\sqrt{2}\sin(x)\sqrt{2-\sqrt{2}}}{\sqrt{2}-2}\right)\sqrt{2-\sqrt{2}}}{4}$$

input
```
int(cos(x)/cos(4*x),x)
```

output
```
(atanh((2*sin(x)*(2^(1/2) + 2)^(1/2) + 2*2^(1/2)*sin(x)*(2^(1/2) + 2)^(1/2
))/(2^(1/2) + 2))*(2^(1/2) + 2)^(1/2))/4 - (atanh((2*sin(x)*(2 - 2^(1/2))^
(1/2) - 2*2^(1/2)*sin(x)*(2 - 2^(1/2))^(1/2))/(2^(1/2) - 2))*(2 - 2^(1/2))
^(1/2))/4
```

3.119 $\int \cos(x)\sec(5x)\,dx$

3.119.1 Optimal result

Integrand size = 7, antiderivative size = 163

$$\begin{aligned}\int \cos(x)\sec(5x)\,dx &= \frac{1}{10}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\log\left(\cos(x)-\sqrt{5-2\sqrt{5}}\sin(x)\right)\\ &-\frac{1}{10}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\log\left(\cos(x)+\sqrt{5-2\sqrt{5}}\sin(x)\right)\\ &-\frac{1}{10}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\log\left(\cos(x)-\sqrt{5+2\sqrt{5}}\sin(x)\right)\\ &+\frac{1}{10}\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\log\left(\cos(x)+\sqrt{5+2\sqrt{5}}\sin(x)\right)\end{aligned}$$

output
```
1/20*ln(cos(x)-sin(x)*(5-2*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2)-1/20*ln(co
s(x)+sin(x)*(5-2*5^(1/2))^(1/2))*(10-2*5^(1/2))^(1/2)-1/20*ln(cos(x)-sin(x
)*(5+2*5^(1/2))^(1/2))*(10+2*5^(1/2))^(1/2)+1/20*ln(cos(x)+sin(x)*(5+2*5^(
1/2))^(1/2))*(10+2*5^(1/2))^(1/2)
```

3.119.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.52

$$\int \cos(x)\sec(5x)\,dx = \frac{\sqrt{5+\sqrt{5}}\text{arctanh}\left(\frac{\left(5+\sqrt{5}\right)\tan(x)}{\sqrt{10-2\sqrt{5}}}\right)+\sqrt{5-\sqrt{5}}\text{arctanh}\left(\frac{\left(-5+\sqrt{5}\right)\tan(x)}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)}{5\sqrt{2}}$$

input
```
Integrate[Cos[x]*Sec[5*x],x]
```

output
```
(Sqrt[5 + Sqrt[5]]*ArcTanh[((5 + Sqrt[5])*Tan[x])/Sqrt[10 - 2*Sqrt[5]]] +
Sqrt[5 - Sqrt[5]]*ArcTanh[((-5 + Sqrt[5])*Tan[x])/Sqrt[2*(5 + Sqrt[5])]])/
(5*Sqrt[2])
```

3.119.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.55, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4889, 1480, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\sec(5x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(x)}{\cos(5x)}dx$$

$$\Big\downarrow \text{ 4889}$$

$$\int \frac{\tan^2(x)+1}{5\tan^4(x)-10\tan^2(x)+1}d\tan(x)$$

$$\Big\downarrow \text{ 1480}$$

$$\frac{1}{2}\left(1+\sqrt{5}\right)\int \frac{1}{5\tan^2(x)-2\sqrt{5}-5}d\tan(x)+\frac{1}{2}\left(1-\sqrt{5}\right)\int \frac{1}{5\tan^2(x)+2\sqrt{5}-5}d\tan(x)$$

$$\Big\downarrow \text{ 220}$$

$$-\frac{(1+\sqrt{5})\operatorname{arctanh}\left(\sqrt{5-2\sqrt{5}}\tan(x)\right)}{2\sqrt{5\left(5+2\sqrt{5}\right)}}-\frac{(1-\sqrt{5})\operatorname{arctanh}\left(\sqrt{5+2\sqrt{5}}\tan(x)\right)}{2\sqrt{5\left(5-2\sqrt{5}\right)}}$$

input
```
Int[Cos[x]*Sec[5*x],x]
```

output
```
-1/2*((1 + Sqrt[5])*ArcTanh[Sqrt[5 - 2*Sqrt[5]]*Tan[x]])/Sqrt[5*(5 + 2*Sqr
t[5])] - ((1 - Sqrt[5])*ArcTanh[Sqrt[5 + 2*Sqrt[5]]*Tan[x]])/(2*Sqrt[5*(5
- 2*Sqrt[5])])
```

3.119.3.1 Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.119.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.95 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.27

method	result	size
risch	$2\left(\sum_{\_R=\operatorname{RootOf}(32000\_Z^4-400\_Z^2+1)}\_R\ln\left(e^{2ix}+4000i\_R^3-200\_R^2-30i\_R+1\right)\right)$	44
default	$-\frac{\sqrt{5}\left(5+\sqrt{5}\right)\operatorname{arctanh}\left(\frac{5\tan(x)}{\sqrt{25+10\sqrt{5}}}\right)}{10\sqrt{25+10\sqrt{5}}}-\frac{\left(-5+\sqrt{5}\right)\sqrt{5}\,\operatorname{arctanh}\left(\frac{5\tan(x)}{\sqrt{25-10\sqrt{5}}}\right)}{10\sqrt{25-10\sqrt{5}}}$	68

input
```
int(cos(x)*sec(5*x),x,method=_RETURNVERBOSE)
```

output
```
2*sum(_R*ln(exp(2*I*x)+4000*I*_R^3-200*_R^2-30*I*_R+1),_R=RootOf(32000*_Z^
4-400*_Z^2+1))
```

3.119.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. 2(111) = 222.

Time = 0.30 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.42

$$\begin{aligned}\int\cos(x)\sec(5x)\,dx = &-\frac{1}{40}\sqrt{2}\sqrt{\sqrt{5}+5}\log\left(\left(\sqrt{5}\sqrt{2}-\sqrt{2}\right)\sqrt{\sqrt{5}+5}\cos(x)\sin(x)\right.\\&\left.+2\left(\sqrt{5}+1\right)\cos(x)^2-\sqrt{5}-5\right)\\&+\frac{1}{40}\sqrt{2}\sqrt{\sqrt{5}+5}\log\left(-\left(\sqrt{5}\sqrt{2}-\sqrt{2}\right)\sqrt{\sqrt{5}+5}\cos(x)\sin(x)\right.\\&\left.+2\left(\sqrt{5}+1\right)\cos(x)^2-\sqrt{5}-5\right)\\&-\frac{1}{40}\sqrt{2}\sqrt{-\sqrt{5}+5}\log\left(\left(\sqrt{5}\sqrt{2}+\sqrt{2}\right)\sqrt{-\sqrt{5}+5}\cos(x)\sin(x)\right.\\&\left.+2\left(\sqrt{5}-1\right)\cos(x)^2-\sqrt{5}+5\right)\\&+\frac{1}{40}\sqrt{2}\sqrt{-\sqrt{5}+5}\log\left(-\left(\sqrt{5}\sqrt{2}+\sqrt{2}\right)\sqrt{-\sqrt{5}+5}\cos(x)\sin(x)\right.\\&\left.+2\left(\sqrt{5}-1\right)\cos(x)^2-\sqrt{5}+5\right)\end{aligned}$$

input
```
integrate(cos(x)*sec(5*x),x, algorithm="fricas")
```

output
```
-1/40*sqrt(2)*sqrt(sqrt(5) + 5)*log((sqrt(5)*sqrt(2) - sqrt(2))*sqrt(sqrt(
5) + 5)*cos(x)*sin(x) + 2*(sqrt(5) + 1)*cos(x)^2 - sqrt(5) - 5) + 1/40*sqr
t(2)*sqrt(sqrt(5) + 5)*log(-(sqrt(5)*sqrt(2) - sqrt(2))*sqrt(sqrt(5) + 5)*
cos(x)*sin(x) + 2*(sqrt(5) + 1)*cos(x)^2 - sqrt(5) - 5) - 1/40*sqrt(2)*sqr
t(-sqrt(5) + 5)*log((sqrt(5)*sqrt(2) + sqrt(2))*sqrt(-sqrt(5) + 5)*cos(x)*
sin(x) + 2*(sqrt(5) - 1)*cos(x)^2 - sqrt(5) + 5) + 1/40*sqrt(2)*sqrt(-sqrt
(5) + 5)*log(-(sqrt(5)*sqrt(2) + sqrt(2))*sqrt(-sqrt(5) + 5)*cos(x)*sin(x)
 + 2*(sqrt(5) - 1)*cos(x)^2 - sqrt(5) + 5)
```

3.119.6 Sympy [F]

$$\int \cos(x)\sec(5x)\,dx = \int \cos{(x)}\sec{(5x)}\,dx$$

input
```
integrate(cos(x)*sec(5*x),x)
```

output
```
Integral(cos(x)*sec(5*x), x)
```

3.119.7 Maxima [F]

$$\int \cos(x)\sec(5x)\,dx = \int \cos(x)\sec(5\,x)\,\,dx$$

input
```
integrate(cos(x)*sec(5*x),x, algorithm="maxima")
```

output
```
integrate(cos(x)*sec(5*x), x)
```

3.119.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.64

$$\int \cos(x)\sec(5x)\,dx = -\frac{1}{20}\sqrt{-2\sqrt{5}+10}\log\left(\left|\sqrt{\frac{2}{5}\sqrt{5}+1}+\tan(x)\right|\right) + \frac{1}{20}\sqrt{-2\sqrt{5}+10}\log\left(\left|-\sqrt{\frac{2}{5}\sqrt{5}+1}+\tan(x)\right|\right) + \frac{1}{20}\sqrt{2\sqrt{5}+10}\log\left(\left|\sqrt{-\frac{2}{5}\sqrt{5}+1}+\tan(x)\right|\right) - \frac{1}{20}\sqrt{2\sqrt{5}+10}\log\left(\left|-\sqrt{-\frac{2}{5}\sqrt{5}+1}+\tan(x)\right|\right)$$

input
```
integrate(cos(x)*sec(5*x),x, algorithm="giac")
```

output
```
-1/20*sqrt(-2*sqrt(5) + 10)*log(abs(sqrt(2/5*sqrt(5) + 1) + tan(x))) + 1/2
0*sqrt(-2*sqrt(5) + 10)*log(abs(-sqrt(2/5*sqrt(5) + 1) + tan(x))) + 1/20*s
qrt(2*sqrt(5) + 10)*log(abs(sqrt(-2/5*sqrt(5) + 1) + tan(x))) - 1/20*sqrt(
2*sqrt(5) + 10)*log(abs(-sqrt(-2/5*sqrt(5) + 1) + tan(x)))
```

3.119.9 Mupad [B] (verification not implemented)

Time = 26.77 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.33

$$\int \cos(x)\sec(5x)\,dx$$

$$= \frac{\sqrt{2}\,\mathrm{atanh}\left(-\frac{34359738368\sqrt{2}\tan\left(\frac{x}{2}\right)\sqrt{5-\sqrt{5}}}{5\left(\frac{124554051584\sqrt{5}}{25}-\frac{124554051584\sqrt{5}\tan\left(\frac{x}{2}\right)^2}{25}-\frac{55834574848\tan\left(\frac{x}{2}\right)^2}{5}+\frac{55834574848}{5}\right)} - \frac{77309411328}{25\left(\frac{124554051584\sqrt{5}}{25}-\frac{124554051584\sqrt{5}}{25}\right.}\right.}{10}$$

$$- \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{77309411328\sqrt{2}\sqrt{5}\tan\left(\frac{x}{2}\right)\sqrt{\sqrt{5}+5}}{25\left(\frac{124554051584\sqrt{5}}{25}-\frac{124554051584\sqrt{5}\tan\left(\frac{x}{2}\right)^2}{25}+\frac{55834574848\tan\left(\frac{x}{2}\right)^2}{5}-\frac{55834574848}{5}\right)} - \frac{3435973838}{5\left(\frac{124554051584\sqrt{5}}{25}-\frac{124554051584\sqrt{}}{25}\right.}\right.}{10}$$

input
```
int(cos(x)/cos(5*x),x)
```

output
```
(2^(1/2)*atanh(- (34359738368*2^(1/2)*tan(x/2)*(5 - 5^(1/2))^(1/2))/(5*((1
24554051584*5^(1/2))/25 - (124554051584*5^(1/2)*tan(x/2)^2)/25 - (55834574
848*tan(x/2)^2)/5 + 55834574848/5)) - (77309411328*2^(1/2)*5^(1/2)*tan(x/2
)*(5 - 5^(1/2))^(1/2))/(25*((124554051584*5^(1/2))/25 - (124554051584*5^(1
/2)*tan(x/2)^2)/25 - (55834574848*tan(x/2)^2)/5 + 55834574848/5)))*(5 - 5^
(1/2))^(1/2))/10 - (2^(1/2)*atanh((77309411328*2^(1/2)*5^(1/2)*tan(x/2)*(5
^(1/2) + 5)^(1/2))/(25*((124554051584*5^(1/2))/25 - (124554051584*5^(1/2)*
tan(x/2)^2)/25 + (55834574848*tan(x/2)^2)/5 - 55834574848/5)) - (343597383
68*2^(1/2)*tan(x/2)*(5^(1/2) + 5)^(1/2))/(5*((124554051584*5^(1/2))/25 - (
124554051584*5^(1/2)*tan(x/2)^2)/25 + (55834574848*tan(x/2)^2)/5 - 5583457
4848/5)))*(5^(1/2) + 5)^(1/2))/10
```

3.119. $\int \cos(x) \sec(5x) \, dx$

3.120 $\int \cos(x)\sec(6x)\,dx$

3.120.1 Optimal result

Integrand size = 7, antiderivative size = 85

$$\int \cos(x)\sec(6x)\,dx=-\frac{\operatorname{arctanh}\left(\sqrt{2}\sin(x)\right)}{3\sqrt{2}}+\frac{\operatorname{arctanh}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{3}}}\right)}{6\sqrt{2-\sqrt{3}}}+\frac{\operatorname{arctanh}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{3}}}\right)}{6\sqrt{2+\sqrt{3}}}$$

output
```
-1/6*arctanh(sin(x)*2^(1/2))*2^(1/2)+1/6*arctanh(2*sin(x)/(1/2*6^(1/2)-1/2
*2^(1/2)))/(1/2*6^(1/2)-1/2*2^(1/2))+1/6*arctanh(2*sin(x)/(1/2*6^(1/2)+1/2
*2^(1/2)))/(1/2*6^(1/2)+1/2*2^(1/2))
```

3.120.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.95

$$\int \cos(x)\sec(6x)\,dx=\frac{1}{6}\left(-\sqrt{2}\operatorname{arctanh}\left(\sqrt{2}\sin(x)\right)+\sqrt{2+\sqrt{3}}\operatorname{arctanh}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{3}}}\right)+\sqrt{2-\sqrt{3}}\operatorname{arctanh}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{3}}}\right)\right)$$

input
```
Integrate[Cos[x]*Sec[6*x],x]
```

output
```
(-(Sqrt[2]*ArcTanh[Sqrt[2]*Sin[x]]) + Sqrt[2 + Sqrt[3]]*ArcTanh[(2*Sin[x])
/Sqrt[2 - Sqrt[3]]] + Sqrt[2 - Sqrt[3]]*ArcTanh[(2*Sin[x])/Sqrt[2 + Sqrt[3
]]])/6
```

3.120.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3042, 4856, 2460, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\sec(6x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(x)}{\cos(6x)}dx$$

$$\Big\downarrow \text{4856}$$

$$\int \frac{1}{-32\sin^6(x)+48\sin^4(x)-18\sin^2(x)+1}d\sin(x)$$

$$\Big\downarrow \text{2460}$$

$$\int \left(\frac{1}{3\left(2\sin^2(x)-1\right)}-\frac{4\left(2\sin^2(x)-1\right)}{3\left(16\sin^4(x)-16\sin^2(x)+1\right)}\right)d\sin(x)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\text{arctanh}\left(\sqrt{2}\sin(x)\right)}{3\sqrt{2}}+\frac{\text{arctanh}\left(\frac{2\sin(x)}{\sqrt{2-\sqrt{3}}}\right)}{6\sqrt{2-\sqrt{3}}}+\frac{\text{arctanh}\left(\frac{2\sin(x)}{\sqrt{2+\sqrt{3}}}\right)}{6\sqrt{2+\sqrt{3}}}$$

input
```
Int[Cos[x]*Sec[6*x],x]
```

output
```
-1/3*ArcTanh[Sqrt[2]*Sin[x]]/Sqrt[2] + ArcTanh[(2*Sin[x])/Sqrt[2 - Sqrt[3]
]]/(6*Sqrt[2 - Sqrt[3]]) + ArcTanh[(2*Sin[x])/Sqrt[2 + Sqrt[3]]]/(6*Sqrt[2
 + Sqrt[3]])
```

3.120.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2460
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px /. x -> Sqrt[x]]},
Int[ExpandIntegrand[u*(Qx /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[Q
x, x]]] /; PolyQ[Px, x^2] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&
 !TrinomialQ[Px, x] && ILtQ[p, 0] && RationalFunctionQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4856
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

3.120.4 Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.94

method	result
default	$-\frac{\operatorname{arctanh}\left(\sin(x)\sqrt{2}\right)\sqrt{2}}{6}+\frac{2\operatorname{arctanh}\left(\frac{8\sin(x)}{2\sqrt{6}+2\sqrt{2}}\right)}{3\left(2\sqrt{6}+2\sqrt{2}\right)}+\frac{2\operatorname{arctanh}\left(\frac{8\sin(x)}{2\sqrt{6}-2\sqrt{2}}\right)}{3\left(2\sqrt{6}-2\sqrt{2}\right)}$
risch	$\frac{\sqrt{2}\,\ln\left(e^{2ix}-i\sqrt{2}\,e^{ix}-1\right)}{12}-\frac{\sqrt{2}\,\ln\left(e^{2ix}+i\sqrt{2}\,e^{ix}-1\right)}{12}+2\left(\sum_{\_R=\operatorname{RootOf}\left(331776\_Z^{4}-2304\_Z^{2}+1\right)}\_R\ln\left(e^{2ix}+\left(-138\right.\right.\right.$

input
```
int(cos(x)*sec(6*x),x,method=_RETURNVERBOSE)
```

output
```
-1/6*arctanh(sin(x)*2^(1/2))*2^(1/2)+2/3/(2*6^(1/2)+2*2^(1/2))*arctanh(8*s
in(x)/(2*6^(1/2)+2*2^(1/2)))+2/3/(2*6^(1/2)-2*2^(1/2))*arctanh(8*sin(x)/(2
*6^(1/2)-2*2^(1/2)))
```

3.120.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. 2(67) = 134.

Time = 0.27 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.81

$$
\begin{aligned}
\int \cos(x)\sec(6x)\,dx = &-\frac{1}{12}\sqrt{\sqrt{3}+2}\log\left(\sqrt{\sqrt{3}+2}\left(\sqrt{3}-2\right)+2\,\sin\left(x\right)\right)\\
&+\frac{1}{12}\sqrt{\sqrt{3}+2}\log\left(\sqrt{\sqrt{3}+2}\left(\sqrt{3}-2\right)-2\,\sin\left(x\right)\right)\\
&+\frac{1}{12}\sqrt{-\sqrt{3}+2}\log\left(\left(\sqrt{3}+2\right)\sqrt{-\sqrt{3}+2}+2\,\sin\left(x\right)\right)\\
&-\frac{1}{12}\sqrt{-\sqrt{3}+2}\log\left(\left(\sqrt{3}+2\right)\sqrt{-\sqrt{3}+2}-2\,\sin\left(x\right)\right)\\
&+\frac{1}{12}\sqrt{2}\log\left(-\frac{2\,\cos\left(x\right)^{2}+2\,\sqrt{2}\sin\left(x\right)-3}{2\,\cos\left(x\right)^{2}-1}\right)
\end{aligned}
$$

input
```
integrate(cos(x)*sec(6*x),x, algorithm="fricas")
```

output
```
-1/12*sqrt(sqrt(3) + 2)*log(sqrt(sqrt(3) + 2)*(sqrt(3) - 2) + 2*sin(x)) +
1/12*sqrt(sqrt(3) + 2)*log(sqrt(sqrt(3) + 2)*(sqrt(3) - 2) - 2*sin(x)) + 1
/12*sqrt(-sqrt(3) + 2)*log((sqrt(3) + 2)*sqrt(-sqrt(3) + 2) + 2*sin(x)) -
1/12*sqrt(-sqrt(3) + 2)*log((sqrt(3) + 2)*sqrt(-sqrt(3) + 2) - 2*sin(x)) +
 1/12*sqrt(2)*log(-(2*cos(x)^2 + 2*sqrt(2)*sin(x) - 3)/(2*cos(x)^2 - 1))
```

3.120.6 Sympy [F]

$$\int \cos(x)\sec(6x)\,dx = \int \cos\left(x\right)\sec\left(6x\right)\,dx$$

input
```
integrate(cos(x)*sec(6*x),x)
```

output
```
Integral(cos(x)*sec(6*x), x)
```

3.120.7 Maxima [F]

$$\int \cos(x)\sec(6x)\,dx = \int \cos\left(x\right)\sec\left(6\,x\right)\,dx$$

input
```
integrate(cos(x)*sec(6*x),x, algorithm="maxima")
```

output
```
-1/24*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*sqrt(2)*s
in(x) + 2) + 1/24*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) -
 2*sqrt(2)*sin(x) + 2) - 1/24*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt
(2)*cos(x) + 2*sqrt(2)*sin(x) + 2) + 1/24*sqrt(2)*log(2*cos(x)^2 + 2*sin(x
)^2 - 2*sqrt(2)*cos(x) - 2*sqrt(2)*sin(x) + 2) + integrate(-1/3*((cos(7*x)
 + cos(5*x) + cos(3*x) + cos(x))*cos(8*x) - (cos(4*x) - 1)*cos(7*x) - (cos
(4*x) - 1)*cos(5*x) - (cos(3*x) + cos(x))*cos(4*x) + (sin(7*x) + sin(5*x)
+ sin(3*x) + sin(x))*sin(8*x) - (sin(3*x) + sin(x))*sin(4*x) - sin(7*x)*si
n(4*x) - sin(5*x)*sin(4*x) + cos(3*x) + cos(x))/(2*(cos(4*x) - 1)*cos(8*x)
 - cos(8*x)^2 - cos(4*x)^2 - sin(8*x)^2 + 2*sin(8*x)*sin(4*x) - sin(4*x)^2
 + 2*cos(4*x) - 1), x)
```

3.120.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.55

$$\begin{aligned}\int \cos(x)\sec(6x)\,dx = &\frac{1}{24}\left(\sqrt{6}-\sqrt{2}\right)\log\left(\left|\frac{1}{4}\sqrt{6}+\frac{1}{4}\sqrt{2}+\sin\left(x\right)\right|\right)\\ &+\frac{1}{24}\left(\sqrt{6}+\sqrt{2}\right)\log\left(\left|\frac{1}{4}\sqrt{6}-\frac{1}{4}\sqrt{2}+\sin\left(x\right)\right|\right)\\ &-\frac{1}{24}\left(\sqrt{6}+\sqrt{2}\right)\log\left(\left|-\frac{1}{4}\sqrt{6}+\frac{1}{4}\sqrt{2}+\sin\left(x\right)\right|\right)\\ &-\frac{1}{24}\left(\sqrt{6}-\sqrt{2}\right)\log\left(\left|-\frac{1}{4}\sqrt{6}-\frac{1}{4}\sqrt{2}+\sin\left(x\right)\right|\right)\\ &+\frac{1}{12}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(x\right)\right|}{\left|2\sqrt{2}+4\sin\left(x\right)\right|}\right)\end{aligned}$$

input
```
integrate(cos(x)*sec(6*x),x, algorithm="giac")
```

output
```
1/24*(sqrt(6) - sqrt(2))*log(abs(1/4*sqrt(6) + 1/4*sqrt(2) + sin(x))) + 1/
24*(sqrt(6) + sqrt(2))*log(abs(1/4*sqrt(6) - 1/4*sqrt(2) + sin(x))) - 1/24
*(sqrt(6) + sqrt(2))*log(abs(-1/4*sqrt(6) + 1/4*sqrt(2) + sin(x))) - 1/24*
(sqrt(6) - sqrt(2))*log(abs(-1/4*sqrt(6) - 1/4*sqrt(2) + sin(x))) + 1/12*s
qrt(2)*log(abs(-2*sqrt(2) + 4*sin(x))/abs(2*sqrt(2) + 4*sin(x)))
```

3.120.9 Mupad [B] (verification not implemented)

Time = 26.14 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.39

$$\int \cos(x)\sec(6x)\,dx = \operatorname{atanh}\left(\frac{5\sqrt{2}\sin(x)}{2097152\left(\frac{\sqrt{2}\sqrt{6}}{4194304}+\frac{1}{1048576}\right)} + \frac{3\sqrt{6}\sin(x)}{2097152\left(\frac{\sqrt{2}\sqrt{6}}{4194304}+\frac{1}{1048576}\right)}\right)\left(\frac{\sqrt{2}}{12}+\frac{\sqrt{6}}{12}\right) - \operatorname{atanh}\left(\frac{5\sqrt{2}\sin(x)}{2097152\left(\frac{\sqrt{2}\sqrt{6}}{4194304}-\frac{1}{1048576}\right)} - \frac{3\sqrt{6}\sin(x)}{2097152\left(\frac{\sqrt{2}\sqrt{6}}{4194304}-\frac{1}{1048576}\right)}\right)\left(\frac{\sqrt{2}}{12}-\frac{\sqrt{6}}{12}\right) - \frac{\sqrt{2}\operatorname{atanh}\left(\sqrt{2}\sin(x)\right)}{6}$$

input
```
int(cos(x)/cos(6*x),x)
```

output
```
atanh((5*2^(1/2)*sin(x))/(2097152*((2^(1/2)*6^(1/2))/4194304 + 1/1048576))
 + (3*6^(1/2)*sin(x))/(2097152*((2^(1/2)*6^(1/2))/4194304 + 1/1048576)))*(
2^(1/2)/12 + 6^(1/2)/12) - atanh((5*2^(1/2)*sin(x))/(2097152*((2^(1/2)*6^(
1/2))/4194304 - 1/1048576)) - (3*6^(1/2)*sin(x))/(2097152*((2^(1/2)*6^(1/2
))/4194304 - 1/1048576)))*(2^(1/2)/12 - 6^(1/2)/12) - (2^(1/2)*atanh(2^(1/
2)*sin(x)))/6
```

3.121 $\int \cos(2x)\sec(x)\,dx$

3.121.1 Optimal result

Integrand size = 7, antiderivative size = 10

$$\int \cos(2x)\sec(x)\,dx = -\operatorname{arctanh}(\sin(x)) + 2\sin(x)$$

output
```
-arctanh(sin(x))+2*sin(x)
```

3.121.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \cos(2x)\sec(x)\,dx = -\operatorname{arctanh}(\sin(x)) + 2\sin(x)$$

input
```
Integrate[Cos[2*x]*Sec[x],x]
```

output
```
-ArcTanh[Sin[x]] + 2*Sin[x]
```

3.121.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3042, 4864, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(2x)\sec(x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(2x)}{\cos(x)}dx$$

$$\Big\downarrow \text{ 4864}$$

$$\int \frac{1-2\sin^2(x)}{1-\sin^2(x)}d\sin(x)$$

$$\Big\downarrow \text{ 299}$$

$$2\sin(x)-\int \frac{1}{1-\sin^2(x)}d\sin(x)$$

$$\Big\downarrow \text{ 219}$$

$$2\sin(x)-\text{arctanh}(\sin(x))$$

input
```
Int[Cos[2*x]*Sec[x],x]
```

output
```
-ArcTanh[Sin[x]] + 2*Sin[x]
```

3.121.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4864
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[(1 - d^2*x
^2)^((n - 1)/2), Sin[c*(a + b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x]
/; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

3.121.4 Maple [A] (verified)

Time = 1.61 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.40

method	result	size
default	$-\ln \left(\sec \left(x \right)+\tan \left(x \right)\right)+2 \sin \left(x \right)$	14
risch	$-i {\mathrm e}^{i x}+i {\mathrm e}^{-i x}+\ln \left({\mathrm e}^{i x}-i\right)-\ln \left(i+{\mathrm e}^{i x}\right)$	38

input
```
int(cos(2*x)*sec(x),x,method=_RETURNVERBOSE)
```

output
```
-ln(sec(x)+tan(x))+2*sin(x)
```

3.121.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(10) = 20$.

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 2.10

$$\int \cos(2x)\sec(x)\,dx = -\frac{1}{2}\,\log\left(\sin\left(x\right)+1\right)+\frac{1}{2}\,\log\left(-\sin\left(x\right)+1\right)+2\,\sin\left(x\right)$$

input
```
integrate(cos(2*x)*sec(x),x, algorithm="fricas")
```

output
```
-1/2*log(sin(x) + 1) + 1/2*log(-sin(x) + 1) + 2*sin(x)
```

3.121.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(8) = 16$.

Time = 0.49 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.00

$$\int \cos(2x)\sec(x)\,dx = \frac{\log(\sin(x) - 1)}{2} - \frac{\log(\sin(x) + 1)}{2} + 2\sin(x)$$

input
```
integrate(cos(2*x)*sec(x),x)
```

output
```
log(sin(x) - 1)/2 - log(sin(x) + 1)/2 + 2*sin(x)
```

3.121.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.90

$$\int \cos(2x)\sec(x)\,dx = -\frac{1}{2}\log(\sin(x) + 1) + \frac{1}{2}\log(\sin(x) - 1) + 2\sin(x)$$

input
```
integrate(cos(2*x)*sec(x),x, algorithm="maxima")
```

output
```
-1/2*log(sin(x) + 1) + 1/2*log(sin(x) - 1) + 2*sin(x)
```

3.121.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(10) = 20$.

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 2.10

$$\int \cos(2x)\sec(x)\,dx = -\frac{1}{2}\log(\sin(x) + 1) + \frac{1}{2}\log(-\sin(x) + 1) + 2\sin(x)$$

input
```
integrate(cos(2*x)*sec(x),x, algorithm="giac")
```

output
```
-1/2*log(sin(x) + 1) + 1/2*log(-sin(x) + 1) + 2*sin(x)
```

3.121.9 Mupad [B] (verification not implemented)

Time = 26.92 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \cos(2x)\sec(x)\,dx = 2\,\sin(x) - \operatorname{atanh}(\sin(x))$$

input
```
int(cos(2*x)/cos(x),x)
```

output
```
2*sin(x) - atanh(sin(x))
```

3.122 $\int \cos(4x)\sec(2x)\,dx$

3.122.1 Optimal result

Integrand size = 9, antiderivative size = 14

$$\int \cos(4x)\sec(2x)\,dx = -\frac{1}{2}\text{arctanh}(\sin(2x)) + \sin(2x)$$

output
```
-1/2*arctanh(sin(2*x))+sin(2*x)
```

3.122.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \cos(4x)\sec(2x)\,dx = -\frac{1}{2}\text{arctanh}(\sin(2x)) + \sin(2x)$$

input
```
Integrate[Cos[4*x]*Sec[2*x],x]
```

output
```
-1/2*ArcTanh[Sin[2*x]] + Sin[2*x]
```

3.122.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3042, 4864, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(4x)\sec(2x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(4x)}{\cos(2x)}dx$$

$$\Big\downarrow \text{4864}$$

$$\frac{1}{2}\int \frac{1-2\sin^2(2x)}{1-\sin^2(2x)}d\sin(2x)$$

$$\Big\downarrow \text{299}$$

$$\frac{1}{2}\left(2\sin(2x)-\int \frac{1}{1-\sin^2(2x)}d\sin(2x)\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}(2\sin(2x)-\text{arctanh}(\sin(2x)))$$

input
```
Int[Cos[4*x]*Sec[2*x],x]
```

output
```
(-ArcTanh[Sin[2*x]] + 2*Sin[2*x])/2
```

3.122.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4864
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[(1 - d^2*x
^2)^((n - 1)/2), Sin[c*(a + b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x]
/; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

3.122.4 Maple [A] (verified)

Time = 1.67 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

method	result	size
default	$-\frac{\ln(\sec(2x)+\tan(2x))}{2}+\sin(2x)$	18
risch	$-\frac{i\,\mathrm{e}^{2ix}}{2}+\frac{i\,\mathrm{e}^{-2ix}}{2}-\frac{\ln(i+\mathrm{e}^{2ix})}{2}+\frac{\ln(\mathrm{e}^{2ix}-i)}{2}$	40

input
```
int(cos(4*x)*sec(2*x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(sec(2*x)+tan(2*x))+sin(2*x)
```

3.122.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(12) = 24.

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.79

$$\int \cos(4x)\sec(2x)\,dx = -\frac{1}{4}\,\log\left(\sin\left(2\,x\right)+1\right)+\frac{1}{4}\,\log\left(-\sin\left(2\,x\right)+1\right)+\sin\left(2\,x\right)$$

input
```
integrate(cos(4*x)*sec(2*x),x, algorithm="fricas")
```

output
```
-1/4*log(sin(2*x) + 1) + 1/4*log(-sin(2*x) + 1) + sin(2*x)
```

3.122.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(12) = 24$.

Time = 1.95 (sec) , antiderivative size = 427, normalized size of antiderivative = 30.50

$$\int \cos(4x)\sec(2x)\,dx = -4x + \frac{32x\tan^4\left(\frac{x}{2}\right)}{8\tan^4\left(\frac{x}{2}\right)+16\tan^2\left(\frac{x}{2}\right)+8} + \frac{64x\tan^2\left(\frac{x}{2}\right)}{8\tan^4\left(\frac{x}{2}\right)+16\tan^2\left(\frac{x}{2}\right)+8} + \frac{32x}{8\tan^4\left(\frac{x}{2}\right)+16\tan^2\left(\frac{x}{2}\right)+8} - \frac{3\log\left(\tan^2\left(\frac{x}{2}\right)-2\tan\left(\frac{x}{2}\right)-1\right)}{2} + \frac{3\log\left(\tan^2\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)-1\right)}{2} + \frac{8\log\left(\tan^2\left(\frac{x}{2}\right)-2\tan\left(\frac{x}{2}\right)-1\right)\tan^4\left(\frac{x}{2}\right)}{8\tan^4\left(\frac{x}{2}\right)+16\tan^2\left(\frac{x}{2}\right)+8} + \frac{16\log\left(\tan^2\left(\frac{x}{2}\right)-2\tan\left(\frac{x}{2}\right)-1\right)\tan^2\left(\frac{x}{2}\right)}{8\tan^4\left(\frac{x}{2}\right)+16\tan^2\left(\frac{x}{2}\right)+8} + \frac{8\log\left(\tan^2\left(\frac{x}{2}\right)-2\tan\left(\frac{x}{2}\right)-1\right)}{8\tan^4\left(\frac{x}{2}\right)+16\tan^2\left(\frac{x}{2}\right)+8} - \frac{8\log\left(\tan^2\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)-1\right)\tan^4\left(\frac{x}{2}\right)}{8\tan^4\left(\frac{x}{2}\right)+16\tan^2\left(\frac{x}{2}\right)+8} - \frac{16\log\left(\tan^2\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)-1\right)\tan^2\left(\frac{x}{2}\right)}{8\tan^4\left(\frac{x}{2}\right)+16\tan^2\left(\frac{x}{2}\right)+8} - \frac{8\log\left(\tan^2\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)-1\right)}{8\tan^4\left(\frac{x}{2}\right)+16\tan^2\left(\frac{x}{2}\right)+8} - \frac{32\tan^3\left(\frac{x}{2}\right)}{8\tan^4\left(\frac{x}{2}\right)+16\tan^2\left(\frac{x}{2}\right)+8} + \frac{32\tan\left(\frac{x}{2}\right)}{8\tan^4\left(\frac{x}{2}\right)+16\tan^2\left(\frac{x}{2}\right)+8}$$

input
```
integrate(cos(4*x)*sec(2*x),x)
```

output
```
-4*x + 32*x*tan(x/2)**4/(8*tan(x/2)**4 + 16*tan(x/2)**2 + 8) + 64*x*tan(x/
2)**2/(8*tan(x/2)**4 + 16*tan(x/2)**2 + 8) + 32*x/(8*tan(x/2)**4 + 16*tan(
x/2)**2 + 8) - 3*log(tan(x/2)**2 - 2*tan(x/2) - 1)/2 + 3*log(tan(x/2)**2 +
 2*tan(x/2) - 1)/2 + 8*log(tan(x/2)**2 - 2*tan(x/2) - 1)*tan(x/2)**4/(8*ta
n(x/2)**4 + 16*tan(x/2)**2 + 8) + 16*log(tan(x/2)**2 - 2*tan(x/2) - 1)*tan
(x/2)**2/(8*tan(x/2)**4 + 16*tan(x/2)**2 + 8) + 8*log(tan(x/2)**2 - 2*tan(
x/2) - 1)/(8*tan(x/2)**4 + 16*tan(x/2)**2 + 8) - 8*log(tan(x/2)**2 + 2*tan
(x/2) - 1)*tan(x/2)**4/(8*tan(x/2)**4 + 16*tan(x/2)**2 + 8) - 16*log(tan(x
/2)**2 + 2*tan(x/2) - 1)*tan(x/2)**2/(8*tan(x/2)**4 + 16*tan(x/2)**2 + 8)
- 8*log(tan(x/2)**2 + 2*tan(x/2) - 1)/(8*tan(x/2)**4 + 16*tan(x/2)**2 + 8)
 - 32*tan(x/2)**3/(8*tan(x/2)**4 + 16*tan(x/2)**2 + 8) + 32*tan(x/2)/(8*ta
n(x/2)**4 + 16*tan(x/2)**2 + 8)
```

3.122.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(12) = 24$.

Time = 0.29 (sec) , antiderivative size = 129, normalized size of antiderivative = 9.21

$$\begin{aligned}\int \cos(4x)\sec(2x)\,dx = {} & \frac{1}{4}\log\left(2\cos(x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) + 2\sqrt{2}\sin(x) + 2\right) \\ & - \frac{1}{4}\log\left(2\cos(x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) - 2\sqrt{2}\sin(x) + 2\right) \\ & - \frac{1}{4}\log\left(2\cos(x)^2 + 2\sin(x)^2 - 2\sqrt{2}\cos(x) + 2\sqrt{2}\sin(x) + 2\right) \\ & + \frac{1}{4}\log\left(2\cos(x)^2 + 2\sin(x)^2 - 2\sqrt{2}\cos(x) - 2\sqrt{2}\sin(x) + 2\right) \\ & + \sin(2x)\end{aligned}$$

input
```
integrate(cos(4*x)*sec(2*x),x, algorithm="maxima")
```

output
```
1/4*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*sqrt(2)*sin(x) + 2)
 - 1/4*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) - 2*sqrt(2)*sin(x) +
 2) - 1/4*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos(x) + 2*sqrt(2)*sin(x
) + 2) + 1/4*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos(x) - 2*sqrt(2)*si
n(x) + 2) + sin(2*x)
```

3.122.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(12) = 24$.

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.79

$$\int \cos(4x)\sec(2x)\,dx = -\frac{1}{4}\log\left(\sin\left(2\,x\right)+1\right)+\frac{1}{4}\log\left(-\sin\left(2\,x\right)+1\right)+\sin\left(2\,x\right)$$

input
```
integrate(cos(4*x)*sec(2*x),x, algorithm="giac")
```

output
```
-1/4*log(sin(2*x) + 1) + 1/4*log(-sin(2*x) + 1) + sin(2*x)
```

3.122.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \cos(4x)\sec(2x)\,dx = \sin\left(2\,x\right)-\frac{\mathrm{atanh}(\sin\left(2\,x\right))}{2}$$

input
```
int(cos(4*x)/cos(2*x),x)
```

output
```
sin(2*x) - atanh(sin(2*x))/2
```

3.123 $\int \cos(x)\csc(2x)\,dx$

3.123.1 Optimal result

Integrand size = 7, antiderivative size = 7

$$\int \cos(x)\csc(2x)\,dx = -\frac{1}{2}\operatorname{arctanh}(\cos(x))$$

output
```
-1/2*arctanh(cos(x))
```

3.123.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 21 vs. 2(7) = 14.

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 3.00

$$\int \cos(x)\csc(2x)\,dx = \frac{1}{2}\left(-\log\left(\cos\left(\frac{x}{2}\right)\right)+\log\left(\sin\left(\frac{x}{2}\right)\right)\right)$$

input
```
Integrate[Cos[x]*Csc[2*x],x]
```

output
```
(-Log[Cos[x/2]] + Log[Sin[x/2]])/2
```

3.123.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 4775, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\csc(2x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(x)}{\sin(2x)}dx$$

$$\Big\downarrow \text{4775}$$

$$\frac{\int \csc(x)dx}{2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc(x)dx}{2}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{1}{2}\text{arctanh}(\cos(x))$$

input
```
Int[Cos[x]*Csc[2*x],x]
```

output
```
-1/2*ArcTanh[Cos[x]]
```

3.123.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4775
```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Simp[2^p/e^p   Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x],
x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && I
ntegerQ[p]
```

3.123.4 Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

method	result	size
default	$-\frac{\ln(\cot(x)+\csc(x))}{2}$	9
risch	$-\frac{\ln(e^{ix}+1)}{2}+\frac{\ln(e^{ix}-1)}{2}$	22

input
```
int(cos(x)*csc(2*x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(cot(x)+csc(x))
```

3.123.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(5) = 10.

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 2.71

$$\int \cos(x)\csc(2x)\,dx = -\frac{1}{4}\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+\frac{1}{4}\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)$$

input
```
integrate(cos(x)*csc(2*x),x, algorithm="fricas")
```

output
```
-1/4*log(1/2*cos(x) + 1/2) + 1/4*log(-1/2*cos(x) + 1/2)
```

3.123.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. $2(7) = 14$.

Time = 0.43 (sec) , antiderivative size = 15, normalized size of antiderivative = 2.14

$$\int \cos(x)\csc(2x)\,dx = \frac{\log(\cos(x)-1)}{4} - \frac{\log(\cos(x)+1)}{4}$$

input
```
integrate(cos(x)*csc(2*x),x)
```

output
```
log(cos(x) - 1)/4 - log(cos(x) + 1)/4
```

3.123.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(5) = 10$.

Time = 0.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 5.00

$$\int \cos(x)\csc(2x)\,dx = -\frac{1}{4}\log\left(\cos(x)^2+\sin(x)^2+2\cos(x)+1\right) + \frac{1}{4}\log\left(\cos(x)^2+\sin(x)^2-2\cos(x)+1\right)$$

input
```
integrate(cos(x)*csc(2*x),x, algorithm="maxima")
```

output
```
-1/4*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/4*log(cos(x)^2 + sin(x)^2
 - 2*cos(x) + 1)
```

3.123.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(5) = 10$.

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.43

$$\int \cos(x)\csc(2x)\,dx = -\frac{1}{4}\log(\cos(x)+1) + \frac{1}{4}\log(-\cos(x)+1)$$

input
```
integrate(cos(x)*csc(2*x),x, algorithm="giac")
```

output
```
-1/4*log(cos(x) + 1) + 1/4*log(-cos(x) + 1)
```

3.123.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \cos(x)\csc(2x)\,dx = -\frac{\operatorname{atanh}(\cos(x))}{2}$$

input
```
int(cos(x)/sin(2*x),x)
```

output
```
-atanh(cos(x))/2
```

3.124 $\int \cos(x)\csc(3x)\,dx$

3.124.1 Optimal result

Integrand size = 7, antiderivative size = 21

$$\int \cos(x)\csc(3x)\,dx = \frac{1}{3}\log(\sin(x)) - \frac{1}{6}\log\left(3-4\sin^2(x)\right)$$

output
```
1/3*ln(sin(x))-1/6*ln(3-4*sin(x)^2)
```

3.124.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \cos(x)\csc(3x)\,dx = \frac{1}{3}\log(\sin(x)) - \frac{1}{6}\log\left(3-4\sin^2(x)\right)$$

input
```
Integrate[Cos[x]*Csc[3*x],x]
```

output
```
Log[Sin[x]]/3 - Log[3 - 4*Sin[x]^2]/6
```

3.124.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {3042, 4856, 243, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\csc(3x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(x)}{\sin(3x)}dx$$

$$\Big\downarrow \text{ 4856}$$

$$\int \frac{\csc(x)}{3-4\sin^2(x)}d\sin(x)$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\int \frac{\csc(x)}{3-4\sin^2(x)}d\sin^2(x)$$

$$\Big\downarrow \text{ 47}$$

$$\frac{1}{2}\left(\frac{4}{3}\int \frac{1}{3-4\sin^2(x)}d\sin^2(x)+\frac{1}{3}\int \csc(x)d\sin^2(x)\right)$$

$$\Big\downarrow \text{ 14}$$

$$\frac{1}{2}\left(\frac{4}{3}\int \frac{1}{3-4\sin^2(x)}d\sin^2(x)+\frac{1}{3}\log\left(\sin^2(x)\right)\right)$$

$$\Big\downarrow \text{ 16}$$

$$\frac{1}{2}\left(\frac{1}{3}\log\left(\sin^2(x)\right)-\frac{1}{3}\log\left(3-4\sin^2(x)\right)\right)$$

input
```
Int[Cos[x]*Csc[3*x],x]
```

output
```
(Log[Sin[x]^2]/3 - Log[3 - 4*Sin[x]^2]/3)/2
```

3.124.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4856
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

3.124.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.68 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

method	result	size
risch	$\frac{\ln\left(e^{2ix}-1\right)}{3}-\frac{\ln\left(e^{4ix}+e^{2ix}+1\right)}{6}$	27
default	$-\frac{\ln(1+2\cos(x))}{6}-\frac{\ln(2\cos(x)-1)}{6}+\frac{\ln(\cos(x)-1)}{6}+\frac{\ln(\cos(x)+1)}{6}$	34

input
```
int(cos(x)*csc(3*x),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(exp(2*I*x)-1)-1/6*ln(exp(4*I*x)+exp(2*I*x)+1)
```

3.124.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \cos(x)\csc(3x)\,dx = -\frac{1}{6}\log\left(4\cos\left(x\right)^2-1\right)+\frac{1}{3}\log\left(\frac{1}{2}\sin\left(x\right)\right)$$

input
```
integrate(cos(x)*csc(3*x),x, algorithm="fricas")
```

output
```
-1/6*log(4*cos(x)^2 - 1) + 1/3*log(1/2*sin(x))
```

3.124.6 Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \cos(x)\csc(3x)\,dx = -\frac{\log\left(4\sin^2\left(x\right)-3\right)}{6}+\frac{\log\left(\sin\left(x\right)\right)}{3}$$

input
```
integrate(cos(x)*csc(3*x),x)
```

output
```
-log(4*sin(x)**2 - 3)/6 + log(sin(x))/3
```

3.124.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(17) = 34.

Time = 0.29 (sec) , antiderivative size = 129, normalized size of antiderivative = 6.14

$$\begin{aligned}\int \cos(x)\csc(3x)\,dx = {} & -\frac{1}{12}\log\left(2\left(\cos\left(x\right)+1\right)\cos\left(2x\right)+\cos\left(2x\right)^2+\cos\left(x\right)^2+\sin\left(2x\right)^2\right.\\ & \left.+2\sin\left(2x\right)\sin\left(x\right)+\sin\left(x\right)^2+2\cos\left(x\right)+1\right)\\ & -\frac{1}{12}\log\left(-2\left(\cos\left(x\right)-1\right)\cos\left(2x\right)+\cos\left(2x\right)^2+\cos\left(x\right)^2+\sin\left(2x\right)^2\right.\\ & \left.-2\sin\left(2x\right)\sin\left(x\right)+\sin\left(x\right)^2-2\cos\left(x\right)+1\right)\\ & +\frac{1}{6}\log\left(\cos\left(x\right)^2+\sin\left(x\right)^2+2\cos\left(x\right)+1\right)\\ & +\frac{1}{6}\log\left(\cos\left(x\right)^2+\sin\left(x\right)^2-2\cos\left(x\right)+1\right)\end{aligned}$$

input
```
integrate(cos(x)*csc(3*x),x, algorithm="maxima")
```

output
```
-1/12*log(2*(cos(x) + 1)*cos(2*x) + cos(2*x)^2 + cos(x)^2 + sin(2*x)^2 + 2
*sin(2*x)*sin(x) + sin(x)^2 + 2*cos(x) + 1) - 1/12*log(-2*(cos(x) - 1)*cos
(2*x) + cos(2*x)^2 + cos(x)^2 + sin(2*x)^2 - 2*sin(2*x)*sin(x) + sin(x)^2
- 2*cos(x) + 1) + 1/6*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/6*log(co
s(x)^2 + sin(x)^2 - 2*cos(x) + 1)
```

3.124.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int \cos(x)\csc(3x)\,dx = \frac{1}{6}\log\left(-\cos(x)^2 + 1\right) - \frac{1}{6}\log\left(\left|4\cos(x)^2 - 1\right|\right)$$

input
```
integrate(cos(x)*csc(3*x),x, algorithm="giac")
```

output
```
1/6*log(-cos(x)^2 + 1) - 1/6*log(abs(4*cos(x)^2 - 1))
```

3.124.9 Mupad [B] (verification not implemented)

Time = 26.32 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \cos(x)\csc(3x)\,dx = \frac{\ln(\sin(x))}{3} - \frac{\ln\left(\frac{1}{4} - \cos(x)^2\right)}{6}$$

input
```
int(cos(x)/sin(3*x),x)
```

output
```
log(sin(x))/3 - log(1/4 - cos(x)^2)/6
```

3.125 $\int \cos(x)\csc(4x)\,dx$

3.125.1 Optimal result

Integrand size = 7, antiderivative size = 26

$$\int \cos(x)\csc(4x)\,dx=-\frac{1}{4}\text{arctanh}(\cos(x))+\frac{\text{arctanh}\left(\sqrt{2}\cos(x)\right)}{2\sqrt{2}}$$

output
```
-1/4*arctanh(cos(x))+1/4*arctanh(cos(x)*2^(1/2))*2^(1/2)
```

3.125.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.54

$$\int \cos(x)\csc(4x)\,dx=\frac{1}{4}\left((1+i)(-1)^{3/4}\text{arctanh}\left(\frac{-1+\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right)+\sqrt{2}\text{arctanh}\left(\frac{1+\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right)-\log\left(\cos\left(\frac{x}{2}\right)\right)+\log\left(\sin\left(\frac{x}{2}\right)\right)\right)$$

input
```
Integrate[Cos[x]*Csc[4*x],x]
```

output
```
((1 + I)*(-1)^(3/4)*ArcTanh[(-1 + Tan[x/2])/Sqrt[2]] + Sqrt[2]*ArcTanh[(1
+ Tan[x/2])/Sqrt[2]] - Log[Cos[x/2]] + Log[Sin[x/2]])/4
```

3.125.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3042, 4879, 1406, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\csc(4x)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\cos(x)}{\sin(4x)}dx$$

$$\Big\downarrow \; 4879$$

$$-\int \frac{1}{-8\cos^4(x)+12\cos^2(x)-4}d\cos(x)$$

$$\Big\downarrow \; 1406$$

$$2\int \frac{1}{4-8\cos^2(x)}d\cos(x)-2\int \frac{1}{8-8\cos^2(x)}d\cos(x)$$

$$\Big\downarrow \; 219$$

$$\frac{\text{arctanh}\left(\sqrt{2}\cos(x)\right)}{2\sqrt{2}}-\frac{1}{4}\text{arctanh}(\cos(x))$$

input
```
Int[Cos[x]*Csc[4*x],x]
```

output
```
-1/4*ArcTanh[Cos[x]] + ArcTanh[Sqrt[2]*Cos[x]]/(2*Sqrt[2])
```

3.125.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4879
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cos[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Cos[v]/d
, u/Sin[v], x], x], x, Cos[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[Nonfree
Factors[Cos[v], x], u/Sin[v], x]]
```

3.125.4 Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

method	result	size
default	$\frac{\operatorname{arctanh}\left(\cos(x)\sqrt{2}\right)\sqrt{2}}{4}-\frac{\ln(\cos(x)+1)}{8}+\frac{\ln(\cos(x)-1)}{8}$	28
risch	$\frac{\ln(e^{ix}-1)}{4}-\frac{\ln(e^{ix}+1)}{4}-\frac{\sqrt{2}\,\ln\left(e^{2ix}-\sqrt{2}\,e^{ix}+1\right)}{8}+\frac{\sqrt{2}\,\ln\left(e^{2ix}+\sqrt{2}\,e^{ix}+1\right)}{8}$	67

input
```
int(cos(x)*csc(4*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*arctanh(cos(x)*2^(1/2))*2^(1/2)-1/8*ln(cos(x)+1)+1/8*ln(cos(x)-1)
```

3.125.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. 2(18) = 36.

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.00

$$\int \cos(x)\csc(4x)\,dx = \frac{1}{8}\sqrt{2}\log\left(-\frac{2\cos(x)^2+2\sqrt{2}\cos(x)+1}{2\cos(x)^2-1}\right) - \frac{1}{8}\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+\frac{1}{8}\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)$$

input
```
integrate(cos(x)*csc(4*x),x, algorithm="fricas")
```

output
```
1/8*sqrt(2)*log(-(2*cos(x)^2 + 2*sqrt(2)*cos(x) + 1)/(2*cos(x)^2 - 1)) - 1
/8*log(1/2*cos(x) + 1/2) + 1/8*log(-1/2*cos(x) + 1/2)
```

3.125.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(22) = 44$.

Time = 2.84 (sec) , antiderivative size = 248, normalized size of antiderivative = 9.54

$$\int \cos(x)\csc(4x)\,dx = -\frac{19601\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)-1+\sqrt{2}\right)}{110880\sqrt{2}+156808} - \frac{27720\log\left(\tan\left(\frac{x}{2}\right)-1+\sqrt{2}\right)}{110880\sqrt{2}+156808} + \frac{27720\log\left(\tan\left(\frac{x}{2}\right)+1+\sqrt{2}\right)}{110880\sqrt{2}+156808} + \frac{19601\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)+1+\sqrt{2}\right)}{110880\sqrt{2}+156808} + \frac{27720\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{2}-1\right)}{110880\sqrt{2}+156808} + \frac{19601\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{2}-1\right)}{110880\sqrt{2}+156808} - \frac{19601\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{2}+1\right)}{110880\sqrt{2}+156808} - \frac{27720\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{2}+1\right)}{110880\sqrt{2}+156808} + \frac{27720\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)\right)}{110880\sqrt{2}+156808} + \frac{39202\log\left(\tan\left(\frac{x}{2}\right)\right)}{110880\sqrt{2}+156808}$$

input
```
integrate(cos(x)*csc(4*x),x)
```

output
```
-19601*sqrt(2)*log(tan(x/2) - 1 + sqrt(2))/(110880*sqrt(2) + 156808) - 277
20*log(tan(x/2) - 1 + sqrt(2))/(110880*sqrt(2) + 156808) + 27720*log(tan(x
/2) + 1 + sqrt(2))/(110880*sqrt(2) + 156808) + 19601*sqrt(2)*log(tan(x/2)
+ 1 + sqrt(2))/(110880*sqrt(2) + 156808) + 27720*log(tan(x/2) - sqrt(2) -
1)/(110880*sqrt(2) + 156808) + 19601*sqrt(2)*log(tan(x/2) - sqrt(2) - 1)/(
110880*sqrt(2) + 156808) - 19601*sqrt(2)*log(tan(x/2) - sqrt(2) + 1)/(1108
80*sqrt(2) + 156808) - 27720*log(tan(x/2) - sqrt(2) + 1)/(110880*sqrt(2) +
 156808) + 27720*sqrt(2)*log(tan(x/2))/(110880*sqrt(2) + 156808) + 39202*l
og(tan(x/2))/(110880*sqrt(2) + 156808)
```

3.125.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(18) = 36$.

Time = 0.32 (sec) , antiderivative size = 163, normalized size of antiderivative = 6.27

$$\int \cos(x)\csc(4x)\,dx = \frac{1}{16}\sqrt{2}\log\left(2\sqrt{2}\sin(2x)\sin(x) + 2\left(\sqrt{2}\cos(x)+1\right)\cos(2x) + \cos(2x)^2 + 2\cos(x)^2 + \sin(2x)^2 + 2\sin(x)^2 + 2\sqrt{2}\cos(x) + 1\right) - \frac{1}{16}\sqrt{2}\log\left(-2\sqrt{2}\sin(2x)\sin(x) - 2\left(\sqrt{2}\cos(x)-1\right)\cos(2x) + \cos(2x)^2 + 2\cos(x)^2 + \sin(2x)^2 + 2\sin(x)^2 - 2\sqrt{2}\cos(x) + 1\right) - \frac{1}{8}\log\left(\cos(x)^2 + \sin(x)^2 + 2\cos(x) + 1\right) + \frac{1}{8}\log\left(\cos(x)^2 + \sin(x)^2 - 2\cos(x) + 1\right)$$

input
```
integrate(cos(x)*csc(4*x),x, algorithm="maxima")
```

output
```
1/16*sqrt(2)*log(2*sqrt(2)*sin(2*x)*sin(x) + 2*(sqrt(2)*cos(x) + 1)*cos(2*
x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x)
+ 1) - 1/16*sqrt(2)*log(-2*sqrt(2)*sin(2*x)*sin(x) - 2*(sqrt(2)*cos(x) - 1
)*cos(2*x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 - 2*sqrt(2)
*cos(x) + 1) - 1/8*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/8*log(cos(x
)^2 + sin(x)^2 - 2*cos(x) + 1)
```

3.125.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(18) = 36$.

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.85

$$\int \cos(x)\csc(4x)\,dx = -\frac{1}{8}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\cos(x)\right|}{\left|2\sqrt{2}+4\cos(x)\right|}\right) - \frac{1}{8}\log(\cos(x)+1) + \frac{1}{8}\log(-\cos(x)+1)$$

input
```
integrate(cos(x)*csc(4*x),x, algorithm="giac")
```

output
```
-1/8*sqrt(2)*log(abs(-2*sqrt(2) + 4*cos(x))/abs(2*sqrt(2) + 4*cos(x))) - 1
/8*log(cos(x) + 1) + 1/8*log(-cos(x) + 1)
```

3.125.9 Mupad [B] (verification not implemented)

Time = 26.56 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.12

$$\int \cos(x)\csc(4x)\,dx = \frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{4} + \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{41\sqrt{2}}{8\left(\frac{169\tan\left(\frac{x}{2}\right)^2}{4}-\frac{29}{4}\right)} - \frac{239\sqrt{2}\tan\left(\frac{x}{2}\right)^2}{8\left(\frac{169\tan\left(\frac{x}{2}\right)^2}{4}-\frac{29}{4}\right)}\right)}{4}$$

input
```
int(cos(x)/sin(4*x),x)
```

output
```
log(tan(x/2))/4 + (2^(1/2)*atanh((41*2^(1/2))/(8*((169*tan(x/2)^2)/4 - 29/
4)) - (239*2^(1/2)*tan(x/2)^2)/(8*((169*tan(x/2)^2)/4 - 29/4))))/4
```

3.126 $\int \cos(x)\csc(5x)\,dx$

3.126.1 Optimal result

Integrand size = 7, antiderivative size = 62

$$\int \cos(x)\csc(5x)\,dx = \frac{1}{5}\log(\sin(x)) - \frac{1}{20}\left(1+\sqrt{5}\right)\log\left(5-\sqrt{5}-8\sin^2(x)\right) - \frac{1}{20}\left(1-\sqrt{5}\right)\log\left(5+\sqrt{5}-8\sin^2(x)\right)$$

output
```
1/5*ln(sin(x))-1/20*ln(5-8*sin(x)^2+5^(1/2))*(-5^(1/2)+1)-1/20*ln(5-8*sin(
x)^2-5^(1/2))*(5^(1/2)+1)
```

3.126.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

$$\int \cos(x)\csc(5x)\,dx = \frac{1}{20}\left(-\left(\left(1+\sqrt{5}\right)\log\left(1-\sqrt{5}+4\cos(2x)\right)\right) + \left(-1+\sqrt{5}\right)\log\left(1+\sqrt{5}+4\cos(2x)\right) + 4\log(\sin(x))\right)$$

input
```
Integrate[Cos[x]*Csc[5*x],x]
```

output
```
(-((1 + Sqrt[5])*Log[1 - Sqrt[5] + 4*Cos[2*x]]) + (-1 + Sqrt[5])*Log[1 + S
qrt[5] + 4*Cos[2*x]] + 4*Log[Sin[x]])/20
```

3.126.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.29, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3042, 4856, 1434, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\csc(5x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(x)}{\sin(5x)}dx$$

$$\Big\downarrow \text{4856}$$

$$\int \frac{\csc(x)}{16\sin^4(x)-20\sin^2(x)+5}d\sin(x)$$

$$\Big\downarrow \text{1434}$$

$$\frac{1}{2}\int \frac{\csc(x)}{16\sin^4(x)-20\sin^2(x)+5}d\sin^2(x)$$

$$\Big\downarrow \text{1141}$$

$$8\int \left(\frac{\csc(x)}{80}+\frac{1}{\sqrt{5}\left(5-\sqrt{5}\right)\left(-8\sin^2(x)-\sqrt{5}+5\right)}-\frac{1}{\sqrt{5}\left(5+\sqrt{5}\right)\left(-8\sin^2(x)+\sqrt{5}+5\right)}\right)d\sin^2(x)$$

$$\Big\downarrow \text{2009}$$

$$8\left(\frac{1}{80}\log\left(\sin^2(x)\right)-\frac{\log\left(-8\sin^2(x)-\sqrt{5}+5\right)}{8\sqrt{5}\left(5-\sqrt{5}\right)}+\frac{\log\left(-8\sin^2(x)+\sqrt{5}+5\right)}{8\sqrt{5}\left(5+\sqrt{5}\right)}\right)$$

input
```
Int[Cos[x]*Csc[5*x],x]
```

output
```
8*(Log[Sin[x]^2]/80 - Log[5 - Sqrt[5] - 8*Sin[x]^2]/(8*Sqrt[5]*(5 - Sqrt[5
])) + Log[5 + Sqrt[5] - 8*Sin[x]^2]/(8*Sqrt[5]*(5 + Sqrt[5])))
```

3.126.3.1 Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1434
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp
[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4856
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

3.126.4 Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.29

method	result
default	$-\frac{\ln\left(4\cos(x)^2+2\cos(x)-1\right)}{20}-\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{(8\cos(x)+2)\sqrt{5}}{10}\right)}{10}+\frac{\ln(\cos(x)-1)}{10}-\frac{\ln\left(4\cos(x)^2-2\cos(x)-1\right)}{20}+\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{(8}{10}\right.}{10}$
risch	$\frac{\ln(e^{2ix}-1)}{5}-\frac{\ln\left(e^{4ix}+\left(\frac{1}{2}-\frac{\sqrt{5}}{2}\right)e^{2ix}+1\right)}{20}-\frac{\ln\left(e^{4ix}+\left(\frac{1}{2}-\frac{\sqrt{5}}{2}\right)e^{2ix}+1\right)\sqrt{5}}{20}-\frac{\ln\left(e^{4ix}+\left(\frac{1}{2}+\frac{\sqrt{5}}{2}\right)e^{2ix}+1\right)}{20}+\frac{\ln\left(e^{4ix}+\left(\frac{1}{2}+\frac{\sqrt{2}}{}\right.\right.}{20}$

input
```
int(cos(x)*csc(5*x),x,method=_RETURNVERBOSE)
```

output
```
-1/20*ln(4*cos(x)^2+2*cos(x)-1)-1/10*5^(1/2)*arctanh(1/10*(8*cos(x)+2)*5^(
1/2))+1/10*ln(cos(x)-1)-1/20*ln(4*cos(x)^2-2*cos(x)-1)+1/10*5^(1/2)*arctan
h(1/10*(8*cos(x)-2)*5^(1/2))+1/10*ln(cos(x)+1)
```

3.126.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.16

$$\int \cos(x)\csc(5x)\,dx = \frac{1}{20}\sqrt{5}\log\left(\frac{32\cos(x)^4 + 8\left(\sqrt{5}-3\right)\cos(x)^2 - 3\sqrt{5} + 7}{16\cos(x)^4 - 12\cos(x)^2 + 1}\right) - \frac{1}{20}\log\left(16\cos(x)^4 - 12\cos(x)^2 + 1\right) + \frac{1}{5}\log\left(\frac{1}{2}\sin(x)\right)$$

input
```
integrate(cos(x)*csc(5*x),x, algorithm="fricas")
```

output
```
1/20*sqrt(5)*log((32*cos(x)^4 + 8*(sqrt(5) - 3)*cos(x)^2 - 3*sqrt(5) + 7)/
(16*cos(x)^4 - 12*cos(x)^2 + 1)) - 1/20*log(16*cos(x)^4 - 12*cos(x)^2 + 1)
 + 1/5*log(1/2*sin(x))
```

3.126.6 Sympy [F]

$$\int \cos(x)\csc(5x)\,dx = \int \cos(x)\csc(5x)\,dx$$

input
```
integrate(cos(x)*csc(5*x),x)
```

output
```
Integral(cos(x)*csc(5*x), x)
```

3.126.7 Maxima [F]

$$\int \cos(x)\csc(5x)\,dx = \int \cos(x)\csc(5\,x)\;dx$$

input
```
integrate(cos(x)*csc(5*x),x, algorithm="maxima")
```

output
```
-1/10*integrate(-(cos(2*x)*sin(4*x) - cos(4*x)*sin(2*x) + cos(3/2*arctan2(
sin(2*x), cos(2*x)))*sin(2*x) + cos(1/2*arctan2(sin(2*x), cos(2*x)))*sin(2
*x) - cos(2*x)*sin(3/2*arctan2(sin(2*x), cos(2*x))) - cos(2*x)*sin(1/2*arc
tan2(sin(2*x), cos(2*x))) - sin(2*x))/(2*(cos(2*x) + 1)*cos(4*x) + cos(4*x
)^2 + cos(2*x)^2 - 2*(cos(4*x) + cos(2*x) - cos(1/2*arctan2(sin(2*x), cos(
2*x))) + 1)*cos(3/2*arctan2(sin(2*x), cos(2*x))) + cos(3/2*arctan2(sin(2*x
), cos(2*x)))^2 - 2*(cos(4*x) + cos(2*x) + 1)*cos(1/2*arctan2(sin(2*x), co
s(2*x))) + cos(1/2*arctan2(sin(2*x), cos(2*x)))^2 + sin(4*x)^2 + 2*sin(4*x
)*sin(2*x) + sin(2*x)^2 - 2*(sin(4*x) + sin(2*x) - sin(1/2*arctan2(sin(2*x
), cos(2*x))))*sin(3/2*arctan2(sin(2*x), cos(2*x))) + sin(3/2*arctan2(sin(
2*x), cos(2*x)))^2 - 2*(sin(4*x) + sin(2*x))*sin(1/2*arctan2(sin(2*x), cos
(2*x))) + sin(1/2*arctan2(sin(2*x), cos(2*x)))^2 + 2*cos(2*x) + 1), x) + 1
/10*integrate((cos(2*x)*sin(4*x) - cos(4*x)*sin(2*x) - cos(3/2*arctan2(sin
(2*x), cos(2*x)))*sin(2*x) - cos(1/2*arctan2(sin(2*x), cos(2*x)))*sin(2*x)
 + cos(2*x)*sin(3/2*arctan2(sin(2*x), cos(2*x))) + cos(2*x)*sin(1/2*arctan
2(sin(2*x), cos(2*x))) - sin(2*x))/(2*(cos(2*x) + 1)*cos(4*x) + cos(4*x)^2
 + cos(2*x)^2 + 2*(cos(4*x) + cos(2*x) + cos(1/2*arctan2(sin(2*x), cos(2*x
))) + 1)*cos(3/2*arctan2(sin(2*x), cos(2*x))) + cos(3/2*arctan2(sin(2*x),
cos(2*x)))^2 + 2*(cos(4*x) + cos(2*x) + 1)*cos(1/2*arctan2(sin(2*x), cos(2
*x))) + cos(1/2*arctan2(sin(2*x), cos(2*x)))^2 + sin(4*x)^2 + 2*sin(4*x...
```

3.126.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.08

$$\int \cos(x)\csc(5x)\,dx = -\frac{1}{20}\sqrt{5}\log\left(\frac{\left|32\,\cos(x)^2-4\sqrt{5}-12\right|}{\left|32\,\cos(x)^2+4\sqrt{5}-12\right|}\right)+\frac{1}{10}\log\left(-\cos(x)^2+1\right) - \frac{1}{20}\log\left(\left|16\,\cos(x)^4-12\,\cos(x)^2+1\right|\right)$$

input
```
integrate(cos(x)*csc(5*x),x, algorithm="giac")
```

output
```
-1/20*sqrt(5)*log(abs(32*cos(x)^2 - 4*sqrt(5) - 12)/abs(32*cos(x)^2 + 4*sq
rt(5) - 12)) + 1/10*log(-cos(x)^2 + 1) - 1/20*log(abs(16*cos(x)^4 - 12*cos
(x)^2 + 1))
```

3.126.9 Mupad [B] (verification not implemented)

Time = 27.45 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.82

$$\int \cos(x)\csc(5x)\,dx = \frac{\ln(\sin(x))}{5} + \ln\left(-\cos(x)^2 - \frac{\sqrt{5}}{8} + \frac{3}{8}\right)\left(\frac{\sqrt{5}}{20} - \frac{1}{20}\right) - \ln\left(-\cos(x)^2 + \frac{\sqrt{5}}{8} + \frac{3}{8}\right)\left(\frac{\sqrt{5}}{20} + \frac{1}{20}\right)$$

input
```
int(cos(x)/sin(5*x),x)
```

output
```
log(sin(x))/5 + log(3/8 - 5^(1/2)/8 - cos(x)^2)*(5^(1/2)/20 - 1/20) - log(
5^(1/2)/8 - cos(x)^2 + 3/8)*(5^(1/2)/20 + 1/20)
```

3.127 $\int \cos(x)\csc(6x)\,dx$

3.127.1 Optimal result

Integrand size = 7, antiderivative size = 36

$$\int \cos(x)\csc(6x)\,dx=-\frac{1}{6}\text{arctanh}(\cos(x))-\frac{1}{6}\text{arctanh}(2\cos(x))+\frac{\text{arctanh}\left(\frac{2\cos(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

output
```
-1/6*arctanh(cos(x))-1/6*arctanh(2*cos(x))+1/6*arctanh(2/3*cos(x)*3^(1/2))
*3^(1/2)
```

3.127.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 83 vs. 2(36) = 72.

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.31

$$\int \cos(x)\csc(6x)\,dx=\frac{1}{12}\left(-2\sqrt{3}\text{arctanh}\left(\frac{-2+\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right)+2\sqrt{3}\text{arctanh}\left(\frac{2+\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right)-2\log\left(\cos\left(\frac{x}{2}\right)\right)+\log(1-2\cos(x))-\log(1+2\cos(x))+2\log\left(\sin\left(\frac{x}{2}\right)\right)\right)$$

input
```
Integrate[Cos[x]*Csc[6*x],x]
```

output
```
(-2*Sqrt[3]*ArcTanh[(-2 + Tan[x/2])/Sqrt[3]] + 2*Sqrt[3]*ArcTanh[(2 + Tan[
x/2])/Sqrt[3]] - 2*Log[Cos[x/2]] + Log[1 - 2*Cos[x]] - Log[1 + 2*Cos[x]] +
 2*Log[Sin[x/2]])/12
```

3.127.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 4879, 27, 2460, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\csc(6x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(x)}{\sin(6x)}dx$$

$$\Big\downarrow \text{4879}$$

$$-\int \frac{1}{2\left(-16\cos^6(x)+32\cos^4(x)-19\cos^2(x)+3\right)}d\cos(x)$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{2}\int \frac{1}{-16\cos^6(x)+32\cos^4(x)-19\cos^2(x)+3}d\cos(x)$$

$$\Big\downarrow \text{2460}$$

$$-\frac{1}{2}\int \left(\frac{2}{4\cos^2(x)-3}-\frac{2}{3\left(4\cos^2(x)-1\right)}-\frac{1}{3\left(\cos^2(x)-1\right)}\right)d\cos(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{1}{3}\text{arctanh}(\cos(x))-\frac{1}{3}\text{arctanh}(2\cos(x))+\frac{\text{arctanh}\left(\frac{2\cos(x)}{\sqrt{3}}\right)}{\sqrt{3}}\right)$$

input
```
Int[Cos[x]*Csc[6*x],x]
```

output
```
(-1/3*ArcTanh[Cos[x]] - ArcTanh[2*Cos[x]]/3 + ArcTanh[(2*Cos[x])/Sqrt[3]]/
Sqrt[3])/2
```

3.127.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2460
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px /. x -> Sqrt[x]]},
Int[ExpandIntegrand[u*(Qx /. x -> x^2)^p, x], x] /;  !SumQ[NonfreeFactors[Q
x, x]]] /; PolyQ[Px, x^2] && GtQ[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&
 !TrinomialQ[Px, x] && ILtQ[p, 0] && RationalFunctionQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4879
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cos[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Cos[v]/d
, u/Sin[v], x], x], x, Cos[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[Nonfree
Factors[Cos[v], x], u/Sin[v], x]]
```

3.127.4 Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.31

method	result	s
default	$\frac{\operatorname{arctanh}\left(\frac{2\cos(x)\sqrt{3}}{3}\right)\sqrt{3}}{6}+\frac{\ln(2\cos(x)-1)}{12}-\frac{\ln(\cos(x)+1)}{12}+\frac{\ln(\cos(x)-1)}{12}-\frac{\ln(1+2\cos(x))}{12}$	4
risch	$-\frac{\ln(\mathrm{e}^{ix}+1)}{6}+\frac{\ln(\mathrm{e}^{ix}-1)}{6}+\frac{\ln(\mathrm{e}^{2ix}-\mathrm{e}^{ix}+1)}{12}-\frac{\sqrt{3}\,\ln\left(\mathrm{e}^{2ix}-\sqrt{3}\,\mathrm{e}^{ix}+1\right)}{12}+\frac{\sqrt{3}\,\ln\left(\mathrm{e}^{2ix}+\sqrt{3}\,\mathrm{e}^{ix}+1\right)}{12}-\frac{\ln(\mathrm{e}^{2ix}+\mathrm{e}^{ix}+1)}{12}$	9

input
```
int(cos(x)*csc(6*x),x,method=_RETURNVERBOSE)
```

output
```
1/6*arctanh(2/3*cos(x)*3^(1/2))*3^(1/2)+1/12*ln(2*cos(x)-1)-1/12*ln(cos(x)
+1)+1/12*ln(cos(x)-1)-1/12*ln(1+2*cos(x))
```

3.127.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(26) = 52.

Time = 0.27 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.94

$$\int \cos(x)\csc(6x)\,dx = \frac{1}{12}\sqrt{3}\log\left(-\frac{4\cos(x)^2+4\sqrt{3}\cos(x)+3}{4\cos(x)^2-3}\right) - \frac{1}{12}\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right) + \frac{1}{12}\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right) + \frac{1}{12}\log(-2\cos(x)+1) - \frac{1}{12}\log(-2\cos(x)-1)$$

input
```
integrate(cos(x)*csc(6*x),x, algorithm="fricas")
```

output
```
1/12*sqrt(3)*log(-(4*cos(x)^2 + 4*sqrt(3)*cos(x) + 3)/(4*cos(x)^2 - 3)) -
1/12*log(1/2*cos(x) + 1/2) + 1/12*log(-1/2*cos(x) + 1/2) + 1/12*log(-2*cos
(x) + 1) - 1/12*log(-2*cos(x) - 1)
```

3.127.6 Sympy [F]

$$\int \cos(x)\csc(6x)\,dx = \int \cos(x)\csc(6x)\,dx$$

input
```
integrate(cos(x)*csc(6*x),x)
```

output
```
Integral(cos(x)*csc(6*x), x)
```

3.127.7 Maxima [F]

$$\int \cos(x)\csc(6x)\,dx = \int \cos(x)\csc(6x)\,dx$$

input
```
integrate(cos(x)*csc(6*x),x, algorithm="maxima")
```

output
```
-integrate(1/2*((sin(3*x) - sin(x))*cos(4*x) - (cos(3*x) - cos(x))*sin(4*x
) - (cos(2*x) - 1)*sin(3*x) + cos(3*x)*sin(2*x) - cos(x)*sin(2*x) + cos(2*
x)*sin(x) - sin(x))/(2*(cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - cos(2*x)^2 -
 sin(4*x)^2 + 2*sin(4*x)*sin(2*x) - sin(2*x)^2 + 2*cos(2*x) - 1), x) - 1/2
4*log(2*(cos(x) + 1)*cos(2*x) + cos(2*x)^2 + cos(x)^2 + sin(2*x)^2 + 2*sin
(2*x)*sin(x) + sin(x)^2 + 2*cos(x) + 1) + 1/24*log(-2*(cos(x) - 1)*cos(2*x
) + cos(2*x)^2 + cos(x)^2 + sin(2*x)^2 - 2*sin(2*x)*sin(x) + sin(x)^2 - 2*
cos(x) + 1) - 1/12*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/12*log(cos(
x)^2 + sin(x)^2 - 2*cos(x) + 1)
```

3.127.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(26) = 52$.

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.89

$$\int \cos(x)\csc(6x)\,dx = -\frac{1}{12}\sqrt{3}\log\left(\frac{\left|-4\sqrt{3}+8\cos(x)\right|}{\left|4\sqrt{3}+8\cos(x)\right|}\right) - \frac{1}{12}\log(\cos(x)+1) + \frac{1}{12}\log(-\cos(x)+1) - \frac{1}{12}\log(|2\cos(x)+1|) + \frac{1}{12}\log(|2\cos(x)-1|)$$

input
```
integrate(cos(x)*csc(6*x),x, algorithm="giac")
```

output
```
-1/12*sqrt(3)*log(abs(-4*sqrt(3) + 8*cos(x))/abs(4*sqrt(3) + 8*cos(x))) -
1/12*log(cos(x) + 1) + 1/12*log(-cos(x) + 1) - 1/12*log(abs(2*cos(x) + 1))
 + 1/12*log(abs(2*cos(x) - 1))
```

3.127.9 Mupad [B] (verification not implemented)

Time = 26.83 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.06

$$\int \cos(x)\csc(6x)\,dx$$
$$= \frac{\operatorname{atanh}\left(\frac{1073741824}{10761687\left(\frac{427973089951744\tan\left(\frac{x}{2}\right)^2}{14348907}-\frac{47552804159488}{4782969}\right)}+\frac{797161}{797162}\right)}{6}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{6}$$
$$+\frac{\sqrt{3}\operatorname{atanh}\left(\frac{4222769432625152\sqrt{3}}{4782969\left(\frac{101871591633190912\tan\left(\frac{x}{2}\right)^2}{4782969}-\frac{7314051205955584}{4782969}\right)}-\frac{19605196950732800\sqrt{3}\tan\left(\frac{x}{2}\right)^2}{1594323\left(\frac{101871591633190912\tan\left(\frac{x}{2}\right)^2}{4782969}-\frac{7314051205955584}{4782969}\right)}\right)}{6}$$

input
```
int(cos(x)/sin(6*x),x)
```

output
```
atanh(1073741824/(10761687*((427973089951744*tan(x/2)^2)/14348907 - 475528
04159488/4782969)) + 797161/797162)/6 + log(tan(x/2))/6 + (3^(1/2)*atanh((
4222769432625152*3^(1/2))/(4782969*((101871591633190912*tan(x/2)^2)/478296
9 - 7314051205955584/4782969)) - (19605196950732800*3^(1/2)*tan(x/2)^2)/(1
594323*((101871591633190912*tan(x/2)^2)/4782969 - 7314051205955584/4782969
))))/6
```

3.128 $\int \cos^3(6x)\sin(x)\,dx$

3.128.1 Optimal result

Integrand size = 9, antiderivative size = 33

$$\int \cos^3(6x)\sin(x)\,dx = \frac{3}{40}\cos(5x) - \frac{3}{56}\cos(7x) + \frac{1}{136}\cos(17x) - \frac{1}{152}\cos(19x)$$

output
```
3/40*cos(5*x)-3/56*cos(7*x)+1/136*cos(17*x)-1/152*cos(19*x)
```

3.128.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \cos^3(6x)\sin(x)\,dx = \frac{3}{40}\cos(5x) - \frac{3}{56}\cos(7x) + \frac{1}{136}\cos(17x) - \frac{1}{152}\cos(19x)$$

input
```
Integrate[Cos[6*x]^3*Sin[x],x]
```

output
```
(3*Cos[5*x])/40 - (3*Cos[7*x])/56 + Cos[17*x]/136 - Cos[19*x]/152
```

3.128.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4854, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\cos^3(6x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x)\cos(6x)^3dx$$

$$\Big\downarrow \text{4854}$$

$$\int \left(-\frac{3}{8}\sin(5x)+\frac{3}{8}\sin(7x)-\frac{1}{8}\sin(17x)+\frac{1}{8}\sin(19x)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{40}\cos(5x)-\frac{3}{56}\cos(7x)+\frac{1}{136}\cos(17x)-\frac{1}{152}\cos(19x)$$

input
```
Int[Cos[6*x]^3*Sin[x],x]
```

output
```
(3*Cos[5*x])/40 - (3*Cos[7*x])/56 + Cos[17*x]/136 - Cos[19*x]/152
```

3.128.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4854
```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol
] :> Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /
; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] ||
 EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]
```

3.128.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{3\cos(5x)}{40}-\frac{3\cos(7x)}{56}+\frac{\cos(17x)}{136}-\frac{\cos(19x)}{152}$	26
risch	$\frac{3\cos(5x)}{40}-\frac{3\cos(7x)}{56}+\frac{\cos(17x)}{136}-\frac{\cos(19x)}{152}$	26
parallelrisch	$-\frac{37}{11305}+\frac{3\cos(5x)}{40}-\frac{3\cos(7x)}{56}+\frac{\cos(17x)}{136}-\frac{\cos(19x)}{152}$	27

input
```
int(cos(6*x)^3*sin(x),x,method=_RETURNVERBOSE)
```

output
```
3/40*cos(5*x)-3/56*cos(7*x)+1/136*cos(17*x)-1/152*cos(19*x)
```

3.128.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(25) = 50.

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.73

$$\int \cos^3(6x)\sin(x)\,dx = -\frac{32768}{19}\cos(x)^{19}+\frac{147456}{17}\cos(x)^{17}-18432\cos(x)^{15}+21504\cos(x)^{13}-14976\cos(x)^{11}+6336\cos(x)^{9}-\frac{11112}{7}\cos(x)^{7}+\frac{1116}{5}\cos(x)^{5}-18\cos(x)^{3}+\cos(x)$$

input
```
integrate(cos(6*x)^3*sin(x),x, algorithm="fricas")
```

output
```
-32768/19*cos(x)^19 + 147456/17*cos(x)^17 - 18432*cos(x)^15 + 21504*cos(x)
^13 - 14976*cos(x)^11 + 6336*cos(x)^9 - 11112/7*cos(x)^7 + 1116/5*cos(x)^5
 - 18*cos(x)^3 + cos(x)
```

3.128.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(29) = 58.

Time = 0.60 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.91

$$\int \cos^3(6x)\sin(x)\,dx = \frac{1296\sin(x)\sin^3(6x)}{11305} + \frac{1926\sin(x)\sin(6x)\cos^2(6x)}{11305} + \frac{216\sin^2(6x)\cos(x)\cos(6x)}{11305} + \frac{251\cos(x)\cos^3(6x)}{11305}$$

input
```
integrate(cos(6*x)**3*sin(x),x)
```

output
```
1296*sin(x)*sin(6*x)**3/11305 + 1926*sin(x)*sin(6*x)*cos(6*x)**2/11305 + 2
16*sin(6*x)**2*cos(x)*cos(6*x)/11305 + 251*cos(x)*cos(6*x)**3/11305
```

3.128.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \cos^3(6x)\sin(x)\,dx = -\frac{1}{152}\cos(19x) + \frac{1}{136}\cos(17x) - \frac{3}{56}\cos(7x) + \frac{3}{40}\cos(5x)$$

input
```
integrate(cos(6*x)^3*sin(x),x, algorithm="maxima")
```

output
```
-1/152*cos(19*x) + 1/136*cos(17*x) - 3/56*cos(7*x) + 3/40*cos(5*x)
```

3.128.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \cos^3(6x)\sin(x)\,dx = -\frac{1}{152}\cos(19x) + \frac{1}{136}\cos(17x) - \frac{3}{56}\cos(7x) + \frac{3}{40}\cos(5x)$$

input
```
integrate(cos(6*x)^3*sin(x),x, algorithm="giac")
```

output
```
-1/152*cos(19*x) + 1/136*cos(17*x) - 3/56*cos(7*x) + 3/40*cos(5*x)
```

3.128.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.73

$$\int \cos^3(6x)\sin(x)\,dx = -\frac{32768\cos(x)^{19}}{19} + \frac{147456\cos(x)^{17}}{17} - 18432\cos(x)^{15} + 21504\cos(x)^{13} - 14976\cos(x)^{11} + 6336\cos(x)^{9} - \frac{11112\cos(x)^{7}}{7} + \frac{1116\cos(x)^{5}}{5} - 18\cos(x)^{3} + \cos(x)$$

input
```
int(cos(6*x)^3*sin(x),x)
```

output
```
cos(x) - 18*cos(x)^3 + (1116*cos(x)^5)/5 - (11112*cos(x)^7)/7 + 6336*cos(x
)^9 - 14976*cos(x)^11 + 21504*cos(x)^13 - 18432*cos(x)^15 + (147456*cos(x)
^17)/17 - (32768*cos(x)^19)/19
```

3.129 $\int \cos^3(6x)\sin(9x)\,dx$

3.129.1 Optimal result

Integrand size = 11, antiderivative size = 33

$$\int \cos^3(6x)\sin(9x)\,dx = -\frac{1}{8}\cos(3x)+\frac{1}{72}\cos(9x)-\frac{1}{40}\cos(15x)-\frac{1}{216}\cos(27x)$$

output
```
-1/8*cos(3*x)+1/72*cos(9*x)-1/40*cos(15*x)-1/216*cos(27*x)
```

3.129.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \cos^3(6x)\sin(9x)\,dx = -\frac{1}{8}\cos(3x)+\frac{1}{72}\cos(9x)-\frac{1}{40}\cos(15x)-\frac{1}{216}\cos(27x)$$

input
```
Integrate[Cos[6*x]^3*Sin[9*x],x]
```

output
```
-1/8*Cos[3*x] + Cos[9*x]/72 - Cos[15*x]/40 - Cos[27*x]/216
```

3.129.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 4854, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(9x)\cos^3(6x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(9x)\cos(6x)^3dx$$

$$\Big\downarrow \text{4854}$$

$$\int \left(\frac{3}{8}\sin(3x)-\frac{1}{8}\sin(9x)+\frac{3}{8}\sin(15x)+\frac{1}{8}\sin(27x)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{8}\cos(3x)+\frac{1}{72}\cos(9x)-\frac{1}{40}\cos(15x)-\frac{1}{216}\cos(27x)$$

input
```
Int[Cos[6*x]^3*Sin[9*x],x]
```

output
```
-1/8*Cos[3*x] + Cos[9*x]/72 - Cos[15*x]/40 - Cos[27*x]/216
```

3.129.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4854
```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol
] :> Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /
; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] ||
 EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]
```

3.129.4 Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

method	result	size
default	$-\frac{\cos(3x)}{8}+\frac{\cos(9x)}{72}-\frac{\cos(15x)}{40}-\frac{\cos(27x)}{216}$	26
risch	$-\frac{\cos(3x)}{8}+\frac{\cos(9x)}{72}-\frac{\cos(15x)}{40}-\frac{\cos(27x)}{216}$	26
parallelrisch	$\frac{13}{135}-\frac{\cos(15x)}{40}+\frac{\cos(9x)}{72}-\frac{\cos(27x)}{216}-\frac{\cos(3x)}{8}$	27

input
```
int(cos(6*x)^3*sin(9*x),x,method=_RETURNVERBOSE)
```

output
```
-1/8*cos(3*x)+1/72*cos(9*x)-1/40*cos(15*x)-1/216*cos(27*x)
```

3.129.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.18

$$\int \cos^3(6x)\sin(9x)\,dx = -\frac{32}{27}\cos(3x)^9+\frac{8}{3}\cos(3x)^7-\frac{12}{5}\cos(3x)^5+\frac{10}{9}\cos(3x)^3-\frac{1}{3}\cos(3x)$$

input
```
integrate(cos(6*x)^3*sin(9*x),x, algorithm="fricas")
```

output
```
-32/27*cos(3*x)^9 + 8/3*cos(3*x)^7 - 12/5*cos(3*x)^5 + 10/9*cos(3*x)^3 - 1
/3*cos(3*x)
```

3.129.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(26) = 52.

Time = 0.60 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.15

$$\int \cos^3(6x)\sin(9x)\,dx = -\frac{16\sin^3(6x)\sin(9x)}{135}-\frac{8\sin^2(6x)\cos(6x)\cos(9x)}{45}-\frac{2\sin(6x)\sin(9x)\cos^2(6x)}{45}-\frac{19\cos^3(6x)\cos(9x)}{135}$$

input
```
integrate(cos(6*x)**3*sin(9*x),x)
```

output
```
-16*sin(6*x)**3*sin(9*x)/135 - 8*sin(6*x)**2*cos(6*x)*cos(9*x)/45 - 2*sin(
6*x)*sin(9*x)*cos(6*x)**2/45 - 19*cos(6*x)**3*cos(9*x)/135
```

3.129.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \cos^3(6x)\sin(9x)\,dx = -\frac{1}{216}\cos(27\,x) - \frac{1}{40}\cos(15\,x) + \frac{1}{72}\cos(9\,x) - \frac{1}{8}\cos(3\,x)$$

input
```
integrate(cos(6*x)^3*sin(9*x),x, algorithm="maxima")
```

output
```
-1/216*cos(27*x) - 1/40*cos(15*x) + 1/72*cos(9*x) - 1/8*cos(3*x)
```

3.129.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \cos^3(6x)\sin(9x)\,dx = -\frac{1}{216}\cos(27\,x) - \frac{1}{40}\cos(15\,x) + \frac{1}{72}\cos(9\,x) - \frac{1}{8}\cos(3\,x)$$

input
```
integrate(cos(6*x)^3*sin(9*x),x, algorithm="giac")
```

output
```
-1/216*cos(27*x) - 1/40*cos(15*x) + 1/72*cos(9*x) - 1/8*cos(3*x)
```

3.129.9 Mupad [B] (verification not implemented)

Time = 27.05 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.36

$$\int \cos^3(6x)\sin(9x)\,dx =$$

$$-\frac{2\left(135\tan\left(\frac{3x}{2}\right)^{16} - 900\tan\left(\frac{3x}{2}\right)^{14} + 5640\tan\left(\frac{3x}{2}\right)^{12} - 13140\tan\left(\frac{3x}{2}\right)^{10} + 15534\tan\left(\frac{3x}{2}\right)^{8} - 4044\tan\left(\right.\right.}{135\left(\tan\left(\frac{3x}{2}\right)^2 + 1\right)^9}$$

input
```
int(cos(6*x)^3*sin(9*x),x)
```

output
```
-(2*(36*tan((3*x)/2)^2 + 1584*tan((3*x)/2)^4 - 4044*tan((3*x)/2)^6 + 15534
*tan((3*x)/2)^8 - 13140*tan((3*x)/2)^10 + 5640*tan((3*x)/2)^12 - 900*tan((
3*x)/2)^14 + 135*tan((3*x)/2)^16 + 19))/(135*(tan((3*x)/2)^2 + 1)^9)
```

3.129. $\int \cos^3(6x)\sin(9x)\,dx$

3.130 $\int \cos(2x)\sin^2(6x)\,dx$

3.130.1 Optimal result

Integrand size = 11, antiderivative size = 25

$$\int \cos(2x)\sin^2(6x)\,dx = \frac{1}{4}\sin(2x) - \frac{1}{40}\sin(10x) - \frac{1}{56}\sin(14x)$$

output
```
1/4*sin(2*x)-1/40*sin(10*x)-1/56*sin(14*x)
```

3.130.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \cos(2x)\sin^2(6x)\,dx = \frac{1}{4}\sin(2x) - \frac{1}{40}\sin(10x) - \frac{1}{56}\sin(14x)$$

input
```
Integrate[Cos[2*x]*Sin[6*x]^2,x]
```

output
```
Sin[2*x]/4 - Sin[10*x]/40 - Sin[14*x]/56
```

3.130.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 4854, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(6x)\cos(2x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(6x)^2\cos(2x)dx$$

$$\Big\downarrow \text{4854}$$

$$\int \left(\frac{1}{2}\cos(2x)-\frac{1}{4}\cos(10x)-\frac{1}{4}\cos(14x)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\sin(2x)-\frac{1}{40}\sin(10x)-\frac{1}{56}\sin(14x)$$

input
```
Int[Cos[2*x]*Sin[6*x]^2,x]
```

output
```
Sin[2*x]/4 - Sin[10*x]/40 - Sin[14*x]/56
```

3.130.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4854
```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol
] :> Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /
; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] ||
 EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]
```

3.130.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{\sin(2x)}{4}-\frac{\sin(10x)}{40}-\frac{\sin(14x)}{56}$	20
risch	$\frac{\sin(2x)}{4}-\frac{\sin(10x)}{40}-\frac{\sin(14x)}{56}$	20
parallelrisch	$\frac{\sin(2x)}{4}-\frac{\sin(10x)}{40}-\frac{\sin(14x)}{56}$	20
norman	$\frac{\frac{6\tan(3x)^3}{35}+\frac{32\tan(x)\tan(3x)^2}{35}+\frac{18\tan(x)\tan(3x)^4}{35}+\frac{6\tan(x)^2\tan(3x)}{35}-\frac{6\tan(x)^2\tan(3x)^3}{35}+\frac{18\tan(x)}{35}-\frac{6\tan(3x)}{35}}{\left(1+\tan(x)^2\right)\left(1+\tan(3x)^2\right)^2}$	81

input
```
int(cos(2*x)*sin(6*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*sin(2*x)-1/40*sin(10*x)-1/56*sin(14*x)
```

3.130.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

$$\int \cos(2x)\sin^2(6x)\,dx = -\frac{1}{70}\left(80\cos(2x)^6-72\cos(2x)^4+9\cos(2x)^2-17\right)\sin(2x)$$

input
```
integrate(cos(2*x)*sin(6*x)^2,x, algorithm="fricas")
```

output
```
-1/70*(80*cos(2*x)^6 - 72*cos(2*x)^4 + 9*cos(2*x)^2 - 17)*sin(2*x)
```

3.130.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(19) = 38.

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.92

$$\int \cos(2x)\sin^2(6x)\,dx = \frac{17\sin(2x)\sin^2(6x)}{70}+\frac{9\sin(2x)\cos^2(6x)}{35}-\frac{3\sin(6x)\cos(2x)\cos(6x)}{35}$$

input
```
integrate(cos(2*x)*sin(6*x)**2,x)
```

output
```
17*sin(2*x)*sin(6*x)**2/70 + 9*sin(2*x)*cos(6*x)**2/35 - 3*sin(6*x)*cos(2*
x)*cos(6*x)/35
```

3.130.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \cos(2x)\sin^2(6x)\,dx = -\frac{1}{56}\sin(14\,x) - \frac{1}{40}\sin(10\,x) + \frac{1}{4}\sin(2\,x)$$

input
```
integrate(cos(2*x)*sin(6*x)^2,x, algorithm="maxima")
```

output
```
-1/56*sin(14*x) - 1/40*sin(10*x) + 1/4*sin(2*x)
```

3.130.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \cos(2x)\sin^2(6x)\,dx = -\frac{1}{56}\sin(14\,x) - \frac{1}{40}\sin(10\,x) + \frac{1}{4}\sin(2\,x)$$

input
```
integrate(cos(2*x)*sin(6*x)^2,x, algorithm="giac")
```

output
```
-1/56*sin(14*x) - 1/40*sin(10*x) + 1/4*sin(2*x)
```

3.130.9 Mupad [B] (verification not implemented)

Time = 27.61 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \cos(2x)\sin^2(6x)\,dx = \frac{8\sin(2\,x)^7}{7} - \frac{12\sin(2\,x)^5}{5} + \frac{3\sin(2\,x)^3}{2}$$

input
```
int(cos(2*x)*sin(6*x)^2,x)
```

output
```
(3*sin(2*x)^3)/2 - (12*sin(2*x)^5)/5 + (8*sin(2*x)^7)/7
```

3.131 $\int \cos(x)\sin^2(6x)\,dx$

3.131.1 Optimal result

Integrand size = 9, antiderivative size = 23

$$\int \cos(x)\sin^2(6x)\,dx = \frac{\sin(x)}{2} - \frac{1}{44}\sin(11x) - \frac{1}{52}\sin(13x)$$

output
```
1/2*sin(x)-1/44*sin(11*x)-1/52*sin(13*x)
```

3.131.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \cos(x)\sin^2(6x)\,dx = \frac{\sin(x)}{2} - \frac{1}{44}\sin(11x) - \frac{1}{52}\sin(13x)$$

input
```
Integrate[Cos[x]*Sin[6*x]^2,x]
```

output
```
Sin[x]/2 - Sin[11*x]/44 - Sin[13*x]/52
```

3.131.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4854, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(6x)\cos(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(6x)^2\cos(x)dx$$

$$\Big\downarrow \text{4854}$$

$$\int \left(\frac{\cos(x)}{2}-\frac{1}{4}\cos(11x)-\frac{1}{4}\cos(13x)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sin(x)}{2}-\frac{1}{44}\sin(11x)-\frac{1}{52}\sin(13x)$$

input
```
Int[Cos[x]*Sin[6*x]^2,x]
```

output
```
Sin[x]/2 - Sin[11*x]/44 - Sin[13*x]/52
```

3.131.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4854
```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol
] :> Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /
; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] ||
 EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]
```

3.131.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	si
default	$\frac{\sin(x)}{2}-\frac{\sin(11x)}{44}-\frac{\sin(13x)}{52}$	18
risch	$\frac{\sin(x)}{2}-\frac{\sin(11x)}{44}-\frac{\sin(13x)}{52}$	18
parallelrisch	$\frac{\sin(x)}{2}-\frac{\sin(11x)}{44}-\frac{\sin(13x)}{52}$	18
norman	$\frac{\frac{24\tan(3x)^3}{143}+\frac{24\tan(3x)\tan\left(\frac{x}{2}\right)^2}{143}+\frac{280\tan(3x)^2\tan\left(\frac{x}{2}\right)}{143}-\frac{24\tan(3x)^3\tan\left(\frac{x}{2}\right)^2}{143}+\frac{144\tan(3x)^4\tan\left(\frac{x}{2}\right)}{143}-\frac{24\tan(3x)}{143}+\frac{144\tan\left(\frac{x}{2}\right)}{143}}{\left(1+\tan\left(\frac{x}{2}\right)^2\right)\left(1+\tan(3x)^2\right)^2}$	93

input
```
int(cos(x)*sin(6*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*sin(x)-1/44*sin(11*x)-1/52*sin(13*x)
```

3.131.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(17) = 34$.

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \cos(x)\sin^2(6x)\,dx =$$
$$-\frac{4}{143}\left(2816\cos(x)^{12}-6912\cos(x)^{10}+6048\cos(x)^{8}-2240\cos(x)^{6}+315\cos(x)^{4}-9\cos(x)^{2}-18\right)$$

input
```
integrate(cos(x)*sin(6*x)^2,x, algorithm="fricas")
```

output
```
-4/143*(2816*cos(x)^12 - 6912*cos(x)^10 + 6048*cos(x)^8 - 2240*cos(x)^6 +
315*cos(x)^4 - 9*cos(x)^2 - 18)*sin(x)
```

3.131.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(17) = 34.

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \cos(x)\sin^2(6x)\,dx = \frac{71\sin(x)\sin^2(6x)}{143} + \frac{72\sin(x)\cos^2(6x)}{143} - \frac{12\sin(6x)\cos(x)\cos(6x)}{143}$$

input
```
integrate(cos(x)*sin(6*x)**2,x)
```

output
```
71*sin(x)*sin(6*x)**2/143 + 72*sin(x)*cos(6*x)**2/143 - 12*sin(6*x)*cos(x)
*cos(6*x)/143
```

3.131.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \cos(x)\sin^2(6x)\,dx = -\frac{1}{52}\sin(13x) - \frac{1}{44}\sin(11x) + \frac{1}{2}\sin(x)$$

input
```
integrate(cos(x)*sin(6*x)^2,x, algorithm="maxima")
```

output
```
-1/52*sin(13*x) - 1/44*sin(11*x) + 1/2*sin(x)
```

3.131.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \cos(x)\sin^2(6x)\,dx = -\frac{1}{52}\sin(13x) - \frac{1}{44}\sin(11x) + \frac{1}{2}\sin(x)$$

input
```
integrate(cos(x)*sin(6*x)^2,x, algorithm="giac")
```

output
```
-1/52*sin(13*x) - 1/44*sin(11*x) + 1/2*sin(x)
```

3.131.9 Mupad [B] (verification not implemented)

Time = 26.96 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \cos(x)\sin^2(6x)\,dx = \frac{\sin(x)}{2} - \frac{\sin(13\,x)}{52} - \frac{\sin(11\,x)}{44}$$

input
```
int(sin(6*x)^2*cos(x),x)
```

output
```
sin(x)/2 - sin(13*x)/52 - sin(11*x)/44
```

3.132 $\int \cos(x)\sin^3(6x)\,dx$

3.132.1 Optimal result

Integrand size = 9, antiderivative size = 33

$$\int \cos(x)\sin^3(6x)\,dx = -\frac{3}{40}\cos(5x)-\frac{3}{56}\cos(7x)+\frac{1}{136}\cos(17x)+\frac{1}{152}\cos(19x)$$

output
```
-3/40*cos(5*x)-3/56*cos(7*x)+1/136*cos(17*x)+1/152*cos(19*x)
```

3.132.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \cos(x)\sin^3(6x)\,dx = -\frac{3}{40}\cos(5x)-\frac{3}{56}\cos(7x)+\frac{1}{136}\cos(17x)+\frac{1}{152}\cos(19x)$$

input
```
Integrate[Cos[x]*Sin[6*x]^3,x]
```

output
```
(-3*Cos[5*x])/40 - (3*Cos[7*x])/56 + Cos[17*x]/136 + Cos[19*x]/152
```

3.132.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4854, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(6x)\cos(x)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin(6x)^3\cos(x)dx$$

$$\Big\downarrow \; 4854$$

$$\int \left(\frac{3}{8}\sin(5x)+\frac{3}{8}\sin(7x)-\frac{1}{8}\sin(17x)-\frac{1}{8}\sin(19x)\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{3}{40}\cos(5x)-\frac{3}{56}\cos(7x)+\frac{1}{136}\cos(17x)+\frac{1}{152}\cos(19x)$$

input
```
Int[Cos[x]*Sin[6*x]^3,x]
```

output
```
(-3*Cos[5*x])/40 - (3*Cos[7*x])/56 + Cos[17*x]/136 + Cos[19*x]/152
```

3.132.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4854
```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol
] :> Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /
; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] ||
 EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]
```

3.132.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

method	result	size
default	$-\frac{3\cos(5x)}{40}-\frac{3\cos(7x)}{56}+\frac{\cos(17x)}{136}+\frac{\cos(19x)}{152}$	26
risch	$-\frac{3\cos(5x)}{40}-\frac{3\cos(7x)}{56}+\frac{\cos(17x)}{136}+\frac{\cos(19x)}{152}$	26
parallelrisch	$\frac{1272}{11305}-\frac{3\cos(5x)}{40}-\frac{3\cos(7x)}{56}+\frac{\cos(17x)}{136}+\frac{\cos(19x)}{152}$	27

input
```
int(cos(x)*sin(6*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/40*cos(5*x)-3/56*cos(7*x)+1/136*cos(17*x)+1/152*cos(19*x)
```

3.132.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.48

$$\int \cos(x)\sin^3(6x)\,dx = \frac{32768}{19}\cos(x)^{19}-\frac{131072}{17}\cos(x)^{17}+14336\cos(x)^{15}-14336\cos(x)^{13}+8320\cos(x)^{11}-2816\cos(x)^{9}+\frac{3672}{7}\cos(x)^{7}-\frac{216}{5}\cos(x)^{5}$$

input
```
integrate(cos(x)*sin(6*x)^3,x, algorithm="fricas")
```

output
```
32768/19*cos(x)^19 - 131072/17*cos(x)^17 + 14336*cos(x)^15 - 14336*cos(x)^
13 + 8320*cos(x)^11 - 2816*cos(x)^9 + 3672/7*cos(x)^7 - 216/5*cos(x)^5
```

3.132.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(29) = 58.

Time = 0.61 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.97

$$\int \cos(x)\sin^3(6x)\,dx = -\frac{251\sin(x)\sin^3(6x)}{11305}-\frac{216\sin(x)\sin(6x)\cos^2(6x)}{11305}-\frac{1926\sin^2(6x)\cos(x)\cos(6x)}{11305}-\frac{1296\cos(x)\cos^3(6x)}{11305}$$

input
```
integrate(cos(x)*sin(6*x)**3,x)
```

output
```
-251*sin(x)*sin(6*x)**3/11305 - 216*sin(x)*sin(6*x)*cos(6*x)**2/11305 - 19
26*sin(6*x)**2*cos(x)*cos(6*x)/11305 - 1296*cos(x)*cos(6*x)**3/11305
```

3.132.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \cos(x)\sin^3(6x)\,dx = \frac{1}{152}\cos(19x) + \frac{1}{136}\cos(17x) - \frac{3}{56}\cos(7x) - \frac{3}{40}\cos(5x)$$

input
```
integrate(cos(x)*sin(6*x)^3,x, algorithm="maxima")
```

output
```
1/152*cos(19*x) + 1/136*cos(17*x) - 3/56*cos(7*x) - 3/40*cos(5*x)
```

3.132.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.48

$$\begin{aligned}\int \cos(x)\sin^3(6x)\,dx = {} & \frac{32768}{19}\cos(x)^{19} - \frac{131072}{17}\cos(x)^{17} + 14336\cos(x)^{15} \\ & - 14336\cos(x)^{13} + 8320\cos(x)^{11} - 2816\cos(x)^{9} + \frac{3672}{7}\cos(x)^{7} \\ & - \frac{216}{5}\cos(x)^{5}\end{aligned}$$

input
```
integrate(cos(x)*sin(6*x)^3,x, algorithm="giac")
```

output
```
32768/19*cos(x)^19 - 131072/17*cos(x)^17 + 14336*cos(x)^15 - 14336*cos(x)^
13 + 8320*cos(x)^11 - 2816*cos(x)^9 + 3672/7*cos(x)^7 - 216/5*cos(x)^5
```

3.132.9 Mupad [B] (verification not implemented)

Time = 27.03 (sec) , antiderivative size = 150, normalized size of antiderivative = 4.55

$$\int \cos(x)\sin^3(6x)\,dx = -\frac{32\left(305235\tan\left(\frac{x}{2}\right)^{34} - 9665775\tan\left(\frac{x}{2}\right)^{32} + 153838440\tan\left(\frac{x}{2}\right)^{30} - 1348695544\tan\left(\frac{x}{2}\right)^{28} + 7083812484\right.}{}$$

input
```
int(sin(6*x)^3*cos(x),x)
```

output
```
-(32*(1539*tan(x/2)^2 - 291384*tan(x/2)^4 + 9744264*tan(x/2)^6 - 153524484
*tan(x/2)^8 + 1349637412*tan(x/2)^10 - 7081614792*tan(x/2)^12 + 2358290959
2*tan(x/2)^14 - 51607368282*tan(x/2)^16 + 75935973762*tan(x/2)^18 - 759284
91144*tan(x/2)^20 + 51613490424*tan(x/2)^22 - 23578828164*tan(x/2)^24 + 70
83812484*tan(x/2)^26 - 1348695544*tan(x/2)^28 + 153838440*tan(x/2)^30 - 96
65775*tan(x/2)^32 + 305235*tan(x/2)^34 + 81))/(11305*(tan(x/2)^2 + 1)^19)
```

3.133 $\int \cos(7x)\sin^3(6x)\,dx$

3.133.1 Optimal result

Integrand size = 11, antiderivative size = 31

$$\int \cos(7x)\sin^3(6x)\,dx = \frac{3\cos(x)}{8} + \frac{1}{88}\cos(11x) - \frac{3}{104}\cos(13x) + \frac{1}{200}\cos(25x)$$

output
```
3/8*cos(x)+1/88*cos(11*x)-3/104*cos(13*x)+1/200*cos(25*x)
```

3.133.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \cos(7x)\sin^3(6x)\,dx = \frac{3\cos(x)}{8} + \frac{1}{88}\cos(11x) - \frac{3}{104}\cos(13x) + \frac{1}{200}\cos(25x)$$

input
```
Integrate[Cos[7*x]*Sin[6*x]^3,x]
```

output
```
(3*Cos[x])/8 + Cos[11*x]/88 - (3*Cos[13*x])/104 + Cos[25*x]/200
```

3.133.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 4854, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(6x)\cos(7x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(6x)^3\cos(7x)dx$$

$$\Big\downarrow \text{4854}$$

$$\int \left(-\frac{3\sin(x)}{8}-\frac{1}{8}\sin(11x)+\frac{3}{8}\sin(13x)-\frac{1}{8}\sin(25x)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\cos(x)}{8}+\frac{1}{88}\cos(11x)-\frac{3}{104}\cos(13x)+\frac{1}{200}\cos(25x)$$

input
```
Int[Cos[7*x]*Sin[6*x]^3,x]
```

output
```
(3*Cos[x])/8 + Cos[11*x]/88 - (3*Cos[13*x])/104 + Cos[25*x]/200
```

3.133.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4854
```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol
] :> Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /
; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] ||
 EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]
```

3.133.4 Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

method	result
default	$\frac{3\cos(x)}{8}+\frac{\cos(11x)}{88}-\frac{3\cos(13x)}{104}+\frac{\cos(25x)}{200}$
risch	$\frac{3\cos(x)}{8}+\frac{\cos(11x)}{88}-\frac{3\cos(13x)}{104}+\frac{\cos(25x)}{200}$
parallelrisch	$\frac{\left(1176\tan(3x)^{6}-720\tan(3x)^{2}-1416\right)\tan\left(\frac{7x}{2}\right)^{2}+\left(6048\tan(3x)^{5}+10640\tan(3x)^{3}+6048\tan(3x)\right)\tan\left(\frac{7x}{2}\right)-1416\tan(3x)^{6}-7}{3575\left(1+\tan\left(\frac{7x}{2}\right)^{2}\right)\left(1+\tan(3x)^{2}\right)^{3}}$

input
```
int(cos(7*x)*sin(6*x)^3,x,method=_RETURNVERBOSE)
```

output
```
3/8*cos(x)+1/88*cos(11*x)-3/104*cos(13*x)+1/200*cos(25*x)
```

3.133.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. 2(23) = 46.

Time = 0.30 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.16

$$\begin{aligned}\int\cos(7x)\sin^3(6x)\,dx=&\ \frac{2097152}{25}\cos(x)^{25}-524288\cos(x)^{23}+1441792\cos(x)^{21}\\&-2293760\cos(x)^{19}+2334720\cos(x)^{17}-\frac{7938048}{5}\cos(x)^{15}\\&+\frac{9503232}{13}\cos(x)^{13}-\frac{2484992}{11}\cos(x)^{11}\\&+45248\cos(x)^{9}-5400\cos(x)^{7}+\frac{1512}{5}\cos(x)^{5}\end{aligned}$$

input
```
integrate(cos(7*x)*sin(6*x)^3,x, algorithm="fricas")
```

output
```
2097152/25*cos(x)^25 - 524288*cos(x)^23 + 1441792*cos(x)^21 - 2293760*cos(
x)^19 + 2334720*cos(x)^17 - 7938048/5*cos(x)^15 + 9503232/13*cos(x)^13 - 2
484992/11*cos(x)^11 + 45248*cos(x)^9 - 5400*cos(x)^7 + 1512/5*cos(x)^5
```

3.133.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(27) = 54$.

Time = 0.61 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.26

$$\int \cos(7x)\sin^3(6x)\,dx = \frac{1421\sin^3(6x)\sin(7x)}{3575} + \frac{1062\sin^2(6x)\cos(6x)\cos(7x)}{3575} + \frac{1512\sin(6x)\sin(7x)\cos^2(6x)}{3575} + \frac{1296\cos^3(6x)\cos(7x)}{3575}$$

input
```
integrate(cos(7*x)*sin(6*x)**3,x)
```

output
```
1421*sin(6*x)**3*sin(7*x)/3575 + 1062*sin(6*x)**2*cos(6*x)*cos(7*x)/3575 +
 1512*sin(6*x)*sin(7*x)*cos(6*x)**2/3575 + 1296*cos(6*x)**3*cos(7*x)/3575
```

3.133.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \cos(7x)\sin^3(6x)\,dx = \frac{1}{200}\cos(25x) - \frac{3}{104}\cos(13x) + \frac{1}{88}\cos(11x) + \frac{3}{8}\cos(x)$$

input
```
integrate(cos(7*x)*sin(6*x)^3,x, algorithm="maxima")
```

output
```
1/200*cos(25*x) - 3/104*cos(13*x) + 1/88*cos(11*x) + 3/8*cos(x)
```

3.133.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int \cos(7x)\sin^3(6x)\,dx = \frac{1}{200}\cos(25x) - \frac{3}{104}\cos(13x) + \frac{1}{88}\cos(11x) + \frac{3}{8}\cos(x)$$

input
```
integrate(cos(7*x)*sin(6*x)^3,x, algorithm="giac")
```

output
```
1/200*cos(25*x) - 3/104*cos(13*x) + 1/88*cos(11*x) + 3/8*cos(x)
```

3.133.9 Mupad [B] (verification not implemented)

Time = 28.18 (sec) , antiderivative size = 198, normalized size of antiderivative = 6.39

$$\int \cos(7x)\sin^3(6x)\,dx$$
$$= \frac{32\left(-96525\tan\left(\frac{x}{2}\right)^{46} + 8655075\tan\left(\frac{x}{2}\right)^{44} - 300482325\tan\left(\frac{x}{2}\right)^{42} + 5743927475\tan\left(\frac{x}{2}\right)^{40} - 67792485475\right.}{}$$

input
```
int(cos(7*x)*sin(6*x)^3,x)
```

output
```
(32*(2025*tan(x/2)^2 + 120825*tan(x/2)^4 - 8468775*tan(x/2)^6 + 301506975*
tan(x/2)^8 - 5739623945*tan(x/2)^10 + 67806830575*tan(x/2)^12 - 5238294762
25*tan(x/2)^14 + 2750536240650*tan(x/2)^16 - 10084340561350*tan(x/2)^18 +
26326043727610*tan(x/2)^20 - 49575456537350*tan(x/2)^22 + 67896209197950*t
an(x/2)^24 - 67895787973650*tan(x/2)^26 + 49575817586750*tan(x/2)^28 - 263
25778958050*tan(x/2)^30 + 10084506042325*tan(x/2)^32 - 2750448633075*tan(x
/2)^34 + 523868412925*tan(x/2)^36 - 67792485475*tan(x/2)^38 + 5743927475*t
an(x/2)^40 - 300482325*tan(x/2)^42 + 8655075*tan(x/2)^44 - 96525*tan(x/2)^
46 + 81))/(3575*(tan(x/2)^2 + 1)^25)
```

3.134 $\int \cos^2(3x)\sin^3(2x)\,dx$

3.134.1 Optimal result

Integrand size = 13, antiderivative size = 41

$$\int \cos^2(3x)\sin^3(2x)\,dx = -\frac{3}{16}\cos(2x)+\frac{3}{64}\cos(4x)+\frac{1}{48}\cos(6x)-\frac{3}{128}\cos(8x)+\frac{1}{192}\cos(12x)$$

output
```
-3/16*cos(2*x)+3/64*cos(4*x)+1/48*cos(6*x)-3/128*cos(8*x)+1/192*cos(12*x)
```

3.134.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \cos^2(3x)\sin^3(2x)\,dx = -\frac{3}{16}\cos(2x)+\frac{3}{64}\cos(4x)+\frac{1}{48}\cos(6x)-\frac{3}{128}\cos(8x)+\frac{1}{192}\cos(12x)$$

input
```
Integrate[Cos[3*x]^2*Sin[2*x]^3,x]
```

output
```
(-3*Cos[2*x])/16 + (3*Cos[4*x])/64 + Cos[6*x]/48 - (3*Cos[8*x])/128 + Cos[
12*x]/192
```

3.134.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 4854, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(2x)\cos^2(3x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(2x)^3\cos(3x)^2dx$$

$$\Big\downarrow \text{4854}$$

$$\int \left(\frac{3}{8}\sin(2x)-\frac{3}{16}\sin(4x)-\frac{1}{8}\sin(6x)+\frac{3}{16}\sin(8x)-\frac{1}{16}\sin(12x)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3}{16}\cos(2x)+\frac{3}{64}\cos(4x)+\frac{1}{48}\cos(6x)-\frac{3}{128}\cos(8x)+\frac{1}{192}\cos(12x)$$

input
```
Int[Cos[3*x]^2*Sin[2*x]^3,x]
```

output
```
(-3*Cos[2*x])/16 + (3*Cos[4*x])/64 + Cos[6*x]/48 - (3*Cos[8*x])/128 + Cos[
12*x]/192
```

3.134.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4854
```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol
] :> Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /
; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] ||
 EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]
```

3.134.4 Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

method	result	size
default	$-\frac{3\cos(2x)}{16}+\frac{3\cos(4x)}{64}+\frac{\cos(6x)}{48}-\frac{3\cos(8x)}{128}+\frac{\cos(12x)}{192}$	32
risch	$-\frac{3\cos(2x)}{16}+\frac{3\cos(4x)}{64}+\frac{\cos(6x)}{48}-\frac{3\cos(8x)}{128}+\frac{\cos(12x)}{192}$	32
parallelrisch	$\frac{377}{1920}-\frac{3\cos(8x)}{128}+\frac{\cos(12x)}{192}-\frac{3\cos(2x)}{16}+\frac{\cos(6x)}{48}+\frac{3\cos(4x)}{64}$	33

input
```
int(cos(3*x)^2*sin(2*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/16*cos(2*x)+3/64*cos(4*x)+1/48*cos(6*x)-3/128*cos(8*x)+1/192*cos(12*x)
```

3.134.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.61

$$\int \cos^2(3x)\sin^3(2x)\,dx = \frac{32}{3}\cos(x)^{12} - 32\cos(x)^{10} + 33\cos(x)^8 - 12\cos(x)^6$$

input
```
integrate(cos(3*x)^2*sin(2*x)^3,x, algorithm="fricas")
```

output
```
32/3*cos(x)^12 - 32*cos(x)^10 + 33*cos(x)^8 - 12*cos(x)^6
```

3.134.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(37) = 74.

Time = 1.51 (sec) , antiderivative size = 226, normalized size of antiderivative = 5.51

$$\int \cos^2(3x)\sin^3(2x)\,dx = -\frac{x\sin^3(2x)\sin^2(3x)}{16} + \frac{x\sin^3(2x)\cos^2(3x)}{16} - \frac{3x\sin^2(2x)\sin(3x)\cos(2x)\cos(3x)}{8} + \frac{3x\sin(2x)\sin^2(3x)\cos^2(2x)}{16} - \frac{3x\sin(2x)\cos^2(2x)\cos^2(3x)}{16} + \frac{x\sin(3x)\cos^3(2x)\cos(3x)}{8} - \frac{\sin^3(2x)\sin(3x)\cos(3x)}{48} - \frac{\sin^2(2x)\cos(2x)\cos^2(3x)}{2} + \frac{5\sin(2x)\sin(3x)\cos^2(2x)\cos(3x)}{8} - \frac{9\sin^2(3x)\cos^3(2x)}{32} - \frac{5\cos^3(2x)\cos^2(3x)}{96}$$

input
```
integrate(cos(3*x)**2*sin(2*x)**3,x)
```

output
```
-x*sin(2*x)**3*sin(3*x)**2/16 + x*sin(2*x)**3*cos(3*x)**2/16 - 3*x*sin(2*x
)**2*sin(3*x)*cos(2*x)*cos(3*x)/8 + 3*x*sin(2*x)*sin(3*x)**2*cos(2*x)**2/1
6 - 3*x*sin(2*x)*cos(2*x)**2*cos(3*x)**2/16 + x*sin(3*x)*cos(2*x)**3*cos(3
*x)/8 - sin(2*x)**3*sin(3*x)*cos(3*x)/48 - sin(2*x)**2*cos(2*x)*cos(3*x)**
2/2 + 5*sin(2*x)*sin(3*x)*cos(2*x)**2*cos(3*x)/8 - 9*sin(3*x)**2*cos(2*x)*
*3/32 - 5*cos(2*x)**3*cos(3*x)**2/96
```

3.134.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \cos^2(3x)\sin^3(2x)\,dx = \frac{1}{192}\cos(12x) - \frac{3}{128}\cos(8x) + \frac{1}{48}\cos(6x) + \frac{3}{64}\cos(4x) - \frac{3}{16}\cos(2x)$$

input
```
integrate(cos(3*x)^2*sin(2*x)^3,x, algorithm="maxima")
```

output
```
1/192*cos(12*x) - 3/128*cos(8*x) + 1/48*cos(6*x) + 3/64*cos(4*x) - 3/16*co
s(2*x)
```

3.134.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \cos^2(3x)\sin^3(2x)\,dx = \frac{1}{192}\cos(12x) - \frac{3}{128}\cos(8x) + \frac{1}{48}\cos(6x) + \frac{3}{64}\cos(4x) - \frac{3}{16}\cos(2x)$$

input
```
integrate(cos(3*x)^2*sin(2*x)^3,x, algorithm="giac")
```

output
```
1/192*cos(12*x) - 3/128*cos(8*x) + 1/48*cos(6*x) + 3/64*cos(4*x) - 3/16*co
s(2*x)
```

3.134.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.61

$$\int \cos^2(3x)\sin^3(2x)\,dx = \frac{32\cos(x)^{12}}{3} - 32\cos(x)^{10} + 33\cos(x)^8 - 12\cos(x)^6$$

input
```
int(cos(3*x)^2*sin(2*x)^3,x)
```

output
```
33*cos(x)^8 - 12*cos(x)^6 - 32*cos(x)^10 + (32*cos(x)^12)/3
```

3.135 $\int \sin(a+bx)\sin(c+bx)\,dx$

3.135.1 Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \sin(a+bx)\sin(c+bx)\,dx = \frac{1}{2}x\cos(a-c) - \frac{\sin(a+c+2bx)}{4b}$$

output
```
1/2*x*cos(a-c)-1/4*sin(2*b*x+a+c)/b
```

3.135.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \sin(a+bx)\sin(c+bx)\,dx = -\frac{-2bx\cos(a-c)+\sin(a+c+2bx)}{4b}$$

input
```
Integrate[Sin[a + b*x]*Sin[c + b*x],x]
```

output
```
-1/4*(-2*b*x*Cos[a - c] + Sin[a + c + 2*b*x])/b
```

3.135.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5080, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sin(bx+c)\,dx$$

$$\Big\downarrow \ 5080$$

$$\int \left(\frac{1}{2}\cos(a-c)-\frac{1}{2}\cos(a+2bx+c)\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}x\cos(a-c)-\frac{\sin(a+2bx+c)}{4b}$$

input
```
Int[Sin[a + b*x]*Sin[c + b*x],x]
```

output
```
(x*Cos[a - c])/2 - Sin[a + c + 2*b*x]/(4*b)
```

3.135.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5080
```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

3.135.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result
default	$\frac{x\cos(a-c)}{2}-\frac{\sin(2xb+a+c)}{4b}$
risch	$\frac{x\cos(a-c)}{2}-\frac{\sin(2xb+a+c)}{4b}$
parallelrisch	$\frac{bx\left(\tan\left(\frac{a}{2}+\frac{xb}{2}\right)-1\right)\left(\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+1\right)\tan\left(\frac{xb}{2}+\frac{c}{2}\right)^{2}+\left(4\tan\left(\frac{a}{2}+\frac{xb}{2}\right)xb+2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}-2\right)\tan\left(\frac{xb}{2}+\frac{c}{2}\right)-\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}xb}{2b\left(1+\tan\left(\frac{xb}{2}+\frac{c}{2}\right)^{2}\right)\left(1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}\right)}$
norman	$\frac{-\frac{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{b}+\frac{x}{2}-\frac{x\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}}{2}-\frac{x\tan\left(\frac{xb}{2}+\frac{c}{2}\right)^{2}}{2}+2x\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\tan\left(\frac{xb}{2}+\frac{c}{2}\right)+\frac{x\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}\tan\left(\frac{xb}{2}+\frac{c}{2}\right)^{2}}{2}+\frac{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\tan}{b}}{\left(1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}\right)\left(1+\tan\left(\frac{xb}{2}+\frac{c}{2}\right)^{2}\right)}$

input
```
int(sin(b*x+a)*sin(b*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*cos(a-c)-1/4*sin(2*b*x+a+c)/b
```

3.135.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(23) = 46$.

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.85

$$\int \sin(a+bx)\sin(c+bx)\,dx$$
$$=\frac{bx\cos(-a+c)-\cos(bx+c)\cos(-a+c)\sin(bx+c)+\cos(bx+c)^{2}\sin(-a+c)}{2\,b}$$

input
```
integrate(sin(b*x+a)*sin(b*x+c),x, algorithm="fricas")
```

output
```
1/2*(b*x*cos(-a + c) - cos(b*x + c)*cos(-a + c)*sin(b*x + c) + cos(b*x + c
)^2*sin(-a + c))/b
```

3.135.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(20) = 40$.

Time = 0.18 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.15

$$\int \sin(a+bx)\sin(c+bx)\,dx = \begin{cases} \frac{x\sin(a+bx)\sin(bx+c)}{2} + \frac{x\cos(a+bx)\cos(bx+c)}{2} - \frac{\sin(bx+c)\cos(a+bx)}{2b} & \text{for } b \neq 0 \\ x\sin(a)\sin(c) & \text{otherwise} \end{cases}$$

input
```
integrate(sin(b*x+a)*sin(b*x+c),x)
```

output
```
Piecewise((x*sin(a + b*x)*sin(b*x + c)/2 + x*cos(a + b*x)*cos(b*x + c)/2 -
 sin(b*x + c)*cos(a + b*x)/(2*b), Ne(b, 0)), (x*sin(a)*sin(c), True))
```

3.135.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \sin(a+bx)\sin(c+bx)\,dx = \frac{1}{2}x\cos(-a+c) - \frac{\sin(2bx+a+c)}{4b}$$

input
```
integrate(sin(b*x+a)*sin(b*x+c),x, algorithm="maxima")
```

output
```
1/2*x*cos(-a + c) - 1/4*sin(2*b*x + a + c)/b
```

3.135.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \sin(a+bx)\sin(c+bx)\,dx = \frac{1}{2}x\cos(a-c) - \frac{\sin(2bx+a+c)}{4b}$$

input
```
integrate(sin(b*x+a)*sin(b*x+c),x, algorithm="giac")
```

output
```
1/2*x*cos(a - c) - 1/4*sin(2*b*x + a + c)/b
```

3.135.9 Mupad [B] (verification not implemented)

Time = 27.00 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.33

$$\int \sin(a+bx)\sin(c+bx)\,dx = \begin{cases} x\,\sin(a)\,\sin(c) & \text{if } b=0 \\ \frac{x\,\cos(a-c)}{2}-\frac{\sin(a+c+2\,b\,x)}{4\,b} & \text{if } b\neq 0 \end{cases}$$

input
```
int(sin(a + b*x)*sin(c + b*x),x)
```

output
```
piecewise(b == 0, x*sin(a)*sin(c), b ~= 0, (x*cos(a - c))/2 - sin(a + c +
2*b*x)/(4*b))
```

3.136 $\int \sin(c-bx)\sin(a+bx)\,dx$

3.136.1 Optimal result

Integrand size = 14, antiderivative size = 27

$$\int \sin(c-bx)\sin(a+bx)\,dx = -\frac{1}{2}x\cos(a+c)+\frac{\sin(a-c+2bx)}{4b}$$

output
```
-1/2*x*cos(a+c)+1/4*sin(2*b*x+a-c)/b
```

3.136.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \sin(c-bx)\sin(a+bx)\,dx = \frac{-2bx\cos(a+c)+\sin(a-c+2bx)}{4b}$$

input
```
Integrate[Sin[c - b*x]*Sin[a + b*x],x]
```

output
```
(-2*b*x*Cos[a + c] + Sin[a - c + 2*b*x])/(4*b)
```

3.136.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5080, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\sin(c-bx)\,dx$$

$$\Big\downarrow \text{5080}$$

$$\int \left(\frac{1}{2}\cos(a+2bx-c)-\frac{1}{2}\cos(a+c)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sin(a+2bx-c)}{4b}-\frac{1}{2}x\cos(a+c)$$

input
```
Int[Sin[c - b*x]*Sin[a + b*x],x]
```

output
```
-1/2*(x*Cos[a + c]) + Sin[a - c + 2*b*x]/(4*b)
```

3.136.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5080
```
Int[Sin[v_]^(p_.)*Sin[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Sin[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

3.136.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result
default	$-\frac{x\cos(a+c)}{2}+\frac{\sin(2xb+a-c)}{4b}$
risch	$-\frac{x\cos(a+c)}{2}+\frac{\sin(2xb+a-c)}{4b}$
parallelrisch	$-\frac{2x\cos(a+c)b-\sin(2xb+a-c)+\sin(a+c)}{4b}$
norman	$\frac{\frac{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{b}-\frac{x}{2}+\frac{x\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}}{2}+\frac{x\tan\left(\frac{xb}{2}-\frac{c}{2}\right)^{2}}{2}-2x\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\tan\left(\frac{xb}{2}-\frac{c}{2}\right)-\frac{x\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}\tan\left(\frac{xb}{2}-\frac{c}{2}\right)^{2}}{2}-\frac{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\tan(}{b}}{\left(1+\tan\left(\frac{xb}{2}-\frac{c}{2}\right)^{2}\right)\left(1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}\right)}$

input
```
int(-sin(b*x-c)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x*cos(a+c)+1/4*sin(2*b*x+a-c)/b
```

3.136.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.63

$$\int \sin(c-bx)\sin(a+bx)\,dx$$
$$=-\frac{bx\cos(a+c)-\cos(bx+a)\cos(a+c)\sin(bx+a)+\cos(bx+a)^{2}\sin(a+c)}{2\,b}$$

input
```
integrate(-sin(b*x-c)*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(b*x*cos(a + c) - cos(b*x + a)*cos(a + c)*sin(b*x + a) + cos(b*x + a)
^2*sin(a + c))/b
```

3.136.6 Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.26

$$\int \sin(c-bx)\sin(a+bx)\,dx = -\begin{cases} \frac{x\sin(a+bx)\sin(bx-c)}{2} + \frac{x\cos(a+bx)\cos(bx-c)}{2} - \frac{\sin(a+bx)\cos(bx-c)}{2b} & \text{for } b\neq 0 \\ -x\sin(a)\sin(c) & \text{otherwise} \end{cases}$$

input
```
integrate(-sin(b*x-c)*sin(b*x+a),x)
```

output
```
-Piecewise((x*sin(a + b*x)*sin(b*x - c)/2 + x*cos(a + b*x)*cos(b*x - c)/2
- sin(a + b*x)*cos(b*x - c)/(2*b), Ne(b, 0)), (-x*sin(a)*sin(c), True))
```

3.136.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \sin(c-bx)\sin(a+bx)\,dx = -\frac{1}{2}\,x\cos(a+c) + \frac{\sin(2\,bx+a-c)}{4\,b}$$

input
```
integrate(-sin(b*x-c)*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*x*cos(a + c) + 1/4*sin(2*b*x + a - c)/b
```

3.136.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \sin(c-bx)\sin(a+bx)\,dx = -\frac{1}{2}\,x\cos(a+c) + \frac{\sin(2\,bx+a-c)}{4\,b}$$

input
```
integrate(-sin(b*x-c)*sin(b*x+a),x, algorithm="giac")
```

output
```
-1/2*x*cos(a + c) + 1/4*sin(2*b*x + a - c)/b
```

3.136.9 Mupad [B] (verification not implemented)

Time = 27.34 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.33

$$\int \sin(c-bx)\sin(a+bx)\,dx = \begin{cases} x\,\sin(a)\,\sin(c) & \text{if } b=0 \\ \frac{\sin(a-c+2\,b\,x)}{4\,b} - \frac{x\,\cos(a+c)}{2} & \text{if } b\neq 0 \end{cases}$$

input
```
int(sin(a + b*x)*sin(c - b*x),x)
```

output
```
piecewise(b == 0, x*sin(a)*sin(c), b ~= 0, sin(a - c + 2*b*x)/(4*b) - (x*c
os(a + c))/2)
```

3.137 $\int \cos(a+bx)\cos(c+bx)\,dx$

3.137.1 Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \cos(a+bx)\cos(c+bx)\,dx = \frac{1}{2}x\cos(a-c) + \frac{\sin(a+c+2bx)}{4b}$$

output
```
1/2*x*cos(a-c)+1/4*sin(2*b*x+a+c)/b
```

3.137.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \cos(a+bx)\cos(c+bx)\,dx = \frac{2bx\cos(a-c)+\sin(a+c+2bx)}{4b}$$

input
```
Integrate[Cos[a + b*x]*Cos[c + b*x],x]
```

output
```
(2*b*x*Cos[a - c] + Sin[a + c + 2*b*x])/(4*b)
```

3.137.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\cos(bx+c)\,dx$$

$$\Big\downarrow \ 5081$$

$$\int \left(\frac{1}{2}\cos(a+2bx+c)+\frac{1}{2}\cos(a-c)\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{\sin(a+2bx+c)}{4b}+\frac{1}{2}x\cos(a-c)$$

input
```
Int[Cos[a + b*x]*Cos[c + b*x],x]
```

output
```
(x*Cos[a - c])/2 + Sin[a + c + 2*b*x]/(4*b)
```

3.137.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5081
```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

3.137.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result
default	$\frac{x\cos(a-c)}{2}+\frac{\sin(2xb+a+c)}{4b}$
risch	$\frac{x\cos(a-c)}{2}+\frac{\sin(2xb+a+c)}{4b}$
parallelrisch	$\frac{bx\left(\tan\left(\frac{a}{2}+\frac{xb}{2}\right)-1\right)\left(\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+1\right)\tan\left(\frac{xb}{2}+\frac{c}{2}\right)^{2}+\left(4\tan\left(\frac{a}{2}+\frac{xb}{2}\right)xb-2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}+2\right)\tan\left(\frac{xb}{2}+\frac{c}{2}\right)-\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}xb}{2b\left(1+\tan\left(\frac{xb}{2}+\frac{c}{2}\right)^{2}\right)\left(1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}\right)}$
norman	$\frac{\frac{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{b}+\frac{x}{2}-\frac{x\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}}{2}-\frac{x\tan\left(\frac{xb}{2}+\frac{c}{2}\right)^{2}}{2}+2x\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\tan\left(\frac{xb}{2}+\frac{c}{2}\right)+\frac{x\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}\tan\left(\frac{xb}{2}+\frac{c}{2}\right)^{2}}{2}-\frac{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\tan(}{b}}{\left(1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}\right)\left(1+\tan\left(\frac{xb}{2}+\frac{c}{2}\right)^{2}\right)}$

input
```
int(cos(b*x+a)*cos(b*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*cos(a-c)+1/4*sin(2*b*x+a+c)/b
```

3.137.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(23) = 46$.

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.85

$$\int \cos(a+bx)\cos(c+bx)\,dx$$
$$=\frac{bx\cos(-a+c)+\cos(bx+c)\cos(-a+c)\sin(bx+c)-\cos(bx+c)^{2}\sin(-a+c)}{2\,b}$$

input
```
integrate(cos(b*x+a)*cos(b*x+c),x, algorithm="fricas")
```

output
```
1/2*(b*x*cos(-a + c) + cos(b*x + c)*cos(-a + c)*sin(b*x + c) - cos(b*x + c
)^2*sin(-a + c))/b
```

3.137.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(20) = 40$.

Time = 0.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.15

$$\int \cos(a+bx)\cos(c+bx)\,dx = \begin{cases} \frac{x\sin(a+bx)\sin(bx+c)}{2} + \frac{x\cos(a+bx)\cos(bx+c)}{2} + \frac{\sin(bx+c)\cos(a+bx)}{2b} & \text{for } b \neq 0 \\ x\cos(a)\cos(c) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(b*x+a)*cos(b*x+c),x)
```

output
```
Piecewise((x*sin(a + b*x)*sin(b*x + c)/2 + x*cos(a + b*x)*cos(b*x + c)/2 +
 sin(b*x + c)*cos(a + b*x)/(2*b), Ne(b, 0)), (x*cos(a)*cos(c), True))
```

3.137.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \cos(a+bx)\cos(c+bx)\,dx = \frac{1}{2}\,x\cos(-a+c) + \frac{\sin(2\,bx+a+c)}{4\,b}$$

input
```
integrate(cos(b*x+a)*cos(b*x+c),x, algorithm="maxima")
```

output
```
1/2*x*cos(-a + c) + 1/4*sin(2*b*x + a + c)/b
```

3.137.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \cos(a+bx)\cos(c+bx)\,dx = \frac{1}{2}\,x\cos(a-c) + \frac{\sin(2\,bx+a+c)}{4\,b}$$

input
```
integrate(cos(b*x+a)*cos(b*x+c),x, algorithm="giac")
```

output
```
1/2*x*cos(a - c) + 1/4*sin(2*b*x + a + c)/b
```

3.137.9 Mupad [B] (verification not implemented)

Time = 27.38 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.33

$$\int \cos(a+bx)\cos(c+bx)\,dx = \begin{cases} x\,\cos(a)\,\cos(c) & \text{if } b=0 \\ \frac{x\,\cos(a-c)}{2}+\frac{\sin(a+c+2\,b\,x)}{4\,b} & \text{if } b\neq 0 \end{cases}$$

input
```
int(cos(a + b*x)*cos(c + b*x),x)
```

output
```
piecewise(b == 0, x*cos(a)*cos(c), b ~= 0, (x*cos(a - c))/2 + sin(a + c +
2*b*x)/(4*b))
```

3.138 $\int \cos(c-bx)\cos(a+bx)\,dx$

3.138.1 Optimal result

Integrand size = 14, antiderivative size = 27

$$\int \cos(c-bx)\cos(a+bx)\,dx=\frac{1}{2}x\cos(a+c)+\frac{\sin(a-c+2bx)}{4b}$$

output
```
1/2*x*cos(a+c)+1/4*sin(2*b*x+a-c)/b
```

3.138.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \cos(c-bx)\cos(a+bx)\,dx=\frac{2bx\cos(a+c)+\sin(a-c+2bx)}{4b}$$

input
```
Integrate[Cos[c - b*x]*Cos[a + b*x],x]
```

output
```
(2*b*x*Cos[a + c] + Sin[a - c + 2*b*x])/(4*b)
```

3.138.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\cos(c-bx)\,dx$$

$$\Big\downarrow \text{5081}$$

$$\int \left(\frac{1}{2}\cos(a+2bx-c)+\frac{1}{2}\cos(a+c)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sin(a+2bx-c)}{4b}+\frac{1}{2}x\cos(a+c)$$

input
```
Int[Cos[c - b*x]*Cos[a + b*x],x]
```

output
```
(x*Cos[a + c])/2 + Sin[a - c + 2*b*x]/(4*b)
```

3.138.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5081
```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.), x_Symbol] :> Int[ExpandTrigReduce[Cos[v]^p
*Cos[w]^q, x], x] /; ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (Binomial
Q[{v, w}, x] && IndependentQ[Cancel[v/w], x])) && IGtQ[p, 0] && IGtQ[q, 0]
```

3.138.4 Maple [A] (verified)

Time = 0.46 (sec), antiderivative size = 24, normalized size of antiderivative = 0.89

method	result
default	$\frac{x\cos(a+c)}{2}+\frac{\sin(2xb+a-c)}{4b}$
risch	$\frac{x\cos(a+c)}{2}+\frac{\sin(2xb+a-c)}{4b}$
parallelrisch	$\frac{2x\cos(a+c)b-\sin(a+c)+\sin(2xb+a-c)}{4b}$
norman	$\frac{\frac{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{b}+\frac{x}{2}-\frac{x\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2}{2}-\frac{x\tan\left(\frac{xb}{2}-\frac{c}{2}\right)^2}{2}+2x\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\tan\left(\frac{xb}{2}-\frac{c}{2}\right)+\frac{x\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2\tan\left(\frac{xb}{2}-\frac{c}{2}\right)^2}{2}-\frac{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\tan(}{b}}{\left(1+\tan\left(\frac{xb}{2}-\frac{c}{2}\right)^2\right)\left(1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2\right)}$

input
```
int(cos(b*x-c)*cos(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*cos(a+c)+1/4*sin(2*b*x+a-c)/b
```

3.138.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec), antiderivative size = 44, normalized size of antiderivative = 1.63

$$\int \cos(c-bx)\cos(a+bx)\,dx = \frac{bx\cos(a+c)+\cos(bx+a)\cos(a+c)\sin(bx+a)-\cos(bx+a)^2\sin(a+c)}{2\,b}$$

input
```
integrate(cos(b*x-c)*cos(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(b*x*cos(a + c) + cos(b*x + a)*cos(a + c)*sin(b*x + a) - cos(b*x + a)^
2*sin(a + c))/b
```

3.138.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(20) = 40$.

Time = 0.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.15

$$\int \cos(c-bx)\cos(a+bx)\,dx = \begin{cases} \frac{x\sin(a+bx)\sin(bx-c)}{2} + \frac{x\cos(a+bx)\cos(bx-c)}{2} + \frac{\sin(bx-c)\cos(a+bx)}{2b} & \text{for } b \neq 0 \\ x\cos(a)\cos(c) & \text{otherwise} \end{cases}$$

input
```
integrate(cos(b*x-c)*cos(b*x+a),x)
```

output
```
Piecewise((x*sin(a + b*x)*sin(b*x - c)/2 + x*cos(a + b*x)*cos(b*x - c)/2 +
 sin(b*x - c)*cos(a + b*x)/(2*b), Ne(b, 0)), (x*cos(a)*cos(c), True))
```

3.138.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \cos(c-bx)\cos(a+bx)\,dx = \frac{1}{2}x\cos(a+c) + \frac{\sin(2bx+a-c)}{4b}$$

input
```
integrate(cos(b*x-c)*cos(b*x+a),x, algorithm="maxima")
```

output
```
1/2*x*cos(a + c) + 1/4*sin(2*b*x + a - c)/b
```

3.138.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \cos(c-bx)\cos(a+bx)\,dx = \frac{1}{2}x\cos(a+c) + \frac{\sin(2bx+a-c)}{4b}$$

input
```
integrate(cos(b*x-c)*cos(b*x+a),x, algorithm="giac")
```

output
```
1/2*x*cos(a + c) + 1/4*sin(2*b*x + a - c)/b
```

3.138.9 Mupad [B] (verification not implemented)

Time = 27.90 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.33

$$\int \cos(c-bx)\cos(a+bx)\,dx = \begin{cases} x\,\cos(a)\,\cos(c) & \text{if } b=0 \\ \frac{\sin(a-c+2\,b\,x)}{4\,b}+\frac{x\,\cos(a+c)}{2} & \text{if } b\neq 0 \end{cases}$$

input
```
int(cos(a + b*x)*cos(c - b*x),x)
```

output
```
piecewise(b == 0, x*cos(a)*cos(c), b ~= 0, sin(a - c + 2*b*x)/(4*b) + (x*c
os(a + c))/2)
```

3.139 $\int \tan(a+bx)\tan(c+bx)\,dx$

3.139.1 Optimal result

Integrand size = 13, antiderivative size = 39

$$\int \tan(a+bx)\tan(c+bx)\,dx = -x-\frac{\cot(a-c)\log(\cos(a+bx))}{b}+\frac{\cot(a-c)\log(\cos(c+bx))}{b}$$

output
```
-x-cot(a-c)*ln(cos(b*x+a))/b+cot(a-c)*ln(cos(b*x+c))/b
```

3.139.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \tan(a+bx)\tan(c+bx)\,dx = -x+\frac{\cot(a-c)(-\log(\cos(a+bx))+\log(\cos(c+bx)))}{b}$$

input
```
Integrate[Tan[a + b*x]*Tan[c + b*x],x]
```

output
```
-x + (Cot[a - c]*(-Log[Cos[a + b*x]] + Log[Cos[c + b*x]]))/b
```

3.139.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.21, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5123, 5121, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(a+bx)\tan(bx+c)\,dx$$

$$\Big\downarrow \text{5123}$$

$$\cos(a-c)\int \sec(a+bx)\sec(c+bx)dx-x$$

$$\Big\downarrow \text{5121}$$

$$\cos(a-c)\left(\csc(a-c)\int \tan(a+bx)dx-\csc(a-c)\int \tan(c+bx)dx\right)-x$$

$$\Big\downarrow \text{3042}$$

$$\cos(a-c)\left(\csc(a-c)\int \tan(a+bx)dx-\csc(a-c)\int \tan(c+bx)dx\right)-x$$

$$\Big\downarrow \text{3956}$$

$$\cos(a-c)\left(\frac{\csc(a-c)\log(\cos(bx+c))}{b}-\frac{\csc(a-c)\log(\cos(a+bx))}{b}\right)-x$$

input
```
Int[Tan[a + b*x]*Tan[c + b*x],x]
```

output
```
-x + Cos[a - c]*(-((Csc[a - c]*Log[Cos[a + b*x]])/b) + (Csc[a - c]*Log[Cos
[c + b*x]])/b)
```

3.139.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 5121
```
Int[Sec[(a_.) + (b_.)*(x_)]*Sec[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[-Csc[
(b*c - a*d)/d]   Int[Tan[a + b*x], x], x] + Simp[Csc[(b*c - a*d)/b]   Int[T
an[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b
*c - a*d, 0]
```

rule 5123
```
Int[Tan[(a_.) + (b_.)*(x_)]*Tan[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[(-b)*
(x/d), x] + Simp[(b/d)*Cos[(b*c - a*d)/d]   Int[Sec[a + b*x]*Sec[c + d*x],
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c - a*d, 0]
```

3.139.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 173, normalized size of antiderivative = 4.44

method	result	size
risch	$-x+\frac{i\ln\left(\mathrm{e}^{2i(xb+a)}+\mathrm{e}^{2i(a-c)}\right)\mathrm{e}^{2ia}}{b\left(\mathrm{e}^{2ia}-\mathrm{e}^{2ic}\right)}+\frac{i\ln\left(\mathrm{e}^{2i(xb+a)}+\mathrm{e}^{2i(a-c)}\right)\mathrm{e}^{2ic}}{b\left(\mathrm{e}^{2ia}-\mathrm{e}^{2ic}\right)}-\frac{i\ln\left(\mathrm{e}^{2i(xb+a)}+1\right)\mathrm{e}^{2ia}}{b\left(\mathrm{e}^{2ia}-\mathrm{e}^{2ic}\right)}-\frac{i\ln\left(\mathrm{e}^{2i(xb+a)}+1\right)\mathrm{e}^{2ic}}{b\left(\mathrm{e}^{2ia}-\mathrm{e}^{2ic}\right)}$	173

input
```
int(tan(b*x+a)*tan(b*x+c),x,method=_RETURNVERBOSE)
```

output
```
-x+I/b/(exp(2*I*a)-exp(2*I*c))*ln(exp(2*I*(b*x+a))+exp(2*I*(a-c)))*exp(2*I
*a)+I/b/(exp(2*I*a)-exp(2*I*c))*ln(exp(2*I*(b*x+a))+exp(2*I*(a-c)))*exp(2*
I*c)-I/b/(exp(2*I*a)-exp(2*I*c))*ln(exp(2*I*(b*x+a))+1)*exp(2*I*a)-I/b/(ex
p(2*I*a)-exp(2*I*c))*ln(exp(2*I*(b*x+a))+1)*exp(2*I*c)
```

3.139.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 145 vs. 2(39) = 78.

Time = 0.25 (sec) , antiderivative size = 145, normalized size of antiderivative = 3.72

$$\int \tan(a+bx)\tan(c+bx)\,dx =$$
$$-\frac{2\,bx\sin\left(-2\,a+2\,c\right)-\left(\cos\left(-2\,a+2\,c\right)+1\right)\log\left(-\frac{\left(\cos\left(-2\,a+2\,c\right)-1\right)\tan\left(bx+c\right)^{2}-2\,\sin\left(-2\,a+2\,c\right)\tan\left(bx+c\right)-\cos\left(-2\,a\right.}{\left(\cos\left(-2\,a+2\,c\right)+1\right)\tan\left(bx+c\right)^{2}+\cos\left(-2\,a+2\,c\right)+1}\right.}{2\,b\sin\left(-2\,a+2\,c\right)}$$

input
```
integrate(tan(b*x+a)*tan(b*x+c),x, algorithm="fricas")
```

output
```
-1/2*(2*b*x*sin(-2*a + 2*c) - (cos(-2*a + 2*c) + 1)*log(-((cos(-2*a + 2*c)
 - 1)*tan(b*x + c)^2 - 2*sin(-2*a + 2*c)*tan(b*x + c) - cos(-2*a + 2*c) -
1)/((cos(-2*a + 2*c) + 1)*tan(b*x + c)^2 + cos(-2*a + 2*c) + 1)) + (cos(-2
*a + 2*c) + 1)*log(1/(tan(b*x + c)^2 + 1)))/(b*sin(-2*a + 2*c))
```

3.139.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2020 vs. $2(31) = 62$.

Time = 2.88 (sec) , antiderivative size = 7672, normalized size of antiderivative = 196.72

$$\int \tan(a+bx)\tan(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(tan(b*x+a)*tan(b*x+c),x)
```

output
```
Piecewise((0, Eq(a, 0) & Eq(b, 0) & Eq(c, 0)), (-2*b*x*tan(c)/(2*b*tan(c)*
*2 + 2*b) - 2*log(tan(b*x) - 1/tan(c))/(2*b*tan(c)**2 + 2*b) + log(tan(b*x
)**2 + 1)/(2*b*tan(c)**2 + 2*b), Eq(a, 0)), (-2*b*x*tan(a)/(2*b*tan(a)**2
+ 2*b) - 2*log(tan(b*x) - 1/tan(a))/(2*b*tan(a)**2 + 2*b) + log(tan(b*x)**
2 + 1)/(2*b*tan(a)**2 + 2*b), Eq(c, 0)), (-4*b*x*tan(c)**2*tan(b*x)/(2*b*t
an(c)**5*tan(b*x) - 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) - 4*b*tan(c)**2
 + 2*b*tan(c)*tan(b*x) - 2*b) + 4*b*x*tan(c)/(2*b*tan(c)**5*tan(b*x) - 2*b
*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) - 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x)
- 2*b) + 2*log(tan(b*x) - 1/tan(c))*tan(c)**3*tan(b*x)/(2*b*tan(c)**5*tan(
b*x) - 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) - 4*b*tan(c)**2 + 2*b*tan(c)
*tan(b*x) - 2*b) - 2*log(tan(b*x) - 1/tan(c))*tan(c)**2/(2*b*tan(c)**5*tan
(b*x) - 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) - 4*b*tan(c)**2 + 2*b*tan(c
)*tan(b*x) - 2*b) - 2*log(tan(b*x) - 1/tan(c))*tan(c)*tan(b*x)/(2*b*tan(c)
**5*tan(b*x) - 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) - 4*b*tan(c)**2 + 2*
b*tan(c)*tan(b*x) - 2*b) + 2*log(tan(b*x) - 1/tan(c))/(2*b*tan(c)**5*tan(b
*x) - 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) - 4*b*tan(c)**2 + 2*b*tan(c)*
tan(b*x) - 2*b) - log(tan(b*x)**2 + 1)*tan(c)**3*tan(b*x)/(2*b*tan(c)**5*t
an(b*x) - 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) - 4*b*tan(c)**2 + 2*b*tan
(c)*tan(b*x) - 2*b) + log(tan(b*x)**2 + 1)*tan(c)**2/(2*b*tan(c)**5*tan(b*
x) - 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) - 4*b*tan(c)**2 + 2*b*tan(c...
```

3.139.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. 2(39) = 78.

Time = 0.22 (sec) , antiderivative size = 371, normalized size of antiderivative = 9.51

$$\int \tan(a+bx)\tan(c+bx)\,dx = -\frac{\left(2\,b\cos\left(2\,a\right)\cos\left(2\,c\right) - b\cos\left(2\,c\right)^{2} + 2\,b\sin\left(2\,a\right)\sin\left(2\,c\right) - b\sin\left(2\,c\right)^{2} - \left(\cos\left(2\,a\right)^{2} + \sin\left(2\,a\right)^{2}\right)b\right)x + \left(\right.}{}$$

input
```
integrate(tan(b*x+a)*tan(b*x+c),x, algorithm="maxima")
```

output
```
-((2*b*cos(2*a)*cos(2*c) - b*cos(2*c)^2 + 2*b*sin(2*a)*sin(2*c) - b*sin(2*
c)^2 - (cos(2*a)^2 + sin(2*a)^2)*b)*x + (cos(2*a)^2 - cos(2*c)^2 + sin(2*a
)^2 - sin(2*c)^2)*arctan2(sin(2*b*x) - sin(2*a), cos(2*b*x) + cos(2*a)) -
(cos(2*a)^2 - cos(2*c)^2 + sin(2*a)^2 - sin(2*c)^2)*arctan2(sin(2*b*x) - s
in(2*c), cos(2*b*x) + cos(2*c)) - (cos(2*c)*sin(2*a) - cos(2*a)*sin(2*c))*
log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) + cos(2*a)^2 + sin(2*b*x)^2 - 2*s
in(2*b*x)*sin(2*a) + sin(2*a)^2) + (cos(2*c)*sin(2*a) - cos(2*a)*sin(2*c))
*log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*c) + cos(2*c)^2 + sin(2*b*x)^2 - 2*
sin(2*b*x)*sin(2*c) + sin(2*c)^2))/(2*b*cos(2*a)*cos(2*c) - b*cos(2*c)^2 +
 2*b*sin(2*a)*sin(2*c) - b*sin(2*c)^2 - (cos(2*a)^2 + sin(2*a)^2)*b)
```

3.139.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. 2(39) = 78.

Time = 0.29 (sec) , antiderivative size = 242, normalized size of antiderivative = 6.21

$$\int \tan(a+bx)\tan(c+bx)\,dx = -\frac{2\,bx + \frac{\left(\tan\left(\frac{1}{2}\,a\right)^{3}\tan\left(\frac{1}{2}\,c\right)^{2} - \tan\left(\frac{1}{2}\,a\right)^{3} + 4\,\tan\left(\frac{1}{2}\,a\right)^{2}\tan\left(\frac{1}{2}\,c\right) - \tan\left(\frac{1}{2}\,a\right)\tan\left(\frac{1}{2}\,c\right)^{2} + \tan\left(\frac{1}{2}\,a\right)\right)\log\left(\left|2\,\tan\left(bx\right)\tan\left(\frac{1}{2}\,a\right) + \tan\left(\frac{1}{2}\,a\right)^{2} - 1\right|\right)}{\tan\left(\frac{1}{2}\,a\right)^{3}\tan\left(\frac{1}{2}\,c\right) - \tan\left(\frac{1}{2}\,a\right)^{2}\tan\left(\frac{1}{2}\,c\right)^{2} + \tan\left(\frac{1}{2}\,a\right)^{2} - \tan\left(\frac{1}{2}\,a\right)\tan\left(\frac{1}{2}\,c\right)}}{2\,b}$$

input
```
integrate(tan(b*x+a)*tan(b*x+c),x, algorithm="giac")
```

output
```
-1/2*(2*b*x + (tan(1/2*a)^3*tan(1/2*c)^2 - tan(1/2*a)^3 + 4*tan(1/2*a)^2*t
an(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a))*log(abs(2*tan(b*x)*tan(1
/2*a) + tan(1/2*a)^2 - 1))/(tan(1/2*a)^3*tan(1/2*c) - tan(1/2*a)^2*tan(1/2
*c)^2 + tan(1/2*a)^2 - tan(1/2*a)*tan(1/2*c)) - (tan(1/2*a)^2*tan(1/2*c)^3
 - tan(1/2*a)^2*tan(1/2*c) + 4*tan(1/2*a)*tan(1/2*c)^2 - tan(1/2*c)^3 + ta
n(1/2*c))*log(abs(2*tan(b*x)*tan(1/2*c) + tan(1/2*c)^2 - 1))/(tan(1/2*a)^2
*tan(1/2*c)^2 - tan(1/2*a)*tan(1/2*c)^3 + tan(1/2*a)*tan(1/2*c) - tan(1/2*
c)^2))/b
```

3.139.9 Mupad [B] (verification not implemented)

Time = 31.89 (sec) , antiderivative size = 207, normalized size of antiderivative = 5.31

$$\int \tan(a+bx)\tan(c+bx)\,dx = -\frac{\frac{x}{2}+x\left(\sin\left(a-c\right)^2-\frac{1}{2}\right)}{\sin\left(a-c\right)^2} - \frac{\frac{\sin\left(2\,a-2\,c\right)\,\ln\left(\sin\left(2\,a-2\,c\right)^2\,2\mathrm{i}-\sin\left(a+b\,x\right)^2\,2\mathrm{i}+\sin\left(3\,a-2\,c+b\,x\right)^2\,2\mathrm{i}+\sin\left(4\,a-4\,c\right)+\sin\left(6\,a-4\,c+2\,b\,x\right)-\sin\left(2\,a+2\,b\,x\right)\right)}{2}-\frac{\sin\left(2\,a-2\,c\right)\,\ln\left(s\right.}{}}{b\sin\left(a-c\right)^2}$$

input
```
int(tan(a + b*x)*tan(c + b*x),x)
```

output
```
- (x/2 + x*(sin(a - c)^2 - 1/2))/sin(a - c)^2 - ((sin(2*a - 2*c)*log(sin(4
*a - 4*c) + sin(6*a - 4*c + 2*b*x) - sin(2*a + 2*b*x) + sin(2*a - 2*c)^2*2
i - sin(a + b*x)^2*2i + sin(3*a - 2*c + b*x)^2*2i))/2 - (sin(2*a - 2*c)*lo
g(sin(4*a - 4*c) + sin(4*a - 2*c + 2*b*x) - sin(2*c + 2*b*x) + sin(2*a - 2
*c)^2*2i - sin(c + b*x)^2*2i + sin(2*a - c + b*x)^2*2i))/2)/(b*sin(a - c)^
2)
```

3.140 $\int \tan(c-bx)\tan(a+bx)\,dx$

3.140.1 Optimal result

Integrand size = 14, antiderivative size = 34

$$\int \tan(c-bx)\tan(a+bx)\,dx = x-\frac{\cot(a+c)\log(\cos(c-bx))}{b}+\frac{\cot(a+c)\log(\cos(a+bx))}{b}$$

output
```
x-cot(a+c)*ln(cos(b*x-c))/b+cot(a+c)*ln(cos(b*x+a))/b
```

3.140.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int \tan(c-bx)\tan(a+bx)\,dx = x+\frac{\cot(a+c)(-\log(\cos(c-bx))+\log(\cos(a+bx)))}{b}$$

input
```
Integrate[Tan[c - b*x]*Tan[a + b*x],x]
```

output
```
x + (Cot[a + c]*(-Log[Cos[c - b*x]] + Log[Cos[a + b*x]]))/b
```

3.140.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.21, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5123, 5121, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(a+bx)\tan(c-bx)\,dx$$

$$\Big\downarrow \text{5123}$$

$$x-\cos(a+c)\int \sec(c-bx)\sec(a+bx)dx$$

$$\Big\downarrow \text{5121}$$

$$x-\cos(a+c)\left(\csc(a+c)\int \tan(c-bx)dx+\csc(a+c)\int \tan(a+bx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$x-\cos(a+c)\left(\csc(a+c)\int \tan(c-bx)dx+\csc(a+c)\int \tan(a+bx)dx\right)$$

$$\Big\downarrow \text{3956}$$

$$x-\cos(a+c)\left(\frac{\csc(a+c)\log(\cos(c-bx))}{b}-\frac{\csc(a+c)\log(\cos(a+bx))}{b}\right)$$

input
```
Int[Tan[c - b*x]*Tan[a + b*x],x]
```

output
```
x - Cos[a + c]*((Csc[a + c]*Log[Cos[c - b*x]])/b - (Csc[a + c]*Log[Cos[a +
 b*x]])/b)
```

3.140.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 5121
```
Int[Sec[(a_.) + (b_.)*(x_)]*Sec[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[-Csc[
(b*c - a*d)/d]   Int[Tan[a + b*x], x], x] + Simp[Csc[(b*c - a*d)/b]   Int[T
an[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b
*c - a*d, 0]
```

rule 5123
```
Int[Tan[(a_.) + (b_.)*(x_)]*Tan[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[(-b)*
(x/d), x] + Simp[(b/d)*Cos[(b*c - a*d)/d]   Int[Sec[a + b*x]*Sec[c + d*x],
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c - a*d, 0]
```

3.140.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 145, normalized size of antiderivative = 4.26

method	result	size
risch	$x+\frac{i\ln\left(\mathrm{e}^{2i(xb+a)}+1\right)\mathrm{e}^{2i(a+c)}}{b\left(\mathrm{e}^{2i(a+c)}-1\right)}+\frac{i\ln\left(\mathrm{e}^{2i(xb+a)}+1\right)}{b\left(\mathrm{e}^{2i(a+c)}-1\right)}-\frac{i\ln\left(\mathrm{e}^{2i(a+c)}+\mathrm{e}^{2i(xb+a)}\right)\mathrm{e}^{2i(a+c)}}{b\left(\mathrm{e}^{2i(a+c)}-1\right)}-\frac{i\ln\left(\mathrm{e}^{2i(a+c)}+\mathrm{e}^{2i(xb+a)}\right)}{b\left(\mathrm{e}^{2i(a+c)}-1\right)}$	145

input
```
int(-tan(b*x-c)*tan(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
x+I/b/(exp(2*I*(a+c))-1)*ln(exp(2*I*(b*x+a))+1)*exp(2*I*(a+c))+I/b/(exp(2*
I*(a+c))-1)*ln(exp(2*I*(b*x+a))+1)-I/b/(exp(2*I*(a+c))-1)*ln(exp(2*I*(a+c)
)+exp(2*I*(b*x+a)))*exp(2*I*(a+c))-I/b/(exp(2*I*(a+c))-1)*ln(exp(2*I*(a+c)
)+exp(2*I*(b*x+a)))
```

3.140.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 145 vs. 2(35) = 70.

Time = 0.27 (sec) , antiderivative size = 145, normalized size of antiderivative = 4.26

$$\int \tan(c-bx)\tan(a+bx)\,dx$$
$$=\frac{2\,bx\sin\left(2\,a+2\,c\right)-\left(\cos\left(2\,a+2\,c\right)+1\right)\log\left(-\frac{(\cos(2\,a+2\,c)-1)\tan(bx+a)^2-2\,\sin(2\,a+2\,c)\tan(bx+a)-\cos(2\,a+2\,c)-1}{(\cos(2\,a+2\,c)+1)\tan(bx+a)^2+\cos(2\,a+2\,c)+1}\right)+}{2\,b\sin\left(2\,a+2\,c\right)}$$

input
```
integrate(-tan(b*x-c)*tan(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(2*b*x*sin(2*a + 2*c) - (cos(2*a + 2*c) + 1)*log(-((cos(2*a + 2*c) - 1
)*tan(b*x + a)^2 - 2*sin(2*a + 2*c)*tan(b*x + a) - cos(2*a + 2*c) - 1)/((c
os(2*a + 2*c) + 1)*tan(b*x + a)^2 + cos(2*a + 2*c) + 1)) + (cos(2*a + 2*c)
 + 1)*log(1/(tan(b*x + a)^2 + 1)))/(b*sin(2*a + 2*c))
```

3.140.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2021 vs. $2(31) = 62$.

Time = 4.07 (sec) , antiderivative size = 7679, normalized size of antiderivative = 225.85

$$\int \tan(c - bx)\tan(a + bx)\,dx = \text{Too large to display}$$

input
```
integrate(-tan(b*x-c)*tan(b*x+a),x)
```

output
```
Piecewise((0, Eq(a, 0) & Eq(b, 0) & Eq(c, 0)), (-2*b*x*tan(c)/(2*b*tan(c)*
*2 + 2*b) + 2*log(tan(b*x) + 1/tan(c))/(2*b*tan(c)**2 + 2*b) - log(tan(b*x
)**2 + 1)/(2*b*tan(c)**2 + 2*b), Eq(a, 0)), (2*b*x*tan(a)/(2*b*tan(a)**2 +
 2*b) + 2*log(tan(b*x) - 1/tan(a))/(2*b*tan(a)**2 + 2*b) - log(tan(b*x)**2
 + 1)/(2*b*tan(a)**2 + 2*b), Eq(c, 0)), (-4*b*x*tan(c)**2*tan(b*x)/(2*b*ta
n(c)**5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2
+ 2*b*tan(c)*tan(b*x) + 2*b) - 4*b*x*tan(c)/(2*b*tan(c)**5*tan(b*x) + 2*b*
tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*tan(b*x) +
 2*b) - 2*log(tan(b*x) + 1/tan(c))*tan(c)**3*tan(b*x)/(2*b*tan(c)**5*tan(b
*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*
tan(b*x) + 2*b) - 2*log(tan(b*x) + 1/tan(c))*tan(c)**2/(2*b*tan(c)**5*tan(
b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)
*tan(b*x) + 2*b) + 2*log(tan(b*x) + 1/tan(c))*tan(c)*tan(b*x)/(2*b*tan(c)*
*5*tan(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b
*tan(c)*tan(b*x) + 2*b) + 2*log(tan(b*x) + 1/tan(c))/(2*b*tan(c)**5*tan(b*
x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)*t
an(b*x) + 2*b) + log(tan(b*x)**2 + 1)*tan(c)**3*tan(b*x)/(2*b*tan(c)**5*ta
n(b*x) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(
c)*tan(b*x) + 2*b) + log(tan(b*x)**2 + 1)*tan(c)**2/(2*b*tan(c)**5*tan(b*x
) + 2*b*tan(c)**4 + 4*b*tan(c)**3*tan(b*x) + 4*b*tan(c)**2 + 2*b*tan(c)...
```

3.140.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 290 vs. 2(35) = 70.

Time = 0.22 (sec) , antiderivative size = 290, normalized size of antiderivative = 8.53

$$\int \tan(c-bx)\tan(a+bx)\,dx$$
$$=\frac{\left(b\cos\left(2\,a+2\,c\right)^2+b\sin\left(2\,a+2\,c\right)^2-2\,b\cos\left(2\,a+2\,c\right)+b\right)x-\left(\cos\left(2\,a+2\,c\right)^2+\sin\left(2\,a+2\,c\right)^2-1\right)a}{}$$

input
```
integrate(-tan(b*x-c)*tan(b*x+a),x, algorithm="maxima")
```

output
```
((b*cos(2*a + 2*c)^2 + b*sin(2*a + 2*c)^2 - 2*b*cos(2*a + 2*c) + b)*x - (c
os(2*a + 2*c)^2 + sin(2*a + 2*c)^2 - 1)*arctan2(sin(2*b*x) - sin(2*a), cos
(2*b*x) + cos(2*a)) + (cos(2*a + 2*c)^2 + sin(2*a + 2*c)^2 - 1)*arctan2(si
n(2*b*x) + sin(2*c), cos(2*b*x) + cos(2*c)) + log(cos(2*b*x)^2 + 2*cos(2*b
*x)*cos(2*a) + cos(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a
)^2)*sin(2*a + 2*c) - log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*c) + cos(2*c)^
2 + sin(2*b*x)^2 + 2*sin(2*b*x)*sin(2*c) + sin(2*c)^2)*sin(2*a + 2*c))/(b*
cos(2*a + 2*c)^2 + b*sin(2*a + 2*c)^2 - 2*b*cos(2*a + 2*c) + b)
```

3.140.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. 2(35) = 70.

Time = 0.29 (sec) , antiderivative size = 245, normalized size of antiderivative = 7.21

$$\int \tan(c-bx)\tan(a+bx)\,dx$$
$$=\frac{2\,bx-\frac{\left(\tan\left(\frac{1}{2}\,a\right)^3\tan\left(\frac{1}{2}\,c\right)^2-\tan\left(\frac{1}{2}\,a\right)^3-4\,\tan\left(\frac{1}{2}\,a\right)^2\tan\left(\frac{1}{2}\,c\right)-\tan\left(\frac{1}{2}\,a\right)\tan\left(\frac{1}{2}\,c\right)^2+\tan\left(\frac{1}{2}\,a\right)\right)\log\left(\left|2\,\tan(bx)\tan\left(\frac{1}{2}\,a\right)+\tan\left(\frac{1}{2}\,a\right)^2-1\right|\right)}{\tan\left(\frac{1}{2}\,a\right)^3\tan\left(\frac{1}{2}\,c\right)+\tan\left(\frac{1}{2}\,a\right)^2\tan\left(\frac{1}{2}\,c\right)^2-\tan\left(\frac{1}{2}\,a\right)^2-\tan\left(\frac{1}{2}\,a\right)\tan\left(\frac{1}{2}\,c\right)}+}{2\,b}$$

input
```
integrate(-tan(b*x-c)*tan(b*x+a),x, algorithm="giac")
```

output
```
1/2*(2*b*x - (tan(1/2*a)^3*tan(1/2*c)^2 - tan(1/2*a)^3 - 4*tan(1/2*a)^2*ta
n(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a))*log(abs(2*tan(b*x)*tan(1/
2*a) + tan(1/2*a)^2 - 1))/(tan(1/2*a)^3*tan(1/2*c) + tan(1/2*a)^2*tan(1/2*
c)^2 - tan(1/2*a)^2 - tan(1/2*a)*tan(1/2*c)) + (tan(1/2*a)^2*tan(1/2*c)^3
- tan(1/2*a)^2*tan(1/2*c) - 4*tan(1/2*a)*tan(1/2*c)^2 - tan(1/2*c)^3 + tan
(1/2*c))*log(abs(2*tan(b*x)*tan(1/2*c) - tan(1/2*c)^2 + 1))/(tan(1/2*a)^2*
tan(1/2*c)^2 + tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*a)*tan(1/2*c) - tan(1/2*c
)^2))/b
```

3.140.9 Mupad [B] (verification not implemented)

Time = 31.88 (sec) , antiderivative size = 196, normalized size of antiderivative = 5.76

$$\int \tan(c-bx)\tan(a+bx)\,dx = \frac{\frac{x}{2}+x\left(\sin(a+c)^2-\frac{1}{2}\right)}{\sin(a+c)^2} + \frac{\frac{\sin(2a+2c)\,\ln\left(\sin(2a+2c)^2\,2\mathrm{i}-\sin(a+bx)^2\,2\mathrm{i}+\sin(3a+2c+bx)^2\,2\mathrm{i}+\sin(4a+4c)+\sin(6a+4c+2bx)-\sin(2a+2bx)\right)}{2}-\frac{\sin(2a+2c)\,\ln\left(\mathrm{s}\right.}{}}{b\sin(a+c)^2}$$

input
```
int(tan(a + b*x)*tan(c - b*x),x)
```

output
```
(x/2 + x*(sin(a + c)^2 - 1/2))/sin(a + c)^2 + ((sin(2*a + 2*c)*log(sin(4*a
 + 4*c) + sin(6*a + 4*c + 2*b*x) - sin(2*a + 2*b*x) + sin(2*a + 2*c)^2*2i
- sin(a + b*x)^2*2i + sin(3*a + 2*c + b*x)^2*2i))/2 - (sin(2*a + 2*c)*log(
sin(4*a + 4*c) + sin(4*a + 2*c + 2*b*x) + sin(2*c - 2*b*x) + sin(2*a + c +
 b*x)^2*2i + sin(2*a + 2*c)^2*2i - sin(c - b*x)^2*2i))/2)/(b*sin(a + c)^2)
```

3.141 $\int \cot(a+bx)\cot(c+bx)\,dx$

3.141.1 Optimal result

Integrand size = 13, antiderivative size = 39

$$\int \cot(a+bx)\cot(c+bx)\,dx = -x-\frac{\cot(a-c)\log(\sin(a+bx))}{b}+\frac{\cot(a-c)\log(\sin(c+bx))}{b}$$

output
```
-x-cot(a-c)*ln(sin(b*x+a))/b+cot(a-c)*ln(sin(b*x+c))/b
```

3.141.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int \cot(a+bx)\cot(c+bx)\,dx = -x+\frac{\cot(a-c)(-\log(\sin(a+bx))+\log(\sin(c+bx)))}{b}$$

input
```
Integrate[Cot[a + b*x]*Cot[c + b*x],x]
```

output
```
-x + (Cot[a - c]*(-Log[Sin[a + b*x]] + Log[Sin[c + b*x]]))/b
```

3.141.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.31, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5124, 5122, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(a+bx)\cot(bx+c)\,dx$$

$$\Big\downarrow \ 5124$$

$$\cos(a-c)\int \csc(a+bx)\csc(c+bx)dx - x$$

$$\Big\downarrow \ 5122$$

$$\cos(a-c)\left(\csc(a-c)\int \cot(c+bx)dx - \csc(a-c)\int \cot(a+bx)dx\right) - x$$

$$\Big\downarrow \ 3042$$

$$\cos(a-c)\left(\csc(a-c)\int -\tan\left(c+bx+\frac{\pi}{2}\right)dx - \csc(a-c)\int -\tan\left(a+bx+\frac{\pi}{2}\right)dx\right) - x$$

$$\Big\downarrow \ 25$$

$$\cos(a-c)\left(\csc(a-c)\int \tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx - \csc(a-c)\int \tan\left(\frac{1}{2}(2c+\pi)+bx\right)dx\right) - x$$

$$\Big\downarrow \ 3956$$

$$\cos(a-c)\left(\frac{\csc(a-c)\log(-\sin(bx+c))}{b} - \frac{\csc(a-c)\log(-\sin(a+bx))}{b}\right) - x$$

input
```
Int[Cot[a + b*x]*Cot[c + b*x],x]
```

output
```
-x + Cos[a - c]*(-((Csc[a - c]*Log[-Sin[a + b*x]])/b) + (Csc[a - c]*Log[-S
in[c + b*x]])/b)
```

3.141.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 5122
```
Int[Csc[(a_.) + (b_.)*(x_)]*Csc[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[Csc[(
b*c - a*d)/b]   Int[Cot[a + b*x], x], x] - Simp[Csc[(b*c - a*d)/d]   Int[Co
t[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*
c - a*d, 0]
```

rule 5124
```
Int[Cot[(a_.) + (b_.)*(x_)]*Cot[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[(-b)*
(x/d), x] + Simp[Cos[(b*c - a*d)/d]   Int[Csc[a + b*x]*Csc[c + d*x], x], x]
 /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c - a*d, 0]
```

3.141.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 177, normalized size of antiderivative = 4.54

method	result	size
risch	$-x-\frac{i\ln\left(\mathrm{e}^{2i(xb+a)}-1\right)\mathrm{e}^{2ia}}{b\left(\mathrm{e}^{2ia}-\mathrm{e}^{2ic}\right)}-\frac{i\ln\left(\mathrm{e}^{2i(xb+a)}-1\right)\mathrm{e}^{2ic}}{b\left(\mathrm{e}^{2ia}-\mathrm{e}^{2ic}\right)}+\frac{i\ln\left(\mathrm{e}^{2i(xb+a)}-\mathrm{e}^{2i(a-c)}\right)\mathrm{e}^{2ia}}{b\left(\mathrm{e}^{2ia}-\mathrm{e}^{2ic}\right)}+\frac{i\ln\left(\mathrm{e}^{2i(xb+a)}-\mathrm{e}^{2i(a-c)}\right)\mathrm{e}^{2ic}}{b\left(\mathrm{e}^{2ia}-\mathrm{e}^{2ic}\right)}$	177

input
```
int(cot(b*x+a)*cot(b*x+c),x,method=_RETURNVERBOSE)
```

output
```
-x-I/b/(exp(2*I*a)-exp(2*I*c))*ln(exp(2*I*(b*x+a))-1)*exp(2*I*a)-I/b/(exp(
2*I*a)-exp(2*I*c))*ln(exp(2*I*(b*x+a))-1)*exp(2*I*c)+I/b/(exp(2*I*a)-exp(2
*I*c))*ln(exp(2*I*(b*x+a))-exp(2*I*(a-c)))*exp(2*I*a)+I/b/(exp(2*I*a)-exp(
2*I*c))*ln(exp(2*I*(b*x+a))-exp(2*I*(a-c)))*exp(2*I*c)
```

3.141.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. 2(39) = 78.

Time = 0.26 (sec) , antiderivative size = 118, normalized size of antiderivative = 3.03

$$\int \cot(a+bx)\cot(c+bx)\,dx =$$

$$-\frac{2\,bx\sin\left(-2\,a+2\,c\right)-\left(\cos\left(-2\,a+2\,c\right)+1\right)\log\left(-\frac{\cos(2\,bx+2\,c)\cos(-2\,a+2\,c)+\sin(2\,bx+2\,c)\sin(-2\,a+2\,c)-1}{\cos(-2\,a+2\,c)+1}\right)+(\mathrm{co}}{2\,b\sin\left(-2\,a+2\,c\right)}$$

input
```
integrate(cot(b*x+a)*cot(b*x+c),x, algorithm="fricas")
```

output
```
-1/2*(2*b*x*sin(-2*a + 2*c) - (cos(-2*a + 2*c) + 1)*log(-(cos(2*b*x + 2*c)
*cos(-2*a + 2*c) + sin(2*b*x + 2*c)*sin(-2*a + 2*c) - 1)/(cos(-2*a + 2*c)
+ 1)) + (cos(-2*a + 2*c) + 1)*log(-1/2*cos(2*b*x + 2*c) + 1/2))/(b*sin(-2*
a + 2*c))
```

3.141.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1982 vs. 2(31) = 62.

Time = 11.46 (sec) , antiderivative size = 7404, normalized size of antiderivative = 189.85

$$\int \cot(a+bx)\cot(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(cot(b*x+a)*cot(b*x+c),x)
```

output
```
Piecewise((x/(zoo*cot(c) + zoo + cot(c)/tan(c) + zoo/tan(c)), Eq(b, 0) & E
q(a, atan(tan(c)) + pi*floor((c - pi/2)/pi))), (-b*x*tan(c)**5/(b*tan(c)**
5 + b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(
c) + b*tan(b*x)) - b*x*tan(c)**4*tan(b*x)/(b*tan(c)**5 + b*tan(c)**4*tan(b
*x) + 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(c) + b*tan(b*x)) + b*
x*tan(c)**3/(b*tan(c)**5 + b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3 + 2*b*tan(
c)**2*tan(b*x) + b*tan(c) + b*tan(b*x)) + b*x*tan(c)**2*tan(b*x)/(b*tan(c)
**5 + b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*ta
n(c) + b*tan(b*x)) - 2*log(tan(c) + tan(b*x))*tan(c)**4/(b*tan(c)**5 + b*t
an(c)**4*tan(b*x) + 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(c) + b*
tan(b*x)) - 2*log(tan(c) + tan(b*x))*tan(c)**3*tan(b*x)/(b*tan(c)**5 + b*t
an(c)**4*tan(b*x) + 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(c) + b*
tan(b*x)) + log(tan(b*x)**2 + 1)*tan(c)**4/(b*tan(c)**5 + b*tan(c)**4*tan(
b*x) + 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(c) + b*tan(b*x)) + l
og(tan(b*x)**2 + 1)*tan(c)**3*tan(b*x)/(b*tan(c)**5 + b*tan(c)**4*tan(b*x)
 + 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(c) + b*tan(b*x)) - tan(c
)**6/(b*tan(c)**5 + b*tan(c)**4*tan(b*x) + 2*b*tan(c)**3 + 2*b*tan(c)**2*t
an(b*x) + b*tan(c) + b*tan(b*x)) - tan(c)**4/(b*tan(c)**5 + b*tan(c)**4*ta
n(b*x) + 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) + b*tan(c) + b*tan(b*x)),
Eq(a, atan(tan(c)) + pi*floor((c - pi/2)/pi))), (x/(cot(a)*cot(c) + zoo...
```

3.141.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 549 vs. $2(39) = 78$.

Time = 0.25 (sec) , antiderivative size = 549, normalized size of antiderivative = 14.08

$$\int \cot(a+bx)\cot(c+bx)\,dx =$$
$$-\frac{\left(2\,b\cos\left(2\,a\right)\cos\left(2\,c\right)-b\cos\left(2\,c\right)^{2}+2\,b\sin\left(2\,a\right)\sin\left(2\,c\right)-b\sin\left(2\,c\right)^{2}-\left(\cos\left(2\,a\right)^{2}+\sin\left(2\,a\right)^{2}\right)b\right)x+\left(\right.}{}$$

input
```
integrate(cot(b*x+a)*cot(b*x+c),x, algorithm="maxima")
```

output
```
-((2*b*cos(2*a)*cos(2*c) - b*cos(2*c)^2 + 2*b*sin(2*a)*sin(2*c) - b*sin(2*
c)^2 - (cos(2*a)^2 + sin(2*a)^2)*b)*x + (cos(2*a)^2 - cos(2*c)^2 + sin(2*a
)^2 - sin(2*c)^2)*arctan2(sin(b*x) + sin(a), cos(b*x) - cos(a)) + (cos(2*a
)^2 - cos(2*c)^2 + sin(2*a)^2 - sin(2*c)^2)*arctan2(sin(b*x) - sin(a), cos
(b*x) + cos(a)) - (cos(2*a)^2 - cos(2*c)^2 + sin(2*a)^2 - sin(2*c)^2)*arct
an2(sin(b*x) + sin(c), cos(b*x) - cos(c)) - (cos(2*a)^2 - cos(2*c)^2 + sin
(2*a)^2 - sin(2*c)^2)*arctan2(sin(b*x) - sin(c), cos(b*x) + cos(c)) - (cos
(2*c)*sin(2*a) - cos(2*a)*sin(2*c))*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + c
os(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - (cos(2*c)*sin(2*a)
- cos(2*a)*sin(2*c))*log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b
*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2) + (cos(2*c)*sin(2*a) - cos(2*a)*sin(
2*c))*log(cos(b*x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*sin(b
*x)*sin(c) + sin(c)^2) + (cos(2*c)*sin(2*a) - cos(2*a)*sin(2*c))*log(cos(b
*x)^2 - 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + si
n(c)^2))/(2*b*cos(2*a)*cos(2*c) - b*cos(2*c)^2 + 2*b*sin(2*a)*sin(2*c) - b
*sin(2*c)^2 - (cos(2*a)^2 + sin(2*a)^2)*b)
```

3.141.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(39) = 78$.

Time = 0.30 (sec) , antiderivative size = 348, normalized size of antiderivative = 8.92

$$\int \cot(a+bx)\cot(c+bx)\,dx =$$

$$-\frac{2\,bx+\frac{\left(\tan\left(\frac{1}{2}\,a\right)^4\tan\left(\frac{1}{2}\,c\right)^2-\tan\left(\frac{1}{2}\,a\right)^4+4\,\tan\left(\frac{1}{2}\,a\right)^3\tan\left(\frac{1}{2}\,c\right)-2\,\tan\left(\frac{1}{2}\,a\right)^2\tan\left(\frac{1}{2}\,c\right)^2+2\,\tan\left(\frac{1}{2}\,a\right)^2-4\,\tan\left(\frac{1}{2}\,a\right)\tan\left(\frac{1}{2}\,c\right)+\tan\left(\frac{1}{2}\,c\right)^2-1\right)}{\tan\left(\frac{1}{2}\,a\right)^4\tan\left(\frac{1}{2}\,c\right)-\tan\left(\frac{1}{2}\,a\right)^3\tan\left(\frac{1}{2}\,c\right)^2+\tan\left(\frac{1}{2}\,a\right)^3-2\,\tan\left(\frac{1}{2}\,a\right)^2\tan\left(\frac{1}{2}\,c\right)+\tan\left(\frac{1}{2}\,a\right)\tan\left(\frac{1}{2}\,c\right)^2-\mathrm{ta}}}{}$$

input
```
integrate(cot(b*x+a)*cot(b*x+c),x, algorithm="giac")
```

output
```
-1/2*(2*b*x + (tan(1/2*a)^4*tan(1/2*c)^2 - tan(1/2*a)^4 + 4*tan(1/2*a)^3*t
an(1/2*c) - 2*tan(1/2*a)^2*tan(1/2*c)^2 + 2*tan(1/2*a)^2 - 4*tan(1/2*a)*ta
n(1/2*c) + tan(1/2*c)^2 - 1)*log(abs(tan(b*x)*tan(1/2*a)^2 - tan(b*x) - 2*
tan(1/2*a)))/(tan(1/2*a)^4*tan(1/2*c) - tan(1/2*a)^3*tan(1/2*c)^2 + tan(1/
2*a)^3 - 2*tan(1/2*a)^2*tan(1/2*c) + tan(1/2*a)*tan(1/2*c)^2 - tan(1/2*a)
+ tan(1/2*c)) - (tan(1/2*a)^2*tan(1/2*c)^4 - 2*tan(1/2*a)^2*tan(1/2*c)^2 +
 4*tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*c)^4 + tan(1/2*a)^2 - 4*tan(1/2*a)*ta
n(1/2*c) + 2*tan(1/2*c)^2 - 1)*log(abs(tan(b*x)*tan(1/2*c)^2 - tan(b*x) -
2*tan(1/2*c)))/(tan(1/2*a)^2*tan(1/2*c)^3 - tan(1/2*a)*tan(1/2*c)^4 - tan(
1/2*a)^2*tan(1/2*c) + 2*tan(1/2*a)*tan(1/2*c)^2 - tan(1/2*c)^3 - tan(1/2*a
) + tan(1/2*c)))/b
```

3.141.9 Mupad [B] (verification not implemented)

Time = 31.69 (sec) , antiderivative size = 207, normalized size of antiderivative = 5.31

$$\int \cot(a+bx)\cot(c+bx)\,dx = -\frac{\frac{x}{2}+x\left(\sin\left(a-c\right)^2-\frac{1}{2}\right)}{\sin\left(a-c\right)^2} - \frac{\frac{\sin(2a-2c)\,\ln\left(\sin(2a-2c)^2\,2\mathrm{i}+\sin(a+bx)^2\,2\mathrm{i}-\sin(3a-2c+bx)^2\,2\mathrm{i}+\sin(4a-4c)-\sin(6a-4c+2bx)+\sin(2a+2bx)\right)}{2} - \frac{\sin(2a-2c)\,\ln\left(s\right.}{}}{b\sin\left(a-c\right)^2}$$

input
```
int(cot(a + b*x)*cot(c + b*x),x)
```

output
```
- (x/2 + x*(sin(a - c)^2 - 1/2))/sin(a - c)^2 - ((sin(2*a - 2*c)*log(sin(4
*a - 4*c) - sin(6*a - 4*c + 2*b*x) + sin(2*a + 2*b*x) + sin(2*a - 2*c)^2*2
i + sin(a + b*x)^2*2i - sin(3*a - 2*c + b*x)^2*2i))/2 - (sin(2*a - 2*c)*lo
g(sin(4*a - 4*c) - sin(4*a - 2*c + 2*b*x) + sin(2*c + 2*b*x) + sin(2*a - 2
*c)^2*2i + sin(c + b*x)^2*2i - sin(2*a - c + b*x)^2*2i))/2)/(b*sin(a - c)^
2)
```

3.142 $\int \cot(c-bx)\cot(a+bx)\,dx$

3.142.1 Optimal result

Integrand size = 14, antiderivative size = 34

$$\int \cot(c-bx)\cot(a+bx)\,dx = x - \frac{\cot(a+c)\log(\sin(c-bx))}{b} + \frac{\cot(a+c)\log(\sin(a+bx))}{b}$$

output
```
x-cot(a+c)*ln(-sin(b*x-c))/b+cot(a+c)*ln(sin(b*x+a))/b
```

3.142.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \cot(c-bx)\cot(a+bx)\,dx = x + \frac{\cot(a+c)(-\log(\sin(c-bx)) + \log(-\sin(a+bx)))}{b}$$

input
```
Integrate[Cot[c - b*x]*Cot[a + b*x],x]
```

output
```
x + (Cot[a + c]*(-Log[Sin[c - b*x]] + Log[-Sin[a + b*x]]))/b
```

3.142.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.29, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5124, 5122, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(a+bx)\cot(c-bx)\,dx$$

$$\Big\downarrow \text{5124}$$

$$\cos(a+c)\int \csc(c-bx)\csc(a+bx)dx+x$$

$$\Big\downarrow \text{5122}$$

$$\cos(a+c)\left(\csc(a+c)\int \cot(c-bx)dx+\csc(a+c)\int \cot(a+bx)dx\right)+x$$

$$\Big\downarrow \text{3042}$$

$$\cos(a+c)\left(\csc(a+c)\int -\tan\left(c-bx+\frac{\pi}{2}\right)dx+\csc(a+c)\int -\tan\left(a+bx+\frac{\pi}{2}\right)dx\right)+x$$

$$\Big\downarrow \text{25}$$

$$\cos(a+c)\left(-\csc(a+c)\int \tan\left(\frac{1}{2}(2c+\pi)-bx\right)dx-\csc(a+c)\int \tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx\right)+x$$

$$\Big\downarrow \text{3956}$$

$$\cos(a+c)\left(\frac{\csc(a+c)\log(-\sin(a+bx))}{b}-\frac{\csc(a+c)\log(-\sin(c-bx))}{b}\right)+x$$

input
```
Int[Cot[c - b*x]*Cot[a + b*x],x]
```

output
```
x + Cos[a + c]*(-((Csc[a + c]*Log[-Sin[c - b*x]])/b) + (Csc[a + c]*Log[-Si
n[a + b*x]])/b)
```

3.142.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 5122
```
Int[Csc[(a_.) + (b_.)*(x_)]*Csc[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[Csc[(
b*c - a*d)/b]   Int[Cot[a + b*x], x], x] - Simp[Csc[(b*c - a*d)/d]   Int[Co
t[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*
c - a*d, 0]
```

rule 5124
```
Int[Cot[(a_.) + (b_.)*(x_)]*Cot[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[(-b)*
(x/d), x] + Simp[Cos[(b*c - a*d)/d]   Int[Csc[a + b*x]*Csc[c + d*x], x], x]
 /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c - a*d, 0]
```

3.142.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 149, normalized size of antiderivative = 4.38

method	result	size
risch	$x-\frac{i \ln\left(-\mathrm{e}^{2 i (a+c)}+\mathrm{e}^{2 i (x b+a)}\right) \mathrm{e}^{2 i (a+c)}}{b \left(\mathrm{e}^{2 i (a+c)}-1\right)}-\frac{i \ln\left(-\mathrm{e}^{2 i (a+c)}+\mathrm{e}^{2 i (x b+a)}\right)}{b \left(\mathrm{e}^{2 i (a+c)}-1\right)}+\frac{i \ln\left(\mathrm{e}^{2 i (x b+a)}-1\right) \mathrm{e}^{2 i (a+c)}}{b \left(\mathrm{e}^{2 i (a+c)}-1\right)}+\frac{i \ln\left(\mathrm{e}^{2 i (x b+a)}-1\right)}{b \left(\mathrm{e}^{2 i (a+c)}-1\right)}$	149

input
```
int(-cot(b*x-c)*cot(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
x-I/b/(exp(2*I*(a+c))-1)*ln(-exp(2*I*(a+c))+exp(2*I*(b*x+a)))*exp(2*I*(a+c
))-I/b/(exp(2*I*(a+c))-1)*ln(-exp(2*I*(a+c))+exp(2*I*(b*x+a)))+I/b/(exp(2*
I*(a+c))-1)*ln(exp(2*I*(b*x+a))-1)*exp(2*I*(a+c))+I/b/(exp(2*I*(a+c))-1)*l
n(exp(2*I*(b*x+a))-1)
```

3.142.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(37) = 74$.

Time = 0.26 (sec) , antiderivative size = 118, normalized size of antiderivative = 3.47

$$\int \cot(c-bx)\cot(a+bx)\,dx$$
$$= \frac{2\,bx\sin\left(2\,a+2\,c\right)-\left(\cos\left(2\,a+2\,c\right)+1\right)\log\left(-\frac{\cos(2\,bx+2\,a)\cos(2\,a+2\,c)+\sin(2\,bx+2\,a)\sin(2\,a+2\,c)-1}{\cos(2\,a+2\,c)+1}\right)+\left(\cos\left(2\,a+2\right.\right.}{2\,b\sin\left(2\,a+2\,c\right)}$$

input
```
integrate(-cot(b*x-c)*cot(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(2*b*x*sin(2*a + 2*c) - (cos(2*a + 2*c) + 1)*log(-(cos(2*b*x + 2*a)*co
s(2*a + 2*c) + sin(2*b*x + 2*a)*sin(2*a + 2*c) - 1)/(cos(2*a + 2*c) + 1))
+ (cos(2*a + 2*c) + 1)*log(-1/2*cos(2*b*x + 2*a) + 1/2))/(b*sin(2*a + 2*c)
)
```

3.142.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1722 vs. $2(32) = 64$.

Time = 12.04 (sec) , antiderivative size = 7417, normalized size of antiderivative = 218.15

$$\int \cot(c-bx)\cot(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(-cot(b*x-c)*cot(b*x+a),x)
```

output
```
-Piecewise((x/(zoo*cot(c) + zoo + cot(c)/tan(c) + zoo/tan(c)), Eq(b, 0) &
Eq(a, -atan(tan(c)) - pi*floor((c - pi/2)/pi))), (b*x*tan(c)**5/(-b*tan(c)
**5 + b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) - b*ta
n(c) + b*tan(b*x)) - b*x*tan(c)**4*tan(b*x)/(-b*tan(c)**5 + b*tan(c)**4*ta
n(b*x) - 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) - b*tan(c) + b*tan(b*x)) -
 b*x*tan(c)**3/(-b*tan(c)**5 + b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + 2*b*
tan(c)**2*tan(b*x) - b*tan(c) + b*tan(b*x)) + b*x*tan(c)**2*tan(b*x)/(-b*t
an(c)**5 + b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) -
 b*tan(c) + b*tan(b*x)) - 2*log(-tan(c) + tan(b*x))*tan(c)**4/(-b*tan(c)**
5 + b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) - b*tan(
c) + b*tan(b*x)) + 2*log(-tan(c) + tan(b*x))*tan(c)**3*tan(b*x)/(-b*tan(c)
**5 + b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) - b*ta
n(c) + b*tan(b*x)) + log(tan(b*x)**2 + 1)*tan(c)**4/(-b*tan(c)**5 + b*tan(
c)**4*tan(b*x) - 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) - b*tan(c) + b*tan
(b*x)) - log(tan(b*x)**2 + 1)*tan(c)**3*tan(b*x)/(-b*tan(c)**5 + b*tan(c)*
*4*tan(b*x) - 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) - b*tan(c) + b*tan(b*
x)) - tan(c)**6/(-b*tan(c)**5 + b*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + 2*b
*tan(c)**2*tan(b*x) - b*tan(c) + b*tan(b*x)) - tan(c)**4/(-b*tan(c)**5 + b
*tan(c)**4*tan(b*x) - 2*b*tan(c)**3 + 2*b*tan(c)**2*tan(b*x) - b*tan(c) +
b*tan(b*x)), Eq(a, -atan(tan(c)) - pi*floor((c - pi/2)/pi))), (x/(-cot(...
```

3.142.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 432 vs. $2(37) = 74$.

Time = 0.24 (sec) , antiderivative size = 432, normalized size of antiderivative = 12.71

$$\int \cot(c-bx)\cot(a+bx)\,dx$$
$$= \frac{(b\cos(2a+2c)^2 + b\sin(2a+2c)^2 - 2b\cos(2a+2c) + b)x - (\cos(2a+2c)^2 + \sin(2a+2c)^2 - 1)\,\mathrm{a}\ldots}{}$$

input
```
integrate(-cot(b*x-c)*cot(b*x+a),x, algorithm="maxima")
```

output
```
((b*cos(2*a + 2*c)^2 + b*sin(2*a + 2*c)^2 - 2*b*cos(2*a + 2*c) + b)*x - (c
os(2*a + 2*c)^2 + sin(2*a + 2*c)^2 - 1)*arctan2(sin(b*x) + sin(a), cos(b*x
) - cos(a)) - (cos(2*a + 2*c)^2 + sin(2*a + 2*c)^2 - 1)*arctan2(sin(b*x) -
 sin(a), cos(b*x) + cos(a)) + (cos(2*a + 2*c)^2 + sin(2*a + 2*c)^2 - 1)*ar
ctan2(sin(b*x) + sin(c), cos(b*x) + cos(c)) + (cos(2*a + 2*c)^2 + sin(2*a
+ 2*c)^2 - 1)*arctan2(sin(b*x) - sin(c), cos(b*x) - cos(c)) + log(cos(b*x)
^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a
)^2)*sin(2*a + 2*c) + log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(a)^2 + sin(
b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2)*sin(2*a + 2*c) - log(cos(b*x)^2 + 2
*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2)*s
in(2*a + 2*c) - log(cos(b*x)^2 - 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2
 - 2*sin(b*x)*sin(c) + sin(c)^2)*sin(2*a + 2*c))/(b*cos(2*a + 2*c)^2 + b*s
in(2*a + 2*c)^2 - 2*b*cos(2*a + 2*c) + b)
```

3.142.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(37) = 74$.

Time = 0.29 (sec) , antiderivative size = 345, normalized size of antiderivative = 10.15

$$\int \cot(c-bx)\cot(a+bx)\,dx$$

$$= \frac{2\,bx - \frac{\left(\tan\left(\frac{1}{2}\,a\right)^{4}\tan\left(\frac{1}{2}\,c\right)^{2} - \tan\left(\frac{1}{2}\,a\right)^{4} - 4\,\tan\left(\frac{1}{2}\,a\right)^{3}\tan\left(\frac{1}{2}\,c\right) - 2\,\tan\left(\frac{1}{2}\,a\right)^{2}\tan\left(\frac{1}{2}\,c\right)^{2} + 2\,\tan\left(\frac{1}{2}\,a\right)^{2} + 4\,\tan\left(\frac{1}{2}\,a\right)\tan\left(\frac{1}{2}\,c\right) + \tan\left(\frac{1}{2}\,c\right)^{2} - 1\right)\log}{\tan\left(\frac{1}{2}\,a\right)^{4}\tan\left(\frac{1}{2}\,c\right) + \tan\left(\frac{1}{2}\,a\right)^{3}\tan\left(\frac{1}{2}\,c\right)^{2} - \tan\left(\frac{1}{2}\,a\right)^{3} - 2\,\tan\left(\frac{1}{2}\,a\right)^{2}\tan\left(\frac{1}{2}\,c\right) - \tan\left(\frac{1}{2}\,a\right)\tan\left(\frac{1}{2}\,c\right)^{2} + \tan\left(\right.}}{}$$

input
```
integrate(-cot(b*x-c)*cot(b*x+a),x, algorithm="giac")
```

output
```
1/2*(2*b*x - (tan(1/2*a)^4*tan(1/2*c)^2 - tan(1/2*a)^4 - 4*tan(1/2*a)^3*ta
n(1/2*c) - 2*tan(1/2*a)^2*tan(1/2*c)^2 + 2*tan(1/2*a)^2 + 4*tan(1/2*a)*tan
(1/2*c) + tan(1/2*c)^2 - 1)*log(abs(tan(b*x)*tan(1/2*a)^2 - tan(b*x) - 2*t
an(1/2*a)))/(tan(1/2*a)^4*tan(1/2*c) + tan(1/2*a)^3*tan(1/2*c)^2 - tan(1/2
*a)^3 - 2*tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) +
 tan(1/2*c)) + (tan(1/2*a)^2*tan(1/2*c)^4 - 2*tan(1/2*a)^2*tan(1/2*c)^2 -
4*tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*c)^4 + tan(1/2*a)^2 + 4*tan(1/2*a)*tan
(1/2*c) + 2*tan(1/2*c)^2 - 1)*log(abs(tan(b*x)*tan(1/2*c)^2 - tan(b*x) + 2
*tan(1/2*c)))/(tan(1/2*a)^2*tan(1/2*c)^3 + tan(1/2*a)*tan(1/2*c)^4 - tan(1
/2*a)^2*tan(1/2*c) - 2*tan(1/2*a)*tan(1/2*c)^2 - tan(1/2*c)^3 + tan(1/2*a)
 + tan(1/2*c)))/b
```

3.142.9 Mupad [B] (verification not implemented)

Time = 31.83 (sec) , antiderivative size = 200, normalized size of antiderivative = 5.88

$$\int \cot(c-bx)\cot(a+bx)\,dx = \frac{\frac{x}{2}+x\left(\sin(a+c)^2-\frac{1}{2}\right)}{\sin(a+c)^2} + \frac{\frac{\sin(2a+2c)\,\ln\left(\sin(2a+2c)^2\,2\mathrm{i}+\sin(a+bx)^2\,2\mathrm{i}-\sin(3a+2c+bx)^2\,2\mathrm{i}+\sin(4a+4c)-\sin(6a+4c+2bx)+\sin(2a+2bx)\right)}{2} - \frac{\sin(2a+2c)\,\ln\left(-\right.}{}}{b\sin(a+c)^2}$$

input
```
int(cot(a + b*x)*cot(c - b*x),x)
```

output
```
(x/2 + x*(sin(a + c)^2 - 1/2))/sin(a + c)^2 + ((sin(2*a + 2*c)*log(sin(4*a
 + 4*c) - sin(6*a + 4*c + 2*b*x) + sin(2*a + 2*b*x) + sin(2*a + 2*c)^2*2i
+ sin(a + b*x)^2*2i - sin(3*a + 2*c + b*x)^2*2i))/2 - (sin(2*a + 2*c)*log(
sin(4*a + 4*c) - sin(4*a + 2*c + 2*b*x) - sin(2*c - 2*b*x) - sin(2*a + c +
 b*x)^2*2i + sin(2*a + 2*c)^2*2i + sin(c - b*x)^2*2i))/2)/(b*sin(a + c)^2)
```

3.143 $\int \sec(a+bx)\sec(c+bx)\,dx$

3.143.1 Optimal result

Integrand size = 13, antiderivative size = 36

$$\int \sec(a+bx)\sec(c+bx)\,dx = -\frac{\csc(a-c)\log(\cos(a+bx))}{b} + \frac{\csc(a-c)\log(\cos(c+bx))}{b}$$

output
```
-csc(a-c)*ln(cos(b*x+a))/b+csc(a-c)*ln(cos(b*x+c))/b
```

3.143.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \sec(a+bx)\sec(c+bx)\,dx = -\frac{\csc(a-c)(\log(\cos(a+bx)) - \log(\cos(c+bx)))}{b}$$

input
```
Integrate[Sec[a + b*x]*Sec[c + b*x],x]
```

output
```
-((Csc[a - c]*(Log[Cos[a + b*x]] - Log[Cos[c + b*x]]))/b)
```

3.143.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {5121, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(a+bx)\sec(bx+c)\,dx$$

$$\Big\downarrow \text{ 5121}$$

$$\csc(a-c)\int \tan(a+bx)dx-\csc(a-c)\int \tan(c+bx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\csc(a-c)\int \tan(a+bx)dx-\csc(a-c)\int \tan(c+bx)dx$$

$$\Big\downarrow \text{ 3956}$$

$$\frac{\csc(a-c)\log(\cos(bx+c))}{b}-\frac{\csc(a-c)\log(\cos(a+bx))}{b}$$

input
```
Int[Sec[a + b*x]*Sec[c + b*x],x]
```

output
```
-((Csc[a - c]*Log[Cos[a + b*x]])/b) + (Csc[a - c]*Log[Cos[c + b*x]])/b
```

3.143.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 5121
```
Int[Sec[(a_.) + (b_.)*(x_)]*Sec[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[-Csc[
(b*c - a*d)/d]   Int[Tan[a + b*x], x], x] + Simp[Csc[(b*c - a*d)/b]   Int[T
an[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b
*c - a*d, 0]
```

3.143.4 Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.50

method	result	size
default	$\frac{\ln(\tan(xb+a)\sin(a)\cos(c)-\tan(xb+a)\cos(a)\sin(c)+\cos(a)\cos(c)+\sin(a)\sin(c))}{b(\sin(a)\cos(c)-\cos(a)\sin(c))}$	54
risch	$\frac{2i\ln\left(e^{2i(xb+a)}+e^{2i(a-c)}\right)e^{i(a+c)}}{(e^{2ia}-e^{2ic})b}-\frac{2i\ln\left(e^{2i(xb+a)}+1\right)e^{i(a+c)}}{(e^{2ia}-e^{2ic})b}$	90

input
```
int(sec(b*x+a)*sec(b*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/b/(sin(a)*cos(c)-cos(a)*sin(c))*ln(tan(b*x+a)*sin(a)*cos(c)-tan(b*x+a)*c
os(a)*sin(c)+cos(a)*cos(c)+sin(a)*sin(c))
```

3.143.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(36) = 72$.

Time = 0.26 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.97

$$\int \sec(a+bx)\sec(c+bx)\,dx = -\frac{\log\left(\cos\left(bx+c\right)^2\right)-\log\left(\frac{4\left(2\cos(bx+c)\cos(-a+c)\sin(bx+c)\sin(-a+c)+\left(2\cos(-a+c)^2-1\right)\cos(bx+c)^2-\cos(-a+c)^2+1\right)}{\cos(-a+c)^2+2\cos(-a+c)+1}\right)}{2\,b\sin\left(-a+c\right)}$$

input
```
integrate(sec(b*x+a)*sec(b*x+c),x, algorithm="fricas")
```

output
```
-1/2*(log(cos(b*x + c)^2) - log(4*(2*cos(b*x + c)*cos(-a + c)*sin(b*x + c)
*sin(-a + c) + (2*cos(-a + c)^2 - 1)*cos(b*x + c)^2 - cos(-a + c)^2 + 1)/(
cos(-a + c)^2 + 2*cos(-a + c) + 1)))/(b*sin(-a + c))
```

3.143.6 Sympy [F]

$$\int \sec(a+bx)\sec(c+bx)\,dx = \int \sec(a+bx)\sec(bx+c)\,dx$$

input
```
integrate(sec(b*x+a)*sec(b*x+c),x)
```

output
```
Integral(sec(a + b*x)*sec(b*x + c), x)
```

3.143.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. $2(36) = 72$.

Time = 0.22 (sec) , antiderivative size = 349, normalized size of antiderivative = 9.69

$$\int \sec(a+bx)\sec(c+bx)\,dx =$$
$$-\frac{2\left((\cos(2a)-\cos(2c))\cos(a+c)+(\sin(2a)-\sin(2c))\sin(a+c)\right)\arctan\left(\sin(2bx)-\sin(2a),\cos\right.}{}$$

input
```
integrate(sec(b*x+a)*sec(b*x+c),x, algorithm="maxima")
```

output
```
-(2*((cos(2*a) - cos(2*c))*cos(a + c) + (sin(2*a) - sin(2*c))*sin(a + c))*
arctan2(sin(2*b*x) - sin(2*a), cos(2*b*x) + cos(2*a)) - 2*((cos(2*a) - cos
(2*c))*cos(a + c) + (sin(2*a) - sin(2*c))*sin(a + c))*arctan2(sin(2*b*x) -
 sin(2*c), cos(2*b*x) + cos(2*c)) - ((sin(2*a) - sin(2*c))*cos(a + c) - (c
os(2*a) - cos(2*c))*sin(a + c))*log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) +
 cos(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2) + ((sin(2
*a) - sin(2*c))*cos(a + c) - (cos(2*a) - cos(2*c))*sin(a + c))*log(cos(2*b
*x)^2 + 2*cos(2*b*x)*cos(2*c) + cos(2*c)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*s
in(2*c) + sin(2*c)^2))/(2*b*cos(2*a)*cos(2*c) - b*cos(2*c)^2 + 2*b*sin(2*a
)*sin(2*c) - b*sin(2*c)^2 - (cos(2*a)^2 + sin(2*a)^2)*b)
```

3.143.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(36) = 72.

Time = 0.30 (sec) , antiderivative size = 171, normalized size of antiderivative = 4.75

$$\int \sec(a+bx)\sec(c+bx)\,dx$$
$$= \frac{\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}a\right)^2+\tan\left(\frac{1}{2}c\right)^2+1\right)\log\left(\left|2\tan\left(bx+a\right)\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)-2\tan\left(bx\right.\right.\right.}{2\left(\tan\left(\frac{1}{2}a\right.\right.}$$

input
```
integrate(sec(b*x+a)*sec(b*x+c),x, algorithm="giac")
```

output
```
1/2*(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1)*log(abs(
2*tan(b*x + a)*tan(1/2*a)^2*tan(1/2*c) - 2*tan(b*x + a)*tan(1/2*a)*tan(1/2
*c)^2 + tan(1/2*a)^2*tan(1/2*c)^2 + 2*tan(b*x + a)*tan(1/2*a) - tan(1/2*a)
^2 - 2*tan(b*x + a)*tan(1/2*c) + 4*tan(1/2*a)*tan(1/2*c) - tan(1/2*c)^2 +
1))/((tan(1/2*a)^2*tan(1/2*c) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) - tan
(1/2*c))*b)
```

3.143.9 Mupad [B] (verification not implemented)

Time = 32.33 (sec) , antiderivative size = 249, normalized size of antiderivative = 6.92

$$\int \sec(a+bx)\sec(c+bx)\,dx$$
$$= \frac{2\sqrt{-e^{a\,2i-c\,2i}}\left(\ln\left(-\frac{2\sqrt{-e^{a\,2i}\,e^{-c\,2i}}\,\left(4\,b\,e^{a\,2i}\,e^{-c\,2i}+2\,b\,e^{a\,2i}\,e^{b\,x\,2i}+2\,b\,e^{a\,4i}\,e^{-c\,2i}\,e^{b\,x\,2i}\right)}{b\,\left(e^{a\,2i}\,e^{-c\,2i}-1\right)}+e^{a\,1i}\,e^{a\,2i}\,e^{-c\,1i}\,e^{b\,x\,2i}\,4i\right)-\ln\left(-\frac{2\cdots}{}\right.\right.}{b\,\left(e^{a\,2i-c\,2i}-1\right)}$$

input
```
int(1/(cos(a + b*x)*cos(c + b*x)),x)
```

output
```
(2*(-exp(a*2i - c*2i))^(1/2)*(log(exp(a*1i)*exp(a*2i)*exp(-c*1i)*exp(b*x*2
i)*4i - (2*(-exp(a*2i)*exp(-c*2i))^(1/2)*(4*b*exp(a*2i)*exp(-c*2i) + 2*b*e
xp(a*2i)*exp(b*x*2i) + 2*b*exp(a*4i)*exp(-c*2i)*exp(b*x*2i)))/(b*(exp(a*2i
)*exp(-c*2i) - 1))) - log(exp(a*1i)*exp(a*2i)*exp(-c*1i)*exp(b*x*2i)*4i -
(2*(-exp(a*2i)*exp(-c*2i))^(1/2)*(4*b*exp(a*2i)*exp(-c*2i) + 2*b*exp(a*2i)
*exp(b*x*2i) + 2*b*exp(a*4i)*exp(-c*2i)*exp(b*x*2i)))/(b - b*exp(a*2i)*exp
(-c*2i)))))/(b*(exp(a*2i - c*2i) - 1))
```

3.144 $\int \sec(c-bx)\sec(a+bx)\,dx$

3.144.1 Optimal result

Integrand size = 14, antiderivative size = 33

$$\int \sec(c-bx)\sec(a+bx)\,dx = \frac{\csc(a+c)\log(\cos(c-bx))}{b} - \frac{\csc(a+c)\log(\cos(a+bx))}{b}$$

output
```
csc(a+c)*ln(cos(b*x-c))/b-csc(a+c)*ln(cos(b*x+a))/b
```

3.144.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int \sec(c-bx)\sec(a+bx)\,dx = \frac{\csc(a+c)(\log(\cos(c-bx))-\log(\cos(a+bx)))}{b}$$

input
```
Integrate[Sec[c - b*x]*Sec[a + b*x],x]
```

output
```
(Csc[a + c]*(Log[Cos[c - b*x]] - Log[Cos[a + b*x]]))/b
```

3.144.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5121, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(a+bx)\sec(c-bx)\,dx$$

$$\Big\downarrow \text{5121}$$

$$\csc(a+c)\int \tan(c-bx)dx+\csc(a+c)\int \tan(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\csc(a+c)\int \tan(c-bx)dx+\csc(a+c)\int \tan(a+bx)dx$$

$$\Big\downarrow \text{3956}$$

$$\frac{\csc(a+c)\log(\cos(c-bx))}{b}-\frac{\csc(a+c)\log(\cos(a+bx))}{b}$$

input
```
Int[Sec[c - b*x]*Sec[a + b*x],x]
```

output
```
(Csc[a + c]*Log[Cos[c - b*x]])/b - (Csc[a + c]*Log[Cos[a + b*x]])/b
```

3.144.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 5121
```
Int[Sec[(a_.) + (b_.)*(x_)]*Sec[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[-Csc[
(b*c - a*d)/d]   Int[Tan[a + b*x], x], x] + Simp[Csc[(b*c - a*d)/b]   Int[T
an[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b
*c - a*d, 0]
```

3.144.4 Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.61

method	result	size
default	$\frac{\ln(\tan(xb+a)\sin(a)\cos(c)+\tan(xb+a)\cos(a)\sin(c)+\cos(a)\cos(c)-\sin(a)\sin(c))}{b(\sin(a)\cos(c)+\cos(a)\sin(c))}$	53
risch	$-\frac{2i\ln\left(\mathrm{e}^{2i(xb+a)}+1\right)\mathrm{e}^{i(a+c)}}{\left(\mathrm{e}^{2i(a+c)}-1\right)b}+\frac{2i\ln\left(\mathrm{e}^{2i(a+c)}+\mathrm{e}^{2i(xb+a)}\right)\mathrm{e}^{i(a+c)}}{\left(\mathrm{e}^{2i(a+c)}-1\right)b}$	80

input
```
int(sec(b*x-c)*sec(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b/(sin(a)*cos(c)+cos(a)*sin(c))*ln(tan(b*x+a)*sin(a)*cos(c)+tan(b*x+a)*c
os(a)*sin(c)+cos(a)*cos(c)-sin(a)*sin(c))
```

3.144.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(34) = 68.

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.82

$$\int \sec(c-bx)\sec(a+bx)\,dx = -\frac{\log\left(\cos\left(bx+a\right)^2\right)-\log\left(\frac{4\left(2\cos(bx+a)\cos(a+c)\sin(bx+a)\sin(a+c)+\left(2\cos(a+c)^2-1\right)\cos(bx+a)^2-\cos(a+c)^2+1\right)}{\cos(a+c)^2+2\cos(a+c)+1}\right)}{2\,b\sin\left(a+c\right)}$$

input
```
integrate(sec(b*x-c)*sec(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(log(cos(b*x + a)^2) - log(4*(2*cos(b*x + a)*cos(a + c)*sin(b*x + a)*
sin(a + c) + (2*cos(a + c)^2 - 1)*cos(b*x + a)^2 - cos(a + c)^2 + 1)/(cos(
a + c)^2 + 2*cos(a + c) + 1)))/(b*sin(a + c))
```

3.144.6 Sympy [F]

$$\int \sec(c-bx)\sec(a+bx)\,dx = \int \sec(a+bx)\sec(bx-c)\,dx$$

input
```
integrate(sec(b*x-c)*sec(b*x+a),x)
```

output
```
Integral(sec(a + b*x)*sec(b*x - c), x)
```

3.144.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(34) = 68$.

Time = 0.22 (sec) , antiderivative size = 322, normalized size of antiderivative = 9.76

$$\int \sec(c-bx)\sec(a+bx)\,dx = \frac{2\left(\cos(2a+2c)\cos(a+c)+\sin(2a+2c)\sin(a+c)-\cos(a+c)\right)\arctan\left(\sin(2bx)-\sin(2a),\cos(2b\right.}{}$$

input
```
integrate(sec(b*x-c)*sec(b*x+a),x, algorithm="maxima")
```

output
```
(2*(cos(2*a + 2*c)*cos(a + c) + sin(2*a + 2*c)*sin(a + c) - cos(a + c))*ar
ctan2(sin(2*b*x) - sin(2*a), cos(2*b*x) + cos(2*a)) - 2*(cos(2*a + 2*c)*co
s(a + c) + sin(2*a + 2*c)*sin(a + c) - cos(a + c))*arctan2(sin(2*b*x) + si
n(2*c), cos(2*b*x) + cos(2*c)) - (cos(a + c)*sin(2*a + 2*c) - cos(2*a + 2*
c)*sin(a + c) + sin(a + c))*log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) + cos
(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2) + (cos(a + c)
*sin(2*a + 2*c) - cos(2*a + 2*c)*sin(a + c) + sin(a + c))*log(cos(2*b*x)^2
 + 2*cos(2*b*x)*cos(2*c) + cos(2*c)^2 + sin(2*b*x)^2 + 2*sin(2*b*x)*sin(2*
c) + sin(2*c)^2))/(b*cos(2*a + 2*c)^2 + b*sin(2*a + 2*c)^2 - 2*b*cos(2*a +
 2*c) + b)
```

3.144.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. 2(34) = 68.

Time = 0.31 (sec) , antiderivative size = 169, normalized size of antiderivative = 5.12

$$\int \sec(c-bx)\sec(a+bx)\,dx = -\frac{\left(\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}a\right)^2+\tan\left(\frac{1}{2}c\right)^2+1\right)\log\left(\left|2\tan(bx+a)\tan\left(\frac{1}{2}a\right)^2\tan\left(\frac{1}{2}c\right)+2\tan(b\right.\right.}{2\left(\tan\left(\right.\right.}$$

input
```
integrate(sec(b*x-c)*sec(b*x+a),x, algorithm="giac")
```

output
```
-1/2*(tan(1/2*a)^2*tan(1/2*c)^2 + tan(1/2*a)^2 + tan(1/2*c)^2 + 1)*log(abs
(2*tan(b*x + a)*tan(1/2*a)^2*tan(1/2*c) + 2*tan(b*x + a)*tan(1/2*a)*tan(1/
2*c)^2 - tan(1/2*a)^2*tan(1/2*c)^2 - 2*tan(b*x + a)*tan(1/2*a) + tan(1/2*a
)^2 - 2*tan(b*x + a)*tan(1/2*c) + 4*tan(1/2*a)*tan(1/2*c) + tan(1/2*c)^2 -
 1))/((tan(1/2*a)^2*tan(1/2*c) + tan(1/2*a)*tan(1/2*c)^2 - tan(1/2*a) - ta
n(1/2*c))*b)
```

3.144.9 Mupad [B] (verification not implemented)

Time = 33.69 (sec) , antiderivative size = 249, normalized size of antiderivative = 7.55

$$\int \sec(c-bx)\sec(a+bx)\,dx = \frac{2\sqrt{-e^{a\,2i+c\,2i}}\left(\ln\left(-\frac{2\sqrt{-e^{a\,2i}\,e^{c\,2i}}\,(4\,b\,e^{a\,2i}\,e^{c\,2i}+2\,b\,e^{a\,2i}\,e^{b\,x\,2i}+2\,b\,e^{a\,4i}\,e^{c\,2i}\,e^{b\,x\,2i})}{b\,(e^{a\,2i}\,e^{c\,2i}-1)}+e^{a\,1i}\,e^{a\,2i}\,e^{c\,1i}\,e^{b\,x\,2i}\,4i\right)-\ln\left(-\frac{2\sqrt{-e^{a\,2}}}{}\right.\right.}{b\,(e^{a\,2i+c\,2i}-1)}$$

input
```
int(1/(cos(a + b*x)*cos(c - b*x)),x)
```

output
```
(2*(-exp(a*2i + c*2i))^(1/2)*(log(exp(a*1i)*exp(a*2i)*exp(c*1i)*exp(b*x*2i
)*4i - (2*(-exp(a*2i)*exp(c*2i))^(1/2)*(4*b*exp(a*2i)*exp(c*2i) + 2*b*exp(
a*2i)*exp(b*x*2i) + 2*b*exp(a*4i)*exp(c*2i)*exp(b*x*2i)))/(b*(exp(a*2i)*ex
p(c*2i) - 1))) - log(exp(a*1i)*exp(a*2i)*exp(c*1i)*exp(b*x*2i)*4i - (2*(-e
xp(a*2i)*exp(c*2i))^(1/2)*(4*b*exp(a*2i)*exp(c*2i) + 2*b*exp(a*2i)*exp(b*x
*2i) + 2*b*exp(a*4i)*exp(c*2i)*exp(b*x*2i)))/(b - b*exp(a*2i)*exp(c*2i))))
)/(b*(exp(a*2i + c*2i) - 1))
```

3.145 $\int \csc(a+bx)\csc(c+bx)\,dx$

3.145.1 Optimal result

Integrand size = 13, antiderivative size = 36

$$\int \csc(a+bx)\csc(c+bx)\,dx = -\frac{\csc(a-c)\log(\sin(a+bx))}{b} + \frac{\csc(a-c)\log(\sin(c+bx))}{b}$$

output
```
-csc(a-c)*ln(sin(b*x+a))/b+csc(a-c)*ln(sin(b*x+c))/b
```

3.145.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \csc(a+bx)\csc(c+bx)\,dx = -\frac{\csc(a-c)(\log(\sin(a+bx)) - \log(\sin(c+bx)))}{b}$$

input
```
Integrate[Csc[a + b*x]*Csc[c + b*x],x]
```

output
```
-((Csc[a - c]*(Log[Sin[a + b*x]] - Log[Sin[c + b*x]]))/b)
```

3.145.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5122, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(a+bx)\csc(bx+c)\,dx$$

$$\Big\downarrow \text{5122}$$

$$\csc(a-c)\int \cot(c+bx)dx-\csc(a-c)\int \cot(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\csc(a-c)\int -\tan\left(c+bx+\frac{\pi}{2}\right)dx-\csc(a-c)\int -\tan\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{25}$$

$$\csc(a-c)\int \tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx-\csc(a-c)\int \tan\left(\frac{1}{2}(2c+\pi)+bx\right)dx$$

$$\Big\downarrow \text{3956}$$

$$\frac{\csc(a-c)\log(-\sin(bx+c))}{b}-\frac{\csc(a-c)\log(-\sin(a+bx))}{b}$$

input
```
Int[Csc[a + b*x]*Csc[c + b*x],x]
```

output
```
-((Csc[a - c]*Log[-Sin[a + b*x]])/b) + (Csc[a - c]*Log[-Sin[c + b*x]])/b
```

3.145.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 5122
```
Int[Csc[(a_.) + (b_.)*(x_)]*Csc[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[Csc[(
b*c - a*d)/b]   Int[Cot[a + b*x], x], x] - Simp[Csc[(b*c - a*d)/d]   Int[Co
t[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*
c - a*d, 0]
```

3.145.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(36) = 72.

Time = 1.49 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.19

method	result	size
default	$\frac{-\frac{\ln(\tan(xb+a))}{\sin(a)\cos(c)-\cos(a)\sin(c)}+\frac{\ln(\tan(xb+a)\cos(a)\cos(c)+\tan(xb+a)\sin(a)\sin(c)-\sin(a)\cos(c)+\cos(a)\sin(c))}{\sin(a)\cos(c)-\cos(a)\sin(c)}}{b}$	79
risch	$\frac{2i\ln\left(e^{2i(xb+a)}-e^{2i(a-c)}\right)e^{i(a+c)}}{(e^{2ia}-e^{2ic})b}-\frac{2i\ln\left(e^{2i(xb+a)}-1\right)e^{i(a+c)}}{(e^{2ia}-e^{2ic})b}$	92

input
```
int(csc(b*x+a)*csc(b*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/(sin(a)*cos(c)-cos(a)*sin(c))*ln(tan(b*x+a))+1/(sin(a)*cos(c)-cos(
a)*sin(c))*ln(tan(b*x+a)*cos(a)*cos(c)+tan(b*x+a)*sin(a)*sin(c)-sin(a)*cos
(c)+cos(a)*sin(c)))
```

3.145.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(36) = 72.

Time = 0.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 3.06

$$\int \csc(a+bx)\csc(c+bx)\,dx =$$

$$-\frac{\log\left(-\frac{1}{4}\cos\left(bx+c\right)^2+\frac{1}{4}\right)-\log\left(-\frac{2\cos(bx+c)\cos(-a+c)\sin(bx+c)\sin(-a+c)+\left(2\cos(-a+c)^2-1\right)\cos(bx+c)^2-\cos(-a+c)}{\cos(-a+c)^2+2\cos(-a+c)+1}\right.}{2\,b\sin\left(-a+c\right)}$$

input
```
integrate(csc(b*x+a)*csc(b*x+c),x, algorithm="fricas")
```

output
```
-1/2*(log(-1/4*cos(b*x + c)^2 + 1/4) - log(-(2*cos(b*x + c)*cos(-a + c)*si
n(b*x + c)*sin(-a + c) + (2*cos(-a + c)^2 - 1)*cos(b*x + c)^2 - cos(-a + c
)^2)/(cos(-a + c)^2 + 2*cos(-a + c) + 1)))/(b*sin(-a + c))
```

3.145.6 Sympy [F]

$$\int \csc(a+bx)\csc(c+bx)\,dx = \int \csc(a+bx)\csc(bx+c)\,dx$$

input
```
integrate(csc(b*x+a)*csc(b*x+c),x)
```

output
```
Integral(csc(a + b*x)*csc(b*x + c), x)
```

3.145.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 564 vs. $2(36) = 72$.

Time = 0.22 (sec) , antiderivative size = 564, normalized size of antiderivative = 15.67

$$\int \csc(a+bx)\csc(c+bx)\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)*csc(b*x+c),x, algorithm="maxima")
```

output
```
-(2*((cos(2*a) - cos(2*c))*cos(a + c) + (sin(2*a) - sin(2*c))*sin(a + c))*
arctan2(sin(b*x) + sin(a), cos(b*x) - cos(a)) + 2*((cos(2*a) - cos(2*c))*c
os(a + c) + (sin(2*a) - sin(2*c))*sin(a + c))*arctan2(sin(b*x) - sin(a), c
os(b*x) + cos(a)) - 2*((cos(2*a) - cos(2*c))*cos(a + c) + (sin(2*a) - sin(
2*c))*sin(a + c))*arctan2(sin(b*x) + sin(c), cos(b*x) - cos(c)) - 2*((cos(
2*a) - cos(2*c))*cos(a + c) + (sin(2*a) - sin(2*c))*sin(a + c))*arctan2(si
n(b*x) - sin(c), cos(b*x) + cos(c)) - ((sin(2*a) - sin(2*c))*cos(a + c) -
(cos(2*a) - cos(2*c))*sin(a + c))*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos
(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - ((sin(2*a) - sin(2*c)
)*cos(a + c) - (cos(2*a) - cos(2*c))*sin(a + c))*log(cos(b*x)^2 - 2*cos(b*
x)*cos(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2) + ((sin(
2*a) - sin(2*c))*cos(a + c) - (cos(2*a) - cos(2*c))*sin(a + c))*log(cos(b*
x)^2 + 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(c) + sin
(c)^2) + ((sin(2*a) - sin(2*c))*cos(a + c) - (cos(2*a) - cos(2*c))*sin(a +
 c))*log(cos(b*x)^2 - 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*
x)*sin(c) + sin(c)^2))/(2*b*cos(2*a)*cos(2*c) - b*cos(2*c)^2 + 2*b*sin(2*a
)*sin(2*c) - b*sin(2*c)^2 - (cos(2*a)^2 + sin(2*a)^2)*b)
```

3.145.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 396 vs. $2(36) = 72$.

Time = 0.29 (sec) , antiderivative size = 396, normalized size of antiderivative = 11.00

$$\int \csc(a+bx)\csc(c+bx)\,dx$$

$$= \frac{\frac{\left(\tan\left(\frac{1}{2}a\right)^4\tan\left(\frac{1}{2}c\right)^4+4\tan\left(\frac{1}{2}a\right)^3\tan\left(\frac{1}{2}c\right)^3-\tan\left(\frac{1}{2}a\right)^4+4\tan\left(\frac{1}{2}a\right)^3\tan\left(\frac{1}{2}c\right)+4\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^3-\tan\left(\frac{1}{2}c\right)^4+4\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)+1\right)\log}{\tan\left(\frac{1}{2}a\right)^4\tan\left(\frac{1}{2}c\right)^3-\tan\left(\frac{1}{2}a\right)^3\tan\left(\frac{1}{2}c\right)^4-\tan\left(\frac{1}{2}a\right)^4\tan\left(\frac{1}{2}c\right)+6\tan\left(\frac{1}{2}a\right)^3\tan\left(\frac{1}{2}c\right.}}{}$$

input
```
integrate(csc(b*x+a)*csc(b*x+c),x, algorithm="giac")
```

output
```
1/2*((tan(1/2*a)^4*tan(1/2*c)^4 + 4*tan(1/2*a)^3*tan(1/2*c)^3 - tan(1/2*a)
^4 + 4*tan(1/2*a)^3*tan(1/2*c) + 4*tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*c)^4
+ 4*tan(1/2*a)*tan(1/2*c) + 1)*log(abs(tan(b*x + a)*tan(1/2*a)^2*tan(1/2*c
)^2 - tan(b*x + a)*tan(1/2*a)^2 + 4*tan(b*x + a)*tan(1/2*a)*tan(1/2*c) - 2
*tan(1/2*a)^2*tan(1/2*c) - tan(b*x + a)*tan(1/2*c)^2 + 2*tan(1/2*a)*tan(1/
2*c)^2 + tan(b*x + a) - 2*tan(1/2*a) + 2*tan(1/2*c)))/(tan(1/2*a)^4*tan(1/
2*c)^3 - tan(1/2*a)^3*tan(1/2*c)^4 - tan(1/2*a)^4*tan(1/2*c) + 6*tan(1/2*a
)^3*tan(1/2*c)^2 - 6*tan(1/2*a)^2*tan(1/2*c)^3 + tan(1/2*a)*tan(1/2*c)^4 -
 tan(1/2*a)^3 + 6*tan(1/2*a)^2*tan(1/2*c) - 6*tan(1/2*a)*tan(1/2*c)^2 + ta
n(1/2*c)^3 + tan(1/2*a) - tan(1/2*c)) - (tan(1/2*a)^2*tan(1/2*c)^2 + tan(1
/2*a)^2 + tan(1/2*c)^2 + 1)*log(abs(tan(b*x + a)))/(tan(1/2*a)^2*tan(1/2*c
) - tan(1/2*a)*tan(1/2*c)^2 + tan(1/2*a) - tan(1/2*c)))/b
```

3.145.9 Mupad [B] (verification not implemented)

Time = 33.33 (sec) , antiderivative size = 249, normalized size of antiderivative = 6.92

$$\int \csc(a+bx)\csc(c+bx)\,dx$$
$$= \frac{2\sqrt{-e^{a\,2i-c\,2i}}\left(\ln\left(\frac{2\sqrt{-e^{a\,2i}\,e^{-c\,2i}}\left(-4\,b\,e^{a\,2i}\,e^{-c\,2i}+2\,b\,e^{a\,2i}\,e^{b\,x\,2i}+2\,b\,e^{a\,4i}\,e^{-c\,2i}\,e^{b\,x\,2i}\right)}{b\left(e^{a\,2i}\,e^{-c\,2i}-1\right)}-e^{a\,1i}\,e^{a\,2i}\,e^{-c\,1i}\,e^{b\,x\,2i}\,4i\right)-\ln\left(\frac{2\sqrt{-}\cdots}{\cdots}\right.\right.}{b\,\left(e^{a\,2i-c\,2i}-1\right)}$$

input
```
int(1/(sin(a + b*x)*sin(c + b*x)),x)
```

output
```
(2*(-exp(a*2i - c*2i))^(1/2)*(log((2*(-exp(a*2i)*exp(-c*2i))^(1/2)*(2*b*ex
p(a*2i)*exp(b*x*2i) - 4*b*exp(a*2i)*exp(-c*2i) + 2*b*exp(a*4i)*exp(-c*2i)*
exp(b*x*2i)))/(b*(exp(a*2i)*exp(-c*2i) - 1)) - exp(a*1i)*exp(a*2i)*exp(-c*
1i)*exp(b*x*2i)*4i) - log((2*(-exp(a*2i)*exp(-c*2i))^(1/2)*(2*b*exp(a*2i)*
exp(b*x*2i) - 4*b*exp(a*2i)*exp(-c*2i) + 2*b*exp(a*4i)*exp(-c*2i)*exp(b*x*
2i)))/(b - b*exp(a*2i)*exp(-c*2i)) - exp(a*1i)*exp(a*2i)*exp(-c*1i)*exp(b*
x*2i)*4i)))/(b*(exp(a*2i - c*2i) - 1))
```

3.146 $\int \csc(c-bx)\csc(a+bx)\,dx$

3.146.1 Optimal result

Integrand size = 14, antiderivative size = 33

$$\int \csc(c-bx)\csc(a+bx)\,dx=-\frac{\csc(a+c)\log(\sin(c-bx))}{b}+\frac{\csc(a+c)\log(\sin(a+bx))}{b}$$

output
```
-csc(a+c)*ln(-sin(b*x-c))/b+csc(a+c)*ln(sin(b*x+a))/b
```

3.146.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \csc(c-bx)\csc(a+bx)\,dx=-\frac{\csc(a+c)(\log(\sin(c-bx))-\log(-\sin(a+bx)))}{b}$$

input
```
Integrate[Csc[c - b*x]*Csc[a + b*x],x]
```

output
```
-((Csc[a + c]*(Log[Sin[c - b*x]] - Log[-Sin[a + b*x]]))/b)
```

3.146.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5122, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(a+bx)\csc(c-bx)\,dx$$

$$\Big\downarrow \text{5122}$$

$$\csc(a+c)\int \cot(c-bx)dx+\csc(a+c)\int \cot(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\csc(a+c)\int -\tan\left(c-bx+\frac{\pi}{2}\right)dx+\csc(a+c)\int -\tan\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\csc(a+c)\int \tan\left(\frac{1}{2}(2c+\pi)-bx\right)dx-\csc(a+c)\int \tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx$$

$$\Big\downarrow \text{3956}$$

$$\frac{\csc(a+c)\log(-\sin(a+bx))}{b}-\frac{\csc(a+c)\log(-\sin(c-bx))}{b}$$

input
```
Int[Csc[c - b*x]*Csc[a + b*x],x]
```

output
```
-((Csc[a + c]*Log[-Sin[c - b*x]])/b) + (Csc[a + c]*Log[-Sin[a + b*x]])/b
```

3.146.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 5122
```
Int[Csc[(a_.) + (b_.)*(x_)]*Csc[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[Csc[(
b*c - a*d)/b]   Int[Cot[a + b*x], x], x] - Simp[Csc[(b*c - a*d)/d]   Int[Co
t[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*
c - a*d, 0]
```

3.146.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(36) = 72$.

Time = 1.48 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.42

method	result	size
default	$-\frac{-\frac{\ln(\tan(xb+a))}{\sin(a)\cos(c)+\cos(a)\sin(c)}+\frac{\ln(\tan(xb+a)\cos(a)\cos(c)-\tan(xb+a)\sin(a)\sin(c)-\sin(a)\cos(c)-\cos(a)\sin(c))}{\sin(a)\cos(c)+\cos(a)\sin(c)}}{b}$	80
risch	$\frac{2i\ln\left(\mathrm{e}^{2i(xb+a)}-1\right)\mathrm{e}^{i(a+c)}}{\left(\mathrm{e}^{2i(a+c)}-1\right)b}-\frac{2i\ln\left(-\mathrm{e}^{2i(a+c)}+\mathrm{e}^{2i(xb+a)}\right)\mathrm{e}^{i(a+c)}}{\left(\mathrm{e}^{2i(a+c)}-1\right)b}$	82

input
```
int(-csc(b*x-c)*csc(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/b*(-1/(sin(a)*cos(c)+cos(a)*sin(c))*ln(tan(b*x+a))+1/(sin(a)*cos(c)+cos
(a)*sin(c))*ln(tan(b*x+a)*cos(a)*cos(c)-tan(b*x+a)*sin(a)*sin(c)-sin(a)*co
s(c)-cos(a)*sin(c)))
```

3.146.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(36) = 72$.

Time = 0.26 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.91

$$\int \csc(c-bx)\csc(a+bx)\,dx$$
$$=\frac{\log\left(-\frac{1}{4}\cos\left(bx+a\right)^2+\frac{1}{4}\right)-\log\left(-\frac{2\cos(bx+a)\cos(a+c)\sin(bx+a)\sin(a+c)+\left(2\cos(a+c)^2-1\right)\cos(bx+a)^2-\cos(a+c)^2}{\cos(a+c)^2+2\cos(a+c)+1}\right)}{2\,b\sin\left(a+c\right)}$$

input
```
integrate(-csc(b*x-c)*csc(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(log(-1/4*cos(b*x + a)^2 + 1/4) - log(-(2*cos(b*x + a)*cos(a + c)*sin(
b*x + a)*sin(a + c) + (2*cos(a + c)^2 - 1)*cos(b*x + a)^2 - cos(a + c)^2)/
(cos(a + c)^2 + 2*cos(a + c) + 1)))/(b*sin(a + c))
```

3.146.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1824 vs. $2(31) = 62$.

Time = 68.55 (sec) , antiderivative size = 1824, normalized size of antiderivative = 55.27

$$\int \csc(c - bx)\csc(a + bx)\,dx = \text{Too large to display}$$

input
```
integrate(-csc(b*x-c)*csc(b*x+a),x)
```

output
```
Piecewise((-log(-tan(c/2) + tan(b*x/2))*tan(c/2)/(2*b) - log(-tan(c/2) + t
an(b*x/2))/(2*b*tan(c/2)) - log(tan(b*x/2) + 1/tan(c/2))*tan(c/2)/(2*b) -
log(tan(b*x/2) + 1/tan(c/2))/(2*b*tan(c/2)) + log(tan(b*x/2))*tan(c/2)/(2*
b) + log(tan(b*x/2))/(2*b*tan(c/2)), Eq(a, 0)), (log(tan(a/2) + tan(b*x/2)
)*tan(a/2)/(2*b) + log(tan(a/2) + tan(b*x/2))/(2*b*tan(a/2)) + log(tan(b*x
/2) - 1/tan(a/2))*tan(a/2)/(2*b) + log(tan(b*x/2) - 1/tan(a/2))/(2*b*tan(a
/2)) - log(tan(b*x/2))*tan(a/2)/(2*b) - log(tan(b*x/2))/(2*b*tan(a/2)), Eq
(c, 0)), (-tan(c/2)**4*tan(b*x/2)/(-2*b*tan(c/2)**3*tan(b*x/2) + 2*b*tan(c
/2)**2*tan(b*x/2)**2 - 2*b*tan(c/2)**2 + 2*b*tan(c/2)*tan(b*x/2)) - 2*tan(
c/2)**2*tan(b*x/2)/(-2*b*tan(c/2)**3*tan(b*x/2) + 2*b*tan(c/2)**2*tan(b*x/
2)**2 - 2*b*tan(c/2)**2 + 2*b*tan(c/2)*tan(b*x/2)) - tan(b*x/2)/(-2*b*tan(
c/2)**3*tan(b*x/2) + 2*b*tan(c/2)**2*tan(b*x/2)**2 - 2*b*tan(c/2)**2 + 2*b
*tan(c/2)*tan(b*x/2)), Eq(a, 2*atan(1/tan(c/2)))), (tan(c/2)**4*tan(b*x/2)
/(-2*b*tan(c/2)**3*tan(b*x/2) + 2*b*tan(c/2)**2*tan(b*x/2)**2 - 2*b*tan(c/
2)**2 + 2*b*tan(c/2)*tan(b*x/2)) + 2*tan(c/2)**2*tan(b*x/2)/(-2*b*tan(c/2)
**3*tan(b*x/2) + 2*b*tan(c/2)**2*tan(b*x/2)**2 - 2*b*tan(c/2)**2 + 2*b*tan
(c/2)*tan(b*x/2)) + tan(b*x/2)/(-2*b*tan(c/2)**3*tan(b*x/2) + 2*b*tan(c/2)
**2*tan(b*x/2)**2 - 2*b*tan(c/2)**2 + 2*b*tan(c/2)*tan(b*x/2)), Eq(a, -2*a
tan(tan(c/2)) - 2*pi*floor((c/2 - pi/2)/pi))), (x/(sin(a)*sin(c)), Eq(b, 0
)), (-log(tan(a/2) + tan(b*x/2))*tan(a/2)**2*tan(c/2)**2/(2*b*tan(a/2)*...
```

3.146.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 536 vs. $2(36) = 72$.

Time = 0.22 (sec) , antiderivative size = 536, normalized size of antiderivative = 16.24

$$\int \csc(c-bx)\csc(a+bx)\,dx = -\frac{2\left(\cos\left(2a+2c\right)\cos\left(a+c\right)+\sin\left(2a+2c\right)\sin\left(a+c\right)-\cos\left(a+c\right)\right)\arctan\left(\sin\left(bx\right)+\sin\left(a\right),\cos\left(bx\right)\right.}{}$$

input
```
integrate(-csc(b*x-c)*csc(b*x+a),x, algorithm="maxima")
```

output
```
-(2*(cos(2*a + 2*c)*cos(a + c) + sin(2*a + 2*c)*sin(a + c) - cos(a + c))*a
rctan2(sin(b*x) + sin(a), cos(b*x) - cos(a)) + 2*(cos(2*a + 2*c)*cos(a + c
) + sin(2*a + 2*c)*sin(a + c) - cos(a + c))*arctan2(sin(b*x) - sin(a), cos
(b*x) + cos(a)) - 2*(cos(2*a + 2*c)*cos(a + c) + sin(2*a + 2*c)*sin(a + c)
 - cos(a + c))*arctan2(sin(b*x) + sin(c), cos(b*x) + cos(c)) - 2*(cos(2*a
+ 2*c)*cos(a + c) + sin(2*a + 2*c)*sin(a + c) - cos(a + c))*arctan2(sin(b*
x) - sin(c), cos(b*x) - cos(c)) - (cos(a + c)*sin(2*a + 2*c) - cos(2*a + 2
*c)*sin(a + c) + sin(a + c))*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2
 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - (cos(a + c)*sin(2*a + 2*c)
 - cos(2*a + 2*c)*sin(a + c) + sin(a + c))*log(cos(b*x)^2 - 2*cos(b*x)*cos
(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2) + (cos(a + c)*
sin(2*a + 2*c) - cos(2*a + 2*c)*sin(a + c) + sin(a + c))*log(cos(b*x)^2 +
2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(c) + sin(c)^2)
+ (cos(a + c)*sin(2*a + 2*c) - cos(2*a + 2*c)*sin(a + c) + sin(a + c))*log
(cos(b*x)^2 - 2*cos(b*x)*cos(c) + cos(c)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(c
) + sin(c)^2))/(b*cos(2*a + 2*c)^2 + b*sin(2*a + 2*c)^2 - 2*b*cos(2*a + 2*
c) + b)
```

3.146.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 397 vs. $2(36) = 72$.

Time = 0.31 (sec) , antiderivative size = 397, normalized size of antiderivative = 12.03

$$\int \csc(c-bx)\csc(a+bx)\,dx = \frac{\frac{\left(\tan\left(\frac{1}{2}a\right)^4\tan\left(\frac{1}{2}c\right)^4-4\tan\left(\frac{1}{2}a\right)^3\tan\left(\frac{1}{2}c\right)^3-\tan\left(\frac{1}{2}a\right)^4-4\tan\left(\frac{1}{2}a\right)^3\tan\left(\frac{1}{2}c\right)-4\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)^3-\tan\left(\frac{1}{2}c\right)^4-4\tan\left(\frac{1}{2}a\right)\tan\left(\frac{1}{2}c\right)+1\right)\log}{\tan\left(\frac{1}{2}a\right)^4\tan\left(\frac{1}{2}c\right)^3+\tan\left(\frac{1}{2}a\right)^3\tan\left(\frac{1}{2}c\right)^4-\tan\left(\frac{1}{2}a\right)^4\tan\left(\frac{1}{2}c\right)-6\tan\left(\frac{1}{2}a\right)^3\tan\left(\frac{1}{2}c\right.}}{}$$

input
```
integrate(-csc(b*x-c)*csc(b*x+a),x, algorithm="giac")
```

output
```
1/2*((tan(1/2*a)^4*tan(1/2*c)^4 - 4*tan(1/2*a)^3*tan(1/2*c)^3 - tan(1/2*a)
^4 - 4*tan(1/2*a)^3*tan(1/2*c) - 4*tan(1/2*a)*tan(1/2*c)^3 - tan(1/2*c)^4
- 4*tan(1/2*a)*tan(1/2*c) + 1)*log(abs(tan(b*x + a)*tan(1/2*a)^2*tan(1/2*c
)^2 - tan(b*x + a)*tan(1/2*a)^2 - 4*tan(b*x + a)*tan(1/2*a)*tan(1/2*c) + 2
*tan(1/2*a)^2*tan(1/2*c) - tan(b*x + a)*tan(1/2*c)^2 + 2*tan(1/2*a)*tan(1/
2*c)^2 + tan(b*x + a) - 2*tan(1/2*a) - 2*tan(1/2*c)))/(tan(1/2*a)^4*tan(1/
2*c)^3 + tan(1/2*a)^3*tan(1/2*c)^4 - tan(1/2*a)^4*tan(1/2*c) - 6*tan(1/2*a
)^3*tan(1/2*c)^2 - 6*tan(1/2*a)^2*tan(1/2*c)^3 - tan(1/2*a)*tan(1/2*c)^4 +
 tan(1/2*a)^3 + 6*tan(1/2*a)^2*tan(1/2*c) + 6*tan(1/2*a)*tan(1/2*c)^2 + ta
n(1/2*c)^3 - tan(1/2*a) - tan(1/2*c)) - (tan(1/2*a)^2*tan(1/2*c)^2 + tan(1
/2*a)^2 + tan(1/2*c)^2 + 1)*log(abs(tan(b*x + a)))/(tan(1/2*a)^2*tan(1/2*c
) + tan(1/2*a)*tan(1/2*c)^2 - tan(1/2*a) - tan(1/2*c)))/b
```

3.146.9 Mupad [B] (verification not implemented)

Time = 32.91 (sec) , antiderivative size = 249, normalized size of antiderivative = 7.55

$$\int \csc(c-bx)\csc(a+bx)\,dx = \frac{2\sqrt{-e^{a\,2i+c\,2i}}\left(\ln\left(\frac{2\sqrt{-e^{a\,2i}\,e^{c\,2i}}\,\left(-4\,b\,e^{a\,2i}\,e^{c\,2i}+2\,b\,e^{a\,2i}\,e^{b\,x\,2i}+2\,b\,e^{a\,4i}\,e^{c\,2i}\,e^{b\,x\,2i}\right)}{b\,(e^{a\,2i}\,e^{c\,2i}-1)}+e^{a\,1i}\,e^{a\,2i}\,e^{c\,1i}\,e^{b\,x\,2i}\,4i\right)-\ln\left(\frac{2\sqrt{-e^{a\,2i}\,e^{c}}}{}\right.\right.}{b\,\left(e^{a\,2i+c\,2i}-1\right)}$$

input
```
int(1/(sin(a + b*x)*sin(c - b*x)),x)
```

output
```
(2*(-exp(a*2i + c*2i))^(1/2)*(log((2*(-exp(a*2i)*exp(c*2i))^(1/2)*(2*b*exp
(a*2i)*exp(b*x*2i) - 4*b*exp(a*2i)*exp(c*2i) + 2*b*exp(a*4i)*exp(c*2i)*exp
(b*x*2i)))/(b*(exp(a*2i)*exp(c*2i) - 1)) + exp(a*1i)*exp(a*2i)*exp(c*1i)*e
xp(b*x*2i)*4i) - log((2*(-exp(a*2i)*exp(c*2i))^(1/2)*(2*b*exp(a*2i)*exp(b*
x*2i) - 4*b*exp(a*2i)*exp(c*2i) + 2*b*exp(a*4i)*exp(c*2i)*exp(b*x*2i)))/(b
 - b*exp(a*2i)*exp(c*2i)) + exp(a*1i)*exp(a*2i)*exp(c*1i)*exp(b*x*2i)*4i))
)/(b*(exp(a*2i + c*2i) - 1))
```

3.147 $\int \sqrt{\sin(x)\tan(x)}\,dx$

3.147.1 Optimal result

Integrand size = 9, antiderivative size = 13

$$\int \sqrt{\sin(x)\tan(x)}\,dx = -2\cot(x)\sqrt{\sin(x)\tan(x)}$$

output
```
-2*cot(x)*(sin(x)*tan(x))^(1/2)
```

3.147.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \sqrt{\sin(x)\tan(x)}\,dx = -2\cot(x)\sqrt{\sin(x)\tan(x)}$$

input
```
Integrate[Sqrt[Sin[x]*Tan[x]],x]
```

output
```
-2*Cot[x]*Sqrt[Sin[x]*Tan[x]]
```

3.147.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3042, 4900, 3042, 3069}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sin(x)\tan(x)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt{\sin(x)\tan(x)}dx$$

$$\Big\downarrow \; 4900$$

$$\frac{\sqrt{\sin(x)\tan(x)}\int \sqrt{\sin(x)}\sqrt{\tan(x)}dx}{\sqrt{\sin(x)}\sqrt{\tan(x)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt{\sin(x)\tan(x)}\int \sqrt{\sin(x)}\sqrt{\tan(x)}dx}{\sqrt{\sin(x)}\sqrt{\tan(x)}}$$

$$\Big\downarrow \; 3069$$

$$-2\cot(x)\sqrt{\sin(x)\tan(x)}$$

input
```
Int[Sqrt[Sin[x]*Tan[x]],x]
```

output
```
-2*Cot[x]*Sqrt[Sin[x]*Tan[x]]
```

3.147.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3069
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

rule 4900
```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Simp[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p]))   Int[uu*vv^(m*p)*ww^(n*p), x]
, x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !
InertTrigFreeQ[w])
```

3.147.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.47 (sec) , antiderivative size = 50, normalized size of antiderivative = 3.85

method	result
risch	$-\frac{i\sqrt{2}\,\sqrt{-\frac{\left(e^{2ix}-1\right)^{2} e^{-ix}}{e^{2ix}+1}}\,\left(e^{2ix}+1\right)}{e^{2ix}-1}$
default	$-\frac{\cot(x)\left(4\cos(x)\sqrt{-\frac{\cos(x)}{(\cos(x)+1)^{2}}}+4\sqrt{-\frac{\cos(x)}{(\cos(x)+1)^{2}}}+\ln\left(\frac{4\cos(x)\sqrt{-\frac{\cos(x)}{(\cos(x)+1)^{2}}}-2\cos(x)+4\sqrt{-\frac{\cos(x)}{(\cos(x)+1)^{2}}}+2}{\cos(x)+1}\right)-\ln\left(\frac{2\cos(x)\sqrt{-\ldots}}{\ldots}\right.\ldots\right.}{4(\cos(x)+1)\sqrt{-\frac{\cos(x)}{(\cos(x)+1)^{2}}}}$

input
```
int((sin(x)*tan(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-I*2^(1/2)*(-(exp(2*I*x)-1)^2*exp(-I*x)/(exp(2*I*x)+1))^(1/2)/(exp(2*I*x)-
1)*(exp(2*I*x)+1)
```

3.147.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int \sqrt{\sin(x)\tan(x)}\,dx = -\frac{2\sqrt{-\frac{\cos(x)^{2}-1}{\cos(x)}}\cos(x)}{\sin(x)}$$

input
```
integrate((sin(x)*tan(x))^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(-(cos(x)^2 - 1)/cos(x))*cos(x)/sin(x)
```

3.147.6 Sympy [F]

$$\int \sqrt{\sin(x)\tan(x)}\,dx = \int \sqrt{\sin(x)\tan(x)}\,dx$$

input
```
integrate((sin(x)*tan(x))**(1/2),x)
```

output
```
Integral(sqrt(sin(x)*tan(x)), x)
```

3.147.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(11) = 22$.

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 4.38

$$\int \sqrt{\sin(x)\tan(x)}\,dx = \frac{2\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}-1\right)}{\sqrt{\frac{\sin(x)}{\cos(x)+1}+1}\sqrt{-\frac{\sin(x)}{\cos(x)+1}+1}\sqrt{\frac{\sin(x)^2}{(\cos(x)+1)^2}+1}}$$

input
```
integrate((sin(x)*tan(x))^(1/2),x, algorithm="maxima")
```

output
```
2*(sin(x)^2/(cos(x) + 1)^2 - 1)/(sqrt(sin(x)/(cos(x) + 1) + 1)*sqrt(-sin(x
)/(cos(x) + 1) + 1)*sqrt(sin(x)^2/(cos(x) + 1)^2 + 1))
```

3.147.8 Giac [F]

$$\int \sqrt{\sin(x)\tan(x)}\,dx = \int \sqrt{\sin(x)\tan(x)}\,dx$$

input
```
integrate((sin(x)*tan(x))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(sin(x)*tan(x)), x)
```

3.147.9 Mupad [B] (verification not implemented)

Time = 27.88 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.54

$$\int \sqrt{\sin(x)\tan(x)}\,dx = -\frac{2\,\sin\left(x\right)}{\sqrt{\frac{1}{\cos(x)}}\,\sqrt{1-{\cos\left(x\right)}^2}}$$

input
```
int((sin(x)*tan(x))^(1/2),x)
```

output
```
-(2*sin(x))/((1/cos(x))^(1/2)*(1 - cos(x)^2)^(1/2))
```

3.148 $\int (\sin(x)\tan(x))^{3/2}\,dx$

3.148.1 Optimal result

Integrand size = 9, antiderivative size = 31

$$\int (\sin(x)\tan(x))^{3/2}\,dx = \frac{8}{3}\csc(x)\sqrt{\sin(x)\tan(x)} - \frac{2}{3}\sin(x)\sqrt{\sin(x)\tan(x)}$$

output
```
8/3*csc(x)*(sin(x)*tan(x))^(1/2)-2/3*sin(x)*(sin(x)*tan(x))^(1/2)
```

3.148.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int (\sin(x)\tan(x))^{3/2}\,dx = \frac{2}{3}\left(-1+4\csc^2(x)\right)\sin(x)\sqrt{\sin(x)\tan(x)}$$

input
```
Integrate[(Sin[x]*Tan[x])^(3/2),x]
```

output
```
(2*(-1 + 4*Csc[x]^2)*Sin[x]*Sqrt[Sin[x]*Tan[x]])/3
```

3.148.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.77, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 4900, 3042, 3078, 3042, 3069}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\sin(x)\tan(x))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (\sin(x)\tan(x))^{3/2}dx$$

$$\Big\downarrow \text{4900}$$

$$\frac{\sqrt{\sin(x)\tan(x)}\int \sin^{\frac{3}{2}}(x)\tan^{\frac{3}{2}}(x)dx}{\sqrt{\sin(x)}\sqrt{\tan(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sin(x)\tan(x)}\int \sin(x)^{3/2}\tan(x)^{3/2}dx}{\sqrt{\sin(x)}\sqrt{\tan(x)}}$$

$$\Big\downarrow \text{3078}$$

$$\frac{\sqrt{\sin(x)\tan(x)}\left(\frac{4}{3}\int \frac{\tan^{\frac{3}{2}}(x)}{\sqrt{\sin(x)}}dx-\frac{2}{3}\sin^{\frac{3}{2}}(x)\sqrt{\tan(x)}\right)}{\sqrt{\sin(x)}\sqrt{\tan(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sin(x)\tan(x)}\left(\frac{4}{3}\int \frac{\tan(x)^{3/2}}{\sqrt{\sin(x)}}dx-\frac{2}{3}\sin^{\frac{3}{2}}(x)\sqrt{\tan(x)}\right)}{\sqrt{\sin(x)}\sqrt{\tan(x)}}$$

$$\Big\downarrow \text{3069}$$

$$\frac{\left(\frac{8\sqrt{\tan(x)}}{3\sqrt{\sin(x)}}-\frac{2}{3}\sin^{\frac{3}{2}}(x)\sqrt{\tan(x)}\right)\sqrt{\sin(x)\tan(x)}}{\sqrt{\sin(x)}\sqrt{\tan(x)}}$$

input
```
Int[(Sin[x]*Tan[x])^(3/2),x]
```

output
```
(((8*Sqrt[Tan[x]])/(3*Sqrt[Sin[x]]) - (2*Sin[x]^(3/2)*Sqrt[Tan[x]])/3)*Sqr
t[Sin[x]*Tan[x]])/(Sqrt[Sin[x]]*Sqrt[Tan[x]])
```

3.148.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3069
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

rule 4900
```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Simp[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p]))   Int[uu*vv^(m*p)*ww^(n*p), x]
, x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !
InertTrigFreeQ[w])
```

3.148.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 521 vs. $2(23) = 46$.

Time = 2.10 (sec) , antiderivative size = 522, normalized size of antiderivative = 16.84

method	result	size
default	Expression too large to display	522

input
```
int((sin(x)*tan(x))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/12*(3*cos(x)^3*ln((2*cos(x)*(-cos(x)/(cos(x)+1)^2)^(1/2)+2*(-cos(x)/(cos
(x)+1)^2)^(1/2)-cos(x)+1)/(cos(x)+1))*(-cos(x)/(cos(x)+1)^2)^(3/2)-3*cos(x
)^3*ln(2*(2*cos(x)*(-cos(x)/(cos(x)+1)^2)^(1/2)+2*(-cos(x)/(cos(x)+1)^2)^(
1/2)-cos(x)+1)/(cos(x)+1))*(-cos(x)/(cos(x)+1)^2)^(3/2)+9*ln((2*cos(x)*(-c
os(x)/(cos(x)+1)^2)^(1/2)+2*(-cos(x)/(cos(x)+1)^2)^(1/2)-cos(x)+1)/(cos(x)
+1))*(-cos(x)/(cos(x)+1)^2)^(3/2)*cos(x)^2-9*ln(2*(2*cos(x)*(-cos(x)/(cos(
x)+1)^2)^(1/2)+2*(-cos(x)/(cos(x)+1)^2)^(1/2)-cos(x)+1)/(cos(x)+1))*(-cos(
x)/(cos(x)+1)^2)^(3/2)*cos(x)^2+9*cos(x)*ln((2*cos(x)*(-cos(x)/(cos(x)+1)^
2)^(1/2)+2*(-cos(x)/(cos(x)+1)^2)^(1/2)-cos(x)+1)/(cos(x)+1))*(-cos(x)/(co
s(x)+1)^2)^(3/2)-9*cos(x)*ln(2*(2*cos(x)*(-cos(x)/(cos(x)+1)^2)^(1/2)+2*(-
cos(x)/(cos(x)+1)^2)^(1/2)-cos(x)+1)/(cos(x)+1))*(-cos(x)/(cos(x)+1)^2)^(3
/2)+3*ln((2*cos(x)*(-cos(x)/(cos(x)+1)^2)^(1/2)+2*(-cos(x)/(cos(x)+1)^2)^(
1/2)-cos(x)+1)/(cos(x)+1))*(-cos(x)/(cos(x)+1)^2)^(3/2)-3*ln(2*(2*cos(x)*(
-cos(x)/(cos(x)+1)^2)^(1/2)+2*(-cos(x)/(cos(x)+1)^2)^(1/2)-cos(x)+1)/(cos(
x)+1))*(-cos(x)/(cos(x)+1)^2)^(3/2)-4*cos(x)^3-12*cos(x))*(sin(x)*tan(x))^
(1/2)*tan(x)/(cos(x)^2-1)*4^(1/2)
```

3.148.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int (\sin(x)\tan(x))^{3/2}\,dx = \frac{2\left(\cos\left(x\right)^{2}+3\right)\sqrt{-\frac{\cos(x)^{2}-1}{\cos(x)}}}{3\,\sin\left(x\right)}$$

input
```
integrate((sin(x)*tan(x))^(3/2),x, algorithm="fricas")
```

output
```
2/3*(cos(x)^2 + 3)*sqrt(-(cos(x)^2 - 1)/cos(x))/sin(x)
```

3.148.6 Sympy [F]

$$\int (\sin(x)\tan(x))^{3/2}\,dx = \int \left(\sin\left(x\right)\tan\left(x\right)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((sin(x)*tan(x))**(3/2),x)
```

output
```
Integral((sin(x)*tan(x))**(3/2), x)
```

3.148.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(23) = 46$.

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.84

$$\int (\sin(x)\tan(x))^{3/2}\,dx = -\frac{8\left(\frac{\sin(x)^6}{(\cos(x)+1)^6}-1\right)}{3\left(\frac{\sin(x)}{\cos(x)+1}+1\right)^{\frac{3}{2}}\left(-\frac{\sin(x)}{\cos(x)+1}+1\right)^{\frac{3}{2}}\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)^{\frac{3}{2}}}$$

input
```
integrate((sin(x)*tan(x))^(3/2),x, algorithm="maxima")
```

output
```
-8/3*(sin(x)^6/(cos(x) + 1)^6 - 1)/((sin(x)/(cos(x) + 1) + 1)^(3/2)*(-sin(
x)/(cos(x) + 1) + 1)^(3/2)*(sin(x)^2/(cos(x) + 1)^2 + 1)^(3/2))
```

3.148.8 Giac [F]

$$\int (\sin(x)\tan(x))^{3/2}\,dx = \int (\sin(x)\tan(x))^{\frac{3}{2}}\,dx$$

input
```
integrate((sin(x)*tan(x))^(3/2),x, algorithm="giac")
```

output
```
integrate((sin(x)*tan(x))^(3/2), x)
```

3.148.9 Mupad [F(-1)]

Timed out.

$$\int (\sin(x)\tan(x))^{3/2}\,dx = \int (\sin(x)\,\tan(x))^{3/2}\,dx$$

input
```
int((sin(x)*tan(x))^(3/2),x)
```

output
```
int((sin(x)*tan(x))^(3/2), x)
```

3.149 $\int (\sin(x)\tan(x))^{5/2}\,dx$

3.149.1 Optimal result

Integrand size = 9, antiderivative size = 50

$$\int (\sin(x)\tan(x))^{5/2}\,dx = \frac{64}{15}\cot(x)\sqrt{\sin(x)\tan(x)} + \frac{16}{15}\tan(x)\sqrt{\sin(x)\tan(x)} - \frac{2}{5}\sin^2(x)\tan(x)\sqrt{\sin(x)\tan(x)}$$

output
```
64/15*cot(x)*(sin(x)*tan(x))^(1/2)+16/15*(sin(x)*tan(x))^(1/2)*tan(x)-2/5*
sin(x)^2*(sin(x)*tan(x))^(1/2)*tan(x)
```

3.149.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.58

$$\int (\sin(x)\tan(x))^{5/2}\,dx = \frac{2}{15}\left(5+3\cos^2(x)+32\cot^2(x)\right)\tan(x)\sqrt{\sin(x)\tan(x)}$$

input
```
Integrate[(Sin[x]*Tan[x])^(5/2),x]
```

output
```
(2*(5 + 3*Cos[x]^2 + 32*Cot[x]^2)*Tan[x]*Sqrt[Sin[x]*Tan[x]])/15
```

3.149.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.52, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.889, Rules used = {3042, 4900, 3042, 3078, 3042, 3074, 3042, 3069}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\sin(x)\tan(x))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (\sin(x)\tan(x))^{5/2}dx$$

$$\Big\downarrow \text{4900}$$

$$\frac{\sqrt{\sin(x)\tan(x)}\int \sin^{\frac{5}{2}}(x)\tan^{\frac{5}{2}}(x)dx}{\sqrt{\sin(x)}\sqrt{\tan(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sin(x)\tan(x)}\int \sin(x)^{5/2}\tan(x)^{5/2}dx}{\sqrt{\sin(x)}\sqrt{\tan(x)}}$$

$$\Big\downarrow \text{3078}$$

$$\frac{\sqrt{\sin(x)\tan(x)}\left(\frac{8}{5}\int \sqrt{\sin(x)}\tan^{\frac{5}{2}}(x)dx-\frac{2}{5}\sin^{\frac{5}{2}}(x)\tan^{\frac{3}{2}}(x)\right)}{\sqrt{\sin(x)}\sqrt{\tan(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sin(x)\tan(x)}\left(\frac{8}{5}\int \sqrt{\sin(x)}\tan(x)^{5/2}dx-\frac{2}{5}\sin^{\frac{5}{2}}(x)\tan^{\frac{3}{2}}(x)\right)}{\sqrt{\sin(x)}\sqrt{\tan(x)}}$$

$$\Big\downarrow \text{3074}$$

$$\frac{\sqrt{\sin(x)\tan(x)}\left(\frac{8}{5}\left(\frac{2}{3}\sqrt{\sin(x)}\tan^{\frac{3}{2}}(x)-\frac{4}{3}\int \sqrt{\sin(x)}\sqrt{\tan(x)}dx\right)-\frac{2}{5}\sin^{\frac{5}{2}}(x)\tan^{\frac{3}{2}}(x)\right)}{\sqrt{\sin(x)}\sqrt{\tan(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sin(x)\tan(x)}\left(\frac{8}{5}\left(\frac{2}{3}\sqrt{\sin(x)}\tan^{\frac{3}{2}}(x)-\frac{4}{3}\int \sqrt{\sin(x)}\sqrt{\tan(x)}dx\right)-\frac{2}{5}\sin^{\frac{5}{2}}(x)\tan^{\frac{3}{2}}(x)\right)}{\sqrt{\sin(x)}\sqrt{\tan(x)}}$$

$$\Big\downarrow \text{3069}$$

$$\frac{\sqrt{\sin(x)\tan(x)}\left(\frac{8}{5}\left(\frac{2}{3}\sqrt{\sin(x)}\tan^{\frac{3}{2}}(x)+\frac{8\sqrt{\sin(x)}}{3\sqrt{\tan(x)}}\right)-\frac{2}{5}\sin^{\frac{5}{2}}(x)\tan^{\frac{3}{2}}(x)\right)}{\sqrt{\sin(x)}\sqrt{\tan(x)}}$$

input
```
Int[(Sin[x]*Tan[x])^(5/2),x]
```

output
```
(Sqrt[Sin[x]*Tan[x]]*((-2*Sin[x]^(5/2)*Tan[x]^(3/2))/5 + (8*((8*Sqrt[Sin[x
]])/(3*Sqrt[Tan[x]]) + (2*Sqrt[Sin[x]]*Tan[x]^(3/2))/3))/5))/(Sqrt[Sin[x]]
*Sqrt[Tan[x]])
```

3.149.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3069
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

rule 3074
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(n
 - 1))), x] - Simp[b^2*((m + n - 1)/(n - 1))   Int[(a*Sin[e + f*x])^m*(b*Ta
n[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && In
tegersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

rule 4900
```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Simp[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p]))   Int[uu*vv^(m*p)*ww^(n*p), x]
, x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !
InertTrigFreeQ[w])
```

3.149.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. 2(38) = 76.

Time = 2.68 (sec) , antiderivative size = 283, normalized size of antiderivative = 5.66

method	result
default	$\tan(x)\left(6\cos(x)^4-15\sqrt{-\frac{\cos(x)}{(\cos(x)+1)^2}}\,\ln\left(\frac{2\cos(x)\sqrt{-\frac{\cos(x)}{(\cos(x)+1)^2}}+2\sqrt{-\frac{\cos(x)}{(\cos(x)+1)^2}}-\cos(x)+1}{\cos(x)+1}\right)\cos(x)^2+15\sqrt{-\frac{\cos(x)}{(\cos(x)+1)^2}}\,\ln\left(\frac{4\cos(\cdots\right.\right.$

input
```
int((sin(x)*tan(x))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/30*tan(x)*(6*cos(x)^4-15*(-cos(x)/(cos(x)+1)^2)^(1/2)*ln((2*cos(x)*(-cos
(x)/(cos(x)+1)^2)^(1/2)+2*(-cos(x)/(cos(x)+1)^2)^(1/2)-cos(x)+1)/(cos(x)+1
))*cos(x)^2+15*(-cos(x)/(cos(x)+1)^2)^(1/2)*ln(2*(2*cos(x)*(-cos(x)/(cos(x
)+1)^2)^(1/2)+2*(-cos(x)/(cos(x)+1)^2)^(1/2)-cos(x)+1)/(cos(x)+1))*cos(x)^
2-15*(-cos(x)/(cos(x)+1)^2)^(1/2)*ln((2*cos(x)*(-cos(x)/(cos(x)+1)^2)^(1/2
)+2*(-cos(x)/(cos(x)+1)^2)^(1/2)-cos(x)+1)/(cos(x)+1))*cos(x)+15*(-cos(x)/
(cos(x)+1)^2)^(1/2)*ln(2*(2*cos(x)*(-cos(x)/(cos(x)+1)^2)^(1/2)+2*(-cos(x)
/(cos(x)+1)^2)^(1/2)-cos(x)+1)/(cos(x)+1))*cos(x)-60*cos(x)^2-10)*(sin(x)*
tan(x))^(1/2)/(cos(x)^2-1)*4^(1/2)
```

3.149.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76

$$\int (\sin(x)\tan(x))^{5/2}\,dx = -\frac{2\left(3\cos(x)^4-30\cos(x)^2-5\right)\sqrt{-\frac{\cos(x)^2-1}{\cos(x)}}}{15\cos(x)\sin(x)}$$

input
```
integrate((sin(x)*tan(x))^(5/2),x, algorithm="fricas")
```

output
```
-2/15*(3*cos(x)^4 - 30*cos(x)^2 - 5)*sqrt(-(cos(x)^2 - 1)/cos(x))/(cos(x)*
sin(x))
```

3.149.6 Sympy [F(-1)]

Timed out.

$$\int (\sin(x)\tan(x))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((sin(x)*tan(x))**(5/2),x)
```

output
```
Timed out
```

3.149.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(38) = 76$.

Time $= 0.30$ (sec) , antiderivative size $= 82$, normalized size of antiderivative $= 1.64$

$$\int (\sin(x)\tan(x))^{5/2}\,dx = -\frac{32\left(\frac{5\sin(x)^4}{(\cos(x)+1)^4} - \frac{5\sin(x)^6}{(\cos(x)+1)^6} + \frac{2\sin(x)^{10}}{(\cos(x)+1)^{10}} - 2\right)}{15\left(\frac{\sin(x)}{\cos(x)+1}+1\right)^{\frac{5}{2}}\left(-\frac{\sin(x)}{\cos(x)+1}+1\right)^{\frac{5}{2}}\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)^{\frac{5}{2}}}$$

input
```
integrate((sin(x)*tan(x))^(5/2),x, algorithm="maxima")
```

output
```
-32/15*(5*sin(x)^4/(cos(x) + 1)^4 - 5*sin(x)^6/(cos(x) + 1)^6 + 2*sin(x)^1
0/(cos(x) + 1)^10 - 2)/((sin(x)/(cos(x) + 1) + 1)^(5/2)*(-sin(x)/(cos(x) +
 1) + 1)^(5/2)*(sin(x)^2/(cos(x) + 1)^2 + 1)^(5/2))
```

3.149.8 Giac [F]

$$\int (\sin(x)\tan(x))^{5/2}\,dx = \int (\sin(x)\tan(x))^{\frac{5}{2}}\,dx$$

input
```
integrate((sin(x)*tan(x))^(5/2),x, algorithm="giac")
```

output
```
integrate((sin(x)*tan(x))^(5/2), x)
```

3.149.9 Mupad [F(-1)]

Timed out.

$$\int (\sin(x)\tan(x))^{5/2}\,dx = \int (\sin(x)\ \tan(x))^{5/2}\,dx$$

input
```
int((sin(x)*tan(x))^(5/2),x)
```

output
```
int((sin(x)*tan(x))^(5/2), x)
```

3.150 $\int \sqrt{\cos(x)\cot(x)}\,dx$

3.150.1 Optimal result

Integrand size = 9, antiderivative size = 13

$$\int \sqrt{\cos(x)\cot(x)}\,dx = 2\sqrt{\cos(x)\cot(x)}\tan(x)$$

output
```
2*(cos(x)*cot(x))^(1/2)*tan(x)
```

3.150.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \sqrt{\cos(x)\cot(x)}\,dx = 2\sqrt{\cos(x)\cot(x)}\tan(x)$$

input
```
Integrate[Sqrt[Cos[x]*Cot[x]],x]
```

output
```
2*Sqrt[Cos[x]*Cot[x]]*Tan[x]
```

3.150.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3042, 4900, 3042, 3069}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(x)\cot(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\cos(x)\cot(x)}dx$$

$$\Big\downarrow \text{4900}$$

$$\frac{\sqrt{\cos(x)\cot(x)}\int \sqrt{\cos(x)}\sqrt{\cot(x)}dx}{\sqrt{\cos(x)}\sqrt{\cot(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(x)\cot(x)}\int \sqrt{\sin\left(x+\frac{\pi}{2}\right)}\sqrt{-\tan\left(x+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(x)}\sqrt{\cot(x)}}$$

$$\Big\downarrow \text{3069}$$

$$2\tan(x)\sqrt{\cos(x)\cot(x)}$$

input
```
Int[Sqrt[Cos[x]*Cot[x]],x]
```

output
```
2*Sqrt[Cos[x]*Cot[x]]*Tan[x]
```

3.150.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3069
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

rule 4900
```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Simp[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p]))   Int[uu*vv^(m*p)*ww^(n*p), x]
, x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !
InertTrigFreeQ[w])
```

3.150.4 Maple [A] (verified)

Time = 1.92 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
default	$2\sqrt{\cot\left(x\right)\cos\left(x\right)}\,\tan\left(x\right)$	12
risch	$-\frac{i\sqrt{2}\,\sqrt{\frac{i\left(\mathrm{e}^{2ix}+1\right)^{2}\mathrm{e}^{-ix}}{\mathrm{e}^{2ix}-1}}\,\left(\mathrm{e}^{2ix}-1\right)}{\mathrm{e}^{2ix}+1}$	51

input
```
int((cot(x)*cos(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(cot(x)*cos(x))^(1/2)*tan(x)
```

3.150.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int\sqrt{\cos(x)\cot(x)}\,dx=\frac{2\,\sqrt{\frac{\cos(x)^2}{\sin(x)}}\,\sin\left(x\right)}{\cos\left(x\right)}$$

input
```
integrate((cos(x)*cot(x))^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(cos(x)^2/sin(x))*sin(x)/cos(x)
```

3.150.6 Sympy [F]

$$\int \sqrt{\cos(x)\cot(x)}\,dx = \int \sqrt{\cos{(x)}\cot{(x)}}\,dx$$

input
```
integrate((cos(x)*cot(x))**(1/2),x)
```

output
```
Integral(sqrt(cos(x)*cot(x)), x)
```

3.150.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(11) = 22$.

Time = 0.35 (sec) , antiderivative size = 188, normalized size of antiderivative = 14.46

$$\int \sqrt{\cos(x)\cot(x)}\,dx$$
$$= \frac{\left(\left(\cos\left(\frac{3}{2}x\right) - \cos\left(\frac{1}{2}x\right) + \sin\left(\frac{3}{2}x\right) + \sin\left(\frac{1}{2}x\right)\right)\cos\left(\frac{1}{2}\arctan\left(\sin\left(x\right),\cos\left(x\right) - 1\right)\right) - \left(\cos\left(\frac{3}{2}x\right) - \cos\left(\frac{1}{2}\right.\right.}{}$$

input
```
integrate((cos(x)*cot(x))^(1/2),x, algorithm="maxima")
```

output
```
(((cos(3/2*x) - cos(1/2*x) + sin(3/2*x) + sin(1/2*x))*cos(1/2*arctan2(sin(
x), cos(x) - 1)) - (cos(3/2*x) - cos(1/2*x) - sin(3/2*x) - sin(1/2*x))*sin
(1/2*arctan2(sin(x), cos(x) - 1)))*cos(1/2*arctan2(sin(x), cos(x) + 1)) -
((cos(3/2*x) - cos(1/2*x) - sin(3/2*x) - sin(1/2*x))*cos(1/2*arctan2(sin(x
), cos(x) - 1)) + (cos(3/2*x) - cos(1/2*x) + sin(3/2*x) + sin(1/2*x))*sin(
1/2*arctan2(sin(x), cos(x) - 1)))*sin(1/2*arctan2(sin(x), cos(x) + 1)))/((
cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)^(1/4)*(cos(x)^2 + sin(x)^2 - 2*cos(x)
+ 1)^(1/4))
```

3.150.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \sqrt{\cos(x)\cot(x)}\,dx = 2\,\text{sgn}(\cos(x))\,\text{sgn}(\sin(x))\,\sqrt{\sin(x)}$$

input
```
integrate((cos(x)*cot(x))^(1/2),x, algorithm="giac")
```

output
```
2*sgn(cos(x))*sgn(sin(x))*sqrt(sin(x))
```

3.150.9 Mupad [B] (verification not implemented)

Time = 25.58 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.38

$$\int \sqrt{\cos(x)\cot(x)}\,dx = \frac{2\,|\cos(x)|\,\sin(x)^{3/2}}{|\sin(x)|\,\cos(x)}$$

input
```
int((cos(x)*cot(x))^(1/2),x)
```

output
```
(2*abs(cos(x))*sin(x)^(3/2))/(abs(sin(x))*cos(x))
```

3.151 $\int (\cos(x)\cot(x))^{3/2}\,dx$

3.151.1 Optimal result

Integrand size = 9, antiderivative size = 31

$$\int (\cos(x)\cot(x))^{3/2}\,dx = \frac{2}{3}\cos(x)\sqrt{\cos(x)\cot(x)} - \frac{8}{3}\sqrt{\cos(x)\cot(x)}\sec(x)$$

output
```
2/3*cos(x)*(cos(x)*cot(x))^(1/2)-8/3*sec(x)*(cos(x)*cot(x))^(1/2)
```

3.151.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68

$$\int (\cos(x)\cot(x))^{3/2}\,dx = \frac{2}{3}\left(-4+\cos^2(x)\right)\sqrt{\cos(x)\cot(x)}\sec(x)$$

input
```
Integrate[(Cos[x]*Cot[x])^(3/2),x]
```

output
```
(2*(-4 + Cos[x]^2)*Sqrt[Cos[x]*Cot[x]]*Sec[x])/3
```

3.151.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.77, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 4900, 3042, 3078, 3042, 3069}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\cos(x)\cot(x))^{3/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (\cos(x)\cot(x))^{3/2}dx$$

$$\Big\downarrow \; 4900$$

$$\frac{\sqrt{\cos(x)\cot(x)}\int \cos^{\frac{3}{2}}(x)\cot^{\frac{3}{2}}(x)dx}{\sqrt{\cos(x)}\sqrt{\cot(x)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt{\cos(x)\cot(x)}\int \sin\left(x+\frac{\pi}{2}\right)^{3/2}\left(-\tan\left(x+\frac{\pi}{2}\right)\right)^{3/2}dx}{\sqrt{\cos(x)}\sqrt{\cot(x)}}$$

$$\Big\downarrow \; 3078$$

$$\frac{\sqrt{\cos(x)\cot(x)}\left(\frac{4}{3}\int \frac{\cot^{\frac{3}{2}}(x)}{\sqrt{\cos(x)}}dx+\frac{2}{3}\cos^{\frac{3}{2}}(x)\sqrt{\cot(x)}\right)}{\sqrt{\cos(x)}\sqrt{\cot(x)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt{\cos(x)\cot(x)}\left(\frac{4}{3}\int \frac{\left(-\tan\left(x+\frac{\pi}{2}\right)\right)^{3/2}}{\sqrt{\sin\left(x+\frac{\pi}{2}\right)}}dx+\frac{2}{3}\cos^{\frac{3}{2}}(x)\sqrt{\cot(x)}\right)}{\sqrt{\cos(x)}\sqrt{\cot(x)}}$$

$$\Big\downarrow \; 3069$$

$$\frac{\left(\frac{2}{3}\cos^{\frac{3}{2}}(x)\sqrt{\cot(x)}-\frac{8\sqrt{\cot(x)}}{3\sqrt{\cos(x)}}\right)\sqrt{\cos(x)\cot(x)}}{\sqrt{\cos(x)}\sqrt{\cot(x)}}$$

input
```
Int[(Cos[x]*Cot[x])^(3/2),x]
```

output
```
(((-8*Sqrt[Cot[x]])/(3*Sqrt[Cos[x]]) + (2*Cos[x]^(3/2)*Sqrt[Cot[x]])/3)*Sq
rt[Cos[x]*Cot[x]])/(Sqrt[Cos[x]]*Sqrt[Cot[x]])
```

3.151.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3069
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

rule 4900
```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Simp[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p]))   Int[uu*vv^(m*p)*ww^(n*p), x]
, x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !
InertTrigFreeQ[w])
```

3.151.4 Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.55

method	result	size
default	$\frac{2\sqrt{\cot(x)\cos(x)}\,(\cos(x)-4\sec(x))}{3}$	17

input
```
int((cot(x)*cos(x))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(cot(x)*cos(x))^(1/2)*(cos(x)-4*sec(x))
```

3.151.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int (\cos(x)\cot(x))^{3/2}\,dx = \frac{2\left(\cos(x)^2-4\right)\sqrt{\frac{\cos(x)^2}{\sin(x)}}}{3\cos(x)}$$

input
```
integrate((cos(x)*cot(x))^(3/2),x, algorithm="fricas")
```

output
```
2/3*(cos(x)^2 - 4)*sqrt(cos(x)^2/sin(x))/cos(x)
```

3.151.6 Sympy [F]

$$\int (\cos(x)\cot(x))^{3/2}\,dx = \int (\cos(x)\cot(x))^{\frac{3}{2}}\,dx$$

input
```
integrate((cos(x)*cot(x))**(3/2),x)
```

output
```
Integral((cos(x)*cot(x))**(3/2), x)
```

3.151.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. 2(23) = 46.

Time = 0.36 (sec) , antiderivative size = 314, normalized size of antiderivative = 10.13

$$\int (\cos(x)\cot(x))^{3/2}\,dx = \frac{\left(\cos(x)^2+\sin(x)^2+2\cos(x)+1\right)^{\frac{1}{4}}\left(\cos(x)^2+\sin(x)^2-2\cos(x)+1\right)^{\frac{1}{4}}(((\cos}{}$$

input
```
integrate((cos(x)*cot(x))^(3/2),x, algorithm="maxima")
```

output
```
1/6*(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)^(1/4)*(cos(x)^2 + sin(x)^2 - 2*co
s(x) + 1)^(1/4)*(((cos(9/2*x) - 15*cos(5/2*x) - cos(3/2*x) + 15*cos(1/2*x)
 - sin(9/2*x) + 15*sin(5/2*x) - sin(3/2*x) - 15*sin(1/2*x))*cos(3/2*arctan
2(sin(x), cos(x) - 1)) + (cos(9/2*x) - 15*cos(5/2*x) - cos(3/2*x) + 15*cos
(1/2*x) + sin(9/2*x) - 15*sin(5/2*x) + sin(3/2*x) + 15*sin(1/2*x))*sin(3/2
*arctan2(sin(x), cos(x) - 1)))*cos(3/2*arctan2(sin(x), cos(x) + 1)) + ((co
s(9/2*x) - 15*cos(5/2*x) - cos(3/2*x) + 15*cos(1/2*x) + sin(9/2*x) - 15*si
n(5/2*x) + sin(3/2*x) + 15*sin(1/2*x))*cos(3/2*arctan2(sin(x), cos(x) - 1)
) - (cos(9/2*x) - 15*cos(5/2*x) - cos(3/2*x) + 15*cos(1/2*x) - sin(9/2*x)
+ 15*sin(5/2*x) - sin(3/2*x) - 15*sin(1/2*x))*sin(3/2*arctan2(sin(x), cos(
x) - 1)))*sin(3/2*arctan2(sin(x), cos(x) + 1)))/(cos(x)^4 + sin(x)^4 + 2*(
cos(x)^2 + 1)*sin(x)^2 - 2*cos(x)^2 + 1)
```

3.151.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

$$\int (\cos(x)\cot(x))^{3/2}\,dx = -\frac{2}{3}\left(\sin(x)^{\frac{3}{2}} + \frac{3}{\sqrt{\sin(x)}}\right)\operatorname{sgn}(\cos(x))\operatorname{sgn}(\sin(x))$$

input
```
integrate((cos(x)*cot(x))^(3/2),x, algorithm="giac")
```

output
```
-2/3*(sin(x)^(3/2) + 3/sqrt(sin(x)))*sgn(cos(x))*sgn(sin(x))
```

3.151.9 Mupad [F(-1)]

Timed out.

$$\int (\cos(x)\cot(x))^{3/2}\,dx = \int (\cos(x)\,\cot(x))^{3/2}\,dx$$

input
```
int((cos(x)*cot(x))^(3/2),x)
```

output
```
int((cos(x)*cot(x))^(3/2), x)
```

3.152 $\int (\cos(x)\cot(x))^{5/2}\,dx$

3.152.1 Optimal result

Integrand size = 9, antiderivative size = 50

$$\int (\cos(x)\cot(x))^{5/2}\,dx = -\frac{16}{15}\cot(x)\sqrt{\cos(x)\cot(x)}+\frac{2}{5}\cos^2(x)\cot(x)\sqrt{\cos(x)\cot(x)}-\frac{64}{15}\sqrt{\cos(x)\cot(x)}\tan(x)$$

output
```
-16/15*cot(x)*(cos(x)*cot(x))^(1/2)+2/5*cos(x)^2*cot(x)*(cos(x)*cot(x))^(1
/2)-64/15*(cos(x)*cot(x))^(1/2)*tan(x)
```

3.152.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.58

$$\int (\cos(x)\cot(x))^{5/2}\,dx = -\frac{2}{15}\sqrt{\cos(x)\cot(x)}\left(32+3\cos^2(x)+5\cot^2(x)\right)\tan(x)$$

input
```
Integrate[(Cos[x]*Cot[x])^(5/2),x]
```

output
```
(-2*Sqrt[Cos[x]*Cot[x]]*(32 + 3*Cos[x]^2 + 5*Cot[x]^2)*Tan[x])/15
```

3.152.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.52, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.889, Rules used = {3042, 4900, 3042, 3078, 3042, 3074, 3042, 3069}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\cos(x)\cot(x))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (\cos(x)\cot(x))^{5/2}dx$$

$$\Big\downarrow \text{4900}$$

$$\frac{\sqrt{\cos(x)\cot(x)}\int \cos^{\frac{5}{2}}(x)\cot^{\frac{5}{2}}(x)dx}{\sqrt{\cos(x)}\sqrt{\cot(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(x)\cot(x)}\int \sin\left(x+\frac{\pi}{2}\right)^{5/2}\left(-\tan\left(x+\frac{\pi}{2}\right)\right)^{5/2}dx}{\sqrt{\cos(x)}\sqrt{\cot(x)}}$$

$$\Big\downarrow \text{3078}$$

$$\frac{\sqrt{\cos(x)\cot(x)}\left(\frac{8}{5}\int \sqrt{\cos(x)}\cot^{\frac{5}{2}}(x)dx+\frac{2}{5}\cos^{\frac{5}{2}}(x)\cot^{\frac{3}{2}}(x)\right)}{\sqrt{\cos(x)}\sqrt{\cot(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(x)\cot(x)}\left(\frac{8}{5}\int \sqrt{\sin\left(x+\frac{\pi}{2}\right)}\left(-\tan\left(x+\frac{\pi}{2}\right)\right)^{5/2}dx+\frac{2}{5}\cos^{\frac{5}{2}}(x)\cot^{\frac{3}{2}}(x)\right)}{\sqrt{\cos(x)}\sqrt{\cot(x)}}$$

$$\Big\downarrow \text{3074}$$

$$\frac{\sqrt{\cos(x)\cot(x)}\left(\frac{8}{5}\left(-\frac{4}{3}\int \sqrt{\cos(x)}\sqrt{\cot(x)}dx-\frac{2}{3}\sqrt{\cos(x)}\cot^{\frac{3}{2}}(x)\right)+\frac{2}{5}\cos^{\frac{5}{2}}(x)\cot^{\frac{3}{2}}(x)\right)}{\sqrt{\cos(x)}\sqrt{\cot(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(x)\cot(x)}\left(\frac{8}{5}\left(-\frac{4}{3}\int \sqrt{\sin\left(x+\frac{\pi}{2}\right)}\sqrt{-\tan\left(x+\frac{\pi}{2}\right)}dx-\frac{2}{3}\sqrt{\cos(x)}\cot^{\frac{3}{2}}(x)\right)+\frac{2}{5}\cos^{\frac{5}{2}}(x)\cot^{\frac{3}{2}}(x)\right)}{\sqrt{\cos(x)}\sqrt{\cot(x)}}$$

$$\Big\downarrow \text{3069}$$

$$\frac{\sqrt{\cos(x)\cot(x)}\left(\frac{2}{5}\cos^{\frac{5}{2}}(x)\cot^{\frac{3}{2}}(x)+\frac{8}{5}\left(-\frac{2}{3}\sqrt{\cos(x)}\cot^{\frac{3}{2}}(x)-\frac{8\sqrt{\cos(x)}}{3\sqrt{\cot(x)}}\right)\right)}{\sqrt{\cos(x)}\sqrt{\cot(x)}}$$

input
```
Int[(Cos[x]*Cot[x])^(5/2),x]
```

output
```
(Sqrt[Cos[x]*Cot[x]]*((2*Cos[x]^(5/2)*Cot[x]^(3/2))/5 + (8*((-8*Sqrt[Cos[x
]])/(3*Sqrt[Cot[x]]) - (2*Sqrt[Cos[x]]*Cot[x]^(3/2))/3))/5))/(Sqrt[Cos[x]]
*Sqrt[Cot[x]])
```

3.152.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3069
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

rule 3074
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(n
 - 1))), x] - Simp[b^2*((m + n - 1)/(n - 1))   Int[(a*Sin[e + f*x])^m*(b*Ta
n[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && In
tegersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

rule 4900
```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Simp[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p]))   Int[uu*vv^(m*p)*ww^(n*p), x]
, x]] /; FreeQ[{m, n, p}, x] && !IntegerQ[p] && ( !InertTrigFreeQ[v] || !
InertTrigFreeQ[w])
```

3.152.4 Maple [A] (verified)

Time = 4.34 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.58

method	result	size
default	$\frac{2\sqrt{\cot(x)\cos(x)}\,\left(3\cos(x)^{2}\cot(x)+24\cot(x)-32\sec(x)\csc(x)\right)}{15}$	29

input
```
int((cot(x)*cos(x))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(cot(x)*cos(x))^(1/2)*(3*cos(x)^2*cot(x)+24*cot(x)-32*sec(x)*csc(x))
```

3.152.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.70

$$\int (\cos(x)\cot(x))^{5/2}\,dx = \frac{2\left(3\cos(x)^{4}+24\cos(x)^{2}-32\right)\sqrt{\frac{\cos(x)^{2}}{\sin(x)}}}{15\cos(x)\sin(x)}$$

input
```
integrate((cos(x)*cot(x))^(5/2),x, algorithm="fricas")
```

output
```
2/15*(3*cos(x)^4 + 24*cos(x)^2 - 32)*sqrt(cos(x)^2/sin(x))/(cos(x)*sin(x))
```

3.152.6 Sympy [F(-1)]

Timed out.

$$\int (\cos(x)\cot(x))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((cos(x)*cot(x))**(5/2),x)
```

output
```
Timed out
```

3.152.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(38) = 76$.

Time = 0.37 (sec) , antiderivative size = 427, normalized size of antiderivative = 8.54

$$\int (\cos(x)\cot(x))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((cos(x)*cot(x))^(5/2),x, algorithm="maxima")
```

output
```
-1/60*(((3*cos(15/2*x) + 105*cos(11/2*x) - 410*cos(7/2*x) - 3*cos(5/2*x) +
 410*cos(3/2*x) - 105*cos(1/2*x) + 3*sin(15/2*x) + 105*sin(11/2*x) - 410*s
in(7/2*x) + 3*sin(5/2*x) + 410*sin(3/2*x) + 105*sin(1/2*x))*cos(5/2*arctan
2(sin(x), cos(x) - 1)) - (3*cos(15/2*x) + 105*cos(11/2*x) - 410*cos(7/2*x)
 - 3*cos(5/2*x) + 410*cos(3/2*x) - 105*cos(1/2*x) - 3*sin(15/2*x) - 105*si
n(11/2*x) + 410*sin(7/2*x) - 3*sin(5/2*x) - 410*sin(3/2*x) - 105*sin(1/2*x
))*sin(5/2*arctan2(sin(x), cos(x) - 1)))*cos(5/2*arctan2(sin(x), cos(x) +
1)) - ((3*cos(15/2*x) + 105*cos(11/2*x) - 410*cos(7/2*x) - 3*cos(5/2*x) +
410*cos(3/2*x) - 105*cos(1/2*x) - 3*sin(15/2*x) - 105*sin(11/2*x) + 410*si
n(7/2*x) - 3*sin(5/2*x) - 410*sin(3/2*x) - 105*sin(1/2*x))*cos(5/2*arctan2
(sin(x), cos(x) - 1)) + (3*cos(15/2*x) + 105*cos(11/2*x) - 410*cos(7/2*x)
- 3*cos(5/2*x) + 410*cos(3/2*x) - 105*cos(1/2*x) + 3*sin(15/2*x) + 105*sin
(11/2*x) - 410*sin(7/2*x) + 3*sin(5/2*x) + 410*sin(3/2*x) + 105*sin(1/2*x)
)*sin(5/2*arctan2(sin(x), cos(x) - 1)))*sin(5/2*arctan2(sin(x), cos(x) + 1
)))/((cos(x)^4 + sin(x)^4 + 2*(cos(x)^2 + 1)*sin(x)^2 - 2*cos(x)^2 + 1)*(c
os(x)^2 + sin(x)^2 + 2*cos(x) + 1)^(1/4)*(cos(x)^2 + sin(x)^2 - 2*cos(x) +
 1)^(1/4))
```

3.152.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.54

$$\int (\cos(x)\cot(x))^{5/2}\,dx = \frac{2}{15}\left(3\sin(x)^{\frac{5}{2}} - 30\sqrt{\sin(x)} - \frac{5}{\sin(x)^{\frac{3}{2}}}\right)\operatorname{sgn}(\cos(x))\operatorname{sgn}(\sin(x))$$

input
```
integrate((cos(x)*cot(x))^(5/2),x, algorithm="giac")
```

output
```
2/15*(3*sin(x)^(5/2) - 30*sqrt(sin(x)) - 5/sin(x)^(3/2))*sgn(cos(x))*sgn(s
in(x))
```

3.152.9 Mupad [F(-1)]

Timed out.

$$\int (\cos(x)\cot(x))^{5/2}\,dx = \int (\cos(x)\cot(x))^{5/2}\,dx$$

input
```
int((cos(x)*cot(x))^(5/2),x)
```

output
```
int((cos(x)*cot(x))^(5/2), x)
```

3.153 $\int \frac{x\cos(x)}{(a+b\sin(x))^2}\,dx$

3.153.1 Optimal result

Integrand size = 12, antiderivative size = 58

$$\int \frac{x\cos(x)}{(a+b\sin(x))^2}\,dx=\frac{2\arctan\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}}-\frac{x}{b(a+b\sin(x))}$$

output
```
-x/b/(a+b*sin(x))+2*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/b/(a^2-b^2)^(
1/2)
```

3.153.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97

$$\int \frac{x\cos(x)}{(a+b\sin(x))^2}\,dx=\frac{\frac{2\arctan\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-\frac{x}{a+b\sin(x)}}{b}$$

input
```
Integrate[(x*Cos[x])/(a + b*Sin[x])^2,x]
```

output
```
((2*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] - x/(a + b*S
in[x]))/b
```

3.153.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {4922, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\cos(x)}{(a+b\sin(x))^2}dx$$

$$\Big\downarrow \text{4922}$$

$$\frac{\int \frac{1}{a+b\sin(x)}dx}{b}-\frac{x}{b(a+b\sin(x))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{a+b\sin(x)}dx}{b}-\frac{x}{b(a+b\sin(x))}$$

$$\Big\downarrow \text{3139}$$

$$\frac{2\int \frac{1}{a\tan^2\left(\frac{x}{2}\right)+2b\tan\left(\frac{x}{2}\right)+a}d\tan\left(\frac{x}{2}\right)}{b}-\frac{x}{b(a+b\sin(x))}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{4\int \frac{1}{-\left(2b+2a\tan\left(\frac{x}{2}\right)\right)^2-4(a^2-b^2)}d\left(2b+2a\tan\left(\frac{x}{2}\right)\right)}{b}-\frac{x}{b(a+b\sin(x))}$$

$$\Big\downarrow \text{217}$$

$$\frac{2\arctan\left(\frac{2a\tan\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}}-\frac{x}{b(a+b\sin(x))}$$

input
```
Int[(x*Cos[x])/(a + b*Sin[x])^2,x]
```

output
```
(2*ArcTan[(2*b + 2*a*Tan[x/2])/(2*Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]) -
 x/(b*(a + b*Sin[x]))
```

3.153.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 4922
```
Int[Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sin[(c
_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sin[c + d*x
])^(n + 1)/(b*d*(n + 1))), x] - Simp[f*(m/(b*d*(n + 1)))   Int[(e + f*x)^(m
 - 1)*(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n},
x] && IGtQ[m, 0] && NeQ[n, -1]
```

3.153.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.74

method	result	size
risch	$-\frac{2ix\,e^{ix}}{b(b\,e^{2ix}-b+2ia\,e^{ix})}-\frac{\ln\left(e^{ix}+\frac{ia\sqrt{-a^2+b^2}-a^2+b^2}{\sqrt{-a^2+b^2}\,b}\right)}{\sqrt{-a^2+b^2}\,b}+\frac{\ln\left(e^{ix}+\frac{ia\sqrt{-a^2+b^2}+a^2-b^2}{\sqrt{-a^2+b^2}\,b}\right)}{\sqrt{-a^2+b^2}\,b}$	159

input
```
int(x*cos(x)/(a+b*sin(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-2*I*x*exp(I*x)/b/(b*exp(2*I*x)-b+2*I*a*exp(I*x))-1/(-a^2+b^2)^(1/2)/b*ln(
exp(I*x)+(I*a*(-a^2+b^2)^(1/2)-a^2+b^2)/(-a^2+b^2)^(1/2)/b)+1/(-a^2+b^2)^(
1/2)/b*ln(exp(I*x)+(I*a*(-a^2+b^2)^(1/2)+a^2-b^2)/(-a^2+b^2)^(1/2)/b)
```

3.153.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 236, normalized size of antiderivative = 4.07

$$\int \frac{x\cos(x)}{(a+b\sin(x))^2}\,dx$$
$$= \left[-\frac{\sqrt{-a^2+b^2}(b\sin(x)+a)\log\left(\frac{(2a^2-b^2)\cos(x)^2-2ab\sin(x)-a^2-b^2+2(a\cos(x)\sin(x)+b\cos(x))\sqrt{-a^2+b^2}}{b^2\cos(x)^2-2ab\sin(x)-a^2-b^2}\right)+2(a^2-b^2)}{2(a^3b-ab^3+(a^2b^2-b^4)\sin(x))}\right.$$
$$\left.-\frac{\sqrt{a^2-b^2}(b\sin(x)+a)\arctan\left(-\frac{a\sin(x)+b}{\sqrt{a^2-b^2}\cos(x)}\right)+(a^2-b^2)x}{a^3b-ab^3+(a^2b^2-b^4)\sin(x)}\right]$$

input
```
integrate(x*cos(x)/(a+b*sin(x))^2,x, algorithm="fricas")
```

output
```
[-1/2*(sqrt(-a^2 + b^2)*(b*sin(x) + a)*log(((2*a^2 - b^2)*cos(x)^2 - 2*a*b
*sin(x) - a^2 - b^2 + 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^
2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2)) + 2*(a^2 - b^2)*x)/(a^3*b - a*b^3
+ (a^2*b^2 - b^4)*sin(x)), -(sqrt(a^2 - b^2)*(b*sin(x) + a)*arctan(-(a*sin
(x) + b)/(sqrt(a^2 - b^2)*cos(x))) + (a^2 - b^2)*x)/(a^3*b - a*b^3 + (a^2*
b^2 - b^4)*sin(x))]
```

3.153.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x\cos(x)}{(a+b\sin(x))^2}\,dx = \text{Timed out}$$

input
```
integrate(x*cos(x)/(a+b*sin(x))**2,x)
```

output
```
Timed out
```

3.153.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x\cos(x)}{(a+b\sin(x))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*cos(x)/(a+b*sin(x))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.153.8 Giac [F]

$$\int \frac{x\cos(x)}{(a+b\sin(x))^2}\,dx = \int \frac{x\cos(x)}{(b\sin(x)+a)^2}\,dx$$

input
```
integrate(x*cos(x)/(a+b*sin(x))^2,x, algorithm="giac")
```

output
```
integrate(x*cos(x)/(b*sin(x) + a)^2, x)
```

3.153.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\cos(x)}{(a+b\sin(x))^2}\,dx = \int \frac{x\,\cos(x)}{(a+b\,\sin(x))^2}\,dx$$

input
```
int((x*cos(x))/(a + b*sin(x))^2,x)
```

output
```
int((x*cos(x))/(a + b*sin(x))^2, x)
```

3.154 $\int \frac{x\cos(x)}{(a+b\sin(x))^3}\,dx$

3.154.1 Optimal result

Integrand size = 12, antiderivative size = 85

$$\int \frac{x\cos(x)}{(a+b\sin(x))^3}\,dx=\frac{a\arctan\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b\,(a^2-b^2)^{3/2}}-\frac{x}{2b(a+b\sin(x))^2}+\frac{\cos(x)}{2\,(a^2-b^2)\,(a+b\sin(x))}$$

output
```
a*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/b/(a^2-b^2)^(3/2)-1/2*x/b/(a+b*
sin(x))^2+1/2*cos(x)/(a^2-b^2)/(a+b*sin(x))
```

3.154.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.99

$$\int \frac{x\cos(x)}{(a+b\sin(x))^3}\,dx=\frac{a\arctan\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b\,(a^2-b^2)^{3/2}}+\frac{-\frac{x}{b}+\frac{\cos(x)(a+b\sin(x))}{(a-b)(a+b)}}{2(a+b\sin(x))^2}$$

input
```
Integrate[(x*Cos[x])/(a + b*Sin[x])^3,x]
```

output
```
(a*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/(b*(a^2 - b^2)^(3/2)) + (-(x/
b) + (Cos[x]*(a + b*Sin[x]))/((a - b)*(a + b)))/(2*(a + b*Sin[x])^2)
```

3.154.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {4922, 3042, 3143, 25, 27, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\cos(x)}{(a+b\sin(x))^3}dx$$

$$\Big\downarrow \text{4922}$$

$$\frac{\int \frac{1}{(a+b\sin(x))^2}dx}{2b}-\frac{x}{2b(a+b\sin(x))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{(a+b\sin(x))^2}dx}{2b}-\frac{x}{2b(a+b\sin(x))^2}$$

$$\Big\downarrow \text{3143}$$

$$\frac{\frac{b\cos(x)}{(a^2-b^2)(a+b\sin(x))}-\frac{\int -\frac{a}{a+b\sin(x)}dx}{a^2-b^2}}{2b}-\frac{x}{2b(a+b\sin(x))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{a}{a+b\sin(x)}dx}{a^2-b^2}+\frac{b\cos(x)}{(a^2-b^2)(a+b\sin(x))}}{2b}-\frac{x}{2b(a+b\sin(x))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{a\int \frac{1}{a+b\sin(x)}dx}{a^2-b^2}+\frac{b\cos(x)}{(a^2-b^2)(a+b\sin(x))}}{2b}-\frac{x}{2b(a+b\sin(x))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{a\int \frac{1}{a+b\sin(x)}dx}{a^2-b^2}+\frac{b\cos(x)}{(a^2-b^2)(a+b\sin(x))}}{2b}-\frac{x}{2b(a+b\sin(x))^2}$$

$$\Big\downarrow \text{3139}$$

$$\frac{\frac{2a\int \frac{1}{a\tan^2\left(\frac{x}{2}\right)+2b\tan\left(\frac{x}{2}\right)+a}d\tan\left(\frac{x}{2}\right)}{a^2-b^2}+\frac{b\cos(x)}{(a^2-b^2)(a+b\sin(x))}}{2b}-\frac{x}{2b(a+b\sin(x))^2}$$

$$\Big\downarrow \text{1083}$$

$$\frac{\frac{b\cos(x)}{(a^2-b^2)(a+b\sin(x))}-\frac{4a\int\frac{1}{-\left(2b+2a\tan\left(\frac{x}{2}\right)\right)^2-4\left(a^2-b^2\right)}d\left(2b+2a\tan\left(\frac{x}{2}\right)\right)}{a^2-b^2}}{2b}-\frac{x}{2b(a+b\sin(x))^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{2a\arctan\left(\frac{2a\tan\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}+\frac{b\cos(x)}{(a^2-b^2)(a+b\sin(x))}}{2b}-\frac{x}{2b(a+b\sin(x))^2}$$

input
```
Int[(x*Cos[x])/(a + b*Sin[x])^3,x]
```

output
```
-1/2*x/(b*(a + b*Sin[x])^2) + ((2*a*ArcTan[(2*b + 2*a*Tan[x/2])/(2*Sqrt[a^
2 - b^2])])/(a^2 - b^2)^(3/2) + (b*Cos[x])/((a^2 - b^2)*(a + b*Sin[x])))/(
2*b)
```

3.154.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 4922
```
Int[Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sin[(c
_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sin[c + d*x
])^(n + 1)/(b*d*(n + 1))), x] - Simp[f*(m/(b*d*(n + 1)))   Int[(e + f*x)^(m
 - 1)*(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n},
x] && IGtQ[m, 0] && NeQ[n, -1]
```

3.154.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.72 (sec) , antiderivative size = 257, normalized size of antiderivative = 3.02

method	result
risch	$\frac{2ia^2 e^{2ix}+ib^2 e^{2ix}+2x a^2 e^{2ix}+ba e^{3ix}-2b^2 x e^{2ix}-ib^2-3ab e^{ix}}{(b e^{2ix}-b+2ia e^{ix})^2 (a^2-b^2) b} - \frac{a \ln\left(e^{ix}+\frac{ia\sqrt{-a^2+b^2}-a^2+b^2}{\sqrt{-a^2+b^2}\, b}\right)}{2\sqrt{-a^2+b^2}\,(a+b)(a-b) b} + \frac{a \ln\left(e^{ix}+\frac{ia\sqrt{-a^2+b^2}+a^2-b^2}{\sqrt{-a^2+b^2}\, b}\right.}{2\sqrt{-a^2+b^2}\,(a+b)(a-b) b}$

input
```
int(x*cos(x)/(a+b*sin(x))^3,x,method=_RETURNVERBOSE)
```

output
```
(2*I*a^2*exp(2*I*x)+I*b^2*exp(2*I*x)+2*x*a^2*exp(2*I*x)+b*a*exp(3*I*x)-2*b
^2*x*exp(2*I*x)-I*b^2-3*a*b*exp(I*x))/(b*exp(2*I*x)-b+2*I*a*exp(I*x))^2/(a
^2-b^2)/b-1/2/(-a^2+b^2)^(1/2)*a/(a+b)/(a-b)/b*ln(exp(I*x)+(I*a*(-a^2+b^2)
^(1/2)-a^2+b^2)/(-a^2+b^2)^(1/2)/b)+1/2/(-a^2+b^2)^(1/2)*a/(a+b)/(a-b)/b*l
n(exp(I*x)+(I*a*(-a^2+b^2)^(1/2)+a^2-b^2)/(-a^2+b^2)^(1/2)/b)
```

3.154.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. 2(75) = 150.

Time = 0.27 (sec) , antiderivative size = 459, normalized size of antiderivative = 5.40

$$\int \frac{x\cos(x)}{(a+b\sin(x))^3}\,dx = \left[\frac{2\,(a^2b^2-b^4)\cos(x)\sin(x)-\left(ab^2\cos(x)^2-2\,a^2b\sin(x)-a^3-ab^2\right)\sqrt{-a^2+b^2}\log\left(-\frac{(2\,a^2-b^2)\cos(x)^2-2\,a}{}\right.}{4\,\left(a^6b-a^4b^3-a^2b^5+b^7-\left(a^4b^3-2\,a^2b^5+b^7\right)\cos(\right.}\right.$$

input
```
integrate(x*cos(x)/(a+b*sin(x))^3,x, algorithm="fricas")
```

output
```
[1/4*(2*(a^2*b^2 - b^4)*cos(x)*sin(x) - (a*b^2*cos(x)^2 - 2*a^2*b*sin(x) -
 a^3 - a*b^2)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x)
 - a^2 - b^2 - 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x
)^2 - 2*a*b*sin(x) - a^2 - b^2)) - 2*(a^4 - 2*a^2*b^2 + b^4)*x + 2*(a^3*b
- a*b^3)*cos(x))/(a^6*b - a^4*b^3 - a^2*b^5 + b^7 - (a^4*b^3 - 2*a^2*b^5 +
 b^7)*cos(x)^2 + 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*sin(x)), 1/2*((a^2*b^2 -
b^4)*cos(x)*sin(x) + (a*b^2*cos(x)^2 - 2*a^2*b*sin(x) - a^3 - a*b^2)*sqrt(
a^2 - b^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x))) - (a^4 - 2*a^2
*b^2 + b^4)*x + (a^3*b - a*b^3)*cos(x))/(a^6*b - a^4*b^3 - a^2*b^5 + b^7 -
 (a^4*b^3 - 2*a^2*b^5 + b^7)*cos(x)^2 + 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*si
n(x))]
```

3.154.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x\cos(x)}{(a+b\sin(x))^3}\,dx = \text{Timed out}$$

input
```
integrate(x*cos(x)/(a+b*sin(x))**3,x)
```

output
```
Timed out
```

3.154.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x\cos(x)}{(a+b\sin(x))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*cos(x)/(a+b*sin(x))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.154.8 Giac [F]

$$\int \frac{x\cos(x)}{(a+b\sin(x))^3}\,dx = \int \frac{x\cos(x)}{(b\sin(x)+a)^3}\,dx$$

input
```
integrate(x*cos(x)/(a+b*sin(x))^3,x, algorithm="giac")
```

output
```
integrate(x*cos(x)/(b*sin(x) + a)^3, x)
```

3.154.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\cos(x)}{(a+b\sin(x))^3}\,dx = \int \frac{x\cos(x)}{(a+b\sin(x))^3}\,dx$$

input
```
int((x*cos(x))/(a + b*sin(x))^3,x)
```

output
```
int((x*cos(x))/(a + b*sin(x))^3, x)
```

3.155 $\int \frac{x\sin(x)}{(a+b\cos(x))^2}\,dx$

3.155.1 Optimal result

Integrand size = 12, antiderivative size = 59

$$\int \frac{x\sin(x)}{(a+b\cos(x))^2}\,dx = -\frac{2\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b\sqrt{a+b}}+\frac{x}{b(a+b\cos(x))}$$

output
```
x/b/(a+b*cos(x))-2*arctan((a-b)^(1/2)*tan(1/2*x)/(a+b)^(1/2))/b/(a-b)^(1/2
)/(a+b)^(1/2)
```

3.155.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.98

$$\int \frac{x\sin(x)}{(a+b\cos(x))^2}\,dx = \frac{2\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{b\sqrt{-a^2+b^2}}+\frac{x}{b(a+b\cos(x))}$$

input
```
Integrate[(x*Sin[x])/(a + b*Cos[x])^2,x]
```

output
```
(2*ArcTanh[((a - b)*Tan[x/2])/Sqrt[-a^2 + b^2]])/(b*Sqrt[-a^2 + b^2]) + x/
(b*(a + b*Cos[x]))
```

3.155.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4923, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sin(x)}{(a+b\cos(x))^2}dx$$

$$\Big\downarrow \text{4923}$$

$$\frac{x}{b(a+b\cos(x))}-\frac{\int \frac{1}{a+b\cos(x)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x}{b(a+b\cos(x))}-\frac{\int \frac{1}{a+b\sin\left(x+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \text{3138}$$

$$\frac{x}{b(a+b\cos(x))}-\frac{2\int \frac{1}{(a-b)\tan^2\left(\frac{x}{2}\right)+a+b}d\tan\left(\frac{x}{2}\right)}{b}$$

$$\Big\downarrow \text{218}$$

$$\frac{x}{b(a+b\cos(x))}-\frac{2\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b\sqrt{a-b}\sqrt{a+b}}$$

input
```
Int[(x*Sin[x])/(a + b*Cos[x])^2,x]
```

output
```
(-2*ArcTan[(Sqrt[a - b]*Tan[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*b*Sqrt[a + b]
) + x/(b*(a + b*Cos[x]))
```

3.155.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4923
```
Int[(Cos[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_.)*((e_.) + (f_.)*(x_))^(m_.)
*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(-(e + f*x)^m)*((a + b*Cos[c +
d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[f*(m/(b*d*(n + 1)))   Int[(e + f*x)
^(m - 1)*(a + b*Cos[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

3.155.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.61

method	result	size
risch	$\frac{2x\,\mathrm{e}^{ix}}{b(b\,\mathrm{e}^{2ix}+2a\,\mathrm{e}^{ix}+b)}-\frac{i\ln\left(\mathrm{e}^{ix}+\frac{a\sqrt{a^2-b^2}+a^2-b^2}{\sqrt{a^2-b^2}\,b}\right)}{\sqrt{a^2-b^2}\,b}+\frac{i\ln\left(\mathrm{e}^{ix}+\frac{a\sqrt{a^2-b^2}-a^2+b^2}{\sqrt{a^2-b^2}\,b}\right)}{\sqrt{a^2-b^2}\,b}$	154

input
```
int(x*sin(x)/(a+b*cos(x))^2,x,method=_RETURNVERBOSE)
```

output
```
2*x*exp(I*x)/b/(b*exp(2*I*x)+2*a*exp(I*x)+b)-I/(a^2-b^2)^(1/2)/b*ln(exp(I*
x)+(a*(a^2-b^2)^(1/2)+a^2-b^2)/(a^2-b^2)^(1/2)/b)+I/(a^2-b^2)^(1/2)/b*ln(e
xp(I*x)+(a*(a^2-b^2)^(1/2)-a^2+b^2)/(a^2-b^2)^(1/2)/b)
```

3.155.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 227, normalized size of antiderivative = 3.85

$$\int \frac{x\sin(x)}{(a+b\cos(x))^2}\,dx$$
$$= \left[-\frac{\sqrt{-a^2+b^2}(b\cos(x)+a)\log\left(\frac{2\,ab\cos(x)+(2\,a^2-b^2)\cos(x)^2-2\sqrt{-a^2+b^2}(a\cos(x)+b)\sin(x)-a^2+2\,b^2}{b^2\cos(x)^2+2\,ab\cos(x)+a^2}\right)-2\,(a^2-b^2)x}{2\,(a^3b-ab^3+(a^2b^2-b^4)\cos(x))}, -\frac{\sqrt{a^2-b^2}(b\cos(x)+a)\arctan\left(-\frac{a\cos(x)+b}{\sqrt{a^2-b^2}\sin(x)}\right)-(a^2-b^2)x}{a^3b-ab^3+(a^2b^2-b^4)\cos(x)}\right]$$

input
```
integrate(x*sin(x)/(a+b*cos(x))^2,x, algorithm="fricas")
```

output
```
[-1/2*(sqrt(-a^2 + b^2)*(b*cos(x) + a)*log((2*a*b*cos(x) + (2*a^2 - b^2)*c
os(x)^2 - 2*sqrt(-a^2 + b^2)*(a*cos(x) + b)*sin(x) - a^2 + 2*b^2)/(b^2*cos
(x)^2 + 2*a*b*cos(x) + a^2)) - 2*(a^2 - b^2)*x)/(a^3*b - a*b^3 + (a^2*b^2
- b^4)*cos(x)), -(sqrt(a^2 - b^2)*(b*cos(x) + a)*arctan(-(a*cos(x) + b)/(s
qrt(a^2 - b^2)*sin(x))) - (a^2 - b^2)*x)/(a^3*b - a*b^3 + (a^2*b^2 - b^4)*
cos(x))]
```

3.155.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1935 vs. 2(46) = 92.

Time = 151.66 (sec) , antiderivative size = 1935, normalized size of antiderivative = 32.80

$$\int \frac{x\sin(x)}{(a+b\cos(x))^2}\,dx = \text{Too large to display}$$

input
```
integrate(x*sin(x)/(a+b*cos(x))**2,x)
```

output
```
Piecewise((zoo*(-x*tan(x/2)**2/(tan(x/2)**2 - 1) - x/(tan(x/2)**2 - 1) + l
og(tan(x/2) - 1)*tan(x/2)**2/(tan(x/2)**2 - 1) - log(tan(x/2) - 1)/(tan(x/
2)**2 - 1) - log(tan(x/2) + 1)*tan(x/2)**2/(tan(x/2)**2 - 1) + log(tan(x/2
) + 1)/(tan(x/2)**2 - 1)), Eq(a, 0) & Eq(b, 0)), (x*tan(x/2)**2/(2*b**2) +
 x/(2*b**2) - tan(x/2)/b**2, Eq(a, b)), (-x/(2*b**2) - x/(2*b**2*tan(x/2)*
*2) - 1/(b**2*tan(x/2)), Eq(a, -b)), ((-x*cos(x) + sin(x))/a**2, Eq(b, 0))
, (a*x*sqrt(-a/(a - b) - b/(a - b))*tan(x/2)**2/(a**2*b*sqrt(-a/(a - b) -
b/(a - b))*tan(x/2)**2 + a**2*b*sqrt(-a/(a - b) - b/(a - b)) - 2*a*b**2*sq
rt(-a/(a - b) - b/(a - b))*tan(x/2)**2 + b**3*sqrt(-a/(a - b) - b/(a - b))
*tan(x/2)**2 - b**3*sqrt(-a/(a - b) - b/(a - b))) + a*x*sqrt(-a/(a - b) -
b/(a - b))/(a**2*b*sqrt(-a/(a - b) - b/(a - b))*tan(x/2)**2 + a**2*b*sqrt(
-a/(a - b) - b/(a - b)) - 2*a*b**2*sqrt(-a/(a - b) - b/(a - b))*tan(x/2)**
2 + b**3*sqrt(-a/(a - b) - b/(a - b))*tan(x/2)**2 - b**3*sqrt(-a/(a - b) -
 b/(a - b))) - a*log(-sqrt(-a/(a - b) - b/(a - b)) + tan(x/2))*tan(x/2)**2
/(a**2*b*sqrt(-a/(a - b) - b/(a - b))*tan(x/2)**2 + a**2*b*sqrt(-a/(a - b)
 - b/(a - b)) - 2*a*b**2*sqrt(-a/(a - b) - b/(a - b))*tan(x/2)**2 + b**3*s
qrt(-a/(a - b) - b/(a - b))*tan(x/2)**2 - b**3*sqrt(-a/(a - b) - b/(a - b)
)) - a*log(-sqrt(-a/(a - b) - b/(a - b)) + tan(x/2))/(a**2*b*sqrt(-a/(a -
b) - b/(a - b))*tan(x/2)**2 + a**2*b*sqrt(-a/(a - b) - b/(a - b)) - 2*a*b*
*2*sqrt(-a/(a - b) - b/(a - b))*tan(x/2)**2 + b**3*sqrt(-a/(a - b) - b/...
```

3.155.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x\sin(x)}{(a+b\cos(x))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*sin(x)/(a+b*cos(x))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.155.8 Giac [F]

$$\int \frac{x\sin(x)}{(a+b\cos(x))^2}\,dx = \int \frac{x\sin(x)}{(b\cos(x)+a)^2}\,dx$$

input
```
integrate(x*sin(x)/(a+b*cos(x))^2,x, algorithm="giac")
```

output
```
integrate(x*sin(x)/(b*cos(x) + a)^2, x)
```

3.155.9 Mupad [B] (verification not implemented)

Time = 29.71 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.24

$$\int \frac{x\sin(x)}{(a+b\cos(x))^2}\,dx = \frac{2\,x\,e^{x\,1i}}{b\,\left(2\,a\,e^{x\,1i}+2\,b\,e^{x\,1i}\cos(x)\right)} + \frac{\ln\left(2\,e^{x\,1i}-\frac{\left(b+a\,e^{x\,1i}\right)\,2i}{\sqrt{a+b}\,\sqrt{b-a}}\right)}{b\,\sqrt{a+b}\,\sqrt{b-a}} - \frac{\ln\left(2\,e^{x\,1i}+\frac{\left(b+a\,e^{x\,1i}\right)\,2i}{\sqrt{a+b}\,\sqrt{b-a}}\right)}{b\,\sqrt{a+b}\,\sqrt{b-a}}$$

input
```
int((x*sin(x))/(a + b*cos(x))^2,x)
```

output
```
(2*x*exp(x*1i))/(b*(2*a*exp(x*1i) + 2*b*exp(x*1i)*cos(x))) + log(2*exp(x*1
i) - ((b + a*exp(x*1i))*2i)/((a + b)^(1/2)*(b - a)^(1/2)))/(b*(a + b)^(1/2
)*(b - a)^(1/2)) - log(2*exp(x*1i) + ((b + a*exp(x*1i))*2i)/((a + b)^(1/2)
*(b - a)^(1/2)))/(b*(a + b)^(1/2)*(b - a)^(1/2))
```

3.156 $\int \frac{x\sin(x)}{(a+b\cos(x))^3}\,dx$

3.156.1 Optimal result

Integrand size = 12, antiderivative size = 88

$$\int \frac{x\sin(x)}{(a+b\cos(x))^3}\,dx = -\frac{a\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b(a+b)^{3/2}}+\frac{x}{2b(a+b\cos(x))^2}+\frac{\sin(x)}{2\left(a^2-b^2\right)(a+b\cos(x))}$$

output
```
-a*arctan((a-b)^(1/2)*tan(1/2*x)/(a+b)^(1/2))/(a-b)^(3/2)/b/(a+b)^(3/2)+1/
2*x/b/(a+b*cos(x))^2+1/2*sin(x)/(a^2-b^2)/(a+b*cos(x))
```

3.156.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.97

$$\int \frac{x\sin(x)}{(a+b\cos(x))^3}\,dx = -\frac{a\text{arctanh}\left(\frac{(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{b\left(-a^2+b^2\right)^{3/2}}+\frac{\frac{x}{b}+\frac{(a+b\cos(x))\sin(x)}{(a-b)(a+b)}}{2(a+b\cos(x))^2}$$

input
```
Integrate[(x*Sin[x])/(a + b*Cos[x])^3,x]
```

output
```
-((a*ArcTanh[((a - b)*Tan[x/2])/Sqrt[-a^2 + b^2]])/(b*(-a^2 + b^2)^(3/2)))
 + (x/b + ((a + b*Cos[x])*Sin[x])/((a - b)*(a + b)))/(2*(a + b*Cos[x])^2)
```

3.156.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.17, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {4923, 3042, 3143, 25, 27, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sin(x)}{(a+b\cos(x))^3}dx$$

$$\Big\downarrow \text{4923}$$

$$\frac{x}{2b(a+b\cos(x))^2}-\frac{\int \frac{1}{(a+b\cos(x))^2}dx}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x}{2b(a+b\cos(x))^2}-\frac{\int \frac{1}{\left(a+b\sin\left(x+\frac{\pi}{2}\right)\right)^2}dx}{2b}$$

$$\Big\downarrow \text{3143}$$

$$\frac{x}{2b(a+b\cos(x))^2}-\frac{-\frac{\int -\frac{a}{a+b\cos(x)}dx}{a^2-b^2}-\frac{b\sin(x)}{(a^2-b^2)(a+b\cos(x))}}{2b}$$

$$\Big\downarrow \text{25}$$

$$\frac{x}{2b(a+b\cos(x))^2}-\frac{\frac{\int \frac{a}{a+b\cos(x)}dx}{a^2-b^2}-\frac{b\sin(x)}{(a^2-b^2)(a+b\cos(x))}}{2b}$$

$$\Big\downarrow \text{27}$$

$$\frac{x}{2b(a+b\cos(x))^2}-\frac{\frac{a\int \frac{1}{a+b\cos(x)}dx}{a^2-b^2}-\frac{b\sin(x)}{(a^2-b^2)(a+b\cos(x))}}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x}{2b(a+b\cos(x))^2}-\frac{\frac{a\int \frac{1}{a+b\sin\left(x+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{b\sin(x)}{(a^2-b^2)(a+b\cos(x))}}{2b}$$

$$\Big\downarrow \text{3138}$$

$$\frac{x}{2b(a+b\cos(x))^2}-\frac{\frac{2a\int \frac{1}{(a-b)\tan^2\left(\frac{x}{2}\right)+a+b}d\tan\left(\frac{x}{2}\right)}{a^2-b^2}-\frac{b\sin(x)}{(a^2-b^2)(a+b\cos(x))}}{2b}$$

$$\Big\downarrow 218$$

$$\frac{x}{2b(a+b\cos(x))^2}-\frac{\frac{2a\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}(a^2-b^2)}-\frac{b\sin(x)}{(a^2-b^2)(a+b\cos(x))}}{2b}$$

input
```
Int[(x*Sin[x])/(a + b*Cos[x])^3,x]
```

output
```
x/(2*b*(a + b*Cos[x])^2) - ((2*a*ArcTan[(Sqrt[a - b]*Tan[x/2])/Sqrt[a + b]
])/(Sqrt[a - b]*Sqrt[a + b]*(a^2 - b^2)) - (b*Sin[x])/((a^2 - b^2)*(a + b*
Cos[x])))/(2*b)
```

3.156.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 4923
```
Int[(Cos[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_.)*((e_.) + (f_.)*(x_))^(m_.)
*Sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(-(e + f*x)^m)*((a + b*Cos[c +
d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[f*(m/(b*d*(n + 1)))   Int[(e + f*x)
^(m - 1)*(a + b*Cos[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

3.156.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.84

method	result
risch	$\frac{i\left(-2ia^2x\,\mathrm{e}^{2ix}+2ib^2x\,\mathrm{e}^{2ix}+ba\,\mathrm{e}^{3ix}+2\,\mathrm{e}^{2ix}a^2+\mathrm{e}^{2ix}b^2+3ab\,\mathrm{e}^{ix}+b^2\right)}{b\left(b\,\mathrm{e}^{2ix}+2a\,\mathrm{e}^{ix}+b\right)^2\left(a^2-b^2\right)}-\frac{ia\ln\left(\mathrm{e}^{ix}+\frac{a\sqrt{a^2-b^2}+a^2-b^2}{\sqrt{a^2-b^2}\,b}\right)}{2\sqrt{a^2-b^2}\,(a+b)(a-b)b}+\frac{ia\ln\left(\mathrm{e}^{ix}+\frac{a\sqrt{a^2-b^2}-a^2+b}{\sqrt{a^2-b^2}\,b}\right.}{2\sqrt{a^2-b^2}\,(a+b)(a-b)b}$

input
```
int(x*sin(x)/(a+b*cos(x))^3,x,method=_RETURNVERBOSE)
```

output
```
I*(-2*I*a^2*x*exp(2*I*x)+2*I*b^2*x*exp(2*I*x)+b*a*exp(3*I*x)+2*exp(2*I*x)*
a^2+exp(2*I*x)*b^2+3*a*b*exp(I*x)+b^2)/b/(b*exp(2*I*x)+2*a*exp(I*x)+b)^2/(
a^2-b^2)-1/2*I/(a^2-b^2)^(1/2)*a/(a+b)/(a-b)/b*ln(exp(I*x)+(a*(a^2-b^2)^(1
/2)+a^2-b^2)/(a^2-b^2)^(1/2)/b)+1/2*I/(a^2-b^2)^(1/2)*a/(a+b)/(a-b)/b*ln(e
xp(I*x)+(a*(a^2-b^2)^(1/2)-a^2+b^2)/(a^2-b^2)^(1/2)/b)
```

3.156.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(74) = 148.

Time = 0.27 (sec) , antiderivative size = 417, normalized size of antiderivative = 4.74

$$\int \frac{x\sin(x)}{(a+b\cos(x))^3}\,dx$$
$$=\left[\frac{\left(ab^2\cos\left(x\right)^2+2\,a^2b\cos\left(x\right)+a^3\right)\sqrt{-a^2+b^2}\log\left(\frac{2\,ab\cos(x)+\left(2\,a^2-b^2\right)\cos(x)^2+2\,\sqrt{-a^2+b^2}(a\cos(x)+b)\sin(x)-a^2+2\,b^2}{b^2\cos(x)^2+2\,ab\cos(x)+a^2}\right.}{4\left(a^6b-2\,a^4b^3+a^2b^5+\left(a^4b^3-2\,a^2b^5+b^7\right)\cos\left(x\right)^2+2\,(a^5b\right.}\right.$$
$$-\frac{\left(ab^2\cos\left(x\right)^2+2\,a^2b\cos\left(x\right)+a^3\right)\sqrt{a^2-b^2}\arctan\left(-\frac{a\cos(x)+b}{\sqrt{a^2-b^2}\sin(x)}\right)-\left(a^4-2\,a^2b^2+b^4\right)x-\left(a^3b-ab^3\right.}{2\left(a^6b-2\,a^4b^3+a^2b^5+\left(a^4b^3-2\,a^2b^5+b^7\right)\cos\left(x\right)^2+2\left(a^5b^2-2\,a^3b^4+ab^6\right)\cos\right.}$$

input
```
integrate(x*sin(x)/(a+b*cos(x))^3,x, algorithm="fricas")
```

output
```
[1/4*((a*b^2*cos(x)^2 + 2*a^2*b*cos(x) + a^3)*sqrt(-a^2 + b^2)*log((2*a*b*
cos(x) + (2*a^2 - b^2)*cos(x)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(x) + b)*sin(x)
 - a^2 + 2*b^2)/(b^2*cos(x)^2 + 2*a*b*cos(x) + a^2)) + 2*(a^4 - 2*a^2*b^2
+ b^4)*x + 2*(a^3*b - a*b^3 + (a^2*b^2 - b^4)*cos(x))*sin(x))/(a^6*b - 2*a
^4*b^3 + a^2*b^5 + (a^4*b^3 - 2*a^2*b^5 + b^7)*cos(x)^2 + 2*(a^5*b^2 - 2*a
^3*b^4 + a*b^6)*cos(x)), -1/2*((a*b^2*cos(x)^2 + 2*a^2*b*cos(x) + a^3)*sqr
t(a^2 - b^2)*arctan(-(a*cos(x) + b)/(sqrt(a^2 - b^2)*sin(x))) - (a^4 - 2*a
^2*b^2 + b^4)*x - (a^3*b - a*b^3 + (a^2*b^2 - b^4)*cos(x))*sin(x))/(a^6*b
- 2*a^4*b^3 + a^2*b^5 + (a^4*b^3 - 2*a^2*b^5 + b^7)*cos(x)^2 + 2*(a^5*b^2
- 2*a^3*b^4 + a*b^6)*cos(x))]
```

3.156.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x\sin(x)}{(a+b\cos(x))^3}\,dx = \text{Timed out}$$

input
```
integrate(x*sin(x)/(a+b*cos(x))**3,x)
```

output
```
Timed out
```

3.156.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x\sin(x)}{(a+b\cos(x))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*sin(x)/(a+b*cos(x))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.156.8 Giac [F]

$$\int \frac{x\sin(x)}{(a+b\cos(x))^3}\,dx = \int \frac{x\sin{(x)}}{(b\cos{(x)}+a)^3}\,dx$$

input
```
integrate(x*sin(x)/(a+b*cos(x))^3,x, algorithm="giac")
```

output
```
integrate(x*sin(x)/(b*cos(x) + a)^3, x)
```

3.156.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\sin(x)}{(a+b\cos(x))^3}\,dx = \int \frac{x\,\sin{(x)}}{(a+b\,\cos{(x)})^3}\,dx$$

input
```
int((x*sin(x))/(a + b*cos(x))^3,x)
```

output
```
int((x*sin(x))/(a + b*cos(x))^3, x)
```

3.157 $\int \frac{x \sec^2(x)}{(a+b\tan(x))^2} \, dx$

3.157.1 Optimal result

Integrand size = 14, antiderivative size = 50

$$\int \frac{x \sec^2(x)}{(a+b\tan(x))^2} \, dx = \frac{ax}{b(a^2+b^2)} + \frac{\log(a\cos(x)+b\sin(x))}{a^2+b^2} - \frac{x}{b(a+b\tan(x))}$$

output
```
a*x/b/(a^2+b^2)+ln(a*cos(x)+b*sin(x))/(a^2+b^2)-x/b/(a+b*tan(x))
```

3.157.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{x \sec^2(x)}{(a+b\tan(x))^2} \, dx = \frac{-bx + a\log(a\cos(x)+b\sin(x))}{a^3+ab^2} + \frac{x\sin(x)}{a^2\cos(x)+ab\sin(x)}$$

input
```
Integrate[(x*Sec[x]^2)/(a + b*Tan[x])^2,x]
```

output
```
(-(b*x) + a*Log[a*Cos[x] + b*Sin[x]])/(a^3 + a*b^2) + (x*Sin[x])/(a^2*Cos[
x] + a*b*Sin[x])
```

3.157.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {4924, 3042, 3965, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sec^2(x)}{(a+b\tan(x))^2}dx$$

$$\Big\downarrow \ 4924$$

$$\frac{\int \frac{1}{a+b\tan(x)}dx}{b}-\frac{x}{b(a+b\tan(x))}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{1}{a+b\tan(x)}dx}{b}-\frac{x}{b(a+b\tan(x))}$$

$$\Big\downarrow \ 3965$$

$$\frac{\frac{b\int \frac{b-a\tan(x)}{a+b\tan(x)}dx}{a^2+b^2}+\frac{ax}{a^2+b^2}}{b}-\frac{x}{b(a+b\tan(x))}$$

$$\Big\downarrow \ 3042$$

$$\frac{\frac{b\int \frac{b-a\tan(x)}{a+b\tan(x)}dx}{a^2+b^2}+\frac{ax}{a^2+b^2}}{b}-\frac{x}{b(a+b\tan(x))}$$

$$\Big\downarrow \ 4013$$

$$\frac{\frac{ax}{a^2+b^2}+\frac{b\log(a\cos(x)+b\sin(x))}{a^2+b^2}}{b}-\frac{x}{b(a+b\tan(x))}$$

input
```
Int[(x*Sec[x]^2)/(a + b*Tan[x])^2,x]
```

output
```
((a*x)/(a^2 + b^2) + (b*Log[a*Cos[x] + b*Sin[x]])/(a^2 + b^2))/b - x/(b*(a
 + b*Tan[x]))
```

3.157.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3965
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[a*(x/(a^
2 + b^2)), x] + Simp[b/(a^2 + b^2)   Int[(b - a*Tan[c + d*x])/(a + b*Tan[c
+ d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4924
```
Int[((e_.) + (f_.)*(x_))^(m_.)*Sec[(c_.) + (d_.)*(x_)]^2*((a_) + (b_.)*Tan[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Tan[c + d
*x])^(n + 1)/(b*d*(n + 1))), x] - Simp[f*(m/(b*d*(n + 1)))   Int[(e + f*x)^
(m - 1)*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && IGtQ[m, 0] && NeQ[n, -1]
```

3.157.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.01 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.72

method	result	size
risch	$-\frac{2ix}{a^2+b^2}+\frac{2ix}{(-ib\,\mathrm{e}^{2ix}+a\,\mathrm{e}^{2ix}+ib+a)(-ib+a)}+\frac{\ln\left(\mathrm{e}^{2ix}-\frac{ib+a}{ib-a}\right)}{a^2+b^2}$	86

input
```
int(x*sec(x)^2/(a+b*tan(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-2*I/(a^2+b^2)*x+2*I*x/(-I*b*exp(2*I*x)+a*exp(2*I*x)+I*b+a)/(-I*b+a)+1/(a^
2+b^2)*ln(exp(2*I*x)-(I*b+a)/(I*b-a))
```

3.157.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.60

$$\int \frac{x\sec^2(x)}{(a+b\tan(x))^2}\,dx = -\frac{2\,bx\cos(x) - 2\,ax\sin(x) - (a\cos(x) + b\sin(x))\log\left(2\,ab\cos(x)\sin(x) + (a^2-b^2)\cos(x)^2 + b^2\right)}{2\left((a^3+ab^2)\cos(x) + (a^2b+b^3)\sin(x)\right)}$$

input
```
integrate(x*sec(x)^2/(a+b*tan(x))^2,x, algorithm="fricas")
```

output
```
-1/2*(2*b*x*cos(x) - 2*a*x*sin(x) - (a*cos(x) + b*sin(x))*log(2*a*b*cos(x)
*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2))/((a^3 + a*b^2)*cos(x) + (a^2*b + b^
3)*sin(x))
```

3.157.6 Sympy [F]

$$\int \frac{x\sec^2(x)}{(a+b\tan(x))^2}\,dx = \int \frac{x\sec^2(x)}{(a+b\tan(x))^2}\,dx$$

input
```
integrate(x*sec(x)**2/(a+b*tan(x))**2,x)
```

output
```
Integral(x*sec(x)**2/(a + b*tan(x))**2, x)
```

3.157.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. 2(50) = 100.

Time = 0.21 (sec) , antiderivative size = 250, normalized size of antiderivative = 5.00

$$\int \frac{x\sec^2(x)}{(a+b\tan(x))^2}\,dx = -\frac{8\,abx\cos(2\,x) - 4\,(a^2-b^2)x\sin(2\,x) - \left((a^2+b^2)\cos(2\,x)^2 + 4\,ab\sin(2\,x) + (a^2+b^2)\sin(2\,x)^2 + a^2 - \cdots\right.}{2\left(a^4+2\,a^2b^2+b^4 + (a^4+2\,a^2b^2+b^4)\cos(2\,x)^2 + (a^4+2\,a^2b^2+b^4 \cdots\right.}$$

input
```
integrate(x*sec(x)^2/(a+b*tan(x))^2,x, algorithm="maxima")
```

output
```
-1/2*(8*a*b*x*cos(2*x) - 4*(a^2 - b^2)*x*sin(2*x) - ((a^2 + b^2)*cos(2*x)^
2 + 4*a*b*sin(2*x) + (a^2 + b^2)*sin(2*x)^2 + a^2 + b^2 + 2*(a^2 - b^2)*co
s(2*x))*log(((a^2 + b^2)*cos(2*x)^2 + 4*a*b*sin(2*x) + (a^2 + b^2)*sin(2*x
)^2 + a^2 + b^2 + 2*(a^2 - b^2)*cos(2*x))/(a^2 + b^2)))/(a^4 + 2*a^2*b^2 +
 b^4 + (a^4 + 2*a^2*b^2 + b^4)*cos(2*x)^2 + (a^4 + 2*a^2*b^2 + b^4)*sin(2*
x)^2 + 2*(a^4 - b^4)*cos(2*x) + 4*(a^3*b + a*b^3)*sin(2*x))
```

3.157.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(50) = 100$.

Time = 0.38 (sec) , antiderivative size = 322, normalized size of antiderivative = 6.44

$$\int \frac{x\sec^2(x)}{(a+b\tan(x))^2}\,dx =$$

$$-\frac{2\,bx\tan\left(\frac{1}{2}\,x\right)^2 - a\log\left(\frac{4\left(a^2\tan\left(\frac{1}{2}\,x\right)^4 - 4\,ab\tan\left(\frac{1}{2}\,x\right)^3 - 2\,a^2\tan\left(\frac{1}{2}\,x\right)^2 + 4\,b^2\tan\left(\frac{1}{2}\,x\right)^2 + 4\,ab\tan\left(\frac{1}{2}\,x\right) + a^2\right)}{\tan\left(\frac{1}{2}\,x\right)^4 + 2\,\tan\left(\frac{1}{2}\,x\right)^2 + 1}\right)\tan\left(\frac{1}{2}\,x\right)^2 + 4}{2}$$

input
```
integrate(x*sec(x)^2/(a+b*tan(x))^2,x, algorithm="giac")
```

output
```
-1/2*(2*b*x*tan(1/2*x)^2 - a*log(4*(a^2*tan(1/2*x)^4 - 4*a*b*tan(1/2*x)^3
- 2*a^2*tan(1/2*x)^2 + 4*b^2*tan(1/2*x)^2 + 4*a*b*tan(1/2*x) + a^2)/(tan(1
/2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x)^2 + 4*a*x*tan(1/2*x) + 2*b*log(4
*(a^2*tan(1/2*x)^4 - 4*a*b*tan(1/2*x)^3 - 2*a^2*tan(1/2*x)^2 + 4*b^2*tan(1
/2*x)^2 + 4*a*b*tan(1/2*x) + a^2)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan
(1/2*x) - 2*b*x + a*log(4*(a^2*tan(1/2*x)^4 - 4*a*b*tan(1/2*x)^3 - 2*a^2*t
an(1/2*x)^2 + 4*b^2*tan(1/2*x)^2 + 4*a*b*tan(1/2*x) + a^2)/(tan(1/2*x)^4 +
 2*tan(1/2*x)^2 + 1)))/(a^3*tan(1/2*x)^2 + a*b^2*tan(1/2*x)^2 - 2*a^2*b*ta
n(1/2*x) - 2*b^3*tan(1/2*x) - a^3 - a*b^2)
```

3.157.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\sec^2(x)}{(a+b\tan(x))^2}\,dx = \int \frac{x}{\cos(x)^2\,(a+b\tan(x))^2}\,dx$$

input
```
int(x/(cos(x)^2*(a + b*tan(x))^2),x)
```

output
```
int(x/(cos(x)^2*(a + b*tan(x))^2), x)
```

3.158 $\int \frac{x\csc^2(x)}{(a+b\cot(x))^2}\,dx$

3.158.1 Optimal result

Integrand size = 14, antiderivative size = 50

$$\int \frac{x\csc^2(x)}{(a+b\cot(x))^2}\,dx = -\frac{ax}{b(a^2+b^2)}+\frac{x}{b(a+b\cot(x))}+\frac{\log(b\cos(x)+a\sin(x))}{a^2+b^2}$$

output
```
-a*x/b/(a^2+b^2)+x/b/(a+b*cot(x))+ln(b*cos(x)+a*sin(x))/(a^2+b^2)
```

3.158.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int \frac{x\csc^2(x)}{(a+b\cot(x))^2}\,dx = \frac{-ax+b\log(b\cos(x)+a\sin(x))}{a^2b+b^3}+\frac{x\sin(x)}{b^2\cos(x)+ab\sin(x)}$$

input
```
Integrate[(x*Csc[x]^2)/(a + b*Cot[x])^2,x]
```

output
```
(-(a*x) + b*Log[b*Cos[x] + a*Sin[x]])/(a^2*b + b^3) + (x*Sin[x])/(b^2*Cos[
x] + a*b*Sin[x])
```

3.158.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4925, 3042, 3965, 25, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\csc^2(x)}{(a+b\cot(x))^2}\,dx$$

$$\Big\downarrow \text{ 4925}$$

$$\frac{x}{b(a+b\cot(x))}-\frac{\int \frac{1}{a+b\cot(x)}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{x}{b(a+b\cot(x))}-\frac{\int \frac{1}{a-b\tan\left(x+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \text{ 3965}$$

$$\frac{x}{b(a+b\cot(x))}-\frac{\frac{ax}{a^2+b^2}-\frac{b\int -\frac{b-a\cot(x)}{a+b\cot(x)}dx}{a^2+b^2}}{b}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{x}{b(a+b\cot(x))}-\frac{\frac{b\int \frac{b-a\cot(x)}{a+b\cot(x)}dx}{a^2+b^2}+\frac{ax}{a^2+b^2}}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{x}{b(a+b\cot(x))}-\frac{\frac{b\int \frac{b+a\tan\left(x+\frac{\pi}{2}\right)}{a-b\tan\left(x+\frac{\pi}{2}\right)}dx}{a^2+b^2}+\frac{ax}{a^2+b^2}}{b}$$

$$\Big\downarrow \text{ 4013}$$

$$\frac{x}{b(a+b\cot(x))}-\frac{\frac{ax}{a^2+b^2}-\frac{b\log(a\sin(x)+b\cos(x))}{a^2+b^2}}{b}$$

input
```
Int[(x*Csc[x]^2)/(a + b*Cot[x])^2,x]
```

output
```
x/(b*(a + b*Cot[x])) - ((a*x)/(a^2 + b^2) - (b*Log[b*Cos[x] + a*Sin[x]])/(
a^2 + b^2))/b
```

3.158.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3965
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[a*(x/(a^
2 + b^2)), x] + Simp[b/(a^2 + b^2)   Int[(b - a*Tan[c + d*x])/(a + b*Tan[c
+ d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4925
```
Int[Csc[(c_.) + (d_.)*(x_)]^2*(Cot[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_.)*
((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(e + f*x)^m)*((a + b*Cot[c
+ d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[f*(m/(b*d*(n + 1)))   Int[(e + f*
x)^(m - 1)*(a + b*Cot[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
 n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

3.158.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.74

method	result	size
risch	$-\frac{2ix}{a^2+b^2}-\frac{2ix}{(ib\,e^{2ix}+a\,e^{2ix}+ib-a)(ib+a)}+\frac{\ln\left(e^{2ix}+\frac{ib-a}{ib+a}\right)}{a^2+b^2}$	87

input
```
int(x*csc(x)^2/(a+b*cot(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-2*I/(a^2+b^2)*x-2*I*x/(I*b*exp(2*I*x)+a*exp(2*I*x)+I*b-a)/(I*b+a)+1/(a^2+
b^2)*ln(exp(2*I*x)+(I*b-a)/(I*b+a))
```

3.158.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.62

$$\int \frac{x\csc^2(x)}{(a+b\cot(x))^2}\,dx = -\frac{2\,ax\cos{(x)} - 2\,bx\sin{(x)} - (b\cos{(x)} + a\sin{(x)})\log\left(2\,ab\cos{(x)}\sin{(x)} - (a^2-b^2)\cos{(x)}^2 + a^2\right)}{2\,((a^2b+b^3)\cos{(x)} + (a^3+ab^2)\sin{(x)})}$$

input
```
integrate(x*csc(x)^2/(a+b*cot(x))^2,x, algorithm="fricas")
```

output
```
-1/2*(2*a*x*cos(x) - 2*b*x*sin(x) - (b*cos(x) + a*sin(x))*log(2*a*b*cos(x)
*sin(x) - (a^2 - b^2)*cos(x)^2 + a^2))/((a^2*b + b^3)*cos(x) + (a^3 + a*b^
2)*sin(x))
```

3.158.6 Sympy [F]

$$\int \frac{x\csc^2(x)}{(a+b\cot(x))^2}\,dx = \int \frac{x\csc^2{(x)}}{(a+b\cot{(x)})^2}\,dx$$

input
```
integrate(x*csc(x)**2/(a+b*cot(x))**2,x)
```

output
```
Integral(x*csc(x)**2/(a + b*cot(x))**2, x)
```

3.158.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(50) = 100$.

Time = 0.22 (sec) , antiderivative size = 250, normalized size of antiderivative = 5.00

$$\int \frac{x\csc^2(x)}{(a+b\cot(x))^2}\,dx = -\frac{8\,abx\cos(2\,x)+4\,(a^2-b^2)x\sin(2\,x)-\left((a^2+b^2)\cos(2\,x)^2+4\,ab\sin(2\,x)+(a^2+b^2)\sin(2\,x)^2+a^2\right.}{2\left(a^4+2\,a^2b^2+b^4+(a^4+2\,a^2b^2+b^4)\cos(2\,x)^2+(a^4+2\,a^2b^2+b^4\right.}$$

input
```
integrate(x*csc(x)^2/(a+b*cot(x))^2,x, algorithm="maxima")
```

output
```
-1/2*(8*a*b*x*cos(2*x) + 4*(a^2 - b^2)*x*sin(2*x) - ((a^2 + b^2)*cos(2*x)^
2 + 4*a*b*sin(2*x) + (a^2 + b^2)*sin(2*x)^2 + a^2 + b^2 - 2*(a^2 - b^2)*co
s(2*x))*log(((a^2 + b^2)*cos(2*x)^2 + 4*a*b*sin(2*x) + (a^2 + b^2)*sin(2*x
)^2 + a^2 + b^2 - 2*(a^2 - b^2)*cos(2*x))/(a^2 + b^2)))/(a^4 + 2*a^2*b^2 +
 b^4 + (a^4 + 2*a^2*b^2 + b^4)*cos(2*x)^2 + (a^4 + 2*a^2*b^2 + b^4)*sin(2*
x)^2 - 2*(a^4 - b^4)*cos(2*x) + 4*(a^3*b + a*b^3)*sin(2*x))
```

3.158.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(50) = 100$.

Time = 0.37 (sec) , antiderivative size = 322, normalized size of antiderivative = 6.44

$$\int \frac{x\csc^2(x)}{(a+b\cot(x))^2}\,dx = -\frac{2\,ax\tan\left(\frac{1}{2}\,x\right)^2-b\log\left(\frac{4\left(b^2\tan\left(\frac{1}{2}\,x\right)^4-4\,ab\tan\left(\frac{1}{2}\,x\right)^3+4\,a^2\tan\left(\frac{1}{2}\,x\right)^2-2\,b^2\tan\left(\frac{1}{2}\,x\right)^2+4\,ab\tan\left(\frac{1}{2}\,x\right)+b^2\right)}{\tan\left(\frac{1}{2}\,x\right)^4+2\,\tan\left(\frac{1}{2}\,x\right)^2+1}\right)\tan\left(\frac{1}{2}\,x\right)^2+4}{2}$$

input
```
integrate(x*csc(x)^2/(a+b*cot(x))^2,x, algorithm="giac")
```

output
```
-1/2*(2*a*x*tan(1/2*x)^2 - b*log(4*(b^2*tan(1/2*x)^4 - 4*a*b*tan(1/2*x)^3
+ 4*a^2*tan(1/2*x)^2 - 2*b^2*tan(1/2*x)^2 + 4*a*b*tan(1/2*x) + b^2)/(tan(1
/2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x)^2 + 4*b*x*tan(1/2*x) + 2*a*log(4
*(b^2*tan(1/2*x)^4 - 4*a*b*tan(1/2*x)^3 + 4*a^2*tan(1/2*x)^2 - 2*b^2*tan(1
/2*x)^2 + 4*a*b*tan(1/2*x) + b^2)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan
(1/2*x) - 2*a*x + b*log(4*(b^2*tan(1/2*x)^4 - 4*a*b*tan(1/2*x)^3 + 4*a^2*t
an(1/2*x)^2 - 2*b^2*tan(1/2*x)^2 + 4*a*b*tan(1/2*x) + b^2)/(tan(1/2*x)^4 +
 2*tan(1/2*x)^2 + 1)))/(a^2*b*tan(1/2*x)^2 + b^3*tan(1/2*x)^2 - 2*a^3*tan(
1/2*x) - 2*a*b^2*tan(1/2*x) - a^2*b - b^3)
```

3.158.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x \csc^2(x)}{(a+b\cot(x))^2} \, dx = \int \frac{x}{\sin(x)^2 (a+b\cot(x))^2} \, dx$$

input
```
int(x/(sin(x)^2*(a + b*cot(x))^2),x)
```

output
```
int(x/(sin(x)^2*(a + b*cot(x))^2), x)
```

3.159 $\int \frac{\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx$

3.159.1 Optimal result

Integrand size = 23, antiderivative size = 32

$$\int \frac{\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{\arctan\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}$$

output
```
arctan(b^(1/2)*tan(d*x+c)/a^(1/2))/d/a^(1/2)/b^(1/2)
```

3.159.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{\arctan\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}$$

input
```
Integrate[Sec[c + d*x]^2/(a + b*Tan[c + d*x]^2),x]
```

output
```
ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)
```

3.159.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3042, 4158, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{a+b\tan^2(c+dx)}dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{\sec(c+dx)^2}{a+b\tan(c+dx)^2}dx$$
$$\Big\downarrow \ 4158$$
$$\frac{\int \frac{1}{b\tan^2(c+dx)+a}d\tan(c+dx)}{d}$$
$$\Big\downarrow \ 218$$
$$\frac{\arctan\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}$$

input
```
Int[Sec[c + d*x]^2/(a + b*Tan[c + d*x]^2),x]
```

output
```
ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)
```

3.159.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

3.159.4 Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{d\sqrt{ab}}$	24
default	$\frac{\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{d\sqrt{ab}}$	24
risch	$-\frac{\ln\left(e^{2i(dx+c)}+\frac{2iba+a\sqrt{-ab}+b\sqrt{-ab}}{\sqrt{-ab}\,(a-b)}\right)}{2\sqrt{-ab}\,d}+\frac{\ln\left(e^{2i(dx+c)}-\frac{2iba-a\sqrt{-ab}-b\sqrt{-ab}}{\sqrt{-ab}\,(a-b)}\right)}{2\sqrt{-ab}\,d}$	121

input
```
int(sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d/(a*b)^(1/2)*arctan(b*tan(d*x+c)/(a*b)^(1/2))
```

3.159.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(24) = 48$.

Time = 0.26 (sec) , antiderivative size = 205, normalized size of antiderivative = 6.41

$$\int \frac{\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx = \left[-\frac{\sqrt{-ab}\log\left(\frac{(a^2+6\,ab+b^2)\cos(dx+c)^4-2\,(3\,ab+b^2)\cos(dx+c)^2+4\,\left((a+b)\cos(dx+c)^3-b\cos(dx+c)\right)\sqrt{-ab}\sin(dx+c)+b^2}{(a^2-2\,ab+b^2)\cos(dx+c)^4+2\,(ab-b^2)\cos(dx+c)^2+b^2}\right)}{4\,abd},\right.$$
$$\left.-\frac{\sqrt{ab}\arctan\left(\frac{\left((a+b)\cos(dx+c)^2-b\right)\sqrt{ab}}{2\,ab\cos(dx+c)\sin(dx+c)}\right)}{2\,abd}\right]$$

input
```
integrate(sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

output
```
[-1/4*sqrt(-a*b)*log(((a^2 + 6*a*b + b^2)*cos(d*x + c)^4 - 2*(3*a*b + b^2)
*cos(d*x + c)^2 + 4*((a + b)*cos(d*x + c)^3 - b*cos(d*x + c))*sqrt(-a*b)*s
in(d*x + c) + b^2)/((a^2 - 2*a*b + b^2)*cos(d*x + c)^4 + 2*(a*b - b^2)*cos
(d*x + c)^2 + b^2))/(a*b*d), -1/2*sqrt(a*b)*arctan(1/2*((a + b)*cos(d*x +
c)^2 - b)*sqrt(a*b)/(a*b*cos(d*x + c)*sin(d*x + c)))/(a*b*d)]
```

3.159.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx = \int \frac{\sec^2{(c+dx)}}{a+b\tan^2{(c+dx)}}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a+b*tan(d*x+c)**2),x)
```

output
```
Integral(sec(c + d*x)**2/(a + b*tan(c + d*x)**2), x)
```

3.159.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.72

$$\int \frac{\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{\sqrt{ab}d}$$

input
```
integrate(sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

output
```
arctan(b*tan(d*x + c)/sqrt(a*b))/(sqrt(a*b)*d)
```

3.159.8 Giac [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.25

$$\int \frac{\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{\pi\left\lfloor \frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor \operatorname{sgn}(b)+\arctan\left(\frac{b\tan(dx+c)}{\sqrt{ab}}\right)}{\sqrt{ab}d}$$

input
```
integrate(sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

output
```
(pi*floor((d*x + c)/pi + 1/2)*sgn(b) + arctan(b*tan(d*x + c)/sqrt(a*b)))/(
sqrt(a*b)*d)
```

3.159.9 Mupad [B] (verification not implemented)

Time = 26.64 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{b}\tan(c+d\,x)}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}\,d}$$

input
```
int(1/(cos(c + d*x)^2*(a + b*tan(c + d*x)^2)),x)
```

output
```
atan((b^(1/2)*tan(c + d*x))/a^(1/2))/(a^(1/2)*b^(1/2)*d)
```

3.160 $\int \frac{x\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx$

3.160.1 Optimal result

Integrand size = 24, antiderivative size = 211

$$\int \frac{x\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx = -\frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}d}+\frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}+\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}d} - \frac{\operatorname{PolyLog}\left(2,-\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{4\sqrt{a}\sqrt{b}d^2}+\frac{\operatorname{PolyLog}\left(2,-\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}+\sqrt{b}\right)^2}\right)}{4\sqrt{a}\sqrt{b}d^2}$$

output
```
-1/2*I*x*ln(1+(a-b)*exp(2*I*(d*x+c))/(a^(1/2)-b^(1/2))^2)/d/a^(1/2)/b^(1/2
)+1/2*I*x*ln(1+(a-b)*exp(2*I*(d*x+c))/(a^(1/2)+b^(1/2))^2)/d/a^(1/2)/b^(1/
2)-1/4*polylog(2,-(a-b)*exp(2*I*(d*x+c))/(a^(1/2)-b^(1/2))^2)/d^2/a^(1/2)/
b^(1/2)+1/4*polylog(2,-(a-b)*exp(2*I*(d*x+c))/(a^(1/2)+b^(1/2))^2)/d^2/a^(
1/2)/b^(1/2)
```

3.160.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 512 vs. 2(211) = 422.

Time = 2.20 (sec) , antiderivative size = 512, normalized size of antiderivative = 2.43

$$\int \frac{x\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx$$
$$= \frac{x\left(4i\sqrt{-a}c\arctan\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a}}\right)-\sqrt{a}\log(1+i\tan(c+dx))\log\left(\frac{\sqrt{-a}-\sqrt{b}\tan(c+dx)}{\sqrt{-a}-i\sqrt{b}}\right)+\sqrt{a}\log(1-i\tan(c\right.}{}$$

input
```
Integrate[(x*Sec[c + d*x]^2)/(a + b*Tan[c + d*x]^2),x]
```

output
```
(x*((4*I)*Sqrt[-a]*c*ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a]] - Sqrt[a]*Log[
1 + I*Tan[c + d*x]]*Log[(Sqrt[-a] - Sqrt[b]*Tan[c + d*x])/(Sqrt[-a] - I*Sq
rt[b])] + Sqrt[a]*Log[1 - I*Tan[c + d*x]]*Log[(Sqrt[-a] - Sqrt[b]*Tan[c +
d*x])/(Sqrt[-a] + I*Sqrt[b])] - Sqrt[a]*Log[1 - I*Tan[c + d*x]]*Log[(Sqrt[
-a] + Sqrt[b]*Tan[c + d*x])/(Sqrt[-a] - I*Sqrt[b])] + Sqrt[a]*Log[1 + I*Ta
n[c + d*x]]*Log[(Sqrt[-a] + Sqrt[b]*Tan[c + d*x])/(Sqrt[-a] + I*Sqrt[b])]
- Sqrt[a]*PolyLog[2, (Sqrt[b]*(1 - I*Tan[c + d*x]))/(I*Sqrt[-a] + Sqrt[b])
] - Sqrt[a]*PolyLog[2, (Sqrt[b]*(1 + I*Tan[c + d*x]))/(I*Sqrt[-a] + Sqrt[b
])] + Sqrt[a]*PolyLog[2, -((Sqrt[b]*(-I + Tan[c + d*x]))/(Sqrt[-a] + I*Sqr
t[b]))] + Sqrt[a]*PolyLog[2, (Sqrt[b]*(I + Tan[c + d*x]))/(Sqrt[-a] + I*Sq
rt[b])]))/(2*Sqrt[-a^2]*Sqrt[b]*d*((2*I)*c + Log[1 - I*Tan[c + d*x]] - Log
[1 + I*Tan[c + d*x]]))
```

3.160.3 Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5099, 3042, 3802, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx$$
$$\Big\downarrow \text{5099}$$

$$2\int \frac{x}{a+b+(a-b)\cos(2c+2dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$2\int \frac{x}{a+b+(a-b)\sin\left(2c+2dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3802}$$

$$4\int \frac{e^{2i(c+dx)}x}{a+2(a+b)e^{2i(c+dx)}+(a-b)e^{4i(c+dx)}-b}dx$$

$$\Big\downarrow \text{2694}$$

$$4\left(\frac{(a-b)\int \frac{e^{2i(c+dx)}x}{2\left(a-2\sqrt{b}\sqrt{a}+(a-b)e^{2i(c+dx)}+b\right)}dx}{2\sqrt{a}\sqrt{b}}-\frac{(a-b)\int \frac{e^{2i(c+dx)}x}{2\left(\left(\sqrt{a}+\sqrt{b}\right)^2+(a-b)e^{2i(c+dx)}\right)}dx}{2\sqrt{a}\sqrt{b}}\right)$$

$$\Big\downarrow \text{27}$$

$$4\left(\frac{(a-b)\int \frac{e^{2i(c+dx)}x}{a-2\sqrt{b}\sqrt{a}+(a-b)e^{2i(c+dx)}+b}dx}{4\sqrt{a}\sqrt{b}}-\frac{(a-b)\int \frac{e^{2i(c+dx)}x}{\left(\sqrt{a}+\sqrt{b}\right)^2+(a-b)e^{2i(c+dx)}}dx}{4\sqrt{a}\sqrt{b}}\right)$$

$$\Big\downarrow \text{2620}$$

$$4\left(\frac{(a-b)\left(\frac{i\int \log\left(\frac{e^{2i(c+dx)}(a-b)}{\left(\sqrt{a}-\sqrt{b}\right)^2}+1\right)dx}{2d(a-b)}-\frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{2d(a-b)}\right)}{4\sqrt{a}\sqrt{b}}-\frac{(a-b)\left(\frac{i\int \log\left(\frac{e^{2i(c+dx)}(a-b)}{\left(\sqrt{a}+\sqrt{b}\right)^2}+1\right)dx}{2d(a-b)}-\frac{ix\log\left(1+\frac{(a\cdots}{\cdots}\right)}{2d(\cdots}\cdots\right)}{4\sqrt{a}\sqrt{b}}\cdots\right.$$

$$\Big\downarrow \text{2715}$$

$$4\left(\frac{(a-b)\left(\frac{\int e^{-2i(c+dx)}\log\left(\frac{e^{2i(c+dx)}(a-b)}{\left(\sqrt{a}-\sqrt{b}\right)^2}+1\right)de^{2i(c+dx)}}{4d^2(a-b)}-\frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{2d(a-b)}\right)}{4\sqrt{a}\sqrt{b}}-\frac{(a-b)\left(\frac{\int e^{-2i(c+dx)}\log\left(\frac{e^{2i(c+dx)}(a\cdots}{\left(\sqrt{a}+\sqrt{b}\right)\cdots}\cdots\right.}{4d^2(a-b)}\cdots\right.}{\cdots}\cdots\right.$$

$$\Big\downarrow \text{2838}$$

3.160. $\int \frac{x\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx$

$$4\left(\frac{(a-b)\left(-\frac{\text{PolyLog}\left(2,-\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{4d^2(a-b)}-\frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{2d(a-b)}\right)}{4\sqrt{a}\sqrt{b}}-\frac{(a-b)\left(-\frac{\text{PolyLog}\left(2,-\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}+\sqrt{b}\right)^2}\right)}{4d^2(a-b)}-\frac{ix\log\left(\right.}{}\right.}{4\sqrt{a}\sqrt{b}}\right.$$

input
```
Int[(x*Sec[c + d*x]^2)/(a + b*Tan[c + d*x]^2),x]
```

output
```
4*(((a - b)*(((-1/2*I)*x*Log[1 + ((a - b)*E^((2*I)*(c + d*x)))/(Sqrt[a] -
Sqrt[b])^2])/((a - b)*d) - PolyLog[2, -(((a - b)*E^((2*I)*(c + d*x)))/(Sqr
t[a] - Sqrt[b])^2)]/(4*(a - b)*d^2)))/(4*Sqrt[a]*Sqrt[b]) - ((a - b)*(((-1
/2*I)*x*Log[1 + ((a - b)*E^((2*I)*(c + d*x)))/(Sqrt[a] + Sqrt[b])^2])/((a
- b)*d) - PolyLog[2, -(((a - b)*E^((2*I)*(c + d*x)))/(Sqrt[a] + Sqrt[b])^2
)]/(4*(a - b)*d^2)))/(4*Sqrt[a]*Sqrt[b]))
```

3.160.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

3.160. $\int \frac{x\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx$

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3802
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*E^(I*Pi*(k - 1/2))*(E^(I*(e +
f*x))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(
e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ
[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 5099
```
Int[(((f_.) + (g_.)*(x_))^(m_.)*Sec[(d_.) + (e_.)*(x_)]^2)/((b_) + (c_.)*Ta
n[(d_.) + (e_.)*(x_)]^2), x_Symbol] :> Simp[2   Int[(f + g*x)^m/(b + c + (b
 - c)*Cos[2*d + 2*e*x]), x], x] /; FreeQ[{b, c, d, e, f, g}, x] && IGtQ[m,
0]
```

3.160.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1002 vs. $2(161) = 322$.

Time = 2.15 (sec) , antiderivative size = 1003, normalized size of antiderivative = 4.75

method	result
risch	$-\frac{a\,c^2}{2d^2\sqrt{ab}\left(-2\sqrt{ab}-a-b\right)}-\frac{c^2 b}{2d^2\sqrt{ab}\left(-2\sqrt{ab}-a-b\right)}-\frac{\operatorname{polylog}\left(2,\frac{(a-b)\mathrm{e}^{2i(dx+c)}}{-2\sqrt{ab}-a-b}\right)a}{4d^2\sqrt{ab}\left(-2\sqrt{ab}-a-b\right)}-\frac{a\,x^2}{2\sqrt{ab}\left(-2\sqrt{ab}-a-b\right)}-\frac{x^2}{2\sqrt{ab}\left(-2\sqrt{\right.}}$

input
```
int(x*sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/d^2/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*a*c^2-1/2/d^2/(a*b)^(1/2)/(-2*(a
*b)^(1/2)-a-b)*c^2*b-1/4/d^2/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*polylog(2,(a
-b)*exp(2*I*(d*x+c))/(-2*(a*b)^(1/2)-a-b))*a-1/4/d^2/(a*b)^(1/2)/(-2*(a*b)
^(1/2)-a-b)*polylog(2,(a-b)*exp(2*I*(d*x+c))/(-2*(a*b)^(1/2)-a-b))*b-I/d^2
/(-2*(a*b)^(1/2)-a-b)*ln(1-(a-b)*exp(2*I*(d*x+c))/(-2*(a*b)^(1/2)-a-b))*c-
1/2*I/d/(a*b)^(1/2)*ln(1-(a-b)*exp(2*I*(d*x+c))/(2*(a*b)^(1/2)-a-b))*x-1/2
*I/d^2/(a*b)^(1/2)*ln(1-(a-b)*exp(2*I*(d*x+c))/(2*(a*b)^(1/2)-a-b))*c-I/d^
2*c/(a*b)^(1/2)*arctanh(1/4*(2*(a-b)*exp(2*I*(d*x+c))+2*a+2*b)/(a*b)^(1/2)
)-I/d/(-2*(a*b)^(1/2)-a-b)*ln(1-(a-b)*exp(2*I*(d*x+c))/(-2*(a*b)^(1/2)-a-b
))*x-1/2/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*a*x^2-1/2/(a*b)^(1/2)/(-2*(a*b)^
(1/2)-a-b)*x^2*b-2/d/(-2*(a*b)^(1/2)-a-b)*c*x-1/d/(a*b)^(1/2)*c*x-1/2*I/d/
(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*ln(1-(a-b)*exp(2*I*(d*x+c))/(-2*(a*b)^(1/
2)-a-b))*a*x-1/2*I/d/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*ln(1-(a-b)*exp(2*I*(
d*x+c))/(-2*(a*b)^(1/2)-a-b))*b*x-1/2*I/d^2/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-
b)*ln(1-(a-b)*exp(2*I*(d*x+c))/(-2*(a*b)^(1/2)-a-b))*a*c-1/2*I/d^2/(a*b)^(
1/2)/(-2*(a*b)^(1/2)-a-b)*ln(1-(a-b)*exp(2*I*(d*x+c))/(-2*(a*b)^(1/2)-a-b)
)*b*c-1/(-2*(a*b)^(1/2)-a-b)*x^2-1/2/(a*b)^(1/2)*x^2-1/d/(a*b)^(1/2)/(-2*(
a*b)^(1/2)-a-b)*a*c*x-1/d/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*c*x*b-1/2/d^2/(
a*b)^(1/2)*c^2-1/4/d^2/(a*b)^(1/2)*polylog(2,(a-b)*exp(2*I*(d*x+c))/(2*(a*
b)^(1/2)-a-b))-1/d^2/(-2*(a*b)^(1/2)-a-b)*c^2-1/2/d^2/(-2*(a*b)^(1/2)-a...
```

3.160.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3284 vs. 2(157) = 314.

Time = 3.66 (sec) , antiderivative size = 3284, normalized size of antiderivative = 15.56

$$\int \frac{x\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(x*sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

output
```
-1/4*(I*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*c*log(2*sqrt(-(2*(a - b)*sqr
t(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) + 2*cos(d*x + c) + 2*I*sin(d*
x + c)) - I*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*c*log(2*sqrt(-(2*(a - b)
*sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) + 2*cos(d*x + c) - 2*I*si
n(d*x + c)) - I*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*c*log(2*sqrt(-(2*(a
- b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) - 2*cos(d*x + c) + 2*
I*sin(d*x + c)) + I*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*c*log(2*sqrt(-(2
*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) - 2*cos(d*x + c)
- 2*I*sin(d*x + c)) - I*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*c*log(2*sqrt
((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) + 2*cos(d*x +
c) + 2*I*sin(d*x + c)) + I*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*c*log(2*s
qrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) + 2*cos(d*x
 + c) - 2*I*sin(d*x + c)) + I*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*c*log(
2*sqrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) - 2*cos(
d*x + c) + 2*I*sin(d*x + c)) - I*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*c*l
og(2*sqrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) - a - b)/(a - b)) - 2*c
os(d*x + c) - 2*I*sin(d*x + c)) - (a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*di
log(-(((a + b)*cos(d*x + c) + (I*a + I*b)*sin(d*x + c) - 2*((a - b)*cos(d*
x + c) + (I*a - I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt(-(2
*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) + a - b)/(a - ...
```

3.160.6 Sympy [F]

$$\int \frac{x\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx = \int \frac{x\sec^2{(c+dx)}}{a+b\tan^2{(c+dx)}}\,dx$$

input
```
integrate(x*sec(d*x+c)**2/(a+b*tan(d*x+c)**2),x)
```

output
```
Integral(x*sec(c + d*x)**2/(a + b*tan(c + d*x)**2), x)
```

3.160.7 Maxima [F]

$$\int \frac{x\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx = \int \frac{x\sec\left(dx+c\right)^2}{b\tan\left(dx+c\right)^2+a}\,dx$$

input
```
integrate(x*sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

output
```
integrate(x*sec(d*x + c)^2/(b*tan(d*x + c)^2 + a), x)
```

3.160.8 Giac [F]

$$\int \frac{x\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx = \int \frac{x\sec\left(dx+c\right)^2}{b\tan\left(dx+c\right)^2+a}\,dx$$

input
```
integrate(x*sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

output
```
integrate(x*sec(d*x + c)^2/(b*tan(d*x + c)^2 + a), x)
```

3.160.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx = \int \frac{x}{\cos\left(c+d\,x\right)^2\,\left(b\tan\left(c+d\,x\right)^2+a\right)}\,dx$$

input
```
int(x/(cos(c + d*x)^2*(a + b*tan(c + d*x)^2)),x)
```

output
```
int(x/(cos(c + d*x)^2*(a + b*tan(c + d*x)^2)), x)
```

3.161 $\int \frac{x^2 \sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx$

3.161.1 Optimal result

Integrand size = 26, antiderivative size = 337

$$\int \frac{x^2 \sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx = -\frac{ix^2 \log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}d} + \frac{ix^2 \log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}+\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}d} - \frac{x\,\text{PolyLog}\left(2,-\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}d^2} + \frac{x\,\text{PolyLog}\left(2,-\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}+\sqrt{b}\right)^2}\right)}{2\sqrt{a}\sqrt{b}d^2} + \frac{i\,\text{PolyLog}\left(3,-\frac{\left(\sqrt{a}-\sqrt{b}\right)e^{2i(c+dx)}}{\sqrt{a}+\sqrt{b}}\right)}{4\sqrt{a}\sqrt{b}d^3} - \frac{i\,\text{PolyLog}\left(3,-\frac{\left(\sqrt{a}+\sqrt{b}\right)e^{2i(c+dx)}}{\sqrt{a}-\sqrt{b}}\right)}{4\sqrt{a}\sqrt{b}d^3}$$

output
```
-1/2*I*x^2*ln(1+(a-b)*exp(2*I*(d*x+c))/(a^(1/2)-b^(1/2))^2)/d/a^(1/2)/b^(1
/2)+1/2*I*x^2*ln(1+(a-b)*exp(2*I*(d*x+c))/(a^(1/2)+b^(1/2))^2)/d/a^(1/2)/b
^(1/2)-1/2*x*polylog(2,-(a-b)*exp(2*I*(d*x+c))/(a^(1/2)-b^(1/2))^2)/d^2/a^
(1/2)/b^(1/2)+1/2*x*polylog(2,-(a-b)*exp(2*I*(d*x+c))/(a^(1/2)+b^(1/2))^2)
/d^2/a^(1/2)/b^(1/2)+1/4*I*polylog(3,-exp(2*I*(d*x+c))*(a^(1/2)-b^(1/2))/(
a^(1/2)+b^(1/2)))/d^3/a^(1/2)/b^(1/2)-1/4*I*polylog(3,-exp(2*I*(d*x+c))*(a
^(1/2)+b^(1/2))/(a^(1/2)-b^(1/2)))/d^3/a^(1/2)/b^(1/2)
```

3.161.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.87

$$\int \frac{x^2\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx = \frac{i\left(2d^2x^2\log\left(1+\frac{\left(\sqrt{a}-\sqrt{b}\right)e^{2i(c+dx)}}{\sqrt{a}+\sqrt{b}}\right)-2d^2x^2\log\left(1+\frac{\left(\sqrt{a}+\sqrt{b}\right)e^{2i(c+dx)}}{\sqrt{a}-\sqrt{b}}\right)-2idx\,\text{PolyLog}\left(2,\frac{\left(-\sqrt{a}+\sqrt{b}\right)e^{2i(c+}}{\sqrt{a}+\sqrt{b}}\right.\right.}{4}$$

input
```
Integrate[(x^2*Sec[c + d*x]^2)/(a + b*Tan[c + d*x]^2),x]
```

output
```
((I/4)*(2*d^2*x^2*Log[1 + ((Sqrt[a] - Sqrt[b])*E^((2*I)*(c + d*x)))/(Sqrt[
a] + Sqrt[b])] - 2*d^2*x^2*Log[1 + ((Sqrt[a] + Sqrt[b])*E^((2*I)*(c + d*x)
))/(Sqrt[a] - Sqrt[b])] - (2*I)*d*x*PolyLog[2, ((-Sqrt[a] + Sqrt[b])*E^((2
*I)*(c + d*x)))/(Sqrt[a] + Sqrt[b])] + (2*I)*d*x*PolyLog[2, -(((Sqrt[a] +
Sqrt[b])*E^((2*I)*(c + d*x)))/(Sqrt[a] - Sqrt[b]))] + PolyLog[3, ((-Sqrt[a
] + Sqrt[b])*E^((2*I)*(c + d*x)))/(Sqrt[a] + Sqrt[b])] - PolyLog[3, -(((Sq
rt[a] + Sqrt[b])*E^((2*I)*(c + d*x)))/(Sqrt[a] - Sqrt[b]))]))/(Sqrt[a]*Sqr
t[b]*d^3)
```

3.161.3 Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {5099, 3042, 3802, 2694, 27, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx$$

$$\Big\downarrow \; 5099$$

$$2\int \frac{x^2}{a+b+(a-b)\cos(2c+2dx)}dx$$

$$\Big\downarrow \; 3042$$

$$2\int \frac{x^2}{a+b+(a-b)\sin\left(2c+2dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 3802$$

$$4\int \frac{e^{2i(c+dx)}x^2}{a+2(a+b)e^{2i(c+dx)}+(a-b)e^{4i(c+dx)}-b}dx$$

$$\Big\downarrow \; 2694$$

$$4\left(\frac{(a-b)\int \frac{e^{2i(c+dx)}x^2}{2\left(a-2\sqrt{b}\sqrt{a}+(a-b)e^{2i(c+dx)}+b\right)}dx}{2\sqrt{a}\sqrt{b}}-\frac{(a-b)\int \frac{e^{2i(c+dx)}x^2}{2\left(\left(\sqrt{a}+\sqrt{b}\right)^2+(a-b)e^{2i(c+dx)}\right)}dx}{2\sqrt{a}\sqrt{b}}\right)$$

$$\Big\downarrow \; 27$$

$$4\left(\frac{(a-b)\int \frac{e^{2i(c+dx)}x^2}{a-2\sqrt{b}\sqrt{a}+(a-b)e^{2i(c+dx)}+b}dx}{4\sqrt{a}\sqrt{b}}-\frac{(a-b)\int \frac{e^{2i(c+dx)}x^2}{\left(\sqrt{a}+\sqrt{b}\right)^2+(a-b)e^{2i(c+dx)}}dx}{4\sqrt{a}\sqrt{b}}\right)$$

$$\Big\downarrow \; 2620$$

$$4\left(\frac{(a-b)\left(\frac{i\int x\log\left(\frac{e^{2i(c+dx)}(a-b)}{\left(\sqrt{a}-\sqrt{b}\right)^2}+1\right)dx}{d(a-b)}-\frac{ix^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{2d(a-b)}\right)}{4\sqrt{a}\sqrt{b}}-\frac{(a-b)\left(\frac{i\int x\log\left(\frac{e^{2i(c+dx)}(a-b)}{\left(\sqrt{a}+\sqrt{b}\right)^2}+1\right)dx}{d(a-b)}-\frac{ix^2\log\left(\right.}{}\right.}{4\sqrt{a}\sqrt{b}}\right.$$

$$\Big\downarrow \; 3011$$

3.161. $\int \frac{x^2\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx$

$$4\left(\frac{(a-b)\left(\frac{i\left(\frac{ix\,\text{PolyLog}\left(2,-\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{2d}-\frac{i\int\text{PolyLog}\left(2,-\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)dx}{2d}\right)}{d(a-b)}-\frac{ix^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{2d(a-b)}\right)}{4\sqrt{a}\sqrt{b}}-\frac{(a-b)\left(i\left(\ldots\right.\right.}{}\right.$$

$$\Big\downarrow \text{ 2720}$$

$$4\left(\frac{(a-b)\left(\frac{i\left(\frac{ix\,\text{PolyLog}\left(2,-\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{2d}-\frac{\int e^{-2i(c+dx)}\,\text{PolyLog}\left(2,-\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)de^{2i(c+dx)}}{4d^2}\right)}{d(a-b)}-\frac{ix^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{2d(a-b)}\right)}{4\sqrt{a}\sqrt{b}}-\right.$$

$$\Big\downarrow \text{ 7143}$$

3.161. $\int \frac{x^2\sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx$

$$4\left(\frac{(a-b)\left(\frac{i\left(\frac{ix\,\mathrm{PolyLog}\left(2,-\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{2d}-\frac{\mathrm{PolyLog}\left(3,-\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{4d^2}\right)}{d(a-b)}-\frac{ix^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)}{2d(a-b)}\right)}{4\sqrt{a}\sqrt{b}}-\frac{(a-b)\left(\frac{i\left(\frac{ix\,\mathrm{Poly}\cdots}{}\right.}{}\right.}{}\right.$$

input
```
Int[(x^2*Sec[c + d*x]^2)/(a + b*Tan[c + d*x]^2),x]
```

output
```
4*(((a - b)*(((-1/2*I)*x^2*Log[1 + ((a - b)*E^((2*I)*(c + d*x)))/(Sqrt[a]
 - Sqrt[b])^2])/((a - b)*d) + (I*(((I/2)*x*PolyLog[2, -(((a - b)*E^((2*I)*(
c + d*x)))/(Sqrt[a] - Sqrt[b])^2)])/d - PolyLog[3, -(((a - b)*E^((2*I)*(c
+ d*x)))/(Sqrt[a] - Sqrt[b])^2)]/(4*d^2)))/((a - b)*d)))/(4*Sqrt[a]*Sqrt[b
]) - ((a - b)*(((-1/2*I)*x^2*Log[1 + ((a - b)*E^((2*I)*(c + d*x)))/(Sqrt[a
] + Sqrt[b])^2])/((a - b)*d) + (I*(((I/2)*x*PolyLog[2, -(((a - b)*E^((2*I)
*(c + d*x)))/(Sqrt[a] + Sqrt[b])^2)])/d - PolyLog[3, -(((a - b)*E^((2*I)*(
c + d*x)))/(Sqrt[a] + Sqrt[b])^2)]/(4*d^2)))/((a - b)*d)))/(4*Sqrt[a]*Sqrt
[b]))
```

3.161.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3802
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*E^(I*Pi*(k - 1/2))*(E^(I*(e +
f*x))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(
e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ
[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 5099
```
Int[(((f_.) + (g_.)*(x_))^(m_.)*Sec[(d_.) + (e_.)*(x_)]^2)/((b_) + (c_.)*Ta
n[(d_.) + (e_.)*(x_)]^2), x_Symbol] :> Simp[2   Int[(f + g*x)^m/(b + c + (b
 - c)*Cos[2*d + 2*e*x]), x], x] /; FreeQ[{b, c, d, e, f, g}, x] && IGtQ[m,
0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.161. $\int \frac{x^2 \sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx$

3.161.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1250 vs. $2(253) = 506$.

Time = 2.10 (sec) , antiderivative size = 1251, normalized size of antiderivative = 3.71

method	result	size
risch	Expression too large to display	1251

input
```
int(x^2*sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*I/d^3/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*a*ln(1-(a-b)*exp(2*I*(d*x+c))/(
-2*(a*b)^(1/2)-a-b))*c^2-1/2*I/d/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*b*ln(1-(
a-b)*exp(2*I*(d*x+c))/(-2*(a*b)^(1/2)-a-b))*x^2-1/2*I/d/(a*b)^(1/2)/(-2*(a
*b)^(1/2)-a-b)*a*ln(1-(a-b)*exp(2*I*(d*x+c))/(-2*(a*b)^(1/2)-a-b))*x^2+1/d
^2/(a*b)^(1/2)*c^2*x-1/2/d^2/(a*b)^(1/2)*polylog(2,(a-b)*exp(2*I*(d*x+c))/
(2*(a*b)^(1/2)-a-b))*x+2/d^2/(-2*(a*b)^(1/2)-a-b)*c^2*x-1/d^2/(-2*(a*b)^(1
/2)-a-b)*polylog(2,(a-b)*exp(2*I*(d*x+c))/(-2*(a*b)^(1/2)-a-b))*x-1/3/(a*b
)^(1/2)/(-2*(a*b)^(1/2)-a-b)*b*x^3-1/3/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*a*
x^3-1/4*I/d^3/(a*b)^(1/2)*polylog(3,(a-b)*exp(2*I*(d*x+c))/(2*(a*b)^(1/2)-
a-b))-1/2*I/d^3/(-2*(a*b)^(1/2)-a-b)*polylog(3,(a-b)*exp(2*I*(d*x+c))/(-2*
(a*b)^(1/2)-a-b))+1/2*I/d^3/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*b*ln(1-(a-b)*
exp(2*I*(d*x+c))/(-2*(a*b)^(1/2)-a-b))*c^2+4/3/d^3/(-2*(a*b)^(1/2)-a-b)*c^
3+2/3/d^3/(a*b)^(1/2)*c^3+I/d^3/(-2*(a*b)^(1/2)-a-b)*ln(1-(a-b)*exp(2*I*(d
*x+c))/(-2*(a*b)^(1/2)-a-b))*c^2+I/d^3*c^2/(a*b)^(1/2)*arctanh(1/4*(2*(a-b
)*exp(2*I*(d*x+c))+2*a+2*b)/(a*b)^(1/2))+2/3/d^3/(a*b)^(1/2)/(-2*(a*b)^(1/
2)-a-b)*a*c^3+2/3/d^3/(a*b)^(1/2)/(-2*(a*b)^(1/2)-a-b)*b*c^3-1/2*I/d/(a*b)
^(1/2)*ln(1-(a-b)*exp(2*I*(d*x+c))/(2*(a*b)^(1/2)-a-b))*x^2-I/d/(-2*(a*b)^
(1/2)-a-b)*ln(1-(a-b)*exp(2*I*(d*x+c))/(-2*(a*b)^(1/2)-a-b))*x^2+1/2*I/d^3
/(a*b)^(1/2)*ln(1-(a-b)*exp(2*I*(d*x+c))/(2*(a*b)^(1/2)-a-b))*c^2-1/3/(a*b
)^(1/2)*x^3-2/3/(-2*(a*b)^(1/2)-a-b)*x^3-1/4*I/d^3/(a*b)^(1/2)/(-2*(a*b...
```

3.161.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4568 vs. $2(247) = 494$.

Time = 2.29 (sec) , antiderivative size = 4568, normalized size of antiderivative = 13.55

$$\int \frac{x^2 \sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/4*(2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*d*x*dilog(-(((a + b)*cos(d*x
+ c) + (I*a + I*b)*sin(d*x + c) - 2*((a - b)*cos(d*x + c) + (I*a - I*b)*si
n(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt(a*b/(a^2
- 2*a*b + b^2)) + a + b)/(a - b)) + a - b)/(a - b) + 1) + 2*(a - b)*sqrt(a
*b/(a^2 - 2*a*b + b^2))*d*x*dilog((((a + b)*cos(d*x + c) - (I*a + I*b)*sin
(d*x + c) - 2*((a - b)*cos(d*x + c) - (I*a - I*b)*sin(d*x + c))*sqrt(a*b/(
a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) + a +
b)/(a - b)) - a + b)/(a - b) + 1) + 2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)
)*d*x*dilog(-(((a + b)*cos(d*x + c) + (-I*a - I*b)*sin(d*x + c) - 2*((a -
b)*cos(d*x + c) + (-I*a + I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2))
)*sqrt(-(2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) + a - b
)/(a - b) + 1) + 2*(a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*d*x*dilog((((a +
b)*cos(d*x + c) - (-I*a - I*b)*sin(d*x + c) - 2*((a - b)*cos(d*x + c) - (-
I*a + I*b)*sin(d*x + c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt(-(2*(a - b)*s
qrt(a*b/(a^2 - 2*a*b + b^2)) + a + b)/(a - b)) - a + b)/(a - b) + 1) - 2*(
a - b)*sqrt(a*b/(a^2 - 2*a*b + b^2))*d*x*dilog(-(((a + b)*cos(d*x + c) + (
I*a + I*b)*sin(d*x + c) + 2*((a - b)*cos(d*x + c) - (-I*a + I*b)*sin(d*x +
 c))*sqrt(a*b/(a^2 - 2*a*b + b^2)))*sqrt((2*(a - b)*sqrt(a*b/(a^2 - 2*a*b
+ b^2)) - a - b)/(a - b)) + a - b)/(a - b) + 1) - 2*(a - b)*sqrt(a*b/(a^2
- 2*a*b + b^2))*d*x*dilog((((a + b)*cos(d*x + c) - (I*a + I*b)*sin(d*x ...
```

3.161.6 Sympy [F]

$$\int \frac{x^2 \sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx = \int \frac{x^2 \sec^2{(c+dx)}}{a+b\tan^2{(c+dx)}}\,dx$$

input
```
integrate(x**2*sec(d*x+c)**2/(a+b*tan(d*x+c)**2),x)
```

output
```
Integral(x**2*sec(c + d*x)**2/(a + b*tan(c + d*x)**2), x)
```

3.161.7 Maxima [F]

$$\int \frac{x^2 \sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx = \int \frac{x^2 \sec(dx+c)^2}{b\tan(dx+c)^2+a}\,dx$$

input
```
integrate(x^2*sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="maxima")
```

output
```
integrate(x^2*sec(d*x + c)^2/(b*tan(d*x + c)^2 + a), x)
```

3.161.8 Giac [F]

$$\int \frac{x^2 \sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx = \int \frac{x^2 \sec(dx+c)^2}{b\tan(dx+c)^2+a}\,dx$$

input
```
integrate(x^2*sec(d*x+c)^2/(a+b*tan(d*x+c)^2),x, algorithm="giac")
```

output
```
integrate(x^2*sec(d*x + c)^2/(b*tan(d*x + c)^2 + a), x)
```

3.161.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \sec^2(c+dx)}{a+b\tan^2(c+dx)}\,dx = \int \frac{x^2}{\cos(c+d\,x)^2\,\left(b\tan(c+d\,x)^2+a\right)}\,dx$$

input
```
int(x^2/(cos(c + d*x)^2*(a + b*tan(c + d*x)^2)),x)
```

output
```
int(x^2/(cos(c + d*x)^2*(a + b*tan(c + d*x)^2)), x)
```

3.162 $\int \frac{\sec^2(c+dx)}{a+c\sec^2(c+dx)+b\tan^2(c+dx)}\,dx$

3.162.1 Optimal result

Integrand size = 33, antiderivative size = 40

$$\int \frac{\sec^2(c+dx)}{a+c\sec^2(c+dx)+b\tan^2(c+dx)}\,dx=\frac{\arctan\left(\frac{\sqrt{b+c}\tan(c+dx)}{\sqrt{a+c}}\right)}{\sqrt{a+c}\sqrt{b+c}d}$$

output
```
arctan((b+c)^(1/2)*tan(d*x+c)/(a+c)^(1/2))/d/(a+c)^(1/2)/(b+c)^(1/2)
```

3.162.2 Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(c+dx)}{a+c\sec^2(c+dx)+b\tan^2(c+dx)}\,dx=\frac{\arctan\left(\frac{\sqrt{b+c}\tan(c+dx)}{\sqrt{a+c}}\right)}{\sqrt{a+c}\sqrt{b+c}d}$$

input
```
Integrate[Sec[c + d*x]^2/(a + c*Sec[c + d*x]^2 + b*Tan[c + d*x]^2),x]
```

output
```
ArcTan[(Sqrt[b + c]*Tan[c + d*x])/Sqrt[a + c]]/(Sqrt[a + c]*Sqrt[b + c]*d)
```

3.162.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3042, 4889, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(c+dx)}{a+b\tan^2(c+dx)+c\sec^2(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sec(c+dx)^2}{a+b\tan(c+dx)^2+c\sec(c+dx)^2}dx$$

$$\Big\downarrow \; 4889$$

$$\frac{\int \frac{1}{(b+c)\tan^2(c+dx)+a+c}d\tan(c+dx)}{d}$$

$$\Big\downarrow \; 218$$

$$\frac{\arctan\left(\frac{\sqrt{b+c}\tan(c+dx)}{\sqrt{a+c}}\right)}{d\sqrt{a+c}\sqrt{b+c}}$$

input
```
Int[Sec[c + d*x]^2/(a + c*Sec[c + d*x]^2 + b*Tan[c + d*x]^2),x]
```

output
```
ArcTan[(Sqrt[b + c]*Tan[c + d*x])/Sqrt[a + c]]/(Sqrt[a + c]*Sqrt[b + c]*d)
```

3.162.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.162.4 Maple [A] (verified)

Time = 2.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{\arctan\left(\frac{(b+c)\tan(dx+c)}{\sqrt{(a+c)(b+c)}}\right)}{d\sqrt{(a+c)(b+c)}}$
default	$\frac{\arctan\left(\frac{(b+c)\tan(dx+c)}{\sqrt{(a+c)(b+c)}}\right)}{d\sqrt{(a+c)(b+c)}}$
risch	$-\frac{\ln\left(e^{2i(dx+c)}+\frac{2iba+2iac+2icb+2ic^2+a\sqrt{-ab-ac-cb-c^2}+b\sqrt{-ab-ac-cb-c^2}+2c\sqrt{-ab-ac-cb-c^2}}{\sqrt{-ab-ac-cb-c^2}\,(a-b)}\right)}{2\sqrt{-ab-ac-cb-c^2}\,d}+\frac{\ln\left(e^{2i(dx+c)}-\frac{2ib}{}\right.}{}$

input
```
int(sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2),x,method=_RETURNVERBOSE
)
```

output
```
1/d/((a+c)*(b+c))^(1/2)*arctan((b+c)*tan(d*x+c)/((a+c)*(b+c))^(1/2))
```

3.162.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(32) = 64.

Time = 0.30 (sec) , antiderivative size = 300, normalized size of antiderivative = 7.50

$$\int \frac{\sec^2(c+dx)}{a+c\sec^2(c+dx)+b\tan^2(c+dx)}\,dx$$
$$= \left[-\frac{\sqrt{-ab-(a+b)c-c^2}\log\left(\frac{(a^2+6ab+b^2+8(a+b)c+8c^2)\cos(dx+c)^4-2(3ab+b^2+(3a+5b)c+4c^2)\cos(dx+c)^2+4\left((a+b+2c)\cos\cdots\right.}{(a^2-2ab+b^2)\cos(dx+c)^4+2(ab-b^2+(a-b)c)\cos\cdots}\right.}{4(ab+(a+b)c+c^2)d}\right.$$
$$\left.-\frac{\arctan\left(\frac{(a+b+2c)\cos(dx+c)^2-b-c}{2\sqrt{ab+(a+b)c+c^2}\cos(dx+c)\sin(dx+c)}\right)}{2\sqrt{ab+(a+b)c+c^2}d}\right]$$

input
```
integrate(sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2),x, algorithm="fri
cas")
```

output
```
[-1/4*sqrt(-a*b - (a + b)*c - c^2)*log(((a^2 + 6*a*b + b^2 + 8*(a + b)*c +
 8*c^2)*cos(d*x + c)^4 - 2*(3*a*b + b^2 + (3*a + 5*b)*c + 4*c^2)*cos(d*x +
 c)^2 + 4*((a + b + 2*c)*cos(d*x + c)^3 - (b + c)*cos(d*x + c))*sqrt(-a*b
- (a + b)*c - c^2)*sin(d*x + c) + b^2 + 2*b*c + c^2)/((a^2 - 2*a*b + b^2)*
cos(d*x + c)^4 + 2*(a*b - b^2 + (a - b)*c)*cos(d*x + c)^2 + b^2 + 2*b*c +
c^2))/((a*b + (a + b)*c + c^2)*d), -1/2*arctan(1/2*((a + b + 2*c)*cos(d*x
+ c)^2 - b - c)/(sqrt(a*b + (a + b)*c + c^2)*cos(d*x + c)*sin(d*x + c)))/(
sqrt(a*b + (a + b)*c + c^2)*d)]
```

3.162.6 Sympy [F]

$$\int \frac{\sec^2(c+dx)}{a+c\sec^2(c+dx)+b\tan^2(c+dx)}\,dx=\int \frac{\sec^2(c+dx)}{a+b\tan^2(c+dx)+c\sec^2(c+dx)}\,dx$$

input
```
integrate(sec(d*x+c)**2/(a+c*sec(d*x+c)**2+b*tan(d*x+c)**2),x)
```

output
```
Integral(sec(c + d*x)**2/(a + b*tan(c + d*x)**2 + c*sec(c + d*x)**2), x)
```

3.162.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.08

$$\int \frac{\sec^2(c+dx)}{a+c\sec^2(c+dx)+b\tan^2(c+dx)}\,dx = \frac{\arctan\left(\frac{(b+c)\tan(dx+c)}{\sqrt{ab+(a+b)c+c^2}}\right)}{\sqrt{ab+(a+b)c+c^2}d}$$

input
```
integrate(sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2),x, algorithm="max
ima")
```

output
```
arctan((b + c)*tan(d*x + c)/sqrt(a*b + (a + b)*c + c^2))/(sqrt(a*b + (a +
b)*c + c^2)*d)
```

3.162.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(32) = 64.

Time = 0.67 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.90

$$\int \frac{\sec^2(c+dx)}{a+c\sec^2(c+dx)+b\tan^2(c+dx)}\,dx$$
$$= \frac{\pi\left\lfloor \frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor \operatorname{sgn}(2\,b+2\,c)+\arctan\left(\frac{b\tan(dx+c)+c\tan(dx+c)}{\sqrt{ab+ac+bc+c^2}}\right)}{\sqrt{ab+ac+bc+c^2}d}$$

input
```
integrate(sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2),x, algorithm="gia
c")
```

output
```
(pi*floor((d*x + c)/pi + 1/2)*sgn(2*b + 2*c) + arctan((b*tan(d*x + c) + c*
tan(d*x + c))/sqrt(a*b + a*c + b*c + c^2)))/(sqrt(a*b + a*c + b*c + c^2)*d
)
```

3.162.9 Mupad [B] (verification not implemented)

Time = 26.76 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.12

$$\int \frac{\sec^2(c+dx)}{a+c\sec^2(c+dx)+b\tan^2(c+dx)}\,dx=\frac{\operatorname{atan}\left(\frac{\tan(c+d\,x)\,(b+c)}{\sqrt{a\,b+a\,c+b\,c+c^2}}\right)}{d\,\sqrt{a\,b+a\,c+b\,c+c^2}}$$

input
```
int(1/(cos(c + d*x)^2*(a + c/cos(c + d*x)^2 + b*tan(c + d*x)^2)),x)
```

output
```
atan((tan(c + d*x)*(b + c))/(a*b + a*c + b*c + c^2)^(1/2))/(d*(a*b + a*c +
 b*c + c^2)^(1/2))
```

3.163 $\int \frac{x\sec^2(c+dx)}{a+c\sec^2(c+dx)+b\tan^2(c+dx)}\,dx$

3.163.1 Optimal result

Integrand size = 34, antiderivative size = 267

$$\int \frac{x\sec^2(c+dx)}{a+c\sec^2(c+dx)+b\tan^2(c+dx)}\,dx = -\frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{a+b+2c-2\sqrt{a+c}\sqrt{b+c}}\right)}{2\sqrt{a+c}\sqrt{b+c}d} + \frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{a+b+2(c+\sqrt{a+c}\sqrt{b+c})}\right)}{2\sqrt{a+c}\sqrt{b+c}d} - \frac{\operatorname{PolyLog}\left(2,-\frac{(a-b)e^{2i(c+dx)}}{a+b+2c-2\sqrt{a+c}\sqrt{b+c}}\right)}{4\sqrt{a+c}\sqrt{b+c}d^2} + \frac{\operatorname{PolyLog}\left(2,-\frac{(a-b)e^{2i(c+dx)}}{a+b+2(c+\sqrt{a+c}\sqrt{b+c})}\right)}{4\sqrt{a+c}\sqrt{b+c}d^2}$$

output
```
-1/2*I*x*ln(1+(a-b)*exp(2*I*(d*x+c))/(a+b+2*c-2*(a+c)^(1/2)*(b+c)^(1/2)))/
d/(a+c)^(1/2)/(b+c)^(1/2)+1/2*I*x*ln(1+(a-b)*exp(2*I*(d*x+c))/(a+b+2*c+2*(
a+c)^(1/2)*(b+c)^(1/2)))/d/(a+c)^(1/2)/(b+c)^(1/2)-1/4*polylog(2,-(a-b)*ex
p(2*I*(d*x+c))/(a+b+2*c-2*(a+c)^(1/2)*(b+c)^(1/2)))/d^2/(a+c)^(1/2)/(b+c)^
(1/2)+1/4*polylog(2,-(a-b)*exp(2*I*(d*x+c))/(a+b+2*c+2*(a+c)^(1/2)*(b+c)^(
1/2)))/d^2/(a+c)^(1/2)/(b+c)^(1/2)
```

3.163.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 751 vs. 2(267) = 534.

Time = 4.10 (sec) , antiderivative size = 751, normalized size of antiderivative = 2.81

$$\int \frac{x\sec^2(c+dx)}{a+c\sec^2(c+dx)+b\tan^2(c+dx)}\,dx$$
$$= \frac{x\left(4\sqrt{-b-c}c\arctan\left(\frac{\sqrt{b+c}\tan(c+dx)}{\sqrt{a+c}}\right)-i\sqrt{b+c}\log(1+i\tan(c+dx))\log\left(\frac{i(\sqrt{a+c}-\sqrt{-b-c}\tan(c+dx))}{\sqrt{-b-c}+i\sqrt{a+c}}\right)+i\sqrt{b}\right.}{}$$

input
```
Integrate[(x*Sec[c + d*x]^2)/(a + c*Sec[c + d*x]^2 + b*Tan[c + d*x]^2),x]
```

output
```
(x*(4*Sqrt[-b - c]*c*ArcTan[(Sqrt[b + c]*Tan[c + d*x])/Sqrt[a + c]] - I*Sq
rt[b + c]*Log[1 + I*Tan[c + d*x]]*Log[(I*(Sqrt[a + c] - Sqrt[-b - c]*Tan[c
 + d*x]))/(Sqrt[-b - c] + I*Sqrt[a + c])] + I*Sqrt[b + c]*Log[1 - I*Tan[c
+ d*x]]*Log[(I*(-Sqrt[a + c] + Sqrt[-b - c]*Tan[c + d*x]))/(Sqrt[-b - c] -
 I*Sqrt[a + c])] + I*Sqrt[b + c]*Log[1 + I*Tan[c + d*x]]*Log[((-I)*(Sqrt[a
 + c] + Sqrt[-b - c]*Tan[c + d*x]))/(Sqrt[-b - c] - I*Sqrt[a + c])] - I*Sq
rt[b + c]*Log[1 - I*Tan[c + d*x]]*Log[(I*(Sqrt[a + c] + Sqrt[-b - c]*Tan[c
 + d*x]))/(Sqrt[-b - c] + I*Sqrt[a + c])] + I*Sqrt[b + c]*PolyLog[2, (Sqrt
[-b - c]*(1 - I*Tan[c + d*x]))/(Sqrt[-b - c] - I*Sqrt[a + c])] - I*Sqrt[b
+ c]*PolyLog[2, (Sqrt[-b - c]*(1 - I*Tan[c + d*x]))/(Sqrt[-b - c] + I*Sqrt
[a + c])] + I*Sqrt[b + c]*PolyLog[2, (Sqrt[-b - c]*(1 + I*Tan[c + d*x]))/(
Sqrt[-b - c] - I*Sqrt[a + c])] - I*Sqrt[b + c]*PolyLog[2, (Sqrt[-b - c]*(1
 + I*Tan[c + d*x]))/(Sqrt[-b - c] + I*Sqrt[a + c])])*(Sqrt[a + c] - Sqrt[-
b - c]*Tan[c + d*x])*(Sqrt[a + c] + Sqrt[-b - c]*Tan[c + d*x]))/(2*Sqrt[a
+ c]*Sqrt[-(b + c)^2]*d*(2*c - I*Log[1 - I*Tan[c + d*x]] + I*Log[1 + I*Tan
[c + d*x]])*(a + c*Sec[c + d*x]^2 + b*Tan[c + d*x]^2))
```

3.163.3 Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {5100, 3042, 3802, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \sec^2(c+dx)}{a+b\tan^2(c+dx)+c\sec^2(c+dx)}dx$$

$$\Big\downarrow \text{5100}$$

$$2\int \frac{x}{a+b+2c+(a-b)\cos(2c+2dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$2\int \frac{x}{a+b+2c+(a-b)\sin\left(2c+2dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3802}$$

$$4\int \frac{e^{2i(c+dx)}x}{a+2(a+b+2c)e^{2i(c+dx)}+(a-b)e^{4i(c+dx)}-b}dx$$

$$\Big\downarrow \text{2694}$$

$$4\left(\frac{(a-b)\int \frac{e^{2i(c+dx)}x}{2\left(a+(a-b)e^{2i(c+dx)}+b+2c-2\sqrt{a+c}\sqrt{b+c}\right)}dx}{2\sqrt{a+c}\sqrt{b+c}}-\frac{(a-b)\int \frac{e^{2i(c+dx)}x}{2\left(a+(a-b)e^{2i(c+dx)}+b+2\left(c+\sqrt{a+c}\sqrt{b+c}\right)\right)}dx}{2\sqrt{a+c}\sqrt{b+c}}\right)$$

$$\Big\downarrow \text{27}$$

$$4\left(\frac{(a-b)\int \frac{e^{2i(c+dx)}x}{a+(a-b)e^{2i(c+dx)}+b+2c-2\sqrt{a+c}\sqrt{b+c}}dx}{4\sqrt{a+c}\sqrt{b+c}}-\frac{(a-b)\int \frac{e^{2i(c+dx)}x}{a+(a-b)e^{2i(c+dx)}+b+2\left(c+\sqrt{a+c}\sqrt{b+c}\right)}dx}{4\sqrt{a+c}\sqrt{b+c}}\right)$$

$$\Big\downarrow \text{2620}$$

$$4\left(\frac{(a-b)\left(\frac{i\int \log\left(\frac{e^{2i(c+dx)}(a-b)}{a+b+2c-2\sqrt{a+c}\sqrt{b+c}}+1\right)dx}{2d(a-b)}-\frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{-2\sqrt{a+c}\sqrt{b+c}+a+b+2c}\right)}{2d(a-b)}\right)}{4\sqrt{a+c}\sqrt{b+c}}-\frac{(a-b)\left(\frac{i\int \log\left(\frac{e^{2i(c+dx)}(a-b)}{a+b+2\left(c+\sqrt{a+c}\sqrt{b+c}\right)}+1\right)d}{2d(a-b)}\right.}{4\sqrt{a+c}\sqrt{}}\right.$$

$$\Big\downarrow \text{2715}$$

$$4\left(\frac{(a-b)\left(\frac{\int e^{-2i(c+dx)}\log\left(\frac{e^{2i(c+dx)}(a-b)}{a+b+2c-2\sqrt{a+c}\sqrt{b+c}}+1\right)de^{2i(c+dx)}}{4d^2(a-b)}-\frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{-2\sqrt{a+c}\sqrt{b+c}+a+b+2c}\right)}{2d(a-b)}\right)}{4\sqrt{a+c}\sqrt{b+c}}-\frac{(a-b)\left(\int e^{-2i(c+dx)}\log\left(\right.\right.}{}\right.$$

$$\Big\downarrow \text{2838}$$

3.163. $\int \frac{x\sec^2(c+dx)}{a+c\sec^2(c+dx)+b\tan^2(c+dx)}dx$

$$4\left(\frac{(a-b)\left(-\frac{\text{PolyLog}\left(2,-\frac{(a-b)e^{2i(c+dx)}}{a+b+2c-2\sqrt{a+c}\sqrt{b+c}}\right)}{4d^2(a-b)}-\frac{ix\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{-2\sqrt{a+c}\sqrt{b+c}+a+b+2c}\right)}{2d(a-b)}\right)}{4\sqrt{a+c}\sqrt{b+c}}-\frac{(a-b)\left(-\frac{\text{PolyLog}\left(2,-\frac{(a-b)e^{2i(c+dx)}}{a+b+2(c+\sqrt{a+c}\sqrt{b+c})}\right)}{4d^2(a-b)}\right.}{4\sqrt{a+}}\right.$$

input
```
Int[(x*Sec[c + d*x]^2)/(a + c*Sec[c + d*x]^2 + b*Tan[c + d*x]^2),x]
```

output
```
4*(((a - b)*(((-1/2*I)*x*Log[1 + ((a - b)*E^((2*I)*(c + d*x)))/(a + b + 2*
c - 2*Sqrt[a + c]*Sqrt[b + c])])/((a - b)*d) - PolyLog[2, -(((a - b)*E^((2
*I)*(c + d*x)))/(a + b + 2*c - 2*Sqrt[a + c]*Sqrt[b + c]))]/(4*(a - b)*d^2
)))/(4*Sqrt[a + c]*Sqrt[b + c]) - ((a - b)*(((-1/2*I)*x*Log[1 + ((a - b)*E
^((2*I)*(c + d*x)))/(a + b + 2*(c + Sqrt[a + c]*Sqrt[b + c]))])/((a - b)*d
) - PolyLog[2, -(((a - b)*E^((2*I)*(c + d*x)))/(a + b + 2*(c + Sqrt[a + c]
*Sqrt[b + c])))]/(4*(a - b)*d^2)))/(4*Sqrt[a + c]*Sqrt[b + c]))
```

3.163.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

3.163. $\int \frac{x\sec^2(c+dx)}{a+c\sec^2(c+dx)+b\tan^2(c+dx)}\,dx$

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3802
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*E^(I*Pi*(k - 1/2))*(E^(I*(e +
f*x))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(
e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ
[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 5100
```
Int[(((f_.) + (g_.)*(x_))^(m_.)*Sec[(d_.) + (e_.)*(x_)]^2)/((b_.) + (a_.)*S
ec[(d_.) + (e_.)*(x_)]^2 + (c_.)*Tan[(d_.) + (e_.)*(x_)]^2), x_Symbol] :> S
imp[2   Int[(f + g*x)^m/(2*a + b + c + (b - c)*Cos[2*d + 2*e*x]), x], x] /;
 FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[m, 0] && NeQ[a + b, 0] && NeQ[a +
c, 0]
```

3.163.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1669 vs. $2(217) = 434$.

Time = 2.66 (sec) , antiderivative size = 1670, normalized size of antiderivative = 6.25

method	result	size
risch	Expression too large to display	1670

input
```
int(x*sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2),x,method=_RETURNVERBO
SE)
```

output
```
-1/2/((a+c)*(b+c))^(1/2)*x^2-1/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*x^2-1/d^2/
(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)/((a+c)*(b+c))^(1/2)*c^3-2/d/(-2*((a+c)*(b
+c))^(1/2)-a-b-2*c)*c*x-1/d/((a+c)*(b+c))^(1/2)*c*x-1/2/(-2*((a+c)*(b+c))^
(1/2)-a-b-2*c)/((a+c)*(b+c))^(1/2)*a*x^2-1/2/(-2*((a+c)*(b+c))^(1/2)-a-b-2
*c)/((a+c)*(b+c))^(1/2)*x^2*b-1/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)/((a+c)*(b
+c))^(1/2)*c*x^2-1/d/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)/((a+c)*(b+c))^(1/2)*
a*c*x-1/d/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)/((a+c)*(b+c))^(1/2)*c*x*b-1/2*I
/d/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)/((a+c)*(b+c))^(1/2)*ln(1-(a-b)*exp(2*I
*(d*x+c))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))*a*x-1/2*I/d/(-2*((a+c)*(b+c))^
(1/2)-a-b-2*c)/((a+c)*(b+c))^(1/2)*ln(1-(a-b)*exp(2*I*(d*x+c))/(-2*((a+c)*
(b+c))^(1/2)-a-b-2*c))*b*x-I/d/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)/((a+c)*(b+
c))^(1/2)*ln(1-(a-b)*exp(2*I*(d*x+c))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))*c*
x-1/2*I/d^2/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)/((a+c)*(b+c))^(1/2)*ln(1-(a-b
)*exp(2*I*(d*x+c))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))*a*c-1/2*I/d^2/(-2*((a
+c)*(b+c))^(1/2)-a-b-2*c)/((a+c)*(b+c))^(1/2)*ln(1-(a-b)*exp(2*I*(d*x+c))/
(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))*b*c-1/2/d^2/((a+c)*(b+c))^(1/2)*c^2-1/4/
d^2/((a+c)*(b+c))^(1/2)*polylog(2,(a-b)*exp(2*I*(d*x+c))/(2*((a+c)*(b+c))^
(1/2)-a-b-2*c))-1/d^2/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*c^2-1/2/d^2/(-2*((a
+c)*(b+c))^(1/2)-a-b-2*c)*polylog(2,(a-b)*exp(2*I*(d*x+c))/(-2*((a+c)*(b+c
))^(1/2)-a-b-2*c))-2/d/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)/((a+c)*(b+c))^(...
```

3.163.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4100 vs. $2(213) = 426$.

Time = 4.60 (sec) , antiderivative size = 4100, normalized size of antiderivative = 15.36

$$\int \frac{x\sec^2(c+dx)}{a+c\sec^2(c+dx)+b\tan^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(x*sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2),x, algorithm="f
ricas")
```

output
```
-1/4*(I*(a - b)*c*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(2*
sqrt(-(2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) + a + b
 + 2*c)/(a - b)) + 2*cos(d*x + c) + 2*I*sin(d*x + c)) - I*(a - b)*c*sqrt((
a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(2*sqrt(-(2*(a - b)*sqrt((a
*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) + a + b + 2*c)/(a - b)) + 2*cos
(d*x + c) - 2*I*sin(d*x + c)) - I*(a - b)*c*sqrt((a*b + (a + b)*c + c^2)/(
a^2 - 2*a*b + b^2))*log(2*sqrt(-(2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a
^2 - 2*a*b + b^2)) + a + b + 2*c)/(a - b)) - 2*cos(d*x + c) + 2*I*sin(d*x
+ c)) + I*(a - b)*c*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(
2*sqrt(-(2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) + a +
 b + 2*c)/(a - b)) - 2*cos(d*x + c) - 2*I*sin(d*x + c)) - I*(a - b)*c*sqrt
((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(2*sqrt((2*(a - b)*sqrt((
a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) - a - b - 2*c)/(a - b)) + 2*co
s(d*x + c) + 2*I*sin(d*x + c)) + I*(a - b)*c*sqrt((a*b + (a + b)*c + c^2)/
(a^2 - 2*a*b + b^2))*log(2*sqrt((2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a
^2 - 2*a*b + b^2)) - a - b - 2*c)/(a - b)) + 2*cos(d*x + c) - 2*I*sin(d*x
+ c)) + I*(a - b)*c*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(
2*sqrt((2*(a - b)*sqrt((a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2)) - a -
b - 2*c)/(a - b)) - 2*cos(d*x + c) + 2*I*sin(d*x + c)) - I*(a - b)*c*sqrt(
(a*b + (a + b)*c + c^2)/(a^2 - 2*a*b + b^2))*log(2*sqrt((2*(a - b)*sqrt...
```

3.163.6 Sympy [F]

$$\int \frac{x\sec^2(c+dx)}{a+c\sec^2(c+dx)+b\tan^2(c+dx)}\,dx = \int \frac{x\sec^2{(c+dx)}}{a+b\tan^2{(c+dx)}+c\sec^2{(c+dx)}}\,dx$$

input
```
integrate(x*sec(d*x+c)**2/(a+c*sec(d*x+c)**2+b*tan(d*x+c)**2),x)
```

output
```
Integral(x*sec(c + d*x)**2/(a + b*tan(c + d*x)**2 + c*sec(c + d*x)**2), x)
```

3.163.7 Maxima [F]

$$\int \frac{x\sec^2(c+dx)}{a+c\sec^2(c+dx)+b\tan^2(c+dx)}\,dx = \int \frac{x\sec{(dx+c)}^2}{c\sec{(dx+c)}^2+b\tan{(dx+c)}^2+a}\,dx$$

input
```
integrate(x*sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2),x, algorithm="m
axima")
```

output
```
integrate(x*sec(d*x + c)^2/(c*sec(d*x + c)^2 + b*tan(d*x + c)^2 + a), x)
```

3.163.8 Giac [F]

$$\int \frac{x\sec^2(c+dx)}{a+c\sec^2(c+dx)+b\tan^2(c+dx)}\,dx = \int \frac{x\sec{(dx+c)}^2}{c\sec{(dx+c)}^2+b\tan{(dx+c)}^2+a}\,dx$$

input
```
integrate(x*sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2),x, algorithm="g
iac")
```

output
```
integrate(x*sec(d*x + c)^2/(c*sec(d*x + c)^2 + b*tan(d*x + c)^2 + a), x)
```

3.163.9 Mupad [F(-1)]

Timed out.

$$\begin{aligned}&\int \frac{x\sec^2(c+dx)}{a+c\sec^2(c+dx)+b\tan^2(c+dx)}\,dx\\&=\int \frac{x}{\cos{(c+d\,x)}^2\,\left(a+\frac{c}{\cos{(c+d\,x)}^2}+b\tan{(c+d\,x)}^2\right)}\,dx\end{aligned}$$

input
```
int(x/(cos(c + d*x)^2*(a + c/cos(c + d*x)^2 + b*tan(c + d*x)^2)),x)
```

output
```
int(x/(cos(c + d*x)^2*(a + c/cos(c + d*x)^2 + b*tan(c + d*x)^2)), x)
```

3.164 $\int \frac{x^2 \sec^2(c+dx)}{a+c\sec^2(c+dx)+b\tan^2(c+dx)}\,dx$

3.164.1 Optimal result

Integrand size = 36, antiderivative size = 407

$$\begin{aligned}\int \frac{x^2 \sec^2(c+dx)}{a+c\sec^2(c+dx)+b\tan^2(c+dx)}\,dx = &-\frac{ix^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{a+b+2c-2\sqrt{a+c}\sqrt{b+c}}\right)}{2\sqrt{a+c}\sqrt{b+c}d} \\ &+\frac{ix^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{a+b+2\left(c+\sqrt{a+c}\sqrt{b+c}\right)}\right)}{2\sqrt{a+c}\sqrt{b+c}d} \\ &-\frac{x\,\text{PolyLog}\left(2,-\frac{(a-b)e^{2i(c+dx)}}{a+b+2c-2\sqrt{a+c}\sqrt{b+c}}\right)}{2\sqrt{a+c}\sqrt{b+c}d^2} \\ &+\frac{x\,\text{PolyLog}\left(2,-\frac{(a-b)e^{2i(c+dx)}}{a+b+2\left(c+\sqrt{a+c}\sqrt{b+c}\right)}\right)}{2\sqrt{a+c}\sqrt{b+c}d^2} \\ &-\frac{i\,\text{PolyLog}\left(3,-\frac{(a-b)e^{2i(c+dx)}}{a+b+2c-2\sqrt{a+c}\sqrt{b+c}}\right)}{4\sqrt{a+c}\sqrt{b+c}d^3} \\ &+\frac{i\,\text{PolyLog}\left(3,-\frac{(a-b)e^{2i(c+dx)}}{a+b+2\left(c+\sqrt{a+c}\sqrt{b+c}\right)}\right)}{4\sqrt{a+c}\sqrt{b+c}d^3}\end{aligned}$$

output
```
-1/2*I*x^2*ln(1+(a-b)*exp(2*I*(d*x+c))/(a+b+2*c-2*(a+c)^(1/2)*(b+c)^(1/2))
)/d/(a+c)^(1/2)/(b+c)^(1/2)+1/2*I*x^2*ln(1+(a-b)*exp(2*I*(d*x+c))/(a+b+2*c
+2*(a+c)^(1/2)*(b+c)^(1/2)))/d/(a+c)^(1/2)/(b+c)^(1/2)-1/2*x*polylog(2,-(a
-b)*exp(2*I*(d*x+c))/(a+b+2*c-2*(a+c)^(1/2)*(b+c)^(1/2)))/d^2/(a+c)^(1/2)/
(b+c)^(1/2)+1/2*x*polylog(2,-(a-b)*exp(2*I*(d*x+c))/(a+b+2*c+2*(a+c)^(1/2)
*(b+c)^(1/2)))/d^2/(a+c)^(1/2)/(b+c)^(1/2)-1/4*I*polylog(3,-(a-b)*exp(2*I*
(d*x+c))/(a+b+2*c-2*(a+c)^(1/2)*(b+c)^(1/2)))/d^3/(a+c)^(1/2)/(b+c)^(1/2)+
1/4*I*polylog(3,-(a-b)*exp(2*I*(d*x+c))/(a+b+2*c+2*(a+c)^(1/2)*(b+c)^(1/2)
))/d^3/(a+c)^(1/2)/(b+c)^(1/2)
```

3.164.2 Mathematica [A] (verified)

Time = 1.72 (sec) , antiderivative size = 314, normalized size of antiderivative = 0.77

$$\int \frac{x^2\sec^2(c+dx)}{a+c\sec^2(c+dx)+b\tan^2(c+dx)}\,dx =$$

$$-\frac{i\left(2d^2x^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{a+b+2c-2\sqrt{a+c}\sqrt{b+c}}\right)-2d^2x^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{a+b+2c+2\sqrt{a+c}\sqrt{b+c}}\right)-2idx\,\text{PolyLog}\left(2,\frac{(-a+b)e^{2}}{a+b+2c-2\sqrt{}}\right.\right.}{4}$$

input
```
Integrate[(x^2*Sec[c + d*x]^2)/(a + c*Sec[c + d*x]^2 + b*Tan[c + d*x]^2),x
]
```

output
```
((-1/4*I)*(2*d^2*x^2*Log[1 + ((a - b)*E^((2*I)*(c + d*x)))/(a + b + 2*c -
2*Sqrt[a + c]*Sqrt[b + c])] - 2*d^2*x^2*Log[1 + ((a - b)*E^((2*I)*(c + d*x
)))/(a + b + 2*c + 2*Sqrt[a + c]*Sqrt[b + c])] - (2*I)*d*x*PolyLog[2, ((-a
 + b)*E^((2*I)*(c + d*x)))/(a + b + 2*c - 2*Sqrt[a + c]*Sqrt[b + c])] + (2
*I)*d*x*PolyLog[2, ((-a + b)*E^((2*I)*(c + d*x)))/(a + b + 2*c + 2*Sqrt[a
+ c]*Sqrt[b + c])] + PolyLog[3, ((-a + b)*E^((2*I)*(c + d*x)))/(a + b + 2*
c - 2*Sqrt[a + c]*Sqrt[b + c])] - PolyLog[3, ((-a + b)*E^((2*I)*(c + d*x))
)/(a + b + 2*c + 2*Sqrt[a + c]*Sqrt[b + c])]))/(Sqrt[a + c]*Sqrt[b + c]*d^
3)
```

3.164.3 Rubi [A] (verified)

Time = 1.57 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5100, 3042, 3802, 2694, 27, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \sec^2(c+dx)}{a+b\tan^2(c+dx)+c\sec^2(c+dx)}dx$$

$$\Big\downarrow \text{5100}$$

$$2\int \frac{x^2}{a+b+2c+(a-b)\cos(2c+2dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$2\int \frac{x^2}{a+b+2c+(a-b)\sin\left(2c+2dx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3802}$$

$$4\int \frac{e^{2i(c+dx)}x^2}{a+2(a+b+2c)e^{2i(c+dx)}+(a-b)e^{4i(c+dx)}-b}dx$$

$$\Big\downarrow \text{2694}$$

$$4\left(\frac{(a-b)\int \frac{e^{2i(c+dx)}x^2}{2\left(a+(a-b)e^{2i(c+dx)}+b+2c-2\sqrt{a+c}\sqrt{b+c}\right)}dx}{2\sqrt{a+c}\sqrt{b+c}}-\frac{(a-b)\int \frac{e^{2i(c+dx)}x^2}{2\left(a+(a-b)e^{2i(c+dx)}+b+2\left(c+\sqrt{a+c}\sqrt{b+c}\right)\right)}dx}{2\sqrt{a+c}\sqrt{b+c}}\right)$$

$$\Big\downarrow \text{27}$$

$$4\left(\frac{(a-b)\int \frac{e^{2i(c+dx)}x^2}{a+(a-b)e^{2i(c+dx)}+b+2c-2\sqrt{a+c}\sqrt{b+c}}dx}{4\sqrt{a+c}\sqrt{b+c}}-\frac{(a-b)\int \frac{e^{2i(c+dx)}x^2}{a+(a-b)e^{2i(c+dx)}+b+2\left(c+\sqrt{a+c}\sqrt{b+c}\right)}dx}{4\sqrt{a+c}\sqrt{b+c}}\right)$$

$$\Big\downarrow \text{2620}$$

$$4\left(\frac{(a-b)\left(\frac{i\int x\log\left(\frac{e^{2i(c+dx)}(a-b)}{a+b+2c-2\sqrt{a+c}\sqrt{b+c}}+1\right)dx}{d(a-b)}-\frac{ix^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{-2\sqrt{a+c}\sqrt{b+c}+a+b+2c}\right)}{2d(a-b)}\right)}{4\sqrt{a+c}\sqrt{b+c}}-\frac{(a-b)\left(\frac{i\int x\log\left(\frac{e^{2i(c+dx)}(a-b)}{a+b+2\left(c+\sqrt{a+c}\sqrt{b+c}\right)}+\right.}{d(a-b)}\right.}{4\sqrt{a+}}\right.$$

$$\Big\downarrow \text{3011}$$

$$4\left(\frac{(a-b)\left(\frac{i\left(\frac{ix\,\text{PolyLog}\left(2,-\frac{(a-b)e^{2i(c+dx)}}{a+b+2c-2\sqrt{a+c}\sqrt{b+c}}\right)}{2d}-\frac{i\int\text{PolyLog}\left(2,-\frac{(a-b)e^{2i(c+dx)}}{a+b+2c-2\sqrt{a+c}\sqrt{b+c}}\right)dx}{2d}\right)}{d(a-b)}-\frac{ix^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{-2\sqrt{a+c}\sqrt{b+c}+a+b+2c}\right)}{2d(a-b)}\right)}{4\sqrt{a+c}\sqrt{b+c}}-\right.$$

$\downarrow$ 2720

$$4\left(\frac{(a-b)\left(\frac{i\left(\frac{ix\,\text{PolyLog}\left(2,-\frac{(a-b)e^{2i(c+dx)}}{a+b+2c-2\sqrt{a+c}\sqrt{b+c}}\right)}{2d}-\frac{\int e^{-2i(c+dx)}\text{PolyLog}\left(2,-\frac{(a-b)e^{2i(c+dx)}}{a+b+2c-2\sqrt{a+c}\sqrt{b+c}}\right)de^{2i(c+dx)}}{4d^2}\right)}{d(a-b)}-\frac{ix^2\log\left(1+\frac{(a-b)e^{2i}}{-2\sqrt{a+c}\sqrt{b+}}\right.}{2d(a-b)}\right.}{4\sqrt{a+c}\sqrt{b+c}}\right.$$

$\downarrow$ 7143

$$4\left(\frac{(a-b)\left(\frac{i\left(\frac{ix\,\text{PolyLog}\left(2,-\frac{(a-b)e^{2i(c+dx)}}{a+b+2c-2\sqrt{a+c}\sqrt{b+c}}\right)}{2d}-\frac{\text{PolyLog}\left(3,-\frac{(a-b)e^{2i(c+dx)}}{a+b+2c-2\sqrt{a+c}\sqrt{b+c}}\right)}{4d^2}\right)}{d(a-b)}-\frac{ix^2\log\left(1+\frac{(a-b)e^{2i(c+dx)}}{-2\sqrt{a+c}\sqrt{b+c}+a+b+2c}\right)}{2d(a-b)}\right)}{4\sqrt{a+c}\sqrt{b+c}}-\frac{(a}{}\right.$$

input
```
Int[(x^2*Sec[c + d*x]^2)/(a + c*Sec[c + d*x]^2 + b*Tan[c + d*x]^2),x]
```

3.164. $\int \frac{x^2\sec^2(c+dx)}{a+c\sec^2(c+dx)+b\tan^2(c+dx)}\,dx$

output
```
4*(((a - b)*(((-1/2*I)*x^2*Log[1 + ((a - b)*E^((2*I)*(c + d*x)))/(a + b +
2*c - 2*Sqrt[a + c]*Sqrt[b + c])])/((a - b)*d) + (I*(((I/2)*x*PolyLog[2, -
(((a - b)*E^((2*I)*(c + d*x)))/(a + b + 2*c - 2*Sqrt[a + c]*Sqrt[b + c]))]
)/d - PolyLog[3, -(((a - b)*E^((2*I)*(c + d*x)))/(a + b + 2*c - 2*Sqrt[a +
 c]*Sqrt[b + c]))]/(4*d^2)))/((a - b)*d)))/(4*Sqrt[a + c]*Sqrt[b + c]) - (
(a - b)*(((-1/2*I)*x^2*Log[1 + ((a - b)*E^((2*I)*(c + d*x)))/(a + b + 2*(c
 + Sqrt[a + c]*Sqrt[b + c]))])/((a - b)*d) + (I*(((I/2)*x*PolyLog[2, -(((a
 - b)*E^((2*I)*(c + d*x)))/(a + b + 2*(c + Sqrt[a + c]*Sqrt[b + c])))])/d
- PolyLog[3, -(((a - b)*E^((2*I)*(c + d*x)))/(a + b + 2*(c + Sqrt[a + c]*S
qrt[b + c])))]/(4*d^2)))/((a - b)*d)))/(4*Sqrt[a + c]*Sqrt[b + c]))
```

3.164.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.164. $\int \frac{x^2 \sec^2(c+dx)}{a+c\sec^2(c+dx)+b\tan^2(c+dx)}\,dx$

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3802
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*E^(I*Pi*(k - 1/2))*(E^(I*(e +
f*x))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(
e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ
[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 5100
```
Int[(((f_.) + (g_.)*(x_))^(m_.)*Sec[(d_.) + (e_.)*(x_)]^2)/((b_.) + (a_.)*S
ec[(d_.) + (e_.)*(x_)]^2 + (c_.)*Tan[(d_.) + (e_.)*(x_)]^2), x_Symbol] :> S
imp[2   Int[(f + g*x)^m/(2*a + b + c + (b - c)*Cos[2*d + 2*e*x]), x], x] /;
 FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[m, 0] && NeQ[a + b, 0] && NeQ[a +
c, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.164.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2060 vs. 2(331) = 662.

Time = 2.41 (sec) , antiderivative size = 2061, normalized size of antiderivative = 5.06

method	result	size
risch	Expression too large to display	2061

input
```
int(x^2*sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2),x,method=_RETURNVER
BOSE)
```

output
```
I/d^3/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)/((a+c)*(b+c))^(1/2)*ln(1-(a-b)*exp(
2*I*(d*x+c))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))*c^3-1/3/((a+c)*(b+c))^(1/2)
*x^3-2/3/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*x^3+2/d^2/(-2*((a+c)*(b+c))^(1/2
)-a-b-2*c)/((a+c)*(b+c))^(1/2)*c^3*x+I/d^3/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c
)*ln(1-(a-b)*exp(2*I*(d*x+c))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))*c^2+I/d^3*
c^2/(a*b+a*c+b*c+c^2)^(1/2)*arctanh(1/4*(2*(a-b)*exp(2*I*(d*x+c))+2*a+2*b+
4*c)/(a*b+a*c+b*c+c^2)^(1/2))+2/3/d^3/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)/((a
+c)*(b+c))^(1/2)*a*c^3+2/3/d^3/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)/((a+c)*(b+
c))^(1/2)*b*c^3+1/2*I/d^3/((a+c)*(b+c))^(1/2)*ln(1-(a-b)*exp(2*I*(d*x+c))/
(2*((a+c)*(b+c))^(1/2)-a-b-2*c))*c^2-I/d/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*
ln(1-(a-b)*exp(2*I*(d*x+c))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))*x^2-1/2*I/d/
((a+c)*(b+c))^(1/2)*ln(1-(a-b)*exp(2*I*(d*x+c))/(2*((a+c)*(b+c))^(1/2)-a-b
-2*c))*x^2-1/3/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)/((a+c)*(b+c))^(1/2)*a*x^3+
4/3/d^3/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)/((a+c)*(b+c))^(1/2)*c^4+2/d^2/(-2
*((a+c)*(b+c))^(1/2)-a-b-2*c)*c^2*x-1/d^2/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)
*polylog(2,(a-b)*exp(2*I*(d*x+c))/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c))*x+1/d^
2/((a+c)*(b+c))^(1/2)*c^2*x-1/2/d^2/((a+c)*(b+c))^(1/2)*polylog(2,(a-b)*ex
p(2*I*(d*x+c))/(2*((a+c)*(b+c))^(1/2)-a-b-2*c))*x-1/3/(-2*((a+c)*(b+c))^(1
/2)-a-b-2*c)/((a+c)*(b+c))^(1/2)*b*x^3-2/3/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c
)/((a+c)*(b+c))^(1/2)*c*x^3-1/2*I/d^3/(-2*((a+c)*(b+c))^(1/2)-a-b-2*c)*...
```

3.164.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 5696 vs. $2(325) = 650$.

Time = 3.42 (sec) , antiderivative size = 5696, normalized size of antiderivative = 14.00

$$\int \frac{x^2 \sec^2(c+dx)}{a+c\sec^2(c+dx)+b\tan^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2),x, algorithm=
"fricas")
```

output
```
Too large to include
```

3.164.6 Sympy [F]

$$\int \frac{x^2 \sec^2(c+dx)}{a+c\sec^2(c+dx)+b\tan^2(c+dx)}\,dx = \int \frac{x^2 \sec^2{(c+dx)}}{a+b\tan^2{(c+dx)}+c\sec^2{(c+dx)}}\,dx$$

input
```
integrate(x**2*sec(d*x+c)**2/(a+c*sec(d*x+c)**2+b*tan(d*x+c)**2),x)
```

output
```
Integral(x**2*sec(c + d*x)**2/(a + b*tan(c + d*x)**2 + c*sec(c + d*x)**2),
 x)
```

3.164.7 Maxima [F]

$$\int \frac{x^2 \sec^2(c+dx)}{a+c\sec^2(c+dx)+b\tan^2(c+dx)}\,dx = \int \frac{x^2 \sec{(dx+c)}^2}{c\sec{(dx+c)}^2+b\tan{(dx+c)}^2+a}\,dx$$

input
```
integrate(x^2*sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2),x, algorithm=
"maxima")
```

output
```
integrate(x^2*sec(d*x + c)^2/(c*sec(d*x + c)^2 + b*tan(d*x + c)^2 + a), x)
```

3.164.8 Giac [F]

$$\int \frac{x^2 \sec^2(c+dx)}{a+c\sec^2(c+dx)+b\tan^2(c+dx)}\,dx = \int \frac{x^2 \sec{(dx+c)}^2}{c\sec{(dx+c)}^2+b\tan{(dx+c)}^2+a}\,dx$$

input
```
integrate(x^2*sec(d*x+c)^2/(a+c*sec(d*x+c)^2+b*tan(d*x+c)^2),x, algorithm=
"giac")
```

output
```
integrate(x^2*sec(d*x + c)^2/(c*sec(d*x + c)^2 + b*tan(d*x + c)^2 + a), x)
```

3.164.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \sec^2(c+dx)}{a+c\sec^2(c+dx)+b\tan^2(c+dx)}\,dx = \int \frac{x^2}{\cos(c+d\,x)^2\,\left(a+\frac{c}{\cos(c+d\,x)^2}+b\tan(c+d\,x)^2\right)}\,dx$$

input
```
int(x^2/(cos(c + d*x)^2*(a + c/cos(c + d*x)^2 + b*tan(c + d*x)^2)),x)
```

output
```
int(x^2/(cos(c + d*x)^2*(a + c/cos(c + d*x)^2 + b*tan(c + d*x)^2)), x)
```

3.165 $\int x^3 \sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\,dx$

3.165.1 Optimal result

Integrand size = 33, antiderivative size = 155

$$\begin{aligned}\int x^3\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\,dx &= -\frac{6\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{f^4}+\frac{3x^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{f^2}\\ &\quad-\frac{6x\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\tan(e+fx)}{f^3}\\ &\quad+\frac{x^3\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\tan(e+fx)}{f}\end{aligned}$$

output
```
-6*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/f^4+3*x^2*(a-a*sin(f*x+e)
)^(1/2)*(c+c*sin(f*x+e))^(1/2)/f^2-6*x*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x
+e))^(1/2)*tan(f*x+e)/f^3+x^3*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2
)*tan(f*x+e)/f
```

3.165.2 Mathematica [A] (verified)

Time = 3.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.39

$$\int x^3\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\,dx = \frac{\sqrt{c(1+\sin(e+fx))}\sqrt{a-a\sin(e+fx)}(-6+3f^2x^2+fx(-6+f^2x^2)\tan(e+fx))}{f^4}$$

input
```
Integrate[x^3*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]],x]
```

output
```
(Sqrt[c*(1 + Sin[e + f*x])]*Sqrt[a - a*Sin[e + f*x]]*(-6 + 3*f^2*x^2 + f*x
*(-6 + f^2*x^2)*Tan[e + f*x]))/f^4
```

3.165.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.63, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5115, 3042, 3777, 25, 3042, 3777, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\,dx$$

$$\Big\downarrow \text{5115}$$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\int x^3\cos(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\int x^3\sin\left(e+fx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3777}$$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\left(\frac{3\int -x^2\sin(e+fx)dx}{f}+\frac{x^3\sin(e+fx)}{f}\right)$$

$$\Big\downarrow \text{25}$$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\left(\frac{x^3\sin(e+fx)}{f}-\frac{3\int x^2\sin(e+fx)dx}{f}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\left(\frac{x^3\sin(e+fx)}{f}-\frac{3\int x^2\sin(e+fx)dx}{f}\right)$$

$$\Big\downarrow \text{3777}$$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\left(\frac{x^3\sin(e+fx)}{f}-\frac{3\left(\frac{2\int x\cos(e+fx)dx}{f}-\frac{x^2\cos(e+fx)}{f}\right)}{f}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\left(\frac{x^3\sin(e+fx)}{f}-\frac{3\left(\frac{2\int x\sin\left(e+fx+\frac{\pi}{2}\right)dx}{f}-\frac{x^2\cos(e+fx)}{f}\right)}{f}\right)$$

$$\Big\downarrow \text{3777}$$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\left(\frac{x^3\sin(e+fx)}{f}-\frac{3\left(\frac{2\left(\frac{\int -\sin(e+fx)dx}{f}+\frac{x\sin(e+fx)}{f}\right)}{f}-\frac{x^2\cos(e+fx)}{f}\right)}{f}\right)$$

$$\Big\downarrow \text{25}$$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\left(\frac{x^3\sin(e+fx)}{f}-\frac{3\left(\frac{2\left(\frac{x\sin(e+fx)}{f}-\frac{\int \sin(e+fx)dx}{f}\right)}{f}-\frac{x^2\cos(e+fx)}{f}\right)}{f}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\left(\frac{x^3\sin(e+fx)}{f}-\frac{3\left(\frac{2\left(\frac{x\sin(e+fx)}{f}-\frac{\int \sin(e+fx)dx}{f}\right)}{f}-\frac{x^2\cos(e+fx)}{f}\right)}{f}\right)$$

$$\Big\downarrow \text{3118}$$

3.165. $\int x^3\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\,dx$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\left(\frac{x^3\sin(e+fx)}{f}-\frac{3\left(\frac{2\left(\frac{\cos(e+fx)}{f^2}+\frac{x\sin(e+fx)}{f}\right)}{f}-\frac{x^2\cos(e+fx)}{f}\right)}{f}\right)$$

input
```
Int[x^3*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]],x]
```

output
```
Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*((x^3*Sin[e
 + f*x])/f - (3*(-((x^2*Cos[e + f*x])/f) + (2*(Cos[e + f*x]/f^2 + (x*Sin[e
 + f*x])/f))/f))/f)
```

3.165.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5115
```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPart[m]
*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPa
rt[m]/Cos[e + f*x]^(2*FracPart[m]))   Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
 + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] &&
IGeQ[n - m, 0]
```

3.165.4 Maple [F]

$$\int x^{3} \sqrt{a-\sin \left(f x +e \right) a}\, \sqrt{c +c \sin \left(f x +e \right)}d x$$

input
```
int(x^3*(a-sin(f*x+e)*a)^(1/2)*(c+c*sin(f*x+e))^(1/2),x)
```

output
```
int(x^3*(a-sin(f*x+e)*a)^(1/2)*(c+c*sin(f*x+e))^(1/2),x)
```

3.165.5 Fricas [F(-2)]

Exception generated.

$$\int x^3 \sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x, algorithm="
fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.165.6 Sympy [F]

$$\int x^3 \sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\,dx = \int x^{3} \sqrt{c \left(\sin{\left(e + f x \right)} + 1\right)} \sqrt{- a \left(\sin{\left(e + f x \right)} - 1\right)}\, dx$$

input
```
integrate(x**3*(a-a*sin(f*x+e))**(1/2)*(c+c*sin(f*x+e))**(1/2),x)
```

output
```
Integral(x**3*sqrt(c*(sin(e + f*x) + 1))*sqrt(-a*(sin(e + f*x) - 1)), x)
```

3.165.7 Maxima [F]

$$\int x^3 \sqrt{a - a\sin(e + fx)}\sqrt{c + c\sin(e + fx)}\, dx = \int \sqrt{-a\sin\left(fx + e\right) + a}\sqrt{c\sin\left(fx + e\right) + c}x^3\, dx$$

input
```
integrate(x^3*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x, algorithm="
maxima")
```

output
```
integrate(sqrt(-a*sin(f*x + e) + a)*sqrt(c*sin(f*x + e) + c)*x^3, x)
```

3.165.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.01

$$\int x^3 \sqrt{a - a\sin(e + fx)}\sqrt{c + c\sin(e + fx)}\, dx = -\sqrt{a}\sqrt{c}\left(\frac{3\left(f^2x^2\text{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right)\text{sgn}\left(\sin\left(-\frac{1}{4}\pi + \frac{1}{2}fx + \frac{1}{2}e\right)\right) - 2\,\text{sgn}\left(\cos\left(-\frac{1}{4}\pi + \frac{1}{2}fx +\right.\right.\right.}{f^4}\right.$$

input
```
integrate(x^3*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x, algorithm="
giac")
```

output
```
-sqrt(a)*sqrt(c)*(3*(f^2*x^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-
1/4*pi + 1/2*f*x + 1/2*e)) - 2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin
(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(f*x + e)/f^4 + (f^3*x^3*sgn(cos(-1/4*pi
+ 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 6*f*x*sgn(cos(-1
/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(f*x + e
)/f^4)
```

3.165.9 Mupad [B] (verification not implemented)

Time = 28.11 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.72

$$\int x^3\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\,dx =$$
$$-\frac{\sqrt{-a\,\left(\sin\left(e+f\,x\right)-1\right)}\,\sqrt{c\,\left(\sin\left(e+f\,x\right)+1\right)}\,\left(6\,\cos\left(2\,e+2\,f\,x\right)-3\,f^2\,x^2+6\,f\,x\,\sin\left(2\,e+2\,f\,x\right)-\right.}{f^4\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}$$

input
```
int(x^3*(a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(1/2),x)
```

output
```
-((-a*(sin(e + f*x) - 1))^(1/2)*(c*(sin(e + f*x) + 1))^(1/2)*(6*cos(2*e +
2*f*x) - 3*f^2*x^2 + 6*f*x*sin(2*e + 2*f*x) - 3*f^2*x^2*cos(2*e + 2*f*x) -
 f^3*x^3*sin(2*e + 2*f*x) + 6))/(f^4*(cos(2*e + 2*f*x) + 1))
```

3.166 $\int x^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\,dx$

3.166.1 Optimal result

Integrand size = 33, antiderivative size = 118

$$\int x^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\,dx = \frac{2x\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{f^2} - \frac{2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\tan(e+fx)}{f^3} + \frac{x^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\tan(e+fx)}{f}$$

output
```
2*x*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/f^2-2*(a-a*sin(f*x+e))^(
1/2)*(c+c*sin(f*x+e))^(1/2)*tan(f*x+e)/f^3+x^2*(a-a*sin(f*x+e))^(1/2)*(c+c
*sin(f*x+e))^(1/2)*tan(f*x+e)/f
```

3.166.2 Mathematica [A] (verified)

Time = 1.51 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.46

$$\int x^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\,dx = \frac{\sqrt{c(1+\sin(e+fx))}\sqrt{a-a\sin(e+fx)}(2fx+(-2+f^2x^2)\tan(e+fx))}{f^3}$$

input
```
Integrate[x^2*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]],x]
```

output
```
(Sqrt[c*(1 + Sin[e + f*x])]*Sqrt[a - a*Sin[e + f*x]]*(2*f*x + (-2 + f^2*x^
2)*Tan[e + f*x]))/f^3
```

3.166.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.66, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {5115, 3042, 3777, 25, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}dx$$

$$\Big\downarrow \text{5115}$$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\int x^2\cos(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\int x^2\sin\left(e+fx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3777}$$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\left(\frac{2\int -x\sin(e+fx)dx}{f}+\frac{x^2\sin(e+fx)}{f}\right)$$

$$\Big\downarrow \text{25}$$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\left(\frac{x^2\sin(e+fx)}{f}-\frac{2\int x\sin(e+fx)dx}{f}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\left(\frac{x^2\sin(e+fx)}{f}-\frac{2\int x\sin(e+fx)dx}{f}\right)$$

$$\Big\downarrow \text{3777}$$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\left(\frac{x^2\sin(e+fx)}{f}-\frac{2\left(\frac{\int\cos(e+fx)dx}{f}-\frac{x\cos(e+fx)}{f}\right)}{f}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\left(\frac{x^2\sin(e+fx)}{f}-\frac{2\left(\frac{\int \sin\left(e+fx+\frac{\pi}{2}\right)dx}{f}-\frac{x\cos(e+fx)}{f}\right)}{f}\right)$$

$$\Big\downarrow \text{3117}$$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\left(\frac{x^2\sin(e+fx)}{f}-\frac{2\left(\frac{\sin(e+fx)}{f^2}-\frac{x\cos(e+fx)}{f}\right)}{f}\right)$$

input
```
Int[x^2*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]],x]
```

output
```
Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*((x^2*Sin[e
 + f*x])/f - (2*(-((x*Cos[e + f*x])/f) + Sin[e + f*x]/f^2))/f)
```

3.166.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.166. $\int x^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\,dx$

rule 5115
```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPart[m]
*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPa
rt[m]/Cos[e + f*x]^(2*FracPart[m]))   Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
 + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] &&
IGeQ[n - m, 0]
```

3.166.4 Maple [F]

$$\int x^2 \sqrt{a-\sin\left(fx+e\right)a}\,\sqrt{c+c\sin\left(fx+e\right)}dx$$

input
```
int(x^2*(a-sin(f*x+e)*a)^(1/2)*(c+c*sin(f*x+e))^(1/2),x)
```

output
```
int(x^2*(a-sin(f*x+e)*a)^(1/2)*(c+c*sin(f*x+e))^(1/2),x)
```

3.166.5 Fricas [F(-2)]

Exception generated.

$$\int x^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x, algorithm="
fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.166.6 Sympy [F]

$$\int x^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\,dx = \int x^2\sqrt{c\left(\sin\left(e+fx\right)+1\right)}\sqrt{-a\left(\sin\left(e+fx\right)-1\right)}\,dx$$

input
```
integrate(x**2*(a-a*sin(f*x+e))**(1/2)*(c+c*sin(f*x+e))**(1/2),x)
```

output
```
Integral(x**2*sqrt(c*(sin(e + f*x) + 1))*sqrt(-a*(sin(e + f*x) - 1)), x)
```

3.166.7 Maxima [F]

$$\int x^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\,dx = \int \sqrt{-a\sin\left(fx+e\right)+a}\sqrt{c\sin\left(fx+e\right)+c}x^2\,dx$$

input
```
integrate(x^2*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x, algorithm="
maxima")
```

output
```
integrate(sqrt(-a*sin(f*x + e) + a)*sqrt(c*sin(f*x + e) + c)*x^2, x)
```

3.166.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.01

$$\int x^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\,dx = -\left(\frac{2\,x\cos\left(fx+e\right)\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)}{f^2}+\frac{\left(f^2x^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\right.\right.\right.}{}$$

input
```
integrate(x^2*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x, algorithm="
giac")
```

output
```
-(2*x*cos(f*x + e)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1
/2*f*x + 1/2*e))/f^2 + (f^2*x^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(si
n(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(
sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(f*x + e)/f^3)*sqrt(a)*sqrt(c)
```

3.166.9 Mupad [B] (verification not implemented)

Time = 27.42 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.73

$$\int x^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\,dx$$
$$=\frac{\sqrt{-a\,(\sin(e+f\,x)-1)}\,\sqrt{c\,(\sin(e+f\,x)+1)}\,\left(2\,f\,x-2\,\sin(2\,e+2\,f\,x)+2\,f\,x\,\left(2\cos(e+f\,x)^2-1\right)-\right.}{2\,f^3\cos(e+f\,x)^2}$$

input
```
int(x^2*(a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(1/2),x)
```

output
```
((-a*(sin(e + f*x) - 1))^(1/2)*(c*(sin(e + f*x) + 1))^(1/2)*(2*f*x - 2*sin
(2*e + 2*f*x) + 2*f*x*(2*cos(e + f*x)^2 - 1) + f^2*x^2*sin(2*e + 2*f*x)))/
(2*f^3*cos(e + f*x)^2)
```

3.167 $\int x\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\,dx$

3.167.1 Optimal result

Integrand size = 31, antiderivative size = 74

$$\int x\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\,dx = \frac{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{f^2} + \frac{x\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\tan(e+fx)}{f}$$

output
```
(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/f^2+x*(a-a*sin(f*x+e))^(1/2)
*(c+c*sin(f*x+e))^(1/2)*tan(f*x+e)/f
```

3.167.2 Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.59

$$\int x\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\,dx = \frac{\sqrt{c(1+\sin(e+fx))}\sqrt{a-a\sin(e+fx)}(1+fx\tan(e+fx))}{f^2}$$

input
```
Integrate[x*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]],x]
```

output
```
(Sqrt[c*(1 + Sin[e + f*x])]*Sqrt[a - a*Sin[e + f*x]]*(1 + f*x*Tan[e + f*x]
))/f^2
```

3.167.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.78, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {5115, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\,dx$$

$$\Big\downarrow \text{5115}$$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\int x\cos(e+fx)dx$$

$$\Big\downarrow \text{3042}$$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\int x\sin\left(e+fx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3777}$$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\left(\frac{\int -\sin(e+fx)dx}{f}+\frac{x\sin(e+fx)}{f}\right)$$

$$\Big\downarrow \text{25}$$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\left(\frac{x\sin(e+fx)}{f}-\frac{\int \sin(e+fx)dx}{f}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\left(\frac{x\sin(e+fx)}{f}-\frac{\int \sin(e+fx)dx}{f}\right)$$

$$\Big\downarrow \text{3118}$$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\left(\frac{\cos(e+fx)}{f^2}+\frac{x\sin(e+fx)}{f}\right)$$

input
```
Int[x*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]],x]
```

output
```
Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*(Cos[e + f*
x]/f^2 + (x*Sin[e + f*x])/f)
```

3.167.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5115
```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPart[m]
*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPa
rt[m]/Cos[e + f*x]^(2*FracPart[m]))   Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
 + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] &&
IGeQ[n - m, 0]
```

3.167.4 Maple [F]

$$\int x\sqrt{a-\sin\left(fx+e\right)a}\,\sqrt{c+c\sin\left(fx+e\right)}dx$$

input
```
int(x*(a-sin(f*x+e)*a)^(1/2)*(c+c*sin(f*x+e))^(1/2),x)
```

output
```
int(x*(a-sin(f*x+e)*a)^(1/2)*(c+c*sin(f*x+e))^(1/2),x)
```

3.167.5 Fricas [F(-2)]

Exception generated.

$$\int x\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x, algorithm="fr
icas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.167.6 Sympy [F]

$$\int x\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\,dx$$
$$=\int x\sqrt{c\,(\sin{(e+fx)}+1)}\sqrt{-a\,(\sin{(e+fx)}-1)}\,dx$$

input
```
integrate(x*(a-a*sin(f*x+e))**(1/2)*(c+c*sin(f*x+e))**(1/2),x)
```

output
```
Integral(x*sqrt(c*(sin(e + f*x) + 1))*sqrt(-a*(sin(e + f*x) - 1)), x)
```

3.167.7 Maxima [F]

$$\int x\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\,dx$$
$$=\int\sqrt{-a\sin{(fx+e)}+a}\sqrt{c\sin{(fx+e)}+c}x\,dx$$

input
```
integrate(x*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(-a*sin(f*x + e) + a)*sqrt(c*sin(f*x + e) + c)*x, x)
```

3.167.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.11

$$\int x\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\,dx = -\left(\frac{x\text{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\text{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin(fx+e)}{f}+\frac{\cos(fx+e)\text{sgn}\left(\cos\left(-\frac{1}{4}\right.\right.}{}\right.$$

input
```
integrate(x*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2),x, algorithm="gi
ac")
```

output
```
-(x*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)
)*sin(f*x + e)/f + cos(f*x + e)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(si
n(-1/4*pi + 1/2*f*x + 1/2*e))/f^2)*sqrt(a)*sqrt(c)
```

3.167.9 Mupad [B] (verification not implemented)

Time = 27.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.82

$$\int x\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\,dx = \frac{\sqrt{-a\left(\sin\left(e+f\,x\right)-1\right)}\left(2\cos\left(e+f\,x\right)^2+f\,x\,\sin\left(2\,e+2\,f\,x\right)\right)\sqrt{c\left(\sin\left(e+f\,x\right)+1\right)}}{2\,f^2\cos\left(e+f\,x\right)^2}$$

input
```
int(x*(a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(1/2),x)
```

output
```
((-a*(sin(e + f*x) - 1))^(1/2)*(2*cos(e + f*x)^2 + f*x*sin(2*e + 2*f*x))*(
c*(sin(e + f*x) + 1))^(1/2))/(2*f^2*cos(e + f*x)^2)
```

3.168 $\int \frac{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{x}\,dx$

3.168.1 Optimal result

Integrand size = 33, antiderivative size = 86

$$\int \frac{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{x}\,dx = \cos(e)\,\text{CosIntegral}(fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)} - \sec(e+fx)\sin(e)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\,\text{Si}(fx)$$

output
```
Ci(f*x)*cos(e)*sec(f*x+e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)-se
c(f*x+e)*Si(f*x)*sin(e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)
```

3.168.2 Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.60

$$\int \frac{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{x}\,dx = \sec(e+fx)\sqrt{c(1+\sin(e+fx))}\sqrt{a-a\sin(e+fx)}(\cos(e)\,\text{CosIntegral}(fx) - \sin(e)\text{Si}(fx))$$

input
```
Integrate[(Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/x,x]
```

output
```
Sec[e + f*x]*Sqrt[c*(1 + Sin[e + f*x])]*Sqrt[a - a*Sin[e + f*x]]*(Cos[e]*C
osIntegral[f*x] - Sin[e]*SinIntegral[f*x])
```

3.168.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.60, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {5115, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}{x}dx$$

$$\Big\downarrow \text{5115}$$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\int \frac{\cos(e+fx)}{x}dx$$

$$\Big\downarrow \text{3042}$$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\int \frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{x}dx$$

$$\Big\downarrow \text{3784}$$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\left(\cos(e)\int \frac{\cos(fx)}{x}dx-\sin(e)\int \frac{\sin(fx)}{x}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\left(\cos(e)\int \frac{\sin\left(fx+\frac{\pi}{2}\right)}{x}dx-\sin(e)\int \frac{\sin(fx)}{x}dx\right)$$

$$\Big\downarrow \text{3780}$$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\left(\cos(e)\int \frac{\sin\left(fx+\frac{\pi}{2}\right)}{x}dx-\sin(e)\text{Si}(fx)\right)$$

$$\Big\downarrow \text{3783}$$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}(\cos(e)\,\text{CosIntegral}(fx)-\sin(e)\text{Si}(fx))$$

input
```
Int[(Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/x,x]
```

output
```
Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*(Cos[e]*Cos
Integral[f*x] - Sin[e]*SinIntegral[f*x])
```

3.168.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 5115
```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPart[m]
*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPa
rt[m]/Cos[e + f*x]^(2*FracPart[m]))   Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
 + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] &&
IGeQ[n - m, 0]
```

3.168.4 Maple [F]

$$\int \frac{\sqrt{a-\sin(fx+e)\,a}\,\sqrt{c+c\sin(fx+e)}}{x}dx$$

input
```
int((a-sin(f*x+e)*a)^(1/2)*(c+c*sin(f*x+e))^(1/2)/x,x)
```

output
```
int((a-sin(f*x+e)*a)^(1/2)*(c+c*sin(f*x+e))^(1/2)/x,x)
```

3.168.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a - a\sin(e + fx)}\sqrt{c + c\sin(e + fx)}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x,x, algorithm="fr
icas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.168.6 Sympy [F]

$$\int \frac{\sqrt{a - a\sin(e + fx)}\sqrt{c + c\sin(e + fx)}}{x}\,dx = \int \frac{\sqrt{c\left(\sin\left(e + fx\right) + 1\right)}\sqrt{-a\left(\sin\left(e + fx\right) - 1\right)}}{x}\,dx$$

input
```
integrate((a-a*sin(f*x+e))**(1/2)*(c+c*sin(f*x+e))**(1/2)/x,x)
```

output
```
Integral(sqrt(c*(sin(e + f*x) + 1))*sqrt(-a*(sin(e + f*x) - 1))/x, x)
```

3.168.7 Maxima [F]

$$\int \frac{\sqrt{a - a\sin(e + fx)}\sqrt{c + c\sin(e + fx)}}{x}\,dx = \int \frac{\sqrt{-a\sin\left(fx + e\right) + a}\sqrt{c\sin\left(fx + e\right) + c}}{x}\,dx$$

input
```
integrate((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(-a*sin(f*x + e) + a)*sqrt(c*sin(f*x + e) + c)/x, x)
```

3.168.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.32 (sec) , antiderivative size = 274, normalized size of antiderivative = 3.19

$$\int \frac{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{x}\,dx$$
$$= \frac{\left(\Re(\mathrm{Ci}\,(fx))\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\tan\left(\frac{1}{2}\,e\right)^{2}+\Re(\mathrm{Ci}\,(-fx))\,\mathrm{sgn}(\mathrm{co}\right.}{}$$

input
```
integrate((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x,x, algorithm="gi
ac")
```

output
```
1/2*(real_part(cos_integral(f*x))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(
sin(-1/4*pi + 1/2*f*x + 1/2*e))*tan(1/2*e)^2 + real_part(cos_integral(-f*x
))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))
*tan(1/2*e)^2 + 2*imag_part(cos_integral(f*x))*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*tan(1/2*e) - 2*imag_part(cos_
integral(-f*x))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*
f*x + 1/2*e))*tan(1/2*e) + 4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-
1/4*pi + 1/2*f*x + 1/2*e))*sin_integral(f*x)*tan(1/2*e) - real_part(cos_in
tegral(f*x))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x
 + 1/2*e)) - real_part(cos_integral(-f*x))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)*sqrt(c)/(tan(1/2*e)^2 +
1)
```

3.168.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{x}\,dx$$
$$= \int \frac{\sqrt{a-a\,\sin\left(e+f\,x\right)}\,\sqrt{c+c\,\sin\left(e+f\,x\right)}}{x}\,dx$$

input
```
int(((a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(1/2))/x,x)
```

output
```
int(((a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(1/2))/x, x)
```

3.169 $\int \frac{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{x^2}\,dx$

3.169.1 Optimal result

Integrand size = 33, antiderivative size = 123

$$\begin{aligned}\int \frac{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{x^2}\,dx &= -\frac{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{x}\\ &\quad - f\,\text{CosIntegral}(fx)\sec(e+fx)\sin(e)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\\ &\quad - f\cos(e)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\,\text{Si}(fx)\end{aligned}$$

output
```
-(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x-f*cos(e)*sec(f*x+e)*Si(f*
x)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)-f*Ci(f*x)*sec(f*x+e)*sin(
e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)
```

3.169.2 Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.53

$$\int \frac{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{x^2}\,dx = -\frac{\sec(e+fx)\sqrt{c(1+\sin(e+fx))}\sqrt{a-a\sin(e+fx)}(\cos(e+fx)+fx\,\text{CosIntegral}(fx)\sin(e)+fx\cos}{x}$$

input
```
Integrate[(Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/x^2,x]
```

output
```
-((Sec[e + f*x]*Sqrt[c*(1 + Sin[e + f*x])]*Sqrt[a - a*Sin[e + f*x]]*(Cos[e
 + f*x] + f*x*CosIntegral[f*x]*Sin[e] + f*x*Cos[e]*SinIntegral[f*x]))/x)
```

3.169.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.54, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {5115, 3042, 3778, 25, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}{x^2}dx$$

$$\Big\downarrow \text{5115}$$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\int \frac{\cos(e+fx)}{x^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\int \frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{x^2}dx$$

$$\Big\downarrow \text{3778}$$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\left(f\int -\frac{\sin(e+fx)}{x}dx-\frac{\cos(e+fx)}{x}\right)$$

$$\Big\downarrow \text{25}$$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\left(-f\int \frac{\sin(e+fx)}{x}dx-\frac{\cos(e+fx)}{x}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\left(-f\int \frac{\sin(e+fx)}{x}dx-\frac{\cos(e+fx)}{x}\right)$$

$$\Big\downarrow \text{3784}$$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\left(-f\left(\sin(e)\int \frac{\cos(fx)}{x}dx+\cos(e)\int \frac{\sin(fx)}{x}dx\right)-\frac{\cos(e+fx)}{x}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\left(-f\left(\sin(e)\int\frac{\sin\left(fx+\frac{\pi}{2}\right)}{x}dx+\cos(e)\int\frac{\sin(fx)}{x}dx\right)-\frac{\cos(e+fx)}{x}\right.$$

$\big\downarrow$ 3780

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\left(-f\left(\sin(e)\int\frac{\sin\left(fx+\frac{\pi}{2}\right)}{x}dx+\cos(e)\text{Si}(fx)\right)-\frac{\cos(e+fx)}{x}\right)$$

$\big\downarrow$ 3783

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\left(-f(\sin(e)\,\text{CosIntegral}(fx)+\cos(e)\text{Si}(fx))-\frac{\cos(e+fx)}{x}\right)$$

input
```
Int[(Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/x^2,x]
```

output
```
Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*(-(Cos[e +
f*x]/x) - f*(CosIntegral[f*x]*Sin[e] + Cos[e]*SinIntegral[f*x]))
```

3.169.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 5115
```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPart[m]
*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPa
rt[m]/Cos[e + f*x]^(2*FracPart[m]))   Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
 + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] &&
IGeQ[n - m, 0]
```

3.169.4 Maple [F]

$$\int \frac{\sqrt{a-\sin (fx+e)\, a}\, \sqrt{c+c\sin (fx+e)}}{x^2}dx$$

input
```
int((a-sin(f*x+e)*a)^(1/2)*(c+c*sin(f*x+e))^(1/2)/x^2,x)
```

output
```
int((a-sin(f*x+e)*a)^(1/2)*(c+c*sin(f*x+e))^(1/2)/x^2,x)
```

3.169.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{x^2}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x^2,x, algorithm="
fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (has polynomial part)
```

3.169.6 Sympy [F]

$$\int \frac{\sqrt{a - a\sin(e + fx)}\sqrt{c + c\sin(e + fx)}}{x^2}\,dx$$
$$= \int \frac{\sqrt{c\left(\sin\left(e + fx\right) + 1\right)}\sqrt{-a\left(\sin\left(e + fx\right) - 1\right)}}{x^2}\,dx$$

input
```
integrate((a-a*sin(f*x+e))**(1/2)*(c+c*sin(f*x+e))**(1/2)/x**2,x)
```

output
```
Integral(sqrt(c*(sin(e + f*x) + 1))*sqrt(-a*(sin(e + f*x) - 1))/x**2, x)
```

3.169.7 Maxima [F]

$$\int \frac{\sqrt{a - a\sin(e + fx)}\sqrt{c + c\sin(e + fx)}}{x^2}\,dx$$
$$= \int \frac{\sqrt{-a\sin\left(fx + e\right) + a}\sqrt{c\sin\left(fx + e\right) + c}}{x^2}\,dx$$

input
```
integrate((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x^2,x, algorithm="
maxima")
```

output
```
integrate(sqrt(-a*sin(f*x + e) + a)*sqrt(c*sin(f*x + e) + c)/x^2, x)
```

3.169.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.41 (sec) , antiderivative size = 886, normalized size of antiderivative = 7.20

$$\int \frac{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{x^2}\,dx = \text{Too large to display}$$

input
```
integrate((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x^2,x, algorithm="
giac")
```

output
```
-1/2*(f*x*imag_part(cos_integral(f*x))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*tan(1/2*f*x)^2*tan(1/2*e)^2 - f*x*ima
g_part(cos_integral(-f*x))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/
4*pi + 1/2*f*x + 1/2*e))*tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*f*x*sgn(cos(-1/4*
pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin_integral(f*
x)*tan(1/2*f*x)^2*tan(1/2*e)^2 - 2*f*x*real_part(cos_integral(f*x))*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*tan(1/2*
f*x)^2*tan(1/2*e) - 2*f*x*real_part(cos_integral(-f*x))*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*tan(1/2*f*x)^2*tan(1
/2*e) - f*x*imag_part(cos_integral(f*x))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*tan(1/2*f*x)^2 + f*x*imag_part(cos_
integral(-f*x))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*
f*x + 1/2*e))*tan(1/2*f*x)^2 - 2*f*x*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*s
gn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin_integral(f*x)*tan(1/2*f*x)^2 + f*x*
imag_part(cos_integral(f*x))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-
1/4*pi + 1/2*f*x + 1/2*e))*tan(1/2*e)^2 - f*x*imag_part(cos_integral(-f*x)
)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*
tan(1/2*e)^2 + 2*f*x*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi +
 1/2*f*x + 1/2*e))*sin_integral(f*x)*tan(1/2*e)^2 - 2*f*x*real_part(cos_in
tegral(f*x))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*...
```

3.169.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{x^2}\,dx = \int \frac{\sqrt{a-a\,\sin\left(e+f\,x\right)}\,\sqrt{c+c\,\sin\left(e+f\,x\right)}}{x^2}\,dx$$

input
```
int(((a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(1/2))/x^2,x)
```

output
```
int(((a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(1/2))/x^2, x)
```

3.169. $\int \frac{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{x^2}\,dx$

3.170 $\int \frac{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{x^3}\,dx$

3.170.1 Optimal result

Integrand size = 33, antiderivative size = 176

$$\begin{aligned}\int \frac{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{x^3}\,dx &= -\frac{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{2x^2} \\ &\quad -\frac{1}{2}f^2\cos(e)\,\mathrm{CosIntegral}(fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)} \\ &\quad +\frac{1}{2}f^2\sec(e+fx)\sin(e)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\,\mathrm{Si}(fx) \\ &\quad +\frac{f\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\tan(e+fx)}{2x}\end{aligned}$$

output
```
-1/2*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x^2-1/2*f^2*Ci(f*x)*cos
(e)*sec(f*x+e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)+1/2*f^2*sec(f
*x+e)*Si(f*x)*sin(e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)+1/2*f*(
a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)*tan(f*x+e)/x
```

3.170.2 Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.49

$$\int \frac{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{x^3}\,dx$$
$$= \frac{\sec(e+fx)\sqrt{c(1+\sin(e+fx))}\sqrt{a-a\sin(e+fx)}(-\cos(e+fx)-f^2x^2\cos(e)\,\text{CosIntegral}(fx)+fx\,\text{s}}{2x^2}$$

input
```
Integrate[(Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/x^3,x]
```

output
```
(Sec[e + f*x]*Sqrt[c*(1 + Sin[e + f*x])]*Sqrt[a - a*Sin[e + f*x]]*(-Cos[e
+ f*x] - f^2*x^2*Cos[e]*CosIntegral[f*x] + f*x*Sin[e + f*x] + f^2*x^2*Sin[
e]*SinIntegral[f*x]))/(2*x^2)
```

3.170.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.48, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5115, 3042, 3778, 25, 3042, 3778, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}{x^3}\,dx$$

$$\Big\downarrow \; 5115$$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\int \frac{\cos(e+fx)}{x^3}dx$$

$$\Big\downarrow \; 3042$$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\int \frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{x^3}dx$$

$$\Big\downarrow \; 3778$$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\left(\frac{1}{2}f\int -\frac{\sin(e+fx)}{x^2}dx-\frac{\cos(e+fx)}{2x^2}\right)$$

$$\Big\downarrow \; 25$$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\left(-\frac{1}{2}f\int\frac{\sin(e+fx)}{x^2}dx-\frac{\cos(e+fx)}{2x^2}\right)$$

$\big\downarrow$ 3042

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\left(-\frac{1}{2}f\int\frac{\sin(e+fx)}{x^2}dx-\frac{\cos(e+fx)}{2x^2}\right)$$

$\big\downarrow$ 3778

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\left(-\frac{1}{2}f\left(f\int\frac{\cos(e+fx)}{x}dx-\frac{\sin(e+fx)}{x}\right)-\frac{\cos(e+fx)}{2x^2}\right)$$

$\big\downarrow$ 3042

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\left(-\frac{1}{2}f\left(f\int\frac{\sin\left(e+fx+\frac{\pi}{2}\right)}{x}dx-\frac{\sin(e+fx)}{x}\right)-\frac{\cos(e+fx)}{2x^2}\right)$$

$\big\downarrow$ 3784

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\left(-\frac{1}{2}f\left(f\left(\cos(e)\int\frac{\cos(fx)}{x}dx-\sin(e)\int\frac{\sin(fx)}{x}dx\right)-\frac{\sin(e+fx)}{x}\right)\right.$$

$\big\downarrow$ 3042

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\left(-\frac{1}{2}f\left(f\left(\cos(e)\int\frac{\sin\left(fx+\frac{\pi}{2}\right)}{x}dx-\sin(e)\int\frac{\sin(fx)}{x}dx\right)-\frac{\sin(e+}{x}\right.\right.$$

$\big\downarrow$ 3780

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\left(-\frac{1}{2}f\left(f\left(\cos(e)\int\frac{\sin\left(fx+\frac{\pi}{2}\right)}{x}dx-\sin(e)\text{Si}(fx)\right)-\frac{\sin(e+fx)}{x}\right)\right.$$

$\big\downarrow$ 3783

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\left(-\frac{1}{2}f\left(f(\cos(e)\,\text{CosIntegral}(fx)-\sin(e)\text{Si}(fx))-\frac{\sin(e+fx)}{x}\right)-\text{co}\right.$$

input
```
Int[(Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])/x^3,x]
```

3.170. $\int\frac{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{x^3}\,dx$

output
```
Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*(-1/2*Cos[e
 + f*x]/x^2 - (f*(-(Sin[e + f*x]/x) + f*(Cos[e]*CosIntegral[f*x] - Sin[e]*
SinIntegral[f*x])))/2)
```

3.170.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 5115
```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPart[m]
*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPa
rt[m]/Cos[e + f*x]^(2*FracPart[m]))   Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
 + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] &&
IGeQ[n - m, 0]
```

3.170. $\int \frac{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{x^3}\,dx$

3.170.4 Maple [F]

$$\int \frac{\sqrt{a-\sin \left(f x +e \right) a}\, \sqrt{c +c \sin \left(f x +e \right)}}{x^{3}}d x$$

input
```
int((a-sin(f*x+e)*a)^(1/2)*(c+c*sin(f*x+e))^(1/2)/x^3,x)
```

output
```
int((a-sin(f*x+e)*a)^(1/2)*(c+c*sin(f*x+e))^(1/2)/x^3,x)
```

3.170.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x^3,x, algorithm="
fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.170.6 Sympy [F]

$$\int \frac{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{x^3}\,dx = \int \frac{\sqrt{c\left(\sin\left(e+fx\right)+1\right)}\sqrt{-a\left(\sin\left(e+fx\right)-1\right)}}{x^3}\,dx$$

input
```
integrate((a-a*sin(f*x+e))**(1/2)*(c+c*sin(f*x+e))**(1/2)/x**3,x)
```

output
```
Integral(sqrt(c*(sin(e + f*x) + 1))*sqrt(-a*(sin(e + f*x) - 1))/x**3, x)
```

3.170.7 Maxima [F]

$$\int \frac{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{x^3}\,dx$$
$$=\int \frac{\sqrt{-a\sin(fx+e)+a}\sqrt{c\sin(fx+e)+c}}{x^3}\,dx$$

input
```
integrate((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x^3,x, algorithm="
maxima")
```

output
```
integrate(sqrt(-a*sin(f*x + e) + a)*sqrt(c*sin(f*x + e) + c)/x^3, x)
```

3.170.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.42 (sec) , antiderivative size = 1022, normalized size of antiderivative = 5.81

$$\int \frac{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{x^3}\,dx = \text{Too large to display}$$

input
```
integrate((a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x^3,x, algorithm="
giac")
```

output
```
-1/4*(f^2*x^2*real_part(cos_integral(f*x))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*tan(1/2*f*x)^2*tan(1/2*e)^2 + f^2
*x^2*real_part(cos_integral(-f*x))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn
(sin(-1/4*pi + 1/2*f*x + 1/2*e))*tan(1/2*f*x)^2*tan(1/2*e)^2 + 2*f^2*x^2*i
mag_part(cos_integral(f*x))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1
/4*pi + 1/2*f*x + 1/2*e))*tan(1/2*f*x)^2*tan(1/2*e) - 2*f^2*x^2*imag_part(
cos_integral(-f*x))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi +
1/2*f*x + 1/2*e))*tan(1/2*f*x)^2*tan(1/2*e) + 4*f^2*x^2*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin_integral(f*x)*ta
n(1/2*f*x)^2*tan(1/2*e) - f^2*x^2*real_part(cos_integral(f*x))*sgn(cos(-1/
4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*tan(1/2*f*x)^
2 - f^2*x^2*real_part(cos_integral(-f*x))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*
e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*tan(1/2*f*x)^2 + f^2*x^2*real_part
(cos_integral(f*x))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi +
1/2*f*x + 1/2*e))*tan(1/2*e)^2 + f^2*x^2*real_part(cos_integral(-f*x))*sgn
(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*tan(1
/2*e)^2 + 2*f^2*x^2*imag_part(cos_integral(f*x))*sgn(cos(-1/4*pi + 1/2*f*x
 + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*tan(1/2*e) - 2*f^2*x^2*imag
_part(cos_integral(-f*x))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4
*pi + 1/2*f*x + 1/2*e))*tan(1/2*e) + 4*f^2*x^2*sgn(cos(-1/4*pi + 1/2*f*...
```

3.170.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{x^3}\,dx = \int \frac{\sqrt{a-a\,\sin\left(e+f\,x\right)}\,\sqrt{c+c\,\sin\left(e+f\,x\right)}}{x^3}\,dx$$

input
```
int(((a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(1/2))/x^3,x)
```

output
```
int(((a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(1/2))/x^3, x)
```

3.171 $\int x^3\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}\,dx$

3.171.1 Optimal result

Integrand size = 33, antiderivative size = 393

$$\begin{aligned}
&\int x^3\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}\,dx = \\
&-\frac{6c\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{f^4} \\
&+\frac{3cx^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{f^2} \\
&+\frac{3cx\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{8f^3} \\
&-\frac{3cx^3\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{4f} \\
&-\frac{3c\sin(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{8f^4} \\
&+\frac{3cx^2\sin(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{4f^2} \\
&+\frac{x^3\sec(e+fx)\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{5/2}}{2cf} \\
&-\frac{6cx\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\tan(e+fx)}{f^3} \\
&-\frac{3cx\sin(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\tan(e+fx)}{4f^3}
\end{aligned}$$

output
```
1/2*x^3*sec(f*x+e)*(c+c*sin(f*x+e))^(5/2)*(a-a*sin(f*x+e))^(1/2)/c/f-6*c*(
a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/f^4+3*c*x^2*(a-a*sin(f*x+e))^
(1/2)*(c+c*sin(f*x+e))^(1/2)/f^2+3/8*c*x*sec(f*x+e)*(a-a*sin(f*x+e))^(1/2)
*(c+c*sin(f*x+e))^(1/2)/f^3-3/4*c*x^3*sec(f*x+e)*(a-a*sin(f*x+e))^(1/2)*(c
+c*sin(f*x+e))^(1/2)/f-3/8*c*sin(f*x+e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*
x+e))^(1/2)/f^4+3/4*c*x^2*sin(f*x+e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e
))^(1/2)/f^2-6*c*x*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)*tan(f*x+e
)/f^3-3/4*c*x*sin(f*x+e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)*tan
(f*x+e)/f^3
```

3.171.2 Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.29

$$\int x^3\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}\,dx=\frac{c\sqrt{c(1+\sin(e+fx))}\sqrt{a-a\sin(e+fx)}(-fx(-3+2f^2x^2)\cos(2(e+fx))\sec(e+fx)+(-}{8f^4}$$

input
```
Integrate[x^3*Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(3/2),x]
```

output
```
(c*Sqrt[c*(1 + Sin[e + f*x])]*Sqrt[a - a*Sin[e + f*x]]*(-(f*x*(-3 + 2*f^2*
x^2)*Cos[2*(e + f*x)]*Sec[e + f*x]) + (-3 + 6*f^2*x^2)*Sin[e + f*x] + 8*(-
6 + 3*f^2*x^2 + f*x*(-6 + f^2*x^2)*Tan[e + f*x])))/(8*f^4)
```

3.171.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.52, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {5115, 4922, 3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{a-a\sin(e+fx)}(c\sin(e+fx)+c)^{3/2}\,dx$$

$$\Big\downarrow \text{ 5115}$$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\int x^3\cos(e+fx)(\sin(e+fx)c+c)dx$$

$$\Big\downarrow \text{4922}$$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\left(\frac{x^3(c\sin(e+fx)+c)^2}{2cf}-\frac{3\int x^2(\sin(e+fx)c+c)^2dx}{2cf}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\left(\frac{x^3(c\sin(e+fx)+c)^2}{2cf}-\frac{3\int x^2(\sin(e+fx)c+c)^2dx}{2cf}\right)$$

$$\Big\downarrow \text{3798}$$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\left(\frac{x^3(c\sin(e+fx)+c)^2}{2cf}-\frac{3\int (c^2x^2+c^2\sin^2(e+fx)x^2+2c^2\sin(e+ \ldots}{2cf}\right.$$

$$\Big\downarrow \text{2009}$$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\left(\frac{x^3(c\sin(e+fx)+c)^2}{2cf}-\frac{3\left(\frac{4c^2\cos(e+fx)}{f^3}+\frac{c^2\sin(e+fx)\cos(e+fx)}{4f^3}+\frac{c^2x\sin\ldots}{}\right.}{}\right.$$

input
```
Int[x^3*Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(3/2),x]
```

output
```
Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*((x^3*(c +
c*Sin[e + f*x])^2)/(2*c*f) - (3*(-1/4*(c^2*x)/f^2 + (c^2*x^3)/2 + (4*c^2*C
os[e + f*x])/f^3 - (2*c^2*x^2*Cos[e + f*x])/f + (4*c^2*x*Sin[e + f*x])/f^2
 + (c^2*Cos[e + f*x]*Sin[e + f*x])/(4*f^3) - (c^2*x^2*Cos[e + f*x]*Sin[e +
 f*x])/(2*f) + (c^2*x*Sin[e + f*x]^2)/(2*f^2)))/(2*c*f))
```

3.171.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

rule 4922
```
Int[Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sin[(c
_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sin[c + d*x
])^(n + 1)/(b*d*(n + 1))), x] - Simp[f*(m/(b*d*(n + 1)))   Int[(e + f*x)^(m
 - 1)*(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n},
x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 5115
```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPart[m]
*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPa
rt[m]/Cos[e + f*x]^(2*FracPart[m]))   Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
 + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] &&
IGeQ[n - m, 0]
```

3.171.4 Maple [F]

$$\int x^{3}(c+c\sin(fx+e))^{\frac{3}{2}}\sqrt{a-\sin(fx+e)\,a}dx$$

input
```
int(x^3*(c+c*sin(f*x+e))^(3/2)*(a-sin(f*x+e)*a)^(1/2),x)
```

output
```
int(x^3*(c+c*sin(f*x+e))^(3/2)*(a-sin(f*x+e)*a)^(1/2),x)
```

3.171.5 Fricas [F(-2)]

Exception generated.

$$\int x^3\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^3*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x, algorithm="
fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (has polynomial part)
```

3.171.6 Sympy [F(-1)]

Timed out.

$$\int x^3\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(x**3*(c+c*sin(f*x+e))**(3/2)*(a-a*sin(f*x+e))**(1/2),x)
```

output
```
Timed out
```

3.171.7 Maxima [F]

$$\int x^3\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}\,dx=\int\sqrt{-a\sin(fx+e)+a}(c\sin(fx+e)+c)^{\frac{3}{2}}x^3\,dx$$

input
```
integrate(x^3*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x, algorithm="
maxima")
```

output
```
integrate(sqrt(-a*sin(f*x + e) + a)*(c*sin(f*x + e) + c)^(3/2)*x^3, x)
```

3.171.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1487 vs. $2(345)=690$.

Time = 0.56 (sec) , antiderivative size = 1487, normalized size of antiderivative = 3.78

$$\int x^3\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}\,dx=\text{Too large to display}$$

input
```
integrate(x^3*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x, algorithm="
giac")
```

output
```
1/32*sqrt(a)*sqrt(c)*((pi^3*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(
-1/4*pi + 1/2*f*x + 1/2*e)) - 3*pi^2*(pi - 2*f*x - 2*e)*c*sgn(cos(-1/4*pi
+ 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 3*pi*(pi - 2*f*x
 - 2*e)^2*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x
+ 1/2*e)) - (pi - 2*f*x - 2*e)^3*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn
(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 6*pi^2*c*e*sgn(cos(-1/4*pi + 1/2*f*x +
1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 12*pi*(pi - 2*f*x - 2*e)*c*e
*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) -
 6*(pi - 2*f*x - 2*e)^2*c*e*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1
/4*pi + 1/2*f*x + 1/2*e)) + 12*pi*c*e^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)
)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 12*(pi - 2*f*x - 2*e)*c*e^2*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 8*c*e^
3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))
- 6*pi*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1
/2*e)) + 6*(pi - 2*f*x - 2*e)*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(si
n(-1/4*pi + 1/2*f*x + 1/2*e)) + 12*c*e*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(2*f*x + 2*e)/f^3 - 24*(pi^2*c*sg
n(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*
pi*(pi - 2*f*x - 2*e)*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*p
i + 1/2*f*x + 1/2*e)) + (pi - 2*f*x - 2*e)^2*c*sgn(cos(-1/4*pi + 1/2*f*...
```

3.171.9 Mupad [B] (verification not implemented)

Time = 29.20 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.55

$$\int x^3\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}\,dx =$$

$$-\frac{c\,\sqrt{-a\,\left(\sin\left(e+f\,x\right)-1\right)}\,\sqrt{c\,\left(\sin\left(e+f\,x\right)+1\right)}\,\left(3\,\sin\left(e+f\,x\right)+96\,\cos\left(2\,e+2\,f\,x\right)+3\,\sin\left(3\,e+3\,f\,x\right.$$

input
```
int(x^3*(a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(3/2),x)
```

output
```
-(c*(-a*(sin(e + f*x) - 1))^(1/2)*(c*(sin(e + f*x) + 1))^(1/2)*(3*sin(e +
f*x) + 96*cos(2*e + 2*f*x) + 3*sin(3*e + 3*f*x) - 48*f^2*x^2 - 6*f*x*cos(3
*e + 3*f*x) + 96*f*x*sin(2*e + 2*f*x) + 4*f^3*x^3*cos(e + f*x) - 6*f^2*x^2
*sin(e + f*x) - 6*f*x*cos(e + f*x) - 48*f^2*x^2*cos(2*e + 2*f*x) + 4*f^3*x
^3*cos(3*e + 3*f*x) - 6*f^2*x^2*sin(3*e + 3*f*x) - 16*f^3*x^3*sin(2*e + 2*
f*x) + 96))/(16*f^4*(cos(2*e + 2*f*x) + 1))
```

3.171. $\int x^3 \sqrt{a - a\sin(e + fx)}(c + c\sin(e + fx))^{3/2}\, dx$

3.172 $\int x^2\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}\,dx$

3.172.1 Optimal result

Integrand size = 33, antiderivative size = 265

$$\int x^2\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}\,dx=\frac{2cx\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{f^2}-\frac{3cx^2\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{4f}+\frac{cx\sin(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{2f^2}+\frac{x^2\sec(e+fx)\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{5/2}}{2cf}-\frac{2c\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\tan(e+fx)}{f^3}-\frac{c\sin(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\tan(e+fx)}{4f^3}$$

output
```
1/2*x^2*sec(f*x+e)*(c+c*sin(f*x+e))^(5/2)*(a-a*sin(f*x+e))^(1/2)/c/f+2*c*x
*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/f^2-3/4*c*x^2*sec(f*x+e)*(a
-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/f+1/2*c*x*sin(f*x+e)*(a-a*sin(
f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/f^2-2*c*(a-a*sin(f*x+e))^(1/2)*(c+c*s
in(f*x+e))^(1/2)*tan(f*x+e)/f^3-1/4*c*sin(f*x+e)*(a-a*sin(f*x+e))^(1/2)*(c
+c*sin(f*x+e))^(1/2)*tan(f*x+e)/f^3
```

3.172.2 Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.36

$$\int x^2\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}\,dx=\frac{c\sqrt{c(1+\sin(e+fx))}\sqrt{a-a\sin(e+fx)}(16fx-(-1+2f^2x^2)\cos(2(e+fx))\sec(e+fx)+}{8f^3}$$

input
```
Integrate[x^2*Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(3/2),x]
```

output
```
(c*Sqrt[c*(1 + Sin[e + f*x])]*Sqrt[a - a*Sin[e + f*x]]*(16*f*x - (-1 + 2*f
^2*x^2)*Cos[2*(e + f*x)]*Sec[e + f*x] + 4*f*x*Sin[e + f*x] + 8*(-2 + f^2*x
^2)*Tan[e + f*x]))/(8*f^3)
```

3.172.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.57, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {5115, 4922, 3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a-a\sin(e+fx)}(c\sin(e+fx)+c)^{3/2}\,dx$$

$$\Big\downarrow \text{5115}$$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\int x^2\cos(e+fx)(\sin(e+fx)c+c)dx$$

$$\Big\downarrow \text{4922}$$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\left(\frac{x^2(c\sin(e+fx)+c)^2}{2cf}-\frac{\int x(\sin(e+fx)c+c)^2dx}{cf}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\left(\frac{x^2(c\sin(e+fx)+c)^2}{2cf}-\frac{\int x(\sin(e+fx)c+c)^2dx}{cf}\right)$$

$$\Big\downarrow \text{3798}$$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\left(\frac{x^2(c\sin(e+fx)+c)^2}{2cf}-\frac{\int\left(x\sin^2(e+fx)c^2+xc^2+2x\sin(e+fx)c\right.}{cf}\right.$$

$\big\downarrow$ 2009

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\left(\frac{x^2(c\sin(e+fx)+c)^2}{2cf}-\frac{\frac{c^2\sin^2(e+fx)}{4f^2}+\frac{2c^2\sin(e+fx)}{f^2}-\frac{2c^2x\cos(e+fx)}{f}-}{cf}\right.$$

input
```
Int[x^2*Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(3/2),x]
```

output
```
Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*((x^2*(c +
c*Sin[e + f*x])^2)/(2*c*f) - ((3*c^2*x^2)/4 - (2*c^2*x*Cos[e + f*x])/f + (
2*c^2*Sin[e + f*x])/f^2 - (c^2*x*Cos[e + f*x]*Sin[e + f*x])/(2*f) + (c^2*S
in[e + f*x]^2)/(4*f^2))/(c*f))
```

3.172.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

rule 4922
```
Int[Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sin[(c
_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sin[c + d*x
])^(n + 1)/(b*d*(n + 1))), x] - Simp[f*(m/(b*d*(n + 1)))   Int[(e + f*x)^(m
 - 1)*(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n},
x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 5115
```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPart[m]
*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPa
rt[m]/Cos[e + f*x]^(2*FracPart[m]))   Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
 + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] &&
IGeQ[n - m, 0]
```

3.172.4 Maple [F]

$$\int x^2(c+c\sin(fx+e))^{\frac{3}{2}}\sqrt{a-\sin(fx+e)\,a}dx$$

input
```
int(x^2*(c+c*sin(f*x+e))^(3/2)*(a-sin(f*x+e)*a)^(1/2),x)
```

output
```
int(x^2*(c+c*sin(f*x+e))^(3/2)*(a-sin(f*x+e)*a)^(1/2),x)
```

3.172.5 Fricas [F(-2)]

Exception generated.

$$\int x^2\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x, algorithm="
fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.172.6 Sympy [F]

$$\int x^2\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}\,dx=\int x^2(c(\sin{(e+fx)}+1))^{\frac{3}{2}}\sqrt{-a(\sin{(e+fx)}-1)}\,dx$$

input
```
integrate(x**2*(c+c*sin(f*x+e))**(3/2)*(a-a*sin(f*x+e))**(1/2),x)
```

output
```
Integral(x**2*(c*(sin(e + f*x) + 1))**(3/2)*sqrt(-a*(sin(e + f*x) - 1)), x
)
```

3.172.7 Maxima [F]

$$\int x^2\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}\,dx=\int\sqrt{-a\sin{(fx+e)}+a}(c\sin{(fx+e)}+c)^{\frac{3}{2}}x^2\,dx$$

input
```
integrate(x^2*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x, algorithm="
maxima")
```

output
```
integrate(sqrt(-a*sin(f*x + e) + a)*(c*sin(f*x + e) + c)^(3/2)*x^2, x)
```

3.172.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 739 vs. $2(233) = 466$.

Time = 0.43 (sec) , antiderivative size = 739, normalized size of antiderivative = 2.79

$$\int x^2\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}\,dx=\text{Too large to display}$$

input
```
integrate(x^2*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x, algorithm="
giac")
```

output
```
1/16*sqrt(a)*sqrt(c)*((pi^2*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(
-1/4*pi + 1/2*f*x + 1/2*e)) - 2*pi*(pi - 2*f*x - 2*e)*c*sgn(cos(-1/4*pi +
1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + (pi - 2*f*x - 2*e)
^2*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e
)) - 4*pi*c*e*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*
x + 1/2*e)) + 4*(pi - 2*f*x - 2*e)*c*e*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 4*c*e^2*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*c*sgn(cos(-1/4*pi + 1/2*f
*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(2*f*x + 2*e)/f^2 - 1
6*(pi*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/
2*e)) - (pi - 2*f*x - 2*e)*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-
1/4*pi + 1/2*f*x + 1/2*e)) - 2*c*e*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn
(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*cos(f*x + e)/f^2 - 2*(pi*c*sgn(cos(-1/4*
pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - (pi - 2*f*x -
 2*e)*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/
2*e)) - 2*c*e*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*
x + 1/2*e)))*sin(2*f*x + 2*e)/f^2 - 4*(pi^2*c*sgn(cos(-1/4*pi + 1/2*f*x +
1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*pi*(pi - 2*f*x - 2*e)*c*sg
n(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + (p
i - 2*f*x - 2*e)^2*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*p...
```

3.172.9 Mupad [B] (verification not implemented)

Time = 28.36 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.60

$$\int x^2\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}\,dx=\frac{c\,\sqrt{-a\,\left(\sin\left(e+f\,x\right)-1\right)}\,\sqrt{c\,\left(\sin\left(e+f\,x\right)+1\right)}\,\left(\cos\left(e+f\,x\right)+\cos\left(3\,e+3\,f\,x\right)-16\,\sin\left(2\right.\right.}{}$$

input
```
int(x^2*(a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(3/2),x)
```

output
```
(c*(-a*(sin(e + f*x) - 1))^(1/2)*(c*(sin(e + f*x) + 1))^(1/2)*(cos(e + f*x
) + cos(3*e + 3*f*x) - 16*sin(2*e + 2*f*x) + 16*f*x + 16*f*x*cos(2*e + 2*f
*x) + 2*f*x*sin(3*e + 3*f*x) - 2*f^2*x^2*cos(e + f*x) + 2*f*x*sin(e + f*x)
 - 2*f^2*x^2*cos(3*e + 3*f*x) + 8*f^2*x^2*sin(2*e + 2*f*x)))/(8*f^3*(cos(2
*e + 2*f*x) + 1))
```

3.173 $\int x\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}\,dx$

3.173.1 Optimal result

Integrand size = 31, antiderivative size = 168

$$\begin{aligned}\int x\sqrt{a-a\sin(e+fx)}(c&+c\sin(e+fx))^{3/2}\,dx=\frac{c\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{f^2}\\&-\frac{3cx\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{4f}\\&+\frac{c\sin(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{4f^2}\\&+\frac{x\sec(e+fx)\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{5/2}}{2cf}\end{aligned}$$

output
```
1/2*x*sec(f*x+e)*(c+c*sin(f*x+e))^(5/2)*(a-a*sin(f*x+e))^(1/2)/c/f+c*(a-a*
sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/f^2-3/4*c*x*sec(f*x+e)*(a-a*sin(f
*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/f+1/4*c*sin(f*x+e)*(a-a*sin(f*x+e))^(1
/2)*(c+c*sin(f*x+e))^(1/2)/f^2
```

3.173.2 Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.43

$$\int x\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}\,dx=\frac{c\sqrt{c(1+\sin(e+fx))}\sqrt{a-a\sin(e+fx)}(4-fx\cos(2(e+fx))\sec(e+fx)+\sin(e+fx)+$$

$$4f^2$$

input
```
Integrate[x*Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(3/2),x]
```

output
```
(c*Sqrt[c*(1 + Sin[e + f*x])]*Sqrt[a - a*Sin[e + f*x]]*(4 - f*x*Cos[2*(e +
 f*x)]*Sec[e + f*x] + Sin[e + f*x] + 4*f*x*Tan[e + f*x]))/(4*f^2)
```

3.173.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.68, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {5115, 4922, 3042, 3123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a-a\sin(e+fx)}(c\sin(e+fx)+c)^{3/2}\,dx$$

$$\Big\downarrow \text{5115}$$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\int x\cos(e+fx)(\sin(e+fx)c+c)dx$$

$$\Big\downarrow \text{4922}$$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\left(\frac{x(c\sin(e+fx)+c)^2}{2cf}-\frac{\int(\sin(e+fx)c+c)^2dx}{2cf}\right)$$

$$\Big\downarrow \text{3042}$$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\left(\frac{x(c\sin(e+fx)+c)^2}{2cf}-\frac{\int(\sin(e+fx)c+c)^2dx}{2cf}\right)$$

$$\Big\downarrow \text{3123}$$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\left(\frac{x(c\sin(e+fx)+c)^2}{2cf}-\frac{-\frac{2c^2\cos(e+fx)}{f}-\frac{c^2\sin(e+fx)\cos(e+fx)}{2f}+\frac{3c^2x}{2}}{2cf}\right)$$

input
```
Int[x*Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(3/2),x]
```

output
```
Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*((x*(c + c*
Sin[e + f*x])^2)/(2*c*f) - ((3*c^2*x)/2 - (2*c^2*Cos[e + f*x])/f - (c^2*Co
s[e + f*x]*Sin[e + f*x])/(2*f))/(2*c*f))
```

3.173.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3123
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(2*a^2 + b^
2)*(x/2), x] + (-Simp[2*a*b*(Cos[c + d*x]/d), x] - Simp[b^2*Cos[c + d*x]*(S
in[c + d*x]/(2*d)), x]) /; FreeQ[{a, b, c, d}, x]
```

rule 4922
```
Int[Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sin[(c
_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(e + f*x)^m*((a + b*Sin[c + d*x
])^(n + 1)/(b*d*(n + 1))), x] - Simp[f*(m/(b*d*(n + 1)))   Int[(e + f*x)^(m
 - 1)*(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n},
x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 5115
```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPart[m]
*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPa
rt[m]/Cos[e + f*x]^(2*FracPart[m]))   Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
 + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] &&
IGeQ[n - m, 0]
```

3.173.4 Maple [F]

$$\int x(c+c\sin(fx+e))^{\frac{3}{2}}\sqrt{a-\sin(fx+e)\,a}dx$$

input
```
int(x*(c+c*sin(f*x+e))^(3/2)*(a-sin(f*x+e)*a)^(1/2),x)
```

output
```
int(x*(c+c*sin(f*x+e))^(3/2)*(a-sin(f*x+e)*a)^(1/2),x)
```

3.173.5 Fricas [F(-2)]

Exception generated.

$$\int x\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x, algorithm="fr
icas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.173.6 Sympy [F]

$$\int x\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}\,dx = \int x(c(\sin(e+fx)+1))^{\frac{3}{2}}\sqrt{-a(\sin(e+fx)-1)}\,dx$$

input
```
integrate(x*(c+c*sin(f*x+e))**(3/2)*(a-a*sin(f*x+e))**(1/2),x)
```

output
```
Integral(x*(c*(sin(e + f*x) + 1))**(3/2)*sqrt(-a*(sin(e + f*x) - 1)), x)
```

3.173.7 Maxima [F]

$$\int x\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}\,dx=\int\sqrt{-a\sin\left(fx+e\right)+a}(c\sin\left(fx+e\right)+c)^{\frac{3}{2}}x\,dx$$

input
```
integrate(x*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(-a*sin(f*x + e) + a)*(c*sin(f*x + e) + c)^(3/2)*x, x)
```

3.173.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(146) = 292.

Time = 0.34 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.85

$$\int x\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}\,dx=$$
$$-\frac{\left(\frac{8\,c\cos(fx+e)\mathrm{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))\mathrm{sgn}(\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))}{f}+\frac{c\mathrm{sgn}(\cos(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))\mathrm{sgn}(\sin(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e))\sin(2fx+2e}{f}\right.}{}$$

input
```
integrate(x*(c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2),x, algorithm="gi
ac")
```

output
```
-1/8*(8*c*cos(f*x + e)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi
 + 1/2*f*x + 1/2*e))/f + c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/
4*pi + 1/2*f*x + 1/2*e))*sin(2*f*x + 2*e)/f - (pi*c*sgn(cos(-1/4*pi + 1/2*
f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - (pi - 2*f*x - 2*e)*c*s
gn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 2
*c*e*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e
)))*cos(2*f*x + 2*e)/f + 4*(pi*c*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(s
in(-1/4*pi + 1/2*f*x + 1/2*e)) - (pi - 2*f*x - 2*e)*c*sgn(cos(-1/4*pi + 1/
2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*c*e*sgn(cos(-1/4*p
i + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sin(f*x + e)/f)
*sqrt(a)*sqrt(c)/f
```

3.173.9 Mupad [B] (verification not implemented)

Time = 29.20 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.73

$$\int x\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}\,dx =$$

$$-\frac{c\sqrt{-a\,(\sin(e+f\,x)-1)}\,\sqrt{c\,(\sin(e+f\,x)+1)}\,\Big(-16\sin(e+f\,x)^2+\sin(e+f\,x)+\sin(3\,e+3\,f\,x)+}{8\,f^2\,\left(2\sin(e+f\,x)^2\right.}$$

input
```
int(x*(a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(3/2),x)
```

output
```
-(c*(-a*(sin(e + f*x) - 1))^(1/2)*(c*(sin(e + f*x) + 1))^(1/2)*(sin(e + f*
x) + sin(3*e + 3*f*x) - 16*sin(e + f*x)^2 + 8*f*x*sin(2*e + 2*f*x) + 2*f*x
*(2*sin(e/2 + (f*x)/2)^2 - 1) + 2*f*x*(2*sin((3*e)/2 + (3*f*x)/2)^2 - 1) +
 16))/(8*f^2*(2*sin(e + f*x)^2 - 2))
```

3.174 $\int \frac{\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}}{x}\,dx$

3.174.1 Optimal result

Integrand size = 33, antiderivative size = 186

$$\begin{aligned}\int \frac{\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}}{x}\,dx &= c\cos(e)\operatorname{CosIntegral}(fx)\sec(e\\ &+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\\ &+\frac{1}{2}c\operatorname{CosIntegral}(2fx)\sec(e+fx)\sin(2e)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\\ &-c\sec(e+fx)\sin(e)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\operatorname{Si}(fx)\\ &+\frac{1}{2}c\cos(2e)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\operatorname{Si}(2fx)\end{aligned}$$

output
```
c*Ci(f*x)*cos(e)*sec(f*x+e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)+
1/2*c*cos(2*e)*sec(f*x+e)*Si(2*f*x)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e)
)^(1/2)-c*sec(f*x+e)*Si(f*x)*sin(e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e)
)^(1/2)+1/2*c*Ci(2*f*x)*sec(f*x+e)*sin(2*e)*(a-a*sin(f*x+e))^(1/2)*(c+c*si
n(f*x+e))^(1/2)
```

3.174.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.57 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}}{x}\,dx = \frac{ce^{-i(e-fx)}\sqrt{-ice^{-i(e+fx)}\left(i+e^{i(e+fx)}\right)^2}(2e^{ie}\,\text{ExpIntegralEi}}{}$$

input
```
Integrate[(Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(3/2))/x,x]
```

output
```
(c*Sqrt[((-I)*c*(I + E^(I*(e + f*x)))^2)/E^(I*(e + f*x))]*(2*E^(I*e)*ExpIn
tegralEi[(-I)*f*x] + 2*E^((3*I)*e)*ExpIntegralEi[I*f*x] + I*(ExpIntegralEi
[(-2*I)*f*x] - E^((4*I)*e)*ExpIntegralEi[(2*I)*f*x]))*Sqrt[a - a*Sin[e + f
*x]])/(2*Sqrt[2]*E^(I*(e - f*x))*(1 + E^((2*I)*(e + f*x))))
```

3.174.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.42, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {5115, 7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-a\sin(e+fx)}(c\sin(e+fx)+c)^{3/2}}{x}\,dx$$

$$\Big\downarrow \; 5115$$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\int \frac{\cos(e+fx)(\sin(e+fx)c+c)}{x}dx$$

$$\Big\downarrow \; 7292$$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\int \frac{c\cos(e+fx)(\sin(e+fx)+1)}{x}dx$$

$$\Big\downarrow \; 27$$

$$c\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\int \frac{\cos(e+fx)(\sin(e+fx)+1)}{x}dx$$

$$\Big\downarrow \; 7293$$

$$c\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\int\left(\frac{\cos(e+fx)}{x}+\frac{\sin(2e+2fx)}{2x}\right)dx$$

$\big\downarrow$ 2009

$$c\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\left(\frac{1}{2}\sin(2e)\operatorname{CosIntegral}(2fx)+\cos(e)\operatorname{CosIntegral}(fx)-\sin(e)\operatorname{Si}(fx)+\right.$$

input
```
Int[(Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(3/2))/x,x]
```

output
```
c*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*(Cos[e]*C
osIntegral[f*x] + (CosIntegral[2*f*x]*Sin[2*e])/2 - Sin[e]*SinIntegral[f*x
] + (Cos[2*e]*SinIntegral[2*f*x])/2)
```

3.174.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5115
```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPart[m]
*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPa
rt[m]/Cos[e + f*x]^(2*FracPart[m]))   Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
 + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] &&
IGeQ[n - m, 0]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.174. $\int \frac{\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}}{x}\,dx$

3.174.4 Maple [F]

$$\int \frac{(c+c\sin(fx+e))^{\frac{3}{2}}\sqrt{a-\sin(fx+e)\,a}}{x}dx$$

input
```
int((c+c*sin(f*x+e))^(3/2)*(a-sin(f*x+e)*a)^(1/2)/x,x)
```

output
```
int((c+c*sin(f*x+e))^(3/2)*(a-sin(f*x+e)*a)^(1/2)/x,x)
```

3.174.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x,x, algorithm="fr
icas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.174.6 Sympy [F]

$$\int \frac{\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}}{x}\,dx = \int \frac{(c(\sin(e+fx)+1))^{\frac{3}{2}}\sqrt{-a(\sin(e+fx)-1)}}{x}\,dx$$

input
```
integrate((c+c*sin(f*x+e))**(3/2)*(a-a*sin(f*x+e))**(1/2)/x,x)
```

output
```
Integral((c*(sin(e + f*x) + 1))**(3/2)*sqrt(-a*(sin(e + f*x) - 1))/x, x)
```

3.174.7 Maxima [F]

$$\int \frac{\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}}{x}\,dx=\int \frac{\sqrt{-a\sin(fx+e)+a}(c\sin(fx+e)+c)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x,x, algorithm="ma
xima")
```

output
```
integrate(sqrt(-a*sin(f*x + e) + a)*(c*sin(f*x + e) + c)^(3/2)/x, x)
```

3.174.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}}{x}\,dx =$$
$$-\frac{\left(2\,cf\cos\left(e\right)\operatorname{Ci}\left(fx\right)\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\operatorname{sgn}\left(\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)+cf\operatorname{Ci}\left(2\,fx\right)\operatorname{sgn}\left(\cos\left(-\right.\right.$$

input
```
integrate((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x,x, algorithm="gi
ac")
```

output
```
-1/2*(2*c*f*cos(e)*cos_integral(f*x)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*s
gn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + c*f*cos_integral(2*f*x)*sgn(cos(-1/4*
pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(2*e) + c*f*
cos(2*e)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1
/2*e))*sin_integral(2*f*x) - 2*c*f*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn
(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(e)*sin_integral(f*x))*sqrt(a)*sqrt(c)
/f
```

3.174.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a - a\sin(e + fx)}(c + c\sin(e + fx))^{3/2}}{x} \, dx = \int \frac{\sqrt{a - a\sin(e + fx)}\,(c + c\sin(e + fx))^{3/2}}{x} \, dx$$

input
```
int(((a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(3/2))/x,x)
```

output
```
int(((a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(3/2))/x, x)
```

3.175 $\int \frac{\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}}{x^2}\,dx$

3.175.1 Optimal result

Integrand size = 33, antiderivative size = 273

$$\begin{aligned}
&\int \frac{\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}}{x^2}\,dx = \\
&-\frac{c\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{x} \\
&+cf\cos(2e)\operatorname{CosIntegral}(2fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)} \\
&-cf\operatorname{CosIntegral}(fx)\sec(e+fx)\sin(e)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)} \\
&-\frac{c\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\sin(2e+2fx)}{2x} \\
&-cf\cos(e)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\operatorname{Si}(fx) \\
&-cf\sec(e+fx)\sin(2e)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\operatorname{Si}(2fx)
\end{aligned}$$

output
```
-c*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x+c*f*Ci(2*f*x)*cos(2*e)*
sec(f*x+e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)-c*f*cos(e)*sec(f*
x+e)*Si(f*x)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)-c*f*Ci(f*x)*sec
(f*x+e)*sin(e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)-c*f*sec(f*x+e
)*Si(2*f*x)*sin(2*e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)-1/2*c*s
ec(f*x+e)*sin(2*f*x+2*e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x
```

3.175.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.71 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}}{x^2}\,dx = \frac{ce^{-i(e+fx)}\sqrt{-ice^{-i(e+fx)}\left(i+e^{i(e+fx)}\right)^2}\left(-i-2e^{i(e+fx)}-2\right.}{}$$

input
```
Integrate[(Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(3/2))/x^2,x]
```

output
```
(c*Sqrt[((-I)*c*(I + E^(I*(e + f*x)))^2)/E^(I*(e + f*x))]*(-I - 2*E^(I*(e
+ f*x)) - 2*E^((3*I)*(e + f*x)) + I*E^((4*I)*(e + f*x)) - (2*I)*E^(I*(e +
2*f*x))*f*x*ExpIntegralEi[(-I)*f*x] + (2*I)*E^((3*I)*e + (2*I)*f*x)*f*x*Ex
pIntegralEi[I*f*x] + 2*E^((2*I)*f*x)*f*x*ExpIntegralEi[(-2*I)*f*x] + 2*E^(
(2*I)*(2*e + f*x))*f*x*ExpIntegralEi[(2*I)*f*x])*Sqrt[a - a*Sin[e + f*x]])
/(2*Sqrt[2]*E^(I*(e + f*x))*(1 + E^((2*I)*(e + f*x)))*x)
```

3.175.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.39, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {5115, 7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-a\sin(e+fx)}(c\sin(e+fx)+c)^{3/2}}{x^2}dx$$
$$\Big\downarrow \text{5115}$$
$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\int \frac{\cos(e+fx)(\sin(e+fx)c+c)}{x^2}dx$$
$$\Big\downarrow \text{7292}$$
$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\int \frac{c\cos(e+fx)(\sin(e+fx)+1)}{x^2}dx$$
$$\Big\downarrow \text{27}$$
$$c\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\int \frac{\cos(e+fx)(\sin(e+fx)+1)}{x^2}dx$$

$$\Big\downarrow \text{7293}$$

$$c\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\int\left(\frac{\cos(e+fx)}{x^2}+\frac{\sin(2e+2fx)}{2x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$c\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\Big(-f\sin(e)\operatorname{CosIntegral}(fx)+f\cos(2e)\operatorname{CosIntegral}(2fx)-f\sin(2e)\mathrm{Si}$$

input
```
Int[(Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(3/2))/x^2,x]
```

output
```
c*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*(-(Cos[e
+ f*x]/x) + f*Cos[2*e]*CosIntegral[2*f*x] - f*CosIntegral[f*x]*Sin[e] - Si
n[2*e + 2*f*x]/(2*x) - f*Cos[e]*SinIntegral[f*x] - f*Sin[2*e]*SinIntegral[
2*f*x])
```

3.175.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5115
```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPart[m]
*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPa
rt[m]/Cos[e + f*x]^(2*FracPart[m]))   Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
 + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] &&
IGeQ[n - m, 0]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.175. $\int \frac{\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}}{x^2}\,dx$

3.175.4 Maple [F]

$$\int \frac{(c+c\sin(fx+e))^{\frac{3}{2}}\sqrt{a-\sin(fx+e)\,a}}{x^2}dx$$

input
```
int((c+c*sin(f*x+e))^(3/2)*(a-sin(f*x+e)*a)^(1/2)/x^2,x)
```

output
```
int((c+c*sin(f*x+e))^(3/2)*(a-sin(f*x+e)*a)^(1/2)/x^2,x)
```

3.175.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x^2,x, algorithm="
fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.175.6 Sympy [F]

$$\int \frac{\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}}{x^2}\,dx = \int \frac{(c(\sin(e+fx)+1))^{\frac{3}{2}}\sqrt{-a(\sin(e+fx)-1)}}{x^2}\,dx$$

input
```
integrate((c+c*sin(f*x+e))**(3/2)*(a-a*sin(f*x+e))**(1/2)/x**2,x)
```

output
```
Integral((c*(sin(e + f*x) + 1))**(3/2)*sqrt(-a*(sin(e + f*x) - 1))/x**2, x
)
```

3.175.7 Maxima [F]

$$\int \frac{\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}}{x^2}\,dx = \int \frac{\sqrt{-a\sin\left(fx+e\right)+a}\left(c\sin\left(fx+e\right)+c\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x^2,x, algorithm="
maxima")
```

output
```
integrate(sqrt(-a*sin(f*x + e) + a)*(c*sin(f*x + e) + c)^(3/2)/x^2, x)
```

3.175.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 608 vs. 2(247) = 494.

Time = 0.38 (sec) , antiderivative size = 608, normalized size of antiderivative = 2.23

$$\int \frac{\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}}{x^2}\,dx = \text{Too large to display}$$

input
```
integrate((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x^2,x, algorithm="
giac")
```

output
```
-1/2*(pi*c*f^2*cos(2*e)*cos_integral(2*f*x)*sgn(cos(-1/4*pi + 1/2*f*x + 1/
2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - (pi - 2*f*x - 2*e)*c*f^2*cos(2
*e)*cos_integral(2*f*x)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*p
i + 1/2*f*x + 1/2*e)) - 2*c*e*f^2*cos(2*e)*cos_integral(2*f*x)*sgn(cos(-1/
4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - pi*c*f^2*co
s_integral(f*x)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*
f*x + 1/2*e))*sin(e) + (pi - 2*f*x - 2*e)*c*f^2*cos_integral(f*x)*sgn(cos(
-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(e) + 2
*c*e*f^2*cos_integral(f*x)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/
4*pi + 1/2*f*x + 1/2*e))*sin(e) - pi*c*f^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(2*e)*sin_integral(2*f*x) + (p
i - 2*f*x - 2*e)*c*f^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi
 + 1/2*f*x + 1/2*e))*sin(2*e)*sin_integral(2*f*x) + 2*c*e*f^2*sgn(cos(-1/4
*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(2*e)*sin_i
ntegral(2*f*x) - pi*c*f^2*cos(e)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(s
in(-1/4*pi + 1/2*f*x + 1/2*e))*sin_integral(f*x) + (pi - 2*f*x - 2*e)*c*f^
2*cos(e)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1
/2*e))*sin_integral(f*x) + 2*c*e*f^2*cos(e)*sgn(cos(-1/4*pi + 1/2*f*x + 1/
2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin_integral(f*x) - 2*c*f^2*cos(
f*x + e)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x ...
```

3.175.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}}{x^2}\,dx = \int \frac{\sqrt{a-a\,\sin\left(e+f\,x\right)}\,\left(c+c\,\sin\left(e+f\,x\right)\right)^{3/2}}{x^2}\,dx$$

input
```
int(((a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(3/2))/x^2,x)
```

output
```
int(((a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(3/2))/x^2, x)
```

3.176 $\int \frac{\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}}{x^3}\,dx$

3.176.1 Optimal result

Integrand size = 33, antiderivative size = 385

$$\begin{aligned}\int \frac{\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}}{x^3}\,dx &= -\frac{c\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{2x^2} \\ &- \frac{cf\cos(2e+2fx)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}{2x} \\ &- \frac{1}{2}cf^2\cos(e)\operatorname{CosIntegral}(fx)\sec(e \\ &+ fx)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)} - cf^2\operatorname{CosIntegral}(2fx)\sec(e \\ &+ fx)\sin(2e)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)} \\ &- \frac{c\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\sin(2e+2fx)}{4x^2} \\ &+ \frac{1}{2}cf^2\sec(e+fx)\sin(e)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\operatorname{Si}(fx) \\ &- cf^2\cos(2e)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\operatorname{Si}(2fx) \\ &+ \frac{cf\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}\tan(e+fx)}{2x}\end{aligned}$$

output
```
-1/2*c*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x^2-1/2*c*f^2*Ci(f*x)
*cos(e)*sec(f*x+e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)-1/2*c*f*c
os(2*f*x+2*e)*sec(f*x+e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))^(1/2)/x-c
*f^2*cos(2*e)*sec(f*x+e)*Si(2*f*x)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x+e))
^(1/2)+1/2*c*f^2*sec(f*x+e)*Si(f*x)*sin(e)*(a-a*sin(f*x+e))^(1/2)*(c+c*sin
(f*x+e))^(1/2)-c*f^2*Ci(2*f*x)*sec(f*x+e)*sin(2*e)*(a-a*sin(f*x+e))^(1/2)*
(c+c*sin(f*x+e))^(1/2)-1/4*c*sec(f*x+e)*sin(2*f*x+2*e)*(a-a*sin(f*x+e))^(1
/2)*(c+c*sin(f*x+e))^(1/2)/x^2+1/2*c*f*(a-a*sin(f*x+e))^(1/2)*(c+c*sin(f*x
+e))^(1/2)*tan(f*x+e)/x
```

3.176.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.00 (sec) , antiderivative size = 317, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}}{x^3}\,dx = \frac{c^2e^{-2i(e+fx)}\left(i+e^{i(e+fx)}\right)\left(-1+2ie^{i(e+fx)}+2ie^{3i(e+fx)}+\cdots\right.}{\cdots}$$

input
```
Integrate[(Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(3/2))/x^3,x]
```

output
```
(c^2*(I + E^(I*(e + f*x)))*(-1 + (2*I)*E^(I*(e + f*x)) + (2*I)*E^((3*I)*(e
 + f*x)) + E^((4*I)*(e + f*x)) + (2*I)*f*x + 2*E^(I*(e + f*x))*f*x - 2*E^(
(3*I)*(e + f*x))*f*x + (2*I)*E^((4*I)*(e + f*x))*f*x + (2*I)*E^(I*(e + 2*f
*x))*f^2*x^2*ExpIntegralEi[(-I)*f*x] + (2*I)*E^((3*I)*e + (2*I)*f*x)*f^2*x
^2*ExpIntegralEi[I*f*x] - 4*E^((2*I)*f*x)*f^2*x^2*ExpIntegralEi[(-2*I)*f*x
] + 4*E^((2*I)*(2*e + f*x))*f^2*x^2*ExpIntegralEi[(2*I)*f*x])*Sqrt[a - a*S
in[e + f*x]])/(4*Sqrt[2]*E^((2*I)*(e + f*x))*(-I + E^(I*(e + f*x)))*Sqrt[(
(-I)*c*(I + E^(I*(e + f*x)))^2)/E^(I*(e + f*x))]*x^2)
```

3.176.3 Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.39, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {5115, 7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-a\sin(e+fx)}(c\sin(e+fx)+c)^{3/2}}{x^3}\,dx$$

$\Big\downarrow$ 5115

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\int \frac{\cos(e+fx)(\sin(e+fx)c+c)}{x^3}dx$$

$\Big\downarrow$ 7292

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\int \frac{c\cos(e+fx)(\sin(e+fx)+1)}{x^3}dx$$

$\Big\downarrow$ 27

$$c\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\int \frac{\cos(e+fx)(\sin(e+fx)+1)}{x^3}dx$$

$\Big\downarrow$ 7293

$$c\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\int \left(\frac{\cos(e+fx)}{x^3}+\frac{\sin(2e+2fx)}{2x^3}\right)dx$$

$\Big\downarrow$ 2009

$$c\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}\left(-f^2\sin(2e)\,\text{CosIntegral}(2fx)-\frac{1}{2}f^2\cos(e)\,\text{CosIntegral}(fx)+\frac{1}{2}f^2\sin$$

input
```
Int[(Sqrt[a - a*Sin[e + f*x]]*(c + c*Sin[e + f*x])^(3/2))/x^3,x]
```

output
```
c*Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]]*(-1/2*Cos
[e + f*x]/x^2 - (f*Cos[2*e + 2*f*x])/(2*x) - (f^2*Cos[e]*CosIntegral[f*x])
/2 - f^2*CosIntegral[2*f*x]*Sin[2*e] + (f*Sin[e + f*x])/(2*x) - Sin[2*e +
2*f*x]/(4*x^2) + (f^2*Sin[e]*SinIntegral[f*x])/2 - f^2*Cos[2*e]*SinIntegra
l[2*f*x])
```

3.176.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5115
```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPart[m]
*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPa
rt[m]/Cos[e + f*x]^(2*FracPart[m]))   Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
 + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] &&
IGeQ[n - m, 0]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.176.4 Maple [F]

$$\int \frac{\left(c+c\sin \left(fx+e\right)\right)^{\frac{3}{2}} \sqrt{a-\sin \left(fx+e\right) a}}{x^{3}}dx$$

input
```
int((c+c*sin(f*x+e))^(3/2)*(a-sin(f*x+e)*a)^(1/2)/x^3,x)
```

output
```
int((c+c*sin(f*x+e))^(3/2)*(a-sin(f*x+e)*a)^(1/2)/x^3,x)
```

3.176.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x^3,x, algorithm="
fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.176.6 Sympy [F]

$$\int \frac{\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}}{x^3}\,dx=\int \frac{(c(\sin(e+fx)+1))^{\frac{3}{2}}\sqrt{-a(\sin(e+fx)-1)}}{x^3}\,dx$$

input
```
integrate((c+c*sin(f*x+e))**(3/2)*(a-a*sin(f*x+e))**(1/2)/x**3,x)
```

output
```
Integral((c*(sin(e + f*x) + 1))**(3/2)*sqrt(-a*(sin(e + f*x) - 1))/x**3, x
)
```

3.176.7 Maxima [F]

$$\int \frac{\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}}{x^3}\,dx=\int \frac{\sqrt{-a\sin(fx+e)+a}(c\sin(fx+e)+c)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x^3,x, algorithm="
maxima")
```

output
```
integrate(sqrt(-a*sin(f*x + e) + a)*(c*sin(f*x + e) + c)^(3/2)/x^3, x)
```

3.176.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1502 vs. 2(341) = 682.

Time = 0.51 (sec) , antiderivative size = 1502, normalized size of antiderivative = 3.90

$$\int \frac{\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}}{x^3}\,dx=\text{Too large to display}$$

input
```
integrate((c+c*sin(f*x+e))^(3/2)*(a-a*sin(f*x+e))^(1/2)/x^3,x, algorithm="
giac")
```

output
```
1/2*(pi^2*c*f^3*cos(e)*cos_integral(f*x)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*pi*(pi - 2*f*x - 2*e)*c*f^3*cos
(e)*cos_integral(f*x)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi
+ 1/2*f*x + 1/2*e)) + (pi - 2*f*x - 2*e)^2*c*f^3*cos(e)*cos_integral(f*x)*
sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) -
4*pi*c*e*f^3*cos(e)*cos_integral(f*x)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*
sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 4*(pi - 2*f*x - 2*e)*c*e*f^3*cos(e)*
cos_integral(f*x)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/
2*f*x + 1/2*e)) + 4*c*e^2*f^3*cos(e)*cos_integral(f*x)*sgn(cos(-1/4*pi + 1
/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) + 2*pi^2*c*f^3*cos_in
tegral(2*f*x)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*
x + 1/2*e))*sin(2*e) - 4*pi*(pi - 2*f*x - 2*e)*c*f^3*cos_integral(2*f*x)*s
gn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin
(2*e) + 2*(pi - 2*f*x - 2*e)^2*c*f^3*cos_integral(2*f*x)*sgn(cos(-1/4*pi +
 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(2*e) - 8*pi*c*e
*f^3*cos_integral(2*f*x)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*
pi + 1/2*f*x + 1/2*e))*sin(2*e) + 8*(pi - 2*f*x - 2*e)*c*e*f^3*cos_integra
l(2*f*x)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1
/2*e))*sin(2*e) + 8*c*e^2*f^3*cos_integral(2*f*x)*sgn(cos(-1/4*pi + 1/2*f*
x + 1/2*e))*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*sin(2*e) + 2*pi^2*c*f^3...
```

3.176.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a-a\sin(e+fx)}(c+c\sin(e+fx))^{3/2}}{x^3}\,dx=\int \frac{\sqrt{a-a\,\sin\left(e+f\,x\right)}\,{\left(c+c\,\sin\left(e+f\,x\right)\right)}^{3/2}}{x^3}\,dx$$

input
```
int(((a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(3/2))/x^3,x)
```

output
```
int(((a - a*sin(e + f*x))^(1/2)*(c + c*sin(e + f*x))^(3/2))/x^3, x)
```

3.177 $\int \frac{(g+hx)^3\sqrt{a-a\sin(e+fx)}}{\sqrt{c+c\sin(e+fx)}}\,dx$

3.177.1 Optimal result

Integrand size = 37, antiderivative size = 767

$$\int \frac{(g+hx)^3\sqrt{a-a\sin(e+fx)}}{\sqrt{c+c\sin(e+fx)}}\,dx = -\frac{ia(g+hx)^4\cos(e+fx)}{4h\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{2ia(g+hx)^3\arctan\left(e^{i(e+fx)}\right)\cos(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} + \frac{a(g+hx)^3\cos(e+fx)\log\left(1+e^{2i(e+fx)}\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} + \frac{3iah(g+hx)^2\cos(e+fx)\operatorname{PolyLog}\left(2,-ie^{i(e+fx)}\right)}{f^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{3iah(g+hx)^2\cos(e+fx)\operatorname{PolyLog}\left(2,ie^{i(e+fx)}\right)}{f^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{3iah(g+hx)^2\cos(e+fx)\operatorname{PolyLog}\left(2,-e^{2i(e+fx)}\right)}{2f^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{6ah^2(g+hx)\cos(e+fx)\operatorname{PolyLog}\left(3,-ie^{i(e+fx)}\right)}{f^3\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} + \frac{6ah^2(g+hx)\cos(e+fx)\operatorname{PolyLog}\left(3,ie^{i(e+fx)}\right)}{f^3\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} + \frac{3ah^2(g+hx)\cos(e+fx)\operatorname{PolyLog}\left(3,-e^{2i(e+fx)}\right)}{2f^3\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{6iah^3\cos(e+fx)\operatorname{PolyLog}\left(4,-ie^{i(e+fx)}\right)}{f^4\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} + \frac{6iah^3\cos(e+fx)\operatorname{PolyLog}\left(4,ie^{i(e+fx)}\right)}{f^4\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} + \frac{3iah^3\cos(e+fx)\operatorname{PolyLog}\left(4,-e^{2i(e+fx)}\right)}{4f^4\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

output
```
-1/4*I*a*(h*x+g)^4*cos(f*x+e)/h/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1
/2)-2*I*a*(h*x+g)^3*arctan(exp(I*(f*x+e)))*cos(f*x+e)/f/(a-a*sin(f*x+e))^(
1/2)/(c+c*sin(f*x+e))^(1/2)+a*(h*x+g)^3*cos(f*x+e)*ln(1+exp(2*I*(f*x+e)))/
f/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)+3*I*a*h*(h*x+g)^2*cos(f*x+
e)*polylog(2,-I*exp(I*(f*x+e)))/f^2/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e)
)^(1/2)-3*I*a*h*(h*x+g)^2*cos(f*x+e)*polylog(2,I*exp(I*(f*x+e)))/f^2/(a-a*
sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)-3/2*I*a*h*(h*x+g)^2*cos(f*x+e)*po
lylog(2,-exp(2*I*(f*x+e)))/f^2/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/
2)-6*a*h^2*(h*x+g)*cos(f*x+e)*polylog(3,-I*exp(I*(f*x+e)))/f^3/(a-a*sin(f*
x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)+6*a*h^2*(h*x+g)*cos(f*x+e)*polylog(3,I*
exp(I*(f*x+e)))/f^3/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)+3/2*a*h^
2*(h*x+g)*cos(f*x+e)*polylog(3,-exp(2*I*(f*x+e)))/f^3/(a-a*sin(f*x+e))^(1/
2)/(c+c*sin(f*x+e))^(1/2)-6*I*a*h^3*cos(f*x+e)*polylog(4,-I*exp(I*(f*x+e))
)/f^4/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)+6*I*a*h^3*cos(f*x+e)*p
olylog(4,I*exp(I*(f*x+e)))/f^4/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/
2)+3/4*I*a*h^3*cos(f*x+e)*polylog(4,-exp(2*I*(f*x+e)))/f^4/(a-a*sin(f*x+e)
)^(1/2)/(c+c*sin(f*x+e))^(1/2)
```

3.177.2 Mathematica [A] (verified)

Time = 2.85 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.32

$$\int \frac{(g+hx)^3\sqrt{a-a\sin(e+fx)}}{\sqrt{c+c\sin(e+fx)}}\,dx$$
$$= \frac{\left(\frac{1}{4}+\frac{i}{4}\right)e^{-\frac{1}{2}i(e+fx)}\left(i+e^{i(e+fx)}\right)\left(\frac{(g+hx)^4}{h}-\frac{8i(g+hx)^3\log\left(1+ie^{-i(e+fx)}\right)}{f}+\frac{24h\left(f^2(g+hx)^2\operatorname{PolyLog}\left(2,-ie^{-i(e+fx)}\right)-2h\left(if(g\right.\right.}{}}{\sqrt{2}\sqrt{-ice^{-i(e+fx)}\left(i+e^{i(e+fx)}\right)^2}\left(\cos\left(\frac{1}{2}(e+fx)\right)-\mathrm{si}\right.}$$

input
```
Integrate[((g + h*x)^3*Sqrt[a - a*Sin[e + f*x]])/Sqrt[c + c*Sin[e + f*x]],
x]
```

output
```
((1/4 + I/4)*(I + E^(I*(e + f*x)))*((g + h*x)^4/h - ((8*I)*(g + h*x)^3*Log
[1 + I/E^(I*(e + f*x))])/f + (24*h*(f^2*(g + h*x)^2*PolyLog[2, (-I)/E^(I*(
e + f*x))] - 2*h*(I*f*(g + h*x)*PolyLog[3, (-I)/E^(I*(e + f*x))] + h*PolyL
og[4, (-I)/E^(I*(e + f*x))])))/f^4)*Sqrt[a - a*Sin[e + f*x]])/(Sqrt[2]*E^(
(I/2)*(e + f*x))*Sqrt[((-I)*c*(I + E^(I*(e + f*x)))^2)/E^(I*(e + f*x))]*(C
os[(e + f*x)/2] - Sin[(e + f*x)/2]))
```

3.177.3 Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 372, normalized size of antiderivative = 0.49, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {5115, 7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(g+hx)^3\sqrt{a-a\sin(e+fx)}}{\sqrt{c\sin(e+fx)+c}}dx$$

$$\Big\downarrow \text{5115}$$

$$\frac{\cos(e+fx)\int (g+hx)^3\sec(e+fx)(a-a\sin(e+fx))dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}$$

$$\Big\downarrow \text{7292}$$

$$\frac{\cos(e+fx)\int a(g+hx)^3\sec(e+fx)(1-\sin(e+fx))dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\cos(e+fx)\int (g+hx)^3\sec(e+fx)(1-\sin(e+fx))dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}$$

$$\Big\downarrow \text{7293}$$

$$\frac{a\cos(e+fx)\int \left((g+hx)^3\sec(e+fx)-(g+hx)^3\tan(e+fx)\right)dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\cos(e+fx)\left(-\frac{2i(g+hx)^3\arctan\left(e^{i(e+fx)}\right)}{f}-\frac{6ih^3\operatorname{PolyLog}\left(4,-ie^{i(e+fx)}\right)}{f^4}+\frac{6ih^3\operatorname{PolyLog}\left(4,ie^{i(e+fx)}\right)}{f^4}+\frac{3ih^3\operatorname{PolyLog}\left(4,-e^{2i(e+fx)}\right)}{4f^4}\right.}{}$$

input
```
Int[((g + h*x)^3*Sqrt[a - a*Sin[e + f*x]])/Sqrt[c + c*Sin[e + f*x]],x]
```

output
```
(a*Cos[e + f*x]*(((-1/4*I)*(g + h*x)^4)/h - ((2*I)*(g + h*x)^3*ArcTan[E^(I
*(e + f*x))])/f + ((g + h*x)^3*Log[1 + E^((2*I)*(e + f*x))])/f + ((3*I)*h*
(g + h*x)^2*PolyLog[2, (-I)*E^(I*(e + f*x))])/f^2 - ((3*I)*h*(g + h*x)^2*P
olyLog[2, I*E^(I*(e + f*x))])/f^2 - (((3*I)/2)*h*(g + h*x)^2*PolyLog[2, -E
^((2*I)*(e + f*x))])/f^2 - (6*h^2*(g + h*x)*PolyLog[3, (-I)*E^(I*(e + f*x)
)])/f^3 + (6*h^2*(g + h*x)*PolyLog[3, I*E^(I*(e + f*x))])/f^3 + (3*h^2*(g
+ h*x)*PolyLog[3, -E^((2*I)*(e + f*x))])/(2*f^3) - ((6*I)*h^3*PolyLog[4, (
-I)*E^(I*(e + f*x))])/f^4 + ((6*I)*h^3*PolyLog[4, I*E^(I*(e + f*x))])/f^4
+ (((3*I)/4)*h^3*PolyLog[4, -E^((2*I)*(e + f*x))])/f^4))/(Sqrt[a - a*Sin[e
 + f*x]]*Sqrt[c + c*Sin[e + f*x]])
```

3.177.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5115
```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPart[m]
*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPa
rt[m]/Cos[e + f*x]^(2*FracPart[m]))   Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
 + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] &&
IGeQ[n - m, 0]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.177. $\int \frac{(g+hx)^3\sqrt{a-a\sin(e+fx)}}{\sqrt{c+c\sin(e+fx)}}\,dx$

3.177.4 Maple [F]

$$\int \frac{(hx+g)^3\sqrt{a-\sin(fx+e)\,a}}{\sqrt{c+c\sin(fx+e)}}dx$$

input
```
int((h*x+g)^3*(a-sin(f*x+e)*a)^(1/2)/(c+c*sin(f*x+e))^(1/2),x)
```

output
```
int((h*x+g)^3*(a-sin(f*x+e)*a)^(1/2)/(c+c*sin(f*x+e))^(1/2),x)
```

3.177.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(g+hx)^3\sqrt{a-a\sin(e+fx)}}{\sqrt{c+c\sin(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((h*x+g)^3*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x, algor
ithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.177.6 Sympy [F]

$$\int \frac{(g+hx)^3\sqrt{a-a\sin(e+fx)}}{\sqrt{c+c\sin(e+fx)}}\,dx = \int \frac{\sqrt{-a(\sin(e+fx)-1)}(g+hx)^3}{\sqrt{c(\sin(e+fx)+1)}}\,dx$$

input
```
integrate((h*x+g)**3*(a-a*sin(f*x+e))**(1/2)/(c+c*sin(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(-a*(sin(e + f*x) - 1))*(g + h*x)**3/sqrt(c*(sin(e + f*x) + 1
)), x)
```

3.177.7 Maxima [F]

$$\int \frac{(g+hx)^3\sqrt{a-a\sin(e+fx)}}{\sqrt{c+c\sin(e+fx)}}\,dx = \int \frac{(hx+g)^3\sqrt{-a\sin(fx+e)+a}}{\sqrt{c\sin(fx+e)+c}}\,dx$$

input
```
integrate((h*x+g)^3*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x, algor
ithm="maxima")
```

output
```
integrate((h*x + g)^3*sqrt(-a*sin(f*x + e) + a)/sqrt(c*sin(f*x + e) + c),
x)
```

3.177.8 Giac [F]

$$\int \frac{(g+hx)^3\sqrt{a-a\sin(e+fx)}}{\sqrt{c+c\sin(e+fx)}}\,dx = \int \frac{(hx+g)^3\sqrt{-a\sin(fx+e)+a}}{\sqrt{c\sin(fx+e)+c}}\,dx$$

input
```
integrate((h*x+g)^3*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x, algor
ithm="giac")
```

output
```
integrate((h*x + g)^3*sqrt(-a*sin(f*x + e) + a)/sqrt(c*sin(f*x + e) + c),
x)
```

3.177.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(g+hx)^3\sqrt{a-a\sin(e+fx)}}{\sqrt{c+c\sin(e+fx)}}\,dx = \int \frac{(g+h\,x)^3\,\sqrt{a-a\,\sin(e+f\,x)}}{\sqrt{c+c\,\sin(e+f\,x)}}\,dx$$

input
```
int(((g + h*x)^3*(a - a*sin(e + f*x))^(1/2))/(c + c*sin(e + f*x))^(1/2),x)
```

output
```
int(((g + h*x)^3*(a - a*sin(e + f*x))^(1/2))/(c + c*sin(e + f*x))^(1/2), x
)
```

3.178 $\int \frac{(g+hx)^2\sqrt{a-a\sin(e+fx)}}{\sqrt{c+c\sin(e+fx)}}\,dx$

3.178.1 Optimal result

Integrand size = 37, antiderivative size = 555

$$\begin{aligned}\int \frac{(g+hx)^2\sqrt{a-a\sin(e+fx)}}{\sqrt{c+c\sin(e+fx)}}\,dx = &-\frac{ia(g+hx)^3\cos(e+fx)}{3h\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} \\ &-\frac{2ia(g+hx)^2\arctan\left(e^{i(e+fx)}\right)\cos(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} \\ &+\frac{a(g+hx)^2\cos(e+fx)\log\left(1+e^{2i(e+fx)}\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} \\ &+\frac{2iah(g+hx)\cos(e+fx)\operatorname{PolyLog}\left(2,-ie^{i(e+fx)}\right)}{f^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} \\ &-\frac{2iah(g+hx)\cos(e+fx)\operatorname{PolyLog}\left(2,ie^{i(e+fx)}\right)}{f^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} \\ &-\frac{iah(g+hx)\cos(e+fx)\operatorname{PolyLog}\left(2,-e^{2i(e+fx)}\right)}{f^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} \\ &-\frac{2ah^2\cos(e+fx)\operatorname{PolyLog}\left(3,-ie^{i(e+fx)}\right)}{f^3\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} \\ &+\frac{2ah^2\cos(e+fx)\operatorname{PolyLog}\left(3,ie^{i(e+fx)}\right)}{f^3\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} \\ &+\frac{ah^2\cos(e+fx)\operatorname{PolyLog}\left(3,-e^{2i(e+fx)}\right)}{2f^3\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}\end{aligned}$$

output
```
-1/3*I*a*(h*x+g)^3*cos(f*x+e)/h/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1
/2)-2*I*a*(h*x+g)^2*arctan(exp(I*(f*x+e)))*cos(f*x+e)/f/(a-a*sin(f*x+e))^(
1/2)/(c+c*sin(f*x+e))^(1/2)+a*(h*x+g)^2*cos(f*x+e)*ln(1+exp(2*I*(f*x+e)))/
f/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)+2*I*a*h*(h*x+g)*cos(f*x+e)
*polylog(2,-I*exp(I*(f*x+e)))/f^2/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^
(1/2)-2*I*a*h*(h*x+g)*cos(f*x+e)*polylog(2,I*exp(I*(f*x+e)))/f^2/(a-a*sin(
f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)-I*a*h*(h*x+g)*cos(f*x+e)*polylog(2,-e
xp(2*I*(f*x+e)))/f^2/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)-2*a*h^2
*cos(f*x+e)*polylog(3,-I*exp(I*(f*x+e)))/f^3/(a-a*sin(f*x+e))^(1/2)/(c+c*s
in(f*x+e))^(1/2)+2*a*h^2*cos(f*x+e)*polylog(3,I*exp(I*(f*x+e)))/f^3/(a-a*s
in(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)+1/2*a*h^2*cos(f*x+e)*polylog(3,-ex
p(2*I*(f*x+e)))/f^3/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)
```

3.178.2 Mathematica [A] (verified)

Time = 2.50 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.35

$$\int \frac{(g+hx)^2\sqrt{a-a\sin(e+fx)}}{\sqrt{c+c\sin(e+fx)}}\,dx$$
$$= \frac{\sqrt{2}\left(i+e^{i(e+fx)}\right)\left(f^2(g+hx)^2\left(f(g+hx)-6ih\log\left(1+ie^{-i(e+fx)}\right)\right)+12fh^2(g+hx)\operatorname{PolyLog}\left(2,-ie^{-i(\cdots}\right.\right.}{3\left(-i+e^{i(e+fx)}\right)\sqrt{-ice^{-i(e+fx)}\left(i+e^{i(e+fx)}\right)^2}f\cdots}$$

input
```
Integrate[((g + h*x)^2*Sqrt[a - a*Sin[e + f*x]])/Sqrt[c + c*Sin[e + f*x]],
x]
```

output
```
(Sqrt[2]*(I + E^(I*(e + f*x)))*(f^2*(g + h*x)^2*(f*(g + h*x) - (6*I)*h*Log
[1 + I/E^(I*(e + f*x))]) + 12*f*h^2*(g + h*x)*PolyLog[2, (-I)/E^(I*(e + f*
x))] - (12*I)*h^3*PolyLog[3, (-I)/E^(I*(e + f*x))])*Sqrt[a - a*Sin[e + f*x
]])/(3*(-I + E^(I*(e + f*x)))*Sqrt[((-I)*c*(I + E^(I*(e + f*x)))^2)/E^(I*(
e + f*x))]*f^3*h)
```

3.178.3 Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.48, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {5115, 7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(g+hx)^2\sqrt{a-a\sin(e+fx)}}{\sqrt{c\sin(e+fx)+c}}dx$$

$$\Big\downarrow \text{5115}$$

$$\frac{\cos(e+fx)\int (g+hx)^2\sec(e+fx)(a-a\sin(e+fx))dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}$$

$$\Big\downarrow \text{7292}$$

$$\frac{\cos(e+fx)\int a(g+hx)^2\sec(e+fx)(1-\sin(e+fx))dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\cos(e+fx)\int (g+hx)^2\sec(e+fx)(1-\sin(e+fx))dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}$$

$$\Big\downarrow \text{7293}$$

$$\frac{a\cos(e+fx)\int \left((g+hx)^2\sec(e+fx)-(g+hx)^2\tan(e+fx)\right)dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\cos(e+fx)\left(-\frac{2i(g+hx)^2\arctan\left(e^{i(e+fx)}\right)}{f}-\frac{2h^2\text{PolyLog}\left(3,-ie^{i(e+fx)}\right)}{f^3}+\frac{2h^2\text{PolyLog}\left(3,ie^{i(e+fx)}\right)}{f^3}+\frac{h^2\text{PolyLog}\left(3,-e^{2i(e+fx)}\right)}{2f^3}+\right.}{\sqrt{a-a\sin(e-}}$$

input
```
Int[((g + h*x)^2*Sqrt[a - a*Sin[e + f*x]])/Sqrt[c + c*Sin[e + f*x]],x]
```

output
```
(a*Cos[e + f*x]*(((-1/3*I)*(g + h*x)^3)/h - ((2*I)*(g + h*x)^2*ArcTan[E^(I
*(e + f*x))])/f + ((g + h*x)^2*Log[1 + E^((2*I)*(e + f*x))])/f + ((2*I)*h*
(g + h*x)*PolyLog[2, (-I)*E^(I*(e + f*x))])/f^2 - ((2*I)*h*(g + h*x)*PolyL
og[2, I*E^(I*(e + f*x))])/f^2 - (I*h*(g + h*x)*PolyLog[2, -E^((2*I)*(e + f
*x))])/f^2 - (2*h^2*PolyLog[3, (-I)*E^(I*(e + f*x))])/f^3 + (2*h^2*PolyLog
[3, I*E^(I*(e + f*x))])/f^3 + (h^2*PolyLog[3, -E^((2*I)*(e + f*x))])/(2*f^
3)))/(Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])
```

3.178.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5115
```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPart[m]
*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPa
rt[m]/Cos[e + f*x]^(2*FracPart[m]))   Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
 + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] &&
IGeQ[n - m, 0]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.178. $\int \frac{(g+hx)^2\sqrt{a-a\sin(e+fx)}}{\sqrt{c+c\sin(e+fx)}}\,dx$

3.178.4 Maple [F]

$$\int \frac{(hx+g)^2 \sqrt{a-\sin(fx+e)\,a}}{\sqrt{c+c\sin(fx+e)}}dx$$

input
```
int((h*x+g)^2*(a-sin(f*x+e)*a)^(1/2)/(c+c*sin(f*x+e))^(1/2),x)
```

output
```
int((h*x+g)^2*(a-sin(f*x+e)*a)^(1/2)/(c+c*sin(f*x+e))^(1/2),x)
```

3.178.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(g+hx)^2\sqrt{a-a\sin(e+fx)}}{\sqrt{c+c\sin(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((h*x+g)^2*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x, algor
ithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.178.6 Sympy [F]

$$\int \frac{(g+hx)^2\sqrt{a-a\sin(e+fx)}}{\sqrt{c+c\sin(e+fx)}}\,dx = \int \frac{\sqrt{-a\left(\sin\left(e+fx\right)-1\right)}\left(g+hx\right)^2}{\sqrt{c\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

input
```
integrate((h*x+g)**2*(a-a*sin(f*x+e))**(1/2)/(c+c*sin(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(-a*(sin(e + f*x) - 1))*(g + h*x)**2/sqrt(c*(sin(e + f*x) + 1
)), x)
```

3.178.7 Maxima [F]

$$\int \frac{(g+hx)^2\sqrt{a-a\sin(e+fx)}}{\sqrt{c+c\sin(e+fx)}}\,dx = \int \frac{(hx+g)^2\sqrt{-a\sin(fx+e)+a}}{\sqrt{c\sin(fx+e)+c}}\,dx$$

input
```
integrate((h*x+g)^2*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x, algor
ithm="maxima")
```

output
```
integrate((h*x + g)^2*sqrt(-a*sin(f*x + e) + a)/sqrt(c*sin(f*x + e) + c),
x)
```

3.178.8 Giac [F]

$$\int \frac{(g+hx)^2\sqrt{a-a\sin(e+fx)}}{\sqrt{c+c\sin(e+fx)}}\,dx = \int \frac{(hx+g)^2\sqrt{-a\sin(fx+e)+a}}{\sqrt{c\sin(fx+e)+c}}\,dx$$

input
```
integrate((h*x+g)^2*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x, algor
ithm="giac")
```

output
```
integrate((h*x + g)^2*sqrt(-a*sin(f*x + e) + a)/sqrt(c*sin(f*x + e) + c),
x)
```

3.178.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(g+hx)^2\sqrt{a-a\sin(e+fx)}}{\sqrt{c+c\sin(e+fx)}}\,dx = \int \frac{(g+h\,x)^2\,\sqrt{a-a\,\sin(e+f\,x)}}{\sqrt{c+c\,\sin(e+f\,x)}}\,dx$$

input
```
int(((g + h*x)^2*(a - a*sin(e + f*x))^(1/2))/(c + c*sin(e + f*x))^(1/2),x)
```

output
```
int(((g + h*x)^2*(a - a*sin(e + f*x))^(1/2))/(c + c*sin(e + f*x))^(1/2), x
)
```

3.179 $\int \frac{(g+hx)\sqrt{a-a\sin(e+fx)}}{\sqrt{c+c\sin(e+fx)}}\,dx$

3.179.1 Optimal result

Integrand size = 35, antiderivative size = 355

$$\begin{aligned}\int \frac{(g+hx)\sqrt{a-a\sin(e+fx)}}{\sqrt{c+c\sin(e+fx)}}\,dx =& -\frac{ia(g+hx)^2\cos(e+fx)}{2h\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} \\ &- \frac{2ia(g+hx)\arctan\left(e^{i(e+fx)}\right)\cos(e+fx)}{f\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} \\ &+ \frac{a(g+hx)\cos(e+fx)\log\left(1+e^{2i(e+fx)}\right)}{f\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} \\ &+ \frac{iah\cos(e+fx)\operatorname{PolyLog}\left(2,-ie^{i(e+fx)}\right)}{f^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} \\ &- \frac{iah\cos(e+fx)\operatorname{PolyLog}\left(2,ie^{i(e+fx)}\right)}{f^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} \\ &- \frac{iah\cos(e+fx)\operatorname{PolyLog}\left(2,-e^{2i(e+fx)}\right)}{2f^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}\end{aligned}$$

output
```
-1/2*I*a*(h*x+g)^2*cos(f*x+e)/h/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1
/2)-2*I*a*(h*x+g)*arctan(exp(I*(f*x+e)))*cos(f*x+e)/f/(a-a*sin(f*x+e))^(1/
2)/(c+c*sin(f*x+e))^(1/2)+a*(h*x+g)*cos(f*x+e)*ln(1+exp(2*I*(f*x+e)))/f/(a
-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)+I*a*h*cos(f*x+e)*polylog(2,-I*
exp(I*(f*x+e)))/f^2/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)-I*a*h*co
s(f*x+e)*polylog(2,I*exp(I*(f*x+e)))/f^2/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f
*x+e))^(1/2)-1/2*I*a*h*cos(f*x+e)*polylog(2,-exp(2*I*(f*x+e)))/f^2/(a-a*si
n(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)
```

3.179.2 Mathematica [A] (verified)

Time = 2.03 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.43

$$\int \frac{(g+hx)\sqrt{a-a\sin(e+fx)}}{\sqrt{c+c\sin(e+fx)}}\,dx$$
$$= \frac{\left(i+e^{i(e+fx)}\right)\left(f\left(fx(2g+hx)-4i(g+hx)\log\left(1+ie^{-i(e+fx)}\right)\right)+4h\,\mathrm{PolyLog}\left(2,-ie^{-i(e+fx)}\right)\right)\sqrt{a-a\,\mathrm{s}}}{\sqrt{2}\left(-i+e^{i(e+fx)}\right)\sqrt{-ice^{-i(e+fx)}\left(i+e^{i(e+fx)}\right)^2}f^2}$$

input
```
Integrate[((g + h*x)*Sqrt[a - a*Sin[e + f*x]])/Sqrt[c + c*Sin[e + f*x]],x]
```

output
```
((I + E^(I*(e + f*x)))*(f*(f*x*(2*g + h*x) - (4*I)*(g + h*x)*Log[1 + I/E^(
I*(e + f*x))]) + 4*h*PolyLog[2, (-I)/E^(I*(e + f*x))])*Sqrt[a - a*Sin[e +
f*x]])/(Sqrt[2]*(-I + E^(I*(e + f*x)))*Sqrt[((-I)*c*(I + E^(I*(e + f*x)))^
2)/E^(I*(e + f*x))]*f^2)
```

3.179.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.50, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5115, 7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(g+hx)\sqrt{a-a\sin(e+fx)}}{\sqrt{c\sin(e+fx)+c}}\,dx$$
$$\Big\downarrow \; 5115$$
$$\frac{\cos(e+fx)\int (g+hx)\sec(e+fx)(a-a\sin(e+fx))dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}$$
$$\Big\downarrow \; 7292$$
$$\frac{\cos(e+fx)\int a(g+hx)\sec(e+fx)(1-\sin(e+fx))dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}$$
$$\Big\downarrow \; 27$$
$$\frac{a\cos(e+fx)\int (g+hx)\sec(e+fx)(1-\sin(e+fx))dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}$$

$$\Big\downarrow \; 7293$$

$$\frac{a\cos(e+fx)\int((g+hx)\sec(e+fx)-(g+hx)\tan(e+fx))dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}$$

$$\Big\downarrow \; 2009$$

$$\frac{a\cos(e+fx)\left(-\frac{2i(g+hx)\arctan\left(e^{i(e+fx)}\right)}{f}+\frac{ih\,\text{PolyLog}\left(2,-ie^{i(e+fx)}\right)}{f^2}-\frac{ih\,\text{PolyLog}\left(2,ie^{i(e+fx)}\right)}{f^2}-\frac{ih\,\text{PolyLog}\left(2,-e^{2i(e+fx)}\right)}{2f^2}+\frac{(g\cdots}{}\right.}{\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}$$

input
```
Int[((g + h*x)*Sqrt[a - a*Sin[e + f*x]])/Sqrt[c + c*Sin[e + f*x]],x]
```

output
```
(a*Cos[e + f*x]*(((-1/2*I)*(g + h*x)^2)/h - ((2*I)*(g + h*x)*ArcTan[E^(I*(
e + f*x))])/f + ((g + h*x)*Log[1 + E^((2*I)*(e + f*x))])/f + (I*h*PolyLog[
2, (-I)*E^(I*(e + f*x))])/f^2 - (I*h*PolyLog[2, I*E^(I*(e + f*x))])/f^2 -
((I/2)*h*PolyLog[2, -E^((2*I)*(e + f*x))])/f^2))/(Sqrt[a - a*Sin[e + f*x]]
*Sqrt[c + c*Sin[e + f*x]])
```

3.179.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5115
```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPart[m]
*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPa
rt[m]/Cos[e + f*x]^(2*FracPart[m]))   Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
 + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] &&
IGeQ[n - m, 0]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

3.179. $\int \frac{(g+hx)\sqrt{a-a\sin(e+fx)}}{\sqrt{c+c\sin(e+fx)}}\,dx$

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.179.4 Maple [F]

$$\int \frac{(hx+g)\sqrt{a-\sin(fx+e)\,a}}{\sqrt{c+c\sin(fx+e)}}dx$$

input
```
int((h*x+g)*(a-sin(f*x+e)*a)^(1/2)/(c+c*sin(f*x+e))^(1/2),x)
```

output
```
int((h*x+g)*(a-sin(f*x+e)*a)^(1/2)/(c+c*sin(f*x+e))^(1/2),x)
```

3.179.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{(g+hx)\sqrt{a-a\sin(e+fx)}}{\sqrt{c+c\sin(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((h*x+g)*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x, algorit
hm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.179.6 Sympy [F]

$$\int \frac{(g+hx)\sqrt{a-a\sin(e+fx)}}{\sqrt{c+c\sin(e+fx)}}\,dx = \int \frac{\sqrt{-a\,(\sin(e+fx)-1)}(g+hx)}{\sqrt{c\,(\sin(e+fx)+1)}}\,dx$$

input
```
integrate((h*x+g)*(a-a*sin(f*x+e))**(1/2)/(c+c*sin(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(-a*(sin(e + f*x) - 1))*(g + h*x)/sqrt(c*(sin(e + f*x) + 1)),
 x)
```

3.179.7 Maxima [F]

$$\int \frac{(g+hx)\sqrt{a-a\sin(e+fx)}}{\sqrt{c+c\sin(e+fx)}}\,dx=\int \frac{(hx+g)\sqrt{-a\sin(fx+e)+a}}{\sqrt{c\sin(fx+e)+c}}\,dx$$

input
```
integrate((h*x+g)*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x, algorit
hm="maxima")
```

output
```
integrate((h*x + g)*sqrt(-a*sin(f*x + e) + a)/sqrt(c*sin(f*x + e) + c), x)
```

3.179.8 Giac [F]

$$\int \frac{(g+hx)\sqrt{a-a\sin(e+fx)}}{\sqrt{c+c\sin(e+fx)}}\,dx=\int \frac{(hx+g)\sqrt{-a\sin(fx+e)+a}}{\sqrt{c\sin(fx+e)+c}}\,dx$$

input
```
integrate((h*x+g)*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2),x, algorit
hm="giac")
```

output
```
integrate((h*x + g)*sqrt(-a*sin(f*x + e) + a)/sqrt(c*sin(f*x + e) + c), x)
```

3.179.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(g+hx)\sqrt{a-a\sin(e+fx)}}{\sqrt{c+c\sin(e+fx)}}\,dx=\int \frac{(g+h\,x)\,\sqrt{a-a\,\sin(e+f\,x)}}{\sqrt{c+c\,\sin(e+f\,x)}}\,dx$$

input
```
int(((g + h*x)*(a - a*sin(e + f*x))^(1/2))/(c + c*sin(e + f*x))^(1/2),x)
```

output
```
int(((g + h*x)*(a - a*sin(e + f*x))^(1/2))/(c + c*sin(e + f*x))^(1/2), x)
```

3.180 $\int \frac{\sqrt{a-a\sin(e+fx)}}{(g+hx)\sqrt{c+c\sin(e+fx)}}\,dx$

3.180.1 Optimal result

Integrand size = 37, antiderivative size = 37

$$\int \frac{\sqrt{a-a\sin(e+fx)}}{(g+hx)\sqrt{c+c\sin(e+fx)}}\,dx = \frac{a\cos(e+fx)\text{Int}\left(\frac{\sec(e+fx)}{g+hx},x\right)}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{a\cos(e+fx)\text{Int}\left(\frac{\tan(e+fx)}{g+hx},x\right)}{\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

output
```
a*cos(f*x+e)*Unintegrable(sec(f*x+e)/(h*x+g),x)/(a-a*sin(f*x+e))^(1/2)/(c+
c*sin(f*x+e))^(1/2)-a*cos(f*x+e)*Unintegrable(tan(f*x+e)/(h*x+g),x)/(a-a*s
in(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)
```

3.180.2 Mathematica [N/A]

Not integrable

Time = 4.39 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{a-a\sin(e+fx)}}{(g+hx)\sqrt{c+c\sin(e+fx)}}\,dx = \int \frac{\sqrt{a-a\sin(e+fx)}}{(g+hx)\sqrt{c+c\sin(e+fx)}}\,dx$$

input
```
Integrate[Sqrt[a - a*Sin[e + f*x]]/((g + h*x)*Sqrt[c + c*Sin[e + f*x]]),x]
```

output
```
Integrate[Sqrt[a - a*Sin[e + f*x]]/((g + h*x)*Sqrt[c + c*Sin[e + f*x]]), x
]
```

3.180.3 Rubi [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5115, 7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a-a\sin(e+fx)}}{(g+hx)\sqrt{c\sin(e+fx)+c}}dx$$

$$\Big\downarrow \text{5115}$$

$$\frac{\cos(e+fx)\int \frac{\sec(e+fx)(a-a\sin(e+fx))}{g+hx}dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}$$

$$\Big\downarrow \text{7292}$$

$$\frac{\cos(e+fx)\int \frac{a\sec(e+fx)(1-\sin(e+fx))}{g+hx}dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\cos(e+fx)\int \frac{\sec(e+fx)(1-\sin(e+fx))}{g+hx}dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}$$

$$\Big\downarrow \text{7293}$$

$$\frac{a\cos(e+fx)\int \left(\frac{\sec(e+fx)}{g+hx}-\frac{\tan(e+fx)}{g+hx}\right)dx}{\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\cos(e+fx)\left(\int \frac{\sec(e+fx)}{g+hx}dx-\int \frac{\tan(e+fx)}{g+hx}dx\right)}{\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}$$

input
```
Int[Sqrt[a - a*Sin[e + f*x]]/((g + h*x)*Sqrt[c + c*Sin[e + f*x]]),x]
```

output
```
$Aborted
```

3.180.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5115
```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPart[m]
*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPa
rt[m]/Cos[e + f*x]^(2*FracPart[m]))   Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
 + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] &&
IGeQ[n - m, 0]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.180.4 Maple [N/A] (verified)

Not integrable

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{a-\sin \left(f x +e \right) a}}{\left(h x +g \right) \sqrt{c +c \sin \left(f x +e \right)}}d x$$

input
```
int((a-sin(f*x+e)*a)^(1/2)/(h*x+g)/(c+c*sin(f*x+e))^(1/2),x)
```

output
```
int((a-sin(f*x+e)*a)^(1/2)/(h*x+g)/(c+c*sin(f*x+e))^(1/2),x)
```

3.180.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a-a\sin(e+fx)}}{(g+hx)\sqrt{c+c\sin(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a-a*sin(f*x+e))^(1/2)/(h*x+g)/(c+c*sin(f*x+e))^(1/2),x, algorit
hm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.180.6 Sympy [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a-a\sin(e+fx)}}{(g+hx)\sqrt{c+c\sin(e+fx)}}\,dx = \int \frac{\sqrt{-a\left(\sin\left(e+fx\right)-1\right)}}{\sqrt{c\left(\sin\left(e+fx\right)+1\right)}\left(g+hx\right)}\,dx$$

input
```
integrate((a-a*sin(f*x+e))**(1/2)/(h*x+g)/(c+c*sin(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(-a*(sin(e + f*x) - 1))/(sqrt(c*(sin(e + f*x) + 1))*(g + h*x)
), x)
```

3.180.7 Maxima [N/A]

Not integrable

Time = 1.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{a-a\sin(e+fx)}}{(g+hx)\sqrt{c+c\sin(e+fx)}}\,dx = \int \frac{\sqrt{-a\sin\left(fx+e\right)+a}}{(hx+g)\sqrt{c\sin\left(fx+e\right)+c}}\,dx$$

input
```
integrate((a-a*sin(f*x+e))^(1/2)/(h*x+g)/(c+c*sin(f*x+e))^(1/2),x, algorit
hm="maxima")
```

output
```
integrate(sqrt(-a*sin(f*x + e) + a)/((h*x + g)*sqrt(c*sin(f*x + e) + c)),
x)
```

3.180.8 Giac [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{a-a\sin(e+fx)}}{(g+hx)\sqrt{c+c\sin(e+fx)}}\,dx = \int \frac{\sqrt{-a\sin\left(fx+e\right)+a}}{(hx+g)\sqrt{c\sin\left(fx+e\right)+c}}\,dx$$

input
```
integrate((a-a*sin(f*x+e))^(1/2)/(h*x+g)/(c+c*sin(f*x+e))^(1/2),x, algorit
hm="giac")
```

output
```
integrate(sqrt(-a*sin(f*x + e) + a)/((h*x + g)*sqrt(c*sin(f*x + e) + c)),
x)
```

3.180.9 Mupad [N/A]

Not integrable

Time = 26.84 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{a-a\sin(e+fx)}}{(g+hx)\sqrt{c+c\sin(e+fx)}}\,dx = \int \frac{\sqrt{a-a\,\sin\left(e+f\,x\right)}}{\left(g+h\,x\right)\,\sqrt{c+c\,\sin\left(e+f\,x\right)}}\,dx$$

input
```
int((a - a*sin(e + f*x))^(1/2)/((g + h*x)*(c + c*sin(e + f*x))^(1/2)),x)
```

output
```
int((a - a*sin(e + f*x))^(1/2)/((g + h*x)*(c + c*sin(e + f*x))^(1/2)), x)
```

3.181 $\int \frac{x^3\sqrt{a-a\sin(e+fx)}}{(c+c\sin(e+fx))^{3/2}}\,dx$

3.181.1 Optimal result

Integrand size = 33, antiderivative size = 536

$$\begin{aligned}\int \frac{x^3\sqrt{a-a\sin(e+fx)}}{(c+c\sin(e+fx))^{3/2}}\,dx &= -\frac{3ax^2}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}\\
&-\frac{3iax^2\cos(e+fx)}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}\\
&-\frac{12iax\arctan\left(e^{i(e+fx)}\right)\cos(e+fx)}{cf^3\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}\\
&+\frac{6ax\cos(e+fx)\log\left(1+e^{2i(e+fx)}\right)}{cf^3\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}\\
&+\frac{6ia\cos(e+fx)\operatorname{PolyLog}\left(2,-ie^{i(e+fx)}\right)}{cf^4\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}\\
&-\frac{6ia\cos(e+fx)\operatorname{PolyLog}\left(2,ie^{i(e+fx)}\right)}{cf^4\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}\\
&-\frac{3ia\cos(e+fx)\operatorname{PolyLog}\left(2,-e^{2i(e+fx)}\right)}{cf^4\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}\\
&-\frac{ax^3\sec(e+fx)}{cf\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}\\
&+\frac{3ax^2\sin(e+fx)}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}\\
&+\frac{ax^3\tan(e+fx)}{cf\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}\end{aligned}$$

output
```
-3*a*x^2/c/f^2/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)-3*I*a*x^2*cos
(f*x+e)/c/f^2/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)-12*I*a*x*arcta
n(exp(I*(f*x+e)))*cos(f*x+e)/c/f^3/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))
^(1/2)+6*a*x*cos(f*x+e)*ln(1+exp(2*I*(f*x+e)))/c/f^3/(a-a*sin(f*x+e))^(1/2
)/(c+c*sin(f*x+e))^(1/2)+6*I*a*cos(f*x+e)*polylog(2,-I*exp(I*(f*x+e)))/c/f
^4/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)-6*I*a*cos(f*x+e)*polylog(
2,I*exp(I*(f*x+e)))/c/f^4/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)-3*
I*a*cos(f*x+e)*polylog(2,-exp(2*I*(f*x+e)))/c/f^4/(a-a*sin(f*x+e))^(1/2)/(
c+c*sin(f*x+e))^(1/2)-a*x^3*sec(f*x+e)/c/f/(a-a*sin(f*x+e))^(1/2)/(c+c*sin
(f*x+e))^(1/2)+3*a*x^2*sin(f*x+e)/c/f^2/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*
x+e))^(1/2)+a*x^3*tan(f*x+e)/c/f/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(
1/2)
```

3.181.2 Mathematica [A] (verified)

Time = 2.30 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.36

$$\int \frac{x^3\sqrt{a-a\sin(e+fx)}}{(c+c\sin(e+fx))^{3/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a-a\sin(e+fx)}\left(12i\,\mathrm{PolyLog}\left(2,-i\right.\right.}{}$$

input
```
Integrate[(x^3*Sqrt[a - a*Sin[e + f*x]])/(c + c*Sin[e + f*x])^(3/2),x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[a - a*Sin[e + f*x]]*((12*I)*Po
lyLog[2, (-I)/E^(I*(e + f*x))]*(1 + Sin[e + f*x]) + f*x*((3*I)*f*x - f^2*x
^2 - 3*f*x*Cos[e + f*x] + 12*Log[1 + I/E^(I*(e + f*x))] + 3*(I*f*x + 4*Log
[1 + I/E^(I*(e + f*x))])*Sin[e + f*x]))/(f^4*(Cos[(e + f*x)/2] - Sin[(e +
 f*x)/2])*(c*(1 + Sin[e + f*x]))^(3/2))
```

3.181.3 Rubi [A] (verified)

Time = 2.95 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.42, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {5115, 7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\sqrt{a-a\sin(e+fx)}}{(c\sin(e+fx)+c)^{3/2}}\,dx$$
$$\Big\downarrow \text{5115}$$
$$\frac{\cos(e+fx)\int x^3\sec^3(e+fx)(a-a\sin(e+fx))^2dx}{ac\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}$$
$$\Big\downarrow \text{7292}$$
$$\frac{\cos(e+fx)\int a^2x^3\sec^3(e+fx)(1-\sin(e+fx))^2dx}{ac\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}$$
$$\Big\downarrow \text{27}$$
$$\frac{a\cos(e+fx)\int x^3\sec^3(e+fx)(1-\sin(e+fx))^2dx}{c\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}$$
$$\Big\downarrow \text{7293}$$
$$\frac{a\cos(e+fx)\int \left(\sec^3(e+fx)x^3+\sec(e+fx)\tan^2(e+fx)x^3-2\sec^2(e+fx)\tan(e+fx)x^3\right)dx}{c\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}$$
$$\Big\downarrow \text{2009}$$
$$\frac{a\cos(e+fx)\left(-\frac{12ix\arctan\left(e^{i(e+fx)}\right)}{f^3}+\frac{6i\,\text{PolyLog}\left(2,-ie^{i(e+fx)}\right)}{f^4}-\frac{6i\,\text{PolyLog}\left(2,ie^{i(e+fx)}\right)}{f^4}-\frac{3i\,\text{PolyLog}\left(2,-e^{2i(e+fx)}\right)}{f^4}+\frac{6x\log(1}{}\right.}{c\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+f}}$$

input
```
Int[(x^3*Sqrt[a - a*Sin[e + f*x]])/(c + c*Sin[e + f*x])^(3/2),x]
```

output
```
(a*Cos[e + f*x]*(((-3*I)*x^2)/f^2 - ((12*I)*x*ArcTan[E^(I*(e + f*x))])/f^3
 + (6*x*Log[1 + E^((2*I)*(e + f*x))])/f^3 + ((6*I)*PolyLog[2, (-I)*E^(I*(e
 + f*x))])/f^4 - ((6*I)*PolyLog[2, I*E^(I*(e + f*x))])/f^4 - ((3*I)*PolyLo
g[2, -E^((2*I)*(e + f*x))])/f^4 - (3*x^2*Sec[e + f*x])/f^2 - (x^3*Sec[e +
f*x]^2)/f + (3*x^2*Tan[e + f*x])/f^2 + (x^3*Sec[e + f*x]*Tan[e + f*x])/f))
/(c*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c + c*Sin[e + f*x]])
```

3.181. $\int \frac{x^3\sqrt{a-a\sin(e+fx)}}{(c+c\sin(e+fx))^{3/2}}\,dx$

3.181.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5115
```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPart[m]
*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPa
rt[m]/Cos[e + f*x]^(2*FracPart[m]))   Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
 + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] &&
IGeQ[n - m, 0]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.181.4 Maple [F]

$$\int \frac{x^3 \sqrt{a-\sin \left(fx+e\right) a}}{\left(c+c \sin \left(fx+e\right)\right)^{\frac{3}{2}}}dx$$

input
```
int(x^3*(a-sin(f*x+e)*a)^(1/2)/(c+c*sin(f*x+e))^(3/2),x)
```

output
```
int(x^3*(a-sin(f*x+e)*a)^(1/2)/(c+c*sin(f*x+e))^(3/2),x)
```

3.181.5 Fricas [F]

$$\int \frac{x^3\sqrt{a-a\sin(e+fx)}}{(c+c\sin(e+fx))^{3/2}}\,dx=\int \frac{\sqrt{-a\sin(fx+e)+a}x^3}{(c\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x, algorithm="
fricas")
```

output
```
integral(-sqrt(-a*sin(f*x + e) + a)*sqrt(c*sin(f*x + e) + c)*x^3/(c^2*cos(
f*x + e)^2 - 2*c^2*sin(f*x + e) - 2*c^2), x)
```

3.181.6 Sympy [F]

$$\int \frac{x^3\sqrt{a-a\sin(e+fx)}}{(c+c\sin(e+fx))^{3/2}}\,dx=\int \frac{x^3\sqrt{-a(\sin(e+fx)-1)}}{(c(\sin(e+fx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3*(a-a*sin(f*x+e))**(1/2)/(c+c*sin(f*x+e))**(3/2),x)
```

output
```
Integral(x**3*sqrt(-a*(sin(e + f*x) - 1))/(c*(sin(e + f*x) + 1))**(3/2), x
)
```

3.181.7 Maxima [F]

$$\int \frac{x^3\sqrt{a-a\sin(e+fx)}}{(c+c\sin(e+fx))^{3/2}}\,dx=\int \frac{\sqrt{-a\sin(fx+e)+a}x^3}{(c\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x, algorithm="
maxima")
```

output
```
integrate(sqrt(-a*sin(f*x + e) + a)*x^3/(c*sin(f*x + e) + c)^(3/2), x)
```

3.181.8 Giac [F]

$$\int \frac{x^3\sqrt{a-a\sin(e+fx)}}{(c+c\sin(e+fx))^{3/2}}\,dx=\int \frac{\sqrt{-a\sin(fx+e)+a}x^3}{(c\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x, algorithm="
giac")
```

output
```
integrate(sqrt(-a*sin(f*x + e) + a)*x^3/(c*sin(f*x + e) + c)^(3/2), x)
```

3.181.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{a-a\sin(e+fx)}}{(c+c\sin(e+fx))^{3/2}}\,dx=\int \frac{x^3\sqrt{a-a\sin(e+fx)}}{(c+c\sin(e+fx))^{3/2}}\,dx$$

input
```
int((x^3*(a - a*sin(e + f*x))^(1/2))/(c + c*sin(e + f*x))^(3/2),x)
```

output
```
int((x^3*(a - a*sin(e + f*x))^(1/2))/(c + c*sin(e + f*x))^(3/2), x)
```

3.182 $\int \frac{x^2\sqrt{a-a\sin(e+fx)}}{(c+c\sin(e+fx))^{3/2}}\,dx$

3.182.1 Optimal result

Integrand size = 33, antiderivative size = 280

$$\begin{aligned}\int \frac{x^2\sqrt{a-a\sin(e+fx)}}{(c+c\sin(e+fx))^{3/2}}\,dx = & -\frac{2ax}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} \\ & + \frac{2a\text{arctanh}(\sin(e+fx))\cos(e+fx)}{cf^3\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} \\ & + \frac{2a\cos(e+fx)\log(\cos(e+fx))}{cf^3\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} \\ & - \frac{ax^2\sec(e+fx)}{cf\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} \\ & + \frac{2ax\sin(e+fx)}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} \\ & + \frac{ax^2\tan(e+fx)}{cf\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}\end{aligned}$$

output
```
-2*a*x/c/f^2/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)+2*a*arctanh(sin
(f*x+e))*cos(f*x+e)/c/f^3/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)+2*
a*cos(f*x+e)*ln(cos(f*x+e))/c/f^3/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^
(1/2)-a*x^2*sec(f*x+e)/c/f/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)+2
*a*x*sin(f*x+e)/c/f^2/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)+a*x^2*
tan(f*x+e)/c/f/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)
```

3.182.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.90 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.55

$$\int \frac{x^2\sqrt{a-a\sin(e+fx)}}{(c+c\sin(e+fx))^{3/2}}\,dx = -\frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\sqrt{a-a\sin(e+fx)}\left(2ifx+f^2x^2+2fx\cos(e+fx)-4\log\left(i+e^{i(e-}\right.\right.}{f^3\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)(c(1+\sin(e+fx)))}$$

input
```
Integrate[(x^2*Sqrt[a - a*Sin[e + f*x]])/(c + c*Sin[e + f*x])^(3/2),x]
```

output
```
-(((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*Sqrt[a - a*Sin[e + f*x]]*((2*I)*f
*x + f^2*x^2 + 2*f*x*Cos[e + f*x] - 4*Log[I + E^(I*(e + f*x))] + ((2*I)*f*
x - 4*Log[I + E^(I*(e + f*x))])*Sin[e + f*x]))/(f^3*(Cos[(e + f*x)/2] - Si
n[(e + f*x)/2])*(c*(1 + Sin[e + f*x]))^(3/2)))
```

3.182.3 Rubi [A] (verified)

Time = 2.05 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.44, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {5115, 7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{a-a\sin(e+fx)}}{(c\sin(e+fx)+c)^{3/2}}dx$$

$$\Big\downarrow \text{5115}$$

$$\frac{\cos(e+fx)\int x^2\sec^3(e+fx)(a-a\sin(e+fx))^2dx}{ac\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}$$

$$\Big\downarrow \text{7292}$$

$$\frac{\cos(e+fx)\int a^2x^2\sec^3(e+fx)(1-\sin(e+fx))^2dx}{ac\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\cos(e+fx)\int x^2\sec^3(e+fx)(1-\sin(e+fx))^2dx}{c\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}$$

$$\Big\downarrow \text{7293}$$

$$\frac{a\cos(e+fx)\int\left(x^2\sec^3(e+fx)-2x^2\tan(e+fx)\sec^2(e+fx)+x^2\tan^2(e+fx)\sec(e+fx)\right)dx}{c\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\cos(e+fx)\left(\frac{2\text{arctanh}(\sin(e+fx))}{f^3}+\frac{2\log(\cos(e+fx))}{f^3}+\frac{2x\tan(e+fx)}{f^2}-\frac{2x\sec(e+fx)}{f^2}-\frac{x^2\sec^2(e+fx)}{f}+\frac{x^2\tan(e+fx)\sec(e+f}{f}\right.}{c\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}$$

input
```
Int[(x^2*Sqrt[a - a*Sin[e + f*x]])/(c + c*Sin[e + f*x])^(3/2),x]
```

output
```
(a*Cos[e + f*x]*((2*ArcTanh[Sin[e + f*x]])/f^3 + (2*Log[Cos[e + f*x]])/f^3
 - (2*x*Sec[e + f*x])/f^2 - (x^2*Sec[e + f*x]^2)/f + (2*x*Tan[e + f*x])/f^
2 + (x^2*Sec[e + f*x]*Tan[e + f*x])/f))/(c*Sqrt[a - a*Sin[e + f*x]]*Sqrt[c
 + c*Sin[e + f*x]])
```

3.182.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5115
```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPart[m]
*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPa
rt[m]/Cos[e + f*x]^(2*FracPart[m]))   Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
 + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] &&
IGeQ[n - m, 0]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

3.182. $\int \frac{x^2\sqrt{a-a\sin(e+fx)}}{(c+c\sin(e+fx))^{3/2}}\,dx$

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.182.4 Maple [F]

$$\int \frac{x^2\sqrt{a-\sin(fx+e)\,a}}{(c+c\sin(fx+e))^{\frac{3}{2}}}dx$$

input
```
int(x^2*(a-sin(f*x+e)*a)^(1/2)/(c+c*sin(f*x+e))^(3/2),x)
```

output
```
int(x^2*(a-sin(f*x+e)*a)^(1/2)/(c+c*sin(f*x+e))^(3/2),x)
```

3.182.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{a-a\sin(e+fx)}}{(c+c\sin(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x, algorithm="
fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.182.6 Sympy [F]

$$\int \frac{x^2\sqrt{a-a\sin(e+fx)}}{(c+c\sin(e+fx))^{3/2}}\,dx = \int \frac{x^2\sqrt{-a\,(\sin(e+fx)-1)}}{(c\,(\sin(e+fx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2*(a-a*sin(f*x+e))**(1/2)/(c+c*sin(f*x+e))**(3/2),x)
```

output
```
Integral(x**2*sqrt(-a*(sin(e + f*x) - 1))/(c*(sin(e + f*x) + 1))**(3/2), x
)
```

3.182.7 Maxima [F]

$$\int \frac{x^2\sqrt{a-a\sin(e+fx)}}{(c+c\sin(e+fx))^{3/2}}\,dx=\int \frac{\sqrt{-a\sin(fx+e)+a}x^2}{(c\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x, algorithm="
maxima")
```

output
```
integrate(sqrt(-a*sin(f*x + e) + a)*x^2/(c*sin(f*x + e) + c)^(3/2), x)
```

3.182.8 Giac [F]

$$\int \frac{x^2\sqrt{a-a\sin(e+fx)}}{(c+c\sin(e+fx))^{3/2}}\,dx=\int \frac{\sqrt{-a\sin(fx+e)+a}x^2}{(c\sin(fx+e)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x, algorithm="
giac")
```

output
```
sage0*x
```

3.182.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{a-a\sin(e+fx)}}{(c+c\sin(e+fx))^{3/2}}\,dx=\int \frac{x^2\sqrt{a-a\sin(e+fx)}}{(c+c\sin(e+fx))^{3/2}}\,dx$$

input
```
int((x^2*(a - a*sin(e + f*x))^(1/2))/(c + c*sin(e + f*x))^(3/2),x)
```

output
```
int((x^2*(a - a*sin(e + f*x))^(1/2))/(c + c*sin(e + f*x))^(3/2), x)
```

3.183 $\int \frac{x\sqrt{a-a\sin(e+fx)}}{(c+c\sin(e+fx))^{3/2}}\,dx$

3.183.1 Optimal result

Integrand size = 31, antiderivative size = 171

$$\int \frac{x\sqrt{a-a\sin(e+fx)}}{(c+c\sin(e+fx))^{3/2}}\,dx = -\frac{a}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} - \frac{ax\sec(e+fx)}{cf\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} + \frac{a\sin(e+fx)}{cf^2\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}} + \frac{ax\tan(e+fx)}{cf\sqrt{a-a\sin(e+fx)}\sqrt{c+c\sin(e+fx)}}$$

output
```
-a/c/f^2/(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)-a*x*sec(f*x+e)/c/f/
(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)+a*sin(f*x+e)/c/f^2/(a-a*sin(
f*x+e))^(1/2)/(c+c*sin(f*x+e))^(1/2)+a*x*tan(f*x+e)/c/f/(a-a*sin(f*x+e))^(
1/2)/(c+c*sin(f*x+e))^(1/2)
```

3.183.2 Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.88

$$\int \frac{x\sqrt{a-a\sin(e+fx)}}{(c+c\sin(e+fx))^{3/2}}\,dx =$$
$$-\frac{\left((-1+fx)\cos\left(\frac{e}{2}\right)+\cos\left(\frac{e}{2}+fx\right)+\sin\left(\frac{e}{2}\right)+fx\sin\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}+fx\right)\right)\sqrt{c(1+\sin(e+fx))}\sqrt{a-a\,\mathrm{si}}}{c^2f^2\left(\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

input
```
Integrate[(x*Sqrt[a - a*Sin[e + f*x]])/(c + c*Sin[e + f*x])^(3/2),x]
```

output
```
-((((-1 + f*x)*Cos[e/2] + Cos[e/2 + f*x] + Sin[e/2] + f*x*Sin[e/2] - Sin[e
/2 + f*x])*Sqrt[c*(1 + Sin[e + f*x])]*Sqrt[a - a*Sin[e + f*x]])/(c^2*f^2*(
Cos[e/2] + Sin[e/2])*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2])^3))
```

3.183.3 Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.54, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {5115, 7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{a-a\sin(e+fx)}}{(c\sin(e+fx)+c)^{3/2}}\,dx$$

$$\Big\downarrow \; 5115$$

$$\frac{\cos(e+fx)\int x\sec^3(e+fx)(a-a\sin(e+fx))^2dx}{ac\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}$$

$$\Big\downarrow \; 7292$$

$$\frac{\cos(e+fx)\int a^2x\sec^3(e+fx)(1-\sin(e+fx))^2dx}{ac\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}$$

$$\Big\downarrow \; 27$$

$$\frac{a\cos(e+fx)\int x\sec^3(e+fx)(1-\sin(e+fx))^2dx}{c\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}$$

$$\Big\downarrow \text{7293}$$

$$\frac{a\cos(e+fx)\int\left(x\sec^3(e+fx)-2x\tan(e+fx)\sec^2(e+fx)+x\tan^2(e+fx)\sec(e+fx)\right)dx}{c\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\cos(e+fx)\left(\frac{\tan(e+fx)}{f^2}-\frac{\sec(e+fx)}{f^2}-\frac{x\sec^2(e+fx)}{f}+\frac{x\tan(e+fx)\sec(e+fx)}{f}\right)}{c\sqrt{a-a\sin(e+fx)}\sqrt{c\sin(e+fx)+c}}$$

input
```
Int[(x*Sqrt[a - a*Sin[e + f*x]])/(c + c*Sin[e + f*x])^(3/2),x]
```

output
```
(a*Cos[e + f*x]*(-(Sec[e + f*x]/f^2) - (x*Sec[e + f*x]^2)/f + Tan[e + f*x]
/f^2 + (x*Sec[e + f*x]*Tan[e + f*x])/f))/(c*Sqrt[a - a*Sin[e + f*x]]*Sqrt[
c + c*Sin[e + f*x]])
```

3.183.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5115
```
Int[((g_.) + (h_.)*(x_))^(p_.)*((a_) + (b_.)*Sin[(e_.) + (f_.)*(x_)])^(m_)*
((c_) + (d_.)*Sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPart[m]
*c^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m]*((c + d*Sin[e + f*x])^FracPa
rt[m]/Cos[e + f*x]^(2*FracPart[m]))   Int[(g + h*x)^p*Cos[e + f*x]^(2*m)*(c
 + d*Sin[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] &&
IGeQ[n - m, 0]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.183. $\int \frac{x\sqrt{a-a\sin(e+fx)}}{(c+c\sin(e+fx))^{3/2}}\,dx$

3.183.4 Maple [F]

$$\int \frac{x\sqrt{a-\sin(fx+e)\,a}}{(c+c\sin(fx+e))^{\frac{3}{2}}}dx$$

input
```
int(x*(a-sin(f*x+e)*a)^(1/2)/(c+c*sin(f*x+e))^(3/2),x)
```

output
```
int(x*(a-sin(f*x+e)*a)^(1/2)/(c+c*sin(f*x+e))^(3/2),x)
```

3.183.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.42

$$\int \frac{x\sqrt{a-a\sin(e+fx)}}{(c+c\sin(e+fx))^{3/2}}\,dx=-\frac{(fx+\cos(fx+e))\sqrt{-a\sin(fx+e)+a}\sqrt{c\sin(fx+e)+c}}{c^2f^2\cos(fx+e)\sin(fx+e)+c^2f^2\cos(fx+e)}$$

input
```
integrate(x*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x, algorithm="fr
icas")
```

output
```
-(f*x + cos(f*x + e))*sqrt(-a*sin(f*x + e) + a)*sqrt(c*sin(f*x + e) + c)/(
c^2*f^2*cos(f*x + e)*sin(f*x + e) + c^2*f^2*cos(f*x + e))
```

3.183.6 Sympy [F]

$$\int \frac{x\sqrt{a-a\sin(e+fx)}}{(c+c\sin(e+fx))^{3/2}}\,dx=\int \frac{x\sqrt{-a(\sin(e+fx)-1)}}{(c(\sin(e+fx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a-a*sin(f*x+e))**(1/2)/(c+c*sin(f*x+e))**(3/2),x)
```

output
```
Integral(x*sqrt(-a*(sin(e + f*x) - 1))/(c*(sin(e + f*x) + 1))**(3/2), x)
```

3.183.7 Maxima [F]

$$\int \frac{x\sqrt{a-a\sin(e+fx)}}{(c+c\sin(e+fx))^{3/2}}\,dx = \int \frac{\sqrt{-a\sin\left(fx+e\right)+a}x}{(c\sin\left(fx+e\right)+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x, algorithm="ma
xima")
```

output
```
integrate(sqrt(-a*sin(f*x + e) + a)*x/(c*sin(f*x + e) + c)^(3/2), x)
```

3.183.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 446 vs. $2(155) = 310$.

Time = 0.38 (sec) , antiderivative size = 446, normalized size of antiderivative = 2.61

$$\int \frac{x\sqrt{a-a\sin(e+fx)}}{(c+c\sin(e+fx))^{3/2}}\,dx = \frac{\sqrt{2}\left(\pi\sqrt{c}\mathrm{sgn}\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\tan\left(-\frac{1}{8}\pi+\frac{1}{4}fx+\frac{1}{4}e\right)^4-(\pi-2fx\right.}{}$$

input
```
integrate(x*(a-a*sin(f*x+e))^(1/2)/(c+c*sin(f*x+e))^(3/2),x, algorithm="gi
ac")
```

output
```
1/4*sqrt(2)*(pi*sqrt(c)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi +
1/4*f*x + 1/4*e)^4 - (pi - 2*f*x - 2*e)*sqrt(c)*sgn(sin(-1/4*pi + 1/2*f*x
+ 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^4 - 2*sqrt(c)*e*sgn(sin(-1/4*pi +
 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^4 + 2*pi*sqrt(c)*sgn(sin
(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 - 2*(pi - 2*
f*x - 2*e)*sqrt(c)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f
*x + 1/4*e)^2 - 4*sqrt(c)*e*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*p
i + 1/4*f*x + 1/4*e)^2 + 8*sqrt(c)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e))*tan
(-1/8*pi + 1/4*f*x + 1/4*e)^3 + pi*sqrt(c)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2
*e)) - (pi - 2*f*x - 2*e)*sqrt(c)*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 2*
sqrt(c)*e*sgn(sin(-1/4*pi + 1/2*f*x + 1/2*e)) - 8*sqrt(c)*sgn(sin(-1/4*pi
+ 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e))*sqrt(a)/((sqrt(2)*c^2*
f*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^4 - 2
*sqrt(2)*c^2*f*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x +
 1/4*e)^2 + sqrt(2)*c^2*f*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*f)
```

3.183.9 Mupad [B] (verification not implemented)

Time = 28.32 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.51

$$\int \frac{x\sqrt{a-a\sin(e+fx)}}{(c+c\sin(e+fx))^{3/2}}\,dx = -\frac{\sqrt{-a\,\left(\sin\left(e+f\,x\right)-1\right)}\,\left(\cos\left(2\,e+2\,f\,x\right)+2\,f\,x\,\cos\left(e+f\,x\right)+1-\cos\left(e+f\,x\right)\,2\mathrm{i}-\sin\left(2\,e+2\,f\,x\right)\,1\right.}{c\,f^2\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)\,\sqrt{c\,\left(\sin\left(e+f\,x\right)+1\right)}}$$

input
```
int((x*(a - a*sin(e + f*x))^(1/2))/(c + c*sin(e + f*x))^(3/2),x)
```

output
```
-((-a*(sin(e + f*x) - 1))^(1/2)*(cos(2*e + 2*f*x) - cos(e + f*x)*2i - sin(
2*e + 2*f*x)*1i + 2*f*x*cos(e + f*x) + 1))/(c*f^2*(cos(2*e + 2*f*x) + 1)*(
c*(sin(e + f*x) + 1))^(1/2))
```

3.184 $\int \frac{z^2\sqrt{1+\cos(z)}}{\sqrt{1-\cos(z)}}\,dz$

3.184.1 Optimal result

Integrand size = 22, antiderivative size = 300

$$\int \frac{z^2\sqrt{1+\cos(z)}}{\sqrt{1-\cos(z)}}\,dz = -\frac{iz^3\sin(z)}{3\sqrt{1-\cos(z)}\sqrt{1+\cos(z)}} - \frac{2z^2\text{arctanh}(e^{iz})\sin(z)}{\sqrt{1-\cos(z)}\sqrt{1+\cos(z)}} + \frac{z^2\log(1-e^{2iz})\sin(z)}{\sqrt{1-\cos(z)}\sqrt{1+\cos(z)}} + \frac{2iz\,\text{PolyLog}(2,-e^{iz})\sin(z)}{\sqrt{1-\cos(z)}\sqrt{1+\cos(z)}} - \frac{2iz\,\text{PolyLog}(2,e^{iz})\sin(z)}{\sqrt{1-\cos(z)}\sqrt{1+\cos(z)}} - \frac{iz\,\text{PolyLog}(2,e^{2iz})\sin(z)}{\sqrt{1-\cos(z)}\sqrt{1+\cos(z)}} - \frac{2\,\text{PolyLog}(3,-e^{iz})\sin(z)}{\sqrt{1-\cos(z)}\sqrt{1+\cos(z)}} + \frac{2\,\text{PolyLog}(3,e^{iz})\sin(z)}{\sqrt{1-\cos(z)}\sqrt{1+\cos(z)}} + \frac{\text{PolyLog}(3,e^{2iz})\sin(z)}{2\sqrt{1-\cos(z)}\sqrt{1+\cos(z)}}$$

output
```
-1/3*I*z^3*sin(z)/(1-cos(z))^(1/2)/(1+cos(z))^(1/2)-2*z^2*arctanh(exp(I*z)
)*sin(z)/(1-cos(z))^(1/2)/(1+cos(z))^(1/2)+z^2*ln(1-exp(2*I*z))*sin(z)/(1-
cos(z))^(1/2)/(1+cos(z))^(1/2)+2*I*z*polylog(2,-exp(I*z))*sin(z)/(1-cos(z)
)^(1/2)/(1+cos(z))^(1/2)-2*I*z*polylog(2,exp(I*z))*sin(z)/(1-cos(z))^(1/2)
/(1+cos(z))^(1/2)-I*z*polylog(2,exp(2*I*z))*sin(z)/(1-cos(z))^(1/2)/(1+cos
(z))^(1/2)-2*polylog(3,-exp(I*z))*sin(z)/(1-cos(z))^(1/2)/(1+cos(z))^(1/2)
+2*polylog(3,exp(I*z))*sin(z)/(1-cos(z))^(1/2)/(1+cos(z))^(1/2)+1/2*polylo
g(3,exp(2*I*z))*sin(z)/(1-cos(z))^(1/2)/(1+cos(z))^(1/2)
```

3.184.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.28

$$\int \frac{z^2\sqrt{1+\cos(z)}}{\sqrt{1-\cos(z)}}\,dz$$
$$= \frac{\sqrt{1+\cos(z)}\left(-i\pi^3+iz^3+6z^2\log\left(1-e^{-iz}\right)+12iz\operatorname{PolyLog}\left(2,e^{-iz}\right)+12\operatorname{PolyLog}\left(3,e^{-iz}\right)\right)\tan\left(\frac{z}{2}\right)}{3\sqrt{1-\cos(z)}}$$

input
```
Integrate[(z^2*Sqrt[1 + Cos[z]])/Sqrt[1 - Cos[z]],z]
```

output
```
(Sqrt[1 + Cos[z]]*((-I)*Pi^3 + I*z^3 + 6*z^2*Log[1 - E^((-I)*z)] + (12*I)*
z*PolyLog[2, E^((-I)*z)] + 12*PolyLog[3, E^((-I)*z)])*Tan[z/2])/(3*Sqrt[1
- Cos[z]])
```

3.184.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.47, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {5116, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{z^2\sqrt{\cos(z)+1}}{\sqrt{1-\cos(z)}}\,dz$$
$$\Big\downarrow \text{ 5116}$$
$$\frac{\sin(z)\int z^2(\cos(z)+1)\csc(z)dz}{\sqrt{1-\cos(z)}\sqrt{\cos(z)+1}}$$
$$\Big\downarrow \text{ 7293}$$
$$\frac{\sin(z)\int \left(\cot(z)z^2+\csc(z)z^2\right)dz}{\sqrt{1-\cos(z)}\sqrt{\cos(z)+1}}$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{\sin(z)\left(-2z^2\text{arctanh}(e^{iz})+2iz\operatorname{PolyLog}\left(2,-e^{iz}\right)-2iz\operatorname{PolyLog}\left(2,e^{iz}\right)-iz\operatorname{PolyLog}\left(2,e^{2iz}\right)-2\operatorname{PolyLog}\left(3,-e\right.\right.}{\sqrt{1-\cos(z)}\sqrt{\cos(z)+1}}$$

input
```
Int[(z^2*Sqrt[1 + Cos[z]])/Sqrt[1 - Cos[z]],z]
```

output
```
(((-1/3*I)*z^3 - 2*z^2*ArcTanh[E^(I*z)] + z^2*Log[1 - E^((2*I)*z)] + (2*I)
*z*PolyLog[2, -E^(I*z)] - (2*I)*z*PolyLog[2, E^(I*z)] - I*z*PolyLog[2, E^(
(2*I)*z)] - 2*PolyLog[3, -E^(I*z)] + 2*PolyLog[3, E^(I*z)] + PolyLog[3, E^
((2*I)*z)]/2)*Sin[z])/(Sqrt[1 - Cos[z]]*Sqrt[1 + Cos[z]])
```

3.184.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5116
```
Int[(Cos[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(Cos[(e_.) + (f_.)*(x_)]*(d
_.) + (c_))^(n_)*((g_.) + (h_.)*(x_))^(p_.), x_Symbol] :> Simp[a^IntPart[m]
*c^IntPart[m]*(a + b*Cos[e + f*x])^FracPart[m]*((c + d*Cos[e + f*x])^FracPa
rt[m]/Sin[e + f*x]^(2*FracPart[m]))   Int[(g + h*x)^p*Sin[e + f*x]^(2*m)*(c
 + d*Cos[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[p] && IntegerQ[2*m] &&
IGeQ[n - m, 0]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.184.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.51

method	result	size
risch	$\frac{\left({\mathrm e}^{i z}-1\right) \sqrt{\left({\mathrm e}^{i z}+1\right)^{2} {\mathrm e}^{-i z}}\, z^{3}}{3 \sqrt{-\left({\mathrm e}^{i z}-1\right)^{2} {\mathrm e}^{-i z}}\, \left({\mathrm e}^{i z}+1\right)}+\frac{2 i \left({\mathrm e}^{i z}-1\right) \sqrt{\left({\mathrm e}^{i z}+1\right)^{2} {\mathrm e}^{-i z}}\, \left(\frac{i z^{3}}{3}-z^{2} \ln \left(-{\mathrm e}^{i z}+1\right)+2 i z \,\operatorname{polylog}\left(2, {\mathrm e}^{i z}\right)-2 \,\operatorname{polylog}\left(3, {\mathrm e}^{i z}\right)\right)}{\sqrt{-\left({\mathrm e}^{i z}-1\right)^{2} {\mathrm e}^{-i z}}\, \left({\mathrm e}^{i z}+1\right)}$	154

input
```
int(z^2*(1+cos(z))^(1/2)/(1-cos(z))^(1/2),z,method=_RETURNVERBOSE)
```

output
```
1/3/(-(exp(I*z)-1)^2*exp(-I*z))^(1/2)*(exp(I*z)-1)*((exp(I*z)+1)^2*exp(-I*
z))^(1/2)/(exp(I*z)+1)*z^3+2*I/(-(exp(I*z)-1)^2*exp(-I*z))^(1/2)*(exp(I*z)
-1)*((exp(I*z)+1)^2*exp(-I*z))^(1/2)/(exp(I*z)+1)*(1/3*I*z^3-z^2*ln(-exp(I
*z)+1)+2*I*z*polylog(2,exp(I*z))-2*polylog(3,exp(I*z)))
```

3.184.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.25

$$\int \frac{z^2\sqrt{1+\cos(z)}}{\sqrt{1-\cos(z)}}\,dz = z^2\log\left(-\cos\left(z\right)+i\,\sin\left(z\right)+1\right)+z^2\log\left(-\cos\left(z\right)-i\,\sin\left(z\right)+1\right) - 2i\,z\mathrm{Li}_2(\cos\left(z\right)+i\,\sin\left(z\right))+2i\,z\mathrm{Li}_2(\cos\left(z\right)-i\,\sin\left(z\right)) + 2\,\mathrm{polylog}(3,\cos\left(z\right)+i\,\sin\left(z\right))+2\,\mathrm{polylog}(3,\cos\left(z\right)-i\,\sin\left(z\right))$$

input
```
integrate(z^2*(1+cos(z))^(1/2)/(1-cos(z))^(1/2),z, algorithm="fricas")
```

output
```
z^2*log(-cos(z) + I*sin(z) + 1) + z^2*log(-cos(z) - I*sin(z) + 1) - 2*I*z*
dilog(cos(z) + I*sin(z)) + 2*I*z*dilog(cos(z) - I*sin(z)) + 2*polylog(3, c
os(z) + I*sin(z)) + 2*polylog(3, cos(z) - I*sin(z))
```

3.184.6 Sympy [F]

$$\int \frac{z^2\sqrt{1+\cos(z)}}{\sqrt{1-\cos(z)}}\,dz = \int \frac{z^2\sqrt{\cos\left(z\right)+1}}{\sqrt{1-\cos\left(z\right)}}\,dz$$

input
```
integrate(z**2*(1+cos(z))**(1/2)/(1-cos(z))**(1/2),z)
```

output
```
Integral(z**2*sqrt(cos(z) + 1)/sqrt(1 - cos(z)), z)
```

3.184.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.19

$$\int \frac{z^2\sqrt{1+\cos(z)}}{\sqrt{1-\cos(z)}}\,dz = \frac{1}{3}i\,z^3 + 2i\,z^2\arctan\left(\sin\left(z\right), -\cos\left(z\right)+1\right) - z^2\log\left(\cos\left(z\right)^2+\sin\left(z\right)^2-2\,\cos\left(z\right)+1\right) + 4i\,z\mathrm{Li}_2\left(e^{(i\,z)}\right) - 4\,\mathrm{Li}_3\left(e^{(i\,z)}\right)$$

input
```
integrate(z^2*(1+cos(z))^(1/2)/(1-cos(z))^(1/2),z, algorithm="maxima")
```

output
```
1/3*I*z^3 + 2*I*z^2*arctan2(sin(z), -cos(z) + 1) - z^2*log(cos(z)^2 + sin(
z)^2 - 2*cos(z) + 1) + 4*I*z*dilog(e^(I*z)) - 4*polylog(3, e^(I*z))
```

3.184.8 Giac [F]

$$\int \frac{z^2\sqrt{1+\cos(z)}}{\sqrt{1-\cos(z)}}\,dz = \int \frac{z^2\sqrt{\cos\left(z\right)+1}}{\sqrt{-\cos\left(z\right)+1}}\,dz$$

input
```
integrate(z^2*(1+cos(z))^(1/2)/(1-cos(z))^(1/2),z, algorithm="giac")
```

output
```
integrate(z^2*sqrt(cos(z) + 1)/sqrt(-cos(z) + 1), z)
```

3.184.9 Mupad [F(-1)]

Timed out.

$$\int \frac{z^2\sqrt{1+\cos(z)}}{\sqrt{1-\cos(z)}}\,dz = \int \frac{z^2\,\sqrt{\cos\left(z\right)+1}}{\sqrt{1-\cos\left(z\right)}}\,dz$$

input
```
int((z^2*(cos(z) + 1)^(1/2))/(1 - cos(z))^(1/2),z)
```

output
```
int((z^2*(cos(z) + 1)^(1/2))/(1 - cos(z))^(1/2), z)
```

3.185 $\int (a+a\cos(x))(A+B\sec(x))\,dx$

3.185.1 Optimal result

Integrand size = 13, antiderivative size = 18

$$\int (a+a\cos(x))(A+B\sec(x))\,dx = a(A+B)x + aB\text{arctanh}(\sin(x)) + aA\sin(x)$$

output
```
a*(A+B)*x+a*B*arctanh(sin(x))+a*A*sin(x)
```

3.185.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (a+a\cos(x))(A+B\sec(x))\,dx = aAx + aBx + aB\text{arctanh}(\sin(x)) + aA\sin(x)$$

input
```
Integrate[(a + a*Cos[x])*(A + B*Sec[x]),x]
```

output
```
a*A*x + a*B*x + a*B*ArcTanh[Sin[x]] + a*A*Sin[x]
```

3.185.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.769, Rules used = {3042, 3307, 3042, 3447, 3042, 3502, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(x)+a)(A+B\sec(x))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin\left(x+\frac{\pi}{2}\right)+a\right)\left(A+B\csc\left(x+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3307}$$

$$\int \sec(x)(a\cos(x)+a)(A\cos(x)+B)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(x+\frac{\pi}{2}\right)+a\right)\left(A\sin\left(x+\frac{\pi}{2}\right)+B\right)}{\sin\left(x+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3447}$$

$$\int \sec(x)\left(\cos(x)(aA+aB)+aA\cos^2(x)+aB\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(x+\frac{\pi}{2}\right)(aA+aB)+aA\sin\left(x+\frac{\pi}{2}\right)^2+aB}{\sin\left(x+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3502}$$

$$\int (aB+a(A+B)\cos(x))\sec(x)dx+aA\sin(x)$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{aB+a(A+B)\sin\left(x+\frac{\pi}{2}\right)}{\sin\left(x+\frac{\pi}{2}\right)}dx+aA\sin(x)$$

$$\Big\downarrow \text{3214}$$

$$aB\int \sec(x)dx+ax(A+B)+aA\sin(x)$$

$$\Big\downarrow \text{3042}$$

$$aB\int \csc\left(x+\frac{\pi}{2}\right)dx + ax(A+B) + aA\sin(x)$$

$$\Big\downarrow \text{4257}$$

$$ax(A+B) + aA\sin(x) + aB\text{arctanh}(\sin(x))$$

input
```
Int[(a + a*Cos[x])*(A + B*Sec[x]),x]
```

output
```
a*(A + B)*x + a*B*ArcTanh[Sin[x]] + a*A*Sin[x]
```

3.185.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3307
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((d + c*Sin[e +
 f*x])^n/Sin[e + f*x]^n), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 && !LtQ[m, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.185.4 Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33

method	result	s
default	$aA\sin(x)+Bax+aAx+Ba\ln(\sec(x)+\tan(x))$	2
parts	$aA\sin(x)+Bax+aAx+Ba\ln(\sec(x)+\tan(x))$	2
parallelrisch	$a(-B\ln(-\cot(x)+\csc(x)-1)+B\ln(\csc(x)-\cot(x)+1)+A\sin(x)+x(A+B))$	3
risch	$aAx+Bax-\frac{iAa\,e^{ix}}{2}+\frac{iAa\,e^{-ix}}{2}+Ba\ln(i+e^{ix})-Ba\ln(e^{ix}-i)$	5
norman	$\frac{(Aa+Ba)x+(Aa+Ba)x\tan(\frac{x}{2})^2+2Aa\tan(\frac{x}{2})}{1+\tan(\frac{x}{2})^2}+Ba\ln(\tan(\frac{x}{2})+1)-Ba\ln(\tan(\frac{x}{2})-1)$	6

input
```
int((a+a*cos(x))*(A+B*sec(x)),x,method=_RETURNVERBOSE)
```

output
```
a*A*sin(x)+B*a*x+a*A*x+B*a*ln(sec(x)+tan(x))
```

3.185.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.78

$$\int(a+a\cos(x))(A+B\sec(x))\,dx=(A+B)ax+\frac{1}{2}Ba\log(\sin(x)+1)-\frac{1}{2}Ba\log(-\sin(x)+1)+Aa\sin(x)$$

input
```
integrate((a+a*cos(x))*(A+B*sec(x)),x, algorithm="fricas")
```

output
```
(A + B)*a*x + 1/2*B*a*log(sin(x) + 1) - 1/2*B*a*log(-sin(x) + 1) + A*a*sin
(x)
```

3.185.6 Sympy [A] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.50

$$\int (a + a\cos(x))(A + B\sec(x))\,dx = Aax + Aa\sin(x) + Bax + Ba\log(\tan(x) + \sec(x))$$

input
```
integrate((a+a*cos(x))*(A+B*sec(x)),x)
```

output
```
A*a*x + A*a*sin(x) + B*a*x + B*a*log(tan(x) + sec(x))
```

3.185.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int (a + a\cos(x))(A + B\sec(x))\,dx = Aax + Bax + Ba\log(\sec(x) + \tan(x)) + Aa\sin(x)$$

input
```
integrate((a+a*cos(x))*(A+B*sec(x)),x, algorithm="maxima")
```

output
```
A*a*x + B*a*x + B*a*log(sec(x) + tan(x)) + A*a*sin(x)
```

3.185.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(18) = 36.

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.83

$$\begin{aligned}\int (a + a\cos(x))(A + B\sec(x))\,dx = {} & Ba\log\left(\left|\tan\left(\frac{1}{2}x\right) + 1\right|\right) \\ & - Ba\log\left(\left|\tan\left(\frac{1}{2}x\right) - 1\right|\right) \\ & + (Aa + Ba)x + \frac{2\,Aa\tan\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)^2 + 1}\end{aligned}$$

input
```
integrate((a+a*cos(x))*(A+B*sec(x)),x, algorithm="giac")
```

output
```
B*a*log(abs(tan(1/2*x) + 1)) - B*a*log(abs(tan(1/2*x) - 1)) + (A*a + B*a)*
x + 2*A*a*tan(1/2*x)/(tan(1/2*x)^2 + 1)
```

3.185.9 Mupad [B] (verification not implemented)

Time = 26.88 (sec) , antiderivative size = 54, normalized size of antiderivative = 3.00

$$\int (a + a\cos(x))(A + B\sec(x))\,dx = 2\,A\,a\,\mathrm{atan}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right) + 2\,B\,a\,\mathrm{atan}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right) + 2\,B\,a\,\mathrm{atanh}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right) + A\,a\,\sin\left(x\right)$$

input
```
int((a + a*cos(x))*(A + B/cos(x)),x)
```

output
```
2*A*a*atan(sin(x/2)/cos(x/2)) + 2*B*a*atan(sin(x/2)/cos(x/2)) + 2*B*a*atan
h(sin(x/2)/cos(x/2)) + A*a*sin(x)
```

3.186 $\int (a+a\cos(x))^2(A+B\sec(x))\,dx$

3.186.1 Optimal result

Integrand size = 15, antiderivative size = 57

$$\int (a+a\cos(x))^2(A+B\sec(x))\,dx = \frac{1}{2}a^2(3A+4B)x+a^2B\text{arctanh}(\sin(x)) + \frac{1}{2}a^2(3A+2B)\sin(x)+\frac{1}{2}A\left(a^2+a^2\cos(x)\right)\sin(x)$$

output
```
1/2*a^2*(3*A+4*B)*x+a^2*B*arctanh(sin(x))+1/2*a^2*(3*A+2*B)*sin(x)+1/2*A*(
a^2+a^2*cos(x))*sin(x)
```

3.186.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.18

$$\int (a+a\cos(x))^2(A+B\sec(x))\,dx = \frac{1}{4}a^2\left(6Ax+8Bx-4B\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right) + 4B\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)+4(2A+B)\sin(x) + A\sin(2x)\right)$$

input
```
Integrate[(a + a*Cos[x])^2*(A + B*Sec[x]),x]
```

output
```
(a^2*(6*A*x + 8*B*x - 4*B*Log[Cos[x/2] - Sin[x/2]] + 4*B*Log[Cos[x/2] + Si
n[x/2]] + 4*(2*A + B)*Sin[x] + A*Sin[2*x]))/4
```

3.186.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {3042, 3307, 3042, 3455, 3042, 3447, 3042, 3502, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(x)+a)^2(A+B\sec(x))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin\left(x+\frac{\pi}{2}\right)+a\right)^2\left(A+B\csc\left(x+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3307}$$

$$\int \sec(x)(a\cos(x)+a)^2(A\cos(x)+B)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(x+\frac{\pi}{2}\right)+a\right)^2\left(A\sin\left(x+\frac{\pi}{2}\right)+B\right)}{\sin\left(x+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3455}$$

$$\frac{1}{2}\int (\cos(x)a+a)(2aB+a(3A+2B)\cos(x))\sec(x)dx+\frac{1}{2}A\sin(x)\left(a^2\cos(x)+a^2\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{\left(\sin\left(x+\frac{\pi}{2}\right)a+a\right)\left(2aB+a(3A+2B)\sin\left(x+\frac{\pi}{2}\right)\right)}{\sin\left(x+\frac{\pi}{2}\right)}dx+\frac{1}{2}A\sin(x)\left(a^2\cos(x)+a^2\right)$$

$$\Big\downarrow \text{3447}$$

$$\frac{1}{2}\int \left((3A+2B)\cos^2(x)a^2+2Ba^2+\left(2Ba^2+(3A+2B)a^2\right)\cos(x)\right)\sec(x)dx+\frac{1}{2}A\sin(x)\left(a^2\cos(x)+a^2\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{(3A+2B)\sin\left(x+\frac{\pi}{2}\right)^2a^2+2Ba^2+\left(2Ba^2+(3A+2B)a^2\right)\sin\left(x+\frac{\pi}{2}\right)}{\sin\left(x+\frac{\pi}{2}\right)}dx+\frac{1}{2}A\sin(x)\left(a^2\cos(x)+a^2\right)$$

$$\Big\downarrow \text{3502}$$

$$\frac{1}{2}\left(\int \left(2Ba^2+(3A+4B)\cos(x)a^2\right)\sec(x)dx+a^2(3A+2B)\sin(x)\right)+\frac{1}{2}A\sin(x)\left(a^2\cos(x)+a^2\right)$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{1}{2}\left(\int \frac{2Ba^2+(3A+4B)\sin\left(x+\frac{\pi}{2}\right)a^2}{\sin\left(x+\frac{\pi}{2}\right)}dx+a^2(3A+2B)\sin(x)\right)+\frac{1}{2}A\sin(x)\left(a^2\cos(x)+a^2\right)$$

$$\Big\downarrow \textbf{3214}$$

$$\frac{1}{2}\left(2a^2B\int \sec(x)dx+a^2x(3A+4B)+a^2(3A+2B)\sin(x)\right)+\frac{1}{2}A\sin(x)\left(a^2\cos(x)+a^2\right)$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{1}{2}\left(2a^2B\int \csc\left(x+\frac{\pi}{2}\right)dx+a^2x(3A+4B)+a^2(3A+2B)\sin(x)\right)+\frac{1}{2}A\sin(x)\left(a^2\cos(x)+a^2\right)$$

$$\Big\downarrow \textbf{4257}$$

$$\frac{1}{2}\left(a^2x(3A+4B)+a^2(3A+2B)\sin(x)+2a^2B\text{arctanh}(\sin(x))\right)+\frac{1}{2}A\sin(x)\left(a^2\cos(x)+a^2\right)$$

input
```
Int[(a + a*Cos[x])^2*(A + B*Sec[x]),x]
```

output
```
(A*(a^2 + a^2*Cos[x])*Sin[x])/2 + (a^2*(3*A + 4*B)*x + 2*a^2*B*ArcTanh[Sin
[x]] + a^2*(3*A + 2*B)*Sin[x])/2
```

3.186.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3307
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((d + c*Sin[e +
 f*x])^n/Sin[e + f*x]^n), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.186.4 Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

method	result
parallelrisch	$a^2\left(-B\ln\left(-\cot\left(x\right)+\csc\left(x\right)-1\right)+\frac{3Ax}{2}+2A\sin\left(x\right)+\frac{A\sin(2x)}{4}+B\ln\left(\csc\left(x\right)-\cot\left(x\right)+\right.\right.$
default	$a^2A\left(\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)+B\,a^2\sin\left(x\right)+2a^2A\sin\left(x\right)+2B\,a^2x+a^2Ax+B\,a^2\ln\left(\sec\left(x\right)+\tan\right.$
parts	$a^2A\left(\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)+B\,a^2\sin\left(x\right)+2a^2A\sin\left(x\right)+2B\,a^2x+a^2Ax+B\,a^2\ln\left(\sec\left(x\right)+\tan\right.$
risch	$\frac{3a^2Ax}{2}+2B\,a^2x-iA\,\mathrm{e}^{ix}a^2-\frac{iB\,\mathrm{e}^{ix}a^2}{2}+iA\,\mathrm{e}^{-ix}a^2+\frac{iB\,\mathrm{e}^{-ix}a^2}{2}+B\,a^2\ln\left(i+\mathrm{e}^{ix}\right)-B\,a^2\ln\left(\mathrm{e}^{ix}\right.$
norman	$\frac{\left(\frac{3}{2}a^2A+2B\,a^2\right)x+\left(3a^2A+2B\,a^2\right)\tan\left(\frac{x}{2}\right)^3+\left(5a^2A+2B\,a^2\right)\tan\left(\frac{x}{2}\right)+\left(\frac{3}{2}a^2A+2B\,a^2\right)x\tan\left(\frac{x}{2}\right)^4+\left(3a^2A+4B\,a^2\right)x\tan\left(\frac{x}{2}\right)^2}{\left(1+\tan\left(\frac{x}{2}\right)^2\right)^2}+$

input
```
int((a+a*cos(x))^2*(A+B*sec(x)),x,method=_RETURNVERBOSE)
```

output
```
a^2*(-B*ln(-cot(x)+csc(x)-1)+3/2*A*x+2*A*sin(x)+1/4*A*sin(2*x)+B*ln(csc(x)
-cot(x)+1)+2*B*x+B*sin(x))
```

3.186.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.05

$$\int (a+a\cos(x))^2(A+B\sec(x))\,dx = \frac{1}{2}\,(3\,A+4\,B)a^2x+\frac{1}{2}\,Ba^2\log\left(\sin\left(x\right)+1\right) - \frac{1}{2}\,Ba^2\log\left(-\sin\left(x\right)+1\right) + \frac{1}{2}\,\left(Aa^2\cos\left(x\right)+2\,(2\,A+B)a^2\right)\sin\left(x\right)$$

input
```
integrate((a+a*cos(x))^2*(A+B*sec(x)),x, algorithm="fricas")
```

output
```
1/2*(3*A + 4*B)*a^2*x + 1/2*B*a^2*log(sin(x) + 1) - 1/2*B*a^2*log(-sin(x)
+ 1) + 1/2*(A*a^2*cos(x) + 2*(2*A + B)*a^2)*sin(x)
```

3.186.6 Sympy [A] (verification not implemented)

Time = 1.24 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07

$$\int (a+a\cos(x))^2(A+B\sec(x))\,dx = \frac{3Aa^2x}{2}+2Aa^2\sin(x)+\frac{Aa^2\sin(2x)}{4}+2Ba^2x + Ba^2\log(\tan(x)+\sec(x))+Ba^2\sin(x)$$

input
```
integrate((a+a*cos(x))**2*(A+B*sec(x)),x)
```

output
```
3*A*a**2*x/2 + 2*A*a**2*sin(x) + A*a**2*sin(2*x)/4 + 2*B*a**2*x + B*a**2*l
og(tan(x) + sec(x)) + B*a**2*sin(x)
```

3.186.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

$$\int (a+a\cos(x))^2(A+B\sec(x))\,dx = \frac{1}{4}Aa^2(2x+\sin(2x))+Aa^2x + 2Ba^2x+Ba^2\log(\sec(x)+\tan(x)) + 2Aa^2\sin(x)+Ba^2\sin(x)$$

input
```
integrate((a+a*cos(x))^2*(A+B*sec(x)),x, algorithm="maxima")
```

output
```
1/4*A*a^2*(2*x + sin(2*x)) + A*a^2*x + 2*B*a^2*x + B*a^2*log(sec(x) + tan(
x)) + 2*A*a^2*sin(x) + B*a^2*sin(x)
```

3.186.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.75

$$\int (a+a\cos(x))^2(A+B\sec(x))\,dx = Ba^2\log\left(\left|\tan\left(\frac{1}{2}x\right)+1\right|\right)-Ba^2\log\left(\left|\tan\left(\frac{1}{2}x\right)-1\right|\right)+\frac{1}{2}(3Aa^2+4Ba^2)x + \frac{3Aa^2\tan\left(\frac{1}{2}x\right)^3+2Ba^2\tan\left(\frac{1}{2}x\right)^3+5Aa^2\tan\left(\frac{1}{2}x\right)+2Ba^2\tan\left(\frac{1}{2}x\right)}{\left(\tan\left(\frac{1}{2}x\right)^2+1\right)^2}$$

input
```
integrate((a+a*cos(x))^2*(A+B*sec(x)),x, algorithm="giac")
```

output
```
B*a^2*log(abs(tan(1/2*x) + 1)) - B*a^2*log(abs(tan(1/2*x) - 1)) + 1/2*(3*A
*a^2 + 4*B*a^2)*x + (3*A*a^2*tan(1/2*x)^3 + 2*B*a^2*tan(1/2*x)^3 + 5*A*a^2
*tan(1/2*x) + 2*B*a^2*tan(1/2*x))/(tan(1/2*x)^2 + 1)^2
```

3.186.9 Mupad [B] (verification not implemented)

Time = 27.83 (sec) , antiderivative size = 403, normalized size of antiderivative = 7.07

$$\int (a+a\cos(x))^2(A+B\sec(x))\,dx$$
$$= \frac{(3\,A\,a^2+2\,B\,a^2)\,\tan\left(\frac{x}{2}\right)^3+(5\,A\,a^2+2\,B\,a^2)\,\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^4+2\tan\left(\frac{x}{2}\right)^2+1}$$
$$+ a^2\,\mathrm{atan}\left(\frac{216\,A^3\,a^6\tan\left(\frac{x}{2}\right)}{216\,A^3\,a^6+864\,A^2\,B\,a^6+1248\,A\,B^2\,a^6+640\,B^3\,a^6} + \frac{640\,B^3\,a^6\tan\left(\frac{x}{2}\right)}{216\,A^3\,a^6+864\,A^2\,B\,a^6+1248\,A\,B^2\,a^6+640\,B^3\,a^6} + \frac{1248\,A\,B^2\,a^6\tan\left(\frac{x}{2}\right)}{216\,A^3\,a^6+864\,A^2\,B\,a^6+1248\,A\,B^2\,a^6+640\,B^3\,a^6} + \frac{864\,A^2\,B\,a^6\tan\left(\frac{x}{2}\right)}{216\,A^3\,a^6+864\,A^2\,B\,a^6+1248\,A\,B^2\,a^6+640\,B^3\,a^6}\right)(3\,A+4\,B)$$
$$+ 2\,B\,a^2\,\mathrm{atanh}\left(\frac{320\,B^3\,a^6\tan\left(\frac{x}{2}\right)}{144\,A^2\,B\,a^6+384\,A\,B^2\,a^6+320\,B^3\,a^6} + \frac{384\,A\,B^2\,a^6\tan\left(\frac{x}{2}\right)}{144\,A^2\,B\,a^6+384\,A\,B^2\,a^6+320\,B^3\,a^6} + \frac{144\,A^2\,B\,a^6\tan\left(\frac{x}{2}\right)}{144\,A^2\,B\,a^6+384\,A\,B^2\,a^6+320\,B^3\,a^6}\right)$$

input
```
int((a + a*cos(x))^2*(A + B/cos(x)),x)
```

output
```
(tan(x/2)^3*(3*A*a^2 + 2*B*a^2) + tan(x/2)*(5*A*a^2 + 2*B*a^2))/(2*tan(x/2
)^2 + tan(x/2)^4 + 1) + a^2*atan((216*A^3*a^6*tan(x/2))/(216*A^3*a^6 + 640
*B^3*a^6 + 1248*A*B^2*a^6 + 864*A^2*B*a^6) + (640*B^3*a^6*tan(x/2))/(216*A
^3*a^6 + 640*B^3*a^6 + 1248*A*B^2*a^6 + 864*A^2*B*a^6) + (1248*A*B^2*a^6*t
an(x/2))/(216*A^3*a^6 + 640*B^3*a^6 + 1248*A*B^2*a^6 + 864*A^2*B*a^6) + (8
64*A^2*B*a^6*tan(x/2))/(216*A^3*a^6 + 640*B^3*a^6 + 1248*A*B^2*a^6 + 864*A
^2*B*a^6))*(3*A + 4*B) + 2*B*a^2*atanh((320*B^3*a^6*tan(x/2))/(320*B^3*a^6
 + 384*A*B^2*a^6 + 144*A^2*B*a^6) + (384*A*B^2*a^6*tan(x/2))/(320*B^3*a^6
+ 384*A*B^2*a^6 + 144*A^2*B*a^6) + (144*A^2*B*a^6*tan(x/2))/(320*B^3*a^6 +
 384*A*B^2*a^6 + 144*A^2*B*a^6))
```

3.186. $\int (a + a\cos(x))^2 (A + B\sec(x))\, dx$

3.187 $\int (a+a\cos(x))^3(A+B\sec(x))\,dx$

3.187.1 Optimal result

Integrand size = 15, antiderivative size = 75

$$\begin{aligned}\int (a+a\cos(x))^3(A+B\sec(x))\,dx &= \frac{1}{2}a^3(5A+7B)x+a^3B\text{arctanh}(\sin(x)) \\ &\quad+\frac{5}{2}a^3(A+B)\sin(x)+\frac{1}{3}aA(a+a\cos(x))^2\sin(x) \\ &\quad+\frac{1}{6}(5A+3B)\left(a^3+a^3\cos(x)\right)\sin(x)\end{aligned}$$

output
```
1/2*a^3*(5*A+7*B)*x+a^3*B*arctanh(sin(x))+5/2*a^3*(A+B)*sin(x)+1/3*a*A*(a+
a*cos(x))^2*sin(x)+1/6*(5*A+3*B)*(a^3+a^3*cos(x))*sin(x)
```

3.187.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.07

$$\begin{aligned}\int (a+a\cos(x))^3(A+B\sec(x))\,dx &= \frac{1}{12}a^3\left(30Ax+42Bx-12B\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)\right. \\ &\quad+12B\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)+9(5A+4B)\sin(x) \\ &\quad\left.+3(3A+B)\sin(2x)+A\sin(3x)\right)\end{aligned}$$

input
```
Integrate[(a + a*Cos[x])^3*(A + B*Sec[x]),x]
```

output
```
(a^3*(30*A*x + 42*B*x - 12*B*Log[Cos[x/2] - Sin[x/2]] + 12*B*Log[Cos[x/2]
+ Sin[x/2]] + 9*(5*A + 4*B)*Sin[x] + 3*(3*A + B)*Sin[2*x] + A*Sin[3*x]))/1
2
```

3.187.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.08, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 3307, 3042, 3455, 3042, 3455, 27, 3042, 3447, 3042, 3502, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(x)+a)^3(A+B\sec(x))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin\left(x+\frac{\pi}{2}\right)+a\right)^3\left(A+B\csc\left(x+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3307}$$

$$\int \sec(x)(a\cos(x)+a)^3(A\cos(x)+B)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(x+\frac{\pi}{2}\right)+a\right)^3\left(A\sin\left(x+\frac{\pi}{2}\right)+B\right)}{\sin\left(x+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3455}$$

$$\frac{1}{3}\int (\cos(x)a+a)^2(3aB+a(5A+3B)\cos(x))\sec(x)dx+\frac{1}{3}aA\sin(x)(a\cos(x)+a)^2$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{\left(\sin\left(x+\frac{\pi}{2}\right)a+a\right)^2\left(3aB+a(5A+3B)\sin\left(x+\frac{\pi}{2}\right)\right)}{\sin\left(x+\frac{\pi}{2}\right)}dx+\frac{1}{3}aA\sin(x)(a\cos(x)+a)^2$$

$$\Big\downarrow \text{3455}$$

$$\frac{1}{3}\left(\frac{1}{2}\int 3(\cos(x)a+a)\left(2Ba^2+5(A+B)\cos(x)a^2\right)\sec(x)dx+\frac{1}{2}(5A+3B)\sin(x)\left(a^3\cos(x)+a^3\right)\right)+\frac{1}{3}aA\sin(x)(a\cos(x)+a)^2$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{3}{2}\int (\cos(x)a+a)\left(2Ba^2+5(A+B)\cos(x)a^2\right)\sec(x)dx+\frac{1}{2}(5A+3B)\sin(x)\left(a^3\cos(x)+a^3\right)\right)+\frac{1}{3}aA\sin(x)(a\cos(x)+a)^2$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\frac{3}{2}\int \frac{\left(\sin\left(x+\frac{\pi}{2}\right)a+a\right)\left(2Ba^2+5(A+B)\sin\left(x+\frac{\pi}{2}\right)a^2\right)}{\sin\left(x+\frac{\pi}{2}\right)}dx+\frac{1}{2}(5A+3B)\sin(x)\left(a^3\cos(x)+a^3\right)\right)+\frac{1}{3}aA\sin(x)(a\cos(x)+a)^2$$

$$\Big\downarrow \text{3447}$$

$$\frac{1}{3}\left(\frac{3}{2}\int \left(5(A+B)\cos^2(x)a^3+2Ba^3+\left(2Ba^3+5(A+B)a^3\right)\cos(x)\right)\sec(x)dx+\frac{1}{2}(5A+3B)\sin(x)\left(a^3\cos(x)+\right.\right.$$
$$\frac{1}{3}aA\sin(x)(a\cos(x)+a)^2$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\frac{3}{2}\int \frac{5(A+B)\sin\left(x+\frac{\pi}{2}\right)^2a^3+2Ba^3+\left(2Ba^3+5(A+B)a^3\right)\sin\left(x+\frac{\pi}{2}\right)}{\sin\left(x+\frac{\pi}{2}\right)}dx+\frac{1}{2}(5A+3B)\sin(x)\left(a^3\cos(x)\right.\right.$$
$$\frac{1}{3}aA\sin(x)(a\cos(x)+a)^2$$

$$\Big\downarrow \text{3502}$$

$$\frac{1}{3}\left(\frac{3}{2}\left(\int \left(2Ba^3+(5A+7B)\cos(x)a^3\right)\sec(x)dx+5a^3(A+B)\sin(x)\right)+\frac{1}{2}(5A+3B)\sin(x)\left(a^3\cos(x)+a^3\right)\right)+\frac{1}{3}aA\sin(x)(a\cos(x)+a)^2$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\frac{3}{2}\left(\int \frac{2Ba^3+(5A+7B)\sin\left(x+\frac{\pi}{2}\right)a^3}{\sin\left(x+\frac{\pi}{2}\right)}dx+5a^3(A+B)\sin(x)\right)+\frac{1}{2}(5A+3B)\sin(x)\left(a^3\cos(x)+a^3\right)\right)+\frac{1}{3}aA\sin(x)(a\cos(x)+a)^2$$

$$\Big\downarrow \text{3214}$$

3.187. $\int (a+a\cos(x))^3(A+B\sec(x))\,dx$

$$\frac{1}{3}\left(\frac{3}{2}\left(2a^3B\int \sec(x)dx+a^3x(5A+7B)+5a^3(A+B)\sin(x)\right)+\frac{1}{2}(5A+3B)\sin(x)\left(a^3\cos(x)+a^3\right)\right)+\frac{1}{3}aA\sin(x)(a\cos(x)+a)^2$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(\frac{3}{2}\left(2a^3B\int \csc\left(x+\frac{\pi}{2}\right)dx+a^3x(5A+7B)+5a^3(A+B)\sin(x)\right)+\frac{1}{2}(5A+3B)\sin(x)\left(a^3\cos(x)+a^3\right)\right)+\frac{1}{3}aA\sin(x)(a\cos(x)+a)^2$$

$\Big\downarrow$ 4257

$$\frac{1}{3}\left(\frac{3}{2}\left(a^3x(5A+7B)+5a^3(A+B)\sin(x)+2a^3B\text{arctanh}(\sin(x))\right)+\frac{1}{2}(5A+3B)\sin(x)\left(a^3\cos(x)+a^3\right)\right)+\frac{1}{3}aA\sin(x)(a\cos(x)+a)^2$$

input
```
Int[(a + a*Cos[x])^3*(A + B*Sec[x]),x]
```

output
```
(a*A*(a + a*Cos[x])^2*Sin[x])/3 + (((5*A + 3*B)*(a^3 + a^3*Cos[x])*Sin[x])
/2 + (3*(a^3*(5*A + 7*B)*x + 2*a^3*B*ArcTanh[Sin[x]] + 5*a^3*(A + B)*Sin[x
]))/2)/3
```

3.187.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3307
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((d + c*Sin[e +
 f*x])^n/Sin[e + f*x]^n), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.187. $\int (a+a\cos(x))^3(A+B\sec(x))\,dx$

3.187.4 Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.89

method	result
parallelrisch	$\frac{5\left(-\frac{2B\ln(-\cot(x)+\csc(x)-1)}{5}+\frac{2B\ln(\csc(x)-\cot(x)+1)}{5}+\left(\frac{3A}{10}+\frac{B}{10}\right)\sin(2x)+\frac{A\sin(3x)}{30}+\left(\frac{3A}{2}+\frac{6B}{5}\right)\sin(x)+x\left(A+\frac{7B}{5}\right)\right)a^3}{2}$
default	$\frac{a^3A\left(2+\cos(x)^2\right)\sin(x)}{3}+a^3B\left(\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)+3a^3A\left(\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)+3a^3B\sin(x)+3a^3A\,\mathrm{s}$
parts	$\frac{a^3A\left(2+\cos(x)^2\right)\sin(x)}{3}+a^3B\left(\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)+3a^3A\left(\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)+3a^3B\sin(x)+3a^3A\,\mathrm{s}$
risch	$\frac{5a^3Ax}{2}+\frac{7a^3Bx}{2}-\frac{15iA\,e^{ix}a^3}{8}-\frac{3iB\,e^{ix}a^3}{2}+\frac{15iA\,e^{-ix}a^3}{8}+\frac{3iB\,e^{-ix}a^3}{2}+a^3B\ln\left(i+e^{ix}\right)-a^3B\ln\left(e^{ix}\right.$
norman	$\frac{\left(\frac{5}{2}a^3A+\frac{7}{2}a^3B\right)x+\left(\frac{40}{3}a^3A+12a^3B\right)\tan\left(\frac{x}{2}\right)^3+\left(5a^3A+5a^3B\right)\tan\left(\frac{x}{2}\right)^5+\left(11a^3A+7a^3B\right)\tan\left(\frac{x}{2}\right)+\left(\frac{5}{2}a^3A+\frac{7}{2}a^3B\right)x\tan\left(\frac{x}{2}\right)^6+}{\left(1+\tan\left(\frac{x}{2}\right)^2\right)^3}$

input
```
int((a+a*cos(x))^3*(A+B*sec(x)),x,method=_RETURNVERBOSE)
```

output
```
5/2*(-2/5*B*ln(-cot(x)+csc(x)-1)+2/5*B*ln(csc(x)-cot(x)+1)+(3/10*A+1/10*B)
*sin(2*x)+1/30*A*sin(3*x)+(3/2*A+6/5*B)*sin(x)+x*(A+7/5*B))*a^3
```

3.187.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.03

$$\int (a+a\cos(x))^3(A+B\sec(x))\,dx$$
$$=\frac{1}{2}(5A+7B)a^3x+\frac{1}{2}Ba^3\log(\sin(x)+1)-\frac{1}{2}Ba^3\log(-\sin(x)+1)$$
$$+\frac{1}{6}\left(2Aa^3\cos(x)^2+3(3A+B)a^3\cos(x)+2(11A+9B)a^3\right)\sin(x)$$

input
```
integrate((a+a*cos(x))^3*(A+B*sec(x)),x, algorithm="fricas")
```

output
```
1/2*(5*A + 7*B)*a^3*x + 1/2*B*a^3*log(sin(x) + 1) - 1/2*B*a^3*log(-sin(x)
+ 1) + 1/6*(2*A*a^3*cos(x)^2 + 3*(3*A + B)*a^3*cos(x) + 2*(11*A + 9*B)*a^3
)*sin(x)
```

3.187.6 Sympy [A] (verification not implemented)

Time = 2.88 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.23

$$\int (a+a\cos(x))^3(A+B\sec(x))\,dx = \frac{5Aa^3x}{2} - \frac{Aa^3\sin^3(x)}{3} + 4Aa^3\sin(x) + \frac{3Aa^3\sin(2x)}{4} + \frac{7Ba^3x}{2} + Ba^3\log(\tan(x)+\sec(x)) + \frac{Ba^3\sin(x)\cos(x)}{2} + 3Ba^3\sin(x)$$

input
```
integrate((a+a*cos(x))**3*(A+B*sec(x)),x)
```

output
```
5*A*a**3*x/2 - A*a**3*sin(x)**3/3 + 4*A*a**3*sin(x) + 3*A*a**3*sin(2*x)/4
+ 7*B*a**3*x/2 + B*a**3*log(tan(x) + sec(x)) + B*a**3*sin(x)*cos(x)/2 + 3*
B*a**3*sin(x)
```

3.187.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.12

$$\int (a+a\cos(x))^3(A+B\sec(x))\,dx = -\frac{1}{3}\left(\sin(x)^3 - 3\sin(x)\right)Aa^3 + \frac{3}{4}Aa^3(2x+\sin(2x)) + \frac{1}{4}Ba^3(2x+\sin(2x)) + Aa^3x + 3Ba^3x + Ba^3\log(\sec(x)+\tan(x)) + 3Aa^3\sin(x) + 3Ba^3\sin(x)$$

input
```
integrate((a+a*cos(x))^3*(A+B*sec(x)),x, algorithm="maxima")
```

output
```
-1/3*(sin(x)^3 - 3*sin(x))*A*a^3 + 3/4*A*a^3*(2*x + sin(2*x)) + 1/4*B*a^3*
(2*x + sin(2*x)) + A*a^3*x + 3*B*a^3*x + B*a^3*log(sec(x) + tan(x)) + 3*A*
a^3*sin(x) + 3*B*a^3*sin(x)
```

3.187.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.67

$$\int (a+a\cos(x))^3(A+B\sec(x))\,dx$$
$$= Ba^3\log\left(\left|\tan\left(\frac{1}{2}x\right)+1\right|\right)-Ba^3\log\left(\left|\tan\left(\frac{1}{2}x\right)-1\right|\right)+\frac{1}{2}\left(5\,Aa^3+7\,Ba^3\right)x$$
$$+\frac{15\,Aa^3\tan\left(\frac{1}{2}x\right)^5+15\,Ba^3\tan\left(\frac{1}{2}x\right)^5+40\,Aa^3\tan\left(\frac{1}{2}x\right)^3+36\,Ba^3\tan\left(\frac{1}{2}x\right)^3+33\,Aa^3\tan\left(\frac{1}{2}x\right)+21}{3\left(\tan\left(\frac{1}{2}x\right)^2+1\right)^3}$$

input
```
integrate((a+a*cos(x))^3*(A+B*sec(x)),x, algorithm="giac")
```

output
```
B*a^3*log(abs(tan(1/2*x) + 1)) - B*a^3*log(abs(tan(1/2*x) - 1)) + 1/2*(5*A
*a^3 + 7*B*a^3)*x + 1/3*(15*A*a^3*tan(1/2*x)^5 + 15*B*a^3*tan(1/2*x)^5 + 4
0*A*a^3*tan(1/2*x)^3 + 36*B*a^3*tan(1/2*x)^3 + 33*A*a^3*tan(1/2*x) + 21*B*
a^3*tan(1/2*x))/(tan(1/2*x)^2 + 1)^3
```

3.187.9 Mupad [B] (verification not implemented)

Time = 28.29 (sec) , antiderivative size = 431, normalized size of antiderivative = 5.75

$$\int (a+a\cos(x))^3(A+B\sec(x))\,dx$$
$$=\frac{\left(5\,A\,a^3+5\,B\,a^3\right)\,\tan\left(\frac{x}{2}\right)^5+\left(\frac{40\,A\,a^3}{3}+12\,B\,a^3\right)\,\tan\left(\frac{x}{2}\right)^3+\left(11\,A\,a^3+7\,B\,a^3\right)\,\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^6+3\tan\left(\frac{x}{2}\right)^4+3\tan\left(\frac{x}{2}\right)^2+1}$$
$$+a^3\,\mathrm{atan}\left(\frac{1000\,A^3\,a^9\,\tan\left(\frac{x}{2}\right)}{1000\,A^3\,a^9+4200\,A^2\,B\,a^9+6040\,A\,B^2\,a^9+2968\,B^3\,a^9}+\frac{2968\,B^3\,a^9\,\tan\left(\frac{x}{2}\right)}{1000\,A^3\,a^9+4200\,A^2\,B\,a^9+6040\,A\,B^2\,a^9+2968\,B^3\,a^9}+\frac{6040\,A\,B^2\,a^9\,\tan\left(\frac{x}{2}\right)}{1000\,A^3\,a^9+4200\,A^2\,B\,a^9+6040\,A\,B^2\,a^9+2968\,B^3\,a^9}+\frac{4200\,A^2\,B\,a^9\,\tan\left(\frac{x}{2}\right)}{1000\,A^3\,a^9+4200\,A^2\,B\,a^9+6040\,A\,B^2\,a^9+2968\,B^3\,a^9}\right)\,(5\,A+7\,B)$$
$$+2\,B\,a^3\,\mathrm{atanh}\left(\frac{848\,B^3\,a^9\,\tan\left(\frac{x}{2}\right)}{400\,A^2\,B\,a^9+1120\,A\,B^2\,a^9+848\,B^3\,a^9}+\frac{1120\,A\,B^2\,a^9\,\tan\left(\frac{x}{2}\right)}{400\,A^2\,B\,a^9+1120\,A\,B^2\,a^9+848\,B^3\,a^9}+\frac{400\,A^2\,B\,a^9\,\tan\left(\frac{x}{2}\right)}{400\,A^2\,B\,a^9+1120\,A\,B^2\,a^9+848\,B^3\,a^9}\right)$$

input
```
int((a + a*cos(x))^3*(A + B/cos(x)),x)
```

output
```
(tan(x/2)^5*(5*A*a^3 + 5*B*a^3) + tan(x/2)^3*((40*A*a^3)/3 + 12*B*a^3) + t
an(x/2)*(11*A*a^3 + 7*B*a^3))/(3*tan(x/2)^2 + 3*tan(x/2)^4 + tan(x/2)^6 +
1) + a^3*atan((1000*A^3*a^9*tan(x/2))/(1000*A^3*a^9 + 2968*B^3*a^9 + 6040*
A*B^2*a^9 + 4200*A^2*B*a^9) + (2968*B^3*a^9*tan(x/2))/(1000*A^3*a^9 + 2968
*B^3*a^9 + 6040*A*B^2*a^9 + 4200*A^2*B*a^9) + (6040*A*B^2*a^9*tan(x/2))/(1
000*A^3*a^9 + 2968*B^3*a^9 + 6040*A*B^2*a^9 + 4200*A^2*B*a^9) + (4200*A^2*
B*a^9*tan(x/2))/(1000*A^3*a^9 + 2968*B^3*a^9 + 6040*A*B^2*a^9 + 4200*A^2*B
*a^9))*(5*A + 7*B) + 2*B*a^3*atanh((848*B^3*a^9*tan(x/2))/(848*B^3*a^9 + 1
120*A*B^2*a^9 + 400*A^2*B*a^9) + (1120*A*B^2*a^9*tan(x/2))/(848*B^3*a^9 +
1120*A*B^2*a^9 + 400*A^2*B*a^9) + (400*A^2*B*a^9*tan(x/2))/(848*B^3*a^9 +
1120*A*B^2*a^9 + 400*A^2*B*a^9))
```

3.187. $\int (a + a\cos(x))^3 (A + B\sec(x))\,dx$

3.188 $\int (a+a\cos(x))^4(A+B\sec(x))\,dx$

3.188.1 Optimal result

Integrand size = 15, antiderivative size = 104

$$\begin{aligned}\int (a+a\cos(x))^4(A+B\sec(x))\,dx &= \frac{1}{8}a^4(35A+48B)x+a^4B\text{arctanh}(\sin(x)) \\ &\quad+\frac{5}{8}a^4(7A+8B)\sin(x)+\frac{1}{4}aA(a+a\cos(x))^3\sin(x) \\ &\quad+\frac{1}{12}(7A+4B)\left(a^2+a^2\cos(x)\right)^2\sin(x) \\ &\quad+\frac{1}{24}(35A+32B)\left(a^4+a^4\cos(x)\right)\sin(x)\end{aligned}$$

output
```
1/8*a^4*(35*A+48*B)*x+a^4*B*arctanh(sin(x))+5/8*a^4*(7*A+8*B)*sin(x)+1/4*a
*A*(a+a*cos(x))^3*sin(x)+1/12*(7*A+4*B)*(a^2+a^2*cos(x))^2*sin(x)+1/24*(35
*A+32*B)*(a^4+a^4*cos(x))*sin(x)
```

3.188.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.93

$$\begin{aligned}\int (a+a\cos(x))^4(A+B\sec(x))\,dx &= \frac{1}{96}a^4\left(420Ax+576Bx-96B\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)\right. \\ &\quad+96B\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right) \\ &\quad+24(28A+27B)\sin(x)+24(7A+4B)\sin(2x) \\ &\quad\left.+32A\sin(3x)+8B\sin(3x)+3A\sin(4x)\right)\end{aligned}$$

input
```
Integrate[(a + a*Cos[x])^4*(A + B*Sec[x]),x]
```

output
```
(a^4*(420*A*x + 576*B*x - 96*B*Log[Cos[x/2] - Sin[x/2]] + 96*B*Log[Cos[x/2
] + Sin[x/2]] + 24*(28*A + 27*B)*Sin[x] + 24*(7*A + 4*B)*Sin[2*x] + 32*A*S
in[3*x] + 8*B*Sin[3*x] + 3*A*Sin[4*x]))/96
```

3.188.3 Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.11, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 1.133$, Rules used = {3042, 3307, 3042, 3455, 3042, 3455, 3042, 3455, 27, 3042, 3447, 3042, 3502, 3042, 3214, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(x)+a)^4(A+B\sec(x))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin\left(x+\frac{\pi}{2}\right)+a\right)^4\left(A+B\csc\left(x+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3307}$$

$$\int \sec(x)(a\cos(x)+a)^4(A\cos(x)+B)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(x+\frac{\pi}{2}\right)+a\right)^4\left(A\sin\left(x+\frac{\pi}{2}\right)+B\right)}{\sin\left(x+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3455}$$

$$\frac{1}{4}\int (\cos(x)a+a)^3(4aB+a(7A+4B)\cos(x))\sec(x)dx+\frac{1}{4}aA\sin(x)(a\cos(x)+a)^3$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \frac{\left(\sin\left(x+\frac{\pi}{2}\right)a+a\right)^3\left(4aB+a(7A+4B)\sin\left(x+\frac{\pi}{2}\right)\right)}{\sin\left(x+\frac{\pi}{2}\right)}dx+\frac{1}{4}aA\sin(x)(a\cos(x)+a)^3$$

$$\Big\downarrow \text{3455}$$

$$\frac{1}{4}\left(\frac{1}{3}\int(\cos(x)a+a)^2\left(12Ba^2+(35A+32B)\cos(x)a^2\right)\sec(x)dx+\frac{1}{3}(7A+4B)\sin(x)\left(a^2\cos(x)+a^2\right)^2\right)+\frac{1}{4}aA\sin(x)(a\cos(x)+a)^3$$

$\Big\downarrow$ 3042

$$\frac{1}{4}\left(\frac{1}{3}\int\frac{\left(\sin\left(x+\frac{\pi}{2}\right)a+a\right)^2\left(12Ba^2+(35A+32B)\sin\left(x+\frac{\pi}{2}\right)a^2\right)}{\sin\left(x+\frac{\pi}{2}\right)}dx+\frac{1}{3}(7A+4B)\sin(x)\left(a^2\cos(x)+a^2\right)^2\right)+\frac{1}{4}aA\sin(x)(a\cos(x)+a)^3$$

$\Big\downarrow$ 3455

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\int3(\cos(x)a+a)\left(8Ba^3+5(7A+8B)\cos(x)a^3\right)\sec(x)dx+\frac{1}{2}(35A+32B)\sin(x)\left(a^4\cos(x)+a^4\right)\right)+\frac{1}{3}\right.$$
$$\frac{1}{4}aA\sin(x)(a\cos(x)+a)^3$$

$\Big\downarrow$ 27

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{3}{2}\int(\cos(x)a+a)\left(8Ba^3+5(7A+8B)\cos(x)a^3\right)\sec(x)dx+\frac{1}{2}(35A+32B)\sin(x)\left(a^4\cos(x)+a^4\right)\right)+\frac{1}{3}(\right.$$
$$\frac{1}{4}aA\sin(x)(a\cos(x)+a)^3$$

$\Big\downarrow$ 3042

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{3}{2}\int\frac{\left(\sin\left(x+\frac{\pi}{2}\right)a+a\right)\left(8Ba^3+5(7A+8B)\sin\left(x+\frac{\pi}{2}\right)a^3\right)}{\sin\left(x+\frac{\pi}{2}\right)}dx+\frac{1}{2}(35A+32B)\sin(x)\left(a^4\cos(x)+a^4\right)\right)\right.$$
$$\frac{1}{4}aA\sin(x)(a\cos(x)+a)^3$$

$\Big\downarrow$ 3447

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{3}{2}\int\left(5(7A+8B)\cos^2(x)a^4+8Ba^4+\left(8Ba^4+5(7A+8B)a^4\right)\cos(x)\right)\sec(x)dx+\frac{1}{2}(35A+32B)\sin(x)\right.\right.(a$$
$$\frac{1}{4}aA\sin(x)(a\cos(x)+a)^3$$

$\Big\downarrow$ 3042

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{3}{2}\int\frac{5(7A+8B)\sin\left(x+\frac{\pi}{2}\right)^2a^4+8Ba^4+\left(8Ba^4+5(7A+8B)a^4\right)\sin\left(x+\frac{\pi}{2}\right)}{\sin\left(x+\frac{\pi}{2}\right)}dx+\frac{1}{2}(35A+32B)\sin(x)\right.\right.$$
$$\frac{1}{4}aA\sin(x)(a\cos(x)+a)^3$$

3.188. $\int(a+a\cos(x))^4(A+B\sec(x))\,dx$

$$\Big\downarrow \text{3502}$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{3}{2}\left(\int (8Ba^4 + (35A + 48B)\cos(x)a^4)\sec(x)dx + 5a^4(7A + 8B)\sin(x)\right) + \frac{1}{2}(35A + 32B)\sin(x)\left(a^4\cos(x\right.\right.\right.$$
$$\frac{1}{4}aA\sin(x)(a\cos(x) + a)^3$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{3}{2}\left(\int \frac{8Ba^4 + (35A + 48B)\sin\left(x + \frac{\pi}{2}\right)a^4}{\sin\left(x + \frac{\pi}{2}\right)}dx + 5a^4(7A + 8B)\sin(x)\right) + \frac{1}{2}(35A + 32B)\sin(x)\left(a^4\cos(x) +\right.\right.\right.$$
$$\frac{1}{4}aA\sin(x)(a\cos(x) + a)^3$$

$$\Big\downarrow \text{3214}$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{3}{2}\left(8a^4B\int \sec(x)dx + a^4x(35A + 48B) + 5a^4(7A + 8B)\sin(x)\right) + \frac{1}{2}(35A + 32B)\sin(x)\left(a^4\cos(x) + a^4\right)\right.\right.$$
$$\frac{1}{4}aA\sin(x)(a\cos(x) + a)^3$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{3}{2}\left(8a^4B\int \csc\left(x + \frac{\pi}{2}\right)dx + a^4x(35A + 48B) + 5a^4(7A + 8B)\sin(x)\right) + \frac{1}{2}(35A + 32B)\sin(x)\left(a^4\cos(x)\right.\right.\right.$$
$$\frac{1}{4}aA\sin(x)(a\cos(x) + a)^3$$

$$\Big\downarrow \text{4257}$$

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{3}{2}(a^4x(35A + 48B) + 5a^4(7A + 8B)\sin(x) + 8a^4B\text{arctanh}(\sin(x))) + \frac{1}{2}(35A + 32B)\sin(x)\left(a^4\cos(x) + a\right.\right.\right.$$
$$\frac{1}{4}aA\sin(x)(a\cos(x) + a)^3$$

input
```
Int[(a + a*Cos[x])^4*(A + B*Sec[x]),x]
```

output
```
(a*A*(a + a*Cos[x])^3*Sin[x])/4 + (((7*A + 4*B)*(a^2 + a^2*Cos[x])^2*Sin[x
])/3 + (((35*A + 32*B)*(a^4 + a^4*Cos[x])*Sin[x])/2 + (3*(a^4*(35*A + 48*B
)*x + 8*a^4*B*ArcTanh[Sin[x]] + 5*a^4*(7*A + 8*B)*Sin[x]))/2)/3)/4
```

3.188. $\int (a + a\cos(x))^4(A + B\sec(x))\,dx$

3.188.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3307
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((d + c*Sin[e +
 f*x])^n/Sin[e + f*x]^n), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

rule 3447
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.188.4 Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.77

method	result
parallelrisch	$\frac{a^4 (-32B\ln(-\cot(x)+\csc(x)-1)+32B\ln(\csc(x)-\cot(x)+1)+8(7A+4B)\sin(2x)+\frac{8(4A+B)\sin(3x)}{3}+A\sin(4x)+8(28A+27B)}{32}$
default	$a^4A\left(\frac{\left(\cos(x)^3+\frac{3\cos(x)}{2}\right)\sin(x)}{4}+\frac{3x}{8}\right)+\frac{a^4B\left(2+\cos(x)^2\right)\sin(x)}{3}+\frac{4a^4A\left(2+\cos(x)^2\right)\sin(x)}{3}+4a^4B(\frac{\cos(x)\sin}{2}$
parts	$a^4A\left(\frac{\left(\cos(x)^3+\frac{3\cos(x)}{2}\right)\sin(x)}{4}+\frac{3x}{8}\right)+\frac{a^4B\left(2+\cos(x)^2\right)\sin(x)}{3}+\frac{4a^4A\left(2+\cos(x)^2\right)\sin(x)}{3}+4a^4B(\frac{\cos(x)\sin}{2}$
risch	$\frac{35a^4Ax}{8}+6a^4Bx-\frac{7iA\,e^{ix}a^4}{2}-\frac{27iB\,e^{ix}a^4}{8}+\frac{7iA\,e^{-ix}a^4}{2}+\frac{27iB\,e^{-ix}a^4}{8}+a^4B\ln\left(i+e^{ix}\right)-a^4B\ln(e$
norman	$\frac{(\frac{35}{4}a^4A+10a^4B)\tan(\frac{x}{2})^7+(\frac{35}{8}a^4A+6a^4B)x+(\frac{93}{4}a^4A+18a^4B)\tan(\frac{x}{2})+(\frac{385}{12}a^4A+\frac{106}{3}a^4B)\tan(\frac{x}{2})^5+(\frac{511}{12}a^4A+\frac{130}{3}a^4B)}{(1+\tan}$

input
```
int((a+a*cos(x))^4*(A+B*sec(x)),x,method=_RETURNVERBOSE)
```

output
```
1/32*a^4*(-32*B*ln(-cot(x)+csc(x)-1)+32*B*ln(csc(x)-cot(x)+1)+8*(7*A+4*B)*
sin(2*x)+8/3*(4*A+B)*sin(3*x)+A*sin(4*x)+8*(28*A+27*B)*sin(x)+140*x*(A+48/
35*B))
```

3.188.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.86

$$\int (a+a\cos(x))^4(A+B\sec(x))\,dx$$
$$=\frac{1}{8}(35A+48B)a^4x+\frac{1}{2}Ba^4\log(\sin(x)+1)-\frac{1}{2}Ba^4\log(-\sin(x)+1)$$
$$+\frac{1}{24}\left(6Aa^4\cos(x)^3+8(4A+B)a^4\cos(x)^2+3(27A+16B)a^4\cos(x)+160(A+B)a^4\right)\sin(x)$$

input
```
integrate((a+a*cos(x))^4*(A+B*sec(x)),x, algorithm="fricas")
```

output
```
1/8*(35*A + 48*B)*a^4*x + 1/2*B*a^4*log(sin(x) + 1) - 1/2*B*a^4*log(-sin(x
) + 1) + 1/24*(6*A*a^4*cos(x)^3 + 8*(4*A + B)*a^4*cos(x)^2 + 3*(27*A + 16*
B)*a^4*cos(x) + 160*(A + B)*a^4)*sin(x)
```

3.188.6 Sympy [A] (verification not implemented)

Time = 6.66 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.12

$$\int (a+a\cos(x))^4(A+B\sec(x))\,dx=\frac{35Aa^4x}{8}-\frac{4Aa^4\sin^3(x)}{3}+8Aa^4\sin(x)+\frac{7Aa^4\sin(2x)}{4}+\frac{Aa^4\sin(4x)}{32}+6Ba^4x+Ba^4\log(\tan(x)+\sec(x))-\frac{Ba^4\sin^3(x)}{3}+2Ba^4\sin(x)\cos(x)+7Ba^4\sin(x)$$

input
```
integrate((a+a*cos(x))**4*(A+B*sec(x)),x)
```

output
```
35*A*a**4*x/8 - 4*A*a**4*sin(x)**3/3 + 8*A*a**4*sin(x) + 7*A*a**4*sin(2*x)
/4 + A*a**4*sin(4*x)/32 + 6*B*a**4*x + B*a**4*log(tan(x) + sec(x)) - B*a**
4*sin(x)**3/3 + 2*B*a**4*sin(x)*cos(x) + 7*B*a**4*sin(x)
```

3.188.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.13

$$\int (a + a\cos(x))^4 (A + B\sec(x))\,dx = -\frac{4}{3}\left(\sin(x)^3 - 3\sin(x)\right)Aa^4 - \frac{1}{3}\left(\sin(x)^3 - 3\sin(x)\right)Ba^4 + \frac{1}{32}Aa^4(12x + \sin(4x) + 8\sin(2x)) + \frac{3}{2}Aa^4(2x + \sin(2x)) + Ba^4(2x + \sin(2x)) + Aa^4x + 4Ba^4x + Ba^4\log(\sec(x) + \tan(x)) + 4Aa^4\sin(x) + 6Ba^4\sin(x)$$

input
```
integrate((a+a*cos(x))^4*(A+B*sec(x)),x, algorithm="maxima")
```

output
```
-4/3*(sin(x)^3 - 3*sin(x))*A*a^4 - 1/3*(sin(x)^3 - 3*sin(x))*B*a^4 + 1/32*
A*a^4*(12*x + sin(4*x) + 8*sin(2*x)) + 3/2*A*a^4*(2*x + sin(2*x)) + B*a^4*
(2*x + sin(2*x)) + A*a^4*x + 4*B*a^4*x + B*a^4*log(sec(x) + tan(x)) + 4*A*
a^4*sin(x) + 6*B*a^4*sin(x)
```

3.188.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.43

$$\int (a + a\cos(x))^4 (A + B\sec(x))\,dx = Ba^4\log\left(\left|\tan\left(\frac{1}{2}x\right) + 1\right|\right) - Ba^4\log\left(\left|\tan\left(\frac{1}{2}x\right) - 1\right|\right) + \frac{1}{8}\left(35Aa^4 + 48Ba^4\right)x + \frac{105Aa^4\tan\left(\frac{1}{2}x\right)^7 + 120Ba^4\tan\left(\frac{1}{2}x\right)^7 + 385Aa^4\tan\left(\frac{1}{2}x\right)^5 + 424Ba^4\tan\left(\frac{1}{2}x\right)^5 + 511Aa^4\tan\left(\frac{1}{2}x\right)}{12\left(\tan\left(\frac{1}{2}x\right)^2 + 1\right)^4}$$

input
```
integrate((a+a*cos(x))^4*(A+B*sec(x)),x, algorithm="giac")
```

output
```
B*a^4*log(abs(tan(1/2*x) + 1)) - B*a^4*log(abs(tan(1/2*x) - 1)) + 1/8*(35*
A*a^4 + 48*B*a^4)*x + 1/12*(105*A*a^4*tan(1/2*x)^7 + 120*B*a^4*tan(1/2*x)^
7 + 385*A*a^4*tan(1/2*x)^5 + 424*B*a^4*tan(1/2*x)^5 + 511*A*a^4*tan(1/2*x)
^3 + 520*B*a^4*tan(1/2*x)^3 + 279*A*a^4*tan(1/2*x) + 216*B*a^4*tan(1/2*x))
/(tan(1/2*x)^2 + 1)^4
```

3.188.9 Mupad [B] (verification not implemented)

Time = 26.38 (sec) , antiderivative size = 460, normalized size of antiderivative = 4.42

$$\int (a+a\cos(x))^4(A+B\sec(x))\,dx$$

$$= \frac{\left(\frac{35\,A\,a^4}{4}+10\,B\,a^4\right)\tan\left(\frac{x}{2}\right)^7+\left(\frac{385\,A\,a^4}{12}+\frac{106\,B\,a^4}{3}\right)\tan\left(\frac{x}{2}\right)^5+\left(\frac{511\,A\,a^4}{12}+\frac{130\,B\,a^4}{3}\right)\tan\left(\frac{x}{2}\right)^3+\left(\frac{93\,A\,a^4}{4}+18\right.}{\tan\left(\frac{x}{2}\right)^8+4\tan\left(\frac{x}{2}\right)^6+6\tan\left(\frac{x}{2}\right)^4+4\tan\left(\frac{x}{2}\right)^2+1}$$

$$+\frac{a^4\,\mathrm{atan}\left(\frac{42875\,A^3\,a^{12}\tan\left(\frac{x}{2}\right)}{8\left(\frac{42875\,A^3\,a^{12}}{8}+22050\,A^2\,B\,a^{12}+30520\,A\,B^2\,a^{12}+14208\,B^3\,a^{12}\right)}+\frac{14208\,B^3\,a^{12}\tan\left(\frac{x}{2}\right)}{\frac{42875\,A^3\,a^{12}}{8}+22050\,A^2\,B\,a^{12}+30520\,A\,B^2\,a^{12}+14208\,B^3\right.}}{}$$

$$+2\,B\,a^4\,\mathrm{atanh}\left(\frac{2368\,B^3\,a^{12}\tan\left(\frac{x}{2}\right)}{1225\,A^2\,B\,a^{12}+3360\,A\,B^2\,a^{12}+2368\,B^3\,a^{12}}+\frac{3360\,A\,B^2\,a^{12}\tan\left(\frac{x}{2}\right)}{1225\,A^2\,B\,a^{12}+3360\,A\,B^2\,a^{12}+2368\,B^3\,a^{12}}+\frac{1225\,A^2\,B\,a^{12}\tan\left(\frac{x}{2}\right)}{1225\,A^2\,B\,a^{12}+3360\,A\,B^2\,a^{12}+2368\,B^3\,a^{12}}\right)$$

input
```
int((a + a*cos(x))^4*(A + B/cos(x)),x)
```

output
```
(tan(x/2)^7*((35*A*a^4)/4 + 10*B*a^4) + tan(x/2)^5*((385*A*a^4)/12 + (106*
B*a^4)/3) + tan(x/2)^3*((511*A*a^4)/12 + (130*B*a^4)/3) + tan(x/2)*((93*A*
a^4)/4 + 18*B*a^4))/(4*tan(x/2)^2 + 6*tan(x/2)^4 + 4*tan(x/2)^6 + tan(x/2)
^8 + 1) + (a^4*atan((42875*A^3*a^12*tan(x/2))/(8*((42875*A^3*a^12)/8 + 142
08*B^3*a^12 + 30520*A*B^2*a^12 + 22050*A^2*B*a^12)) + (14208*B^3*a^12*tan(
x/2))/((42875*A^3*a^12)/8 + 14208*B^3*a^12 + 30520*A*B^2*a^12 + 22050*A^2*
B*a^12) + (30520*A*B^2*a^12*tan(x/2))/((42875*A^3*a^12)/8 + 14208*B^3*a^12
 + 30520*A*B^2*a^12 + 22050*A^2*B*a^12) + (22050*A^2*B*a^12*tan(x/2))/((42
875*A^3*a^12)/8 + 14208*B^3*a^12 + 30520*A*B^2*a^12 + 22050*A^2*B*a^12))*(
35*A + 48*B))/4 + 2*B*a^4*atanh((2368*B^3*a^12*tan(x/2))/(2368*B^3*a^12 +
3360*A*B^2*a^12 + 1225*A^2*B*a^12) + (3360*A*B^2*a^12*tan(x/2))/(2368*B^3*
a^12 + 3360*A*B^2*a^12 + 1225*A^2*B*a^12) + (1225*A^2*B*a^12*tan(x/2))/(23
68*B^3*a^12 + 3360*A*B^2*a^12 + 1225*A^2*B*a^12))
```

3.189 $\int \frac{A+B\sec(x)}{a+a\cos(x)}\,dx$

3.189.1 Optimal result

Integrand size = 15, antiderivative size = 25

$$\int \frac{A+B\sec(x)}{a+a\cos(x)}\,dx=\frac{B\text{arctanh}(\sin(x))}{a}+\frac{(A-B)\sin(x)}{a+a\cos(x)}$$

output
```
B*arctanh(sin(x))/a+(A-B)*sin(x)/(a+a*cos(x))
```

3.189.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 71 vs. 2(25) = 50.

Time = 0.22 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.84

$$\int \frac{A+B\sec(x)}{a+a\cos(x)}\,dx = -\frac{2\cos\left(\frac{x}{2}\right)\left(B\cos\left(\frac{x}{2}\right)\left(\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\right)+(-A+B)\sin\left(\frac{x}{2}\right)\right)}{a(1+\cos(x))}$$

input
```
Integrate[(A + B*Sec[x])/(a + a*Cos[x]),x]
```

output
```
(-2*Cos[x/2]*(B*Cos[x/2]*(Log[Cos[x/2] - Sin[x/2]] - Log[Cos[x/2] + Sin[x/
2]]) + (-A + B)*Sin[x/2]))/(a*(1 + Cos[x]))
```

3.189.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3042, 3307, 3042, 3457, 27, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sec(x)}{a\cos(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\csc\left(x+\frac{\pi}{2}\right)}{a\sin\left(x+\frac{\pi}{2}\right)+a}dx$$

$$\Big\downarrow \text{3307}$$

$$\int \frac{\sec(x)(A\cos(x)+B)}{a\cos(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A\sin\left(x+\frac{\pi}{2}\right)+B}{\sin\left(x+\frac{\pi}{2}\right)\left(a\sin\left(x+\frac{\pi}{2}\right)+a\right)}dx$$

$$\Big\downarrow \text{3457}$$

$$\frac{\int aB\sec(x)dx}{a^2}+\frac{(A-B)\sin(x)}{a\cos(x)+a}$$

$$\Big\downarrow \text{27}$$

$$\frac{B\int \sec(x)dx}{a}+\frac{(A-B)\sin(x)}{a\cos(x)+a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{B\int \csc\left(x+\frac{\pi}{2}\right)dx}{a}+\frac{(A-B)\sin(x)}{a\cos(x)+a}$$

$$\Big\downarrow \text{4257}$$

$$\frac{(A-B)\sin(x)}{a\cos(x)+a}+\frac{B\text{arctanh}(\sin(x))}{a}$$

input
```
Int[(A + B*Sec[x])/(a + a*Cos[x]),x]
```

output
```
(B*ArcTanh[Sin[x]])/a + ((A - B)*Sin[x])/(a + a*Cos[x])
```

3.189.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3307
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((d + c*Sin[e +
 f*x])^n/Sin[e + f*x]^n), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.189.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.40

method	result	size
parallelrisch	$\frac{-B\ln\left(\tan\left(\frac{x}{2}\right)-1\right)+B\ln\left(\tan\left(\frac{x}{2}\right)+1\right)+\tan\left(\frac{x}{2}\right)(A-B)}{a}$	35
default	$\frac{A\tan\left(\frac{x}{2}\right)-B\tan\left(\frac{x}{2}\right)-B\ln\left(\tan\left(\frac{x}{2}\right)-1\right)+B\ln\left(\tan\left(\frac{x}{2}\right)+1\right)}{a}$	38
norman	$\frac{(A-B)\tan\left(\frac{x}{2}\right)}{a}+\frac{B\ln\left(\tan\left(\frac{x}{2}\right)+1\right)}{a}-\frac{B\ln\left(\tan\left(\frac{x}{2}\right)-1\right)}{a}$	40
risch	$\frac{2iA}{(e^{ix}+1)a}-\frac{2iB}{(e^{ix}+1)a}+\frac{B\ln(i+e^{ix})}{a}-\frac{B\ln(e^{ix}-i)}{a}$	63

input
```
int((A+B*sec(x))/(a+a*cos(x)),x,method=_RETURNVERBOSE)
```

output
```
(-B*ln(tan(1/2*x)-1)+B*ln(tan(1/2*x)+1)+tan(1/2*x)*(A-B))/a
```

3.189.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.88

$$\int \frac{A+B\sec(x)}{a+a\cos(x)}\,dx = \frac{(B\cos(x)+B)\log(\sin(x)+1)-(B\cos(x)+B)\log(-\sin(x)+1)+2(A-B)\sin(x)}{2(a\cos(x)+a)}$$

input
```
integrate((A+B*sec(x))/(a+a*cos(x)),x, algorithm="fricas")
```

output
```
1/2*((B*cos(x) + B)*log(sin(x) + 1) - (B*cos(x) + B)*log(-sin(x) + 1) + 2*
(A - B)*sin(x))/(a*cos(x) + a)
```

3.189.6 Sympy [F]

$$\int \frac{A+B\sec(x)}{a+a\cos(x)}\,dx = \frac{\int \frac{A}{\cos(x)+1}\,dx + \int \frac{B\sec(x)}{\cos(x)+1}\,dx}{a}$$

input
```
integrate((A+B*sec(x))/(a+a*cos(x)),x)
```

output
```
(Integral(A/(cos(x) + 1), x) + Integral(B*sec(x)/(cos(x) + 1), x))/a
```

3.189.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(25) = 50$.

Time = 0.20 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.52

$$\int \frac{A+B\sec(x)}{a+a\cos(x)}\,dx = B\left(\frac{\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right)}{a} - \frac{\log\left(\frac{\sin(x)}{\cos(x)+1}-1\right)}{a} - \frac{\sin(x)}{a(\cos(x)+1)}\right) + \frac{A\sin(x)}{a(\cos(x)+1)}$$

input
```
integrate((A+B*sec(x))/(a+a*cos(x)),x, algorithm="maxima")
```

output
```
B*(log(sin(x)/(cos(x) + 1) + 1)/a - log(sin(x)/(cos(x) + 1) - 1)/a - sin(x
)/(a*(cos(x) + 1))) + A*sin(x)/(a*(cos(x) + 1))
```

3.189.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.84

$$\int \frac{A+B\sec(x)}{a+a\cos(x)}\,dx = \frac{B\log\left(\left|\tan\left(\frac{1}{2}x\right)+1\right|\right)}{a} - \frac{B\log\left(\left|\tan\left(\frac{1}{2}x\right)-1\right|\right)}{a} + \frac{A\tan\left(\frac{1}{2}x\right) - B\tan\left(\frac{1}{2}x\right)}{a}$$

input
```
integrate((A+B*sec(x))/(a+a*cos(x)),x, algorithm="giac")
```

output
```
B*log(abs(tan(1/2*x) + 1))/a - B*log(abs(tan(1/2*x) - 1))/a + (A*tan(1/2*x
) - B*tan(1/2*x))/a
```

3.189.9 Mupad [B] (verification not implemented)

Time = 26.66 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{A+B\sec(x)}{a+a\cos(x)}\,dx=\frac{2\,B\,\mathrm{atanh}\left(\tan\left(\frac{x}{2}\right)\right)}{a}+\frac{\tan\left(\frac{x}{2}\right)\,\left(A-B\right)}{a}$$

input
```
int((A + B/cos(x))/(a + a*cos(x)),x)
```

output
```
(2*B*atanh(tan(x/2)))/a + (tan(x/2)*(A - B))/a
```

3.190 $\int \frac{A+B\sec(x)}{(a+a\cos(x))^2}\,dx$

3.190.1 Optimal result

Integrand size = 15, antiderivative size = 48

$$\int \frac{A+B\sec(x)}{(a+a\cos(x))^2}\,dx=\frac{B\text{arctanh}(\sin(x))}{a^2}+\frac{(A-4B)\sin(x)}{3a^2(1+\cos(x))}+\frac{(A-B)\sin(x)}{3(a+a\cos(x))^2}$$

output
```
B*arctanh(sin(x))/a^2+1/3*(A-4*B)*sin(x)/a^2/(1+cos(x))+1/3*(A-B)*sin(x)/(
a+a*cos(x))^2
```

3.190.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.58

$$\int \frac{A+B\sec(x)}{(a+a\cos(x))^2}\,dx=\frac{-12B\cos^4\left(\frac{x}{2}\right)\left(\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\right)+(2A-5B+(A-4B)\cos(x))\sin(x)}{3a^2(1+\cos(x))^2}$$

input
```
Integrate[(A + B*Sec[x])/(a + a*Cos[x])^2,x]
```

output
```
(-12*B*Cos[x/2]^4*(Log[Cos[x/2] - Sin[x/2]] - Log[Cos[x/2] + Sin[x/2]]) +
(2*A - 5*B + (A - 4*B)*Cos[x])*Sin[x])/(3*a^2*(1 + Cos[x])^2)
```

3.190.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 3307, 3042, 3457, 3042, 3457, 27, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sec(x)}{(a\cos(x)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\csc\left(x+\frac{\pi}{2}\right)}{\left(a\sin\left(x+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{3307}$$

$$\int \frac{\sec(x)(A\cos(x)+B)}{(a\cos(x)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A\sin\left(x+\frac{\pi}{2}\right)+B}{\sin\left(x+\frac{\pi}{2}\right)\left(a\sin\left(x+\frac{\pi}{2}\right)+a\right)^2}dx$$

$$\Big\downarrow \text{3457}$$

$$\frac{\int \frac{(3aB+a(A-B)\cos(x))\sec(x)}{\cos(x)a+a}dx}{3a^2}+\frac{(A-B)\sin(x)}{3(a\cos(x)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{3aB+a(A-B)\sin\left(x+\frac{\pi}{2}\right)}{\sin\left(x+\frac{\pi}{2}\right)\left(\sin\left(x+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}+\frac{(A-B)\sin(x)}{3(a\cos(x)+a)^2}$$

$$\Big\downarrow \text{3457}$$

$$\frac{\frac{\int 3a^2B\sec(x)dx}{a^2}+\frac{(A-4B)\sin(x)}{\cos(x)+1}}{3a^2}+\frac{(A-B)\sin(x)}{3(a\cos(x)+a)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{3B\int \sec(x)dx+\frac{(A-4B)\sin(x)}{\cos(x)+1}}{3a^2}+\frac{(A-B)\sin(x)}{3(a\cos(x)+a)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3B\int \csc\left(x+\frac{\pi}{2}\right)dx+\frac{(A-4B)\sin(x)}{\cos(x)+1}}{3a^2}+\frac{(A-B)\sin(x)}{3(a\cos(x)+a)^2}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\frac{(A-4B)\sin(x)}{\cos(x)+1}+3B\text{arctanh}(\sin(x))}{3a^2}+\frac{(A-B)\sin(x)}{3(a\cos(x)+a)^2}$$

input
```
Int[(A + B*Sec[x])/(a + a*Cos[x])^2,x]
```

output
```
((A - B)*Sin[x])/(3*(a + a*Cos[x])^2) + (3*B*ArcTanh[Sin[x]] + ((A - 4*B)*
Sin[x])/(1 + Cos[x]))/(3*a^2)
```

3.190.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3307
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((d + c*Sin[e +
 f*x])^n/Sin[e + f*x]^n), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.190.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.06

method	result	size
parallelrisch	$\frac{-6B\ln\left(\tan\left(\frac{x}{2}\right)-1\right)+6B\ln\left(\tan\left(\frac{x}{2}\right)+1\right)+\left((A-B)\tan\left(\frac{x}{2}\right)^{2}+3A-9B\right)\tan\left(\frac{x}{2}\right)}{6a^{2}}$	51
default	$\frac{\frac{A\tan\left(\frac{x}{2}\right)^{3}}{3}-\frac{B\tan\left(\frac{x}{2}\right)^{3}}{3}+A\tan\left(\frac{x}{2}\right)-3B\tan\left(\frac{x}{2}\right)-2B\ln\left(\tan\left(\frac{x}{2}\right)-1\right)+2B\ln\left(\tan\left(\frac{x}{2}\right)+1\right)}{2a^{2}}$	58
norman	$\frac{\frac{(A-B)\tan\left(\frac{x}{2}\right)^{3}}{6a}+\frac{(-3B+A)\tan\left(\frac{x}{2}\right)}{2a}}{a}+\frac{B\ln\left(\tan\left(\frac{x}{2}\right)+1\right)}{a^{2}}-\frac{B\ln\left(\tan\left(\frac{x}{2}\right)-1\right)}{a^{2}}$	62
risch	$-\frac{2i\left(3B\,e^{2ix}-3A\,e^{ix}+9B\,e^{ix}-A+4B\right)}{3\left(e^{ix}+1\right)^{3}a^{2}}+\frac{B\ln\left(i+e^{ix}\right)}{a^{2}}-\frac{B\ln\left(e^{ix}-i\right)}{a^{2}}$	77

input
```
int((A+B*sec(x))/(a+a*cos(x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*(-6*B*ln(tan(1/2*x)-1)+6*B*ln(tan(1/2*x)+1)+((A-B)*tan(1/2*x)^2+3*A-9*
B)*tan(1/2*x))/a^2
```

3.190.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.77

$$\int \frac{A+B\sec(x)}{(a+a\cos(x))^2}\,dx$$
$$= \frac{3\left(B\cos(x)^2+2B\cos(x)+B\right)\log\left(\sin(x)+1\right)-3\left(B\cos(x)^2+2B\cos(x)+B\right)\log\left(-\sin(x)+1\right)+}{6\left(a^2\cos(x)^2+2a^2\cos(x)+a^2\right)}$$

input
```
integrate((A+B*sec(x))/(a+a*cos(x))^2,x, algorithm="fricas")
```

output
```
1/6*(3*(B*cos(x)^2 + 2*B*cos(x) + B)*log(sin(x) + 1) - 3*(B*cos(x)^2 + 2*B
*cos(x) + B)*log(-sin(x) + 1) + 2*((A - 4*B)*cos(x) + 2*A - 5*B)*sin(x))/(
a^2*cos(x)^2 + 2*a^2*cos(x) + a^2)
```

3.190.6 Sympy [F]

$$\int \frac{A+B\sec(x)}{(a+a\cos(x))^2}\,dx = \frac{\int \frac{A}{\cos^2(x)+2\cos(x)+1}\,dx + \int \frac{B\sec(x)}{\cos^2(x)+2\cos(x)+1}\,dx}{a^2}$$

input
```
integrate((A+B*sec(x))/(a+a*cos(x))**2,x)
```

output
```
(Integral(A/(cos(x)**2 + 2*cos(x) + 1), x) + Integral(B*sec(x)/(cos(x)**2
+ 2*cos(x) + 1), x))/a**2
```

3.190.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(44) = 88$.

Time = 0.21 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.94

$$\int \frac{A+B\sec(x)}{(a+a\cos(x))^2}\,dx = -\frac{1}{6}B\left(\frac{\frac{9\sin(x)}{\cos(x)+1}+\frac{\sin(x)^3}{(\cos(x)+1)^3}}{a^2} - \frac{6\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right)}{a^2} + \frac{6\log\left(\frac{\sin(x)}{\cos(x)+1}-1\right)}{a^2}\right) + \frac{A\left(\frac{3\sin(x)}{\cos(x)+1}+\frac{\sin(x)^3}{(\cos(x)+1)^3}\right)}{6a^2}$$

input
```
integrate((A+B*sec(x))/(a+a*cos(x))^2,x, algorithm="maxima")
```

output
```
-1/6*B*((9*sin(x)/(cos(x) + 1) + sin(x)^3/(cos(x) + 1)^3)/a^2 - 6*log(sin(
x)/(cos(x) + 1) + 1)/a^2 + 6*log(sin(x)/(cos(x) + 1) - 1)/a^2) + 1/6*A*(3*
sin(x)/(cos(x) + 1) + sin(x)^3/(cos(x) + 1)^3)/a^2
```

3.190.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.60

$$\int \frac{A+B\sec(x)}{(a+a\cos(x))^2}\,dx = \frac{B\log\left(\left|\tan\left(\frac{1}{2}x\right)+1\right|\right)}{a^2} - \frac{B\log\left(\left|\tan\left(\frac{1}{2}x\right)-1\right|\right)}{a^2} + \frac{Aa^4\tan\left(\frac{1}{2}x\right)^3 - Ba^4\tan\left(\frac{1}{2}x\right)^3 + 3\,Aa^4\tan\left(\frac{1}{2}x\right) - 9\,Ba^4\tan\left(\frac{1}{2}x\right)}{6\,a^6}$$

input
```
integrate((A+B*sec(x))/(a+a*cos(x))^2,x, algorithm="giac")
```

output
```
B*log(abs(tan(1/2*x) + 1))/a^2 - B*log(abs(tan(1/2*x) - 1))/a^2 + 1/6*(A*a
^4*tan(1/2*x)^3 - B*a^4*tan(1/2*x)^3 + 3*A*a^4*tan(1/2*x) - 9*B*a^4*tan(1/
2*x))/a^6
```

3.190.9 Mupad [B] (verification not implemented)

Time = 27.59 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

$$\int \frac{A+B\sec(x)}{(a+a\cos(x))^2}\,dx = \tan\left(\frac{x}{2}\right)\left(\frac{A-B}{2\,a^2} - \frac{B}{a^2}\right) + \frac{\tan\left(\frac{x}{2}\right)^3(A-B)}{6\,a^2} + \frac{2\,B\,\mathrm{atanh}\left(\tan\left(\frac{x}{2}\right)\right)}{a^2}$$

input
```
int((A + B/cos(x))/(a + a*cos(x))^2,x)
```

output
```
tan(x/2)*((A - B)/(2*a^2) - B/a^2) + (tan(x/2)^3*(A - B))/(6*a^2) + (2*B*a
tanh(tan(x/2)))/a^2
```

3.191 $\int \frac{A+B\sec(x)}{(a+a\cos(x))^3}\,dx$

3.191.1 Optimal result

Integrand size = 15, antiderivative size = 75

$$\int \frac{A+B\sec(x)}{(a+a\cos(x))^3}\,dx = \frac{B\operatorname{arctanh}(\sin(x))}{a^3} + \frac{(A-B)\sin(x)}{5(a+a\cos(x))^3} + \frac{(2A-7B)\sin(x)}{15a(a+a\cos(x))^2} + \frac{2(A-11B)\sin(x)}{15\left(a^3+a^3\cos(x)\right)}$$

output
```
B*arctanh(sin(x))/a^3+1/5*(A-B)*sin(x)/(a+a*cos(x))^3+1/15*(2*A-7*B)*sin(x
)/a/(a+a*cos(x))^2+2/15*(A-11*B)*sin(x)/(a^3+a^3*cos(x))
```

3.191.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.17

$$\int \frac{A+B\sec(x)}{(a+a\cos(x))^3}\,dx = \frac{-120B\cos^6\left(\frac{x}{2}\right)\left(\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\right)+(8A-43B+(6A-51B)\cos(x)+\cdots}{15a^3(1+\cos(x))^3}$$

input
```
Integrate[(A + B*Sec[x])/(a + a*Cos[x])^3,x]
```

output
```
(-120*B*Cos[x/2]^6*(Log[Cos[x/2] - Sin[x/2]] - Log[Cos[x/2] + Sin[x/2]]) +
 (8*A - 43*B + (6*A - 51*B)*Cos[x] + (A - 11*B)*Cos[2*x])*Sin[x])/(15*a^3*
(1 + Cos[x])^3)
```

3.191.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.13, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.733, Rules used = {3042, 3307, 3042, 3457, 3042, 3457, 3042, 3457, 27, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sec(x)}{(a\cos(x)+a)^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\csc\left(x+\frac{\pi}{2}\right)}{\left(a\sin\left(x+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{ 3307}$$

$$\int \frac{\sec(x)(A\cos(x)+B)}{(a\cos(x)+a)^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A\sin\left(x+\frac{\pi}{2}\right)+B}{\sin\left(x+\frac{\pi}{2}\right)\left(a\sin\left(x+\frac{\pi}{2}\right)+a\right)^3}dx$$

$$\Big\downarrow \text{ 3457}$$

$$\frac{\int \frac{(5aB+2a(A-B)\cos(x))\sec(x)}{(\cos(x)a+a)^2}dx}{5a^2}+\frac{(A-B)\sin(x)}{5(a\cos(x)+a)^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{5aB+2a(A-B)\sin\left(x+\frac{\pi}{2}\right)}{\sin\left(x+\frac{\pi}{2}\right)\left(\sin\left(x+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}+\frac{(A-B)\sin(x)}{5(a\cos(x)+a)^3}$$

$$\Big\downarrow \text{ 3457}$$

$$\frac{\frac{\int \frac{\left(15Ba^2+(2A-7B)\cos(x)a^2\right)\sec(x)}{\cos(x)a+a}dx}{3a^2}+\frac{a(2A-7B)\sin(x)}{3(a\cos(x)+a)^2}}{5a^2}+\frac{(A-B)\sin(x)}{5(a\cos(x)+a)^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{\int \frac{15Ba^2+(2A-7B)\sin\left(x+\frac{\pi}{2}\right)a^2}{\sin\left(x+\frac{\pi}{2}\right)\left(\sin\left(x+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}+\frac{a(2A-7B)\sin(x)}{3(a\cos(x)+a)^2}}{5a^2}+\frac{(A-B)\sin(x)}{5(a\cos(x)+a)^3}$$

$$\Big\downarrow \textbf{3457}$$

$$\frac{\frac{\frac{\int 15a^3B\sec(x)dx}{a^2}+\frac{2a^2(A-11B)\sin(x)}{a\cos(x)+a}}{3a^2}+\frac{a(2A-7B)\sin(x)}{3(a\cos(x)+a)^2}}{5a^2}+\frac{(A-B)\sin(x)}{5(a\cos(x)+a)^3}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\frac{15aB\int\sec(x)dx+\frac{2a^2(A-11B)\sin(x)}{a\cos(x)+a}}{3a^2}+\frac{a(2A-7B)\sin(x)}{3(a\cos(x)+a)^2}}{5a^2}+\frac{(A-B)\sin(x)}{5(a\cos(x)+a)^3}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\frac{15aB\int\csc\left(x+\frac{\pi}{2}\right)dx+\frac{2a^2(A-11B)\sin(x)}{a\cos(x)+a}}{3a^2}+\frac{a(2A-7B)\sin(x)}{3(a\cos(x)+a)^2}}{5a^2}+\frac{(A-B)\sin(x)}{5(a\cos(x)+a)^3}$$

$$\Big\downarrow \textbf{4257}$$

$$\frac{\frac{\frac{2a^2(A-11B)\sin(x)}{a\cos(x)+a}+15aB\text{arctanh}(\sin(x))}{3a^2}+\frac{a(2A-7B)\sin(x)}{3(a\cos(x)+a)^2}}{5a^2}+\frac{(A-B)\sin(x)}{5(a\cos(x)+a)^3}$$

input
```
Int[(A + B*Sec[x])/(a + a*Cos[x])^3,x]
```

output
```
((A - B)*Sin[x])/(5*(a + a*Cos[x])^3) + ((a*(2*A - 7*B)*Sin[x])/(3*(a + a*
Cos[x])^2) + (15*a*B*ArcTanh[Sin[x]] + (2*a^2*(A - 11*B)*Sin[x])/(a + a*Co
s[x]))/(3*a^2))/(5*a^2)
```

3.191.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3307
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((d + c*Sin[e +
 f*x])^n/Sin[e + f*x]^n), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.191.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.85

method	result	size
parallelrisch	$\frac{-20B\ln\left(\tan\left(\frac{x}{2}\right)-1\right)+20B\ln\left(\tan\left(\frac{x}{2}\right)+1\right)+\tan\left(\frac{x}{2}\right)\left((A-B)\tan\left(\frac{x}{2}\right)^{4}+\frac{10(-2B+A)\tan\left(\frac{x}{2}\right)^{2}}{3}+5A-35B\right)}{20a^{3}}$	64
default	$\frac{4B\ln\left(\tan\left(\frac{x}{2}\right)+1\right)+\frac{A\tan\left(\frac{x}{2}\right)^{5}}{5}-\frac{B\tan\left(\frac{x}{2}\right)^{5}}{5}-\frac{4B\tan\left(\frac{x}{2}\right)^{3}}{3}+\frac{2A\tan\left(\frac{x}{2}\right)^{3}}{3}-4B\ln\left(\tan\left(\frac{x}{2}\right)-1\right)+A\tan\left(\frac{x}{2}\right)-7B\tan\left(\frac{x}{2}\right)}{4a^{3}}$	76
norman	$\frac{\frac{(A-7B)\tan\left(\frac{x}{2}\right)}{4a}+\frac{(A-B)\tan\left(\frac{x}{2}\right)^{5}}{20a}+\frac{(-2B+A)\tan\left(\frac{x}{2}\right)^{3}}{6a}}{a^{2}}+\frac{B\ln\left(\tan\left(\frac{x}{2}\right)+1\right)}{a^{3}}-\frac{B\ln\left(\tan\left(\frac{x}{2}\right)-1\right)}{a^{3}}$	78
risch	$-\frac{2i\left(15B\,e^{4ix}+75B\,e^{3ix}-20A\,e^{2ix}+145B\,e^{2ix}-10A\,e^{ix}+95B\,e^{ix}-2A+22B\right)}{15\left(e^{ix}+1\right)^{5}a^{3}}+\frac{B\ln\left(i+e^{ix}\right)}{a^{3}}-\frac{B\ln\left(e^{ix}-i\right)}{a^{3}}$	101

input
```
int((A+B*sec(x))/(a+a*cos(x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/20*(-20*B*ln(tan(1/2*x)-1)+20*B*ln(tan(1/2*x)+1)+tan(1/2*x)*((A-B)*tan(1
/2*x)^4+10/3*(-2*B+A)*tan(1/2*x)^2+5*A-35*B))/a^3
```

3.191.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.63

$$\int \frac{A+B\sec(x)}{(a+a\cos(x))^3}\,dx = \frac{15\left(B\cos(x)^3+3\,B\cos(x)^2+3\,B\cos(x)+B\right)\log(\sin(x)+1)-15\left(B\cos(x)^3+3\,B\cos(x)^2+3\,B\cos\right.}{30\left(a^3\cos(x)^3+3\,a^3\cos\right.}$$

input
```
integrate((A+B*sec(x))/(a+a*cos(x))^3,x, algorithm="fricas")
```

output
```
1/30*(15*(B*cos(x)^3 + 3*B*cos(x)^2 + 3*B*cos(x) + B)*log(sin(x) + 1) - 15
*(B*cos(x)^3 + 3*B*cos(x)^2 + 3*B*cos(x) + B)*log(-sin(x) + 1) + 2*(2*(A -
 11*B)*cos(x)^2 + 3*(2*A - 17*B)*cos(x) + 7*A - 32*B)*sin(x))/(a^3*cos(x)^
3 + 3*a^3*cos(x)^2 + 3*a^3*cos(x) + a^3)
```

3.191.6 Sympy [F]

$$\int \frac{A+B\sec(x)}{(a+a\cos(x))^3}\,dx = \frac{\int \frac{A}{\cos^3(x)+3\cos^2(x)+3\cos(x)+1}\,dx+\int \frac{B\sec(x)}{\cos^3(x)+3\cos^2(x)+3\cos(x)+1}\,dx}{a^3}$$

input
```
integrate((A+B*sec(x))/(a+a*cos(x))**3,x)
```

output
```
(Integral(A/(cos(x)**3 + 3*cos(x)**2 + 3*cos(x) + 1), x) + Integral(B*sec(
x)/(cos(x)**3 + 3*cos(x)**2 + 3*cos(x) + 1), x))/a**3
```

3.191.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.59

$$\int \frac{A+B\sec(x)}{(a+a\cos(x))^3}\,dx = -\frac{1}{60}\,B\left(\frac{\frac{105\sin(x)}{\cos(x)+1}+\frac{20\sin(x)^3}{(\cos(x)+1)^3}+\frac{3\sin(x)^5}{(\cos(x)+1)^5}}{a^3}-\frac{60\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right)}{a^3}+\frac{60\log\left(\frac{\sin(x)}{\cos(x)+1}-1\right)}{a^3}\right)+\frac{A\left(\frac{15\sin(x)}{\cos(x)+1}+\frac{10\sin(x)^3}{(\cos(x)+1)^3}+\frac{3\sin(x)^5}{(\cos(x)+1)^5}\right)}{60\,a^3}$$

input
```
integrate((A+B*sec(x))/(a+a*cos(x))^3,x, algorithm="maxima")
```

output
```
-1/60*B*((105*sin(x)/(cos(x) + 1) + 20*sin(x)^3/(cos(x) + 1)^3 + 3*sin(x)^
5/(cos(x) + 1)^5)/a^3 - 60*log(sin(x)/(cos(x) + 1) + 1)/a^3 + 60*log(sin(x
)/(cos(x) + 1) - 1)/a^3) + 1/60*A*(15*sin(x)/(cos(x) + 1) + 10*sin(x)^3/(c
os(x) + 1)^3 + 3*sin(x)^5/(cos(x) + 1)^5)/a^3
```

3.191.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.36

$$\int \frac{A+B\sec(x)}{(a+a\cos(x))^3}\,dx = \frac{B\log\left(\left|\tan\left(\frac{1}{2}\,x\right)+1\right|\right)}{a^3} - \frac{B\log\left(\left|\tan\left(\frac{1}{2}\,x\right)-1\right|\right)}{a^3} + \frac{3\,Aa^{12}\tan\left(\frac{1}{2}\,x\right)^5 - 3\,Ba^{12}\tan\left(\frac{1}{2}\,x\right)^5 + 10\,Aa^{12}\tan\left(\frac{1}{2}\,x\right)^3 - 20\,Ba^{12}\tan\left(\frac{1}{2}\,x\right)^3 + 15\,Aa^{12}\tan\left(\frac{1}{2}\,x\right) - 1}{60\,a^{15}}$$

input
```
integrate((A+B*sec(x))/(a+a*cos(x))^3,x, algorithm="giac")
```

output
```
B*log(abs(tan(1/2*x) + 1))/a^3 - B*log(abs(tan(1/2*x) - 1))/a^3 + 1/60*(3*
A*a^12*tan(1/2*x)^5 - 3*B*a^12*tan(1/2*x)^5 + 10*A*a^12*tan(1/2*x)^3 - 20*
B*a^12*tan(1/2*x)^3 + 15*A*a^12*tan(1/2*x) - 105*B*a^12*tan(1/2*x))/a^15
```

3.191.9 Mupad [B] (verification not implemented)

Time = 26.18 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.23

$$\int \frac{A+B\sec(x)}{(a+a\cos(x))^3}\,dx = \tan\left(\frac{x}{2}\right)^3\left(\frac{A-B}{12\,a^3}+\frac{A-3\,B}{12\,a^3}\right) + \tan\left(\frac{x}{2}\right)\left(\frac{A-B}{4\,a^3}+\frac{A-3\,B}{4\,a^3}-\frac{A+3\,B}{4\,a^3}\right) + \frac{\tan\left(\frac{x}{2}\right)^5(A-B)}{20\,a^3} + \frac{2\,B\,\mathrm{atanh}\left(\tan\left(\frac{x}{2}\right)\right)}{a^3}$$

input
```
int((A + B/cos(x))/(a + a*cos(x))^3,x)
```

output
```
tan(x/2)^3*((A - B)/(12*a^3) + (A - 3*B)/(12*a^3)) + tan(x/2)*((A - B)/(4*
a^3) + (A - 3*B)/(4*a^3) - (A + 3*B)/(4*a^3)) + (tan(x/2)^5*(A - B))/(20*a
^3) + (2*B*atanh(tan(x/2)))/a^3
```

3.192 $\int \frac{A+B\sec(x)}{(a+a\cos(x))^4}\,dx$

3.192.1 Optimal result

Integrand size = 15, antiderivative size = 96

$$\int \frac{A+B\sec(x)}{(a+a\cos(x))^4}\,dx = \frac{B\operatorname{arctanh}(\sin(x))}{a^4} + \frac{(6A-55B)\sin(x)}{105a^4(1+\cos(x))^2} + \frac{2(3A-80B)\sin(x)}{105a^4(1+\cos(x))} + \frac{(A-B)\sin(x)}{7(a+a\cos(x))^4} + \frac{(3A-10B)\sin(x)}{35a(a+a\cos(x))^3}$$

output
```
B*arctanh(sin(x))/a^4+1/105*(6*A-55*B)*sin(x)/a^4/(1+cos(x))^2+2/105*(3*A-
80*B)*sin(x)/a^4/(1+cos(x))+1/7*(A-B)*sin(x)/(a+a*cos(x))^4+1/35*(3*A-10*B
)*sin(x)/a/(a+a*cos(x))^3
```

3.192.2 Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.08

$$\int \frac{A+B\sec(x)}{(a+a\cos(x))^4}\,dx = \frac{-3360B\cos^8\left(\frac{x}{2}\right)\left(\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\right)+(96A-1055B+(87A-1480B)\cos}{210a^4(1+\cos(x))^4}$$

input
```
Integrate[(A + B*Sec[x])/(a + a*Cos[x])^4,x]
```

output
```
(-3360*B*Cos[x/2]^8*(Log[Cos[x/2] - Sin[x/2]] - Log[Cos[x/2] + Sin[x/2]])
+ (96*A - 1055*B + (87*A - 1480*B)*Cos[x] + (24*A - 535*B)*Cos[2*x] + 3*A*
Cos[3*x] - 80*B*Cos[3*x])*Sin[x])/(210*a^4*(1 + Cos[x])^4)
```

3.192.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.21, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.867, Rules used = {3042, 3307, 3042, 3457, 3042, 3457, 3042, 3457, 3042, 3457, 27, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sec(x)}{(a\cos(x)+a)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\csc\left(x+\frac{\pi}{2}\right)}{\left(a\sin\left(x+\frac{\pi}{2}\right)+a\right)^4}dx$$

$$\Big\downarrow \text{3307}$$

$$\int \frac{\sec(x)(A\cos(x)+B)}{(a\cos(x)+a)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A\sin\left(x+\frac{\pi}{2}\right)+B}{\sin\left(x+\frac{\pi}{2}\right)\left(a\sin\left(x+\frac{\pi}{2}\right)+a\right)^4}dx$$

$$\Big\downarrow \text{3457}$$

$$\frac{\int \frac{(7aB+3a(A-B)\cos(x))\sec(x)}{(\cos(x)a+a)^3}dx}{7a^2}+\frac{(A-B)\sin(x)}{7(a\cos(x)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{7aB+3a(A-B)\sin\left(x+\frac{\pi}{2}\right)}{\sin\left(x+\frac{\pi}{2}\right)\left(\sin\left(x+\frac{\pi}{2}\right)a+a\right)^3}dx}{7a^2}+\frac{(A-B)\sin(x)}{7(a\cos(x)+a)^4}$$

$$\Big\downarrow \text{3457}$$

$$\frac{\frac{\int \frac{\left(35Ba^2+2(3A-10B)\cos(x)a^2\right)\sec(x)}{(\cos(x)a+a)^2}dx}{5a^2}+\frac{a(3A-10B)\sin(x)}{5(a\cos(x)+a)^3}}{7a^2}+\frac{(A-B)\sin(x)}{7(a\cos(x)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{35Ba^2+2(3A-10B)\sin\left(x+\frac{\pi}{2}\right)a^2}{\sin\left(x+\frac{\pi}{2}\right)\left(\sin\left(x+\frac{\pi}{2}\right)a+a\right)^2}dx}{5a^2}+\frac{a(3A-10B)\sin(x)}{5(a\cos(x)+a)^3}}{7a^2}+\frac{(A-B)\sin(x)}{7(a\cos(x)+a)^4}$$

$$\Big\downarrow \text{3457}$$

$$\frac{\frac{\frac{\int \frac{\left(105Ba^3+(6A-55B)\cos(x)a^3\right)\sec(x)}{\cos(x)a+a}dx}{3a^2}+\frac{(6A-55B)\sin(x)}{3(\cos(x)+1)^2}}{5a^2}+\frac{a(3A-10B)\sin(x)}{5(a\cos(x)+a)^3}}{7a^2}+\frac{(A-B)\sin(x)}{7(a\cos(x)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{\int \frac{105Ba^3+(6A-55B)\sin\left(x+\frac{\pi}{2}\right)a^3}{\sin\left(x+\frac{\pi}{2}\right)\left(\sin\left(x+\frac{\pi}{2}\right)a+a\right)}dx}{3a^2}+\frac{(6A-55B)\sin(x)}{3(\cos(x)+1)^2}}{5a^2}+\frac{a(3A-10B)\sin(x)}{5(a\cos(x)+a)^3}}{7a^2}+\frac{(A-B)\sin(x)}{7(a\cos(x)+a)^4}$$

$$\Big\downarrow \text{3457}$$

$$\frac{\frac{\frac{\frac{\int 105a^4B\sec(x)dx}{a^2}+\frac{2a^3(3A-80B)\sin(x)}{a\cos(x)+a}}{3a^2}+\frac{(6A-55B)\sin(x)}{3(\cos(x)+1)^2}}{5a^2}+\frac{a(3A-10B)\sin(x)}{5(a\cos(x)+a)^3}}{7a^2}+\frac{(A-B)\sin(x)}{7(a\cos(x)+a)^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{105a^2B\int \sec(x)dx+\frac{2a^3(3A-80B)\sin(x)}{a\cos(x)+a}}{3a^2}+\frac{(6A-55B)\sin(x)}{3(\cos(x)+1)^2}}{5a^2}+\frac{a(3A-10B)\sin(x)}{5(a\cos(x)+a)^3}}{7a^2}+\frac{(A-B)\sin(x)}{7(a\cos(x)+a)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{105a^2B\int \csc\left(x+\frac{\pi}{2}\right)dx+\frac{2a^3(3A-80B)\sin(x)}{a\cos(x)+a}}{3a^2}+\frac{(6A-55B)\sin(x)}{3(\cos(x)+1)^2}}{5a^2}+\frac{a(3A-10B)\sin(x)}{5(a\cos(x)+a)^3}}{7a^2}+\frac{(A-B)\sin(x)}{7(a\cos(x)+a)^4}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\frac{\frac{\frac{2a^3(3A-80B)\sin(x)}{a\cos(x)+a}+105a^2B\text{arctanh}(\sin(x))}{3a^2}+\frac{(6A-55B)\sin(x)}{3(\cos(x)+1)^2}}{5a^2}+\frac{a(3A-10B)\sin(x)}{5(a\cos(x)+a)^3}}{7a^2}+\frac{(A-B)\sin(x)}{7(a\cos(x)+a)^4}$$

input
```
Int[(A + B*Sec[x])/(a + a*Cos[x])^4,x]
```

output
```
((A - B)*Sin[x])/(7*(a + a*Cos[x])^4) + ((a*(3*A - 10*B)*Sin[x])/(5*(a + a
*Cos[x])^3) + (((6*A - 55*B)*Sin[x])/(3*(1 + Cos[x])^2) + (105*a^2*B*ArcTa
nh[Sin[x]] + (2*a^3*(3*A - 80*B)*Sin[x])/(a + a*Cos[x]))/(3*a^2))/(5*a^2))
/(7*a^2)
```

3.192. $\int \frac{A+B\sec(x)}{(a+a\cos(x))^4}\,dx$

3.192.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3307
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((d + c*Sin[e +
 f*x])^n/Sin[e + f*x]^n), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.192. $\int \frac{A+B\sec(x)}{(a+a\cos(x))^4}\,dx$

3.192.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.82

method	result
parallelrisch	$\frac{-56B\ln(\tan(\frac{x}{2})-1)+56B\ln(\tan(\frac{x}{2})+1)+\tan(\frac{x}{2})\left((A-B)\tan(\frac{x}{2})^6+7\left(\frac{3A}{5}-B\right)\tan(\frac{x}{2})^4+7\left(A-\frac{11B}{3}\right)\tan(\frac{x}{2})^2+7A-105B\right.}{56a^4}$
default	$\frac{-8B\ln(\tan(\frac{x}{2})-1)+A\tan(\frac{x}{2})-15B\tan(\frac{x}{2})+8B\ln(\tan(\frac{x}{2})+1)+A\tan(\frac{x}{2})^3-\frac{11B\tan(\frac{x}{2})^3}{3}+\frac{3A\tan(\frac{x}{2})^5}{5}-B\tan(\frac{x}{2})^5+\frac{\tan(}{7}}{8a^4}$
norman	$\frac{\frac{(A-15B)\tan(\frac{x}{2})}{8a}+\frac{(A-B)\tan(\frac{x}{2})^7}{56a}+\frac{(3A-11B)\tan(\frac{x}{2})^3}{24a}+\frac{(3A-5B)\tan(\frac{x}{2})^5}{40a}}{a^3}+\frac{B\ln(\tan(\frac{x}{2})+1)}{a^4}-\frac{B\ln(\tan(\frac{x}{2})-1)}{a^4}$
risch	$-\frac{2i(105B\,e^{6ix}+735B\,e^{5ix}+2170B\,e^{4ix}-210A\,e^{3ix}+3430B\,e^{3ix}-126A\,e^{2ix}+2625B\,e^{2ix}-42A\,e^{ix}+1015B\,e^{ix}-6A+160B)}{105(e^{ix}+1)^7a^4}$

input
```
int((A+B*sec(x))/(a+a*cos(x))^4,x,method=_RETURNVERBOSE)
```

output
```
1/56*(-56*B*ln(tan(1/2*x)-1)+56*B*ln(tan(1/2*x)+1)+tan(1/2*x)*((A-B)*tan(1
/2*x)^6+7*(3/5*A-B)*tan(1/2*x)^4+7*(A-11/3*B)*tan(1/2*x)^2+7*A-105*B))/a^4
```

3.192.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.65

$$\int \frac{A+B\sec(x)}{(a+a\cos(x))^4}\,dx$$
$$=\frac{105\left(B\cos(x)^4+4B\cos(x)^3+6B\cos(x)^2+4B\cos(x)+B\right)\log(\sin(x)+1)-105\left(B\cos(x)^4+4B\right.}{210}$$

input
```
integrate((A+B*sec(x))/(a+a*cos(x))^4,x, algorithm="fricas")
```

output
```
1/210*(105*(B*cos(x)^4 + 4*B*cos(x)^3 + 6*B*cos(x)^2 + 4*B*cos(x) + B)*log
(sin(x) + 1) - 105*(B*cos(x)^4 + 4*B*cos(x)^3 + 6*B*cos(x)^2 + 4*B*cos(x)
+ B)*log(-sin(x) + 1) + 2*(2*(3*A - 80*B)*cos(x)^3 + (24*A - 535*B)*cos(x)
^2 + (39*A - 620*B)*cos(x) + 36*A - 260*B)*sin(x))/(a^4*cos(x)^4 + 4*a^4*c
os(x)^3 + 6*a^4*cos(x)^2 + 4*a^4*cos(x) + a^4)
```

3.192.6 Sympy [F]

$$\int \frac{A+B\sec(x)}{(a+a\cos(x))^4}\,dx = \frac{\int \frac{A}{\cos^4(x)+4\cos^3(x)+6\cos^2(x)+4\cos(x)+1}\,dx+\int \frac{B\sec(x)}{\cos^4(x)+4\cos^3(x)+6\cos^2(x)+4\cos(x)+1}\,dx}{a^4}$$

input
```
integrate((A+B*sec(x))/(a+a*cos(x))**4,x)
```

output
```
(Integral(A/(cos(x)**4 + 4*cos(x)**3 + 6*cos(x)**2 + 4*cos(x) + 1), x) + I
ntegral(B*sec(x)/(cos(x)**4 + 4*cos(x)**3 + 6*cos(x)**2 + 4*cos(x) + 1), x
))/a**4
```

3.192.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.49

$$\int \frac{A+B\sec(x)}{(a+a\cos(x))^4}\,dx = -\frac{1}{168}B\left(\frac{\frac{315\sin(x)}{\cos(x)+1}+\frac{77\sin(x)^3}{(\cos(x)+1)^3}+\frac{21\sin(x)^5}{(\cos(x)+1)^5}+\frac{3\sin(x)^7}{(\cos(x)+1)^7}}{a^4}-\frac{168\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right)}{a^4}+\frac{168\log\left(\frac{\sin(x)}{\cos(x)+1}-1\right.}{a^4}\right. + \frac{A\left(\frac{35\sin(x)}{\cos(x)+1}+\frac{35\sin(x)^3}{(\cos(x)+1)^3}+\frac{21\sin(x)^5}{(\cos(x)+1)^5}+\frac{5\sin(x)^7}{(\cos(x)+1)^7}\right)}{280\,a^4}$$

input
```
integrate((A+B*sec(x))/(a+a*cos(x))^4,x, algorithm="maxima")
```

output
```
-1/168*B*((315*sin(x)/(cos(x) + 1) + 77*sin(x)^3/(cos(x) + 1)^3 + 21*sin(x
)^5/(cos(x) + 1)^5 + 3*sin(x)^7/(cos(x) + 1)^7)/a^4 - 168*log(sin(x)/(cos(
x) + 1) + 1)/a^4 + 168*log(sin(x)/(cos(x) + 1) - 1)/a^4) + 1/280*A*(35*sin
(x)/(cos(x) + 1) + 35*sin(x)^3/(cos(x) + 1)^3 + 21*sin(x)^5/(cos(x) + 1)^5
 + 5*sin(x)^7/(cos(x) + 1)^7)/a^4
```

3.192.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.31

$$\int \frac{A+B\sec(x)}{(a+a\cos(x))^4}\,dx = \frac{B\log\left(\left|\tan\left(\frac{1}{2}\,x\right)+1\right|\right)}{a^4} - \frac{B\log\left(\left|\tan\left(\frac{1}{2}\,x\right)-1\right|\right)}{a^4} + \frac{15\,Aa^{24}\tan\left(\frac{1}{2}\,x\right)^7 - 15\,Ba^{24}\tan\left(\frac{1}{2}\,x\right)^7 + 63\,Aa^{24}\tan\left(\frac{1}{2}\,x\right)^5 - 105\,Ba^{24}\tan\left(\frac{1}{2}\,x\right)^5 + 105\,Aa^{24}\tan\left(\frac{1}{2}\,x\right.}{840\,a^{28}}$$

input
```
integrate((A+B*sec(x))/(a+a*cos(x))^4,x, algorithm="giac")
```

output
```
B*log(abs(tan(1/2*x) + 1))/a^4 - B*log(abs(tan(1/2*x) - 1))/a^4 + 1/840*(1
5*A*a^24*tan(1/2*x)^7 - 15*B*a^24*tan(1/2*x)^7 + 63*A*a^24*tan(1/2*x)^5 -
105*B*a^24*tan(1/2*x)^5 + 105*A*a^24*tan(1/2*x)^3 - 385*B*a^24*tan(1/2*x)^
3 + 105*A*a^24*tan(1/2*x) - 1575*B*a^24*tan(1/2*x))/a^28
```

3.192.9 Mupad [B] (verification not implemented)

Time = 26.96 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.46

$$\int \frac{A+B\sec(x)}{(a+a\cos(x))^4}\,dx = \tan\left(\frac{x}{2}\right)\left(\frac{A-B}{8\,a^4} - \frac{3\,B}{4\,a^4} + \frac{2\,A-4\,B}{8\,a^4} - \frac{2\,A+4\,B}{8\,a^4}\right) + \tan\left(\frac{x}{2}\right)^5\left(\frac{A-B}{40\,a^4} + \frac{2\,A-4\,B}{40\,a^4}\right) + \tan\left(\frac{x}{2}\right)^3\left(\frac{A-B}{24\,a^4} - \frac{B}{4\,a^4} + \frac{2\,A-4\,B}{24\,a^4}\right) + \frac{\tan\left(\frac{x}{2}\right)^7(A-B)}{56\,a^4} + \frac{2\,B\,\mathrm{atanh}\left(\tan\left(\frac{x}{2}\right)\right)}{a^4}$$

input
```
int((A + B/cos(x))/(a + a*cos(x))^4,x)
```

output
```
tan(x/2)*((A - B)/(8*a^4) - (3*B)/(4*a^4) + (2*A - 4*B)/(8*a^4) - (2*A + 4
*B)/(8*a^4)) + tan(x/2)^5*((A - B)/(40*a^4) + (2*A - 4*B)/(40*a^4)) + tan(
x/2)^3*((A - B)/(24*a^4) - B/(4*a^4) + (2*A - 4*B)/(24*a^4)) + (tan(x/2)^7
*(A - B))/(56*a^4) + (2*B*atanh(tan(x/2)))/a^4
```

3.193 $\int (a+a\cos(x))^{5/2}(A+B\sec(x))\,dx$

3.193.1 Optimal result

Integrand size = 17, antiderivative size = 98

$$\int (a+a\cos(x))^{5/2}(A+B\sec(x))\,dx = 2a^{5/2}B\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{a+a\cos(x)}}\right) + \frac{2a^3(32A+35B)\sin(x)}{15\sqrt{a+a\cos(x)}} + \frac{2}{15}a^2(8A+5B)\sqrt{a+a\cos(x)}\sin(x) + \frac{2}{5}aA(a+a\cos(x))^{3/2}\sin(x)$$

output
```
2*a^(5/2)*B*arctanh(sin(x)*a^(1/2)/(a+a*cos(x))^(1/2))+2/5*a*A*(a+a*cos(x)
)^(3/2)*sin(x)+2/15*a^3*(32*A+35*B)*sin(x)/(a+a*cos(x))^(1/2)+2/15*a^2*(8*
A+5*B)*sin(x)*(a+a*cos(x))^(1/2)
```

3.193.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.80

$$\int (a+a\cos(x))^{5/2}(A+B\sec(x))\,dx = \frac{1}{30}a^2\sqrt{a(1+\cos(x))}\sec\left(\frac{x}{2}\right)\left(30\sqrt{2}B\operatorname{arctanh}\left(\sqrt{2}\sin\left(\frac{x}{2}\right)\right) + 2(89A+80B+2(14A+5B)\cos(x)+3A\cos(2x))\sin\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[(a + a*Cos[x])^(5/2)*(A + B*Sec[x]),x]
```

output
```
(a^2*Sqrt[a*(1 + Cos[x])]*Sec[x/2]*(30*Sqrt[2]*B*ArcTanh[Sqrt[2]*Sin[x/2]]
 + 2*(89*A + 80*B + 2*(14*A + 5*B)*Cos[x] + 3*A*Cos[2*x])*Sin[x/2]))/30
```

3.193.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.08, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.765, Rules used = {3042, 3307, 3042, 3455, 27, 3042, 3455, 27, 3042, 3460, 3042, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(x)+a)^{5/2}(A+B\sec(x))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin\left(x+\frac{\pi}{2}\right)+a\right)^{5/2}\left(A+B\csc\left(x+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3307}$$

$$\int \sec(x)(a\cos(x)+a)^{5/2}(A\cos(x)+B)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(x+\frac{\pi}{2}\right)+a\right)^{5/2}\left(A\sin\left(x+\frac{\pi}{2}\right)+B\right)}{\sin\left(x+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3455}$$

$$\frac{2}{5}\int \frac{1}{2}(\cos(x)a+a)^{3/2}(5aB+a(8A+5B)\cos(x))\sec(x)dx+\frac{2}{5}aA\sin(x)(a\cos(x)+a)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\int (\cos(x)a+a)^{3/2}(5aB+a(8A+5B)\cos(x))\sec(x)dx+\frac{2}{5}aA\sin(x)(a\cos(x)+a)^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\int \frac{\left(\sin\left(x+\frac{\pi}{2}\right)a+a\right)^{3/2}\left(5aB+a(8A+5B)\sin\left(x+\frac{\pi}{2}\right)\right)}{\sin\left(x+\frac{\pi}{2}\right)}dx+\frac{2}{5}aA\sin(x)(a\cos(x)+a)^{3/2}$$

$$\Big\downarrow \text{3455}$$

$$\frac{1}{5}\left(\frac{2}{3}\int \frac{1}{2}\sqrt{\cos(x)a+a}\left(15Ba^2+(32A+35B)\cos(x)a^2\right)\sec(x)dx+\frac{2}{3}a^2(8A+5B)\sin(x)\sqrt{a\cos(x)+a}\right)+\frac{2}{5}aA\sin(x)(a\cos(x)+a)^{3/2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{5}\left(\frac{1}{3}\int \sqrt{\cos(x)a+a}\left(15Ba^2+(32A+35B)\cos(x)a^2\right)\sec(x)dx+\frac{2}{3}a^2(8A+5B)\sin(x)\sqrt{a\cos(x)+a}\right)+\frac{2}{5}aA\sin(x)(a\cos(x)+a)^{3/2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{\sqrt{\sin\left(x+\frac{\pi}{2}\right)a+a}\left(15Ba^2+(32A+35B)\sin\left(x+\frac{\pi}{2}\right)a^2\right)}{\sin\left(x+\frac{\pi}{2}\right)}dx+\frac{2}{3}a^2(8A+5B)\sin(x)\sqrt{a\cos(x)+a}\right)+\frac{2}{5}aA\sin(x)(a\cos(x)+a)^{3/2}$$

$$\Big\downarrow \text{ 3460}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(15a^2B\int \sqrt{\cos(x)a+a}\sec(x)dx+\frac{2a^3(32A+35B)\sin(x)}{\sqrt{a\cos(x)+a}}\right)+\frac{2}{3}a^2(8A+5B)\sin(x)\sqrt{a\cos(x)+a}\right)+\frac{2}{5}aA\sin(x)(a\cos(x)+a)^{3/2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(15a^2B\int \frac{\sqrt{\sin\left(x+\frac{\pi}{2}\right)a+a}}{\sin\left(x+\frac{\pi}{2}\right)}dx+\frac{2a^3(32A+35B)\sin(x)}{\sqrt{a\cos(x)+a}}\right)+\frac{2}{3}a^2(8A+5B)\sin(x)\sqrt{a\cos(x)+a}\right)+\frac{2}{5}aA\sin(x)(a\cos(x)+a)^{3/2}$$

$$\Big\downarrow \text{ 3252}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{2a^3(32A+35B)\sin(x)}{\sqrt{a\cos(x)+a}}-30a^3B\int \frac{1}{a-\frac{a^2\sin^2(x)}{\cos(x)a+a}}d\left(-\frac{a\sin(x)}{\sqrt{\cos(x)a+a}}\right)\right)+\frac{2}{3}a^2(8A+5B)\sin(x)\sqrt{a\cos(x\right.$$
$$\frac{2}{5}aA\sin(x)(a\cos(x)+a)^{3/2}$$

$$\Big\downarrow \text{ 219}$$

3.193. $\int (a+a\cos(x))^{5/2}(A+B\sec(x))\,dx$

$$\frac{1}{5}\left(\frac{2}{3}a^2(8A+5B)\sin(x)\sqrt{a\cos(x)+a}+\frac{1}{3}\left(30a^{5/2}B\text{arctanh}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{a\cos(x)+a}}\right)+\frac{2a^3(32A+35B)\sin(x)}{\sqrt{a\cos(x)+a}}\right)\right)+\frac{2}{5}aA\sin(x)(a\cos(x)+a)^{3/2}$$

input
```
Int[(a + a*Cos[x])^(5/2)*(A + B*Sec[x]),x]
```

output
```
(2*a*A*(a + a*Cos[x])^(3/2)*Sin[x])/5 + ((2*a^2*(8*A + 5*B)*Sqrt[a + a*Cos
[x]]*Sin[x])/3 + (30*a^(5/2)*B*ArcTanh[(Sqrt[a]*Sin[x])/Sqrt[a + a*Cos[x]]
] + (2*a^3*(32*A + 35*B)*Sin[x])/Sqrt[a + a*Cos[x]])/3)/5
```

3.193.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3307
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((d + c*Sin[e +
 f*x])^n/Sin[e + f*x]^n), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

3.193.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(80) = 160$.

Time = 3.62 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.30

method	result
parts	$\frac{8A\,a^{3}\cos\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)\left(3\cos\left(\frac{x}{2}\right)^{4}+4\cos\left(\frac{x}{2}\right)^{2}+8\right)\sqrt{2}}{15\sqrt{a\cos\left(\frac{x}{2}\right)^{2}}}+\frac{B\,a^{\frac{3}{2}}\cos\left(\frac{x}{2}\right)\sqrt{\sin\left(\frac{x}{2}\right)^{2}a}\,\left(-4\sqrt{a}\,\sqrt{2}\,\sqrt{\sin\left(\frac{x}{2}\right)^{2}a}\,\sin\left(\frac{x}{2}\right)^{2}+18\sqrt{a}\,\sqrt{2}\,\sqrt{\sin\left(\frac{x}{2}\right.}\right.}{}$
default	$\frac{a^{\frac{3}{2}}\cos\left(\frac{x}{2}\right)\sqrt{\sin\left(\frac{x}{2}\right)^{2}a}\,\left(24A\sqrt{2}\,\sqrt{\sin\left(\frac{x}{2}\right)^{2}a}\,\sqrt{a}\,\sin\left(\frac{x}{2}\right)^{4}-20\sqrt{a}\,\sqrt{2}\,\sqrt{\sin\left(\frac{x}{2}\right)^{2}a}\,(4A+B)\sin\left(\frac{x}{2}\right)^{2}+120A\sqrt{2}\,\sqrt{\sin\left(\frac{x}{2}\right)^{2}a}\,\sqrt{a}+90B\sqrt{2}\right.}{15\sin\left(\frac{x}{2}\right)\sqrt{a\cos}}$

input
```
int((a+a*cos(x))^(5/2)*(A+B*sec(x)),x,method=_RETURNVERBOSE)
```

output
```
8/15*A*a^3*cos(1/2*x)*sin(1/2*x)*(3*cos(1/2*x)^4+4*cos(1/2*x)^2+8)*2^(1/2)
/(a*cos(1/2*x)^2)^(1/2)+1/3*B*a^(3/2)*cos(1/2*x)*(sin(1/2*x)^2*a)^(1/2)*(-
4*a^(1/2)*2^(1/2)*(sin(1/2*x)^2*a)^(1/2)*sin(1/2*x)^2+18*a^(1/2)*2^(1/2)*(
sin(1/2*x)^2*a)^(1/2)+3*ln(4/(2*cos(1/2*x)+2^(1/2))*(a*2^(1/2)*cos(1/2*x)+
a^(1/2)*2^(1/2)*(sin(1/2*x)^2*a)^(1/2)+2*a))*a+3*ln(-4/(2*cos(1/2*x)-2^(1/
2))*(a*2^(1/2)*cos(1/2*x)-a^(1/2)*2^(1/2)*(sin(1/2*x)^2*a)^(1/2)-2*a))*a)/
sin(1/2*x)/(a*cos(1/2*x)^2)^(1/2)
```

3.193.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.26

$$\int (a+a\cos(x))^{5/2}(A+B\sec(x))\,dx = \frac{15\,(Ba^2\cos(x)+Ba^2)\sqrt{a}\log\left(\frac{a\cos(x)^3-7\,a\cos(x)^2-4\,\sqrt{a\cos(x)+a}\sqrt{a}(\cos(x)-2)\sin(x)+8\,a}{\cos(x)^3+\cos(x)^2}\right)+4\,(3\,Aa^2}{30\,(\cos(x)+1)}$$

input
```
integrate((a+a*cos(x))^(5/2)*(A+B*sec(x)),x, algorithm="fricas")
```

output
```
1/30*(15*(B*a^2*cos(x) + B*a^2)*sqrt(a)*log((a*cos(x)^3 - 7*a*cos(x)^2 - 4
*sqrt(a*cos(x) + a)*sqrt(a)*(cos(x) - 2)*sin(x) + 8*a)/(cos(x)^3 + cos(x)^
2)) + 4*(3*A*a^2*cos(x)^2 + (14*A + 5*B)*a^2*cos(x) + (43*A + 40*B)*a^2)*s
qrt(a*cos(x) + a)*sin(x))/(cos(x) + 1)
```

3.193.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\cos(x))^{5/2}(A+B\sec(x))\,dx = \text{Timed out}$$

input
```
integrate((a+a*cos(x))**(5/2)*(A+B*sec(x)),x)
```

output
```
Timed out
```

3.193.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.44

$$\int (a+a\cos(x))^{5/2}(A+B\sec(x))\,dx = \frac{1}{30}\left(3\sqrt{2}a^2\sin\left(\frac{5}{2}x\right)+25\sqrt{2}a^2\sin\left(\frac{3}{2}x\right)+150\sqrt{2}a^2\sin\left(\frac{1}{2}x\right)\right)A\sqrt{a}$$

input
```
integrate((a+a*cos(x))^(5/2)*(A+B*sec(x)),x, algorithm="maxima")
```

output
```
1/30*(3*sqrt(2)*a^2*sin(5/2*x) + 25*sqrt(2)*a^2*sin(3/2*x) + 150*sqrt(2)*a
^2*sin(1/2*x))*A*sqrt(a)
```

3.193.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.37

$$\int (a+a\cos(x))^{5/2}(A+B\sec(x))\,dx = \frac{1}{30}\sqrt{2}\left(48\,Aa^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)\sin\left(\frac{1}{2}x\right)^5-160\,Aa^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)\sin\left(\frac{1}{2}x\right)^3-40\,B\right.$$

input
```
integrate((a+a*cos(x))^(5/2)*(A+B*sec(x)),x, algorithm="giac")
```

output
```
1/30*sqrt(2)*(48*A*a^2*sgn(cos(1/2*x))*sin(1/2*x)^5 - 160*A*a^2*sgn(cos(1/
2*x))*sin(1/2*x)^3 - 40*B*a^2*sgn(cos(1/2*x))*sin(1/2*x)^3 - 15*sqrt(2)*B*
a^2*log(abs(-2*sqrt(2) + 4*sin(1/2*x))/abs(2*sqrt(2) + 4*sin(1/2*x)))*sgn(
cos(1/2*x)) + 240*A*a^2*sgn(cos(1/2*x))*sin(1/2*x) + 180*B*a^2*sgn(cos(1/2
*x))*sin(1/2*x))*sqrt(a)
```

3.193.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\cos(x))^{5/2}(A + B\sec(x))\,dx = \int (a + a\ \cos(x))^{5/2} \left(A + \frac{B}{\cos(x)}\right) dx$$

input
```
int((a + a*cos(x))^(5/2)*(A + B/cos(x)),x)
```

output
```
int((a + a*cos(x))^(5/2)*(A + B/cos(x)), x)
```

3.194 $\int (a+a\cos(x))^{3/2}(A+B\sec(x))\,dx$

3.194.1 Optimal result

Integrand size = 17, antiderivative size = 72

$$\int (a+a\cos(x))^{3/2}(A+B\sec(x))\,dx = 2a^{3/2}B\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{a+a\cos(x)}}\right) + \frac{2a^2(4A+3B)\sin(x)}{3\sqrt{a+a\cos(x)}} + \frac{2}{3}aA\sqrt{a+a\cos(x)}\sin(x)$$

output
```
2*a^(3/2)*B*arctanh(sin(x)*a^(1/2)/(a+a*cos(x))^(1/2))+2/3*a^2*(4*A+3*B)*s
in(x)/(a+a*cos(x))^(1/2)+2/3*a*A*sin(x)*(a+a*cos(x))^(1/2)
```

3.194.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.86

$$\int (a+a\cos(x))^{3/2}(A+B\sec(x))\,dx = \frac{1}{3}a\sqrt{a(1+\cos(x))}\sec\left(\frac{x}{2}\right)\left(3\sqrt{2}B\operatorname{arctanh}\left(\sqrt{2}\sin\left(\frac{x}{2}\right)\right) + 2(5A+3B+A\cos(x))\sin\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[(a + a*Cos[x])^(3/2)*(A + B*Sec[x]),x]
```

output
```
(a*Sqrt[a*(1 + Cos[x])]*Sec[x/2]*(3*Sqrt[2]*B*ArcTanh[Sqrt[2]*Sin[x/2]] +
2*(5*A + 3*B + A*Cos[x])*Sin[x/2]))/3
```

3.194.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules used = {3042, 3307, 3042, 3455, 27, 3042, 3460, 3042, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(x)+a)^{3/2}(A+B\sec(x))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin\left(x+\frac{\pi}{2}\right)+a\right)^{3/2}\left(A+B\csc\left(x+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3307}$$

$$\int \sec(x)(a\cos(x)+a)^{3/2}(A\cos(x)+B)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a\sin\left(x+\frac{\pi}{2}\right)+a\right)^{3/2}\left(A\sin\left(x+\frac{\pi}{2}\right)+B\right)}{\sin\left(x+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3455}$$

$$\frac{2}{3}\int \frac{1}{2}\sqrt{\cos(x)a+a}(3aB+a(4A+3B)\cos(x))\sec(x)dx+\frac{2}{3}aA\sin(x)\sqrt{a\cos(x)+a}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \sqrt{\cos(x)a+a}(3aB+a(4A+3B)\cos(x))\sec(x)dx+\frac{2}{3}aA\sin(x)\sqrt{a\cos(x)+a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{\sqrt{\sin\left(x+\frac{\pi}{2}\right)a+a}\left(3aB+a(4A+3B)\sin\left(x+\frac{\pi}{2}\right)\right)}{\sin\left(x+\frac{\pi}{2}\right)}dx+\frac{2}{3}aA\sin(x)\sqrt{a\cos(x)+a}$$

$$\Big\downarrow \text{3460}$$

$$\frac{1}{3}\left(3aB\int \sqrt{\cos(x)a+a}\sec(x)dx+\frac{2a^2(4A+3B)\sin(x)}{\sqrt{a\cos(x)+a}}\right)+\frac{2}{3}aA\sin(x)\sqrt{a\cos(x)+a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(3aB\int \frac{\sqrt{\sin\left(x+\frac{\pi}{2}\right)a+a}}{\sin\left(x+\frac{\pi}{2}\right)}dx+\frac{2a^2(4A+3B)\sin(x)}{\sqrt{a\cos(x)+a}}\right)+\frac{2}{3}aA\sin(x)\sqrt{a\cos(x)+a}$$

$$\Big\downarrow \text{ 3252}$$

$$\frac{1}{3}\left(\frac{2a^2(4A+3B)\sin(x)}{\sqrt{a\cos(x)+a}}-6a^2B\int \frac{1}{a-\frac{a^2\sin^2(x)}{\cos(x)a+a}}d\left(-\frac{a\sin(x)}{\sqrt{\cos(x)a+a}}\right)\right)+\frac{2}{3}aA\sin(x)\sqrt{a\cos(x)+a}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{3}\left(6a^{3/2}B\text{arctanh}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{a\cos(x)+a}}\right)+\frac{2a^2(4A+3B)\sin(x)}{\sqrt{a\cos(x)+a}}\right)+\frac{2}{3}aA\sin(x)\sqrt{a\cos(x)+a}$$

input
```
Int[(a + a*Cos[x])^(3/2)*(A + B*Sec[x]),x]
```

output
```
(2*a*A*Sqrt[a + a*Cos[x]]*Sin[x])/3 + (6*a^(3/2)*B*ArcTanh[(Sqrt[a]*Sin[x]
)/Sqrt[a + a*Cos[x]]] + (2*a^2*(4*A + 3*B)*Sin[x])/Sqrt[a + a*Cos[x]])/3
```

3.194.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3307
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((d + c*Sin[e +
 f*x])^n/Sin[e + f*x]^n), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

3.194.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(58) = 116$.

Time = 2.11 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.61

method	result
parts	$\frac{4Aa^2\cos(\frac{x}{2})\sin(\frac{x}{2})\left(2+\cos(\frac{x}{2})^2\right)\sqrt{2}}{3\sqrt{a\cos(\frac{x}{2})^2}}+\frac{B\sqrt{a}\cos(\frac{x}{2})\sqrt{\sin(\frac{x}{2})^2 a}\left(2\sqrt{a}\sqrt{2}\sqrt{\sin(\frac{x}{2})^2 a}+\ln\left(\frac{4a\sqrt{2}\cos(\frac{x}{2})+4\sqrt{a}\sqrt{2}\sqrt{\sin(\frac{x}{2})^2 a}+8a\cdots}{2\cos(\frac{x}{2})+\sqrt{2}}\right.\right.\cdots}{\sin(\frac{x}{2})\sqrt{a\cos(\frac{x}{2})^2}}$
default	$\frac{\sqrt{a}\cos(\frac{x}{2})\sqrt{\sin(\frac{x}{2})^2 a}\left(-4A\sqrt{a}\sqrt{2}\sqrt{\sin(\frac{x}{2})^2 a}\sin(\frac{x}{2})^2+12A\sqrt{2}\sqrt{\sin(\frac{x}{2})^2 a}\sqrt{a}+6B\sqrt{2}\sqrt{\sin(\frac{x}{2})^2 a}\sqrt{a}+3B\ln\left(\frac{4a\sqrt{2}\cos(\frac{x}{2})+4\cdots}{2\cos\cdots}\right.\right.\cdots}{3\sin(\frac{x}{2})\sqrt{a\cos(\frac{x}{2})^2}}$

input
```
int((a+a*cos(x))^(3/2)*(A+B*sec(x)),x,method=_RETURNVERBOSE)
```

output
```
4/3*A*a^2*cos(1/2*x)*sin(1/2*x)*(2+cos(1/2*x)^2)*2^(1/2)/(a*cos(1/2*x)^2)^
(1/2)+B*a^(1/2)*cos(1/2*x)*(sin(1/2*x)^2*a)^(1/2)*(2*a^(1/2)*2^(1/2)*(sin(
1/2*x)^2*a)^(1/2)+ln(4/(2*cos(1/2*x)+2^(1/2))*(a*2^(1/2)*cos(1/2*x)+a^(1/2
)*2^(1/2)*(sin(1/2*x)^2*a)^(1/2)+2*a))*a+ln(-4/(2*cos(1/2*x)-2^(1/2))*(a*2
^(1/2)*cos(1/2*x)-a^(1/2)*2^(1/2)*(sin(1/2*x)^2*a)^(1/2)-2*a))*a)/sin(1/2*
x)/(a*cos(1/2*x)^2)^(1/2)
```

3.194.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.38

$$\int (a+a\cos(x))^{3/2}(A+B\sec(x))\,dx = \frac{3\,(Ba\cos(x)+Ba)\sqrt{a}\log\left(\frac{a\cos(x)^3-7\,a\cos(x)^2-4\,\sqrt{a\cos(x)+a}\sqrt{a}(\cos(x)-2)\sin(x)+8\,a}{\cos(x)^3+\cos(x)^2}\right)+4\,(Aa\cos(x$$
$$6\,(\cos(x)+1)$$

input
```
integrate((a+a*cos(x))^(3/2)*(A+B*sec(x)),x, algorithm="fricas")
```

output
```
1/6*(3*(B*a*cos(x) + B*a)*sqrt(a)*log((a*cos(x)^3 - 7*a*cos(x)^2 - 4*sqrt(
a*cos(x) + a)*sqrt(a)*(cos(x) - 2)*sin(x) + 8*a)/(cos(x)^3 + cos(x)^2)) +
4*(A*a*cos(x) + (5*A + 3*B)*a)*sqrt(a*cos(x) + a)*sin(x))/(cos(x) + 1)
```

3.194.6 Sympy [F]

$$\int (a+a\cos(x))^{3/2}(A+B\sec(x))\,dx = \int (a(\cos(x)+1))^{\frac{3}{2}}\,(A+B\sec(x))\,dx$$

input
```
integrate((a+a*cos(x))**(3/2)*(A+B*sec(x)),x)
```

output
```
Integral((a*(cos(x) + 1))**(3/2)*(A + B*sec(x)), x)
```

3.194.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.36

$$\int (a+a\cos(x))^{3/2}(A+B\sec(x))\,dx = \frac{1}{3}\left(\sqrt{2}a\sin\left(\frac{3}{2}\,x\right)+9\,\sqrt{2}a\sin\left(\frac{1}{2}\,x\right)\right)A\sqrt{a}$$

input
```
integrate((a+a*cos(x))^(3/2)*(A+B*sec(x)),x, algorithm="maxima")
```

output
```
1/3*(sqrt(2)*a*sin(3/2*x) + 9*sqrt(2)*a*sin(1/2*x))*A*sqrt(a)
```

3.194.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.28

$$\int (a+a\cos(x))^{3/2}(A+B\sec(x))\,dx =$$

$$-\frac{1}{6}\sqrt{2}\left(8\,A a\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)\sin\left(\frac{1}{2}\,x\right)^{3}+3\,\sqrt{2}Ba\log\left(\frac{\left|-2\,\sqrt{2}+4\,\sin\left(\frac{1}{2}\,x\right)\right|}{\left|2\,\sqrt{2}+4\,\sin\left(\frac{1}{2}\,x\right)\right|}\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)-24\,$$

input
```
integrate((a+a*cos(x))^(3/2)*(A+B*sec(x)),x, algorithm="giac")
```

output
```
-1/6*sqrt(2)*(8*A*a*sgn(cos(1/2*x))*sin(1/2*x)^3 + 3*sqrt(2)*B*a*log(abs(-
2*sqrt(2) + 4*sin(1/2*x))/abs(2*sqrt(2) + 4*sin(1/2*x)))*sgn(cos(1/2*x)) -
 24*A*a*sgn(cos(1/2*x))*sin(1/2*x) - 12*B*a*sgn(cos(1/2*x))*sin(1/2*x))*sq
rt(a)
```

3.194.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\cos(x))^{3/2}(A+B\sec(x))\,dx = \int (a+a\,\cos(x))^{3/2}\left(A+\frac{B}{\cos(x)}\right)\,dx$$

input
```
int((a + a*cos(x))^(3/2)*(A + B/cos(x)),x)
```

output
```
int((a + a*cos(x))^(3/2)*(A + B/cos(x)), x)
```

3.195 $\int \sqrt{a+a\cos(x)}(A+B\sec(x))\,dx$

3.195.1 Optimal result

Integrand size = 17, antiderivative size = 44

$$\int \sqrt{a+a\cos(x)}(A+B\sec(x))\,dx=2\sqrt{a}B\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{a+a\cos(x)}}\right)+\frac{2aA\sin(x)}{\sqrt{a+a\cos(x)}}$$

output
```
2*B*arctanh(sin(x)*a^(1/2)/(a+a*cos(x))^(1/2))*a^(1/2)+2*a*A*sin(x)/(a+a*c
os(x))^(1/2)
```

3.195.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.07

$$\int \sqrt{a+a\cos(x)}(A+B\sec(x))\,dx=\sqrt{a(1+\cos(x))}\sec\left(\frac{x}{2}\right)\left(\sqrt{2}B\operatorname{arctanh}\left(\sqrt{2}\sin\left(\frac{x}{2}\right)\right)+2A\sin\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[Sqrt[a + a*Cos[x]]*(A + B*Sec[x]),x]
```

output
```
Sqrt[a*(1 + Cos[x])]*Sec[x/2]*(Sqrt[2]*B*ArcTanh[Sqrt[2]*Sin[x/2]] + 2*A*S
in[x/2])
```

3.195.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {3042, 3307, 3042, 3460, 3042, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\cos(x)+a}(A+B\sec(x))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt{a\sin\left(x+\frac{\pi}{2}\right)+a}\left(A+B\csc\left(x+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{ 3307}$$

$$\int \sec(x)\sqrt{a\cos(x)+a}(A\cos(x)+B)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{a\sin\left(x+\frac{\pi}{2}\right)+a}\left(A\sin\left(x+\frac{\pi}{2}\right)+B\right)}{\sin\left(x+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 3460}$$

$$B\int \sqrt{\cos(x)a+a}\sec(x)dx+\frac{2aA\sin(x)}{\sqrt{a\cos(x)+a}}$$

$$\Big\downarrow \text{ 3042}$$

$$B\int \frac{\sqrt{\sin\left(x+\frac{\pi}{2}\right)a+a}}{\sin\left(x+\frac{\pi}{2}\right)}dx+\frac{2aA\sin(x)}{\sqrt{a\cos(x)+a}}$$

$$\Big\downarrow \text{ 3252}$$

$$\frac{2aA\sin(x)}{\sqrt{a\cos(x)+a}}-2aB\int \frac{1}{a-\frac{a^2\sin^2(x)}{\cos(x)a+a}}d\left(-\frac{a\sin(x)}{\sqrt{\cos(x)a+a}}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{2aA\sin(x)}{\sqrt{a\cos(x)+a}}+2\sqrt{a}B\text{arctanh}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{a\cos(x)+a}}\right)$$

input
```
Int[Sqrt[a + a*Cos[x]]*(A + B*Sec[x]),x]
```

output
```
2*Sqrt[a]*B*ArcTanh[(Sqrt[a]*Sin[x])/Sqrt[a + a*Cos[x]]] + (2*a*A*Sin[x])/
Sqrt[a + a*Cos[x]]
```

3.195.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3307
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((d + c*Sin[e +
 f*x])^n/Sin[e + f*x]^n), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

3.195.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(36) = 72$.

Time = 2.10 (sec) , antiderivative size = 154, normalized size of antiderivative = 3.50

method	result
default	$\frac{\cos\left(\frac{x}{2}\right)\sqrt{\sin\left(\frac{x}{2}\right)^{2} a}\left(2A\sqrt{2}\,\sqrt{\sin\left(\frac{x}{2}\right)^{2} a}\,\sqrt{a}+B\ln\left(-\frac{4\left(a\sqrt{2}\,\cos\left(\frac{x}{2}\right)-\sqrt{a}\,\sqrt{2}\,\sqrt{\sin\left(\frac{x}{2}\right)^{2} a}-2a\right)}{2\cos\left(\frac{x}{2}\right)-\sqrt{2}}\right)a+B\ln\left(\frac{4a\sqrt{2}\,\cos\left(\frac{x}{2}\right)+4\sqrt{a}\,\sqrt{2}\,\sqrt{\sin\left(\frac{x}{2}\right.}}{2\cos\left(\frac{x}{2}\right)+\sqrt{2}}\right.\right.}{\sqrt{a}\,\sin\left(\frac{x}{2}\right)\sqrt{a\cos\left(\frac{x}{2}\right)^{2}}}$
parts	$\frac{2Aa\cos\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)\sqrt{2}}{\sqrt{a\cos\left(\frac{x}{2}\right)^{2}}}+\frac{B\sqrt{a}\,\cos\left(\frac{x}{2}\right)\sqrt{\sin\left(\frac{x}{2}\right)^{2} a}\left(\ln\left(\frac{4a\sqrt{2}\,\cos\left(\frac{x}{2}\right)+4\sqrt{a}\,\sqrt{2}\,\sqrt{\sin\left(\frac{x}{2}\right)^{2} a}+8a}{2\cos\left(\frac{x}{2}\right)+\sqrt{2}}\right)+\ln\left(-\frac{4\left(a\sqrt{2}\,\cos\left(\frac{x}{2}\right)-\sqrt{a}\,\sqrt{2}\,\sqrt{\sin\left(\right.}\right.}{2\cos\left(\frac{x}{2}\right)-\sqrt{2}}\right.\right.}{\sin\left(\frac{x}{2}\right)\sqrt{a\cos\left(\frac{x}{2}\right)^{2}}}$

input

```
int((a+a*cos(x))^(1/2)*(A+B*sec(x)),x,method=_RETURNVERBOSE)
```

output

```
1/a^(1/2)*cos(1/2*x)*(sin(1/2*x)^2*a)^(1/2)*(2*A*2^(1/2)*(sin(1/2*x)^2*a)^
(1/2)*a^(1/2)+B*ln(-4/(2*cos(1/2*x)-2^(1/2))*(a*2^(1/2)*cos(1/2*x)-a^(1/2)
*2^(1/2)*(sin(1/2*x)^2*a)^(1/2)-2*a))*a+B*ln(4/(2*cos(1/2*x)+2^(1/2))*(a*2
^(1/2)*cos(1/2*x)+a^(1/2)*2^(1/2)*(sin(1/2*x)^2*a)^(1/2)+2*a))*a)/sin(1/2*
x)/(a*cos(1/2*x)^2)^(1/2)
```

3.195.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(36) = 72$.

Time = 0.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.84

$$\int \sqrt{a+a\cos(x)}(A+B\sec(x))\,dx$$
$$=\frac{(B\cos(x)+B)\sqrt{a}\log\left(\frac{a\cos(x)^3-7a\cos(x)^2-4\sqrt{a\cos(x)+a}\sqrt{a}(\cos(x)-2)\sin(x)+8a}{\cos(x)^3+\cos(x)^2}\right)+4\sqrt{a\cos(x)+a}A\sin(x)}{2(\cos(x)+1)}$$

input

```
integrate((a+a*cos(x))^(1/2)*(A+B*sec(x)),x, algorithm="fricas")
```

output

```
1/2*((B*cos(x) + B)*sqrt(a)*log((a*cos(x)^3 - 7*a*cos(x)^2 - 4*sqrt(a*cos(
x) + a)*sqrt(a)*(cos(x) - 2)*sin(x) + 8*a)/(cos(x)^3 + cos(x)^2)) + 4*sqrt
(a*cos(x) + a)*A*sin(x))/(cos(x) + 1)
```

3.195.6 Sympy [F]

$$\int \sqrt{a+a\cos(x)}(A+B\sec(x))\,dx=\int \sqrt{a\left(\cos\left(x\right)+1\right)}\left(A+B\sec\left(x\right)\right)\,dx$$

input
```
integrate((a+a*cos(x))**(1/2)*(A+B*sec(x)),x)
```

output
```
Integral(sqrt(a*(cos(x) + 1))*(A + B*sec(x)), x)
```

3.195.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.30

$$\int \sqrt{a+a\cos(x)}(A+B\sec(x))\,dx=2\sqrt{2}A\sqrt{a}\sin\left(\frac{1}{2}\,x\right)$$

input
```
integrate((a+a*cos(x))^(1/2)*(A+B*sec(x)),x, algorithm="maxima")
```

output
```
2*sqrt(2)*A*sqrt(a)*sin(1/2*x)
```

3.195.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.39

$$\int \sqrt{a+a\cos(x)}(A+B\sec(x))\,dx=$$
$$-\frac{1}{2}\sqrt{2}\left(\sqrt{2}B\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(\frac{1}{2}\,x\right)\right|}{\left|2\sqrt{2}+4\sin\left(\frac{1}{2}\,x\right)\right|}\right)\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)-4\,A\mathrm{sgn}\left(\cos\left(\frac{1}{2}\,x\right)\right)\sin\left(\frac{1}{2}\,x\right)\right)\sqrt{a}$$

input
```
integrate((a+a*cos(x))^(1/2)*(A+B*sec(x)),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*(sqrt(2)*B*log(abs(-2*sqrt(2) + 4*sin(1/2*x))/abs(2*sqrt(2) +
 4*sin(1/2*x)))*sgn(cos(1/2*x)) - 4*A*sgn(cos(1/2*x))*sin(1/2*x))*sqrt(a)
```

3.195.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\cos(x)}(A+B\sec(x))\,dx=\int \sqrt{a+a\,\cos\left(x\right)}\,\left(A+\frac{B}{\cos\left(x\right)}\right)\,dx$$

input
```
int((a + a*cos(x))^(1/2)*(A + B/cos(x)),x)
```

output
```
int((a + a*cos(x))^(1/2)*(A + B/cos(x)), x)
```

3.196 $\int \frac{A+B\sec(x)}{\sqrt{a+a\cos(x)}}\,dx$

3.196.1 Optimal result

Integrand size = 17, antiderivative size = 68

$$\int \frac{A+B\sec(x)}{\sqrt{a+a\cos(x)}}\,dx=\frac{2B\text{arctanh}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{a+a\cos(x)}}\right)}{\sqrt{a}}+\frac{\sqrt{2}(A-B)\text{arctanh}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{2}\sqrt{a+a\cos(x)}}\right)}{\sqrt{a}}$$

output
```
2*B*arctanh(sin(x)*a^(1/2)/(a+a*cos(x))^(1/2))/a^(1/2)+(A-B)*arctanh(1/2*s
in(x)*a^(1/2)*2^(1/2)/(a+a*cos(x))^(1/2))*2^(1/2)/a^(1/2)
```

3.196.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.76

$$\int \frac{A+B\sec(x)}{\sqrt{a+a\cos(x)}}\,dx=\frac{2\left((A-B)\text{arctanh}\left(\sin\left(\frac{x}{2}\right)\right)+\sqrt{2}B\text{arctanh}\left(\sqrt{2}\sin\left(\frac{x}{2}\right)\right)\right)\cos\left(\frac{x}{2}\right)}{\sqrt{a(1+\cos(x))}}$$

input
```
Integrate[(A + B*Sec[x])/Sqrt[a + a*Cos[x]],x]
```

output
```
(2*((A - B)*ArcTanh[Sin[x/2]] + Sqrt[2]*B*ArcTanh[Sqrt[2]*Sin[x/2]])*Cos[x
/2])/Sqrt[a*(1 + Cos[x])]
```

3.196.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {3042, 3307, 3042, 3464, 3042, 3128, 219, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sec(x)}{\sqrt{a\cos(x)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\csc\left(x+\frac{\pi}{2}\right)}{\sqrt{a\sin\left(x+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{3307}$$

$$\int \frac{\sec(x)(A\cos(x)+B)}{\sqrt{a\cos(x)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A\sin\left(x+\frac{\pi}{2}\right)+B}{\sin\left(x+\frac{\pi}{2}\right)\sqrt{a\sin\left(x+\frac{\pi}{2}\right)+a}}dx$$

$$\Big\downarrow \text{3464}$$

$$(A-B)\int \frac{1}{\sqrt{\cos(x)a+a}}dx+\frac{B\int \sqrt{\cos(x)a+a}\sec(x)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$(A-B)\int \frac{1}{\sqrt{\sin\left(x+\frac{\pi}{2}\right)a+a}}dx+\frac{B\int \frac{\sqrt{\sin\left(x+\frac{\pi}{2}\right)a+a}}{\sin\left(x+\frac{\pi}{2}\right)}dx}{a}$$

$$\Big\downarrow \text{3128}$$

$$\frac{B\int \frac{\sqrt{\sin\left(x+\frac{\pi}{2}\right)a+a}}{\sin\left(x+\frac{\pi}{2}\right)}dx}{a}-2(A-B)\int \frac{1}{2a-\frac{a^2\sin^2(x)}{\cos(x)a+a}}d\left(-\frac{a\sin(x)}{\sqrt{\cos(x)a+a}}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{B\int \frac{\sqrt{\sin\left(x+\frac{\pi}{2}\right)a+a}}{\sin\left(x+\frac{\pi}{2}\right)}dx}{a}+\frac{\sqrt{2}(A-B)\text{arctanh}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{2}\sqrt{a\cos(x)+a}}\right)}{\sqrt{a}}$$

$$\Big\downarrow \text{3252}$$

$$\frac{\sqrt{2}(A-B)\text{arctanh}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{2}\sqrt{a\cos(x)+a}}\right)}{\sqrt{a}}-2B\int\frac{1}{a-\frac{a^2\sin^2(x)}{\cos(x)a+a}}d\left(-\frac{a\sin(x)}{\sqrt{\cos(x)a+a}}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{2}(A-B)\text{arctanh}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{2}\sqrt{a\cos(x)+a}}\right)}{\sqrt{a}}+\frac{2B\text{arctanh}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{a\cos(x)+a}}\right)}{\sqrt{a}}$$

input
```
Int[(A + B*Sec[x])/Sqrt[a + a*Cos[x]],x]
```

output
```
(2*B*ArcTanh[(Sqrt[a]*Sin[x])/Sqrt[a + a*Cos[x]]])/Sqrt[a] + (Sqrt[2]*(A -
 B)*ArcTanh[(Sqrt[a]*Sin[x])/(Sqrt[2]*Sqrt[a + a*Cos[x]])])/Sqrt[a]
```

3.196.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3307
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((d + c*Sin[e +
 f*x])^n/Sin[e + f*x]^n), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

rule 3464
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A
*b - a*B)/(b*c - a*d)   Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Simp[(B*c
- A*d)/(b*c - a*d)   Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.196.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(53) = 106$.

Time = 2.05 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.85

method	result
default	$\frac{\cos(\frac{x}{2})\sqrt{\sin(\frac{x}{2})^2 a}\,(\sqrt{2}\ln(\frac{4\sqrt{a}\sqrt{\sin(\frac{x}{2})^2 a}+4a}{\cos(\frac{x}{2})})A-\sqrt{2}\ln(\frac{4\sqrt{a}\sqrt{\sin(\frac{x}{2})^2 a}+4a}{\cos(\frac{x}{2})})B+B\ln(\frac{4a\sqrt{2}\cos(\frac{x}{2})+4\sqrt{a}\sqrt{2}\sqrt{\sin(\frac{x}{2})^2 a}+8a}{2\cos(\frac{x}{2})+\sqrt{2}})}{\sqrt{a}\sin(\frac{x}{2})\sqrt{a\cos(\frac{x}{2})^2}}$
parts	$\frac{A\sqrt{2}\,\text{InverseJacobiAM}(\frac{x}{2},1)}{\sec(\frac{x}{2})\sqrt{a\cos(\frac{x}{2})^2}\,\text{csgn}(\cos(\frac{x}{2}))}-\frac{B\cos(\frac{x}{2})\sqrt{\sin(\frac{x}{2})^2 a}\,(\sqrt{2}\ln(\frac{4\sqrt{a}\sqrt{\sin(\frac{x}{2})^2 a}+4a}{\cos(\frac{x}{2})})-\ln(\frac{4a\sqrt{2}\cos(\frac{x}{2})+4\sqrt{a}\sqrt{2}\sqrt{\sin(\frac{x}{2})^2 a}+8}{2\cos(\frac{x}{2})+\sqrt{2}}}{\sqrt{a}\sin(\frac{x}{2})\sqrt{a\cos(\frac{x}{2})^2}}$

input
```
int((A+B*sec(x))/(a+a*cos(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
cos(1/2*x)*(sin(1/2*x)^2*a)^(1/2)*(2^(1/2)*ln(4/cos(1/2*x)*(a^(1/2)*(sin(1
/2*x)^2*a)^(1/2)+a))*A-2^(1/2)*ln(4/cos(1/2*x)*(a^(1/2)*(sin(1/2*x)^2*a)^(
1/2)+a))*B+B*ln(4/(2*cos(1/2*x)+2^(1/2))*(a*2^(1/2)*cos(1/2*x)+a^(1/2)*2^(
1/2)*(sin(1/2*x)^2*a)^(1/2)+2*a))+B*ln(-4/(2*cos(1/2*x)-2^(1/2))*(a*2^(1/2
)*cos(1/2*x)-a^(1/2)*2^(1/2)*(sin(1/2*x)^2*a)^(1/2)-2*a)))/a^(1/2)/sin(1/2
*x)/(a*cos(1/2*x)^2)^(1/2)
```

3.196.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(53) = 106.

Time = 0.26 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.71

$$\int \frac{A+B\sec(x)}{\sqrt{a+a\cos(x)}}\,dx = -\frac{\sqrt{2}(A-B)\sqrt{a}\log\left(-\frac{\cos(x)^2+\frac{2\sqrt{2}\sqrt{a\cos(x)+a}\sin(x)}{\sqrt{a}}-2\cos(x)-3}{\cos(x)^2+2\cos(x)+1}\right)-B\sqrt{a}\log\left(\frac{a\cos(x)^3-7a\cos(x)^2-4\sqrt{a\cos(x)+a}\sqrt{a}(\cos(x)-2)\sin(x)+8a}{\cos(x)^3+\cos(x)^2}\right)}{2a}$$

input
```
integrate((A+B*sec(x))/(a+a*cos(x))^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(2)*(A - B)*sqrt(a)*log(-(cos(x)^2 + 2*sqrt(2)*sqrt(a*cos(x) + a
)*sin(x)/sqrt(a) - 2*cos(x) - 3)/(cos(x)^2 + 2*cos(x) + 1)) - B*sqrt(a)*lo
g((a*cos(x)^3 - 7*a*cos(x)^2 - 4*sqrt(a*cos(x) + a)*sqrt(a)*(cos(x) - 2)*s
in(x) + 8*a)/(cos(x)^3 + cos(x)^2)))/a
```

3.196.6 Sympy [F]

$$\int \frac{A+B\sec(x)}{\sqrt{a+a\cos(x)}}\,dx = \int \frac{A+B\sec(x)}{\sqrt{a(\cos(x)+1)}}\,dx$$

input
```
integrate((A+B*sec(x))/(a+a*cos(x))**(1/2),x)
```

output
```
Integral((A + B*sec(x))/sqrt(a*(cos(x) + 1)), x)
```

3.196.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.85

$$\int \frac{A+B\sec(x)}{\sqrt{a+a\cos(x)}}\,dx = \frac{\left(\sqrt{2}\log\left(\cos\left(\frac{1}{2}x\right)^2+\sin\left(\frac{1}{2}x\right)^2+2\sin\left(\frac{1}{2}x\right)+1\right)-\sqrt{2}\log\left(\cos\left(\frac{1}{2}x\right)^2+\sin\left(\frac{1}{2}x\right)^2-2\sin\left(\frac{1}{2}x\right)+1\right)\right)}{2\sqrt{a}}$$

input
```
integrate((A+B*sec(x))/(a+a*cos(x))^(1/2),x, algorithm="maxima")
```

output
```
1/2*(sqrt(2)*log(cos(1/2*x)^2 + sin(1/2*x)^2 + 2*sin(1/2*x) + 1) - sqrt(2)
*log(cos(1/2*x)^2 + sin(1/2*x)^2 - 2*sin(1/2*x) + 1))*A/sqrt(a)
```

3.196.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(53) = 106$.

Time = 0.29 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.76

$$\int \frac{A+B\sec(x)}{\sqrt{a+a\cos(x)}}\,dx = \frac{\sqrt{2}(A\sqrt{a}-B\sqrt{a})\log\left(\sin\left(\frac{1}{2}x\right)+1\right)}{2\,a\mathrm{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)} - \frac{\sqrt{2}(A\sqrt{a}-B\sqrt{a})\log\left(-\sin\left(\frac{1}{2}x\right)+1\right)}{2\,a\mathrm{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)} + \frac{B\log\left(\left|\frac{1}{2}\sqrt{2}+\sin\left(\frac{1}{2}x\right)\right|\right)}{\sqrt{a}\mathrm{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)} - \frac{B\log\left(\left|-\frac{1}{2}\sqrt{2}+\sin\left(\frac{1}{2}x\right)\right|\right)}{\sqrt{a}\mathrm{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)}$$

input
```
integrate((A+B*sec(x))/(a+a*cos(x))^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*(A*sqrt(a) - B*sqrt(a))*log(sin(1/2*x) + 1)/(a*sgn(cos(1/2*x))
) - 1/2*sqrt(2)*(A*sqrt(a) - B*sqrt(a))*log(-sin(1/2*x) + 1)/(a*sgn(cos(1/
2*x))) + B*log(abs(1/2*sqrt(2) + sin(1/2*x)))/(sqrt(a)*sgn(cos(1/2*x))) -
B*log(abs(-1/2*sqrt(2) + sin(1/2*x)))/(sqrt(a)*sgn(cos(1/2*x)))
```

3.196.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sec(x)}{\sqrt{a+a\cos(x)}}\,dx = \int \frac{A+\frac{B}{\cos(x)}}{\sqrt{a+a\,\cos(x)}}\,dx$$

input
```
int((A + B/cos(x))/(a + a*cos(x))^(1/2),x)
```

output
```
int((A + B/cos(x))/(a + a*cos(x))^(1/2), x)
```

3.197 $\int \frac{A+B\sec(x)}{(a+a\cos(x))^{3/2}}\,dx$

3.197.1 Optimal result

Integrand size = 17, antiderivative size = 92

$$\int \frac{A+B\sec(x)}{(a+a\cos(x))^{3/2}}\,dx = \frac{2B\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{a+a\cos(x)}}\right)}{a^{3/2}} + \frac{(A-5B)\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{2}\sqrt{a+a\cos(x)}}\right)}{2\sqrt{2}a^{3/2}} + \frac{(A-B)\sin(x)}{2(a+a\cos(x))^{3/2}}$$

output
```
2*B*arctanh(sin(x)*a^(1/2)/(a+a*cos(x))^(1/2))/a^(3/2)+1/2*(A-B)*sin(x)/(a
+a*cos(x))^(3/2)+1/4*(A-5*B)*arctanh(1/2*sin(x)*a^(1/2)*2^(1/2)/(a+a*cos(x
))^(1/2))/a^(3/2)*2^(1/2)
```

3.197.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.79

$$\int \frac{A+B\sec(x)}{(a+a\cos(x))^{3/2}}\,dx = \frac{(A-5B)\operatorname{arctanh}\left(\sin\left(\frac{x}{2}\right)\right)\cos^3\left(\frac{x}{2}\right)+4\sqrt{2}B\operatorname{arctanh}\left(\sqrt{2}\sin\left(\frac{x}{2}\right)\right)\cos^3\left(\frac{x}{2}\right)+\frac{1}{2}(A -}{(a(1+\cos(x)))^{3/2}}$$

input
```
Integrate[(A + B*Sec[x])/(a + a*Cos[x])^(3/2),x]
```

output
```
((A - 5*B)*ArcTanh[Sin[x/2]]*Cos[x/2]^3 + 4*Sqrt[2]*B*ArcTanh[Sqrt[2]*Sin[
x/2]]*Cos[x/2]^3 + ((A - B)*Sin[x])/2)/(a*(1 + Cos[x]))^(3/2)
```

3.197.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.05, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.706, Rules used = {3042, 3307, 3042, 3457, 27, 3042, 3464, 3042, 3128, 219, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sec(x)}{(a\cos(x)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\csc\left(x+\frac{\pi}{2}\right)}{\left(a\sin\left(x+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{3307}$$

$$\int \frac{\sec(x)(A\cos(x)+B)}{(a\cos(x)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A\sin\left(x+\frac{\pi}{2}\right)+B}{\sin\left(x+\frac{\pi}{2}\right)\left(a\sin\left(x+\frac{\pi}{2}\right)+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{3457}$$

$$\frac{\int \frac{(4aB+a(A-B)\cos(x))\sec(x)}{2\sqrt{\cos(x)a+a}}dx}{2a^2}+\frac{(A-B)\sin(x)}{2(a\cos(x)+a)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(4aB+a(A-B)\cos(x))\sec(x)}{\sqrt{\cos(x)a+a}}dx}{4a^2}+\frac{(A-B)\sin(x)}{2(a\cos(x)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{4aB+a(A-B)\sin\left(x+\frac{\pi}{2}\right)}{\sin\left(x+\frac{\pi}{2}\right)\sqrt{\sin\left(x+\frac{\pi}{2}\right)a+a}}dx}{4a^2}+\frac{(A-B)\sin(x)}{2(a\cos(x)+a)^{3/2}}$$

$$\Big\downarrow \text{3464}$$

$$\frac{a(A-5B)\int \frac{1}{\sqrt{\cos(x)a+a}}dx+4B\int \sqrt{\cos(x)a+a}\sec(x)dx}{4a^2}+\frac{(A-B)\sin(x)}{2(a\cos(x)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a(A-5B)\int \frac{1}{\sqrt{\sin(x+\frac{\pi}{2})a+a}}dx+4B\int \frac{\sqrt{\sin(x+\frac{\pi}{2})a+a}}{\sin(x+\frac{\pi}{2})}dx}{4a^2}+\frac{(A-B)\sin(x)}{2(a\cos(x)+a)^{3/2}}$$

$$\Big\downarrow \text{3128}$$

$$\frac{4B\int \frac{\sqrt{\sin(x+\frac{\pi}{2})a+a}}{\sin(x+\frac{\pi}{2})}dx-2a(A-5B)\int \frac{1}{2a-\frac{a^2\sin^2(x)}{\cos(x)a+a}}d\left(-\frac{a\sin(x)}{\sqrt{\cos(x)a+a}}\right)}{4a^2}+\frac{(A-B)\sin(x)}{2(a\cos(x)+a)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{4B\int \frac{\sqrt{\sin(x+\frac{\pi}{2})a+a}}{\sin(x+\frac{\pi}{2})}dx+\sqrt{2}\sqrt{a}(A-5B)\text{arctanh}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{2}\sqrt{a\cos(x)+a}}\right)}{4a^2}+\frac{(A-B)\sin(x)}{2(a\cos(x)+a)^{3/2}}$$

$$\Big\downarrow \text{3252}$$

$$\frac{\sqrt{2}\sqrt{a}(A-5B)\text{arctanh}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{2}\sqrt{a\cos(x)+a}}\right)-8aB\int \frac{1}{a-\frac{a^2\sin^2(x)}{\cos(x)a+a}}d\left(-\frac{a\sin(x)}{\sqrt{\cos(x)a+a}}\right)}{4a^2}+\frac{(A-B)\sin(x)}{2(a\cos(x)+a)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{2}\sqrt{a}(A-5B)\text{arctanh}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{2}\sqrt{a\cos(x)+a}}\right)+8\sqrt{a}B\text{arctanh}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{a\cos(x)+a}}\right)}{4a^2}+\frac{(A-B)\sin(x)}{2(a\cos(x)+a)^{3/2}}$$

input
```
Int[(A + B*Sec[x])/(a + a*Cos[x])^(3/2),x]
```

output
```
(8*Sqrt[a]*B*ArcTanh[(Sqrt[a]*Sin[x])/Sqrt[a + a*Cos[x]]] + Sqrt[2]*Sqrt[a
]*(A - 5*B)*ArcTanh[(Sqrt[a]*Sin[x])/(Sqrt[2]*Sqrt[a + a*Cos[x]])])/(4*a^2
) + ((A - B)*Sin[x])/(2*(a + a*Cos[x])^(3/2))
```

3.197.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3307
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((d + c*Sin[e +
 f*x])^n/Sin[e + f*x]^n), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3464
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A
*b - a*B)/(b*c - a*d)   Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Simp[(B*c
- A*d)/(b*c - a*d)   Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.197. $\int \frac{A+B\sec(x)}{(a+a\cos(x))^{3/2}}\,dx$

3.197.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(71) = 142$.

Time = 2.57 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.93

method	result
default	$\frac{\sqrt{\sin(\frac{x}{2})^2 a}\left(A\sqrt{2}\,\ln\left(\frac{4\sqrt{a}\,\sqrt{\sin(\frac{x}{2})^2 a}+4a}{\cos(\frac{x}{2})}\right)\cos(\frac{x}{2})^2 a-5B\sqrt{2}\,\ln\left(\frac{4\sqrt{a}\,\sqrt{\sin(\frac{x}{2})^2 a}+4a}{\cos(\frac{x}{2})}\right)a\cos(\frac{x}{2})^2+4B\ln\left(\frac{4a\sqrt{2}\,\cos(\frac{x}{2})+4\sqrt{a}\,\sqrt{2}\,\sqrt{}}{2\cos(\frac{x}{2})+\sqrt{}}\right.}{4a^{\frac{5}{2}}\cos(\frac{x}{2})\sin}$
parts	$\frac{A\sqrt{\sin(\frac{x}{2})^2 a}\left(\sqrt{2}\,\ln\left(\frac{4\sqrt{a}\,\sqrt{\sin(\frac{x}{2})^2 a}+4a}{\cos(\frac{x}{2})}\right)a\cos(\frac{x}{2})^2+\sqrt{a}\,\sqrt{2}\,\sqrt{\sin(\frac{x}{2})^2 a}\right)}{4a^{\frac{5}{2}}\cos(\frac{x}{2})\sin(\frac{x}{2})\sqrt{a\cos(\frac{x}{2})^2}}-\frac{B\sqrt{\sin(\frac{x}{2})^2 a}\left(5\sqrt{2}\,\ln\left(\frac{4\sqrt{a}\,\sqrt{\sin(\frac{x}{2})^2 a}+4a}{\cos(\frac{x}{2})}\right)a\,c\right.}{}$

input
```
int((A+B*sec(x))/(a+a*cos(x))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/a^(5/2)/cos(1/2*x)*(sin(1/2*x)^2*a)^(1/2)*(A*2^(1/2)*ln(2*(2*a^(1/2)*(
sin(1/2*x)^2*a)^(1/2)+2*a)/cos(1/2*x))*cos(1/2*x)^2*a-5*B*2^(1/2)*ln(2*(2*
a^(1/2)*(sin(1/2*x)^2*a)^(1/2)+2*a)/cos(1/2*x))*a*cos(1/2*x)^2+4*B*ln(4/(2
*cos(1/2*x)+2^(1/2))*(a*2^(1/2)*cos(1/2*x)+a^(1/2)*2^(1/2)*(sin(1/2*x)^2*a
)^(1/2)+2*a))*a*cos(1/2*x)^2+4*B*ln(-4/(2*cos(1/2*x)-2^(1/2))*(a*2^(1/2)*c
os(1/2*x)-a^(1/2)*2^(1/2)*(sin(1/2*x)^2*a)^(1/2)-2*a))*a*cos(1/2*x)^2+A*2^
(1/2)*(sin(1/2*x)^2*a)^(1/2)*a^(1/2)-B*2^(1/2)*(sin(1/2*x)^2*a)^(1/2)*a^(1
/2))/sin(1/2*x)/(a*cos(1/2*x)^2)^(1/2)
```

3.197.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(71) = 142$.

Time = 0.26 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.03

$$\int \frac{A+B\sec(x)}{(a+a\cos(x))^{3/2}}\,dx =$$
$$-\frac{\sqrt{2}\left((A-5\,B)\cos(x)^2+2\,(A-5\,B)\cos(x)+A-5\,B\right)\sqrt{a}\log\left(-\frac{a\cos(x)^2+2\sqrt{2}\sqrt{a\cos(x)+a}\sqrt{a}\sin(x)-2\,a\cos(x)-3}{\cos(x)^2+2\cos(x)+1}\right.}{8\,(a^2}$$

input
```
integrate((A+B*sec(x))/(a+a*cos(x))^(3/2),x, algorithm="fricas")
```

output
```
-1/8*(sqrt(2)*((A - 5*B)*cos(x)^2 + 2*(A - 5*B)*cos(x) + A - 5*B)*sqrt(a)*
log(-(a*cos(x)^2 + 2*sqrt(2)*sqrt(a*cos(x) + a)*sqrt(a)*sin(x) - 2*a*cos(x
) - 3*a)/(cos(x)^2 + 2*cos(x) + 1)) - 4*(B*cos(x)^2 + 2*B*cos(x) + B)*sqrt
(a)*log((a*cos(x)^3 - 7*a*cos(x)^2 - 4*sqrt(a*cos(x) + a)*sqrt(a)*(cos(x)
- 2)*sin(x) + 8*a)/(cos(x)^3 + cos(x)^2)) - 4*sqrt(a*cos(x) + a)*(A - B)*s
in(x))/(a^2*cos(x)^2 + 2*a^2*cos(x) + a^2)
```

3.197.6 Sympy [F]

$$\int \frac{A+B\sec(x)}{(a+a\cos(x))^{3/2}}\,dx = \int \frac{A+B\sec{(x)}}{\left(a\left(\cos{(x)}+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*sec(x))/(a+a*cos(x))**(3/2),x)
```

output
```
Integral((A + B*sec(x))/(a*(cos(x) + 1))**(3/2), x)
```

3.197.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9153 vs. 2(71) = 142.

Time = 0.79 (sec) , antiderivative size = 9153, normalized size of antiderivative = 99.49

$$\int \frac{A+B\sec(x)}{(a+a\cos(x))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sec(x))/(a+a*cos(x))^(3/2),x, algorithm="maxima")
```

output
```
1/4*(32*(cos(3/2*x)*sin(2*x) + cos(2*x)*sin(3/2*x) + cos(x)*sin(3/2*x) + c
os(3/2*x)*sin(x))*cos(3*x)^2 + 96*(cos(3/2*x)*sin(3*x) + 3*cos(3/2*x)*sin(
2*x) - (3*cos(x) + 1)*sin(3/2*x) - cos(3*x)*sin(3/2*x) - 3*cos(2*x)*sin(3/
2*x) + 3*cos(3/2*x)*sin(x))*cos(4/3*arctan2(sin(3/2*x), cos(3/2*x)))^2 + 9
6*(cos(3/2*x)*sin(3*x) + 3*cos(3/2*x)*sin(2*x) - (3*cos(x) + 1)*sin(3/2*x)
 - cos(3*x)*sin(3/2*x) - 3*cos(2*x)*sin(3/2*x) + 3*cos(3/2*x)*sin(x))*cos(
2/3*arctan2(sin(3/2*x), cos(3/2*x)))^2 - 32*(cos(3/2*x)*sin(2*x) + cos(2*x
)*sin(3/2*x) + cos(x)*sin(3/2*x) + cos(3/2*x)*sin(x))*sin(3*x)^2 + 32*(6*c
os(x) + 1)*cos(2*x)*sin(3/2*x) + 96*cos(2*x)^2*sin(3/2*x) + 96*sin(2*x)^2*
sin(3/2*x) + 96*(cos(3/2*x)*sin(3*x) + 3*cos(3/2*x)*sin(2*x) - (3*cos(x) +
 1)*sin(3/2*x) - cos(3*x)*sin(3/2*x) - 3*cos(2*x)*sin(3/2*x) + 3*cos(3/2*x
)*sin(x))*sin(4/3*arctan2(sin(3/2*x), cos(3/2*x)))^2 + 96*(cos(3/2*x)*sin(
3*x) + 3*cos(3/2*x)*sin(2*x) - (3*cos(x) + 1)*sin(3/2*x) - cos(3*x)*sin(3/
2*x) - 3*cos(2*x)*sin(3/2*x) + 3*cos(3/2*x)*sin(x))*sin(2/3*arctan2(sin(3/
2*x), cos(3/2*x)))^2 + 32*(2*(3*cos(x) + 1)*cos(2*x)*sin(3/2*x) + 3*cos(2*
x)^2*sin(3/2*x) + 3*sin(2*x)^2*sin(3/2*x) + 2*(3*sin(3/2*x)*sin(x) + cos(3
/2*x))*sin(2*x) + (3*cos(x)^2 + 3*sin(x)^2 + 2*cos(x))*sin(3/2*x) + 2*cos(
3/2*x)*sin(x))*cos(3*x) - 4*(6*(sin(2*x) + sin(x))*sin(3*x)^2 + sin(3*x)^3
 + (2*(3*cos(2*x) + 3*cos(x) + 1)*cos(3*x) + cos(3*x)^2 + 6*(3*cos(x) + 1)
*cos(2*x) + 9*cos(2*x)^2 + 9*cos(x)^2 + 9*sin(2*x)^2 + 18*sin(2*x)*sin(...
```

3.197.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(71) = 142$.

Time = 0.28 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.72

$$\int \frac{A+B\sec(x)}{(a+a\cos(x))^{3/2}}\,dx = \frac{B\log\left(\frac{\left|-2\sqrt{2}-4\sin\left(\frac{1}{2}x\right)\right|}{\left|2\sqrt{2}-4\sin\left(\frac{1}{2}x\right)\right|}\right)}{a^{\frac{3}{2}}\mathrm{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)} + \frac{\sqrt{2}(A\sqrt{a}-5B\sqrt{a})\log\left(\sin\left(\frac{1}{2}x\right)+1\right)}{8a^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)} - \frac{\sqrt{2}(A\sqrt{a}-5B\sqrt{a})\log\left(-\sin\left(\frac{1}{2}x\right)+1\right)}{8a^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)} - \frac{\sqrt{2}\left(A\sqrt{a}\sin\left(\frac{1}{2}x\right)-B\sqrt{a}\sin\left(\frac{1}{2}x\right)\right)}{4\left(\sin\left(\frac{1}{2}x\right)^2-1\right)a^2\mathrm{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)}$$

input
```
integrate((A+B*sec(x))/(a+a*cos(x))^(3/2),x, algorithm="giac")
```

output
```
B*log(abs(-2*sqrt(2) - 4*sin(1/2*x))/abs(2*sqrt(2) - 4*sin(1/2*x)))/(a^(3/
2)*sgn(cos(1/2*x))) + 1/8*sqrt(2)*(A*sqrt(a) - 5*B*sqrt(a))*log(sin(1/2*x)
 + 1)/(a^2*sgn(cos(1/2*x))) - 1/8*sqrt(2)*(A*sqrt(a) - 5*B*sqrt(a))*log(-s
in(1/2*x) + 1)/(a^2*sgn(cos(1/2*x))) - 1/4*sqrt(2)*(A*sqrt(a)*sin(1/2*x) -
 B*sqrt(a)*sin(1/2*x))/((sin(1/2*x)^2 - 1)*a^2*sgn(cos(1/2*x)))
```

3.197.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sec(x)}{(a+a\cos(x))^{3/2}}\,dx = \int \frac{A+\frac{B}{\cos(x)}}{(a+a\,\cos(x))^{3/2}}\,dx$$

input
```
int((A + B/cos(x))/(a + a*cos(x))^(3/2),x)
```

output
```
int((A + B/cos(x))/(a + a*cos(x))^(3/2), x)
```

3.198 $\int \frac{A+B\sec(x)}{(a+a\cos(x))^{5/2}}\,dx$

3.198.1 Optimal result

Integrand size = 17, antiderivative size = 120

$$\int \frac{A+B\sec(x)}{(a+a\cos(x))^{5/2}}\,dx = \frac{2B\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{a+a\cos(x)}}\right)}{a^{5/2}} + \frac{(3A-43B)\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{2}\sqrt{a+a\cos(x)}}\right)}{16\sqrt{2}a^{5/2}} + \frac{(A-B)\sin(x)}{4(a+a\cos(x))^{5/2}} + \frac{(3A-11B)\sin(x)}{16a(a+a\cos(x))^{3/2}}$$

output
```
2*B*arctanh(sin(x)*a^(1/2)/(a+a*cos(x))^(1/2))/a^(5/2)+1/4*(A-B)*sin(x)/(a
+a*cos(x))^(5/2)+1/16*(3*A-11*B)*sin(x)/a/(a+a*cos(x))^(3/2)+1/32*(3*A-43*
B)*arctanh(1/2*sin(x)*a^(1/2)*2^(1/2)/(a+a*cos(x))^(1/2))/a^(5/2)*2^(1/2)
```

3.198.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.79

$$\int \frac{A+B\sec(x)}{(a+a\cos(x))^{5/2}}\,dx = \frac{2(3A-43B)\operatorname{arctanh}\left(\sin\left(\frac{x}{2}\right)\right)\cos^3\left(\frac{x}{2}\right) + 64\sqrt{2}B\operatorname{arctanh}\left(\sqrt{2}\sin\left(\frac{x}{2}\right)\right)\cos^3\left(\frac{x}{2}\right) + (}{16a(a(1+\cos(x)))^{3/2}}$$

input
```
Integrate[(A + B*Sec[x])/(a + a*Cos[x])^(5/2),x]
```

output
```
(2*(3*A - 43*B)*ArcTanh[Sin[x/2]]*Cos[x/2]^3 + 64*Sqrt[2]*B*ArcTanh[Sqrt[2
]*Sin[x/2]]*Cos[x/2]^3 + (7*A - 15*B + 3*A*Cos[x] - 11*B*Cos[x])*Tan[x/2])
/(16*a*(a*(1 + Cos[x]))^(3/2))
```

3.198.3 Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.09, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.882, Rules used = {3042, 3307, 3042, 3457, 27, 3042, 3457, 27, 3042, 3464, 3042, 3128, 219, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sec(x)}{(a\cos(x)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\csc\left(x+\frac{\pi}{2}\right)}{\left(a\sin\left(x+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{3307}$$

$$\int \frac{\sec(x)(A\cos(x)+B)}{(a\cos(x)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A\sin\left(x+\frac{\pi}{2}\right)+B}{\sin\left(x+\frac{\pi}{2}\right)\left(a\sin\left(x+\frac{\pi}{2}\right)+a\right)^{5/2}}dx$$

$$\Big\downarrow \text{3457}$$

$$\frac{\int \frac{(8aB+3a(A-B)\cos(x))\sec(x)}{2(\cos(x)a+a)^{3/2}}dx}{4a^2}+\frac{(A-B)\sin(x)}{4(a\cos(x)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(8aB+3a(A-B)\cos(x))\sec(x)}{(\cos(x)a+a)^{3/2}}dx}{8a^2}+\frac{(A-B)\sin(x)}{4(a\cos(x)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{8aB+3a(A-B)\sin\left(x+\frac{\pi}{2}\right)}{\sin\left(x+\frac{\pi}{2}\right)\left(\sin\left(x+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a^2}+\frac{(A-B)\sin(x)}{4(a\cos(x)+a)^{5/2}}$$

$$\Big\downarrow \text{3457}$$

$$\frac{\frac{\int \frac{\left(32Ba^2+(3A-11B)\cos(x)a^2\right)\sec(x)}{2\sqrt{\cos(x)a+a}}dx}{2a^2}+\frac{a(3A-11B)\sin(x)}{2(a\cos(x)+a)^{3/2}}}{8a^2}+\frac{(A-B)\sin(x)}{4(a\cos(x)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\left(32Ba^2+(3A-11B)\cos(x)a^2\right)\sec(x)}{\sqrt{\cos(x)a+a}}dx}{4a^2}+\frac{a(3A-11B)\sin(x)}{2(a\cos(x)+a)^{3/2}}}{8a^2}+\frac{(A-B)\sin(x)}{4(a\cos(x)+a)^{5/2}}$$

$\big\downarrow$ 3042

$$\frac{\frac{\int \frac{32Ba^2+(3A-11B)\sin\left(x+\frac{\pi}{2}\right)a^2}{\sin\left(x+\frac{\pi}{2}\right)\sqrt{\sin\left(x+\frac{\pi}{2}\right)a+a}}dx}{4a^2}+\frac{a(3A-11B)\sin(x)}{2(a\cos(x)+a)^{3/2}}}{8a^2}+\frac{(A-B)\sin(x)}{4(a\cos(x)+a)^{5/2}}$$

$\big\downarrow$ 3464

$$\frac{\frac{a^2(3A-43B)\int \frac{1}{\sqrt{\cos(x)a+a}}dx+32aB\int \sqrt{\cos(x)a+a}\sec(x)dx}{4a^2}+\frac{a(3A-11B)\sin(x)}{2(a\cos(x)+a)^{3/2}}}{8a^2}+\frac{(A-B)\sin(x)}{4(a\cos(x)+a)^{5/2}}$$

$\big\downarrow$ 3042

$$\frac{\frac{a^2(3A-43B)\int \frac{1}{\sqrt{\sin\left(x+\frac{\pi}{2}\right)a+a}}dx+32aB\int \frac{\sqrt{\sin\left(x+\frac{\pi}{2}\right)a+a}}{\sin\left(x+\frac{\pi}{2}\right)}dx}{4a^2}+\frac{a(3A-11B)\sin(x)}{2(a\cos(x)+a)^{3/2}}}{8a^2}+\frac{(A-B)\sin(x)}{4(a\cos(x)+a)^{5/2}}$$

$\big\downarrow$ 3128

$$\frac{\frac{32aB\int \frac{\sqrt{\sin\left(x+\frac{\pi}{2}\right)a+a}}{\sin\left(x+\frac{\pi}{2}\right)}dx-2a^2(3A-43B)\int \frac{1}{2a-\frac{a^2\sin^2(x)}{\cos(x)a+a}}d\left(-\frac{a\sin(x)}{\sqrt{\cos(x)a+a}}\right)}{4a^2}+\frac{a(3A-11B)\sin(x)}{2(a\cos(x)+a)^{3/2}}}{8a^2}+\frac{(A-B)\sin(x)}{4(a\cos(x)+a)^{5/2}}$$

$\big\downarrow$ 219

$$\frac{\frac{32aB\int \frac{\sqrt{\sin\left(x+\frac{\pi}{2}\right)a+a}}{\sin\left(x+\frac{\pi}{2}\right)}dx+\sqrt{2}a^{3/2}(3A-43B)\text{arctanh}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{2}\sqrt{a\cos(x)+a}}\right)}{4a^2}+\frac{a(3A-11B)\sin(x)}{2(a\cos(x)+a)^{3/2}}}{8a^2}+\frac{(A-B)\sin(x)}{4(a\cos(x)+a)^{5/2}}$$

$\big\downarrow$ 3252

$$\frac{\frac{\sqrt{2}a^{3/2}(3A-43B)\text{arctanh}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{2}\sqrt{a\cos(x)+a}}\right)-64a^2B\int \frac{1}{a-\frac{a^2\sin^2(x)}{\cos(x)a+a}}d\left(-\frac{a\sin(x)}{\sqrt{\cos(x)a+a}}\right)}{4a^2}+\frac{a(3A-11B)\sin(x)}{2(a\cos(x)+a)^{3/2}}}{8a^2}+\frac{(A-B)\sin(x)}{4(a\cos(x)+a)^{5/2}}$$

$\big\downarrow$ 219

3.198. $\int \frac{A+B\sec(x)}{(a+a\cos(x))^{5/2}}\,dx$

$$\frac{\frac{\sqrt{2}a^{3/2}(3A-43B)\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{2}\sqrt{a\cos(x)+a}}\right)+64a^{3/2}B\operatorname{arctanh}\left(\frac{\sqrt{a}\sin(x)}{\sqrt{a\cos(x)+a}}\right)}{4a^2}+\frac{a(3A-11B)\sin(x)}{2(a\cos(x)+a)^{3/2}}}{8a^2}+\frac{(A-B)\sin(x)}{4(a\cos(x)+a)^{5/2}}$$

input
```
Int[(A + B*Sec[x])/(a + a*Cos[x])^(5/2),x]
```

output
```
((A - B)*Sin[x])/(4*(a + a*Cos[x])^(5/2)) + ((64*a^(3/2)*B*ArcTanh[(Sqrt[a
]*Sin[x])/Sqrt[a + a*Cos[x]]] + Sqrt[2]*a^(3/2)*(3*A - 43*B)*ArcTanh[(Sqrt
[a]*Sin[x])/(Sqrt[2]*Sqrt[a + a*Cos[x]])])/(4*a^2) + (a*(3*A - 11*B)*Sin[x
])/(2*(a + a*Cos[x])^(3/2)))/(8*a^2)
```

3.198.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NegQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.198. $\int \frac{A+B\sec(x)}{(a+a\cos(x))^{5/2}}\,dx$

rule 3307
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((d + c*Sin[e +
 f*x])^n/Sin[e + f*x]^n), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3464
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A
*b - a*B)/(b*c - a*d)   Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Simp[(B*c
- A*d)/(b*c - a*d)   Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.198.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(95) = 190$.

Time = 2.79 (sec) , antiderivative size = 322, normalized size of antiderivative = 2.68

method	result
default	$\frac{\sqrt{\sin(\frac{x}{2})^2 a}\left(3A\sqrt{2}\ln\left(\frac{4\sqrt{a}\sqrt{\sin(\frac{x}{2})^2 a}+4a}{\cos(\frac{x}{2})}\right)\cos(\frac{x}{2})^4 a-43B\sqrt{2}\ln\left(\frac{4\sqrt{a}\sqrt{\sin(\frac{x}{2})^2 a}+4a}{\cos(\frac{x}{2})}\right)a\cos(\frac{x}{2})^4+32B\ln\left(-\frac{4\left(a\sqrt{2}\cos(\frac{x}{2})-\sqrt{\cdots}\right.}{2\cos(\cdots}\right.\right.}{\cdots}$
parts	$\frac{A\sqrt{\sin(\frac{x}{2})^2 a}\left(3\sqrt{2}\ln\left(\frac{4\sqrt{a}\sqrt{\sin(\frac{x}{2})^2 a}+4a}{\cos(\frac{x}{2})}\right)a\cos(\frac{x}{2})^4+3\sqrt{a}\sqrt{2}\sqrt{\sin(\frac{x}{2})^2 a}\cos(\frac{x}{2})^2+2\sqrt{a}\sqrt{2}\sqrt{\sin(\frac{x}{2})^2 a}\right)}{32a^{\frac{7}{2}}\cos(\frac{x}{2})^3\sin(\frac{x}{2})\sqrt{a\cos(\frac{x}{2})^2}}-\frac{B\sqrt{\sin(\frac{x}{2})^2 a}\left(\cdots\right.}{\cdots}$

input
```
int((A+B*sec(x))/(a+a*cos(x))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/32/a^(7/2)/cos(1/2*x)^3*(sin(1/2*x)^2*a)^(1/2)*(3*A*2^(1/2)*ln(2*(2*a^(1
/2)*(sin(1/2*x)^2*a)^(1/2)+2*a)/cos(1/2*x))*cos(1/2*x)^4*a-43*B*2^(1/2)*ln
(2*(2*a^(1/2)*(sin(1/2*x)^2*a)^(1/2)+2*a)/cos(1/2*x))*a*cos(1/2*x)^4+32*B*
ln(-4/(2*cos(1/2*x)-2^(1/2))*(a*2^(1/2)*cos(1/2*x)-a^(1/2)*2^(1/2)*(sin(1/
2*x)^2*a)^(1/2)-2*a))*a*cos(1/2*x)^4+32*B*ln(4/(2*cos(1/2*x)+2^(1/2))*(a*2
^(1/2)*cos(1/2*x)+a^(1/2)*2^(1/2)*(sin(1/2*x)^2*a)^(1/2)+2*a))*a*cos(1/2*x
)^4+3*A*2^(1/2)*a^(1/2)*(sin(1/2*x)^2*a)^(1/2)*cos(1/2*x)^2-11*B*a^(1/2)*2
^(1/2)*(sin(1/2*x)^2*a)^(1/2)*cos(1/2*x)^2+2*A*2^(1/2)*(sin(1/2*x)^2*a)^(1
/2)*a^(1/2)-2*B*2^(1/2)*(sin(1/2*x)^2*a)^(1/2)*a^(1/2))/sin(1/2*x)/(a*cos(
1/2*x)^2)^(1/2)
```

3.198.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(95) = 190$.

Time = 0.26 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.95

$$\int \frac{A+B\sec(x)}{(a+a\cos(x))^{5/2}}\,dx =$$
$$-\frac{\sqrt{2}\left((3\,A-43\,B)\cos\left(x\right)^{3}+3\,(3\,A-43\,B)\cos\left(x\right)^{2}+3\,(3\,A-43\,B)\cos\left(x\right)+3\,A-43\,B\right)\sqrt{a}\log\left(-\frac{a\cos(x}{}\right.}{}$$

input
```
integrate((A+B*sec(x))/(a+a*cos(x))^(5/2),x, algorithm="fricas")
```

output
```
-1/64*(sqrt(2)*((3*A - 43*B)*cos(x)^3 + 3*(3*A - 43*B)*cos(x)^2 + 3*(3*A -
 43*B)*cos(x) + 3*A - 43*B)*sqrt(a)*log(-(a*cos(x)^2 + 2*sqrt(2)*sqrt(a*co
s(x) + a)*sqrt(a)*sin(x) - 2*a*cos(x) - 3*a)/(cos(x)^2 + 2*cos(x) + 1)) -
32*(B*cos(x)^3 + 3*B*cos(x)^2 + 3*B*cos(x) + B)*sqrt(a)*log((a*cos(x)^3 -
7*a*cos(x)^2 - 4*sqrt(a*cos(x) + a)*sqrt(a)*(cos(x) - 2)*sin(x) + 8*a)/(co
s(x)^3 + cos(x)^2)) - 4*((3*A - 11*B)*cos(x) + 7*A - 15*B)*sqrt(a*cos(x) +
 a)*sin(x))/(a^3*cos(x)^3 + 3*a^3*cos(x)^2 + 3*a^3*cos(x) + a^3)
```

3.198.6 Sympy [F]

$$\int \frac{A+B\sec(x)}{(a+a\cos(x))^{5/2}}\,dx = \int \frac{A+B\sec{(x)}}{(a(\cos{(x)}+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*sec(x))/(a+a*cos(x))**(5/2),x)
```

output
```
Integral((A + B*sec(x))/(a*(cos(x) + 1))**(5/2), x)
```

3.198.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50224 vs. 2(95) = 190.

Time = 14.43 (sec) , antiderivative size = 50224, normalized size of antiderivative = 418.53

$$\int \frac{A+B\sec(x)}{(a+a\cos(x))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sec(x))/(a+a*cos(x))^(5/2),x, algorithm="maxima")
```

output
```
1/32*(512*((2*sin(2*x) + sin(x))*cos(5/2*x) + cos(5/2*x)*sin(4*x) + 2*cos(
5/2*x)*sin(3*x) + (2*cos(2*x) + cos(x))*sin(5/2*x) + cos(4*x)*sin(5/2*x) +
 2*cos(3*x)*sin(5/2*x))*cos(5*x)^2 + 2560*(5*(2*sin(2*x) + sin(x))*cos(5/2
*x) + cos(5/2*x)*sin(5*x) + 5*cos(5/2*x)*sin(4*x) + 10*cos(5/2*x)*sin(3*x)
 - (10*cos(2*x) + 5*cos(x) + 1)*sin(5/2*x) - cos(5*x)*sin(5/2*x) - 5*cos(4
*x)*sin(5/2*x) - 10*cos(3*x)*sin(5/2*x))*cos(8/5*arctan2(sin(5/2*x), cos(5
/2*x)))^2 + 10240*(5*(2*sin(2*x) + sin(x))*cos(5/2*x) + cos(5/2*x)*sin(5*x
) + 5*cos(5/2*x)*sin(4*x) + 10*cos(5/2*x)*sin(3*x) - (10*cos(2*x) + 5*cos(
x) + 1)*sin(5/2*x) - cos(5*x)*sin(5/2*x) - 5*cos(4*x)*sin(5/2*x) - 10*cos(
3*x)*sin(5/2*x))*cos(6/5*arctan2(sin(5/2*x), cos(5/2*x)))^2 + 10240*(5*(2*
sin(2*x) + sin(x))*cos(5/2*x) + cos(5/2*x)*sin(5*x) + 5*cos(5/2*x)*sin(4*x
) + 10*cos(5/2*x)*sin(3*x) - (10*cos(2*x) + 5*cos(x) + 1)*sin(5/2*x) - cos
(5*x)*sin(5/2*x) - 5*cos(4*x)*sin(5/2*x) - 10*cos(3*x)*sin(5/2*x))*cos(4/5
*arctan2(sin(5/2*x), cos(5/2*x)))^2 + 2560*(5*(2*sin(2*x) + sin(x))*cos(5/
2*x) + cos(5/2*x)*sin(5*x) + 5*cos(5/2*x)*sin(4*x) + 10*cos(5/2*x)*sin(3*x
) - (10*cos(2*x) + 5*cos(x) + 1)*sin(5/2*x) - cos(5*x)*sin(5/2*x) - 5*cos(
4*x)*sin(5/2*x) - 10*cos(3*x)*sin(5/2*x))*cos(2/5*arctan2(sin(5/2*x), cos(
5/2*x)))^2 - 512*((2*sin(2*x) + sin(x))*cos(5/2*x) + cos(5/2*x)*sin(4*x) +
 2*cos(5/2*x)*sin(3*x) + (2*cos(2*x) + cos(x))*sin(5/2*x) + cos(4*x)*sin(5
/2*x) + 2*cos(3*x)*sin(5/2*x))*sin(5*x)^2 + 2560*cos(4*x)^2*sin(5/2*x) ...
```

3.198.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.52

$$\int \frac{A+B\sec(x)}{(a+a\cos(x))^{5/2}}\,dx = -\frac{B\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(\frac{1}{2}x\right)\right|}{\left|2\sqrt{2}+4\sin\left(\frac{1}{2}x\right)\right|}\right)}{a^{\frac{5}{2}}\mathrm{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)} + \frac{\sqrt{2}(3A\sqrt{a}-43B\sqrt{a})\log\left(\sin\left(\frac{1}{2}x\right)+1\right)}{64a^3\mathrm{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)} - \frac{\sqrt{2}(3A\sqrt{a}-43B\sqrt{a})\log\left(-\sin\left(\frac{1}{2}x\right)+1\right)}{64a^3\mathrm{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)} - \frac{3\sqrt{2}A\sin\left(\frac{1}{2}x\right)^3-11\sqrt{2}B\sin\left(\frac{1}{2}x\right)^3-5\sqrt{2}A\sin\left(\frac{1}{2}x\right)+13\sqrt{2}B\sin\left(\frac{1}{2}x\right)}{32\left(\sin\left(\frac{1}{2}x\right)^2-1\right)^2a^{\frac{5}{2}}\mathrm{sgn}\left(\cos\left(\frac{1}{2}x\right)\right)}$$

input
```
integrate((A+B*sec(x))/(a+a*cos(x))^(5/2),x, algorithm="giac")
```

output
```
-B*log(abs(-2*sqrt(2) + 4*sin(1/2*x))/abs(2*sqrt(2) + 4*sin(1/2*x)))/(a^(5
/2)*sgn(cos(1/2*x))) + 1/64*sqrt(2)*(3*A*sqrt(a) - 43*B*sqrt(a))*log(sin(1
/2*x) + 1)/(a^3*sgn(cos(1/2*x))) - 1/64*sqrt(2)*(3*A*sqrt(a) - 43*B*sqrt(a
))*log(-sin(1/2*x) + 1)/(a^3*sgn(cos(1/2*x))) - 1/32*(3*sqrt(2)*A*sin(1/2*
x)^3 - 11*sqrt(2)*B*sin(1/2*x)^3 - 5*sqrt(2)*A*sin(1/2*x) + 13*sqrt(2)*B*s
in(1/2*x))/((sin(1/2*x)^2 - 1)^2*a^(5/2)*sgn(cos(1/2*x)))
```

3.198.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\sec(x)}{(a+a\cos(x))^{5/2}}\,dx = \int \frac{A+\frac{B}{\cos(x)}}{(a+a\cos(x))^{5/2}}\,dx$$

input
```
int((A + B/cos(x))/(a + a*cos(x))^(5/2),x)
```

output
```
int((A + B/cos(x))/(a + a*cos(x))^(5/2), x)
```

3.199 $\int \frac{x(b+a\sin(x))}{(a+b\sin(x))^2}\,dx$

3.199.1 Optimal result

Integrand size = 16, antiderivative size = 25

$$\int \frac{x(b+a\sin(x))}{(a+b\sin(x))^2}\,dx=\frac{\log(a+b\sin(x))}{b}-\frac{x\cos(x)}{a+b\sin(x)}$$

output
```
ln(a+b*sin(x))/b-x*cos(x)/(a+b*sin(x))
```

3.199.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{x(b+a\sin(x))}{(a+b\sin(x))^2}\,dx=\frac{\log(a+b\sin(x))}{b}-\frac{x\cos(x)}{a+b\sin(x)}$$

input
```
Integrate[(x*(b + a*Sin[x]))/(a + b*Sin[x])^2,x]
```

output
```
Log[a + b*Sin[x]]/b - (x*Cos[x])/(a + b*Sin[x])
```

3.199.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5103, 3042, 3147, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a\sin(x)+b)}{(a+b\sin(x))^2}dx$$

$$\Big\downarrow \text{ 5103}$$

$$\int \frac{\cos(x)}{a+b\sin(x)}dx-\frac{x\cos(x)}{a+b\sin(x)}$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(x)}{a+b\sin(x)}dx-\frac{x\cos(x)}{a+b\sin(x)}$$

$$\Big\downarrow \text{ 3147}$$

$$\frac{\int \frac{1}{a+b\sin(x)}d(b\sin(x))}{b}-\frac{x\cos(x)}{a+b\sin(x)}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{\log(a+b\sin(x))}{b}-\frac{x\cos(x)}{a+b\sin(x)}$$

input
```
Int[(x*(b + a*Sin[x]))/(a + b*Sin[x])^2,x]
```

output
```
Log[a + b*Sin[x]]/b - (x*Cos[x])/(a + b*Sin[x])
```

3.199.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 5103
```
Int[(((e_.) + (f_.)*(x_))*((A_) + (B_.)*Sin[(c_.) + (d_.)*(x_)]))/((a_) + (
b_.)*Sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(-B)*(e + f*x)*(Cos[c +
d*x]/(a*d*(a + b*Sin[c + d*x]))), x] + Simp[B*(f/(a*d))   Int[Cos[c + d*x]/
(a + b*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && EqQ[a
*A - b*B, 0]
```

3.199.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(25) = 50$.

Time = 0.71 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.32

method	result	size
parallelrisch	$\frac{(a+b\sin(x))\ln\left(\frac{a+b\sin(x)}{\cos(x)+1}\right)+(-b\sin(x)-a)\ln\left(\frac{1}{\cos(x)+1}\right)-bx\cos(x)}{b(a+b\sin(x))}$	58
risch	$-\frac{2ix}{b}-\frac{2x(ib+a\,e^{ix})}{b(b\,e^{2ix}-b+2ia\,e^{ix})}+\frac{\ln\left(e^{2ix}+\frac{2ia\,e^{ix}}{b}-1\right)}{b}$	73
norman	$\frac{x\tan\left(\frac{x}{2}\right)^4-x}{\left(1+\tan\left(\frac{x}{2}\right)^2\right)\left(\tan\left(\frac{x}{2}\right)^2a+2b\tan\left(\frac{x}{2}\right)+a\right)}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)^2a+2b\tan\left(\frac{x}{2}\right)+a\right)}{b}-\frac{\ln\left(1+\tan\left(\frac{x}{2}\right)^2\right)}{b}$	80

input
```
int(x*(a*sin(x)+b)/(a+b*sin(x))^2,x,method=_RETURNVERBOSE)
```

output
```
((a+b*sin(x))*ln((a+b*sin(x))/(cos(x)+1))+(-b*sin(x)-a)*ln(1/(cos(x)+1))-b
*x*cos(x))/b/(a+b*sin(x))
```

3.199.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.40

$$\int \frac{x(b+a\sin(x))}{(a+b\sin(x))^2}\,dx = -\frac{bx\cos(x) - (b\sin(x)+a)\log(b\sin(x)+a)}{b^2\sin(x)+ab}$$

input
```
integrate(x*(b+a*sin(x))/(a+b*sin(x))^2,x, algorithm="fricas")
```

output
```
-(b*x*cos(x) - (b*sin(x) + a)*log(b*sin(x) + a))/(b^2*sin(x) + a*b)
```

3.199.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x(b+a\sin(x))}{(a+b\sin(x))^2}\,dx = \text{Timed out}$$

input
```
integrate(x*(b+a*sin(x))/(a+b*sin(x))**2,x)
```

output
```
Timed out
```

3.199.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x(b+a\sin(x))}{(a+b\sin(x))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(b+a*sin(x))/(a+b*sin(x))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.199.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. 2(25) = 50.

Time = 0.37 (sec) , antiderivative size = 283, normalized size of antiderivative = 11.32

$$\int \frac{x(b+a\sin(x))}{(a+b\sin(x))^2}\,dx = \frac{4\,bx\tan\left(\frac{1}{2}\,x\right)^2 + a\log\left(\frac{4\left(a^2\tan\left(\frac{1}{2}\,x\right)^4+4\,ab\tan\left(\frac{1}{2}\,x\right)^3+2\,a^2\tan\left(\frac{1}{2}\,x\right)^2+4\,b^2\tan\left(\frac{1}{2}\,x\right)^2+4\,ab\tan\left(\frac{1}{2}\,x\right)+a^2\right)}{\tan\left(\frac{1}{2}\,x\right)^4+2\,\tan\left(\frac{1}{2}\,x\right)^2+1}\right)\tan\left(\frac{1}{2}\,x\right)^2 + 2\,b\,\mathrm{l}}{}$$

input
```
integrate(x*(b+a*sin(x))/(a+b*sin(x))^2,x, algorithm="giac")
```

output
```
1/2*(4*b*x*tan(1/2*x)^2 + a*log(4*(a^2*tan(1/2*x)^4 + 4*a*b*tan(1/2*x)^3 +
 2*a^2*tan(1/2*x)^2 + 4*b^2*tan(1/2*x)^2 + 4*a*b*tan(1/2*x) + a^2)/(tan(1/
2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x)^2 + 2*b*log(4*(a^2*tan(1/2*x)^4 +
 4*a*b*tan(1/2*x)^3 + 2*a^2*tan(1/2*x)^2 + 4*b^2*tan(1/2*x)^2 + 4*a*b*tan(
1/2*x) + a^2)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x) - 4*b*x + a*
log(4*(a^2*tan(1/2*x)^4 + 4*a*b*tan(1/2*x)^3 + 2*a^2*tan(1/2*x)^2 + 4*b^2*
tan(1/2*x)^2 + 4*a*b*tan(1/2*x) + a^2)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1)
))/(a*b*tan(1/2*x)^2 + 2*b^2*tan(1/2*x) + a*b)
```

3.199.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(b+a\sin(x))}{(a+b\sin(x))^2}\,dx = \int \frac{x\,(b+a\,\sin(x))}{(a+b\,\sin(x))^2}\,dx$$

input
```
int((x*(b + a*sin(x)))/(a + b*sin(x))^2,x)
```

output
```
int((x*(b + a*sin(x)))/(a + b*sin(x))^2, x)
```

3.200 $\int \frac{x(b+a\cos(x))}{(a+b\cos(x))^2}\,dx$

3.200.1 Optimal result

Integrand size = 16, antiderivative size = 24

$$\int \frac{x(b+a\cos(x))}{(a+b\cos(x))^2}\,dx = \frac{\log(a+b\cos(x))}{b}+\frac{x\sin(x)}{a+b\cos(x)}$$

output
```
ln(a+b*cos(x))/b+x*sin(x)/(a+b*cos(x))
```

3.200.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x(b+a\cos(x))}{(a+b\cos(x))^2}\,dx = \frac{\log(a+b\cos(x))}{b}+\frac{x\sin(x)}{a+b\cos(x)}$$

input
```
Integrate[(x*(b + a*Cos[x]))/(a + b*Cos[x])^2,x]
```

output
```
Log[a + b*Cos[x]]/b + (x*Sin[x])/(a + b*Cos[x])
```

3.200.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5104, 3042, 3147, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a\cos(x)+b)}{(a+b\cos(x))^2}dx$$

$$\Big\downarrow \; 5104$$

$$\frac{x\sin(x)}{a+b\cos(x)}-\int \frac{\sin(x)}{a+b\cos(x)}dx$$

$$\Big\downarrow \; 3042$$

$$\frac{x\sin(x)}{a+b\cos(x)}-\int \frac{\cos\left(x-\frac{\pi}{2}\right)}{a-b\sin\left(x-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 3147$$

$$\frac{\int \frac{1}{a+b\cos(x)}d(b\cos(x))}{b}+\frac{x\sin(x)}{a+b\cos(x)}$$

$$\Big\downarrow \; 16$$

$$\frac{\log(a+b\cos(x))}{b}+\frac{x\sin(x)}{a+b\cos(x)}$$

input
```
Int[(x*(b + a*Cos[x]))/(a + b*Cos[x])^2,x]
```

output
```
Log[a + b*Cos[x]]/b + (x*Sin[x])/(a + b*Cos[x])
```

3.200.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 5104
```
Int[((Cos[(c_.) + (d_.)*(x_)]*(B_.) + (A_))*((e_.) + (f_.)*(x_)))/(Cos[(c_.
) + (d_.)*(x_)]*(b_.) + (a_))^2, x_Symbol] :> Simp[B*(e + f*x)*(Sin[c + d*x
]/(a*d*(a + b*Cos[c + d*x]))), x] - Simp[B*(f/(a*d))   Int[Sin[c + d*x]/(a
+ b*Cos[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && EqQ[a*A
- b*B, 0]
```

3.200.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(24) = 48$.

Time = 0.62 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.38

method	result	size
parallelrisch	$\frac{(a+b\cos(x))\ln\left(\frac{a+b\cos(x)}{\cos(x)+1}\right)+(-a-b\cos(x))\ln\left(\frac{1}{\cos(x)+1}\right)+bx\sin(x)}{b(a+b\cos(x))}$	57
risch	$-\frac{2ix}{b}+\frac{2ix(a\,e^{ix}+b)}{b(b\,e^{2ix}+2a\,e^{ix}+b)}+\frac{\ln\left(e^{2ix}+1+\frac{2a\,e^{ix}}{b}\right)}{b}$	67
norman	$\frac{2x\tan\left(\frac{x}{2}\right)+2x\tan\left(\frac{x}{2}\right)^{3}}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)\left(\tan\left(\frac{x}{2}\right)^{2}a-\tan\left(\frac{x}{2}\right)^{2}b+a+b\right)}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)^{2}a-\tan\left(\frac{x}{2}\right)^{2}b+a+b\right)}{b}-\frac{\ln\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)}{b}$	91

input
```
int(x*(b+a*cos(x))/(a+b*cos(x))^2,x,method=_RETURNVERBOSE)
```

output
```
((a+b*cos(x))*ln((a+b*cos(x))/(cos(x)+1))+(-a-b*cos(x))*ln(1/(cos(x)+1))+b
*x*sin(x))/b/(a+b*cos(x))
```

3.200.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.50

$$\int \frac{x(b+a\cos(x))}{(a+b\cos(x))^2}\,dx = \frac{bx\sin{(x)} + (b\cos{(x)} + a)\log{(-b\cos{(x)} - a)}}{b^2\cos{(x)} + ab}$$

input
```
integrate(x*(b+a*cos(x))/(a+b*cos(x))^2,x, algorithm="fricas")
```

output
```
(b*x*sin(x) + (b*cos(x) + a)*log(-b*cos(x) - a))/(b^2*cos(x) + a*b)
```

3.200.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x(b+a\cos(x))}{(a+b\cos(x))^2}\,dx = \text{Timed out}$$

input
```
integrate(x*(b+a*cos(x))/(a+b*cos(x))**2,x)
```

output
```
Timed out
```

3.200.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x(b+a\cos(x))}{(a+b\cos(x))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(b+a*cos(x))/(a+b*cos(x))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.200.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 397 vs. $2(24) = 48$.

Time = 0.36 (sec) , antiderivative size = 397, normalized size of antiderivative = 16.54

$$\int \frac{x(b+a\cos(x))}{(a+b\cos(x))^2}\,dx = \frac{a\log\left(\frac{4\left(a^2\tan\left(\frac{1}{2}x\right)^4-2\,ab\tan\left(\frac{1}{2}x\right)^4+b^2\tan\left(\frac{1}{2}x\right)^4+2\,a^2\tan\left(\frac{1}{2}x\right)^2-2\,b^2\tan\left(\frac{1}{2}x\right)^2+a^2+2\,ab+b^2\right)}{\tan\left(\frac{1}{2}x\right)^4+2\tan\left(\frac{1}{2}x\right)^2+1}\right)\tan\left(\frac{1}{2}x\right)^2-b\log\left(\frac{4\left(a^2\tan(\right.}{}\right.}{}$$

input
```
integrate(x*(b+a*cos(x))/(a+b*cos(x))^2,x, algorithm="giac")
```

output
```
1/2*(a*log(4*(a^2*tan(1/2*x)^4 - 2*a*b*tan(1/2*x)^4 + b^2*tan(1/2*x)^4 + 2
*a^2*tan(1/2*x)^2 - 2*b^2*tan(1/2*x)^2 + a^2 + 2*a*b + b^2)/(tan(1/2*x)^4
+ 2*tan(1/2*x)^2 + 1))*tan(1/2*x)^2 - b*log(4*(a^2*tan(1/2*x)^4 - 2*a*b*ta
n(1/2*x)^4 + b^2*tan(1/2*x)^4 + 2*a^2*tan(1/2*x)^2 - 2*b^2*tan(1/2*x)^2 +
a^2 + 2*a*b + b^2)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x)^2 + 8*b
*x*tan(1/2*x) + a*log(4*(a^2*tan(1/2*x)^4 - 2*a*b*tan(1/2*x)^4 + b^2*tan(1
/2*x)^4 + 2*a^2*tan(1/2*x)^2 - 2*b^2*tan(1/2*x)^2 + a^2 + 2*a*b + b^2)/(ta
n(1/2*x)^4 + 2*tan(1/2*x)^2 + 1)) + b*log(4*(a^2*tan(1/2*x)^4 - 2*a*b*tan(
1/2*x)^4 + b^2*tan(1/2*x)^4 + 2*a^2*tan(1/2*x)^2 - 2*b^2*tan(1/2*x)^2 + a^
2 + 2*a*b + b^2)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1)))/(a*b*tan(1/2*x)^2 -
 b^2*tan(1/2*x)^2 + a*b + b^2)
```

3.200.9 Mupad [B] (verification not implemented)

Time = 26.22 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.83

$$\int \frac{x(b+a\cos(x))}{(a+b\cos(x))^2}\,dx = \frac{\ln\left(b+2\,a\,\mathrm{e}^{x\,1\mathrm{i}}+b\,\mathrm{e}^{x\,2\mathrm{i}}\right)}{b}-\frac{x\,2\mathrm{i}}{b}+\frac{x\,2\mathrm{i}+\frac{a\,x\,\mathrm{e}^{x\,1\mathrm{i}}\,2\mathrm{i}}{b}}{b+2\,a\,\mathrm{e}^{x\,1\mathrm{i}}+b\,\mathrm{e}^{x\,2\mathrm{i}}}$$

input
```
int((x*(b + a*cos(x)))/(a + b*cos(x))^2,x)
```

output
```
log(b + 2*a*exp(x*1i) + b*exp(x*2i))/b - (x*2i)/b + (x*2i + (a*x*exp(x*1i)
*2i)/b)/(b + 2*a*exp(x*1i) + b*exp(x*2i))
```

3.201 $\int \frac{1+\sin^2(x)}{1-\sin^2(x)} \, dx$

3.201.1 Optimal result

Integrand size = 17, antiderivative size = 8

$$\int \frac{1+\sin^2(x)}{1-\sin^2(x)} \, dx = -x+2\tan(x)$$

output
```
-x+2*tan(x)
```

3.201.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{1+\sin^2(x)}{1-\sin^2(x)} \, dx = -\arctan(\tan(x))+2\tan(x)$$

input
```
Integrate[(1 + Sin[x]^2)/(1 - Sin[x]^2),x]
```

output
```
-ArcTan[Tan[x]] + 2*Tan[x]
```

3.201.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {3042, 3650, 3042, 3654, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(x)+1}{1-\sin^2(x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin(x)^2+1}{1-\sin(x)^2}dx$$
$$\Big\downarrow \text{3650}$$
$$2\int \frac{1}{1-\sin^2(x)}dx-x$$
$$\Big\downarrow \text{3042}$$
$$2\int \frac{1}{1-\sin(x)^2}dx-x$$
$$\Big\downarrow \text{3654}$$
$$2\int \sec^2(x)dx-x$$
$$\Big\downarrow \text{3042}$$
$$2\int \csc\left(x+\frac{\pi}{2}\right)^2dx-x$$
$$\Big\downarrow \text{4254}$$
$$-2\int 1d(-\tan(x))-x$$
$$\Big\downarrow \text{24}$$
$$2\tan(x)-x$$

input
```
Int[(1 + Sin[x]^2)/(1 - Sin[x]^2),x]
```

output
```
-x + 2*Tan[x]
```

3.201.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3650
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[B*(x/b), x] + Simp[(A*b - a*B)/b   Int[1/(a
+ b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.201.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

method	result	size
parallelrisch	$-x+2\tan\left(x\right)$	9
default	$2\tan\left(x\right)-\arctan\left(\tan\left(x\right)\right)$	11
risch	$-x+\frac{4i}{e^{2ix}+1}$	17
norman	$\frac{x+x\tan\left(\frac{x}{2}\right)^{2}-8\tan\left(\frac{x}{2}\right)^{3}-4\tan\left(\frac{x}{2}\right)^{5}-x\tan\left(\frac{x}{2}\right)^{4}-x\tan\left(\frac{x}{2}\right)^{6}-4\tan\left(\frac{x}{2}\right)}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)^{2}\left(\tan\left(\frac{x}{2}\right)^{2}-1\right)}$	72

input
```
int((sin(x)^2+1)/(1-sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-x+2*tan(x)
```

3.201.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.88

$$\int \frac{1+\sin^2(x)}{1-\sin^2(x)}\,dx = -\frac{x\cos(x) - 2\sin(x)}{\cos(x)}$$

input
```
integrate((1+sin(x)^2)/(1-sin(x)^2),x, algorithm="fricas")
```

output
```
-(x*cos(x) - 2*sin(x))/cos(x)
```

3.201.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(5) = 10$.

Time = 0.44 (sec) , antiderivative size = 41, normalized size of antiderivative = 5.12

$$\int \frac{1+\sin^2(x)}{1-\sin^2(x)}\,dx = -\frac{x\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1} + \frac{x}{\tan^2\left(\frac{x}{2}\right)-1} - \frac{4\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1}$$

input
```
integrate((1+sin(x)**2)/(1-sin(x)**2),x)
```

output
```
-x*tan(x/2)**2/(tan(x/2)**2 - 1) + x/(tan(x/2)**2 - 1) - 4*tan(x/2)/(tan(x
/2)**2 - 1)
```

3.201.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1+\sin^2(x)}{1-\sin^2(x)}\,dx = -x + 2\tan(x)$$

input
```
integrate((1+sin(x)^2)/(1-sin(x)^2),x, algorithm="maxima")
```

output
```
-x + 2*tan(x)
```

3.201.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1+\sin^2(x)}{1-\sin^2(x)}\,dx = -x + 2\,\tan(x)$$

input
```
integrate((1+sin(x)^2)/(1-sin(x)^2),x, algorithm="giac")
```

output
```
-x + 2*tan(x)
```

3.201.9 Mupad [B] (verification not implemented)

Time = 27.28 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1+\sin^2(x)}{1-\sin^2(x)}\,dx = 2\tan(x) - x$$

input
```
int(-(sin(x)^2 + 1)/(sin(x)^2 - 1),x)
```

output
```
2*tan(x) - x
```

3.202 $\int \frac{1-\sin^2(x)}{1+\sin^2(x)}\,dx$

3.202.1 Optimal result

Integrand size = 17, antiderivative size = 36

$$\int \frac{1-\sin^2(x)}{1+\sin^2(x)}\,dx = -x+\sqrt{2}x+\sqrt{2}\arctan\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\sin^2(x)}\right)$$

output
```
-x+x*2^(1/2)+arctan(cos(x)*sin(x)/(1+sin(x)^2+2^(1/2)))*2^(1/2)
```

3.202.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \frac{1-\sin^2(x)}{1+\sin^2(x)}\,dx = -2\left(\frac{x}{2}-\frac{\arctan\left(\sqrt{2}\tan(x)\right)}{\sqrt{2}}\right)$$

input
```
Integrate[(1 - Sin[x]^2)/(1 + Sin[x]^2),x]
```

output
```
-2*(x/2 - ArcTan[Sqrt[2]*Tan[x]]/Sqrt[2])
```

3.202.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.53, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3042, 3650, 3042, 3660, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-\sin^2(x)}{\sin^2(x)+1} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1-\sin(x)^2}{\sin(x)^2+1}dx$$

$$\Big\downarrow \text{3650}$$

$$2\int \frac{1}{\sin^2(x)+1}dx-x$$

$$\Big\downarrow \text{3042}$$

$$2\int \frac{1}{\sin(x)^2+1}dx-x$$

$$\Big\downarrow \text{3660}$$

$$2\int \frac{1}{2\tan^2(x)+1}d\tan(x)-x$$

$$\Big\downarrow \text{216}$$

$$\sqrt{2}\arctan\left(\sqrt{2}\tan(x)\right)-x$$

input
```
Int[(1 - Sin[x]^2)/(1 + Sin[x]^2),x]
```

output
```
-x + Sqrt[2]*ArcTan[Sqrt[2]*Tan[x]]
```

3.202.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3650
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[B*(x/b), x] + Simp[(A*b - a*B)/b   Int[1/(a
+ b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

3.202.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.50

method	result	size
default	$-\arctan\left(\tan\left(x\right)\right)+\sqrt{2}\,\arctan\left(\sqrt{2}\,\tan\left(x\right)\right)$	18
risch	$-x+\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}-2\sqrt{2}-3\right)}{2}-\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}+2\sqrt{2}-3\right)}{2}$	43

input
```
int((1-sin(x)^2)/(sin(x)^2+1),x,method=_RETURNVERBOSE)
```

output
```
-arctan(tan(x))+2^(1/2)*arctan(2^(1/2)*tan(x))
```

3.202.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int \frac{1-\sin^2(x)}{1+\sin^2(x)}\,dx = -\frac{1}{2}\,\sqrt{2}\arctan\left(\frac{3\,\sqrt{2}\cos{(x)}^2-2\,\sqrt{2}}{4\,\cos{(x)}\sin{(x)}}\right)-x$$

input
```
integrate((1-sin(x)^2)/(1+sin(x)^2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - 2*sqrt(2))/(cos(x)*sin(x)))
- x
```

3.202.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. 2(32) = 64.

Time = 21.85 (sec) , antiderivative size = 248, normalized size of antiderivative = 6.89

$$\begin{aligned}\int \frac{1-\sin^2(x)}{1+\sin^2(x)}\,dx &= -\frac{22619537x}{15994428\sqrt{2}+22619537}-\frac{15994428\sqrt{2}x}{15994428\sqrt{2}+22619537}\\&+\frac{54608393\sqrt{2}\sqrt{3-2\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{15994428\sqrt{2}+22619537}\\&+\frac{77227930\sqrt{3-2\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{15994428\sqrt{2}+22619537}\\&+\frac{9369319\sqrt{2}\sqrt{2\sqrt{2}+3}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{2\sqrt{2}+3}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{15994428\sqrt{2}+22619537}\\&+\frac{13250218\sqrt{2\sqrt{2}+3}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{2\sqrt{2}+3}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{15994428\sqrt{2}+22619537}\end{aligned}$$

input
```
integrate((1-sin(x)**2)/(1+sin(x)**2),x)
```

output
```
-22619537*x/(15994428*sqrt(2) + 22619537) - 15994428*sqrt(2)*x/(15994428*s
qrt(2) + 22619537) + 54608393*sqrt(2)*sqrt(3 - 2*sqrt(2))*(atan(tan(x/2)/s
qrt(3 - 2*sqrt(2))) + pi*floor((x/2 - pi/2)/pi))/(15994428*sqrt(2) + 22619
537) + 77227930*sqrt(3 - 2*sqrt(2))*(atan(tan(x/2)/sqrt(3 - 2*sqrt(2))) +
pi*floor((x/2 - pi/2)/pi))/(15994428*sqrt(2) + 22619537) + 9369319*sqrt(2)
*sqrt(2*sqrt(2) + 3)*(atan(tan(x/2)/sqrt(2*sqrt(2) + 3)) + pi*floor((x/2 -
 pi/2)/pi))/(15994428*sqrt(2) + 22619537) + 13250218*sqrt(2*sqrt(2) + 3)*(
atan(tan(x/2)/sqrt(2*sqrt(2) + 3)) + pi*floor((x/2 - pi/2)/pi))/(15994428*
sqrt(2) + 22619537)
```

3.202.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.42

$$\int \frac{1-\sin^2(x)}{1+\sin^2(x)}\,dx = \sqrt{2}\arctan\left(\sqrt{2}\tan(x)\right) - x$$

input
```
integrate((1-sin(x)^2)/(1+sin(x)^2),x, algorithm="maxima")
```

output
```
sqrt(2)*arctan(sqrt(2)*tan(x)) - x
```

3.202.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.36

$$\int \frac{1-\sin^2(x)}{1+\sin^2(x)}\,dx = \sqrt{2}\left(x + \arctan\left(-\frac{\sqrt{2}\sin(2x) - 2\sin(2x)}{\sqrt{2}\cos(2x) + \sqrt{2} - 2\cos(2x) + 2}\right)\right) - x$$

input
```
integrate((1-sin(x)^2)/(1+sin(x)^2),x, algorithm="giac")
```

output
```
sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - 2*sin(2*x))/(sqrt(2)*cos(2*x) + s
qrt(2) - 2*cos(2*x) + 2))) - x
```

3.202.9 Mupad [B] (verification not implemented)

Time = 26.93 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int \frac{1-\sin^2(x)}{1+\sin^2(x)}\,dx = \sqrt{2}\,(x-\operatorname{atan}(\tan(x)))-x+\sqrt{2}\operatorname{atan}\left(\sqrt{2}\tan(x)\right)$$

input
```
int(-(sin(x)^2 - 1)/(sin(x)^2 + 1),x)
```

output
```
2^(1/2)*(x - atan(tan(x))) - x + 2^(1/2)*atan(2^(1/2)*tan(x))
```

3.203 $\int \frac{1+\cos^2(x)}{1-\cos^2(x)}\,dx$

3.203.1 Optimal result

Integrand size = 17, antiderivative size = 8

$$\int \frac{1+\cos^2(x)}{1-\cos^2(x)}\,dx = -x-2\cot(x)$$

output
```
-x-2*cot(x)
```

3.203.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.88

$$\int \frac{1+\cos^2(x)}{1-\cos^2(x)}\,dx = -\cot(x)-\cot(x)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(x)\right)$$

input
```
Integrate[(1 + Cos[x]^2)/(1 - Cos[x]^2),x]
```

output
```
-Cot[x] - Cot[x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[x]^2]
```

3.203.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {3042, 3650, 3042, 3654, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(x)+1}{1-\cos^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(x+\frac{\pi}{2}\right)^2+1}{1-\sin\left(x+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3650}$$

$$2\int \frac{1}{1-\cos^2(x)}dx-x$$

$$\Big\downarrow \text{3042}$$

$$2\int \frac{1}{1-\sin\left(x+\frac{\pi}{2}\right)^2}dx-x$$

$$\Big\downarrow \text{3654}$$

$$2\int \csc^2(x)dx-x$$

$$\Big\downarrow \text{3042}$$

$$2\int \csc(x)^2dx-x$$

$$\Big\downarrow \text{4254}$$

$$-2\int 1d\cot(x)-x$$

$$\Big\downarrow \text{24}$$

$$-x-2\cot(x)$$

input
```
Int[(1 + Cos[x]^2)/(1 - Cos[x]^2),x]
```

output
```
-x - 2*Cot[x]
```

3.203.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3650
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[B*(x/b), x] + Simp[(A*b - a*B)/b   Int[1/(a
+ b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.203.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

method	result	size
parallelrisch	$-x-2\cot\left(x\right)$	9
default	$-\frac{2}{\tan\left(x\right)}-\arctan\left(\tan\left(x\right)\right)$	13
risch	$-x-\frac{4i}{{\mathrm e}^{2ix}-1}$	17
norman	$\frac{-1+\tan\left(\frac{x}{2}\right)^{4}+\tan\left(\frac{x}{2}\right)^{6}-\tan\left(\frac{x}{2}\right)^{2}-x\tan\left(\frac{x}{2}\right)-2x\tan\left(\frac{x}{2}\right)^{3}-x\tan\left(\frac{x}{2}\right)^{5}}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)^{2}\tan\left(\frac{x}{2}\right)}$	65

input
```
int((cos(x)^2+1)/(1-cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-x-2*cot(x)
```

3.203.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.88

$$\int \frac{1+\cos^2(x)}{1-\cos^2(x)}\,dx = -\frac{x\sin(x)+2\cos(x)}{\sin(x)}$$

input
```
integrate((1+cos(x)^2)/(1-cos(x)^2),x, algorithm="fricas")
```

output
```
-(x*sin(x) + 2*cos(x))/sin(x)
```

3.203.6 Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.50

$$\int \frac{1+\cos^2(x)}{1-\cos^2(x)}\,dx = -x+\tan\left(\frac{x}{2}\right)-\frac{1}{\tan\left(\frac{x}{2}\right)}$$

input
```
integrate((1+cos(x)**2)/(1-cos(x)**2),x)
```

output
```
-x + tan(x/2) - 1/tan(x/2)
```

3.203.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{1+\cos^2(x)}{1-\cos^2(x)}\,dx = -x-\frac{2}{\tan(x)}$$

input
```
integrate((1+cos(x)^2)/(1-cos(x)^2),x, algorithm="maxima")
```

output
```
-x - 2/tan(x)
```

3.203.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.00

$$\int \frac{1+\cos^2(x)}{1-\cos^2(x)}\,dx = -x - \frac{1}{\tan\left(\frac{1}{2}\,x\right)} + \tan\left(\frac{1}{2}\,x\right)$$

input
```
integrate((1+cos(x)^2)/(1-cos(x)^2),x, algorithm="giac")
```

output
```
-x - 1/tan(1/2*x) + tan(1/2*x)
```

3.203.9 Mupad [B] (verification not implemented)

Time = 25.79 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1+\cos^2(x)}{1-\cos^2(x)}\,dx = -x - 2\cot(x)$$

input
```
int(-(cos(x)^2 + 1)/(cos(x)^2 - 1),x)
```

output
```
- x - 2*cot(x)
```

3.204 $\int \frac{1-\cos^2(x)}{1+\cos^2(x)}\,dx$

3.204.1 Optimal result

Integrand size = 17, antiderivative size = 37

$$\int \frac{1-\cos^2(x)}{1+\cos^2(x)}\,dx = -x+\sqrt{2}x-\sqrt{2}\arctan\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)$$

output
```
-x+x*2^(1/2)-arctan(cos(x)*sin(x)/(1+cos(x)^2+2^(1/2)))*2^(1/2)
```

3.204.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.62

$$\int \frac{1-\cos^2(x)}{1+\cos^2(x)}\,dx = 2\left(-\frac{x}{2}+\frac{\arctan\left(\frac{\tan(x)}{\sqrt{2}}\right)}{\sqrt{2}}\right)$$

input
```
Integrate[(1 - Cos[x]^2)/(1 + Cos[x]^2),x]
```

output
```
2*(-1/2*x + ArcTan[Tan[x]/Sqrt[2]]/Sqrt[2])
```

3.204.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.54, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3042, 3650, 3042, 3660, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-\cos^2(x)}{\cos^2(x)+1}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1-\sin\left(x+\frac{\pi}{2}\right)^2}{\sin\left(x+\frac{\pi}{2}\right)^2+1}dx$$

$$\Big\downarrow \text{3650}$$

$$2\int \frac{1}{\cos^2(x)+1}dx-x$$

$$\Big\downarrow \text{3042}$$

$$2\int \frac{1}{\sin\left(x+\frac{\pi}{2}\right)^2+1}dx-x$$

$$\Big\downarrow \text{3660}$$

$$-2\int \frac{1}{2\cot^2(x)+1}d\cot(x)-x$$

$$\Big\downarrow \text{216}$$

$$-\sqrt{2}\arctan\left(\sqrt{2}\cot(x)\right)-x$$

input
```
Int[(1 - Cos[x]^2)/(1 + Cos[x]^2),x]
```

output
```
-x - Sqrt[2]*ArcTan[Sqrt[2]*Cot[x]]
```

3.204.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3650
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[B*(x/b), x] + Simp[(A*b - a*B)/b   Int[1/(a
+ b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

rule 3660
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Simp[ff/f   Subst[Int[1/(a + (a + b)*ff^2*x^
2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

3.204.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.51

method	result	size
default	$-\arctan\left(\tan\left(x\right)\right)+\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\tan\left(x\right)}{2}\right)$	19
risch	$-x+\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}+2\sqrt{2}+3\right)}{2}-\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}-2\sqrt{2}+3\right)}{2}$	43

input
```
int((1-cos(x)^2)/(cos(x)^2+1),x,method=_RETURNVERBOSE)
```

output
```
-arctan(tan(x))+2^(1/2)*arctan(1/2*2^(1/2)*tan(x))
```

3.204.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{1-\cos^2(x)}{1+\cos^2(x)}\,dx = -\frac{1}{2}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(x)^2-\sqrt{2}}{4\cos(x)\sin(x)}\right)-x$$

input
```
integrate((1-cos(x)^2)/(1+cos(x)^2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - sqrt(2))/(cos(x)*sin(x))) -
x
```

3.204.6 Sympy [A] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.65

$$\int \frac{1-\cos^2(x)}{1+\cos^2(x)}\,dx = -x+\sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)-1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right) + \sqrt{2}\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)+1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)$$

input
```
integrate((1-cos(x)**2)/(1+cos(x)**2),x)
```

output
```
-x + sqrt(2)*(atan(sqrt(2)*tan(x/2) - 1) + pi*floor((x/2 - pi/2)/pi)) + sq
rt(2)*(atan(sqrt(2)*tan(x/2) + 1) + pi*floor((x/2 - pi/2)/pi))
```

3.204.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.43

$$\int \frac{1-\cos^2(x)}{1+\cos^2(x)}\,dx = \sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\tan(x)\right)-x$$

input
```
integrate((1-cos(x)^2)/(1+cos(x)^2),x, algorithm="maxima")
```

output
```
sqrt(2)*arctan(1/2*sqrt(2)*tan(x)) - x
```

3.204.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.32

$$\int \frac{1-\cos^2(x)}{1+\cos^2(x)}\,dx = \sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\sin(2x)-\sin(2x)}{\sqrt{2}\cos(2x)+\sqrt{2}-\cos(2x)+1}\right)\right)-x$$

input
```
integrate((1-cos(x)^2)/(1+cos(x)^2),x, algorithm="giac")
```

output
```
sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - sin(2*x))/(sqrt(2)*cos(2*x) + sqr
t(2) - cos(2*x) + 1))) - x
```

3.204.9 Mupad [B] (verification not implemented)

Time = 26.54 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{1-\cos^2(x)}{1+\cos^2(x)}\,dx = \sqrt{2}\,(x-\operatorname{atan}(\tan(x)))-x+\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\tan(x)}{2}\right)$$

input
```
int(-(cos(x)^2 - 1)/(cos(x)^2 + 1),x)
```

output
```
2^(1/2)*(x - atan(tan(x))) - x + 2^(1/2)*atan((2^(1/2)*tan(x))/2)
```

3.205 $\int \frac{-1+\frac{c^2}{d^2}+\sin^2(x)}{c+d\cos(x)}\,dx$

3.205.1 Optimal result

Integrand size = 22, antiderivative size = 14

$$\int \frac{-1+\frac{c^2}{d^2}+\sin^2(x)}{c+d\cos(x)}\,dx=\frac{cx}{d^2}-\frac{\sin(x)}{d}$$

output
```
c*x/d^2-sin(x)/d
```

3.205.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{-1+\frac{c^2}{d^2}+\sin^2(x)}{c+d\cos(x)}\,dx=\frac{cx}{d^2}-\frac{\sin(x)}{d}$$

input
```
Integrate[(-1 + c^2/d^2 + Sin[x]^2)/(c + d*Cos[x]),x]
```

output
```
(c*x)/d^2 - Sin[x]/d
```

3.205.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {3042, 4897, 3042, 3495, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\frac{c^2}{d^2}+\sin^2(x)-1}{c+d\cos(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\frac{c^2}{d^2}+\sin(x)^2-1}{c+d\cos(x)}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \frac{\frac{c^2}{d^2}-\cos^2(x)}{c+d\cos(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\frac{c^2}{d^2}-\sin\left(x+\frac{\pi}{2}\right)^2}{c+d\sin\left(x+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3495}$$

$$-\frac{\int (d\cos(x)-c)dx}{d^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d\sin(x)-cx}{d^2}$$

input
```
Int[(-1 + c^2/d^2 + Sin[x]^2)/(c + d*Cos[x]),x]
```

output
```
-((-(c*x) + d*Sin[x])/d^2)
```

3.205.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3495
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[C/b^2   Int[(a + b*Sin[e + f*x])^(m + 1)
*Simp[-a + b*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] &&
 EqQ[A*b^2 + a^2*C, 0]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.205.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$\frac{-d \sin(x)+c x}{d^2}$	14
risch	$\frac{c x}{d^2}-\frac{\sin(x)}{d}$	15
default	$\frac{-\frac{2 d \tan\left(\frac{x}{2}\right)}{1+\tan\left(\frac{x}{2}\right)^2}+2 c \arctan\left(\tan\left(\frac{x}{2}\right)\right)}{d^2}$	31
norman	$\frac{\frac{c x}{d}+\frac{c x \tan\left(\frac{x}{2}\right)^4}{d}-2 \tan\left(\frac{x}{2}\right)^3+\frac{2 c x \tan\left(\frac{x}{2}\right)^2}{d}-2 \tan\left(\frac{x}{2}\right)}{d\left(1+\tan\left(\frac{x}{2}\right)^2\right)^2}$	61

input
```
int((-1+c^2/d^2+sin(x)^2)/(c+d*cos(x)),x,method=_RETURNVERBOSE)
```

output
```
(-d*sin(x)+c*x)/d^2
```

3.205.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{-1+\frac{c^2}{d^2}+\sin^2(x)}{c+d\cos(x)}\,dx = \frac{cx-d\sin(x)}{d^2}$$

input
```
integrate((-1+c^2/d^2+sin(x)^2)/(c+d*cos(x)),x, algorithm="fricas")
```

output
```
(c*x - d*sin(x))/d^2
```

3.205.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(10) = 20.

Time = 28.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 4.36

$$\int \frac{-1+\frac{c^2}{d^2}+\sin^2(x)}{c+d\cos(x)}\,dx = \frac{cx\tan^2\left(\frac{x}{2}\right)}{d^2\tan^2\left(\frac{x}{2}\right)+d^2} + \frac{cx}{d^2\tan^2\left(\frac{x}{2}\right)+d^2} - \frac{2d\tan\left(\frac{x}{2}\right)}{d^2\tan^2\left(\frac{x}{2}\right)+d^2}$$

input
```
integrate((-1+c**2/d**2+sin(x)**2)/(c+d*cos(x)),x)
```

output
```
c*x*tan(x/2)**2/(d**2*tan(x/2)**2 + d**2) + c*x/(d**2*tan(x/2)**2 + d**2)
- 2*d*tan(x/2)/(d**2*tan(x/2)**2 + d**2)
```

3.205.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{-1+\frac{c^2}{d^2}+\sin^2(x)}{c+d\cos(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((-1+c^2/d^2+sin(x)^2)/(c+d*cos(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` f
or more de
```

3.205.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.86

$$\int \frac{-1+\frac{c^2}{d^2}+\sin^2(x)}{c+d\cos(x)}\,dx = \frac{cx}{d^2} - \frac{2\tan\left(\frac{1}{2}x\right)}{\left(\tan\left(\frac{1}{2}x\right)^2+1\right)d}$$

input
```
integrate((-1+c^2/d^2+sin(x)^2)/(c+d*cos(x)),x, algorithm="giac")
```

output
```
c*x/d^2 - 2*tan(1/2*x)/((tan(1/2*x)^2 + 1)*d)
```

3.205.9 Mupad [B] (verification not implemented)

Time = 27.95 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{-1+\frac{c^2}{d^2}+\sin^2(x)}{c+d\cos(x)}\,dx = \frac{c\,x-d\,\sin(x)}{d^2}$$

input
```
int((sin(x)^2 + c^2/d^2 - 1)/(c + d*cos(x)),x)
```

output
```
(c*x - d*sin(x))/d^2
```

3.206 $\int \frac{a+b\sin^2(x)}{c+d\cos(x)}\,dx$

3.206.1 Optimal result

Integrand size = 17, antiderivative size = 105

$$\int \frac{a+b\sin^2(x)}{c+d\cos(x)}\,dx=\frac{bcx}{d^2}+\frac{2a\arctan\left(\frac{\sqrt{c-d}\tan\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{\sqrt{c-d}\sqrt{c+d}}-\frac{2b\sqrt{c-d}\sqrt{c+d}\arctan\left(\frac{\sqrt{c-d}\tan\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{d^2}-\frac{b\sin(x)}{d}$$

output
```
b*c*x/d^2-b*sin(x)/d+2*a*arctan((c-d)^(1/2)*tan(1/2*x)/(c+d)^(1/2))/(c-d)^
(1/2)/(c+d)^(1/2)-2*b*arctan((c-d)^(1/2)*tan(1/2*x)/(c+d)^(1/2))*(c-d)^(1/
2)*(c+d)^(1/2)/d^2
```

3.206.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.70

$$\int \frac{a+b\sin^2(x)}{c+d\cos(x)}\,dx=\frac{bcx-\frac{2(ad^2+b(-c^2+d^2))\text{arctanh}\left(\frac{(c-d)\tan\left(\frac{x}{2}\right)}{\sqrt{-c^2+d^2}}\right)}{\sqrt{-c^2+d^2}}-bd\sin(x)}{d^2}$$

input
```
Integrate[(a + b*Sin[x]^2)/(c + d*Cos[x]),x]
```

output
```
(b*c*x - (2*(a*d^2 + b*(-c^2 + d^2))*ArcTanh[((c - d)*Tan[x/2])/Sqrt[-c^2
+ d^2]])/Sqrt[-c^2 + d^2] - b*d*Sin[x])/d^2
```

3.206.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3042, 4901, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sin^2(x)}{c+d\cos(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\sin(x)^2}{c+d\cos(x)}dx$$

$$\Big\downarrow \text{4901}$$

$$\int \left(\frac{a}{c+d\cos(x)}+\frac{b\sin^2(x)}{c+d\cos(x)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a\arctan\left(\frac{\sqrt{c-d}\tan\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{\sqrt{c-d}\sqrt{c+d}}-\frac{2b\sqrt{c-d}\sqrt{c+d}\arctan\left(\frac{\sqrt{c-d}\tan\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{d^2}+\frac{bcx}{d^2}-\frac{b\sin(x)}{d}$$

input
```
Int[(a + b*Sin[x]^2)/(c + d*Cos[x]),x]
```

output
```
(b*c*x)/d^2 + (2*a*ArcTan[(Sqrt[c - d]*Tan[x/2])/Sqrt[c + d]])/(Sqrt[c - d
]*Sqrt[c + d]) - (2*b*Sqrt[c - d]*Sqrt[c + d]*ArcTan[(Sqrt[c - d]*Tan[x/2]
)/Sqrt[c + d]])/d^2 - (b*Sin[x])/d
```

3.206.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4901
```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

3.206.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.84

method	result
default	$\frac{2b\left(-\frac{d\tan\left(\frac{x}{2}\right)}{1+\tan\left(\frac{x}{2}\right)^{2}}+c\arctan\left(\tan\left(\frac{x}{2}\right)\right)\right)}{d^{2}}+\frac{2\left(a\,d^{2}-c^{2}b+b\,d^{2}\right)\arctan\left(\frac{(c-d)\tan\left(\frac{x}{2}\right)}{\sqrt{(c+d)(c-d)}}\right)}{d^{2}\sqrt{(c+d)(c-d)}}$
risch	$\frac{bcx}{d^{2}}+\frac{ib\,\mathrm{e}^{ix}}{2d}-\frac{ib\,\mathrm{e}^{-ix}}{2d}-\frac{\ln\left(\mathrm{e}^{ix}+\frac{ic^{2}-id^{2}+\sqrt{-c^{2}+d^{2}}\,c}{d\sqrt{-c^{2}+d^{2}}}\right)a}{\sqrt{-c^{2}+d^{2}}}+\frac{\ln\left(\mathrm{e}^{ix}+\frac{ic^{2}-id^{2}+\sqrt{-c^{2}+d^{2}}\,c}{d\sqrt{-c^{2}+d^{2}}}\right)c^{2}b}{\sqrt{-c^{2}+d^{2}}\,d^{2}}-\frac{\ln\left(\mathrm{e}^{ix}+\frac{ic^{2}-id^{2}+\sqrt{-c^{2}+d^{2}}}{d\sqrt{-c^{2}+d^{2}}}\right.}{\sqrt{-c^{2}+d^{2}}}$

input
```
int((a+sin(x)^2*b)/(c+d*cos(x)),x,method=_RETURNVERBOSE)
```

output
```
2*b/d^2*(-d*tan(1/2*x)/(1+tan(1/2*x)^2)+c*arctan(tan(1/2*x)))+2*(a*d^2-b*c
^2+b*d^2)/d^2/((c+d)*(c-d))^(1/2)*arctan((c-d)*tan(1/2*x)/((c+d)*(c-d))^(1
/2))
```

3.206.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 254, normalized size of antiderivative = 2.42

$$\int \frac{a+b\sin^2(x)}{c+d\cos(x)}\,dx$$
$$=\left[\frac{(bc^2-(a+b)d^2)\sqrt{-c^2+d^2}\log\left(\frac{2\,cd\cos(x)+(2\,c^2-d^2)\cos(x)^2+2\,\sqrt{-c^2+d^2}(c\cos(x)+d)\sin(x)-c^2+2\,d^2}{d^2\cos(x)^2+2\,cd\cos(x)+c^2}\right)+2\,(bc^3-bcd^2}{2\,(c^2d^2-d^4)}\right.$$
$$\left.-\frac{(bc^2-(a+b)d^2)\sqrt{c^2-d^2}\arctan\left(-\frac{c\cos(x)+d}{\sqrt{c^2-d^2}\sin(x)}\right)-(bc^3-bcd^2)x+(bc^2d-bd^3)\sin(x)}{c^2d^2-d^4}\right]$$

input
```
integrate((a+b*sin(x)^2)/(c+d*cos(x)),x, algorithm="fricas")
```

output
```
[1/2*((b*c^2 - (a + b)*d^2)*sqrt(-c^2 + d^2)*log((2*c*d*cos(x) + (2*c^2 -
d^2)*cos(x)^2 + 2*sqrt(-c^2 + d^2)*(c*cos(x) + d)*sin(x) - c^2 + 2*d^2)/(d
^2*cos(x)^2 + 2*c*d*cos(x) + c^2)) + 2*(b*c^3 - b*c*d^2)*x - 2*(b*c^2*d -
b*d^3)*sin(x))/(c^2*d^2 - d^4), -((b*c^2 - (a + b)*d^2)*sqrt(c^2 - d^2)*ar
ctan(-(c*cos(x) + d)/(sqrt(c^2 - d^2)*sin(x))) - (b*c^3 - b*c*d^2)*x + (b*
c^2*d - b*d^3)*sin(x))/(c^2*d^2 - d^4)]
```

3.206.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2608 vs. $2(92) = 184$.

Time = 53.37 (sec) , antiderivative size = 2608, normalized size of antiderivative = 24.84

$$\int \frac{a+b\sin^2(x)}{c+d\cos(x)}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sin(x)**2)/(c+d*cos(x)),x)
```

output
```
Piecewise((zoo*(-a*log(tan(x/2) - 1)*tan(x/2)**2/(tan(x/2)**2 + 1) - a*log
(tan(x/2) - 1)/(tan(x/2)**2 + 1) + a*log(tan(x/2) + 1)*tan(x/2)**2/(tan(x/
2)**2 + 1) + a*log(tan(x/2) + 1)/(tan(x/2)**2 + 1) - b*log(tan(x/2) - 1)*t
an(x/2)**2/(tan(x/2)**2 + 1) - b*log(tan(x/2) - 1)/(tan(x/2)**2 + 1) + b*l
og(tan(x/2) + 1)*tan(x/2)**2/(tan(x/2)**2 + 1) + b*log(tan(x/2) + 1)/(tan(
x/2)**2 + 1) - 2*b*tan(x/2)/(tan(x/2)**2 + 1)), Eq(c, 0) & Eq(d, 0)), (a*t
an(x/2)**3/(d*tan(x/2)**2 + d) + a*tan(x/2)/(d*tan(x/2)**2 + d) + b*x*tan(
x/2)**2/(d*tan(x/2)**2 + d) + b*x/(d*tan(x/2)**2 + d) - 2*b*tan(x/2)/(d*ta
n(x/2)**2 + d), Eq(c, d)), (a*tan(x/2)**2/(d*tan(x/2)**3 + d*tan(x/2)) + a
/(d*tan(x/2)**3 + d*tan(x/2)) - b*x*tan(x/2)**3/(d*tan(x/2)**3 + d*tan(x/2
)) - b*x*tan(x/2)/(d*tan(x/2)**3 + d*tan(x/2)) - 2*b*tan(x/2)**2/(d*tan(x/
2)**3 + d*tan(x/2)), Eq(c, -d)), ((a*x + b*x*sin(x)**2/2 + b*x*cos(x)**2/2
 - b*sin(x)*cos(x)/2)/c, Eq(d, 0)), (a*d**2*log(-sqrt(-c/(c - d) - d/(c -
d)) + tan(x/2))*tan(x/2)**2/(c*d**2*sqrt(-c/(c - d) - d/(c - d))*tan(x/2)*
*2 + c*d**2*sqrt(-c/(c - d) - d/(c - d)) - d**3*sqrt(-c/(c - d) - d/(c - d
))*tan(x/2)**2 - d**3*sqrt(-c/(c - d) - d/(c - d))) + a*d**2*log(-sqrt(-c/
(c - d) - d/(c - d)) + tan(x/2))/(c*d**2*sqrt(-c/(c - d) - d/(c - d))*tan(
x/2)**2 + c*d**2*sqrt(-c/(c - d) - d/(c - d)) - d**3*sqrt(-c/(c - d) - d/(
c - d))*tan(x/2)**2 - d**3*sqrt(-c/(c - d) - d/(c - d))) - a*d**2*log(sqrt
(-c/(c - d) - d/(c - d)) + tan(x/2))*tan(x/2)**2/(c*d**2*sqrt(-c/(c - d...
```

3.206.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\sin^2(x)}{c + d\cos(x)}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*sin(x)^2)/(c+d*cos(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` f
or more de
```

3.206.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.05

$$\int \frac{a + b\sin^2(x)}{c + d\cos(x)}\, dx = \frac{bcx}{d^2} - \frac{2\, b\tan\left(\frac{1}{2}\, x\right)}{\left(\tan\left(\frac{1}{2}\, x\right)^2 + 1\right)d} + \frac{2\,(bc^2 - ad^2 - bd^2)\left(\pi\left\lfloor\frac{x}{2\,\pi} + \frac{1}{2}\right\rfloor \mathrm{sgn}(-2\,c + 2\,d) + \arctan\left(-\frac{c\tan\left(\frac{1}{2}\,x\right) - d\tan\left(\frac{1}{2}\,x\right)}{\sqrt{c^2 - d^2}}\right)\right)}{\sqrt{c^2 - d^2}\,d^2}$$

input
```
integrate((a+b*sin(x)^2)/(c+d*cos(x)),x, algorithm="giac")
```

output
```
b*c*x/d^2 - 2*b*tan(1/2*x)/((tan(1/2*x)^2 + 1)*d) + 2*(b*c^2 - a*d^2 - b*d
^2)*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*x) - d*
tan(1/2*x))/sqrt(c^2 - d^2)))/(sqrt(c^2 - d^2)*d^2)
```

3.206.9 Mupad [B] (verification not implemented)

Time = 29.12 (sec) , antiderivative size = 2429, normalized size of antiderivative = 23.13

$$\int \frac{a+b\sin^2(x)}{c+d\cos(x)}\,dx = \text{Too large to display}$$

input
```
int((a + b*sin(x)^2)/(c + d*cos(x)),x)
```

output
```
(b*c^2*d*sin(x))/(d^4 - c^2*d^2) - (b*d^3*sin(x))/(d^4 - c^2*d^2) - (2*b*c
^3*atan(sin(x/2)/cos(x/2)))/(d^4 - c^2*d^2) - (a*d^2*atan((a^2*d^7*sin(x/2
)*(d^2 - c^2)^(1/2)*1i - b^2*c^5*sin(x/2)*(d^2 - c^2)^(3/2)*2i - b^2*c^7*s
in(x/2)*(d^2 - c^2)^(1/2)*2i + b^2*d^7*sin(x/2)*(d^2 - c^2)^(1/2)*1i - a^2
*c*d^4*sin(x/2)*(d^2 - c^2)^(3/2)*2i + a^2*c*d^6*sin(x/2)*(d^2 - c^2)^(1/2
)*1i - b^2*c*d^4*sin(x/2)*(d^2 - c^2)^(3/2)*2i + b^2*c*d^6*sin(x/2)*(d^2 -
 c^2)^(1/2)*1i - a^2*c^2*d^5*sin(x/2)*(d^2 - c^2)^(1/2)*1i - a^2*c^3*d^4*s
in(x/2)*(d^2 - c^2)^(1/2)*1i - b^2*c^2*d^5*sin(x/2)*(d^2 - c^2)^(1/2)*2i +
 b^2*c^3*d^2*sin(x/2)*(d^2 - c^2)^(3/2)*4i - b^2*c^3*d^4*sin(x/2)*(d^2 - c
^2)^(1/2)*4i + b^2*c^4*d^3*sin(x/2)*(d^2 - c^2)^(1/2)*1i + b^2*c^5*d^2*sin
(x/2)*(d^2 - c^2)^(1/2)*5i + a*b*d^7*sin(x/2)*(d^2 - c^2)^(1/2)*2i - a*b*c
*d^4*sin(x/2)*(d^2 - c^2)^(3/2)*4i + a*b*c*d^6*sin(x/2)*(d^2 - c^2)^(1/2)*
2i - a*b*c^2*d^5*sin(x/2)*(d^2 - c^2)^(1/2)*4i + a*b*c^3*d^2*sin(x/2)*(d^2
 - c^2)^(3/2)*4i - a*b*c^3*d^4*sin(x/2)*(d^2 - c^2)^(1/2)*4i + a*b*c^4*d^3
*sin(x/2)*(d^2 - c^2)^(1/2)*2i + a*b*c^5*d^2*sin(x/2)*(d^2 - c^2)^(1/2)*2i
)/(a^2*d^8*cos(x/2) + b^2*d^8*cos(x/2) + 2*a*b*d^8*cos(x/2) - 2*a^2*c^2*d^
6*cos(x/2) + a^2*c^4*d^4*cos(x/2) - 3*b^2*c^2*d^6*cos(x/2) + 3*b^2*c^4*d^4
*cos(x/2) - b^2*c^6*d^2*cos(x/2) - 6*a*b*c^2*d^6*cos(x/2) + 6*a*b*c^4*d^4*
cos(x/2) - 2*a*b*c^6*d^2*cos(x/2)))*(d^2 - c^2)^(1/2)*2i)/(d^4 - c^2*d^2)
+ (b*c^2*atan((a^2*d^7*sin(x/2)*(d^2 - c^2)^(1/2)*1i - b^2*c^5*sin(x/2)...
```

3.207 $\int \frac{a+b\sin^2(x)}{c+c\cos^2(x)}\,dx$

3.207.1 Optimal result

Integrand size = 19, antiderivative size = 57

$$\int \frac{a+b\sin^2(x)}{c+c\cos^2(x)}\,dx = -\frac{bx}{c}+\frac{(a+2b)x}{\sqrt{2}c}-\frac{(a+2b)\arctan\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)}{\sqrt{2}c}$$

output
```
-b*x/c+1/2*(a+2*b)*x/c*2^(1/2)-1/2*(a+2*b)*arctan(cos(x)*sin(x)/(1+cos(x)^
2+2^(1/2)))/c*2^(1/2)
```

3.207.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.60

$$\int \frac{a+b\sin^2(x)}{c+c\cos^2(x)}\,dx = -\frac{bx}{c}-\frac{(-a-2b)\arctan\left(\frac{\tan(x)}{\sqrt{2}}\right)}{\sqrt{2}c}$$

input
```
Integrate[(a + b*Sin[x]^2)/(c + c*Cos[x]^2),x]
```

output
```
-((b*x)/c) - ((-a - 2*b)*ArcTan[Tan[x]/Sqrt[2]])/(Sqrt[2]*c)
```

3.207.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.54, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3042, 4889, 27, 1480, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sin^2(x)}{c\cos^2(x)+c}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{a+b\sin(x)^2}{c\cos(x)^2+c}dx$$

$$\Big\downarrow \ 4889$$

$$\int \frac{(a+b)\tan^2(x)+a}{c\left(\tan^4(x)+3\tan^2(x)+2\right)}d\tan(x)$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{(a+b)\tan^2(x)+a}{\tan^4(x)+3\tan^2(x)+2}d\tan(x)}{c}$$

$$\Big\downarrow \ 1480$$

$$\frac{(a+2b)\int \frac{1}{\tan^2(x)+2}d\tan(x)-b\int \frac{1}{\tan^2(x)+1}d\tan(x)}{c}$$

$$\Big\downarrow \ 216$$

$$\frac{\frac{(a+2b)\arctan\left(\frac{\tan(x)}{\sqrt{2}}\right)}{\sqrt{2}}-b\arctan(\tan(x))}{c}$$

input
```
Int[(a + b*Sin[x]^2)/(c + c*Cos[x]^2),x]
```

output
```
(-(b*ArcTan[Tan[x]]) + ((a + 2*b)*ArcTan[Tan[x]/Sqrt[2]])/Sqrt[2])/c
```

3.207.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.207.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.53

method	result	size
default	$\frac{-b\arctan(\tan(x))+\frac{(a+2b)\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\tan(x)}{2}\right)}{2}}{c}$	30
risch	$-\frac{bx}{c}+\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}+2\sqrt{2}+3\right)a}{4c}+\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}+2\sqrt{2}+3\right)b}{2c}-\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}-2\sqrt{2}+3\right)a}{4c}-\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}-2\sqrt{2}+3\right)b}{2c}$	101

input
```
int((a+sin(x)^2*b)/(c+c*cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/c*(-b*arctan(tan(x))+1/2*(a+2*b)*2^(1/2)*arctan(1/2*2^(1/2)*tan(x)))
```

3.207.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.79

$$\int \frac{a+b\sin^2(x)}{c+c\cos^2(x)}\,dx=-\frac{\sqrt{2}(a+2\,b)\arctan\left(\frac{3\,\sqrt{2}\cos(x)^2-\sqrt{2}}{4\,\cos(x)\sin(x)}\right)+4\,bx}{4\,c}$$

input
```
integrate((a+b*sin(x)^2)/(c+c*cos(x)^2),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(2)*(a + 2*b)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - sqrt(2))/(cos(x)*
sin(x))) + 4*b*x)/c
```

3.207.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(49) = 98.

Time = 4.14 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.51

$$\begin{aligned}\int \frac{a+b\sin^2(x)}{c+c\cos^2(x)}\,dx=&\frac{\sqrt{2}a\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)-1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{2c}\\&+\frac{\sqrt{2}a\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)+1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{2c}\\&-\frac{bx}{c}+\frac{\sqrt{2}b\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)-1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{c}\\&+\frac{\sqrt{2}b\left(\operatorname{atan}\left(\sqrt{2}\tan\left(\frac{x}{2}\right)+1\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{c}\end{aligned}$$

input
```
integrate((a+b*sin(x)**2)/(c+c*cos(x)**2),x)
```

output
```
sqrt(2)*a*(atan(sqrt(2)*tan(x/2) - 1) + pi*floor((x/2 - pi/2)/pi))/(2*c) +
 sqrt(2)*a*(atan(sqrt(2)*tan(x/2) + 1) + pi*floor((x/2 - pi/2)/pi))/(2*c)
- b*x/c + sqrt(2)*b*(atan(sqrt(2)*tan(x/2) - 1) + pi*floor((x/2 - pi/2)/pi
))/c + sqrt(2)*b*(atan(sqrt(2)*tan(x/2) + 1) + pi*floor((x/2 - pi/2)/pi))/
c
```

3.207.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.51

$$\int \frac{a+b\sin^2(x)}{c+c\cos^2(x)}\,dx = \frac{\sqrt{2}(a+2\,b)\arctan\left(\frac{1}{2}\,\sqrt{2}\tan\left(x\right)\right)}{2\,c} - \frac{bx}{c}$$

input
```
integrate((a+b*sin(x)^2)/(c+c*cos(x)^2),x, algorithm="maxima")
```

output
```
1/2*sqrt(2)*(a + 2*b)*arctan(1/2*sqrt(2)*tan(x))/c - b*x/c
```

3.207.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.09

$$\int \frac{a+b\sin^2(x)}{c+c\cos^2(x)}\,dx = \frac{\sqrt{2}(a+2\,b)\left(x+\arctan\left(-\frac{\sqrt{2}\sin(2\,x)-\sin(2\,x)}{\sqrt{2}\cos(2\,x)+\sqrt{2}-\cos(2\,x)+1}\right)\right)}{2\,c} - \frac{bx}{c}$$

input
```
integrate((a+b*sin(x)^2)/(c+c*cos(x)^2),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*(a + 2*b)*(x + arctan(-(sqrt(2)*sin(2*x) - sin(2*x))/(sqrt(2)*
cos(2*x) + sqrt(2) - cos(2*x) + 1)))/c - b*x/c
```

3.207.9 Mupad [B] (verification not implemented)

Time = 26.73 (sec) , antiderivative size = 242, normalized size of antiderivative = 4.25

$$\int \frac{a+b\sin^2(x)}{c+c\cos^2(x)}\,dx$$
$$= \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,a^3\tan(x)}{2\,(a^3+6\,a^2\,b+10\,a\,b^2+4\,b^3)} + \frac{2\,\sqrt{2}\,b^3\tan(x)}{a^3+6\,a^2\,b+10\,a\,b^2+4\,b^3} + \frac{5\,\sqrt{2}\,a\,b^2\tan(x)}{a^3+6\,a^2\,b+10\,a\,b^2+4\,b^3} + \frac{3\,\sqrt{2}\,a^2\,b\tan(x)}{a^3+6\,a^2\,b+10\,a\,b^2+4\,b^3}\right)(a+2\,b)}{2\,c} - \frac{b\,\mathrm{atan}\left(\frac{4\,b^3\tan(x)}{2\,a^2\,b+8\,a\,b^2+4\,b^3} + \frac{8\,a\,b^2\tan(x)}{2\,a^2\,b+8\,a\,b^2+4\,b^3} + \frac{2\,a^2\,b\tan(x)}{2\,a^2\,b+8\,a\,b^2+4\,b^3}\right)}{c}$$

input
```
int((a + b*sin(x)^2)/(c + c*cos(x)^2),x)
```

output
```
(2^(1/2)*atan((2^(1/2)*a^3*tan(x))/(2*(10*a*b^2 + 6*a^2*b + a^3 + 4*b^3))
+ (2*2^(1/2)*b^3*tan(x))/(10*a*b^2 + 6*a^2*b + a^3 + 4*b^3) + (5*2^(1/2)*a
*b^2*tan(x))/(10*a*b^2 + 6*a^2*b + a^3 + 4*b^3) + (3*2^(1/2)*a^2*b*tan(x))
/(10*a*b^2 + 6*a^2*b + a^3 + 4*b^3))*(a + 2*b))/(2*c) - (b*atan((4*b^3*tan
(x))/(8*a*b^2 + 2*a^2*b + 4*b^3) + (8*a*b^2*tan(x))/(8*a*b^2 + 2*a^2*b + 4
*b^3) + (2*a^2*b*tan(x))/(8*a*b^2 + 2*a^2*b + 4*b^3)))/c
```

3.207. $\int \frac{a+b\sin^2(x)}{c+c\cos^2(x)}\,dx$

3.208 $\int \frac{a+b\sin^2(x)}{c-c\cos^2(x)}\,dx$

3.208.1 Optimal result

Integrand size = 20, antiderivative size = 15

$$\int \frac{a+b\sin^2(x)}{c-c\cos^2(x)}\,dx=\frac{bx}{c}-\frac{a\cot(x)}{c}$$

output
```
b*x/c-a*cot(x)/c
```

3.208.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sin^2(x)}{c-c\cos^2(x)}\,dx=\frac{bx}{c}-\frac{a\cot(x)}{c}$$

input
```
Integrate[(a + b*Sin[x]^2)/(c - c*Cos[x]^2),x]
```

output
```
(b*x)/c - (a*Cot[x])/c
```

3.208.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 4889, 27, 359, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sin^2(x)}{c-c\cos^2(x)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{a+b\sin(x)^2}{c-c\cos(x)^2}dx$$

$$\Big\downarrow \text{ 4889}$$

$$\int \frac{\cot^2(x)\left((a+b)\tan^2(x)+a\right)}{c\left(\tan^2(x)+1\right)}d\tan(x)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\cot^2(x)((a+b)\tan^2(x)+a)}{\tan^2(x)+1}d\tan(x)}{c}$$

$$\Big\downarrow \text{ 359}$$

$$\frac{b\int \frac{1}{\tan^2(x)+1}d\tan(x)-a\cot(x)}{c}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{b\arctan(\tan(x))-a\cot(x)}{c}$$

input
```
Int[(a + b*Sin[x]^2)/(c - c*Cos[x]^2),x]
```

output
```
(b*ArcTan[Tan[x]] - a*Cot[x])/c
```

3.208.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.208.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
parallelrisch	$\frac{xb-a\cot(x)}{c}$	14
default	$\frac{-\frac{a}{\tan(x)}+b\arctan(\tan(x))}{c}$	18
risch	$\frac{bx}{c}-\frac{2ia}{(e^{2ix}-1)c}$	24
norman	$\frac{\frac{bx\tan(\frac{x}{2})}{c}+\frac{bx\tan(\frac{x}{2})^5}{c}-\frac{a}{2c}-\frac{a\tan(\frac{x}{2})^2}{2c}+\frac{a\tan(\frac{x}{2})^4}{2c}+\frac{a\tan(\frac{x}{2})^6}{2c}+\frac{2bx\tan(\frac{x}{2})^3}{c}}{\left(1+\tan(\frac{x}{2})^2\right)^2\tan(\frac{x}{2})}$	96

input
```
int((a+sin(x)^2*b)/(c-c*cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
(x*b-a*cot(x))/c
```

3.208.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int \frac{a+b\sin^2(x)}{c-c\cos^2(x)}\,dx = \frac{bx\sin{(x)}-a\cos{(x)}}{c\sin{(x)}}$$

input
```
integrate((a+b*sin(x)^2)/(c-c*cos(x)^2),x, algorithm="fricas")
```

output
```
(b*x*sin(x) - a*cos(x))/(c*sin(x))
```

3.208.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. 2(10) = 20.

Time = 0.42 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.60

$$\int \frac{a+b\sin^2(x)}{c-c\cos^2(x)}\,dx = \frac{a\tan{\left(\frac{x}{2}\right)}}{2c} - \frac{a}{2c\tan{\left(\frac{x}{2}\right)}} + \frac{bx}{c}$$

input
```
integrate((a+b*sin(x)**2)/(c-c*cos(x)**2),x)
```

output
```
a*tan(x/2)/(2*c) - a/(2*c*tan(x/2)) + b*x/c
```

3.208.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{a+b\sin^2(x)}{c-c\cos^2(x)}\,dx = \frac{bx}{c} - \frac{a}{c\tan{(x)}}$$

input
```
integrate((a+b*sin(x)^2)/(c-c*cos(x)^2),x, algorithm="maxima")
```

output
```
b*x/c - a/(c*tan(x))
```

3.208.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.93

$$\int \frac{a+b\sin^2(x)}{c-c\cos^2(x)}\,dx = \frac{bx}{c} + \frac{a\tan\left(\frac{1}{2}x\right)}{2c} - \frac{a}{2c\tan\left(\frac{1}{2}x\right)}$$

input
```
integrate((a+b*sin(x)^2)/(c-c*cos(x)^2),x, algorithm="giac")
```

output
```
b*x/c + 1/2*a*tan(1/2*x)/c - 1/2*a/(c*tan(1/2*x))
```

3.208.9 Mupad [B] (verification not implemented)

Time = 26.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{a+b\sin^2(x)}{c-c\cos^2(x)}\,dx = \frac{b\,x-a\cot(x)}{c}$$

input
```
int((a + b*sin(x)^2)/(c - c*cos(x)^2),x)
```

output
```
(b*x - a*cot(x))/c
```

3.209 $\int \frac{a+b\sin^2(x)}{c+d\cos^2(x)}\,dx$

3.209.1 Optimal result

Integrand size = 19, antiderivative size = 49

$$\int \frac{a+b\sin^2(x)}{c+d\cos^2(x)}\,dx = -\frac{bx}{d}+\frac{(ad+b(c+d))\arctan\left(\frac{\sqrt{c}\tan(x)}{\sqrt{c+d}}\right)}{\sqrt{c}d\sqrt{c+d}}$$

output
```
-b*x/d+(a*d+b*(c+d))*arctan(c^(1/2)*tan(x)/(c+d)^(1/2))/d/c^(1/2)/(c+d)^(1
/2)
```

3.209.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.96

$$\int \frac{a+b\sin^2(x)}{c+d\cos^2(x)}\,dx = \frac{-bx+\frac{(ad+b(c+d))\arctan\left(\frac{\sqrt{c}\tan(x)}{\sqrt{c+d}}\right)}{\sqrt{c}\sqrt{c+d}}}{d}$$

input
```
Integrate[(a + b*Sin[x]^2)/(c + d*Cos[x]^2),x]
```

output
```
(-(b*x) + ((a*d + b*(c + d))*ArcTan[(Sqrt[c]*Tan[x])/Sqrt[c + d]])/(Sqrt[c
]*Sqrt[c + d]))/d
```

3.209.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3042, 4889, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sin^2(x)}{c+d\cos^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\sin(x)^2}{c+d\cos(x)^2}dx$$

$$\Big\downarrow \text{4889}$$

$$\int \frac{(a+b)\tan^2(x)+a}{\left(\tan^2(x)+1\right)\left(c\tan^2(x)+c+d\right)}d\tan(x)$$

$$\Big\downarrow \text{397}$$

$$\frac{(ad+b(c+d))\int \frac{1}{c\tan^2(x)+c+d}d\tan(x)}{d}-\frac{b\int \frac{1}{\tan^2(x)+1}d\tan(x)}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{(ad+b(c+d))\int \frac{1}{c\tan^2(x)+c+d}d\tan(x)}{d}-\frac{b\arctan(\tan(x))}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{(ad+b(c+d))\arctan\left(\frac{\sqrt{c}\tan(x)}{\sqrt{c+d}}\right)}{\sqrt{c}d\sqrt{c+d}}-\frac{b\arctan(\tan(x))}{d}$$

input
```
Int[(a + b*Sin[x]^2)/(c + d*Cos[x]^2),x]
```

output
```
-((b*ArcTan[Tan[x]])/d) + ((a*d + b*(c + d))*ArcTan[(Sqrt[c]*Tan[x])/Sqrt[
c + d]])/(Sqrt[c]*d*Sqrt[c + d])
```

3.209.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.209.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

method	result
default	$\frac{(ad+cb+bd)\arctan\left(\frac{c\tan(x)}{\sqrt{(c+d)c}}\right)}{d\sqrt{(c+d)c}}-\frac{b\arctan(\tan(x))}{d}$
risch	$-\frac{bx}{d}-\frac{\ln\left(\mathrm{e}^{2ix}+\frac{2ic^2+2idc+2\sqrt{-c^2-cd}\,c+\sqrt{-c^2-cd}\,d}{\sqrt{-c^2-cd}\,d}\right)a}{2\sqrt{-c^2-cd}}-\frac{\ln\left(\mathrm{e}^{2ix}+\frac{2ic^2+2idc+2\sqrt{-c^2-cd}\,c+\sqrt{-c^2-cd}\,d}{\sqrt{-c^2-cd}\,d}\right)cb}{2\sqrt{-c^2-cd}\,d}-\frac{\ln\left(\mathrm{e}^{2ix}+\frac{2ic^2+2i}{}\right.}{}$

input
```
int((a+sin(x)^2*b)/(c+d*cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
(a*d+b*c+b*d)/d/((c+d)*c)^(1/2)*arctan(c*tan(x)/((c+d)*c)^(1/2))-b/d*arcta
n(tan(x))
```

3.209.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 228, normalized size of antiderivative = 4.65

$$\int \frac{a+b\sin^2(x)}{c+d\cos^2(x)}\,dx = \left[-\frac{(bc+(a+b)d)\sqrt{-c^2-cd}\log\left(\frac{(8c^2+8cd+d^2)\cos(x)^4-2(4c^2+3cd)\cos(x)^2+4\left((2c+d)\cos(x)^3-c\cos(x)\right)\sqrt{-c^2-cd}\sin(x)+}{d^2\cos(x)^4+2cd\cos(x)^2+c^2}\right.}{4(c^2d+cd^2)}\right.$$

$$\left.-\frac{(bc+(a+b)d)\sqrt{c^2+cd}\arctan\left(\frac{(2c+d)\cos(x)^2-c}{2\sqrt{c^2+cd}\cos(x)\sin(x)}\right)+2(bc^2+bcd)x}{2(c^2d+cd^2)}\right]$$

input
```
integrate((a+b*sin(x)^2)/(c+d*cos(x)^2),x, algorithm="fricas")
```

output
```
[-1/4*((b*c + (a + b)*d)*sqrt(-c^2 - c*d)*log(((8*c^2 + 8*c*d + d^2)*cos(x
)^4 - 2*(4*c^2 + 3*c*d)*cos(x)^2 + 4*((2*c + d)*cos(x)^3 - c*cos(x))*sqrt(
-c^2 - c*d)*sin(x) + c^2)/(d^2*cos(x)^4 + 2*c*d*cos(x)^2 + c^2)) + 4*(b*c^
2 + b*c*d)*x)/(c^2*d + c*d^2), -1/2*((b*c + (a + b)*d)*sqrt(c^2 + c*d)*arc
tan(1/2*((2*c + d)*cos(x)^2 - c)/(sqrt(c^2 + c*d)*cos(x)*sin(x))) + 2*(b*c
^2 + b*c*d)*x)/(c^2*d + c*d^2)]
```

3.209.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\sin^2(x)}{c+d\cos^2(x)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sin(x)**2)/(c+d*cos(x)**2),x)
```

output
```
Timed out
```

3.209.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int \frac{a+b\sin^2(x)}{c+d\cos^2(x)}\,dx = -\frac{bx}{d} + \frac{(bc+(a+b)d)\arctan\left(\frac{c\tan(x)}{\sqrt{(c+d)c}}\right)}{\sqrt{(c+d)c}d}$$

input
```
integrate((a+b*sin(x)^2)/(c+d*cos(x)^2),x, algorithm="maxima")
```

output
```
-b*x/d + (b*c + (a + b)*d)*arctan(c*tan(x)/sqrt((c + d)*c))/(sqrt((c + d)*
c)*d)
```

3.209.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.18

$$\int \frac{a+b\sin^2(x)}{c+d\cos^2(x)}\,dx = -\frac{bx}{d} + \frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(c)+\arctan\left(\frac{c\tan(x)}{\sqrt{c^2+cd}}\right)\right)(bc+ad+bd)}{\sqrt{c^2+cd}d}$$

input
```
integrate((a+b*sin(x)^2)/(c+d*cos(x)^2),x, algorithm="giac")
```

output
```
-b*x/d + (pi*floor(x/pi + 1/2)*sgn(c) + arctan(c*tan(x)/sqrt(c^2 + c*d)))*
(b*c + a*d + b*d)/(sqrt(c^2 + c*d)*d)
```

3.209.9 Mupad [B] (verification not implemented)

Time = 27.56 (sec) , antiderivative size = 1987, normalized size of antiderivative = 40.55

$$\int \frac{a+b\sin^2(x)}{c+d\cos^2(x)}\,dx = \text{Too large to display}$$

input
```
int((a + b*sin(x)^2)/(c + d*cos(x)^2),x)
```

output
```
- (b*c^2*x)/(c*d^2 + c^2*d) - (a*d*atan((a^2*d^3*tan(x)*(- c*d - c^2)^(3/2
)*1i + b^2*c^3*tan(x)*(- c*d - c^2)^(3/2)*2i + b^2*c^5*tan(x)*(- c*d - c^2
)^(1/2)*2i + b^2*d^3*tan(x)*(- c*d - c^2)^(3/2)*1i + a^2*c*d^2*tan(x)*(- c
*d - c^2)^(3/2)*2i + a^2*c*d^4*tan(x)*(- c*d - c^2)^(1/2)*1i + b^2*c*d^2*t
an(x)*(- c*d - c^2)^(3/2)*4i + b^2*c*d^4*tan(x)*(- c*d - c^2)^(1/2)*1i + b
^2*c^2*d*tan(x)*(- c*d - c^2)^(3/2)*5i + b^2*c^4*d*tan(x)*(- c*d - c^2)^(1
/2)*6i + a^2*c^2*d^3*tan(x)*(- c*d - c^2)^(1/2)*2i + a^2*c^3*d^2*tan(x)*(-
 c*d - c^2)^(1/2)*1i + b^2*c^2*d^3*tan(x)*(- c*d - c^2)^(1/2)*4i + b^2*c^3
*d^2*tan(x)*(- c*d - c^2)^(1/2)*7i + a*b*d^3*tan(x)*(- c*d - c^2)^(3/2)*2i
 + a*b*c*d^2*tan(x)*(- c*d - c^2)^(3/2)*6i + a*b*c*d^4*tan(x)*(- c*d - c^2
)^(1/2)*2i + a*b*c^2*d*tan(x)*(- c*d - c^2)^(3/2)*4i + a*b*c^4*d*tan(x)*(-
 c*d - c^2)^(1/2)*2i + a*b*c^2*d^3*tan(x)*(- c*d - c^2)^(1/2)*6i + a*b*c^3
*d^2*tan(x)*(- c*d - c^2)^(1/2)*6i)/(b^2*c^5*d + a^2*c^2*d^4 + 2*a^2*c^3*d
^3 + a^2*c^4*d^2 + b^2*c^2*d^4 + 3*b^2*c^3*d^3 + 3*b^2*c^4*d^2 + 2*a*b*c^5
*d + 2*a*b*c^2*d^4 + 6*a*b*c^3*d^3 + 6*a*b*c^4*d^2))*(- c*d - c^2)^(1/2)*1
i)/(c*d^2 + c^2*d) - (b*c*atan((a^2*d^3*tan(x)*(- c*d - c^2)^(3/2)*1i + b^
2*c^3*tan(x)*(- c*d - c^2)^(3/2)*2i + b^2*c^5*tan(x)*(- c*d - c^2)^(1/2)*2
i + b^2*d^3*tan(x)*(- c*d - c^2)^(3/2)*1i + a^2*c*d^2*tan(x)*(- c*d - c^2)
^(3/2)*2i + a^2*c*d^4*tan(x)*(- c*d - c^2)^(1/2)*1i + b^2*c*d^2*tan(x)*(-
c*d - c^2)^(3/2)*4i + b^2*c*d^4*tan(x)*(- c*d - c^2)^(1/2)*1i + b^2*c^2...
```

3.209. $\int \frac{a+b\sin^2(x)}{c+d\cos^2(x)}\,dx$

3.210 $\int \frac{-1+\frac{c^2}{d^2}+\cos^2(x)}{c+d\sin(x)}\,dx$

3.210.1 Optimal result

Integrand size = 22, antiderivative size = 13

$$\int \frac{-1+\frac{c^2}{d^2}+\cos^2(x)}{c+d\sin(x)}\,dx=\frac{cx}{d^2}+\frac{\cos(x)}{d}$$

output
```
c*x/d^2+cos(x)/d
```

3.210.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{-1+\frac{c^2}{d^2}+\cos^2(x)}{c+d\sin(x)}\,dx=\frac{cx}{d^2}+\frac{\cos(x)}{d}$$

input
```
Integrate[(-1 + c^2/d^2 + Cos[x]^2)/(c + d*Sin[x]),x]
```

output
```
(c*x)/d^2 + Cos[x]/d
```

3.210.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {3042, 4897, 3042, 3495, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\frac{c^2}{d^2}+\cos^2(x)-1}{c+d\sin(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\frac{c^2}{d^2}+\cos(x)^2-1}{c+d\sin(x)}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \frac{\frac{c^2}{d^2}-\sin^2(x)}{c+d\sin(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\frac{c^2}{d^2}-\sin(x)^2}{c+d\sin(x)}dx$$

$$\Big\downarrow \text{3495}$$

$$-\frac{\int (d\sin(x)-c)dx}{d^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-cx-d\cos(x)}{d^2}$$

input
```
Int[(-1 + c^2/d^2 + Cos[x]^2)/(c + d*Sin[x]),x]
```

output
```
-((-(c*x) - d*Cos[x])/d^2)
```

3.210.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3495
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[C/b^2   Int[(a + b*Sin[e + f*x])^(m + 1)
*Simp[-a + b*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] &&
 EqQ[A*b^2 + a^2*C, 0]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.210.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

method	result	size
risch	$\frac{cx}{d^2}+\frac{\cos(x)}{d}$	14
parallelrisch	$\frac{d\cos(x)+cx+d}{d^2}$	14
default	$\frac{\frac{2d}{1+\tan\left(\frac{x}{2}\right)^2}+2c\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{d^2}$	26
norman	$\frac{2\tan\left(\frac{x}{2}\right)^2+\frac{cx}{d}+\frac{cx\tan\left(\frac{x}{2}\right)^4}{d}+\frac{2cx\tan\left(\frac{x}{2}\right)^2}{d}+2}{d\left(1+\tan\left(\frac{x}{2}\right)^2\right)^2}$	56
parts	$\frac{\frac{2d}{1+\tan\left(\frac{x}{2}\right)^2}+2c\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{d^2}+\frac{2(-c^2+d^2)\arctan\left(\frac{2c\tan\left(\frac{x}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{d^2\sqrt{c^2-d^2}}+\frac{2\sqrt{c^2-d^2}\,\arctan\left(\frac{2c\tan\left(\frac{x}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{d^2}$	118

input
```
int((-1+c^2/d^2+cos(x)^2)/(c+d*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
c*x/d^2+cos(x)/d
```

3.210.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{-1+\frac{c^2}{d^2}+\cos^2(x)}{c+d\sin(x)}\,dx=\frac{cx+d\cos(x)}{d^2}$$

input
```
integrate((-1+c^2/d^2+cos(x)^2)/(c+d*sin(x)),x, algorithm="fricas")
```

output
```
(c*x + d*cos(x))/d^2
```

3.210.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(10) = 20$.

Time = 37.14 (sec) , antiderivative size = 56, normalized size of antiderivative = 4.31

$$\int \frac{-1+\frac{c^2}{d^2}+\cos^2(x)}{c+d\sin(x)}\,dx=\frac{cx\tan^2\left(\frac{x}{2}\right)}{d^2\tan^2\left(\frac{x}{2}\right)+d^2}+\frac{cx}{d^2\tan^2\left(\frac{x}{2}\right)+d^2}+\frac{2d}{d^2\tan^2\left(\frac{x}{2}\right)+d^2}$$

input
```
integrate((-1+c**2/d**2+cos(x)**2)/(c+d*sin(x)),x)
```

output
```
c*x*tan(x/2)**2/(d**2*tan(x/2)**2 + d**2) + c*x/(d**2*tan(x/2)**2 + d**2)
+ 2*d/(d**2*tan(x/2)**2 + d**2)
```

3.210.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{-1+\frac{c^2}{d^2}+\cos^2(x)}{c+d\sin(x)}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((-1+c^2/d^2+cos(x)^2)/(c+d*sin(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` f
or more de
```

3.210.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int \frac{-1+\frac{c^2}{d^2}+\cos^2(x)}{c+d\sin(x)}\,dx = \frac{cx}{d^2}+\frac{2}{\left(\tan\left(\frac{1}{2}\,x\right)^2+1\right)d}$$

input
```
integrate((-1+c^2/d^2+cos(x)^2)/(c+d*sin(x)),x, algorithm="giac")
```

output
```
c*x/d^2 + 2/((tan(1/2*x)^2 + 1)*d)
```

3.210.9 Mupad [B] (verification not implemented)

Time = 27.43 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{-1+\frac{c^2}{d^2}+\cos^2(x)}{c+d\sin(x)}\,dx = \frac{\cos(x)}{d}+\frac{c\,x}{d^2}$$

input
```
int((cos(x)^2 + c^2/d^2 - 1)/(c + d*sin(x)),x)
```

output
```
cos(x)/d + (c*x)/d^2
```

3.211 $\int \frac{a+b\cos^2(x)}{c+d\sin(x)}\,dx$

3.211.1 Optimal result

Integrand size = 17, antiderivative size = 100

$$\int \frac{a+b\cos^2(x)}{c+d\sin(x)}\,dx=\frac{bcx}{d^2}+\frac{2a\arctan\left(\frac{d+c\tan\left(\frac{x}{2}\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}}-\frac{2b\sqrt{c^2-d^2}\arctan\left(\frac{d+c\tan\left(\frac{x}{2}\right)}{\sqrt{c^2-d^2}}\right)}{d^2}+\frac{b\cos(x)}{d}$$

output
```
b*c*x/d^2+b*cos(x)/d+2*a*arctan((d+c*tan(1/2*x))/(c^2-d^2)^(1/2))/(c^2-d^2
)^(1/2)-2*b*arctan((d+c*tan(1/2*x))/(c^2-d^2)^(1/2))*(c^2-d^2)^(1/2)/d^2
```

3.211.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.72

$$\int \frac{a+b\cos^2(x)}{c+d\sin(x)}\,dx=\frac{\frac{2(ad^2+b(-c^2+d^2))\arctan\left(\frac{d+c\tan\left(\frac{x}{2}\right)}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}}+b(cx+d\cos(x))}{d^2}$$

input
```
Integrate[(a + b*Cos[x]^2)/(c + d*Sin[x]),x]
```

output
```
((2*(a*d^2 + b*(-c^2 + d^2))*ArcTan[(d + c*Tan[x/2])/Sqrt[c^2 - d^2]])/Sqr
t[c^2 - d^2] + b*(c*x + d*Cos[x]))/d^2
```

3.211.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3042, 4901, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\cos^2(x)}{c+d\sin(x)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{a+b\cos(x)^2}{c+d\sin(x)}dx$$

$$\Big\downarrow \text{ 4901}$$

$$\int \left(\frac{a}{c+d\sin(x)}+\frac{b\cos^2(x)}{c+d\sin(x)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2a\arctan\left(\frac{c\tan\left(\frac{x}{2}\right)+d}{\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}}-\frac{2b\sqrt{c^2-d^2}\arctan\left(\frac{c\tan\left(\frac{x}{2}\right)+d}{\sqrt{c^2-d^2}}\right)}{d^2}+\frac{bcx}{d^2}+\frac{b\cos(x)}{d}$$

input
```
Int[(a + b*Cos[x]^2)/(c + d*Sin[x]),x]
```

output
```
(b*c*x)/d^2 + (2*a*ArcTan[(d + c*Tan[x/2])/Sqrt[c^2 - d^2]])/Sqrt[c^2 - d^
2] - (2*b*Sqrt[c^2 - d^2]*ArcTan[(d + c*Tan[x/2])/Sqrt[c^2 - d^2]])/d^2 +
(b*Cos[x])/d
```

3.211.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4901
```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

3.211.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.86

method	result
default	$\frac{2(a\,d^2-c^2b+b\,d^2)\arctan\left(\frac{2c\tan\left(\frac{x}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{d^2\sqrt{c^2-d^2}}+\frac{2b\left(\frac{d}{1+\tan\left(\frac{x}{2}\right)^2}+c\arctan\left(\tan\left(\frac{x}{2}\right)\right)\right)}{d^2}$
parts	$\frac{2a\arctan\left(\frac{2c\tan\left(\frac{x}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{\sqrt{c^2-d^2}}+b\left(\frac{\frac{2d}{1+\tan\left(\frac{x}{2}\right)^2}+2c\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{d^2}+\frac{2(-c^2+d^2)\arctan\left(\frac{2c\tan\left(\frac{x}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{d^2\sqrt{c^2-d^2}}\right)$
risch	$\frac{bcx}{d^2}+\frac{b\,e^{ix}}{2d}+\frac{b\,e^{-ix}}{2d}-\frac{\ln\left(e^{ix}+\frac{i\sqrt{-c^2+d^2}\,c-c^2+d^2}{d\sqrt{-c^2+d^2}}\right)a}{\sqrt{-c^2+d^2}}+\frac{\ln\left(e^{ix}+\frac{i\sqrt{-c^2+d^2}\,c-c^2+d^2}{d\sqrt{-c^2+d^2}}\right)c^2b}{\sqrt{-c^2+d^2}\,d^2}-\frac{\ln\left(e^{ix}+\frac{i\sqrt{-c^2+d^2}\,c-c^2+d^2}{d\sqrt{-c^2+d^2}}\right)b}{\sqrt{-c^2+d^2}}$

input
```
int((a+b*cos(x)^2)/(c+d*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
2*(a*d^2-b*c^2+b*d^2)/d^2/(c^2-d^2)^(1/2)*arctan(1/2*(2*c*tan(1/2*x)+2*d)/
(c^2-d^2)^(1/2))+2*b/d^2*(d/(1+tan(1/2*x)^2)+c*arctan(tan(1/2*x)))
```

3.211.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 262, normalized size of antiderivative = 2.62

$$\int \frac{a+b\cos^2(x)}{c+d\sin(x)}\,dx$$
$$=\left[\frac{(bc^2-(a+b)d^2)\sqrt{-c^2+d^2}\log\left(\frac{(2c^2-d^2)\cos(x)^2-2cd\sin(x)-c^2-d^2+2(c\cos(x)\sin(x)+d\cos(x))\sqrt{-c^2+d^2}}{d^2\cos(x)^2-2cd\sin(x)-c^2-d^2}\right)+2(bc^3-b}{2(c^2d^2-d^4)}\right.$$

input
```
integrate((a+b*cos(x)^2)/(c+d*sin(x)),x, algorithm="fricas")
```

output
```
[1/2*((b*c^2 - (a + b)*d^2)*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(x)^2 -
 2*c*d*sin(x) - c^2 - d^2 + 2*(c*cos(x)*sin(x) + d*cos(x))*sqrt(-c^2 + d^2
))/(d^2*cos(x)^2 - 2*c*d*sin(x) - c^2 - d^2)) + 2*(b*c^3 - b*c*d^2)*x + 2*
(b*c^2*d - b*d^3)*cos(x))/(c^2*d^2 - d^4), ((b*c^2 - (a + b)*d^2)*sqrt(c^2
 - d^2)*arctan(-(c*sin(x) + d)/(sqrt(c^2 - d^2)*cos(x))) + (b*c^3 - b*c*d^
2)*x + (b*c^2*d - b*d^3)*cos(x))/(c^2*d^2 - d^4)]
```

3.211.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2035 vs. $2(85) = 170$.

Time = 110.01 (sec) , antiderivative size = 2035, normalized size of antiderivative = 20.35

$$\int \frac{a + b\cos^2(x)}{c + d\sin(x)}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*cos(x)**2)/(c+d*sin(x)),x)
```

output
```
Piecewise((zoo*(a*log(tan(x/2))*tan(x/2)**2/(tan(x/2)**2 + 1) + a*log(tan(
x/2))/(tan(x/2)**2 + 1) + b*log(tan(x/2))*tan(x/2)**2/(tan(x/2)**2 + 1) +
b*log(tan(x/2))/(tan(x/2)**2 + 1) + 2*b/(tan(x/2)**2 + 1)), Eq(c, 0) & Eq(
d, 0)), ((a*log(tan(x/2))*tan(x/2)**2/(tan(x/2)**2 + 1) + a*log(tan(x/2))/
(tan(x/2)**2 + 1) + b*log(tan(x/2))*tan(x/2)**2/(tan(x/2)**2 + 1) + b*log(
tan(x/2))/(tan(x/2)**2 + 1) + 2*b/(tan(x/2)**2 + 1))/d, Eq(c, 0)), (2*a*sq
rt(d**2)*tan(x/2)**2/(d**2*tan(x/2)**3 + d**2*tan(x/2) - d*sqrt(d**2)*tan(
x/2)**2 - d*sqrt(d**2)) + 2*a*sqrt(d**2)/(d**2*tan(x/2)**3 + d**2*tan(x/2)
 - d*sqrt(d**2)*tan(x/2)**2 - d*sqrt(d**2)) + b*d*x*tan(x/2)**2/(d**2*tan(
x/2)**3 + d**2*tan(x/2) - d*sqrt(d**2)*tan(x/2)**2 - d*sqrt(d**2)) + b*d*x
/(d**2*tan(x/2)**3 + d**2*tan(x/2) - d*sqrt(d**2)*tan(x/2)**2 - d*sqrt(d**
2)) + 2*b*d*tan(x/2)/(d**2*tan(x/2)**3 + d**2*tan(x/2) - d*sqrt(d**2)*tan(
x/2)**2 - d*sqrt(d**2)) - b*x*sqrt(d**2)*tan(x/2)**3/(d**2*tan(x/2)**3 + d
**2*tan(x/2) - d*sqrt(d**2)*tan(x/2)**2 - d*sqrt(d**2)) - b*x*sqrt(d**2)*t
an(x/2)/(d**2*tan(x/2)**3 + d**2*tan(x/2) - d*sqrt(d**2)*tan(x/2)**2 - d*s
qrt(d**2)) - 2*b*sqrt(d**2)/(d**2*tan(x/2)**3 + d**2*tan(x/2) - d*sqrt(d**
2)*tan(x/2)**2 - d*sqrt(d**2)), Eq(c, -sqrt(d**2))), (-2*a*sqrt(d**2)*tan(
x/2)**2/(d**2*tan(x/2)**3 + d**2*tan(x/2) + d*sqrt(d**2)*tan(x/2)**2 + d*s
qrt(d**2)) - 2*a*sqrt(d**2)/(d**2*tan(x/2)**3 + d**2*tan(x/2) + d*sqrt(d**
2)*tan(x/2)**2 + d*sqrt(d**2)) + b*d*x*tan(x/2)**2/(d**2*tan(x/2)**3 + ...
```

3.211.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\cos^2(x)}{c + d\sin(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*cos(x)^2)/(c+d*sin(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` f
or more de
```

3.211.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.93

$$\int \frac{a+b\cos^2(x)}{c+d\sin(x)}\,dx = \frac{bcx}{d^2} - \frac{2\left(bc^2-ad^2-bd^2\right)\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}x\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\sqrt{c^2-d^2}\,d^2} + \frac{2\,b}{\left(\tan\left(\frac{1}{2}x\right)^2+1\right)d}$$

input
```
integrate((a+b*cos(x)^2)/(c+d*sin(x)),x, algorithm="giac")
```

output
```
b*c*x/d^2 - 2*(b*c^2 - a*d^2 - b*d^2)*(pi*floor(1/2*x/pi + 1/2)*sgn(c) + a
rctan((c*tan(1/2*x) + d)/sqrt(c^2 - d^2)))/(sqrt(c^2 - d^2)*d^2) + 2*b/((t
an(1/2*x)^2 + 1)*d)
```

3.211.9 Mupad [B] (verification not implemented)

Time = 30.62 (sec) , antiderivative size = 1646, normalized size of antiderivative = 16.46

$$\int \frac{a+b\cos^2(x)}{c+d\sin(x)}\,dx = \text{Too large to display}$$

input
```
int((a + b*cos(x)^2)/(c + d*sin(x)),x)
```

output
```
(2*b)/(d*(tan(x/2)^2 + 1)) - (atan((((-(c + d)*(c - d))^(1/2)*((32*b^2*c^4
)/d - (32*tan(x/2)*(a^2*c*d^5 + b^2*c*d^5 + 2*b^2*c^5*d - 4*b^2*c^3*d^3 +
2*a*b*c*d^5 - 2*a*b*c^3*d^3))/d^3 + ((-(c + d)*(c - d))^(1/2)*(32*a*c^2*d^
2 + (32*tan(x/2)*(2*a*c*d^6 - 2*b*c^3*d^4 + 2*b*c*d^6))/d^3 + ((-(c + d)*(
c - d))^(1/2)*(32*c^2*d^3 + (32*tan(x/2)*(3*c*d^7 - 2*c^3*d^5))/d^3)*(a*d^
2 - b*c^2 + b*d^2))/(d^4 - c^2*d^2))*(a*d^2 - b*c^2 + b*d^2))/(d^4 - c^2*d
^2))*(a*d^2 - b*c^2 + b*d^2)*1i)/(d^4 - c^2*d^2) - ((-(c + d)*(c - d))^(1/
2)*((32*tan(x/2)*(a^2*c*d^5 + b^2*c*d^5 + 2*b^2*c^5*d - 4*b^2*c^3*d^3 + 2*
a*b*c*d^5 - 2*a*b*c^3*d^3))/d^3 - (32*b^2*c^4)/d + ((-(c + d)*(c - d))^(1/
2)*(32*a*c^2*d^2 + (32*tan(x/2)*(2*a*c*d^6 - 2*b*c^3*d^4 + 2*b*c*d^6))/d^3
 - ((-(c + d)*(c - d))^(1/2)*(32*c^2*d^3 + (32*tan(x/2)*(3*c*d^7 - 2*c^3*d
^5))/d^3)*(a*d^2 - b*c^2 + b*d^2))/(d^4 - c^2*d^2))*(a*d^2 - b*c^2 + b*d^2
))/(d^4 - c^2*d^2))*(a*d^2 - b*c^2 + b*d^2)*1i)/(d^4 - c^2*d^2))/((64*(b^3
*c^2*d^2 - a*b^2*c^4 - b^3*c^4 + 2*a*b^2*c^2*d^2 + a^2*b*c^2*d^2))/d^2 + (
64*tan(x/2)*(2*b^3*c^3*d^2 - 2*b^3*c^5 + 2*a*b^2*c^3*d^2))/d^3 + ((-(c + d
)*(c - d))^(1/2)*((32*b^2*c^4)/d - (32*tan(x/2)*(a^2*c*d^5 + b^2*c*d^5 + 2
*b^2*c^5*d - 4*b^2*c^3*d^3 + 2*a*b*c*d^5 - 2*a*b*c^3*d^3))/d^3 + ((-(c + d
)*(c - d))^(1/2)*(32*a*c^2*d^2 + (32*tan(x/2)*(2*a*c*d^6 - 2*b*c^3*d^4 + 2
*b*c*d^6))/d^3 + ((-(c + d)*(c - d))^(1/2)*(32*c^2*d^3 + (32*tan(x/2)*(3*c
*d^7 - 2*c^3*d^5))/d^3)*(a*d^2 - b*c^2 + b*d^2))/(d^4 - c^2*d^2))*(a*d^...
```

3.211. $\int \frac{a+b\cos^2(x)}{c+d\sin(x)}\,dx$

3.212 $\int \frac{a+b\cos^2(x)}{c+c\sin^2(x)}\,dx$

3.212.1 Optimal result

Integrand size = 19, antiderivative size = 56

$$\int \frac{a+b\cos^2(x)}{c+c\sin^2(x)}\,dx=-\frac{bx}{c}+\frac{(a+2b)x}{\sqrt{2}c}+\frac{(a+2b)\arctan\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\sin^2(x)}\right)}{\sqrt{2}c}$$

output
```
-b*x/c+1/2*(a+2*b)*x/c*2^(1/2)+1/2*(a+2*b)*arctan(cos(x)*sin(x)/(1+sin(x)^
2+2^(1/2)))/c*2^(1/2)
```

3.212.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.55

$$\int \frac{a+b\cos^2(x)}{c+c\sin^2(x)}\,dx=-\frac{bx}{c}+\frac{(a+2b)\arctan\left(\sqrt{2}\tan(x)\right)}{\sqrt{2}c}$$

input
```
Integrate[(a + b*Cos[x]^2)/(c + c*Sin[x]^2),x]
```

output
```
-((b*x)/c) + ((a + 2*b)*ArcTan[Sqrt[2]*Tan[x]])/(Sqrt[2]*c)
```

3.212.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.59, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 4889, 1480, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\cos^2(x)}{c\sin^2(x)+c}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{a+b\cos(x)^2}{c\sin(x)^2+c}dx$$

$$\Big\downarrow \; 4889$$

$$\int \frac{a\tan^2(x)+a+b}{2c\tan^4(x)+3c\tan^2(x)+c}d\tan(x)$$

$$\Big\downarrow \; 1480$$

$$(a+2b)\int \frac{1}{2c\tan^2(x)+c}d\tan(x)-2b\int \frac{1}{2c\tan^2(x)+2c}d\tan(x)$$

$$\Big\downarrow \; 218$$

$$\frac{(a+2b)\arctan\left(\sqrt{2}\tan(x)\right)}{\sqrt{2}c}-\frac{b\arctan(\tan(x))}{c}$$

input
```
Int[(a + b*Cos[x]^2)/(c + c*Sin[x]^2),x]
```

output
```
-((b*ArcTan[Tan[x]])/c) + ((a + 2*b)*ArcTan[Sqrt[2]*Tan[x]])/(Sqrt[2]*c)
```

3.212.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.212.4 Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.52

method	result	size
default	$\frac{\frac{(a+2b)\sqrt{2}\,\arctan\left(\sqrt{2}\,\tan(x)\right)}{2}-b\arctan(\tan(x))}{c}$	29
parts	$\frac{a\sqrt{2}\,\arctan\left(\sqrt{2}\,\tan(x)\right)}{2c}+\frac{b\left(-\arctan(\tan(x))+\sqrt{2}\,\arctan\left(\sqrt{2}\,\tan(x)\right)\right)}{c}$	40
risch	$-\frac{bx}{c}+\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}-2\sqrt{2}-3\right)a}{4c}+\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}-2\sqrt{2}-3\right)b}{2c}-\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}+2\sqrt{2}-3\right)a}{4c}-\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}+2\sqrt{2}-3\right)b}{2c}$	101

input
```
int((a+b*cos(x)^2)/(c+c*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/c*(1/2*(a+2*b)*2^(1/2)*arctan(2^(1/2)*tan(x))-b*arctan(tan(x)))
```

3.212.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.80

$$\int \frac{a+b\cos^2(x)}{c+c\sin^2(x)}\,dx = -\frac{\sqrt{2}(a+2\,b)\arctan\left(\frac{3\,\sqrt{2}\cos(x)^2-2\,\sqrt{2}}{4\,\cos(x)\sin(x)}\right)+4\,bx}{4\,c}$$

input
```
integrate((a+b*cos(x)^2)/(c+c*sin(x)^2),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(2)*(a + 2*b)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - 2*sqrt(2))/(cos(x
)*sin(x))) + 4*b*x)/c
```

3.212.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 520 vs. 2(49) = 98.

Time = 22.71 (sec) , antiderivative size = 520, normalized size of antiderivative = 9.29

$$\begin{aligned}\int \frac{a+b\cos^2(x)}{c+c\sin^2(x)}\,dx &= \frac{54608393\sqrt{2}a\sqrt{3-2\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{31988856\sqrt{2}c+45239074c} \\
&+ \frac{77227930a\sqrt{3-2\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{31988856\sqrt{2}c+45239074c} \\
&+ \frac{9369319\sqrt{2}a\sqrt{2\sqrt{2}+3}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{2\sqrt{2}+3}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{31988856\sqrt{2}c+45239074c} \\
&+ \frac{13250218a\sqrt{2\sqrt{2}+3}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{2\sqrt{2}+3}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{31988856\sqrt{2}c+45239074c} \\
&- \frac{45239074bx}{31988856\sqrt{2}c+45239074c} - \frac{31988856\sqrt{2}bx}{31988856\sqrt{2}c+45239074c} \\
&+ \frac{109216786\sqrt{2}b\sqrt{3-2\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{31988856\sqrt{2}c+45239074c} \\
&+ \frac{154455860b\sqrt{3-2\sqrt{2}}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3-2\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{31988856\sqrt{2}c+45239074c} \\
&+ \frac{18738638\sqrt{2}b\sqrt{2\sqrt{2}+3}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{2\sqrt{2}+3}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{31988856\sqrt{2}c+45239074c} \\
&+ \frac{26500436b\sqrt{2\sqrt{2}+3}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{2\sqrt{2}+3}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{31988856\sqrt{2}c+45239074c}\end{aligned}$$

input
```
integrate((a+b*cos(x)**2)/(c+c*sin(x)**2),x)
```

output
```
54608393*sqrt(2)*a*sqrt(3 - 2*sqrt(2))*(atan(tan(x/2)/sqrt(3 - 2*sqrt(2)))
 + pi*floor((x/2 - pi/2)/pi))/(31988856*sqrt(2)*c + 45239074*c) + 77227930
*a*sqrt(3 - 2*sqrt(2))*(atan(tan(x/2)/sqrt(3 - 2*sqrt(2))) + pi*floor((x/2
 - pi/2)/pi))/(31988856*sqrt(2)*c + 45239074*c) + 9369319*sqrt(2)*a*sqrt(2
*sqrt(2) + 3)*(atan(tan(x/2)/sqrt(2*sqrt(2) + 3)) + pi*floor((x/2 - pi/2)/
pi))/(31988856*sqrt(2)*c + 45239074*c) + 13250218*a*sqrt(2*sqrt(2) + 3)*(a
tan(tan(x/2)/sqrt(2*sqrt(2) + 3)) + pi*floor((x/2 - pi/2)/pi))/(31988856*s
qrt(2)*c + 45239074*c) - 45239074*b*x/(31988856*sqrt(2)*c + 45239074*c) -
31988856*sqrt(2)*b*x/(31988856*sqrt(2)*c + 45239074*c) + 109216786*sqrt(2)
*b*sqrt(3 - 2*sqrt(2))*(atan(tan(x/2)/sqrt(3 - 2*sqrt(2))) + pi*floor((x/2
 - pi/2)/pi))/(31988856*sqrt(2)*c + 45239074*c) + 154455860*b*sqrt(3 - 2*s
qrt(2))*(atan(tan(x/2)/sqrt(3 - 2*sqrt(2))) + pi*floor((x/2 - pi/2)/pi))/(
31988856*sqrt(2)*c + 45239074*c) + 18738638*sqrt(2)*b*sqrt(2*sqrt(2) + 3)*
(atan(tan(x/2)/sqrt(2*sqrt(2) + 3)) + pi*floor((x/2 - pi/2)/pi))/(31988856
*sqrt(2)*c + 45239074*c) + 26500436*b*sqrt(2*sqrt(2) + 3)*(atan(tan(x/2)/s
qrt(2*sqrt(2) + 3)) + pi*floor((x/2 - pi/2)/pi))/(31988856*sqrt(2)*c + 452
39074*c)
```

3.212.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.50

$$\int \frac{a+b\cos^2(x)}{c+c\sin^2(x)}\,dx = \frac{\sqrt{2}(a+2\,b)\arctan\left(\sqrt{2}\tan\left(x\right)\right)}{2\,c} - \frac{bx}{c}$$

input
```
integrate((a+b*cos(x)^2)/(c+c*sin(x)^2),x, algorithm="maxima")
```

output
```
1/2*sqrt(2)*(a + 2*b)*arctan(sqrt(2)*tan(x))/c - b*x/c
```

3.212.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.11

$$\int \frac{a+b\cos^2(x)}{c+c\sin^2(x)}\,dx = \frac{\sqrt{2}(a+2b)\left(x+\arctan\left(-\frac{\sqrt{2}\sin(2x)-2\sin(2x)}{\sqrt{2}\cos(2x)+\sqrt{2}-2\cos(2x)+2}\right)\right)}{2c} - \frac{bx}{c}$$

input
```
integrate((a+b*cos(x)^2)/(c+c*sin(x)^2),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*(a + 2*b)*(x + arctan(-(sqrt(2)*sin(2*x) - 2*sin(2*x))/(sqrt(2
)*cos(2*x) + sqrt(2) - 2*cos(2*x) + 2)))/c - b*x/c
```

3.212.9 Mupad [B] (verification not implemented)

Time = 25.78 (sec) , antiderivative size = 249, normalized size of antiderivative = 4.45

$$\int \frac{a+b\cos^2(x)}{c+c\sin^2(x)}\,dx$$
$$= \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{4\sqrt{2}\,a^3\tan(x)}{4a^3+24a^2b+40ab^2+16b^3}+\frac{16\sqrt{2}\,b^3\tan(x)}{4a^3+24a^2b+40ab^2+16b^3}+\frac{40\sqrt{2}\,a\,b^2\tan(x)}{4a^3+24a^2b+40ab^2+16b^3}+\frac{24\sqrt{2}\,a^2\,b\tan(x)}{4a^3+24a^2b+40ab^2+16b^3}\right)(a}{2c}$$
$$- \frac{b\,\mathrm{atan}\left(\frac{8b^3\tan(x)}{4a^2b+16ab^2+8b^3}+\frac{16ab^2\tan(x)}{4a^2b+16ab^2+8b^3}+\frac{4a^2b\tan(x)}{4a^2b+16ab^2+8b^3}\right)}{c}$$

input
```
int((a + b*cos(x)^2)/(c + c*sin(x)^2),x)
```

output
```
(2^(1/2)*atan((4*2^(1/2)*a^3*tan(x))/(40*a*b^2 + 24*a^2*b + 4*a^3 + 16*b^3
) + (16*2^(1/2)*b^3*tan(x))/(40*a*b^2 + 24*a^2*b + 4*a^3 + 16*b^3) + (40*2
^(1/2)*a*b^2*tan(x))/(40*a*b^2 + 24*a^2*b + 4*a^3 + 16*b^3) + (24*2^(1/2)*
a^2*b*tan(x))/(40*a*b^2 + 24*a^2*b + 4*a^3 + 16*b^3))*(a + 2*b))/(2*c) - (
b*atan((8*b^3*tan(x))/(16*a*b^2 + 4*a^2*b + 8*b^3) + (16*a*b^2*tan(x))/(16
*a*b^2 + 4*a^2*b + 8*b^3) + (4*a^2*b*tan(x))/(16*a*b^2 + 4*a^2*b + 8*b^3))
)/c
```

3.213 $\int \frac{a+b\cos^2(x)}{c-c\sin^2(x)}\,dx$

3.213.1 Optimal result

Integrand size = 20, antiderivative size = 14

$$\int \frac{a+b\cos^2(x)}{c-c\sin^2(x)}\,dx=\frac{bx}{c}+\frac{a\tan(x)}{c}$$

output
```
b*x/c+a*tan(x)/c
```

3.213.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{a+b\cos^2(x)}{c-c\sin^2(x)}\,dx=\frac{bx}{c}+\frac{a\tan(x)}{c}$$

input
```
Integrate[(a + b*Cos[x]^2)/(c - c*Sin[x]^2),x]
```

output
```
(b*x)/c + (a*Tan[x])/c
```

3.213.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3654, 3042, 3491, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\cos^2(x)}{c-c\sin^2(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\cos(x)^2}{c-c\sin(x)^2}dx$$

$$\Big\downarrow \text{3654}$$

$$\frac{\int \left(b\cos^2(x)+a\right)\sec^2(x)dx}{c}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{b\sin\left(x+\frac{\pi}{2}\right)^2+a}{\sin\left(x+\frac{\pi}{2}\right)^2}dx}{c}$$

$$\Big\downarrow \text{3491}$$

$$\frac{b\int 1dx+a\tan(x)}{c}$$

$$\Big\downarrow \text{24}$$

$$\frac{a\tan(x)+bx}{c}$$

input
```
Int[(a + b*Cos[x]^2)/(c - c*Sin[x]^2),x]
```

output
```
(b*x + a*Tan[x])/c
```

3.213.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.213.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
parallelrisch	$\frac{xb+a\tan(x)}{c}$	13
default	$\frac{a\tan(x)+b\arctan(\tan(x))}{c}$	15
parts	$\frac{a\tan(x)}{c}+\frac{b\arctan(\tan(x))}{c}$	17
risch	$\frac{bx}{c}+\frac{2ia}{(e^{2ix}+1)c}$	24
norman	$\frac{\frac{bx\tan(\frac{x}{2})^4}{c}+\frac{bx\tan(\frac{x}{2})^6}{c}-\frac{2a\tan(\frac{x}{2})}{c}-\frac{4a\tan(\frac{x}{2})^3}{c}-\frac{2a\tan(\frac{x}{2})^5}{c}-\frac{bx}{c}-\frac{bx\tan(\frac{x}{2})^2}{c}}{\left(1+\tan(\frac{x}{2})^2\right)^2\left(\tan(\frac{x}{2})^2-1\right)}$	101

input
```
int((a+b*cos(x)^2)/(c-c*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
(x*b+a*tan(x))/c
```

3.213.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \frac{a + b\cos^2(x)}{c - c\sin^2(x)}\, dx = \frac{bx\cos(x) + a\sin(x)}{c\cos(x)}$$

input
```
integrate((a+b*cos(x)^2)/(c-c*sin(x)^2),x, algorithm="fricas")
```

output
```
(b*x*cos(x) + a*sin(x))/(c*cos(x))
```

3.213.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(10) = 20.

Time = 0.44 (sec) , antiderivative size = 51, normalized size of antiderivative = 3.64

$$\int \frac{a + b\cos^2(x)}{c - c\sin^2(x)}\, dx = -\frac{2a\tan\left(\frac{x}{2}\right)}{c\tan^2\left(\frac{x}{2}\right) - c} + \frac{bx\tan^2\left(\frac{x}{2}\right)}{c\tan^2\left(\frac{x}{2}\right) - c} - \frac{bx}{c\tan^2\left(\frac{x}{2}\right) - c}$$

input
```
integrate((a+b*cos(x)**2)/(c-c*sin(x)**2),x)
```

output
```
-2*a*tan(x/2)/(c*tan(x/2)**2 - c) + b*x*tan(x/2)**2/(c*tan(x/2)**2 - c) -
b*x/(c*tan(x/2)**2 - c)
```

3.213.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{a + b\cos^2(x)}{c - c\sin^2(x)}\, dx = \frac{bx}{c} + \frac{a\tan(x)}{c}$$

input
```
integrate((a+b*cos(x)^2)/(c-c*sin(x)^2),x, algorithm="maxima")
```

output
```
b*x/c + a*tan(x)/c
```

3.213.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.64

$$\int \frac{a+b\cos^2(x)}{c-c\sin^2(x)}\,dx=\frac{b\arctan\left(\frac{|c|\tan(x)}{c}\right)}{|c|}+\frac{a\tan(x)}{c}$$

input
```
integrate((a+b*cos(x)^2)/(c-c*sin(x)^2),x, algorithm="giac")
```

output
```
b*arctan(abs(c)*tan(x)/c)/abs(c) + a*tan(x)/c
```

3.213.9 Mupad [B] (verification not implemented)

Time = 26.72 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{a+b\cos^2(x)}{c-c\sin^2(x)}\,dx=\frac{b\,x+a\tan(x)}{c}$$

input
```
int((a + b*cos(x)^2)/(c - c*sin(x)^2),x)
```

output
```
(b*x + a*tan(x))/c
```

3.214 $\int \frac{a+b\cos^2(x)}{c+d\sin^2(x)}\,dx$

3.214.1 Optimal result

Integrand size = 19, antiderivative size = 49

$$\int \frac{a+b\cos^2(x)}{c+d\sin^2(x)}\,dx=-\frac{bx}{d}+\frac{(ad+b(c+d))\arctan\left(\frac{\sqrt{c+d}\tan(x)}{\sqrt{c}}\right)}{\sqrt{c}d\sqrt{c+d}}$$

output
```
-b*x/d+(a*d+b*(c+d))*arctan((c+d)^(1/2)*tan(x)/c^(1/2))/d/c^(1/2)/(c+d)^(1
/2)
```

3.214.2 Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.96

$$\int \frac{a+b\cos^2(x)}{c+d\sin^2(x)}\,dx=\frac{-bx+\frac{(ad+b(c+d))\arctan\left(\frac{\sqrt{c+d}\tan(x)}{\sqrt{c}}\right)}{\sqrt{c}\sqrt{c+d}}}{d}$$

input
```
Integrate[(a + b*Cos[x]^2)/(c + d*Sin[x]^2),x]
```

output
```
(-(b*x) + ((a*d + b*(c + d))*ArcTan[(Sqrt[c + d]*Tan[x])/Sqrt[c]])/(Sqrt[c
]*Sqrt[c + d]))/d
```

3.214.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3042, 4889, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\cos^2(x)}{c+d\sin^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\cos(x)^2}{c+d\sin(x)^2}dx$$

$$\Big\downarrow \text{4889}$$

$$\int \frac{a\tan^2(x)+a+b}{\left(\tan^2(x)+1\right)\left((c+d)\tan^2(x)+c\right)}d\tan(x)$$

$$\Big\downarrow \text{397}$$

$$\frac{(ad+b(c+d))\int \frac{1}{(c+d)\tan^2(x)+c}d\tan(x)}{d}-\frac{b\int \frac{1}{\tan^2(x)+1}d\tan(x)}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{(ad+b(c+d))\int \frac{1}{(c+d)\tan^2(x)+c}d\tan(x)}{d}-\frac{b\arctan(\tan(x))}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{(ad+b(c+d))\arctan\left(\frac{\sqrt{c+d}\tan(x)}{\sqrt{c}}\right)}{\sqrt{c}d\sqrt{c+d}}-\frac{b\arctan(\tan(x))}{d}$$

input
```
Int[(a + b*Cos[x]^2)/(c + d*Sin[x]^2),x]
```

output
```
-((b*ArcTan[Tan[x]])/d) + ((a*d + b*(c + d))*ArcTan[(Sqrt[c + d]*Tan[x])/S
qrt[c]])/(Sqrt[c]*d*Sqrt[c + d])
```

3.214.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.214.4 Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

method	result
default	$\frac{(ad+cb+bd)\arctan\left(\frac{(c+d)\tan(x)}{\sqrt{(c+d)c}}\right)}{d\sqrt{(c+d)c}}-\frac{b\arctan(\tan(x))}{d}$
parts	$\frac{a\arctan\left(\frac{(c+d)\tan(x)}{\sqrt{(c+d)c}}\right)}{\sqrt{(c+d)c}}+b\left(\frac{(c+d)\arctan\left(\frac{(c+d)\tan(x)}{\sqrt{(c+d)c}}\right)}{d\sqrt{(c+d)c}}-\frac{\arctan(\tan(x))}{d}\right)$
risch	$-\frac{bx}{d}-\frac{\ln\left(e^{2ix}-\frac{2ic^2+2idc+2\sqrt{-c^2-cd}\,c+\sqrt{-c^2-cd}\,d}{\sqrt{-c^2-cd}\,d}\right)a}{2\sqrt{-c^2-cd}}-\frac{\ln\left(e^{2ix}-\frac{2ic^2+2idc+2\sqrt{-c^2-cd}\,c+\sqrt{-c^2-cd}\,d}{\sqrt{-c^2-cd}\,d}\right)cb}{2\sqrt{-c^2-cd}\,d}-\frac{\ln\left(e^{2ix}-\frac{2ic^2+2i\cdots}{\cdots}\right.}{\cdots}$

3.214. $\int \frac{a+b\cos^2(x)}{c+d\sin^2(x)}\,dx$

input
```
int((a+b*cos(x)^2)/(c+d*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
(a*d+b*c+b*d)/d/((c+d)*c)^(1/2)*arctan((c+d)*tan(x)/((c+d)*c)^(1/2))-b/d*a
rctan(tan(x))
```

3.214.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(41) = 82$.

Time = 0.27 (sec) , antiderivative size = 255, normalized size of antiderivative = 5.20

$$\int \frac{a+b\cos^2(x)}{c+d\sin^2(x)}\,dx$$
$$= \left[-\frac{(bc+(a+b)d)\sqrt{-c^2-cd}\log\left(\frac{(8\,c^2+8\,cd+d^2)\cos(x)^4-2\,(4\,c^2+5\,cd+d^2)\cos(x)^2+4\,\left((2\,c+d)\cos(x)^3-(c+d)\cos(x)\right)\sqrt{-c^2-cd}}{d^2\cos(x)^4-2\,(cd+d^2)\cos(x)^2+c^2+2\,cd+d^2}\right.}{4\,(c^2d+cd^2)}\right.$$
$$\left.-\frac{(bc+(a+b)d)\sqrt{c^2+cd}\arctan\left(\frac{(2\,c+d)\cos(x)^2-c-d}{2\,\sqrt{c^2+cd}\cos(x)\sin(x)}\right)+2\,(bc^2+bcd)x}{2\,(c^2d+cd^2)}\right]$$

input
```
integrate((a+b*cos(x)^2)/(c+d*sin(x)^2),x, algorithm="fricas")
```

output
```
[-1/4*((b*c + (a + b)*d)*sqrt(-c^2 - c*d)*log(((8*c^2 + 8*c*d + d^2)*cos(x
)^4 - 2*(4*c^2 + 5*c*d + d^2)*cos(x)^2 + 4*((2*c + d)*cos(x)^3 - (c + d)*c
os(x))*sqrt(-c^2 - c*d)*sin(x) + c^2 + 2*c*d + d^2)/(d^2*cos(x)^4 - 2*(c*d
 + d^2)*cos(x)^2 + c^2 + 2*c*d + d^2)) + 4*(b*c^2 + b*c*d)*x)/(c^2*d + c*d
^2), -1/2*((b*c + (a + b)*d)*sqrt(c^2 + c*d)*arctan(1/2*((2*c + d)*cos(x)^
2 - c - d)/(sqrt(c^2 + c*d)*cos(x)*sin(x))) + 2*(b*c^2 + b*c*d)*x)/(c^2*d
+ c*d^2)]
```

3.214.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\cos^2(x)}{c+d\sin^2(x)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(x)**2)/(c+d*sin(x)**2),x)
```

output
```
Timed out
```

3.214.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \frac{a+b\cos^2(x)}{c+d\sin^2(x)}\,dx = -\frac{bx}{d} + \frac{(bc+(a+b)d)\arctan\left(\frac{(c+d)\tan(x)}{\sqrt{(c+d)c}}\right)}{\sqrt{(c+d)c}d}$$

input
```
integrate((a+b*cos(x)^2)/(c+d*sin(x)^2),x, algorithm="maxima")
```

output
```
-b*x/d + (b*c + (a + b)*d)*arctan((c + d)*tan(x)/sqrt((c + d)*c))/(sqrt((c
 + d)*c)*d)
```

3.214.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.43

$$\int \frac{a+b\cos^2(x)}{c+d\sin^2(x)}\,dx = -\frac{bx}{d} + \frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2c+2d)+\arctan\left(\frac{c\tan(x)+d\tan(x)}{\sqrt{c^2+cd}}\right)\right)(bc+ad+bd)}{\sqrt{c^2+cd}d}$$

input
```
integrate((a+b*cos(x)^2)/(c+d*sin(x)^2),x, algorithm="giac")
```

output
```
-b*x/d + (pi*floor(x/pi + 1/2)*sgn(2*c + 2*d) + arctan((c*tan(x) + d*tan(x
))/sqrt(c^2 + c*d)))*(b*c + a*d + b*d)/(sqrt(c^2 + c*d)*d)
```

3.214.9 Mupad [B] (verification not implemented)

Time = 28.67 (sec) , antiderivative size = 1774, normalized size of antiderivative = 36.20

$$\int \frac{a + b\cos^2(x)}{c + d\sin^2(x)}\,dx = \text{Too large to display}$$

input
```
int((a + b*cos(x)^2)/(c + d*sin(x)^2),x)
```

output
```
- (b*c^2*x)/(c*d^2 + c^2*d) - (a*d*atan((a^2*d^3*tan(x)*(- c*d - c^2)^(3/2
)*1i + b^2*c^3*tan(x)*(- c*d - c^2)^(3/2)*2i + b^2*c^5*tan(x)*(- c*d - c^2
)^(1/2)*2i + b^2*d^3*tan(x)*(- c*d - c^2)^(3/2)*1i + a^2*c*d^2*tan(x)*(- c
*d - c^2)^(3/2)*2i + b^2*c*d^2*tan(x)*(- c*d - c^2)^(3/2)*4i + b^2*c^2*d*t
an(x)*(- c*d - c^2)^(3/2)*5i + b^2*c^4*d*tan(x)*(- c*d - c^2)^(1/2)*6i + a
^2*c^2*d^3*tan(x)*(- c*d - c^2)^(1/2)*1i + a^2*c^3*d^2*tan(x)*(- c*d - c^2
)^(1/2)*1i + b^2*c^2*d^3*tan(x)*(- c*d - c^2)^(1/2)*2i + b^2*c^3*d^2*tan(x
)*(- c*d - c^2)^(1/2)*6i + a*b*d^3*tan(x)*(- c*d - c^2)^(3/2)*2i + a*b*c*d
^2*tan(x)*(- c*d - c^2)^(3/2)*6i + a*b*c^2*d*tan(x)*(- c*d - c^2)^(3/2)*4i
 + a*b*c^4*d*tan(x)*(- c*d - c^2)^(1/2)*2i + a*b*c^2*d^3*tan(x)*(- c*d - c
^2)^(1/2)*2i + a*b*c^3*d^2*tan(x)*(- c*d - c^2)^(1/2)*4i)/(b^2*c^5*d + a^2
*c^2*d^4 + 2*a^2*c^3*d^3 + a^2*c^4*d^2 + b^2*c^2*d^4 + 3*b^2*c^3*d^3 + 3*b
^2*c^4*d^2 + 2*a*b*c^5*d + 2*a*b*c^2*d^4 + 6*a*b*c^3*d^3 + 6*a*b*c^4*d^2))
*(- c*d - c^2)^(1/2)*1i)/(c*d^2 + c^2*d) - (b*c*atan((a^2*d^3*tan(x)*(- c*
d - c^2)^(3/2)*1i + b^2*c^3*tan(x)*(- c*d - c^2)^(3/2)*2i + b^2*c^5*tan(x)
*(- c*d - c^2)^(1/2)*2i + b^2*d^3*tan(x)*(- c*d - c^2)^(3/2)*1i + a^2*c*d^
2*tan(x)*(- c*d - c^2)^(3/2)*2i + b^2*c*d^2*tan(x)*(- c*d - c^2)^(3/2)*4i
+ b^2*c^2*d*tan(x)*(- c*d - c^2)^(3/2)*5i + b^2*c^4*d*tan(x)*(- c*d - c^2)
^(1/2)*6i + a^2*c^2*d^3*tan(x)*(- c*d - c^2)^(1/2)*1i + a^2*c^3*d^2*tan(x)
*(- c*d - c^2)^(1/2)*1i + b^2*c^2*d^3*tan(x)*(- c*d - c^2)^(1/2)*2i + b...
```

3.215 $\int \frac{a+b\sec^2(x)}{c+d\cos(x)}\,dx$

3.215.1 Optimal result

Integrand size = 17, antiderivative size = 74

$$\int \frac{a+b\sec^2(x)}{c+d\cos(x)}\,dx = \frac{2(ac^2+bd^2)\arctan\left(\frac{\sqrt{c-d}\tan\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{c^2\sqrt{c-d}\sqrt{c+d}} - \frac{bd\text{arctanh}(\sin(x))}{c^2} + \frac{b\tan(x)}{c}$$

output
```
-b*d*arctanh(sin(x))/c^2+2*(a*c^2+b*d^2)*arctan((c-d)^(1/2)*tan(1/2*x)/(c+
d)^(1/2))/c^2/(c-d)^(1/2)/(c+d)^(1/2)+b*tan(x)/c
```

3.215.2 Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.32

$$\int \frac{a+b\sec^2(x)}{c+d\cos(x)}\,dx$$
$$= \frac{-\frac{2(ac^2+bd^2)\text{arctanh}\left(\frac{(c-d)\tan\left(\frac{x}{2}\right)}{\sqrt{-c^2+d^2}}\right)}{\sqrt{-c^2+d^2}} + bd\left(\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\right)+bc\tan(x)}{c^2}$$

input
```
Integrate[(a + b*Sec[x]^2)/(c + d*Cos[x]),x]
```

output
```
((-2*(a*c^2 + b*d^2)*ArcTanh[((c - d)*Tan[x/2])/Sqrt[-c^2 + d^2]])/Sqrt[-c
^2 + d^2] + b*d*(Log[Cos[x/2] - Sin[x/2]] - Log[Cos[x/2] + Sin[x/2]]) + b*
c*Tan[x])/c^2
```

3.215.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$, Rules used = {3042, 4722, 3042, 3535, 25, 3042, 3480, 3042, 3138, 218, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec^2(x)}{c+d\cos(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\sec(x)^2}{c+d\cos(x)}dx$$

$$\Big\downarrow \text{4722}$$

$$\int \frac{\sec^2(x)\left(a\cos^2(x)+b\right)}{c+d\cos(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\sin\left(x+\frac{\pi}{2}\right)^2+b}{\sin\left(x+\frac{\pi}{2}\right)^2\left(c+d\sin\left(x+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3535}$$

$$\frac{\int -\frac{(bd-ac\cos(x))\sec(x)}{c+d\cos(x)}dx}{c}+\frac{b\tan(x)}{c}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\tan(x)}{c}-\frac{\int \frac{(bd-ac\cos(x))\sec(x)}{c+d\cos(x)}dx}{c}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\tan(x)}{c}-\frac{\int \frac{bd-ac\sin\left(x+\frac{\pi}{2}\right)}{\sin\left(x+\frac{\pi}{2}\right)\left(c+d\sin\left(x+\frac{\pi}{2}\right)\right)}dx}{c}$$

$$\Big\downarrow \text{3480}$$

$$\frac{b\tan(x)}{c}-\frac{\frac{bd\int \sec(x)dx}{c}-\frac{(ac^2+bd^2)\int \frac{1}{c+d\cos(x)}dx}{c}}{c}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\tan(x)}{c}-\frac{\frac{bd\int\csc\left(x+\frac{\pi}{2}\right)dx}{c}-\frac{\left(ac^2+bd^2\right)\int\frac{1}{c+d\sin\left(x+\frac{\pi}{2}\right)}dx}{c}}{c}$$

$$\Big\downarrow \text{3138}$$

$$\frac{b\tan(x)}{c}-\frac{\frac{bd\int\csc\left(x+\frac{\pi}{2}\right)dx}{c}-\frac{2\left(ac^2+bd^2\right)\int\frac{1}{(c-d)\tan^2\left(\frac{x}{2}\right)+c+d}d\tan\left(\frac{x}{2}\right)}{c}}{c}$$

$$\Big\downarrow \text{218}$$

$$\frac{b\tan(x)}{c}-\frac{\frac{bd\int\csc\left(x+\frac{\pi}{2}\right)dx}{c}-\frac{2\left(ac^2+bd^2\right)\arctan\left(\frac{\sqrt{c-d}\tan\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{c\sqrt{c-d}\sqrt{c+d}}}{c}$$

$$\Big\downarrow \text{4257}$$

$$\frac{b\tan(x)}{c}-\frac{\frac{bd\text{arctanh}(\sin(x))}{c}-\frac{2\left(ac^2+bd^2\right)\arctan\left(\frac{\sqrt{c-d}\tan\left(\frac{x}{2}\right)}{\sqrt{c+d}}\right)}{c\sqrt{c-d}\sqrt{c+d}}}{c}$$

input
```
Int[(a + b*Sec[x]^2)/(c + d*Cos[x]),x]
```

output
```
-(((-2*(a*c^2 + b*d^2)*ArcTan[(Sqrt[c - d]*Tan[x/2])/Sqrt[c + d]])/(c*Sqrt
[c - d]*Sqrt[c + d]) + (b*d*ArcTanh[Sin[x]])/c)/c) + (b*Tan[x])/c
```

3.215.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3535
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(A*b^2 + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*S
in[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m
+ 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n
+ 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d
*(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||
 !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a,
 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4722
```
Int[(u_)*((A_) + (C_.)*sec[(a_.) + (b_.)*(x_)]^2), x_Symbol] :> Int[Activat
eTrig[u]*((C + A*Cos[a + b*x]^2)/Cos[a + b*x]^2), x] /; FreeQ[{a, b, A, C},
 x] && KnownSineIntegrandQ[u, x]
```

3.215.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.43

method	result
default	$-\frac{b}{c(\tan(\frac{x}{2})-1)}+\frac{bd\ln(\tan(\frac{x}{2})-1)}{c^2}+\frac{2(a\,c^2+b\,d^2)\arctan\left(\frac{(c-d)\tan(\frac{x}{2})}{\sqrt{(c+d)(c-d)}}\right)}{c^2\sqrt{(c+d)(c-d)}}-\frac{b}{c(\tan(\frac{x}{2})+1)}-\frac{bd\ln(\tan(\frac{x}{2})+1)}{c^2}$
risch	$\frac{2ib}{c(e^{2ix}+1)}-\frac{\ln\left(e^{ix}+\frac{ic^2-id^2+\sqrt{-c^2+d^2}\,c}{d\sqrt{-c^2+d^2}}\right)a}{\sqrt{-c^2+d^2}}-\frac{\ln\left(e^{ix}+\frac{ic^2-id^2+\sqrt{-c^2+d^2}\,c}{d\sqrt{-c^2+d^2}}\right)b\,d^2}{\sqrt{-c^2+d^2}\,c^2}+\frac{\ln\left(e^{ix}-\frac{ic^2-id^2-\sqrt{-c^2+d^2}\,c}{d\sqrt{-c^2+d^2}}\right)a}{\sqrt{-c^2+d^2}}+\ln\Big(e$

3.215. $\int \frac{a+b\sec^2(x)}{c+d\cos(x)}\,dx$

input
```
int((a+b*sec(x)^2)/(c+d*cos(x)),x,method=_RETURNVERBOSE)
```

output
```
-b/c/(tan(1/2*x)-1)+b*d/c^2*ln(tan(1/2*x)-1)+2*(a*c^2+b*d^2)/c^2/((c+d)*(c
-d))^(1/2)*arctan((c-d)*tan(1/2*x)/((c+d)*(c-d))^(1/2))-b/c/(tan(1/2*x)+1)
-b*d/c^2*ln(tan(1/2*x)+1)
```

3.215.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(64) = 128$.

Time = 0.63 (sec) , antiderivative size = 318, normalized size of antiderivative = 4.30

$$\int \frac{a+b\sec^2(x)}{c+d\cos(x)}\,dx = \left[-\frac{(ac^2+bd^2)\sqrt{-c^2+d^2}\cos(x)\log\left(\frac{2\,cd\cos(x)+(2\,c^2-d^2)\cos(x)^2+2\,\sqrt{-c^2+d^2}(c\cos(x)+d)\sin(x)-c^2+2\,d^2}{d^2\cos(x)^2+2\,cd\cos(x)+c^2}\right)+(bc^2d-bc}{2\,(c^4-c^2d^2)\cos}\right.$$

input
```
integrate((a+b*sec(x)^2)/(c+d*cos(x)),x, algorithm="fricas")
```

output
```
[-1/2*((a*c^2 + b*d^2)*sqrt(-c^2 + d^2)*cos(x)*log((2*c*d*cos(x) + (2*c^2
- d^2)*cos(x)^2 + 2*sqrt(-c^2 + d^2)*(c*cos(x) + d)*sin(x) - c^2 + 2*d^2)/
(d^2*cos(x)^2 + 2*c*d*cos(x) + c^2)) + (b*c^2*d - b*d^3)*cos(x)*log(sin(x)
 + 1) - (b*c^2*d - b*d^3)*cos(x)*log(-sin(x) + 1) - 2*(b*c^3 - b*c*d^2)*si
n(x))/((c^4 - c^2*d^2)*cos(x)), 1/2*(2*(a*c^2 + b*d^2)*sqrt(c^2 - d^2)*arc
tan(-(c*cos(x) + d)/(sqrt(c^2 - d^2)*sin(x)))*cos(x) - (b*c^2*d - b*d^3)*c
os(x)*log(sin(x) + 1) + (b*c^2*d - b*d^3)*cos(x)*log(-sin(x) + 1) + 2*(b*c
^3 - b*c*d^2)*sin(x))/((c^4 - c^2*d^2)*cos(x))]
```

3.215.6 Sympy [F]

$$\int \frac{a+b\sec^2(x)}{c+d\cos(x)}\,dx = \int \frac{a+b\sec^2{(x)}}{c+d\cos{(x)}}\,dx$$

input
```
integrate((a+b*sec(x)**2)/(c+d*cos(x)),x)
```

output
```
Integral((a + b*sec(x)**2)/(c + d*cos(x)), x)
```

3.215.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\sec^2(x)}{c + d\cos(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*sec(x)^2)/(c+d*cos(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` f
or more de
```

3.215.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.69

$$\begin{aligned}\int \frac{a + b\sec^2(x)}{c + d\cos(x)}\,dx &= -\frac{bd\log\left(\left|\tan\left(\frac{1}{2}\,x\right) + 1\right|\right)}{c^2} + \frac{bd\log\left(\left|\tan\left(\frac{1}{2}\,x\right) - 1\right|\right)}{c^2} - \frac{2\,b\tan\left(\frac{1}{2}\,x\right)}{\left(\tan\left(\frac{1}{2}\,x\right)^2 - 1\right)c} \\ &\quad - \frac{2\,(ac^2 + bd^2)\left(\pi\left\lfloor\frac{x}{2\,\pi} + \frac{1}{2}\right\rfloor\operatorname{sgn}(-2\,c + 2\,d) + \arctan\left(-\frac{c\tan\left(\frac{1}{2}\,x\right) - d\tan\left(\frac{1}{2}\,x\right)}{\sqrt{c^2 - d^2}}\right)\right)}{\sqrt{c^2 - d^2}\,c^2}\end{aligned}$$

input
```
integrate((a+b*sec(x)^2)/(c+d*cos(x)),x, algorithm="giac")
```

output
```
-b*d*log(abs(tan(1/2*x) + 1))/c^2 + b*d*log(abs(tan(1/2*x) - 1))/c^2 - 2*b
*tan(1/2*x)/((tan(1/2*x)^2 - 1)*c) - 2*(a*c^2 + b*d^2)*(pi*floor(1/2*x/pi
+ 1/2)*sgn(-2*c + 2*d) + arctan(-(c*tan(1/2*x) - d*tan(1/2*x))/sqrt(c^2 -
d^2)))/(sqrt(c^2 - d^2)*c^2)
```

3.215.9 Mupad [B] (verification not implemented)

Time = 28.61 (sec) , antiderivative size = 1302, normalized size of antiderivative = 17.59

$$\int \frac{a+b\sec^2(x)}{c+d\cos(x)}\,dx = \text{Too large to display}$$

input
```
int((a + b/cos(x)^2)/(c + d*cos(x)),x)
```

output
```
(b*c^3*sin(x))/(c^4*cos(x) - c^2*d^2*cos(x)) - (b*c*d^2*sin(x))/(c^4*cos(x
) - c^2*d^2*cos(x)) + (2*b*d^3*atanh(sin(x/2)/cos(x/2))*cos(x))/(c^4*cos(x
) - c^2*d^2*cos(x)) + (a*c^2*atan((a^2*c^7*sin(x/2)*(d^2 - c^2)^(1/2)*1i +
 b^2*d^5*sin(x/2)*(d^2 - c^2)^(3/2)*2i - b^2*d^7*sin(x/2)*(d^2 - c^2)^(1/2
)*2i + a^2*c^4*d*sin(x/2)*(d^2 - c^2)^(3/2)*2i + a^2*c^6*d*sin(x/2)*(d^2 -
 c^2)^(1/2)*1i - a^2*c^4*d^3*sin(x/2)*(d^2 - c^2)^(1/2)*1i - a^2*c^5*d^2*s
in(x/2)*(d^2 - c^2)^(1/2)*1i + b^2*c^2*d^5*sin(x/2)*(d^2 - c^2)^(1/2)*3i -
 b^2*c^3*d^4*sin(x/2)*(d^2 - c^2)^(1/2)*1i - b^2*c^4*d^3*sin(x/2)*(d^2 - c
^2)^(1/2)*1i + b^2*c^5*d^2*sin(x/2)*(d^2 - c^2)^(1/2)*1i + a*b*c^2*d^3*sin
(x/2)*(d^2 - c^2)^(3/2)*4i - a*b*c^2*d^5*sin(x/2)*(d^2 - c^2)^(1/2)*2i - a
*b*c^3*d^4*sin(x/2)*(d^2 - c^2)^(1/2)*2i + a*b*c^4*d^3*sin(x/2)*(d^2 - c^2
)^(1/2)*2i + a*b*c^5*d^2*sin(x/2)*(d^2 - c^2)^(1/2)*2i)/(a^2*c^8*cos(x/2)
+ a^2*c^4*d^4*cos(x/2) - 2*a^2*c^6*d^2*cos(x/2) + b^2*c^2*d^6*cos(x/2) - 2
*b^2*c^4*d^4*cos(x/2) + b^2*c^6*d^2*cos(x/2) + 2*a*b*c^2*d^6*cos(x/2) - 4*
a*b*c^4*d^4*cos(x/2) + 2*a*b*c^6*d^2*cos(x/2)))*cos(x)*(d^2 - c^2)^(1/2)*2
i)/(c^4*cos(x) - c^2*d^2*cos(x)) + (b*d^2*atan((a^2*c^7*sin(x/2)*(d^2 - c^
2)^(1/2)*1i + b^2*d^5*sin(x/2)*(d^2 - c^2)^(3/2)*2i - b^2*d^7*sin(x/2)*(d^
2 - c^2)^(1/2)*2i + a^2*c^4*d*sin(x/2)*(d^2 - c^2)^(3/2)*2i + a^2*c^6*d*si
n(x/2)*(d^2 - c^2)^(1/2)*1i - a^2*c^4*d^3*sin(x/2)*(d^2 - c^2)^(1/2)*1i -
a^2*c^5*d^2*sin(x/2)*(d^2 - c^2)^(1/2)*1i + b^2*c^2*d^5*sin(x/2)*(d^2 -...
```

3.216 $\int \frac{a+b\csc^2(x)}{c+d\sin(x)}\,dx$

3.216.1 Optimal result

Integrand size = 17, antiderivative size = 72

$$\int \frac{a+b\csc^2(x)}{c+d\sin(x)}\,dx=\frac{2(ac^2+bd^2)\arctan\left(\frac{d+c\tan\left(\frac{x}{2}\right)}{\sqrt{c^2-d^2}}\right)}{c^2\sqrt{c^2-d^2}}+\frac{bd\text{arctanh}(\cos(x))}{c^2}-\frac{b\cot(x)}{c}$$

output
```
b*d*arctanh(cos(x))/c^2-b*cot(x)/c+2*(a*c^2+b*d^2)*arctan((d+c*tan(1/2*x))
/(c^2-d^2)^(1/2))/c^2/(c^2-d^2)^(1/2)
```

3.216.2 Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.42

$$\int \frac{a+b\csc^2(x)}{c+d\sin(x)}\,dx$$
$$=\frac{\csc\left(\frac{x}{2}\right)\sec\left(\frac{x}{2}\right)\left(\frac{2(ac^2+bd^2)\arctan\left(\frac{d+c\tan\left(\frac{x}{2}\right)}{\sqrt{c^2-d^2}}\right)\sin(x)}{\sqrt{c^2-d^2}}-b\left(c\cos(x)+d\left(-\log\left(\cos\left(\frac{x}{2}\right)\right)+\log\left(\sin\left(\frac{x}{2}\right)\right)\right)\sin(x)\right)\right)}{2c^2}$$

input
```
Integrate[(a + b*Csc[x]^2)/(c + d*Sin[x]),x]
```

output
```
(Csc[x/2]*Sec[x/2]*((2*(a*c^2 + b*d^2)*ArcTan[(d + c*Tan[x/2])/Sqrt[c^2 -
d^2]]*Sin[x])/Sqrt[c^2 - d^2] - b*(c*Cos[x] + d*(-Log[Cos[x/2]] + Log[Sin[
x/2]])*Sin[x])))/(2*c^2)
```

3.216.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.18, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.706, Rules used = {3042, 4721, 3042, 3535, 25, 3042, 3480, 3042, 3139, 1083, 217, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\csc^2(x)}{c+d\sin(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\csc(x)^2}{c+d\sin(x)}dx$$

$$\Big\downarrow \text{4721}$$

$$\int \frac{\csc^2(x)\left(a\sin^2(x)+b\right)}{c+d\sin(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\sin(x)^2+b}{\sin(x)^2(c+d\sin(x))}dx$$

$$\Big\downarrow \text{3535}$$

$$\frac{\int -\frac{\csc(x)(bd-ac\sin(x))}{c+d\sin(x)}dx}{c}-\frac{b\cot(x)}{c}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\csc(x)(bd-ac\sin(x))}{c+d\sin(x)}dx}{c}-\frac{b\cot(x)}{c}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{bd-ac\sin(x)}{\sin(x)(c+d\sin(x))}dx}{c}-\frac{b\cot(x)}{c}$$

$$\Big\downarrow \text{3480}$$

$$-\frac{\frac{bd\int \csc(x)dx}{c}-\frac{(ac^2+bd^2)\int \frac{1}{c+d\sin(x)}dx}{c}}{c}-\frac{b\cot(x)}{c}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{bd\int \csc(x)dx}{c}-\frac{(ac^2+bd^2)\int \frac{1}{c+d\sin(x)}dx}{c}}{c}-\frac{b\cot(x)}{c}$$

$$\Big\downarrow \text{3139}$$

$$-\frac{\frac{bd\int \csc(x)dx}{c}-\frac{2(ac^2+bd^2)\int \frac{1}{c\tan^2\left(\frac{x}{2}\right)+2d\tan\left(\frac{x}{2}\right)+c}d\tan\left(\frac{x}{2}\right)}{c}}{c}-\frac{b\cot(x)}{c}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{\frac{4(ac^2+bd^2)\int \frac{1}{-\left(2d+2c\tan\left(\frac{x}{2}\right)\right)^2-4(c^2-d^2)}d\left(2d+2c\tan\left(\frac{x}{2}\right)\right)}{c}+\frac{bd\int \csc(x)dx}{c}}{c}-\frac{b\cot(x)}{c}$$

$$\Big\downarrow \text{217}$$

$$-\frac{\frac{bd\int \csc(x)dx}{c}-\frac{2(ac^2+bd^2)\arctan\left(\frac{2c\tan\left(\frac{x}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{c\sqrt{c^2-d^2}}}{c}-\frac{b\cot(x)}{c}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{-\frac{2(ac^2+bd^2)\arctan\left(\frac{2c\tan\left(\frac{x}{2}\right)+2d}{2\sqrt{c^2-d^2}}\right)}{c\sqrt{c^2-d^2}}-\frac{bd\text{arctanh}(\cos(x))}{c}}{c}-\frac{b\cot(x)}{c}$$

input
```
Int[(a + b*Csc[x]^2)/(c + d*Sin[x]),x]
```

output
```
-(((-2*(a*c^2 + b*d^2)*ArcTan[(2*d + 2*c*Tan[x/2])/(2*Sqrt[c^2 - d^2])])/(
c*Sqrt[c^2 - d^2]) - (b*d*ArcTanh[Cos[x]])/c)/c) - (b*Cot[x])/c
```

3.216.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

3.216. $\int \frac{a+b\csc^2(x)}{c+d\sin(x)}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3535
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(A*b^2 + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*S
in[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m
+ 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n
+ 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d
*(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||
 !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a,
 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4721
```
Int[(csc[(a_.) + (b_.)*(x_)]^2*(C_.) + (A_))*(u_), x_Symbol] :> Int[Activat
eTrig[u]*((C + A*Sin[a + b*x]^2)/Sin[a + b*x]^2), x] /; FreeQ[{a, b, A, C},
 x] && KnownSineIntegrandQ[u, x]
```

3.216. $\int \frac{a+b\csc^2(x)}{c+d\sin(x)}\,dx$

3.216.4 Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.25

method	result
default	$\frac{\tan\left(\frac{x}{2}\right)b}{2c}+\frac{\left(4a\,c^{2}+4b\,d^{2}\right)\arctan\left(\frac{2c\tan\left(\frac{x}{2}\right)+2d}{2\sqrt{c^{2}-d^{2}}}\right)}{2c^{2}\sqrt{c^{2}-d^{2}}}-\frac{b}{2c\tan\left(\frac{x}{2}\right)}-\frac{bd\ln\left(\tan\left(\frac{x}{2}\right)\right)}{c^{2}}$
parts	$\frac{2a\arctan\left(\frac{2c\tan\left(\frac{x}{2}\right)+2d}{2\sqrt{c^{2}-d^{2}}}\right)}{\sqrt{c^{2}-d^{2}}}+b\left(\frac{\tan\left(\frac{x}{2}\right)}{2c}-\frac{1}{2c\tan\left(\frac{x}{2}\right)}-\frac{d\ln\left(\tan\left(\frac{x}{2}\right)\right)}{c^{2}}+\frac{2d^{2}\arctan\left(\frac{2c\tan\left(\frac{x}{2}\right)+2d}{2\sqrt{c^{2}-d^{2}}}\right)}{c^{2}\sqrt{c^{2}-d^{2}}}\right)$
risch	$-\frac{2ib}{c\left(e^{2ix}-1\right)}-\frac{\ln\left(e^{ix}+\frac{i\sqrt{-c^{2}+d^{2}}\,c-c^{2}+d^{2}}{d\sqrt{-c^{2}+d^{2}}}\right)a}{\sqrt{-c^{2}+d^{2}}}-\frac{\ln\left(e^{ix}+\frac{i\sqrt{-c^{2}+d^{2}}\,c-c^{2}+d^{2}}{d\sqrt{-c^{2}+d^{2}}}\right)b\,d^{2}}{\sqrt{-c^{2}+d^{2}}\,c^{2}}+\frac{\ln\left(e^{ix}+\frac{i\sqrt{-c^{2}+d^{2}}\,c+c^{2}-d^{2}}{d\sqrt{-c^{2}+d^{2}}}\right)a}{\sqrt{-c^{2}+d^{2}}}+\frac{\ln(e}{}$

input
```
int((a+b*csc(x)^2)/(c+d*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*tan(1/2*x)*b/c+1/2/c^2*(4*a*c^2+4*b*d^2)/(c^2-d^2)^(1/2)*arctan(1/2*(2
*c*tan(1/2*x)+2*d)/(c^2-d^2)^(1/2))-1/2*b/c/tan(1/2*x)-b*d/c^2*ln(tan(1/2*
x))
```

3.216.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. 2(66) = 132.

Time = 0.59 (sec) , antiderivative size = 332, normalized size of antiderivative = 4.61

$$\int \frac{a+b\csc^2(x)}{c+d\sin(x)}\,dx$$
$$= \left[-\frac{(ac^2+bd^2)\sqrt{-c^2+d^2}\log\left(\frac{(2\,c^2-d^2)\cos(x)^2-2\,cd\sin(x)-c^2-d^2+2\,(c\cos(x)\sin(x)+d\cos(x))\sqrt{-c^2+d^2}}{d^2\cos(x)^2-2\,cd\sin(x)-c^2-d^2}\right)\sin(x)-(bc^2d-}{2\,(c^4-c^2d^2}\right.$$
$$-\frac{2\,(ac^2+bd^2)\sqrt{c^2-d^2}\arctan\left(-\frac{c\sin(x)+d}{\sqrt{c^2-d^2}\cos(x)}\right)\sin(x)-(bc^2d-bd^3)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)\sin(x)+(bc^2d-}{2\,(c^4-c^2d^2)\sin(x)}$$

input
```
integrate((a+b*csc(x)^2)/(c+d*sin(x)),x, algorithm="fricas")
```

output
```
[-1/2*((a*c^2 + b*d^2)*sqrt(-c^2 + d^2)*log(((2*c^2 - d^2)*cos(x)^2 - 2*c*
d*sin(x) - c^2 - d^2 + 2*(c*cos(x)*sin(x) + d*cos(x))*sqrt(-c^2 + d^2))/(d
^2*cos(x)^2 - 2*c*d*sin(x) - c^2 - d^2))*sin(x) - (b*c^2*d - b*d^3)*log(1/
2*cos(x) + 1/2)*sin(x) + (b*c^2*d - b*d^3)*log(-1/2*cos(x) + 1/2)*sin(x) +
 2*(b*c^3 - b*c*d^2)*cos(x))/((c^4 - c^2*d^2)*sin(x)), -1/2*(2*(a*c^2 + b*
d^2)*sqrt(c^2 - d^2)*arctan(-(c*sin(x) + d)/(sqrt(c^2 - d^2)*cos(x)))*sin(
x) - (b*c^2*d - b*d^3)*log(1/2*cos(x) + 1/2)*sin(x) + (b*c^2*d - b*d^3)*lo
g(-1/2*cos(x) + 1/2)*sin(x) + 2*(b*c^3 - b*c*d^2)*cos(x))/((c^4 - c^2*d^2)
*sin(x))]
```

3.216.6 Sympy [F]

$$\int \frac{a + b\csc^2(x)}{c + d\sin(x)}\,dx = \int \frac{a + b\csc^2{(x)}}{c + d\sin{(x)}}\,dx$$

input
```
integrate((a+b*csc(x)**2)/(c+d*sin(x)),x)
```

output
```
Integral((a + b*csc(x)**2)/(c + d*sin(x)), x)
```

3.216.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\csc^2(x)}{c + d\sin(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*csc(x)^2)/(c+d*sin(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d^2-4*c^2>0)', see `assume?` f
or more de
```

3.216.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.53

$$\int \frac{a+b\csc^2(x)}{c+d\sin(x)}\,dx = -\frac{bd\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)}{c^2}+\frac{b\tan\left(\frac{1}{2}\,x\right)}{2\,c}+\frac{2\left(ac^2+bd^2\right)\left(\pi\left\lfloor\frac{x}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(c)+\arctan\left(\frac{c\tan\left(\frac{1}{2}\,x\right)+d}{\sqrt{c^2-d^2}}\right)\right)}{\sqrt{c^2-d^2}c^2}+\frac{2\,bd\tan\left(\frac{1}{2}\,x\right)-bc}{2\,c^2\tan\left(\frac{1}{2}\,x\right)}$$

input
```
integrate((a+b*csc(x)^2)/(c+d*sin(x)),x, algorithm="giac")
```

output
```
-b*d*log(abs(tan(1/2*x)))/c^2 + 1/2*b*tan(1/2*x)/c + 2*(a*c^2 + b*d^2)*(pi
*floor(1/2*x/pi + 1/2)*sgn(c) + arctan((c*tan(1/2*x) + d)/sqrt(c^2 - d^2))
)/(sqrt(c^2 - d^2)*c^2) + 1/2*(2*b*d*tan(1/2*x) - b*c)/(c^2*tan(1/2*x))
```

3.216.9 Mupad [B] (verification not implemented)

Time = 27.90 (sec) , antiderivative size = 463, normalized size of antiderivative = 6.43

$$\int \frac{a+b\csc^2(x)}{c+d\sin(x)}\,dx = \frac{b\,d^3\,\ln\left(\tan\left(\frac{x}{2}\right)\right)-b\,c^2\,d\,\ln\left(\tan\left(\frac{x}{2}\right)\right)+a\,c^2\,\mathrm{atan}\left(\frac{a\,c^3\,\sqrt{d^2-c^2}\,1\mathrm{i}+b\,d^3\,\tan\left(\frac{x}{2}\right)\sqrt{d^2-c^2}\,4\mathrm{i}+b\,c\,d^2\,\sqrt{d^2-c^2}\,2\mathrm{i}+a\,c^2\,d\tan\left(\frac{x}{2}\right)\sqrt{d^2}}{4\,b\,d^4\,\tan\left(\frac{x}{2}\right)-a\,c^4\,\tan\left(\frac{x}{2}\right)+a\,c^3\,d+2\,b\,c\,d^3-b\,c^3\,d+2\,a\,c^2\,d^2\,\tan(}\right.}{} - \frac{b\,c^3-b\,c\,d^2}{c^4\tan\left(x\right)-c^2\,d^2\tan\left(x\right)}$$

input
```
int((a + b/sin(x)^2)/(c + d*sin(x)),x)
```

output
```
(b*d^3*log(tan(x/2)) + a*c^2*atan((a*c^3*(d^2 - c^2)^(1/2)*1i + b*d^3*tan(
x/2)*(d^2 - c^2)^(1/2)*4i + b*c*d^2*(d^2 - c^2)^(1/2)*2i + a*c^2*d*tan(x/2
)*(d^2 - c^2)^(1/2)*2i - b*c^2*d*tan(x/2)*(d^2 - c^2)^(1/2)*1i)/(4*b*d^4*t
an(x/2) - a*c^4*tan(x/2) + a*c^3*d + 2*b*c*d^3 - b*c^3*d + 2*a*c^2*d^2*tan
(x/2) - 3*b*c^2*d^2*tan(x/2)))*(d^2 - c^2)^(1/2)*2i + b*d^2*atan((a*c^3*(d
^2 - c^2)^(1/2)*1i + b*d^3*tan(x/2)*(d^2 - c^2)^(1/2)*4i + b*c*d^2*(d^2 -
c^2)^(1/2)*2i + a*c^2*d*tan(x/2)*(d^2 - c^2)^(1/2)*2i - b*c^2*d*tan(x/2)*(
d^2 - c^2)^(1/2)*1i)/(4*b*d^4*tan(x/2) - a*c^4*tan(x/2) + a*c^3*d + 2*b*c*
d^3 - b*c^3*d + 2*a*c^2*d^2*tan(x/2) - 3*b*c^2*d^2*tan(x/2)))*(d^2 - c^2)^
(1/2)*2i - b*c^2*d*log(tan(x/2)))/(c^4 - c^2*d^2) - (b*c^3 - b*c*d^2)/(c^4
*tan(x) - c^2*d^2*tan(x))
```

3.216. $\int \frac{a+b\csc^2(x)}{c+d\sin(x)}\,dx$

3.217 $\int (a\cos(c+dx)+b\sin(c+dx))^n\,dx$

3.217.1 Optimal result

Integrand size = 19, antiderivative size = 136

$$\int (a\cos(c+dx)+b\sin(c+dx))^n\,dx =$$

$$-\frac{\cos^{1+n}\left(c+dx-\tan^{-1}(a,b)\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2\left(c+dx-\tan^{-1}(a,b)\right)\right)(a\cos(c+d}{d(1+n)\sqrt{\sin^2\left(c+dx-\tan^{-1}(a,\right.}}$$

output
```
-cos(c+d*x-arctan(a,b))^(1+n)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],cos(c
+d*x-arctan(a,b))^2)*(a*cos(d*x+c)+b*sin(d*x+c))^n*sin(c+d*x-arctan(a,b))/
d/(1+n)/(((a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^(1/2))^n)/(sin(c+d*x-arcta
n(a,b))^2)^(1/2)
```

3.217.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.69

$$\int (a\cos(c+dx)+b\sin(c+dx))^n\,dx =$$

$$-\frac{\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3}{2},\cos^2\left(c+dx+\arctan\left(\frac{a}{b}\right)\right)\right)(a\cos(c+dx)+b\sin(c+dx))^n\sin^2\left(c+d\right.}{2d}$$

input
```
Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^n,x]
```

output
```
-1/2*(Hypergeometric2F1[1/2, (1 - n)/2, 3/2, Cos[c + d*x + ArcTan[a/b]]^2]
*(a*Cos[c + d*x] + b*Sin[c + d*x])^n*(Sin[c + d*x + ArcTan[a/b]]^2)^(-1/2
- n/2)*Sin[2*(c + d*x + ArcTan[a/b])])/d
```

3.217.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 3557, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(c+dx)+b\sin(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\cos(c+dx)+b\sin(c+dx))^n dx$$

$$\Big\downarrow \text{3557}$$

$$(a\cos(c+dx)+b\sin(c+dx))^n\left(\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}\right)^{-n}\int \cos^n\left(c+dx-\tan^{-1}(a,b)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$(a\cos(c+dx)+b\sin(c+dx))^n\left(\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}\right)^{-n}\int \sin\left(c+dx-\tan^{-1}(a,b)+\frac{\pi}{2}\right)^n dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{\sin\left(-\tan^{-1}(a,b)+c+dx\right)(a\cos(c+dx)+b\sin(c+dx))^n\left(\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}\right)^{-n}\cos^{n+1}\left(-\tan^{-1}(a,b)+c\right.}{d(n+1)\sqrt{\sin^2\left(-\tan^{-1}(a,b)+c+dx\right)}}$$

input
```
Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^n,x]
```

output
```
-((Cos[c + d*x - ArcTan[a, b]]^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (
3 + n)/2, Cos[c + d*x - ArcTan[a, b]]^2]*(a*Cos[c + d*x] + b*Sin[c + d*x])
^n*Sin[c + d*x - ArcTan[a, b]])/(d*(1 + n)*((a*Cos[c + d*x] + b*Sin[c + d*
x])/Sqrt[a^2 + b^2])^n*Sqrt[Sin[c + d*x - ArcTan[a, b]]^2]))
```

3.217.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

rule 3557
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
in[c + d*x])/Sqrt[a^2 + b^2])^n   Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
 /; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) &&  !(GtQ[a^2
+ b^2, 0] || EqQ[a^2 + b^2, 0])
```

3.217.4 Maple [F]

$$\int \left(\cos \left(d x +c \right) a +b \sin \left(d x +c \right)\right)^{n}d x$$

input
```
int((cos(d*x+c)*a+b*sin(d*x+c))^n,x)
```

output
```
int((cos(d*x+c)*a+b*sin(d*x+c))^n,x)
```

3.217.5 Fricas [F]

$$\int (a\cos(c+dx)+b\sin(c+dx))^n\,dx=\int \left(a\cos\left(dx+c\right)+b\sin\left(dx+c\right)\right)^{n}\,dx$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((a*cos(d*x + c) + b*sin(d*x + c))^n, x)
```

3.217.6 Sympy [F]

$$\int (a\cos(c+dx)+b\sin(c+dx))^n\,dx=\int (a\cos{(c+dx)}+b\sin{(c+dx)})^n\,dx$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))**n,x)
```

output
```
Integral((a*cos(c + d*x) + b*sin(c + d*x))**n, x)
```

3.217.7 Maxima [F]

$$\int (a\cos(c+dx)+b\sin(c+dx))^n\,dx=\int (a\cos{(dx+c)}+b\sin{(dx+c)})^n\,dx$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + b*sin(d*x + c))^n, x)
```

3.217.8 Giac [F]

$$\int (a\cos(c+dx)+b\sin(c+dx))^n\,dx=\int (a\cos{(dx+c)}+b\sin{(dx+c)})^n\,dx$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + b*sin(d*x + c))^n, x)
```

3.217.9 Mupad [F(-1)]

Timed out.

$$\int (a\cos(c+dx)+b\sin(c+dx))^n\,dx = \int \left(a\,\cos\left(c+d\,x\right)+b\,\sin\left(c+d\,x\right)\right)^n\,dx$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))^n,x)
```

output
```
int((a*cos(c + d*x) + b*sin(c + d*x))^n, x)
```

3.218 $\int (2\cos(c+dx)+3\sin(c+dx))^n\,dx$

3.218.1 Optimal result

Integrand size = 19, antiderivative size = 95

$$\int (2\cos(c+dx)+3\sin(c+dx))^n\,dx = -\frac{13^{n/2}\cos^{1+n}\left(c+dx-\arctan\left(\frac{3}{2}\right)\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2\left(c+dx-\arctan\left(\frac{3}{2}\right)\right)\right)\sin\left(c\right.}{d(1+n)\sqrt{\sin^2\left(c+dx-\arctan\left(\frac{3}{2}\right)\right)}}$$

output
```
-13^(1/2*n)*cos(c+d*x-arctan(3/2))^(1+n)*hypergeom([1/2, 1/2+1/2*n],[3/2+1
/2*n],cos(c+d*x-arctan(3/2))^2)*sin(c+d*x-arctan(3/2))/d/(1+n)/(sin(c+d*x-
arctan(3/2))^2)^(1/2)
```

3.218.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.93

$$\int (2\cos(c+dx)+3\sin(c+dx))^n\,dx = -\frac{\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-n}{2},\frac{3}{2},\cos^2\left(c+dx+\arctan\left(\frac{2}{3}\right)\right)\right)(2\cos(c+dx)+3\sin(c+dx))^n\sin^2\left(c+d\right.}{2d}$$

input
```
Integrate[(2*Cos[c + d*x] + 3*Sin[c + d*x])^n,x]
```

output
```
-1/2*(Hypergeometric2F1[1/2, (1 - n)/2, 3/2, Cos[c + d*x + ArcTan[2/3]]^2]
*(2*Cos[c + d*x] + 3*Sin[c + d*x])^n*(Sin[c + d*x + ArcTan[2/3]]^2)^(-1/2
- n/2)*Sin[2*(c + d*x + ArcTan[2/3])])/d
```

3.218.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3556, 3042, 3122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (3\sin(c+dx)+2\cos(c+dx))^n\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (3\sin(c+dx)+2\cos(c+dx))^n dx$$

$$\Big\downarrow \; 3556$$

$$13^{n/2}\int \cos^n\left(c+dx-\arctan\left(\frac{3}{2}\right)\right)dx$$

$$\Big\downarrow \; 3042$$

$$13^{n/2}\int \sin\left(c+dx-\arctan\left(\frac{3}{2}\right)+\frac{\pi}{2}\right)^n dx$$

$$\Big\downarrow \; 3122$$

$$-\frac{13^{n/2}\sin\left(-\arctan\left(\frac{3}{2}\right)+c+dx\right)\cos^{n+1}\left(-\arctan\left(\frac{3}{2}\right)+c+dx\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\cos^2\left(c+dx\right.\right.}{d(n+1)\sqrt{\sin^2\left(-\arctan\left(\frac{3}{2}\right)+c+dx\right)}}$$

input
```
Int[(2*Cos[c + d*x] + 3*Sin[c + d*x])^n,x]
```

output
```
-((13^(n/2)*Cos[c + d*x - ArcTan[3/2]]^(1 + n)*Hypergeometric2F1[1/2, (1 +
 n)/2, (3 + n)/2, Cos[c + d*x - ArcTan[3/2]]^2]*Sin[c + d*x - ArcTan[3/2]]
)/(d*(1 + n)*Sqrt[Sin[c + d*x - ArcTan[3/2]]^2]))
```

3.218.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

rule 3556
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a^2 + b^2)^(n/2)   Int[Cos[c + d*x - ArcTan[a, b]]^n, x],
 x] /; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) && GtQ[a^2
+ b^2, 0]
```

3.218.4 Maple [F]

$$\int (2\cos(dx+c)+3\sin(dx+c))^n\, dx$$

input
```
int((2*cos(d*x+c)+3*sin(d*x+c))^n,x)
```

output
```
int((2*cos(d*x+c)+3*sin(d*x+c))^n,x)
```

3.218.5 Fricas [F]

$$\int (2\cos(c+dx)+3\sin(c+dx))^n\, dx = \int (2\cos(dx+c)+3\sin(dx+c))^n\, dx$$

input
```
integrate((2*cos(d*x+c)+3*sin(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((2*cos(d*x + c) + 3*sin(d*x + c))^n, x)
```

3.218.6 Sympy [F]

$$\int (2\cos(c+dx)+3\sin(c+dx))^n\,dx = \int (3\sin(c+dx)+2\cos(c+dx))^n\,dx$$

input
```
integrate((2*cos(d*x+c)+3*sin(d*x+c))**n,x)
```

output
```
Integral((3*sin(c + d*x) + 2*cos(c + d*x))**n, x)
```

3.218.7 Maxima [F]

$$\int (2\cos(c+dx)+3\sin(c+dx))^n\,dx = \int (2\cos(dx+c)+3\sin(dx+c))^n\,dx$$

input
```
integrate((2*cos(d*x+c)+3*sin(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((2*cos(d*x + c) + 3*sin(d*x + c))^n, x)
```

3.218.8 Giac [F]

$$\int (2\cos(c+dx)+3\sin(c+dx))^n\,dx = \int (2\cos(dx+c)+3\sin(dx+c))^n\,dx$$

input
```
integrate((2*cos(d*x+c)+3*sin(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((2*cos(d*x + c) + 3*sin(d*x + c))^n, x)
```

3.218.9 Mupad [F(-1)]

Timed out.

$$\int (2\cos(c+dx)+3\sin(c+dx))^n\,dx = \int (2\ \cos\left(c+d\,x\right)+3\ \sin\left(c+d\,x\right))^n\,dx$$

input
```
int((2*cos(c + d*x) + 3*sin(c + d*x))^n,x)
```

output
```
int((2*cos(c + d*x) + 3*sin(c + d*x))^n, x)
```

3.219 $\int (a\cos(c+dx)+b\sin(c+dx))^7\,dx$

3.219.1 Optimal result

Integrand size = 19, antiderivative size = 127

$$\int (a\cos(c+dx)+b\sin(c+dx))^7\,dx = -\frac{(a^2+b^2)^3(b\cos(c+dx)-a\sin(c+dx))}{d} + \frac{(a^2+b^2)^2(b\cos(c+dx)-a\sin(c+dx))^3}{d} - \frac{3(a^2+b^2)(b\cos(c+dx)-a\sin(c+dx))^5}{5d} + \frac{(b\cos(c+dx)-a\sin(c+dx))^7}{7d}$$

output
```
-(a^2+b^2)^3*(b*cos(d*x+c)-a*sin(d*x+c))/d+(a^2+b^2)^2*(b*cos(d*x+c)-a*sin
(d*x+c))^3/d-3/5*(a^2+b^2)*(b*cos(d*x+c)-a*sin(d*x+c))^5/d+1/7*(b*cos(d*x+
c)-a*sin(d*x+c))^7/d
```

3.219.2 Mathematica [A] (verified)

Time = 2.18 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.94

$$\int (a\cos(c+dx)+b\sin(c+dx))^7\,dx$$
$$= \frac{-1225b(a^2+b^2)^3\cos(c+dx)+245b(-3a^2+b^2)(a^2+b^2)^2\cos(3(c+dx))-49b(5a^6-5a^4b^2-9a^2b^4+b}{}$$

input
```
Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^7,x]
```

output
```
(-1225*b*(a^2 + b^2)^3*Cos[c + d*x] + 245*b*(-3*a^2 + b^2)*(a^2 + b^2)^2*C
os[3*(c + d*x)] - 49*b*(5*a^6 - 5*a^4*b^2 - 9*a^2*b^4 + b^6)*Cos[5*(c + d*
x)] + 5*b*(-7*a^6 + 35*a^4*b^2 - 21*a^2*b^4 + b^6)*Cos[7*(c + d*x)] + 1225
*a*(a^2 + b^2)^3*Sin[c + d*x] + 245*a*(a^2 - 3*b^2)*(a^2 + b^2)^2*Sin[3*(c
 + d*x)] + 49*a*(a^6 - 9*a^4*b^2 - 5*a^2*b^4 + 5*b^6)*Sin[5*(c + d*x)] + 5
*a*(a^6 - 21*a^4*b^2 + 35*a^2*b^4 - 7*b^6)*Sin[7*(c + d*x)])/(2240*d)
```

3.219.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 3551, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(c+dx)+b\sin(c+dx))^7\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\cos(c+dx)+b\sin(c+dx))^7dx$$

$$\Big\downarrow \text{3551}$$

$$-\frac{\int \left(a^2+b^2-(b\cos(c+dx)-a\sin(c+dx))^2\right)^3 d(b\cos(c+dx)-a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{210}$$

$$-\frac{\int \left(\left(\frac{b^6+3a^2b^4+3a^4b^2}{a^6}+1\right)a^6-3\left(\frac{b^4+2a^2b^2}{a^4}+1\right)(b\cos(c+dx)-a\sin(c+dx))^2a^4+3\left(\frac{b^2}{a^2}+1\right)(b\cos(c+dx)-a\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{3}{5}\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))^5-\left(a^2+b^2\right)^2(b\cos(c+dx)-a\sin(c+dx))^3+\left(a^2+b^2\right)^3(b\cos(c+}{d}$$

input
```
Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^7,x]
```

output
```
-(((a^2 + b^2)^3*(b*Cos[c + d*x] - a*Sin[c + d*x]) - (a^2 + b^2)^2*(b*Cos[
c + d*x] - a*Sin[c + d*x])^3 + (3*(a^2 + b^2)*(b*Cos[c + d*x] - a*Sin[c +
d*x])^5)/5 - (b*Cos[c + d*x] - a*Sin[c + d*x])^7/7)/d)
```

3.219.3.1 Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3551
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[(a^2 + b^2 - x^2)^((n - 1)/2), x], x,
b*Cos[c + d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
 b^2, 0] && IGtQ[(n - 1)/2, 0]
```

3.219.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(123) = 246$.

Time = 3.19 (sec) , antiderivative size = 271, normalized size of antiderivative = 2.13

method	result
parts	$\frac{a^7\left(\frac{16}{5}+\cos(dx+c)^6+\frac{6\cos(dx+c)^4}{5}+\frac{8\cos(dx+c)^2}{5}\right)\sin(dx+c)}{7d}-\frac{b^7\left(\frac{16}{5}+\sin(dx+c)^6+\frac{6\sin(dx+c)^4}{5}+\frac{8\sin(dx+c)^2}{5}\right)\cos(}{7d}$
derivativedivides	$\frac{\frac{a^7\left(\frac{16}{5}+\cos(dx+c)^6+\frac{6\cos(dx+c)^4}{5}+\frac{8\cos(dx+c)^2}{5}\right)\sin(dx+c)}{7}-a^6b\cos(dx+c)^7+21a^5b^2\left(-\frac{\sin(dx+c)\cos(dx+c)^6}{7}+\frac{\left(\frac{8}{3}+\cos(d\right.}{}\right.}{}$
default	$\frac{\frac{a^7\left(\frac{16}{5}+\cos(dx+c)^6+\frac{6\cos(dx+c)^4}{5}+\frac{8\cos(dx+c)^2}{5}\right)\sin(dx+c)}{7}-a^6b\cos(dx+c)^7+21a^5b^2\left(-\frac{\sin(dx+c)\cos(dx+c)^6}{7}+\frac{\left(\frac{8}{3}+\cos(d\right.}{}\right.}{}$
parallelrisch	$\frac{70a^7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{13}-490\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}a^6b+(140a^7+1960a^5b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{11}-4900\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}a^4b^3+(602a^7-15}{}$
norman	$\frac{-\frac{70a^6b+140a^4b^3+112a^2b^5+32b^7}{35d}+\frac{2a^7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2a^7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{13}}{d}-\frac{140a^4b^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{d}-\frac{14a^6b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}}{d}-\frac{(14}{}}{}$
risch	$-\frac{7a^5\sin(3dx+3c)b^2}{64d}-\frac{35a^3\sin(3dx+3c)b^4}{64d}-\frac{21a\sin(3dx+3c)b^6}{64d}+\frac{b^7\cos(7dx+7c)}{448d}+\frac{35a^7\sin(dx+c)}{64d}+\frac{a^7\sin}{}$

input
```
int((cos(d*x+c)*a+b*sin(d*x+c))^7,x,method=_RETURNVERBOSE)
```

output
```
1/7*a^7/d*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c)
-1/7*b^7/d*(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c
)+21*a^5*b^2/d*(1/7*sin(d*x+c)^7-2/5*sin(d*x+c)^5+1/3*sin(d*x+c)^3)+35*a^3
*b^4/d*(-1/7*sin(d*x+c)^7+1/5*sin(d*x+c)^5)+a*b^6/d*sin(d*x+c)^7-a^6*b/d*c
os(d*x+c)^7+35*a^4*b^3/d*(1/7*cos(d*x+c)^7-1/5*cos(d*x+c)^5)-21*a^2*b^5/d*
(1/7*cos(d*x+c)^7-2/5*cos(d*x+c)^5+1/3*cos(d*x+c)^3)
```

3.219.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(123) = 246$.

Time = 0.27 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.02

$$\int (a\cos(c+dx)+b\sin(c+dx))^7\,dx =$$

$$-\frac{35\,b^7\cos\left(dx+c\right)+5\left(7\,a^6b-35\,a^4b^3+21\,a^2b^5-b^7\right)\cos\left(dx+c\right)^7+7\left(35\,a^4b^3-42\,a^2b^5+3\,b^7\right)\cos\left(dx\right.}{}$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))^7,x, algorithm="fricas")
```

output
```
-1/35*(35*b^7*cos(d*x + c) + 5*(7*a^6*b - 35*a^4*b^3 + 21*a^2*b^5 - b^7)*c
os(d*x + c)^7 + 7*(35*a^4*b^3 - 42*a^2*b^5 + 3*b^7)*cos(d*x + c)^5 + 35*(7
*a^2*b^5 - b^7)*cos(d*x + c)^3 - (16*a^7 + 56*a^5*b^2 + 70*a^3*b^4 + 35*a*
b^6 + 5*(a^7 - 21*a^5*b^2 + 35*a^3*b^4 - 7*a*b^6)*cos(d*x + c)^6 + (6*a^7
+ 21*a^5*b^2 - 280*a^3*b^4 + 105*a*b^6)*cos(d*x + c)^4 + (8*a^7 + 28*a^5*b
^2 + 35*a^3*b^4 - 105*a*b^6)*cos(d*x + c)^2)*sin(d*x + c))/d
```

3.219.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(110) = 220$.

Time = 0.51 (sec) , antiderivative size = 461, normalized size of antiderivative = 3.63

$$\int (a\cos(c+dx)+b\sin(c+dx))^7\,dx$$
$$= \begin{cases} \frac{16a^7\sin^7(c+dx)}{35d} + \frac{8a^7\sin^5(c+dx)\cos^2(c+dx)}{5d} + \frac{2a^7\sin^3(c+dx)\cos^4(c+dx)}{d} + \frac{a^7\sin(c+dx)\cos^6(c+dx)}{d} - \frac{a^6b\cos^7(c+dx)}{d} + \frac{8a^5b \\ x(a\cos(c)+b\sin(c))^7 \end{cases}$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))**7,x)
```

output
```
Piecewise((16*a**7*sin(c + d*x)**7/(35*d) + 8*a**7*sin(c + d*x)**5*cos(c +
 d*x)**2/(5*d) + 2*a**7*sin(c + d*x)**3*cos(c + d*x)**4/d + a**7*sin(c + d
*x)*cos(c + d*x)**6/d - a**6*b*cos(c + d*x)**7/d + 8*a**5*b**2*sin(c + d*x
)**7/(5*d) + 28*a**5*b**2*sin(c + d*x)**5*cos(c + d*x)**2/(5*d) + 7*a**5*b
**2*sin(c + d*x)**3*cos(c + d*x)**4/d - 7*a**4*b**3*sin(c + d*x)**2*cos(c
+ d*x)**5/d - 2*a**4*b**3*cos(c + d*x)**7/d + 2*a**3*b**4*sin(c + d*x)**7/
d + 7*a**3*b**4*sin(c + d*x)**5*cos(c + d*x)**2/d - 7*a**2*b**5*sin(c + d*
x)**4*cos(c + d*x)**3/d - 28*a**2*b**5*sin(c + d*x)**2*cos(c + d*x)**5/(5*
d) - 8*a**2*b**5*cos(c + d*x)**7/(5*d) + a*b**6*sin(c + d*x)**7/d - b**7*s
in(c + d*x)**6*cos(c + d*x)/d - 2*b**7*sin(c + d*x)**4*cos(c + d*x)**3/d -
 8*b**7*sin(c + d*x)**2*cos(c + d*x)**5/(5*d) - 16*b**7*cos(c + d*x)**7/(3
5*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**7, True))
```

3.219.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(123) = 246$.

Time = 0.22 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.02

$$\int (a\cos(c+dx)+b\sin(c+dx))^7\,dx = -\frac{35\,a^6b\cos{(dx+c)}^7 - 35\,ab^6\sin{(dx+c)}^7 + \left(5\,\sin{(dx+c)}^7 - 21\,\sin{(dx+c)}^5 + 35\,\sin{(dx+c)}^3 - 35\right.}{}$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))^7,x, algorithm="maxima")
```

output
```
-1/35*(35*a^6*b*cos(d*x + c)^7 - 35*a*b^6*sin(d*x + c)^7 + (5*sin(d*x + c)
^7 - 21*sin(d*x + c)^5 + 35*sin(d*x + c)^3 - 35*sin(d*x + c))*a^7 - 7*(15*
sin(d*x + c)^7 - 42*sin(d*x + c)^5 + 35*sin(d*x + c)^3)*a^5*b^2 - 35*(5*co
s(d*x + c)^7 - 7*cos(d*x + c)^5)*a^4*b^3 + 35*(5*sin(d*x + c)^7 - 7*sin(d*
x + c)^5)*a^3*b^4 + 7*(15*cos(d*x + c)^7 - 42*cos(d*x + c)^5 + 35*cos(d*x
+ c)^3)*a^2*b^5 - (5*cos(d*x + c)^7 - 21*cos(d*x + c)^5 + 35*cos(d*x + c)^
3 - 35*cos(d*x + c))*b^7)/d
```

3.219.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(123) = 246$.

Time = 0.50 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.49

$$\begin{aligned}\int (a\cos(c+dx)+b\sin(c+dx))^7\,dx = &-\frac{(7\,a^6b - 35\,a^4b^3 + 21\,a^2b^5 - b^7)\cos{(7\,dx + 7\,c)}}{448\,d}\\ &-\frac{7\,(5\,a^6b - 5\,a^4b^3 - 9\,a^2b^5 + b^7)\cos{(5\,dx + 5\,c)}}{320\,d}\\ &-\frac{7\,(3\,a^6b + 5\,a^4b^3 + a^2b^5 - b^7)\cos{(3\,dx + 3\,c)}}{64\,d}\\ &-\frac{35\,(a^6b + 3\,a^4b^3 + 3\,a^2b^5 + b^7)\cos{(dx + c)}}{64\,d}\\ &+\frac{(a^7 - 21\,a^5b^2 + 35\,a^3b^4 - 7\,ab^6)\sin{(7\,dx + 7\,c)}}{448\,d}\\ &+\frac{7\,(a^7 - 9\,a^5b^2 - 5\,a^3b^4 + 5\,ab^6)\sin{(5\,dx + 5\,c)}}{320\,d}\\ &+\frac{7\,(a^7 - a^5b^2 - 5\,a^3b^4 - 3\,ab^6)\sin{(3\,dx + 3\,c)}}{64\,d}\\ &+\frac{35\,(a^7 + 3\,a^5b^2 + 3\,a^3b^4 + ab^6)\sin{(dx + c)}}{64\,d}\end{aligned}$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))^7,x, algorithm="giac")
```

output
```
-1/448*(7*a^6*b - 35*a^4*b^3 + 21*a^2*b^5 - b^7)*cos(7*d*x + 7*c)/d - 7/32
0*(5*a^6*b - 5*a^4*b^3 - 9*a^2*b^5 + b^7)*cos(5*d*x + 5*c)/d - 7/64*(3*a^6
*b + 5*a^4*b^3 + a^2*b^5 - b^7)*cos(3*d*x + 3*c)/d - 35/64*(a^6*b + 3*a^4*
b^3 + 3*a^2*b^5 + b^7)*cos(d*x + c)/d + 1/448*(a^7 - 21*a^5*b^2 + 35*a^3*b
^4 - 7*a*b^6)*sin(7*d*x + 7*c)/d + 7/320*(a^7 - 9*a^5*b^2 - 5*a^3*b^4 + 5*
a*b^6)*sin(5*d*x + 5*c)/d + 7/64*(a^7 - a^5*b^2 - 5*a^3*b^4 - 3*a*b^6)*sin
(3*d*x + 3*c)/d + 35/64*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*sin(d*x + c)
/d
```

3.219.9 Mupad [B] (verification not implemented)

Time = 32.37 (sec) , antiderivative size = 422, normalized size of antiderivative = 3.32

$$\int (a\cos(c+dx)+b\sin(c+dx))^7\,dx =$$

$$-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8\left(70\,a^6\,b-140\,a^4\,b^3+224\,a^2\,b^5\right)-2\,a^7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7\left(\frac{424\,a^7}{35}+\frac{912\,a^5\,b^2}{5}-\right.}{}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))^7,x)
```

output
```
-(tan(c/2 + (d*x)/2)^8*(70*a^6*b + 224*a^2*b^5 - 140*a^4*b^3) - 2*a^7*tan(
c/2 + (d*x)/2)^13 - tan(c/2 + (d*x)/2)^7*(128*a*b^6 + (424*a^7)/35 - 192*a
^3*b^4 + (912*a^5*b^2)/5) + tan(c/2 + (d*x)/2)^4*(42*a^6*b + (96*b^7)/5 +
(336*a^2*b^5)/5 - 56*a^4*b^3) + 2*a^6*b + (32*b^7)/35 - tan(c/2 + (d*x)/2)
^5*((86*a^7)/5 + 224*a^3*b^4 - (224*a^5*b^2)/5) - tan(c/2 + (d*x)/2)^9*((8
6*a^7)/5 + 224*a^3*b^4 - (224*a^5*b^2)/5) + tan(c/2 + (d*x)/2)^2*((32*b^7)
/5 + (112*a^2*b^5)/5 + 28*a^4*b^3) + tan(c/2 + (d*x)/2)^6*(32*b^7 - 112*a^
2*b^5 + 280*a^4*b^3) + (16*a^2*b^5)/5 + 4*a^4*b^3 - tan(c/2 + (d*x)/2)^3*(
4*a^7 + 56*a^5*b^2) - tan(c/2 + (d*x)/2)^11*(4*a^7 + 56*a^5*b^2) - 2*a^7*t
an(c/2 + (d*x)/2) + 140*a^4*b^3*tan(c/2 + (d*x)/2)^10 + 14*a^6*b*tan(c/2 +
 (d*x)/2)^12)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^7)
```

3.220 $\int (a\cos(c+dx)+b\sin(c+dx))^6\,dx$

3.220.1 Optimal result

Integrand size = 19, antiderivative size = 161

$$\begin{aligned}
&\int (a\cos(c+dx)+b\sin(c+dx))^6\,dx \\
&= \frac{5}{16}\left(a^2+b^2\right)^3 x \\
&\quad - \frac{5(a^2+b^2)^2\,(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))}{16d} \\
&\quad - \frac{5(a^2+b^2)\,(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^3}{24d} \\
&\quad - \frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^5}{6d}
\end{aligned}$$

output
```
5/16*(a^2+b^2)^3*x-5/16*(a^2+b^2)^2*(b*cos(d*x+c)-a*sin(d*x+c))*(a*cos(d*x
+c)+b*sin(d*x+c))/d-5/24*(a^2+b^2)*(b*cos(d*x+c)-a*sin(d*x+c))*(a*cos(d*x+
c)+b*sin(d*x+c))^3/d-1/6*(b*cos(d*x+c)-a*sin(d*x+c))*(a*cos(d*x+c)+b*sin(d
*x+c))^5/d
```

3.220.2 Mathematica [A] (verified)

Time = 2.22 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.19

$$\int (a\cos(c+dx)+b\sin(c+dx))^6\,dx$$
$$= \frac{60(a^2+b^2)^3(c+dx)-90ab(a^2+b^2)^2\cos(2(c+dx))-36ab(a^4-b^4)\cos(4(c+dx))-2ab(3a^4-10a^2b^2}{}$$

input
```
Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^6,x]
```

output
```
(60*(a^2 + b^2)^3*(c + d*x) - 90*a*b*(a^2 + b^2)^2*Cos[2*(c + d*x)] - 36*a
*b*(a^4 - b^4)*Cos[4*(c + d*x)] - 2*a*b*(3*a^4 - 10*a^2*b^2 + 3*b^4)*Cos[6
*(c + d*x)] + 45*(a^2 - b^2)*(a^2 + b^2)^2*Sin[2*(c + d*x)] + 9*(a^6 - 5*a
^4*b^2 - 5*a^2*b^4 + b^6)*Sin[4*(c + d*x)] + (a^6 - 15*a^4*b^2 + 15*a^2*b^
4 - b^6)*Sin[6*(c + d*x)])/(192*d)
```

3.220.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3042, 3552, 3042, 3552, 3042, 3552, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(c+dx)+b\sin(c+dx))^6\,dx$$
$$\Big\downarrow \; 3042$$
$$\int (a\cos(c+dx)+b\sin(c+dx))^6dx$$
$$\Big\downarrow \; 3552$$
$$\frac{5}{6}(a^2+b^2)\int (a\cos(c+dx)+b\sin(c+dx))^4dx-\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^5}{6d}$$
$$\Big\downarrow \; 3042$$

$$\frac{5}{6}(a^2+b^2)\int (a\cos(c+dx)+b\sin(c+dx))^4dx-\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^5}{6d}$$

$\big\downarrow$ 3552

$$\frac{5}{6}(a^2+b^2)\left(\frac{3}{4}(a^2+b^2)\int (a\cos(c+dx)+b\sin(c+dx))^2dx-\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx)}{4d}\right.$$
$$-\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^5}{6d}$$

$\big\downarrow$ 3042

$$\frac{5}{6}(a^2+b^2)\left(\frac{3}{4}(a^2+b^2)\int (a\cos(c+dx)+b\sin(c+dx))^2dx-\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx)}{4d}\right.$$
$$-\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^5}{6d}$$

$\big\downarrow$ 3552

$$\frac{5}{6}(a^2+b^2)\left(\frac{3}{4}(a^2+b^2)\left(\frac{1}{2}(a^2+b^2)\int 1dx-\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))}{2d}\right)-\right.$$
$$-\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^5}{6d}$$

$\big\downarrow$ 24

$$\frac{5}{6}(a^2+b^2)\left(\frac{3}{4}(a^2+b^2)\left(\frac{1}{2}x(a^2+b^2)-\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))}{2d}\right)-\frac{(b\cos(}{}\right.$$
$$-\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^5}{6d}$$

input
```
Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^6,x]
```

output
```
-1/6*((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin[c + d*x])^
5)/d + (5*(a^2 + b^2)*(-1/4*((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c +
d*x] + b*Sin[c + d*x])^3)/d + (3*(a^2 + b^2)*(((a^2 + b^2)*x)/2 - ((b*Cos[
c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin[c + d*x]))/(2*d)))/4))/
6
```

3.220. $\int (a\cos(c+dx)+b\sin(c+dx))^6\,dx$

3.220.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3552
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(-(b*Cos[c + d*x] - a*Sin[c + d*x]))*((a*Cos[c + d*x] + b*
Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[(n - 1)*((a^2 + b^2)/n)   Int[(a*Co
s[c + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

3.220.4 Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.46

method	result
parallelrisch	$\frac{45(a-b)(a+b)(a^2+b^2)^2\sin(2dx+2c)+9(a^6-5a^4b^2-5a^2b^4+b^6)\sin(4dx+4c)+(a^6-15a^4b^2+15a^2b^4-b^6)\sin(6dx+6c)-}{}$
derivativedivides	$\frac{a^6\left(\frac{\left(\cos(dx+c)^5+\frac{5\cos(dx+c)^3}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)-a^5b\cos(dx+c)^6+15a^4b^2\left(-\frac{\cos(dx+c)^5\sin(dx+c)}{6}+\right.}{}$
default	$\frac{a^6\left(\frac{\left(\cos(dx+c)^5+\frac{5\cos(dx+c)^3}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)-a^5b\cos(dx+c)^6+15a^4b^2\left(-\frac{\cos(dx+c)^5\sin(dx+c)}{6}+\right.}{}$
parts	$\frac{a^6\left(\frac{\left(\cos(dx+c)^5+\frac{5\cos(dx+c)^3}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)}{d}+\frac{b^6\left(-\frac{\left(\sin(dx+c)^5+\frac{5\sin(dx+c)^3}{4}+\frac{15\sin(dx+c)}{8}\right)}{6}\right.}{d}$
risch	$\frac{5x\,a^6}{16}+\frac{15x\,a^4b^2}{16}+\frac{15x\,a^2b^4}{16}+\frac{5b^6x}{16}-\frac{a^5b\cos(6dx+6c)}{32d}+\frac{5a^3b^3\cos(6dx+6c)}{48d}-\frac{a\,b^5\cos(6dx+6c)}{32d}+\frac{\sin(6dx}{19}$
norman	$\frac{(\frac{5}{16}a^6+\frac{15}{16}a^4b^2+\frac{15}{16}a^2b^4+\frac{5}{16}b^6)x+(\frac{5}{16}a^6+\frac{15}{16}a^4b^2+\frac{15}{16}a^2b^4+\frac{5}{16}b^6)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}+(\frac{15}{8}a^6+\frac{45}{8}a^4b^2+\frac{45}{8}a^2b^4+\frac{15}{8}b^6)x}{}$

input
```
int((cos(d*x+c)*a+b*sin(d*x+c))^6,x,method=_RETURNVERBOSE)
```

output
```
1/192*(45*(a-b)*(a+b)*(a^2+b^2)^2*sin(2*d*x+2*c)+9*(a^6-5*a^4*b^2-5*a^2*b^
4+b^6)*sin(4*d*x+4*c)+(a^6-15*a^4*b^2+15*a^2*b^4-b^6)*sin(6*d*x+6*c)-90*b*
a*(a^2+b^2)^2*cos(2*d*x+2*c)+2*(-3*a^5*b+10*a^3*b^3-3*a*b^5)*cos(6*d*x+6*c
)+36*(-a^5*b+a*b^5)*cos(4*d*x+4*c)+60*a^6*d*x+180*a^4*b^2*d*x+180*a^2*b^4*
d*x+60*b^6*d*x+132*a^5*b+160*a^3*b^3+60*a*b^5)/d
```

3.220.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.36

$$\int (a\cos(c+dx)+b\sin(c+dx))^6\,dx = -\frac{144\,ab^5\cos\left(dx+c\right)^2+16\left(3\,a^5b-10\,a^3b^3+3\,ab^5\right)\cos\left(dx+c\right)^6+48\left(5\,a^3b^3-3\,ab^5\right)\cos\left(dx+c\right)^4-15}{}$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))^6,x, algorithm="fricas")
```

output
```
-1/48*(144*a*b^5*cos(d*x + c)^2 + 16*(3*a^5*b - 10*a^3*b^3 + 3*a*b^5)*cos(
d*x + c)^6 + 48*(5*a^3*b^3 - 3*a*b^5)*cos(d*x + c)^4 - 15*(a^6 + 3*a^4*b^2
 + 3*a^2*b^4 + b^6)*d*x - (8*(a^6 - 15*a^4*b^2 + 15*a^2*b^4 - b^6)*cos(d*x
 + c)^5 + 2*(5*a^6 + 15*a^4*b^2 - 105*a^2*b^4 + 13*b^6)*cos(d*x + c)^3 + 3
*(5*a^6 + 15*a^4*b^2 + 15*a^2*b^4 - 11*b^6)*cos(d*x + c))*sin(d*x + c))/d
```

3.220.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 770 vs. $2(151) = 302$.

Time = 0.41 (sec) , antiderivative size = 770, normalized size of antiderivative = 4.78

$$\int (a\cos(c+dx)+b\sin(c+dx))^6\,dx = \begin{cases} \frac{5a^6x\sin^6{(c+dx)}}{16}+\frac{15a^6x\sin^4{(c+dx)}\cos^2{(c+dx)}}{16}+\frac{15a^6x\sin^2{(c+dx)}\cos^4{(c+dx)}}{16}+\frac{5a^6x\cos^6{(c+dx)}}{16}+\frac{5a^6\sin^5{(c+dx)}\cos{(c+dx)}}{16d} \\ x(a\cos{(c)}+b\sin{(c)})^6 \end{cases}$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))**6,x)
```

output
```
Piecewise((5*a**6*x*sin(c + d*x)**6/16 + 15*a**6*x*sin(c + d*x)**4*cos(c +
 d*x)**2/16 + 15*a**6*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*a**6*x*cos(
c + d*x)**6/16 + 5*a**6*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 5*a**6*sin(c
 + d*x)**3*cos(c + d*x)**3/(6*d) + 11*a**6*sin(c + d*x)*cos(c + d*x)**5/(1
6*d) - a**5*b*cos(c + d*x)**6/d + 15*a**4*b**2*x*sin(c + d*x)**6/16 + 45*a
**4*b**2*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 45*a**4*b**2*x*sin(c + d*x
)**2*cos(c + d*x)**4/16 + 15*a**4*b**2*x*cos(c + d*x)**6/16 + 15*a**4*b**2
*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 5*a**4*b**2*sin(c + d*x)**3*cos(c +
 d*x)**3/(2*d) - 15*a**4*b**2*sin(c + d*x)*cos(c + d*x)**5/(16*d) + 5*a**3
*b**3*sin(c + d*x)**6/(3*d) + 5*a**3*b**3*sin(c + d*x)**4*cos(c + d*x)**2/
d + 15*a**2*b**4*x*sin(c + d*x)**6/16 + 45*a**2*b**4*x*sin(c + d*x)**4*cos
(c + d*x)**2/16 + 45*a**2*b**4*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 15*a
**2*b**4*x*cos(c + d*x)**6/16 + 15*a**2*b**4*sin(c + d*x)**5*cos(c + d*x)/
(16*d) - 5*a**2*b**4*sin(c + d*x)**3*cos(c + d*x)**3/(2*d) - 15*a**2*b**4*
sin(c + d*x)*cos(c + d*x)**5/(16*d) + a*b**5*sin(c + d*x)**6/d + 5*b**6*x*
sin(c + d*x)**6/16 + 15*b**6*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 15*b**
6*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*b**6*x*cos(c + d*x)**6/16 - 11*
b**6*sin(c + d*x)**5*cos(c + d*x)/(16*d) - 5*b**6*sin(c + d*x)**3*cos(c +
d*x)**3/(6*d) - 5*b**6*sin(c + d*x)*cos(c + d*x)**5/(16*d), Ne(d, 0)), (x*
(a*cos(c) + b*sin(c))**6, True))
```

3.220.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.48

$$\int (a\cos(c+dx)+b\sin(c+dx))^6\,dx =$$
$$-\frac{192\,a^5b\cos{(dx+c)}^6-192\,ab^5\sin{(dx+c)}^6+\left(4\,\sin{(2\,dx+2\,c)}^3-60\,dx-60\,c-9\,\sin{(4\,dx+4\,c)}-\right.}{}$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))^6,x, algorithm="maxima")
```

output
```
-1/192*(192*a^5*b*cos(d*x + c)^6 - 192*a*b^5*sin(d*x + c)^6 + (4*sin(2*d*x
 + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a^6
- 15*(4*sin(2*d*x + 2*c)^3 + 12*d*x + 12*c - 3*sin(4*d*x + 4*c))*a^4*b^2 +
 320*(2*sin(d*x + c)^6 - 3*sin(d*x + c)^4)*a^3*b^3 + 15*(4*sin(2*d*x + 2*c
)^3 - 12*d*x - 12*c + 3*sin(4*d*x + 4*c))*a^2*b^4 - (4*sin(2*d*x + 2*c)^3
+ 60*d*x + 60*c + 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*b^6)/d
```

3.220.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.46

$$\int (a\cos(c+dx)+b\sin(c+dx))^6\,dx = \frac{5}{16}\left(a^6+3a^4b^2+3a^2b^4+b^6\right)x - \frac{\left(3a^5b-10a^3b^3+3ab^5\right)\cos\left(6dx+6c\right)}{96d} - \frac{3\left(a^5b-ab^5\right)\cos\left(4dx+4c\right)}{16d} - \frac{15\left(a^5b+2a^3b^3+ab^5\right)\cos\left(2dx+2c\right)}{32d} + \frac{\left(a^6-15a^4b^2+15a^2b^4-b^6\right)\sin\left(6dx+6c\right)}{192d} + \frac{3\left(a^6-5a^4b^2-5a^2b^4+b^6\right)\sin\left(4dx+4c\right)}{64d} + \frac{15\left(a^6+a^4b^2-a^2b^4-b^6\right)\sin\left(2dx+2c\right)}{64d}$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))^6,x, algorithm="giac")
```

output
```
5/16*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*x - 1/96*(3*a^5*b - 10*a^3*b^3 +
3*a*b^5)*cos(6*d*x + 6*c)/d - 3/16*(a^5*b - a*b^5)*cos(4*d*x + 4*c)/d - 15
/32*(a^5*b + 2*a^3*b^3 + a*b^5)*cos(2*d*x + 2*c)/d + 1/192*(a^6 - 15*a^4*b
^2 + 15*a^2*b^4 - b^6)*sin(6*d*x + 6*c)/d + 3/64*(a^6 - 5*a^4*b^2 - 5*a^2*
b^4 + b^6)*sin(4*d*x + 4*c)/d + 15/64*(a^6 + a^4*b^2 - a^2*b^4 - b^6)*sin(
2*d*x + 2*c)/d
```

3.220.9 Mupad [B] (verification not implemented)

Time = 28.25 (sec) , antiderivative size = 519, normalized size of antiderivative = 3.22

$$\int (a\cos(c+dx)+b\sin(c+dx))^6\,dx = \frac{5\,\mathrm{atan}\left(\frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(a^2+b^2\right)^3}{8\left(\frac{5a^6}{8}+\frac{15a^4b^2}{8}+\frac{15a^2b^4}{8}+\frac{5b^6}{8}\right)}\right)\left(a^2+b^2\right)^3}{8d} - \frac{5\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)-\frac{dx}{2}\right)\left(a^2+b^2\right)^3}{8d} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(40a^5b-\frac{160a^3b^3}{3}+64ab^5\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(-\frac{11a^6}{8}+\frac{15a^4b^2}{8}+\frac{15a^2b^4}{8}+\frac{5b^6}{8}\right)+\tan\left(\frac{c}{2}+\right.}{}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))^6,x)
```

output
```
(5*atan((5*tan(c/2 + (d*x)/2)*(a^2 + b^2)^3)/(8*((5*a^6)/8 + (5*b^6)/8 + (
15*a^2*b^4)/8 + (15*a^4*b^2)/8)))*(a^2 + b^2)^3)/(8*d) - (5*(atan(tan(c/2
+ (d*x)/2)) - (d*x)/2)*(a^2 + b^2)^3)/(8*d) + (tan(c/2 + (d*x)/2)^6*(64*a*
b^5 + 40*a^5*b - (160*a^3*b^3)/3) - tan(c/2 + (d*x)/2)*((5*b^6)/8 - (11*a^
6)/8 + (15*a^2*b^4)/8 + (15*a^4*b^2)/8) + tan(c/2 + (d*x)/2)^11*((5*b^6)/8
 - (11*a^6)/8 + (15*a^2*b^4)/8 + (15*a^4*b^2)/8) - tan(c/2 + (d*x)/2)^3*((
5*a^6)/24 + (85*b^6)/24 + (85*a^2*b^4)/8 - (235*a^4*b^2)/8) + tan(c/2 + (d
*x)/2)^9*((5*a^6)/24 + (85*b^6)/24 + (85*a^2*b^4)/8 - (235*a^4*b^2)/8) + t
an(c/2 + (d*x)/2)^5*((15*a^6)/4 - (33*b^6)/4 + (285*a^2*b^4)/4 - (195*a^4*
b^2)/4) - tan(c/2 + (d*x)/2)^7*((15*a^6)/4 - (33*b^6)/4 + (285*a^2*b^4)/4
- (195*a^4*b^2)/4) + 80*a^3*b^3*tan(c/2 + (d*x)/2)^4 + 80*a^3*b^3*tan(c/2
+ (d*x)/2)^8 + 12*a^5*b*tan(c/2 + (d*x)/2)^2 + 12*a^5*b*tan(c/2 + (d*x)/2)
^10)/(d*(6*tan(c/2 + (d*x)/2)^2 + 15*tan(c/2 + (d*x)/2)^4 + 20*tan(c/2 + (
d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 + 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 +
(d*x)/2)^12 + 1))
```

3.220. $\int (a\cos(c+dx)+b\sin(c+dx))^6\,dx$

3.221 $\int (a\cos(c+dx)+b\sin(c+dx))^5\,dx$

3.221.1 Optimal result

Integrand size = 19, antiderivative size = 94

$$\int (a\cos(c+dx)+b\sin(c+dx))^5\,dx=-\frac{(a^2+b^2)^2(b\cos(c+dx)-a\sin(c+dx))}{d}+\frac{2(a^2+b^2)(b\cos(c+dx)-a\sin(c+dx))^3}{3d}-\frac{(b\cos(c+dx)-a\sin(c+dx))^5}{5d}$$

output
```
-(a^2+b^2)^2*(b*cos(d*x+c)-a*sin(d*x+c))/d+2/3*(a^2+b^2)*(b*cos(d*x+c)-a*s
in(d*x+c))^3/d-1/5*(b*cos(d*x+c)-a*sin(d*x+c))^5/d
```

3.221.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.66

$$\int (a\cos(c+dx)+b\sin(c+dx))^5\,dx$$
$$=\frac{-150b(a^2+b^2)^2\cos(c+dx)+25b(-3a^4-2a^2b^2+b^4)\cos(3(c+dx))-3b(5a^4-10a^2b^2+b^4)\cos(5(c+ \ldots}{\ldots}$$

input
```
Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

output
```
(-150*b*(a^2 + b^2)^2*Cos[c + d*x] + 25*b*(-3*a^4 - 2*a^2*b^2 + b^4)*Cos[3
*(c + d*x)] - 3*b*(5*a^4 - 10*a^2*b^2 + b^4)*Cos[5*(c + d*x)] + 150*a*(a^2
 + b^2)^2*Sin[c + d*x] + 25*a*(a^4 - 2*a^2*b^2 - 3*b^4)*Sin[3*(c + d*x)] +
 3*a*(a^4 - 10*a^2*b^2 + 5*b^4)*Sin[5*(c + d*x)])/(240*d)
```

3.221.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3551, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(c+dx)+b\sin(c+dx))^5\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\cos(c+dx)+b\sin(c+dx))^5dx$$

$$\Big\downarrow \text{3551}$$

$$-\frac{\int \left(a^2+b^2-(b\cos(c+dx)-a\sin(c+dx))^2\right)^2 d(b\cos(c+dx)-a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{210}$$

$$-\frac{\int \left(\left(\frac{b^4+2a^2b^2}{a^4}+1\right)a^4-2\left(\frac{b^2}{a^2}+1\right)(b\cos(c+dx)-a\sin(c+dx))^2a^2+(b\cos(c+dx)-a\sin(c+dx))^4\right)d(b\cos(}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{2}{3}\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))^3+\left(a^2+b^2\right)^2(b\cos(c+dx)-a\sin(c+dx))+\frac{1}{5}(b\cos(c+dx)-a\,s}{d}$$

input
```
Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

output
```
-(((a^2 + b^2)^2*(b*Cos[c + d*x] - a*Sin[c + d*x]) - (2*(a^2 + b^2)*(b*Cos
[c + d*x] - a*Sin[c + d*x])^3)/3 + (b*Cos[c + d*x] - a*Sin[c + d*x])^5/5)/
d)
```

3.221.3.1 Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3551
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[(a^2 + b^2 - x^2)^((n - 1)/2), x], x,
b*Cos[c + d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
 b^2, 0] && IGtQ[(n - 1)/2, 0]
```

3.221.4 Maple [A] (verified)

Time = 2.44 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.78

method	result
parts	$\frac{a^5\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5d}-\frac{b^5\left(\frac{8}{3}+\sin(dx+c)^4+\frac{4\sin(dx+c)^2}{3}\right)\cos(dx+c)}{5d}+\frac{a\,b^4\sin(dx+c)^5}{d}-$
derivativedivides	$\frac{\frac{a^5\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}-\cos(dx+c)^5a^4b+10a^3b^2\left(-\frac{\cos(dx+c)^4\sin(dx+c)}{5}+\frac{\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{15}\right.}{d}$
default	$\frac{\frac{a^5\left(\frac{8}{3}+\cos(dx+c)^4+\frac{4\cos(dx+c)^2}{3}\right)\sin(dx+c)}{5}-\cos(dx+c)^5a^4b+10a^3b^2\left(-\frac{\cos(dx+c)^4\sin(dx+c)}{5}+\frac{\left(2+\cos(dx+c)^2\right)\sin(dx+c)}{15}\right.}{d}$
parallelrisch	$2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9a^5-10\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8a^4b+\frac{8\left(a^5+10a^3b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{3}-40\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6a^2b^3+\frac{4\left(29a^5-40a^3b^2+120a\,b^4\right)\text{ta}}{15}$
norman	$-\frac{30a^4b+40a^2b^3+16b^5}{15d}+\frac{2a^5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2a^5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^9}{d}-\frac{40a^2b^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{d}-\frac{10a^4b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{d}-\frac{\left(40a^2b^3+16b^5\right)\text{tan}}{3d}$
risch	$-\frac{5a^4b\cos(dx+c)}{8d}-\frac{5a^2b^3\cos(dx+c)}{4d}-\frac{5b^5\cos(dx+c)}{8d}+\frac{5a^5\sin(dx+c)}{8d}+\frac{5a^3b^2\sin(dx+c)}{4d}+\frac{5a\,b^4\sin(dx+c)}{8d}$

input
```
int((cos(d*x+c)*a+b*sin(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
1/5*a^5/d*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)-1/5*b^5/d*(8/3+si
n(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)+a*b^4*sin(d*x+c)^5/d-a^4*b/d*cos(d
*x+c)^5+10*a^3*b^2/d*(-1/5*sin(d*x+c)^5+1/3*sin(d*x+c)^3)+10*a^2*b^3/d*(1/
5*cos(d*x+c)^5-1/3*cos(d*x+c)^3)
```

3.221.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.65

$$\int (a\cos(c+dx)+b\sin(c+dx))^5\,dx =$$
$$-\frac{15\,b^5\cos{(dx+c)}+3\,(5\,a^4b-10\,a^2b^3+b^5)\cos{(dx+c)}^5+10\,(5\,a^2b^3-b^5)\cos{(dx+c)}^3-\left(8\,a^5+20\,a^3\right.}{1}$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas")
```

output
```
-1/15*(15*b^5*cos(d*x + c) + 3*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c)^5
 + 10*(5*a^2*b^3 - b^5)*cos(d*x + c)^3 - (8*a^5 + 20*a^3*b^2 + 15*a*b^4 +
3*(a^5 - 10*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^4 + 2*(2*a^5 + 5*a^3*b^2 - 15*
a*b^4)*cos(d*x + c)^2)*sin(d*x + c))/d
```

3.221.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. 2(82) = 164.

Time = 0.27 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.84

$$\int (a\cos(c+dx)+b\sin(c+dx))^5\,dx$$
$$=\begin{cases}\frac{8a^5\sin^5{(c+dx)}}{15d}+\frac{4a^5\sin^3{(c+dx)}\cos^2{(c+dx)}}{3d}+\frac{a^5\sin{(c+dx)}\cos^4{(c+dx)}}{d}-\frac{a^4b\cos^5{(c+dx)}}{d}+\frac{4a^3b^2\sin^5{(c+dx)}}{3d}+\frac{10a^3b^2\sin^3{(c+}}{3}\\ x(a\cos{(c)}+b\sin{(c)})^5\end{cases}$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

output
```
Piecewise((8*a**5*sin(c + d*x)**5/(15*d) + 4*a**5*sin(c + d*x)**3*cos(c +
d*x)**2/(3*d) + a**5*sin(c + d*x)*cos(c + d*x)**4/d - a**4*b*cos(c + d*x)*
*5/d + 4*a**3*b**2*sin(c + d*x)**5/(3*d) + 10*a**3*b**2*sin(c + d*x)**3*co
s(c + d*x)**2/(3*d) - 10*a**2*b**3*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) -
 4*a**2*b**3*cos(c + d*x)**5/(3*d) + a*b**4*sin(c + d*x)**5/d - b**5*sin(c
 + d*x)**4*cos(c + d*x)/d - 4*b**5*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) -
 8*b**5*cos(c + d*x)**5/(15*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**5, Tr
ue))
```

3.221.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.83

$$\int (a\cos(c+dx)+b\sin(c+dx))^5\,dx$$
$$= -\frac{a^4b\cos{(dx+c)}^5}{d} + \frac{ab^4\sin{(dx+c)}^5}{d} + \frac{\left(3\,\sin{(dx+c)}^5 - 10\,\sin{(dx+c)}^3 + 15\,\sin{(dx+c)}\right)a^5}{15\,d} - \frac{2\left(3\,\sin{(dx+c)}^5 - 5\,\sin{(dx+c)}^3\right)a^3b^2}{3\,d} + \frac{2\left(3\,\cos{(dx+c)}^5 - 5\,\cos{(dx+c)}^3\right)a^2b^3}{3\,d} - \frac{\left(3\,\cos{(dx+c)}^5 - 10\,\cos{(dx+c)}^3 + 15\,\cos{(dx+c)}\right)b^5}{15\,d}$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima")
```

output
```
-a^4*b*cos(d*x + c)^5/d + a*b^4*sin(d*x + c)^5/d + 1/15*(3*sin(d*x + c)^5
- 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^5/d - 2/3*(3*sin(d*x + c)^5 - 5*s
in(d*x + c)^3)*a^3*b^2/d + 2/3*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a^2*b
^3/d - 1/15*(3*cos(d*x + c)^5 - 10*cos(d*x + c)^3 + 15*cos(d*x + c))*b^5/d
```

3.221.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(90) = 180.

Time = 0.34 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.99

$$\int (a\cos(c+dx)+b\sin(c+dx))^5\,dx = -\frac{(5\,a^4b-10\,a^2b^3+b^5)\cos(5\,dx+5\,c)}{80\,d} - \frac{5\,(3\,a^4b+2\,a^2b^3-b^5)\cos(3\,dx+3\,c)}{48\,d} - \frac{5\,(a^4b+2\,a^2b^3+b^5)\cos(dx+c)}{8\,d} + \frac{(a^5-10\,a^3b^2+5\,ab^4)\sin(5\,dx+5\,c)}{80\,d} + \frac{5\,(a^5-2\,a^3b^2-3\,ab^4)\sin(3\,dx+3\,c)}{48\,d} + \frac{5\,(a^5+2\,a^3b^2+ab^4)\sin(dx+c)}{8\,d}$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

output
```
-1/80*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(5*d*x + 5*c)/d - 5/48*(3*a^4*b + 2*
a^2*b^3 - b^5)*cos(3*d*x + 3*c)/d - 5/8*(a^4*b + 2*a^2*b^3 + b^5)*cos(d*x
+ c)/d + 1/80*(a^5 - 10*a^3*b^2 + 5*a*b^4)*sin(5*d*x + 5*c)/d + 5/48*(a^5
- 2*a^3*b^2 - 3*a*b^4)*sin(3*d*x + 3*c)/d + 5/8*(a^5 + 2*a^3*b^2 + a*b^4)*
sin(d*x + c)/d
```

3.221.9 Mupad [B] (verification not implemented)

Time = 27.39 (sec) , antiderivative size = 248, normalized size of antiderivative = 2.64

$$\int (a\cos(c+dx)+b\sin(c+dx))^5\,dx = \frac{2\left(\frac{3\,\sin(c+d\,x)\,a^5\cos(c+d\,x)^4}{2}+2\,\sin(c+d\,x)\,a^5\cos(c+d\,x)^2+4\,\sin(c+d\,x)\,a^5-\frac{15\,a^4\,b\cos(c+d\,x)^5}{2}-15\,\sin(c\right.}{}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))^5,x)
```

output
```
(2*(4*a^5*sin(c + d*x) - (15*b^5*cos(c + d*x))/2 + 5*b^5*cos(c + d*x)^3 -
(3*b^5*cos(c + d*x)^5)/2 - (15*a^4*b*cos(c + d*x)^5)/2 + 2*a^5*cos(c + d*x
)^2*sin(c + d*x) + (3*a^5*cos(c + d*x)^4*sin(c + d*x))/2 + 10*a^3*b^2*sin(
c + d*x) - 25*a^2*b^3*cos(c + d*x)^3 + 15*a^2*b^3*cos(c + d*x)^5 + (15*a*b
^4*sin(c + d*x))/2 + 5*a^3*b^2*cos(c + d*x)^2*sin(c + d*x) - 15*a^3*b^2*co
s(c + d*x)^4*sin(c + d*x) - 15*a*b^4*cos(c + d*x)^2*sin(c + d*x) + (15*a*b
^4*cos(c + d*x)^4*sin(c + d*x))/2))/(15*d)
```

3.221. $\int (a\cos(c+dx)+b\sin(c+dx))^5\,dx$

3.222 $\int (a\cos(c+dx)+b\sin(c+dx))^4\,dx$

3.222.1 Optimal result

Integrand size = 19, antiderivative size = 108

$$\int (a\cos(c+dx)+b\sin(c+dx))^4\,dx = \frac{3}{8}\left(a^2+b^2\right)^2 x-\frac{3(a^2+b^2)\left(b\cos(c+dx)-a\sin(c+dx)\right)(a\cos(c+dx)+b\sin(c+dx))}{8d} - \frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^3}{4d}$$

output
```
3/8*(a^2+b^2)^2*x-3/8*(a^2+b^2)*(b*cos(d*x+c)-a*sin(d*x+c))*(a*cos(d*x+c)+
b*sin(d*x+c))/d-1/4*(b*cos(d*x+c)-a*sin(d*x+c))*(a*cos(d*x+c)+b*sin(d*x+c)
)^3/d
```

3.222.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.99

$$\int (a\cos(c+dx)+b\sin(c+dx))^4\,dx = \frac{12(a^2+b^2)^2(c+dx)-16ab(a^2+b^2)\cos(2(c+dx))-4ab(a^2-b^2)\cos(4(c+dx))+8(a^4-b^4)\sin(2(c+dx))}{32d}$$

input
```
Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

output
```
(12*(a^2 + b^2)^2*(c + d*x) - 16*a*b*(a^2 + b^2)*Cos[2*(c + d*x)] - 4*a*b*
(a^2 - b^2)*Cos[4*(c + d*x)] + 8*(a^4 - b^4)*Sin[2*(c + d*x)] + (a^4 - 6*a
^2*b^2 + b^4)*Sin[4*(c + d*x)])/(32*d)
```

3.222.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 3552, 3042, 3552, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(c+dx)+b\sin(c+dx))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\cos(c+dx)+b\sin(c+dx))^4dx$$

$$\Big\downarrow \text{3552}$$

$$\frac{3}{4}(a^2+b^2)\int (a\cos(c+dx)+b\sin(c+dx))^2dx-\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^3}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{4}(a^2+b^2)\int (a\cos(c+dx)+b\sin(c+dx))^2dx-\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^3}{4d}$$

$$\Big\downarrow \text{3552}$$

$$\frac{3}{4}(a^2+b^2)\left(\frac{1}{2}(a^2+b^2)\int 1dx-\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))}{2d}\right)-\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^3}{4d}$$

$$\Big\downarrow \text{24}$$

$$\frac{3}{4}(a^2+b^2)\left(\frac{1}{2}x(a^2+b^2)-\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))}{2d}\right)-\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^3}{4d}$$

input
```
Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

output
```
-1/4*((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin[c + d*x])^
3)/d + (3*(a^2 + b^2)*(((a^2 + b^2)*x)/2 - ((b*Cos[c + d*x] - a*Sin[c + d*
x])*(a*Cos[c + d*x] + b*Sin[c + d*x]))/(2*d)))/4
```

3.222.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3552
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(-(b*Cos[c + d*x] - a*Sin[c + d*x]))*((a*Cos[c + d*x] + b*
Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[(n - 1)*((a^2 + b^2)/n)   Int[(a*Co
s[c + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

3.222.4 Maple [A] (verified)

Time = 1.70 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.25

method	result
parallelrisch	$\frac{(a^4-6a^2b^2+b^4)\sin(4dx+4c)+16(-a^3b-a\,b^3)\cos(2dx+2c)+4(-a^3b+a\,b^3)\cos(4dx+4c)+8(a^4-b^4)\sin(2dx+2c)+12a^4}{32d}$
derivativedivides	$\frac{a^4\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)-a^3b\cos(dx+c)^4+6a^2b^2\left(-\frac{\sin(dx+c)\cos(dx+c)^3}{4}+\frac{\sin(dx+c)\cos(dx+}{8}\right.}{d}$
default	$\frac{a^4\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)-a^3b\cos(dx+c)^4+6a^2b^2\left(-\frac{\sin(dx+c)\cos(dx+c)^3}{4}+\frac{\sin(dx+c)\cos(dx+}{8}\right.}{d}$
parts	$\frac{a^4\left(\frac{\left(\cos(dx+c)^3+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}+\frac{b^4\left(-\frac{\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}+\frac{a\,b^3}{}$
risch	$\frac{3a^4x}{8}+\frac{3a^2b^2x}{4}+\frac{3b^4x}{8}-\frac{a^3b\cos(4dx+4c)}{8d}+\frac{a\,b^3\cos(4dx+4c)}{8d}+\frac{\sin(4dx+4c)a^4}{32d}-\frac{3\sin(4dx+4c)a^2b^2}{16d}+\sin$
norman	$\frac{(\frac{3}{8}a^4+\frac{3}{4}a^2b^2+\frac{3}{8}b^4)x+(\frac{3}{2}a^4+3a^2b^2+\frac{3}{2}b^4)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+(\frac{3}{2}a^4+3a^2b^2+\frac{3}{2}b^4)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+(\frac{3}{8}a^4+\frac{3}{4}a^2b^2+\frac{3}{8}b^4)x\,t}{}$

input
```
int((cos(d*x+c)*a+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/32*((a^4-6*a^2*b^2+b^4)*sin(4*d*x+4*c)+16*(-a^3*b-a*b^3)*cos(2*d*x+2*c)+
4*(-a^3*b+a*b^3)*cos(4*d*x+4*c)+8*(a^4-b^4)*sin(2*d*x+2*c)+12*a^4*d*x+24*a
^2*b^2*d*x+12*b^4*d*x+20*a^3*b+12*a*b^3)/d
```

3.222.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.12

$$\int (a\cos(c+dx)+b\sin(c+dx))^4\,dx = -\frac{16\,ab^3\cos(dx+c)^2+8\,(a^3b-ab^3)\cos(dx+c)^4-3\,(a^4+2\,a^2b^2+b^4)dx-\left(2\,(a^4-6\,a^2b^2+b^4)\cos(d\right.}{8\,d}$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

output
```
-1/8*(16*a*b^3*cos(d*x + c)^2 + 8*(a^3*b - a*b^3)*cos(d*x + c)^4 - 3*(a^4
+ 2*a^2*b^2 + b^4)*d*x - (2*(a^4 - 6*a^2*b^2 + b^4)*cos(d*x + c)^3 + (3*a^
4 + 6*a^2*b^2 - 5*b^4)*cos(d*x + c))*sin(d*x + c))/d
```

3.222.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(102) = 204$.

Time = 0.20 (sec) , antiderivative size = 381, normalized size of antiderivative = 3.53

$$\int (a\cos(c+dx)+b\sin(c+dx))^4\,dx$$
$$=\begin{cases} \frac{3a^4x\sin^4(c+dx)}{8}+\frac{3a^4x\sin^2(c+dx)\cos^2(c+dx)}{4}+\frac{3a^4x\cos^4(c+dx)}{8}+\frac{3a^4\sin^3(c+dx)\cos(c+dx)}{8d}+\frac{5a^4\sin(c+dx)\cos^3(c+dx)}{8d}-\frac{a}{} \\ x(a\cos(c)+b\sin(c))^4 \end{cases}$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

output
```
Piecewise((3*a**4*x*sin(c + d*x)**4/8 + 3*a**4*x*sin(c + d*x)**2*cos(c + d
*x)**2/4 + 3*a**4*x*cos(c + d*x)**4/8 + 3*a**4*sin(c + d*x)**3*cos(c + d*x
)/(8*d) + 5*a**4*sin(c + d*x)*cos(c + d*x)**3/(8*d) - a**3*b*cos(c + d*x)*
*4/d + 3*a**2*b**2*x*sin(c + d*x)**4/4 + 3*a**2*b**2*x*sin(c + d*x)**2*cos
(c + d*x)**2/2 + 3*a**2*b**2*x*cos(c + d*x)**4/4 + 3*a**2*b**2*sin(c + d*x
)**3*cos(c + d*x)/(4*d) - 3*a**2*b**2*sin(c + d*x)*cos(c + d*x)**3/(4*d) +
 a*b**3*sin(c + d*x)**4/d + 3*b**4*x*sin(c + d*x)**4/8 + 3*b**4*x*sin(c +
d*x)**2*cos(c + d*x)**2/4 + 3*b**4*x*cos(c + d*x)**4/8 - 5*b**4*sin(c + d*
x)**3*cos(c + d*x)/(8*d) - 3*b**4*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d
, 0)), (x*(a*cos(c) + b*sin(c))**4, True))
```

3.222.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.26

$$\int (a\cos(c+dx)+b\sin(c+dx))^4\,dx$$
$$= -\frac{a^3b\cos(dx+c)^4}{d}+\frac{ab^3\sin(dx+c)^4}{d} + \frac{(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))a^4}{32\,d} + \frac{3\,(4\,dx+4\,c-\sin(4\,dx+4\,c))a^2b^2}{16\,d} + \frac{(12\,dx+12\,c+\sin(4\,dx+4\,c)-8\,\sin(2\,dx+2\,c))b^4}{32\,d}$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

output
```
-a^3*b*cos(d*x + c)^4/d + a*b^3*sin(d*x + c)^4/d + 1/32*(12*d*x + 12*c + s
in(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^4/d + 3/16*(4*d*x + 4*c - sin(4*d*
x + 4*c))*a^2*b^2/d + 1/32*(12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*d*x
 + 2*c))*b^4/d
```

3.222.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.13

$$\int (a\cos(c+dx)+b\sin(c+dx))^4\,dx = \frac{3}{8}\left(a^4+2\,a^2b^2+b^4\right)x - \frac{(a^3b-ab^3)\cos(4\,dx+4\,c)}{8\,d} - \frac{(a^3b+ab^3)\cos(2\,dx+2\,c)}{2\,d} + \frac{(a^4-6\,a^2b^2+b^4)\sin(4\,dx+4\,c)}{32\,d} + \frac{(a^4-b^4)\sin(2\,dx+2\,c)}{4\,d}$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

output
```
3/8*(a^4 + 2*a^2*b^2 + b^4)*x - 1/8*(a^3*b - a*b^3)*cos(4*d*x + 4*c)/d - 1
/2*(a^3*b + a*b^3)*cos(2*d*x + 2*c)/d + 1/32*(a^4 - 6*a^2*b^2 + b^4)*sin(4
*d*x + 4*c)/d + 1/4*(a^4 - b^4)*sin(2*d*x + 2*c)/d
```

3.222.9 Mupad [B] (verification not implemented)

Time = 27.99 (sec) , antiderivative size = 320, normalized size of antiderivative = 2.96

$$\int (a\cos(c+dx)+b\sin(c+dx))^4\,dx = \frac{3\,\mathrm{atan}\left(\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,{\left(a^2+b^2\right)}^2}{4\,\left(\frac{3\,a^4}{4}+\frac{3\,a^2\,b^2}{2}+\frac{3\,b^4}{4}\right)}\right)\,{\left(a^2+b^2\right)}^2}{4\,d} + \frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7\,\left(-\frac{5\,a^4}{4}+\frac{3\,a^2\,b^2}{2}+\frac{3\,b^4}{4}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(\frac{3\,a^4}{4}-\frac{21\,a^2\,b^2}{2}+\frac{11\,b^4}{4}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(\frac{3\,a^4}{4}-\frac{21\,a\ldots}{2}\right.}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+4\tan\left(\frac{c}{2}+\ldots\right.\right.} - \frac{3\,\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)\,{\left(a^2+b^2\right)}^2}{4\,d}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))^4,x)
```

output
```
(3*atan((3*tan(c/2 + (d*x)/2)*(a^2 + b^2)^2)/(4*((3*a^4)/4 + (3*b^4)/4 + (
3*a^2*b^2)/2)))*(a^2 + b^2)^2)/(4*d) + (tan(c/2 + (d*x)/2)^7*((3*b^4)/4 -
(5*a^4)/4 + (3*a^2*b^2)/2) - tan(c/2 + (d*x)/2)^3*((3*a^4)/4 + (11*b^4)/4
- (21*a^2*b^2)/2) + tan(c/2 + (d*x)/2)^5*((3*a^4)/4 + (11*b^4)/4 - (21*a^2
*b^2)/2) - tan(c/2 + (d*x)/2)*((3*b^4)/4 - (5*a^4)/4 + (3*a^2*b^2)/2) + 8*
a^3*b*tan(c/2 + (d*x)/2)^2 + 16*a*b^3*tan(c/2 + (d*x)/2)^4 + 8*a^3*b*tan(c
/2 + (d*x)/2)^6)/(d*(4*tan(c/2 + (d*x)/2)^2 + 6*tan(c/2 + (d*x)/2)^4 + 4*t
an(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) - (3*(atan(tan(c/2 + (d*x
)/2)) - (d*x)/2)*(a^2 + b^2)^2)/(4*d)
```

3.222. $\int (a\cos(c+dx)+b\sin(c+dx))^4\,dx$

3.223 $\int (a\cos(c+dx)+b\sin(c+dx))^3\,dx$

3.223.1 Optimal result

Integrand size = 19, antiderivative size = 58

$$\int (a\cos(c+dx)+b\sin(c+dx))^3\,dx = -\frac{(a^2+b^2)\,(b\cos(c+dx)-a\sin(c+dx))}{d} + \frac{(b\cos(c+dx)-a\sin(c+dx))^3}{3d}$$

output
```
-(a^2+b^2)*(b*cos(d*x+c)-a*sin(d*x+c))/d+1/3*(b*cos(d*x+c)-a*sin(d*x+c))^3
/d
```

3.223.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.40

$$\int (a\cos(c+dx)+b\sin(c+dx))^3\,dx = \frac{-9b(a^2+b^2)\cos(c+dx)+(-3a^2b+b^3)\cos(3(c+dx))+2a(5a^2+3b^2+(a^2-3b^2)\cos(2(c+dx)))\sin(c}{12d}$$

input
```
Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

output
```
(-9*b*(a^2 + b^2)*Cos[c + d*x] + (-3*a^2*b + b^3)*Cos[3*(c + d*x)] + 2*a*(
5*a^2 + 3*b^2 + (a^2 - 3*b^2)*Cos[2*(c + d*x)])*Sin[c + d*x])/(12*d)
```

3.223.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3042, 3551, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(c+dx)+b\sin(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\cos(c+dx)+b\sin(c+dx))^3dx$$

$$\Big\downarrow \text{3551}$$

$$-\frac{\int \left(a^2+b^2-(b\cos(c+dx)-a\sin(c+dx))^2\right)d(b\cos(c+dx)-a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(a^2+b^2)\,(b\cos(c+dx)-a\sin(c+dx))-\frac{1}{3}(b\cos(c+dx)-a\sin(c+dx))^3}{d}$$

input
```
Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

output
```
-(((a^2 + b^2)*(b*Cos[c + d*x] - a*Sin[c + d*x]) - (b*Cos[c + d*x] - a*Sin
[c + d*x])^3/3)/d)
```

3.223.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3551
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[(a^2 + b^2 - x^2)^((n - 1)/2), x], x,
b*Cos[c + d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
 b^2, 0] && IGtQ[(n - 1)/2, 0]
```

3.223.4 Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.29

method	result
derivativedivides	$\frac{\frac{a^{3}\left(2+\cos(dx+c)^{2}\right)\sin(dx+c)}{3}-a^{2}b\cos(dx+c)^{3}+a\,b^{2}\sin(dx+c)^{3}-\frac{b^{3}\left(2+\sin(dx+c)^{2}\right)\cos(dx+c)}{3}}{d}$
default	$\frac{\frac{a^{3}\left(2+\cos(dx+c)^{2}\right)\sin(dx+c)}{3}-a^{2}b\cos(dx+c)^{3}+a\,b^{2}\sin(dx+c)^{3}-\frac{b^{3}\left(2+\sin(dx+c)^{2}\right)\cos(dx+c)}{3}}{d}$
parts	$\frac{a^{3}\left(2+\cos(dx+c)^{2}\right)\sin(dx+c)}{3d}-\frac{b^{3}\left(2+\sin(dx+c)^{2}\right)\cos(dx+c)}{3d}+\frac{a\,b^{2}\sin(dx+c)^{3}}{d}-\frac{a^{2}b\cos(dx+c)^{3}}{d}$
parallelrisch	$\frac{2a^{3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}-6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a^{2}b+\frac{4\left(a^{3}+6a\,b^{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3}-4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b^{3}+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)a^{3}-2a^{2}b-\frac{4b^{3}}{3}}{d\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{3}}$
risch	$-\frac{3a^{2}b\cos(dx+c)}{4d}-\frac{3b^{3}\cos(dx+c)}{4d}+\frac{3a^{3}\sin(dx+c)}{4d}+\frac{3a\,b^{2}\sin(dx+c)}{4d}-\frac{b\cos(3dx+3c)a^{2}}{4d}+\frac{b^{3}\cos(3dx+3c)}{12d}-$
norman	$\frac{-\frac{6a^{2}b+4b^{3}}{3d}+\frac{2a^{3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2a^{3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{d}-\frac{4b^{3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{d}-\frac{6a^{2}b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{d}+\frac{4a\left(a^{2}+6b^{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{3}}$

input
```
int((cos(d*x+c)*a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/3*a^3*(2+cos(d*x+c)^2)*sin(d*x+c)-a^2*b*cos(d*x+c)^3+a*b^2*sin(d*x+
c)^3-1/3*b^3*(2+sin(d*x+c)^2)*cos(d*x+c))
```

3.223.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.33

$$\int (a\cos(c+dx)+b\sin(c+dx))^3\,dx = -\frac{3\,b^{3}\cos\left(dx+c\right)+\left(3\,a^{2}b-b^{3}\right)\cos\left(dx+c\right)^{3}-\left(2\,a^{3}+3\,ab^{2}+\left(a^{3}-3\,ab^{2}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)}{3\,d}$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/3*(3*b^3*cos(d*x + c) + (3*a^2*b - b^3)*cos(d*x + c)^3 - (2*a^3 + 3*a*b
^2 + (a^3 - 3*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/d
```

3.223.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(48) = 96$.

Time = 0.13 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.02

$$\int (a\cos(c+dx)+b\sin(c+dx))^3\,dx$$
$$= \begin{cases} \frac{2a^3\sin^3(c+dx)}{3d} + \frac{a^3\sin(c+dx)\cos^2(c+dx)}{d} - \frac{a^2b\cos^3(c+dx)}{d} + \frac{ab^2\sin^3(c+dx)}{d} - \frac{b^3\sin^2(c+dx)\cos(c+dx)}{d} - \frac{2b^3\cos^3(c+dx)}{3d} & \text{fo} \\ x(a\cos(c)+b\sin(c))^3 & \end{cases}$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

output
```
Piecewise((2*a**3*sin(c + d*x)**3/(3*d) + a**3*sin(c + d*x)*cos(c + d*x)**
2/d - a**2*b*cos(c + d*x)**3/d + a*b**2*sin(c + d*x)**3/d - b**3*sin(c + d
*x)**2*cos(c + d*x)/d - 2*b**3*cos(c + d*x)**3/(3*d), Ne(d, 0)), (x*(a*cos
(c) + b*sin(c))**3, True))
```

3.223.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.45

$$\int (a\cos(c+dx)+b\sin(c+dx))^3\,dx = -\frac{a^2b\cos(dx+c)^3}{d} + \frac{ab^2\sin(dx+c)^3}{d} - \frac{(\sin(dx+c)^3 - 3\sin(dx+c))a^3}{3\,d} + \frac{(\cos(dx+c)^3 - 3\cos(dx+c))b^3}{3\,d}$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
-a^2*b*cos(d*x + c)^3/d + a*b^2*sin(d*x + c)^3/d - 1/3*(sin(d*x + c)^3 - 3
*sin(d*x + c))*a^3/d + 1/3*(cos(d*x + c)^3 - 3*cos(d*x + c))*b^3/d
```

3.223.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.57

$$\int (a\cos(c+dx)+b\sin(c+dx))^3\,dx = -\frac{(3\,a^2b-b^3)\cos(3\,dx+3\,c)}{12\,d} - \frac{3\,(a^2b+b^3)\cos(dx+c)}{4\,d} + \frac{(a^3-3\,ab^2)\sin(3\,dx+3\,c)}{12\,d} + \frac{3\,(a^3+ab^2)\sin(dx+c)}{4\,d}$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/12*(3*a^2*b - b^3)*cos(3*d*x + 3*c)/d - 3/4*(a^2*b + b^3)*cos(d*x + c)/
d + 1/12*(a^3 - 3*a*b^2)*sin(3*d*x + 3*c)/d + 3/4*(a^3 + a*b^2)*sin(d*x +
c)/d
```

3.223.9 Mupad [B] (verification not implemented)

Time = 27.64 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.79

$$\int (a\cos(c+dx)+b\sin(c+dx))^3\,dx$$

$$= \frac{\frac{\sin(c+d\,x)\,a^3\cos(c+d\,x)^2}{3} + \frac{2\,\sin(c+d\,x)\,a^3}{3} - a^2\,b\cos\left(c+d\,x\right)^3 - \sin\left(c+d\,x\right)\,a\,b^2\cos\left(c+d\,x\right)^2 + \sin\left(c+d\,x\right)\,a\,b}{d}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))^3,x)
```

output
```
((2*a^3*sin(c + d*x))/3 - b^3*cos(c + d*x) + (b^3*cos(c + d*x)^3)/3 - a^2*
b*cos(c + d*x)^3 + (a^3*cos(c + d*x)^2*sin(c + d*x))/3 + a*b^2*sin(c + d*x
) - a*b^2*cos(c + d*x)^2*sin(c + d*x))/d
```

3.224 $\int (a\cos(c+dx)+b\sin(c+dx))^2\,dx$

3.224.1 Optimal result

Integrand size = 19, antiderivative size = 55

$$\int (a\cos(c+dx)+b\sin(c+dx))^2\,dx = \frac{1}{2}\left(a^2+b^2\right)x-\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))}{2d}$$

output
```
1/2*(a^2+b^2)*x-1/2*(b*cos(d*x+c)-a*sin(d*x+c))*(a*cos(d*x+c)+b*sin(d*x+c)
)/d
```

3.224.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

$$\int (a\cos(c+dx)+b\sin(c+dx))^2\,dx = \frac{2(a^2+b^2)(c+dx)-2ab\cos(2(c+dx))+(a^2-b^2)\sin(2(c+dx))}{4d}$$

input
```
Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

output
```
(2*(a^2 + b^2)*(c + d*x) - 2*a*b*Cos[2*(c + d*x)] + (a^2 - b^2)*Sin[2*(c +
 d*x)])/(4*d)
```

3.224.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3042, 3552, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(c+dx)+b\sin(c+dx))^2\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (a\cos(c+dx)+b\sin(c+dx))^2dx$$

$$\Big\downarrow \; 3552$$

$$\frac{1}{2}\left(a^2+b^2\right)\int 1dx-\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))}{2d}$$

$$\Big\downarrow \; 24$$

$$\frac{1}{2}x\left(a^2+b^2\right)-\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))}{2d}$$

input
```
Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

output
```
((a^2 + b^2)*x)/2 - ((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b
*Sin[c + d*x]))/(2*d)
```

3.224.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3552
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(-(b*Cos[c + d*x] - a*Sin[c + d*x]))*((a*Cos[c + d*x] + b*
Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[(n - 1)*((a^2 + b^2)/n)   Int[(a*Co
s[c + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

3.224.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.04

method	result
parallelrisch	$\frac{\sin(2dx+2c)\left(a^2-b^2\right)+2a^2dx+2b^2dx-2ab\cos(2dx+2c)+2ab}{4d}$
risch	$\frac{a^2x}{2}+\frac{x\,b^2}{2}-\frac{ab\cos(2dx+2c)}{2d}+\frac{\sin(2dx+2c)a^2}{4d}-\frac{\sin(2dx+2c)b^2}{4d}$
derivativedivides	$\frac{a^2\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-\cos(dx+c)^2ab+b^2\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
default	$\frac{a^2\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-\cos(dx+c)^2ab+b^2\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
parts	$\frac{a^2\left(\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{b^2\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{ab\sin(dx+c)^2}{d}$
norman	$\frac{\left(\frac{a^2}{2}+\frac{b^2}{2}\right)x+\left(\frac{a^2}{2}+\frac{b^2}{2}\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+\frac{\left(a^2-b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\left(a^2+b^2\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2-\frac{\left(a^2-b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{d}+4ab\tan}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}$

input
```
int((cos(d*x+c)*a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(sin(2*d*x+2*c)*(a^2-b^2)+2*a^2*d*x+2*b^2*d*x-2*a*b*cos(2*d*x+2*c)+2*a
*b)/d
```

3.224.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

$$\int (a\cos(c+dx)+b\sin(c+dx))^2\,dx = -\frac{2\,ab\cos\left(dx+c\right)^2-\left(a^2+b^2\right)dx-\left(a^2-b^2\right)\cos\left(dx+c\right)\sin\left(dx+c\right)}{2\,d}$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/2*(2*a*b*cos(d*x + c)^2 - (a^2 + b^2)*d*x - (a^2 - b^2)*cos(d*x + c)*si
n(d*x + c))/d
```

3.224.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(49) = 98.

Time = 0.09 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.33

$$\int (a\cos(c+dx)+b\sin(c+dx))^2\,dx = \begin{cases} \frac{a^2x\sin^2(c+dx)}{2}+\frac{a^2x\cos^2(c+dx)}{2}+\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}-\frac{ab\cos^2(c+dx)}{d}+\frac{b^2x\sin^2(c+dx)}{2}+\frac{b^2x\cos^2(c+dx)}{2}-\frac{b^2\sin(c+dx)}{2} \\ x(a\cos(c)+b\sin(c))^2 \end{cases}$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

output
```
Piecewise((a**2*x*sin(c + d*x)**2/2 + a**2*x*cos(c + d*x)**2/2 + a**2*sin(
c + d*x)*cos(c + d*x)/(2*d) - a*b*cos(c + d*x)**2/d + b**2*x*sin(c + d*x)*
*2/2 + b**2*x*cos(c + d*x)**2/2 - b**2*sin(c + d*x)*cos(c + d*x)/(2*d), Ne
(d, 0)), (x*(a*cos(c) + b*sin(c))**2, True))
```

3.224.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.24

$$\int (a\cos(c+dx)+b\sin(c+dx))^2\,dx = -\frac{ab\cos(dx+c)^2}{d}+\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))a^2}{4\,d}+\frac{(2\,dx+2\,c-\sin(2\,dx+2\,c))b^2}{4\,d}$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
-a*b*cos(d*x + c)^2/d + 1/4*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^2/d + 1/4*(
2*d*x + 2*c - sin(2*d*x + 2*c))*b^2/d
```

3.224.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int (a\cos(c+dx)+b\sin(c+dx))^2\,dx = \frac{1}{2}\,(a^2+b^2)x - \frac{ab\cos(2\,dx+2\,c)}{2\,d} + \frac{(a^2-b^2)\sin(2\,dx+2\,c)}{4\,d}$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
1/2*(a^2 + b^2)*x - 1/2*a*b*cos(2*d*x + 2*c)/d + 1/4*(a^2 - b^2)*sin(2*d*x
 + 2*c)/d
```

3.224.9 Mupad [B] (verification not implemented)

Time = 25.83 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.15

$$\int (a\cos(c+dx)+b\sin(c+dx))^2\,dx = \frac{a^2\,x}{2} + \frac{b^2\,x}{2} + \frac{a^2\,\sin(2\,c+2\,d\,x)}{4\,d} - \frac{b^2\,\sin(2\,c+2\,d\,x)}{4\,d} - \frac{a\,b\,\cos(2\,c+2\,d\,x)}{2\,d}$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))^2,x)
```

output
```
(a^2*x)/2 + (b^2*x)/2 + (a^2*sin(2*c + 2*d*x))/(4*d) - (b^2*sin(2*c + 2*d*
x))/(4*d) - (a*b*cos(2*c + 2*d*x))/(2*d)
```

3.225 $\int (a\cos(c+dx)+b\sin(c+dx))\,dx$

3.225.1 Optimal result

Integrand size = 17, antiderivative size = 24

$$\int (a\cos(c+dx)+b\sin(c+dx))\,dx=-\frac{b\cos(c+dx)}{d}+\frac{a\sin(c+dx)}{d}$$

output
```
-b*cos(d*x+c)/d+a*sin(d*x+c)/d
```

3.225.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.92

$$\int (a\cos(c+dx)+b\sin(c+dx))\,dx=-\frac{b\cos(c)\cos(dx)}{d}+\frac{a\cos(dx)\sin(c)}{d}+\frac{a\cos(c)\sin(dx)}{d}+\frac{b\sin(c)\sin(dx)}{d}$$

input
```
Integrate[a*Cos[c + d*x] + b*Sin[c + d*x],x]
```

output
```
-((b*Cos[c]*Cos[d*x])/d) + (a*Cos[d*x]*Sin[c])/d + (a*Cos[c]*Sin[d*x])/d +
 (b*Sin[c]*Sin[d*x])/d
```

3.225.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(c+dx)+b\sin(c+dx))\,dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a\sin(c+dx)}{d}-\frac{b\cos(c+dx)}{d}$$

input
```
Int[a*Cos[c + d*x] + b*Sin[c + d*x],x]
```

output
```
-((b*Cos[c + d*x])/d) + (a*Sin[c + d*x])/d
```

3.225.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.225.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{-\cos(dx+c)b+\sin(dx+c)a}{d}$	23
parallelrisch	$\frac{b-\cos(dx+c)b+\sin(dx+c)a}{d}$	24
default	$-\frac{b\cos(dx+c)}{d}+\frac{a\sin(dx+c)}{d}$	25
risch	$-\frac{b\cos(dx+c)}{d}+\frac{a\sin(dx+c)}{d}$	25
parts	$-\frac{b\cos(dx+c)}{d}+\frac{a\sin(dx+c)}{d}$	25
norman	$\frac{\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{d}+\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}$	50
meijerg	$\frac{(\sqrt{\pi}\,\cos(c)a+\sqrt{\pi}\,\sin(c)b)\sin(dx)}{\sqrt{\pi}\,d}+\frac{(\sqrt{\pi}\,\cos(c)b-\sqrt{\pi}\,\sin(c)a)\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(dx)}{\sqrt{\pi}}\right)}{d}$	61

input
```
int(cos(d*x+c)*a+b*sin(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-cos(d*x+c)*b+sin(d*x+c)*a)
```

3.225.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int (a\cos(c+dx)+b\sin(c+dx))\,dx=-\frac{b\cos(dx+c)-a\sin(dx+c)}{d}$$

input
```
integrate(a*cos(d*x+c)+b*sin(d*x+c),x, algorithm="fricas")
```

output
```
-(b*cos(d*x + c) - a*sin(d*x + c))/d
```

3.225.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.29

$$\int (a\cos(c+dx)+b\sin(c+dx))\,dx=a\left(\begin{cases}\frac{\sin(c+dx)}{d} & \text{for } d\neq 0\\ x\cos(c) & \text{otherwise}\end{cases}\right)+b\left(\begin{cases}-\frac{\cos(c+dx)}{d} & \text{for } d\neq 0\\ x\sin(c) & \text{otherwise}\end{cases}\right)$$

input
```
integrate(a*cos(d*x+c)+b*sin(d*x+c),x)
```

output
```
a*Piecewise((sin(c + d*x)/d, Ne(d, 0)), (x*cos(c), True)) + b*Piecewise((-
cos(c + d*x)/d, Ne(d, 0)), (x*sin(c), True))
```

3.225.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int (a\cos(c+dx)+b\sin(c+dx))\,dx=-\frac{b\cos{(dx+c)}}{d}+\frac{a\sin{(dx+c)}}{d}$$

input
```
integrate(a*cos(d*x+c)+b*sin(d*x+c),x, algorithm="maxima")
```

output
```
-b*cos(d*x + c)/d + a*sin(d*x + c)/d
```

3.225.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int (a\cos(c+dx)+b\sin(c+dx))\,dx=-\frac{b\cos{(dx+c)}}{d}+\frac{a\sin{(dx+c)}}{d}$$

input
```
integrate(a*cos(d*x+c)+b*sin(d*x+c),x, algorithm="giac")
```

output
```
-b*cos(d*x + c)/d + a*sin(d*x + c)/d
```

3.225.9 Mupad [B] (verification not implemented)

Time = 25.83 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.58

$$\int (a\cos(c+dx)+b\sin(c+dx))\,dx=-\frac{2\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(b\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

input
```
int(a*cos(c + d*x) + b*sin(c + d*x),x)
```

output
```
-(2*cos(c/2 + (d*x)/2)*(b*cos(c/2 + (d*x)/2) - a*sin(c/2 + (d*x)/2)))/d
```

3.226 $\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

3.226.1 Optimal result

Integrand size = 19, antiderivative size = 47

$$\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}\,dx = -\frac{\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,d}$$

output
```
-arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/d/(a^2+b^2)^(1/2)
```

3.226.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96

$$\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \frac{2\operatorname{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,d}$$

input
```
Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-1),x]
```

output
```
(2*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(Sqrt[a^2 + b^2]*d)
```

3.226.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 3553, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{3553}$$

$$-\frac{\int \frac{1}{a^2+b^2-(b\cos(c+dx)-a\sin(c+dx))^2}d(b\cos(c+dx)-a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$$

input
```
Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-1),x]
```

output
```
-(ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]]/(Sqrt[a^2 + b
^2]*d))
```

3.226.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

3.226.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{2\,\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$	43
default	$\frac{2\,\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$	43
risch	$\frac{\ln\left(e^{i(dx+c)}+\frac{ia-b}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,d}-\frac{\ln\left(e^{i(dx+c)}-\frac{ia-b}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,d}$	88

input
```
int(1/(cos(d*x+c)*a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2/d/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/
2))
```

3.226.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(43) = 86$.

Time = 0.25 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.79

$$\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \frac{\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2}\right)}{2\,\sqrt{a^2+b^2}d}$$

input
```
integrate(1/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2
*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*c
os(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2))/(sqrt(a^2 +
b^2)*d)
```

3.226.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.92 (sec) , antiderivative size = 163, normalized size of antiderivative = 3.47

$$\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \begin{cases} \frac{\tilde{\infty}x}{\sin(c)} & \text{for } a=0 \wedge b=0 \wedge d=0 \\ \frac{\log\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{bd} & \text{for } a=0 \\ \frac{x}{a\cos(c)+b\sin(c)} & \text{for } d=0 \\ -\frac{1}{ibd\sin(c+dx)+bd\cos(c+dx)} & \text{for } a=-ib \\ -\frac{1}{-ibd\sin(c+dx)+bd\cos(c+dx)} & \text{for } a=ib \\ -\frac{\log\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-\frac{b}{a}-\frac{\sqrt{a^2+b^2}}{a}\right)}{d\sqrt{a^2+b^2}}+\frac{\log\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-\frac{b}{a}+\frac{\sqrt{a^2+b^2}}{a}\right)}{d\sqrt{a^2+b^2}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

output
```
Piecewise((zoo*x/sin(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (log(tan(c/2 + d
*x/2))/(b*d), Eq(a, 0)), (x/(a*cos(c) + b*sin(c)), Eq(d, 0)), (-1/(I*b*d*s
in(c + d*x) + b*d*cos(c + d*x)), Eq(a, -I*b)), (-1/(-I*b*d*sin(c + d*x) +
b*d*cos(c + d*x)), Eq(a, I*b)), (-log(tan(c/2 + d*x/2) - b/a - sqrt(a**2 +
 b**2)/a)/(d*sqrt(a**2 + b**2)) + log(tan(c/2 + d*x/2) - b/a + sqrt(a**2 +
 b**2)/a)/(d*sqrt(a**2 + b**2)), True))
```

3.226.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.70

$$\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}\,dx = -\frac{\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}d}$$

input
```
integrate(1/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*sin(
d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*d)
```

3.226.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.57

$$\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}\,dx = -\frac{\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}d}$$

input
```
integrate(1/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
-log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1
/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*d)
```

3.226.9 Mupad [B] (verification not implemented)

Time = 28.70 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{b-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{d\,\sqrt{a^2+b^2}}$$

input
```
int(1/(a*cos(c + d*x) + b*sin(c + d*x)),x)
```

output
```
-(2*atanh((b - a*tan(c/2 + (d*x)/2))/(a^2 + b^2)^(1/2)))/(d*(a^2 + b^2)^(1
/2))
```

3.227 $\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$

3.227.1 Optimal result

Integrand size = 19, antiderivative size = 32

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx=\frac{\sin(c+dx)}{ad(a\cos(c+dx)+b\sin(c+dx))}$$

output
```
sin(d*x+c)/a/d/(a*cos(d*x+c)+b*sin(d*x+c))
```

3.227.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx=\frac{\sin(c+dx)}{ad(a\cos(c+dx)+b\sin(c+dx))}$$

input
```
Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-2),x]
```

output
```
Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x]))
```

3.227.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3042, 3554}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{3554}$$

$$\frac{\sin(c+dx)}{ad(a\cos(c+dx)+b\sin(c+dx))}$$

input
```
Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-2),x]
```

output
```
Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x]))
```

3.227.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3554
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

3.227.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.66

method	result	size
derivativedivides	$-\frac{1}{db(a+b\tan(dx+c))}$	21
default	$-\frac{1}{db(a+b\tan(dx+c))}$	21
risch	$\frac{2i}{d(-ib+a)\left(-ib\,\mathrm{e}^{2i(dx+c)}+\mathrm{e}^{2i(dx+c)}a+ib+a\right)}$	47
parallelrisch	$\frac{1-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{bd\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}$	54
norman	$\frac{\frac{1}{bd}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{bd}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a}$	60

input
```
int(1/(cos(d*x+c)*a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/d/b/(a+b*tan(d*x+c))
```

3.227.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.78

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = -\frac{b\cos(dx+c)-a\sin(dx+c)}{(a^3+ab^2)d\cos(dx+c)+(a^2b+b^3)d\sin(dx+c)}$$

input
```
integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
-(b*cos(d*x + c) - a*sin(d*x + c))/((a^3 + a*b^2)*d*cos(d*x + c) + (a^2*b
+ b^3)*d*sin(d*x + c))
```

3.227.6 Sympy [F]

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = \int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$$

input
```
integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

output
```
Integral((a*cos(c + d*x) + b*sin(c + d*x))**(-2), x)
```

3.227.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.66

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = -\frac{1}{(b^2\tan(dx+c)+ab)d}$$

input
```
integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/((b^2*tan(d*x + c) + a*b)*d)
```

3.227.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.62

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = -\frac{1}{(b\tan(dx+c)+a)bd}$$

input
```
integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/((b*tan(d*x + c) + a)*b*d)
```

3.227.9 Mupad [B] (verification not implemented)

Time = 28.65 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.47

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = \frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a\,d\,\left(-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)}$$

input

```
int(1/(a*cos(c + d*x) + b*sin(c + d*x))^2,x)
```

output

```
(2*tan(c/2 + (d*x)/2))/(a*d*(a + 2*b*tan(c/2 + (d*x)/2) - a*tan(c/2 + (d*x
)/2)^2))
```

3.228 $\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

3.228.1 Optimal result

Integrand size = 19, antiderivative size = 103

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx=-\frac{\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{3/2}d}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^2}$$

output
```
-1/2*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)/
d+1/2*(-b*cos(d*x+c)+a*sin(d*x+c))/(a^2+b^2)/d/(a*cos(d*x+c)+b*sin(d*x+c))
^2
```

3.228.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.28

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx=\frac{\left(a^2+b^2\right)(-b\cos(c+dx)+a\sin(c+dx))+2\sqrt{a^2+b^2}\operatorname{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)(a\cos(c+dx)+b\sin(c}{2(a-ib)^2(a+ib)^2d(a\cos(c+dx)+b\sin(c+dx))^2}$$

input
```
Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-3),x]
```

output
```
((a^2 + b^2)*(-(b*Cos[c + d*x]) + a*Sin[c + d*x]) + 2*Sqrt[a^2 + b^2]*ArcT
anh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]]*(a*Cos[c + d*x] + b*Sin[c +
 d*x])^2)/(2*(a - I*b)^2*(a + I*b)^2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2
)
```

3.228.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3042, 3555, 3042, 3553, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^3}dx$$

$$\Big\downarrow \text{3555}$$

$$\frac{\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{2(a^2+b^2)}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{2(a^2+b^2)}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^2}$$

$$\Big\downarrow \text{3553}$$

$$-\frac{\int \frac{1}{a^2+b^2-(b\cos(c+dx)-a\sin(c+dx))^2}d(b\cos(c+dx)-a\sin(c+dx))}{2d(a^2+b^2)}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^2}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2d(a^2+b^2)^{3/2}}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^2}$$

input
```
Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-3),x]
```

output
```
-1/2*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]]/((a^2 + b^
2)^(3/2)*d) - (b*Cos[c + d*x] - a*Sin[c + d*x])/(2*(a^2 + b^2)*d*(a*Cos[c
+ d*x] + b*Sin[c + d*x])^2)
```

3.228.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3555
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(b*Cos[c + d*x] - a*Sin[c + d*x])*((a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Simp[(n + 2)/((n + 1)*(a^
2 + b^2))   Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

3.228.4 Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.85

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{\left(a^{2}+2b^{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{2\left(a^{2}+b^{2}\right)a}-\frac{b\left(a^{2}-2b^{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{2\left(a^{2}+b^{2}\right)a^{2}}-\frac{\left(a^{2}-2b^{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2\left(a^{2}+b^{2}\right)a}+\frac{b}{2a^{2}+2b^{2}}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^{2}}+\frac{\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}}{d}$
default	$\frac{-\frac{2\left(-\frac{\left(a^{2}+2b^{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{2\left(a^{2}+b^{2}\right)a}-\frac{b\left(a^{2}-2b^{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{2\left(a^{2}+b^{2}\right)a^{2}}-\frac{\left(a^{2}-2b^{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2\left(a^{2}+b^{2}\right)a}+\frac{b}{2a^{2}+2b^{2}}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^{2}}+\frac{\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right)}{\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}}{d}$
risch	$\frac{e^{i(dx+c)}\left(ia\,e^{2i(dx+c)}+b\,e^{2i(dx+c)}-ia+b\right)}{\left(b\,e^{2i(dx+c)}+ia\,e^{2i(dx+c)}-b+ia\right)^{2}(-ia+b)d(ia+b)}+\frac{\ln\left(e^{i(dx+c)}+\frac{ia^{3}+ia\,b^{2}-a^{2}b-b^{3}}{\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}\right)}{2(a^{2}+b^{2})^{\frac{3}{2}}d}-\frac{\ln\left(e^{i(dx+c)}-\frac{ia^{3}+ia\,b^{2}-}{\left(a^{2}+b^{2}\right.}\right.}{2(a^{2}+b^{2})^{\frac{3}{2}}d}$

input
```
int(1/(cos(d*x+c)*a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*(-1/2*(a^2+2*b^2)/(a^2+b^2)/a*tan(1/2*d*x+1/2*c)^3-1/2*b*(a^2-2*b^
2)/(a^2+b^2)/a^2*tan(1/2*d*x+1/2*c)^2-1/2*(a^2-2*b^2)/(a^2+b^2)/a*tan(1/2*
d*x+1/2*c)+1/2*b/(a^2+b^2))/(tan(1/2*d*x+1/2*c)^2*a-2*b*tan(1/2*d*x+1/2*c)
-a)^2+1/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)
^(1/2)))
```

3.228.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(95) = 190$.

Time = 0.27 (sec) , antiderivative size = 294, normalized size of antiderivative = 2.85

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$$
$$= \frac{\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^{2}-b^{2}\right)\cos\left(dx+c\right)^{2}+b^{2}\right)\sqrt{a^{2}+b^{2}}\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^{2}-b^{2})\cos}{2\,ab\cos(dx+c)\sin}\right.}{4\left(\left(a^{6}+a^{4}b^{2}-a^{2}b^{4}-b^{6}\right)d\cos\left(dx+c\right)^{2}+2\left(a^{5}b+2\,a^{3}b^{3}+a\right.\right.}$$

input
```
integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/4*((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)*
sqrt(a^2 + b^2)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*
x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c
)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2))
- 2*(a^2*b + b^3)*cos(d*x + c) + 2*(a^3 + a*b^2)*sin(d*x + c))/((a^6 + a^4
*b^2 - a^2*b^4 - b^6)*d*cos(d*x + c)^2 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d*c
os(d*x + c)*sin(d*x + c) + (a^4*b^2 + 2*a^2*b^4 + b^6)*d)
```

3.228.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

output
```
Timed out
```

3.228.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 326 vs. 2(95) = 190.

Time = 0.32 (sec) , antiderivative size = 326, normalized size of antiderivative = 3.17

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx =$$

$$-\frac{\frac{2\left(a^2b-\frac{(a^3-2ab^2)\sin(dx+c)}{\cos(dx+c)+1}-\frac{(a^2b-2b^3)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{(a^3+2ab^2)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^6+a^4b^2+\frac{4(a^5b+a^3b^3)\sin(dx+c)}{\cos(dx+c)+1}-\frac{2(a^6-a^4b^2-2a^2b^4)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{4(a^5b+a^3b^3)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{(a^6+a^4b^2)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\cdots}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\cdots}\right)}{(a^2+b^2)^{\frac{3}{2}}}}{2d}$$

input
```
integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/2*(2*(a^2*b - (a^3 - 2*a*b^2)*sin(d*x + c)/(cos(d*x + c) + 1) - (a^2*b
- 2*b^3)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - (a^3 + 2*a*b^2)*sin(d*x + c
)^3/(cos(d*x + c) + 1)^3)/(a^6 + a^4*b^2 + 4*(a^5*b + a^3*b^3)*sin(d*x + c
)/(cos(d*x + c) + 1) - 2*(a^6 - a^4*b^2 - 2*a^2*b^4)*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 - 4*(a^5*b + a^3*b^3)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 +
 (a^6 + a^4*b^2)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) + log((b - a*sin(d*x
 + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d*x +
 c) + 1) - sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2))/d
```

3.228.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(95) = 190$.

Time = 0.31 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.15

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = -\frac{\frac{\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^2+b^2)^{\frac{3}{2}}} - \frac{2\left(a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a^3\,\mathrm{ta}\right.}{(a^4+a^2b^2)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)}}{2\,d}$$

input
```
integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/2*(log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*
tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(a^
3*tan(1/2*d*x + 1/2*c)^3 + 2*a*b^2*tan(1/2*d*x + 1/2*c)^3 + a^2*b*tan(1/2*
d*x + 1/2*c)^2 - 2*b^3*tan(1/2*d*x + 1/2*c)^2 + a^3*tan(1/2*d*x + 1/2*c) -
 2*a*b^2*tan(1/2*d*x + 1/2*c) - a^2*b)/((a^4 + a^2*b^2)*(a*tan(1/2*d*x + 1
/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c) - a)^2))/d
```

3.228.9 Mupad [B] (verification not implemented)

Time = 31.50 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.52

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = \frac{\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(a^2-2b^2)}{a(a^2+b^2)}-\frac{b}{a^2+b^2}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(a^2+2b^2)}{a(a^2+b^2)}+\frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(a^2-2b^2)}{a^2(a^2+b^2)}}{d\left(a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(2a^2-4b^2)+a^2-4ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+4ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)} + \frac{\operatorname{atanh}\left(\frac{\left(2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-\frac{2a^2b+2b^3}{a^2+b^2}\right)\left(\frac{a^2}{2}+\frac{b^2}{2}\right)}{(a^2+b^2)^{3/2}}\right)}{d(a^2+b^2)^{3/2}}$$

input
```
int(1/(a*cos(c + d*x) + b*sin(c + d*x))^3,x)
```

output
```
((tan(c/2 + (d*x)/2)*(a^2 - 2*b^2))/(a*(a^2 + b^2)) - b/(a^2 + b^2) + (tan
(c/2 + (d*x)/2)^3*(a^2 + 2*b^2))/(a*(a^2 + b^2)) + (b*tan(c/2 + (d*x)/2)^2
*(a^2 - 2*b^2))/(a^2*(a^2 + b^2)))/(d*(a^2*tan(c/2 + (d*x)/2)^4 - tan(c/2
+ (d*x)/2)^2*(2*a^2 - 4*b^2) + a^2 - 4*a*b*tan(c/2 + (d*x)/2)^3 + 4*a*b*ta
n(c/2 + (d*x)/2))) + atanh(((2*a*tan(c/2 + (d*x)/2) - (2*a^2*b + 2*b^3)/(a
^2 + b^2))*(a^2/2 + b^2/2))/(a^2 + b^2)^(3/2))/(d*(a^2 + b^2)^(3/2))
```

3.229 $\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

3.229.1 Optimal result

Integrand size = 19, antiderivative size = 98

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = -\frac{b\cos(c+dx)-a\sin(c+dx)}{3(a^2+b^2)\,d(a\cos(c+dx)+b\sin(c+dx))^3} + \frac{2\sin(c+dx)}{3a(a^2+b^2)\,d(a\cos(c+dx)+b\sin(c+dx))}$$

output
```
1/3*(-b*cos(d*x+c)+a*sin(d*x+c))/(a^2+b^2)/d/(a*cos(d*x+c)+b*sin(d*x+c))^3
+2/3*sin(d*x+c)/a/(a^2+b^2)/d/(a*cos(d*x+c)+b*sin(d*x+c))
```

3.229.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.87

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = \frac{-ab\cos(3(c+dx))+(2a^2+b^2+(a^2-b^2)\cos(2(c+dx)))\sin(c+dx)}{3a(a^2+b^2)\,d(a\cos(c+dx)+b\sin(c+dx))^3}$$

input
```
Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-4),x]
```

output
```
(-(a*b*Cos[3*(c + d*x)]) + (2*a^2 + b^2 + (a^2 - b^2)*Cos[2*(c + d*x)])*Si
n[c + d*x])/(3*a*(a^2 + b^2)*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3)
```

3.229.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 3555, 3042, 3554}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^4}dx$$

$$\Big\downarrow \text{3555}$$

$$\frac{2\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^2}dx}{3(a^2+b^2)}-\frac{b\cos(c+dx)-a\sin(c+dx)}{3d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^2}dx}{3(a^2+b^2)}-\frac{b\cos(c+dx)-a\sin(c+dx)}{3d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^3}$$

$$\Big\downarrow \text{3554}$$

$$\frac{2\sin(c+dx)}{3ad(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))}-\frac{b\cos(c+dx)-a\sin(c+dx)}{3d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^3}$$

input
```
Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-4),x]
```

output
```
-1/3*(b*Cos[c + d*x] - a*Sin[c + d*x])/((a^2 + b^2)*d*(a*Cos[c + d*x] + b*
Sin[c + d*x])^3) + (2*Sin[c + d*x])/(3*a*(a^2 + b^2)*d*(a*Cos[c + d*x] + b
*Sin[c + d*x]))
```

3.229.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3554
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3555
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(b*Cos[c + d*x] - a*Sin[c + d*x])*((a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Simp[(n + 2)/((n + 1)*(a^
2 + b^2))   Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

3.229.4 Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.65

method	result	size
derivativedivides	$\frac{-\frac{a^2+b^2}{3b^3(a+b\tan(dx+c))^3}+\frac{a}{b^3(a+b\tan(dx+c))^2}-\frac{1}{b^3(a+b\tan(dx+c))}}{d}$	64
default	$\frac{-\frac{a^2+b^2}{3b^3(a+b\tan(dx+c))^3}+\frac{a}{b^3(a+b\tan(dx+c))^2}-\frac{1}{b^3(a+b\tan(dx+c))}}{d}$	64
risch	$\frac{4i\left(3ia\,e^{2i(dx+c)}+3b\,e^{2i(dx+c)}+ia-b\right)}{3\left(b\,e^{2i(dx+c)}+ia\,e^{2i(dx+c)}-b+ia\right)^3 d(ia+b)^2}$	82
norman	$\frac{\frac{1}{3bd}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{3bd}-\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{bd}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{db}}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^3}$	117
parallelrisch	$-\frac{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a^2-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3 ab+\frac{2\left(-a^2+2b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{3}+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)ab+a^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,a^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^3}$	120

input
```
int(1/(cos(d*x+c)*a+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/3*(a^2+b^2)/b^3/(a+b*tan(d*x+c))^3+a/b^3/(a+b*tan(d*x+c))^2-1/b^3/
(a+b*tan(d*x+c)))
```

3.229.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. 2(94) = 188.

Time = 0.26 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.21

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = -\frac{2\,(3\,a^2b-b^3)\cos{(dx+c)}^3-3\,(a^2b-b^3)\cos{(dx+c)}-(a^3+3\,ab^2+2\,(a}{3\,((a^7-a^5b^2-5\,a^3b^4-3\,ab^6)d\cos{(dx+c)}^3+3\,(a^5b^2+2\,a^3b^4+ab^6)d\cos{(dx+c)}+((3\,a^6b+5\,a^4b^3}$$

input
```
integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

output
```
-1/3*(2*(3*a^2*b - b^3)*cos(d*x + c)^3 - 3*(a^2*b - b^3)*cos(d*x + c) - (a
^3 + 3*a*b^2 + 2*(a^3 - 3*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/((a^7 - a^5
*b^2 - 5*a^3*b^4 - 3*a*b^6)*d*cos(d*x + c)^3 + 3*(a^5*b^2 + 2*a^3*b^4 + a*
b^6)*d*cos(d*x + c) + ((3*a^6*b + 5*a^4*b^3 + a^2*b^5 - b^7)*d*cos(d*x + c
)^2 + (a^4*b^3 + 2*a^2*b^5 + b^7)*d)*sin(d*x + c))
```

3.229.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

output
```
Timed out
```

3.229.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.87

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = -\frac{3\,b^2\tan{(dx+c)}^2+3\,ab\tan{(dx+c)}+a^2+b^2}{3\,\left(b^6\tan{(dx+c)}^3+3\,ab^5\tan{(dx+c)}^2+3\,a^2b^4\tan{(dx+c)}+a^3b^3\right)d}$$

input
```
integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/3*(3*b^2*tan(d*x + c)^2 + 3*a*b*tan(d*x + c) + a^2 + b^2)/((b^6*tan(d*x
 + c)^3 + 3*a*b^5*tan(d*x + c)^2 + 3*a^2*b^4*tan(d*x + c) + a^3*b^3)*d)
```

3.229.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.51

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = -\frac{3\,b^2\tan\left(dx+c\right)^2+3\,ab\tan\left(dx+c\right)+a^2+b^2}{3\left(b\tan\left(dx+c\right)+a\right)^3 b^3 d}$$

input
```
integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/3*(3*b^2*tan(d*x + c)^2 + 3*a*b*tan(d*x + c) + a^2 + b^2)/((b*tan(d*x +
 c) + a)^3*b^3*d)
```

3.229.9 Mupad [B] (verification not implemented)

Time = 27.89 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.27

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$$

$$= \frac{\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{a}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a}-\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(a^2-2\,b^2\right)}{3\,a^3}+\frac{4\,b\tan\left(\frac{c}{2}+\frac{d}{2}\right.}{a^2}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(12\,a\,b^2-3\,a^3\right)-a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(12\,a\,b^2-3\,a^3\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(12\right.\right.}$$

input
```
int(1/(a*cos(c + d*x) + b*sin(c + d*x))^4,x)
```

output
```
((2*tan(c/2 + (d*x)/2)^5)/a + (2*tan(c/2 + (d*x)/2))/a - (4*tan(c/2 + (d*x
)/2)^3*(a^2 - 2*b^2))/(3*a^3) + (4*b*tan(c/2 + (d*x)/2)^2)/a^2 - (4*b*tan(
c/2 + (d*x)/2)^4)/a^2)/(d*(tan(c/2 + (d*x)/2)^2*(12*a*b^2 - 3*a^3) - a^3*t
an(c/2 + (d*x)/2)^6 - tan(c/2 + (d*x)/2)^4*(12*a*b^2 - 3*a^3) - tan(c/2 +
(d*x)/2)^3*(12*a^2*b - 8*b^3) + a^3 + 6*a^2*b*tan(c/2 + (d*x)/2) + 6*a^2*b
*tan(c/2 + (d*x)/2)^5))
```

3.230 $\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^5}\,dx$

3.230.1 Optimal result

Integrand size = 19, antiderivative size = 156

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^5}\,dx = -\frac{3\text{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{8\left(a^2+b^2\right)^{5/2}d} - \frac{b\cos(c+dx)-a\sin(c+dx)}{4\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^4} - \frac{3(b\cos(c+dx)-a\sin(c+dx))}{8\left(a^2+b^2\right)^2d(a\cos(c+dx)+b\sin(c+dx))^2}$$

output
```
-3/8*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2)/
d+1/4*(-b*cos(d*x+c)+a*sin(d*x+c))/(a^2+b^2)/d/(a*cos(d*x+c)+b*sin(d*x+c))
^4-3/8*(b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^2/d/(a*cos(d*x+c)+b*sin(d*x+c
))^2
```

3.230.2 Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.01

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^5}\,dx = \frac{\frac{6\text{arctanh}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}} + \frac{-11b\left(a^2+b^2\right)\cos(c+dx)+\left(-9a^2b+3b^3\right)\cos(3(c+dx))+2a\left(7a^2+b^2+3\left(a^2-3b^2\right)\cos(2(c+dx))\right)\sin(c+dx)}{4\left(a^2+b^2\right)^2(a\cos(c+dx)+b\sin(c+dx))^4}}{8d}$$

input
```
Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-5),x]
```

output
```
((6*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2)
+ (-11*b*(a^2 + b^2)*Cos[c + d*x] + (-9*a^2*b + 3*b^3)*Cos[3*(c + d*x)] +
2*a*(7*a^2 + b^2 + 3*(a^2 - 3*b^2)*Cos[2*(c + d*x)])*Sin[c + d*x])/(4*(a^2
 + b^2)^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^4))/(8*d)
```

3.230.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3042, 3555, 3042, 3555, 3042, 3553, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^5}dx$$

$$\Big\downarrow \text{3555}$$

$$\frac{3\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^3}dx}{4(a^2+b^2)}-\frac{b\cos(c+dx)-a\sin(c+dx)}{4d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^3}dx}{4(a^2+b^2)}-\frac{b\cos(c+dx)-a\sin(c+dx)}{4d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^4}$$

$$\Big\downarrow \text{3555}$$

$$\frac{3\left(\frac{\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{2(a^2+b^2)}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^2}\right)}{4(a^2+b^2)}-\frac{b\cos(c+dx)-a\sin(c+dx)}{4d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}dx}{2(a^2+b^2)}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^2}\right)}{4\left(a^2+b^2\right)}-\frac{b\cos(c+dx)-a\sin(c+dx)}{4d\left(a^2+b^2\right)\left(a\cos(c+dx)+b\sin(c+dx)\right)^4}$$

$$\Big\downarrow \text{3553}$$

$$\frac{3\left(-\frac{\int \frac{1}{a^2+b^2-(b\cos(c+dx)-a\sin(c+dx))^2}d(b\cos(c+dx)-a\sin(c+dx))}{2d(a^2+b^2)}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^2}\right)}{4\left(a^2+b^2\right)}-\frac{b\cos(c+dx)-a\sin(c+dx)}{4d\left(a^2+b^2\right)\left(a\cos(c+dx)+b\sin(c+dx)\right)^4}$$

$$\Big\downarrow \text{219}$$

$$\frac{3\left(-\frac{\operatorname{arctanh}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2d(a^2+b^2)^{3/2}}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^2}\right)}{4\left(a^2+b^2\right)}-\frac{b\cos(c+dx)-a\sin(c+dx)}{4d\left(a^2+b^2\right)\left(a\cos(c+dx)+b\sin(c+dx)\right)^4}$$

input
```
Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-5),x]
```

output
```
-1/4*(b*Cos[c + d*x] - a*Sin[c + d*x])/((a^2 + b^2)*d*(a*Cos[c + d*x] + b*
Sin[c + d*x])^4) + (3*(-1/2*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt
[a^2 + b^2]]/((a^2 + b^2)^(3/2)*d) - (b*Cos[c + d*x] - a*Sin[c + d*x])/(2*
(a^2 + b^2)*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)))/(4*(a^2 + b^2))
```

3.230.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.230. $\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^5}\,dx$

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3555
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(b*Cos[c + d*x] - a*Sin[c + d*x])*((a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Simp[(n + 2)/((n + 1)*(a^
2 + b^2))   Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

3.230.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.22 (sec) , antiderivative size = 454, normalized size of antiderivative = 2.91

method	result
risch	$\frac{e^{i(dx+c)}\left(3ia^3-9ia\,b^2-9a^2b\,e^{6i(dx+c)}+3b^3e^{6i(dx+c)}-3ia^3e^{6i(dx+c)}-11ia\,b^2e^{4i(dx+c)}-11a^2b\,e^{4i(dx+c)}-11b^3e^{4i(dx+c)}-\right.}{4(-ia+b)^2\left(b\,e^{2i(dx+c)}+ia\,e^{2i(\right.}}$
derivativedivides	$-\frac{2\left(-\frac{\left(5a^4+16a^2b^2+8b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{8a\left(a^4+2a^2b^2+b^4\right)}+\frac{3b\left(a^4+16a^2b^2+8b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{8a^2\left(a^4+2a^2b^2+b^4\right)}-\frac{\left(3a^6-36a^4b^2+56a^2b^4+32b^6\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{8a^3\left(a^4+2a^2b^2+b^4\right)}+\frac{b(\ldots}{}\right.}{}$
default	$-\frac{2\left(-\frac{\left(5a^4+16a^2b^2+8b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{8a\left(a^4+2a^2b^2+b^4\right)}+\frac{3b\left(a^4+16a^2b^2+8b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{8a^2\left(a^4+2a^2b^2+b^4\right)}-\frac{\left(3a^6-36a^4b^2+56a^2b^4+32b^6\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{8a^3\left(a^4+2a^2b^2+b^4\right)}+\frac{b(\ldots}{}\right.}{}$

input
```
int(1/(cos(d*x+c)*a+b*sin(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output
```
1/4*exp(I*(d*x+c))*(3*I*a^3-9*I*a*b^2-9*a^2*b*exp(I*(d*x+c))^6+3*b^3*exp(I
*(d*x+c))^6+11*I*a^3*exp(I*(d*x+c))^2+9*I*a*b^2*exp(I*(d*x+c))^6-11*a^2*b*
exp(I*(d*x+c))^4-11*b^3*exp(I*(d*x+c))^4-11*I*a^3*exp(I*(d*x+c))^4+11*I*a*
b^2*exp(I*(d*x+c))^2-11*a^2*b*exp(I*(d*x+c))^2-11*b^3*exp(I*(d*x+c))^2-3*I
*a^3*exp(I*(d*x+c))^6-11*I*a*b^2*exp(I*(d*x+c))^4-9*a^2*b+3*b^3)/(b-I*a)^2
/(b*exp(I*(d*x+c))^2+I*a*exp(I*(d*x+c))^2-b+I*a)^4/d/(b+I*a)^2+3/8/(a^2+b^
2)^(5/2)/d*ln(exp(I*(d*x+c))+(I*a^5+2*I*a^3*b^2+I*a*b^4-a^4*b-2*a^2*b^3-b^
5)/(a^2+b^2)^(5/2))-3/8/(a^2+b^2)^(5/2)/d*ln(exp(I*(d*x+c))-(I*a^5+2*I*a^3
*b^2+I*a*b^4-a^4*b-2*a^2*b^3-b^5)/(a^2+b^2)^(5/2))
```

3.230.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 544 vs. $2(146) = 292$.

Time = 0.29 (sec) , antiderivative size = 544, normalized size of antiderivative = 3.49

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^5}\,dx =$$
$$-\frac{6\left(3\,a^4b+2\,a^2b^3-b^5\right)\cos\left(dx+c\right)^3-3\left(\left(a^4-6\,a^2b^2+b^4\right)\cos\left(dx+c\right)^4+b^4+2\left(3\,a^2b^2-b^4\right)\cos\left(dx+\right.\right.}{16\left(\left(a^{10}-3\,a^8b^2-14\,a^6b^4-14\,a^4b^6-3\,a^2b^8\right.\right.}$$

input
```
integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas")
```

output
```
-1/16*(6*(3*a^4*b + 2*a^2*b^3 - b^5)*cos(d*x + c)^3 - 3*((a^4 - 6*a^2*b^2
+ b^4)*cos(d*x + c)^4 + b^4 + 2*(3*a^2*b^2 - b^4)*cos(d*x + c)^2 + 4*(a*b^
3*cos(d*x + c) + (a^3*b - a*b^3)*cos(d*x + c)^3)*sin(d*x + c))*sqrt(a^2 +
b^2)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 -
2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*
cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) - 2*(4*a^4*
b - a^2*b^3 - 5*b^5)*cos(d*x + c) - 2*(2*a^5 + 7*a^3*b^2 + 5*a*b^4 + 3*(a^
5 - 2*a^3*b^2 - 3*a*b^4)*cos(d*x + c)^2)*sin(d*x + c))/((a^10 - 3*a^8*b^2
- 14*a^6*b^4 - 14*a^4*b^6 - 3*a^2*b^8 + b^10)*d*cos(d*x + c)^4 + 2*(3*a^8*
b^2 + 8*a^6*b^4 + 6*a^4*b^6 - b^10)*d*cos(d*x + c)^2 + (a^6*b^4 + 3*a^4*b^
6 + 3*a^2*b^8 + b^10)*d + 4*((a^9*b + 2*a^7*b^3 - 2*a^3*b^7 - a*b^9)*d*cos
(d*x + c)^3 + (a^7*b^3 + 3*a^5*b^5 + 3*a^3*b^7 + a*b^9)*d*cos(d*x + c))*si
n(d*x + c))
```

3.230.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^5}\,dx=\text{Timed out}$$

input
```
integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

output
```
Timed out
```

3.230.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 822 vs. $2(146) = 292$.

Time = 0.33 (sec) , antiderivative size = 822, normalized size of antiderivative = 5.27

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^5}\,dx=$$

$$-\frac{2\left(5\,a^6b+2\,a^4b^3-\frac{\left(5\,a^7-24\,a^5b^2-8\,a^3b^4\right)\sin(dx+c)}{\cos(dx+c)+1}-\frac{\left(23\,a^6b-64\,a^4b^3-24\,a^2b^5\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{\left(3\,a^7+84\,a^5b^2-56\,a^3b^4\cdots\right.}{(\cos(dx+c\cdots}\right.}{a^{12}+2\,a^{10}b^2+a^8b^4+\frac{8\left(a^{11}b+2\,a^9b^3+a^7b^5\right)\sin(dx+c)}{\cos(dx+c)+1}-\frac{4\left(a^{12}-4\,a^{10}b^2-11\,a^8b^4-6\,a^6b^6\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{8\left(3\,a^{11}b+2\,a^9b^3-5\,a^7b^5-4\,a^5b^7\right)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\cdots}$$

input
```
integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima")
```

output
```
-1/8*(2*(5*a^6*b + 2*a^4*b^3 - (5*a^7 - 24*a^5*b^2 - 8*a^3*b^4)*sin(d*x +
c)/(cos(d*x + c) + 1) - (23*a^6*b - 64*a^4*b^3 - 24*a^2*b^5)*sin(d*x + c)^
2/(cos(d*x + c) + 1)^2 - (3*a^7 + 84*a^5*b^2 - 56*a^3*b^4 - 32*a*b^6)*sin(
d*x + c)^3/(cos(d*x + c) + 1)^3 + (15*a^6*b - 114*a^4*b^3 - 8*a^2*b^5 + 16
*b^7)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - (3*a^7 - 36*a^5*b^2 + 56*a^3*b
^4 + 32*a*b^6)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 3*(a^6*b + 16*a^4*b^3
 + 8*a^2*b^5)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - (5*a^7 + 16*a^5*b^2 +
8*a^3*b^4)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/(a^12 + 2*a^10*b^2 + a^8*b
^4 + 8*(a^11*b + 2*a^9*b^3 + a^7*b^5)*sin(d*x + c)/(cos(d*x + c) + 1) - 4*
(a^12 - 4*a^10*b^2 - 11*a^8*b^4 - 6*a^6*b^6)*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 - 8*(3*a^11*b + 2*a^9*b^3 - 5*a^7*b^5 - 4*a^5*b^7)*sin(d*x + c)^3/(
cos(d*x + c) + 1)^3 + 2*(3*a^12 - 18*a^10*b^2 - 37*a^8*b^4 - 8*a^6*b^6 + 8
*a^4*b^8)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 8*(3*a^11*b + 2*a^9*b^3 -
5*a^7*b^5 - 4*a^5*b^7)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 4*(a^12 - 4*a
^10*b^2 - 11*a^8*b^4 - 6*a^6*b^6)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 8*
(a^11*b + 2*a^9*b^3 + a^7*b^5)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + (a^12
 + 2*a^10*b^2 + a^8*b^4)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) + 3*log((b -
 a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*sin(d*x + c)/
(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 +
 b^2)))/d
```

3.230.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 588 vs. $2(146) = 292$.

Time = 0.35 (sec) , antiderivative size = 588, normalized size of antiderivative = 3.77

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^5}\,dx =$$

$$-\frac{\frac{3\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}} - \frac{2\left(5\,a^7\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+16\,a^5b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+8\,a^3b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-3\,a^6b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{}}{}$$

input
```
integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

output
```
-1/8*(3*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*
a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4
)*sqrt(a^2 + b^2)) - 2*(5*a^7*tan(1/2*d*x + 1/2*c)^7 + 16*a^5*b^2*tan(1/2*
d*x + 1/2*c)^7 + 8*a^3*b^4*tan(1/2*d*x + 1/2*c)^7 - 3*a^6*b*tan(1/2*d*x +
1/2*c)^6 - 48*a^4*b^3*tan(1/2*d*x + 1/2*c)^6 - 24*a^2*b^5*tan(1/2*d*x + 1/
2*c)^6 + 3*a^7*tan(1/2*d*x + 1/2*c)^5 - 36*a^5*b^2*tan(1/2*d*x + 1/2*c)^5
+ 56*a^3*b^4*tan(1/2*d*x + 1/2*c)^5 + 32*a*b^6*tan(1/2*d*x + 1/2*c)^5 - 15
*a^6*b*tan(1/2*d*x + 1/2*c)^4 + 114*a^4*b^3*tan(1/2*d*x + 1/2*c)^4 + 8*a^2
*b^5*tan(1/2*d*x + 1/2*c)^4 - 16*b^7*tan(1/2*d*x + 1/2*c)^4 + 3*a^7*tan(1/
2*d*x + 1/2*c)^3 + 84*a^5*b^2*tan(1/2*d*x + 1/2*c)^3 - 56*a^3*b^4*tan(1/2*
d*x + 1/2*c)^3 - 32*a*b^6*tan(1/2*d*x + 1/2*c)^3 + 23*a^6*b*tan(1/2*d*x +
1/2*c)^2 - 64*a^4*b^3*tan(1/2*d*x + 1/2*c)^2 - 24*a^2*b^5*tan(1/2*d*x + 1/
2*c)^2 + 5*a^7*tan(1/2*d*x + 1/2*c) - 24*a^5*b^2*tan(1/2*d*x + 1/2*c) - 8*
a^3*b^4*tan(1/2*d*x + 1/2*c) - 5*a^6*b - 2*a^4*b^3)/((a^8 + 2*a^6*b^2 + a^
4*b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c) - a)^4))/d
```

3.230.9 Mupad [B] (verification not implemented)

Time = 32.33 (sec) , antiderivative size = 719, normalized size of antiderivative = 4.61

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^5}\,dx =$$

$$-\frac{\frac{5\,a^2\,b+2\,b^3}{4\,(a^4+2\,a^2\,b^2+b^4)}+\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,(a^4\,b+16\,a^2\,b^3+8\,b^5)}{4\,a^2\,(a^4+2\,a^2\,b^2+b^4)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,(-23\,a^4\,b+64\,a^2\,b^3+24\,b^5)}{4\,a^2\,(a^4+2\,a^2\,b^2+b^4)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(-5\,a^4+24\,a^2\,b^2}{4\,a\,(a^4+2\,a^2\,b^2+b^4)}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,(6\,a^4-48\,a^2\,b^2+16\,b^4)+a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+a^4-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,(4\,a^4-24\,a^2\,b^2)-\right.}$$

$$+\frac{\mathrm{atan}\left(\frac{-1\mathrm{i}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^5+a^4\,b\,1\mathrm{i}-2\mathrm{i}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^3\,b^2+a^2\,b^3\,2\mathrm{i}-1\mathrm{i}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b^4+b^5\,1\mathrm{i}}{(a^2+b^2)^{5/2}}\right)\,3\mathrm{i}}{4\,d\,(a^2+b^2)^{5/2}}$$

input
```
int(1/(a*cos(c + d*x) + b*sin(c + d*x))^5,x)
```

output
```
(atan((a^4*b*1i + b^5*1i + a^2*b^3*2i - a^5*tan(c/2 + (d*x)/2)*1i - a*b^4*
tan(c/2 + (d*x)/2)*1i - a^3*b^2*tan(c/2 + (d*x)/2)*2i)/(a^2 + b^2)^(5/2))*
3i)/(4*d*(a^2 + b^2)^(5/2)) - ((5*a^2*b + 2*b^3)/(4*(a^4 + b^4 + 2*a^2*b^2
)) + (3*tan(c/2 + (d*x)/2)^6*(a^4*b + 8*b^5 + 16*a^2*b^3))/(4*a^2*(a^4 + b
^4 + 2*a^2*b^2)) + (tan(c/2 + (d*x)/2)^2*(24*b^5 - 23*a^4*b + 64*a^2*b^3))
/(4*a^2*(a^4 + b^4 + 2*a^2*b^2)) + (tan(c/2 + (d*x)/2)*(8*b^4 - 5*a^4 + 24
*a^2*b^2))/(4*a*(a^4 + b^4 + 2*a^2*b^2)) - (tan(c/2 + (d*x)/2)^5*(3*a^6 +
32*b^6 + 56*a^2*b^4 - 36*a^4*b^2))/(4*a^3*(a^4 + b^4 + 2*a^2*b^2)) - (tan(
c/2 + (d*x)/2)^3*(3*a^6 - 32*b^6 - 56*a^2*b^4 + 84*a^4*b^2))/(4*a^3*(a^4 +
 b^4 + 2*a^2*b^2)) - (tan(c/2 + (d*x)/2)^7*(5*a^4 + 8*b^4 + 16*a^2*b^2))/(
4*a*(a^4 + b^4 + 2*a^2*b^2)) + (tan(c/2 + (d*x)/2)^4*(5*a^2*b + 2*b^3)*(3*
a^4 + 8*b^4 - 24*a^2*b^2))/(4*a^4*(a^4 + b^4 + 2*a^2*b^2)))/(d*(tan(c/2 +
(d*x)/2)^4*(6*a^4 + 16*b^4 - 48*a^2*b^2) + a^4*tan(c/2 + (d*x)/2)^8 + a^4
- tan(c/2 + (d*x)/2)^2*(4*a^4 - 24*a^2*b^2) - tan(c/2 + (d*x)/2)^6*(4*a^4
- 24*a^2*b^2) + tan(c/2 + (d*x)/2)^3*(32*a*b^3 - 24*a^3*b) - tan(c/2 + (d*
x)/2)^5*(32*a*b^3 - 24*a^3*b) + 8*a^3*b*tan(c/2 + (d*x)/2) - 8*a^3*b*tan(c
/2 + (d*x)/2)^7))
```

3.230. $\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^5}\,dx$

3.231 $\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^6}\,dx$

3.231.1 Optimal result

Integrand size = 19, antiderivative size = 151

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^6}\,dx = -\frac{b\cos(c+dx)-a\sin(c+dx)}{5(a^2+b^2)\,d(a\cos(c+dx)+b\sin(c+dx))^5} - \frac{4(b\cos(c+dx)-a\sin(c+dx))}{15(a^2+b^2)^2\,d(a\cos(c+dx)+b\sin(c+dx))^3} + \frac{8\sin(c+dx)}{15a(a^2+b^2)^2\,d(a\cos(c+dx)+b\sin(c+dx))}$$

output
```
1/5*(-b*cos(d*x+c)+a*sin(d*x+c))/(a^2+b^2)/d/(a*cos(d*x+c)+b*sin(d*x+c))^5
-4/15*(b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^2/d/(a*cos(d*x+c)+b*sin(d*x+c)
)^3+8/15*sin(d*x+c)/a/(a^2+b^2)^2/d/(a*cos(d*x+c)+b*sin(d*x+c))
```

3.231.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.21

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^6}\,dx = \frac{-10ab(a^2+b^2)\cos(3(c+dx))+(-4a^3b+4ab^3)\cos(5(c+dx))+10a^4\sin(c+dx)+20a^2b^2\sin(c+dx) -}{30a(a^2+b^2)^2\,d}$$

input
```
Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-6),x]
```

output
```
(-10*a*b*(a^2 + b^2)*Cos[3*(c + d*x)] + (-4*a^3*b + 4*a*b^3)*Cos[5*(c + d*
x)] + 10*a^4*Sin[c + d*x] + 20*a^2*b^2*Sin[c + d*x] + 10*b^4*Sin[c + d*x]
+ 5*a^4*Sin[3*(c + d*x)] - 5*b^4*Sin[3*(c + d*x)] + a^4*Sin[5*(c + d*x)] -
 6*a^2*b^2*Sin[5*(c + d*x)] + b^4*Sin[5*(c + d*x)])/(30*a*(a^2 + b^2)^2*d*
(a*Cos[c + d*x] + b*Sin[c + d*x])^5)
```

3.231.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 3555, 3042, 3555, 3042, 3554}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^6}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^6}dx$$

$$\Big\downarrow \; 3555$$

$$\frac{4\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^4}dx}{5(a^2+b^2)}-\frac{b\cos(c+dx)-a\sin(c+dx)}{5d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^5}$$

$$\Big\downarrow \; 3042$$

$$\frac{4\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^4}dx}{5(a^2+b^2)}-\frac{b\cos(c+dx)-a\sin(c+dx)}{5d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^5}$$

$$\Big\downarrow \; 3555$$

$$\frac{4\left(\frac{2\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^2}dx}{3(a^2+b^2)}-\frac{b\cos(c+dx)-a\sin(c+dx)}{3d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^3}\right)}{5(a^2+b^2)}-\frac{b\cos(c+dx)-a\sin(c+dx)}{5d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^5}$$

$$\Big\downarrow \; 3042$$

$$\frac{4\left(\frac{2\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^2}dx}{3(a^2+b^2)}-\frac{b\cos(c+dx)-a\sin(c+dx)}{3d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^3}\right)}{5\left(a^2+b^2\right)}-\frac{b\cos(c+dx)-a\sin(c+dx)}{5d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^5}$$

$$\Big\downarrow \text{ 3554}$$

$$\frac{4\left(\frac{2\sin(c+dx)}{3ad(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))}-\frac{b\cos(c+dx)-a\sin(c+dx)}{3d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^3}\right)}{5\left(a^2+b^2\right)}-\frac{b\cos(c+dx)-a\sin(c+dx)}{5d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^5}$$

input
```
Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-6),x]
```

output
```
-1/5*(b*Cos[c + d*x] - a*Sin[c + d*x])/((a^2 + b^2)*d*(a*Cos[c + d*x] + b*
Sin[c + d*x])^5) + (4*(-1/3*(b*Cos[c + d*x] - a*Sin[c + d*x])/((a^2 + b^2)
*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) + (2*Sin[c + d*x])/(3*a*(a^2 + b^2
)*d*(a*Cos[c + d*x] + b*Sin[c + d*x]))))/(5*(a^2 + b^2))
```

3.231.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3554
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3555
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(b*Cos[c + d*x] - a*Sin[c + d*x])*((a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Simp[(n + 2)/((n + 1)*(a^
2 + b^2))   Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

3.231.4 Maple [A] (verified)

Time = 2.31 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{\frac{(a^2+b^2)a}{b^5(a+b\tan(dx+c))^4}-\frac{6a^2+2b^2}{3b^5(a+b\tan(dx+c))^3}-\frac{a^4+2a^2b^2+b^4}{5b^5(a+b\tan(dx+c))^5}-\frac{1}{b^5(a+b\tan(dx+c))}+\frac{2a}{b^5(a+b\tan(dx+c))^2}}{d}$
default	$\frac{\frac{(a^2+b^2)a}{b^5(a+b\tan(dx+c))^4}-\frac{6a^2+2b^2}{3b^5(a+b\tan(dx+c))^3}-\frac{a^4+2a^2b^2+b^4}{5b^5(a+b\tan(dx+c))^5}-\frac{1}{b^5(a+b\tan(dx+c))}+\frac{2a}{b^5(a+b\tan(dx+c))^2}}{d}$
risch	$\frac{16i(-20iab\,e^{4i(dx+c)}+10a^2e^{4i(dx+c)}-10b^2e^{4i(dx+c)}+5a^2e^{2i(dx+c)}+5b^2e^{2i(dx+c)}+2iba+a^2-b^2)}{15d(-ib+a)^3(-ib\,e^{2i(dx+c)}+e^{2i(dx+c)}a+ib+a)^5}$
norman	$\frac{\frac{1}{5bd}+\frac{2(32a^3-16ab^2)\tan(\frac{dx}{2}+\frac{c}{2})^5}{15da^4}-\frac{16\tan(\frac{dx}{2}+\frac{c}{2})^3}{3ad}-\frac{16\tan(\frac{dx}{2}+\frac{c}{2})^7}{3ad}-\frac{\tan(\frac{dx}{2}+\frac{c}{2})^{10}}{5bd}+\frac{(6a^3-16ab^2)\tan(\frac{dx}{2}+\frac{c}{2})^4}{3da^3b}-\frac{(6a^3}{}}{(\tan(\frac{dx}{2}+\frac{c}{2})^2a-2b\tan(\frac{dx}{2}+\frac{c}{2})-a)^5}$
parallelrisch	$-\frac{2\tan(\frac{dx}{2}+\frac{c}{2})(15\tan(\frac{dx}{2}+\frac{c}{2})^8a^4-60\tan(\frac{dx}{2}+\frac{c}{2})^7a^3b-20a^4\tan(\frac{dx}{2}+\frac{c}{2})^6+120\tan(\frac{dx}{2}+\frac{c}{2})^6a^2b^2+140\tan(\frac{dx}{2}+}{}$

input
```
int(1/(cos(d*x+c)*a+b*sin(d*x+c))^6,x,method=_RETURNVERBOSE)
```

output
```
1/d*((a^2+b^2)*a/b^5/(a+b*tan(d*x+c))^4-1/3*(6*a^2+2*b^2)/b^5/(a+b*tan(d*x
+c))^3-1/5*(a^4+2*a^2*b^2+b^4)/b^5/(a+b*tan(d*x+c))^5-1/b^5/(a+b*tan(d*x+c
))+2*a/b^5/(a+b*tan(d*x+c))^2)
```

3.231.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 441 vs. $2(145) = 290$.

Time = 0.28 (sec) , antiderivative size = 441, normalized size of antiderivative = 2.92

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^6}\,dx =$$
$$-\frac{8\,(5\,a^4b-10\,a^2b^3+b^5)\cos(dx+c)^5-20\,(a^4b}{15\,((a^{11}-7\,a^9b^2-22\,a^7b^4-14\,a^5b^6+5\,a^3b^8+5\,ab^{10})d\cos(dx+c)^5+10\,(a^9b^2+2\,a^7b^4-2\,a^3b^8-ab^1}$$

input
```
integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^6,x, algorithm="fricas")
```

output
```
-1/15*(8*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c)^5 - 20*(a^4*b - 6*a^2*b
^3 + b^5)*cos(d*x + c)^3 - 5*(a^4*b + 6*a^2*b^3 - 3*b^5)*cos(d*x + c) - (3
*a^5 + 10*a^3*b^2 + 15*a*b^4 + 8*(a^5 - 10*a^3*b^2 + 5*a*b^4)*cos(d*x + c)
^4 + 4*(a^5 + 10*a^3*b^2 - 15*a*b^4)*cos(d*x + c)^2)*sin(d*x + c))/((a^11
- 7*a^9*b^2 - 22*a^7*b^4 - 14*a^5*b^6 + 5*a^3*b^8 + 5*a*b^10)*d*cos(d*x +
c)^5 + 10*(a^9*b^2 + 2*a^7*b^4 - 2*a^3*b^8 - a*b^10)*d*cos(d*x + c)^3 + 5*
(a^7*b^4 + 3*a^5*b^6 + 3*a^3*b^8 + a*b^10)*d*cos(d*x + c) + ((5*a^10*b + 5
*a^8*b^3 - 14*a^6*b^5 - 22*a^4*b^7 - 7*a^2*b^9 + b^11)*d*cos(d*x + c)^4 +
2*(5*a^8*b^3 + 14*a^6*b^5 + 12*a^4*b^7 + 2*a^2*b^9 - b^11)*d*cos(d*x + c)^
2 + (a^6*b^5 + 3*a^4*b^7 + 3*a^2*b^9 + b^11)*d)*sin(d*x + c))
```

3.231.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^6}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))**6,x)
```

output
```
Timed out
```

3.231.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.15

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^6}\,dx = -\frac{15\,b^4\tan(dx+c)^4+30\,ab^3\tan(dx+c)^3+3\,a^4+a^2b^2+3\,b^4+10\,(3\,a^2b^2+b^4)\tan(dx+c)^2+5\,(3\,a^3b}{15\,\left(b^{10}\tan(dx+c)^5+5\,ab^9\tan(dx+c)^4+10\,a^2b^8\tan(dx+c)^3+10\,a^3b^7\tan(dx+c)^2+5\,a^4b^6\tan}$$

input
```
integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^6,x, algorithm="maxima")
```

output
```
-1/15*(15*b^4*tan(d*x + c)^4 + 30*a*b^3*tan(d*x + c)^3 + 3*a^4 + a^2*b^2 +
 3*b^4 + 10*(3*a^2*b^2 + b^4)*tan(d*x + c)^2 + 5*(3*a^3*b + a*b^3)*tan(d*x
 + c))/((b^10*tan(d*x + c)^5 + 5*a*b^9*tan(d*x + c)^4 + 10*a^2*b^8*tan(d*x
 + c)^3 + 10*a^3*b^7*tan(d*x + c)^2 + 5*a^4*b^6*tan(d*x + c) + a^5*b^5)*d)
```

3.231.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.78

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^6}\,dx = -\frac{15\,b^4\tan(dx+c)^4+30\,ab^3\tan(dx+c)^3+30\,a^2b^2\tan(dx+c)^2+10\,b^4\tan(dx+c)^2+15\,a^3b\tan(dx+\cdots}{15\,(b\tan(dx+c)+a)^5b^5d}$$

input
```
integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^6,x, algorithm="giac")
```

output
```
-1/15*(15*b^4*tan(d*x + c)^4 + 30*a*b^3*tan(d*x + c)^3 + 30*a^2*b^2*tan(d*
x + c)^2 + 10*b^4*tan(d*x + c)^2 + 15*a^3*b*tan(d*x + c) + 5*a*b^3*tan(d*x
 + c) + 3*a^4 + a^2*b^2 + 3*b^4)/((b*tan(d*x + c) + a)^5*b^5*d)
```

3.231.9 Mupad [B] (verification not implemented)

Time = 31.23 (sec) , antiderivative size = 470, normalized size of antiderivative = 3.11

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^6}\,dx$$

$$= \frac{\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9}{a}+\frac{2\tan\left(\frac{c}{2}+\frac{d\cdots}{2}\right)}{a}\cdots}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(10\,a^5-120\,a^3\,b^2+80\,a\,b^4\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(40\,a^4\,b-80\,a^2\,b^3\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7\left(40\,a^4\cdots\right.\right.}$$

input
```
int(1/(a*cos(c + d*x) + b*sin(c + d*x))^6,x)
```

output
```
((2*tan(c/2 + (d*x)/2)^9)/a + (2*tan(c/2 + (d*x)/2))/a - (8*tan(c/2 + (d*x
)/2)^4*(7*a^2*b - 6*b^3))/(3*a^4) + (8*tan(c/2 + (d*x)/2)^6*(7*a^2*b - 6*b
^3))/(3*a^4) - (8*tan(c/2 + (d*x)/2)^3*(a^2 - 6*b^2))/(3*a^3) - (8*tan(c/2
 + (d*x)/2)^7*(a^2 - 6*b^2))/(3*a^3) + (8*b*tan(c/2 + (d*x)/2)^2)/a^2 - (8
*b*tan(c/2 + (d*x)/2)^8)/a^2 + (4*tan(c/2 + (d*x)/2)^5*(29*a^4 + 24*b^4 -
112*a^2*b^2))/(15*a^5))/(d*(tan(c/2 + (d*x)/2)^4*(80*a*b^4 + 10*a^5 - 120*
a^3*b^2) - tan(c/2 + (d*x)/2)^3*(40*a^4*b - 80*a^2*b^3) - tan(c/2 + (d*x)/
2)^7*(40*a^4*b - 80*a^2*b^3) - a^5*tan(c/2 + (d*x)/2)^10 - tan(c/2 + (d*x)
/2)^6*(80*a*b^4 + 10*a^5 - 120*a^3*b^2) + tan(c/2 + (d*x)/2)^5*(60*a^4*b +
 32*b^5 - 160*a^2*b^3) + a^5 - tan(c/2 + (d*x)/2)^2*(5*a^5 - 40*a^3*b^2) +
 tan(c/2 + (d*x)/2)^8*(5*a^5 - 40*a^3*b^2) + 10*a^4*b*tan(c/2 + (d*x)/2) +
 10*a^4*b*tan(c/2 + (d*x)/2)^9))
```

3.232 $\int (a\cos(c+dx)+b\sin(c+dx))^{7/2}\,dx$

3.232.1 Optimal result

Integrand size = 21, antiderivative size = 186

$$\begin{aligned}\int (a\cos(c+dx)+b\sin(c+dx))^{7/2}\,dx &= -\frac{10(a^2+b^2)\,(b\cos(c+dx)-a\sin(c+dx))\sqrt{a\cos(c+dx)+b\sin(c+dx)}}{21d}\\ &\quad-\frac{2(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^{5/2}}{7d}\\ &\quad+\frac{10(a^2+b^2)^2\,\text{EllipticF}\left(\frac{1}{2}(c+dx-\tan^{-1}(a,b)),2\right)\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}}{21d\sqrt{a\cos(c+dx)+b\sin(c+dx)}}\end{aligned}$$

output
```
-2/7*(b*cos(d*x+c)-a*sin(d*x+c))*(a*cos(d*x+c)+b*sin(d*x+c))^(5/2)/d-10/21
*(a^2+b^2)*(b*cos(d*x+c)-a*sin(d*x+c))*(a*cos(d*x+c)+b*sin(d*x+c))^(1/2)/d
+10/21*(a^2+b^2)^2*(cos(1/2*c+1/2*d*x-1/2*arctan(a,b))^2)^(1/2)/cos(1/2*c+
1/2*d*x-1/2*arctan(a,b))*EllipticF(sin(1/2*c+1/2*d*x-1/2*arctan(a,b)),2^(1
/2))*((a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^(1/2))^(1/2)/d/(a*cos(d*x+c)+b
*sin(d*x+c))^(1/2)
```

3.232.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.43 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.10

$$\int (a\cos(c+dx)+b\sin(c+dx))^{7/2}\,dx = \frac{\sqrt{a\cos(c+dx)+b\sin(c+dx)}(-23b(a^2+b^2)\cos(c+dx)+(-9a^2b+3b^3)\cos(3(c+dx))+ \cdots}{\cdots}$$

input
```
Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(7/2),x]
```

output
```
(Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]]*(-23*b*(a^2 + b^2)*Cos[c + d*x] + (
-9*a^2*b + 3*b^3)*Cos[3*(c + d*x)] + 2*a*(13*a^2 + 7*b^2 + 3*(a^2 - 3*b^2)
*Cos[2*(c + d*x)])*Sin[c + d*x]) + (20*(a^2 + b^2)^2*Sqrt[Cos[c + d*x + Ar
cTan[a/b]]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[c + d*x + ArcTan[a/
b]]^2]*Tan[c + d*x + ArcTan[a/b]])/Sqrt[Sqrt[1 + a^2/b^2]*b*Sin[c + d*x +
ArcTan[a/b]]])/(42*d)
```

3.232.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 3552, 3042, 3552, 3042, 3557, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(c+dx)+b\sin(c+dx))^{7/2}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int (a\cos(c+dx)+b\sin(c+dx))^{7/2}dx$$

$$\Big\downarrow \ 3552$$

$$\frac{5}{7}(a^2+b^2)\int (a\cos(c+dx)+b\sin(c+dx))^{3/2}dx-\frac{2(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{7}\left(a^2+b^2\right)\int (a\cos(c+dx)+b\sin(c+dx))^{3/2}dx-\frac{2(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{3552}$$

$$\frac{5}{7}\left(a^2+b^2\right)\left(\frac{1}{3}\left(a^2+b^2\right)\int \frac{1}{\sqrt{a\cos(c+dx)+b\sin(c+dx)}}dx-\frac{2(b\cos(c+dx)-a\sin(c+dx))\sqrt{a\cos(c+dx)+}}{3d}\right.$$
$$\frac{2(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{7}\left(a^2+b^2\right)\left(\frac{1}{3}\left(a^2+b^2\right)\int \frac{1}{\sqrt{a\cos(c+dx)+b\sin(c+dx)}}dx-\frac{2(b\cos(c+dx)-a\sin(c+dx))\sqrt{a\cos(c+dx)+}}{3d}\right.$$
$$\frac{2(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{3557}$$

$$\frac{5}{7}\left(a^2+b^2\right)\left(\frac{\left(a^2+b^2\right)\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}\int \frac{1}{\sqrt{\cos(c+dx-\tan^{-1}(a,b))}}dx}{3\sqrt{a\cos(c+dx)+b\sin(c+dx)}}-\frac{2(b\cos(c+dx)-a\sin(c+dx))\sqrt{a\cos(}}{3d}\right.$$
$$\frac{2(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{7}\left(a^2+b^2\right)\left(\frac{\left(a^2+b^2\right)\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}\int \frac{1}{\sqrt{\sin\left(c+dx-\tan^{-1}(a,b)+\frac{\pi}{2}\right)}}dx}{3\sqrt{a\cos(c+dx)+b\sin(c+dx)}}-\frac{2(b\cos(c+dx)-a\sin(c+dx))\sqrt{a\,c}}{3d}\right.$$
$$\frac{2(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{3120}$$

$$\frac{5}{7}\left(a^2+b^2\right)\left(\frac{2\left(a^2+b^2\right)\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\tan^{-1}(a,b)\right),2\right)}{3d\sqrt{a\cos(c+dx)+b\sin(c+dx)}}-\frac{2(b\cos(c+dx)-a\sin(c}{}\right.$$
$$\frac{2(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^{5/2}}{7d}$$

3.232. $\int (a\cos(c+dx)+b\sin(c+dx))^{7/2}\,dx$

input
```
Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(7/2),x]
```

output
```
(-2*(b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin[c + d*x])^(5
/2))/(7*d) + (5*(a^2 + b^2)*((-2*(b*Cos[c + d*x] - a*Sin[c + d*x])*Sqrt[a*
Cos[c + d*x] + b*Sin[c + d*x]])/(3*d) + (2*(a^2 + b^2)*EllipticF[(c + d*x
- ArcTan[a, b])/2, 2]*Sqrt[(a*Cos[c + d*x] + b*Sin[c + d*x])/Sqrt[a^2 + b^
2]])/(3*d*Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]])))/7
```

3.232.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3552
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(-(b*Cos[c + d*x] - a*Sin[c + d*x]))*((a*Cos[c + d*x] + b*
Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[(n - 1)*((a^2 + b^2)/n)   Int[(a*Co
s[c + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

rule 3557
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
in[c + d*x])/Sqrt[a^2 + b^2])^n   Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
 /; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) &&  !(GtQ[a^2
+ b^2, 0] || EqQ[a^2 + b^2, 0])
```

3.232.4 Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.99

method	result
default	$-\frac{(a^2+b^2)^2\left(-6\sin(dx+c-\arctan(-a,b))^5+5\sqrt{-\sin(dx+c-\arctan(-a,b))+1}\,\sqrt{2\sin(dx+c-\arctan(-a,b))+2}\,\sqrt{\sin(dx+c-\arctan}\right.}{21\cos(dx+c-\arctan(-a,b))\sqrt{\sin(dx+}}$

input
```
int((cos(d*x+c)*a+b*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/21*(a^2+b^2)^2*(-6*sin(d*x+c-arctan(-a,b))^5+5*(-sin(d*x+c-arctan(-a,b)
)+1)^(1/2)*(2*sin(d*x+c-arctan(-a,b))+2)^(1/2)*sin(d*x+c-arctan(-a,b))^(1/
2)*EllipticF((-sin(d*x+c-arctan(-a,b))+1)^(1/2),1/2*2^(1/2))-4*sin(d*x+c-a
rctan(-a,b))^3+10*sin(d*x+c-arctan(-a,b)))/cos(d*x+c-arctan(-a,b))/(sin(d*
x+c-arctan(-a,b))*(a^2+b^2)^(1/2))^(1/2)/d
```

3.232.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.40

$$\int (a\cos(c+dx)+b\sin(c+dx))^{7/2}\,dx =$$

$$-\frac{5\sqrt{2}(i\,a^3-a^2b+i\,ab^2-b^3)\sqrt{a-i\,b}\,\text{weierstrassPInverse}\left(-\frac{4\,(a^2+2i\,ab-b^2)}{a^2+b^2},0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)}{}$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
-1/21*(5*sqrt(2)*(I*a^3 - a^2*b + I*a*b^2 - b^3)*sqrt(a - I*b)*weierstrass
PInverse(-4*(a^2 + 2*I*a*b - b^2)/(a^2 + b^2), 0, cos(d*x + c) + I*sin(d*x
 + c)) + 5*sqrt(2)*(-I*a^3 - a^2*b - I*a*b^2 - b^3)*sqrt(a + I*b)*weierstr
assPInverse(-4*(a^2 - 2*I*a*b - b^2)/(a^2 + b^2), 0, cos(d*x + c) - I*sin(
d*x + c)) + 2*(3*(3*a^2*b - b^3)*cos(d*x + c)^3 - (a^2*b - 8*b^3)*cos(d*x
+ c) - (5*a^3 + 8*a*b^2 + 3*(a^3 - 3*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))*
sqrt(a*cos(d*x + c) + b*sin(d*x + c)))/d
```

3.232.6 Sympy [F(-1)]

Timed out.

$$\int (a\cos(c+dx)+b\sin(c+dx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.232.7 Maxima [F]

$$\int (a\cos(c+dx)+b\sin(c+dx))^{7/2}\,dx = \int (a\cos(dx+c)+b\sin(dx+c))^{\frac{7}{2}}\,dx$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + b*sin(d*x + c))^(7/2), x)
```

3.232.8 Giac [F]

$$\int (a\cos(c+dx)+b\sin(c+dx))^{7/2}\,dx = \int (a\cos(dx+c)+b\sin(dx+c))^{\frac{7}{2}}\,dx$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + b*sin(d*x + c))^(7/2), x)
```

3.232.9 Mupad [F(-1)]

Timed out.

$$\int (a\cos(c+dx)+b\sin(c+dx))^{7/2}\,dx=\int (a\ \cos\left(c+d\,x\right)+b\ \sin\left(c+d\,x\right))^{7/2}\,dx$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))^(7/2),x)
```

output
```
int((a*cos(c + d*x) + b*sin(c + d*x))^(7/2), x)
```

3.233 $\int (a\cos(c+dx)+b\sin(c+dx))^{5/2}\,dx$

3.233.1 Optimal result

Integrand size = 21, antiderivative size = 131

$$\int (a\cos(c+dx)+b\sin(c+dx))^{5/2}\,dx = -\frac{2(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^{3/2}}{5d} + \frac{6(a^2+b^2)\,E\left(\frac{1}{2}(c+dx-\tan^{-1}(a,b))\middle|\,2\right)\sqrt{a\cos(c+dx)+b\sin(c+dx)}}{5d\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}}$$

output
```
-2/5*(b*cos(d*x+c)-a*sin(d*x+c))*(a*cos(d*x+c)+b*sin(d*x+c))^(3/2)/d+6/5*(
a^2+b^2)*(cos(1/2*c+1/2*d*x-1/2*arctan(a,b))^2)^(1/2)/cos(1/2*c+1/2*d*x-1/
2*arctan(a,b))*EllipticE(sin(1/2*c+1/2*d*x-1/2*arctan(a,b)),2^(1/2))*(a*co
s(d*x+c)+b*sin(d*x+c))^(1/2)/d/((a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^(1/2
))^(1/2)
```

3.233.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.29 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.95

$$\int (a\cos(c+dx)+b\sin(c+dx))^{5/2}\,dx = \frac{\sqrt{a\cos(c+dx)+b\sin(c+dx)}(6a(a^2+b^2)-2ab^2\cos(2(c+dx))+b(a^2-b^2)\sin(2(c+dx))}{}$$

input
```
Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(5/2),x]
```

output
```
(Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]]*(6*a*(a^2 + b^2) - 2*a*b^2*Cos[2*(c
 + d*x)] + b*(a^2 - b^2)*Sin[2*(c + d*x)]) - (3*(a^2 + b^2)^2*Cos[c + d*x
- ArcTan[b/a]]*(b*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[c + d*x - Arc
Tan[b/a]]^2]*Sin[c + d*x - ArcTan[b/a]] + Sqrt[Sin[c + d*x - ArcTan[b/a]]^
2]*(2*a*Cos[c + d*x - ArcTan[b/a]] - b*Sin[c + d*x - ArcTan[b/a]])))/((a*S
qrt[1 + b^2/a^2]*Cos[c + d*x - ArcTan[b/a]])^(3/2)*Sqrt[Sin[c + d*x - ArcT
an[b/a]]^2]))/(5*b*d)
```

3.233.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3552, 3042, 3557, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(c+dx)+b\sin(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\cos(c+dx)+b\sin(c+dx))^{5/2}dx$$

$$\Big\downarrow \text{3552}$$

$$\frac{3}{5}(a^2+b^2)\int \sqrt{a\cos(c+dx)+b\sin(c+dx)}dx-\frac{2(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{5}(a^2+b^2)\int \sqrt{a\cos(c+dx)+b\sin(c+dx)}dx-\frac{2(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3557}$$

$$\frac{3(a^2+b^2)\sqrt{a\cos(c+dx)+b\sin(c+dx)}\int\sqrt{\cos\left(c+dx-\tan^{-1}(a,b)\right)}dx}{5\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}}-\frac{2(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3(a^2+b^2)\sqrt{a\cos(c+dx)+b\sin(c+dx)}\int\sqrt{\sin\left(c+dx-\tan^{-1}(a,b)+\frac{\pi}{2}\right)}dx}{5\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}}-\frac{2(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3119}$$

$$\frac{6(a^2+b^2)\sqrt{a\cos(c+dx)+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\tan^{-1}(a,b)\right)\middle|2\right)}{5d\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}}-\frac{2(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^{3/2}}{5d}$$

input
```
Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(5/2),x]
```

output
```
(-2*(b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin[c + d*x])^(3
/2))/(5*d) + (6*(a^2 + b^2)*EllipticE[(c + d*x - ArcTan[a, b])/2, 2]*Sqrt[
a*Cos[c + d*x] + b*Sin[c + d*x]])/(5*d*Sqrt[(a*Cos[c + d*x] + b*Sin[c + d*
x])/Sqrt[a^2 + b^2]])
```

3.233.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.233. $\int (a\cos(c+dx)+b\sin(c+dx))^{5/2}\,dx$

rule 3552
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(-(b*Cos[c + d*x] - a*Sin[c + d*x]))*((a*Cos[c + d*x] + b*
Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[(n - 1)*((a^2 + b^2)/n)   Int[(a*Co
s[c + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

rule 3557
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
in[c + d*x])/Sqrt[a^2 + b^2])^n   Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
 /; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) &&  !(GtQ[a^2
+ b^2, 0] || EqQ[a^2 + b^2, 0])
```

3.233.4 Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.91

method	result
default	$-\frac{(a^2+b^2)^{\frac{3}{2}}\left(6\sqrt{-\sin(dx+c-\arctan(-a,b))+1}\,\sqrt{2\sin(dx+c-\arctan(-a,b))+2}\,\sqrt{\sin(dx+c-\arctan(-a,b))}\,\operatorname{EllipticE}\left(\sqrt{-\sin(dx+}\right.\right.}{}$

input
```
int((cos(d*x+c)*a+b*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*(a^2+b^2)^(3/2)*(6*(-sin(d*x+c-arctan(-a,b))+1)^(1/2)*(2*sin(d*x+c-ar
ctan(-a,b))+2)^(1/2)*sin(d*x+c-arctan(-a,b))^(1/2)*EllipticE((-sin(d*x+c-a
rctan(-a,b))+1)^(1/2),1/2*2^(1/2))-3*(-sin(d*x+c-arctan(-a,b))+1)^(1/2)*(2
*sin(d*x+c-arctan(-a,b))+2)^(1/2)*sin(d*x+c-arctan(-a,b))^(1/2)*EllipticF(
(-sin(d*x+c-arctan(-a,b))+1)^(1/2),1/2*2^(1/2))-2*sin(d*x+c-arctan(-a,b))^
4+2*sin(d*x+c-arctan(-a,b))^2)/cos(d*x+c-arctan(-a,b))/(sin(d*x+c-arctan(-
a,b))*(a^2+b^2)^(1/2))^(1/2)/d
```

3.233.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.89

$$\int (a\cos(c+dx)+b\sin(c+dx))^{5/2}\,dx =$$
$$-\frac{3\sqrt{2}(-i\,a^2-i\,b^2)\sqrt{a-i\,b}\,\text{weierstrassZeta}\left(-\frac{4\,(a^2+2i\,ab-b^2)}{a^2+b^2},0,\text{weierstrassPInverse}\left(-\frac{4\,(a^2+2i\,ab-b^2)}{a^2+b^2},0,\cos\left(\right.\right.\right.}{}$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-1/5*(3*sqrt(2)*(-I*a^2 - I*b^2)*sqrt(a - I*b)*weierstrassZeta(-4*(a^2 + 2
*I*a*b - b^2)/(a^2 + b^2), 0, weierstrassPInverse(-4*(a^2 + 2*I*a*b - b^2)
/(a^2 + b^2), 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt(2)*(I*a^2 + I*b^
2)*sqrt(a + I*b)*weierstrassZeta(-4*(a^2 - 2*I*a*b - b^2)/(a^2 + b^2), 0,
weierstrassPInverse(-4*(a^2 - 2*I*a*b - b^2)/(a^2 + b^2), 0, cos(d*x + c)
- I*sin(d*x + c))) + 2*(2*a*b*cos(d*x + c)^2 - (a^2 - b^2)*cos(d*x + c)*si
n(d*x + c) - a*b)*sqrt(a*cos(d*x + c) + b*sin(d*x + c)))/d
```

3.233.6 Sympy [F(-1)]

Timed out.

$$\int (a\cos(c+dx)+b\sin(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.233.7 Maxima [F]

$$\int (a\cos(c+dx)+b\sin(c+dx))^{5/2}\,dx=\int (a\cos(dx+c)+b\sin(dx+c))^{\frac{5}{2}}\,dx$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + b*sin(d*x + c))^(5/2), x)
```

3.233.8 Giac [F]

$$\int (a\cos(c+dx)+b\sin(c+dx))^{5/2}\,dx=\int (a\cos(dx+c)+b\sin(dx+c))^{\frac{5}{2}}\,dx$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + b*sin(d*x + c))^(5/2), x)
```

3.233.9 Mupad [F(-1)]

Timed out.

$$\int (a\cos(c+dx)+b\sin(c+dx))^{5/2}\,dx=\int (a\cos(c+d\,x)+b\sin(c+d\,x))^{5/2}\,dx$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))^(5/2),x)
```

output
```
int((a*cos(c + d*x) + b*sin(c + d*x))^(5/2), x)
```

3.234 $\int (a\cos(c+dx)+b\sin(c+dx))^{3/2}\,dx$

3.234.1 Optimal result

Integrand size = 21, antiderivative size = 131

$$\int (a\cos(c+dx)+b\sin(c+dx))^{3/2}\,dx = -\frac{2(b\cos(c+dx)-a\sin(c+dx))\sqrt{a\cos(c+dx)+b\sin(c+dx)}}{3d} + \frac{2(a^2+b^2)\,\text{EllipticF}\left(\frac{1}{2}(c+dx-\tan^{-1}(a,b)),2\right)\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}}{3d\sqrt{a\cos(c+dx)+b\sin(c+dx)}}$$

output
```
-2/3*(b*cos(d*x+c)-a*sin(d*x+c))*(a*cos(d*x+c)+b*sin(d*x+c))^(1/2)/d+2/3*(
a^2+b^2)*(cos(1/2*c+1/2*d*x-1/2*arctan(a,b))^2)^(1/2)/cos(1/2*c+1/2*d*x-1/
2*arctan(a,b))*EllipticF(sin(1/2*c+1/2*d*x-1/2*arctan(a,b)),2^(1/2))*((a*c
os(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^(1/2))^(1/2)/d/(a*cos(d*x+c)+b*sin(d*x+c
))^(1/2)
```

3.234.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.12 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.09

$$\int (a\cos(c+dx)+b\sin(c+dx))^{3/2}\,dx = \frac{2\left((-b\cos(c+dx)+a\sin(c+dx))\sqrt{a\cos(c+dx)+b\sin(c+dx)}+\frac{(a^2+b^2)\sqrt{\cos^2(c+dx+\arctan(\dots$$

input
```
Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(3/2),x]
```

output
```
(2*((-(b*Cos[c + d*x]) + a*Sin[c + d*x])*Sqrt[a*Cos[c + d*x] + b*Sin[c + d
*x]] + ((a^2 + b^2)*Sqrt[Cos[c + d*x + ArcTan[a/b]]^2]*HypergeometricPFQ[{
1/4, 1/2}, {5/4}, Sin[c + d*x + ArcTan[a/b]]^2]*Tan[c + d*x + ArcTan[a/b]]
)/Sqrt[Sqrt[1 + a^2/b^2]*b*Sin[c + d*x + ArcTan[a/b]]]))/(3*d)
```

3.234.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3552, 3042, 3557, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(c+dx)+b\sin(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (a\cos(c+dx)+b\sin(c+dx))^{3/2}dx$$

$$\Big\downarrow \text{ 3552}$$

$$\frac{1}{3}\left(a^2+b^2\right)\int \frac{1}{\sqrt{a\cos(c+dx)+b\sin(c+dx)}}dx-\frac{2(b\cos(c+dx)-a\sin(c+dx))\sqrt{a\cos(c+dx)+b\sin(c+dx)}}{3d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{3}\left(a^2+b^2\right)\int \frac{1}{\sqrt{a\cos(c+dx)+b\sin(c+dx)}}dx-\frac{2(b\cos(c+dx)-a\sin(c+dx))\sqrt{a\cos(c+dx)+b\sin(c+dx)}}{3d}$$

$$\Big\downarrow \text{ 3557}$$

$$\frac{\left(a^2+b^2\right)\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}\int \frac{1}{\sqrt{\cos\left(c+dx-\tan^{-1}(a,b)\right)}}dx}{3\sqrt{a\cos(c+dx)+b\sin(c+dx)}}-\frac{2(b\cos(c+dx)-a\sin(c+dx))\sqrt{a\cos(c+dx)+b\sin(c+dx)}}{3d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(a^2+b^2)\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}\int\frac{1}{\sqrt{\sin\left(c+dx-\tan^{-1}(a,b)+\frac{\pi}{2}\right)}}dx}{3\sqrt{a\cos(c+dx)+b\sin(c+dx)}}-\frac{2(b\cos(c+dx)-a\sin(c+dx))\sqrt{a\cos(c+dx)+b\sin(c+dx)}}{3d}$$

$$\Big\downarrow \text{ 3120}$$

$$\frac{2(a^2+b^2)\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\tan^{-1}(a,b)\right),2\right)}{3d\sqrt{a\cos(c+dx)+b\sin(c+dx)}}-\frac{2(b\cos(c+dx)-a\sin(c+dx))\sqrt{a\cos(c+dx)+b\sin(c+dx)}}{3d}$$

input
```
Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(3/2),x]
```

output
```
(-2*(b*Cos[c + d*x] - a*Sin[c + d*x])*Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]
])/(3*d) + (2*(a^2 + b^2)*EllipticF[(c + d*x - ArcTan[a, b])/2, 2]*Sqrt[(a
*Cos[c + d*x] + b*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(3*d*Sqrt[a*Cos[c + d*x]
 + b*Sin[c + d*x]])
```

3.234.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3552
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(-(b*Cos[c + d*x] - a*Sin[c + d*x]))*((a*Cos[c + d*x] + b*
Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[(n - 1)*((a^2 + b^2)/n)   Int[(a*Co
s[c + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

rule 3557
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
in[c + d*x])/Sqrt[a^2 + b^2])^n   Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
 /; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) &&  !(GtQ[a^2
+ b^2, 0] || EqQ[a^2 + b^2, 0])
```

3.234.4 Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.26

method	result
default	$-\frac{(a^2+b^2)\left(\sqrt{-\sin(dx+c-\arctan(-a,b))+1}\,\sqrt{2\sin(dx+c-\arctan(-a,b))+2}\,\sqrt{\sin(dx+c-\arctan(-a,b))}\ \mathrm{EllipticF}\left(\sqrt{-\sin(dx+c-\right.\right.}{3\cos(dx+c-\arctan(-a,b))\sqrt{\sin(dx+c-\arctan(-a,b))\sqrt{}}}$

input
```
int((cos(d*x+c)*a+b*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(a^2+b^2)*((-sin(d*x+c-arctan(-a,b))+1)^(1/2)*(2*sin(d*x+c-arctan(-a,
b))+2)^(1/2)*sin(d*x+c-arctan(-a,b))^(1/2)*EllipticF((-sin(d*x+c-arctan(-a
,b))+1)^(1/2),1/2*2^(1/2))-2*sin(d*x+c-arctan(-a,b))^3+2*sin(d*x+c-arctan(
-a,b)))/cos(d*x+c-arctan(-a,b))/(sin(d*x+c-arctan(-a,b))*(a^2+b^2)^(1/2))^
(1/2)/d
```

3.234.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.21

$$\int (a\cos(c+dx)+b\sin(c+dx))^{3/2}\,dx = \frac{\sqrt{2}\sqrt{a-i\,b}(-i\,a+b)\mathrm{weierstrassPInverse}\left(-\frac{4\,(a^2+2i\,ab-b^2)}{a^2+b^2},0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)+}{}$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/3*(sqrt(2)*sqrt(a - I*b)*(-I*a + b)*weierstrassPInverse(-4*(a^2 + 2*I*a*
b - b^2)/(a^2 + b^2), 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*sqrt(a +
 I*b)*(I*a + b)*weierstrassPInverse(-4*(a^2 - 2*I*a*b - b^2)/(a^2 + b^2),
0, cos(d*x + c) - I*sin(d*x + c)) - 2*sqrt(a*cos(d*x + c) + b*sin(d*x + c)
)*(b*cos(d*x + c) - a*sin(d*x + c)))/d
```

3.234.6 Sympy [F]

$$\int (a\cos(c+dx)+b\sin(c+dx))^{3/2}\,dx=\int (a\cos{(c+dx)}+b\sin{(c+dx)})^{\frac{3}{2}}\,dx$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))**(3/2),x)
```

output
```
Integral((a*cos(c + d*x) + b*sin(c + d*x))**(3/2), x)
```

3.234.7 Maxima [F]

$$\int (a\cos(c+dx)+b\sin(c+dx))^{3/2}\,dx=\int (a\cos{(dx+c)}+b\sin{(dx+c)})^{\frac{3}{2}}\,dx$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + b*sin(d*x + c))^(3/2), x)
```

3.234.8 Giac [F]

$$\int (a\cos(c+dx)+b\sin(c+dx))^{3/2}\,dx=\int (a\cos{(dx+c)}+b\sin{(dx+c)})^{\frac{3}{2}}\,dx$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + b*sin(d*x + c))^(3/2), x)
```

3.234.9 Mupad [F(-1)]

Timed out.

$$\int (a\cos(c+dx)+b\sin(c+dx))^{3/2}\,dx = \int {\left(a\,\cos\left(c+d\,x\right)+b\,\sin\left(c+d\,x\right)\right)}^{3/2}\,dx$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))^(3/2),x)
```

output
```
int((a*cos(c + d*x) + b*sin(c + d*x))^(3/2), x)
```

3.235 $\int \sqrt{a\cos(c+dx)+b\sin(c+dx)}\,dx$

3.235.1 Optimal result

Integrand size = 21, antiderivative size = 75

$$\int \sqrt{a\cos(c+dx)+b\sin(c+dx)}\,dx = \frac{2E\left(\frac{1}{2}(c+dx-\tan^{-1}(a,b))\middle|2\right)\sqrt{a\cos(c+dx)+b\sin(c+dx)}}{d\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}}$$

output
```
2*(cos(1/2*c+1/2*d*x-1/2*arctan(a,b))^2)^(1/2)/cos(1/2*c+1/2*d*x-1/2*arcta
n(a,b))*EllipticE(sin(1/2*c+1/2*d*x-1/2*arctan(a,b)),2^(1/2))*(a*cos(d*x+c
)+b*sin(d*x+c))^(1/2)/d/((a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^(1/2))^(1/2
)
```

3.235.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.88 (sec) , antiderivative size = 268, normalized size of antiderivative = 3.57

$$\int \sqrt{a\cos(c+dx)+b\sin(c+dx)}\,dx = \frac{\cos\left(c+dx-\arctan\left(\frac{b}{a}\right)\right)\left(-b(a^2+b^2)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2\left(c+dx-\arctan\left(\frac{b}{a}\right)\right)\right)\sin\left(c+dx-\arctan\right.\right.}{}$$

input
```
Integrate[Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]],x]
```

output
```
(Cos[c + d*x - ArcTan[b/a]]*(-(b*(a^2 + b^2)*HypergeometricPFQ[{-1/2, -1/4
}, {3/4}, Cos[c + d*x - ArcTan[b/a]]^2]*Sin[c + d*x - ArcTan[b/a]]) + Sqrt
[Sin[c + d*x - ArcTan[b/a]]^2]*(-2*a*(a^2 + b^2)*Cos[c + d*x - ArcTan[b/a]
] + 2*a^2*Sqrt[1 + b^2/a^2]*Sqrt[a*Sqrt[1 + b^2/a^2]*Cos[c + d*x - ArcTan[
b/a]]]*Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]] + b*(a^2 + b^2)*Sin[c + d*x -
 ArcTan[b/a]])))/(b*d*(a*Sqrt[1 + b^2/a^2]*Cos[c + d*x - ArcTan[b/a]])^(3/
2)*Sqrt[Sin[c + d*x - ArcTan[b/a]]^2])
```

3.235.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3557, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\cos(c+dx)+b\sin(c+dx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt{a\cos(c+dx)+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{ 3557}$$

$$\frac{\sqrt{a\cos(c+dx)+b\sin(c+dx)}\int \sqrt{\cos\left(c+dx-\tan^{-1}(a,b)\right)}dx}{\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{a\cos(c+dx)+b\sin(c+dx)}\int \sqrt{\sin\left(c+dx-\tan^{-1}(a,b)+\frac{\pi}{2}\right)}dx}{\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}}$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{2\sqrt{a\cos(c+dx)+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\tan^{-1}(a,b)\right)\middle|2\right)}{d\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}}$$

input
```
Int[Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]],x]
```

output
```
(2*EllipticE[(c + d*x - ArcTan[a, b])/2, 2]*Sqrt[a*Cos[c + d*x] + b*Sin[c
+ d*x]])/(d*Sqrt[(a*Cos[c + d*x] + b*Sin[c + d*x])/Sqrt[a^2 + b^2]])
```

3.235.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3557
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
in[c + d*x])/Sqrt[a^2 + b^2])^n   Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
 /; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) &&  !(GtQ[a^2
+ b^2, 0] || EqQ[a^2 + b^2, 0])
```

3.235.4 Maple [A] (verified)

Time = 3.51 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.17

method	result
default	$-\frac{\sqrt{a^2+b^2}\,\sqrt{-\sin(dx+c-\arctan(-a,b))+1}\,\sqrt{2\sin(dx+c-\arctan(-a,b))+2}\,\sqrt{\sin(dx+c-\arctan(-a,b))}\left(2\,\mathrm{EllipticE}\left(\sqrt{-\sin(dx+c}\right.\right.}{\cos(dx+c-\arctan(-a,b))\sqrt{\sin(dx+c-\arctan(-a,b))\sqrt{a^2+}}}$
risch	Expression too large to display

input
```
int((cos(d*x+c)*a+b*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(a^2+b^2)^(1/2)*(-sin(d*x+c-arctan(-a,b))+1)^(1/2)*(2*sin(d*x+c-arctan(-a
,b))+2)^(1/2)*sin(d*x+c-arctan(-a,b))^(1/2)*(2*EllipticE((-sin(d*x+c-arcta
n(-a,b))+1)^(1/2),1/2*2^(1/2))-EllipticF((-sin(d*x+c-arctan(-a,b))+1)^(1/2
),1/2*2^(1/2)))/cos(d*x+c-arctan(-a,b))/(sin(d*x+c-arctan(-a,b))*(a^2+b^2)
^(1/2))^(1/2)/d
```

3.235.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.17

$$\int \sqrt{a\cos(c+dx)+b\sin(c+dx)}\,dx = \frac{i\sqrt{2}\sqrt{a-ib}\,\text{weierstrassZeta}\left(-\frac{4\,(a^2+2i\,ab-b^2)}{a^2+b^2},0,\text{weierstrassPInverse}\left(-\frac{4\,(a^2+2i\,ab-b^2)}{a^2+b^2},0,\cos\left(dx+c\right)+i\sin\ldots\right.\right.}{\ldots}$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
(I*sqrt(2)*sqrt(a - I*b)*weierstrassZeta(-4*(a^2 + 2*I*a*b - b^2)/(a^2 + b
^2), 0, weierstrassPInverse(-4*(a^2 + 2*I*a*b - b^2)/(a^2 + b^2), 0, cos(d
*x + c) + I*sin(d*x + c))) - I*sqrt(2)*sqrt(a + I*b)*weierstrassZeta(-4*(a
^2 - 2*I*a*b - b^2)/(a^2 + b^2), 0, weierstrassPInverse(-4*(a^2 - 2*I*a*b
- b^2)/(a^2 + b^2), 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

3.235.6 Sympy [F]

$$\int \sqrt{a\cos(c+dx)+b\sin(c+dx)}\,dx = \int \sqrt{a\cos\left(c+dx\right)+b\sin\left(c+dx\right)}\,dx$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a*cos(c + d*x) + b*sin(c + d*x)), x)
```

3.235.7 Maxima [F]

$$\int \sqrt{a\cos(c+dx)+b\sin(c+dx)}\,dx = \int \sqrt{a\cos\left(dx+c\right)+b\sin\left(dx+c\right)}\,dx$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*cos(d*x + c) + b*sin(d*x + c)), x)
```

3.235.8 Giac [F]

$$\int \sqrt{a\cos(c+dx)+b\sin(c+dx)}\,dx=\int \sqrt{a\cos\left(dx+c\right)+b\sin\left(dx+c\right)}\,dx$$

input
```
integrate((a*cos(d*x+c)+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*cos(d*x + c) + b*sin(d*x + c)), x)
```

3.235.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a\cos(c+dx)+b\sin(c+dx)}\,dx=\int \sqrt{a\ \cos\left(c+d\,x\right)+b\ \sin\left(c+d\,x\right)}\,dx$$

input
```
int((a*cos(c + d*x) + b*sin(c + d*x))^(1/2),x)
```

output
```
int((a*cos(c + d*x) + b*sin(c + d*x))^(1/2), x)
```

3.236 $\int \frac{1}{\sqrt{a\cos(c+dx)+b\sin(c+dx)}}\,dx$

3.236.1 Optimal result

Integrand size = 21, antiderivative size = 75

$$\int \frac{1}{\sqrt{a\cos(c+dx)+b\sin(c+dx)}}\,dx = \frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx-\tan^{-1}(a,b)),2\right)\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}}{d\sqrt{a\cos(c+dx)+b\sin(c+dx)}}$$

output
```
2*(cos(1/2*c+1/2*d*x-1/2*arctan(a,b))^2)^(1/2)/cos(1/2*c+1/2*d*x-1/2*arcta
n(a,b))*EllipticF(sin(1/2*c+1/2*d*x-1/2*arctan(a,b)),2^(1/2))*((a*cos(d*x+
c)+b*sin(d*x+c))/(a^2+b^2)^(1/2))^(1/2)/d/(a*cos(d*x+c)+b*sin(d*x+c))^(1/2
)
```

3.236.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.21 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.23

$$\int \frac{1}{\sqrt{a\cos(c+dx)+b\sin(c+dx)}}\,dx = \frac{2\sqrt{\cos^2\left(c+dx+\arctan\left(\frac{a}{b}\right)\right)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2\left(c+dx+\arctan\left(\frac{a}{b}\right)\right)\right)\tan\left(c+dx+\arctan\left(\frac{a}{b}\right)\right)}{d\sqrt{\sqrt{1+\frac{a^2}{b^2}}\,b\sin\left(c+dx+\arctan\left(\frac{a}{b}\right)\right)}}$$

input
```
Integrate[1/Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]],x]
```

output
```
(2*Sqrt[Cos[c + d*x + ArcTan[a/b]]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4},
 Sin[c + d*x + ArcTan[a/b]]^2]*Tan[c + d*x + ArcTan[a/b]])/(d*Sqrt[Sqrt[1
+ a^2/b^2]*b*Sin[c + d*x + ArcTan[a/b]]])
```

3.236.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3557, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\cos(c+dx)+b\sin(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a\cos(c+dx)+b\sin(c+dx)}}dx$$

$$\Big\downarrow \text{3557}$$

$$\frac{\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}\int \frac{1}{\sqrt{\cos(c+dx-\tan^{-1}(a,b))}}dx}{\sqrt{a\cos(c+dx)+b\sin(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}\int \frac{1}{\sqrt{\sin\left(c+dx-\tan^{-1}(a,b)+\frac{\pi}{2}\right)}}dx}{\sqrt{a\cos(c+dx)+b\sin(c+dx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\tan^{-1}(a,b)\right),2\right)}{d\sqrt{a\cos(c+dx)+b\sin(c+dx)}}$$

input
```
Int[1/Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]],x]
```

output
```
(2*EllipticF[(c + d*x - ArcTan[a, b])/2, 2]*Sqrt[(a*Cos[c + d*x] + b*Sin[c
 + d*x])/Sqrt[a^2 + b^2]])/(d*Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]])
```

3.236.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3557
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
in[c + d*x])/Sqrt[a^2 + b^2])^n   Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
 /; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) &&  !(GtQ[a^2
+ b^2, 0] || EqQ[a^2 + b^2, 0])
```

3.236.4 Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.65

method	result
default	$-\frac{\sqrt{-\sin(dx+c-\arctan(-a,b))+1}\,\sqrt{2\sin(dx+c-\arctan(-a,b))+2}\,\sqrt{\sin(dx+c-\arctan(-a,b))}\,\mathrm{EllipticF}\left(\sqrt{-\sin(dx+c-\arctan(-\right.}{\cos(dx+c-\arctan(-a,b))\sqrt{\sin(dx+c-\arctan(-a,b))\sqrt{a^2+b^2}}\,d}$

input
```
int(1/(cos(d*x+c)*a+b*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-sin(d*x+c-arctan(-a,b))+1)^(1/2)*(2*sin(d*x+c-arctan(-a,b))+2)^(1/2)*si
n(d*x+c-arctan(-a,b))^(1/2)*EllipticF((-sin(d*x+c-arctan(-a,b))+1)^(1/2),1
/2*2^(1/2))/cos(d*x+c-arctan(-a,b))/(sin(d*x+c-arctan(-a,b))*(a^2+b^2)^(1/
2))^(1/2)/d
```

3.236.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.71

$$\int \frac{1}{\sqrt{a\cos(c+dx)+b\sin(c+dx)}}\,dx = \frac{\sqrt{2}\sqrt{a-i\,b}(-i\,a+b)\text{weierstrassPInverse}\left(-\frac{4\,(a^2+2i\,ab-b^2)}{a^2+b^2},0,\cos\,(dx+c)+i\,\sin\,(dx+c)\right)+\sqrt{2}\sqrt{a+i\,b}\ldots}{(a^2+b^2)d}$$

input
```
integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(2)*sqrt(a - I*b)*(-I*a + b)*weierstrassPInverse(-4*(a^2 + 2*I*a*b -
b^2)/(a^2 + b^2), 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*sqrt(a + I*b
)*(I*a + b)*weierstrassPInverse(-4*(a^2 - 2*I*a*b - b^2)/(a^2 + b^2), 0, c
os(d*x + c) - I*sin(d*x + c)))/((a^2 + b^2)*d)
```

3.236.6 Sympy [F]

$$\int \frac{1}{\sqrt{a\cos(c+dx)+b\sin(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\cos\,(c+dx)+b\sin\,(c+dx)}}\,dx$$

input
```
integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))**(1/2),x)
```

output
```
Integral(1/sqrt(a*cos(c + d*x) + b*sin(c + d*x)), x)
```

3.236.7 Maxima [F]

$$\int \frac{1}{\sqrt{a\cos(c+dx)+b\sin(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\cos\,(dx+c)+b\sin\,(dx+c)}}\,dx$$

input
```
integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(a*cos(d*x + c) + b*sin(d*x + c)), x)
```

3.236.8 Giac [F]

$$\int \frac{1}{\sqrt{a\cos(c+dx)+b\sin(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\cos\left(dx+c\right)+b\sin\left(dx+c\right)}}\,dx$$

input
```
integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(a*cos(d*x + c) + b*sin(d*x + c)), x)
```

3.236.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a\cos(c+dx)+b\sin(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\,\cos\left(c+d\,x\right)+b\,\sin\left(c+d\,x\right)}}\,dx$$

input
```
int(1/(a*cos(c + d*x) + b*sin(c + d*x))^(1/2),x)
```

output
```
int(1/(a*cos(c + d*x) + b*sin(c + d*x))^(1/2), x)
```

3.237 $\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^{3/2}}\,dx$

3.237.1 Optimal result

Integrand size = 21, antiderivative size = 138

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^{3/2}}\,dx = -\frac{2(b\cos(c+dx)-a\sin(c+dx))}{(a^2+b^2)\,d\sqrt{a\cos(c+dx)+b\sin(c+dx)}} - \frac{2E\left(\frac{1}{2}(c+dx-\tan^{-1}(a,b))\middle|2\right)\sqrt{a\cos(c+dx)+b\sin(c+dx)}}{(a^2+b^2)\,d\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}}$$

output
```
-2*(b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)/d/(a*cos(d*x+c)+b*sin(d*x+c))^(1/
2)-2*(cos(1/2*c+1/2*d*x-1/2*arctan(a,b))^2)^(1/2)/cos(1/2*c+1/2*d*x-1/2*ar
ctan(a,b))*EllipticE(sin(1/2*c+1/2*d*x-1/2*arctan(a,b)),2^(1/2))*(a*cos(d*
x+c)+b*sin(d*x+c))^(1/2)/(a^2+b^2)/d/((a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2
)^(1/2))^(1/2)
```

3.237.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.38 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.59

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^{3/2}}\,dx = \frac{-\frac{2b\cos(c+dx)}{\sqrt{a\cos(c+dx)+b\sin(c+dx)}} + \frac{2a\sin(c+dx)}{\sqrt{a\cos(c+dx)+b\sin(c+dx)}} - \sqrt{a\sqrt{1+\frac{b^2}{a^2}}\cos}}{}$$

input
```
Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-3/2),x]
```

output
```
((-2*b*Cos[c + d*x])/Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]] + (2*a*Sin[c +
d*x])/Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]] - Sqrt[a*Sqrt[1 + b^2/a^2]*Cos
[c + d*x - ArcTan[b/a]]]*Tan[c + d*x - ArcTan[b/a]] + (Sqrt[a*Sqrt[1 + b^2
/a^2]*Cos[c + d*x - ArcTan[b/a]]]*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, C
os[c + d*x - ArcTan[b/a]]^2]*Tan[c + d*x - ArcTan[b/a]])/Sqrt[Sin[c + d*x
- ArcTan[b/a]]^2])/((a^2 + b^2)*d)
```

3.237.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3555, 3042, 3557, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^{3/2}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^{3/2}}dx$$

$$\Big\downarrow \ 3555$$

$$-\frac{\int \sqrt{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}-\frac{2(b\cos(c+dx)-a\sin(c+dx))}{d\left(a^2+b^2\right)\sqrt{a\cos(c+dx)+b\sin(c+dx)}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\int \sqrt{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}-\frac{2(b\cos(c+dx)-a\sin(c+dx))}{d\left(a^2+b^2\right)\sqrt{a\cos(c+dx)+b\sin(c+dx)}}$$

$$\Big\downarrow \ 3557$$

$$-\frac{\sqrt{a\cos(c+dx)+b\sin(c+dx)}\int \sqrt{\cos\left(c+dx-\tan^{-1}(a,b)\right)}dx}{\left(a^2+b^2\right)\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}}-\frac{2(b\cos(c+dx)-a\sin(c+dx))}{d\left(a^2+b^2\right)\sqrt{a\cos(c+dx)+b\sin(c+dx)}}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\sqrt{a\cos(c+dx)+b\sin(c+dx)}\int\sqrt{\sin\left(c+dx-\tan^{-1}(a,b)+\frac{\pi}{2}\right)}dx}{(a^2+b^2)\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}}-\frac{2(b\cos(c+dx)-a\sin(c+dx))}{d(a^2+b^2)\sqrt{a\cos(c+dx)+b\sin(c+dx)}}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{2\sqrt{a\cos(c+dx)+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\tan^{-1}(a,b)\right)\middle|2\right)}{d(a^2+b^2)\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}}-\frac{2(b\cos(c+dx)-a\sin(c+dx))}{d(a^2+b^2)\sqrt{a\cos(c+dx)+b\sin(c+dx)}}$$

input
```
Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-3/2),x]
```

output
```
(-2*(b*Cos[c + d*x] - a*Sin[c + d*x]))/((a^2 + b^2)*d*Sqrt[a*Cos[c + d*x]
+ b*Sin[c + d*x]]) - (2*EllipticE[(c + d*x - ArcTan[a, b])/2, 2]*Sqrt[a*Co
s[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)*d*Sqrt[(a*Cos[c + d*x] + b*Sin[
c + d*x])/Sqrt[a^2 + b^2]])
```

3.237.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3555
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(b*Cos[c + d*x] - a*Sin[c + d*x])*((a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Simp[(n + 2)/((n + 1)*(a^
2 + b^2))   Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

rule 3557
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
in[c + d*x])/Sqrt[a^2 + b^2])^n   Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
 /; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) &&  !(GtQ[a^2
+ b^2, 0] || EqQ[a^2 + b^2, 0])
```

3.237.4 Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.68

method	result
default	$\frac{2\sqrt{-\sin(dx+c-\arctan(-a,b))+1}\,\sqrt{2\sin(dx+c-\arctan(-a,b))+2}\,\sqrt{\sin(dx+c-\arctan(-a,b))}\,\mathrm{EllipticE}\left(\sqrt{-\sin(dx+c-\arctan(-a,\right.}{\sqrt{a^2+b^2}}$

input
```
int(1/(cos(d*x+c)*a+b*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(2*(-sin(d*x+c-arctan(-a,b))+1)^(1/2)*(2*sin(d*x+c-arctan(-a,b))+2)^(1/2)*
sin(d*x+c-arctan(-a,b))^(1/2)*EllipticE((-sin(d*x+c-arctan(-a,b))+1)^(1/2)
,1/2*2^(1/2))-(-sin(d*x+c-arctan(-a,b))+1)^(1/2)*(2*sin(d*x+c-arctan(-a,b)
)+2)^(1/2)*sin(d*x+c-arctan(-a,b))^(1/2)*EllipticF((-sin(d*x+c-arctan(-a,b
))+1)^(1/2),1/2*2^(1/2))-2*cos(d*x+c-arctan(-a,b))^2)/(a^2+b^2)^(1/2)/cos(
d*x+c-arctan(-a,b))/(sin(d*x+c-arctan(-a,b))*(a^2+b^2)^(1/2))^(1/2)/d
```

3.237.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.01

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^{3/2}}\,dx = \frac{\left(-i\sqrt{2}a\cos\left(dx+c\right)-i\sqrt{2}b\sin\left(dx+c\right)\right)\sqrt{a-i\,b}\,\mathrm{weierstrassZet}}{}$$

input
```
integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
((-I*sqrt(2)*a*cos(d*x + c) - I*sqrt(2)*b*sin(d*x + c))*sqrt(a - I*b)*weie
rstrassZeta(-4*(a^2 + 2*I*a*b - b^2)/(a^2 + b^2), 0, weierstrassPInverse(-
4*(a^2 + 2*I*a*b - b^2)/(a^2 + b^2), 0, cos(d*x + c) + I*sin(d*x + c))) +
(I*sqrt(2)*a*cos(d*x + c) + I*sqrt(2)*b*sin(d*x + c))*sqrt(a + I*b)*weiers
trassZeta(-4*(a^2 - 2*I*a*b - b^2)/(a^2 + b^2), 0, weierstrassPInverse(-4*
(a^2 - 2*I*a*b - b^2)/(a^2 + b^2), 0, cos(d*x + c) - I*sin(d*x + c))) - 2*
sqrt(a*cos(d*x + c) + b*sin(d*x + c))*(b*cos(d*x + c) - a*sin(d*x + c)))/(
(a^3 + a*b^2)*d*cos(d*x + c) + (a^2*b + b^3)*d*sin(d*x + c))
```

3.237.6 Sympy [F]

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a\cos\left(c+dx\right)+b\sin\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))**(3/2),x)
```

output
```
Integral((a*cos(c + d*x) + b*sin(c + d*x))**(-3/2), x)
```

3.237.7 Maxima [F]

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a\cos\left(dx+c\right)+b\sin\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + b*sin(d*x + c))^(-3/2), x)
```

3.237.8 Giac [F]

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^{3/2}}\,dx = \int \frac{1}{(a\cos(dx+c)+b\sin(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + b*sin(d*x + c))^(-3/2), x)
```

3.237.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^{3/2}}\,dx = \int \frac{1}{(a\,\cos(c+d\,x)+b\,\sin(c+d\,x))^{3/2}}\,dx$$

input
```
int(1/(a*cos(c + d*x) + b*sin(c + d*x))^(3/2),x)
```

output
```
int(1/(a*cos(c + d*x) + b*sin(c + d*x))^(3/2), x)
```

3.238 $\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^{5/2}}\,dx$

3.238.1 Optimal result

Integrand size = 21, antiderivative size = 142

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^{5/2}}\,dx = -\frac{2(b\cos(c+dx)-a\sin(c+dx))}{3(a^2+b^2)\,d(a\cos(c+dx)+b\sin(c+dx))^{3/2}} + \frac{2\,\text{EllipticF}\left(\frac{1}{2}(c+dx-\tan^{-1}(a,b)),2\right)\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}}{3(a^2+b^2)\,d\sqrt{a\cos(c+dx)+b\sin(c+dx)}}$$

output
```
-2/3*(b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)/d/(a*cos(d*x+c)+b*sin(d*x+c))^(
3/2)+2/3*(cos(1/2*c+1/2*d*x-1/2*arctan(a,b))^2)^(1/2)/cos(1/2*c+1/2*d*x-1/
2*arctan(a,b))*EllipticF(sin(1/2*c+1/2*d*x-1/2*arctan(a,b)),2^(1/2))*((a*c
os(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^(1/2))^(1/2)/(a^2+b^2)/d/(a*cos(d*x+c)+b
*sin(d*x+c))^(1/2)
```

3.238.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.42 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.02

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^{5/2}}\,dx = \frac{2\left(\frac{-b\cos(c+dx)+a\sin(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^{3/2}} + \frac{\sqrt{\cos^2\left(c+dx+\arctan\left(\frac{a}{b}\right)\right)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2(c+dx\ldots\right.}{\sqrt{\sqrt{1+\frac{a^2}{b^2}}\,b\sin(c+dx+\ldots}}\right.}{3(a^2+b^2)\,d}$$

input
```
Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-5/2),x]
```

output
```
(2*((-(b*Cos[c + d*x]) + a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x])
^(3/2) + (Sqrt[Cos[c + d*x + ArcTan[a/b]]^2]*HypergeometricPFQ[{1/4, 1/2},
 {5/4}, Sin[c + d*x + ArcTan[a/b]]^2]*Tan[c + d*x + ArcTan[a/b]])/Sqrt[Sqr
t[1 + a^2/b^2]*b*Sin[c + d*x + ArcTan[a/b]]]))/(3*(a^2 + b^2)*d)
```

3.238.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3555, 3042, 3557, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3555}$$

$$\frac{\int \frac{1}{\sqrt{a\cos(c+dx)+b\sin(c+dx)}}dx}{3(a^2+b^2)}-\frac{2(b\cos(c+dx)-a\sin(c+dx))}{3d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\sqrt{a\cos(c+dx)+b\sin(c+dx)}}dx}{3(a^2+b^2)}-\frac{2(b\cos(c+dx)-a\sin(c+dx))}{3d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3557}$$

$$\frac{\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}\int \frac{1}{\sqrt{\cos(c+dx-\tan^{-1}(a,b))}}dx}{3(a^2+b^2)\sqrt{a\cos(c+dx)+b\sin(c+dx)}}-\frac{2(b\cos(c+dx)-a\sin(c+dx))}{3d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}\int \frac{1}{\sqrt{\sin\left(c+dx-\tan^{-1}(a,b)+\frac{\pi}{2}\right)}}dx}{3(a^2+b^2)\sqrt{a\cos(c+dx)+b\sin(c+dx)}}-\frac{2(b\cos(c+dx)-a\sin(c+dx))}{3d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\tan^{-1}(a,b)\right),2\right)}{3d\left(a^2+b^2\right)\sqrt{a\cos(c+dx)+b\sin(c+dx)}}-\frac{2(b\cos(c+dx)-a\sin(c+dx))}{3d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^{3/2}}$$

input
```
Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-5/2),x]
```

output
```
(-2*(b*Cos[c + d*x] - a*Sin[c + d*x]))/(3*(a^2 + b^2)*d*(a*Cos[c + d*x] +
b*Sin[c + d*x])^(3/2)) + (2*EllipticF[(c + d*x - ArcTan[a, b])/2, 2]*Sqrt[
(a*Cos[c + d*x] + b*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(3*(a^2 + b^2)*d*Sqrt[
a*Cos[c + d*x] + b*Sin[c + d*x]])
```

3.238.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3555
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(b*Cos[c + d*x] - a*Sin[c + d*x])*((a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Simp[(n + 2)/((n + 1)*(a^
2 + b^2))   Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

rule 3557
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
in[c + d*x])/Sqrt[a^2 + b^2])^n   Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
 /; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) &&  !(GtQ[a^2
+ b^2, 0] || EqQ[a^2 + b^2, 0])
```

3.238. $\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^{5/2}}\,dx$

3.238.4 Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.28

method	result
default	$-\frac{\sqrt{-\sin(dx+c-\arctan(-a,b))+1}\,\sqrt{2\sin(dx+c-\arctan(-a,b))+2}\,\sin(dx+c-\arctan(-a,b))^{\frac{5}{2}}\,\mathrm{EllipticF}\left(\sqrt{-\sin(dx+c-\arctan(-a}\right.}{3(a^2+b^2)\sin(dx+c-\arctan(-a,b))^2\cos(dx+c-\arctan(-a,b))\sqrt{\sin(dx+c-\mathrm{arct}}}$

input
```
int(1/(cos(d*x+c)*a+b*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/(a^2+b^2)*((-sin(d*x+c-arctan(-a,b))+1)^(1/2)*(2*sin(d*x+c-arctan(-a,
b))+2)^(1/2)*sin(d*x+c-arctan(-a,b))^(5/2)*EllipticF((-sin(d*x+c-arctan(-a
,b))+1)^(1/2),1/2*2^(1/2))-2*sin(d*x+c-arctan(-a,b))^3+2*sin(d*x+c-arctan(
-a,b)))/sin(d*x+c-arctan(-a,b))^2/cos(d*x+c-arctan(-a,b))/(sin(d*x+c-arcta
n(-a,b))*(a^2+b^2)^(1/2))^(1/2)/d
```

3.238.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 406, normalized size of antiderivative = 2.86

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^{5/2}}\,dx = \frac{\left(\sqrt{2}(-i\,a^3+a^2b+i\,ab^2-b^3)\cos{(dx+c)}^2-2\,\sqrt{2}(i\,a^2b-ab^2)\,\mathrm{co}\right.}{}$$

input
```
integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/3*((sqrt(2)*(-I*a^3 + a^2*b + I*a*b^2 - b^3)*cos(d*x + c)^2 - 2*sqrt(2)*
(I*a^2*b - a*b^2)*cos(d*x + c)*sin(d*x + c) + sqrt(2)*(-I*a*b^2 + b^3))*sq
rt(a - I*b)*weierstrassPInverse(-4*(a^2 + 2*I*a*b - b^2)/(a^2 + b^2), 0, c
os(d*x + c) + I*sin(d*x + c)) + (sqrt(2)*(I*a^3 + a^2*b - I*a*b^2 - b^3)*c
os(d*x + c)^2 - 2*sqrt(2)*(-I*a^2*b - a*b^2)*cos(d*x + c)*sin(d*x + c) + s
qrt(2)*(I*a*b^2 + b^3))*sqrt(a + I*b)*weierstrassPInverse(-4*(a^2 - 2*I*a*
b - b^2)/(a^2 + b^2), 0, cos(d*x + c) - I*sin(d*x + c)) - 2*((a^2*b + b^3)
*cos(d*x + c) - (a^3 + a*b^2)*sin(d*x + c))*sqrt(a*cos(d*x + c) + b*sin(d*
x + c)))/((a^6 + a^4*b^2 - a^2*b^4 - b^6)*d*cos(d*x + c)^2 + 2*(a^5*b + 2*
a^3*b^3 + a*b^5)*d*cos(d*x + c)*sin(d*x + c) + (a^4*b^2 + 2*a^2*b^4 + b^6)
*d)
```

3.238.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.238.7 Maxima [F]

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^{5/2}}\,dx = \int \frac{1}{(a\cos(dx+c)+b\sin(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + b*sin(d*x + c))^(-5/2), x)
```

3.238.8 Giac [F]

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^{5/2}}\,dx = \int \frac{1}{(a\cos(dx+c)+b\sin(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + b*sin(d*x + c))^(-5/2), x)
```

3.238.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(a\,\cos\left(c+d\,x\right)+b\,\sin\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int(1/(a*cos(c + d*x) + b*sin(c + d*x))^(5/2),x)
```

output
```
int(1/(a*cos(c + d*x) + b*sin(c + d*x))^(5/2), x)
```

3.239 $\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^{7/2}}\,dx$

3.239.1 Optimal result

Integrand size = 21, antiderivative size = 197

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^{7/2}}\,dx = -\frac{2(b\cos(c+dx)-a\sin(c+dx))}{5(a^2+b^2)\,d(a\cos(c+dx)+b\sin(c+dx))^{5/2}} - \frac{6(b\cos(c+dx)-a\sin(c+dx))}{5(a^2+b^2)^2\,d\sqrt{a\cos(c+dx)+b\sin(c+dx)}} - \frac{6E\left(\frac{1}{2}(c+dx-\tan^{-1}(a,b))\middle|2\right)\sqrt{a\cos(c+dx)+b\sin(c+dx)}}{5(a^2+b^2)^2\,d\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}}$$

output
```
-2/5*(b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)/d/(a*cos(d*x+c)+b*sin(d*x+c))^(
5/2)-6/5*(b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^2/d/(a*cos(d*x+c)+b*sin(d*x
+c))^(1/2)-6/5*(cos(1/2*c+1/2*d*x-1/2*arctan(a,b))^2)^(1/2)/cos(1/2*c+1/2*
d*x-1/2*arctan(a,b))*EllipticE(sin(1/2*c+1/2*d*x-1/2*arctan(a,b)),2^(1/2))
*(a*cos(d*x+c)+b*sin(d*x+c))^(1/2)/(a^2+b^2)^2/d/((a*cos(d*x+c)+b*sin(d*x+
c))/(a^2+b^2)^(1/2))^(1/2)
```

3.239.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.93 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.41

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^{7/2}}\,dx = \frac{-\frac{2(3a^2\cos^3(c+dx)-ab\sin(c+dx)+6ab\cos^2(c+dx)\sin(c+dx)+b^2\cos(c+dx)(1+3\sin^2(c\cdots}{(a\cos(c+dx)+b\sin(c+dx))^{5/2}}}{\cdots}$$

input
```
Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-7/2),x]
```

output
```
((-2*(3*a^2*Cos[c + d*x]^3 - a*b*Sin[c + d*x] + 6*a*b*Cos[c + d*x]^2*Sin[c
 + d*x] + b^2*Cos[c + d*x]*(1 + 3*Sin[c + d*x]^2)))/(a*Cos[c + d*x] + b*Si
n[c + d*x])^(5/2) + (Cos[c + d*x - ArcTan[b/a]]*(3*b*HypergeometricPFQ[{-1
/2, -1/4}, {3/4}, Cos[c + d*x - ArcTan[b/a]]^2]*Sin[c + d*x - ArcTan[b/a]]
 - 3*Sqrt[Sin[c + d*x - ArcTan[b/a]]^2]*(-2*a*Cos[c + d*x - ArcTan[b/a]] +
 b*Sin[c + d*x - ArcTan[b/a]])))/((a*Sqrt[1 + b^2/a^2]*Cos[c + d*x - ArcTa
n[b/a]])^(3/2)*Sqrt[Sin[c + d*x - ArcTan[b/a]]^2]))/(5*b*(a^2 + b^2)*d)
```

3.239.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 3555, 3042, 3555, 3042, 3557, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^{7/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3555}$$

$$\frac{3\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^{3/2}}dx}{5(a^2+b^2)}-\frac{2(b\cos(c+dx)-a\sin(c+dx))}{5d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^{3/2}}dx}{5(a^2+b^2)}-\frac{2(b\cos(c+dx)-a\sin(c+dx))}{5d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3555}$$

$$\frac{3\left(-\frac{\int \sqrt{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}-\frac{2(b\cos(c+dx)-a\sin(c+dx))}{d(a^2+b^2)\sqrt{a\cos(c+dx)+b\sin(c+dx)}}\right)}{5(a^2+b^2)}-\frac{2(b\cos(c+dx)-a\sin(c+dx))}{5d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(-\frac{\int \sqrt{a\cos(c+dx)+b\sin(c+dx)}dx}{a^2+b^2}-\frac{2(b\cos(c+dx)-a\sin(c+dx))}{d(a^2+b^2)\sqrt{a\cos(c+dx)+b\sin(c+dx)}}\right)}{5(a^2+b^2)}-\frac{2(b\cos(c+dx)-a\sin(c+dx))}{5d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3557}$$

$$\frac{3\left(-\frac{\sqrt{a\cos(c+dx)+b\sin(c+dx)}\int \sqrt{\cos(c+dx-\tan^{-1}(a,b))}dx}{(a^2+b^2)\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}}-\frac{2(b\cos(c+dx)-a\sin(c+dx))}{d(a^2+b^2)\sqrt{a\cos(c+dx)+b\sin(c+dx)}}\right)}{5(a^2+b^2)}-\frac{2(b\cos(c+dx)-a\sin(c+dx))}{5d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(-\frac{\sqrt{a\cos(c+dx)+b\sin(c+dx)}\int \sqrt{\sin\left(c+dx-\tan^{-1}(a,b)+\frac{\pi}{2}\right)}dx}{(a^2+b^2)\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}}-\frac{2(b\cos(c+dx)-a\sin(c+dx))}{d(a^2+b^2)\sqrt{a\cos(c+dx)+b\sin(c+dx)}}\right)}{5(a^2+b^2)}-\frac{2(b\cos(c+dx)-a\sin(c+dx))}{5d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{3\left(-\frac{2\sqrt{a\cos(c+dx)+b\sin(c+dx)}E\left(\frac{1}{2}\left(c+dx-\tan^{-1}(a,b)\right)|2\right)}{d(a^2+b^2)\sqrt{\frac{a\cos(c+dx)+b\sin(c+dx)}{\sqrt{a^2+b^2}}}}-\frac{2(b\cos(c+dx)-a\sin(c+dx))}{d(a^2+b^2)\sqrt{a\cos(c+dx)+b\sin(c+dx)}}\right)}{5(a^2+b^2)}-\frac{2(b\cos(c+dx)-a\sin(c+dx))}{5d(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^{5/2}}$$

input
```
Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-7/2),x]
```

3.239. $\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^{7/2}}\,dx$

output
```
(-2*(b*Cos[c + d*x] - a*Sin[c + d*x]))/(5*(a^2 + b^2)*d*(a*Cos[c + d*x] +
b*Sin[c + d*x])^(5/2)) + (3*((-2*(b*Cos[c + d*x] - a*Sin[c + d*x]))/((a^2
+ b^2)*d*Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]]) - (2*EllipticE[(c + d*x -
ArcTan[a, b])/2, 2]*Sqrt[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)*d*
Sqrt[(a*Cos[c + d*x] + b*Sin[c + d*x])/Sqrt[a^2 + b^2]])))/(5*(a^2 + b^2))
```

3.239.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3555
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(b*Cos[c + d*x] - a*Sin[c + d*x])*((a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Simp[(n + 2)/((n + 1)*(a^
2 + b^2))   Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

rule 3557
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*Cos[c + d*x] + b*Sin[c + d*x])^n/((a*Cos[c + d*x] + b*S
in[c + d*x])/Sqrt[a^2 + b^2])^n   Int[Cos[c + d*x - ArcTan[a, b]]^n, x], x]
 /; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) &&  !(GtQ[a^2
+ b^2, 0] || EqQ[a^2 + b^2, 0])
```

3.239.4 Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.51

method	result
default	$\frac{\sqrt{a^2+b^2}\left(6\sqrt{-\sin(dx+c-\arctan(-a,b))+1}\,\sqrt{2\sin(dx+c-\arctan(-a,b))+2}\,\sin(dx+c-\arctan(-a,b))^{\frac{7}{2}}\operatorname{EllipticE}\left(\sqrt{-\sin(dx+c-a\right.\right.}{}$

input
```
int(1/(cos(d*x+c)*a+b*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*(a^2+b^2)^(1/2)*(6*(-sin(d*x+c-arctan(-a,b))+1)^(1/2)*(2*sin(d*x+c-arc
tan(-a,b))+2)^(1/2)*sin(d*x+c-arctan(-a,b))^(7/2)*EllipticE((-sin(d*x+c-ar
ctan(-a,b))+1)^(1/2),1/2*2^(1/2))-3*(-sin(d*x+c-arctan(-a,b))+1)^(1/2)*(2*
sin(d*x+c-arctan(-a,b))+2)^(1/2)*sin(d*x+c-arctan(-a,b))^(7/2)*EllipticF((
-sin(d*x+c-arctan(-a,b))+1)^(1/2),1/2*2^(1/2))+6*sin(d*x+c-arctan(-a,b))^5
-4*sin(d*x+c-arctan(-a,b))^3-2*sin(d*x+c-arctan(-a,b)))/sin(d*x+c-arctan(-
a,b))^3/(a^4+2*a^2*b^2+b^4)/cos(d*x+c-arctan(-a,b))/(sin(d*x+c-arctan(-a,b
))*(a^2+b^2)^(1/2))^(1/2)/d
```

3.239.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 552, normalized size of antiderivative = 2.80

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^{7/2}}\,dx = \frac{3\left(-3i\sqrt{2}ab^2\cos\left(dx+c\right)+\sqrt{2}(-i\,a^3+3i\,ab^2)\cos\left(dx+c\right)^3+(\cdots\right.}{}$$

input
```
integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/5*(3*(-3*I*sqrt(2)*a*b^2*cos(d*x + c) + sqrt(2)*(-I*a^3 + 3*I*a*b^2)*cos
(d*x + c)^3 + (-I*sqrt(2)*b^3 + sqrt(2)*(-3*I*a^2*b + I*b^3)*cos(d*x + c)^
2)*sin(d*x + c))*sqrt(a - I*b)*weierstrassZeta(-4*(a^2 + 2*I*a*b - b^2)/(a
^2 + b^2), 0, weierstrassPInverse(-4*(a^2 + 2*I*a*b - b^2)/(a^2 + b^2), 0,
 cos(d*x + c) + I*sin(d*x + c))) + 3*(3*I*sqrt(2)*a*b^2*cos(d*x + c) + sqr
t(2)*(I*a^3 - 3*I*a*b^2)*cos(d*x + c)^3 + (I*sqrt(2)*b^3 + sqrt(2)*(3*I*a^
2*b - I*b^3)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a + I*b)*weierstrassZeta(-
4*(a^2 - 2*I*a*b - b^2)/(a^2 + b^2), 0, weierstrassPInverse(-4*(a^2 - 2*I*
a*b - b^2)/(a^2 + b^2), 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(3*(3*a^2*b
 - b^3)*cos(d*x + c)^3 - (5*a^2*b - 4*b^3)*cos(d*x + c) - (a^3 + 4*a*b^2 +
 3*(a^3 - 3*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a*cos(d*x + c) + b*s
in(d*x + c)))/((a^7 - a^5*b^2 - 5*a^3*b^4 - 3*a*b^6)*d*cos(d*x + c)^3 + 3*
(a^5*b^2 + 2*a^3*b^4 + a*b^6)*d*cos(d*x + c) + ((3*a^6*b + 5*a^4*b^3 + a^2
*b^5 - b^7)*d*cos(d*x + c)^2 + (a^4*b^3 + 2*a^2*b^5 + b^7)*d)*sin(d*x + c)
)
```

3.239.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.239.7 Maxima [F]

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^{7/2}}\,dx = \int \frac{1}{(a\cos(dx+c)+b\sin(dx+c))^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((a*cos(d*x + c) + b*sin(d*x + c))^(-7/2), x)
```

3.239.8 Giac [F]

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^{7/2}}\,dx = \int \frac{1}{(a\cos(dx+c)+b\sin(dx+c))^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((a*cos(d*x + c) + b*sin(d*x + c))^(-7/2), x)
```

3.239.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^{7/2}}\,dx = \int \frac{1}{(a\,\cos\left(c+d\,x\right)+b\,\sin\left(c+d\,x\right))^{7/2}}\,dx$$

input
```
int(1/(a*cos(c + d*x) + b*sin(c + d*x))^(7/2),x)
```

output
```
int(1/(a*cos(c + d*x) + b*sin(c + d*x))^(7/2), x)
```

3.240 $\int (2\cos(c+dx)+3\sin(c+dx))^{7/2}\,dx$

3.240.1 Optimal result

Integrand size = 21, antiderivative size = 120

$$\int (2\cos(c+dx)+3\sin(c+dx))^{7/2}\,dx=\frac{130\ 13^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\arctan\left(\frac{3}{2}\right)\right),2\right)}{21d}-\frac{130(3\cos(c+dx)-2\sin(c+dx))\sqrt{2\cos(c+dx)+3\sin(c+dx)}}{21d}-\frac{2(3\cos(c+dx)-2\sin(c+dx))(2\cos(c+dx)+3\sin(c+dx))^{5/2}}{7d}$$

output
```
130/21*13^(3/4)*(cos(1/2*c+1/2*d*x-1/2*arctan(3/2))^2)^(1/2)/cos(1/2*c+1/2
*d*x-1/2*arctan(3/2))*EllipticF(sin(1/2*c+1/2*d*x-1/2*arctan(3/2)),2^(1/2)
)/d-2/7*(3*cos(d*x+c)-2*sin(d*x+c))*(2*cos(d*x+c)+3*sin(d*x+c))^(5/2)/d-13
0/21*(3*cos(d*x+c)-2*sin(d*x+c))*(2*cos(d*x+c)+3*sin(d*x+c))^(1/2)/d
```

3.240.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.46 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.28

$$\int (2\cos(c+dx)+3\sin(c+dx))^{7/2}\,dx=\frac{-\sqrt{2\cos(c+dx)+3\sin(c+dx)}(897\cos(c+dx)+27\cos(3(c+dx))-598\sin(c+dx)+13\cdots}{\cdots}$$

input
```
Integrate[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(7/2),x]
```

output
```
(-(Sqrt[2*Cos[c + d*x] + 3*Sin[c + d*x]]*(897*Cos[c + d*x] + 27*Cos[3*(c +
 d*x)] - 598*Sin[c + d*x] + 138*Sin[3*(c + d*x)])) + 260*13^(3/4)*Hypergeo
metricPFQ[{1/4, 1/2}, {5/4}, Sin[c + d*x + ArcTan[2/3]]^2]*Sec[c + d*x + A
rcTan[2/3]]*Sqrt[-((-1 + Sin[c + d*x + ArcTan[2/3]])*Sin[c + d*x + ArcTan[
2/3]])]*Sqrt[1 + Sin[c + d*x + ArcTan[2/3]]])/(42*d)
```

3.240.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3042, 3552, 3042, 3552, 3042, 3556, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (3\sin(c+dx)+2\cos(c+dx))^{7/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (3\sin(c+dx)+2\cos(c+dx))^{7/2}dx$$

$$\Big\downarrow \text{3552}$$

$$\frac{65}{7}\int (2\cos(c+dx)+3\sin(c+dx))^{3/2}dx-\frac{2(3\cos(c+dx)-2\sin(c+dx))(3\sin(c+dx)+2\cos(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{65}{7}\int (2\cos(c+dx)+3\sin(c+dx))^{3/2}dx-\frac{2(3\cos(c+dx)-2\sin(c+dx))(3\sin(c+dx)+2\cos(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{3552}$$

$$\frac{65}{7}\left(\frac{13}{3}\int \frac{1}{\sqrt{2\cos(c+dx)+3\sin(c+dx)}}dx-\frac{2(3\cos(c+dx)-2\sin(c+dx))\sqrt{3\sin(c+dx)+2\cos(c+dx)}}{3d}\right)-\frac{2(3\cos(c+dx)-2\sin(c+dx))(3\sin(c+dx)+2\cos(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{65}{7}\left(\frac{13}{3}\int \frac{1}{\sqrt{2\cos(c+dx)+3\sin(c+dx)}}dx-\frac{2(3\cos(c+dx)-2\sin(c+dx))\sqrt{3\sin(c+dx)+2\cos(c+dx)}}{3d}\right)-$$
$$\frac{2(3\cos(c+dx)-2\sin(c+dx))(3\sin(c+dx)+2\cos(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{3556}$$

$$\frac{65}{7}\left(\frac{1}{3}13^{3/4}\int \frac{1}{\sqrt{\cos\left(c+dx-\arctan\left(\frac{3}{2}\right)\right)}}dx-\frac{2(3\cos(c+dx)-2\sin(c+dx))\sqrt{3\sin(c+dx)+2\cos(c+dx)}}{3d}\right)$$
$$\frac{2(3\cos(c+dx)-2\sin(c+dx))(3\sin(c+dx)+2\cos(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{65}{7}\left(\frac{1}{3}13^{3/4}\int \frac{1}{\sqrt{\sin\left(c+dx-\arctan\left(\frac{3}{2}\right)+\frac{\pi}{2}\right)}}dx-\frac{2(3\cos(c+dx)-2\sin(c+dx))\sqrt{3\sin(c+dx)+2\cos(c+d}}{3d}\right.$$
$$\frac{2(3\cos(c+dx)-2\sin(c+dx))(3\sin(c+dx)+2\cos(c+dx))^{5/2}}{7d}$$

$$\Big\downarrow \text{3120}$$

$$\frac{65}{7}\left(\frac{2\ 13^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\arctan\left(\frac{3}{2}\right)\right),2\right)}{3d}-\frac{2(3\cos(c+dx)-2\sin(c+dx))\sqrt{3\sin(c+dx)+2\cos(c+d}}{3d}\right.$$
$$\frac{2(3\cos(c+dx)-2\sin(c+dx))(3\sin(c+dx)+2\cos(c+dx))^{5/2}}{7d}$$

input
```
Int[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(7/2),x]
```

output
```
(-2*(3*Cos[c + d*x] - 2*Sin[c + d*x])*(2*Cos[c + d*x] + 3*Sin[c + d*x])^(5
/2))/(7*d) + (65*((2*13^(3/4)*EllipticF[(c + d*x - ArcTan[3/2])/2, 2])/(3*
d) - (2*(3*Cos[c + d*x] - 2*Sin[c + d*x])*Sqrt[2*Cos[c + d*x] + 3*Sin[c +
d*x]])/(3*d)))/7
```

3.240. $\int (2\cos(c+dx)+3\sin(c+dx))^{7/2}\,dx$

3.240.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3552
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(-(b*Cos[c + d*x] - a*Sin[c + d*x]))*((a*Cos[c + d*x] + b*
Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[(n - 1)*((a^2 + b^2)/n)   Int[(a*Co
s[c + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

rule 3556
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a^2 + b^2)^(n/2)   Int[Cos[c + d*x - ArcTan[a, b]]^n, x],
 x] /; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) && GtQ[a^2
+ b^2, 0]
```

3.240.4 Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.07

method	result
default	$\frac{\frac{338 \cos\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)^{4} \sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)}{7}+\frac{845 \sqrt{\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)+1}\, \sqrt{-2 \sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)+2}\, \sqrt{-\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)} \cdots}{21} \cdots}{\cos\left(dx+c+\arctan\left(\frac{2}{3}\right)\right) \sqrt{\sqrt{13}\, \sin\left(dx+c+\arctan \cdots\right.}}$

input
```
int((2*cos(d*x+c)+3*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
(338/7*cos(d*x+c+arctan(2/3))^4*sin(d*x+c+arctan(2/3))+845/21*(sin(d*x+c+a
rctan(2/3))+1)^(1/2)*(-2*sin(d*x+c+arctan(2/3))+2)^(1/2)*(-sin(d*x+c+arcta
n(2/3)))^(1/2)*EllipticF((sin(d*x+c+arctan(2/3))+1)^(1/2),1/2*2^(1/2))-270
4/21*cos(d*x+c+arctan(2/3))^2*sin(d*x+c+arctan(2/3)))/cos(d*x+c+arctan(2/3
))/(13^(1/2)*sin(d*x+c+arctan(2/3)))^(1/2)/d
```

3.240.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.98

$$\int (2\cos(c+dx)+3\sin(c+dx))^{7/2}\,dx = \frac{(130i+195)\ \sqrt{3i+2}\sqrt{2}\text{weierstrassPInverse}\left(\frac{48}{13}i+\frac{20}{13},0,\cos\left(dx+c\right)-i\ \sin\left(dx+c\right)\right)-(130\cdots}{}$$

input
```
integrate((2*cos(d*x+c)+3*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/21*((130*I + 195)*sqrt(3*I + 2)*sqrt(2)*weierstrassPInverse(48/13*I + 20
/13, 0, cos(d*x + c) - I*sin(d*x + c)) - (130*I - 195)*sqrt(2)*sqrt(-3*I +
 2)*weierstrassPInverse(-48/13*I + 20/13, 0, cos(d*x + c) + I*sin(d*x + c)
) - 2*(27*cos(d*x + c)^3 + 46*(3*cos(d*x + c)^2 - 4)*sin(d*x + c) + 204*co
s(d*x + c))*sqrt(2*cos(d*x + c) + 3*sin(d*x + c)))/d
```

3.240.6 Sympy [F(-1)]

Timed out.

$$\int (2\cos(c+dx)+3\sin(c+dx))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate((2*cos(d*x+c)+3*sin(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.240.7 Maxima [F]

$$\int (2\cos(c+dx)+3\sin(c+dx))^{7/2}\,dx = \int \left(2\ \cos\left(dx+c\right)+3\ \sin\left(dx+c\right)\right)^{\frac{7}{2}}\,dx$$

input
```
integrate((2*cos(d*x+c)+3*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(7/2), x)
```

3.240.8 Giac [F]

$$\int (2\cos(c+dx)+3\sin(c+dx))^{7/2}\,dx = \int (2\cos(dx+c)+3\sin(dx+c))^{\frac{7}{2}}\,dx$$

input
```
integrate((2*cos(d*x+c)+3*sin(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(7/2), x)
```

3.240.9 Mupad [F(-1)]

Timed out.

$$\int (2\cos(c+dx)+3\sin(c+dx))^{7/2}\,dx = \int (2\cos(c+d\,x)+3\sin(c+d\,x))^{7/2}\,dx$$

input
```
int((2*cos(c + d*x) + 3*sin(c + d*x))^(7/2),x)
```

output
```
int((2*cos(c + d*x) + 3*sin(c + d*x))^(7/2), x)
```

3.241 $\int (2\cos(c+dx)+3\sin(c+dx))^{5/2}\,dx$

3.241.1 Optimal result

Integrand size = 21, antiderivative size = 75

$$\int (2\cos(c+dx)+3\sin(c+dx))^{5/2}\,dx = \frac{78\sqrt[4]{13}E\left(\frac{1}{2}\left(c+dx-\arctan\left(\frac{3}{2}\right)\right)\middle|2\right)}{5d} - \frac{2(3\cos(c+dx)-2\sin(c+dx))(2\cos(c+dx)+3\sin(c+dx))^{3/2}}{5d}$$

output
```
78/5*13^(1/4)*(cos(1/2*c+1/2*d*x-1/2*arctan(3/2))^2)^(1/2)/cos(1/2*c+1/2*d
*x-1/2*arctan(3/2))*EllipticE(sin(1/2*c+1/2*d*x-1/2*arctan(3/2)),2^(1/2))/
d-2/5*(3*cos(d*x+c)-2*sin(d*x+c))*(2*cos(d*x+c)+3*sin(d*x+c))^(3/2)/d
```

3.241.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.93 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.65

$$\int (2\cos(c+dx)+3\sin(c+dx))^{5/2}\,dx = \frac{\sqrt{2\cos(c+dx)+3\sin(c+dx)}(52-12\cos(2(c+dx))-5\sin(2(c+dx))) - \frac{13\sqrt[4]{13}(4\cos(c+dx\cdots}{\sqrt{c\cdots}}}{\cdots}$$

input
```
Integrate[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(5/2),x]
```

output
```
(Sqrt[2*Cos[c + d*x] + 3*Sin[c + d*x]]*(52 - 12*Cos[2*(c + d*x)] - 5*Sin[2
*(c + d*x)]) - (13*13^(1/4)*(4*Cos[c + d*x - ArcTan[3/2]] - 3*Sin[c + d*x
- ArcTan[3/2]]))/Sqrt[Cos[c + d*x - ArcTan[3/2]]] - (39*13^(1/4)*Hypergeom
etricPFQ[{-1/2, -1/4}, {3/4}, Cos[c + d*x - ArcTan[3/2]]^2]*Sin[c + d*x -
ArcTan[3/2]])/(Sqrt[-((-1 + Cos[c + d*x - ArcTan[3/2]])*Cos[c + d*x - ArcT
an[3/2]])]*Sqrt[1 + Cos[c + d*x - ArcTan[3/2]]]))/(5*d)
```

3.241.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3552, 3042, 3556, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (3\sin(c+dx)+2\cos(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (3\sin(c+dx)+2\cos(c+dx))^{5/2}dx$$

$$\Big\downarrow \text{3552}$$

$$\frac{39}{5}\int \sqrt{2\cos(c+dx)+3\sin(c+dx)}dx-\frac{2(3\cos(c+dx)-2\sin(c+dx))(3\sin(c+dx)+2\cos(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{39}{5}\int \sqrt{2\cos(c+dx)+3\sin(c+dx)}dx-\frac{2(3\cos(c+dx)-2\sin(c+dx))(3\sin(c+dx)+2\cos(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3556}$$

$$\frac{39}{5}\sqrt[4]{13}\int \sqrt{\cos\left(c+dx-\arctan\left(\frac{3}{2}\right)\right)}dx-\frac{2(3\cos(c+dx)-2\sin(c+dx))(3\sin(c+dx)+2\cos(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{39}{5}\sqrt[4]{13}\int \sqrt{\sin\left(c+dx-\arctan\left(\frac{3}{2}\right)+\frac{\pi}{2}\right)}dx-\frac{2(3\cos(c+dx)-2\sin(c+dx))(3\sin(c+dx)+2\cos(c+dx))^{3/2}}{5d}$$

$\Big\downarrow$ 3119

$$\frac{78\sqrt[4]{13}E\left(\frac{1}{2}\left(c+dx-\arctan\left(\frac{3}{2}\right)\right)\middle|2\right)}{5d}-\frac{2(3\cos(c+dx)-2\sin(c+dx))(3\sin(c+dx)+2\cos(c+dx))^{3/2}}{5d}$$

input
```
Int[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(5/2),x]
```

output
```
(78*13^(1/4)*EllipticE[(c + d*x - ArcTan[3/2])/2, 2])/(5*d) - (2*(3*Cos[c
+ d*x] - 2*Sin[c + d*x])*(2*Cos[c + d*x] + 3*Sin[c + d*x])^(3/2))/(5*d)
```

3.241.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3552
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(-(b*Cos[c + d*x] - a*Sin[c + d*x]))*((a*Cos[c + d*x] + b*
Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[(n - 1)*((a^2 + b^2)/n)   Int[(a*Co
s[c + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

rule 3556
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a^2 + b^2)^(n/2)   Int[Cos[c + d*x - ArcTan[a, b]]^n, x],
 x] /; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) && GtQ[a^2
+ b^2, 0]
```

3.241.4 Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.32

method	result
default	$-\frac{13\sqrt{13}\left(6\sqrt{\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)+1}\,\sqrt{-2\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)+2}\,\sqrt{-\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)}\ \mathrm{EllipticE}\left(\sqrt{\sin\left(dx+c+\arctan\right.}\right.\right.}{}$

input
```
int((2*cos(d*x+c)+3*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-13/5*13^(1/2)*(6*(sin(d*x+c+arctan(2/3))+1)^(1/2)*(-2*sin(d*x+c+arctan(2/
3))+2)^(1/2)*(-sin(d*x+c+arctan(2/3)))^(1/2)*EllipticE((sin(d*x+c+arctan(2
/3))+1)^(1/2),1/2*2^(1/2))-3*(sin(d*x+c+arctan(2/3))+1)^(1/2)*(-2*sin(d*x+
c+arctan(2/3))+2)^(1/2)*(-sin(d*x+c+arctan(2/3)))^(1/2)*EllipticF((sin(d*x
+c+arctan(2/3))+1)^(1/2),1/2*2^(1/2))-2*sin(d*x+c+arctan(2/3))^4+2*sin(d*x
+c+arctan(2/3))^2)/cos(d*x+c+arctan(2/3))/(13^(1/2)*sin(d*x+c+arctan(2/3))
)^(1/2)/d
```

3.241.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.48

$$\int (2\cos(c+dx)+3\sin(c+dx))^{5/2}\,dx = \frac{-39i\sqrt{3i+2}\sqrt{2}\mathrm{weierstrassZeta}\left(\frac{48}{13}i+\frac{20}{13},0,\mathrm{weierstrassPInverse}\left(\frac{48}{13}i+\frac{20}{13},0,\cos\left(dx+c\right)-i\right.\right.}{}$$

input
```
integrate((2*cos(d*x+c)+3*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/5*(-39*I*sqrt(3*I + 2)*sqrt(2)*weierstrassZeta(48/13*I + 20/13, 0, weier
strassPInverse(48/13*I + 20/13, 0, cos(d*x + c) - I*sin(d*x + c))) + 39*I*
sqrt(2)*sqrt(-3*I + 2)*weierstrassZeta(-48/13*I + 20/13, 0, weierstrassPIn
verse(-48/13*I + 20/13, 0, cos(d*x + c) + I*sin(d*x + c))) - 2*(12*cos(d*x
 + c)^2 + 5*cos(d*x + c)*sin(d*x + c) - 6)*sqrt(2*cos(d*x + c) + 3*sin(d*x
 + c)))/d
```

3.241.6 Sympy [F(-1)]

Timed out.

$$\int (2\cos(c+dx)+3\sin(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((2*cos(d*x+c)+3*sin(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.241.7 Maxima [F]

$$\int (2\cos(c+dx)+3\sin(c+dx))^{5/2}\,dx = \int (2\cos(dx+c)+3\sin(dx+c))^{\frac{5}{2}}\,dx$$

input
```
integrate((2*cos(d*x+c)+3*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(5/2), x)
```

3.241.8 Giac [F]

$$\int (2\cos(c+dx)+3\sin(c+dx))^{5/2}\,dx = \int (2\cos(dx+c)+3\sin(dx+c))^{\frac{5}{2}}\,dx$$

input
```
integrate((2*cos(d*x+c)+3*sin(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(5/2), x)
```

3.241.9 Mupad [F(-1)]

Timed out.

$$\int (2\cos(c+dx)+3\sin(c+dx))^{5/2}\,dx = \int (2\,\cos\left(c+d\,x\right)+3\,\sin\left(c+d\,x\right))^{5/2}\,dx$$

input
```
int((2*cos(c + d*x) + 3*sin(c + d*x))^(5/2),x)
```

output
```
int((2*cos(c + d*x) + 3*sin(c + d*x))^(5/2), x)
```

3.242 $\int (2\cos(c+dx)+3\sin(c+dx))^{3/2}\,dx$

3.242.1 Optimal result

Integrand size = 21, antiderivative size = 75

$$\int (2\cos(c+dx)+3\sin(c+dx))^{3/2}\,dx=\frac{2\ 13^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\arctan\left(\frac{3}{2}\right)\right),2\right)}{3d}-\frac{2(3\cos(c+dx)-2\sin(c+dx))\sqrt{2\cos(c+dx)+3\sin(c+dx)}}{3d}$$

output
```
2/3*13^(3/4)*(cos(1/2*c+1/2*d*x-1/2*arctan(3/2))^2)^(1/2)/cos(1/2*c+1/2*d*
x-1/2*arctan(3/2))*EllipticF(sin(1/2*c+1/2*d*x-1/2*arctan(3/2)),2^(1/2))/d
-2/3*(3*cos(d*x+c)-2*sin(d*x+c))*(2*cos(d*x+c)+3*sin(d*x+c))^(1/2)/d
```

3.242.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.50 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.77

$$\int (2\cos(c+dx)+3\sin(c+dx))^{3/2}\,dx=\frac{2(-3\cos(c+dx)+2\sin(c+dx))\sqrt{2\cos(c+dx)+3\sin(c+dx)}+2\ 13^{3/4}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2\left(c\right.\right.}{}$$

input
```
Integrate[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(3/2),x]
```

output
```
(2*(-3*Cos[c + d*x] + 2*Sin[c + d*x])*Sqrt[2*Cos[c + d*x] + 3*Sin[c + d*x]
] + 2*13^(3/4)*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[c + d*x + ArcTan[2
/3]]^2]*Sec[c + d*x + ArcTan[2/3]]*Sqrt[-((-1 + Sin[c + d*x + ArcTan[2/3]]
)*Sin[c + d*x + ArcTan[2/3]])]*Sqrt[1 + Sin[c + d*x + ArcTan[2/3]]])/(3*d)
```

3.242.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3552, 3042, 3556, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (3\sin(c+dx)+2\cos(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (3\sin(c+dx)+2\cos(c+dx))^{3/2}dx$$

$$\Big\downarrow \text{ 3552}$$

$$\frac{13}{3}\int \frac{1}{\sqrt{2\cos(c+dx)+3\sin(c+dx)}}dx-\frac{2(3\cos(c+dx)-2\sin(c+dx))\sqrt{3\sin(c+dx)+2\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{13}{3}\int \frac{1}{\sqrt{2\cos(c+dx)+3\sin(c+dx)}}dx-\frac{2(3\cos(c+dx)-2\sin(c+dx))\sqrt{3\sin(c+dx)+2\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{ 3556}$$

$$\frac{1}{3}13^{3/4}\int \frac{1}{\sqrt{\cos\left(c+dx-\arctan\left(\frac{3}{2}\right)\right)}}dx-\frac{2(3\cos(c+dx)-2\sin(c+dx))\sqrt{3\sin(c+dx)+2\cos(c+dx)}}{3d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{3}13^{3/4}\int \frac{1}{\sqrt{\sin \left(c+dx-\arctan \left(\frac{3}{2}\right)+\frac{\pi}{2}\right)}}dx-\frac{2(3\cos(c+dx)-2\sin(c+dx))\sqrt{3\sin(c+dx)+2\cos(c+dx)}}{3d}$$

$$\Big\downarrow \; 3120$$

$$\frac{2\ 13^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\arctan\left(\frac{3}{2}\right)\right),2\right)}{3d}-\frac{2(3\cos(c+dx)-2\sin(c+dx))\sqrt{3\sin(c+dx)+2\cos(c+dx)}}{3d}$$

input
```
Int[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(3/2),x]
```

output
```
(2*13^(3/4)*EllipticF[(c + d*x - ArcTan[3/2])/2, 2])/(3*d) - (2*(3*Cos[c +
 d*x] - 2*Sin[c + d*x])*Sqrt[2*Cos[c + d*x] + 3*Sin[c + d*x]])/(3*d)
```

3.242.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3552
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(-(b*Cos[c + d*x] - a*Sin[c + d*x]))*((a*Cos[c + d*x] + b*
Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[(n - 1)*((a^2 + b^2)/n)   Int[(a*Co
s[c + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

rule 3556
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a^2 + b^2)^(n/2)   Int[Cos[c + d*x - ArcTan[a, b]]^n, x],
 x] /; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) && GtQ[a^2
+ b^2, 0]
```

3.242.4 Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.44

method	result
default	$\frac{\frac{13\sqrt{\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)+1}\,\sqrt{-2\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)+2}\,\sqrt{-\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)}\,\operatorname{EllipticF}\left(\sqrt{\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)+1},\frac{\sqrt{2}}{2}\right)}{3}-\frac{26\cos\left(dx\right.}{}}{\cos\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)\sqrt{\sqrt{13}\,\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)}\,d}$

input
```
int((2*cos(d*x+c)+3*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(13/3*(sin(d*x+c+arctan(2/3))+1)^(1/2)*(-2*sin(d*x+c+arctan(2/3))+2)^(1/2)
*(-sin(d*x+c+arctan(2/3)))^(1/2)*EllipticF((sin(d*x+c+arctan(2/3))+1)^(1/2
),1/2*2^(1/2))-26/3*cos(d*x+c+arctan(2/3))^2*sin(d*x+c+arctan(2/3)))/cos(d
*x+c+arctan(2/3))/(13^(1/2)*sin(d*x+c+arctan(2/3)))^(1/2)/d
```

3.242.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.28

$$\int (2\cos(c+dx)+3\sin(c+dx))^{3/2}\,dx = \frac{(2i+3)\,\sqrt{3i+2}\sqrt{2}\text{weierstrassPInverse}\left(\frac{48}{13}i+\frac{20}{13},0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)-(2i-3)}{}$$

input
```
integrate((2*cos(d*x+c)+3*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/3*((2*I + 3)*sqrt(3*I + 2)*sqrt(2)*weierstrassPInverse(48/13*I + 20/13,
0, cos(d*x + c) - I*sin(d*x + c)) - (2*I - 3)*sqrt(2)*sqrt(-3*I + 2)*weier
strassPInverse(-48/13*I + 20/13, 0, cos(d*x + c) + I*sin(d*x + c)) - 2*(3*
cos(d*x + c) - 2*sin(d*x + c))*sqrt(2*cos(d*x + c) + 3*sin(d*x + c)))/d
```

3.242.6 Sympy [F(-1)]

Timed out.

$$\int (2\cos(c+dx)+3\sin(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((2*cos(d*x+c)+3*sin(d*x+c))**(3/2),x)
```

output
```
Timed out
```

3.242.7 Maxima [F]

$$\int (2\cos(c+dx)+3\sin(c+dx))^{3/2}\,dx = \int (2\cos(dx+c)+3\sin(dx+c))^{\frac{3}{2}}\,dx$$

input
```
integrate((2*cos(d*x+c)+3*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(3/2), x)
```

3.242.8 Giac [F]

$$\int (2\cos(c+dx)+3\sin(c+dx))^{3/2}\,dx = \int (2\cos(dx+c)+3\sin(dx+c))^{\frac{3}{2}}\,dx$$

input
```
integrate((2*cos(d*x+c)+3*sin(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(3/2), x)
```

3.242.9 Mupad [F(-1)]

Timed out.

$$\int (2\cos(c+dx)+3\sin(c+dx))^{3/2}\,dx=\int (2\,\cos\left(c+d\,x\right)+3\,\sin\left(c+d\,x\right))^{3/2}\,dx$$

input
```
int((2*cos(c + d*x) + 3*sin(c + d*x))^(3/2),x)
```

output
```
int((2*cos(c + d*x) + 3*sin(c + d*x))^(3/2), x)
```

3.243 $\int \sqrt{2\cos(c+dx)+3\sin(c+dx)}\,dx$

3.243.1 Optimal result

Integrand size = 21, antiderivative size = 27

$$\int \sqrt{2\cos(c+dx)+3\sin(c+dx)}\,dx=\frac{2\sqrt[4]{13}E\left(\frac{1}{2}\left(c+dx-\arctan\left(\frac{3}{2}\right)\right)\middle|2\right)}{d}$$

output
```
2*13^(1/4)*(cos(1/2*c+1/2*d*x-1/2*arctan(3/2))^2)^(1/2)/cos(1/2*c+1/2*d*x-
1/2*arctan(3/2))*EllipticE(sin(1/2*c+1/2*d*x-1/2*arctan(3/2)),2^(1/2))/d
```

3.243.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.82 (sec) , antiderivative size = 184, normalized size of antiderivative = 6.81

$$\int \sqrt{2\cos(c+dx)+3\sin(c+dx)}\,dx$$
$$=\frac{-4\sqrt[4]{13}\sqrt{\cos\left(c+dx-\arctan\left(\frac{3}{2}\right)\right)}+4\sqrt{2\cos(c+dx)+3\sin(c+dx)}+\frac{3\sqrt[4]{13}\sin\left(c+dx-\arctan\left(\frac{3}{2}\right)\right)}{\sqrt{\cos\left(c+dx-\arctan\left(\frac{3}{2}\right)\right)}}-\frac{\cdots}{\sqrt{-((\cdots}}}{3d}$$

input
```
Integrate[Sqrt[2*Cos[c + d*x] + 3*Sin[c + d*x]],x]
```

output
```
(-4*13^(1/4)*Sqrt[Cos[c + d*x - ArcTan[3/2]]] + 4*Sqrt[2*Cos[c + d*x] + 3*
Sin[c + d*x]] + (3*13^(1/4)*Sin[c + d*x - ArcTan[3/2]])/Sqrt[Cos[c + d*x -
 ArcTan[3/2]]] - (3*13^(1/4)*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[c
+ d*x - ArcTan[3/2]]^2]*Sin[c + d*x - ArcTan[3/2]])/(Sqrt[-((-1 + Cos[c +
d*x - ArcTan[3/2]])*Cos[c + d*x - ArcTan[3/2]])]*Sqrt[1 + Cos[c + d*x - Ar
cTan[3/2]]]))/(3*d)
```

3.243.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3556, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{3\sin(c+dx)+2\cos(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{3\sin(c+dx)+2\cos(c+dx)}dx$$

$$\Big\downarrow \text{3556}$$

$$\sqrt[4]{13}\int \sqrt{\cos\left(c+dx-\arctan\left(\frac{3}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3042}$$

$$\sqrt[4]{13}\int \sqrt{\sin\left(c+dx-\arctan\left(\frac{3}{2}\right)+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3119}$$

$$\frac{2\sqrt[4]{13}E\left(\frac{1}{2}\left(c+dx-\arctan\left(\frac{3}{2}\right)\right)\middle|2\right)}{d}$$

input
```
Int[Sqrt[2*Cos[c + d*x] + 3*Sin[c + d*x]],x]
```

output
```
(2*13^(1/4)*EllipticE[(c + d*x - ArcTan[3/2])/2, 2])/d
```

3.243.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3556
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a^2 + b^2)^(n/2)   Int[Cos[c + d*x - ArcTan[a, b]]^n, x],
 x] /; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) && GtQ[a^2
+ b^2, 0]
```

3.243.4 Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 112, normalized size of antiderivative = 4.15

method	result
default	$-\frac{\sqrt{13}\,\sqrt{\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)+1}\,\sqrt{-2\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)+2}\,\sqrt{-\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)}\,\left(2\,\mathrm{EllipticE}\left(\sqrt{\sin\left(dx+c+\arctan\left(\frac{2}{3}\right.\right.}\right.\right.}{\cos\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)\sqrt{\sqrt{13}\,\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)}\,d}$
risch	$-\frac{i\sqrt{2}\,\sqrt{-\left(3i\mathrm{e}^{2i(dx+c)}-2\,\mathrm{e}^{2i(dx+c)}-2-3i\right)\mathrm{e}^{-i(dx+c)}}}{d}+\frac{(12+5i)\left(\frac{\left(-\frac{4}{2197}+\frac{6i}{2197}\right)\left(-507i\mathrm{e}^{2i(dx+c)}+338\,\mathrm{e}^{2i(dx+c)}+338+507i\right)}{\sqrt{\mathrm{e}^{i(dx+c)}\left(-507i\mathrm{e}^{2i(dx+c)}+338\,\mathrm{e}^{2i(dx+c)}+338+507i\right)}}+\frac{\left(-\frac{6}{16}\right.}{}\right.}{}$

input
```
int((2*cos(d*x+c)+3*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-13^(1/2)*(sin(d*x+c+arctan(2/3))+1)^(1/2)*(-2*sin(d*x+c+arctan(2/3))+2)^(
1/2)*(-sin(d*x+c+arctan(2/3)))^(1/2)*(2*EllipticE((sin(d*x+c+arctan(2/3))+
1)^(1/2),1/2*2^(1/2))-EllipticF((sin(d*x+c+arctan(2/3))+1)^(1/2),1/2*2^(1/
2)))/cos(d*x+c+arctan(2/3))/(13^(1/2)*sin(d*x+c+arctan(2/3)))^(1/2)/d
```

3.243.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.33

$$\int \sqrt{2\cos(c+dx)+3\sin(c+dx)}\,dx = \frac{-i\,\sqrt{3i+2}\sqrt{2}\text{weierstrassZeta}\left(\frac{48}{13}i+\frac{20}{13},0,\text{weierstrassPInverse}\left(\frac{48}{13}i+\frac{20}{13},0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)}{}$$

input
```
integrate((2*cos(d*x+c)+3*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
(-I*sqrt(3*I + 2)*sqrt(2)*weierstrassZeta(48/13*I + 20/13, 0, weierstrassP
Inverse(48/13*I + 20/13, 0, cos(d*x + c) - I*sin(d*x + c))) + I*sqrt(2)*sq
rt(-3*I + 2)*weierstrassZeta(-48/13*I + 20/13, 0, weierstrassPInverse(-48/
13*I + 20/13, 0, cos(d*x + c) + I*sin(d*x + c))))/d
```

3.243.6 Sympy [F]

$$\int \sqrt{2\cos(c+dx)+3\sin(c+dx)}\,dx = \int \sqrt{3\sin\left(c+dx\right)+2\cos\left(c+dx\right)}\,dx$$

input
```
integrate((2*cos(d*x+c)+3*sin(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(3*sin(c + d*x) + 2*cos(c + d*x)), x)
```

3.243.7 Maxima [F]

$$\int \sqrt{2\cos(c+dx)+3\sin(c+dx)}\,dx = \int \sqrt{2\,\cos\left(dx+c\right)+3\,\sin\left(dx+c\right)}\,dx$$

input
```
integrate((2*cos(d*x+c)+3*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(2*cos(d*x + c) + 3*sin(d*x + c)), x)
```

3.243.8 Giac [F]

$$\int \sqrt{2\cos(c+dx)+3\sin(c+dx)}\,dx = \int \sqrt{2\,\cos(dx+c)+3\,\sin(dx+c)}\,dx$$

input
```
integrate((2*cos(d*x+c)+3*sin(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(2*cos(d*x + c) + 3*sin(d*x + c)), x)
```

3.243.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{2\cos(c+dx)+3\sin(c+dx)}\,dx = \int \sqrt{2\,\cos(c+d\,x)+3\,\sin(c+d\,x)}\,dx$$

input
```
int((2*cos(c + d*x) + 3*sin(c + d*x))^(1/2),x)
```

output
```
int((2*cos(c + d*x) + 3*sin(c + d*x))^(1/2), x)
```

3.244 $\int \frac{1}{\sqrt{2\cos(c+dx)+3\sin(c+dx)}}\,dx$

3.244.1 Optimal result

Integrand size = 21, antiderivative size = 27

$$\int \frac{1}{\sqrt{2\cos(c+dx)+3\sin(c+dx)}}\,dx=\frac{2\operatorname{EllipticF}\left(\frac{1}{2}\left(c+dx-\arctan\left(\frac{3}{2}\right)\right),2\right)}{\sqrt[4]{13}d}$$

output
```
2/13*13^(3/4)*(cos(1/2*c+1/2*d*x-1/2*arctan(3/2))^2)^(1/2)/cos(1/2*c+1/2*d
*x-1/2*arctan(3/2))*EllipticF(sin(1/2*c+1/2*d*x-1/2*arctan(3/2)),2^(1/2))/
d
```

3.244.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.19 (sec) , antiderivative size = 88, normalized size of antiderivative = 3.26

$$\int \frac{1}{\sqrt{2\cos(c+dx)+3\sin(c+dx)}}\,dx$$
$$=\frac{2\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2\left(c+dx+\arctan\left(\frac{2}{3}\right)\right)\right)\sec\left(c+dx+\arctan\left(\frac{2}{3}\right)\right)\sqrt{-\left(\left(-1+\sin\left(c+dx+\arctan\left(\frac{2}{3}\right)\right)\right.\right.}}{\sqrt[4]{13}d}$$

input
```
Integrate[1/Sqrt[2*Cos[c + d*x] + 3*Sin[c + d*x]],x]
```

output
```
(2*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[c + d*x + ArcTan[2/3]]^2]*Sec[
c + d*x + ArcTan[2/3]]*Sqrt[-((-1 + Sin[c + d*x + ArcTan[2/3]])*Sin[c + d*
x + ArcTan[2/3]])]*Sqrt[1 + Sin[c + d*x + ArcTan[2/3]]])/(13^(1/4)*d)
```

3.244.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3556, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3\sin(c+dx)+2\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{3\sin(c+dx)+2\cos(c+dx)}}dx$$

$$\Big\downarrow \text{3556}$$

$$\frac{\int \frac{1}{\sqrt{\cos\left(c+dx-\arctan\left(\frac{3}{2}\right)\right)}}dx}{\sqrt[4]{13}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\sqrt{\sin\left(c+dx-\arctan\left(\frac{3}{2}\right)+\frac{\pi}{2}\right)}}dx}{\sqrt[4]{13}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\arctan\left(\frac{3}{2}\right)\right),2\right)}{\sqrt[4]{13}d}$$

input
```
Int[1/Sqrt[2*Cos[c + d*x] + 3*Sin[c + d*x]],x]
```

output
```
(2*EllipticF[(c + d*x - ArcTan[3/2])/2, 2])/(13^(1/4)*d)
```

3.244.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3556
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a^2 + b^2)^(n/2)   Int[Cos[c + d*x - ArcTan[a, b]]^n, x],
 x] /; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) && GtQ[a^2
+ b^2, 0]
```

3.244.4 Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 85, normalized size of antiderivative = 3.15

method	result
default	$\frac{\sqrt{\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)+1}\,\sqrt{-2\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)+2}\,\sqrt{-\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)}\,\operatorname{EllipticF}\left(\sqrt{\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)+1},\frac{\sqrt{2}}{2}\right)}{\cos\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)\sqrt{\sqrt{13}\,\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)}\,d}$

input
```
int(1/(2*cos(d*x+c)+3*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(sin(d*x+c+arctan(2/3))+1)^(1/2)*(-2*sin(d*x+c+arctan(2/3))+2)^(1/2)*(-sin
(d*x+c+arctan(2/3)))^(1/2)*EllipticF((sin(d*x+c+arctan(2/3))+1)^(1/2),1/2*
2^(1/2))/cos(d*x+c+arctan(2/3))/(13^(1/2)*sin(d*x+c+arctan(2/3)))^(1/2)/d
```

3.244.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.15

$$\int \frac{1}{\sqrt{2\cos(c+dx)+3\sin(c+dx)}}\,dx$$
$$= \frac{(2i+3)\ \sqrt{3i+2}\sqrt{2}\text{weierstrassPInverse}\left(\frac{48}{13}i+\frac{20}{13},0,\cos\left(dx+c\right)-i\ \sin\left(dx+c\right)\right)-(2i-3)\ \sqrt{2}\sqrt{-3i+}}{13\,d}$$

input
```
integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
1/13*((2*I + 3)*sqrt(3*I + 2)*sqrt(2)*weierstrassPInverse(48/13*I + 20/13,
 0, cos(d*x + c) - I*sin(d*x + c)) - (2*I - 3)*sqrt(2)*sqrt(-3*I + 2)*weie
rstrassPInverse(-48/13*I + 20/13, 0, cos(d*x + c) + I*sin(d*x + c)))/d
```

3.244.6 Sympy [F]

$$\int \frac{1}{\sqrt{2\cos(c+dx)+3\sin(c+dx)}}\,dx = \int \frac{1}{\sqrt{3\sin{(c+dx)}+2\cos{(c+dx)}}}\,dx$$

input
```
integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))**(1/2),x)
```

output
```
Integral(1/sqrt(3*sin(c + d*x) + 2*cos(c + d*x)), x)
```

3.244.7 Maxima [F]

$$\int \frac{1}{\sqrt{2\cos(c+dx)+3\sin(c+dx)}}\,dx = \int \frac{1}{\sqrt{2\,\cos{(dx+c)}+3\,\sin{(dx+c)}}}\,dx$$

input
```
integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(2*cos(d*x + c) + 3*sin(d*x + c)), x)
```

3.244.8 Giac [F]

$$\int \frac{1}{\sqrt{2\cos(c+dx)+3\sin(c+dx)}}\,dx = \int \frac{1}{\sqrt{2\,\cos{(dx+c)}+3\,\sin{(dx+c)}}}\,dx$$

input
```
integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(2*cos(d*x + c) + 3*sin(d*x + c)), x)
```

3.244.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2\cos(c+dx)+3\sin(c+dx)}}\,dx = \int \frac{1}{\sqrt{2\,\cos\left(c+d\,x\right)+3\,\sin\left(c+d\,x\right)}}\,dx$$

input
```
int(1/(2*cos(c + d*x) + 3*sin(c + d*x))^(1/2),x)
```

output
```
int(1/(2*cos(c + d*x) + 3*sin(c + d*x))^(1/2), x)
```

3.245 $\int \frac{1}{(2\cos(c+dx)+3\sin(c+dx))^{3/2}}\,dx$

3.245.1 Optimal result

Integrand size = 21, antiderivative size = 73

$$\int \frac{1}{(2\cos(c+dx)+3\sin(c+dx))^{3/2}}\,dx = -\frac{2E\left(\frac{1}{2}\left(c+dx-\arctan\left(\frac{3}{2}\right)\right)\middle|2\right)}{13^{3/4}d} - \frac{2(3\cos(c+dx)-2\sin(c+dx))}{13d\sqrt{2\cos(c+dx)+3\sin(c+dx)}}$$

output
```
-2/13*13^(1/4)*(cos(1/2*c+1/2*d*x-1/2*arctan(3/2))^2)^(1/2)/cos(1/2*c+1/2*
d*x-1/2*arctan(3/2))*EllipticE(sin(1/2*c+1/2*d*x-1/2*arctan(3/2)),2^(1/2))
/d-2/13*(3*cos(d*x+c)-2*sin(d*x+c))/d/(2*cos(d*x+c)+3*sin(d*x+c))^(1/2)
```

3.245.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.02 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.60

$$\int \frac{1}{(2\cos(c+dx)+3\sin(c+dx))^{3/2}}\,dx = \frac{\frac{4\sqrt{\cos\left(c+dx-\arctan\left(\frac{3}{2}\right)\right)}}{13^{3/4}} - \frac{2\cos(c+dx)}{\sqrt{2\cos(c+dx)+3\sin(c+dx)}} - \frac{3\sin\left(c+dx-\arctan\left(\frac{3}{2}\right.\right.}{13^{3/4}\sqrt{\cos\left(c+dx-\arctan\right.}}}{}$$

input
```
Integrate[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(-3/2),x]
```

output
```
((4*Sqrt[Cos[c + d*x - ArcTan[3/2]]])/13^(3/4) - (2*Cos[c + d*x])/Sqrt[2*C
os[c + d*x] + 3*Sin[c + d*x]] - (3*Sin[c + d*x - ArcTan[3/2]])/(13^(3/4)*S
qrt[Cos[c + d*x - ArcTan[3/2]]]) + (3*HypergeometricPFQ[{-1/2, -1/4}, {3/4
}, Cos[c + d*x - ArcTan[3/2]]^2]*Sin[c + d*x - ArcTan[3/2]])/(13^(3/4)*Sqr
t[-((-1 + Cos[c + d*x - ArcTan[3/2]])*Cos[c + d*x - ArcTan[3/2]])]*Sqrt[1
+ Cos[c + d*x - ArcTan[3/2]]]))/(3*d)
```

3.245.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3555, 3042, 3556, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3\sin(c+dx)+2\cos(c+dx))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(3\sin(c+dx)+2\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3555}$$

$$-\frac{1}{13}\int \sqrt{2\cos(c+dx)+3\sin(c+dx)}dx-\frac{2(3\cos(c+dx)-2\sin(c+dx))}{13d\sqrt{3\sin(c+dx)+2\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{13}\int \sqrt{2\cos(c+dx)+3\sin(c+dx)}dx-\frac{2(3\cos(c+dx)-2\sin(c+dx))}{13d\sqrt{3\sin(c+dx)+2\cos(c+dx)}}$$

$$\Big\downarrow \text{3556}$$

$$-\frac{\int \sqrt{\cos\left(c+dx-\arctan\left(\frac{3}{2}\right)\right)}dx}{13^{3/4}}-\frac{2(3\cos(c+dx)-2\sin(c+dx))}{13d\sqrt{3\sin(c+dx)+2\cos(c+dx)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \sqrt{\sin\left(c+dx-\arctan\left(\frac{3}{2}\right)+\frac{\pi}{2}\right)}dx}{13^{3/4}}-\frac{2(3\cos(c+dx)-2\sin(c+dx))}{13d\sqrt{3\sin(c+dx)+2\cos(c+dx)}}$$

$$\Big\downarrow \text{3119}$$

$$-\frac{2E\left(\frac{1}{2}\left(c+dx-\arctan\left(\frac{3}{2}\right)\right)\middle|2\right)}{13^{3/4}d}-\frac{2(3\cos(c+dx)-2\sin(c+dx))}{13d\sqrt{3\sin(c+dx)+2\cos(c+dx)}}$$

input
```
Int[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(-3/2),x]
```

output
```
(-2*EllipticE[(c + d*x - ArcTan[3/2])/2, 2])/(13^(3/4)*d) - (2*(3*Cos[c +
d*x] - 2*Sin[c + d*x]))/(13*d*Sqrt[2*Cos[c + d*x] + 3*Sin[c + d*x]])
```

3.245.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3555
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(b*Cos[c + d*x] - a*Sin[c + d*x])*((a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Simp[(n + 2)/((n + 1)*(a^
2 + b^2))   Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

rule 3556
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a^2 + b^2)^(n/2)   Int[Cos[c + d*x - ArcTan[a, b]]^n, x],
 x] /; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) && GtQ[a^2
+ b^2, 0]
```

3.245.4 Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.22

method	result
default	$\frac{\sqrt{13}\left(2\sqrt{\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)+1}\,\sqrt{-2\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)+2}\,\sqrt{-\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)}\ \text{EllipticE}\left(\sqrt{\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)-}\right.\right.}{13\cos(dx}$

input
```
int(1/(2*cos(d*x+c)+3*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/13*13^(1/2)*(2*(sin(d*x+c+arctan(2/3))+1)^(1/2)*(-2*sin(d*x+c+arctan(2/3
))+2)^(1/2)*(-sin(d*x+c+arctan(2/3)))^(1/2)*EllipticE((sin(d*x+c+arctan(2/
3))+1)^(1/2),1/2*2^(1/2))-(sin(d*x+c+arctan(2/3))+1)^(1/2)*(-2*sin(d*x+c+a
rctan(2/3))+2)^(1/2)*(-sin(d*x+c+arctan(2/3)))^(1/2)*EllipticF((sin(d*x+c+
arctan(2/3))+1)^(1/2),1/2*2^(1/2))-2*cos(d*x+c+arctan(2/3))^2)/cos(d*x+c+a
rctan(2/3))/(13^(1/2)*sin(d*x+c+arctan(2/3)))^(1/2)/d
```

3.245.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.16

$$\int \frac{1}{(2\cos(c+dx)+3\sin(c+dx))^{3/2}}\,dx = \frac{\sqrt{3i+2}\left(2i\sqrt{2}\cos\left(dx+c\right)+3i\sqrt{2}\sin\left(dx+c\right)\right)\text{weierstrassZeta}(}{}$$

input
```
integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/13*(sqrt(3*I + 2)*(2*I*sqrt(2)*cos(d*x + c) + 3*I*sqrt(2)*sin(d*x + c))*
weierstrassZeta(48/13*I + 20/13, 0, weierstrassPInverse(48/13*I + 20/13, 0
, cos(d*x + c) - I*sin(d*x + c))) + sqrt(-3*I + 2)*(-2*I*sqrt(2)*cos(d*x +
 c) - 3*I*sqrt(2)*sin(d*x + c))*weierstrassZeta(-48/13*I + 20/13, 0, weier
strassPInverse(-48/13*I + 20/13, 0, cos(d*x + c) + I*sin(d*x + c))) - 2*(3
*cos(d*x + c) - 2*sin(d*x + c))*sqrt(2*cos(d*x + c) + 3*sin(d*x + c)))/(2*
d*cos(d*x + c) + 3*d*sin(d*x + c))
```

3.245.6 Sympy [F]

$$\int \frac{1}{(2\cos(c+dx)+3\sin(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(3\sin\left(c+dx\right)+2\cos\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))**(3/2),x)
```

output
```
Integral((3*sin(c + d*x) + 2*cos(c + d*x))**(-3/2), x)
```

3.245.7 Maxima [F]

$$\int \frac{1}{(2\cos(c+dx)+3\sin(c+dx))^{3/2}}\,dx = \int \frac{1}{(2\cos(dx+c)+3\sin(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(-3/2), x)
```

3.245.8 Giac [F]

$$\int \frac{1}{(2\cos(c+dx)+3\sin(c+dx))^{3/2}}\,dx = \int \frac{1}{(2\cos(dx+c)+3\sin(dx+c))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(-3/2), x)
```

3.245.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2\cos(c+dx)+3\sin(c+dx))^{3/2}}\,dx = \int \frac{1}{(2\cos(c+d\,x)+3\sin(c+d\,x))^{3/2}}\,dx$$

input
```
int(1/(2*cos(c + d*x) + 3*sin(c + d*x))^(3/2),x)
```

output
```
int(1/(2*cos(c + d*x) + 3*sin(c + d*x))^(3/2), x)
```

3.246 $\int \frac{1}{(2\cos(c+dx)+3\sin(c+dx))^{5/2}}\,dx$

3.246.1 Optimal result

Integrand size = 21, antiderivative size = 75

$$\int \frac{1}{(2\cos(c+dx)+3\sin(c+dx))^{5/2}}\,dx = \frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\arctan\left(\frac{3}{2}\right)\right),2\right)}{39\sqrt[4]{13}d} - \frac{2(3\cos(c+dx)-2\sin(c+dx))}{39d(2\cos(c+dx)+3\sin(c+dx))^{3/2}}$$

output
```
2/507*13^(3/4)*(cos(1/2*c+1/2*d*x-1/2*arctan(3/2))^2)^(1/2)/cos(1/2*c+1/2*
d*x-1/2*arctan(3/2))*EllipticF(sin(1/2*c+1/2*d*x-1/2*arctan(3/2)),2^(1/2))
/d-2/39*(3*cos(d*x+c)-2*sin(d*x+c))/d/(2*cos(d*x+c)+3*sin(d*x+c))^(3/2)
```

3.246.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.63 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.09

$$\int \frac{1}{(2\cos(c+dx)+3\sin(c+dx))^{5/2}}\,dx = \frac{-78\cos(c+dx)+52\sin(c+dx)+\sqrt{2}13^{3/4}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2\left(c+\right.\right.}{}$$

input
```
Integrate[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(-5/2),x]
```

output
```
(-78*Cos[c + d*x] + 52*Sin[c + d*x] + Sqrt[2]*13^(3/4)*HypergeometricPFQ[{
1/4, 1/2}, {5/4}, Sin[c + d*x + ArcTan[2/3]]^2]*Sec[c + d*x + ArcTan[2/3]]
*(2*Cos[c + d*x] + 3*Sin[c + d*x])^(3/2)*Sqrt[1 + Sin[c + d*x + ArcTan[2/3
]]]*Sqrt[-1 + Cos[2*(c + d*x + ArcTan[2/3])] + 2*Sin[c + d*x + ArcTan[2/3]
]])/(507*d*(2*Cos[c + d*x] + 3*Sin[c + d*x])^(3/2))
```

3.246.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3555, 3042, 3556, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3\sin(c+dx)+2\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(3\sin(c+dx)+2\cos(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3555}$$

$$\frac{1}{39}\int \frac{1}{\sqrt{2\cos(c+dx)+3\sin(c+dx)}}dx-\frac{2(3\cos(c+dx)-2\sin(c+dx))}{39d(3\sin(c+dx)+2\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{39}\int \frac{1}{\sqrt{2\cos(c+dx)+3\sin(c+dx)}}dx-\frac{2(3\cos(c+dx)-2\sin(c+dx))}{39d(3\sin(c+dx)+2\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3556}$$

$$\frac{\int \frac{1}{\sqrt{\cos\left(c+dx-\arctan\left(\frac{3}{2}\right)\right)}}dx}{39\sqrt[4]{13}}-\frac{2(3\cos(c+dx)-2\sin(c+dx))}{39d(3\sin(c+dx)+2\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\sqrt{\sin\left(c+dx-\arctan\left(\frac{3}{2}\right)+\frac{\pi}{2}\right)}}dx}{39\sqrt[4]{13}}-\frac{2(3\cos(c+dx)-2\sin(c+dx))}{39d(3\sin(c+dx)+2\cos(c+dx))^{3/2}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\arctan\left(\frac{3}{2}\right)\right),2\right)}{39\sqrt[4]{13}d}-\frac{2(3\cos(c+dx)-2\sin(c+dx))}{39d(3\sin(c+dx)+2\cos(c+dx))^{3/2}}$$

input
```
Int[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(-5/2),x]
```

output
```
(2*EllipticF[(c + d*x - ArcTan[3/2])/2, 2])/(39*13^(1/4)*d) - (2*(3*Cos[c
+ d*x] - 2*Sin[c + d*x]))/(39*d*(2*Cos[c + d*x] + 3*Sin[c + d*x])^(3/2))
```

3.246.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3555
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(b*Cos[c + d*x] - a*Sin[c + d*x])*((a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Simp[(n + 2)/((n + 1)*(a^
2 + b^2))   Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

rule 3556
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a^2 + b^2)^(n/2)   Int[Cos[c + d*x - ArcTan[a, b]]^n, x],
 x] /; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) && GtQ[a^2
+ b^2, 0]
```

3.246.4 Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.57

method	result
default	$\frac{\sqrt{\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)+1}\,\sqrt{-2\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)+2}\,\sqrt{-\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)}\ \operatorname{EllipticF}\left(\sqrt{\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)+1},\frac{\sqrt{2}}{2}\right)}{39\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)\cos\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)\sqrt{\sqrt{13}\,\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)}\,d}$

input
```
int(1/(2*cos(d*x+c)+3*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/39/sin(d*x+c+arctan(2/3))*((sin(d*x+c+arctan(2/3))+1)^(1/2)*(-2*sin(d*x+
c+arctan(2/3))+2)^(1/2)*(-sin(d*x+c+arctan(2/3)))^(1/2)*EllipticF((sin(d*x
+c+arctan(2/3))+1)^(1/2),1/2*2^(1/2))*sin(d*x+c+arctan(2/3))-2*cos(d*x+c+a
rctan(2/3))^2)/cos(d*x+c+arctan(2/3))/(13^(1/2)*sin(d*x+c+arctan(2/3)))^(1
/2)/d
```

3.246.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.52

$$\int \frac{1}{(2\cos(c+dx)+3\sin(c+dx))^{5/2}}\,dx =$$
$$-\frac{\sqrt{3i+2}\left(-(10i+15)\ \sqrt{2}\cos{(dx+c)}^{2}+(24i+36)\ \sqrt{2}\cos{(dx+c)}\sin{(dx+c)}+(18i+27)\ \sqrt{2}\right)\text{weierst}}{}$$

input
```
integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-1/507*(sqrt(3*I + 2)*(-(10*I + 15)*sqrt(2)*cos(d*x + c)^2 + (24*I + 36)*s
qrt(2)*cos(d*x + c)*sin(d*x + c) + (18*I + 27)*sqrt(2))*weierstrassPInvers
e(48/13*I + 20/13, 0, cos(d*x + c) - I*sin(d*x + c)) + sqrt(-3*I + 2)*((10
*I - 15)*sqrt(2)*cos(d*x + c)^2 - (24*I - 36)*sqrt(2)*cos(d*x + c)*sin(d*x
 + c) - (18*I - 27)*sqrt(2))*weierstrassPInverse(-48/13*I + 20/13, 0, cos(
d*x + c) + I*sin(d*x + c)) - 26*(3*cos(d*x + c) - 2*sin(d*x + c))*sqrt(2*c
os(d*x + c) + 3*sin(d*x + c)))/(5*d*cos(d*x + c)^2 - 12*d*cos(d*x + c)*sin
(d*x + c) - 9*d)
```

3.246.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(2\cos(c+dx)+3\sin(c+dx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.246.7 Maxima [F]

$$\int \frac{1}{(2\cos(c+dx)+3\sin(c+dx))^{5/2}}\,dx = \int \frac{1}{(2\cos(dx+c)+3\sin(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(-5/2), x)
```

3.246.8 Giac [F]

$$\int \frac{1}{(2\cos(c+dx)+3\sin(c+dx))^{5/2}}\,dx = \int \frac{1}{(2\cos(dx+c)+3\sin(dx+c))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(-5/2), x)
```

3.246.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2\cos(c+dx)+3\sin(c+dx))^{5/2}}\,dx = \int \frac{1}{(2\cos(c+d\,x)+3\sin(c+d\,x))^{5/2}}\,dx$$

input
```
int(1/(2*cos(c + d*x) + 3*sin(c + d*x))^(5/2),x)
```

output
```
int(1/(2*cos(c + d*x) + 3*sin(c + d*x))^(5/2), x)
```

3.247 $\int \frac{1}{(2\cos(c+dx)+3\sin(c+dx))^{7/2}}\,dx$

3.247.1 Optimal result

Integrand size = 21, antiderivative size = 120

$$\int \frac{1}{(2\cos(c+dx)+3\sin(c+dx))^{7/2}}\,dx = -\frac{6E\left(\frac{1}{2}\left(c+dx-\arctan\left(\frac{3}{2}\right)\right)\middle|2\right)}{65\ 13^{3/4}d} - \frac{2(3\cos(c+dx)-2\sin(c+dx))}{65d(2\cos(c+dx)+3\sin(c+dx))^{5/2}} - \frac{6(3\cos(c+dx)-2\sin(c+dx))}{845d\sqrt{2\cos(c+dx)+3\sin(c+dx)}}$$

output
```
-6/845*13^(1/4)*(cos(1/2*c+1/2*d*x-1/2*arctan(3/2))^2)^(1/2)/cos(1/2*c+1/2
*d*x-1/2*arctan(3/2))*EllipticE(sin(1/2*c+1/2*d*x-1/2*arctan(3/2)),2^(1/2)
)/d-2/65*(3*cos(d*x+c)-2*sin(d*x+c))/d/(2*cos(d*x+c)+3*sin(d*x+c))^(5/2)-6
/845*(3*cos(d*x+c)-2*sin(d*x+c))/d/(2*cos(d*x+c)+3*sin(d*x+c))^(1/2)
```

3.247.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.51 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.87

$$\int \frac{1}{(2\cos(c+dx)+3\sin(c+dx))^{7/2}}\,dx = \frac{\frac{4\sqrt{\cos\left(c+dx-\arctan\left(\frac{3}{2}\right)\right)}}{13^{3/4}} + \frac{-33\cos(c+dx)+5\cos(3(c+dx))-4(\sin(c+dx)+3\sin(3(c+\ldots}{2(2\cos(c+dx)+3\sin(c+dx))^{5/2}}}{\ldots}$$

input
```
Integrate[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(-7/2),x]
```

output
```
((4*Sqrt[Cos[c + d*x - ArcTan[3/2]]])/13^(3/4) + (-33*Cos[c + d*x] + 5*Cos
[3*(c + d*x)] - 4*(Sin[c + d*x] + 3*Sin[3*(c + d*x)]))/(2*(2*Cos[c + d*x]
+ 3*Sin[c + d*x])^(5/2)) - (3*Sin[c + d*x - ArcTan[3/2]])/(13^(3/4)*Sqrt[C
os[c + d*x - ArcTan[3/2]]]) + (3*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Co
s[c + d*x - ArcTan[3/2]]^2]*Sin[c + d*x - ArcTan[3/2]])/(13^(3/4)*Sqrt[-((
-1 + Cos[c + d*x - ArcTan[3/2]])*Cos[c + d*x - ArcTan[3/2]])]*Sqrt[1 + Cos
[c + d*x - ArcTan[3/2]]]))/(65*d)
```

3.247.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 3555, 3042, 3555, 3042, 3556, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3\sin(c+dx)+2\cos(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(3\sin(c+dx)+2\cos(c+dx))^{7/2}}dx$$

$$\Big\downarrow \text{3555}$$

$$\frac{3}{65}\int \frac{1}{(2\cos(c+dx)+3\sin(c+dx))^{3/2}}dx-\frac{2(3\cos(c+dx)-2\sin(c+dx))}{65d(3\sin(c+dx)+2\cos(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{65}\int \frac{1}{(2\cos(c+dx)+3\sin(c+dx))^{3/2}}dx-\frac{2(3\cos(c+dx)-2\sin(c+dx))}{65d(3\sin(c+dx)+2\cos(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3555}$$

$$\frac{3}{65}\left(-\frac{1}{13}\int \sqrt{2\cos(c+dx)+3\sin(c+dx)}dx-\frac{2(3\cos(c+dx)-2\sin(c+dx))}{13d\sqrt{3\sin(c+dx)+2\cos(c+dx)}}\right)-\frac{2(3\cos(c+dx)-2\sin(c+dx))}{65d(3\sin(c+dx)+2\cos(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{65}\left(-\frac{1}{13}\int \sqrt{2\cos(c+dx)+3\sin(c+dx)}dx-\frac{2(3\cos(c+dx)-2\sin(c+dx))}{13d\sqrt{3\sin(c+dx)+2\cos(c+dx)}}\right)-\frac{2(3\cos(c+dx)-2\sin(c+dx))}{65d(3\sin(c+dx)+2\cos(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3556}$$

$$\frac{3}{65}\left(-\frac{\int \sqrt{\cos\left(c+dx-\arctan\left(\frac{3}{2}\right)\right)}dx}{13^{3/4}}-\frac{2(3\cos(c+dx)-2\sin(c+dx))}{13d\sqrt{3\sin(c+dx)+2\cos(c+dx)}}\right)-\frac{2(3\cos(c+dx)-2\sin(c+dx))}{65d(3\sin(c+dx)+2\cos(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{65}\left(-\frac{\int \sqrt{\sin\left(c+dx-\arctan\left(\frac{3}{2}\right)+\frac{\pi}{2}\right)}dx}{13^{3/4}}-\frac{2(3\cos(c+dx)-2\sin(c+dx))}{13d\sqrt{3\sin(c+dx)+2\cos(c+dx)}}\right)-\frac{2(3\cos(c+dx)-2\sin(c+dx))}{65d(3\sin(c+dx)+2\cos(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{3}{65}\left(-\frac{2E\left(\frac{1}{2}\left(c+dx-\arctan\left(\frac{3}{2}\right)\right)\middle|2\right)}{13^{3/4}d}-\frac{2(3\cos(c+dx)-2\sin(c+dx))}{13d\sqrt{3\sin(c+dx)+2\cos(c+dx)}}\right)-\frac{2(3\cos(c+dx)-2\sin(c+dx))}{65d(3\sin(c+dx)+2\cos(c+dx))^{5/2}}$$

input
```
Int[(2*Cos[c + d*x] + 3*Sin[c + d*x])^(-7/2),x]
```

output
```
(-2*(3*Cos[c + d*x] - 2*Sin[c + d*x]))/(65*d*(2*Cos[c + d*x] + 3*Sin[c + d
*x])^(5/2)) + (3*((-2*EllipticE[(c + d*x - ArcTan[3/2])/2, 2])/(13^(3/4)*d
) - (2*(3*Cos[c + d*x] - 2*Sin[c + d*x]))/(13*d*Sqrt[2*Cos[c + d*x] + 3*Si
n[c + d*x]])))/65
```

3.247.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.247. $\int \frac{1}{(2\cos(c+dx)+3\sin(c+dx))^{7/2}}\,dx$

rule 3555
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(b*Cos[c + d*x] - a*Sin[c + d*x])*((a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Simp[(n + 2)/((n + 1)*(a^
2 + b^2))   Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

rule 3556
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a^2 + b^2)^(n/2)   Int[Cos[c + d*x - ArcTan[a, b]]^n, x],
 x] /; FreeQ[{a, b, c, d, n}, x] &&  !(GeQ[n, 1] || LeQ[n, -1]) && GtQ[a^2
+ b^2, 0]
```

3.247.4 Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.71

method	result
default	$\frac{\sqrt{13}\left(6\sqrt{\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)+1}\,\sqrt{-2\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)+2}\,\sqrt{-\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)}\,\sin\left(dx+c+\arctan\left(\frac{2}{3}\right)\right)^{2}\operatorname{EllipticE}\left(\sqrt{\right.\right.}{}$

input
```
int(1/(2*cos(d*x+c)+3*sin(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/845*13^(1/2)/sin(d*x+c+arctan(2/3))^2*(6*(sin(d*x+c+arctan(2/3))+1)^(1/2
)*(-2*sin(d*x+c+arctan(2/3))+2)^(1/2)*(-sin(d*x+c+arctan(2/3)))^(1/2)*sin(
d*x+c+arctan(2/3))^2*EllipticE((sin(d*x+c+arctan(2/3))+1)^(1/2),1/2*2^(1/2
))-3*(sin(d*x+c+arctan(2/3))+1)^(1/2)*(-2*sin(d*x+c+arctan(2/3))+2)^(1/2)*
(-sin(d*x+c+arctan(2/3)))^(1/2)*sin(d*x+c+arctan(2/3))^2*EllipticF((sin(d*
x+c+arctan(2/3))+1)^(1/2),1/2*2^(1/2))+6*sin(d*x+c+arctan(2/3))^4-4*sin(d*
x+c+arctan(2/3))^2-2)/cos(d*x+c+arctan(2/3))/(13^(1/2)*sin(d*x+c+arctan(2/
3)))^(1/2)/d
```

3.247.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 264, normalized size of antiderivative = 2.20

$$\int \frac{1}{(2\cos(c+dx)+3\sin(c+dx))^{7/2}}\,dx = \frac{3\sqrt{3i+2}\left(46i\sqrt{2}\cos\left(dx+c\right)^{3}+9\left(-i\sqrt{2}\cos\left(dx+c\right)^{2}-3i\sqrt{2}\right)\right.}{}$$

input
```
integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

output
```
1/845*(3*sqrt(3*I + 2)*(46*I*sqrt(2)*cos(d*x + c)^3 + 9*(-I*sqrt(2)*cos(d*
x + c)^2 - 3*I*sqrt(2))*sin(d*x + c) - 54*I*sqrt(2)*cos(d*x + c))*weierstr
assZeta(48/13*I + 20/13, 0, weierstrassPInverse(48/13*I + 20/13, 0, cos(d*
x + c) - I*sin(d*x + c))) + 3*sqrt(-3*I + 2)*(-46*I*sqrt(2)*cos(d*x + c)^3
 + 9*(I*sqrt(2)*cos(d*x + c)^2 + 3*I*sqrt(2))*sin(d*x + c) + 54*I*sqrt(2)*
cos(d*x + c))*weierstrassZeta(-48/13*I + 20/13, 0, weierstrassPInverse(-48
/13*I + 20/13, 0, cos(d*x + c) + I*sin(d*x + c))) + 2*(27*cos(d*x + c)^3 +
 2*(69*cos(d*x + c)^2 - 40)*sin(d*x + c) + 48*cos(d*x + c))*sqrt(2*cos(d*x
 + c) + 3*sin(d*x + c)))/(46*d*cos(d*x + c)^3 - 54*d*cos(d*x + c) - 9*(d*c
os(d*x + c)^2 + 3*d)*sin(d*x + c))
```

3.247.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(2\cos(c+dx)+3\sin(c+dx))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))**(7/2),x)
```

output
```
Timed out
```

3.247.7 Maxima [F]

$$\int \frac{1}{(2\cos(c+dx)+3\sin(c+dx))^{7/2}}\,dx = \int \frac{1}{(2\cos(dx+c)+3\sin(dx+c))^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

output
```
integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(-7/2), x)
```

3.247.8 Giac [F]

$$\int \frac{1}{(2\cos(c+dx)+3\sin(c+dx))^{7/2}}\,dx = \int \frac{1}{(2\cos(dx+c)+3\sin(dx+c))^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(2*cos(d*x+c)+3*sin(d*x+c))^(7/2),x, algorithm="giac")
```

output
```
integrate((2*cos(d*x + c) + 3*sin(d*x + c))^(-7/2), x)
```

3.247.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2\cos(c+dx)+3\sin(c+dx))^{7/2}}\,dx = \int \frac{1}{(2\cos(c+d\,x)+3\sin(c+d\,x))^{7/2}}\,dx$$

input
```
int(1/(2*cos(c + d*x) + 3*sin(c + d*x))^(7/2),x)
```

output
```
int(1/(2*cos(c + d*x) + 3*sin(c + d*x))^(7/2), x)
```

3.248 $\int (a\cos(c+dx)+ia\sin(c+dx))^n\,dx$

3.248.1 Optimal result

Integrand size = 22, antiderivative size = 32

$$\int (a\cos(c+dx)+ia\sin(c+dx))^n\,dx=-\frac{i(a\cos(c+dx)+ia\sin(c+dx))^n}{dn}$$

output
```
-I*(a*cos(d*x+c)+I*a*sin(d*x+c))^n/d/n
```

3.248.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int (a\cos(c+dx)+ia\sin(c+dx))^n\,dx=-\frac{i(a(\cos(c+dx)+i\sin(c+dx)))^n}{dn}$$

input
```
Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^n,x]
```

output
```
((-I)*(a*(Cos[c + d*x] + I*Sin[c + d*x]))^n)/(d*n)
```

3.248.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3042, 3550}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(c+dx)+ia\sin(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\cos(c+dx)+ia\sin(c+dx))^n dx$$

$$\Big\downarrow \text{3550}$$

$$-\frac{i(a\cos(c+dx)+ia\sin(c+dx))^n}{dn}$$

input
```
Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^n,x]
```

output
```
((-I)*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^n)/(d*n)
```

3.248.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3550
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

3.248.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

method	result	size
derivativedivides	$-\frac{i(\cos(dx+c)a+ia\sin(dx+c))^n}{dn}$	31
default	$-\frac{i(\cos(dx+c)a+ia\sin(dx+c))^n}{dn}$	31
norman	$-\frac{i\,\mathrm{e}^{n\ln\left(\frac{\left(1-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)a}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{2ia\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\right)}}{nd}$	75
risch	$-\frac{ia^n\left(\mathrm{e}^{i(dx+c)}\right)^n\mathrm{e}^{-\frac{i\,\mathrm{csgn}\left(ia\,\mathrm{e}^{i(dx+c)}\right)\pi n\left(-\mathrm{csgn}\left(ia\,\mathrm{e}^{i(dx+c)}\right)+\mathrm{csgn}\left(i\mathrm{e}^{i(dx+c)}\right)\right)\left(-\mathrm{csgn}\left(ia\,\mathrm{e}^{i(dx+c)}\right)+\mathrm{csgn}(ia)\right)}{2}}}{nd}$	96

input
```
int((cos(d*x+c)*a+I*a*sin(d*x+c))^n,x,method=_RETURNVERBOSE)
```

output
```
-I*(cos(d*x+c)*a+I*a*sin(d*x+c))^n/d/n
```

3.248.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.72

$$\int (a\cos(c+dx)+ia\sin(c+dx))^n\,dx = -\frac{i\,e^{(i\,dnx+i\,cn+n\log(a))}}{dn}$$

input
```
integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^n,x, algorithm="fricas")
```

output
```
-I*e^(I*d*n*x + I*c*n + n*log(a))/(d*n)
```

3.248.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.31

$$\int (a\cos(c+dx)+ia\sin(c+dx))^n\,dx = \begin{cases} x & \text{for } d=0\wedge n=0 \\ x(ia\sin(c)+a\cos(c))^n & \text{for } d=0 \\ x & \text{for } n=0 \\ -\frac{i(ia\sin(c+dx)+a\cos(c+dx))^n}{dn} & \text{otherwise} \end{cases}$$

input
```
integrate((a*cos(d*x+c)+I*a*sin(d*x+c))**n,x)
```

output
```
Piecewise((x, Eq(d, 0) & Eq(n, 0)), (x*(I*a*sin(c) + a*cos(c))**n, Eq(d, 0
)), (x, Eq(n, 0)), (-I*(I*a*sin(c + d*x) + a*cos(c + d*x))**n/(d*n), True)
)
```

3.248.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(28) = 56.

Time = 0.33 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.84

$$\int (a\cos(c+dx)+ia\sin(c+dx))^n\,dx = -\frac{i\,a^n e^{\left(-n\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+i\right)+n\log\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+i\right)\right)}}{dn}$$

input
```
integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^n,x, algorithm="maxima")
```

output
```
-I*a^n*e^(-n*log(sin(d*x + c)/(cos(d*x + c) + 1) + I) + n*log(-sin(d*x + c
)/(cos(d*x + c) + 1) + I))/(d*n)
```

3.248.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.72

$$\int (a\cos(c+dx)+ia\sin(c+dx))^n\,dx = -\frac{i\,e^{(i\,dnx+i\,cn+n\log(a))}}{dn}$$

input
```
integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^n,x, algorithm="giac")
```

output
```
-I*e^(I*d*n*x + I*c*n + n*log(a))/(d*n)
```

3.248.9 Mupad [F(-1)]

Timed out.

$$\int (a\cos(c+dx)+ia\sin(c+dx))^n\,dx=\int (a\ \cos\left(c+d\,x\right)+a\ \sin\left(c+d\,x\right)\ 1\mathrm{i})^n\,dx$$

input
```
int((a*cos(c + d*x) + a*sin(c + d*x)*1i)^n,x)
```

output
```
int((a*cos(c + d*x) + a*sin(c + d*x)*1i)^n, x)
```

3.249 $\int (a\cos(c+dx)+ia\sin(c+dx))^4\,dx$

3.249.1 Optimal result

Integrand size = 22, antiderivative size = 31

$$\int (a\cos(c+dx)+ia\sin(c+dx))^4\,dx=-\frac{i(a\cos(c+dx)+ia\sin(c+dx))^4}{4d}$$

output
```
-1/4*I*(a*cos(d*x+c)+I*a*sin(d*x+c))^4/d
```

3.249.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int (a\cos(c+dx)+ia\sin(c+dx))^4\,dx=-\frac{i(a\cos(c+dx)+ia\sin(c+dx))^4}{4d}$$

input
```
Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^4,x]
```

output
```
((-1/4*I)*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^4)/d
```

3.249.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3042, 3550}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(c+dx)+ia\sin(c+dx))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\cos(c+dx)+ia\sin(c+dx))^4dx$$

$$\Big\downarrow \text{3550}$$

$$-\frac{i(a\cos(c+dx)+ia\sin(c+dx))^4}{4d}$$

input
```
Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^4,x]
```

output
```
((-1/4*I)*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^4)/d
```

3.249.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3550
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

3.249.4 Maple [A] (verified)

Time = 1.95 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

method	result
risch	$-\frac{i a^{4} {\mathrm e}^{4 i (d x +c)}}{4 d}$
parallelrisch	$-\frac{a^{4} (i \cos (4 d x +4 c)-i-\sin (4 d x +4 c))}{4 d}$
derivativedivides	$\frac{a^{4} \left(-\frac{\left(\sin (d x +c)^{3}+\frac{3 \sin (d x +c)}{2}\right) \cos (d x +c)}{4}+\frac{3 d x}{8}+\frac{3 c}{8}\right)-i a^{4} \sin (d x +c)^{4}-6 a^{4} \left(-\frac{\sin (d x +c) \cos (d x +c)^{3}}{4}+\frac{\sin (d x +c) \cos (d x +c}{8}\right.}{d}$
default	$\frac{a^{4} \left(-\frac{\left(\sin (d x +c)^{3}+\frac{3 \sin (d x +c)}{2}\right) \cos (d x +c)}{4}+\frac{3 d x}{8}+\frac{3 c}{8}\right)-i a^{4} \sin (d x +c)^{4}-6 a^{4} \left(-\frac{\sin (d x +c) \cos (d x +c)^{3}}{4}+\frac{\sin (d x +c) \cos (d x +c}{8}\right.}{d}$
norman	$\frac{\frac{2 a^{4} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)}{d}-\frac{14 a^{4} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{3}}{d}+\frac{14 a^{4} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{5}}{d}-\frac{2 a^{4} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{7}}{d}-\frac{16 i a^{4} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{4}}{d}+\frac{8 i a^{4} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}}{d}+\frac{8 i a^{4}}{}}{\left(1+\tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}\right)^{4}}$
parts	$\frac{a^{4} \left(-\frac{\left(\sin (d x +c)^{3}+\frac{3 \sin (d x +c)}{2}\right) \cos (d x +c)}{4}+\frac{3 d x}{8}+\frac{3 c}{8}\right)}{d}+\frac{a^{4} \left(\frac{\left(\cos (d x +c)^{3}+\frac{3 \cos (d x +c)}{2}\right) \sin (d x +c)}{4}+\frac{3 d x}{8}+\frac{3 c}{8}\right)}{d}-\frac{i a^{4}}{}$

input
```
int((cos(d*x+c)*a+I*a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
-1/4*I*a^4/d*exp(4*I*(d*x+c))
```

3.249.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.55

$$\int (a\cos(c+dx)+ia\sin(c+dx))^4\,dx = -\frac{i\,a^4e^{(4i\,dx+4i\,c)}}{4\,d}$$

input
```
integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^4,x, algorithm="fricas")
```

output
```
-1/4*I*a^4*e^(4*I*d*x + 4*I*c)/d
```

3.249.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int (a\cos(c+dx)+ia\sin(c+dx))^4\,dx=\begin{cases}-\frac{ia^4e^{4ic}e^{4idx}}{4d} & \text{for } d\neq 0\\ a^4xe^{4ic} & \text{otherwise}\end{cases}$$

input
```
integrate((a*cos(d*x+c)+I*a*sin(d*x+c))**4,x)
```

output
```
Piecewise((-I*a**4*exp(4*I*c)*exp(4*I*d*x)/(4*d), Ne(d, 0)), (a**4*x*exp(4
*I*c), True))
```

3.249.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(25) = 50.

Time = 0.23 (sec) , antiderivative size = 132, normalized size of antiderivative = 4.26

$$\begin{aligned}\int (a\cos(c+dx)+ia\sin(c+dx))^4\,dx\\ =-\frac{i\,a^4\cos(dx+c)^4}{d}-\frac{i\,a^4\sin(dx+c)^4}{d}\\ +\frac{(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))a^4}{32\,d}\\ +\frac{(12\,dx+12\,c+\sin(4\,dx+4\,c)-8\,\sin(2\,dx+2\,c))a^4}{32\,d}\\ -\frac{3\,(4\,dx+4\,c-\sin(4\,dx+4\,c))a^4}{16\,d}\end{aligned}$$

input
```
integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^4,x, algorithm="maxima")
```

output
```
-I*a^4*cos(d*x + c)^4/d - I*a^4*sin(d*x + c)^4/d + 1/32*(12*d*x + 12*c + s
in(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^4/d + 1/32*(12*d*x + 12*c + sin(4*
d*x + 4*c) - 8*sin(2*d*x + 2*c))*a^4/d - 3/16*(4*d*x + 4*c - sin(4*d*x + 4
*c))*a^4/d
```

3.249.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(25) = 50$.

Time = 0.31 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.68

$$\int (a\cos(c+dx)+ia\sin(c+dx))^4\,dx = -\frac{i\,a^4e^{(4i\,dx+4i\,c)}}{8\,d} - \frac{i\,a^4e^{(-4i\,dx-4i\,c)}}{8\,d} + \frac{a^4\sin\left(4\,dx+4\,c\right)}{4\,d}$$

input
```
integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/8*I*a^4*e^(4*I*d*x + 4*I*c)/d - 1/8*I*a^4*e^(-4*I*d*x - 4*I*c)/d + 1/4*
a^4*sin(4*d*x + 4*c)/d
```

3.249.9 Mupad [B] (verification not implemented)

Time = 27.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.71

$$\int (a\cos(c+dx)+ia\sin(c+dx))^4\,dx$$
$$= -\frac{2\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,4\mathrm{i}-6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,4\mathrm{i}+1\right)}$$

input
```
int((a*cos(c + d*x) + a*sin(c + d*x)*1i)^4,x)
```

output
```
-(2*a^4*tan(c/2 + (d*x)/2)*(tan(c/2 + (d*x)/2)^2 - 1))/(d*(tan(c/2 + (d*x)
/2)^3*4i - 6*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)*4i + tan(c/2 + (d*x
)/2)^4 + 1))
```

3.250 $\int (a\cos(c+dx)+ia\sin(c+dx))^3\,dx$

3.250.1 Optimal result

Integrand size = 22, antiderivative size = 31

$$\int (a\cos(c+dx)+ia\sin(c+dx))^3\,dx=-\frac{i(a\cos(c+dx)+ia\sin(c+dx))^3}{3d}$$

output
```
-1/3*I*(a*cos(d*x+c)+I*a*sin(d*x+c))^3/d
```

3.250.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int (a\cos(c+dx)+ia\sin(c+dx))^3\,dx=-\frac{i(a\cos(c+dx)+ia\sin(c+dx))^3}{3d}$$

input
```
Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

output
```
((-1/3*I)*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3)/d
```

3.250.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3042, 3550}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(c+dx)+ia\sin(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\cos(c+dx)+ia\sin(c+dx))^3dx$$

$$\Big\downarrow \text{3550}$$

$$-\frac{i(a\cos(c+dx)+ia\sin(c+dx))^3}{3d}$$

input
```
Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

output
```
((-1/3*I)*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3)/d
```

3.250.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3550
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

3.250.4 Maple [A] (verified)

Time = 1.93 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

method	result	size
risch	$-\frac{i a^{3} {\mathrm e}^{3 i \left(d x +c \right)}}{3 d}$	19
parallelrisch	$-\frac{a^{3} \left(i \cos \left(3 d x +3 c \right)+i-\sin \left(3 d x +3 c \right)\right)}{3 d}$	35
derivativedivides	$\frac{\frac{i a^{3} \left(2+\sin \left(d x +c \right)^{2}\right) \cos \left(d x +c \right)}{3}-a^{3} \sin \left(d x +c \right)^{3}-i a^{3} \cos \left(d x +c \right)^{3}+\frac{a^{3} \left(2+\cos \left(d x +c \right)^{2}\right) \sin \left(d x +c \right)}{3}}{d}$	76
default	$\frac{\frac{i a^{3} \left(2+\sin \left(d x +c \right)^{2}\right) \cos \left(d x +c \right)}{3}-a^{3} \sin \left(d x +c \right)^{3}-i a^{3} \cos \left(d x +c \right)^{3}+\frac{a^{3} \left(2+\cos \left(d x +c \right)^{2}\right) \sin \left(d x +c \right)}{3}}{d}$	76
parts	$\frac{a^{3} \left(2+\cos \left(d x +c \right)^{2}\right) \sin \left(d x +c \right)}{3 d}+\frac{i a^{3} \left(2+\sin \left(d x +c \right)^{2}\right) \cos \left(d x +c \right)}{3 d}-\frac{i a^{3} \cos \left(d x +c \right)^{3}}{d}-\frac{a^{3} \sin \left(d x +c \right)^{3}}{d}$	84
norman	$\frac{\frac{4 i a^{3} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}}{d}-\frac{2 i a^{3}}{3 d}+\frac{2 a^{3} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)}{d}-\frac{20 a^{3} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{3}}{3 d}+\frac{2 a^{3} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{5}}{d}-\frac{6 i a^{3} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{4}}{d}}{\left(1+\tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}\right)^{3}}$	122

input
```
int((cos(d*x+c)*a+I*a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3*I*a^3/d*exp(3*I*(d*x+c))
```

3.250.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.55

$$\int (a\cos(c+dx)+ia\sin(c+dx))^3\,dx = -\frac{i\,a^3e^{(3i\,dx+3i\,c)}}{3\,d}$$

input
```
integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/3*I*a^3*e^(3*I*d*x + 3*I*c)/d
```

3.250.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int (a\cos(c+dx)+ia\sin(c+dx))^3\,dx = \begin{cases} -\frac{ia^3e^{3ic}e^{3idx}}{3d} & \text{for } d\neq 0 \\ a^3xe^{3ic} & \text{otherwise} \end{cases}$$

input
```
integrate((a*cos(d*x+c)+I*a*sin(d*x+c))**3,x)
```

output
```
Piecewise((-I*a**3*exp(3*I*c)*exp(3*I*d*x)/(3*d), Ne(d, 0)), (a**3*x*exp(3
*I*c), True))
```

3.250.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(25) = 50.

Time = 0.23 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.68

$$\begin{aligned}\int (a\cos(c+dx)+ia\sin(c+dx))^3\,dx = & -\frac{i\,a^3\cos(dx+c)^3}{d}-\frac{a^3\sin(dx+c)^3}{d} \\ & -\frac{i\left(\cos(dx+c)^3-3\,\cos(dx+c)\right)a^3}{3\,d} \\ & -\frac{\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)a^3}{3\,d}\end{aligned}$$

input
```
integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
-I*a^3*cos(d*x + c)^3/d - a^3*sin(d*x + c)^3/d - 1/3*I*(cos(d*x + c)^3 - 3
*cos(d*x + c))*a^3/d - 1/3*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^3/d
```

3.250.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(25) = 50$.

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.68

$$\int (a\cos(c+dx)+ia\sin(c+dx))^3\,dx = -\frac{i\,a^3e^{(3i\,dx+3i\,c)}}{6\,d} - \frac{i\,a^3e^{(-3i\,dx-3i\,c)}}{6\,d} + \frac{a^3\sin(3\,dx+3\,c)}{3\,d}$$

input
```
integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/6*I*a^3*e^(3*I*d*x + 3*I*c)/d - 1/6*I*a^3*e^(-3*I*d*x - 3*I*c)/d + 1/3*
a^3*sin(3*d*x + 3*c)/d
```

3.250.9 Mupad [B] (verification not implemented)

Time = 26.30 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.13

$$\int (a\cos(c+dx)+ia\sin(c+dx))^3\,dx = -\frac{2\,a^3\left(3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}{3\,d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,3\mathrm{i}+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)}$$

input
```
int((a*cos(c + d*x) + a*sin(c + d*x)*1i)^3,x)
```

output
```
-(2*a^3*(3*tan(c/2 + (d*x)/2)^2 - 1))/(3*d*(3*tan(c/2 + (d*x)/2) - tan(c/2
 + (d*x)/2)^2*3i - tan(c/2 + (d*x)/2)^3 + 1i))
```

3.251 $\int (a\cos(c+dx)+ia\sin(c+dx))^2\,dx$

3.251.1 Optimal result

Integrand size = 22, antiderivative size = 31

$$\int (a\cos(c+dx)+ia\sin(c+dx))^2\,dx=-\frac{i(a\cos(c+dx)+ia\sin(c+dx))^2}{2d}$$

output
```
-1/2*I*(a*cos(d*x+c)+I*a*sin(d*x+c))^2/d
```

3.251.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int (a\cos(c+dx)+ia\sin(c+dx))^2\,dx=-\frac{i(a\cos(c+dx)+ia\sin(c+dx))^2}{2d}$$

input
```
Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

output
```
((-1/2*I)*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2)/d
```

3.251.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3042, 3550}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(c+dx)+ia\sin(c+dx))^2\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (a\cos(c+dx)+ia\sin(c+dx))^2dx$$

$$\Big\downarrow \; 3550$$

$$-\frac{i(a\cos(c+dx)+ia\sin(c+dx))^2}{2d}$$

input
```
Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

output
```
((-1/2*I)*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2)/d
```

3.251.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3550
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

3.251.4 Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

method	result	size
risch	$-\frac{i a^{2} {\mathrm e}^{2 i \left(d x +c \right)}}{2 d}$	19
parallelrisch	$\frac{2 a^{2} \left(2 i \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}-\tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{3}+\tan \left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{d \left(1+\tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}\right)^{2}}$	61
derivativedivides	$\frac{-a^{2} \left(-\frac{\sin \left(d x +c \right) \cos \left(d x +c \right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)-i a^{2} \cos \left(d x +c \right)^{2}+a^{2} \left(\frac{\sin \left(d x +c \right) \cos \left(d x +c \right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)}{d}$	73
default	$\frac{-a^{2} \left(-\frac{\sin \left(d x +c \right) \cos \left(d x +c \right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)-i a^{2} \cos \left(d x +c \right)^{2}+a^{2} \left(\frac{\sin \left(d x +c \right) \cos \left(d x +c \right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)}{d}$	73
norman	$\frac{\frac{2 a^{2} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)}{d}-\frac{2 a^{2} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{3}}{d}+\frac{4 i a^{2} \tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}}{d}}{\left(1+\tan \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}\right)^{2}}$	74
parts	$\frac{a^{2} \left(\frac{\sin \left(d x +c \right) \cos \left(d x +c \right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)}{d}+\frac{i a^{2} \sin \left(d x +c \right)^{2}}{d}-\frac{a^{2} \left(-\frac{\sin \left(d x +c \right) \cos \left(d x +c \right)}{2}+\frac{d x}{2}+\frac{c}{2}\right)}{d}$	78

input
```
int((cos(d*x+c)*a+I*a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*a^2/d*exp(2*I*(d*x+c))
```

3.251.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.55

$$\int (a\cos(c+dx)+ia\sin(c+dx))^2\,dx=-\frac{i\,a^2e^{(2i\,dx+2i\,c)}}{2\,d}$$

input
```
integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/2*I*a^2*e^(2*I*d*x + 2*I*c)/d
```

3.251.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int (a\cos(c+dx)+ia\sin(c+dx))^2\,dx = \begin{cases} -\frac{ia^2e^{2ic}e^{2idx}}{2d} & \text{for } d\neq 0\\ a^2xe^{2ic} & \text{otherwise}\end{cases}$$

input
```
integrate((a*cos(d*x+c)+I*a*sin(d*x+c))**2,x)
```

output
```
Piecewise((-I*a**2*exp(2*I*c)*exp(2*I*d*x)/(2*d), Ne(d, 0)), (a**2*x*exp(2
*I*c), True))
```

3.251.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(25) = 50.

Time = 0.21 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.23

$$\int (a\cos(c+dx)+ia\sin(c+dx))^2\,dx = -\frac{i\,a^2\cos(dx+c)^2}{d} + \frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))a^2}{4\,d} - \frac{(2\,dx+2\,c-\sin(2\,dx+2\,c))a^2}{4\,d}$$

input
```
integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
-I*a^2*cos(d*x + c)^2/d + 1/4*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^2/d - 1/4
*(2*d*x + 2*c - sin(2*d*x + 2*c))*a^2/d
```

3.251.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 52 vs. 2(25) = 50.

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.68

$$\int (a\cos(c+dx)+ia\sin(c+dx))^2\,dx = -\frac{i\,a^2e^{(2i\,dx+2i\,c)}}{4\,d}-\frac{i\,a^2e^{(-2i\,dx-2i\,c)}}{4\,d}+\frac{a^2\sin\left(2\,dx+2\,c\right)}{2\,d}$$

input
```
integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/4*I*a^2*e^(2*I*d*x + 2*I*c)/d - 1/4*I*a^2*e^(-2*I*d*x - 2*I*c)/d + 1/2*
a^2*sin(2*d*x + 2*c)/d
```

3.251.9 Mupad [B] (verification not implemented)

Time = 26.86 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.42

$$\int (a\cos(c+dx)+ia\sin(c+dx))^2\,dx = -\frac{2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,2\mathrm{i}-1\right)}$$

input
```
int((a*cos(c + d*x) + a*sin(c + d*x)*1i)^2,x)
```

output
```
-(2*a^2*tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)/2)*2i + tan(c/2 + (d*x)/2)
^2 - 1))
```

3.252 $\int (a\cos(c+dx)+ia\sin(c+dx))\,dx$

3.252.1 Optimal result

Integrand size = 20, antiderivative size = 26

$$\int (a\cos(c+dx)+ia\sin(c+dx))\,dx=-\frac{ia\cos(c+dx)}{d}+\frac{a\sin(c+dx)}{d}$$

output
```
-I*a*cos(d*x+c)/d+a*sin(d*x+c)/d
```

3.252.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.96

$$\int (a\cos(c+dx)+ia\sin(c+dx))\,dx=-\frac{ia\cos(c)\cos(dx)}{d}+\frac{a\cos(dx)\sin(c)}{d}+\frac{a\cos(c)\sin(dx)}{d}+\frac{ia\sin(c)\sin(dx)}{d}$$

input
```
Integrate[a*Cos[c + d*x] + I*a*Sin[c + d*x],x]
```

output
```
((-I)*a*Cos[c]*Cos[d*x])/d + (a*Cos[d*x]*Sin[c])/d + (a*Cos[c]*Sin[d*x])/d
 + (I*a*Sin[c]*Sin[d*x])/d
```

3.252.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(c+dx)+ia\sin(c+dx))\,dx$$
$$\Big\downarrow \ 2009$$
$$\frac{a\sin(c+dx)}{d}-\frac{ia\cos(c+dx)}{d}$$

input
```
Int[a*Cos[c + d*x] + I*a*Sin[c + d*x],x]
```

output
```
((-I)*a*Cos[c + d*x])/d + (a*Sin[c + d*x])/d
```

3.252.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.252.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

method	result	size
risch	$-\frac{ia\,e^{i(dx+c)}}{d}$	17
derivativedivides	$\frac{\sin(dx+c)a-i\cos(dx+c)a}{d}$	24
default	$-\frac{ia\cos(dx+c)}{d}+\frac{a\sin(dx+c)}{d}$	26
parts	$-\frac{ia\cos(dx+c)}{d}+\frac{a\sin(dx+c)}{d}$	26
parallelrisch	$-\frac{2a\left(i-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)}$	36
norman	$\frac{\frac{2ia\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{d}+\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}$	51
meijerg	$\frac{(i\sqrt{\pi}\,\sin(c)a+\sqrt{\pi}\,\cos(c)a)\sin(dx)}{\sqrt{\pi}\,d}+\frac{(i\sqrt{\pi}\,\cos(c)a-\sqrt{\pi}\,\sin(c)a)\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(dx)}{\sqrt{\pi}}\right)}{d}$	65

input
```
int(cos(d*x+c)*a+I*a*sin(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-I*a/d*exp(I*(d*x+c))
```

3.252.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.58

$$\int (a\cos(c+dx)+ia\sin(c+dx))\,dx=-\frac{i\,ae^{(i\,dx+i\,c)}}{d}$$

input
```
integrate(a*cos(d*x+c)+I*a*sin(d*x+c),x, algorithm="fricas")
```

output
```
-I*a*e^(I*d*x + I*c)/d
```

3.252.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int (a\cos(c+dx)+ia\sin(c+dx))\,dx = \begin{cases} -\frac{iae^{ic}e^{idx}}{d} & \text{for } d \neq 0 \\ axe^{ic} & \text{otherwise} \end{cases}$$

input
```
integrate(a*cos(d*x+c)+I*a*sin(d*x+c),x)
```

output
```
Piecewise((-I*a*exp(I*c)*exp(I*d*x)/d, Ne(d, 0)), (a*x*exp(I*c), True))
```

3.252.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int (a\cos(c+dx)+ia\sin(c+dx))\,dx = -\frac{i\,a\cos(dx+c)}{d}+\frac{a\sin(dx+c)}{d}$$

input
```
integrate(a*cos(d*x+c)+I*a*sin(d*x+c),x, algorithm="maxima")
```

output
```
-I*a*cos(d*x + c)/d + a*sin(d*x + c)/d
```

3.252.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int (a\cos(c+dx)+ia\sin(c+dx))\,dx = -\frac{i\,a\cos(dx+c)}{d}+\frac{a\sin(dx+c)}{d}$$

input
```
integrate(a*cos(d*x+c)+I*a*sin(d*x+c),x, algorithm="giac")
```

output
```
-I*a*cos(d*x + c)/d + a*sin(d*x + c)/d
```

3.252.9 Mupad [B] (verification not implemented)

Time = 26.90 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int (a\cos(c+dx)+ia\sin(c+dx))\,dx=\frac{2\,a}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)}$$

input
```
int(a*cos(c + d*x) + a*sin(c + d*x)*1i,x)
```

output
```
(2*a)/(d*(tan(c/2 + (d*x)/2) + 1i))
```

3.253 $\int \frac{1}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

3.253.1 Optimal result

Integrand size = 22, antiderivative size = 29

$$\int \frac{1}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \frac{i}{d(a\cos(c+dx)+ia\sin(c+dx))}$$

output
```
I/d/(a*cos(d*x+c)+I*a*sin(d*x+c))
```

3.253.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{1}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \frac{i}{d(a\cos(c+dx)+ia\sin(c+dx))}$$

input
```
Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-1),x]
```

output
```
I/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x]))
```

3.253.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3042, 3550}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{1}{a\cos(c+dx)+ia\sin(c+dx)}dx$$
$$\Big\downarrow \ 3550$$
$$\frac{i}{d(a\cos(c+dx)+ia\sin(c+dx))}$$

input
```
Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-1),x]
```

output
```
I/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x]))
```

3.253.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3550
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

3.253.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.66

method	result	size
risch	$\frac{i\mathrm{e}^{-i(dx+c)}}{ad}$	19
derivativedivides	$\frac{2}{da\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)}$	23
default	$\frac{2}{da\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)}$	23
norman	$\frac{-\frac{2i\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{ad}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}$	55

input
```
int(1/(cos(d*x+c)*a+I*a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
I/a/d*exp(-I*(d*x+c))
```

3.253.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.59

$$\int \frac{1}{a\cos(c+dx)+ia\sin(c+dx)}\,dx=\frac{i\,e^{(-i\,dx-i\,c)}}{ad}$$

input
```
integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
I*e^(-I*d*x - I*c)/(a*d)
```

3.253.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int \frac{1}{a\cos(c+dx)+ia\sin(c+dx)}\,dx=\begin{cases}\frac{ie^{-ic}e^{-idx}}{ad} & \text{for } ade^{ic}\neq 0\\ \frac{xe^{-ic}}{a} & \text{otherwise}\end{cases}$$

input
```
integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

output
```
Piecewise((I*exp(-I*c)*exp(-I*d*x)/(a*d), Ne(a*d*exp(I*c), 0)), (x*exp(-I*
c)/a, True))
```

3.253.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{1}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \frac{2}{\left(-i\,a+\frac{a\sin(dx+c)}{\cos(dx+c)+1}\right)d}$$

input
```
integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
2/((-I*a + a*sin(d*x + c)/(cos(d*x + c) + 1))*d)
```

3.253.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.72

$$\int \frac{1}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \frac{2}{ad\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)}$$

input
```
integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="giac")
```

output
```
2/(a*d*(tan(1/2*d*x + 1/2*c) - I))
```

3.253.9 Mupad [B] (verification not implemented)

Time = 26.92 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{1}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \frac{2\mathrm{i}}{a\,d\,\left(1+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}\right)}$$

input
```
int(1/(a*cos(c + d*x) + a*sin(c + d*x)*1i),x)
```

output
```
2i/(a*d*(tan(c/2 + (d*x)/2)*1i + 1))
```

3.254 $\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$

3.254.1 Optimal result

Integrand size = 22, antiderivative size = 31

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \frac{i}{2d(a\cos(c+dx)+ia\sin(c+dx))^2}$$

output
```
1/2*I/d/(a*cos(d*x+c)+I*a*sin(d*x+c))^2
```

3.254.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \frac{i}{2d(a\cos(c+dx)+ia\sin(c+dx))^2}$$

input
```
Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-2),x]
```

output
```
(I/2)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2)
```

3.254.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3042, 3550}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{ 3550}$$

$$\frac{i}{2d(a\cos(c+dx)+ia\sin(c+dx))^2}$$

input
```
Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-2),x]
```

output
```
(I/2)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2)
```

3.254.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3550
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

3.254.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

method	result	size
risch	$\frac{i\mathrm{e}^{-2i(dx+c)}}{2a^2d}$	19
derivativedivides	$\frac{i}{d\,a^2(i\tan(dx+c)+1)}$	23
default	$\frac{i}{d\,a^2(i\tan(dx+c)+1)}$	23
norman	$\frac{\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{ad}-\frac{4i\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{ad}}{a\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}$	77

input
```
int(1/(cos(d*x+c)*a+I*a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*I/a^2/d*exp(-2*I*(d*x+c))
```

3.254.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.55

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx=\frac{i\,e^{(-2i\,dx-2i\,c)}}{2\,a^2d}$$

input
```
integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/2*I*e^(-2*I*d*x - 2*I*c)/(a^2*d)
```

3.254.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.42

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx=\begin{cases}\frac{ie^{-2ic}e^{-2idx}}{2a^2d} & \text{for } a^2de^{2ic}\neq 0\\ \frac{xe^{-2ic}}{a^2} & \text{otherwise}\end{cases}$$

input
```
integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))**2,x)
```

output
```
Piecewise((I*exp(-2*I*c)*exp(-2*I*d*x)/(2*a**2*d), Ne(a**2*d*exp(2*I*c), 0
)), (x*exp(-2*I*c)/a**2, True))
```

3.254.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \frac{1}{(a^2\tan{(dx+c)}-i\,a^2)d}$$

input
```
integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/((a^2*tan(d*x + c) - I*a^2)*d)
```

3.254.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = -\frac{2\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^2}$$

input
```
integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
-2*tan(1/2*d*x + 1/2*c)/(a^2*d*(tan(1/2*d*x + 1/2*c) - I)^2)
```

3.254.9 Mupad [B] (verification not implemented)

Time = 27.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = -\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^2\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\mathrm{i}\right)^2}$$

input
```
int(1/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^2,x)
```

output
```
-(2*tan(c/2 + (d*x)/2))/(a^2*d*(tan(c/2 + (d*x)/2) - 1i)^2)
```

3.255 $\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$

3.255.1 Optimal result

Integrand size = 22, antiderivative size = 31

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{i}{3d(a\cos(c+dx)+ia\sin(c+dx))^3}$$

output
```
1/3*I/d/(a*cos(d*x+c)+I*a*sin(d*x+c))^3
```

3.255.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{i}{3d(a\cos(c+dx)+ia\sin(c+dx))^3}$$

input
```
Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-3),x]
```

output
```
(I/3)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3)
```

3.255.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3042, 3550}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^3}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^3}dx$$
$$\Big\downarrow \text{ 3550}$$
$$\frac{i}{3d(a\cos(c+dx)+ia\sin(c+dx))^3}$$

input
```
Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-3),x]
```

output
```
(I/3)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3)
```

3.255.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3550
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

3.255.4 Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

method	result	size
risch	$\frac{i\mathrm{e}^{-3i(dx+c)}}{3d\,a^{3}}$	19
derivativedivides	$\frac{\frac{4i}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^{2}}+\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i}-\frac{8}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^{3}}}{d\,a^{3}}$	57
default	$\frac{\frac{4i}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^{2}}+\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i}-\frac{8}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-i\right)^{3}}}{d\,a^{3}}$	57
norman	$\frac{-\frac{4i\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{ad}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{20\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{3ad}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{ad}+\frac{2i}{3ad}+\frac{6i\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{ad}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{3}a^{2}}$	125

input
```
int(1/(cos(d*x+c)*a+I*a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*I/d/a^3*exp(-3*I*(d*x+c))
```

3.255.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.55

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{i\,e^{(-3i\,dx-3i\,c)}}{3\,a^3d}$$

input
```
integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/3*I*e^(-3*I*d*x - 3*I*c)/(a^3*d)
```

3.255.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.42

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \begin{cases} \frac{ie^{-3ic}e^{-3idx}}{3a^3d} & \text{for}\ a^3de^{3ic}\neq 0 \\ \frac{xe^{-3ic}}{a^3} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))**3,x)
```

output
```
Piecewise((I*exp(-3*I*c)*exp(-3*I*d*x)/(3*a**3*d), Ne(a**3*d*exp(3*I*c), 0
)), (x*exp(-3*I*c)/a**3, True))
```

3.255.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{i\,\cos\left(3\,dx+3\,c\right)+\sin\left(3\,dx+3\,c\right)}{3\,a^3d}$$

input
```
integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/3*(I*cos(3*d*x + 3*c) + sin(3*d*x + 3*c))/(a^3*d)
```

3.255.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.16

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{2\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}{3\,a^3d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^3}$$

input
```
integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
2/3*(3*tan(1/2*d*x + 1/2*c)^2 - 1)/(a^3*d*(tan(1/2*d*x + 1/2*c) - I)^3)
```

3.255.9 Mupad [B] (verification not implemented)

Time = 28.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.19

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$$
$$= -\frac{2\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,3\mathrm{i}-\mathrm{i}\right)}{3\,a^3\,d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,1\mathrm{i}-3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,3\mathrm{i}+1\right)}$$

input
```
int(1/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^3,x)
```

output
```
-(2*(tan(c/2 + (d*x)/2)^2*3i - 1i))/(3*a^3*d*(tan(c/2 + (d*x)/2)*3i - 3*ta
n(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)^3*1i + 1))
```

3.255. $\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$

3.256 $\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^4}\,dx$

3.256.1 Optimal result

Integrand size = 22, antiderivative size = 31

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^4}\,dx = \frac{i}{4d(a\cos(c+dx)+ia\sin(c+dx))^4}$$

output
```
1/4*I/d/(a*cos(d*x+c)+I*a*sin(d*x+c))^4
```

3.256.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^4}\,dx = \frac{i}{4d(a\cos(c+dx)+ia\sin(c+dx))^4}$$

input
```
Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-4),x]
```

output
```
(I/4)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^4)
```

3.256.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3042, 3550}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^4}dx$$

$$\Big\downarrow \text{3550}$$

$$\frac{i}{4d(a\cos(c+dx)+ia\sin(c+dx))^4}$$

input
```
Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-4),x]
```

output
```
(I/4)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^4)
```

3.256.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3550
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

3.256.4 Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

method	result
risch	$\frac{i\mathrm{e}^{-4i(dx+c)}}{4d\,a^{4}}$
derivativedivides	$\frac{-\frac{1}{\tan(dx+c)-i}-\frac{i}{(\tan(dx+c)-i)^{2}}}{d\,a^{4}}$
default	$\frac{-\frac{1}{\tan(dx+c)-i}-\frac{i}{(\tan(dx+c)-i)^{2}}}{d\,a^{4}}$
norman	$\frac{\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{14\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{ad}+\frac{14\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{ad}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{ad}-\frac{8i\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{ad}-\frac{8i\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}}{ad}+\frac{16i\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{ad}}{a^{3}\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\right)^{4}}$

input
```
int(1/(cos(d*x+c)*a+I*a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/4*I/d/a^4*exp(-4*I*(d*x+c))
```

3.256.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.55

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^4}\,dx = \frac{i\,e^{(-4i\,dx-4i\,c)}}{4\,a^4d}$$

input
```
integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/4*I*e^(-4*I*d*x - 4*I*c)/(a^4*d)
```

3.256.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.42

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^4}\,dx = \begin{cases} \frac{ie^{-4ic}e^{-4idx}}{4a^4d} & \text{for } a^4de^{4ic}\neq 0 \\ \frac{xe^{-4ic}}{a^4} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))**4,x)
```

output
```
Piecewise((I*exp(-4*I*c)*exp(-4*I*d*x)/(4*a**4*d), Ne(a**4*d*exp(4*I*c), 0
)), (x*exp(-4*I*c)/a**4, True))
```

3.256.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^4}\,dx = \frac{i\,\cos\left(4\,dx+4\,c\right)+\sin\left(4\,dx+4\,c\right)}{4\,a^4d}$$

input
```
integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/4*(I*cos(4*d*x + 4*c) + sin(4*d*x + 4*c))/(a^4*d)
```

3.256.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.42

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^4}\,dx = -\frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a^4d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^4}$$

input
```
integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^4,x, algorithm="giac")
```

output
```
-2*(tan(1/2*d*x + 1/2*c)^3 - tan(1/2*d*x + 1/2*c))/(a^4*d*(tan(1/2*d*x + 1
/2*c) - I)^4)
```

3.256.9 Mupad [B] (verification not implemented)

Time = 27.70 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.94

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^4}\,dx$$
$$=-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,1\mathrm{i}-\mathrm{i}\right)}{a^4\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,1\mathrm{i}+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,6\mathrm{i}-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)}$$

input
```
int(1/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^4,x)
```

output
```
-(2*tan(c/2 + (d*x)/2)*(tan(c/2 + (d*x)/2)^2*1i - 1i))/(a^4*d*(4*tan(c/2 +
 (d*x)/2)^3 - tan(c/2 + (d*x)/2)^2*6i - 4*tan(c/2 + (d*x)/2) + tan(c/2 + (
d*x)/2)^4*1i + 1i))
```

3.257 $\int (a\cos(c+dx)+ia\sin(c+dx))^{5/2}\,dx$

3.257.1 Optimal result

Integrand size = 24, antiderivative size = 33

$$\int (a\cos(c+dx)+ia\sin(c+dx))^{5/2}\,dx=-\frac{2i(a\cos(c+dx)+ia\sin(c+dx))^{5/2}}{5d}$$

output
```
-2/5*I*(a*cos(d*x+c)+I*a*sin(d*x+c))^(5/2)/d
```

3.257.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int (a\cos(c+dx)+ia\sin(c+dx))^{5/2}\,dx=-\frac{2i(a(\cos(c+dx)+i\sin(c+dx)))^{5/2}}{5d}$$

input
```
Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(5/2),x]
```

output
```
(((-2*I)/5)*(a*(Cos[c + d*x] + I*Sin[c + d*x]))^(5/2))/d
```

3.257.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3042, 3550}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(c+dx)+ia\sin(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\cos(c+dx)+ia\sin(c+dx))^{5/2}dx$$

$$\Big\downarrow \text{3550}$$

$$-\frac{2i(a\cos(c+dx)+ia\sin(c+dx))^{5/2}}{5d}$$

input
```
Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(5/2),x]
```

output
```
(((-2*I)/5)*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(5/2))/d
```

3.257.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3550
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

3.257.4 Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$-\frac{2i(\cos(dx+c)a+ia\sin(dx+c))^{\frac{5}{2}}}{5d}$	28
default	$-\frac{2i(\cos(dx+c)a+ia\sin(dx+c))^{\frac{5}{2}}}{5d}$	28
risch	$-\frac{2ia^2\sqrt{a\,\mathrm{e}^{i(dx+c)}}\,\mathrm{e}^{2i(dx+c)}}{5d}$	32

input
```
int((cos(d*x+c)*a+I*a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*I*(cos(d*x+c)*a+I*a*sin(d*x+c))^(5/2)/d
```

3.257.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.52

$$\int (a\cos(c+dx)+ia\sin(c+dx))^{5/2}\,dx = -\frac{2i\,a^{\frac{5}{2}}e^{\left(\frac{5}{2}i\,dx+\frac{5}{2}i\,c\right)}}{5\,d}$$

input
```
integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-2/5*I*a^(5/2)*e^(5/2*I*d*x + 5/2*I*c)/d
```

3.257.6 Sympy [F(-1)]

Timed out.

$$\int (a\cos(c+dx)+ia\sin(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((a*cos(d*x+c)+I*a*sin(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.257.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(25) = 50$.

Time = 0.32 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.55

$$\int (a\cos(c+dx)+ia\sin(c+dx))^{5/2}\,dx = -\frac{2i\,a^{\frac{5}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+i\right)^{\frac{5}{2}}}{5\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+i\right)^{\frac{5}{2}}}$$

input
```
integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
-2/5*I*a^(5/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + I)^(5/2)/(d*(sin(d*x +
c)/(cos(d*x + c) + 1) + I)^(5/2))
```

3.257.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.52

$$\int (a\cos(c+dx)+ia\sin(c+dx))^{5/2}\,dx = -\frac{2i\,a^{\frac{5}{2}}e^{\left(\frac{5}{2}i\,dx+\frac{5}{2}i\,c\right)}}{5\,d}$$

input
```
integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
-2/5*I*a^(5/2)*e^(5/2*I*d*x + 5/2*I*c)/d
```

3.257.9 Mupad [B] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int (a\cos(c+dx)+ia\sin(c+dx))^{5/2}\,dx = -\frac{a^2\,\mathrm{e}^{c\,2\mathrm{i}}\,\mathrm{e}^{d\,x\,2\mathrm{i}}\,\sqrt{a\,\mathrm{e}^{c\,1\mathrm{i}}\,\mathrm{e}^{d\,x\,1\mathrm{i}}}\,2\mathrm{i}}{5\,d}$$

input
```
int((a*cos(c + d*x) + a*sin(c + d*x)*1i)^(5/2),x)
```

output
```
-(a^2*exp(c*2i)*exp(d*x*2i)*(a*exp(c*1i)*exp(d*x*1i))^(1/2)*2i)/(5*d)
```

3.258 $\int (a\cos(c+dx)+ia\sin(c+dx))^{3/2}\,dx$

3.258.1 Optimal result

Integrand size = 24, antiderivative size = 33

$$\int (a\cos(c+dx)+ia\sin(c+dx))^{3/2}\,dx = -\frac{2i(a\cos(c+dx)+ia\sin(c+dx))^{3/2}}{3d}$$

output
```
-2/3*I*(a*cos(d*x+c)+I*a*sin(d*x+c))^(3/2)/d
```

3.258.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int (a\cos(c+dx)+ia\sin(c+dx))^{3/2}\,dx = -\frac{2i(a(\cos(c+dx)+i\sin(c+dx)))^{3/2}}{3d}$$

input
```
Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(3/2),x]
```

output
```
(((-2*I)/3)*(a*(Cos[c + d*x] + I*Sin[c + d*x]))^(3/2))/d
```

3.258.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3042, 3550}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cos(c+dx)+ia\sin(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\cos(c+dx)+ia\sin(c+dx))^{3/2}dx$$

$$\Big\downarrow \text{3550}$$

$$-\frac{2i(a\cos(c+dx)+ia\sin(c+dx))^{3/2}}{3d}$$

input
```
Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(3/2),x]
```

output
```
(((-2*I)/3)*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(3/2))/d
```

3.258.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3550
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

3.258.4 Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$-\frac{2i(\cos(dx+c)a+ia\sin(dx+c))^{\frac{3}{2}}}{3d}$	28
default	$-\frac{2i(\cos(dx+c)a+ia\sin(dx+c))^{\frac{3}{2}}}{3d}$	28
risch	$-\frac{2ia\sqrt{a\,\mathrm{e}^{i(dx+c)}}\,\mathrm{e}^{i(dx+c)}}{3d}$	30

input
```
int((cos(d*x+c)*a+I*a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*I*(cos(d*x+c)*a+I*a*sin(d*x+c))^(3/2)/d
```

3.258.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.52

$$\int (a\cos(c+dx)+ia\sin(c+dx))^{3/2}\,dx = -\frac{2i\,a^{\frac{3}{2}}e^{\left(\frac{3}{2}i\,dx+\frac{3}{2}i\,c\right)}}{3\,d}$$

input
```
integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-2/3*I*a^(3/2)*e^(3/2*I*d*x + 3/2*I*c)/d
```

3.258.6 Sympy [F]

$$\int (a\cos(c+dx)+ia\sin(c+dx))^{3/2}\,dx = \int (ia\sin(c+dx)+a\cos(c+dx))^{\frac{3}{2}}\,dx$$

input
```
integrate((a*cos(d*x+c)+I*a*sin(d*x+c))**(3/2),x)
```

output
```
Integral((I*a*sin(c + d*x) + a*cos(c + d*x))**(3/2), x)
```

3.258.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(25) = 50$.

Time = 0.32 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.55

$$\int (a\cos(c+dx)+ia\sin(c+dx))^{3/2}\,dx = -\frac{2i\,a^{\frac{3}{2}}\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+i\right)^{\frac{3}{2}}}{3\,d\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+i\right)^{\frac{3}{2}}}$$

input
```
integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
-2/3*I*a^(3/2)*(-sin(d*x + c)/(cos(d*x + c) + 1) + I)^(3/2)/(d*(sin(d*x +
c)/(cos(d*x + c) + 1) + I)^(3/2))
```

3.258.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int (a\cos(c+dx)+ia\sin(c+dx))^{3/2}\,dx = -\frac{2i\,(a\cos(dx+c)+i\,a\sin(dx+c))^{\frac{3}{2}}}{3\,d}$$

input
```
integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
-2/3*I*(a*cos(d*x + c) + I*a*sin(d*x + c))^(3/2)/d
```

3.258.9 Mupad [B] (verification not implemented)

Time = 28.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int (a\cos(c+dx)+ia\sin(c+dx))^{3/2}\,dx = -\frac{a\,e^{c\,1i}\,e^{d\,x\,1i}\,\sqrt{a\,e^{c\,1i}\,e^{d\,x\,1i}}\,2i}{3\,d}$$

input
```
int((a*cos(c + d*x) + a*sin(c + d*x)*1i)^(3/2),x)
```

output
```
-(a*exp(c*1i)*exp(d*x*1i)*(a*exp(c*1i)*exp(d*x*1i))^(1/2)*2i)/(3*d)
```

3.259 $\int \sqrt{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

3.259.1 Optimal result

Integrand size = 24, antiderivative size = 31

$$\int \sqrt{a\cos(c+dx)+ia\sin(c+dx)}\,dx=-\frac{2i\sqrt{a\cos(c+dx)+ia\sin(c+dx)}}{d}$$

output
```
-2*I*(a*cos(d*x+c)+I*a*sin(d*x+c))^(1/2)/d
```

3.259.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \sqrt{a\cos(c+dx)+ia\sin(c+dx)}\,dx=-\frac{2i\sqrt{a(\cos(c+dx)+i\sin(c+dx))}}{d}$$

input
```
Integrate[Sqrt[a*Cos[c + d*x] + I*a*Sin[c + d*x]],x]
```

output
```
((-2*I)*Sqrt[a*(Cos[c + d*x] + I*Sin[c + d*x])])/d
```

3.259.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3042, 3550}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\cos(c+dx)+ia\sin(c+dx)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sqrt{a\cos(c+dx)+ia\sin(c+dx)}dx$$

$$\Big\downarrow \ 3550$$

$$-\frac{2i\sqrt{a\cos(c+dx)+ia\sin(c+dx)}}{d}$$

input
```
Int[Sqrt[a*Cos[c + d*x] + I*a*Sin[c + d*x]],x]
```

output
```
((-2*I)*Sqrt[a*Cos[c + d*x] + I*a*Sin[c + d*x]])/d
```

3.259.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3550
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

3.259.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

method	result	size
risch	$-\frac{2i\sqrt{a\,e^{i(dx+c)}}}{d}$	20
derivativedivides	$-\frac{2i\sqrt{\cos(dx+c)a+ia\sin(dx+c)}}{d}$	28
default	$-\frac{2i\sqrt{\cos(dx+c)a+ia\sin(dx+c)}}{d}$	28

input
```
int((cos(d*x+c)*a+I*a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*I*(a*exp(I*(d*x+c)))^(1/2)/d
```

3.259.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.55

$$\int \sqrt{a\cos(c+dx)+ia\sin(c+dx)}\,dx = -\frac{2i\,\sqrt{a}e^{\left(\frac{1}{2}i\,dx+\frac{1}{2}i\,c\right)}}{d}$$

input
```
integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-2*I*sqrt(a)*e^(1/2*I*d*x + 1/2*I*c)/d
```

3.259.6 Sympy [F]

$$\int \sqrt{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \int \sqrt{ia\sin{(c+dx)}+a\cos{(c+dx)}}\,dx$$

input
```
integrate((a*cos(d*x+c)+I*a*sin(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(I*a*sin(c + d*x) + a*cos(c + d*x)), x)
```

3.259.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(25) = 50.

Time = 0.32 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.65

$$\int \sqrt{a\cos(c+dx)+ia\sin(c+dx)}\,dx = -\frac{2i\sqrt{a}\sqrt{-\frac{\sin(dx+c)}{\cos(dx+c)+1}+i}}{d\sqrt{\frac{\sin(dx+c)}{\cos(dx+c)+1}+i}}$$

input
```
integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-2*I*sqrt(a)*sqrt(-sin(d*x + c)/(cos(d*x + c) + 1) + I)/(d*sqrt(sin(d*x +
c)/(cos(d*x + c) + 1) + I))
```

3.259.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \sqrt{a\cos(c+dx)+ia\sin(c+dx)}\,dx = -\frac{2i\sqrt{a\cos(dx+c)+i\,a\sin(dx+c)}}{d}$$

input
```
integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
-2*I*sqrt(a*cos(d*x + c) + I*a*sin(d*x + c))/d
```

3.259.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \int \sqrt{a\,\cos(c+d\,x)+a\,\sin(c+d\,x)\,1\mathrm{i}}\,dx$$

input
```
int((a*cos(c + d*x) + a*sin(c + d*x)*1i)^(1/2),x)
```

output
```
int((a*cos(c + d*x) + a*sin(c + d*x)*1i)^(1/2), x)
```

3.260 $\int \frac{1}{\sqrt{a\cos(c+dx)+ia\sin(c+dx)}}\,dx$

3.260.1 Optimal result

Integrand size = 24, antiderivative size = 31

$$\int \frac{1}{\sqrt{a\cos(c+dx)+ia\sin(c+dx)}}\,dx=\frac{2i}{d\sqrt{a\cos(c+dx)+ia\sin(c+dx)}}$$

output
```
2*I/d/(a*cos(d*x+c)+I*a*sin(d*x+c))^(1/2)
```

3.260.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \frac{1}{\sqrt{a\cos(c+dx)+ia\sin(c+dx)}}\,dx=\frac{2i}{d\sqrt{a(\cos(c+dx)+i\sin(c+dx))}}$$

input
```
Integrate[1/Sqrt[a*Cos[c + d*x] + I*a*Sin[c + d*x]],x]
```

output
```
(2*I)/(d*Sqrt[a*(Cos[c + d*x] + I*Sin[c + d*x])])
```

3.260.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3042, 3550}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\cos(c+dx)+ia\sin(c+dx)}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sqrt{a\cos(c+dx)+ia\sin(c+dx)}}dx$$

$$\Big\downarrow \; 3550$$

$$\frac{2i}{d\sqrt{a\cos(c+dx)+ia\sin(c+dx)}}$$

input
```
Int[1/Sqrt[a*Cos[c + d*x] + I*a*Sin[c + d*x]],x]
```

output
```
(2*I)/(d*Sqrt[a*Cos[c + d*x] + I*a*Sin[c + d*x]])
```

3.260.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3550
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

3.260.4 Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.65

method	result	size
risch	$\frac{2i}{\sqrt{a\,e^{i(dx+c)}}\,d}$	20
derivativedivides	$\frac{2i}{d\sqrt{\cos(dx+c)a+ia\sin(dx+c)}}$	28
default	$\frac{2i}{d\sqrt{\cos(dx+c)a+ia\sin(dx+c)}}$	28

input
```
int(1/(cos(d*x+c)*a+I*a*sin(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*I/(a*exp(I*(d*x+c)))^(1/2)/d
```

3.260.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.55

$$\int \frac{1}{\sqrt{a\cos(c+dx)+ia\sin(c+dx)}}\,dx=\frac{2i\,e^{\left(-\frac{1}{2}i\,dx-\frac{1}{2}i\,c\right)}}{\sqrt{a}d}$$

input
```
integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
2*I*e^(-1/2*I*d*x - 1/2*I*c)/(sqrt(a)*d)
```

3.260.6 Sympy [F]

$$\int \frac{1}{\sqrt{a\cos(c+dx)+ia\sin(c+dx)}}\,dx=\int \frac{1}{\sqrt{ia\sin{(c+dx)}+a\cos{(c+dx)}}}\,dx$$

input
```
integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))**(1/2),x)
```

output
```
Integral(1/sqrt(I*a*sin(c + d*x) + a*cos(c + d*x)), x)
```

3.260.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(25) = 50.

Time = 0.33 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.65

$$\int \frac{1}{\sqrt{a\cos(c+dx)+ia\sin(c+dx)}}\,dx = \frac{2i\sqrt{\frac{\sin(dx+c)}{\cos(dx+c)+1}+i}}{\sqrt{a}d\sqrt{-\frac{\sin(dx+c)}{\cos(dx+c)+1}+i}}$$

input
```
integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
2*I*sqrt(sin(d*x + c)/(cos(d*x + c) + 1) + I)/(sqrt(a)*d*sqrt(-sin(d*x + c
)/(cos(d*x + c) + 1) + I))
```

3.260.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{a\cos(c+dx)+ia\sin(c+dx)}}\,dx = \frac{2i}{\sqrt{a\cos(dx+c)+i\,a\sin(dx+c)}d}$$

input
```
integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
2*I/(sqrt(a*cos(d*x + c) + I*a*sin(d*x + c))*d)
```

3.260.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a\cos(c+dx)+ia\sin(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\,\cos(c+d\,x)+a\,\sin(c+d\,x)\,1\mathrm{i}}}\,dx$$

input
```
int(1/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^(1/2),x)
```

output
```
int(1/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^(1/2), x)
```

3.261 $\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^{3/2}}\,dx$

3.261.1 Optimal result

Integrand size = 24, antiderivative size = 33

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^{3/2}}\,dx = \frac{2i}{3d(a\cos(c+dx)+ia\sin(c+dx))^{3/2}}$$

output
```
2/3*I/d/(a*cos(d*x+c)+I*a*sin(d*x+c))^(3/2)
```

3.261.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^{3/2}}\,dx = \frac{2i}{3d(a(\cos(c+dx)+i\sin(c+dx)))^{3/2}}$$

input
```
Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-3/2),x]
```

output
```
((2*I)/3)/(d*(a*(Cos[c + d*x] + I*Sin[c + d*x]))^(3/2))
```

3.261.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3042, 3550}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^{3/2}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^{3/2}}dx$$
$$\Big\downarrow \text{3550}$$
$$\frac{2i}{3d(a\cos(c+dx)+ia\sin(c+dx))^{3/2}}$$

input
```
Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-3/2),x]
```

output
```
((2*I)/3)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(3/2))
```

3.261.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3550
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

3.261.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{2i}{3d(\cos(dx+c)a+ia\sin(dx+c))^{\frac{3}{2}}}$	28
default	$\frac{2i}{3d(\cos(dx+c)a+ia\sin(dx+c))^{\frac{3}{2}}}$	28
risch	$\frac{2i\,\mathrm{e}^{-i(dx+c)}}{3a\sqrt{a\,\mathrm{e}^{i(dx+c)}}\,d}$	32

input
```
int(1/(cos(d*x+c)*a+I*a*sin(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*I/d/(cos(d*x+c)*a+I*a*sin(d*x+c))^(3/2)
```

3.261.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.52

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^{3/2}}\,dx = \frac{2i\,e^{\left(-\frac{3}{2}i\,dx-\frac{3}{2}i\,c\right)}}{3\,a^{\frac{3}{2}}d}$$

input
```
integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
2/3*I*e^(-3/2*I*d*x - 3/2*I*c)/(a^(3/2)*d)
```

3.261.6 Sympy [F]

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^{3/2}}\,dx = \int \frac{1}{(ia\sin(c+dx)+a\cos(c+dx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))**(3/2),x)
```

output
```
Integral((I*a*sin(c + d*x) + a*cos(c + d*x))**(-3/2), x)
```

3.261.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(25) = 50$.

Time = 0.31 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.55

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^{3/2}}\,dx = \frac{2i\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+i\right)^{\frac{3}{2}}}{3\,a^{\frac{3}{2}}d\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+i\right)^{\frac{3}{2}}}$$

input
```
integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
2/3*I*(sin(d*x + c)/(cos(d*x + c) + 1) + I)^(3/2)/(a^(3/2)*d*(-sin(d*x + c
)/(cos(d*x + c) + 1) + I)^(3/2))
```

3.261.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(25) = 50$.

Time = 0.80 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.97

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^{3/2}}\,dx = -\frac{2i\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)}{3\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\,a\right)d\sqrt{-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i}}}$$

input
```
integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
-2/3*I*(tan(1/2*d*x + 1/2*c) + I)/((a*tan(1/2*d*x + 1/2*c) - I*a)*d*sqrt(-
(a*tan(1/2*d*x + 1/2*c) - I*a)/(tan(1/2*d*x + 1/2*c) + I)))
```

3.261.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a\ \cos\left(c+d\,x\right)+a\ \sin\left(c+d\,x\right)\ 1\mathrm{i}\right)^{3/2}}\,dx$$

input
```
int(1/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^(3/2),x)
```

output
```
int(1/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^(3/2), x)
```

3.262 $\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^{5/2}}\,dx$

3.262.1 Optimal result

Integrand size = 24, antiderivative size = 33

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^{5/2}}\,dx = \frac{2i}{5d(a\cos(c+dx)+ia\sin(c+dx))^{5/2}}$$

output
```
2/5*I/d/(a*cos(d*x+c)+I*a*sin(d*x+c))^(5/2)
```

3.262.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^{5/2}}\,dx = \frac{2i}{5d(a(\cos(c+dx)+i\sin(c+dx)))^{5/2}}$$

input
```
Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-5/2),x]
```

output
```
((2*I)/5)/(d*(a*(Cos[c + d*x] + I*Sin[c + d*x]))^(5/2))
```

3.262.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3042, 3550}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^{5/2}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^{5/2}}dx$$
$$\Big\downarrow \text{3550}$$
$$\frac{2i}{5d(a\cos(c+dx)+ia\sin(c+dx))^{5/2}}$$

input
```
Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-5/2),x]
```

output
```
((2*I)/5)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(5/2))
```

3.262.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3550
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

3.262.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{2i}{5d(\cos(dx+c)a+ia\sin(dx+c))^{\frac{5}{2}}}$	28
default	$\frac{2i}{5d(\cos(dx+c)a+ia\sin(dx+c))^{\frac{5}{2}}}$	28

input
```
int(1/(cos(d*x+c)*a+I*a*sin(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*I/d/(cos(d*x+c)*a+I*a*sin(d*x+c))^(5/2)
```

3.262.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.52

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^{5/2}}\,dx = \frac{2i\,e^{\left(-\frac{5}{2}i\,dx-\frac{5}{2}i\,c\right)}}{5\,a^{\frac{5}{2}}d}$$

input
```
integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
2/5*I*e^(-5/2*I*d*x - 5/2*I*c)/(a^(5/2)*d)
```

3.262.6 Sympy [F]

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^{5/2}}\,dx = \int \frac{1}{(ia\sin{(c+dx)}+a\cos{(c+dx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))**(5/2),x)
```

output
```
Integral((I*a*sin(c + d*x) + a*cos(c + d*x))**(-5/2), x)
```

3.262.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(25) = 50$.

Time = 0.33 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.55

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^{5/2}}\,dx = \frac{2i\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+i\right)^{\frac{5}{2}}}{5\,a^{\frac{5}{2}}d\left(-\frac{\sin(dx+c)}{\cos(dx+c)+1}+i\right)^{\frac{5}{2}}}$$

input
```
integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
2/5*I*(sin(d*x + c)/(cos(d*x + c) + 1) + I)^(5/2)/(a^(5/2)*d*(-sin(d*x + c
)/(cos(d*x + c) + 1) + I)^(5/2))
```

3.262.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(25) = 50$.

Time = 0.67 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.03

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^{5/2}}\,dx = \frac{2i\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)^{2}}{5\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\,a\right)^{2}d\sqrt{-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\,a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i}}}$$

input
```
integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
2/5*I*(tan(1/2*d*x + 1/2*c) + I)^2/((a*tan(1/2*d*x + 1/2*c) - I*a)^2*d*sqr
t(-(a*tan(1/2*d*x + 1/2*c) - I*a)/(tan(1/2*d*x + 1/2*c) + I)))
```

3.262.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^{5/2}}\,dx = \int \frac{1}{(a\,\cos\left(c+d\,x\right)+a\,\sin\left(c+d\,x\right)\,1\mathrm{i})^{5/2}}\,dx$$

input
```
int(1/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^(5/2),x)
```

output
```
int(1/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^(5/2), x)
```

3.263 $\int (a\sec(x)+b\tan(x))^5\,dx$

3.263.1 Optimal result

Integrand size = 11, antiderivative size = 149

$$\begin{aligned}\int (a\sec(x)+b\tan(x))^5\,dx = &-\frac{1}{16}(a+b)^3\left(3a^2-9ab+8b^2\right)\log(1-\sin(x))\\ &+\frac{1}{16}(a-b)^3\left(3a^2+9ab+8b^2\right)\log(1+\sin(x))\\ &-\frac{1}{8}a\left(7-\frac{3a^2}{b^2}\right)b^4\sin(x)+\frac{1}{4}\sec^4(x)(b+a\sin(x))(a+b\sin(x))^4\\ &+\frac{1}{8}\sec^2(x)(a+b\sin(x))^2\left(2b\left(a^2-2b^2\right)+a\left(3a^2-5b^2\right)\sin(x)\right)\end{aligned}$$

output
```
-1/16*(a+b)^3*(3*a^2-9*a*b+8*b^2)*ln(1-sin(x))+1/16*(a-b)^3*(3*a^2+9*a*b+8
*b^2)*ln(1+sin(x))-1/8*a*(7-3*a^2/b^2)*b^4*sin(x)+1/4*sec(x)^4*(b+a*sin(x)
)*(a+b*sin(x))^4+1/8*sec(x)^2*(a+b*sin(x))^2*(2*b*(a^2-2*b^2)+a*(3*a^2-5*b
^2)*sin(x))
```

3.263.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 303 vs. 2(149) = 298.

Time = 1.01 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.03

$$\int (a\sec(x)+b\tan(x))^5\,dx =$$

$$-\frac{\left(a^2-b^2\right)^2\left(\left((a+b)^3\left(3a^2-9ab+8b^2\right)\log(1-\sin(x))-(a-b)^3\left(3a^2+9ab+8b^2\right)\log(1+\sin(x))\right)-10\right.}{}$$

input
```
Integrate[(a*Sec[x] + b*Tan[x])^5,x]
```

output
```
-1/16*((a^2 - b^2)^2*((a + b)^3*(3*a^2 - 9*a*b + 8*b^2)*Log[1 - Sin[x]] -
(a - b)^3*(3*a^2 + 9*a*b + 8*b^2)*Log[1 + Sin[x]]) - 10*a*b^2*(9*a^6 - 6*a
^4*b^2 + 8*a^2*b^4 - 3*b^6)*Sin[x] + 8*b^3*(-15*a^6 - 4*a^4*b^2 - 2*a^2*b^
4 + b^6)*Sin[x]^2 - 10*a*b^4*(9*a^4 + 8*a^2*b^2 - b^4)*Sin[x]^3 + 4*b^5*(-
9*a^4 - 12*a^2*b^2 + b^4)*Sin[x]^4 - 2*a*b^6*(3*a^2 + 5*b^2)*Sin[x]^5 + 4*
(-a^2 + b^2)*Sec[x]^4*(-b + a*Sin[x])*(a + b*Sin[x])^6 + 2*Sec[x]^2*(a + b
*Sin[x])^6*(6*a^2*b + 2*b^3 - 3*a^3*Sin[x] - 5*a*b^2*Sin[x]))/(a^2 - b^2)^
2
```

3.263.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {3042, 4891, 3042, 3147, 477, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(x)+b\tan(x))^5\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (a\sec(x)+b\tan(x))^5dx$$

$$\Big\downarrow \; 4891$$

$$\int \sec^5(x)(a+b\sin(x))^5dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a+b\sin(x))^5}{\cos(x)^5}dx$$

$$\Big\downarrow \; 3147$$

$$b^5\int \frac{(a+b\sin(x))^5}{\left(b^2-b^2\sin^2(x)\right)^3}d(b\sin(x))$$

$$\Big\downarrow \; 477$$

$$\frac{\int \left(\frac{b^3(a-b)^5}{8(\sin(x)b+b)^3}+\frac{b^2(3a+7b)(a-b)^4}{16(\sin(x)b+b)^2}+\frac{b(3a^2+9ba+8b^2)(a-b)^3}{16(\sin(x)b+b)}+\frac{b(a+b)^3(3a^2-9ba+8b^2)}{16(b-b\sin(x))}+\frac{(3a-7b)b^2(a+b)^4}{16(b-b\sin(x))^2}+\frac{b^3(a+b)^5}{8(b-b\sin(x))^3}\right)d(b\sin}{b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{1}{16}b(3a^2+9ab+8b^2)(a-b)^3\log(b\sin(x)+b)-\frac{1}{16}b(a+b)^3(3a^2-9ab+8b^2)\log(b-b\sin(x))-\frac{b^3(a-b)^5}{16(b\sin(x)+b)^2}+}{b}$$

input
```
Int[(a*Sec[x] + b*Tan[x])^5,x]
```

output
```
(-1/16*(b*(a + b)^3*(3*a^2 - 9*a*b + 8*b^2)*Log[b - b*Sin[x]]) + ((a - b)^
3*b*(3*a^2 + 9*a*b + 8*b^2)*Log[b + b*Sin[x]])/16 + (b^3*(a + b)^5)/(16*(b
 - b*Sin[x])^2) + ((3*a - 7*b)*b^2*(a + b)^4)/(16*(b - b*Sin[x])) - ((a -
b)^5*b^3)/(16*(b + b*Sin[x])^2) - ((a - b)^4*b^2*(3*a + 7*b))/(16*(b + b*S
in[x])))/b
```

3.263.3.1 Defintions of rubi rules used

rule 477
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ExpandIntegrand[(c + d*x)^n*(1 - Rt[-b/a, 2]*x)^p*(1 + Rt[-b/a, 2
]*x)^p, x], x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[n] &
& NiceSqrtQ[-b/a] &&  !FractionalPowerFactorQ[Rt[-b/a, 2]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.263. $\int (a\sec(x)+b\tan(x))^5\,dx$

rule 4891
```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.263.4 Maple [A] (verified)

Time = 60.63 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.10

method	result
default	$a^5\left(-\left(-\frac{\sec(x)^3}{4}-\frac{3\sec(x)}{8}\right)\tan(x)+\frac{3\ln(\sec(x)+\tan(x))}{8}\right)+\frac{5a^4b}{4\cos(x)^4}+10a^3b^2\left(\frac{\sin(x)^3}{4\cos(x)^4}+\frac{\sin(x)^3}{8\cos(x)^2}+\frac{\sin(x}{8}\right.$
parts	$a^5\left(-\left(-\frac{\sec(x)^3}{4}-\frac{3\sec(x)}{8}\right)\tan(x)+\frac{3\ln(\sec(x)+\tan(x))}{8}\right)+b^5\left(\frac{\tan(x)^4}{4}-\frac{\tan(x)^2}{2}+\frac{\ln\left(1+\tan(x)^2\right)}{2}\right)+\frac{5a^4}{}$
risch	$ib^5x-\frac{e^{ix}(-3ia^5+3ia^5e^{6ix}-15ia\,b^4e^{2ix}+15ia\,b^4e^{4ix}-10ia^3b^2e^{6ix}-11ia^5e^{2ix}+80a^2b^3e^{5ix}+16b^5e^{5ix}-25ia\,b^4e^{6ix}+70ia^3b^2e^{4ix}-}{4(e^{2ix}+1)^4}$

input
```
int((a*sec(x)+b*tan(x))^5,x,method=_RETURNVERBOSE)
```

output
```
a^5*(-(-1/4*sec(x)^3-3/8*sec(x))*tan(x)+3/8*ln(sec(x)+tan(x)))+5/4*a^4*b/c
os(x)^4+10*a^3*b^2*(1/4*sin(x)^3/cos(x)^4+1/8*sin(x)^3/cos(x)^2+1/8*sin(x)
-1/8*ln(sec(x)+tan(x)))+5/2*a^2*b^3*sin(x)^4/cos(x)^4+5*a*b^4*(1/4*sin(x)^
5/cos(x)^4-1/8*sin(x)^5/cos(x)^2-1/8*sin(x)^3-3/8*sin(x)+3/8*ln(sec(x)+tan
(x)))+b^5*(1/4*tan(x)^4-1/2*tan(x)^2-ln(cos(x)))
```

3.263.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.11

$$\int (a\sec(x)+b\tan(x))^5\,dx$$
$$=\frac{(3\,a^5-10\,a^3b^2+15\,ab^4-8\,b^5)\cos(x)^4\log(\sin(x)+1)-(3\,a^5-10\,a^3b^2+15\,ab^4+8\,b^5)\cos(x)^4\log(-\mathrm{s}}{}$$

input
```
integrate((a*sec(x)+b*tan(x))^5,x, algorithm="fricas")
```

output
```
1/16*((3*a^5 - 10*a^3*b^2 + 15*a*b^4 - 8*b^5)*cos(x)^4*log(sin(x) + 1) - (
3*a^5 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*cos(x)^4*log(-sin(x) + 1) + 20*a^4*
b + 40*a^2*b^3 + 4*b^5 - 16*(5*a^2*b^3 + b^5)*cos(x)^2 + 2*(2*a^5 + 20*a^3
*b^2 + 10*a*b^4 + (3*a^5 - 10*a^3*b^2 - 25*a*b^4)*cos(x)^2)*sin(x))/cos(x)
^4
```

3.263.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(141) = 282$.

Time = 2.12 (sec) , antiderivative size = 308, normalized size of antiderivative = 2.07

$$\begin{aligned}\int (a\sec(x)+b\tan(x))^5\,dx = &-\frac{3a^5\log(\sin(x)-1)}{16}+\frac{3a^5\log(\sin(x)+1)}{16}\\&-\frac{3a^5\sin^3(x)}{8\sin^4(x)-16\sin^2(x)+8}+\frac{5a^5\sin(x)}{8\sin^4(x)-16\sin^2(x)+8}\\&+\frac{5a^4b\sec^4(x)}{4}+\frac{5a^3b^2\log(\sin(x)-1)}{8}\\&-\frac{5a^3b^2\log(\sin(x)+1)}{8}+\frac{10a^3b^2\sin^3(x)}{8\sin^4(x)-16\sin^2(x)+8}\\&+\frac{10a^3b^2\sin(x)}{8\sin^4(x)-16\sin^2(x)+8}+\frac{5a^2b^3\tan^4(x)}{2}\\&-\frac{15ab^4\log(\sin(x)-1)}{16}+\frac{15ab^4\log(\sin(x)+1)}{16}\\&+\frac{25ab^4\sin^3(x)}{8\sin^4(x)-16\sin^2(x)+8}-\frac{15ab^4\sin(x)}{8\sin^4(x)-16\sin^2(x)+8}\\&+\frac{b^5\log(\sec^2(x))}{2}+\frac{b^5\sec^4(x)}{4}-b^5\sec^2(x)\end{aligned}$$

input
```
integrate((a*sec(x)+b*tan(x))**5,x)
```

output
```
-3*a**5*log(sin(x) - 1)/16 + 3*a**5*log(sin(x) + 1)/16 - 3*a**5*sin(x)**3/
(8*sin(x)**4 - 16*sin(x)**2 + 8) + 5*a**5*sin(x)/(8*sin(x)**4 - 16*sin(x)*
*2 + 8) + 5*a**4*b*sec(x)**4/4 + 5*a**3*b**2*log(sin(x) - 1)/8 - 5*a**3*b*
*2*log(sin(x) + 1)/8 + 10*a**3*b**2*sin(x)**3/(8*sin(x)**4 - 16*sin(x)**2
+ 8) + 10*a**3*b**2*sin(x)/(8*sin(x)**4 - 16*sin(x)**2 + 8) + 5*a**2*b**3*
tan(x)**4/2 - 15*a*b**4*log(sin(x) - 1)/16 + 15*a*b**4*log(sin(x) + 1)/16
+ 25*a*b**4*sin(x)**3/(8*sin(x)**4 - 16*sin(x)**2 + 8) - 15*a*b**4*sin(x)/
(8*sin(x)**4 - 16*sin(x)**2 + 8) + b**5*log(sec(x)**2)/2 + b**5*sec(x)**4/
4 - b**5*sec(x)**2
```

3.263.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.37

$$\int (a\sec(x)+b\tan(x))^5\,dx$$
$$= \frac{5}{2}a^2b^3\tan(x)^4$$
$$+\frac{5}{16}ab^4\left(\frac{2\left(5\sin(x)^3-3\sin(x)\right)}{\sin(x)^4-2\sin(x)^2+1}+3\log(\sin(x)+1)-3\log(\sin(x)-1)\right)$$
$$-\frac{1}{16}a^5\left(\frac{2\left(3\sin(x)^3-5\sin(x)\right)}{\sin(x)^4-2\sin(x)^2+1}-3\log(\sin(x)+1)+3\log(\sin(x)-1)\right)$$
$$+\frac{5}{8}a^3b^2\left(\frac{2\left(\sin(x)^3+\sin(x)\right)}{\sin(x)^4-2\sin(x)^2+1}-\log(\sin(x)+1)+\log(\sin(x)-1)\right)$$
$$+\frac{1}{4}b^5\left(\frac{4\sin(x)^2-3}{\sin(x)^4-2\sin(x)^2+1}-2\log\left(\sin(x)^2-1\right)\right)+\frac{5a^4b}{4\left(\sin(x)^2-1\right)^2}$$

input
```
integrate((a*sec(x)+b*tan(x))^5,x, algorithm="maxima")
```

output
```
5/2*a^2*b^3*tan(x)^4 + 5/16*a*b^4*(2*(5*sin(x)^3 - 3*sin(x))/(sin(x)^4 - 2
*sin(x)^2 + 1) + 3*log(sin(x) + 1) - 3*log(sin(x) - 1)) - 1/16*a^5*(2*(3*s
in(x)^3 - 5*sin(x))/(sin(x)^4 - 2*sin(x)^2 + 1) - 3*log(sin(x) + 1) + 3*lo
g(sin(x) - 1)) + 5/8*a^3*b^2*(2*(sin(x)^3 + sin(x))/(sin(x)^4 - 2*sin(x)^2
 + 1) - log(sin(x) + 1) + log(sin(x) - 1)) + 1/4*b^5*((4*sin(x)^2 - 3)/(si
n(x)^4 - 2*sin(x)^2 + 1) - 2*log(sin(x)^2 - 1)) + 5/4*a^4*b/(sin(x)^2 - 1)
^2
```

3.263.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.19

$$\int (a\sec(x)+b\tan(x))^5\,dx = \frac{1}{16}\left(3a^5-10a^3b^2+15ab^4-8b^5\right)\log(\sin(x)+1)$$
$$-\frac{1}{16}\left(3a^5-10a^3b^2+15ab^4+8b^5\right)\log(-\sin(x)+1)$$
$$+\frac{6b^5\sin(x)^4-3a^5\sin(x)^3+10a^3b^2\sin(x)^3+25ab^4\sin(x)^3+40a^2b^3\sin(x)^2-4b^5\sin(x)^2+5a^5\sin}{8\left(\sin(x)^2-1\right)^2}$$

input
```
integrate((a*sec(x)+b*tan(x))^5,x, algorithm="giac")
```

output
```
1/16*(3*a^5 - 10*a^3*b^2 + 15*a*b^4 - 8*b^5)*log(sin(x) + 1) - 1/16*(3*a^5
 - 10*a^3*b^2 + 15*a*b^4 + 8*b^5)*log(-sin(x) + 1) + 1/8*(6*b^5*sin(x)^4 -
 3*a^5*sin(x)^3 + 10*a^3*b^2*sin(x)^3 + 25*a*b^4*sin(x)^3 + 40*a^2*b^3*sin
(x)^2 - 4*b^5*sin(x)^2 + 5*a^5*sin(x) + 10*a^3*b^2*sin(x) - 15*a*b^4*sin(x
) + 10*a^4*b - 20*a^2*b^3)/(sin(x)^2 - 1)^2
```

3.263.9 Mupad [B] (verification not implemented)

Time = 29.64 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.83

$$\int (a\sec(x)+b\tan(x))^5\,dx$$
$$= \frac{\left(\frac{5a^5}{4}+\frac{5a^3b^2}{2}-\frac{15ab^4}{4}\right)\tan\left(\frac{x}{2}\right)^7+(10a^4b-2b^5)\tan\left(\frac{x}{2}\right)^6+\left(\frac{3a^5}{4}+\frac{35a^3b^2}{2}+\frac{55ab^4}{4}\right)\tan\left(\frac{x}{2}\right)^5+(40a^2b^3\cdots}{\tan\left(\frac{x}{2}\right)^8-4\tan\left(\frac{x}{2}\right)^6+\cdots}$$
$$+b^5\ln\left(\tan\left(\frac{x}{2}\right)^2+1\right)-\frac{\ln\left(\tan\left(\frac{x}{2}\right)-1\right)(a+b)^3(3a^2-9ab+8b^2)}{8}$$
$$+\ln\left(\tan\left(\frac{x}{2}\right)+1\right)(a-b)^3\left(\frac{3a^2}{8}+\frac{9ab}{8}+b^2\right)$$

input
```
int((b*tan(x) + a/cos(x))^5,x)
```

output
```
(tan(x/2)^2*(10*a^4*b - 2*b^5) + tan(x/2)^6*(10*a^4*b - 2*b^5) + tan(x/2)^
4*(8*b^5 + 40*a^2*b^3) + tan(x/2)*((5*a^5)/4 - (15*a*b^4)/4 + (5*a^3*b^2)/
2) + tan(x/2)^7*((5*a^5)/4 - (15*a*b^4)/4 + (5*a^3*b^2)/2) + tan(x/2)^3*((
55*a*b^4)/4 + (3*a^5)/4 + (35*a^3*b^2)/2) + tan(x/2)^5*((55*a*b^4)/4 + (3*
a^5)/4 + (35*a^3*b^2)/2))/(6*tan(x/2)^4 - 4*tan(x/2)^2 - 4*tan(x/2)^6 + ta
n(x/2)^8 + 1) + b^5*log(tan(x/2)^2 + 1) - (log(tan(x/2) - 1)*(a + b)^3*(3*
a^2 - 9*a*b + 8*b^2))/8 + log(tan(x/2) + 1)*(a - b)^3*((9*a*b)/8 + (3*a^2)
/8 + b^2)
```

3.264 $\int (a\sec(x)+b\tan(x))^4\,dx$

3.264.1 Optimal result

Integrand size = 11, antiderivative size = 100

$$\int (a\sec(x)+b\tan(x))^4\,dx = b^4x+\frac{4}{3}ab\left(a^2-2b^2\right)\cos(x)+\frac{1}{3}b^2\left(2a^2-3b^2\right)\cos(x)\sin(x) + \frac{1}{3}\sec^3(x)(b+a\sin(x))(a+b\sin(x))^3 - \frac{1}{3}\sec(x)(a+b\sin(x))^2\left(ab-\left(2a^2-3b^2\right)\sin(x)\right)$$

output
```
b^4*x+4/3*a*b*(a^2-2*b^2)*cos(x)+1/3*b^2*(2*a^2-3*b^2)*cos(x)*sin(x)+1/3*s
ec(x)^3*(b+a*sin(x))*(a+b*sin(x))^3-1/3*sec(x)*(a+b*sin(x))^2*(a*b-(2*a^2-
3*b^2)*sin(x))
```

3.264.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.96

$$\int (a\sec(x)+b\tan(x))^4\,dx = \frac{1}{12}\sec^3(x)\left(16a^3b-8ab^3+9b^4x\cos(x)-24ab^3\cos(2x) + 3b^4x\cos(3x)+6a^4\sin(x)+18a^2b^2\sin(x)+2a^4\sin(3x) - 6a^2b^2\sin(3x)-4b^4\sin(3x)\right)$$

input
```
Integrate[(a*Sec[x] + b*Tan[x])^4,x]
```

output
```
(Sec[x]^3*(16*a^3*b - 8*a*b^3 + 9*b^4*x*Cos[x] - 24*a*b^3*Cos[2*x] + 3*b^4
*x*Cos[3*x] + 6*a^4*Sin[x] + 18*a^2*b^2*Sin[x] + 2*a^4*Sin[3*x] - 6*a^2*b^
2*Sin[3*x] - 4*b^4*Sin[3*x]))/12
```

3.264.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.909$, Rules used = {3042, 4891, 3042, 3170, 25, 3042, 3340, 27, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(x)+b\tan(x))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sec(x)+b\tan(x))^4dx$$

$$\Big\downarrow \text{4891}$$

$$\int \sec^4(x)(a+b\sin(x))^4dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\sin(x))^4}{\cos(x)^4}dx$$

$$\Big\downarrow \text{3170}$$

$$\frac{1}{3}\sec^3(x)(a\sin(x)+b)(a+b\sin(x))^3-\frac{1}{3}\int -\sec^2(x)(a+b\sin(x))^2\left(2a^2-b\sin(x)a-3b^2\right)dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\int \sec^2(x)(a+b\sin(x))^2\left(2a^2-b\sin(x)a-3b^2\right)dx+\frac{1}{3}\sec^3(x)(a\sin(x)+b)(a+b\sin(x))^3$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\int \frac{(a+b\sin(x))^2\left(2a^2-b\sin(x)a-3b^2\right)}{\cos(x)^2}dx+\frac{1}{3}\sec^3(x)(a\sin(x)+b)(a+b\sin(x))^3$$

$$\Big\downarrow \text{3340}$$

$$\frac{1}{3}\left(-\int -2(a+b\sin(x))\left(ab^2-b\left(2a^2-3b^2\right)\sin(x)\right)dx-\sec(x)\left(ab-\left(2a^2-3b^2\right)\sin(x)\right)(a+b\sin(x))^2\right)+\frac{1}{3}\sec^3(x)(a\sin(x)+b)(a+b\sin(x))^3$$

$\Big\downarrow$ 27

$$\frac{1}{3}\left(2\int (a+b\sin(x))\left(ab^2-b\left(2a^2-3b^2\right)\sin(x)\right)dx-\sec(x)(a+b\sin(x))^2\left(ab-\left(2a^2-3b^2\right)\sin(x)\right)\right)+\frac{1}{3}\sec^3(x)(a\sin(x)+b)(a+b\sin(x))^3$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(2\int (a+b\sin(x))\left(ab^2-b\left(2a^2-3b^2\right)\sin(x)\right)dx-\sec(x)(a+b\sin(x))^2\left(ab-\left(2a^2-3b^2\right)\sin(x)\right)\right)+\frac{1}{3}\sec^3(x)(a\sin(x)+b)(a+b\sin(x))^3$$

$\Big\downarrow$ 3213

$$\frac{1}{3}\left(2\left(2ab\left(a^2-2b^2\right)\cos(x)+\frac{1}{2}b^2\left(2a^2-3b^2\right)\sin(x)\cos(x)+\frac{3b^4x}{2}\right)-\sec(x)(a+b\sin(x))^2\left(ab-\left(2a^2-3b^2\right)\sin\right.\right.$$
$$\frac{1}{3}\sec^3(x)(a\sin(x)+b)(a+b\sin(x))^3$$

input
```
Int[(a*Sec[x] + b*Tan[x])^4,x]
```

output
```
(Sec[x]^3*(b + a*Sin[x])*(a + b*Sin[x])^3)/3 + (-(Sec[x]*(a + b*Sin[x])^2*
(a*b - (2*a^2 - 3*b^2)*Sin[x])) + 2*((3*b^4*x)/2 + 2*a*b*(a^2 - 2*b^2)*Cos
[x] + (b^2*(2*a^2 - 3*b^2)*Cos[x]*Sin[x])/2))/3
```

3.264.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3170
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Simp[1/(g^2*(p + 1))
  Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) +
a^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g
}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*
p] || IntegerQ[m])
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3340
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(g*
Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x])^m*((d + c*Sin[e + f*x])/(f*g*(p
 + 1))), x] + Simp[1/(g^2*(p + 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Si
n[e + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ
[m, 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0]
&& SimplerQ[c + d*x, a + b*x])
```

rule 4891
```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.264. $\int (a\sec(x) + b\tan(x))^4\, dx$

3.264.4 Maple [A] (verified)

Time = 17.59 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.74

method	result
parts	$-a^4\left(-\frac{2}{3}-\frac{\sec(x)^2}{3}\right)\tan(x)+b^4\left(\frac{\tan(x)^3}{3}-\tan(x)+\arctan(\tan(x))\right)+\frac{4a^3b\sec(x)^3}{3}+2a^2b^2\tan(x$
default	$-a^4\left(-\frac{2}{3}-\frac{\sec(x)^2}{3}\right)\tan(x)+\frac{4a^3b}{3\cos(x)^3}+\frac{2a^2b^2\sin(x)^3}{\cos(x)^3}+4a\,b^3\left(\frac{\sin(x)^4}{3\cos(x)^3}-\frac{\sin(x)^4}{3\cos(x)}-\frac{\left(2+\sin(x)^2\right)\cos(x)}{3}\right)+$
risch	$b^4x-\frac{4\left(9ia^2b^2\mathrm{e}^{4ix}+3ib^4\mathrm{e}^{4ix}+6a\,b^3\mathrm{e}^{5ix}-3ia^4\mathrm{e}^{2ix}+3ib^4\mathrm{e}^{2ix}-8a^3b\,\mathrm{e}^{3ix}+4a\,b^3\mathrm{e}^{3ix}-ia^4+3ia^2b^2+2ib^4+6a\,b^3\mathrm{e}^{ix}\right)}{3\left(\mathrm{e}^{2ix}+1\right)^3}$

input
```
int((a*sec(x)+b*tan(x))^4,x,method=_RETURNVERBOSE)
```

output
```
-a^4*(-2/3-1/3*sec(x)^2)*tan(x)+b^4*(1/3*tan(x)^3-tan(x)+arctan(tan(x)))+4
/3*a^3*b*sec(x)^3+2*a^2*b^2*tan(x)^3+4*a*b^3*(1/3*sec(x)^3-sec(x))
```

3.264.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.80

$$\int (a\sec(x)+b\tan(x))^4\,dx$$
$$=\frac{3\,b^4x\cos(x)^3-12\,ab^3\cos(x)^2+4\,a^3b+4\,ab^3+\left(a^4+6\,a^2b^2+b^4+2\left(a^4-3\,a^2b^2-2\,b^4\right)\cos(x)^2\right)\sin(x)}{3\,\cos(x)^3}$$

input
```
integrate((a*sec(x)+b*tan(x))^4,x, algorithm="fricas")
```

output
```
1/3*(3*b^4*x*cos(x)^3 - 12*a*b^3*cos(x)^2 + 4*a^3*b + 4*a*b^3 + (a^4 + 6*a
^2*b^2 + b^4 + 2*(a^4 - 3*a^2*b^2 - 2*b^4)*cos(x)^2)*sin(x))/cos(x)^3
```

3.264.6 Sympy [A] (verification not implemented)

Time = 1.19 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.97

$$\int (a\sec(x)+b\tan(x))^4\,dx = \frac{a^4\tan^3(x)}{3}+a^4\tan(x)+\frac{4a^3b\sec^3(x)}{3}+2a^2b^2\tan^3(x)+\frac{4ab^3\sec^3(x)}{3}-4ab^3\sec(x)+b^4x+\frac{b^4\sin^3(x)}{3\cos^3(x)}-\frac{b^4\sin(x)}{\cos(x)}$$

input
```
integrate((a*sec(x)+b*tan(x))**4,x)
```

output
```
a**4*tan(x)**3/3 + a**4*tan(x) + 4*a**3*b*sec(x)**3/3 + 2*a**2*b**2*tan(x)
**3 + 4*a*b**3*sec(x)**3/3 - 4*a*b**3*sec(x) + b**4*x + b**4*sin(x)**3/(3*
cos(x)**3) - b**4*sin(x)/cos(x)
```

3.264.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.72

$$\int (a\sec(x)+b\tan(x))^4\,dx = 2\,a^2b^2\tan(x)^3+\frac{1}{3}\left(\tan(x)^3+3\,\tan(x)\right)a^4+\frac{1}{3}\left(\tan(x)^3+3\,x-3\,\tan(x)\right)b^4-\frac{4\left(3\,\cos(x)^2-1\right)ab^3}{3\,\cos(x)^3}+\frac{4\,a^3b}{3\,\cos(x)^3}$$

input
```
integrate((a*sec(x)+b*tan(x))^4,x, algorithm="maxima")
```

output
```
2*a^2*b^2*tan(x)^3 + 1/3*(tan(x)^3 + 3*tan(x))*a^4 + 1/3*(tan(x)^3 + 3*x -
 3*tan(x))*b^4 - 4/3*(3*cos(x)^2 - 1)*a*b^3/cos(x)^3 + 4/3*a^3*b/cos(x)^3
```

3.264.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.31

$$\int (a\sec(x)+b\tan(x))^4\,dx = b^4x - \frac{2\left(3\,a^4\tan\left(\frac{1}{2}\,x\right)^5 - 3\,b^4\tan\left(\frac{1}{2}\,x\right)^5 + 12\,a^3b\tan\left(\frac{1}{2}\,x\right)^4 - 2\,a^4\tan\left(\frac{1}{2}\,x\right)^3 + 24\,a^2b^2\tan\left(\frac{1}{2}\,x\right)^3 + 10\,b^4\tan\right.}{3\left(\tan\left(\frac{1}{2}\,x\right)^2 - 1\right)^3}$$

input
```
integrate((a*sec(x)+b*tan(x))^4,x, algorithm="giac")
```

output
```
b^4*x - 2/3*(3*a^4*tan(1/2*x)^5 - 3*b^4*tan(1/2*x)^5 + 12*a^3*b*tan(1/2*x)
^4 - 2*a^4*tan(1/2*x)^3 + 24*a^2*b^2*tan(1/2*x)^3 + 10*b^4*tan(1/2*x)^3 +
24*a*b^3*tan(1/2*x)^2 + 3*a^4*tan(1/2*x) - 3*b^4*tan(1/2*x) + 4*a^3*b - 8*
a*b^3)/(tan(1/2*x)^2 - 1)^3
```

3.264.9 Mupad [B] (verification not implemented)

Time = 28.72 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.15

$$\int (a\sec(x)+b\tan(x))^4\,dx = b^4\,x - \frac{\tan\left(\frac{x}{2}\right)\,\left(2\,a^4-2\,b^4\right) - \frac{16\,a\,b^3}{3} + \frac{8\,a^3\,b}{3} + \tan\left(\frac{x}{2}\right)^3\,\left(-\frac{4\,a^4}{3} + 16\,a^2\,b^2 + \frac{20\,b^4}{3}\right) + \tan\left(\frac{x}{2}\right)^5\,\left(2\,a^4-2\,b^4\right) + 16\,a}{\left(\tan\left(\frac{x}{2}\right)^2 - 1\right)^3}$$

input
```
int((b*tan(x) + a/cos(x))^4,x)
```

output
```
b^4*x - (tan(x/2)*(2*a^4 - 2*b^4) - (16*a*b^3)/3 + (8*a^3*b)/3 + tan(x/2)^
3*((20*b^4)/3 - (4*a^4)/3 + 16*a^2*b^2) + tan(x/2)^5*(2*a^4 - 2*b^4) + 16*
a*b^3*tan(x/2)^2 + 8*a^3*b*tan(x/2)^4)/(tan(x/2)^2 - 1)^3
```

3.265 $\int (a\sec(x)+b\tan(x))^3\,dx$

3.265.1 Optimal result

Integrand size = 11, antiderivative size = 75

$$\begin{aligned}\int (a\sec(x)+b\tan(x))^3\,dx = & -\frac{1}{4}(a-2b)(a+b)^2\log(1-\sin(x)) \\ & +\frac{1}{4}(a-b)^2(a+2b)\log(1+\sin(x))+\frac{1}{2}ab^2\sin(x) \\ & +\frac{1}{2}\sec^2(x)(b+a\sin(x))(a+b\sin(x))^2\end{aligned}$$

output
```
-1/4*(a-2*b)*(a+b)^2*ln(1-sin(x))+1/4*(a-b)^2*(a+2*b)*ln(1+sin(x))+1/2*a*b
^2*sin(x)+1/2*sec(x)^2*(b+a*sin(x))*(a+b*sin(x))^2
```

3.265.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.64

$$\int (a\sec(x)+b\tan(x))^3\,dx$$
$$=\frac{(a^2-b^2)\left((a-2b)(a+b)^2\log(1-\sin(x))-(a-b)^2(a+2b)\log(1+\sin(x))\right)+2a^4b\sec^2(x)-2a(a^4+2a}{4\left(-a^2+b^2\right)}$$

input
```
Integrate[(a*Sec[x] + b*Tan[x])^3,x]
```

output
```
((a^2 - b^2)*((a - 2*b)*(a + b)^2*Log[1 - Sin[x]] - (a - b)^2*(a + 2*b)*Lo
g[1 + Sin[x]]) + 2*a^4*b*Sec[x]^2 - 2*a*(a^4 + 2*a^2*b^2 - 3*b^4)*Sec[x]*T
an[x] + (-8*a^4*b + 4*a^2*b^3 + 2*b^5)*Tan[x]^2)/(4*(-a^2 + b^2))
```

3.265.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.27, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {3042, 4891, 3042, 3147, 477, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(x)+b\tan(x))^3\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (a\sec(x)+b\tan(x))^3dx$$

$$\Big\downarrow \text{ 4891}$$

$$\int \sec^3(x)(a+b\sin(x))^3dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+b\sin(x))^3}{\cos(x)^3}dx$$

$$\Big\downarrow \text{ 3147}$$

$$b^3\int \frac{(a+b\sin(x))^3}{\left(b^2-b^2\sin^2(x)\right)^2}d(b\sin(x))$$

$$\Big\downarrow \text{ 477}$$

$$\frac{\int \left(\frac{b^2(a-b)^3}{4(\sin(x)b+b)^2}+\frac{b(a+2b)(a-b)^2}{4(\sin(x)b+b)}+\frac{(a-2b)b(a+b)^2}{4(b-b\sin(x))}+\frac{b^2(a+b)^3}{4(b-b\sin(x))^2}\right)d(b\sin(x))}{b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{b^2(a-b)^3}{4(b\sin(x)+b)}+\frac{b^2(a+b)^3}{4(b-b\sin(x))}+\frac{1}{4}b(a+2b)(a-b)^2\log(b\sin(x)+b)-\frac{1}{4}b(a-2b)(a+b)^2\log(b-b\sin(x))}{b}$$

input
```
Int[(a*Sec[x] + b*Tan[x])^3,x]
```

output
```
(-1/4*((a - 2*b)*b*(a + b)^2*Log[b - b*Sin[x]]) + ((a - b)^2*b*(a + 2*b)*L
og[b + b*Sin[x]])/4 + (b^2*(a + b)^3)/(4*(b - b*Sin[x])) - ((a - b)^3*b^2)
/(4*(b + b*Sin[x])))/b
```

3.265.3.1 Defintions of rubi rules used

rule 477
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ExpandIntegrand[(c + d*x)^n*(1 - Rt[-b/a, 2]*x)^p*(1 + Rt[-b/a, 2
]*x)^p, x], x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[n] &
& NiceSqrtQ[-b/a] &&  !FractionalPowerFactorQ[Rt[-b/a, 2]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 4891
```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.265.4 Maple [A] (verified)

Time = 4.86 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.99

method	result
default	$a^3\left(\frac{\sec(x)\tan(x)}{2}+\frac{\ln(\sec(x)+\tan(x))}{2}\right)+\frac{3a^2b}{2\cos(x)^2}+3a\,b^2\left(\frac{\sin(x)^3}{2\cos(x)^2}+\frac{\sin(x)}{2}-\frac{\ln(\sec(x)+\tan(x))}{2}\right)+b^3\left(\frac{\tan(x)^2}{2}\right.$
parts	$a^3\left(\frac{\sec(x)\tan(x)}{2}+\frac{\ln(\sec(x)+\tan(x))}{2}\right)+b^3\left(\frac{\tan(x)^2}{2}-\frac{\ln\left(1+\tan(x)^2\right)}{2}\right)+3a\,b^2\left(\frac{\sin(x)^3}{2\cos(x)^2}+\frac{\sin(x)}{2}-\frac{\ln(\sec(x)+}{2}\right.$
risch	$-ix\,b^3+\frac{e^{ix}\left(-ia^3e^{2ix}-3ia\,b^2e^{2ix}+ia^3+3ia\,b^2+6a^2b\,e^{ix}+2b^3e^{ix}\right)}{\left(e^{2ix}+1\right)^2}-\frac{\ln\left(e^{ix}-i\right)a^3}{2}+\frac{3\ln\left(e^{ix}-i\right)a\,b^2}{2}+\ln\left(e^{ix}-i\right)b^3+$

input
```
int((a*sec(x)+b*tan(x))^3,x,method=_RETURNVERBOSE)
```

output
```
a^3*(1/2*sec(x)*tan(x)+1/2*ln(sec(x)+tan(x)))+3/2*a^2*b/cos(x)^2+3*a*b^2*(
1/2*sin(x)^3/cos(x)^2+1/2*sin(x)-1/2*ln(sec(x)+tan(x)))+b^3*(1/2*tan(x)^2+
ln(cos(x)))
```

3.265.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.13

$$\int (a\sec(x)+b\tan(x))^3\,dx = \frac{(a^3-3\,ab^2+2\,b^3)\cos{(x)}^2\log{(\sin{(x)}+1)}-(a^3-3\,ab^2-2\,b^3)\cos{(x)}^2\log{(-\sin{(x)}+1)}+6\,a^2b+2\,b^3}{4\,\cos{(x)}^2}$$

input
```
integrate((a*sec(x)+b*tan(x))^3,x, algorithm="fricas")
```

output
```
1/4*((a^3 - 3*a*b^2 + 2*b^3)*cos(x)^2*log(sin(x) + 1) - (a^3 - 3*a*b^2 - 2
*b^3)*cos(x)^2*log(-sin(x) + 1) + 6*a^2*b + 2*b^3 + 2*(a^3 + 3*a*b^2)*sin(
x))/cos(x)^2
```

3.265.6 Sympy [A] (verification not implemented)

Time = 1.27 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.63

$$\int (a\sec(x)+b\tan(x))^3\,dx = -\frac{a^3\log{(\sin{(x)}-1)}}{4}+\frac{a^3\log{(\sin{(x)}+1)}}{4}-\frac{a^3\sin{(x)}}{2\sin^2{(x)}-2}+\frac{3a^2b\sec^2{(x)}}{2}+\frac{3ab^2\log{(\sin{(x)}-1)}}{4}-\frac{3ab^2\log{(\sin{(x)}+1)}}{4}-\frac{3ab^2\sin{(x)}}{2\sin^2{(x)}-2}-\frac{b^3\log{(\sec^2{(x)})}}{2}+\frac{b^3\sec^2{(x)}}{2}$$

input
```
integrate((a*sec(x)+b*tan(x))**3,x)
```

output
```
-a**3*log(sin(x) - 1)/4 + a**3*log(sin(x) + 1)/4 - a**3*sin(x)/(2*sin(x)**
2 - 2) + 3*a**2*b*sec(x)**2/2 + 3*a*b**2*log(sin(x) - 1)/4 - 3*a*b**2*log(
sin(x) + 1)/4 - 3*a*b**2*sin(x)/(2*sin(x)**2 - 2) - b**3*log(sec(x)**2)/2
+ b**3*sec(x)**2/2
```

3.265.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.27

$$\int (a\sec(x)+b\tan(x))^3\,dx = \frac{3}{2}\,a^2b\tan\left(x\right)^2 - \frac{3}{4}\,ab^2\left(\frac{2\,\sin\left(x\right)}{\sin\left(x\right)^2-1}+\log\left(\sin\left(x\right)+1\right)-\log\left(\sin\left(x\right)-1\right)\right) - \frac{1}{4}\,a^3\left(\frac{2\,\sin\left(x\right)}{\sin\left(x\right)^2-1}-\log\left(\sin\left(x\right)+1\right)+\log\left(\sin\left(x\right)-1\right)\right) - \frac{1}{2}\,b^3\left(\frac{1}{\sin\left(x\right)^2-1}-\log\left(\sin\left(x\right)^2-1\right)\right)$$

input
```
integrate((a*sec(x)+b*tan(x))^3,x, algorithm="maxima")
```

output
```
3/2*a^2*b*tan(x)^2 - 3/4*a*b^2*(2*sin(x)/(sin(x)^2 - 1) + log(sin(x) + 1)
- log(sin(x) - 1)) - 1/4*a^3*(2*sin(x)/(sin(x)^2 - 1) - log(sin(x) + 1) +
log(sin(x) - 1)) - 1/2*b^3*(1/(sin(x)^2 - 1) - log(sin(x)^2 - 1))
```

3.265.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.15

$$\int (a\sec(x)+b\tan(x))^3\,dx = \frac{1}{4}\left(a^3-3\,ab^2+2\,b^3\right)\log\left(\sin\left(x\right)+1\right) - \frac{1}{4}\left(a^3-3\,ab^2-2\,b^3\right)\log\left(-\sin\left(x\right)+1\right) - \frac{b^3\sin\left(x\right)^2+a^3\sin\left(x\right)+3\,ab^2\sin\left(x\right)+3\,a^2b}{2\left(\sin\left(x\right)^2-1\right)}$$

input
```
integrate((a*sec(x)+b*tan(x))^3,x, algorithm="giac")
```

output
```
1/4*(a^3 - 3*a*b^2 + 2*b^3)*log(sin(x) + 1) - 1/4*(a^3 - 3*a*b^2 - 2*b^3)*
log(-sin(x) + 1) - 1/2*(b^3*sin(x)^2 + a^3*sin(x) + 3*a*b^2*sin(x) + 3*a^2
*b)/(sin(x)^2 - 1)
```

3.265.9 Mupad [B] (verification not implemented)

Time = 28.61 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.68

$$\int (a\sec(x)+b\tan(x))^3\,dx = \frac{(a^3+3\,a\,b^2)\,\tan\left(\frac{x}{2}\right)^3+(6\,a^2\,b+2\,b^3)\,\tan\left(\frac{x}{2}\right)^2+(a^3+3\,a\,b^2)\,\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^4-2\tan\left(\frac{x}{2}\right)^2+1} - b^3\,\ln\left(\tan\left(\frac{x}{2}\right)^2+1\right) - \frac{\ln\left(\tan\left(\frac{x}{2}\right)-1\right)\,(a+b)^2\,(a-2\,b)}{2} + \frac{\ln\left(\tan\left(\frac{x}{2}\right)+1\right)\,(a-b)^2\,(a+2\,b)}{2}$$

input
```
int((b*tan(x) + a/cos(x))^3,x)
```

output
```
(tan(x/2)^2*(6*a^2*b + 2*b^3) + tan(x/2)*(3*a*b^2 + a^3) + tan(x/2)^3*(3*a
*b^2 + a^3))/(tan(x/2)^4 - 2*tan(x/2)^2 + 1) - b^3*log(tan(x/2)^2 + 1) - (
log(tan(x/2) - 1)*(a + b)^2*(a - 2*b))/2 + (log(tan(x/2) + 1)*(a - b)^2*(a
 + 2*b))/2
```

3.266 $\int (a\sec(x)+b\tan(x))^2\,dx$

3.266.1 Optimal result

Integrand size = 11, antiderivative size = 27

$$\int (a\sec(x)+b\tan(x))^2\,dx = -b^2x+ab\cos(x)+\sec(x)(b+a\sin(x))(a+b\sin(x))$$

output
```
-b^2*x+a*b*cos(x)+sec(x)*(b+a*sin(x))*(a+b*sin(x))
```

3.266.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int (a\sec(x)+b\tan(x))^2\,dx = -b^2\arctan(\tan(x))+2ab\sec(x)+\left(a^2+b^2\right)\tan(x)$$

input
```
Integrate[(a*Sec[x] + b*Tan[x])^2,x]
```

output
```
-(b^2*ArcTan[Tan[x]]) + 2*a*b*Sec[x] + (a^2 + b^2)*Tan[x]
```

3.266.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {3042, 4891, 3042, 3170, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(x)+b\tan(x))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sec(x)+b\tan(x))^2dx$$

$$\Big\downarrow \text{4891}$$

$$\int \sec^2(x)(a+b\sin(x))^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\sin(x))^2}{\cos(x)^2}dx$$

$$\Big\downarrow \text{3170}$$

$$\sec(x)(a\sin(x)+b)(a+b\sin(x))-\int \left(b^2+a\sin(x)b\right)dx$$

$$\Big\downarrow \text{2009}$$

$$ab\cos(x)+\sec(x)(a\sin(x)+b)(a+b\sin(x))+b^2(-x)$$

input
```
Int[(a*Sec[x] + b*Tan[x])^2,x]
```

output
```
-(b^2*x) + a*b*Cos[x] + Sec[x]*(b + a*Sin[x])*(a + b*Sin[x])
```

3.266.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3170
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Simp[1/(g^2*(p + 1))
  Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) +
a^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g
}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*
p] || IntegerQ[m])
```

rule 4891
```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.266.4 Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

method	result	size
default	$a^2\tan(x)+\frac{2ab}{\cos(x)}+b^2(\tan(x)-x)$	26
parts	$a^2\tan(x)+b^2(\tan(x)-\arctan(\tan(x)))+2ab\sec(x)$	26
risch	$-x\,b^2+\frac{4ab\,e^{ix}+2ia^2+2ib^2}{e^{2ix}+1}$	41

input
```
int((a*sec(x)+b*tan(x))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*tan(x)+2*a*b/cos(x)+b^2*(tan(x)-x)
```

3.266.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int (a\sec(x)+b\tan(x))^2\,dx = -\frac{b^2x\cos(x)-2\,ab-(a^2+b^2)\sin(x)}{\cos(x)}$$

input
```
integrate((a*sec(x)+b*tan(x))^2,x, algorithm="fricas")
```

output
```
-(b^2*x*cos(x) - 2*a*b - (a^2 + b^2)*sin(x))/cos(x)
```

3.266.6 Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int (a\sec(x)+b\tan(x))^2\,dx = a^2\tan(x)+2ab\sec(x)+b^2(-x+\tan(x))$$

input
```
integrate((a*sec(x)+b*tan(x))**2,x)
```

output
```
a**2*tan(x) + 2*a*b*sec(x) + b**2*(-x + tan(x))
```

3.266.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int (a\sec(x)+b\tan(x))^2\,dx = -b^2(x-\tan(x))+a^2\tan(x)+\frac{2\,ab}{\cos(x)}$$

input
```
integrate((a*sec(x)+b*tan(x))^2,x, algorithm="maxima")
```

output
```
-b^2*(x - tan(x)) + a^2*tan(x) + 2*a*b/cos(x)
```

3.266.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.48

$$\int (a\sec(x)+b\tan(x))^2\,dx=-b^2x-\frac{2\left(a^2\tan\left(\frac{1}{2}\,x\right)+b^2\tan\left(\frac{1}{2}\,x\right)+2\,ab\right)}{\tan\left(\frac{1}{2}\,x\right)^2-1}$$

input
```
integrate((a*sec(x)+b*tan(x))^2,x, algorithm="giac")
```

output
```
-b^2*x - 2*(a^2*tan(1/2*x) + b^2*tan(1/2*x) + 2*a*b)/(tan(1/2*x)^2 - 1)
```

3.266.9 Mupad [B] (verification not implemented)

Time = 29.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.48

$$\int (a\sec(x)+b\tan(x))^2\,dx=-\frac{4\,a\,b+\tan\left(\frac{x}{2}\right)\,\left(2\,a^2+2\,b^2\right)}{\tan\left(\frac{x}{2}\right)^2-1}-b^2\,x$$

input
```
int((b*tan(x) + a/cos(x))^2,x)
```

output
```
- (4*a*b + tan(x/2)*(2*a^2 + 2*b^2))/(tan(x/2)^2 - 1) - b^2*x
```

3.267 $\int (a\sec(x)+b\tan(x))\,dx$

3.267.1 Optimal result

Integrand size = 9, antiderivative size = 12

$$\int (a\sec(x)+b\tan(x))\,dx = a\text{arctanh}(\sin(x))-b\log(\cos(x))$$

output
```
a*arctanh(sin(x))-b*ln(cos(x))
```

3.267.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (a\sec(x)+b\tan(x))\,dx = a\text{arctanh}(\sin(x))-b\log(\cos(x))$$

input
```
Integrate[a*Sec[x] + b*Tan[x],x]
```

output
```
a*ArcTanh[Sin[x]] - b*Log[Cos[x]]
```

3.267.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sec(x)+b\tan(x))\,dx$$

$$\Big\downarrow \; 2009$$

$$a\text{arctanh}(\sin(x))-b\log(\cos(x))$$

input
```
Int[a*Sec[x] + b*Tan[x],x]
```

output
```
a*ArcTanh[Sin[x]] - b*Log[Cos[x]]
```

3.267.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.267.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

method	result	size
default	$a\ln\left(\sec\left(x\right)+\tan\left(x\right)\right)-b\ln\left(\cos\left(x\right)\right)$	16
parts	$a\ln\left(\sec\left(x\right)+\tan\left(x\right)\right)-b\ln\left(\cos\left(x\right)\right)$	16
norman	$a\ln\left(\tan\left(\frac{x}{2}\right)+1\right)-a\ln\left(\tan\left(\frac{x}{2}\right)-1\right)+\frac{b\ln\left(1+\tan\left(x\right)^{2}\right)}{2}$	31
parallelrisch	$-a(-\ln\left(\csc\left(x\right)-\cot\left(x\right)+1\right)+\ln\left(-\cot\left(x\right)+\csc\left(x\right)-1\right))+b\ln\left(\sqrt{\sec\left(x\right)^{2}}\right)$	35
risch	$-a\ln\left(\mathrm{e}^{ix}-i\right)+a\ln\left(i+\mathrm{e}^{ix}\right)+ibx-b\ln\left(\mathrm{e}^{2ix}+1\right)$	41

input
```
int(a*sec(x)+b*tan(x),x,method=_RETURNVERBOSE)
```

output
```
a*ln(sec(x)+tan(x))-b*ln(cos(x))
```

3.267.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(12) = 24$.

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.08

$$\int (a\sec(x)+b\tan(x))\,dx=\frac{1}{2}\,(a-b)\log\left(\sin\left(x\right)+1\right)-\frac{1}{2}\,(a+b)\log\left(-\sin\left(x\right)+1\right)$$

input
```
integrate(a*sec(x)+b*tan(x),x, algorithm="fricas")
```

output
```
1/2*(a - b)*log(sin(x) + 1) - 1/2*(a + b)*log(-sin(x) + 1)
```

3.267.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.00

$$\int (a\sec(x)+b\tan(x))\,dx=a\left(-\frac{\log\left(\sin\left(x\right)-1\right)}{2}+\frac{\log\left(\sin\left(x\right)+1\right)}{2}\right)-b\log\left(\cos\left(x\right)\right)$$

input
```
integrate(a*sec(x)+b*tan(x),x)
```

output
```
a*(-log(sin(x) - 1)/2 + log(sin(x) + 1)/2) - b*log(cos(x))
```

3.267.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int (a\sec(x)+b\tan(x))\,dx=a\log\left(\sec\left(x\right)+\tan\left(x\right)\right)+b\log\left(\sec\left(x\right)\right)$$

input
```
integrate(a*sec(x)+b*tan(x),x, algorithm="maxima")
```

output
```
a*log(sec(x) + tan(x)) + b*log(sec(x))
```

3.267.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(12) = 24$.

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.83

$$\int (a\sec(x) + b\tan(x))\,dx = \frac{1}{4}a\left(\log\left(\left|\frac{1}{\sin(x)} + \sin(x) + 2\right|\right) - \log\left(\left|\frac{1}{\sin(x)} + \sin(x) - 2\right|\right)\right) - b\log(|\cos(x)|)$$

input
```
integrate(a*sec(x)+b*tan(x),x, algorithm="giac")
```

output
```
1/4*a*(log(abs(1/sin(x) + sin(x) + 2)) - log(abs(1/sin(x) + sin(x) - 2)))
- b*log(abs(cos(x)))
```

3.267.9 Mupad [B] (verification not implemented)

Time = 29.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 3.08

$$\int (a\sec(x) + b\tan(x))\,dx = b\,\ln\left(\tan\left(\frac{x}{2}\right)^2 + 1\right) - \ln\left(\tan\left(\frac{x}{2}\right) - 1\right)\,(a+b) + \ln\left(\tan\left(\frac{x}{2}\right) + 1\right)\,(a-b)$$

input
```
int(b*tan(x) + a/cos(x),x)
```

output
```
b*log(tan(x/2)^2 + 1) - log(tan(x/2) - 1)*(a + b) + log(tan(x/2) + 1)*(a -
 b)
```

3.268 $\int \frac{1}{a\sec(x)+b\tan(x)}\,dx$

3.268.1 Optimal result

Integrand size = 11, antiderivative size = 11

$$\int \frac{1}{a\sec(x)+b\tan(x)}\,dx=\frac{\log(a+b\sin(x))}{b}$$

output
```
ln(a+b*sin(x))/b
```

3.268.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{a\sec(x)+b\tan(x)}\,dx=\frac{\log(a+b\sin(x))}{b}$$

input
```
Integrate[(a*Sec[x] + b*Tan[x])^(-1),x]
```

output
```
Log[a + b*Sin[x]]/b
```

3.268.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {3042, 3638, 3042, 3147, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a\sec(x)+b\tan(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a\sec(x)+b\tan(x)}dx$$

$$\Big\downarrow \text{3638}$$

$$\int \frac{\cos(x)}{a+b\sin(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(x)}{a+b\sin(x)}dx$$

$$\Big\downarrow \text{3147}$$

$$\frac{\int \frac{1}{a+b\sin(x)}d(b\sin(x))}{b}$$

$$\Big\downarrow \text{16}$$

$$\frac{\log(a+b\sin(x))}{b}$$

input
```
Int[(a*Sec[x] + b*Tan[x])^(-1),x]
```

output
```
Log[a + b*Sin[x]]/b
```

3.268.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 3638
```
Int[((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)])
^(-1), x_Symbol] :> Int[Cos[d + e*x]/(b + a*Cos[d + e*x] + c*Sin[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

3.268.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
default	$\frac{\ln(a+b\sin(x))}{b}$	12
risch	$-\frac{ix}{b}+\frac{\ln\left(\mathrm{e}^{2ix}+\frac{2ia\,\mathrm{e}^{ix}}{b}-1\right)}{b}$	33

input
```
int(1/(a*sec(x)+b*tan(x)),x,method=_RETURNVERBOSE)
```

output
```
ln(a+b*sin(x))/b
```

3.268.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{a\sec(x)+b\tan(x)}\,dx=\frac{\log\left(b\sin\left(x\right)+a\right)}{b}$$

input
```
integrate(1/(a*sec(x)+b*tan(x)),x, algorithm="fricas")
```

output
```
log(b*sin(x) + a)/b
```

3.268.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(8) = 16.

Time = 0.13 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.91

$$\int \frac{1}{a\sec(x)+b\tan(x)}\,dx=\begin{cases}\frac{\log\left(\frac{a\sec\left(x\right)}{b}+\tan\left(x\right)\right)}{b}-\frac{\log\left(\tan^{2}\left(x\right)+1\right)}{2b} & \text{for } b\neq 0\\ \frac{\tan\left(x\right)}{a\sec\left(x\right)} & \text{otherwise}\end{cases}$$

input
```
integrate(1/(a*sec(x)+b*tan(x)),x)
```

output
```
Piecewise((log(a*sec(x)/b + tan(x))/b - log(tan(x)**2 + 1)/(2*b), Ne(b, 0)
), (tan(x)/(a*sec(x)), True))
```

3.268.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. 2(11) = 22.

Time = 0.30 (sec) , antiderivative size = 50, normalized size of antiderivative = 4.55

$$\int \frac{1}{a\sec(x)+b\tan(x)}\,dx=\frac{\log\left(a+\frac{2\,b\sin(x)}{\cos(x)+1}+\frac{a\sin(x)^{2}}{(\cos(x)+1)^{2}}\right)}{b}-\frac{\log\left(\frac{\sin(x)^{2}}{(\cos(x)+1)^{2}}+1\right)}{b}$$

input
```
integrate(1/(a*sec(x)+b*tan(x)),x, algorithm="maxima")
```

output
```
log(a + 2*b*sin(x)/(cos(x) + 1) + a*sin(x)^2/(cos(x) + 1)^2)/b - log(sin(x
)^2/(cos(x) + 1)^2 + 1)/b
```

3.268.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{1}{a\sec(x) + b\tan(x)}\,dx = \frac{\log(|b\sin(x) + a|)}{b}$$

input
```
integrate(1/(a*sec(x)+b*tan(x)),x, algorithm="giac")
```

output
```
log(abs(b*sin(x) + a))/b
```

3.268.9 Mupad [B] (verification not implemented)

Time = 31.00 (sec) , antiderivative size = 55, normalized size of antiderivative = 5.00

$$\int \frac{1}{a\sec(x) + b\tan(x)}\,dx = \frac{2\,\mathrm{atanh}\left(\frac{b\left(2\,a^3\,\sin(x)+\frac{5\,a^2\,b}{2}-b^3-\frac{a^2\,b\,\cos(2\,x)}{2}\right)}{(2\,a^2+\sin(x)\,a\,b-b^2)^2}\right)}{b}$$

input
```
int(1/(b*tan(x) + a/cos(x)),x)
```

output
```
(2*atanh((b*(2*a^3*sin(x) + (5*a^2*b)/2 - b^3 - (a^2*b*cos(2*x))/2))/(2*a^
2 - b^2 + a*b*sin(x))^2))/b
```

3.269 $\int \frac{1}{(a\sec(x)+b\tan(x))^2}\,dx$

3.269.1 Optimal result

Integrand size = 11, antiderivative size = 66

$$\int \frac{1}{(a\sec(x)+b\tan(x))^2}\,dx=-\frac{x}{b^2}+\frac{2a\arctan\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b^2\sqrt{a^2-b^2}}-\frac{\cos(x)}{b(a+b\sin(x))}$$

output
```
-x/b^2-cos(x)/b/(a+b*sin(x))+2*a*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/
b^2/(a^2-b^2)^(1/2)
```

3.269.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 344 vs. 2(66) = 132.

Time = 1.61 (sec) , antiderivative size = 344, normalized size of antiderivative = 5.21

$$\int \frac{1}{(a\sec(x)+b\tan(x))^2}\,dx$$
$$=\frac{\cos(x)(1+\sin(x))\left(2a(a-b)\text{arctanh}\left(\frac{\sqrt{a-b}\sqrt{-\frac{b(1+\sin(x))}{a-b}}}{\sqrt{a+b}\sqrt{-\frac{b(-1+\sin(x))}{a+b}}}\right)\sqrt{1-\sin(x)}(a+b\sin(x))+\sqrt{a+b}\left(-2a\sqrt{a\right.}}{(a-}$$

input
```
Integrate[(a*Sec[x] + b*Tan[x])^(-2),x]
```

output
```
(Cos[x]*(1 + Sin[x])*(2*a*(a - b)*ArcTanh[(Sqrt[a - b]*Sqrt[-((b*(1 + Sin[
x]))/(a - b))])/(Sqrt[a + b]*Sqrt[-((b*(-1 + Sin[x]))/(a + b))])]*Sqrt[1 -
 Sin[x]]*(a + b*Sin[x]) + Sqrt[a + b]*(-2*a*Sqrt[a - b]*ArcTanh[Sqrt[(b*(1
 + Sin[x]))/(-a + b)]/Sqrt[(b - b*Sin[x])/(a + b)]]*Sqrt[1 - Sin[x]]*(a +
b*Sin[x]) - (-a + b)*Sqrt[(b - b*Sin[x])/(a + b)]*(Sqrt[a - b]*(a + b)*Sqr
t[1 - Sin[x]]*Sqrt[-((b*(1 + Sin[x]))/(a - b))] + 2*Sqrt[b]*ArcSinh[(Sqrt[
a - b]*Sqrt[-((b*(1 + Sin[x]))/(a - b))])/(Sqrt[2]*Sqrt[b])]*(a + b*Sin[x]
)))))/((a - b)^(5/2)*(a + b)^(3/2)*Sqrt[1 - Sin[x]]*(-((b*(1 + Sin[x]))/(a
 - b)))^(3/2)*Sqrt[(b - b*Sin[x])/(a + b)]*(a + b*Sin[x]))
```

3.269.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.17, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.909$, Rules used = {3042, 4891, 3042, 3172, 3042, 3214, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(x)+b\tan(x))^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(a\sec(x)+b\tan(x))^2}dx$$
$$\Big\downarrow \text{4891}$$
$$\int \frac{\cos^2(x)}{(a+b\sin(x))^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos(x)^2}{(a+b\sin(x))^2}dx$$
$$\Big\downarrow \text{3172}$$
$$-\frac{\int \frac{\sin(x)}{a+b\sin(x)}dx}{b}-\frac{\cos(x)}{b(a+b\sin(x))}$$
$$\Big\downarrow \text{3042}$$
$$-\frac{\int \frac{\sin(x)}{a+b\sin(x)}dx}{b}-\frac{\cos(x)}{b(a+b\sin(x))}$$

$$\Big\downarrow \text{3214}$$

$$-\frac{\frac{x}{b}-\frac{a\int \frac{1}{a+b\sin(x)}dx}{b}}{b}-\frac{\cos(x)}{b(a+b\sin(x))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{x}{b}-\frac{a\int \frac{1}{a+b\sin(x)}dx}{b}}{b}-\frac{\cos(x)}{b(a+b\sin(x))}$$

$$\Big\downarrow \text{3139}$$

$$-\frac{\frac{x}{b}-\frac{2a\int \frac{1}{a\tan^2\left(\frac{x}{2}\right)+2b\tan\left(\frac{x}{2}\right)+a}d\tan\left(\frac{x}{2}\right)}{b}}{b}-\frac{\cos(x)}{b(a+b\sin(x))}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{\frac{4a\int \frac{1}{-\left(2b+2a\tan\left(\frac{x}{2}\right)\right)^2-4\left(a^2-b^2\right)}d\left(2b+2a\tan\left(\frac{x}{2}\right)\right)}{b}+\frac{x}{b}}{b}-\frac{\cos(x)}{b(a+b\sin(x))}$$

$$\Big\downarrow \text{217}$$

$$-\frac{\frac{x}{b}-\frac{2a\arctan\left(\frac{2a\tan\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}}}{b}-\frac{\cos(x)}{b(a+b\sin(x))}$$

input
```
Int[(a*Sec[x] + b*Tan[x])^(-2),x]
```

output
```
-((x/b - (2*a*ArcTan[(2*b + 2*a*Tan[x/2])/(2*Sqrt[a^2 - b^2])])/(b*Sqrt[a^
2 - b^2]))/b) - Cos[x]/(b*(a + b*Sin[x]))
```

3.269.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

3.269. $\int \frac{1}{(a\sec(x)+b\tan(x))^2}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3172
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Simp[g^2*((p - 1)/(b*(m + 1)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Fre
eQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && I
ntegersQ[2*m, 2*p]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 4891
```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.269.4 Maple [A] (verified)

Time = 1.68 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.39

method	result	size
default	$-\frac{2\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{b^{2}}+\frac{\frac{2\left(-\frac{b^{2}\tan\left(\frac{x}{2}\right)}{a}-b\right)}{\tan\left(\frac{x}{2}\right)^{2}a+2b\tan\left(\frac{x}{2}\right)+a}+\frac{2a\arctan\left(\frac{2a\tan\left(\frac{x}{2}\right)+2b}{2\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}}}{b^{2}}$	92
risch	$-\frac{x}{b^{2}}-\frac{2\left(ib+a\,{\mathrm e}^{ix}\right)}{b^{2}\left(b\,{\mathrm e}^{2ix}-b+2ia\,{\mathrm e}^{ix}\right)}-\frac{a\ln\left({\mathrm e}^{ix}+\frac{ia\sqrt{-a^{2}+b^{2}}-a^{2}+b^{2}}{\sqrt{-a^{2}+b^{2}}\,b}\right)}{\sqrt{-a^{2}+b^{2}}\,b^{2}}+\frac{a\ln\left({\mathrm e}^{ix}+\frac{ia\sqrt{-a^{2}+b^{2}}+a^{2}-b^{2}}{\sqrt{-a^{2}+b^{2}}\,b}\right)}{\sqrt{-a^{2}+b^{2}}\,b^{2}}$	172

input
```
int(1/(a*sec(x)+b*tan(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/b^2*arctan(tan(1/2*x))+2/b^2*((-b^2/a*tan(1/2*x)-b)/(tan(1/2*x)^2*a+2*b
*tan(1/2*x)+a)+a/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)
^(1/2)))
```

3.269.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(60) = 120.

Time = 0.29 (sec) , antiderivative size = 308, normalized size of antiderivative = 4.67

$$\int \frac{1}{(a\sec(x)+b\tan(x))^2}\,dx$$
$$= \left[-\frac{2\,(a^2b-b^3)x\sin(x)+(ab\sin(x)+a^2)\sqrt{-a^2+b^2}\log\left(\frac{(2\,a^2-b^2)\cos(x)^2-2\,ab\sin(x)-a^2-b^2+2\,(a\cos(x)\sin(x)+b\cos(}{b^2\cos(x)^2-2\,ab\sin(x)-a^2-b^2}\right.}{2\,(a^3b^2-ab^4+(a^2b^3-b^5)\sin(x))}\right.$$
$$-\frac{(a^2b-b^3)x\sin(x)+(ab\sin(x)+a^2)\sqrt{a^2-b^2}\arctan\left(-\frac{a\sin(x)+b}{\sqrt{a^2-b^2}\cos(x)}\right)+(a^3-ab^2)x+(a^2b-b^3)\cos(}{a^3b^2-ab^4+(a^2b^3-b^5)\sin(x)}$$

input
```
integrate(1/(a*sec(x)+b*tan(x))^2,x, algorithm="fricas")
```

output
```
[-1/2*(2*(a^2*b - b^3)*x*sin(x) + (a*b*sin(x) + a^2)*sqrt(-a^2 + b^2)*log(
((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2 + 2*(a*cos(x)*sin(x) +
b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2)) + 2
*(a^3 - a*b^2)*x + 2*(a^2*b - b^3)*cos(x))/(a^3*b^2 - a*b^4 + (a^2*b^3 - b
^5)*sin(x)), -((a^2*b - b^3)*x*sin(x) + (a*b*sin(x) + a^2)*sqrt(a^2 - b^2)
*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)*cos(x))) + (a^3 - a*b^2)*x + (a^2
*b - b^3)*cos(x))/(a^3*b^2 - a*b^4 + (a^2*b^3 - b^5)*sin(x))]
```

3.269.6 Sympy [F]

$$\int \frac{1}{(a\sec(x)+b\tan(x))^2}\,dx = \int \frac{1}{(a\sec(x)+b\tan(x))^2}\,dx$$

input
```
integrate(1/(a*sec(x)+b*tan(x))**2,x)
```

output
```
Integral((a*sec(x) + b*tan(x))**(-2), x)
```

3.269.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a\sec(x)+b\tan(x))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a*sec(x)+b*tan(x))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.269.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.42

$$\int \frac{1}{(a\sec(x)+b\tan(x))^2}\,dx = \frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^2-b^2}}\right)\right)a}{\sqrt{a^2-b^2}b^2} - \frac{x}{b^2} - \frac{2\left(b\tan\left(\frac{1}{2}x\right)+a\right)}{\left(a\tan\left(\frac{1}{2}x\right)^2+2\,b\tan\left(\frac{1}{2}x\right)+a\right)ab}$$

input
```
integrate(1/(a*sec(x)+b*tan(x))^2,x, algorithm="giac")
```

output
```
2*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x) + b)/sqrt(a^2 -
b^2)))*a/(sqrt(a^2 - b^2)*b^2) - x/b^2 - 2*(b*tan(1/2*x) + a)/((a*tan(1/2*
x)^2 + 2*b*tan(1/2*x) + a)*a*b)
```

3.269.9 Mupad [B] (verification not implemented)

Time = 29.68 (sec) , antiderivative size = 604, normalized size of antiderivative = 9.15

$$\int \frac{1}{(a\sec(x)+b\tan(x))^2}\,dx = -\frac{x}{b^2} - \frac{\frac{2\tan\left(\frac{x}{2}\right)}{a}+\frac{2}{b}}{a\tan\left(\frac{x}{2}\right)^2+2b\tan\left(\frac{x}{2}\right)+a}$$

$$-\frac{a\,\mathrm{atan}\left(\frac{\frac{a\left(\frac{32a^2}{b}+\frac{32\tan\left(\frac{x}{2}\right)\left(2ab^3-2a^3b\right)}{b^3}+\frac{a\left(32ab^2+64a^2b\tan\left(\frac{x}{2}\right)+\frac{a\left(32a^2b^3+\frac{32\tan\left(\frac{x}{2}\right)\left(3ab^7-2a^3b^5\right)}{b^3}\right)}{b^2\sqrt{b^2-a^2}}\right)}{b^2\sqrt{b^2-a^2}}\right)1i}{b^2\sqrt{b^2-a^2}}+\frac{a\left(\frac{32a^2}{b}+\cdots\right)\cdots}{\cdots}}{\frac{128a^2\tan\left(\frac{x}{2}\right)}{b^3}+\frac{a\left(\frac{32a^2}{b}+\frac{32\tan\left(\frac{x}{2}\right)\left(2ab^3-2a^3b\right)}{b^3}+\frac{a\left(32ab^2+64a^2b\tan\left(\frac{x}{2}\right)+\frac{a\left(32a^2b^3+\frac{32\tan\left(\frac{x}{2}\right)\left(3ab^7-2a^3b^5\right)}{b^3}\right)}{b^2\sqrt{b^2-a^2}}\right)}{b^2\sqrt{b^2-a^2}}\right)}{b^2\sqrt{b^2-a^2}}-\frac{a\left(\frac{32a^2}{b}\cdots\right)}{\cdots}}\right)\cdots}{b^2\sqrt{b^2-a^2}}$$

input
```
int(1/(b*tan(x) + a/cos(x))^2,x)
```

output
```
- x/b^2 - ((2*tan(x/2))/a + 2/b)/(a + 2*b*tan(x/2) + a*tan(x/2)^2) - (a*at
an(((a*((32*a^2)/b + (32*tan(x/2)*(2*a*b^3 - 2*a^3*b))/b^3 + (a*(32*a*b^2
+ 64*a^2*b*tan(x/2) + (a*(32*a^2*b^3 + (32*tan(x/2)*(3*a*b^7 - 2*a^3*b^5))
/b^3))/(b^2*(b^2 - a^2)^(1/2))))/(b^2*(b^2 - a^2)^(1/2)))*1i)/(b^2*(b^2 -
a^2)^(1/2)) + (a*((32*a^2)/b + (32*tan(x/2)*(2*a*b^3 - 2*a^3*b))/b^3 - (a*
(32*a*b^2 + 64*a^2*b*tan(x/2) - (a*(32*a^2*b^3 + (32*tan(x/2)*(3*a*b^7 - 2
*a^3*b^5))/b^3))/(b^2*(b^2 - a^2)^(1/2))))/(b^2*(b^2 - a^2)^(1/2)))*1i)/(b
^2*(b^2 - a^2)^(1/2)))/((128*a^2*tan(x/2))/b^3 + (a*((32*a^2)/b + (32*tan(
x/2)*(2*a*b^3 - 2*a^3*b))/b^3 + (a*(32*a*b^2 + 64*a^2*b*tan(x/2) + (a*(32*
a^2*b^3 + (32*tan(x/2)*(3*a*b^7 - 2*a^3*b^5))/b^3))/(b^2*(b^2 - a^2)^(1/2)
)))/(b^2*(b^2 - a^2)^(1/2))))/(b^2*(b^2 - a^2)^(1/2)) - (a*((32*a^2)/b + (
32*tan(x/2)*(2*a*b^3 - 2*a^3*b))/b^3 - (a*(32*a*b^2 + 64*a^2*b*tan(x/2) -
(a*(32*a^2*b^3 + (32*tan(x/2)*(3*a*b^7 - 2*a^3*b^5))/b^3))/(b^2*(b^2 - a^2
)^(1/2))))/(b^2*(b^2 - a^2)^(1/2))))/(b^2*(b^2 - a^2)^(1/2))))*2i)/(b^2*(b
^2 - a^2)^(1/2))
```

3.270 $\int \frac{1}{(a\sec(x)+b\tan(x))^3}\,dx$

3.270.1 Optimal result

Integrand size = 11, antiderivative size = 51

$$\int \frac{1}{(a\sec(x)+b\tan(x))^3}\,dx=-\frac{\log(a+b\sin(x))}{b^3}+\frac{a^2-b^2}{2b^3(a+b\sin(x))^2}-\frac{2a}{b^3(a+b\sin(x))}$$

output
```
-ln(a+b*sin(x))/b^3+1/2*(a^2-b^2)/b^3/(a+b*sin(x))^2-2*a/b^3/(a+b*sin(x))
```

3.270.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int \frac{1}{(a\sec(x)+b\tan(x))^3}\,dx=-\frac{\log(a+b\sin(x))+\frac{3a^2+b^2+4ab\sin(x)}{2(a+b\sin(x))^2}}{b^3}$$

input
```
Integrate[(a*Sec[x] + b*Tan[x])^(-3),x]
```

output
```
-((Log[a + b*Sin[x]] + (3*a^2 + b^2 + 4*a*b*Sin[x])/(2*(a + b*Sin[x])^2))/
b^3)
```

3.270.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {3042, 4891, 3042, 3147, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(x)+b\tan(x))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a\sec(x)+b\tan(x))^3}dx$$

$$\Big\downarrow \text{4891}$$

$$\int \frac{\cos^3(x)}{(a+b\sin(x))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(x)^3}{(a+b\sin(x))^3}dx$$

$$\Big\downarrow \text{3147}$$

$$\frac{\int \frac{b^2-b^2\sin^2(x)}{(a+b\sin(x))^3}d(b\sin(x))}{b^3}$$

$$\Big\downarrow \text{476}$$

$$\frac{\int \left(\frac{2a}{(a+b\sin(x))^2}+\frac{1}{-a-b\sin(x)}+\frac{b^2-a^2}{(a+b\sin(x))^3}\right)d(b\sin(x))}{b^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{a^2-b^2}{2(a+b\sin(x))^2}-\frac{2a}{a+b\sin(x)}-\log(a+b\sin(x))}{b^3}$$

input
```
Int[(a*Sec[x] + b*Tan[x])^(-3),x]
```

output
```
(-Log[a + b*Sin[x]] + (a^2 - b^2)/(2*(a + b*Sin[x])^2) - (2*a)/(a + b*Sin[
x]))/b^3
```

3.270.3.1 Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 4891
```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.270.4 Maple [A] (verified)

Time = 7.00 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.98

method	result	size
default	$-\frac{\ln(a+b\sin(x))}{b^3}-\frac{-a^2+b^2}{2b^3(a+b\sin(x))^2}-\frac{2a}{b^3(a+b\sin(x))}$	50
risch	$\frac{ix}{b^3}-\frac{2i(3ia^2\mathrm{e}^{2ix}+ib^2\mathrm{e}^{2ix}+2ba\,\mathrm{e}^{3ix}-2ab\,\mathrm{e}^{ix})}{(b\,\mathrm{e}^{2ix}-b+2ia\,\mathrm{e}^{ix})^2 b^3}-\frac{\ln\left(\mathrm{e}^{2ix}+\frac{2ia\,\mathrm{e}^{ix}}{b}-1\right)}{b^3}$	103

input
```
int(1/(a*sec(x)+b*tan(x))^3,x,method=_RETURNVERBOSE)
```

output
```
-ln(a+b*sin(x))/b^3-1/2*(-a^2+b^2)/b^3/(a+b*sin(x))^2-2*a/b^3/(a+b*sin(x))
```

3.270.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.63

$$\int \frac{1}{(a\sec(x)+b\tan(x))^3}\,dx = \frac{4\,ab\sin\left(x\right)+3\,a^2+b^2-2\,\left(b^2\cos\left(x\right)^2-2\,ab\sin\left(x\right)-a^2-b^2\right)\log\left(b\sin\left(x\right)+a\right)}{2\,\left(b^5\cos\left(x\right)^2-2\,ab^4\sin\left(x\right)-a^2b^3-b^5\right)}$$

input
```
integrate(1/(a*sec(x)+b*tan(x))^3,x, algorithm="fricas")
```

output
```
1/2*(4*a*b*sin(x) + 3*a^2 + b^2 - 2*(b^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b
^2)*log(b*sin(x) + a))/(b^5*cos(x)^2 - 2*a*b^4*sin(x) - a^2*b^3 - b^5)
```

3.270.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 503 vs. $2(46) = 92$.

Time = 1.06 (sec) , antiderivative size = 503, normalized size of antiderivative = 9.86

$$\int \frac{1}{(a\sec(x)+b\tan(x))^3}\,dx = \begin{cases} -\frac{2a^2\log\left(\frac{a\sec\left(x\right)}{b}+\tan\left(x\right)\right)\sec^2\left(x\right)}{2a^2b^3\sec^2\left(x\right)+4ab^4\tan\left(x\right)\sec\left(x\right)+2b^5\tan^2\left(x\right)}+\frac{a^2\log\left(\tan^2\left(x\right)+1\right)\sec^2\left(x\right)}{2a^2b^3\sec^2\left(x\right)+4ab^4\tan\left(x\right)\sec\left(x\right)+2b^5\tan^2\left(x\right)}-\frac{a^2\sec^2\left(x\right)}{2a^2b^3\sec^2\left(x\right)+4ab^4\tan\left(x\right)\sec\left(x\right.} \\ \frac{\frac{2\tan^3\left(x\right)}{3\sec^3\left(x\right)}+\frac{\tan\left(x\right)}{\sec^3\left(x\right)}}{a^3} \end{cases}$$

input
```
integrate(1/(a*sec(x)+b*tan(x))**3,x)
```

output
```
Piecewise((-2*a**2*log(a*sec(x)/b + tan(x))*sec(x)**2/(2*a**2*b**3*sec(x)*
*2 + 4*a*b**4*tan(x)*sec(x) + 2*b**5*tan(x)**2) + a**2*log(tan(x)**2 + 1)*
sec(x)**2/(2*a**2*b**3*sec(x)**2 + 4*a*b**4*tan(x)*sec(x) + 2*b**5*tan(x)*
*2) - a**2*sec(x)**2/(2*a**2*b**3*sec(x)**2 + 4*a*b**4*tan(x)*sec(x) + 2*b
**5*tan(x)**2) - 4*a*b*log(a*sec(x)/b + tan(x))*tan(x)*sec(x)/(2*a**2*b**3
*sec(x)**2 + 4*a*b**4*tan(x)*sec(x) + 2*b**5*tan(x)**2) + 2*a*b*log(tan(x)
**2 + 1)*tan(x)*sec(x)/(2*a**2*b**3*sec(x)**2 + 4*a*b**4*tan(x)*sec(x) + 2
*b**5*tan(x)**2) - 2*b**2*log(a*sec(x)/b + tan(x))*tan(x)**2/(2*a**2*b**3*
sec(x)**2 + 4*a*b**4*tan(x)*sec(x) + 2*b**5*tan(x)**2) + b**2*log(tan(x)**
2 + 1)*tan(x)**2/(2*a**2*b**3*sec(x)**2 + 4*a*b**4*tan(x)*sec(x) + 2*b**5*
tan(x)**2) + b**2*tan(x)**2/(2*a**2*b**3*sec(x)**2 + 4*a*b**4*tan(x)*sec(x
) + 2*b**5*tan(x)**2) - b**2/(2*a**2*b**3*sec(x)**2 + 4*a*b**4*tan(x)*sec(
x) + 2*b**5*tan(x)**2), Ne(b, 0)), ((2*tan(x)**3/(3*sec(x)**3) + tan(x)/se
c(x)**3)/a**3, True))
```

3.270.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(49) = 98$.

Time = 0.31 (sec) , antiderivative size = 201, normalized size of antiderivative = 3.94

$$\int \frac{1}{(a\sec(x)+b\tan(x))^3}\,dx = \frac{2\left(\frac{(a^3+ab^2)\sin(x)}{\cos(x)+1}+\frac{(3a^2b+b^3)\sin(x)^2}{(\cos(x)+1)^2}+\frac{(a^3+ab^2)\sin(x)^3}{(\cos(x)+1)^3}\right)}{a^4b^2+\frac{4a^3b^3\sin(x)}{\cos(x)+1}+\frac{4a^3b^3\sin(x)^3}{(\cos(x)+1)^3}+\frac{a^4b^2\sin(x)^4}{(\cos(x)+1)^4}+\frac{2(a^4b^2+2a^2b^4)\sin(x)^2}{(\cos(x)+1)^2}} - \frac{\log\left(a+\frac{2b\sin(x)}{\cos(x)+1}+\frac{a\sin(x)^2}{(\cos(x)+1)^2}\right)}{b^3}+\frac{\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{b^3}$$

input
```
integrate(1/(a*sec(x)+b*tan(x))^3,x, algorithm="maxima")
```

output
```
2*((a^3 + a*b^2)*sin(x)/(cos(x) + 1) + (3*a^2*b + b^3)*sin(x)^2/(cos(x) +
1)^2 + (a^3 + a*b^2)*sin(x)^3/(cos(x) + 1)^3)/(a^4*b^2 + 4*a^3*b^3*sin(x)/
(cos(x) + 1) + 4*a^3*b^3*sin(x)^3/(cos(x) + 1)^3 + a^4*b^2*sin(x)^4/(cos(x
) + 1)^4 + 2*(a^4*b^2 + 2*a^2*b^4)*sin(x)^2/(cos(x) + 1)^2) - log(a + 2*b*
sin(x)/(cos(x) + 1) + a*sin(x)^2/(cos(x) + 1)^2)/b^3 + log(sin(x)^2/(cos(x
) + 1)^2 + 1)/b^3
```

3.270.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int \frac{1}{(a\sec(x)+b\tan(x))^3}\,dx = -\frac{\log(|b\sin(x)+a|)}{b^3}+\frac{3\,b\sin(x)^2+2\,a\sin(x)-b}{2\,(b\sin(x)+a)^2b^2}$$

input
```
integrate(1/(a*sec(x)+b*tan(x))^3,x, algorithm="giac")
```

output
```
-log(abs(b*sin(x) + a))/b^3 + 1/2*(3*b*sin(x)^2 + 2*a*sin(x) - b)/((b*sin(
x) + a)^2*b^2)
```

3.270.9 Mupad [B] (verification not implemented)

Time = 29.63 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.08

$$\int \frac{1}{(a\sec(x)+b\tan(x))^3}\,dx = \frac{2\,a^3\,b\,\sin(x)+3\,a^2\,b^2\sin(x)^2+2\,a\,b^3\,\sin(x)+b^4\sin(x)^2}{2\,a^4\,b^3+4\,a^3\,b^4\,\sin(x)+2\,a^2\,b^5\sin(x)^2} - \frac{2\,\mathrm{atanh}\left(\frac{b^2+a\,\sin(x)\,b}{2\,a^2+\sin(x)\,a\,b-b^2}\right)}{b^3}$$

input
```
int(1/(b*tan(x) + a/cos(x))^3,x)
```

output
```
(b^4*sin(x)^2 + 3*a^2*b^2*sin(x)^2 + 2*a*b^3*sin(x) + 2*a^3*b*sin(x))/(2*a
^4*b^3 + 2*a^2*b^5*sin(x)^2 + 4*a^3*b^4*sin(x)) - (2*atanh((b^2 + a*b*sin(
x))/(2*a^2 - b^2 + a*b*sin(x))))/b^3
```

3.271 $\int \frac{1}{(a\sec(x)+b\tan(x))^4}\,dx$

3.271.1 Optimal result

Integrand size = 11, antiderivative size = 156

$$\int \frac{1}{(a\sec(x)+b\tan(x))^4}\,dx = \frac{x}{b^4} - \frac{a(2a^2-3b^2)\arctan\left(\frac{b+a\tan(\frac{x}{2})}{\sqrt{a^2-b^2}}\right)}{b^4(a^2-b^2)^{3/2}} - \frac{\cos^3(x)}{3b(a+b\sin(x))^3} + \frac{a\cos^3(x)}{2b(a^2-b^2)(a+b\sin(x))^2} + \frac{\cos(x)\left(2(a^2-b^2)+ab\sin(x)\right)}{2b^3(a^2-b^2)(a+b\sin(x))}$$

output
```
x/b^4-a*(2*a^2-3*b^2)*arctan((b+a*tan(1/2*x))/(a^2-b^2)^(1/2))/b^4/(a^2-b^
2)^(3/2)-1/3*cos(x)^3/b/(a+b*sin(x))^3+1/2*a*cos(x)^3/b/(a^2-b^2)/(a+b*sin
(x))^2+1/2*cos(x)*(2*a^2-2*b^2+a*b*sin(x))/b^3/(a^2-b^2)/(a+b*sin(x))
```

3.271.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2661 vs. 2(156) = 312.

Time = 6.85 (sec) , antiderivative size = 2661, normalized size of antiderivative = 17.06

$$\int \frac{1}{(a\sec(x)+b\tan(x))^4}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a*Sec[x] + b*Tan[x])^(-4),x]
```

output
```
(Sec[x]*(a + b*Sin[x])^4*(-1/3*(b*(-(b/(a - b)) - (b*Sin[x])/(a - b))^(5/2
)*(b/(a + b) - (b*Sin[x])/(a + b))^(5/2))/(((a*b)/(a - b) - b^2/(a - b))*(
(a*b)/(a + b) + b^2/(a + b))*(a + b*Sin[x])^3) - ((a*b^3*(-(b/(a - b)) - (
b*Sin[x])/(a - b))^(5/2)*(b/(a + b) - (b*Sin[x])/(a + b))^(5/2))/(2*(a^2 -
 b^2)*((a*b)/(a - b) - b^2/(a - b))*((a*b)/(a + b) + b^2/(a + b))*(a + b*S
in[x])^2) - (-((((-3*a^2*b^5)/((a - b)^2*(a + b)^2) + (2*b^5*(3*a^2 - 2*b^
2))/((a - b)^2*(a + b)^2))*(-(b/(a - b)) - (b*Sin[x])/(a - b))^(5/2)*(b/(a
 + b) - (b*Sin[x])/(a + b))^(5/2))/(((a*b)/(a - b) - b^2/(a - b))*((a*b)/(
a + b) + b^2/(a + b))*(a + b*Sin[x]))) - ((16*Sqrt[2]*b^6*(3*a^2 - 4*b^2)*
(-(b/(a - b)) - (b*Sin[x])/(a - b))^(5/2)*Sqrt[b/(a + b) - (b*Sin[x])/(a +
 b)]*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[x])/(a - b)))/(2*b))^(5/2)*((5*(
1/(2*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[x])/(a - b)))/(2*b))^2) + (1 + (
(a - b)*(-(b/(a - b)) - (b*Sin[x])/(a - b)))/(2*b))^(-1)))/8 - (15*b^3*(((
a - b)*(-(b/(a - b)) - (b*Sin[x])/(a - b)))/b - ((a - b)^2*(-(b/(a - b)) -
 (b*Sin[x])/(a - b))^2)/(3*b^2) - (Sqrt[2]*Sqrt[a - b]*ArcSinh[(Sqrt[a - b
]*Sqrt[-(b/(a - b)) - (b*Sin[x])/(a - b)])/(Sqrt[2]*Sqrt[b])]*Sqrt[-(b/(a
- b)) - (b*Sin[x])/(a - b)])/(Sqrt[b]*Sqrt[1 + ((a - b)*(-(b/(a - b)) - (b
*Sin[x])/(a - b)))/(2*b)])))/(32*(a - b)^3*(-(b/(a - b)) - (b*Sin[x])/(a -
 b))^3*(1 + ((a - b)*(-(b/(a - b)) - (b*Sin[x])/(a - b)))/(2*b))^2)))/(5*(
a - b)^2*(a + b)^4*Sqrt[((a + b)*(b/(a + b) - (b*Sin[x])/(a + b)))/b]) ...
```

3.271.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.17, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.364$, Rules used = {3042, 4891, 3042, 3172, 3042, 3343, 3042, 3342, 25, 3042, 3214, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(x)+b\tan(x))^4}\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{1}{(a\sec(x)+b\tan(x))^4}dx$$
$$\Big\downarrow \ 4891$$
$$\int \frac{\cos^4(x)}{(a+b\sin(x))^4}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\cos(x)^4}{(a+b\sin(x))^4}dx$$

$$\Big\downarrow \; 3172$$

$$-\frac{\int \frac{\cos^2(x)\sin(x)}{(a+b\sin(x))^3}dx}{b}-\frac{\cos^3(x)}{3b(a+b\sin(x))^3}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\int \frac{\cos(x)^2\sin(x)}{(a+b\sin(x))^3}dx}{b}-\frac{\cos^3(x)}{3b(a+b\sin(x))^3}$$

$$\Big\downarrow \; 3343$$

$$-\frac{-\frac{\int \frac{\cos^2(x)(2b+a\sin(x))}{(a+b\sin(x))^2}dx}{2(a^2-b^2)}-\frac{a\cos^3(x)}{2(a^2-b^2)(a+b\sin(x))^2}}{b}-\frac{\cos^3(x)}{3b(a+b\sin(x))^3}$$

$$\Big\downarrow \; 3042$$

$$-\frac{-\frac{\int \frac{\cos(x)^2(2b+a\sin(x))}{(a+b\sin(x))^2}dx}{2(a^2-b^2)}-\frac{a\cos^3(x)}{2(a^2-b^2)(a+b\sin(x))^2}}{b}-\frac{\cos^3(x)}{3b(a+b\sin(x))^3}$$

$$\Big\downarrow \; 3342$$

$$-\frac{-\frac{\frac{\cos(x)\left(2\left(a^2-b^2\right)+ab\sin(x)\right)}{b^2(a+b\sin(x))}-\frac{\int -\frac{ab+2\left(a^2-b^2\right)\sin(x)}{a+b\sin(x)}dx}{b^2}}{2(a^2-b^2)}-\frac{a\cos^3(x)}{2(a^2-b^2)(a+b\sin(x))^2}}{b}-\frac{\cos^3(x)}{3b(a+b\sin(x))^3}$$

$$\Big\downarrow \; 25$$

$$-\frac{-\frac{\frac{\int \frac{ab+2\left(a^2-b^2\right)\sin(x)}{a+b\sin(x)}dx}{b^2}+\frac{\cos(x)\left(2\left(a^2-b^2\right)+ab\sin(x)\right)}{b^2(a+b\sin(x))}}{2(a^2-b^2)}-\frac{a\cos^3(x)}{2(a^2-b^2)(a+b\sin(x))^2}}{b}-\frac{\cos^3(x)}{3b(a+b\sin(x))^3}$$

$$\Big\downarrow \; 3042$$

$$-\frac{-\frac{\frac{\int \frac{ab+2\left(a^2-b^2\right)\sin(x)}{a+b\sin(x)}dx}{b^2}+\frac{\cos(x)\left(2\left(a^2-b^2\right)+ab\sin(x)\right)}{b^2(a+b\sin(x))}}{2(a^2-b^2)}-\frac{a\cos^3(x)}{2(a^2-b^2)(a+b\sin(x))^2}}{b}-\frac{\cos^3(x)}{3b(a+b\sin(x))^3}$$

$$\Big\downarrow \; 3214$$

3.271. $\int \frac{1}{(a\sec(x)+b\tan(x))^4}\,dx$

$$-\frac{-\frac{\frac{\frac{2x(a^2-b^2)}{b}-\frac{a(2a^2-3b^2)\int\frac{1}{a+b\sin(x)}dx}{b}}{b^2}+\frac{\cos(x)(2(a^2-b^2)+ab\sin(x))}{b^2(a+b\sin(x))}}{2(a^2-b^2)}-\frac{a\cos^3(x)}{2(a^2-b^2)(a+b\sin(x))^2}}{b}-\frac{\cos^3(x)}{3b(a+b\sin(x))^3}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{-\frac{\frac{\frac{2x(a^2-b^2)}{b}-\frac{a(2a^2-3b^2)\int\frac{1}{a+b\sin(x)}dx}{b}}{b^2}+\frac{\cos(x)(2(a^2-b^2)+ab\sin(x))}{b^2(a+b\sin(x))}}{2(a^2-b^2)}-\frac{a\cos^3(x)}{2(a^2-b^2)(a+b\sin(x))^2}}{b}-\frac{\cos^3(x)}{3b(a+b\sin(x))^3}$$

$$\Big\downarrow \text{ 3139}$$

$$-\frac{-\frac{\frac{\frac{2x(a^2-b^2)}{b}-\frac{2a(2a^2-3b^2)\int\frac{1}{a\tan^2\left(\frac{x}{2}\right)+2b\tan\left(\frac{x}{2}\right)+a}d\tan\left(\frac{x}{2}\right)}{b}}{b^2}+\frac{\cos(x)(2(a^2-b^2)+ab\sin(x))}{b^2(a+b\sin(x))}}{2(a^2-b^2)}-\frac{a\cos^3(x)}{2(a^2-b^2)(a+b\sin(x))^2}}{b}-\frac{\cos^3(x)}{3b(a+b\sin(x))^3}$$

$$\Big\downarrow \text{ 1083}$$

$$-\frac{-\frac{\frac{\frac{4a(2a^2-3b^2)\int\frac{1}{-\left(2b+2a\tan\left(\frac{x}{2}\right)\right)^2-4(a^2-b^2)}d\left(2b+2a\tan\left(\frac{x}{2}\right)\right)}{b}+\frac{2x(a^2-b^2)}{b}}{b^2}+\frac{\cos(x)(2(a^2-b^2)+ab\sin(x))}{b^2(a+b\sin(x))}}{2(a^2-b^2)}-\frac{a\cos^3(x)}{2(a^2-b^2)(a+b\sin(x))^2}}{b}-\frac{\cos^3(x)}{3b(a+b\sin(x))^3}$$

$$\Big\downarrow \text{ 217}$$

$$-\frac{-\frac{\frac{\frac{2x(a^2-b^2)}{b}-\frac{2a(2a^2-3b^2)\arctan\left(\frac{2a\tan\left(\frac{x}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}}}{b^2}+\frac{\cos(x)(2(a^2-b^2)+ab\sin(x))}{b^2(a+b\sin(x))}}{2(a^2-b^2)}-\frac{a\cos^3(x)}{2(a^2-b^2)(a+b\sin(x))^2}}{b}-\frac{\cos^3(x)}{3b(a+b\sin(x))^3}$$

input
```
Int[(a*Sec[x] + b*Tan[x])^(-4),x]
```

output
```
-1/3*Cos[x]^3/(b*(a + b*Sin[x])^3) - (-1/2*(a*Cos[x]^3)/((a^2 - b^2)*(a +
b*Sin[x])^2) - (((2*(a^2 - b^2)*x)/b - (2*a*(2*a^2 - 3*b^2)*ArcTan[(2*b +
2*a*Tan[x/2])/(2*Sqrt[a^2 - b^2])])/(b*Sqrt[a^2 - b^2]))/b^2 + (Cos[x]*(2*
(a^2 - b^2) + a*b*Sin[x]))/(b^2*(a + b*Sin[x])))/(2*(a^2 - b^2)))/b
```

3.271. $\int\frac{1}{(a\sec(x)+b\tan(x))^4}\,dx$

3.271.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3172
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Simp[g^2*((p - 1)/(b*(m + 1)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Fre
eQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && I
ntegersQ[2*m, 2*p]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

3.271. $\int \frac{1}{(a\sec(x)+b\tan(x))^4}\,dx$

rule 3342
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*C
os[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x])/(b^2*f*(m + 1)*(m + p + 1))), x] + Simp[g^2*((
p - 1)/(b^2*(m + 1)*(m + p + 1)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

rule 3343
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Simp[1/((a^2 - b^2)*(m + 1))   Int[(g*Cos[e + f*x])^p
*(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ
[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 4891
```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.271.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(143) = 286$.

Time = 28.20 (sec) , antiderivative size = 358, normalized size of antiderivative = 2.29

method	result
default	$\frac{2\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{b^4} - \frac{2\left(\frac{-\frac{b^2\left(a^4-2a^2b^2+2b^4\right)\tan\left(\frac{x}{2}\right)^5}{2a\left(a^2-b^2\right)} - \frac{b\left(2a^6+3a^4b^2-4a^2b^4+4b^6\right)\tan\left(\frac{x}{2}\right)^4}{2\left(a^2-b^2\right)a^2} - \frac{b^2\left(18a^6-3a^4b^2-4a^2b^4+4b^6\right)\tan\left(\frac{x}{2}\right)^3}{3a^3\left(a^2-b^2\right)} - b}{\left(\tan\left(\frac{x}{2}\right)^2a+2b\tan\left(\frac{x}{2}\right)+a\right.}}{}$
risch	$\frac{x}{b^4} - \frac{i(-54ib\,a^4e^{4ix}+27ib^3a^2e^{4ix}+12ib^5e^{4ix}-18b^2a^3e^{5ix}+15b^4a\,e^{5ix}+78ia^4b\,e^{2ix}-36ia^2b^3e^{2ix}-12ib^5e^{2ix}+44a^5e^{3ix}+34a^3b^2e^{3ix}}{3(-ib\,e^{2ix}+ib+2a\,e^{ix})^3(a^2-b^2)b^4}$

input
```
int(1/(a*sec(x)+b*tan(x))^4,x,method=_RETURNVERBOSE)
```

output
```
2/b^4*arctan(tan(1/2*x))-2/b^4*((-1/2*b^2*(a^4-2*a^2*b^2+2*b^4)/a/(a^2-b^2
)*tan(1/2*x)^5-1/2*b*(2*a^6+3*a^4*b^2-4*a^2*b^4+4*b^6)/(a^2-b^2)/a^2*tan(1
/2*x)^4-1/3/a^3*b^2*(18*a^6-3*a^4*b^2-4*a^2*b^4+4*b^6)/(a^2-b^2)*tan(1/2*x
)^3-b*(2*a^6+8*a^4*b^2-7*a^2*b^4+2*b^6)/(a^2-b^2)/a^2*tan(1/2*x)^2-1/2*b^2
*(11*a^4-8*a^2*b^2+2*b^4)/a/(a^2-b^2)*tan(1/2*x)-1/6*(6*a^4-5*a^2*b^2+2*b^
4)*b/(a^2-b^2))/(tan(1/2*x)^2*a+2*b*tan(1/2*x)+a)^3+1/2*a*(2*a^2-3*b^2)/(a
^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2)))
```

3.271.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 433 vs. $2(143) = 286$.

Time = 0.32 (sec) , antiderivative size = 931, normalized size of antiderivative = 5.97

$$\int \frac{1}{(a\sec(x) + b\tan(x))^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*sec(x)+b*tan(x))^4,x, algorithm="fricas")
```

output
```
[-1/12*(36*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*x*cos(x)^2 + 2*(11*a^4*b^3 - 19*a
^2*b^5 + 8*b^7)*cos(x)^3 + 3*(2*a^6 + 3*a^4*b^2 - 9*a^2*b^4 - 3*(2*a^4*b^2
 - 3*a^2*b^4)*cos(x)^2 + (6*a^5*b - 7*a^3*b^3 - 3*a*b^5 - (2*a^3*b^3 - 3*a
*b^5)*cos(x)^2)*sin(x))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(x)^2 - 2*
a*b*sin(x) - a^2 - b^2 - 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/
(b^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2)) - 12*(a^7 + a^5*b^2 - 5*a^3*b^4
 + 3*a*b^6)*x - 12*(a^6*b - 2*a^2*b^5 + b^7)*cos(x) + 6*(2*(a^4*b^3 - 2*a^
2*b^5 + b^7)*x*cos(x)^2 - 2*(3*a^6*b - 5*a^4*b^3 + a^2*b^5 + b^7)*x - (5*a
^5*b^2 - 8*a^3*b^4 + 3*a*b^6)*cos(x))*sin(x))/(a^7*b^4 + a^5*b^6 - 5*a^3*b
^8 + 3*a*b^10 - 3*(a^5*b^6 - 2*a^3*b^8 + a*b^10)*cos(x)^2 + (3*a^6*b^5 - 5
*a^4*b^7 + a^2*b^9 + b^11 - (a^4*b^7 - 2*a^2*b^9 + b^11)*cos(x)^2)*sin(x))
, -1/6*(18*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*x*cos(x)^2 + (11*a^4*b^3 - 19*a^2
*b^5 + 8*b^7)*cos(x)^3 - 3*(2*a^6 + 3*a^4*b^2 - 9*a^2*b^4 - 3*(2*a^4*b^2 -
 3*a^2*b^4)*cos(x)^2 + (6*a^5*b - 7*a^3*b^3 - 3*a*b^5 - (2*a^3*b^3 - 3*a*b
^5)*cos(x)^2)*sin(x))*sqrt(a^2 - b^2)*arctan(-(a*sin(x) + b)/(sqrt(a^2 - b
^2)*cos(x))) - 6*(a^7 + a^5*b^2 - 5*a^3*b^4 + 3*a*b^6)*x - 6*(a^6*b - 2*a^
2*b^5 + b^7)*cos(x) + 3*(2*(a^4*b^3 - 2*a^2*b^5 + b^7)*x*cos(x)^2 - 2*(3*a
^6*b - 5*a^4*b^3 + a^2*b^5 + b^7)*x - (5*a^5*b^2 - 8*a^3*b^4 + 3*a*b^6)*co
s(x))*sin(x))/(a^7*b^4 + a^5*b^6 - 5*a^3*b^8 + 3*a*b^10 - 3*(a^5*b^6 - 2*a
^3*b^8 + a*b^10)*cos(x)^2 + (3*a^6*b^5 - 5*a^4*b^7 + a^2*b^9 + b^11 - (...
```

3.271.6 Sympy [F]

$$\int \frac{1}{(a\sec(x)+b\tan(x))^4}\,dx = \int \frac{1}{(a\sec{(x)}+b\tan{(x)})^4}\,dx$$

input
```
integrate(1/(a*sec(x)+b*tan(x))**4,x)
```

output
```
Integral((a*sec(x) + b*tan(x))**(-4), x)
```

3.271.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a\sec(x)+b\tan(x))^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a*sec(x)+b*tan(x))^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.271.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 369 vs. 2(143) = 286.

Time = 0.28 (sec) , antiderivative size = 369, normalized size of antiderivative = 2.37

$$\int \frac{1}{(a\sec(x)+b\tan(x))^4}\,dx = -\frac{(2a^3-3ab^2)\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{(a^2b^4-b^6)\sqrt{a^2-b^2}} + \frac{3a^6b\tan\left(\frac{1}{2}x\right)^5-6a^4b^3\tan\left(\frac{1}{2}x\right)^5+6a^2b^5\tan\left(\frac{1}{2}x\right)^5+6a^7\tan\left(\frac{1}{2}x\right)^4+9a^5b^2\tan\left(\frac{1}{2}x\right)^4-12a^3b^4\tan}{} + \frac{x}{b^4}$$

input
```
integrate(1/(a*sec(x)+b*tan(x))^4,x, algorithm="giac")
```

output
```
-(2*a^3 - 3*a*b^2)*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x)
 + b)/sqrt(a^2 - b^2)))/((a^2*b^4 - b^6)*sqrt(a^2 - b^2)) + 1/3*(3*a^6*b*t
an(1/2*x)^5 - 6*a^4*b^3*tan(1/2*x)^5 + 6*a^2*b^5*tan(1/2*x)^5 + 6*a^7*tan(
1/2*x)^4 + 9*a^5*b^2*tan(1/2*x)^4 - 12*a^3*b^4*tan(1/2*x)^4 + 12*a*b^6*tan
(1/2*x)^4 + 36*a^6*b*tan(1/2*x)^3 - 6*a^4*b^3*tan(1/2*x)^3 - 8*a^2*b^5*tan
(1/2*x)^3 + 8*b^7*tan(1/2*x)^3 + 12*a^7*tan(1/2*x)^2 + 48*a^5*b^2*tan(1/2*
x)^2 - 42*a^3*b^4*tan(1/2*x)^2 + 12*a*b^6*tan(1/2*x)^2 + 33*a^6*b*tan(1/2*
x) - 24*a^4*b^3*tan(1/2*x) + 6*a^2*b^5*tan(1/2*x) + 6*a^7 - 5*a^5*b^2 + 2*
a^3*b^4)/((a^5*b^3 - a^3*b^5)*(a*tan(1/2*x)^2 + 2*b*tan(1/2*x) + a)^3) + x
/b^4
```

3.271.9 Mupad [B] (verification not implemented)

Time = 33.16 (sec) , antiderivative size = 2782, normalized size of antiderivative = 17.83

$$\int \frac{1}{(a\sec(x)+b\tan(x))^4}\,dx = \text{Too large to display}$$

input
```
int(1/(b*tan(x) + a/cos(x))^4,x)
```

output
```
((6*a^4 + 2*b^4 - 5*a^2*b^2)/(3*b^3*(a^2 - b^2)) + (tan(x/2)*(11*a^4 + 2*b
^4 - 8*a^2*b^2))/(a*b^2*(a^2 - b^2)) + (tan(x/2)^5*(a^4 + 2*b^4 - 2*a^2*b^
2))/(a*b^2*(a^2 - b^2)) + (tan(x/2)^4*(2*a^6 + 4*b^6 - 4*a^2*b^4 + 3*a^4*b
^2))/(a^2*b^3*(a^2 - b^2)) + (2*tan(x/2)^2*(2*a^6 + 2*b^6 - 7*a^2*b^4 + 8*
a^4*b^2))/(a^2*b^3*(a^2 - b^2)) + (2*tan(x/2)^3*(3*a^2 + 2*b^2)*(6*a^4 + 2
*b^4 - 5*a^2*b^2))/(3*a^3*b^2*(a^2 - b^2)))/(tan(x/2)^2*(12*a*b^2 + 3*a^3)
 + tan(x/2)^4*(12*a*b^2 + 3*a^3) + tan(x/2)^3*(12*a^2*b + 8*b^3) + a^3 + a
^3*tan(x/2)^6 + 6*a^2*b*tan(x/2)^5 + 6*a^2*b*tan(x/2)) + (2*atan((48*a^3*b
^3*tan(x/2))/((176*a^3*b^15)/(b^12 - 2*a^2*b^10 + a^4*b^8) - (160*a^5*b^13
)/(b^12 - 2*a^2*b^10 + a^4*b^8) + (48*a^7*b^11)/(b^12 - 2*a^2*b^10 + a^4*b
^8) - (64*a*b^17)/(b^12 - 2*a^2*b^10 + a^4*b^8)) - (64*a*b^5*tan(x/2))/((1
76*a^3*b^15)/(b^12 - 2*a^2*b^10 + a^4*b^8) - (160*a^5*b^13)/(b^12 - 2*a^2*
b^10 + a^4*b^8) + (48*a^7*b^11)/(b^12 - 2*a^2*b^10 + a^4*b^8) - (64*a*b^17
)/(b^12 - 2*a^2*b^10 + a^4*b^8))))/b^4 + (a*atan(((a*(2*a^2 - 3*b^2)*(-(a
+ b)^3*(a - b)^3)^(1/2)*((8*(4*a^2*b^7 - 8*a^4*b^5 + 4*a^6*b^3))/(b^12 - 2
*a^2*b^10 + a^4*b^8) + (8*tan(x/2)*(8*a*b^9 - 29*a^3*b^7 + 28*a^5*b^5 - 8*
a^7*b^3))/(b^13 - 2*a^2*b^11 + a^4*b^9) - (a*(2*a^2 - 3*b^2)*(-(a + b)^3*(
a - b)^3)^(1/2)*((8*(4*a*b^12 - 6*a^3*b^10 + 2*a^5*b^8))/(b^12 - 2*a^2*b^1
0 + a^4*b^8) + (8*tan(x/2)*(12*a^2*b^12 - 20*a^4*b^10 + 8*a^6*b^8))/(b^13
- 2*a^2*b^11 + a^4*b^9) - (a*((8*(4*a^2*b^15 - 8*a^4*b^13 + 4*a^6*b^11)...
```

3.271. $\int \frac{1}{(a\sec(x)+b\tan(x))^4}\,dx$

3.272 $\int \frac{1}{(a\sec(x)+b\tan(x))^5}\,dx$

3.272.1 Optimal result

Integrand size = 11, antiderivative size = 101

$$\int \frac{1}{(a\sec(x)+b\tan(x))^5}\,dx=\frac{\log(a+b\sin(x))}{b^5}-\frac{(a^2-b^2)^2}{4b^5(a+b\sin(x))^4}+\frac{4a(a^2-b^2)}{3b^5(a+b\sin(x))^3}-\frac{3a^2-b^2}{b^5(a+b\sin(x))^2}+\frac{4a}{b^5(a+b\sin(x))}$$

output
```
ln(a+b*sin(x))/b^5-1/4*(a^2-b^2)^2/b^5/(a+b*sin(x))^4+4/3*a*(a^2-b^2)/b^5/
(a+b*sin(x))^3+(-3*a^2+b^2)/b^5/(a+b*sin(x))^2+4*a/b^5/(a+b*sin(x))
```

3.272.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.85

$$\int \frac{1}{(a\sec(x)+b\tan(x))^5}\,dx=\frac{\log(a+b\sin(x))+\frac{25a^4+2a^2b^2-3b^4+8ab(11a^2+b^2)\sin(x)+12b^2(9a^2+b^2)\sin^2(x)+48ab^3\sin^3(x)}{12(a+b\sin(x))^4}}{b^5}$$

input
```
Integrate[(a*Sec[x] + b*Tan[x])^(-5),x]
```

output
```
(Log[a + b*Sin[x]] + (25*a^4 + 2*a^2*b^2 - 3*b^4 + 8*a*b*(11*a^2 + b^2)*Si
n[x] + 12*b^2*(9*a^2 + b^2)*Sin[x]^2 + 48*a*b^3*Sin[x]^3)/(12*(a + b*Sin[x
])^4))/b^5
```

3.272.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.88, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {3042, 4891, 3042, 3147, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\sec(x)+b\tan(x))^5}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{(a\sec(x)+b\tan(x))^5}dx$$

$$\Big\downarrow \text{ 4891}$$

$$\int \frac{\cos^5(x)}{(a+b\sin(x))^5}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(x)^5}{(a+b\sin(x))^5}dx$$

$$\Big\downarrow \text{ 3147}$$

$$\frac{\int \frac{(b^2-b^2\sin^2(x))^2}{(a+b\sin(x))^5}d(b\sin(x))}{b^5}$$

$$\Big\downarrow \text{ 476}$$

$$\frac{\int \left(\frac{(a^2-b^2)^2}{(a+b\sin(x))^5}+\frac{1}{a+b\sin(x)}-\frac{4a}{(a+b\sin(x))^2}+\frac{2(3a^2-b^2)}{(a+b\sin(x))^3}-\frac{4(a^3-ab^2)}{(a+b\sin(x))^4}\right)d(b\sin(x))}{b^5}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{(a^2-b^2)^2}{4(a+b\sin(x))^4}+\frac{4a(a^2-b^2)}{3(a+b\sin(x))^3}-\frac{3a^2-b^2}{(a+b\sin(x))^2}+\frac{4a}{a+b\sin(x)}+\log(a+b\sin(x))}{b^5}$$

input
```
Int[(a*Sec[x] + b*Tan[x])^(-5),x]
```

output
```
(Log[a + b*Sin[x]] - (a^2 - b^2)^2/(4*(a + b*Sin[x])^4) + (4*a*(a^2 - b^2)
)/(3*(a + b*Sin[x])^3) - (3*a^2 - b^2)/(a + b*Sin[x])^2 + (4*a)/(a + b*Sin
[x]))/b^5
```

3.272.3.1 Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 4891
```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.272.4 Maple [A] (verified)

Time = 163.53 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.01

method	result
default	$\frac{\ln(a+b\sin(x))}{b^5}+\frac{4a(a^2-b^2)}{3b^5(a+b\sin(x))^3}-\frac{a^4-2a^2b^2+b^4}{4b^5(a+b\sin(x))^4}+\frac{4a}{b^5(a+b\sin(x))}-\frac{6a^2-2b^2}{2b^5(a+b\sin(x))^2}$
risch	$-\frac{ix}{b^5}+\frac{8iab^3e^{7ix}-\frac{176ia^3b\,e^{5ix}}{3}-\frac{88iab^3e^{5ix}}{3}-36a^2b^2e^{6ix}-4b^4e^{6ix}+\frac{176ia^3b\,e^{3ix}}{3}+\frac{88iab^3e^{3ix}}{3}+\frac{100a^4e^{4ix}}{3}+\frac{224a^2b^2e^{4ix}}{3}+4b^4e^{4ix}-8}{(-ib\,e^{2ix}+ib+2a\,e^{ix})^4b^5}$

input
```
int(1/(a*sec(x)+b*tan(x))^5,x,method=_RETURNVERBOSE)
```

output
```
ln(a+b*sin(x))/b^5+4/3*a*(a^2-b^2)/b^5/(a+b*sin(x))^3-1/4*(a^4-2*a^2*b^2+b
^4)/b^5/(a+b*sin(x))^4+4*a/b^5/(a+b*sin(x))-1/2*(6*a^2-2*b^2)/b^5/(a+b*sin
(x))^2
```

3.272.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. 2(97) = 194.

Time = 0.27 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.15

$$\int \frac{1}{(a\sec(x)+b\tan(x))^5}\,dx$$
$$= \frac{25\,a^4+110\,a^2b^2+9\,b^4-12\,(9\,a^2b^2+b^4)\cos{(x)}^2+12\,\left(b^4\cos{(x)}^4+a^4+6\,a^2b^2+b^4-2\,(3\,a^2b^2+b^4)\cos\right.}{12\,\left(b^9\cos{(x)}^4+a^4b^5+6\,a^2b^7+b^9-2\,(3\,a^2b^7+b^9\right.}$$

input
```
integrate(1/(a*sec(x)+b*tan(x))^5,x, algorithm="fricas")
```

output
```
1/12*(25*a^4 + 110*a^2*b^2 + 9*b^4 - 12*(9*a^2*b^2 + b^4)*cos(x)^2 + 12*(b
^4*cos(x)^4 + a^4 + 6*a^2*b^2 + b^4 - 2*(3*a^2*b^2 + b^4)*cos(x)^2 - 4*(a*
b^3*cos(x)^2 - a^3*b - a*b^3)*sin(x))*log(b*sin(x) + a) - 8*(6*a*b^3*cos(x
)^2 - 11*a^3*b - 7*a*b^3)*sin(x))/(b^9*cos(x)^4 + a^4*b^5 + 6*a^2*b^7 + b^
9 - 2*(3*a^2*b^7 + b^9)*cos(x)^2 - 4*(a*b^8*cos(x)^2 - a^3*b^6 - a*b^8)*si
n(x))
```

3.272.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1719 vs. 2(90) = 180.

Time = 6.00 (sec) , antiderivative size = 1719, normalized size of antiderivative = 17.02

$$\int \frac{1}{(a\sec(x)+b\tan(x))^5}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*sec(x)+b*tan(x))**5,x)
```

output
```
Piecewise((36*a**4*log(a*sec(x)/b + tan(x))*sec(x)**4/(36*a**4*b**5*sec(x)
**4 + 144*a**3*b**6*tan(x)*sec(x)**3 + 216*a**2*b**7*tan(x)**2*sec(x)**2 +
 144*a*b**8*tan(x)**3*sec(x) + 36*b**9*tan(x)**4) - 18*a**4*log(tan(x)**2
+ 1)*sec(x)**4/(36*a**4*b**5*sec(x)**4 + 144*a**3*b**6*tan(x)*sec(x)**3 +
216*a**2*b**7*tan(x)**2*sec(x)**2 + 144*a*b**8*tan(x)**3*sec(x) + 36*b**9*
tan(x)**4) + 20*a**4*sec(x)**4/(36*a**4*b**5*sec(x)**4 + 144*a**3*b**6*tan
(x)*sec(x)**3 + 216*a**2*b**7*tan(x)**2*sec(x)**2 + 144*a*b**8*tan(x)**3*s
ec(x) + 36*b**9*tan(x)**4) + 144*a**3*b*log(a*sec(x)/b + tan(x))*tan(x)*se
c(x)**3/(36*a**4*b**5*sec(x)**4 + 144*a**3*b**6*tan(x)*sec(x)**3 + 216*a**
2*b**7*tan(x)**2*sec(x)**2 + 144*a*b**8*tan(x)**3*sec(x) + 36*b**9*tan(x)*
*4) - 72*a**3*b*log(tan(x)**2 + 1)*tan(x)*sec(x)**3/(36*a**4*b**5*sec(x)**
4 + 144*a**3*b**6*tan(x)*sec(x)**3 + 216*a**2*b**7*tan(x)**2*sec(x)**2 + 1
44*a*b**8*tan(x)**3*sec(x) + 36*b**9*tan(x)**4) + 44*a**3*b*tan(x)*sec(x)*
*3/(36*a**4*b**5*sec(x)**4 + 144*a**3*b**6*tan(x)*sec(x)**3 + 216*a**2*b**
7*tan(x)**2*sec(x)**2 + 144*a*b**8*tan(x)**3*sec(x) + 36*b**9*tan(x)**4) +
 216*a**2*b**2*log(a*sec(x)/b + tan(x))*tan(x)**2*sec(x)**2/(36*a**4*b**5*
sec(x)**4 + 144*a**3*b**6*tan(x)*sec(x)**3 + 216*a**2*b**7*tan(x)**2*sec(x
)**2 + 144*a*b**8*tan(x)**3*sec(x) + 36*b**9*tan(x)**4) - 108*a**2*b**2*lo
g(tan(x)**2 + 1)*tan(x)**2*sec(x)**2/(36*a**4*b**5*sec(x)**4 + 144*a**3*b*
*6*tan(x)*sec(x)**3 + 216*a**2*b**7*tan(x)**2*sec(x)**2 + 144*a*b**8*ta...
```

3.272.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 483 vs. $2(97) = 194$.

Time = 0.34 (sec) , antiderivative size = 483, normalized size of antiderivative = 4.78

$$\int \frac{1}{(a\sec(x)+b\tan(x))^5}\,dx = -\frac{2\left(\frac{3(a^7-a^3b^4)\sin(x)}{\cos(x)+1}+\frac{3(7a^6b-3a^2b^5)\sin(x)^2}{(\cos(x)+1)^2}+\frac{(9a^7+52a^5b^2-a^3b^4-12ab^6)\sin(x)^3}{(\cos(x)+1)^3}+\frac{2(21a^6b+25a^4b^3-7a^2b^5-3b^7)\sin(x)^4}{(\cos(x)+1)^4} - \cdots\right.}{3\left(a^8b^4+\frac{8a^7b^5\sin(x)}{\cos(x)+1}+\frac{8a^7b^5\sin(x)^7}{(\cos(x)+1)^7}+\frac{a^8b^4\sin(x)^8}{(\cos(x)+1)^8}+\frac{4(a^8b^4+6a^6b^6)\sin(x)^2}{(\cos(x)+1)^2}+\frac{8(3a^7b^5+4a^5b^7)\sin(x)^3}{(\cos(x)+1)^3}+\frac{2(3a^8b\cdots}{\cdots}\right.} + \frac{\log\left(a+\frac{2b\sin(x)}{\cos(x)+1}+\frac{a\sin(x)^2}{(\cos(x)+1)^2}\right)}{b^5} - \frac{\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{b^5}$$

input
```
integrate(1/(a*sec(x)+b*tan(x))^5,x, algorithm="maxima")
```

output
```
-2/3*(3*(a^7 - a^3*b^4)*sin(x)/(cos(x) + 1) + 3*(7*a^6*b - 3*a^2*b^5)*sin(
x)^2/(cos(x) + 1)^2 + (9*a^7 + 52*a^5*b^2 - a^3*b^4 - 12*a*b^6)*sin(x)^3/(
cos(x) + 1)^3 + 2*(21*a^6*b + 25*a^4*b^3 - 7*a^2*b^5 - 3*b^7)*sin(x)^4/(co
s(x) + 1)^4 + (9*a^7 + 52*a^5*b^2 - a^3*b^4 - 12*a*b^6)*sin(x)^5/(cos(x) +
 1)^5 + 3*(7*a^6*b - 3*a^2*b^5)*sin(x)^6/(cos(x) + 1)^6 + 3*(a^7 - a^3*b^4
)*sin(x)^7/(cos(x) + 1)^7)/(a^8*b^4 + 8*a^7*b^5*sin(x)/(cos(x) + 1) + 8*a^
7*b^5*sin(x)^7/(cos(x) + 1)^7 + a^8*b^4*sin(x)^8/(cos(x) + 1)^8 + 4*(a^8*b
^4 + 6*a^6*b^6)*sin(x)^2/(cos(x) + 1)^2 + 8*(3*a^7*b^5 + 4*a^5*b^7)*sin(x)
^3/(cos(x) + 1)^3 + 2*(3*a^8*b^4 + 24*a^6*b^6 + 8*a^4*b^8)*sin(x)^4/(cos(x
) + 1)^4 + 8*(3*a^7*b^5 + 4*a^5*b^7)*sin(x)^5/(cos(x) + 1)^5 + 4*(a^8*b^4
+ 6*a^6*b^6)*sin(x)^6/(cos(x) + 1)^6) + log(a + 2*b*sin(x)/(cos(x) + 1) +
a*sin(x)^2/(cos(x) + 1)^2)/b^5 - log(sin(x)^2/(cos(x) + 1)^2 + 1)/b^5
```

3.272.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.90

$$\int \frac{1}{(a\sec(x)+b\tan(x))^5}\,dx = \frac{\log\left(|b\sin(x)+a|\right)}{b^5} - \frac{25\,b^3\sin(x)^4 + 52\,ab^2\sin(x)^3 + 42\,a^2b\sin(x)^2 - 12\,b^3\sin(x)^2 + 12\,a^3\sin(x) - 8\,ab^2\sin(x) - 2\,a^2b + 3}{12\,(b\sin(x)+a)^4b^4}$$

input
```
integrate(1/(a*sec(x)+b*tan(x))^5,x, algorithm="giac")
```

output
```
log(abs(b*sin(x) + a))/b^5 - 1/12*(25*b^3*sin(x)^4 + 52*a*b^2*sin(x)^3 + 4
2*a^2*b*sin(x)^2 - 12*b^3*sin(x)^2 + 12*a^3*sin(x) - 8*a*b^2*sin(x) - 2*a^
2*b + 3*b^3)/((b*sin(x) + a)^4*b^4)
```

3.272.9 Mupad [B] (verification not implemented)

Time = 30.44 (sec) , antiderivative size = 541, normalized size of antiderivative = 5.36

$$\int \frac{1}{(a\sec(x)+b\tan(x))^5}\,dx = \frac{2\,\mathrm{atanh}\left(\frac{16\,a}{\frac{32\,a^3}{b^2}-16\,a\tan\left(\frac{x}{2}\right)^2-16\,a+\frac{32\,a^2\tan\left(\frac{x}{2}\right)}{b}+\frac{32\,a^3\tan\left(\frac{x}{2}\right)^2}{b^2}} + \frac{16\,a\tan\left(\frac{x}{2}\right)^2}{\frac{32\,a^3}{b^2}-16\,a\tan\left(\frac{x}{2}\right)^2-16\,a+\frac{32\,a^2\tan\left(\frac{x}{2}\right)}{b}+\frac{32\,a^3\tan\left(\frac{x}{2}\right)^2}{b^2}} + \frac{}{32\,a^2\tan(}\right.}{b^5} - \frac{\frac{2\tan\left(\frac{x}{2}\right)^2\left(7\,a^4-3\,b^4\right)}{a^2\,b^3} + \frac{2\tan\left(\frac{x}{2}\right)^6\left(7\,a^4-3\,b^4\right)}{a^2\,b^3} + \frac{2\tan\left(\frac{x}{2}\right)\left(a^4-b^4\right)}{a\,b^4} + \frac{2\tan\left(\frac{x}{2}\right)^7\left(a^4-b^4\right)}{a\,b^4} + \frac{4\tan\left(\frac{x}{2}\right)^4\left(21\,a^6+\right.}{3}}{\tan\left(\frac{x}{2}\right)^2\left(4\,a^4+24\,a^2\,b^2\right)+\tan\left(\frac{x}{2}\right)^6\left(4\,a^4+24\,a^2\,b^2\right)+\tan\left(\frac{x}{2}\right)^3\left(24\,a^3\,b+32\,a\,b^3\right)+\tan\left(\frac{x}{2}\right)^5\left(24\,a^3\,b\right.}$$

input
```
int(1/(b*tan(x) + a/cos(x))^5,x)
```

output
```
(2*atanh((16*a)/((32*a^3)/b^2 - 16*a*tan(x/2)^2 - 16*a + (32*a^2*tan(x/2))
/b + (32*a^3*tan(x/2)^2)/b^2) + (16*a*tan(x/2)^2)/((32*a^3)/b^2 - 16*a*tan
(x/2)^2 - 16*a + (32*a^2*tan(x/2))/b + (32*a^3*tan(x/2)^2)/b^2) + (32*a^2*
tan(x/2))/(32*a^2*tan(x/2) - 16*a*b + (32*a^3)/b + (32*a^3*tan(x/2)^2)/b -
 16*a*b*tan(x/2)^2)))/b^5 - ((2*tan(x/2)^2*(7*a^4 - 3*b^4))/(a^2*b^3) + (2
*tan(x/2)^6*(7*a^4 - 3*b^4))/(a^2*b^3) + (2*tan(x/2)*(a^4 - b^4))/(a*b^4)
+ (2*tan(x/2)^7*(a^4 - b^4))/(a*b^4) + (4*tan(x/2)^4*(21*a^6 - 3*b^6 - 7*a
^2*b^4 + 25*a^4*b^2))/(3*a^4*b^3) + (2*tan(x/2)^3*(9*a^6 - 12*b^6 - a^2*b^
4 + 52*a^4*b^2))/(3*a^3*b^4) + (2*tan(x/2)^5*(9*a^6 - 12*b^6 - a^2*b^4 + 5
2*a^4*b^2))/(3*a^3*b^4))/(tan(x/2)^2*(4*a^4 + 24*a^2*b^2) + tan(x/2)^6*(4*
a^4 + 24*a^2*b^2) + tan(x/2)^3*(32*a*b^3 + 24*a^3*b) + tan(x/2)^5*(32*a*b^
3 + 24*a^3*b) + tan(x/2)^4*(6*a^4 + 16*b^4 + 48*a^2*b^2) + a^4 + a^4*tan(x
/2)^8 + 8*a^3*b*tan(x/2)^7 + 8*a^3*b*tan(x/2))
```

3.272. $\int \frac{1}{(a\sec(x)+b\tan(x))^5}\,dx$

3.273 $\int (\sec(x)+\tan(x))^5 \, dx$

3.273.1 Optimal result

Integrand size = 7, antiderivative size = 30

$$\int (\sec(x)+\tan(x))^5 \, dx = -\log(1-\sin(x)) + \frac{2}{(1-\sin(x))^2} - \frac{4}{1-\sin(x)}$$

output
```
-ln(1-sin(x))+2/(1-sin(x))^2-4/(1-sin(x))
```

3.273.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.80

$$\int (\sec(x)+\tan(x))^5 \, dx = \operatorname{arctanh}(\sin(x)) - \log(\cos(x)) + \frac{5\sec^4(x)}{4} + \sec(x)\tan(x) - \sec^3(x)\tan(x) - \frac{\tan^2(x)}{2} + 5\sec(x)\tan^3(x) + \frac{11\tan^4(x)}{4}$$

input
```
Integrate[(Sec[x] + Tan[x])^5,x]
```

output
```
ArcTanh[Sin[x]] - Log[Cos[x]] + (5*Sec[x]^4)/4 + Sec[x]*Tan[x] - Sec[x]^3*
Tan[x] - Tan[x]^2/2 + 5*Sec[x]*Tan[x]^3 + (11*Tan[x]^4)/4
```

3.273.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {3042, 4891, 3042, 3146, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\tan(x)+\sec(x))^5\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (\tan(x)+\sec(x))^5dx$$

$$\Big\downarrow \text{4891}$$

$$\int (\sin(x)+1)^5\sec^5(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(\sin(x)+1)^5}{\cos(x)^5}dx$$

$$\Big\downarrow \text{3146}$$

$$\int \frac{(\sin(x)+1)^2}{(1-\sin(x))^3}d\sin(x)$$

$$\Big\downarrow \text{49}$$

$$\int \left(-\frac{4}{(\sin(x)-1)^2}-\frac{4}{(\sin(x)-1)^3}+\frac{1}{1-\sin(x)}\right)d\sin(x)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4}{1-\sin(x)}+\frac{2}{(1-\sin(x))^2}-\log(1-\sin(x))$$

input
```
Int[(Sec[x] + Tan[x])^5,x]
```

output
```
-Log[1 - Sin[x]] + 2/(1 - Sin[x])^2 - 4/(1 - Sin[x])
```

3.273.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 4891
```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.273.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 8.77 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.70

method	result
risch	$ix+\frac{8i\left(-i\mathrm{e}^{2ix}+\mathrm{e}^{3ix}-\mathrm{e}^{ix}\right)}{\left(\mathrm{e}^{ix}-i\right)^{4}}-2\ln\left(\mathrm{e}^{ix}-i\right)$
parts	$-\left(-\frac{\sec(x)^{3}}{4}-\frac{3\sec(x)}{8}\right)\tan(x)+\ln(\sec(x)+\tan(x))+\frac{11\tan(x)^{4}}{4}-\frac{\tan(x)^{2}}{2}+\frac{\ln\left(1+\tan(x)^{2}\right)}{2}+\frac{5\sin(x)}{2\cos(x)}$
default	$-\left(-\frac{\sec(x)^{3}}{4}-\frac{3\sec(x)}{8}\right)\tan(x)+\ln(\sec(x)+\tan(x))+\frac{5}{4\cos(x)^{4}}+\frac{5\sin(x)^{3}}{2\cos(x)^{4}}+\frac{5\sin(x)^{3}}{4\cos(x)^{2}}-\frac{5\sin(x)}{8}+$

input
```
int((sec(x)+tan(x))^5,x,method=_RETURNVERBOSE)
```

output
```
I*x+8*I/(exp(I*x)-I)^4*(-I*exp(2*I*x)+exp(3*I*x)-exp(I*x))-2*ln(exp(I*x)-I
)
```

3.273.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.27

$$\int (\sec(x)+\tan(x))^5\,dx = -\frac{(\cos(x)^2+2\sin(x)-2)\log(-\sin(x)+1)+4\sin(x)-2}{\cos(x)^2+2\sin(x)-2}$$

input
```
integrate((sec(x)+tan(x))^5,x, algorithm="fricas")
```

output
```
-((cos(x)^2 + 2*sin(x) - 2)*log(-sin(x) + 1) + 4*sin(x) - 2)/(cos(x)^2 + 2
*sin(x) - 2)
```

3.273.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(20) = 40.

Time = 1.98 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.27

$$\int (\sec(x)+\tan(x))^5\,dx = -\frac{\log(\sin(x)-1)}{2}+\frac{\log(\sin(x)+1)}{2}+\frac{\log(\sec^2(x))}{2} + \frac{5\tan^4(x)}{2}+\frac{3\sec^4(x)}{2}-\sec^2(x)+\frac{32\sin^3(x)}{8\sin^4(x)-16\sin^2(x)+8}$$

input
```
integrate((sec(x)+tan(x))**5,x)
```

output
```
-log(sin(x) - 1)/2 + log(sin(x) + 1)/2 + log(sec(x)**2)/2 + 5*tan(x)**4/2
+ 3*sec(x)**4/2 - sec(x)**2 + 32*sin(x)**3/(8*sin(x)**4 - 16*sin(x)**2 + 8
)
```

3.273.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(26) = 52.

Time = 0.22 (sec) , antiderivative size = 141, normalized size of antiderivative = 4.70

$$\int (\sec(x)+\tan(x))^5\,dx = \frac{5}{2}\tan(x)^4 + \frac{5\left(5\sin(x)^3 - 3\sin(x)\right)}{8\left(\sin(x)^4 - 2\sin(x)^2 + 1\right)} - \frac{3\sin(x)^3 - 5\sin(x)}{8\left(\sin(x)^4 - 2\sin(x)^2 + 1\right)} + \frac{5\left(\sin(x)^3 + \sin(x)\right)}{4\left(\sin(x)^4 - 2\sin(x)^2 + 1\right)} + \frac{4\sin(x)^2 - 3}{4\left(\sin(x)^4 - 2\sin(x)^2 + 1\right)} + \frac{5}{4\left(\sin(x)^2 - 1\right)^2} - \frac{1}{2}\log\left(\sin(x)^2 - 1\right) + \frac{1}{2}\log\left(\sin(x) + 1\right) - \frac{1}{2}\log\left(\sin(x) - 1\right)$$

input
```
integrate((sec(x)+tan(x))^5,x, algorithm="maxima")
```

output
```
5/2*tan(x)^4 + 5/8*(5*sin(x)^3 - 3*sin(x))/(sin(x)^4 - 2*sin(x)^2 + 1) - 1
/8*(3*sin(x)^3 - 5*sin(x))/(sin(x)^4 - 2*sin(x)^2 + 1) + 5/4*(sin(x)^3 + s
in(x))/(sin(x)^4 - 2*sin(x)^2 + 1) + 1/4*(4*sin(x)^2 - 3)/(sin(x)^4 - 2*si
n(x)^2 + 1) + 5/4/(sin(x)^2 - 1)^2 - 1/2*log(sin(x)^2 - 1) + 1/2*log(sin(x
) + 1) - 1/2*log(sin(x) - 1)
```

3.273.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. 2(26) = 52.

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.07

$$\int (\sec(x)+\tan(x))^5\,dx = \frac{25\tan\left(\frac{1}{2}x\right)^4 - 100\tan\left(\frac{1}{2}x\right)^3 + 198\tan\left(\frac{1}{2}x\right)^2 - 100\tan\left(\frac{1}{2}x\right) + 25}{6\left(\tan\left(\frac{1}{2}x\right) - 1\right)^4} + \log\left(\tan\left(\frac{1}{2}x\right)^2 + 1\right) - 2\log\left(\left|\tan\left(\frac{1}{2}x\right) - 1\right|\right)$$

input
```
integrate((sec(x)+tan(x))^5,x, algorithm="giac")
```

output
```
1/6*(25*tan(1/2*x)^4 - 100*tan(1/2*x)^3 + 198*tan(1/2*x)^2 - 100*tan(1/2*x
) + 25)/(tan(1/2*x) - 1)^4 + log(tan(1/2*x)^2 + 1) - 2*log(abs(tan(1/2*x)
- 1))
```

3.273.9 Mupad [B] (verification not implemented)

Time = 29.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.97

$$\int (\sec(x)+\tan(x))^5\,dx = \ln\left(\tan\left(\frac{x}{2}\right)^2+1\right) - 2\ \ln\left(\tan\left(\frac{x}{2}\right)-1\right) + \frac{8\tan\left(\frac{x}{2}\right)^2}{\tan\left(\frac{x}{2}\right)^4-4\tan\left(\frac{x}{2}\right)^3+6\tan\left(\frac{x}{2}\right)^2-4\tan\left(\frac{x}{2}\right)+1}$$

input
```
int((tan(x) + 1/cos(x))^5,x)
```

output
```
log(tan(x/2)^2 + 1) - 2*log(tan(x/2) - 1) + (8*tan(x/2)^2)/(6*tan(x/2)^2 -
 4*tan(x/2) - 4*tan(x/2)^3 + tan(x/2)^4 + 1)
```

3.274 $\int (\sec(x)+\tan(x))^4 \, dx$

3.274.1 Optimal result

Integrand size = 7, antiderivative size = 30

$$\int (\sec(x)+\tan(x))^4 \, dx = x+\frac{2\cos^3(x)}{3(1-\sin(x))^3}-\frac{2\cos(x)}{1-\sin(x)}$$

output
```
x+2/3*cos(x)^3/(1-sin(x))^3-2*cos(x)/(1-sin(x))
```

3.274.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 64 vs. 2(30) = 60.

Time = 0.13 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.13

$$\int (\sec(x)+\tan(x))^4 \, dx = -\frac{-3(8+3x)\cos\left(\frac{x}{2}\right)+(16+3x)\cos\left(\frac{3x}{2}\right)+6(4+2x+x\cos(x))\sin\left(\frac{x}{2}\right)}{6\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)^3}$$

input
```
Integrate[(Sec[x] + Tan[x])^4,x]
```

output
```
-1/6*(-3*(8 + 3*x)*Cos[x/2] + (16 + 3*x)*Cos[(3*x)/2] + 6*(4 + 2*x + x*Cos
[x])*Sin[x/2])/(Cos[x/2] - Sin[x/2])^3
```

3.274.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.286, Rules used = {3042, 4891, 3042, 3149, 3042, 3159, 3042, 3159, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\tan(x)+\sec(x))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (\tan(x)+\sec(x))^4dx$$

$$\Big\downarrow \text{4891}$$

$$\int (\sin(x)+1)^4\sec^4(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(\sin(x)+1)^4}{\cos(x)^4}dx$$

$$\Big\downarrow \text{3149}$$

$$\int \frac{\cos^4(x)}{(1-\sin(x))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(x)^4}{(1-\sin(x))^4}dx$$

$$\Big\downarrow \text{3159}$$

$$\frac{2\cos^3(x)}{3(1-\sin(x))^3}-\int \frac{\cos^2(x)}{(1-\sin(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\cos^3(x)}{3(1-\sin(x))^3}-\int \frac{\cos(x)^2}{(1-\sin(x))^2}dx$$

$$\Big\downarrow \text{3159}$$

$$\int 1dx+\frac{2\cos^3(x)}{3(1-\sin(x))^3}-\frac{2\cos(x)}{1-\sin(x)}$$

$$\Big\downarrow \text{24}$$

$$x + \frac{2\cos^3(x)}{3(1-\sin(x))^3} - \frac{2\cos(x)}{1-\sin(x)}$$

input
```
Int[(Sec[x] + Tan[x])^4,x]
```

output
```
x + (2*Cos[x]^3)/(3*(1 - Sin[x])^3) - (2*Cos[x])/(1 - Sin[x])
```

3.274.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3149
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(a/g)^(2*m)   Int[(g*Cos[e + f*x])^(2*m + p)/(
a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 4891
```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.274.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.33 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

method	result
risch	$x-\frac{8\left(-2-3 i \mathrm{e}^{i x}+3 \,\mathrm{e}^{2 i x}\right)}{3\left(\mathrm{e}^{i x}-i\right)^{3}}$
parts	$-\left(-\frac{2}{3}-\frac{\sec(x)^{2}}{3}\right)\tan\left(x\right)+\frac{7\tan(x)^{3}}{3}-\tan\left(x\right)+\arctan\left(\tan\left(x\right)\right)+\frac{8\sec(x)^{3}}{3}-4\sec\left(x\right)$
default	$-\left(-\frac{2}{3}-\frac{\sec(x)^{2}}{3}\right)\tan\left(x\right)+\frac{4}{3\cos(x)^{3}}+\frac{2\sin(x)^{3}}{\cos(x)^{3}}+\frac{4\sin(x)^{4}}{3\cos(x)^{3}}-\frac{4\sin(x)^{4}}{3\cos(x)}-\frac{4\left(2+\sin(x)^{2}\right)\cos(x)}{3}+\frac{\tan(x)^{3}}{3}-$ ta

input
```
int((sec(x)+tan(x))^4,x,method=_RETURNVERBOSE)
```

output
```
x-8/3*(-2-3*I*exp(I*x)+3*exp(2*I*x))/(exp(I*x)-I)^3
```

3.274.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(24) = 48.

Time = 0.24 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.03

$$\int (\sec(x)+\tan(x))^4\,dx = \frac{(3\,x+8)\cos\left(x\right)^{2}-(3\,x-4)\cos\left(x\right)+((3\,x-8)\cos\left(x\right)+6\,x-4)\sin\left(x\right)-6\,x-4}{3\left(\cos\left(x\right)^{2}+(\cos\left(x\right)+2)\sin\left(x\right)-\cos\left(x\right)-2\right)}$$

input
```
integrate((sec(x)+tan(x))^4,x, algorithm="fricas")
```

output
```
1/3*((3*x + 8)*cos(x)^2 - (3*x - 4)*cos(x) + ((3*x - 8)*cos(x) + 6*x - 4)*
sin(x) - 6*x - 4)/(cos(x)^2 + (cos(x) + 2)*sin(x) - cos(x) - 2)
```

3.274.6 Sympy [A] (verification not implemented)

Time = 1.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.47

$$\int (\sec(x)+\tan(x))^4\,dx = x+\frac{\sin^3(x)}{3\cos^3(x)}-\frac{\sin(x)}{\cos(x)}+\frac{7\tan^3(x)}{3}+\tan(x)+\frac{8\sec^3(x)}{3}-4\sec(x)$$

input
```
integrate((sec(x)+tan(x))**4,x)
```

output
```
x + sin(x)**3/(3*cos(x)**3) - sin(x)/cos(x) + 7*tan(x)**3/3 + tan(x) + 8*s
ec(x)**3/3 - 4*sec(x)
```

3.274.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int (\sec(x)+\tan(x))^4\,dx = \frac{8}{3}\tan(x)^3+x-\frac{4\left(3\cos(x)^2-1\right)}{3\cos(x)^3}+\frac{4}{3\cos(x)^3}$$

input
```
integrate((sec(x)+tan(x))^4,x, algorithm="maxima")
```

output
```
8/3*tan(x)^3 + x - 4/3*(3*cos(x)^2 - 1)/cos(x)^3 + 4/3/cos(x)^3
```

3.274.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

$$\int (\sec(x)+\tan(x))^4\,dx = x-\frac{8\left(3\tan\left(\frac{1}{2}x\right)-1\right)}{3\left(\tan\left(\frac{1}{2}x\right)-1\right)^3}$$

input
```
integrate((sec(x)+tan(x))^4,x, algorithm="giac")
```

output
```
x - 8/3*(3*tan(1/2*x) - 1)/(tan(1/2*x) - 1)^3
```

3.274.9 Mupad [B] (verification not implemented)

Time = 28.81 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

$$\int (\sec(x)+\tan(x))^4\,dx = x-\frac{8\tan\left(\frac{x}{2}\right)-\frac{8}{3}}{\left(\tan\left(\frac{x}{2}\right)-1\right)^3}$$

input
```
int((tan(x) + 1/cos(x))^4,x)
```

output
```
x - (8*tan(x/2) - 8/3)/(tan(x/2) - 1)^3
```

3.275 $\int (\sec(x)+\tan(x))^3 \, dx$

3.275.1 Optimal result

Integrand size = 7, antiderivative size = 18

$$\int (\sec(x)+\tan(x))^3 \, dx = \log(1-\sin(x)) + \frac{2}{1-\sin(x)}$$

output
```
ln(1-sin(x))+2/(1-sin(x))
```

3.275.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.72

$$\int (\sec(x)+\tan(x))^3 \, dx = -\text{arctanh}(\sin(x)) + \log(\cos(x)) + \frac{3\sec^2(x)}{2} + 2\sec(x)\tan(x) + \frac{\tan^2(x)}{2}$$

input
```
Integrate[(Sec[x] + Tan[x])^3,x]
```

output
```
-ArcTanh[Sin[x]] + Log[Cos[x]] + (3*Sec[x]^2)/2 + 2*Sec[x]*Tan[x] + Tan[x]
^2/2
```

3.275.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {3042, 4891, 3042, 3146, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\tan(x) + \sec(x))^3 \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int (\tan(x) + \sec(x))^3 dx$$

$$\Big\downarrow \text{4891}$$

$$\int (\sin(x) + 1)^3 \sec^3(x) dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(\sin(x) + 1)^3}{\cos(x)^3} dx$$

$$\Big\downarrow \text{3146}$$

$$\int \frac{\sin(x) + 1}{(1 - \sin(x))^2} d\sin(x)$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{1}{\sin(x) - 1} + \frac{2}{(\sin(x) - 1)^2}\right) d\sin(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{1 - \sin(x)} + \log(1 - \sin(x))$$

input
```
Int[(Sec[x] + Tan[x])^3,x]
```

output
```
Log[1 - Sin[x]] + 2/(1 - Sin[x])
```

3.275.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 4891
```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.275.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.64 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.94

method	result	size
risch	$-ix-\frac{4i\,\mathrm{e}^{ix}}{\left(\mathrm{e}^{ix}-i\right)^{2}}+2\ln\left(\mathrm{e}^{ix}-i\right)$	35
default	$\frac{\sec(x)\tan(x)}{2}-\ln\left(\sec\left(x\right)+\tan\left(x\right)\right)+\frac{3}{2\cos(x)^{2}}+\frac{3\sin(x)^{3}}{2\cos(x)^{2}}+\frac{3\sin(x)}{2}+\frac{\tan(x)^{2}}{2}+\ln\left(\cos\left(x\right)\right)$	45
parts	$\frac{\sec(x)\tan(x)}{2}-\ln\left(\sec\left(x\right)+\tan\left(x\right)\right)+\frac{\tan(x)^{2}}{2}-\frac{\ln\left(1+\tan(x)^{2}\right)}{2}+\frac{3\sin(x)^{3}}{2\cos(x)^{2}}+\frac{3\sin(x)}{2}+\frac{3\sec(x)^{2}}{2}$	51

input
```
int((sec(x)+tan(x))^3,x,method=_RETURNVERBOSE)
```

output
```
-I*x-4*I*exp(I*x)/(exp(I*x)-I)^2+2*ln(exp(I*x)-I)
```

3.275.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17

$$\int (\sec(x)+\tan(x))^3\,dx = \frac{(\sin(x)-1)\log(-\sin(x)+1)-2}{\sin(x)-1}$$

input
```
integrate((sec(x)+tan(x))^3,x, algorithm="fricas")
```

output
```
((sin(x) - 1)*log(-sin(x) + 1) - 2)/(sin(x) - 1)
```

3.275.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. 2(12) = 24.

Time = 1.20 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.44

$$\int (\sec(x)+\tan(x))^3\,dx = \frac{\log(\sin(x)-1)}{2} - \frac{\log(\sin(x)+1)}{2} - \frac{\log(\sec^2(x))}{2} + 2\sec^2(x) - \frac{4\sin(x)}{2\sin^2(x)-2}$$

input
```
integrate((sec(x)+tan(x))**3,x)
```

output
```
log(sin(x) - 1)/2 - log(sin(x) + 1)/2 - log(sec(x)**2)/2 + 2*sec(x)**2 - 4
*sin(x)/(2*sin(x)**2 - 2)
```

3.275.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. 2(16) = 32.

Time = 0.22 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.89

$$\int (\sec(x)+\tan(x))^3\,dx = \frac{3}{2}\tan(x)^2 - \frac{2\sin(x)}{\sin(x)^2-1} - \frac{1}{2(\sin(x)^2-1)} + \frac{1}{2}\log(\sin(x)^2-1) - \frac{1}{2}\log(\sin(x)+1) + \frac{1}{2}\log(\sin(x)-1)$$

input
```
integrate((sec(x)+tan(x))^3,x, algorithm="maxima")
```

output
```
3/2*tan(x)^2 - 2*sin(x)/(sin(x)^2 - 1) - 1/2/(sin(x)^2 - 1) + 1/2*log(sin(
x)^2 - 1) - 1/2*log(sin(x) + 1) + 1/2*log(sin(x) - 1)
```

3.275.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(16) = 32$.

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.67

$$\int (\sec(x) + \tan(x))^3 \, dx = -\frac{3 \tan\left(\frac{1}{2}x\right)^2 - 10 \tan\left(\frac{1}{2}x\right) + 3}{\left(\tan\left(\frac{1}{2}x\right) - 1\right)^2} - \log\left(\tan\left(\frac{1}{2}x\right)^2 + 1\right) + 2 \log\left(\left|\tan\left(\frac{1}{2}x\right) - 1\right|\right)$$

input
```
integrate((sec(x)+tan(x))^3,x, algorithm="giac")
```

output
```
-(3*tan(1/2*x)^2 - 10*tan(1/2*x) + 3)/(tan(1/2*x) - 1)^2 - log(tan(1/2*x)^
2 + 1) + 2*log(abs(tan(1/2*x) - 1))
```

3.275.9 Mupad [B] (verification not implemented)

Time = 28.30 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.39

$$\int (\sec(x)+\tan(x))^3 \, dx = 2 \ln\left(\tan\left(\frac{x}{2}\right)-1\right) - \ln\left(\tan\left(\frac{x}{2}\right)^2+1\right) + \frac{4 \tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^2 - 2 \tan\left(\frac{x}{2}\right) + 1}$$

input
```
int((tan(x) + 1/cos(x))^3,x)
```

output
```
2*log(tan(x/2) - 1) - log(tan(x/2)^2 + 1) + (4*tan(x/2))/(tan(x/2)^2 - 2*t
an(x/2) + 1)
```

3.276 $\int (\sec(x)+\tan(x))^2\,dx$

3.276.1 Optimal result

Integrand size = 7, antiderivative size = 16

$$\int (\sec(x)+\tan(x))^2\,dx=-x+\frac{2\cos(x)}{1-\sin(x)}$$

output
```
-x+2*cos(x)/(1-sin(x))
```

3.276.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int (\sec(x)+\tan(x))^2\,dx=-\arctan(\tan(x))+2\sec(x)+2\tan(x)$$

input
```
Integrate[(Sec[x] + Tan[x])^2,x]
```

output
```
-ArcTan[Tan[x]] + 2*Sec[x] + 2*Tan[x]
```

3.276.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 4891, 3042, 3149, 3042, 3159, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\tan(x)+\sec(x))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (\tan(x)+\sec(x))^2dx$$

$$\Big\downarrow \text{4891}$$

$$\int (\sin(x)+1)^2\sec^2(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(\sin(x)+1)^2}{\cos(x)^2}dx$$

$$\Big\downarrow \text{3149}$$

$$\int \frac{\cos^2(x)}{(1-\sin(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(x)^2}{(1-\sin(x))^2}dx$$

$$\Big\downarrow \text{3159}$$

$$\frac{2\cos(x)}{1-\sin(x)}-\int 1dx$$

$$\Big\downarrow \text{24}$$

$$\frac{2\cos(x)}{1-\sin(x)}-x$$

input
```
Int[(Sec[x] + Tan[x])^2,x]
```

output
```
-x + (2*Cos[x])/(1 - Sin[x])
```

3.276.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3149
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(a/g)^(2*m)   Int[(g*Cos[e + f*x])^(2*m + p)/(
a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 4891
```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.276.4 Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
default	$2\tan\left(x\right)+\frac{2}{\cos\left(x\right)}-x$	15
parts	$2\tan\left(x\right)-\arctan\left(\tan\left(x\right)\right)+2\sec\left(x\right)$	15
risch	$-x+\frac{4}{{\mathrm e}^{i x}-i}$	17

input
```
int((sec(x)+tan(x))^2,x,method=_RETURNVERBOSE)
```

output
```
2*tan(x)+2/cos(x)-x
```

3.276.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.75

$$\int (\sec(x)+\tan(x))^2\,dx = -\frac{(x-2)\cos(x)-(x+2)\sin(x)+x-2}{\cos(x)-\sin(x)+1}$$

input
```
integrate((sec(x)+tan(x))^2,x, algorithm="fricas")
```

output
```
-((x - 2)*cos(x) - (x + 2)*sin(x) + x - 2)/(cos(x) - sin(x) + 1)
```

3.276.6 Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int (\sec(x)+\tan(x))^2\,dx = -x+2\tan(x)+2\sec(x)$$

input
```
integrate((sec(x)+tan(x))**2,x)
```

output
```
-x + 2*tan(x) + 2*sec(x)
```

3.276.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int (\sec(x)+\tan(x))^2\,dx = -x+\frac{2}{\cos(x)}+2\,\tan(x)$$

input
```
integrate((sec(x)+tan(x))^2,x, algorithm="maxima")
```

output
```
-x + 2/cos(x) + 2*tan(x)
```

3.276.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int (\sec(x) + \tan(x))^2 \, dx = -x - \frac{4}{\tan\left(\frac{1}{2}\,x\right) - 1}$$

input
```
integrate((sec(x)+tan(x))^2,x, algorithm="giac")
```

output
```
-x - 4/(tan(1/2*x) - 1)
```

3.276.9 Mupad [B] (verification not implemented)

Time = 28.58 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int (\sec(x) + \tan(x))^2 \, dx = -x - \frac{4}{\tan\left(\frac{x}{2}\right) - 1}$$

input
```
int((tan(x) + 1/cos(x))^2,x)
```

output
```
- x - 4/(tan(x/2) - 1)
```

3.277 $\int (\sec(x) + \tan(x))\,dx$

3.277.1 Optimal result

Integrand size = 5, antiderivative size = 13

$$\int (\sec(x) + \tan(x))\,dx = -2\log\left(\cos\left(\frac{1}{4}(\pi + 2x)\right)\right)$$

output
```
-2*ln(cos(1/4*Pi+1/2*x))
```

3.277.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int (\sec(x) + \tan(x))\,dx = \operatorname{arctanh}(\sin(x)) - \log(\cos(x))$$

input
```
Integrate[Sec[x] + Tan[x],x]
```

output
```
ArcTanh[Sin[x]] - Log[Cos[x]]
```

3.277.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\tan(x)+\sec(x))\,dx$$

$$\Big\downarrow \; 2009$$

$$\text{arctanh}(\sin(x))-\log(\cos(x))$$

input
```
Int[Sec[x] + Tan[x],x]
```

output
```
ArcTanh[Sin[x]] - Log[Cos[x]]
```

3.277.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.277.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

method	result	size
default	$\ln(\sec(x)+\tan(x))-\ln(\cos(x))$	13
parts	$\ln(\sec(x)+\tan(x))-\ln(\cos(x))$	13
norman	$-\ln\left(\tan\left(\frac{x}{2}\right)-1\right)+\frac{\ln\left(1+\tan(x)^2\right)}{2}+\ln\left(\tan\left(\frac{x}{2}\right)+1\right)$	27
parallelrisch	$-\ln(-\cot(x)+\csc(x)-1)+\ln\left(\sqrt{\sec(x)^2}\right)+\ln(\csc(x)-\cot(x)+1)$	29
risch	$\ln(i+e^{ix})-\ln(e^{ix}-i)+ix-\ln(e^{2ix}+1)$	36

input
```
int(sec(x)+tan(x),x,method=_RETURNVERBOSE)
```

output
```
ln(sec(x)+tan(x))-ln(cos(x))
```

3.277.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int (\sec(x)+\tan(x))\,dx=-\log\left(-\sin\left(x\right)+1\right)$$

input
```
integrate(sec(x)+tan(x),x, algorithm="fricas")
```

output
```
-log(-sin(x) + 1)
```

3.277.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.54

$$\int (\sec(x)+\tan(x))\,dx=-\frac{\log\left(\sin\left(x\right)-1\right)}{2}+\frac{\log\left(\sin\left(x\right)+1\right)}{2}-\log\left(\cos\left(x\right)\right)$$

input
```
integrate(sec(x)+tan(x),x)
```

output
```
-log(sin(x) - 1)/2 + log(sin(x) + 1)/2 - log(cos(x))
```

3.277.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int (\sec(x)+\tan(x))\,dx=\log\left(\sec\left(x\right)+\tan\left(x\right)\right)+\log\left(\sec\left(x\right)\right)$$

input
```
integrate(sec(x)+tan(x),x, algorithm="maxima")
```

output
```
log(sec(x) + tan(x)) + log(sec(x))
```

3.277.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(11) = 22$.

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.38

$$\int (\sec(x) + \tan(x))\, dx = \frac{1}{4} \log\left(\left|\frac{1}{\sin(x)} + \sin(x) + 2\right|\right) - \frac{1}{4} \log\left(\left|\frac{1}{\sin(x)} + \sin(x) - 2\right|\right) - \log(|\cos(x)|)$$

input
```
integrate(sec(x)+tan(x),x, algorithm="giac")
```

output
```
1/4*log(abs(1/sin(x) + sin(x) + 2)) - 1/4*log(abs(1/sin(x) + sin(x) - 2))
- log(abs(cos(x)))
```

3.277.9 Mupad [B] (verification not implemented)

Time = 30.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int (\sec(x) + \tan(x))\, dx = \ln\left(\tan\left(\frac{x}{2}\right)^2 + 1\right) - 2 \ln\left(\tan\left(\frac{x}{2}\right) - 1\right)$$

input
```
int(tan(x) + 1/cos(x),x)
```

output
```
log(tan(x/2)^2 + 1) - 2*log(tan(x/2) - 1)
```

3.278 $\int \frac{1}{\sec(x)+\tan(x)}\,dx$

3.278.1 Optimal result

Integrand size = 7, antiderivative size = 5

$$\int \frac{1}{\sec(x)+\tan(x)}\,dx = \log(1+\sin(x))$$

output
```
ln(1+sin(x))
```

3.278.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 16 vs. 2(5) = 10.

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 3.20

$$\int \frac{1}{\sec(x)+\tan(x)}\,dx = 2\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[(Sec[x] + Tan[x])^(-1),x]
```

output
```
2*Log[Cos[x/2] + Sin[x/2]]
```

3.278.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3042, 3638, 3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\tan(x)+\sec(x)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\tan(x)+\sec(x)}dx$$
$$\Big\downarrow \; 3638$$
$$\int \frac{\cos(x)}{\sin(x)+1}dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\cos(x)}{\sin(x)+1}dx$$
$$\Big\downarrow \; 3146$$
$$\int \frac{1}{\sin(x)+1}d\sin(x)$$
$$\Big\downarrow \; 16$$
$$\log(\sin(x)+1)$$

input
```
Int[(Sec[x] + Tan[x])^(-1),x]
```

output
```
Log[1 + Sin[x]]
```

3.278.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + 
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3638
```
Int[((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)])
^(-1), x_Symbol] :> Int[Cos[d + e*x]/(b + a*Cos[d + e*x] + c*Sin[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

3.278.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

method	result	size
default	$\ln\left(1+\sin\left(x\right)\right)$	6
risch	$-ix+2\ln\left(i+{\mathrm e}^{ix}\right)$	17

input
```
int(1/(sec(x)+tan(x)),x,method=_RETURNVERBOSE)
```

output
```
ln(1+sin(x))
```

3.278.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sec(x)+\tan(x)}\,dx = \log\left(\sin\left(x\right)+1\right)$$

input
```
integrate(1/(sec(x)+tan(x)),x, algorithm="fricas")
```

output
```
log(sin(x) + 1)
```

3.278.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(5) = 10.

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 3.40

$$\int \frac{1}{\sec(x)+\tan(x)}\,dx = \log\left(\tan\left(x\right)+\sec\left(x\right)\right) - \frac{\log\left(\tan^{2}\left(x\right)+1\right)}{2}$$

input
```
integrate(1/(sec(x)+tan(x)),x)
```

output
```
log(tan(x) + sec(x)) - log(tan(x)**2 + 1)/2
```

3.278.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(5) = 10.

Time = 0.22 (sec) , antiderivative size = 31, normalized size of antiderivative = 6.20

$$\int \frac{1}{\sec(x)+\tan(x)}\,dx = 2\,\log\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}+1\right) - \log\left(\frac{\sin\left(x\right)^{2}}{\left(\cos\left(x\right)+1\right)^{2}}+1\right)$$

input
```
integrate(1/(sec(x)+tan(x)),x, algorithm="maxima")
```

output
```
2*log(sin(x)/(cos(x) + 1) + 1) - log(sin(x)^2/(cos(x) + 1)^2 + 1)
```

3.278.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(5) = 10.

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 4.40

$$\int \frac{1}{\sec(x)+\tan(x)}\,dx = -\log\left(\tan\left(\frac{1}{2}\,x\right)^2+1\right)+2\,\log\left(\left|\tan\left(\frac{1}{2}\,x\right)+1\right|\right)$$

input
```
integrate(1/(sec(x)+tan(x)),x, algorithm="giac")
```

output
```
-log(tan(1/2*x)^2 + 1) + 2*log(abs(tan(1/2*x) + 1))
```

3.278.9 Mupad [B] (verification not implemented)

Time = 30.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 4.20

$$\int \frac{1}{\sec(x)+\tan(x)}\,dx = 2\,\ln\left(\tan\left(\frac{x}{2}\right)+1\right)-\ln\left(\tan\left(\frac{x}{2}\right)^2+1\right)$$

input
```
int(1/(tan(x) + 1/cos(x)),x)
```

output
```
2*log(tan(x/2) + 1) - log(tan(x/2)^2 + 1)
```

3.279 $\int \frac{1}{(\sec(x)+\tan(x))^2} dx$

3.279.1 Optimal result

Integrand size = 7, antiderivative size = 14

$$\int \frac{1}{(\sec(x)+\tan(x))^2} dx = -x - \frac{2\cos(x)}{1+\sin(x)}$$

output
```
-x-2*cos(x)/(1+sin(x))
```

3.279.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.93

$$\int \frac{1}{(\sec(x)+\tan(x))^2} dx = -x + \frac{4\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}$$

input
```
Integrate[(Sec[x] + Tan[x])^(-2),x]
```

output
```
-x + (4*Sin[x/2])/(Cos[x/2] + Sin[x/2])
```

3.279.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3042, 4891, 3042, 3159, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\tan(x)+\sec(x))^2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{(\tan(x)+\sec(x))^2}dx$$

$$\Big\downarrow \; 4891$$

$$\int \frac{\cos^2(x)}{(\sin(x)+1)^2}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\cos(x)^2}{(\sin(x)+1)^2}dx$$

$$\Big\downarrow \; 3159$$

$$-\int 1dx-\frac{2\cos(x)}{\sin(x)+1}$$

$$\Big\downarrow \; 24$$

$$-x-\frac{2\cos(x)}{\sin(x)+1}$$

input
```
Int[(Sec[x] + Tan[x])^(-2),x]
```

output
```
-x - (2*Cos[x])/(1 + Sin[x])
```

3.279.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 4891
```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.279.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.21

method	result	size
risch	$-x-\frac{4}{i+e^{i x}}$	17
default	$-\frac{4}{\tan\left(\frac{x}{2}\right)+1}-2\arctan\left(\tan\left(\frac{x}{2}\right)\right)$	19

input
```
int(1/(sec(x)+tan(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-x-4/(I+exp(I*x))
```

3.279.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.79

$$\int \frac{1}{(\sec(x)+\tan(x))^2}\,dx = -\frac{(x+2)\cos(x)+(x-2)\sin(x)+x+2}{\cos(x)+\sin(x)+1}$$

input
```
integrate(1/(sec(x)+tan(x))^2,x, algorithm="fricas")
```

output
```
-((x + 2)*cos(x) + (x - 2)*sin(x) + x + 2)/(cos(x) + sin(x) + 1)
```

3.279.6 Sympy [F]

$$\int \frac{1}{(\sec(x)+\tan(x))^2}\,dx = \int \frac{1}{(\tan(x)+\sec(x))^2}\,dx$$

input
```
integrate(1/(sec(x)+tan(x))**2,x)
```

output
```
Integral((tan(x) + sec(x))**(-2), x)
```

3.279.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.00

$$\int \frac{1}{(\sec(x)+\tan(x))^2}\,dx = -\frac{4}{\frac{\sin(x)}{\cos(x)+1}+1} - 2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

input
```
integrate(1/(sec(x)+tan(x))^2,x, algorithm="maxima")
```

output
```
-4/(sin(x)/(cos(x) + 1) + 1) - 2*arctan(sin(x)/(cos(x) + 1))
```

3.279.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(\sec(x)+\tan(x))^2}\,dx = -x - \frac{4}{\tan\left(\frac{1}{2}x\right)+1}$$

input
```
integrate(1/(sec(x)+tan(x))^2,x, algorithm="giac")
```

output
```
-x - 4/(tan(1/2*x) + 1)
```

3.279.9 Mupad [B] (verification not implemented)

Time = 30.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(\sec(x)+\tan(x))^2}\,dx = -x - \frac{4}{\tan\left(\frac{x}{2}\right)+1}$$

input
```
int(1/(tan(x) + 1/cos(x))^2,x)
```

output
```
- x - 4/(tan(x/2) + 1)
```

3.280 $\int \frac{1}{(\sec(x)+\tan(x))^3} dx$

3.280.1 Optimal result

Integrand size = 7, antiderivative size = 16

$$\int \frac{1}{(\sec(x)+\tan(x))^3} dx = -\log(1+\sin(x)) - \frac{2}{1+\sin(x)}$$

output
```
-ln(1+sin(x))-2/(1+sin(x))
```

3.280.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 34 vs. 2(16) = 32.

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.12

$$\int \frac{1}{(\sec(x)+\tan(x))^3} dx = -2\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right) - \frac{2}{\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^2}$$

input
```
Integrate[(Sec[x] + Tan[x])^(-3),x]
```

output
```
-2*Log[Cos[x/2] + Sin[x/2]] - 2/(Cos[x/2] + Sin[x/2])^2
```

3.280.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {3042, 4891, 3042, 3146, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\tan(x)+\sec(x))^3}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(\tan(x)+\sec(x))^3}dx$$
$$\Big\downarrow \text{4891}$$
$$\int \frac{\cos^3(x)}{(\sin(x)+1)^3}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos(x)^3}{(\sin(x)+1)^3}dx$$
$$\Big\downarrow \text{3146}$$
$$\int \frac{1-\sin(x)}{(\sin(x)+1)^2}d\sin(x)$$
$$\Big\downarrow \text{49}$$
$$\int \left(\frac{2}{(\sin(x)+1)^2}+\frac{1}{-\sin(x)-1}\right)d\sin(x)$$
$$\Big\downarrow \text{2009}$$
$$-\frac{2}{\sin(x)+1}-\log(\sin(x)+1)$$

input
```
Int[(Sec[x] + Tan[x])^(-3),x]
```

output
```
-Log[1 + Sin[x]] - 2/(1 + Sin[x])
```

3.280.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 4891
```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.280.4 Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

method	result	size
default	$-\ln\left(1+\sin\left(x\right)\right)-\frac{2}{1+\sin\left(x\right)}$	17
risch	$ix-\frac{4i\mathrm{e}^{ix}}{\left(i+\mathrm{e}^{ix}\right)^{2}}-2\ln\left(i+\mathrm{e}^{ix}\right)$	35

input
```
int(1/(sec(x)+tan(x))^3,x,method=_RETURNVERBOSE)
```

output
```
-ln(1+sin(x))-2/(1+sin(x))
```

3.280.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int \frac{1}{(\sec(x)+\tan(x))^3}\,dx = -\frac{(\sin(x)+1)\log(\sin(x)+1)+2}{\sin(x)+1}$$

input
```
integrate(1/(sec(x)+tan(x))^3,x, algorithm="fricas")
```

output
```
-((sin(x) + 1)*log(sin(x) + 1) + 2)/(sin(x) + 1)
```

3.280.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. 2(14) = 28.

Time = 0.46 (sec) , antiderivative size = 301, normalized size of antiderivative = 18.81

$$\begin{aligned}\int \frac{1}{(\sec(x)+\tan(x))^3}\,dx = &-\frac{2\log(\tan(x)+\sec(x))\tan^2(x)}{2\tan^2(x)+4\tan(x)\sec(x)+2\sec^2(x)}\\ &-\frac{4\log(\tan(x)+\sec(x))\tan(x)\sec(x)}{2\tan^2(x)+4\tan(x)\sec(x)+2\sec^2(x)}\\ &-\frac{2\log(\tan(x)+\sec(x))\sec^2(x)}{2\tan^2(x)+4\tan(x)\sec(x)+2\sec^2(x)}\\ &+\frac{\log(\tan^2(x)+1)\tan^2(x)}{2\tan^2(x)+4\tan(x)\sec(x)+2\sec^2(x)}\\ &+\frac{2\log(\tan^2(x)+1)\tan(x)\sec(x)}{2\tan^2(x)+4\tan(x)\sec(x)+2\sec^2(x)}\\ &+\frac{\log(\tan^2(x)+1)\sec^2(x)}{2\tan^2(x)+4\tan(x)\sec(x)+2\sec^2(x)}\\ &+\frac{\tan^2(x)}{2\tan^2(x)+4\tan(x)\sec(x)+2\sec^2(x)}\\ &-\frac{\sec^2(x)}{2\tan^2(x)+4\tan(x)\sec(x)+2\sec^2(x)}\\ &-\frac{1}{2\tan^2(x)+4\tan(x)\sec(x)+2\sec^2(x)}\end{aligned}$$

input
```
integrate(1/(sec(x)+tan(x))**3,x)
```

output
```
-2*log(tan(x) + sec(x))*tan(x)**2/(2*tan(x)**2 + 4*tan(x)*sec(x) + 2*sec(x
)**2) - 4*log(tan(x) + sec(x))*tan(x)*sec(x)/(2*tan(x)**2 + 4*tan(x)*sec(x
) + 2*sec(x)**2) - 2*log(tan(x) + sec(x))*sec(x)**2/(2*tan(x)**2 + 4*tan(x
)*sec(x) + 2*sec(x)**2) + log(tan(x)**2 + 1)*tan(x)**2/(2*tan(x)**2 + 4*ta
n(x)*sec(x) + 2*sec(x)**2) + 2*log(tan(x)**2 + 1)*tan(x)*sec(x)/(2*tan(x)*
*2 + 4*tan(x)*sec(x) + 2*sec(x)**2) + log(tan(x)**2 + 1)*sec(x)**2/(2*tan(
x)**2 + 4*tan(x)*sec(x) + 2*sec(x)**2) + tan(x)**2/(2*tan(x)**2 + 4*tan(x)
*sec(x) + 2*sec(x)**2) - sec(x)**2/(2*tan(x)**2 + 4*tan(x)*sec(x) + 2*sec(
x)**2) - 1/(2*tan(x)**2 + 4*tan(x)*sec(x) + 2*sec(x)**2)
```

3.280.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(16) = 32$.

Time = 0.32 (sec) , antiderivative size = 64, normalized size of antiderivative = 4.00

$$\int \frac{1}{(\sec(x)+\tan(x))^3}\,dx = \frac{4\,\sin\left(x\right)}{\left(\frac{2\,\sin\left(x\right)}{\cos\left(x\right)+1}+\frac{\sin\left(x\right)^2}{\left(\cos\left(x\right)+1\right)^2}+1\right)\left(\cos\left(x\right)+1\right)} - 2\,\log\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}+1\right)+\log\left(\frac{\sin\left(x\right)^2}{\left(\cos\left(x\right)+1\right)^2}+1\right)$$

input
```
integrate(1/(sec(x)+tan(x))^3,x, algorithm="maxima")
```

output
```
4*sin(x)/((2*sin(x)/(cos(x) + 1) + sin(x)^2/(cos(x) + 1)^2 + 1)*(cos(x) +
1)) - 2*log(sin(x)/(cos(x) + 1) + 1) + log(sin(x)^2/(cos(x) + 1)^2 + 1)
```

3.280.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(16) = 32$.

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.81

$$\int \frac{1}{(\sec(x)+\tan(x))^3}\,dx = \frac{3\,\tan\left(\frac{1}{2}\,x\right)^2+10\,\tan\left(\frac{1}{2}\,x\right)+3}{\left(\tan\left(\frac{1}{2}\,x\right)+1\right)^2} + \log\left(\tan\left(\frac{1}{2}\,x\right)^2+1\right) - 2\,\log\left(\left|\tan\left(\frac{1}{2}\,x\right)+1\right|\right)$$

input
```
integrate(1/(sec(x)+tan(x))^3,x, algorithm="giac")
```

output
```
(3*tan(1/2*x)^2 + 10*tan(1/2*x) + 3)/(tan(1/2*x) + 1)^2 + log(tan(1/2*x)^2
 + 1) - 2*log(abs(tan(1/2*x) + 1))
```

3.280.9 Mupad [B] (verification not implemented)

Time = 30.53 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.56

$$\int \frac{1}{(\sec(x)+\tan(x))^3}\,dx = \ln\left(\tan\left(\frac{x}{2}\right)^2+1\right)-2\,\ln\left(\tan\left(\frac{x}{2}\right)+1\right)+\frac{4\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^2+2\tan\left(\frac{x}{2}\right)+1}$$

input
```
int(1/(tan(x) + 1/cos(x))^3,x)
```

output
```
log(tan(x/2)^2 + 1) - 2*log(tan(x/2) + 1) + (4*tan(x/2))/(2*tan(x/2) + tan
(x/2)^2 + 1)
```

3.281 $\int \frac{1}{(\sec(x)+\tan(x))^4}\,dx$

3.281.1 Optimal result

Integrand size = 7, antiderivative size = 26

$$\int \frac{1}{(\sec(x)+\tan(x))^4}\,dx = x-\frac{2\cos^3(x)}{3(1+\sin(x))^3}+\frac{2\cos(x)}{1+\sin(x)}$$

output
```
x-2/3*cos(x)^3/(1+sin(x))^3+2*cos(x)/(1+sin(x))
```

3.281.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 62 vs. 2(26) = 52.

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.38

$$\int \frac{1}{(\sec(x)+\tan(x))^4}\,dx = \frac{3(-8+3x)\cos\left(\frac{x}{2}\right)+(16-3x)\cos\left(\frac{3x}{2}\right)+6(-4+2x+x\cos(x))\sin\left(\frac{x}{2}\right)}{6\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^3}$$

input
```
Integrate[(Sec[x] + Tan[x])^(-4),x]
```

output
```
(3*(-8 + 3*x)*Cos[x/2] + (16 - 3*x)*Cos[(3*x)/2] + 6*(-4 + 2*x + x*Cos[x])
*Sin[x/2])/(6*(Cos[x/2] + Sin[x/2])^3)
```

3.281.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 4891, 3042, 3159, 3042, 3159, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\tan(x)+\sec(x))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(\tan(x)+\sec(x))^4}dx$$

$$\Big\downarrow \text{4891}$$

$$\int \frac{\cos^4(x)}{(\sin(x)+1)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(x)^4}{(\sin(x)+1)^4}dx$$

$$\Big\downarrow \text{3159}$$

$$-\int \frac{\cos^2(x)}{(\sin(x)+1)^2}dx-\frac{2\cos^3(x)}{3(\sin(x)+1)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{\cos(x)^2}{(\sin(x)+1)^2}dx-\frac{2\cos^3(x)}{3(\sin(x)+1)^3}$$

$$\Big\downarrow \text{3159}$$

$$\int 1dx-\frac{2\cos^3(x)}{3(\sin(x)+1)^3}+\frac{2\cos(x)}{\sin(x)+1}$$

$$\Big\downarrow \text{24}$$

$$x-\frac{2\cos^3(x)}{3(\sin(x)+1)^3}+\frac{2\cos(x)}{\sin(x)+1}$$

input
```
Int[(Sec[x] + Tan[x])^(-4),x]
```

output
```
x - (2*Cos[x]^3)/(3*(1 + Sin[x])^3) + (2*Cos[x])/(1 + Sin[x])
```

3.281.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 4891
```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.281.4 Maple [A] (verified)

Time = 2.80 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.12

method	result	size
default	$2\arctan\left(\tan\left(\frac{x}{2}\right)\right)-\frac{16}{3\left(\tan\left(\frac{x}{2}\right)+1\right)^{3}}+\frac{8}{\left(\tan\left(\frac{x}{2}\right)+1\right)^{2}}$	29
risch	$x+\frac{-\frac{16}{3}+8i\mathrm{e}^{ix}+8\,\mathrm{e}^{2ix}}{\left(i+\mathrm{e}^{ix}\right)^{3}}$	32

input
```
int(1/(sec(x)+tan(x))^4,x,method=_RETURNVERBOSE)
```

output
```
2*arctan(tan(1/2*x))-16/3/(tan(1/2*x)+1)^3+8/(tan(1/2*x)+1)^2
```

3.281.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(24) = 48$.

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.42

$$\int \frac{1}{(\sec(x)+\tan(x))^4}\,dx = \frac{(3\,x-8)\cos\left(x\right)^2-(3\,x+4)\cos\left(x\right)-((3\,x+8)\cos\left(x\right)+6\,x+4)\sin\left(x\right)-6\,x+4}{3\left(\cos\left(x\right)^2-(\cos\left(x\right)+2)\sin\left(x\right)-\cos\left(x\right)-2\right)}$$

input
```
integrate(1/(sec(x)+tan(x))^4,x, algorithm="fricas")
```

output
```
1/3*((3*x - 8)*cos(x)^2 - (3*x + 4)*cos(x) - ((3*x + 8)*cos(x) + 6*x + 4)*
sin(x) - 6*x + 4)/(cos(x)^2 - (cos(x) + 2)*sin(x) - cos(x) - 2)
```

3.281.6 Sympy [F]

$$\int \frac{1}{(\sec(x)+\tan(x))^4}\,dx = \int \frac{1}{\left(\tan\left(x\right)+\sec\left(x\right)\right)^4}\,dx$$

input
```
integrate(1/(sec(x)+tan(x))**4,x)
```

output
```
Integral((tan(x) + sec(x))**(-4), x)
```

3.281.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(24) = 48$.

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.46

$$\int \frac{1}{(\sec(x)+\tan(x))^4}\,dx = \frac{8\left(\frac{3\sin(x)}{\cos(x)+1}+1\right)}{3\left(\frac{3\sin(x)}{\cos(x)+1}+\frac{3\sin(x)^2}{(\cos(x)+1)^2}+\frac{\sin(x)^3}{(\cos(x)+1)^3}+1\right)} + 2\arctan\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}\right)$$

input
```
integrate(1/(sec(x)+tan(x))^4,x, algorithm="maxima")
```

output
```
8/3*(3*sin(x)/(cos(x) + 1) + 1)/(3*sin(x)/(cos(x) + 1) + 3*sin(x)^2/(cos(x
) + 1)^2 + sin(x)^3/(cos(x) + 1)^3 + 1) + 2*arctan(sin(x)/(cos(x) + 1))
```

3.281.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \frac{1}{(\sec(x)+\tan(x))^4}\,dx = x + \frac{8\left(3\tan\left(\frac{1}{2}x\right)+1\right)}{3\left(\tan\left(\frac{1}{2}x\right)+1\right)^3}$$

input
```
integrate(1/(sec(x)+tan(x))^4,x, algorithm="giac")
```

output
```
x + 8/3*(3*tan(1/2*x) + 1)/(tan(1/2*x) + 1)^3
```

3.281.9 Mupad [B] (verification not implemented)

Time = 31.76 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

$$\int \frac{1}{(\sec(x)+\tan(x))^4}\,dx = x + \frac{8\tan\left(\frac{x}{2}\right)+\frac{8}{3}}{\left(\tan\left(\frac{x}{2}\right)+1\right)^3}$$

input
```
int(1/(tan(x) + 1/cos(x))^4,x)
```

output
```
x + (8*tan(x/2) + 8/3)/(tan(x/2) + 1)^3
```

3.282 $\int \frac{1}{(\sec(x)+\tan(x))^5} dx$

3.282.1 Optimal result

Integrand size = 7, antiderivative size = 22

$$\int \frac{1}{(\sec(x)+\tan(x))^5} dx = \log(1+\sin(x)) - \frac{2}{(1+\sin(x))^2} + \frac{4}{1+\sin(x)}$$

output
```
ln(1+sin(x))-2/(1+sin(x))^2+4/(1+sin(x))
```

3.282.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

$$\int \frac{1}{(\sec(x)+\tan(x))^5} dx = 2\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right) + \frac{2+4\sin(x)}{\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^4}$$

input
```
Integrate[(Sec[x] + Tan[x])^(-5),x]
```

output
```
2*Log[Cos[x/2] + Sin[x/2]] + (2 + 4*Sin[x])/(Cos[x/2] + Sin[x/2])^4
```

3.282.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {3042, 4891, 3042, 3146, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\tan(x)+\sec(x))^5}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(\tan(x)+\sec(x))^5}dx$$

$$\Big\downarrow \text{4891}$$

$$\int \frac{\cos^5(x)}{(\sin(x)+1)^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(x)^5}{(\sin(x)+1)^5}dx$$

$$\Big\downarrow \text{3146}$$

$$\int \frac{(1-\sin(x))^2}{(\sin(x)+1)^3}d\sin(x)$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{1}{\sin(x)+1}-\frac{4}{(\sin(x)+1)^2}+\frac{4}{(\sin(x)+1)^3}\right)d\sin(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{4}{\sin(x)+1}-\frac{2}{(\sin(x)+1)^2}+\log(\sin(x)+1)$$

input
```
Int[(Sec[x] + Tan[x])^(-5),x]
```

output
```
Log[1 + Sin[x]] - 2/(1 + Sin[x])^2 + 4/(1 + Sin[x])
```

3.282.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 4891
```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.282.4 Maple [A] (verified)

Time = 6.85 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

method	result	size
default	$\ln\left(1+\sin\left(x\right)\right)-\frac{2}{\left(1+\sin\left(x\right)\right)^{2}}+\frac{4}{1+\sin\left(x\right)}$	23
risch	$-ix+\frac{8i\left(i\,{\mathrm e}^{2ix}+{\mathrm e}^{3ix}-{\mathrm e}^{ix}\right)}{\left(i+{\mathrm e}^{ix}\right)^{4}}+2\ln\left(i+{\mathrm e}^{ix}\right)$	51

input
```
int(1/(sec(x)+tan(x))^5,x,method=_RETURNVERBOSE)
```

output
```
ln(1+sin(x))-2/(1+sin(x))^2+4/(1+sin(x))
```

3.282.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \frac{1}{(\sec(x)+\tan(x))^5}\,dx = \frac{\left(\cos\left(x\right)^2-2\,\sin\left(x\right)-2\right)\log\left(\sin\left(x\right)+1\right)-4\,\sin\left(x\right)-2}{\cos\left(x\right)^2-2\,\sin\left(x\right)-2}$$

input
```
integrate(1/(sec(x)+tan(x))^5,x, algorithm="fricas")
```

output
```
((cos(x)^2 - 2*sin(x) - 2)*log(sin(x) + 1) - 4*sin(x) - 2)/(cos(x)^2 - 2*s
in(x) - 2)
```

3.282.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1059 vs. 2(20) = 40.

Time = 1.66 (sec) , antiderivative size = 1059, normalized size of antiderivative = 48.14

$$\int \frac{1}{(\sec(x)+\tan(x))^5}\,dx = \text{Too large to display}$$

input
```
integrate(1/(sec(x)+tan(x))**5,x)
```

output
```
36*log(tan(x) + sec(x))*tan(x)**4/(36*tan(x)**4 + 144*tan(x)**3*sec(x) + 2
16*tan(x)**2*sec(x)**2 + 144*tan(x)*sec(x)**3 + 36*sec(x)**4) + 144*log(ta
n(x) + sec(x))*tan(x)**3*sec(x)/(36*tan(x)**4 + 144*tan(x)**3*sec(x) + 216
*tan(x)**2*sec(x)**2 + 144*tan(x)*sec(x)**3 + 36*sec(x)**4) + 216*log(tan(
x) + sec(x))*tan(x)**2*sec(x)**2/(36*tan(x)**4 + 144*tan(x)**3*sec(x) + 21
6*tan(x)**2*sec(x)**2 + 144*tan(x)*sec(x)**3 + 36*sec(x)**4) + 144*log(tan
(x) + sec(x))*tan(x)*sec(x)**3/(36*tan(x)**4 + 144*tan(x)**3*sec(x) + 216*
tan(x)**2*sec(x)**2 + 144*tan(x)*sec(x)**3 + 36*sec(x)**4) + 36*log(tan(x)
 + sec(x))*sec(x)**4/(36*tan(x)**4 + 144*tan(x)**3*sec(x) + 216*tan(x)**2*
sec(x)**2 + 144*tan(x)*sec(x)**3 + 36*sec(x)**4) - 18*log(tan(x)**2 + 1)*t
an(x)**4/(36*tan(x)**4 + 144*tan(x)**3*sec(x) + 216*tan(x)**2*sec(x)**2 +
144*tan(x)*sec(x)**3 + 36*sec(x)**4) - 72*log(tan(x)**2 + 1)*tan(x)**3*sec
(x)/(36*tan(x)**4 + 144*tan(x)**3*sec(x) + 216*tan(x)**2*sec(x)**2 + 144*t
an(x)*sec(x)**3 + 36*sec(x)**4) - 108*log(tan(x)**2 + 1)*tan(x)**2*sec(x)*
*2/(36*tan(x)**4 + 144*tan(x)**3*sec(x) + 216*tan(x)**2*sec(x)**2 + 144*ta
n(x)*sec(x)**3 + 36*sec(x)**4) - 72*log(tan(x)**2 + 1)*tan(x)*sec(x)**3/(3
6*tan(x)**4 + 144*tan(x)**3*sec(x) + 216*tan(x)**2*sec(x)**2 + 144*tan(x)*
sec(x)**3 + 36*sec(x)**4) - 18*log(tan(x)**2 + 1)*sec(x)**4/(36*tan(x)**4
+ 144*tan(x)**3*sec(x) + 216*tan(x)**2*sec(x)**2 + 144*tan(x)*sec(x)**3 +
36*sec(x)**4) - 28*tan(x)**4/(36*tan(x)**4 + 144*tan(x)**3*sec(x) + 216...
```

3.282.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(22) = 44$.

Time = 0.31 (sec) , antiderivative size = 92, normalized size of antiderivative = 4.18

$$\int \frac{1}{(\sec(x)+\tan(x))^5}\,dx = -\frac{8\,\sin(x)^2}{\left(\frac{4\,\sin(x)}{\cos(x)+1}+\frac{6\,\sin(x)^2}{(\cos(x)+1)^2}+\frac{4\,\sin(x)^3}{(\cos(x)+1)^3}+\frac{\sin(x)^4}{(\cos(x)+1)^4}+1\right)(\cos(x)+1)^2} + 2\,\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right) - \log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)$$

input
```
integrate(1/(sec(x)+tan(x))^5,x, algorithm="maxima")
```

output
```
-8*sin(x)^2/((4*sin(x)/(cos(x) + 1) + 6*sin(x)^2/(cos(x) + 1)^2 + 4*sin(x)
^3/(cos(x) + 1)^3 + sin(x)^4/(cos(x) + 1)^4 + 1)*(cos(x) + 1)^2) + 2*log(s
in(x)/(cos(x) + 1) + 1) - log(sin(x)^2/(cos(x) + 1)^2 + 1)
```

3.282.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(22) = 44$.

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.91

$$\int \frac{1}{(\sec(x)+\tan(x))^5}\,dx = -\frac{25\,\tan\left(\frac{1}{2}\,x\right)^4+100\,\tan\left(\frac{1}{2}\,x\right)^3+198\,\tan\left(\frac{1}{2}\,x\right)^2+100\,\tan\left(\frac{1}{2}\,x\right)+25}{6\left(\tan\left(\frac{1}{2}\,x\right)+1\right)^4} - \log\left(\tan\left(\frac{1}{2}\,x\right)^2+1\right)+2\,\log\left(\left|\tan\left(\frac{1}{2}\,x\right)+1\right|\right)$$

input
```
integrate(1/(sec(x)+tan(x))^5,x, algorithm="giac")
```

output
```
-1/6*(25*tan(1/2*x)^4 + 100*tan(1/2*x)^3 + 198*tan(1/2*x)^2 + 100*tan(1/2*
x) + 25)/(tan(1/2*x) + 1)^4 - log(tan(1/2*x)^2 + 1) + 2*log(abs(tan(1/2*x)
 + 1))
```

3.282.9 Mupad [B] (verification not implemented)

Time = 30.56 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.77

$$\int \frac{1}{(\sec(x)+\tan(x))^5}\,dx = 2\,\ln\left(\tan\left(\frac{x}{2}\right)+1\right)-\ln\left(\tan\left(\frac{x}{2}\right)^2+1\right) - \frac{8\tan\left(\frac{x}{2}\right)^2}{\tan\left(\frac{x}{2}\right)^4+4\tan\left(\frac{x}{2}\right)^3+6\tan\left(\frac{x}{2}\right)^2+4\tan\left(\frac{x}{2}\right)+1}$$

input
```
int(1/(tan(x) + 1/cos(x))^5,x)
```

output
```
2*log(tan(x/2) + 1) - log(tan(x/2)^2 + 1) - (8*tan(x/2)^2)/(4*tan(x/2) + 6
*tan(x/2)^2 + 4*tan(x/2)^3 + tan(x/2)^4 + 1)
```

3.283 $\int (a\cot(x)+b\csc(x))^5\,dx$

3.283.1 Optimal result

Integrand size = 11, antiderivative size = 152

$$\begin{aligned}\int (a\cot(x)+b\csc(x))^5\,dx =& \frac{1}{8}a^2b(7a^2-3b^2)\cos(x)\\&+\frac{1}{8}(b+a\cos(x))^2\left(2a(2a^2-b^2)+b(5a^2-3b^2)\cos(x)\right)\csc^2(x)\\&-\frac{1}{4}(b+a\cos(x))^4(a+b\cos(x))\csc^4(x)\\&+\frac{1}{16}(a+b)^3\left(8a^2-9ab+3b^2\right)\log(1-\cos(x))\\&+\frac{1}{16}(a-b)^3\left(8a^2+9ab+3b^2\right)\log(1+\cos(x))\end{aligned}$$

output
```
1/8*a^2*b*(7*a^2-3*b^2)*cos(x)+1/8*(b+a*cos(x))^2*(2*a*(2*a^2-b^2)+b*(5*a^
2-3*b^2)*cos(x))*csc(x)^2-1/4*(b+a*cos(x))^4*(a+b*cos(x))*csc(x)^4+1/16*(a
+b)^3*(8*a^2-9*a*b+3*b^2)*ln(1-cos(x))+1/16*(a-b)^3*(8*a^2+9*a*b+3*b^2)*ln
(1+cos(x))
```

3.283.2 Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.94

$$\int (a\cot(x)+b\csc(x))^5\,dx=\frac{1}{64}\Big(2(7a-3b)(a+b)^4\csc^2\left(\frac{x}{2}\right)-(a+b)^5\csc^4\left(\frac{x}{2}\right)+8(a-b)^3\left(8a^2+9ab+3b^2\right)\log\left(\cos\left(\frac{x}{2}\right)\right)+8(a+b)^3\left(8a^2-9ab+3b^2\right)\log\left(\sin\left(\frac{x}{2}\right)\right)+2(a-b)^4(7a+3b)\sec^2\left(\frac{x}{2}\right)-(a-b)^5\sec^4\left(\frac{x}{2}\right)\Big)$$

input
```
Integrate[(a*Cot[x] + b*Csc[x])^5,x]
```

output
```
(2*(7*a - 3*b)*(a + b)^4*Csc[x/2]^2 - (a + b)^5*Csc[x/2]^4 + 8*(a - b)^3*(
8*a^2 + 9*a*b + 3*b^2)*Log[Cos[x/2]] + 8*(a + b)^3*(8*a^2 - 9*a*b + 3*b^2)
*Log[Sin[x/2]] + 2*(a - b)^4*(7*a + 3*b)*Sec[x/2]^2 - (a - b)^5*Sec[x/2]^4
)/64
```

3.283.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {3042, 4892, 3042, 3147, 477, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cot(x)+b\csc(x))^5\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\cot(x)+b\csc(x))^5dx$$

$$\Big\downarrow \text{4892}$$

$$\int \csc^5(x)(a\cos(x)+b)^5dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b-a\sin\left(x-\frac{\pi}{2}\right)\right)^5}{\cos\left(x-\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{3147}$$

$$-a^5\int \frac{(b+a\cos(x))^5}{\left(a^2-a^2\cos^2(x)\right)^3}d(a\cos(x))$$

$$\Big\downarrow \text{477}$$

$$-\frac{\int \left(-\frac{a^3(a-b)^5}{8(\cos(x)a+a)^3}+\frac{a^2(7a+3b)(a-b)^4}{16(\cos(x)a+a)^2}-\frac{a(8a^2+9ba+3b^2)(a-b)^3}{16(\cos(x)a+a)}+\frac{a(a+b)^3(8a^2-9ba+3b^2)}{16(a-a\cos(x))}-\frac{a^2(7a-3b)(a+b)^4}{16(a-a\cos(x))^2}+\frac{a^3(a+b)^5}{8(a-a\cos(x))^3}\right.}{a}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{a^3(a-b)^5}{16(a\cos(x)+a)^2}+\frac{a^3(a+b)^5}{16(a-a\cos(x))^2}-\frac{1}{16}a\left(8a^2+9ab+3b^2\right)(a-b)^3\log(a\cos(x)+a)-\frac{1}{16}a(a+b)^3\left(8a^2-9ab+3b^2\right.}{a}$$

input
```
Int[(a*Cot[x] + b*Csc[x])^5,x]
```

output
```
-(((a^3*(a + b)^5)/(16*(a - a*Cos[x])^2) - (a^2*(7*a - 3*b)*(a + b)^4)/(16
*(a - a*Cos[x])) + (a^3*(a - b)^5)/(16*(a + a*Cos[x])^2) - (a^2*(a - b)^4*
(7*a + 3*b))/(16*(a + a*Cos[x])) - (a*(a + b)^3*(8*a^2 - 9*a*b + 3*b^2)*Lo
g[a - a*Cos[x]])/16 - (a*(a - b)^3*(8*a^2 + 9*a*b + 3*b^2)*Log[a + a*Cos[x
]])/16)/a)
```

3.283.3.1 Defintions of rubi rules used

rule 477
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ExpandIntegrand[(c + d*x)^n*(1 - Rt[-b/a, 2]*x)^p*(1 + Rt[-b/a, 2
]*x)^p, x], x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[n] &
& NiceSqrtQ[-b/a] &&  !FractionalPowerFactorQ[Rt[-b/a, 2]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 4892
```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.283.4 Maple [A] (verified)

Time = 30.27 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.10

method	result
default	$b^5\left(\left(-\frac{\csc(x)^3}{4}-\frac{3\csc(x)}{8}\right)\cot(x)+\frac{3\ln(\csc(x)-\cot(x))}{8}\right)-\frac{5a\,b^4}{4\sin(x)^4}+10a^2b^3\left(-\frac{\cos(x)^3}{4\sin(x)^4}-\frac{\cos(x)^3}{8\sin(x)^2}-\frac{\cos(x)}{8}\right.$
parts	$a^5\left(-\frac{\cot(x)^4}{4}+\frac{\cot(x)^2}{2}-\frac{\ln\left(1+\cot(x)^2\right)}{2}\right)+b^5\left(\left(-\frac{\csc(x)^3}{4}-\frac{3\csc(x)}{8}\right)\cot(x)+\frac{3\ln(\csc(x)-\cot(x))}{8}\right)+5a^4b$
risch	$-ia^5x-\frac{e^{ix}(25a^4b\,e^{6ix}+10a^2b^3e^{6ix}-3b^5e^{6ix}+16a^5e^{5ix}+80b^2a^3e^{5ix}+15a^4b\,e^{4ix}+70a^2b^3e^{4ix}+11b^5e^{4ix}-16a^5e^{3ix}+80a\,b^4e^{3ix}+}{4(e^{2ix}-1)^4}$

input
```
int((a*cot(x)+b*csc(x))^5,x,method=_RETURNVERBOSE)
```

output
```
b^5*((-1/4*csc(x)^3-3/8*csc(x))*cot(x)+3/8*ln(csc(x)-cot(x)))-5/4*a*b^4/si
n(x)^4+10*a^2*b^3*(-1/4/sin(x)^4*cos(x)^3-1/8/sin(x)^2*cos(x)^3-1/8*cos(x)
-1/8*ln(csc(x)-cot(x)))-5/2*a^3*b^2/sin(x)^4*cos(x)^4+5*a^4*b*(-1/4/sin(x)
^4*cos(x)^5+1/8/sin(x)^2*cos(x)^5+1/8*cos(x)^3+3/8*cos(x)+3/8*ln(csc(x)-co
t(x)))+a^5*(-1/4*cot(x)^4+1/2*cot(x)^2+ln(sin(x)))
```

3.283.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. 2(142) = 284.

Time = 0.27 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.92

$$\int (a\cot(x)+b\csc(x))^5\,dx$$
$$=\frac{12\,a^5+40\,a^3b^2-20\,ab^4-2\,(25\,a^4b+10\,a^2b^3-3\,b^5)\cos(x)^3-16\,(a^5+5\,a^3b^2)\cos(x)^2+10\,(3\,a^4b-2\,a^2}{}$$

input
```
integrate((a*cot(x)+b*csc(x))^5,x, algorithm="fricas")
```

output
```
1/16*(12*a^5 + 40*a^3*b^2 - 20*a*b^4 - 2*(25*a^4*b + 10*a^2*b^3 - 3*b^5)*c
os(x)^3 - 16*(a^5 + 5*a^3*b^2)*cos(x)^2 + 10*(3*a^4*b - 2*a^2*b^3 - b^5)*c
os(x) + (8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5 + (8*a^5 - 15*a^4*b + 10*a^
2*b^3 - 3*b^5)*cos(x)^4 - 2*(8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*cos(x)
^2)*log(1/2*cos(x) + 1/2) + (8*a^5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5 + (8*a^
5 + 15*a^4*b - 10*a^2*b^3 + 3*b^5)*cos(x)^4 - 2*(8*a^5 + 15*a^4*b - 10*a^2
*b^3 + 3*b^5)*cos(x)^2)*log(-1/2*cos(x) + 1/2))/(cos(x)^4 - 2*cos(x)^2 + 1
)
```

3.283.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(141) = 282$.

Time = 51.39 (sec) , antiderivative size = 308, normalized size of antiderivative = 2.03

$$\begin{aligned}\int (a\cot(x)+b\csc(x))^5\,dx = &-\frac{a^5\log\left(\csc^2\left(x\right)\right)}{2}-\frac{a^5\csc^4\left(x\right)}{4}+a^5\csc^2\left(x\right)\\&+\frac{15a^4b\log\left(\cos\left(x\right)-1\right)}{16}-\frac{15a^4b\log\left(\cos\left(x\right)+1\right)}{16}\\&-\frac{25a^4b\cos^3\left(x\right)}{8\cos^4\left(x\right)-16\cos^2\left(x\right)+8}+\frac{15a^4b\cos\left(x\right)}{8\cos^4\left(x\right)-16\cos^2\left(x\right)+8}\\&-\frac{5a^3b^2\cot^4\left(x\right)}{2}-\frac{5a^2b^3\log\left(\cos\left(x\right)-1\right)}{8}\\&+\frac{5a^2b^3\log\left(\cos\left(x\right)+1\right)}{8}-\frac{10a^2b^3\cos^3\left(x\right)}{8\cos^4\left(x\right)-16\cos^2\left(x\right)+8}\\&-\frac{10a^2b^3\cos\left(x\right)}{8\cos^4\left(x\right)-16\cos^2\left(x\right)+8}-\frac{5ab^4\csc^4\left(x\right)}{4}\\&+\frac{3b^5\log\left(\cos\left(x\right)-1\right)}{16}-\frac{3b^5\log\left(\cos\left(x\right)+1\right)}{16}\\&+\frac{3b^5\cos^3\left(x\right)}{8\cos^4\left(x\right)-16\cos^2\left(x\right)+8}-\frac{5b^5\cos\left(x\right)}{8\cos^4\left(x\right)-16\cos^2\left(x\right)+8}\end{aligned}$$

input
```
integrate((a*cot(x)+b*csc(x))**5,x)
```

output
```
-a**5*log(csc(x)**2)/2 - a**5*csc(x)**4/4 + a**5*csc(x)**2 + 15*a**4*b*log
(cos(x) - 1)/16 - 15*a**4*b*log(cos(x) + 1)/16 - 25*a**4*b*cos(x)**3/(8*co
s(x)**4 - 16*cos(x)**2 + 8) + 15*a**4*b*cos(x)/(8*cos(x)**4 - 16*cos(x)**2
 + 8) - 5*a**3*b**2*cot(x)**4/2 - 5*a**2*b**3*log(cos(x) - 1)/8 + 5*a**2*b
**3*log(cos(x) + 1)/8 - 10*a**2*b**3*cos(x)**3/(8*cos(x)**4 - 16*cos(x)**2
 + 8) - 10*a**2*b**3*cos(x)/(8*cos(x)**4 - 16*cos(x)**2 + 8) - 5*a*b**4*cs
c(x)**4/4 + 3*b**5*log(cos(x) - 1)/16 - 3*b**5*log(cos(x) + 1)/16 + 3*b**5
*cos(x)**3/(8*cos(x)**4 - 16*cos(x)**2 + 8) - 5*b**5*cos(x)/(8*cos(x)**4 -
 16*cos(x)**2 + 8)
```

3.283.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.24

$$\int (a\cot(x)+b\csc(x))^5\,dx = -\frac{5}{2}a^3b^2\cot(x)^4 - \frac{5}{16}a^4b\left(\frac{2\left(5\cos(x)^3-3\cos(x)\right)}{\cos(x)^4-2\cos(x)^2+1}+3\log(\cos(x)+1)-3\log(\cos(x)-1)\right) + \frac{1}{16}b^5\left(\frac{2\left(3\cos(x)^3-5\cos(x)\right)}{\cos(x)^4-2\cos(x)^2+1}-3\log(\cos(x)+1)+3\log(\cos(x)-1)\right) - \frac{5}{8}a^2b^3\left(\frac{2\left(\cos(x)^3+\cos(x)\right)}{\cos(x)^4-2\cos(x)^2+1}-\log(\cos(x)+1)+\log(\cos(x)-1)\right) + \frac{1}{4}a^5\left(\frac{4\sin(x)^2-1}{\sin(x)^4}+2\log\left(\sin(x)^2\right)\right) - \frac{5ab^4}{4\sin(x)^4}$$

input
```
integrate((a*cot(x)+b*csc(x))^5,x, algorithm="maxima")
```

output
```
-5/2*a^3*b^2*cot(x)^4 - 5/16*a^4*b*(2*(5*cos(x)^3 - 3*cos(x))/(cos(x)^4 -
2*cos(x)^2 + 1) + 3*log(cos(x) + 1) - 3*log(cos(x) - 1)) + 1/16*b^5*(2*(3*
cos(x)^3 - 5*cos(x))/(cos(x)^4 - 2*cos(x)^2 + 1) - 3*log(cos(x) + 1) + 3*l
og(cos(x) - 1)) - 5/8*a^2*b^3*(2*(cos(x)^3 + cos(x))/(cos(x)^4 - 2*cos(x)^
2 + 1) - log(cos(x) + 1) + log(cos(x) - 1)) + 1/4*a^5*((4*sin(x)^2 - 1)/si
n(x)^4 + 2*log(sin(x)^2)) - 5/4*a*b^4/sin(x)^4
```

3.283.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.11

$$\int (a\cot(x)+b\csc(x))^5\,dx = \frac{1}{16}\left(8\,a^5-15\,a^4b+10\,a^2b^3-3\,b^5\right)\log\left(\cos\left(x\right)+1\right) + \frac{1}{16}\left(8\,a^5+15\,a^4b-10\,a^2b^3+3\,b^5\right)\log\left(-\cos\left(x\right)+1\right) + \frac{6\,a^5+20\,a^3b^2-10\,ab^4-\left(25\,a^4b+10\,a^2b^3-3\,b^5\right)\cos\left(x\right)^3-8\left(a^5+5\,a^3b^2\right)\cos\left(x\right)^2+5\left(3\,a^4b-2\,a^2b^3\right.}{8\left(\cos\left(x\right)+1\right)^2\left(\cos\left(x\right)-1\right)^2}$$

input
```
integrate((a*cot(x)+b*csc(x))^5,x, algorithm="giac")
```

output
```
1/16*(8*a^5 - 15*a^4*b + 10*a^2*b^3 - 3*b^5)*log(cos(x) + 1) + 1/16*(8*a^5
 + 15*a^4*b - 10*a^2*b^3 + 3*b^5)*log(-cos(x) + 1) + 1/8*(6*a^5 + 20*a^3*b
^2 - 10*a*b^4 - (25*a^4*b + 10*a^2*b^3 - 3*b^5)*cos(x)^3 - 8*(a^5 + 5*a^3*
b^2)*cos(x)^2 + 5*(3*a^4*b - 2*a^2*b^3 - b^5)*cos(x))/((cos(x) + 1)^2*(cos
(x) - 1)^2)
```

3.283.9 Mupad [B] (verification not implemented)

Time = 28.98 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.14

$$\int (a\cot(x)+b\csc(x))^5\,dx = \tan\left(\frac{x}{2}\right)^2\left(\frac{5\,(a+b)\,(a-b)^4}{32}+\frac{(a-b)^5}{32}\right) - \frac{\frac{5\,a\,b^4}{4}+\frac{5\,a^4\,b}{4}-\tan\left(\frac{x}{2}\right)^2\left(3\,a^5+10\,a^4\,b+10\,a^3\,b^2-5\,a\,b^4-2\,b^5\right)+\frac{a^5}{4}+\frac{b^5}{4}+\frac{5\,a^2\,b^3}{2}+\frac{5\,a^3\,b^2}{2}}{16\tan\left(\frac{x}{2}\right)^4} - a^5\,\ln\left(\tan\left(\frac{x}{2}\right)^2+1\right)+\ln\left(\tan\left(\frac{x}{2}\right)\right)\left(a^5+\frac{15\,a^4\,b}{8}-\frac{5\,a^2\,b^3}{4}+\frac{3\,b^5}{8}\right)-\frac{\tan\left(\frac{x}{2}\right)^4\,(a-b)^5}{64}$$

input
```
int((b/sin(x) + a*cot(x))^5,x)
```

output
```
tan(x/2)^2*((5*(a + b)*(a - b)^4)/32 + (a - b)^5/32) - ((5*a*b^4)/4 + (5*a
^4*b)/4 - tan(x/2)^2*(10*a^4*b - 5*a*b^4 + 3*a^5 - 2*b^5 + 10*a^3*b^2) + a
^5/4 + b^5/4 + (5*a^2*b^3)/2 + (5*a^3*b^2)/2)/(16*tan(x/2)^4) - a^5*log(ta
n(x/2)^2 + 1) + log(tan(x/2))*((15*a^4*b)/8 + a^5 + (3*b^5)/8 - (5*a^2*b^3
)/4) - (tan(x/2)^4*(a - b)^5)/64
```

3.284 $\int (a\cot(x)+b\csc(x))^4\,dx$

3.284.1 Optimal result

Integrand size = 11, antiderivative size = 101

$$\int (a\cot(x)+b\csc(x))^4\,dx = a^4x+\frac{1}{3}(b+a\cos(x))^2\left(ab+\left(3a^2-2b^2\right)\cos(x)\right)\csc(x) -\frac{1}{3}(b+a\cos(x))^3(a+b\cos(x))\csc^3(x) +\frac{4}{3}ab\left(2a^2-b^2\right)\sin(x)+\frac{1}{3}a^2\left(3a^2-2b^2\right)\cos(x)\sin(x)$$

output
```
a^4*x+1/3*(b+a*cos(x))^2*(a*b+(3*a^2-2*b^2)*cos(x))*csc(x)-1/3*(b+a*cos(x)
)^3*(a+b*cos(x))*csc(x)^3+4/3*a*b*(2*a^2-b^2)*sin(x)+1/3*a^2*(3*a^2-2*b^2)
*cos(x)*sin(x)
```

3.284.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.94

$$\int (a\cot(x)+b\csc(x))^4\,dx = -\frac{1}{12}\csc^3(x)\left(-8a^3b+16ab^3+6b^2\left(3a^2+b^2\right)\cos(x) +24a^3b\cos(2x)+4a^4\cos(3x)+6a^2b^2\cos(3x)-2b^4\cos(3x) -9a^4x\sin(x)+3a^4x\sin(3x)\right)$$

input
```
Integrate[(a*Cot[x] + b*Csc[x])^4,x]
```

output
```
-1/12*(Csc[x]^3*(-8*a^3*b + 16*a*b^3 + 6*b^2*(3*a^2 + b^2)*Cos[x] + 24*a^3
*b*Cos[2*x] + 4*a^4*Cos[3*x] + 6*a^2*b^2*Cos[3*x] - 2*b^4*Cos[3*x] - 9*a^4
*x*Sin[x] + 3*a^4*x*Sin[3*x]))
```

3.284.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.818$, Rules used = {3042, 4892, 3042, 3170, 3042, 3340, 27, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cot(x)+b\csc(x))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\cot(x)+b\csc(x))^4dx$$

$$\Big\downarrow \text{4892}$$

$$\int \csc^4(x)(a\cos(x)+b)^4dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b-a\sin\left(x-\frac{\pi}{2}\right)\right)^4}{\cos\left(x-\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{3170}$$

$$-\frac{1}{3}\int (b+a\cos(x))^2\left(3a^2+b\cos(x)a-2b^2\right)\csc^2(x)dx-\frac{1}{3}\csc^3(x)(a\cos(x)+b)^3(a+b\cos(x))$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{3}\int \frac{\left(b-a\sin\left(x-\frac{\pi}{2}\right)\right)^2\left(3a^2-b\sin\left(x-\frac{\pi}{2}\right)a-2b^2\right)}{\cos\left(x-\frac{\pi}{2}\right)^2}dx-\frac{1}{3}\csc^3(x)(a\cos(x)+b)^3(a+b\cos(x))$$

$$\Big\downarrow \text{3340}$$

$$\frac{1}{3}\left(\int 2(b+a\cos(x))\left(ba^2+\left(3a^2-2b^2\right)\cos(x)a\right)dx+\csc(x)\left(\left(3a^2-2b^2\right)\cos(x)+ab\right)\left(a\cos(x)+b\right)^2\right)-\frac{1}{3}\csc^3(x)(a\cos(x)+b)^3(a+b\cos(x))$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(2\int(b+a\cos(x))\left(ba^2+\left(3a^2-2b^2\right)\cos(x)a\right)dx+\csc(x)\left(\left(3a^2-2b^2\right)\cos(x)+ab\right)(a\cos(x)+b)^2\right)-\frac{1}{3}\csc^3(x)(a\cos(x)+b)^3(a+b\cos(x))$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(2\int\left(b+a\sin\left(x+\frac{\pi}{2}\right)\right)\left(ba^2+\left(3a^2-2b^2\right)\sin\left(x+\frac{\pi}{2}\right)a\right)dx+\csc(x)\left(\left(3a^2-2b^2\right)\cos(x)+ab\right)(a\cos(x)+\right.$$
$$\frac{1}{3}\csc^3(x)(a\cos(x)+b)^3(a+b\cos(x))$$

$$\Big\downarrow \text{3213}$$

$$\frac{1}{3}\left(\csc(x)\left(\left(3a^2-2b^2\right)\cos(x)+ab\right)(a\cos(x)+b)^2+2\left(\frac{3a^4x}{2}+2ab\left(2a^2-b^2\right)\sin(x)+\frac{1}{2}a^2\left(3a^2-2b^2\right)\sin(x)\cos\right.\right.$$
$$\frac{1}{3}\csc^3(x)(a\cos(x)+b)^3(a+b\cos(x))$$

input
```
Int[(a*Cot[x] + b*Csc[x])^4,x]
```

output
```
-1/3*((b + a*Cos[x])^3*(a + b*Cos[x])*Csc[x]^3) + ((b + a*Cos[x])^2*(a*b +
 (3*a^2 - 2*b^2)*Cos[x])*Csc[x] + 2*((3*a^4*x)/2 + 2*a*b*(2*a^2 - b^2)*Sin
[x] + (a^2*(3*a^2 - 2*b^2)*Cos[x]*Sin[x])/2))/3
```

3.284.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3170
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Simp[1/(g^2*(p + 1))
  Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) +
a^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g
}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*
p] || IntegerQ[m])
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3340
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(g*
Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x])^m*((d + c*Sin[e + f*x])/(f*g*(p
 + 1))), x] + Simp[1/(g^2*(p + 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Si
n[e + f*x])^(m - 1)*Simp[a*c*(p + 2) + b*d*m + b*c*(m + p + 2)*Sin[e + f*x]
, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && GtQ
[m, 0] && LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0]
&& SimplerQ[c + d*x, a + b*x])
```

rule 4892
```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.284.4 Maple [A] (verified)

Time = 10.31 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.71

method	result
parts	$a^4\left(-\frac{\cot(x)^3}{3}+\cot(x)-\frac{\pi}{2}+\operatorname{arccot}(\cot(x))\right)+b^4\left(-\frac{2}{3}-\frac{\csc(x)^2}{3}\right)\cot(x)+4a^3b\left(-\frac{\csc(x)^3}{3}+\csc(\right.$
default	$b^4\left(-\frac{2}{3}-\frac{\csc(x)^2}{3}\right)\cot(x)-\frac{4a\,b^3}{3\sin(x)^3}-\frac{2a^2b^2\cos(x)^3}{\sin(x)^3}+4a^3b\left(-\frac{\cos(x)^4}{3\sin(x)^3}+\frac{\cos(x)^4}{3\sin(x)}+\frac{\left(2+\cos(x)^2\right)\sin(x)}{3}\right)+$
risch	$a^4x+\frac{4i\left(6a^3b\,e^{5ix}+3a^4e^{4ix}+9a^2b^2e^{4ix}-4a^3b\,e^{3ix}+8a\,b^3e^{3ix}-3a^4e^{2ix}+3b^4e^{2ix}+6a^3b\,e^{ix}+2a^4+3a^2b^2-b^4\right)}{3\left(e^{2ix}-1\right)^3}$

input
```
int((a*cot(x)+b*csc(x))^4,x,method=_RETURNVERBOSE)
```

output
```
a^4*(-1/3*cot(x)^3+cot(x)-1/2*Pi+arccot(cot(x)))+b^4*(-2/3-1/3*csc(x)^2)*c
ot(x)+4*a^3*b*(-1/3*csc(x)^3+csc(x))-2*a^2*b^2*cot(x)^3-4/3*b^3*csc(x)^3*a
```

3.284.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.94

$$\int (a\cot(x)+b\csc(x))^4\,dx = \frac{12\,a^3b\cos{(x)}^2-8\,a^3b+4\,ab^3+2\,(2\,a^4+3\,a^2b^2-b^4)\cos{(x)}^3-3\,(a^4-b^4)\cos{(x)}+3\,\left(a^4x\cos{(x)}^2-a^4x\right.}{3\,\left(\cos{(x)}^2-1\right)\sin{(x)}}$$

input
```
integrate((a*cot(x)+b*csc(x))^4,x, algorithm="fricas")
```

output
```
1/3*(12*a^3*b*cos(x)^2 - 8*a^3*b + 4*a*b^3 + 2*(2*a^4 + 3*a^2*b^2 - b^4)*c
os(x)^3 - 3*(a^4 - b^4)*cos(x) + 3*(a^4*x*cos(x)^2 - a^4*x)*sin(x))/((cos(
x)^2 - 1)*sin(x))
```

3.284.6 Sympy [A] (verification not implemented)

Time = 16.15 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.96

$$\int (a\cot(x)+b\csc(x))^4\,dx = a^4x+\frac{a^4\cos{(x)}}{\sin{(x)}}-\frac{a^4\cos^3{(x)}}{3\sin^3{(x)}}-\frac{4a^3b\csc^3{(x)}}{3}+4a^3b\csc{(x)} - 2a^2b^2\cot^3{(x)}-\frac{4ab^3\csc^3{(x)}}{3}-\frac{b^4\cot^3{(x)}}{3}-b^4\cot{(x)}$$

input
```
integrate((a*cot(x)+b*csc(x))**4,x)
```

output
```
a**4*x + a**4*cos(x)/sin(x) - a**4*cos(x)**3/(3*sin(x)**3) - 4*a**3*b*csc(
x)**3/3 + 4*a**3*b*csc(x) - 2*a**2*b**2*cot(x)**3 - 4*a*b**3*csc(x)**3/3 -
 b**4*cot(x)**3/3 - b**4*cot(x)
```

3.284.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.79

$$\int (a\cot(x)+b\csc(x))^4\,dx = -2\,a^2b^2\cot{(x)}^3 + \frac{1}{3}\,a^4\left(3\,x + \frac{3\,\tan{(x)}^2-1}{\tan{(x)}^3}\right) + \frac{4\,(3\,\sin{(x)}^2-1)a^3b}{3\,\sin{(x)}^3} - \frac{(3\,\tan{(x)}^2+1)b^4}{3\,\tan{(x)}^3} - \frac{4\,ab^3}{3\,\sin{(x)}^3}$$

input
```
integrate((a*cot(x)+b*csc(x))^4,x, algorithm="maxima")
```

output
```
-2*a^2*b^2*cot(x)^3 + 1/3*a^4*(3*x + (3*tan(x)^2 - 1)/tan(x)^3) + 4/3*(3*s
in(x)^2 - 1)*a^3*b/sin(x)^3 - 1/3*(3*tan(x)^2 + 1)*b^4/tan(x)^3 - 4/3*a*b^
3/sin(x)^3
```

3.284.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. 2(93) = 186.

Time = 0.27 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.13

$$\int (a\cot(x)+b\csc(x))^4\,dx = \frac{1}{24}\,a^4\tan\left(\frac{1}{2}\,x\right)^3 - \frac{1}{6}\,a^3b\tan\left(\frac{1}{2}\,x\right)^3 + \frac{1}{4}\,a^2b^2\tan\left(\frac{1}{2}\,x\right)^3 - \frac{1}{6}\,ab^3\tan\left(\frac{1}{2}\,x\right)^3 + \frac{1}{24}\,b^4\tan\left(\frac{1}{2}\,x\right)^3 + a^4x - \frac{5}{8}\,a^4\tan\left(\frac{1}{2}\,x\right) + \frac{3}{2}\,a^3b\tan\left(\frac{1}{2}\,x\right) - \frac{3}{4}\,a^2b^2\tan\left(\frac{1}{2}\,x\right) - \frac{1}{2}\,ab^3\tan\left(\frac{1}{2}\,x\right) + \frac{3}{8}\,b^4\tan\left(\frac{1}{2}\,x\right) + \frac{15\,a^4\tan\left(\frac{1}{2}\,x\right)^2 + 36\,a^3b\tan\left(\frac{1}{2}\,x\right)^2 + 18\,a^2b^2\tan\left(\frac{1}{2}\,x\right)^2 - 12\,ab^3\tan\left(\frac{1}{2}\,x\right)^2 - 9\,b^4\tan\left(\frac{1}{2}\,x\right)^2 - a^4 - 4\,a^3 \cdots}{24\,\tan\left(\frac{1}{2}\,x\right)^3}$$

input
```
integrate((a*cot(x)+b*csc(x))^4,x, algorithm="giac")
```

output
```
1/24*a^4*tan(1/2*x)^3 - 1/6*a^3*b*tan(1/2*x)^3 + 1/4*a^2*b^2*tan(1/2*x)^3
- 1/6*a*b^3*tan(1/2*x)^3 + 1/24*b^4*tan(1/2*x)^3 + a^4*x - 5/8*a^4*tan(1/2
*x) + 3/2*a^3*b*tan(1/2*x) - 3/4*a^2*b^2*tan(1/2*x) - 1/2*a*b^3*tan(1/2*x)
 + 3/8*b^4*tan(1/2*x) + 1/24*(15*a^4*tan(1/2*x)^2 + 36*a^3*b*tan(1/2*x)^2
+ 18*a^2*b^2*tan(1/2*x)^2 - 12*a*b^3*tan(1/2*x)^2 - 9*b^4*tan(1/2*x)^2 - a
^4 - 4*a^3*b - 6*a^2*b^2 - 4*a*b^3 - b^4)/tan(1/2*x)^3
```

3.284.9 Mupad [B] (verification not implemented)

Time = 29.14 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.26

$$\int (a\cot(x)+b\csc(x))^4\,dx$$
$$=a^4\,x-\frac{\frac{4\,a\,b^3}{3}+\frac{4\,a^3\,b}{3}-\tan\left(\frac{x}{2}\right)^2\,\left(5\,a^4+12\,a^3\,b+6\,a^2\,b^2-4\,a\,b^3-3\,b^4\right)+\frac{a^4}{3}+\frac{b^4}{3}+2\,a^2\,b^2}{8\,\tan\left(\frac{x}{2}\right)^3}$$
$$-\tan\left(\frac{x}{2}\right)\,\left(\frac{(a+b)\,\left(a-b\right)^3}{2}+\frac{\left(a-b\right)^4}{8}\right)+\frac{\tan\left(\frac{x}{2}\right)^3\,\left(a-b\right)^4}{24}$$

input
```
int((b/sin(x) + a*cot(x))^4,x)
```

output
```
a^4*x - ((4*a*b^3)/3 + (4*a^3*b)/3 - tan(x/2)^2*(12*a^3*b - 4*a*b^3 + 5*a^
4 - 3*b^4 + 6*a^2*b^2) + a^4/3 + b^4/3 + 2*a^2*b^2)/(8*tan(x/2)^3) - tan(x
/2)*(((a + b)*(a - b)^3)/2 + (a - b)^4/8) + (tan(x/2)^3*(a - b)^4)/24
```

3.285 $\int (a\cot(x)+b\csc(x))^3\,dx$

3.285.1 Optimal result

Integrand size = 11, antiderivative size = 77

$$\int (a\cot(x)+b\csc(x))^3\,dx = -\frac{1}{2}a^2b\cos(x)-\frac{1}{2}(b+a\cos(x))^2(a+b\cos(x))\csc^2(x) -\frac{1}{4}(2a-b)(a+b)^2\log(1-\cos(x)) -\frac{1}{4}(a-b)^2(2a+b)\log(1+\cos(x))$$

output
```
-1/2*a^2*b*cos(x)-1/2*(b+a*cos(x))^2*(a+b*cos(x))*csc(x)^2-1/4*(2*a-b)*(a+
b)^2*ln(1-cos(x))-1/4*(a-b)^2*(2*a+b)*ln(1+cos(x))
```

3.285.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.03

$$\int (a\cot(x)+b\csc(x))^3\,dx = \frac{1}{8}\left(-(a+b)^3\csc^2\left(\frac{x}{2}\right)-4(a-b)^2(2a+b)\log\left(\cos\left(\frac{x}{2}\right)\right) -4(2a-b)(a+b)^2\log\left(\sin\left(\frac{x}{2}\right)\right)-(a-b)^3\sec^2\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[(a*Cot[x] + b*Csc[x])^3,x]
```

output
```
(-((a + b)^3*Csc[x/2]^2) - 4*(a - b)^2*(2*a + b)*Log[Cos[x/2]] - 4*(2*a -
b)*(a + b)^2*Log[Sin[x/2]] - (a - b)^3*Sec[x/2]^2)/8
```

3.285.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.27, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {3042, 4892, 3042, 3147, 477, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cot(x)+b\csc(x))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\cot(x)+b\csc(x))^3dx$$

$$\Big\downarrow \text{4892}$$

$$\int \csc^3(x)(a\cos(x)+b)^3dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b-a\sin\left(x-\frac{\pi}{2}\right)\right)^3}{\cos\left(x-\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{3147}$$

$$-a^3\int \frac{(b+a\cos(x))^3}{(a^2-a^2\cos^2(x))^2}d(a\cos(x))$$

$$\Big\downarrow \text{477}$$

$$-\frac{\int \left(-\frac{a^2(a-b)^3}{4(\cos(x)a+a)^2}+\frac{a(2a+b)(a-b)^2}{4(\cos(x)a+a)}-\frac{a(2a-b)(a+b)^2}{4(a-a\cos(x))}+\frac{a^2(a+b)^3}{4(a-a\cos(x))^2}\right)d(a\cos(x))}{a}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{a^2(a-b)^3}{4(a\cos(x)+a)}+\frac{a^2(a+b)^3}{4(a-a\cos(x))}+\frac{1}{4}a(2a+b)(a-b)^2\log(a\cos(x)+a)+\frac{1}{4}a(2a-b)(a+b)^2\log(a-a\cos(x))}{a}$$

input
```
Int[(a*Cot[x] + b*Csc[x])^3,x]
```

output
```
-(((a^2*(a + b)^3)/(4*(a - a*Cos[x])) + (a^2*(a - b)^3)/(4*(a + a*Cos[x]))
 + (a*(2*a - b)*(a + b)^2*Log[a - a*Cos[x]])/4 + (a*(a - b)^2*(2*a + b)*Lo
g[a + a*Cos[x]])/4)/a)
```

3.285.3.1 Defintions of rubi rules used

rule 477
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ExpandIntegrand[(c + d*x)^n*(1 - Rt[-b/a, 2]*x)^p*(1 + Rt[-b/a, 2
]*x)^p, x], x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[n] &
& NiceSqrtQ[-b/a] &&  !FractionalPowerFactorQ[Rt[-b/a, 2]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 4892
```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.285.4 Maple [A] (verified)

Time = 3.15 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.04

method	result
default	$b^3\left(-\frac{\csc(x)\cot(x)}{2}+\frac{\ln(\csc(x)-\cot(x))}{2}\right)-\frac{3a\,b^2}{2\sin(x)^2}+3a^2b\left(-\frac{\cos(x)^3}{2\sin(x)^2}-\frac{\cos(x)}{2}-\frac{\ln(\csc(x)-\cot(x))}{2}\right)+a^3\left(-\frac{c}{}\right.$
parts	$a^3\left(-\frac{\cot(x)^2}{2}+\frac{\ln\left(1+\cot(x)^2\right)}{2}\right)+b^3\left(-\frac{\csc(x)\cot(x)}{2}+\frac{\ln(\csc(x)-\cot(x))}{2}\right)+3a^2b\left(-\frac{\cos(x)^3}{2\sin(x)^2}-\frac{\cos(x)}{2}-\frac{\ln(cs}{}\right.$
risch	$ia^3x+\frac{e^{ix}\left(3a^2b\,e^{2ix}+b^3e^{2ix}+2a^3e^{ix}+6a\,b^2e^{ix}+3a^2b+b^3\right)}{\left(e^{2ix}-1\right)^2}-\ln\left(e^{ix}+1\right)a^3+\frac{3\ln\left(e^{ix}+1\right)a^2b}{2}-\frac{\ln\left(e^{ix}+1\right)b^3}{2}-\ln\left(e^{\right.$

input
```
int((a*cot(x)+b*csc(x))^3,x,method=_RETURNVERBOSE)
```

output
```
b^3*(-1/2*csc(x)*cot(x)+1/2*ln(csc(x)-cot(x)))-3/2*a*b^2/sin(x)^2+3*a^2*b*
(-1/2/sin(x)^2*cos(x)^3-1/2*cos(x)-1/2*ln(csc(x)-cot(x)))+a^3*(-1/2*cot(x)
^2-ln(sin(x)))
```

3.285.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.66

$$\int (a\cot(x) + b\csc(x))^3\,dx$$
$$= \frac{2\,a^3 + 6\,ab^2 + 2\,(3\,a^2b + b^3)\cos{(x)} + \left(2\,a^3 - 3\,a^2b + b^3 - (2\,a^3 - 3\,a^2b + b^3)\cos{(x)}^2\right)\log\left(\frac{1}{2}\,\cos{(x)} + \frac{1}{2}\right)}{4\left(\cos{(x)}^2 - 1\right)}$$

input
```
integrate((a*cot(x)+b*csc(x))^3,x, algorithm="fricas")
```

output
```
1/4*(2*a^3 + 6*a*b^2 + 2*(3*a^2*b + b^3)*cos(x) + (2*a^3 - 3*a^2*b + b^3 -
 (2*a^3 - 3*a^2*b + b^3)*cos(x)^2)*log(1/2*cos(x) + 1/2) + (2*a^3 + 3*a^2*
b - b^3 - (2*a^3 + 3*a^2*b - b^3)*cos(x)^2)*log(-1/2*cos(x) + 1/2))/(cos(x
)^2 - 1)
```

3.285.6 Sympy [A] (verification not implemented)

Time = 5.85 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.61

$$\int (a\cot(x) + b\csc(x))^3\,dx = \frac{a^3\log\left(-\csc^2{(x)}\right)}{2} - \frac{a^3\csc^2{(x)}}{2} - \frac{3a^2b\log\left(\cos{(x)} - 1\right)}{4} + \frac{3a^2b\log\left(\cos{(x)} + 1\right)}{4} + \frac{3a^2b\cos{(x)}}{2\cos^2{(x)} - 2} - \frac{3ab^2\csc^2{(x)}}{2} + \frac{b^3\log\left(\cos{(x)} - 1\right)}{4} - \frac{b^3\log\left(\cos{(x)} + 1\right)}{4} + \frac{b^3\cos{(x)}}{2\cos^2{(x)} - 2}$$

input
```
integrate((a*cot(x)+b*csc(x))**3,x)
```

output
```
a**3*log(-csc(x)**2)/2 - a**3*csc(x)**2/2 - 3*a**2*b*log(cos(x) - 1)/4 + 3
*a**2*b*log(cos(x) + 1)/4 + 3*a**2*b*cos(x)/(2*cos(x)**2 - 2) - 3*a*b**2*c
sc(x)**2/2 + b**3*log(cos(x) - 1)/4 - b**3*log(cos(x) + 1)/4 + b**3*cos(x)
/(2*cos(x)**2 - 2)
```

3.285.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.13

$$\int (a\cot(x)+b\csc(x))^3\,dx = -\frac{3}{2}\,ab^2\cot(x)^2 + \frac{3}{4}\,a^2b\left(\frac{2\cos(x)}{\cos(x)^2-1}+\log(\cos(x)+1)-\log(\cos(x)-1)\right) + \frac{1}{4}\,b^3\left(\frac{2\cos(x)}{\cos(x)^2-1}-\log(\cos(x)+1)+\log(\cos(x)-1)\right) - \frac{1}{2}\,a^3\left(\frac{1}{\sin(x)^2}+\log\left(\sin(x)^2\right)\right)$$

input
```
integrate((a*cot(x)+b*csc(x))^3,x, algorithm="maxima")
```

output
```
-3/2*a*b^2*cot(x)^2 + 3/4*a^2*b*(2*cos(x)/(cos(x)^2 - 1) + log(cos(x) + 1)
 - log(cos(x) - 1)) + 1/4*b^3*(2*cos(x)/(cos(x)^2 - 1) - log(cos(x) + 1) +
 log(cos(x) - 1)) - 1/2*a^3*(1/sin(x)^2 + log(sin(x)^2))
```

3.285.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.12

$$\int (a\cot(x)+b\csc(x))^3\,dx = -\frac{1}{4}\left(2\,a^3-3\,a^2b+b^3\right)\log(\cos(x)+1) - \frac{1}{4}\left(2\,a^3+3\,a^2b-b^3\right)\log(-\cos(x)+1) + \frac{a^3+3\,ab^2+\left(3\,a^2b+b^3\right)\cos(x)}{2\,(\cos(x)+1)(\cos(x)-1)}$$

input
```
integrate((a*cot(x)+b*csc(x))^3,x, algorithm="giac")
```

output
```
-1/4*(2*a^3 - 3*a^2*b + b^3)*log(cos(x) + 1) - 1/4*(2*a^3 + 3*a^2*b - b^3)
*log(-cos(x) + 1) + 1/2*(a^3 + 3*a*b^2 + (3*a^2*b + b^3)*cos(x))/((cos(x)
+ 1)*(cos(x) - 1))
```

3.285.9 Mupad [B] (verification not implemented)

Time = 29.61 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.06

$$\int (a\cot(x)+b\csc(x))^3\,dx = a^3\,\ln\left(\tan\left(\frac{x}{2}\right)^2+1\right)-\frac{\frac{a^3}{8}+\frac{3\,a^2\,b}{8}+\frac{3\,a\,b^2}{8}+\frac{b^3}{8}}{\tan\left(\frac{x}{2}\right)^2} - \ln\left(\tan\left(\frac{x}{2}\right)\right)\,\left(a^3+\frac{3\,a^2\,b}{2}-\frac{b^3}{2}\right)-\frac{\tan\left(\frac{x}{2}\right)^2\,(a-b)^3}{8}$$

input
```
int((b/sin(x) + a*cot(x))^3,x)
```

output
```
a^3*log(tan(x/2)^2 + 1) - ((3*a*b^2)/8 + (3*a^2*b)/8 + a^3/8 + b^3/8)/tan(
x/2)^2 - log(tan(x/2))*((3*a^2*b)/2 + a^3 - b^3/2) - (tan(x/2)^2*(a - b)^3
)/8
```

3.286 $\int (a\cot(x)+b\csc(x))^2\,dx$

3.286.1 Optimal result

Integrand size = 11, antiderivative size = 29

$$\int (a\cot(x)+b\csc(x))^2\,dx = -a^2x-(b+a\cos(x))(a+b\cos(x))\csc(x)-ab\sin(x)$$

output
```
-a^2*x-(b+a*cos(x))*(a+b*cos(x))*csc(x)-a*b*sin(x)
```

3.286.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int (a\cot(x)+b\csc(x))^2\,dx = -\left(\left(a^2+b^2\right)\cot(x)\right)-a(ax+2b\csc(x))$$

input
```
Integrate[(a*Cot[x] + b*Csc[x])^2,x]
```

output
```
-((a^2 + b^2)*Cot[x]) - a*(a*x + 2*b*Csc[x])
```

3.286.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {3042, 4892, 3042, 3170, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cot(x)+b\csc(x))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\cot(x)+b\csc(x))^2dx$$

$$\Big\downarrow \text{4892}$$

$$\int \csc^2(x)(a\cos(x)+b)^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(b-a\sin\left(x-\frac{\pi}{2}\right)\right)^2}{\cos\left(x-\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3170}$$

$$-\int \left(a^2+b\cos(x)a\right)dx-(\csc(x)(a\cos(x)+b)(a+b\cos(x)))$$

$$\Big\downarrow \text{2009}$$

$$a^2(-x)-ab\sin(x)-\csc(x)(a\cos(x)+b)(a+b\cos(x))$$

input
```
Int[(a*Cot[x] + b*Csc[x])^2,x]
```

output
```
-(a^2*x) - (b + a*Cos[x])*(a + b*Cos[x])*Csc[x] - a*b*Sin[x]
```

3.286.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3170
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-(g*Cos[e + f*x])^(p + 1))*(a + b*Sin[e + f*x
])^(m - 1)*((b + a*Sin[e + f*x])/(f*g*(p + 1))), x] + Simp[1/(g^2*(p + 1))
  Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) +
a^2*(p + 2) + a*b*(m + p + 1)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g
}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] && LtQ[p, -1] && (IntegersQ[2*m, 2*
p] || IntegerQ[m])
```

rule 4892
```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.286.4 Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

method	result	size
default	$-b^{2} \cot \left(x \right)-\frac{2 a b}{\sin \left(x \right)}+a^{2} \left(-\cot \left(x \right)-x \right)$	29
parts	$a^{2} \left(-\cot \left(x \right)+\frac{\pi}{2}-\operatorname{arccot}\left(\cot \left(x \right)\right)\right)-b^{2} \cot \left(x \right)-2 b \csc \left(x \right) a$	32
risch	$-a^{2} x -\frac{2 i \left(2 a b \,{\mathrm e}^{i x}+a^{2}+b^{2}\right)}{{\mathrm e}^{2 i x}-1}$	36

input
```
int((a*cot(x)+b*csc(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-b^2*cot(x)-2*a*b/sin(x)+a^2*(-cot(x)-x)
```

3.286.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int (a\cot(x)+b\csc(x))^2\,dx = -\frac{a^2x\sin(x)+2\,ab+(a^2+b^2)\cos(x)}{\sin(x)}$$

input
```
integrate((a*cot(x)+b*csc(x))^2,x, algorithm="fricas")
```

output
```
-(a^2*x*sin(x) + 2*a*b + (a^2 + b^2)*cos(x))/sin(x)
```

3.286.6 Sympy [A] (verification not implemented)

Time = 1.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int (a\cot(x)+b\csc(x))^2\,dx = -a^2x-\frac{a^2\cos(x)}{\sin(x)}-2ab\csc(x)-b^2\cot(x)$$

input
```
integrate((a*cot(x)+b*csc(x))**2,x)
```

output
```
-a**2*x - a**2*cos(x)/sin(x) - 2*a*b*csc(x) - b**2*cot(x)
```

3.286.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int (a\cot(x)+b\csc(x))^2\,dx = -a^2\left(x+\frac{1}{\tan(x)}\right)-\frac{2\,ab}{\sin(x)}-\frac{b^2}{\tan(x)}$$

input
```
integrate((a*cot(x)+b*csc(x))^2,x, algorithm="maxima")
```

output
```
-a^2*(x + 1/tan(x)) - 2*a*b/sin(x) - b^2/tan(x)
```

3.286.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.79

$$\int (a\cot(x)+b\csc(x))^2\,dx = -a^2x + \frac{1}{2}\,a^2\tan\left(\frac{1}{2}\,x\right) - ab\tan\left(\frac{1}{2}\,x\right) + \frac{1}{2}\,b^2\tan\left(\frac{1}{2}\,x\right) - \frac{a^2+2\,ab+b^2}{2\,\tan\left(\frac{1}{2}\,x\right)}$$

input
```
integrate((a*cot(x)+b*csc(x))^2,x, algorithm="giac")
```

output
```
-a^2*x + 1/2*a^2*tan(1/2*x) - a*b*tan(1/2*x) + 1/2*b^2*tan(1/2*x) - 1/2*(a
^2 + 2*a*b + b^2)/tan(1/2*x)
```

3.286.9 Mupad [B] (verification not implemented)

Time = 30.31 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int (a\cot(x)+b\csc(x))^2\,dx = -\frac{\cos(x)\ a^2+2\,a\,b+\cos(x)\ b^2}{\sin(x)} - a^2\,x$$

input
```
int((b/sin(x) + a*cot(x))^2,x)
```

output
```
- (2*a*b + a^2*cos(x) + b^2*cos(x))/sin(x) - a^2*x
```

3.287 $\int (a\cot(x)+b\csc(x))\,dx$

3.287.1 Optimal result

Integrand size = 9, antiderivative size = 12

$$\int (a\cot(x)+b\csc(x))\,dx=-b\,\text{arctanh}(\cos(x))+a\log(\sin(x))$$

output
```
-b*arctanh(cos(x))+a*ln(sin(x))
```

3.287.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 30 vs. 2(12) = 24.

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.50

$$\int (a\cot(x)+b\csc(x))\,dx=-b\log\left(\cos\left(\frac{x}{2}\right)\right)+a\log(\cos(x))+b\log\left(\sin\left(\frac{x}{2}\right)\right)+a\log(\tan(x))$$

input
```
Integrate[a*Cot[x] + b*Csc[x],x]
```

output
```
-(b*Log[Cos[x/2]]) + a*Log[Cos[x]] + b*Log[Sin[x/2]] + a*Log[Tan[x]]
```

3.287.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cot(x)+b\csc(x))\,dx$$

$$\Big\downarrow \; 2009$$

$$a\log(\sin(x)) - b\text{arctanh}(\cos(x))$$

input
```
Int[a*Cot[x] + b*Csc[x],x]
```

output
```
-(b*ArcTanh[Cos[x]]) + a*Log[Sin[x]]
```

3.287.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.287.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

method	result	size
default	$a\ln(\sin(x)) - b\ln(\cot(x)+\csc(x))$	16
parts	$a\ln(\sin(x)) - b\ln(\cot(x)+\csc(x))$	16
norman	$a\ln(\tan(x)) + b\ln\left(\tan\left(\frac{x}{2}\right)\right) - \frac{a\ln\left(1+\tan(x)^2\right)}{2}$	24
parallelrisch	$a\left(\ln(\tan(x)) + \ln\left(\frac{1}{\sqrt{\sec(x)^2}}\right)\right) + b\ln(\csc(x)-\cot(x))$	25
risch	$-iax + a\ln\left(\mathrm{e}^{2ix}-1\right) + b\ln\left(\mathrm{e}^{ix}-1\right) - b\ln\left(\mathrm{e}^{ix}+1\right)$	38

input
```
int(a*cot(x)+b*csc(x),x,method=_RETURNVERBOSE)
```

output
```
a*ln(sin(x))-b*ln(cot(x)+csc(x))
```

3.287.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(12) = 24.

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.25

$$\int (a\cot(x)+b\csc(x))\,dx=\frac{1}{2}\,(a-b)\log\left(\frac{1}{2}\,\cos\left(x\right)+\frac{1}{2}\right)+\frac{1}{2}\,(a+b)\log\left(-\frac{1}{2}\,\cos\left(x\right)+\frac{1}{2}\right)$$

input
```
integrate(a*cot(x)+b*csc(x),x, algorithm="fricas")
```

output
```
1/2*(a - b)*log(1/2*cos(x) + 1/2) + 1/2*(a + b)*log(-1/2*cos(x) + 1/2)
```

3.287.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.00

$$\int (a\cot(x)+b\csc(x))\,dx=a\log\left(\sin\left(x\right)\right)+b\left(\frac{\log\left(\cos\left(x\right)-1\right)}{2}-\frac{\log\left(\cos\left(x\right)+1\right)}{2}\right)$$

input
```
integrate(a*cot(x)+b*csc(x),x)
```

output
```
a*log(sin(x)) + b*(log(cos(x) - 1)/2 - log(cos(x) + 1)/2)
```

3.287.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int (a\cot(x)+b\csc(x))\,dx=-b\log\left(\cot\left(x\right)+\csc\left(x\right)\right)+a\log\left(\sin\left(x\right)\right)$$

input
```
integrate(a*cot(x)+b*csc(x),x, algorithm="maxima")
```

output
```
-b*log(cot(x) + csc(x)) + a*log(sin(x))
```

3.287.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int (a\cot(x)+b\csc(x))\,dx=a\log\left(|\sin\left(x\right)|\right)+b\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)$$

input
```
integrate(a*cot(x)+b*csc(x),x, algorithm="giac")
```

output
```
a*log(abs(sin(x))) + b*log(abs(tan(1/2*x)))
```

3.287.9 Mupad [B] (verification not implemented)

Time = 30.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.25

$$\int (a\cot(x)+b\csc(x))\,dx=a\,\ln\left(\tan\left(\frac{x}{2}\right)\right)-a\,\ln\left({\tan\left(\frac{x}{2}\right)}^2+1\right)+b\,\ln\left(\tan\left(\frac{x}{2}\right)\right)$$

input
```
int(b/sin(x) + a*cot(x),x)
```

output
```
a*log(tan(x/2)) - a*log(tan(x/2)^2 + 1) + b*log(tan(x/2))
```

3.288 $\int \frac{1}{a\cot(x)+b\csc(x)}\,dx$

3.288.1 Optimal result

Integrand size = 11, antiderivative size = 12

$$\int \frac{1}{a\cot(x)+b\csc(x)}\,dx=-\frac{\log(b+a\cos(x))}{a}$$

output
```
-ln(b+a*cos(x))/a
```

3.288.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{a\cot(x)+b\csc(x)}\,dx=-\frac{\log(b+a\cos(x))}{a}$$

input
```
Integrate[(a*Cot[x] + b*Csc[x])^(-1),x]
```

output
```
-(Log[b + a*Cos[x]]/a)
```

3.288.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {3042, 3639, 3042, 3147, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a\cot(x)+b\csc(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a\cot(x)+b\csc(x)}dx$$

$$\Big\downarrow \text{3639}$$

$$\int \frac{\sin(x)}{a\cos(x)+b}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)}{b-a\sin\left(x-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3147}$$

$$-\frac{\int \frac{1}{b+a\cos(x)}d(a\cos(x))}{a}$$

$$\Big\downarrow \text{16}$$

$$-\frac{\log(a\cos(x)+b)}{a}$$

input
```
Int[(a*Cot[x] + b*Csc[x])^(-1),x]
```

output
```
-(Log[b + a*Cos[x]]/a)
```

3.288.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 3639
```
Int[((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) + cot[(d_.) + (e_.)*(x_)]*(c_.))
^(-1), x_Symbol] :> Int[Sin[d + e*x]/(b + a*Sin[d + e*x] + c*Cos[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

3.288.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
default	$-\frac{\ln(b+a\cos(x))}{a}$	13
risch	$\frac{ix}{a}-\frac{\ln\left(\mathrm{e}^{2ix}+\frac{2b\,\mathrm{e}^{ix}}{a}+1\right)}{a}$	33

input
```
int(1/(a*cot(x)+b*csc(x)),x,method=_RETURNVERBOSE)
```

output
```
-ln(b+a*cos(x))/a
```

3.288.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{a\cot(x) + b\csc(x)}\,dx = -\frac{\log\left(a\cos\left(x\right) + b\right)}{a}$$

input
```
integrate(1/(a*cot(x)+b*csc(x)),x, algorithm="fricas")
```

output
```
-log(a*cos(x) + b)/a
```

3.288.6 Sympy [F]

$$\int \frac{1}{a\cot(x) + b\csc(x)}\,dx = \int \frac{1}{a\cot\left(x\right) + b\csc\left(x\right)}\,dx$$

input
```
integrate(1/(a*cot(x)+b*csc(x)),x)
```

output
```
Integral(1/(a*cot(x) + b*csc(x)), x)
```

3.288.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(12) = 24.

Time = 0.31 (sec) , antiderivative size = 45, normalized size of antiderivative = 3.75

$$\int \frac{1}{a\cot(x) + b\csc(x)}\,dx = -\frac{\log\left(a + b - \frac{(a-b)\sin(x)^2}{(\cos(x)+1)^2}\right)}{a} + \frac{\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2} + 1\right)}{a}$$

input
```
integrate(1/(a*cot(x)+b*csc(x)),x, algorithm="maxima")
```

output
```
-log(a + b - (a - b)*sin(x)^2/(cos(x) + 1)^2)/a + log(sin(x)^2/(cos(x) + 1
)^2 + 1)/a
```

3.288.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{1}{a\cot(x)+b\csc(x)}\,dx = -\frac{\log\left(|a\cos\left(x\right)+b|\right)}{a}$$

input
```
integrate(1/(a*cot(x)+b*csc(x)),x, algorithm="giac")
```

output
```
-log(abs(a*cos(x) + b))/a
```

3.288.9 Mupad [B] (verification not implemented)

Time = 30.74 (sec) , antiderivative size = 36, normalized size of antiderivative = 3.00

$$\int \frac{1}{a\cot(x)+b\csc(x)}\,dx = \frac{\mathrm{atan}\left(\frac{a\sin\left(\frac{x}{2}\right)^2}{-1\mathrm{i}\,a\sin\left(\frac{x}{2}\right)^2+a\,1\mathrm{i}+b\,1\mathrm{i}}\right)\,2\mathrm{i}}{a}$$

input
```
int(1/(b/sin(x) + a*cot(x)),x)
```

output
```
(atan((a*sin(x/2)^2)/(a*1i + b*1i - a*sin(x/2)^2*1i))*2i)/a
```

3.289 $\int \frac{1}{(a\cot(x)+b\csc(x))^2}\,dx$

3.289.1 Optimal result

Integrand size = 11, antiderivative size = 67

$$\int \frac{1}{(a\cot(x)+b\csc(x))^2}\,dx=-\frac{x}{a^2}+\frac{2b\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\sqrt{a+b}}+\frac{\sin(x)}{a(b+a\cos(x))}$$

output
```
-x/a^2+sin(x)/a/(b+a*cos(x))+2*b*arctanh((a-b)^(1/2)*tan(1/2*x)/(a+b)^(1/2
))/a^2/(a-b)^(1/2)/(a+b)^(1/2)
```

3.289.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.06

$$\int \frac{1}{(a\cot(x)+b\csc(x))^2}\,dx=-\frac{\frac{2b\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{bx+ax\cos(x)-a\sin(x)}{b+a\cos(x)}}{a^2}$$

input
```
Integrate[(a*Cot[x] + b*Csc[x])^(-2),x]
```

output
```
-(((2*b*ArcTanh[((-a + b)*Tan[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + (b
*x + a*x*Cos[x] - a*Sin[x])/(b + a*Cos[x]))/a^2)
```

3.289.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.909$, Rules used = {3042, 4892, 3042, 3172, 25, 3042, 3214, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cot(x)+b\csc(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a\cot(x)+b\csc(x))^2}dx$$

$$\Big\downarrow \text{4892}$$

$$\int \frac{\sin^2(x)}{(a\cos(x)+b)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)^2}{\left(b-a\sin\left(x-\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3172}$$

$$\frac{\int -\frac{\cos(x)}{b+a\cos(x)}dx}{a}+\frac{\sin(x)}{a(a\cos(x)+b)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sin(x)}{a(a\cos(x)+b)}-\frac{\int \frac{\cos(x)}{b+a\cos(x)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(x)}{a(a\cos(x)+b)}-\frac{\int \frac{\sin\left(x+\frac{\pi}{2}\right)}{b+a\sin\left(x+\frac{\pi}{2}\right)}dx}{a}$$

$$\Big\downarrow \text{3214}$$

$$\frac{\sin(x)}{a(a\cos(x)+b)}-\frac{\frac{x}{a}-\frac{b\int \frac{1}{b+a\cos(x)}dx}{a}}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin(x)}{a(a\cos(x)+b)}-\frac{\frac{x}{a}-\frac{b\int\frac{1}{b+a\sin\left(x+\frac{\pi}{2}\right)}dx}{a}}{a}$$

$$\Big\downarrow \text{3138}$$

$$\frac{\sin(x)}{a(a\cos(x)+b)}-\frac{\frac{x}{a}-\frac{2b\int\frac{1}{-\left((a-b)\tan^2\left(\frac{x}{2}\right)\right)+a+b}d\tan\left(\frac{x}{2}\right)}{a}}{a}$$

$$\Big\downarrow \text{221}$$

$$\frac{\sin(x)}{a(a\cos(x)+b)}-\frac{\frac{x}{a}-\frac{2b\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}}{a}$$

input
```
Int[(a*Cot[x] + b*Csc[x])^(-2),x]
```

output
```
-((x/a - (2*b*ArcTanh[(Sqrt[a - b]*Tan[x/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*
Sqrt[a + b]))/a) + Sin[x]/(a*(b + a*Cos[x]))
```

3.289.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

3.289. $\int \frac{1}{(a\cot(x)+b\csc(x))^2}\,dx$

rule 3172
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Simp[g^2*((p - 1)/(b*(m + 1)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Fre
eQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && I
ntegersQ[2*m, 2*p]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 4892
```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.289.4 Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.28

method	result	size
default	$\frac{-\frac{2a\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^{2}a-\tan\left(\frac{x}{2}\right)^{2}b-a-b}+\frac{2b\,\operatorname{arctanh}\left(\frac{\tan\left(\frac{x}{2}\right)(a-b)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}}{a^{2}}-\frac{2\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{a^{2}}$	86
risch	$-\frac{x}{a^{2}}+\frac{2i(b\,e^{ix}+a)}{a^{2}(a\,e^{2ix}+2b\,e^{ix}+a)}+\frac{b\ln\left(e^{ix}+\frac{ia^{2}-ib^{2}+\sqrt{a^{2}-b^{2}}\,b}{a\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,a^{2}}-\frac{b\ln\left(e^{ix}+\frac{-ia^{2}+ib^{2}+\sqrt{a^{2}-b^{2}}\,b}{a\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,a^{2}}$	171

input
```
int(1/(a*cot(x)+b*csc(x))^2,x,method=_RETURNVERBOSE)
```

output
```
2/a^2*(-a*tan(1/2*x)/(tan(1/2*x)^2*a-tan(1/2*x)^2*b-a-b)+b/((a+b)*(a-b))^(
1/2)*arctanh(tan(1/2*x)*(a-b)/((a+b)*(a-b))^(1/2)))-2/a^2*arctan(tan(1/2*x
))
```

3.289.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(57) = 114$.

Time = 0.28 (sec) , antiderivative size = 307, normalized size of antiderivative = 4.58

$$\int \frac{1}{(a\cot(x)+b\csc(x))^2}\,dx$$
$$= \left[-\frac{2\left(a^3-ab^2\right)x\cos(x)-\left(ab\cos(x)+b^2\right)\sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos(x)-\left(a^2-2\,b^2\right)\cos(x)^2+2\,\sqrt{a^2-b^2}(b\cos(x)+a)\sin(x)+2\,a}{a^2\cos(x)^2+2\,ab\cos(x)+b^2}\right.}{2\left(a^4b-a^2b^3+\left(a^5-a^3b^2\right)\cos(x)\right)}\right.$$
$$-\frac{\left(a^3-ab^2\right)x\cos(x)-\left(ab\cos(x)+b^2\right)\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cos(x)+a)}{(a^2-b^2)\sin(x)}\right)+\left(a^2b-b^3\right)x-\left(a^3-a\right.}{a^4b-a^2b^3+\left(a^5-a^3b^2\right)\cos(x)}$$

input
```
integrate(1/(a*cot(x)+b*csc(x))^2,x, algorithm="fricas")
```

output
```
[-1/2*(2*(a^3 - a*b^2)*x*cos(x) - (a*b*cos(x) + b^2)*sqrt(a^2 - b^2)*log((
2*a*b*cos(x) - (a^2 - 2*b^2)*cos(x)^2 + 2*sqrt(a^2 - b^2)*(b*cos(x) + a)*s
in(x) + 2*a^2 - b^2)/(a^2*cos(x)^2 + 2*a*b*cos(x) + b^2)) + 2*(a^2*b - b^3
)*x - 2*(a^3 - a*b^2)*sin(x))/(a^4*b - a^2*b^3 + (a^5 - a^3*b^2)*cos(x)),
-((a^3 - a*b^2)*x*cos(x) - (a*b*cos(x) + b^2)*sqrt(-a^2 + b^2)*arctan(-sqr
t(-a^2 + b^2)*(b*cos(x) + a)/((a^2 - b^2)*sin(x))) + (a^2*b - b^3)*x - (a^
3 - a*b^2)*sin(x))/(a^4*b - a^2*b^3 + (a^5 - a^3*b^2)*cos(x))]
```

3.289.6 Sympy [F]

$$\int \frac{1}{(a\cot(x)+b\csc(x))^2}\,dx = \int \frac{1}{\left(a\cot(x)+b\csc(x)\right)^2}\,dx$$

input
```
integrate(1/(a*cot(x)+b*csc(x))**2,x)
```

output
```
Integral((a*cot(x) + b*csc(x))**(-2), x)
```

3.289.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a\cot(x) + b\csc(x))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a*cot(x)+b*csc(x))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.289.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.60

$$\int \frac{1}{(a\cot(x) + b\csc(x))^2}\,dx = \frac{2\left(\pi\left\lfloor\frac{x}{2\pi} + \frac{1}{2}\right\rfloor \operatorname{sgn}(-2a + 2b) + \arctan\left(-\frac{a\tan\left(\frac{1}{2}x\right) - b\tan\left(\frac{1}{2}x\right)}{\sqrt{-a^2+b^2}}\right)\right)b}{\sqrt{-a^2+b^2}\,a^2} - \frac{x}{a^2} - \frac{2\tan\left(\frac{1}{2}x\right)}{\left(a\tan\left(\frac{1}{2}x\right)^2 - b\tan\left(\frac{1}{2}x\right)^2 - a - b\right)a}$$

input
```
integrate(1/(a*cot(x)+b*csc(x))^2,x, algorithm="giac")
```

output
```
2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*ta
n(1/2*x))/sqrt(-a^2 + b^2)))*b/(sqrt(-a^2 + b^2)*a^2) - x/a^2 - 2*tan(1/2*
x)/((a*tan(1/2*x)^2 - b*tan(1/2*x)^2 - a - b)*a)
```

3.289.9 Mupad [B] (verification not implemented)

Time = 29.27 (sec) , antiderivative size = 440, normalized size of antiderivative = 6.57

$$\int \frac{1}{(a\cot(x)+b\csc(x))^2}\,dx$$
$$= \frac{a^3\,\sin(x)+b^2\left(-a\,\sin(x)+\operatorname{atan}\left(\frac{-a^5\,\sin\left(\frac{x}{2}\right)\sqrt{a^2-b^2}\,1\mathrm{i}+b^3\,\sin\left(\frac{x}{2}\right)\left(a^2-b^2\right)^{3/2}2\mathrm{i}+b^5\,\sin\left(\frac{x}{2}\right)\sqrt{a^2-b^2}\,2\mathrm{i}+a^4\,b\,\sin\left(\frac{x}{2}\right)\sqrt{a^2-b^2}\,1\right.}{\cos\left(\frac{x}{2}\right)a^6-2\,\cos\left(\frac{x}{2}\right)a^4\,b^2+\cos\left(\frac{x}{2}\right)a^2\,b^4}}{}$$
$$-\frac{2\operatorname{atan}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{a^2}$$

input
```
int(1/(b/sin(x) + a*cot(x))^2,x)
```

output
```
(a^3*sin(x) + b^2*(atan((b^3*sin(x/2)*(a^2 - b^2)^(3/2)*2i - a^5*sin(x/2)*
(a^2 - b^2)^(1/2)*1i + b^5*sin(x/2)*(a^2 - b^2)^(1/2)*2i + a^4*b*sin(x/2)*
(a^2 - b^2)^(1/2)*1i - a^2*b^3*sin(x/2)*(a^2 - b^2)^(1/2)*3i + a^3*b^2*sin
(x/2)*(a^2 - b^2)^(1/2)*1i)/(a^6*cos(x/2) + a^2*b^4*cos(x/2) - 2*a^4*b^2*c
os(x/2)))*(a^2 - b^2)^(1/2)*2i - a*sin(x)) + a*b*atan((b^3*sin(x/2)*(a^2 -
 b^2)^(3/2)*2i - a^5*sin(x/2)*(a^2 - b^2)^(1/2)*1i + b^5*sin(x/2)*(a^2 - b
^2)^(1/2)*2i + a^4*b*sin(x/2)*(a^2 - b^2)^(1/2)*1i - a^2*b^3*sin(x/2)*(a^2
 - b^2)^(1/2)*3i + a^3*b^2*sin(x/2)*(a^2 - b^2)^(1/2)*1i)/(a^6*cos(x/2) +
a^2*b^4*cos(x/2) - 2*a^4*b^2*cos(x/2)))*cos(x)*(a^2 - b^2)^(1/2)*2i)/(a^4*
b - a^2*b^3 + a^5*cos(x) - a^3*b^2*cos(x)) - (2*atan(sin(x/2)/cos(x/2)))/a
^2
```

3.290 $\int \frac{1}{(a\cot(x)+b\csc(x))^3}\,dx$

3.290.1 Optimal result

Integrand size = 11, antiderivative size = 50

$$\int \frac{1}{(a\cot(x)+b\csc(x))^3}\,dx=\frac{a^2-b^2}{2a^3(b+a\cos(x))^2}+\frac{2b}{a^3(b+a\cos(x))}+\frac{\log(b+a\cos(x))}{a^3}$$

output
```
1/2*(a^2-b^2)/a^3/(b+a*cos(x))^2+2*b/a^3/(b+a*cos(x))+ln(b+a*cos(x))/a^3
```

3.290.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.54

$$\int \frac{1}{(a\cot(x)+b\csc(x))^3}\,dx = \frac{a^2+3b^2+a^2\log(b+a\cos(x))+2b^2\log(b+a\cos(x))+a^2\cos(2x)\log(b+a\cos(x))+4ab\cos(x)(1+\log}{2a^3(b+a\cos(x))^2}$$

input
```
Integrate[(a*Cot[x] + b*Csc[x])^(-3),x]
```

output
```
(a^2 + 3*b^2 + a^2*Log[b + a*Cos[x]] + 2*b^2*Log[b + a*Cos[x]] + a^2*Cos[2
*x]*Log[b + a*Cos[x]] + 4*a*b*Cos[x]*(1 + Log[b + a*Cos[x]]))/(2*a^3*(b +
a*Cos[x])^2)
```

3.290.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {3042, 4892, 3042, 3147, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cot(x)+b\csc(x))^3}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(a\cot(x)+b\csc(x))^3}dx$$
$$\Big\downarrow \text{4892}$$
$$\int \frac{\sin^3(x)}{(a\cos(x)+b)^3}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)^3}{\left(b-a\sin\left(x-\frac{\pi}{2}\right)\right)^3}dx$$
$$\Big\downarrow \text{3147}$$
$$-\frac{\int \frac{a^2-a^2\cos^2(x)}{(b+a\cos(x))^3}d(a\cos(x))}{a^3}$$
$$\Big\downarrow \text{476}$$
$$-\frac{\int \left(\frac{2b}{(b+a\cos(x))^2}+\frac{1}{-b-a\cos(x)}+\frac{a^2-b^2}{(b+a\cos(x))^3}\right)d(a\cos(x))}{a^3}$$
$$\Big\downarrow \text{2009}$$
$$-\frac{-\frac{a^2-b^2}{2(a\cos(x)+b)^2}-\frac{2b}{a\cos(x)+b}-\log(a\cos(x)+b)}{a^3}$$

input
```
Int[(a*Cot[x] + b*Csc[x])^(-3),x]
```

output
```
-((-1/2*(a^2 - b^2)/(b + a*Cos[x])^2 - (2*b)/(b + a*Cos[x]) - Log[b + a*Co
s[x]])/a^3)
```

3.290.3.1 Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 4892
```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.290.4 Maple [A] (verified)

Time = 5.56 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{2b}{a^3(b+a\cos(x))}+\frac{\ln(b+a\cos(x))}{a^3}-\frac{-a^2+b^2}{2a^3(b+a\cos(x))^2}$	49
risch	$-\frac{ix}{a^3}+\frac{4ba\,e^{3ix}+2\,e^{2ix}a^2+6\,e^{2ix}b^2+4ab\,e^{ix}}{a^3(a\,e^{2ix}+2b\,e^{ix}+a)^2}+\frac{\ln\left(e^{2ix}+\frac{2b\,e^{ix}}{a}+1\right)}{a^3}$	94

input
```
int(1/(a*cot(x)+b*csc(x))^3,x,method=_RETURNVERBOSE)
```

output
```
2*b/a^3/(b+a*cos(x))+ln(b+a*cos(x))/a^3-1/2*(-a^2+b^2)/a^3/(b+a*cos(x))^2
```

3.290.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.40

$$\int \frac{1}{(a\cot(x)+b\csc(x))^3}\,dx = \frac{4\,ab\cos(x)+a^2+3\,b^2+2\,(a^2\cos(x)^2+2\,ab\cos(x)+b^2)\log(a\cos(x)+b)}{2\,(a^5\cos(x)^2+2\,a^4b\cos(x)+a^3b^2)}$$

input
```
integrate(1/(a*cot(x)+b*csc(x))^3,x, algorithm="fricas")
```

output
```
1/2*(4*a*b*cos(x) + a^2 + 3*b^2 + 2*(a^2*cos(x)^2 + 2*a*b*cos(x) + b^2)*lo
g(a*cos(x) + b))/(a^5*cos(x)^2 + 2*a^4*b*cos(x) + a^3*b^2)
```

3.290.6 Sympy [F]

$$\int \frac{1}{(a\cot(x)+b\csc(x))^3}\,dx = \int \frac{1}{(a\cot(x)+b\csc(x))^3}\,dx$$

input
```
integrate(1/(a*cot(x)+b*csc(x))**3,x)
```

output
```
Integral((a*cot(x) + b*csc(x))**(-3), x)
```

3.290.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. 2(48) = 96.

Time = 0.31 (sec) , antiderivative size = 177, normalized size of antiderivative = 3.54

$$\int \frac{1}{(a\cot(x)+b\csc(x))^3}\,dx = \frac{2\left(ab+b^2+\frac{(a^2-2\,ab+b^2)\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^5+a^4b-a^3b^2-a^2b^3-\frac{2\,(a^5-a^4b-a^3b^2+a^2b^3)\sin(x)^2}{(\cos(x)+1)^2}+\frac{(a^5-3\,a^4b+3\,a^3b^2-a^2b^3)\sin(x)^4}{(\cos(x)+1)^4}} + \frac{\log\left(a+b-\frac{(a-b)\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^3} - \frac{\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{a^3}$$

input
```
integrate(1/(a*cot(x)+b*csc(x))^3,x, algorithm="maxima")
```

output
```
2*(a*b + b^2 + (a^2 - 2*a*b + b^2)*sin(x)^2/(cos(x) + 1)^2)/(a^5 + a^4*b -
 a^3*b^2 - a^2*b^3 - 2*(a^5 - a^4*b - a^3*b^2 + a^2*b^3)*sin(x)^2/(cos(x)
+ 1)^2 + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*sin(x)^4/(cos(x) + 1)^4) +
log(a + b - (a - b)*sin(x)^2/(cos(x) + 1)^2)/a^3 - log(sin(x)^2/(cos(x) +
1)^2 + 1)/a^3
```

3.290.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

$$\int \frac{1}{(a\cot(x)+b\csc(x))^3}\,dx = \frac{\log\left(|a\cos(x)+b|\right)}{a^3} + \frac{4\,b\cos(x)+\frac{a^2+3\,b^2}{a}}{2\,(a\cos(x)+b)^2a^2}$$

input
```
integrate(1/(a*cot(x)+b*csc(x))^3,x, algorithm="giac")
```

output
```
log(abs(a*cos(x) + b))/a^3 + 1/2*(4*b*cos(x) + (a^2 + 3*b^2)/a)/((a*cos(x)
 + b)^2*a^2)
```

3.290.9 Mupad [B] (verification not implemented)

Time = 28.50 (sec) , antiderivative size = 311, normalized size of antiderivative = 6.22

$$\int \frac{1}{(a\cot(x)+b\csc(x))^3}\,dx$$

$$= \frac{\frac{2\,(b^2+a\,b)}{a^2\,(a-b)}+\frac{2\tan\left(\frac{x}{2}\right)^2\,(a-b)}{a^2}}{\tan\left(\frac{x}{2}\right)^4\,(a^2-2\,a\,b+b^2)+2\,a\,b-\tan\left(\frac{x}{2}\right)^2\,(2\,a^2-2\,b^2)+a^2+b^2} - \frac{2\,\mathrm{atanh}\left(\frac{32\tan\left(\frac{x}{2}\right)^2}{\frac{32\,b^3}{a^3}-\frac{32\,b^2}{a^2}-\frac{32\,b}{a}+\frac{32\,b\tan\left(\frac{x}{2}\right)^2}{a}-\frac{64\,b^2\tan\left(\frac{x}{2}\right)^2}{a^2}+\frac{32\,b^3\tan\left(\frac{x}{2}\right)^2}{a^3}+32}-\frac{64\,b\tan\left(\frac{x}{2}\right)^2}{32\,a-32\,b+32\,b\tan\left(\frac{x}{2}\right)^2-\frac{32\,b^2}{a}+\frac{32\,b^3}{a^2}-\frac{64\,b^2\tan\left(\frac{x}{2}\right)^2}{a}+\cdots}\right.}{a^3}$$

input
```
int(1/(b/sin(x) + a*cot(x))^3,x)
```

output
```
((2*(a*b + b^2))/(a^2*(a - b)) + (2*tan(x/2)^2*(a - b))/a^2)/(tan(x/2)^4*(
a^2 - 2*a*b + b^2) + 2*a*b - tan(x/2)^2*(2*a^2 - 2*b^2) + a^2 + b^2) - (2*
atanh((32*tan(x/2)^2)/((32*b^3)/a^3 - (32*b^2)/a^2 - (32*b)/a + (32*b*tan(
x/2)^2)/a - (64*b^2*tan(x/2)^2)/a^2 + (32*b^3*tan(x/2)^2)/a^3 + 32) - (64*
b*tan(x/2)^2)/(32*a - 32*b + 32*b*tan(x/2)^2 - (32*b^2)/a + (32*b^3)/a^2 -
 (64*b^2*tan(x/2)^2)/a + (32*b^3*tan(x/2)^2)/a^2) + (32*b^2*tan(x/2)^2)/(3
2*a^2 - 32*a*b - 32*b^2 - 64*b^2*tan(x/2)^2 + (32*b^3)/a + (32*b^3*tan(x/2
)^2)/a + 32*a*b*tan(x/2)^2)))/a^3
```

3.290. $\int \frac{1}{(a\cot(x)+b\csc(x))^3}\,dx$

3.291 $\int \frac{1}{(a\cot(x)+b\csc(x))^4}\,dx$

3.291.1 Optimal result

Integrand size = 11, antiderivative size = 159

$$\int \frac{1}{(a\cot(x)+b\csc(x))^4}\,dx = \frac{x}{a^4}-\frac{b(3a^2-2b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{3/2}(a+b)^{3/2}} - \frac{(2(a^2-b^2)-ab\cos(x))\sin(x)}{2a^3(a^2-b^2)(b+a\cos(x))} + \frac{\sin^3(x)}{3a(b+a\cos(x))^3}+\frac{b\sin^3(x)}{2a(a^2-b^2)(b+a\cos(x))^2}$$

output
```
x/a^4-b*(3*a^2-2*b^2)*arctanh((a-b)^(1/2)*tan(1/2*x)/(a+b)^(1/2))/a^4/(a-b
)^(3/2)/(a+b)^(3/2)-1/2*(2*a^2-2*b^2-a*b*cos(x))*sin(x)/a^3/(a^2-b^2)/(b+a
*cos(x))+1/3*sin(x)^3/a/(b+a*cos(x))^3+1/2*b*sin(x)^3/a/(a^2-b^2)/(b+a*cos
(x))^2
```

3.291.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.94

$$\int \frac{1}{(a\cot(x)+b\csc(x))^4}\,dx = \frac{\left(2a(a^2-b^2)+7ab(b+a\cos(x))-\frac{a(8a^2-11b^2)(b+a\cos(x))^2}{(a-b)(a+b)}+6x(b+a\cos(x))^3\csc(x)-\frac{6b(-3a^2+2b^2)\operatorname{arctanh}}{\cdots}\right.}{6a^4(b+a\cos(x))^3}$$

input
```
Integrate[(a*Cot[x] + b*Csc[x])^(-4),x]
```

output
```
((2*a*(a^2 - b^2) + 7*a*b*(b + a*Cos[x]) - (a*(8*a^2 - 11*b^2)*(b + a*Cos[
x])^2)/((a - b)*(a + b)) + 6*x*(b + a*Cos[x])^3*Csc[x] - (6*b*(-3*a^2 + 2*
b^2)*ArcTanh[((-a + b)*Tan[x/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[x])^3*Csc[x])
/(a^2 - b^2)^(3/2))*Sin[x])/(6*a^4*(b + a*Cos[x])^3)
```

3.291.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.13, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.364$, Rules used = {3042, 4892, 3042, 3172, 25, 3042, 3343, 3042, 3342, 25, 3042, 3214, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cot(x)+b\csc(x))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a\cot(x)+b\csc(x))^4}dx$$

$$\Big\downarrow \text{4892}$$

$$\int \frac{\sin^4(x)}{(a\cos(x)+b)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)^4}{\left(b-a\sin\left(x-\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \text{3172}$$

$$\frac{\int -\frac{\cos(x)\sin^2(x)}{(b+a\cos(x))^3}dx}{a}+\frac{\sin^3(x)}{3a(a\cos(x)+b)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sin^3(x)}{3a(a\cos(x)+b)^3}-\frac{\int \frac{\cos(x)\sin^2(x)}{(b+a\cos(x))^3}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin^3(x)}{3a(a\cos(x)+b)^3}-\frac{\int \frac{\cos\left(x+\frac{\pi}{2}\right)^2\sin\left(x+\frac{\pi}{2}\right)}{\left(b+a\sin\left(x+\frac{\pi}{2}\right)\right)^3}dx}{a}$$

$\Big\downarrow$ 3343

$$\frac{\sin^3(x)}{3a(a\cos(x)+b)^3}-\frac{\frac{\int \frac{(2a+b\cos(x))\sin^2(x)}{(b+a\cos(x))^2}dx}{2(a^2-b^2)}-\frac{b\sin^3(x)}{2(a^2-b^2)(a\cos(x)+b)^2}}{a}$$

$\Big\downarrow$ 3042

$$\frac{\sin^3(x)}{3a(a\cos(x)+b)^3}-\frac{\frac{\int \frac{\cos\left(x+\frac{\pi}{2}\right)^2\left(2a+b\sin\left(x+\frac{\pi}{2}\right)\right)}{\left(b+a\sin\left(x+\frac{\pi}{2}\right)\right)^2}dx}{2(a^2-b^2)}-\frac{b\sin^3(x)}{2(a^2-b^2)(a\cos(x)+b)^2}}{a}$$

$\Big\downarrow$ 3342

$$\frac{\sin^3(x)}{3a(a\cos(x)+b)^3}-\frac{\frac{\frac{\sin(x)\left(2\left(a^2-b^2\right)-ab\cos(x)\right)}{a^2(a\cos(x)+b)}-\frac{\int -\frac{ab-2\left(a^2-b^2\right)\cos(x)}{b+a\cos(x)}dx}{a^2}}{2(a^2-b^2)}-\frac{b\sin^3(x)}{2(a^2-b^2)(a\cos(x)+b)^2}}{a}$$

$\Big\downarrow$ 25

$$\frac{\sin^3(x)}{3a(a\cos(x)+b)^3}-\frac{\frac{\frac{\int \frac{ab-2\left(a^2-b^2\right)\cos(x)}{b+a\cos(x)}dx}{a^2}+\frac{\sin(x)\left(2\left(a^2-b^2\right)-ab\cos(x)\right)}{a^2(a\cos(x)+b)}}{2(a^2-b^2)}-\frac{b\sin^3(x)}{2(a^2-b^2)(a\cos(x)+b)^2}}{a}$$

$\Big\downarrow$ 3042

$$\frac{\sin^3(x)}{3a(a\cos(x)+b)^3}-\frac{\frac{\frac{\int \frac{ab-2\left(a^2-b^2\right)\sin\left(x+\frac{\pi}{2}\right)}{b+a\sin\left(x+\frac{\pi}{2}\right)}dx}{a^2}+\frac{\sin(x)\left(2\left(a^2-b^2\right)-ab\cos(x)\right)}{a^2(a\cos(x)+b)}}{2(a^2-b^2)}-\frac{b\sin^3(x)}{2(a^2-b^2)(a\cos(x)+b)^2}}{a}$$

$\Big\downarrow$ 3214

$$\frac{\sin^3(x)}{3a(a\cos(x)+b)^3}-\frac{\frac{\frac{\frac{b\left(3a^2-2b^2\right)\int \frac{1}{b+a\cos(x)}dx}{a}-\frac{2x\left(a^2-b^2\right)}{a}}{a^2}+\frac{\sin(x)\left(2\left(a^2-b^2\right)-ab\cos(x)\right)}{a^2(a\cos(x)+b)}}{2(a^2-b^2)}-\frac{b\sin^3(x)}{2(a^2-b^2)(a\cos(x)+b)^2}}{a}$$

$\Big\downarrow$ 3042

$$\frac{\sin^3(x)}{3a(a\cos(x)+b)^3}-\frac{\frac{\frac{\frac{b\left(3a^2-2b^2\right)\int \frac{1}{b+a\sin\left(x+\frac{\pi}{2}\right)}dx}{a}-\frac{2x\left(a^2-b^2\right)}{a}}{a^2}+\frac{\sin(x)\left(2\left(a^2-b^2\right)-ab\cos(x)\right)}{a^2(a\cos(x)+b)}}{2(a^2-b^2)}-\frac{b\sin^3(x)}{2(a^2-b^2)(a\cos(x)+b)^2}}{a}$$

$\Big\downarrow$ 3138

3.291. $\int \frac{1}{(a\cot(x)+b\csc(x))^4}\,dx$

$$\frac{\frac{\sin^3(x)}{3a(a\cos(x)+b)^3}-\frac{\frac{\frac{\frac{2b(3a^2-2b^2)\int \frac{1}{-\left((a-b)\tan^2\left(\frac{x}{2}\right)\right)+a+b}d\tan\left(\frac{x}{2}\right)}{a}-\frac{2x(a^2-b^2)}{a}}{a^2}+\frac{\sin(x)\left(2\left(a^2-b^2\right)-ab\cos(x)\right)}{a^2(a\cos(x)+b)}}{2(a^2-b^2)}-\frac{b\sin^3(x)}{2(a^2-b^2)(a\cos(x)+b)^2}}{a}}{}$$

$\Big\downarrow$ 221

$$\frac{\sin^3(x)}{3a(a\cos(x)+b)^3}-\frac{\frac{\frac{\frac{2b(3a^2-2b^2)\text{arctanh}\left(\frac{\sqrt{a-b}\tan\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}-\frac{2x(a^2-b^2)}{a}}{a^2}+\frac{\sin(x)\left(2\left(a^2-b^2\right)-ab\cos(x)\right)}{a^2(a\cos(x)+b)}}{2(a^2-b^2)}-\frac{b\sin^3(x)}{2(a^2-b^2)(a\cos(x)+b)^2}}{a}$$

input
```
Int[(a*Cot[x] + b*Csc[x])^(-4),x]
```

output
```
Sin[x]^3/(3*a*(b + a*Cos[x])^3) - (-1/2*(b*Sin[x]^3)/((a^2 - b^2)*(b + a*C
os[x])^2) + (((-2*(a^2 - b^2)*x)/a + (2*b*(3*a^2 - 2*b^2)*ArcTanh[(Sqrt[a
- b]*Tan[x/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b]))/a^2 + ((2*(a^2 -
 b^2) - a*b*Cos[x])*Sin[x])/(a^2*(b + a*Cos[x])))/(2*(a^2 - b^2)))/a
```

3.291.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

3.291. $\int \frac{1}{(a\cot(x)+b\csc(x))^4}\,dx$

rule 3172
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Simp[g^2*((p - 1)/(b*(m + 1)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Fre
eQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && I
ntegersQ[2*m, 2*p]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3342
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*C
os[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*p
 + b*d*(m + 1)*Sin[e + f*x])/(b^2*f*(m + 1)*(m + p + 1))), x] + Simp[g^2*((
p - 1)/(b^2*(m + 1)*(m + p + 1)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^(m + 1)*Simp[b*d*(m + 1) + (b*c*(m + p + 1) - a*d*p)*Sin[e + f*x
], x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[m, -1] && GtQ[p, 1] && NeQ[m + p + 1, 0] && IntegerQ[2*m]
```

rule 3343
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Simp[1/((a^2 - b^2)*(m + 1))   Int[(g*Cos[e + f*x])^p
*(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
 + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ
[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 4892
```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.291. $\int \frac{1}{(a\cot(x)+b\csc(x))^4}\,dx$

3.291.4 Maple [A] (verified)

Time = 20.36 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.24

method	result
default	$\frac{2\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{a^4} - \frac{2\left(\frac{-\frac{\left(2a^3-a^2b-3a\,b^2+2b^3\right)a\tan\left(\frac{x}{2}\right)^5}{2(a+b)}+\frac{2a\left(5a^2-3b^2\right)\tan\left(\frac{x}{2}\right)^3}{3}-\frac{\left(2a^3+a^2b-3a\,b^2-2b^3\right)a\tan\left(\frac{x}{2}\right)}{2(a-b)}}{\left(\tan\left(\frac{x}{2}\right)^2a-\tan\left(\frac{x}{2}\right)^2b-a-b\right)^3}+\frac{b\left(3a^2-2b^2\right)\text{arct}}{2\left(a^2-b^2\right.}\right.}{a^4}$
risch	$\frac{x}{a^4} - \frac{i(-15a^4b\,e^{5ix}+18a^2b^3e^{5ix}-12a^5e^{4ix}-27a^3b^2e^{4ix}+54a\,b^4e^{4ix}-48a^4b\,e^{3ix}+34a^2b^3e^{3ix}+44b^5e^{3ix}-12a^5e^{2ix}-36a^3b^2e^{2ix}+7}{3a^4(a\,e^{2ix}+2b\,e^{ix}+a)^3(-a^2+b^2)}$

input
```
int(1/(a*cot(x)+b*csc(x))^4,x,method=_RETURNVERBOSE)
```

output
```
2/a^4*arctan(tan(1/2*x))-2/a^4*((-1/2*(2*a^3-a^2*b-3*a*b^2+2*b^3)*a/(a+b)*
tan(1/2*x)^5+2/3*a*(5*a^2-3*b^2)*tan(1/2*x)^3-1/2*(2*a^3+a^2*b-3*a*b^2-2*b
^3)*a/(a-b)*tan(1/2*x))/(tan(1/2*x)^2*a-tan(1/2*x)^2*b-a-b)^3+1/2*b*(3*a^2
-2*b^2)/(a^2-b^2)/((a+b)*(a-b))^(1/2)*arctanh(tan(1/2*x)*(a-b)/((a+b)*(a-b
))^(1/2)))
```

3.291.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 418 vs. 2(141) = 282.

Time = 0.31 (sec) , antiderivative size = 878, normalized size of antiderivative = 5.52

$$\int \frac{1}{(a\cot(x)+b\csc(x))^4}\,dx$$
$$= \left[\frac{12\,(a^7-2\,a^5b^2+a^3b^4)x\cos{(x)}^3+36\,(a^6b-2\,a^4b^3+a^2b^5)x\cos{(x)}^2+36\,(a^5b^2-2\,a^3b^4+ab^6)x\cos{(x)}}{}\right.$$

input
```
integrate(1/(a*cot(x)+b*csc(x))^4,x, algorithm="fricas")
```

output
```
[1/12*(12*(a^7 - 2*a^5*b^2 + a^3*b^4)*x*cos(x)^3 + 36*(a^6*b - 2*a^4*b^3 +
 a^2*b^5)*x*cos(x)^2 + 36*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*x*cos(x) + 3*(3*a^
2*b^4 - 2*b^6 + (3*a^5*b - 2*a^3*b^3)*cos(x)^3 + 3*(3*a^4*b^2 - 2*a^2*b^4)
*cos(x)^2 + 3*(3*a^3*b^3 - 2*a*b^5)*cos(x))*sqrt(a^2 - b^2)*log((2*a*b*cos
(x) - (a^2 - 2*b^2)*cos(x)^2 - 2*sqrt(a^2 - b^2)*(b*cos(x) + a)*sin(x) + 2
*a^2 - b^2)/(a^2*cos(x)^2 + 2*a*b*cos(x) + b^2)) + 12*(a^4*b^3 - 2*a^2*b^5
 + b^7)*x + 2*(2*a^7 - 7*a^5*b^2 + 11*a^3*b^4 - 6*a*b^6 - (8*a^7 - 19*a^5*
b^2 + 11*a^3*b^4)*cos(x)^2 - 3*(3*a^6*b - 8*a^4*b^3 + 5*a^2*b^5)*cos(x))*s
in(x))/(a^8*b^3 - 2*a^6*b^5 + a^4*b^7 + (a^11 - 2*a^9*b^2 + a^7*b^4)*cos(x
)^3 + 3*(a^10*b - 2*a^8*b^3 + a^6*b^5)*cos(x)^2 + 3*(a^9*b^2 - 2*a^7*b^4 +
 a^5*b^6)*cos(x)), 1/6*(6*(a^7 - 2*a^5*b^2 + a^3*b^4)*x*cos(x)^3 + 18*(a^6
*b - 2*a^4*b^3 + a^2*b^5)*x*cos(x)^2 + 18*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*x*
cos(x) - 3*(3*a^2*b^4 - 2*b^6 + (3*a^5*b - 2*a^3*b^3)*cos(x)^3 + 3*(3*a^4*
b^2 - 2*a^2*b^4)*cos(x)^2 + 3*(3*a^3*b^3 - 2*a*b^5)*cos(x))*sqrt(-a^2 + b^
2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(x) + a)/((a^2 - b^2)*sin(x))) + 6*(a^4*
b^3 - 2*a^2*b^5 + b^7)*x + (2*a^7 - 7*a^5*b^2 + 11*a^3*b^4 - 6*a*b^6 - (8*
a^7 - 19*a^5*b^2 + 11*a^3*b^4)*cos(x)^2 - 3*(3*a^6*b - 8*a^4*b^3 + 5*a^2*b
^5)*cos(x))*sin(x))/(a^8*b^3 - 2*a^6*b^5 + a^4*b^7 + (a^11 - 2*a^9*b^2 + a
^7*b^4)*cos(x)^3 + 3*(a^10*b - 2*a^8*b^3 + a^6*b^5)*cos(x)^2 + 3*(a^9*b^2
- 2*a^7*b^4 + a^5*b^6)*cos(x))]
```

3.291.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a\cot(x)+b\csc(x))^4}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*cot(x)+b*csc(x))**4,x)
```

output
```
Timed out
```

3.291.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a\cot(x)+b\csc(x))^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a*cot(x)+b*csc(x))^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.291.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.77

$$\int \frac{1}{(a\cot(x)+b\csc(x))^4}\,dx$$
$$= -\frac{(3a^2b-2b^3)\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}x\right)-b\tan\left(\frac{1}{2}x\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^6-a^4b^2)\sqrt{-a^2+b^2}}$$
$$+\frac{6a^4\tan\left(\frac{1}{2}x\right)^5-9a^3b\tan\left(\frac{1}{2}x\right)^5-6a^2b^2\tan\left(\frac{1}{2}x\right)^5+15ab^3\tan\left(\frac{1}{2}x\right)^5-6b^4\tan\left(\frac{1}{2}x\right)^5-20a^4\tan\left(\frac{1}{2}\right.}{3(a^5-a^3b^2)}$$
$$+\frac{x}{a^4}$$

input
```
integrate(1/(a*cot(x)+b*csc(x))^4,x, algorithm="giac")
```

output
```
-(3*a^2*b - 2*b^3)*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*
tan(1/2*x) - b*tan(1/2*x))/sqrt(-a^2 + b^2)))/((a^6 - a^4*b^2)*sqrt(-a^2 +
 b^2)) + 1/3*(6*a^4*tan(1/2*x)^5 - 9*a^3*b*tan(1/2*x)^5 - 6*a^2*b^2*tan(1/
2*x)^5 + 15*a*b^3*tan(1/2*x)^5 - 6*b^4*tan(1/2*x)^5 - 20*a^4*tan(1/2*x)^3
+ 32*a^2*b^2*tan(1/2*x)^3 - 12*b^4*tan(1/2*x)^3 + 6*a^4*tan(1/2*x) + 9*a^3
*b*tan(1/2*x) - 6*a^2*b^2*tan(1/2*x) - 15*a*b^3*tan(1/2*x) - 6*b^4*tan(1/2
*x))/((a^5 - a^3*b^2)*(a*tan(1/2*x)^2 - b*tan(1/2*x)^2 - a - b)^3) + x/a^4
```

3.291.9 Mupad [B] (verification not implemented)

Time = 33.92 (sec) , antiderivative size = 3068, normalized size of antiderivative = 19.30

$$\int \frac{1}{(a\cot(x) + b\csc(x))^4}\,dx = \text{Too large to display}$$

input
```
int(1/(b/sin(x) + a*cot(x))^4,x)
```

output
```
(2*atan((((((8*(6*a^12*b - 4*a^13 + 4*a^8*b^5 - 2*a^9*b^4 - 10*a^10*b^3 +
6*a^11*b^2))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) - (tan(x/2)*(8*a^13*b -
8*a^8*b^6 + 8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2)*8i)/(a^4*(
a^8*b + a^9 - a^6*b^3 - a^7*b^2)))*1i)/a^4 + (8*tan(x/2)*(4*a^6 - 8*a^5*b
- 8*a*b^5 + 8*b^6 - 16*a^2*b^4 + 16*a^3*b^3 + 5*a^4*b^2))/(a^8*b + a^9 - a
^6*b^3 - a^7*b^2))/a^4 - ((((8*(6*a^12*b - 4*a^13 + 4*a^8*b^5 - 2*a^9*b^4
- 10*a^10*b^3 + 6*a^11*b^2))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) + (tan(x
/2)*(8*a^13*b - 8*a^8*b^6 + 8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12
*b^2)*8i)/(a^4*(a^8*b + a^9 - a^6*b^3 - a^7*b^2)))*1i)/a^4 - (8*tan(x/2)*(
4*a^6 - 8*a^5*b - 8*a*b^5 + 8*b^6 - 16*a^2*b^4 + 16*a^3*b^3 + 5*a^4*b^2))/
(a^8*b + a^9 - a^6*b^3 - a^7*b^2))/a^4)/((16*(6*a^4*b - 2*a*b^4 + 4*b^5 -
10*a^2*b^3 + 3*a^3*b^2))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) + (((((8*(6*
a^12*b - 4*a^13 + 4*a^8*b^5 - 2*a^9*b^4 - 10*a^10*b^3 + 6*a^11*b^2))/(a^11
*b + a^12 - a^9*b^3 - a^10*b^2) - (tan(x/2)*(8*a^13*b - 8*a^8*b^6 + 8*a^9*
b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2)*8i)/(a^4*(a^8*b + a^9 - a^6*
b^3 - a^7*b^2)))*1i)/a^4 + (8*tan(x/2)*(4*a^6 - 8*a^5*b - 8*a*b^5 + 8*b^6
- 16*a^2*b^4 + 16*a^3*b^3 + 5*a^4*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2))
*1i)/a^4 + (((((8*(6*a^12*b - 4*a^13 + 4*a^8*b^5 - 2*a^9*b^4 - 10*a^10*b^3
 + 6*a^11*b^2))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) + (tan(x/2)*(8*a^13*b
 - 8*a^8*b^6 + 8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2)*8i)/...
```

3.292 $\int \frac{1}{(a\cot(x)+b\csc(x))^5}\,dx$

3.292.1 Optimal result

Integrand size = 11, antiderivative size = 100

$$\int \frac{1}{(a\cot(x)+b\csc(x))^5}\,dx = \frac{(a^2-b^2)^2}{4a^5(b+a\cos(x))^4}+\frac{4b(a^2-b^2)}{3a^5(b+a\cos(x))^3} - \frac{a^2-3b^2}{a^5(b+a\cos(x))^2}-\frac{4b}{a^5(b+a\cos(x))}-\frac{\log(b+a\cos(x))}{a^5}$$

output
```
1/4*(a^2-b^2)^2/a^5/(b+a*cos(x))^4+4/3*b*(a^2-b^2)/a^5/(b+a*cos(x))^3+(-a^
2+3*b^2)/a^5/(b+a*cos(x))^2-4*b/a^5/(b+a*cos(x))-ln(b+a*cos(x))/a^5
```

3.292.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.38

$$\int \frac{1}{(a\cot(x)+b\csc(x))^5}\,dx = -\frac{-3a^4+2a^2b^2+25b^4+12b^4\log(b+a\cos(x))+12a^4\cos^4(x)\log(b+a\cos(x))+48a^3b\cos^3(x)(1+\log(\ldots}{12a^5(\ldots}$$

input
```
Integrate[(a*Cot[x] + b*Csc[x])^(-5),x]
```

output
```
-1/12*(-3*a^4 + 2*a^2*b^2 + 25*b^4 + 12*b^4*Log[b + a*Cos[x]] + 12*a^4*Cos
[x]^4*Log[b + a*Cos[x]] + 48*a^3*b*Cos[x]^3*(1 + Log[b + a*Cos[x]]) + 12*a
^2*Cos[x]^2*(a^2 + 9*b^2 + 6*b^2*Log[b + a*Cos[x]]) + 8*a*b*Cos[x]*(a^2 +
11*b^2 + 6*b^2*Log[b + a*Cos[x]]))/(a^5*(b + a*Cos[x])^4)
```

3.292.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.87, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {3042, 4892, 3042, 3147, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cot(x)+b\csc(x))^5}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a\cot(x)+b\csc(x))^5}dx$$

$$\Big\downarrow \text{4892}$$

$$\int \frac{\sin^5(x)}{(a\cos(x)+b)^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)^5}{\left(b-a\sin\left(x-\frac{\pi}{2}\right)\right)^5}dx$$

$$\Big\downarrow \text{3147}$$

$$-\frac{\int \frac{\left(a^2-a^2\cos^2(x)\right)^2}{(b+a\cos(x))^5}d(a\cos(x))}{a^5}$$

$$\Big\downarrow \text{476}$$

$$-\frac{\int \left(\frac{\left(a^2-b^2\right)^2}{(b+a\cos(x))^5}+\frac{1}{b+a\cos(x)}-\frac{4b}{(b+a\cos(x))^2}-\frac{2\left(a^2-3b^2\right)}{(b+a\cos(x))^3}-\frac{4b\left(b^2-a^2\right)}{(b+a\cos(x))^4}\right)d(a\cos(x))}{a^5}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{\left(a^2-b^2\right)^2}{4(a\cos(x)+b)^4}-\frac{4b\left(a^2-b^2\right)}{3(a\cos(x)+b)^3}+\frac{a^2-3b^2}{(a\cos(x)+b)^2}+\frac{4b}{a\cos(x)+b}+\log(a\cos(x)+b)}{a^5}$$

input
```
Int[(a*Cot[x] + b*Csc[x])^(-5),x]
```

output
```
-((-1/4*(a^2 - b^2)^2/(b + a*Cos[x])^4 - (4*b*(a^2 - b^2))/(3*(b + a*Cos[x
])^3) + (a^2 - 3*b^2)/(b + a*Cos[x])^2 + (4*b)/(b + a*Cos[x]) + Log[b + a*
Cos[x]])/a^5)
```

3.292.3.1 Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 4892
```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.292.4 Maple [A] (verified)

Time = 64.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.07

method	result
default	$-\frac{-a^{4}+2a^{2}b^{2}-b^{4}}{4a^{5}\left(b+a\cos\left(x\right)\right)^{4}}-\frac{4b}{a^{5}\left(b+a\cos\left(x\right)\right)}-\frac{\ln\left(b+a\cos\left(x\right)\right)}{a^{5}}+\frac{4b\left(a^{2}-b^{2}\right)}{3a^{5}\left(b+a\cos\left(x\right)\right)^{3}}-\frac{2a^{2}-6b^{2}}{2a^{5}\left(b+a\cos\left(x\right)\right)^{2}}$
risch	$\frac{ix}{a^{5}}-\frac{4\left(6a^{3}b\,e^{7ix}+3a^{4}e^{6ix}+27a^{2}b^{2}e^{6ix}+22a^{3}b\,e^{5ix}+44a\,b^{3}e^{5ix}+3a^{4}e^{4ix}+56a^{2}b^{2}e^{4ix}+25b^{4}e^{4ix}+22a^{3}b\,e^{3ix}+44a\,b^{3}e^{3ix}+3a^{4}e^{2ix}\right.}{3a^{5}\left(a\,e^{2ix}+2b\,e^{ix}+a\right)^{4}}$

input
```
int(1/(a*cot(x)+b*csc(x))^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-a^4+2*a^2*b^2-b^4)/a^5/(b+a*cos(x))^4-4*b/a^5/(b+a*cos(x))-ln(b+a*c
os(x))/a^5+4/3*b*(a^2-b^2)/a^5/(b+a*cos(x))^3-1/2*(2*a^2-6*b^2)/a^5/(b+a*c
os(x))^2
```

3.292.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.66

$$\int \frac{1}{(a\cot(x)+b\csc(x))^5}\,dx = -\frac{48\,a^3b\cos(x)^3 - 3\,a^4 + 2\,a^2b^2 + 25\,b^4 + 12\,(a^4 + 9\,a^2b^2)\cos(x)^2 + 8\,(a^3b + 11\,ab^3)\cos(x) + 12\,(a^4\cos(}{12\,(a^9\cos(x)^4 + 4\,a^8b\cos(x)^3 + 6\,a^7b^2\cos(x)^2 +}$$

input
```
integrate(1/(a*cot(x)+b*csc(x))^5,x, algorithm="fricas")
```

output
```
-1/12*(48*a^3*b*cos(x)^3 - 3*a^4 + 2*a^2*b^2 + 25*b^4 + 12*(a^4 + 9*a^2*b^
2)*cos(x)^2 + 8*(a^3*b + 11*a*b^3)*cos(x) + 12*(a^4*cos(x)^4 + 4*a^3*b*cos
(x)^3 + 6*a^2*b^2*cos(x)^2 + 4*a*b^3*cos(x) + b^4)*log(a*cos(x) + b))/(a^9
*cos(x)^4 + 4*a^8*b*cos(x)^3 + 6*a^7*b^2*cos(x)^2 + 4*a^6*b^3*cos(x) + a^5
*b^4)
```

3.292.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a\cot(x)+b\csc(x))^5}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*cot(x)+b*csc(x))**5,x)
```

output
```
Timed out
```

3.292.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 497 vs. 2(96) = 192.

Time = 0.34 (sec) , antiderivative size = 497, normalized size of antiderivative = 4.97

$$\int \frac{1}{(a\cot(x)+b\csc(x))^5}\,dx = -\frac{2\left(5\,a^4b + 10\,a^3b^2 + 2\,a^2b^3 - 6\,ab^4 - 3\,b^5 + \frac{(3\,a^5-17\,a^4b-6\,a^3b^2+26\,a^2b^3+3\,ab^4-9\,b^5)\,s}{(\cos(x)+1)^2}\right.}{3\left(a^{10} + 2\,a^9b - a^8b^2 - 4\,a^7b^3 - a^6b^4 + 2\,a^5b^5 + a^4b^6 - \frac{4\,(a^{10}-3\,a^8b^2+3\,a^6b^4-a^4b^6)\sin(x)^2}{(\cos(x)+1)^2} + \frac{6\,(a^{10}-2\,a^9b-a^8b^2+4\,a}{(c}\right.} - \frac{\log\left(a+b-\frac{(a-b)\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^5} + \frac{\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{a^5}$$

input
```
integrate(1/(a*cot(x)+b*csc(x))^5,x, algorithm="maxima")
```

output
```
-2/3*(5*a^4*b + 10*a^3*b^2 + 2*a^2*b^3 - 6*a*b^4 - 3*b^5 + (3*a^5 - 17*a^4
*b - 6*a^3*b^2 + 26*a^2*b^3 + 3*a*b^4 - 9*b^5)*sin(x)^2/(cos(x) + 1)^2 - 3
*(4*a^5 - 13*a^4*b + 12*a^3*b^2 + 2*a^2*b^3 - 8*a*b^4 + 3*b^5)*sin(x)^4/(c
os(x) + 1)^4 + 3*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)
*sin(x)^6/(cos(x) + 1)^6)/(a^10 + 2*a^9*b - a^8*b^2 - 4*a^7*b^3 - a^6*b^4
+ 2*a^5*b^5 + a^4*b^6 - 4*(a^10 - 3*a^8*b^2 + 3*a^6*b^4 - a^4*b^6)*sin(x)^
2/(cos(x) + 1)^2 + 6*(a^10 - 2*a^9*b - a^8*b^2 + 4*a^7*b^3 - a^6*b^4 - 2*a
^5*b^5 + a^4*b^6)*sin(x)^4/(cos(x) + 1)^4 - 4*(a^10 - 4*a^9*b + 5*a^8*b^2
- 5*a^6*b^4 + 4*a^5*b^5 - a^4*b^6)*sin(x)^6/(cos(x) + 1)^6 + (a^10 - 6*a^9
*b + 15*a^8*b^2 - 20*a^7*b^3 + 15*a^6*b^4 - 6*a^5*b^5 + a^4*b^6)*sin(x)^8/
(cos(x) + 1)^8) - log(a + b - (a - b)*sin(x)^2/(cos(x) + 1)^2)/a^5 + log(s
in(x)^2/(cos(x) + 1)^2 + 1)/a^5
```

3.292.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.93

$$\int \frac{1}{(a\cot(x)+b\csc(x))^5}\,dx = -\frac{\log\left(|a\cos\left(x\right)+b|\right)}{a^5} - \frac{48\,a^2b\cos\left(x\right)^3+12\left(a^3+9\,ab^2\right)\cos\left(x\right)^2+8\left(a^2b+11\,b^3\right)\cos\left(x\right)-\frac{3\,a^4-2\,a^2b^2-25\,b^4}{a}}{12\left(a\cos\left(x\right)+b\right)^4a^4}$$

input
```
integrate(1/(a*cot(x)+b*csc(x))^5,x, algorithm="giac")
```

output
```
-log(abs(a*cos(x) + b))/a^5 - 1/12*(48*a^2*b*cos(x)^3 + 12*(a^3 + 9*a*b^2)
*cos(x)^2 + 8*(a^2*b + 11*b^3)*cos(x) - (3*a^4 - 2*a^2*b^2 - 25*b^4)/a)/((
a*cos(x) + b)^4*a^4)
```

3.292.9 Mupad [B] (verification not implemented)

Time = 29.44 (sec) , antiderivative size = 538, normalized size of antiderivative = 5.38

$$\int \frac{1}{(a\cot(x)+b\csc(x))^5}\,dx$$

$$= \frac{2\,\mathrm{atanh}\left(\frac{32\tan\left(\frac{x}{2}\right)^2}{\frac{32\,b^3}{a^3}-\frac{32\,b^2}{a^2}-\frac{32\,b}{a}+\frac{32\,b\tan\left(\frac{x}{2}\right)^2}{a}-\frac{64\,b^2\tan\left(\frac{x}{2}\right)^2}{a^2}+\frac{32\,b^3\tan\left(\frac{x}{2}\right)^2}{a^3}+32}-\frac{64\,b\tan\left(\frac{x}{2}\right)^2}{32\,a-32\,b+32\,b\tan\left(\frac{x}{2}\right)^2-\frac{32\,b^2}{a}+\frac{32\,b^3}{a^2}-\frac{64\,b^2\tan\left(\frac{x}{2}\right)^2}{a}+\frac{32\,b^3}{}\cdots}\right.}{a^5}$$

$$-\frac{\frac{2\tan\left(\frac{x}{2}\right)^6(a^3-3\,a^2\,b+3\,a\,b^2-b^3)}{a^4}+\frac{2\,(5\,a^4\,b+10\,a^3\,b^2+2\,a^2\,b^3-6\,a\,b^4-3\,b^5)}{3\,a^4\,(a-b)^2}+\frac{2\tan\left(\frac{x}{2}\right)^4}{}\cdots}{4\,a\,b^3+4\,a^3\,b+\tan\left(\frac{x}{2}\right)^4(6\,a^4-12\,a^2\,b^2+6\,b^4)+\tan\left(\frac{x}{2}\right)^2(-4\,a^4-8\,a^3\,b+8\,a\,b^3+4\,b^4)-\tan\left(\frac{x}{2}\right)^6\,(\cdots}$$

input
```
int(1/(b/sin(x) + a*cot(x))^5,x)
```

output
```
(2*atanh((32*tan(x/2)^2)/((32*b^3)/a^3 - (32*b^2)/a^2 - (32*b)/a + (32*b*t
an(x/2)^2)/a - (64*b^2*tan(x/2)^2)/a^2 + (32*b^3*tan(x/2)^2)/a^3 + 32) - (
64*b*tan(x/2)^2)/(32*a - 32*b + 32*b*tan(x/2)^2 - (32*b^2)/a + (32*b^3)/a^
2 - (64*b^2*tan(x/2)^2)/a + (32*b^3*tan(x/2)^2)/a^2) + (32*b^2*tan(x/2)^2)
/(32*a^2 - 32*a*b - 32*b^2 - 64*b^2*tan(x/2)^2 + (32*b^3)/a + (32*b^3*tan(
x/2)^2)/a + 32*a*b*tan(x/2)^2)))/a^5 - ((2*tan(x/2)^6*(3*a*b^2 - 3*a^2*b +
 a^3 - b^3))/a^4 + (2*(5*a^4*b - 6*a*b^4 - 3*b^5 + 2*a^2*b^3 + 10*a^3*b^2)
)/(3*a^4*(a - b)^2) + (2*tan(x/2)^4*(2*a*b^2 + 5*a^2*b - 4*a^3 - 3*b^3))/a
^4 + (2*tan(x/2)^2*(6*a*b^3 - 14*a^3*b + 3*a^4 + 9*b^4 - 20*a^2*b^2))/(3*a
^4*(a - b)))/(4*a*b^3 + 4*a^3*b + tan(x/2)^4*(6*a^4 + 6*b^4 - 12*a^2*b^2)
+ tan(x/2)^2*(8*a*b^3 - 8*a^3*b - 4*a^4 + 4*b^4) - tan(x/2)^6*(8*a*b^3 - 8
*a^3*b + 4*a^4 - 4*b^4) + a^4 + b^4 + tan(x/2)^8*(a^4 - 4*a^3*b - 4*a*b^3
+ b^4 + 6*a^2*b^2) + 6*a^2*b^2)
```

3.293 $\int (\cot(x)+\csc(x))^5\,dx$

3.293.1 Optimal result

Integrand size = 7, antiderivative size = 28

$$\int (\cot(x)+\csc(x))^5\,dx = -\frac{2}{(1-\cos(x))^2}+\frac{4}{1-\cos(x)}+\log(1-\cos(x))$$

output
```
-2/(1-cos(x))^2+4/(1-cos(x))+ln(1-cos(x))
```

3.293.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int (\cot(x)+\csc(x))^5\,dx = 2\csc^2\left(\frac{x}{2}\right)-\frac{1}{2}\csc^4\left(\frac{x}{2}\right)+2\log\left(\sin\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[(Cot[x] + Csc[x])^5,x]
```

output
```
2*Csc[x/2]^2 - Csc[x/2]^4/2 + 2*Log[Sin[x/2]]
```

3.293.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {3042, 4892, 3042, 3146, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\cot(x)+\csc(x))^5\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int (\cot(x)+\csc(x))^5dx$$
$$\Big\downarrow \text{ 4892}$$
$$\int (\cos(x)+1)^5\csc^5(x)dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\left(1-\sin\left(x-\frac{\pi}{2}\right)\right)^5}{\cos\left(x-\frac{\pi}{2}\right)^5}dx$$
$$\Big\downarrow \text{ 3146}$$
$$-\int \frac{(\cos(x)+1)^2}{(1-\cos(x))^3}d\cos(x)$$
$$\Big\downarrow \text{ 49}$$
$$-\int \left(-\frac{4}{(\cos(x)-1)^2}-\frac{4}{(\cos(x)-1)^3}+\frac{1}{1-\cos(x)}\right)d\cos(x)$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{4}{1-\cos(x)}-\frac{2}{(1-\cos(x))^2}+\log(1-\cos(x))$$

input
```
Int[(Cot[x] + Csc[x])^5,x]
```

output
```
-2/(1 - Cos[x])^2 + 4/(1 - Cos[x]) + Log[1 - Cos[x]]
```

3.293.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 4892
```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.293.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.44 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.61

method	result
risch	$-ix-\frac{8\left(\mathrm{e}^{3ix}-\mathrm{e}^{2ix}+\mathrm{e}^{ix}\right)}{\left(\mathrm{e}^{ix}-1\right)^{4}}+2\ln\left(\mathrm{e}^{ix}-1\right)$
parts	$-\frac{11\cot(x)^{4}}{4}+\frac{\cot(x)^{2}}{2}-\frac{\ln\left(1+\cot(x)^{2}\right)}{2}+\left(-\frac{\csc(x)^{3}}{4}-\frac{3\csc(x)}{8}\right)\cot\left(x\right)+\ln\left(\csc\left(x\right)-\cot\left(x\right)\right)-\frac{5\csc(x)^{4}}{4}$
default	$-\frac{\cot(x)^{4}}{4}+\frac{\cot(x)^{2}}{2}+\ln\left(\sin\left(x\right)\right)-\frac{5\cos(x)^{5}}{4\sin(x)^{4}}+\frac{5\cos(x)^{5}}{8\sin(x)^{2}}+\frac{5\cos(x)^{3}}{8}+\frac{5\cos(x)}{8}+\ln\left(\csc\left(x\right)-\cot\left(x\right)\right)-$

input
```
int((cot(x)+csc(x))^5,x,method=_RETURNVERBOSE)
```

output
```
-I*x-8/(exp(I*x)-1)^4*(exp(3*I*x)-exp(2*I*x)+exp(I*x))+2*ln(exp(I*x)-1)
```

3.293.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.32

$$\int (\cot(x)+\csc(x))^5\,dx = \frac{\left(\cos\left(x\right)^2 - 2\,\cos\left(x\right) + 1\right)\log\left(-\frac{1}{2}\,\cos\left(x\right) + \frac{1}{2}\right) - 4\,\cos\left(x\right) + 2}{\cos\left(x\right)^2 - 2\,\cos\left(x\right) + 1}$$

input
```
integrate((cot(x)+csc(x))^5,x, algorithm="fricas")
```

output
```
((cos(x)^2 - 2*cos(x) + 1)*log(-1/2*cos(x) + 1/2) - 4*cos(x) + 2)/(cos(x)^
2 - 2*cos(x) + 1)
```

3.293.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(20) = 40.

Time = 131.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.43

$$\int (\cot(x)+\csc(x))^5\,dx = \frac{\log\left(\cos\left(x\right)-1\right)}{2} - \frac{\log\left(\cos\left(x\right)+1\right)}{2} - \frac{\log\left(\csc^2\left(x\right)\right)}{2} - \frac{5\cot^4\left(x\right)}{2} - \frac{3\csc^4\left(x\right)}{2} + \csc^2\left(x\right) - \frac{32\cos^3\left(x\right)}{8\cos^4\left(x\right) - 16\cos^2\left(x\right) + 8}$$

input
```
integrate((cot(x)+csc(x))**5,x)
```

output
```
log(cos(x) - 1)/2 - log(cos(x) + 1)/2 - log(csc(x)**2)/2 - 5*cot(x)**4/2 -
 3*csc(x)**4/2 + csc(x)**2 - 32*cos(x)**3/(8*cos(x)**4 - 16*cos(x)**2 + 8)
```

3.293.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. 2(24) = 48.

Time = 0.22 (sec) , antiderivative size = 125, normalized size of antiderivative = 4.46

$$\int (\cot(x)+\csc(x))^5\,dx = -\frac{5}{2}\cot(x)^4 - \frac{5\left(5\cos(x)^3-3\cos(x)\right)}{8\left(\cos(x)^4-2\cos(x)^2+1\right)} + \frac{3\cos(x)^3-5\cos(x)}{8\left(\cos(x)^4-2\cos(x)^2+1\right)} - \frac{5\left(\cos(x)^3+\cos(x)\right)}{4\left(\cos(x)^4-2\cos(x)^2+1\right)} + \frac{4\sin(x)^2-1}{4\sin(x)^4} - \frac{5}{4\sin(x)^4} + \frac{1}{2}\log\left(\sin(x)^2\right) - \frac{1}{2}\log(\cos(x)+1) + \frac{1}{2}\log(\cos(x)-1)$$

input
```
integrate((cot(x)+csc(x))^5,x, algorithm="maxima")
```

output
```
-5/2*cot(x)^4 - 5/8*(5*cos(x)^3 - 3*cos(x))/(cos(x)^4 - 2*cos(x)^2 + 1) +
1/8*(3*cos(x)^3 - 5*cos(x))/(cos(x)^4 - 2*cos(x)^2 + 1) - 5/4*(cos(x)^3 +
cos(x))/(cos(x)^4 - 2*cos(x)^2 + 1) + 1/4*(4*sin(x)^2 - 1)/sin(x)^4 - 5/4/
sin(x)^4 + 1/2*log(sin(x)^2) - 1/2*log(cos(x) + 1) + 1/2*log(cos(x) - 1)
```

3.293.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int (\cot(x)+\csc(x))^5\,dx = -\frac{2\,(2\cos(x)-1)}{(\cos(x)-1)^2} + \log(-\cos(x)+1)$$

input
```
integrate((cot(x)+csc(x))^5,x, algorithm="giac")
```

output
```
-2*(2*cos(x) - 1)/(cos(x) - 1)^2 + log(-cos(x) + 1)
```

3.293.9 Mupad [B] (verification not implemented)

Time = 27.53 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.21

$$\int (\cot(x)+\csc(x))^5\,dx = 2\,\ln\left(\tan\left(\frac{x}{2}\right)\right)-\ln\left(\tan\left(\frac{x}{2}\right)^2+1\right)+\frac{\tan\left(\frac{x}{2}\right)^2-\frac{1}{2}}{\tan\left(\frac{x}{2}\right)^4}$$

input
```
int((cot(x) + 1/sin(x))^5,x)
```

output
```
2*log(tan(x/2)) - log(tan(x/2)^2 + 1) + (tan(x/2)^2 - 1/2)/tan(x/2)^4
```

3.294 $\int (\cot(x)+\csc(x))^4\,dx$

3.294.1 Optimal result

Integrand size = 7, antiderivative size = 30

$$\int (\cot(x)+\csc(x))^4\,dx = x+\frac{2\sin(x)}{1-\cos(x)}-\frac{2\sin^3(x)}{3(1-\cos(x))^3}$$

output
```
x+2*sin(x)/(1-cos(x))-2/3*sin(x)^3/(1-cos(x))^3
```

3.294.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (\cot(x)+\csc(x))^4\,dx = x+\frac{8}{3}\cot\left(\frac{x}{2}\right)-\frac{2}{3}\cot\left(\frac{x}{2}\right)\csc^2\left(\frac{x}{2}\right)$$

input
```
Integrate[(Cot[x] + Csc[x])^4,x]
```

output
```
x + (8*Cot[x/2])/3 - (2*Cot[x/2]*Csc[x/2]^2)/3
```

3.294.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.286, Rules used = {3042, 4892, 3042, 3149, 3042, 3159, 3042, 3159, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\cot(x)+\csc(x))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (\cot(x)+\csc(x))^4dx$$

$$\Big\downarrow \text{4892}$$

$$\int (\cos(x)+1)^4\csc^4(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(1-\sin\left(x-\frac{\pi}{2}\right)\right)^4}{\cos\left(x-\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{3149}$$

$$\int \frac{\sin^4(x)}{(1-\cos(x))^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)^4}{\left(\sin\left(x-\frac{\pi}{2}\right)+1\right)^4}dx$$

$$\Big\downarrow \text{3159}$$

$$-\int \frac{\sin^2(x)}{(1-\cos(x))^2}dx-\frac{2\sin^3(x)}{3(1-\cos(x))^3}$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{\cos\left(x-\frac{\pi}{2}\right)^2}{\left(\sin\left(x-\frac{\pi}{2}\right)+1\right)^2}dx-\frac{2\sin^3(x)}{3(1-\cos(x))^3}$$

$$\Big\downarrow \text{3159}$$

$$\int 1dx-\frac{2\sin^3(x)}{3(1-\cos(x))^3}+\frac{2\sin(x)}{1-\cos(x)}$$

$$\Big\downarrow \; 24$$

$$x-\frac{2\sin^3(x)}{3(1-\cos(x))^3}+\frac{2\sin(x)}{1-\cos(x)}$$

input
```
Int[(Cot[x] + Csc[x])^4,x]
```

output
```
x + (2*Sin[x])/(1 - Cos[x]) - (2*Sin[x]^3)/(3*(1 - Cos[x])^3)
```

3.294.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3149
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(a/g)^(2*m)   Int[(g*Cos[e + f*x])^(2*m + p)/(
a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 4892
```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.294.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.66 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

method	result
risch	$x+\frac{8i(3\,\mathrm{e}^{2ix}-3\,\mathrm{e}^{ix}+2)}{3(\mathrm{e}^{ix}-1)^3}$
parts	$-\frac{7\cot(x)^3}{3}+\cot(x)-\frac{\pi}{2}+\operatorname{arccot}(\cot(x))+\left(-\frac{2}{3}-\frac{\csc(x)^2}{3}\right)\cot(x)-\frac{8\csc(x)^3}{3}+4\csc(x)$
default	$-\frac{\cot(x)^3}{3}+\cot(x)+x-\frac{4\cos(x)^4}{3\sin(x)^3}+\frac{4\cos(x)^4}{3\sin(x)}+\frac{4\left(2+\cos(x)^2\right)\sin(x)}{3}-\frac{2\cos(x)^3}{\sin(x)^3}-\frac{4}{3\sin(x)^3}+\left(-\frac{2}{3}-\frac{\csc(x)^2}{3}\right.$

input
```
int((cot(x)+csc(x))^4,x,method=_RETURNVERBOSE)
```

output
```
x+8/3*I*(3*exp(2*I*x)-3*exp(I*x)+2)/(exp(I*x)-1)^3
```

3.294.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20

$$\int(\cot(x)+\csc(x))^4\,dx=\frac{8\cos(x)^2+3(x\cos(x)-x)\sin(x)+4\cos(x)-4}{3(\cos(x)-1)\sin(x)}$$

input
```
integrate((cot(x)+csc(x))^4,x, algorithm="fricas")
```

output
```
1/3*(8*cos(x)^2 + 3*(x*cos(x) - x)*sin(x) + 4*cos(x) - 4)/((cos(x) - 1)*si
n(x))
```

3.294.6 Sympy [A] (verification not implemented)

Time = 34.61 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.47

$$\int(\cot(x)+\csc(x))^4\,dx=x-\frac{7\cot^3(x)}{3}-\cot(x)-\frac{8\csc^3(x)}{3}+4\csc(x)+\frac{\cos(x)}{\sin(x)}-\frac{\cos^3(x)}{3\sin^3(x)}$$

input
```
integrate((cot(x)+csc(x))**4,x)
```

output
```
x - 7*cot(x)**3/3 - cot(x) - 8*csc(x)**3/3 + 4*csc(x) + cos(x)/sin(x) - co
s(x)**3/(3*sin(x)**3)
```

3.294.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(24) = 48$.

Time = 0.30 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.87

$$\int (\cot(x)+\csc(x))^4\,dx = -2\cot(x)^3 + x + \frac{4\,(3\sin(x)^2-1)}{3\sin(x)^3} - \frac{3\tan(x)^2+1}{3\tan(x)^3} + \frac{3\tan(x)^2-1}{3\tan(x)^3} - \frac{4}{3\sin(x)^3}$$

input
```
integrate((cot(x)+csc(x))^4,x, algorithm="maxima")
```

output
```
-2*cot(x)^3 + x + 4/3*(3*sin(x)^2 - 1)/sin(x)^3 - 1/3*(3*tan(x)^2 + 1)/tan
(x)^3 + 1/3*(3*tan(x)^2 - 1)/tan(x)^3 - 4/3/sin(x)^3
```

3.294.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

$$\int (\cot(x)+\csc(x))^4\,dx = x + \frac{2\left(3\tan\left(\frac{1}{2}x\right)^2-1\right)}{3\tan\left(\frac{1}{2}x\right)^3}$$

input
```
integrate((cot(x)+csc(x))^4,x, algorithm="giac")
```

output
```
x + 2/3*(3*tan(1/2*x)^2 - 1)/tan(1/2*x)^3
```

3.294.9 Mupad [B] (verification not implemented)

Time = 27.68 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.53

$$\int (\cot(x)+\csc(x))^4\,dx = -\frac{2\cot\left(\frac{x}{2}\right)^3}{3}+2\cot\left(\frac{x}{2}\right)+x$$

input
```
int((cot(x) + 1/sin(x))^4,x)
```

output
```
x + 2*cot(x/2) - (2*cot(x/2)^3)/3
```

3.295 $\int (\cot(x)+\csc(x))^3\,dx$

3.295.1 Optimal result

Integrand size = 7, antiderivative size = 20

$$\int (\cot(x)+\csc(x))^3\,dx=-\frac{2}{1-\cos(x)}-\log(1-\cos(x))$$

output
```
-2/(1-cos(x))-ln(1-cos(x))
```

3.295.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (\cot(x)+\csc(x))^3\,dx=-\csc^2\left(\frac{x}{2}\right)-2\log\left(\sin\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[(Cot[x] + Csc[x])^3,x]
```

output
```
-Csc[x/2]^2 - 2*Log[Sin[x/2]]
```

3.295.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {3042, 4892, 3042, 3146, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\cot(x)+\csc(x))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (\cot(x)+\csc(x))^3dx$$

$$\Big\downarrow \text{4892}$$

$$\int (\cos(x)+1)^3\csc^3(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(1-\sin\left(x-\frac{\pi}{2}\right)\right)^3}{\cos\left(x-\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{3146}$$

$$-\int \frac{\cos(x)+1}{(1-\cos(x))^2}d\cos(x)$$

$$\Big\downarrow \text{49}$$

$$-\int \left(\frac{1}{\cos(x)-1}+\frac{2}{(\cos(x)-1)^2}\right)d\cos(x)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2}{1-\cos(x)}-\log(1-\cos(x))$$

input
```
Int[(Cot[x] + Csc[x])^3,x]
```

output
```
-2/(1 - Cos[x]) - Log[1 - Cos[x]]
```

3.295.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 4892
```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.295.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.99 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.60

method	result	size
risch	$ix+\frac{4\,\mathrm{e}^{ix}}{\left(\mathrm{e}^{ix}-1\right)^{2}}-2\ln\left(\mathrm{e}^{ix}-1\right)$	32
parts	$-2\cot\left(x\right)^{2}+\frac{\ln\left(1+\cot\left(x\right)^{2}\right)}{2}-\frac{\csc\left(x\right)\cot\left(x\right)}{2}-\ln\left(\csc\left(x\right)-\cot\left(x\right)\right)-\frac{3\cos\left(x\right)^{3}}{2\sin\left(x\right)^{2}}-\frac{3\cos\left(x\right)}{2}$	47
default	$-\frac{\cot\left(x\right)^{2}}{2}-\ln\left(\sin\left(x\right)\right)-\frac{3\cos\left(x\right)^{3}}{2\sin\left(x\right)^{2}}-\frac{3\cos\left(x\right)}{2}-\ln\left(\csc\left(x\right)-\cot\left(x\right)\right)-\frac{3}{2\sin\left(x\right)^{2}}-\frac{\csc\left(x\right)\cot\left(x\right)}{2}$	49

input
```
int((cot(x)+csc(x))^3,x,method=_RETURNVERBOSE)
```

output
```
I*x+4*exp(I*x)/(exp(I*x)-1)^2-2*ln(exp(I*x)-1)
```

3.295.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (\cot(x) + \csc(x))^3 \, dx = -\frac{(\cos(x) - 1)\log\left(-\frac{1}{2}\cos(x) + \frac{1}{2}\right) - 2}{\cos(x) - 1}$$

input
```
integrate((cot(x)+csc(x))^3,x, algorithm="fricas")
```

output
```
-((cos(x) - 1)*log(-1/2*cos(x) + 1/2) - 2)/(cos(x) - 1)
```

3.295.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(14) = 28.

Time = 9.00 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.30

$$\int (\cot(x) + \csc(x))^3 \, dx = -\frac{\log(\cos(x) - 1)}{2} + \frac{\log(\cos(x) + 1)}{2} + \frac{\log(-\csc^2(x))}{2} - 2\csc^2(x) + \frac{4\cos(x)}{2\cos^2(x) - 2}$$

input
```
integrate((cot(x)+csc(x))**3,x)
```

output
```
-log(cos(x) - 1)/2 + log(cos(x) + 1)/2 + log(-csc(x)**2)/2 - 2*csc(x)**2 +
 4*cos(x)/(2*cos(x)**2 - 2)
```

3.295.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(18) = 36$.

Time = 0.22 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.30

$$\int (\cot(x) + \csc(x))^3 \, dx = -\frac{3}{2} \cot(x)^2 + \frac{2 \cos(x)}{\cos(x)^2 - 1} - \frac{1}{2 \sin(x)^2} - \frac{1}{2} \log(\sin(x)^2) + \frac{1}{2} \log(\cos(x) + 1) - \frac{1}{2} \log(\cos(x) - 1)$$

input
```
integrate((cot(x)+csc(x))^3,x, algorithm="maxima")
```

output
```
-3/2*cot(x)^2 + 2*cos(x)/(cos(x)^2 - 1) - 1/2/sin(x)^2 - 1/2*log(sin(x)^2)
 + 1/2*log(cos(x) + 1) - 1/2*log(cos(x) - 1)
```

3.295.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int (\cot(x) + \csc(x))^3 \, dx = \frac{2}{\cos(x) - 1} - \log(-\cos(x) + 1)$$

input
```
integrate((cot(x)+csc(x))^3,x, algorithm="giac")
```

output
```
2/(cos(x) - 1) - log(-cos(x) + 1)
```

3.295.9 Mupad [B] (verification not implemented)

Time = 28.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int (\cot(x) + \csc(x))^3 \, dx = \ln\left(\tan\left(\frac{x}{2}\right)^2 + 1\right) - 2 \ln\left(\tan\left(\frac{x}{2}\right)\right) - \frac{1}{\tan\left(\frac{x}{2}\right)^2}$$

input
```
int((cot(x) + 1/sin(x))^3,x)
```

output
```
log(tan(x/2)^2 + 1) - 2*log(tan(x/2)) - 1/tan(x/2)^2
```

3.296 $\int (\cot(x)+\csc(x))^2\,dx$

3.296.1 Optimal result

Integrand size = 7, antiderivative size = 16

$$\int (\cot(x)+\csc(x))^2\,dx = -x-\frac{2\sin(x)}{1-\cos(x)}$$

output
```
-x-2*sin(x)/(1-cos(x))
```

3.296.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int (\cot(x)+\csc(x))^2\,dx = -x-2\cot\left(\frac{x}{2}\right)$$

input
```
Integrate[(Cot[x] + Csc[x])^2,x]
```

output
```
-x - 2*Cot[x/2]
```

3.296.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 4892, 3042, 3149, 3042, 3159, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\cot(x)+\csc(x))^2 \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int (\cot(x)+\csc(x))^2dx$$

$$\Big\downarrow \text{4892}$$

$$\int (\cos(x)+1)^2 \csc^2(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(1-\sin\left(x-\frac{\pi}{2}\right)\right)^2}{\cos\left(x-\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3149}$$

$$\int \frac{\sin^2(x)}{(1-\cos(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)^2}{\left(\sin\left(x-\frac{\pi}{2}\right)+1\right)^2}dx$$

$$\Big\downarrow \text{3159}$$

$$-\int 1dx-\frac{2\sin(x)}{1-\cos(x)}$$

$$\Big\downarrow \text{24}$$

$$-x-\frac{2\sin(x)}{1-\cos(x)}$$

input
```
Int[(Cot[x] + Csc[x])^2,x]
```

output
```
-x - (2*Sin[x])/(1 - Cos[x])
```

3.296.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3149
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(a/g)^(2*m)   Int[(g*Cos[e + f*x])^(2*m + p)/(
a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 4892
```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.296.4 Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
default	$-2\cot\left(x\right)-x-\frac{2}{\sin\left(x\right)}$	15
risch	$-x-\frac{4i}{{\mathrm e}^{i x}-1}$	17
parts	$-2\cot\left(x\right)+\frac{\pi}{2}-\operatorname{arccot}\left(\cot\left(x\right)\right)-2\csc\left(x\right)$	18

input
```
int((cot(x)+csc(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-2*cot(x)-x-2/sin(x)
```

3.296.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (\cot(x) + \csc(x))^2 \, dx = -\frac{x \sin{(x)} + 2 \cos{(x)} + 2}{\sin{(x)}}$$

input
```
integrate((cot(x)+csc(x))^2,x, algorithm="fricas")
```

output
```
-(x*sin(x) + 2*cos(x) + 2)/sin(x)
```

3.296.6 Sympy [A] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int (\cot(x) + \csc(x))^2 \, dx = -x - \cot{(x)} - 2 \csc{(x)} - \frac{\cos{(x)}}{\sin{(x)}}$$

input
```
integrate((cot(x)+csc(x))**2,x)
```

output
```
-x - cot(x) - 2*csc(x) - cos(x)/sin(x)
```

3.296.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (\cot(x) + \csc(x))^2 \, dx = -x - \frac{2}{\sin{(x)}} - \frac{2}{\tan{(x)}}$$

input
```
integrate((cot(x)+csc(x))^2,x, algorithm="maxima")
```

output
```
-x - 2/sin(x) - 2/tan(x)
```

3.296.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int (\cot(x) + \csc(x))^2 \, dx = -x - \frac{2}{\tan\left(\frac{1}{2}\,x\right)}$$

input
```
integrate((cot(x)+csc(x))^2,x, algorithm="giac")
```

output
```
-x - 2/tan(1/2*x)
```

3.296.9 Mupad [B] (verification not implemented)

Time = 27.55 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int (\cot(x) + \csc(x))^2 \, dx = -x - 2\cot\left(\frac{x}{2}\right)$$

input
```
int((cot(x) + 1/sin(x))^2,x)
```

output
```
- x - 2*cot(x/2)
```

3.297 $\int (\cot(x) + \csc(x)) \, dx$

3.297.1 Optimal result

Integrand size = 5, antiderivative size = 9

$$\int (\cot(x) + \csc(x)) \, dx = -\operatorname{arctanh}(\cos(x)) + \log(\sin(x))$$

output
```
-arctanh(cos(x))+ln(sin(x))
```

3.297.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 23 vs. 2(9) = 18.

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.56

$$\int (\cot(x) + \csc(x)) \, dx = -\log\left(\cos\left(\frac{x}{2}\right)\right) + \log(\cos(x)) + \log\left(\sin\left(\frac{x}{2}\right)\right) + \log(\tan(x))$$

input
```
Integrate[Cot[x] + Csc[x],x]
```

output
```
-Log[Cos[x/2]] + Log[Cos[x]] + Log[Sin[x/2]] + Log[Tan[x]]
```

3.297.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\cot(x) + \csc(x)) \, dx$$

$$\Big\downarrow \text{ 2009}$$

$$\log(\sin(x)) - \operatorname{arctanh}(\cos(x))$$

input
```
Int[Cot[x] + Csc[x],x]
```

output
```
-ArcTanh[Cos[x]] + Log[Sin[x]]
```

3.297.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.297.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.44

method	result	size
default	$\ln(\sin(x)) - \ln(\cot(x) + \csc(x))$	13
parts	$\ln(\sin(x)) - \ln(\cot(x) + \csc(x))$	13
norman	$-\frac{\ln\left(1+\tan(x)^2\right)}{2} + \ln(\tan(x)) + \ln\left(\tan\left(\frac{x}{2}\right)\right)$	19
parallelrisch	$\ln\left(\frac{1}{\sqrt{\sec(x)^2}}\right) + \ln(\tan(x)) + \ln(\csc(x) - \cot(x))$	20
risch	$-ix + \ln(e^{2ix} - 1) - \ln(e^{ix} + 1) + \ln(e^{ix} - 1)$	32

input
```
int(cot(x)+csc(x),x,method=_RETURNVERBOSE)
```

output
```
ln(sin(x))-ln(cot(x)+csc(x))
```

3.297.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int (\cot(x) + \csc(x))\, dx = \log\left(-\frac{1}{2}\cos(x) + \frac{1}{2}\right)$$

input
```
integrate(cot(x)+csc(x),x, algorithm="fricas")
```

output
```
log(-1/2*cos(x) + 1/2)
```

3.297.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. 2(8) = 16.

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.22

$$\int (\cot(x) + \csc(x))\, dx = \frac{\log(\cos(x) - 1)}{2} - \frac{\log(\cos(x) + 1)}{2} + \log(\sin(x))$$

input
```
integrate(cot(x)+csc(x),x)
```

output
```
log(cos(x) - 1)/2 - log(cos(x) + 1)/2 + log(sin(x))
```

3.297.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.33

$$\int (\cot(x) + \csc(x))\, dx = -\log(\cot(x) + \csc(x)) + \log(\sin(x))$$

input
```
integrate(cot(x)+csc(x),x, algorithm="maxima")
```

output
```
-log(cot(x) + csc(x)) + log(sin(x))
```

3.297.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int (\cot(x) + \csc(x))\, dx = \log(|\sin(x)|) + \log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)$$

input
```
integrate(cot(x)+csc(x),x, algorithm="giac")
```

output
```
log(abs(sin(x))) + log(abs(tan(1/2*x)))
```

3.297.9 Mupad [B] (verification not implemented)

Time = 27.42 (sec) , antiderivative size = 19, normalized size of antiderivative = 2.11

$$\int (\cot(x) + \csc(x))\, dx = 2\ln\left(\tan\left(\frac{x}{2}\right)\right) - \ln\left(\tan\left(\frac{x}{2}\right)^2 + 1\right)$$

input
```
int(cot(x) + 1/sin(x),x)
```

output
```
2*log(tan(x/2)) - log(tan(x/2)^2 + 1)
```

3.298 $\int \frac{1}{\cot(x)+\csc(x)}\,dx$

3.298.1 Optimal result

Integrand size = 7, antiderivative size = 7

$$\int \frac{1}{\cot(x)+\csc(x)}\,dx = -\log(1+\cos(x))$$

output
```
-ln(1+cos(x))
```

3.298.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \frac{1}{\cot(x)+\csc(x)}\,dx = -2\log\left(\cos\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[(Cot[x] + Csc[x])^(-1),x]
```

output
```
-2*Log[Cos[x/2]]
```

3.298.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3042, 3639, 3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cot(x)+\csc(x)} \, dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\cot(x)+\csc(x)}dx$$
$$\Big\downarrow \text{3639}$$
$$\int \frac{\sin(x)}{\cos(x)+1}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)}{1-\sin\left(x-\frac{\pi}{2}\right)}dx$$
$$\Big\downarrow \text{3146}$$
$$-\int \frac{1}{\cos(x)+1}d\cos(x)$$
$$\Big\downarrow \text{16}$$
$$-\log(\cos(x)+1)$$

input
```
Int[(Cot[x] + Csc[x])^(-1),x]
```

output
```
-Log[1 + Cos[x]]
```

3.298.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 3639
```
Int[((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) + cot[(d_.) + (e_.)*(x_)]*(c_.))
^(-1), x_Symbol] :> Int[Sin[d + e*x]/(b + a*Sin[d + e*x] + c*Cos[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

3.298.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

method	result	size
default	$-\ln\left(\cos\left(x\right)+1\right)$	8
risch	$ix-2\ln\left({\mathrm e}^{ix}+1\right)$	16

input
```
int(1/(cot(x)+csc(x)),x,method=_RETURNVERBOSE)
```

output
```
-ln(cos(x)+1)
```

3.298.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \frac{1}{\cot(x)+\csc(x)}\,dx = -\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)$$

input
```
integrate(1/(cot(x)+csc(x)),x, algorithm="fricas")
```

output
```
-log(1/2*cos(x) + 1/2)
```

3.298.6 Sympy [F]

$$\int \frac{1}{\cot(x)+\csc(x)}\,dx = \int \frac{1}{\cot(x)+\csc(x)}\,dx$$

input
```
integrate(1/(cot(x)+csc(x)),x)
```

output
```
Integral(1/(cot(x) + csc(x)), x)
```

3.298.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 14, normalized size of antiderivative = 2.00

$$\int \frac{1}{\cot(x)+\csc(x)}\,dx = \log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)$$

input
```
integrate(1/(cot(x)+csc(x)),x, algorithm="maxima")
```

output
```
log(sin(x)^2/(cos(x) + 1)^2 + 1)
```

3.298.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{1}{\cot(x) + \csc(x)}\,dx = -\log\left(\cos\left(x\right) + 1\right)$$

input
```
integrate(1/(cot(x)+csc(x)),x, algorithm="giac")
```

output
```
-log(cos(x) + 1)
```

3.298.9 Mupad [B] (verification not implemented)

Time = 29.58 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \frac{1}{\cot(x) + \csc(x)}\,dx = \ln\left(\tan\left(\frac{x}{2}\right)^2 + 1\right)$$

input
```
int(1/(cot(x) + 1/sin(x)),x)
```

output
```
log(tan(x/2)^2 + 1)
```

3.299 $\int \frac{1}{(\cot(x)+\csc(x))^2} dx$

3.299.1 Optimal result

Integrand size = 7, antiderivative size = 14

$$\int \frac{1}{(\cot(x)+\csc(x))^2} dx = -x + \frac{2\sin(x)}{1+\cos(x)}$$

output
```
-x+2*sin(x)/(1+cos(x))
```

3.299.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{(\cot(x)+\csc(x))^2} dx = -x + 2\tan\left(\frac{x}{2}\right)$$

input
```
Integrate[(Cot[x] + Csc[x])^(-2),x]
```

output
```
-x + 2*Tan[x/2]
```

3.299.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 4892, 3042, 3159, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\cot(x)+\csc(x))^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(\cot(x)+\csc(x))^2}dx$$
$$\Big\downarrow \text{4892}$$
$$\int \frac{\sin^2(x)}{(\cos(x)+1)^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)^2}{\left(1-\sin\left(x-\frac{\pi}{2}\right)\right)^2}dx$$
$$\Big\downarrow \text{3159}$$
$$\frac{2\sin(x)}{\cos(x)+1}-\int 1dx$$
$$\Big\downarrow \text{24}$$
$$\frac{2\sin(x)}{\cos(x)+1}-x$$

input
```
Int[(Cot[x] + Csc[x])^(-2),x]
```

output
```
-x + (2*Sin[x])/(1 + Cos[x])
```

3.299.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 4892
```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.299.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result	size
default	$2\tan\left(\frac{x}{2}\right)-2\arctan\left(\tan\left(\frac{x}{2}\right)\right)$	15
risch	$-x+\frac{4i}{{\rm e}^{ix}+1}$	17

input
```
int(1/(cot(x)+csc(x))^2,x,method=_RETURNVERBOSE)
```

output
```
2*tan(1/2*x)-2*arctan(tan(1/2*x))
```

3.299.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \frac{1}{(\cot(x)+\csc(x))^2}\,dx = -\frac{x\cos(x)+x-2\sin(x)}{\cos(x)+1}$$

input
```
integrate(1/(cot(x)+csc(x))^2,x, algorithm="fricas")
```

output
```
-(x*cos(x) + x - 2*sin(x))/(cos(x) + 1)
```

3.299.6 Sympy [F]

$$\int \frac{1}{(\cot(x)+\csc(x))^2}\,dx = \int \frac{1}{(\cot(x)+\csc(x))^2}\,dx$$

input
```
integrate(1/(cot(x)+csc(x))**2,x)
```

output
```
Integral((cot(x) + csc(x))**(-2), x)
```

3.299.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.64

$$\int \frac{1}{(\cot(x)+\csc(x))^2}\,dx = \frac{2\sin(x)}{\cos(x)+1} - 2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

input
```
integrate(1/(cot(x)+csc(x))^2,x, algorithm="maxima")
```

output
```
2*sin(x)/(cos(x) + 1) - 2*arctan(sin(x)/(cos(x) + 1))
```

3.299.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{(\cot(x)+\csc(x))^2}\,dx = -x + 2\,\tan\left(\frac{1}{2}\,x\right)$$

input
```
integrate(1/(cot(x)+csc(x))^2,x, algorithm="giac")
```

output
```
-x + 2*tan(1/2*x)
```

3.299.9 Mupad [B] (verification not implemented)

Time = 28.43 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{(\cot(x)+\csc(x))^2}\,dx = 2\tan\left(\frac{x}{2}\right) - x$$

input
```
int(1/(cot(x) + 1/sin(x))^2,x)
```

output
```
2*tan(x/2) - x
```

3.300 $\int \frac{1}{(\cot(x)+\csc(x))^3}\,dx$

3.300.1 Optimal result

Integrand size = 7, antiderivative size = 14

$$\int \frac{1}{(\cot(x)+\csc(x))^3}\,dx=\frac{2}{1+\cos(x)}+\log(1+\cos(x))$$

output
```
2/(1+cos(x))+ln(1+cos(x))
```

3.300.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \frac{1}{(\cot(x)+\csc(x))^3}\,dx=2\log\left(\cos\left(\frac{x}{2}\right)\right)+\sec^2\left(\frac{x}{2}\right)$$

input
```
Integrate[(Cot[x] + Csc[x])^(-3),x]
```

output
```
2*Log[Cos[x/2]] + Sec[x/2]^2
```

3.300.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {3042, 4892, 3042, 3146, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\cot(x)+\csc(x))^3}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(\cot(x)+\csc(x))^3}dx$$
$$\Big\downarrow \text{4892}$$
$$\int \frac{\sin^3(x)}{(\cos(x)+1)^3}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)^3}{\left(1-\sin\left(x-\frac{\pi}{2}\right)\right)^3}dx$$
$$\Big\downarrow \text{3146}$$
$$-\int \frac{1-\cos(x)}{(\cos(x)+1)^2}d\cos(x)$$
$$\Big\downarrow \text{49}$$
$$-\int \left(\frac{2}{(\cos(x)+1)^2}+\frac{1}{-\cos(x)-1}\right)d\cos(x)$$
$$\Big\downarrow \text{2009}$$
$$\frac{2}{\cos(x)+1}+\log(\cos(x)+1)$$

input
```
Int[(Cot[x] + Csc[x])^(-3),x]
```

output
```
2/(1 + Cos[x]) + Log[1 + Cos[x]]
```

3.300.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 4892
```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.300.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result	size
default	$\frac{2}{\cos(x)+1}+\ln\left(\cos\left(x\right)+1\right)$	15
risch	$-ix+\frac{4\,\mathrm{e}^{ix}}{\left(\mathrm{e}^{ix}+1\right)^{2}}+2\ln\left(\mathrm{e}^{ix}+1\right)$	32

input
```
int(1/(cot(x)+csc(x))^3,x,method=_RETURNVERBOSE)
```

output
```
2/(cos(x)+1)+ln(cos(x)+1)
```

3.300.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.50

$$\int \frac{1}{(\cot(x)+\csc(x))^3}\,dx = \frac{(\cos(x)+1)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+2}{\cos(x)+1}$$

input
```
integrate(1/(cot(x)+csc(x))^3,x, algorithm="fricas")
```

output
```
((cos(x) + 1)*log(1/2*cos(x) + 1/2) + 2)/(cos(x) + 1)
```

3.300.6 Sympy [F]

$$\int \frac{1}{(\cot(x)+\csc(x))^3}\,dx = \int \frac{1}{(\cot(x)+\csc(x))^3}\,dx$$

input
```
integrate(1/(cot(x)+csc(x))**3,x)
```

output
```
Integral((cot(x) + csc(x))**(-3), x)
```

3.300.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.00

$$\int \frac{1}{(\cot(x)+\csc(x))^3}\,dx = \frac{\sin(x)^2}{(\cos(x)+1)^2} - \log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)$$

input
```
integrate(1/(cot(x)+csc(x))^3,x, algorithm="maxima")
```

output
```
sin(x)^2/(cos(x) + 1)^2 - log(sin(x)^2/(cos(x) + 1)^2 + 1)
```

3.300.8 Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(\cot(x)+\csc(x))^3}\,dx = \frac{2}{\cos(x)+1} + \log(\cos(x)+1)$$

input
```
integrate(1/(cot(x)+csc(x))^3,x, algorithm="giac")
```

output
```
2/(cos(x) + 1) + log(cos(x) + 1)
```

3.300.9 Mupad [B] (verification not implemented)

Time = 29.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \frac{1}{(\cot(x)+\csc(x))^3}\,dx = \tan\left(\frac{x}{2}\right)^2 - \ln\left(\tan\left(\frac{x}{2}\right)^2+1\right)$$

input
```
int(1/(cot(x) + 1/sin(x))^3,x)
```

output
```
tan(x/2)^2 - log(tan(x/2)^2 + 1)
```

3.301 $\int \frac{1}{(\cot(x)+\csc(x))^4} dx$

3.301.1 Optimal result

Integrand size = 7, antiderivative size = 26

$$\int \frac{1}{(\cot(x)+\csc(x))^4} dx = x - \frac{2\sin(x)}{1+\cos(x)} + \frac{2\sin^3(x)}{3(1+\cos(x))^3}$$

output
```
x-2*sin(x)/(1+cos(x))+2/3*sin(x)^3/(1+cos(x))^3
```

3.301.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{1}{(\cot(x)+\csc(x))^4} dx = x - \frac{8}{3}\tan\left(\frac{x}{2}\right) + \frac{2}{3}\sec^2\left(\frac{x}{2}\right)\tan\left(\frac{x}{2}\right)$$

input
```
Integrate[(Cot[x] + Csc[x])^(-4),x]
```

output
```
x - (8*Tan[x/2])/3 + (2*Sec[x/2]^2*Tan[x/2])/3
```

3.301.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 4892, 3042, 3159, 3042, 3159, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\cot(x)+\csc(x))^4}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(\cot(x)+\csc(x))^4}dx$$
$$\Big\downarrow \text{4892}$$
$$\int \frac{\sin^4(x)}{(\cos(x)+1)^4}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)^4}{\left(1-\sin\left(x-\frac{\pi}{2}\right)\right)^4}dx$$
$$\Big\downarrow \text{3159}$$
$$\frac{2\sin^3(x)}{3(\cos(x)+1)^3}-\int \frac{\sin^2(x)}{(\cos(x)+1)^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\frac{2\sin^3(x)}{3(\cos(x)+1)^3}-\int \frac{\cos\left(x-\frac{\pi}{2}\right)^2}{\left(1-\sin\left(x-\frac{\pi}{2}\right)\right)^2}dx$$
$$\Big\downarrow \text{3159}$$
$$\int 1dx+\frac{2\sin^3(x)}{3(\cos(x)+1)^3}-\frac{2\sin(x)}{\cos(x)+1}$$
$$\Big\downarrow \text{24}$$
$$x+\frac{2\sin^3(x)}{3(\cos(x)+1)^3}-\frac{2\sin(x)}{\cos(x)+1}$$

input
```
Int[(Cot[x] + Csc[x])^(-4),x]
```

output
```
x - (2*Sin[x])/(1 + Cos[x]) + (2*Sin[x]^3)/(3*(1 + Cos[x])^3)
```

3.301.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

rule 4892
```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.301.4 Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{2\tan\left(\frac{x}{2}\right)^{3}}{3}-2\tan\left(\frac{x}{2}\right)+2\arctan\left(\tan\left(\frac{x}{2}\right)\right)$	23
risch	$x-\frac{8i\left(3\,{\mathrm e}^{2ix}+3\,{\mathrm e}^{ix}+2\right)}{3\left({\mathrm e}^{ix}+1\right)^{3}}$	31

input
```
int(1/(cot(x)+csc(x))^4,x,method=_RETURNVERBOSE)
```

output
```
2/3*tan(1/2*x)^3-2*tan(1/2*x)+2*arctan(tan(1/2*x))
```

3.301.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.54

$$\int \frac{1}{(\cot(x)+\csc(x))^4}\,dx = \frac{3\,x\cos\left(x\right)^2+6\,x\cos\left(x\right)-4\left(2\,\cos\left(x\right)+1\right)\sin\left(x\right)+3\,x}{3\left(\cos\left(x\right)^2+2\,\cos\left(x\right)+1\right)}$$

input
```
integrate(1/(cot(x)+csc(x))^4,x, algorithm="fricas")
```

output
```
1/3*(3*x*cos(x)^2 + 6*x*cos(x) - 4*(2*cos(x) + 1)*sin(x) + 3*x)/(cos(x)^2
+ 2*cos(x) + 1)
```

3.301.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(\cot(x)+\csc(x))^4}\,dx = \text{Timed out}$$

input
```
integrate(1/(cot(x)+csc(x))**4,x)
```

output
```
Timed out
```

3.301.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.35

$$\int \frac{1}{(\cot(x)+\csc(x))^4}\,dx = -\frac{2\,\sin\left(x\right)}{\cos\left(x\right)+1}+\frac{2\,\sin\left(x\right)^3}{3\left(\cos\left(x\right)+1\right)^3}+2\,\arctan\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}\right)$$

input
```
integrate(1/(cot(x)+csc(x))^4,x, algorithm="maxima")
```

output
```
-2*sin(x)/(cos(x) + 1) + 2/3*sin(x)^3/(cos(x) + 1)^3 + 2*arctan(sin(x)/(co
s(x) + 1))
```

3.301.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.62

$$\int \frac{1}{(\cot(x)+\csc(x))^4}\,dx = \frac{2}{3}\tan\left(\frac{1}{2}x\right)^3 + x - 2\tan\left(\frac{1}{2}x\right)$$

input
```
integrate(1/(cot(x)+csc(x))^4,x, algorithm="giac")
```

output
```
2/3*tan(1/2*x)^3 + x - 2*tan(1/2*x)
```

3.301.9 Mupad [B] (verification not implemented)

Time = 28.95 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.62

$$\int \frac{1}{(\cot(x)+\csc(x))^4}\,dx = \frac{2\tan\left(\frac{x}{2}\right)^3}{3} - 2\tan\left(\frac{x}{2}\right) + x$$

input
```
int(1/(cot(x) + 1/sin(x))^4,x)
```

output
```
x - 2*tan(x/2) + (2*tan(x/2)^3)/3
```

3.302 $\int \frac{1}{(\cot(x)+\csc(x))^5} dx$

3.302.1 Optimal result

Integrand size = 7, antiderivative size = 24

$$\int \frac{1}{(\cot(x)+\csc(x))^5} dx = \frac{2}{(1+\cos(x))^2} - \frac{4}{1+\cos(x)} - \log(1+\cos(x))$$

output
```
2/(1+cos(x))^2-4/(1+cos(x))-ln(1+cos(x))
```

3.302.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

$$\int \frac{1}{(\cot(x)+\csc(x))^5} dx = -2\log\left(\cos\left(\frac{x}{2}\right)\right) - 2\sec^2\left(\frac{x}{2}\right) + \frac{1}{2}\sec^4\left(\frac{x}{2}\right)$$

input
```
Integrate[(Cot[x] + Csc[x])^(-5),x]
```

output
```
-2*Log[Cos[x/2]] - 2*Sec[x/2]^2 + Sec[x/2]^4/2
```

3.302.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {3042, 4892, 3042, 3146, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\cot(x)+\csc(x))^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(\cot(x)+\csc(x))^5}dx$$

$$\Big\downarrow \text{4892}$$

$$\int \frac{\sin^5(x)}{(\cos(x)+1)^5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)^5}{\left(1-\sin\left(x-\frac{\pi}{2}\right)\right)^5}dx$$

$$\Big\downarrow \text{3146}$$

$$-\int \frac{(1-\cos(x))^2}{(\cos(x)+1)^3}d\cos(x)$$

$$\Big\downarrow \text{49}$$

$$-\int \left(\frac{1}{\cos(x)+1}-\frac{4}{(\cos(x)+1)^2}+\frac{4}{(\cos(x)+1)^3}\right)d\cos(x)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4}{\cos(x)+1}+\frac{2}{(\cos(x)+1)^2}-\log(\cos(x)+1)$$

input
```
Int[(Cot[x] + Csc[x])^(-5),x]
```

output
```
2/(1 + Cos[x])^2 - 4/(1 + Cos[x]) - Log[1 + Cos[x]]
```

3.302.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

rule 4892
```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

3.302.4 Maple [A] (verified)

Time = 3.73 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.08

method	result	size
parallelrisch	0	2
default	$\frac{2}{(\cos(x)+1)^2}-\frac{4}{\cos(x)+1}-\ln(\cos(x)+1)$	25
risch	$ix-\frac{8(e^{3ix}+e^{2ix}+e^{ix})}{(e^{ix}+1)^4}-2\ln(e^{ix}+1)$	43

input
```
int(1/(cot(x)+csc(x))^5,x,method=_RETURNVERBOSE)
```

output
```
0
```

3.302.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.58

$$\int \frac{1}{(\cot(x)+\csc(x))^5}\,dx = -\frac{(\cos(x)^2+2\cos(x)+1)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+4\cos(x)+2}{\cos(x)^2+2\cos(x)+1}$$

input
```
integrate(1/(cot(x)+csc(x))^5,x, algorithm="fricas")
```

output
```
-((cos(x)^2 + 2*cos(x) + 1)*log(1/2*cos(x) + 1/2) + 4*cos(x) + 2)/(cos(x)^
2 + 2*cos(x) + 1)
```

3.302.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(\cot(x)+\csc(x))^5}\,dx = \text{Timed out}$$

input
```
integrate(1/(cot(x)+csc(x))**5,x)
```

output
```
Timed out
```

3.302.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.62

$$\int \frac{1}{(\cot(x)+\csc(x))^5}\,dx = -\frac{\sin(x)^2}{(\cos(x)+1)^2}+\frac{\sin(x)^4}{2(\cos(x)+1)^4}+\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)$$

input
```
integrate(1/(cot(x)+csc(x))^5,x, algorithm="maxima")
```

output
```
-sin(x)^2/(cos(x) + 1)^2 + 1/2*sin(x)^4/(cos(x) + 1)^4 + log(sin(x)^2/(cos
(x) + 1)^2 + 1)
```

3.302.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{(\cot(x)+\csc(x))^5}\,dx = -\frac{2\,(2\,\cos\left(x\right)+1)}{\left(\cos\left(x\right)+1\right)^2} - \log\left(\cos\left(x\right)+1\right)$$

input
```
integrate(1/(cot(x)+csc(x))^5,x, algorithm="giac")
```

output
```
-2*(2*cos(x) + 1)/(cos(x) + 1)^2 - log(cos(x) + 1)
```

3.302.9 Mupad [B] (verification not implemented)

Time = 28.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{(\cot(x)+\csc(x))^5}\,dx = \ln\left(\tan\left(\frac{x}{2}\right)^2+1\right) - \tan\left(\frac{x}{2}\right)^2 + \frac{\tan\left(\frac{x}{2}\right)^4}{2}$$

input
```
int(1/(cot(x) + 1/sin(x))^5,x)
```

output
```
log(tan(x/2)^2 + 1) - tan(x/2)^2 + tan(x/2)^4/2
```

3.303 $\int (\csc(x)-\sin(x))^4\,dx$

3.303.1 Optimal result

Integrand size = 9, antiderivative size = 44

$$\int (\csc(x)-\sin(x))^4\,dx=\frac{35x}{8}+\frac{35\cot(x)}{8}-\frac{35\cot^3(x)}{24}+\frac{7}{8}\cos^2(x)\cot^3(x)+\frac{1}{4}\cos^4(x)\cot^3(x)$$

output
```
35/8*x+35/8*cot(x)-35/24*cot(x)^3+7/8*cos(x)^2*cot(x)^3+1/4*cos(x)^4*cot(x
)^3
```

3.303.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int (\csc(x)-\sin(x))^4\,dx=\frac{35x}{8}+\frac{10\cot(x)}{3}-\frac{1}{3}\cot(x)\csc^2(x)+\frac{3}{4}\sin(2x)+\frac{1}{32}\sin(4x)$$

input
```
Integrate[(Csc[x] - Sin[x])^4,x]
```

output
```
(35*x)/8 + (10*Cot[x])/3 - (Cot[x]*Csc[x]^2)/3 + (3*Sin[2*x])/4 + Sin[4*x]
/32
```

3.303.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.27, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {3042, 4889, 253, 253, 264, 264, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\csc(x)-\sin(x))^4\,dx$$
$$\Big\downarrow \; 3042$$
$$\int (\csc(x)-\sin(x))^4dx$$
$$\Big\downarrow \; 4889$$
$$\int \frac{\cot^4(x)}{\left(\tan^2(x)+1\right)^3}d\tan(x)$$
$$\Big\downarrow \; 253$$
$$\frac{7}{4}\int \frac{\cot^4(x)}{\left(\tan^2(x)+1\right)^2}d\tan(x)+\frac{\cot^3(x)}{4\left(\tan^2(x)+1\right)^2}$$
$$\Big\downarrow \; 253$$
$$\frac{7}{4}\left(\frac{5}{2}\int \frac{\cot^4(x)}{\tan^2(x)+1}d\tan(x)+\frac{\cot^3(x)}{2\left(\tan^2(x)+1\right)}\right)+\frac{\cot^3(x)}{4\left(\tan^2(x)+1\right)^2}$$
$$\Big\downarrow \; 264$$
$$\frac{7}{4}\left(\frac{5}{2}\left(-\int \frac{\cot^2(x)}{\tan^2(x)+1}d\tan(x)-\frac{1}{3}\cot^3(x)\right)+\frac{\cot^3(x)}{2\left(\tan^2(x)+1\right)}\right)+\frac{\cot^3(x)}{4\left(\tan^2(x)+1\right)^2}$$
$$\Big\downarrow \; 264$$
$$\frac{7}{4}\left(\frac{5}{2}\left(\int \frac{1}{\tan^2(x)+1}d\tan(x)-\frac{1}{3}\cot^3(x)+\cot(x)\right)+\frac{\cot^3(x)}{2\left(\tan^2(x)+1\right)}\right)+\frac{\cot^3(x)}{4\left(\tan^2(x)+1\right)^2}$$
$$\Big\downarrow \; 216$$
$$\frac{7}{4}\left(\frac{5}{2}\left(\arctan(\tan(x))-\frac{1}{3}\cot^3(x)+\cot(x)\right)+\frac{\cot^3(x)}{2\left(\tan^2(x)+1\right)}\right)+\frac{\cot^3(x)}{4\left(\tan^2(x)+1\right)^2}$$

input
```
Int[(Csc[x] - Sin[x])^4,x]
```

output
```
Cot[x]^3/(4*(1 + Tan[x]^2)^2) + (7*((5*(ArcTan[Tan[x]] + Cot[x] - Cot[x]^3
/3))/2 + Cot[x]^3/(2*(1 + Tan[x]^2))))/4
```

3.303.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.303.4 Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

method	result
default	$-\frac{\left(\sin(x)^3+\frac{3\sin(x)}{2}\right)\cos(x)}{4}+\frac{35x}{8}+2\cos(x)\sin(x)+4\cot(x)+\left(-\frac{2}{3}-\frac{\csc(x)^2}{3}\right)\cot(x)$
parts	$-\frac{\left(\sin(x)^3+\frac{3\sin(x)}{2}\right)\cos(x)}{4}+\frac{35x}{8}+2\cos(x)\sin(x)+4\cot(x)+\left(-\frac{2}{3}-\frac{\csc(x)^2}{3}\right)\cot(x)$
parallelrisch	$\frac{\csc(x)^3(2520x\sin(x)-840x\sin(3x)+525\cos(x)+3\cos(7x)+63\cos(5x)-847\cos(3x))}{768}$
risch	$\frac{35x}{8}-\frac{i e^{4ix}}{64}-\frac{3i e^{2ix}}{8}+\frac{3i e^{-2ix}}{8}+\frac{i e^{-4ix}}{64}+\frac{4i(6\,e^{4ix}-9\,e^{2ix}+5)}{3(e^{2ix}-1)^3}$
norman	$\frac{-\frac{1}{24}+\frac{35\tan(\frac{x}{2})^2}{24}+\frac{63\tan(\frac{x}{2})^4}{8}+\frac{35\tan(\frac{x}{2})^6}{8}-\frac{35\tan(\frac{x}{2})^8}{8}-\frac{63\tan(\frac{x}{2})^{10}}{8}-\frac{35\tan(\frac{x}{2})^{12}}{24}+\frac{\tan(\frac{x}{2})^{14}}{24}+\frac{35x\tan(\frac{x}{2})^3}{8}+\frac{35x\tan(\frac{x}{2})^5}{2} \cdots}{\tan(\frac{x}{2})^3\left(1+\tan(\frac{x}{2})^2\right)^4}$

input
```
int((csc(x)-sin(x))^4,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(sin(x)^3+3/2*sin(x))*cos(x)+35/8*x+2*cos(x)*sin(x)+4*cot(x)+(-2/3-1/
3*csc(x)^2)*cot(x)
```

3.303.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.16

$$\int(\csc(x)-\sin(x))^4\,dx$$
$$=-\frac{6\cos(x)^7+21\cos(x)^5-140\cos(x)^3-105\left(x\cos(x)^2-x\right)\sin(x)+105\cos(x)}{24\left(\cos(x)^2-1\right)\sin(x)}$$

input
```
integrate((csc(x)-sin(x))^4,x, algorithm="fricas")
```

output
```
-1/24*(6*cos(x)^7 + 21*cos(x)^5 - 140*cos(x)^3 - 105*(x*cos(x)^2 - x)*sin(
x) + 105*cos(x))/((cos(x)^2 - 1)*sin(x))
```

3.303.6 Sympy [A] (verification not implemented)

Time = 3.45 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int (\csc(x)-\sin(x))^4\,dx = \frac{35x}{8}+2\sin(x)\cos(x)-\frac{\sin(2x)}{4}+\frac{\sin(4x)}{32}-\frac{\cot^3(x)}{3}-\cot(x)+\frac{4\cos(x)}{\sin(x)}$$

input
```
integrate((csc(x)-sin(x))**4,x)
```

output
```
35*x/8 + 2*sin(x)*cos(x) - sin(2*x)/4 + sin(4*x)/32 - cot(x)**3/3 - cot(x)
 + 4*cos(x)/sin(x)
```

3.303.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int (\csc(x)-\sin(x))^4\,dx = \frac{35}{8}\,x+\frac{4}{\tan(x)}-\frac{3\,\tan(x)^2+1}{3\,\tan(x)^3}+\frac{1}{32}\,\sin(4\,x)+\frac{3}{4}\,\sin(2\,x)$$

input
```
integrate((csc(x)-sin(x))^4,x, algorithm="maxima")
```

output
```
35/8*x + 4/tan(x) - 1/3*(3*tan(x)^2 + 1)/tan(x)^3 + 1/32*sin(4*x) + 3/4*si
n(2*x)
```

3.303.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int (\csc(x)-\sin(x))^4\,dx = \frac{35}{8}\,x+\frac{11\,\tan(x)^3+13\,\tan(x)}{8\,\left(\tan(x)^2+1\right)^2}+\frac{9\,\tan(x)^2-1}{3\,\tan(x)^3}$$

input
```
integrate((csc(x)-sin(x))^4,x, algorithm="giac")
```

output
```
35/8*x + 1/8*(11*tan(x)^3 + 13*tan(x))/(tan(x)^2 + 1)^2 + 1/3*(9*tan(x)^2
- 1)/tan(x)^3
```

3.303.9 Mupad [B] (verification not implemented)

Time = 29.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.34

$$\int (\csc(x)-\sin(x))^4\,dx = \frac{\frac{\cos(x)^7}{4}+\frac{7\cos(x)^5}{8}-\frac{35\cos(x)^3}{6}+\frac{35\,\cos(x)}{8}}{\sin(x)-\cos(x)^2\sin(x)}-\frac{\frac{35\,x}{8}-\frac{35\,x\cos(x)^2}{8}}{\cos(x)^2-1}$$

input
```
int((sin(x) - 1/sin(x))^4,x)
```

output
```
((35*cos(x))/8 - (35*cos(x)^3)/6 + (7*cos(x)^5)/8 + cos(x)^7/4)/(sin(x) -
cos(x)^2*sin(x)) - ((35*x)/8 - (35*x*cos(x)^2)/8)/(cos(x)^2 - 1)
```

3.304 $\int (\csc(x) - \sin(x))^3 \, dx$

3.304.1 Optimal result

Integrand size = 9, antiderivative size = 34

$$\int (\csc(x) - \sin(x))^3 \, dx = \frac{5}{2}\text{arctanh}(\cos(x)) - \frac{5\cos(x)}{2} - \frac{5\cos^3(x)}{6} - \frac{1}{2}\cos^3(x)\cot^2(x)$$

output
```
5/2*arctanh(cos(x))-5/2*cos(x)-5/6*cos(x)^3-1/2*cos(x)^3*cot(x)^2
```

3.304.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.79

$$\int (\csc(x) - \sin(x))^3 \, dx = -\frac{9\cos(x)}{4} - \frac{1}{12}\cos(3x) - \frac{1}{8}\csc^2\left(\frac{x}{2}\right) + \frac{5}{2}\log\left(\cos\left(\frac{x}{2}\right)\right) - \frac{5}{2}\log\left(\sin\left(\frac{x}{2}\right)\right) + \frac{1}{8}\sec^2\left(\frac{x}{2}\right)$$

input
```
Integrate[(Csc[x] - Sin[x])^3,x]
```

output
```
(-9*Cos[x])/4 - Cos[3*x]/12 - Csc[x/2]^2/8 + (5*Log[Cos[x/2]])/2 - (5*Log[
Sin[x/2]])/2 + Sec[x/2]^2/8
```

3.304.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.15, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.889, Rules used = {3042, 4897, 3042, 25, 3072, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\csc(x)-\sin(x))^3\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int (\csc(x)-\sin(x))^3dx$$
$$\Big\downarrow \text{4897}$$
$$\int \cos^3(x)\cot^3(x)dx$$
$$\Big\downarrow \text{3042}$$
$$\int -\sin\left(x+\frac{\pi}{2}\right)^3\tan\left(x+\frac{\pi}{2}\right)^3dx$$
$$\Big\downarrow \text{25}$$
$$-\int \sin\left(x+\frac{\pi}{2}\right)^3\tan\left(x+\frac{\pi}{2}\right)^3dx$$
$$\Big\downarrow \text{3072}$$
$$-\int \frac{\cos^6(x)}{(1-\cos^2(x))^2}d\cos(x)$$
$$\Big\downarrow \text{252}$$
$$\frac{5}{2}\int \frac{\cos^4(x)}{1-\cos^2(x)}d\cos(x)-\frac{\cos^5(x)}{2(1-\cos^2(x))}$$
$$\Big\downarrow \text{254}$$
$$\frac{5}{2}\int \left(-\cos^2(x)+\frac{1}{1-\cos^2(x)}-1\right)d\cos(x)-\frac{\cos^5(x)}{2(1-\cos^2(x))}$$
$$\Big\downarrow \text{2009}$$
$$\frac{5}{2}\left(\text{arctanh}(\cos(x))-\frac{1}{3}\cos^3(x)-\cos(x)\right)-\frac{\cos^5(x)}{2(1-\cos^2(x))}$$

input
```
Int[(Csc[x] - Sin[x])^3,x]
```

output
```
-1/2*Cos[x]^5/(1 - Cos[x]^2) + (5*(ArcTanh[Cos[x]] - Cos[x] - Cos[x]^3/3))
/2
```

3.304.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.304.4 Maple [A] (verified)

Time = 1.57 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

method	result	size
parallelrisch	$\frac{5}{12}-\frac{5\ln(\csc(x)-\cot(x))}{2}-\frac{\cos(3x)}{12}-\frac{9\cos(x)}{4}-\frac{\csc(x)\cot(x)}{2}$	29
default	$\frac{\left(2+\sin(x)^2\right)\cos(x)}{3}-3\cos\left(x\right)-\frac{5\ln(\csc(x)-\cot(x))}{2}-\frac{\csc(x)\cot(x)}{2}$	32
parts	$\frac{\left(2+\sin(x)^2\right)\cos(x)}{3}-3\cos\left(x\right)-\frac{5\ln(\csc(x)-\cot(x))}{2}-\frac{\csc(x)\cot(x)}{2}$	32
norman	$\frac{-\frac{1}{8}-\frac{75\tan\left(\frac{x}{2}\right)^4}{8}-\frac{65\tan\left(\frac{x}{2}\right)^2}{12}-\frac{55\tan\left(\frac{x}{2}\right)^6}{8}+\frac{\tan\left(\frac{x}{2}\right)^{10}}{8}}{\tan\left(\frac{x}{2}\right)^2\left(1+\tan\left(\frac{x}{2}\right)^2\right)^3}-\frac{5\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2}$	60
risch	$-\frac{e^{3ix}}{24}-\frac{9\,e^{ix}}{8}-\frac{9\,e^{-ix}}{8}-\frac{e^{-3ix}}{24}+\frac{e^{3ix}+e^{ix}}{\left(e^{2ix}-1\right)^2}-\frac{5\ln\left(e^{ix}-1\right)}{2}+\frac{5\ln\left(e^{ix}+1\right)}{2}$	71

input
```
int((csc(x)-sin(x))^3,x,method=_RETURNVERBOSE)
```

output
```
5/12-5/2*ln(csc(x)-cot(x))-1/12*cos(3*x)-9/4*cos(x)-1/2*csc(x)*cot(x)
```

3.304.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(26) = 52$.

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.68

$$\int (\csc(x)-\sin(x))^3\,dx = -\frac{4\cos\left(x\right)^5+20\cos\left(x\right)^3-15\left(\cos\left(x\right)^2-1\right)\log\left(\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)+15\left(\cos\left(x\right)^2-1\right)\log\left(-\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)}{12\left(\cos\left(x\right)^2-1\right)}$$

input
```
integrate((csc(x)-sin(x))^3,x, algorithm="fricas")
```

output
```
-1/12*(4*cos(x)^5 + 20*cos(x)^3 - 15*(cos(x)^2 - 1)*log(1/2*cos(x) + 1/2)
+ 15*(cos(x)^2 - 1)*log(-1/2*cos(x) + 1/2) - 30*cos(x))/(cos(x)^2 - 1)
```

3.304.6 Sympy [A] (verification not implemented)

Time = 1.41 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

$$\int (\csc(x)-\sin(x))^3\,dx = -\frac{5\log(\cos(x)-1)}{4}+\frac{5\log(\cos(x)+1)}{4} - \frac{\cos^3(x)}{3}-2\cos(x)+\frac{\cos(x)}{2\cos^2(x)-2}$$

input
```
integrate((csc(x)-sin(x))**3,x)
```

output
```
-5*log(cos(x) - 1)/4 + 5*log(cos(x) + 1)/4 - cos(x)**3/3 - 2*cos(x) + cos(
x)/(2*cos(x)**2 - 2)
```

3.304.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

$$\int (\csc(x)-\sin(x))^3\,dx = -\frac{1}{3}\cos(x)^3+\frac{\cos(x)}{2(\cos(x)^2-1)}-2\cos(x) + \frac{5}{4}\log(\cos(x)+1)-\frac{5}{4}\log(\cos(x)-1)$$

input
```
integrate((csc(x)-sin(x))^3,x, algorithm="maxima")
```

output
```
-1/3*cos(x)^3 + 1/2*cos(x)/(cos(x)^2 - 1) - 2*cos(x) + 5/4*log(cos(x) + 1)
 - 5/4*log(cos(x) - 1)
```

3.304.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(26) = 52.

Time = 0.26 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.91

$$\int (\csc(x)-\sin(x))^3\,dx = \frac{\left(\frac{10(\cos(x)-1)}{\cos(x)+1}+1\right)(\cos(x)+1)}{8(\cos(x)-1)}-\frac{\cos(x)-1}{8(\cos(x)+1)} - \frac{2\left(\frac{12(\cos(x)-1)}{\cos(x)+1}-\frac{9(\cos(x)-1)^2}{(\cos(x)+1)^2}-7\right)}{3\left(\frac{\cos(x)-1}{\cos(x)+1}-1\right)^3}-\frac{5}{4}\log\left(-\frac{\cos(x)-1}{\cos(x)+1}\right)$$

input
```
integrate((csc(x)-sin(x))^3,x, algorithm="giac")
```

output
```
1/8*(10*(cos(x) - 1)/(cos(x) + 1) + 1)*(cos(x) + 1)/(cos(x) - 1) - 1/8*(co
s(x) - 1)/(cos(x) + 1) - 2/3*(12*(cos(x) - 1)/(cos(x) + 1) - 9*(cos(x) - 1
)^2/(cos(x) + 1)^2 - 7)/((cos(x) - 1)/(cos(x) + 1) - 1)^3 - 5/4*log(-(cos(
x) - 1)/(cos(x) + 1))
```

3.304.9 Mupad [B] (verification not implemented)

Time = 28.56 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.21

$$\int (\csc(x)-\sin(x))^3\,dx = \frac{\tan\left(\frac{x}{2}\right)^2}{8} - \frac{\frac{49\tan\left(\frac{x}{2}\right)^6}{8} + \frac{67\tan\left(\frac{x}{2}\right)^4}{8} + \frac{121\tan\left(\frac{x}{2}\right)^2}{24} + \frac{1}{8}}{\tan\left(\frac{x}{2}\right)^8 + 3\tan\left(\frac{x}{2}\right)^6 + 3\tan\left(\frac{x}{2}\right)^4 + \tan\left(\frac{x}{2}\right)^2} - \frac{5\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2}$$

input
```
int(-(sin(x) - 1/sin(x))^3,x)
```

output
```
tan(x/2)^2/8 - ((121*tan(x/2)^2)/24 + (67*tan(x/2)^4)/8 + (49*tan(x/2)^6)/
8 + 1/8)/(tan(x/2)^2 + 3*tan(x/2)^4 + 3*tan(x/2)^6 + tan(x/2)^8) - (5*log(
tan(x/2)))/2
```

3.305 $\int (\csc(x)-\sin(x))^2 \, dx$

3.305.1 Optimal result

Integrand size = 9, antiderivative size = 22

$$\int (\csc(x)-\sin(x))^2 \, dx = -\frac{3x}{2}-\frac{3\cot(x)}{2}+\frac{1}{2}\cos^2(x)\cot(x)$$

output
```
-3/2*x-3/2*cot(x)+1/2*cos(x)^2*cot(x)
```

3.305.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int (\csc(x)-\sin(x))^2 \, dx = -\frac{3x}{2}-\cot(x)-\frac{1}{4}\sin(2x)$$

input
```
Integrate[(Csc[x] - Sin[x])^2,x]
```

output
```
(-3*x)/2 - Cot[x] - Sin[2*x]/4
```

3.305.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.32, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {3042, 4889, 253, 264, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\csc(x)-\sin(x))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (\csc(x)-\sin(x))^2dx$$

$$\Big\downarrow \text{4889}$$

$$\int \frac{\cot^2(x)}{\left(\tan^2(x)+1\right)^2}d\tan(x)$$

$$\Big\downarrow \text{253}$$

$$\frac{3}{2}\int \frac{\cot^2(x)}{\tan^2(x)+1}d\tan(x)+\frac{\cot(x)}{2\left(\tan^2(x)+1\right)}$$

$$\Big\downarrow \text{264}$$

$$\frac{3}{2}\left(-\int \frac{1}{\tan^2(x)+1}d\tan(x)-\cot(x)\right)+\frac{\cot(x)}{2\left(\tan^2(x)+1\right)}$$

$$\Big\downarrow \text{216}$$

$$\frac{3}{2}(-\arctan(\tan(x))-\cot(x))+\frac{\cot(x)}{2\left(\tan^2(x)+1\right)}$$

input
```
Int[(Csc[x] - Sin[x])^2,x]
```

output
```
(3*(-ArcTan[Tan[x]] - Cot[x]))/2 + Cot[x]/(2*(1 + Tan[x]^2))
```

3.305.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.305.4 Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

method	result	size
default	$-\frac{\cos(x)\sin(x)}{2}-\frac{3x}{2}-\cot(x)$	15
parts	$-\frac{\cos(x)\sin(x)}{2}-\frac{3x}{2}-\cot(x)$	15
parallelrisch	$\frac{\cot(x)\cos(2x)}{4}-\frac{5\cot(x)}{4}-\frac{3x}{2}$	17
risch	$-\frac{3x}{2}+\frac{i\mathrm{e}^{2ix}}{8}-\frac{i\mathrm{e}^{-2ix}}{8}-\frac{2i}{\mathrm{e}^{2ix}-1}$	33
norman	$\frac{-\frac{1}{2}-\frac{3\tan\left(\frac{x}{2}\right)^{2}}{2}+\frac{3\tan\left(\frac{x}{2}\right)^{4}}{2}+\frac{\tan\left(\frac{x}{2}\right)^{6}}{2}-\frac{3x\tan\left(\frac{x}{2}\right)}{2}-3x\tan\left(\frac{x}{2}\right)^{3}-\frac{3x\tan\left(\frac{x}{2}\right)^{5}}{2}}{\tan\left(\frac{x}{2}\right)\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)^{2}}$	69

input
```
int((csc(x)-sin(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*cos(x)*sin(x)-3/2*x-cot(x)
```

3.305.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int (\csc(x)-\sin(x))^2\,dx=\frac{\cos(x)^3-3\,x\sin(x)-3\,\cos(x)}{2\,\sin(x)}$$

input
```
integrate((csc(x)-sin(x))^2,x, algorithm="fricas")
```

output
```
1/2*(cos(x)^3 - 3*x*sin(x) - 3*cos(x))/sin(x)
```

3.305.6 Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.68

$$\int (\csc(x)-\sin(x))^2\,dx=-\frac{3x}{2}-\frac{\sin(2x)}{4}-\cot(x)$$

input
```
integrate((csc(x)-sin(x))**2,x)
```

output
```
-3*x/2 - sin(2*x)/4 - cot(x)
```

3.305.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int (\csc(x) - \sin(x))^2 \, dx = -\frac{3}{2}\,x - \frac{1}{\tan(x)} - \frac{1}{4}\,\sin(2\,x)$$

input
```
integrate((csc(x)-sin(x))^2,x, algorithm="maxima")
```

output
```
-3/2*x - 1/tan(x) - 1/4*sin(2*x)
```

3.305.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int (\csc(x) - \sin(x))^2 \, dx = -\frac{3}{2}\,x - \frac{3\,\tan(x)^2 + 2}{2\,(\tan(x)^3 + \tan(x))}$$

input
```
integrate((csc(x)-sin(x))^2,x, algorithm="giac")
```

output
```
-3/2*x - 1/2*(3*tan(x)^2 + 2)/(tan(x)^3 + tan(x))
```

3.305.9 Mupad [B] (verification not implemented)

Time = 28.88 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

$$\int (\csc(x) - \sin(x))^2 \, dx = -\frac{3\,x}{2} - \frac{\frac{3\,\cos(x)}{2} - \frac{\cos(x)^3}{2}}{\sin(x)}$$

input
```
int((sin(x) - 1/sin(x))^2,x)
```

output
```
- (3*x)/2 - ((3*cos(x))/2 - cos(x)^3/2)/sin(x)
```

3.306 $\int (\csc(x) - \sin(x))\,dx$

3.306.1 Optimal result

Integrand size = 7, antiderivative size = 8

$$\int (\csc(x) - \sin(x))\,dx = -\text{arctanh}(\cos(x)) + \cos(x)$$

output
```
-arctanh(cos(x))+cos(x)
```

3.306.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 19 vs. 2(8) = 16.

Time = 0.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 2.38

$$\int (\csc(x) - \sin(x))\,dx = \cos(x) - \log\left(\cos\left(\frac{x}{2}\right)\right) + \log\left(\sin\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[Csc[x] - Sin[x],x]
```

output
```
Cos[x] - Log[Cos[x/2]] + Log[Sin[x/2]]
```

3.306.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\csc(x)-\sin(x))\,dx$$

$$\Big\downarrow \text{2009}$$

$$\cos(x)-\text{arctanh}(\cos(x))$$

input
```
Int[Csc[x] - Sin[x],x]
```

output
```
-ArcTanh[Cos[x]] + Cos[x]
```

3.306.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.306.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.50

method	result	size
default	$\cos(x)-\ln(\cot(x)+\csc(x))$	12
parts	$\cos(x)-\ln(\cot(x)+\csc(x))$	12
parallelrisch	$\cos(x)+\ln(\csc(x)-\cot(x))+1$	13
norman	$\frac{2}{1+\tan\left(\frac{x}{2}\right)^{2}}+\ln\left(\tan\left(\frac{x}{2}\right)\right)$	19
risch	$-\ln\left(e^{ix}+1\right)+\ln\left(e^{ix}-1\right)+\frac{e^{ix}}{2}+\frac{e^{-ix}}{2}$	34

input
```
int(csc(x)-sin(x),x,method=_RETURNVERBOSE)
```

output
```
cos(x)-ln(cot(x)+csc(x))
```

3.306.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(8) = 16.

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 2.62

$$\int (\csc(x) - \sin(x))\,dx = \cos(x) - \frac{1}{2}\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right) + \frac{1}{2}\log\left(-\frac{1}{2}\cos(x) + \frac{1}{2}\right)$$

input
```
integrate(csc(x)-sin(x),x, algorithm="fricas")
```

output
```
cos(x) - 1/2*log(1/2*cos(x) + 1/2) + 1/2*log(-1/2*cos(x) + 1/2)
```

3.306.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(7) = 14.

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 2.38

$$\int (\csc(x) - \sin(x))\,dx = \frac{\log(\cos(x) - 1)}{2} - \frac{\log(\cos(x) + 1)}{2} + \cos(x)$$

input
```
integrate(csc(x)-sin(x),x)
```

output
```
log(cos(x) - 1)/2 - log(cos(x) + 1)/2 + cos(x)
```

3.306.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.38

$$\int (\csc(x) - \sin(x))\,dx = \cos(x) - \log(\cot(x) + \csc(x))$$

input
```
integrate(csc(x)-sin(x),x, algorithm="maxima")
```

output
```
cos(x) - log(cot(x) + csc(x))
```

3.306.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

$$\int (\csc(x) - \sin(x))\,dx = \cos(x) + \log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)$$

input
```
integrate(csc(x)-sin(x),x, algorithm="giac")
```

output
```
cos(x) + log(abs(tan(1/2*x)))
```

3.306.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int (\csc(x) - \sin(x))\,dx = \ln\left(\tan\left(\frac{x}{2}\right)\right) + \cos(x)$$

input
```
int(1/sin(x) - sin(x),x)
```

output
```
log(tan(x/2)) + cos(x)
```

3.307 $\int \frac{1}{\csc(x)-\sin(x)} \, dx$

3.307.1 Optimal result

Integrand size = 9, antiderivative size = 2

$$\int \frac{1}{\csc(x)-\sin(x)} \, dx = \sec(x)$$

output
```
sec(x)
```

3.307.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00

$$\int \frac{1}{\csc(x)-\sin(x)} \, dx = \sec(x)$$

input
```
Integrate[(Csc[x] - Sin[x])^(-1),x]
```

output
```
Sec[x]
```

3.307.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {3042, 4897, 3042, 3086, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\csc(x)-\sin(x)}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{\csc(x)-\sin(x)}dx$$
$$\Big\downarrow \text{ 4897}$$
$$\int \tan(x)\sec(x)dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \tan(x)\sec(x)dx$$
$$\Big\downarrow \text{ 3086}$$
$$\int 1d\sec(x)$$
$$\Big\downarrow \text{ 24}$$
$$\sec(x)$$

input
```
Int[(Csc[x] - Sin[x])^(-1),x]
```

output
```
Sec[x]
```

3.307.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.307.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 5, normalized size of antiderivative = 2.50

method	result	size
default	$\frac{1}{\cos(x)}$	5
parallelrisch	$1+\sec(x)$	5
norman	$-\frac{2}{\tan(\frac{x}{2})^2-1}$	13
risch	$\frac{2\,\mathrm{e}^{ix}}{\mathrm{e}^{2ix}+1}$	17

input
```
int(1/(csc(x)-sin(x)),x,method=_RETURNVERBOSE)
```

output
```
1/cos(x)
```

3.307.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 4, normalized size of antiderivative = 2.00

$$\int \frac{1}{\csc(x) - \sin(x)} \, dx = \frac{1}{\cos(x)}$$

input
```
integrate(1/(csc(x)-sin(x)),x, algorithm="fricas")
```

output
```
1/cos(x)
```

3.307.6 Sympy [F]

$$\int \frac{1}{\csc(x) - \sin(x)} \, dx = \int \frac{1}{-\sin(x) + \csc(x)} \, dx$$

input
```
integrate(1/(csc(x)-sin(x)),x)
```

output
```
Integral(1/(-sin(x) + csc(x)), x)
```

3.307.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(2) = 4.

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 8.50

$$\int \frac{1}{\csc(x) - \sin(x)} \, dx = -\frac{2}{\frac{\sin(x)^2}{(\cos(x)+1)^2} - 1}$$

input
```
integrate(1/(csc(x)-sin(x)),x, algorithm="maxima")
```

output
```
-2/(sin(x)^2/(cos(x) + 1)^2 - 1)
```

3.307.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(2) = 4$.

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 8.50

$$\int \frac{1}{\csc(x) - \sin(x)}\, dx = \frac{2}{\frac{\cos(x)-1}{\cos(x)+1} + 1}$$

input
```
integrate(1/(csc(x)-sin(x)),x, algorithm="giac")
```

output
```
2/((cos(x) - 1)/(cos(x) + 1) + 1)
```

3.307.9 Mupad [B] (verification not implemented)

Time = 29.31 (sec) , antiderivative size = 12, normalized size of antiderivative = 6.00

$$\int \frac{1}{\csc(x) - \sin(x)}\, dx = -\frac{2}{\tan\left(\frac{x}{2}\right)^2 - 1}$$

input
```
int(-1/(sin(x) - 1/sin(x)),x)
```

output
```
-2/(tan(x/2)^2 - 1)
```

3.308 $\int \frac{1}{(\csc(x)-\sin(x))^2}\,dx$

3.308.1 Optimal result

Integrand size = 9, antiderivative size = 8

$$\int \frac{1}{(\csc(x)-\sin(x))^2}\,dx = \frac{\tan^3(x)}{3}$$

output
```
1/3*tan(x)^3
```

3.308.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1}{(\csc(x)-\sin(x))^2}\,dx = \frac{\tan^3(x)}{3}$$

input
```
Integrate[(Csc[x] - Sin[x])^(-2),x]
```

output
```
Tan[x]^3/3
```

3.308.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4889, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\csc(x)-\sin(x))^2} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(\csc(x)-\sin(x))^2} dx$$

$$\Big\downarrow \text{4889}$$

$$\int \tan^2(x) d\tan(x)$$

$$\Big\downarrow \text{15}$$

$$\frac{\tan^3(x)}{3}$$

input
```
Int[(Csc[x] - Sin[x])^(-2),x]
```

output
```
Tan[x]^3/3
```

3.308.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.308.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{\tan(x)^3}{3}$	7
parallelrisch	$\frac{\tan(x)^3}{3}$	7
norman	$-\frac{8\tan\left(\frac{x}{2}\right)^3}{3\left(\tan\left(\frac{x}{2}\right)^2-1\right)^3}$	19
risch	$-\frac{2i\left(3\,e^{4ix}+1\right)}{3\left(e^{2ix}+1\right)^3}$	22

input
```
int(1/(csc(x)-sin(x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*tan(x)^3
```

3.308.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14 vs. 2(6) = 12.

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.75

$$\int \frac{1}{(\csc(x)-\sin(x))^2}\,dx = -\frac{\left(\cos(x)^2-1\right)\sin(x)}{3\cos(x)^3}$$

input
```
integrate(1/(csc(x)-sin(x))^2,x, algorithm="fricas")
```

output
```
-1/3*(cos(x)^2 - 1)*sin(x)/cos(x)^3
```

3.308.6 Sympy [F]

$$\int \frac{1}{(\csc(x) - \sin(x))^2}\, dx = \int \frac{1}{(-\sin{(x)} + \csc{(x)})^2}\, dx$$

input
```
integrate(1/(csc(x)-sin(x))**2,x)
```

output
```
Integral((-sin(x) + csc(x))**(-2), x)
```

3.308.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{(\csc(x) - \sin(x))^2}\, dx = \frac{1}{3}\,\tan{(x)}^3$$

input
```
integrate(1/(csc(x)-sin(x))^2,x, algorithm="maxima")
```

output
```
1/3*tan(x)^3
```

3.308.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{(\csc(x) - \sin(x))^2}\, dx = \frac{1}{3}\,\tan{(x)}^3$$

input
```
integrate(1/(csc(x)-sin(x))^2,x, algorithm="giac")
```

output
```
1/3*tan(x)^3
```

3.308.9 Mupad [B] (verification not implemented)

Time = 29.13 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{(\csc(x)-\sin(x))^2}\,dx=\frac{\tan(x)^3}{3}$$

input
```
int(1/(sin(x) - 1/sin(x))^2,x)
```

output
```
tan(x)^3/3
```

3.309 $\int \frac{1}{(\csc(x)-\sin(x))^3} \, dx$

3.309.1 Optimal result

Integrand size = 9, antiderivative size = 17

$$\int \frac{1}{(\csc(x)-\sin(x))^3} \, dx=-\frac{1}{3}\sec^3(x)+\frac{\sec^5(x)}{5}$$

output
```
-1/3*sec(x)^3+1/5*sec(x)^5
```

3.309.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{(\csc(x)-\sin(x))^3} \, dx=-\frac{1}{3}\sec^3(x)+\frac{\sec^5(x)}{5}$$

input
```
Integrate[(Csc[x] - Sin[x])^(-3),x]
```

output
```
-1/3*Sec[x]^3 + Sec[x]^5/5
```

3.309.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {3042, 4897, 3042, 3086, 25, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\csc(x)-\sin(x))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(\csc(x)-\sin(x))^3}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \tan^3(x)\sec^3(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(x)^3\sec(x)^3dx$$

$$\Big\downarrow \text{3086}$$

$$\int -\sec^2(x)\left(1-\sec^2(x)\right)d\sec(x)$$

$$\Big\downarrow \text{25}$$

$$-\int \sec^2(x)\left(1-\sec^2(x)\right)d\sec(x)$$

$$\Big\downarrow \text{244}$$

$$-\int \left(\sec^2(x)-\sec^4(x)\right)d\sec(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sec^5(x)}{5}-\frac{\sec^3(x)}{3}$$

input
```
Int[(Csc[x] - Sin[x])^(-3),x]
```

output
```
-1/3*Sec[x]^3 + Sec[x]^5/5
```

3.309.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.309.4 Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{1}{3\cos(x)^3}+\frac{1}{5\cos(x)^5}$	14
parallelrisch	$\frac{4}{15}-\frac{\sec(x)^3}{3}+\frac{\sec(x)^5}{5}$	15
risch	$-\frac{8(5\,e^{7ix}-2\,e^{5ix}+5\,e^{3ix})}{15(e^{2ix}+1)^5}$	34
norman	$\frac{-4\tan\left(\frac{x}{2}\right)^6-\frac{4\tan\left(\frac{x}{2}\right)^4}{3}-\frac{4\tan\left(\frac{x}{2}\right)^2}{3}+\frac{4}{15}}{\left(\tan\left(\frac{x}{2}\right)^2-1\right)^5}$	38

input
```
int(1/(csc(x)-sin(x))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3/cos(x)^3+1/5/cos(x)^5
```

3.309.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{1}{(\csc(x)-\sin(x))^3}\,dx = -\frac{5\,\cos\left(x\right)^2-3}{15\,\cos\left(x\right)^5}$$

input
```
integrate(1/(csc(x)-sin(x))^3,x, algorithm="fricas")
```

output
```
-1/15*(5*cos(x)^2 - 3)/cos(x)^5
```

3.309.6 Sympy [F]

$$\int \frac{1}{(\csc(x)-\sin(x))^3}\,dx = \int \frac{1}{\left(-\sin\left(x\right)+\csc\left(x\right)\right)^3}\,dx$$

input
```
integrate(1/(csc(x)-sin(x))**3,x)
```

output
```
Integral((-sin(x) + csc(x))**(-3), x)
```

3.309.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(13) = 26.

Time = 0.23 (sec) , antiderivative size = 103, normalized size of antiderivative = 6.06

$$\int \frac{1}{(\csc(x)-\sin(x))^3}\,dx = -\frac{4\left(\frac{5\,\sin(x)^2}{(\cos(x)+1)^2}+\frac{5\,\sin(x)^4}{(\cos(x)+1)^4}+\frac{15\,\sin(x)^6}{(\cos(x)+1)^6}-1\right)}{15\left(\frac{5\,\sin(x)^2}{(\cos(x)+1)^2}-\frac{10\,\sin(x)^4}{(\cos(x)+1)^4}+\frac{10\,\sin(x)^6}{(\cos(x)+1)^6}-\frac{5\,\sin(x)^8}{(\cos(x)+1)^8}+\frac{\sin(x)^{10}}{(\cos(x)+1)^{10}}-1\right)}$$

input
```
integrate(1/(csc(x)-sin(x))^3,x, algorithm="maxima")
```

output
```
-4/15*(5*sin(x)^2/(cos(x) + 1)^2 + 5*sin(x)^4/(cos(x) + 1)^4 + 15*sin(x)^6
/(cos(x) + 1)^6 - 1)/(5*sin(x)^2/(cos(x) + 1)^2 - 10*sin(x)^4/(cos(x) + 1)
^4 + 10*sin(x)^6/(cos(x) + 1)^6 - 5*sin(x)^8/(cos(x) + 1)^8 + sin(x)^10/(c
os(x) + 1)^10 - 1)
```

3.309.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(13) = 26$.

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 3.47

$$\int \frac{1}{(\csc(x) - \sin(x))^3}\, dx = -\frac{4\left(\frac{5(\cos(x)-1)}{\cos(x)+1} - \frac{5(\cos(x)-1)^2}{(\cos(x)+1)^2} + \frac{15(\cos(x)-1)^3}{(\cos(x)+1)^3} + 1\right)}{15\left(\frac{\cos(x)-1}{\cos(x)+1} + 1\right)^5}$$

input
```
integrate(1/(csc(x)-sin(x))^3,x, algorithm="giac")
```

output
```
-4/15*(5*(cos(x) - 1)/(cos(x) + 1) - 5*(cos(x) - 1)^2/(cos(x) + 1)^2 + 15*
(cos(x) - 1)^3/(cos(x) + 1)^3 + 1)/((cos(x) - 1)/(cos(x) + 1) + 1)^5
```

3.309.9 Mupad [B] (verification not implemented)

Time = 28.04 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{1}{(\csc(x) - \sin(x))^3}\, dx = \frac{1}{5\cos(x)^5} - \frac{1}{3\cos(x)^3}$$

input
```
int(-1/(sin(x) - 1/sin(x))^3,x)
```

output
```
1/(5*cos(x)^5) - 1/(3*cos(x)^3)
```

3.310 $\int \frac{1}{(\csc(x)-\sin(x))^4} dx$

3.310.1 Optimal result

Integrand size = 9, antiderivative size = 17

$$\int \frac{1}{(\csc(x)-\sin(x))^4} dx = \frac{\tan^5(x)}{5} + \frac{\tan^7(x)}{7}$$

output
```
1/5*tan(x)^5+1/7*tan(x)^7
```

3.310.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 37 vs. 2(17) = 34.

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.18

$$\int \frac{1}{(\csc(x)-\sin(x))^4} dx = \frac{2\tan(x)}{35} + \frac{1}{35}\sec^2(x)\tan(x) - \frac{8}{35}\sec^4(x)\tan(x) + \frac{1}{7}\sec^6(x)\tan(x)$$

input
```
Integrate[(Csc[x] - Sin[x])^(-4),x]
```

output
```
(2*Tan[x])/35 + (Sec[x]^2*Tan[x])/35 - (8*Sec[x]^4*Tan[x])/35 + (Sec[x]^6*
Tan[x])/7
```

3.310.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4889, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\csc(x)-\sin(x))^4} \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{(\csc(x)-\sin(x))^4}dx$$

$$\Big\downarrow \text{ 4889}$$

$$\int \left(\tan^6(x)+\tan^4(x)\right) d\tan(x)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\tan^7(x)}{7}+\frac{\tan^5(x)}{5}$$

input
```
Int[(Csc[x] - Sin[x])^(-4),x]
```

output
```
Tan[x]^5/5 + Tan[x]^7/7
```

3.310.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.310.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{\tan(x)^5}{5}+\frac{\tan(x)^7}{7}$	14
parallelrisch	$\frac{2\tan(x)^5}{35}+\frac{\sec(x)^2\tan(x)^5}{7}$	18
norman	$\frac{-\frac{32\tan\left(\frac{x}{2}\right)^5}{5}-\frac{192\tan\left(\frac{x}{2}\right)^7}{35}-\frac{32\tan\left(\frac{x}{2}\right)^9}{5}}{\left(\tan\left(\frac{x}{2}\right)^2-1\right)^7}$	37
risch	$\frac{4i(35\,e^{10ix}-35\,e^{8ix}+70\,e^{6ix}-14\,e^{4ix}+7\,e^{2ix}+1)}{35(e^{2ix}+1)^7}$	50

input
```
int(1/(csc(x)-sin(x))^4,x,method=_RETURNVERBOSE)
```

output
```
1/5*tan(x)^5+1/7*tan(x)^7
```

3.310.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int \frac{1}{(\csc(x)-\sin(x))^4}\,dx = \frac{\left(2\,\cos\left(x\right)^6+\cos\left(x\right)^4-8\,\cos\left(x\right)^2+5\right)\sin\left(x\right)}{35\,\cos\left(x\right)^7}$$

input
```
integrate(1/(csc(x)-sin(x))^4,x, algorithm="fricas")
```

output
```
1/35*(2*cos(x)^6 + cos(x)^4 - 8*cos(x)^2 + 5)*sin(x)/cos(x)^7
```

3.310.6 Sympy [F]

$$\int \frac{1}{(\csc(x)-\sin(x))^4}\,dx=\int \frac{1}{(-\sin{(x)}+\csc{(x)})^4}\,dx$$

input
```
integrate(1/(csc(x)-sin(x))**4,x)
```

output
```
Integral((-sin(x) + csc(x))**(-4), x)
```

3.310.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{1}{(\csc(x)-\sin(x))^4}\,dx=\frac{1}{7}\tan{(x)}^7+\frac{1}{5}\tan{(x)}^5$$

input
```
integrate(1/(csc(x)-sin(x))^4,x, algorithm="maxima")
```

output
```
1/7*tan(x)^7 + 1/5*tan(x)^5
```

3.310.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{1}{(\csc(x)-\sin(x))^4}\,dx=\frac{1}{7}\tan{(x)}^7+\frac{1}{5}\tan{(x)}^5$$

input
```
integrate(1/(csc(x)-sin(x))^4,x, algorithm="giac")
```

output
```
1/7*tan(x)^7 + 1/5*tan(x)^5
```

3.310.9 Mupad [B] (verification not implemented)

Time = 28.67 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.35

$$\int \frac{1}{(\csc(x)-\sin(x))^4}\,dx = \frac{2\cos(x)^2\sin(x)^5+5\sin(x)^5}{35\cos(x)^7}$$

input
```
int(1/(sin(x) - 1/sin(x))^4,x)
```

output
```
(5*sin(x)^5 + 2*cos(x)^2*sin(x)^5)/(35*cos(x)^7)
```

3.311 $\int \frac{1}{(\csc(x)-\sin(x))^5} \, dx$

3.311.1 Optimal result

Integrand size = 9, antiderivative size = 25

$$\int \frac{1}{(\csc(x)-\sin(x))^5} \, dx = \frac{\sec^5(x)}{5} - \frac{2\sec^7(x)}{7} + \frac{\sec^9(x)}{9}$$

output
```
1/5*sec(x)^5-2/7*sec(x)^7+1/9*sec(x)^9
```

3.311.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(\csc(x)-\sin(x))^5} \, dx = \frac{\sec^5(x)}{5} - \frac{2\sec^7(x)}{7} + \frac{\sec^9(x)}{9}$$

input
```
Integrate[(Csc[x] - Sin[x])^(-5),x]
```

output
```
Sec[x]^5/5 - (2*Sec[x]^7)/7 + Sec[x]^9/9
```

3.311.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 4897, 3042, 3086, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\csc(x)-\sin(x))^5} \, dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(\csc(x)-\sin(x))^5}dx$$
$$\Big\downarrow \text{4897}$$
$$\int \tan^5(x)\sec^5(x)dx$$
$$\Big\downarrow \text{3042}$$
$$\int \tan(x)^5\sec(x)^5dx$$
$$\Big\downarrow \text{3086}$$
$$\int \sec^4(x)\left(1-\sec^2(x)\right)^2d\sec(x)$$
$$\Big\downarrow \text{244}$$
$$\int \left(\sec^8(x)-2\sec^6(x)+\sec^4(x)\right)d\sec(x)$$
$$\Big\downarrow \text{2009}$$
$$\frac{\sec^9(x)}{9}-\frac{2\sec^7(x)}{7}+\frac{\sec^5(x)}{5}$$

input
```
Int[(Csc[x] - Sin[x])^(-5),x]
```

output
```
Sec[x]^5/5 - (2*Sec[x]^7)/7 + Sec[x]^9/9
```

3.311.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.311.4 Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{1}{9\cos\left(x\right)^{9}}-\frac{2}{7\cos\left(x\right)^{7}}+\frac{1}{5\cos\left(x\right)^{5}}$	20
parallelrisch	$\frac{32}{315}+\frac{\sec\left(x\right)^{5}}{5}-\frac{2\sec\left(x\right)^{7}}{7}+\frac{\sec\left(x\right)^{9}}{9}$	21
risch	$\frac{\frac{32\,{\mathrm e}^{13 i x}}{5}-\frac{384\,{\mathrm e}^{11 i x}}{35}+\frac{6976\,{\mathrm e}^{9 i x}}{315}-\frac{384\,{\mathrm e}^{7 i x}}{35}+\frac{32\,{\mathrm e}^{5 i x}}{5}}{\left({\mathrm e}^{2 i x}+1\right)^{9}}$	48

input
```
int(1/(csc(x)-sin(x))^5,x,method=_RETURNVERBOSE)
```

output
```
1/9/cos(x)^9-2/7/cos(x)^7+1/5/cos(x)^5
```

3.311.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{1}{(\csc(x)-\sin(x))^5}\,dx = \frac{63\cos(x)^4-90\cos(x)^2+35}{315\cos(x)^9}$$

input
```
integrate(1/(csc(x)-sin(x))^5,x, algorithm="fricas")
```

output
```
1/315*(63*cos(x)^4 - 90*cos(x)^2 + 35)/cos(x)^9
```

3.311.6 Sympy [F]

$$\int \frac{1}{(\csc(x)-\sin(x))^5}\,dx = \int \frac{1}{(-\sin(x)+\csc(x))^5}\,dx$$

input
```
integrate(1/(csc(x)-sin(x))**5,x)
```

output
```
Integral((-sin(x) + csc(x))**(-5), x)
```

3.311.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(19) = 38.

Time = 0.23 (sec) , antiderivative size = 187, normalized size of antiderivative = 7.48

$$\int \frac{1}{(\csc(x)-\sin(x))^5}\,dx = \frac{16\left(\frac{9\sin(x)^2}{(\cos(x)+1)^2}-\frac{36\sin(x)^4}{(\cos(x)+1)^4}-\frac{126\sin(x)^6}{(\cos(x)+1)^6}-\frac{441\sin(x)^8}{(\cos(x)+1)^8}-\frac{315\sin(x)^{10}}{(\cos(x)+1)^{10}}-\frac{210\sin(x)^{12}}{(\cos(x)+1)^{12}}-1\right)}{315\left(\frac{9\sin(x)^2}{(\cos(x)+1)^2}-\frac{36\sin(x)^4}{(\cos(x)+1)^4}+\frac{84\sin(x)^6}{(\cos(x)+1)^6}-\frac{126\sin(x)^8}{(\cos(x)+1)^8}+\frac{126\sin(x)^{10}}{(\cos(x)+1)^{10}}-\frac{84\sin(x)^{12}}{(\cos(x)+1)^{12}}+\frac{36\sin(x)^{14}}{(\cos(x)+1)^{14}}-\frac{9\sin(x)^{16}}{(\cos(x)+1)^{16}}\cdots}$$

input
```
integrate(1/(csc(x)-sin(x))^5,x, algorithm="maxima")
```

output
```
16/315*(9*sin(x)^2/(cos(x) + 1)^2 - 36*sin(x)^4/(cos(x) + 1)^4 - 126*sin(x
)^6/(cos(x) + 1)^6 - 441*sin(x)^8/(cos(x) + 1)^8 - 315*sin(x)^10/(cos(x) +
 1)^10 - 210*sin(x)^12/(cos(x) + 1)^12 - 1)/(9*sin(x)^2/(cos(x) + 1)^2 - 3
6*sin(x)^4/(cos(x) + 1)^4 + 84*sin(x)^6/(cos(x) + 1)^6 - 126*sin(x)^8/(cos
(x) + 1)^8 + 126*sin(x)^10/(cos(x) + 1)^10 - 84*sin(x)^12/(cos(x) + 1)^12
+ 36*sin(x)^14/(cos(x) + 1)^14 - 9*sin(x)^16/(cos(x) + 1)^16 + sin(x)^18/(
cos(x) + 1)^18 - 1)
```

3.311.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(19) = 38$.

Time = 0.29 (sec) , antiderivative size = 101, normalized size of antiderivative = 4.04

$$\int \frac{1}{(\csc(x)-\sin(x))^5}\,dx = \frac{16\left(\frac{9(\cos(x)-1)}{\cos(x)+1}+\frac{36(\cos(x)-1)^2}{(\cos(x)+1)^2}-\frac{126(\cos(x)-1)^3}{(\cos(x)+1)^3}+\frac{441(\cos(x)-1)^4}{(\cos(x)+1)^4}-\frac{315(\cos(x)-1)^5}{(\cos(x)+1)^5}+\frac{210(\cos(x)-1)^6}{(\cos(x)+1)^6}+1\right)}{315\left(\frac{\cos(x)-1}{\cos(x)+1}+1\right)^9}$$

input
```
integrate(1/(csc(x)-sin(x))^5,x, algorithm="giac")
```

output
```
16/315*(9*(cos(x) - 1)/(cos(x) + 1) + 36*(cos(x) - 1)^2/(cos(x) + 1)^2 - 1
26*(cos(x) - 1)^3/(cos(x) + 1)^3 + 441*(cos(x) - 1)^4/(cos(x) + 1)^4 - 315
*(cos(x) - 1)^5/(cos(x) + 1)^5 + 210*(cos(x) - 1)^6/(cos(x) + 1)^6 + 1)/((
cos(x) - 1)/(cos(x) + 1) + 1)^9
```

3.311.9 Mupad [B] (verification not implemented)

Time = 29.59 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1}{(\csc(x)-\sin(x))^5}\,dx = \frac{1}{5\cos(x)^5}-\frac{2}{7\cos(x)^7}+\frac{1}{9\cos(x)^9}$$

input
```
int(-1/(sin(x) - 1/sin(x))^5,x)
```

output
```
1/(5*cos(x)^5) - 2/(7*cos(x)^7) + 1/(9*cos(x)^9)
```

3.312 $\int \frac{1}{(\csc(x)-\sin(x))^6} dx$

3.312.1 Optimal result

Integrand size = 9, antiderivative size = 25

$$\int \frac{1}{(\csc(x)-\sin(x))^6} dx = \frac{\tan^7(x)}{7} + \frac{2\tan^9(x)}{9} + \frac{\tan^{11}(x)}{11}$$

output
```
1/7*tan(x)^7+2/9*tan(x)^9+1/11*tan(x)^11
```

3.312.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 57 vs. 2(25) = 50.

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.28

$$\int \frac{1}{(\csc(x)-\sin(x))^6} dx = -\frac{8\tan(x)}{693} - \frac{4}{693}\sec^2(x)\tan(x) - \frac{1}{231}\sec^4(x)\tan(x) + \frac{113}{693}\sec^6(x)\tan(x) - \frac{23}{99}\sec^8(x)\tan(x) + \frac{1}{11}\sec^{10}(x)\tan(x)$$

input
```
Integrate[(Csc[x] - Sin[x])^(-6),x]
```

output
```
(-8*Tan[x])/693 - (4*Sec[x]^2*Tan[x])/693 - (Sec[x]^4*Tan[x])/231 + (113*S
ec[x]^6*Tan[x])/693 - (23*Sec[x]^8*Tan[x])/99 + (Sec[x]^10*Tan[x])/11
```

3.312.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3042, 4889, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\csc(x)-\sin(x))^6}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{(\csc(x)-\sin(x))^6}dx$$
$$\Big\downarrow \text{ 4889}$$
$$\int \tan^6(x)\left(\tan^2(x)+1\right)^2d\tan(x)$$
$$\Big\downarrow \text{ 244}$$
$$\int \left(\tan^{10}(x)+2\tan^8(x)+\tan^6(x)\right)d\tan(x)$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{\tan^{11}(x)}{11}+\frac{2\tan^9(x)}{9}+\frac{\tan^7(x)}{7}$$

input
```
Int[(Csc[x] - Sin[x])^(-6),x]
```

output
```
Tan[x]^7/7 + (2*Tan[x]^9)/9 + Tan[x]^11/11
```

3.312.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.312.4 Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{\tan(x)^7}{7}+\frac{2\tan(x)^9}{9}+\frac{\tan(x)^{11}}{11}$	20
parallelrisch	$\frac{\tan(x)^7\sec(x)^4(80+\cos(4x)+18\cos(2x))}{693}$	23
risch	$-\frac{16i(462\,\mathrm{e}^{16ix}-1155\,\mathrm{e}^{14ix}+2541\,\mathrm{e}^{12ix}-2079\,\mathrm{e}^{10ix}+1485\,\mathrm{e}^{8ix}-297\,\mathrm{e}^{6ix}+55\,\mathrm{e}^{4ix}+11\,\mathrm{e}^{2ix}+1)}{693(\mathrm{e}^{2ix}+1)^{11}}$	71

input
```
int(1/(csc(x)-sin(x))^6,x,method=_RETURNVERBOSE)
```

output
```
1/7*tan(x)^7+2/9*tan(x)^9+1/11*tan(x)^11
```

3.312.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(19) = 38.

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.60

$$\int \frac{1}{(\csc(x)-\sin(x))^6}\,dx = -\frac{\left(8\cos(x)^{10}+4\cos(x)^8+3\cos(x)^6-113\cos(x)^4+161\cos(x)^2-63\right)\sin(x)}{693\cos(x)^{11}}$$

input
```
integrate(1/(csc(x)-sin(x))^6,x, algorithm="fricas")
```

output
```
-1/693*(8*cos(x)^10 + 4*cos(x)^8 + 3*cos(x)^6 - 113*cos(x)^4 + 161*cos(x)^
2 - 63)*sin(x)/cos(x)^11
```

3.312.6 Sympy [F]

$$\int \frac{1}{(\csc(x)-\sin(x))^6}\,dx = \int \frac{1}{(-\sin{(x)}+\csc{(x)})^6}\,dx$$

input
```
integrate(1/(csc(x)-sin(x))**6,x)
```

output
```
Integral((-sin(x) + csc(x))**(-6), x)
```

3.312.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1}{(\csc(x)-\sin(x))^6}\,dx = \frac{1}{11}\tan{(x)}^{11} + \frac{2}{9}\tan{(x)}^9 + \frac{1}{7}\tan{(x)}^7$$

input
```
integrate(1/(csc(x)-sin(x))^6,x, algorithm="maxima")
```

output
```
1/11*tan(x)^11 + 2/9*tan(x)^9 + 1/7*tan(x)^7
```

3.312.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{1}{(\csc(x)-\sin(x))^6}\,dx = \frac{1}{11}\tan{(x)}^{11} + \frac{2}{9}\tan{(x)}^9 + \frac{1}{7}\tan{(x)}^7$$

input
```
integrate(1/(csc(x)-sin(x))^6,x, algorithm="giac")
```

output
```
1/11*tan(x)^11 + 2/9*tan(x)^9 + 1/7*tan(x)^7
```

3.312.9 Mupad [B] (verification not implemented)

Time = 28.92 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

$$\int \frac{1}{(\csc(x)-\sin(x))^6}\,dx=\frac{8\cos\left(x\right)^4\sin\left(x\right)^7+28\cos\left(x\right)^2\sin\left(x\right)^7+63\sin\left(x\right)^7}{693\cos\left(x\right)^{11}}$$

input
```
int(1/(sin(x) - 1/sin(x))^6,x)
```

output
```
(63*sin(x)^7 + 28*cos(x)^2*sin(x)^7 + 8*cos(x)^4*sin(x)^7)/(693*cos(x)^11)
```

3.313 $\int \frac{1}{(\csc(x)-\sin(x))^7} dx$

3.313.1 Optimal result

Integrand size = 9, antiderivative size = 33

$$\int \frac{1}{(\csc(x)-\sin(x))^7} dx = -\frac{1}{7}\sec^7(x)+\frac{\sec^9(x)}{3}-\frac{3\sec^{11}(x)}{11}+\frac{\sec^{13}(x)}{13}$$

output
```
-1/7*sec(x)^7+1/3*sec(x)^9-3/11*sec(x)^11+1/13*sec(x)^13
```

3.313.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{1}{(\csc(x)-\sin(x))^7} dx = -\frac{1}{7}\sec^7(x)+\frac{\sec^9(x)}{3}-\frac{3\sec^{11}(x)}{11}+\frac{\sec^{13}(x)}{13}$$

input
```
Integrate[(Csc[x] - Sin[x])^(-7),x]
```

output
```
-1/7*Sec[x]^7 + Sec[x]^9/3 - (3*Sec[x]^11)/11 + Sec[x]^13/13
```

3.313.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {3042, 4897, 3042, 3086, 25, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\csc(x)-\sin(x))^7} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(\csc(x)-\sin(x))^7}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \tan^7(x)\sec^7(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(x)^7\sec(x)^7dx$$

$$\Big\downarrow \text{3086}$$

$$\int -\sec^6(x)\left(1-\sec^2(x)\right)^3d\sec(x)$$

$$\Big\downarrow \text{25}$$

$$-\int \sec^6(x)\left(1-\sec^2(x)\right)^3d\sec(x)$$

$$\Big\downarrow \text{244}$$

$$-\int \left(-\sec^{12}(x)+3\sec^{10}(x)-3\sec^8(x)+\sec^6(x)\right)d\sec(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sec^{13}(x)}{13}-\frac{3\sec^{11}(x)}{11}+\frac{\sec^9(x)}{3}-\frac{\sec^7(x)}{7}$$

input
```
Int[(Csc[x] - Sin[x])^(-7),x]
```

output
```
-1/7*Sec[x]^7 + Sec[x]^9/3 - (3*Sec[x]^11)/11 + Sec[x]^13/13
```

3.313.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.313.4 Maple [A] (verified)

Time = 1.87 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

method	result	size
default	$\frac{1}{13\cos(x)^{13}}-\frac{3}{11\cos(x)^{11}}+\frac{1}{3\cos(x)^{9}}-\frac{1}{7\cos(x)^{7}}$	26
parallelrisch	$\frac{592}{15015}-\frac{\sec(x)^{7}}{7}+\frac{\sec(x)^{9}}{3}-\frac{3\sec(x)^{11}}{11}+\frac{\sec(x)^{13}}{13}$	27
risch	$-\frac{128\left(429\,\mathrm{e}^{19ix}-1430\,\mathrm{e}^{17ix}+3523\,\mathrm{e}^{15ix}-4020\,\mathrm{e}^{13ix}+3523\,\mathrm{e}^{11ix}-1430\,\mathrm{e}^{9ix}+429\,\mathrm{e}^{7ix}\right)}{3003\left(\mathrm{e}^{2ix}+1\right)^{13}}$	62

input
```
int(1/(csc(x)-sin(x))^7,x,method=_RETURNVERBOSE)
```

output
```
1/13/cos(x)^13-3/11/cos(x)^11+1/3/cos(x)^9-1/7/cos(x)^7
```

3.313.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int \frac{1}{(\csc(x)-\sin(x))^7}\,dx = -\frac{429 \cos{(x)}^6 - 1001 \cos{(x)}^4 + 819 \cos{(x)}^2 - 231}{3003 \cos{(x)}^{13}}$$

input
```
integrate(1/(csc(x)-sin(x))^7,x, algorithm="fricas")
```

output
```
-1/3003*(429*cos(x)^6 - 1001*cos(x)^4 + 819*cos(x)^2 - 231)/cos(x)^13
```

3.313.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(\csc(x)-\sin(x))^7}\,dx = \text{Timed out}$$

input
```
integrate(1/(csc(x)-sin(x))**7,x)
```

output
```
Timed out
```

3.313.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. 2(25) = 50.

Time = 0.23 (sec) , antiderivative size = 271, normalized size of antiderivative = 8.21

$$\int \frac{1}{(\csc(x)-\sin(x))^7}\,dx = -\frac{32\left(\frac{13\sin(x)^2}{(\cos(x)+1)^2} - \frac{78\sin(x)^4}{(\cos(x)+1)^4} + \frac{286\sin(x)^6}{(\cos(x)+1)^6} + \frac{2288\sin(x)^8}{(\cos(x)+1)^8} + \frac{10296\sin(x)^{10}}{(\cos(x)+1)^{10}} + \frac{16302\sin(x}{(\cos(x)+1)}\right.}{3003\left(\frac{13\sin(x)^2}{(\cos(x)+1)^2} - \frac{78\sin(x)^4}{(\cos(x)+1)^4} + \frac{286\sin(x)^6}{(\cos(x)+1)^6} - \frac{715\sin(x)^8}{(\cos(x)+1)^8} + \frac{1287\sin(x)^{10}}{(\cos(x)+1)^{10}} - \frac{1716\sin(x)^{12}}{(\cos(x)+1)^{12}} + \frac{1716\sin(x)^{14}}{(\cos(x)+1)^{14}} - \frac{1287\sin}{(\cos(x)+}\right.}$$

input
```
integrate(1/(csc(x)-sin(x))^7,x, algorithm="maxima")
```

output
```
-32/3003*(13*sin(x)^2/(cos(x) + 1)^2 - 78*sin(x)^4/(cos(x) + 1)^4 + 286*si
n(x)^6/(cos(x) + 1)^6 + 2288*sin(x)^8/(cos(x) + 1)^8 + 10296*sin(x)^10/(co
s(x) + 1)^10 + 16302*sin(x)^12/(cos(x) + 1)^12 + 18018*sin(x)^14/(cos(x) +
 1)^14 + 9009*sin(x)^16/(cos(x) + 1)^16 + 3003*sin(x)^18/(cos(x) + 1)^18 -
 1)/(13*sin(x)^2/(cos(x) + 1)^2 - 78*sin(x)^4/(cos(x) + 1)^4 + 286*sin(x)^
6/(cos(x) + 1)^6 - 715*sin(x)^8/(cos(x) + 1)^8 + 1287*sin(x)^10/(cos(x) +
1)^10 - 1716*sin(x)^12/(cos(x) + 1)^12 + 1716*sin(x)^14/(cos(x) + 1)^14 -
1287*sin(x)^16/(cos(x) + 1)^16 + 715*sin(x)^18/(cos(x) + 1)^18 - 286*sin(x
)^20/(cos(x) + 1)^20 + 78*sin(x)^22/(cos(x) + 1)^22 - 13*sin(x)^24/(cos(x)
 + 1)^24 + sin(x)^26/(cos(x) + 1)^26 - 1)
```

3.313.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(25) = 50$.

Time = 0.26 (sec) , antiderivative size = 143, normalized size of antiderivative = 4.33

$$\int \frac{1}{(\csc(x)-\sin(x))^7}\,dx = -\frac{32\left(\frac{13(\cos(x)-1)}{\cos(x)+1}+\frac{78(\cos(x)-1)^2}{(\cos(x)+1)^2}+\frac{286(\cos(x)-1)^3}{(\cos(x)+1)^3}-\frac{2288(\cos(x)-1)^4}{(\cos(x)+1)^4}+\frac{10296(\cos(x)-1)^5}{(\cos(x)+1)^5}-\frac{16302(\cos(x)-1)^6}{(\cos(x)+1)^6}+\frac{18018(\cdots}{(\cos\cdots}\right.}{3003\left(\frac{\cos(x)-1}{\cos(x)+1}+1\right)^{13}}$$

input
```
integrate(1/(csc(x)-sin(x))^7,x, algorithm="giac")
```

output
```
-32/3003*(13*(cos(x) - 1)/(cos(x) + 1) + 78*(cos(x) - 1)^2/(cos(x) + 1)^2
+ 286*(cos(x) - 1)^3/(cos(x) + 1)^3 - 2288*(cos(x) - 1)^4/(cos(x) + 1)^4 +
 10296*(cos(x) - 1)^5/(cos(x) + 1)^5 - 16302*(cos(x) - 1)^6/(cos(x) + 1)^6
 + 18018*(cos(x) - 1)^7/(cos(x) + 1)^7 - 9009*(cos(x) - 1)^8/(cos(x) + 1)^
8 + 3003*(cos(x) - 1)^9/(cos(x) + 1)^9 + 1)/((cos(x) - 1)/(cos(x) + 1) + 1
)^13
```

3.313.9 Mupad [B] (verification not implemented)

Time = 28.92 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{1}{(\csc(x)-\sin(x))^7}\,dx = \frac{1}{3\cos(x)^9}-\frac{1}{7\cos(x)^7}-\frac{3}{11\cos(x)^{11}}+\frac{1}{13\cos(x)^{13}}$$

input
```
int(-1/(sin(x) - 1/sin(x))^7,x)
```

output
```
1/(3*cos(x)^9) - 1/(7*cos(x)^7) - 3/(11*cos(x)^11) + 1/(13*cos(x)^13)
```

3.314 $\int (\csc(x) - \sin(x))^{7/2} \, dx$

3.314.1 Optimal result

Integrand size = 11, antiderivative size = 73

$$\int (\csc(x) - \sin(x))^{7/2} \, dx = \frac{8}{7} \cos(x) \cot^2(x) \sqrt{\cos(x) \cot(x)} + \frac{2}{7} \cos^3(x) \cot^2(x) \sqrt{\cos(x) \cot(x)} - \frac{64}{35} \cot(x) \sqrt{\cos(x) \cot(x)} \csc(x) + \frac{256}{35} \sqrt{\cos(x) \cot(x)} \sec(x)$$

output
```
8/7*cos(x)*cot(x)^2*(cos(x)*cot(x))^(1/2)+2/7*cos(x)^3*cot(x)^2*(cos(x)*co
t(x))^(1/2)-64/35*cot(x)*csc(x)*(cos(x)*cot(x))^(1/2)+256/35*sec(x)*(cos(x
)*cot(x))^(1/2)
```

3.314.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.51

$$\int (\csc(x) - \sin(x))^{7/2} \, dx = -\frac{1}{70} \sqrt{\cos(x) \cot(x)} \left(-512 + 115 \cos^2(x) + 5 \cos(x) \cos(3x) + 28 \cot^2(x)\right) \sec(x)$$

input
```
Integrate[(Csc[x] - Sin[x])^(7/2),x]
```

output
```
-1/70*(Sqrt[Cos[x]*Cot[x]]*(-512 + 115*Cos[x]^2 + 5*Cos[x]*Cos[3*x] + 28*C
ot[x]^2)*Sec[x])
```

3.314.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.33, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.091, Rules used = {3042, 4897, 3042, 4900, 3042, 3078, 3042, 3078, 3042, 3074, 3042, 3069}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\csc(x)-\sin(x))^{7/2} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int (\csc(x)-\sin(x))^{7/2}dx$$

$$\Big\downarrow \text{4897}$$

$$\int (\cos(x)\cot(x))^{7/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int (\cos(x)\cot(x))^{7/2}dx$$

$$\Big\downarrow \text{4900}$$

$$\frac{\sqrt{\cos(x)\cot(x)}\int \cos^{\frac{7}{2}}(x)\cot^{\frac{7}{2}}(x)dx}{\sqrt{\cos(x)}\sqrt{\cot(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(x)\cot(x)}\int \sin\left(x+\frac{\pi}{2}\right)^{7/2}\left(-\tan\left(x+\frac{\pi}{2}\right)\right)^{7/2}dx}{\sqrt{\cos(x)}\sqrt{\cot(x)}}$$

$$\Big\downarrow \text{3078}$$

$$\frac{\sqrt{\cos(x)\cot(x)}\left(\frac{12}{7}\int \cos^{\frac{3}{2}}(x)\cot^{\frac{7}{2}}(x)dx+\frac{2}{7}\cos^{\frac{7}{2}}(x)\cot^{\frac{5}{2}}(x)\right)}{\sqrt{\cos(x)}\sqrt{\cot(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(x)\cot(x)}\left(\frac{12}{7}\int \sin\left(x+\frac{\pi}{2}\right)^{3/2}\left(-\tan\left(x+\frac{\pi}{2}\right)\right)^{7/2}dx+\frac{2}{7}\cos^{\frac{7}{2}}(x)\cot^{\frac{5}{2}}(x)\right)}{\sqrt{\cos(x)}\sqrt{\cot(x)}}$$

$$\Big\downarrow \text{3078}$$

$$\frac{\sqrt{\cos(x)\cot(x)}\left(\frac{12}{7}\left(\frac{8}{3}\int \frac{\cot^{\frac{7}{2}}(x)}{\sqrt{\cos(x)}}dx + \frac{2}{3}\cos^{\frac{3}{2}}(x)\cot^{\frac{5}{2}}(x)\right) + \frac{2}{7}\cos^{\frac{7}{2}}(x)\cot^{\frac{5}{2}}(x)\right)}{\sqrt{\cos(x)}\sqrt{\cot(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(x)\cot(x)}\left(\frac{12}{7}\left(\frac{8}{3}\int \frac{\left(-\tan\left(x+\frac{\pi}{2}\right)\right)^{7/2}}{\sqrt{\sin\left(x+\frac{\pi}{2}\right)}}dx + \frac{2}{3}\cos^{\frac{3}{2}}(x)\cot^{\frac{5}{2}}(x)\right) + \frac{2}{7}\cos^{\frac{7}{2}}(x)\cot^{\frac{5}{2}}(x)\right)}{\sqrt{\cos(x)}\sqrt{\cot(x)}}$$

$$\Big\downarrow \text{3074}$$

$$\frac{\sqrt{\cos(x)\cot(x)}\left(\frac{12}{7}\left(\frac{8}{3}\left(-\frac{4}{5}\int \frac{\cot^{\frac{3}{2}}(x)}{\sqrt{\cos(x)}}dx - \frac{2\cot^{\frac{5}{2}}(x)}{5\sqrt{\cos(x)}}\right) + \frac{2}{3}\cos^{\frac{3}{2}}(x)\cot^{\frac{5}{2}}(x)\right) + \frac{2}{7}\cos^{\frac{7}{2}}(x)\cot^{\frac{5}{2}}(x)\right)}{\sqrt{\cos(x)}\sqrt{\cot(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(x)\cot(x)}\left(\frac{12}{7}\left(\frac{8}{3}\left(-\frac{4}{5}\int \frac{\left(-\tan\left(x+\frac{\pi}{2}\right)\right)^{3/2}}{\sqrt{\sin\left(x+\frac{\pi}{2}\right)}}dx - \frac{2\cot^{\frac{5}{2}}(x)}{5\sqrt{\cos(x)}}\right) + \frac{2}{3}\cos^{\frac{3}{2}}(x)\cot^{\frac{5}{2}}(x)\right) + \frac{2}{7}\cos^{\frac{7}{2}}(x)\cot^{\frac{5}{2}}(x)\right)}{\sqrt{\cos(x)}\sqrt{\cot(x)}}$$

$$\Big\downarrow \text{3069}$$

$$\frac{\sqrt{\cos(x)\cot(x)}\left(\frac{2}{7}\cos^{\frac{7}{2}}(x)\cot^{\frac{5}{2}}(x) + \frac{12}{7}\left(\frac{2}{3}\cos^{\frac{3}{2}}(x)\cot^{\frac{5}{2}}(x) + \frac{8}{3}\left(\frac{8\sqrt{\cot(x)}}{5\sqrt{\cos(x)}} - \frac{2\cot^{\frac{5}{2}}(x)}{5\sqrt{\cos(x)}}\right)\right)\right)}{\sqrt{\cos(x)}\sqrt{\cot(x)}}$$

input
```
Int[(Csc[x] - Sin[x])^(7/2),x]
```

output
```
(Sqrt[Cos[x]*Cot[x]]*((2*Cos[x]^(7/2)*Cot[x]^(5/2))/7 + (12*((2*Cos[x]^(3/
2)*Cot[x]^(5/2))/3 + (8*((8*Sqrt[Cot[x]])/(5*Sqrt[Cos[x]]) - (2*Cot[x]^(5/
2))/(5*Sqrt[Cos[x]])))/3))/7))/(Sqrt[Cos[x]]*Sqrt[Cot[x]])
```

3.314. $\int (\csc(x) - \sin(x))^{7/2}\,dx$

3.314.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3069
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

rule 3074
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(n
 - 1))), x] - Simp[b^2*((m + n - 1)/(n - 1))   Int[(a*Sin[e + f*x])^m*(b*Ta
n[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && In
tegersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

rule 4900
```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Simp[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p]))   Int[uu*vv^(m*p)*ww^(n*p), x]
, x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !
InertTrigFreeQ[w])
```

3.314.4 Maple [A] (verified)

Time = 2.60 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.49

method	result	size
default	$\frac{2\sec(x)\csc(x)^{2}\left(5\cos(x)^{6}+20\cos(x)^{4}-160\cos(x)^{2}+128\right)\sqrt{\cot(x)\cos(x)}}{35}$	36

input
```
int((csc(x)-sin(x))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/35*sec(x)*csc(x)^2*(5*cos(x)^6+20*cos(x)^4-160*cos(x)^2+128)*(cot(x)*cos
(x))^(1/2)
```

3.314.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.60

$$\int (\csc(x)-\sin(x))^{7/2}\,dx = -\frac{2\left(5\cos(x)^{6}+20\cos(x)^{4}-160\cos(x)^{2}+128\right)\sqrt{\frac{\cos(x)^{2}}{\sin(x)}}}{35\left(\cos(x)^{3}-\cos(x)\right)}$$

input
```
integrate((csc(x)-sin(x))^(7/2),x, algorithm="fricas")
```

output
```
-2/35*(5*cos(x)^6 + 20*cos(x)^4 - 160*cos(x)^2 + 128)*sqrt(cos(x)^2/sin(x)
)/(cos(x)^3 - cos(x))
```

3.314.6 Sympy [F(-1)]

Timed out.

$$\int (\csc(x)-\sin(x))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate((csc(x)-sin(x))**(7/2),x)
```

output
```
Timed out
```

3.314.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 578 vs. $2(57) = 114$.

Time = 0.43 (sec) , antiderivative size = 578, normalized size of antiderivative = 7.92

$$\int (\csc(x) - \sin(x))^{7/2} \, dx = \text{Too large to display}$$

input
```
integrate((csc(x)-sin(x))^(7/2),x, algorithm="maxima")
```

output
```
-1/280*(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)^(1/4)*(cos(x)^2 + sin(x)^2 - 2
*cos(x) + 1)^(1/4)*(((5*cos(21/2*x) + 105*cos(17/2*x) - 2275*cos(13/2*x) +
 5817*cos(9/2*x) - 5*cos(7/2*x) - 5817*cos(5/2*x) - 105*cos(3/2*x) + 2275*
cos(1/2*x) - 5*sin(21/2*x) - 105*sin(17/2*x) + 2275*sin(13/2*x) - 5817*sin
(9/2*x) - 5*sin(7/2*x) + 5817*sin(5/2*x) - 105*sin(3/2*x) - 2275*sin(1/2*x
))*cos(7/2*arctan2(sin(x), cos(x) - 1)) + (5*cos(21/2*x) + 105*cos(17/2*x)
 - 2275*cos(13/2*x) + 5817*cos(9/2*x) - 5*cos(7/2*x) - 5817*cos(5/2*x) - 1
05*cos(3/2*x) + 2275*cos(1/2*x) + 5*sin(21/2*x) + 105*sin(17/2*x) - 2275*s
in(13/2*x) + 5817*sin(9/2*x) + 5*sin(7/2*x) - 5817*sin(5/2*x) + 105*sin(3/
2*x) + 2275*sin(1/2*x))*sin(7/2*arctan2(sin(x), cos(x) - 1)))*cos(7/2*arct
an2(sin(x), cos(x) + 1)) + ((5*cos(21/2*x) + 105*cos(17/2*x) - 2275*cos(13
/2*x) + 5817*cos(9/2*x) - 5*cos(7/2*x) - 5817*cos(5/2*x) - 105*cos(3/2*x)
+ 2275*cos(1/2*x) + 5*sin(21/2*x) + 105*sin(17/2*x) - 2275*sin(13/2*x) + 5
817*sin(9/2*x) + 5*sin(7/2*x) - 5817*sin(5/2*x) + 105*sin(3/2*x) + 2275*si
n(1/2*x))*cos(7/2*arctan2(sin(x), cos(x) - 1)) - (5*cos(21/2*x) + 105*cos(
17/2*x) - 2275*cos(13/2*x) + 5817*cos(9/2*x) - 5*cos(7/2*x) - 5817*cos(5/2
*x) - 105*cos(3/2*x) + 2275*cos(1/2*x) - 5*sin(21/2*x) - 105*sin(17/2*x) +
 2275*sin(13/2*x) - 5817*sin(9/2*x) - 5*sin(7/2*x) + 5817*sin(5/2*x) - 105
*sin(3/2*x) - 2275*sin(1/2*x))*sin(7/2*arctan2(sin(x), cos(x) - 1)))*sin(7
/2*arctan2(sin(x), cos(x) + 1)))/(cos(x)^8 + sin(x)^8 + 4*(cos(x)^2 + 1...
```

3.314.8 Giac [F]

$$\int (\csc(x) - \sin(x))^{7/2} \, dx = \int (\csc(x) - \sin(x))^{\frac{7}{2}} \, dx$$

input
```
integrate((csc(x)-sin(x))^(7/2),x, algorithm="giac")
```

output
```
integrate((csc(x) - sin(x))^(7/2), x)
```

3.314.9 Mupad [F(-1)]

Timed out.

$$\int (\csc(x) - \sin(x))^{7/2} \, dx = \int \left(\frac{1}{\sin(x)} - \sin(x)\right)^{7/2} \, dx$$

input
```
int((1/sin(x) - sin(x))^(7/2),x)
```

output
```
int((1/sin(x) - sin(x))^(7/2), x)
```

3.315 $\int (\csc(x)-\sin(x))^{5/2}\,dx$

3.315.1 Optimal result

Integrand size = 11, antiderivative size = 50

$$\int (\csc(x)-\sin(x))^{5/2}\,dx = -\frac{16}{15}\cot(x)\sqrt{\cos(x)\cot(x)}+\frac{2}{5}\cos^2(x)\cot(x)\sqrt{\cos(x)\cot(x)}-\frac{64}{15}\sqrt{\cos(x)\cot(x)}\tan(x)$$

output
```
-16/15*cot(x)*(cos(x)*cot(x))^(1/2)+2/5*cos(x)^2*cot(x)*(cos(x)*cot(x))^(1
/2)-64/15*(cos(x)*cot(x))^(1/2)*tan(x)
```

3.315.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.58

$$\int (\csc(x)-\sin(x))^{5/2}\,dx = -\frac{2}{15}\sqrt{\cos(x)\cot(x)}\left(32+3\cos^2(x)+5\cot^2(x)\right)\tan(x)$$

input
```
Integrate[(Csc[x] - Sin[x])^(5/2),x]
```

output
```
(-2*Sqrt[Cos[x]*Cot[x]]*(32 + 3*Cos[x]^2 + 5*Cot[x]^2)*Tan[x])/15
```

3.315.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.52, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.909, Rules used = {3042, 4897, 3042, 4900, 3042, 3078, 3042, 3074, 3042, 3069}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\csc(x)-\sin(x))^{5/2} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int (\csc(x)-\sin(x))^{5/2}dx$$

$$\Big\downarrow \text{4897}$$

$$\int (\cos(x)\cot(x))^{5/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int (\cos(x)\cot(x))^{5/2}dx$$

$$\Big\downarrow \text{4900}$$

$$\frac{\sqrt{\cos(x)\cot(x)}\int \cos^{\frac{5}{2}}(x)\cot^{\frac{5}{2}}(x)dx}{\sqrt{\cos(x)}\sqrt{\cot(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(x)\cot(x)}\int \sin\left(x+\frac{\pi}{2}\right)^{5/2}\left(-\tan\left(x+\frac{\pi}{2}\right)\right)^{5/2}dx}{\sqrt{\cos(x)}\sqrt{\cot(x)}}$$

$$\Big\downarrow \text{3078}$$

$$\frac{\sqrt{\cos(x)\cot(x)}\left(\frac{8}{5}\int \sqrt{\cos(x)}\cot^{\frac{5}{2}}(x)dx+\frac{2}{5}\cos^{\frac{5}{2}}(x)\cot^{\frac{3}{2}}(x)\right)}{\sqrt{\cos(x)}\sqrt{\cot(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(x)\cot(x)}\left(\frac{8}{5}\int \sqrt{\sin\left(x+\frac{\pi}{2}\right)}\left(-\tan\left(x+\frac{\pi}{2}\right)\right)^{5/2}dx+\frac{2}{5}\cos^{\frac{5}{2}}(x)\cot^{\frac{3}{2}}(x)\right)}{\sqrt{\cos(x)}\sqrt{\cot(x)}}$$

$$\Big\downarrow \text{3074}$$

$$\frac{\sqrt{\cos(x)\cot(x)}\left(\frac{8}{5}\left(-\frac{4}{3}\int\sqrt{\cos(x)}\sqrt{\cot(x)}dx-\frac{2}{3}\sqrt{\cos(x)}\cot^{\frac{3}{2}}(x)\right)+\frac{2}{5}\cos^{\frac{5}{2}}(x)\cot^{\frac{3}{2}}(x)\right)}{\sqrt{\cos(x)}\sqrt{\cot(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(x)\cot(x)}\left(\frac{8}{5}\left(-\frac{4}{3}\int\sqrt{\sin\left(x+\frac{\pi}{2}\right)}\sqrt{-\tan\left(x+\frac{\pi}{2}\right)}dx-\frac{2}{3}\sqrt{\cos(x)}\cot^{\frac{3}{2}}(x)\right)+\frac{2}{5}\cos^{\frac{5}{2}}(x)\cot^{\frac{3}{2}}(x)\right)}{\sqrt{\cos(x)}\sqrt{\cot(x)}}$$

$$\Big\downarrow \text{3069}$$

$$\frac{\sqrt{\cos(x)\cot(x)}\left(\frac{2}{5}\cos^{\frac{5}{2}}(x)\cot^{\frac{3}{2}}(x)+\frac{8}{5}\left(-\frac{2}{3}\sqrt{\cos(x)}\cot^{\frac{3}{2}}(x)-\frac{8\sqrt{\cos(x)}}{3\sqrt{\cot(x)}}\right)\right)}{\sqrt{\cos(x)}\sqrt{\cot(x)}}$$

input
```
Int[(Csc[x] - Sin[x])^(5/2),x]
```

output
```
(Sqrt[Cos[x]*Cot[x]]*((2*Cos[x]^(5/2)*Cot[x]^(3/2))/5 + (8*((-8*Sqrt[Cos[x
]])/(3*Sqrt[Cot[x]]) - (2*Sqrt[Cos[x]]*Cot[x]^(3/2))/3))/5))/(Sqrt[Cos[x]]
*Sqrt[Cot[x]])
```

3.315.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3069
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

rule 3074
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(n
 - 1))), x] - Simp[b^2*((m + n - 1)/(n - 1))   Int[(a*Sin[e + f*x])^m*(b*Ta
n[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && In
tegersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

3.315. $\int(\csc(x)-\sin(x))^{5/2}\,dx$

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

rule 4900
```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Simp[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p]))   Int[uu*vv^(m*p)*ww^(n*p), x]
, x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !
InertTrigFreeQ[w])
```

3.315.4 Maple [A] (verified)

Time = 2.59 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.58

method	result	size
default	$\frac{2\sqrt{\cot(x)\cos(x)}\,\left(3\cos(x)^{2}\cot(x)+24\cot(x)-32\sec(x)\csc(x)\right)}{15}$	29

input
```
int((csc(x)-sin(x))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(cot(x)*cos(x))^(1/2)*(3*cos(x)^2*cot(x)+24*cot(x)-32*sec(x)*csc(x))
```

3.315.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.70

$$\int (\csc(x)-\sin(x))^{5/2}\,dx=\frac{2\left(3\,\cos\left(x\right)^{4}+24\,\cos\left(x\right)^{2}-32\right)\sqrt{\frac{\cos(x)^{2}}{\sin(x)}}}{15\,\cos\left(x\right)\sin\left(x\right)}$$

input
```
integrate((csc(x)-sin(x))^(5/2),x, algorithm="fricas")
```

output
```
2/15*(3*cos(x)^4 + 24*cos(x)^2 - 32)*sqrt(cos(x)^2/sin(x))/(cos(x)*sin(x))
```

3.315.6 Sympy [F(-1)]

Timed out.

$$\int (\csc(x)-\sin(x))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((csc(x)-sin(x))**(5/2),x)
```

output
```
Timed out
```

3.315.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(38) = 76$.

Time = 0.42 (sec) , antiderivative size = 427, normalized size of antiderivative = 8.54

$$\int (\csc(x)-\sin(x))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((csc(x)-sin(x))^(5/2),x, algorithm="maxima")
```

output
```
-1/60*(((3*cos(15/2*x) + 105*cos(11/2*x) - 410*cos(7/2*x) - 3*cos(5/2*x) +
 410*cos(3/2*x) - 105*cos(1/2*x) + 3*sin(15/2*x) + 105*sin(11/2*x) - 410*s
in(7/2*x) + 3*sin(5/2*x) + 410*sin(3/2*x) + 105*sin(1/2*x))*cos(5/2*arctan
2(sin(x), cos(x) - 1)) - (3*cos(15/2*x) + 105*cos(11/2*x) - 410*cos(7/2*x)
 - 3*cos(5/2*x) + 410*cos(3/2*x) - 105*cos(1/2*x) - 3*sin(15/2*x) - 105*si
n(11/2*x) + 410*sin(7/2*x) - 3*sin(5/2*x) - 410*sin(3/2*x) - 105*sin(1/2*x
))*sin(5/2*arctan2(sin(x), cos(x) - 1)))*cos(5/2*arctan2(sin(x), cos(x) +
1)) - ((3*cos(15/2*x) + 105*cos(11/2*x) - 410*cos(7/2*x) - 3*cos(5/2*x) +
410*cos(3/2*x) - 105*cos(1/2*x) - 3*sin(15/2*x) - 105*sin(11/2*x) + 410*si
n(7/2*x) - 3*sin(5/2*x) - 410*sin(3/2*x) - 105*sin(1/2*x))*cos(5/2*arctan2
(sin(x), cos(x) - 1)) + (3*cos(15/2*x) + 105*cos(11/2*x) - 410*cos(7/2*x)
- 3*cos(5/2*x) + 410*cos(3/2*x) - 105*cos(1/2*x) + 3*sin(15/2*x) + 105*sin
(11/2*x) - 410*sin(7/2*x) + 3*sin(5/2*x) + 410*sin(3/2*x) + 105*sin(1/2*x)
)*sin(5/2*arctan2(sin(x), cos(x) - 1)))*sin(5/2*arctan2(sin(x), cos(x) + 1
)))/((cos(x)^4 + sin(x)^4 + 2*(cos(x)^2 + 1)*sin(x)^2 - 2*cos(x)^2 + 1)*(c
os(x)^2 + sin(x)^2 + 2*cos(x) + 1)^(1/4)*(cos(x)^2 + sin(x)^2 - 2*cos(x) +
 1)^(1/4))
```

3.315.8 Giac [F]

$$\int (\csc(x) - \sin(x))^{5/2} \, dx = \int (\csc(x) - \sin(x))^{\frac{5}{2}} \, dx$$

input
```
integrate((csc(x)-sin(x))^(5/2),x, algorithm="giac")
```

output
```
integrate((csc(x) - sin(x))^(5/2), x)
```

3.315.9 Mupad [F(-1)]

Timed out.

$$\int (\csc(x) - \sin(x))^{5/2} \, dx = \int \left(\frac{1}{\sin(x)} - \sin(x)\right)^{5/2} \, dx$$

input
```
int((1/sin(x) - sin(x))^(5/2),x)
```

output
```
int((1/sin(x) - sin(x))^(5/2), x)
```

3.316 $\int (\csc(x)-\sin(x))^{3/2}\,dx$

3.316.1 Optimal result

Integrand size = 11, antiderivative size = 31

$$\int (\csc(x)-\sin(x))^{3/2}\,dx=\frac{2}{3}\cos(x)\sqrt{\cos(x)\cot(x)}-\frac{8}{3}\sqrt{\cos(x)\cot(x)}\sec(x)$$

output
```
2/3*cos(x)*(cos(x)*cot(x))^(1/2)-8/3*sec(x)*(cos(x)*cot(x))^(1/2)
```

3.316.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68

$$\int (\csc(x)-\sin(x))^{3/2}\,dx=\frac{2}{3}\left(-4+\cos^2(x)\right)\sqrt{\cos(x)\cot(x)}\sec(x)$$

input
```
Integrate[(Csc[x] - Sin[x])^(3/2),x]
```

output
```
(2*(-4 + Cos[x]^2)*Sqrt[Cos[x]*Cot[x]]*Sec[x])/3
```

3.316.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.77, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {3042, 4897, 3042, 4900, 3042, 3078, 3042, 3069}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\csc(x)-\sin(x))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (\csc(x)-\sin(x))^{3/2}dx$$

$$\Big\downarrow \text{4897}$$

$$\int (\cos(x)\cot(x))^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int (\cos(x)\cot(x))^{3/2}dx$$

$$\Big\downarrow \text{4900}$$

$$\frac{\sqrt{\cos(x)\cot(x)}\int \cos^{\frac{3}{2}}(x)\cot^{\frac{3}{2}}(x)dx}{\sqrt{\cos(x)}\sqrt{\cot(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(x)\cot(x)}\int \sin\left(x+\frac{\pi}{2}\right)^{3/2}\left(-\tan\left(x+\frac{\pi}{2}\right)\right)^{3/2}dx}{\sqrt{\cos(x)}\sqrt{\cot(x)}}$$

$$\Big\downarrow \text{3078}$$

$$\frac{\sqrt{\cos(x)\cot(x)}\left(\frac{4}{3}\int \frac{\cot^{\frac{3}{2}}(x)}{\sqrt{\cos(x)}}dx+\frac{2}{3}\cos^{\frac{3}{2}}(x)\sqrt{\cot(x)}\right)}{\sqrt{\cos(x)}\sqrt{\cot(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(x)\cot(x)}\left(\frac{4}{3}\int \frac{\left(-\tan\left(x+\frac{\pi}{2}\right)\right)^{3/2}}{\sqrt{\sin\left(x+\frac{\pi}{2}\right)}}dx+\frac{2}{3}\cos^{\frac{3}{2}}(x)\sqrt{\cot(x)}\right)}{\sqrt{\cos(x)}\sqrt{\cot(x)}}$$

$$\Big\downarrow \text{3069}$$

$$\frac{\left(\frac{2}{3}\cos^{\frac{3}{2}}(x)\sqrt{\cot(x)}-\frac{8\sqrt{\cot(x)}}{3\sqrt{\cos(x)}}\right)\sqrt{\cos(x)\cot(x)}}{\sqrt{\cos(x)}\sqrt{\cot(x)}}$$

input
```
Int[(Csc[x] - Sin[x])^(3/2),x]
```

output
```
(((-8*Sqrt[Cot[x]])/(3*Sqrt[Cos[x]]) + (2*Cos[x]^(3/2)*Sqrt[Cot[x]])/3)*Sq
rt[Cos[x]*Cot[x]])/(Sqrt[Cos[x]]*Sqrt[Cot[x]])
```

3.316.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3069
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

rule 4900
```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Simp[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p]))   Int[uu*vv^(m*p)*ww^(n*p), x]
, x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !
InertTrigFreeQ[w])
```

3.316.4 Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.55

method	result	size
default	$\frac{2\sqrt{\cot(x)\cos(x)}\,(\cos(x)-4\sec(x))}{3}$	17

input
```
int((csc(x)-sin(x))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(cot(x)*cos(x))^(1/2)*(cos(x)-4*sec(x))
```

3.316.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int (\csc(x)-\sin(x))^{3/2}\,dx = \frac{2\left(\cos\left(x\right)^{2}-4\right)\sqrt{\frac{\cos(x)^{2}}{\sin(x)}}}{3\,\cos\left(x\right)}$$

input
```
integrate((csc(x)-sin(x))^(3/2),x, algorithm="fricas")
```

output
```
2/3*(cos(x)^2 - 4)*sqrt(cos(x)^2/sin(x))/cos(x)
```

3.316.6 Sympy [F]

$$\int (\csc(x)-\sin(x))^{3/2}\,dx = \int \left(-\sin\left(x\right)+\csc\left(x\right)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((csc(x)-sin(x))**(3/2),x)
```

output
```
Integral((-sin(x) + csc(x))**(3/2), x)
```

3.316.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(23) = 46$.

Time = 0.39 (sec) , antiderivative size = 314, normalized size of antiderivative = 10.13

$$\int (\csc(x) - \sin(x))^{3/2}\,dx = \frac{\left(\cos(x)^2 + \sin(x)^2 + 2\cos(x) + 1\right)^{\frac{1}{4}}\left(\cos(x)^2 + \sin(x)^2 - 2\cos(x) + 1\right)^{\frac{1}{4}}\left(\left(\left(\cos\left(\frac{9}{2}x\right) - \cdots\right.\right.\right.}{\cdots}$$

input
```
integrate((csc(x)-sin(x))^(3/2),x, algorithm="maxima")
```

output
```
1/6*(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)^(1/4)*(cos(x)^2 + sin(x)^2 - 2*co
s(x) + 1)^(1/4)*(((cos(9/2*x) - 15*cos(5/2*x) - cos(3/2*x) + 15*cos(1/2*x)
 - sin(9/2*x) + 15*sin(5/2*x) - sin(3/2*x) - 15*sin(1/2*x))*cos(3/2*arctan
2(sin(x), cos(x) - 1)) + (cos(9/2*x) - 15*cos(5/2*x) - cos(3/2*x) + 15*cos
(1/2*x) + sin(9/2*x) - 15*sin(5/2*x) + sin(3/2*x) + 15*sin(1/2*x))*sin(3/2
*arctan2(sin(x), cos(x) - 1)))*cos(3/2*arctan2(sin(x), cos(x) + 1)) + ((co
s(9/2*x) - 15*cos(5/2*x) - cos(3/2*x) + 15*cos(1/2*x) + sin(9/2*x) - 15*si
n(5/2*x) + sin(3/2*x) + 15*sin(1/2*x))*cos(3/2*arctan2(sin(x), cos(x) - 1)
) - (cos(9/2*x) - 15*cos(5/2*x) - cos(3/2*x) + 15*cos(1/2*x) - sin(9/2*x)
+ 15*sin(5/2*x) - sin(3/2*x) - 15*sin(1/2*x))*sin(3/2*arctan2(sin(x), cos(
x) - 1)))*sin(3/2*arctan2(sin(x), cos(x) + 1)))/(cos(x)^4 + sin(x)^4 + 2*(
cos(x)^2 + 1)*sin(x)^2 - 2*cos(x)^2 + 1)
```

3.316.8 Giac [F]

$$\int (\csc(x) - \sin(x))^{3/2}\,dx = \int (\csc(x) - \sin(x))^{\frac{3}{2}}\,dx$$

input
```
integrate((csc(x)-sin(x))^(3/2),x, algorithm="giac")
```

output
```
integrate((csc(x) - sin(x))^(3/2), x)
```

3.316.9 Mupad [F(-1)]

Timed out.

$$\int (\csc(x) - \sin(x))^{3/2} \, dx = \int \left(\frac{1}{\sin(x)} - \sin(x)\right)^{3/2} \, dx$$

input
```
int((1/sin(x) - sin(x))^(3/2),x)
```

output
```
int((1/sin(x) - sin(x))^(3/2), x)
```

3.317 $\int \sqrt{\csc(x)-\sin(x)}\,dx$

3.317.1 Optimal result

Integrand size = 11, antiderivative size = 13

$$\int \sqrt{\csc(x)-\sin(x)}\,dx=2\sqrt{\cos(x)\cot(x)}\tan(x)$$

output
```
2*(cos(x)*cot(x))^(1/2)*tan(x)
```

3.317.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \sqrt{\csc(x)-\sin(x)}\,dx=2\sqrt{\cos(x)\cot(x)}\tan(x)$$

input
```
Integrate[Sqrt[Csc[x] - Sin[x]],x]
```

output
```
2*Sqrt[Cos[x]*Cot[x]]*Tan[x]
```

3.317.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {3042, 4897, 3042, 4900, 3042, 3069}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\csc(x)-\sin(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\csc(x)-\sin(x)}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \sqrt{\cos(x)\cot(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\cos(x)\cot(x)}dx$$

$$\Big\downarrow \text{4900}$$

$$\frac{\sqrt{\cos(x)\cot(x)}\int \sqrt{\cos(x)}\sqrt{\cot(x)}dx}{\sqrt{\cos(x)}\sqrt{\cot(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(x)\cot(x)}\int \sqrt{\sin\left(x+\frac{\pi}{2}\right)}\sqrt{-\tan\left(x+\frac{\pi}{2}\right)}dx}{\sqrt{\cos(x)}\sqrt{\cot(x)}}$$

$$\Big\downarrow \text{3069}$$

$$2\tan(x)\sqrt{\cos(x)\cot(x)}$$

input
```
Int[Sqrt[Csc[x] - Sin[x]],x]
```

output
```
2*Sqrt[Cos[x]*Cot[x]]*Tan[x]
```

3.317.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3069
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

rule 4900
```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Simp[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p]))   Int[uu*vv^(m*p)*ww^(n*p), x]
, x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !
InertTrigFreeQ[w])
```

3.317.4 Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
default	$2\sqrt{\cot\left(x\right)\cos\left(x\right)}\,\tan\left(x\right)$	12
risch	$-\frac{i\sqrt{2}\,\sqrt{\frac{i\left({e}^{2 i x}+1\right)^{2}{e}^{-i x}}{{e}^{2 i x}-1}}\,\left({e}^{2 i x}-1\right)}{{e}^{2 i x}+1}$	51

input
```
int((csc(x)-sin(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(cot(x)*cos(x))^(1/2)*tan(x)
```

3.317.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46

$$\int \sqrt{\csc(x)-\sin(x)}\,dx = \frac{2\sqrt{\frac{\cos(x)^2}{\sin(x)}}\sin(x)}{\cos(x)}$$

input
```
integrate((csc(x)-sin(x))^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(cos(x)^2/sin(x))*sin(x)/cos(x)
```

3.317.6 Sympy [F]

$$\int \sqrt{\csc(x)-\sin(x)}\,dx = \int \sqrt{-\sin(x)+\csc(x)}\,dx$$

input
```
integrate((csc(x)-sin(x))**(1/2),x)
```

output
```
Integral(sqrt(-sin(x) + csc(x)), x)
```

3.317.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(11) = 22.

Time = 0.41 (sec) , antiderivative size = 188, normalized size of antiderivative = 14.46

$$\int \sqrt{\csc(x)-\sin(x)}\,dx$$
$$= \frac{\left(\left(\cos\left(\frac{3}{2}x\right)-\cos\left(\frac{1}{2}x\right)+\sin\left(\frac{3}{2}x\right)+\sin\left(\frac{1}{2}x\right)\right)\cos\left(\frac{1}{2}\arctan\left(\sin(x),\cos(x)-1\right)\right)-\left(\cos\left(\frac{3}{2}x\right)-\cos\left(\frac{1}{2}\right.\right.\right.}{}$$

input
```
integrate((csc(x)-sin(x))^(1/2),x, algorithm="maxima")
```

output
```
(((cos(3/2*x) - cos(1/2*x) + sin(3/2*x) + sin(1/2*x))*cos(1/2*arctan2(sin(
x), cos(x) - 1)) - (cos(3/2*x) - cos(1/2*x) - sin(3/2*x) - sin(1/2*x))*sin
(1/2*arctan2(sin(x), cos(x) - 1)))*cos(1/2*arctan2(sin(x), cos(x) + 1)) -
((cos(3/2*x) - cos(1/2*x) - sin(3/2*x) - sin(1/2*x))*cos(1/2*arctan2(sin(x
), cos(x) - 1)) + (cos(3/2*x) - cos(1/2*x) + sin(3/2*x) + sin(1/2*x))*sin(
1/2*arctan2(sin(x), cos(x) - 1)))*sin(1/2*arctan2(sin(x), cos(x) + 1)))/((
cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)^(1/4)*(cos(x)^2 + sin(x)^2 - 2*cos(x)
+ 1)^(1/4))
```

3.317.8 Giac [F]

$$\int \sqrt{\csc(x)-\sin(x)}\,dx=\int \sqrt{\csc\left(x\right)-\sin\left(x\right)}\,dx$$

input
```
integrate((csc(x)-sin(x))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(csc(x) - sin(x)), x)
```

3.317.9 Mupad [B] (verification not implemented)

Time = 27.75 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \sqrt{\csc(x)-\sin(x)}\,dx=\frac{2\left|\cos\left(x\right)\right|}{\cos\left(x\right)\,\sqrt{\frac{1}{\sin(x)}}}$$

input
```
int((1/sin(x) - sin(x))^(1/2),x)
```

output
```
(2*abs(cos(x)))/(cos(x)*(1/sin(x))^(1/2))
```

3.318 $\int \frac{1}{\sqrt{\csc(x)-\sin(x)}}\,dx$

3.318.1 Optimal result

Integrand size = 11, antiderivative size = 60

$$\int \frac{1}{\sqrt{\csc(x)-\sin(x)}}\,dx = \frac{\arctan\left(\sqrt{-\sin(x)}\right)\cos(x)}{\sqrt{\cos(x)\cot(x)}\sqrt{-\sin(x)}} - \frac{\operatorname{arctanh}\left(\sqrt{-\sin(x)}\right)\cos(x)}{\sqrt{\cos(x)\cot(x)}\sqrt{-\sin(x)}}$$

output
```
arctan((-sin(x))^(1/2))*cos(x)/(cos(x)*cot(x))^(1/2)/(-sin(x))^(1/2)-arcta
nh((-sin(x))^(1/2))*cos(x)/(cos(x)*cot(x))^(1/2)/(-sin(x))^(1/2)
```

3.318.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.73

$$\int \frac{1}{\sqrt{\csc(x)-\sin(x)}}\,dx = -\frac{\left(\arctan\left(\sqrt[4]{\sin^2(x)}\right) - \operatorname{arctanh}\left(\sqrt[4]{\sin^2(x)}\right)\right)\sqrt{\cos(x)\cot(x)}\sin(x)\tan(x)}{\sin^2(x)^{3/4}}$$

input
```
Integrate[1/Sqrt[Csc[x] - Sin[x]],x]
```

output
```
-(((ArcTan[(Sin[x]^2)^(1/4)] - ArcTanh[(Sin[x]^2)^(1/4)])*Sqrt[Cos[x]*Cot[
x]]*Sin[x]*Tan[x])/(Sin[x]^2)^(3/4))
```

3.318.3 Rubi [A] (warning: unable to verify)

Time = 0.38 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.60, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.091, Rules used = {3042, 4897, 3042, 4900, 3042, 3081, 3042, 3044, 266, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\csc(x)-\sin(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\csc(x)-\sin(x)}}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \frac{1}{\sqrt{\cos(x)\cot(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\cos(x)\cot(x)}}dx$$

$$\Big\downarrow \text{4900}$$

$$\frac{\sqrt{\cos(x)}\sqrt{\cot(x)}\int \frac{1}{\sqrt{\cos(x)}\sqrt{\cot(x)}}dx}{\sqrt{\cos(x)\cot(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(x)}\sqrt{\cot(x)}\int \frac{1}{\sqrt{\sin\left(x+\frac{\pi}{2}\right)}\sqrt{-\tan\left(x+\frac{\pi}{2}\right)}}dx}{\sqrt{\cos(x)\cot(x)}}$$

$$\Big\downarrow \text{3081}$$

$$\frac{\cos(x)\int \sec(x)\sqrt{-\sin(x)}dx}{\sqrt{-\sin(x)}\sqrt{\cos(x)\cot(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos(x)\int \frac{\sqrt{-\sin(x)}}{\cos(x)}dx}{\sqrt{-\sin(x)}\sqrt{\cos(x)\cot(x)}}$$

$$\Big\downarrow \text{3044}$$

$$-\frac{\cos(x)\int \frac{\sqrt{-\sin(x)}}{1-\sin^2(x)}d(-\sin(x))}{\sqrt{-\sin(x)}\sqrt{\cos(x)\cot(x)}}$$

$$\Big\downarrow \; 266$$

$$-\frac{2\cos(x)\int \frac{\sin^2(x)}{1-\sin^4(x)}d\sqrt{-\sin(x)}}{\sqrt{-\sin(x)}\sqrt{\cos(x)\cot(x)}}$$

$$\Big\downarrow \; 827$$

$$-\frac{2\cos(x)\left(\frac{1}{2}\int \frac{1}{1-\sin^2(x)}d\sqrt{-\sin(x)}-\frac{1}{2}\int \frac{1}{\sin^2(x)+1}d\sqrt{-\sin(x)}\right)}{\sqrt{-\sin(x)}\sqrt{\cos(x)\cot(x)}}$$

$$\Big\downarrow \; 216$$

$$-\frac{2\cos(x)\left(\frac{1}{2}\int \frac{1}{1-\sin^2(x)}d\sqrt{-\sin(x)}+\frac{1}{2}\arctan(\sin(x))\right)}{\sqrt{-\sin(x)}\sqrt{\cos(x)\cot(x)}}$$

$$\Big\downarrow \; 219$$

$$-\frac{2\cos(x)\left(\frac{1}{2}\arctan(\sin(x))-\frac{1}{2}\text{arctanh}(\sin(x))\right)}{\sqrt{-\sin(x)}\sqrt{\cos(x)\cot(x)}}$$

input
```
Int[1/Sqrt[Csc[x] - Sin[x]],x]
```

output
```
(-2*(ArcTan[Sin[x]]/2 - ArcTanh[Sin[x]]/2)*Cos[x])/(Sqrt[Cos[x]*Cot[x]]*Sq
rt[-Sin[x]])
```

3.318.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.318. $\int \frac{1}{\sqrt{\csc(x)-\sin(x)}}\,dx$

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

rule 4900
```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Simp[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p]))   Int[uu*vv^(m*p)*ww^(n*p), x]
, x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !
InertTrigFreeQ[w])
```

3.318. $\int \frac{1}{\sqrt{\csc(x)-\sin(x)}}\,dx$

3.318.4 Maple [F]

$$\int \frac{1}{\sqrt{\csc(x)-\sin(x)}}dx$$

input
```
int(1/(csc(x)-sin(x))^(1/2),x)
```

output
```
int(1/(csc(x)-sin(x))^(1/2),x)
```

3.318.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(48) = 96$.

Time = 0.27 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.07

$$\int \frac{1}{\sqrt{\csc(x)-\sin(x)}}\,dx = \frac{1}{2}\arctan\left(\frac{2\sqrt{\frac{\cos(x)^2}{\sin(x)}}\sin(x)}{\cos(x)\sin(x)-\cos(x)}\right) + \frac{1}{4}\log\left(\frac{\cos(x)^3-5\cos(x)^2-\left(\cos(x)^2+6\cos(x)+4\right)\sin(x)+4\left(\cos(x)^2-(\cos(x)+1)\sin(x)-\right.}{\cos(x)^3+3\cos(x)^2-\left(\cos(x)^2-2\cos(x)-4\right)\sin(x)-2\cos(x)-}\right.$$

input
```
integrate(1/(csc(x)-sin(x))^(1/2),x, algorithm="fricas")
```

output
```
1/2*arctan(2*sqrt(cos(x)^2/sin(x))*sin(x)/(cos(x)*sin(x) - cos(x))) + 1/4*
log((cos(x)^3 - 5*cos(x)^2 - (cos(x)^2 + 6*cos(x) + 4)*sin(x) + 4*(cos(x)^
2 - (cos(x) + 1)*sin(x) - 1)*sqrt(cos(x)^2/sin(x)) - 2*cos(x) + 4)/(cos(x)
^3 + 3*cos(x)^2 - (cos(x)^2 - 2*cos(x) - 4)*sin(x) - 2*cos(x) - 4))
```

3.318.6 Sympy [F]

$$\int \frac{1}{\sqrt{\csc(x)-\sin(x)}}\,dx = \int \frac{1}{\sqrt{-\sin(x)+\csc(x)}}\,dx$$

input
```
integrate(1/(csc(x)-sin(x))**(1/2),x)
```

output
```
Integral(1/sqrt(-sin(x) + csc(x)), x)
```

3.318.7 Maxima [F]

$$\int \frac{1}{\sqrt{\csc(x) - \sin(x)}}\, dx = \int \frac{1}{\sqrt{\csc\left(x\right) - \sin\left(x\right)}}\, dx$$

input
```
integrate(1/(csc(x)-sin(x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(csc(x) - sin(x)), x)
```

3.318.8 Giac [F]

$$\int \frac{1}{\sqrt{\csc(x) - \sin(x)}}\, dx = \int \frac{1}{\sqrt{\csc\left(x\right) - \sin\left(x\right)}}\, dx$$

input
```
integrate(1/(csc(x)-sin(x))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(csc(x) - sin(x)), x)
```

3.318.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\csc(x) - \sin(x)}}\, dx = \int \frac{1}{\sqrt{\frac{1}{\sin(x)} - \sin\left(x\right)}}\, dx$$

input
```
int(1/(1/sin(x) - sin(x))^(1/2),x)
```

output
```
int(1/(1/sin(x) - sin(x))^(1/2), x)
```

3.319 $\int \frac{1}{(\csc(x)-\sin(x))^{3/2}}\,dx$

3.319.1 Optimal result

Integrand size = 11, antiderivative size = 80

$$\int \frac{1}{(\csc(x)-\sin(x))^{3/2}}\,dx = \frac{\sec(x)}{2\sqrt{\cos(x)\cot(x)}} + \frac{\arctan\left(\sqrt{-\sin(x)}\right)\cot(x)\sqrt{-\sin(x)}}{4\sqrt{\cos(x)\cot(x)}} + \frac{\operatorname{arctanh}\left(\sqrt{-\sin(x)}\right)\cot(x)\sqrt{-\sin(x)}}{4\sqrt{\cos(x)\cot(x)}}$$

output
```
1/2*sec(x)/(cos(x)*cot(x))^(1/2)+1/4*arctan((-sin(x))^(1/2))*cot(x)*(-sin(
x))^(1/2)/(cos(x)*cot(x))^(1/2)+1/4*arctanh((-sin(x))^(1/2))*cot(x)*(-sin(
x))^(1/2)/(cos(x)*cot(x))^(1/2)
```

3.319.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.75

$$\int \frac{1}{(\csc(x)-\sin(x))^{3/2}}\,dx = \frac{-\arctan\left(\sqrt[4]{\sin^2(x)}\right)\cos(x)-\operatorname{arctanh}\left(\sqrt[4]{\sin^2(x)}\right)\cos(x)+2\sec(x)\sqrt[4]{\sin^2(x)}}{4\sqrt{\cos(x)\cot(x)}\sqrt[4]{\sin^2(x)}}$$

input
```
Integrate[(Csc[x] - Sin[x])^(-3/2),x]
```

output
```
(-(ArcTan[(Sin[x]^2)^(1/4)]*Cos[x]) - ArcTanh[(Sin[x]^2)^(1/4)]*Cos[x] + 2
*Sec[x]*(Sin[x]^2)^(1/4))/(4*Sqrt[Cos[x]*Cot[x]]*(Sin[x]^2)^(1/4))
```

3.319.3 Rubi [A] (warning: unable to verify)

Time = 0.49 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.98, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.273, Rules used = {3042, 4897, 3042, 4900, 3042, 3077, 3042, 3081, 3042, 3044, 266, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\csc(x)-\sin(x))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(\csc(x)-\sin(x))^{3/2}}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \frac{1}{(\cos(x)\cot(x))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(\cos(x)\cot(x))^{3/2}}dx$$

$$\Big\downarrow \text{4900}$$

$$\frac{\sqrt{\cos(x)}\sqrt{\cot(x)}\int \frac{1}{\cos^{\frac{3}{2}}(x)\cot^{\frac{3}{2}}(x)}dx}{\sqrt{\cos(x)\cot(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(x)}\sqrt{\cot(x)}\int \frac{1}{\sin\left(x+\frac{\pi}{2}\right)^{3/2}\left(-\tan\left(x+\frac{\pi}{2}\right)\right)^{3/2}}dx}{\sqrt{\cos(x)\cot(x)}}$$

$$\Big\downarrow \text{3077}$$

$$\frac{\sqrt{\cos(x)}\sqrt{\cot(x)}\left(\frac{1}{2\cos^{\frac{3}{2}}(x)\sqrt{\cot(x)}}-\frac{1}{4}\int \frac{\sqrt{\cot(x)}}{\cos^{\frac{3}{2}}(x)}dx\right)}{\sqrt{\cos(x)\cot(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(x)}\sqrt{\cot(x)}\left(\frac{1}{2\cos^{\frac{3}{2}}(x)\sqrt{\cot(x)}}-\frac{1}{4}\int \frac{\sqrt{-\tan\left(x+\frac{\pi}{2}\right)}}{\sin\left(x+\frac{\pi}{2}\right)^{3/2}}dx\right)}{\sqrt{\cos(x)\cot(x)}}$$

$$\Big\downarrow \text{3081}$$

$$\frac{\sqrt{\cos(x)}\sqrt{\cot(x)}\left(\frac{1}{2\cos^{\frac{3}{2}}(x)\sqrt{\cot(x)}}-\frac{\sqrt{-\sin(x)}\sqrt{\cot(x)}\int\frac{\sec(x)}{\sqrt{-\sin(x)}}dx}{4\sqrt{\cos(x)}}\right)}{\sqrt{\cos(x)\cot(x)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt{\cos(x)}\sqrt{\cot(x)}\left(\frac{1}{2\cos^{\frac{3}{2}}(x)\sqrt{\cot(x)}}-\frac{\sqrt{-\sin(x)}\sqrt{\cot(x)}\int\frac{1}{\cos(x)\sqrt{-\sin(x)}}dx}{4\sqrt{\cos(x)}}\right)}{\sqrt{\cos(x)\cot(x)}}$$

$$\Big\downarrow \; 3044$$

$$\frac{\sqrt{\cos(x)}\sqrt{\cot(x)}\left(\frac{\sqrt{-\sin(x)}\sqrt{\cot(x)}\int\frac{1}{\sqrt{-\sin(x)}\left(1-\sin^2(x)\right)}d(-\sin(x))}{4\sqrt{\cos(x)}}+\frac{1}{2\cos^{\frac{3}{2}}(x)\sqrt{\cot(x)}}\right)}{\sqrt{\cos(x)\cot(x)}}$$

$$\Big\downarrow \; 266$$

$$\frac{\sqrt{\cos(x)}\sqrt{\cot(x)}\left(\frac{\sqrt{-\sin(x)}\sqrt{\cot(x)}\int\frac{1}{1-\sin^4(x)}d\sqrt{-\sin(x)}}{2\sqrt{\cos(x)}}+\frac{1}{2\cos^{\frac{3}{2}}(x)\sqrt{\cot(x)}}\right)}{\sqrt{\cos(x)\cot(x)}}$$

$$\Big\downarrow \; 756$$

$$\frac{\sqrt{\cos(x)}\sqrt{\cot(x)}\left(\frac{\sqrt{-\sin(x)}\sqrt{\cot(x)}\left(\frac{1}{2}\int\frac{1}{1-\sin^2(x)}d\sqrt{-\sin(x)}+\frac{1}{2}\int\frac{1}{\sin^2(x)+1}d\sqrt{-\sin(x)}\right)}{2\sqrt{\cos(x)}}+\frac{1}{2\cos^{\frac{3}{2}}(x)\sqrt{\cot(x)}}\right)}{\sqrt{\cos(x)\cot(x)}}$$

$$\Big\downarrow \; 216$$

$$\frac{\sqrt{\cos(x)}\sqrt{\cot(x)}\left(\frac{\sqrt{-\sin(x)}\sqrt{\cot(x)}\left(\frac{1}{2}\int\frac{1}{1-\sin^2(x)}d\sqrt{-\sin(x)}-\frac{1}{2}\arctan(\sin(x))\right)}{2\sqrt{\cos(x)}}+\frac{1}{2\cos^{\frac{3}{2}}(x)\sqrt{\cot(x)}}\right)}{\sqrt{\cos(x)\cot(x)}}$$

$$\Big\downarrow \; 219$$

$$\frac{\sqrt{\cos(x)}\sqrt{\cot(x)}\left(\frac{\sqrt{-\sin(x)}\sqrt{\cot(x)}\left(-\frac{1}{2}\arctan(\sin(x))-\frac{1}{2}\text{arctanh}(\sin(x))\right)}{2\sqrt{\cos(x)}}+\frac{1}{2\cos^{\frac{3}{2}}(x)\sqrt{\cot(x)}}\right)}{\sqrt{\cos(x)\cot(x)}}$$

input
```
Int[(Csc[x] - Sin[x])^(-3/2),x]
```

3.319. $\int \frac{1}{(\csc(x)-\sin(x))^{3/2}}\,dx$

output
```
(Sqrt[Cos[x]]*Sqrt[Cot[x]]*(1/(2*Cos[x]^(3/2)*Sqrt[Cot[x]]) + ((-1/2*ArcTa
n[Sin[x]] - ArcTanh[Sin[x]]/2)*Sqrt[Cot[x]]*Sqrt[-Sin[x]])/(2*Sqrt[Cos[x]]
)))/Sqrt[Cos[x]*Cot[x]]
```

3.319.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

3.319. $\int \frac{1}{(\csc(x)-\sin(x))^{3/2}} dx$

rule 3077
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(m
 + n + 1))), x] - Simp[(n + 1)/(b^2*(m + n + 1))   Int[(a*Sin[e + f*x])^m*(
b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1]
&& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1]
)
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

rule 4900
```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Simp[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p]))   Int[uu*vv^(m*p)*ww^(n*p), x]
, x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !
InertTrigFreeQ[w])
```

3.319.4 Maple [A] (verified)

Time = 15.63 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.24

method	result
default	$\frac{\cos(x)\arctan\left(\sqrt{\frac{\sin(x)}{(\cos(x)+1)^2}}\,(\cot(x)+\csc(x))\right)-\cos(x)\,\mathrm{arctanh}\left(\sqrt{\frac{\sin(x)}{(\cos(x)+1)^2}}\,(\cot(x)+\csc(x))\right)+2\sqrt{\frac{\sin(x)}{(\cos(x)+1)^2}}+2\sec(x)\sqrt{\frac{\sin}{(\cos(x}}}{4(\cos(x)+1)\sqrt{\frac{\sin(x)}{(\cos(x)+1)^2}}\,\sqrt{\cot(x)\cos(x)}}$

input
```
int(1/(csc(x)-sin(x))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/(cos(x)+1)/(sin(x)/(cos(x)+1)^2)^(1/2)/(cot(x)*cos(x))^(1/2)*(cos(x)*a
rctan((sin(x)/(cos(x)+1)^2)^(1/2)*(cot(x)+csc(x)))-cos(x)*arctanh((sin(x)/
(cos(x)+1)^2)^(1/2)*(cot(x)+csc(x)))+2*(sin(x)/(cos(x)+1)^2)^(1/2)+2*sec(x
)*(sin(x)/(cos(x)+1)^2)^(1/2))
```

3.319.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. 2(60) = 120.

Time = 0.29 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.90

$$\int \frac{1}{(\csc(x)-\sin(x))^{3/2}}\,dx = \frac{2\arctan\left(\frac{2\sqrt{\frac{\cos(x)^2}{\sin(x)}}\sin(x)}{\cos(x)\sin(x)-\cos(x)}\right)\cos(x)^3+\cos(x)^3\log\left(\frac{\cos(x)^3-5\cos(x)^2-\left(\cos(x)^2+6\,\mathrm{c}\right.}{\cos(x)^3+3\,}\right.}{16\cos}$$

input
```
integrate(1/(csc(x)-sin(x))^(3/2),x, algorithm="fricas")
```

output
```
1/16*(2*arctan(2*sqrt(cos(x)^2/sin(x))*sin(x)/(cos(x)*sin(x) - cos(x)))*co
s(x)^3 + cos(x)^3*log((cos(x)^3 - 5*cos(x)^2 - (cos(x)^2 + 6*cos(x) + 4)*s
in(x) - 4*(cos(x)^2 - (cos(x) + 1)*sin(x) - 1)*sqrt(cos(x)^2/sin(x)) - 2*c
os(x) + 4)/(cos(x)^3 + 3*cos(x)^2 - (cos(x)^2 - 2*cos(x) - 4)*sin(x) - 2*c
os(x) - 4)) + 8*sqrt(cos(x)^2/sin(x))*sin(x))/cos(x)^3
```

3.319.6 Sympy [F]

$$\int \frac{1}{(\csc(x)-\sin(x))^{3/2}}\,dx = \int \frac{1}{(-\sin(x)+\csc(x))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(csc(x)-sin(x))**(3/2),x)
```

output
```
Integral((-sin(x) + csc(x))**(-3/2), x)
```

3.319.7 Maxima [F]

$$\int \frac{1}{(\csc(x)-\sin(x))^{3/2}}\,dx = \int \frac{1}{(\csc(x)-\sin(x))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(csc(x)-sin(x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((csc(x) - sin(x))^(-3/2), x)
```

3.319.8 Giac [F]

$$\int \frac{1}{(\csc(x)-\sin(x))^{3/2}}\,dx = \int \frac{1}{(\csc{(x)}-\sin{(x)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(csc(x)-sin(x))^(3/2),x, algorithm="giac")
```

output
```
integrate((csc(x) - sin(x))^(-3/2), x)
```

3.319.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(\csc(x)-\sin(x))^{3/2}}\,dx = \int \frac{1}{\left(\frac{1}{\sin(x)}-\sin{(x)}\right)^{3/2}}\,dx$$

input
```
int(1/(1/sin(x) - sin(x))^(3/2),x)
```

output
```
int(1/(1/sin(x) - sin(x))^(3/2), x)
```

3.320 $\int \frac{1}{(\csc(x)-\sin(x))^{5/2}}\,dx$

3.320.1 Optimal result

Integrand size = 11, antiderivative size = 99

$$\int \frac{1}{(\csc(x)-\sin(x))^{5/2}}\,dx = -\frac{3\arctan\left(\sqrt{-\sin(x)}\right)\cos(x)}{32\sqrt{\cos(x)\cot(x)}\sqrt{-\sin(x)}} + \frac{3\text{arctanh}\left(\sqrt{-\sin(x)}\right)\cos(x)}{32\sqrt{\cos(x)\cot(x)}\sqrt{-\sin(x)}} - \frac{3\tan(x)}{16\sqrt{\cos(x)\cot(x)}} + \frac{\sec^2(x)\tan(x)}{4\sqrt{\cos(x)\cot(x)}}$$

output
```
-3/32*arctan((-sin(x))^(1/2))*cos(x)/(cos(x)*cot(x))^(1/2)/(-sin(x))^(1/2)
+3/32*arctanh((-sin(x))^(1/2))*cos(x)/(cos(x)*cot(x))^(1/2)/(-sin(x))^(1/2
)-3/16*tan(x)/(cos(x)*cot(x))^(1/2)+1/4*sec(x)^2*tan(x)/(cos(x)*cot(x))^(1
/2)
```

3.320.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.70

$$\int \frac{1}{(\csc(x)-\sin(x))^{5/2}}\,dx = -\frac{\sqrt{\cos(x)\cot(x)}\sin(x)\left(-3\arctan\left(\sqrt[4]{\sin^2(x)}\right)+3\text{arctanh}\left(\sqrt[4]{\sin^2(x)}\right)+(-5+3\cos(2x))\sec^4(x)\sin^2(\right.}{32\sin^2(x)^{3/4}}$$

input
```
Integrate[(Csc[x] - Sin[x])^(-5/2),x]
```

output
```
-1/32*(Sqrt[Cos[x]*Cot[x]]*Sin[x]*(-3*ArcTan[(Sin[x]^2)^(1/4)] + 3*ArcTanh
[(Sin[x]^2)^(1/4)] + (-5 + 3*Cos[2*x])*Sec[x]^4*(Sin[x]^2)^(3/4))*Tan[x])/
(Sin[x]^2)^(3/4)
```

3.320.3 Rubi [A] (warning: unable to verify)

Time = 0.58 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.455$, Rules used = {3042, 4897, 3042, 4900, 3042, 3077, 3042, 3079, 3042, 3081, 3042, 3044, 266, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\csc(x)-\sin(x))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(\csc(x)-\sin(x))^{5/2}}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \frac{1}{(\cos(x)\cot(x))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(\cos(x)\cot(x))^{5/2}}dx$$

$$\Big\downarrow \text{4900}$$

$$\frac{\sqrt{\cos(x)}\sqrt{\cot(x)}\int \frac{1}{\cos^{\frac{5}{2}}(x)\cot^{\frac{5}{2}}(x)}dx}{\sqrt{\cos(x)\cot(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(x)}\sqrt{\cot(x)}\int \frac{1}{\sin\left(x+\frac{\pi}{2}\right)^{5/2}\left(-\tan\left(x+\frac{\pi}{2}\right)\right)^{5/2}}dx}{\sqrt{\cos(x)\cot(x)}}$$

$$\Big\downarrow \text{3077}$$

$$\frac{\sqrt{\cos(x)}\sqrt{\cot(x)}\left(\frac{1}{4\cos^{\frac{5}{2}}(x)\cot^{\frac{3}{2}}(x)}-\frac{3}{8}\int\frac{1}{\cos^{\frac{5}{2}}(x)\sqrt{\cot(x)}}dx\right)}{\sqrt{\cos(x)\cot(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(x)}\sqrt{\cot(x)}\left(\frac{1}{4\cos^{\frac{5}{2}}(x)\cot^{\frac{3}{2}}(x)}-\frac{3}{8}\int\frac{1}{\sin\left(x+\frac{\pi}{2}\right)^{5/2}\sqrt{-\tan\left(x+\frac{\pi}{2}\right)}}dx\right)}{\sqrt{\cos(x)\cot(x)}}$$

$$\Big\downarrow \text{3079}$$

$$\frac{\sqrt{\cos(x)}\sqrt{\cot(x)}\left(\frac{1}{4\cos^{\frac{5}{2}}(x)\cot^{\frac{3}{2}}(x)}-\frac{3}{8}\left(\frac{1}{4}\int\frac{1}{\sqrt{\cos(x)}\sqrt{\cot(x)}}dx+\frac{1}{2\sqrt{\cos(x)}\cot^{\frac{3}{2}}(x)}\right)\right)}{\sqrt{\cos(x)\cot(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(x)}\sqrt{\cot(x)}\left(\frac{1}{4\cos^{\frac{5}{2}}(x)\cot^{\frac{3}{2}}(x)}-\frac{3}{8}\left(\frac{1}{4}\int\frac{1}{\sqrt{\sin\left(x+\frac{\pi}{2}\right)}\sqrt{-\tan\left(x+\frac{\pi}{2}\right)}}dx+\frac{1}{2\sqrt{\cos(x)}\cot^{\frac{3}{2}}(x)}\right)\right)}{\sqrt{\cos(x)\cot(x)}}$$

$$\Big\downarrow \text{3081}$$

$$\frac{\sqrt{\cos(x)}\sqrt{\cot(x)}\left(\frac{1}{4\cos^{\frac{5}{2}}(x)\cot^{\frac{3}{2}}(x)}-\frac{3}{8}\left(\frac{\sqrt{\cos(x)}\int\sec(x)\sqrt{-\sin(x)}dx}{4\sqrt{-\sin(x)}\sqrt{\cot(x)}}+\frac{1}{2\sqrt{\cos(x)}\cot^{\frac{3}{2}}(x)}\right)\right)}{\sqrt{\cos(x)\cot(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(x)}\sqrt{\cot(x)}\left(\frac{1}{4\cos^{\frac{5}{2}}(x)\cot^{\frac{3}{2}}(x)}-\frac{3}{8}\left(\frac{\sqrt{\cos(x)}\int\frac{\sqrt{-\sin(x)}}{\cos(x)}dx}{4\sqrt{-\sin(x)}\sqrt{\cot(x)}}+\frac{1}{2\sqrt{\cos(x)}\cot^{\frac{3}{2}}(x)}\right)\right)}{\sqrt{\cos(x)\cot(x)}}$$

$$\Big\downarrow \text{3044}$$

$$\frac{\sqrt{\cos(x)}\sqrt{\cot(x)}\left(\frac{1}{4\cos^{\frac{5}{2}}(x)\cot^{\frac{3}{2}}(x)}-\frac{3}{8}\left(\frac{1}{2\sqrt{\cos(x)}\cot^{\frac{3}{2}}(x)}-\frac{\sqrt{\cos(x)}\int\frac{\sqrt{-\sin(x)}}{1-\sin^2(x)}d(-\sin(x))}{4\sqrt{-\sin(x)}\sqrt{\cot(x)}}\right)\right)}{\sqrt{\cos(x)\cot(x)}}$$

$$\Big\downarrow \text{266}$$

$$\frac{\sqrt{\cos(x)}\sqrt{\cot(x)}\left(\frac{1}{4\cos^{\frac{5}{2}}(x)\cot^{\frac{3}{2}}(x)}-\frac{3}{8}\left(\frac{1}{2\sqrt{\cos(x)}\cot^{\frac{3}{2}}(x)}-\frac{\sqrt{\cos(x)}\int\frac{\sin^2(x)}{1-\sin^4(x)}d\sqrt{-\sin(x)}}{2\sqrt{-\sin(x)}\sqrt{\cot(x)}}\right)\right)}{\sqrt{\cos(x)\cot(x)}}$$

$$\Big\downarrow \text{827}$$

3.320. $\int\frac{1}{(\csc(x)-\sin(x))^{5/2}}\,dx$

$$\frac{\sqrt{\cos(x)}\sqrt{\cot(x)}\left(\frac{1}{4\cos^{\frac{5}{2}}(x)\cot^{\frac{3}{2}}(x)}-\frac{3}{8}\left(\frac{1}{2\sqrt{\cos(x)}\cot^{\frac{3}{2}}(x)}-\frac{\sqrt{\cos(x)}\left(\frac{1}{2}\int\frac{1}{1-\sin^2(x)}d\sqrt{-\sin(x)}-\frac{1}{2}\int\frac{1}{\sin^2(x)+1}d\sqrt{-\sin(x)}\right)}{2\sqrt{-\sin(x)}\sqrt{\cot(x)}}\right)\right)}{\sqrt{\cos(x)\cot(x)}}$$

$$\Big\downarrow \textbf{216}$$

$$\frac{\sqrt{\cos(x)}\sqrt{\cot(x)}\left(\frac{1}{4\cos^{\frac{5}{2}}(x)\cot^{\frac{3}{2}}(x)}-\frac{3}{8}\left(\frac{1}{2\sqrt{\cos(x)}\cot^{\frac{3}{2}}(x)}-\frac{\sqrt{\cos(x)}\left(\frac{1}{2}\int\frac{1}{1-\sin^2(x)}d\sqrt{-\sin(x)}+\frac{1}{2}\arctan(\sin(x))\right)}{2\sqrt{-\sin(x)}\sqrt{\cot(x)}}\right)\right)}{\sqrt{\cos(x)\cot(x)}}$$

$$\Big\downarrow \textbf{219}$$

$$\frac{\sqrt{\cos(x)}\sqrt{\cot(x)}\left(\frac{1}{4\cos^{\frac{5}{2}}(x)\cot^{\frac{3}{2}}(x)}-\frac{3}{8}\left(\frac{1}{2\sqrt{\cos(x)}\cot^{\frac{3}{2}}(x)}-\frac{\sqrt{\cos(x)}\left(\frac{1}{2}\arctan(\sin(x))-\frac{1}{2}\text{arctanh}(\sin(x))\right)}{2\sqrt{-\sin(x)}\sqrt{\cot(x)}}\right)\right)}{\sqrt{\cos(x)\cot(x)}}$$

input
```
Int[(Csc[x] - Sin[x])^(-5/2),x]
```

output
```
(Sqrt[Cos[x]]*Sqrt[Cot[x]]*(1/(4*Cos[x]^(5/2)*Cot[x]^(3/2)) - (3*(1/(2*Sqr
t[Cos[x]]*Cot[x]^(3/2)) - ((ArcTan[Sin[x]]/2 - ArcTanh[Sin[x]]/2)*Sqrt[Cos
[x]])/(2*Sqrt[Cot[x]]*Sqrt[-Sin[x]])))/8))/Sqrt[Cos[x]*Cot[x]]
```

3.320.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3077
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(m
 + n + 1))), x] - Simp[(n + 1)/(b^2*(m + n + 1))   Int[(a*Sin[e + f*x])^m*(
b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1]
&& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1]
)
```

rule 3079
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)
/(a^2*f*(m + n + 1))), x] + Simp[(m + 2)/(a^2*(m + n + 1))   Int[(a*Sin[e +
 f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && L
tQ[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.320. $\int \frac{1}{(\csc(x)-\sin(x))^{5/2}} dx$

rule 4900
```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Simp[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p]))   Int[uu*vv^(m*p)*ww^(n*p), x]
, x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !
InertTrigFreeQ[w])
```

3.320.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(75) = 150$.

Time = 16.58 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.60

method	result
default	$\frac{3\cos(x)\sin(x)^2\operatorname{arctanh}\left(\sqrt{\frac{\sin(x)}{(\cos(x)+1)^2}}\,(\cot(x)+\csc(x))\right)+3\cos(x)\sin(x)^2\arctan\left(\sqrt{\frac{\sin(x)}{(\cos(x)+1)^2}}\,(\cot(x)+\csc(x))\right)+6\sqrt{\frac{\sin(x)}{(\cos(x)+1)^2}}}{32(\cos(x)-1)(\cos(x)+1)^2\sqrt{\frac{\sin(x)}{(\cos(x)+1)^2}}\,\sqrt{\cot}}$

input
```
int(1/(csc(x)-sin(x))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/32/(cos(x)-1)/(cos(x)+1)^2/(sin(x)/(cos(x)+1)^2)^(1/2)/(cot(x)*cos(x))^(
1/2)*(3*cos(x)*sin(x)^2*arctanh((sin(x)/(cos(x)+1)^2)^(1/2)*(cot(x)+csc(x)
))+3*cos(x)*sin(x)^2*arctan((sin(x)/(cos(x)+1)^2)^(1/2)*(cot(x)+csc(x)))+6
*(sin(x)/(cos(x)+1)^2)^(1/2)*sin(x)^3+6*sin(x)^2*tan(x)*(sin(x)/(cos(x)+1)
^2)^(1/2)-8*sin(x)*tan(x)^2*(sin(x)/(cos(x)+1)^2)^(1/2)-8*tan(x)^3*(sin(x)
/(cos(x)+1)^2)^(1/2))
```

3.320.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(75) = 150$.

Time = 0.29 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.67

$$\int \frac{1}{(\csc(x)-\sin(x))^{5/2}}\,dx = -\frac{6\arctan\left(\frac{2\sqrt{\frac{\cos(x)^2}{\sin(x)}}\sin(x)}{\cos(x)\sin(x)-\cos(x)}\right)\cos(x)^5-3\cos(x)^5\log\left(\frac{\cos(x)^3-5\cos(x)^2-\left(\cos(x)^2+6\cos(x)+4\right)\sin(x)-4\left(\cos(x)^2-(\cos\right.}{\cos(x)^3+3\cos(x)^2-\left(\cos(x)^2-2\cos(x)-4\right)\sin}\right.}{128\cos(x)^5}$$

input
```
integrate(1/(csc(x)-sin(x))^(5/2),x, algorithm="fricas")
```

output
```
-1/128*(6*arctan(2*sqrt(cos(x)^2/sin(x))*sin(x)/(cos(x)*sin(x) - cos(x)))*
cos(x)^5 - 3*cos(x)^5*log((cos(x)^3 - 5*cos(x)^2 - (cos(x)^2 + 6*cos(x) +
4)*sin(x) - 4*(cos(x)^2 - (cos(x) + 1)*sin(x) - 1)*sqrt(cos(x)^2/sin(x)) -
 2*cos(x) + 4)/(cos(x)^3 + 3*cos(x)^2 - (cos(x)^2 - 2*cos(x) - 4)*sin(x) -
 2*cos(x) - 4)) - 8*(3*cos(x)^4 - 7*cos(x)^2 + 4)*sqrt(cos(x)^2/sin(x)))/c
os(x)^5
```

3.320.6 Sympy [F]

$$\int \frac{1}{(\csc(x)-\sin(x))^{5/2}}\,dx = \int \frac{1}{\left(-\sin\left(x\right)+\csc\left(x\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(csc(x)-sin(x))**(5/2),x)
```

output
```
Integral((-sin(x) + csc(x))**(-5/2), x)
```

3.320.7 Maxima [F]

$$\int \frac{1}{(\csc(x)-\sin(x))^{5/2}}\,dx = \int \frac{1}{\left(\csc\left(x\right)-\sin\left(x\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(csc(x)-sin(x))^(5/2),x, algorithm="maxima")
```

output
```
integrate((csc(x) - sin(x))^(-5/2), x)
```

3.320.8 Giac [F]

$$\int \frac{1}{(\csc(x)-\sin(x))^{5/2}}\,dx = \int \frac{1}{\left(\csc\left(x\right)-\sin\left(x\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(csc(x)-sin(x))^(5/2),x, algorithm="giac")
```

output
```
integrate((csc(x) - sin(x))^(-5/2), x)
```

3.320.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(\csc(x) - \sin(x))^{5/2}}\,dx = \int \frac{1}{\left(\frac{1}{\sin(x)} - \sin(x)\right)^{5/2}}\,dx$$

input
```
int(1/(1/sin(x) - sin(x))^(5/2),x)
```

output
```
int(1/(1/sin(x) - sin(x))^(5/2), x)
```

3.321 $\int \frac{1}{(\csc(x)-\sin(x))^{7/2}}\,dx$

3.321.1 Optimal result

Integrand size = 11, antiderivative size = 118

$$\int \frac{1}{(\csc(x)-\sin(x))^{7/2}}\,dx = \frac{5\sec(x)}{192\sqrt{\cos(x)\cot(x)}} - \frac{5\sec^3(x)}{48\sqrt{\cos(x)\cot(x)}} - \frac{5\arctan\left(\sqrt{-\sin(x)}\right)\cot(x)\sqrt{-\sin(x)}}{128\sqrt{\cos(x)\cot(x)}} - \frac{5\text{arctanh}\left(\sqrt{-\sin(x)}\right)\cot(x)\sqrt{-\sin(x)}}{128\sqrt{\cos(x)\cot(x)}} + \frac{\sec^3(x)\tan^2(x)}{6\sqrt{\cos(x)\cot(x)}}$$

output
```
5/192*sec(x)/(cos(x)*cot(x))^(1/2)-5/48*sec(x)^3/(cos(x)*cot(x))^(1/2)-5/1
28*arctan((-sin(x))^(1/2))*cot(x)*(-sin(x))^(1/2)/(cos(x)*cot(x))^(1/2)-5/
128*arctanh((-sin(x))^(1/2))*cot(x)*(-sin(x))^(1/2)/(cos(x)*cot(x))^(1/2)+
1/6*sec(x)^3*tan(x)^2/(cos(x)*cot(x))^(1/2)
```

3.321.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.63

$$\int \frac{1}{(\csc(x)-\sin(x))^{7/2}}\,dx = \frac{15\arctan\left(\sqrt[4]{\sin^2(x)}\right)\cos(x) + 15\text{arctanh}\left(\sqrt[4]{\sin^2(x)}\right)\cos(x) + 2\sec(x)\,(5-}{384\sqrt{\cos(x)\cot(x)}\sqrt[4]{\sin^2(x)}}$$

input
```
Integrate[(Csc[x] - Sin[x])^(-7/2),x]
```

output
```
(15*ArcTan[(Sin[x]^2)^(1/4)]*Cos[x] + 15*ArcTanh[(Sin[x]^2)^(1/4)]*Cos[x]
+ 2*Sec[x]*(5 - 52*Sec[x]^2 + 32*Sec[x]^4)*(Sin[x]^2)^(1/4))/(384*Sqrt[Cos
[x]*Cot[x]]*(Sin[x]^2)^(1/4))
```

3.321.3 Rubi [A] (warning: unable to verify)

Time = 0.68 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.02, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 1.636$, Rules used = {3042, 4897, 3042, 4900, 3042, 3077, 3042, 3077, 3042, 3079, 3042, 3081, 3042, 3044, 266, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\csc(x)-\sin(x))^{7/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{(\csc(x)-\sin(x))^{7/2}}dx$$

$$\Big\downarrow \; 4897$$

$$\int \frac{1}{(\cos(x)\cot(x))^{7/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{(\cos(x)\cot(x))^{7/2}}dx$$

$$\Big\downarrow \; 4900$$

$$\frac{\sqrt{\cos(x)}\sqrt{\cot(x)}\int \frac{1}{\cos^{\frac{7}{2}}(x)\cot^{\frac{7}{2}}(x)}dx}{\sqrt{\cos(x)\cot(x)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt{\cos(x)}\sqrt{\cot(x)}\int \frac{1}{\sin\left(x+\frac{\pi}{2}\right)^{7/2}\left(-\tan\left(x+\frac{\pi}{2}\right)\right)^{7/2}}dx}{\sqrt{\cos(x)\cot(x)}}$$

$$\Big\downarrow \; 3077$$

$$\frac{\sqrt{\cos(x)}\sqrt{\cot(x)}\left(\frac{1}{6\cos^{\frac{7}{2}}(x)\cot^{\frac{5}{2}}(x)}-\frac{5}{12}\int\frac{1}{\cos^{\frac{7}{2}}(x)\cot^{\frac{3}{2}}(x)}dx\right)}{\sqrt{\cos(x)\cot(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(x)}\sqrt{\cot(x)}\left(\frac{1}{6\cos^{\frac{7}{2}}(x)\cot^{\frac{5}{2}}(x)}-\frac{5}{12}\int\frac{1}{\sin\left(x+\frac{\pi}{2}\right)^{7/2}\left(-\tan\left(x+\frac{\pi}{2}\right)\right)^{3/2}}dx\right)}{\sqrt{\cos(x)\cot(x)}}$$

$$\Big\downarrow \text{3077}$$

$$\frac{\sqrt{\cos(x)}\sqrt{\cot(x)}\left(\frac{1}{6\cos^{\frac{7}{2}}(x)\cot^{\frac{5}{2}}(x)}-\frac{5}{12}\left(\frac{1}{4\cos^{\frac{7}{2}}(x)\sqrt{\cot(x)}}-\frac{1}{8}\int\frac{\sqrt{\cot(x)}}{\cos^{\frac{7}{2}}(x)}dx\right)\right)}{\sqrt{\cos(x)\cot(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(x)}\sqrt{\cot(x)}\left(\frac{1}{6\cos^{\frac{7}{2}}(x)\cot^{\frac{5}{2}}(x)}-\frac{5}{12}\left(\frac{1}{4\cos^{\frac{7}{2}}(x)\sqrt{\cot(x)}}-\frac{1}{8}\int\frac{\sqrt{-\tan\left(x+\frac{\pi}{2}\right)}}{\sin\left(x+\frac{\pi}{2}\right)^{7/2}}dx\right)\right)}{\sqrt{\cos(x)\cot(x)}}$$

$$\Big\downarrow \text{3079}$$

$$\frac{\sqrt{\cos(x)}\sqrt{\cot(x)}\left(\frac{1}{6\cos^{\frac{7}{2}}(x)\cot^{\frac{5}{2}}(x)}-\frac{5}{12}\left(\frac{1}{8}\left(-\frac{3}{4}\int\frac{\sqrt{\cot(x)}}{\cos^{\frac{3}{2}}(x)}dx-\frac{1}{2\cos^{\frac{3}{2}}(x)\sqrt{\cot(x)}}\right)+\frac{1}{4\cos^{\frac{7}{2}}(x)\sqrt{\cot(x)}}\right)\right)}{\sqrt{\cos(x)\cot(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(x)}\sqrt{\cot(x)}\left(\frac{1}{6\cos^{\frac{7}{2}}(x)\cot^{\frac{5}{2}}(x)}-\frac{5}{12}\left(\frac{1}{8}\left(-\frac{3}{4}\int\frac{\sqrt{-\tan\left(x+\frac{\pi}{2}\right)}}{\sin\left(x+\frac{\pi}{2}\right)^{3/2}}dx-\frac{1}{2\cos^{\frac{3}{2}}(x)\sqrt{\cot(x)}}\right)+\frac{1}{4\cos^{\frac{7}{2}}(x)\sqrt{\cot(x)}}\right)\right)}{\sqrt{\cos(x)\cot(x)}}$$

$$\Big\downarrow \text{3081}$$

$$\frac{\sqrt{\cos(x)}\sqrt{\cot(x)}\left(\frac{1}{6\cos^{\frac{7}{2}}(x)\cot^{\frac{5}{2}}(x)}-\frac{5}{12}\left(\frac{1}{8}\left(-\frac{3\sqrt{-\sin(x)}\sqrt{\cot(x)}\int\frac{\sec(x)}{\sqrt{-\sin(x)}}dx}{4\sqrt{\cos(x)}}-\frac{1}{2\cos^{\frac{3}{2}}(x)\sqrt{\cot(x)}}\right)+\frac{1}{4\cos^{\frac{7}{2}}(x)\sqrt{\cot(x)}}\right)\right)}{\sqrt{\cos(x)\cot(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(x)}\sqrt{\cot(x)}\left(\frac{1}{6\cos^{\frac{7}{2}}(x)\cot^{\frac{5}{2}}(x)}-\frac{5}{12}\left(\frac{1}{8}\left(-\frac{3\sqrt{-\sin(x)}\sqrt{\cot(x)}\int\frac{1}{\cos(x)\sqrt{-\sin(x)}}dx}{4\sqrt{\cos(x)}}-\frac{1}{2\cos^{\frac{3}{2}}(x)\sqrt{\cot(x)}}\right)+\frac{1}{4\cos^{\frac{7}{2}}(x)\sqrt{\cot(}}\right.\right.}{\sqrt{\cos(x)\cot(x)}}$$

3.321. $\int\frac{1}{(\csc(x)-\sin(x))^{7/2}}\,dx$

$\Big\downarrow$ 3044

$$\frac{\sqrt{\cos(x)}\sqrt{\cot(x)}\left(\frac{1}{6\cos^{\frac{7}{2}}(x)\cot^{\frac{5}{2}}(x)}-\frac{5}{12}\left(\frac{1}{8}\left(\frac{3\sqrt{-\sin(x)}\sqrt{\cot(x)}\int\frac{1}{\sqrt{-\sin(x)}\left(1-\sin^2(x)\right)}d(-\sin(x))}{4\sqrt{\cos(x)}}-\frac{1}{2\cos^{\frac{3}{2}}(x)\sqrt{\cot(x)}}\right)+\frac{1}{4\cos\right.\right.}{\sqrt{\cos(x)\cot(x)}}$$

$\Big\downarrow$ 266

$$\frac{\sqrt{\cos(x)}\sqrt{\cot(x)}\left(\frac{1}{6\cos^{\frac{7}{2}}(x)\cot^{\frac{5}{2}}(x)}-\frac{5}{12}\left(\frac{1}{8}\left(\frac{3\sqrt{-\sin(x)}\sqrt{\cot(x)}\int\frac{1}{1-\sin^4(x)}d\sqrt{-\sin(x)}}{2\sqrt{\cos(x)}}-\frac{1}{2\cos^{\frac{3}{2}}(x)\sqrt{\cot(x)}}\right)+\frac{1}{4\cos^{\frac{7}{2}}(x)\sqrt{\cot(\right.\right.}{\sqrt{\cos(x)\cot(x)}}$$

$\Big\downarrow$ 756

$$\frac{\sqrt{\cos(x)}\sqrt{\cot(x)}\left(\frac{1}{6\cos^{\frac{7}{2}}(x)\cot^{\frac{5}{2}}(x)}-\frac{5}{12}\left(\frac{1}{8}\left(\frac{3\sqrt{-\sin(x)}\sqrt{\cot(x)}\left(\frac{1}{2}\int\frac{1}{1-\sin^2(x)}d\sqrt{-\sin(x)}+\frac{1}{2}\int\frac{1}{\sin^2(x)+1}d\sqrt{-\sin(x)}\right)}{2\sqrt{\cos(x)}}-\frac{}{2\cos^{\frac{3}{2}}(\right.\right.\right.}{\sqrt{\cos(x)\cot(x)}}$$

$\Big\downarrow$ 216

$$\frac{\sqrt{\cos(x)}\sqrt{\cot(x)}\left(\frac{1}{6\cos^{\frac{7}{2}}(x)\cot^{\frac{5}{2}}(x)}-\frac{5}{12}\left(\frac{1}{8}\left(\frac{3\sqrt{-\sin(x)}\sqrt{\cot(x)}\left(\frac{1}{2}\int\frac{1}{1-\sin^2(x)}d\sqrt{-\sin(x)}-\frac{1}{2}\arctan(\sin(x))\right)}{2\sqrt{\cos(x)}}-\frac{1}{2\cos^{\frac{3}{2}}(x)\sqrt{\cot(\right.\right.\right.}{\sqrt{\cos(x)\cot(x)}}$$

$\Big\downarrow$ 219

$$\frac{\sqrt{\cos(x)}\sqrt{\cot(x)}\left(\frac{1}{6\cos^{\frac{7}{2}}(x)\cot^{\frac{5}{2}}(x)}-\frac{5}{12}\left(\frac{1}{8}\left(\frac{3\sqrt{-\sin(x)}\sqrt{\cot(x)}\left(-\frac{1}{2}\arctan(\sin(x))-\frac{1}{2}\text{arctanh}(\sin(x))\right)}{2\sqrt{\cos(x)}}-\frac{1}{2\cos^{\frac{3}{2}}(x)\sqrt{\cot(x)}}\right)\right.\right.}{\sqrt{\cos(x)\cot(x)}}$$

input
```
Int[(Csc[x] - Sin[x])^(-7/2),x]
```

output
```
(Sqrt[Cos[x]]*Sqrt[Cot[x]]*(1/(6*Cos[x]^(7/2)*Cot[x]^(5/2)) - (5*(1/(4*Cos
[x]^(7/2)*Sqrt[Cot[x]]) + (-1/2*1/(Cos[x]^(3/2)*Sqrt[Cot[x]]) + (3*(-1/2*A
rcTan[Sin[x]] - ArcTanh[Sin[x]]/2)*Sqrt[Cot[x]]*Sqrt[-Sin[x]])/(2*Sqrt[Cos
[x]]))/8))/12))/Sqrt[Cos[x]*Cot[x]]
```

3.321. $\int\frac{1}{(\csc(x)-\sin(x))^{7/2}}\,dx$

3.321.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3077
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(m
 + n + 1))), x] - Simp[(n + 1)/(b^2*(m + n + 1))   Int[(a*Sin[e + f*x])^m*(
b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1]
&& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1]
)
```

3.321. $\int \frac{1}{(\csc(x)-\sin(x))^{7/2}}\,dx$

rule 3079
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)
/(a^2*f*(m + n + 1))), x] + Simp[(m + 2)/(a^2*(m + n + 1))   Int[(a*Sin[e +
 f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && L
tQ[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

rule 4900
```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Simp[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p]))   Int[uu*vv^(m*p)*ww^(n*p), x]
, x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !
InertTrigFreeQ[w])
```

3.321.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(90) = 180$.

Time = 16.42 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.58

method	result
default	$\frac{\tan(x)^2\sec(x)^3\left(15\cos(x)^6\arctan\left(\sqrt{\frac{\sin(x)}{(\cos(x)+1)^2}}\,(\cot(x)+\csc(x))\right)-15\cos(x)^6\operatorname{arctanh}\left(\sqrt{\frac{\sin(x)}{(\cos(x)+1)^2}}\,(\cot(x)+\csc(x))\right)-10\sqrt{\frac{}{(\mathrm{co}}}\right.}{384(\cos(x)-1)(\cos(x)+}$

input
```
int(1/(csc(x)-sin(x))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/384*tan(x)^2*sec(x)^3*(15*cos(x)^6*arctan((sin(x)/(cos(x)+1)^2)^(1/2)*(c
ot(x)+csc(x)))-15*cos(x)^6*arctanh((sin(x)/(cos(x)+1)^2)^(1/2)*(cot(x)+csc
(x)))-10*(sin(x)/(cos(x)+1)^2)^(1/2)*cos(x)^5-10*(sin(x)/(cos(x)+1)^2)^(1/
2)*cos(x)^4+104*(sin(x)/(cos(x)+1)^2)^(1/2)*cos(x)^3+104*(sin(x)/(cos(x)+1
)^2)^(1/2)*cos(x)^2-64*(sin(x)/(cos(x)+1)^2)^(1/2)*cos(x)-64*(sin(x)/(cos(
x)+1)^2)^(1/2))/(cos(x)-1)/(cos(x)+1)^2/(sin(x)/(cos(x)+1)^2)^(1/2)/(cot(x
)*cos(x))^(1/2)
```

3.321.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.42

$$\int \frac{1}{(\csc(x)-\sin(x))^{7/2}}\,dx =$$

$$-\frac{30 \arctan\left(\frac{2\sqrt{\frac{\cos(x)^2}{\sin(x)}}\sin(x)}{\cos(x)\sin(x)-\cos(x)}\right)\cos(x)^7 - 15\cos(x)^7\log\left(\frac{\cos(x)^3-5\cos(x)^2-\left(\cos(x)^2+6\cos(x)+4\right)\sin(x)+4\left(\cos(x)^2-(\cdots\right.}{\cos(x)^3+3\cos(x)^2-\left(\cos(x)^2-2\cos(x)-4\cdots\right.}\right.\cdots}{1536\cos(x)^7}$$

input
```
integrate(1/(csc(x)-sin(x))^(7/2),x, algorithm="fricas")
```

output
```
-1/1536*(30*arctan(2*sqrt(cos(x)^2/sin(x))*sin(x)/(cos(x)*sin(x) - cos(x))
)*cos(x)^7 - 15*cos(x)^7*log((cos(x)^3 - 5*cos(x)^2 - (cos(x)^2 + 6*cos(x)
 + 4)*sin(x) + 4*(cos(x)^2 - (cos(x) + 1)*sin(x) - 1)*sqrt(cos(x)^2/sin(x)
) - 2*cos(x) + 4)/(cos(x)^3 + 3*cos(x)^2 - (cos(x)^2 - 2*cos(x) - 4)*sin(x
) - 2*cos(x) - 4)) - 8*(5*cos(x)^4 - 52*cos(x)^2 + 32)*sqrt(cos(x)^2/sin(x
))*sin(x))/cos(x)^7
```

3.321.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(\csc(x)-\sin(x))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(csc(x)-sin(x))**(7/2),x)
```

output
```
Timed out
```

3.321.7 Maxima [F]

$$\int \frac{1}{(\csc(x)-\sin(x))^{7/2}}\,dx=\int \frac{1}{(\csc(x)-\sin(x))^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(csc(x)-sin(x))^(7/2),x, algorithm="maxima")
```

output
```
integrate((csc(x) - sin(x))^(-7/2), x)
```

3.321.8 Giac [F]

$$\int \frac{1}{(\csc(x)-\sin(x))^{7/2}}\,dx=\int \frac{1}{(\csc(x)-\sin(x))^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(csc(x)-sin(x))^(7/2),x, algorithm="giac")
```

output
```
integrate((csc(x) - sin(x))^(-7/2), x)
```

3.321.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(\csc(x)-\sin(x))^{7/2}}\,dx=\int \frac{1}{\left(\frac{1}{\sin(x)}-\sin(x)\right)^{7/2}}\,dx$$

input
```
int(1/(1/sin(x) - sin(x))^(7/2),x)
```

output
```
int(1/(1/sin(x) - sin(x))^(7/2), x)
```

3.322 $\int (-\cos(x) + \sec(x))^4 \, dx$

3.322.1 Optimal result

Integrand size = 9, antiderivative size = 44

$$\int (-\cos(x) + \sec(x))^4 \, dx = \frac{35x}{8} - \frac{35\tan(x)}{8} + \frac{35\tan^3(x)}{24} - \frac{7}{8}\sin^2(x)\tan^3(x) - \frac{1}{4}\sin^4(x)\tan^3(x)$$

output
```
35/8*x-35/8*tan(x)+35/24*tan(x)^3-7/8*sin(x)^2*tan(x)^3-1/4*sin(x)^4*tan(x
)^3
```

3.322.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int (-\cos(x) + \sec(x))^4 \, dx = \frac{35x}{8} - \frac{3}{4}\sin(2x) + \frac{1}{32}\sin(4x) - \frac{10\tan(x)}{3} + \frac{1}{3}\sec^2(x)\tan(x)$$

input
```
Integrate[(-Cos[x] + Sec[x])^4,x]
```

output
```
(35*x)/8 - (3*Sin[2*x])/4 + Sin[4*x]/32 - (10*Tan[x])/3 + (Sec[x]^2*Tan[x]
)/3
```

3.322.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.32, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 4889, 252, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\sec(x)-\cos(x))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (\sec(x)-\cos(x))^4dx$$

$$\Big\downarrow \text{4889}$$

$$\int \frac{\tan^8(x)}{\left(\tan^2(x)+1\right)^3}d\tan(x)$$

$$\Big\downarrow \text{252}$$

$$\frac{7}{4}\int \frac{\tan^6(x)}{\left(\tan^2(x)+1\right)^2}d\tan(x)-\frac{\tan^7(x)}{4\left(\tan^2(x)+1\right)^2}$$

$$\Big\downarrow \text{252}$$

$$\frac{7}{4}\left(\frac{5}{2}\int \frac{\tan^4(x)}{\tan^2(x)+1}d\tan(x)-\frac{\tan^5(x)}{2\left(\tan^2(x)+1\right)}\right)-\frac{\tan^7(x)}{4\left(\tan^2(x)+1\right)^2}$$

$$\Big\downarrow \text{254}$$

$$\frac{7}{4}\left(\frac{5}{2}\int \left(\tan^2(x)+\frac{1}{\tan^2(x)+1}-1\right)d\tan(x)-\frac{\tan^5(x)}{2\left(\tan^2(x)+1\right)}\right)-\frac{\tan^7(x)}{4\left(\tan^2(x)+1\right)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{7}{4}\left(\frac{5}{2}\left(\arctan(\tan(x))+\frac{\tan^3(x)}{3}-\tan(x)\right)-\frac{\tan^5(x)}{2\left(\tan^2(x)+1\right)}\right)-\frac{\tan^7(x)}{4\left(\tan^2(x)+1\right)^2}$$

input
```
Int[(-Cos[x] + Sec[x])^4,x]
```

output
```
-1/4*Tan[x]^7/(1 + Tan[x]^2)^2 + (7*(-1/2*Tan[x]^5/(1 + Tan[x]^2) + (5*(Ar
cTan[Tan[x]] - Tan[x] + Tan[x]^3/3))/2))/4
```

3.322.3.1 Defintions of rubi rules used

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.322.4 Maple [A] (verified)

Time = 2.32 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.66

method	result
parallelrisch	$\frac{35x}{8}-\frac{3\sin(2x)}{4}+\frac{\sin(4x)}{32}-\frac{10\tan(x)}{3}+\frac{\tan(x)\sec(x)^2}{3}$
default	$\frac{\left(\cos(x)^3+\frac{3\cos(x)}{2}\right)\sin(x)}{4}+\frac{35x}{8}-2\cos\left(x\right)\sin\left(x\right)-4\tan\left(x\right)-\left(-\frac{2}{3}-\frac{\sec(x)^2}{3}\right)\tan\left(x\right)$
parts	$\frac{\left(\cos(x)^3+\frac{3\cos(x)}{2}\right)\sin(x)}{4}+\frac{35x}{8}-2\cos\left(x\right)\sin\left(x\right)-4\tan\left(x\right)-\left(-\frac{2}{3}-\frac{\sec(x)^2}{3}\right)\tan\left(x\right)$
risch	$\frac{35x}{8}-\frac{i e^{4ix}}{64}+\frac{3i e^{2ix}}{8}-\frac{3i e^{-2ix}}{8}+\frac{i e^{-4ix}}{64}-\frac{4i(6\,e^{4ix}+9\,e^{2ix}+5)}{3(e^{2ix}+1)^3}$
norman	$\frac{-\frac{35x}{8}+\frac{35\tan\left(\frac{x}{2}\right)^3}{6}-\frac{329\tan\left(\frac{x}{2}\right)^5}{12}-17\tan\left(\frac{x}{2}\right)^7-\frac{329\tan\left(\frac{x}{2}\right)^9}{12}+\frac{35\tan\left(\frac{x}{2}\right)^{11}}{6}+\frac{35\tan\left(\frac{x}{2}\right)^{13}}{4}-\frac{35x\tan\left(\frac{x}{2}\right)^2}{8}+\frac{105x\tan\left(\frac{x}{2}\right)^4}{8}+\frac{105x\,\mathrm{t}}{}}{\left(1+\tan\left(\frac{x}{2}\right)^2\right)^4\left(\tan\left(\frac{x}{2}\right)^2-1\right)^3}$

input
```
int((-cos(x)+sec(x))^4,x,method=_RETURNVERBOSE)
```

output
```
35/8*x-3/4*sin(2*x)+1/32*sin(4*x)-10/3*tan(x)+1/3*tan(x)*sec(x)^2
```

3.322.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

$$\int(-\cos(x)+\sec(x))^4\,dx$$
$$=\frac{105\,x\cos\left(x\right)^3+\left(6\,\cos\left(x\right)^6-39\,\cos\left(x\right)^4-80\,\cos\left(x\right)^2+8\right)\sin\left(x\right)}{24\,\cos\left(x\right)^3}$$

input
```
integrate((-cos(x)+sec(x))^4,x, algorithm="fricas")
```

output
```
1/24*(105*x*cos(x)^3 + (6*cos(x)^6 - 39*cos(x)^4 - 80*cos(x)^2 + 8)*sin(x)
)/cos(x)^3
```

3.322.6 Sympy [A] (verification not implemented)

Time = 4.65 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int (-\cos(x)+\sec(x))^4\,dx = \frac{35x}{8} - 2\sin(x)\cos(x) - \frac{4\sin(x)}{\cos(x)} + \frac{\sin(2x)}{4} + \frac{\sin(4x)}{32} + \frac{\tan^3(x)}{3} + \tan(x)$$

input
```
integrate((-cos(x)+sec(x))**4,x)
```

output
```
35*x/8 - 2*sin(x)*cos(x) - 4*sin(x)/cos(x) + sin(2*x)/4 + sin(4*x)/32 + ta
n(x)**3/3 + tan(x)
```

3.322.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.59

$$\int (-\cos(x)+\sec(x))^4\,dx = \frac{1}{3}\tan(x)^3 + \frac{35}{8}x + \frac{1}{32}\sin(4x) - \frac{3}{4}\sin(2x) - 3\tan(x)$$

input
```
integrate((-cos(x)+sec(x))^4,x, algorithm="maxima")
```

output
```
1/3*tan(x)^3 + 35/8*x + 1/32*sin(4*x) - 3/4*sin(2*x) - 3*tan(x)
```

3.322.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.80

$$\int (-\cos(x)+\sec(x))^4\,dx = \frac{1}{3}\tan(x)^3 + \frac{35}{8}x - \frac{13\tan(x)^3 + 11\tan(x)}{8\left(\tan(x)^2+1\right)^2} - 3\tan(x)$$

input
```
integrate((-cos(x)+sec(x))^4,x, algorithm="giac")
```

output
```
1/3*tan(x)^3 + 35/8*x - 1/8*(13*tan(x)^3 + 11*tan(x))/(tan(x)^2 + 1)^2 - 3
*tan(x)
```

3.322.9 Mupad [B] (verification not implemented)

Time = 29.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.82

$$\int (-\cos(x)+\sec(x))^4\,dx$$
$$=\frac{35\,x}{8}+\frac{\frac{35\tan\left(\frac{x}{2}\right)^{13}}{4}+\frac{35\tan\left(\frac{x}{2}\right)^{11}}{6}-\frac{329\tan\left(\frac{x}{2}\right)^{9}}{12}-17\tan\left(\frac{x}{2}\right)^{7}-\frac{329\tan\left(\frac{x}{2}\right)^{5}}{12}+\frac{35\tan\left(\frac{x}{2}\right)^{3}}{6}+\frac{35\tan\left(\frac{x}{2}\right)}{4}}{\left(\tan\left(\frac{x}{2}\right)^2-1\right)^3\left(\tan\left(\frac{x}{2}\right)^2+1\right)^4}$$

input
```
int((cos(x) - 1/cos(x))^4,x)
```

output
```
(35*x)/8 + ((35*tan(x/2))/4 + (35*tan(x/2)^3)/6 - (329*tan(x/2)^5)/12 - 17
*tan(x/2)^7 - (329*tan(x/2)^9)/12 + (35*tan(x/2)^11)/6 + (35*tan(x/2)^13)/
4)/((tan(x/2)^2 - 1)^3*(tan(x/2)^2 + 1)^4)
```

3.323 $\int (-\cos(x)+\sec(x))^3\,dx$

3.323.1 Optimal result

Integrand size = 9, antiderivative size = 34

$$\int (-\cos(x)+\sec(x))^3\,dx=-\frac{5}{2}\text{arctanh}(\sin(x))+\frac{5\sin(x)}{2}+\frac{5\sin^3(x)}{6}+\frac{1}{2}\sin^3(x)\tan^2(x)$$

output
```
-5/2*arctanh(sin(x))+5/2*sin(x)+5/6*sin(x)^3+1/2*sin(x)^3*tan(x)^2
```

3.323.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.12

$$\int (-\cos(x)+\sec(x))^3\,dx=-\frac{5}{2}\text{arctanh}(\sin(x))+\frac{5}{2}\sec(x)\tan(x)-\frac{5}{3}\sin(x)\tan^2(x)-\frac{1}{3}\sin^3(x)\tan^2(x)$$

input
```
Integrate[(-Cos[x] + Sec[x])^3,x]
```

output
```
(-5*ArcTanh[Sin[x]])/2 + (5*Sec[x]*Tan[x])/2 - (5*Sin[x]*Tan[x]^2)/3 - (Si
n[x]^3*Tan[x]^2)/3
```

3.323.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {3042, 4897, 3042, 3072, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\sec(x)-\cos(x))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (\sec(x)-\cos(x))^3dx$$

$$\Big\downarrow \text{4897}$$

$$\int \sin^3(x)\tan^3(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x)^3\tan(x)^3dx$$

$$\Big\downarrow \text{3072}$$

$$\int \frac{\sin^6(x)}{\left(1-\sin^2(x)\right)^2}d\sin(x)$$

$$\Big\downarrow \text{252}$$

$$\frac{\sin^5(x)}{2\left(1-\sin^2(x)\right)}-\frac{5}{2}\int \frac{\sin^4(x)}{1-\sin^2(x)}d\sin(x)$$

$$\Big\downarrow \text{254}$$

$$\frac{\sin^5(x)}{2\left(1-\sin^2(x)\right)}-\frac{5}{2}\int \left(-\sin^2(x)+\frac{1}{1-\sin^2(x)}-1\right)d\sin(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sin^5(x)}{2\left(1-\sin^2(x)\right)}-\frac{5}{2}\left(\text{arctanh}(\sin(x))-\frac{1}{3}\sin^3(x)-\sin(x)\right)$$

input
```
Int[(-Cos[x] + Sec[x])^3,x]
```

output
```
Sin[x]^5/(2*(1 - Sin[x]^2)) - (5*(ArcTanh[Sin[x]] - Sin[x] - Sin[x]^3/3))/
2
```

3.323.3.1 Defintions of rubi rules used

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.323. $\int (-\cos(x)+\sec(x))^3 \, dx$

3.323.4 Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{\left(2+\cos \left(x \right)^{2}\right) \sin \left(x \right)}{3}+3 \sin \left(x \right)-\frac{5 \ln \left(\sec \left(x \right)+\tan \left(x \right)\right)}{2}+\frac{\sec \left(x \right) \tan \left(x \right)}{2}$	30
parts	$-\frac{\left(2+\cos \left(x \right)^{2}\right) \sin \left(x \right)}{3}+3 \sin \left(x \right)-\frac{5 \ln \left(\sec \left(x \right)+\tan \left(x \right)\right)}{2}+\frac{\sec \left(x \right) \tan \left(x \right)}{2}$	30
parallelrisch	$-\frac{5 \ln \left(\csc \left(x \right)-\cot \left(x \right)+1\right)}{2}+\frac{5 \ln \left(-\cot \left(x \right)+\csc \left(x \right)-1\right)}{2}-\frac{\sin \left(3 x \right)}{12}+\frac{9 \sin \left(x \right)}{4}+\frac{\sec \left(x \right) \tan \left(x \right)}{2}$	40
norman	$\frac{\frac{20 \tan \left(\frac{x}{2}\right)^{3}}{3}-\frac{22 \tan \left(\frac{x}{2}\right)^{5}}{3}+\frac{20 \tan \left(\frac{x}{2}\right)^{7}}{3}+5 \tan \left(\frac{x}{2}\right)^{9}+5 \tan \left(\frac{x}{2}\right)}{\left(1+\tan \left(\frac{x}{2}\right)^{2}\right)^{3} \left(\tan \left(\frac{x}{2}\right)^{2}-1\right)^{2}}+\frac{5 \ln \left(\tan \left(\frac{x}{2}\right)-1\right)}{2}-\frac{5 \ln \left(\tan \left(\frac{x}{2}\right)+1\right)}{2}$	80
risch	$\frac{i {\mathrm e}^{3 i x}}{24}-\frac{9 i {\mathrm e}^{i x}}{8}+\frac{9 i {\mathrm e}^{-i x}}{8}-\frac{i {\mathrm e}^{-3 i x}}{24}-\frac{i \left({\mathrm e}^{3 i x}-{\mathrm e}^{i x}\right)}{\left({\mathrm e}^{2 i x}+1\right)^{2}}+\frac{5 \ln \left({\mathrm e}^{i x}-i\right)}{2}-\frac{5 \ln \left(i+{\mathrm e}^{i x}\right)}{2}$	81

input
```
int((-cos(x)+sec(x))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(2+cos(x)^2)*sin(x)+3*sin(x)-5/2*ln(sec(x)+tan(x))+1/2*sec(x)*tan(x)
```

3.323.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.44

$$\int (-\cos(x)+\sec(x))^3\,dx = -\frac{15\,\cos\left(x\right)^{2}\log\left(\sin\left(x\right)+1\right)-15\,\cos\left(x\right)^{2}\log\left(-\sin\left(x\right)+1\right)+2\,\left(2\,\cos\left(x\right)^{4}-14\,\cos\left(x\right)^{2}-3\right)\sin\left(x\right)}{12\,\cos\left(x\right)^{2}}$$

input
```
integrate((-cos(x)+sec(x))^3,x, algorithm="fricas")
```

output
```
-1/12*(15*cos(x)^2*log(sin(x) + 1) - 15*cos(x)^2*log(-sin(x) + 1) + 2*(2*c
os(x)^4 - 14*cos(x)^2 - 3)*sin(x))/cos(x)^2
```

3.323.6 Sympy [A] (verification not implemented)

Time = 1.63 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

$$\int (-\cos(x)+\sec(x))^3\,dx = \frac{5\log(\sin(x)-1)}{4} - \frac{5\log(\sin(x)+1)}{4} + \frac{\sin^3(x)}{3} + 2\sin(x) - \frac{\sin(x)}{2\sin^2(x)-2}$$

input
```
integrate((-cos(x)+sec(x))**3,x)
```

output
```
5*log(sin(x) - 1)/4 - 5*log(sin(x) + 1)/4 + sin(x)**3/3 + 2*sin(x) - sin(x
)/(2*sin(x)**2 - 2)
```

3.323.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09

$$\int (-\cos(x)+\sec(x))^3\,dx = \frac{1}{3}\sin(x)^3 - \frac{\sin(x)}{2\left(\sin(x)^2-1\right)} - \frac{5}{4}\log(\sin(x)+1) + \frac{5}{4}\log(\sin(x)-1) + 2\sin(x)$$

input
```
integrate((-cos(x)+sec(x))^3,x, algorithm="maxima")
```

output
```
1/3*sin(x)^3 - 1/2*sin(x)/(sin(x)^2 - 1) - 5/4*log(sin(x) + 1) + 5/4*log(s
in(x) - 1) + 2*sin(x)
```

3.323.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.15

$$\int (-\cos(x)+\sec(x))^3\,dx = \frac{1}{3}\sin(x)^3 - \frac{\sin(x)}{2\left(\sin(x)^2-1\right)} - \frac{5}{4}\log(\sin(x)+1) + \frac{5}{4}\log(-\sin(x)+1) + 2\sin(x)$$

input
```
integrate((-cos(x)+sec(x))^3,x, algorithm="giac")
```

output
```
1/3*sin(x)^3 - 1/2*sin(x)/(sin(x)^2 - 1) - 5/4*log(sin(x) + 1) + 5/4*log(-
sin(x) + 1) + 2*sin(x)
```

3.323.9 Mupad [B] (verification not implemented)

Time = 26.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.00

$$\int (-\cos(x)+\sec(x))^3\,dx = \frac{5\tan\left(\frac{x}{2}\right)^9+\frac{20\tan\left(\frac{x}{2}\right)^7}{3}-\frac{22\tan\left(\frac{x}{2}\right)^5}{3}+\frac{20\tan\left(\frac{x}{2}\right)^3}{3}+5\tan\left(\frac{x}{2}\right)}{\left(\tan\left(\frac{x}{2}\right)^2-1\right)^2\left(\tan\left(\frac{x}{2}\right)^2+1\right)^3} - 5\,\mathrm{atanh}\left(\tan\left(\frac{x}{2}\right)\right)$$

input
```
int(-(cos(x) - 1/cos(x))^3,x)
```

output
```
(5*tan(x/2) + (20*tan(x/2)^3)/3 - (22*tan(x/2)^5)/3 + (20*tan(x/2)^7)/3 +
5*tan(x/2)^9)/((tan(x/2)^2 - 1)^2*(tan(x/2)^2 + 1)^3) - 5*atanh(tan(x/2))
```

3.324 $\int (-\cos(x)+\sec(x))^2\,dx$

3.324.1 Optimal result

Integrand size = 9, antiderivative size = 22

$$\int (-\cos(x)+\sec(x))^2\,dx = -\frac{3x}{2}+\frac{3\tan(x)}{2}-\frac{1}{2}\sin^2(x)\tan(x)$$

output
```
-3/2*x+3/2*tan(x)-1/2*sin(x)^2*tan(x)
```

3.324.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int (-\cos(x)+\sec(x))^2\,dx = -\frac{3x}{2}+\frac{1}{4}\sin(2x)+\tan(x)$$

input
```
Integrate[(-Cos[x] + Sec[x])^2,x]
```

output
```
(-3*x)/2 + Sin[2*x]/4 + Tan[x]
```

3.324.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.32, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {3042, 4889, 252, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\sec(x)-\cos(x))^2 dx$$

$$\Big\downarrow \text{3042}$$

$$\int (\sec(x)-\cos(x))^2dx$$

$$\Big\downarrow \text{4889}$$

$$\int \frac{\tan^4(x)}{(\tan^2(x)+1)^2}d\tan(x)$$

$$\Big\downarrow \text{252}$$

$$\frac{3}{2}\int \frac{\tan^2(x)}{\tan^2(x)+1}d\tan(x)-\frac{\tan^3(x)}{2(\tan^2(x)+1)}$$

$$\Big\downarrow \text{262}$$

$$\frac{3}{2}\left(\tan(x)-\int \frac{1}{\tan^2(x)+1}d\tan(x)\right)-\frac{\tan^3(x)}{2(\tan^2(x)+1)}$$

$$\Big\downarrow \text{216}$$

$$\frac{3}{2}(\tan(x)-\arctan(\tan(x)))-\frac{\tan^3(x)}{2(\tan^2(x)+1)}$$

input
```
Int[(-Cos[x] + Sec[x])^2,x]
```

output
```
(3*(-ArcTan[Tan[x]] + Tan[x]))/2 - Tan[x]^3/(2*(1 + Tan[x]^2))
```

3.324.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.324. $\int (-\cos(x) + \sec(x))^2 \, dx$

3.324.4 Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.59

method	result	size
default	$\frac{\cos(x)\sin(x)}{2}-\frac{3x}{2}+\tan(x)$	13
parallelrisch	$-\frac{3x}{2}+\frac{\sin(2x)}{4}+\tan(x)$	13
parts	$\frac{\cos(x)\sin(x)}{2}-\frac{3x}{2}+\tan(x)$	13
risch	$-\frac{3x}{2}-\frac{i e^{2ix}}{8}+\frac{i e^{-2ix}}{8}+\frac{2i}{e^{2ix}+1}$	33
norman	$\frac{\frac{3x}{2}-2\tan\left(\frac{x}{2}\right)^{3}-3\tan\left(\frac{x}{2}\right)^{5}+\frac{3x\tan\left(\frac{x}{2}\right)^{2}}{2}-\frac{3x\tan\left(\frac{x}{2}\right)^{4}}{2}-\frac{3x\tan\left(\frac{x}{2}\right)^{6}}{2}-3\tan\left(\frac{x}{2}\right)}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)^{2}\left(\tan\left(\frac{x}{2}\right)^{2}-1\right)}$	75

input
```
int((-cos(x)+sec(x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*cos(x)*sin(x)-3/2*x+tan(x)
```

3.324.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int(-\cos(x)+\sec(x))^2\,dx=-\frac{3\,x\cos(x)-\left(\cos(x)^2+2\right)\sin(x)}{2\,\cos(x)}$$

input
```
integrate((-cos(x)+sec(x))^2,x, algorithm="fricas")
```

output
```
-1/2*(3*x*cos(x) - (cos(x)^2 + 2)*sin(x))/cos(x)
```

3.324.6 Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.64

$$\int(-\cos(x)+\sec(x))^2\,dx=-\frac{3x}{2}+\frac{\sin(2x)}{4}+\tan(x)$$

input
```
integrate((-cos(x)+sec(x))**2,x)
```

output
```
-3*x/2 + sin(2*x)/4 + tan(x)
```

3.324.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.55

$$\int (-\cos(x) + \sec(x))^2 \, dx = -\frac{3}{2}x + \frac{1}{4}\sin(2x) + \tan(x)$$

input
```
integrate((-cos(x)+sec(x))^2,x, algorithm="maxima")
```

output
```
-3/2*x + 1/4*sin(2*x) + tan(x)
```

3.324.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int (-\cos(x) + \sec(x))^2 \, dx = -\frac{3}{2}x + \frac{\tan(x)}{2(\tan(x)^2 + 1)} + \tan(x)$$

input
```
integrate((-cos(x)+sec(x))^2,x, algorithm="giac")
```

output
```
-3/2*x + 1/2*tan(x)/(tan(x)^2 + 1) + tan(x)
```

3.324.9 Mupad [B] (verification not implemented)

Time = 26.34 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.23

$$\int (-\cos(x) + \sec(x))^2 \, dx = -\frac{3x}{2} - \frac{3\tan\left(\frac{x}{2}\right)^5 + 2\tan\left(\frac{x}{2}\right)^3 + 3\tan\left(\frac{x}{2}\right)}{\left(\tan\left(\frac{x}{2}\right)^2 - 1\right)\left(\tan\left(\frac{x}{2}\right)^2 + 1\right)^2}$$

input
```
int((cos(x) - 1/cos(x))^2,x)
```

output
```
- (3*x)/2 - (3*tan(x/2) + 2*tan(x/2)^3 + 3*tan(x/2)^5)/((tan(x/2)^2 - 1)*(
tan(x/2)^2 + 1)^2)
```

3.325 $\int (-\cos(x) + \sec(x))\, dx$

3.325.1 Optimal result

Integrand size = 7, antiderivative size = 8

$$\int (-\cos(x) + \sec(x))\, dx = \operatorname{arctanh}(\sin(x)) - \sin(x)$$

output
```
arctanh(sin(x))-sin(x)
```

3.325.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int (-\cos(x) + \sec(x))\, dx = \operatorname{arctanh}(\sin(x)) - \sin(x)$$

input
```
Integrate[-Cos[x] + Sec[x],x]
```

output
```
ArcTanh[Sin[x]] - Sin[x]
```

3.325.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\sec(x) - \cos(x))\,dx$$

$$\Big\downarrow \text{ 2009}$$

$$\operatorname{arctanh}(\sin(x)) - \sin(x)$$

input
```
Int[-Cos[x] + Sec[x],x]
```

output
```
ArcTanh[Sin[x]] - Sin[x]
```

3.325.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.325.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.50

method	result	size
default	$-\sin(x)+\ln(\sec(x)+\tan(x))$	12
parts	$-\sin(x)+\ln(\sec(x)+\tan(x))$	12
parallelrisch	$-\sin(x)-\ln(-\cot(x)+\csc(x)-1)+\ln(\csc(x)-\cot(x)+1)$	26
norman	$-\frac{2\tan(\frac{x}{2})}{1+\tan(\frac{x}{2})^2}-\ln\left(\tan\left(\frac{x}{2}\right)-1\right)+\ln\left(\tan\left(\frac{x}{2}\right)+1\right)$	34
risch	$\frac{i e^{ix}}{2}-\frac{i e^{-ix}}{2}+\ln(i+e^{ix})-\ln(e^{ix}-i)$	38

input
```
int(-cos(x)+sec(x),x,method=_RETURNVERBOSE)
```

output
```
-sin(x)+ln(sec(x)+tan(x))
```

3.325.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(8) = 16.

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 2.62

$$\int (-\cos(x) + \sec(x))\,dx = \frac{1}{2}\log(\sin(x) + 1) - \frac{1}{2}\log(-\sin(x) + 1) - \sin(x)$$

input
```
integrate(-cos(x)+sec(x),x, algorithm="fricas")
```

output
```
1/2*log(sin(x) + 1) - 1/2*log(-sin(x) + 1) - sin(x)
```

3.325.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(7) = 14.

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 2.38

$$\int (-\cos(x) + \sec(x))\,dx = -\frac{\log(\sin(x) - 1)}{2} + \frac{\log(\sin(x) + 1)}{2} - \sin(x)$$

input
```
integrate(-cos(x)+sec(x),x)
```

output
```
-log(sin(x) - 1)/2 + log(sin(x) + 1)/2 - sin(x)
```

3.325.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.38

$$\int (-\cos(x) + \sec(x))\,dx = \log(\sec(x) + \tan(x)) - \sin(x)$$

input
```
integrate(-cos(x)+sec(x),x, algorithm="maxima")
```

output
```
log(sec(x) + tan(x)) - sin(x)
```

3.325.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(8) = 16$.

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 3.62

$$\int (-\cos(x) + \sec(x))\,dx = \frac{1}{4}\log\left(\left|\frac{1}{\sin(x)} + \sin(x) + 2\right|\right) - \frac{1}{4}\log\left(\left|\frac{1}{\sin(x)} + \sin(x) - 2\right|\right) - \sin(x)$$

input
```
integrate(-cos(x)+sec(x),x, algorithm="giac")
```

output
```
1/4*log(abs(1/sin(x) + sin(x) + 2)) - 1/4*log(abs(1/sin(x) + sin(x) - 2))
- sin(x)
```

3.325.9 Mupad [B] (verification not implemented)

Time = 27.76 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.75

$$\int (-\cos(x) + \sec(x))\,dx = \ln\left(\tan\left(\frac{x}{2} + \frac{\pi}{4}\right)\right) - \sin(x)$$

input
```
int(1/cos(x) - cos(x),x)
```

output
```
log(tan(x/2 + pi/4)) - sin(x)
```

3.326 $\int \frac{1}{-\cos(x)+\sec(x)} dx$

3.326.1 Optimal result

Integrand size = 9, antiderivative size = 4

$$\int \frac{1}{-\cos(x)+\sec(x)} dx = -\csc(x)$$

output
```
-csc(x)
```

3.326.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{1}{-\cos(x)+\sec(x)} dx = -\csc(x)$$

input
```
Integrate[(-Cos[x] + Sec[x])^(-1),x]
```

output
```
-Csc[x]
```

3.326.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 4897, 3042, 25, 3086, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec(x)-\cos(x)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sec(x)-\cos(x)}dx$$
$$\Big\downarrow \text{4897}$$
$$\int \cot(x)\csc(x)dx$$
$$\Big\downarrow \text{3042}$$
$$\int \tan\left(x-\frac{\pi}{2}\right)\left(-\sec\left(x-\frac{\pi}{2}\right)\right)dx$$
$$\Big\downarrow \text{25}$$
$$-\int \sec\left(x-\frac{\pi}{2}\right)\tan\left(x-\frac{\pi}{2}\right)dx$$
$$\Big\downarrow \text{3086}$$
$$-\int 1d\csc(x)$$
$$\Big\downarrow \text{24}$$
$$-\csc(x)$$

input
```
Int[(-Cos[x] + Sec[x])^(-1),x]
```

output
```
-Csc[x]
```

3.326.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.326.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

method	result	size
parallelrisch	$-\csc \left(x \right)$	5
default	$-\frac{1}{\sin \left(x \right)}$	7
norman	$\frac{-\frac{1}{2}-\frac{\tan \left(\frac{x}{2}\right)^{2}}{2}}{\tan \left(\frac{x}{2}\right)}$	18
risch	$-\frac{2 i {\mathrm e}^{i x}}{{\mathrm e}^{2 i x}-1}$	18

input
```
int(1/(-cos(x)+sec(x)),x,method=_RETURNVERBOSE)
```

output
```
-csc(x)
```

3.326.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.50

$$\int \frac{1}{-\cos(x)+\sec(x)}\,dx = -\frac{1}{\sin(x)}$$

input
```
integrate(1/(-cos(x)+sec(x)),x, algorithm="fricas")
```

output
```
-1/sin(x)
```

3.326.6 Sympy [F]

$$\int \frac{1}{-\cos(x)+\sec(x)}\,dx = -\int \frac{1}{\cos(x)-\sec(x)}\,dx$$

input
```
integrate(1/(-cos(x)+sec(x)),x)
```

output
```
-Integral(1/(cos(x) - sec(x)), x)
```

3.326.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(4) = 8.

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 5.25

$$\int \frac{1}{-\cos(x)+\sec(x)}\,dx = -\frac{\cos(x)+1}{2\sin(x)} - \frac{\sin(x)}{2(\cos(x)+1)}$$

input
```
integrate(1/(-cos(x)+sec(x)),x, algorithm="maxima")
```

output
```
-1/2*(cos(x) + 1)/sin(x) - 1/2*sin(x)/(cos(x) + 1)
```

3.326.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.50

$$\int \frac{1}{-\cos(x)+\sec(x)}\,dx = -\frac{1}{\sin(x)}$$

input
```
integrate(1/(-cos(x)+sec(x)),x, algorithm="giac")
```

output
```
-1/sin(x)
```

3.326.9 Mupad [B] (verification not implemented)

Time = 25.78 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.50

$$\int \frac{1}{-\cos(x)+\sec(x)}\,dx = -\frac{1}{\sin(x)}$$

input
```
int(-1/(cos(x) - 1/cos(x)),x)
```

output
```
-1/sin(x)
```

3.327 $\int \frac{1}{(-\cos(x)+\sec(x))^2} \, dx$

3.327.1 Optimal result

Integrand size = 9, antiderivative size = 8

$$\int \frac{1}{(-\cos(x)+\sec(x))^2} \, dx = -\frac{1}{3}\cot^3(x)$$

output
```
-1/3*cot(x)^3
```

3.327.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{1}{(-\cos(x)+\sec(x))^2} \, dx = -\frac{1}{3}\cot^3(x)$$

input
```
Integrate[(-Cos[x] + Sec[x])^(-2),x]
```

output
```
-1/3*Cot[x]^3
```

3.327.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4889, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\sec(x)-\cos(x))^2} \, dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{1}{(\sec(x)-\cos(x))^2}dx$$
$$\Big\downarrow \ 4889$$
$$\int \cot^4(x)d\tan(x)$$
$$\Big\downarrow \ 15$$
$$-\frac{1}{3}\cot^3(x)$$

input
```
Int[(-Cos[x] + Sec[x])^(-2),x]
```

output
```
-1/3*Cot[x]^3
```

3.327.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.327.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{1}{3\tan\left(x\right)^{3}}$	7
parallelrisch	$-\frac{\cot\left(x\right)^{3}}{3}$	7
risch	$\frac{2i\left(3\,{\mathrm e}^{4 i x}+1\right)}{3\left({\mathrm e}^{2 i x}-1\right)^{3}}$	22
norman	$\frac{-\frac{1}{24}+\frac{\tan\left(\frac{x}{2}\right)^{2}}{8}-\frac{\tan\left(\frac{x}{2}\right)^{4}}{8}+\frac{\tan\left(\frac{x}{2}\right)^{6}}{24}}{\tan\left(\frac{x}{2}\right)^{3}}$	34

input
```
int(1/(-cos(x)+sec(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3/tan(x)^3
```

3.327.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. 2(6) = 12.

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.25

$$\int \frac{1}{(-\cos(x)+\sec(x))^2}\,dx = \frac{\cos(x)^3}{3\left(\cos(x)^2-1\right)\sin(x)}$$

input
```
integrate(1/(-cos(x)+sec(x))^2,x, algorithm="fricas")
```

output
```
1/3*cos(x)^3/((cos(x)^2 - 1)*sin(x))
```

3.327.6 Sympy [F]

$$\int \frac{1}{(-\cos(x)+\sec(x))^2}\,dx = \int \frac{1}{\left(-\cos{(x)}+\sec{(x)}\right)^2}\,dx$$

input
```
integrate(1/(-cos(x)+sec(x))**2,x)
```

output
```
Integral((-cos(x) + sec(x))**(-2), x)
```

3.327.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{(-\cos(x)+\sec(x))^2}\,dx = -\frac{1}{3\tan{(x)}^3}$$

input
```
integrate(1/(-cos(x)+sec(x))^2,x, algorithm="maxima")
```

output
```
-1/3/tan(x)^3
```

3.327.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{(-\cos(x)+\sec(x))^2}\,dx = -\frac{1}{3\tan{(x)}^3}$$

input
```
integrate(1/(-cos(x)+sec(x))^2,x, algorithm="giac")
```

output
```
-1/3/tan(x)^3
```

3.327.9 Mupad [B] (verification not implemented)

Time = 27.90 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{1}{(-\cos(x)+\sec(x))^2}\,dx = -\frac{\cot(x)^3}{3}$$

input
```
int(1/(cos(x) - 1/cos(x))^2,x)
```

output
```
-cot(x)^3/3
```

3.328 $\int \frac{1}{(-\cos(x)+\sec(x))^3} dx$

3.328.1 Optimal result

Integrand size = 9, antiderivative size = 17

$$\int \frac{1}{(-\cos(x)+\sec(x))^3} dx = \frac{\csc^3(x)}{3} - \frac{\csc^5(x)}{5}$$

output
```
1/3*csc(x)^3-1/5*csc(x)^5
```

3.328.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{(-\cos(x)+\sec(x))^3} dx = \frac{\csc^3(x)}{3} - \frac{\csc^5(x)}{5}$$

input
```
Integrate[(-Cos[x] + Sec[x])^(-3),x]
```

output
```
Csc[x]^3/3 - Csc[x]^5/5
```

3.328.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.889$, Rules used = {3042, 4897, 3042, 25, 3086, 25, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\sec(x)-\cos(x))^3}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(\sec(x)-\cos(x))^3}dx$$
$$\Big\downarrow \text{4897}$$
$$\int \cot^3(x)\csc^3(x)dx$$
$$\Big\downarrow \text{3042}$$
$$\int \tan\left(x-\frac{\pi}{2}\right)^3\left(-\sec\left(x-\frac{\pi}{2}\right)^3\right)dx$$
$$\Big\downarrow \text{25}$$
$$-\int \sec\left(x-\frac{\pi}{2}\right)^3\tan\left(x-\frac{\pi}{2}\right)^3dx$$
$$\Big\downarrow \text{3086}$$
$$-\int -\csc^2(x)\left(1-\csc^2(x)\right)d\csc(x)$$
$$\Big\downarrow \text{25}$$
$$\int \csc^2(x)\left(1-\csc^2(x)\right)d\csc(x)$$
$$\Big\downarrow \text{244}$$
$$\int \left(\csc^2(x)-\csc^4(x)\right)d\csc(x)$$
$$\Big\downarrow \text{2009}$$
$$\frac{\csc^3(x)}{3}-\frac{\csc^5(x)}{5}$$

input
```
Int[(-Cos[x] + Sec[x])^(-3),x]
```

output
```
Csc[x]^3/3 - Csc[x]^5/5
```

3.328.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.328. $\int \frac{1}{(-\cos(x)+\sec(x))^3} \, dx$

3.328.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{1}{3\sin(x)^3}-\frac{1}{5\sin(x)^5}$	14
parallelrisch	$-\frac{\csc(x)^5(1+5\cos(2x))}{30}$	15
risch	$-\frac{8i(5\,e^{7ix}+2\,e^{5ix}+5\,e^{3ix})}{15(e^{2ix}-1)^5}$	35
norman	$\frac{-\frac{1}{160}+\frac{\tan(\frac{x}{2})^2}{96}+\frac{\tan(\frac{x}{2})^4}{16}+\frac{\tan(\frac{x}{2})^6}{16}+\frac{\tan(\frac{x}{2})^8}{96}-\frac{\tan(\frac{x}{2})^{10}}{160}}{\tan(\frac{x}{2})^5}$	50

input
```
int(1/(-cos(x)+sec(x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/3/sin(x)^3-1/5/sin(x)^5
```

3.328.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. 2(13) = 26.

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.65

$$\int \frac{1}{(-\cos(x)+\sec(x))^3}\,dx = -\frac{5\,\cos(x)^2-2}{15\,(\cos(x)^4-2\,\cos(x)^2+1)\sin(x)}$$

input
```
integrate(1/(-cos(x)+sec(x))^3,x, algorithm="fricas")
```

output
```
-1/15*(5*cos(x)^2 - 2)/((cos(x)^4 - 2*cos(x)^2 + 1)*sin(x))
```

3.328.6 Sympy [F]

$$\int \frac{1}{(-\cos(x)+\sec(x))^3}\,dx$$
$$= -\int \frac{1}{\cos^3(x)-3\cos^2(x)\sec(x)+3\cos(x)\sec^2(x)-\sec^3(x)}\,dx$$

input
```
integrate(1/(-cos(x)+sec(x))**3,x)
```

output
```
-Integral(1/(cos(x)**3 - 3*cos(x)**2*sec(x) + 3*cos(x)*sec(x)**2 - sec(x)*
*3), x)
```

3.328.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(13) = 26$.

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 4.29

$$\int \frac{1}{(-\cos(x)+\sec(x))^3}\,dx = \frac{\left(\frac{5\sin(x)^2}{(\cos(x)+1)^2} + \frac{30\sin(x)^4}{(\cos(x)+1)^4} - 3\right)(\cos(x)+1)^5}{480\sin(x)^5} + \frac{\sin(x)}{16(\cos(x)+1)} + \frac{\sin(x)^3}{96(\cos(x)+1)^3} - \frac{\sin(x)^5}{160(\cos(x)+1)^5}$$

input
```
integrate(1/(-cos(x)+sec(x))^3,x, algorithm="maxima")
```

output
```
1/480*(5*sin(x)^2/(cos(x) + 1)^2 + 30*sin(x)^4/(cos(x) + 1)^4 - 3)*(cos(x)
 + 1)^5/sin(x)^5 + 1/16*sin(x)/(cos(x) + 1) + 1/96*sin(x)^3/(cos(x) + 1)^3
 - 1/160*sin(x)^5/(cos(x) + 1)^5
```

3.328.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{1}{(-\cos(x)+\sec(x))^3}\,dx = \frac{5\sin(x)^2-3}{15\sin(x)^5}$$

input
```
integrate(1/(-cos(x)+sec(x))^3,x, algorithm="giac")
```

output
```
1/15*(5*sin(x)^2 - 3)/sin(x)^5
```

3.328.9 Mupad [B] (verification not implemented)

Time = 28.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{1}{(-\cos(x)+\sec(x))^3}\,dx = \frac{5\sin(x)^2-3}{15\sin(x)^5}$$

input
```
int(-1/(cos(x) - 1/cos(x))^3,x)
```

output
```
(5*sin(x)^2 - 3)/(15*sin(x)^5)
```

3.329 $\int \frac{1}{(-\cos(x)+\sec(x))^4} dx$

3.329.1 Optimal result

Integrand size = 9, antiderivative size = 17

$$\int \frac{1}{(-\cos(x)+\sec(x))^4} dx = -\frac{1}{5}\cot^5(x) - \frac{\cot^7(x)}{7}$$

output
```
-1/5*cot(x)^5-1/7*cot(x)^7
```

3.329.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 37 vs. 2(17) = 34.

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.18

$$\int \frac{1}{(-\cos(x)+\sec(x))^4} dx = -\frac{2\cot(x)}{35} - \frac{1}{35}\cot(x)\csc^2(x) + \frac{8}{35}\cot(x)\csc^4(x) - \frac{1}{7}\cot(x)\csc^6(x)$$

input
```
Integrate[(-Cos[x] + Sec[x])^(-4),x]
```

output
```
(-2*Cot[x])/35 - (Cot[x]*Csc[x]^2)/35 + (8*Cot[x]*Csc[x]^4)/35 - (Cot[x]*C
sc[x]^6)/7
```

3.329.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4889, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\sec(x)-\cos(x))^4} \, dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{(\sec(x)-\cos(x))^4}dx$$
$$\Big\downarrow \text{ 4889}$$
$$\int \left(\cot^8(x)+\cot^6(x)\right)d\tan(x)$$
$$\Big\downarrow \text{ 2009}$$
$$-\frac{1}{7}\cot^7(x)-\frac{\cot^5(x)}{5}$$

input
```
Int[(-Cos[x] + Sec[x])^(-4),x]
```

output
```
-1/5*Cot[x]^5 - Cot[x]^7/7
```

3.329.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.329.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
default	$-\frac{1}{5\tan(x)^5}-\frac{1}{7\tan(x)^7}$	14
parallelrisch	$\frac{2\cot(x)^7}{35}-\frac{\cot(x)^5\csc(x)^2}{5}$	18
risch	$\frac{4i(35\,\mathrm{e}^{10ix}+35\,\mathrm{e}^{8ix}+70\,\mathrm{e}^{6ix}+14\,\mathrm{e}^{4ix}+7\,\mathrm{e}^{2ix}-1)}{35(\mathrm{e}^{2ix}-1)^7}$	50
norman	$\frac{-\frac{1}{896}+\frac{\tan(\frac{x}{2})^2}{640}+\frac{\tan(\frac{x}{2})^4}{128}-\frac{3\tan(\frac{x}{2})^6}{128}+\frac{3\tan(\frac{x}{2})^8}{128}-\frac{\tan(\frac{x}{2})^{10}}{128}-\frac{\tan(\frac{x}{2})^{12}}{640}+\frac{\tan(\frac{x}{2})^{14}}{896}}{\tan(\frac{x}{2})^7}$	66

input
```
int(1/(-cos(x)+sec(x))^4,x,method=_RETURNVERBOSE)
```

output
```
-1/5/tan(x)^5-1/7/tan(x)^7
```

3.329.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(13) = 26.

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.29

$$\int \frac{1}{(-\cos(x)+\sec(x))^4}\,dx = -\frac{2\,\cos(x)^7-7\,\cos(x)^5}{35\left(\cos(x)^6-3\,\cos(x)^4+3\,\cos(x)^2-1\right)\sin(x)}$$

input
```
integrate(1/(-cos(x)+sec(x))^4,x, algorithm="fricas")
```

output
```
-1/35*(2*cos(x)^7 - 7*cos(x)^5)/((cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 - 1)*
sin(x))
```

3.329.6 Sympy [F]

$$\int \frac{1}{(-\cos(x)+\sec(x))^4}\,dx=\int \frac{1}{(-\cos{(x)}+\sec{(x)})^4}\,dx$$

input
```
integrate(1/(-cos(x)+sec(x))**4,x)
```

output
```
Integral((-cos(x) + sec(x))**(-4), x)
```

3.329.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{1}{(-\cos(x)+\sec(x))^4}\,dx=-\frac{7\tan{(x)}^2+5}{35\tan{(x)}^7}$$

input
```
integrate(1/(-cos(x)+sec(x))^4,x, algorithm="maxima")
```

output
```
-1/35*(7*tan(x)^2 + 5)/tan(x)^7
```

3.329.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{1}{(-\cos(x)+\sec(x))^4}\,dx=-\frac{7\tan{(x)}^2+5}{35\tan{(x)}^7}$$

input
```
integrate(1/(-cos(x)+sec(x))^4,x, algorithm="giac")
```

output
```
-1/35*(7*tan(x)^2 + 5)/tan(x)^7
```

3.329.9 Mupad [B] (verification not implemented)

Time = 28.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{1}{(-\cos(x)+\sec(x))^4}\,dx = \frac{\cos(x)^5\,(\cos(2\,x)-6)}{35\sin(x)^7}$$

input
```
int(1/(cos(x) - 1/cos(x))^4,x)
```

output
```
(cos(x)^5*(cos(2*x) - 6))/(35*sin(x)^7)
```

3.330 $\int \frac{1}{(-\cos(x)+\sec(x))^5} \, dx$

3.330.1 Optimal result

Integrand size = 9, antiderivative size = 25

$$\int \frac{1}{(-\cos(x)+\sec(x))^5} \, dx = -\frac{1}{5}\csc^5(x)+\frac{2\csc^7(x)}{7}-\frac{\csc^9(x)}{9}$$

output
```
-1/5*csc(x)^5+2/7*csc(x)^7-1/9*csc(x)^9
```

3.330.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{1}{(-\cos(x)+\sec(x))^5} \, dx = -\frac{1}{5}\csc^5(x)+\frac{2\csc^7(x)}{7}-\frac{\csc^9(x)}{9}$$

input
```
Integrate[(-Cos[x] + Sec[x])^(-5),x]
```

output
```
-1/5*Csc[x]^5 + (2*Csc[x]^7)/7 - Csc[x]^9/9
```

3.330.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {3042, 4897, 3042, 25, 3086, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\sec(x)-\cos(x))^5}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{(\sec(x)-\cos(x))^5}dx$$

$$\Big\downarrow \text{ 4897}$$

$$\int \cot^5(x)\csc^5(x)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan\left(x-\frac{\pi}{2}\right)^5\left(-\sec\left(x-\frac{\pi}{2}\right)^5\right)dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \sec\left(x-\frac{\pi}{2}\right)^5\tan\left(x-\frac{\pi}{2}\right)^5dx$$

$$\Big\downarrow \text{ 3086}$$

$$-\int \csc^4(x)\left(1-\csc^2(x)\right)^2d\csc(x)$$

$$\Big\downarrow \text{ 244}$$

$$-\int \left(\csc^8(x)-2\csc^6(x)+\csc^4(x)\right)d\csc(x)$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{1}{9}\csc^9(x)+\frac{2\csc^7(x)}{7}-\frac{\csc^5(x)}{5}$$

input
```
Int[(-Cos[x] + Sec[x])^(-5),x]
```

output
```
-1/5*Csc[x]^5 + (2*Csc[x]^7)/7 - Csc[x]^9/9
```

3.330. $\int \frac{1}{(-\cos(x)+\sec(x))^5}dx$

3.330.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.330.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
default	$\frac{2}{7\sin(x)^7}-\frac{1}{9\sin(x)^9}-\frac{1}{5\sin(x)^5}$	20
parallelrisch	$-\frac{\csc(x)^9(109+63\cos(4x)+108\cos(2x))}{2520}$	21
risch	$-\frac{32i(63\,e^{13ix}+108\,e^{11ix}+218\,e^{9ix}+108\,e^{7ix}+63\,e^{5ix})}{315(e^{2ix}-1)^9}$	49

input
```
int(1/(-cos(x)+sec(x))^5,x,method=_RETURNVERBOSE)
```

output
```
2/7/sin(x)^7-1/9/sin(x)^9-1/5/sin(x)^5
```

3.330.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(19) = 38$.

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.84

$$\int \frac{1}{(-\cos(x)+\sec(x))^5}\,dx = -\frac{63\cos(x)^4-36\cos(x)^2+8}{315\left(\cos(x)^8-4\cos(x)^6+6\cos(x)^4-4\cos(x)^2+1\right)\sin(x)}$$

input
```
integrate(1/(-cos(x)+sec(x))^5,x, algorithm="fricas")
```

output
```
-1/315*(63*cos(x)^4 - 36*cos(x)^2 + 8)/((cos(x)^8 - 4*cos(x)^6 + 6*cos(x)^
4 - 4*cos(x)^2 + 1)*sin(x))
```

3.330.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(-\cos(x)+\sec(x))^5}\,dx = \text{Timed out}$$

input
```
integrate(1/(-cos(x)+sec(x))**5,x)
```

output
```
Timed out
```

3.330.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(19) = 38$.

Time = 0.22 (sec) , antiderivative size = 121, normalized size of antiderivative = 4.84

$$\int \frac{1}{(-\cos(x)+\sec(x))^5}\,dx = \frac{\left(\frac{45\sin(x)^2}{(\cos(x)+1)^2}+\frac{252\sin(x)^4}{(\cos(x)+1)^4}-\frac{420\sin(x)^6}{(\cos(x)+1)^6}-\frac{1890\sin(x)^8}{(\cos(x)+1)^8}-35\right)(\cos(x)+1)^9}{161280\sin(x)^9}-\frac{3\sin(x)}{256(\cos(x)+1)} - \frac{\sin(x)^3}{384(\cos(x)+1)^3}+\frac{\sin(x)^5}{640(\cos(x)+1)^5}+\frac{\sin(x)^7}{3584(\cos(x)+1)^7}-\frac{\sin(x)^9}{4608(\cos(x)+1)^9}$$

input
```
integrate(1/(-cos(x)+sec(x))^5,x, algorithm="maxima")
```

output
```
1/161280*(45*sin(x)^2/(cos(x) + 1)^2 + 252*sin(x)^4/(cos(x) + 1)^4 - 420*s
in(x)^6/(cos(x) + 1)^6 - 1890*sin(x)^8/(cos(x) + 1)^8 - 35)*(cos(x) + 1)^9
/sin(x)^9 - 3/256*sin(x)/(cos(x) + 1) - 1/384*sin(x)^3/(cos(x) + 1)^3 + 1/
640*sin(x)^5/(cos(x) + 1)^5 + 1/3584*sin(x)^7/(cos(x) + 1)^7 - 1/4608*sin(
x)^9/(cos(x) + 1)^9
```

3.330.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{1}{(-\cos(x)+\sec(x))^5}\,dx = -\frac{63\,\sin\left(x\right)^4-90\,\sin\left(x\right)^2+35}{315\,\sin\left(x\right)^9}$$

input
```
integrate(1/(-cos(x)+sec(x))^5,x, algorithm="giac")
```

output
```
-1/315*(63*sin(x)^4 - 90*sin(x)^2 + 35)/sin(x)^9
```

3.330.9 Mupad [B] (verification not implemented)

Time = 28.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{1}{(-\cos(x)+\sec(x))^5}\,dx = -\frac{63\sin\left(x\right)^4-90\sin\left(x\right)^2+35}{315\sin\left(x\right)^9}$$

input
```
int(-1/(cos(x) - 1/cos(x))^5,x)
```

output
```
-(63*sin(x)^4 - 90*sin(x)^2 + 35)/(315*sin(x)^9)
```

3.331 $\int \frac{1}{(-\cos(x)+\sec(x))^6} dx$

3.331.1 Optimal result

Integrand size = 9, antiderivative size = 25

$$\int \frac{1}{(-\cos(x)+\sec(x))^6} dx = -\frac{1}{7}\cot^7(x) - \frac{2\cot^9(x)}{9} - \frac{\cot^{11}(x)}{11}$$

output
```
-1/7*cot(x)^7-2/9*cot(x)^9-1/11*cot(x)^11
```

3.331.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 57 vs. 2(25) = 50.

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.28

$$\int \frac{1}{(-\cos(x)+\sec(x))^6} dx = \frac{8\cot(x)}{693} + \frac{4}{693}\cot(x)\csc^2(x) + \frac{1}{231}\cot(x)\csc^4(x) - \frac{113}{693}\cot(x)\csc^6(x) + \frac{23}{99}\cot(x)\csc^8(x) - \frac{1}{11}\cot(x)\csc^{10}(x)$$

input
```
Integrate[(-Cos[x] + Sec[x])^(-6),x]
```

output
```
(8*Cot[x])/693 + (4*Cot[x]*Csc[x]^2)/693 + (Cot[x]*Csc[x]^4)/231 - (113*Co
t[x]*Csc[x]^6)/693 + (23*Cot[x]*Csc[x]^8)/99 - (Cot[x]*Csc[x]^10)/11
```

3.331.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3042, 4889, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\sec(x)-\cos(x))^6}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(\sec(x)-\cos(x))^6}dx$$

$$\Big\downarrow \text{4889}$$

$$\int \left(\tan^2(x)+1\right)^2\cot^{12}(x)d\tan(x)$$

$$\Big\downarrow \text{244}$$

$$\int \left(\cot^{12}(x)+2\cot^{10}(x)+\cot^8(x)\right)d\tan(x)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{11}\cot^{11}(x)-\frac{2\cot^9(x)}{9}-\frac{\cot^7(x)}{7}$$

input
```
Int[(-Cos[x] + Sec[x])^(-6),x]
```

output
```
-1/7*Cot[x]^7 - (2*Cot[x]^9)/9 - Cot[x]^11/11
```

3.331.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.331.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{1}{11\tan(x)^{11}}-\frac{2}{9\tan(x)^{9}}-\frac{1}{7\tan(x)^{7}}$	20
parallelrisch	$-\frac{\cot(x)^{7}\csc(x)^{4}(80+\cos(4x)-18\cos(2x))}{693}$	23
risch	$\frac{16i(462\,e^{16ix}+1155\,e^{14ix}+2541\,e^{12ix}+2079\,e^{10ix}+1485\,e^{8ix}+297\,e^{6ix}+55\,e^{4ix}-11\,e^{2ix}+1)}{693(e^{2ix}-1)^{11}}$	71

input
```
int(1/(-cos(x)+sec(x))^6,x,method=_RETURNVERBOSE)
```

output
```
-1/11/tan(x)^11-2/9/tan(x)^9-1/7/tan(x)^7
```

3.331.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(19) = 38.

Time = 0.23 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.28

$$\int \frac{1}{(-\cos(x)+\sec(x))^6}\,dx = \frac{8\cos(x)^{11}-44\cos(x)^{9}+99\cos(x)^{7}}{693\left(\cos(x)^{10}-5\cos(x)^{8}+10\cos(x)^{6}-10\cos(x)^{4}+5\cos(x)^{2}-1\right)\sin(x)}$$

input
```
integrate(1/(-cos(x)+sec(x))^6,x, algorithm="fricas")
```

output
```
1/693*(8*cos(x)^11 - 44*cos(x)^9 + 99*cos(x)^7)/((cos(x)^10 - 5*cos(x)^8 +
 10*cos(x)^6 - 10*cos(x)^4 + 5*cos(x)^2 - 1)*sin(x))
```

3.331.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(-\cos(x)+\sec(x))^6}\,dx = \text{Timed out}$$

input
```
integrate(1/(-cos(x)+sec(x))**6,x)
```

output
```
Timed out
```

3.331.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{1}{(-\cos(x)+\sec(x))^6}\,dx = -\frac{99\tan(x)^4+154\tan(x)^2+63}{693\tan(x)^{11}}$$

input
```
integrate(1/(-cos(x)+sec(x))^6,x, algorithm="maxima")
```

output
```
-1/693*(99*tan(x)^4 + 154*tan(x)^2 + 63)/tan(x)^11
```

3.331.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{1}{(-\cos(x)+\sec(x))^6}\,dx = -\frac{99\tan(x)^4+154\tan(x)^2+63}{693\tan(x)^{11}}$$

input
```
integrate(1/(-cos(x)+sec(x))^6,x, algorithm="giac")
```

output
```
-1/693*(99*tan(x)^4 + 154*tan(x)^2 + 63)/tan(x)^11
```

3.331.9 Mupad [B] (verification not implemented)

Time = 28.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.84

$$\int \frac{1}{(-\cos(x)+\sec(x))^6}\,dx = -\frac{80\cos(x)^7-18\cos(x)^7\left(2\cos(x)^2-1\right)+\cos(x)^7\left(2\left(2\cos(x)^2-1\right)^2-1\right)}{693\sin(x)^{11}}$$

input
```
int(1/(cos(x) - 1/cos(x))^6,x)
```

output
```
-(80*cos(x)^7 - 18*cos(x)^7*(2*cos(x)^2 - 1) + cos(x)^7*(2*(2*cos(x)^2 - 1
)^2 - 1))/(693*sin(x)^11)
```

3.332 $\int \frac{1}{(-\cos(x)+\sec(x))^7} dx$

3.332.1 Optimal result

Integrand size = 9, antiderivative size = 33

$$\int \frac{1}{(-\cos(x)+\sec(x))^7} dx = \frac{\csc^7(x)}{7} - \frac{\csc^9(x)}{3} + \frac{3\csc^{11}(x)}{11} - \frac{\csc^{13}(x)}{13}$$

output
```
1/7*csc(x)^7-1/3*csc(x)^9+3/11*csc(x)^11-1/13*csc(x)^13
```

3.332.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{1}{(-\cos(x)+\sec(x))^7} dx = \frac{\csc^7(x)}{7} - \frac{\csc^9(x)}{3} + \frac{3\csc^{11}(x)}{11} - \frac{\csc^{13}(x)}{13}$$

input
```
Integrate[(-Cos[x] + Sec[x])^(-7),x]
```

output
```
Csc[x]^7/7 - Csc[x]^9/3 + (3*Csc[x]^11)/11 - Csc[x]^13/13
```

3.332.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.889, Rules used = {3042, 4897, 3042, 25, 3086, 25, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\sec(x)-\cos(x))^7}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(\sec(x)-\cos(x))^7}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \cot^7(x)\csc^7(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(x-\frac{\pi}{2}\right)^7\left(-\sec\left(x-\frac{\pi}{2}\right)^7\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sec\left(x-\frac{\pi}{2}\right)^7\tan\left(x-\frac{\pi}{2}\right)^7dx$$

$$\Big\downarrow \text{3086}$$

$$-\int -\csc^6(x)\left(1-\csc^2(x)\right)^3d\csc(x)$$

$$\Big\downarrow \text{25}$$

$$\int \csc^6(x)\left(1-\csc^2(x)\right)^3d\csc(x)$$

$$\Big\downarrow \text{244}$$

$$\int \left(-\csc^{12}(x)+3\csc^{10}(x)-3\csc^8(x)+\csc^6(x)\right)d\csc(x)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{13}\csc^{13}(x)+\frac{3\csc^{11}(x)}{11}-\frac{\csc^9(x)}{3}+\frac{\csc^7(x)}{7}$$

input
```
Int[(-Cos[x] + Sec[x])^(-7),x]
```

output
```
Csc[x]^7/7 - Csc[x]^9/3 + (3*Csc[x]^11)/11 - Csc[x]^13/13
```

3.332.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.332.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

method	result	size
default	$-\frac{1}{13\sin(x)^{13}}+\frac{1}{7\sin(x)^{7}}+\frac{3}{11\sin(x)^{11}}-\frac{1}{3\sin(x)^{9}}$	26
parallelrisch	$-\frac{\csc(x)^{13}(2010+429\cos(6x)+1430\cos(4x)+3523\cos(2x))}{96096}$	27
risch	$-\frac{128i(429\,\mathrm{e}^{19ix}+1430\,\mathrm{e}^{17ix}+3523\,\mathrm{e}^{15ix}+4020\,\mathrm{e}^{13ix}+3523\,\mathrm{e}^{11ix}+1430\,\mathrm{e}^{9ix}+429\,\mathrm{e}^{7ix})}{3003(\mathrm{e}^{2ix}-1)^{13}}$	63

input
```
int(1/(-cos(x)+sec(x))^7,x,method=_RETURNVERBOSE)
```

output
```
-1/13/sin(x)^13+1/7/sin(x)^7+3/11/sin(x)^11-1/3/sin(x)^9
```

3.332.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(25) = 50$.

Time = 0.23 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.94

$$\int \frac{1}{(-\cos(x)+\sec(x))^7}\,dx = -\frac{429\cos(x)^6-286\cos(x)^4+104\cos(x)^2-16}{3003\left(\cos(x)^{12}-6\cos(x)^{10}+15\cos(x)^8-20\cos(x)^6+15\cos(x)^4-6\cos(x)^2+1\right)\sin(x)}$$

input
```
integrate(1/(-cos(x)+sec(x))^7,x, algorithm="fricas")
```

output
```
-1/3003*(429*cos(x)^6 - 286*cos(x)^4 + 104*cos(x)^2 - 16)/((cos(x)^12 - 6*
cos(x)^10 + 15*cos(x)^8 - 20*cos(x)^6 + 15*cos(x)^4 - 6*cos(x)^2 + 1)*sin(
x))
```

3.332.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(-\cos(x)+\sec(x))^7}\,dx = \text{Timed out}$$

input
```
integrate(1/(-cos(x)+sec(x))**7,x)
```

output
```
Timed out
```

3.332.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. 2(25) = 50.

Time = 0.22 (sec) , antiderivative size = 169, normalized size of antiderivative = 5.12

$$\int \frac{1}{(-\cos(x)+\sec(x))^7}\,dx = \frac{\left(\frac{273\sin(x)^2}{(\cos(x)+1)^2}+\frac{2002\sin(x)^4}{(\cos(x)+1)^4}-\frac{2574\sin(x)^6}{(\cos(x)+1)^6}-\frac{9009\sin(x)^8}{(\cos(x)+1)^8}+\frac{15015\sin(x)^{10}}{(\cos(x)+1)^{10}}+\frac{60060\sin(x)^{12}}{(\cos(x)+1)^{12}}-231\right)(\cos(x)+1)^{13}}{24600576\sin(x)^{13}} + \frac{5\sin(x)}{2048(\cos(x)+1)} + \frac{5\sin(x)^3}{8192(\cos(x)+1)^3} - \frac{3\sin(x)^5}{8192(\cos(x)+1)^5} - \frac{3\sin(x)^7}{28672(\cos(x)+1)^7} + \frac{\sin(x)^9}{12288(\cos(x)+1)^9} + \frac{\sin(x)^{11}}{90112(\cos(x)+1)^{11}} - \frac{\sin(x)^{13}}{106496(\cos(x)+1)^{13}}$$

input
```
integrate(1/(-cos(x)+sec(x))^7,x, algorithm="maxima")
```

output
```
1/24600576*(273*sin(x)^2/(cos(x) + 1)^2 + 2002*sin(x)^4/(cos(x) + 1)^4 - 2
574*sin(x)^6/(cos(x) + 1)^6 - 9009*sin(x)^8/(cos(x) + 1)^8 + 15015*sin(x)^
10/(cos(x) + 1)^10 + 60060*sin(x)^12/(cos(x) + 1)^12 - 231)*(cos(x) + 1)^1
3/sin(x)^13 + 5/2048*sin(x)/(cos(x) + 1) + 5/8192*sin(x)^3/(cos(x) + 1)^3
- 3/8192*sin(x)^5/(cos(x) + 1)^5 - 3/28672*sin(x)^7/(cos(x) + 1)^7 + 1/122
88*sin(x)^9/(cos(x) + 1)^9 + 1/90112*sin(x)^11/(cos(x) + 1)^11 - 1/106496*
sin(x)^13/(cos(x) + 1)^13
```

3.332.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int \frac{1}{(-\cos(x)+\sec(x))^7}\,dx = \frac{429\sin(x)^6-1001\sin(x)^4+819\sin(x)^2-231}{3003\sin(x)^{13}}$$

input
```
integrate(1/(-cos(x)+sec(x))^7,x, algorithm="giac")
```

output
```
1/3003*(429*sin(x)^6 - 1001*sin(x)^4 + 819*sin(x)^2 - 231)/sin(x)^13
```

3.332.9 Mupad [B] (verification not implemented)

Time = 27.34 (sec) , antiderivative size = 109, normalized size of antiderivative = 3.30

$$\int \frac{1}{(-\cos(x)+\sec(x))^7}\,dx = -\frac{\cot\left(\frac{x}{2}\right)^{13}}{106496} + \frac{\cot\left(\frac{x}{2}\right)^{11}}{90112} + \frac{\cot\left(\frac{x}{2}\right)^{9}}{12288} - \frac{3\cot\left(\frac{x}{2}\right)^{7}}{28672} - \frac{3\cot\left(\frac{x}{2}\right)^{5}}{8192} + \frac{5\cot\left(\frac{x}{2}\right)^{3}}{8192} + \frac{5\cot\left(\frac{x}{2}\right)}{2048} - \frac{\tan\left(\frac{x}{2}\right)^{13}}{106496} + \frac{\tan\left(\frac{x}{2}\right)^{11}}{90112} + \frac{\tan\left(\frac{x}{2}\right)^{9}}{12288} - \frac{3\tan\left(\frac{x}{2}\right)^{7}}{28672} - \frac{3\tan\left(\frac{x}{2}\right)^{5}}{8192} + \frac{5\tan\left(\frac{x}{2}\right)^{3}}{8192} + \frac{5\tan\left(\frac{x}{2}\right)}{2048}$$

input
```
int(-1/(cos(x) - 1/cos(x))^7,x)
```

output
```
(5*cot(x/2))/2048 + (5*tan(x/2))/2048 + (5*cot(x/2)^3)/8192 - (3*cot(x/2)^
5)/8192 - (3*cot(x/2)^7)/28672 + cot(x/2)^9/12288 + cot(x/2)^11/90112 - co
t(x/2)^13/106496 + (5*tan(x/2)^3)/8192 - (3*tan(x/2)^5)/8192 - (3*tan(x/2)
^7)/28672 + tan(x/2)^9/12288 + tan(x/2)^11/90112 - tan(x/2)^13/106496
```

3.333 $\int (-\cos(x)+\sec(x))^{7/2}\,dx$

3.333.1 Optimal result

Integrand size = 11, antiderivative size = 73

$$\int (-\cos(x)+\sec(x))^{7/2}\,dx = -\frac{256}{35}\csc(x)\sqrt{\sin(x)\tan(x)}+\frac{64}{35}\sec(x)\tan(x)\sqrt{\sin(x)\tan(x)} -\frac{8}{7}\sin(x)\tan^2(x)\sqrt{\sin(x)\tan(x)}-\frac{2}{7}\sin^3(x)\tan^2(x)\sqrt{\sin(x)\tan(x)}$$

output
```
-256/35*csc(x)*(sin(x)*tan(x))^(1/2)+64/35*sec(x)*(sin(x)*tan(x))^(1/2)*ta
n(x)-8/7*sin(x)*(sin(x)*tan(x))^(1/2)*tan(x)^2-2/7*sin(x)^3*(sin(x)*tan(x)
)^(1/2)*tan(x)^2
```

3.333.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.51

$$\int (-\cos(x)+\sec(x))^{7/2}\,dx = \frac{1}{70}\sec(x)\sqrt{\sin(x)\tan(x)}(-512\cot(x) -5\cos(x)(-23\sin(x)+\sin(3x))+28\tan(x))$$

input
```
Integrate[(-Cos[x] + Sec[x])^(7/2),x]
```

output
```
(Sec[x]*Sqrt[Sin[x]*Tan[x]]*(-512*Cot[x] - 5*Cos[x]*(-23*Sin[x] + Sin[3*x]
) + 28*Tan[x]))/70
```

3.333.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.33, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.091, Rules used = {3042, 4897, 3042, 4900, 3042, 3078, 3042, 3078, 3042, 3074, 3042, 3069}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\sec(x)-\cos(x))^{7/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (\sec(x)-\cos(x))^{7/2}dx$$

$$\Big\downarrow \text{4897}$$

$$\int (\sin(x)\tan(x))^{7/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int (\sin(x)\tan(x))^{7/2}dx$$

$$\Big\downarrow \text{4900}$$

$$\frac{\sqrt{\sin(x)\tan(x)}\int \sin^{\frac{7}{2}}(x)\tan^{\frac{7}{2}}(x)dx}{\sqrt{\sin(x)}\sqrt{\tan(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sin(x)\tan(x)}\int \sin(x)^{7/2}\tan(x)^{7/2}dx}{\sqrt{\sin(x)}\sqrt{\tan(x)}}$$

$$\Big\downarrow \text{3078}$$

$$\frac{\sqrt{\sin(x)\tan(x)}\left(\frac{12}{7}\int \sin^{\frac{3}{2}}(x)\tan^{\frac{7}{2}}(x)dx-\frac{2}{7}\sin^{\frac{7}{2}}(x)\tan^{\frac{5}{2}}(x)\right)}{\sqrt{\sin(x)}\sqrt{\tan(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sin(x)\tan(x)}\left(\frac{12}{7}\int \sin(x)^{3/2}\tan(x)^{7/2}dx-\frac{2}{7}\sin^{\frac{7}{2}}(x)\tan^{\frac{5}{2}}(x)\right)}{\sqrt{\sin(x)}\sqrt{\tan(x)}}$$

$$\Big\downarrow \text{3078}$$

$$\frac{\sqrt{\sin(x)\tan(x)}\left(\frac{12}{7}\left(\frac{8}{3}\int\frac{\tan^{\frac{7}{2}}(x)}{\sqrt{\sin(x)}}dx-\frac{2}{3}\sin^{\frac{3}{2}}(x)\tan^{\frac{5}{2}}(x)\right)-\frac{2}{7}\sin^{\frac{7}{2}}(x)\tan^{\frac{5}{2}}(x)\right)}{\sqrt{\sin(x)}\sqrt{\tan(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sin(x)\tan(x)}\left(\frac{12}{7}\left(\frac{8}{3}\int\frac{\tan(x)^{7/2}}{\sqrt{\sin(x)}}dx-\frac{2}{3}\sin^{\frac{3}{2}}(x)\tan^{\frac{5}{2}}(x)\right)-\frac{2}{7}\sin^{\frac{7}{2}}(x)\tan^{\frac{5}{2}}(x)\right)}{\sqrt{\sin(x)}\sqrt{\tan(x)}}$$

$$\Big\downarrow \text{3074}$$

$$\frac{\sqrt{\sin(x)\tan(x)}\left(\frac{12}{7}\left(\frac{8}{3}\left(\frac{2\tan^{\frac{5}{2}}(x)}{5\sqrt{\sin(x)}}-\frac{4}{5}\int\frac{\tan^{\frac{3}{2}}(x)}{\sqrt{\sin(x)}}dx\right)-\frac{2}{3}\sin^{\frac{3}{2}}(x)\tan^{\frac{5}{2}}(x)\right)-\frac{2}{7}\sin^{\frac{7}{2}}(x)\tan^{\frac{5}{2}}(x)\right)}{\sqrt{\sin(x)}\sqrt{\tan(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sin(x)\tan(x)}\left(\frac{12}{7}\left(\frac{8}{3}\left(\frac{2\tan^{\frac{5}{2}}(x)}{5\sqrt{\sin(x)}}-\frac{4}{5}\int\frac{\tan(x)^{3/2}}{\sqrt{\sin(x)}}dx\right)-\frac{2}{3}\sin^{\frac{3}{2}}(x)\tan^{\frac{5}{2}}(x)\right)-\frac{2}{7}\sin^{\frac{7}{2}}(x)\tan^{\frac{5}{2}}(x)\right)}{\sqrt{\sin(x)}\sqrt{\tan(x)}}$$

$$\Big\downarrow \text{3069}$$

$$\frac{\sqrt{\sin(x)\tan(x)}\left(\frac{12}{7}\left(\frac{8}{3}\left(\frac{2\tan^{\frac{5}{2}}(x)}{5\sqrt{\sin(x)}}-\frac{8\sqrt{\tan(x)}}{5\sqrt{\sin(x)}}\right)-\frac{2}{3}\sin^{\frac{3}{2}}(x)\tan^{\frac{5}{2}}(x)\right)-\frac{2}{7}\sin^{\frac{7}{2}}(x)\tan^{\frac{5}{2}}(x)\right)}{\sqrt{\sin(x)}\sqrt{\tan(x)}}$$

input
```
Int[(-Cos[x] + Sec[x])^(7/2),x]
```

output
```
(Sqrt[Sin[x]*Tan[x]]*((-2*Sin[x]^(7/2)*Tan[x]^(5/2))/7 + (12*((-2*Sin[x]^(
3/2)*Tan[x]^(5/2))/3 + (8*((-8*Sqrt[Tan[x]])/(5*Sqrt[Sin[x]]) + (2*Tan[x]^
(5/2))/(5*Sqrt[Sin[x]])))/3))/7))/(Sqrt[Sin[x]*Sqrt[Tan[x]])
```

3.333.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3069
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

rule 3074
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(n
 - 1))), x] - Simp[b^2*((m + n - 1)/(n - 1))   Int[(a*Sin[e + f*x])^m*(b*Ta
n[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && In
tegersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

rule 4900
```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Simp[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p]))   Int[uu*vv^(m*p)*ww^(n*p), x]
, x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !
InertTrigFreeQ[w])
```

3.333.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(57) = 114$.

Time = 11.32 (sec) , antiderivative size = 286, normalized size of antiderivative = 3.92

method	result
default	$\sec(x)^2 \csc(x) \left(20 \cos(x)^6 + 105 \ln\left(\frac{4 \cos(x) \sqrt{-\frac{\cos(x)}{(\cos(x)+1)^2}} - 2 \cos(x) + 4 \sqrt{-\frac{\cos(x)}{(\cos(x)+1)^2}} + 2}{\cos(x)+1}\right) \cos(x)^3 \sqrt{-\frac{\cos(x)}{(\cos(x)+1)^2}} - 105 \ln\left(\frac{2 \cos(x) \sqrt{\cdots}}{\cdots}\right.\right.$

input
```
int((-cos(x)+sec(x))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/70*sec(x)^2*csc(x)*(20*cos(x)^6+105*ln(2*(2*cos(x)*(-cos(x)/(cos(x)+1)^2
)^(1/2)+2*(-cos(x)/(cos(x)+1)^2)^(1/2)-cos(x)+1)/(cos(x)+1))*cos(x)^3*(-co
s(x)/(cos(x)+1)^2)^(1/2)-105*ln((2*cos(x)*(-cos(x)/(cos(x)+1)^2)^(1/2)+2*(
-cos(x)/(cos(x)+1)^2)^(1/2)-cos(x)+1)/(cos(x)+1))*cos(x)^3*(-cos(x)/(cos(x
)+1)^2)^(1/2)-140*cos(x)^4+105*(-cos(x)/(cos(x)+1)^2)^(1/2)*ln(2*(2*cos(x)
*(-cos(x)/(cos(x)+1)^2)^(1/2)+2*(-cos(x)/(cos(x)+1)^2)^(1/2)-cos(x)+1)/(co
s(x)+1))*cos(x)^2-105*(-cos(x)/(cos(x)+1)^2)^(1/2)*ln((2*cos(x)*(-cos(x)/(
cos(x)+1)^2)^(1/2)+2*(-cos(x)/(cos(x)+1)^2)^(1/2)-cos(x)+1)/(cos(x)+1))*co
s(x)^2-420*cos(x)^2+28)*(sin(x)*tan(x))^(1/2)
```

3.333.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.60

$$\int (-\cos(x)+\sec(x))^{7/2}\,dx = \frac{2\left(5\,\cos\left(x\right)^{6}-35\,\cos\left(x\right)^{4}-105\,\cos\left(x\right)^{2}+7\right)\sqrt{-\frac{\cos(x)^{2}-1}{\cos(x)}}}{35\,\cos\left(x\right)^{2}\sin\left(x\right)}$$

input
```
integrate((-cos(x)+sec(x))^(7/2),x, algorithm="fricas")
```

output
```
2/35*(5*cos(x)^6 - 35*cos(x)^4 - 105*cos(x)^2 + 7)*sqrt(-(cos(x)^2 - 1)/co
s(x))/(cos(x)^2*sin(x))
```

3.333.6 Sympy [F(-1)]

Timed out.

$$\int (-\cos(x)+\sec(x))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate((-cos(x)+sec(x))**(7/2),x)
```

output
```
Timed out
```

3.333.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.12

$$\int (-\cos(x)+\sec(x))^{7/2}\,dx = \frac{128\left(\frac{7\sin(x)^4}{(\cos(x)+1)^4}-\frac{7\sin(x)^{10}}{(\cos(x)+1)^{10}}+\frac{2\sin(x)^{14}}{(\cos(x)+1)^{14}}-2\right)}{35\left(\frac{\sin(x)}{\cos(x)+1}+1\right)^{\frac{7}{2}}\left(-\frac{\sin(x)}{\cos(x)+1}+1\right)^{\frac{7}{2}}\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)^{\frac{7}{2}}}$$

input
```
integrate((-cos(x)+sec(x))^(7/2),x, algorithm="maxima")
```

output
```
128/35*(7*sin(x)^4/(cos(x) + 1)^4 - 7*sin(x)^10/(cos(x) + 1)^10 + 2*sin(x)
^14/(cos(x) + 1)^14 - 2)/((sin(x)/(cos(x) + 1) + 1)^(7/2)*(-sin(x)/(cos(x)
 + 1) + 1)^(7/2)*(sin(x)^2/(cos(x) + 1)^2 + 1)^(7/2))
```

3.333.8 Giac [F]

$$\int (-\cos(x)+\sec(x))^{7/2}\,dx = \int (-\cos(x)+\sec(x))^{\frac{7}{2}}\,dx$$

input
```
integrate((-cos(x)+sec(x))^(7/2),x, algorithm="giac")
```

output
```
integrate((-cos(x) + sec(x))^(7/2), x)
```

3.333.9 Mupad [F(-1)]

Timed out.

$$\int (-\cos(x)+\sec(x))^{7/2}\,dx = \int \left(\frac{1}{\cos(x)}-\cos(x)\right)^{7/2}\,dx$$

input
```
int((1/cos(x) - cos(x))^(7/2),x)
```

output
```
int((1/cos(x) - cos(x))^(7/2), x)
```

3.334 $\int (-\cos(x)+\sec(x))^{5/2}\,dx$

3.334.1 Optimal result

Integrand size = 11, antiderivative size = 50

$$\int (-\cos(x)+\sec(x))^{5/2}\,dx = \frac{64}{15}\cot(x)\sqrt{\sin(x)\tan(x)} + \frac{16}{15}\tan(x)\sqrt{\sin(x)\tan(x)} - \frac{2}{5}\sin^2(x)\tan(x)\sqrt{\sin(x)\tan(x)}$$

output
```
64/15*cot(x)*(sin(x)*tan(x))^(1/2)+16/15*(sin(x)*tan(x))^(1/2)*tan(x)-2/5*
sin(x)^2*(sin(x)*tan(x))^(1/2)*tan(x)
```

3.334.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.58

$$\int (-\cos(x)+\sec(x))^{5/2}\,dx = \frac{2}{15}\left(5+3\cos^2(x)+32\cot^2(x)\right)\tan(x)\sqrt{\sin(x)\tan(x)}$$

input
```
Integrate[(-Cos[x] + Sec[x])^(5/2),x]
```

output
```
(2*(5 + 3*Cos[x]^2 + 32*Cot[x]^2)*Tan[x]*Sqrt[Sin[x]*Tan[x]])/15
```

3.334.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.52, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.909, Rules used = {3042, 4897, 3042, 4900, 3042, 3078, 3042, 3074, 3042, 3069}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\sec(x)-\cos(x))^{5/2} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int (\sec(x)-\cos(x))^{5/2}dx$$

$$\Big\downarrow \text{4897}$$

$$\int (\sin(x)\tan(x))^{5/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int (\sin(x)\tan(x))^{5/2}dx$$

$$\Big\downarrow \text{4900}$$

$$\frac{\sqrt{\sin(x)\tan(x)}\int \sin^{\frac{5}{2}}(x)\tan^{\frac{5}{2}}(x)dx}{\sqrt{\sin(x)}\sqrt{\tan(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sin(x)\tan(x)}\int \sin(x)^{5/2}\tan(x)^{5/2}dx}{\sqrt{\sin(x)}\sqrt{\tan(x)}}$$

$$\Big\downarrow \text{3078}$$

$$\frac{\sqrt{\sin(x)\tan(x)}\left(\frac{8}{5}\int \sqrt{\sin(x)}\tan^{\frac{5}{2}}(x)dx-\frac{2}{5}\sin^{\frac{5}{2}}(x)\tan^{\frac{3}{2}}(x)\right)}{\sqrt{\sin(x)}\sqrt{\tan(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sin(x)\tan(x)}\left(\frac{8}{5}\int \sqrt{\sin(x)}\tan(x)^{5/2}dx-\frac{2}{5}\sin^{\frac{5}{2}}(x)\tan^{\frac{3}{2}}(x)\right)}{\sqrt{\sin(x)}\sqrt{\tan(x)}}$$

$$\Big\downarrow \text{3074}$$

$$\frac{\sqrt{\sin(x)\tan(x)}\left(\frac{8}{5}\left(\frac{2}{3}\sqrt{\sin(x)}\tan^{\frac{3}{2}}(x)-\frac{4}{3}\int\sqrt{\sin(x)}\sqrt{\tan(x)}dx\right)-\frac{2}{5}\sin^{\frac{5}{2}}(x)\tan^{\frac{3}{2}}(x)\right)}{\sqrt{\sin(x)}\sqrt{\tan(x)}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\sqrt{\sin(x)\tan(x)}\left(\frac{8}{5}\left(\frac{2}{3}\sqrt{\sin(x)}\tan^{\frac{3}{2}}(x)-\frac{4}{3}\int\sqrt{\sin(x)}\sqrt{\tan(x)}dx\right)-\frac{2}{5}\sin^{\frac{5}{2}}(x)\tan^{\frac{3}{2}}(x)\right)}{\sqrt{\sin(x)}\sqrt{\tan(x)}}$$

$$\Big\downarrow \ 3069$$

$$\frac{\sqrt{\sin(x)\tan(x)}\left(\frac{8}{5}\left(\frac{2}{3}\sqrt{\sin(x)}\tan^{\frac{3}{2}}(x)+\frac{8\sqrt{\sin(x)}}{3\sqrt{\tan(x)}}\right)-\frac{2}{5}\sin^{\frac{5}{2}}(x)\tan^{\frac{3}{2}}(x)\right)}{\sqrt{\sin(x)}\sqrt{\tan(x)}}$$

input
```
Int[(-Cos[x] + Sec[x])^(5/2),x]
```

output
```
(Sqrt[Sin[x]*Tan[x]]*((-2*Sin[x]^(5/2)*Tan[x]^(3/2))/5 + (8*((8*Sqrt[Sin[x
]])/(3*Sqrt[Tan[x]]) + (2*Sqrt[Sin[x]]*Tan[x]^(3/2))/3))/5))/(Sqrt[Sin[x]]
*Sqrt[Tan[x]])
```

3.334.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3069
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

rule 3074
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(n
 - 1))), x] - Simp[b^2*((m + n - 1)/(n - 1))   Int[(a*Sin[e + f*x])^m*(b*Ta
n[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && In
tegersQ[2*m, 2*n] &&  !(GtQ[m, 1] &&  !IntegerQ[(m - 1)/2])
```

3.334. $\int (-\cos(x)+\sec(x))^{5/2}\,dx$

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

rule 4900
```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Simp[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p]))   Int[uu*vv^(m*p)*ww^(n*p), x]
, x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !
InertTrigFreeQ[w])
```

3.334.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(38) = 76$.

Time = 9.23 (sec) , antiderivative size = 280, normalized size of antiderivative = 5.60

method	result
default	$\tan(x)\left(6\cos(x)^4-15\sqrt{-\frac{\cos(x)}{(\cos(x)+1)^2}}\ \ln\left(\frac{2\cos(x)\sqrt{-\frac{\cos(x)}{(\cos(x)+1)^2}}+2\sqrt{-\frac{\cos(x)}{(\cos(x)+1)^2}}-\cos(x)+1}{\cos(x)+1}\right)\cos(x)^2+15\sqrt{-\frac{\cos(x)}{(\cos(x)+1)^2}}\ \ln\left(\frac{4\cos(\right.\right.$

input
```
int((-cos(x)+sec(x))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*tan(x)*(6*cos(x)^4-15*(-cos(x)/(cos(x)+1)^2)^(1/2)*ln((2*cos(x)*(-cos
(x)/(cos(x)+1)^2)^(1/2)+2*(-cos(x)/(cos(x)+1)^2)^(1/2)-cos(x)+1)/(cos(x)+1
))*cos(x)^2+15*(-cos(x)/(cos(x)+1)^2)^(1/2)*ln(2*(2*cos(x)*(-cos(x)/(cos(x
)+1)^2)^(1/2)+2*(-cos(x)/(cos(x)+1)^2)^(1/2)-cos(x)+1)/(cos(x)+1))*cos(x)^
2-15*(-cos(x)/(cos(x)+1)^2)^(1/2)*ln((2*cos(x)*(-cos(x)/(cos(x)+1)^2)^(1/2
)+2*(-cos(x)/(cos(x)+1)^2)^(1/2)-cos(x)+1)/(cos(x)+1))*cos(x)+15*(-cos(x)/
(cos(x)+1)^2)^(1/2)*ln(2*(2*cos(x)*(-cos(x)/(cos(x)+1)^2)^(1/2)+2*(-cos(x)
/(cos(x)+1)^2)^(1/2)-cos(x)+1)/(cos(x)+1))*cos(x)-60*cos(x)^2-10)*(sin(x)*
tan(x))^(1/2)/(cos(x)^2-1)
```

3.334.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76

$$\int (-\cos(x)+\sec(x))^{5/2}\,dx = -\frac{2\,(3\,\cos(x)^4-30\,\cos(x)^2-5)\sqrt{-\frac{\cos(x)^2-1}{\cos(x)}}}{15\,\cos(x)\sin(x)}$$

input
```
integrate((-cos(x)+sec(x))^(5/2),x, algorithm="fricas")
```

output
```
-2/15*(3*cos(x)^4 - 30*cos(x)^2 - 5)*sqrt(-(cos(x)^2 - 1)/cos(x))/(cos(x)*
sin(x))
```

3.334.6 Sympy [F(-1)]

Timed out.

$$\int (-\cos(x)+\sec(x))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((-cos(x)+sec(x))**(5/2),x)
```

output
```
Timed out
```

3.334.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(38) = 76.

Time = 0.32 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.64

$$\int (-\cos(x)+\sec(x))^{5/2}\,dx = -\frac{32\left(\frac{5\,\sin(x)^4}{(\cos(x)+1)^4}-\frac{5\,\sin(x)^6}{(\cos(x)+1)^6}+\frac{2\,\sin(x)^{10}}{(\cos(x)+1)^{10}}-2\right)}{15\left(\frac{\sin(x)}{\cos(x)+1}+1\right)^{\frac{5}{2}}\left(-\frac{\sin(x)}{\cos(x)+1}+1\right)^{\frac{5}{2}}\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)^{\frac{5}{2}}}$$

input
```
integrate((-cos(x)+sec(x))^(5/2),x, algorithm="maxima")
```

output
```
-32/15*(5*sin(x)^4/(cos(x) + 1)^4 - 5*sin(x)^6/(cos(x) + 1)^6 + 2*sin(x)^1
0/(cos(x) + 1)^10 - 2)/((sin(x)/(cos(x) + 1) + 1)^(5/2)*(-sin(x)/(cos(x) +
 1) + 1)^(5/2)*(sin(x)^2/(cos(x) + 1)^2 + 1)^(5/2))
```

3.334.8 Giac [F]

$$\int (-\cos(x) + \sec(x))^{5/2} \, dx = \int (-\cos(x) + \sec(x))^{\frac{5}{2}} \, dx$$

input
```
integrate((-cos(x)+sec(x))^(5/2),x, algorithm="giac")
```

output
```
integrate((-cos(x) + sec(x))^(5/2), x)
```

3.334.9 Mupad [F(-1)]

Timed out.

$$\int (-\cos(x) + \sec(x))^{5/2} \, dx = \int \left(\frac{1}{\cos(x)} - \cos(x)\right)^{5/2} dx$$

input
```
int((1/cos(x) - cos(x))^(5/2),x)
```

output
```
int((1/cos(x) - cos(x))^(5/2), x)
```

3.335 $\int (-\cos(x)+\sec(x))^{3/2}\,dx$

3.335.1 Optimal result

Integrand size = 11, antiderivative size = 31

$$\int (-\cos(x)+\sec(x))^{3/2}\,dx=\frac{8}{3}\csc(x)\sqrt{\sin(x)\tan(x)}-\frac{2}{3}\sin(x)\sqrt{\sin(x)\tan(x)}$$

output
```
8/3*csc(x)*(sin(x)*tan(x))^(1/2)-2/3*sin(x)*(sin(x)*tan(x))^(1/2)
```

3.335.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.74

$$\int (-\cos(x)+\sec(x))^{3/2}\,dx=\frac{2}{3}\left(-1+4\csc^2(x)\right)\sin(x)\sqrt{\sin(x)\tan(x)}$$

input
```
Integrate[(-Cos[x] + Sec[x])^(3/2),x]
```

output
```
(2*(-1 + 4*Csc[x]^2)*Sin[x]*Sqrt[Sin[x]*Tan[x]])/3
```

3.335.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.77, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {3042, 4897, 3042, 4900, 3042, 3078, 3042, 3069}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\sec(x)-\cos(x))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (\sec(x)-\cos(x))^{3/2}dx$$

$$\Big\downarrow \text{4897}$$

$$\int (\sin(x)\tan(x))^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int (\sin(x)\tan(x))^{3/2}dx$$

$$\Big\downarrow \text{4900}$$

$$\frac{\sqrt{\sin(x)\tan(x)}\int \sin^{\frac{3}{2}}(x)\tan^{\frac{3}{2}}(x)dx}{\sqrt{\sin(x)}\sqrt{\tan(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sin(x)\tan(x)}\int \sin(x)^{3/2}\tan(x)^{3/2}dx}{\sqrt{\sin(x)}\sqrt{\tan(x)}}$$

$$\Big\downarrow \text{3078}$$

$$\frac{\sqrt{\sin(x)\tan(x)}\left(\frac{4}{3}\int \frac{\tan^{\frac{3}{2}}(x)}{\sqrt{\sin(x)}}dx-\frac{2}{3}\sin^{\frac{3}{2}}(x)\sqrt{\tan(x)}\right)}{\sqrt{\sin(x)}\sqrt{\tan(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sin(x)\tan(x)}\left(\frac{4}{3}\int \frac{\tan(x)^{3/2}}{\sqrt{\sin(x)}}dx-\frac{2}{3}\sin^{\frac{3}{2}}(x)\sqrt{\tan(x)}\right)}{\sqrt{\sin(x)}\sqrt{\tan(x)}}$$

$$\Big\downarrow \text{3069}$$

$$\frac{\left(\frac{8\sqrt{\tan(x)}}{3\sqrt{\sin(x)}}-\frac{2}{3}\sin^{\frac{3}{2}}(x)\sqrt{\tan(x)}\right)\sqrt{\sin(x)\tan(x)}}{\sqrt{\sin(x)}\sqrt{\tan(x)}}$$

input
```
Int[(-Cos[x] + Sec[x])^(3/2),x]
```

output
```
(((8*Sqrt[Tan[x]])/(3*Sqrt[Sin[x]]) - (2*Sin[x]^(3/2)*Sqrt[Tan[x]])/3)*Sqr
t[Sin[x]*Tan[x]])/(Sqrt[Sin[x]]*Sqrt[Tan[x]])
```

3.335.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3069
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

rule 3078
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(
f*m)), x] + Simp[a^2*((m + n - 1)/m)   Int[(a*Sin[e + f*x])^(m - 2)*(b*Tan[
e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && (GtQ[m, 1] || (EqQ[m, 1
] && EqQ[n, 1/2])) && IntegersQ[2*m, 2*n]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

rule 4900
```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Simp[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p]))   Int[uu*vv^(m*p)*ww^(n*p), x]
, x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !
InertTrigFreeQ[w])
```

3.335.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 518 vs. $2(23) = 46$.

Time = 2.21 (sec) , antiderivative size = 519, normalized size of antiderivative = 16.74

method	result	size
default	Expression too large to display	519

input
```
int((-cos(x)+sec(x))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(3*cos(x)^3*ln((2*cos(x)*(-cos(x)/(cos(x)+1)^2)^(1/2)+2*(-cos(x)/(cos(
x)+1)^2)^(1/2)-cos(x)+1)/(cos(x)+1))*(-cos(x)/(cos(x)+1)^2)^(3/2)-3*cos(x)
^3*ln(2*(2*cos(x)*(-cos(x)/(cos(x)+1)^2)^(1/2)+2*(-cos(x)/(cos(x)+1)^2)^(1
/2)-cos(x)+1)/(cos(x)+1))*(-cos(x)/(cos(x)+1)^2)^(3/2)+9*ln((2*cos(x)*(-co
s(x)/(cos(x)+1)^2)^(1/2)+2*(-cos(x)/(cos(x)+1)^2)^(1/2)-cos(x)+1)/(cos(x)+
1))*(-cos(x)/(cos(x)+1)^2)^(3/2)*cos(x)^2-9*ln(2*(2*cos(x)*(-cos(x)/(cos(x
)+1)^2)^(1/2)+2*(-cos(x)/(cos(x)+1)^2)^(1/2)-cos(x)+1)/(cos(x)+1))*(-cos(x
)/(cos(x)+1)^2)^(3/2)*cos(x)^2+9*cos(x)*ln((2*cos(x)*(-cos(x)/(cos(x)+1)^2
)^(1/2)+2*(-cos(x)/(cos(x)+1)^2)^(1/2)-cos(x)+1)/(cos(x)+1))*(-cos(x)/(cos
(x)+1)^2)^(3/2)-9*cos(x)*ln(2*(2*cos(x)*(-cos(x)/(cos(x)+1)^2)^(1/2)+2*(-c
os(x)/(cos(x)+1)^2)^(1/2)-cos(x)+1)/(cos(x)+1))*(-cos(x)/(cos(x)+1)^2)^(3/
2)+3*ln((2*cos(x)*(-cos(x)/(cos(x)+1)^2)^(1/2)+2*(-cos(x)/(cos(x)+1)^2)^(1
/2)-cos(x)+1)/(cos(x)+1))*(-cos(x)/(cos(x)+1)^2)^(3/2)-3*ln(2*(2*cos(x)*(-
cos(x)/(cos(x)+1)^2)^(1/2)+2*(-cos(x)/(cos(x)+1)^2)^(1/2)-cos(x)+1)/(cos(x
)+1))*(-cos(x)/(cos(x)+1)^2)^(3/2)-4*cos(x)^3-12*cos(x))*(sin(x)*tan(x))^(
1/2)*tan(x)/(cos(x)^2-1)
```

3.335.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int (-\cos(x)+\sec(x))^{3/2}\,dx = \frac{2\left(\cos\left(x\right)^{2}+3\right)\sqrt{-\frac{\cos(x)^{2}-1}{\cos(x)}}}{3\,\sin\left(x\right)}$$

input
```
integrate((-cos(x)+sec(x))^(3/2),x, algorithm="fricas")
```

output
```
2/3*(cos(x)^2 + 3)*sqrt(-(cos(x)^2 - 1)/cos(x))/sin(x)
```

3.335.6 Sympy [F]

$$\int (-\cos(x) + \sec(x))^{3/2} \, dx = \int (-\cos(x) + \sec(x))^{\frac{3}{2}} \, dx$$

input
```
integrate((-cos(x)+sec(x))**(3/2),x)
```

output
```
Integral((-cos(x) + sec(x))**(3/2), x)
```

3.335.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(23) = 46.

Time = 0.31 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.84

$$\int (-\cos(x) + \sec(x))^{3/2} \, dx = -\frac{8\left(\frac{\sin(x)^6}{(\cos(x)+1)^6} - 1\right)}{3\left(\frac{\sin(x)}{\cos(x)+1} + 1\right)^{\frac{3}{2}}\left(-\frac{\sin(x)}{\cos(x)+1} + 1\right)^{\frac{3}{2}}\left(\frac{\sin(x)^2}{(\cos(x)+1)^2} + 1\right)^{\frac{3}{2}}}$$

input
```
integrate((-cos(x)+sec(x))^(3/2),x, algorithm="maxima")
```

output
```
-8/3*(sin(x)^6/(cos(x) + 1)^6 - 1)/((sin(x)/(cos(x) + 1) + 1)^(3/2)*(-sin(
x)/(cos(x) + 1) + 1)^(3/2)*(sin(x)^2/(cos(x) + 1)^2 + 1)^(3/2))
```

3.335.8 Giac [F]

$$\int (-\cos(x) + \sec(x))^{3/2} \, dx = \int (-\cos(x) + \sec(x))^{\frac{3}{2}} \, dx$$

input
```
integrate((-cos(x)+sec(x))^(3/2),x, algorithm="giac")
```

output
```
integrate((-cos(x) + sec(x))^(3/2), x)
```

3.335.9 Mupad [F(-1)]

Timed out.

$$\int (-\cos(x)+\sec(x))^{3/2}\,dx=\int \left(\frac{1}{\cos\left(x\right)}-\cos\left(x\right)\right)^{3/2}\,dx$$

input
```
int((1/cos(x) - cos(x))^(3/2),x)
```

output
```
int((1/cos(x) - cos(x))^(3/2), x)
```

3.336 $\int \sqrt{-\cos(x)+\sec(x)}\,dx$

3.336.1 Optimal result

Integrand size = 11, antiderivative size = 13

$$\int \sqrt{-\cos(x)+\sec(x)}\,dx = -2\cot(x)\sqrt{\sin(x)\tan(x)}$$

output
```
-2*cot(x)*(sin(x)*tan(x))^(1/2)
```

3.336.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \sqrt{-\cos(x)+\sec(x)}\,dx = -2\cot(x)\sqrt{\sin(x)\tan(x)}$$

input
```
Integrate[Sqrt[-Cos[x] + Sec[x]],x]
```

output
```
-2*Cot[x]*Sqrt[Sin[x]*Tan[x]]
```

3.336.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {3042, 4897, 3042, 4900, 3042, 3069}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sec(x)-\cos(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sec(x)-\cos(x)}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \sqrt{\sin(x)\tan(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin(x)\tan(x)}dx$$

$$\Big\downarrow \text{4900}$$

$$\frac{\sqrt{\sin(x)\tan(x)}\int \sqrt{\sin(x)}\sqrt{\tan(x)}dx}{\sqrt{\sin(x)}\sqrt{\tan(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sin(x)\tan(x)}\int \sqrt{\sin(x)}\sqrt{\tan(x)}dx}{\sqrt{\sin(x)}\sqrt{\tan(x)}}$$

$$\Big\downarrow \text{3069}$$

$$-2\cot(x)\sqrt{\sin(x)\tan(x)}$$

input
```
Int[Sqrt[-Cos[x] + Sec[x]],x]
```

output
```
-2*Cot[x]*Sqrt[Sin[x]*Tan[x]]
```

3.336.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3069
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b)*(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*
m)), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n - 1, 0]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

rule 4900
```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Simp[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p]))   Int[uu*vv^(m*p)*ww^(n*p), x]
, x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !
InertTrigFreeQ[w])
```

3.336.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.77 (sec) , antiderivative size = 50, normalized size of antiderivative = 3.85

method	result
risch	$-\frac{i\sqrt{2}\,\sqrt{-\frac{\left(e^{2ix}-1\right)^{2} e^{-ix}}{e^{2ix}+1}}\,\left(e^{2ix}+1\right)}{e^{2ix}-1}$
default	$-\frac{\cot(x)\left(4\cos(x)\sqrt{-\frac{\cos(x)}{(\cos(x)+1)^{2}}}+4\sqrt{-\frac{\cos(x)}{(\cos(x)+1)^{2}}}+\ln\left(\frac{4\cos(x)\sqrt{-\frac{\cos(x)}{(\cos(x)+1)^{2}}}-2\cos(x)+4\sqrt{-\frac{\cos(x)}{(\cos(x)+1)^{2}}}+2}{\cos(x)+1}\right)-\ln\left(\frac{2\cos(x)\sqrt{-\cdots}}{\cdots}\right.\cdots}{2(\cos(x)+1)\sqrt{-\frac{\cos(x)}{(\cos(x)+1)^{2}}}}$

input
```
int((-cos(x)+sec(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-I*2^(1/2)*(-(exp(2*I*x)-1)^2*exp(-I*x)/(exp(2*I*x)+1))^(1/2)/(exp(2*I*x)-
1)*(exp(2*I*x)+1)
```

3.336.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int \sqrt{-\cos(x)+\sec(x)}\,dx = -\frac{2\sqrt{-\frac{\cos(x)^2-1}{\cos(x)}}\cos(x)}{\sin(x)}$$

input
```
integrate((-cos(x)+sec(x))^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(-(cos(x)^2 - 1)/cos(x))*cos(x)/sin(x)
```

3.336.6 Sympy [F]

$$\int \sqrt{-\cos(x)+\sec(x)}\,dx = \int \sqrt{-\cos(x)+\sec(x)}\,dx$$

input
```
integrate((-cos(x)+sec(x))**(1/2),x)
```

output
```
Integral(sqrt(-cos(x) + sec(x)), x)
```

3.336.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(11) = 22.

Time = 0.31 (sec) , antiderivative size = 57, normalized size of antiderivative = 4.38

$$\int \sqrt{-\cos(x)+\sec(x)}\,dx = \frac{2\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}-1\right)}{\sqrt{\frac{\sin(x)}{\cos(x)+1}+1}\sqrt{-\frac{\sin(x)}{\cos(x)+1}+1}\sqrt{\frac{\sin(x)^2}{(\cos(x)+1)^2}+1}}$$

input
```
integrate((-cos(x)+sec(x))^(1/2),x, algorithm="maxima")
```

output
```
2*(sin(x)^2/(cos(x) + 1)^2 - 1)/(sqrt(sin(x)/(cos(x) + 1) + 1)*sqrt(-sin(x
)/(cos(x) + 1) + 1)*sqrt(sin(x)^2/(cos(x) + 1)^2 + 1))
```

3.336.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(11) = 22$.

Time = 0.32 (sec) , antiderivative size = 46, normalized size of antiderivative = 3.54

$$\int \sqrt{-\cos(x)+\sec(x)}\,dx = -\frac{4\,\mathrm{sgn}\left(-\tan\left(\frac{1}{2}\,x\right)^3-\tan\left(\frac{1}{2}\,x\right)\right)\mathrm{sgn}(\cos(x))}{\frac{\sqrt{-\tan\left(\frac{1}{2}\,x\right)^4+1}-1}{\tan\left(\frac{1}{2}\,x\right)^2}-1}$$

input
```
integrate((-cos(x)+sec(x))^(1/2),x, algorithm="giac")
```

output
```
-4*sgn(-tan(1/2*x)^3 - tan(1/2*x))*sgn(cos(x))/((sqrt(-tan(1/2*x)^4 + 1) -
 1)/tan(1/2*x)^2 - 1)
```

3.336.9 Mupad [B] (verification not implemented)

Time = 28.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.54

$$\int \sqrt{-\cos(x)+\sec(x)}\,dx = -\frac{2\,\sin(x)}{\sqrt{\frac{1}{\cos(x)}}\,\sqrt{1-\cos(x)^2}}$$

input
```
int((1/cos(x) - cos(x))^(1/2),x)
```

output
```
-(2*sin(x))/((1/cos(x))^(1/2)*(1 - cos(x)^2)^(1/2))
```

3.337 $\int \frac{1}{\sqrt{-\cos(x)+\sec(x)}}\,dx$

3.337.1 Optimal result

Integrand size = 11, antiderivative size = 52

$$\int \frac{1}{\sqrt{-\cos(x)+\sec(x)}}\,dx = \frac{\arctan\left(\sqrt{\cos(x)}\right)\sin(x)}{\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}} - \frac{\operatorname{arctanh}\left(\sqrt{\cos(x)}\right)\sin(x)}{\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}$$

output
```
arctan(cos(x)^(1/2))*sin(x)/cos(x)^(1/2)/(sin(x)*tan(x))^(1/2)-arctanh(cos
(x)^(1/2))*sin(x)/cos(x)^(1/2)/(sin(x)*tan(x))^(1/2)
```

3.337.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{-\cos(x)+\sec(x)}}\,dx = \frac{\left(\arctan\left(\sqrt[4]{\cos^2(x)}\right) - \operatorname{arctanh}\left(\sqrt[4]{\cos^2(x)}\right)\right)\cos(x)\cot(x)\sqrt{\sin(x)\tan(x)}}{\cos^2(x)^{3/4}}$$

input
```
Integrate[1/Sqrt[-Cos[x] + Sec[x]],x]
```

output
```
((ArcTan[(Cos[x]^2)^(1/4)] - ArcTanh[(Cos[x]^2)^(1/4)])*Cos[x]*Cot[x]*Sqrt
[Sin[x]*Tan[x]])/(Cos[x]^2)^(3/4)
```

3.337.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.091$, Rules used = {3042, 4897, 3042, 4900, 3042, 3081, 3042, 3045, 266, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sec(x)-\cos(x)}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sqrt{\sec(x)-\cos(x)}}dx$$

$$\Big\downarrow \; 4897$$

$$\int \frac{1}{\sqrt{\sin(x)\tan(x)}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\sqrt{\sin(x)\tan(x)}}dx$$

$$\Big\downarrow \; 4900$$

$$\frac{\sqrt{\sin(x)}\sqrt{\tan(x)}\int \frac{1}{\sqrt{\sin(x)}\sqrt{\tan(x)}}dx}{\sqrt{\sin(x)\tan(x)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt{\sin(x)}\sqrt{\tan(x)}\int \frac{1}{\sqrt{\sin(x)}\sqrt{\tan(x)}}dx}{\sqrt{\sin(x)\tan(x)}}$$

$$\Big\downarrow \; 3081$$

$$\frac{\sin(x)\int \sqrt{\cos(x)}\csc(x)dx}{\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sin(x)\int \frac{\sqrt{\cos(x)}}{\sin(x)}dx}{\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}$$

$$\Big\downarrow \; 3045$$

$$-\frac{\sin(x)\int \frac{\sqrt{\cos(x)}}{1-\cos^2(x)}d\cos(x)}{\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}$$

$$\Big\downarrow \text{266}$$

$$-\frac{2\sin(x)\int \frac{\cos(x)}{1-\cos^2(x)}d\sqrt{\cos(x)}}{\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}$$

$$\Big\downarrow \text{827}$$

$$-\frac{2\sin(x)\left(\frac{1}{2}\int \frac{1}{1-\cos(x)}d\sqrt{\cos(x)}-\frac{1}{2}\int \frac{1}{\cos(x)+1}d\sqrt{\cos(x)}\right)}{\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}$$

$$\Big\downarrow \text{216}$$

$$-\frac{2\sin(x)\left(\frac{1}{2}\int \frac{1}{1-\cos(x)}d\sqrt{\cos(x)}-\frac{1}{2}\arctan\left(\sqrt{\cos(x)}\right)\right)}{\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2\sin(x)\left(\frac{1}{2}\text{arctanh}\left(\sqrt{\cos(x)}\right)-\frac{1}{2}\arctan\left(\sqrt{\cos(x)}\right)\right)}{\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}$$

input
```
Int[1/Sqrt[-Cos[x] + Sec[x]],x]
```

output
```
(-2*(-1/2*ArcTan[Sqrt[Cos[x]]] + ArcTanh[Sqrt[Cos[x]]]/2)*Sin[x])/(Sqrt[Co
s[x]]*Sqrt[Sin[x]*Tan[x]])
```

3.337.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.337. $\int \frac{1}{\sqrt{-\cos(x)+\sec(x)}}\,dx$

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

rule 4900
```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Simp[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p]))   Int[uu*vv^(m*p)*ww^(n*p), x]
, x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !
InertTrigFreeQ[w])
```

3.337. $\int \frac{1}{\sqrt{-\cos(x)+\sec(x)}}\,dx$

3.337.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(40) = 80$.

Time = 1.14 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.73

method	result	size
default	$\frac{\sin(x)\left(\arctan\left(\frac{1}{2\sqrt{-\frac{\cos(x)}{(\cos(x)+1)^2}}}\right)+\ln\left(\frac{2\cos(x)\sqrt{-\frac{\cos(x)}{(\cos(x)+1)^2}}+2\sqrt{-\frac{\cos(x)}{(\cos(x)+1)^2}}-\cos(x)+1}{\cos(x)+1}\right)\right)}{2(\cos(x)+1)\sqrt{\sin(x)\tan(x)}\,\sqrt{-\frac{\cos(x)}{(\cos(x)+1)^2}}}$	90

input
```
int(1/(-cos(x)+sec(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*sin(x)*(arctan(1/2/(-cos(x)/(cos(x)+1)^2)^(1/2))+ln((2*cos(x)*(-cos(x)
/(cos(x)+1)^2)^(1/2)+2*(-cos(x)/(cos(x)+1)^2)^(1/2)-cos(x)+1)/(cos(x)+1)))
/(cos(x)+1)/(sin(x)*tan(x))^(1/2)/(-cos(x)/(cos(x)+1)^2)^(1/2)
```

3.337.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.38

$$\int \frac{1}{\sqrt{-\cos(x)+\sec(x)}}\,dx = -\frac{1}{2}\arctan\left(\frac{2\sqrt{-\frac{\cos(x)^2-1}{\cos(x)}}\cos(x)}{(\cos(x)-1)\sin(x)}\right) + \frac{1}{2}\log\left(\frac{(\cos(x)+1)\sin(x)-2\sqrt{-\frac{\cos(x)^2-1}{\cos(x)}}\cos(x)}{(\cos(x)-1)\sin(x)}\right)$$

input
```
integrate(1/(-cos(x)+sec(x))^(1/2),x, algorithm="fricas")
```

output
```
-1/2*arctan(2*sqrt(-(cos(x)^2 - 1)/cos(x))*cos(x)/((cos(x) - 1)*sin(x))) +
 1/2*log(((cos(x) + 1)*sin(x) - 2*sqrt(-(cos(x)^2 - 1)/cos(x))*cos(x))/((c
os(x) - 1)*sin(x)))
```

3.337.6 Sympy [F]

$$\int \frac{1}{\sqrt{-\cos(x)+\sec(x)}}\,dx = \int \frac{1}{\sqrt{-\cos{(x)}+\sec{(x)}}}\,dx$$

input
```
integrate(1/(-cos(x)+sec(x))**(1/2),x)
```

output
```
Integral(1/sqrt(-cos(x) + sec(x)), x)
```

3.337.7 Maxima [F]

$$\int \frac{1}{\sqrt{-\cos(x)+\sec(x)}}\,dx = \int \frac{1}{\sqrt{-\cos{(x)}+\sec{(x)}}}\,dx$$

input
```
integrate(1/(-cos(x)+sec(x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-cos(x) + sec(x)), x)
```

3.337.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{-\cos(x)+\sec(x)}}\,dx = \frac{1}{2}\arcsin\left(\tan\left(\frac{1}{2}x\right)^2\right) - \frac{1}{2}\log\left(-\frac{\sqrt{-\tan\left(\frac{1}{2}x\right)^4+1}-1}{\tan\left(\frac{1}{2}x\right)^2}\right)$$

input
```
integrate(1/(-cos(x)+sec(x))^(1/2),x, algorithm="giac")
```

output
```
1/2*arcsin(tan(1/2*x)^2) - 1/2*log(-(sqrt(-tan(1/2*x)^4 + 1) - 1)/tan(1/2*
x)^2)
```

3.337.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-\cos(x)+\sec(x)}}\,dx = \int \frac{1}{\sqrt{\frac{1}{\cos(x)}-\cos(x)}}\,dx$$

input
```
int(1/(1/cos(x) - cos(x))^(1/2),x)
```

output
```
int(1/(1/cos(x) - cos(x))^(1/2), x)
```

3.338 $\int \frac{1}{(-\cos(x)+\sec(x))^{3/2}}\,dx$

3.338.1 Optimal result

Integrand size = 11, antiderivative size = 72

$$\int \frac{1}{(-\cos(x)+\sec(x))^{3/2}}\,dx = -\frac{\csc(x)}{2\sqrt{\sin(x)\tan(x)}} + \frac{\arctan\left(\sqrt{\cos(x)}\right)\sin(x)}{4\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}} + \frac{\operatorname{arctanh}\left(\sqrt{\cos(x)}\right)\sin(x)}{4\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}$$

output
```
-1/2*csc(x)/(sin(x)*tan(x))^(1/2)+1/4*arctan(cos(x)^(1/2))*sin(x)/cos(x)^(
1/2)/(sin(x)*tan(x))^(1/2)+1/4*arctanh(cos(x)^(1/2))*sin(x)/cos(x)^(1/2)/(
sin(x)*tan(x))^(1/2)
```

3.338.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int \frac{1}{(-\cos(x)+\sec(x))^{3/2}}\,dx = \frac{\cot(x)\left(\arctan\left(\sqrt[4]{\cos^2(x)}\right)+\operatorname{arctanh}\left(\sqrt[4]{\cos^2(x)}\right)-2\sqrt[4]{\cos^2(x)}\csc^2(x)\right)\sqrt{}}{4\sqrt[4]{\cos^2(x)}}$$

input
```
Integrate[(-Cos[x] + Sec[x])^(-3/2),x]
```

output
```
(Cot[x]*(ArcTan[(Cos[x]^2)^(1/4)] + ArcTanh[(Cos[x]^2)^(1/4)] - 2*(Cos[x]^
2)^(1/4)*Csc[x]^2)*Sqrt[Sin[x]*Tan[x]])/(4*(Cos[x]^2)^(1/4))
```

3.338.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.17, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.273, Rules used = {3042, 4897, 3042, 4900, 3042, 3077, 3042, 3081, 3042, 3045, 266, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\sec(x)-\cos(x))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(\sec(x)-\cos(x))^{3/2}}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \frac{1}{(\sin(x)\tan(x))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(\sin(x)\tan(x))^{3/2}}dx$$

$$\Big\downarrow \text{4900}$$

$$\frac{\sqrt{\sin(x)}\sqrt{\tan(x)}\int \frac{1}{\sin^{\frac{3}{2}}(x)\tan^{\frac{3}{2}}(x)}dx}{\sqrt{\sin(x)\tan(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sin(x)}\sqrt{\tan(x)}\int \frac{1}{\sin(x)^{3/2}\tan(x)^{3/2}}dx}{\sqrt{\sin(x)\tan(x)}}$$

$$\Big\downarrow \text{3077}$$

$$\frac{\sqrt{\sin(x)}\sqrt{\tan(x)}\left(-\frac{1}{4}\int \frac{\sqrt{\tan(x)}}{\sin^{\frac{3}{2}}(x)}dx-\frac{1}{2\sin^{\frac{3}{2}}(x)\sqrt{\tan(x)}}\right)}{\sqrt{\sin(x)\tan(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sin(x)}\sqrt{\tan(x)}\left(-\frac{1}{4}\int \frac{\sqrt{\tan(x)}}{\sin(x)^{3/2}}dx-\frac{1}{2\sin^{\frac{3}{2}}(x)\sqrt{\tan(x)}}\right)}{\sqrt{\sin(x)\tan(x)}}$$

$$\Big\downarrow \text{3081}$$

$$\frac{\sqrt{\sin(x)}\sqrt{\tan(x)}\left(-\frac{\sqrt{\cos(x)}\sqrt{\tan(x)}\int\frac{\csc(x)}{\sqrt{\cos(x)}}dx}{4\sqrt{\sin(x)}}-\frac{1}{2\sin^{\frac{3}{2}}(x)\sqrt{\tan(x)}}\right)}{\sqrt{\sin(x)\tan(x)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt{\sin(x)}\sqrt{\tan(x)}\left(-\frac{\sqrt{\cos(x)}\sqrt{\tan(x)}\int\frac{1}{\sqrt{\cos(x)}\sin(x)}dx}{4\sqrt{\sin(x)}}-\frac{1}{2\sin^{\frac{3}{2}}(x)\sqrt{\tan(x)}}\right)}{\sqrt{\sin(x)\tan(x)}}$$

$$\Big\downarrow \; 3045$$

$$\frac{\sqrt{\sin(x)}\sqrt{\tan(x)}\left(\frac{\sqrt{\cos(x)}\sqrt{\tan(x)}\int\frac{1}{\sqrt{\cos(x)}\left(1-\cos^2(x)\right)}d\cos(x)}{4\sqrt{\sin(x)}}-\frac{1}{2\sin^{\frac{3}{2}}(x)\sqrt{\tan(x)}}\right)}{\sqrt{\sin(x)\tan(x)}}$$

$$\Big\downarrow \; 266$$

$$\frac{\sqrt{\sin(x)}\sqrt{\tan(x)}\left(\frac{\sqrt{\cos(x)}\sqrt{\tan(x)}\int\frac{1}{1-\cos^2(x)}d\sqrt{\cos(x)}}{2\sqrt{\sin(x)}}-\frac{1}{2\sin^{\frac{3}{2}}(x)\sqrt{\tan(x)}}\right)}{\sqrt{\sin(x)\tan(x)}}$$

$$\Big\downarrow \; 756$$

$$\frac{\sqrt{\sin(x)}\sqrt{\tan(x)}\left(\frac{\sqrt{\cos(x)}\sqrt{\tan(x)}\left(\frac{1}{2}\int\frac{1}{1-\cos(x)}d\sqrt{\cos(x)}+\frac{1}{2}\int\frac{1}{\cos(x)+1}d\sqrt{\cos(x)}\right)}{2\sqrt{\sin(x)}}-\frac{1}{2\sin^{\frac{3}{2}}(x)\sqrt{\tan(x)}}\right)}{\sqrt{\sin(x)\tan(x)}}$$

$$\Big\downarrow \; 216$$

$$\frac{\sqrt{\sin(x)}\sqrt{\tan(x)}\left(\frac{\sqrt{\cos(x)}\sqrt{\tan(x)}\left(\frac{1}{2}\int\frac{1}{1-\cos(x)}d\sqrt{\cos(x)}+\frac{1}{2}\arctan\left(\sqrt{\cos(x)}\right)\right)}{2\sqrt{\sin(x)}}-\frac{1}{2\sin^{\frac{3}{2}}(x)\sqrt{\tan(x)}}\right)}{\sqrt{\sin(x)\tan(x)}}$$

$$\Big\downarrow \; 219$$

$$\frac{\sqrt{\sin(x)}\sqrt{\tan(x)}\left(\frac{\sqrt{\cos(x)}\sqrt{\tan(x)}\left(\frac{1}{2}\arctan\left(\sqrt{\cos(x)}\right)+\frac{1}{2}\text{arctanh}\left(\sqrt{\cos(x)}\right)\right)}{2\sqrt{\sin(x)}}-\frac{1}{2\sin^{\frac{3}{2}}(x)\sqrt{\tan(x)}}\right)}{\sqrt{\sin(x)\tan(x)}}$$

input
```
Int[(-Cos[x] + Sec[x])^(-3/2),x]
```

3.338. $\int \frac{1}{(-\cos(x)+\sec(x))^{3/2}}\,dx$

output
```
(Sqrt[Sin[x]]*(-1/2*1/(Sin[x]^(3/2)*Sqrt[Tan[x]]) + ((ArcTan[Sqrt[Cos[x]]]
/2 + ArcTanh[Sqrt[Cos[x]]]/2)*Sqrt[Cos[x]*Sqrt[Tan[x]])/(2*Sqrt[Sin[x]]))
*Sqrt[Tan[x]])/Sqrt[Sin[x]*Tan[x]]
```

3.338.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

3.338. $\int \frac{1}{(-\cos(x)+\sec(x))^{3/2}}\,dx$

rule 3077
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(m
 + n + 1))), x] - Simp[(n + 1)/(b^2*(m + n + 1))   Int[(a*Sin[e + f*x])^m*(
b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1]
&& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1]
)
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

rule 4900
```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Simp[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p]))   Int[uu*vv^(m*p)*ww^(n*p), x]
, x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !
InertTrigFreeQ[w])
```

3.338.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(52) = 104$.

Time = 1.32 (sec) , antiderivative size = 166, normalized size of antiderivative = 2.31

method	result
default	$\frac{\csc(x)\left(\cos(x)\arctan\left(\frac{1}{2\sqrt{-\frac{\cos(x)}{(\cos(x)+1)^2}}}\right)-\cos(x)\ln\left(\frac{2\cos(x)\sqrt{-\frac{\cos(x)}{(\cos(x)+1)^2}}+2\sqrt{-\frac{\cos(x)}{(\cos(x)+1)^2}}-\cos(x)+1}{\cos(x)+1}\right)-\arctan\left(\frac{1}{2\sqrt{-\frac{\cos(x)}{(\cos(x)+1)}}}\right.\right.}{8\sqrt{\sin(x)\tan(x)}\sqrt{-\frac{\cos(x)}{(\cos(x)+1)^2}}}$

input
```
int(1/(-cos(x)+sec(x))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*csc(x)*(cos(x)*arctan(1/2/(-cos(x)/(cos(x)+1)^2)^(1/2))-cos(x)*ln((2*c
os(x)*(-cos(x)/(cos(x)+1)^2)^(1/2)+2*(-cos(x)/(cos(x)+1)^2)^(1/2)-cos(x)+1
)/(cos(x)+1))-arctan(1/2/(-cos(x)/(cos(x)+1)^2)^(1/2))+ln((2*cos(x)*(-cos(
x)/(cos(x)+1)^2)^(1/2)+2*(-cos(x)/(cos(x)+1)^2)^(1/2)-cos(x)+1)/(cos(x)+1)
)-4*(-cos(x)/(cos(x)+1)^2)^(1/2))/(sin(x)*tan(x))^(1/2)/(-cos(x)/(cos(x)+1
)^2)^(1/2)
```

3.338.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(52) = 104$.

Time = 0.27 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.65

$$\int \frac{1}{(-\cos(x)+\sec(x))^{3/2}}\,dx =$$
$$-\frac{(\cos(x)^2-1)\arctan\left(\frac{2\sqrt{-\frac{\cos(x)^2-1}{\cos(x)}}\cos(x)}{(\cos(x)-1)\sin(x)}\right)\sin(x)-(\cos(x)^2-1)\log\left(\frac{(\cos(x)+1)\sin(x)+2\sqrt{-\frac{\cos(x)^2-1}{\cos(x)}}\cos(x)}{(\cos(x)-1)\sin(x)}\right)}{8(\cos(x)^2-1)\sin(x)}$$

input
```
integrate(1/(-cos(x)+sec(x))^(3/2),x, algorithm="fricas")
```

output
```
-1/8*((cos(x)^2 - 1)*arctan(2*sqrt(-(cos(x)^2 - 1)/cos(x))*cos(x)/((cos(x)
 - 1)*sin(x)))*sin(x) - (cos(x)^2 - 1)*log(((cos(x) + 1)*sin(x) + 2*sqrt(-
(cos(x)^2 - 1)/cos(x))*cos(x))/((cos(x) - 1)*sin(x)))*sin(x) - 4*sqrt(-(co
s(x)^2 - 1)/cos(x))*cos(x))/((cos(x)^2 - 1)*sin(x))
```

3.338.6 Sympy [F]

$$\int \frac{1}{(-\cos(x)+\sec(x))^{3/2}}\,dx = \int \frac{1}{(-\cos(x)+\sec(x))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-cos(x)+sec(x))**(3/2),x)
```

output
```
Integral((-cos(x) + sec(x))**(-3/2), x)
```

3.338.7 Maxima [F]

$$\int \frac{1}{(-\cos(x)+\sec(x))^{3/2}}\,dx = \int \frac{1}{(-\cos{(x)}+\sec{(x)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-cos(x)+sec(x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((-cos(x) + sec(x))^(-3/2), x)
```

3.338.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.32

$$\begin{aligned}\int \frac{1}{(-\cos(x)+\sec(x))^{3/2}}\,dx &= -\frac{\tan\left(\frac{1}{2}\,x\right)^2}{16\left(\sqrt{-\tan\left(\frac{1}{2}\,x\right)^4+1}-1\right)} \\ &\quad+\frac{1}{8}\sqrt{-\tan\left(\frac{1}{2}\,x\right)^4+1}+\frac{\sqrt{-\tan\left(\frac{1}{2}\,x\right)^4+1}-1}{16\,\tan\left(\frac{1}{2}\,x\right)^2} \\ &\quad+\frac{1}{8}\,\arcsin\left(\tan\left(\frac{1}{2}\,x\right)^2\right)+\frac{1}{8}\,\log\left(-\frac{\sqrt{-\tan\left(\frac{1}{2}\,x\right)^4+1}-1}{\tan\left(\frac{1}{2}\,x\right)^2}\right)\end{aligned}$$

input
```
integrate(1/(-cos(x)+sec(x))^(3/2),x, algorithm="giac")
```

output
```
-1/16*tan(1/2*x)^2/(sqrt(-tan(1/2*x)^4 + 1) - 1) + 1/8*sqrt(-tan(1/2*x)^4
+ 1) + 1/16*(sqrt(-tan(1/2*x)^4 + 1) - 1)/tan(1/2*x)^2 + 1/8*arcsin(tan(1/
2*x)^2) + 1/8*log(-(sqrt(-tan(1/2*x)^4 + 1) - 1)/tan(1/2*x)^2)
```

3.338.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-\cos(x)+\sec(x))^{3/2}}\,dx = \int \frac{1}{\left(\frac{1}{\cos(x)}-\cos(x)\right)^{3/2}}\,dx$$

input
```
int(1/(1/cos(x) - cos(x))^(3/2),x)
```

output
```
int(1/(1/cos(x) - cos(x))^(3/2), x)
```

3.339 $\int \frac{1}{(-\cos(x)+\sec(x))^{5/2}}\,dx$

3.339.1 Optimal result

Integrand size = 11, antiderivative size = 91

$$\int \frac{1}{(-\cos(x)+\sec(x))^{5/2}}\,dx = \frac{3\cot(x)}{16\sqrt{\sin(x)\tan(x)}} - \frac{\cot(x)\csc^2(x)}{4\sqrt{\sin(x)\tan(x)}} - \frac{3\arctan\left(\sqrt{\cos(x)}\right)\sin(x)}{32\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}} + \frac{3\text{arctanh}\left(\sqrt{\cos(x)}\right)\sin(x)}{32\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}$$

output
```
3/16*cot(x)/(sin(x)*tan(x))^(1/2)-1/4*cot(x)*csc(x)^2/(sin(x)*tan(x))^(1/2
)-3/32*arctan(cos(x)^(1/2))*sin(x)/cos(x)^(1/2)/(sin(x)*tan(x))^(1/2)+3/32
*arctanh(cos(x)^(1/2))*sin(x)/cos(x)^(1/2)/(sin(x)*tan(x))^(1/2)
```

3.339.2 Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.80

$$\int \frac{1}{(-\cos(x)+\sec(x))^{5/2}}\,dx = -\frac{\cot(x)\left(3\arctan\left(\sqrt[4]{\cos^2(x)}\right)\cos(x) - 3\text{arctanh}\left(\sqrt[4]{\cos^2(x)}\right)\cos(x) + \cos^2(x)^{3/4}(5+3\cos(2x))\cot(x)\csc}{32\cos^2(x)^{3/4}}$$

input
```
Integrate[(-Cos[x] + Sec[x])^(-5/2),x]
```

output
```
-1/32*(Cot[x]*(3*ArcTan[(Cos[x]^2)^(1/4)]*Cos[x] - 3*ArcTanh[(Cos[x]^2)^(1
/4)]*Cos[x] + (Cos[x]^2)^(3/4)*(5 + 3*Cos[2*x])*Cot[x]*Csc[x]^3)*Sqrt[Sin[
x]*Tan[x]])/(Cos[x]^2)^(3/4)
```

3.339.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.15, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.455$, Rules used = {3042, 4897, 3042, 4900, 3042, 3077, 3042, 3079, 3042, 3081, 3042, 3045, 266, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\sec(x)-\cos(x))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(\sec(x)-\cos(x))^{5/2}}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \frac{1}{(\sin(x)\tan(x))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(\sin(x)\tan(x))^{5/2}}dx$$

$$\Big\downarrow \text{4900}$$

$$\frac{\sqrt{\sin(x)}\sqrt{\tan(x)}\int \frac{1}{\sin^{\frac{5}{2}}(x)\tan^{\frac{5}{2}}(x)}dx}{\sqrt{\sin(x)\tan(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sin(x)}\sqrt{\tan(x)}\int \frac{1}{\sin(x)^{5/2}\tan(x)^{5/2}}dx}{\sqrt{\sin(x)\tan(x)}}$$

$$\Big\downarrow \text{3077}$$

$$\frac{\sqrt{\sin(x)}\sqrt{\tan(x)}\left(-\frac{3}{8}\int \frac{1}{\sin^{\frac{5}{2}}(x)\sqrt{\tan(x)}}dx-\frac{1}{4\sin^{\frac{5}{2}}(x)\tan^{\frac{3}{2}}(x)}\right)}{\sqrt{\sin(x)\tan(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sin(x)}\sqrt{\tan(x)}\left(-\frac{3}{8}\int \frac{1}{\sin(x)^{5/2}\sqrt{\tan(x)}}dx-\frac{1}{4\sin^{\frac{5}{2}}(x)\tan^{\frac{3}{2}}(x)}\right)}{\sqrt{\sin(x)\tan(x)}}$$

$$\Big\downarrow \text{3079}$$

$$\frac{\sqrt{\sin(x)}\sqrt{\tan(x)}\left(-\frac{3}{8}\left(\frac{1}{4}\int \frac{1}{\sqrt{\sin(x)}\sqrt{\tan(x)}}dx-\frac{1}{2\sqrt{\sin(x)}\tan^{\frac{3}{2}}(x)}\right)-\frac{1}{4\sin^{\frac{5}{2}}(x)\tan^{\frac{3}{2}}(x)}\right)}{\sqrt{\sin(x)\tan(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sin(x)}\sqrt{\tan(x)}\left(-\frac{3}{8}\left(\frac{1}{4}\int \frac{1}{\sqrt{\sin(x)}\sqrt{\tan(x)}}dx-\frac{1}{2\sqrt{\sin(x)}\tan^{\frac{3}{2}}(x)}\right)-\frac{1}{4\sin^{\frac{5}{2}}(x)\tan^{\frac{3}{2}}(x)}\right)}{\sqrt{\sin(x)\tan(x)}}$$

$$\Big\downarrow \text{3081}$$

$$\frac{\sqrt{\sin(x)}\sqrt{\tan(x)}\left(-\frac{3}{8}\left(\frac{\sqrt{\sin(x)}\int \sqrt{\cos(x)}\csc(x)dx}{4\sqrt{\cos(x)}\sqrt{\tan(x)}}-\frac{1}{2\sqrt{\sin(x)}\tan^{\frac{3}{2}}(x)}\right)-\frac{1}{4\sin^{\frac{5}{2}}(x)\tan^{\frac{3}{2}}(x)}\right)}{\sqrt{\sin(x)\tan(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sin(x)}\sqrt{\tan(x)}\left(-\frac{3}{8}\left(\frac{\sqrt{\sin(x)}\int \frac{\sqrt{\cos(x)}}{\sin(x)}dx}{4\sqrt{\cos(x)}\sqrt{\tan(x)}}-\frac{1}{2\sqrt{\sin(x)}\tan^{\frac{3}{2}}(x)}\right)-\frac{1}{4\sin^{\frac{5}{2}}(x)\tan^{\frac{3}{2}}(x)}\right)}{\sqrt{\sin(x)\tan(x)}}$$

$$\Big\downarrow \text{3045}$$

$$\frac{\sqrt{\sin(x)}\sqrt{\tan(x)}\left(-\frac{3}{8}\left(-\frac{\sqrt{\sin(x)}\int \frac{\sqrt{\cos(x)}}{1-\cos^2(x)}d\cos(x)}{4\sqrt{\cos(x)}\sqrt{\tan(x)}}-\frac{1}{2\sqrt{\sin(x)}\tan^{\frac{3}{2}}(x)}\right)-\frac{1}{4\sin^{\frac{5}{2}}(x)\tan^{\frac{3}{2}}(x)}\right)}{\sqrt{\sin(x)\tan(x)}}$$

$$\Big\downarrow \text{266}$$

$$\frac{\sqrt{\sin(x)}\sqrt{\tan(x)}\left(-\frac{3}{8}\left(-\frac{\sqrt{\sin(x)}\int \frac{\cos(x)}{1-\cos^2(x)}d\sqrt{\cos(x)}}{2\sqrt{\cos(x)}\sqrt{\tan(x)}}-\frac{1}{2\sqrt{\sin(x)}\tan^{\frac{3}{2}}(x)}\right)-\frac{1}{4\sin^{\frac{5}{2}}(x)\tan^{\frac{3}{2}}(x)}\right)}{\sqrt{\sin(x)\tan(x)}}$$

$$\Big\downarrow \text{827}$$

$$\frac{\sqrt{\sin(x)}\sqrt{\tan(x)}\left(-\frac{3}{8}\left(-\frac{\sqrt{\sin(x)}\left(\frac{1}{2}\int \frac{1}{1-\cos(x)}d\sqrt{\cos(x)}-\frac{1}{2}\int \frac{1}{\cos(x)+1}d\sqrt{\cos(x)}\right)}{2\sqrt{\cos(x)}\sqrt{\tan(x)}}-\frac{1}{2\sqrt{\sin(x)}\tan^{\frac{3}{2}}(x)}\right)-\frac{1}{4\sin^{\frac{5}{2}}(x)\tan^{\frac{3}{2}}(x)}\right)}{\sqrt{\sin(x)\tan(x)}}$$

3.339. $\int \frac{1}{(-\cos(x)+\sec(x))^{5/2}}\,dx$

$$\Big\downarrow \text{216}$$

$$\frac{\sqrt{\sin(x)}\sqrt{\tan(x)}\left(-\frac{3}{8}\left(-\frac{\sqrt{\sin(x)}\left(\frac{1}{2}\int\frac{1}{1-\cos(x)}d\sqrt{\cos(x)}-\frac{1}{2}\arctan\left(\sqrt{\cos(x)}\right)\right)}{2\sqrt{\cos(x)}\sqrt{\tan(x)}}-\frac{1}{2\sqrt{\sin(x)}\tan^{\frac{3}{2}}(x)}\right)-\frac{1}{4\sin^{\frac{5}{2}}(x)\tan^{\frac{3}{2}}(x)}\right)}{\sqrt{\sin(x)\tan(x)}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{\sin(x)}\sqrt{\tan(x)}\left(-\frac{3}{8}\left(-\frac{\sqrt{\sin(x)}\left(\frac{1}{2}\text{arctanh}\left(\sqrt{\cos(x)}\right)-\frac{1}{2}\arctan\left(\sqrt{\cos(x)}\right)\right)}{2\sqrt{\cos(x)}\sqrt{\tan(x)}}-\frac{1}{2\sqrt{\sin(x)}\tan^{\frac{3}{2}}(x)}\right)-\frac{1}{4\sin^{\frac{5}{2}}(x)\tan^{\frac{3}{2}}(x)}\right)}{\sqrt{\sin(x)\tan(x)}}$$

input
```
Int[(-Cos[x] + Sec[x])^(-5/2),x]
```

output
```
(Sqrt[Sin[x]]*((-3*(-1/2*1/(Sqrt[Sin[x]]*Tan[x]^(3/2)) - ((-1/2*ArcTan[Sqr
t[Cos[x]]] + ArcTanh[Sqrt[Cos[x]]/2)*Sqrt[Sin[x]])/(2*Sqrt[Cos[x]]*Sqrt[T
an[x]])))/8 - 1/(4*Sin[x]^(5/2)*Tan[x]^(3/2)))*Sqrt[Tan[x]])/Sqrt[Sin[x]*T
an[x]]
```

3.339.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

3.339. $\int \frac{1}{(-\cos(x)+\sec(x))^{5/2}}\,dx$

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 3077
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(m
 + n + 1))), x] - Simp[(n + 1)/(b^2*(m + n + 1))   Int[(a*Sin[e + f*x])^m*(
b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1]
&& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1]
)
```

rule 3079
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)
/(a^2*f*(m + n + 1))), x] + Simp[(m + 2)/(a^2*(m + n + 1))   Int[(a*Sin[e +
 f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && L
tQ[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.339. $\int \frac{1}{(-\cos(x)+\sec(x))^{5/2}}\,dx$

rule 4900
```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Simp[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p]))   Int[uu*vv^(m*p)*ww^(n*p), x]
, x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !
InertTrigFreeQ[w])
```

3.339.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(67) = 134$.

Time = 1.23 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.14

method	result
default	$-\frac{12\cot(x)^3\sqrt{-\frac{\cos(x)}{(\cos(x)+1)^2}}-3\cot(x)\arctan\left(\frac{1}{2\sqrt{-\frac{\cos(x)}{(\cos(x)+1)^2}}}\right)-3\cot(x)\ln\left(\frac{2\cos(x)\sqrt{-\frac{\cos(x)}{(\cos(x)+1)^2}}+2\sqrt{-\frac{\cos(x)}{(\cos(x)+1)^2}}-\cos(x)+1}{\cos(x)+1}\right)}{64\sqrt{\sin(x)\tan}}$

input
```
int(1/(-cos(x)+sec(x))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/64/(sin(x)*tan(x))^(1/2)/(-cos(x)/(cos(x)+1)^2)^(1/2)*(12*cot(x)^3*(-co
s(x)/(cos(x)+1)^2)^(1/2)-3*cot(x)*arctan(1/2/(-cos(x)/(cos(x)+1)^2)^(1/2))
-3*cot(x)*ln((2*cos(x)*(-cos(x)/(cos(x)+1)^2)^(1/2)+2*(-cos(x)/(cos(x)+1)^
2)^(1/2)-cos(x)+1)/(cos(x)+1))+3*csc(x)*arctan(1/2/(-cos(x)/(cos(x)+1)^2)^
(1/2))+3*csc(x)*ln((2*cos(x)*(-cos(x)/(cos(x)+1)^2)^(1/2)+2*(-cos(x)/(cos(
x)+1)^2)^(1/2)-cos(x)+1)/(cos(x)+1))+4*csc(x)^2*cot(x)*(-cos(x)/(cos(x)+1)
^2)^(1/2))
```

3.339.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(67) = 134$.

Time = 0.27 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.62

$$\int \frac{1}{(-\cos(x)+\sec(x))^{5/2}}\,dx = \frac{3\left(\cos(x)^4-2\cos(x)^2+1\right)\arctan\left(\frac{2\sqrt{-\frac{\cos(x)^2-1}{\cos(x)}}\cos(x)}{(\cos(x)-1)\sin(x)}\right)\sin(x)+3\left(\cos(x)^4\right.}{64\,(c}$$

input
```
integrate(1/(-cos(x)+sec(x))^(5/2),x, algorithm="fricas")
```

output
```
1/64*(3*(cos(x)^4 - 2*cos(x)^2 + 1)*arctan(2*sqrt(-(cos(x)^2 - 1)/cos(x))*
cos(x)/((cos(x) - 1)*sin(x)))*sin(x) + 3*(cos(x)^4 - 2*cos(x)^2 + 1)*log((
(cos(x) + 1)*sin(x) + 2*sqrt(-(cos(x)^2 - 1)/cos(x))*cos(x))/((cos(x) - 1)
*sin(x)))*sin(x) - 4*(3*cos(x)^4 + cos(x)^2)*sqrt(-(cos(x)^2 - 1)/cos(x)))
/((cos(x)^4 - 2*cos(x)^2 + 1)*sin(x))
```

3.339.6 Sympy [F]

$$\int \frac{1}{(-\cos(x)+\sec(x))^{5/2}}\,dx = \int \frac{1}{(-\cos{(x)}+\sec{(x)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(-cos(x)+sec(x))**(5/2),x)
```

output
```
Integral((-cos(x) + sec(x))**(-5/2), x)
```

3.339.7 Maxima [F]

$$\int \frac{1}{(-\cos(x)+\sec(x))^{5/2}}\,dx = \int \frac{1}{(-\cos{(x)}+\sec{(x)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(-cos(x)+sec(x))^(5/2),x, algorithm="maxima")
```

output
```
integrate((-cos(x) + sec(x))^(-5/2), x)
```

3.339.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(67) = 134.

Time = 0.38 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.66

$$\int \frac{1}{(-\cos(x)+\sec(x))^{5/2}}\,dx = \frac{\left(\frac{4\left(\sqrt{-\tan\left(\frac{1}{2}x\right)^4+1}-1\right)}{\tan\left(\frac{1}{2}x\right)^2}+1\right)\tan\left(\frac{1}{2}x\right)^4}{256\left(\sqrt{-\tan\left(\frac{1}{2}x\right)^4+1}-1\right)^2} - \frac{1}{64}\sqrt{-\tan\left(\frac{1}{2}x\right)^4+1}\left(\tan\left(\frac{1}{2}x\right)^2-2\right) - \frac{\sqrt{-\tan\left(\frac{1}{2}x\right)^4+1}-1}{64\tan\left(\frac{1}{2}x\right)^2} - \frac{\left(\sqrt{-\tan\left(\frac{1}{2}x\right)^4+1}-1\right)^2}{256\tan\left(\frac{1}{2}x\right)^4} - \frac{3}{64}\arcsin\left(\tan\left(\frac{1}{2}x\right)^2\right) + \frac{3}{64}\log\left(-\frac{\sqrt{-\tan\left(\frac{1}{2}x\right)^4+1}-1}{\tan\left(\frac{1}{2}x\right)^2}\right)$$

input
```
integrate(1/(-cos(x)+sec(x))^(5/2),x, algorithm="giac")
```

output
```
1/256*(4*(sqrt(-tan(1/2*x)^4 + 1) - 1)/tan(1/2*x)^2 + 1)*tan(1/2*x)^4/(sqr
t(-tan(1/2*x)^4 + 1) - 1)^2 - 1/64*sqrt(-tan(1/2*x)^4 + 1)*(tan(1/2*x)^2 -
 2) - 1/64*(sqrt(-tan(1/2*x)^4 + 1) - 1)/tan(1/2*x)^2 - 1/256*(sqrt(-tan(1
/2*x)^4 + 1) - 1)^2/tan(1/2*x)^4 - 3/64*arcsin(tan(1/2*x)^2) + 3/64*log(-(
sqrt(-tan(1/2*x)^4 + 1) - 1)/tan(1/2*x)^2)
```

3.339.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-\cos(x)+\sec(x))^{5/2}}\,dx = \int \frac{1}{\left(\frac{1}{\cos(x)}-\cos(x)\right)^{5/2}}\,dx$$

input
```
int(1/(1/cos(x) - cos(x))^(5/2),x)
```

output
```
int(1/(1/cos(x) - cos(x))^(5/2), x)
```

3.340 $\int \frac{1}{(-\cos(x)+\sec(x))^{7/2}}\,dx$

3.340.1 Optimal result

Integrand size = 11, antiderivative size = 110

$$\int \frac{1}{(-\cos(x)+\sec(x))^{7/2}}\,dx = -\frac{5\csc(x)}{192\sqrt{\sin(x)\tan(x)}}+\frac{5\csc^3(x)}{48\sqrt{\sin(x)\tan(x)}} - \frac{\cot^2(x)\csc^3(x)}{6\sqrt{\sin(x)\tan(x)}} - \frac{5\arctan\left(\sqrt{\cos(x)}\right)\sin(x)}{128\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}} - \frac{5\operatorname{arctanh}\left(\sqrt{\cos(x)}\right)\sin(x)}{128\sqrt{\cos(x)}\sqrt{\sin(x)\tan(x)}}$$

output
```
-5/192*csc(x)/(sin(x)*tan(x))^(1/2)+5/48*csc(x)^3/(sin(x)*tan(x))^(1/2)-1/
6*cot(x)^2*csc(x)^3/(sin(x)*tan(x))^(1/2)-5/128*arctan(cos(x)^(1/2))*sin(x
)/cos(x)^(1/2)/(sin(x)*tan(x))^(1/2)-5/128*arctanh(cos(x)^(1/2))*sin(x)/co
s(x)^(1/2)/(sin(x)*tan(x))^(1/2)
```

3.340.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.67

$$\int \frac{1}{(-\cos(x)+\sec(x))^{7/2}}\,dx = -\frac{\cot(x)\left(15\arctan\left(\sqrt[4]{\cos^2(x)}\right)+15\operatorname{arctanh}\left(\sqrt[4]{\cos^2(x)}\right)+2\sqrt[4]{\cos^2(x)}\csc^2(x)\left(5-52\csc^2(x)+32\csc^4(x\right.\right.}{384\sqrt[4]{\cos^2(x)}}$$

input
```
Integrate[(-Cos[x] + Sec[x])^(-7/2),x]
```

output
```
-1/384*(Cot[x]*(15*ArcTan[(Cos[x]^2)^(1/4)] + 15*ArcTanh[(Cos[x]^2)^(1/4)]
 + 2*(Cos[x]^2)^(1/4)*Csc[x]^2*(5 - 52*Csc[x]^2 + 32*Csc[x]^4))*Sqrt[Sin[x
]*Tan[x]])/(Cos[x]^2)^(1/4)
```

3.340.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.15, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 1.636$, Rules used = {3042, 4897, 3042, 4900, 3042, 3077, 3042, 3077, 3042, 3079, 3042, 3081, 3042, 3045, 266, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\sec(x)-\cos(x))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(\sec(x)-\cos(x))^{7/2}}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \frac{1}{(\sin(x)\tan(x))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(\sin(x)\tan(x))^{7/2}}dx$$

$$\Big\downarrow \text{4900}$$

$$\frac{\sqrt{\sin(x)}\sqrt{\tan(x)}\int \frac{1}{\sin^{\frac{7}{2}}(x)\tan^{\frac{7}{2}}(x)}dx}{\sqrt{\sin(x)\tan(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sin(x)}\sqrt{\tan(x)}\int \frac{1}{\sin(x)^{7/2}\tan(x)^{7/2}}dx}{\sqrt{\sin(x)\tan(x)}}$$

$$\Big\downarrow \text{3077}$$

$$\frac{\sqrt{\sin(x)}\sqrt{\tan(x)}\left(-\frac{5}{12}\int \frac{1}{\sin^{\frac{7}{2}}(x)\tan^{\frac{3}{2}}(x)}dx-\frac{1}{6\sin^{\frac{7}{2}}(x)\tan^{\frac{5}{2}}(x)}\right)}{\sqrt{\sin(x)\tan(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sin(x)}\sqrt{\tan(x)}\left(-\frac{5}{12}\int \frac{1}{\sin(x)^{7/2}\tan(x)^{3/2}}dx-\frac{1}{6\sin^{\frac{7}{2}}(x)\tan^{\frac{5}{2}}(x)}\right)}{\sqrt{\sin(x)\tan(x)}}$$

$$\Big\downarrow \text{3077}$$

$$\frac{\sqrt{\sin(x)}\sqrt{\tan(x)}\left(-\frac{5}{12}\left(-\frac{1}{8}\int \frac{\sqrt{\tan(x)}}{\sin^{\frac{7}{2}}(x)}dx-\frac{1}{4\sin^{\frac{7}{2}}(x)\sqrt{\tan(x)}}\right)-\frac{1}{6\sin^{\frac{7}{2}}(x)\tan^{\frac{5}{2}}(x)}\right)}{\sqrt{\sin(x)\tan(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sin(x)}\sqrt{\tan(x)}\left(-\frac{5}{12}\left(-\frac{1}{8}\int \frac{\sqrt{\tan(x)}}{\sin(x)^{7/2}}dx-\frac{1}{4\sin^{\frac{7}{2}}(x)\sqrt{\tan(x)}}\right)-\frac{1}{6\sin^{\frac{7}{2}}(x)\tan^{\frac{5}{2}}(x)}\right)}{\sqrt{\sin(x)\tan(x)}}$$

$$\Big\downarrow \text{3079}$$

$$\frac{\sqrt{\sin(x)}\sqrt{\tan(x)}\left(-\frac{5}{12}\left(\frac{1}{8}\left(\frac{1}{2\sin^{\frac{3}{2}}(x)\sqrt{\tan(x)}}-\frac{3}{4}\int \frac{\sqrt{\tan(x)}}{\sin^{\frac{3}{2}}(x)}dx\right)-\frac{1}{4\sin^{\frac{7}{2}}(x)\sqrt{\tan(x)}}\right)-\frac{1}{6\sin^{\frac{7}{2}}(x)\tan^{\frac{5}{2}}(x)}\right)}{\sqrt{\sin(x)\tan(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sin(x)}\sqrt{\tan(x)}\left(-\frac{5}{12}\left(\frac{1}{8}\left(\frac{1}{2\sin^{\frac{3}{2}}(x)\sqrt{\tan(x)}}-\frac{3}{4}\int \frac{\sqrt{\tan(x)}}{\sin(x)^{3/2}}dx\right)-\frac{1}{4\sin^{\frac{7}{2}}(x)\sqrt{\tan(x)}}\right)-\frac{1}{6\sin^{\frac{7}{2}}(x)\tan^{\frac{5}{2}}(x)}\right)}{\sqrt{\sin(x)\tan(x)}}$$

$$\Big\downarrow \text{3081}$$

$$\frac{\sqrt{\sin(x)}\sqrt{\tan(x)}\left(-\frac{5}{12}\left(\frac{1}{8}\left(\frac{1}{2\sin^{\frac{3}{2}}(x)\sqrt{\tan(x)}}-\frac{3\sqrt{\cos(x)}\sqrt{\tan(x)}\int \frac{\csc(x)}{\sqrt{\cos(x)}}dx}{4\sqrt{\sin(x)}}\right)-\frac{1}{4\sin^{\frac{7}{2}}(x)\sqrt{\tan(x)}}\right)-\frac{1}{6\sin^{\frac{7}{2}}(x)\tan^{\frac{5}{2}}(x)}\right)}{\sqrt{\sin(x)\tan(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sin(x)}\sqrt{\tan(x)}\left(-\frac{5}{12}\left(\frac{1}{8}\left(\frac{1}{2\sin^{\frac{3}{2}}(x)\sqrt{\tan(x)}}-\frac{3\sqrt{\cos(x)}\sqrt{\tan(x)}\int \frac{1}{\sqrt{\cos(x)}\sin(x)}dx}{4\sqrt{\sin(x)}}\right)-\frac{1}{4\sin^{\frac{7}{2}}(x)\sqrt{\tan(x)}}\right)-\frac{1}{6\sin^{\frac{7}{2}}(x)\tan^{\frac{5}{2}}(x)}\right)}{\sqrt{\sin(x)\tan(x)}}$$

$$\Big\downarrow \text{3045}$$

3.340. $\int \frac{1}{(-\cos(x)+\sec(x))^{7/2}}\,dx$

$$\frac{\sqrt{\sin(x)}\sqrt{\tan(x)}\left(-\frac{5}{12}\left(\frac{1}{8}\left(\frac{3\sqrt{\cos(x)}\sqrt{\tan(x)}\int\frac{1}{\sqrt{\cos(x)}\left(1-\cos^2(x)\right)}d\cos(x)}{4\sqrt{\sin(x)}}+\frac{1}{2\sin^{\frac{3}{2}}(x)\sqrt{\tan(x)}}\right)-\frac{1}{4\sin^{\frac{7}{2}}(x)\sqrt{\tan(x)}}\right)-\frac{1}{6\sin^{\frac{7}{2}}\cdots}\right.}{\sqrt{\sin(x)\tan(x)}}$$

$\downarrow$ 266

$$\frac{\sqrt{\sin(x)}\sqrt{\tan(x)}\left(-\frac{5}{12}\left(\frac{1}{8}\left(\frac{3\sqrt{\cos(x)}\sqrt{\tan(x)}\int\frac{1}{1-\cos^2(x)}d\sqrt{\cos(x)}}{2\sqrt{\sin(x)}}+\frac{1}{2\sin^{\frac{3}{2}}(x)\sqrt{\tan(x)}}\right)-\frac{1}{4\sin^{\frac{7}{2}}(x)\sqrt{\tan(x)}}\right)-\frac{1}{6\sin^{\frac{7}{2}}(x)\tan^{\frac{5}{2}}\cdots}\right.}{\sqrt{\sin(x)\tan(x)}}$$

$\downarrow$ 756

$$\frac{\sqrt{\sin(x)}\sqrt{\tan(x)}\left(-\frac{5}{12}\left(\frac{1}{8}\left(\frac{3\sqrt{\cos(x)}\sqrt{\tan(x)}\left(\frac{1}{2}\int\frac{1}{1-\cos(x)}d\sqrt{\cos(x)}+\frac{1}{2}\int\frac{1}{\cos(x)+1}d\sqrt{\cos(x)}\right)}{2\sqrt{\sin(x)}}+\frac{1}{2\sin^{\frac{3}{2}}(x)\sqrt{\tan(x)}}\right)-\frac{1}{4\sin^{\frac{7}{2}}(x)\sqrt{\cdots}}\right.\right.}{\sqrt{\sin(x)\tan(x)}}$$

$\downarrow$ 216

$$\frac{\sqrt{\sin(x)}\sqrt{\tan(x)}\left(-\frac{5}{12}\left(\frac{1}{8}\left(\frac{3\sqrt{\cos(x)}\sqrt{\tan(x)}\left(\frac{1}{2}\int\frac{1}{1-\cos(x)}d\sqrt{\cos(x)}+\frac{1}{2}\arctan\left(\sqrt{\cos(x)}\right)\right)}{2\sqrt{\sin(x)}}+\frac{1}{2\sin^{\frac{3}{2}}(x)\sqrt{\tan(x)}}\right)-\frac{1}{4\sin^{\frac{7}{2}}(x)\sqrt{\cdots}}\right.\right.}{\sqrt{\sin(x)\tan(x)}}$$

$\downarrow$ 219

$$\frac{\sqrt{\sin(x)}\sqrt{\tan(x)}\left(-\frac{5}{12}\left(\frac{1}{8}\left(\frac{3\sqrt{\cos(x)}\sqrt{\tan(x)}\left(\frac{1}{2}\arctan\left(\sqrt{\cos(x)}\right)+\frac{1}{2}\text{arctanh}\left(\sqrt{\cos(x)}\right)\right)}{2\sqrt{\sin(x)}}+\frac{1}{2\sin^{\frac{3}{2}}(x)\sqrt{\tan(x)}}\right)-\frac{1}{4\sin^{\frac{7}{2}}(x)\sqrt{\cdots}}\right.\right.}{\sqrt{\sin(x)\tan(x)}}$$

input
```
Int[(-Cos[x] + Sec[x])^(-7/2),x]
```

output
```
(Sqrt[Sin[x]]*((-5*((1/(2*Sin[x]^(3/2)*Sqrt[Tan[x]]) + (3*(ArcTan[Sqrt[Cos
[x]]]/2 + ArcTanh[Sqrt[Cos[x]]]/2)*Sqrt[Cos[x]]*Sqrt[Tan[x]])/(2*Sqrt[Sin[
x]]))/8 - 1/(4*Sin[x]^(7/2)*Sqrt[Tan[x]])))/12 - 1/(6*Sin[x]^(7/2)*Tan[x]^
(5/2))*Sqrt[Tan[x]])/Sqrt[Sin[x]*Tan[x]]
```

3.340. $\int\frac{1}{(-\cos(x)+\sec(x))^{7/2}}\,dx$

3.340.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

rule 3077
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sin[e + f*x])^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(m
 + n + 1))), x] - Simp[(n + 1)/(b^2*(m + n + 1))   Int[(a*Sin[e + f*x])^m*(
b*Tan[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1]
&& NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n] &&  !(EqQ[n, -3/2] && EqQ[m, 1]
)
```

3.340. $\int \frac{1}{(-\cos(x)+\sec(x))^{7/2}} dx$

rule 3079
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n
_.), x_Symbol] :> Simp[b*(a*Sin[e + f*x])^(m + 2)*((b*Tan[e + f*x])^(n - 1)
/(a^2*f*(m + n + 1))), x] + Simp[(m + 2)/(a^2*(m + n + 1))   Int[(a*Sin[e +
 f*x])^(m + 2)*(b*Tan[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, n}, x] && L
tQ[m, -1] && NeQ[m + n + 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3081
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[Cos[e + f*x]^n*((b*Tan[e + f*x])^n/(a*Sin[e + f*x])^
n)   Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x], x] /; FreeQ[{a, b, e,
 f, m, n}, x] &&  !IntegerQ[n] && (ILtQ[m, 0] || (EqQ[m, 1] && EqQ[n, -2^(-
1)]) || IntegersQ[m - 1/2, n - 1/2])
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

rule 4900
```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Simp[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p]))   Int[uu*vv^(m*p)*ww^(n*p), x]
, x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !
InertTrigFreeQ[w])
```

3.340.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(82) = 164$.

Time = 1.22 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.97

method	result
default	$-\frac{15\cot(x)\arctan\left(\frac{1}{2\sqrt{-\frac{\cos(x)}{(\cos(x)+1)^2}}}\right)-15\cot(x)\ln\left(\frac{2\cos(x)\sqrt{-\frac{\cos(x)}{(\cos(x)+1)^2}}+2\sqrt{-\frac{\cos(x)}{(\cos(x)+1)^2}}-\cos(x)+1}{\cos(x)+1}\right)+20\cot(x)^4\csc(x)\sqrt{-\frac{\cdots}{(\text{co}\cdots}}}{}$

input
```
int(1/(-cos(x)+sec(x))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/768/(-cos(x)/(cos(x)+1)^2)^(1/2)/(sin(x)*tan(x))^(1/2)*(15*cot(x)*arcta
n(1/2/(-cos(x)/(cos(x)+1)^2)^(1/2))-15*cot(x)*ln((2*cos(x)*(-cos(x)/(cos(x
)+1)^2)^(1/2)+2*(-cos(x)/(cos(x)+1)^2)^(1/2)-cos(x)+1)/(cos(x)+1))+20*cot(
x)^4*csc(x)*(-cos(x)/(cos(x)+1)^2)^(1/2)-15*csc(x)*arctan(1/2/(-cos(x)/(co
s(x)+1)^2)^(1/2))+15*csc(x)*ln((2*cos(x)*(-cos(x)/(cos(x)+1)^2)^(1/2)+2*(-
cos(x)/(cos(x)+1)^2)^(1/2)-cos(x)+1)/(cos(x)+1))+168*cot(x)^2*csc(x)^3*(-c
os(x)/(cos(x)+1)^2)^(1/2)-60*csc(x)^5*(-cos(x)/(cos(x)+1)^2)^(1/2))
```

3.340.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(82) = 164$.

Time = 0.28 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.55

$$\int \frac{1}{(-\cos(x)+\sec(x))^{7/2}}\,dx = \frac{15\left(\cos\left(x\right)^{6}-3\,\cos\left(x\right)^{4}+3\,\cos\left(x\right)^{2}-1\right)\arctan\left(\frac{2\sqrt{-\frac{\cos(x)^{2}-1}{\cos(x)}}\cos(x)}{(\cos(x)-1)\sin(x)}\right)\sin\left(x\right.}{}$$

input
```
integrate(1/(-cos(x)+sec(x))^(7/2),x, algorithm="fricas")
```

output
```
1/768*(15*(cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 - 1)*arctan(2*sqrt(-(cos(x)^
2 - 1)/cos(x))*cos(x)/((cos(x) - 1)*sin(x)))*sin(x) + 15*(cos(x)^6 - 3*cos
(x)^4 + 3*cos(x)^2 - 1)*log(((cos(x) + 1)*sin(x) - 2*sqrt(-(cos(x)^2 - 1)/
cos(x))*cos(x))/((cos(x) - 1)*sin(x)))*sin(x) + 4*(5*cos(x)^5 + 42*cos(x)^
3 - 15*cos(x))*sqrt(-(cos(x)^2 - 1)/cos(x)))/((cos(x)^6 - 3*cos(x)^4 + 3*c
os(x)^2 - 1)*sin(x))
```

3.340.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(-\cos(x)+\sec(x))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(-cos(x)+sec(x))**(7/2),x)
```

output
```
Timed out
```

3.340.7 Maxima [F]

$$\int \frac{1}{(-\cos(x)+\sec(x))^{7/2}}\,dx = \int \frac{1}{(-\cos{(x)}+\sec{(x)})^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(-cos(x)+sec(x))^(7/2),x, algorithm="maxima")
```

output
```
integrate((-cos(x) + sec(x))^(-7/2), x)
```

3.340.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(82) = 164$.

Time = 0.46 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.91

$$\int \frac{1}{(-\cos(x)+\sec(x))^{7/2}}\,dx = -\frac{\left(\frac{3\left(\sqrt{-\tan\left(\frac{1}{2}x\right)^4+1}-1\right)}{\tan\left(\frac{1}{2}x\right)^2}-\frac{27\left(\sqrt{-\tan\left(\frac{1}{2}x\right)^4+1}-1\right)^2}{\tan\left(\frac{1}{2}x\right)^4}+1\right)\tan\left(\frac{1}{2}x\right)^6}{3072\left(\sqrt{-\tan\left(\frac{1}{2}x\right)^4+1}-1\right)^3} + \frac{1}{768}\sqrt{-\tan\left(\frac{1}{2}x\right)^4+1}\left(\left(2\tan\left(\frac{1}{2}x\right)^2-3\right)\tan\left(\frac{1}{2}x\right)^2-14\right) - \frac{9\left(\sqrt{-\tan\left(\frac{1}{2}x\right)^4+1}-1\right)}{1024\tan\left(\frac{1}{2}x\right)^2} + \frac{\left(\sqrt{-\tan\left(\frac{1}{2}x\right)^4+1}-1\right)^2}{1024\tan\left(\frac{1}{2}x\right)^4} + \frac{\left(\sqrt{-\tan\left(\frac{1}{2}x\right)^4+1}-1\right)^3}{3072\tan\left(\frac{1}{2}x\right)^6} - \frac{5}{256}\arcsin\left(\tan\left(\frac{1}{2}x\right)^2\right) - \frac{5}{256}\log\left(-\frac{\sqrt{-\tan\left(\frac{1}{2}x\right)^4+1}-1}{\tan\left(\frac{1}{2}x\right)^2}\right)$$

input
```
integrate(1/(-cos(x)+sec(x))^(7/2),x, algorithm="giac")
```

output
```
-1/3072*(3*(sqrt(-tan(1/2*x)^4 + 1) - 1)/tan(1/2*x)^2 - 27*(sqrt(-tan(1/2*
x)^4 + 1) - 1)^2/tan(1/2*x)^4 + 1)*tan(1/2*x)^6/(sqrt(-tan(1/2*x)^4 + 1) -
 1)^3 + 1/768*sqrt(-tan(1/2*x)^4 + 1)*((2*tan(1/2*x)^2 - 3)*tan(1/2*x)^2 -
 14) - 9/1024*(sqrt(-tan(1/2*x)^4 + 1) - 1)/tan(1/2*x)^2 + 1/1024*(sqrt(-t
an(1/2*x)^4 + 1) - 1)^2/tan(1/2*x)^4 + 1/3072*(sqrt(-tan(1/2*x)^4 + 1) - 1
)^3/tan(1/2*x)^6 - 5/256*arcsin(tan(1/2*x)^2) - 5/256*log(-(sqrt(-tan(1/2*
x)^4 + 1) - 1)/tan(1/2*x)^2)
```

3.340.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-\cos(x)+\sec(x))^{7/2}}\,dx=\int \frac{1}{\left(\frac{1}{\cos(x)}-\cos\left(x\right)\right)^{7/2}}\,dx$$

input
```
int(1/(1/cos(x) - cos(x))^(7/2),x)
```

output
```
int(1/(1/cos(x) - cos(x))^(7/2), x)
```

3.341 $\int (\sin(x) + \tan(x))^4 \, dx$

3.341.1 Optimal result

Integrand size = 7, antiderivative size = 55

$$\int (\sin(x) + \tan(x))^4 \, dx = -\frac{61x}{8} - 2\text{arctanh}(\sin(x)) + \frac{19}{8}\cos(x)\sin(x) + \frac{1}{4}\cos^3(x)\sin(x) - \frac{4\sin^3(x)}{3} + 5\tan(x) + 2\sec(x)\tan(x) + \frac{\tan^3(x)}{3}$$

output
```
-61/8*x-2*arctanh(sin(x))+19/8*cos(x)*sin(x)+1/4*cos(x)^3*sin(x)-4/3*sin(x
)^3+5*tan(x)+2*sec(x)*tan(x)+1/3*tan(x)^3
```

3.341.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 129 vs. 2(55) = 110.

Time = 0.15 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.35

$$\int (\sin(x) + \tan(x))^4 \, dx = \frac{1}{768}\sec^3(x)\left(-72\cos(x)\left(61x - 16\log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) + 16\log\left(\cos\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right)\right)\right) - 24\cos(3x)\left(61x - 16\log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) + 16\log\left(\cos\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right)\right)\right) + 1395\sin(x) + 672\sin(2x) + 1265\sin(3x) + 129\sin(5x) + 32\sin(6x) + 3\sin(7x)\right)$$

input
```
Integrate[(Sin[x] + Tan[x])^4,x]
```

output
```
(Sec[x]^3*(-72*Cos[x]*(61*x - 16*Log[Cos[x/2] - Sin[x/2]] + 16*Log[Cos[x/2
] + Sin[x/2]]) - 24*Cos[3*x]*(61*x - 16*Log[Cos[x/2] - Sin[x/2]] + 16*Log[
Cos[x/2] + Sin[x/2]]) + 1395*Sin[x] + 672*Sin[2*x] + 1265*Sin[3*x] + 129*S
in[5*x] + 32*Sin[6*x] + 3*Sin[7*x]))/768
```

3.341.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 4897, 3042, 3188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\sin(x)+\tan(x))^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (\sin(x)+\tan(x))^4dx$$

$$\Big\downarrow \text{4897}$$

$$\int (\cos(x)+1)^4\tan^4(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(1-\sin\left(x-\frac{\pi}{2}\right)\right)^4}{\tan\left(x-\frac{\pi}{2}\right)^4}dx$$

$$\Big\downarrow \text{3188}$$

$$\int \left(\cos^4(x)+4\cos^3(x)+4\cos^2(x)-4\cos(x)+\sec^4(x)+4\sec^3(x)+4\sec^2(x)-4\sec(x)-10\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-2\text{arctanh}(\sin(x))-\frac{61x}{8}-\frac{4\sin^3(x)}{3}+\frac{\tan^3(x)}{3}+5\tan(x)+\frac{1}{4}\sin(x)\cos^3(x)+\frac{19}{8}\sin(x)\cos(x)+2\tan(x)\sec(x)$$

input
```
Int[(Sin[x] + Tan[x])^4,x]
```

output
```
(-61*x)/8 - 2*ArcTanh[Sin[x]] + (19*Cos[x]*Sin[x])/8 + (Cos[x]^3*Sin[x])/4
 - (4*Sin[x]^3)/3 + 5*Tan[x] + 2*Sec[x]*Tan[x] + Tan[x]^3/3
```

3.341.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3188
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Simp[a^p   Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
 + f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b,
e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m
- p/2, 0])
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.341.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.20

$$\frac{23\left(\sin\left(x\right)^{3}+\frac{3\sin(x)}{2}\right)\cos\left(x\right)}{4}-\frac{61x}{8}+\frac{2\sin\left(x\right)^{3}}{3}+2\sin\left(x\right)-2\ln\left(\sec\left(x\right)+\tan\left(x\right)\right)+\frac{6\sin\left(x\right)^{5}}{\cos\left(x\right)}+\frac{2\sin\left(x\right)^{5}}{\cos\left(x\right)^{2}}+\text{ta}$$

input
```
int((sin(x)+tan(x))^4,x)
```

output
```
23/4*(sin(x)^3+3/2*sin(x))*cos(x)-61/8*x+2/3*sin(x)^3+2*sin(x)-2*ln(sec(x)
+tan(x))+6*sin(x)^5/cos(x)+2*sin(x)^5/cos(x)^2+1/3*tan(x)^3-tan(x)
```

3.341.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.42

$$\int (\sin(x) + \tan(x))^4 \, dx =$$
$$-\frac{183\,x\cos(x)^3 + 24\cos(x)^3\log(\sin(x)+1) - 24\cos(x)^3\log(-\sin(x)+1) - (6\cos(x)^6 + 32\cos(x)^{}}{24\cos(x)^3}$$

input
```
integrate((sin(x)+tan(x))^4,x, algorithm="fricas")
```

output
```
-1/24*(183*x*cos(x)^3 + 24*cos(x)^3*log(sin(x) + 1) - 24*cos(x)^3*log(-sin
(x) + 1) - (6*cos(x)^6 + 32*cos(x)^5 + 57*cos(x)^4 - 32*cos(x)^3 + 112*cos
(x)^2 + 48*cos(x) + 8)*sin(x))/cos(x)^3
```

3.341.6 Sympy [A] (verification not implemented)

Time = 1.63 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.64

$$\int (\sin(x) + \tan(x))^4 \, dx = -\frac{61x}{8} + \log(\sin(x) - 1) - \log(\sin(x) + 1) - \frac{4\sin^3(x)}{3} + \frac{6\sin^3(x)}{\cos(x)} + \frac{\sin^3(x)}{3\cos^3(x)} + 9\sin(x)\cos(x) - \frac{\sin(x)}{\cos(x)} - \frac{\sin(2x)}{4} + \frac{\sin(4x)}{32} - \frac{4\sin(x)}{2\sin^2(x) - 2}$$

input
```
integrate((sin(x)+tan(x))**4,x)
```

output
```
-61*x/8 + log(sin(x) - 1) - log(sin(x) + 1) - 4*sin(x)**3/3 + 6*sin(x)**3/
cos(x) + sin(x)**3/(3*cos(x)**3) + 9*sin(x)*cos(x) - sin(x)/cos(x) - sin(2
*x)/4 + sin(4*x)/32 - 4*sin(x)/(2*sin(x)**2 - 2)
```

3.341.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.24

$$\int (\sin(x) + \tan(x))^4 \, dx = -\frac{4}{3} \sin(x)^3 + \frac{1}{3} \tan(x)^3 - \frac{61}{8} x - \frac{2 \sin(x)}{\sin(x)^2 - 1} + \frac{3 \tan(x)}{\tan(x)^2 + 1} - \log(\sin(x) + 1) + \log(\sin(x) - 1) + \frac{1}{32} \sin(4x) - \frac{1}{4} \sin(2x) + 5 \tan(x)$$

input
```
integrate((sin(x)+tan(x))^4,x, algorithm="maxima")
```

output
```
-4/3*sin(x)^3 + 1/3*tan(x)^3 - 61/8*x - 2*sin(x)/(sin(x)^2 - 1) + 3*tan(x)
/(tan(x)^2 + 1) - log(sin(x) + 1) + log(sin(x) - 1) + 1/32*sin(4*x) - 1/4*
sin(2*x) + 5*tan(x)
```

3.341.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1375 vs. 2(45) = 90.

Time = 3.10 (sec) , antiderivative size = 1375, normalized size of antiderivative = 25.00

$$\int (\sin(x) + \tan(x))^4 \, dx = \text{Too large to display}$$

input
```
integrate((sin(x)+tan(x))^4,x, algorithm="giac")
```

output
```
1/24*(8*tan(1/2*x)^10*tan(x)^5 - 183*x*tan(1/2*x)^10*tan(x)^2 - 24*log(2*(
tan(1/2*x)^2 + 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^10*tan(x)^
2 + 24*log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2
*x)^10*tan(x)^2 + 128*tan(1/2*x)^10*tan(x)^3 + 8*tan(1/2*x)^8*tan(x)^5 - 1
83*x*tan(1/2*x)^10 - 24*log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)/(tan(1/2*x
)^2 + 1))*tan(1/2*x)^10 + 24*log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(
1/2*x)^2 + 1))*tan(1/2*x)^10 + 180*tan(1/2*x)^10*tan(x) - 183*x*tan(1/2*x)
^8*tan(x)^2 - 24*log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1
))*tan(1/2*x)^8*tan(x)^2 + 24*log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan
(1/2*x)^2 + 1))*tan(1/2*x)^8*tan(x)^2 + 96*tan(1/2*x)^9*tan(x)^2 + 128*tan
(1/2*x)^8*tan(x)^3 - 16*tan(1/2*x)^6*tan(x)^5 - 183*x*tan(1/2*x)^8 - 24*lo
g(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^8 + 2
4*log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^8
 + 96*tan(1/2*x)^9 + 180*tan(1/2*x)^8*tan(x) + 366*x*tan(1/2*x)^6*tan(x)^2
 + 48*log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*
x)^6*tan(x)^2 - 48*log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 +
 1))*tan(1/2*x)^6*tan(x)^2 + 128*tan(1/2*x)^7*tan(x)^2 - 256*tan(1/2*x)^6*
tan(x)^3 - 16*tan(1/2*x)^4*tan(x)^5 + 366*x*tan(1/2*x)^6 + 48*log(2*(tan(1
/2*x)^2 + 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^6 - 48*log(2*(t
an(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^6 + 128*...
```

3.341.9 Mupad [B] (verification not implemented)

Time = 29.10 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.60

$$\int (\sin(x)+\tan(x))^4\,dx$$
$$= -\frac{61\,x}{8} - 4\,\mathrm{atanh}\left(\tan\left(\frac{x}{2}\right)\right) - \frac{\frac{45\tan\left(\frac{x}{2}\right)^{13}}{4} + \frac{29\tan\left(\frac{x}{2}\right)^{11}}{6} - \frac{455\tan\left(\frac{x}{2}\right)^{9}}{12} - 15\tan\left(\frac{x}{2}\right)^{7} + \frac{179\tan\left(\frac{x}{2}\right)^{5}}{4} + \frac{31\tan\left(\frac{x}{2}\right)^{3}}{2} + \frac{77\tan\left(\frac{x}{2}\right)}{4}}{\left(\tan\left(\frac{x}{2}\right)^2-1\right)^3\left(\tan\left(\frac{x}{2}\right)^2+1\right)^4}$$

input
```
int((sin(x) + tan(x))^4,x)
```

output
```
- (61*x)/8 - 4*atanh(tan(x/2)) - ((77*tan(x/2))/4 + (31*tan(x/2)^3)/2 + (1
79*tan(x/2)^5)/4 - 15*tan(x/2)^7 - (455*tan(x/2)^9)/12 + (29*tan(x/2)^11)/
6 + (45*tan(x/2)^13)/4)/((tan(x/2)^2 - 1)^3*(tan(x/2)^2 + 1)^4)
```

3.342 $\int (\sin(x) + \tan(x))^3 \, dx$

3.342.1 Optimal result

Integrand size = 7, antiderivative size = 38

$$\int (\sin(x) + \tan(x))^3 \, dx = 2\cos(x) + \frac{3\cos^2(x)}{2} + \frac{\cos^3(x)}{3} - 2\log(\cos(x)) + 3\sec(x) + \frac{\sec^2(x)}{2}$$

output
```
2*cos(x)+3/2*cos(x)^2+1/3*cos(x)^3-2*ln(cos(x))+3*sec(x)+1/2*sec(x)^2
```

3.342.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05

$$\int (\sin(x) + \tan(x))^3 \, dx = \frac{9\cos(x)}{4} + \frac{3}{4}\cos(2x) + \frac{1}{12}\cos(3x) - 2\log(\cos(x)) + 3\sec(x) + \frac{\sec^2(x)}{2}$$

input
```
Integrate[(Sin[x] + Tan[x])^3,x]
```

output
```
(9*Cos[x])/4 + (3*Cos[2*x])/4 + Cos[3*x]/12 - 2*Log[Cos[x]] + 3*Sec[x] + S
ec[x]^2/2
```

3.342.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 4897, 3042, 25, 3186, 84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\sin(x)+\tan(x))^3 \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int (\sin(x)+\tan(x))^3 dx$$

$$\Big\downarrow \text{4897}$$

$$\int (\cos(x)+1)^3 \tan^3(x) dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\left(1-\sin\left(x-\frac{\pi}{2}\right)\right)^3}{\tan\left(x-\frac{\pi}{2}\right)^3} dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\left(1-\sin\left(x-\frac{\pi}{2}\right)\right)^3}{\tan\left(x-\frac{\pi}{2}\right)^3} dx$$

$$\Big\downarrow \text{3186}$$

$$-\int (1-\cos(x))(\cos(x)+1)^4 \sec^3(x) d\cos(x)$$

$$\Big\downarrow \text{84}$$

$$-\int \left(\sec^3(x)+3\sec^2(x)+2\sec(x)-\cos^2(x)-3\cos(x)-2\right) d\cos(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{\cos^3(x)}{3}+\frac{3\cos^2(x)}{2}+2\cos(x)+\frac{\sec^2(x)}{2}+3\sec(x)-2\log(\cos(x))$$

input
```
Int[(Sin[x] + Tan[x])^3,x]
```

output
```
2*Cos[x] + (3*Cos[x]^2)/2 + Cos[x]^3/3 - 2*Log[Cos[x]] + 3*Sec[x] + Sec[x]
^2/2
```

3.342.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.342.4 Maple [A] (verified)

Time = 10.22 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

method	result	size
default	$\frac{8\left(2+\sin(x)^2\right)\cos(x)}{3}-\frac{3\sin(x)^2}{2}-2\ln\left(\cos\left(x\right)\right)+\frac{3\sin(x)^4}{\cos(x)}+\frac{\tan(x)^2}{2}$	39
parts	$\frac{8\left(2+\sin(x)^2\right)\cos(x)}{3}+\frac{\tan(x)^2}{2}-\frac{\ln\left(1+\tan(x)^2\right)}{2}+\frac{3\sin(x)^4}{\cos(x)}-\frac{3\sin(x)^2}{2}-3\ln\left(\cos\left(x\right)\right)$	48
risch	$2ix+\frac{e^{3ix}}{24}+\frac{3\,e^{2ix}}{8}+\frac{9\,e^{ix}}{8}+\frac{9\,e^{-ix}}{8}+\frac{3\,e^{-2ix}}{8}+\frac{e^{-3ix}}{24}+\frac{6\,e^{3ix}+2\,e^{2ix}+6\,e^{ix}}{\left(e^{2ix}+1\right)^2}-2\ln\left(e^{2ix}+1\right)$	89

input
```
int((sin(x)+tan(x))^3,x,method=_RETURNVERBOSE)
```

output
```
8/3*(2+sin(x)^2)*cos(x)-3/2*sin(x)^2-2*ln(cos(x))+3*sin(x)^4/cos(x)+1/2*ta
n(x)^2
```

3.342.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.24

$$\int (\sin(x) + \tan(x))^3 \, dx = \frac{4 \cos(x)^5 + 18 \cos(x)^4 + 24 \cos(x)^3 - 24 \cos(x)^2 \log(-\cos(x)) - 9 \cos(x)^2 + 36 \cos(x) + 6}{12 \cos(x)^2}$$

input
```
integrate((sin(x)+tan(x))^3,x, algorithm="fricas")
```

output
```
1/12*(4*cos(x)^5 + 18*cos(x)^4 + 24*cos(x)^3 - 24*cos(x)^2*log(-cos(x)) -
9*cos(x)^2 + 36*cos(x) + 6)/cos(x)^2
```

3.342.6 Sympy [A] (verification not implemented)

Time = 1.65 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.21

$$\int (\sin(x) + \tan(x))^3 \, dx = -3 \log(\cos(x)) - \frac{\log(\sec^2(x))}{2} + \frac{\cos^3(x)}{3} + \frac{3\cos^2(x)}{2} + 2\cos(x) + \frac{\sec^2(x)}{2} + \frac{3}{\cos(x)}$$

input
```
integrate((sin(x)+tan(x))**3,x)
```

output
```
-3*log(cos(x)) - log(sec(x)**2)/2 + cos(x)**3/3 + 3*cos(x)**2/2 + 2*cos(x)
 + sec(x)**2/2 + 3/cos(x)
```

3.342.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

$$\int (\sin(x)+\tan(x))^3\,dx = \frac{1}{3}\cos(x)^3 - \frac{3}{2}\sin(x)^2 - \frac{1}{2\left(\sin(x)^2-1\right)} + \frac{3}{\cos(x)} + 2\cos(x) - \log\left(\sin(x)^2-1\right)$$

input
```
integrate((sin(x)+tan(x))^3,x, algorithm="maxima")
```

output
```
1/3*cos(x)^3 - 3/2*sin(x)^2 - 1/2/(sin(x)^2 - 1) + 3/cos(x) + 2*cos(x) - l
og(sin(x)^2 - 1)
```

3.342.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(32) = 64.

Time = 0.46 (sec) , antiderivative size = 173, normalized size of antiderivative = 4.55

$$\int (\sin(x)+\tan(x))^3\,dx = \frac{\tan\left(\frac{1}{2}x\right)^4\tan(x)^4 - 2\log\left(\frac{4}{\tan(x)^2+1}\right)\tan\left(\frac{1}{2}x\right)^4\tan(x)^2 - 10\tan\left(\frac{1}{2}x\right)^4\tan(x)^2 - 2\log\left(\frac{4}{\tan(x)^2+1}\right)\tan\cdots}{2\left(\tan\left(\frac{1}{2}\cdots\right.\right.} + \frac{1}{12}\cos(3x)$$

input
```
integrate((sin(x)+tan(x))^3,x, algorithm="giac")
```

output
```
1/2*(tan(1/2*x)^4*tan(x)^4 - 2*log(4/(tan(x)^2 + 1))*tan(1/2*x)^4*tan(x)^2
 - 10*tan(1/2*x)^4*tan(x)^2 - 2*log(4/(tan(x)^2 + 1))*tan(1/2*x)^4 - 8*tan
(1/2*x)^4 - 3*tan(1/2*x)^2*tan(x)^2 - tan(x)^4 + 2*log(4/(tan(x)^2 + 1))*t
an(x)^2 - 3*tan(1/2*x)^2 - 11*tan(x)^2 + 2*log(4/(tan(x)^2 + 1)) - 13)/(ta
n(1/2*x)^4*tan(x)^2 + tan(1/2*x)^4 - tan(x)^2 - 1) + 1/12*cos(3*x)
```

3.342.9 Mupad [B] (verification not implemented)

Time = 28.72 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.71

$$\int (\sin(x)+\tan(x))^3\,dx = 4\,\mathrm{atanh}\left(\tan\left(\frac{x}{2}\right)^2\right) + \frac{-4\tan\left(\frac{x}{2}\right)^8-4\tan\left(\frac{x}{2}\right)^6+\frac{20\tan\left(\frac{x}{2}\right)^4}{3}+\frac{20\tan\left(\frac{x}{2}\right)^2}{3}+\frac{32}{3}}{\left(\tan\left(\frac{x}{2}\right)^2-1\right)^2\left(\tan\left(\frac{x}{2}\right)^2+1\right)^3}$$

input
```
int((sin(x) + tan(x))^3,x)
```

output
```
4*atanh(tan(x/2)^2) + ((20*tan(x/2)^2)/3 + (20*tan(x/2)^4)/3 - 4*tan(x/2)^
6 - 4*tan(x/2)^8 + 32/3)/((tan(x/2)^2 - 1)^2*(tan(x/2)^2 + 1)^3)
```

3.343 $\int (\sin(x) + \tan(x))^2 \, dx$

3.343.1 Optimal result

Integrand size = 7, antiderivative size = 25

$$\int (\sin(x) + \tan(x))^2 \, dx = -\frac{x}{2} + 2\text{arctanh}(\sin(x)) - 2\sin(x) - \frac{1}{2}\cos(x)\sin(x) + \tan(x)$$

output
```
-1/2*x+2*arctanh(sin(x))-2*sin(x)-1/2*cos(x)*sin(x)+tan(x)
```

3.343.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 60 vs. 2(25) = 50.

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.40

$$\int (\sin(x) + \tan(x))^2 \, dx = -\frac{x}{2} - 2\log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) + 2\log\left(\cos\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right)\right) - 2\sin(x) - \frac{1}{8}\sec(x)\sin(3x) + \frac{7\tan(x)}{8}$$

input
```
Integrate[(Sin[x] + Tan[x])^2,x]
```

output
```
-1/2*x - 2*Log[Cos[x/2] - Sin[x/2]] + 2*Log[Cos[x/2] + Sin[x/2]] - 2*Sin[x
] - (Sec[x]*Sin[3*x])/8 + (7*Tan[x])/8
```

3.343.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 4897, 3042, 3188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\sin(x)+\tan(x))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (\sin(x)+\tan(x))^2dx$$

$$\Big\downarrow \text{4897}$$

$$\int (\cos(x)+1)^2\tan^2(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(1-\sin\left(x-\frac{\pi}{2}\right)\right)^2}{\tan\left(x-\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3188}$$

$$\int \left(-\cos^2(x)-2\cos(x)+\sec^2(x)+2\sec(x)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$2\text{arctanh}(\sin(x))-\frac{x}{2}-2\sin(x)+\tan(x)-\frac{1}{2}\sin(x)\cos(x)$$

input
```
Int[(Sin[x] + Tan[x])^2,x]
```

output
```
-1/2*x + 2*ArcTanh[Sin[x]] - 2*Sin[x] - (Cos[x]*Sin[x])/2 + Tan[x]
```

3.343.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3188
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Simp[a^p   Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
 + f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b,
e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m
- p/2, 0])
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.343.4 Maple [A] (verified)

Time = 2.95 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

method	result	size
default	$-\frac{\cos(x)\sin(x)}{2}-\frac{x}{2}-2\sin(x)+2\ln(\sec(x)+\tan(x))+\tan(x)$	25
parts	$-\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}+\tan(x)-\arctan(\tan(x))-2\sin(x)+2\ln(\sec(x)+\tan(x))$	30
risch	$-\frac{x}{2}+\frac{i\mathrm{e}^{2ix}}{8}+i\mathrm{e}^{ix}-i\mathrm{e}^{-ix}-\frac{i\mathrm{e}^{-2ix}}{8}+\frac{2i}{\mathrm{e}^{2ix}+1}+2\ln(i+\mathrm{e}^{ix})-2\ln(\mathrm{e}^{ix}-i)$	71

input
```
int((sin(x)+tan(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*cos(x)*sin(x)-1/2*x-2*sin(x)+2*ln(sec(x)+tan(x))+tan(x)
```

3.343.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. 2(21) = 42.

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.76

$$\int (\sin(x)+\tan(x))^2\,dx = -\frac{x\cos(x) - 2\cos(x)\log(\sin(x)+1) + 2\cos(x)\log(-\sin(x)+1) + \left(\cos(x)^2 + 4\cos(x) - 2\right)\sin(x)}{2\cos(x)}$$

input
```
integrate((sin(x)+tan(x))^2,x, algorithm="fricas")
```

output
```
-1/2*(x*cos(x) - 2*cos(x)*log(sin(x) + 1) + 2*cos(x)*log(-sin(x) + 1) + (c
os(x)^2 + 4*cos(x) - 2)*sin(x))/cos(x)
```

3.343.6 Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.24

$$\int (\sin(x)+\tan(x))^2\,dx = -\frac{x}{2} - \log(\sin(x)-1) + \log(\sin(x)+1) - 2\sin(x) - \frac{\sin(2x)}{4} + \tan(x)$$

input
```
integrate((sin(x)+tan(x))**2,x)
```

output
```
-x/2 - log(sin(x) - 1) + log(sin(x) + 1) - 2*sin(x) - sin(2*x)/4 + tan(x)
```

3.343.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int (\sin(x)+\tan(x))^2\,dx = -\frac{1}{2}\,x + \log(\sin(x)+1) - \log(\sin(x)-1) - \frac{1}{4}\,\sin(2\,x) - 2\,\sin(x) + \tan(x)$$

input
```
integrate((sin(x)+tan(x))^2,x, algorithm="maxima")
```

output
```
-1/2*x + log(sin(x) + 1) - log(sin(x) - 1) - 1/4*sin(2*x) - 2*sin(x) + tan
(x)
```

3.343.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(21) = 42$.

Time = 0.33 (sec) , antiderivative size = 177, normalized size of antiderivative = 7.08

$$\int (\sin(x)+\tan(x))^2\,dx = \frac{1}{2}x - \frac{x\tan\left(\frac{1}{2}x\right)^2 - \log\left(\frac{2\left(\tan\left(\frac{1}{2}x\right)^2+2\tan\left(\frac{1}{2}x\right)+1\right)}{\tan\left(\frac{1}{2}x\right)^2+1}\right)\tan\left(\frac{1}{2}x\right)^2 + \log\left(\frac{2\left(\tan\left(\frac{1}{2}x\right)^2-2\tan\left(\frac{1}{2}x\right)+1\right)}{\tan\left(\frac{1}{2}x\right)^2+1}\right)\tan\left(\frac{1}{2}x\right)^2 - \mathrm{ta}}{\tan(} - \frac{1}{4}\sin(2x)$$

input
```
integrate((sin(x)+tan(x))^2,x, algorithm="giac")
```

output
```
1/2*x - (x*tan(1/2*x)^2 - log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)/(tan(1/2
*x)^2 + 1))*tan(1/2*x)^2 + log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/
2*x)^2 + 1))*tan(1/2*x)^2 - tan(1/2*x)^2*tan(x) + x - log(2*(tan(1/2*x)^2
+ 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1)) + log(2*(tan(1/2*x)^2 - 2*tan(1/2*
x) + 1)/(tan(1/2*x)^2 + 1)) + 4*tan(1/2*x) - tan(x))/(tan(1/2*x)^2 + 1) -
1/4*sin(2*x)
```

3.343.9 Mupad [B] (verification not implemented)

Time = 28.60 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.44

$$\int (\sin(x)+\tan(x))^2\,dx = 4\,\mathrm{atanh}\left(\tan\left(\frac{x}{2}\right)\right) - \frac{x}{2} + \frac{5\tan\left(\frac{x}{2}\right)^5 + 6\tan\left(\frac{x}{2}\right)^3 - 3\tan\left(\frac{x}{2}\right)}{-\tan\left(\frac{x}{2}\right)^6 - \tan\left(\frac{x}{2}\right)^4 + \tan\left(\frac{x}{2}\right)^2 + 1}$$

input
```
int((sin(x) + tan(x))^2,x)
```

output
```
4*atanh(tan(x/2)) - x/2 + (6*tan(x/2)^3 - 3*tan(x/2) + 5*tan(x/2)^5)/(tan(
x/2)^2 - tan(x/2)^4 - tan(x/2)^6 + 1)
```

3.344 $\int (\sin(x) + \tan(x))\,dx$

3.344.1 Optimal result

Integrand size = 5, antiderivative size = 10

$$\int (\sin(x) + \tan(x))\,dx = -\cos(x) - \log(\cos(x))$$

output
```
-cos(x)-ln(cos(x))
```

3.344.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int (\sin(x) + \tan(x))\,dx = -\cos(x) - \log(\cos(x))$$

input
```
Integrate[Sin[x] + Tan[x],x]
```

output
```
-Cos[x] - Log[Cos[x]]
```

3.344.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\sin(x)+\tan(x))\,dx$$
$$\Big\downarrow \ 2009$$
$$-\cos(x)-\log(\cos(x))$$

input
```
Int[Sin[x] + Tan[x],x]
```

output
```
-Cos[x] - Log[Cos[x]]
```

3.344.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.344.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
default	$-\cos\left(x\right)-\ln\left(\cos\left(x\right)\right)$	11
parts	$-\cos\left(x\right)-\ln\left(\cos\left(x\right)\right)$	11
parallelrisch	$-\cos\left(x\right)+\ln\left(\sqrt{\sec\left(x\right)^{2}}\right)-1$	14
risch	$ix-\ln\left({\mathrm e}^{2ix}+1\right)-\cos\left(x\right)$	20
norman	$-\frac{2}{1+\tan\left(\frac{x}{2}\right)^{2}}+\frac{\ln\left(1+\tan\left(x\right)^{2}\right)}{2}$	23

input
```
int(sin(x)+tan(x),x,method=_RETURNVERBOSE)
```

output
```
-cos(x)-ln(cos(x))
```

3.344.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int (\sin(x) + \tan(x))\, dx = -\cos(x) - \log(-\cos(x))$$

input
```
integrate(sin(x)+tan(x),x, algorithm="fricas")
```

output
```
-cos(x) - log(-cos(x))
```

3.344.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int (\sin(x) + \tan(x))\, dx = -\log(\cos(x)) - \cos(x)$$

input
```
integrate(sin(x)+tan(x),x)
```

output
```
-log(cos(x)) - cos(x)
```

3.344.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int (\sin(x) + \tan(x))\, dx = -\cos(x) + \log(\sec(x))$$

input
```
integrate(sin(x)+tan(x),x, algorithm="maxima")
```

output
```
-cos(x) + log(sec(x))
```

3.344.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int (\sin(x) + \tan(x))\, dx = -\cos(x) - \log(|\cos(x)|)$$

input
```
integrate(sin(x)+tan(x),x, algorithm="giac")
```

output
```
-cos(x) - log(abs(cos(x)))
```

3.344.9 Mupad [B] (verification not implemented)

Time = 27.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.20

$$\int (\sin(x) + \tan(x))\, dx = 2\,\mathrm{atanh}\left(\tan\left(\frac{x}{2}\right)^2\right) - \frac{2}{\tan\left(\frac{x}{2}\right)^2 + 1}$$

input
```
int(sin(x) + tan(x),x)
```

output
```
2*atanh(tan(x/2)^2) - 2/(tan(x/2)^2 + 1)
```

3.345 $\int \frac{1}{\sin(x)+\tan(x)} \, dx$

3.345.1 Optimal result

Integrand size = 7, antiderivative size = 24

$$\int \frac{1}{\sin(x)+\tan(x)} \, dx = -\frac{1}{2}\operatorname{arctanh}(\cos(x)) + \frac{1}{2}\cot(x)\csc(x) - \frac{\csc^2(x)}{2}$$

output
```
-1/2*arctanh(cos(x))+1/2*cot(x)*csc(x)-1/2*csc(x)^2
```

3.345.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \frac{1}{\sin(x)+\tan(x)} \, dx = -\frac{1}{2}\log\left(\cos\left(\frac{x}{2}\right)\right) + \frac{1}{2}\log\left(\sin\left(\frac{x}{2}\right)\right) - \frac{1}{4}\sec^2\left(\frac{x}{2}\right)$$

input
```
Integrate[(Sin[x] + Tan[x])^(-1),x]
```

output
```
-1/2*Log[Cos[x/2]] + Log[Sin[x/2]]/2 - Sec[x/2]^2/4
```

3.345.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.857$, Rules used = {3042, 4897, 3042, 25, 3185, 25, 3042, 25, 3086, 15, 3091, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sin(x)+\tan(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(x)+\tan(x)}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \frac{\cot(x)}{\cos(x)+1}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\tan\left(x-\frac{\pi}{2}\right)}{1-\sin\left(x-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\tan\left(x-\frac{\pi}{2}\right)}{1-\sin\left(x-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3185}$$

$$-\int \cot^2(x)\csc(x)dx-\int -\cot(x)\csc^2(x)dx$$

$$\Big\downarrow \text{25}$$

$$\int \cot(x)\csc^2(x)dx-\int \cot^2(x)\csc(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\sec\left(x-\frac{\pi}{2}\right)^2\tan\left(x-\frac{\pi}{2}\right)dx-\int \sec\left(x-\frac{\pi}{2}\right)\tan\left(x-\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sec\left(x-\frac{\pi}{2}\right)^2\tan\left(x-\frac{\pi}{2}\right)dx-\int \sec\left(x-\frac{\pi}{2}\right)\tan\left(x-\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{3086}$$

$$-\int \csc(x)d\csc(x)-\int \sec\left(x-\frac{\pi}{2}\right)\tan\left(x-\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \ 15$$

$$-\int \sec\left(x-\frac{\pi}{2}\right)\tan\left(x-\frac{\pi}{2}\right)^2dx-\frac{1}{2}\csc^2(x)$$

$$\Big\downarrow \ 3091$$

$$\frac{\int \csc(x)dx}{2}-\frac{1}{2}\csc^2(x)+\frac{1}{2}\cot(x)\csc(x)$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \csc(x)dx}{2}-\frac{1}{2}\csc^2(x)+\frac{1}{2}\cot(x)\csc(x)$$

$$\Big\downarrow \ 4257$$

$$-\frac{1}{2}\text{arctanh}(\cos(x))-\frac{1}{2}\csc^2(x)+\frac{1}{2}\cot(x)\csc(x)$$

input
```
Int[(Sin[x] + Tan[x])^(-1),x]
```

output
```
-1/2*ArcTanh[Cos[x]] + (Cot[x]*Csc[x])/2 - Csc[x]^2/2
```

3.345.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3185
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[1/a   Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x
] - Simp[1/(b*g)   Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; Fre
eQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.345.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

method	result	size
default	$-\frac{1}{2(\cos(x)+1)}-\frac{\ln(\cos(x)+1)}{4}+\frac{\ln(\cos(x)-1)}{4}$	24
risch	$-\frac{e^{ix}}{(e^{ix}+1)^2}+\frac{\ln(e^{ix}-1)}{2}-\frac{\ln(e^{ix}+1)}{2}$	38

input
```
int(1/(sin(x)+tan(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(cos(x)+1)-1/4*ln(cos(x)+1)+1/4*ln(cos(x)-1)
```

3.345.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \frac{1}{\sin(x)+\tan(x)}\,dx = -\frac{(\cos(x)+1)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)-(\cos(x)+1)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)+2}{4(\cos(x)+1)}$$

input
```
integrate(1/(sin(x)+tan(x)),x, algorithm="fricas")
```

output
```
-1/4*((cos(x) + 1)*log(1/2*cos(x) + 1/2) - (cos(x) + 1)*log(-1/2*cos(x) +
1/2) + 2)/(cos(x) + 1)
```

3.345.6 Sympy [F]

$$\int \frac{1}{\sin(x)+\tan(x)}\,dx = \int \frac{1}{\sin(x)+\tan(x)}\,dx$$

input
```
integrate(1/(sin(x)+tan(x)),x)
```

output
```
Integral(1/(sin(x) + tan(x)), x)
```

3.345.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sin(x)+\tan(x)}\,dx = -\frac{\sin(x)^2}{4(\cos(x)+1)^2}+\frac{1}{2}\log\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

input
```
integrate(1/(sin(x)+tan(x)),x, algorithm="maxima")
```

output
```
-1/4*sin(x)^2/(cos(x) + 1)^2 + 1/2*log(sin(x)/(cos(x) + 1))
```

3.345.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17

$$\int \frac{1}{\sin(x)+\tan(x)}\,dx = \frac{\cos(x)-1}{4(\cos(x)+1)} + \frac{1}{4}\log\left(-\frac{\cos(x)-1}{\cos(x)+1}\right)$$

input
```
integrate(1/(sin(x)+tan(x)),x, algorithm="giac")
```

output
```
1/4*(cos(x) - 1)/(cos(x) + 1) + 1/4*log(-(cos(x) - 1)/(cos(x) + 1))
```

3.345.9 Mupad [B] (verification not implemented)

Time = 27.93 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sin(x)+\tan(x)}\,dx = \frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2} - \frac{\tan\left(\frac{x}{2}\right)^2}{4}$$

input
```
int(1/(sin(x) + tan(x)),x)
```

output
```
log(tan(x/2))/2 - tan(x/2)^2/4
```

3.346 $\int \frac{1}{(\sin(x)+\tan(x))^2}\,dx$

3.346.1 Optimal result

Integrand size = 7, antiderivative size = 33

$$\int \frac{1}{(\sin(x)+\tan(x))^2}\,dx=-\frac{1}{3}\cot^3(x)-\frac{2\cot^5(x)}{5}-\frac{2\csc^3(x)}{3}+\frac{2\csc^5(x)}{5}$$

output
```
-1/3*cot(x)^3-2/5*cot(x)^5-2/3*csc(x)^3+2/5*csc(x)^5
```

3.346.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.73

$$\int \frac{1}{(\sin(x)+\tan(x))^2}\,dx=-\frac{1}{8}\cot\left(\frac{x}{2}\right)-\frac{7}{120}\tan\left(\frac{x}{2}\right)-\frac{11}{120}\sec^2\left(\frac{x}{2}\right)\tan\left(\frac{x}{2}\right)+\frac{1}{40}\sec^4\left(\frac{x}{2}\right)\tan\left(\frac{x}{2}\right)$$

input
```
Integrate[(Sin[x] + Tan[x])^(-2),x]
```

output
```
-1/8*Cot[x/2] - (7*Tan[x/2])/120 - (11*Sec[x/2]^2*Tan[x/2])/120 + (Sec[x/2
]^4*Tan[x/2])/40
```

3.346.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {3042, 4897, 3042, 3190, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\sin(x)+\tan(x))^2} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(\sin(x)+\tan(x))^2}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \frac{\cot^2(x)}{(\cos(x)+1)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan\left(x-\frac{\pi}{2}\right)^2}{\left(1-\sin\left(x-\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3190}$$

$$\int \left(\cot^4(x)\csc^2(x)-2\cot^3(x)\csc^3(x)+\cot^2(x)\csc^4(x)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2}{5}\cot^5(x)-\frac{\cot^3(x)}{3}+\frac{2\csc^5(x)}{5}-\frac{2\csc^3(x)}{3}$$

input
```
Int[(Sin[x] + Tan[x])^(-2),x]
```

output
```
-1/3*Cot[x]^3 - (2*Cot[x]^5)/5 - (2*Csc[x]^3)/3 + (2*Csc[x]^5)/5
```

3.346.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3190
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((g_.)*tan[(e_.) + (f_.)*(x
_)])^(p_.), x_Symbol] :> Simp[a^(2*m)   Int[ExpandIntegrand[(g*Tan[e + f*x]
)^p/Sec[e + f*x]^m, (a*Sec[e + f*x] - b*Tan[e + f*x])^(-m), x], x], x] /; F
reeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[m, 0]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.346.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{\tan\left(\frac{x}{2}\right)^{5}}{40}-\frac{\tan\left(\frac{x}{2}\right)^{3}}{24}-\frac{\tan\left(\frac{x}{2}\right)}{8}-\frac{1}{8\tan\left(\frac{x}{2}\right)}$	32
risch	$-\frac{2i\left(15\,{\rm e}^{4ix}+20\,{\rm e}^{3ix}+20\,{\rm e}^{2ix}+4\,{\rm e}^{ix}+1\right)}{15\left({\rm e}^{ix}+1\right)^{5}\left({\rm e}^{ix}-1\right)}$	52

input
```
int(1/(sin(x)+tan(x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/40*tan(1/2*x)^5-1/24*tan(1/2*x)^3-1/8*tan(1/2*x)-1/8/tan(1/2*x)
```

3.346.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

$$\int \frac{1}{(\sin(x)+\tan(x))^2}\,dx = -\frac{\cos(x)^3 + 2\cos(x)^2 + 8\cos(x) + 4}{15\left(\cos(x)^2 + 2\cos(x) + 1\right)\sin(x)}$$

input
```
integrate(1/(sin(x)+tan(x))^2,x, algorithm="fricas")
```

output
```
-1/15*(cos(x)^3 + 2*cos(x)^2 + 8*cos(x) + 4)/((cos(x)^2 + 2*cos(x) + 1)*si
n(x))
```

3.346.6 Sympy [F]

$$\int \frac{1}{(\sin(x)+\tan(x))^2}\,dx = \int \frac{1}{(\sin(x)+\tan(x))^2}\,dx$$

input
```
integrate(1/(sin(x)+tan(x))**2,x)
```

output
```
Integral((sin(x) + tan(x))**(-2), x)
```

3.346.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.36

$$\int \frac{1}{(\sin(x)+\tan(x))^2}\,dx = -\frac{\cos(x)+1}{8\sin(x)} - \frac{\sin(x)}{8(\cos(x)+1)} - \frac{\sin(x)^3}{24(\cos(x)+1)^3} + \frac{\sin(x)^5}{40(\cos(x)+1)^5}$$

input
```
integrate(1/(sin(x)+tan(x))^2,x, algorithm="maxima")
```

output
```
-1/8*(cos(x) + 1)/sin(x) - 1/8*sin(x)/(cos(x) + 1) - 1/24*sin(x)^3/(cos(x)
 + 1)^3 + 1/40*sin(x)^5/(cos(x) + 1)^5
```

3.346.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{1}{(\sin(x)+\tan(x))^2}\,dx = \frac{1}{40}\tan\left(\frac{1}{2}x\right)^5 - \frac{1}{24}\tan\left(\frac{1}{2}x\right)^3 - \frac{1}{8\tan\left(\frac{1}{2}x\right)} - \frac{1}{8}\tan\left(\frac{1}{2}x\right)$$

input
```
integrate(1/(sin(x)+tan(x))^2,x, algorithm="giac")
```

output
```
1/40*tan(1/2*x)^5 - 1/24*tan(1/2*x)^3 - 1/8/tan(1/2*x) - 1/8*tan(1/2*x)
```

3.346.9 Mupad [B] (verification not implemented)

Time = 29.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.21

$$\int \frac{1}{(\sin(x)+\tan(x))^2}\,dx = -\frac{8\cos\left(\frac{x}{2}\right)^6 - 4\cos\left(\frac{x}{2}\right)^4 + 14\cos\left(\frac{x}{2}\right)^2 - 3}{120\cos\left(\frac{x}{2}\right)^5\sin\left(\frac{x}{2}\right)}$$

input
```
int(1/(sin(x) + tan(x))^2,x)
```

output
```
-(14*cos(x/2)^2 - 4*cos(x/2)^4 + 8*cos(x/2)^6 - 3)/(120*cos(x/2)^5*sin(x/2
))
```

3.347 $\int \frac{1}{(\sin(x)+\tan(x))^3} dx$

3.347.1 Optimal result

Integrand size = 7, antiderivative size = 60

$$\int \frac{1}{(\sin(x)+\tan(x))^3} dx = \frac{1}{32}\text{arctanh}(\cos(x)) - \frac{1}{32(1-\cos(x))} - \frac{1}{16(1+\cos(x))^4} + \frac{1}{6(1+\cos(x))^3} - \frac{3}{32(1+\cos(x))^2} - \frac{1}{16(1+\cos(x))}$$

output
```
1/32*arctanh(cos(x))-1/32/(1-cos(x))-1/16/(1+cos(x))^4+1/6/(1+cos(x))^3-3/
32/(1+cos(x))^2-1/16/(1+cos(x))
```

3.347.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.38

$$\int \frac{1}{(\sin(x)+\tan(x))^3} dx = -\frac{1}{64}\csc^2\left(\frac{x}{2}\right) + \frac{1}{32}\log\left(\cos\left(\frac{x}{2}\right)\right) - \frac{1}{32}\log\left(\sin\left(\frac{x}{2}\right)\right) - \frac{1}{32}\sec^2\left(\frac{x}{2}\right) - \frac{3}{128}\sec^4\left(\frac{x}{2}\right) + \frac{1}{48}\sec^6\left(\frac{x}{2}\right) - \frac{1}{256}\sec^8\left(\frac{x}{2}\right)$$

input
```
Integrate[(Sin[x] + Tan[x])^(-3),x]
```

output
```
-1/64*Csc[x/2]^2 + Log[Cos[x/2]]/32 - Log[Sin[x/2]]/32 - Sec[x/2]^2/32 - (
3*Sec[x/2]^4)/128 + Sec[x/2]^6/48 - Sec[x/2]^8/256
```

3.347.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 4897, 3042, 25, 3186, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\sin(x)+\tan(x))^3} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(\sin(x)+\tan(x))^3}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \frac{\cot^3(x)}{(\cos(x)+1)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\tan\left(x-\frac{\pi}{2}\right)^3}{\left(1-\sin\left(x-\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\tan\left(x-\frac{\pi}{2}\right)^3}{\left(1-\sin\left(x-\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3186}$$

$$-\int \frac{\cos^3(x)}{(1-\cos(x))^2(\cos(x)+1)^5}d\cos(x)$$

$$\Big\downarrow \text{99}$$

$$-\int \left(\frac{1}{32(\cos(x)-1)^2}-\frac{1}{16(\cos(x)+1)^2}-\frac{3}{16(\cos(x)+1)^3}+\frac{1}{2(\cos(x)+1)^4}-\frac{1}{4(\cos(x)+1)^5}+\frac{1}{32\left(\cos^2(x)-\right.}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{32}\text{arctanh}(\cos(x))-\frac{1}{32(1-\cos(x))}-\frac{1}{16(\cos(x)+1)}-\frac{3}{32(\cos(x)+1)^2}+\frac{1}{6(\cos(x)+1)^3}-\frac{1}{16(\cos(x)+1)^4}$$

input
```
Int[(Sin[x] + Tan[x])^(-3),x]
```

output
```
ArcTanh[Cos[x]]/32 - 1/(32*(1 - Cos[x])) - 1/(16*(1 + Cos[x])^4) + 1/(6*(1
 + Cos[x])^3) - 3/(32*(1 + Cos[x])^2) - 1/(16*(1 + Cos[x]))
```

3.347.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.347.4 Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

method	result	s
default	$-\frac{1}{16(\cos(x)+1)^4}+\frac{1}{6(\cos(x)+1)^3}-\frac{3}{32(\cos(x)+1)^2}-\frac{1}{16(\cos(x)+1)}+\frac{\ln(\cos(x)+1)}{64}+\frac{1}{32\cos(x)-32}-\frac{\ln(\cos(x)-1)}{64}$	5
risch	$-\frac{3\,e^{9ix}+18\,e^{8ix}-88\,e^{7ix}-162\,e^{6ix}-310\,e^{5ix}-162\,e^{4ix}-88\,e^{3ix}+18\,e^{2ix}+3\,e^{ix}}{48(e^{ix}+1)^8(e^{ix}-1)^2}+\frac{\ln(e^{ix}+1)}{32}-\frac{\ln(e^{ix}-1)}{32}$	1

input
```
int(1/(sin(x)+tan(x))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/16/(cos(x)+1)^4+1/6/(cos(x)+1)^3-3/32/(cos(x)+1)^2-1/16/(cos(x)+1)+1/64
*ln(cos(x)+1)+1/32/(cos(x)-1)-1/64*ln(cos(x)-1)
```

3.347.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. 2(46) = 92.

Time = 0.26 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.17

$$\int \frac{1}{(\sin(x)+\tan(x))^3}\,dx = -\frac{6\,\cos(x)^4+18\,\cos(x)^3-50\,\cos(x)^2-3\left(\cos(x)^5+3\,\cos(x)^4+2\,\cos(x)^3-2\,\cos(x)^2-3\,\cos(x)-\right.}{192\left(\cos(x)^5+3\,\cos\right.}$$

input
```
integrate(1/(sin(x)+tan(x))^3,x, algorithm="fricas")
```

output
```
-1/192*(6*cos(x)^4 + 18*cos(x)^3 - 50*cos(x)^2 - 3*(cos(x)^5 + 3*cos(x)^4
+ 2*cos(x)^3 - 2*cos(x)^2 - 3*cos(x) - 1)*log(1/2*cos(x) + 1/2) + 3*(cos(x
)^5 + 3*cos(x)^4 + 2*cos(x)^3 - 2*cos(x)^2 - 3*cos(x) - 1)*log(-1/2*cos(x)
 + 1/2) - 54*cos(x) - 16)/(cos(x)^5 + 3*cos(x)^4 + 2*cos(x)^3 - 2*cos(x)^2
 - 3*cos(x) - 1)
```

3.347.6 Sympy [F]

$$\int \frac{1}{(\sin(x)+\tan(x))^3}\,dx = \int \frac{1}{(\sin(x)+\tan(x))^3}\,dx$$

input
```
integrate(1/(sin(x)+tan(x))**3,x)
```

output
```
Integral((sin(x) + tan(x))**(-3), x)
```

3.347.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.22

$$\int \frac{1}{(\sin(x)+\tan(x))^3}\,dx = -\frac{(\cos(x)+1)^2}{64\sin(x)^2} - \frac{\sin(x)^2}{32(\cos(x)+1)^2} + \frac{\sin(x)^4}{64(\cos(x)+1)^4} + \frac{\sin(x)^6}{192(\cos(x)+1)^6} - \frac{\sin(x)^8}{256(\cos(x)+1)^8} - \frac{1}{32}\log\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

input
```
integrate(1/(sin(x)+tan(x))^3,x, algorithm="maxima")
```

output
```
-1/64*(cos(x) + 1)^2/sin(x)^2 - 1/32*sin(x)^2/(cos(x) + 1)^2 + 1/64*sin(x)
^4/(cos(x) + 1)^4 + 1/192*sin(x)^6/(cos(x) + 1)^6 - 1/256*sin(x)^8/(cos(x)
 + 1)^8 - 1/32*log(sin(x)/(cos(x) + 1))
```

3.347.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(46) = 92.

Time = 0.28 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.58

$$\int \frac{1}{(\sin(x)+\tan(x))^3}\,dx = \frac{\left(\frac{\cos(x)-1}{\cos(x)+1}+1\right)(\cos(x)+1)}{64(\cos(x)-1)} + \frac{\cos(x)-1}{32(\cos(x)+1)} + \frac{(\cos(x)-1)^2}{64(\cos(x)+1)^2} - \frac{(\cos(x)-1)^3}{192(\cos(x)+1)^3} - \frac{(\cos(x)-1)^4}{256(\cos(x)+1)^4} - \frac{1}{64}\log\left(-\frac{\cos(x)-1}{\cos(x)+1}\right)$$

input
```
integrate(1/(sin(x)+tan(x))^3,x, algorithm="giac")
```

output
```
1/64*((cos(x) - 1)/(cos(x) + 1) + 1)*(cos(x) + 1)/(cos(x) - 1) + 1/32*(cos
(x) - 1)/(cos(x) + 1) + 1/64*(cos(x) - 1)^2/(cos(x) + 1)^2 - 1/192*(cos(x)
 - 1)^3/(cos(x) + 1)^3 - 1/256*(cos(x) - 1)^4/(cos(x) + 1)^4 - 1/64*log(-(
cos(x) - 1)/(cos(x) + 1))
```

3.347.9 Mupad [B] (verification not implemented)

Time = 29.39 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.80

$$\int \frac{1}{(\sin(x)+\tan(x))^3}\,dx = \frac{\tan\left(\frac{x}{2}\right)^4}{64} - \frac{1}{64\tan\left(\frac{x}{2}\right)^2} - \frac{\tan\left(\frac{x}{2}\right)^2}{32} - \frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{32} + \frac{\tan\left(\frac{x}{2}\right)^6}{192} - \frac{\tan\left(\frac{x}{2}\right)^8}{256}$$

input
```
int(1/(sin(x) + tan(x))^3,x)
```

output
```
tan(x/2)^4/64 - 1/(64*tan(x/2)^2) - tan(x/2)^2/32 - log(tan(x/2))/32 + tan
(x/2)^6/192 - tan(x/2)^8/256
```

3.348 $\int \frac{1}{(\sin(x)+\tan(x))^4} \, dx$

3.348.1 Optimal result

Integrand size = 7, antiderivative size = 65

$$\int \frac{1}{(\sin(x)+\tan(x))^4} \, dx = -\frac{1}{5}\cot^5(x)-\frac{9\cot^7(x)}{7}-\frac{16\cot^9(x)}{9}-\frac{8\cot^{11}(x)}{11}-\frac{4\csc^5(x)}{5}+\frac{16\csc^7(x)}{7}-\frac{20\csc^9(x)}{9}+\frac{8\csc^{11}(x)}{11}$$

output
```
-1/5*cot(x)^5-9/7*cot(x)^7-16/9*cot(x)^9-8/11*cot(x)^11-4/5*csc(x)^5+16/7*
csc(x)^7-20/9*csc(x)^9+8/11*csc(x)^11
```

3.348.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.98

$$\int \frac{1}{(\sin(x)+\tan(x))^4} \, dx = \frac{1}{96}\cot\left(\frac{x}{2}\right)-\frac{1}{384}\cot\left(\frac{x}{2}\right)\csc^2\left(\frac{x}{2}\right)-\frac{2749\tan\left(\frac{x}{2}\right)}{110880}-\frac{2033\sec^2\left(\frac{x}{2}\right)\tan\left(\frac{x}{2}\right)}{443520}+\frac{179\sec^4\left(\frac{x}{2}\right)\tan\left(\frac{x}{2}\right)}{73920}+\frac{641\sec^6\left(\frac{x}{2}\right)\tan\left(\frac{x}{2}\right)}{88704}-\frac{7\sec^8\left(\frac{x}{2}\right)\tan\left(\frac{x}{2}\right)}{1584}+\frac{\sec^{10}\left(\frac{x}{2}\right)\tan\left(\frac{x}{2}\right)}{1408}$$

input
```
Integrate[(Sin[x] + Tan[x])^(-4),x]
```

output
```
Cot[x/2]/96 - (Cot[x/2]*Csc[x/2]^2)/384 - (2749*Tan[x/2])/110880 - (2033*S
ec[x/2]^2*Tan[x/2])/443520 + (179*Sec[x/2]^4*Tan[x/2])/73920 + (641*Sec[x/
2]^6*Tan[x/2])/88704 - (7*Sec[x/2]^8*Tan[x/2])/1584 + (Sec[x/2]^10*Tan[x/2
])/1408
```

3.348.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 4897, 3042, 3190, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\sin(x)+\tan(x))^4}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{(\sin(x)+\tan(x))^4}dx$$

$$\Big\downarrow \; 4897$$

$$\int \frac{\cot^4(x)}{(\cos(x)+1)^4}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\tan\left(x-\frac{\pi}{2}\right)^4}{\left(1-\sin\left(x-\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \; 3190$$

$$\int \left(\cot^8(x)\csc^4(x)-4\cot^7(x)\csc^5(x)+6\cot^6(x)\csc^6(x)-4\cot^5(x)\csc^7(x)+\cot^4(x)\csc^8(x)\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{8}{11}\cot^{11}(x)-\frac{16\cot^9(x)}{9}-\frac{9\cot^7(x)}{7}-\frac{\cot^5(x)}{5}+\frac{8\csc^{11}(x)}{11}-\frac{20\csc^9(x)}{9}+\frac{16\csc^7(x)}{7}-\frac{4\csc^5(x)}{5}$$

input
```
Int[(Sin[x] + Tan[x])^(-4),x]
```

output
```
-1/5*Cot[x]^5 - (9*Cot[x]^7)/7 - (16*Cot[x]^9)/9 - (8*Cot[x]^11)/11 - (4*C
sc[x]^5)/5 + (16*Csc[x]^7)/7 - (20*Csc[x]^9)/9 + (8*Csc[x]^11)/11
```

3.348.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3190
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((g_.)*tan[(e_.) + (f_.)*(x
_)])^(p_.), x_Symbol] :> Simp[a^(2*m)   Int[ExpandIntegrand[(g*Tan[e + f*x]
)^p/Sec[e + f*x]^m, (a*Sec[e + f*x] - b*Tan[e + f*x])^(-m), x], x], x] /; F
reeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[m, 0]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.348.4 Maple [A] (verified)

Time = 1.66 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{\tan\left(\frac{x}{2}\right)^{11}}{1408}-\frac{\tan\left(\frac{x}{2}\right)^{9}}{1152}-\frac{3\tan\left(\frac{x}{2}\right)^{7}}{896}+\frac{3\tan\left(\frac{x}{2}\right)^{5}}{640}+\frac{\tan\left(\frac{x}{2}\right)^{3}}{128}-\frac{3\tan\left(\frac{x}{2}\right)}{128}+\frac{1}{128\tan\left(\frac{x}{2}\right)}-\frac{1}{384\tan\left(\frac{x}{2}\right)^{3}}$	64
risch	$\frac{4i\left(3465\,{\mathrm e}^{10ix}+5544\,{\mathrm e}^{9ix}+10857\,{\mathrm e}^{8ix}+5280\,{\mathrm e}^{7ix}+4818\,{\mathrm e}^{6ix}+176\,{\mathrm e}^{5ix}+2794\,{\mathrm e}^{4ix}+1952\,{\mathrm e}^{3ix}+1525\,{\mathrm e}^{2ix}+488\,{\mathrm e}^{ix}+61\right)}{3465\left({\mathrm e}^{ix}+1\right)^{11}\left({\mathrm e}^{ix}-1\right)^{3}}$	94

input
```
int(1/(sin(x)+tan(x))^4,x,method=_RETURNVERBOSE)
```

output
```
1/1408*tan(1/2*x)^11-1/1152*tan(1/2*x)^9-3/896*tan(1/2*x)^7+3/640*tan(1/2*
x)^5+1/128*tan(1/2*x)^3-3/128*tan(1/2*x)+1/128/tan(1/2*x)-1/384/tan(1/2*x)
^3
```

3.348.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.20

$$\int \frac{1}{(\sin(x)+\tan(x))^4}\,dx = \frac{122\cos(x)^7+488\cos(x)^6+549\cos(x)^5-244\cos(x)^4-64\cos(x)^3+144\cos(x)^2+128\cos(x)+32}{3465\left(\cos(x)^6+4\cos(x)^5+5\cos(x)^4-5\cos(x)^2-4\cos(x)-1\right)\sin(x)}$$

input
```
integrate(1/(sin(x)+tan(x))^4,x, algorithm="fricas")
```

output
```
1/3465*(122*cos(x)^7 + 488*cos(x)^6 + 549*cos(x)^5 - 244*cos(x)^4 - 64*cos
(x)^3 + 144*cos(x)^2 + 128*cos(x) + 32)/((cos(x)^6 + 4*cos(x)^5 + 5*cos(x)
^4 - 5*cos(x)^2 - 4*cos(x) - 1)*sin(x))
```

3.348.6 Sympy [F]

$$\int \frac{1}{(\sin(x)+\tan(x))^4}\,dx = \int \frac{1}{(\sin(x)+\tan(x))^4}\,dx$$

input
```
integrate(1/(sin(x)+tan(x))**4,x)
```

output
```
Integral((sin(x) + tan(x))**(-4), x)
```

3.348.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.49

$$\int \frac{1}{(\sin(x)+\tan(x))^4}\,dx = \frac{\left(\frac{3\sin(x)^2}{(\cos(x)+1)^2}-1\right)(\cos(x)+1)^3}{384\sin(x)^3} - \frac{3\sin(x)}{128(\cos(x)+1)} + \frac{\sin(x)^3}{128(\cos(x)+1)^3} + \frac{3\sin(x)^5}{640(\cos(x)+1)^5} - \frac{3\sin(x)^7}{896(\cos(x)+1)^7} - \frac{\sin(x)^9}{1152(\cos(x)+1)^9} + \frac{\sin(x)^{11}}{1408(\cos(x)+1)^{11}}$$

input
```
integrate(1/(sin(x)+tan(x))^4,x, algorithm="maxima")
```

output
```
1/384*(3*sin(x)^2/(cos(x) + 1)^2 - 1)*(cos(x) + 1)^3/sin(x)^3 - 3/128*sin(
x)/(cos(x) + 1) + 1/128*sin(x)^3/(cos(x) + 1)^3 + 3/640*sin(x)^5/(cos(x) +
 1)^5 - 3/896*sin(x)^7/(cos(x) + 1)^7 - 1/1152*sin(x)^9/(cos(x) + 1)^9 + 1
/1408*sin(x)^11/(cos(x) + 1)^11
```

3.348.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int \frac{1}{(\sin(x)+\tan(x))^4}\,dx = \frac{1}{1408}\tan\left(\frac{1}{2}x\right)^{11} - \frac{1}{1152}\tan\left(\frac{1}{2}x\right)^{9} - \frac{3}{896}\tan\left(\frac{1}{2}x\right)^{7} + \frac{3}{640}\tan\left(\frac{1}{2}x\right)^{5} + \frac{1}{128}\tan\left(\frac{1}{2}x\right)^{3} + \frac{3\tan\left(\frac{1}{2}x\right)^{2}-1}{384\tan\left(\frac{1}{2}x\right)^{3}} - \frac{3}{128}\tan\left(\frac{1}{2}x\right)$$

input
```
integrate(1/(sin(x)+tan(x))^4,x, algorithm="giac")
```

output
```
1/1408*tan(1/2*x)^11 - 1/1152*tan(1/2*x)^9 - 3/896*tan(1/2*x)^7 + 3/640*ta
n(1/2*x)^5 + 1/128*tan(1/2*x)^3 + 1/384*(3*tan(1/2*x)^2 - 1)/tan(1/2*x)^3
- 3/128*tan(1/2*x)
```

3.348.9 Mupad [B] (verification not implemented)

Time = 29.11 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.34

$$\int \frac{1}{(\sin(x)+\tan(x))^4}\,dx = -\frac{15616\cos\left(\frac{x}{2}\right)^{14} - 23424\cos\left(\frac{x}{2}\right)^{12} + 5856\cos\left(\frac{x}{2}\right)^{10} + 976\cos\left(\frac{x}{2}\right)^{8} + 7296\cos\left(\frac{x}{2}\right)^{6} - 7440\cos\left(\frac{x}{2}\right)^{4} + 25\cdots}{443520\left(\cos\left(\frac{x}{2}\right)^{11}\sin\left(\frac{x}{2}\right) - \cos\left(\frac{x}{2}\right)^{13}\sin\left(\frac{x}{2}\right)\right)}$$

input
```
int(1/(sin(x) + tan(x))^4,x)
```

output
```
-(2590*cos(x/2)^2 - 7440*cos(x/2)^4 + 7296*cos(x/2)^6 + 976*cos(x/2)^8 + 5
856*cos(x/2)^10 - 23424*cos(x/2)^12 + 15616*cos(x/2)^14 - 315)/(443520*(co
s(x/2)^11*sin(x/2) - cos(x/2)^13*sin(x/2)))
```

3.348. $\int \frac{1}{(\sin(x)+\tan(x))^4}\,dx$

3.349 $\int \frac{A+C\sin(x)}{b\cos(x)+c\sin(x)}\,dx$

3.349.1 Optimal result

Integrand size = 18, antiderivative size = 74

$$\int \frac{A+C\sin(x)}{b\cos(x)+c\sin(x)}\,dx=\frac{cCx}{b^2+c^2}-\frac{A\text{arctanh}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}}-\frac{bC\log(b\cos(x)+c\sin(x))}{b^2+c^2}$$

output
```
c*C*x/(b^2+c^2)-b*C*ln(b*cos(x)+c*sin(x))/(b^2+c^2)-A*arctanh((c*cos(x)-b*
sin(x))/(b^2+c^2)^(1/2))/(b^2+c^2)^(1/2)
```

3.349.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.92

$$\int \frac{A+C\sin(x)}{b\cos(x)+c\sin(x)}\,dx=\frac{2A\text{arctanh}\left(\frac{-c+b\tan\left(\frac{x}{2}\right)}{\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}}+\frac{C(cx-b\log(b\cos(x)+c\sin(x)))}{b^2+c^2}$$

input
```
Integrate[(A + C*Sin[x])/(b*Cos[x] + c*Sin[x]),x]
```

output
```
(2*A*ArcTanh[(-c + b*Tan[x/2])/Sqrt[b^2 + c^2]])/Sqrt[b^2 + c^2] + (C*(c*x
 - b*Log[b*Cos[x] + c*Sin[x]]))/(b^2 + c^2)
```

3.349.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3042, 3616, 3042, 3553, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+C\sin(x)}{b\cos(x)+c\sin(x)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{A+C\sin(x)}{b\cos(x)+c\sin(x)}dx$$

$$\Big\downarrow \; 3616$$

$$A\int \frac{1}{b\cos(x)+c\sin(x)}dx+\frac{cCx}{b^2+c^2}-\frac{bC\log(b\cos(x)+c\sin(x))}{b^2+c^2}$$

$$\Big\downarrow \; 3042$$

$$A\int \frac{1}{b\cos(x)+c\sin(x)}dx+\frac{cCx}{b^2+c^2}-\frac{bC\log(b\cos(x)+c\sin(x))}{b^2+c^2}$$

$$\Big\downarrow \; 3553$$

$$-A\int \frac{1}{b^2+c^2-(c\cos(x)-b\sin(x))^2}d(c\cos(x)-b\sin(x))+\frac{cCx}{b^2+c^2}-\frac{bC\log(b\cos(x)+c\sin(x))}{b^2+c^2}$$

$$\Big\downarrow \; 219$$

$$-\frac{A\text{arctanh}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}}+\frac{cCx}{b^2+c^2}-\frac{bC\log(b\cos(x)+c\sin(x))}{b^2+c^2}$$

input
```
Int[(A + C*Sin[x])/(b*Cos[x] + c*Sin[x]),x]
```

output
```
(c*C*x)/(b^2 + c^2) - (A*ArcTanh[(c*Cos[x] - b*Sin[x])/Sqrt[b^2 + c^2]])/S
qrt[b^2 + c^2] - (b*C*Log[b*Cos[x] + c*Sin[x]])/(b^2 + c^2)
```

3.349.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3616
```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[c*C*((d + e*x)/
(e*(b^2 + c^2))), x] + (-Simp[b*C*(Log[a + b*Cos[d + e*x] + c*Sin[d + e*x]]
/(e*(b^2 + c^2))), x] + Simp[(A*(b^2 + c^2) - a*c*C)/(b^2 + c^2)   Int[1/(a
 + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x]) /; FreeQ[{a, b, c, d, e, A, C}
, x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*c*C, 0]
```

3.349.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.58

method	result
default	$\frac{2C\left(\frac{b\ln\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)}{2}+c\arctan\left(\tan\left(\frac{x}{2}\right)\right)\right)}{b^{2}+c^{2}}+\frac{-bC\ln\left(\tan\left(\frac{x}{2}\right)^{2}b-2c\tan\left(\frac{x}{2}\right)-b\right)-\frac{2\left(-A\,b^{2}-A\,c^{2}\right)\operatorname{arctanh}\left(\frac{2b\tan\left(\frac{x}{2}\right)-2c}{2\sqrt{b^{2}+c^{2}}}\right)}{\sqrt{b^{2}+c^{2}}}}{b^{2}+c^{2}}$
risch	$\frac{ixC}{ic-b}+\frac{2iCx\,b^{3}}{b^{4}+2b^{2}c^{2}+c^{4}}+\frac{2iCx\,c^{2}b}{b^{4}+2b^{2}c^{2}+c^{4}}-\frac{\ln\left(e^{ix}+\frac{(ib-c)\sqrt{A^{2}b^{2}+A^{2}c^{2}}}{A\left(b^{2}+c^{2}\right)}\right)bC}{b^{2}+c^{2}}+\frac{\ln\left(e^{ix}+\frac{(ib-c)\sqrt{A^{2}b^{2}+A^{2}c^{2}}}{A\left(b^{2}+c^{2}\right)}\right)\sqrt{A^{2}b^{2}+A^{2}c^{2}}}{b^{2}+c^{2}}-$

input
```
int((A+C*sin(x))/(b*cos(x)+c*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
2*C/(b^2+c^2)*(1/2*b*ln(1+tan(1/2*x)^2)+c*arctan(tan(1/2*x)))+2/(b^2+c^2)*
(-1/2*b*C*ln(tan(1/2*x)^2*b-2*c*tan(1/2*x)-b)-(-A*b^2-A*c^2)/(b^2+c^2)^(1/
2)*arctanh(1/2*(2*b*tan(1/2*x)-2*c)/(b^2+c^2)^(1/2)))
```

3.349.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(70) = 140.

Time = 0.26 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.95

$$\int \frac{A+C\sin(x)}{b\cos(x)+c\sin(x)}\,dx = \frac{2\,Ccx - Cb\log\left(2\,bc\cos(x)\sin(x) + (b^2-c^2)\cos(x)^2 + c^2\right) + \sqrt{b^2+c^2}A\log\left(-\frac{2\,bc\cos(x)\sin(x)+(b^2-c^2)\cos(x)}{2\,bc\cos(x)\sin(x}\right.}{2\,(b^2+c^2)}$$

input
```
integrate((A+C*sin(x))/(b*cos(x)+c*sin(x)),x, algorithm="fricas")
```

output
```
1/2*(2*C*c*x - C*b*log(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 + c^2) +
 sqrt(b^2 + c^2)*A*log(-(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 - 2*b^
2 - c^2 + 2*sqrt(b^2 + c^2)*(c*cos(x) - b*sin(x)))/(2*b*c*cos(x)*sin(x) +
(b^2 - c^2)*cos(x)^2 + c^2)))/(b^2 + c^2)
```

3.349.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.68 (sec) , antiderivative size = 634, normalized size of antiderivative = 8.57

$$\int \frac{A+C\sin(x)}{b\cos(x)+c\sin(x)}\,dx = \begin{cases} \tilde{\infty}\left(A\log\left(\tan\left(\frac{x}{2}\right)\right)+Cx\right) \\ \frac{A\log\left(\tan\left(\frac{x}{2}\right)\right)+Cx}{c} \\ -\frac{2A}{2ic\sin(x)+2c\cos(x)} + \frac{iCx\sin(x)}{2ic\sin(x)+2c\cos(x)} + \frac{Cx\cos(x)}{2ic\sin(x)+2c\cos(x)} - \frac{C\sin(x)}{2ic\sin(x)+2c\cos(x)} \\ -\frac{2A}{-2ic\sin(x)+2c\cos(x)} - \frac{iCx\sin(x)}{-2ic\sin(x)+2c\cos(x)} + \frac{Cx\cos(x)}{-2ic\sin(x)+2c\cos(x)} - \frac{C\sin(x)}{-2ic\sin(x)+2c\cos(x)} \\ -\frac{Ab^2\log\left(\tan\left(\frac{x}{2}\right)-\frac{c}{b}-\frac{\sqrt{b^2+c^2}}{b}\right)}{b^2\sqrt{b^2+c^2}+c^2\sqrt{b^2+c^2}} + \frac{Ab^2\log\left(\tan\left(\frac{x}{2}\right)-\frac{c}{b}+\frac{\sqrt{b^2+c^2}}{b}\right)}{b^2\sqrt{b^2+c^2}+c^2\sqrt{b^2+c^2}} - \frac{Ac^2\log\left(\tan\left(\frac{x}{2}\right)-\frac{c}{b}-\frac{\sqrt{b^2+c^2}}{b}\right)}{b^2\sqrt{b^2+c^2}+c^2\sqrt{b^2+c^2}} + \frac{Ac^2\log\left(\tan\left(\frac{x}{2}\right)-\frac{c}{b}+\frac{\sqrt{b^2+}}{b}\right.}{b^2\sqrt{b^2+c^2}+c^2\sqrt{b^2+c^2}} \end{cases}$$

input
```
integrate((A+C*sin(x))/(b*cos(x)+c*sin(x)),x)
```

output
```
Piecewise((zoo*(A*log(tan(x/2)) + C*x), Eq(b, 0) & Eq(c, 0)), ((A*log(tan(
x/2)) + C*x)/c, Eq(b, 0)), (-2*A/(2*I*c*sin(x) + 2*c*cos(x)) + I*C*x*sin(x
)/(2*I*c*sin(x) + 2*c*cos(x)) + C*x*cos(x)/(2*I*c*sin(x) + 2*c*cos(x)) - C
*sin(x)/(2*I*c*sin(x) + 2*c*cos(x)), Eq(b, -I*c)), (-2*A/(-2*I*c*sin(x) +
2*c*cos(x)) - I*C*x*sin(x)/(-2*I*c*sin(x) + 2*c*cos(x)) + C*x*cos(x)/(-2*I
*c*sin(x) + 2*c*cos(x)) - C*sin(x)/(-2*I*c*sin(x) + 2*c*cos(x)), Eq(b, I*c
)), (-A*b**2*log(tan(x/2) - c/b - sqrt(b**2 + c**2)/b)/(b**2*sqrt(b**2 + c
**2) + c**2*sqrt(b**2 + c**2)) + A*b**2*log(tan(x/2) - c/b + sqrt(b**2 + c
**2)/b)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)) - A*c**2*log(tan
(x/2) - c/b - sqrt(b**2 + c**2)/b)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**
2 + c**2)) + A*c**2*log(tan(x/2) - c/b + sqrt(b**2 + c**2)/b)/(b**2*sqrt(b
**2 + c**2) + c**2*sqrt(b**2 + c**2)) + C*b*sqrt(b**2 + c**2)*log(tan(x/2)
**2 + 1)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)) - C*b*sqrt(b**2
 + c**2)*log(tan(x/2) - c/b - sqrt(b**2 + c**2)/b)/(b**2*sqrt(b**2 + c**2)
 + c**2*sqrt(b**2 + c**2)) - C*b*sqrt(b**2 + c**2)*log(tan(x/2) - c/b + sq
rt(b**2 + c**2)/b)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)) + C*c
*x*sqrt(b**2 + c**2)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)), Tr
ue))
```

3.349.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(70) = 140$.

Time = 0.32 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.07

$$\int \frac{A + C\sin(x)}{b\cos(x) + c\sin(x)}\,dx = C\left(\frac{2\,c\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{b^2+c^2} - \frac{b\log\left(-b - \frac{2\,c\sin(x)}{\cos(x)+1} + \frac{b\sin(x)^2}{(\cos(x)+1)^2}\right)}{b^2+c^2} + \frac{b\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{b^2+c^2}\right) - \frac{A\log\left(\frac{c - \frac{b\sin(x)}{\cos(x)+1} + \sqrt{b^2+c^2}}{c - \frac{b\sin(x)}{\cos(x)+1} - \sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}}$$

input
```
integrate((A+C*sin(x))/(b*cos(x)+c*sin(x)),x, algorithm="maxima")
```

output
```
C*(2*c*arctan(sin(x)/(cos(x) + 1))/(b^2 + c^2) - b*log(-b - 2*c*sin(x)/(co
s(x) + 1) + b*sin(x)^2/(cos(x) + 1)^2)/(b^2 + c^2) + b*log(sin(x)^2/(cos(x
) + 1)^2 + 1)/(b^2 + c^2)) - A*log((c - b*sin(x)/(cos(x) + 1) + sqrt(b^2 +
 c^2))/(c - b*sin(x)/(cos(x) + 1) - sqrt(b^2 + c^2)))/sqrt(b^2 + c^2)
```

3.349.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.77

$$\int \frac{A+C\sin(x)}{b\cos(x)+c\sin(x)}\,dx = \frac{Ccx}{b^2+c^2} + \frac{Cb\log\left(\tan\left(\frac{1}{2}x\right)^2+1\right)}{b^2+c^2} - \frac{Cb\log\left(\left|b\tan\left(\frac{1}{2}x\right)^2-2c\tan\left(\frac{1}{2}x\right)-b\right|\right)}{b^2+c^2} - \frac{A\log\left(\frac{\left|2b\tan\left(\frac{1}{2}x\right)-2c-2\sqrt{b^2+c^2}\right|}{\left|2b\tan\left(\frac{1}{2}x\right)-2c+2\sqrt{b^2+c^2}\right|}\right)}{\sqrt{b^2+c^2}}$$

input
```
integrate((A+C*sin(x))/(b*cos(x)+c*sin(x)),x, algorithm="giac")
```

output
```
C*c*x/(b^2 + c^2) + C*b*log(tan(1/2*x)^2 + 1)/(b^2 + c^2) - C*b*log(abs(b*
tan(1/2*x)^2 - 2*c*tan(1/2*x) - b))/(b^2 + c^2) - A*log(abs(2*b*tan(1/2*x)
 - 2*c - 2*sqrt(b^2 + c^2))/abs(2*b*tan(1/2*x) - 2*c + 2*sqrt(b^2 + c^2)))
/sqrt(b^2 + c^2)
```

3.349.9 Mupad [B] (verification not implemented)

Time = 35.82 (sec) , antiderivative size = 695, normalized size of antiderivative = 9.39

$$\int \frac{A+C\sin(x)}{b\cos(x)+c\sin(x)}\,dx = -\ln\left(-32\,A\,C^2\,b^2\right.$$

$$-\frac{\left(A\,\sqrt{{\left(b^2+c^2\right)}^3}+C\,b^3+C\,b\,c^2\right)\,\left(64\,A^2\,b^2\,c+32\,C^2\,b^2\,c-32\,b\,\tan\left(\frac{x}{2}\right)\,\left(A^2\,b^2-A^2\,c^2-4\,A\,C\,b\,c+2\right.\right.}{}$$

$$\left.-\,32\,A^2\,C\,b\,c-32\,C\,b\,\tan\left(\frac{x}{2}\right)\,\left(-b\,A^2+2\,c\,A\,C+2\,b\,C^2\right)\right)\,\left(\frac{C\,b}{b^2+c^2}+\frac{A\,\sqrt{{\left(b^2+c^2\right)}^3}}{{\left(b^2+c^2\right)}^2}\right)$$

$$-\ln\left(-32\,A\,C^2\,b^2\right.$$

$$-\frac{\left(C\,b^3-A\,\sqrt{{\left(b^2+c^2\right)}^3}+C\,b\,c^2\right)\,\left(64\,A^2\,b^2\,c+32\,C^2\,b^2\,c-32\,b\,\tan\left(\frac{x}{2}\right)\,\left(A^2\,b^2-A^2\,c^2-4\,A\,C\,b\,c+2\right.\right.}{}$$

input
```
int((A + C*sin(x))/(b*cos(x) + c*sin(x)),x)
```

output
```
(C*log(tan(x/2) + 1i))/(b - c*1i) - log(- 32*A*C^2*b^2 - ((C*b^3 - A*((b^2
 + c^2)^3)^(1/2) + C*b*c^2)*(64*A^2*b^2*c + 32*C^2*b^2*c - 32*b*tan(x/2)*(
A^2*b^2 - A^2*c^2 + 2*C^2*c^2 - 4*A*C*b*c) + 64*A*C*b^3 + ((C*b^3 - A*((b^
2 + c^2)^3)^(1/2) + C*b*c^2)*(32*A*b^4 + 32*A*b^2*c^2 + 32*b*tan(x/2)*(2*A
*c^3 - 2*C*b^3 + 2*A*b^2*c + C*b*c^2) - 32*C*b*c^3 + 64*C*b^3*c - (96*b*c*
(b + c*tan(x/2))*(C*b^3 - A*((b^2 + c^2)^3)^(1/2) + C*b*c^2))/(b^2 + c^2))
)/(b^2 + c^2)^2))/(b^2 + c^2)^2 - 32*A^2*C*b*c - 32*C*b*tan(x/2)*(2*C^2*b
- A^2*b + 2*A*C*c))*((C*b)/(b^2 + c^2) - (A*((b^2 + c^2)^3)^(1/2))/(b^2 +
c^2)^2) - log(- 32*A*C^2*b^2 - ((A*((b^2 + c^2)^3)^(1/2) + C*b^3 + C*b*c^2
)*(64*A^2*b^2*c + 32*C^2*b^2*c - 32*b*tan(x/2)*(A^2*b^2 - A^2*c^2 + 2*C^2*
c^2 - 4*A*C*b*c) + 64*A*C*b^3 + ((A*((b^2 + c^2)^3)^(1/2) + C*b^3 + C*b*c^
2)*(32*A*b^4 + 32*A*b^2*c^2 + 32*b*tan(x/2)*(2*A*c^3 - 2*C*b^3 + 2*A*b^2*c
 + C*b*c^2) - 32*C*b*c^3 + 64*C*b^3*c - (96*b*c*(b + c*tan(x/2))*(A*((b^2
+ c^2)^3)^(1/2) + C*b^3 + C*b*c^2))/(b^2 + c^2)))/(b^2 + c^2)^2))/(b^2 + c
^2)^2 - 32*A^2*C*b*c - 32*C*b*tan(x/2)*(2*C^2*b - A^2*b + 2*A*C*c))*((C*b)
/(b^2 + c^2) + (A*((b^2 + c^2)^3)^(1/2))/(b^2 + c^2)^2) + (C*log(tan(x/2)
- 1i)*1i)/(b*1i - c)
```

3.349. $\int \frac{A+C\sin(x)}{b\cos(x)+c\sin(x)}\,dx$

3.350 $\int \frac{A+C\sin(x)}{(b\cos(x)+c\sin(x))^2}\,dx$

3.350.1 Optimal result

Integrand size = 18, antiderivative size = 75

$$\int \frac{A+C\sin(x)}{(b\cos(x)+c\sin(x))^2}\,dx=-\frac{cC\text{arctanh}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{3/2}}+\frac{bC-Ac\cos(x)+Ab\sin(x)}{(b^2+c^2)\,(b\cos(x)+c\sin(x))}$$

output
```
-c*C*arctanh((c*cos(x)-b*sin(x))/(b^2+c^2)^(1/2))/(b^2+c^2)^(3/2)+(b*C-A*c
*cos(x)+A*b*sin(x))/(b^2+c^2)/(b*cos(x)+c*sin(x))
```

3.350.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.09

$$\int \frac{A+C\sin(x)}{(b\cos(x)+c\sin(x))^2}\,dx=\frac{2cC\text{arctanh}\left(\frac{-c+b\tan\left(\frac{x}{2}\right)}{\sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{3/2}}+\frac{b^2C+A(b^2+c^2)\sin(x)}{b\,(b^2+c^2)\,(b\cos(x)+c\sin(x))}$$

input
```
Integrate[(A + C*Sin[x])/(b*Cos[x] + c*Sin[x])^2,x]
```

output
```
(2*c*C*ArcTanh[(-c + b*Tan[x/2])/Sqrt[b^2 + c^2]])/(b^2 + c^2)^(3/2) + (b^
2*C + A*(b^2 + c^2)*Sin[x])/(b*(b^2 + c^2)*(b*Cos[x] + c*Sin[x]))
```

3.350.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3042, 3633, 3042, 3553, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+C\sin(x)}{(b\cos(x)+c\sin(x))^2}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{A+C\sin(x)}{(b\cos(x)+c\sin(x))^2}dx$$
$$\Big\downarrow \text{ 3633}$$
$$\frac{cC\int \frac{1}{b\cos(x)+c\sin(x)}dx}{b^2+c^2}+\frac{Ab\sin(x)-Ac\cos(x)+bC}{(b^2+c^2)\,(b\cos(x)+c\sin(x))}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{cC\int \frac{1}{b\cos(x)+c\sin(x)}dx}{b^2+c^2}+\frac{Ab\sin(x)-Ac\cos(x)+bC}{(b^2+c^2)\,(b\cos(x)+c\sin(x))}$$
$$\Big\downarrow \text{ 3553}$$
$$\frac{Ab\sin(x)-Ac\cos(x)+bC}{(b^2+c^2)\,(b\cos(x)+c\sin(x))}-\frac{cC\int \frac{1}{b^2+c^2-(c\cos(x)-b\sin(x))^2}d(c\cos(x)-b\sin(x))}{b^2+c^2}$$
$$\Big\downarrow \text{ 219}$$
$$\frac{Ab\sin(x)-Ac\cos(x)+bC}{(b^2+c^2)\,(b\cos(x)+c\sin(x))}-\frac{cC\text{arctanh}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{3/2}}$$

input
```
Int[(A + C*Sin[x])/(b*Cos[x] + c*Sin[x])^2,x]
```

output
```
-((c*C*ArcTanh[(c*Cos[x] - b*Sin[x])/Sqrt[b^2 + c^2]])/(b^2 + c^2)^(3/2))
+ (b*C - A*c*Cos[x] + A*b*Sin[x])/((b^2 + c^2)*(b*Cos[x] + c*Sin[x]))
```

3.350.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3633
```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[-(b*C + (a*C
- c*A)*Cos[d + e*x] + b*A*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
 e*x] + c*Sin[d + e*x])), x] + Simp[(a*A - c*C)/(a^2 - b^2 - c^2)   Int[1/(
a + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, C}
, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - c*C, 0]
```

3.350.4 Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.44

method	result	size
default	$\frac{-\frac{2\left(A\,b^{2}+A\,c^{2}+Cbc\right)\tan\left(\frac{x}{2}\right)}{b\left(b^{2}+c^{2}\right)}-\frac{2Cb}{b^{2}+c^{2}}}{\tan\left(\frac{x}{2}\right)^{2}b-2c\tan\left(\frac{x}{2}\right)-b}+\frac{2Cc\,\operatorname{arctanh}\left(\frac{2b\tan\left(\frac{x}{2}\right)-2c}{2\sqrt{b^{2}+c^{2}}}\right)}{\left(b^{2}+c^{2}\right)^{\frac{3}{2}}}$	108
risch	$-\frac{2i\left(-iAb-Cb\,\mathrm{e}^{ix}+cA\right)}{(-ib+c)(ib+c)\left(c\,\mathrm{e}^{2ix}+ib\,\mathrm{e}^{2ix}-c+ib\right)}+\frac{cC\ln\left(\mathrm{e}^{ix}+\frac{i b^{3}+i b\,c^{2}-b^{2}c-c^{3}}{\left(b^{2}+c^{2}\right)^{\frac{3}{2}}}\right)}{\left(b^{2}+c^{2}\right)^{\frac{3}{2}}}-\frac{cC\ln\left(\mathrm{e}^{ix}-\frac{i b^{3}+i b\,c^{2}-b^{2}c-c^{3}}{\left(b^{2}+c^{2}\right)^{\frac{3}{2}}}\right)}{\left(b^{2}+c^{2}\right)^{\frac{3}{2}}}$	175

input
```
int((A+C*sin(x))/(b*cos(x)+c*sin(x))^2,x,method=_RETURNVERBOSE)
```

output
```
2*(-(A*b^2+A*c^2+C*b*c)/b/(b^2+c^2)*tan(1/2*x)-C*b/(b^2+c^2))/(tan(1/2*x)^
2*b-2*c*tan(1/2*x)-b)+2*C*c/(b^2+c^2)^(3/2)*arctanh(1/2*(2*b*tan(1/2*x)-2*
c)/(b^2+c^2)^(1/2))
```

3.350.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(73) = 146$.

Time = 0.25 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.67

$$\int \frac{A+C\sin(x)}{(b\cos(x)+c\sin(x))^2}\,dx$$
$$= \frac{2\,Cb^3+2\,Cbc^2+(Cbc\cos(x)+Cc^2\sin(x))\sqrt{b^2+c^2}\log\left(-\frac{2\,bc\cos(x)\sin(x)+(b^2-c^2)\cos(x)^2-2\,b^2-c^2+2\,\sqrt{b^2+c^2}(c\cos}{2\,bc\cos(x)\sin(x)+(b^2-c^2)\cos(x)^2+c^2}\right.}{2\,((b^5+2\,b^3c^2+bc^4)\cos(x)+(b^4c+2\,b^2c^3+c^5)\,\mathrm{si}}$$

input
```
integrate((A+C*sin(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="fricas")
```

output
```
1/2*(2*C*b^3 + 2*C*b*c^2 + (C*b*c*cos(x) + C*c^2*sin(x))*sqrt(b^2 + c^2)*l
og(-(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 - 2*b^2 - c^2 + 2*sqrt(b^2
 + c^2)*(c*cos(x) - b*sin(x)))/(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2
 + c^2)) - 2*(A*b^2*c + A*c^3)*cos(x) + 2*(A*b^3 + A*b*c^2)*sin(x))/((b^5
+ 2*b^3*c^2 + b*c^4)*cos(x) + (b^4*c + 2*b^2*c^3 + c^5)*sin(x))
```

3.350.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{A+C\sin(x)}{(b\cos(x)+c\sin(x))^2}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate((A+C*sin(x))/(b*cos(x)+c*sin(x))**2,x)
```

output
```
Exception raised: AttributeError >> 'NoneType' object has no attribute 'pr
imitive'
```

3.350.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.95

$$\int \frac{A + C\sin(x)}{(b\cos(x) + c\sin(x))^2}\,dx = -C\left(\frac{c\log\left(\frac{c - \frac{b\sin(x)}{\cos(x)+1} + \sqrt{b^2+c^2}}{c - \frac{b\sin(x)}{\cos(x)+1} - \sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{\frac{3}{2}}} - \frac{2\left(b + \frac{c\sin(x)}{\cos(x)+1}\right)}{b^3 + bc^2 + \frac{2(b^2c+c^3)\sin(x)}{\cos(x)+1} - \frac{(b^3+bc^2)\sin(x)^2}{(\cos(x)+1)^2}}\right) - \frac{A}{c^2\tan(x) + bc}$$

input
```
integrate((A+C*sin(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="maxima")
```

output
```
-C*(c*log((c - b*sin(x)/(cos(x) + 1) + sqrt(b^2 + c^2))/(c - b*sin(x)/(cos
(x) + 1) - sqrt(b^2 + c^2)))/(b^2 + c^2)^(3/2) - 2*(b + c*sin(x)/(cos(x) +
 1))/(b^3 + b*c^2 + 2*(b^2*c + c^3)*sin(x)/(cos(x) + 1) - (b^3 + b*c^2)*si
n(x)^2/(cos(x) + 1)^2)) - A/(c^2*tan(x) + b*c)
```

3.350.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.73

$$\int \frac{A + C\sin(x)}{(b\cos(x) + c\sin(x))^2}\,dx = -\frac{Cc\log\left(\frac{\left|2\,b\tan\left(\frac{1}{2}\,x\right) - 2\,c - 2\,\sqrt{b^2+c^2}\right|}{\left|2\,b\tan\left(\frac{1}{2}\,x\right) - 2\,c + 2\,\sqrt{b^2+c^2}\right|}\right)}{(b^2+c^2)^{\frac{3}{2}}} - \frac{2\left(Ab^2\tan\left(\frac{1}{2}\,x\right) + Cbc\tan\left(\frac{1}{2}\,x\right) + Ac^2\tan\left(\frac{1}{2}\,x\right) + Cb^2\right)}{(b^3+bc^2)\left(b\tan\left(\frac{1}{2}\,x\right)^2 - 2\,c\tan\left(\frac{1}{2}\,x\right) - b\right)}$$

input
```
integrate((A+C*sin(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="giac")
```

output
```
-C*c*log(abs(2*b*tan(1/2*x) - 2*c - 2*sqrt(b^2 + c^2))/abs(2*b*tan(1/2*x)
- 2*c + 2*sqrt(b^2 + c^2)))/(b^2 + c^2)^(3/2) - 2*(A*b^2*tan(1/2*x) + C*b*
c*tan(1/2*x) + A*c^2*tan(1/2*x) + C*b^2)/((b^3 + b*c^2)*(b*tan(1/2*x)^2 -
2*c*tan(1/2*x) - b))
```

3.350.9 Mupad [B] (verification not implemented)

Time = 29.62 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.40

$$\int \frac{A+C\sin(x)}{(b\cos(x)+c\sin(x))^2}\,dx = \frac{\frac{2\,C\,b}{b^2+c^2}+\frac{2\tan\left(\frac{x}{2}\right)\,\left(A\,b^2+C\,b\,c+A\,c^2\right)}{b\,\left(b^2+c^2\right)}}{-b\tan\left(\frac{x}{2}\right)^2+2\,c\tan\left(\frac{x}{2}\right)+b}-\frac{2\,C\,c\,\mathrm{atanh}\left(\frac{2\,c-2\,b\tan\left(\frac{x}{2}\right)}{2\,\sqrt{b^2+c^2}}\right)}{\left(b^2+c^2\right)^{3/2}}$$

input
```
int((A + C*sin(x))/(b*cos(x) + c*sin(x))^2,x)
```

output
```
((2*C*b)/(b^2 + c^2) + (2*tan(x/2)*(A*b^2 + A*c^2 + C*b*c))/(b*(b^2 + c^2)
))/(b + 2*c*tan(x/2) - b*tan(x/2)^2) - (2*C*c*atanh((2*c - 2*b*tan(x/2))/(
2*(b^2 + c^2)^(1/2))))/(b^2 + c^2)^(3/2)
```

3.351 $\int \frac{A+C\sin(x)}{(b\cos(x)+c\sin(x))^3}\,dx$

3.351.1 Optimal result

Integrand size = 18, antiderivative size = 116

$$\int \frac{A+C\sin(x)}{(b\cos(x)+c\sin(x))^3}\,dx = -\frac{A\text{arctanh}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{2\left(b^2+c^2\right)^{3/2}}+\frac{bC-Ac\cos(x)+Ab\sin(x)}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2} - \frac{c^2C\cos(x)-bcC\sin(x)}{\left(b^2+c^2\right)^2(b\cos(x)+c\sin(x))}$$

output
```
-1/2*A*arctanh((c*cos(x)-b*sin(x))/(b^2+c^2)^(1/2))/(b^2+c^2)^(3/2)+1/2*(b
*C-A*c*cos(x)+A*b*sin(x))/(b^2+c^2)/(b*cos(x)+c*sin(x))^2+(-c^2*C*cos(x)+b
*c*C*sin(x))/(b^2+c^2)^2/(b*cos(x)+c*sin(x))
```

3.351.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.14

$$\int \frac{A+C\sin(x)}{(b\cos(x)+c\sin(x))^3}\,dx$$
$$= \frac{2Ab\sqrt{b^2+c^2}\text{arctanh}\left(\frac{-c+b\tan\left(\frac{x}{2}\right)}{\sqrt{b^2+c^2}}\right)(b\cos(x)+c\sin(x))^2+\left(b^2+c^2\right)\left(-Abc\cos(x)+Ab^2\sin(x)+2c^2C\sin\right.}{2b(b-ic)^2(b+ic)^2(b\cos(x)+c\sin(x))^2}$$

input
```
Integrate[(A + C*Sin[x])/(b*Cos[x] + c*Sin[x])^3,x]
```

output
```
(2*A*b*Sqrt[b^2 + c^2]*ArcTanh[(-c + b*Tan[x/2])/Sqrt[b^2 + c^2]]*(b*Cos[x
] + c*Sin[x])^2 + (b^2 + c^2)*(-(A*b*c*Cos[x]) + A*b^2*Sin[x] + 2*c^2*C*Si
n[x]^2 + b*C*(b + c*Sin[2*x])))/(2*b*(b - I*c)^2*(b + I*c)^2*(b*Cos[x] + c
*Sin[x])^2)
```

3.351.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3042, 3636, 3042, 3632, 3042, 3553, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+C\sin(x)}{(b\cos(x)+c\sin(x))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+C\sin(x)}{(b\cos(x)+c\sin(x))^3}dx$$

$$\Big\downarrow \text{3636}$$

$$\frac{\int \frac{2cC+Ab\cos(x)+Ac\sin(x)}{(b\cos(x)+c\sin(x))^2}dx}{2(b^2+c^2)}+\frac{Ab\sin(x)-Ac\cos(x)+bC}{2(b^2+c^2)(b\cos(x)+c\sin(x))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{2cC+Ab\cos(x)+Ac\sin(x)}{(b\cos(x)+c\sin(x))^2}dx}{2(b^2+c^2)}+\frac{Ab\sin(x)-Ac\cos(x)+bC}{2(b^2+c^2)(b\cos(x)+c\sin(x))^2}$$

$$\Big\downarrow \text{3632}$$

$$\frac{A\int \frac{1}{b\cos(x)+c\sin(x)}dx-\frac{2(c^2C\cos(x)-bcC\sin(x))}{(b^2+c^2)(b\cos(x)+c\sin(x))}}{2(b^2+c^2)}+\frac{Ab\sin(x)-Ac\cos(x)+bC}{2(b^2+c^2)(b\cos(x)+c\sin(x))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{A\int \frac{1}{b\cos(x)+c\sin(x)}dx-\frac{2(c^2C\cos(x)-bcC\sin(x))}{(b^2+c^2)(b\cos(x)+c\sin(x))}}{2(b^2+c^2)}+\frac{Ab\sin(x)-Ac\cos(x)+bC}{2(b^2+c^2)(b\cos(x)+c\sin(x))^2}$$

$$\Big\downarrow \text{3553}$$

$$\frac{-A\int \frac{1}{b^2+c^2-(c\cos(x)-b\sin(x))^2}d(c\cos(x)-b\sin(x))-\frac{2(c^2C\cos(x)-bcC\sin(x))}{(b^2+c^2)(b\cos(x)+c\sin(x))}}{2\left(b^2+c^2\right)}+\frac{Ab\sin(x)-Ac\cos(x)+bC}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{-\frac{A\text{arctanh}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}}-\frac{2(c^2C\cos(x)-bcC\sin(x))}{(b^2+c^2)(b\cos(x)+c\sin(x))}}{2\left(b^2+c^2\right)}+\frac{Ab\sin(x)-Ac\cos(x)+bC}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}$$

input
```
Int[(A + C*Sin[x])/(b*Cos[x] + c*Sin[x])^3,x]
```

output
```
(b*C - A*c*Cos[x] + A*b*Sin[x])/(2*(b^2 + c^2)*(b*Cos[x] + c*Sin[x])^2) +
(-((A*ArcTanh[(c*Cos[x] - b*Sin[x])/Sqrt[b^2 + c^2]])/Sqrt[b^2 + c^2]) - (
2*(c^2*C*Cos[x] - b*c*C*Sin[x]))/((b^2 + c^2)*(b*Cos[x] + c*Sin[x])))/(2*(
b^2 + c^2))
```

3.351.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3632
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Simp[(a*A - b*B - c*C)/(a^2 - b^2 - c^2)   Int[1/(a + b*Cos[d + e*x] + c*S
in[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
 - c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

rule 3636
```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(b*C + (a*
C - c*A)*Cos[d + e*x] + b*A*Sin[d + e*x])*((a + b*Cos[d + e*x] + c*Sin[d +
e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Simp[1/((n + 1)*(a^2 - b
^2 - c^2))   Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)
*(a*A - c*C) - (n + 2)*b*A*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, A, C}, x] && LtQ[n, -1] && NeQ[a^2 - b
^2 - c^2, 0] && NeQ[n, -2]
```

3.351.4 Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.53

method	result
default	$-\frac{2\left(-\frac{A\left(b^{2}+2c^{2}\right)\tan\left(\frac{x}{2}\right)^{3}}{2\left(b^{2}+c^{2}\right)b}-\frac{\left(A\,b^{2}c-2A\,c^{3}+2C\,b^{3}+2Cb\,c^{2}\right)\tan\left(\frac{x}{2}\right)^{2}}{2\left(b^{2}+c^{2}\right)b^{2}}-\frac{A\left(b^{2}-2c^{2}\right)\tan\left(\frac{x}{2}\right)}{2\left(b^{2}+c^{2}\right)b}+\frac{cA}{2b^{2}+2c^{2}}\right)}{\left(\tan\left(\frac{x}{2}\right)^{2}b-2c\tan\left(\frac{x}{2}\right)-b\right)^{2}}+\frac{A\,\operatorname{arctanh}\left(\frac{2b\tan\left(\frac{x}{2}\right)-2c}{2\sqrt{b^{2}+c^{2}}}\right)}{\left(b^{2}+c^{2}\right)^{\frac{3}{2}}}$
risch	$-\frac{i\left(-A\,c^{2}e^{ix}-2Cbc-A\,b^{2}e^{ix}-A\,c^{2}e^{3ix}-2iAbc\,e^{3ix}+A\,b^{2}e^{3ix}+2iC\,b^{2}e^{2ix}+2iC\,c^{2}e^{2ix}-2iC\,c^{2}\right)}{\left(-ic\,e^{2ix}+b\,e^{2ix}+ic+b\right)^{2}\left(ic+b\right)\left(-ic+b\right)^{2}}+\frac{A\ln\left(e^{ix}+\frac{ib^{3}+ib\,c^{2}-b^{2}c-c^{3}}{\left(b^{2}+c^{2}\right)^{\frac{3}{2}}}\right)}{2\left(b^{2}+c^{2}\right)^{\frac{3}{2}}}$

input
```
int((A+C*sin(x))/(b*cos(x)+c*sin(x))^3,x,method=_RETURNVERBOSE)
```

output
```
-2*(-1/2*A*(b^2+2*c^2)/(b^2+c^2)/b*tan(1/2*x)^3-1/2*(A*b^2*c-2*A*c^3+2*C*b
^3+2*C*b*c^2)/(b^2+c^2)/b^2*tan(1/2*x)^2-1/2*A*(b^2-2*c^2)/(b^2+c^2)/b*tan
(1/2*x)+1/2*c*A/(b^2+c^2))/(tan(1/2*x)^2*b-2*c*tan(1/2*x)-b)^2+A/(b^2+c^2)
^(3/2)*arctanh(1/2*(2*b*tan(1/2*x)-2*c)/(b^2+c^2)^(1/2))
```

3.351.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(109) = 218$.

Time = 0.27 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.41

$$\int \frac{A+C\sin(x)}{(b\cos(x)+c\sin(x))^3}\,dx =$$
$$-\frac{8\,Cbc^{2}\cos\left(x\right)^{2}-2\,Cb^{3}-6\,Cbc^{2}-\left(2\,Abc\cos\left(x\right)\sin\left(x\right)+Ac^{2}+\left(Ab^{2}-Ac^{2}\right)\cos\left(x\right)^{2}\right)\sqrt{b^{2}+c^{2}}\log\left(-\right.}{4\left(b^{4}c^{2}+2\,b^{2}c^{4}+c^{6}+\left(b^{6}+b^{4}c^{2}\right.\right.}$$

input
```
integrate((A+C*sin(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="fricas")
```

output
```
-1/4*(8*C*b*c^2*cos(x)^2 - 2*C*b^3 - 6*C*b*c^2 - (2*A*b*c*cos(x)*sin(x) +
A*c^2 + (A*b^2 - A*c^2)*cos(x)^2)*sqrt(b^2 + c^2)*log(-(2*b*c*cos(x)*sin(x
) + (b^2 - c^2)*cos(x)^2 - 2*b^2 - c^2 + 2*sqrt(b^2 + c^2)*(c*cos(x) - b*s
in(x)))/(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 + c^2)) + 2*(A*b^2*c +
 A*c^3)*cos(x) - 2*(A*b^3 + A*b*c^2 + 2*(C*b^2*c - C*c^3)*cos(x))*sin(x))/
(b^4*c^2 + 2*b^2*c^4 + c^6 + (b^6 + b^4*c^2 - b^2*c^4 - c^6)*cos(x)^2 + 2*
(b^5*c + 2*b^3*c^3 + b*c^5)*cos(x)*sin(x))
```

3.351.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + C\sin(x)}{(b\cos(x) + c\sin(x))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+C*sin(x))/(b*cos(x)+c*sin(x))**3,x)
```

output
```
Timed out
```

3.351.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(109) = 218$.

Time = 0.33 (sec) , antiderivative size = 338, normalized size of antiderivative = 2.91

$$\int \frac{A + C\sin(x)}{(b\cos(x) + c\sin(x))^3}\,dx =$$

$$-\frac{1}{2}A\left(\frac{2\left(b^2c - \frac{(b^3-2bc^2)\sin(x)}{\cos(x)+1} - \frac{(b^2c-2c^3)\sin(x)^2}{(\cos(x)+1)^2} - \frac{(b^3+2bc^2)\sin(x)^3}{(\cos(x)+1)^3}\right)}{b^6 + b^4c^2 + \frac{4(b^5c+b^3c^3)\sin(x)}{\cos(x)+1} - \frac{2(b^6-b^4c^2-2b^2c^4)\sin(x)^2}{(\cos(x)+1)^2} - \frac{4(b^5c+b^3c^3)\sin(x)^3}{(\cos(x)+1)^3} + \frac{(b^6+b^4c^2)\sin(x)^4}{(\cos(x)+1)^4}} + \frac{\log\left(\frac{c-\frac{b\,s}{\cos}}{c-\frac{b\,s}{\cos}}\right.}{(b^2}\right.$$

$$+ \frac{2C\sin(x)^2}{\left(b^3 + \frac{4b^2c\sin(x)}{\cos(x)+1} - \frac{4b^2c\sin(x)^3}{(\cos(x)+1)^3} + \frac{b^3\sin(x)^4}{(\cos(x)+1)^4} - \frac{2(b^3-2bc^2)\sin(x)^2}{(\cos(x)+1)^2}\right)(\cos(x)+1)^2}$$

input
```
integrate((A+C*sin(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="maxima")
```

output
```
-1/2*A*(2*(b^2*c - (b^3 - 2*b*c^2)*sin(x)/(cos(x) + 1) - (b^2*c - 2*c^3)*s
in(x)^2/(cos(x) + 1)^2 - (b^3 + 2*b*c^2)*sin(x)^3/(cos(x) + 1)^3)/(b^6 + b
^4*c^2 + 4*(b^5*c + b^3*c^3)*sin(x)/(cos(x) + 1) - 2*(b^6 - b^4*c^2 - 2*b^
2*c^4)*sin(x)^2/(cos(x) + 1)^2 - 4*(b^5*c + b^3*c^3)*sin(x)^3/(cos(x) + 1)
^3 + (b^6 + b^4*c^2)*sin(x)^4/(cos(x) + 1)^4) + log((c - b*sin(x)/(cos(x)
+ 1) + sqrt(b^2 + c^2))/(c - b*sin(x)/(cos(x) + 1) - sqrt(b^2 + c^2)))/(b^
2 + c^2)^(3/2)) + 2*C*sin(x)^2/((b^3 + 4*b^2*c*sin(x)/(cos(x) + 1) - 4*b^2
*c*sin(x)^3/(cos(x) + 1)^3 + b^3*sin(x)^4/(cos(x) + 1)^4 - 2*(b^3 - 2*b*c^
2)*sin(x)^2/(cos(x) + 1)^2)*(cos(x) + 1)^2)
```

3.351.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.72

$$\int \frac{A+C\sin(x)}{(b\cos(x)+c\sin(x))^3}\,dx = \frac{A\log\left(\frac{\left|-2\,b\tan\left(\frac{1}{2}\,x\right)+2\,c-2\,\sqrt{b^2+c^2}\right|}{\left|-2\,b\tan\left(\frac{1}{2}\,x\right)+2\,c+2\,\sqrt{b^2+c^2}\right|}\right)}{2\,(b^2+c^2)^{\frac{3}{2}}} + \frac{Ab^3\tan\left(\frac{1}{2}\,x\right)^3+2\,Abc^2\tan\left(\frac{1}{2}\,x\right)^3+2\,Cb^3\tan\left(\frac{1}{2}\,x\right)^2+Ab^2c\tan\left(\frac{1}{2}\,x\right)^2+2\,Cbc^2\tan\left(\frac{1}{2}\,x\right)^2-2\,Ac^3\tan}{(b^4+b^2c^2)\left(b\tan\left(\frac{1}{2}\,x\right)^2-2\,c\tan\left(\frac{1}{2}\,x\right)-b\right)^2}$$

input
```
integrate((A+C*sin(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="giac")
```

output
```
1/2*A*log(abs(-2*b*tan(1/2*x) + 2*c - 2*sqrt(b^2 + c^2))/abs(-2*b*tan(1/2*
x) + 2*c + 2*sqrt(b^2 + c^2)))/(b^2 + c^2)^(3/2) + (A*b^3*tan(1/2*x)^3 + 2
*A*b*c^2*tan(1/2*x)^3 + 2*C*b^3*tan(1/2*x)^2 + A*b^2*c*tan(1/2*x)^2 + 2*C*
b*c^2*tan(1/2*x)^2 - 2*A*c^3*tan(1/2*x)^2 + A*b^3*tan(1/2*x) - 2*A*b*c^2*t
an(1/2*x) - A*b^2*c)/((b^4 + b^2*c^2)*(b*tan(1/2*x)^2 - 2*c*tan(1/2*x) - b
)^2)
```

3.351.9 Mupad [B] (verification not implemented)

Time = 30.33 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.96

$$\int \frac{A+C\sin(x)}{(b\cos(x)+c\sin(x))^3}\,dx = \frac{\frac{\tan\left(\frac{x}{2}\right)^3\,(A\,b^2+2\,A\,c^2)}{b\,(b^2+c^2)}-\frac{A\,c}{b^2+c^2}+\frac{\tan\left(\frac{x}{2}\right)^2\,(2\,C\,b^3+A\,b^2\,c+2\,C\,b\,c^2-2\,A\,c^3)}{b^2\,(b^2+c^2)}+\frac{\tan\left(\frac{x}{2}\right)\,(A\,b^2-2\,A\,c^2)}{b\,(b^2+c^2)}}{b^2-\tan\left(\frac{x}{2}\right)^2\,(2\,b^2-4\,c^2)+b^2\tan\left(\frac{x}{2}\right)^4+4\,b\,c\tan\left(\frac{x}{2}\right)-4\,b\,c\tan\left(\frac{x}{2}\right)^3} + \frac{A\,\mathrm{atan}\left(\frac{b^2\,c\,1\mathrm{i}+c^3\,1\mathrm{i}-b\tan\left(\frac{x}{2}\right)\,(b^2+c^2)\,1\mathrm{i}}{(b^2+c^2)^{3/2}}\right)\,1\mathrm{i}}{(b^2+c^2)^{3/2}}$$

input
```
int((A + C*sin(x))/(b*cos(x) + c*sin(x))^3,x)
```

output
```
((tan(x/2)^3*(A*b^2 + 2*A*c^2))/(b*(b^2 + c^2)) - (A*c)/(b^2 + c^2) + (tan
(x/2)^2*(2*C*b^3 - 2*A*c^3 + A*b^2*c + 2*C*b*c^2))/(b^2*(b^2 + c^2)) + (ta
n(x/2)*(A*b^2 - 2*A*c^2))/(b*(b^2 + c^2)))/(b^2 - tan(x/2)^2*(2*b^2 - 4*c^
2) + b^2*tan(x/2)^4 + 4*b*c*tan(x/2) - 4*b*c*tan(x/2)^3) + (A*atan((b^2*c*
1i + c^3*1i - b*tan(x/2)*(b^2 + c^2)*1i)/(b^2 + c^2)^(3/2))*1i)/(b^2 + c^2
)^(3/2)
```

3.352 $\int \frac{A+B\cos(x)}{b\cos(x)+c\sin(x)}\,dx$

3.352.1 Optimal result

Integrand size = 18, antiderivative size = 73

$$\int \frac{A+B\cos(x)}{b\cos(x)+c\sin(x)}\,dx=\frac{bBx}{b^2+c^2}-\frac{A\,\text{arctanh}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}}+\frac{Bc\log(b\cos(x)+c\sin(x))}{b^2+c^2}$$

output
```
b*B*x/(b^2+c^2)+B*c*ln(b*cos(x)+c*sin(x))/(b^2+c^2)-A*arctanh((c*cos(x)-b*
sin(x))/(b^2+c^2)^(1/2))/(b^2+c^2)^(1/2)
```

3.352.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.92

$$\int \frac{A+B\cos(x)}{b\cos(x)+c\sin(x)}\,dx=\frac{2A\,\text{arctanh}\left(\frac{-c+b\tan\left(\frac{x}{2}\right)}{\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}}+\frac{B(bx+c\log(b\cos(x)+c\sin(x)))}{b^2+c^2}$$

input
```
Integrate[(A + B*Cos[x])/(b*Cos[x] + c*Sin[x]),x]
```

output
```
(2*A*ArcTanh[(-c + b*Tan[x/2])/Sqrt[b^2 + c^2]])/Sqrt[b^2 + c^2] + (B*(b*x
 + c*Log[b*Cos[x] + c*Sin[x]]))/(b^2 + c^2)
```

3.352.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3042, 3617, 3042, 3553, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(x)}{b\cos(x)+c\sin(x)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\cos(x)}{b\cos(x)+c\sin(x)}dx$$

$$\Big\downarrow \text{ 3617}$$

$$A\int \frac{1}{b\cos(x)+c\sin(x)}dx+\frac{bBx}{b^2+c^2}+\frac{Bc\log(b\cos(x)+c\sin(x))}{b^2+c^2}$$

$$\Big\downarrow \text{ 3042}$$

$$A\int \frac{1}{b\cos(x)+c\sin(x)}dx+\frac{bBx}{b^2+c^2}+\frac{Bc\log(b\cos(x)+c\sin(x))}{b^2+c^2}$$

$$\Big\downarrow \text{ 3553}$$

$$-A\int \frac{1}{b^2+c^2-(c\cos(x)-b\sin(x))^2}d(c\cos(x)-b\sin(x))+\frac{bBx}{b^2+c^2}+\frac{Bc\log(b\cos(x)+c\sin(x))}{b^2+c^2}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{A\text{arctanh}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}}+\frac{bBx}{b^2+c^2}+\frac{Bc\log(b\cos(x)+c\sin(x))}{b^2+c^2}$$

input
```
Int[(A + B*Cos[x])/(b*Cos[x] + c*Sin[x]),x]
```

output
```
(b*B*x)/(b^2 + c^2) - (A*ArcTanh[(c*Cos[x] - b*Sin[x])/Sqrt[b^2 + c^2]])/S
qrt[b^2 + c^2] + (B*c*Log[b*Cos[x] + c*Sin[x]])/(b^2 + c^2)
```

3.352.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3617
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[b*B*((d + e*x)/
(e*(b^2 + c^2))), x] + (Simp[c*B*(Log[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/
(e*(b^2 + c^2))), x] + Simp[(A*(b^2 + c^2) - a*b*B)/(b^2 + c^2)   Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x]) /; FreeQ[{a, b, c, d, e, A, B},
 x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*b*B, 0]
```

3.352.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.60

method	result
default	$\frac{2B\left(-\frac{c\ln\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)}{2}+b\arctan\left(\tan\left(\frac{x}{2}\right)\right)\right)}{b^{2}+c^{2}}+\frac{Bc\ln\left(\tan\left(\frac{x}{2}\right)^{2}b-2c\tan\left(\frac{x}{2}\right)-b\right)-\frac{2\left(-A\,b^{2}-A\,c^{2}\right)\operatorname{arctanh}\left(\frac{2b\tan\left(\frac{x}{2}\right)-2c}{2\sqrt{b^{2}+c^{2}}}\right)}{\sqrt{b^{2}+c^{2}}}}{b^{2}+c^{2}}$
risch	$-\frac{Bx}{ic-b}+\frac{2iBx\,b^{2}c}{-b^{4}-2b^{2}c^{2}-c^{4}}+\frac{2iBx\,c^{3}}{-b^{4}-2b^{2}c^{2}-c^{4}}+\frac{\ln\left(\mathrm{e}^{ix}+\frac{(ib-c)\sqrt{A^{2}b^{2}+A^{2}c^{2}}}{A\left(b^{2}+c^{2}\right)}\right)Bc}{b^{2}+c^{2}}+\frac{\ln\left(\mathrm{e}^{ix}+\frac{(ib-c)\sqrt{A^{2}b^{2}+A^{2}c^{2}}}{A\left(b^{2}+c^{2}\right)}\right)\sqrt{A^{2}b^{2}+A^{2}c}}{b^{2}+c^{2}}$

input
```
int((A+B*cos(x))/(b*cos(x)+c*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
2*B/(b^2+c^2)*(-1/2*c*ln(1+tan(1/2*x)^2)+b*arctan(tan(1/2*x)))+2/(b^2+c^2)
*(1/2*B*c*ln(tan(1/2*x)^2*b-2*c*tan(1/2*x)-b)-(-A*b^2-A*c^2)/(b^2+c^2)^(1/
2)*arctanh(1/2*(2*b*tan(1/2*x)-2*c)/(b^2+c^2)^(1/2)))
```

3.352.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(69) = 138.

Time = 0.27 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.96

$$\int \frac{A+B\cos(x)}{b\cos(x)+c\sin(x)}\,dx = \frac{2\,Bbx + Bc\log\left(2\,bc\cos(x)\sin(x) + (b^2-c^2)\cos(x)^2 + c^2\right) + \sqrt{b^2+c^2}A\log\left(-\frac{2\,bc\cos(x)\sin(x)+(b^2-c^2)\cos(x)}{2\,bc\cos(x)\sin(x}\right.}{2\,(b^2+c^2)}$$

input
```
integrate((A+B*cos(x))/(b*cos(x)+c*sin(x)),x, algorithm="fricas")
```

output
```
1/2*(2*B*b*x + B*c*log(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 + c^2) +
 sqrt(b^2 + c^2)*A*log(-(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 - 2*b^
2 - c^2 + 2*sqrt(b^2 + c^2)*(c*cos(x) - b*sin(x)))/(2*b*c*cos(x)*sin(x) +
(b^2 - c^2)*cos(x)^2 + c^2)))/(b^2 + c^2)
```

3.352.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.60 (sec) , antiderivative size = 673, normalized size of antiderivative = 9.22

$$\int \frac{A+B\cos(x)}{b\cos(x)+c\sin(x)}\,dx = \begin{cases} \tilde{\infty}\left(A\log\left(\tan\left(\frac{x}{2}\right)\right) - B\log\left(\tan^2\left(\frac{x}{2}\right)+1\right) + B\log\left(\tan\left(\frac{x}{2}\right)\right)\right) \\ \frac{A\log\left(\tan\left(\frac{x}{2}\right)\right) - B\log\left(\tan^2\left(\frac{x}{2}\right)+1\right) + B\log\left(\tan\left(\frac{x}{2}\right)\right)}{c} \\ -\frac{2A}{2ic\sin(x)+2c\cos(x)} - \frac{Bx\sin(x)}{2ic\sin(x)+2c\cos(x)} + \frac{iBx\cos(x)}{2ic\sin(x)+2c\cos(x)} + \frac{iB\sin(x)}{2ic\sin(x)+2c\cos(x)} \\ -\frac{2A}{-2ic\sin(x)+2c\cos(x)} - \frac{Bx\sin(x)}{-2ic\sin(x)+2c\cos(x)} - \frac{iBx\cos(x)}{-2ic\sin(x)+2c\cos(x)} - \frac{iB\sin(x)}{-2ic\sin(x)+2c\cos(x)} \\ -\frac{Ab^2\log\left(\tan\left(\frac{x}{2}\right)-\frac{c}{b}-\frac{\sqrt{b^2+c^2}}{b}\right)}{b^2\sqrt{b^2+c^2}+c^2\sqrt{b^2+c^2}} + \frac{Ab^2\log\left(\tan\left(\frac{x}{2}\right)-\frac{c}{b}+\frac{\sqrt{b^2+c^2}}{b}\right)}{b^2\sqrt{b^2+c^2}+c^2\sqrt{b^2+c^2}} - \frac{Ac^2\log\left(\tan\left(\frac{x}{2}\right)-\frac{c}{b}-\frac{\sqrt{b^2+c^2}}{b}\right)}{b^2\sqrt{b^2+c^2}+c^2\sqrt{b^2+c^2}} + \frac{Ac^2\log\left(\tan\left(\frac{x}{2}\right)-\frac{c}{b}+\frac{\sqrt{b^2+}}{b}\right.}{b^2\sqrt{b^2+c^2}+c^2\sqrt{b^2+c^2}} \end{cases}$$

input
```
integrate((A+B*cos(x))/(b*cos(x)+c*sin(x)),x)
```

output
```
Piecewise((zoo*(A*log(tan(x/2)) - B*log(tan(x/2)**2 + 1) + B*log(tan(x/2))
), Eq(b, 0) & Eq(c, 0)), ((A*log(tan(x/2)) - B*log(tan(x/2)**2 + 1) + B*lo
g(tan(x/2)))/c, Eq(b, 0)), (-2*A/(2*I*c*sin(x) + 2*c*cos(x)) - B*x*sin(x)/
(2*I*c*sin(x) + 2*c*cos(x)) + I*B*x*cos(x)/(2*I*c*sin(x) + 2*c*cos(x)) + I
*B*sin(x)/(2*I*c*sin(x) + 2*c*cos(x)), Eq(b, -I*c)), (-2*A/(-2*I*c*sin(x)
+ 2*c*cos(x)) - B*x*sin(x)/(-2*I*c*sin(x) + 2*c*cos(x)) - I*B*x*cos(x)/(-2
*I*c*sin(x) + 2*c*cos(x)) - I*B*sin(x)/(-2*I*c*sin(x) + 2*c*cos(x)), Eq(b,
 I*c)), (-A*b**2*log(tan(x/2) - c/b - sqrt(b**2 + c**2)/b)/(b**2*sqrt(b**2
 + c**2) + c**2*sqrt(b**2 + c**2)) + A*b**2*log(tan(x/2) - c/b + sqrt(b**2
 + c**2)/b)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)) - A*c**2*log
(tan(x/2) - c/b - sqrt(b**2 + c**2)/b)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt
(b**2 + c**2)) + A*c**2*log(tan(x/2) - c/b + sqrt(b**2 + c**2)/b)/(b**2*sq
rt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)) + B*b*x*sqrt(b**2 + c**2)/(b**2*
sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)) - B*c*sqrt(b**2 + c**2)*log(ta
n(x/2)**2 + 1)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)) + B*c*sqr
t(b**2 + c**2)*log(tan(x/2) - c/b - sqrt(b**2 + c**2)/b)/(b**2*sqrt(b**2 +
 c**2) + c**2*sqrt(b**2 + c**2)) + B*c*sqrt(b**2 + c**2)*log(tan(x/2) - c/
b + sqrt(b**2 + c**2)/b)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2))
, True))
```

3.352.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(69) = 138$.

Time = 0.31 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.10

$$\int \frac{A + B\cos(x)}{b\cos(x) + c\sin(x)}\,dx = B\left(\frac{2\,b\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{b^2+c^2} + \frac{c\log\left(-b - \frac{2\,c\sin(x)}{\cos(x)+1} + \frac{b\sin(x)^2}{(\cos(x)+1)^2}\right)}{b^2+c^2} - \frac{c\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{b^2+c^2}\right) - \frac{A\log\left(\frac{c - \frac{b\sin(x)}{\cos(x)+1} + \sqrt{b^2+c^2}}{c - \frac{b\sin(x)}{\cos(x)+1} - \sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}}$$

input
```
integrate((A+B*cos(x))/(b*cos(x)+c*sin(x)),x, algorithm="maxima")
```

output
```
B*(2*b*arctan(sin(x)/(cos(x) + 1))/(b^2 + c^2) + c*log(-b - 2*c*sin(x)/(co
s(x) + 1) + b*sin(x)^2/(cos(x) + 1)^2)/(b^2 + c^2) - c*log(sin(x)^2/(cos(x
) + 1)^2 + 1)/(b^2 + c^2)) - A*log((c - b*sin(x)/(cos(x) + 1) + sqrt(b^2 +
 c^2))/(c - b*sin(x)/(cos(x) + 1) - sqrt(b^2 + c^2)))/sqrt(b^2 + c^2)
```

3.352.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.79

$$\int \frac{A+B\cos(x)}{b\cos(x)+c\sin(x)}\,dx = \frac{Bbx}{b^2+c^2} - \frac{Bc\log\left(\tan\left(\frac{1}{2}x\right)^2+1\right)}{b^2+c^2} + \frac{Bc\log\left(\left|b\tan\left(\frac{1}{2}x\right)^2-2c\tan\left(\frac{1}{2}x\right)-b\right|\right)}{b^2+c^2} - \frac{A\log\left(\frac{\left|2b\tan\left(\frac{1}{2}x\right)-2c-2\sqrt{b^2+c^2}\right|}{\left|2b\tan\left(\frac{1}{2}x\right)-2c+2\sqrt{b^2+c^2}\right|}\right)}{\sqrt{b^2+c^2}}$$

input
```
integrate((A+B*cos(x))/(b*cos(x)+c*sin(x)),x, algorithm="giac")
```

output
```
B*b*x/(b^2 + c^2) - B*c*log(tan(1/2*x)^2 + 1)/(b^2 + c^2) + B*c*log(abs(b*
tan(1/2*x)^2 - 2*c*tan(1/2*x) - b))/(b^2 + c^2) - A*log(abs(2*b*tan(1/2*x)
 - 2*c - 2*sqrt(b^2 + c^2))/abs(2*b*tan(1/2*x) - 2*c + 2*sqrt(b^2 + c^2)))
/sqrt(b^2 + c^2)
```

3.352.9 Mupad [B] (verification not implemented)

Time = 33.80 (sec) , antiderivative size = 692, normalized size of antiderivative = 9.48

$$\int \frac{A+B\cos(x)}{b\cos(x)+c\sin(x)}\,dx = \ln\left(32\,A^2\,B\,b^2-32\,A\,B^2\,b^2\right.$$

$$-\frac{\left(A\,\sqrt{{\left(b^2+c^2\right)}^3}+B\,c^3+B\,b^2\,c\right)\,\left(32\,b\tan\left(\frac{x}{2}\right)\,\left(A^2\,b^2-A^2\,c^2+4\,A\,B\,c^2+B^2\,b^2-3\,B^2\,c^2\right)-64\,A^2\,b\right.}{}$$

$$\left.+32\,B\,b\,c\tan\left(\frac{x}{2}\right)\,{\left(A-B\right)}^2\right)\,\left(\frac{B\,c}{b^2+c^2}+\frac{A\,\sqrt{{\left(b^2+c^2\right)}^3}}{{\left(b^2+c^2\right)}^2}\right)+\ln\left(32\,A^2\,B\,b^2-32\,A\,B^2\,b^2\right.$$

$$-\frac{\left(B\,c^3-A\,\sqrt{{\left(b^2+c^2\right)}^3}+B\,b^2\,c\right)\,\left(32\,b\tan\left(\frac{x}{2}\right)\,\left(A^2\,b^2-A^2\,c^2+4\,A\,B\,c^2+B^2\,b^2-3\,B^2\,c^2\right)-64\,A^2\,b\right.}{}$$

$$\left.+32\,B\,b\,c\tan\left(\frac{x}{2}\right)\,{\left(A-B\right)}^2\right)\,\left(\frac{B\,c}{b^2+c^2}-\frac{A\,\sqrt{{\left(b^2+c^2\right)}^3}}{{\left(b^2+c^2\right)}^2}\right)$$

$$-\frac{B\,\ln\left(\tan\left(\frac{x}{2}\right)-1{}\mathrm{i}\right)}{b+c\,1{}\mathrm{i}}-\frac{B\,\ln\left(\tan\left(\frac{x}{2}\right)+1{}\mathrm{i}\right)}{c+b\,1{}\mathrm{i}}$$

input
```
int((A + B*cos(x))/(b*cos(x) + c*sin(x)),x)
```

output
```
log(32*A^2*B*b^2 - 32*A*B^2*b^2 - ((A*((b^2 + c^2)^3)^(1/2) + B*c^3 + B*b^
2*c)*(32*b*tan(x/2)*(A^2*b^2 - A^2*c^2 + B^2*b^2 - 3*B^2*c^2 + 4*A*B*c^2)
- 64*A^2*b^2*c - 32*B^2*b^2*c + ((A*((b^2 + c^2)^3)^(1/2) + B*c^3 + B*b^2*
c)*(32*A*b^4 + 32*B*b^4 + 32*A*b^2*c^2 - 64*B*b^2*c^2 + 32*b*c*tan(x/2)*(2
*A*b^2 + 2*A*c^2 + 4*B*b^2 + B*c^2) + (96*b*c*(b + c*tan(x/2))*(A*((b^2 +
c^2)^3)^(1/2) + B*c^3 + B*b^2*c))/(b^2 + c^2)))/(b^2 + c^2)^2 + 64*A*B*b^2
*c))/(b^2 + c^2)^2 + 32*B*b*c*tan(x/2)*(A - B)^2)*((B*c)/(b^2 + c^2) + (A*
((b^2 + c^2)^3)^(1/2))/(b^2 + c^2)^2) + log(32*A^2*B*b^2 - 32*A*B^2*b^2 -
((B*c^3 - A*((b^2 + c^2)^3)^(1/2) + B*b^2*c)*(32*b*tan(x/2)*(A^2*b^2 - A^2
*c^2 + B^2*b^2 - 3*B^2*c^2 + 4*A*B*c^2) - 64*A^2*b^2*c - 32*B^2*b^2*c + ((
B*c^3 - A*((b^2 + c^2)^3)^(1/2) + B*b^2*c)*(32*A*b^4 + 32*B*b^4 + 32*A*b^2
*c^2 - 64*B*b^2*c^2 + 32*b*c*tan(x/2)*(2*A*b^2 + 2*A*c^2 + 4*B*b^2 + B*c^2
) + (96*b*c*(b + c*tan(x/2))*(B*c^3 - A*((b^2 + c^2)^3)^(1/2) + B*b^2*c))/
(b^2 + c^2)))/(b^2 + c^2)^2 + 64*A*B*b^2*c))/(b^2 + c^2)^2 + 32*B*b*c*tan(
x/2)*(A - B)^2)*((B*c)/(b^2 + c^2) - (A*((b^2 + c^2)^3)^(1/2))/(b^2 + c^2)
^2) - (B*log(tan(x/2) - 1i)*1i)/(b + c*1i) - (B*log(tan(x/2) + 1i))/(b*1i
+ c)
```

3.352. $\int \frac{A+B\cos(x)}{b\cos(x)+c\sin(x)}\,dx$

3.353 $\int \frac{A+B\cos(x)}{(b\cos(x)+c\sin(x))^2}\,dx$

3.353.1 Optimal result

Integrand size = 18, antiderivative size = 76

$$\int \frac{A+B\cos(x)}{(b\cos(x)+c\sin(x))^2}\,dx=-\frac{bB\text{arctanh}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{3/2}}-\frac{Bc+Ac\cos(x)-Ab\sin(x)}{(b^2+c^2)\,(b\cos(x)+c\sin(x))}$$

output
```
-b*B*arctanh((c*cos(x)-b*sin(x))/(b^2+c^2)^(1/2))/(b^2+c^2)^(3/2)+(-B*c-A*
c*cos(x)+A*b*sin(x))/(b^2+c^2)/(b*cos(x)+c*sin(x))
```

3.353.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.08

$$\int \frac{A+B\cos(x)}{(b\cos(x)+c\sin(x))^2}\,dx=\frac{2bB\text{arctanh}\left(\frac{-c+b\tan\left(\frac{x}{2}\right)}{\sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{3/2}}+\frac{-bBc+A(b^2+c^2)\sin(x)}{b\,(b^2+c^2)\,(b\cos(x)+c\sin(x))}$$

input
```
Integrate[(A + B*Cos[x])/(b*Cos[x] + c*Sin[x])^2,x]
```

output
```
(2*b*B*ArcTanh[(-c + b*Tan[x/2])/Sqrt[b^2 + c^2]])/(b^2 + c^2)^(3/2) + (-(
b*B*c) + A*(b^2 + c^2)*Sin[x])/(b*(b^2 + c^2)*(b*Cos[x] + c*Sin[x]))
```

3.353.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3042, 3634, 3042, 3553, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(x)}{(b\cos(x)+c\sin(x))^2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{A+B\cos(x)}{(b\cos(x)+c\sin(x))^2}dx$$

$$\Big\downarrow \; 3634$$

$$\frac{bB\int \frac{1}{b\cos(x)+c\sin(x)}dx}{b^2+c^2}-\frac{-Ab\sin(x)+Ac\cos(x)+Bc}{(b^2+c^2)\,(b\cos(x)+c\sin(x))}$$

$$\Big\downarrow \; 3042$$

$$\frac{bB\int \frac{1}{b\cos(x)+c\sin(x)}dx}{b^2+c^2}-\frac{-Ab\sin(x)+Ac\cos(x)+Bc}{(b^2+c^2)\,(b\cos(x)+c\sin(x))}$$

$$\Big\downarrow \; 3553$$

$$-\frac{bB\int \frac{1}{b^2+c^2-(c\cos(x)-b\sin(x))^2}d(c\cos(x)-b\sin(x))}{b^2+c^2}-\frac{-Ab\sin(x)+Ac\cos(x)+Bc}{(b^2+c^2)\,(b\cos(x)+c\sin(x))}$$

$$\Big\downarrow \; 219$$

$$-\frac{-Ab\sin(x)+Ac\cos(x)+Bc}{(b^2+c^2)\,(b\cos(x)+c\sin(x))}-\frac{bB\text{arctanh}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{3/2}}$$

input
```
Int[(A + B*Cos[x])/(b*Cos[x] + c*Sin[x])^2,x]
```

output
```
-((b*B*ArcTanh[(c*Cos[x] - b*Sin[x])/Sqrt[b^2 + c^2]])/(b^2 + c^2)^(3/2))
- (B*c + A*c*Cos[x] - A*b*Sin[x])/((b^2 + c^2)*(b*Cos[x] + c*Sin[x]))
```

3.353.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3634
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[(c*B + c*A*Co
s[d + e*x] + (a*B - b*A)*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])), x] + Simp[(a*A - b*B)/(a^2 - b^2 - c^2)   Int[1/(a
 + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B},
 x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - b*B, 0]
```

3.353.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.43

method	result	size
default	$\frac{-\frac{2\left(A\,b^{2}+A\,c^{2}-B\,c^{2}\right)\tan\left(\frac{x}{2}\right)}{b\left(b^{2}+c^{2}\right)}+\frac{2Bc}{b^{2}+c^{2}}}{\tan\left(\frac{x}{2}\right)^{2}b-2c\tan\left(\frac{x}{2}\right)-b}+\frac{2bB\,\operatorname{arctanh}\left(\frac{2b\tan\left(\frac{x}{2}\right)-2c}{2\sqrt{b^{2}+c^{2}}}\right)}{\left(b^{2}+c^{2}\right)^{\frac{3}{2}}}$	109
risch	$-\frac{2i\left(cA-iAb+Bc\,{\mathrm e}^{ix}\right)}{\left(-ib+c\right)\left(ib+c\right)\left(c\,{\mathrm e}^{2ix}+ib\,{\mathrm e}^{2ix}-c+ib\right)}+\frac{bB\ln\left({\mathrm e}^{ix}+\frac{ib^{3}+ib\,c^{2}-b^{2}c-c^{3}}{\left(b^{2}+c^{2}\right)^{\frac{3}{2}}}\right)}{\left(b^{2}+c^{2}\right)^{\frac{3}{2}}}-\frac{bB\ln\left({\mathrm e}^{ix}-\frac{ib^{3}+ib\,c^{2}-b^{2}c-c^{3}}{\left(b^{2}+c^{2}\right)^{\frac{3}{2}}}\right)}{\left(b^{2}+c^{2}\right)^{\frac{3}{2}}}$	174

input
```
int((A+B*cos(x))/(b*cos(x)+c*sin(x))^2,x,method=_RETURNVERBOSE)
```

output
```
2*(-(A*b^2+A*c^2-B*c^2)/b/(b^2+c^2)*tan(1/2*x)+B*c/(b^2+c^2))/(tan(1/2*x)^
2*b-2*c*tan(1/2*x)-b)+2*b*B/(b^2+c^2)^(3/2)*arctanh(1/2*(2*b*tan(1/2*x)-2*
c)/(b^2+c^2)^(1/2))
```

3.353.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(72) = 144.

Time = 0.25 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.64

$$\int \frac{A+B\cos(x)}{(b\cos(x)+c\sin(x))^2}\,dx = -\frac{2\,Bb^2c+2\,Bc^3-(Bb^2\cos(x)+Bbc\sin(x))\sqrt{b^2+c^2}\log\left(-\frac{2\,bc\cos(x)\sin(x)+(b^2-c^2)\cos(x)^2-2\,b^2-c^2+2\,\sqrt{b^2+c^2}(c}{2\,bc\cos(x)\sin(x)+(b^2-c^2)\cos(x)^2+c^2}\right.}{2\,((b^5+2\,b^3c^2+bc^4)\cos(x)+(b^4c+2\,b^2c^3+c^5}$$

input
```
integrate((A+B*cos(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="fricas")
```

output
```
-1/2*(2*B*b^2*c + 2*B*c^3 - (B*b^2*cos(x) + B*b*c*sin(x))*sqrt(b^2 + c^2)*
log(-(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 - 2*b^2 - c^2 + 2*sqrt(b^
2 + c^2)*(c*cos(x) - b*sin(x)))/(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^
2 + c^2)) + 2*(A*b^2*c + A*c^3)*cos(x) - 2*(A*b^3 + A*b*c^2)*sin(x))/((b^5
 + 2*b^3*c^2 + b*c^4)*cos(x) + (b^4*c + 2*b^2*c^3 + c^5)*sin(x))
```

3.353.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{A+B\cos(x)}{(b\cos(x)+c\sin(x))^2}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate((A+B*cos(x))/(b*cos(x)+c*sin(x))**2,x)
```

output
```
Exception raised: AttributeError >> 'NoneType' object has no attribute 'pr
imitive'
```

3.353.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(72) = 144$.

Time = 0.32 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.05

$$\int \frac{A+B\cos(x)}{(b\cos(x)+c\sin(x))^2}\,dx = -B\left(\frac{b\log\left(\frac{c-\frac{b\sin(x)}{\cos(x)+1}+\sqrt{b^2+c^2}}{c-\frac{b\sin(x)}{\cos(x)+1}-\sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{\frac{3}{2}}}+\frac{2\left(bc+\frac{c^2\sin(x)}{\cos(x)+1}\right)}{b^4+b^2c^2+\frac{2(b^3c+bc^3)\sin(x)}{\cos(x)+1}-\frac{(b^4+b^2c^2)\sin(x)^2}{(\cos(x)+1)^2}}\right) - \frac{A}{c^2\tan(x)+bc}$$

input
```
integrate((A+B*cos(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="maxima")
```

output
```
-B*(b*log((c - b*sin(x)/(cos(x) + 1) + sqrt(b^2 + c^2))/(c - b*sin(x)/(cos
(x) + 1) - sqrt(b^2 + c^2)))/(b^2 + c^2)^(3/2) + 2*(b*c + c^2*sin(x)/(cos(
x) + 1))/(b^4 + b^2*c^2 + 2*(b^3*c + b*c^3)*sin(x)/(cos(x) + 1) - (b^4 + b
^2*c^2)*sin(x)^2/(cos(x) + 1)^2)) - A/(c^2*tan(x) + b*c)
```

3.353.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.74

$$\int \frac{A+B\cos(x)}{(b\cos(x)+c\sin(x))^2}\,dx = -\frac{Bb\log\left(\frac{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,c-2\,\sqrt{b^2+c^2}\right|}{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,c+2\,\sqrt{b^2+c^2}\right|}\right)}{(b^2+c^2)^{\frac{3}{2}}} - \frac{2\left(Ab^2\tan\left(\frac{1}{2}\,x\right)+Ac^2\tan\left(\frac{1}{2}\,x\right)-Bc^2\tan\left(\frac{1}{2}\,x\right)-Bbc\right)}{(b^3+bc^2)\left(b\tan\left(\frac{1}{2}\,x\right)^2-2\,c\tan\left(\frac{1}{2}\,x\right)-b\right)}$$

input
```
integrate((A+B*cos(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="giac")
```

output
```
-B*b*log(abs(2*b*tan(1/2*x) - 2*c - 2*sqrt(b^2 + c^2))/abs(2*b*tan(1/2*x)
- 2*c + 2*sqrt(b^2 + c^2)))/(b^2 + c^2)^(3/2) - 2*(A*b^2*tan(1/2*x) + A*c^
2*tan(1/2*x) - B*c^2*tan(1/2*x) - B*b*c)/((b^3 + b*c^2)*(b*tan(1/2*x)^2 -
2*c*tan(1/2*x) - b))
```

3.353.9 Mupad [B] (verification not implemented)

Time = 27.85 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.66

$$\int \frac{A+B\cos(x)}{(b\cos(x)+c\sin(x))^2}\,dx = -\frac{\frac{2\,B\,c}{b^2+c^2}-\frac{2\tan\left(\frac{x}{2}\right)\,\left(A\,b^2+A\,c^2-B\,c^2\right)}{b\,\left(b^2+c^2\right)}}{-b\tan\left(\frac{x}{2}\right)^2+2\,c\tan\left(\frac{x}{2}\right)+b} + \frac{B\,b\,\mathrm{atan}\left(\frac{b^2\,c\,1\mathrm{i}+c^3\,1\mathrm{i}-b\tan\left(\frac{x}{2}\right)\,\left(b^2+c^2\right)\,1\mathrm{i}}{\left(b^2+c^2\right)^{3/2}}\right)\,2\mathrm{i}}{\left(b^2+c^2\right)^{3/2}}$$

input
```
int((A + B*cos(x))/(b*cos(x) + c*sin(x))^2,x)
```

output
```
(B*b*atan((b^2*c*1i + c^3*1i - b*tan(x/2)*(b^2 + c^2)*1i)/(b^2 + c^2)^(3/2
))*2i)/(b^2 + c^2)^(3/2) - ((2*B*c)/(b^2 + c^2) - (2*tan(x/2)*(A*b^2 + A*c
^2 - B*c^2))/(b*(b^2 + c^2)))/(b + 2*c*tan(x/2) - b*tan(x/2)^2)
```

3.354 $\int \frac{A+B\cos(x)}{(b\cos(x)+c\sin(x))^3}\,dx$

3.354.1 Optimal result

Integrand size = 18, antiderivative size = 116

$$\int \frac{A+B\cos(x)}{(b\cos(x)+c\sin(x))^3}\,dx = -\frac{A\operatorname{arctanh}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{2\left(b^2+c^2\right)^{3/2}} - \frac{Bc+Ac\cos(x)-Ab\sin(x)}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2} - \frac{bBc\cos(x)-b^2B\sin(x)}{\left(b^2+c^2\right)^2(b\cos(x)+c\sin(x))}$$

output
```
-1/2*A*arctanh((c*cos(x)-b*sin(x))/(b^2+c^2)^(1/2))/(b^2+c^2)^(3/2)+1/2*(-
B*c-A*c*cos(x)+A*b*sin(x))/(b^2+c^2)/(b*cos(x)+c*sin(x))^2+(-b*B*c*cos(x)+
b^2*B*sin(x))/(b^2+c^2)^2/(b*cos(x)+c*sin(x))
```

3.354.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.02

$$\int \frac{A+B\cos(x)}{(b\cos(x)+c\sin(x))^3}\,dx = \frac{2A\sqrt{b^2+c^2}\operatorname{arctanh}\left(\frac{-c+b\tan\left(\frac{x}{2}\right)}{\sqrt{b^2+c^2}}\right)(b\cos(x)+c\sin(x))^2+\left(b^2+c^2\right)\left(-Ac\cos(x)-Bc\cos(2x)+b(A+2B\right.}{2(b-ic)^2(b+ic)^2(b\cos(x)+c\sin(x))^2}$$

input
```
Integrate[(A + B*Cos[x])/(b*Cos[x] + c*Sin[x])^3,x]
```

output
```
(2*A*Sqrt[b^2 + c^2]*ArcTanh[(-c + b*Tan[x/2])/Sqrt[b^2 + c^2]]*(b*Cos[x]
+ c*Sin[x])^2 + (b^2 + c^2)*(-(A*c*Cos[x]) - B*c*Cos[2*x] + b*(A + 2*B*Cos
[x])*Sin[x]))/(2*(b - I*c)^2*(b + I*c)^2*(b*Cos[x] + c*Sin[x])^2)
```

3.354.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3042, 3637, 3042, 3632, 3042, 3553, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(x)}{(b\cos(x)+c\sin(x))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\cos(x)}{(b\cos(x)+c\sin(x))^3}dx$$

$$\Big\downarrow \text{3637}$$

$$\frac{\int \frac{2bB+Ab\cos(x)+Ac\sin(x)}{(b\cos(x)+c\sin(x))^2}dx}{2(b^2+c^2)}-\frac{-Ab\sin(x)+Ac\cos(x)+Bc}{2(b^2+c^2)(b\cos(x)+c\sin(x))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{2bB+Ab\cos(x)+Ac\sin(x)}{(b\cos(x)+c\sin(x))^2}dx}{2(b^2+c^2)}-\frac{-Ab\sin(x)+Ac\cos(x)+Bc}{2(b^2+c^2)(b\cos(x)+c\sin(x))^2}$$

$$\Big\downarrow \text{3632}$$

$$\frac{A\int \frac{1}{b\cos(x)+c\sin(x)}dx-\frac{2(bBc\cos(x)-b^2B\sin(x))}{(b^2+c^2)(b\cos(x)+c\sin(x))}}{2(b^2+c^2)}-\frac{-Ab\sin(x)+Ac\cos(x)+Bc}{2(b^2+c^2)(b\cos(x)+c\sin(x))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{A\int \frac{1}{b\cos(x)+c\sin(x)}dx-\frac{2(bBc\cos(x)-b^2B\sin(x))}{(b^2+c^2)(b\cos(x)+c\sin(x))}}{2(b^2+c^2)}-\frac{-Ab\sin(x)+Ac\cos(x)+Bc}{2(b^2+c^2)(b\cos(x)+c\sin(x))^2}$$

$$\Big\downarrow \text{3553}$$

$$\frac{-A\int \frac{1}{b^2+c^2-(c\cos(x)-b\sin(x))^2}d(c\cos(x)-b\sin(x))-\frac{2(bBc\cos(x)-b^2B\sin(x))}{(b^2+c^2)(b\cos(x)+c\sin(x))}}{2(b^2+c^2)}-\frac{-Ab\sin(x)+Ac\cos(x)+Bc}{2(b^2+c^2)(b\cos(x)+c\sin(x))^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{-\frac{A\text{arctanh}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}}-\frac{2(bBc\cos(x)-b^2B\sin(x))}{(b^2+c^2)(b\cos(x)+c\sin(x))}}{2(b^2+c^2)}-\frac{-Ab\sin(x)+Ac\cos(x)+Bc}{2(b^2+c^2)(b\cos(x)+c\sin(x))^2}$$

input
```
Int[(A + B*Cos[x])/(b*Cos[x] + c*Sin[x])^3,x]
```

output
```
-1/2*(B*c + A*c*Cos[x] - A*b*Sin[x])/((b^2 + c^2)*(b*Cos[x] + c*Sin[x])^2)
 + (-((A*ArcTanh[(c*Cos[x] - b*Sin[x])/Sqrt[b^2 + c^2]])/Sqrt[b^2 + c^2])
- (2*(b*B*c*Cos[x] - b^2*B*Sin[x]))/((b^2 + c^2)*(b*Cos[x] + c*Sin[x])))/(
2*(b^2 + c^2))
```

3.354.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3632
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Simp[(a*A - b*B - c*C)/(a^2 - b^2 - c^2)   Int[1/(a + b*Cos[d + e*x] + c*S
in[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
 - c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

3.354. $\int \frac{A+B\cos(x)}{(b\cos(x)+c\sin(x))^3}\,dx$

rule 3637
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))*((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(c*B + c
*A*Cos[d + e*x] + (a*B - b*A)*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d
 + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Simp[1/((n + 1)*(a^2
- b^2 - c^2))   Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n +
 1)*(a*A - b*B) + (n + 2)*(a*B - b*A)*Cos[d + e*x] - (n + 2)*c*A*Sin[d + e*
x], x], x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && LtQ[n, -1] && NeQ[a^2
- b^2 - c^2, 0] && NeQ[n, -2]
```

3.354.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.76

method	result
default	$-\frac{2\left(-\frac{\left(A\,b^2+2A\,c^2-2B\,b^2-2B\,c^2\right)\tan\left(\frac{x}{2}\right)^3}{2b\left(b^2+c^2\right)}-\frac{c\left(A\,b^2-2A\,c^2+2B\,b^2+2B\,c^2\right)\tan\left(\frac{x}{2}\right)^2}{2\left(b^2+c^2\right)b^2}-\frac{\left(A\,b^2-2A\,c^2+2B\,b^2+2B\,c^2\right)\tan\left(\frac{x}{2}\right)}{2\left(b^2+c^2\right)b}+\frac{cA}{2b^2+2c^2}\right)}{\left(\tan\left(\frac{x}{2}\right)^2b-2c\tan\left(\frac{x}{2}\right)-b\right)^2}+$
risch	$\frac{-A\,b^2\mathrm{e}^{3ix}+A\,c^2\mathrm{e}^{3ix}+2iAbc\,\mathrm{e}^{3ix}+2B\,b^2\mathrm{e}^{2ix}+2B\,c^2\mathrm{e}^{2ix}+A\,b^2\mathrm{e}^{ix}+A\,c^2\mathrm{e}^{ix}+2B\,b^2+2iBbc}{(-ib+c)(c\,\mathrm{e}^{2ix}+ib\,\mathrm{e}^{2ix}-c+ib)^2(ib+c)^2}+\frac{A\ln\left(\mathrm{e}^{ix}+\frac{ib^3+ib\,c^2-b^2c-c^3}{\left(b^2+c^2\right)^{\frac{3}{2}}}\right)}{2(b^2+c^2)^{\frac{3}{2}}}-$

input
```
int((A+B*cos(x))/(b*cos(x)+c*sin(x))^3,x,method=_RETURNVERBOSE)
```

output
```
-2*(-1/2*(A*b^2+2*A*c^2-2*B*b^2-2*B*c^2)/b/(b^2+c^2)*tan(1/2*x)^3-1/2*c*(A
*b^2-2*A*c^2+2*B*b^2+2*B*c^2)/(b^2+c^2)/b^2*tan(1/2*x)^2-1/2*(A*b^2-2*A*c^
2+2*B*b^2+2*B*c^2)/(b^2+c^2)/b*tan(1/2*x)+1/2*c*A/(b^2+c^2))/(tan(1/2*x)^2
*b-2*c*tan(1/2*x)-b)^2+A/(b^2+c^2)^(3/2)*arctanh(1/2*(2*b*tan(1/2*x)-2*c)/
(b^2+c^2)^(1/2))
```

3.354.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. 2(108) = 216.

Time = 0.26 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.41

$$\int \frac{A+B\cos(x)}{(b\cos(x)+c\sin(x))^3}\,dx = -\frac{8\,Bb^2c\cos(x)^2-2\,Bb^2c+2\,Bc^3-\left(2\,Abc\cos(x)\sin(x)+Ac^2+(Ab^2-Ac^2)\cos(x)^2\right)\sqrt{b^2+c^2}\log\left(-\right.}{4\left(b^4c^2+2\,b^2c^4+c^6+(b^6+b^4c\right.}$$

input
```
integrate((A+B*cos(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="fricas")
```

output
```
-1/4*(8*B*b^2*c*cos(x)^2 - 2*B*b^2*c + 2*B*c^3 - (2*A*b*c*cos(x)*sin(x) +
A*c^2 + (A*b^2 - A*c^2)*cos(x)^2)*sqrt(b^2 + c^2)*log(-(2*b*c*cos(x)*sin(x
) + (b^2 - c^2)*cos(x)^2 - 2*b^2 - c^2 + 2*sqrt(b^2 + c^2)*(c*cos(x) - b*s
in(x)))/(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 + c^2)) + 2*(A*b^2*c +
 A*c^3)*cos(x) - 2*(A*b^3 + A*b*c^2 + 2*(B*b^3 - B*b*c^2)*cos(x))*sin(x))/
(b^4*c^2 + 2*b^2*c^4 + c^6 + (b^6 + b^4*c^2 - b^2*c^4 - c^6)*cos(x)^2 + 2*
(b^5*c + 2*b^3*c^3 + b*c^5)*cos(x)*sin(x))
```

3.354.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cos(x)}{(b\cos(x) + c\sin(x))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(x))/(b*cos(x)+c*sin(x))**3,x)
```

output
```
Timed out
```

3.354.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(108) = 216$.

Time = 0.34 (sec) , antiderivative size = 366, normalized size of antiderivative = 3.16

$$\int \frac{A + B\cos(x)}{(b\cos(x) + c\sin(x))^3}\,dx =$$

$$-\frac{1}{2}A\left(\frac{2\left(b^2c - \frac{(b^3-2bc^2)\sin(x)}{\cos(x)+1} - \frac{(b^2c-2c^3)\sin(x)^2}{(\cos(x)+1)^2} - \frac{(b^3+2bc^2)\sin(x)^3}{(\cos(x)+1)^3}\right)}{b^6 + b^4c^2 + \frac{4(b^5c+b^3c^3)\sin(x)}{\cos(x)+1} - \frac{2(b^6-b^4c^2-2b^2c^4)\sin(x)^2}{(\cos(x)+1)^2} - \frac{4(b^5c+b^3c^3)\sin(x)^3}{(\cos(x)+1)^3} + \frac{(b^6+b^4c^2)\sin(x)^4}{(\cos(x)+1)^4}} + \frac{\log\left(\frac{c-\frac{b\,s}{\cos}}{c-\frac{b\,s}{\cos}}\right.}{(b^2}\right.$$

$$+\frac{2B\left(\frac{b\sin(x)}{\cos(x)+1} + \frac{c\sin(x)^2}{(\cos(x)+1)^2} - \frac{b\sin(x)^3}{(\cos(x)+1)^3}\right)}{b^4 + \frac{4b^3c\sin(x)}{\cos(x)+1} - \frac{4b^3c\sin(x)^3}{(\cos(x)+1)^3} + \frac{b^4\sin(x)^4}{(\cos(x)+1)^4} - \frac{2(b^4-2b^2c^2)\sin(x)^2}{(\cos(x)+1)^2}}$$

input
```
integrate((A+B*cos(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="maxima")
```

output
```
-1/2*A*(2*(b^2*c - (b^3 - 2*b*c^2)*sin(x)/(cos(x) + 1) - (b^2*c - 2*c^3)*s
in(x)^2/(cos(x) + 1)^2 - (b^3 + 2*b*c^2)*sin(x)^3/(cos(x) + 1)^3)/(b^6 + b
^4*c^2 + 4*(b^5*c + b^3*c^3)*sin(x)/(cos(x) + 1) - 2*(b^6 - b^4*c^2 - 2*b^
2*c^4)*sin(x)^2/(cos(x) + 1)^2 - 4*(b^5*c + b^3*c^3)*sin(x)^3/(cos(x) + 1)
^3 + (b^6 + b^4*c^2)*sin(x)^4/(cos(x) + 1)^4) + log((c - b*sin(x)/(cos(x)
+ 1) + sqrt(b^2 + c^2))/(c - b*sin(x)/(cos(x) + 1) - sqrt(b^2 + c^2)))/(b^
2 + c^2)^(3/2)) + 2*B*(b*sin(x)/(cos(x) + 1) + c*sin(x)^2/(cos(x) + 1)^2 -
 b*sin(x)^3/(cos(x) + 1)^3)/(b^4 + 4*b^3*c*sin(x)/(cos(x) + 1) - 4*b^3*c*s
in(x)^3/(cos(x) + 1)^3 + b^4*sin(x)^4/(cos(x) + 1)^4 - 2*(b^4 - 2*b^2*c^2)
*sin(x)^2/(cos(x) + 1)^2)
```

3.354.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(108) = 216$.

Time = 0.34 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.11

$$\int \frac{A + B\cos(x)}{(b\cos(x) + c\sin(x))^3}\,dx = \frac{A\log\left(\frac{\left|-2\,b\tan\left(\frac{1}{2}\,x\right)+2\,c-2\,\sqrt{b^2+c^2}\right|}{\left|-2\,b\tan\left(\frac{1}{2}\,x\right)+2\,c+2\,\sqrt{b^2+c^2}\right|}\right)}{2\,(b^2+c^2)^{\frac{3}{2}}} + \frac{Ab^3\tan\left(\frac{1}{2}\,x\right)^3 - 2\,Bb^3\tan\left(\frac{1}{2}\,x\right)^3 + 2\,Abc^2\tan\left(\frac{1}{2}\,x\right)^3 - 2\,Bbc^2\tan\left(\frac{1}{2}\,x\right)^3 + Ab^2c\tan\left(\frac{1}{2}\,x\right)^2 + 2\,Bb^2c\,\mathrm{ta}}{(b^4+b^2c^2)\,(}$$

input
```
integrate((A+B*cos(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="giac")
```

output
```
1/2*A*log(abs(-2*b*tan(1/2*x) + 2*c - 2*sqrt(b^2 + c^2))/abs(-2*b*tan(1/2*
x) + 2*c + 2*sqrt(b^2 + c^2)))/(b^2 + c^2)^(3/2) + (A*b^3*tan(1/2*x)^3 - 2
*B*b^3*tan(1/2*x)^3 + 2*A*b*c^2*tan(1/2*x)^3 - 2*B*b*c^2*tan(1/2*x)^3 + A*
b^2*c*tan(1/2*x)^2 + 2*B*b^2*c*tan(1/2*x)^2 - 2*A*c^3*tan(1/2*x)^2 + 2*B*c
^3*tan(1/2*x)^2 + A*b^3*tan(1/2*x) + 2*B*b^3*tan(1/2*x) - 2*A*b*c^2*tan(1/
2*x) + 2*B*b*c^2*tan(1/2*x) - A*b^2*c)/((b^4 + b^2*c^2)*(b*tan(1/2*x)^2 -
2*c*tan(1/2*x) - b)^2)
```

3.354.9 Mupad [B] (verification not implemented)

Time = 27.08 (sec) , antiderivative size = 251, normalized size of antiderivative = 2.16

$$\int \frac{A+B\cos(x)}{(b\cos(x)+c\sin(x))^3}\,dx = \frac{\frac{\tan\left(\frac{x}{2}\right)\,(A\,b^2-2\,A\,c^2+2\,B\,b^2+2\,B\,c^2)}{b\,(b^2+c^2)}-\frac{A\,c}{b^2+c^2}+\frac{\tan\left(\frac{x}{2}\right)^2\,(2\,B\,c^3-2\,A\,c^3+A\,b^2\,c+2\,B\,b^2\,c)}{b^2\,(b^2+c^2)}+\frac{\tan\left(\frac{x}{2}\right)^3\,(A\,b^2+2\,A\,c^2-2\,B\,b^2-2\,B\,c^2)}{b\,(b^2+c^2)}}{b^2-\tan\left(\frac{x}{2}\right)^2\,(2\,b^2-4\,c^2)+b^2\tan\left(\frac{x}{2}\right)^4+4\,b\,c\tan\left(\frac{x}{2}\right)-4\,b\,c\tan\left(\frac{x}{2}\right)^3} + \frac{A\,\mathrm{atan}\left(\frac{b^2\,c\,1\mathrm{i}+c^3\,1\mathrm{i}-b\tan\left(\frac{x}{2}\right)\,(b^2+c^2)\,1\mathrm{i}}{(b^2+c^2)^{3/2}}\right)\,1\mathrm{i}}{(b^2+c^2)^{3/2}}$$

input
```
int((A + B*cos(x))/(b*cos(x) + c*sin(x))^3,x)
```

output
```
((tan(x/2)*(A*b^2 - 2*A*c^2 + 2*B*b^2 + 2*B*c^2))/(b*(b^2 + c^2)) - (A*c)/
(b^2 + c^2) + (tan(x/2)^2*(2*B*c^3 - 2*A*c^3 + A*b^2*c + 2*B*b^2*c))/(b^2*
(b^2 + c^2)) + (tan(x/2)^3*(A*b^2 + 2*A*c^2 - 2*B*b^2 - 2*B*c^2))/(b*(b^2
+ c^2)))/(b^2 - tan(x/2)^2*(2*b^2 - 4*c^2) + b^2*tan(x/4)^4 + 4*b*c*tan(x/
2) - 4*b*c*tan(x/2)^3) + (A*atan((b^2*c*1i + c^3*1i - b*tan(x/2)*(b^2 + c^
2)*1i)/(b^2 + c^2)^(3/2))*1i)/(b^2 + c^2)^(3/2)
```

3.355 $\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^4 dx$

3.355.1 Optimal result

Integrand size = 30, antiderivative size = 246

$$\begin{aligned}
&\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^4 dx \\
&= \frac{35}{8}\left(b^2+c^2\right)^2 x-\frac{35c(b^2+c^2)^{3/2}\cos(d+ex)}{8e}+\frac{35b(b^2+c^2)^{3/2}\sin(d+ex)}{8e} \\
&\quad -\frac{35(b^2+c^2)\left(c\cos(d+ex)-b\sin(d+ex)\right)\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)}{24e} \\
&\quad -\frac{7\sqrt{b^2+c^2}(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2}{12e} \\
&\quad -\frac{(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3}{4e}
\end{aligned}$$

output
```
35/8*(b^2+c^2)^2*x-35/8*c*(b^2+c^2)^(3/2)*cos(e*x+d)/e+35/8*b*(b^2+c^2)^(3
/2)*sin(e*x+d)/e-35/24*(b^2+c^2)*(c*cos(e*x+d)-b*sin(e*x+d))*(b*cos(e*x+d)
+c*sin(e*x+d)+(b^2+c^2)^(1/2))/e-7/12*(c*cos(e*x+d)-b*sin(e*x+d))*(b^2+c^2
)^(1/2)*(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^2/e-1/4*(c*cos(e*x+d)-
b*sin(e*x+d))*(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^3/e
```

3.355.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.08 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.97

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^4 dx$$
$$= \frac{420(b^2+c^2)^2(d+ex)-672(b-ic)(b+ic)c\sqrt{b^2+c^2}\cos(d+ex)-336bc(b^2+c^2)\cos(2(d+ex))+32c(}{}$$

input
```
Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^4,x]
```

output
```
(420*(b^2 + c^2)^2*(d + e*x) - 672*(b - I*c)*(b + I*c)*c*Sqrt[b^2 + c^2]*C
os[d + e*x] - 336*b*c*(b^2 + c^2)*Cos[2*(d + e*x)] + 32*c*(-3*b^2 + c^2)*S
qrt[b^2 + c^2]*Cos[3*(d + e*x)] - 12*b*c*(b^2 - c^2)*Cos[4*(d + e*x)] + 67
2*b*(b - I*c)*(b + I*c)*Sqrt[b^2 + c^2]*Sin[d + e*x] + 168*(b^4 - c^4)*Sin
[2*(d + e*x)] + 32*b*(b^2 - 3*c^2)*Sqrt[b^2 + c^2]*Sin[3*(d + e*x)] + 3*(b
^4 - 6*b^2*c^2 + c^4)*Sin[4*(d + e*x)])/(96*e)
```

3.355.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {3042, 3592, 3042, 3592, 3042, 3592, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^4 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^4 dx$$

$$\Big\downarrow \text{3592}$$

$$\frac{7}{4}\sqrt{b^2+c^2}\int \left(b\cos(d+ex)+c\sin(d+ex)+\sqrt{b^2+c^2}\right)^3 dx - \frac{(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3}{4e}$$

$$\big\downarrow \text{3042}$$

$$\frac{7}{4}\sqrt{b^2+c^2}\int \left(b\cos(d+ex)+c\sin(d+ex)+\sqrt{b^2+c^2}\right)^3dx-\frac{(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3}{4e}$$

$$\big\downarrow \text{3592}$$

$$\frac{7}{4}\sqrt{b^2+c^2}\left(\frac{5}{3}\sqrt{b^2+c^2}\int \left(b\cos(d+ex)+c\sin(d+ex)+\sqrt{b^2+c^2}\right)^2dx-\frac{(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2\right.}}{}\right.$$
$$-\frac{(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3}{4e}$$

$$\big\downarrow \text{3042}$$

$$\frac{7}{4}\sqrt{b^2+c^2}\left(\frac{5}{3}\sqrt{b^2+c^2}\int \left(b\cos(d+ex)+c\sin(d+ex)+\sqrt{b^2+c^2}\right)^2dx-\frac{(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2\right.}}{}\right.$$
$$-\frac{(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3}{4e}$$

$$\big\downarrow \text{3592}$$

$$\frac{7}{4}\sqrt{b^2+c^2}\left(\frac{5}{3}\sqrt{b^2+c^2}\left(\frac{3}{2}\sqrt{b^2+c^2}\int \left(b\cos(d+ex)+c\sin(d+ex)+\sqrt{b^2+c^2}\right)dx-\frac{(c\cos(d+ex)-b\sin(d}{}\right.\right.$$
$$-\frac{(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3}{4e}$$

$$\big\downarrow \text{2009}$$

$$\frac{7}{4}\sqrt{b^2+c^2}\left(\frac{5}{3}\sqrt{b^2+c^2}\left(\frac{3}{2}\sqrt{b^2+c^2}\left(x\sqrt{b^2+c^2}+\frac{b\sin(d+ex)}{e}-\frac{c\cos(d+ex)}{e}\right)-\frac{(c\cos(d+ex)-b\sin(d+e}{}\right.\right.$$
$$-\frac{(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3}{4e}$$

input
```
Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^4,x]
```

3.355. $\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^4 dx$

output
```
-1/4*((c*Cos[d + e*x] - b*Sin[d + e*x])*(Sqrt[b^2 + c^2] + b*Cos[d + e*x]
+ c*Sin[d + e*x])^3)/e + (7*Sqrt[b^2 + c^2]*(-1/3*((c*Cos[d + e*x] - b*Sin
[d + e*x])*(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^2)/e + (5*S
qrt[b^2 + c^2]*(-1/2*((c*Cos[d + e*x] - b*Sin[d + e*x])*(Sqrt[b^2 + c^2] +
 b*Cos[d + e*x] + c*Sin[d + e*x]))/e + (3*Sqrt[b^2 + c^2]*(Sqrt[b^2 + c^2]
*x - (c*Cos[d + e*x])/e + (b*Sin[d + e*x])/e))/2))/3))/4
```

3.355.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3592
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Simp[a*((2*n - 1)/n)   Int[(a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

3.355.4 Maple [A] (verified)

Time = 3.27 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.35

method	result
risch	$\frac{35b^4x}{8}+\frac{35x\,b^2c^2}{4}+\frac{35c^4x}{8}-\frac{7c(b^2+c^2)^{\frac{3}{2}}\cos(ex+d)}{e}+\frac{7b(b^2+c^2)^{\frac{3}{2}}\sin(ex+d)}{e}-\frac{c\,b^3\cos(4ex+4d)}{8e}+\frac{c^3b\cos(\cdots}{8\cdots}$
parts	$(b^2+c^2)^2x-\frac{4b^3\left(\frac{c\sin(ex+d)^4}{4}+\frac{\sqrt{b^2+c^2}\sin(ex+d)^3}{3}-\frac{\sin(ex+d)^2c}{2}-\sin(ex+d)\sqrt{b^2+c^2}\right)}{e}+\frac{6b^2c^2\left(-\frac{\sin(ex+d)\cos\cdots}{4}\cdots\right.}{\cdots}$
derivativedivides	$\frac{c^4(ex+d)+2b^2c^2(ex+d)+b^4(ex+d)+6b^2c^2\left(-\frac{\sin(ex+d)\cos(ex+d)^3}{4}+\frac{\cos(ex+d)\sin(ex+d)}{8}+\frac{ex}{8}+\frac{d}{8}\right)+c^3b\sin(ex+d)^4-6b^3\cdots}{\cdots}$
default	$\frac{c^4(ex+d)+2b^2c^2(ex+d)+b^4(ex+d)+6b^2c^2\left(-\frac{\sin(ex+d)\cos(ex+d)^3}{4}+\frac{\cos(ex+d)\sin(ex+d)}{8}+\frac{ex}{8}+\frac{d}{8}\right)+c^3b\sin(ex+d)^4-6b^3\cdots}{\cdots}$
norman	$\frac{(\frac{35}{8}b^4+\frac{35}{4}b^2c^2+\frac{35}{8}c^4)x+(\frac{35}{2}b^4+35b^2c^2+\frac{35}{2}c^4)x\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2+(\frac{35}{2}b^4+35b^2c^2+\frac{35}{2}c^4)x\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^6+(\frac{35}{8}b^4+\frac{35}{4}b^2c^2\cdots}{\cdots}$

input
```
int((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^4,x,method=_RETURNVERBOSE)
```

output
```
35/8*b^4*x+35/4*x*b^2*c^2+35/8*c^4*x-7*c*(b^2+c^2)^(3/2)*cos(e*x+d)/e+7*b*
(b^2+c^2)^(3/2)*sin(e*x+d)/e-1/8*c*b^3/e*cos(4*e*x+4*d)+1/8*c^3*b/e*cos(4*
e*x+4*d)+1/32/e*sin(4*e*x+4*d)*b^4-3/16/e*sin(4*e*x+4*d)*b^2*c^2+1/32/e*si
n(4*e*x+4*d)*c^4-(b^2+c^2)^(1/2)*c/e*cos(3*e*x+3*d)*b^2+1/3*(b^2+c^2)^(1/2
)*c^3/e*cos(3*e*x+3*d)+1/3*(b^2+c^2)^(1/2)*b^3/e*sin(3*e*x+3*d)-(b^2+c^2)^
(1/2)*b/e*sin(3*e*x+3*d)*c^2-7/2*c*b^3/e*cos(2*e*x+2*d)-7/2*c^3*b/e*cos(2*
e*x+2*d)+7/4/e*sin(2*e*x+2*d)*b^4-7/4/e*sin(2*e*x+2*d)*c^4
```

3.355.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.90

$$\int\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^4\,dx =$$
$$-\frac{24\,(b^3c-bc^3)\cos\left(ex+d\right)^4-105\,(b^4+2\,b^2c^2+c^4)ex+48\,(3\,b^3c+4\,bc^3)\cos\left(ex+d\right)^2-3\,\left(2\,(b^4-6\,b^2\cdots\right.}{\cdots}$$

input
```
integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^4,x, algorithm="fric
as")
```

output
```
-1/24*(24*(b^3*c - b*c^3)*cos(e*x + d)^4 - 105*(b^4 + 2*b^2*c^2 + c^4)*e*x
 + 48*(3*b^3*c + 4*b*c^3)*cos(e*x + d)^2 - 3*(2*(b^4 - 6*b^2*c^2 + c^4)*co
s(e*x + d)^3 + (27*b^4 + 6*b^2*c^2 - 29*c^4)*cos(e*x + d))*sin(e*x + d) +
32*((3*b^2*c - c^3)*cos(e*x + d)^3 + 3*(b^2*c + 2*c^3)*cos(e*x + d) - (5*b
^3 + 6*b*c^2 + (b^3 - 3*b*c^2)*cos(e*x + d)^2)*sin(e*x + d))*sqrt(b^2 + c^
2))/e
```

3.355.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 857 vs. $2(235) = 470$.

Time = 0.56 (sec) , antiderivative size = 857, normalized size of antiderivative = 3.48

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^4 dx$$
$$= \begin{cases} \frac{3b^4x\sin^4(d+ex)}{8} + \frac{3b^4x\sin^2(d+ex)\cos^2(d+ex)}{4} + 3b^4x\sin^2(d+ex) + \frac{3b^4x\cos^4(d+ex)}{8} + 3b^4x\cos^2(d+ex) + b^4x + \\ x\left(b\cos(d)+c\sin(d)+\sqrt{b^2+c^2}\right)^4 \end{cases}$$

input
```
integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**4,x)
```

output
```
Piecewise((3*b**4*x*sin(d + e*x)**4/8 + 3*b**4*x*sin(d + e*x)**2*cos(d + e
*x)**2/4 + 3*b**4*x*sin(d + e*x)**2 + 3*b**4*x*cos(d + e*x)**4/8 + 3*b**4*
x*cos(d + e*x)**2 + b**4*x + 3*b**4*sin(d + e*x)**3*cos(d + e*x)/(8*e) + 5
*b**4*sin(d + e*x)*cos(d + e*x)**3/(8*e) + 3*b**4*sin(d + e*x)*cos(d + e*x
)/e + 6*b**3*c*sin(d + e*x)**2/e - b**3*c*cos(d + e*x)**4/e + 8*b**3*sqrt(
b**2 + c**2)*sin(d + e*x)**3/(3*e) + 4*b**3*sqrt(b**2 + c**2)*sin(d + e*x)
*cos(d + e*x)**2/e + 4*b**3*sqrt(b**2 + c**2)*sin(d + e*x)/e + 3*b**2*c**2
*x*sin(d + e*x)**4/4 + 3*b**2*c**2*x*sin(d + e*x)**2*cos(d + e*x)**2/2 + 6
*b**2*c**2*x*sin(d + e*x)**2 + 3*b**2*c**2*x*cos(d + e*x)**4/4 + 6*b**2*c*
*2*x*cos(d + e*x)**2 + 2*b**2*c**2*x + 3*b**2*c**2*sin(d + e*x)**3*cos(d +
 e*x)/(4*e) - 3*b**2*c**2*sin(d + e*x)*cos(d + e*x)**3/(4*e) - 4*b**2*c*sq
rt(b**2 + c**2)*cos(d + e*x)**3/e - 4*b**2*c*sqrt(b**2 + c**2)*cos(d + e*x
)/e + b*c**3*sin(d + e*x)**4/e + 6*b*c**3*sin(d + e*x)**2/e + 4*b*c**2*sqr
t(b**2 + c**2)*sin(d + e*x)**3/e + 4*b*c**2*sqrt(b**2 + c**2)*sin(d + e*x)
/e + 3*c**4*x*sin(d + e*x)**4/8 + 3*c**4*x*sin(d + e*x)**2*cos(d + e*x)**2
/4 + 3*c**4*x*sin(d + e*x)**2 + 3*c**4*x*cos(d + e*x)**4/8 + 3*c**4*x*cos(
d + e*x)**2 + c**4*x - 5*c**4*sin(d + e*x)**3*cos(d + e*x)/(8*e) - 3*c**4*
sin(d + e*x)*cos(d + e*x)**3/(8*e) - 3*c**4*sin(d + e*x)*cos(d + e*x)/e -
4*c**3*sqrt(b**2 + c**2)*sin(d + e*x)**2*cos(d + e*x)/e - 8*c**3*sqrt(b**2
 + c**2)*cos(d + e*x)**3/(3*e) - 4*c**3*sqrt(b**2 + c**2)*cos(d + e*x)/...
```

3.355.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.44

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^4\,dx = -\frac{b^3c\cos\left(ex+d\right)^4}{e} + \frac{bc^3\sin\left(ex+d\right)^4}{e} + \frac{(12\,ex+12\,d+\sin\left(4\,ex+4\,d\right)+8\,\sin\left(2\,ex+2\,d\right))b^4}{32\,e} + \frac{3\,(4\,ex+4\,d-\sin\left(4\,ex+4\,d\right))b^2c^2}{16\,e} + \frac{(12\,ex+12\,d+\sin\left(4\,ex+4\,d\right)-8\,\sin\left(2\,ex+2\,d\right))c^4}{32\,e} + \left(b^2+c^2\right)^2x - 4\left(b^2+c^2\right)^{\frac{3}{2}}\left(\frac{c\cos\left(ex+d\right)}{e}-\frac{b\sin\left(ex+d\right)}{e}\right) - \frac{3}{2}\left(\frac{4\,bc\cos\left(ex+d\right)^2}{e}-\frac{(2\,ex+2\,d+\sin\left(2\,ex+2\,d\right))b^2}{e}-\frac{(2\,ex+2\,d-\sin\left(2\,ex+2\,d\right))c^2}{e}\right)\left(b^2+c^2\right) - \frac{4}{3}\left(\frac{3\,b^2c\cos\left(ex+d\right)^3}{e}-\frac{3\,bc^2\sin\left(ex+d\right)^3}{e}+\frac{(\sin\left(ex+d\right)^3-3\,\sin\left(ex+d\right))b^3}{e}-\frac{(\cos\left(ex+d\right)^3-3\,c}{e}\right.$$

input
```
integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^4,x, algorithm="maxi
ma")
```

output
```
-b^3*c*cos(e*x + d)^4/e + b*c^3*sin(e*x + d)^4/e + 1/32*(12*e*x + 12*d + s
in(4*e*x + 4*d) + 8*sin(2*e*x + 2*d))*b^4/e + 3/16*(4*e*x + 4*d - sin(4*e*
x + 4*d))*b^2*c^2/e + 1/32*(12*e*x + 12*d + sin(4*e*x + 4*d) - 8*sin(2*e*x
 + 2*d))*c^4/e + (b^2 + c^2)^2*x - 4*(b^2 + c^2)^(3/2)*(c*cos(e*x + d)/e -
 b*sin(e*x + d)/e) - 3/2*(4*b*c*cos(e*x + d)^2/e - (2*e*x + 2*d + sin(2*e*
x + 2*d))*b^2/e - (2*e*x + 2*d - sin(2*e*x + 2*d))*c^2/e)*(b^2 + c^2) - 4/
3*(3*b^2*c*cos(e*x + d)^3/e - 3*b*c^2*sin(e*x + d)^3/e + (sin(e*x + d)^3 -
 3*sin(e*x + d))*b^3/e - (cos(e*x + d)^3 - 3*cos(e*x + d))*c^3/e)*sqrt(b^2
 + c^2)
```

3.355.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.17

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^4 dx$$
$$= \frac{35}{8}\,(b^4+2\,b^2c^2+c^4)x-\frac{(b^3c-bc^3)\cos(4\,ex+4\,d)}{8\,e}$$
$$-\frac{\left(3\,\sqrt{b^2+c^2}b^2c-\sqrt{b^2+c^2}c^3\right)\cos(3\,ex+3\,d)}{3\,e}$$
$$-\frac{7\,(b^3c+bc^3)\cos(2\,ex+2\,d)}{2\,e}-\frac{7\left(\sqrt{b^2+c^2}b^2c+\sqrt{b^2+c^2}c^3\right)\cos(ex+d)}{e}$$
$$+\frac{(b^4-6\,b^2c^2+c^4)\sin(4\,ex+4\,d)}{32\,e}+\frac{\left(\sqrt{b^2+c^2}b^3-3\,\sqrt{b^2+c^2}bc^2\right)\sin(3\,ex+3\,d)}{3\,e}$$
$$+\frac{7\,(b^4-c^4)\sin(2\,ex+2\,d)}{4\,e}+\frac{7\left(\sqrt{b^2+c^2}b^3+\sqrt{b^2+c^2}bc^2\right)\sin(ex+d)}{e}$$

input
```
integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^4,x, algorithm="giac
")
```

output
```
35/8*(b^4 + 2*b^2*c^2 + c^4)*x - 1/8*(b^3*c - b*c^3)*cos(4*e*x + 4*d)/e -
1/3*(3*sqrt(b^2 + c^2)*b^2*c - sqrt(b^2 + c^2)*c^3)*cos(3*e*x + 3*d)/e - 7
/2*(b^3*c + b*c^3)*cos(2*e*x + 2*d)/e - 7*(sqrt(b^2 + c^2)*b^2*c + sqrt(b^
2 + c^2)*c^3)*cos(e*x + d)/e + 1/32*(b^4 - 6*b^2*c^2 + c^4)*sin(4*e*x + 4*
d)/e + 1/3*(sqrt(b^2 + c^2)*b^3 - 3*sqrt(b^2 + c^2)*b*c^2)*sin(3*e*x + 3*d
)/e + 7/4*(b^4 - c^4)*sin(2*e*x + 2*d)/e + 7*(sqrt(b^2 + c^2)*b^3 + sqrt(b
^2 + c^2)*b*c^2)*sin(e*x + d)/e
```

3.355.9 Mupad [B] (verification not implemented)

Time = 31.39 (sec) , antiderivative size = 522, normalized size of antiderivative = 2.12

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^4 dx$$
$$= \frac{35\,\mathrm{atan}\left(\frac{35\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,(b^2+c^2)^2}{4\left(\frac{35\,b^4}{4}+\frac{35\,b^2\,c^2}{2}+\frac{35\,c^4}{4}\right)}\right)\,(b^2+c^2)^2}{4\,e}-\frac{35\,\left(\mathrm{atan}\left(\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\right)-\frac{e\,x}{2}\right)\,(b^2+c^2)^2}{4\,e}$$
$$+\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\left(\left(16\,b^3+8\,b\,c^2\right)\sqrt{b^2+c^2}+\frac{29\,b^4}{4}-\frac{27\,c^4}{4}-\frac{3\,b^2\,c^2}{2}\right)+\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^6\,\left(24\,b\,c^3+32\,b^3\,c-(32\,b\right.}{}$$

input
```
int((b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^4,x)
```

output
```
(35*atan((35*tan(d/2 + (e*x)/2)*(b^2 + c^2)^2)/(4*((35*b^4)/4 + (35*c^4)/4
 + (35*b^2*c^2)/2)))*(b^2 + c^2)^2)/(4*e) - (35*(atan(tan(d/2 + (e*x)/2))
- (e*x)/2)*(b^2 + c^2)^2)/(4*e) + (tan(d/2 + (e*x)/2)*((8*b*c^2 + 16*b^3)*
(b^2 + c^2)^(1/2) + (29*b^4)/4 - (27*c^4)/4 - (3*b^2*c^2)/2) + tan(d/2 + (
e*x)/2)^6*(24*b*c^3 + 32*b^3*c - (32*b^2*c + 8*c^3)*(b^2 + c^2)^(1/2)) + t
an(d/2 + (e*x)/2)^4*(64*b*c^3 + 48*b^3*c - (48*b^2*c + 40*c^3)*(b^2 + c^2)
^(1/2)) + tan(d/2 + (e*x)/2)^2*(24*b*c^3 + 32*b^3*c - (32*b^2*c + (136*c^3
)/3)*(b^2 + c^2)^(1/2)) - (16*b^2*c + (40*c^3)/3)*(b^2 + c^2)^(1/2) + tan(
d/2 + (e*x)/2)^7*((8*b*c^2 + 16*b^3)*(b^2 + c^2)^(1/2) - (29*b^4)/4 + (27*
c^4)/4 + (3*b^2*c^2)/2) + tan(d/2 + (e*x)/2)^3*((56*b*c^2 + (112*b^3)/3)*(
b^2 + c^2)^(1/2) + (21*b^4)/4 - (35*c^4)/4 + (21*b^2*c^2)/2) + tan(d/2 + (
e*x)/2)^5*((56*b*c^2 + (112*b^3)/3)*(b^2 + c^2)^(1/2) - (21*b^4)/4 + (35*c
^4)/4 - (21*b^2*c^2)/2))/(e*(4*tan(d/2 + (e*x)/2)^2 + 6*tan(d/2 + (e*x)/2)
^4 + 4*tan(d/2 + (e*x)/2)^6 + tan(d/2 + (e*x)/2)^8 + 1))
```

3.355. $\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^4\,dx$

3.356 $\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3 dx$

3.356.1 Optimal result

Integrand size = 30, antiderivative size = 178

$$\begin{aligned}
&\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3 dx \\
&= \frac{5}{2}\left(b^2+c^2\right)^{3/2}x-\frac{5c(b^2+c^2)\cos(d+ex)}{2e}+\frac{5b(b^2+c^2)\sin(d+ex)}{2e} \\
&\quad -\frac{5\sqrt{b^2+c^2}(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)}{6e} \\
&\quad -\frac{(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2}{3e}
\end{aligned}$$

output
```
5/2*(b^2+c^2)^(3/2)*x-5/2*c*(b^2+c^2)*cos(e*x+d)/e+5/2*b*(b^2+c^2)*sin(e*x
+d)/e-5/6*(c*cos(e*x+d)-b*sin(e*x+d))*(b^2+c^2)^(1/2)*(b*cos(e*x+d)+c*sin(
e*x+d)+(b^2+c^2)^(1/2))/e-1/3*(c*cos(e*x+d)-b*sin(e*x+d))*(b*cos(e*x+d)+c*
sin(e*x+d)+(b^2+c^2)^(1/2))^2/e
```

3.356.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.51 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.92

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3 dx$$
$$= \frac{30(b-ic)(b+ic)\sqrt{b^2+c^2}(d+ex)-45c(b^2+c^2)\cos(d+ex)-18bc\sqrt{b^2+c^2}\cos(2(d+ex))+c(-3b^2+}{}$$

input
```
Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^3,x]
```

output
```
(30*(b - I*c)*(b + I*c)*Sqrt[b^2 + c^2]*(d + e*x) - 45*c*(b^2 + c^2)*Cos[d
 + e*x] - 18*b*c*Sqrt[b^2 + c^2]*Cos[2*(d + e*x)] + c*(-3*b^2 + c^2)*Cos[3
*(d + e*x)] + 45*b*(b^2 + c^2)*Sin[d + e*x] + 9*(b^2 - c^2)*Sqrt[b^2 + c^2
]*Sin[2*(d + e*x)] + b*(b^2 - 3*c^2)*Sin[3*(d + e*x)])/(12*e)
```

3.356.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3592, 3042, 3592, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3 dx$$

$$\Big\downarrow \text{3592}$$

$$\frac{5}{3}\sqrt{b^2+c^2}\int \left(b\cos(d+ex)+c\sin(d+ex)+\sqrt{b^2+c^2}\right)^2 dx-\frac{(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2}{3e}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5}{3}\sqrt{b^2+c^2}\int \left(b\cos(d+ex)+c\sin(d+ex)+\sqrt{b^2+c^2}\right)^2dx-\frac{(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2}{3e}$$

$$\Big\downarrow \text{3592}$$

$$\frac{5}{3}\sqrt{b^2+c^2}\left(\frac{3}{2}\sqrt{b^2+c^2}\int \left(b\cos(d+ex)+c\sin(d+ex)+\sqrt{b^2+c^2}\right)dx-\frac{(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+\right.}{2}\right.-\frac{(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2}{3e}$$

$$\Big\downarrow \text{2009}$$

$$\frac{5}{3}\sqrt{b^2+c^2}\left(\frac{3}{2}\sqrt{b^2+c^2}\left(x\sqrt{b^2+c^2}+\frac{b\sin(d+ex)}{e}-\frac{c\cos(d+ex)}{e}\right)-\frac{(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c\right.}{2e}\right.-\frac{(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2}{3e}$$

input
```
Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^3,x]
```

output
```
-1/3*((c*Cos[d + e*x] - b*Sin[d + e*x])*(Sqrt[b^2 + c^2] + b*Cos[d + e*x]
+ c*Sin[d + e*x])^2)/e + (5*Sqrt[b^2 + c^2]*(-1/2*((c*Cos[d + e*x] - b*Sin
[d + e*x])*(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]))/e + (3*Sqr
t[b^2 + c^2]*(Sqrt[b^2 + c^2]*x - (c*Cos[d + e*x])/e + (b*Sin[d + e*x])/e)
)/2))/3
```

3.356.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.356. $\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3dx$

rule 3592
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Simp[a*((2*n - 1)/n)   Int[(a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

3.356.4 Maple [A] (verified)

Time = 2.35 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.23

method	result
risch	$\frac{5(b^2+c^2)^{\frac{3}{2}}x}{2} - \frac{15c\cos(ex+d)b^2}{4e} - \frac{15c^3\cos(ex+d)}{4e} + \frac{15b^3\sin(ex+d)}{4e} + \frac{15b\sin(ex+d)c^2}{4e} - \frac{c\cos(3ex+3d)b^2}{4e} +$
parts	$(b^2+c^2)^{\frac{3}{2}}x + \frac{-b^2c\cos(ex+d)^3+3\sqrt{b^2+c^2}\,b^2\left(\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)}{e} + \frac{3b\left(\frac{\sin(ex+d)^3c^2}{3}+\sqrt{b^2+c^2}\,\sin(ex\right.}{}$
derivativedivides	$\frac{\frac{b^3(2+\cos(ex+d)^2)\sin(ex+d)}{3} - b^2c\cos(ex+d)^3+3\sqrt{b^2+c^2}\,b^2\left(\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)+c^2b\sin(ex+d)^3-3\sqrt{b^2+c^2}\,bc\,c}{}$
default	$\frac{\frac{b^3(2+\cos(ex+d)^2)\sin(ex+d)}{3} - b^2c\cos(ex+d)^3+3\sqrt{b^2+c^2}\,b^2\left(\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)+c^2b\sin(ex+d)^3-3\sqrt{b^2+c^2}\,bc\,c}{}$
norman	$\frac{\frac{(-16c^3+12\sqrt{b^2+c^2}\,bc-12b^2c)\tan(\frac{ex}{2}+\frac{d}{2})^2}{e}+\frac{(-3\sqrt{b^2+c^2}\,b^2+3\sqrt{b^2+c^2}\,c^2+8b^3+6c^2b)\tan(\frac{ex}{2}+\frac{d}{2})^5}{e}+\frac{(3\sqrt{b^2+c^2}\,b^2-3\sqrt{b^2+c^2}}{}}{}$

input
```
int((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^3,x,method=_RETURNVERBOSE)
```

output
```
5/2*(b^2+c^2)^(3/2)*x-15/4*c*cos(e*x+d)/e*b^2-15/4*c^3*cos(e*x+d)/e+15/4*b
^3*sin(e*x+d)/e+15/4*b*sin(e*x+d)/e*c^2-1/4*c/e*cos(3*e*x+3*d)*b^2+1/12*c^
3/e*cos(3*e*x+3*d)+1/12*b^3/e*sin(3*e*x+3*d)-1/4*b/e*sin(3*e*x+3*d)*c^2-3/
2*(b^2+c^2)^(1/2)*b*c/e*cos(2*e*x+2*d)+3/4*(b^2+c^2)^(1/2)/e*sin(2*e*x+2*d
)*b^2-3/4*(b^2+c^2)^(1/2)/e*sin(2*e*x+2*d)*c^2
```

3.356.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.81

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3\,dx =$$
$$-\frac{2\,(3\,b^2c-c^3)\cos{(ex+d)}^3+6\,(3\,b^2c+4\,c^3)\cos{(ex+d)}-2\,\big(11\,b^3+12\,bc^2+(b^3-3\,bc^2)\cos{(ex+d)}^2}{6\,e}$$

input
```
integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^3,x, algorithm="fric
as")
```

output
```
-1/6*(2*(3*b^2*c - c^3)*cos(e*x + d)^3 + 6*(3*b^2*c + 4*c^3)*cos(e*x + d)
- 2*(11*b^3 + 12*b*c^2 + (b^3 - 3*b*c^2)*cos(e*x + d)^2)*sin(e*x + d) + 3*
(6*b*c*cos(e*x + d)^2 - 5*(b^2 + c^2)*e*x - 3*(b^2 - c^2)*cos(e*x + d)*sin
(e*x + d))*sqrt(b^2 + c^2))/e
```

3.356.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 415 vs. $2(168) = 336$.

Time = 0.34 (sec) , antiderivative size = 415, normalized size of antiderivative = 2.33

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3\,dx$$
$$= \begin{cases} \frac{2b^3\sin^3{(d+ex)}}{3e}+\frac{b^3\sin{(d+ex)}\cos^2{(d+ex)}}{e}+\frac{3b^3\sin{(d+ex)}}{e}-\frac{b^2c\cos^3{(d+ex)}}{e}-\frac{3b^2c\cos{(d+ex)}}{e}+\frac{3b^2x\sqrt{b^2+c^2}\sin^2{(d+ex)}}{2}+\frac{3b}{} \\ x\left(b\cos{(d)}+c\sin{(d)}+\sqrt{b^2+c^2}\right)^3 \end{cases}$$

input
```
integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**3,x)
```

output
```
Piecewise((2*b**3*sin(d + e*x)**3/(3*e) + b**3*sin(d + e*x)*cos(d + e*x)**
2/e + 3*b**3*sin(d + e*x)/e - b**2*c*cos(d + e*x)**3/e - 3*b**2*c*cos(d +
e*x)/e + 3*b**2*x*sqrt(b**2 + c**2)*sin(d + e*x)**2/2 + 3*b**2*x*sqrt(b**2
 + c**2)*cos(d + e*x)**2/2 + b**2*x*sqrt(b**2 + c**2) + 3*b**2*sqrt(b**2 +
 c**2)*sin(d + e*x)*cos(d + e*x)/(2*e) + b*c**2*sin(d + e*x)**3/e + 3*b*c*
*2*sin(d + e*x)/e + 3*b*c*sqrt(b**2 + c**2)*sin(d + e*x)**2/e - c**3*sin(d
 + e*x)**2*cos(d + e*x)/e - 2*c**3*cos(d + e*x)**3/(3*e) - 3*c**3*cos(d +
e*x)/e + 3*c**2*x*sqrt(b**2 + c**2)*sin(d + e*x)**2/2 + 3*c**2*x*sqrt(b**2
 + c**2)*cos(d + e*x)**2/2 + c**2*x*sqrt(b**2 + c**2) - 3*c**2*sqrt(b**2 +
 c**2)*sin(d + e*x)*cos(d + e*x)/(2*e), Ne(e, 0)), (x*(b*cos(d) + c*sin(d)
 + sqrt(b**2 + c**2))**3, True))
```

3.356.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.16

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3\,dx = -\frac{b^2c\cos\left(ex+d\right)^3}{e}+\frac{bc^2\sin\left(ex+d\right)^3}{e} - \frac{\left(\sin\left(ex+d\right)^3-3\,\sin\left(ex+d\right)\right)b^3}{3\,e}+\frac{\left(\cos\left(ex+d\right)^3-3\,\cos\left(ex+d\right)\right)c^3}{3\,e} + \left(b^2+c^2\right)^{\frac{3}{2}}x-3\left(b^2+c^2\right)\left(\frac{c\cos\left(ex+d\right)}{e}-\frac{b\sin\left(ex+d\right)}{e}\right) - \frac{3}{4}\left(\frac{4\,bc\cos\left(ex+d\right)^2}{e}-\frac{\left(2\,ex+2\,d+\sin\left(2\,ex+2\,d\right)\right)b^2}{e}-\frac{\left(2\,ex+2\,d-\sin\left(2\,ex+2\,d\right)\right)c^2}{e}\right)\sqrt{b^2+c^2}$$

input
```
integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^3,x, algorithm="maxi
ma")
```

output
```
-b^2*c*cos(e*x + d)^3/e + b*c^2*sin(e*x + d)^3/e - 1/3*(sin(e*x + d)^3 - 3
*sin(e*x + d))*b^3/e + 1/3*(cos(e*x + d)^3 - 3*cos(e*x + d))*c^3/e + (b^2
+ c^2)^(3/2)*x - 3*(b^2 + c^2)*(c*cos(e*x + d)/e - b*sin(e*x + d)/e) - 3/4
*(4*b*c*cos(e*x + d)^2/e - (2*e*x + 2*d + sin(2*e*x + 2*d))*b^2/e - (2*e*x
 + 2*d - sin(2*e*x + 2*d))*c^2/e)*sqrt(b^2 + c^2)
```

3.356.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.12

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3 dx$$
$$= \left(b^2+c^2\right)^{\frac{3}{2}}x - \frac{3\sqrt{b^2+c^2}bc\cos\left(2\,ex+2\,d\right)}{2\,e} + \frac{3}{2}\left(\sqrt{b^2+c^2}b^2+\sqrt{b^2+c^2}c^2\right)x$$
$$- \frac{\left(3\,b^2c-c^3\right)\cos\left(3\,ex+3\,d\right)}{12\,e} - \frac{15\left(b^2c+c^3\right)\cos\left(ex+d\right)}{4\,e} + \frac{\left(b^3-3\,bc^2\right)\sin\left(3\,ex+3\,d\right)}{12\,e}$$
$$+ \frac{3\left(\sqrt{b^2+c^2}b^2-\sqrt{b^2+c^2}c^2\right)\sin\left(2\,ex+2\,d\right)}{4\,e} + \frac{15\left(b^3+bc^2\right)\sin\left(ex+d\right)}{4\,e}$$

input
```
integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^3,x, algorithm="giac
")
```

output
```
(b^2 + c^2)^(3/2)*x - 3/2*sqrt(b^2 + c^2)*b*c*cos(2*e*x + 2*d)/e + 3/2*(sq
rt(b^2 + c^2)*b^2 + sqrt(b^2 + c^2)*c^2)*x - 1/12*(3*b^2*c - c^3)*cos(3*e*
x + 3*d)/e - 15/4*(b^2*c + c^3)*cos(e*x + d)/e + 1/12*(b^3 - 3*b*c^2)*sin(
3*e*x + 3*d)/e + 3/4*(sqrt(b^2 + c^2)*b^2 - sqrt(b^2 + c^2)*c^2)*sin(2*e*x
 + 2*d)/e + 15/4*(b^3 + b*c^2)*sin(e*x + d)/e
```

3.356.9 Mupad [B] (verification not implemented)

Time = 32.70 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.47

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3 dx = \frac{5\,x\left(b^2+c^2\right)^{3/2}}{2}$$
$$- \frac{8\,b^2\,c-\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\left(\left(3\,b^2-3\,c^2\right)\,\sqrt{b^2+c^2}+6\,b\,c^2+8\,b^3\right)-\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^3\,\left(\frac{40\,b^3}{3}+20\,b\,c^2\right)+\frac{22\,c^3}{3}-\mathrm{t}}{e\,\left(\mathrm{tan}\right.}$$

input
```
int((b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^3,x)
```

output
```
(5*x*(b^2 + c^2)^(3/2))/2 - (8*b^2*c - tan(d/2 + (e*x)/2)*((3*b^2 - 3*c^2)
*(b^2 + c^2)^(1/2) + 6*b*c^2 + 8*b^3) - tan(d/2 + (e*x)/2)^3*(20*b*c^2 + (
40*b^3)/3) + (22*c^3)/3 - tan(d/2 + (e*x)/2)^5*(6*b*c^2 - (3*b^2 - 3*c^2)*
(b^2 + c^2)^(1/2) + 8*b^3) + tan(d/2 + (e*x)/2)^4*(12*b^2*c + 6*c^3 - 12*b
*c*(b^2 + c^2)^(1/2)) + tan(d/2 + (e*x)/2)^2*(12*b^2*c + 16*c^3 - 12*b*c*(
b^2 + c^2)^(1/2)))/(e*(3*tan(d/2 + (e*x)/2)^2 + 3*tan(d/2 + (e*x)/2)^4 + t
an(d/2 + (e*x)/2)^6 + 1))
```

3.356. $\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3 \, dx$

3.357 $\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2\,dx$

3.357.1 Optimal result

Integrand size = 30, antiderivative size = 116

$$\begin{aligned}&\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2\,dx\\&=\frac{3}{2}\left(b^2+c^2\right)x-\frac{3c\sqrt{b^2+c^2}\cos(d+ex)}{2e}+\frac{3b\sqrt{b^2+c^2}\sin(d+ex)}{2e}\\&\quad-\frac{(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)}{2e}\end{aligned}$$

output
```
3/2*(b^2+c^2)*x-3/2*c*cos(e*x+d)*(b^2+c^2)^(1/2)/e+3/2*b*sin(e*x+d)*(b^2+c
^2)^(1/2)/e-1/2*(c*cos(e*x+d)-b*sin(e*x+d))*(b*cos(e*x+d)+c*sin(e*x+d)+(b^
2+c^2)^(1/2))/e
```

3.357.2 Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.96

$$\begin{aligned}&\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2\,dx\\&=\frac{6b^2d+6c^2d+6b^2ex+6c^2ex-8c\sqrt{b^2+c^2}\cos(d+ex)-2bc\cos(2(d+ex))+8b\sqrt{b^2+c^2}\sin(d+ex)+b}{4e}\end{aligned}$$

input
```
Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^2,x]
```

output
```
(6*b^2*d + 6*c^2*d + 6*b^2*e*x + 6*c^2*e*x - 8*c*Sqrt[b^2 + c^2]*Cos[d + e
*x] - 2*b*c*Cos[2*(d + e*x)] + 8*b*Sqrt[b^2 + c^2]*Sin[d + e*x] + b^2*Sin[
2*(d + e*x)] - c^2*Sin[2*(d + e*x)])/(4*e)
```

3.357.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {3042, 3592, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2dx$$

$$\Big\downarrow \text{3592}$$

$$\frac{3}{2}\sqrt{b^2+c^2}\int \left(b\cos(d+ex)+c\sin(d+ex)+\sqrt{b^2+c^2}\right)dx-\frac{(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)}{2e}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{2}\sqrt{b^2+c^2}\left(x\sqrt{b^2+c^2}+\frac{b\sin(d+ex)}{e}-\frac{c\cos(d+ex)}{e}\right)-\frac{(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)}{2e}$$

input
```
Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^2,x]
```

output
```
-1/2*((c*Cos[d + e*x] - b*Sin[d + e*x])*(Sqrt[b^2 + c^2] + b*Cos[d + e*x]
+ c*Sin[d + e*x]))/e + (3*Sqrt[b^2 + c^2]*(Sqrt[b^2 + c^2]*x - (c*Cos[d +
e*x])/e + (b*Sin[d + e*x])/e))/2
```

3.357.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3592
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Simp[a*((2*n - 1)/n)   Int[(a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

3.357.4 Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.91

method	result
risch	$\frac{3x\,b^2}{2}+\frac{3x\,c^2}{2}-\frac{2c\cos(ex+d)\sqrt{b^2+c^2}}{e}+\frac{2b\sin(ex+d)\sqrt{b^2+c^2}}{e}-\frac{cb\cos(2ex+2d)}{2e}+\frac{\sin(2ex+2d)b^2}{4e}-\frac{\sin(2ex+}{4e}$
derivativedivides	$\frac{b^2\left(\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)-cb\cos(ex+d)^2+c^2\left(-\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)+2\sqrt{b^2+c^2}\,b\sin(ex+d)-2\sqrt{b^2+c^2}\,c}{e}$
default	$\frac{b^2\left(\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)-cb\cos(ex+d)^2+c^2\left(-\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)+2\sqrt{b^2+c^2}\,b\sin(ex+d)-2\sqrt{b^2+c^2}\,c}{e}$
parts	$x\,b^2+x\,c^2+\frac{2b\left(\frac{\sin(ex+d)^2c}{2}+\sin(ex+d)\sqrt{b^2+c^2}\right)}{e}+\frac{b^2\left(\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)}{e}+\frac{c^2\left(-\frac{\cos(ex+d)\sin(ex+}{2}\right.}{e}$
norman	$\frac{\left(\frac{3b^2}{2}+\frac{3c^2}{2}\right)x-\frac{4c\sqrt{b^2+c^2}}{e}+\left(\frac{3b^2}{2}+\frac{3c^2}{2}\right)x\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^4+\frac{\left(4\sqrt{b^2+c^2}\,b+b^2-c^2\right)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{e}+(3b^2+3c^2)x\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2-}{\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2\right)^2}$

input
```
int((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^2,x,method=_RETURNVERBOSE)
```

output
```
3/2*x*b^2+3/2*x*c^2-2*c*cos(e*x+d)*(b^2+c^2)^(1/2)/e+2*b*sin(e*x+d)*(b^2+c
^2)^(1/2)/e-1/2*c*b/e*cos(2*e*x+2*d)+1/4/e*sin(2*e*x+2*d)*b^2-1/4/e*sin(2*
e*x+2*d)*c^2
```

3.357.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.70

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2\,dx =$$
$$-\frac{2\,bc\cos\left(ex+d\right)^2-3\,(b^2+c^2)ex-(b^2-c^2)\cos\left(ex+d\right)\sin\left(ex+d\right)+4\sqrt{b^2+c^2}(c\cos\left(ex+d\right)-b\,\mathrm{si}}{2\,e}$$

input
```
integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^2,x, algorithm="fric
as")
```

output
```
-1/2*(2*b*c*cos(e*x + d)^2 - 3*(b^2 + c^2)*e*x - (b^2 - c^2)*cos(e*x + d)*
sin(e*x + d) + 4*sqrt(b^2 + c^2)*(c*cos(e*x + d) - b*sin(e*x + d)))/e
```

3.357.6 Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.66

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2\,dx$$
$$=\begin{cases}\frac{b^2x\sin^2\left(d+ex\right)}{2}+\frac{b^2x\cos^2\left(d+ex\right)}{2}+b^2x+\frac{b^2\sin\left(d+ex\right)\cos\left(d+ex\right)}{2e}+\frac{bc\sin^2\left(d+ex\right)}{e}+\frac{2b\sqrt{b^2+c^2}\sin\left(d+ex\right)}{e}+\frac{c^2x\sin^2\left(d+ex\right)}{2}+ \\ x\left(b\cos\left(d\right)+c\sin\left(d\right)+\sqrt{b^2+c^2}\right)^2\end{cases}$$

input
```
integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**2,x)
```

output
```
Piecewise((b**2*x*sin(d + e*x)**2/2 + b**2*x*cos(d + e*x)**2/2 + b**2*x +
b**2*sin(d + e*x)*cos(d + e*x)/(2*e) + b*c*sin(d + e*x)**2/e + 2*b*sqrt(b*
*2 + c**2)*sin(d + e*x)/e + c**2*x*sin(d + e*x)**2/2 + c**2*x*cos(d + e*x)
**2/2 + c**2*x - c**2*sin(d + e*x)*cos(d + e*x)/(2*e) - 2*c*sqrt(b**2 + c*
*2)*cos(d + e*x)/e, Ne(e, 0)), (x*(b*cos(d) + c*sin(d) + sqrt(b**2 + c**2)
)**2, True))
```

3.357.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.97

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2\,dx$$
$$=b^2x+c^2x-\frac{bc\cos(ex+d)^2}{e}+\frac{(2\,ex+2\,d+\sin(2\,ex+2\,d))b^2}{4\,e}+\frac{(2\,ex+2\,d-\sin(2\,ex+2\,d))c^2}{4\,e}-2\,\sqrt{b^2+c^2}\left(\frac{c\cos(ex+d)}{e}-\frac{b\sin(ex+d)}{e}\right)$$

input
```
integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^2,x, algorithm="maxi
ma")
```

output
```
b^2*x + c^2*x - b*c*cos(e*x + d)^2/e + 1/4*(2*e*x + 2*d + sin(2*e*x + 2*d)
)*b^2/e + 1/4*(2*e*x + 2*d - sin(2*e*x + 2*d))*c^2/e - 2*sqrt(b^2 + c^2)*(
c*cos(e*x + d)/e - b*sin(e*x + d)/e)
```

3.357.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.79

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2\,dx$$
$$=\frac{3}{2}\,(b^2+c^2)x-\frac{bc\cos(2\,ex+2\,d)}{2\,e}-\frac{2\,\sqrt{b^2+c^2}c\cos(ex+d)}{e}+\frac{2\,\sqrt{b^2+c^2}b\sin(ex+d)}{e}+\frac{(b^2-c^2)\sin(2\,ex+2\,d)}{4\,e}$$

input
```
integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^2,x, algorithm="giac
")
```

output
```
3/2*(b^2 + c^2)*x - 1/2*b*c*cos(2*e*x + 2*d)/e - 2*sqrt(b^2 + c^2)*c*cos(e
*x + d)/e + 2*sqrt(b^2 + c^2)*b*sin(e*x + d)/e + 1/4*(b^2 - c^2)*sin(2*e*x
 + 2*d)/e
```

3.357.9 Mupad [B] (verification not implemented)

Time = 28.23 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.86

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2\,dx$$
$$=\frac{b^2\,\sin\left(2\,d+2\,e\,x\right)-c^2\,\sin\left(2\,d+2\,e\,x\right)+16\,c\sin\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2\sqrt{b^2+c^2}+8\,b\,\sin\left(d+e\,x\right)\,\sqrt{b^2+c^2}+4\,b\,c\,\mathrm{s}}{4\,e}$$

input
```
int((b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^2,x)
```

output
```
(b^2*sin(2*d + 2*e*x) - c^2*sin(2*d + 2*e*x) + 16*c*sin(d/2 + (e*x)/2)^2*(
b^2 + c^2)^(1/2) + 8*b*sin(d + e*x)*(b^2 + c^2)^(1/2) + 4*b*c*sin(d + e*x)
^2 + 6*b^2*e*x + 6*c^2*e*x)/(4*e)
```

3.358 $\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)\,dx$

3.358.1 Optimal result

Integrand size = 28, antiderivative size = 37

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)\,dx=\sqrt{b^2+c^2}x-\frac{c\cos(d+ex)}{e}+\frac{b\sin(d+ex)}{e}$$

output
```
-c*cos(e*x+d)/e+b*sin(e*x+d)/e+x*(b^2+c^2)^(1/2)
```

3.358.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)\,dx=\frac{\sqrt{b^2+c^2}ex-c\cos(d+ex)+b\sin(d+ex)}{e}$$

input
```
Integrate[Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x],x]
```

output
```
(Sqrt[b^2 + c^2]*e*x - c*Cos[d + e*x] + b*Sin[d + e*x])/e
```

3.358.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)\,dx$$
$$\Big\downarrow \text{ 2009}$$
$$x\sqrt{b^2+c^2}+\frac{b\sin(d+ex)}{e}-\frac{c\cos(d+ex)}{e}$$

input
```
Int[Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x],x]
```

output
```
Sqrt[b^2 + c^2]*x - (c*Cos[d + e*x])/e + (b*Sin[d + e*x])/e
```

3.358.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.358.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

method	result	size
default	$-\frac{c\cos(ex+d)}{e}+\frac{b\sin(ex+d)}{e}+x\sqrt{b^2+c^2}$	36
parallelrisch	$\frac{c-c\cos(ex+d)+b\sin(ex+d)}{e}+x\sqrt{b^2+c^2}$	36
parts	$-\frac{c\cos(ex+d)}{e}+\frac{b\sin(ex+d)}{e}+x\sqrt{b^2+c^2}$	36
derivativedivides	$\frac{(ex+d)\sqrt{b^2+c^2}+b\sin(ex+d)-c\cos(ex+d)}{e}$	38
norman	$\frac{x\sqrt{b^2+c^2}+\sqrt{b^2+c^2}\,x\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2-\frac{2c}{e}+\frac{2b\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{e}}{1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}$	72

input
```
int(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-c*cos(e*x+d)/e+b*sin(e*x+d)/e+x*(b^2+c^2)^(1/2)
```

3.358.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)\,dx=\frac{\sqrt{b^2+c^2}ex-c\cos(ex+d)+b\sin(ex+d)}{e}$$

input
```
integrate(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(b^2 + c^2)*e*x - c*cos(e*x + d) + b*sin(e*x + d))/e
```

3.358.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)\,dx$$
$$=b\left(\begin{cases}\frac{\sin(d+ex)}{e} & \text{for } e\neq 0\\ x\cos(d) & \text{otherwise}\end{cases}\right)+c\left(\begin{cases}-\frac{\cos(d+ex)}{e} & \text{for } e\neq 0\\ x\sin(d) & \text{otherwise}\end{cases}\right)+x\sqrt{b^2+c^2}$$

input
```
integrate(b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2),x)
```

output
```
b*Piecewise((sin(d + e*x)/e, Ne(e, 0)), (x*cos(d), True)) + c*Piecewise((-
cos(d + e*x)/e, Ne(e, 0)), (x*sin(d), True)) + x*sqrt(b**2 + c**2)
```

3.358.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)\,dx=\sqrt{b^2+c^2}x-\frac{c\cos(ex+d)}{e}+\frac{b\sin(ex+d)}{e}$$

input
```
integrate(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(b^2 + c^2)*x - c*cos(e*x + d)/e + b*sin(e*x + d)/e
```

3.358.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right) dx = \sqrt{b^2+c^2}x - \frac{c\cos(ex+d)}{e} + \frac{b\sin(ex+d)}{e}$$

input
```
integrate(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2),x, algorithm="giac")
```

output
```
sqrt(b^2 + c^2)*x - c*cos(e*x + d)/e + b*sin(e*x + d)/e
```

3.358.9 Mupad [B] (verification not implemented)

Time = 28.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.30

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right) dx = x\sqrt{b^2+c^2} - \frac{2c-2b\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{e\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)^2+1\right)}$$

input
```
int(b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2),x)
```

output
```
x*(b^2 + c^2)^(1/2) - (2*c - 2*b*tan(d/2 + (e*x)/2))/(e*(tan(d/2 + (e*x)/2
)^2 + 1))
```

3.359 $\int \frac{1}{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}\,dx$

3.359.1 Optimal result

Integrand size = 30, antiderivative size = 49

$$\int \frac{1}{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}\,dx=-\frac{c-\sqrt{b^2+c^2}\sin(d+ex)}{ce(c\cos(d+ex)-b\sin(d+ex))}$$

output
```
(-c+sin(e*x+d)*(b^2+c^2)^(1/2))/c/e/(c*cos(e*x+d)-b*sin(e*x+d))
```

3.359.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}\,dx=\frac{-c+\sqrt{b^2+c^2}\sin(d+ex)}{ce(c\cos(d+ex)-b\sin(d+ex))}$$

input
```
Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-1),x]
```

output
```
(-c + Sqrt[b^2 + c^2]*Sin[d + e*x])/(c*e*(c*Cos[d + e*x] - b*Sin[d + e*x])
)
```

3.359.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3042, 3593}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}dx$$

$$\Big\downarrow \text{3593}$$

$$-\frac{c-\sqrt{b^2+c^2}\sin(d+ex)}{ce(c\cos(d+ex)-b\sin(d+ex))}$$

input
```
Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-1),x]
```

output
```
-((c - Sqrt[b^2 + c^2]*Sin[d + e*x])/(c*e*(c*Cos[d + e*x] - b*Sin[d + e*x]
)))
```

3.359.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3593
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Simp[-(c - a*Sin[d + e*x])/(c*e*(c*Cos[d + e*x] - b*Sin[
d + e*x])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

3.359.4 Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02

method	result	size
derivativedivides	$-\frac{2\left(\sqrt{b^{2}+c^{2}}+b\right)}{e\,c^{2}\left(\tan\left(\frac{e x}{2}+\frac{d}{2}\right)+\frac{\sqrt{b^{2}+c^{2}}}{c}+\frac{b}{c}\right)}$	50
default	$-\frac{2\left(\sqrt{b^{2}+c^{2}}+b\right)}{e\,c^{2}\left(\tan\left(\frac{e x}{2}+\frac{d}{2}\right)+\frac{\sqrt{b^{2}+c^{2}}}{c}+\frac{b}{c}\right)}$	50
risch	$\frac{2 i b}{\left(i\sqrt{b^{2}+c^{2}}\,c+b^{2}{\mathrm e}^{i(e x+d)}+c^{2}{\mathrm e}^{i(e x+d)}+\sqrt{b^{2}+c^{2}}\,b\right)e}-\frac{2 c}{\left(i\sqrt{b^{2}+c^{2}}\,c+b^{2}{\mathrm e}^{i(e x+d)}+c^{2}{\mathrm e}^{i(e x+d)}+\sqrt{b^{2}+c^{2}}\,b\right)e}$	121

input
```
int(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-2/e*((b^2+c^2)^(1/2)+b)/c^2/(tan(1/2*e*x+1/2*d)+1/c*(b^2+c^2)^(1/2)+b/c)
```

3.359.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.53

$$\int \frac{1}{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}\,dx = -\frac{b^{2}+c^{2}-\sqrt{b^{2}+c^{2}}(b\cos\left(ex+d\right)+c\sin\left(ex+d\right))}{(b^{2}c+c^{3})e\cos\left(ex+d\right)-(b^{3}+bc^{2})e\sin\left(ex+d\right)}$$

input
```
integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2)),x, algorithm="fric
as")
```

output
```
-(b^2 + c^2 - sqrt(b^2 + c^2)*(b*cos(e*x + d) + c*sin(e*x + d)))/((b^2*c +
 c^3)*e*cos(e*x + d) - (b^3 + b*c^2)*e*sin(e*x + d))
```

3.359.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2)),x)
```

output
```
Timed out
```

3.359.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int \frac{1}{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}\,dx = -\frac{2}{\left(c-\frac{\left(b-\sqrt{b^2+c^2}\right)\sin(ex+d)}{\cos(ex+d)+1}\right)e}$$

input
```
integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2)),x, algorithm="maxi
ma")
```

output
```
-2/((c - (b - sqrt(b^2 + c^2))*sin(e*x + d)/(cos(e*x + d) + 1))*e)
```

3.359.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}\,dx = -\frac{2\left(b+\sqrt{b^2+c^2}\right)}{\left(c\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+b+\sqrt{b^2+c^2}\right)ce}$$

input
```
integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2)),x, algorithm="giac
")
```

output
```
-2*(b + sqrt(b^2 + c^2))/((c*tan(1/2*e*x + 1/2*d) + b + sqrt(b^2 + c^2))*c
*e)
```

3.359.9 Mupad [B] (verification not implemented)

Time = 27.23 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}\,dx=\frac{2\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{e\,\left(b+\sqrt{b^2+c^2}+c\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\right)}$$

input
```
int(1/(b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2)),x)
```

output
```
(2*tan(d/2 + (e*x)/2))/(e*(b + (b^2 + c^2)^(1/2) + c*tan(d/2 + (e*x)/2)))
```

3.360 $\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2}\,dx$

3.360.1 Optimal result

Integrand size = 30, antiderivative size = 129

$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2}\,dx = \frac{-c\cos(d+ex)+b\sin(d+ex)}{3\sqrt{b^2+c^2}e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2} - \frac{c-\sqrt{b^2+c^2}\sin(d+ex)}{3c\sqrt{b^2+c^2}e(c\cos(d+ex)-b\sin(d+ex))}$$

output
```
1/3*(-c*cos(e*x+d)+b*sin(e*x+d))/e/(b^2+c^2)^(1/2)/(b*cos(e*x+d)+c*sin(e*x
+d)+(b^2+c^2)^(1/2))^2+1/3*(-c+sin(e*x+d)*(b^2+c^2)^(1/2))/c/e/(c*cos(e*x+
d)-b*sin(e*x+d))/(b^2+c^2)^(1/2)
```

3.360.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.76

$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2}\,dx = \frac{-2c\sqrt{b^2+c^2}+2bc\cos^3(d+ex)+2c^2\sin(d+ex)+c^2\cos^2(d+ex)\sin(d+ex)+b^2\sin^3(d+ex)}{3ce(c\cos(d+ex)-b\sin(d+ex))^3}$$

input
```
Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-2),x]
```

output
```
(-2*c*Sqrt[b^2 + c^2] + 2*b*c*Cos[d + e*x]^3 + 2*c^2*Sin[d + e*x] + c^2*Co
s[d + e*x]^2*Sin[d + e*x] + b^2*Sin[d + e*x]^3)/(3*c*e*(c*Cos[d + e*x] - b
*Sin[d + e*x])^3)
```

3.360.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3042, 3595, 3042, 3593}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2}dx$$

$$\Big\downarrow \text{3595}$$

$$\frac{\int \frac{1}{b\cos(d+ex)+c\sin(d+ex)+\sqrt{b^2+c^2}}dx}{3\sqrt{b^2+c^2}}-\frac{c\cos(d+ex)-b\sin(d+ex)}{3e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{b\cos(d+ex)+c\sin(d+ex)+\sqrt{b^2+c^2}}dx}{3\sqrt{b^2+c^2}}-\frac{c\cos(d+ex)-b\sin(d+ex)}{3e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2}$$

$$\Big\downarrow \text{3593}$$

$$-\frac{c\cos(d+ex)-b\sin(d+ex)}{3e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2}-\frac{c-\sqrt{b^2+c^2}\sin(d+ex)}{3ce\sqrt{b^2+c^2}(c\cos(d+ex)-b\sin(d+ex))}$$

input
```
Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-2),x]
```

output
```
-1/3*(c*Cos[d + e*x] - b*Sin[d + e*x])/(Sqrt[b^2 + c^2]*e*(Sqrt[b^2 + c^2]
 + b*Cos[d + e*x] + c*Sin[d + e*x])^2) - (c - Sqrt[b^2 + c^2]*Sin[d + e*x]
)/(3*c*Sqrt[b^2 + c^2]*e*(c*Cos[d + e*x] - b*Sin[d + e*x]))
```

3.360.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3593
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Simp[-(c - a*Sin[d + e*x])/(c*e*(c*Cos[d + e*x] - b*Sin[
d + e*x])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

rule 3595
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(c*Cos[d + e*x] - b*Sin[d + e*x])*((a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n/(a*e*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n + 1))
 Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b,
c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

3.360.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.38 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00

method	result
risch	$\frac{2\left(i\sqrt{b^{2}+c^{2}}\, c+3b^{2}\mathrm{e}^{i(ex+d)}+3c^{2}\mathrm{e}^{i(ex+d)}+\sqrt{b^{2}+c^{2}}\, b\right)\left(ib^{2}-ic^{2}-2cb\right)}{3\left(i\sqrt{b^{2}+c^{2}}\, c+b^{2}\mathrm{e}^{i(ex+d)}+c^{2}\mathrm{e}^{i(ex+d)}+\sqrt{b^{2}+c^{2}}\, b\right)^{3} e}$
derivativedivides	$\frac{2\left(\sqrt{b^{2}+c^{2}}+b\right)\left(-\frac{\left(\sqrt{b^{2}+c^{2}}+b\right)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}}{c^{2}}-\frac{\left(2b^{2}+c^{2}+2\sqrt{b^{2}+c^{2}}\, b\right)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{c^{3}}-\frac{2\left(2\sqrt{b^{2}+c^{2}}\, b^{2}+\sqrt{b^{2}+c^{2}}\, c^{2}+2b^{3}+2c^{2}b\right)}{3c^{4}}\right)}{e\, c^{2}\left(\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}+\frac{2\sqrt{b^{2}+c^{2}}\, \tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{c}+\frac{2b\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{c}+\frac{2b\sqrt{b^{2}+c^{2}}}{c^{2}}+\frac{2b^{2}}{c^{2}}+1\right)\left(\tan\left(\frac{ex}{2}+\frac{d}{2}\right)+\frac{\sqrt{b^{2}+c^{2}}}{c}+\frac{b}{c}\right)}$
default	$\frac{2\left(\sqrt{b^{2}+c^{2}}+b\right)\left(-\frac{\left(\sqrt{b^{2}+c^{2}}+b\right)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}}{c^{2}}-\frac{\left(2b^{2}+c^{2}+2\sqrt{b^{2}+c^{2}}\, b\right)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{c^{3}}-\frac{2\left(2\sqrt{b^{2}+c^{2}}\, b^{2}+\sqrt{b^{2}+c^{2}}\, c^{2}+2b^{3}+2c^{2}b\right)}{3c^{4}}\right)}{e\, c^{2}\left(\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}+\frac{2\sqrt{b^{2}+c^{2}}\, \tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{c}+\frac{2b\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{c}+\frac{2b\sqrt{b^{2}+c^{2}}}{c^{2}}+\frac{2b^{2}}{c^{2}}+1\right)\left(\tan\left(\frac{ex}{2}+\frac{d}{2}\right)+\frac{\sqrt{b^{2}+c^{2}}}{c}+\frac{b}{c}\right)}$

3.360. $\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2}\, dx$

input
```
int(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^2,x,method=_RETURNVERBOS
E)
```

output
```
2/3*(I*(b^2+c^2)^(1/2)*c+3*b^2*exp(I*(e*x+d))+3*c^2*exp(I*(e*x+d))+(b^2+c^
2)^(1/2)*b)*(I*b^2-I*c^2-2*c*b)/(I*(b^2+c^2)^(1/2)*c+b^2*exp(I*(e*x+d))+c^
2*exp(I*(e*x+d))+(b^2+c^2)^(1/2)*b)^3/e
```

3.360.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.49

$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2}\,dx =$$
$$-\frac{3\,b^3\cos\left(ex+d\right)-\left(b^3-3\,bc^2\right)\cos\left(ex+d\right)^3+\left(3\,b^2c+2\,c^3-\left(3\,b^2c-c^3\right)\cos\left(ex+d\right)^2\right)\sin\left(e\right.}{3\left(\left(3\,b^4c+2\,b^2c^3-c^5\right)e\cos\left(ex+d\right)^3-3\left(b^4c+b^2c^3\right)e\cos\left(ex+d\right)-\left(\left(b^5-2\,b^3c^2-3\,bc^4\right)e\cos\left(ex+d\right.\right.\right.}$$

input
```
integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^2,x, algorithm="fr
icas")
```

output
```
-1/3*(3*b^3*cos(e*x + d) - (b^3 - 3*b*c^2)*cos(e*x + d)^3 + (3*b^2*c + 2*c
^3 - (3*b^2*c - c^3)*cos(e*x + d)^2)*sin(e*x + d) - 2*(b^2 + c^2)^(3/2))/(
(3*b^4*c + 2*b^2*c^3 - c^5)*e*cos(e*x + d)^3 - 3*(b^4*c + b^2*c^3)*e*cos(e
*x + d) - ((b^5 - 2*b^3*c^2 - 3*b*c^4)*e*cos(e*x + d)^2 - (b^5 + b^3*c^2)*
e)*sin(e*x + d))
```

3.360.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**2,x)
```

output
```
Timed out
```

3.360.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^2,x, algorithm="ma
xima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.360.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.22

$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2}\,dx =$$
$$-\frac{2\left(8\,b^4+10\,b^2c^2+2\,c^4+3\left(2\,b^2c^2+c^4+2\,\sqrt{b^2+c^2}bc^2\right)\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^2+3\left(4\,b^3c+3\,bc^3+\left(4\,b^2c+\right.\right.\right.}{3\left(c\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+b+\sqrt{b^2+c^2}\right)^3c^3e}$$

input
```
integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^2,x, algorithm="gi
ac")
```

output
```
-2/3*(8*b^4 + 10*b^2*c^2 + 2*c^4 + 3*(2*b^2*c^2 + c^4 + 2*sqrt(b^2 + c^2)*
b*c^2)*tan(1/2*e*x + 1/2*d)^2 + 3*(4*b^3*c + 3*b*c^3 + (4*b^2*c + c^3)*sqr
t(b^2 + c^2))*tan(1/2*e*x + 1/2*d) + 2*(4*b^3 + 3*b*c^2)*sqrt(b^2 + c^2))/
((c*tan(1/2*e*x + 1/2*d) + b + sqrt(b^2 + c^2))^3*c^3*e)
```

3.360.9 Mupad [B] (verification not implemented)

Time = 29.11 (sec) , antiderivative size = 274, normalized size of antiderivative = 2.12

$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2}\,dx =$$

$$-\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2\left(\frac{4\,b^2+2\,c^2}{c^4}+\frac{4\,b\,\sqrt{b^2+c^2}}{c^4}\right)+\frac{\frac{16\,b^4}{3}+\frac{20\,b^2\,c^2}{3}+\frac{4\,c^4}{3}}{c^6}+\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\left(\frac{8\,b^3+6\,b\,c^2}{c^5}+\frac{\left(8\,b^2+2\,c^2\right)\sqrt{b^2+c^2}}{c^5}\right)+\ \cdots}{e\left(\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\left(\frac{6\,b^2+3\,c^2}{c^2}+\frac{6\,b\,\sqrt{b^2+c^2}}{c^2}\right)+\frac{4\,b^3+3\,b\,c^2}{c^3}+\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^3+\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2\left(\frac{3\,\sqrt{b^2+c^2}}{c}+\frac{3\,b}{c}\right)+\ \cdots\right)}$$

input
```
int(1/(b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^2,x)
```

output
```
-(tan(d/2 + (e*x)/2)^2*((4*b^2 + 2*c^2)/c^4 + (4*b*(b^2 + c^2)^(1/2))/c^4)
 + ((16*b^4)/3 + (4*c^4)/3 + (20*b^2*c^2)/3)/c^6 + tan(d/2 + (e*x)/2)*((6*
b*c^2 + 8*b^3)/c^5 + ((8*b^2 + 2*c^2)*(b^2 + c^2)^(1/2))/c^5) + ((4*b*c^2
+ (16*b^3)/3)*(b^2 + c^2)^(1/2))/c^6)/(e*(tan(d/2 + (e*x)/2)*((6*b^2 + 3*c
^2)/c^2 + (6*b*(b^2 + c^2)^(1/2))/c^2) + (3*b*c^2 + 4*b^3)/c^3 + tan(d/2 +
 (e*x)/2)^3 + tan(d/2 + (e*x)/2)^2*((3*(b^2 + c^2)^(1/2))/c + (3*b)/c) + (
(4*b^2 + c^2)*(b^2 + c^2)^(1/2))/c^3))
```

3.361 $\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3}\,dx$

3.361.1 Optimal result

Integrand size = 30, antiderivative size = 191

$$\begin{aligned}\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3}\,dx &= \frac{-c\cos(d+ex)+b\sin(d+ex)}{5\sqrt{b^2+c^2}e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3} \\ &\quad - \frac{2(c\cos(d+ex)-b\sin(d+ex))}{15\left(b^2+c^2\right)e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2} \\ &\quad - \frac{2\left(c-\sqrt{b^2+c^2}\sin(d+ex)\right)}{15c\left(b^2+c^2\right)e(c\cos(d+ex)-b\sin(d+ex))}\end{aligned}$$

output
```
1/5*(-c*cos(e*x+d)+b*sin(e*x+d))/e/(b^2+c^2)^(1/2)/(b*cos(e*x+d)+c*sin(e*x
+d)+(b^2+c^2)^(1/2))^3-2/15*(c*cos(e*x+d)-b*sin(e*x+d))/(b^2+c^2)/e/(b*cos
(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^2-2/15*(c-sin(e*x+d)*(b^2+c^2)^(1/2)
)/c/(b^2+c^2)/e/(c*cos(e*x+d)-b*sin(e*x+d))
```

3.361.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 420 vs. 2(191) = 382.

Time = 1.84 (sec) , antiderivative size = 420, normalized size of antiderivative = 2.20

$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3}\,dx$$
$$= \frac{-76b^4c-152b^2c^3-76c^5+90bc(b^2+c^2)^{3/2}\cos(d+ex)+20c(-b^4+c^4)\cos(2(d+ex))+10b^3c\sqrt{b^2+c^2}\,c}{}$$

input
```
Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-3),x]
```

output
```
(-76*b^4*c - 152*b^2*c^3 - 76*c^5 + 90*b*c*(b^2 + c^2)^(3/2)*Cos[d + e*x]
+ 20*c*(-b^4 + c^4)*Cos[2*(d + e*x)] + 10*b^3*c*Sqrt[b^2 + c^2]*Cos[3*(d +
 e*x)] + 10*b*c^3*Sqrt[b^2 + c^2]*Cos[3*(d + e*x)] - 4*b^3*c*Sqrt[b^2 + c^
2]*Cos[5*(d + e*x)] + 4*b*c^3*Sqrt[b^2 + c^2]*Cos[5*(d + e*x)] + 10*b^4*Sq
rt[b^2 + c^2]*Sin[d + e*x] + 110*b^2*c^2*Sqrt[b^2 + c^2]*Sin[d + e*x] + 10
0*c^4*Sqrt[b^2 + c^2]*Sin[d + e*x] - 40*b^3*c^2*Sin[2*(d + e*x)] - 40*b*c^
4*Sin[2*(d + e*x)] - 5*b^4*Sqrt[b^2 + c^2]*Sin[3*(d + e*x)] + 5*c^4*Sqrt[b
^2 + c^2]*Sin[3*(d + e*x)] + b^4*Sqrt[b^2 + c^2]*Sin[5*(d + e*x)] - 6*b^2*
c^2*Sqrt[b^2 + c^2]*Sin[5*(d + e*x)] + c^4*Sqrt[b^2 + c^2]*Sin[5*(d + e*x)
])/(120*c*(b^2 + c^2)*e*(c*Cos[d + e*x] - b*Sin[d + e*x])^5)
```

3.361.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3595, 3042, 3595, 3042, 3593}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3}dx$$

$$\Big\downarrow \text{3595}$$

$$\frac{2\int \frac{1}{\left(b\cos(d+ex)+c\sin(d+ex)+\sqrt{b^2+c^2}\right)^2}dx}{5\sqrt{b^2+c^2}}-\frac{c\cos(d+ex)-b\sin(d+ex)}{5e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \frac{1}{\left(b\cos(d+ex)+c\sin(d+ex)+\sqrt{b^2+c^2}\right)^2}dx}{5\sqrt{b^2+c^2}}-\frac{c\cos(d+ex)-b\sin(d+ex)}{5e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3}$$

$$\Big\downarrow \text{3595}$$

$$\frac{2\left(\frac{\int \frac{1}{b\cos(d+ex)+c\sin(d+ex)+\sqrt{b^2+c^2}}dx}{3\sqrt{b^2+c^2}}-\frac{c\cos(d+ex)-b\sin(d+ex)}{3e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2}\right)}{5\sqrt{b^2+c^2}}-\frac{c\cos(d+ex)-b\sin(d+ex)}{5e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(\frac{\int \frac{1}{b\cos(d+ex)+c\sin(d+ex)+\sqrt{b^2+c^2}}dx}{3\sqrt{b^2+c^2}}-\frac{c\cos(d+ex)-b\sin(d+ex)}{3e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2}\right)}{5\sqrt{b^2+c^2}}-\frac{c\cos(d+ex)-b\sin(d+ex)}{5e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3}$$

$$\Big\downarrow \text{3593}$$

$$\frac{2\left(-\frac{c\cos(d+ex)-b\sin(d+ex)}{3e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2}-\frac{c-\sqrt{b^2+c^2}\sin(d+ex)}{3ce\sqrt{b^2+c^2}(c\cos(d+ex)-b\sin(d+ex))}\right)}{5\sqrt{b^2+c^2}}-\frac{c\cos(d+ex)-b\sin(d+ex)}{5e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3}$$

input
```
Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-3),x]
```

3.361. $\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3}\,dx$

output
```
-1/5*(c*Cos[d + e*x] - b*Sin[d + e*x])/(Sqrt[b^2 + c^2]*e*(Sqrt[b^2 + c^2]
 + b*Cos[d + e*x] + c*Sin[d + e*x])^3) + (2*(-1/3*(c*Cos[d + e*x] - b*Sin[
d + e*x])/(Sqrt[b^2 + c^2]*e*(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d +
 e*x])^2) - (c - Sqrt[b^2 + c^2]*Sin[d + e*x])/(3*c*Sqrt[b^2 + c^2]*e*(c*C
os[d + e*x] - b*Sin[d + e*x]))))/(5*Sqrt[b^2 + c^2])
```

3.361.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3593
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Simp[-(c - a*Sin[d + e*x])/(c*e*(c*Cos[d + e*x] - b*Sin[
d + e*x])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

rule 3595
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(c*Cos[d + e*x] - b*Sin[d + e*x])*((a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n/(a*e*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n + 1))
 Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b,
c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

3.361.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.98 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.29

method	result
risch	$\frac{4\left(5i\sqrt{b^2+c^2}\,b^2c\,e^{i(ex+d)}+5i\sqrt{b^2+c^2}\,c^3e^{i(ex+d)}+10b^4e^{2i(ex+d)}+20b^2c^2e^{2i(ex+d)}+10c^4e^{2i(ex+d)}+2ib^3c+2ib\,c^3+5\sqrt{b^2-\right.}{15e\left(i\sqrt{b^2+c^2}\,c+b^2e^{i(ex+d)}+c^2e^{i(ex+d)}+\sqrt{b^2+c^2}\,b\right.}$
derivativedivides	$\frac{-\frac{2\left(4\sqrt{b^2+c^2}\,b^2+\sqrt{b^2+c^2}\,c^2+4b^3+3c^2b\right)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^4}{c^2}-\frac{4\left(8\sqrt{b^2+c^2}\,b^3+4\sqrt{b^2+c^2}\,b\,c^2+8b^4+8b^2c^2+c^4\right)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^3}{c^3}-\frac{8\left(24\sqrt{b\right.}{}}{}$
default	$\frac{-\frac{2\left(4\sqrt{b^2+c^2}\,b^2+\sqrt{b^2+c^2}\,c^2+4b^3+3c^2b\right)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^4}{c^2}-\frac{4\left(8\sqrt{b^2+c^2}\,b^3+4\sqrt{b^2+c^2}\,b\,c^2+8b^4+8b^2c^2+c^4\right)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^3}{c^3}-\frac{8\left(24\sqrt{b\right.}{}}{}$

3.361. $\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3}\,dx$

input
```
int(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^3,x,method=_RETURNVERBOS
E)
```

output
```
4/15*(5*I*(b^2+c^2)^(1/2)*b^2*c*exp(I*(e*x+d))+5*I*(b^2+c^2)^(1/2)*c^3*exp
(I*(e*x+d))+10*b^4*exp(2*I*(e*x+d))+20*b^2*c^2*exp(2*I*(e*x+d))+10*c^4*exp
(2*I*(e*x+d))+2*I*b^3*c+2*I*b*c^3+5*(b^2+c^2)^(1/2)*b^3*exp(I*(e*x+d))+5*(
b^2+c^2)^(1/2)*b*c^2*exp(I*(e*x+d))+b^4-c^4)*(I*b^3-3*I*b*c^2-3*b^2*c+c^3)
/e/(I*(b^2+c^2)^(1/2)*c+b^2*exp(I*(e*x+d))+c^2*exp(I*(e*x+d))+(b^2+c^2)^(1
/2)*b)^5
```

3.361.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 490 vs. $2(178) = 356$.

Time = 0.30 (sec) , antiderivative size = 490, normalized size of antiderivative = 2.57

$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3}\,dx = -\frac{7\,b^6+26\,b^4c^2+31\,b^2c^4+12\,c^6+5\,(b^6+b^4c^2-b^2c^4-c^6)\cos{(ex+d)}^2+10\,(b^5c+2\,b^3c^3+bc^5)\cos{(ex}}{15\,\left((5\,b^8c-14\,b^4c^5-8\,b^2c^7+c^9)e\cos{(ex+d)}^5-10\,(\right.}$$

input
```
integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^3,x, algorithm="fr
icas")
```

output
```
-1/15*(7*b^6 + 26*b^4*c^2 + 31*b^2*c^4 + 12*c^6 + 5*(b^6 + b^4*c^2 - b^2*c
^4 - c^6)*cos(e*x + d)^2 + 10*(b^5*c + 2*b^3*c^3 + b*c^5)*cos(e*x + d)*sin
(e*x + d) - (2*(b^5 - 10*b^3*c^2 + 5*b*c^4)*cos(e*x + d)^5 - 5*(b^5 - 6*b^
3*c^2 + b*c^4)*cos(e*x + d)^3 + 5*(3*b^5 + 3*b^3*c^2 + 2*b*c^4)*cos(e*x +
d) + (15*b^4*c + 25*b^2*c^3 + 12*c^5 + 2*(5*b^4*c - 10*b^2*c^3 + c^5)*cos(
e*x + d)^4 - (15*b^4*c - 10*b^2*c^3 - c^5)*cos(e*x + d)^2)*sin(e*x + d))*s
qrt(b^2 + c^2))/((5*b^8*c - 14*b^4*c^5 - 8*b^2*c^7 + c^9)*e*cos(e*x + d)^5
 - 10*(b^8*c + b^6*c^3 - b^4*c^5 - b^2*c^7)*e*cos(e*x + d)^3 + 5*(b^8*c +
2*b^6*c^3 + b^4*c^5)*e*cos(e*x + d) - ((b^9 - 8*b^7*c^2 - 14*b^5*c^4 + 5*b
*c^8)*e*cos(e*x + d)^4 - 2*(b^9 - 3*b^7*c^2 - 9*b^5*c^4 - 5*b^3*c^6)*e*cos
(e*x + d)^2 + (b^9 + 2*b^7*c^2 + b^5*c^4)*e)*sin(e*x + d))
```

3.361.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**3,x)
```

output
```
Timed out
```

3.361.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^3,x, algorithm="ma
xima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.361.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.79

$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3}\,dx =$$

$$-\frac{2\left(192\,b^7+352\,b^5c^2+200\,b^3c^4+35\,bc^6+15\left(4\,b^3c^4+3\,bc^6+\left(4\,b^2c^4+c^6\right)\sqrt{b^2+c^2}\right)\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^4\right.}{}$$

input
```
integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^3,x, algorithm="gi
ac")
```

output
```
-2/15*(192*b^7 + 352*b^5*c^2 + 200*b^3*c^4 + 35*b*c^6 + 15*(4*b^3*c^4 + 3*
b*c^6 + (4*b^2*c^4 + c^6)*sqrt(b^2 + c^2))*tan(1/2*e*x + 1/2*d)^4 + 30*(8*
b^4*c^3 + 8*b^2*c^5 + c^7 + 4*(2*b^3*c^3 + b*c^5)*sqrt(b^2 + c^2))*tan(1/2
*e*x + 1/2*d)^3 + 20*(24*b^5*c^2 + 32*b^3*c^4 + 9*b*c^6 + 2*(12*b^4*c^2 +
10*b^2*c^4 + c^6)*sqrt(b^2 + c^2))*tan(1/2*e*x + 1/2*d)^2 + 10*(48*b^6*c +
 76*b^4*c^3 + 31*b^2*c^5 + 2*c^7 + (48*b^5*c + 52*b^3*c^3 + 11*b*c^5)*sqrt
(b^2 + c^2))*tan(1/2*e*x + 1/2*d) + (192*b^6 + 256*b^4*c^2 + 96*b^2*c^4 +
7*c^6)*sqrt(b^2 + c^2))/((c*tan(1/2*e*x + 1/2*d) + b + sqrt(b^2 + c^2))^5*
c^5*e)
```

3.361.9 Mupad [B] (verification not implemented)

Time = 33.51 (sec) , antiderivative size = 592, normalized size of antiderivative = 3.10

$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3}\,dx =$$

$$-\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^3\left(\frac{32\,b^4+32\,b^2\,c^2+4\,c^4}{c^7}+\frac{\left(32\,b^3+16\,b\,c^2\right)\sqrt{b^2+c^2}}{c^7}\right)+\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^4\left(\frac{8\,b^3+6\,b\,c^2}{c^6}+\frac{\left(8\,b^2+2\,c^2\right)\sqrt{b^2+c^2}}{c^6}\right)+\mathrm{ta}}{e\left(\frac{16\,b^5+20\,b^3\,c^2+5\,b\,c^4}{c^5}+\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2\left(\frac{40\,b^3+30\,b\,c^2}{c^3}+\frac{\left(40\,b^2+10\,c^2\right)\sqrt{b^2+c^2}}{c^3}\right)+\mathrm{ta}\right.}$$

input
```
int(1/(b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^3,x)
```

output
```
-(tan(d/2 + (e*x)/2)^3*((32*b^4 + 4*c^4 + 32*b^2*c^2)/c^7 + ((16*b*c^2 + 3
2*b^3)*(b^2 + c^2)^(1/2))/c^7) + tan(d/2 + (e*x)/2)^4*((6*b*c^2 + 8*b^3)/c
^6 + ((8*b^2 + 2*c^2)*(b^2 + c^2)^(1/2))/c^6) + tan(d/2 + (e*x)/2)*((64*b^
6 + (8*c^6)/3 + (124*b^2*c^4)/3 + (304*b^4*c^2)/3)/c^9 + ((b^2 + c^2)^(1/2
)*((44*b*c^4)/3 + 64*b^5 + (208*b^3*c^2)/3))/c^9) + tan(d/2 + (e*x)/2)^2*(
(24*b*c^4 + 64*b^5 + (256*b^3*c^2)/3)/c^8 + ((b^2 + c^2)^(1/2)*(64*b^4 + (
16*c^4)/3 + (160*b^2*c^2)/3))/c^8) + ((14*b*c^6)/3 + (128*b^7)/5 + (80*b^3
*c^4)/3 + (704*b^5*c^2)/15)/c^10 + ((b^2 + c^2)^(1/2)*((128*b^6)/5 + (14*c
^6)/15 + (64*b^2*c^4)/5 + (512*b^4*c^2)/15))/c^10)/(e*((5*b*c^4 + 16*b^5 +
 20*b^3*c^2)/c^5 + tan(d/2 + (e*x)/2)^2*((30*b*c^2 + 40*b^3)/c^3 + ((40*b^
2 + 10*c^2)*(b^2 + c^2)^(1/2))/c^3) + tan(d/2 + (e*x)/2)^5 + tan(d/2 + (e*
x)/2)^3*((20*b^2 + 10*c^2)/c^2 + (20*b*(b^2 + c^2)^(1/2))/c^2) + tan(d/2 +
 (e*x)/2)*((40*b^4 + 5*c^4 + 40*b^2*c^2)/c^4 + ((20*b*c^2 + 40*b^3)*(b^2 +
 c^2)^(1/2))/c^4) + tan(d/2 + (e*x)/2)^4*((5*(b^2 + c^2)^(1/2))/c + (5*b)/
c) + ((b^2 + c^2)^(1/2)*(16*b^4 + c^4 + 12*b^2*c^2))/c^5))
```

3.362 $\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^4}\,dx$

3.362.1 Optimal result

Integrand size = 30, antiderivative size = 259

$$\begin{aligned}
&\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^4}\,dx \\
&= \frac{-c\cos(d+ex)+b\sin(d+ex)}{7\sqrt{b^2+c^2}e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^4} \\
&\quad - \frac{3(c\cos(d+ex)-b\sin(d+ex))}{35\left(b^2+c^2\right)e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3} \\
&\quad - \frac{2(c\cos(d+ex)-b\sin(d+ex))}{35\left(b^2+c^2\right)^{3/2}e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2} \\
&\quad - \frac{2\left(c-\sqrt{b^2+c^2}\sin(d+ex)\right)}{35c\left(b^2+c^2\right)^{3/2}e(c\cos(d+ex)-b\sin(d+ex))}
\end{aligned}$$

output
```
1/7*(-c*cos(e*x+d)+b*sin(e*x+d))/e/(b^2+c^2)^(1/2)/(b*cos(e*x+d)+c*sin(e*x
+d)+(b^2+c^2)^(1/2))^4-3/35*(c*cos(e*x+d)-b*sin(e*x+d))/(b^2+c^2)/e/(b*cos
(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^3-2/35*(c*cos(e*x+d)-b*sin(e*x+d))/(
b^2+c^2)^(3/2)/e/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^2-2/35*(c-sin
(e*x+d)*(b^2+c^2)^(1/2))/c/(b^2+c^2)^(3/2)/e/(c*cos(e*x+d)-b*sin(e*x+d))
```

3.362.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 533 vs. 2(259) = 518.

Time = 1.74 (sec) , antiderivative size = 533, normalized size of antiderivative = 2.06

$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^4}\,dx$$
$$= \frac{832b^4c\sqrt{b^2+c^2}+1664b^2c^3\sqrt{b^2+c^2}+832c^5\sqrt{b^2+c^2}-1190bc(b^2+c^2)^2\cos(d+ex)+448c\sqrt{b^2+c^2}(b^4-\cdots}{\cdots}$$

input
```
Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-4),x]
```

output
```
(832*b^4*c*Sqrt[b^2 + c^2] + 1664*b^2*c^3*Sqrt[b^2 + c^2] + 832*c^5*Sqrt[b
^2 + c^2] - 1190*b*c*(b^2 + c^2)^2*Cos[d + e*x] + 448*c*Sqrt[b^2 + c^2]*(b
^4 - c^4)*Cos[2*(d + e*x)] - 112*b^5*c*Cos[3*(d + e*x)] + 56*b^3*c^3*Cos[3
*(d + e*x)] + 168*b*c^5*Cos[3*(d + e*x)] + 28*b^5*c*Cos[5*(d + e*x)] - 28*
b*c^5*Cos[5*(d + e*x)] - 6*b^5*c*Cos[7*(d + e*x)] + 20*b^3*c^3*Cos[7*(d +
e*x)] - 6*b*c^5*Cos[7*(d + e*x)] - 35*b^6*Sin[d + e*x] - 1295*b^4*c^2*Sin[
d + e*x] - 2485*b^2*c^4*Sin[d + e*x] - 1225*c^6*Sin[d + e*x] + 896*b^3*c^2
*Sqrt[b^2 + c^2]*Sin[2*(d + e*x)] + 896*b*c^4*Sqrt[b^2 + c^2]*Sin[2*(d + e
*x)] + 21*b^6*Sin[3*(d + e*x)] - 189*b^4*c^2*Sin[3*(d + e*x)] - 161*b^2*c^
4*Sin[3*(d + e*x)] + 49*c^6*Sin[3*(d + e*x)] - 7*b^6*Sin[5*(d + e*x)] + 35
*b^4*c^2*Sin[5*(d + e*x)] + 35*b^2*c^4*Sin[5*(d + e*x)] - 7*c^6*Sin[5*(d +
 e*x)] + b^6*Sin[7*(d + e*x)] - 15*b^4*c^2*Sin[7*(d + e*x)] + 15*b^2*c^4*S
in[7*(d + e*x)] - c^6*Sin[7*(d + e*x)])/(1120*c*(b^2 + c^2)*e*(-(c*Cos[d +
 e*x]) + b*Sin[d + e*x])^7)
```

3.362.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3042, 3595, 3042, 3595, 3042, 3595, 3042, 3593}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^4}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^4}dx$$

$$\Big\downarrow \; 3595$$

$$\frac{3\int \frac{1}{\left(b\cos(d+ex)+c\sin(d+ex)+\sqrt{b^2+c^2}\right)^3}dx}{7\sqrt{b^2+c^2}}-\frac{c\cos(d+ex)-b\sin(d+ex)}{7e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^4}$$

$$\Big\downarrow \; 3042$$

$$\frac{3\int \frac{1}{\left(b\cos(d+ex)+c\sin(d+ex)+\sqrt{b^2+c^2}\right)^3}dx}{7\sqrt{b^2+c^2}}-\frac{c\cos(d+ex)-b\sin(d+ex)}{7e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^4}$$

$$\Big\downarrow \; 3595$$

$$\frac{3\left(\frac{2\int \frac{1}{\left(b\cos(d+ex)+c\sin(d+ex)+\sqrt{b^2+c^2}\right)^2}dx}{5\sqrt{b^2+c^2}}-\frac{c\cos(d+ex)-b\sin(d+ex)}{5e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3}\right)}{7\sqrt{b^2+c^2}}-\frac{c\cos(d+ex)-b\sin(d+ex)}{7e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^4}$$

$$\Big\downarrow \; 3042$$

$$\frac{3\left(\frac{2\int \frac{1}{\left(b\cos(d+ex)+c\sin(d+ex)+\sqrt{b^2+c^2}\right)^2}dx}{5\sqrt{b^2+c^2}}-\frac{c\cos(d+ex)-b\sin(d+ex)}{5e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3}\right)}{7\sqrt{b^2+c^2}}-\frac{c\cos(d+ex)-b\sin(d+ex)}{7e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^4}$$

$$\Big\downarrow \; 3595$$

$$\frac{3\left(\frac{2\left(\frac{\int \frac{1}{b\cos(d+ex)+c\sin(d+ex)+\sqrt{b^2+c^2}}dx}{3\sqrt{b^2+c^2}}-\frac{c\cos(d+ex)-b\sin(d+ex)}{3e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2}\right)}{5\sqrt{b^2+c^2}}-\frac{c\cos(d+ex)-b\sin(d+ex)}{5e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3}\right)}{7\sqrt{b^2+c^2}}-\frac{c\cos(d+ex)-b\sin(d+ex)}{7e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^4}$$

$$\Big\downarrow \; 3042$$

3.362. $\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^4}\,dx$

$$\frac{3\left(\frac{2\left(\frac{\int \frac{1}{b\cos(d+ex)+c\sin(d+ex)+\sqrt{b^2+c^2}}dx}{3\sqrt{b^2+c^2}}-\frac{c\cos(d+ex)-b\sin(d+ex)}{3e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2}\right)}{5\sqrt{b^2+c^2}}-\frac{c\cos(d+ex)-b\sin(d+ex)}{5e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3}\right)}{7\sqrt{b^2+c^2}}$$

$$\frac{c\cos(d+ex)-b\sin(d+ex)}{7e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^4}$$

$$\Big\downarrow \; 3593$$

$$\frac{3\left(\frac{2\left(-\frac{c\cos(d+ex)-b\sin(d+ex)}{3e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^2}-\frac{c-\sqrt{b^2+c^2}\sin(d+ex)}{3ce\sqrt{b^2+c^2}(c\cos(d+ex)-b\sin(d+ex))}\right)}{5\sqrt{b^2+c^2}}-\frac{c\cos(d+ex)-b\sin(d+ex)}{5e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^3}\right)}{7\sqrt{b^2+c^2}}$$

$$\frac{c\cos(d+ex)-b\sin(d+ex)}{7e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^4}$$

input
```
Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-4),x]
```

output
```
-1/7*(c*Cos[d + e*x] - b*Sin[d + e*x])/(Sqrt[b^2 + c^2]*e*(Sqrt[b^2 + c^2]
 + b*Cos[d + e*x] + c*Sin[d + e*x])^4) + (3*(-1/5*(c*Cos[d + e*x] - b*Sin[
d + e*x])/(Sqrt[b^2 + c^2]*e*(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d +
 e*x])^3) + (2*(-1/3*(c*Cos[d + e*x] - b*Sin[d + e*x])/(Sqrt[b^2 + c^2]*e*
(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^2) - (c - Sqrt[b^2 + c
^2]*Sin[d + e*x])/(3*c*Sqrt[b^2 + c^2]*e*(c*Cos[d + e*x] - b*Sin[d + e*x])
)))/(5*Sqrt[b^2 + c^2])))/(7*Sqrt[b^2 + c^2])
```

3.362.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3593
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Simp[-(c - a*Sin[d + e*x])/(c*e*(c*Cos[d + e*x] - b*Sin[
d + e*x])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

3.362. $\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^4}\,dx$

rule 3595
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(c*Cos[d + e*x] - b*Sin[d + e*x])*((a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n/(a*e*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n + 1))
 Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b,
c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

3.362.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.78 (sec) , antiderivative size = 511, normalized size of antiderivative = 1.97

method	result
risch	$\frac{4\left(3i\sqrt{b^2+c^2}\,b^4c+14ibc^5e^{i(ex+d)}+28ib^3c^3e^{i(ex+d)}+35b^6e^{3i(ex+d)}+105b^4c^2e^{3i(ex+d)}+105b^2c^4e^{3i(ex+d)}+35c^6e^{3i(ex+d}\right.}{}$
derivativedivides	$-\frac{2\left(\frac{\left(8\sqrt{b^2+c^2}\,b^3+4\sqrt{b^2+c^2}\,bc^2+8b^4+8b^2c^2+c^4\right)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^6}{c^2}+\frac{3\left(16\sqrt{b^2+c^2}\,b^4+12\sqrt{b^2+c^2}\,b^2c^2+\sqrt{b^2+c^2}\,c^4+16b^5+20b^3c^2-\right.}{c^3}\right.}{}$
default	$-\frac{2\left(\frac{\left(8\sqrt{b^2+c^2}\,b^3+4\sqrt{b^2+c^2}\,bc^2+8b^4+8b^2c^2+c^4\right)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^6}{c^2}+\frac{3\left(16\sqrt{b^2+c^2}\,b^4+12\sqrt{b^2+c^2}\,b^2c^2+\sqrt{b^2+c^2}\,c^4+16b^5+20b^3c^2-\right.}{c^3}\right.}{}$

input
```
int(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^4,x,method=_RETURNVERBOS
E)
```

output
```
4/35*(3*I*(b^2+c^2)^(1/2)*b^4*c+14*I*b*c^5*exp(I*(e*x+d))+28*I*b^3*c^3*exp
(I*(e*x+d))+35*b^6*exp(3*I*(e*x+d))+105*b^4*c^2*exp(3*I*(e*x+d))+105*b^2*c
^4*exp(3*I*(e*x+d))+35*c^6*exp(3*I*(e*x+d))-I*(b^2+c^2)^(1/2)*c^5+14*I*b^5
*c*exp(I*(e*x+d))+2*I*(b^2+c^2)^(1/2)*b^2*c^3+21*(b^2+c^2)^(1/2)*b^5*exp(2
*I*(e*x+d))+42*(b^2+c^2)^(1/2)*b^3*c^2*exp(2*I*(e*x+d))+21*(b^2+c^2)^(1/2)
*b*c^4*exp(2*I*(e*x+d))+42*I*(b^2+c^2)^(1/2)*b^2*c^3*exp(2*I*(e*x+d))+21*I
*(b^2+c^2)^(1/2)*c^5*exp(2*I*(e*x+d))+21*I*(b^2+c^2)^(1/2)*b^4*c*exp(2*I*(
e*x+d))+7*b^6*exp(I*(e*x+d))+7*b^4*c^2*exp(I*(e*x+d))-7*b^2*c^4*exp(I*(e*x
+d))-7*c^6*exp(I*(e*x+d))+(b^2+c^2)^(1/2)*b^5-2*(b^2+c^2)^(1/2)*b^3*c^2-3*
(b^2+c^2)^(1/2)*b*c^4)*(I*b^4-6*I*b^2*c^2+I*c^4-4*b^3*c+4*c^3*b)/(I*(b^2+c
^2)^(1/2)*c+b^2*exp(I*(e*x+d))+c^2*exp(I*(e*x+d))+(b^2+c^2)^(1/2)*b)^7/e
```

3.362.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 739 vs. 2(238) = 476.

Time = 0.48 (sec) , antiderivative size = 739, normalized size of antiderivative = 2.85

$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^4}\,dx$$
$$=\frac{2\,(b^7-21\,b^5c^2+35\,b^3c^4-7\,bc^6)\cos\left(ex+d\right)^7-7\,(b^7-15\,b^5c^2+15\,b^3c^4-bc^6)\cos\left(ex+d\right)^5-14\,(5\,b^5c^2}{35\,\left((7\,b^{10}c-21\,b^8c^3-42\,b^6c^5+6\,b^4c^7+19\,b^2c^9-c^{11})e\cos\left(ex+d\right)^7\right.}$$

input
```
integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^4,x, algorithm="fr
icas")
```

output
```
1/35*(2*(b^7 - 21*b^5*c^2 + 35*b^3*c^4 - 7*b*c^6)*cos(e*x + d)^7 - 7*(b^7
- 15*b^5*c^2 + 15*b^3*c^4 - b*c^6)*cos(e*x + d)^5 - 14*(5*b^5*c^2 - 5*b^3*
c^4 - 2*b*c^6)*cos(e*x + d)^3 - 7*(5*b^7 + 15*b^5*c^2 + 20*b^3*c^4 + 8*b*c
^6)*cos(e*x + d) - (35*b^6*c + 105*b^4*c^3 + 112*b^2*c^5 + 40*c^7 - 2*(7*b
^6*c - 35*b^4*c^3 + 21*b^2*c^5 - c^7)*cos(e*x + d)^6 + (35*b^6*c - 105*b^4
*c^3 + 21*b^2*c^5 + c^7)*cos(e*x + d)^4 + 2*(35*b^4*c^3 + 7*b^2*c^5 - 4*c^
7)*cos(e*x + d)^2)*sin(e*x + d) + 4*(3*b^6 + 16*b^4*c^2 + 23*b^2*c^4 + 10*
c^6 + 7*(b^6 + b^4*c^2 - b^2*c^4 - c^6)*cos(e*x + d)^2 + 14*(b^5*c + 2*b^3
*c^3 + b*c^5)*cos(e*x + d)*sin(e*x + d))*sqrt(b^2 + c^2))/((7*b^10*c - 21*
b^8*c^3 - 42*b^6*c^5 + 6*b^4*c^7 + 19*b^2*c^9 - c^11)*e*cos(e*x + d)^7 - 7
*(3*b^10*c - 4*b^8*c^3 - 14*b^6*c^5 - 4*b^4*c^7 + 3*b^2*c^9)*e*cos(e*x + d
)^5 + 7*(3*b^10*c + b^8*c^3 - 7*b^6*c^5 - 5*b^4*c^7)*e*cos(e*x + d)^3 - 7*
(b^10*c + 2*b^8*c^3 + b^6*c^5)*e*cos(e*x + d) - ((b^11 - 19*b^9*c^2 - 6*b^
7*c^4 + 42*b^5*c^6 + 21*b^3*c^8 - 7*b*c^10)*e*cos(e*x + d)^6 - (3*b^11 - 3
6*b^9*c^2 - 46*b^7*c^4 + 28*b^5*c^6 + 35*b^3*c^8)*e*cos(e*x + d)^4 + 3*(b^
11 - 5*b^9*c^2 - 13*b^7*c^4 - 7*b^5*c^6)*e*cos(e*x + d)^2 - (b^11 + 2*b^9*
c^2 + b^7*c^4)*e)*sin(e*x + d))
```

3.362.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^4}\,dx=\text{Timed out}$$

input
```
integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**4,x)
```

output
```
Timed out
```

3.362.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^4}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^4,x, algorithm="ma
xima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.362.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 593 vs. $2(238) = 476$.

Time = 0.68 (sec) , antiderivative size = 593, normalized size of antiderivative = 2.29

$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^4}\,dx =$$
$$-\frac{2\left(2560\,b^{10}+6528\,b^8c^2+5888\,b^6c^4+2248\,b^4c^6+340\,b^2c^8+12\,c^{10}+35\left(8\,b^4c^6+8\,b^2c^8+c^{10}+4\,(2\,b^3c\right.\right.}{}$$

input
```
integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^4,x, algorithm="gi
ac")
```

output
```
-2/35*(2560*b^10 + 6528*b^8*c^2 + 5888*b^6*c^4 + 2248*b^4*c^6 + 340*b^2*c^
8 + 12*c^10 + 35*(8*b^4*c^6 + 8*b^2*c^8 + c^10 + 4*(2*b^3*c^6 + b*c^8)*sqr
t(b^2 + c^2))*tan(1/2*e*x + 1/2*d)^6 + 105*(16*b^5*c^5 + 20*b^3*c^7 + 5*b*
c^9 + (16*b^4*c^5 + 12*b^2*c^7 + c^9)*sqrt(b^2 + c^2))*tan(1/2*e*x + 1/2*d
)^5 + 70*(80*b^6*c^4 + 124*b^4*c^6 + 49*b^2*c^8 + 3*c^10 + (80*b^5*c^4 + 8
4*b^3*c^6 + 17*b*c^8)*sqrt(b^2 + c^2))*tan(1/2*e*x + 1/2*d)^4 + 70*(160*b^
7*c^3 + 288*b^5*c^5 + 150*b^3*c^7 + 20*b*c^9 + (160*b^6*c^3 + 208*b^4*c^5
+ 66*b^2*c^7 + 3*c^9)*sqrt(b^2 + c^2))*tan(1/2*e*x + 1/2*d)^3 + 21*(640*b^
8*c^2 + 1312*b^6*c^4 + 856*b^4*c^6 + 186*b^2*c^8 + 7*c^10 + 2*(320*b^7*c^2
 + 496*b^5*c^4 + 220*b^3*c^6 + 25*b*c^8)*sqrt(b^2 + c^2))*tan(1/2*e*x + 1/
2*d)^2 + 7*(1280*b^9*c + 2944*b^7*c^3 + 2288*b^5*c^5 + 676*b^3*c^7 + 57*b*
c^9 + (1280*b^8*c + 2304*b^6*c^3 + 1296*b^4*c^5 + 236*b^2*c^7 + 7*c^9)*sqr
t(b^2 + c^2))*tan(1/2*e*x + 1/2*d) + 4*(640*b^9 + 1312*b^7*c^2 + 896*b^5*c
^4 + 238*b^3*c^6 + 21*b*c^8)*sqrt(b^2 + c^2))/((c*tan(1/2*e*x + 1/2*d) + b
 + sqrt(b^2 + c^2))^7*c^7*e)
```

3.362.9 Mupad [B] (verification not implemented)

Time = 40.36 (sec) , antiderivative size = 1004, normalized size of antiderivative = 3.88

$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^4}\,dx =$$

$$-\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^6\left(\frac{16\,b^4+16\,b^2\,c^2+2\,c^4}{c^8}+\frac{\left(16\,b^3+8\,b\,c^2\right)\sqrt{b^2+c^2}}{c^8}\right)+\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\left(\frac{512\,b^9+\frac{5888\,b^7\,c^2}{5}+\frac{4576\,b^5\,c^4}{5}+\frac{1352\,b^3\,c^6}{5}+\frac{114}{}}{c^{13}}\right.}{e\left(\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^3\left(\frac{280\,b^4}{}\right.\right.}$$

input
```
int(1/(b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^4,x)
```

output
```
-(tan(d/2 + (e*x)/2)^6*((16*b^4 + 2*c^4 + 16*b^2*c^2)/c^8 + ((8*b*c^2 + 16
*b^3)*(b^2 + c^2)^(1/2))/c^8) + tan(d/2 + (e*x)/2)*(((114*b*c^8)/5 + 512*b
^9 + (1352*b^3*c^6)/5 + (4576*b^5*c^4)/5 + (5888*b^7*c^2)/5)/c^13 + ((b^2
+ c^2)^(1/2)*(512*b^8 + (14*c^8)/5 + (472*b^2*c^6)/5 + (2592*b^4*c^4)/5 +
(4608*b^6*c^2)/5))/c^13) + ((1024*b^10)/7 + (24*c^10)/35 + (136*b^2*c^8)/7
 + (4496*b^4*c^6)/35 + (11776*b^6*c^4)/35 + (13056*b^8*c^2)/35)/c^14 + tan
(d/2 + (e*x)/2)^2*((768*b^8 + (42*c^8)/5 + (1116*b^2*c^6)/5 + (5136*b^4*c^
4)/5 + (7872*b^6*c^2)/5)/c^12 + ((b^2 + c^2)^(1/2)*(60*b*c^6 + 768*b^7 + 5
28*b^3*c^4 + (5952*b^5*c^2)/5))/c^12) + tan(d/2 + (e*x)/2)^3*((80*b*c^6 +
640*b^7 + 600*b^3*c^4 + 1152*b^5*c^2)/c^11 + ((b^2 + c^2)^(1/2)*(640*b^6 +
 12*c^6 + 264*b^2*c^4 + 832*b^4*c^2))/c^11) + tan(d/2 + (e*x)/2)^4*((320*b
^6 + 12*c^6 + 196*b^2*c^4 + 496*b^4*c^2)/c^10 + ((b^2 + c^2)^(1/2)*(68*b*c
^4 + 320*b^5 + 336*b^3*c^2))/c^10) + tan(d/2 + (e*x)/2)^5*((30*b*c^4 + 96*
b^5 + 120*b^3*c^2)/c^9 + ((b^2 + c^2)^(1/2)*(96*b^4 + 6*c^4 + 72*b^2*c^2))
/c^9) + ((b^2 + c^2)^(1/2)*((24*b*c^8)/5 + (1024*b^9)/7 + (272*b^3*c^6)/5
+ (1024*b^5*c^4)/5 + (10496*b^7*c^2)/35))/c^14)/(e*(tan(d/2 + (e*x)/2)^3*(
(280*b^4 + 35*c^4 + 280*b^2*c^2)/c^4 + ((140*b*c^2 + 280*b^3)*(b^2 + c^2)^
(1/2))/c^4) + tan(d/2 + (e*x)/2)^4*((105*b*c^2 + 140*b^3)/c^3 + ((140*b^2
+ 35*c^2)*(b^2 + c^2)^(1/2))/c^3) + tan(d/2 + (e*x)/2)^7 + tan(d/2 + (e*x)
/2)*((224*b^6 + 7*c^6 + 126*b^2*c^4 + 336*b^4*c^2)/c^6 + ((b^2 + c^2)^(...
```

3.362. $\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^4}\,dx$

3.363 $\int (2a+2a\cos(d+ex)+2c\sin(d+ex))^3\,dx$

3.363.1 Optimal result

Integrand size = 24, antiderivative size = 157

$$\begin{aligned}
&\int (2a+2a\cos(d+ex)+2c\sin(d+ex))^3\,dx \\
&= 4a(5a^2+3c^2)\,x-\frac{4c(15a^2+4c^2)\cos(d+ex)}{3e}+\frac{4a(15a^2+4c^2)\sin(d+ex)}{3e} \\
&\quad -\frac{20(ac\cos(d+ex)-a^2\sin(d+ex))\,(a+a\cos(d+ex)+c\sin(d+ex))}{3e} \\
&\quad -\frac{8(c\cos(d+ex)-a\sin(d+ex))(a+a\cos(d+ex)+c\sin(d+ex))^2}{3e}
\end{aligned}$$

output
```
4*a*(5*a^2+3*c^2)*x-4/3*c*(15*a^2+4*c^2)*cos(e*x+d)/e+4/3*a*(15*a^2+4*c^2)
*sin(e*x+d)/e-20/3*(a*c*cos(e*x+d)-a^2*sin(e*x+d))*(a+a*cos(e*x+d)+c*sin(e
*x+d))/e-8/3*(c*cos(e*x+d)-a*sin(e*x+d))*(a+a*cos(e*x+d)+c*sin(e*x+d))^2/e
```

3.363.2 Mathematica [A] (verified)

Time = 1.46 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.86

$$\begin{aligned}
&\int (2a+2a\cos(d+ex)+2c\sin(d+ex))^3\,dx \\
&= \frac{2(6a(5a^2+3c^2)\,(d+ex)-9c(5a^2+c^2)\cos(d+ex)-18a^2c\cos(2(d+ex))+c(-3a^2+c^2)\cos(3(d+ex)}{3e}
\end{aligned}$$

input
```
Integrate[(2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^3,x]
```

output
```
(2*(6*a*(5*a^2 + 3*c^2)*(d + e*x) - 9*c*(5*a^2 + c^2)*Cos[d + e*x] - 18*a^
2*c*Cos[2*(d + e*x)] + c*(-3*a^2 + c^2)*Cos[3*(d + e*x)] + 9*a*(5*a^2 + c^
2)*Sin[d + e*x] + 9*a*(a^2 - c^2)*Sin[2*(d + e*x)] + a*(a^2 - 3*c^2)*Sin[3
*(d + e*x)]))/(3*e)
```

3.363.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3599, 27, 3042, 3625, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2a\cos(d+ex)+2a+2c\sin(d+ex))^3\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (2a\cos(d+ex)+2a+2c\sin(d+ex))^3dx$$

$$\Big\downarrow \text{ 3599}$$

$$\frac{1}{3}\int 8(\cos(d+ex)a+a+c\sin(d+ex))\left(5\cos(d+ex)a^2+5a^2+5c\sin(d+ex)a+2c^2\right)dx-\frac{8(c\cos(d+ex)-a\sin(d+ex))(a\cos(d+ex)+a+c\sin(d+ex))^2}{3e}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{8}{3}\int (\cos(d+ex)a+a+c\sin(d+ex))\left(5\cos(d+ex)a^2+5a^2+5c\sin(d+ex)a+2c^2\right)dx-\frac{8(c\cos(d+ex)-a\sin(d+ex))(a\cos(d+ex)+a+c\sin(d+ex))^2}{3e}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{8}{3}\int (\cos(d+ex)a+a+c\sin(d+ex))\left(5\cos(d+ex)a^2+5a^2+5c\sin(d+ex)a+2c^2\right)dx-\frac{8(c\cos(d+ex)-a\sin(d+ex))(a\cos(d+ex)+a+c\sin(d+ex))^2}{3e}$$

$$\Big\downarrow \text{ 3625}$$

$$\frac{8}{3}\left(\frac{\int \left(3\left(5a^2+3c^2\right)a^2+\left(15a^2+4c^2\right)\cos(d+ex)a^2+c\left(15a^2+4c^2\right)\sin(d+ex)a\right)dx}{2a}-\frac{5\left(ac\cos(d+ex)-a^2\sin\right.}{}\right.$$
$$\frac{8(c\cos(d+ex)-a\sin(d+ex))(a\cos(d+ex)+a+c\sin(d+ex))^2}{3e}$$

$\Big\downarrow$ 2009

$$\frac{8}{3}\left(\frac{\frac{a^2(15a^2+4c^2)\sin(d+ex)}{e}-\frac{ac(15a^2+4c^2)\cos(d+ex)}{e}+3a^2x\left(5a^2+3c^2\right)}{2a}-\frac{5\left(ac\cos(d+ex)-a^2\sin(d+ex)\right)\left(a\cos(d+\right.}{2e}\right.$$
$$\frac{8(c\cos(d+ex)-a\sin(d+ex))(a\cos(d+ex)+a+c\sin(d+ex))^2}{3e}$$

input
```
Int[(2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^3,x]
```

output
```
(-8*(c*Cos[d + e*x] - a*Sin[d + e*x])*(a + a*Cos[d + e*x] + c*Sin[d + e*x]
)^2)/(3*e) + (8*((-5*(a*c*Cos[d + e*x] - a^2*Sin[d + e*x])*(a + a*Cos[d +
e*x] + c*Sin[d + e*x]))/(2*e) + (3*a^2*(5*a^2 + 3*c^2)*x - (a*c*(15*a^2 +
4*c^2)*Cos[d + e*x])/e + (a^2*(15*a^2 + 4*c^2)*Sin[d + e*x])/e)/(2*a)))/3
```

3.363.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3599
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Simp[1/n   Int[Simp[n*a^2 + (
n - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x
], x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

rule 3625
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*((a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^n/(a*e*(n + 1))), x] + Simp[1/(a*(n + 1
))   Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
 + a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
 + (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

3.363.4 Maple [A] (verified)

Time = 2.04 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.97

method	result
parallelrisch	$\frac{\left(-6a^2c+2c^3\right)\cos(3ex+3d)+\left(18a^3-18a\,c^2\right)\sin(2ex+2d)+\left(2a^3-6a\,c^2\right)\sin(3ex+3d)-36a^2c\cos(2ex+2d)+\left(-90a^2c-18\right.}{3e}$
parts	$\frac{-8a^2c\cos(ex+d)^3+24a^3\left(\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)}{e}+\frac{8a(a+c\sin(ex+d))^3}{ec}+8a^3x+\frac{8a^3\left(2+\cos(ex+d)^2\right)\sin}{3e}$
derivativedivides	$\frac{\frac{8a^3\left(2+\cos(ex+d)^2\right)\sin(ex+d)}{3}-8a^2c\cos(ex+d)^3+24a^3\left(\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)+8a\,c^2\sin(ex+d)^3-24a^2c\cos(ex+d}{}$
default	$\frac{\frac{8a^3\left(2+\cos(ex+d)^2\right)\sin(ex+d)}{3}-8a^2c\cos(ex+d)^3+24a^3\left(\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)+8a\,c^2\sin(ex+d)^3-24a^2c\cos(ex+d}{}$
risch	$20a^3x+12a\,c^2x-\frac{30c\cos(ex+d)a^2}{e}-\frac{6c^3\cos(ex+d)}{e}+\frac{30a^3\sin(ex+d)}{e}+\frac{6a\sin(ex+d)c^2}{e}-\frac{2c\cos(3ex+3}{e}$
norman	$\frac{-\frac{192a^2c+32c^3}{3e}+4a\left(5a^2+3c^2\right)x-\frac{32c^3\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}{e}+\frac{64a\left(5a^2+3c^2\right)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^3}{3e}+\frac{8a\left(5a^2+3c^2\right)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^5}{e}+12a\left(5a^2+\right.}{\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right.\right.}$

input
```
int((2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*((-6*a^2*c+2*c^3)*cos(3*e*x+3*d)+(18*a^3-18*a*c^2)*sin(2*e*x+2*d)+(2*a
^3-6*a*c^2)*sin(3*e*x+3*d)-36*a^2*c*cos(2*e*x+2*d)+(-90*a^2*c-18*c^3)*cos(
e*x+d)+(90*a^3+18*a*c^2)*sin(e*x+d)+60*a^3*e*x+36*a*c^2*e*x-60*a^2*c-16*c^
3)/e
```

3.363.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.85

$$\int (2a + 2a\cos(d+ex) + 2c\sin(d+ex))^3\,dx =$$
$$-\frac{4\left(18\,a^2c\cos\left(ex+d\right)^2 + 2\left(3\,a^2c - c^3\right)\cos\left(ex+d\right)^3 - 3\left(5\,a^3 + 3\,ac^2\right)ex + 6\left(3\,a^2c + c^3\right)\cos\left(ex+d\right) - \right.}{3\,e}$$

input
```
integrate((2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="fricas")
```

output
```
-4/3*(18*a^2*c*cos(e*x + d)^2 + 2*(3*a^2*c - c^3)*cos(e*x + d)^3 - 3*(5*a^
3 + 3*a*c^2)*e*x + 6*(3*a^2*c + c^3)*cos(e*x + d) - (22*a^3 + 6*a*c^2 + 2*
(a^3 - 3*a*c^2)*cos(e*x + d)^2 + 9*(a^3 - a*c^2)*cos(e*x + d))*sin(e*x + d
))/e
```

3.363.6 Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.85

$$\int (2a + 2a\cos(d+ex) + 2c\sin(d+ex))^3\,dx$$
$$= \begin{cases} 12a^3x\sin^2\left(d+ex\right) + 12a^3x\cos^2\left(d+ex\right) + 8a^3x + \frac{16a^3\sin^3\left(d+ex\right)}{3e} + \frac{8a^3\sin\left(d+ex\right)\cos^2\left(d+ex\right)}{e} + \frac{12a^3\sin\left(d+ex\right)\cdots}{e} \\ x(2a\cos\left(d\right) + 2a + 2c\sin\left(d\right))^3 \end{cases}$$

input
```
integrate((2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))**3,x)
```

output
```
Piecewise((12*a**3*x*sin(d + e*x)**2 + 12*a**3*x*cos(d + e*x)**2 + 8*a**3*
x + 16*a**3*sin(d + e*x)**3/(3*e) + 8*a**3*sin(d + e*x)*cos(d + e*x)**2/e
+ 12*a**3*sin(d + e*x)*cos(d + e*x)/e + 24*a**3*sin(d + e*x)/e + 24*a**2*c
*sin(d + e*x)**2/e - 8*a**2*c*cos(d + e*x)**3/e - 24*a**2*c*cos(d + e*x)/e
 + 12*a*c**2*x*sin(d + e*x)**2 + 12*a*c**2*x*cos(d + e*x)**2 + 8*a*c**2*si
n(d + e*x)**3/e - 12*a*c**2*sin(d + e*x)*cos(d + e*x)/e - 8*c**3*sin(d + e
*x)**2*cos(d + e*x)/e - 16*c**3*cos(d + e*x)**3/(3*e), Ne(e, 0)), (x*(2*a*
cos(d) + 2*a + 2*c*sin(d))**3, True))
```

3.363.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.22

$$\int (2a+2a\cos(d+ex)+2c\sin(d+ex))^3\,dx$$
$$= -\frac{8\,a^2c\cos{(ex+d)}^3}{e} + \frac{8\,ac^2\sin{(ex+d)}^3}{e} + 8\,a^3x - \frac{8\left(\sin{(ex+d)}^3 - 3\,\sin{(ex+d)}\right)a^3}{3\,e} + \frac{8\left(\cos{(ex+d)}^3 - 3\,\cos{(ex+d)}\right)c^3}{3\,e} - 24\,a^2\left(\frac{c\cos{(ex+d)}}{e} - \frac{a\sin{(ex+d)}}{e}\right) - 6\left(\frac{4\,ac\cos{(ex+d)}^2}{e} - \frac{(2\,ex+2\,d+\sin{(2\,ex+2\,d)})a^2}{e} - \frac{(2\,ex+2\,d-\sin{(2\,ex+2\,d)})c^2}{e}\right)a$$

input
```
integrate((2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="maxima")
```

output
```
-8*a^2*c*cos(e*x + d)^3/e + 8*a*c^2*sin(e*x + d)^3/e + 8*a^3*x - 8/3*(sin(
e*x + d)^3 - 3*sin(e*x + d))*a^3/e + 8/3*(cos(e*x + d)^3 - 3*cos(e*x + d))
*c^3/e - 24*a^2*(c*cos(e*x + d)/e - a*sin(e*x + d)/e) - 6*(4*a*c*cos(e*x +
 d)^2/e - (2*e*x + 2*d + sin(2*e*x + 2*d))*a^2/e - (2*e*x + 2*d - sin(2*e*
x + 2*d))*c^2/e)*a
```

3.363.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.96

$$\int (2a+2a\cos(d+ex)+2c\sin(d+ex))^3\,dx$$
$$= -\frac{12\,a^2c\cos{(2\,ex+2\,d)}}{e} + 4\left(5\,a^3+3\,ac^2\right)x - \frac{2\left(3\,a^2c-c^3\right)\cos{(3\,ex+3\,d)}}{3\,e} - \frac{6\left(5\,a^2c+c^3\right)\cos{(ex+d)}}{e} + \frac{2\left(a^3-3\,ac^2\right)\sin{(3\,ex+3\,d)}}{3\,e} + \frac{6\left(a^3-ac^2\right)\sin{(2\,ex+2\,d)}}{e} + \frac{6\left(5\,a^3+ac^2\right)\sin{(ex+d)}}{e}$$

input
```
integrate((2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="giac")
```

output
```
-12*a^2*c*cos(2*e*x + 2*d)/e + 4*(5*a^3 + 3*a*c^2)*x - 2/3*(3*a^2*c - c^3)
*cos(3*e*x + 3*d)/e - 6*(5*a^2*c + c^3)*cos(e*x + d)/e + 2/3*(a^3 - 3*a*c^
2)*sin(3*e*x + 3*d)/e + 6*(a^3 - a*c^2)*sin(2*e*x + 2*d)/e + 6*(5*a^3 + a*
c^2)*sin(e*x + d)/e
```

3.363.9 Mupad [B] (verification not implemented)

Time = 27.66 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.52

$$\int (2a+2a\cos(d+ex)+2c\sin(d+ex))^3\,dx = 20\,a^3\,x - \frac{32\,c^3\cos\left(\frac{d}{2}+\frac{e\,x}{2}\right)^4}{e} + \frac{64\,c^3\cos\left(\frac{d}{2}+\frac{e\,x}{2}\right)^6}{3\,e} + 12\,a\,c^2\,x - \frac{64\,a^2\,c\cos\left(\frac{d}{2}+\frac{e\,x}{2}\right)^6}{e} + \frac{40\,a^3\,\cos\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\sin\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{e} + \frac{80\,a^3\cos\left(\frac{d}{2}+\frac{e\,x}{2}\right)^3\,\sin\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{3\,e} + \frac{64\,a^3\cos\left(\frac{d}{2}+\frac{e\,x}{2}\right)^5\,\sin\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{3\,e} + \frac{16\,a\,c^2\cos\left(\frac{d}{2}+\frac{e\,x}{2}\right)^3\,\sin\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{e} - \frac{64\,a\,c^2\cos\left(\frac{d}{2}+\frac{e\,x}{2}\right)^5\,\sin\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{e} + \frac{24\,a\,c^2\,\cos\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\sin\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{e}$$

input
```
int((2*a + 2*a*cos(d + e*x) + 2*c*sin(d + e*x))^3,x)
```

output
```
20*a^3*x - (32*c^3*cos(d/2 + (e*x)/2)^4)/e + (64*c^3*cos(d/2 + (e*x)/2)^6)
/(3*e) + 12*a*c^2*x - (64*a^2*c*cos(d/2 + (e*x)/2)^6)/e + (40*a^3*cos(d/2
+ (e*x)/2)*sin(d/2 + (e*x)/2))/e + (80*a^3*cos(d/2 + (e*x)/2)^3*sin(d/2 +
(e*x)/2))/(3*e) + (64*a^3*cos(d/2 + (e*x)/2)^5*sin(d/2 + (e*x)/2))/(3*e) +
 (16*a*c^2*cos(d/2 + (e*x)/2)^3*sin(d/2 + (e*x)/2))/e - (64*a*c^2*cos(d/2
+ (e*x)/2)^5*sin(d/2 + (e*x)/2))/e + (24*a*c^2*cos(d/2 + (e*x)/2)*sin(d/2
+ (e*x)/2))/e
```

3.364 $\int (2a+2a\cos(d+ex)+2c\sin(d+ex))^2\,dx$

3.364.1 Optimal result

Integrand size = 24, antiderivative size = 81

$$\int (2a+2a\cos(d+ex)+2c\sin(d+ex))^2\,dx = 2(3a^2+c^2)\,x-\frac{6ac\cos(d+ex)}{e}+\frac{6a^2\sin(d+ex)}{e}-\frac{2(c\cos(d+ex)-a\sin(d+ex))(a+a\cos(d+ex)+c\sin(d+ex))}{e}$$

output
```
2*(3*a^2+c^2)*x-6*a*c*cos(e*x+d)/e+6*a^2*sin(e*x+d)/e-2*(c*cos(e*x+d)-a*si
n(e*x+d))*(a+a*cos(e*x+d)+c*sin(e*x+d))/e
```

3.364.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.14

$$\int (2a+2a\cos(d+ex)+2c\sin(d+ex))^2\,dx = 4\left(\frac{(3a^2+c^2)(d+ex)}{2e}-\frac{2ac\cos(d+ex)}{e}-\frac{ac\cos(2(d+ex))}{2e}+\frac{2a^2\sin(d+ex)}{e}+\frac{(a^2-c^2)\sin(2(d+ex))}{4e}\right)$$

input
```
Integrate[(2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^2,x]
```

output
```
4*(((3*a^2 + c^2)*(d + e*x))/(2*e) - (2*a*c*Cos[d + e*x])/e - (a*c*Cos[2*(
d + e*x)])/(2*e) + (2*a^2*Sin[d + e*x])/e + ((a^2 - c^2)*Sin[2*(d + e*x)])
/(4*e))
```

3.364.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3042, 3599, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2a\cos(d+ex)+2a+2c\sin(d+ex))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (2a\cos(d+ex)+2a+2c\sin(d+ex))^2dx$$

$$\Big\downarrow \text{3599}$$

$$\frac{1}{2}\int \left(12\cos(d+ex)a^2+12c\sin(d+ex)a+4\left(3a^2+c^2\right)\right)dx-\frac{2(c\cos(d+ex)-a\sin(d+ex))(a\cos(d+ex)+a+c\sin(d+ex))}{e}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(4x\left(3a^2+c^2\right)+\frac{12a^2\sin(d+ex)}{e}-\frac{12ac\cos(d+ex)}{e}\right)-\frac{2(c\cos(d+ex)-a\sin(d+ex))(a\cos(d+ex)+a+c\sin(d+ex))}{e}$$

input
```
Int[(2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^2,x]
```

output
```
(-2*(c*Cos[d + e*x] - a*Sin[d + e*x])*(a + a*Cos[d + e*x] + c*Sin[d + e*x]
))/e + (4*(3*a^2 + c^2)*x - (12*a*c*Cos[d + e*x])/e + (12*a^2*Sin[d + e*x]
)/e)/2
```

3.364.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3599
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Simp[1/n   Int[Simp[n*a^2 + (
n - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x
], x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

3.364.4 Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

method	result
parallelrisch	$\frac{(a^2-c^2)\sin(2ex+2d)+6a^2ex+2c^2ex+8a^2\sin(ex+d)-8ac\cos(ex+d)-2ac\cos(2ex+2d)-6ac}{e}$
risch	$6a^2x+2x\,c^2-\frac{8ac\cos(ex+d)}{e}+\frac{8a^2\sin(ex+d)}{e}-\frac{2ac\cos(2ex+2d)}{e}+\frac{\sin(2ex+2d)a^2}{e}-\frac{\sin(2ex+2d)c^2}{e}$
derivativedivides	$\frac{4a^2\left(\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)-4ac\cos(ex+d)^2+8a^2\sin(ex+d)+4c^2\left(-\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)-8ac\cos(ex+d}{e}$
default	$\frac{4a^2\left(\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)-4ac\cos(ex+d)^2+8a^2\sin(ex+d)+4c^2\left(-\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)-8ac\cos(ex+d}{e}$
parts	$\frac{8a\left(\frac{\sin(ex+d)^2c}{2}+a\sin(ex+d)\right)}{e}+4a^2x+\frac{4a^2\left(\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)}{e}+\frac{4c^2\left(-\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right.}{e}$
norman	$\frac{(6a^2+2c^2)x+(6a^2+2c^2)x\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^4+(12a^2+4c^2)x\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2-\frac{16ac}{e}+\frac{4(3a^2+c^2)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^3}{e}+\frac{4(5a^2-c^2)\tan(}{e}}{\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2\right)^2}$

input
```
int((2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x,method=_RETURNVERBOSE)
```

output
```
((a^2-c^2)*sin(2*e*x+2*d)+6*a^2*e*x+2*c^2*e*x+8*a^2*sin(e*x+d)-8*a*c*cos(e
*x+d)-2*a*c*cos(2*e*x+2*d)-6*a*c)/e
```

3.364.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.88

$$\int (2a+2a\cos(d+ex)+2c\sin(d+ex))^2\,dx =$$
$$-\frac{2\left(2\,ac\cos\left(ex+d\right)^2-(3\,a^2+c^2)ex+4\,ac\cos\left(ex+d\right)-\left(4\,a^2+\left(a^2-c^2\right)\cos\left(ex+d\right)\right)\sin\left(ex+d\right)\right)}{e}$$

input
```
integrate((2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="fricas")
```

output
```
-2*(2*a*c*cos(e*x + d)^2 - (3*a^2 + c^2)*e*x + 4*a*c*cos(e*x + d) - (4*a^2
 + (a^2 - c^2)*cos(e*x + d))*sin(e*x + d))/e
```

3.364.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(78) = 156.

Time = 0.10 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.10

$$\int (2a+2a\cos(d+ex)+2c\sin(d+ex))^2\,dx$$
$$=\begin{cases} 2a^2x\sin^2\left(d+ex\right)+2a^2x\cos^2\left(d+ex\right)+4a^2x+\frac{2a^2\sin\left(d+ex\right)\cos\left(d+ex\right)}{e}+\frac{8a^2\sin\left(d+ex\right)}{e}+\frac{4ac\sin^2\left(d+ex\right)}{e}-\frac{8ac}{} \\ x(2a\cos\left(d\right)+2a+2c\sin\left(d\right))^2 \end{cases}$$

input
```
integrate((2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))**2,x)
```

output
```
Piecewise((2*a**2*x*sin(d + e*x)**2 + 2*a**2*x*cos(d + e*x)**2 + 4*a**2*x
+ 2*a**2*sin(d + e*x)*cos(d + e*x)/e + 8*a**2*sin(d + e*x)/e + 4*a*c*sin(d
 + e*x)**2/e - 8*a*c*cos(d + e*x)/e + 2*c**2*x*sin(d + e*x)**2 + 2*c**2*x*
cos(d + e*x)**2 - 2*c**2*sin(d + e*x)*cos(d + e*x)/e, Ne(e, 0)), (x*(2*a*c
os(d) + 2*a + 2*c*sin(d))**2, True))
```

3.364.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.22

$$\int (2a+2a\cos(d+ex)+2c\sin(d+ex))^2\,dx = 4\,a^2x - \frac{4\,ac\cos\left(ex+d\right)^2}{e} + \frac{(2\,ex+2\,d+\sin\left(2\,ex+2\,d\right))a^2}{e} + \frac{(2\,ex+2\,d-\sin\left(2\,ex+2\,d\right))c^2}{e} - 8\,a\left(\frac{c\cos\left(ex+d\right)}{e} - \frac{a\sin\left(ex+d\right)}{e}\right)$$

input
```
integrate((2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="maxima")
```

output
```
4*a^2*x - 4*a*c*cos(e*x + d)^2/e + (2*e*x + 2*d + sin(2*e*x + 2*d))*a^2/e
+ (2*e*x + 2*d - sin(2*e*x + 2*d))*c^2/e - 8*a*(c*cos(e*x + d)/e - a*sin(e
*x + d)/e)
```

3.364.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.96

$$\int (2a+2a\cos(d+ex)+2c\sin(d+ex))^2\,dx = 2\left(3\,a^2+c^2\right)x - \frac{2\,ac\cos\left(2\,ex+2\,d\right)}{e} - \frac{8\,ac\cos\left(ex+d\right)}{e} + \frac{8\,a^2\sin\left(ex+d\right)}{e} + \frac{(a^2-c^2)\sin\left(2\,ex+2\,d\right)}{e}$$

input
```
integrate((2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="giac")
```

output
```
2*(3*a^2 + c^2)*x - 2*a*c*cos(2*e*x + 2*d)/e - 8*a*c*cos(e*x + d)/e + 8*a^
2*sin(e*x + d)/e + (a^2 - c^2)*sin(2*e*x + 2*d)/e
```

3.364.9 Mupad [B] (verification not implemented)

Time = 27.30 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.19

$$\int (2a+2a\cos(d+ex)+2c\sin(d+ex))^2\,dx$$
$$=\frac{x\,(12\,a^2+4\,c^2)}{2}+\frac{(12\,a^2+4\,c^2)\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^3+(20\,a^2-4\,c^2)\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)-16\,a\,c}{e\,\left(\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^4+2\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2+1\right)}$$

input
```
int((2*a + 2*a*cos(d + e*x) + 2*c*sin(d + e*x))^2,x)
```

output
```
(x*(12*a^2 + 4*c^2))/2 + (tan(d/2 + (e*x)/2)^3*(12*a^2 + 4*c^2) - 16*a*c +
 tan(d/2 + (e*x)/2)*(20*a^2 - 4*c^2))/(e*(2*tan(d/2 + (e*x)/2)^2 + tan(d/2
 + (e*x)/2)^4 + 1))
```

3.365 $\int (2a+2a\cos(d+ex)+2c\sin(d+ex))\,dx$

3.365.1 Optimal result

Integrand size = 22, antiderivative size = 29

$$\int (2a+2a\cos(d+ex)+2c\sin(d+ex))\,dx=2ax-\frac{2c\cos(d+ex)}{e}+\frac{2a\sin(d+ex)}{e}$$

output
```
2*a*x-2*c*cos(e*x+d)/e+2*a*sin(e*x+d)/e
```

3.365.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.83

$$\int (2a+2a\cos(d+ex)+2c\sin(d+ex))\,dx=2ax-\frac{2c\cos(d)\cos(ex)}{e}+\frac{2a\cos(ex)\sin(d)}{e}+\frac{2a\cos(d)\sin(ex)}{e}+\frac{2c\sin(d)\sin(ex)}{e}$$

input
```
Integrate[2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x],x]
```

output
```
2*a*x - (2*c*Cos[d]*Cos[e*x])/e + (2*a*Cos[e*x]*Sin[d])/e + (2*a*Cos[d]*Si
n[e*x])/e + (2*c*Sin[d]*Sin[e*x])/e
```

3.365.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2a\cos(d+ex)+2a+2c\sin(d+ex))\,dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a\sin(d+ex)}{e}+2ax-\frac{2c\cos(d+ex)}{e}$$

input
```
Int[2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x],x]
```

output
```
2*a*x - (2*c*Cos[d + e*x])/e + (2*a*Sin[d + e*x])/e
```

3.365.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.365.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

method	result	size
default	$2ax-\frac{2c\cos(ex+d)}{e}+\frac{2a\sin(ex+d)}{e}$	30
risch	$2ax-\frac{2c\cos(ex+d)}{e}+\frac{2a\sin(ex+d)}{e}$	30
parts	$2ax-\frac{2c\cos(ex+d)}{e}+\frac{2a\sin(ex+d)}{e}$	30
derivativedivides	$\frac{2(ex+d)a+2a\sin(ex+d)-2c\cos(ex+d)}{e}$	31
parallelrisch	$\frac{2a\sin(ex+d)-2c\cos(ex+d)+2c}{e}+2ax$	32
norman	$\frac{\frac{4c\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}}{e}+2ax+\frac{4a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{e}+2ax\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}}{1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}}$	69

input
```
int(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
2*a*x-2*c*cos(e*x+d)/e+2*a*sin(e*x+d)/e
```

3.365.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int (2a + 2a\cos(d + ex) + 2c\sin(d + ex))\,dx = \frac{2\,(aex - c\cos{(ex + d)} + a\sin{(ex + d)})}{e}$$

input
```
integrate(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d),x, algorithm="fricas")
```

output
```
2*(a*e*x - c*cos(e*x + d) + a*sin(e*x + d))/e
```

3.365.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.34

$$\int (2a + 2a\cos(d + ex) + 2c\sin(d + ex))\,dx = 2ax + 2a\left(\begin{cases} \frac{\sin{(d+ex)}}{e} & \text{for}\ e \neq 0 \\ x\cos{(d)} & \text{otherwise} \end{cases}\right) + 2c\left(\begin{cases} -\frac{\cos{(d+ex)}}{e} & \text{for}\ e \neq 0 \\ x\sin{(d)} & \text{otherwise} \end{cases}\right)$$

input
```
integrate(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d),x)
```

output
```
2*a*x + 2*a*Piecewise((sin(d + e*x)/e, Ne(e, 0)), (x*cos(d), True)) + 2*c*
Piecewise((-cos(d + e*x)/e, Ne(e, 0)), (x*sin(d), True))
```

3.365.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int (2a + 2a\cos(d + ex) + 2c\sin(d + ex))\,dx = 2\,ax - \frac{2\,c\cos(ex + d)}{e} + \frac{2\,a\sin(ex + d)}{e}$$

input
```
integrate(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d),x, algorithm="maxima")
```

output
```
2*a*x - 2*c*cos(e*x + d)/e + 2*a*sin(e*x + d)/e
```

3.365.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int (2a + 2a\cos(d + ex) + 2c\sin(d + ex))\,dx = 2\,ax - \frac{2\,c\cos(ex + d)}{e} + \frac{2\,a\sin(ex + d)}{e}$$

input
```
integrate(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d),x, algorithm="giac")
```

output
```
2*a*x - 2*c*cos(e*x + d)/e + 2*a*sin(e*x + d)/e
```

3.365.9 Mupad [B] (verification not implemented)

Time = 27.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int (2a + 2a\cos(d + ex) + 2c\sin(d + ex))\,dx = 2\,a\,x - \frac{2\,c\,\cos(d + e\,x)}{e} + \frac{2\,a\,\sin(d + e\,x)}{e}$$

input
```
int(2*a + 2*a*cos(d + e*x) + 2*c*sin(d + e*x),x)
```

output
```
2*a*x - (2*c*cos(d + e*x))/e + (2*a*sin(d + e*x))/e
```

3.366 $\int \frac{1}{2a+2a\cos(d+ex)+2c\sin(d+ex)}\,dx$

3.366.1 Optimal result

Integrand size = 24, antiderivative size = 25

$$\int \frac{1}{2a+2a\cos(d+ex)+2c\sin(d+ex)}\,dx=\frac{\log\left(a+c\tan\left(\frac{1}{2}(d+ex)\right)\right)}{2ce}$$

output
```
1/2*ln(a+c*tan(1/2*e*x+1/2*d))/c/e
```

3.366.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 57 vs. 2(25) = 50.

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.28

$$\int \frac{1}{2a+2a\cos(d+ex)+2c\sin(d+ex)}\,dx=\frac{1}{2}\left(-\frac{\log\left(\cos\left(\frac{1}{2}(d+ex)\right)\right)}{ce}+\frac{\log\left(a\cos\left(\frac{1}{2}(d+ex)\right)+c\sin\left(\frac{1}{2}(d+ex)\right)\right)}{ce}\right)$$

input
```
Integrate[(2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-1),x]
```

output
```
(-(Log[Cos[(d + e*x)/2]]/(c*e)) + Log[a*Cos[(d + e*x)/2] + c*Sin[(d + e*x)
/2]]/(c*e))/2
```

3.366.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3042, 3603, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{2a\cos(d+ex)+2a+2c\sin(d+ex)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{2a\cos(d+ex)+2a+2c\sin(d+ex)}dx$$

$$\Big\downarrow \text{3603}$$

$$\frac{2\int \frac{1}{4a+4c\tan\left(\frac{1}{2}(d+ex)\right)}d\tan\left(\frac{1}{2}(d+ex)\right)}{e}$$

$$\Big\downarrow \text{16}$$

$$\frac{\log\left(a+c\tan\left(\frac{1}{2}(d+ex)\right)\right)}{2ce}$$

input
```
Int[(2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-1),x]
```

output
```
Log[a + c*Tan[(d + e*x)/2]]/(2*c*e)
```

3.366.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

3.366.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{\ln\left(a+c\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}{2ce}$	23
default	$\frac{\ln\left(a+c\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}{2ce}$	23
parallelrisch	$\frac{\ln\left(\sqrt{a+c\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}\right)}{ce}$	24
norman	$\frac{\ln\left(4c\tan\left(\frac{ex}{2}+\frac{d}{2}\right)+4a\right)}{2ce}$	26
risch	$-\frac{\ln\left(e^{i(ex+d)}+1\right)}{2ce}+\frac{\ln\left(e^{i(ex+d)}-\frac{ic+a}{ic-a}\right)}{2ce}$	59

input
```
int(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d)),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(a+c*tan(1/2*e*x+1/2*d))/c/e
```

3.366.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(22) = 44$.

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.40

$$\int \frac{1}{2a+2a\cos(d+ex)+2c\sin(d+ex)}\,dx = \frac{\log\left(ac\sin\left(ex+d\right)+\frac{1}{2}\,a^2+\frac{1}{2}\,c^2+\frac{1}{2}\,(a^2-c^2)\cos\left(ex+d\right)\right)-\log\left(\frac{1}{2}\,\cos\left(ex+d\right)+\frac{1}{2}\right)}{4\,ce}$$

input
```
integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d)),x, algorithm="fricas")
```

output
```
1/4*(log(a*c*sin(e*x + d) + 1/2*a^2 + 1/2*c^2 + 1/2*(a^2 - c^2)*cos(e*x +
d)) - log(1/2*cos(e*x + d) + 1/2))/(c*e)
```

3.366.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(19) = 38$.

Time = 0.63 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.52

$$\int \frac{1}{2a + 2a\cos(d+ex) + 2c\sin(d+ex)}\,dx = \begin{cases} \frac{x}{2a\cos(d)+2a} & \text{for } c = 0 \wedge e = 0 \\ \frac{\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{2ae} & \text{for } c = 0 \\ \frac{x}{2a\cos(d)+2a+2c\sin(d)} & \text{for } e = 0 \\ \frac{\log\left(\frac{a}{c}+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{2ce} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d)),x)
```

output
```
Piecewise((x/(2*a*cos(d) + 2*a), Eq(c, 0) & Eq(e, 0)), (tan(d/2 + e*x/2)/(
2*a*e), Eq(c, 0)), (x/(2*a*cos(d) + 2*a + 2*c*sin(d)), Eq(e, 0)), (log(a/c
 + tan(d/2 + e*x/2))/(2*c*e), True))
```

3.366.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \frac{1}{2a + 2a\cos(d+ex) + 2c\sin(d+ex)}\,dx = \frac{\log\left(a + \frac{c\sin(ex+d)}{\cos(ex+d)+1}\right)}{2\,ce}$$

input
```
integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d)),x, algorithm="maxima")
```

output
```
1/2*log(a + c*sin(e*x + d)/(cos(e*x + d) + 1))/(c*e)
```

3.366.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{1}{2a+2a\cos(d+ex)+2c\sin(d+ex)}\,dx = \frac{\log\left(\left|c\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+a\right|\right)}{2\,ce}$$

input
```
integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d)),x, algorithm="giac")
```

output
```
1/2*log(abs(c*tan(1/2*e*x + 1/2*d) + a))/(c*e)
```

3.366.9 Mupad [B] (verification not implemented)

Time = 26.76 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \frac{1}{2a+2a\cos(d+ex)+2c\sin(d+ex)}\,dx = \frac{\ln\left(a+c\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\right)}{2\,c\,e}$$

input
```
int(1/(2*a + 2*a*cos(d + e*x) + 2*c*sin(d + e*x)),x)
```

output
```
log(a + c*tan(d/2 + (e*x)/2))/(2*c*e)
```

3.367 $\int \frac{1}{(2a+2a\cos(d+ex)+2c\sin(d+ex))^2}\,dx$

3.367.1 Optimal result

Integrand size = 24, antiderivative size = 75

$$\int \frac{1}{(2a+2a\cos(d+ex)+2c\sin(d+ex))^2}\,dx = -\frac{a\log\left(a+c\tan\left(\frac{1}{2}(d+ex)\right)\right)}{4c^3e} - \frac{c\cos(d+ex)-a\sin(d+ex)}{4c^2e(a+a\cos(d+ex)+c\sin(d+ex))}$$

output
```
-1/4*a*ln(a+c*tan(1/2*e*x+1/2*d))/c^3/e+1/4*(-c*cos(e*x+d)+a*sin(e*x+d))/c
^2/e/(a+a*cos(e*x+d)+c*sin(e*x+d))
```

3.367.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.53

$$\int \frac{1}{(2a+2a\cos(d+ex)+2c\sin(d+ex))^2}\,dx = \frac{2a\left(\log\left(\cos\left(\frac{1}{2}(d+ex)\right)\right)-\log\left(a\cos\left(\frac{1}{2}(d+ex)\right)+c\sin\left(\frac{1}{2}(d+ex)\right)\right)\right)+\frac{c(a^2+c^2)\sin\left(\frac{1}{2}(d+ex)\right)}{a\left(a\cos\left(\frac{1}{2}(d+ex)\right)+c\sin\left(\frac{1}{2}(d+ex)\right)\right)}+c}{8c^3e}$$

input
```
Integrate[(2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-2),x]
```

output
```
(2*a*(Log[Cos[(d + e*x)/2]] - Log[a*Cos[(d + e*x)/2] + c*Sin[(d + e*x)/2]]
) + (c*(a^2 + c^2)*Sin[(d + e*x)/2])/(a*(a*Cos[(d + e*x)/2] + c*Sin[(d + e
*x)/2])) + c*Tan[(d + e*x)/2])/(8*c^3*e)
```

3.367.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {3042, 3608, 25, 27, 3042, 3603, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2a\cos(d+ex)+2a+2c\sin(d+ex))^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(2a\cos(d+ex)+2a+2c\sin(d+ex))^2}dx$$
$$\Big\downarrow \text{3608}$$
$$\frac{\int -\frac{a}{\cos(d+ex)a+a+c\sin(d+ex)}dx}{4c^2}-\frac{c\cos(d+ex)-a\sin(d+ex)}{4c^2e(a\cos(d+ex)+a+c\sin(d+ex))}$$
$$\Big\downarrow \text{25}$$
$$-\frac{\int \frac{a}{\cos(d+ex)a+a+c\sin(d+ex)}dx}{4c^2}-\frac{c\cos(d+ex)-a\sin(d+ex)}{4c^2e(a\cos(d+ex)+a+c\sin(d+ex))}$$
$$\Big\downarrow \text{27}$$
$$-\frac{a\int \frac{1}{\cos(d+ex)a+a+c\sin(d+ex)}dx}{4c^2}-\frac{c\cos(d+ex)-a\sin(d+ex)}{4c^2e(a\cos(d+ex)+a+c\sin(d+ex))}$$
$$\Big\downarrow \text{3042}$$
$$-\frac{a\int \frac{1}{\cos(d+ex)a+a+c\sin(d+ex)}dx}{4c^2}-\frac{c\cos(d+ex)-a\sin(d+ex)}{4c^2e(a\cos(d+ex)+a+c\sin(d+ex))}$$
$$\Big\downarrow \text{3603}$$
$$-\frac{a\int \frac{1}{2a+2c\tan\left(\frac{1}{2}(d+ex)\right)}d\tan\left(\frac{1}{2}(d+ex)\right)}{2c^2e}-\frac{c\cos(d+ex)-a\sin(d+ex)}{4c^2e(a\cos(d+ex)+a+c\sin(d+ex))}$$
$$\Big\downarrow \text{16}$$
$$-\frac{a\log\left(a+c\tan\left(\frac{1}{2}(d+ex)\right)\right)}{4c^3e}-\frac{c\cos(d+ex)-a\sin(d+ex)}{4c^2e(a\cos(d+ex)+a+c\sin(d+ex))}$$

input
```
Int[(2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-2),x]
```

output
```
-1/4*(a*Log[a + c*Tan[(d + e*x)/2]])/(c^3*e) - (c*Cos[d + e*x] - a*Sin[d +
 e*x])/(4*c^2*e*(a + a*Cos[d + e*x] + c*Sin[d + e*x]))
```

3.367.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

rule 3608
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Simp[
1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c
*(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x
] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] &&
NeQ[n, -3/2]
```

3.367.4 Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{2c^2}-\frac{a\ln\left(a+c\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}{c^3}-\frac{a^2+c^2}{2c^3\left(a+c\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}}{4e}$
default	$\frac{\frac{\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{2c^2}-\frac{a\ln\left(a+c\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}{c^3}-\frac{a^2+c^2}{2c^3\left(a+c\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}}{4e}$
norman	$\frac{-\frac{2a^2+c^2}{8c^3e}+\frac{\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}{8ce}}{a+c\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}-\frac{a\ln\left(a+c\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}{4c^3e}$
parallelrisch	$\frac{-2\ln\left(a+c\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)a^2c+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2ac^2-2\ln\left(a+c\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)a^3+2\tan\left(\frac{ex}{2}+\frac{d}{2}\right)a^2c+\tan\left(\frac{ex}{2}+\right.}{8ac^3e\left(a+c\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}$
risch	$-\frac{i\left(-ia\,e^{i(ex+d)}-ia+c\right)}{2c^2e\left(c\,e^{2i(ex+d)}+ia\,e^{2i(ex+d)}-c+2ia\,e^{i(ex+d)}+ia\right)}+\frac{a\ln\left(e^{i(ex+d)}+1\right)}{4c^3e}-\frac{a\ln\left(e^{i(ex+d)}-\frac{ic+a}{ic-a}\right)}{4c^3e}$

input
```
int(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4/e*(1/2*tan(1/2*e*x+1/2*d)/c^2-1/c^3*a*ln(a+c*tan(1/2*e*x+1/2*d))-1/2/c
^3*(a^2+c^2)/(a+c*tan(1/2*e*x+1/2*d)))
```

3.367.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(70) = 140$.

Time = 0.26 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.05

$$\int \frac{1}{(2a+2a\cos(d+ex)+2c\sin(d+ex))^2}\,dx =$$
$$-\frac{2\,c^2\cos(ex+d)-2\,ac\sin(ex+d)+(a^2\cos(ex+d)+ac\sin(ex+d)+a^2)\log\left(ac\sin(ex+d)+\frac{1}{2}\,a^2\right.}{8\,(ac^3e\cos(ex+d)+c^4}$$

input
```
integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="fricas")
```

output
```
-1/8*(2*c^2*cos(e*x + d) - 2*a*c*sin(e*x + d) + (a^2*cos(e*x + d) + a*c*si
n(e*x + d) + a^2)*log(a*c*sin(e*x + d) + 1/2*a^2 + 1/2*c^2 + 1/2*(a^2 - c^
2)*cos(e*x + d)) - (a^2*cos(e*x + d) + a*c*sin(e*x + d) + a^2)*log(1/2*cos
(e*x + d) + 1/2))/(a*c^3*e*cos(e*x + d) + c^4*e*sin(e*x + d) + a*c^3*e)
```

3.367.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(2a+2a\cos(d+ex)+2c\sin(d+ex))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))**2,x)
```

output
```
Timed out
```

3.367.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.20

$$\int \frac{1}{(2a+2a\cos(d+ex)+2c\sin(d+ex))^2}\,dx = -\frac{\frac{a^2+c^2}{ac^3+\frac{c^4\sin(ex+d)}{\cos(ex+d)+1}} + \frac{2\,a\log\left(a+\frac{c\sin(ex+d)}{\cos(ex+d)+1}\right)}{c^3} - \frac{\sin(ex+d)}{c^2(\cos(ex+d)+1)}}{8\,e}$$

input
```
integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="maxima")
```

output
```
-1/8*((a^2 + c^2)/(a*c^3 + c^4*sin(e*x + d)/(cos(e*x + d) + 1)) + 2*a*log(
a + c*sin(e*x + d)/(cos(e*x + d) + 1))/c^3 - sin(e*x + d)/(c^2*(cos(e*x +
d) + 1)))/e
```

3.367.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.11

$$\int \frac{1}{(2a+2a\cos(d+ex)+2c\sin(d+ex))^2}\,dx = -\frac{\frac{2\,a\log\left(\left|c\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+a\right|\right)}{c^3} - \frac{\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)}{c^2} - \frac{2\,ac\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+a^2-c^2}{\left(c\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+a\right)c^3}}{8\,e}$$

input
```
integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="giac")
```

output
```
-1/8*(2*a*log(abs(c*tan(1/2*e*x + 1/2*d) + a))/c^3 - tan(1/2*e*x + 1/2*d)/
c^2 - (2*a*c*tan(1/2*e*x + 1/2*d) + a^2 - c^2)/((c*tan(1/2*e*x + 1/2*d) +
a)*c^3))/e
```

3.367.9 Mupad [B] (verification not implemented)

Time = 26.77 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.05

$$\int \frac{1}{(2a+2a\cos(d+ex)+2c\sin(d+ex))^2}\,dx = \frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{8\,c^2\,e}-\frac{a\,\ln\left(a+c\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\right)}{4\,c^3\,e}-\frac{a^2+c^2}{c\,e\,\left(8\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,c^3+8\,a\,c^2\right)}$$

input
```
int(1/(2*a + 2*a*cos(d + e*x) + 2*c*sin(d + e*x))^2,x)
```

output
```
tan(d/2 + (e*x)/2)/(8*c^2*e) - (a*log(a + c*tan(d/2 + (e*x)/2)))/(4*c^3*e)
 - (a^2 + c^2)/(c*e*(8*a*c^2 + 8*c^3*tan(d/2 + (e*x)/2)))
```

3.368 $\int \frac{1}{(2a+2a\cos(d+ex)+2c\sin(d+ex))^3}\,dx$

3.368.1 Optimal result

Integrand size = 24, antiderivative size = 134

$$\int \frac{1}{(2a+2a\cos(d+ex)+2c\sin(d+ex))^3}\,dx = \frac{(3a^2+c^2)\log\left(a+c\tan\left(\frac{1}{2}(d+ex)\right)\right)}{16c^5e} - \frac{c\cos(d+ex)-a\sin(d+ex)}{16c^2e(a+a\cos(d+ex)+c\sin(d+ex))^2} + \frac{3(ac\cos(d+ex)-a^2\sin(d+ex))}{16c^4e(a+a\cos(d+ex)+c\sin(d+ex))}$$

output
```
1/16*(3*a^2+c^2)*ln(a+c*tan(1/2*e*x+1/2*d))/c^5/e+1/16*(-c*cos(e*x+d)+a*si
n(e*x+d))/c^2/e/(a+a*cos(e*x+d)+c*sin(e*x+d))^2+3/16*(a*c*cos(e*x+d)-a^2*s
in(e*x+d))/c^4/e/(a+a*cos(e*x+d)+c*sin(e*x+d))
```

3.368.2 Mathematica [A] (verified)

Time = 2.23 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.39

$$\int \frac{1}{(2a+2a\cos(d+ex)+2c\sin(d+ex))^3}\,dx = -\frac{4(3a^2+c^2)\log\left(\cos\left(\frac{1}{2}(d+ex)\right)\right)-4(3a^2+c^2)\log\left(a\cos\left(\frac{1}{2}(d+ex)\right)+c\sin\left(\frac{1}{2}(d+ex)\right)\right)-c^2\sec^2\left(\frac{1}{2}(\right.}{64c^5e}$$

input
```
Integrate[(2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-3),x]
```

output
```
-1/64*(4*(3*a^2 + c^2)*Log[Cos[(d + e*x)/2]] - 4*(3*a^2 + c^2)*Log[a*Cos[(
d + e*x)/2] + c*Sin[(d + e*x)/2]] - c^2*Sec[(d + e*x)/2]^2 + (c^2*(a^2 + c
^2))/(a*Cos[(d + e*x)/2] + c*Sin[(d + e*x)/2])^2 + (6*c*(a^2 + c^2)*Sin[(d
 + e*x)/2])/(a*Cos[(d + e*x)/2] + c*Sin[(d + e*x)/2]) + 6*a*c*Tan[(d + e*x
)/2])/(c^5*e)
```

3.368.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3608, 27, 3042, 3632, 3042, 3603, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2a\cos(d+ex)+2a+2c\sin(d+ex))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(2a\cos(d+ex)+2a+2c\sin(d+ex))^3}dx$$

$$\Big\downarrow \text{3608}$$

$$\frac{\int -\frac{-\cos(d+ex)a+2a-c\sin(d+ex)}{2(\cos(d+ex)a+a+c\sin(d+ex))^2}dx}{8c^2}-\frac{c\cos(d+ex)-a\sin(d+ex)}{16c^2e(a\cos(d+ex)+a+c\sin(d+ex))^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{-\cos(d+ex)a+2a-c\sin(d+ex)}{(\cos(d+ex)a+a+c\sin(d+ex))^2}dx}{16c^2}-\frac{c\cos(d+ex)-a\sin(d+ex)}{16c^2e(a\cos(d+ex)+a+c\sin(d+ex))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{-\cos(d+ex)a+2a-c\sin(d+ex)}{(\cos(d+ex)a+a+c\sin(d+ex))^2}dx}{16c^2}-\frac{c\cos(d+ex)-a\sin(d+ex)}{16c^2e(a\cos(d+ex)+a+c\sin(d+ex))^2}$$

$$\Big\downarrow \text{3632}$$

$$-\frac{-\left(\frac{3a^2}{c^2}+1\right)\int \frac{1}{\cos(d+ex)a+a+c\sin(d+ex)}dx-\frac{3(ac\cos(d+ex)-a^2\sin(d+ex))}{c^2e(a\cos(d+ex)+a+c\sin(d+ex))}}{16c^2}-\frac{c\cos(d+ex)-a\sin(d+ex)}{16c^2e(a\cos(d+ex)+a+c\sin(d+ex))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\left(\frac{3a^2}{c^2}+1\right)\int\frac{1}{\cos(d+ex)a+a+c\sin(d+ex)}dx-\frac{3(ac\cos(d+ex)-a^2\sin(d+ex))}{c^2e(a\cos(d+ex)+a+c\sin(d+ex))}}{16c^2}-\frac{c\cos(d+ex)-a\sin(d+ex)}{16c^2e(a\cos(d+ex)+a+c\sin(d+ex))^2}$$

$$\Big\downarrow \ 3603$$

$$-\frac{-\frac{2\left(\frac{3a^2}{c^2}+1\right)\int\frac{1}{2a+2c\tan\left(\frac{1}{2}(d+ex)\right)}d\tan\left(\frac{1}{2}(d+ex)\right)}{e}-\frac{3(ac\cos(d+ex)-a^2\sin(d+ex))}{c^2e(a\cos(d+ex)+a+c\sin(d+ex))}}{16c^2}-\frac{c\cos(d+ex)-a\sin(d+ex)}{16c^2e(a\cos(d+ex)+a+c\sin(d+ex))^2}$$

$$\Big\downarrow \ 16$$

$$-\frac{-\frac{\left(\frac{3a^2}{c^2}+1\right)\log\left(a+c\tan\left(\frac{1}{2}(d+ex)\right)\right)}{ce}-\frac{3(ac\cos(d+ex)-a^2\sin(d+ex))}{c^2e(a\cos(d+ex)+a+c\sin(d+ex))}}{16c^2}-\frac{c\cos(d+ex)-a\sin(d+ex)}{16c^2e(a\cos(d+ex)+a+c\sin(d+ex))^2}$$

input
```
Int[(2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-3),x]
```

output
```
-1/16*(c*Cos[d + e*x] - a*Sin[d + e*x])/(c^2*e*(a + a*Cos[d + e*x] + c*Sin
[d + e*x])^2) - (-(((1 + (3*a^2)/c^2)*Log[a + c*Tan[(d + e*x)/2]])/(c*e))
- (3*(a*c*Cos[d + e*x] - a^2*Sin[d + e*x]))/(c^2*e*(a + a*Cos[d + e*x] + c
*Sin[d + e*x])))/(16*c^2)
```

3.368.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.368. $\int\frac{1}{(2a+2a\cos(d+ex)+2c\sin(d+ex))^3}\,dx$

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

rule 3608
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Simp[
1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c
*(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x
] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] &&
NeQ[n, -3/2]
```

rule 3632
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Simp[(a*A - b*B - c*C)/(a^2 - b^2 - c^2)   Int[1/(a + b*Cos[d + e*x] + c*S
in[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
 - c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

3.368.4 Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{-\frac{-\frac{c\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}}{2}+3a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{4c^{4}}-\frac{a^{4}+2a^{2}c^{2}+c^{4}}{8c^{5}\left(a+c\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)^{2}}+\frac{a\left(a^{2}+c^{2}\right)}{c^{5}\left(a+c\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}+\frac{\left(6a^{2}+2c^{2}\right)\ln\left(a+c\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}{4c^{5}}}{8e}$
default	$\frac{-\frac{-\frac{c\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}}{2}+3a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{4c^{4}}-\frac{a^{4}+2a^{2}c^{2}+c^{4}}{8c^{5}\left(a+c\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)^{2}}+\frac{a\left(a^{2}+c^{2}\right)}{c^{5}\left(a+c\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}+\frac{\left(6a^{2}+2c^{2}\right)\ln\left(a+c\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}{4c^{5}}}{8e}$
parallelrisch	$\frac{12\left(a^{2}+\frac{c^{2}}{3}\right)\left(a+c\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)^{2}\ln\left(a+c\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{4}c^{4}-4a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{3}c^{3}+8\left(3a^{3}c+a\,c^{3}\right)\tan\left(\frac{ex}{2}+\right.}{64c^{5}e\left(a+c\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)^{2}}$
norman	$\frac{\frac{18a^{4}+6a^{2}c^{2}-c^{4}}{64c^{5}e}+\frac{\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{4}}{64ce}-\frac{a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{3}}{16c^{2}e}+\frac{\left(3a^{2}+c^{2}\right)a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{8c^{4}e}}{\left(a+c\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)^{2}}+\frac{\left(3a^{2}+c^{2}\right)\ln\left(a+c\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}{16c^{5}e}$
risch	$\frac{3ia^{3}e^{3i(ex+d)}+ia\,c^{2}e^{3i(ex+d)}+9ia^{3}e^{2i(ex+d)}+3a^{2}c\,e^{3i(ex+d)}+3ia\,c^{2}e^{2i(ex+d)}+c^{3}e^{3i(ex+d)}+9ia^{3}e^{i(ex+d)}-ia\,c^{2}e^{i(ex+}}{8\left(c\,e^{2i(ex+d)}+ia\,e^{2i(ex+d)}-c+2ia\,e^{i(ex+d)}+ia\right)^{2}c^{4}e}$

input
```
int(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x,method=_RETURNVERBOSE)
```

output
```
1/8/e*(-1/4/c^4*(-1/2*c*tan(1/2*e*x+1/2*d)^2+3*a*tan(1/2*e*x+1/2*d))-1/8/c
^5*(a^4+2*a^2*c^2+c^4)/(a+c*tan(1/2*e*x+1/2*d))^2+a/c^5*(a^2+c^2)/(a+c*tan
(1/2*e*x+1/2*d))+1/4*(6*a^2+2*c^2)/c^5*ln(a+c*tan(1/2*e*x+1/2*d)))
```

3.368.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 433 vs. $2(127) = 254$.

Time = 0.26 (sec) , antiderivative size = 433, normalized size of antiderivative = 3.23

$$\int \frac{1}{(2a+2a\cos(d+ex)+2c\sin(d+ex))^3}\,dx$$
$$= \frac{12\,a^2c^2\cos{(ex+d)}^2-6\,a^2c^2+2\left(3\,a^2c^2-c^4\right)\cos{(ex+d)}+\left(3\,a^4+4\,a^2c^2+c^4+\left(3\,a^4-2\,a^2c^2-c^4\right)\cos\right.}{}$$

input
```
integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="fricas")
```

output
```
1/32*(12*a^2*c^2*cos(e*x + d)^2 - 6*a^2*c^2 + 2*(3*a^2*c^2 - c^4)*cos(e*x
+ d) + (3*a^4 + 4*a^2*c^2 + c^4 + (3*a^4 - 2*a^2*c^2 - c^4)*cos(e*x + d)^2
 + 2*(3*a^4 + a^2*c^2)*cos(e*x + d) + 2*(3*a^3*c + a*c^3 + (3*a^3*c + a*c^
3)*cos(e*x + d))*sin(e*x + d))*log(a*c*sin(e*x + d) + 1/2*a^2 + 1/2*c^2 +
1/2*(a^2 - c^2)*cos(e*x + d)) - (3*a^4 + 4*a^2*c^2 + c^4 + (3*a^4 - 2*a^2*
c^2 - c^4)*cos(e*x + d)^2 + 2*(3*a^4 + a^2*c^2)*cos(e*x + d) + 2*(3*a^3*c
+ a*c^3 + (3*a^3*c + a*c^3)*cos(e*x + d))*sin(e*x + d))*log(1/2*cos(e*x +
d) + 1/2) - 2*(3*a^3*c - a*c^3 + 3*(a^3*c - a*c^3)*cos(e*x + d))*sin(e*x +
 d))/(2*a^2*c^5*e*cos(e*x + d) + (a^2*c^5 - c^7)*e*cos(e*x + d)^2 + (a^2*c
^5 + c^7)*e + 2*(a*c^6*e*cos(e*x + d) + a*c^6*e)*sin(e*x + d))
```

3.368.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(2a + 2a\cos(d + ex) + 2c\sin(d + ex))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))**3,x)
```

output
```
Timed out
```

3.368.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.42

$$\int \frac{1}{(2a + 2a\cos(d + ex) + 2c\sin(d + ex))^3}\,dx = \frac{\frac{7\,a^4+6\,a^2c^2-c^4+\frac{8\,\left(a^3c+ac^3\right)\sin(ex+d)}{\cos(ex+d)+1}}{a^2c^5+\frac{2\,ac^6\sin(ex+d)}{\cos(ex+d)+1}+\frac{c^7\sin(ex+d)^2}{(\cos(ex+d)+1)^2}} - \frac{\frac{6\,a\sin(ex+d)}{\cos(ex+d)+1}-\frac{c\sin(ex+d)^2}{(\cos(ex+d)+1)^2}}{c^4} + \frac{4\,(3\,a^2+c^2)\log\left(a+\frac{c\sin(ex+d)}{\cos(ex+d)+1}\right)}{c^5}}{64\,e}$$

input
```
integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="maxima")
```

output
```
1/64*((7*a^4 + 6*a^2*c^2 - c^4 + 8*(a^3*c + a*c^3)*sin(e*x + d)/(cos(e*x +
 d) + 1))/(a^2*c^5 + 2*a*c^6*sin(e*x + d)/(cos(e*x + d) + 1) + c^7*sin(e*x
 + d)^2/(cos(e*x + d) + 1)^2) - (6*a*sin(e*x + d)/(cos(e*x + d) + 1) - c*s
in(e*x + d)^2/(cos(e*x + d) + 1)^2)/c^4 + 4*(3*a^2 + c^2)*log(a + c*sin(e*
x + d)/(cos(e*x + d) + 1))/c^5)/e
```

3.368.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.22

$$\int \frac{1}{(2a+2a\cos(d+ex)+2c\sin(d+ex))^3}\,dx = \frac{\frac{4\,(3\,a^2+c^2)\log\left(\left|c\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+a\right|\right)}{c^5}+\frac{c^3\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^2-6\,ac^2\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)}{c^6}-\frac{18\,a^2c^2\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^2+6\,c^4\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^2+28\,a^3c\,\mathrm{t}}{(c\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,a\right.}}{64\,e}$$

input
```
integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="giac")
```

output
```
1/64*(4*(3*a^2 + c^2)*log(abs(c*tan(1/2*e*x + 1/2*d) + a))/c^5 + (c^3*tan(
1/2*e*x + 1/2*d)^2 - 6*a*c^2*tan(1/2*e*x + 1/2*d))/c^6 - (18*a^2*c^2*tan(1
/2*e*x + 1/2*d)^2 + 6*c^4*tan(1/2*e*x + 1/2*d)^2 + 28*a^3*c*tan(1/2*e*x +
1/2*d) + 4*a*c^3*tan(1/2*e*x + 1/2*d) + 11*a^4 + c^4)/((c*tan(1/2*e*x + 1/
2*d) + a)^2*c^5))/e
```

3.368.9 Mupad [B] (verification not implemented)

Time = 26.35 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.21

$$\begin{aligned}\int \frac{1}{(2a+2a\cos(d+ex)+2c\sin(d+ex))^3}\,dx &= \frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2}{64\,c^3\,e}+\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\left(4\,a^3+4\,a\,c^2\right)+\frac{7\,a^4+6\,a^2\,c^2-c^4}{2\,c}}{e\,\left(32\,a^2\,c^4+64\,a\,c^5\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)+32\,c^6\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2\right)}\\&\quad-\frac{3\,a\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{32\,c^4\,e}+\frac{\ln\left(a+c\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\right)\,\left(3\,a^2+c^2\right)}{16\,c^5\,e}\end{aligned}$$

input
```
int(1/(2*a + 2*a*cos(d + e*x) + 2*c*sin(d + e*x))^3,x)
```

output
```
tan(d/2 + (e*x)/2)^2/(64*c^3*e) + (tan(d/2 + (e*x)/2)*(4*a*c^2 + 4*a^3) +
(7*a^4 - c^4 + 6*a^2*c^2)/(2*c))/(e*(32*c^6*tan(d/2 + (e*x)/2)^2 + 32*a^2*
c^4 + 64*a*c^5*tan(d/2 + (e*x)/2))) - (3*a*tan(d/2 + (e*x)/2))/(32*c^4*e)
+ (log(a + c*tan(d/2 + (e*x)/2))*(3*a^2 + c^2))/(16*c^5*e)
```

3.369 $\int \frac{1}{(2a+2a\cos(d+ex)+2c\sin(d+ex))^4}\,dx$

3.369.1 Optimal result

Integrand size = 24, antiderivative size = 207

$$\begin{aligned}\int \frac{1}{(2a+2a\cos(d+ex)+2c\sin(d+ex))^4}\,dx &= -\frac{a(5a^2+3c^2)\log\left(a+c\tan\left(\frac{1}{2}(d+ex)\right)\right)}{32c^7e} - \frac{c\cos(d+ex)-a\sin(d+ex)}{48c^2e(a+a\cos(d+ex)+c\sin(d+ex))^3} \\ &\quad + \frac{5(ac\cos(d+ex)-a^2\sin(d+ex))}{96c^4e(a+a\cos(d+ex)+c\sin(d+ex))^2} \\ &\quad - \frac{c(15a^2+4c^2)\cos(d+ex)-a(15a^2+4c^2)\sin(d+ex)}{96c^6e(a+a\cos(d+ex)+c\sin(d+ex))}\end{aligned}$$

output
```
-1/32*a*(5*a^2+3*c^2)*ln(a+c*tan(1/2*e*x+1/2*d))/c^7/e+1/48*(-c*cos(e*x+d)
+a*sin(e*x+d))/c^2/e/(a+a*cos(e*x+d)+c*sin(e*x+d))^3+5/96*(a*c*cos(e*x+d)-
a^2*sin(e*x+d))/c^4/e/(a+a*cos(e*x+d)+c*sin(e*x+d))^2+1/96*(-c*(15*a^2+4*c
^2)*cos(e*x+d)+a*(15*a^2+4*c^2)*sin(e*x+d))/c^6/e/(a+a*cos(e*x+d)+c*sin(e*
x+d))
```

3.369.2 Mathematica [A] (verified)

Time = 4.44 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.92

$$\int \frac{1}{(2a+2a\cos(d+ex)+2c\sin(d+ex))^4}\,dx$$
$$= \frac{\cos^8\left(\frac{1}{2}(d+ex)\right)\left(a+c\tan\left(\frac{1}{2}(d+ex)\right)\right)\left(3c^4(a^2+2c^2)\sec^4\left(\frac{1}{2}(d+ex)\right)+c^6\sec^6\left(\frac{1}{2}(d+ex)\right)-2(37a^6+\right.}{}$$

input
```
Integrate[(2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-4),x]
```

output
```
(Cos[(d + e*x)/2]^8*(a + c*Tan[(d + e*x)/2])*(3*c^4*(a^2 + 2*c^2)*Sec[(d +
 e*x)/2]^4 + c^6*Sec[(d + e*x)/2]^6 - 2*(37*a^6 + 36*a^4*c^2 + 3*a^2*c^4 +
 4*c^6 + 30*a^6*Log[a + c*Tan[(d + e*x)/2]] + 18*a^4*c^2*Log[a + c*Tan[(d
+ e*x)/2]] + 48*a*c^5*Csc[d + e*x]^5*Sin[(d + e*x)/2]^6 + 3*a*c*(27*a^4 +
30*a^2*c^2 + c^4 + 6*a^2*(5*a^2 + 3*c^2)*Log[a + c*Tan[(d + e*x)/2]])*Tan[
(d + e*x)/2] + 6*c^2*(6*a^4 + 11*a^2*c^2 + 2*c^4 + 3*a^2*(5*a^2 + 3*c^2)*L
og[a + c*Tan[(d + e*x)/2]])*Tan[(d + e*x)/2]^2 + 6*a*c^3*(-3*a^2 + (5*a^2
+ 3*c^2)*Log[a + c*Tan[(d + e*x)/2]])*Tan[(d + e*x)/2]^3 - 6*a^2*c^4*Tan[(
d + e*x)/2]^4)))/(24*c^7*e*(a + a*Cos[d + e*x] + c*Sin[d + e*x])^4)
```

3.369.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {3042, 3608, 27, 3042, 3635, 25, 3042, 3632, 3042, 3603, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2a\cos(d+ex)+2a+2c\sin(d+ex))^4}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(2a\cos(d+ex)+2a+2c\sin(d+ex))^4}dx$$
$$\Big\downarrow \text{3608}$$
$$\frac{\int -\frac{-2\cos(d+ex)a+3a-2c\sin(d+ex)}{4(\cos(d+ex)a+a+c\sin(d+ex))^3}dx}{12c^2}-\frac{c\cos(d+ex)-a\sin(d+ex)}{48c^2e(a\cos(d+ex)+a+c\sin(d+ex))^3}$$

$$\Big\downarrow \ 27$$

$$-\frac{\int \frac{-2\cos(d+ex)a+3a-2c\sin(d+ex)}{(\cos(d+ex)a+a+c\sin(d+ex))^3}dx}{48c^2}-\frac{c\cos(d+ex)-a\sin(d+ex)}{48c^2e(a\cos(d+ex)+a+c\sin(d+ex))^3}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\int \frac{-2\cos(d+ex)a+3a-2c\sin(d+ex)}{(\cos(d+ex)a+a+c\sin(d+ex))^3}dx}{48c^2}-\frac{c\cos(d+ex)-a\sin(d+ex)}{48c^2e(a\cos(d+ex)+a+c\sin(d+ex))^3}$$

$$\Big\downarrow \ 3635$$

$$-\frac{\frac{\int -\frac{-5\cos(d+ex)a^2-5c\sin(d+ex)a+2\left(5a^2+2c^2\right)}{(\cos(d+ex)a+a+c\sin(d+ex))^2}dx}{2c^2}-\frac{5\left(ac\cos(d+ex)-a^2\sin(d+ex)\right)}{2c^2e(a\cos(d+ex)+a+c\sin(d+ex))^2}}{48c^2}-\frac{c\cos(d+ex)-a\sin(d+ex)}{48c^2e(a\cos(d+ex)+a+c\sin(d+ex))^3}$$

$$\Big\downarrow \ 25$$

$$-\frac{-\frac{\int \frac{-5\cos(d+ex)a^2-5c\sin(d+ex)a+2\left(5a^2+2c^2\right)}{(\cos(d+ex)a+a+c\sin(d+ex))^2}dx}{2c^2}-\frac{5\left(ac\cos(d+ex)-a^2\sin(d+ex)\right)}{2c^2e(a\cos(d+ex)+a+c\sin(d+ex))^2}}{48c^2}-\frac{c\cos(d+ex)-a\sin(d+ex)}{48c^2e(a\cos(d+ex)+a+c\sin(d+ex))^3}$$

$$\Big\downarrow \ 3042$$

$$-\frac{-\frac{\int \frac{-5\cos(d+ex)a^2-5c\sin(d+ex)a+2\left(5a^2+2c^2\right)}{(\cos(d+ex)a+a+c\sin(d+ex))^2}dx}{2c^2}-\frac{5\left(ac\cos(d+ex)-a^2\sin(d+ex)\right)}{2c^2e(a\cos(d+ex)+a+c\sin(d+ex))^2}}{48c^2}-\frac{c\cos(d+ex)-a\sin(d+ex)}{48c^2e(a\cos(d+ex)+a+c\sin(d+ex))^3}$$

$$\Big\downarrow \ 3632$$

$$-\frac{-\frac{-3a\left(\frac{5a^2}{c^2}+3\right)\int \frac{1}{\cos(d+ex)a+a+c\sin(d+ex)}dx-\frac{c\left(15a^2+4c^2\right)\cos(d+ex)-a\left(15a^2+4c^2\right)\sin(d+ex)}{c^2e(a\cos(d+ex)+a+c\sin(d+ex))}}{2c^2}-\frac{5\left(ac\cos(d+ex)-a^2\sin(d+ex)\right)}{2c^2e(a\cos(d+ex)+a+c\sin(d+ex))^2}}{48c^2}-\frac{c\cos(d+ex)-a\sin(d+ex)}{48c^2e(a\cos(d+ex)+a+c\sin(d+ex))^3}$$

$$\Big\downarrow \ 3042$$

$$-\frac{-\frac{-3a\left(\frac{5a^2}{c^2}+3\right)\int \frac{1}{\cos(d+ex)a+a+c\sin(d+ex)}dx-\frac{c\left(15a^2+4c^2\right)\cos(d+ex)-a\left(15a^2+4c^2\right)\sin(d+ex)}{c^2e(a\cos(d+ex)+a+c\sin(d+ex))}}{2c^2}-\frac{5\left(ac\cos(d+ex)-a^2\sin(d+ex)\right)}{2c^2e(a\cos(d+ex)+a+c\sin(d+ex))^2}}{48c^2}-\frac{c\cos(d+ex)-a\sin(d+ex)}{48c^2e(a\cos(d+ex)+a+c\sin(d+ex))^3}$$

3.369. $\int \frac{1}{(2a+2a\cos(d+ex)+2c\sin(d+ex))^4}\,dx$

$$\Big\downarrow \text{3603}$$

$$-\frac{-\frac{-\frac{6a\left(\frac{5a^2}{c^2}+3\right)\int \frac{1}{2a+2c\tan\left(\frac{1}{2}(d+ex)\right)}d\tan\left(\frac{1}{2}(d+ex)\right)}{e}-\frac{c\left(15a^2+4c^2\right)\cos(d+ex)-a\left(15a^2+4c^2\right)\sin(d+ex)}{c^2e(a\cos(d+ex)+a+c\sin(d+ex))}}{2c^2}-\frac{5(ac\cos(d+ex)-a^2\sin(d+ex))}{2c^2e(a\cos(d+ex)+a+c\sin(d+ex))^2}}{48c^2}-\frac{c\cos(d+ex)-a\sin(d+ex)}{48c^2e(a\cos(d+ex)+a+c\sin(d+ex))^3}$$

$$\Big\downarrow \text{16}$$

$$-\frac{-\frac{5(ac\cos(d+ex)-a^2\sin(d+ex))}{2c^2e(a\cos(d+ex)+a+c\sin(d+ex))^2}-\frac{-\frac{3a\left(\frac{5a^2}{c^2}+3\right)\log\left(a+c\tan\left(\frac{1}{2}(d+ex)\right)\right)}{ce}-\frac{c\left(15a^2+4c^2\right)\cos(d+ex)-a\left(15a^2+4c^2\right)\sin(d+ex)}{c^2e(a\cos(d+ex)+a+c\sin(d+ex))}}{2c^2}}{48c^2}-\frac{c\cos(d+ex)-a\sin(d+ex)}{48c^2e(a\cos(d+ex)+a+c\sin(d+ex))^3}$$

input
```
Int[(2*a + 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-4),x]
```

output
```
-1/48*(c*Cos[d + e*x] - a*Sin[d + e*x])/(c^2*e*(a + a*Cos[d + e*x] + c*Sin
[d + e*x])^3) - ((-5*(a*c*Cos[d + e*x] - a^2*Sin[d + e*x]))/(2*c^2*e*(a +
a*Cos[d + e*x] + c*Sin[d + e*x])^2) - ((-3*a*(3 + (5*a^2)/c^2)*Log[a + c*T
an[(d + e*x)/2]])/(c*e) - (c*(15*a^2 + 4*c^2)*Cos[d + e*x] - a*(15*a^2 + 4
*c^2)*Sin[d + e*x])/(c^2*e*(a + a*Cos[d + e*x] + c*Sin[d + e*x])))/(2*c^2)
)/(48*c^2)
```

3.369.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.369. $\int \frac{1}{(2a+2a\cos(d+ex)+2c\sin(d+ex))^4}\,dx$

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

rule 3608
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Simp[
1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c
*(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x
] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] &&
NeQ[n, -3/2]
```

rule 3632
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Simp[(a*A - b*B - c*C)/(a^2 - b^2 - c^2)   Int[1/(a + b*Cos[d + e*x] + c*S
in[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
 - c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

rule 3635
```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(-(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(
a^2 - b^2 - c^2))), x] + Simp[1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a + b*Co
s[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n +
2)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x]
/; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[n, -2]
```

3.369. $\int \frac{1}{(2a+2a\cos(d+ex)+2c\sin(d+ex))^4}\,dx$

3.369.4 Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.04

method	result
parallelrisch	$\left(-60a\left(a^2+\frac{3c^2}{5}\right)\left(a+c\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)^3\ln\left(a+c\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^6c^6-3a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^5c^5+\left(15a^2c^4+9c^6\right)\tan\left(\cdots\right)\right) / \left(384c^7e\left(a+c\cdots\right)\right)$
derivativedivides	$\left(\frac{\frac{\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^3c^2}{3}-2a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2c+10\tan\left(\frac{ex}{2}+\frac{d}{2}\right)a^2+3\tan\left(\frac{ex}{2}+\frac{d}{2}\right)c^2}{8c^6}+\frac{3a\left(a^4+2a^2c^2+c^4\right)}{8c^7\left(a+c\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)^2}-\frac{a\left(5a^2+3c^2\right)\ln\left(a+c\tan\left(\frac{ex}{2}\cdots\right)\right)}{2c^7}\cdots\right) / (16e)$
default	$\left(\frac{\frac{\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^3c^2}{3}-2a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2c+10\tan\left(\frac{ex}{2}+\frac{d}{2}\right)a^2+3\tan\left(\frac{ex}{2}+\frac{d}{2}\right)c^2}{8c^6}+\frac{3a\left(a^4+2a^2c^2+c^4\right)}{8c^7\left(a+c\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)^2}-\frac{a\left(5a^2+3c^2\right)\ln\left(a+c\tan\left(\frac{ex}{2}\cdots\right)\right)}{2c^7}\cdots\right) / (16e)$
norman	$\left(-\frac{110a^6+66a^4c^2+3a^2c^4+c^6}{384c^7e}+\frac{\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^6}{384ce}-\frac{a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^5}{128c^2e}+\frac{\left(5a^2+3c^2\right)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^4}{128c^3e}-\frac{3\left(20a^4+12a^2c^2+c^4\right)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}{128c^5e}-\cdots\right) / \left(a+c\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)^3$
risch	$i\left(-4c^5+45a^4c-15ia^5-3a^2c^3-130ia^3c^2e^{3i(ex+d)}-24iac^4e^{3i(ex+d)}+12c^5e^{2i(ex+d)}-75ia^5e^{4i(ex+d)}-150ia^5e^{3i(ex+d)}+\cdots\right)$

input
```
int(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^4,x,method=_RETURNVERBOSE)
```

output
```
1/384*(-60*a*(a^2+3/5*c^2)*(a+c*tan(1/2*e*x+1/2*d))^3*ln(a+c*tan(1/2*e*x+1
/2*d))+tan(1/2*e*x+1/2*d)^6*c^6-3*a*tan(1/2*e*x+1/2*d)^5*c^5+(15*a^2*c^4+9
*c^6)*tan(1/2*e*x+1/2*d)^4+(-180*a^4*c^2-108*a^2*c^4-9*c^6)*tan(1/2*e*x+1/
2*d)^2+(-270*a^5*c-162*a^3*c^3-9*a*c^5)*tan(1/2*e*x+1/2*d)-110*a^6-66*a^4*
c^2-3*a^2*c^4-c^6)/c^7/e/(a+c*tan(1/2*e*x+1/2*d))^3
```

3.369.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 791 vs. 2(198) = 396.

Time = 0.30 (sec) , antiderivative size = 791, normalized size of antiderivative = 3.82

$$\int \frac{1}{(2a+2a\cos(d+ex)+2c\sin(d+ex))^4}\,dx$$
$$= \frac{60\,a^4c^2+6\,a^2c^4-2\left(45\,a^4c^2-3\,a^2c^4-4\,c^6\right)\cos\left(ex+d\right)^3-12\left(10\,a^4c^2+a^2c^4\right)\cos\left(ex+d\right)^2+6\left(5\,a^4c^2-\cdots\right.}{\cdots}$$

input
```
integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^4,x, algorithm="fricas")
```

output
```
1/192*(60*a^4*c^2 + 6*a^2*c^4 - 2*(45*a^4*c^2 - 3*a^2*c^4 - 4*c^6)*cos(e*x
 + d)^3 - 12*(10*a^4*c^2 + a^2*c^4)*cos(e*x + d)^2 + 6*(5*a^4*c^2 - 2*a^2*
c^4 - 2*c^6)*cos(e*x + d) - 3*(5*a^6 + 18*a^4*c^2 + 9*a^2*c^4 + (5*a^6 - 1
2*a^4*c^2 - 9*a^2*c^4)*cos(e*x + d)^3 + 3*(5*a^6 - 2*a^4*c^2 - 3*a^2*c^4)*
cos(e*x + d)^2 + 3*(5*a^6 + 8*a^4*c^2 + 3*a^2*c^4)*cos(e*x + d) + (15*a^5*
c + 14*a^3*c^3 + 3*a*c^5 + (15*a^5*c + 4*a^3*c^3 - 3*a*c^5)*cos(e*x + d)^2
 + 6*(5*a^5*c + 3*a^3*c^3)*cos(e*x + d))*sin(e*x + d))*log(a*c*sin(e*x + d
) + 1/2*a^2 + 1/2*c^2 + 1/2*(a^2 - c^2)*cos(e*x + d)) + 3*(5*a^6 + 18*a^4*
c^2 + 9*a^2*c^4 + (5*a^6 - 12*a^4*c^2 - 9*a^2*c^4)*cos(e*x + d)^3 + 3*(5*a
^6 - 2*a^4*c^2 - 3*a^2*c^4)*cos(e*x + d)^2 + 3*(5*a^6 + 8*a^4*c^2 + 3*a^2*
c^4)*cos(e*x + d) + (15*a^5*c + 14*a^3*c^3 + 3*a*c^5 + (15*a^5*c + 4*a^3*c
^3 - 3*a*c^5)*cos(e*x + d)^2 + 6*(5*a^5*c + 3*a^3*c^3)*cos(e*x + d))*sin(e
*x + d))*log(1/2*cos(e*x + d) + 1/2) + 2*(15*a^5*c + 14*a^3*c^3 + 6*a*c^5
+ (15*a^5*c - 41*a^3*c^3 - 12*a*c^5)*cos(e*x + d)^2 + 3*(10*a^5*c - 9*a^3*
c^3 - a*c^5)*cos(e*x + d))*sin(e*x + d))/((a^3*c^7 - 3*a*c^9)*e*cos(e*x +
d)^3 + 3*(a^3*c^7 - a*c^9)*e*cos(e*x + d)^2 + 3*(a^3*c^7 + a*c^9)*e*cos(e*
x + d) + (a^3*c^7 + 3*a*c^9)*e + (6*a^2*c^8*e*cos(e*x + d) + (3*a^2*c^8 -
c^10)*e*cos(e*x + d)^2 + (3*a^2*c^8 + c^10)*e)*sin(e*x + d))
```

3.369.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(2a + 2a\cos(d + ex) + 2c\sin(d + ex))^4}\, dx = \text{Timed out}$$

input
```
integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))**4,x)
```

output
```
Timed out
```

3.369.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.48

$$\int \frac{1}{(2a + 2a\cos(d + ex) + 2c\sin(d + ex))^4}\, dx = -\frac{\frac{37\,a^6+39\,a^4c^2+3\,a^2c^4+c^6+\frac{9\left(9\,a^5c+10\,a^3c^3+ac^5\right)\sin(ex+d)}{\cos(ex+d)+1}+\frac{9\left(5\,a^4c^2+6\,a^2c^4+c^6\right)\sin(ex+d)^2}{(\cos(ex+d)+1)^2}}{a^3c^7+\frac{3\,a^2c^8\sin(ex+d)}{\cos(ex+d)+1}+\frac{3\,ac^9\sin(ex+d)^2}{(\cos(ex+d)+1)^2}+\frac{c^{10}\sin(ex+d)^3}{(\cos(ex+d)+1)^3}} + \frac{\frac{6\,ac\sin(ex+d)^2}{(\cos(ex+d)+1)^2}-\frac{c^2\sin(ex+d)^3}{(\cos(ex+d)+1)^3}-\frac{3\left(10\,a^2+3 \cdots\right.}{\cos(e \cdots}}{c^6}\cdots}{384\,e}$$

input
```
integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^4,x, algorithm="maxima")
```

output
```
-1/384*((37*a^6 + 39*a^4*c^2 + 3*a^2*c^4 + c^6 + 9*(9*a^5*c + 10*a^3*c^3 +
 a*c^5)*sin(e*x + d)/(cos(e*x + d) + 1) + 9*(5*a^4*c^2 + 6*a^2*c^4 + c^6)*
sin(e*x + d)^2/(cos(e*x + d) + 1)^2)/(a^3*c^7 + 3*a^2*c^8*sin(e*x + d)/(co
s(e*x + d) + 1) + 3*a*c^9*sin(e*x + d)^2/(cos(e*x + d) + 1)^2 + c^10*sin(e
*x + d)^3/(cos(e*x + d) + 1)^3) + (6*a*c*sin(e*x + d)^2/(cos(e*x + d) + 1)
^2 - c^2*sin(e*x + d)^3/(cos(e*x + d) + 1)^3 - 3*(10*a^2 + 3*c^2)*sin(e*x
+ d)/(cos(e*x + d) + 1))/c^6 + 12*(5*a^3 + 3*a*c^2)*log(a + c*sin(e*x + d)
/(cos(e*x + d) + 1))/c^7)/e
```

3.369.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.41

$$\int \frac{1}{(2a+2a\cos(d+ex)+2c\sin(d+ex))^4}\,dx =$$

$$-\frac{\frac{12\,(5\,a^3+3\,ac^2)\log\left(\left|c\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+a\right|\right)}{c^7} - \frac{110\,a^3c^3\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^3+66\,ac^5\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^3+285\,a^4c^2\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^2+144\,a^2c^4\tan\left(\frac{1}{2}\right.}{}}{}$$

input
```
integrate(1/(2*a+2*a*cos(e*x+d)+2*c*sin(e*x+d))^4,x, algorithm="giac")
```

output
```
-1/384*(12*(5*a^3 + 3*a*c^2)*log(abs(c*tan(1/2*e*x + 1/2*d) + a))/c^7 - (1
10*a^3*c^3*tan(1/2*e*x + 1/2*d)^3 + 66*a*c^5*tan(1/2*e*x + 1/2*d)^3 + 285*
a^4*c^2*tan(1/2*e*x + 1/2*d)^2 + 144*a^2*c^4*tan(1/2*e*x + 1/2*d)^2 - 9*c^
6*tan(1/2*e*x + 1/2*d)^2 + 249*a^5*c*tan(1/2*e*x + 1/2*d) + 108*a^3*c^3*ta
n(1/2*e*x + 1/2*d) - 9*a*c^5*tan(1/2*e*x + 1/2*d) + 73*a^6 + 27*a^4*c^2 -
3*a^2*c^4 - c^6)/((c*tan(1/2*e*x + 1/2*d) + a)^3*c^7) - (c^8*tan(1/2*e*x +
 1/2*d)^3 - 6*a*c^7*tan(1/2*e*x + 1/2*d)^2 + 30*a^2*c^6*tan(1/2*e*x + 1/2*
d) + 9*c^8*tan(1/2*e*x + 1/2*d))/c^12)/e
```

3.369.9 Mupad [B] (verification not implemented)

Time = 26.99 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.26

$$\int \frac{1}{(2a+2a\cos(d+ex)+2c\sin(d+ex))^4}\,dx = \frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^3}{384\,c^4\,e}+\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\left(\frac{3}{128\,c^4}+\frac{5\,a^2}{64\,c^6}\right)}{e}$$
$$-\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\left(27\,a^5+30\,a^3\,c^2+3\,a\,c^4\right)+\frac{37\,a^6+39\,a^4\,c^2+3\,a^2\,c^4+c^6}{3\,c}+\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2\,\left(15\,a^4\,c+18\,a^2\,c^3+3\,c^5\right)}{e\,\left(128\,a^3\,c^6+384\,a^2\,c^7\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)+384\,a\,c^8\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2+128\,c^9\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^3\right)}$$
$$-\frac{a\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2}{64\,c^5\,e}-\frac{\ln\left(a+c\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\right)\,\left(5\,a^3+3\,a\,c^2\right)}{32\,c^7\,e}$$

input
```
int(1/(2*a + 2*a*cos(d + e*x) + 2*c*sin(d + e*x))^4,x)
```

output
```
tan(d/2 + (e*x)/2)^3/(384*c^4*e) + (tan(d/2 + (e*x)/2)*(3/(128*c^4) + (5*a
^2)/(64*c^6)))/e - (tan(d/2 + (e*x)/2)*(3*a*c^4 + 27*a^5 + 30*a^3*c^2) + (
37*a^6 + c^6 + 3*a^2*c^4 + 39*a^4*c^2)/(3*c) + tan(d/2 + (e*x)/2)^2*(15*a^
4*c + 3*c^5 + 18*a^2*c^3))/(e*(128*c^9*tan(d/2 + (e*x)/2)^3 + 128*a^3*c^6
+ 384*a^2*c^7*tan(d/2 + (e*x)/2) + 384*a*c^8*tan(d/2 + (e*x)/2)^2)) - (a*t
an(d/2 + (e*x)/2)^2)/(64*c^5*e) - (log(a + c*tan(d/2 + (e*x)/2))*(3*a*c^2
+ 5*a^3))/(32*c^7*e)
```

3.370 $\int \frac{1}{2a+2a\cos(d+ex)+2a\sin(d+ex)}\,dx$

3.370.1 Optimal result

Integrand size = 24, antiderivative size = 23

$$\int \frac{1}{2a+2a\cos(d+ex)+2a\sin(d+ex)}\,dx = \frac{\log\left(1+\tan\left(\frac{1}{2}(d+ex)\right)\right)}{2ae}$$

output
```
1/2*ln(1+tan(1/2*e*x+1/2*d))/a/e
```

3.370.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 50 vs. 2(23) = 46.

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.17

$$\int \frac{1}{2a+2a\cos(d+ex)+2a\sin(d+ex)}\,dx = \frac{-\frac{\log\left(\cos\left(\frac{1}{2}(d+ex)\right)\right)}{e}+\frac{\log\left(\cos\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{1}{2}(d+ex)\right)\right)}{e}}{2a}$$

input
```
Integrate[(2*a + 2*a*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-1),x]
```

output
```
(-(Log[Cos[(d + e*x)/2]]/e) + Log[Cos[(d + e*x)/2] + Sin[(d + e*x)/2]]/e)/
(2*a)
```

3.370.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3042, 3603, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{2a\sin(d+ex)+2a\cos(d+ex)+2a}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{2a\sin(d+ex)+2a\cos(d+ex)+2a}dx$$

$$\Big\downarrow \; 3603$$

$$\frac{2\int \frac{1}{4\tan\left(\frac{1}{2}(d+ex)\right)a+4a}d\tan\left(\frac{1}{2}(d+ex)\right)}{e}$$

$$\Big\downarrow \; 16$$

$$\frac{\log\left(\tan\left(\frac{1}{2}(d+ex)\right)+1\right)}{2ae}$$

input
```
Int[(2*a + 2*a*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-1),x]
```

output
```
Log[1 + Tan[(d + e*x)/2]]/(2*a*e)
```

3.370.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

3.370.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{\ln\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}{2ae}$	21
default	$\frac{\ln\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}{2ae}$	21
norman	$\frac{\ln\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}{2ae}$	21
parallelrisch	$\frac{\ln\left(\sqrt{1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}\right)}{ae}$	22
risch	$-\frac{\ln\left(e^{i(ex+d)}+1\right)}{2ae}+\frac{\ln\left(e^{i(ex+d)}+i\right)}{2ae}$	43

input
```
int(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d)),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(1+tan(1/2*e*x+1/2*d))/a/e
```

3.370.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int \frac{1}{2a+2a\cos(d+ex)+2a\sin(d+ex)}\,dx$$
$$=-\frac{\log\left(\frac{1}{2}\cos\left(ex+d\right)+\frac{1}{2}\right)-\log\left(\sin\left(ex+d\right)+1\right)}{4\,ae}$$

input
```
integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d)),x, algorithm="fricas")
```

output
```
-1/4*(log(1/2*cos(e*x + d) + 1/2) - log(sin(e*x + d) + 1))/(a*e)
```

3.370.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(17) = 34.

Time = 0.32 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \frac{1}{2a+2a\cos(d+ex)+2a\sin(d+ex)}\,dx = \begin{cases} \frac{\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+1\right)}{2ae} & \text{for } e\neq 0 \\ \frac{x}{2a\sin(d)+2a\cos(d)+2a} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d)),x)
```

output
```
Piecewise((log(tan(d/2 + e*x/2) + 1)/(2*a*e), Ne(e, 0)), (x/(2*a*sin(d) +
2*a*cos(d) + 2*a), True))
```

3.370.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.22

$$\int \frac{1}{2a+2a\cos(d+ex)+2a\sin(d+ex)}\,dx = \frac{\log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}+1\right)}{2\,ae}$$

input
```
integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d)),x, algorithm="maxima")
```

output
```
1/2*log(sin(e*x + d)/(cos(e*x + d) + 1) + 1)/(a*e)
```

3.370.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{2a+2a\cos(d+ex)+2a\sin(d+ex)}\,dx = \frac{\log\left(\left|\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+1\right|\right)}{2\,ae}$$

input
```
integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d)),x, algorithm="giac")
```

output
```
1/2*log(abs(tan(1/2*e*x + 1/2*d) + 1))/(a*e)
```

3.370.9 Mupad [B] (verification not implemented)

Time = 27.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{1}{2a+2a\cos(d+ex)+2a\sin(d+ex)}\,dx=\frac{\ln\left(\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)+1\right)}{2\,a\,e}$$

input
```
int(1/(2*a + 2*a*cos(d + e*x) + 2*a*sin(d + e*x)),x)
```

output
```
log(tan(d/2 + (e*x)/2) + 1)/(2*a*e)
```

3.371 $\int \frac{1}{(2a+2a\cos(d+ex)+2a\sin(d+ex))^2}\,dx$

3.371.1 Optimal result

Integrand size = 24, antiderivative size = 75

$$\int \frac{1}{(2a+2a\cos(d+ex)+2a\sin(d+ex))^2}\,dx = -\frac{\log\left(1+\tan\left(\frac{1}{2}(d+ex)\right)\right)}{4a^2e} - \frac{a\cos(d+ex)-a\sin(d+ex)}{4e\left(a^3+a^3\cos(d+ex)+a^3\sin(d+ex)\right)}$$

output
```
-1/4*ln(1+tan(1/2*e*x+1/2*d))/a^2/e+1/4*(-a*cos(e*x+d)+a*sin(e*x+d))/e/(a^
3+a^3*cos(e*x+d)+a^3*sin(e*x+d))
```

3.371.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.24

$$\int \frac{1}{(2a+2a\cos(d+ex)+2a\sin(d+ex))^2}\,dx = \frac{2\log\left(\cos\left(\frac{1}{2}(d+ex)\right)\right) - 2\log\left(\cos\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{1}{2}(d+ex)\right)\right) + \frac{2\sin\left(\frac{1}{2}(d+ex)\right)}{\cos\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{1}{2}(d+ex)\right)} + \tan\left(\frac{1}{2}(d+ex)\right)}{8a^2e}$$

input
```
Integrate[(2*a + 2*a*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-2),x]
```

output
```
(2*Log[Cos[(d + e*x)/2]] - 2*Log[Cos[(d + e*x)/2] + Sin[(d + e*x)/2]] + (2
*Sin[(d + e*x)/2])/(Cos[(d + e*x)/2] + Sin[(d + e*x)/2]) + Tan[(d + e*x)/2
])/(8*a^2*e)
```

3.371.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3608, 25, 3042, 3603, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2a\sin(d+ex)+2a\cos(d+ex)+2a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(2a\sin(d+ex)+2a\cos(d+ex)+2a)^2}dx$$

$$\Big\downarrow \text{3608}$$

$$\frac{\int -\frac{1}{\cos(d+ex)+\sin(d+ex)+1}dx}{4a^2}-\frac{a\cos(d+ex)-a\sin(d+ex)}{4e\left(a^3\sin(d+ex)+a^3\cos(d+ex)+a^3\right)}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{1}{\cos(d+ex)+\sin(d+ex)+1}dx}{4a^2}-\frac{a\cos(d+ex)-a\sin(d+ex)}{4e\left(a^3\sin(d+ex)+a^3\cos(d+ex)+a^3\right)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{1}{\cos(d+ex)+\sin(d+ex)+1}dx}{4a^2}-\frac{a\cos(d+ex)-a\sin(d+ex)}{4e\left(a^3\sin(d+ex)+a^3\cos(d+ex)+a^3\right)}$$

$$\Big\downarrow \text{3603}$$

$$-\frac{\int \frac{1}{2\tan\left(\frac{1}{2}(d+ex)\right)+2}d\tan\left(\frac{1}{2}(d+ex)\right)}{2a^2e}-\frac{a\cos(d+ex)-a\sin(d+ex)}{4e\left(a^3\sin(d+ex)+a^3\cos(d+ex)+a^3\right)}$$

$$\Big\downarrow \text{16}$$

$$-\frac{a\cos(d+ex)-a\sin(d+ex)}{4e\left(a^3\sin(d+ex)+a^3\cos(d+ex)+a^3\right)}-\frac{\log\left(\tan\left(\frac{1}{2}(d+ex)\right)+1\right)}{4a^2e}$$

input
```
Int[(2*a + 2*a*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-2),x]
```

output
```
-1/4*Log[1 + Tan[(d + e*x)/2]]/(a^2*e) - (a*Cos[d + e*x] - a*Sin[d + e*x])
/(4*e*(a^3 + a^3*Cos[d + e*x] + a^3*Sin[d + e*x]))
```

3.371.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

rule 3608
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Simp[
1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c
*(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x
] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] &&
NeQ[n, -3/2]
```

3.371.4 Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.64

method	result	size
derivativedivides	$\frac{\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-2\ln\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)-\frac{2}{1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}}{8e\,a^{2}}$	48
default	$\frac{\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-2\ln\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)-\frac{2}{1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}}{8e\,a^{2}}$	48
norman	$\frac{\frac{\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}}{8ae}-\frac{3}{8ae}}{a\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}-\frac{\ln\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}{4a^{2}e}$	67
parallelrisch	$\frac{\left(-2\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-2\right)\ln\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}+3\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{8e\,a^{2}\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}$	71
risch	$\frac{\left(-\frac{1}{4}+\frac{i}{4}\right)\left(e^{i(ex+d)}+1+i\right)}{a^{2}e\left(ie^{i(ex+d)}+e^{2i(ex+d)}+i+e^{i(ex+d)}\right)}-\frac{\ln\left(e^{i(ex+d)}+i\right)}{4a^{2}e}+\frac{\ln\left(e^{i(ex+d)}+1\right)}{4a^{2}e}$	101

input
```
int(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))^2,x,method=_RETURNVERBOSE)
```

output
```
1/8/e/a^2*(tan(1/2*e*x+1/2*d)-2*ln(1+tan(1/2*e*x+1/2*d))-2/(1+tan(1/2*e*x+
1/2*d)))
```

3.371.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.33

$$\int \frac{1}{(2a+2a\cos(d+ex)+2a\sin(d+ex))^2}\,dx$$
$$= \frac{(\cos(ex+d)+\sin(ex+d)+1)\log\left(\frac{1}{2}\cos(ex+d)+\frac{1}{2}\right)-(\cos(ex+d)+\sin(ex+d)+1)\log(\sin(ex-}{8\left(a^2e\cos(ex+d)+a^2e\sin(ex+d)+a^2e\right)}$$

input
```
integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))^2,x, algorithm="fricas")
```

output
```
1/8*((cos(e*x + d) + sin(e*x + d) + 1)*log(1/2*cos(e*x + d) + 1/2) - (cos(
e*x + d) + sin(e*x + d) + 1)*log(sin(e*x + d) + 1) - 2*cos(e*x + d) + 2*si
n(e*x + d))/(a^2*e*cos(e*x + d) + a^2*e*sin(e*x + d) + a^2*e)
```

3.371.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(65) = 130.

Time = 0.83 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.24

$$\int \frac{1}{(2a+2a\cos(d+ex)+2a\sin(d+ex))^2}\,dx = \begin{cases} -\frac{2\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+1\right)\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{8a^2e\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+8a^2e} - \frac{2\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+1\right)}{8a^2e\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+8a^2e} + \frac{\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)}{8a^2e\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+8a^2e} - \frac{3}{8a^2e\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+8a^2e} & \text{for } e \neq 0 \\ \frac{x}{(2a\sin(d)+2a\cos(d)+2a)^2} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))**2,x)
```

output
```
Piecewise((-2*log(tan(d/2 + e*x/2) + 1)*tan(d/2 + e*x/2)/(8*a**2*e*tan(d/2
 + e*x/2) + 8*a**2*e) - 2*log(tan(d/2 + e*x/2) + 1)/(8*a**2*e*tan(d/2 + e*
x/2) + 8*a**2*e) + tan(d/2 + e*x/2)**2/(8*a**2*e*tan(d/2 + e*x/2) + 8*a**2
*e) - 3/(8*a**2*e*tan(d/2 + e*x/2) + 8*a**2*e), Ne(e, 0)), (x/(2*a*sin(d)
+ 2*a*cos(d) + 2*a)**2, True))
```

3.371.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.07

$$\int \frac{1}{(2a+2a\cos(d+ex)+2a\sin(d+ex))^2}\,dx = -\frac{\frac{2}{a^2+\frac{a^2\sin(ex+d)}{\cos(ex+d)+1}} + \frac{2\log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}+1\right)}{a^2} - \frac{\sin(ex+d)}{a^2(\cos(ex+d)+1)}}{8e}$$

input
```
integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))^2,x, algorithm="maxima")
```

output
```
-1/8*(2/(a^2 + a^2*sin(e*x + d)/(cos(e*x + d) + 1)) + 2*log(sin(e*x + d)/(
cos(e*x + d) + 1) + 1)/a^2 - sin(e*x + d)/(a^2*(cos(e*x + d) + 1)))/e
```

3.371.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.87

$$\int \frac{1}{(2a + 2a\cos(d + ex) + 2a\sin(d + ex))^2}\,dx = -\frac{\frac{2\log\left(\left|\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+1\right|\right)}{a^2} - \frac{\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)}{a^2} - \frac{2\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)}{a^2\left(\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+1\right)}}{8e}$$

input
```
integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))^2,x, algorithm="giac")
```

output
```
-1/8*(2*log(abs(tan(1/2*e*x + 1/2*d) + 1))/a^2 - tan(1/2*e*x + 1/2*d)/a^2
- 2*tan(1/2*e*x + 1/2*d)/(a^2*(tan(1/2*e*x + 1/2*d) + 1)))/e
```

3.371.9 Mupad [B] (verification not implemented)

Time = 27.88 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.79

$$\int \frac{1}{(2a + 2a\cos(d + ex) + 2a\sin(d + ex))^2}\,dx = \frac{\tan\left(\frac{d}{2} + \frac{e\,x}{2}\right)}{8\,a^2\,e} - \frac{\ln\left(\tan\left(\frac{d}{2} + \frac{e\,x}{2}\right) + 1\right)}{4\,a^2\,e} - \frac{1}{4\,a^2\,e\,\left(\tan\left(\frac{d}{2} + \frac{e\,x}{2}\right) + 1\right)}$$

input
```
int(1/(2*a + 2*a*cos(d + e*x) + 2*a*sin(d + e*x))^2,x)
```

output
```
tan(d/2 + (e*x)/2)/(8*a^2*e) - log(tan(d/2 + (e*x)/2) + 1)/(4*a^2*e) - 1/(
4*a^2*e*(tan(d/2 + (e*x)/2) + 1))
```

3.372 $\int \frac{1}{(2a+2a\cos(d+ex)+2a\sin(d+ex))^3}\,dx$

3.372.1 Optimal result

Integrand size = 24, antiderivative size = 123

$$\int \frac{1}{(2a+2a\cos(d+ex)+2a\sin(d+ex))^3}\,dx = \frac{\log\left(1+\tan\left(\frac{1}{2}(d+ex)\right)\right)}{4a^3e} - \frac{a\cos(d+ex)-a\sin(d+ex)}{16e\left(a^2+a^2\cos(d+ex)+a^2\sin(d+ex)\right)^2} + \frac{3(\cos(d+ex)-\sin(d+ex))}{16e\left(a^3+a^3\cos(d+ex)+a^3\sin(d+ex)\right)}$$

output
```
1/4*ln(1+tan(1/2*e*x+1/2*d))/a^3/e+1/16*(-a*cos(e*x+d)+a*sin(e*x+d))/e/(a^
2+a^2*cos(e*x+d)+a^2*sin(e*x+d))^2+3/16*(cos(e*x+d)-sin(e*x+d))/e/(a^3+a^3
*cos(e*x+d)+a^3*sin(e*x+d))
```

3.372.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.10

$$\int \frac{1}{(2a+2a\cos(d+ex)+2a\sin(d+ex))^3}\,dx = \frac{\sec^2\left(\frac{1}{2}(d+ex)\right)+2\Big(-8\log\left(\cos\left(\frac{1}{2}(d+ex)\right)\right)+8\log\left(\cos\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{1}{2}(d+ex)\right)\right)-\frac{\quad}{(\cos\left(\frac{1}{2}(d+ex)\right)}}{64a^3e}$$

input
```
Integrate[(2*a + 2*a*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-3),x]
```

output
```
(Sec[(d + e*x)/2]^2 + 2*(-8*Log[Cos[(d + e*x)/2]] + 8*Log[Cos[(d + e*x)/2]
 + Sin[(d + e*x)/2]] - (Cos[(d + e*x)/2] + Sin[(d + e*x)/2])^(-2) - (6*Sin
[(d + e*x)/2])/(Cos[(d + e*x)/2] + Sin[(d + e*x)/2]) - 3*Tan[(d + e*x)/2])
)/(64*a^3*e)
```

3.372.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3608, 27, 3042, 3632, 3042, 3603, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2a\sin(d+ex)+2a\cos(d+ex)+2a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(2a\sin(d+ex)+2a\cos(d+ex)+2a)^3}dx$$

$$\Big\downarrow \text{3608}$$

$$\frac{\int -\frac{-\cos(d+ex)a-\sin(d+ex)a+2a}{2(\cos(d+ex)a+\sin(d+ex)a+a)^2}dx}{8a^2}-\frac{a\cos(d+ex)-a\sin(d+ex)}{16e\left(a^2\sin(d+ex)+a^2\cos(d+ex)+a^2\right)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{-\cos(d+ex)a-\sin(d+ex)a+2a}{(\cos(d+ex)a+\sin(d+ex)a+a)^2}dx}{16a^2}-\frac{a\cos(d+ex)-a\sin(d+ex)}{16e\left(a^2\sin(d+ex)+a^2\cos(d+ex)+a^2\right)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{-\cos(d+ex)a-\sin(d+ex)a+2a}{(\cos(d+ex)a+\sin(d+ex)a+a)^2}dx}{16a^2}-\frac{a\cos(d+ex)-a\sin(d+ex)}{16e\left(a^2\sin(d+ex)+a^2\cos(d+ex)+a^2\right)^2}$$

$$\Big\downarrow \text{3632}$$

$$-\frac{-4\int \frac{1}{\cos(d+ex)a+\sin(d+ex)a+a}dx-\frac{3\left(a^2\cos(d+ex)-a^2\sin(d+ex)\right)}{e\left(a^3\sin(d+ex)+a^3\cos(d+ex)+a^3\right)}}{16a^2}-\frac{a\cos(d+ex)-a\sin(d+ex)}{16e\left(a^2\sin(d+ex)+a^2\cos(d+ex)+a^2\right)^2}$$

$$\big\downarrow \text{3042}$$

$$-\frac{-4\int \frac{1}{\cos(d+ex)a+\sin(d+ex)a+a}dx - \frac{3(a^2\cos(d+ex)-a^2\sin(d+ex))}{e(a^3\sin(d+ex)+a^3\cos(d+ex)+a^3)}}{16a^2} - \frac{a\cos(d+ex)-a\sin(d+ex)}{16e\left(a^2\sin(d+ex)+a^2\cos(d+ex)+a^2\right)^2}$$

$$\big\downarrow \text{3603}$$

$$-\frac{-\frac{8\int \frac{1}{2\tan\left(\frac{1}{2}(d+ex)\right)a+2a}d\tan\left(\frac{1}{2}(d+ex)\right)}{e} - \frac{3(a^2\cos(d+ex)-a^2\sin(d+ex))}{e(a^3\sin(d+ex)+a^3\cos(d+ex)+a^3)}}{16a^2} - \frac{a\cos(d+ex)-a\sin(d+ex)}{16e\left(a^2\sin(d+ex)+a^2\cos(d+ex)+a^2\right)^2}$$

$$\big\downarrow \text{16}$$

$$-\frac{a\cos(d+ex)-a\sin(d+ex)}{16e\left(a^2\sin(d+ex)+a^2\cos(d+ex)+a^2\right)^2} - \frac{-\frac{3(a^2\cos(d+ex)-a^2\sin(d+ex))}{e(a^3\sin(d+ex)+a^3\cos(d+ex)+a^3)} - \frac{4\log\left(\tan\left(\frac{1}{2}(d+ex)\right)+1\right)}{ae}}{16a^2}$$

input
```
Int[(2*a + 2*a*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-3),x]
```

output
```
-1/16*(a*Cos[d + e*x] - a*Sin[d + e*x])/(e*(a^2 + a^2*Cos[d + e*x] + a^2*S
in[d + e*x])^2) - ((-4*Log[1 + Tan[(d + e*x)/2]])/(a*e) - (3*(a^2*Cos[d +
e*x] - a^2*Sin[d + e*x]))/(e*(a^3 + a^3*Cos[d + e*x] + a^3*Sin[d + e*x])))
/(16*a^2)
```

3.372.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.372. $\int \frac{1}{(2a+2a\cos(d+ex)+2a\sin(d+ex))^3}\,dx$

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

rule 3608
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Simp[
1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c
*(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x
] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] &&
NeQ[n, -3/2]
```

rule 3632
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Simp[(a*A - b*B - c*C)/(a^2 - b^2 - c^2)   Int[1/(a + b*Cos[d + e*x] + c*S
in[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
 - c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

3.372.4 Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.63

method	result	size
derivativedivides	$\frac{\frac{\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}}{2}-3\tan\left(\frac{ex}{2}+\frac{d}{2}\right)+\frac{8}{1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}+8\ln\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)-\frac{2}{\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)^{2}}}{32e\,a^{3}}$	78
default	$\frac{\frac{\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}}{2}-3\tan\left(\frac{ex}{2}+\frac{d}{2}\right)+\frac{8}{1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}+8\ln\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)-\frac{2}{\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)^{2}}}{32e\,a^{3}}$	78
parallelrisch	$\frac{-12+16(\sin(ex+d)+1)\ln\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}+(-5\cos(ex+d)-9)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)+12\cos(ex+d)}{64(\sin(ex+d)+1)a^{3}e}$	82
norman	$\frac{-\frac{\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{3}}{16ae}+\frac{\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{4}}{64ae}+\frac{23}{64ae}+\frac{\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{2ae}}{a^{2}\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)^{2}}+\frac{\ln\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}{4a^{3}e}$	103
risch	$\frac{\left(\frac{1}{16}-\frac{i}{16}\right)\left(6i\,e^{2i(ex+d)}+4\,e^{3i(ex+d)}+8i\,e^{i(ex+d)}+6\,e^{2i(ex+d)}-3+3i\right)}{a^{3}e\left(i\,e^{i(ex+d)}+e^{2i(ex+d)}+i+e^{i(ex+d)}\right)^{2}}-\frac{\ln\left(e^{i(ex+d)}+1\right)}{4a^{3}e}+\frac{\ln\left(e^{i(ex+d)}+i\right)}{4a^{3}e}$	138

3.372. $\int\frac{1}{(2a+2a\cos(d+ex)+2a\sin(d+ex))^{3}}\,dx$

input
```
int(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))^3,x,method=_RETURNVERBOSE)
```

output
```
1/32/e/a^3*(1/2*tan(1/2*e*x+1/2*d)^2-3*tan(1/2*e*x+1/2*d)+8/(1+tan(1/2*e*x
+1/2*d))+8*ln(1+tan(1/2*e*x+1/2*d))-2/(1+tan(1/2*e*x+1/2*d))^2)
```

3.372.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.16

$$\int \frac{1}{(2a+2a\cos(d+ex)+2a\sin(d+ex))^3}\,dx$$
$$= \frac{6\cos(ex+d)^2 - 4\left((\cos(ex+d)+1)\sin(ex+d)+\cos(ex+d)+1\right)\log\left(\frac{1}{2}\cos(ex+d)+\frac{1}{2}\right)+4\left((\cos\right.}{32\left(a^3e\cos(ex+d)+a^3e+(a^3e\right.}$$

input
```
integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))^3,x, algorithm="fricas")
```

output
```
1/32*(6*cos(e*x + d)^2 - 4*((cos(e*x + d) + 1)*sin(e*x + d) + cos(e*x + d)
 + 1)*log(1/2*cos(e*x + d) + 1/2) + 4*((cos(e*x + d) + 1)*sin(e*x + d) + c
os(e*x + d) + 1)*log(sin(e*x + d) + 1) + 2*cos(e*x + d) - 2*sin(e*x + d) -
 3)/(a^3*e*cos(e*x + d) + a^3*e + (a^3*e*cos(e*x + d) + a^3*e)*sin(e*x + d
))
```

3.372.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 423 vs. 2(112) = 224.

Time = 2.61 (sec) , antiderivative size = 423, normalized size of antiderivative = 3.44

$$\int \frac{1}{(2a+2a\cos(d+ex)+2a\sin(d+ex))^3}\,dx$$
$$= \begin{cases} \frac{16\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+1\right)\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)}{64a^3e\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)+128a^3e\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+64a^3e} + \frac{32\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+1\right)\tan\left(\frac{d}{2}+\frac{ex}{2}\right)}{64a^3e\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)+128a^3e\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+64a^3e} + \frac{16\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right.\right.}{64a^3e\tan^2\left(\frac{d}{2}+\frac{ex}{2}\right)+128a^3e\tan\right.} \\ \frac{x}{(2a\sin(d)+2a\cos(d)+2a)^3} \end{cases}$$

input
```
integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))**3,x)
```

output
```
Piecewise((16*log(tan(d/2 + e*x/2) + 1)*tan(d/2 + e*x/2)**2/(64*a**3*e*tan
(d/2 + e*x/2)**2 + 128*a**3*e*tan(d/2 + e*x/2) + 64*a**3*e) + 32*log(tan(d
/2 + e*x/2) + 1)*tan(d/2 + e*x/2)/(64*a**3*e*tan(d/2 + e*x/2)**2 + 128*a**
3*e*tan(d/2 + e*x/2) + 64*a**3*e) + 16*log(tan(d/2 + e*x/2) + 1)/(64*a**3*
e*tan(d/2 + e*x/2)**2 + 128*a**3*e*tan(d/2 + e*x/2) + 64*a**3*e) + tan(d/2
 + e*x/2)**4/(64*a**3*e*tan(d/2 + e*x/2)**2 + 128*a**3*e*tan(d/2 + e*x/2)
+ 64*a**3*e) - 4*tan(d/2 + e*x/2)**3/(64*a**3*e*tan(d/2 + e*x/2)**2 + 128*
a**3*e*tan(d/2 + e*x/2) + 64*a**3*e) + 32*tan(d/2 + e*x/2)/(64*a**3*e*tan(
d/2 + e*x/2)**2 + 128*a**3*e*tan(d/2 + e*x/2) + 64*a**3*e) + 23/(64*a**3*e
*tan(d/2 + e*x/2)**2 + 128*a**3*e*tan(d/2 + e*x/2) + 64*a**3*e), Ne(e, 0))
, (x/(2*a*sin(d) + 2*a*cos(d) + 2*a)**3, True))
```

3.372.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.19

$$\int \frac{1}{(2a+2a\cos(d+ex)+2a\sin(d+ex))^3}\,dx = \frac{\frac{4\left(\frac{4\sin(ex+d)}{\cos(ex+d)+1}+3\right)}{a^3+\frac{2a^3\sin(ex+d)}{\cos(ex+d)+1}+\frac{a^3\sin(ex+d)^2}{(\cos(ex+d)+1)^2}} - \frac{\frac{6\sin(ex+d)}{\cos(ex+d)+1}-\frac{\sin(ex+d)^2}{(\cos(ex+d)+1)^2}}{a^3} + \frac{16\log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}+1\right)}{a^3}}{64\,e}$$

input
```
integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))^3,x, algorithm="maxima")
```

output
```
1/64*(4*(4*sin(e*x + d)/(cos(e*x + d) + 1) + 3)/(a^3 + 2*a^3*sin(e*x + d)/
(cos(e*x + d) + 1) + a^3*sin(e*x + d)^2/(cos(e*x + d) + 1)^2) - (6*sin(e*x
 + d)/(cos(e*x + d) + 1) - sin(e*x + d)^2/(cos(e*x + d) + 1)^2)/a^3 + 16*l
og(sin(e*x + d)/(cos(e*x + d) + 1) + 1)/a^3)/e
```

3.372.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.83

$$\int \frac{1}{(2a+2a\cos(d+ex)+2a\sin(d+ex))^3}\,dx = \frac{\frac{16\log\left(\left|\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+1\right|\right)}{a^3} - \frac{4\left(6\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^2+8\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+3\right)}{a^3\left(\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+1\right)^2} + \frac{a^3\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^2-6a^3\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)}{a^6}}{64\,e}$$

input
```
integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))^3,x, algorithm="giac")
```

output
```
1/64*(16*log(abs(tan(1/2*e*x + 1/2*d) + 1))/a^3 - 4*(6*tan(1/2*e*x + 1/2*d
)^2 + 8*tan(1/2*e*x + 1/2*d) + 3)/(a^3*(tan(1/2*e*x + 1/2*d) + 1)^2) + (a^
3*tan(1/2*e*x + 1/2*d)^2 - 6*a^3*tan(1/2*e*x + 1/2*d))/a^6)/e
```

3.372.9 Mupad [B] (verification not implemented)

Time = 27.89 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.73

$$\int \frac{1}{(2a+2a\cos(d+ex)+2a\sin(d+ex))^3}\,dx$$
$$=\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2}{64\,a^3\,e}+\frac{\ln\left(\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)+1\right)}{4\,a^3\,e}-\frac{3\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{32\,a^3\,e}+\frac{\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{4}+\frac{3}{16}}{a^3\,e\left(\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)+1\right)^2}$$

input
```
int(1/(2*a + 2*a*cos(d + e*x) + 2*a*sin(d + e*x))^3,x)
```

output
```
tan(d/2 + (e*x)/2)^2/(64*a^3*e) + log(tan(d/2 + (e*x)/2) + 1)/(4*a^3*e) -
(3*tan(d/2 + (e*x)/2))/(32*a^3*e) + (tan(d/2 + (e*x)/2)/4 + 3/16)/(a^3*e*(
tan(d/2 + (e*x)/2) + 1)^2)
```

3.373 $\int \frac{1}{(2a+2a\cos(d+ex)+2a\sin(d+ex))^4}\,dx$

3.373.1 Optimal result

Integrand size = 24, antiderivative size = 168

$$\begin{aligned}\int \frac{1}{(2a+2a\cos(d+ex)+2a\sin(d+ex))^4}\,dx &= -\frac{\log\left(1+\tan\left(\frac{1}{2}(d+ex)\right)\right)}{4a^4e} - \frac{\cos(d+ex)-\sin(d+ex)}{48ae(a+a\cos(d+ex)+a\sin(d+ex))^3} \\ &\quad + \frac{5(\cos(d+ex)-\sin(d+ex))}{96e\left(a^2+a^2\cos(d+ex)+a^2\sin(d+ex)\right)^2} \\ &\quad - \frac{19(a\cos(d+ex)-a\sin(d+ex))}{96e\left(a^5+a^5\cos(d+ex)+a^5\sin(d+ex)\right)}\end{aligned}$$

output
```
-1/4*ln(1+tan(1/2*e*x+1/2*d))/a^4/e+1/48*(-cos(e*x+d)+sin(e*x+d))/a/e/(a+a
*cos(e*x+d)+a*sin(e*x+d))^3+5/96*(cos(e*x+d)-sin(e*x+d))/e/(a^2+a^2*cos(e*
x+d)+a^2*sin(e*x+d))^2-19/96*(a*cos(e*x+d)-a*sin(e*x+d))/e/(a^5+a^5*cos(e*
x+d)+a^5*sin(e*x+d))
```

3.373.2 Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.47

$$\int \frac{1}{(2a+2a\cos(d+ex)+2a\sin(d+ex))^4}\,dx = \frac{\log\left(\cos\left(\frac{1}{2}(d+ex)\right)\right)}{4a^4e} - \frac{\log\left(\cos\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{1}{2}(d+ex)\right)\right)}{4a^4e} - \frac{\sec^2\left(\frac{1}{2}(d+ex)\right)}{64a^4e} + \frac{\sin\left(\frac{1}{2}(d+ex)\right)}{96a^4e\left(\cos\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{1}{2}(d+ex)\right)\right)^3} + \frac{5}{192a^4e\left(\cos\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{1}{2}(d+ex)\right)\right)^2} + \frac{19\sin\left(\frac{1}{2}(d+ex)\right)}{96a^4e\left(\cos\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{1}{2}(d+ex)\right)\right)} + \frac{19\tan\left(\frac{1}{2}(d+ex)\right)}{192a^4e} + \frac{\sec^2\left(\frac{1}{2}(d+ex)\right)\tan\left(\frac{1}{2}(d+ex)\right)}{384a^4e}$$

input
```
Integrate[(2*a + 2*a*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-4),x]
```

output
```
Log[Cos[(d + e*x)/2]]/(4*a^4*e) - Log[Cos[(d + e*x)/2] + Sin[(d + e*x)/2]]
/(4*a^4*e) - Sec[(d + e*x)/2]^2/(64*a^4*e) + Sin[(d + e*x)/2]/(96*a^4*e*(C
os[(d + e*x)/2] + Sin[(d + e*x)/2])^3) + 5/(192*a^4*e*(Cos[(d + e*x)/2] +
Sin[(d + e*x)/2])^2) + (19*Sin[(d + e*x)/2])/(96*a^4*e*(Cos[(d + e*x)/2] +
 Sin[(d + e*x)/2])) + (19*Tan[(d + e*x)/2])/(192*a^4*e) + (Sec[(d + e*x)/2
]^2*Tan[(d + e*x)/2])/(384*a^4*e)
```

3.373.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.12, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {3042, 3608, 27, 3042, 3635, 25, 3042, 3632, 3042, 3603, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2a\sin(d+ex)+2a\cos(d+ex)+2a)^4}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(2a\sin(d+ex)+2a\cos(d+ex)+2a)^4}dx$$

$\Big\downarrow$ 3608

$$\frac{\int -\frac{-2\cos(d+ex)a-2\sin(d+ex)a+3a}{4(\cos(d+ex)a+\sin(d+ex)a+a)^3}dx}{12a^2}-\frac{\cos(d+ex)-\sin(d+ex)}{48ae(a\sin(d+ex)+a\cos(d+ex)+a)^3}$$

$\Big\downarrow$ 27

$$-\frac{\int \frac{-2\cos(d+ex)a-2\sin(d+ex)a+3a}{(\cos(d+ex)a+\sin(d+ex)a+a)^3}dx}{48a^2}-\frac{\cos(d+ex)-\sin(d+ex)}{48ae(a\sin(d+ex)+a\cos(d+ex)+a)^3}$$

$\Big\downarrow$ 3042

$$-\frac{\int \frac{-2\cos(d+ex)a-2\sin(d+ex)a+3a}{(\cos(d+ex)a+\sin(d+ex)a+a)^3}dx}{48a^2}-\frac{\cos(d+ex)-\sin(d+ex)}{48ae(a\sin(d+ex)+a\cos(d+ex)+a)^3}$$

$\Big\downarrow$ 3635

$$-\frac{\frac{\int -\frac{-5\cos(d+ex)a^2-5\sin(d+ex)a^2+14a^2}{(\cos(d+ex)a+\sin(d+ex)a+a)^2}dx}{2a^2}-\frac{5(a^2\cos(d+ex)-a^2\sin(d+ex))}{2e(a^2\sin(d+ex)+a^2\cos(d+ex)+a^2)^2}}{48a^2}-\frac{\cos(d+ex)-\sin(d+ex)}{48ae(a\sin(d+ex)+a\cos(d+ex)+a)^3}$$

$\Big\downarrow$ 25

$$-\frac{-\frac{\int \frac{-5\cos(d+ex)a^2-5\sin(d+ex)a^2+14a^2}{(\cos(d+ex)a+\sin(d+ex)a+a)^2}dx}{2a^2}-\frac{5(a^2\cos(d+ex)-a^2\sin(d+ex))}{2e(a^2\sin(d+ex)+a^2\cos(d+ex)+a^2)^2}}{48a^2}-\frac{\cos(d+ex)-\sin(d+ex)}{48ae(a\sin(d+ex)+a\cos(d+ex)+a)^3}$$

$\Big\downarrow$ 3042

$$-\frac{-\frac{\int \frac{-5\cos(d+ex)a^2-5\sin(d+ex)a^2+14a^2}{(\cos(d+ex)a+\sin(d+ex)a+a)^2}dx}{2a^2}-\frac{5(a^2\cos(d+ex)-a^2\sin(d+ex))}{2e(a^2\sin(d+ex)+a^2\cos(d+ex)+a^2)^2}}{48a^2}-\frac{\cos(d+ex)-\sin(d+ex)}{48ae(a\sin(d+ex)+a\cos(d+ex)+a)^3}$$

$\Big\downarrow$ 3632

$$-\frac{-\frac{-24a\int \frac{1}{\cos(d+ex)a+\sin(d+ex)a+a}dx-\frac{19\left(a^3\cos(d+ex)-a^3\sin(d+ex)\right)}{e\left(a^3\sin(d+ex)+a^3\cos(d+ex)+a^3\right)}}{2a^2}-\frac{5(a^2\cos(d+ex)-a^2\sin(d+ex))}{2e(a^2\sin(d+ex)+a^2\cos(d+ex)+a^2)^2}}{48a^2}-\frac{\cos(d+ex)-\sin(d+ex)}{48ae(a\sin(d+ex)+a\cos(d+ex)+a)^3}$$

$\Big\downarrow$ 3042

3.373. $\int \frac{1}{(2a+2a\cos(d+ex)+2a\sin(d+ex))^4}\,dx$

$$-\frac{-\frac{-24a\int \frac{1}{\cos(d+ex)a+\sin(d+ex)a+a}dx-\frac{19\left(a^3\cos(d+ex)-a^3\sin(d+ex)\right)}{e\left(a^3\sin(d+ex)+a^3\cos(d+ex)+a^3\right)}}{2a^2}-\frac{5(a^2\cos(d+ex)-a^2\sin(d+ex))}{2e(a^2\sin(d+ex)+a^2\cos(d+ex)+a^2)^2}}{48a^2}-\frac{\cos(d+ex)-\sin(d+ex)}{48ae(a\sin(d+ex)+a\cos(d+ex)+a)^3}$$

$$\Big\downarrow \; 3603$$

$$-\frac{-\frac{-\frac{48a\int \frac{1}{2\tan\left(\frac{1}{2}(d+ex)\right)a+2a}d\tan\left(\frac{1}{2}(d+ex)\right)}{e}-\frac{19\left(a^3\cos(d+ex)-a^3\sin(d+ex)\right)}{e\left(a^3\sin(d+ex)+a^3\cos(d+ex)+a^3\right)}}{2a^2}-\frac{5(a^2\cos(d+ex)-a^2\sin(d+ex))}{2e(a^2\sin(d+ex)+a^2\cos(d+ex)+a^2)^2}}{48a^2}-\frac{\cos(d+ex)-\sin(d+ex)}{48ae(a\sin(d+ex)+a\cos(d+ex)+a)^3}$$

$$\Big\downarrow \; 16$$

$$-\frac{-\frac{5(a^2\cos(d+ex)-a^2\sin(d+ex))}{2e(a^2\sin(d+ex)+a^2\cos(d+ex)+a^2)^2}-\frac{-\frac{19\left(a^3\cos(d+ex)-a^3\sin(d+ex)\right)}{e\left(a^3\sin(d+ex)+a^3\cos(d+ex)+a^3\right)}-\frac{24\log\left(\tan\left(\frac{1}{2}(d+ex)\right)+1\right)}{e}}{2a^2}}{48a^2}-\frac{\cos(d+ex)-\sin(d+ex)}{48ae(a\sin(d+ex)+a\cos(d+ex)+a)^3}$$

input
```
Int[(2*a + 2*a*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-4),x]
```

output
```
-1/48*(Cos[d + e*x] - Sin[d + e*x])/(a*e*(a + a*Cos[d + e*x] + a*Sin[d + e
*x])^3) - ((-5*(a^2*Cos[d + e*x] - a^2*Sin[d + e*x]))/(2*e*(a^2 + a^2*Cos[
d + e*x] + a^2*Sin[d + e*x])^2) - ((-24*Log[1 + Tan[(d + e*x)/2]])/e - (19
*(a^3*Cos[d + e*x] - a^3*Sin[d + e*x]))/(e*(a^3 + a^3*Cos[d + e*x] + a^3*S
in[d + e*x])))/(2*a^2))/(48*a^2)
```

3.373.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

rule 3608
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Simp[
1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c
*(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x
] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] &&
NeQ[n, -3/2]
```

rule 3632
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Simp[(a*A - b*B - c*C)/(a^2 - b^2 - c^2)   Int[1/(a + b*Cos[d + e*x] + c*S
in[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
 - c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

rule 3635
```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(-(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(
a^2 - b^2 - c^2))), x] + Simp[1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a + b*Co
s[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n +
2)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x]
/; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[n, -2]
```

3.373. $\int \frac{1}{(2a+2a\cos(d+ex)+2a\sin(d+ex))^4}\,dx$

3.373.4 Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.63

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^3}{3}-2\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2+13\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-32\ln\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)+\frac{12}{\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)^2}-\frac{36}{1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}-\frac{}{3\left(1+\tan\right.}}{128e\,a^4}$
default	$\frac{\frac{\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^3}{3}-2\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2+13\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-32\ln\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)+\frac{12}{\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)^2}-\frac{36}{1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}-\frac{}{3\left(1+\tan\right.}}{128e\,a^4}$
parallelrisch	$\frac{-96\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)^3\ln\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^6-3\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^5+24\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^4-297\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2-44}{384e\,a^4\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)^3}$
norman	$\frac{\frac{\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^4}{16ae}-\frac{\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^5}{128ae}+\frac{\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^6}{384ae}-\frac{15}{32ae}-\frac{147\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{128ae}-\frac{99\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}{128ae}}{a^3\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)^3}-\frac{\ln\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}{4a^4e}$
risch	$\frac{\left(-\frac{1}{96}+\frac{i}{96}\right)\left(60ie^{4i(ex+d)}+24\,e^{5i(ex+d)}+152ie^{3i(ex+d)}+60\,e^{4i(ex+d)}+111ie^{2i(ex+d)}-111\,e^{2i(ex+d)}-19-19i-90\,e^{i(ex+}\right.}{a^4e\left(ie^{i(ex+d)}+e^{2i(ex+d)}+i+e^{i(ex+d)}\right)^3}$

input
```
int(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))^4,x,method=_RETURNVERBOSE)
```

output
```
1/128/e/a^4*(1/3*tan(1/2*e*x+1/2*d)^3-2*tan(1/2*e*x+1/2*d)^2+13*tan(1/2*e*
x+1/2*d)-32*ln(1+tan(1/2*e*x+1/2*d))+12/(1+tan(1/2*e*x+1/2*d))^2-36/(1+tan
(1/2*e*x+1/2*d))-8/3/(1+tan(1/2*e*x+1/2*d))^3)
```

3.373.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.41

$$\int \frac{1}{(2a+2a\cos(d+ex)+2a\sin(d+ex))^4}\,dx$$
$$= \frac{38\cos(ex+d)^3+66\cos(ex+d)^2+24\left(\cos(ex+d)^3-\left(\cos(ex+d)^2+3\cos(ex+d)+2\right)\sin(ex+d\right.}{}$$

input
```
integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))^4,x, algorithm="fricas")
```

output
```
1/192*(38*cos(e*x + d)^3 + 66*cos(e*x + d)^2 + 24*(cos(e*x + d)^3 - (cos(e
*x + d)^2 + 3*cos(e*x + d) + 2)*sin(e*x + d) - 3*cos(e*x + d) - 2)*log(1/2
*cos(e*x + d) + 1/2) - 24*(cos(e*x + d)^3 - (cos(e*x + d)^2 + 3*cos(e*x +
d) + 2)*sin(e*x + d) - 3*cos(e*x + d) - 2)*log(sin(e*x + d) + 1) + (38*cos
(e*x + d)^2 - 35)*sin(e*x + d) - 3*cos(e*x + d) - 33)/(a^4*e*cos(e*x + d)^
3 - 3*a^4*e*cos(e*x + d) - 2*a^4*e - (a^4*e*cos(e*x + d)^2 + 3*a^4*e*cos(e
*x + d) + 2*a^4*e)*sin(e*x + d))
```

3.373.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 792 vs. $2(156) = 312$.

Time = 9.73 (sec) , antiderivative size = 792, normalized size of antiderivative = 4.71

$$\int \frac{1}{(2a + 2a\cos(d + ex) + 2a\sin(d + ex))^4} \, dx = \text{Too large to display}$$

input
```
integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))**4,x)
```

output
```
Piecewise((-96*log(tan(d/2 + e*x/2) + 1)*tan(d/2 + e*x/2)**3/(384*a**4*e*t
an(d/2 + e*x/2)**3 + 1152*a**4*e*tan(d/2 + e*x/2)**2 + 1152*a**4*e*tan(d/2
 + e*x/2) + 384*a**4*e) - 288*log(tan(d/2 + e*x/2) + 1)*tan(d/2 + e*x/2)**
2/(384*a**4*e*tan(d/2 + e*x/2)**3 + 1152*a**4*e*tan(d/2 + e*x/2)**2 + 1152
*a**4*e*tan(d/2 + e*x/2) + 384*a**4*e) - 288*log(tan(d/2 + e*x/2) + 1)*tan
(d/2 + e*x/2)/(384*a**4*e*tan(d/2 + e*x/2)**3 + 1152*a**4*e*tan(d/2 + e*x/
2)**2 + 1152*a**4*e*tan(d/2 + e*x/2) + 384*a**4*e) - 96*log(tan(d/2 + e*x/
2) + 1)/(384*a**4*e*tan(d/2 + e*x/2)**3 + 1152*a**4*e*tan(d/2 + e*x/2)**2
+ 1152*a**4*e*tan(d/2 + e*x/2) + 384*a**4*e) + tan(d/2 + e*x/2)**6/(384*a*
*4*e*tan(d/2 + e*x/2)**3 + 1152*a**4*e*tan(d/2 + e*x/2)**2 + 1152*a**4*e*t
an(d/2 + e*x/2) + 384*a**4*e) - 3*tan(d/2 + e*x/2)**5/(384*a**4*e*tan(d/2
+ e*x/2)**3 + 1152*a**4*e*tan(d/2 + e*x/2)**2 + 1152*a**4*e*tan(d/2 + e*x/
2) + 384*a**4*e) + 24*tan(d/2 + e*x/2)**4/(384*a**4*e*tan(d/2 + e*x/2)**3
+ 1152*a**4*e*tan(d/2 + e*x/2)**2 + 1152*a**4*e*tan(d/2 + e*x/2) + 384*a**
4*e) - 297*tan(d/2 + e*x/2)**2/(384*a**4*e*tan(d/2 + e*x/2)**3 + 1152*a**4
*e*tan(d/2 + e*x/2)**2 + 1152*a**4*e*tan(d/2 + e*x/2) + 384*a**4*e) - 441*
tan(d/2 + e*x/2)/(384*a**4*e*tan(d/2 + e*x/2)**3 + 1152*a**4*e*tan(d/2 + e
*x/2)**2 + 1152*a**4*e*tan(d/2 + e*x/2) + 384*a**4*e) - 180/(384*a**4*e*ta
n(d/2 + e*x/2)**3 + 1152*a**4*e*tan(d/2 + e*x/2)**2 + 1152*a**4*e*tan(d/2
+ e*x/2) + 384*a**4*e), Ne(e, 0)), (x/(2*a*sin(d) + 2*a*cos(d) + 2*a)**...
```

3.373.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.24

$$\int \frac{1}{(2a+2a\cos(d+ex)+2a\sin(d+ex))^4}\,dx = -\frac{\frac{4\left(\frac{45\sin(ex+d)}{\cos(ex+d)+1}+\frac{27\sin(ex+d)^2}{(\cos(ex+d)+1)^2}+20\right)}{a^4+\frac{3a^4\sin(ex+d)}{\cos(ex+d)+1}+\frac{3a^4\sin(ex+d)^2}{(\cos(ex+d)+1)^2}+\frac{a^4\sin(ex+d)^3}{(\cos(ex+d)+1)^3}}-\frac{\frac{39\sin(ex+d)}{\cos(ex+d)+1}-\frac{6\sin(ex+d)^2}{(\cos(ex+d)+1)^2}+\frac{\sin(ex+d)^3}{(\cos(ex+d)+1)^3}}{a^4}+\frac{96\log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}+1\right)}{a^4}}{384\,e}$$

input
```
integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))^4,x, algorithm="maxima")
```

output
```
-1/384*(4*(45*sin(e*x + d)/(cos(e*x + d) + 1) + 27*sin(e*x + d)^2/(cos(e*x
 + d) + 1)^2 + 20)/(a^4 + 3*a^4*sin(e*x + d)/(cos(e*x + d) + 1) + 3*a^4*si
n(e*x + d)^2/(cos(e*x + d) + 1)^2 + a^4*sin(e*x + d)^3/(cos(e*x + d) + 1)^
3) - (39*sin(e*x + d)/(cos(e*x + d) + 1) - 6*sin(e*x + d)^2/(cos(e*x + d)
+ 1)^2 + sin(e*x + d)^3/(cos(e*x + d) + 1)^3)/a^4 + 96*log(sin(e*x + d)/(c
os(e*x + d) + 1) + 1)/a^4)/e
```

3.373.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.79

$$\int \frac{1}{(2a+2a\cos(d+ex)+2a\sin(d+ex))^4}\,dx = -\frac{\frac{96\log\left(\left|\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+1\right|\right)}{a^4}-\frac{4\left(44\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^3+105\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^2+87\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+24\right)}{a^4\left(\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+1\right)^3}-\frac{a^8\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^3-6a^8\tan\left(\frac{1}{2}e\right.}{a^{1}}}{384\,e}$$

input
```
integrate(1/(2*a+2*a*cos(e*x+d)+2*a*sin(e*x+d))^4,x, algorithm="giac")
```

output
```
-1/384*(96*log(abs(tan(1/2*e*x + 1/2*d) + 1))/a^4 - 4*(44*tan(1/2*e*x + 1/
2*d)^3 + 105*tan(1/2*e*x + 1/2*d)^2 + 87*tan(1/2*e*x + 1/2*d) + 24)/(a^4*(
tan(1/2*e*x + 1/2*d) + 1)^3) - (a^8*tan(1/2*e*x + 1/2*d)^3 - 6*a^8*tan(1/2
*e*x + 1/2*d)^2 + 39*a^8*tan(1/2*e*x + 1/2*d))/a^12)/e
```

3.373.9 Mupad [B] (verification not implemented)

Time = 27.12 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.96

$$\int \frac{1}{(2a+2a\cos(d+ex)+2a\sin(d+ex))^4}\,dx = \frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^3}{384\,a^4\,e}-\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2}{64\,a^4\,e}-\frac{\ln\left(\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)+1\right)}{4\,a^4\,e}+\frac{13\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{128\,a^4\,e}-\frac{9\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2+15\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)+\frac{20}{3}}{e\left(32\,a^4\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^3+96\,a^4\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2+96\,a^4\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)+32\,a^4\right)}$$

input
```
int(1/(2*a + 2*a*cos(d + e*x) + 2*a*sin(d + e*x))^4,x)
```

output
```
tan(d/2 + (e*x)/2)^3/(384*a^4*e) - tan(d/2 + (e*x)/2)^2/(64*a^4*e) - log(t
an(d/2 + (e*x)/2) + 1)/(4*a^4*e) + (13*tan(d/2 + (e*x)/2))/(128*a^4*e) - (
15*tan(d/2 + (e*x)/2) + 9*tan(d/2 + (e*x)/2)^2 + 20/3)/(e*(96*a^4*tan(d/2
+ (e*x)/2)^2 + 32*a^4*tan(d/2 + (e*x)/2)^3 + 32*a^4 + 96*a^4*tan(d/2 + (e*
x)/2)))
```

3.374 $\int (2a - 2a\cos(d+ex) + 2c\sin(d+ex))^3 \, dx$

3.374.1 Optimal result

Integrand size = 24, antiderivative size = 157

$$\begin{aligned}
&\int (2a - 2a\cos(d+ex) + 2c\sin(d+ex))^3 \, dx \\
&= 4a(5a^2+3c^2)\,x - \frac{4c(15a^2+4c^2)\cos(d+ex)}{3e} - \frac{4a(15a^2+4c^2)\sin(d+ex)}{3e} \\
&\quad - \frac{20(ac\cos(d+ex)+a^2\sin(d+ex))\,(a-a\cos(d+ex)+c\sin(d+ex))}{3e} \\
&\quad - \frac{8(c\cos(d+ex)+a\sin(d+ex))(a-a\cos(d+ex)+c\sin(d+ex))^2}{3e}
\end{aligned}$$

output
```
4*a*(5*a^2+3*c^2)*x-4/3*c*(15*a^2+4*c^2)*cos(e*x+d)/e-4/3*a*(15*a^2+4*c^2)
*sin(e*x+d)/e-20/3*(a*c*cos(e*x+d)+a^2*sin(e*x+d))*(a-a*cos(e*x+d)+c*sin(e
*x+d))/e-8/3*(c*cos(e*x+d)+a*sin(e*x+d))*(a-a*cos(e*x+d)+c*sin(e*x+d))^2/e
```

3.374.2 Mathematica [A] (verified)

Time = 1.73 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.87

$$\begin{aligned}
&\int (2a - 2a\cos(d+ex) + 2c\sin(d+ex))^3 \, dx \\
&= \frac{2(6a(5a^2+3c^2)(d+ex) - 9c(5a^2+c^2)\cos(d+ex) + 18a^2c\cos(2(d+ex)) + c(-3a^2+c^2)\cos(3(d+ex)}{3e}
\end{aligned}$$

input
```
Integrate[(2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^3,x]
```

output
```
(2*(6*a*(5*a^2 + 3*c^2)*(d + e*x) - 9*c*(5*a^2 + c^2)*Cos[d + e*x] + 18*a^
2*c*Cos[2*(d + e*x)] + c*(-3*a^2 + c^2)*Cos[3*(d + e*x)] - 9*a*(5*a^2 + c^
2)*Sin[d + e*x] + 9*a*(a^2 - c^2)*Sin[2*(d + e*x)] - a*(a^2 - 3*c^2)*Sin[3
*(d + e*x)]))/(3*e)
```

3.374.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3599, 27, 3042, 3625, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (-2a\cos(d+ex)+2a+2c\sin(d+ex))^3\,dx$$

$$\Big\downarrow \ 3042$$

$$\int (-2a\cos(d+ex)+2a+2c\sin(d+ex))^3dx$$

$$\Big\downarrow \ 3599$$

$$\frac{1}{3}\int 8(-\cos(d+ex)a+a+c\sin(d+ex))\left(-5\cos(d+ex)a^2+5a^2+5c\sin(d+ex)a+2c^2\right)dx-\frac{8(a\sin(d+ex)+c\cos(d+ex))(a(-\cos(d+ex))+a+c\sin(d+ex))^2}{3e}$$

$$\Big\downarrow \ 27$$

$$\frac{8}{3}\int (-\cos(d+ex)a+a+c\sin(d+ex))\left(-5\cos(d+ex)a^2+5a^2+5c\sin(d+ex)a+2c^2\right)dx-\frac{8(a\sin(d+ex)+c\cos(d+ex))(a(-\cos(d+ex))+a+c\sin(d+ex))^2}{3e}$$

$$\Big\downarrow \ 3042$$

$$\frac{8}{3}\int (-\cos(d+ex)a+a+c\sin(d+ex))\left(-5\cos(d+ex)a^2+5a^2+5c\sin(d+ex)a+2c^2\right)dx-\frac{8(a\sin(d+ex)+c\cos(d+ex))(a(-\cos(d+ex))+a+c\sin(d+ex))^2}{3e}$$

$$\Big\downarrow \ 3625$$

$$\frac{8}{3}\left(\frac{\int \left(3\left(5a^2+3c^2\right)a^2-\left(15a^2+4c^2\right)\cos(d+ex)a^2+c\left(15a^2+4c^2\right)\sin(d+ex)a\right)dx}{2a}-\frac{5\left(a^2\sin(d+ex)+ac\cos\right.}{}\right.$$
$$\frac{8(a\sin(d+ex)+c\cos(d+ex))(a(-\cos(d+ex))+a+c\sin(d+ex))^2}{3e}$$

$\Big\downarrow$ 2009

$$\frac{8}{3}\left(\frac{-\frac{a^2\left(15a^2+4c^2\right)\sin(d+ex)}{e}-\frac{ac\left(15a^2+4c^2\right)\cos(d+ex)}{e}+3a^2x\left(5a^2+3c^2\right)}{2a}-\frac{5\left(a^2\sin(d+ex)+ac\cos(d+ex)\right)\left(a(-\cos\right.}{2e}\right.$$
$$\frac{8(a\sin(d+ex)+c\cos(d+ex))(a(-\cos(d+ex))+a+c\sin(d+ex))^2}{3e}$$

input
```
Int[(2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^3,x]
```

output
```
(-8*(c*Cos[d + e*x] + a*Sin[d + e*x])*(a - a*Cos[d + e*x] + c*Sin[d + e*x]
)^2)/(3*e) + (8*((-5*(a*c*Cos[d + e*x] + a^2*Sin[d + e*x])*(a - a*Cos[d +
e*x] + c*Sin[d + e*x]))/(2*e) + (3*a^2*(5*a^2 + 3*c^2)*x - (a*c*(15*a^2 +
4*c^2)*Cos[d + e*x])/e - (a^2*(15*a^2 + 4*c^2)*Sin[d + e*x])/e)/(2*a)))/3
```

3.374.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3599
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Simp[1/n   Int[Simp[n*a^2 + (
n - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x
], x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

rule 3625
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*((a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^n/(a*e*(n + 1))), x] + Simp[1/(a*(n + 1
))   Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
 + a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
 + (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

3.374.4 Maple [A] (verified)

Time = 2.09 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.97

method	result
parallelrisch	$\frac{\left(-6a^{2}c+2c^{3}\right)\cos\left(3ex+3d\right)+\left(18a^{3}-18a\,c^{2}\right)\sin\left(2ex+2d\right)+\left(-2a^{3}+6a\,c^{2}\right)\sin\left(3ex+3d\right)+36a^{2}c\cos\left(2ex+2d\right)+\left(-90a^{2}c-\right.}{3e}$
parts	$\frac{-8a^{2}c\cos\left(ex+d\right)^{3}+24a^{3}\left(\frac{\cos\left(ex+d\right)\sin\left(ex+d\right)}{2}+\frac{ex}{2}+\frac{d}{2}\right)}{e}-\frac{8a\left(a+c\sin\left(ex+d\right)\right)^{3}}{ec}+8a^{3}x-\frac{8a^{3}\left(2+\cos\left(ex+d\right)^{2}\right)\sin}{3e}$
derivativedivides	$\frac{-\frac{8a^{3}\left(2+\cos\left(ex+d\right)^{2}\right)\sin\left(ex+d\right)}{3}-8a^{2}c\cos\left(ex+d\right)^{3}+24a^{3}\left(\frac{\cos\left(ex+d\right)\sin\left(ex+d\right)}{2}+\frac{ex}{2}+\frac{d}{2}\right)-8a\,c^{2}\sin\left(ex+d\right)^{3}+24a^{2}c\cos\left(ex-\right.}{}$
default	$\frac{-\frac{8a^{3}\left(2+\cos\left(ex+d\right)^{2}\right)\sin\left(ex+d\right)}{3}-8a^{2}c\cos\left(ex+d\right)^{3}+24a^{3}\left(\frac{\cos\left(ex+d\right)\sin\left(ex+d\right)}{2}+\frac{ex}{2}+\frac{d}{2}\right)-8a\,c^{2}\sin\left(ex+d\right)^{3}+24a^{2}c\cos\left(ex-\right.}{}$
risch	$20a^{3}x+12a\,c^{2}x-\frac{30c\cos\left(ex+d\right)a^{2}}{e}-\frac{6c^{3}\cos\left(ex+d\right)}{e}-\frac{30a^{3}\sin\left(ex+d\right)}{e}-\frac{6a\sin\left(ex+d\right)c^{2}}{e}-\frac{2c\cos\left(3ex+\right.}{e}$
norman	$\frac{-\frac{192a^{2}c+32c^{3}}{3e}+4a\left(5a^{2}+3c^{2}\right)x-\frac{192a^{2}c\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{4}}{e}-\frac{\left(192a^{2}c+32c^{3}\right)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}}{e}-\frac{8a\left(5a^{2}+3c^{2}\right)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{e}-\frac{64a\left(5a^{2}\right.}{}}{}$

input
```
int((2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*((-6*a^2*c+2*c^3)*cos(3*e*x+3*d)+(18*a^3-18*a*c^2)*sin(2*e*x+2*d)+(-2*
a^3+6*a*c^2)*sin(3*e*x+3*d)+36*a^2*c*cos(2*e*x+2*d)+(-90*a^2*c-18*c^3)*cos
(e*x+d)+(-90*a^3-18*a*c^2)*sin(e*x+d)+60*a^3*e*x+36*a*c^2*e*x-132*a^2*c-16
*c^3)/e
```

3.374.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.85

$$\int (2a-2a\cos(d+ex)+2c\sin(d+ex))^3\,dx$$
$$= \frac{4\left(18\,a^2c\cos\left(ex+d\right)^2-2\left(3\,a^2c-c^3\right)\cos\left(ex+d\right)^3+3\left(5\,a^3+3\,ac^2\right)ex-6\left(3\,a^2c+c^3\right)\cos\left(ex+d\right)-(2\right.}{3\,e}$$

input
```
integrate((2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="fricas")
```

output
```
4/3*(18*a^2*c*cos(e*x + d)^2 - 2*(3*a^2*c - c^3)*cos(e*x + d)^3 + 3*(5*a^3
 + 3*a*c^2)*e*x - 6*(3*a^2*c + c^3)*cos(e*x + d) - (22*a^3 + 6*a*c^2 + 2*(
a^3 - 3*a*c^2)*cos(e*x + d)^2 - 9*(a^3 - a*c^2)*cos(e*x + d))*sin(e*x + d)
)/e
```

3.374.6 Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.85

$$\int (2a-2a\cos(d+ex)+2c\sin(d+ex))^3\,dx$$
$$= \begin{cases} 12a^3x\sin^2\left(d+ex\right)+12a^3x\cos^2\left(d+ex\right)+8a^3x-\frac{16a^3\sin^3\left(d+ex\right)}{3e}-\frac{8a^3\sin\left(d+ex\right)\cos^2\left(d+ex\right)}{e}+\frac{12a^3\sin\left(d+ex\right)\mathrm{c}}{e} \\ x(-2a\cos\left(d\right)+2a+2c\sin\left(d\right))^3 \end{cases}$$

input
```
integrate((2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))**3,x)
```

output
```
Piecewise((12*a**3*x*sin(d + e*x)**2 + 12*a**3*x*cos(d + e*x)**2 + 8*a**3*
x - 16*a**3*sin(d + e*x)**3/(3*e) - 8*a**3*sin(d + e*x)*cos(d + e*x)**2/e
+ 12*a**3*sin(d + e*x)*cos(d + e*x)/e - 24*a**3*sin(d + e*x)/e - 24*a**2*c
*sin(d + e*x)**2/e - 8*a**2*c*cos(d + e*x)**3/e - 24*a**2*c*cos(d + e*x)/e
 + 12*a*c**2*x*sin(d + e*x)**2 + 12*a*c**2*x*cos(d + e*x)**2 - 8*a*c**2*si
n(d + e*x)**3/e - 12*a*c**2*sin(d + e*x)*cos(d + e*x)/e - 8*c**3*sin(d + e
*x)**2*cos(d + e*x)/e - 16*c**3*cos(d + e*x)**3/(3*e), Ne(e, 0)), (x*(-2*a
*cos(d) + 2*a + 2*c*sin(d))**3, True))
```

3.374.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.20

$$\int (2a-2a\cos(d+ex)+2c\sin(d+ex))^3\,dx$$
$$=-\frac{8\,a^2c\cos\left(ex+d\right)^3}{e}-\frac{8\,ac^2\sin\left(ex+d\right)^3}{e}+8\,a^3x+\frac{8\left(\sin\left(ex+d\right)^3-3\,\sin\left(ex+d\right)\right)a^3}{3\,e}+\frac{8\left(\cos\left(ex+d\right)^3-3\,\cos\left(ex+d\right)\right)c^3}{3\,e}-24\,a^2\left(\frac{c\cos\left(ex+d\right)}{e}+\frac{a\sin\left(ex+d\right)}{e}\right)+6\left(\frac{4\,ac\cos\left(ex+d\right)^2}{e}+\frac{\left(2\,ex+2\,d+\sin\left(2\,ex+2\,d\right)\right)a^2}{e}+\frac{\left(2\,ex+2\,d-\sin\left(2\,ex+2\,d\right)\right)c^2}{e}\right)a$$

input
```
integrate((2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="maxima")
```

output
```
-8*a^2*c*cos(e*x + d)^3/e - 8*a*c^2*sin(e*x + d)^3/e + 8*a^3*x + 8/3*(sin(
e*x + d)^3 - 3*sin(e*x + d))*a^3/e + 8/3*(cos(e*x + d)^3 - 3*cos(e*x + d))
*c^3/e - 24*a^2*(c*cos(e*x + d)/e + a*sin(e*x + d)/e) + 6*(4*a*c*cos(e*x +
 d)^2/e + (2*e*x + 2*d + sin(2*e*x + 2*d))*a^2/e + (2*e*x + 2*d - sin(2*e*
x + 2*d))*c^2/e)*a
```

3.374.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.96

$$\int (2a-2a\cos(d+ex)+2c\sin(d+ex))^3\,dx$$
$$=\frac{12\,a^2c\cos\left(2\,ex+2\,d\right)}{e}+4\left(5\,a^3+3\,ac^2\right)x-\frac{2\left(3\,a^2c-c^3\right)\cos\left(3\,ex+3\,d\right)}{3\,e}-\frac{6\left(5\,a^2c+c^3\right)\cos\left(ex+d\right)}{e}-\frac{2\left(a^3-3\,ac^2\right)\sin\left(3\,ex+3\,d\right)}{3\,e}+\frac{6\left(a^3-ac^2\right)\sin\left(2\,ex+2\,d\right)}{e}-\frac{6\left(5\,a^3+ac^2\right)\sin\left(ex+d\right)}{e}$$

input
```
integrate((2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="giac")
```

output
```
12*a^2*c*cos(2*e*x + 2*d)/e + 4*(5*a^3 + 3*a*c^2)*x - 2/3*(3*a^2*c - c^3)*
cos(3*e*x + 3*d)/e - 6*(5*a^2*c + c^3)*cos(e*x + d)/e - 2/3*(a^3 - 3*a*c^2
)*sin(3*e*x + 3*d)/e + 6*(a^3 - a*c^2)*sin(2*e*x + 2*d)/e - 6*(5*a^3 + a*c
^2)*sin(e*x + d)/e
```

3.374.9 Mupad [B] (verification not implemented)

Time = 28.03 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.64

$$\int (2a-2a\cos(d+ex)+2c\sin(d+ex))^3\,dx = \frac{8\,a\,\mathrm{atan}\left(\frac{8\,a\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,(5\,a^2+3\,c^2)}{40\,a^3+24\,a\,c^2}\right)\,(5\,a^2+3\,c^2)}{e}$$
$$-\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,(40\,a^3+24\,a\,c^2)+64\,a^2\,c-\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^5\,(24\,a\,c^2-88\,a^3)+\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^3\,\left(\frac{320\,a^3}{3}+64\,a\,c\right.}{e\,\left(\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^6+3\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^4+3\tan\left(\frac{d}{2}\right.\right.}$$
$$-\frac{8\,a\,(5\,a^2+3\,c^2)\,\left(\mathrm{atan}\left(\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\right)-\frac{e\,x}{2}\right)}{e}$$

input
```
int((2*a - 2*a*cos(d + e*x) + 2*c*sin(d + e*x))^3,x)
```

output
```
(8*a*atan((8*a*tan(d/2 + (e*x)/2)*(5*a^2 + 3*c^2))/(24*a*c^2 + 40*a^3))*(5
*a^2 + 3*c^2))/e - (tan(d/2 + (e*x)/2)*(24*a*c^2 + 40*a^3) + 64*a^2*c - ta
n(d/2 + (e*x)/2)^5*(24*a*c^2 - 88*a^3) + tan(d/2 + (e*x)/2)^3*(64*a*c^2 +
(320*a^3)/3) + tan(d/2 + (e*x)/2)^2*(192*a^2*c + 32*c^3) + (32*c^3)/3 + 19
2*a^2*c*tan(d/2 + (e*x)/2)^4)/(e*(3*tan(d/2 + (e*x)/2)^2 + 3*tan(d/2 + (e*
x)/2)^4 + tan(d/2 + (e*x)/2)^6 + 1)) - (8*a*(5*a^2 + 3*c^2)*(atan(tan(d/2
+ (e*x)/2)) - (e*x)/2))/e
```

3.375 $\int (2a-2a\cos(d+ex)+2c\sin(d+ex))^2\,dx$

3.375.1 Optimal result

Integrand size = 24, antiderivative size = 81

$$\int (2a-2a\cos(d+ex)+2c\sin(d+ex))^2\,dx = 2(3a^2+c^2)\,x-\frac{6ac\cos(d+ex)}{e}-\frac{6a^2\sin(d+ex)}{e}-\frac{2(c\cos(d+ex)+a\sin(d+ex))(a-a\cos(d+ex)+c\sin(d+ex))}{e}$$

output
```
2*(3*a^2+c^2)*x-6*a*c*cos(e*x+d)/e-6*a^2*sin(e*x+d)/e-2*(c*cos(e*x+d)+a*si
n(e*x+d))*(a-a*cos(e*x+d)+c*sin(e*x+d))/e
```

3.375.2 Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.14

$$\int (2a-2a\cos(d+ex)+2c\sin(d+ex))^2\,dx = 4\left(\frac{(3a^2+c^2)\,(d+ex)}{2e}-\frac{2ac\cos(d+ex)}{e}+\frac{ac\cos(2(d+ex))}{2e}-\frac{2a^2\sin(d+ex)}{e}+\frac{(a^2-c^2)\sin(2(d+ex))}{4e}\right)$$

input
```
Integrate[(2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^2,x]
```

output
```
4*(((3*a^2 + c^2)*(d + e*x))/(2*e) - (2*a*c*Cos[d + e*x])/e + (a*c*Cos[2*(
d + e*x)])/(2*e) - (2*a^2*Sin[d + e*x])/e + ((a^2 - c^2)*Sin[2*(d + e*x)])
/(4*e))
```

3.375.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3042, 3599, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (-2a\cos(d+ex)+2a+2c\sin(d+ex))^2\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (-2a\cos(d+ex)+2a+2c\sin(d+ex))^2dx$$

$$\Big\downarrow \; 3599$$

$$\frac{1}{2}\int \left(-12\cos(d+ex)a^2+12c\sin(d+ex)a+4\left(3a^2+c^2\right)\right)dx-\frac{2(a\sin(d+ex)+c\cos(d+ex))(a(-\cos(d+ex))+a+c\sin(d+ex))}{e}$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(4x\left(3a^2+c^2\right)-\frac{12a^2\sin(d+ex)}{e}-\frac{12ac\cos(d+ex)}{e}\right)-\frac{2(a\sin(d+ex)+c\cos(d+ex))(a(-\cos(d+ex))+a+c\sin(d+ex))}{e}$$

input
```
Int[(2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^2,x]
```

output
```
(-2*(c*Cos[d + e*x] + a*Sin[d + e*x])*(a - a*Cos[d + e*x] + c*Sin[d + e*x]
))/e + (4*(3*a^2 + c^2)*x - (12*a*c*Cos[d + e*x])/e - (12*a^2*Sin[d + e*x]
)/e)/2
```

3.375.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3599
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Simp[1/n   Int[Simp[n*a^2 + (
n - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x
], x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

3.375.4 Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

method	result
parallelrisch	$\frac{(a^2-c^2)\sin(2ex+2d)+6a^2ex+2c^2ex-8a^2\sin(ex+d)-8ac\cos(ex+d)+2ac\cos(2ex+2d)-10ac}{e}$
risch	$6a^2x+2x\,c^2-\frac{8ac\cos(ex+d)}{e}-\frac{8a^2\sin(ex+d)}{e}+\frac{2ac\cos(2ex+2d)}{e}+\frac{\sin(2ex+2d)a^2}{e}-\frac{\sin(2ex+2d)c^2}{e}$
derivativedivides	$\frac{4a^2\left(\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)+4ac\cos(ex+d)^2-8a^2\sin(ex+d)+4c^2\left(-\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)-8ac\cos(ex+d}{e}$
default	$\frac{4a^2\left(\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)+4ac\cos(ex+d)^2-8a^2\sin(ex+d)+4c^2\left(-\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)-8ac\cos(ex+d}{e}$
parts	$-\frac{8a\left(\frac{\sin(ex+d)^2c}{2}+a\sin(ex+d)\right)}{e}+4a^2x+\frac{4a^2\left(\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)}{e}+\frac{4c^2\left(-\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\right.}{e}$
norman	$\frac{(6a^2+2c^2)x+(6a^2+2c^2)x\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^4+(12a^2+4c^2)x\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2+\frac{16ac\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^4}{e}-\frac{4(3a^2+c^2)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{e}-\frac{4(5}{}}{\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2\right)^2}$

input
```
int((2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x,method=_RETURNVERBOSE)
```

output
```
((a^2-c^2)*sin(2*e*x+2*d)+6*a^2*e*x+2*c^2*e*x-8*a^2*sin(e*x+d)-8*a*c*cos(e
*x+d)+2*a*c*cos(2*e*x+2*d)-10*a*c)/e
```

3.375.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.88

$$\int (2a-2a\cos(d+ex)+2c\sin(d+ex))^2\,dx$$
$$=\frac{2\left(2\,ac\cos\left(ex+d\right)^2+(3\,a^2+c^2)ex-4\,ac\cos\left(ex+d\right)-\left(4\,a^2-(a^2-c^2)\cos\left(ex+d\right)\right)\sin\left(ex+d\right)\right)}{e}$$

input
```
integrate((2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="fricas")
```

output
```
2*(2*a*c*cos(e*x + d)^2 + (3*a^2 + c^2)*e*x - 4*a*c*cos(e*x + d) - (4*a^2
- (a^2 - c^2)*cos(e*x + d))*sin(e*x + d))/e
```

3.375.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(78) = 156.

Time = 0.10 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.10

$$\int (2a-2a\cos(d+ex)+2c\sin(d+ex))^2\,dx$$
$$=\begin{cases}2a^2x\sin^2\left(d+ex\right)+2a^2x\cos^2\left(d+ex\right)+4a^2x+\frac{2a^2\sin\left(d+ex\right)\cos\left(d+ex\right)}{e}-\frac{8a^2\sin\left(d+ex\right)}{e}-\frac{4ac\sin^2\left(d+ex\right)}{e}-\frac{8ac}{}\\ x(-2a\cos\left(d\right)+2a+2c\sin\left(d\right))^2\end{cases}$$

input
```
integrate((2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))**2,x)
```

output
```
Piecewise((2*a**2*x*sin(d + e*x)**2 + 2*a**2*x*cos(d + e*x)**2 + 4*a**2*x
+ 2*a**2*sin(d + e*x)*cos(d + e*x)/e - 8*a**2*sin(d + e*x)/e - 4*a*c*sin(d
 + e*x)**2/e - 8*a*c*cos(d + e*x)/e + 2*c**2*x*sin(d + e*x)**2 + 2*c**2*x*
cos(d + e*x)**2 - 2*c**2*sin(d + e*x)*cos(d + e*x)/e, Ne(e, 0)), (x*(-2*a*
cos(d) + 2*a + 2*c*sin(d))**2, True))
```

3.375.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.21

$$\int (2a-2a\cos(d+ex)+2c\sin(d+ex))^2\,dx = 4\,a^2x+\frac{4\,ac\cos\left(ex+d\right)^2}{e}+\frac{(2\,ex+2\,d+\sin\left(2\,ex+2\,d\right))a^2}{e}+\frac{(2\,ex+2\,d-\sin\left(2\,ex+2\,d\right))c^2}{e}-8\,a\left(\frac{c\cos\left(ex+d\right)}{e}+\frac{a\sin\left(ex+d\right)}{e}\right)$$

input
```
integrate((2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="maxima")
```

output
```
4*a^2*x + 4*a*c*cos(e*x + d)^2/e + (2*e*x + 2*d + sin(2*e*x + 2*d))*a^2/e
+ (2*e*x + 2*d - sin(2*e*x + 2*d))*c^2/e - 8*a*(c*cos(e*x + d)/e + a*sin(e
*x + d)/e)
```

3.375.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.96

$$\int (2a-2a\cos(d+ex)+2c\sin(d+ex))^2\,dx = 2\left(3\,a^2+c^2\right)x+\frac{2\,ac\cos\left(2\,ex+2\,d\right)}{e}-\frac{8\,ac\cos\left(ex+d\right)}{e}-\frac{8\,a^2\sin\left(ex+d\right)}{e}+\frac{\left(a^2-c^2\right)\sin\left(2\,ex+2\,d\right)}{e}$$

input
```
integrate((2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="giac")
```

output
```
2*(3*a^2 + c^2)*x + 2*a*c*cos(2*e*x + 2*d)/e - 8*a*c*cos(e*x + d)/e - 8*a^
2*sin(e*x + d)/e + (a^2 - c^2)*sin(2*e*x + 2*d)/e
```

3.375.9 Mupad [B] (verification not implemented)

Time = 26.00 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.04

$$\int (2a-2a\cos(d+ex)+2c\sin(d+ex))^2\,dx$$
$$=\frac{a^2\,\sin\left(2\,d+2\,e\,x\right)-8\,a^2\,\sin\left(d+e\,x\right)-c^2\,\sin\left(2\,d+2\,e\,x\right)+16\,a\,c\sin\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2-4\,a\,c\sin\left(d+e\,x\right)^2+6}{e}$$

input
```
int((2*a - 2*a*cos(d + e*x) + 2*c*sin(d + e*x))^2,x)
```

output
```
(a^2*sin(2*d + 2*e*x) - 8*a^2*sin(d + e*x) - c^2*sin(2*d + 2*e*x) + 16*a*c
*sin(d/2 + (e*x)/2)^2 - 4*a*c*sin(d + e*x)^2 + 6*a^2*e*x + 2*c^2*e*x)/e
```

3.376 $\int (2a - 2a\cos(d+ex) + 2c\sin(d+ex))\,dx$

3.376.1 Optimal result

Integrand size = 22, antiderivative size = 29

$$\int (2a - 2a\cos(d+ex) + 2c\sin(d+ex))\,dx = 2ax - \frac{2c\cos(d+ex)}{e} - \frac{2a\sin(d+ex)}{e}$$

output
```
2*a*x-2*c*cos(e*x+d)/e-2*a*sin(e*x+d)/e
```

3.376.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.83

$$\int (2a - 2a\cos(d+ex) + 2c\sin(d+ex))\,dx = 2ax - \frac{2c\cos(d)\cos(ex)}{e} - \frac{2a\cos(ex)\sin(d)}{e} - \frac{2a\cos(d)\sin(ex)}{e} + \frac{2c\sin(d)\sin(ex)}{e}$$

input
```
Integrate[2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x],x]
```

output
```
2*a*x - (2*c*Cos[d]*Cos[e*x])/e - (2*a*Cos[e*x]*Sin[d])/e - (2*a*Cos[d]*Si
n[e*x])/e + (2*c*Sin[d]*Sin[e*x])/e
```

3.376.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (-2a\cos(d+ex)+2a+2c\sin(d+ex))\,dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2a\sin(d+ex)}{e}+2ax-\frac{2c\cos(d+ex)}{e}$$

input
```
Int[2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x],x]
```

output
```
2*a*x - (2*c*Cos[d + e*x])/e - (2*a*Sin[d + e*x])/e
```

3.376.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.376.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

method	result	size
default	$2ax-\frac{2c\cos(ex+d)}{e}-\frac{2a\sin(ex+d)}{e}$	30
risch	$2ax-\frac{2c\cos(ex+d)}{e}-\frac{2a\sin(ex+d)}{e}$	30
parts	$2ax-\frac{2c\cos(ex+d)}{e}-\frac{2a\sin(ex+d)}{e}$	30
derivativedivides	$\frac{2(ex+d)a-2c\cos(ex+d)-2a\sin(ex+d)}{e}$	32
parallelrisch	$\frac{-2a\sin(ex+d)-2c\cos(ex+d)+2c}{e}+2ax$	32
norman	$\frac{\frac{4c\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}{e}+2ax-\frac{4a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{e}+2ax\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}{1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}$	69

input
```
int(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
2*a*x-2*c*cos(e*x+d)/e-2*a*sin(e*x+d)/e
```

3.376.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

$$\int (2a-2a\cos(d+ex)+2c\sin(d+ex))\,dx=\frac{2\,(aex-c\cos{(ex+d)}-a\sin{(ex+d)})}{e}$$

input
```
integrate(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d),x, algorithm="fricas")
```

output
```
2*(a*e*x - c*cos(e*x + d) - a*sin(e*x + d))/e
```

3.376.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.34

$$\int (2a-2a\cos(d+ex)+2c\sin(d+ex))\,dx=2ax-2a\left(\begin{cases}\frac{\sin{(d+ex)}}{e} & \text{for } e\neq 0\\ x\cos{(d)} & \text{otherwise}\end{cases}\right)+2c\left(\begin{cases}-\frac{\cos{(d+ex)}}{e} & \text{for } e\neq 0\\ x\sin{(d)} & \text{otherwise}\end{cases}\right)$$

input
```
integrate(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d),x)
```

output
```
2*a*x - 2*a*Piecewise((sin(d + e*x)/e, Ne(e, 0)), (x*cos(d), True)) + 2*c*
Piecewise((-cos(d + e*x)/e, Ne(e, 0)), (x*sin(d), True))
```

3.376.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int (2a - 2a\cos(d+ex) + 2c\sin(d+ex))\,dx = 2\,ax - \frac{2\,c\cos(ex+d)}{e} - \frac{2\,a\sin(ex+d)}{e}$$

input
```
integrate(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d),x, algorithm="maxima")
```

output
```
2*a*x - 2*c*cos(e*x + d)/e - 2*a*sin(e*x + d)/e
```

3.376.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int (2a - 2a\cos(d+ex) + 2c\sin(d+ex))\,dx = 2\,ax - \frac{2\,c\cos(ex+d)}{e} - \frac{2\,a\sin(ex+d)}{e}$$

input
```
integrate(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d),x, algorithm="giac")
```

output
```
2*a*x - 2*c*cos(e*x + d)/e - 2*a*sin(e*x + d)/e
```

3.376.9 Mupad [B] (verification not implemented)

Time = 27.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int (2a - 2a\cos(d+ex) + 2c\sin(d+ex))\,dx = 2\,a\,x - \frac{2\,c\,\cos(d+e\,x)}{e} - \frac{2\,a\,\sin(d+e\,x)}{e}$$

input
```
int(2*a - 2*a*cos(d + e*x) + 2*c*sin(d + e*x),x)
```

output
```
2*a*x - (2*c*cos(d + e*x))/e - (2*a*sin(d + e*x))/e
```

3.377 $\int \frac{1}{2a-2a\cos(d+ex)+2c\sin(d+ex)}\,dx$

3.377.1 Optimal result

Integrand size = 24, antiderivative size = 25

$$\int \frac{1}{2a-2a\cos(d+ex)+2c\sin(d+ex)}\,dx=-\frac{\log\left(a+c\cot\left(\frac{1}{2}(d+ex)\right)\right)}{2ce}$$

output
```
-1/2*ln(a+c*cot(1/2*e*x+1/2*d))/c/e
```

3.377.2 Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.00

$$\int \frac{1}{2a-2a\cos(d+ex)+2c\sin(d+ex)}\,dx =\frac{\log\left(\sin\left(\frac{1}{2}(d+ex)\right)\right)-\log\left(c\cos\left(\frac{1}{2}(d+ex)\right)+a\sin\left(\frac{1}{2}(d+ex)\right)\right)}{2ce}$$

input
```
Integrate[(2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-1),x]
```

output
```
(Log[Sin[(d + e*x)/2]] - Log[c*Cos[(d + e*x)/2] + a*Sin[(d + e*x)/2]])/(2*
c*e)
```

3.377.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3042, 3600, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{-2a\cos(d+ex)+2a+2c\sin(d+ex)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{-2a\cos(d+ex)+2a+2c\sin(d+ex)}dx$$
$$\Big\downarrow \text{3600}$$
$$-\frac{\int \frac{1}{2a+2c\cot\left(\frac{1}{2}(d+ex)\right)}d\cot\left(\frac{1}{2}(d+ex)\right)}{e}$$
$$\Big\downarrow \text{16}$$
$$-\frac{\log\left(a+c\cot\left(\frac{1}{2}(d+ex)\right)\right)}{2ce}$$

input
```
Int[(2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-1),x]
```

output
```
-1/2*Log[a + c*Cot[(d + e*x)/2]]/(c*e)
```

3.377.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + 
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3600
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Cot[(d + e*x)/2], x]}, Simp[-f/e
   Subst[Int[1/(a + c*f*x), x], x, Cot[(d + e*x)/2]/f], x]] /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[a + b, 0]
```

3.377.4 Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.44

method	result	size
parallelrisch	$\frac{\ln\left(\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)-\ln\left(c+a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}{2ce}$	36
derivativedivides	$\frac{-\frac{\ln\left(c+a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}{c}+\frac{\ln\left(\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}{c}}{2e}$	40
default	$\frac{-\frac{\ln\left(c+a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}{c}+\frac{\ln\left(\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}{c}}{2e}$	40
norman	$\frac{\ln\left(\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}{2ce}-\frac{\ln\left(c+a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}{2ce}$	42
risch	$\frac{\ln\left(e^{i(ex+d)}-1\right)}{2ce}-\frac{\ln\left(e^{i(ex+d)}+\frac{ic-a}{ic+a}\right)}{2ce}$	58

input
```
int(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d)),x,method=_RETURNVERBOSE)
```

output
```
1/2*(ln(tan(1/2*e*x+1/2*d))-ln(c+a*tan(1/2*e*x+1/2*d)))/c/e
```

3.377.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(22) = 44$.

Time = 0.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.40

$$\int \frac{1}{2a-2a\cos(d+ex)+2c\sin(d+ex)}\,dx$$
$$= -\frac{\log\left(ac\sin\left(ex+d\right)+\frac{1}{2}\,a^2+\frac{1}{2}\,c^2-\frac{1}{2}\left(a^2-c^2\right)\cos\left(ex+d\right)\right)-\log\left(-\frac{1}{2}\,\cos\left(ex+d\right)+\frac{1}{2}\right)}{4\,ce}$$

input
```
integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d)),x, algorithm="fricas")
```

output
```
-1/4*(log(a*c*sin(e*x + d) + 1/2*a^2 + 1/2*c^2 - 1/2*(a^2 - c^2)*cos(e*x +
 d)) - log(-1/2*cos(e*x + d) + 1/2))/(c*e)
```

3.377.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(20) = 40$.

Time = 0.71 (sec) , antiderivative size = 95, normalized size of antiderivative = 3.80

$$\int \frac{1}{2a - 2a\cos(d + ex) + 2c\sin(d + ex)}\,dx = \begin{cases} \frac{\tilde{\infty}x}{\sin(d)} & \text{for } a = 0 \wedge c = 0 \wedge e = 0 \\ \frac{\log\left(\tan\left(\frac{d}{2} + \frac{ex}{2}\right)\right)}{2ce} & \text{for } a = 0 \\ -\frac{1}{2ae\tan\left(\frac{d}{2} + \frac{ex}{2}\right)} & \text{for } c = 0 \\ \frac{x}{-2a\cos(d) + 2a + 2c\sin(d)} & \text{for } e = 0 \\ -\frac{\log\left(\tan\left(\frac{d}{2} + \frac{ex}{2}\right) + \frac{c}{a}\right)}{2ce} + \frac{\log\left(\tan\left(\frac{d}{2} + \frac{ex}{2}\right)\right)}{2ce} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d)),x)
```

output
```
Piecewise((zoo*x/sin(d), Eq(a, 0) & Eq(c, 0) & Eq(e, 0)), (log(tan(d/2 + e
*x/2))/(2*c*e), Eq(a, 0)), (-1/(2*a*e*tan(d/2 + e*x/2)), Eq(c, 0)), (x/(-2
*a*cos(d) + 2*a + 2*c*sin(d)), Eq(e, 0)), (-log(tan(d/2 + e*x/2) + c/a)/(2
*c*e) + log(tan(d/2 + e*x/2))/(2*c*e), True))
```

3.377.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(22) = 44$.

Time = 0.21 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.16

$$\int \frac{1}{2a - 2a\cos(d + ex) + 2c\sin(d + ex)}\,dx = -\frac{\frac{\log\left(c + \frac{a\sin(ex+d)}{\cos(ex+d)+1}\right)}{c} - \frac{\log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}\right)}{c}}{2\,e}$$

input
```
integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d)),x, algorithm="maxima")
```

output
```
-1/2*(log(c + a*sin(e*x + d)/(cos(e*x + d) + 1))/c - log(sin(e*x + d)/(cos
(e*x + d) + 1))/c)/e
```

3.377.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.64

$$\int \frac{1}{2a-2a\cos(d+ex)+2c\sin(d+ex)}\,dx = -\frac{\frac{\log\left(\left|a\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+c\right|\right)}{c}-\frac{\log\left(\left|\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)\right|\right)}{c}}{2e}$$

input
```
integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d)),x, algorithm="giac")
```

output
```
-1/2*(log(abs(a*tan(1/2*e*x + 1/2*d) + c))/c - log(abs(tan(1/2*e*x + 1/2*d
)))/c)/e
```

3.377.9 Mupad [B] (verification not implemented)

Time = 27.44 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{1}{2a-2a\cos(d+ex)+2c\sin(d+ex)}\,dx = -\frac{\operatorname{atanh}\left(\frac{2\,a\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{c}+1\right)}{c\,e}$$

input
```
int(1/(2*a - 2*a*cos(d + e*x) + 2*c*sin(d + e*x)),x)
```

output
```
-atanh((2*a*tan(d/2 + (e*x)/2))/c + 1)/(c*e)
```

3.378 $\int \frac{1}{(2a-2a\cos(d+ex)+2c\sin(d+ex))^2}\,dx$

3.378.1 Optimal result

Integrand size = 24, antiderivative size = 75

$$\int \frac{1}{(2a-2a\cos(d+ex)+2c\sin(d+ex))^2}\,dx = \frac{a\log\left(a+c\cot\left(\frac{1}{2}(d+ex)\right)\right)}{4c^3e} - \frac{c\cos(d+ex)+a\sin(d+ex)}{4c^2e(a-a\cos(d+ex)+c\sin(d+ex))}$$

output
```
1/4*a*ln(a+c*cot(1/2*e*x+1/2*d))/c^3/e+1/4*(-c*cos(e*x+d)-a*sin(e*x+d))/c^
2/e/(a-a*cos(e*x+d)+c*sin(e*x+d))
```

3.378.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 229 vs. 2(75) = 150.

Time = 5.41 (sec) , antiderivative size = 229, normalized size of antiderivative = 3.05

$$\int \frac{1}{(2a-2a\cos(d+ex)+2c\sin(d+ex))^2}\,dx = -\frac{\sin\left(\frac{1}{2}(d+ex)\right)\left(c\cos\left(\frac{1}{2}(d+ex)\right)+a\sin\left(\frac{1}{2}(d+ex)\right)\right)\left(\cos(d+ex)\left(a^2+2c^2-2a^2\log\left(\sin\left(\frac{1}{2}(d+ex\right.\right.\right.\right.}{}$$

input
```
Integrate[(2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-2),x]
```

output
```
-1/4*(Sin[(d + e*x)/2]*(c*Cos[(d + e*x)/2] + a*Sin[(d + e*x)/2])*(Cos[d +
e*x]*(a^2 + 2*c^2 - 2*a^2*Log[Sin[(d + e*x)/2]] + 2*a^2*Log[c*Cos[(d + e*x
)/2] + a*Sin[(d + e*x)/2]]) + a*(a*(-1 + 2*Log[Sin[(d + e*x)/2]] - 2*Log[c
*Cos[(d + e*x)/2] + a*Sin[(d + e*x)/2]]) + c*(1 + 2*Log[Sin[(d + e*x)/2]]
- 2*Log[c*Cos[(d + e*x)/2] + a*Sin[(d + e*x)/2]])*Sin[d + e*x])))/(c^3*e*(
a - a*Cos[d + e*x] + c*Sin[d + e*x])^2)
```

3.378.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {3042, 3608, 25, 27, 3042, 3600, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-2a\cos(d+ex)+2a+2c\sin(d+ex))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(-2a\cos(d+ex)+2a+2c\sin(d+ex))^2}dx$$

$$\Big\downarrow \text{3608}$$

$$\frac{\int -\frac{a}{-\cos(d+ex)a+a+c\sin(d+ex)}dx}{4c^2}-\frac{a\sin(d+ex)+c\cos(d+ex)}{4c^2e(a(-\cos(d+ex))+a+c\sin(d+ex))}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{a}{-\cos(d+ex)a+a+c\sin(d+ex)}dx}{4c^2}-\frac{a\sin(d+ex)+c\cos(d+ex)}{4c^2e(a(-\cos(d+ex))+a+c\sin(d+ex))}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\int \frac{1}{-\cos(d+ex)a+a+c\sin(d+ex)}dx}{4c^2}-\frac{a\sin(d+ex)+c\cos(d+ex)}{4c^2e(a(-\cos(d+ex))+a+c\sin(d+ex))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{1}{-\cos(d+ex)a+a+c\sin(d+ex)}dx}{4c^2}-\frac{a\sin(d+ex)+c\cos(d+ex)}{4c^2e(a(-\cos(d+ex))+a+c\sin(d+ex))}$$

$$\Big\downarrow \text{3600}$$

$$\frac{a\int \frac{1}{a+c\cot\left(\frac{1}{2}(d+ex)\right)}d\cot\left(\frac{1}{2}(d+ex)\right)}{4c^2e}-\frac{a\sin(d+ex)+c\cos(d+ex)}{4c^2e(a(-\cos(d+ex))+a+c\sin(d+ex))}$$

$$\Big\downarrow \text{16}$$

$$\frac{a\log\left(a+c\cot\left(\frac{1}{2}(d+ex)\right)\right)}{4c^3e}-\frac{a\sin(d+ex)+c\cos(d+ex)}{4c^2e(a(-\cos(d+ex))+a+c\sin(d+ex))}$$

input
```
Int[(2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-2),x]
```

output
```
(a*Log[a + c*Cot[(d + e*x)/2]])/(4*c^3*e) - (c*Cos[d + e*x] + a*Sin[d + e*
x])/(4*c^2*e*(a - a*Cos[d + e*x] + c*Sin[d + e*x]))
```

3.378.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3600
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Cot[(d + e*x)/2], x]}, Simp[-f/e
   Subst[Int[1/(a + c*f*x), x], x, Cot[(d + e*x)/2]/f], x]] /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[a + b, 0]
```

rule 3608
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Simp[
1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c
*(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x
] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] &&
NeQ[n, -3/2]
```

3.378.4 Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.17

method	result
derivativedivides	$\frac{-\frac{a^{2}+c^{2}}{2 c^{2} a \left(c+a \tan \left(\frac{e x}{2}+\frac{d}{2}\right)\right)}+\frac{a \ln \left(c+a \tan \left(\frac{e x}{2}+\frac{d}{2}\right)\right)}{c^{3}}-\frac{1}{2 c^{2} \tan \left(\frac{e x}{2}+\frac{d}{2}\right)}-\frac{a \ln \left(\tan \left(\frac{e x}{2}+\frac{d}{2}\right)\right)}{c^{3}}}{4 e}$
default	$\frac{-\frac{a^{2}+c^{2}}{2 c^{2} a \left(c+a \tan \left(\frac{e x}{2}+\frac{d}{2}\right)\right)}+\frac{a \ln \left(c+a \tan \left(\frac{e x}{2}+\frac{d}{2}\right)\right)}{c^{3}}-\frac{1}{2 c^{2} \tan \left(\frac{e x}{2}+\frac{d}{2}\right)}-\frac{a \ln \left(\tan \left(\frac{e x}{2}+\frac{d}{2}\right)\right)}{c^{3}}}{4 e}$
norman	$\frac{-\frac{1}{8 c e}-\frac{\left(2 a^{2}+c^{2}\right) \tan \left(\frac{e x}{2}+\frac{d}{2}\right)}{8 a \,c^{2} e}}{\tan \left(\frac{e x}{2}+\frac{d}{2}\right) \left(c+a \tan \left(\frac{e x}{2}+\frac{d}{2}\right)\right)}-\frac{a \ln \left(\tan \left(\frac{e x}{2}+\frac{d}{2}\right)\right)}{4 c^{3} e}+\frac{a \ln \left(c+a \tan \left(\frac{e x}{2}+\frac{d}{2}\right)\right)}{4 c^{3} e}$
risch	$-\frac{i \left(-i a \,{\mathrm e}^{i \left(e x +d \right)}+i a +c \right)}{2 c^{2} e \left(c \,{\mathrm e}^{2 i \left(e x +d \right)}-i a \,{\mathrm e}^{2 i \left(e x +d \right)}-c +2 i a \,{\mathrm e}^{i \left(e x +d \right)}-i a \right)}+\frac{a \ln \left({\mathrm e}^{i \left(e x +d \right)}+\frac{i c -a}{i c +a}\right)}{4 c^{3} e}-\frac{a \ln \left({\mathrm e}^{i \left(e x +d \right)}-1\right)}{4 c^{3} e}$
parallelrisch	$\frac{-2 \tan \left(\frac{e x}{2}+\frac{d}{2}\right) \ln \left(\tan \left(\frac{e x}{2}+\frac{d}{2}\right)\right) a^{2}+2 \tan \left(\frac{e x}{2}+\frac{d}{2}\right) \ln \left(c+a \tan \left(\frac{e x}{2}+\frac{d}{2}\right)\right) a^{2}-2 \ln \left(\tan \left(\frac{e x}{2}+\frac{d}{2}\right)\right) a c+2 \ln \left(c+a \tan \left(\frac{e x}{2}\right.\right.}{8 \left(c+a \tan \left(\frac{e x}{2}+\frac{d}{2}\right)\right) c^{3} e}$

input
```
int(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4/e*(-1/2*(a^2+c^2)/c^2/a/(c+a*tan(1/2*e*x+1/2*d))+1/c^3*a*ln(c+a*tan(1/
2*e*x+1/2*d))-1/2/c^2/tan(1/2*e*x+1/2*d)-1/c^3*a*ln(tan(1/2*e*x+1/2*d)))
```

3.378.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. 2(72) = 144.

Time = 0.26 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.16

$$\int \frac{1}{(2a-2a\cos(d+ex)+2c\sin(d+ex))^2}\,dx$$
$$= \frac{2\,c^2\cos\left(ex+d\right)+2\,ac\sin\left(ex+d\right)+\left(a^2\cos\left(ex+d\right)-ac\sin\left(ex+d\right)-a^2\right)\log\left(ac\sin\left(ex+d\right)+\frac{1}{2}\,a^2+\right.}{8\left(ac^3e\cos\left(ex+d\right)-c^4e\right.}$$

input
```
integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="fricas")
```

output
```
1/8*(2*c^2*cos(e*x + d) + 2*a*c*sin(e*x + d) + (a^2*cos(e*x + d) - a*c*sin
(e*x + d) - a^2)*log(a*c*sin(e*x + d) + 1/2*a^2 + 1/2*c^2 - 1/2*(a^2 - c^2
)*cos(e*x + d)) - (a^2*cos(e*x + d) - a*c*sin(e*x + d) - a^2)*log(-1/2*cos
(e*x + d) + 1/2))/(a*c^3*e*cos(e*x + d) - c^4*e*sin(e*x + d) - a*c^3*e)
```

3.378.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(2a-2a\cos(d+ex)+2c\sin(d+ex))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))**2,x)
```

output
```
Timed out
```

3.378.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.83

$$\int \frac{1}{(2a-2a\cos(d+ex)+2c\sin(d+ex))^2}\,dx$$
$$= -\frac{\frac{ac+\frac{\left(2\,a^2+c^2\right)\sin(ex+d)}{\cos(ex+d)+1}}{\frac{ac^3\sin(ex+d)}{\cos(ex+d)+1}+\frac{a^2c^2\sin(ex+d)^2}{(\cos(ex+d)+1)^2}}-\frac{2\,a\log\left(c+\frac{a\sin(ex+d)}{\cos(ex+d)+1}\right)}{c^3}+\frac{2\,a\log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}\right)}{c^3}}{8\,e}$$

input
```
integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="maxima")
```

output
```
-1/8*((a*c + (2*a^2 + c^2)*sin(e*x + d)/(cos(e*x + d) + 1))/(a*c^3*sin(e*x
 + d)/(cos(e*x + d) + 1) + a^2*c^2*sin(e*x + d)^2/(cos(e*x + d) + 1)^2) -
2*a*log(c + a*sin(e*x + d)/(cos(e*x + d) + 1))/c^3 + 2*a*log(sin(e*x + d)/
(cos(e*x + d) + 1))/c^3)/e
```

3.378.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.47

$$\int \frac{1}{(2a-2a\cos(d+ex)+2c\sin(d+ex))^2}\,dx = \frac{\frac{2\,a\log\left(\left|a\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+c\right|\right)}{c^3}-\frac{2\,a\log\left(\left|\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)\right|\right)}{c^3}-\frac{2\,a^2\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+c^2\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+ac}{\left(a\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^2+c\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)\right)ac^2}}{8\,e}$$

input
```
integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^2,x, algorithm="giac")
```

output
```
1/8*(2*a*log(abs(a*tan(1/2*e*x + 1/2*d) + c))/c^3 - 2*a*log(abs(tan(1/2*e*
x + 1/2*d)))/c^3 - (2*a^2*tan(1/2*e*x + 1/2*d) + c^2*tan(1/2*e*x + 1/2*d)
+ a*c)/((a*tan(1/2*e*x + 1/2*d)^2 + c*tan(1/2*e*x + 1/2*d))*a*c^2))/e
```

3.378.9 Mupad [B] (verification not implemented)

Time = 27.32 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.21

$$\int \frac{1}{(2a-2a\cos(d+ex)+2c\sin(d+ex))^2}\,dx = \frac{a\,\mathrm{atanh}\left(\frac{2\,a\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{c}+1\right)}{2\,c^3\,e} - \frac{\frac{1}{c}+\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,(2\,a^2+c^2)}{a\,c^2}}{e\,\left(8\,a\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2+8\,c\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\right)}$$

input
```
int(1/(2*a - 2*a*cos(d + e*x) + 2*c*sin(d + e*x))^2,x)
```

output
```
(a*atanh((2*a*tan(d/2 + (e*x)/2))/c + 1))/(2*c^3*e) - (1/c + (tan(d/2 + (e
*x)/2)*(2*a^2 + c^2))/(a*c^2))/(e*(8*c*tan(d/2 + (e*x)/2) + 8*a*tan(d/2 +
(e*x)/2)^2))
```

3.378. $\int \frac{1}{(2a-2a\cos(d+ex)+2c\sin(d+ex))^2}\,dx$

3.379 $\int \frac{1}{(2a-2a\cos(d+ex)+2c\sin(d+ex))^3}\,dx$

3.379.1 Optimal result

Integrand size = 24, antiderivative size = 134

$$\int \frac{1}{(2a-2a\cos(d+ex)+2c\sin(d+ex))^3}\,dx = -\frac{(3a^2+c^2)\log\left(a+c\cot\left(\frac{1}{2}(d+ex)\right)\right)}{16c^5e} - \frac{c\cos(d+ex)+a\sin(d+ex)}{16c^2e(a-a\cos(d+ex)+c\sin(d+ex))^2} + \frac{3(ac\cos(d+ex)+a^2\sin(d+ex))}{16c^4e(a-a\cos(d+ex)+c\sin(d+ex))}$$

output
```
-1/16*(3*a^2+c^2)*ln(a+c*cot(1/2*e*x+1/2*d))/c^5/e+1/16*(-c*cos(e*x+d)-a*s
in(e*x+d))/c^2/e/(a-a*cos(e*x+d)+c*sin(e*x+d))^2+3/16*(a*c*cos(e*x+d)+a^2*
sin(e*x+d))/c^4/e/(a-a*cos(e*x+d)+c*sin(e*x+d))
```

3.379.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.66 (sec) , antiderivative size = 350, normalized size of antiderivative = 2.61

$$\int \frac{1}{(2a-2a\cos(d+ex)+2c\sin(d+ex))^3}\,dx = \frac{\sin\left(\frac{1}{2}(d+ex)\right)\left(c\cos\left(\frac{1}{2}(d+ex)\right)+a\sin\left(\frac{1}{2}(d+ex)\right)\right)\left(c^2(-ia+c)(ia+c)\sin^2\left(\frac{1}{2}(d+ex)\right)-6a(a^2+c\right.}{}$$

input
```
Integrate[(2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-3),x]
```

output
```
(Sin[(d + e*x)/2]*(c*Cos[(d + e*x)/2] + a*Sin[(d + e*x)/2])*(c^2*((-I)*a +
 c)*(I*a + c)*Sin[(d + e*x)/2]^2 - 6*a*(a^2 + c^2)*Sin[(d + e*x)/2]^3*(c*C
os[(d + e*x)/2] + a*Sin[(d + e*x)/2]) - c^2*(c*Cos[(d + e*x)/2] + a*Sin[(d
 + e*x)/2])^2 + 4*(3*a^2 + c^2)*Log[Sin[(d + e*x)/2]]*Sin[(d + e*x)/2]^2*(
c*Cos[(d + e*x)/2] + a*Sin[(d + e*x)/2])^2 - 4*(3*a^2 + c^2)*Log[c*Cos[(d
+ e*x)/2] + a*Sin[(d + e*x)/2]]*Sin[(d + e*x)/2]^2*(c*Cos[(d + e*x)/2] + a
*Sin[(d + e*x)/2])^2 + 3*a*c*(c*Cos[(d + e*x)/2] + a*Sin[(d + e*x)/2])^2*S
in[d + e*x]))/(8*c^5*e*(a - a*Cos[d + e*x] + c*Sin[d + e*x])^3)
```

3.379.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3608, 27, 3042, 3632, 3042, 3600, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-2a\cos(d+ex)+2a+2c\sin(d+ex))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(-2a\cos(d+ex)+2a+2c\sin(d+ex))^3}dx$$

$$\Big\downarrow \text{3608}$$

$$\frac{\int -\frac{\cos(d+ex)a+2a-c\sin(d+ex)}{2(-\cos(d+ex)a+a+c\sin(d+ex))^2}dx}{8c^2}-\frac{a\sin(d+ex)+c\cos(d+ex)}{16c^2e(a(-\cos(d+ex))+a+c\sin(d+ex))^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\cos(d+ex)a+2a-c\sin(d+ex)}{(-\cos(d+ex)a+a+c\sin(d+ex))^2}dx}{16c^2}-\frac{a\sin(d+ex)+c\cos(d+ex)}{16c^2e(a(-\cos(d+ex))+a+c\sin(d+ex))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\cos(d+ex)a+2a-c\sin(d+ex)}{(-\cos(d+ex)a+a+c\sin(d+ex))^2}dx}{16c^2}-\frac{a\sin(d+ex)+c\cos(d+ex)}{16c^2e(a(-\cos(d+ex))+a+c\sin(d+ex))^2}$$

$$\Big\downarrow \text{3632}$$

$$-\frac{-\left(\frac{3a^2}{c^2}+1\right)\int \frac{1}{-\cos(d+ex)a+a+c\sin(d+ex)}dx-\frac{3(a^2\sin(d+ex)+ac\cos(d+ex))}{c^2e(a(-\cos(d+ex))+a+c\sin(d+ex))}}{16c^2}-\frac{a\sin(d+ex)+c\cos(d+ex)}{16c^2e(a(-\cos(d+ex))+a+c\sin(d+ex))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\left(\frac{3a^2}{c^2}+1\right)\int \frac{1}{-\cos(d+ex)a+a+c\sin(d+ex)}dx-\frac{3(a^2\sin(d+ex)+ac\cos(d+ex))}{c^2e(a(-\cos(d+ex))+a+c\sin(d+ex))}}{16c^2}-\frac{a\sin(d+ex)+c\cos(d+ex)}{16c^2e(a(-\cos(d+ex))+a+c\sin(d+ex))^2}$$

$$\Big\downarrow \text{3600}$$

$$-\frac{\frac{\left(\frac{3a^2}{c^2}+1\right)\int \frac{1}{a+c\cot\left(\frac{1}{2}(d+ex)\right)}d\cot\left(\frac{1}{2}(d+ex)\right)}{e}-\frac{3(a^2\sin(d+ex)+ac\cos(d+ex))}{c^2e(a(-\cos(d+ex))+a+c\sin(d+ex))}}{16c^2}-\frac{a\sin(d+ex)+c\cos(d+ex)}{16c^2e(a(-\cos(d+ex))+a+c\sin(d+ex))^2}$$

$$\Big\downarrow \text{16}$$

$$-\frac{\frac{\left(\frac{3a^2}{c^2}+1\right)\log\left(a+c\cot\left(\frac{1}{2}(d+ex)\right)\right)}{ce}-\frac{3(a^2\sin(d+ex)+ac\cos(d+ex))}{c^2e(a(-\cos(d+ex))+a+c\sin(d+ex))}}{16c^2}-\frac{a\sin(d+ex)+c\cos(d+ex)}{16c^2e(a(-\cos(d+ex))+a+c\sin(d+ex))^2}$$

input
```
Int[(2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-3),x]
```

output
```
-1/16*(c*Cos[d + e*x] + a*Sin[d + e*x])/(c^2*e*(a - a*Cos[d + e*x] + c*Sin
[d + e*x])^2) - (((1 + (3*a^2)/c^2)*Log[a + c*Cot[(d + e*x)/2]])/(c*e) - (
3*(a*c*Cos[d + e*x] + a^2*Sin[d + e*x]))/(c^2*e*(a - a*Cos[d + e*x] + c*Si
n[d + e*x])))/(16*c^2)
```

3.379.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.379. $\int \frac{1}{(2a-2a\cos(d+ex)+2c\sin(d+ex))^3}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3600
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Cot[(d + e*x)/2], x]}, Simp[-f/e
   Subst[Int[1/(a + c*f*x), x], x, Cot[(d + e*x)/2]/f], x]] /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[a + b, 0]
```

rule 3608
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Simp[
1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c
*(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x
] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] &&
NeQ[n, -3/2]
```

rule 3632
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Simp[(a*A - b*B - c*C)/(a^2 - b^2 - c^2)   Int[1/(a + b*Cos[d + e*x] + c*S
in[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
 - c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

3.379.4 Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.31

method	result
derivativedivides	$\frac{-\frac{1}{8c^3\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}+\frac{\left(6a^2+2c^2\right)\ln\left(\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}{4c^5}+\frac{3a}{4c^4\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}-\frac{-3a^4-2a^2c^2+c^4}{4a^2c^4\left(c+a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}-\frac{-a^4-2a^2c^2-c^4}{8a^2c^3\left(c+a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)^2}-}{8e}$
default	$\frac{-\frac{1}{8c^3\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}+\frac{\left(6a^2+2c^2\right)\ln\left(\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}{4c^5}+\frac{3a}{4c^4\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}-\frac{-3a^4-2a^2c^2+c^4}{4a^2c^4\left(c+a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}-\frac{-a^4-2a^2c^2-c^4}{8a^2c^3\left(c+a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)^2}-}{8e}$
norman	$\frac{-\frac{1}{64ce}+\frac{a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{16c^2e}-\frac{\left(18a^4+6a^2c^2-c^4\right)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^4}{64c^5e}-\frac{\left(3a^2+c^2\right)a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^3}{8c^4e}}{\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2\left(c+a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)^2}+\frac{\left(3a^2+c^2\right)\ln\left(\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}{16c^5e}-$
parallelrisch	$\frac{-12\left(c+a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)^2\left(a^2+\frac{c^2}{3}\right)\ln\left(c+a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)+12\left(c+a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)^2\left(a^2+\frac{c^2}{3}\right)\ln\left(\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)+\left(-18a^4\right.}{64c^5e\left(c+a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}$
risch	$\frac{-3ia^3e^{3i(ex+d)}-iac^2e^{3i(ex+d)}+9ia^3e^{2i(ex+d)}+3a^2ce^{3i(ex+d)}+3iac^2e^{2i(ex+d)}+c^3e^{3i(ex+d)}-9ia^3e^{i(ex+d)}+iac^2e^{i(ex}}{8\left(ce^{2i(ex+d)}-iae^{2i(ex+d)}-c+2iae^{i(ex+d)}-ia\right)^2c^4e}$

input
```
int(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x,method=_RETURNVERBOSE)
```

output
```
1/8/e*(-1/8/c^3/tan(1/2*e*x+1/2*d)^2+1/4*(6*a^2+2*c^2)/c^5*ln(tan(1/2*e*x+
1/2*d))+3/4/c^4*a/tan(1/2*e*x+1/2*d)-1/4*(-3*a^4-2*a^2*c^2+c^4)/a^2/c^4/(c
+a*tan(1/2*e*x+1/2*d))-1/8*(-a^4-2*a^2*c^2-c^4)/a^2/c^3/(c+a*tan(1/2*e*x+1
/2*d))^2-1/2*(3*a^2+c^2)/c^5*ln(c+a*tan(1/2*e*x+1/2*d)))
```

3.379.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 438 vs. $2(131) = 262$.

Time = 0.27 (sec) , antiderivative size = 438, normalized size of antiderivative = 3.27

$$\int \frac{1}{(2a-2a\cos(d+ex)+2c\sin(d+ex))^3}\,dx$$
$$=\frac{12\,a^2c^2\cos\left(ex+d\right)^2-6\,a^2c^2-2\left(3\,a^2c^2-c^4\right)\cos\left(ex+d\right)+\left(3\,a^4+4\,a^2c^2+c^4+\left(3\,a^4-2\,a^2c^2-c^4\right)\cos\right.}{}$$

input
```
integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="fricas")
```

output
```
1/32*(12*a^2*c^2*cos(e*x + d)^2 - 6*a^2*c^2 - 2*(3*a^2*c^2 - c^4)*cos(e*x
+ d) + (3*a^4 + 4*a^2*c^2 + c^4 + (3*a^4 - 2*a^2*c^2 - c^4)*cos(e*x + d)^2
 - 2*(3*a^4 + a^2*c^2)*cos(e*x + d) + 2*(3*a^3*c + a*c^3 - (3*a^3*c + a*c^
3)*cos(e*x + d))*sin(e*x + d))*log(a*c*sin(e*x + d) + 1/2*a^2 + 1/2*c^2 -
1/2*(a^2 - c^2)*cos(e*x + d)) - (3*a^4 + 4*a^2*c^2 + c^4 + (3*a^4 - 2*a^2*
c^2 - c^4)*cos(e*x + d)^2 - 2*(3*a^4 + a^2*c^2)*cos(e*x + d) + 2*(3*a^3*c
+ a*c^3 - (3*a^3*c + a*c^3)*cos(e*x + d))*sin(e*x + d))*log(-1/2*cos(e*x +
 d) + 1/2) - 2*(3*a^3*c - a*c^3 - 3*(a^3*c - a*c^3)*cos(e*x + d))*sin(e*x
+ d))/(2*a^2*c^5*e*cos(e*x + d) - (a^2*c^5 - c^7)*e*cos(e*x + d)^2 - (a^2*
c^5 + c^7)*e + 2*(a*c^6*e*cos(e*x + d) - a*c^6*e)*sin(e*x + d))
```

3.379.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(2a-2a\cos(d+ex)+2c\sin(d+ex))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))**3,x)
```

output
```
Timed out
```

3.379.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(131) = 262$.

Time = 0.24 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.98

$$\int \frac{1}{(2a-2a\cos(d+ex)+2c\sin(d+ex))^3}\,dx = -\frac{\frac{a^2c^3-\frac{4a^3c^2\sin(ex+d)}{\cos(ex+d)+1}-\frac{(18a^4c+6a^2c^3-c^5)\sin(ex+d)^2}{(\cos(ex+d)+1)^2}-\frac{2(6a^5+2a^3c^2-ac^4)\sin(ex+d)^3}{(\cos(ex+d)+1)^3}}{\frac{a^2c^6\sin(ex+d)^2}{(\cos(ex+d)+1)^2}+\frac{2a^3c^5\sin(ex+d)^3}{(\cos(ex+d)+1)^3}+\frac{a^4c^4\sin(ex+d)^4}{(\cos(ex+d)+1)^4}}+\frac{4(3a^2+c^2)\log\left(c+\frac{a\sin(ex+d)}{\cos(ex+d)+1}\right)}{c^5}-\frac{4(3a^2+c^2)\ldots}{\ldots}}{64e}$$

input
```
integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="maxima")
```

output
```
-1/64*((a^2*c^3 - 4*a^3*c^2*sin(e*x + d)/(cos(e*x + d) + 1) - (18*a^4*c +
6*a^2*c^3 - c^5)*sin(e*x + d)^2/(cos(e*x + d) + 1)^2 - 2*(6*a^5 + 2*a^3*c^
2 - a*c^4)*sin(e*x + d)^3/(cos(e*x + d) + 1)^3)/(a^2*c^6*sin(e*x + d)^2/(c
os(e*x + d) + 1)^2 + 2*a^3*c^5*sin(e*x + d)^3/(cos(e*x + d) + 1)^3 + a^4*c
^4*sin(e*x + d)^4/(cos(e*x + d) + 1)^4) + 4*(3*a^2 + c^2)*log(c + a*sin(e*
x + d)/(cos(e*x + d) + 1))/c^5 - 4*(3*a^2 + c^2)*log(sin(e*x + d)/(cos(e*x
 + d) + 1))/c^5)/e
```

3.379.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.71

$$\int \frac{1}{(2a-2a\cos(d+ex)+2c\sin(d+ex))^3}\,dx = \frac{\frac{4\,(3\,a^2+c^2)\log\left(\left|\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)\right|\right)}{c^5} - \frac{4\,(3\,a^3+ac^2)\log\left(\left|a\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+c\right|\right)}{ac^5} + \frac{12\,a^5\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^3+4\,a^3c^2\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^3-2\,ac^4\tan\left(\frac{1}{2}\,e\right.}{}}{64\,e}$$

input
```
integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^3,x, algorithm="giac")
```

output
```
1/64*(4*(3*a^2 + c^2)*log(abs(tan(1/2*e*x + 1/2*d)))/c^5 - 4*(3*a^3 + a*c^
2)*log(abs(a*tan(1/2*e*x + 1/2*d) + c))/(a*c^5) + (12*a^5*tan(1/2*e*x + 1/
2*d)^3 + 4*a^3*c^2*tan(1/2*e*x + 1/2*d)^3 - 2*a*c^4*tan(1/2*e*x + 1/2*d)^3
 + 18*a^4*c*tan(1/2*e*x + 1/2*d)^2 + 6*a^2*c^3*tan(1/2*e*x + 1/2*d)^2 - c^
5*tan(1/2*e*x + 1/2*d)^2 + 4*a^3*c^2*tan(1/2*e*x + 1/2*d) - a^2*c^3)/((a*t
an(1/2*e*x + 1/2*d)^2 + c*tan(1/2*e*x + 1/2*d))^2*a^2*c^4))/e
```

3.379.9 Mupad [B] (verification not implemented)

Time = 28.77 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.39

$$\int \frac{1}{(2a-2a\cos(d+ex)+2c\sin(d+ex))^3}\,dx = \frac{\frac{2\,a\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{c^2}-\frac{1}{2\,c}+\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^3\,(6\,a^4+2\,a^2\,c^2-c^4)}{a\,c^4}+\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2\,(18\,a^4+6\,a^2\,c^2-c^4)}{2\,a^2\,c^3}}{e\,\left(32\,a^2\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^4+64\,a\,c\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^3+32\,c^2\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2\right)} - \frac{\mathrm{atanh}\left(\frac{2\,a\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{c}+1\right)\,(3\,a^2+c^2)}{8\,c^5\,e}$$

input
```
int(1/(2*a - 2*a*cos(d + e*x) + 2*c*sin(d + e*x))^3,x)
```

output
```
((2*a*tan(d/2 + (e*x)/2))/c^2 - 1/(2*c) + (tan(d/2 + (e*x)/2)^3*(6*a^4 - c
^4 + 2*a^2*c^2))/(a*c^4) + (tan(d/2 + (e*x)/2)^2*(18*a^4 - c^4 + 6*a^2*c^2
))/(2*a^2*c^3))/(e*(32*a^2*tan(d/2 + (e*x)/2)^4 + 32*c^2*tan(d/2 + (e*x)/2
)^2 + 64*a*c*tan(d/2 + (e*x)/2)^3)) - (atanh((2*a*tan(d/2 + (e*x)/2))/c +
1)*(3*a^2 + c^2))/(8*c^5*e)
```

3.379. $\int \frac{1}{(2a-2a\cos(d+ex)+2c\sin(d+ex))^3}\,dx$

3.380 $\int \frac{1}{(2a-2a\cos(d+ex)+2c\sin(d+ex))^4}\,dx$

3.380.1 Optimal result

Integrand size = 24, antiderivative size = 207

$$\begin{aligned}\int \frac{1}{(2a-2a\cos(d+ex)+2c\sin(d+ex))^4}\,dx &= \frac{a(5a^2+3c^2)\log\left(a+c\cot\left(\frac{1}{2}(d+ex)\right)\right)}{32c^7e} - \frac{c\cos(d+ex)+a\sin(d+ex)}{48c^2e(a-a\cos(d+ex)+c\sin(d+ex))^3} \\ &\quad + \frac{5(ac\cos(d+ex)+a^2\sin(d+ex))}{96c^4e(a-a\cos(d+ex)+c\sin(d+ex))^2} \\ &\quad - \frac{c(15a^2+4c^2)\cos(d+ex)+a(15a^2+4c^2)\sin(d+ex)}{96c^6e(a-a\cos(d+ex)+c\sin(d+ex))}\end{aligned}$$

output
```
1/32*a*(5*a^2+3*c^2)*ln(a+c*cot(1/2*e*x+1/2*d))/c^7/e+1/48*(-c*cos(e*x+d)-
a*sin(e*x+d))/c^2/e/(a-a*cos(e*x+d)+c*sin(e*x+d))^3+5/96*(a*c*cos(e*x+d)+a
^2*sin(e*x+d))/c^4/e/(a-a*cos(e*x+d)+c*sin(e*x+d))^2+1/96*(-c*(15*a^2+4*c^
2)*cos(e*x+d)-a*(15*a^2+4*c^2)*sin(e*x+d))/c^6/e/(a-a*cos(e*x+d)+c*sin(e*x
+d))
```

3.380.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 494 vs. $2(207) = 414$.

Time = 7.97 (sec) , antiderivative size = 494, normalized size of antiderivative = 2.39

$$\int \frac{1}{(2a-2a\cos(d+ex)+2c\sin(d+ex))^4}\,dx$$
$$= \frac{\sin\left(\frac{1}{2}(d+ex)\right)\left(c\cos\left(\frac{1}{2}(d+ex)\right)+a\sin\left(\frac{1}{2}(d+ex)\right)\right)\left(150a^6+130a^4c^2+24a^2c^4-225a^6\cos(d+ex)-\right.}{}$$

input
```
Integrate[(2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-4),x]
```

output
```
(Sin[(d + e*x)/2]*(c*Cos[(d + e*x)/2] + a*Sin[(d + e*x)/2])*(150*a^6 + 130
*a^4*c^2 + 24*a^2*c^4 - 225*a^6*Cos[d + e*x] - 255*a^4*c^2*Cos[d + e*x] -
42*a^2*c^4*Cos[d + e*x] - 24*c^6*Cos[d + e*x] + 90*a^6*Cos[2*(d + e*x)] +
174*a^4*c^2*Cos[2*(d + e*x)] - 15*a^6*Cos[3*(d + e*x)] - 49*a^4*c^2*Cos[3*
(d + e*x)] + 18*a^2*c^4*Cos[3*(d + e*x)] + 8*c^6*Cos[3*(d + e*x)] - 192*(5
*a^3 + 3*a*c^2)*Log[Sin[(d + e*x)/2]]*Sin[(d + e*x)/2]^3*(c*Cos[(d + e*x)/
2] + a*Sin[(d + e*x)/2])^3 + 192*(5*a^3 + 3*a*c^2)*Log[c*Cos[(d + e*x)/2]
 + a*Sin[(d + e*x)/2]]*Sin[(d + e*x)/2]^3*(c*Cos[(d + e*x)/2] + a*Sin[(d +
e*x)/2])^3 + 75*a^5*c*Sin[d + e*x] + 75*a^3*c^3*Sin[d + e*x] - 12*a*c^5*Si
n[d + e*x] - 60*a^5*c*Sin[2*(d + e*x)] - 156*a^3*c^3*Sin[2*(d + e*x)] - 12
*a*c^5*Sin[2*(d + e*x)] + 15*a^5*c*Sin[3*(d + e*x)] + 79*a^3*c^3*Sin[3*(d
 + e*x)] + 20*a*c^5*Sin[3*(d + e*x)]))/(384*c^7*e*(a - a*Cos[d + e*x] + c*S
in[d + e*x])^4)
```

3.380.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {3042, 3608, 27, 3042, 3635, 25, 3042, 3632, 3042, 3600, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-2a\cos(d+ex)+2a+2c\sin(d+ex))^4}\,dx$$
$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(-2a\cos(d+ex)+2a+2c\sin(d+ex))^4}dx$$

$$\Big\downarrow \text{3608}$$

$$\frac{\int -\frac{2\cos(d+ex)a+3a-2c\sin(d+ex)}{4(-\cos(d+ex)a+a+c\sin(d+ex))^3}dx}{12c^2}-\frac{a\sin(d+ex)+c\cos(d+ex)}{48c^2e(a(-\cos(d+ex))+a+c\sin(d+ex))^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{2\cos(d+ex)a+3a-2c\sin(d+ex)}{(-\cos(d+ex)a+a+c\sin(d+ex))^3}dx}{48c^2}-\frac{a\sin(d+ex)+c\cos(d+ex)}{48c^2e(a(-\cos(d+ex))+a+c\sin(d+ex))^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{2\cos(d+ex)a+3a-2c\sin(d+ex)}{(-\cos(d+ex)a+a+c\sin(d+ex))^3}dx}{48c^2}-\frac{a\sin(d+ex)+c\cos(d+ex)}{48c^2e(a(-\cos(d+ex))+a+c\sin(d+ex))^3}$$

$$\Big\downarrow \text{3635}$$

$$-\frac{\frac{\int -\frac{5\cos(d+ex)a^2-5c\sin(d+ex)a+2(5a^2+2c^2)}{(-\cos(d+ex)a+a+c\sin(d+ex))^2}dx}{2c^2}-\frac{5(a^2\sin(d+ex)+ac\cos(d+ex))}{2c^2e(a(-\cos(d+ex))+a+c\sin(d+ex))^2}}{48c^2}-\frac{a\sin(d+ex)+c\cos(d+ex)}{48c^2e(a(-\cos(d+ex))+a+c\sin(d+ex))^3}$$

$$\Big\downarrow \text{25}$$

$$-\frac{-\frac{\int \frac{5\cos(d+ex)a^2-5c\sin(d+ex)a+2(5a^2+2c^2)}{(-\cos(d+ex)a+a+c\sin(d+ex))^2}dx}{2c^2}-\frac{5(a^2\sin(d+ex)+ac\cos(d+ex))}{2c^2e(a(-\cos(d+ex))+a+c\sin(d+ex))^2}}{48c^2}-\frac{a\sin(d+ex)+c\cos(d+ex)}{48c^2e(a(-\cos(d+ex))+a+c\sin(d+ex))^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{\int \frac{5\cos(d+ex)a^2-5c\sin(d+ex)a+2(5a^2+2c^2)}{(-\cos(d+ex)a+a+c\sin(d+ex))^2}dx}{2c^2}-\frac{5(a^2\sin(d+ex)+ac\cos(d+ex))}{2c^2e(a(-\cos(d+ex))+a+c\sin(d+ex))^2}}{48c^2}-\frac{a\sin(d+ex)+c\cos(d+ex)}{48c^2e(a(-\cos(d+ex))+a+c\sin(d+ex))^3}$$

$$\Big\downarrow \text{3632}$$

$$-\frac{-\frac{-3a\left(\frac{5a^2}{c^2}+3\right)\int \frac{1}{-\cos(d+ex)a+a+c\sin(d+ex)}dx-\frac{a(15a^2+4c^2)\sin(d+ex)+c(15a^2+4c^2)\cos(d+ex)}{c^2e(a(-\cos(d+ex))+a+c\sin(d+ex))}}{2c^2}-\frac{5(a^2\sin(d+ex)+ac\cos(d+ex))}{2c^2e(a(-\cos(d+ex))+a+c\sin(d+ex))^2}}{48c^2}-\frac{a\sin(d+ex)+c\cos(d+ex)}{48c^2e(a(-\cos(d+ex))+a+c\sin(d+ex))^3}$$

3.380. $\int \frac{1}{(2a-2a\cos(d+ex)+2c\sin(d+ex))^4}\,dx$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{-3a\left(\frac{5a^2}{c^2}+3\right)\int \frac{1}{-\cos(d+ex)a+a+c\sin(d+ex)}dx-\frac{a\left(15a^2+4c^2\right)\sin(d+ex)+c\left(15a^2+4c^2\right)\cos(d+ex)}{c^2e(a(-\cos(d+ex))+a+c\sin(d+ex))}}{2c^2}-\frac{5(a^2\sin(d+ex)+ac\cos(d+ex))}{2c^2e(a(-\cos(d+ex))+a+c\sin(d+ex))^2}}{48c^2}-\frac{a\sin(d+ex)+c\cos(d+ex)}{48c^2e(a(-\cos(d+ex))+a+c\sin(d+ex))^3}$$

$$\Big\downarrow \text{3600}$$

$$-\frac{-\frac{\frac{3a\left(\frac{5a^2}{c^2}+3\right)\int \frac{1}{a+c\cot\left(\frac{1}{2}(d+ex)\right)}d\cot\left(\frac{1}{2}(d+ex)\right)}{e}-\frac{a\left(15a^2+4c^2\right)\sin(d+ex)+c\left(15a^2+4c^2\right)\cos(d+ex)}{c^2e(a(-\cos(d+ex))+a+c\sin(d+ex))}}{2c^2}-\frac{5(a^2\sin(d+ex)+ac\cos(d+ex))}{2c^2e(a(-\cos(d+ex))+a+c\sin(d+ex))^2}}{48c^2}-\frac{a\sin(d+ex)+c\cos(d+ex)}{48c^2e(a(-\cos(d+ex))+a+c\sin(d+ex))^3}$$

$$\Big\downarrow \text{16}$$

$$-\frac{-\frac{5(a^2\sin(d+ex)+ac\cos(d+ex))}{2c^2e(a(-\cos(d+ex))+a+c\sin(d+ex))^2}-\frac{\frac{3a\left(\frac{5a^2}{c^2}+3\right)\log\left(a+c\cot\left(\frac{1}{2}(d+ex)\right)\right)}{ce}-\frac{a\left(15a^2+4c^2\right)\sin(d+ex)+c\left(15a^2+4c^2\right)\cos(d+ex)}{c^2e(a(-\cos(d+ex))+a+c\sin(d+ex))}}{2c^2}}{48c^2}-\frac{a\sin(d+ex)+c\cos(d+ex)}{48c^2e(a(-\cos(d+ex))+a+c\sin(d+ex))^3}$$

input
```
Int[(2*a - 2*a*Cos[d + e*x] + 2*c*Sin[d + e*x])^(-4),x]
```

output
```
-1/48*(c*Cos[d + e*x] + a*Sin[d + e*x])/(c^2*e*(a - a*Cos[d + e*x] + c*Sin
[d + e*x])^3) - ((-5*(a*c*Cos[d + e*x] + a^2*Sin[d + e*x]))/(2*c^2*e*(a -
a*Cos[d + e*x] + c*Sin[d + e*x])^2) - ((3*a*(3 + (5*a^2)/c^2)*Log[a + c*Co
t[(d + e*x)/2]])/(c*e) - (c*(15*a^2 + 4*c^2)*Cos[d + e*x] + a*(15*a^2 + 4*
c^2)*Sin[d + e*x])/(c^2*e*(a - a*Cos[d + e*x] + c*Sin[d + e*x])))/(2*c^2))
/(48*c^2)
```

3.380. $\int \frac{1}{(2a-2a\cos(d+ex)+2c\sin(d+ex))^4}\,dx$

3.380.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3600
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Cot[(d + e*x)/2], x]}, Simp[-f/e
   Subst[Int[1/(a + c*f*x), x], x, Cot[(d + e*x)/2]/f], x]] /; FreeQ[{a, b,
 c, d, e}, x] && EqQ[a + b, 0]
```

rule 3608
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Simp[
1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c
*(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x
] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] &&
NeQ[n, -3/2]
```

rule 3632
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Simp[(a*A - b*B - c*C)/(a^2 - b^2 - c^2)   Int[1/(a + b*Cos[d + e*x] + c*S
in[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
 - c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

3.380. $\int \frac{1}{(2a-2a\cos(d+ex)+2c\sin(d+ex))^4}\,dx$

rule 3635
```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(-(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(
a^2 - b^2 - c^2))), x] + Simp[1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a + b*Co
s[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n +
2)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x]
/; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[n, -2]
```

3.380.4 Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.22

method	result
derivativedivides	$\frac{-\frac{1}{24c^4\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^3}-\frac{10a^2+3c^2}{8c^6\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}+\frac{a}{4c^5\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}-\frac{a\left(5a^2+3c^2\right)\ln\left(\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}{2c^7}-\frac{4a^6+6a^4c^2-2c^6}{16a^3c^5\left(c+a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)^2}-\cdots}{16e}$
default	$\frac{-\frac{1}{24c^4\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^3}-\frac{10a^2+3c^2}{8c^6\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}+\frac{a}{4c^5\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}-\frac{a\left(5a^2+3c^2\right)\ln\left(\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}{2c^7}-\frac{4a^6+6a^4c^2-2c^6}{16a^3c^5\left(c+a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)^2}-\cdots}{16e}$
norman	$\frac{-\frac{1}{384ce}+\frac{a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{128c^2e}+\frac{\left(50a^6+30a^4c^2+c^6\right)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^6}{384c^7e}-\frac{\left(5a^2+3c^2\right)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}{128c^3e}+\frac{a\left(15a^4+9a^2c^2\right)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^5}{64c^6e}-\cdots}{\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^3\left(c+a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)^3}$
parallelrisch	$60\left(a^2+\frac{3c^2}{5}\right)\left(c+a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)^3a\ln\left(c+a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)-60\left(a^2+\frac{3c^2}{5}\right)\left(c+a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)^3a\ln\left(\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)+(110\cdots$
risch	$i(-4c^5+45a^4c+15ia^5+15ia\,c^4e^{i(ex+d)}+12ia\,c^4e^{2i(ex+d)}+60ia^3c^2e^{2i(ex+d)}+45ia^3c^2e^{4i(ex+d)}-3a^2c^3-130ia^3c^2e^{3i(ex\cdots$

input
```
int(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^4,x,method=_RETURNVERBOSE)
```

output
```
1/16/e*(-1/24/c^4/tan(1/2*e*x+1/2*d)^3-1/8*(10*a^2+3*c^2)/c^6/tan(1/2*e*x+
1/2*d)+1/4/c^5*a/tan(1/2*e*x+1/2*d)^2-1/2*a*(5*a^2+3*c^2)/c^7*ln(tan(1/2*e
*x+1/2*d))-1/16*(4*a^6+6*a^4*c^2-2*c^6)/a^3/c^5/(c+a*tan(1/2*e*x+1/2*d))^2
-1/8*(10*a^6+9*a^4*c^2+c^6)/c^6/a^3/(c+a*tan(1/2*e*x+1/2*d))-1/24*(a^6+3*a
^4*c^2+3*a^2*c^4+c^6)/a^3/c^4/(c+a*tan(1/2*e*x+1/2*d))^3+1/2*a*(5*a^2+3*c^
2)/c^7*ln(c+a*tan(1/2*e*x+1/2*d)))
```

3.380.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 796 vs. 2(204) = 408.

Time = 0.28 (sec) , antiderivative size = 796, normalized size of antiderivative = 3.85

$$\int \frac{1}{(2a-2a\cos(d+ex)+2c\sin(d+ex))^4}\,dx$$
$$= \frac{60\,a^4c^2+6\,a^2c^4+2\,(45\,a^4c^2-3\,a^2c^4-4\,c^6)\cos{(ex+d)}^3-12\,(10\,a^4c^2+a^2c^4)\cos{(ex+d)}^2-6\,(5\,a^4c^2-}{}$$

input
```
integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^4,x, algorithm="fricas")
```

output
```
1/192*(60*a^4*c^2 + 6*a^2*c^4 + 2*(45*a^4*c^2 - 3*a^2*c^4 - 4*c^6)*cos(e*x
 + d)^3 - 12*(10*a^4*c^2 + a^2*c^4)*cos(e*x + d)^2 - 6*(5*a^4*c^2 - 2*a^2*
c^4 - 2*c^6)*cos(e*x + d) - 3*(5*a^6 + 18*a^4*c^2 + 9*a^2*c^4 - (5*a^6 - 1
2*a^4*c^2 - 9*a^2*c^4)*cos(e*x + d)^3 + 3*(5*a^6 - 2*a^4*c^2 - 3*a^2*c^4)*
cos(e*x + d)^2 - 3*(5*a^6 + 8*a^4*c^2 + 3*a^2*c^4)*cos(e*x + d) + (15*a^5*
c + 14*a^3*c^3 + 3*a*c^5 + (15*a^5*c + 4*a^3*c^3 - 3*a*c^5)*cos(e*x + d)^2
 - 6*(5*a^5*c + 3*a^3*c^3)*cos(e*x + d))*sin(e*x + d))*log(a*c*sin(e*x + d
) + 1/2*a^2 + 1/2*c^2 - 1/2*(a^2 - c^2)*cos(e*x + d)) + 3*(5*a^6 + 18*a^4*
c^2 + 9*a^2*c^4 - (5*a^6 - 12*a^4*c^2 - 9*a^2*c^4)*cos(e*x + d)^3 + 3*(5*a
^6 - 2*a^4*c^2 - 3*a^2*c^4)*cos(e*x + d)^2 - 3*(5*a^6 + 8*a^4*c^2 + 3*a^2*
c^4)*cos(e*x + d) + (15*a^5*c + 14*a^3*c^3 + 3*a*c^5 + (15*a^5*c + 4*a^3*c
^3 - 3*a*c^5)*cos(e*x + d)^2 - 6*(5*a^5*c + 3*a^3*c^3)*cos(e*x + d))*sin(e
*x + d))*log(-1/2*cos(e*x + d) + 1/2) + 2*(15*a^5*c + 14*a^3*c^3 + 6*a*c^5
 + (15*a^5*c - 41*a^3*c^3 - 12*a*c^5)*cos(e*x + d)^2 - 3*(10*a^5*c - 9*a^3
*c^3 - a*c^5)*cos(e*x + d))*sin(e*x + d))/((a^3*c^7 - 3*a*c^9)*e*cos(e*x +
 d)^3 - 3*(a^3*c^7 - a*c^9)*e*cos(e*x + d)^2 + 3*(a^3*c^7 + a*c^9)*e*cos(e
*x + d) - (a^3*c^7 + 3*a*c^9)*e + (6*a^2*c^8*e*cos(e*x + d) - (3*a^2*c^8 -
 c^10)*e*cos(e*x + d)^2 - (3*a^2*c^8 + c^10)*e)*sin(e*x + d))
```

3.380.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(2a-2a\cos(d+ex)+2c\sin(d+ex))^4}\,dx = \text{Timed out}$$

input
```
integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))**4,x)
```

output
```
Timed out
```

3.380.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.85

$$\int \frac{1}{(2a-2a\cos(d+ex)+2c\sin(d+ex))^4}\,dx =$$

$$-\frac{\frac{a^3c^5-\frac{3a^4c^4\sin(ex+d)}{\cos(ex+d)+1}+\frac{3\left(5a^5c^3+3a^3c^5\right)\sin(ex+d)^2}{(\cos(ex+d)+1)^2}+\frac{\left(110a^6c^2+66a^4c^4+3a^2c^6+c^8\right)\sin(ex+d)^3}{(\cos(ex+d)+1)^3}+\frac{3\left(50a^7c+30a^5c^3+ac^7\right)\sin(ex+d)^4}{(\cos(ex+d)+1)^4}+\frac{3\left(20a^8+1\right.}{(}}{\frac{a^3c^9\sin(ex+d)^3}{(\cos(ex+d)+1)^3}+\frac{3a^4c^8\sin(ex+d)^4}{(\cos(ex+d)+1)^4}+\frac{3a^5c^7\sin(ex+d)^5}{(\cos(ex+d)+1)^5}+\frac{a^6c^6\sin(ex+d)^6}{(\cos(ex+d)+1)^6}}}{384\,e}$$

input
```
integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^4,x, algorithm="maxima")
```

output
```
-1/384*((a^3*c^5 - 3*a^4*c^4*sin(e*x + d)/(cos(e*x + d) + 1) + 3*(5*a^5*c^
3 + 3*a^3*c^5)*sin(e*x + d)^2/(cos(e*x + d) + 1)^2 + (110*a^6*c^2 + 66*a^4
*c^4 + 3*a^2*c^6 + c^8)*sin(e*x + d)^3/(cos(e*x + d) + 1)^3 + 3*(50*a^7*c
+ 30*a^5*c^3 + a*c^7)*sin(e*x + d)^4/(cos(e*x + d) + 1)^4 + 3*(20*a^8 + 12
*a^6*c^2 + a^2*c^6)*sin(e*x + d)^5/(cos(e*x + d) + 1)^5)/(a^3*c^9*sin(e*x
+ d)^3/(cos(e*x + d) + 1)^3 + 3*a^4*c^8*sin(e*x + d)^4/(cos(e*x + d) + 1)^
4 + 3*a^5*c^7*sin(e*x + d)^5/(cos(e*x + d) + 1)^5 + a^6*c^6*sin(e*x + d)^6
/(cos(e*x + d) + 1)^6) - 12*(5*a^3 + 3*a*c^2)*log(c + a*sin(e*x + d)/(cos(
e*x + d) + 1))/c^7 + 12*(5*a^3 + 3*a*c^2)*log(sin(e*x + d)/(cos(e*x + d) +
 1))/c^7)/e
```

3.380.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.68

$$\int \frac{1}{(2a-2a\cos(d+ex)+2c\sin(d+ex))^4}\,dx =$$

$$-\frac{\frac{12\left(5a^3+3ac^2\right)\log\left(\left|\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)\right|\right)}{c^7}-\frac{12\left(5a^4+3a^2c^2\right)\log\left(\left|a\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+c\right|\right)}{ac^7}+\frac{60a^8\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^5+36a^6c^2\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^5+\cdots}{}}{}$$

input
```
integrate(1/(2*a-2*a*cos(e*x+d)+2*c*sin(e*x+d))^4,x, algorithm="giac")
```

output
```
-1/384*(12*(5*a^3 + 3*a*c^2)*log(abs(tan(1/2*e*x + 1/2*d)))/c^7 - 12*(5*a^
4 + 3*a^2*c^2)*log(abs(a*tan(1/2*e*x + 1/2*d) + c))/(a*c^7) + (60*a^8*tan(
1/2*e*x + 1/2*d)^5 + 36*a^6*c^2*tan(1/2*e*x + 1/2*d)^5 + 3*a^2*c^6*tan(1/2
*e*x + 1/2*d)^5 + 150*a^7*c*tan(1/2*e*x + 1/2*d)^4 + 90*a^5*c^3*tan(1/2*e*
x + 1/2*d)^4 + 3*a*c^7*tan(1/2*e*x + 1/2*d)^4 + 110*a^6*c^2*tan(1/2*e*x +
1/2*d)^3 + 66*a^4*c^4*tan(1/2*e*x + 1/2*d)^3 + 3*a^2*c^6*tan(1/2*e*x + 1/2
*d)^3 + c^8*tan(1/2*e*x + 1/2*d)^3 + 15*a^5*c^3*tan(1/2*e*x + 1/2*d)^2 + 9
*a^3*c^5*tan(1/2*e*x + 1/2*d)^2 - 3*a^4*c^4*tan(1/2*e*x + 1/2*d) + a^3*c^5
)/((a*tan(1/2*e*x + 1/2*d)^2 + c*tan(1/2*e*x + 1/2*d))^3*a^3*c^6))/e
```

3.380.9 Mupad [B] (verification not implemented)

Time = 30.81 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.45

$$\int \frac{1}{(2a-2a\cos(d+ex)+2c\sin(d+ex))^4}\,dx = \frac{a\,\mathrm{atanh}\left(\frac{a\left(c+2\,a\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\right)\left(5\,a^2+3\,c^2\right)}{c\left(5\,a^3+3\,a\,c^2\right)}\right)\left(5\,a^2+3\,c^2\right)}{16\,c^7\,e} - \frac{\frac{1}{3\,c}-\frac{a\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{c^2}+\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2\left(5\,a^2+3\,c^2\right)}{c^3}+\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^3\left(110\,a^6+66\,a^4\,c^2+3\,a^2\,c^4+c^6\right)}{3\,a^3\,c^4}+\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^5\left(20\,a^6+12\,a^4\,c^2+c^6\right.}{a\,c^6}}{e\left(128\,a^3\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^6+384\,a^2\,c\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^5+384\,a\,c^2\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^4+128\,c^3\tan\right.}$$

input
```
int(1/(2*a - 2*a*cos(d + e*x) + 2*c*sin(d + e*x))^4,x)
```

output
```
(a*atanh((a*(c + 2*a*tan(d/2 + (e*x)/2))*(5*a^2 + 3*c^2))/(c*(3*a*c^2 + 5*
a^3)))*(5*a^2 + 3*c^2))/(16*c^7*e) - (1/(3*c) - (a*tan(d/2 + (e*x)/2))/c^2
 + (tan(d/2 + (e*x)/2)^2*(5*a^2 + 3*c^2))/c^3 + (tan(d/2 + (e*x)/2)^3*(110
*a^6 + c^6 + 3*a^2*c^4 + 66*a^4*c^2))/(3*a^3*c^4) + (tan(d/2 + (e*x)/2)^5*
(20*a^6 + c^6 + 12*a^4*c^2))/(a*c^6) + (tan(d/2 + (e*x)/2)^4*(50*a^6 + c^6
 + 30*a^4*c^2))/(a^2*c^5))/(e*(128*a^3*tan(d/2 + (e*x)/2)^6 + 128*c^3*tan(
d/2 + (e*x)/2)^3 + 384*a*c^2*tan(d/2 + (e*x)/2)^4 + 384*a^2*c*tan(d/2 + (e
*x)/2)^5))
```

3.381 $\int (2a+2b\cos(d+ex)+2a\sin(d+ex))^3\,dx$

3.381.1 Optimal result

Integrand size = 24, antiderivative size = 157

$$\begin{aligned}
&\int (2a+2b\cos(d+ex)+2a\sin(d+ex))^3\,dx \\
&= 4a\left(5a^2+3b^2\right)x-\frac{4a(15a^2+4b^2)\cos(d+ex)}{3e}+\frac{4b(15a^2+4b^2)\sin(d+ex)}{3e} \\
&\quad-\frac{8(a+b\cos(d+ex)+a\sin(d+ex))^2(a\cos(d+ex)-b\sin(d+ex))}{3e} \\
&\quad-\frac{20(a+b\cos(d+ex)+a\sin(d+ex))\left(a^2\cos(d+ex)-ab\sin(d+ex)\right)}{3e}
\end{aligned}$$

output
```
4*a*(5*a^2+3*b^2)*x-4/3*a*(15*a^2+4*b^2)*cos(e*x+d)/e+4/3*b*(15*a^2+4*b^2)
*sin(e*x+d)/e-8/3*(a+b*cos(e*x+d)+a*sin(e*x+d))^2*(a*cos(e*x+d)-b*sin(e*x+
d))/e-20/3*(a+b*cos(e*x+d)+a*sin(e*x+d))*(a^2*cos(e*x+d)-a*b*sin(e*x+d))/e
```

3.381.2 Mathematica [A] (verified)

Time = 1.45 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.86

$$\begin{aligned}
&\int (2a+2b\cos(d+ex)+2a\sin(d+ex))^3\,dx \\
&= \frac{2\left(6a\left(5a^2+3b^2\right)(d+ex)-9a\left(5a^2+b^2\right)\cos(d+ex)-18a^2b\cos(2(d+ex))+a\left(a^2-3b^2\right)\cos(3(d+ex))\right.}{3e}
\end{aligned}$$

input
```
Integrate[(2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x])^3,x]
```

output
```
(2*(6*a*(5*a^2 + 3*b^2)*(d + e*x) - 9*a*(5*a^2 + b^2)*Cos[d + e*x] - 18*a^
2*b*Cos[2*(d + e*x)] + a*(a^2 - 3*b^2)*Cos[3*(d + e*x)] + 9*b*(5*a^2 + b^2
)*Sin[d + e*x] - 9*a*(a^2 - b^2)*Sin[2*(d + e*x)] + b*(-3*a^2 + b^2)*Sin[3
*(d + e*x)]))/(3*e)
```

3.381.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3599, 27, 3042, 3625, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2a\sin(d+ex)+2a+2b\cos(d+ex))^3\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (2a\sin(d+ex)+2a+2b\cos(d+ex))^3dx$$

$$\Big\downarrow \text{ 3599}$$

$$\frac{1}{3}\int 8(\sin(d+ex)a+a+b\cos(d+ex))\left(5\sin(d+ex)a^2+5a^2+5b\cos(d+ex)a+2b^2\right)dx-\frac{8(a\sin(d+ex)+a+b\cos(d+ex))^2(a\cos(d+ex)-b\sin(d+ex))}{3e}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{8}{3}\int (\sin(d+ex)a+a+b\cos(d+ex))\left(5\sin(d+ex)a^2+5a^2+5b\cos(d+ex)a+2b^2\right)dx-\frac{8(a\sin(d+ex)+a+b\cos(d+ex))^2(a\cos(d+ex)-b\sin(d+ex))}{3e}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{8}{3}\int (\sin(d+ex)a+a+b\cos(d+ex))\left(5\sin(d+ex)a^2+5a^2+5b\cos(d+ex)a+2b^2\right)dx-\frac{8(a\sin(d+ex)+a+b\cos(d+ex))^2(a\cos(d+ex)-b\sin(d+ex))}{3e}$$

$$\Big\downarrow \text{ 3625}$$

$$\frac{8}{3}\left(\frac{\int \left(3\left(5a^2+3b^2\right)a^2+\left(15a^2+4b^2\right)\sin(d+ex)a^2+b\left(15a^2+4b^2\right)\cos(d+ex)a\right)dx}{2a}-\frac{5(a\sin(d+ex)+a+b\cos\right.$$
$$\frac{8(a\sin(d+ex)+a+b\cos(d+ex))^2(a\cos(d+ex)-b\sin(d+ex))}{3e}$$

$\Big\downarrow$ 2009

$$\frac{8}{3}\left(\frac{\frac{ab(15a^2+4b^2)\sin(d+ex)}{e}-\frac{a^2(15a^2+4b^2)\cos(d+ex)}{e}+3a^2x\left(5a^2+3b^2\right)}{2a}-\frac{5(a\sin(d+ex)+a+b\cos(d+ex))\left(a^2\cos(\right.}{2e}\right.$$
$$\frac{8(a\sin(d+ex)+a+b\cos(d+ex))^2(a\cos(d+ex)-b\sin(d+ex))}{3e}$$

input
```
Int[(2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x])^3,x]
```

output
```
(-8*(a + b*Cos[d + e*x] + a*Sin[d + e*x])^2*(a*Cos[d + e*x] - b*Sin[d + e*
x]))/(3*e) + (8*((-5*(a + b*Cos[d + e*x] + a*Sin[d + e*x])*(a^2*Cos[d + e*
x] - a*b*Sin[d + e*x]))/(2*e) + (3*a^2*(5*a^2 + 3*b^2)*x - (a^2*(15*a^2 +
4*b^2)*Cos[d + e*x])/e + (a*b*(15*a^2 + 4*b^2)*Sin[d + e*x])/e)/(2*a)))/3
```

3.381.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3599
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Simp[1/n   Int[Simp[n*a^2 + (
n - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x
], x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

rule 3625
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*((a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^n/(a*e*(n + 1))), x] + Simp[1/(a*(n + 1
))   Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
 + a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
 + (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

3.381.4 Maple [A] (verified)

Time = 2.19 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.96

method	result
parallelrisch	$\frac{\frac{2\left(a^3-3a\,b^2\right)\cos(3ex+3d)}{3}+6\left(-a^3+a\,b^2\right)\sin(2ex+2d)+\frac{2\left(-3a^2b+b^3\right)\sin(3ex+3d)}{3}-12a^2b\cos(2ex+2d)+6\left(-5a^3-a\,b^2\right)\cos}{e}$
parts	$\frac{-8a\,b^2\cos(ex+d)^3+24a\,b^2\left(\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)}{e}+\frac{8a^2b(\sin(ex+d)+1)^3}{e}+8a^3x-\frac{24a^3\cos(ex+d)}{e}+\frac{2}{}$
derivativedivides	$\frac{\frac{8b^3\left(2+\cos(ex+d)^2\right)\sin(ex+d)}{3}-8a\,b^2\cos(ex+d)^3+24a\,b^2\left(\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)+8a^2b\sin(ex+d)^3-24a^2b\cos(ex+}{}$
default	$\frac{\frac{8b^3\left(2+\cos(ex+d)^2\right)\sin(ex+d)}{3}-8a\,b^2\cos(ex+d)^3+24a\,b^2\left(\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)+8a^2b\sin(ex+d)^3-24a^2b\cos(ex+}{}$
risch	$20a^3x+12a\,b^2x-\frac{30a^3\cos(ex+d)}{e}-\frac{6a\cos(ex+d)b^2}{e}+\frac{30b\sin(ex+d)a^2}{e}+\frac{6b^3\sin(ex+d)}{e}+\frac{2a^3\cos(3ex+}{3e}$
norman	$\frac{\left(20a^3+12a\,b^2\right)x+\left(20a^3+12a\,b^2\right)x\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^6+\left(60a^3+36a\,b^2\right)x\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2+\left(60a^3+36a\,b^2\right)x\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^4-\frac{176a}{}}{}$

input
```
int((2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^3,x,method=_RETURNVERBOSE)
```

output
```
2/3*((a^3-3*a*b^2)*cos(3*e*x+3*d)+9*(-a^3+a*b^2)*sin(2*e*x+2*d)+(-3*a^2*b+
b^3)*sin(3*e*x+3*d)-18*a^2*b*cos(2*e*x+2*d)+9*(-5*a^3-a*b^2)*cos(e*x+d)+9*
(5*a^2*b+b^3)*sin(e*x+d)+2*(15*e*x-22)*a^3+18*a^2*b+6*(3*e*x-2)*b^2*a)/e
```

3.381.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.81

$$\int (2a + 2b\cos(d + ex) + 2a\sin(d + ex))^3 \, dx =$$
$$-\frac{4\left(18\,a^2b\cos\left(ex+d\right)^2 + 24\,a^3\cos\left(ex+d\right) - 2\left(a^3 - 3\,ab^2\right)\cos\left(ex+d\right)^3 - 3\left(5\,a^3 + 3\,ab^2\right)ex - \left(24\,a^2b \cdots\right.\right.}{3\,e}$$

input
```
integrate((2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^3,x, algorithm="fricas")
```

output
```
-4/3*(18*a^2*b*cos(e*x + d)^2 + 24*a^3*cos(e*x + d) - 2*(a^3 - 3*a*b^2)*co
s(e*x + d)^3 - 3*(5*a^3 + 3*a*b^2)*e*x - (24*a^2*b + 4*b^3 - 2*(3*a^2*b -
b^3)*cos(e*x + d)^2 - 9*(a^3 - a*b^2)*cos(e*x + d))*sin(e*x + d))/e
```

3.381.6 Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.85

$$\int (2a + 2b\cos(d + ex) + 2a\sin(d + ex))^3 \, dx$$
$$= \begin{cases} 12a^3x\sin^2(d+ex) + 12a^3x\cos^2(d+ex) + 8a^3x - \frac{8a^3\sin^2(d+ex)\cos(d+ex)}{e} - \frac{12a^3\sin(d+ex)\cos(d+ex)}{e} - \frac{16a^3\cos \cdots}{} \\ x(2a\sin(d) + 2a + 2b\cos(d))^3 \end{cases}$$

input
```
integrate((2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))**3,x)
```

output
```
Piecewise((12*a**3*x*sin(d + e*x)**2 + 12*a**3*x*cos(d + e*x)**2 + 8*a**3*
x - 8*a**3*sin(d + e*x)**2*cos(d + e*x)/e - 12*a**3*sin(d + e*x)*cos(d + e
*x)/e - 16*a**3*cos(d + e*x)**3/(3*e) - 24*a**3*cos(d + e*x)/e + 8*a**2*b*
sin(d + e*x)**3/e + 24*a**2*b*sin(d + e*x)**2/e + 24*a**2*b*sin(d + e*x)/e
 + 12*a*b**2*x*sin(d + e*x)**2 + 12*a*b**2*x*cos(d + e*x)**2 + 12*a*b**2*s
in(d + e*x)*cos(d + e*x)/e - 8*a*b**2*cos(d + e*x)**3/e + 16*b**3*sin(d +
e*x)**3/(3*e) + 8*b**3*sin(d + e*x)*cos(d + e*x)**2/e, Ne(e, 0)), (x*(2*a*
sin(d) + 2*a + 2*b*cos(d))**3, True))
```

3.381.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.22

$$\int (2a+2b\cos(d+ex)+2a\sin(d+ex))^3\,dx$$
$$= -\frac{8\,ab^2\cos(ex+d)^3}{e} + \frac{8\,a^2b\sin(ex+d)^3}{e} + 8\,a^3x + \frac{8\left(\cos(ex+d)^3 - 3\,\cos(ex+d)\right)a^3}{3\,e} - \frac{8\left(\sin(ex+d)^3 - 3\,\sin(ex+d)\right)b^3}{3\,e} - 24\,a^2\left(\frac{a\cos(ex+d)}{e} - \frac{b\sin(ex+d)}{e}\right) - 6\left(\frac{4\,ab\cos(ex+d)^2}{e} - \frac{(2\,ex+2\,d-\sin(2\,ex+2\,d))a^2}{e} - \frac{(2\,ex+2\,d+\sin(2\,ex+2\,d))b^2}{e}\right)a$$

input
```
integrate((2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^3,x, algorithm="maxima")
```

output
```
-8*a*b^2*cos(e*x + d)^3/e + 8*a^2*b*sin(e*x + d)^3/e + 8*a^3*x + 8/3*(cos(
e*x + d)^3 - 3*cos(e*x + d))*a^3/e - 8/3*(sin(e*x + d)^3 - 3*sin(e*x + d))
*b^3/e - 24*a^2*(a*cos(e*x + d)/e - b*sin(e*x + d)/e) - 6*(4*a*b*cos(e*x +
 d)^2/e - (2*e*x + 2*d - sin(2*e*x + 2*d))*a^2/e - (2*e*x + 2*d + sin(2*e*
x + 2*d))*b^2/e)*a
```

3.381.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.96

$$\int (2a+2b\cos(d+ex)+2a\sin(d+ex))^3\,dx$$
$$= -\frac{12\,a^2b\cos(2\,ex+2\,d)}{e} + 4\left(5\,a^3+3\,ab^2\right)x + \frac{2\left(a^3-3\,ab^2\right)\cos(3\,ex+3\,d)}{3\,e} - \frac{6\left(5\,a^3+ab^2\right)\cos(ex+d)}{e} - \frac{2\left(3\,a^2b-b^3\right)\sin(3\,ex+3\,d)}{3\,e} - \frac{6\left(a^3-ab^2\right)\sin(2\,ex+2\,d)}{e} + \frac{6\left(5\,a^2b+b^3\right)\sin(ex+d)}{e}$$

input
```
integrate((2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^3,x, algorithm="giac")
```

output
```
-12*a^2*b*cos(2*e*x + 2*d)/e + 4*(5*a^3 + 3*a*b^2)*x + 2/3*(a^3 - 3*a*b^2)
*cos(3*e*x + 3*d)/e - 6*(5*a^3 + a*b^2)*cos(e*x + d)/e - 2/3*(3*a^2*b - b^
3)*sin(3*e*x + 3*d)/e - 6*(a^3 - a*b^2)*sin(2*e*x + 2*d)/e + 6*(5*a^2*b +
b^3)*sin(e*x + d)/e
```

3.381.9 Mupad [B] (verification not implemented)

Time = 28.53 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.86

$$\int (2a+2b\cos(d+ex)+2a\sin(d+ex))^3\,dx$$
$$= \frac{\tan\left(\frac{d}{2}+\frac{ex}{2}\right)^5 (24\,a^3+48\,a^2\,b-24\,a\,b^2+16\,b^3)-\tan\left(\frac{d}{2}+\frac{ex}{2}\right)^4 (48\,a^3-96\,a^2\,b+48\,a\,b^2)-16\,a\,b^2+\tan(}{e\,\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)^6+3\,t}$$
$$+\frac{8\,a\,\mathrm{atan}\left(\frac{8\,a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\,(5\,a^2+3\,b^2)}{40\,a^3+24\,a\,b^2}\right)\,(5\,a^2+3\,b^2)}{e}$$
$$-\frac{8\,a\,(5\,a^2+3\,b^2)\,\left(\mathrm{atan}\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)-\frac{ex}{2}\right)}{e}$$

input
```
int((2*a + 2*b*cos(d + e*x) + 2*a*sin(d + e*x))^3,x)
```

output
```
(tan(d/2 + (e*x)/2)^5*(48*a^2*b - 24*a*b^2 + 24*a^3 + 16*b^3) - tan(d/2 +
(e*x)/2)^4*(48*a*b^2 - 96*a^2*b + 48*a^3) - 16*a*b^2 + tan(d/2 + (e*x)/2)^
2*(96*a^2*b - 128*a^3) + tan(d/2 + (e*x)/2)^3*(160*a^2*b + (32*b^3)/3) - (
176*a^3)/3 + tan(d/2 + (e*x)/2)*(24*a*b^2 + 48*a^2*b - 24*a^3 + 16*b^3))/(
e*(3*tan(d/2 + (e*x)/2)^2 + 3*tan(d/2 + (e*x)/2)^4 + tan(d/2 + (e*x)/2)^6
+ 1)) + (8*a*atan((8*a*tan(d/2 + (e*x)/2)*(5*a^2 + 3*b^2))/(24*a*b^2 + 40*
a^3))*(5*a^2 + 3*b^2))/e - (8*a*(5*a^2 + 3*b^2)*(atan(tan(d/2 + (e*x)/2))
- (e*x)/2))/e
```

3.382 $\int (2a+2b\cos(d+ex)+2a\sin(d+ex))^2\,dx$

3.382.1 Optimal result

Integrand size = 24, antiderivative size = 81

$$\int (2a+2b\cos(d+ex)+2a\sin(d+ex))^2\,dx = 2(3a^2+b^2)\,x-\frac{6a^2\cos(d+ex)}{e}+\frac{6ab\sin(d+ex)}{e} - \frac{2(a+b\cos(d+ex)+a\sin(d+ex))(a\cos(d+ex)-b\sin(d+ex))}{e}$$

output
```
2*(3*a^2+b^2)*x-6*a^2*cos(e*x+d)/e+6*a*b*sin(e*x+d)/e-2*(a+b*cos(e*x+d)+a*
sin(e*x+d))*(a*cos(e*x+d)-b*sin(e*x+d))/e
```

3.382.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.14

$$\int (2a+2b\cos(d+ex)+2a\sin(d+ex))^2\,dx = 4\left(\frac{(3a^2+b^2)\,(d+ex)}{2e}-\frac{2a^2\cos(d+ex)}{e} - \frac{ab\cos(2(d+ex))}{2e}+\frac{2ab\sin(d+ex)}{e} - \frac{(a^2-b^2)\sin(2(d+ex))}{4e}\right)$$

input
```
Integrate[(2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x])^2,x]
```

output
```
4*(((3*a^2 + b^2)*(d + e*x))/(2*e) - (2*a^2*Cos[d + e*x])/e - (a*b*Cos[2*(
d + e*x)])/(2*e) + (2*a*b*Sin[d + e*x])/e - ((a^2 - b^2)*Sin[2*(d + e*x)])
/(4*e))
```

3.382.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3042, 3599, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2a\sin(d+ex)+2a+2b\cos(d+ex))^2\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (2a\sin(d+ex)+2a+2b\cos(d+ex))^2dx$$

$$\Big\downarrow \; 3599$$

$$\frac{1}{2}\int \left(12\sin(d+ex)a^2+12b\cos(d+ex)a+4\left(3a^2+b^2\right)\right)dx-\frac{2(a\sin(d+ex)+a+b\cos(d+ex))(a\cos(d+ex)-b\sin(d+ex))}{e}$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(4x\left(3a^2+b^2\right)-\frac{12a^2\cos(d+ex)}{e}+\frac{12ab\sin(d+ex)}{e}\right)-\frac{2(a\sin(d+ex)+a+b\cos(d+ex))(a\cos(d+ex)-b\sin(d+ex))}{e}$$

input
```
Int[(2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x])^2,x]
```

output
```
(-2*(a + b*Cos[d + e*x] + a*Sin[d + e*x])*(a*Cos[d + e*x] - b*Sin[d + e*x]
))/e + (4*(3*a^2 + b^2)*x - (12*a^2*Cos[d + e*x])/e + (12*a*b*Sin[d + e*x]
)/e)/2
```

3.382.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3599
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Simp[1/n   Int[Simp[n*a^2 + (
n - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x
], x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

3.382.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.99

method	result
parallelrisch	$\frac{(-a^2+b^2)\sin(2ex+2d)-2ab\cos(2ex+2d)-8a^2\cos(ex+d)+8ab\sin(ex+d)+(6ex+8)a^2+2ab+2b^2ex}{e}$
risch	$6a^2x+2x\,b^2-\frac{8a^2\cos(ex+d)}{e}+\frac{8ab\sin(ex+d)}{e}-\frac{2ab\cos(2ex+2d)}{e}-\frac{\sin(2ex+2d)a^2}{e}+\frac{\sin(2ex+2d)b^2}{e}$
derivativedivides	$\frac{4b^2\left(\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)-4ab\cos(ex+d)^2+8ab\sin(ex+d)+4a^2\left(-\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)-8a^2\cos(ex+d}{e}$
default	$\frac{4b^2\left(\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)-4ab\cos(ex+d)^2+8ab\sin(ex+d)+4a^2\left(-\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)-8a^2\cos(ex+d}{e}$
parts	$\frac{8ab\left(\frac{\sin(ex+d)^2}{2}+\sin(ex+d)\right)}{e}+4a^2x-\frac{8a^2\cos(ex+d)}{e}+\frac{4a^2\left(-\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)}{e}+\frac{4b^2\left(\frac{\cos(ex+d)\,\mathrm{s}}{2}\right.}{}$
norman	$\frac{(6a^2+2b^2)x+(6a^2+2b^2)x\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^4+(12a^2+4b^2)x\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2+\frac{16a^2\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^4}{e}-\frac{4(a^2-4ab-b^2)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{e}+}{\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2\right)^2}$

input
```
int((2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^2,x,method=_RETURNVERBOSE)
```

output
```
((-a^2+b^2)*sin(2*e*x+2*d)-2*a*b*cos(2*e*x+2*d)-8*a^2*cos(e*x+d)+8*a*b*sin
(e*x+d)+(6*e*x+8)*a^2+2*a*b+2*b^2*e*x)/e
```

3.382.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.89

$$\int (2a+2b\cos(d+ex)+2a\sin(d+ex))^2\,dx = -\frac{2\left(2\,ab\cos\left(ex+d\right)^2-(3\,a^2+b^2)ex+4\,a^2\cos\left(ex+d\right)-\left(4\,ab-(a^2-b^2)\cos\left(ex+d\right)\right)\sin\left(ex+d\right)\right)}{e}$$

input
```
integrate((2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^2,x, algorithm="fricas")
```

output
```
-2*(2*a*b*cos(e*x + d)^2 - (3*a^2 + b^2)*e*x + 4*a^2*cos(e*x + d) - (4*a*b
 - (a^2 - b^2)*cos(e*x + d))*sin(e*x + d))/e
```

3.382.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(80) = 160.

Time = 0.11 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.10

$$\int (2a+2b\cos(d+ex)+2a\sin(d+ex))^2\,dx = \begin{cases} 2a^2x\sin^2{(d+ex)}+2a^2x\cos^2{(d+ex)}+4a^2x-\frac{2a^2\sin{(d+ex)}\cos{(d+ex)}}{e}-\frac{8a^2\cos{(d+ex)}}{e}+\frac{4ab\sin^2{(d+ex)}}{e}+\frac{8ab}{} \\ x(2a\sin{(d)}+2a+2b\cos{(d)})^2 \end{cases}$$

input
```
integrate((2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))**2,x)
```

output
```
Piecewise((2*a**2*x*sin(d + e*x)**2 + 2*a**2*x*cos(d + e*x)**2 + 4*a**2*x
- 2*a**2*sin(d + e*x)*cos(d + e*x)/e - 8*a**2*cos(d + e*x)/e + 4*a*b*sin(d
 + e*x)**2/e + 8*a*b*sin(d + e*x)/e + 2*b**2*x*sin(d + e*x)**2 + 2*b**2*x*
cos(d + e*x)**2 + 2*b**2*sin(d + e*x)*cos(d + e*x)/e, Ne(e, 0)), (x*(2*a*s
in(d) + 2*a + 2*b*cos(d))**2, True))
```

3.382.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.22

$$\int (2a+2b\cos(d+ex)+2a\sin(d+ex))^2\,dx = 4\,a^2x - \frac{4\,ab\cos{(ex+d)}^2}{e} + \frac{(2\,ex+2\,d-\sin{(2\,ex+2\,d)})a^2}{e} + \frac{(2\,ex+2\,d+\sin{(2\,ex+2\,d)})b^2}{e} - 8\,a\left(\frac{a\cos{(ex+d)}}{e} - \frac{b\sin{(ex+d)}}{e}\right)$$

input
```
integrate((2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^2,x, algorithm="maxima")
```

output
```
4*a^2*x - 4*a*b*cos(e*x + d)^2/e + (2*e*x + 2*d - sin(2*e*x + 2*d))*a^2/e
+ (2*e*x + 2*d + sin(2*e*x + 2*d))*b^2/e - 8*a*(a*cos(e*x + d)/e - b*sin(e
*x + d)/e)
```

3.382.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.98

$$\int (2a+2b\cos(d+ex)+2a\sin(d+ex))^2\,dx = 2\left(3\,a^2+b^2\right)x - \frac{2\,ab\cos{(2\,ex+2\,d)}}{e} - \frac{8\,a^2\cos{(ex+d)}}{e} + \frac{8\,ab\sin{(ex+d)}}{e} - \frac{(a^2-b^2)\sin{(2\,ex+2\,d)}}{e}$$

input
```
integrate((2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^2,x, algorithm="giac")
```

output
```
2*(3*a^2 + b^2)*x - 2*a*b*cos(2*e*x + 2*d)/e - 8*a^2*cos(e*x + d)/e + 8*a*
b*sin(e*x + d)/e - (a^2 - b^2)*sin(2*e*x + 2*d)/e
```

3.382.9 Mupad [B] (verification not implemented)

Time = 27.88 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.57

$$\int (2a+2b\cos(d+ex)+2a\sin(d+ex))^2\,dx = \frac{x\,(12\,a^2+4\,b^2)}{2} + \frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2(16\,a\,b-16\,a^2)+\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^3(4\,a^2+16\,a\,b-4\,b^2)-16\,a^2+\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,(-4\,a^2+16\,a\ldots}{e\,\left(\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^4+2\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2+1\right)}$$

input
```
int((2*a + 2*b*cos(d + e*x) + 2*a*sin(d + e*x))^2,x)
```

output
```
(x*(12*a^2 + 4*b^2))/2 + (tan(d/2 + (e*x)/2)^2*(16*a*b - 16*a^2) + tan(d/2
 + (e*x)/2)^3*(16*a*b + 4*a^2 - 4*b^2) - 16*a^2 + tan(d/2 + (e*x)/2)*(16*a
*b - 4*a^2 + 4*b^2))/(e*(2*tan(d/2 + (e*x)/2)^2 + tan(d/2 + (e*x)/2)^4 + 1
))
```

3.383 $\int (2a+2b\cos(d+ex)+2a\sin(d+ex))\,dx$

3.383.1 Optimal result

Integrand size = 22, antiderivative size = 29

$$\int (2a+2b\cos(d+ex)+2a\sin(d+ex))\,dx=2ax-\frac{2a\cos(d+ex)}{e}+\frac{2b\sin(d+ex)}{e}$$

output
```
2*a*x-2*a*cos(e*x+d)/e+2*b*sin(e*x+d)/e
```

3.383.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.83

$$\int (2a+2b\cos(d+ex)+2a\sin(d+ex))\,dx=2ax-\frac{2a\cos(d)\cos(ex)}{e}+\frac{2b\cos(ex)\sin(d)}{e}+\frac{2b\cos(d)\sin(ex)}{e}+\frac{2a\sin(d)\sin(ex)}{e}$$

input
```
Integrate[2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x],x]
```

output
```
2*a*x - (2*a*Cos[d]*Cos[e*x])/e + (2*b*Cos[e*x]*Sin[d])/e + (2*b*Cos[d]*Si
n[e*x])/e + (2*a*Sin[d]*Sin[e*x])/e
```

3.383.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2a\sin(d+ex)+2a+2b\cos(d+ex))\,dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{2a\cos(d+ex)}{e}+2ax+\frac{2b\sin(d+ex)}{e}$$

input
```
Int[2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x],x]
```

output
```
2*a*x - (2*a*Cos[d + e*x])/e + (2*b*Sin[d + e*x])/e
```

3.383.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.383.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

method	result	size
default	$2ax-\frac{2a\cos(ex+d)}{e}+\frac{2b\sin(ex+d)}{e}$	30
risch	$2ax-\frac{2a\cos(ex+d)}{e}+\frac{2b\sin(ex+d)}{e}$	30
parts	$2ax-\frac{2a\cos(ex+d)}{e}+\frac{2b\sin(ex+d)}{e}$	30
derivativedivides	$\frac{2(ex+d)a+2b\sin(ex+d)-2a\cos(ex+d)}{e}$	31
parallelrisch	$\frac{2b\sin(ex+d)-2a\cos(ex+d)+2a}{e}+2ax$	32
norman	$\frac{\frac{4a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}}{e}+2ax+2ax\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}+\frac{4b\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{e}}{1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}}$	69

input
```
int(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
2*a*x-2*a*cos(e*x+d)/e+2*b*sin(e*x+d)/e
```

3.383.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int (2a + 2b\cos(d+ex) + 2a\sin(d+ex))\,dx = \frac{2\,(aex - a\cos{(ex+d)} + b\sin{(ex+d)})}{e}$$

input
```
integrate(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d),x, algorithm="fricas")
```

output
```
2*(a*e*x - a*cos(e*x + d) + b*sin(e*x + d))/e
```

3.383.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.34

$$\int (2a + 2b\cos(d+ex) + 2a\sin(d+ex))\,dx = 2ax + 2a\left(\begin{cases} -\frac{\cos{(d+ex)}}{e} & \text{for}\ e\neq 0 \\ x\sin{(d)} & \text{otherwise} \end{cases}\right) + 2b\left(\begin{cases} \frac{\sin{(d+ex)}}{e} & \text{for}\ e\neq 0 \\ x\cos{(d)} & \text{otherwise} \end{cases}\right)$$

input
```
integrate(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d),x)
```

output
```
2*a*x + 2*a*Piecewise((-cos(d + e*x)/e, Ne(e, 0)), (x*sin(d), True)) + 2*b
*Piecewise((sin(d + e*x)/e, Ne(e, 0)), (x*cos(d), True))
```

3.383.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int (2a + 2b\cos(d + ex) + 2a\sin(d + ex))\, dx = 2\,ax - \frac{2\,a\cos(ex + d)}{e} + \frac{2\,b\sin(ex + d)}{e}$$

input
```
integrate(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d),x, algorithm="maxima")
```

output
```
2*a*x - 2*a*cos(e*x + d)/e + 2*b*sin(e*x + d)/e
```

3.383.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int (2a + 2b\cos(d + ex) + 2a\sin(d + ex))\, dx = 2\,ax - \frac{2\,a\cos(ex + d)}{e} + \frac{2\,b\sin(ex + d)}{e}$$

input
```
integrate(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d),x, algorithm="giac")
```

output
```
2*a*x - 2*a*cos(e*x + d)/e + 2*b*sin(e*x + d)/e
```

3.383.9 Mupad [B] (verification not implemented)

Time = 27.69 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int (2a + 2b\cos(d + ex) + 2a\sin(d + ex))\, dx = 2\,a\,x - \frac{2\,a\,\cos(d + e\,x)}{e} + \frac{2\,b\,\sin(d + e\,x)}{e}$$

input
```
int(2*a + 2*b*cos(d + e*x) + 2*a*sin(d + e*x),x)
```

output
```
2*a*x - (2*a*cos(d + e*x))/e + (2*b*sin(d + e*x))/e
```

3.384 $\int \frac{1}{2a+2b\cos(d+ex)+2a\sin(d+ex)}\,dx$

3.384.1 Optimal result

Integrand size = 24, antiderivative size = 33

$$\int \frac{1}{2a+2b\cos(d+ex)+2a\sin(d+ex)}\,dx=-\frac{\log\left(a+b\cot\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)\right)}{2be}$$

output
```
-1/2*ln(a+b*cot(1/2*d+1/4*Pi+1/2*e*x))/b/e
```

3.384.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 93 vs. 2(33) = 66.

Time = 0.12 (sec) , antiderivative size = 93, normalized size of antiderivative = 2.82

$$\begin{aligned}&\int \frac{1}{2a+2b\cos(d+ex)+2a\sin(d+ex)}\,dx\\&=\frac{1}{2}\left(\frac{\log\left(\cos\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{1}{2}(d+ex)\right)\right)}{be}\right.\\&\left.-\frac{\log\left(a\cos\left(\frac{1}{2}(d+ex)\right)+b\cos\left(\frac{1}{2}(d+ex)\right)+a\sin\left(\frac{1}{2}(d+ex)\right)-b\sin\left(\frac{1}{2}(d+ex)\right)\right)}{be}\right)\end{aligned}$$

input
```
Integrate[(2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-1),x]
```

output
```
(Log[Cos[(d + e*x)/2] + Sin[(d + e*x)/2]]/(b*e) - Log[a*Cos[(d + e*x)/2] +
 b*Cos[(d + e*x)/2] + a*Sin[(d + e*x)/2] - b*Sin[(d + e*x)/2]]/(b*e))/2
```

3.384.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3042, 3602, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{2a\sin(d+ex)+2a+2b\cos(d+ex)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{2a\sin(d+ex)+2a+2b\cos(d+ex)}dx$$

$$\Big\downarrow \text{3602}$$

$$-\frac{\int \frac{1}{2a+2b\cot\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)}d\cot\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)}{e}$$

$$\Big\downarrow \text{16}$$

$$-\frac{\log\left(a+b\cot\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)\right)}{2be}$$

input
```
Int[(2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-1),x]
```

output
```
-1/2*Log[a + b*Cot[d/2 + Pi/4 + (e*x)/2]]/(b*e)
```

3.384.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3602
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Cot[(d + e*x)/2 + Pi/4], x]}, Si
mp[-f/e   Subst[Int[1/(a + b*f*x), x], x, Cot[(d + e*x)/2 + Pi/4]/f], x]] /
; FreeQ[{a, b, c, d, e}, x] && EqQ[a - c, 0] && NeQ[a - b, 0]
```

3.384.4 Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.30

method	result	size
parallelrisch	$\frac{\ln\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)-\ln\left(a+b+(a-b)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}{2be}$	43
norman	$\frac{\ln\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}{2be}-\frac{\ln\left(a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-b\tan\left(\frac{ex}{2}+\frac{d}{2}\right)+a+b\right)}{2be}$	57
risch	$\frac{\ln\left(\mathrm{e}^{i(ex+d)}+i\right)}{2be}-\frac{\ln\left(\mathrm{e}^{i(ex+d)}+\frac{ia+b}{ib+a}\right)}{2be}$	57
derivativedivides	$\frac{\frac{\ln\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}{b}+\frac{(-a+b)\ln\left(a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-b\tan\left(\frac{ex}{2}+\frac{d}{2}\right)+a+b\right)}{b(a-b)}}{2e}$	66
default	$\frac{\frac{\ln\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}{b}+\frac{(-a+b)\ln\left(a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-b\tan\left(\frac{ex}{2}+\frac{d}{2}\right)+a+b\right)}{b(a-b)}}{2e}$	66

input
```
int(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d)),x,method=_RETURNVERBOSE)
```

output
```
1/2*(ln(1+tan(1/2*e*x+1/2*d))-ln(a+b+(a-b)*tan(1/2*e*x+1/2*d)))/b/e
```

3.384.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(25) = 50.

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.64

$$\int \frac{1}{2a+2b\cos(d+ex)+2a\sin(d+ex)}\,dx = -\frac{\log\left(2\,ab\cos\left(ex+d\right)+a^2+b^2+\left(a^2-b^2\right)\sin\left(ex+d\right)\right)-\log\left(\sin\left(ex+d\right)+1\right)}{4\,be}$$

input
```
integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d)),x, algorithm="fricas")
```

output
```
-1/4*(log(2*a*b*cos(e*x + d) + a^2 + b^2 + (a^2 - b^2)*sin(e*x + d)) - log
(sin(e*x + d) + 1))/(b*e)
```

3.384.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(24) = 48$.

Time = 1.00 (sec) , antiderivative size = 107, normalized size of antiderivative = 3.24

$$\int \frac{1}{2a + 2b\cos(d + ex) + 2a\sin(d + ex)}\,dx = \begin{cases} \frac{\tilde{\infty}x}{\cos(d)} & \text{for } a = 0 \wedge b = 0 \wedge e = 0 \\ -\frac{1}{ae\tan\left(\frac{d}{2} + \frac{ex}{2}\right) + ae} & \text{for } b = 0 \\ \frac{x}{2a\sin(d) + 2a + 2b\cos(d)} & \text{for } e = 0 \\ \frac{\log\left(\tan\left(\frac{d}{2} + \frac{ex}{2}\right) + 1\right)}{2be} & \text{for } a = b \\ \frac{\log\left(\tan\left(\frac{d}{2} + \frac{ex}{2}\right) + 1\right)}{2be} - \frac{\log\left(\frac{a}{a-b} + \frac{b}{a-b} + \tan\left(\frac{d}{2} + \frac{ex}{2}\right)\right)}{2be} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d)),x)
```

output
```
Piecewise((zoo*x/cos(d), Eq(a, 0) & Eq(b, 0) & Eq(e, 0)), (-1/(a*e*tan(d/2
 + e*x/2) + a*e), Eq(b, 0)), (x/(2*a*sin(d) + 2*a + 2*b*cos(d)), Eq(e, 0))
, (log(tan(d/2 + e*x/2) + 1)/(2*b*e), Eq(a, b)), (log(tan(d/2 + e*x/2) + 1
)/(2*b*e) - log(a/(a - b) + b/(a - b) + tan(d/2 + e*x/2))/(2*b*e), True))
```

3.384.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(25) = 50$.

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.00

$$\int \frac{1}{2a + 2b\cos(d + ex) + 2a\sin(d + ex)}\,dx = -\frac{\frac{\log\left(-a - b - \frac{(a-b)\sin(ex+d)}{\cos(ex+d)+1}\right)}{b} - \frac{\log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1} + 1\right)}{b}}{2\,e}$$

input
```
integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d)),x, algorithm="maxima")
```

output
```
-1/2*(log(-a - b - (a - b)*sin(e*x + d)/(cos(e*x + d) + 1))/b - log(sin(e*
x + d)/(cos(e*x + d) + 1) + 1)/b)/e
```

3.384.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(25) = 50$.

Time = 0.29 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.39

$$\int \frac{1}{2a + 2b\cos(d + ex) + 2a\sin(d + ex)}\,dx = \frac{\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)-2\,b\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+2\,a-2\,|b|\right|}{\left|2\,a\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)-2\,b\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+2\,a+2\,|b|\right|}\right)}{2\,e|b|}$$

input
```
integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d)),x, algorithm="giac")
```

output
```
1/2*log(abs(2*a*tan(1/2*e*x + 1/2*d) - 2*b*tan(1/2*e*x + 1/2*d) + 2*a - 2*
abs(b))/abs(2*a*tan(1/2*e*x + 1/2*d) - 2*b*tan(1/2*e*x + 1/2*d) + 2*a + 2*
abs(b)))/(e*abs(b))
```

3.384.9 Mupad [B] (verification not implemented)

Time = 27.37 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{1}{2a + 2b\cos(d + ex) + 2a\sin(d + ex)}\,dx = -\frac{\operatorname{atanh}\left(\frac{a+\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,(2\,a-2\,b)}{2}}{b}\right)}{b\,e}$$

input
```
int(1/(2*a + 2*b*cos(d + e*x) + 2*a*sin(d + e*x)),x)
```

output
```
-atanh((a + (tan(d/2 + (e*x)/2)*(2*a - 2*b))/2)/b)/(b*e)
```

3.385 $\int \frac{1}{(2a+2b\cos(d+ex)+2a\sin(d+ex))^2}\,dx$

3.385.1 Optimal result

Integrand size = 24, antiderivative size = 83

$$\int \frac{1}{(2a+2b\cos(d+ex)+2a\sin(d+ex))^2}\,dx = \frac{a\log\left(a+b\cot\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)\right)}{4b^3e} - \frac{a\cos(d+ex)-b\sin(d+ex)}{4b^2e(a+b\cos(d+ex)+a\sin(d+ex))}$$

output
```
1/4*a*ln(a+b*cot(1/2*d+1/4*Pi+1/2*e*x))/b^3/e+1/4*(-a*cos(e*x+d)+b*sin(e*x
+d))/b^2/e/(a+b*cos(e*x+d)+a*sin(e*x+d))
```

3.385.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.95

$$\int \frac{1}{(2a+2b\cos(d+ex)+2a\sin(d+ex))^2}\,dx = \frac{-a\log\left(\cos\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{1}{2}(d+ex)\right)\right)+a\log\left((a+b)\cos\left(\frac{1}{2}(d+ex)\right)+(a-b)\sin\left(\frac{1}{2}(d+ex)\right)\right)+\cdots}{4b^3e}$$

input
```
Integrate[(2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-2),x]
```

output
```
(-(a*Log[Cos[(d + e*x)/2] + Sin[(d + e*x)/2]]) + a*Log[(a + b)*Cos[(d + e*
x)/2] + (a - b)*Sin[(d + e*x)/2]] + (b*Sin[(d + e*x)/2])/(Cos[(d + e*x)/2]
 + Sin[(d + e*x)/2]) + (b*(a^2 + b^2)*Sin[(d + e*x)/2])/((a + b)*((a + b)*
Cos[(d + e*x)/2] + (a - b)*Sin[(d + e*x)/2])))/(4*b^3*e)
```

3.385.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {3042, 3608, 25, 27, 3042, 3602, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2a\sin(d+ex)+2a+2b\cos(d+ex))^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(2a\sin(d+ex)+2a+2b\cos(d+ex))^2}dx$$
$$\Big\downarrow \text{3608}$$
$$\frac{\int -\frac{a}{\sin(d+ex)a+a+b\cos(d+ex)}dx}{4b^2}-\frac{a\cos(d+ex)-b\sin(d+ex)}{4b^2e(a\sin(d+ex)+a+b\cos(d+ex))}$$
$$\Big\downarrow \text{25}$$
$$-\frac{\int \frac{a}{\sin(d+ex)a+a+b\cos(d+ex)}dx}{4b^2}-\frac{a\cos(d+ex)-b\sin(d+ex)}{4b^2e(a\sin(d+ex)+a+b\cos(d+ex))}$$
$$\Big\downarrow \text{27}$$
$$-\frac{a\int \frac{1}{\sin(d+ex)a+a+b\cos(d+ex)}dx}{4b^2}-\frac{a\cos(d+ex)-b\sin(d+ex)}{4b^2e(a\sin(d+ex)+a+b\cos(d+ex))}$$
$$\Big\downarrow \text{3042}$$
$$-\frac{a\int \frac{1}{\sin(d+ex)a+a+b\cos(d+ex)}dx}{4b^2}-\frac{a\cos(d+ex)-b\sin(d+ex)}{4b^2e(a\sin(d+ex)+a+b\cos(d+ex))}$$
$$\Big\downarrow \text{3602}$$
$$\frac{a\int \frac{1}{a+b\cot\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)}d\cot\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)}{4b^2e}-\frac{a\cos(d+ex)-b\sin(d+ex)}{4b^2e(a\sin(d+ex)+a+b\cos(d+ex))}$$

$$\Big\downarrow 16$$

$$\frac{a\log\left(a+b\cot\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)\right)}{4b^3e}-\frac{a\cos(d+ex)-b\sin(d+ex)}{4b^2e(a\sin(d+ex)+a+b\cos(d+ex))}$$

input
```
Int[(2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-2),x]
```

output
```
(a*Log[a + b*Cot[d/2 + Pi/4 + (e*x)/2]])/(4*b^3*e) - (a*Cos[d + e*x] - b*S
in[d + e*x])/(4*b^2*e*(a + b*Cos[d + e*x] + a*Sin[d + e*x]))
```

3.385.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3602
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Cot[(d + e*x)/2 + Pi/4], x]}, Si
mp[-f/e   Subst[Int[1/(a + b*f*x), x], x, Cot[(d + e*x)/2 + Pi/4]/f], x]] /
; FreeQ[{a, b, c, d, e}, x] && EqQ[a - c, 0] && NeQ[a - b, 0]
```

rule 3608
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Simp[
1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c
*(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x
] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] &&
NeQ[n, -3/2]
```

3.385. $\int \frac{1}{(2a+2b\cos(d+ex)+2a\sin(d+ex))^2}\,dx$

3.385.4 Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.47

method	result
derivativedivides	$\frac{-\frac{a^2+b^2}{b^2(a-b)\left(a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-b\tan\left(\frac{ex}{2}+\frac{d}{2}\right)+a+b\right)}+\frac{a\ln\left(a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-b\tan\left(\frac{ex}{2}+\frac{d}{2}\right)+a+b\right)}{b^3}-\frac{1}{b^2\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}-\frac{a\ln\left(1+\tan\left(\right.\right.}{b^3}}{4e}$
default	$\frac{-\frac{a^2+b^2}{b^2(a-b)\left(a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-b\tan\left(\frac{ex}{2}+\frac{d}{2}\right)+a+b\right)}+\frac{a\ln\left(a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-b\tan\left(\frac{ex}{2}+\frac{d}{2}\right)+a+b\right)}{b^3}-\frac{1}{b^2\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}-\frac{a\ln\left(1+\tan\left(\right.\right.}{b^3}}{4e}$
risch	$\frac{i\left(ia+b+a\,e^{i(ex+d)}\right)}{2b^2e\left(-ia\,e^{2i(ex+d)}+b\,e^{2i(ex+d)}+ia+2a\,e^{i(ex+d)}+b\right)}+\frac{a\ln\left(e^{i(ex+d)}+\frac{ia+b}{ib+a}\right)}{4b^3e}-\frac{a\ln\left(e^{i(ex+d)}+i\right)}{4b^3e}$
parallelrisch	$\frac{a^2(a(\sin(ex+d)+1)+b\cos(ex+d))\ln\left(a+b+(a-b)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)+\left(-\cos(ex+d)a^2b-a^3(\sin(ex+d)+1)\right)\ln\left(1+\tan\left(\frac{ex}{2}+\right.\right.}{4b^3ea(a(\sin(ex+d)+1)+b\cos(ex+d))}$
norman	$\frac{-\frac{a^2+ab+b^2}{4a\,b^2e}+\frac{\left(a^2-ab+b^2\right)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}{4a\,b^2e}}{\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)\left(a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-b\tan\left(\frac{ex}{2}+\frac{d}{2}\right)+a+b\right)}-\frac{a\ln\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}{4b^3e}+\frac{a\ln\left(a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-b\tan\left(\frac{ex}{2}\right.\right.}{4b^3e}$

input
```
int(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4/e*(-(a^2+b^2)/b^2/(a-b)/(a*tan(1/2*e*x+1/2*d)-b*tan(1/2*e*x+1/2*d)+a+b
)+a/b^3*ln(a*tan(1/2*e*x+1/2*d)-b*tan(1/2*e*x+1/2*d)+a+b)-1/b^2/(1+tan(1/2
*e*x+1/2*d))-a/b^3*ln(1+tan(1/2*e*x+1/2*d)))
```

3.385.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(73) = 146.

Time = 0.27 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.78

$$\int \frac{1}{(2a+2b\cos(d+ex)+2a\sin(d+ex))^2}\,dx = -\frac{2\,ab\cos(ex+d)-2\,b^2\sin(ex+d)-\left(ab\cos(ex+d)+a^2\sin(ex+d)+a^2\right)\log\left(2\,ab\cos(ex+d)+a^2\right.}{8\left(b^4e\cos(ex+d)+ab^3e\sin\right.}$$

input
```
integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^2,x, algorithm="fricas")
```

output
```
-1/8*(2*a*b*cos(e*x + d) - 2*b^2*sin(e*x + d) - (a*b*cos(e*x + d) + a^2*si
n(e*x + d) + a^2)*log(2*a*b*cos(e*x + d) + a^2 + b^2 + (a^2 - b^2)*sin(e*x
 + d)) + (a*b*cos(e*x + d) + a^2*sin(e*x + d) + a^2)*log(sin(e*x + d) + 1)
)/(b^4*e*cos(e*x + d) + a*b^3*e*sin(e*x + d) + a*b^3*e)
```

3.385.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(2a+2b\cos(d+ex)+2a\sin(d+ex))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))**2,x)
```

output
```
Timed out
```

3.385.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(73) = 146$.

Time = 0.23 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.23

$$\int \frac{1}{(2a+2b\cos(d+ex)+2a\sin(d+ex))^2}\,dx = -\frac{\frac{2\left(a^2+\frac{(a^2-ab+b^2)\sin(ex+d)}{\cos(ex+d)+1}\right)}{a^2b^2-b^4+\frac{2(a^2b^2-ab^3)\sin(ex+d)}{\cos(ex+d)+1}+\frac{(a^2b^2-2ab^3+b^4)\sin(ex+d)^2}{(\cos(ex+d)+1)^2}} - \frac{a\log\left(-a-b-\frac{(a-b)\sin(ex+d)}{\cos(ex+d)+1}\right)}{b^3} + \frac{a\log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}+1\right)}{b^3}}{4e}$$

input
```
integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^2,x, algorithm="maxima")
```

output
```
-1/4*(2*(a^2 + (a^2 - a*b + b^2)*sin(e*x + d)/(cos(e*x + d) + 1))/(a^2*b^2
 - b^4 + 2*(a^2*b^2 - a*b^3)*sin(e*x + d)/(cos(e*x + d) + 1) + (a^2*b^2 -
2*a*b^3 + b^4)*sin(e*x + d)^2/(cos(e*x + d) + 1)^2) - a*log(-a - b - (a -
b)*sin(e*x + d)/(cos(e*x + d) + 1))/b^3 + a*log(sin(e*x + d)/(cos(e*x + d)
 + 1) + 1)/b^3)/e
```

3.385.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(73) = 146.

Time = 0.29 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.25

$$\int \frac{1}{(2a+2b\cos(d+ex)+2a\sin(d+ex))^2}\,dx = -\frac{\frac{2\left(a^2\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)-ab\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+b^2\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+a^2\right)}{(ab^2-b^3)\left(a\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^2-b\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^2+2a\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+a+b\right)}+\frac{a\log\left(\frac{\left|2a\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)-2b\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+2a-2|b|\right|}{\left|2a\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)-2b\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+2a+2|b|\right|}\right)}{b^2|b|}}{4e}$$

input
```
integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^2,x, algorithm="giac")
```

output
```
-1/4*(2*(a^2*tan(1/2*e*x + 1/2*d) - a*b*tan(1/2*e*x + 1/2*d) + b^2*tan(1/2
*e*x + 1/2*d) + a^2)/((a*b^2 - b^3)*(a*tan(1/2*e*x + 1/2*d)^2 - b*tan(1/2*
e*x + 1/2*d)^2 + 2*a*tan(1/2*e*x + 1/2*d) + a + b)) + a*log(abs(2*a*tan(1/
2*e*x + 1/2*d) - 2*b*tan(1/2*e*x + 1/2*d) + 2*a - 2*abs(b))/abs(2*a*tan(1/
2*e*x + 1/2*d) - 2*b*tan(1/2*e*x + 1/2*d) + 2*a + 2*abs(b)))/(b^2*abs(b)))
/e
```

3.385.9 Mupad [B] (verification not implemented)

Time = 26.78 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.52

$$\int \frac{1}{(2a+2b\cos(d+ex)+2a\sin(d+ex))^2}\,dx = \frac{a\,\mathrm{atanh}\left(\frac{a+\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)(2\,a-2\,b)}{2}}{b}\right)}{2\,b^3\,e} - \frac{\frac{a^2}{b^2\,(a-b)}+\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\left(a^2-a\,b+b^2\right)}{b^2\,(a-b)}}{e\,\left((2\,a-2\,b)\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2+4\,a\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)+2\,a+2\,b\right)}$$

input
```
int(1/(2*a + 2*b*cos(d + e*x) + 2*a*sin(d + e*x))^2,x)
```

output
```
(a*atanh((a + (tan(d/2 + (e*x)/2)*(2*a - 2*b))/2)/b))/(2*b^3*e) - (a^2/(b^
2*(a - b)) + (tan(d/2 + (e*x)/2)*(a^2 - a*b + b^2))/(b^2*(a - b)))/(e*(2*a
 + 2*b + tan(d/2 + (e*x)/2)^2*(2*a - 2*b) + 4*a*tan(d/2 + (e*x)/2)))
```

3.386 $\int \frac{1}{(2a+2b\cos(d+ex)+2a\sin(d+ex))^3}\,dx$

3.386.1 Optimal result

Integrand size = 24, antiderivative size = 142

$$\int \frac{1}{(2a+2b\cos(d+ex)+2a\sin(d+ex))^3}\,dx = -\frac{(3a^2+b^2)\log\left(a+b\cot\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)\right)}{16b^5e} - \frac{a\cos(d+ex)-b\sin(d+ex)}{16b^2e(a+b\cos(d+ex)+a\sin(d+ex))^2} + \frac{3(a^2\cos(d+ex)-ab\sin(d+ex))}{16b^4e(a+b\cos(d+ex)+a\sin(d+ex))}$$

output
```
-1/16*(3*a^2+b^2)*ln(a+b*cot(1/2*d+1/4*Pi+1/2*e*x))/b^5/e+1/16*(-a*cos(e*x
+d)+b*sin(e*x+d))/b^2/e/(a+b*cos(e*x+d)+a*sin(e*x+d))^2+3/16*(a^2*cos(e*x+
d)-a*b*sin(e*x+d))/b^4/e/(a+b*cos(e*x+d)+a*sin(e*x+d))
```

3.386.2 Mathematica [A] (verified)

Time = 1.92 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.80

$$\int \frac{1}{(2a+2b\cos(d+ex)+2a\sin(d+ex))^3}\,dx = -\frac{-2(3a^2+b^2)\log\left(\cos\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{1}{2}(d+ex)\right)\right)+2(3a^2+b^2)\log\left((a+b)\cos\left(\frac{1}{2}(d+ex)\right)+(a-\right.}{}$$

input
```
Integrate[(2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-3),x]
```

output
```
-1/32*(-2*(3*a^2 + b^2)*Log[Cos[(d + e*x)/2] + Sin[(d + e*x)/2]] + 2*(3*a^
2 + b^2)*Log[(a + b)*Cos[(d + e*x)/2] + (a - b)*Sin[(d + e*x)/2]] + b^2/(C
os[(d + e*x)/2] + Sin[(d + e*x)/2])^2 + (6*a*b*Sin[(d + e*x)/2])/(Cos[(d +
 e*x)/2] + Sin[(d + e*x)/2]) - (b^2*(a^2 + b^2))/((a + b)*Cos[(d + e*x)/2]
 + (a - b)*Sin[(d + e*x)/2])^2 + (6*a*b*(a^2 + b^2)*Sin[(d + e*x)/2])/((a
+ b)*((a + b)*Cos[(d + e*x)/2] + (a - b)*Sin[(d + e*x)/2])))/(b^5*e)
```

3.386.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3608, 27, 3042, 3632, 3042, 3602, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2a\sin(d+ex)+2a+2b\cos(d+ex))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(2a\sin(d+ex)+2a+2b\cos(d+ex))^3}dx$$

$$\Big\downarrow \text{3608}$$

$$\frac{\int -\frac{-\sin(d+ex)a+2a-b\cos(d+ex)}{2(\sin(d+ex)a+a+b\cos(d+ex))^2}dx}{8b^2}-\frac{a\cos(d+ex)-b\sin(d+ex)}{16b^2e(a\sin(d+ex)+a+b\cos(d+ex))^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{-\sin(d+ex)a+2a-b\cos(d+ex)}{(\sin(d+ex)a+a+b\cos(d+ex))^2}dx}{16b^2}-\frac{a\cos(d+ex)-b\sin(d+ex)}{16b^2e(a\sin(d+ex)+a+b\cos(d+ex))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{-\sin(d+ex)a+2a-b\cos(d+ex)}{(\sin(d+ex)a+a+b\cos(d+ex))^2}dx}{16b^2}-\frac{a\cos(d+ex)-b\sin(d+ex)}{16b^2e(a\sin(d+ex)+a+b\cos(d+ex))^2}$$

$$\Big\downarrow \text{3632}$$

$$-\frac{-\left(\frac{3a^2}{b^2}+1\right)\int \frac{1}{\sin(d+ex)a+a+b\cos(d+ex)}dx-\frac{3(a^2\cos(d+ex)-ab\sin(d+ex))}{b^2e(a\sin(d+ex)+a+b\cos(d+ex))}}{16b^2}-\frac{a\cos(d+ex)-b\sin(d+ex)}{16b^2e(a\sin(d+ex)+a+b\cos(d+ex))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\left(\frac{3a^2}{b^2}+1\right)\int \frac{1}{\sin(d+ex)a+a+b\cos(d+ex)}dx-\frac{3(a^2\cos(d+ex)-ab\sin(d+ex))}{b^2e(a\sin(d+ex)+a+b\cos(d+ex))}}{16b^2}-\frac{a\cos(d+ex)-b\sin(d+ex)}{16b^2e(a\sin(d+ex)+a+b\cos(d+ex))^2}$$

$$\Big\downarrow \text{3602}$$

$$-\frac{\frac{\left(\frac{3a^2}{b^2}+1\right)\int \frac{1}{a+b\cot\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)}d\cot\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)}{e}-\frac{3(a^2\cos(d+ex)-ab\sin(d+ex))}{b^2e(a\sin(d+ex)+a+b\cos(d+ex))}}{16b^2}-\frac{a\cos(d+ex)-b\sin(d+ex)}{16b^2e(a\sin(d+ex)+a+b\cos(d+ex))^2}$$

$$\Big\downarrow \text{16}$$

$$-\frac{\frac{\left(\frac{3a^2}{b^2}+1\right)\log\left(a+b\cot\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)\right)}{be}-\frac{3(a^2\cos(d+ex)-ab\sin(d+ex))}{b^2e(a\sin(d+ex)+a+b\cos(d+ex))}}{16b^2}-\frac{a\cos(d+ex)-b\sin(d+ex)}{16b^2e(a\sin(d+ex)+a+b\cos(d+ex))^2}$$

input
```
Int[(2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-3),x]
```

output
```
-1/16*(a*Cos[d + e*x] - b*Sin[d + e*x])/(b^2*e*(a + b*Cos[d + e*x] + a*Sin
[d + e*x])^2) - (((1 + (3*a^2)/b^2)*Log[a + b*Cot[d/2 + Pi/4 + (e*x)/2]])/
(b*e) - (3*(a^2*Cos[d + e*x] - a*b*Sin[d + e*x]))/(b^2*e*(a + b*Cos[d + e*
x] + a*Sin[d + e*x])))/(16*b^2)
```

3.386.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.386. $\int \frac{1}{(2a+2b\cos(d+ex)+2a\sin(d+ex))^3}\,dx$

rule 3602
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Cot[(d + e*x)/2 + Pi/4], x]}, Si
mp[-f/e   Subst[Int[1/(a + b*f*x), x], x, Cot[(d + e*x)/2 + Pi/4]/f], x]] /
; FreeQ[{a, b, c, d, e}, x] && EqQ[a - c, 0] && NeQ[a - b, 0]
```

rule 3608
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Simp[
1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c
*(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x
] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] &&
NeQ[n, -3/2]
```

rule 3632
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Simp[(a*A - b*B - c*C)/(a^2 - b^2 - c^2)   Int[1/(a + b*Cos[d + e*x] + c*S
in[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
 - c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

3.386.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(130) = 260$.

Time = 1.95 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.86

method	result
derivativedivides	$\frac{-\frac{1}{2b^3\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)^2}-\frac{-3a-b}{2b^4\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}+\frac{\left(3a^2+b^2\right)\ln\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}{2b^5}+\frac{\left(-3a^3+3a^2b-a\,b^2+b^3\right)\ln\left(a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-b\tan\right.}{2b^5(a-b)}}{8e}$
default	$\frac{-\frac{1}{2b^3\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)^2}-\frac{-3a-b}{2b^4\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}+\frac{\left(3a^2+b^2\right)\ln\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}{2b^5}+\frac{\left(-3a^3+3a^2b-a\,b^2+b^3\right)\ln\left(a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-b\tan\right.}{2b^5(a-b)}}{8e}$
risch	$-\frac{i\left(3a^2b\,e^{3i(ex+d)}+b^3e^{3i(ex+d)}+9a^3e^{2i(ex+d)}+3a\,b^2e^{2i(ex+d)}-3ia^3e^{3i(ex+d)}-ia\,b^2e^{3i(ex+d)}+9a^2b\,e^{i(ex+d)}-b^3e^{i(ex+\right.}{8\left(-ia\,e^{2i(ex+d)}+b\,e^{2i(ex+d)}+ia+2a\,e^{i(ex+d)}+b\right)^2b^4e}$
norman	$\frac{\frac{9a^5+18a^4b+12a^3b^2+6a^2b^3+a\,b^4}{16b^4e\left(3a^2-b^2\right)}-\frac{\left(9a^5-9a^4b+6a^3b^2-6a^2b^3+3a\,b^4+b^5\right)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^3}{8b^4e\left(3a^2-b^2\right)}+\frac{\left(9a^5+9a^4b+6a^3b^2+6a^2b^3+3a\,b^4-b^5\right)}{8b^4e\left(3a^2-b^2\right)}}{\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)^2\left(a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-b\tan\left(\frac{ex}{2}+\frac{d}{2}\right)+a+b\right)^2}$
parallelrisch	$\frac{-9\left(a^2+\frac{b^2}{3}\right)\left(\left(a^2-b^2\right)\cos(2ex+2d)-4a^2\sin(ex+d)-4ab\cos(ex+d)-2ab\sin(2ex+2d)-3a^2-b^2\right)\left(a^2-\frac{b^2}{3}\right)\ln\left(a+b+(a-\right.}{}$

input
```
int(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^3,x,method=_RETURNVERBOSE)
```

output
```
1/8/e*(-1/2/b^3/(1+tan(1/2*e*x+1/2*d))^2-1/2*(-3*a-b)/b^4/(1+tan(1/2*e*x+1
/2*d))+1/2*(3*a^2+b^2)/b^5*ln(1+tan(1/2*e*x+1/2*d))+1/2*(-3*a^3+3*a^2*b-a*
b^2+b^3)/b^5/(a-b)*ln(a*tan(1/2*e*x+1/2*d)-b*tan(1/2*e*x+1/2*d)+a+b)-1/2*(
-a^4-2*a^2*b^2-b^4)/b^3/(a-b)^2/(a*tan(1/2*e*x+1/2*d)-b*tan(1/2*e*x+1/2*d)
+a+b)^2-1/2*(-3*a^4+4*a^3*b-2*a^2*b^2+4*a*b^3+b^4)/b^4/(a-b)^2/(a*tan(1/2*
e*x+1/2*d)-b*tan(1/2*e*x+1/2*d)+a+b))
```

3.386.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 420 vs. $2(130) = 260$.

Time = 0.26 (sec) , antiderivative size = 420, normalized size of antiderivative = 2.96

$$\int \frac{1}{(2a+2b\cos(d+ex)+2a\sin(d+ex))^3}\,dx$$
$$= \frac{12\,a^2b^2\cos(ex+d)^2-6\,a^2b^2+2\left(3\,a^3b-ab^3\right)\cos(ex+d)-\left(6\,a^4+2\,a^2b^2-\left(3\,a^4-2\,a^2b^2-b^4\right)\cos(ex\right.}{}$$

input
```
integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^3,x, algorithm="fricas")
```

output
```
1/32*(12*a^2*b^2*cos(e*x + d)^2 - 6*a^2*b^2 + 2*(3*a^3*b - a*b^3)*cos(e*x
+ d) - (6*a^4 + 2*a^2*b^2 - (3*a^4 - 2*a^2*b^2 - b^4)*cos(e*x + d)^2 + 2*(
3*a^3*b + a*b^3)*cos(e*x + d) + 2*(3*a^4 + a^2*b^2 + (3*a^3*b + a*b^3)*cos
(e*x + d))*sin(e*x + d))*log(2*a*b*cos(e*x + d) + a^2 + b^2 + (a^2 - b^2)*
sin(e*x + d)) + (6*a^4 + 2*a^2*b^2 - (3*a^4 - 2*a^2*b^2 - b^4)*cos(e*x + d
)^2 + 2*(3*a^3*b + a*b^3)*cos(e*x + d) + 2*(3*a^4 + a^2*b^2 + (3*a^3*b + a
*b^3)*cos(e*x + d))*sin(e*x + d))*log(sin(e*x + d) + 1) - 2*(3*a^2*b^2 - b
^4 - 3*(a^3*b - a*b^3)*cos(e*x + d))*sin(e*x + d))/(2*a*b^6*e*cos(e*x + d)
 + 2*a^2*b^5*e - (a^2*b^5 - b^7)*e*cos(e*x + d)^2 + 2*(a*b^6*e*cos(e*x + d
) + a^2*b^5*e)*sin(e*x + d))
```

3.386.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(2a+2b\cos(d+ex)+2a\sin(d+ex))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))**3,x)
```

output
```
Timed out
```

3.386.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 493 vs. $2(130) = 260$.

Time = 0.23 (sec) , antiderivative size = 493, normalized size of antiderivative = 3.47

$$\int \frac{1}{(2a+2b\cos(d+ex)+2a\sin(d+ex))^3}\,dx$$
$$= \frac{\frac{2\left(3a^5-4a^3b^2-ab^4+\frac{\left(9a^5-9a^4b-2a^3b^2+2a^2b^3-5ab^4+b^5\right)\sin(ex+d)}{\cos(ex+d)+1}+\frac{\left(9a^5-18a^4b+12a^3b^2-6a^2b^3+ab^4\right)\sin(ex+d)^2}{(\cos(ex+d)+1)^2}+\frac{(3a^5-9a^4b+10a^3b}{(co}}{a^4b^4-2a^2b^6+b^8+\frac{4\left(a^4b^4-a^3b^5-a^2b^6+ab^7\right)\sin(ex+d)}{\cos(ex+d)+1}+\frac{2\left(3a^4b^4-6a^3b^5+2a^2b^6+2ab^7-b^8\right)\sin(ex+d)^2}{(\cos(ex+d)+1)^2}+\frac{4\left(a^4b^4-3a^3b^5+3a^2b^6-ab^7\right)\sin(ex+d)^3}{(\cos(ex+d)+1)^3}+\frac{(a}{}}}{16e}$$

input
```
integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^3,x, algorithm="maxima")
```

output
```
1/16*(2*(3*a^5 - 4*a^3*b^2 - a*b^4 + (9*a^5 - 9*a^4*b - 2*a^3*b^2 + 2*a^2*
b^3 - 5*a*b^4 + b^5)*sin(e*x + d)/(cos(e*x + d) + 1) + (9*a^5 - 18*a^4*b +
 12*a^3*b^2 - 6*a^2*b^3 + a*b^4)*sin(e*x + d)^2/(cos(e*x + d) + 1)^2 + (3*
a^5 - 9*a^4*b + 10*a^3*b^2 - 6*a^2*b^3 + a*b^4 + b^5)*sin(e*x + d)^3/(cos(
e*x + d) + 1)^3)/(a^4*b^4 - 2*a^2*b^6 + b^8 + 4*(a^4*b^4 - a^3*b^5 - a^2*b
^6 + a*b^7)*sin(e*x + d)/(cos(e*x + d) + 1) + 2*(3*a^4*b^4 - 6*a^3*b^5 + 2
*a^2*b^6 + 2*a*b^7 - b^8)*sin(e*x + d)^2/(cos(e*x + d) + 1)^2 + 4*(a^4*b^4
 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*sin(e*x + d)^3/(cos(e*x + d) + 1)^3 + (a
^4*b^4 - 4*a^3*b^5 + 6*a^2*b^6 - 4*a*b^7 + b^8)*sin(e*x + d)^4/(cos(e*x +
d) + 1)^4) - (3*a^2 + b^2)*log(-a - b - (a - b)*sin(e*x + d)/(cos(e*x + d)
 + 1))/b^5 + (3*a^2 + b^2)*log(sin(e*x + d)/(cos(e*x + d) + 1) + 1)/b^5)/e
```

3.386.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(130) = 260$.

Time = 0.30 (sec) , antiderivative size = 458, normalized size of antiderivative = 3.23

$$\int \frac{1}{(2a + 2b\cos(d + ex) + 2a\sin(d + ex))^3}\,dx$$

$$= \frac{2\left(3a^5\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^3-9a^4b\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^3+10a^3b^2\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^3-6a^2b^3\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^3+ab^4\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^3+b^5\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^3+9a\right.}{}$$

input
```
integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^3,x, algorithm="giac")
```

output
```
1/16*(2*(3*a^5*tan(1/2*e*x + 1/2*d)^3 - 9*a^4*b*tan(1/2*e*x + 1/2*d)^3 + 1
0*a^3*b^2*tan(1/2*e*x + 1/2*d)^3 - 6*a^2*b^3*tan(1/2*e*x + 1/2*d)^3 + a*b^
4*tan(1/2*e*x + 1/2*d)^3 + b^5*tan(1/2*e*x + 1/2*d)^3 + 9*a^5*tan(1/2*e*x
+ 1/2*d)^2 - 18*a^4*b*tan(1/2*e*x + 1/2*d)^2 + 12*a^3*b^2*tan(1/2*e*x + 1/
2*d)^2 - 6*a^2*b^3*tan(1/2*e*x + 1/2*d)^2 + a*b^4*tan(1/2*e*x + 1/2*d)^2 +
 9*a^5*tan(1/2*e*x + 1/2*d) - 9*a^4*b*tan(1/2*e*x + 1/2*d) - 2*a^3*b^2*tan
(1/2*e*x + 1/2*d) + 2*a^2*b^3*tan(1/2*e*x + 1/2*d) - 5*a*b^4*tan(1/2*e*x +
 1/2*d) + b^5*tan(1/2*e*x + 1/2*d) + 3*a^5 - 4*a^3*b^2 - a*b^4)/((a^2*b^4
- 2*a*b^5 + b^6)*(a*tan(1/2*e*x + 1/2*d)^2 - b*tan(1/2*e*x + 1/2*d)^2 + 2*
a*tan(1/2*e*x + 1/2*d) + a + b)^2) + (3*a^2 + b^2)*log(abs(2*a*tan(1/2*e*x
 + 1/2*d) - 2*b*tan(1/2*e*x + 1/2*d) + 2*a - 2*abs(b))/abs(2*a*tan(1/2*e*x
 + 1/2*d) - 2*b*tan(1/2*e*x + 1/2*d) + 2*a + 2*abs(b)))/(b^4*abs(b)))/e
```

3.386.9 Mupad [B] (verification not implemented)

Time = 30.88 (sec) , antiderivative size = 360, normalized size of antiderivative = 2.54

$$\int \frac{1}{(2a+2b\cos(d+ex)+2a\sin(d+ex))^3}\,dx =$$

$$-\frac{\frac{-3a^5+4a^3b^2+ab^4}{2b^4(a-b)^2}+\frac{\tan\left(\frac{d}{2}+\frac{ex}{2}\right)(-9a^5+9a^4b+2a^3b^2-2a^2b^3+5ab^4-b^5)}{2b^4(a-b)^2}+\frac{\tan\left(\frac{d}{2}+\frac{ex}{2}\right)^3(-3a^4+6a^3b-4a^2b^2+2ab^3+}{2b^4(a-b)}}{e\left(8ab+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)^2(24a^2-8b^2)-\tan\left(\frac{d}{2}+\frac{ex}{2}\right)^3(16ab-16a^2)+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)^4(4a^2-8ab+\right.}$$

$$-\frac{\operatorname{atanh}\left(\frac{2a+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)(2a-2b)}{2b}\right)(3a^2+b^2)}{8b^5e}$$

input
```
int(1/(2*a + 2*b*cos(d + e*x) + 2*a*sin(d + e*x))^3,x)
```

output
```
- ((a*b^4 - 3*a^5 + 4*a^3*b^2)/(2*b^4*(a - b)^2) + (tan(d/2 + (e*x)/2)*(5*
a*b^4 + 9*a^4*b - 9*a^5 - b^5 - 2*a^2*b^3 + 2*a^3*b^2))/(2*b^4*(a - b)^2)
+ (tan(d/2 + (e*x)/2)^3*(2*a*b^3 + 6*a^3*b - 3*a^4 + b^4 - 4*a^2*b^2))/(2*
b^4*(a - b)) - (tan(d/2 + (e*x)/2)^2*(a*b^4 - 18*a^4*b + 9*a^5 - 6*a^2*b^3
 + 12*a^3*b^2))/(2*b^4*(a - b)^2))/(e*(8*a*b + tan(d/2 + (e*x)/2)^2*(24*a^
2 - 8*b^2) - tan(d/2 + (e*x)/2)^3*(16*a*b - 16*a^2) + tan(d/2 + (e*x)/2)^4
*(4*a^2 - 8*a*b + 4*b^2) + 4*a^2 + 4*b^2 + tan(d/2 + (e*x)/2)*(16*a*b + 16
*a^2))) - (atanh((2*a + tan(d/2 + (e*x)/2)*(2*a - 2*b))/(2*b))*(3*a^2 + b^
2))/(8*b^5*e)
```

3.387 $\int \frac{1}{(2a+2b\cos(d+ex)+2a\sin(d+ex))^4}\,dx$

3.387.1 Optimal result

Integrand size = 24, antiderivative size = 215

$$\int \frac{1}{(2a+2b\cos(d+ex)+2a\sin(d+ex))^4}\,dx = \frac{a(5a^2+3b^2)\log\left(a+b\cot\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)\right)}{32b^7e} - \frac{a\cos(d+ex)-b\sin(d+ex)}{48b^2e(a+b\cos(d+ex)+a\sin(d+ex))^3} + \frac{5(a^2\cos(d+ex)-ab\sin(d+ex))}{96b^4e(a+b\cos(d+ex)+a\sin(d+ex))^2} - \frac{a(15a^2+4b^2)\cos(d+ex)-b(15a^2+4b^2)\sin(d+ex)}{96b^6e(a+b\cos(d+ex)+a\sin(d+ex))}$$

output
```
1/32*a*(5*a^2+3*b^2)*ln(a+b*cot(1/2*d+1/4*Pi+1/2*e*x))/b^7/e+1/48*(-a*cos(
e*x+d)+b*sin(e*x+d))/b^2/e/(a+b*cos(e*x+d)+a*sin(e*x+d))^3+5/96*(a^2*cos(e
*x+d)-a*b*sin(e*x+d))/b^4/e/(a+b*cos(e*x+d)+a*sin(e*x+d))^2+1/96*(-a*(15*a
^2+4*b^2)*cos(e*x+d)+b*(15*a^2+4*b^2)*sin(e*x+d))/b^6/e/(a+b*cos(e*x+d)+a*
sin(e*x+d))
```

3.387.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 632 vs. $2(215) = 430$.

Time = 1.75 (sec) , antiderivative size = 632, normalized size of antiderivative = 2.94

$$\int \frac{1}{(2a+2b\cos(d+ex)+2a\sin(d+ex))^4}\,dx$$
$$= \frac{-12a(5a^2+3b^2)\log\left(\cos\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{1}{2}(d+ex)\right)\right)+12a(5a^2+3b^2)\log\left((a+b)\cos\left(\frac{1}{2}(d+ex)\right)+\right.}{}$$

input
```
Integrate[(2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-4),x]
```

output
```
(-12*a*(5*a^2 + 3*b^2)*Log[Cos[(d + e*x)/2] + Sin[(d + e*x)/2]] + 12*a*(5*
a^2 + 3*b^2)*Log[(a + b)*Cos[(d + e*x)/2] + (a - b)*Sin[(d + e*x)/2]] + (b
*(150*a^6 + 130*a^4*b^2 + 24*a^2*b^4 - 3*a^2*(25*a^4 - 50*a^3*b + 5*a^2*b^
2 - 30*a*b^3 + 4*b^4)*Cos[d + e*x] - 6*a^2*(15*a^4 + 20*a^3*b + 9*a^2*b^2
+ 2*a*b^3 - 2*b^4)*Cos[2*(d + e*x)] + 15*a^6*Cos[3*(d + e*x)] - 30*a^5*b*C
os[3*(d + e*x)] - 41*a^4*b^2*Cos[3*(d + e*x)] - 38*a^3*b^3*Cos[3*(d + e*x)
] - 12*a^2*b^4*Cos[3*(d + e*x)] - 8*a*b^5*Cos[3*(d + e*x)] + 225*a^6*Sin[d
 + e*x] + 75*a^5*b*Sin[d + e*x] + 180*a^4*b^2*Sin[d + e*x] + 15*a^3*b^3*Si
n[d + e*x] + 27*a^2*b^4*Sin[d + e*x] + 12*a*b^5*Sin[d + e*x] + 12*b^6*Sin[
d + e*x] - 60*a^6*Sin[2*(d + e*x)] + 120*a^5*b*Sin[2*(d + e*x)] + 54*a^4*b
^2*Sin[2*(d + e*x)] + 102*a^3*b^3*Sin[2*(d + e*x)] + 6*a^2*b^4*Sin[2*(d +
e*x)] + 6*a*b^5*Sin[2*(d + e*x)] - 15*a^6*Sin[3*(d + e*x)] - 45*a^5*b*Sin[
3*(d + e*x)] - 4*a^4*b^2*Sin[3*(d + e*x)] + 3*a^3*b^3*Sin[3*(d + e*x)] + 1
5*a^2*b^4*Sin[3*(d + e*x)] + 4*a*b^5*Sin[3*(d + e*x)] + 4*b^6*Sin[3*(d + e
*x)]))/((a + b)*(Cos[(d + e*x)/2] + Sin[(d + e*x)/2])^3*((a + b)*Cos[(d +
e*x)/2] + (a - b)*Sin[(d + e*x)/2])^3))/(384*b^7*e)
```

3.387.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {3042, 3608, 27, 3042, 3635, 25, 3042, 3632, 3042, 3602, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2a\sin(d+ex)+2a+2b\cos(d+ex))^4}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{(2a\sin(d+ex)+2a+2b\cos(d+ex))^4}dx$$

$$\Big\downarrow \ 3608$$

$$\frac{\int -\frac{-2\sin(d+ex)a+3a-2b\cos(d+ex)}{4(\sin(d+ex)a+a+b\cos(d+ex))^3}dx}{12b^2}-\frac{a\cos(d+ex)-b\sin(d+ex)}{48b^2e(a\sin(d+ex)+a+b\cos(d+ex))^3}$$

$$\Big\downarrow \ 27$$

$$-\frac{\int \frac{-2\sin(d+ex)a+3a-2b\cos(d+ex)}{(\sin(d+ex)a+a+b\cos(d+ex))^3}dx}{48b^2}-\frac{a\cos(d+ex)-b\sin(d+ex)}{48b^2e(a\sin(d+ex)+a+b\cos(d+ex))^3}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\int \frac{-2\sin(d+ex)a+3a-2b\cos(d+ex)}{(\sin(d+ex)a+a+b\cos(d+ex))^3}dx}{48b^2}-\frac{a\cos(d+ex)-b\sin(d+ex)}{48b^2e(a\sin(d+ex)+a+b\cos(d+ex))^3}$$

$$\Big\downarrow \ 3635$$

$$-\frac{\frac{\int -\frac{-5\sin(d+ex)a^2-5b\cos(d+ex)a+2\left(5a^2+2b^2\right)}{(\sin(d+ex)a+a+b\cos(d+ex))^2}dx}{2b^2}-\frac{5\left(a^2\cos(d+ex)-ab\sin(d+ex)\right)}{2b^2e(a\sin(d+ex)+a+b\cos(d+ex))^2}}{48b^2}-\frac{a\cos(d+ex)-b\sin(d+ex)}{48b^2e(a\sin(d+ex)+a+b\cos(d+ex))^3}$$

$$\Big\downarrow \ 25$$

$$-\frac{-\frac{\int \frac{-5\sin(d+ex)a^2-5b\cos(d+ex)a+2\left(5a^2+2b^2\right)}{(\sin(d+ex)a+a+b\cos(d+ex))^2}dx}{2b^2}-\frac{5\left(a^2\cos(d+ex)-ab\sin(d+ex)\right)}{2b^2e(a\sin(d+ex)+a+b\cos(d+ex))^2}}{48b^2}-\frac{a\cos(d+ex)-b\sin(d+ex)}{48b^2e(a\sin(d+ex)+a+b\cos(d+ex))^3}$$

$$\Big\downarrow \ 3042$$

$$-\frac{-\frac{\int \frac{-5\sin(d+ex)a^2-5b\cos(d+ex)a+2\left(5a^2+2b^2\right)}{(\sin(d+ex)a+a+b\cos(d+ex))^2}dx}{2b^2}-\frac{5\left(a^2\cos(d+ex)-ab\sin(d+ex)\right)}{2b^2e(a\sin(d+ex)+a+b\cos(d+ex))^2}}{48b^2}-\frac{a\cos(d+ex)-b\sin(d+ex)}{48b^2e(a\sin(d+ex)+a+b\cos(d+ex))^3}$$

$$\Big\downarrow \ 3632$$

3.387. $\int \frac{1}{(2a+2b\cos(d+ex)+2a\sin(d+ex))^4}\,dx$

$$-\frac{-\frac{-3a\left(\frac{5a^2}{b^2}+3\right)\int\frac{1}{\sin(d+ex)a+a+b\cos(d+ex)}dx-\frac{a\left(15a^2+4b^2\right)\cos(d+ex)-b\left(15a^2+4b^2\right)\sin(d+ex)}{b^2e(a\sin(d+ex)+a+b\cos(d+ex))}}{2b^2}-\frac{5\left(a^2\cos(d+ex)-ab\sin(d+ex)\right)}{2b^2e(a\sin(d+ex)+a+b\cos(d+ex))^2}}{48b^2}-\frac{a\cos(d+ex)-b\sin(d+ex)}{48b^2e(a\sin(d+ex)+a+b\cos(d+ex))^3}$$

$$\Big\downarrow \; 3042$$

$$-\frac{-\frac{-3a\left(\frac{5a^2}{b^2}+3\right)\int\frac{1}{\sin(d+ex)a+a+b\cos(d+ex)}dx-\frac{a\left(15a^2+4b^2\right)\cos(d+ex)-b\left(15a^2+4b^2\right)\sin(d+ex)}{b^2e(a\sin(d+ex)+a+b\cos(d+ex))}}{2b^2}-\frac{5\left(a^2\cos(d+ex)-ab\sin(d+ex)\right)}{2b^2e(a\sin(d+ex)+a+b\cos(d+ex))^2}}{48b^2}-\frac{a\cos(d+ex)-b\sin(d+ex)}{48b^2e(a\sin(d+ex)+a+b\cos(d+ex))^3}$$

$$\Big\downarrow \; 3602$$

$$-\frac{-\frac{\frac{3a\left(\frac{5a^2}{b^2}+3\right)\int\frac{1}{a+b\cot\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)}d\cot\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)}{e}-\frac{a\left(15a^2+4b^2\right)\cos(d+ex)-b\left(15a^2+4b^2\right)\sin(d+ex)}{b^2e(a\sin(d+ex)+a+b\cos(d+ex))}}{2b^2}-\frac{5\left(a^2\cos(d+ex)-ab\sin(d+ex)\right)}{2b^2e(a\sin(d+ex)+a+b\cos(d+ex))^2}}{48b^2}-\frac{a\cos(d+ex)-b\sin(d+ex)}{48b^2e(a\sin(d+ex)+a+b\cos(d+ex))^3}$$

$$\Big\downarrow \; 16$$

$$-\frac{-\frac{5\left(a^2\cos(d+ex)-ab\sin(d+ex)\right)}{2b^2e(a\sin(d+ex)+a+b\cos(d+ex))^2}-\frac{\frac{3a\left(\frac{5a^2}{b^2}+3\right)\log\left(a+b\cot\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)\right)}{be}-\frac{a\left(15a^2+4b^2\right)\cos(d+ex)-b\left(15a^2+4b^2\right)\sin(d+ex)}{b^2e(a\sin(d+ex)+a+b\cos(d+ex))}}{2b^2}}{48b^2}-\frac{a\cos(d+ex)-b\sin(d+ex)}{48b^2e(a\sin(d+ex)+a+b\cos(d+ex))^3}$$

input
```
Int[(2*a + 2*b*Cos[d + e*x] + 2*a*Sin[d + e*x])^(-4),x]
```

output
```
-1/48*(a*Cos[d + e*x] - b*Sin[d + e*x])/(b^2*e*(a + b*Cos[d + e*x] + a*Sin
[d + e*x])^3) - ((-5*(a^2*Cos[d + e*x] - a*b*Sin[d + e*x]))/(2*b^2*e*(a +
b*Cos[d + e*x] + a*Sin[d + e*x])^2) - ((3*a*(3 + (5*a^2)/b^2)*Log[a + b*Co
t[d/2 + Pi/4 + (e*x)/2]])/(b*e) - (a*(15*a^2 + 4*b^2)*Cos[d + e*x] - b*(15
*a^2 + 4*b^2)*Sin[d + e*x])/(b^2*e*(a + b*Cos[d + e*x] + a*Sin[d + e*x])))
/(2*b^2))/(48*b^2)
```

3.387. $\int\frac{1}{(2a+2b\cos(d+ex)+2a\sin(d+ex))^4}\,dx$

3.387.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3602
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Cot[(d + e*x)/2 + Pi/4], x]}, Si
mp[-f/e   Subst[Int[1/(a + b*f*x), x], x, Cot[(d + e*x)/2 + Pi/4]/f], x]] /
; FreeQ[{a, b, c, d, e}, x] && EqQ[a - c, 0] && NeQ[a - b, 0]
```

rule 3608
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Simp[
1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c
*(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x
] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] &&
NeQ[n, -3/2]
```

rule 3632
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Simp[(a*A - b*B - c*C)/(a^2 - b^2 - c^2)   Int[1/(a + b*Cos[d + e*x] + c*S
in[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
 - c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

3.387. $\int \frac{1}{(2a+2b\cos(d+ex)+2a\sin(d+ex))^4}\,dx$

rule 3635
```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(-(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(
a^2 - b^2 - c^2))), x] + Simp[1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a + b*Co
s[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n +
2)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x]
/; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[n, -2]
```

3.387.4 Maple [A] (verified)

Time = 3.18 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.85

method	result
derivativedivides	$\frac{-\frac{1}{3b^4\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)^3}-\frac{-2a-b}{2b^5\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)^2}-\frac{5a^2+2ab+2b^2}{2b^6\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}-\frac{a\left(5a^2+3b^2\right)\ln\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}{2b^7}+\frac{\left(5a^3-5a^2b+3a\,b^2\right.}{}}{}$
default	$\frac{-\frac{1}{3b^4\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)^3}-\frac{-2a-b}{2b^5\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)^2}-\frac{5a^2+2ab+2b^2}{2b^6\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}-\frac{a\left(5a^2+3b^2\right)\ln\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}{2b^7}+\frac{\left(5a^3-5a^2b+3a\,b^2\right.}{}}{}$
risch	$\frac{i\left(-15a^5e^{5i(ex+d)}-75a^5e^{i(ex+d)}+150a^5e^{3i(ex+d)}+12b^5e^{2i(ex+d)}+4b^5-45a^4b+3a^2b^3+6a^3b^2e^{5i(ex+d)}+24a\,b^4e^{3i(ex+d}\right.}{}$
norman	$\frac{-\frac{\left(50a^7+100a^6b+75a^5b^2+50a^4b^3+22a^3b^4-6a^2b^5-a\,b^6+4b^7\right)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{16e\,b^6\left(5a^2-3b^2\right)}+\frac{\left(50a^7-100a^6b+75a^5b^2-50a^4b^3+22a^3b^4+6a^2b^5-a\,b\right.}{16e\,b^6\left(5a^2-3b^2\right)}}{}$
parallelrisch	$\frac{15\left(a^2+\frac{3b^2}{5}\right)\left(\left(6a^3-6a\,b^2\right)\cos(2ex+2d)+\left(3a^2b-b^3\right)\cos(3ex+3d)+\left(a^3-3a\,b^2\right)\sin(3ex+3d)-12a^2b\sin(2ex+2d)+\left(-15\right.\right.}{}$

input
```
int(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^4,x,method=_RETURNVERBOSE)
```

output
```
1/16/e*(-1/3/b^4/(1+tan(1/2*e*x+1/2*d))^3-1/2*(-2*a-b)/b^5/(1+tan(1/2*e*x+
1/2*d))^2-1/2*(5*a^2+2*a*b+2*b^2)/b^6/(1+tan(1/2*e*x+1/2*d))-1/2*a*(5*a^2+
3*b^2)/b^7*ln(1+tan(1/2*e*x+1/2*d))+1/2*(5*a^3-5*a^2*b+3*a*b^2-3*b^3)*a/b^
7/(a-b)*ln(a*tan(1/2*e*x+1/2*d)-b*tan(1/2*e*x+1/2*d)+a+b)-1/3*(a^6+3*a^4*b
^2+3*a^2*b^4+b^6)/b^4/(a-b)^3/(a*tan(1/2*e*x+1/2*d)-b*tan(1/2*e*x+1/2*d)+a
+b)^3-1/2*(2*a^6-3*a^5*b+3*a^4*b^2-6*a^3*b^3-3*a*b^5-b^6)/b^5/(a-b)^3/(a*t
an(1/2*e*x+1/2*d)-b*tan(1/2*e*x+1/2*d)+a+b)^2-1/2*(5*a^6-12*a^5*b+12*a^4*b
^2-12*a^3*b^3+9*a^2*b^4+2*b^6)/b^6/(a-b)^3/(a*tan(1/2*e*x+1/2*d)-b*tan(1/2
*e*x+1/2*d)+a+b))
```

3.387.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 729 vs. $2(201) = 402$.

Time = 0.30 (sec) , antiderivative size = 729, normalized size of antiderivative = 3.39

$$\int \frac{1}{(2a+2b\cos(d+ex)+2a\sin(d+ex))^4}\,dx = \frac{60\,a^4b^2+6\,a^2b^4+2\,(15\,a^5b-41\,a^3b^3-12\,ab^5)\cos{(ex+d)}^3-12\,(10\,a^4b^2+a^2b^4)\cos{(ex+d)}^2-6\,(10\,a^5}{}$$

input
```
integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^4,x, algorithm="fricas")
```

output
```
1/192*(60*a^4*b^2 + 6*a^2*b^4 + 2*(15*a^5*b - 41*a^3*b^3 - 12*a*b^5)*cos(e
*x + d)^3 - 12*(10*a^4*b^2 + a^2*b^4)*cos(e*x + d)^2 - 6*(10*a^5*b - 9*a^3
*b^3 - 2*a*b^5)*cos(e*x + d) + 3*(20*a^6 + 12*a^4*b^2 - (15*a^5*b + 4*a^3*
b^3 - 3*a*b^5)*cos(e*x + d)^3 - 3*(5*a^6 - 2*a^4*b^2 - 3*a^2*b^4)*cos(e*x
+ d)^2 + 6*(5*a^5*b + 3*a^3*b^3)*cos(e*x + d) + (20*a^6 + 12*a^4*b^2 - (5*
a^6 - 12*a^4*b^2 - 9*a^2*b^4)*cos(e*x + d)^2 + 6*(5*a^5*b + 3*a^3*b^3)*cos
(e*x + d))*sin(e*x + d))*log(2*a*b*cos(e*x + d) + a^2 + b^2 + (a^2 - b^2)*
sin(e*x + d)) - 3*(20*a^6 + 12*a^4*b^2 - (15*a^5*b + 4*a^3*b^3 - 3*a*b^5)*
cos(e*x + d)^3 - 3*(5*a^6 - 2*a^4*b^2 - 3*a^2*b^4)*cos(e*x + d)^2 + 6*(5*a
^5*b + 3*a^3*b^3)*cos(e*x + d) + (20*a^6 + 12*a^4*b^2 - (5*a^6 - 12*a^4*b^
2 - 9*a^2*b^4)*cos(e*x + d)^2 + 6*(5*a^5*b + 3*a^3*b^3)*cos(e*x + d))*sin(
e*x + d))*log(sin(e*x + d) + 1) + 2*(30*a^4*b^2 + 3*a^2*b^4 + 2*b^6 - (45*
a^4*b^2 - 3*a^2*b^4 - 4*b^6)*cos(e*x + d)^2 - 3*(10*a^5*b - 9*a^3*b^3 - a*
b^5)*cos(e*x + d))*sin(e*x + d))/(6*a^2*b^8*e*cos(e*x + d) + 4*a^3*b^7*e -
 (3*a^2*b^8 - b^10)*e*cos(e*x + d)^3 - 3*(a^3*b^7 - a*b^9)*e*cos(e*x + d)^
2 + (6*a^2*b^8*e*cos(e*x + d) + 4*a^3*b^7*e - (a^3*b^7 - 3*a*b^9)*e*cos(e*
x + d)^2)*sin(e*x + d))
```

3.387.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(2a + 2b\cos(d + ex) + 2a\sin(d + ex))^4}\,dx = \text{Timed out}$$

input
```
integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))**4,x)
```

output
```
Timed out
```

3.387.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 963 vs. $2(201) = 402$.

Time = 0.27 (sec) , antiderivative size = 963, normalized size of antiderivative = 4.48

$$\int \frac{1}{(2a + 2b\cos(d + ex) + 2a\sin(d + ex))^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^4,x, algorithm="maxima")
```

output
```
-1/96*(2*(15*a^8 - 31*a^6*b^2 + 9*a^4*b^4 + 15*a^2*b^6 + 3*(25*a^8 - 25*a^
7*b - 25*a^6*b^2 + 25*a^5*b^3 - 13*a^4*b^4 + 13*a^3*b^5 + 11*a^2*b^6 - 5*a
*b^7 + 2*b^8)*sin(e*x + d)/(cos(e*x + d) + 1) + 6*(25*a^8 - 50*a^7*b + 20*
a^6*b^2 + 10*a^5*b^3 - 17*a^4*b^4 + 24*a^3*b^5 - 10*a^2*b^6 + 2*a*b^7)*sin
(e*x + d)^2/(cos(e*x + d) + 1)^2 + 2*(75*a^8 - 225*a^7*b + 250*a^6*b^2 - 1
50*a^5*b^3 + 63*a^4*b^4 + 11*a^3*b^5 - 24*a^2*b^6 + 6*a*b^7 - 2*b^8)*sin(e
*x + d)^3/(cos(e*x + d) + 1)^3 + 3*(25*a^8 - 100*a^7*b + 165*a^6*b^2 - 160
*a^5*b^3 + 115*a^4*b^4 - 60*a^3*b^5 + 19*a^2*b^6 - 4*a*b^7)*sin(e*x + d)^4
/(cos(e*x + d) + 1)^4 + 3*(5*a^8 - 25*a^7*b + 53*a^6*b^2 - 65*a^5*b^3 + 55
*a^4*b^4 - 35*a^3*b^5 + 17*a^2*b^6 - 7*a*b^7 + 2*b^8)*sin(e*x + d)^5/(cos(
e*x + d) + 1)^5)/(a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10 - b^12 + 6*(a^6*b^6 - a
^5*b^7 - 2*a^4*b^8 + 2*a^3*b^9 + a^2*b^10 - a*b^11)*sin(e*x + d)/(cos(e*x
+ d) + 1) + 3*(5*a^6*b^6 - 10*a^5*b^7 - a^4*b^8 + 12*a^3*b^9 - 5*a^2*b^10
- 2*a*b^11 + b^12)*sin(e*x + d)^2/(cos(e*x + d) + 1)^2 + 4*(5*a^6*b^6 - 15
*a^5*b^7 + 12*a^4*b^8 + 4*a^3*b^9 - 9*a^2*b^10 + 3*a*b^11)*sin(e*x + d)^3/
(cos(e*x + d) + 1)^3 + 3*(5*a^6*b^6 - 20*a^5*b^7 + 29*a^4*b^8 - 16*a^3*b^9
 - a^2*b^10 + 4*a*b^11 - b^12)*sin(e*x + d)^4/(cos(e*x + d) + 1)^4 + 6*(a^
6*b^6 - 5*a^5*b^7 + 10*a^4*b^8 - 10*a^3*b^9 + 5*a^2*b^10 - a*b^11)*sin(e*x
 + d)^5/(cos(e*x + d) + 1)^5 + (a^6*b^6 - 6*a^5*b^7 + 15*a^4*b^8 - 20*a^3*
b^9 + 15*a^2*b^10 - 6*a*b^11 + b^12)*sin(e*x + d)^6/(cos(e*x + d) + 1)^...
```

3.387.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 957 vs. $2(201) = 402$.

Time = 0.33 (sec) , antiderivative size = 957, normalized size of antiderivative = 4.45

$$\int \frac{1}{(2a + 2b\cos(d + ex) + 2a\sin(d + ex))^4} \, dx = \text{Too large to display}$$

input
```
integrate(1/(2*a+2*b*cos(e*x+d)+2*a*sin(e*x+d))^4,x, algorithm="giac")
```

output
```
-1/96*(2*(15*a^8*tan(1/2*e*x + 1/2*d)^5 - 75*a^7*b*tan(1/2*e*x + 1/2*d)^5
+ 159*a^6*b^2*tan(1/2*e*x + 1/2*d)^5 - 195*a^5*b^3*tan(1/2*e*x + 1/2*d)^5
+ 165*a^4*b^4*tan(1/2*e*x + 1/2*d)^5 - 105*a^3*b^5*tan(1/2*e*x + 1/2*d)^5
+ 51*a^2*b^6*tan(1/2*e*x + 1/2*d)^5 - 21*a*b^7*tan(1/2*e*x + 1/2*d)^5 + 6*
b^8*tan(1/2*e*x + 1/2*d)^5 + 75*a^8*tan(1/2*e*x + 1/2*d)^4 - 300*a^7*b*tan
(1/2*e*x + 1/2*d)^4 + 495*a^6*b^2*tan(1/2*e*x + 1/2*d)^4 - 480*a^5*b^3*tan
(1/2*e*x + 1/2*d)^4 + 345*a^4*b^4*tan(1/2*e*x + 1/2*d)^4 - 180*a^3*b^5*tan
(1/2*e*x + 1/2*d)^4 + 57*a^2*b^6*tan(1/2*e*x + 1/2*d)^4 - 12*a*b^7*tan(1/2
*e*x + 1/2*d)^4 + 150*a^8*tan(1/2*e*x + 1/2*d)^3 - 450*a^7*b*tan(1/2*e*x +
 1/2*d)^3 + 500*a^6*b^2*tan(1/2*e*x + 1/2*d)^3 - 300*a^5*b^3*tan(1/2*e*x +
 1/2*d)^3 + 126*a^4*b^4*tan(1/2*e*x + 1/2*d)^3 + 22*a^3*b^5*tan(1/2*e*x +
1/2*d)^3 - 48*a^2*b^6*tan(1/2*e*x + 1/2*d)^3 + 12*a*b^7*tan(1/2*e*x + 1/2*
d)^3 - 4*b^8*tan(1/2*e*x + 1/2*d)^3 + 150*a^8*tan(1/2*e*x + 1/2*d)^2 - 300
*a^7*b*tan(1/2*e*x + 1/2*d)^2 + 120*a^6*b^2*tan(1/2*e*x + 1/2*d)^2 + 60*a^
5*b^3*tan(1/2*e*x + 1/2*d)^2 - 102*a^4*b^4*tan(1/2*e*x + 1/2*d)^2 + 144*a^
3*b^5*tan(1/2*e*x + 1/2*d)^2 - 60*a^2*b^6*tan(1/2*e*x + 1/2*d)^2 + 12*a*b^
7*tan(1/2*e*x + 1/2*d)^2 + 75*a^8*tan(1/2*e*x + 1/2*d) - 75*a^7*b*tan(1/2*
e*x + 1/2*d) - 75*a^6*b^2*tan(1/2*e*x + 1/2*d) + 75*a^5*b^3*tan(1/2*e*x +
1/2*d) - 39*a^4*b^4*tan(1/2*e*x + 1/2*d) + 39*a^3*b^5*tan(1/2*e*x + 1/2*d)
 + 33*a^2*b^6*tan(1/2*e*x + 1/2*d) - 15*a*b^7*tan(1/2*e*x + 1/2*d) + 6*...
```

3.387.9 Mupad [B] (verification not implemented)

Time = 32.74 (sec) , antiderivative size = 730, normalized size of antiderivative = 3.40

$$\int \frac{1}{(2a+2b\cos(d+ex)+2a\sin(d+ex))^4}\,dx$$
$$= \frac{a\,\mathrm{atanh}\left(\frac{a\left(2\,a+\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,(2\,a-2\,b)\right)\,(5\,a^2+3\,b^2)}{2\,b\,(5\,a^3+3\,a\,b^2)}\right)\,(5\,a^2+3\,b^2)}{16\,b^7\,e} - \frac{\frac{15\,a^8-31\,a^6\,b^2+9\,a^4\,b^4+15\,a^2\,b^6}{6\,b^6\,(a-b)^3}+\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2\,(25\,a^8-50\,a^7\,b+20\,a^6\,b^2+10\,a^5\,b^3-17\,a^4\,b^4+24\,a^3\,b^5-10\,a^2\,b^6+2\,a\,b^7)}{b^6\,(a-b)^3}+\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^4}{}}{e\,\left(\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^5\,(48\,a^3-96\,a^2\,b+48\,a\,b^2)+\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^6\,(8\,a^3-24\,a^2\,b+24\,a\,b^2-8\right.}$$

input
```
int(1/(2*a + 2*b*cos(d + e*x) + 2*a*sin(d + e*x))^4,x)
```

output
```
(a*atanh((a*(2*a + tan(d/2 + (e*x)/2)*(2*a - 2*b))*(5*a^2 + 3*b^2))/(2*b*(
3*a*b^2 + 5*a^3)))*(5*a^2 + 3*b^2))/(16*b^7*e) - ((15*a^8 + 15*a^2*b^6 + 9
*a^4*b^4 - 31*a^6*b^2)/(6*b^6*(a - b)^3) + (tan(d/2 + (e*x)/2)^2*(2*a*b^7
- 50*a^7*b + 25*a^8 - 10*a^2*b^6 + 24*a^3*b^5 - 17*a^4*b^4 + 10*a^5*b^3 +
20*a^6*b^2))/(b^6*(a - b)^3) + (tan(d/2 + (e*x)/2)^4*(4*a*b^6 - 75*a^6*b +
 25*a^7 - 15*a^2*b^5 + 45*a^3*b^4 - 70*a^4*b^3 + 90*a^5*b^2))/(2*b^6*(a -
b)^2) + (tan(d/2 + (e*x)/2)^3*(6*a*b^7 - 225*a^7*b + 75*a^8 - 2*b^8 - 24*a
^2*b^6 + 11*a^3*b^5 + 63*a^4*b^4 - 150*a^5*b^3 + 250*a^6*b^2))/(3*b^6*(a -
 b)^3) + (tan(d/2 + (e*x)/2)^5*(5*a^6 - 15*a^5*b - 3*a*b^5 + 2*b^6 + 9*a^2
*b^4 - 14*a^3*b^3 + 18*a^4*b^2))/(2*b^6*(a - b)) + (tan(d/2 + (e*x)/2)*(25
*a^8 - 25*a^7*b - 5*a*b^7 + 2*b^8 + 11*a^2*b^6 + 13*a^3*b^5 - 13*a^4*b^4 +
 25*a^5*b^3 - 25*a^6*b^2))/(2*b^6*(a - b)^3))/(e*(tan(d/2 + (e*x)/2)^5*(48
*a*b^2 - 96*a^2*b + 48*a^3) + tan(d/2 + (e*x)/2)^6*(24*a*b^2 - 24*a^2*b +
8*a^3 - 8*b^3) - tan(d/2 + (e*x)/2)^2*(24*a*b^2 - 120*a^2*b - 120*a^3 + 24
*b^3) - tan(d/2 + (e*x)/2)^4*(24*a*b^2 + 120*a^2*b - 120*a^3 - 24*b^3) + 2
4*a*b^2 + 24*a^2*b - tan(d/2 + (e*x)/2)^3*(96*a*b^2 - 160*a^3) + tan(d/2 +
 (e*x)/2)*(48*a*b^2 + 96*a^2*b + 48*a^3) + 8*a^3 + 8*b^3))
```

3.388 $\int (2a+2b\cos(d+ex)-2a\sin(d+ex))^3\,dx$

3.388.1 Optimal result

Integrand size = 24, antiderivative size = 157

$$\begin{aligned}\int (2a+2b\cos(d+ex)-2a\sin(d+ex))^3\,dx &= 4a(5a^2+3b^2)\,x+\frac{4a(15a^2+4b^2)\cos(d+ex)}{3e}+\frac{4b(15a^2+4b^2)\sin(d+ex)}{3e}\\&\quad+\frac{8(a+b\cos(d+ex)-a\sin(d+ex))^2(a\cos(d+ex)+b\sin(d+ex))}{3e}\\&\quad+\frac{20(a+b\cos(d+ex)-a\sin(d+ex))\,(a^2\cos(d+ex)+ab\sin(d+ex))}{3e}\end{aligned}$$

output
```
4*a*(5*a^2+3*b^2)*x+4/3*a*(15*a^2+4*b^2)*cos(e*x+d)/e+4/3*b*(15*a^2+4*b^2)
*sin(e*x+d)/e+8/3*(a+b*cos(e*x+d)-a*sin(e*x+d))^2*(a*cos(e*x+d)+b*sin(e*x+
d))/e+20/3*(a+b*cos(e*x+d)-a*sin(e*x+d))*(a^2*cos(e*x+d)+a*b*sin(e*x+d))/e
```

3.388.2 Mathematica [A] (verified)

Time = 1.67 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.87

$$\int (2a+2b\cos(d+ex)-2a\sin(d+ex))^3\,dx = \frac{2(6a(5a^2+3b^2)\,(d+ex)+9a(5a^2+b^2)\cos(d+ex)+18a^2b\cos(2(d+ex))-a(a^2-3b^2)\cos(3(d+ex))}{3e}$$

input
```
Integrate[(2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x])^3,x]
```

output
```
(2*(6*a*(5*a^2 + 3*b^2)*(d + e*x) + 9*a*(5*a^2 + b^2)*Cos[d + e*x] + 18*a^
2*b*Cos[2*(d + e*x)] - a*(a^2 - 3*b^2)*Cos[3*(d + e*x)] + 9*b*(5*a^2 + b^2
)*Sin[d + e*x] - 9*a*(a^2 - b^2)*Sin[2*(d + e*x)] + b*(-3*a^2 + b^2)*Sin[3
*(d + e*x)]))/(3*e)
```

3.388.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3599, 27, 3042, 3625, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (-2a\sin(d+ex)+2a+2b\cos(d+ex))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (-2a\sin(d+ex)+2a+2b\cos(d+ex))^3dx$$

$$\Big\downarrow \text{3599}$$

$$\frac{1}{3}\int 8(-\sin(d+ex)a+a+b\cos(d+ex))\left(-5\sin(d+ex)a^2+5a^2+5b\cos(d+ex)a+2b^2\right)dx+\frac{8(a\cos(d+ex)+b\sin(d+ex))(a(-\sin(d+ex))+a+b\cos(d+ex))^2}{3e}$$

$$\Big\downarrow \text{27}$$

$$\frac{8}{3}\int (-\sin(d+ex)a+a+b\cos(d+ex))\left(-5\sin(d+ex)a^2+5a^2+5b\cos(d+ex)a+2b^2\right)dx+\frac{8(a\cos(d+ex)+b\sin(d+ex))(a(-\sin(d+ex))+a+b\cos(d+ex))^2}{3e}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{3}\int (-\sin(d+ex)a+a+b\cos(d+ex))\left(-5\sin(d+ex)a^2+5a^2+5b\cos(d+ex)a+2b^2\right)dx+\frac{8(a\cos(d+ex)+b\sin(d+ex))(a(-\sin(d+ex))+a+b\cos(d+ex))^2}{3e}$$

$$\Big\downarrow \text{3625}$$

$$\frac{8}{3}\left(\frac{\int \left(3\left(5a^2+3b^2\right)a^2-\left(15a^2+4b^2\right)\sin(d+ex)a^2+b\left(15a^2+4b^2\right)\cos(d+ex)a\right)dx}{2a}+\frac{5(a(-\sin(d+ex))+a+\right.$$
$$\frac{8(a\cos(d+ex)+b\sin(d+ex))(a(-\sin(d+ex))+a+b\cos(d+ex))^2}{3e}$$

$\big\downarrow$ 2009

$$\frac{8}{3}\left(\frac{\frac{ab\left(15a^2+4b^2\right)\sin(d+ex)}{e}+\frac{a^2\left(15a^2+4b^2\right)\cos(d+ex)}{e}+3a^2x\left(5a^2+3b^2\right)}{2a}+\frac{5(a(-\sin(d+ex))+a+b\cos(d+ex))\left(a^2\right.}{2e}\right.$$
$$\frac{8(a\cos(d+ex)+b\sin(d+ex))(a(-\sin(d+ex))+a+b\cos(d+ex))^2}{3e}$$

input
```
Int[(2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x])^3,x]
```

output
```
(8*(a + b*Cos[d + e*x] - a*Sin[d + e*x])^2*(a*Cos[d + e*x] + b*Sin[d + e*x
]))/(3*e) + (8*((5*(a + b*Cos[d + e*x] - a*Sin[d + e*x])*(a^2*Cos[d + e*x]
 + a*b*Sin[d + e*x]))/(2*e) + (3*a^2*(5*a^2 + 3*b^2)*x + (a^2*(15*a^2 + 4*
b^2)*Cos[d + e*x])/e + (a*b*(15*a^2 + 4*b^2)*Sin[d + e*x])/e)/(2*a)))/3
```

3.388.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3599
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Simp[1/n   Int[Simp[n*a^2 + (
n - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x
], x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

rule 3625
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*((a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^n/(a*e*(n + 1))), x] + Simp[1/(a*(n + 1
))   Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
 + a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
 + (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

3.388.4 Maple [A] (verified)

Time = 2.20 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.97

method	result
parallelrisch	$\frac{\frac{2(-a^3+3ab^2)\cos(3ex+3d)}{3}+6(-a^3+ab^2)\sin(2ex+2d)+\frac{2(-3a^2b+b^3)\sin(3ex+3d)}{3}+12a^2b\cos(2ex+2d)+6(5a^3+ab^2)\cos\ldots}{e}$
parts	$\frac{8ab^2\cos(ex+d)^3+24ab^2\left(\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)}{e}+\frac{8a^2b(\sin(ex+d)-1)^3}{e}+8a^3x+\frac{24a^3\cos(ex+d)}{e}+\frac{24a\ldots}{}$
derivativedivides	$\frac{\frac{8b^3(2+\cos(ex+d)^2)\sin(ex+d)}{3}+8ab^2\cos(ex+d)^3+24ab^2\left(\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)+8a^2b\sin(ex+d)^3+24a^2b\cos(ex+\ldots}{}$
default	$\frac{\frac{8b^3(2+\cos(ex+d)^2)\sin(ex+d)}{3}+8ab^2\cos(ex+d)^3+24ab^2\left(\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)+8a^2b\sin(ex+d)^3+24a^2b\cos(ex+\ldots}{}$
risch	$20a^3x+12ab^2x+\frac{30a^3\cos(ex+d)}{e}+\frac{6a\cos(ex+d)b^2}{e}+\frac{30b\sin(ex+d)a^2}{e}+\frac{6b^3\sin(ex+d)}{e}-\frac{2a^3\cos(3ex+\ldots}{3e}$
norman	$\frac{(20a^3+12ab^2)x+(20a^3+12ab^2)x\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^6+(60a^3+36ab^2)x\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2+(60a^3+36ab^2)x\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^4+\frac{(128\ldots}{}}{}$

input
```
int((2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^3,x,method=_RETURNVERBOSE)
```

output
```
2/3*((-a^3+3*a*b^2)*cos(3*e*x+3*d)+9*(-a^3+a*b^2)*sin(2*e*x+2*d)+(-3*a^2*b
+b^3)*sin(3*e*x+3*d)+18*a^2*b*cos(2*e*x+2*d)+9*(5*a^3+a*b^2)*cos(e*x+d)+9*
(5*a^2*b+b^3)*sin(e*x+d)+2*(15*e*x+22)*a^3-18*a^2*b+6*(3*e*x+2)*b^2*a)/e
```

3.388.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.80

$$\int (2a+2b\cos(d+ex)-2a\sin(d+ex))^3\,dx$$
$$= \frac{4\left(18\,a^2b\cos\left(ex+d\right)^2+24\,a^3\cos\left(ex+d\right)-2\left(a^3-3\,ab^2\right)\cos\left(ex+d\right)^3+3\left(5\,a^3+3\,ab^2\right)ex+\left(24\,a^2b+\right.\right.}{3\,e}$$

input
```
integrate((2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^3,x, algorithm="fricas")
```

output
```
4/3*(18*a^2*b*cos(e*x + d)^2 + 24*a^3*cos(e*x + d) - 2*(a^3 - 3*a*b^2)*cos
(e*x + d)^3 + 3*(5*a^3 + 3*a*b^2)*e*x + (24*a^2*b + 4*b^3 - 2*(3*a^2*b - b
^3)*cos(e*x + d)^2 - 9*(a^3 - a*b^2)*cos(e*x + d))*sin(e*x + d))/e
```

3.388.6 Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.85

$$\int (2a+2b\cos(d+ex)-2a\sin(d+ex))^3\,dx$$
$$= \begin{cases} 12a^3x\sin^2\left(d+ex\right)+12a^3x\cos^2\left(d+ex\right)+8a^3x+\frac{8a^3\sin^2\left(d+ex\right)\cos\left(d+ex\right)}{e}-\frac{12a^3\sin\left(d+ex\right)\cos\left(d+ex\right)}{e}+\frac{16a^3\cos}{} \\ x(-2a\sin\left(d\right)+2a+2b\cos\left(d\right))^3 \end{cases}$$

input
```
integrate((2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))**3,x)
```

output
```
Piecewise((12*a**3*x*sin(d + e*x)**2 + 12*a**3*x*cos(d + e*x)**2 + 8*a**3*
x + 8*a**3*sin(d + e*x)**2*cos(d + e*x)/e - 12*a**3*sin(d + e*x)*cos(d + e
*x)/e + 16*a**3*cos(d + e*x)**3/(3*e) + 24*a**3*cos(d + e*x)/e + 8*a**2*b*
sin(d + e*x)**3/e - 24*a**2*b*sin(d + e*x)**2/e + 24*a**2*b*sin(d + e*x)/e
 + 12*a*b**2*x*sin(d + e*x)**2 + 12*a*b**2*x*cos(d + e*x)**2 + 12*a*b**2*s
in(d + e*x)*cos(d + e*x)/e + 8*a*b**2*cos(d + e*x)**3/e + 16*b**3*sin(d +
e*x)**3/(3*e) + 8*b**3*sin(d + e*x)*cos(d + e*x)**2/e, Ne(e, 0)), (x*(-2*a
*sin(d) + 2*a + 2*b*cos(d))**3, True))
```

3.388.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.20

$$\int (2a+2b\cos(d+ex)-2a\sin(d+ex))^3\,dx$$
$$= \frac{8\,ab^2\cos{(ex+d)}^3}{e} + \frac{8\,a^2b\sin{(ex+d)}^3}{e} + 8\,a^3x - \frac{8\left(\cos{(ex+d)}^3 - 3\,\cos{(ex+d)}\right)a^3}{3\,e} - \frac{8\left(\sin{(ex+d)}^3 - 3\,\sin{(ex+d)}\right)b^3}{3\,e} + 24\,a^2\left(\frac{a\cos{(ex+d)}}{e} + \frac{b\sin{(ex+d)}}{e}\right) + 6\left(\frac{4\,ab\cos{(ex+d)}^2}{e} + \frac{(2\,ex+2\,d-\sin{(2\,ex+2\,d)})a^2}{e} + \frac{(2\,ex+2\,d+\sin{(2\,ex+2\,d)})b^2}{e}\right)a$$

input
```
integrate((2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^3,x, algorithm="maxima")
```

output
```
8*a*b^2*cos(e*x + d)^3/e + 8*a^2*b*sin(e*x + d)^3/e + 8*a^3*x - 8/3*(cos(e
*x + d)^3 - 3*cos(e*x + d))*a^3/e - 8/3*(sin(e*x + d)^3 - 3*sin(e*x + d))*
b^3/e + 24*a^2*(a*cos(e*x + d)/e + b*sin(e*x + d)/e) + 6*(4*a*b*cos(e*x +
d)^2/e + (2*e*x + 2*d - sin(2*e*x + 2*d))*a^2/e + (2*e*x + 2*d + sin(2*e*x
 + 2*d))*b^2/e)*a
```

3.388.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.96

$$\int (2a+2b\cos(d+ex)-2a\sin(d+ex))^3\,dx$$
$$= \frac{12\,a^2b\cos{(2\,ex+2\,d)}}{e} + 4\left(5\,a^3+3\,ab^2\right)x - \frac{2\left(a^3-3\,ab^2\right)\cos{(3\,ex+3\,d)}}{3\,e} + \frac{6\left(5\,a^3+ab^2\right)\cos{(ex+d)}}{e} - \frac{2\left(3\,a^2b-b^3\right)\sin{(3\,ex+3\,d)}}{3\,e} - \frac{6\left(a^3-ab^2\right)\sin{(2\,ex+2\,d)}}{e} + \frac{6\left(5\,a^2b+b^3\right)\sin{(ex+d)}}{e}$$

input
```
integrate((2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^3,x, algorithm="giac")
```

output
```
12*a^2*b*cos(2*e*x + 2*d)/e + 4*(5*a^3 + 3*a*b^2)*x - 2/3*(a^3 - 3*a*b^2)*
cos(3*e*x + 3*d)/e + 6*(5*a^3 + a*b^2)*cos(e*x + d)/e - 2/3*(3*a^2*b - b^3
)*sin(3*e*x + 3*d)/e - 6*(a^3 - a*b^2)*sin(2*e*x + 2*d)/e + 6*(5*a^2*b + b
^3)*sin(e*x + d)/e
```

3.388.9 Mupad [B] (verification not implemented)

Time = 27.16 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.86

$$\int (2a+2b\cos(d+ex)-2a\sin(d+ex))^3\,dx$$
$$= \frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^4\,(48\,a^3-96\,a^2\,b+48\,a\,b^2)+\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^5\,(24\,a^3+48\,a^2\,b-24\,a\,b^2+16\,b^3)+16\,a\,b^2-\tan(}{e\,\left(\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^6+3\,t}$$
$$+\frac{8\,a\,\mathrm{atan}\left(\frac{8\,a\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,(5\,a^2+3\,b^2)}{40\,a^3+24\,a\,b^2}\right)\,(5\,a^2+3\,b^2)}{e}$$
$$-\frac{8\,a\,(5\,a^2+3\,b^2)\,\left(\mathrm{atan}\left(\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\right)-\frac{e\,x}{2}\right)}{e}$$

input
```
int((2*a + 2*b*cos(d + e*x) - 2*a*sin(d + e*x))^3,x)
```

output
```
(tan(d/2 + (e*x)/2)^4*(48*a*b^2 - 96*a^2*b + 48*a^3) + tan(d/2 + (e*x)/2)^
5*(48*a^2*b - 24*a*b^2 + 24*a^3 + 16*b^3) + 16*a*b^2 - tan(d/2 + (e*x)/2)^
2*(96*a^2*b - 128*a^3) + tan(d/2 + (e*x)/2)^3*(160*a^2*b + (32*b^3)/3) + (
176*a^3)/3 + tan(d/2 + (e*x)/2)*(24*a*b^2 + 48*a^2*b - 24*a^3 + 16*b^3))/(
e*(3*tan(d/2 + (e*x)/2)^2 + 3*tan(d/2 + (e*x)/2)^4 + tan(d/2 + (e*x)/2)^6
+ 1)) + (8*a*atan((8*a*tan(d/2 + (e*x)/2)*(5*a^2 + 3*b^2))/(24*a*b^2 + 40*
a^3))*(5*a^2 + 3*b^2))/e - (8*a*(5*a^2 + 3*b^2)*(atan(tan(d/2 + (e*x)/2))
- (e*x)/2))/e
```

3.389 $\int (2a+2b\cos(d+ex)-2a\sin(d+ex))^2\,dx$

3.389.1 Optimal result

Integrand size = 24, antiderivative size = 81

$$\int (2a+2b\cos(d+ex)-2a\sin(d+ex))^2\,dx = 2(3a^2+b^2)\,x+\frac{6a^2\cos(d+ex)}{e}+\frac{6ab\sin(d+ex)}{e} + \frac{2(a+b\cos(d+ex)-a\sin(d+ex))(a\cos(d+ex)+b\sin(d+ex))}{e}$$

output
```
2*(3*a^2+b^2)*x+6*a^2*cos(e*x+d)/e+6*a*b*sin(e*x+d)/e+2*(a+b*cos(e*x+d)-a*
sin(e*x+d))*(a*cos(e*x+d)+b*sin(e*x+d))/e
```

3.389.2 Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.14

$$\int (2a+2b\cos(d+ex)-2a\sin(d+ex))^2\,dx = 4\left(\frac{(3a^2+b^2)\,(d+ex)}{2e}+\frac{2a^2\cos(d+ex)}{e} + \frac{ab\cos(2(d+ex))}{2e}+\frac{2ab\sin(d+ex)}{e} - \frac{(a^2-b^2)\sin(2(d+ex))}{4e}\right)$$

input
```
Integrate[(2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x])^2,x]
```

output
```
4*(((3*a^2 + b^2)*(d + e*x))/(2*e) + (2*a^2*Cos[d + e*x])/e + (a*b*Cos[2*(
d + e*x)])/(2*e) + (2*a*b*Sin[d + e*x])/e - ((a^2 - b^2)*Sin[2*(d + e*x)])
/(4*e))
```

3.389.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3042, 3599, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (-2a\sin(d+ex)+2a+2b\cos(d+ex))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (-2a\sin(d+ex)+2a+2b\cos(d+ex))^2dx$$

$$\Big\downarrow \text{3599}$$

$$\frac{1}{2}\int \left(-12\sin(d+ex)a^2+12b\cos(d+ex)a+4\left(3a^2+b^2\right)\right)dx+\frac{2(a(-\sin(d+ex))+a+b\cos(d+ex))(a\cos(d+ex)+b\sin(d+ex))}{e}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(4x\left(3a^2+b^2\right)+\frac{12a^2\cos(d+ex)}{e}+\frac{12ab\sin(d+ex)}{e}\right)+\frac{2(a(-\sin(d+ex))+a+b\cos(d+ex))(a\cos(d+ex)+b\sin(d+ex))}{e}$$

input
```
Int[(2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x])^2,x]
```

output
```
(2*(a + b*Cos[d + e*x] - a*Sin[d + e*x])*(a*Cos[d + e*x] + b*Sin[d + e*x])
)/e + (4*(3*a^2 + b^2)*x + (12*a^2*Cos[d + e*x])/e + (12*a*b*Sin[d + e*x])
/e)/2
```

3.389.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3599
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Simp[1/n   Int[Simp[n*a^2 + (
n - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x
], x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

3.389.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.99

method	result
parallelrisch	$\frac{\left(-a^{2}+b^{2}\right) \sin \left(2 e x+2 d\right)+2 a b \cos \left(2 e x+2 d\right)+8 a^{2} \cos \left(e x+d\right)+8 a b \sin \left(e x+d\right)+\left(6 e x-8\right) a^{2}-2 a b+2 b^{2} e x}{e}$
risch	$6 a^{2} x+2 x\, b^{2}+\frac{8 a^{2} \cos \left(e x+d\right)}{e}+\frac{8 a b \sin \left(e x+d\right)}{e}+\frac{2 a b \cos \left(2 e x+2 d\right)}{e}-\frac{\sin \left(2 e x+2 d\right) a^{2}}{e}+\frac{\sin \left(2 e x+2 d\right) b^{2}}{e}$
derivativedivides	$\frac{4 b^{2} \left(\frac{\cos \left(e x+d\right) \sin \left(e x+d\right)}{2}+\frac{e x}{2}+\frac{d}{2}\right)+4 a b \cos \left(e x+d\right)^{2}+8 a b \sin \left(e x+d\right)+4 a^{2} \left(-\frac{\cos \left(e x+d\right) \sin \left(e x+d\right)}{2}+\frac{e x}{2}+\frac{d}{2}\right)+8 a^{2} \cos \left(e x+d\right.}{e}$
default	$\frac{4 b^{2} \left(\frac{\cos \left(e x+d\right) \sin \left(e x+d\right)}{2}+\frac{e x}{2}+\frac{d}{2}\right)+4 a b \cos \left(e x+d\right)^{2}+8 a b \sin \left(e x+d\right)+4 a^{2} \left(-\frac{\cos \left(e x+d\right) \sin \left(e x+d\right)}{2}+\frac{e x}{2}+\frac{d}{2}\right)+8 a^{2} \cos \left(e x+d\right.}{e}$
parts	$-\frac{8 a b \left(\frac{\sin \left(e x+d\right)^{2}}{2}-\sin \left(e x+d\right)\right)}{e}+4 a^{2} x+\frac{8 a^{2} \cos \left(e x+d\right)}{e}+\frac{4 a^{2} \left(-\frac{\cos \left(e x+d\right) \sin \left(e x+d\right)}{2}+\frac{e x}{2}+\frac{d}{2}\right)}{e}+\frac{4 b^{2} \left(\frac{\cos (e x+}{}\right.}{}$
norman	$\frac{\left(6 a^{2}+2 b^{2}\right) x+\left(6 a^{2}+2 b^{2}\right) x \tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{4}+\left(12 a^{2}+4 b^{2}\right) x \tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{2}-\frac{16 a^{2} \tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{4}}{e}-\frac{4 \left(a^{2}-4 a b-b^{2}\right) \tan \left(\frac{e x}{2}+\frac{d}{2}\right)}{e}+}{\left(1+\tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{2}\right)^{2}}$

input
```
int((2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^2,x,method=_RETURNVERBOSE)
```

output
```
((-a^2+b^2)*sin(2*e*x+2*d)+2*a*b*cos(2*e*x+2*d)+8*a^2*cos(e*x+d)+8*a*b*sin
(e*x+d)+(6*e*x-8)*a^2-2*a*b+2*b^2*e*x)/e
```

3.389.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

$$\int (2a+2b\cos(d+ex)-2a\sin(d+ex))^2\,dx$$
$$=\frac{2\left(2\,ab\cos\left(ex+d\right)^2+\left(3\,a^2+b^2\right)ex+4\,a^2\cos\left(ex+d\right)+\left(4\,ab-\left(a^2-b^2\right)\cos\left(ex+d\right)\right)\sin\left(ex+d\right)\right)}{e}$$

input
```
integrate((2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^2,x, algorithm="fricas")
```

output
```
2*(2*a*b*cos(e*x + d)^2 + (3*a^2 + b^2)*e*x + 4*a^2*cos(e*x + d) + (4*a*b
- (a^2 - b^2)*cos(e*x + d))*sin(e*x + d))/e
```

3.389.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(80) = 160.

Time = 0.11 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.10

$$\int (2a+2b\cos(d+ex)-2a\sin(d+ex))^2\,dx$$
$$=\begin{cases}2a^2x\sin^2\left(d+ex\right)+2a^2x\cos^2\left(d+ex\right)+4a^2x-\frac{2a^2\sin\left(d+ex\right)\cos\left(d+ex\right)}{e}+\frac{8a^2\cos\left(d+ex\right)}{e}-\frac{4ab\sin^2\left(d+ex\right)}{e}+\frac{8ab}{} \\ x(-2a\sin\left(d\right)+2a+2b\cos\left(d\right))^2\end{cases}$$

input
```
integrate((2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))**2,x)
```

output
```
Piecewise((2*a**2*x*sin(d + e*x)**2 + 2*a**2*x*cos(d + e*x)**2 + 4*a**2*x
- 2*a**2*sin(d + e*x)*cos(d + e*x)/e + 8*a**2*cos(d + e*x)/e - 4*a*b*sin(d
 + e*x)**2/e + 8*a*b*sin(d + e*x)/e + 2*b**2*x*sin(d + e*x)**2 + 2*b**2*x*
cos(d + e*x)**2 + 2*b**2*sin(d + e*x)*cos(d + e*x)/e, Ne(e, 0)), (x*(-2*a*
sin(d) + 2*a + 2*b*cos(d))**2, True))
```

3.389.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.21

$$\int (2a+2b\cos(d+ex)-2a\sin(d+ex))^2\,dx = 4\,a^2x + \frac{4\,ab\cos\left(ex+d\right)^2}{e} + \frac{(2\,ex+2\,d-\sin\left(2\,ex+2\,d\right))a^2}{e} + \frac{(2\,ex+2\,d+\sin\left(2\,ex+2\,d\right))b^2}{e} + 8\,a\left(\frac{a\cos\left(ex+d\right)}{e}+\frac{b\sin\left(ex+d\right)}{e}\right)$$

input

```
integrate((2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^2,x, algorithm="maxima")
```

output

```
4*a^2*x + 4*a*b*cos(e*x + d)^2/e + (2*e*x + 2*d - sin(2*e*x + 2*d))*a^2/e
+ (2*e*x + 2*d + sin(2*e*x + 2*d))*b^2/e + 8*a*(a*cos(e*x + d)/e + b*sin(e
*x + d)/e)
```

3.389.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.98

$$\int (2a+2b\cos(d+ex)-2a\sin(d+ex))^2\,dx = 2\left(3\,a^2+b^2\right)x + \frac{2\,ab\cos\left(2\,ex+2\,d\right)}{e} + \frac{8\,a^2\cos\left(ex+d\right)}{e} + \frac{8\,ab\sin\left(ex+d\right)}{e} - \frac{\left(a^2-b^2\right)\sin\left(2\,ex+2\,d\right)}{e}$$

input

```
integrate((2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^2,x, algorithm="giac")
```

output

```
2*(3*a^2 + b^2)*x + 2*a*b*cos(2*e*x + 2*d)/e + 8*a^2*cos(e*x + d)/e + 8*a*
b*sin(e*x + d)/e - (a^2 - b^2)*sin(2*e*x + 2*d)/e
```

3.389.9 Mupad [B] (verification not implemented)

Time = 27.04 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.58

$$\int (2a+2b\cos(d+ex)-2a\sin(d+ex))^2\,dx = \frac{x\,(12\,a^2+4\,b^2)}{2} + \frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^3(4\,a^2+16\,a\,b-4\,b^2)-\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2(16\,a\,b-16\,a^2)+16\,a^2+\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,(-4\,a^2+16\,a}{e\,\left(\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^4+2\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2+1\right)}$$

input
```
int((2*a + 2*b*cos(d + e*x) - 2*a*sin(d + e*x))^2,x)
```

output
```
(x*(12*a^2 + 4*b^2))/2 + (tan(d/2 + (e*x)/2)^3*(16*a*b + 4*a^2 - 4*b^2) -
tan(d/2 + (e*x)/2)^2*(16*a*b - 16*a^2) + 16*a^2 + tan(d/2 + (e*x)/2)*(16*a
*b - 4*a^2 + 4*b^2))/(e*(2*tan(d/2 + (e*x)/2)^2 + tan(d/2 + (e*x)/2)^4 + 1
))
```

3.390 $\int (2a+2b\cos(d+ex)-2a\sin(d+ex))\,dx$

3.390.1 Optimal result

Integrand size = 22, antiderivative size = 29

$$\int (2a+2b\cos(d+ex)-2a\sin(d+ex))\,dx = 2ax+\frac{2a\cos(d+ex)}{e}+\frac{2b\sin(d+ex)}{e}$$

output
```
2*a*x+2*a*cos(e*x+d)/e+2*b*sin(e*x+d)/e
```

3.390.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.83

$$\int (2a+2b\cos(d+ex)-2a\sin(d+ex))\,dx = 2ax+\frac{2a\cos(d)\cos(ex)}{e}+\frac{2b\cos(ex)\sin(d)}{e}+\frac{2b\cos(d)\sin(ex)}{e}-\frac{2a\sin(d)\sin(ex)}{e}$$

input
```
Integrate[2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x],x]
```

output
```
2*a*x + (2*a*Cos[d]*Cos[e*x])/e + (2*b*Cos[e*x]*Sin[d])/e + (2*b*Cos[d]*Si
n[e*x])/e - (2*a*Sin[d]*Sin[e*x])/e
```

3.390.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (-2a\sin(d+ex)+2a+2b\cos(d+ex))\,dx$$

$$\Big\downarrow \ 2009$$

$$\frac{2a\cos(d+ex)}{e}+2ax+\frac{2b\sin(d+ex)}{e}$$

input
```
Int[2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x],x]
```

output
```
2*a*x + (2*a*Cos[d + e*x])/e + (2*b*Sin[d + e*x])/e
```

3.390.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.390.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

method	result	size
derivativedivides	$\frac{2(ex+d)a+2b\sin(ex+d)+2a\cos(ex+d)}{e}$	30
default	$2ax+\frac{2a\cos(ex+d)}{e}+\frac{2b\sin(ex+d)}{e}$	30
risch	$2ax+\frac{2a\cos(ex+d)}{e}+\frac{2b\sin(ex+d)}{e}$	30
parts	$2ax+\frac{2a\cos(ex+d)}{e}+\frac{2b\sin(ex+d)}{e}$	30
parallelrisch	$\frac{2b\sin(ex+d)+2a\cos(ex+d)-2a}{e}+2ax$	32
norman	$\frac{2ax+2ax\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2+\frac{4b\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{e}-\frac{4a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}{e}}{1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}$	69

input
```
int(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
2/e*((e*x+d)*a+b*sin(e*x+d)+a*cos(e*x+d))
```

3.390.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int (2a+2b\cos(d+ex)-2a\sin(d+ex))\,dx = \frac{2\,(aex+a\cos{(ex+d)}+b\sin{(ex+d)})}{e}$$

input
```
integrate(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d),x, algorithm="fricas")
```

output
```
2*(a*e*x + a*cos(e*x + d) + b*sin(e*x + d))/e
```

3.390.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.34

$$\int (2a+2b\cos(d+ex)-2a\sin(d+ex))\,dx = 2ax-2a\left(\begin{cases}-\frac{\cos{(d+ex)}}{e} & \text{for}\ e\neq 0\\ x\sin{(d)} & \text{otherwise}\end{cases}\right) + 2b\left(\begin{cases}\frac{\sin{(d+ex)}}{e} & \text{for}\ e\neq 0\\ x\cos{(d)} & \text{otherwise}\end{cases}\right)$$

input
```
integrate(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d),x)
```

output
```
2*a*x - 2*a*Piecewise((-cos(d + e*x)/e, Ne(e, 0)), (x*sin(d), True)) + 2*b
*Piecewise((sin(d + e*x)/e, Ne(e, 0)), (x*cos(d), True))
```

3.390.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int (2a + 2b\cos(d + ex) - 2a\sin(d + ex))\,dx = 2\,ax + \frac{2\,a\cos(ex + d)}{e} + \frac{2\,b\sin(ex + d)}{e}$$

input
```
integrate(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d),x, algorithm="maxima")
```

output
```
2*a*x + 2*a*cos(e*x + d)/e + 2*b*sin(e*x + d)/e
```

3.390.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int (2a + 2b\cos(d + ex) - 2a\sin(d + ex))\,dx = 2\,ax + \frac{2\,a\cos(ex + d)}{e} + \frac{2\,b\sin(ex + d)}{e}$$

input
```
integrate(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d),x, algorithm="giac")
```

output
```
2*a*x + 2*a*cos(e*x + d)/e + 2*b*sin(e*x + d)/e
```

3.390.9 Mupad [B] (verification not implemented)

Time = 26.38 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int (2a + 2b\cos(d + ex) - 2a\sin(d + ex))\,dx = 2\,a\,x + \frac{2\,a\,\cos(d + e\,x)}{e} + \frac{2\,b\,\sin(d + e\,x)}{e}$$

input
```
int(2*a + 2*b*cos(d + e*x) - 2*a*sin(d + e*x),x)
```

output
```
2*a*x + (2*a*cos(d + e*x))/e + (2*b*sin(d + e*x))/e
```

3.391 $\int \frac{1}{2a+2b\cos(d+ex)-2a\sin(d+ex)}\,dx$

3.391.1 Optimal result

Integrand size = 24, antiderivative size = 33

$$\int \frac{1}{2a+2b\cos(d+ex)-2a\sin(d+ex)}\,dx = \frac{\log\left(a+b\tan\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)\right)}{2be}$$

output
```
1/2*ln(a+b*tan(1/2*d+1/4*Pi+1/2*e*x))/b/e
```

3.391.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 96 vs. 2(33) = 66.

Time = 0.41 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.91

$$\int \frac{1}{2a+2b\cos(d+ex)-2a\sin(d+ex)}\,dx = -\frac{\log\left(\cos\left(\frac{1}{2}(d+ex)\right)-\sin\left(\frac{1}{2}(d+ex)\right)\right)}{2be} + \frac{\log\left(a\cos\left(\frac{1}{2}(d+ex)\right)+b\cos\left(\frac{1}{2}(d+ex)\right)-a\sin\left(\frac{1}{2}(d+ex)\right)+b\sin\left(\frac{1}{2}(d+ex)\right)\right)}{2be}$$

input
```
Integrate[(2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x])^(-1),x]
```

output
```
-1/2*Log[Cos[(d + e*x)/2] - Sin[(d + e*x)/2]]/(b*e) + Log[a*Cos[(d + e*x)/
2] + b*Cos[(d + e*x)/2] - a*Sin[(d + e*x)/2] + b*Sin[(d + e*x)/2]]/(2*b*e)
```

3.391.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3042, 3601, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{-2a\sin(d+ex)+2a+2b\cos(d+ex)}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{-2a\sin(d+ex)+2a+2b\cos(d+ex)}dx$$
$$\Big\downarrow \text{ 3601}$$
$$\frac{\int \frac{1}{2a+2b\tan\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)}d\tan\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)}{e}$$
$$\Big\downarrow \text{ 16}$$
$$\frac{\log\left(a+b\tan\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)\right)}{2be}$$

input
```
Int[(2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x])^(-1),x]
```

output
```
Log[a + b*Tan[d/2 + Pi/4 + (e*x)/2]]/(2*b*e)
```

3.391.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3601
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2 + Pi/4], x]}, Si
mp[f/e   Subst[Int[1/(a + b*f*x), x], x, Tan[(d + e*x)/2 + Pi/4]/f], x]] /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[a + c, 0]
```

3.391.4 Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.42

method	result	size
parallelrisch	$\frac{-\ln\left(\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-1\right)+\ln\left(-a-b+(a-b)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}{2be}$	47
derivativedivides	$\frac{-\frac{\ln\left(\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-1\right)}{b}+\frac{\ln\left(a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-b\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-a-b\right)}{b}}{2e}$	59
default	$\frac{-\frac{\ln\left(\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-1\right)}{b}+\frac{\ln\left(a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-b\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-a-b\right)}{b}}{2e}$	59
norman	$-\frac{\ln\left(\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-1\right)}{2be}+\frac{\ln\left(a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-b\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-a-b\right)}{2be}$	61
risch	$\frac{\ln\left(e^{i(ex+d)}+\frac{ia-b}{ib-a}\right)}{2be}-\frac{\ln\left(e^{i(ex+d)}-i\right)}{2be}$	61

input
```
int(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d)),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-ln(tan(1/2*e*x+1/2*d)-1)+ln(-a-b+(a-b)*tan(1/2*e*x+1/2*d)))/b/e
```

3.391.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(25) = 50.

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.73

$$\int \frac{1}{2a+2b\cos(d+ex)-2a\sin(d+ex)}\,dx = \frac{\log\left(2\,ab\cos\left(ex+d\right)+a^2+b^2-\left(a^2-b^2\right)\sin\left(ex+d\right)\right)-\log\left(-\sin\left(ex+d\right)+1\right)}{4\,be}$$

input
```
integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d)),x, algorithm="fricas")
```

output
```
1/4*(log(2*a*b*cos(e*x + d) + a^2 + b^2 - (a^2 - b^2)*sin(e*x + d)) - log(
-sin(e*x + d) + 1))/(b*e)
```

3.391.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(22) = 44$.

Time = 0.98 (sec) , antiderivative size = 109, normalized size of antiderivative = 3.30

$$\int \frac{1}{2a + 2b\cos(d + ex) - 2a\sin(d + ex)}\,dx = \begin{cases} \frac{\tilde{\infty}x}{\cos(d)} & \text{for } a = 0 \wedge b = 0 \wedge e = 0 \\ -\frac{1}{ae\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-ae} & \text{for } b = 0 \\ \frac{x}{-2a\sin(d)+2a+2b\cos(d)} & \text{for } e = 0 \\ -\frac{\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-1\right)}{2be} & \text{for } a = b \\ -\frac{\log\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-1\right)}{2be} + \frac{\log\left(-\frac{a}{a-b}-\frac{b}{a-b}+\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\right)}{2be} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d)),x)
```

output
```
Piecewise((zoo*x/cos(d), Eq(a, 0) & Eq(b, 0) & Eq(e, 0)), (-1/(a*e*tan(d/2
 + e*x/2) - a*e), Eq(b, 0)), (x/(-2*a*sin(d) + 2*a + 2*b*cos(d)), Eq(e, 0)
), (-log(tan(d/2 + e*x/2) - 1)/(2*b*e), Eq(a, b)), (-log(tan(d/2 + e*x/2)
- 1)/(2*b*e) + log(-a/(a - b) - b/(a - b) + tan(d/2 + e*x/2))/(2*b*e), Tru
e))
```

3.391.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(25) = 50$.

Time = 0.23 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.88

$$\int \frac{1}{2a + 2b\cos(d + ex) - 2a\sin(d + ex)}\,dx = \frac{\frac{\log\left(a+b-\frac{(a-b)\sin(ex+d)}{\cos(ex+d)+1}\right)}{b} - \frac{\log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}-1\right)}{b}}{2e}$$

input
```
integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d)),x, algorithm="maxima")
```

output
```
1/2*(log(a + b - (a - b)*sin(e*x + d)/(cos(e*x + d) + 1))/b - log(sin(e*x
+ d)/(cos(e*x + d) + 1) - 1)/b)/e
```

3.391.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(25) = 50$.

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.39

$$\int \frac{1}{2a + 2b\cos(d + ex) - 2a\sin(d + ex)}\,dx = \frac{\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)-2\,b\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)-2\,a-2\,|b|\right|}{\left|2\,a\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)-2\,b\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)-2\,a+2\,|b|\right|}\right)}{2\,e|b|}$$

input
```
integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d)),x, algorithm="giac")
```

output
```
1/2*log(abs(2*a*tan(1/2*e*x + 1/2*d) - 2*b*tan(1/2*e*x + 1/2*d) - 2*a - 2*
abs(b))/abs(2*a*tan(1/2*e*x + 1/2*d) - 2*b*tan(1/2*e*x + 1/2*d) - 2*a + 2*
abs(b)))/(e*abs(b))
```

3.391.9 Mupad [B] (verification not implemented)

Time = 27.43 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \frac{1}{2a + 2b\cos(d + ex) - 2a\sin(d + ex)}\,dx = \frac{\mathrm{atanh}\left(\frac{a-\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,(2\,a-2\,b)}{2}}{b}\right)}{b\,e}$$

input
```
int(1/(2*a + 2*b*cos(d + e*x) - 2*a*sin(d + e*x)),x)
```

output
```
atanh((a - (tan(d/2 + (e*x)/2)*(2*a - 2*b))/2)/b)/(b*e)
```

3.392 $\int \frac{1}{(2a+2b\cos(d+ex)-2a\sin(d+ex))^2}\,dx$

3.392.1 Optimal result

Integrand size = 24, antiderivative size = 83

$$\int \frac{1}{(2a+2b\cos(d+ex)-2a\sin(d+ex))^2}\,dx=-\frac{a\log\left(a+b\tan\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)\right)}{4b^3e}+\frac{a\cos(d+ex)+b\sin(d+ex)}{4b^2e(a+b\cos(d+ex)-a\sin(d+ex))}$$

output
```
-1/4*a*ln(a+b*tan(1/2*d+1/4*Pi+1/2*e*x))/b^3/e+1/4*(a*cos(e*x+d)+b*sin(e*x
+d))/b^2/e/(a+b*cos(e*x+d)-a*sin(e*x+d))
```

3.392.2 Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 166, normalized size of antiderivative = 2.00

$$\int \frac{1}{(2a+2b\cos(d+ex)-2a\sin(d+ex))^2}\,dx$$
$$=\frac{a\log\left(\cos\left(\frac{1}{2}(d+ex)\right)-\sin\left(\frac{1}{2}(d+ex)\right)\right)-a\log\left((a+b)\cos\left(\frac{1}{2}(d+ex)\right)+(-a+b)\sin\left(\frac{1}{2}(d+ex)\right)\right)+}{4b^3e}$$

input
```
Integrate[(2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x])^(-2),x]
```

output
```
(a*Log[Cos[(d + e*x)/2] - Sin[(d + e*x)/2]] - a*Log[(a + b)*Cos[(d + e*x)/
2] + (-a + b)*Sin[(d + e*x)/2]] + (b*Sin[(d + e*x)/2])/(Cos[(d + e*x)/2] -
 Sin[(d + e*x)/2]) + (b*(a^2 + b^2)*Sin[(d + e*x)/2])/((a + b)*((a + b)*Co
s[(d + e*x)/2] + (-a + b)*Sin[(d + e*x)/2])))/(4*b^3*e)
```

3.392.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {3042, 3608, 25, 27, 3042, 3601, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-2a\sin(d+ex)+2a+2b\cos(d+ex))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(-2a\sin(d+ex)+2a+2b\cos(d+ex))^2}dx$$

$$\Big\downarrow \text{3608}$$

$$\frac{\int -\frac{a}{-\sin(d+ex)a+a+b\cos(d+ex)}dx}{4b^2}+\frac{a\cos(d+ex)+b\sin(d+ex)}{4b^2e(a(-\sin(d+ex))+a+b\cos(d+ex))}$$

$$\Big\downarrow \text{25}$$

$$\frac{a\cos(d+ex)+b\sin(d+ex)}{4b^2e(a(-\sin(d+ex))+a+b\cos(d+ex))}-\frac{\int \frac{a}{-\sin(d+ex)a+a+b\cos(d+ex)}dx}{4b^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\cos(d+ex)+b\sin(d+ex)}{4b^2e(a(-\sin(d+ex))+a+b\cos(d+ex))}-\frac{a\int \frac{1}{-\sin(d+ex)a+a+b\cos(d+ex)}dx}{4b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\cos(d+ex)+b\sin(d+ex)}{4b^2e(a(-\sin(d+ex))+a+b\cos(d+ex))}-\frac{a\int \frac{1}{-\sin(d+ex)a+a+b\cos(d+ex)}dx}{4b^2}$$

$$\Big\downarrow \text{3601}$$

$$\frac{a\cos(d+ex)+b\sin(d+ex)}{4b^2e(a(-\sin(d+ex))+a+b\cos(d+ex))}-\frac{a\int \frac{1}{a+b\tan\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)}d\tan\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)}{4b^2e}$$

$$\Big\downarrow 16$$

$$\frac{a\cos(d+ex)+b\sin(d+ex)}{4b^2e(a(-\sin(d+ex))+a+b\cos(d+ex))}-\frac{a\log\left(a+b\tan\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)\right)}{4b^3e}$$

input
```
Int[(2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x])^(-2),x]
```

output
```
-1/4*(a*Log[a + b*Tan[d/2 + Pi/4 + (e*x)/2]])/(b^3*e) + (a*Cos[d + e*x] +
b*Sin[d + e*x])/(4*b^2*e*(a + b*Cos[d + e*x] - a*Sin[d + e*x]))
```

3.392.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3601
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2 + Pi/4], x]}, Si
mp[f/e   Subst[Int[1/(a + b*f*x), x], x, Tan[(d + e*x)/2 + Pi/4]/f], x]] /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[a + c, 0]
```

rule 3608
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Simp[
1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c
*(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x
] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] &&
NeQ[n, -3/2]
```

3.392.4 Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.57

method	result
derivativedivides	$\frac{-\frac{a^2+b^2}{b^2(a-b)\left(a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-b\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-a-b\right)}-\frac{a\ln\left(a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-b\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-a-b\right)}{b^3}-\frac{1}{b^2\left(\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-1\right)}+\frac{a\ln\left(\tan\left(\frac{ex}{2}\right.\right.}{b^3}}{4e}$
default	$\frac{-\frac{a^2+b^2}{b^2(a-b)\left(a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-b\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-a-b\right)}-\frac{a\ln\left(a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-b\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-a-b\right)}{b^3}-\frac{1}{b^2\left(\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-1\right)}+\frac{a\ln\left(\tan\left(\frac{ex}{2}\right.\right.}{b^3}}{4e}$
risch	$\frac{i\left(-ia+b+a\,e^{i(ex+d)}\right)}{2b^2e\left(ia\,e^{2i(ex+d)}+b\,e^{2i(ex+d)}-ia+2a\,e^{i(ex+d)}+b\right)}+\frac{a\ln\left(e^{i(ex+d)}-i\right)}{4b^3e}-\frac{a\ln\left(e^{i(ex+d)}+\frac{ia-b}{ib-a}\right)}{4b^3e}$
parallelrisch	$\frac{(\cos(ex+d)a^2b-a^3(\sin(ex+d)-1))\ln\left(-a-b+(a-b)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)+a^2(a(\sin(ex+d)-1)-b\cos(ex+d))\ln\left(\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right.}{4b^3ea(a(\sin(ex+d)-1)-b\cos(ex+d))}$
norman	$\frac{\frac{a^2+ab+b^2}{4a\,b^2e}-\frac{(a^2-ab+b^2)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}{4a\,b^2e}}{\left(\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-1\right)\left(a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-b\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-a-b\right)}+\frac{a\ln\left(\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-1\right)}{4b^3e}-\frac{a\ln\left(a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-b\tan\left(\frac{ex}{2}\right.\right.}{4b^3e}$

input
```
int(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4/e*(-(a^2+b^2)/b^2/(a-b)/(a*tan(1/2*e*x+1/2*d)-b*tan(1/2*e*x+1/2*d)-a-b
)-a/b^3*ln(a*tan(1/2*e*x+1/2*d)-b*tan(1/2*e*x+1/2*d)-a-b)-1/b^2/(tan(1/2*e
*x+1/2*d)-1)+a/b^3*ln(tan(1/2*e*x+1/2*d)-1))
```

3.392.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. 2(73) = 146.

Time = 0.26 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.86

$$\int \frac{1}{(2a+2b\cos(d+ex)-2a\sin(d+ex))^2}\,dx$$
$$= \frac{2\,ab\cos(ex+d)+2\,b^2\sin(ex+d)-(ab\cos(ex+d)-a^2\sin(ex+d)+a^2)\log(2\,ab\cos(ex+d)+a^2+\ldots}{8\,(b^4e\cos(ex+d)-ab^3e\sin\ldots}$$

input
```
integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^2,x, algorithm="fricas")
```

output
```
1/8*(2*a*b*cos(e*x + d) + 2*b^2*sin(e*x + d) - (a*b*cos(e*x + d) - a^2*sin
(e*x + d) + a^2)*log(2*a*b*cos(e*x + d) + a^2 + b^2 - (a^2 - b^2)*sin(e*x
+ d)) + (a*b*cos(e*x + d) - a^2*sin(e*x + d) + a^2)*log(-sin(e*x + d) + 1)
)/(b^4*e*cos(e*x + d) - a*b^3*e*sin(e*x + d) + a*b^3*e)
```

3.392.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(2a + 2b\cos(d + ex) - 2a\sin(d + ex))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))**2,x)
```

output
```
Timed out
```

3.392.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(73) = 146$.

Time = 0.24 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.19

$$\int \frac{1}{(2a + 2b\cos(d + ex) - 2a\sin(d + ex))^2}\,dx = \frac{\frac{2\left(a^2-\frac{(a^2-ab+b^2)\sin(ex+d)}{\cos(ex+d)+1}\right)}{a^2b^2-b^4-\frac{2(a^2b^2-ab^3)\sin(ex+d)}{\cos(ex+d)+1}+\frac{(a^2b^2-2\,ab^3+b^4)\sin(ex+d)^2}{(\cos(ex+d)+1)^2}} - \frac{a\log\left(a+b-\frac{(a-b)\sin(ex+d)}{\cos(ex+d)+1}\right)}{b^3} + \frac{a\log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}-1\right)}{b^3}}{4\,e}$$

input
```
integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^2,x, algorithm="maxima")
```

output
```
1/4*(2*(a^2 - (a^2 - a*b + b^2)*sin(e*x + d)/(cos(e*x + d) + 1))/(a^2*b^2
 - b^4 - 2*(a^2*b^2 - a*b^3)*sin(e*x + d)/(cos(e*x + d) + 1) + (a^2*b^2 - 2
*a*b^3 + b^4)*sin(e*x + d)^2/(cos(e*x + d) + 1)^2) - a*log(a + b - (a - b)
*sin(e*x + d)/(cos(e*x + d) + 1))/b^3 + a*log(sin(e*x + d)/(cos(e*x + d) +
 1) - 1)/b^3)/e
```

3.392.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(73) = 146.

Time = 0.30 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.28

$$\int \frac{1}{(2a + 2b\cos(d + ex) - 2a\sin(d + ex))^2}\,dx = -\frac{\frac{2\left(a^2\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)-ab\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+b^2\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)-a^2\right)}{(ab^2-b^3)\left(a\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^2-b\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^2-2a\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+a+b\right)}+\frac{a\log\left(\frac{\left|2a\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)-2b\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)-2a-2|b|\right|}{\left|2a\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)-2b\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)-2a+2|b|\right|}\right)}{b^2|b|}}{4e}$$

input
```
integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^2,x, algorithm="giac")
```

output
```
-1/4*(2*(a^2*tan(1/2*e*x + 1/2*d) - a*b*tan(1/2*e*x + 1/2*d) + b^2*tan(1/2
*e*x + 1/2*d) - a^2)/((a*b^2 - b^3)*(a*tan(1/2*e*x + 1/2*d)^2 - b*tan(1/2*
e*x + 1/2*d)^2 - 2*a*tan(1/2*e*x + 1/2*d) + a + b)) + a*log(abs(2*a*tan(1/
2*e*x + 1/2*d) - 2*b*tan(1/2*e*x + 1/2*d) - 2*a - 2*abs(b))/abs(2*a*tan(1/
2*e*x + 1/2*d) - 2*b*tan(1/2*e*x + 1/2*d) - 2*a + 2*abs(b)))/(b^2*abs(b)))
/e
```

3.392.9 Mupad [B] (verification not implemented)

Time = 27.15 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.52

$$\int \frac{1}{(2a + 2b\cos(d + ex) - 2a\sin(d + ex))^2}\,dx = \frac{\frac{a^2}{b^2\,(a-b)}-\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,(a^2-a\,b+b^2)}{b^2\,(a-b)}}{e\,\left((2\,a-2\,b)\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2-4\,a\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)+2\,a+2\,b\right)} - \frac{a\,\mathrm{atanh}\left(\frac{a-\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,(2\,a-2\,b)}{2}}{b}\right)}{2\,b^3\,e}$$

input
```
int(1/(2*a + 2*b*cos(d + e*x) - 2*a*sin(d + e*x))^2,x)
```

output
```
(a^2/(b^2*(a - b)) - (tan(d/2 + (e*x)/2)*(a^2 - a*b + b^2))/(b^2*(a - b)))
/(e*(2*a + 2*b + tan(d/2 + (e*x)/2)^2*(2*a - 2*b) - 4*a*tan(d/2 + (e*x)/2)
)) - (a*atanh((a - (tan(d/2 + (e*x)/2)*(2*a - 2*b))/2)/b))/(2*b^3*e)
```

3.393 $\int \frac{1}{(2a+2b\cos(d+ex)-2a\sin(d+ex))^3}\,dx$

3.393.1 Optimal result

Integrand size = 24, antiderivative size = 142

$$\int \frac{1}{(2a+2b\cos(d+ex)-2a\sin(d+ex))^3}\,dx = \frac{(3a^2+b^2)\log\left(a+b\tan\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)\right)}{16b^5e} + \frac{a\cos(d+ex)+b\sin(d+ex)}{16b^2e(a+b\cos(d+ex)-a\sin(d+ex))^2} - \frac{3(a^2\cos(d+ex)+ab\sin(d+ex))}{16b^4e(a+b\cos(d+ex)-a\sin(d+ex))}$$

output
```
1/16*(3*a^2+b^2)*ln(a+b*tan(1/2*d+1/4*Pi+1/2*e*x))/b^5/e+1/16*(a*cos(e*x+d
)+b*sin(e*x+d))/b^2/e/(a+b*cos(e*x+d)-a*sin(e*x+d))^2-3/16*(a^2*cos(e*x+d)
+a*b*sin(e*x+d))/b^4/e/(a+b*cos(e*x+d)-a*sin(e*x+d))
```

3.393.2 Mathematica [A] (verified)

Time = 2.33 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.84

$$\int \frac{1}{(2a+2b\cos(d+ex)-2a\sin(d+ex))^3}\,dx = -\frac{2(3a^2+b^2)\log\left(\cos\left(\frac{1}{2}(d+ex)\right)-\sin\left(\frac{1}{2}(d+ex)\right)\right)-2(3a^2+b^2)\log\left((a+b)\cos\left(\frac{1}{2}(d+ex)\right)+(-a+\right.}{}$$

input
```
Integrate[(2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x])^(-3),x]
```

output
```
-1/32*(2*(3*a^2 + b^2)*Log[Cos[(d + e*x)/2] - Sin[(d + e*x)/2]] - 2*(3*a^2
 + b^2)*Log[(a + b)*Cos[(d + e*x)/2] + (-a + b)*Sin[(d + e*x)/2]] - b^2/(C
os[(d + e*x)/2] - Sin[(d + e*x)/2])^2 + (6*a*b*Sin[(d + e*x)/2])/(Cos[(d +
 e*x)/2] - Sin[(d + e*x)/2]) + (b^2*(a^2 + b^2))/((a + b)*Cos[(d + e*x)/2]
 + (-a + b)*Sin[(d + e*x)/2])^2 + (6*a*b*(a^2 + b^2)*Sin[(d + e*x)/2])/((a
 + b)*((a + b)*Cos[(d + e*x)/2] + (-a + b)*Sin[(d + e*x)/2])))/(b^5*e)
```

3.393.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3608, 27, 3042, 3632, 3042, 3601, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-2a\sin(d+ex)+2a+2b\cos(d+ex))^3}dx$$

$$\Big\downarrow \textbf{3042}$$

$$\int \frac{1}{(-2a\sin(d+ex)+2a+2b\cos(d+ex))^3}dx$$

$$\Big\downarrow \textbf{3608}$$

$$\frac{\int -\frac{\sin(d+ex)a+2a-b\cos(d+ex)}{2(-\sin(d+ex)a+a+b\cos(d+ex))^2}dx}{8b^2}+\frac{a\cos(d+ex)+b\sin(d+ex)}{16b^2e(a(-\sin(d+ex))+a+b\cos(d+ex))^2}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{a\cos(d+ex)+b\sin(d+ex)}{16b^2e(a(-\sin(d+ex))+a+b\cos(d+ex))^2}-\frac{\int \frac{\sin(d+ex)a+2a-b\cos(d+ex)}{(-\sin(d+ex)a+a+b\cos(d+ex))^2}dx}{16b^2}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{a\cos(d+ex)+b\sin(d+ex)}{16b^2e(a(-\sin(d+ex))+a+b\cos(d+ex))^2}-\frac{\int \frac{\sin(d+ex)a+2a-b\cos(d+ex)}{(-\sin(d+ex)a+a+b\cos(d+ex))^2}dx}{16b^2}$$

$$\Big\downarrow \textbf{3632}$$

$$\frac{a\cos(d+ex)+b\sin(d+ex)}{16b^2e(a(-\sin(d+ex))+a+b\cos(d+ex))^2}-\frac{\frac{3(a^2\cos(d+ex)+ab\sin(d+ex))}{b^2e(a(-\sin(d+ex))+a+b\cos(d+ex))}-\left(\frac{3a^2}{b^2}+1\right)\int \frac{1}{-\sin(d+ex)a+a+b\cos(d+ex)}dx}{16b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\cos(d+ex)+b\sin(d+ex)}{16b^2e(a(-\sin(d+ex))+a+b\cos(d+ex))^2}-\frac{\frac{3(a^2\cos(d+ex)+ab\sin(d+ex))}{b^2e(a(-\sin(d+ex))+a+b\cos(d+ex))}-\left(\frac{3a^2}{b^2}+1\right)\int\frac{1}{-\sin(d+ex)a+a+b\cos(d+ex)}dx}{16b^2}$$

$$\Big\downarrow \text{3601}$$

$$\frac{a\cos(d+ex)+b\sin(d+ex)}{16b^2e(a(-\sin(d+ex))+a+b\cos(d+ex))^2}-\frac{\frac{3(a^2\cos(d+ex)+ab\sin(d+ex))}{b^2e(a(-\sin(d+ex))+a+b\cos(d+ex))}-\frac{\left(\frac{3a^2}{b^2}+1\right)\int\frac{1}{a+b\tan\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)}d\tan\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)}{e}}{16b^2}$$

$$\Big\downarrow \text{16}$$

$$\frac{a\cos(d+ex)+b\sin(d+ex)}{16b^2e(a(-\sin(d+ex))+a+b\cos(d+ex))^2}-\frac{\frac{3(a^2\cos(d+ex)+ab\sin(d+ex))}{b^2e(a(-\sin(d+ex))+a+b\cos(d+ex))}-\frac{\left(\frac{3a^2}{b^2}+1\right)\log\left(a+b\tan\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)\right)}{be}}{16b^2}$$

input
```
Int[(2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x])^(-3),x]
```

output
```
(a*Cos[d + e*x] + b*Sin[d + e*x])/(16*b^2*e*(a + b*Cos[d + e*x] - a*Sin[d
+ e*x])^2) - (-(((1 + (3*a^2)/b^2)*Log[a + b*Tan[d/2 + Pi/4 + (e*x)/2]])/(
b*e)) + (3*(a^2*Cos[d + e*x] + a*b*Sin[d + e*x]))/(b^2*e*(a + b*Cos[d + e*
x] - a*Sin[d + e*x])))/(16*b^2)
```

3.393.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.393. $\int\frac{1}{(2a+2b\cos(d+ex)-2a\sin(d+ex))^3}\,dx$

rule 3601
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2 + Pi/4], x]}, Si
mp[f/e   Subst[Int[1/(a + b*f*x), x], x, Tan[(d + e*x)/2 + Pi/4]/f], x]] /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[a + c, 0]
```

rule 3608
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Simp[
1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c
*(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x
] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] &&
NeQ[n, -3/2]
```

rule 3632
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Simp[(a*A - b*B - c*C)/(a^2 - b^2 - c^2)   Int[1/(a + b*Cos[d + e*x] + c*S
in[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
 - c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

3.393.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(130) = 260$.

Time = 1.98 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.94

3.393. $\int \frac{1}{(2a+2b\cos(d+ex)-2a\sin(d+ex))^3}\,dx$

method	result
derivativedivides	$\frac{\frac{(3a^3-3a^2b+a\,b^2-b^3)\ln\left(a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-b\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-a-b\right)}{2b^5(a-b)}-\frac{a^4+2a^2b^2+b^4}{2b^3(a-b)^2\left(a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-b\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-a-b\right)^2}-\frac{-3a}{2b^4(a-b)^2(a}}{8e}$
default	$\frac{\frac{(3a^3-3a^2b+a\,b^2-b^3)\ln\left(a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-b\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-a-b\right)}{2b^5(a-b)}-\frac{a^4+2a^2b^2+b^4}{2b^3(a-b)^2\left(a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-b\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-a-b\right)^2}-\frac{-3a}{2b^4(a-b)^2(a}}{8e}$
risch	$-\frac{i(3a^2b\,e^{3i(ex+d)}+b^3e^{3i(ex+d)}+9a^3e^{2i(ex+d)}+3a\,b^2e^{2i(ex+d)}+3ia^3e^{3i(ex+d)}+ia\,b^2e^{3i(ex+d)}+9a^2b\,e^{i(ex+d)}-b^3e^{i(ex+}}{8\left(ia\,e^{2i(ex+d)}+b\,e^{2i(ex+d)}-ia+2a\,e^{i(ex+d)}+b\right)^2b^4e}$
norman	$\frac{-\frac{9a^5+18a^4b+12a^3b^2+6a^2b^3+a\,b^4}{16b^4e(3a^2-b^2)}+\frac{(9a^5+9a^4b+6a^3b^2+6a^2b^3+3a\,b^4-b^5)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{8b^4e(3a^2-b^2)}-\frac{(9a^5-9a^4b+6a^3b^2-6a^2b^3+3a\,b^4+b^5)}{8b^4e(3a^2-b^2)}}{\left(\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-1\right)^2\left(a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-b\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-a-b\right)^2}$
parallelrisch	$9\left(a^2-\frac{b^2}{3}\right)\left(a^2+\frac{b^2}{3}\right)\left((a^2-b^2)\cos(2ex+2d)+4a^2\sin(ex+d)-4ab\cos(ex+d)+2ab\sin(2ex+2d)-3a^2-b^2\right)\ln\left(-a-b+(a-\right.$

input
```
int(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^3,x,method=_RETURNVERBOSE)
```

output
```
1/8/e*(1/2*(3*a^3-3*a^2*b+a*b^2-b^3)/b^5/(a-b)*ln(a*tan(1/2*e*x+1/2*d)-b*t
an(1/2*e*x+1/2*d)-a-b)-1/2*(a^4+2*a^2*b^2+b^4)/b^3/(a-b)^2/(a*tan(1/2*e*x+
1/2*d)-b*tan(1/2*e*x+1/2*d)-a-b)^2-1/2*(-3*a^4+4*a^3*b-2*a^2*b^2+4*a*b^3+b
^4)/b^4/(a-b)^2/(a*tan(1/2*e*x+1/2*d)-b*tan(1/2*e*x+1/2*d)-a-b)+1/2/b^3/(t
an(1/2*e*x+1/2*d)-1)^2-1/2*(-3*a-b)/b^4/(tan(1/2*e*x+1/2*d)-1)+1/2/b^5*(-3
*a^2-b^2)*ln(tan(1/2*e*x+1/2*d)-1))
```

3.393.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 423 vs. $2(130) = 260$.

Time = 0.26 (sec) , antiderivative size = 423, normalized size of antiderivative = 2.98

$$\int \frac{1}{(2a+2b\cos(d+ex)-2a\sin(d+ex))^3}\,dx =$$
$$-\frac{12\,a^2b^2\cos\left(ex+d\right)^2-6\,a^2b^2+2\left(3\,a^3b-ab^3\right)\cos\left(ex+d\right)-\left(6\,a^4+2\,a^2b^2-\left(3\,a^4-2\,a^2b^2-b^4\right)\cos\left(e\right.\right.}{}$$

input
```
integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^3,x, algorithm="fricas")
```

output
```
-1/32*(12*a^2*b^2*cos(e*x + d)^2 - 6*a^2*b^2 + 2*(3*a^3*b - a*b^3)*cos(e*x
 + d) - (6*a^4 + 2*a^2*b^2 - (3*a^4 - 2*a^2*b^2 - b^4)*cos(e*x + d)^2 + 2*
(3*a^3*b + a*b^3)*cos(e*x + d) - 2*(3*a^4 + a^2*b^2 + (3*a^3*b + a*b^3)*co
s(e*x + d))*sin(e*x + d))*log(2*a*b*cos(e*x + d) + a^2 + b^2 - (a^2 - b^2)
*sin(e*x + d)) + (6*a^4 + 2*a^2*b^2 - (3*a^4 - 2*a^2*b^2 - b^4)*cos(e*x +
d)^2 + 2*(3*a^3*b + a*b^3)*cos(e*x + d) - 2*(3*a^4 + a^2*b^2 + (3*a^3*b +
a*b^3)*cos(e*x + d))*sin(e*x + d))*log(-sin(e*x + d) + 1) + 2*(3*a^2*b^2 -
 b^4 - 3*(a^3*b - a*b^3)*cos(e*x + d))*sin(e*x + d))/(2*a*b^6*e*cos(e*x +
d) + 2*a^2*b^5*e - (a^2*b^5 - b^7)*e*cos(e*x + d)^2 - 2*(a*b^6*e*cos(e*x +
 d) + a^2*b^5*e)*sin(e*x + d))
```

3.393.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(2a + 2b\cos(d + ex) - 2a\sin(d + ex))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))**3,x)
```

output
```
Timed out
```

3.393.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 491 vs. $2(130) = 260$.

Time = 0.25 (sec) , antiderivative size = 491, normalized size of antiderivative = 3.46

$$\int \frac{1}{(2a + 2b\cos(d + ex) - 2a\sin(d + ex))^3}\,dx =$$

$$-\frac{\frac{2\left(3a^5-4a^3b^2-ab^4-\frac{(9a^5-9a^4b-2a^3b^2+2a^2b^3-5ab^4+b^5)\sin(ex+d)}{\cos(ex+d)+1}+\frac{(9a^5-18a^4b+12a^3b^2-6a^2b^3+ab^4)\sin(ex+d)^2}{(\cos(ex+d)+1)^2}-\frac{(3a^5-9a^4b+10\cdots}{\cdots}\right.}{a^4b^4-2a^2b^6+b^8-\frac{4(a^4b^4-a^3b^5-a^2b^6+ab^7)\sin(ex+d)}{\cos(ex+d)+1}+\frac{2(3a^4b^4-6a^3b^5+2a^2b^6+2ab^7-b^8)\sin(ex+d)^2}{(\cos(ex+d)+1)^2}-\frac{4(a^4b^4-3a^3b^5+3a^2b^6-ab^7)\sin(ex+d)^3}{(\cos(ex+d)+1)^3}+\cdots}\cdots}{16e}$$

input
```
integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^3,x, algorithm="maxima")
```

output
```
-1/16*(2*(3*a^5 - 4*a^3*b^2 - a*b^4 - (9*a^5 - 9*a^4*b - 2*a^3*b^2 + 2*a^2
*b^3 - 5*a*b^4 + b^5)*sin(e*x + d)/(cos(e*x + d) + 1) + (9*a^5 - 18*a^4*b
+ 12*a^3*b^2 - 6*a^2*b^3 + a*b^4)*sin(e*x + d)^2/(cos(e*x + d) + 1)^2 - (3
*a^5 - 9*a^4*b + 10*a^3*b^2 - 6*a^2*b^3 + a*b^4 + b^5)*sin(e*x + d)^3/(cos
(e*x + d) + 1)^3)/(a^4*b^4 - 2*a^2*b^6 + b^8 - 4*(a^4*b^4 - a^3*b^5 - a^2*
b^6 + a*b^7)*sin(e*x + d)/(cos(e*x + d) + 1) + 2*(3*a^4*b^4 - 6*a^3*b^5 +
2*a^2*b^6 + 2*a*b^7 - b^8)*sin(e*x + d)^2/(cos(e*x + d) + 1)^2 - 4*(a^4*b^
4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*sin(e*x + d)^3/(cos(e*x + d) + 1)^3 + (
a^4*b^4 - 4*a^3*b^5 + 6*a^2*b^6 - 4*a*b^7 + b^8)*sin(e*x + d)^4/(cos(e*x +
 d) + 1)^4) - (3*a^2 + b^2)*log(a + b - (a - b)*sin(e*x + d)/(cos(e*x + d)
 + 1))/b^5 + (3*a^2 + b^2)*log(sin(e*x + d)/(cos(e*x + d) + 1) - 1)/b^5)/e
```

3.393.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(130) = 260$.

Time = 0.32 (sec) , antiderivative size = 458, normalized size of antiderivative = 3.23

$$\int \frac{1}{(2a + 2b\cos(d + ex) - 2a\sin(d + ex))^3}\,dx$$

$$= \frac{2\left(3\,a^5\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^3-9\,a^4b\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^3+10\,a^3b^2\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^3-6\,a^2b^3\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^3+ab^4\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^3+b^5\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^3-9\,a\right.}{}$$

input
```
integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^3,x, algorithm="giac")
```

output
```
1/16*(2*(3*a^5*tan(1/2*e*x + 1/2*d)^3 - 9*a^4*b*tan(1/2*e*x + 1/2*d)^3 + 1
0*a^3*b^2*tan(1/2*e*x + 1/2*d)^3 - 6*a^2*b^3*tan(1/2*e*x + 1/2*d)^3 + a*b^
4*tan(1/2*e*x + 1/2*d)^3 + b^5*tan(1/2*e*x + 1/2*d)^3 - 9*a^5*tan(1/2*e*x
+ 1/2*d)^2 + 18*a^4*b*tan(1/2*e*x + 1/2*d)^2 - 12*a^3*b^2*tan(1/2*e*x + 1/
2*d)^2 + 6*a^2*b^3*tan(1/2*e*x + 1/2*d)^2 - a*b^4*tan(1/2*e*x + 1/2*d)^2 +
 9*a^5*tan(1/2*e*x + 1/2*d) - 9*a^4*b*tan(1/2*e*x + 1/2*d) - 2*a^3*b^2*tan
(1/2*e*x + 1/2*d) + 2*a^2*b^3*tan(1/2*e*x + 1/2*d) - 5*a*b^4*tan(1/2*e*x +
 1/2*d) + b^5*tan(1/2*e*x + 1/2*d) - 3*a^5 + 4*a^3*b^2 + a*b^4)/((a^2*b^4
- 2*a*b^5 + b^6)*(a*tan(1/2*e*x + 1/2*d)^2 - b*tan(1/2*e*x + 1/2*d)^2 - 2*
a*tan(1/2*e*x + 1/2*d) + a + b)^2) + (3*a^2 + b^2)*log(abs(2*a*tan(1/2*e*x
 + 1/2*d) - 2*b*tan(1/2*e*x + 1/2*d) - 2*a - 2*abs(b))/abs(2*a*tan(1/2*e*x
 + 1/2*d) - 2*b*tan(1/2*e*x + 1/2*d) - 2*a + 2*abs(b)))/(b^4*abs(b)))/e
```

3.393.9 Mupad [B] (verification not implemented)

Time = 32.50 (sec) , antiderivative size = 361, normalized size of antiderivative = 2.54

$$\int \frac{1}{(2a+2b\cos(d+ex)-2a\sin(d+ex))^3}\,dx = \frac{\operatorname{atanh}\left(\frac{2\,a-\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,(2\,a-2\,b)}{2\,b}\right)\,(3\,a^2+b^2)}{8\,b^5\,e}$$

$$-\frac{\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,(-9\,a^5+9\,a^4\,b+2\,a^3\,b^2-2\,a^2\,b^3+5\,a\,b^4-b^5)}{2\,b^4\,(a-b)^2}-\frac{-3\,a^5+4\,a^3\,b^2+a\,b^4}{2\,b^4\,(a-b)^2}+\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^3\,(-3\,a^4+6\,a^3\,b-4\,a^2\,b^2+2\,a\,b^3+}{2\,b^4\,(a-b)}}{e\,\left(8\,a\,b+\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2\,(24\,a^2-8\,b^2)+\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^3\,(16\,a\,b-16\,a^2)+\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^4\,(4\,a^2-8\,a\,b+\right.}$$

input
```
int(1/(2*a + 2*b*cos(d + e*x) - 2*a*sin(d + e*x))^3,x)
```

output
```
(atanh((2*a - tan(d/2 + (e*x)/2)*(2*a - 2*b))/(2*b))*(3*a^2 + b^2))/(8*b^5
*e) - ((tan(d/2 + (e*x)/2)*(5*a*b^4 + 9*a^4*b - 9*a^5 - b^5 - 2*a^2*b^3 +
2*a^3*b^2))/(2*b^4*(a - b)^2) - (a*b^4 - 3*a^5 + 4*a^3*b^2)/(2*b^4*(a - b)
^2) + (tan(d/2 + (e*x)/2)^3*(2*a*b^3 + 6*a^3*b - 3*a^4 + b^4 - 4*a^2*b^2))
/(2*b^4*(a - b)) + (tan(d/2 + (e*x)/2)^2*(a*b^4 - 18*a^4*b + 9*a^5 - 6*a^2
*b^3 + 12*a^3*b^2))/(2*b^4*(a - b)^2))/(e*(8*a*b + tan(d/2 + (e*x)/2)^2*(2
4*a^2 - 8*b^2) + tan(d/2 + (e*x)/2)^3*(16*a*b - 16*a^2) + tan(d/2 + (e*x)/
2)^4*(4*a^2 - 8*a*b + 4*b^2) + 4*a^2 + 4*b^2 - tan(d/2 + (e*x)/2)*(16*a*b
+ 16*a^2)))
```

3.394 $\int \frac{1}{(2a+2b\cos(d+ex)-2a\sin(d+ex))^4}\,dx$

3.394.1 Optimal result

Integrand size = 24, antiderivative size = 215

$$\begin{aligned}\int \frac{1}{(2a+2b\cos(d+ex)-2a\sin(d+ex))^4}\,dx &= -\frac{a(5a^2+3b^2)\log\left(a+b\tan\left(\frac{d}{2}+\frac{\pi}{4}+\frac{ex}{2}\right)\right)}{32b^7e}+\frac{a\cos(d+ex)+b\sin(d+ex)}{48b^2e(a+b\cos(d+ex)-a\sin(d+ex))^3}\\ &\quad -\frac{5(a^2\cos(d+ex)+ab\sin(d+ex))}{96b^4e(a+b\cos(d+ex)-a\sin(d+ex))^2}\\ &\quad +\frac{a(15a^2+4b^2)\cos(d+ex)+b(15a^2+4b^2)\sin(d+ex)}{96b^6e(a+b\cos(d+ex)-a\sin(d+ex))}\end{aligned}$$

output
```
-1/32*a*(5*a^2+3*b^2)*ln(a+b*tan(1/2*d+1/4*Pi+1/2*e*x))/b^7/e+1/48*(a*cos(
e*x+d)+b*sin(e*x+d))/b^2/e/(a+b*cos(e*x+d)-a*sin(e*x+d))^3-5/96*(a^2*cos(e
*x+d)+a*b*sin(e*x+d))/b^4/e/(a+b*cos(e*x+d)-a*sin(e*x+d))^2+1/96*(a*(15*a^
2+4*b^2)*cos(e*x+d)+b*(15*a^2+4*b^2)*sin(e*x+d))/b^6/e/(a+b*cos(e*x+d)-a*s
in(e*x+d))
```

3.394.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 636 vs. 2(215) = 430.

Time = 1.87 (sec) , antiderivative size = 636, normalized size of antiderivative = 2.96

$$\int \frac{1}{(2a+2b\cos(d+ex)-2a\sin(d+ex))^4}\,dx$$
$$= \frac{12a(5a^2+3b^2)\log\left(\cos\left(\frac{1}{2}(d+ex)\right)-\sin\left(\frac{1}{2}(d+ex)\right)\right)-12a(5a^2+3b^2)\log\left((a+b)\cos\left(\frac{1}{2}(d+ex)\right)+(-\right.}{}$$

input
```
Integrate[(2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x])^(-4),x]
```

output
```
(12*a*(5*a^2 + 3*b^2)*Log[Cos[(d + e*x)/2] - Sin[(d + e*x)/2]] - 12*a*(5*a
^2 + 3*b^2)*Log[(a + b)*Cos[(d + e*x)/2] + (-a + b)*Sin[(d + e*x)/2]] + (b
*(-150*a^6 - 130*a^4*b^2 - 24*a^2*b^4 + 3*a^2*(25*a^4 - 50*a^3*b + 5*a^2*b
^2 - 30*a*b^3 + 4*b^4)*Cos[d + e*x] + 6*a^2*(15*a^4 + 20*a^3*b + 9*a^2*b^2
 + 2*a*b^3 - 2*b^4)*Cos[2*(d + e*x)] - 15*a^6*Cos[3*(d + e*x)] + 30*a^5*b*
Cos[3*(d + e*x)] + 41*a^4*b^2*Cos[3*(d + e*x)] + 38*a^3*b^3*Cos[3*(d + e*x
)] + 12*a^2*b^4*Cos[3*(d + e*x)] + 8*a*b^5*Cos[3*(d + e*x)] + 225*a^6*Sin[
d + e*x] + 75*a^5*b*Sin[d + e*x] + 180*a^4*b^2*Sin[d + e*x] + 15*a^3*b^3*S
in[d + e*x] + 27*a^2*b^4*Sin[d + e*x] + 12*a*b^5*Sin[d + e*x] + 12*b^6*Sin
[d + e*x] - 60*a^6*Sin[2*(d + e*x)] + 120*a^5*b*Sin[2*(d + e*x)] + 54*a^4*
b^2*Sin[2*(d + e*x)] + 102*a^3*b^3*Sin[2*(d + e*x)] + 6*a^2*b^4*Sin[2*(d +
 e*x)] + 6*a*b^5*Sin[2*(d + e*x)] - 15*a^6*Sin[3*(d + e*x)] - 45*a^5*b*Sin
[3*(d + e*x)] - 4*a^4*b^2*Sin[3*(d + e*x)] + 3*a^3*b^3*Sin[3*(d + e*x)] +
15*a^2*b^4*Sin[3*(d + e*x)] + 4*a*b^5*Sin[3*(d + e*x)] + 4*b^6*Sin[3*(d +
e*x)]))/((a + b)*(Cos[(d + e*x)/2] - Sin[(d + e*x)/2])^3*((a + b)*Cos[(d +
 e*x)/2] + (-a + b)*Sin[(d + e*x)/2])^3))/(384*b^7*e)
```

3.394.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {3042, 3608, 27, 3042, 3635, 25, 3042, 3632, 3042, 3601, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-2a\sin(d+ex)+2a+2b\cos(d+ex))^4}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{(-2a\sin(d+ex)+2a+2b\cos(d+ex))^4}dx$$

$$\Big\downarrow \; 3608$$

$$\frac{\int -\frac{2\sin(d+ex)a+3a-2b\cos(d+ex)}{4(-\sin(d+ex)a+a+b\cos(d+ex))^3}dx}{12b^2}+\frac{a\cos(d+ex)+b\sin(d+ex)}{48b^2e(a(-\sin(d+ex))+a+b\cos(d+ex))^3}$$

$$\Big\downarrow \; 27$$

$$\frac{a\cos(d+ex)+b\sin(d+ex)}{48b^2e(a(-\sin(d+ex))+a+b\cos(d+ex))^3}-\frac{\int \frac{2\sin(d+ex)a+3a-2b\cos(d+ex)}{(-\sin(d+ex)a+a+b\cos(d+ex))^3}dx}{48b^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{a\cos(d+ex)+b\sin(d+ex)}{48b^2e(a(-\sin(d+ex))+a+b\cos(d+ex))^3}-\frac{\int \frac{2\sin(d+ex)a+3a-2b\cos(d+ex)}{(-\sin(d+ex)a+a+b\cos(d+ex))^3}dx}{48b^2}$$

$$\Big\downarrow \; 3635$$

$$\frac{a\cos(d+ex)+b\sin(d+ex)}{48b^2e(a(-\sin(d+ex))+a+b\cos(d+ex))^3}-\frac{\frac{\int -\frac{5\sin(d+ex)a^2-5b\cos(d+ex)a+2\left(5a^2+2b^2\right)}{(-\sin(d+ex)a+a+b\cos(d+ex))^2}dx}{2b^2}+\frac{5\left(a^2\cos(d+ex)+ab\sin(d+ex)\right)}{2b^2e(a(-\sin(d+ex))+a+b\cos(d+ex))^2}}{48b^2}$$

$$\Big\downarrow \; 25$$

$$\frac{a\cos(d+ex)+b\sin(d+ex)}{48b^2e(a(-\sin(d+ex))+a+b\cos(d+ex))^3}-\frac{\frac{5\left(a^2\cos(d+ex)+ab\sin(d+ex)\right)}{2b^2e(a(-\sin(d+ex))+a+b\cos(d+ex))^2}-\frac{\int \frac{5\sin(d+ex)a^2-5b\cos(d+ex)a+2\left(5a^2+2b^2\right)}{(-\sin(d+ex)a+a+b\cos(d+ex))^2}dx}{2b^2}}{48b^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{a\cos(d+ex)+b\sin(d+ex)}{48b^2e(a(-\sin(d+ex))+a+b\cos(d+ex))^3}-\frac{\frac{5\left(a^2\cos(d+ex)+ab\sin(d+ex)\right)}{2b^2e(a(-\sin(d+ex))+a+b\cos(d+ex))^2}-\frac{\int \frac{5\sin(d+ex)a^2-5b\cos(d+ex)a+2\left(5a^2+2b^2\right)}{(-\sin(d+ex)a+a+b\cos(d+ex))^2}dx}{2b^2}}{48b^2}$$

$$\Big\downarrow \; 3632$$

$$\frac{a\cos(d+ex)+b\sin(d+ex)}{48b^2e(a(-\sin(d+ex))+a+b\cos(d+ex))^3}-\frac{\frac{5\left(a^2\cos(d+ex)+ab\sin(d+ex)\right)}{2b^2e(a(-\sin(d+ex))+a+b\cos(d+ex))^2}-\frac{\frac{b\left(15a^2+4b^2\right)\sin(d+ex)+a\left(15a^2+4b^2\right)\cos(d+ex)}{b^2e(a(-\sin(d+ex))+a+b\cos(d+ex))}-3a\left(\frac{5a^2}{b^2}+3\right)\int \frac{1}{-\sin(d+ex)a+a+b\cos(d+ex)}dx}{2b^2}}{48b^2}$$

3.394. $\int \frac{1}{(2a+2b\cos(d+ex)-2a\sin(d+ex))^4}\,dx$

$$\Big\downarrow \; 3042$$

$$\frac{a\cos(d+ex)+b\sin(d+ex)}{48b^2e(a(-\sin(d+ex))+a+b\cos(d+ex))^3}-\frac{\frac{5(a^2\cos(d+ex)+ab\sin(d+ex))}{2b^2e(a(-\sin(d+ex))+a+b\cos(d+ex))^2}-\frac{\frac{b(15a^2+4b^2)\sin(d+ex)+a(15a^2+4b^2)\cos(d+ex)}{b^2e(a(-\sin(d+ex))+a+b\cos(d+ex))}-3a\left(\frac{5a^2}{b^2}+3\right)\int\frac{1}{-\sin(d+ex)a+a+b\cos(d+ex)}dx}{2b^2}}{48b^2}$$

$$\Big\downarrow \; 3601$$

$$\frac{a\cos(d+ex)+b\sin(d+ex)}{48b^2e(a(-\sin(d+ex))+a+b\cos(d+ex))^3}-\frac{\frac{5(a^2\cos(d+ex)+ab\sin(d+ex))}{2b^2e(a(-\sin(d+ex))+a+b\cos(d+ex))^2}-\frac{\frac{b(15a^2+4b^2)\sin(d+ex)+a(15a^2+4b^2)\cos(d+ex)}{b^2e(a(-\sin(d+ex))+a+b\cos(d+ex))}-\frac{3a\left(\frac{5a^2}{b^2}+3\right)\int\frac{1}{a+b\tan\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)}d\tan\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)}{e}}{2b^2}}{48b^2}$$

$$\Big\downarrow \; 16$$

$$\frac{a\cos(d+ex)+b\sin(d+ex)}{48b^2e(a(-\sin(d+ex))+a+b\cos(d+ex))^3}-\frac{\frac{5(a^2\cos(d+ex)+ab\sin(d+ex))}{2b^2e(a(-\sin(d+ex))+a+b\cos(d+ex))^2}-\frac{\frac{b(15a^2+4b^2)\sin(d+ex)+a(15a^2+4b^2)\cos(d+ex)}{b^2e(a(-\sin(d+ex))+a+b\cos(d+ex))}-\frac{3a\left(\frac{5a^2}{b^2}+3\right)\log\left(a+b\tan\left(\frac{d}{2}+\frac{ex}{2}+\frac{\pi}{4}\right)\right)}{be}}{2b^2}}{48b^2}$$

input
```
Int[(2*a + 2*b*Cos[d + e*x] - 2*a*Sin[d + e*x])^(-4),x]
```

output
```
(a*Cos[d + e*x] + b*Sin[d + e*x])/(48*b^2*e*(a + b*Cos[d + e*x] - a*Sin[d
+ e*x])^3) - ((5*(a^2*Cos[d + e*x] + a*b*Sin[d + e*x]))/(2*b^2*e*(a + b*Co
s[d + e*x] - a*Sin[d + e*x])^2) - ((-3*a*(3 + (5*a^2)/b^2)*Log[a + b*Tan[d
/2 + Pi/4 + (e*x)/2]])/(b*e) + (a*(15*a^2 + 4*b^2)*Cos[d + e*x] + b*(15*a^
2 + 4*b^2)*Sin[d + e*x])/(b^2*e*(a + b*Cos[d + e*x] - a*Sin[d + e*x])))/(2
*b^2))/(48*b^2)
```

3.394.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

3.394. $\int \frac{1}{(2a+2b\cos(d+ex)-2a\sin(d+ex))^4}\,dx$

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3601
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2 + Pi/4], x]}, Si
mp[f/e   Subst[Int[1/(a + b*f*x), x], x, Tan[(d + e*x)/2 + Pi/4]/f], x]] /;
 FreeQ[{a, b, c, d, e}, x] && EqQ[a + c, 0]
```

rule 3608
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Simp[
1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c
*(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x
] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] &&
NeQ[n, -3/2]
```

rule 3632
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Simp[(a*A - b*B - c*C)/(a^2 - b^2 - c^2)   Int[1/(a + b*Cos[d + e*x] + c*S
in[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
 - c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

rule 3635
```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(-(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(
a^2 - b^2 - c^2))), x] + Simp[1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a + b*Co
s[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n +
2)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x]
/; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[n, -2]
```

3.394. $\int \frac{1}{(2a+2b\cos(d+ex)-2a\sin(d+ex))^4}\,dx$

3.394.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 409 vs. $2(201) = 402$.

Time = 3.04 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.91

method	result
derivativedivides	$\frac{-\frac{\left(5a^3-5a^2b+3a\,b^2-3b^3\right)a\ln\left(a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-b\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-a-b\right)}{2b^7(a-b)}-\frac{a^6+3a^4b^2+3a^2b^4+b^6}{3b^4(a-b)^3\left(a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-b\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-a-b\right)^3}-\frac{-}{2b^5(a-}}{}$
default	$\frac{-\frac{\left(5a^3-5a^2b+3a\,b^2-3b^3\right)a\ln\left(a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-b\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-a-b\right)}{2b^7(a-b)}-\frac{a^6+3a^4b^2+3a^2b^4+b^6}{3b^4(a-b)^3\left(a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-b\tan\left(\frac{ex}{2}+\frac{d}{2}\right)-a-b\right)^3}-\frac{-}{2b^5(a-}}{}$
risch	$\frac{i\left(-15a^5\mathrm{e}^{5i(ex+d)}-75a^5\mathrm{e}^{i(ex+d)}+150a^5\mathrm{e}^{3i(ex+d)}+12b^5\mathrm{e}^{2i(ex+d)}+4b^5-45a^4b+3a^2b^3+6a^3b^2\mathrm{e}^{5i(ex+d)}+24a\,b^4\mathrm{e}^{3i(ex+d}\right.}{}$
norman	$\frac{-\frac{\left(50a^7+100a^6b+75a^5b^2+50a^4b^3+22a^3b^4-6a^2b^5-a\,b^6+4b^7\right)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{16e\,b^6\left(5a^2-3b^2\right)}+\frac{\left(50a^7-100a^6b+75a^5b^2-50a^4b^3+22a^3b^4+6a^2b^5-a\,b\right.}{16e\,b^6\left(5a^2-3b^2\right)}}{}$
parallelrisch	$\frac{-\frac{5\left(\left(-3a^2b+b^3\right)\cos(3ex+3d)+6\left(-a^3+a\,b^2\right)\cos(2ex+2d)+\left(a^3-3a\,b^2\right)\sin(3ex+3d)-12a^2b\sin(2ex+2d)+3\left(5a^2b+b^3\right)\cos(ex+d)+3\right.}{32}}{}$

input
```
int(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^4,x,method=_RETURNVERBOSE)
```

output
```
1/16/e*(-1/2*(5*a^3-5*a^2*b+3*a*b^2-3*b^3)*a/b^7/(a-b)*ln(a*tan(1/2*e*x+1/
2*d)-b*tan(1/2*e*x+1/2*d)-a-b)-1/3*(a^6+3*a^4*b^2+3*a^2*b^4+b^6)/b^4/(a-b)
^3/(a*tan(1/2*e*x+1/2*d)-b*tan(1/2*e*x+1/2*d)-a-b)^3-1/2*(-2*a^6+3*a^5*b-3
*a^4*b^2+6*a^3*b^3+3*a*b^5+b^6)/b^5/(a-b)^3/(a*tan(1/2*e*x+1/2*d)-b*tan(1/
2*e*x+1/2*d)-a-b)^2-1/2*(5*a^6-12*a^5*b+12*a^4*b^2-12*a^3*b^3+9*a^2*b^4+2*
b^6)/b^6/(a-b)^3/(a*tan(1/2*e*x+1/2*d)-b*tan(1/2*e*x+1/2*d)-a-b)-1/3/b^4/(
tan(1/2*e*x+1/2*d)-1)^3-1/2*(2*a+b)/b^5/(tan(1/2*e*x+1/2*d)-1)^2-1/2*(5*a^
2+2*a*b+2*b^2)/b^6/(tan(1/2*e*x+1/2*d)-1)+1/2*a*(5*a^2+3*b^2)/b^7*ln(tan(1
/2*e*x+1/2*d)-1))
```

3.394.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 735 vs. 2(201) = 402.

Time = 0.29 (sec) , antiderivative size = 735, normalized size of antiderivative = 3.42

$$\int \frac{1}{(2a+2b\cos(d+ex)-2a\sin(d+ex))^4}\,dx =$$
$$-\frac{60\,a^4b^2+6\,a^2b^4+2\,(15\,a^5b-41\,a^3b^3-12\,ab^5)\cos{(ex+d)}^3-12\,(10\,a^4b^2+a^2b^4)\cos{(ex+d)}^2-6\,(10}{}$$

input
```
integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^4,x, algorithm="fricas")
```

output
```
-1/192*(60*a^4*b^2 + 6*a^2*b^4 + 2*(15*a^5*b - 41*a^3*b^3 - 12*a*b^5)*cos(
e*x + d)^3 - 12*(10*a^4*b^2 + a^2*b^4)*cos(e*x + d)^2 - 6*(10*a^5*b - 9*a^
3*b^3 - 2*a*b^5)*cos(e*x + d) + 3*(20*a^6 + 12*a^4*b^2 - (15*a^5*b + 4*a^3
*b^3 - 3*a*b^5)*cos(e*x + d)^3 - 3*(5*a^6 - 2*a^4*b^2 - 3*a^2*b^4)*cos(e*x
 + d)^2 + 6*(5*a^5*b + 3*a^3*b^3)*cos(e*x + d) - (20*a^6 + 12*a^4*b^2 - (5
*a^6 - 12*a^4*b^2 - 9*a^2*b^4)*cos(e*x + d)^2 + 6*(5*a^5*b + 3*a^3*b^3)*co
s(e*x + d))*sin(e*x + d))*log(2*a*b*cos(e*x + d) + a^2 + b^2 - (a^2 - b^2)
*sin(e*x + d)) - 3*(20*a^6 + 12*a^4*b^2 - (15*a^5*b + 4*a^3*b^3 - 3*a*b^5)
*cos(e*x + d)^3 - 3*(5*a^6 - 2*a^4*b^2 - 3*a^2*b^4)*cos(e*x + d)^2 + 6*(5*
a^5*b + 3*a^3*b^3)*cos(e*x + d) - (20*a^6 + 12*a^4*b^2 - (5*a^6 - 12*a^4*b
^2 - 9*a^2*b^4)*cos(e*x + d)^2 + 6*(5*a^5*b + 3*a^3*b^3)*cos(e*x + d))*sin
(e*x + d))*log(-sin(e*x + d) + 1) - 2*(30*a^4*b^2 + 3*a^2*b^4 + 2*b^6 - (4
5*a^4*b^2 - 3*a^2*b^4 - 4*b^6)*cos(e*x + d)^2 - 3*(10*a^5*b - 9*a^3*b^3 -
a*b^5)*cos(e*x + d))*sin(e*x + d))/(6*a^2*b^8*e*cos(e*x + d) + 4*a^3*b^7*e
 - (3*a^2*b^8 - b^10)*e*cos(e*x + d)^3 - 3*(a^3*b^7 - a*b^9)*e*cos(e*x + d
)^2 - (6*a^2*b^8*e*cos(e*x + d) + 4*a^3*b^7*e - (a^3*b^7 - 3*a*b^9)*e*cos(
e*x + d)^2)*sin(e*x + d))
```

3.394.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(2a+2b\cos(d+ex)-2a\sin(d+ex))^4}\,dx = \text{Timed out}$$

input
```
integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))**4,x)
```

output
```
Timed out
```

3.394.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 959 vs. 2(201) = 402.

Time = 0.27 (sec) , antiderivative size = 959, normalized size of antiderivative = 4.46

$$\int \frac{1}{(2a + 2b\cos(d + ex) - 2a\sin(d + ex))^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^4,x, algorithm="maxima")
```

output
```
1/96*(2*(15*a^8 - 31*a^6*b^2 + 9*a^4*b^4 + 15*a^2*b^6 - 3*(25*a^8 - 25*a^7
*b - 25*a^6*b^2 + 25*a^5*b^3 - 13*a^4*b^4 + 13*a^3*b^5 + 11*a^2*b^6 - 5*a*
b^7 + 2*b^8)*sin(e*x + d)/(cos(e*x + d) + 1) + 6*(25*a^8 - 50*a^7*b + 20*a
^6*b^2 + 10*a^5*b^3 - 17*a^4*b^4 + 24*a^3*b^5 - 10*a^2*b^6 + 2*a*b^7)*sin(
e*x + d)^2/(cos(e*x + d) + 1)^2 - 2*(75*a^8 - 225*a^7*b + 250*a^6*b^2 - 15
0*a^5*b^3 + 63*a^4*b^4 + 11*a^3*b^5 - 24*a^2*b^6 + 6*a*b^7 - 2*b^8)*sin(e*
x + d)^3/(cos(e*x + d) + 1)^3 + 3*(25*a^8 - 100*a^7*b + 165*a^6*b^2 - 160*
a^5*b^3 + 115*a^4*b^4 - 60*a^3*b^5 + 19*a^2*b^6 - 4*a*b^7)*sin(e*x + d)^4/
(cos(e*x + d) + 1)^4 - 3*(5*a^8 - 25*a^7*b + 53*a^6*b^2 - 65*a^5*b^3 + 55*
a^4*b^4 - 35*a^3*b^5 + 17*a^2*b^6 - 7*a*b^7 + 2*b^8)*sin(e*x + d)^5/(cos(e
*x + d) + 1)^5)/(a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10 - b^12 - 6*(a^6*b^6 - a^
5*b^7 - 2*a^4*b^8 + 2*a^3*b^9 + a^2*b^10 - a*b^11)*sin(e*x + d)/(cos(e*x +
 d) + 1) + 3*(5*a^6*b^6 - 10*a^5*b^7 - a^4*b^8 + 12*a^3*b^9 - 5*a^2*b^10 -
 2*a*b^11 + b^12)*sin(e*x + d)^2/(cos(e*x + d) + 1)^2 - 4*(5*a^6*b^6 - 15*
a^5*b^7 + 12*a^4*b^8 + 4*a^3*b^9 - 9*a^2*b^10 + 3*a*b^11)*sin(e*x + d)^3/(
cos(e*x + d) + 1)^3 + 3*(5*a^6*b^6 - 20*a^5*b^7 + 29*a^4*b^8 - 16*a^3*b^9
- a^2*b^10 + 4*a*b^11 - b^12)*sin(e*x + d)^4/(cos(e*x + d) + 1)^4 - 6*(a^6
*b^6 - 5*a^5*b^7 + 10*a^4*b^8 - 10*a^3*b^9 + 5*a^2*b^10 - a*b^11)*sin(e*x
+ d)^5/(cos(e*x + d) + 1)^5 + (a^6*b^6 - 6*a^5*b^7 + 15*a^4*b^8 - 20*a^3*b
^9 + 15*a^2*b^10 - 6*a*b^11 + b^12)*sin(e*x + d)^6/(cos(e*x + d) + 1)^6...
```

3.394.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 957 vs. 2(201) = 402.

Time = 0.36 (sec) , antiderivative size = 957, normalized size of antiderivative = 4.45

$$\int \frac{1}{(2a + 2b\cos(d + ex) - 2a\sin(d + ex))^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(2*a+2*b*cos(e*x+d)-2*a*sin(e*x+d))^4,x, algorithm="giac")
```

output
```
-1/96*(2*(15*a^8*tan(1/2*e*x + 1/2*d)^5 - 75*a^7*b*tan(1/2*e*x + 1/2*d)^5
+ 159*a^6*b^2*tan(1/2*e*x + 1/2*d)^5 - 195*a^5*b^3*tan(1/2*e*x + 1/2*d)^5
+ 165*a^4*b^4*tan(1/2*e*x + 1/2*d)^5 - 105*a^3*b^5*tan(1/2*e*x + 1/2*d)^5
+ 51*a^2*b^6*tan(1/2*e*x + 1/2*d)^5 - 21*a*b^7*tan(1/2*e*x + 1/2*d)^5 + 6*
b^8*tan(1/2*e*x + 1/2*d)^5 - 75*a^8*tan(1/2*e*x + 1/2*d)^4 + 300*a^7*b*tan
(1/2*e*x + 1/2*d)^4 - 495*a^6*b^2*tan(1/2*e*x + 1/2*d)^4 + 480*a^5*b^3*tan
(1/2*e*x + 1/2*d)^4 - 345*a^4*b^4*tan(1/2*e*x + 1/2*d)^4 + 180*a^3*b^5*tan
(1/2*e*x + 1/2*d)^4 - 57*a^2*b^6*tan(1/2*e*x + 1/2*d)^4 + 12*a*b^7*tan(1/2
*e*x + 1/2*d)^4 + 150*a^8*tan(1/2*e*x + 1/2*d)^3 - 450*a^7*b*tan(1/2*e*x +
 1/2*d)^3 + 500*a^6*b^2*tan(1/2*e*x + 1/2*d)^3 - 300*a^5*b^3*tan(1/2*e*x +
 1/2*d)^3 + 126*a^4*b^4*tan(1/2*e*x + 1/2*d)^3 + 22*a^3*b^5*tan(1/2*e*x +
1/2*d)^3 - 48*a^2*b^6*tan(1/2*e*x + 1/2*d)^3 + 12*a*b^7*tan(1/2*e*x + 1/2*
d)^3 - 4*b^8*tan(1/2*e*x + 1/2*d)^3 - 150*a^8*tan(1/2*e*x + 1/2*d)^2 + 300
*a^7*b*tan(1/2*e*x + 1/2*d)^2 - 120*a^6*b^2*tan(1/2*e*x + 1/2*d)^2 - 60*a^
5*b^3*tan(1/2*e*x + 1/2*d)^2 + 102*a^4*b^4*tan(1/2*e*x + 1/2*d)^2 - 144*a^
3*b^5*tan(1/2*e*x + 1/2*d)^2 + 60*a^2*b^6*tan(1/2*e*x + 1/2*d)^2 - 12*a*b^
7*tan(1/2*e*x + 1/2*d)^2 + 75*a^8*tan(1/2*e*x + 1/2*d) - 75*a^7*b*tan(1/2*
e*x + 1/2*d) - 75*a^6*b^2*tan(1/2*e*x + 1/2*d) + 75*a^5*b^3*tan(1/2*e*x +
1/2*d) - 39*a^4*b^4*tan(1/2*e*x + 1/2*d) + 39*a^3*b^5*tan(1/2*e*x + 1/2*d)
 + 33*a^2*b^6*tan(1/2*e*x + 1/2*d) - 15*a*b^7*tan(1/2*e*x + 1/2*d) + 6*...
```

3.394.9 Mupad [B] (verification not implemented)

Time = 33.67 (sec) , antiderivative size = 731, normalized size of antiderivative = 3.40

$$\int \frac{1}{(2a+2b\cos(d+ex)-2a\sin(d+ex))^4}\,dx$$
$$= \frac{\frac{15\,a^8-31\,a^6\,b^2+9\,a^4\,b^4+15\,a^2\,b^6}{6\,b^6\,(a-b)^3}+\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2\,(25\,a^8-50\,a^7\,b+20\,a^6\,b^2+10\,a^5\,b^3-17\,a^4\,b^4+24\,a^3\,b^5-10\,a^2\,b^6+2\,a\,b^7)}{b^6\,(a-b)^3}+\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^4\,(25}{}}{e\,\left(\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^6\,(8\,a^3-24\,a^2\,b+24\,a\,b^2-8\,b^3)-\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^5\,(48\,a^3-96\,a^2\,b+48\,a\,b}$$
$$-\frac{a\,\mathrm{atanh}\left(\frac{a\,\left(2\,a-\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,(2\,a-2\,b)\right)\,(5\,a^2+3\,b^2)}{2\,b\,(5\,a^3+3\,a\,b^2)}\right)\,(5\,a^2+3\,b^2)}{16\,b^7\,e}$$

input
```
int(1/(2*a + 2*b*cos(d + e*x) - 2*a*sin(d + e*x))^4,x)
```

output
```
((15*a^8 + 15*a^2*b^6 + 9*a^4*b^4 - 31*a^6*b^2)/(6*b^6*(a - b)^3) + (tan(d
/2 + (e*x)/2)^2*(2*a*b^7 - 50*a^7*b + 25*a^8 - 10*a^2*b^6 + 24*a^3*b^5 - 1
7*a^4*b^4 + 10*a^5*b^3 + 20*a^6*b^2))/(b^6*(a - b)^3) + (tan(d/2 + (e*x)/2
)^4*(4*a*b^6 - 75*a^6*b + 25*a^7 - 15*a^2*b^5 + 45*a^3*b^4 - 70*a^4*b^3 +
90*a^5*b^2))/(2*b^6*(a - b)^2) - (tan(d/2 + (e*x)/2)^3*(6*a*b^7 - 225*a^7*
b + 75*a^8 - 2*b^8 - 24*a^2*b^6 + 11*a^3*b^5 + 63*a^4*b^4 - 150*a^5*b^3 +
250*a^6*b^2))/(3*b^6*(a - b)^3) - (tan(d/2 + (e*x)/2)^5*(5*a^6 - 15*a^5*b
- 3*a*b^5 + 2*b^6 + 9*a^2*b^4 - 14*a^3*b^3 + 18*a^4*b^2))/(2*b^6*(a - b))
- (tan(d/2 + (e*x)/2)*(25*a^8 - 25*a^7*b - 5*a*b^7 + 2*b^8 + 11*a^2*b^6 +
13*a^3*b^5 - 13*a^4*b^4 + 25*a^5*b^3 - 25*a^6*b^2))/(2*b^6*(a - b)^3))/(e*
(tan(d/2 + (e*x)/2)^6*(24*a*b^2 - 24*a^2*b + 8*a^3 - 8*b^3) - tan(d/2 + (e
*x)/2)^5*(48*a*b^2 - 96*a^2*b + 48*a^3) - tan(d/2 + (e*x)/2)^2*(24*a*b^2 -
 120*a^2*b - 120*a^3 + 24*b^3) - tan(d/2 + (e*x)/2)^4*(24*a*b^2 + 120*a^2*
b - 120*a^3 - 24*b^3) + 24*a*b^2 + 24*a^2*b + tan(d/2 + (e*x)/2)^3*(96*a*b
^2 - 160*a^3) - tan(d/2 + (e*x)/2)*(48*a*b^2 + 96*a^2*b + 48*a^3) + 8*a^3
+ 8*b^3)) - (a*atanh((a*(2*a - tan(d/2 + (e*x)/2)*(2*a - 2*b))*(5*a^2 + 3*
b^2))/(2*b*(3*a*b^2 + 5*a^3)))*(5*a^2 + 3*b^2))/(16*b^7*e)
```

3.394. $\int \frac{1}{(2a+2b\cos(d+ex)-2a\sin(d+ex))^4}\,dx$

3.395 $\int (a+b\cos(d+ex)+c\sin(d+ex))^4\,dx$

3.395.1 Optimal result

Integrand size = 20, antiderivative size = 260

$$\int (a+b\cos(d+ex)+c\sin(d+ex))^4\,dx=\frac{1}{8}\left(8a^4+24a^2(b^2+c^2)+3(b^2+c^2)^2\right)x-\frac{5ac(10a^2+11(b^2+c^2))\cos(d+ex)}{24e}+\frac{5ab(10a^2+11(b^2+c^2))\sin(d+ex)}{24e}-\frac{7(ac\cos(d+ex)-ab\sin(d+ex))(a+b\cos(d+ex)+c\sin(d+ex))^2}{12e}-\frac{(c\cos(d+ex)-b\sin(d+ex))(a+b\cos(d+ex)+c\sin(d+ex))^3}{4e}-\frac{(a+b\cos(d+ex)+c\sin(d+ex))\left(c(26a^2+9(b^2+c^2))\cos(d+ex)-b(26a^2+9(b^2+c^2))\sin(d+ex)\right)}{24e}$$

output
```
1/8*(8*a^4+24*a^2*(b^2+c^2)+3*(b^2+c^2)^2)*x-5/24*a*c*(10*a^2+11*b^2+11*c^
2)*cos(e*x+d)/e+5/24*a*b*(10*a^2+11*b^2+11*c^2)*sin(e*x+d)/e-7/12*(a*c*cos
(e*x+d)-a*b*sin(e*x+d))*(a+b*cos(e*x+d)+c*sin(e*x+d))^2/e-1/4*(c*cos(e*x+d
)-b*sin(e*x+d))*(a+b*cos(e*x+d)+c*sin(e*x+d))^3/e-1/24*(a+b*cos(e*x+d)+c*s
in(e*x+d))*(c*(26*a^2+9*b^2+9*c^2)*cos(e*x+d)-b*(26*a^2+9*b^2+9*c^2)*sin(e
*x+d))/e
```

3.395.2 Mathematica [A] (verified)

Time = 1.99 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.91

$$\int (a + b\cos(d + ex) + c\sin(d + ex))^4 \, dx$$
$$= \frac{12\left(8a^4 + 24a^2(b^2 + c^2) + 3(b^2 + c^2)^2\right)(d + ex) - 96ac(4a^2 + 3(b^2 + c^2))\cos(d + ex) - 48bc(6a^2 + b^2 + c}{}$$

input
```
Integrate[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^4,x]
```

output
```
(12*(8*a^4 + 24*a^2*(b^2 + c^2) + 3*(b^2 + c^2)^2)*(d + e*x) - 96*a*c*(4*a
^2 + 3*(b^2 + c^2))*Cos[d + e*x] - 48*b*c*(6*a^2 + b^2 + c^2)*Cos[2*(d + e
*x)] + 32*a*c*(-3*b^2 + c^2)*Cos[3*(d + e*x)] - 12*b*c*(b^2 - c^2)*Cos[4*(
d + e*x)] + 96*a*b*(4*a^2 + 3*(b^2 + c^2))*Sin[d + e*x] + 24*(b^2 - c^2)*(
6*a^2 + b^2 + c^2)*Sin[2*(d + e*x)] + 32*a*b*(b^2 - 3*c^2)*Sin[3*(d + e*x)
] + 3*(b^4 - 6*b^2*c^2 + c^4)*Sin[4*(d + e*x)])/(96*e)
```

3.395.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {3042, 3599, 3042, 3625, 3042, 3625, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + b\cos(d + ex) + c\sin(d + ex))^4 \, dx$$

$$\Big\downarrow \; 3042$$

$$\int (a + b\cos(d + ex) + c\sin(d + ex))^4 dx$$

$$\Big\downarrow \; 3599$$

$$\frac{1}{4}\int (a + b\cos(d + ex) + c\sin(d + ex))^2 \left(4a^2 + 7b\cos(d + ex)a + 7c\sin(d + ex)a + 3(b^2 + c^2)\right) dx - \frac{(c\cos(d + ex) - b\sin(d + ex))(a + b\cos(d + ex) + c\sin(d + ex))^3}{4e}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{4}\int (a+b\cos(d+ex)+c\sin(d+ex))^2 \left(4a^2+7b\cos(d+ex)a+7c\sin(d+ex)a+3\left(b^2+c^2\right)\right)dx-$$
$$\frac{(c\cos(d+ex)-b\sin(d+ex))(a+b\cos(d+ex)+c\sin(d+ex))^3}{4e}$$

$\big\downarrow$ 3625

$$\frac{1}{4}\Bigg(\frac{\int (a+b\cos(d+ex)+c\sin(d+ex))\left(\left(12a^2+23\left(b^2+c^2\right)\right)a^2+b\left(26a^2+9\left(b^2+c^2\right)\right)\cos(d+ex)a+c(26a^2+}{3a}$$
$$\frac{(c\cos(d+ex)-b\sin(d+ex))(a+b\cos(d+ex)+c\sin(d+ex))^3}{4e}$$

$\big\downarrow$ 3042

$$\frac{1}{4}\Bigg(\frac{\int (a+b\cos(d+ex)+c\sin(d+ex))\left(\left(12a^2+23\left(b^2+c^2\right)\right)a^2+b\left(26a^2+9\left(b^2+c^2\right)\right)\cos(d+ex)a+c(26a^2+}{3a}$$
$$\frac{(c\cos(d+ex)-b\sin(d+ex))(a+b\cos(d+ex)+c\sin(d+ex))^3}{4e}$$

$\big\downarrow$ 3625

$$\frac{1}{4}\Bigg(\frac{\frac{\int \left(5b\left(10a^2+11\left(b^2+c^2\right)\right)\cos(d+ex)a^3+5c\left(10a^2+11\left(b^2+c^2\right)\right)\sin(d+ex)a^3+3\left(8a^4+24\left(b^2+c^2\right)a^2+3\left(b^2+c^2\right)^2\right)a^2\right)dx}{2a}-\frac{(a+b\cos(d+ex)+c}{}}{3a}$$
$$\frac{(c\cos(d+ex)-b\sin(d+ex))(a+b\cos(d+ex)+c\sin(d+ex))^3}{4e}$$

$\big\downarrow$ 2009

$$\frac{1}{4}\Bigg(\frac{\frac{3a^2x\left(8a^4+24a^2\left(b^2+c^2\right)+3\left(b^2+c^2\right)^2\right)+\frac{5a^3b\left(10a^2+11\left(b^2+c^2\right)\right)\sin(d+ex)}{e}-\frac{5a^3c\left(10a^2+11\left(b^2+c^2\right)\right)\cos(d+ex)}{e}}{2a}-\frac{(a+b\cos(d+ex)+c\sin(d+ex)}{}}{3a}$$
$$\frac{(c\cos(d+ex)-b\sin(d+ex))(a+b\cos(d+ex)+c\sin(d+ex))^3}{4e}$$

input
```
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^4,x]
```

3.395. $\int (a+b\cos(d+ex)+c\sin(d+ex))^4\,dx$

output
```
-1/4*((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*
x])^3)/e + ((-7*(a*c*Cos[d + e*x] - a*b*Sin[d + e*x])*(a + b*Cos[d + e*x]
+ c*Sin[d + e*x])^2)/(3*e) + (-1/2*((a + b*Cos[d + e*x] + c*Sin[d + e*x])*
(a*c*(26*a^2 + 9*(b^2 + c^2))*Cos[d + e*x] - a*b*(26*a^2 + 9*(b^2 + c^2))*
Sin[d + e*x]))/e + (3*a^2*(8*a^4 + 24*a^2*(b^2 + c^2) + 3*(b^2 + c^2)^2)*x
 - (5*a^3*c*(10*a^2 + 11*(b^2 + c^2))*Cos[d + e*x])/e + (5*a^3*b*(10*a^2 +
 11*(b^2 + c^2))*Sin[d + e*x])/e)/(2*a))/(3*a))/4
```

3.395.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3599
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Simp[1/n   Int[Simp[n*a^2 + (
n - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x
], x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

rule 3625
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*((a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^n/(a*e*(n + 1))), x] + Simp[1/(a*(n + 1
))   Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
 + a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
 + (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

3.395.4 Maple [A] (verified)

Time = 3.20 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.13

method	result
parallelrisch	$\frac{24(6a^2b^2-6a^2c^2+b^4-c^4)\sin(2ex+2d)-288b\left(a^2+\frac{b^2}{6}+\frac{c^2}{6}\right)c\cos(2ex+2d)+3(b^4-6b^2c^2+c^4)\sin(4ex+4d)+32a(-3b^2c+}{}$
parts	$a^4x-\frac{4b^3\left(\frac{c\sin(ex+d)^4}{4}+\frac{\sin(ex+d)^3a}{3}-\frac{\sin(ex+d)^2c}{2}-a\sin(ex+d)\right)}{e}+\frac{6b^2c^2\left(-\frac{\sin(ex+d)\cos(ex+d)^3}{4}+\frac{\cos(ex+d)\sin(ex}{8}\right.}{}$
derivativedivides	$\frac{b^4\left(\frac{\left(\cos(ex+d)^3+\frac{3\cos(ex+d)}{2}\right)\sin(ex+d)}{4}+\frac{3ex}{8}+\frac{3d}{8}\right)+c^4\left(-\frac{\left(\sin(ex+d)^3+\frac{3\sin(ex+d)}{2}\right)\cos(ex+d)}{4}+\frac{3ex}{8}+\frac{3d}{8}\right)+a^4(ex+d)}{}$
default	$\frac{b^4\left(\frac{\left(\cos(ex+d)^3+\frac{3\cos(ex+d)}{2}\right)\sin(ex+d)}{4}+\frac{3ex}{8}+\frac{3d}{8}\right)+c^4\left(-\frac{\left(\sin(ex+d)^3+\frac{3\sin(ex+d)}{2}\right)\cos(ex+d)}{4}+\frac{3ex}{8}+\frac{3d}{8}\right)+a^4(ex+d)}{}$
risch	$a^4x-\frac{c\,b^3\cos(4ex+4d)}{8e}+\frac{c^3b\cos(4ex+4d)}{8e}-\frac{3\sin(4ex+4d)b^2c^2}{16e}+\frac{4a^3b\sin(ex+d)}{e}+\frac{3a\,b^3\sin(ex+d)}{e}+\frac{a\,c^3}{}$
norman	$\frac{(a^4+3a^2b^2+\frac{3}{8}b^4+3a^2c^2+\frac{3}{4}b^2c^2+\frac{3}{8}c^4)x+(a^4+3a^2b^2+\frac{3}{8}b^4+3a^2c^2+\frac{3}{4}b^2c^2+\frac{3}{8}c^4)x\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^8+(4a^4+12a^2b^2+\frac{3}{2}b^4+}{}$

input
```
int((a+b*cos(e*x+d)+c*sin(e*x+d))^4,x,method=_RETURNVERBOSE)
```

output
```
1/96*(24*(6*a^2*b^2-6*a^2*c^2+b^4-c^4)*sin(2*e*x+2*d)-288*b*(a^2+1/6*b^2+1
/6*c^2)*c*cos(2*e*x+2*d)+3*(b^4-6*b^2*c^2+c^4)*sin(4*e*x+4*d)+32*a*(-3*b^2
*c+c^3)*cos(3*e*x+3*d)+12*(-b^3*c+b*c^3)*cos(4*e*x+4*d)+32*b*a*(b^2-3*c^2)
*sin(3*e*x+3*d)-384*(a^2+3/4*b^2+3/4*c^2)*c*a*cos(e*x+d)+384*b*(a^2+3/4*b^
2+3/4*c^2)*a*sin(e*x+d)+36*c^4*e*x+4*(-64*a+9*b)*c^3+288*(a^2+1/4*b^2)*e*x
*c^2+12*(-32*a^3+24*a^2*b-32*a*b^2+5*b^3)*c+96*(a^4+3*a^2*b^2+3/8*b^4)*e*x
)/e
```

3.395.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.98

$$\int (a+b\cos(d+ex)+c\sin(d+ex))^4\,dx =$$
$$-\frac{24\,(b^3c-bc^3)\cos{(ex+d)}^4+32\,(3\,ab^2c-ac^3)\cos{(ex+d)}^3-3\,(8\,a^4+24\,a^2b^2+3\,b^4+3\,c^4+6\,(4\,a^2+}{}$$

input
```
integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^4,x, algorithm="fricas")
```

output
```
-1/24*(24*(b^3*c - b*c^3)*cos(e*x + d)^4 + 32*(3*a*b^2*c - a*c^3)*cos(e*x
+ d)^3 - 3*(8*a^4 + 24*a^2*b^2 + 3*b^4 + 3*c^4 + 6*(4*a^2 + b^2)*c^2)*e*x
+ 48*(3*a^2*b*c + b*c^3)*cos(e*x + d)^2 + 96*(a^3*c + a*c^3)*cos(e*x + d)
- (96*a^3*b + 64*a*b^3 + 96*a*b*c^2 + 6*(b^4 - 6*b^2*c^2 + c^4)*cos(e*x +
d)^3 + 32*(a*b^3 - 3*a*b*c^2)*cos(e*x + d)^2 + 3*(24*a^2*b^2 + 3*b^4 - 5*c
^4 - 6*(4*a^2 - b^2)*c^2)*cos(e*x + d))*sin(e*x + d))/e
```

3.395.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 682 vs. $2(253) = 506$.

Time = 0.29 (sec) , antiderivative size = 682, normalized size of antiderivative = 2.62

$$\int (a + b\cos(d+ex) + c\sin(d+ex))^4\,dx$$
$$= \begin{cases} a^4x + \frac{4a^3b\sin(d+ex)}{e} - \frac{4a^3c\cos(d+ex)}{e} + 3a^2b^2x\sin^2(d+ex) + 3a^2b^2x\cos^2(d+ex) + \frac{3a^2b^2\sin(d+ex)\cos(d+ex)}{e} \\ x(a+b\cos(d)+c\sin(d))^4 \end{cases}$$

input
```
integrate((a+b*cos(e*x+d)+c*sin(e*x+d))**4,x)
```

output
```
Piecewise((a**4*x + 4*a**3*b*sin(d + e*x)/e - 4*a**3*c*cos(d + e*x)/e + 3*
a**2*b**2*x*sin(d + e*x)**2 + 3*a**2*b**2*x*cos(d + e*x)**2 + 3*a**2*b**2*
sin(d + e*x)*cos(d + e*x)/e + 6*a**2*b*c*sin(d + e*x)**2/e + 3*a**2*c**2*x
*sin(d + e*x)**2 + 3*a**2*c**2*x*cos(d + e*x)**2 - 3*a**2*c**2*sin(d + e*x
)*cos(d + e*x)/e + 8*a*b**3*sin(d + e*x)**3/(3*e) + 4*a*b**3*sin(d + e*x)*
cos(d + e*x)**2/e - 4*a*b**2*c*cos(d + e*x)**3/e + 4*a*b*c**2*sin(d + e*x)
**3/e - 4*a*c**3*sin(d + e*x)**2*cos(d + e*x)/e - 8*a*c**3*cos(d + e*x)**3
/(3*e) + 3*b**4*x*sin(d + e*x)**4/8 + 3*b**4*x*sin(d + e*x)**2*cos(d + e*x
)**2/4 + 3*b**4*x*cos(d + e*x)**4/8 + 3*b**4*sin(d + e*x)**3*cos(d + e*x)/
(8*e) + 5*b**4*sin(d + e*x)*cos(d + e*x)**3/(8*e) - b**3*c*cos(d + e*x)**4
/e + 3*b**2*c**2*x*sin(d + e*x)**4/4 + 3*b**2*c**2*x*sin(d + e*x)**2*cos(d
 + e*x)**2/2 + 3*b**2*c**2*x*cos(d + e*x)**4/4 + 3*b**2*c**2*sin(d + e*x)*
*3*cos(d + e*x)/(4*e) - 3*b**2*c**2*sin(d + e*x)*cos(d + e*x)**3/(4*e) + b
*c**3*sin(d + e*x)**4/e + 3*c**4*x*sin(d + e*x)**4/8 + 3*c**4*x*sin(d + e*
x)**2*cos(d + e*x)**2/4 + 3*c**4*x*cos(d + e*x)**4/8 - 5*c**4*sin(d + e*x)
**3*cos(d + e*x)/(8*e) - 3*c**4*sin(d + e*x)*cos(d + e*x)**3/(8*e), Ne(e,
0)), (x*(a + b*cos(d) + c*sin(d))**4, True))
```

3.395.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.27

$$\int (a+b\cos(d+ex)+c\sin(d+ex))^4\,dx = -\frac{b^3c\cos(ex+d)^4}{e}+\frac{bc^3\sin(ex+d)^4}{e}+a^4x+\frac{(12\,ex+12\,d+\sin(4\,ex+4\,d)+8\,\sin(2\,ex+2\,d))b^4}{32\,e}+\frac{3\,(4\,ex+4\,d-\sin(4\,ex+4\,d))b^2c^2}{16\,e}+\frac{(12\,ex+12\,d+\sin(4\,ex+4\,d)-8\,\sin(2\,ex+2\,d))c^4}{32\,e}-4\,a^3\left(\frac{c\cos(ex+d)}{e}-\frac{b\sin(ex+d)}{e}\right)-\frac{3}{2}\left(\frac{4\,bc\cos(ex+d)^2}{e}-\frac{(2\,ex+2\,d+\sin(2\,ex+2\,d))b^2}{e}-\frac{(2\,ex+2\,d-\sin(2\,ex+2\,d))c^2}{e}\right)a^2-\frac{4}{3}\left(\frac{3\,b^2c\cos(ex+d)^3}{e}-\frac{3\,bc^2\sin(ex+d)^3}{e}+\frac{(\sin(ex+d)^3-3\,\sin(ex+d))b^3}{e}-\frac{(\cos(ex+d)^3-3\,\mathrm{c}}{e}\right.$$

input
```
integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^4,x, algorithm="maxima")
```

output
```
-b^3*c*cos(e*x + d)^4/e + b*c^3*sin(e*x + d)^4/e + a^4*x + 1/32*(12*e*x +
12*d + sin(4*e*x + 4*d) + 8*sin(2*e*x + 2*d))*b^4/e + 3/16*(4*e*x + 4*d -
sin(4*e*x + 4*d))*b^2*c^2/e + 1/32*(12*e*x + 12*d + sin(4*e*x + 4*d) - 8*s
in(2*e*x + 2*d))*c^4/e - 4*a^3*(c*cos(e*x + d)/e - b*sin(e*x + d)/e) - 3/2
*(4*b*c*cos(e*x + d)^2/e - (2*e*x + 2*d + sin(2*e*x + 2*d))*b^2/e - (2*e*x
 + 2*d - sin(2*e*x + 2*d))*c^2/e)*a^2 - 4/3*(3*b^2*c*cos(e*x + d)^3/e - 3*
b*c^2*sin(e*x + d)^3/e + (sin(e*x + d)^3 - 3*sin(e*x + d))*b^3/e - (cos(e*
x + d)^3 - 3*cos(e*x + d))*c^3/e)*a
```

3.395.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.10

$$\int (a+b\cos(d+ex)+c\sin(d+ex))^4\,dx$$
$$= \frac{1}{8}\left(8a^4+24a^2b^2+3b^4+24a^2c^2+6b^2c^2+3c^4\right)x - \frac{(b^3c-bc^3)\cos(4ex+4d)}{8e} - \frac{(3ab^2c-ac^3)\cos(3ex+3d)}{3e} - \frac{(6a^2bc+b^3c+bc^3)\cos(2ex+2d)}{2e} - \frac{(4a^3c+3ab^2c+3ac^3)\cos(ex+d)}{e} + \frac{(b^4-6b^2c^2+c^4)\sin(4ex+4d)}{32e} + \frac{(ab^3-3abc^2)\sin(3ex+3d)}{3e} + \frac{(6a^2b^2+b^4-6a^2c^2-c^4)\sin(2ex+2d)}{4e} + \frac{(4a^3b+3ab^3+3abc^2)\sin(ex+d)}{e}$$

input
```
integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^4,x, algorithm="giac")
```

output
```
1/8*(8*a^4 + 24*a^2*b^2 + 3*b^4 + 24*a^2*c^2 + 6*b^2*c^2 + 3*c^4)*x - 1/8*
(b^3*c - b*c^3)*cos(4*e*x + 4*d)/e - 1/3*(3*a*b^2*c - a*c^3)*cos(3*e*x + 3
*d)/e - 1/2*(6*a^2*b*c + b^3*c + b*c^3)*cos(2*e*x + 2*d)/e - (4*a^3*c + 3*
a*b^2*c + 3*a*c^3)*cos(e*x + d)/e + 1/32*(b^4 - 6*b^2*c^2 + c^4)*sin(4*e*x
 + 4*d)/e + 1/3*(a*b^3 - 3*a*b*c^2)*sin(3*e*x + 3*d)/e + 1/4*(6*a^2*b^2 +
b^4 - 6*a^2*c^2 - c^4)*sin(2*e*x + 2*d)/e + (4*a^3*b + 3*a*b^3 + 3*a*b*c^2
)*sin(e*x + d)/e
```

3.395.9 Mupad [B] (verification not implemented)

Time = 28.62 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.45

$$\int (a+b\cos(d+ex)+c\sin(d+ex))^4\,dx$$
$$= \frac{6b^4\sin(2d+2ex)+\frac{3b^4\sin(4d+4ex)}{4}-6c^4\sin(2d+2ex)+\frac{3c^4\sin(4d+4ex)}{4}+8ac^3\cos(3d+3ex)-12}{}$$

input
```
int((a + b*cos(d + e*x) + c*sin(d + e*x))^4,x)
```

output
```
(6*b^4*sin(2*d + 2*e*x) + (3*b^4*sin(4*d + 4*e*x))/4 - 6*c^4*sin(2*d + 2*e
*x) + (3*c^4*sin(4*d + 4*e*x))/4 + 8*a*c^3*cos(3*d + 3*e*x) - 12*b*c^3*cos
(2*d + 2*e*x) - 12*b^3*c*cos(2*d + 2*e*x) + 3*b*c^3*cos(4*d + 4*e*x) - 3*b
^3*c*cos(4*d + 4*e*x) + 8*a*b^3*sin(3*d + 3*e*x) + 36*a^2*b^2*sin(2*d + 2*
e*x) - 36*a^2*c^2*sin(2*d + 2*e*x) - (9*b^2*c^2*sin(4*d + 4*e*x))/2 - 72*a
*c^3*cos(d + e*x) - 96*a^3*c*cos(d + e*x) + 72*a*b^3*sin(d + e*x) + 96*a^3
*b*sin(d + e*x) + 24*a^4*e*x + 9*b^4*e*x + 9*c^4*e*x - 72*a*b^2*c*cos(d +
e*x) + 72*a*b*c^2*sin(d + e*x) - 72*a^2*b*c*cos(2*d + 2*e*x) - 24*a*b^2*c*
cos(3*d + 3*e*x) - 24*a*b*c^2*sin(3*d + 3*e*x) + 72*a^2*b^2*e*x + 72*a^2*c
^2*e*x + 18*b^2*c^2*e*x)/(24*e)
```

3.395. $\int (a + b\cos(d + ex) + c\sin(d + ex))^4 \, dx$

3.396 $\int (a+b\cos(d+ex)+c\sin(d+ex))^3\,dx$

3.396.1 Optimal result

Integrand size = 20, antiderivative size = 170

$$\begin{aligned}\int (a+b\cos(d+ex)+c\sin(d+ex))^3\,dx &= \frac{1}{2}a\left(2a^2+3\left(b^2+c^2\right)\right)x-\frac{c(11a^2+4(b^2+c^2))\cos(d+ex)}{6e}\\ &\quad+\frac{b(11a^2+4(b^2+c^2))\sin(d+ex)}{6e}\\ &\quad-\frac{5(ac\cos(d+ex)-ab\sin(d+ex))(a+b\cos(d+ex)+c\sin(d+ex))}{6e}\\ &\quad-\frac{(c\cos(d+ex)-b\sin(d+ex))(a+b\cos(d+ex)+c\sin(d+ex))^2}{3e}\end{aligned}$$

output
```
1/2*a*(2*a^2+3*b^2+3*c^2)*x-1/6*c*(11*a^2+4*b^2+4*c^2)*cos(e*x+d)/e+1/6*b*
(11*a^2+4*b^2+4*c^2)*sin(e*x+d)/e-5/6*(a*c*cos(e*x+d)-a*b*sin(e*x+d))*(a+b
*cos(e*x+d)+c*sin(e*x+d))/e-1/3*(c*cos(e*x+d)-b*sin(e*x+d))*(a+b*cos(e*x+d
)+c*sin(e*x+d))^2/e
```

3.396.2 Mathematica [A] (verified)

Time = 1.37 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.85

$$\int (a+b\cos(d+ex)+c\sin(d+ex))^3\,dx$$
$$=\frac{6a(2a^2+3(b^2+c^2))\,(d+ex)-9c(4a^2+b^2+c^2)\cos(d+ex)-18abc\cos(2(d+ex))+c(-3b^2+c^2)\cos(}{12e}$$

input
```
Integrate[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^3,x]
```

output
```
(6*a*(2*a^2 + 3*(b^2 + c^2))*(d + e*x) - 9*c*(4*a^2 + b^2 + c^2)*Cos[d + e
*x] - 18*a*b*c*Cos[2*(d + e*x)] + c*(-3*b^2 + c^2)*Cos[3*(d + e*x)] + 9*b*
(4*a^2 + b^2 + c^2)*Sin[d + e*x] + 9*a*(b^2 - c^2)*Sin[2*(d + e*x)] + b*(b
^2 - 3*c^2)*Sin[3*(d + e*x)])/(12*e)
```

3.396.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3599, 3042, 3625, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cos(d+ex)+c\sin(d+ex))^3\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int (a+b\cos(d+ex)+c\sin(d+ex))^3dx$$
$$\Big\downarrow \text{3599}$$
$$\frac{1}{3}\int (a+b\cos(d+ex)+c\sin(d+ex))\left(3a^2+5b\cos(d+ex)a+5c\sin(d+ex)a+2\left(b^2+c^2\right)\right)dx-\frac{(c\cos(d+ex)-b\sin(d+ex))(a+b\cos(d+ex)+c\sin(d+ex))^2}{3e}$$
$$\Big\downarrow \text{3042}$$
$$\frac{1}{3}\int (a+b\cos(d+ex)+c\sin(d+ex))\left(3a^2+5b\cos(d+ex)a+5c\sin(d+ex)a+2\left(b^2+c^2\right)\right)dx-\frac{(c\cos(d+ex)-b\sin(d+ex))(a+b\cos(d+ex)+c\sin(d+ex))^2}{3e}$$

$$\Big\downarrow \text{3625}$$

$$\frac{1}{3}\left(\frac{\int \left(3\left(2a^2+3\left(b^2+c^2\right)\right)a^2+b\left(11a^2+4\left(b^2+c^2\right)\right)\cos(d+ex)a+c\left(11a^2+4\left(b^2+c^2\right)\right)\sin(d+ex)a\right)dx}{2a}-\frac{5(a\right.$$

$$\frac{(c\cos(d+ex)-b\sin(d+ex))(a+b\cos(d+ex)+c\sin(d+ex))^2}{3e}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{\frac{ab\left(11a^2+4\left(b^2+c^2\right)\right)\sin(d+ex)}{e}-\frac{ac\left(11a^2+4\left(b^2+c^2\right)\right)\cos(d+ex)}{e}+3a^2x\left(2a^2+3\left(b^2+c^2\right)\right)}{2a}-\frac{5(ac\cos(d+ex)-ab\sin(d\right.$$

$$\frac{(c\cos(d+ex)-b\sin(d+ex))(a+b\cos(d+ex)+c\sin(d+ex))^2}{3e}$$

input
```
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^3,x]
```

output
```
-1/3*((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*
x])^2)/e + ((-5*(a*c*Cos[d + e*x] - a*b*Sin[d + e*x])*(a + b*Cos[d + e*x]
+ c*Sin[d + e*x]))/(2*e) + (3*a^2*(2*a^2 + 3*(b^2 + c^2))*x - (a*c*(11*a^2
 + 4*(b^2 + c^2))*Cos[d + e*x])/e + (a*b*(11*a^2 + 4*(b^2 + c^2))*Sin[d +
e*x])/e)/(2*a))/3
```

3.396.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3599
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Simp[1/n   Int[Simp[n*a^2 + (
n - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x
], x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

rule 3625
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*((a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^n/(a*e*(n + 1))), x] + Simp[1/(a*(n + 1
))   Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
 + a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
 + (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

3.396.4 Maple [A] (verified)

Time = 1.98 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.99

method	result
parts	$a^{3} x +\frac{-b^{2} c \cos \left(e x +d \right)^{3}+3 a \,b^{2} \left(\frac{\cos \left(e x +d \right) \sin \left(e x +d \right)}{2}+\frac{e x}{2}+\frac{d}{2}\right)}{e}+\frac{b \left(a +c \sin \left(e x +d \right)\right)^{3}}{e c}+\frac{b^{3} \left(2+\cos \left(e x +d \right)^{2}\right) \sin \left(e x +\right.}{3 e}$
derivativedivides	$\frac{a^{3} \left(e x +d \right)+3 \sin \left(e x +d \right) a^{2} b -3 a^{2} c \cos \left(e x +d \right)+3 a \,b^{2} \left(\frac{\cos \left(e x +d \right) \sin \left(e x +d \right)}{2}+\frac{e x}{2}+\frac{d}{2}\right)-3 a b c \cos \left(e x +d \right)^{2}+3 a \,c^{2} \left(-\frac{\cos \left(e x +d \right)}{2}\right.}{e}$
default	$\frac{a^{3} \left(e x +d \right)+3 \sin \left(e x +d \right) a^{2} b -3 a^{2} c \cos \left(e x +d \right)+3 a \,b^{2} \left(\frac{\cos \left(e x +d \right) \sin \left(e x +d \right)}{2}+\frac{e x}{2}+\frac{d}{2}\right)-3 a b c \cos \left(e x +d \right)^{2}+3 a \,c^{2} \left(-\frac{\cos \left(e x +d \right)}{2}\right.}{e}$
parallelrisch	$\frac{\left(-3 b^{2} c +c^{3}\right) \cos \left(3 e x +3 d \right)+9 a \left(b^{2}-c^{2}\right) \sin \left(2 e x +2 d \right)+\left(b^{3}-3 c^{2} b \right) \sin \left(3 e x +3 d \right)-18 a c b \cos \left(2 e x +2 d \right)+9 \left(-c^{3}+\left(-4 a^{2}-b^{2}\right)\right.}{12 e}$
risch	$a^{3} x +\frac{3 a \,b^{2} x}{2}+\frac{3 a \,c^{2} x}{2}-\frac{3 c \cos \left(e x +d \right) a^{2}}{e}-\frac{3 c \cos \left(e x +d \right) b^{2}}{4 e}-\frac{3 c^{3} \cos \left(e x +d \right)}{4 e}+\frac{3 b \sin \left(e x +d \right) a^{2}}{e}+\frac{3 b^{3} \sin \left(e\right.}{4 e}$
norman	$\frac{\left(a^{3}+\frac{3}{2} a \,b^{2}+\frac{3}{2} a \,c^{2}\right) x +\left(a^{3}+\frac{3}{2} a \,b^{2}+\frac{3}{2} a \,c^{2}\right) x \tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{6}+\left(3 a^{3}+\frac{9}{2} a \,b^{2}+\frac{9}{2} a \,c^{2}\right) x \tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{2}+\left(3 a^{3}+\frac{9}{2} a \,b^{2}+\frac{9}{2} a \,c^{2}\right) x}{}$

input
```
int((a+b*cos(e*x+d)+c*sin(e*x+d))^3,x,method=_RETURNVERBOSE)
```

output
```
a^3*x+3/e*(-1/3*b^2*c*cos(e*x+d)^3+a*b^2*(1/2*cos(e*x+d)*sin(e*x+d)+1/2*e*
x+1/2*d))+1/e*b*(a+c*sin(e*x+d))^3/c+1/3*b^3/e*(2+cos(e*x+d)^2)*sin(e*x+d)
-1/3*c^3/e*(2+sin(e*x+d)^2)*cos(e*x+d)-3*c/e*cos(e*x+d)*a^2+3*a*c^2/e*(-1/
2*cos(e*x+d)*sin(e*x+d)+1/2*e*x+1/2*d)
```

3.396.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.86

$$\int (a+b\cos(d+ex)+c\sin(d+ex))^3\,dx =$$
$$-\frac{18\,abc\cos{(ex+d)}^2+2\,(3\,b^2c-c^3)\cos{(ex+d)}^3-3\,(2\,a^3+3\,ab^2+3\,ac^2)ex+6\,(3\,a^2c+c^3)\cos{(ex+}}{6\,e}$$

input
```
integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^3,x, algorithm="fricas")
```

output
```
-1/6*(18*a*b*c*cos(e*x + d)^2 + 2*(3*b^2*c - c^3)*cos(e*x + d)^3 - 3*(2*a^
3 + 3*a*b^2 + 3*a*c^2)*e*x + 6*(3*a^2*c + c^3)*cos(e*x + d) - (18*a^2*b +
4*b^3 + 6*b*c^2 + 2*(b^3 - 3*b*c^2)*cos(e*x + d)^2 + 9*(a*b^2 - a*c^2)*cos
(e*x + d))*sin(e*x + d))/e
```

3.396.6 Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.73

$$\int (a+b\cos(d+ex)+c\sin(d+ex))^3\,dx$$
$$=\begin{cases} a^3x+\frac{3a^2b\sin{(d+ex)}}{e}-\frac{3a^2c\cos{(d+ex)}}{e}+\frac{3ab^2x\sin^2{(d+ex)}}{2}+\frac{3ab^2x\cos^2{(d+ex)}}{2}+\frac{3ab^2\sin{(d+ex)}\cos{(d+ex)}}{2e}+\frac{3abc\sin^2{(d+ex}}{e} \\ x(a+b\cos{(d)}+c\sin{(d)})^3 \end{cases}$$

input
```
integrate((a+b*cos(e*x+d)+c*sin(e*x+d))**3,x)
```

output
```
Piecewise((a**3*x + 3*a**2*b*sin(d + e*x)/e - 3*a**2*c*cos(d + e*x)/e + 3*
a*b**2*x*sin(d + e*x)**2/2 + 3*a*b**2*x*cos(d + e*x)**2/2 + 3*a*b**2*sin(d
 + e*x)*cos(d + e*x)/(2*e) + 3*a*b*c*sin(d + e*x)**2/e + 3*a*c**2*x*sin(d
+ e*x)**2/2 + 3*a*c**2*x*cos(d + e*x)**2/2 - 3*a*c**2*sin(d + e*x)*cos(d +
 e*x)/(2*e) + 2*b**3*sin(d + e*x)**3/(3*e) + b**3*sin(d + e*x)*cos(d + e*x
)**2/e - b**2*c*cos(d + e*x)**3/e + b*c**2*sin(d + e*x)**3/e - c**3*sin(d
+ e*x)**2*cos(d + e*x)/e - 2*c**3*cos(d + e*x)**3/(3*e), Ne(e, 0)), (x*(a
+ b*cos(d) + c*sin(d))**3, True))
```

3.396.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.11

$$\int (a+b\cos(d+ex)+c\sin(d+ex))^3\,dx$$
$$= -\frac{b^2c\cos{(ex+d)}^3}{e}+\frac{bc^2\sin{(ex+d)}^3}{e}+a^3x-\frac{\left(\sin{(ex+d)}^3-3\,\sin{(ex+d)}\right)b^3}{3\,e}+\frac{\left(\cos{(ex+d)}^3-3\,\cos{(ex+d)}\right)c^3}{3\,e}-3\,a^2\left(\frac{c\cos{(ex+d)}}{e}-\frac{b\sin{(ex+d)}}{e}\right)-\frac{3}{4}\left(\frac{4\,bc\cos{(ex+d)}^2}{e}-\frac{(2\,ex+2\,d+\sin{(2\,ex+2\,d)})b^2}{e}-\frac{(2\,ex+2\,d-\sin{(2\,ex+2\,d)})c^2}{e}\right)a$$

input
```
integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^3,x, algorithm="maxima")
```

output
```
-b^2*c*cos(e*x + d)^3/e + b*c^2*sin(e*x + d)^3/e + a^3*x - 1/3*(sin(e*x +
d)^3 - 3*sin(e*x + d))*b^3/e + 1/3*(cos(e*x + d)^3 - 3*cos(e*x + d))*c^3/e
 - 3*a^2*(c*cos(e*x + d)/e - b*sin(e*x + d)/e) - 3/4*(4*b*c*cos(e*x + d)^2
/e - (2*e*x + 2*d + sin(2*e*x + 2*d))*b^2/e - (2*e*x + 2*d - sin(2*e*x + 2
*d))*c^2/e)*a
```

3.396.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.98

$$\int (a+b\cos(d+ex)+c\sin(d+ex))^3\,dx$$
$$= -\frac{3\,abc\cos{(2\,ex+2\,d)}}{2\,e}+\frac{1}{2}\left(2\,a^3+3\,ab^2+3\,ac^2\right)x-\frac{\left(3\,b^2c-c^3\right)\cos{(3\,ex+3\,d)}}{12\,e}-\frac{3\left(4\,a^2c+b^2c+c^3\right)\cos{(ex+d)}}{4\,e}+\frac{\left(b^3-3\,bc^2\right)\sin{(3\,ex+3\,d)}}{12\,e}+\frac{3\left(ab^2-ac^2\right)\sin{(2\,ex+2\,d)}}{4\,e}+\frac{3\left(4\,a^2b+b^3+bc^2\right)\sin{(ex+d)}}{4\,e}$$

input
```
integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^3,x, algorithm="giac")
```

output
```
-3/2*a*b*c*cos(2*e*x + 2*d)/e + 1/2*(2*a^3 + 3*a*b^2 + 3*a*c^2)*x - 1/12*(
3*b^2*c - c^3)*cos(3*e*x + 3*d)/e - 3/4*(4*a^2*c + b^2*c + c^3)*cos(e*x +
d)/e + 1/12*(b^3 - 3*b*c^2)*sin(3*e*x + 3*d)/e + 3/4*(a*b^2 - a*c^2)*sin(2
*e*x + 2*d)/e + 3/4*(4*a^2*b + b^3 + b*c^2)*sin(e*x + d)/e
```

3.396.9 Mupad [B] (verification not implemented)

Time = 29.07 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.96

$$\int (a + b\cos(d + ex) + c\sin(d + ex))^3 \, dx$$

$$= \frac{a \operatorname{atan}\left(\frac{a \tan\left(\frac{d}{2}+\frac{e x}{2}\right) (2 a^2+3 b^2+3 c^2)}{2 a^3+3 a b^2+3 a c^2}\right) (2 a^2 + 3 b^2 + 3 c^2)}{e} - \frac{a \left(\operatorname{atan}\left(\tan\left(\frac{d}{2} + \frac{e x}{2}\right)\right) - \frac{e x}{2}\right) (2 a^2 + 3 b^2 + 3 c^2)}{e} - \frac{\tan\left(\frac{d}{2} + \frac{e x}{2}\right)^2 (12 a^2 c - 12 b a c + 4 c^3) - \tan\left(\frac{d}{2} + \frac{e x}{2}\right)^3 \left(12 a^2 b + \frac{4 b^3}{3} + 8 b c^2\right) - \tan\left(\frac{d}{2} + \frac{e x}{2}\right) (6 a^2 b +}{e \left(\tan\left(\frac{d}{2} + \frac{e x}{2}\right.\right.}$$

input
```
int((a + b*cos(d + e*x) + c*sin(d + e*x))^3,x)
```

output
```
(a*atan((a*tan(d/2 + (e*x)/2)*(2*a^2 + 3*b^2 + 3*c^2))/(3*a*b^2 + 3*a*c^2
+ 2*a^3))*(2*a^2 + 3*b^2 + 3*c^2))/e - (a*(atan(tan(d/2 + (e*x)/2)) - (e*x
)/2)*(2*a^2 + 3*b^2 + 3*c^2))/e - (tan(d/2 + (e*x)/2)^2*(12*a^2*c + 4*c^3
- 12*a*b*c) - tan(d/2 + (e*x)/2)^3*(12*a^2*b + 8*b*c^2 + (4*b^3)/3) - tan(
d/2 + (e*x)/2)*(3*a*b^2 + 6*a^2*b - 3*a*c^2 + 2*b^3) + tan(d/2 + (e*x)/2)^
4*(6*a^2*c + 6*b^2*c - 12*a*b*c) + 6*a^2*c + 2*b^2*c - tan(d/2 + (e*x)/2)^
5*(6*a^2*b - 3*a*b^2 + 3*a*c^2 + 2*b^3) + (4*c^3)/3)/(e*(3*tan(d/2 + (e*x)
/2)^2 + 3*tan(d/2 + (e*x)/2)^4 + tan(d/2 + (e*x)/2)^6 + 1))
```

3.397 $\int (a+b\cos(d+ex)+c\sin(d+ex))^2\,dx$

3.397.1 Optimal result

Integrand size = 20, antiderivative size = 91

$$\int (a+b\cos(d+ex)+c\sin(d+ex))^2\,dx = \frac{1}{2}\left(2a^2+b^2+c^2\right)x-\frac{3ac\cos(d+ex)}{2e}+\frac{3ab\sin(d+ex)}{2e} - \frac{(c\cos(d+ex)-b\sin(d+ex))(a+b\cos(d+ex)+c\sin(d+ex))}{2e}$$

output
```
1/2*(2*a^2+b^2+c^2)*x-3/2*a*c*cos(e*x+d)/e+3/2*a*b*sin(e*x+d)/e-1/2*(c*cos
(e*x+d)-b*sin(e*x+d))*(a+b*cos(e*x+d)+c*sin(e*x+d))/e
```

3.397.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.85

$$\int (a+b\cos(d+ex)+c\sin(d+ex))^2\,dx = \frac{2(2a^2+b^2+c^2)(d+ex)-8ac\cos(d+ex)-2bc\cos(2(d+ex))+8ab\sin(d+ex)+(b^2-c^2)\sin(2(d+e}{4e}$$

input
```
Integrate[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^2,x]
```

output
```
(2*(2*a^2 + b^2 + c^2)*(d + e*x) - 8*a*c*Cos[d + e*x] - 2*b*c*Cos[2*(d + e
*x)] + 8*a*b*Sin[d + e*x] + (b^2 - c^2)*Sin[2*(d + e*x)])/(4*e)
```

3.397.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {3042, 3599, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cos(d+ex)+c\sin(d+ex))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\cos(d+ex)+c\sin(d+ex))^2dx$$

$$\Big\downarrow \text{3599}$$

$$\frac{1}{2}\int \left(2a^2+3b\cos(d+ex)a+3c\sin(d+ex)a+b^2+c^2\right)dx-\frac{(c\cos(d+ex)-b\sin(d+ex))(a+b\cos(d+ex)+c\sin(d+ex))}{2e}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(x\left(2a^2+b^2+c^2\right)+\frac{3ab\sin(d+ex)}{e}-\frac{3ac\cos(d+ex)}{e}\right)-\frac{(c\cos(d+ex)-b\sin(d+ex))(a+b\cos(d+ex)+c\sin(d+ex))}{2e}$$

input
```
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^2,x]
```

output
```
-1/2*((c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*
x]))/e + ((2*a^2 + b^2 + c^2)*x - (3*a*c*Cos[d + e*x])/e + (3*a*b*Sin[d +
e*x])/e)/2
```

3.397.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3599
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Simp[1/n   Int[Simp[n*a^2 + (
n - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x
], x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

3.397.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.97

method	result
parallelrisch	$\frac{(b^2-c^2)\sin(2ex+2d)-2bc\cos(2ex+2d)-8ac\cos(ex+d)+8ab\sin(ex+d)+2c^2ex+(8a+2b)c+4\left(a^2+\frac{b^2}{2}\right)ex}{4e}$
risch	$a^2x+\frac{x b^2}{2}+\frac{x c^2}{2}-\frac{2ac\cos(ex+d)}{e}+\frac{2ab\sin(ex+d)}{e}-\frac{bc\cos(2ex+2d)}{2e}+\frac{\sin(2ex+2d)b^2}{4e}-\frac{\sin(2ex+2d)c^2}{4e}$
derivativedivides	$\frac{a^2(ex+d)+2ab\sin(ex+d)-2ac\cos(ex+d)+b^2\left(\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)-cb\cos(ex+d)^2+c^2\left(-\frac{\cos(ex+d)\sin(ex+d)}{2}+\right.}{e}$
default	$\frac{a^2(ex+d)+2ab\sin(ex+d)-2ac\cos(ex+d)+b^2\left(\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)-cb\cos(ex+d)^2+c^2\left(-\frac{\cos(ex+d)\sin(ex+d)}{2}+\right.}{e}$
parts	$a^2x+\frac{2b\left(\frac{\sin(ex+d)^2c}{2}+a\sin(ex+d)\right)}{e}+\frac{b^2\left(\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)}{e}+\frac{c^2\left(-\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)}{e}-$
norman	$\frac{\left(a^2+\frac{b^2}{2}+\frac{c^2}{2}\right)x+\left(a^2+\frac{b^2}{2}+\frac{c^2}{2}\right)x\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^4+\frac{4ac\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^4}{e}+\frac{\left(4ab-b^2+c^2\right)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^3}{e}+\frac{\left(4ab+b^2-c^2\right)\tan\left(\frac{ex}{2}+\frac{d}{2}\right.}{e}}{\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2\right)^2}$

input
```
int((a+b*cos(e*x+d)+c*sin(e*x+d))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*((b^2-c^2)*sin(2*e*x+2*d)-2*b*c*cos(2*e*x+2*d)-8*a*c*cos(e*x+d)+8*a*b*
sin(e*x+d)+2*c^2*e*x+(8*a+2*b)*c+4*(a^2+1/2*b^2)*e*x)/e
```

3.397.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.80

$$\int (a+b\cos(d+ex)+c\sin(d+ex))^2\,dx = -\frac{2\,bc\cos{(ex+d)}^2-(2\,a^2+b^2+c^2)ex+4\,ac\cos{(ex+d)}-(4\,ab+(b^2-c^2)\cos{(ex+d)})\sin{(ex+d)}}{2\,e}$$

input
```
integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^2,x, algorithm="fricas")
```

output
```
-1/2*(2*b*c*cos(e*x + d)^2 - (2*a^2 + b^2 + c^2)*e*x + 4*a*c*cos(e*x + d)
- (4*a*b + (b^2 - c^2)*cos(e*x + d))*sin(e*x + d))/e
```

3.397.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.78

$$\int (a + b\cos(d + ex) + c\sin(d + ex))^2\,dx$$
$$= \begin{cases} a^2x + \frac{2ab\sin(d+ex)}{e} - \frac{2ac\cos(d+ex)}{e} + \frac{b^2x\sin^2(d+ex)}{2} + \frac{b^2x\cos^2(d+ex)}{2} + \frac{b^2\sin(d+ex)\cos(d+ex)}{2e} + \frac{bc\sin^2(d+ex)}{e} + \frac{c^2x\sin}{} \\ x(a + b\cos(d) + c\sin(d))^2 \end{cases}$$

input
```
integrate((a+b*cos(e*x+d)+c*sin(e*x+d))**2,x)
```

output
```
Piecewise((a**2*x + 2*a*b*sin(d + e*x)/e - 2*a*c*cos(d + e*x)/e + b**2*x*s
in(d + e*x)**2/2 + b**2*x*cos(d + e*x)**2/2 + b**2*sin(d + e*x)*cos(d + e*
x)/(2*e) + b*c*sin(d + e*x)**2/e + c**2*x*sin(d + e*x)**2/2 + c**2*x*cos(d
 + e*x)**2/2 - c**2*sin(d + e*x)*cos(d + e*x)/(2*e), Ne(e, 0)), (x*(a + b*
cos(d) + c*sin(d))**2, True))
```

3.397.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.10

$$\int (a + b\cos(d + ex) + c\sin(d + ex))^2\,dx = a^2x - \frac{bc\cos(ex + d)^2}{e} + \frac{(2\,ex + 2\,d + \sin(2\,ex + 2\,d))b^2}{4\,e} + \frac{(2\,ex + 2\,d - \sin(2\,ex + 2\,d))c^2}{4\,e} - 2\,a\left(\frac{c\cos(ex + d)}{e} - \frac{b\sin(ex + d)}{e}\right)$$

input
```
integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^2,x, algorithm="maxima")
```

output
```
a^2*x - b*c*cos(e*x + d)^2/e + 1/4*(2*e*x + 2*d + sin(2*e*x + 2*d))*b^2/e
+ 1/4*(2*e*x + 2*d - sin(2*e*x + 2*d))*c^2/e - 2*a*(c*cos(e*x + d)/e - b*s
in(e*x + d)/e)
```

3.397.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.89

$$\int (a+b\cos(d+ex)+c\sin(d+ex))^2\,dx = \frac{1}{2}\left(2\,a^2+b^2+c^2\right)x - \frac{bc\cos\left(2\,ex+2\,d\right)}{2\,e} - \frac{2\,ac\cos\left(ex+d\right)}{e} + \frac{2\,ab\sin\left(ex+d\right)}{e} + \frac{\left(b^2-c^2\right)\sin\left(2\,ex+2\,d\right)}{4\,e}$$

input
```
integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^2,x, algorithm="giac")
```

output
```
1/2*(2*a^2 + b^2 + c^2)*x - 1/2*b*c*cos(2*e*x + 2*d)/e - 2*a*c*cos(e*x + d
)/e + 2*a*b*sin(e*x + d)/e + 1/4*(b^2 - c^2)*sin(2*e*x + 2*d)/e
```

3.397.9 Mupad [B] (verification not implemented)

Time = 29.42 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.37

$$\int (a+b\cos(d+ex)+c\sin(d+ex))^2\,dx = \frac{x\left(2\,a^2+b^2+c^2\right)}{2} - \frac{\left(b^2-4\,a\,b-c^2\right)\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^3+\left(4\,a\,c-4\,b\,c\right)\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2+\left(-b^2-4\,a\,b+c^2\right)\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)+4\,a\,c}{e\,\left(\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^4+2\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2+1\right)}$$

input
```
int((a + b*cos(d + e*x) + c*sin(d + e*x))^2,x)
```

output
```
(x*(2*a^2 + b^2 + c^2))/2 - (4*a*c + tan(d/2 + (e*x)/2)^2*(4*a*c - 4*b*c)
- tan(d/2 + (e*x)/2)*(4*a*b + b^2 - c^2) - tan(d/2 + (e*x)/2)^3*(4*a*b - b
^2 + c^2))/(e*(2*tan(d/2 + (e*x)/2)^2 + tan(d/2 + (e*x)/2)^4 + 1))
```

3.398 $\int (a+b\cos(d+ex)+c\sin(d+ex))\,dx$

3.398.1 Optimal result

Integrand size = 18, antiderivative size = 27

$$\int (a+b\cos(d+ex)+c\sin(d+ex))\,dx=ax-\frac{c\cos(d+ex)}{e}+\frac{b\sin(d+ex)}{e}$$

output
```
a*x-c*cos(e*x+d)/e+b*sin(e*x+d)/e
```

3.398.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.81

$$\int (a+b\cos(d+ex)+c\sin(d+ex))\,dx=ax-\frac{c\cos(d)\cos(ex)}{e}+\frac{b\cos(ex)\sin(d)}{e}+\frac{b\cos(d)\sin(ex)}{e}+\frac{c\sin(d)\sin(ex)}{e}$$

input
```
Integrate[a + b*Cos[d + e*x] + c*Sin[d + e*x],x]
```

output
```
a*x - (c*Cos[d]*Cos[e*x])/e + (b*Cos[e*x]*Sin[d])/e + (b*Cos[d]*Sin[e*x])/
e + (c*Sin[d]*Sin[e*x])/e
```

3.398.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + b\cos(d + ex) + c\sin(d + ex))\, dx$$

$$\Big\downarrow \; 2009$$

$$ax + \frac{b\sin(d + ex)}{e} - \frac{c\cos(d + ex)}{e}$$

input
```
Int[a + b*Cos[d + e*x] + c*Sin[d + e*x],x]
```

output
```
a*x - (c*Cos[d + e*x])/e + (b*Sin[d + e*x])/e
```

3.398.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.398.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

method	result	size
default	$ax - \frac{c\cos(ex+d)}{e} + \frac{b\sin(ex+d)}{e}$	28
risch	$ax - \frac{c\cos(ex+d)}{e} + \frac{b\sin(ex+d)}{e}$	28
parallelrisch	$\frac{c-c\cos(ex+d)+b\sin(ex+d)}{e} + ax$	28
parts	$ax - \frac{c\cos(ex+d)}{e} + \frac{b\sin(ex+d)}{e}$	28
derivativedivides	$\frac{(ex+d)a+b\sin(ex+d)-c\cos(ex+d)}{e}$	30
norman	$\frac{ax+ax\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2+\frac{2c\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}{e}+\frac{2b\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{e}}{1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2}$	67

input
```
int(a+b*cos(e*x+d)+c*sin(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
a*x-c*cos(e*x+d)/e+b*sin(e*x+d)/e
```

3.398.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int (a+b\cos(d+ex)+c\sin(d+ex))\,dx=\frac{aex-c\cos{(ex+d)}+b\sin{(ex+d)}}{e}$$

input
```
integrate(a+b*cos(e*x+d)+c*sin(e*x+d),x, algorithm="fricas")
```

output
```
(a*e*x - c*cos(e*x + d) + b*sin(e*x + d))/e
```

3.398.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$\int (a+b\cos(d+ex)+c\sin(d+ex))\,dx=ax+b\left(\begin{cases}\frac{\sin{(d+ex)}}{e} & \text{for}\ e\neq 0\\ x\cos{(d)} & \text{otherwise}\end{cases}\right)+c\left(\begin{cases}-\frac{\cos{(d+ex)}}{e} & \text{for}\ e\neq 0\\ x\sin{(d)} & \text{otherwise}\end{cases}\right)$$

input
```
integrate(a+b*cos(e*x+d)+c*sin(e*x+d),x)
```

output
```
a*x + b*Piecewise((sin(d + e*x)/e, Ne(e, 0)), (x*cos(d), True)) + c*Piecew
ise((-cos(d + e*x)/e, Ne(e, 0)), (x*sin(d), True))
```

3.398.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int (a + b\cos(d + ex) + c\sin(d + ex))\,dx = ax - \frac{c\cos(ex + d)}{e} + \frac{b\sin(ex + d)}{e}$$

input
```
integrate(a+b*cos(e*x+d)+c*sin(e*x+d),x, algorithm="maxima")
```

output
```
a*x - c*cos(e*x + d)/e + b*sin(e*x + d)/e
```

3.398.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int (a + b\cos(d + ex) + c\sin(d + ex))\,dx = ax - \frac{c\cos(ex + d)}{e} + \frac{b\sin(ex + d)}{e}$$

input
```
integrate(a+b*cos(e*x+d)+c*sin(e*x+d),x, algorithm="giac")
```

output
```
a*x - c*cos(e*x + d)/e + b*sin(e*x + d)/e
```

3.398.9 Mupad [B] (verification not implemented)

Time = 26.44 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.48

$$\int (a + b\cos(d + ex) + c\sin(d + ex))\,dx = a\,x - \frac{2\,c - 2\,b\tan\left(\frac{d}{2} + \frac{e\,x}{2}\right)}{e\,\left(\tan\left(\frac{d}{2} + \frac{e\,x}{2}\right)^2 + 1\right)}$$

input
```
int(a + b*cos(d + e*x) + c*sin(d + e*x),x)
```

output
```
a*x - (2*c - 2*b*tan(d/2 + (e*x)/2))/(e*(tan(d/2 + (e*x)/2)^2 + 1))
```

3.399 $\int \frac{1}{a+b\cos(d+ex)+c\sin(d+ex)}\,dx$

3.399.1 Optimal result

Integrand size = 20, antiderivative size = 61

$$\int \frac{1}{a+b\cos(d+ex)+c\sin(d+ex)}\,dx = \frac{2\arctan\left(\frac{c+(a-b)\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}\,e}$$

output
```
2*arctan((c+(a-b)*tan(1/2*e*x+1/2*d))/(a^2-b^2-c^2)^(1/2))/e/(a^2-b^2-c^2)
^(1/2)
```

3.399.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.93

$$\int \frac{1}{a+b\cos(d+ex)+c\sin(d+ex)}\,dx = -\frac{2\text{arctanh}\left(\frac{c+(a-b)\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{-a^2+b^2+c^2}}\right)}{\sqrt{-a^2+b^2+c^2}\,e}$$

input
```
Integrate[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-1),x]
```

output
```
(-2*ArcTanh[(c + (a - b)*Tan[(d + e*x)/2])/Sqrt[-a^2 + b^2 + c^2]])/(Sqrt[
-a^2 + b^2 + c^2]*e)
```

3.399.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3603, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\cos(d+ex)+c\sin(d+ex)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a+b\cos(d+ex)+c\sin(d+ex)}dx$$

$$\Big\downarrow \text{3603}$$

$$\frac{2\int \frac{1}{(a-b)\tan^2\left(\frac{1}{2}(d+ex)\right)+2c\tan\left(\frac{1}{2}(d+ex)\right)+a+b}d\tan\left(\frac{1}{2}(d+ex)\right)}{e}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{4\int \frac{1}{-\left(2c+2(a-b)\tan\left(\frac{1}{2}(d+ex)\right)\right)^2-4(a^2-b^2-c^2)}d\left(2c+2(a-b)\tan\left(\frac{1}{2}(d+ex)\right)\right)}{e}$$

$$\Big\downarrow \text{217}$$

$$\frac{2\arctan\left(\frac{2(a-b)\tan\left(\frac{1}{2}(d+ex)\right)+2c}{2\sqrt{a^2-b^2-c^2}}\right)}{e\sqrt{a^2-b^2-c^2}}$$

input
```
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-1),x]
```

output
```
(2*ArcTan[(2*c + 2*(a - b)*Tan[(d + e*x)/2])/(2*Sqrt[a^2 - b^2 - c^2])])/(
Sqrt[a^2 - b^2 - c^2]*e)
```

3.399.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

3.399.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

method	result
derivativedivides	$\frac{2\arctan\left(\frac{2(a-b)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)+2c}{2\sqrt{a^2-b^2-c^2}}\right)}{e\sqrt{a^2-b^2-c^2}}$
default	$\frac{2\arctan\left(\frac{2(a-b)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)+2c}{2\sqrt{a^2-b^2-c^2}}\right)}{e\sqrt{a^2-b^2-c^2}}$
risch	$-\frac{\ln\left(e^{i(ex+d)}+\frac{iac\sqrt{-a^2+b^2+c^2}+iba^2-ib^3-ibc^2+ab\sqrt{-a^2+b^2+c^2}-a^2c+b^2c+c^3}{\left(b^2+c^2\right)\sqrt{-a^2+b^2+c^2}}\right)}{\sqrt{-a^2+b^2+c^2}\,e}+\frac{\ln\left(e^{i(ex+d)}+\frac{iac\sqrt{-a^2+b^2+c^2}-ib}{}\right.}{\sqrt{-}}$

input
```
int(1/(a+b*cos(e*x+d)+c*sin(e*x+d)),x,method=_RETURNVERBOSE)
```

output
```
2/e/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*e*x+1/2*d)+2*c)/(a^2-b
^2-c^2)^(1/2))
```

3.399.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(56) = 112$.

Time = 0.30 (sec) , antiderivative size = 434, normalized size of antiderivative = 7.11

$$\int \frac{1}{a+b\cos(d+ex)+c\sin(d+ex)}\,dx$$
$$= \left[-\frac{\sqrt{-a^2+b^2+c^2}\log\left(-\frac{a^2b^2-2\,b^4-c^4-(a^2+3\,b^2)c^2-(2\,a^2b^2-b^4-2\,a^2c^2+c^4)\cos(ex+d)^2-2\,(ab^3+abc^2)\cos(ex+d)-2\,(ab^2c+ac^3-}{2\,ab\cos(ex+d)+}\right.}{2\,(}\right.$$

input
```
integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d)),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-a^2 + b^2 + c^2)*log(-(a^2*b^2 - 2*b^4 - c^4 - (a^2 + 3*b^2)*c
^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(e*x + d)^2 - 2*(a*b^3 + a*b*c
^2)*cos(e*x + d) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*cos(e*
x + d))*sin(e*x + d) + 2*(2*a*b*c*cos(e*x + d)^2 - a*b*c + (b^2*c + c^3)*c
os(e*x + d) - (b^3 + b*c^2 + (a*b^2 - a*c^2)*cos(e*x + d))*sin(e*x + d))*s
qrt(-a^2 + b^2 + c^2))/(2*a*b*cos(e*x + d) + (b^2 - c^2)*cos(e*x + d)^2 +
a^2 + c^2 + 2*(b*c*cos(e*x + d) + a*c)*sin(e*x + d)))/((a^2 - b^2 - c^2)*e
), arctan(-(a*b*cos(e*x + d) + a*c*sin(e*x + d) + b^2 + c^2)*sqrt(a^2 - b^
2 - c^2)/((c^3 - (a^2 - b^2)*c)*cos(e*x + d) + (a^2*b - b^3 - b*c^2)*sin(e
*x + d)))/(sqrt(a^2 - b^2 - c^2)*e)]
```

3.399.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3179 vs. $2(46) = 92$.

Time = 113.18 (sec) , antiderivative size = 3179, normalized size of antiderivative = 52.11

$$\int \frac{1}{a+b\cos(d+ex)+c\sin(d+ex)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d)),x)
```

output
```
Piecewise((x/(a + b*cos(d) + c*sin(d)), Eq(e, 0)), (log(b/c + tan(d/2 + e*
x/2))/(c*e), Eq(a, b)), (32*b**5/(32*b**6*e*tan(d/2 + e*x/2) - 16*b**5*c*e
 + 32*b**5*e*sqrt(b**2 + c**2)*tan(d/2 + e*x/2) + 48*b**4*c**2*e*tan(d/2 +
 e*x/2) - 16*b**4*c*e*sqrt(b**2 + c**2) - 20*b**3*c**3*e + 32*b**3*c**2*e*
sqrt(b**2 + c**2)*tan(d/2 + e*x/2) + 18*b**2*c**4*e*tan(d/2 + e*x/2) - 12*
b**2*c**3*e*sqrt(b**2 + c**2) - 5*b*c**5*e + 6*b*c**4*e*sqrt(b**2 + c**2)*
tan(d/2 + e*x/2) + c**6*e*tan(d/2 + e*x/2) - c**5*e*sqrt(b**2 + c**2)) + 3
2*b**4*sqrt(b**2 + c**2)/(32*b**6*e*tan(d/2 + e*x/2) - 16*b**5*c*e + 32*b*
*5*e*sqrt(b**2 + c**2)*tan(d/2 + e*x/2) + 48*b**4*c**2*e*tan(d/2 + e*x/2)
- 16*b**4*c*e*sqrt(b**2 + c**2) - 20*b**3*c**3*e + 32*b**3*c**2*e*sqrt(b**
2 + c**2)*tan(d/2 + e*x/2) + 18*b**2*c**4*e*tan(d/2 + e*x/2) - 12*b**2*c**
3*e*sqrt(b**2 + c**2) - 5*b*c**5*e + 6*b*c**4*e*sqrt(b**2 + c**2)*tan(d/2
+ e*x/2) + c**6*e*tan(d/2 + e*x/2) - c**5*e*sqrt(b**2 + c**2)) + 40*b**3*c
**2/(32*b**6*e*tan(d/2 + e*x/2) - 16*b**5*c*e + 32*b**5*e*sqrt(b**2 + c**2
)*tan(d/2 + e*x/2) + 48*b**4*c**2*e*tan(d/2 + e*x/2) - 16*b**4*c*e*sqrt(b*
*2 + c**2) - 20*b**3*c**3*e + 32*b**3*c**2*e*sqrt(b**2 + c**2)*tan(d/2 + e
*x/2) + 18*b**2*c**4*e*tan(d/2 + e*x/2) - 12*b**2*c**3*e*sqrt(b**2 + c**2)
 - 5*b*c**5*e + 6*b*c**4*e*sqrt(b**2 + c**2)*tan(d/2 + e*x/2) + c**6*e*tan
(d/2 + e*x/2) - c**5*e*sqrt(b**2 + c**2)) + 24*b**2*c**2*sqrt(b**2 + c**2)
/(32*b**6*e*tan(d/2 + e*x/2) - 16*b**5*c*e + 32*b**5*e*sqrt(b**2 + c**2...
```

3.399.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a + b\cos(d + ex) + c\sin(d + ex)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` f
or more de
```

3.399.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.46

$$\int \frac{1}{a+b\cos(d+ex)+c\sin(d+ex)}\,dx = -\frac{2\left(\pi\left\lfloor\frac{ex+d}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)-b\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+c}{\sqrt{a^2-b^2-c^2}}\right)\right)}{\sqrt{a^2-b^2-c^2}e}$$

input
```
integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d)),x, algorithm="giac")
```

output
```
-2*(pi*floor(1/2*(e*x + d)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*
e*x + 1/2*d) - b*tan(1/2*e*x + 1/2*d) + c)/sqrt(a^2 - b^2 - c^2)))/(sqrt(a
^2 - b^2 - c^2)*e)
```

3.399.9 Mupad [B] (verification not implemented)

Time = 28.84 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.23

$$\int \frac{1}{a+b\cos(d+ex)+c\sin(d+ex)}\,dx = \frac{2\,\mathrm{atan}\left(\frac{c}{\sqrt{a^2-b^2-c^2}}+\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)(2a-2b)}{2\sqrt{a^2-b^2-c^2}}\right)}{e\sqrt{a^2-b^2-c^2}}$$

input
```
int(1/(a + b*cos(d + e*x) + c*sin(d + e*x)),x)
```

output
```
(2*atan(c/(a^2 - b^2 - c^2)^(1/2) + (tan(d/2 + (e*x)/2)*(2*a - 2*b))/(2*(a
^2 - b^2 - c^2)^(1/2))))/(e*(a^2 - b^2 - c^2)^(1/2))
```

3.400 $\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^2}\,dx$

3.400.1 Optimal result

Integrand size = 20, antiderivative size = 121

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^2}\,dx = \frac{2a\arctan\left(\frac{c+(a-b)\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-b^2-c^2}}\right)}{(a^2-b^2-c^2)^{3/2}e} + \frac{c\cos(d+ex)-b\sin(d+ex)}{(a^2-b^2-c^2)e(a+b\cos(d+ex)+c\sin(d+ex))}$$

output
```
2*a*arctan((c+(a-b)*tan(1/2*e*x+1/2*d))/(a^2-b^2-c^2)^(1/2))/(a^2-b^2-c^2)
^(3/2)/e+(c*cos(e*x+d)-b*sin(e*x+d))/(a^2-b^2-c^2)/e/(a+b*cos(e*x+d)+c*sin
(e*x+d))
```

3.400.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.96

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^2}\,dx = \frac{\frac{2a\,\mathrm{arctanh}\left(\frac{c+(a-b)\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{-a^2+b^2+c^2}}\right)}{(-a^2+b^2+c^2)^{3/2}} + \frac{ac+(b^2+c^2)\sin(d+ex)}{b(-a^2+b^2+c^2)(a+b\cos(d+ex)+c\sin(d+ex))}}{e}$$

input
```
Integrate[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-2),x]
```

output
```
((2*a*ArcTanh[(c + (a - b)*Tan[(d + e*x)/2])/Sqrt[-a^2 + b^2 + c^2]])/(-a^
2 + b^2 + c^2)^(3/2) + (a*c + (b^2 + c^2)*Sin[d + e*x])/(b*(-a^2 + b^2 + c
^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])))/e
```

3.400.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3608, 25, 27, 3042, 3603, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^2}dx$$

$$\Big\downarrow \text{3608}$$

$$\frac{c\cos(d+ex)-b\sin(d+ex)}{e(a^2-b^2-c^2)(a+b\cos(d+ex)+c\sin(d+ex))}-\frac{\int -\frac{a}{a+b\cos(d+ex)+c\sin(d+ex)}dx}{a^2-b^2-c^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{a}{a+b\cos(d+ex)+c\sin(d+ex)}dx}{a^2-b^2-c^2}+\frac{c\cos(d+ex)-b\sin(d+ex)}{e(a^2-b^2-c^2)(a+b\cos(d+ex)+c\sin(d+ex))}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\int \frac{1}{a+b\cos(d+ex)+c\sin(d+ex)}dx}{a^2-b^2-c^2}+\frac{c\cos(d+ex)-b\sin(d+ex)}{e(a^2-b^2-c^2)(a+b\cos(d+ex)+c\sin(d+ex))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \frac{1}{a+b\cos(d+ex)+c\sin(d+ex)}dx}{a^2-b^2-c^2}+\frac{c\cos(d+ex)-b\sin(d+ex)}{e(a^2-b^2-c^2)(a+b\cos(d+ex)+c\sin(d+ex))}$$

$$\Big\downarrow \text{3603}$$

$$\frac{2a\int \frac{1}{(a-b)\tan^2\left(\frac{1}{2}(d+ex)\right)+2c\tan\left(\frac{1}{2}(d+ex)\right)+a+b}d\tan\left(\frac{1}{2}(d+ex)\right)}{e(a^2-b^2-c^2)}+\frac{c\cos(d+ex)-b\sin(d+ex)}{e(a^2-b^2-c^2)(a+b\cos(d+ex)+c\sin(d+ex))}$$

$$\Big\downarrow \; 1083$$

$$\frac{c\cos(d+ex)-b\sin(d+ex)}{e\left(a^2-b^2-c^2\right)\left(a+b\cos(d+ex)+c\sin(d+ex)\right)}-\frac{4a\int \frac{1}{-\left(2c+2(a-b)\tan\left(\frac{1}{2}(d+ex)\right)\right)^2-4\left(a^2-b^2-c^2\right)}d\left(2c+2(a-b)\tan\left(\frac{1}{2}(d+ex)\right)\right)}{e\left(a^2-b^2-c^2\right)}$$

$$\Big\downarrow \; 217$$

$$\frac{2a\arctan\left(\frac{2(a-b)\tan\left(\frac{1}{2}(d+ex)\right)+2c}{2\sqrt{a^2-b^2-c^2}}\right)}{e\left(a^2-b^2-c^2\right)^{3/2}}+\frac{c\cos(d+ex)-b\sin(d+ex)}{e\left(a^2-b^2-c^2\right)\left(a+b\cos(d+ex)+c\sin(d+ex)\right)}$$

input
```
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-2),x]
```

output
```
(2*a*ArcTan[(2*c + 2*(a - b)*Tan[(d + e*x)/2])/(2*Sqrt[a^2 - b^2 - c^2])])
/((a^2 - b^2 - c^2)^(3/2)*e) + (c*Cos[d + e*x] - b*Sin[d + e*x])/((a^2 - b
^2 - c^2)*e*(a + b*Cos[d + e*x] + c*Sin[d + e*x]))
```

3.400.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.400. $\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^2}\,dx$

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

rule 3608
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Simp[
1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c
*(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x
] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] &&
NeQ[n, -3/2]
```

3.400.4 Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.68

method	result
derivativedivides	$\frac{\frac{-\frac{2\left(ab-b^2-c^2\right)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{a^3-a^2b-a\,b^2-a\,c^2+b^3+c^2b}+\frac{2ac}{a^3-a^2b-a\,b^2-a\,c^2+b^3+c^2b}}{\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2a-\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2b+2c\tan\left(\frac{ex}{2}+\frac{d}{2}\right)+a+b}+\frac{2a\arctan\left(\frac{2(a-b)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)+2c}{2\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{\frac{3}{2}}}}{e}$
default	$\frac{\frac{-\frac{2\left(ab-b^2-c^2\right)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{a^3-a^2b-a\,b^2-a\,c^2+b^3+c^2b}+\frac{2ac}{a^3-a^2b-a\,b^2-a\,c^2+b^3+c^2b}}{\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2a-\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2b+2c\tan\left(\frac{ex}{2}+\frac{d}{2}\right)+a+b}+\frac{2a\arctan\left(\frac{2(a-b)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)+2c}{2\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{\frac{3}{2}}}}{e}$
risch	$-\frac{2i\left(-ia\,\mathrm{e}^{i(ex+d)}-ib+c\right)}{(-a^2+b^2+c^2)e\left(c\,\mathrm{e}^{2i(ex+d)}+ib\,\mathrm{e}^{2i(ex+d)}-c+2ia\,\mathrm{e}^{i(ex+d)}+ib\right)}-\frac{a\ln\left(\mathrm{e}^{i(ex+d)}+\frac{iac\sqrt{-a^2+b^2+c^2}+ib\,a^2-ib^3-ib\,c^2+a\cdots}{\left(b^2+c^2\right)\sqrt{-a^2\cdots}}\right.}{\sqrt{-a^2+b^2+c^2}\,(a^2-b^2-c\cdots}$

input
```
int(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^2,x,method=_RETURNVERBOSE)
```

output
```
1/e*(2*(-(a*b-b^2-c^2)/(a^3-a^2*b-a*b^2-a*c^2+b^3+b*c^2)*tan(1/2*e*x+1/2*d
)+a*c/(a^3-a^2*b-a*b^2-a*c^2+b^3+b*c^2))/(tan(1/2*e*x+1/2*d)^2*a-tan(1/2*e
*x+1/2*d)^2*b+2*c*tan(1/2*e*x+1/2*d)+a+b)+2*a/(a^2-b^2-c^2)^(3/2)*arctan(1
/2*(2*(a-b)*tan(1/2*e*x+1/2*d)+2*c)/(a^2-b^2-c^2)^(1/2)))
```

3.400.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(116) = 232$.

Time = 0.30 (sec) , antiderivative size = 819, normalized size of antiderivative = 6.77

$$\int \frac{1}{(a + b\cos(d + ex)) + c\sin(d + ex))^2}\,dx$$

$$= \left[\frac{(ab\cos(ex + d) + ac\sin(ex + d) + a^2)\sqrt{-a^2 + b^2 + c^2}\log\left(\frac{a^2b^2 - 2\,b^4 - c^4 - (a^2 + 3\,b^2)c^2 - (2\,a^2b^2 - b^4 - 2\,a^2c^2 + c^4)\cos(\cdots}{\cdots}\right.}{2\,((a^4b - 2\,a^2b^3 + b^5 + bc\cdots} \right.$$

input
```
integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^2,x, algorithm="fricas")
```

output
```
[1/2*((a*b*cos(e*x + d) + a*c*sin(e*x + d) + a^2)*sqrt(-a^2 + b^2 + c^2)*l
og((a^2*b^2 - 2*b^4 - c^4 - (a^2 + 3*b^2)*c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c
^2 + c^4)*cos(e*x + d)^2 - 2*(a*b^3 + a*b*c^2)*cos(e*x + d) - 2*(a*b^2*c +
 a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*cos(e*x + d))*sin(e*x + d) - 2*(2*a*b
*c*cos(e*x + d)^2 - a*b*c + (b^2*c + c^3)*cos(e*x + d) - (b^3 + b*c^2 + (a
*b^2 - a*c^2)*cos(e*x + d))*sin(e*x + d))*sqrt(-a^2 + b^2 + c^2))/(2*a*b*c
os(e*x + d) + (b^2 - c^2)*cos(e*x + d)^2 + a^2 + c^2 + 2*(b*c*cos(e*x + d)
 + a*c)*sin(e*x + d))) - 2*(c^3 - (a^2 - b^2)*c)*cos(e*x + d) - 2*(a^2*b -
 b^3 - b*c^2)*sin(e*x + d))/((a^4*b - 2*a^2*b^3 + b^5 + b*c^4 - 2*(a^2*b -
 b^3)*c^2)*e*cos(e*x + d) + (c^5 - 2*(a^2 - b^2)*c^3 + (a^4 - 2*a^2*b^2 +
b^4)*c)*e*sin(e*x + d) + (a^5 - 2*a^3*b^2 + a*b^4 + a*c^4 - 2*(a^3 - a*b^2
)*c^2)*e), ((a*b*cos(e*x + d) + a*c*sin(e*x + d) + a^2)*sqrt(a^2 - b^2 - c
^2)*arctan(-(a*b*cos(e*x + d) + a*c*sin(e*x + d) + b^2 + c^2)*sqrt(a^2 - b
^2 - c^2)/((c^3 - (a^2 - b^2)*c)*cos(e*x + d) + (a^2*b - b^3 - b*c^2)*sin(
e*x + d))) - (c^3 - (a^2 - b^2)*c)*cos(e*x + d) - (a^2*b - b^3 - b*c^2)*si
n(e*x + d))/((a^4*b - 2*a^2*b^3 + b^5 + b*c^4 - 2*(a^2*b - b^3)*c^2)*e*cos
(e*x + d) + (c^5 - 2*(a^2 - b^2)*c^3 + (a^4 - 2*a^2*b^2 + b^4)*c)*e*sin(e*
x + d) + (a^5 - 2*a^3*b^2 + a*b^4 + a*c^4 - 2*(a^3 - a*b^2)*c^2)*e)]
```

3.400.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))**2,x)
```

output
```
Timed out
```

3.400.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` f
or more de
```

3.400.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.77

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^2}\,dx = -\frac{2\left(\frac{\left(\pi\left\lfloor\frac{ex+d}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)-b\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+c}{\sqrt{a^2-b^2-c^2}}\right)\right)a}{(a^2-b^2-c^2)^{\frac{3}{2}}}+\frac{ab\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)-b^2\tan\left(\frac{1}{2}ex+\right.}{(a^3-a^2b-ab^2+b^3-ac^2+bc^2)\left(a\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^2-\right.}\right.}{e}$$

input
```
integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^2,x, algorithm="giac")
```

output
```
-2*((pi*floor(1/2*(e*x + d)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2
*e*x + 1/2*d) - b*tan(1/2*e*x + 1/2*d) + c)/sqrt(a^2 - b^2 - c^2)))*a/(a^2
 - b^2 - c^2)^(3/2) + (a*b*tan(1/2*e*x + 1/2*d) - b^2*tan(1/2*e*x + 1/2*d)
 - c^2*tan(1/2*e*x + 1/2*d) - a*c)/((a^3 - a^2*b - a*b^2 + b^3 - a*c^2 + b
*c^2)*(a*tan(1/2*e*x + 1/2*d)^2 - b*tan(1/2*e*x + 1/2*d)^2 + 2*c*tan(1/2*e
*x + 1/2*d) + a + b)))/e
```

3.400.9 Mupad [B] (verification not implemented)

Time = 27.54 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.61

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^2}\,dx = \frac{2\,a\,\mathrm{atanh}\left(\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,(2\,a-2\,b)+\frac{2\,\left(-a^2\,c+b^2\,c+c^3\right)}{-a^2+b^2+c^2}}{2\,\sqrt{-a^2+b^2+c^2}}\right)}{e\,(-a^2+b^2+c^2)^{3/2}} - \frac{\frac{2\,a\,c}{(a-b)\,(-a^2+b^2+c^2)}+\frac{2\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,(b^2-a\,b+c^2)}{(a-b)\,(-a^2+b^2+c^2)}}{e\,\left((a-b)\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2+2\,c\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)+a+b\right)}$$

input
```
int(1/(a + b*cos(d + e*x) + c*sin(d + e*x))^2,x)
```

output
```
(2*a*atanh((tan(d/2 + (e*x)/2)*(2*a - 2*b) + (2*(b^2*c - a^2*c + c^3))/(b^
2 - a^2 + c^2))/(2*(b^2 - a^2 + c^2)^(1/2))))/(e*(b^2 - a^2 + c^2)^(3/2))
- ((2*a*c)/((a - b)*(b^2 - a^2 + c^2)) + (2*tan(d/2 + (e*x)/2)*(b^2 - a*b
+ c^2))/((a - b)*(b^2 - a^2 + c^2)))/(e*(a + b + tan(d/2 + (e*x)/2)^2*(a -
 b) + 2*c*tan(d/2 + (e*x)/2)))
```

3.401 $\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^3}\,dx$

3.401.1 Optimal result

Integrand size = 20, antiderivative size = 197

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^3}\,dx = \frac{(2a^2+b^2+c^2)\arctan\left(\frac{c+(a-b)\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-b^2-c^2}}\right)}{(a^2-b^2-c^2)^{5/2}e} + \frac{c\cos(d+ex)-b\sin(d+ex)}{2(a^2-b^2-c^2)e(a+b\cos(d+ex)+c\sin(d+ex))^2} + \frac{3(ac\cos(d+ex)-ab\sin(d+ex))}{2(a^2-b^2-c^2)^2e(a+b\cos(d+ex)+c\sin(d+ex))}$$

output
```
(2*a^2+b^2+c^2)*arctan((c+(a-b)*tan(1/2*e*x+1/2*d))/(a^2-b^2-c^2)^(1/2))/(
a^2-b^2-c^2)^(5/2)/e+1/2*(c*cos(e*x+d)-b*sin(e*x+d))/(a^2-b^2-c^2)/e/(a+b*
cos(e*x+d)+c*sin(e*x+d))^2+3/2*(a*c*cos(e*x+d)-a*b*sin(e*x+d))/(a^2-b^2-c^
2)^2/e/(a+b*cos(e*x+d)+c*sin(e*x+d))
```

3.401.2 Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.02

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^3}\,dx$$
$$=\frac{-\frac{2(2a^2+b^2+c^2)\operatorname{arctanh}\left(\frac{c+(a-b)\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{-a^2+b^2+c^2}}\right)}{(-a^2+b^2+c^2)^{5/2}}+\frac{ac+(b^2+c^2)\sin(d+ex)}{b(-a^2+b^2+c^2)(a+b\cos(d+ex)+c\sin(d+ex))^2}-\frac{c(2a^2+b^2+c^2)+3a(b^2+c^2)\sin(d+ex)}{b(-a^2+b^2+c^2)^2(a+b\cos(d+ex)+c\sin(d+ex))}}{2e}$$

input
```
Integrate[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-3),x]
```

output
```
((-2*(2*a^2 + b^2 + c^2)*ArcTanh[(c + (a - b)*Tan[(d + e*x)/2])/Sqrt[-a^2
+ b^2 + c^2]])/(-a^2 + b^2 + c^2)^(5/2) + (a*c + (b^2 + c^2)*Sin[d + e*x])
/(b*(-a^2 + b^2 + c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^2) - (c*(2*a^
2 + b^2 + c^2) + 3*a*(b^2 + c^2)*Sin[d + e*x])/(b*(-a^2 + b^2 + c^2)^2*(a
+ b*Cos[d + e*x] + c*Sin[d + e*x])))/(2*e)
```

3.401.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.13, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {3042, 3608, 25, 3042, 3632, 3042, 3603, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^3}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^3}dx$$
$$\Big\downarrow \; 3608$$
$$\frac{c\cos(d+ex)-b\sin(d+ex)}{2e\left(a^2-b^2-c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))^2}-\frac{\int -\frac{2a-b\cos(d+ex)-c\sin(d+ex)}{(a+b\cos(d+ex)+c\sin(d+ex))^2}dx}{2\left(a^2-b^2-c^2\right)}$$
$$\Big\downarrow \; 25$$

$$\frac{\int \frac{2a-b\cos(d+ex)-c\sin(d+ex)}{(a+b\cos(d+ex)+c\sin(d+ex))^2}dx}{2\left(a^2-b^2-c^2\right)}+\frac{c\cos(d+ex)-b\sin(d+ex)}{2e\left(a^2-b^2-c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{2a-b\cos(d+ex)-c\sin(d+ex)}{(a+b\cos(d+ex)+c\sin(d+ex))^2}dx}{2\left(a^2-b^2-c^2\right)}+\frac{c\cos(d+ex)-b\sin(d+ex)}{2e\left(a^2-b^2-c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))^2}$$

$$\Big\downarrow \text{ 3632}$$

$$\frac{\frac{\left(2a^2+b^2+c^2\right)\int \frac{1}{a+b\cos(d+ex)+c\sin(d+ex)}dx}{a^2-b^2-c^2}+\frac{3(ac\cos(d+ex)-ab\sin(d+ex))}{e(a^2-b^2-c^2)(a+b\cos(d+ex)+c\sin(d+ex))}}{2\left(a^2-b^2-c^2\right)}+\frac{c\cos(d+ex)-b\sin(d+ex)}{2e\left(a^2-b^2-c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{\left(2a^2+b^2+c^2\right)\int \frac{1}{a+b\cos(d+ex)+c\sin(d+ex)}dx}{a^2-b^2-c^2}+\frac{3(ac\cos(d+ex)-ab\sin(d+ex))}{e(a^2-b^2-c^2)(a+b\cos(d+ex)+c\sin(d+ex))}}{2\left(a^2-b^2-c^2\right)}+\frac{c\cos(d+ex)-b\sin(d+ex)}{2e\left(a^2-b^2-c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))^2}$$

$$\Big\downarrow \text{ 3603}$$

$$\frac{\frac{2\left(2a^2+b^2+c^2\right)\int \frac{1}{(a-b)\tan^2\left(\frac{1}{2}(d+ex)\right)+2c\tan\left(\frac{1}{2}(d+ex)\right)+a+b}d\tan\left(\frac{1}{2}(d+ex)\right)}{e(a^2-b^2-c^2)}+\frac{3(ac\cos(d+ex)-ab\sin(d+ex))}{e(a^2-b^2-c^2)(a+b\cos(d+ex)+c\sin(d+ex))}}{2\left(a^2-b^2-c^2\right)}+\frac{c\cos(d+ex)-b\sin(d+ex)}{2e\left(a^2-b^2-c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))^2}$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{\frac{3(ac\cos(d+ex)-ab\sin(d+ex))}{e(a^2-b^2-c^2)(a+b\cos(d+ex)+c\sin(d+ex))}-\frac{4\left(2a^2+b^2+c^2\right)\int \frac{1}{-\left(2c+2(a-b)\tan\left(\frac{1}{2}(d+ex)\right)\right)^2-4\left(a^2-b^2-c^2\right)}d\left(2c+2(a-b)\tan\left(\frac{1}{2}(d+ex)\right)\right)}{e(a^2-b^2-c^2)}}{2\left(a^2-b^2-c^2\right)}+\frac{c\cos(d+ex)-b\sin(d+ex)}{2e\left(a^2-b^2-c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))^2}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{\frac{2\left(2a^2+b^2+c^2\right)\arctan\left(\frac{2(a-b)\tan\left(\frac{1}{2}(d+ex)\right)+2c}{2\sqrt{a^2-b^2-c^2}}\right)}{e(a^2-b^2-c^2)^{3/2}}+\frac{3(ac\cos(d+ex)-ab\sin(d+ex))}{e(a^2-b^2-c^2)(a+b\cos(d+ex)+c\sin(d+ex))}}{2\left(a^2-b^2-c^2\right)}+\frac{c\cos(d+ex)-b\sin(d+ex)}{2e\left(a^2-b^2-c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))^2}$$

3.401. $\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^3}\,dx$

input
```
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-3),x]
```

output
```
(c*Cos[d + e*x] - b*Sin[d + e*x])/(2*(a^2 - b^2 - c^2)*e*(a + b*Cos[d + e*
x] + c*Sin[d + e*x])^2) + ((2*(2*a^2 + b^2 + c^2)*ArcTan[(2*c + 2*(a - b)*
Tan[(d + e*x)/2])/(2*Sqrt[a^2 - b^2 - c^2])])/((a^2 - b^2 - c^2)^(3/2)*e)
+ (3*(a*c*Cos[d + e*x] - a*b*Sin[d + e*x]))/((a^2 - b^2 - c^2)*e*(a + b*Co
s[d + e*x] + c*Sin[d + e*x])))/(2*(a^2 - b^2 - c^2))
```

3.401.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

rule 3608
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Simp[
1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c
*(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x
] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] &&
NeQ[n, -3/2]
```

rule 3632
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Simp[(a*A - b*B - c*C)/(a^2 - b^2 - c^2)   Int[1/(a + b*Cos[d + e*x] + c*S
in[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
 - c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

3.401.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 610 vs. $2(188) = 376$.

Time = 1.72 (sec) , antiderivative size = 611, normalized size of antiderivative = 3.10

method	result
derivativedivides	$\frac{-\frac{\left(4a^3b-7a^2b^2-5a^2c^2+2a\,b^3+2ab\,c^2+b^4+3b^2c^2+2c^4\right)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^3}{(a-b)\left(a^4-2a^2b^2-2a^2c^2+b^4+2b^2c^2+c^4\right)}+\frac{c\left(4a^4-12a^3b+13a^2b^2+7a^2c^2-6a\,b^3-6ab\,c^2+b^4-b^2c^2-2c\right.}{\left(a^4-2a^2b^2-2a^2c^2+b^4+2b^2c^2+c^4\right)\left(a^2-2ab+\right.}}{\left(\tan\left(\frac{ex}{2}+\frac{d}{2}\right.\right.}$
default	$\frac{-\frac{\left(4a^3b-7a^2b^2-5a^2c^2+2a\,b^3+2ab\,c^2+b^4+3b^2c^2+2c^4\right)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^3}{(a-b)\left(a^4-2a^2b^2-2a^2c^2+b^4+2b^2c^2+c^4\right)}+\frac{c\left(4a^4-12a^3b+13a^2b^2+7a^2c^2-6a\,b^3-6ab\,c^2+b^4-b^2c^2-2c\right.}{\left(a^4-2a^2b^2-2a^2c^2+b^4+2b^2c^2+c^4\right)\left(a^2-2ab+\right.}}{\left(\tan\left(\frac{ex}{2}+\frac{d}{2}\right.\right.}$
risch	Expression too large to display

input
```
int(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^3,x,method=_RETURNVERBOSE)
```

output
```
1/e*(2*(-1/2*(4*a^3*b-7*a^2*b^2-5*a^2*c^2+2*a*b^3+2*a*b*c^2+b^4+3*b^2*c^2+
2*c^4)/(a-b)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)*tan(1/2*e*x+1/2*d
)^3+1/2*c*(4*a^4-12*a^3*b+13*a^2*b^2+7*a^2*c^2-6*a*b^3-6*a*b*c^2+b^4-b^2*c
^2-2*c^4)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tan(
1/2*e*x+1/2*d)^2-1/2*(4*a^4*b-5*a^3*b^2-11*a^3*c^2-3*a^2*b^3+3*a^2*b*c^2+5
*a*b^4+7*a*b^2*c^2+2*a*c^4-b^5+b^3*c^2+2*b*c^4)/(a^4-2*a^2*b^2-2*a^2*c^2+b
^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tan(1/2*e*x+1/2*d)+1/2*c*(4*a^4-3*a^2*b^
2-a^2*c^2-b^4-b^2*c^2)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*
a*b+b^2))/(tan(1/2*e*x+1/2*d)^2*a-tan(1/2*e*x+1/2*d)^2*b+2*c*tan(1/2*e*x+1
/2*d)+a+b)^2+(2*a^2+b^2+c^2)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(
a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*e*x+1/2*d)+2*c)/(a^2-b^2-c^
2)^(1/2)))
```

3.401.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 878 vs. $2(188) = 376$.

Time = 0.35 (sec) , antiderivative size = 1947, normalized size of antiderivative = 9.88

$$\int \frac{1}{(a + b\cos(d + ex) + c\sin(d + ex))^3} dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^3,x, algorithm="fricas")
```

output
```
[1/4*(6*a*b*c^3 - 12*(a*b*c^3 - (a^3*b - a*b^3)*c)*cos(e*x + d)^2 - (2*a^4
 + a^2*b^2 + c^4 + (3*a^2 + b^2)*c^2 + (2*a^2*b^2 + b^4 - 2*a^2*c^2 - c^4)
*cos(e*x + d)^2 + 2*(2*a^3*b + a*b^3 + a*b*c^2)*cos(e*x + d) + 2*(a*c^3 +
(2*a^3 + a*b^2)*c + (b*c^3 + (2*a^2*b + b^3)*c)*cos(e*x + d))*sin(e*x + d)
)*sqrt(-a^2 + b^2 + c^2)*log(-(a^2*b^2 - 2*b^4 - c^4 - (a^2 + 3*b^2)*c^2 -
 (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(e*x + d)^2 - 2*(a*b^3 + a*b*c^2)*
cos(e*x + d) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*cos(e*x +
d))*sin(e*x + d) + 2*(2*a*b*c*cos(e*x + d)^2 - a*b*c + (b^2*c + c^3)*cos(e
*x + d) - (b^3 + b*c^2 + (a*b^2 - a*c^2)*cos(e*x + d))*sin(e*x + d))*sqrt(
-a^2 + b^2 + c^2))/(2*a*b*cos(e*x + d) + (b^2 - c^2)*cos(e*x + d)^2 + a^2
+ c^2 + 2*(b*c*cos(e*x + d) + a*c)*sin(e*x + d))) - 6*(a^3*b - a*b^3)*c +
2*(c^5 - (5*a^2 - 2*b^2)*c^3 + (4*a^4 - 5*a^2*b^2 + b^4)*c)*cos(e*x + d) -
 2*(4*a^4*b - 5*a^2*b^3 + b^5 + b*c^4 - (5*a^2*b - 2*b^3)*c^2 + 3*(a^3*b^2
 - a*b^4 - a^3*c^2 + a*c^4)*cos(e*x + d))*sin(e*x + d))/((a^6*b^2 - 3*a^4*
b^4 + 3*a^2*b^6 - b^8 + c^8 - (3*a^2 - 2*b^2)*c^6 + 3*(a^4 - a^2*b^2)*c^4
- (a^6 - 3*a^2*b^4 + 2*b^6)*c^2)*e*cos(e*x + d)^2 + 2*(a^7*b - 3*a^5*b^3 +
 3*a^3*b^5 - a*b^7 - a*b*c^6 + 3*(a^3*b - a*b^3)*c^4 - 3*(a^5*b - 2*a^3*b^
3 + a*b^5)*c^2)*e*cos(e*x + d) + (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6 -
c^8 + (2*a^2 - 3*b^2)*c^6 + 3*(a^2*b^2 - b^4)*c^4 - (2*a^6 - 3*a^4*b^2 + b
^6)*c^2)*e - 2*((b*c^7 - 3*(a^2*b - b^3)*c^5 + 3*(a^4*b - 2*a^2*b^3 + b...
```

3.401.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a + b\cos(d + ex) + c\sin(d + ex))^3} dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))**3,x)
```

output
```
Timed out
```

3.401.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` f
or more de
```

3.401.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 856 vs. $2(188) = 376$.

Time = 0.32 (sec) , antiderivative size = 856, normalized size of antiderivative = 4.35

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^3,x, algorithm="giac")
```

output
```
-((pi*floor(1/2*(e*x + d)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*e
*x + 1/2*d) - b*tan(1/2*e*x + 1/2*d) + c)/sqrt(a^2 - b^2 - c^2)))*(2*a^2 +
 b^2 + c^2)/((a^4 - 2*a^2*b^2 + b^4 - 2*a^2*c^2 + 2*b^2*c^2 + c^4)*sqrt(a^
2 - b^2 - c^2)) + (4*a^4*b*tan(1/2*e*x + 1/2*d)^3 - 11*a^3*b^2*tan(1/2*e*x
 + 1/2*d)^3 + 9*a^2*b^3*tan(1/2*e*x + 1/2*d)^3 - a*b^4*tan(1/2*e*x + 1/2*d
)^3 - b^5*tan(1/2*e*x + 1/2*d)^3 - 5*a^3*c^2*tan(1/2*e*x + 1/2*d)^3 + 7*a^
2*b*c^2*tan(1/2*e*x + 1/2*d)^3 + a*b^2*c^2*tan(1/2*e*x + 1/2*d)^3 - 3*b^3*
c^2*tan(1/2*e*x + 1/2*d)^3 + 2*a*c^4*tan(1/2*e*x + 1/2*d)^3 - 2*b*c^4*tan(
1/2*e*x + 1/2*d)^3 - 4*a^4*c*tan(1/2*e*x + 1/2*d)^2 + 12*a^3*b*c*tan(1/2*e
*x + 1/2*d)^2 - 13*a^2*b^2*c*tan(1/2*e*x + 1/2*d)^2 + 6*a*b^3*c*tan(1/2*e*
x + 1/2*d)^2 - b^4*c*tan(1/2*e*x + 1/2*d)^2 - 7*a^2*c^3*tan(1/2*e*x + 1/2*
d)^2 + 6*a*b*c^3*tan(1/2*e*x + 1/2*d)^2 + b^2*c^3*tan(1/2*e*x + 1/2*d)^2 +
 2*c^5*tan(1/2*e*x + 1/2*d)^2 + 4*a^4*b*tan(1/2*e*x + 1/2*d) - 5*a^3*b^2*t
an(1/2*e*x + 1/2*d) - 3*a^2*b^3*tan(1/2*e*x + 1/2*d) + 5*a*b^4*tan(1/2*e*x
 + 1/2*d) - b^5*tan(1/2*e*x + 1/2*d) - 11*a^3*c^2*tan(1/2*e*x + 1/2*d) + 3
*a^2*b*c^2*tan(1/2*e*x + 1/2*d) + 7*a*b^2*c^2*tan(1/2*e*x + 1/2*d) + b^3*c
^2*tan(1/2*e*x + 1/2*d) + 2*a*c^4*tan(1/2*e*x + 1/2*d) + 2*b*c^4*tan(1/2*e
*x + 1/2*d) - 4*a^4*c + 3*a^2*b^2*c + b^4*c + a^2*c^3 + b^2*c^3)/((a^6 - 2
*a^5*b - a^4*b^2 + 4*a^3*b^3 - a^2*b^4 - 2*a*b^5 + b^6 - 2*a^4*c^2 + 4*a^3
*b*c^2 - 4*a*b^3*c^2 + 2*b^4*c^2 + a^2*c^4 - 2*a*b*c^4 + b^2*c^4)*(a*ta...
```

3.401.9 Mupad [B] (verification not implemented)

Time = 32.72 (sec) , antiderivative size = 700, normalized size of antiderivative = 3.55

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^3}\,dx =$$

$$-\frac{\frac{-4\,a^4\,c+3\,a^2\,b^2\,c+a^2\,c^3+b^4\,c+b^2\,c^3}{(a-b)^2\,(a^4-2\,a^2\,b^2-2\,a^2\,c^2+b^4+2\,b^2\,c^2+c^4)}+\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,(4\,a^4\,b-5\,a^3\,b^2-11\,a^3\,c^2-3\,a^2\,b^3+3\,a^2\,b\,c^2+5\,a\,b^4+7\,a\,b^2\,c^2+2\,a\,c^4-b^5+b^3\,c^2+}{(a-b)^2\,(a^4-2\,a^2\,b^2-2\,a^2\,c^2+b^4+2\,b^2\,c^2+c^4)}}{e\,\left(2\,a\,b+\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^3\,(4\,a\,c-4\,b\,c)+\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2\,(2\,a^2-\right.}$$

$$-\frac{\mathrm{atanh}\left(\frac{a^4\,c-2\,a^2\,b^2\,c-2\,a^2\,c^3+b^4\,c+2\,b^2\,c^3+c^5}{(-a^2+b^2+c^2)^{5/2}}+\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,(2\,a-2\,b)\,(a^4-2\,a^2\,b^2-2\,a^2\,c^2+b^4+2\,b^2\,c^2+c^4)}{2\,(-a^2+b^2+c^2)^{5/2}}\right)\,(2\,a^2+b^2+c^2)}{e\,(-a^2+b^2+c^2)^{5/2}}$$

input
```
int(1/(a + b*cos(d + e*x) + c*sin(d + e*x))^3,x)
```

output
```
- ((b^4*c - 4*a^4*c + a^2*c^3 + b^2*c^3 + 3*a^2*b^2*c)/((a - b)^2*(a^4 + b
^4 + c^4 - 2*a^2*b^2 - 2*a^2*c^2 + 2*b^2*c^2)) + (tan(d/2 + (e*x)/2)*(5*a*
b^4 + 4*a^4*b + 2*a*c^4 + 2*b*c^4 - b^5 - 3*a^2*b^3 - 5*a^3*b^2 - 11*a^3*c
^2 + b^3*c^2 + 7*a*b^2*c^2 + 3*a^2*b*c^2))/((a - b)^2*(a^4 + b^4 + c^4 - 2
*a^2*b^2 - 2*a^2*c^2 + 2*b^2*c^2)) + (tan(d/2 + (e*x)/2)^2*(2*c^5 - b^4*c
- 4*a^4*c - 7*a^2*c^3 + b^2*c^3 - 13*a^2*b^2*c + 6*a*b*c^3 + 6*a*b^3*c + 1
2*a^3*b*c))/((a - b)^2*(a^4 + b^4 + c^4 - 2*a^2*b^2 - 2*a^2*c^2 + 2*b^2*c^
2)) + (tan(d/2 + (e*x)/2)^3*(2*a*b^3 + 4*a^3*b + b^4 + 2*c^4 - 7*a^2*b^2 -
 5*a^2*c^2 + 3*b^2*c^2 + 2*a*b*c^2))/((a - b)*(a^4 + b^4 + c^4 - 2*a^2*b^2
 - 2*a^2*c^2 + 2*b^2*c^2)))/(e*(2*a*b + tan(d/2 + (e*x)/2)^3*(4*a*c - 4*b*
c) + tan(d/2 + (e*x)/2)^2*(2*a^2 - 2*b^2 + 4*c^2) + tan(d/2 + (e*x)/2)^4*(
a^2 - 2*a*b + b^2) + a^2 + b^2 + tan(d/2 + (e*x)/2)*(4*a*c + 4*b*c))) - (a
tanh((a^4*c + b^4*c + c^5 - 2*a^2*c^3 + 2*b^2*c^3 - 2*a^2*b^2*c)/(b^2 - a^
2 + c^2)^(5/2) + (tan(d/2 + (e*x)/2)*(2*a - 2*b)*(a^4 + b^4 + c^4 - 2*a^2*
b^2 - 2*a^2*c^2 + 2*b^2*c^2))/(2*(b^2 - a^2 + c^2)^(5/2)))*(2*a^2 + b^2 +
c^2))/(e*(b^2 - a^2 + c^2)^(5/2))
```

3.401. $\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^3}\,dx$

3.402 $\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^4}\,dx$

3.402.1 Optimal result

Integrand size = 20, antiderivative size = 292

$$\begin{aligned}\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^4}\,dx &= \frac{a(2a^2+3(b^2+c^2))\arctan\left(\frac{c+(a-b)\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-b^2-c^2}}\right)}{(a^2-b^2-c^2)^{7/2}e} \\ &+ \frac{c\cos(d+ex)-b\sin(d+ex)}{3(a^2-b^2-c^2)e(a+b\cos(d+ex)+c\sin(d+ex))^3} \\ &+ \frac{5(ac\cos(d+ex)-ab\sin(d+ex))}{6(a^2-b^2-c^2)^2e(a+b\cos(d+ex)+c\sin(d+ex))^2} \\ &+ \frac{c(11a^2+4(b^2+c^2))\cos(d+ex)-b(11a^2+4(b^2+c^2))\sin(d+ex)}{6(a^2-b^2-c^2)^3e(a+b\cos(d+ex)+c\sin(d+ex))}\end{aligned}$$

output
```
a*(2*a^2+3*b^2+3*c^2)*arctan((c+(a-b)*tan(1/2*e*x+1/2*d))/(a^2-b^2-c^2)^(1
/2))/(a^2-b^2-c^2)^(7/2)/e+1/3*(c*cos(e*x+d)-b*sin(e*x+d))/(a^2-b^2-c^2)/e
/(a+b*cos(e*x+d)+c*sin(e*x+d))^3+5/6*(a*c*cos(e*x+d)-a*b*sin(e*x+d))/(a^2-
b^2-c^2)^2/e/(a+b*cos(e*x+d)+c*sin(e*x+d))^2+1/6*(c*(11*a^2+4*b^2+4*c^2)*c
os(e*x+d)-b*(11*a^2+4*b^2+4*c^2)*sin(e*x+d))/(a^2-b^2-c^2)^3/e/(a+b*cos(e*
x+d)+c*sin(e*x+d))
```

3.402.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 606 vs. $2(292) = 584$.

Time = 1.71 (sec) , antiderivative size = 606, normalized size of antiderivative = 2.08

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^4}\,dx$$
$$= \frac{\frac{24a(2a^2+3(b^2+c^2))\operatorname{arctanh}\left(\frac{c+(a-b)\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{-a^2+b^2+c^2}}\right)}{(-a^2+b^2+c^2)^{7/2}} + \frac{44a^5c+82a^3b^2c+24ab^4c+82a^3c^3+48ab^2c^3+24ac^5+30a^2bc(2a^2+3(b^2+c^2))\cos(d-}{}}{}$$

input
```
Integrate[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-4),x]
```

output
```
((24*a*(2*a^2 + 3*(b^2 + c^2))*ArcTanh[(c + (a - b)*Tan[(d + e*x)/2])/Sqrt
[-a^2 + b^2 + c^2]])/(-a^2 + b^2 + c^2)^(7/2) + (44*a^5*c + 82*a^3*b^2*c +
 24*a*b^4*c + 82*a^3*c^3 + 48*a*b^2*c^3 + 24*a*c^5 + 30*a^2*b*c*(2*a^2 + 3
*(b^2 + c^2))*Cos[d + e*x] - 6*a*c*(-2*b^4 + 2*b^2*c^2 + 4*c^4 + a^2*(7*b^
2 + 11*c^2))*Cos[2*(d + e*x)] - 22*a^2*b^3*c*Cos[3*(d + e*x)] - 8*b^5*c*Co
s[3*(d + e*x)] - 22*a^2*b*c^3*Cos[3*(d + e*x)] - 16*b^3*c^3*Cos[3*(d + e*x
)] - 8*b*c^5*Cos[3*(d + e*x)] + 72*a^4*b^2*Sin[d + e*x] - 9*a^2*b^4*Sin[d
+ e*x] + 12*b^6*Sin[d + e*x] + 132*a^4*c^2*Sin[d + e*x] + 72*a^2*b^2*c^2*S
in[d + e*x] + 36*b^4*c^2*Sin[d + e*x] + 81*a^2*c^4*Sin[d + e*x] + 36*b^2*c
^4*Sin[d + e*x] + 12*c^6*Sin[d + e*x] + 54*a^3*b^3*Sin[2*(d + e*x)] + 6*a*
b^5*Sin[2*(d + e*x)] + 78*a^3*b*c^2*Sin[2*(d + e*x)] + 48*a*b^3*c^2*Sin[2*
(d + e*x)] + 42*a*b*c^4*Sin[2*(d + e*x)] + 11*a^2*b^4*Sin[3*(d + e*x)] + 4
*b^6*Sin[3*(d + e*x)] + 4*b^4*c^2*Sin[3*(d + e*x)] - 11*a^2*c^4*Sin[3*(d +
 e*x)] - 4*b^2*c^4*Sin[3*(d + e*x)] - 4*c^6*Sin[3*(d + e*x)])/(b*(-a^2 + b
^2 + c^2)^3*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^3))/(24*e)
```

3.402.3 Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.16, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 3608, 25, 3042, 3635, 25, 3042, 3632, 3042, 3603, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^4}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^4}dx$$

$$\Big\downarrow \; 3608$$

$$\frac{c\cos(d+ex)-b\sin(d+ex)}{3e\left(a^2-b^2-c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))^3}-\frac{\int -\frac{3a-2b\cos(d+ex)-2c\sin(d+ex)}{(a+b\cos(d+ex)+c\sin(d+ex))^3}dx}{3\left(a^2-b^2-c^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{3a-2b\cos(d+ex)-2c\sin(d+ex)}{(a+b\cos(d+ex)+c\sin(d+ex))^3}dx}{3\left(a^2-b^2-c^2\right)}+\frac{c\cos(d+ex)-b\sin(d+ex)}{3e\left(a^2-b^2-c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{3a-2b\cos(d+ex)-2c\sin(d+ex)}{(a+b\cos(d+ex)+c\sin(d+ex))^3}dx}{3\left(a^2-b^2-c^2\right)}+\frac{c\cos(d+ex)-b\sin(d+ex)}{3e\left(a^2-b^2-c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))^3}$$

$$\Big\downarrow \; 3635$$

$$\frac{\frac{5(ac\cos(d+ex)-ab\sin(d+ex))}{2e(a^2-b^2-c^2)(a+b\cos(d+ex)+c\sin(d+ex))^2}-\frac{\int -\frac{2\left(3a^2+2\left(b^2+c^2\right)\right)-5ab\cos(d+ex)-5ac\sin(d+ex)}{(a+b\cos(d+ex)+c\sin(d+ex))^2}dx}{2(a^2-b^2-c^2)}}{3\left(a^2-b^2-c^2\right)}+\frac{c\cos(d+ex)-b\sin(d+ex)}{3e\left(a^2-b^2-c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))^3}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{2\left(3a^2+2\left(b^2+c^2\right)\right)-5ab\cos(d+ex)-5ac\sin(d+ex)}{(a+b\cos(d+ex)+c\sin(d+ex))^2}dx}{2(a^2-b^2-c^2)}+\frac{5(ac\cos(d+ex)-ab\sin(d+ex))}{2e(a^2-b^2-c^2)(a+b\cos(d+ex)+c\sin(d+ex))^2}}{3\left(a^2-b^2-c^2\right)}+\frac{c\cos(d+ex)-b\sin(d+ex)}{3e\left(a^2-b^2-c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{2\left(3a^2+2\left(b^2+c^2\right)\right)-5ab\cos(d+ex)-5ac\sin(d+ex)}{(a+b\cos(d+ex)+c\sin(d+ex))^2}dx}{2(a^2-b^2-c^2)}+\frac{5(ac\cos(d+ex)-ab\sin(d+ex))}{2e(a^2-b^2-c^2)(a+b\cos(d+ex)+c\sin(d+ex))^2}}{3\left(a^2-b^2-c^2\right)}+\frac{c\cos(d+ex)-b\sin(d+ex)}{3e\left(a^2-b^2-c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))^3}$$

$$\Big\downarrow \; 3632$$

3.402. $\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^4}\,dx$

$$\frac{\frac{\frac{3a\left(2a^2+3\left(b^2+c^2\right)\right)\int \frac{1}{a+b\cos(d+ex)+c\sin(d+ex)}dx}{a^2-b^2-c^2}+\frac{c\left(11a^2+4\left(b^2+c^2\right)\right)\cos(d+ex)-b\left(11a^2+4\left(b^2+c^2\right)\right)\sin(d+ex)}{e\left(a^2-b^2-c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))}}{2(a^2-b^2-c^2)}+\frac{5(ac\cos(d+ex)-ab\sin(d+}{2e(a^2-b^2-c^2)(a+b\cos(d+ex)+c}}{3\left(a^2-b^2-c^2\right)}$$

$$\frac{c\cos(d+ex)-b\sin(d+ex)}{3e\left(a^2-b^2-c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))^3}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{3a\left(2a^2+3\left(b^2+c^2\right)\right)\int \frac{1}{a+b\cos(d+ex)+c\sin(d+ex)}dx}{a^2-b^2-c^2}+\frac{c\left(11a^2+4\left(b^2+c^2\right)\right)\cos(d+ex)-b\left(11a^2+4\left(b^2+c^2\right)\right)\sin(d+ex)}{e\left(a^2-b^2-c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))}}{2(a^2-b^2-c^2)}+\frac{5(ac\cos(d+ex)-ab\sin(d+}{2e(a^2-b^2-c^2)(a+b\cos(d+ex)+c}}{3\left(a^2-b^2-c^2\right)}$$

$$\frac{c\cos(d+ex)-b\sin(d+ex)}{3e\left(a^2-b^2-c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))^3}$$

$\Big\downarrow$ 3603

$$\frac{\frac{\frac{6a\left(2a^2+3\left(b^2+c^2\right)\right)\int \frac{1}{(a-b)\tan^2\left(\frac{1}{2}(d+ex)\right)+2c\tan\left(\frac{1}{2}(d+ex)\right)+a+b}d\tan\left(\frac{1}{2}(d+ex)\right)}{e\left(a^2-b^2-c^2\right)}+\frac{c\left(11a^2+4\left(b^2+c^2\right)\right)\cos(d+ex)-b\left(11a^2+4\left(b^2+c^2\right)\right)\sin(d+ex)}{e\left(a^2-b^2-c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))}}{2(a^2-b^2-c^2)}+\frac{}{2e(a}}{3\left(a^2-b^2-c^2\right)}$$

$$\frac{c\cos(d+ex)-b\sin(d+ex)}{3e\left(a^2-b^2-c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))^3}$$

$\Big\downarrow$ 1083

$$\frac{\frac{\frac{c\left(11a^2+4\left(b^2+c^2\right)\right)\cos(d+ex)-b\left(11a^2+4\left(b^2+c^2\right)\right)\sin(d+ex)}{e\left(a^2-b^2-c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))}-\frac{12a\left(2a^2+3\left(b^2+c^2\right)\right)\int \frac{1}{-\left(2c+2(a-b)\tan\left(\frac{1}{2}(d+ex)\right)\right)^2-4\left(a^2-b^2-c^2\right)}d\left(2c+2(a-b)\tan\left(\frac{1}{2}(d+\right.\right.}{e\left(a^2-b^2-c^2\right)}}{2(a^2-b^2-c^2)}}{3\left(a^2-b^2-c^2\right)}$$

$$\frac{c\cos(d+ex)-b\sin(d+ex)}{3e\left(a^2-b^2-c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))^3}$$

$\Big\downarrow$ 217

$$\frac{\frac{\frac{6a\left(2a^2+3\left(b^2+c^2\right)\right)\arctan\left(\frac{2(a-b)\tan\left(\frac{1}{2}(d+ex)\right)+2c}{2\sqrt{a^2-b^2-c^2}}\right)}{e\left(a^2-b^2-c^2\right)^{3/2}}+\frac{c\left(11a^2+4\left(b^2+c^2\right)\right)\cos(d+ex)-b\left(11a^2+4\left(b^2+c^2\right)\right)\sin(d+ex)}{e\left(a^2-b^2-c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))}}{2(a^2-b^2-c^2)}+\frac{5(ac\cos(d+ex)-ab\sin}{2e(a^2-b^2-c^2)(a+b\cos(d+ex)}}{3\left(a^2-b^2-c^2\right)}$$

$$\frac{c\cos(d+ex)-b\sin(d+ex)}{3e\left(a^2-b^2-c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))^3}$$

input

```
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-4),x]
```

3.402. $\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^4}\,dx$

output
```
(c*Cos[d + e*x] - b*Sin[d + e*x])/(3*(a^2 - b^2 - c^2)*e*(a + b*Cos[d + e*
x] + c*Sin[d + e*x])^3) + ((5*(a*c*Cos[d + e*x] - a*b*Sin[d + e*x]))/(2*(a
^2 - b^2 - c^2)*e*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^2) + ((6*a*(2*a^2
+ 3*(b^2 + c^2))*ArcTan[(2*c + 2*(a - b)*Tan[(d + e*x)/2])/(2*Sqrt[a^2 - b
^2 - c^2])])/((a^2 - b^2 - c^2)^(3/2)*e) + (c*(11*a^2 + 4*(b^2 + c^2))*Cos
[d + e*x] - b*(11*a^2 + 4*(b^2 + c^2))*Sin[d + e*x])/((a^2 - b^2 - c^2)*e*
(a + b*Cos[d + e*x] + c*Sin[d + e*x])))/(2*(a^2 - b^2 - c^2)))/(3*(a^2 - b
^2 - c^2))
```

3.402.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

rule 3608
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Simp[
1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c
*(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x
] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] &&
NeQ[n, -3/2]
```

rule 3632
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Simp[(a*A - b*B - c*C)/(a^2 - b^2 - c^2)   Int[1/(a + b*Cos[d + e*x] + c*S
in[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
 - c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

rule 3635
```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(-(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(
a^2 - b^2 - c^2))), x] + Simp[1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a + b*Co
s[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n +
2)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x]
/; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[n, -2]
```

3.402.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.41 (sec) , antiderivative size = 1655, normalized size of antiderivative = 5.67

method	result	size
risch	Expression too large to display	1655
derivativedivides	Expression too large to display	1656
default	Expression too large to display	1656

input
```
int(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^4,x,method=_RETURNVERBOSE)
```

output
```
1/3*I*(-12*I*b*c^4*exp(2*I*(e*x+d))-82*I*a^3*b^2*exp(3*I*(e*x+d))-24*I*a*b
^4*exp(3*I*(e*x+d))+120*a^3*b*c*exp(I*(e*x+d))+8*I*b^3*c^2+12*I*b*c^4+30*a
*b^3*c*exp(I*(e*x+d))-24*I*b^3*c^2*exp(2*I*(e*x+d))+33*a^2*b^2*c+60*I*a^3*
c^2*exp(I*(e*x+d))+15*I*a*c^4*exp(I*(e*x+d))-4*c^5-30*a^4*c*exp(4*I*(e*x+d
))+102*a^4*c*exp(2*I*(e*x+d))-45*a^2*c^3*exp(4*I*(e*x+d))+36*a^2*c^3*exp(2
*I*(e*x+d))-6*I*a^3*b^2*exp(5*I*(e*x+d))-30*I*a^4*b*exp(4*I*(e*x+d))-24*I*
a*c^4*exp(3*I*(e*x+d))-44*I*exp(3*I*(e*x+d))*a^5-12*I*b^5*exp(2*I*(e*x+d))
-48*I*a*b^2*c^2*exp(3*I*(e*x+d))-11*a^2*c^3+9*I*a*c^4*exp(5*I*(e*x+d))-60*
I*a^3*b^2*exp(I*(e*x+d))-15*I*a*b^4*exp(I*(e*x+d))+24*b^2*c^3*exp(2*I*(e*x
+d))+12*b^4*c*exp(2*I*(e*x+d))-11*I*a^2*b^3-4*I*b^5+12*c^5*exp(2*I*(e*x+d)
)+12*b^4*c+8*b^2*c^3+6*I*a^3*c^2*exp(5*I*(e*x+d))+33*I*a^2*b*c^2-82*I*a^3*
c^2*exp(3*I*(e*x+d))-9*I*a*b^4*exp(5*I*(e*x+d))-36*I*a^2*b^3*exp(2*I*(e*x+
d))-12*a^3*b*c*exp(5*I*(e*x+d))-36*I*a^2*b*c^2*exp(2*I*(e*x+d))+36*a^2*b^2
*c*exp(2*I*(e*x+d))-102*I*a^4*b*exp(2*I*(e*x+d))-45*I*a^2*b*c^2*exp(4*I*(e
*x+d))-45*a^2*b^2*c*exp(4*I*(e*x+d))+30*a*b*c^3*exp(I*(e*x+d))-45*I*a^2*b^
3*exp(4*I*(e*x+d))-18*a*b*c^3*exp(5*I*(e*x+d))-18*a*b^3*c*exp(5*I*(e*x+d))
)/(c*exp(2*I*(e*x+d))+I*b*exp(2*I*(e*x+d))-c+2*I*a*exp(I*(e*x+d))+I*b)^3/(
-a^2+b^2+c^2)^3/e-1/(-a^2+b^2+c^2)^(1/2)*a^3/(a^2-b^2-c^2)^3/e*ln(exp(I*(e
*x+d))+(I*a*c*(-a^2+b^2+c^2)^(1/2)+I*a^2*b-I*b^3-I*b*c^2+a*b*(-a^2+b^2+c^2
)^(1/2)-a^2*c+b^2*c+c^3)/(b^2+c^2)/(-a^2+b^2+c^2)^(1/2))-3/2/(-a^2+b^2+...
```

3.402.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1940 vs. $2(284) = 568$.

Time = 0.47 (sec) , antiderivative size = 4069, normalized size of antiderivative = 13.93

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^4,x, algorithm="fricas")
```

output
```
[1/12*(6*a*b*c^5 + 12*(4*a^3*b + a*b^3)*c^3 + 2*(4*c^7 + (7*a^2 - 4*b^2)*c
^5 - (11*a^4 + 14*a^2*b^2 + 20*b^4)*c^3 + 3*(11*a^4*b^2 - 7*a^2*b^4 - 4*b^
6)*c)*cos(e*x + d)^3 - 12*(a*b*c^5 + 2*(4*a^3*b + a*b^3)*c^3 - (9*a^5*b -
8*a^3*b^3 - a*b^5)*c)*cos(e*x + d)^2 + 3*(2*a^6 + 3*a^4*b^2 + 9*a^2*c^4 +
(2*a^3*b^3 + 3*a*b^5 - 9*a*b*c^4 - 6*(a^3*b + a*b^3)*c^2)*cos(e*x + d)^3 +
 9*(a^4 + a^2*b^2)*c^2 + 3*(2*a^4*b^2 + 3*a^2*b^4 - 2*a^4*c^2 - 3*a^2*c^4)
*cos(e*x + d)^2 + 3*(2*a^5*b + 3*a^3*b^3 + 3*a*b*c^4 + (5*a^3*b + 3*a*b^3)
*c^2)*cos(e*x + d) + (3*a*c^5 + (11*a^3 + 3*a*b^2)*c^3 - (3*a*c^5 + 2*(a^3
 - 3*a*b^2)*c^3 - 3*(2*a^3*b^2 + 3*a*b^4)*c)*cos(e*x + d)^2 + 3*(2*a^5 + 3
*a^3*b^2)*c + 6*(3*a^2*b*c^3 + (2*a^4*b + 3*a^2*b^3)*c)*cos(e*x + d))*sin(
e*x + d))*sqrt(-a^2 + b^2 + c^2)*log((a^2*b^2 - 2*b^4 - c^4 - (a^2 + 3*b^2
)*c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(e*x + d)^2 - 2*(a*b^3 + a*
b*c^2)*cos(e*x + d) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*cos
(e*x + d))*sin(e*x + d) - 2*(2*a*b*c*cos(e*x + d)^2 - a*b*c + (b^2*c + c^3
)*cos(e*x + d) - (b^3 + b*c^2 + (a*b^2 - a*c^2)*cos(e*x + d))*sin(e*x + d)
)*sqrt(-a^2 + b^2 + c^2))/(2*a*b*cos(e*x + d) + (b^2 - c^2)*cos(e*x + d)^2
 + a^2 + c^2 + 2*(b*c*cos(e*x + d) + a*c)*sin(e*x + d))) - 6*(9*a^5*b - 8*
a^3*b^3 - a*b^5)*c - 6*(2*b^2*c^5 + 2*c^7 + (4*a^4 - 7*a^2*b^2 - 2*b^4)*c^
3 - (6*a^6 - 15*a^4*b^2 + 7*a^2*b^4 + 2*b^6)*c)*cos(e*x + d) - 2*(18*a^6*b
 - 23*a^4*b^3 + 7*a^2*b^5 - 2*b^7 - 14*b^3*c^4 - 6*b*c^6 - (12*a^4*b - ...
```

3.402.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a + b\cos(d + ex) + c\sin(d + ex))^4}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))**4,x)
```

output
```
Timed out
```

3.402.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` f
or more de
```

3.402.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2558 vs. $2(284) = 568$.

Time = 0.42 (sec) , antiderivative size = 2558, normalized size of antiderivative = 8.76

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^4,x, algorithm="giac")
```

output
```
-1/3*(3*(2*a^3 + 3*a*b^2 + 3*a*c^2)*(pi*floor(1/2*(e*x + d)/pi + 1/2)*sgn(
-2*a + 2*b) + arctan(-(a*tan(1/2*e*x + 1/2*d) - b*tan(1/2*e*x + 1/2*d) + c
)/sqrt(a^2 - b^2 - c^2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6 - 3*a^4*c^2
+ 6*a^2*b^2*c^2 - 3*b^4*c^2 + 3*a^2*c^4 - 3*b^2*c^4 - c^6)*sqrt(a^2 - b^2
- c^2)) + (18*a^7*b*tan(1/2*e*x + 1/2*d)^5 - 81*a^6*b^2*tan(1/2*e*x + 1/2*
d)^5 + 141*a^5*b^3*tan(1/2*e*x + 1/2*d)^5 - 120*a^4*b^4*tan(1/2*e*x + 1/2*
d)^5 + 60*a^3*b^5*tan(1/2*e*x + 1/2*d)^5 - 33*a^2*b^6*tan(1/2*e*x + 1/2*d)
^5 + 21*a*b^7*tan(1/2*e*x + 1/2*d)^5 - 6*b^8*tan(1/2*e*x + 1/2*d)^5 - 27*a
^6*c^2*tan(1/2*e*x + 1/2*d)^5 + 81*a^5*b*c^2*tan(1/2*e*x + 1/2*d)^5 - 72*a
^4*b^2*c^2*tan(1/2*e*x + 1/2*d)^5 + 18*a^3*b^3*c^2*tan(1/2*e*x + 1/2*d)^5
- 27*a^2*b^4*c^2*tan(1/2*e*x + 1/2*d)^5 + 45*a*b^5*c^2*tan(1/2*e*x + 1/2*d
)^5 - 18*b^6*c^2*tan(1/2*e*x + 1/2*d)^5 + 18*a^4*c^4*tan(1/2*e*x + 1/2*d)^
5 - 36*a^3*b*c^4*tan(1/2*e*x + 1/2*d)^5 + 36*a*b^3*c^4*tan(1/2*e*x + 1/2*d
)^5 - 18*b^4*c^4*tan(1/2*e*x + 1/2*d)^5 - 6*a^2*c^6*tan(1/2*e*x + 1/2*d)^5
 + 12*a*b*c^6*tan(1/2*e*x + 1/2*d)^5 - 6*b^2*c^6*tan(1/2*e*x + 1/2*d)^5 -
18*a^7*c*tan(1/2*e*x + 1/2*d)^4 + 108*a^6*b*c*tan(1/2*e*x + 1/2*d)^4 - 261
*a^5*b^2*c*tan(1/2*e*x + 1/2*d)^4 + 336*a^4*b^3*c*tan(1/2*e*x + 1/2*d)^4 -
 264*a^3*b^4*c*tan(1/2*e*x + 1/2*d)^4 + 144*a^2*b^5*c*tan(1/2*e*x + 1/2*d)
^4 - 57*a*b^6*c*tan(1/2*e*x + 1/2*d)^4 + 12*b^7*c*tan(1/2*e*x + 1/2*d)^4 -
 81*a^5*c^3*tan(1/2*e*x + 1/2*d)^4 + 216*a^4*b*c^3*tan(1/2*e*x + 1/2*d)...
```

3.402.9 Mupad [B] (verification not implemented)

Time = 29.88 (sec) , antiderivative size = 1946, normalized size of antiderivative = 6.66

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^4}\,dx = \text{Too large to display}$$

input
```
int(1/(a + b*cos(d + e*x) + c*sin(d + e*x))^4,x)
```

output
```
(a*atanh((a*(2*a^2 + 3*b^2 + 3*c^2)*(2*b^6*c - 2*a^6*c + 2*c^7 - 6*a^2*c^5
 + 6*a^4*c^3 + 6*b^2*c^5 + 6*b^4*c^3 - 6*a^2*b^4*c + 6*a^4*b^2*c - 12*a^2*
b^2*c^3))/(2*(b^2 - a^2 + c^2)^(7/2)*(3*a*b^2 + 3*a*c^2 + 2*a^3)) + (a*tan
(d/2 + (e*x)/2)*(2*a - 2*b)*(2*a^2 + 3*b^2 + 3*c^2)*(b^6 - a^6 + c^6 - 3*a
^2*b^4 + 3*a^4*b^2 - 3*a^2*c^4 + 3*a^4*c^2 + 3*b^2*c^4 + 3*b^4*c^2 - 6*a^2
*b^2*c^2))/(2*(b^2 - a^2 + c^2)^(7/2)*(3*a*b^2 + 3*a*c^2 + 2*a^3)))*(2*a^2
 + 3*b^2 + 3*c^2))/(e*(b^2 - a^2 + c^2)^(7/2)) - ((18*a^7*c + 2*a^3*c^5 -
5*a^5*c^3 + 6*a*b^2*c^5 + 21*a*b^4*c^3 - 12*a^3*b^4*c - 21*a^5*b^2*c - 16*
a^3*b^2*c^3 + 15*a*b^6*c)/(3*(a - b)^3*(b^6 - a^6 + c^6 - 3*a^2*b^4 + 3*a^
4*b^2 - 3*a^2*c^4 + 3*a^4*c^2 + 3*b^2*c^4 + 3*b^4*c^2 - 6*a^2*b^2*c^2)) +
(tan(d/2 + (e*x)/2)*(2*b^8 - 6*a^7*b - 5*a*b^7 + 5*a^2*b^6 + 4*a^3*b^5 - 1
6*a^4*b^4 + 7*a^5*b^3 + 9*a^6*b^2 + 2*a^2*c^6 - 4*a^4*c^4 + 27*a^6*c^2 + 2
*b^2*c^6 + 6*b^4*c^4 + 6*b^6*c^2 + 18*a*b^3*c^4 + 9*a*b^5*c^2 - 14*a^3*b*c
^4 - 9*a^5*b*c^2 - 6*a^2*b^2*c^4 - 3*a^2*b^4*c^2 - 30*a^4*b^2*c^2 + 4*a*b*
c^6))/((a - b)^3*(b^6 - a^6 + c^6 - 3*a^2*b^4 + 3*a^4*b^2 - 3*a^2*c^4 + 3*
a^4*c^2 + 3*b^2*c^4 + 3*b^4*c^2 - 6*a^2*b^2*c^2)) + (tan(d/2 + (e*x)/2)^4*
(6*a^6*c + 4*b^6*c + 4*c^7 - 12*a^2*c^5 + 27*a^4*c^3 + 12*b^2*c^5 + 12*b^4
*c^3 - 15*a*b^3*c^3 + 33*a^2*b^4*c - 45*a^3*b*c^3 - 55*a^3*b^3*c + 57*a^4*
b^2*c + 21*a^2*b^2*c^3 - 15*a*b^5*c - 30*a^5*b*c))/((a - b)^2*(b^6 - a^6 +
 c^6 - 3*a^2*b^4 + 3*a^4*b^2 - 3*a^2*c^4 + 3*a^4*c^2 + 3*b^2*c^4 + 3*b^...
```

3.402. $\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^4}\,dx$

3.403 $\int (2+3\cos(d+ex)+5\sin(d+ex))^{5/2}\,dx$

3.403.1 Optimal result

Integrand size = 22, antiderivative size = 185

$$\int (2+3\cos(d+ex)+5\sin(d+ex))^{5/2}\,dx = \frac{796\sqrt{2+\sqrt{34}}\,E\left(\frac{1}{2}\left(d+ex-\arctan\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{15e} + \frac{64\,\text{EllipticF}\left(\frac{1}{2}\left(d+ex-\arctan\left(\frac{5}{3}\right)\right),\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{\sqrt{2+\sqrt{34}}\,e} - \frac{32(5\cos(d+ex)-3\sin(d+ex))\sqrt{2+3\cos(d+ex)+5\sin(d+ex)}}{15e} - \frac{2(5\cos(d+ex)-3\sin(d+ex))(2+3\cos(d+ex)+5\sin(d+ex))^{3/2}}{5e}$$

output
```
-2/5*(5*cos(e*x+d)-3*sin(e*x+d))*(2+3*cos(e*x+d)+5*sin(e*x+d))^(3/2)/e-32/
15*(5*cos(e*x+d)-3*sin(e*x+d))*(2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2)/e+64*(c
os(1/2*d+1/2*e*x-1/2*arctan(5/3))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(5/
3))*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan(5/3)),1/15*(510-30*34^(1/2))^(1
/2))/e/(2+34^(1/2))^(1/2)+796/15*(cos(1/2*d+1/2*e*x-1/2*arctan(5/3))^2)^(1
/2)/cos(1/2*d+1/2*e*x-1/2*arctan(5/3))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arc
tan(5/3)),1/15*(510-30*34^(1/2))^(1/2))*(2+34^(1/2))^(1/2)/e
```

3.403.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 4.94 (sec) , antiderivative size = 399, normalized size of antiderivative = 2.16

$$\int (2+3\cos(d+ex)+5\sin(d+ex))^{5/2}\,dx = \frac{-2388\sqrt{2+\sqrt{34}\cos\left(d+ex-\arctan\left(\frac{5}{3}\right)\right)}-2\sqrt{2+3\cos(d+ex)+5\sin(d+ex)}(550\cos(d}{}$$

input
```
Integrate[(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(5/2),x]
```

output
```
(-2388*Sqrt[2 + Sqrt[34]*Cos[d + e*x - ArcTan[5/3]]] - 2*Sqrt[2 + 3*Cos[d
+ e*x] + 5*Sin[d + e*x]]*(550*Cos[d + e*x] + 3*(-398 + 75*Cos[2*(d + e*x)]
 - 110*Sin[d + e*x] + 40*Sin[2*(d + e*x)])) + 1276*Sqrt[10/3]*AppellF1[1/2
, 1/2, 1/2, 3/2, (Sqrt[34] + 17*Sin[d + e*x + ArcTan[3/5]])/(-17 + Sqrt[34
]), (Sqrt[34] + 17*Sin[d + e*x + ArcTan[3/5]])/(17 + Sqrt[34])]*Sqrt[Cos[d
 + e*x + ArcTan[3/5]]^2]*Sec[d + e*x + ArcTan[3/5]]*Sqrt[2 + Sqrt[34]*Sin[
d + e*x + ArcTan[3/5]]] + (1990*Sin[d + e*x - ArcTan[5/3]])/Sqrt[1/17 + Co
s[d + e*x - ArcTan[5/3]]/Sqrt[34]] - (1990*Sqrt[30]*AppellF1[-1/2, -1/2, -
1/2, 1/2, (Sqrt[34] + 17*Cos[d + e*x - ArcTan[5/3]])/(-17 + Sqrt[34]), (Sq
rt[34] + 17*Cos[d + e*x - ArcTan[5/3]])/(17 + Sqrt[34])]*Csc[d + e*x - Arc
Tan[5/3]]*Sqrt[Sin[d + e*x - ArcTan[5/3]]^2])/Sqrt[2 + Sqrt[34]*Cos[d + e*
x - ArcTan[5/3]]])/(75*e)
```

3.403.3 Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.04, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {3042, 3599, 3042, 3625, 3042, 3628, 3042, 3597, 3042, 3132, 3605, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (5\sin(d+ex)+3\cos(d+ex)+2)^{5/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (5\sin(d+ex)+3\cos(d+ex)+2)^{5/2}dx$$

$$\Big\downarrow \text{3599}$$

$$\frac{2}{5}\int \sqrt{3\cos(d+ex)+5\sin(d+ex)+2}(24\cos(d+ex)+40\sin(d+ex)+61)dx - \frac{2(5\cos(d+ex)-3\sin(d+ex))(5\sin(d+ex)+3\cos(d+ex)+2)^{3/2}}{5e}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}\int \sqrt{3\cos(d+ex)+5\sin(d+ex)+2}(24\cos(d+ex)+40\sin(d+ex)+61)dx - \frac{2(5\cos(d+ex)-3\sin(d+ex))(5\sin(d+ex)+3\cos(d+ex)+2)^{3/2}}{5e}$$

$$\Big\downarrow \text{3625}$$

$$\frac{2}{5}\left(\frac{1}{3}\int \frac{597\cos(d+ex)+995\sin(d+ex)+638}{\sqrt{3\cos(d+ex)+5\sin(d+ex)+2}}dx - \frac{16(5\cos(d+ex)-3\sin(d+ex))\sqrt{5\sin(d+ex)+3\cos(d-}}{3e}\right. \frac{2(5\cos(d+ex)-3\sin(d+ex))(5\sin(d+ex)+3\cos(d+ex)+2)^{3/2}}{5e}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}\left(\frac{1}{3}\int \frac{597\cos(d+ex)+995\sin(d+ex)+638}{\sqrt{3\cos(d+ex)+5\sin(d+ex)+2}}dx - \frac{16(5\cos(d+ex)-3\sin(d+ex))\sqrt{5\sin(d+ex)+3\cos(d-}}{3e}\right. \frac{2(5\cos(d+ex)-3\sin(d+ex))(5\sin(d+ex)+3\cos(d+ex)+2)^{3/2}}{5e}$$

$$\Big\downarrow \text{3628}$$

$$\frac{2}{5}\left(\frac{1}{3}\left(240\int \frac{1}{\sqrt{3\cos(d+ex)+5\sin(d+ex)+2}}dx+199\int \sqrt{3\cos(d+ex)+5\sin(d+ex)+2}dx\right) - \frac{16(5\cos(}{}\right. \frac{2(5\cos(d+ex)-3\sin(d+ex))(5\sin(d+ex)+3\cos(d+ex)+2)^{3/2}}{5e}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}\left(\frac{1}{3}\left(240\int \frac{1}{\sqrt{3\cos(d+ex)+5\sin(d+ex)+2}}dx+199\int \sqrt{3\cos(d+ex)+5\sin(d+ex)+2}dx\right) - \frac{16(5\cos(}{}\right. \frac{2(5\cos(d+ex)-3\sin(d+ex))(5\sin(d+ex)+3\cos(d+ex)+2)^{3/2}}{5e}$$

$$\Big\downarrow \text{3597}$$

3.403. $\int (2+3\cos(d+ex)+5\sin(d+ex))^{5/2}\,dx$

$$\frac{2}{5}\left(\frac{1}{3}\left(199\int\sqrt{\sqrt{34}\cos\left(d+ex-\arctan\left(\frac{5}{3}\right)\right)+2}dx+240\int\frac{1}{\sqrt{3\cos(d+ex)+5\sin(d+ex)+2}}dx\right)-\frac{16(5\right.$$
$$\frac{2(5\cos(d+ex)-3\sin(d+ex))(5\sin(d+ex)+3\cos(d+ex)+2)^{3/2}}{5e}$$

$\downarrow$ 3042

$$\frac{2}{5}\left(\frac{1}{3}\left(199\int\sqrt{\sqrt{34}\sin\left(d+ex-\arctan\left(\frac{5}{3}\right)+\frac{\pi}{2}\right)+2}dx+240\int\frac{1}{\sqrt{3\cos(d+ex)+5\sin(d+ex)+2}}dx\right)-\right.$$
$$\frac{2(5\cos(d+ex)-3\sin(d+ex))(5\sin(d+ex)+3\cos(d+ex)+2)^{3/2}}{5e}$$

$\downarrow$ 3132

$$\frac{2}{5}\left(\frac{1}{3}\left(240\int\frac{1}{\sqrt{3\cos(d+ex)+5\sin(d+ex)+2}}dx+\frac{398\sqrt{2+\sqrt{34}}E\left(\frac{1}{2}\left(d+ex-\arctan\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{e}\right)\right.$$
$$\frac{2(5\cos(d+ex)-3\sin(d+ex))(5\sin(d+ex)+3\cos(d+ex)+2)^{3/2}}{5e}$$

$\downarrow$ 3605

$$\frac{2}{5}\left(\frac{1}{3}\left(240\int\frac{1}{\sqrt{\sqrt{34}\cos\left(d+ex-\arctan\left(\frac{5}{3}\right)\right)+2}}dx+\frac{398\sqrt{2+\sqrt{34}}E\left(\frac{1}{2}\left(d+ex-\arctan\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{e}\right.\right.$$
$$\frac{2(5\cos(d+ex)-3\sin(d+ex))(5\sin(d+ex)+3\cos(d+ex)+2)^{3/2}}{5e}$$

$\downarrow$ 3042

$$\frac{2}{5}\left(\frac{1}{3}\left(240\int\frac{1}{\sqrt{\sqrt{34}\sin\left(d+ex-\arctan\left(\frac{5}{3}\right)+\frac{\pi}{2}\right)+2}}dx+\frac{398\sqrt{2+\sqrt{34}}E\left(\frac{1}{2}\left(d+ex-\arctan\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{\right.\right.\right.}{e}$$
$$\frac{2(5\cos(d+ex)-3\sin(d+ex))(5\sin(d+ex)+3\cos(d+ex)+2)^{3/2}}{5e}$$

$\downarrow$ 3140

$$\frac{2}{5}\left(\frac{1}{3}\left(\frac{480\,\text{EllipticF}\left(\frac{1}{2}\left(d+ex-\arctan\left(\frac{5}{3}\right)\right),\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{\sqrt{2+\sqrt{34}}e}+\frac{398\sqrt{2+\sqrt{34}}E\left(\frac{1}{2}\left(d+ex-\arctan\left(\frac{5}{3}\right)\right)|\frac{2}{15}(17\right.}{e}\right.\right.$$
$$\frac{2(5\cos(d+ex)-3\sin(d+ex))(5\sin(d+ex)+3\cos(d+ex)+2)^{3/2}}{5e}$$

3.403. $\int(2+3\cos(d+ex)+5\sin(d+ex))^{5/2}\,dx$

input
```
Int[(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(5/2),x]
```

output
```
(-2*(5*Cos[d + e*x] - 3*Sin[d + e*x])*(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x]
)^(3/2))/(5*e) + (2*(((398*Sqrt[2 + Sqrt[34]]*EllipticE[(d + e*x - ArcTan[
5/3])/2, (2*(17 - Sqrt[34]))/15])/e + (480*EllipticF[(d + e*x - ArcTan[5/3
])/2, (2*(17 - Sqrt[34]))/15])/(Sqrt[2 + Sqrt[34]]*e))/3 - (16*(5*Cos[d +
e*x] - 3*Sin[d + e*x])*Sqrt[2 + 3*Cos[d + e*x] + 5*Sin[d + e*x]])/(3*e)))/
5
```

3.403.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3597
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Int[Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 + c^2, 0] && GtQ[a + Sqrt[b^2
+ c^2], 0]
```

rule 3599
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Simp[1/n   Int[Simp[n*a^2 + (
n - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x
], x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

rule 3605
```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 + c^2, 0] && GtQ[a + Sqrt[
b^2 + c^2], 0]
```

rule 3625
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*((a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^n/(a*e*(n + 1))), x] + Simp[1/(a*(n + 1
))   Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
 + a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
 + (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

rule 3628
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]
, x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[
A*b - a*B, 0]
```

3.403.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.04 (sec) , antiderivative size = 821, normalized size of antiderivative = 4.44

method	result	size
default	Expression too large to display	821

input
```
int((2+3*cos(e*x+d)+5*sin(e*x+d))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(424/17*((17*sin(e*x+d+arctan(3/5))+34^(1/2))/(34^(1/2)+17))^(1/2)*17^(1/2
)*((sin(e*x+d+arctan(3/5))+1)/(-34^(1/2)+17))^(1/2)*(-17*(sin(e*x+d+arctan
(3/5))-1)/(34^(1/2)+17))^(1/2)*EllipticF(((17*sin(e*x+d+arctan(3/5))+34^(1
/2))/(34^(1/2)+17))^(1/2),I*(1/(-34^(1/2)+17)*(34^(1/2)+17))^(1/2))*34^(1/
2)+184*((17*sin(e*x+d+arctan(3/5))+34^(1/2))/(34^(1/2)+17))^(1/2)*17^(1/2)
*((sin(e*x+d+arctan(3/5))+1)/(-34^(1/2)+17))^(1/2)*(-17*(sin(e*x+d+arctan(
3/5))-1)/(34^(1/2)+17))^(1/2)*EllipticF(((17*sin(e*x+d+arctan(3/5))+34^(1/
2))/(34^(1/2)+17))^(1/2),I*(1/(-34^(1/2)+17)*(34^(1/2)+17))^(1/2))+1904/15
*sin(e*x+d+arctan(3/5))^3-1904/15*sin(e*x+d+arctan(3/5))-116/15*34^(1/2)*s
in(e*x+d+arctan(3/5))^2-88/15*34^(1/2)-240/17*34^(1/2)*((17*sin(e*x+d+arct
an(3/5))+34^(1/2))/(34^(1/2)+17))^(1/2)*17^(1/2)*((sin(e*x+d+arctan(3/5))+
1)/(-34^(1/2)+17))^(1/2)*(-17*(sin(e*x+d+arctan(3/5))-1)/(34^(1/2)+17))^(1
/2)*EllipticE(((17*sin(e*x+d+arctan(3/5))+34^(1/2))/(34^(1/2)+17))^(1/2),I
*(1/(-34^(1/2)+17)*(34^(1/2)+17))^(1/2))+1036/17*34^(1/2)*17^(1/2)*((sin(e
*x+d+arctan(3/5))+1)/(-34^(1/2)+17))^(1/2)*(-17*(sin(e*x+d+arctan(3/5))-1)
/(34^(1/2)+17))^(1/2)*(-(17*sin(e*x+d+arctan(3/5))+34^(1/2))/(-34^(1/2)+17
))^(1/2)*EllipticE((-(17*sin(e*x+d+arctan(3/5))+34^(1/2))/(-34^(1/2)+17))^
(1/2),I*((-34^(1/2)+17)/(34^(1/2)+17))^(1/2))+120*(-(17*sin(e*x+d+arctan(3
/5))+34^(1/2))/(-34^(1/2)+17))^(1/2)*(-17*(sin(e*x+d+arctan(3/5))-1)/(34^(
1/2)+17))^(1/2)*17^(1/2)*((sin(e*x+d+arctan(3/5))+1)/(-34^(1/2)+17))^(1...
```

3.403.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.97

$$\int (2+3\cos(d+ex)+5\sin(d+ex))^{5/2}\,dx =$$

$$-\frac{2\left(-(1677i+2795)\,\sqrt{5i+3}\sqrt{2}\text{weierstrassPInverse}\left(\frac{860}{289}i+\frac{1376}{867},-\frac{5480}{132651}i-\frac{12056}{14739},\cos\left(ex+d\right)-i\,\sin\left(ex\right.\right.\right.}{}$$

input
```
integrate((2+3*cos(e*x+d)+5*sin(e*x+d))^(5/2),x, algorithm="fricas")
```

output
```
-2/765*(-(1677*I + 2795)*sqrt(5*I + 3)*sqrt(2)*weierstrassPInverse(860/289
*I + 1376/867, -5480/132651*I - 12056/14739, cos(e*x + d) - I*sin(e*x + d)
 - 10/51*I + 2/17) + (1677*I - 2795)*sqrt(-5*I + 3)*sqrt(2)*weierstrassPIn
verse(-860/289*I + 1376/867, 5480/132651*I - 12056/14739, cos(e*x + d) + I
*sin(e*x + d) + 10/51*I + 2/17) + 10149*I*sqrt(5*I + 3)*sqrt(2)*weierstras
sZeta(860/289*I + 1376/867, -5480/132651*I - 12056/14739, weierstrassPInve
rse(860/289*I + 1376/867, -5480/132651*I - 12056/14739, cos(e*x + d) - I*s
in(e*x + d) - 10/51*I + 2/17)) - 10149*I*sqrt(-5*I + 3)*sqrt(2)*weierstras
sZeta(-860/289*I + 1376/867, 5480/132651*I - 12056/14739, weierstrassPInve
rse(-860/289*I + 1376/867, 5480/132651*I - 12056/14739, cos(e*x + d) + I*s
in(e*x + d) + 10/51*I + 2/17)) + 51*(90*cos(e*x + d)^2 + 6*(8*cos(e*x + d)
 - 11)*sin(e*x + d) + 110*cos(e*x + d) - 45)*sqrt(3*cos(e*x + d) + 5*sin(e
*x + d) + 2))/e
```

3.403.6 Sympy [F(-1)]

Timed out.

$$\int (2+3\cos(d+ex)+5\sin(d+ex))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((2+3*cos(e*x+d)+5*sin(e*x+d))**(5/2),x)
```

output
```
Timed out
```

3.403.7 Maxima [F]

$$\int (2+3\cos(d+ex)+5\sin(d+ex))^{5/2}\,dx = \int (3\,\cos(ex+d)+5\,\sin(ex+d)+2)^{\frac{5}{2}}\,dx$$

input
```
integrate((2+3*cos(e*x+d)+5*sin(e*x+d))^(5/2),x, algorithm="maxima")
```

output
```
integrate((3*cos(e*x + d) + 5*sin(e*x + d) + 2)^(5/2), x)
```

3.403.8 Giac [F]

$$\int (2+3\cos(d+ex)+5\sin(d+ex))^{5/2}\,dx=\int (3\,\cos(ex+d)+5\,\sin(ex+d)+2)^{\frac{5}{2}}\,dx$$

input
```
integrate((2+3*cos(e*x+d)+5*sin(e*x+d))^(5/2),x, algorithm="giac")
```

output
```
integrate((3*cos(e*x + d) + 5*sin(e*x + d) + 2)^(5/2), x)
```

3.403.9 Mupad [F(-1)]

Timed out.

$$\int (2+3\cos(d+ex)+5\sin(d+ex))^{5/2}\,dx=\int (3\,\cos(d+e\,x)+5\,\sin(d+e\,x)+2)^{5/2}\,dx$$

input
```
int((3*cos(d + e*x) + 5*sin(d + e*x) + 2)^(5/2),x)
```

output
```
int((3*cos(d + e*x) + 5*sin(d + e*x) + 2)^(5/2), x)
```

3.404 $\int (2+3\cos(d+ex)+5\sin(d+ex))^{3/2}\,dx$

3.404.1 Optimal result

Integrand size = 22, antiderivative size = 139

$$\int (2+3\cos(d+ex)+5\sin(d+ex))^{3/2}\,dx = \frac{16\sqrt{2+\sqrt{34}}E\left(\frac{1}{2}\left(d+ex-\arctan\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{3e} + \frac{20\,\text{EllipticF}\left(\frac{1}{2}\left(d+ex-\arctan\left(\frac{5}{3}\right)\right),\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{\sqrt{2+\sqrt{34}}e} - \frac{2(5\cos(d+ex)-3\sin(d+ex))\sqrt{2+3\cos(d+ex)+5\sin(d+ex)}}{3e}$$

output
```
-2/3*(5*cos(e*x+d)-3*sin(e*x+d))*(2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2)/e+20*
(cos(1/2*d+1/2*e*x-1/2*arctan(5/3))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(
5/3))*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan(5/3)),1/15*(510-30*34^(1/2))^
(1/2))/e/(2+34^(1/2))^(1/2)+16/3*(cos(1/2*d+1/2*e*x-1/2*arctan(5/3))^2)^(1
/2)/cos(1/2*d+1/2*e*x-1/2*arctan(5/3))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arc
tan(5/3)),1/15*(510-30*34^(1/2))^(1/2))*(2+34^(1/2))^(1/2)/e
```

3.404.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 2.86 (sec) , antiderivative size = 349, normalized size of antiderivative = 2.51

$$\int (2+3\cos(d+ex)+5\sin(d+ex))^{3/2}\,dx = \frac{2\left(-60\sqrt{30}\,\text{AppellF1}\left(-\frac{1}{2},-\frac{1}{2},-\frac{1}{2},\frac{1}{2},\frac{\sqrt{34}+17\cos\left(d+ex-\arctan\left(\frac{5}{3}\right)\right)}{-17+\sqrt{34}},\frac{\sqrt{34}+17\cos\left(d+ex-\arctan\left(\frac{5}{3}\right)\right)}{17+\sqrt{34}}\right)\sin(\cdots}{\cdots}$$

input
```
Integrate[(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(3/2),x]
```

output
```
(2*(-60*Sqrt[30]*AppellF1[-1/2, -1/2, -1/2, 1/2, (Sqrt[34] + 17*Cos[d + e*
x - ArcTan[5/3]])/(-17 + Sqrt[34]), (Sqrt[34] + 17*Cos[d + e*x - ArcTan[5/
3]])/(17 + Sqrt[34])]*Sin[d + e*x - ArcTan[5/3]] + (-15*(30*Cos[d + e*x] +
 15*Cos[2*(d + e*x)] - 18*Sin[d + e*x] + 8*Sin[2*(d + e*x)]) + 23*Sqrt[30]
*AppellF1[1/2, 1/2, 1/2, 3/2, (Sqrt[34] + 17*Sin[d + e*x + ArcTan[3/5]])/(
-17 + Sqrt[34]), (Sqrt[34] + 17*Sin[d + e*x + ArcTan[3/5]])/(17 + Sqrt[34]
)]*Sqrt[Cos[d + e*x + ArcTan[3/5]]^2]*Sqrt[2 + Sqrt[34]*Cos[d + e*x - ArcT
an[5/3]]]*Sec[d + e*x + ArcTan[3/5]]*Sqrt[2 + Sqrt[34]*Sin[d + e*x + ArcTa
n[3/5]]])*Sqrt[Sin[d + e*x - ArcTan[5/3]]^2]))/(45*e*Sqrt[2 + Sqrt[34]*Cos
[d + e*x - ArcTan[5/3]]]*Sqrt[Sin[d + e*x - ArcTan[5/3]]^2])
```

3.404.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3599, 3042, 3628, 3042, 3597, 3042, 3132, 3605, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (5\sin(d+ex)+3\cos(d+ex)+2)^{3/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (5\sin(d+ex)+3\cos(d+ex)+2)^{3/2}dx$$

$$\Big\downarrow \text{3599}$$

$$\frac{2}{3}\int \frac{12\cos(d+ex)+20\sin(d+ex)+23}{\sqrt{3\cos(d+ex)+5\sin(d+ex)+2}}dx-\frac{2(5\cos(d+ex)-3\sin(d+ex))\sqrt{5\sin(d+ex)+3\cos(d+ex)+2}}{3e}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\int \frac{12\cos(d+ex)+20\sin(d+ex)+23}{\sqrt{3\cos(d+ex)+5\sin(d+ex)+2}}dx-\frac{2(5\cos(d+ex)-3\sin(d+ex))\sqrt{5\sin(d+ex)+3\cos(d+ex)+2}}{3e}$$

$$\Big\downarrow \text{3628}$$

$$\frac{2}{3}\left(15\int \frac{1}{\sqrt{3\cos(d+ex)+5\sin(d+ex)+2}}dx+4\int \sqrt{3\cos(d+ex)+5\sin(d+ex)+2}dx\right)-\frac{2(5\cos(d+ex)-3\sin(d+ex))\sqrt{5\sin(d+ex)+3\cos(d+ex)+2}}{3e}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\left(15\int \frac{1}{\sqrt{3\cos(d+ex)+5\sin(d+ex)+2}}dx+4\int \sqrt{3\cos(d+ex)+5\sin(d+ex)+2}dx\right)-\frac{2(5\cos(d+ex)-3\sin(d+ex))\sqrt{5\sin(d+ex)+3\cos(d+ex)+2}}{3e}$$

$$\Big\downarrow \text{3597}$$

$$\frac{2}{3}\left(4\int \sqrt{\sqrt{34}\cos\left(d+ex-\arctan\left(\frac{5}{3}\right)\right)+2}dx+15\int \frac{1}{\sqrt{3\cos(d+ex)+5\sin(d+ex)+2}}dx\right)-\frac{2(5\cos(d+ex)-3\sin(d+ex))\sqrt{5\sin(d+ex)+3\cos(d+ex)+2}}{3e}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\left(4\int \sqrt{\sqrt{34}\sin\left(d+ex-\arctan\left(\frac{5}{3}\right)+\frac{\pi}{2}\right)+2}dx+15\int \frac{1}{\sqrt{3\cos(d+ex)+5\sin(d+ex)+2}}dx\right)-\frac{2(5\cos(d+ex)-3\sin(d+ex))\sqrt{5\sin(d+ex)+3\cos(d+ex)+2}}{3e}$$

$$\Big\downarrow \text{3132}$$

3.404. $\int (2+3\cos(d+ex)+5\sin(d+ex))^{3/2}\,dx$

$$\frac{2}{3}\left(15\int \frac{1}{\sqrt{3\cos(d+ex)+5\sin(d+ex)+2}}dx+\frac{8\sqrt{2+\sqrt{34}}E\left(\frac{1}{2}\left(d+ex-\arctan\left(\frac{5}{3}\right)\right)|\frac{2}{15}(17-\sqrt{34})\right)}{e}\right)-\frac{2(5\cos(d+ex)-3\sin(d+ex))\sqrt{5\sin(d+ex)+3\cos(d+ex)+2}}{3e}$$

$$\Big\downarrow \text{3605}$$

$$\frac{2}{3}\left(15\int \frac{1}{\sqrt{\sqrt{34}\cos\left(d+ex-\arctan\left(\frac{5}{3}\right)\right)+2}}dx+\frac{8\sqrt{2+\sqrt{34}}E\left(\frac{1}{2}\left(d+ex-\arctan\left(\frac{5}{3}\right)\right)|\frac{2}{15}(17-\sqrt{34})\right)}{e}\right)-\frac{2(5\cos(d+ex)-3\sin(d+ex))\sqrt{5\sin(d+ex)+3\cos(d+ex)+2}}{3e}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\left(15\int \frac{1}{\sqrt{\sqrt{34}\sin\left(d+ex-\arctan\left(\frac{5}{3}\right)+\frac{\pi}{2}\right)+2}}dx+\frac{8\sqrt{2+\sqrt{34}}E\left(\frac{1}{2}\left(d+ex-\arctan\left(\frac{5}{3}\right)\right)|\frac{2}{15}(17-\sqrt{34})\right)}{e}\right)-\frac{2(5\cos(d+ex)-3\sin(d+ex))\sqrt{5\sin(d+ex)+3\cos(d+ex)+2}}{3e}$$

$$\Big\downarrow \text{3140}$$

$$\frac{2}{3}\left(\frac{30\,\text{EllipticF}\left(\frac{1}{2}\left(d+ex-\arctan\left(\frac{5}{3}\right)\right),\frac{2}{15}(17-\sqrt{34})\right)}{\sqrt{2+\sqrt{34}}e}+\frac{8\sqrt{2+\sqrt{34}}E\left(\frac{1}{2}\left(d+ex-\arctan\left(\frac{5}{3}\right)\right)|\frac{2}{15}(17-\sqrt{34})\right)}{e}\right)-\frac{2(5\cos(d+ex)-3\sin(d+ex))\sqrt{5\sin(d+ex)+3\cos(d+ex)+2}}{3e}$$

input
```
Int[(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(3/2),x]
```

output
```
(2*((8*Sqrt[2 + Sqrt[34]]*EllipticE[(d + e*x - ArcTan[5/3])/2, (2*(17 - Sq
rt[34]))/15])/e + (30*EllipticF[(d + e*x - ArcTan[5/3])/2, (2*(17 - Sqrt[3
4]))/15])/(Sqrt[2 + Sqrt[34]]*e)))/3 - (2*(5*Cos[d + e*x] - 3*Sin[d + e*x]
)*Sqrt[2 + 3*Cos[d + e*x] + 5*Sin[d + e*x]])/(3*e)
```

3.404. $\int (2+3\cos(d+ex)+5\sin(d+ex))^{3/2}\,dx$

3.404.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3597
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Int[Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 + c^2, 0] && GtQ[a + Sqrt[b^2
+ c^2], 0]
```

rule 3599
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Simp[1/n   Int[Simp[n*a^2 + (
n - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x
], x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

rule 3605
```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 + c^2, 0] && GtQ[a + Sqrt[
b^2 + c^2], 0]
```

rule 3628
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]
, x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[
A*b - a*B, 0]
```

3.404.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.34 (sec) , antiderivative size = 806, normalized size of antiderivative = 5.80

method	result	size
default	Expression too large to display	806

input
```
int((2+3*cos(e*x+d)+5*sin(e*x+d))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(76/17*((17*sin(e*x+d+arctan(3/5))+34^(1/2))/(34^(1/2)+17))^(1/2)*17^(1/2)
*((sin(e*x+d+arctan(3/5))+1)/(-34^(1/2)+17))^(1/2)*(-17*(sin(e*x+d+arctan(
3/5))-1)/(34^(1/2)+17))^(1/2)*EllipticF(((17*sin(e*x+d+arctan(3/5))+34^(1/
2))/(34^(1/2)+17))^(1/2),I*(1/(-34^(1/2)+17)*(34^(1/2)+17))^(1/2))*34^(1/2
)+36*((17*sin(e*x+d+arctan(3/5))+34^(1/2))/(34^(1/2)+17))^(1/2)*17^(1/2)*(
(sin(e*x+d+arctan(3/5))+1)/(-34^(1/2)+17))^(1/2)*(-17*(sin(e*x+d+arctan(3/
5))-1)/(34^(1/2)+17))^(1/2)*EllipticF(((17*sin(e*x+d+arctan(3/5))+34^(1/2)
)/(34^(1/2)+17))^(1/2),I*(1/(-34^(1/2)+17)*(34^(1/2)+17))^(1/2))+68/3*sin(
e*x+d+arctan(3/5))^3-68/3*sin(e*x+d+arctan(3/5))+4/3*34^(1/2)*sin(e*x+d+ar
ctan(3/5))^2-4/3*34^(1/2)-40/17*34^(1/2)*((17*sin(e*x+d+arctan(3/5))+34^(1
/2))/(34^(1/2)+17))^(1/2)*17^(1/2)*((sin(e*x+d+arctan(3/5))+1)/(-34^(1/2)+
17))^(1/2)*(-17*(sin(e*x+d+arctan(3/5))-1)/(34^(1/2)+17))^(1/2)*EllipticE(
((17*sin(e*x+d+arctan(3/5))+34^(1/2))/(34^(1/2)+17))^(1/2),I*(1/(-34^(1/2)
+17)*(34^(1/2)+17))^(1/2))+120/17*34^(1/2)*17^(1/2)*((sin(e*x+d+arctan(3/5
))+1)/(-34^(1/2)+17))^(1/2)*(-17*(sin(e*x+d+arctan(3/5))-1)/(34^(1/2)+17))
^(1/2)*(-(17*sin(e*x+d+arctan(3/5))+34^(1/2))/(-34^(1/2)+17))^(1/2)*Ellipt
icE((-(17*sin(e*x+d+arctan(3/5))+34^(1/2))/(-34^(1/2)+17))^(1/2),I*((-34^(
1/2)+17)/(34^(1/2)+17))^(1/2))+16*(-(17*sin(e*x+d+arctan(3/5))+34^(1/2))/(
-34^(1/2)+17))^(1/2)*(-17*(sin(e*x+d+arctan(3/5))-1)/(34^(1/2)+17))^(1/2)*
17^(1/2)*((sin(e*x+d+arctan(3/5))+1)/(-34^(1/2)+17))^(1/2)*EllipticF((-...
```

3.404.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.14

$$\int (2+3\cos(d+ex)+5\sin(d+ex))^{3/2}\,dx = \frac{(159i+265)\,\sqrt{5i+3}\sqrt{2}\text{weierstrassPInverse}\left(\frac{860}{289}i+\frac{1376}{867},-\frac{5480}{132651}i-\frac{12056}{14739},\cos(ex+d)-i\sin\right.}{}$$

input
```
integrate((2+3*cos(e*x+d)+5*sin(e*x+d))^(3/2),x, algorithm="fricas")
```

output
```
1/153*((159*I + 265)*sqrt(5*I + 3)*sqrt(2)*weierstrassPInverse(860/289*I +
 1376/867, -5480/132651*I - 12056/14739, cos(e*x + d) - I*sin(e*x + d) - 1
0/51*I + 2/17) - (159*I - 265)*sqrt(-5*I + 3)*sqrt(2)*weierstrassPInverse(
-860/289*I + 1376/867, 5480/132651*I - 12056/14739, cos(e*x + d) + I*sin(e
*x + d) + 10/51*I + 2/17) - 408*I*sqrt(5*I + 3)*sqrt(2)*weierstrassZeta(86
0/289*I + 1376/867, -5480/132651*I - 12056/14739, weierstrassPInverse(860/
289*I + 1376/867, -5480/132651*I - 12056/14739, cos(e*x + d) - I*sin(e*x +
 d) - 10/51*I + 2/17)) + 408*I*sqrt(-5*I + 3)*sqrt(2)*weierstrassZeta(-860
/289*I + 1376/867, 5480/132651*I - 12056/14739, weierstrassPInverse(-860/2
89*I + 1376/867, 5480/132651*I - 12056/14739, cos(e*x + d) + I*sin(e*x + d
) + 10/51*I + 2/17)) - 102*(5*cos(e*x + d) - 3*sin(e*x + d))*sqrt(3*cos(e*
x + d) + 5*sin(e*x + d) + 2))/e
```

3.404.6 Sympy [F]

$$\int (2+3\cos(d+ex)+5\sin(d+ex))^{3/2}\,dx=\int (5\sin{(d+ex)}+3\cos{(d+ex)}+2)^{\frac{3}{2}}\,dx$$

input
```
integrate((2+3*cos(e*x+d)+5*sin(e*x+d))**(3/2),x)
```

output
```
Integral((5*sin(d + e*x) + 3*cos(d + e*x) + 2)**(3/2), x)
```

3.404.7 Maxima [F]

$$\int (2+3\cos(d+ex)+5\sin(d+ex))^{3/2}\,dx=\int (3\cos{(ex+d)}+5\sin{(ex+d)}+2)^{\frac{3}{2}}\,dx$$

input
```
integrate((2+3*cos(e*x+d)+5*sin(e*x+d))^(3/2),x, algorithm="maxima")
```

output
```
integrate((3*cos(e*x + d) + 5*sin(e*x + d) + 2)^(3/2), x)
```

3.404.8 Giac [F]

$$\int (2+3\cos(d+ex)+5\sin(d+ex))^{3/2}\,dx = \int (3\,\cos\left(ex+d\right)+5\,\sin\left(ex+d\right)+2)^{\frac{3}{2}}\,dx$$

input
```
integrate((2+3*cos(e*x+d)+5*sin(e*x+d))^(3/2),x, algorithm="giac")
```

output
```
integrate((3*cos(e*x + d) + 5*sin(e*x + d) + 2)^(3/2), x)
```

3.404.9 Mupad [F(-1)]

Timed out.

$$\int (2+3\cos(d+ex)+5\sin(d+ex))^{3/2}\,dx = \int (3\,\cos\left(d+e\,x\right)+5\,\sin\left(d+e\,x\right)+2)^{3/2}\,dx$$

input
```
int((3*cos(d + e*x) + 5*sin(d + e*x) + 2)^(3/2),x)
```

output
```
int((3*cos(d + e*x) + 5*sin(d + e*x) + 2)^(3/2), x)
```

3.405 $\int \sqrt{2+3\cos(d+ex)+5\sin(d+ex)}\,dx$

3.405.1 Optimal result

Integrand size = 22, antiderivative size = 45

$$\int \sqrt{2+3\cos(d+ex)+5\sin(d+ex)}\,dx = \frac{2\sqrt{2+\sqrt{34}}E\left(\frac{1}{2}\left(d+ex-\arctan\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{e}$$

output
```
2*(cos(1/2*d+1/2*e*x-1/2*arctan(5/3))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arcta
n(5/3))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(5/3)),1/15*(510-30*34^(1/2)
)^(1/2))*(2+34^(1/2))^(1/2)/e
```

3.405.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 1.94 (sec) , antiderivative size = 326, normalized size of antiderivative = 7.24

$$\int \sqrt{2+3\cos(d+ex)+5\sin(d+ex)}\,dx = \frac{-15\sqrt{30}\,\text{AppellF1}\left(-\frac{1}{2},-\frac{1}{2},-\frac{1}{2},\frac{1}{2},\frac{\sqrt{34}+17\cos\left(d+ex-\arctan\left(\frac{5}{3}\right)\right)}{-17+\sqrt{34}},\frac{\sqrt{34}+17\cos\left(d+ex-\arctan\left(\frac{5}{3}\right)\right)}{17+\sqrt{34}}\right)\sin\left(d+ex-\text{arcta}\right.}{}$$

input
```
Integrate[Sqrt[2 + 3*Cos[d + e*x] + 5*Sin[d + e*x]],x]
```

output
```
(-15*Sqrt[30]*AppellF1[-1/2, -1/2, -1/2, 1/2, (Sqrt[34] + 17*Cos[d + e*x -
 ArcTan[5/3]])/(-17 + Sqrt[34]), (Sqrt[34] + 17*Cos[d + e*x - ArcTan[5/3]]
)/(17 + Sqrt[34])]*Sin[d + e*x - ArcTan[5/3]] + (-75*Cos[d + e*x] + 45*Sin
[d + e*x] + 2*Sqrt[30]*AppellF1[1/2, 1/2, 1/2, 3/2, (Sqrt[34] + 17*Sin[d +
 e*x + ArcTan[3/5]])/(-17 + Sqrt[34]), (Sqrt[34] + 17*Sin[d + e*x + ArcTan
[3/5]])/(17 + Sqrt[34])]*Sqrt[Cos[d + e*x + ArcTan[3/5]]^2]*Sqrt[2 + Sqrt[
34]*Cos[d + e*x - ArcTan[5/3]]]*Sec[d + e*x + ArcTan[3/5]]*Sqrt[2 + Sqrt[3
4]*Sin[d + e*x + ArcTan[3/5]]])*Sqrt[Sin[d + e*x - ArcTan[5/3]]^2])/(15*e*
Sqrt[2 + Sqrt[34]*Cos[d + e*x - ArcTan[5/3]]]*Sqrt[Sin[d + e*x - ArcTan[5/
3]]^2])
```

3.405.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3042, 3597, 3042, 3132}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{5\sin(d+ex)+3\cos(d+ex)+2}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sqrt{5\sin(d+ex)+3\cos(d+ex)+2}dx$$

$$\Big\downarrow \ 3597$$

$$\int \sqrt{\sqrt{34}\cos\left(-\arctan\left(\frac{5}{3}\right)+d+ex\right)+2}dx$$

$$\Big\downarrow \ 3042$$

$$\int \sqrt{\sqrt{34}\sin\left(-\arctan\left(\frac{5}{3}\right)+d+ex+\frac{\pi}{2}\right)+2}dx$$

$$\Big\downarrow \ 3132$$

$$\frac{2\sqrt{2+\sqrt{34}}E\left(\frac{1}{2}\left(d+ex-\arctan\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{e}$$

input
```
Int[Sqrt[2 + 3*Cos[d + e*x] + 5*Sin[d + e*x]],x]
```

output
```
(2*Sqrt[2 + Sqrt[34]]*EllipticE[(d + e*x - ArcTan[5/3])/2, (2*(17 - Sqrt[3
4]))/15])/e
```

3.405.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3597
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Int[Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 + c^2, 0] && GtQ[a + Sqrt[b^2
+ c^2], 0]
```

3.405.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.76 (sec) , antiderivative size = 461, normalized size of antiderivative = 10.24

method	result
default	$\frac{2\sqrt{17}\sqrt{\frac{\sin\left(ex+d+\arctan\left(\frac{3}{5}\right)\right)+1}{-\sqrt{34}+17}}\sqrt{-\frac{17\left(\sin\left(ex+d+\arctan\left(\frac{3}{5}\right)\right)-1\right)}{\sqrt{34}+17}}\left(2\sqrt{\frac{17\sin\left(ex+d+\arctan\left(\frac{3}{5}\right)\right)+\sqrt{34}}{\sqrt{34}+17}}\ \mathrm{EllipticF}\left(\sqrt{\frac{17\sin\left(ex+d+\arctan\right.}{\sqrt{34}+17}}\right.\right.}{}$
risch	Expression too large to display

input
```
int((2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/17*17^(1/2)*((sin(e*x+d+arctan(3/5))+1)/(-34^(1/2)+17))^(1/2)*(-17*(sin(
e*x+d+arctan(3/5))-1)/(34^(1/2)+17))^(1/2)*(2*((17*sin(e*x+d+arctan(3/5))+
34^(1/2))/(34^(1/2)+17))^(1/2)*EllipticF(((17*sin(e*x+d+arctan(3/5))+34^(1
/2))/(34^(1/2)+17))^(1/2),I*(1/(-34^(1/2)+17)*(34^(1/2)+17))^(1/2))*34^(1/
2)+15*(-(17*sin(e*x+d+arctan(3/5))+34^(1/2))/(-34^(1/2)+17))^(1/2)*Ellipti
cE((-(17*sin(e*x+d+arctan(3/5))+34^(1/2))/(-34^(1/2)+17))^(1/2),I*((-34^(1
/2)+17)/(34^(1/2)+17))^(1/2))*34^(1/2)-17*(-(17*sin(e*x+d+arctan(3/5))+34^
(1/2))/(-34^(1/2)+17))^(1/2)*EllipticF((-(17*sin(e*x+d+arctan(3/5))+34^(1/
2))/(-34^(1/2)+17))^(1/2),I*((-34^(1/2)+17)/(34^(1/2)+17))^(1/2))*34^(1/2)
+34*((17*sin(e*x+d+arctan(3/5))+34^(1/2))/(34^(1/2)+17))^(1/2)*EllipticF((
(17*sin(e*x+d+arctan(3/5))+34^(1/2))/(34^(1/2)+17))^(1/2),I*(1/(-34^(1/2)+
17)*(34^(1/2)+17))^(1/2))+34*(-(17*sin(e*x+d+arctan(3/5))+34^(1/2))/(-34^(
1/2)+17))^(1/2)*EllipticF((-(17*sin(e*x+d+arctan(3/5))+34^(1/2))/(-34^(1/2
)+17))^(1/2),I*((-34^(1/2)+17)/(34^(1/2)+17))^(1/2)))/cos(e*x+d+arctan(3/5
))/(34^(1/2)*sin(e*x+d+arctan(3/5))+2)^(1/2)/e
```

3.405.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.67

$$\int \sqrt{2+3\cos(d+ex)+5\sin(d+ex)}\,dx$$
$$= \frac{(3i+5)\ \sqrt{5i+3}\sqrt{2}\text{weierstrassPInverse}\left(\frac{860}{289}i+\frac{1376}{867},-\frac{5480}{132651}i-\frac{12056}{14739},\cos\left(ex+d\right)-i\,\sin\left(ex+d\right)-\frac{10}{51}i\right.}{}$$

input
```
integrate((2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2),x, algorithm="fricas")
```

output
```
1/51*((3*I + 5)*sqrt(5*I + 3)*sqrt(2)*weierstrassPInverse(860/289*I + 1376
/867, -5480/132651*I - 12056/14739, cos(e*x + d) - I*sin(e*x + d) - 10/51*
I + 2/17) - (3*I - 5)*sqrt(-5*I + 3)*sqrt(2)*weierstrassPInverse(-860/289*
I + 1376/867, 5480/132651*I - 12056/14739, cos(e*x + d) + I*sin(e*x + d) +
 10/51*I + 2/17) - 51*I*sqrt(5*I + 3)*sqrt(2)*weierstrassZeta(860/289*I +
1376/867, -5480/132651*I - 12056/14739, weierstrassPInverse(860/289*I + 13
76/867, -5480/132651*I - 12056/14739, cos(e*x + d) - I*sin(e*x + d) - 10/5
1*I + 2/17)) + 51*I*sqrt(-5*I + 3)*sqrt(2)*weierstrassZeta(-860/289*I + 13
76/867, 5480/132651*I - 12056/14739, weierstrassPInverse(-860/289*I + 1376
/867, 5480/132651*I - 12056/14739, cos(e*x + d) + I*sin(e*x + d) + 10/51*I
 + 2/17)))/e
```

3.405.6 Sympy [F]

$$\int \sqrt{2+3\cos(d+ex)+5\sin(d+ex)}\,dx = \int \sqrt{5\sin{(d+ex)}+3\cos{(d+ex)}+2}\,dx$$

input
```
integrate((2+3*cos(e*x+d)+5*sin(e*x+d))**(1/2),x)
```

output
```
Integral(sqrt(5*sin(d + e*x) + 3*cos(d + e*x) + 2), x)
```

3.405.7 Maxima [F]

$$\int \sqrt{2+3\cos(d+ex)+5\sin(d+ex)}\,dx = \int \sqrt{3\cos{(ex+d)}+5\sin{(ex+d)}+2}\,dx$$

input
```
integrate((2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(3*cos(e*x + d) + 5*sin(e*x + d) + 2), x)
```

3.405.8 Giac [F]

$$\int \sqrt{2+3\cos(d+ex)+5\sin(d+ex)}\,dx = \int \sqrt{3\cos{(ex+d)}+5\sin{(ex+d)}+2}\,dx$$

input
```
integrate((2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(3*cos(e*x + d) + 5*sin(e*x + d) + 2), x)
```

3.405.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{2+3\cos(d+ex)+5\sin(d+ex)}\,dx=\int \sqrt{3\,\cos\left(d+e\,x\right)+5\,\sin\left(d+e\,x\right)+2}\,dx$$

input
```
int((3*cos(d + e*x) + 5*sin(d + e*x) + 2)^(1/2),x)
```

output
```
int((3*cos(d + e*x) + 5*sin(d + e*x) + 2)^(1/2), x)
```

3.406 $\int \frac{1}{\sqrt{2+3\cos(d+ex)+5\sin(d+ex)}}\,dx$

3.406.1 Optimal result

Integrand size = 22, antiderivative size = 45

$$\int \frac{1}{\sqrt{2+3\cos(d+ex)+5\sin(d+ex)}}\,dx = \frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(d+ex-\arctan\left(\frac{5}{3}\right)\right),\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{\sqrt{2+\sqrt{34}}e}$$

output
```
2*(cos(1/2*d+1/2*e*x-1/2*arctan(5/3))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arcta
n(5/3))*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan(5/3)),1/15*(510-30*34^(1/2)
)^(1/2))/e/(2+34^(1/2))^(1/2)
```

3.406.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 0.33 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.84

$$\int \frac{1}{\sqrt{2+3\cos(d+ex)+5\sin(d+ex)}}\,dx = \frac{\sqrt{\frac{2}{15}}\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{3}{2},\frac{\sqrt{34}+17\sin\left(d+ex+\arctan\left(\frac{3}{5}\right)\right)}{-17+\sqrt{34}},\frac{\sqrt{34}+17\sin\left(d+ex+\arctan\left(\frac{3}{5}\right)\right)}{17+\sqrt{34}}\right)\sqrt{\cos^2\left(d+ex+\arctan\left(\frac{3}{5}\right)\right)}}{e}$$

input
```
Integrate[1/Sqrt[2 + 3*Cos[d + e*x] + 5*Sin[d + e*x]],x]
```

output
```
(Sqrt[2/15]*AppellF1[1/2, 1/2, 1/2, 3/2, (Sqrt[34] + 17*Sin[d + e*x + ArcT
an[3/5]])/(-17 + Sqrt[34]), (Sqrt[34] + 17*Sin[d + e*x + ArcTan[3/5]])/(17
 + Sqrt[34])]*Sqrt[Cos[d + e*x + ArcTan[3/5]]^2]*Sec[d + e*x + ArcTan[3/5]
]*Sqrt[2 + Sqrt[34]*Sin[d + e*x + ArcTan[3/5]]])/e
```

3.406.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3042, 3605, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{5\sin(d+ex)+3\cos(d+ex)+2}}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{\sqrt{5\sin(d+ex)+3\cos(d+ex)+2}}dx$$
$$\Big\downarrow \text{ 3605}$$
$$\int \frac{1}{\sqrt{\sqrt{34}\cos\left(-\arctan\left(\frac{5}{3}\right)+d+ex\right)+2}}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{\sqrt{\sqrt{34}\sin\left(-\arctan\left(\frac{5}{3}\right)+d+ex+\frac{\pi}{2}\right)+2}}dx$$
$$\Big\downarrow \text{ 3140}$$
$$\frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(d+ex-\arctan\left(\frac{5}{3}\right)\right),\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{\sqrt{2+\sqrt{34}}e}$$

input
```
Int[1/Sqrt[2 + 3*Cos[d + e*x] + 5*Sin[d + e*x]],x]
```

output
```
(2*EllipticF[(d + e*x - ArcTan[5/3])/2, (2*(17 - Sqrt[34]))/15])/(Sqrt[2 +
 Sqrt[34]]*e)
```

3.406.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3605
```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 + c^2, 0] && GtQ[a + Sqrt[
b^2 + c^2], 0]
```

3.406.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.59 (sec) , antiderivative size = 152, normalized size of antiderivative = 3.38

method	result
default	$\frac{2\left(\sqrt{34}+17\right)\sqrt{\frac{17\sin\left(ex+d+\arctan\left(\frac{3}{5}\right)\right)+\sqrt{34}}{\sqrt{34}+17}}\,\sqrt{17}\,\sqrt{\frac{\sin\left(ex+d+\arctan\left(\frac{3}{5}\right)\right)+1}{-\sqrt{34}+17}}\,\sqrt{-\frac{17\left(\sin\left(ex+d+\arctan\left(\frac{3}{5}\right)\right)-1\right)}{\sqrt{34}+17}}\,\mathrm{EllipticF}\left(\sqrt{\frac{17\sin\left(ex+\right.}{}}\right.}{17\cos\left(ex+d+\arctan\left(\frac{3}{5}\right)\right)\sqrt{\sqrt{34}\,\sin\left(ex+d+\arctan\left(\frac{3}{5}\right)\right)+2}\,e}$

input
```
int(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/17*(34^(1/2)+17)*((17*sin(e*x+d+arctan(3/5))+34^(1/2))/(34^(1/2)+17))^(1
/2)*17^(1/2)*((sin(e*x+d+arctan(3/5))+1)/(-34^(1/2)+17))^(1/2)*(-17*(sin(e
*x+d+arctan(3/5))-1)/(34^(1/2)+17))^(1/2)*EllipticF(((17*sin(e*x+d+arctan(
3/5))+34^(1/2))/(34^(1/2)+17))^(1/2),I*(1/(-34^(1/2)+17)*(34^(1/2)+17))^(1
/2))/cos(e*x+d+arctan(3/5))/(34^(1/2)*sin(e*x+d+arctan(3/5))+2)^(1/2)/e
```

3.406.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.33

$$\int \frac{1}{\sqrt{2+3\cos(d+ex)+5\sin(d+ex)}}\,dx$$
$$= \frac{(3i+5)\sqrt{5i+3}\sqrt{2}\text{weierstrassPInverse}\left(\frac{860}{289}i+\frac{1376}{867},-\frac{5480}{132651}i-\frac{12056}{14739},\cos(ex+d)-i\sin(ex+d)-\frac{10}{51}i\right.}{}$$

input
```
integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2),x, algorithm="fricas")
```

output
```
1/34*((3*I + 5)*sqrt(5*I + 3)*sqrt(2)*weierstrassPInverse(860/289*I + 1376
/867, -5480/132651*I - 12056/14739, cos(e*x + d) - I*sin(e*x + d) - 10/51*
I + 2/17) - (3*I - 5)*sqrt(-5*I + 3)*sqrt(2)*weierstrassPInverse(-860/289*
I + 1376/867, 5480/132651*I - 12056/14739, cos(e*x + d) + I*sin(e*x + d) +
 10/51*I + 2/17))/e
```

3.406.6 Sympy [F]

$$\int \frac{1}{\sqrt{2+3\cos(d+ex)+5\sin(d+ex)}}\,dx = \int \frac{1}{\sqrt{5\sin(d+ex)+3\cos(d+ex)+2}}\,dx$$

input
```
integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))**(1/2),x)
```

output
```
Integral(1/sqrt(5*sin(d + e*x) + 3*cos(d + e*x) + 2), x)
```

3.406.7 Maxima [F]

$$\int \frac{1}{\sqrt{2+3\cos(d+ex)+5\sin(d+ex)}}\,dx = \int \frac{1}{\sqrt{3\cos(ex+d)+5\sin(ex+d)+2}}\,dx$$

input
```
integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(3*cos(e*x + d) + 5*sin(e*x + d) + 2), x)
```

3.406.8 Giac [F]

$$\int \frac{1}{\sqrt{2+3\cos(d+ex)+5\sin(d+ex)}}\,dx=\int \frac{1}{\sqrt{3\,\cos\left(ex+d\right)+5\,\sin\left(ex+d\right)+2}}\,dx$$

input
```
integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(3*cos(e*x + d) + 5*sin(e*x + d) + 2), x)
```

3.406.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{2+3\cos(d+ex)+5\sin(d+ex)}}\,dx=\int \frac{1}{\sqrt{3\,\cos\left(d+e\,x\right)+5\,\sin\left(d+e\,x\right)+2}}\,dx$$

input
```
int(1/(3*cos(d + e*x) + 5*sin(d + e*x) + 2)^(1/2),x)
```

output
```
int(1/(3*cos(d + e*x) + 5*sin(d + e*x) + 2)^(1/2), x)
```

3.407 $\int \frac{1}{(2+3\cos(d+ex)+5\sin(d+ex))^{3/2}}\,dx$

3.407.1 Optimal result

Integrand size = 22, antiderivative size = 94

$$\int \frac{1}{(2+3\cos(d+ex)+5\sin(d+ex))^{3/2}}\,dx = -\frac{\sqrt{2+\sqrt{34}}E\left(\frac{1}{2}\left(d+ex-\arctan\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{15e} - \frac{5\cos(d+ex)-3\sin(d+ex)}{15e\sqrt{2+3\cos(d+ex)+5\sin(d+ex)}}$$

output
```
1/15*(-5*cos(e*x+d)+3*sin(e*x+d))/e/(2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2)-1/
15*(cos(1/2*d+1/2*e*x-1/2*arctan(5/3))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arct
an(5/3))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(5/3)),1/15*(510-30*34^(1/2
))^(1/2))*(2+34^(1/2))^(1/2)/e
```

3.407.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 4.79 (sec) , antiderivative size = 390, normalized size of antiderivative = 4.15

$$\int \frac{1}{(2+3\cos(d+ex)+5\sin(d+ex))^{3/2}}\,dx = \frac{18\sqrt{2+\sqrt{34}\cos\left(d+ex-\arctan\left(\frac{5}{3}\right)\right)} - 68\sqrt{2+3\cos(d+}{}$$

input
```
Integrate[(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(-3/2),x]
```

output
```
(18*Sqrt[2 + Sqrt[34]*Cos[d + e*x - ArcTan[5/3]]] - 68*Sqrt[2 + 3*Cos[d +
e*x] + 5*Sin[d + e*x]] + (20*(5 + 17*Sin[d + e*x]))/Sqrt[2 + 3*Cos[d + e*x
] + 5*Sin[d + e*x]] - 2*Sqrt[30]*AppellF1[1/2, 1/2, 1/2, 3/2, (Sqrt[34] +
17*Sin[d + e*x + ArcTan[3/5]])/(-17 + Sqrt[34]), (Sqrt[34] + 17*Sin[d + e*
x + ArcTan[3/5]])/(17 + Sqrt[34])]*Sqrt[Cos[d + e*x + ArcTan[3/5]]^2]*Sec[
d + e*x + ArcTan[3/5]]*Sqrt[2 + Sqrt[34]*Sin[d + e*x + ArcTan[3/5]]] - (15
*Sin[d + e*x - ArcTan[5/3]])/Sqrt[1/17 + Cos[d + e*x - ArcTan[5/3]]/Sqrt[3
4]] + (15*Sqrt[30]*AppellF1[-1/2, -1/2, -1/2, 1/2, (Sqrt[34] + 17*Cos[d +
e*x - ArcTan[5/3]])/(-17 + Sqrt[34]), (Sqrt[34] + 17*Cos[d + e*x - ArcTan[
5/3]])/(17 + Sqrt[34])]*Csc[d + e*x - ArcTan[5/3]]*Sqrt[Sin[d + e*x - ArcT
an[5/3]]^2])/Sqrt[2 + Sqrt[34]*Cos[d + e*x - ArcTan[5/3]]])/(450*e)
```

3.407.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 3607, 3042, 3597, 3042, 3132}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(5\sin(d+ex)+3\cos(d+ex)+2)^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(5\sin(d+ex)+3\cos(d+ex)+2)^{3/2}}dx$$

$$\Big\downarrow \text{3607}$$

$$-\frac{1}{30}\int \sqrt{3\cos(d+ex)+5\sin(d+ex)+2}dx-\frac{5\cos(d+ex)-3\sin(d+ex)}{15e\sqrt{5\sin(d+ex)+3\cos(d+ex)+2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{30}\int \sqrt{3\cos(d+ex)+5\sin(d+ex)+2}dx-\frac{5\cos(d+ex)-3\sin(d+ex)}{15e\sqrt{5\sin(d+ex)+3\cos(d+ex)+2}}$$

$$\Big\downarrow \text{3597}$$

$$-\frac{1}{30}\int \sqrt{\sqrt{34}\cos\left(d+ex-\arctan\left(\frac{5}{3}\right)\right)+2}dx-\frac{5\cos(d+ex)-3\sin(d+ex)}{15e\sqrt{5\sin(d+ex)+3\cos(d+ex)+2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{30}\int \sqrt{\sqrt{34}\sin\left(d+ex-\arctan\left(\frac{5}{3}\right)+\frac{\pi}{2}\right)+2}dx-\frac{5\cos(d+ex)-3\sin(d+ex)}{15e\sqrt{5\sin(d+ex)+3\cos(d+ex)+2}}$$

$$\Big\downarrow \text{3132}$$

$$-\frac{\sqrt{2+\sqrt{34}}E\left(\frac{1}{2}\left(d+ex-\arctan\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{15e}-\frac{5\cos(d+ex)-3\sin(d+ex)}{15e\sqrt{5\sin(d+ex)+3\cos(d+ex)+2}}$$

input
```
Int[(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(-3/2),x]
```

output
```
-1/15*(Sqrt[2 + Sqrt[34]]*EllipticE[(d + e*x - ArcTan[5/3])/2, (2*(17 - Sq
rt[34]))/15])/e - (5*Cos[d + e*x] - 3*Sin[d + e*x])/(15*e*Sqrt[2 + 3*Cos[d
 + e*x] + 5*Sin[d + e*x]])
```

3.407.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3597
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Int[Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 + c^2, 0] && GtQ[a + Sqrt[b^2
+ c^2], 0]
```

rule 3607
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-3/2), x_Symbol] :> Simp[2*((c*Cos[d + e*x] - b*Sin[d + e*x])/(e*(a^2 - b^
2 - c^2)*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]])), x] + Simp[1/(a^2 - b^
2 - c^2)   Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] /; FreeQ[{
a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

3.407. $\int \frac{1}{(2+3\cos(d+ex)+5\sin(d+ex))^{3/2}}\,dx$

3.407.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.35 (sec) , antiderivative size = 425, normalized size of antiderivative = 4.52

method	result
default	$\frac{\sqrt{34}\left(255\sqrt{\left(17\sin\left(ex+d+\arctan\left(\frac{3}{5}\right)\right)+\sqrt{34}\right)\sqrt{34}\,\cos\left(ex+d+\arctan\left(\frac{3}{5}\right)\right)^{2}}\,\sqrt{\frac{17\sin\left(ex+d+\arctan\left(\frac{3}{5}\right)\right)+\sqrt{34}}{\sqrt{34}+17}}\,\sqrt{\frac{\sin\left(ex+d+\arctan\left(\frac{3}{5}\right)\right)}{-\sqrt{34}+17}}}{}$

input
```
int(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4335*34^(1/2)*(255*((17*sin(e*x+d+arctan(3/5))+34^(1/2))*34^(1/2)*cos(e*
x+d+arctan(3/5))^2)^(1/2)*((17*sin(e*x+d+arctan(3/5))+34^(1/2))/(34^(1/2)+
17))^(1/2)*((sin(e*x+d+arctan(3/5))+1)/(-34^(1/2)+17))^(1/2)*(-17*(sin(e*x
+d+arctan(3/5))-1)/(34^(1/2)+17))^(1/2)*EllipticF(((17*sin(e*x+d+arctan(3/
5))+34^(1/2))/(34^(1/2)+17))^(1/2),I*(1/(-34^(1/2)+17)*(34^(1/2)+17))^(1/2
))-255*((17*sin(e*x+d+arctan(3/5))+34^(1/2))*34^(1/2)*cos(e*x+d+arctan(3/5
))^2)^(1/2)*((17*sin(e*x+d+arctan(3/5))+34^(1/2))/(34^(1/2)+17))^(1/2)*((s
in(e*x+d+arctan(3/5))+1)/(-34^(1/2)+17))^(1/2)*(-17*(sin(e*x+d+arctan(3/5)
)-1)/(34^(1/2)+17))^(1/2)*EllipticE(((17*sin(e*x+d+arctan(3/5))+34^(1/2))/
(34^(1/2)+17))^(1/2),I*(1/(-34^(1/2)+17)*(34^(1/2)+17))^(1/2))+289*((34^(1
/2)*sin(e*x+d+arctan(3/5))+2)*cos(e*x+d+arctan(3/5))^2)^(1/2)*sin(e*x+d+ar
ctan(3/5))^2-289*((34^(1/2)*sin(e*x+d+arctan(3/5))+2)*cos(e*x+d+arctan(3/5
))^2)^(1/2))*17^(1/2)/((17*sin(e*x+d+arctan(3/5))+34^(1/2))*34^(1/2)*cos(e
*x+d+arctan(3/5))^2)^(1/2)/cos(e*x+d+arctan(3/5))/(34^(1/2)*sin(e*x+d+arct
an(3/5))+2)^(1/2)/e
```

3.407.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.96

$$\int \frac{1}{(2+3\cos(d+ex)+5\sin(d+ex))^{3/2}}\,dx = \frac{\sqrt{5i+3}\left(-(9i+15)\sqrt{2}\cos\left(ex+d\right)-(15i+25)\sqrt{2}\sin\left(ex\right.\right.}{}$$

input
```
integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(3/2),x, algorithm="fricas")
```

output
```
1/1530*(sqrt(5*I + 3)*(-(9*I + 15)*sqrt(2)*cos(e*x + d) - (15*I + 25)*sqrt
(2)*sin(e*x + d) - (6*I + 10)*sqrt(2))*weierstrassPInverse(860/289*I + 137
6/867, -5480/132651*I - 12056/14739, cos(e*x + d) - I*sin(e*x + d) - 10/51
*I + 2/17) + sqrt(-5*I + 3)*((9*I - 15)*sqrt(2)*cos(e*x + d) + (15*I - 25)
*sqrt(2)*sin(e*x + d) + (6*I - 10)*sqrt(2))*weierstrassPInverse(-860/289*I
 + 1376/867, 5480/132651*I - 12056/14739, cos(e*x + d) + I*sin(e*x + d) +
10/51*I + 2/17) - 51*sqrt(5*I + 3)*(-3*I*sqrt(2)*cos(e*x + d) - 5*I*sqrt(2
)*sin(e*x + d) - 2*I*sqrt(2))*weierstrassZeta(860/289*I + 1376/867, -5480/
132651*I - 12056/14739, weierstrassPInverse(860/289*I + 1376/867, -5480/13
2651*I - 12056/14739, cos(e*x + d) - I*sin(e*x + d) - 10/51*I + 2/17)) - 5
1*sqrt(-5*I + 3)*(3*I*sqrt(2)*cos(e*x + d) + 5*I*sqrt(2)*sin(e*x + d) + 2*
I*sqrt(2))*weierstrassZeta(-860/289*I + 1376/867, 5480/132651*I - 12056/14
739, weierstrassPInverse(-860/289*I + 1376/867, 5480/132651*I - 12056/1473
9, cos(e*x + d) + I*sin(e*x + d) + 10/51*I + 2/17)) - 102*(5*cos(e*x + d)
- 3*sin(e*x + d))*sqrt(3*cos(e*x + d) + 5*sin(e*x + d) + 2))/(3*e*cos(e*x
+ d) + 5*e*sin(e*x + d) + 2*e)
```

3.407.6 Sympy [F]

$$\int \frac{1}{(2+3\cos(d+ex)+5\sin(d+ex))^{3/2}}\,dx = \int \frac{1}{(5\sin(d+ex)+3\cos(d+ex)+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))**(3/2),x)
```

output
```
Integral((5*sin(d + e*x) + 3*cos(d + e*x) + 2)**(-3/2), x)
```

3.407.7 Maxima [F]

$$\int \frac{1}{(2+3\cos(d+ex)+5\sin(d+ex))^{3/2}}\,dx = \int \frac{1}{(3\cos(ex+d)+5\sin(ex+d)+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(3/2),x, algorithm="maxima")
```

output
```
integrate((3*cos(e*x + d) + 5*sin(e*x + d) + 2)^(-3/2), x)
```

3.407.8 Giac [F]

$$\int \frac{1}{(2+3\cos(d+ex)+5\sin(d+ex))^{3/2}}\,dx = \int \frac{1}{(3\,\cos(ex+d)+5\,\sin(ex+d)+2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(3/2),x, algorithm="giac")
```

output
```
integrate((3*cos(e*x + d) + 5*sin(e*x + d) + 2)^(-3/2), x)
```

3.407.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2+3\cos(d+ex)+5\sin(d+ex))^{3/2}}\,dx = \int \frac{1}{(3\,\cos(d+e\,x)+5\,\sin(d+e\,x)+2)^{3/2}}\,dx$$

input
```
int(1/(3*cos(d + e*x) + 5*sin(d + e*x) + 2)^(3/2),x)
```

output
```
int(1/(3*cos(d + e*x) + 5*sin(d + e*x) + 2)^(3/2), x)
```

3.408 $\int \frac{1}{(2+3\cos(d+ex)+5\sin(d+ex))^{5/2}}\,dx$

3.408.1 Optimal result

Integrand size = 22, antiderivative size = 187

$$\int \frac{1}{(2+3\cos(d+ex)+5\sin(d+ex))^{5/2}}\,dx=\frac{4\sqrt{2+\sqrt{34}}E\left(\frac{1}{2}\left(d+ex-\arctan\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{675e}+\frac{\text{EllipticF}\left(\frac{1}{2}\left(d+ex-\arctan\left(\frac{5}{3}\right)\right),\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{45\sqrt{2+\sqrt{34}}e}-\frac{5\cos(d+ex)-3\sin(d+ex)}{45e(2+3\cos(d+ex)+5\sin(d+ex))^{3/2}}+\frac{4(5\cos(d+ex)-3\sin(d+ex))}{675e\sqrt{2+3\cos(d+ex)+5\sin(d+ex)}}$$

output
```
1/45*(-5*cos(e*x+d)+3*sin(e*x+d))/e/(2+3*cos(e*x+d)+5*sin(e*x+d))^(3/2)+4/
675*(5*cos(e*x+d)-3*sin(e*x+d))/e/(2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2)+1/45
*(cos(1/2*d+1/2*e*x-1/2*arctan(5/3))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan
(5/3))*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan(5/3)),1/15*(510-30*34^(1/2))
^(1/2))/e/(2+34^(1/2))^(1/2)+4/675*(cos(1/2*d+1/2*e*x-1/2*arctan(5/3))^2)^
(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(5/3))*EllipticE(sin(1/2*d+1/2*e*x-1/2*a
rctan(5/3)),1/15*(510-30*34^(1/2))^(1/2))*(2+34^(1/2))^(1/2)/e
```

3.408.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 2.47 (sec) , antiderivative size = 430, normalized size of antiderivative = 2.30

$$\int \frac{1}{(2+3\cos(d+ex)+5\sin(d+ex))^{5/2}}\,dx = \frac{-24\sqrt{2+\sqrt{34}\cos\left(d+ex-\arctan\left(\frac{5}{3}\right)\right)}+\frac{272}{3}\sqrt{2+3\cos(d}{}$$

input
```
Integrate[(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(-5/2),x]
```

output
```
(-24*Sqrt[2 + Sqrt[34]*Cos[d + e*x - ArcTan[5/3]]] + (272*Sqrt[2 + 3*Cos[d
 + e*x] + 5*Sin[d + e*x]])/3 + (100*(5 + 17*Sin[d + e*x]))/(2 + 3*Cos[d +
e*x] + 5*Sin[d + e*x])^(3/2) - (10*(115 + 136*Sin[d + e*x]))/(3*Sqrt[2 + 3
*Cos[d + e*x] + 5*Sin[d + e*x]]) + 23*Sqrt[10/3]*AppellF1[1/2, 1/2, 1/2, 3
/2, (Sqrt[34] + 17*Sin[d + e*x + ArcTan[3/5]])/(-17 + Sqrt[34]), (Sqrt[34]
 + 17*Sin[d + e*x + ArcTan[3/5]])/(17 + Sqrt[34])]*Sqrt[Cos[d + e*x + ArcT
an[3/5]]^2]*Sec[d + e*x + ArcTan[3/5]]*Sqrt[2 + Sqrt[34]*Sin[d + e*x + Arc
Tan[3/5]]] + (20*Sin[d + e*x - ArcTan[5/3]])/Sqrt[1/17 + Cos[d + e*x - Arc
Tan[5/3]]/Sqrt[34]] - (20*Sqrt[30]*AppellF1[-1/2, -1/2, -1/2, 1/2, (Sqrt[3
4] + 17*Cos[d + e*x - ArcTan[5/3]])/(-17 + Sqrt[34]), (Sqrt[34] + 17*Cos[d
 + e*x - ArcTan[5/3]])/(17 + Sqrt[34])]*Csc[d + e*x - ArcTan[5/3]]*Sqrt[Si
n[d + e*x - ArcTan[5/3]]^2])/Sqrt[2 + Sqrt[34]*Cos[d + e*x - ArcTan[5/3]]]
)/(6750*e)
```

3.408.3 Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.03, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.682$, Rules used = {3042, 3608, 27, 3042, 3635, 25, 3042, 3628, 3042, 3597, 3042, 3132, 3605, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(5\sin(d+ex)+3\cos(d+ex)+2)^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(5\sin(d+ex)+3\cos(d+ex)+2)^{5/2}}dx$$

$$\Big\downarrow \text{3608}$$

$$\frac{1}{45}\int -\frac{-3\cos(d+ex)-5\sin(d+ex)+6}{2(3\cos(d+ex)+5\sin(d+ex)+2)^{3/2}}dx-\frac{5\cos(d+ex)-3\sin(d+ex)}{45e(5\sin(d+ex)+3\cos(d+ex)+2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{90}\int \frac{-3\cos(d+ex)-5\sin(d+ex)+6}{(3\cos(d+ex)+5\sin(d+ex)+2)^{3/2}}dx-\frac{5\cos(d+ex)-3\sin(d+ex)}{45e(5\sin(d+ex)+3\cos(d+ex)+2)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{90}\int \frac{-3\cos(d+ex)-5\sin(d+ex)+6}{(3\cos(d+ex)+5\sin(d+ex)+2)^{3/2}}dx-\frac{5\cos(d+ex)-3\sin(d+ex)}{45e(5\sin(d+ex)+3\cos(d+ex)+2)^{3/2}}$$

$$\Big\downarrow \text{3635}$$

$$\frac{1}{90}\left(\frac{8(5\cos(d+ex)-3\sin(d+ex))}{15e\sqrt{5\sin(d+ex)+3\cos(d+ex)+2}}-\frac{1}{15}\int -\frac{12\cos(d+ex)+20\sin(d+ex)+23}{\sqrt{3\cos(d+ex)+5\sin(d+ex)+2}}dx\right)-\frac{5\cos(d+ex)-3\sin(d+ex)}{45e(5\sin(d+ex)+3\cos(d+ex)+2)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{90}\left(\frac{1}{15}\int \frac{12\cos(d+ex)+20\sin(d+ex)+23}{\sqrt{3\cos(d+ex)+5\sin(d+ex)+2}}dx+\frac{8(5\cos(d+ex)-3\sin(d+ex))}{15e\sqrt{5\sin(d+ex)+3\cos(d+ex)+2}}\right)-\frac{5\cos(d+ex)-3\sin(d+ex)}{45e(5\sin(d+ex)+3\cos(d+ex)+2)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{90}\left(\frac{1}{15}\int \frac{12\cos(d+ex)+20\sin(d+ex)+23}{\sqrt{3\cos(d+ex)+5\sin(d+ex)+2}}dx+\frac{8(5\cos(d+ex)-3\sin(d+ex))}{15e\sqrt{5\sin(d+ex)+3\cos(d+ex)+2}}\right)-\frac{5\cos(d+ex)-3\sin(d+ex)}{45e(5\sin(d+ex)+3\cos(d+ex)+2)^{3/2}}$$

$$\Big\downarrow \text{3628}$$

$$\frac{1}{90}\left(\frac{1}{15}\left(15\int \frac{1}{\sqrt{3\cos(d+ex)+5\sin(d+ex)+2}}dx+4\int \sqrt{3\cos(d+ex)+5\sin(d+ex)+2}dx\right)+\frac{8(5\cos}{15e\sqrt{5\sin}}\right.$$
$$\frac{5\cos(d+ex)-3\sin(d+ex)}{45e(5\sin(d+ex)+3\cos(d+ex)+2)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

3.408. $\int \frac{1}{(2+3\cos(d+ex)+5\sin(d+ex))^{5/2}}\,dx$

$$\frac{1}{90}\left(\frac{1}{15}\left(15\int\frac{1}{\sqrt{3\cos(d+ex)+5\sin(d+ex)+2}}dx+4\int\sqrt{3\cos(d+ex)+5\sin(d+ex)+2}dx\right)+\frac{8(5\cos}{15e\sqrt{5\sin}}\right.$$
$$\frac{5\cos(d+ex)-3\sin(d+ex)}{45e(5\sin(d+ex)+3\cos(d+ex)+2)^{3/2}}$$

$\downarrow$ 3597

$$\frac{1}{90}\left(\frac{1}{15}\left(4\int\sqrt{\sqrt{34}\cos\left(d+ex-\arctan\left(\frac{5}{3}\right)\right)+2}dx+15\int\frac{1}{\sqrt{3\cos(d+ex)+5\sin(d+ex)+2}}dx\right)+\frac{8(5}{15e\sqrt{}}\right.$$
$$\frac{5\cos(d+ex)-3\sin(d+ex)}{45e(5\sin(d+ex)+3\cos(d+ex)+2)^{3/2}}$$

$\downarrow$ 3042

$$\frac{1}{90}\left(\frac{1}{15}\left(4\int\sqrt{\sqrt{34}\sin\left(d+ex-\arctan\left(\frac{5}{3}\right)+\frac{\pi}{2}\right)+2}dx+15\int\frac{1}{\sqrt{3\cos(d+ex)+5\sin(d+ex)+2}}dx\right)+\frac{}{1}\right.$$
$$\frac{5\cos(d+ex)-3\sin(d+ex)}{45e(5\sin(d+ex)+3\cos(d+ex)+2)^{3/2}}$$

$\downarrow$ 3132

$$\frac{1}{90}\left(\frac{1}{15}\left(15\int\frac{1}{\sqrt{3\cos(d+ex)+5\sin(d+ex)+2}}dx+\frac{8\sqrt{2+\sqrt{34}}E\left(\frac{1}{2}\left(d+ex-\arctan\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{e}\right)\right.$$
$$\frac{5\cos(d+ex)-3\sin(d+ex)}{45e(5\sin(d+ex)+3\cos(d+ex)+2)^{3/2}}$$

$\downarrow$ 3605

$$\frac{1}{90}\left(\frac{1}{15}\left(15\int\frac{1}{\sqrt{\sqrt{34}\cos\left(d+ex-\arctan\left(\frac{5}{3}\right)\right)+2}}dx+\frac{8\sqrt{2+\sqrt{34}}E\left(\frac{1}{2}\left(d+ex-\arctan\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{e}\right)\right.$$
$$\frac{5\cos(d+ex)-3\sin(d+ex)}{45e(5\sin(d+ex)+3\cos(d+ex)+2)^{3/2}}$$

$\downarrow$ 3042

$$\frac{1}{90}\left(\frac{1}{15}\left(15\int\frac{1}{\sqrt{\sqrt{34}\sin\left(d+ex-\arctan\left(\frac{5}{3}\right)+\frac{\pi}{2}\right)+2}}dx+\frac{8\sqrt{2+\sqrt{34}}E\left(\frac{1}{2}\left(d+ex-\arctan\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\sqrt{3}\right.\right.}{e}\right.\right.$$
$$\frac{5\cos(d+ex)-3\sin(d+ex)}{45e(5\sin(d+ex)+3\cos(d+ex)+2)^{3/2}}$$

$\downarrow$ 3140

3.408. $\int\frac{1}{(2+3\cos(d+ex)+5\sin(d+ex))^{5/2}}\,dx$

$$\frac{1}{90}\left(\frac{1}{15}\left(\frac{30\,\text{EllipticF}\left(\frac{1}{2}\left(d+ex-\arctan\left(\frac{5}{3}\right)\right),\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{\sqrt{2+\sqrt{34}}e}+\frac{8\sqrt{2+\sqrt{34}}E\left(\frac{1}{2}\left(d+ex-\arctan\left(\frac{5}{3}\right)\right)|\frac{2}{15}\left(17-\right.\right.}{e}\right.\right.$$

$$\frac{5\cos(d+ex)-3\sin(d+ex)}{45e(5\sin(d+ex)+3\cos(d+ex)+2)^{3/2}}$$

input
```
Int[(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(-5/2),x]
```

output
```
-1/45*(5*Cos[d + e*x] - 3*Sin[d + e*x])/(e*(2 + 3*Cos[d + e*x] + 5*Sin[d +
 e*x])^(3/2)) + (((8*Sqrt[2 + Sqrt[34]]*EllipticE[(d + e*x - ArcTan[5/3])/
2, (2*(17 - Sqrt[34]))/15])/e + (30*EllipticF[(d + e*x - ArcTan[5/3])/2, (
2*(17 - Sqrt[34]))/15])/(Sqrt[2 + Sqrt[34]]*e))/15 + (8*(5*Cos[d + e*x] -
3*Sin[d + e*x]))/(15*e*Sqrt[2 + 3*Cos[d + e*x] + 5*Sin[d + e*x]]))/90
```

3.408.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3597
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Int[Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 + c^2, 0] && GtQ[a + Sqrt[b^2
+ c^2], 0]
```

rule 3605
```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 + c^2, 0] && GtQ[a + Sqrt[
b^2 + c^2], 0]
```

rule 3608
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Simp[
1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c
*(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x
] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] &&
NeQ[n, -3/2]
```

rule 3628
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]
, x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[
A*b - a*B, 0]
```

rule 3635
```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(-(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(
a^2 - b^2 - c^2))), x] + Simp[1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a + b*Co
s[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n +
2)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x]
/; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[n, -2]
```

3.408. $\int \frac{1}{(2+3\cos(d+ex)+5\sin(d+ex))^{5/2}}\,dx$

3.408.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.62 (sec) , antiderivative size = 586, normalized size of antiderivative = 3.13

method	result
default	$\frac{17\sqrt{-\left(-\sqrt{34}\,\sin\left(ex+d+\arctan\left(\frac{3}{5}\right)\right)-2\right)\cos\left(ex+d+\arctan\left(\frac{3}{5}\right)\right)^{2}}\,\left(17\sin\left(ex+d+\arctan\left(\frac{3}{5}\right)\right)^{2}+2\sqrt{34}\,\sin\left(ex+d+\arctan\left(\frac{3}{5}\right)\right)+2\right)\left(\right.}{}$

input
```
int(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
17/2*(-(-34^(1/2)*sin(e*x+d+arctan(3/5))-2)*cos(e*x+d+arctan(3/5))^2)^(1/2
)/(17*sin(e*x+d+arctan(3/5))+34^(1/2))^2*(17*sin(e*x+d+arctan(3/5))^2+2*34
^(1/2)*sin(e*x+d+arctan(3/5))+2)*(-1/765*34^(1/2)*(-(-34^(1/2)*sin(e*x+d+a
rctan(3/5))-2)*cos(e*x+d+arctan(3/5))^2)^(1/2)/(sin(e*x+d+arctan(3/5))+1/1
7*34^(1/2))^2+136/675*34^(1/2)*cos(e*x+d+arctan(3/5))^2/(-cos(e*x+d+arctan
(3/5))^2*34^(1/2)*(-289*sin(e*x+d+arctan(3/5))-17*34^(1/2)))^(1/2)+46/675*
(-1+1/17*34^(1/2))*(1/(-34^(1/2)+17)*(-17*sin(e*x+d+arctan(3/5))-34^(1/2))
)^(1/2)*((-17*sin(e*x+d+arctan(3/5))+17)/(34^(1/2)+17))^(1/2)*(1/(-34^(1/2
)+17)*(17*sin(e*x+d+arctan(3/5))+17))^(1/2)/(-(-34^(1/2)*sin(e*x+d+arctan(
3/5))-2)*cos(e*x+d+arctan(3/5))^2)^(1/2)*EllipticF((1/(-34^(1/2)+17)*(-17*
sin(e*x+d+arctan(3/5))-34^(1/2)))^(1/2),I*((-34^(1/2)+17)/(34^(1/2)+17))^(
1/2))+8/675*34^(1/2)*(-1+1/17*34^(1/2))*(1/(-34^(1/2)+17)*(-17*sin(e*x+d+a
rctan(3/5))-34^(1/2)))^(1/2)*((-17*sin(e*x+d+arctan(3/5))+17)/(34^(1/2)+17
))^(1/2)*(1/(-34^(1/2)+17)*(17*sin(e*x+d+arctan(3/5))+17))^(1/2)/(-(-34^(1
/2)*sin(e*x+d+arctan(3/5))-2)*cos(e*x+d+arctan(3/5))^2)^(1/2)*((-1/17*34^(
1/2)-1)*EllipticE((1/(-34^(1/2)+17)*(-17*sin(e*x+d+arctan(3/5))-34^(1/2)))
^(1/2),I*((-34^(1/2)+17)/(34^(1/2)+17))^(1/2))+EllipticF((1/(-34^(1/2)+17)
*(-17*sin(e*x+d+arctan(3/5))-34^(1/2)))^(1/2),I*((-34^(1/2)+17)/(34^(1/2)+
17))^(1/2))))/cos(e*x+d+arctan(3/5))/(34^(1/2)*sin(e*x+d+arctan(3/5))+2)^(
1/2)/e
```

3.408.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 431, normalized size of antiderivative = 2.30

$$\int \frac{1}{(2+3\cos(d+ex)+5\sin(d+ex))^{5/2}}\,dx = \frac{53\sqrt{5i+3}\left((48i+80)\sqrt{2}\cos(ex+d)^2+10\left(-(9i+15)\sqrt{2}\right.\right.}{}$$

input
```
integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(5/2),x, algorithm="fricas")
```

output
```
1/137700*(53*sqrt(5*I + 3)*((48*I + 80)*sqrt(2)*cos(e*x + d)^2 + 10*(-(9*I
 + 15)*sqrt(2)*cos(e*x + d) - (6*I + 10)*sqrt(2))*sin(e*x + d) - (36*I + 6
0)*sqrt(2)*cos(e*x + d) - (87*I + 145)*sqrt(2))*weierstrassPInverse(860/28
9*I + 1376/867, -5480/132651*I - 12056/14739, cos(e*x + d) - I*sin(e*x + d
) - 10/51*I + 2/17) + 53*sqrt(-5*I + 3)*(-(48*I - 80)*sqrt(2)*cos(e*x + d)
^2 + 10*((9*I - 15)*sqrt(2)*cos(e*x + d) + (6*I - 10)*sqrt(2))*sin(e*x + d
) + (36*I - 60)*sqrt(2)*cos(e*x + d) + (87*I - 145)*sqrt(2))*weierstrassPI
nverse(-860/289*I + 1376/867, 5480/132651*I - 12056/14739, cos(e*x + d) +
I*sin(e*x + d) + 10/51*I + 2/17) + 408*sqrt(5*I + 3)*(-16*I*sqrt(2)*cos(e*
x + d)^2 + 10*(3*I*sqrt(2)*cos(e*x + d) + 2*I*sqrt(2))*sin(e*x + d) + 12*I
*sqrt(2)*cos(e*x + d) + 29*I*sqrt(2))*weierstrassZeta(860/289*I + 1376/867
, -5480/132651*I - 12056/14739, weierstrassPInverse(860/289*I + 1376/867,
-5480/132651*I - 12056/14739, cos(e*x + d) - I*sin(e*x + d) - 10/51*I + 2/
17)) + 408*sqrt(-5*I + 3)*(16*I*sqrt(2)*cos(e*x + d)^2 + 10*(-3*I*sqrt(2)*
cos(e*x + d) - 2*I*sqrt(2))*sin(e*x + d) - 12*I*sqrt(2)*cos(e*x + d) - 29*
I*sqrt(2))*weierstrassZeta(-860/289*I + 1376/867, 5480/132651*I - 12056/14
739, weierstrassPInverse(-860/289*I + 1376/867, 5480/132651*I - 12056/1473
9, cos(e*x + d) + I*sin(e*x + d) + 10/51*I + 2/17)) - 204*(120*cos(e*x + d
)^2 + (64*cos(e*x + d) + 21)*sin(e*x + d) - 35*cos(e*x + d) - 60)*sqrt(3*c
os(e*x + d) + 5*sin(e*x + d) + 2))/(16*e*cos(e*x + d)^2 - 12*e*cos(e*x ...
```

3.408.6 Sympy [F]

$$\int \frac{1}{(2+3\cos(d+ex)+5\sin(d+ex))^{5/2}}\,dx = \int \frac{1}{(5\sin(d+ex)+3\cos(d+ex)+2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))**(5/2),x)
```

output
```
Integral((5*sin(d + e*x) + 3*cos(d + e*x) + 2)**(-5/2), x)
```

3.408.7 Maxima [F]

$$\int \frac{1}{(2+3\cos(d+ex)+5\sin(d+ex))^{5/2}}\,dx = \int \frac{1}{(3\cos(ex+d)+5\sin(ex+d)+2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(5/2),x, algorithm="maxima")
```

output
```
integrate((3*cos(e*x + d) + 5*sin(e*x + d) + 2)^(-5/2), x)
```

3.408.8 Giac [F]

$$\int \frac{1}{(2+3\cos(d+ex)+5\sin(d+ex))^{5/2}}\,dx = \int \frac{1}{(3\cos(ex+d)+5\sin(ex+d)+2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(5/2),x, algorithm="giac")
```

output
```
integrate((3*cos(e*x + d) + 5*sin(e*x + d) + 2)^(-5/2), x)
```

3.408.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2+3\cos(d+ex)+5\sin(d+ex))^{5/2}}\,dx = \int \frac{1}{(3\cos(d+e\,x)+5\sin(d+e\,x)+2)^{5/2}}\,dx$$

input
```
int(1/(3*cos(d + e*x) + 5*sin(d + e*x) + 2)^(5/2),x)
```

output
```
int(1/(3*cos(d + e*x) + 5*sin(d + e*x) + 2)^(5/2), x)
```

3.409 $\int \frac{1}{(2+3\cos(d+ex)+5\sin(d+ex))^{7/2}}\,dx$

3.409.1 Optimal result

Integrand size = 22, antiderivative size = 233

$$\begin{aligned}\int \frac{1}{(2+3\cos(d+ex)+5\sin(d+ex))^{7/2}}\,dx = &-\frac{199\sqrt{2+\sqrt{34}}E\left(\frac{1}{2}\left(d+ex-\arctan\left(\frac{5}{3}\right)\right)|\frac{2}{15}(17-\sqrt{34})\right)}{101250e}\\ &-\frac{8\,\text{EllipticF}\left(\frac{1}{2}\left(d+ex-\arctan\left(\frac{5}{3}\right)\right),\frac{2}{15}(17-\sqrt{34})\right)}{3375\sqrt{2+\sqrt{34}}e}\\ &-\frac{5\cos(d+ex)-3\sin(d+ex)}{75e(2+3\cos(d+ex)+5\sin(d+ex))^{5/2}}\\ &+\frac{8(5\cos(d+ex)-3\sin(d+ex))}{3375e(2+3\cos(d+ex)+5\sin(d+ex))^{3/2}}\\ &-\frac{199(5\cos(d+ex)-3\sin(d+ex))}{101250e\sqrt{2+3\cos(d+ex)+5\sin(d+ex)}}\end{aligned}$$

output
```
1/75*(-5*cos(e*x+d)+3*sin(e*x+d))/e/(2+3*cos(e*x+d)+5*sin(e*x+d))^(5/2)+8/
3375*(5*cos(e*x+d)-3*sin(e*x+d))/e/(2+3*cos(e*x+d)+5*sin(e*x+d))^(3/2)-199
/101250*(5*cos(e*x+d)-3*sin(e*x+d))/e/(2+3*cos(e*x+d)+5*sin(e*x+d))^(1/2)-
8/3375*(cos(1/2*d+1/2*e*x-1/2*arctan(5/3))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*
arctan(5/3))*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan(5/3)),1/15*(510-30*34^
(1/2))^(1/2))/e/(2+34^(1/2))^(1/2)-199/101250*(cos(1/2*d+1/2*e*x-1/2*arcta
n(5/3))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(5/3))*EllipticE(sin(1/2*d+1/
2*e*x-1/2*arctan(5/3)),1/15*(510-30*34^(1/2))^(1/2))*(2+34^(1/2))^(1/2)/e
```

3.409.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 3.00 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.87

$$\int \frac{1}{(2+3\cos(d+ex)+5\sin(d+ex))^{7/2}}\,dx = \frac{-13532\sqrt{2+3\cos(d+ex)+5\sin(d+ex)}+\frac{597(12+43\cos(d+e}{\sqrt{2+\sqrt{34}\cos(d+}}}{}$$

input
```
Integrate[(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(-7/2),x]
```

output
```
(-13532*Sqrt[2 + 3*Cos[d + e*x] + 5*Sin[d + e*x]] + (597*(12 + 43*Cos[d +
e*x] + 15*Sin[d + e*x]))/Sqrt[2 + Sqrt[34]*Cos[d + e*x - ArcTan[5/3]]] + (
27000*(5 + 17*Sin[d + e*x]))/(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(5/2) -
 (300*(305 + 272*Sin[d + e*x]))/(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(3/2
) + (20*(1595 + 3383*Sin[d + e*x]))/Sqrt[2 + 3*Cos[d + e*x] + 5*Sin[d + e*
x]] - 638*Sqrt[30]*AppellF1[1/2, 1/2, 1/2, 3/2, (Sqrt[34] + 17*Sin[d + e*x
 + ArcTan[3/5]])/(-17 + Sqrt[34]), (Sqrt[34] + 17*Sin[d + e*x + ArcTan[3/5
]])/(17 + Sqrt[34])]*Sqrt[Cos[d + e*x + ArcTan[3/5]]^2]*Sec[d + e*x + ArcT
an[3/5]]*Sqrt[2 + Sqrt[34]*Sin[d + e*x + ArcTan[3/5]]] + (2985*Sqrt[30]*Ap
pellF1[-1/2, -1/2, -1/2, 1/2, (Sqrt[34] + 17*Cos[d + e*x - ArcTan[5/3]])/(
-17 + Sqrt[34]), (Sqrt[34] + 17*Cos[d + e*x - ArcTan[5/3]])/(17 + Sqrt[34]
)]*Csc[d + e*x - ArcTan[5/3]]*Sqrt[Sin[d + e*x - ArcTan[5/3]]^2])/Sqrt[2 +
 Sqrt[34]*Cos[d + e*x - ArcTan[5/3]]])/(3037500*e)
```

3.409.3 Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.05, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.818$, Rules used = {3042, 3608, 27, 3042, 3635, 25, 3042, 3635, 27, 3042, 3628, 3042, 3597, 3042, 3132, 3605, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(5\sin(d+ex)+3\cos(d+ex)+2)^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(5\sin(d+ex)+3\cos(d+ex)+2)^{7/2}}dx$$

$$\Big\downarrow \text{ 3608}$$

$$\frac{1}{75}\int -\frac{-9\cos(d+ex)-15\sin(d+ex)+10}{2(3\cos(d+ex)+5\sin(d+ex)+2)^{5/2}}dx-\frac{5\cos(d+ex)-3\sin(d+ex)}{75e(5\sin(d+ex)+3\cos(d+ex)+2)^{5/2}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{1}{150}\int \frac{-9\cos(d+ex)-15\sin(d+ex)+10}{(3\cos(d+ex)+5\sin(d+ex)+2)^{5/2}}dx-\frac{5\cos(d+ex)-3\sin(d+ex)}{75e(5\sin(d+ex)+3\cos(d+ex)+2)^{5/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{1}{150}\int \frac{-9\cos(d+ex)-15\sin(d+ex)+10}{(3\cos(d+ex)+5\sin(d+ex)+2)^{5/2}}dx-\frac{5\cos(d+ex)-3\sin(d+ex)}{75e(5\sin(d+ex)+3\cos(d+ex)+2)^{5/2}}$$

$$\Big\downarrow \text{ 3635}$$

$$\frac{1}{150}\left(\frac{16(5\cos(d+ex)-3\sin(d+ex))}{45e(5\sin(d+ex)+3\cos(d+ex)+2)^{3/2}}-\frac{1}{45}\int -\frac{-24\cos(d+ex)-40\sin(d+ex)+183}{(3\cos(d+ex)+5\sin(d+ex)+2)^{3/2}}dx\right)-\frac{5\cos(d+ex)-3\sin(d+ex)}{75e(5\sin(d+ex)+3\cos(d+ex)+2)^{5/2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{150}\left(\frac{1}{45}\int \frac{-24\cos(d+ex)-40\sin(d+ex)+183}{(3\cos(d+ex)+5\sin(d+ex)+2)^{3/2}}dx+\frac{16(5\cos(d+ex)-3\sin(d+ex))}{45e(5\sin(d+ex)+3\cos(d+ex)+2)^{3/2}}\right)-\frac{5\cos(d+ex)-3\sin(d+ex)}{75e(5\sin(d+ex)+3\cos(d+ex)+2)^{5/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{150}\left(\frac{1}{45}\int \frac{-24\cos(d+ex)-40\sin(d+ex)+183}{(3\cos(d+ex)+5\sin(d+ex)+2)^{3/2}}dx+\frac{16(5\cos(d+ex)-3\sin(d+ex))}{45e(5\sin(d+ex)+3\cos(d+ex)+2)^{3/2}}\right)-\frac{5\cos(d+ex)-3\sin(d+ex)}{75e(5\sin(d+ex)+3\cos(d+ex)+2)^{5/2}}$$

$$\Big\downarrow \text{ 3635}$$

$$\frac{1}{150}\left(\frac{1}{45}\left(\frac{1}{15}\int -\frac{597\cos(d+ex)+995\sin(d+ex)+638}{2\sqrt{3\cos(d+ex)+5\sin(d+ex)+2}}dx-\frac{199(5\cos(d+ex)-3\sin(d+ex))}{15e\sqrt{5\sin(d+ex)+3\cos(d+ex)+2}}\right)+\frac{1}{45e(5}\right.$$
$$-\frac{5\cos(d+ex)-3\sin(d+ex)}{75e(5\sin(d+ex)+3\cos(d+ex)+2)^{5/2}}$$

$$\Big\downarrow \text{ 27}$$

3.409. $\int \frac{1}{(2+3\cos(d+ex)+5\sin(d+ex))^{7/2}}\,dx$

$$\frac{1}{150}\left(\frac{1}{45}\left(-\frac{1}{30}\int\frac{597\cos(d+ex)+995\sin(d+ex)+638}{\sqrt{3\cos(d+ex)+5\sin(d+ex)+2}}dx-\frac{199(5\cos(d+ex)-3\sin(d+ex))}{15e\sqrt{5\sin(d+ex)+3\cos(d+ex)+2}}\right)+\frac{1}{45e(5$$

$$\frac{5\cos(d+ex)-3\sin(d+ex)}{75e(5\sin(d+ex)+3\cos(d+ex)+2)^{5/2}}$$

$\downarrow$ 3042

$$\frac{1}{150}\left(\frac{1}{45}\left(-\frac{1}{30}\int\frac{597\cos(d+ex)+995\sin(d+ex)+638}{\sqrt{3\cos(d+ex)+5\sin(d+ex)+2}}dx-\frac{199(5\cos(d+ex)-3\sin(d+ex))}{15e\sqrt{5\sin(d+ex)+3\cos(d+ex)+2}}\right)+\frac{1}{45e(5$$

$$\frac{5\cos(d+ex)-3\sin(d+ex)}{75e(5\sin(d+ex)+3\cos(d+ex)+2)^{5/2}}$$

$\downarrow$ 3628

$$\frac{1}{150}\left(\frac{1}{45}\left(\frac{1}{30}\left(-240\int\frac{1}{\sqrt{3\cos(d+ex)+5\sin(d+ex)+2}}dx-199\int\sqrt{3\cos(d+ex)+5\sin(d+ex)+2}dx\right)-$$

$$\frac{5\cos(d+ex)-3\sin(d+ex)}{75e(5\sin(d+ex)+3\cos(d+ex)+2)^{5/2}}$$

$\downarrow$ 3042

$$\frac{1}{150}\left(\frac{1}{45}\left(\frac{1}{30}\left(-240\int\frac{1}{\sqrt{3\cos(d+ex)+5\sin(d+ex)+2}}dx-199\int\sqrt{3\cos(d+ex)+5\sin(d+ex)+2}dx\right)-$$

$$\frac{5\cos(d+ex)-3\sin(d+ex)}{75e(5\sin(d+ex)+3\cos(d+ex)+2)^{5/2}}$$

$\downarrow$ 3597

$$\frac{1}{150}\left(\frac{1}{45}\left(\frac{1}{30}\left(-199\int\sqrt{\sqrt{34}\cos\left(d+ex-\arctan\left(\frac{5}{3}\right)\right)+2}dx-240\int\frac{1}{\sqrt{3\cos(d+ex)+5\sin(d+ex)+2}}dx$$

$$\frac{5\cos(d+ex)-3\sin(d+ex)}{75e(5\sin(d+ex)+3\cos(d+ex)+2)^{5/2}}$$

$\downarrow$ 3042

$$\frac{1}{150}\left(\frac{1}{45}\left(\frac{1}{30}\left(-199\int\sqrt{\sqrt{34}\sin\left(d+ex-\arctan\left(\frac{5}{3}\right)+\frac{\pi}{2}\right)+2}dx-240\int\frac{1}{\sqrt{3\cos(d+ex)+5\sin(d+ex)+}}$$

$$\frac{5\cos(d+ex)-3\sin(d+ex)}{75e(5\sin(d+ex)+3\cos(d+ex)+2)^{5/2}}$$

$\downarrow$ 3132

3.409. $\int\frac{1}{(2+3\cos(d+ex)+5\sin(d+ex))^{7/2}}\,dx$

$$\frac{1}{150}\left(\frac{1}{45}\left(\frac{1}{30}\left(-240\int\frac{1}{\sqrt{3\cos(d+ex)+5\sin(d+ex)+2}}dx-\frac{398\sqrt{2+\sqrt{34}}E\left(\frac{1}{2}\left(d+ex-\arctan\left(\frac{5}{3}\right)\right)|\frac{2}{15}(17\right.}{e}\right.\right.\right.$$
$$\frac{5\cos(d+ex)-3\sin(d+ex)}{75e(5\sin(d+ex)+3\cos(d+ex)+2)^{5/2}}$$

$\downarrow$ 3605

$$\frac{1}{150}\left(\frac{1}{45}\left(\frac{1}{30}\left(-240\int\frac{1}{\sqrt{\sqrt{34}\cos\left(d+ex-\arctan\left(\frac{5}{3}\right)\right)+2}}dx-\frac{398\sqrt{2+\sqrt{34}}E\left(\frac{1}{2}\left(d+ex-\arctan\left(\frac{5}{3}\right)\right)|\frac{2}{15}(1\right.}{e}\right.\right.\right.$$
$$\frac{5\cos(d+ex)-3\sin(d+ex)}{75e(5\sin(d+ex)+3\cos(d+ex)+2)^{5/2}}$$

$\downarrow$ 3042

$$\frac{1}{150}\left(\frac{1}{45}\left(\frac{1}{30}\left(-240\int\frac{1}{\sqrt{\sqrt{34}\sin\left(d+ex-\arctan\left(\frac{5}{3}\right)+\frac{\pi}{2}\right)+2}}dx-\frac{398\sqrt{2+\sqrt{34}}E\left(\frac{1}{2}\left(d+ex-\arctan\left(\frac{5}{3}\right)\right)|\right.}{e}\right.\right.\right.$$
$$\frac{5\cos(d+ex)-3\sin(d+ex)}{75e(5\sin(d+ex)+3\cos(d+ex)+2)^{5/2}}$$

$\downarrow$ 3140

$$\frac{1}{150}\left(\frac{1}{45}\left(\frac{1}{30}\left(-\frac{480\,\text{EllipticF}\left(\frac{1}{2}\left(d+ex-\arctan\left(\frac{5}{3}\right)\right),\frac{2}{15}\left(17-\sqrt{34}\right)\right)}{\sqrt{2+\sqrt{34}}e}-\frac{398\sqrt{2+\sqrt{34}}E\left(\frac{1}{2}\left(d+ex-\arctan\left(\frac{5}{3}\right.\right.\right.}{e}\right.\right.\right.$$
$$\frac{5\cos(d+ex)-3\sin(d+ex)}{75e(5\sin(d+ex)+3\cos(d+ex)+2)^{5/2}}$$

input
```
Int[(2 + 3*Cos[d + e*x] + 5*Sin[d + e*x])^(-7/2),x]
```

output
```
-1/75*(5*Cos[d + e*x] - 3*Sin[d + e*x])/(e*(2 + 3*Cos[d + e*x] + 5*Sin[d +
 e*x])^(5/2)) + ((16*(5*Cos[d + e*x] - 3*Sin[d + e*x]))/(45*e*(2 + 3*Cos[d
 + e*x] + 5*Sin[d + e*x])^(3/2)) + (((-398*Sqrt[2 + Sqrt[34]]*EllipticE[(d
 + e*x - ArcTan[5/3])/2, (2*(17 - Sqrt[34]))/15])/e - (480*EllipticF[(d +
e*x - ArcTan[5/3])/2, (2*(17 - Sqrt[34]))/15])/(Sqrt[2 + Sqrt[34]]*e))/30
- (199*(5*Cos[d + e*x] - 3*Sin[d + e*x]))/(15*e*Sqrt[2 + 3*Cos[d + e*x] +
5*Sin[d + e*x]]))/45)/150
```

3.409. $\int \frac{1}{(2+3\cos(d+ex)+5\sin(d+ex))^{7/2}}\,dx$

3.409.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3597
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Int[Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 + c^2, 0] && GtQ[a + Sqrt[b^2
+ c^2], 0]
```

rule 3605
```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 + c^2, 0] && GtQ[a + Sqrt[
b^2 + c^2], 0]
```

rule 3608
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Simp[
1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c
*(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x
] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] &&
NeQ[n, -3/2]
```

3.409. $\int \frac{1}{(2+3\cos(d+ex)+5\sin(d+ex))^{7/2}}\,dx$

rule 3628
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]
, x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[
A*b - a*B, 0]
```

rule 3635
```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(-(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(
a^2 - b^2 - c^2))), x] + Simp[1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a + b*Co
s[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n +
2)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x]
/; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[n, -2]
```

3.409.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.70 (sec) , antiderivative size = 649, normalized size of antiderivative = 2.79

method	result	size
default	Expression too large to display	649

input
```
int(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
17/4*(-(-34^(1/2)*sin(e*x+d+arctan(3/5))-2)*cos(e*x+d+arctan(3/5))^2)^(1/2
)/(17*sin(e*x+d+arctan(3/5))+34^(1/2))^3*(51*34^(1/2)*sin(e*x+d+arctan(3/5
))^2+289*sin(e*x+d+arctan(3/5))^3+2*34^(1/2)+102*sin(e*x+d+arctan(3/5)))*(
-2/1275*(-(-34^(1/2)*sin(e*x+d+arctan(3/5))-2)*cos(e*x+d+arctan(3/5))^2)^(
1/2)/(sin(e*x+d+arctan(3/5))+1/17*34^(1/2))^3+16/57375*34^(1/2)*(-(-34^(1/
2)*sin(e*x+d+arctan(3/5))-2)*cos(e*x+d+arctan(3/5))^2)^(1/2)/(sin(e*x+d+ar
ctan(3/5))+1/17*34^(1/2))^2-6766/50625*34^(1/2)*cos(e*x+d+arctan(3/5))^2/(
-cos(e*x+d+arctan(3/5))^2*34^(1/2)*(-289*sin(e*x+d+arctan(3/5))-17*34^(1/2
)))^(1/2)-1276/50625*(-1+1/17*34^(1/2))*(1/(-34^(1/2)+17)*(-17*sin(e*x+d+a
rctan(3/5))-34^(1/2)))^(1/2)*((-17*sin(e*x+d+arctan(3/5))+17)/(34^(1/2)+17
))^(1/2)*(1/(-34^(1/2)+17)*(17*sin(e*x+d+arctan(3/5))+17))^(1/2)/(-(-34^(1
/2)*sin(e*x+d+arctan(3/5))-2)*cos(e*x+d+arctan(3/5))^2)^(1/2)*EllipticF((1
/(-34^(1/2)+17)*(-17*sin(e*x+d+arctan(3/5))-34^(1/2)))^(1/2),I*((-34^(1/2)
+17)/(34^(1/2)+17))^(1/2))-398/50625*34^(1/2)*(-1+1/17*34^(1/2))*(1/(-34^(
1/2)+17)*(-17*sin(e*x+d+arctan(3/5))-34^(1/2)))^(1/2)*((-17*sin(e*x+d+arct
an(3/5))+17)/(34^(1/2)+17))^(1/2)*(1/(-34^(1/2)+17)*(17*sin(e*x+d+arctan(3
/5))+17))^(1/2)/(-(-34^(1/2)*sin(e*x+d+arctan(3/5))-2)*cos(e*x+d+arctan(3/
5))^2)^(1/2)*((-1/17*34^(1/2)-1)*EllipticE((1/(-34^(1/2)+17)*(-17*sin(e*x+
d+arctan(3/5))-34^(1/2)))^(1/2),I*((-34^(1/2)+17)/(34^(1/2)+17))^(1/2))+El
lipticF((1/(-34^(1/2)+17)*(-17*sin(e*x+d+arctan(3/5))-34^(1/2)))^(1/2),...
```

3.409.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 577, normalized size of antiderivative = 2.48

$$\int \frac{1}{(2+3\cos(d+ex)+5\sin(d+ex))^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(7/2),x, algorithm="fricas")
```

output
```
1/10327500*(559*sqrt(5*I + 3)*(-(594*I + 990)*sqrt(2)*cos(e*x + d)^3 - (28
8*I + 480)*sqrt(2)*cos(e*x + d)^2 + 5*((6*I + 10)*sqrt(2)*cos(e*x + d)^2 +
 (108*I + 180)*sqrt(2)*cos(e*x + d) + (111*I + 185)*sqrt(2))*sin(e*x + d)
+ (783*I + 1305)*sqrt(2)*cos(e*x + d) + (474*I + 790)*sqrt(2))*weierstrass
PInverse(860/289*I + 1376/867, -5480/132651*I - 12056/14739, cos(e*x + d)
- I*sin(e*x + d) - 10/51*I + 2/17) + 559*sqrt(-5*I + 3)*((594*I - 990)*sqr
t(2)*cos(e*x + d)^3 + (288*I - 480)*sqrt(2)*cos(e*x + d)^2 + 5*(-(6*I - 10
)*sqrt(2)*cos(e*x + d)^2 - (108*I - 180)*sqrt(2)*cos(e*x + d) - (111*I - 1
85)*sqrt(2))*sin(e*x + d) - (783*I - 1305)*sqrt(2)*cos(e*x + d) - (474*I -
 790)*sqrt(2))*weierstrassPInverse(-860/289*I + 1376/867, 5480/132651*I -
12056/14739, cos(e*x + d) + I*sin(e*x + d) + 10/51*I + 2/17) + 10149*sqrt(
5*I + 3)*(198*I*sqrt(2)*cos(e*x + d)^3 + 96*I*sqrt(2)*cos(e*x + d)^2 + 5*(
-2*I*sqrt(2)*cos(e*x + d)^2 - 36*I*sqrt(2)*cos(e*x + d) - 37*I*sqrt(2))*si
n(e*x + d) - 261*I*sqrt(2)*cos(e*x + d) - 158*I*sqrt(2))*weierstrassZeta(8
60/289*I + 1376/867, -5480/132651*I - 12056/14739, weierstrassPInverse(860
/289*I + 1376/867, -5480/132651*I - 12056/14739, cos(e*x + d) - I*sin(e*x
+ d) - 10/51*I + 2/17)) + 10149*sqrt(-5*I + 3)*(-198*I*sqrt(2)*cos(e*x + d
)^3 - 96*I*sqrt(2)*cos(e*x + d)^2 + 5*(2*I*sqrt(2)*cos(e*x + d)^2 + 36*I*s
qrt(2)*cos(e*x + d) + 37*I*sqrt(2))*sin(e*x + d) + 261*I*sqrt(2)*cos(e*x +
 d) + 158*I*sqrt(2))*weierstrassZeta(-860/289*I + 1376/867, 5480/132651...
```

3.409.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(2+3\cos(d+ex)+5\sin(d+ex))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))**(7/2),x)
```

output
```
Timed out
```

3.409.7 Maxima [F]

$$\int \frac{1}{(2+3\cos(d+ex)+5\sin(d+ex))^{7/2}}\,dx = \int \frac{1}{(3\,\cos\left(ex+d\right)+5\,\sin\left(ex+d\right)+2)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(7/2),x, algorithm="maxima")
```

output
```
integrate((3*cos(e*x + d) + 5*sin(e*x + d) + 2)^(-7/2), x)
```

3.409.8 Giac [F]

$$\int \frac{1}{(2+3\cos(d+ex)+5\sin(d+ex))^{7/2}}\,dx = \int \frac{1}{(3\,\cos\left(ex+d\right)+5\,\sin\left(ex+d\right)+2)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(2+3*cos(e*x+d)+5*sin(e*x+d))^(7/2),x, algorithm="giac")
```

output
```
integrate((3*cos(e*x + d) + 5*sin(e*x + d) + 2)^(-7/2), x)
```

3.409.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2+3\cos(d+ex)+5\sin(d+ex))^{7/2}}\,dx = \int \frac{1}{(3\,\cos\left(d+e\,x\right)+5\,\sin\left(d+e\,x\right)+2)^{7/2}}\,dx$$

input
```
int(1/(3*cos(d + e*x) + 5*sin(d + e*x) + 2)^(7/2),x)
```

output
```
int(1/(3*cos(d + e*x) + 5*sin(d + e*x) + 2)^(7/2), x)
```

3.410 $\int (a+b\cos(d+ex)+c\sin(d+ex))^{5/2}\,dx$

3.410.1 Optimal result

Integrand size = 22, antiderivative size = 347

$$\int (a+b\cos(d+ex)+c\sin(d+ex))^{5/2}\,dx = -\frac{16(ac\cos(d+ex)-ab\sin(d+ex))\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}{15e} - \frac{2(c\cos(d+ex)-b\sin(d+ex))(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}{5e} + \frac{2(23a^2+9(b^2+c^2))\,E\left(\frac{1}{2}(d+ex-\tan^{-1}(b,c))\,|\,\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}{15e\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}} - \frac{16a(a^2-b^2-c^2)\,\mathrm{EllipticF}\left(\frac{1}{2}(d+ex-\tan^{-1}(b,c)),\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}{15e\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}$$

output
```
-2/5*(c*cos(e*x+d)-b*sin(e*x+d))*(a+b*cos(e*x+d)+c*sin(e*x+d))^(3/2)/e-16/
15*(a*c*cos(e*x+d)-a*b*sin(e*x+d))*(a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2)/e+2
/15*(23*a^2+9*b^2+9*c^2)*(cos(1/2*d+1/2*e*x-1/2*arctan(b,c))^2)^(1/2)/cos(
1/2*d+1/2*e*x-1/2*arctan(b,c))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(b,c)
),2^(1/2)*((b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2))*(a+b*cos(e*x+d)+c*s
in(e*x+d))^(1/2)/e/((a+b*cos(e*x+d)+c*sin(e*x+d))/(a+(b^2+c^2)^(1/2)))^(1/
2)-16/15*a*(a^2-b^2-c^2)*(cos(1/2*d+1/2*e*x-1/2*arctan(b,c))^2)^(1/2)/cos(
1/2*d+1/2*e*x-1/2*arctan(b,c))*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan(b,c)
),2^(1/2)*((b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2))*((a+b*cos(e*x+d)+c*
sin(e*x+d))/(a+(b^2+c^2)^(1/2)))^(1/2)/e/(a+b*cos(e*x+d)+c*sin(e*x+d))^(1/
2)
```

3.410.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 6.76 (sec) , antiderivative size = 3767, normalized size of antiderivative = 10.86

$$\int (a+b\cos(d+ex)+c\sin(d+ex))^{5/2}\,dx=\text{Result too large to show}$$

input
```
Integrate[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(5/2),x]
```

output
```
(Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]*((2*b*(23*a^2 + 9*b^2 + 9*c^2))
/(15*c) - (22*a*c*Cos[d + e*x])/15 - (2*b*c*Cos[2*(d + e*x)])/5 + (22*a*b*
Sin[d + e*x])/15 + ((b^2 - c^2)*Sin[2*(d + e*x)])/5))/e + (2*a^3*AppellF1[
1/2, 1/2, 1/2, 3/2, -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])
/(Sqrt[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1 + b^2
/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b
^2/c^2]*c))*c))]*Sec[d + e*x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2]
- c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c
^2)/c^2])]*Sqrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]]]*Sq
rt[(c*Sqrt[(b^2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan
[b/c]])/(-a + c*Sqrt[(b^2 + c^2)/c^2])]])/(Sqrt[1 + b^2/c^2]*c*e) + (34*a*b
^2*AppellF1[1/2, 1/2, 1/2, 3/2, -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + A
rcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a +
Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a
/(Sqrt[1 + b^2/c^2]*c))*c))]*Sec[d + e*x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2
+ c^2)/c^2] - c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(a + c*S
qrt[(b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcT
an[b/c]]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d +
e*x + ArcTan[b/c]])/(-a + c*Sqrt[(b^2 + c^2)/c^2])]])/(15*Sqrt[1 + b^2/c^2]
*c*e) + (34*a*c*AppellF1[1/2, 1/2, 1/2, 3/2, -((a + Sqrt[1 + b^2/c^2]*c...
```

3.410.3 Rubi [A] (verified)

Time = 1.62 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.03, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}=0.682$, Rules used = {3042, 3599, 27, 3042, 3625, 27, 3042, 3628, 3042, 3598, 3042, 3132, 3606, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cos(d+ex)+c\sin(d+ex))^{5/2}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int (a+b\cos(d+ex)+c\sin(d+ex))^{5/2}dx$$

$$\Big\downarrow \ 3599$$

$$\frac{2}{5}\int \frac{1}{2}\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}\left(5a^2+8b\cos(d+ex)a+8c\sin(d+ex)a+3\left(b^2+c^2\right)\right)dx-\frac{2(c\cos(d+ex)-b\sin(d+ex))(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}{5e}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{5}\int \sqrt{a+b\cos(d+ex)+c\sin(d+ex)}\left(5a^2+8b\cos(d+ex)a+8c\sin(d+ex)a+3\left(b^2+c^2\right)\right)dx-\frac{2(c\cos(d+ex)-b\sin(d+ex))(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}{5e}$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{5}\int \sqrt{a+b\cos(d+ex)+c\sin(d+ex)}\left(5a^2+8b\cos(d+ex)a+8c\sin(d+ex)a+3\left(b^2+c^2\right)\right)dx-\frac{2(c\cos(d+ex)-b\sin(d+ex))(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}{5e}$$

$$\Big\downarrow \ 3625$$

$$\frac{1}{5}\left(\frac{2\int \frac{\left(15a^2+17\left(b^2+c^2\right)\right)a^2+b\left(23a^2+9\left(b^2+c^2\right)\right)\cos(d+ex)a+c\left(23a^2+9\left(b^2+c^2\right)\right)\sin(d+ex)a}{2\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}dx}{3a}-\frac{16(ac\cos(d+ex)-ab\sin(d+e}{}\right.$$
$$-\frac{2(c\cos(d+ex)-b\sin(d+ex))(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}{5e}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{5}\left(\frac{\int \frac{\left(15a^2+17\left(b^2+c^2\right)\right)a^2+b\left(23a^2+9\left(b^2+c^2\right)\right)\cos(d+ex)a+c\left(23a^2+9\left(b^2+c^2\right)\right)\sin(d+ex)a}{\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}dx}{3a}-\frac{16(ac\cos(d+ex)-ab\sin(d+ex}{}\right.$$
$$-\frac{2(c\cos(d+ex)-b\sin(d+ex))(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}{5e}$$

$$\Big\downarrow \ 3042$$

3.410. $\int (a+b\cos(d+ex)+c\sin(d+ex))^{5/2}\,dx$

$$\frac{1}{5}\left(\frac{\int \frac{(15a^2+17(b^2+c^2))a^2+b(23a^2+9(b^2+c^2))\cos(d+ex)a+c(23a^2+9(b^2+c^2))\sin(d+ex)a}{\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}dx}{3a} - \frac{16(ac\cos(d+ex)-ab\sin(d+ex$$

$$\frac{2(c\cos(d+ex)-b\sin(d+ex))(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}{5e}$$

$\downarrow$ 3628

$$\frac{1}{5}\left(\frac{a\left(23a^2+9\left(b^2+c^2\right)\right)\int\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}dx-8a^2\left(a^2-b^2-c^2\right)\int\frac{1}{\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}d}{3a}\right.$$

$$\frac{2(c\cos(d+ex)-b\sin(d+ex))(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}{5e}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{a\left(23a^2+9\left(b^2+c^2\right)\right)\int\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}dx-8a^2\left(a^2-b^2-c^2\right)\int\frac{1}{\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}d}{3a}\right.$$

$$\frac{2(c\cos(d+ex)-b\sin(d+ex))(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}{5e}$$

$\downarrow$ 3598

$$\frac{1}{5}\left(\frac{\frac{a\left(23a^2+9\left(b^2+c^2\right)\right)\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}\int\sqrt{\frac{a}{a+\sqrt{b^2+c^2}}+\frac{\sqrt{b^2+c^2}\cos\left(d+ex-\tan^{-1}(b,c)\right)}{a+\sqrt{b^2+c^2}}}dx}{\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}-8a^2\left(a^2-b^2-c^2\right)\int\frac{}{\sqrt{a+b\cos(}}}{3a}\right.$$

$$\frac{2(c\cos(d+ex)-b\sin(d+ex))(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}{5e}$$

$\downarrow$ 3042

$$\frac{1}{5}\left(\frac{\frac{a\left(23a^2+9\left(b^2+c^2\right)\right)\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}\int\sqrt{\frac{a}{a+\sqrt{b^2+c^2}}+\frac{\sqrt{b^2+c^2}\sin\left(d+ex-\tan^{-1}(b,c)+\frac{\pi}{2}\right)}{a+\sqrt{b^2+c^2}}}dx}{\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}-8a^2\left(a^2-b^2-c^2\right)\int\frac{}{\sqrt{a+b\,c}}}{3a}\right.$$

$$\frac{2(c\cos(d+ex)-b\sin(d+ex))(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}{5e}$$

3.410. $\int(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}\,dx$

$$\Big\downarrow \text{ 3132}$$

$$\frac{1}{5}\left(\frac{\frac{2a(23a^2+9(b^2+c^2))\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}E\left(\frac{1}{2}(d+ex-\tan^{-1}(b,c))|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{e\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}-8a^2\left(a^2-b^2-c^2\right)\int\frac{1}{\sqrt{a+b\cos(d+ex)+c\ldots}}}{3a}\right.$$

$$\frac{2(c\cos(d+ex)-b\sin(d+ex))(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}{5e}$$

$$\Big\downarrow \text{ 3606}$$

$$\frac{1}{5}\left(\frac{\frac{2a(23a^2+9(b^2+c^2))\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}E\left(\frac{1}{2}(d+ex-\tan^{-1}(b,c))|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{e\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}-\frac{8a^2(a^2-b^2-c^2)\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}\int\ldots}{\sqrt{a+b\cos(d+e\ldots}}}{3a}\right.$$

$$\frac{2(c\cos(d+ex)-b\sin(d+ex))(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}{5e}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{5}\left(\frac{\frac{2a(23a^2+9(b^2+c^2))\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}E\left(\frac{1}{2}(d+ex-\tan^{-1}(b,c))|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{e\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}-\frac{8a^2(a^2-b^2-c^2)\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}\int\ldots}{\sqrt{a+b\cos(d+\ldots}}}{3a}\right.$$

$$\frac{2(c\cos(d+ex)-b\sin(d+ex))(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}{5e}$$

$$\Big\downarrow \text{ 3140}$$

3.410. $\int(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}\,dx$

$$\frac{1}{5}\left(\frac{\frac{2a(23a^2+9(b^2+c^2))\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}E\left(\frac{1}{2}(d+ex-\tan^{-1}(b,c))|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{e\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}-\frac{16a^2(a^2-b^2-c^2)\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}\,\mathrm{E}}{e\sqrt{a+b\cos(d+ex}}}{3a}\right.$$

$$\frac{2(c\cos(d+ex)-b\sin(d+ex))(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}{5e}$$

input
```
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(5/2),x]
```

output
```
(-2*(c*Cos[d + e*x] - b*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x]
)^(3/2))/(5*e) + ((-16*(a*c*Cos[d + e*x] - a*b*Sin[d + e*x])*Sqrt[a + b*Co
s[d + e*x] + c*Sin[d + e*x]])/(3*e) + ((2*a*(23*a^2 + 9*(b^2 + c^2))*Ellip
ticE[(d + e*x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c^2])
]*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*Sqrt[(a + b*Cos[d + e*x] +
 c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]) - (16*a^2*(a^2 - b^2 - c^2)*Ellip
ticF[(d + e*x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c^2])
]*Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])])/(e*Sq
rt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]))/(3*a))/5
```

3.410.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3598
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]   Int[Sqrt[a/(a + S
qrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - Arc
Tan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

rule 3599
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Simp[1/n   Int[Simp[n*a^2 + (
n - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x
], x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

rule 3606
```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Simp[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]   Int[1/Sqrt[a/(a
 + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x -
 ArcTan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2
, 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

rule 3625
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*((a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^n/(a*e*(n + 1))), x] + Simp[1/(a*(n + 1
))   Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
 + a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
 + (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

rule 3628
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]
, x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[
A*b - a*B, 0]
```

3.410. $\int (a+b\cos(d+ex)+c\sin(d+ex))^{5/2}\,dx$

3.410.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2269 vs. $2(394) = 788$.

Time = 3.02 (sec) , antiderivative size = 2270, normalized size of antiderivative = 6.54

method	result	size
default	Expression too large to display	2270

input
```
int((a+b*cos(e*x+d)+c*sin(e*x+d))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(-(-b^2*sin(e*x+d-arctan(-b,c))-c^2*sin(e*x+d-arctan(-b,c))-a*(b^2+c^2)^(1
/2))*cos(e*x+d-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)*(2*a^3*(1/(b^2+c^2)^
(1/2)*a+1)*(((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)
))^(1/2)*((sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2))
)^(1/2)*((-sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))
^(1/2)/(-(-b^2*sin(e*x+d-arctan(-b,c))-c^2*sin(e*x+d-arctan(-b,c))-a*(b^2+
c^2)^(1/2))*cos(e*x+d-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)*EllipticF(((
b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2),((-a-
(b^2+c^2)^(1/2))/(-a+(b^2+c^2)^(1/2)))^(1/2))+(b^2+c^2)^(3/2)*(-2/5/(b^2+c
^2)^(1/2)*sin(e*x+d-arctan(-b,c))*(((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c)
)+a)*cos(e*x+d-arctan(-b,c))^2)^(1/2)+8/15/(b^2+c^2)*a*(((b^2+c^2)^(1/2)*s
in(e*x+d-arctan(-b,c))+a)*cos(e*x+d-arctan(-b,c))^2)^(1/2)+4/15/(b^2+c^2)^
(1/2)*a*(1/(b^2+c^2)^(1/2)*a+1)*(((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+
a)/(a+(b^2+c^2)^(1/2)))^(1/2)*((sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)
/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)
/(a+(b^2+c^2)^(1/2)))^(1/2)/(((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a)*c
os(e*x+d-arctan(-b,c))^2)^(1/2)*EllipticF((((b^2+c^2)^(1/2)*sin(e*x+d-arct
an(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2),((-a-(b^2+c^2)^(1/2))/(-a+(b^2+c^2
)^(1/2)))^(1/2))+2*(3/5+8/15/(b^2+c^2)*a^2)*(1/(b^2+c^2)^(1/2)*a+1)*(((b^2
+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2)*((sin...
```

3.410.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 1595, normalized size of antiderivative = 4.60

$$\int (a + b\cos(d + ex) + c\sin(d + ex))^{5/2} \, dx = \text{Too large to display}$$

input
```
integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^(5/2),x, algorithm="fricas")
```

output
```
1/45*(sqrt(2)*(-I*a^3*b + 33*I*a*b^3 + 33*I*a*b*c^2 + 33*a*c^3 - (a^3 - 33
*a*b^2)*c)*sqrt(b + I*c)*weierstrassPInverse(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^
2*c^2 + 6*I*b*c^3 + 3*c^4 - 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^
4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 - 9*I*a*c^5 + 2*I*(4*a^3 + 9*a
*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 - 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6
+ 3*b^4*c^2 + 3*b^2*c^4 + c^6), 1/3*(2*a*b - 2*I*a*c + 3*(b^2 + c^2)*cos(e
*x + d) - 3*(I*b^2 + I*c^2)*sin(e*x + d))/(b^2 + c^2)) + sqrt(2)*(I*a^3*b
- 33*I*a*b^3 - 33*I*a*b*c^2 + 33*a*c^3 - (a^3 - 33*a*b^2)*c)*sqrt(b - I*c)
*weierstrassPInverse(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2*c^2 - 6*I*b*c^3 + 3*c^
4 + 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4), -8/27*(8*a^3*b^3 - 9
*a*b^5 + 27*a*b*c^4 + 9*I*a*c^5 - 2*I*(4*a^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b -
 3*a*b^3)*c^2 + 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4
+ c^6), 1/3*(2*a*b + 2*I*a*c + 3*(b^2 + c^2)*cos(e*x + d) - 3*(-I*b^2 - I*
c^2)*sin(e*x + d))/(b^2 + c^2)) - 3*sqrt(2)*(23*I*a^2*b^2 + 9*I*b^4 + 9*I*
c^4 + I*(23*a^2 + 18*b^2)*c^2)*sqrt(b + I*c)*weierstrassZeta(4/3*(4*a^2*b^
2 - 3*b^4 - 4*a^2*c^2 + 6*I*b*c^3 + 3*c^4 - 2*I*(4*a^2*b - 3*b^3)*c)/(b^4
+ 2*b^2*c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 - 9*I*a*c^5 +
2*I*(4*a^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 - 3*I*(8*a^3*b^2 - 9
*a*b^4)*c)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), weierstrassPInverse(4/3*(4
*a^2*b^2 - 3*b^4 - 4*a^2*c^2 + 6*I*b*c^3 + 3*c^4 - 2*I*(4*a^2*b - 3*b^3...
```

3.410.6 Sympy [F(-1)]

Timed out.

$$\int (a + b\cos(d + ex) + c\sin(d + ex))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(e*x+d)+c*sin(e*x+d))**(5/2),x)
```

output
```
Timed out
```

3.410.7 Maxima [F]

$$\int (a+b\cos(d+ex)+c\sin(d+ex))^{5/2}\,dx=\int (b\cos(ex+d)+c\sin(ex+d)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(e*x + d) + c*sin(e*x + d) + a)^(5/2), x)
```

3.410.8 Giac [F]

$$\int (a+b\cos(d+ex)+c\sin(d+ex))^{5/2}\,dx=\int (b\cos(ex+d)+c\sin(ex+d)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*cos(e*x + d) + c*sin(e*x + d) + a)^(5/2), x)
```

3.410.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(d+ex)+c\sin(d+ex))^{5/2}\,dx=\int \left(a+b\,\cos\left(d+e\,x\right)+c\,\sin\left(d+e\,x\right)\right)^{5/2}\,dx$$

input
```
int((a + b*cos(d + e*x) + c*sin(d + e*x))^(5/2),x)
```

output
```
int((a + b*cos(d + e*x) + c*sin(d + e*x))^(5/2), x)
```

3.411 $\int (a+b\cos(d+ex)+c\sin(d+ex))^{3/2}\,dx$

3.411.1 Optimal result

Integrand size = 22, antiderivative size = 283

$$\int (a+b\cos(d+ex)+c\sin(d+ex))^{3/2}\,dx = -\frac{2(c\cos(d+ex)-b\sin(d+ex))\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}{3e} + \frac{8aE\left(\frac{1}{2}(d+ex-\tan^{-1}(b,c))\,|\,\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}{3e\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}} - \frac{2(a^2-b^2-c^2)\,\mathrm{EllipticF}\left(\frac{1}{2}(d+ex-\tan^{-1}(b,c)),\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}{3e\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}$$

output
```
-2/3*(c*cos(e*x+d)-b*sin(e*x+d))*(a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2)/e+8/3
*a*(cos(1/2*d+1/2*e*x-1/2*arctan(b,c))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arct
an(b,c))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(b,c)),2^(1/2)*((b^2+c^2)^(
1/2)/(a+(b^2+c^2)^(1/2)))^(1/2))*(a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2)/e/((a
+b*cos(e*x+d)+c*sin(e*x+d))/(a+(b^2+c^2)^(1/2)))^(1/2)-2/3*(a^2-b^2-c^2)*(
cos(1/2*d+1/2*e*x-1/2*arctan(b,c))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(b
,c))*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan(b,c)),2^(1/2)*((b^2+c^2)^(1/2)
/(a+(b^2+c^2)^(1/2)))^(1/2))*((a+b*cos(e*x+d)+c*sin(e*x+d))/(a+(b^2+c^2)^(
1/2)))^(1/2)/e/(a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2)
```

3.411.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 6.34 (sec) , antiderivative size = 2190, normalized size of antiderivative = 7.74

$$\int (a + b\cos(d + ex) + c\sin(d + ex))^{3/2} \, dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(3/2),x]
```

output
```
(((8*a*b)/(3*c) - (2*c*Cos[d + e*x])/3 + (2*b*Sin[d + e*x])/3)*Sqrt[a + b*
Cos[d + e*x] + c*Sin[d + e*x]])/e + (2*a^2*AppellF1[1/2, 1/2, 1/2, 3/2, -(
(a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(1
 - a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x +
ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c))*c))]*Sec[d
 + e*x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqrt[(b^2 + c^2)/c
^2]*Sin[d + e*x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*Sqrt[a + c*
Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[(b^2 + c^2)
/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(-a + c*Sqrt[(
b^2 + c^2)/c^2])])/(Sqrt[1 + b^2/c^2]*c*e) + (2*b^2*AppellF1[1/2, 1/2, 1/2
, 3/2, -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^
2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1 + b^2/c^2]*c*Sin[d
 + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c))*c
))]*Sec[d + e*x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqrt[(b^2
 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*Sq
rt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[(b
^2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(-a +
 c*Sqrt[(b^2 + c^2)/c^2])])/(3*Sqrt[1 + b^2/c^2]*c*e) + (2*c*AppellF1[1/2,
 1/2, 1/2, 3/2, -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sq
rt[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1 + b^2/...
```

3.411.3 Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {3042, 3599, 27, 3042, 3628, 3042, 3598, 3042, 3132, 3606, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cos(d+ex)+c\sin(d+ex))^{3/2}\,dx$$

$\Big\downarrow$ 3042

$$\int (a+b\cos(d+ex)+c\sin(d+ex))^{3/2}dx$$

$\Big\downarrow$ 3599

$$\frac{2}{3}\int \frac{3a^2+4b\cos(d+ex)a+4c\sin(d+ex)a+b^2+c^2}{2\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}dx-\frac{2(c\cos(d+ex)-b\sin(d+ex))\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}{3e}$$

$\Big\downarrow$ 27

$$\frac{1}{3}\int \frac{3a^2+4b\cos(d+ex)a+4c\sin(d+ex)a+b^2+c^2}{\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}dx-\frac{2(c\cos(d+ex)-b\sin(d+ex))\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}{3e}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\int \frac{3a^2+4b\cos(d+ex)a+4c\sin(d+ex)a+b^2+c^2}{\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}dx-\frac{2(c\cos(d+ex)-b\sin(d+ex))\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}{3e}$$

$\Big\downarrow$ 3628

$$\frac{1}{3}\left(4a\int \sqrt{a+b\cos(d+ex)+c\sin(d+ex)}dx-\left(a^2-b^2-c^2\right)\int \frac{1}{\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}dx\right)-\frac{2(c\cos(d+ex)-b\sin(d+ex))\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}{3e}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(4a\int \sqrt{a+b\cos(d+ex)+c\sin(d+ex)}dx-\left(a^2-b^2-c^2\right)\int \frac{1}{\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}dx\right)-\frac{2(c\cos(d+ex)-b\sin(d+ex))\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}{3e}$$

$\Big\downarrow$ 3598

3.411. $\int (a+b\cos(d+ex)+c\sin(d+ex))^{3/2}\,dx$

$$\frac{1}{3}\left(\frac{4a\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}\int\sqrt{\frac{a}{a+\sqrt{b^2+c^2}}+\frac{\sqrt{b^2+c^2}\cos(d+ex-\tan^{-1}(b,c))}{a+\sqrt{b^2+c^2}}}dx}{\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}-\left(a^2-b^2-c^2\right)\int\frac{}{\sqrt{a+b}}\right.$$

$$\frac{2(c\cos(d+ex)-b\sin(d+ex))\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}{3e}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(\frac{4a\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}\int\sqrt{\frac{a}{a+\sqrt{b^2+c^2}}+\frac{\sqrt{b^2+c^2}\sin\left(d+ex-\tan^{-1}(b,c)+\frac{\pi}{2}\right)}{a+\sqrt{b^2+c^2}}}dx}{\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}-\left(a^2-b^2-c^2\right)\int\frac{}{\sqrt{a}}\right.$$

$$\frac{2(c\cos(d+ex)-b\sin(d+ex))\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}{3e}$$

$\Big\downarrow$ 3132

$$\frac{1}{3}\left(\frac{8a\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{e\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}-\left(a^2-b^2-c^2\right)\int\frac{}{\sqrt{a+b\cos(d}}\right.$$

$$\frac{2(c\cos(d+ex)-b\sin(d+ex))\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}{3e}$$

$\Big\downarrow$ 3606

$$\frac{1}{3}\left(\frac{8a\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{e\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}-\frac{\left(a^2-b^2-c^2\right)\sqrt{\frac{a+b\cos(d+ex)+}{a+\sqrt{b^2+}}}}{\sqrt{a+b}}\right.$$

$$\frac{2(c\cos(d+ex)-b\sin(d+ex))\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}{3e}$$

$\Big\downarrow$ 3042

$$\frac{1}{3}\left(\frac{8a\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{e\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}-\frac{\left(a^2-b^2-c^2\right)\sqrt{\frac{a+b\cos(d+ex)+}{a+\sqrt{b^2+}}}}{\sqrt{a+}}\right.$$

$$\frac{2(c\cos(d+ex)-b\sin(d+ex))\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}{3e}$$

$\Big\downarrow$ 3140

3.411. $\int(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}\,dx$

$$\frac{1}{3}\left(\frac{8a\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{e\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}-\frac{2(a^2-b^2-c^2)\sqrt{\frac{a+b\cos(d+ex)}{a+\sqrt{b^2}}}}{e\sqrt{a}}\right.$$
$$\frac{2(c\cos(d+ex)-b\sin(d+ex))\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}{3e}$$

input
```
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(3/2),x]
```

output
```
(-2*(c*Cos[d + e*x] - b*Sin[d + e*x])*Sqrt[a + b*Cos[d + e*x] + c*Sin[d +
e*x]])/(3*e) + ((8*a*EllipticE[(d + e*x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c
^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*
Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]) - (2*(a
^2 - b^2 - c^2)*EllipticF[(d + e*x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/
(a + Sqrt[b^2 + c^2])]*Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqr
t[b^2 + c^2])])/(e*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]))/3
```

3.411.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3598
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]   Int[Sqrt[a/(a + S
qrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - Arc
Tan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

rule 3599
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Simp[1/n   Int[Simp[n*a^2 + (
n - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x
], x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

rule 3606
```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Simp[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]   Int[1/Sqrt[a/(a
 + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x -
 ArcTan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2
, 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

rule 3628
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]
, x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[
A*b - a*B, 0]
```

3.411.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1493 vs. $2(333) = 666$.

Time = 1.98 (sec) , antiderivative size = 1494, normalized size of antiderivative = 5.28

method	result	size
default	Expression too large to display	1494

input
```
int((a+b*cos(e*x+d)+c*sin(e*x+d))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(-(-b^2*sin(e*x+d-arctan(-b,c))-c^2*sin(e*x+d-arctan(-b,c))-a*(b^2+c^2)^(1
/2))*cos(e*x+d-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)*(2*a^2*(1/(b^2+c^2)^
(1/2)*a+1)*(((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)
))^(1/2)*((sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2))
)^(1/2)*((-sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))
^(1/2)/(-(-b^2*sin(e*x+d-arctan(-b,c))-c^2*sin(e*x+d-arctan(-b,c))-a*(b^2+
c^2)^(1/2))*cos(e*x+d-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)*EllipticF(((
b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2),((-a-
(b^2+c^2)^(1/2))/(-a+(b^2+c^2)^(1/2)))^(1/2))+(b^2+c^2)*(-2/3/(b^2+c^2)^(1
/2)*(((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a)*cos(e*x+d-arctan(-b,c))^2
)^(1/2)+2/3*(1/(b^2+c^2)^(1/2)*a+1)*(((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,
c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2)*((sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(
1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(
1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)/(((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+
a)*cos(e*x+d-arctan(-b,c))^2)^(1/2)*EllipticF((((b^2+c^2)^(1/2)*sin(e*x+d-
arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2),((-a-(b^2+c^2)^(1/2))/(-a+(b^2
+c^2)^(1/2)))^(1/2))-4/3/(b^2+c^2)^(1/2)*a*(1/(b^2+c^2)^(1/2)*a+1)*(((b^2+
c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2)*((sin(e*x
+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(e*x
+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)/(((b^2+c...
```

3.411.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 1483, normalized size of antiderivative = 5.24

$$\int (a+b\cos(d+ex)+c\sin(d+ex))^{3/2}\,dx=\text{Too large to display}$$

input
```
integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^(3/2),x, algorithm="fricas")
```

output
```
1/9*(sqrt(2)*(I*a^2*b + 3*I*b^3 + 3*I*b*c^2 + 3*c^3 + (a^2 + 3*b^2)*c)*sqr
t(b + I*c)*weierstrassPInverse(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2*c^2 + 6*I*b*
c^3 + 3*c^4 - 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4), -8/27*(8*a
^3*b^3 - 9*a*b^5 + 27*a*b*c^4 - 9*I*a*c^5 + 2*I*(4*a^3 + 9*a*b^2)*c^3 - 6*
(4*a^3*b - 3*a*b^3)*c^2 - 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^4*c^2 +
3*b^2*c^4 + c^6), 1/3*(2*a*b - 2*I*a*c + 3*(b^2 + c^2)*cos(e*x + d) - 3*(I
*b^2 + I*c^2)*sin(e*x + d))/(b^2 + c^2)) + sqrt(2)*(-I*a^2*b - 3*I*b^3 - 3
*I*b*c^2 + 3*c^3 + (a^2 + 3*b^2)*c)*sqrt(b - I*c)*weierstrassPInverse(4/3*
(4*a^2*b^2 - 3*b^4 - 4*a^2*c^2 - 6*I*b*c^3 + 3*c^4 + 2*I*(4*a^2*b - 3*b^3)
*c)/(b^4 + 2*b^2*c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 + 9*I
*a*c^5 - 2*I*(4*a^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 + 3*I*(8*a^
3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), 1/3*(2*a*b + 2*I*
a*c + 3*(b^2 + c^2)*cos(e*x + d) - 3*(-I*b^2 - I*c^2)*sin(e*x + d))/(b^2 +
 c^2)) - 12*sqrt(2)*(I*a*b^2 + I*a*c^2)*sqrt(b + I*c)*weierstrassZeta(4/3*
(4*a^2*b^2 - 3*b^4 - 4*a^2*c^2 + 6*I*b*c^3 + 3*c^4 - 2*I*(4*a^2*b - 3*b^3)
*c)/(b^4 + 2*b^2*c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 - 9*I
*a*c^5 + 2*I*(4*a^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 - 3*I*(8*a^
3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), weierstrassPInver
se(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2*c^2 + 6*I*b*c^3 + 3*c^4 - 2*I*(4*a^2*b -
 3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b...
```

3.411.6 Sympy [F]

$$\int (a+b\cos(d+ex)+c\sin(d+ex))^{3/2}\,dx=\int \left(a+b\cos\left(d+ex\right)+c\sin\left(d+ex\right)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cos(e*x+d)+c*sin(e*x+d))**(3/2),x)
```

output
```
Integral((a + b*cos(d + e*x) + c*sin(d + e*x))**(3/2), x)
```

3.411.7 Maxima [F]

$$\int (a+b\cos(d+ex)+c\sin(d+ex))^{3/2}\,dx=\int (b\cos(ex+d)+c\sin(ex+d)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(e*x + d) + c*sin(e*x + d) + a)^(3/2), x)
```

3.411.8 Giac [F]

$$\int (a+b\cos(d+ex)+c\sin(d+ex))^{3/2}\,dx=\int (b\cos(ex+d)+c\sin(ex+d)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*cos(e*x + d) + c*sin(e*x + d) + a)^(3/2), x)
```

3.411.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(d+ex)+c\sin(d+ex))^{3/2}\,dx=\int (a+b\,\cos(d+e\,x)+c\,\sin(d+e\,x))^{3/2}\,dx$$

input
```
int((a + b*cos(d + e*x) + c*sin(d + e*x))^(3/2),x)
```

output
```
int((a + b*cos(d + e*x) + c*sin(d + e*x))^(3/2), x)
```

3.412 $\int \sqrt{a+b\cos(d+ex)+c\sin(d+ex)}\,dx$

3.412.1 Optimal result

Integrand size = 22, antiderivative size = 108

$$\int \sqrt{a+b\cos(d+ex)+c\sin(d+ex)}\,dx = \frac{2E\left(\frac{1}{2}(d+ex-\tan^{-1}(b,c))\,|\,\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}{e\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}$$

output
```
2*(cos(1/2*d+1/2*e*x-1/2*arctan(b,c))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arcta
n(b,c))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(b,c)),2^(1/2)*((b^2+c^2)^(1
/2)/(a+(b^2+c^2)^(1/2)))^(1/2))*(a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2)/e/((a+
b*cos(e*x+d)+c*sin(e*x+d))/(a+(b^2+c^2)^(1/2)))^(1/2)
```

3.412.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 6.31 (sec) , antiderivative size = 1408, normalized size of antiderivative = 13.04

$$\int \sqrt{a+b\cos(d+ex)+c\sin(d+ex)}\,dx = \text{Too large to display}$$

input
```
Integrate[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],x]
```

output
```
(2*b*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]])/(c*e) + (2*a*AppellF1[1/2,
 1/2, 1/2, 3/2, -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sq
rt[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1 + b^2/c^2
]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c
^2]*c))*c))]*Sec[d + e*x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*
Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/
c^2])]*Sqrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]]]*Sqrt[(
c*Sqrt[(b^2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c
]])/(-a + c*Sqrt[(b^2 + c^2)/c^2])])/(Sqrt[1 + b^2/c^2]*c*e) + (b^2*(-((c*
AppellF1[-1/2, -1/2, -1/2, 1/2, -((a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - A
rcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2/b^2])))), -((a +
b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(-1 -
 a/(b*Sqrt[1 + c^2/b^2]))))]*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b
^2]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] - b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x -
ArcTan[c/b]])/(a + b*Sqrt[(b^2 + c^2)/b^2])]*Sqrt[a + b*Sqrt[(b^2 + c^2)/b
^2]*Cos[d + e*x - ArcTan[c/b]]]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] + b*Sqrt[(b^
2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(-a + b*Sqrt[(b^2 + c^2)/b^2])])
) - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]))/(b^2 + c^2
) - (c*Sin[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]))/Sqrt[a + b*Sqrt[
1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]]])/(c*e) + (c*(-((c*AppellF1[-1...
```

3.412.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3042, 3598, 3042, 3132}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\cos(d+ex)+c\sin(d+ex)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a+b\cos(d+ex)+c\sin(d+ex)}dx$$

$$\Big\downarrow \text{3598}$$

$$\frac{\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}\int \sqrt{\frac{a}{a+\sqrt{b^2+c^2}}+\frac{\sqrt{b^2+c^2}\cos(d+ex-\tan^{-1}(b,c))}{a+\sqrt{b^2+c^2}}}dx}{\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}\int\sqrt{\frac{a}{a+\sqrt{b^2+c^2}}+\frac{\sqrt{b^2+c^2}\sin\left(d+ex-\tan^{-1}(b,c)+\frac{\pi}{2}\right)}{a+\sqrt{b^2+c^2}}}dx}{\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}$$

$$\Big\downarrow \; 3132$$

$$\frac{2\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{e\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}$$

input
```
Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],x]
```

output
```
(2*EllipticE[(d + e*x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2
 + c^2])]*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*Sqrt[(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])])
```

3.412.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3598
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]   Int[Sqrt[a/(a + S
qrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - Arc
Tan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

3.412.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 690 vs. $2(137) = 274$.

Time = 5.31 (sec) , antiderivative size = 691, normalized size of antiderivative = 6.40

method	result
default	$\frac{2\left(a+\sqrt{b^2+c^2}\right)\sqrt{\frac{\sqrt{b^2+c^2}\,\sin(ex+d-\arctan(-b,c))+a}{a+\sqrt{b^2+c^2}}}\sqrt{\frac{(\sin(ex+d-\arctan(-b,c))+1)\sqrt{b^2+c^2}}{-a+\sqrt{b^2+c^2}}}\sqrt{-\frac{(\sin(ex+d-\arctan(-b,c))-1)\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}}\sqrt{\sqrt{}}}{}$
risch	Expression too large to display

input
```
int((a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(a+(b^2+c^2)^(1/2))*(((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a)/(a+(b^2
+c^2)^(1/2)))^(1/2)*((sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(-a+(b^2+
c^2)^(1/2)))^(1/2)*(-(sin(e*x+d-arctan(-b,c))-1)*(b^2+c^2)^(1/2)/(a+(b^2+c
^2)^(1/2)))^(1/2)/(b^2+c^2)^(1/2)*((b^2+c^2)^(1/2)*cos(e*x+d-arctan(-b,c))
^2*sin(e*x+d-arctan(-b,c))+cos(e*x+d-arctan(-b,c))^2*a)^(1/2)*((b^2+c^2)^(
1/2)*EllipticE((1/(a+(b^2+c^2)^(1/2))*(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,
c))+1/(a+(b^2+c^2)^(1/2))*a)^(1/2),(-(a+(b^2+c^2)^(1/2))/(-a+(b^2+c^2)^(1/
2)))^(1/2))-(b^2+c^2)^(1/2)*EllipticF((1/(a+(b^2+c^2)^(1/2))*(b^2+c^2)^(1/
2)*sin(e*x+d-arctan(-b,c))+1/(a+(b^2+c^2)^(1/2))*a)^(1/2),(-(a+(b^2+c^2)^(
1/2))/(-a+(b^2+c^2)^(1/2)))^(1/2))-EllipticE((1/(a+(b^2+c^2)^(1/2))*(b^2+c
^2)^(1/2)*sin(e*x+d-arctan(-b,c))+1/(a+(b^2+c^2)^(1/2))*a)^(1/2),(-(a+(b^2
+c^2)^(1/2))/(-a+(b^2+c^2)^(1/2)))^(1/2))*a+EllipticF((1/(a+(b^2+c^2)^(1/2
))*(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+1/(a+(b^2+c^2)^(1/2))*a)^(1/2),
(-(a+(b^2+c^2)^(1/2))/(-a+(b^2+c^2)^(1/2)))^(1/2))*a)/(((b^2+c^2)^(1/2)*si
n(e*x+d-arctan(-b,c))+a)*cos(e*x+d-arctan(-b,c))^2)^(1/2)/cos(e*x+d-arctan
(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-arctan(-b,c))+a*(b^2+c
^2)^(1/2))/(b^2+c^2)^(1/2))^(1/2)/e
```

3.412.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 1371, normalized size of antiderivative = 12.69

$$\int \sqrt{a+b\cos(d+ex)+c\sin(d+ex)}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2),x, algorithm="fricas")
```

output
```
1/3*(sqrt(2)*(I*a*b + a*c)*sqrt(b + I*c)*weierstrassPInverse(4/3*(4*a^2*b^
2 - 3*b^4 - 4*a^2*c^2 + 6*I*b*c^3 + 3*c^4 - 2*I*(4*a^2*b - 3*b^3)*c)/(b^4
+ 2*b^2*c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 - 9*I*a*c^5 +
2*I*(4*a^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 - 3*I*(8*a^3*b^2 - 9
*a*b^4)*c)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), 1/3*(2*a*b - 2*I*a*c + 3*(
b^2 + c^2)*cos(e*x + d) - 3*(I*b^2 + I*c^2)*sin(e*x + d))/(b^2 + c^2)) + s
qrt(2)*(-I*a*b + a*c)*sqrt(b - I*c)*weierstrassPInverse(4/3*(4*a^2*b^2 - 3
*b^4 - 4*a^2*c^2 - 6*I*b*c^3 + 3*c^4 + 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b
^2*c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 + 9*I*a*c^5 - 2*I*(
4*a^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 + 3*I*(8*a^3*b^2 - 9*a*b^
4)*c)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), 1/3*(2*a*b + 2*I*a*c + 3*(b^2 +
 c^2)*cos(e*x + d) - 3*(-I*b^2 - I*c^2)*sin(e*x + d))/(b^2 + c^2)) - 3*sqr
t(2)*(I*b^2 + I*c^2)*sqrt(b + I*c)*weierstrassZeta(4/3*(4*a^2*b^2 - 3*b^4
- 4*a^2*c^2 + 6*I*b*c^3 + 3*c^4 - 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^
2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 - 9*I*a*c^5 + 2*I*(4*a^3
 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 - 3*I*(8*a^3*b^2 - 9*a*b^4)*c)
/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), weierstrassPInverse(4/3*(4*a^2*b^2 -
 3*b^4 - 4*a^2*c^2 + 6*I*b*c^3 + 3*c^4 - 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2
*b^2*c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 - 9*I*a*c^5 + 2*I
*(4*a^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 - 3*I*(8*a^3*b^2 - 9...
```

3.412.6 Sympy [F]

$$\int \sqrt{a + b\cos(d + ex) + c\sin(d + ex)}\, dx = \int \sqrt{a + b\cos{(d + ex)} + c\sin{(d + ex)}}\, dx$$

input
```
integrate((a+b*cos(e*x+d)+c*sin(e*x+d))**(1/2),x)
```

output
```
Integral(sqrt(a + b*cos(d + e*x) + c*sin(d + e*x)), x)
```

3.412.7 Maxima [F]

$$\int \sqrt{a+b\cos(d+ex)+c\sin(d+ex)}\,dx = \int \sqrt{b\cos(ex+d)+c\sin(ex+d)+a}\,dx$$

input
```
integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*cos(e*x + d) + c*sin(e*x + d) + a), x)
```

3.412.8 Giac [F]

$$\int \sqrt{a+b\cos(d+ex)+c\sin(d+ex)}\,dx = \int \sqrt{b\cos(ex+d)+c\sin(ex+d)+a}\,dx$$

input
```
integrate((a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*cos(e*x + d) + c*sin(e*x + d) + a), x)
```

3.412.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\cos(d+ex)+c\sin(d+ex)}\,dx = \int \sqrt{a+b\,\cos(d+e\,x)+c\,\sin(d+e\,x)}\,dx$$

input
```
int((a + b*cos(d + e*x) + c*sin(d + e*x))^(1/2),x)
```

output
```
int((a + b*cos(d + e*x) + c*sin(d + e*x))^(1/2), x)
```

3.413 $\int \frac{1}{\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}\,dx$

3.413.1 Optimal result

Integrand size = 22, antiderivative size = 108

$$\int \frac{1}{\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}\,dx = \frac{2\,\text{EllipticF}\left(\frac{1}{2}(d+ex-\tan^{-1}(b,c)),\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}{e\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}$$

output
```
2*(cos(1/2*d+1/2*e*x-1/2*arctan(b,c))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arcta
n(b,c))*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan(b,c)),2^(1/2)*((b^2+c^2)^(1
/2)/(a+(b^2+c^2)^(1/2)))^(1/2))*((a+b*cos(e*x+d)+c*sin(e*x+d))/(a+(b^2+c^2
)^(1/2)))^(1/2)/e/(a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2)
```

3.413.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 0.68 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.64

$$\int \frac{1}{\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}\,dx = \frac{2\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{3}{2},\frac{a+\sqrt{1+\frac{b^2}{c^2}}c\sin\left(d+ex+\arctan\left(\frac{b}{c}\right)\right)}{a-\sqrt{1+\frac{b^2}{c^2}}c},\frac{a+\sqrt{1+\frac{b^2}{c^2}}c\sin\left(d+ex+\arctan\left(\frac{b}{c}\right)\right)}{a+\sqrt{1+\frac{b^2}{c^2}}c}\right)\sec\left(d+ex+\arctan\left(\frac{b}{c}\right)\right)\cdots}{\sqrt{1\cdots}}$$

input
```
Integrate[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],x]
```

output
```
(2*AppellF1[1/2, 1/2, 1/2, 3/2, (a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + Arc
Tan[b/c]])/(a - Sqrt[1 + b^2/c^2]*c), (a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x
 + ArcTan[b/c]])/(a + Sqrt[1 + b^2/c^2]*c)]*Sec[d + e*x + ArcTan[b/c]]*Sqr
t[-((Sqrt[1 + b^2/c^2]*c*(-1 + Sin[d + e*x + ArcTan[b/c]]))/(a + Sqrt[1 +
b^2/c^2]*c))]*Sqrt[(Sqrt[1 + b^2/c^2]*c*(1 + Sin[d + e*x + ArcTan[b/c]]))/
(-a + Sqrt[1 + b^2/c^2]*c)]*Sqrt[a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + Arc
Tan[b/c]]])/(Sqrt[1 + b^2/c^2]*c*e)
```

3.413.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3042, 3606, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}dx$$

$$\Big\downarrow \text{3606}$$

$$\frac{\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}\int \frac{1}{\sqrt{\frac{a}{a+\sqrt{b^2+c^2}}+\frac{\sqrt{b^2+c^2}\cos\left(d+ex-\tan^{-1}(b,c)\right)}{a+\sqrt{b^2+c^2}}}}dx}{\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}\int \frac{1}{\sqrt{\frac{a}{a+\sqrt{b^2+c^2}}+\frac{\sqrt{b^2+c^2}\sin\left(d+ex-\tan^{-1}(b,c)+\frac{\pi}{2}\right)}{a+\sqrt{b^2+c^2}}}}dx}{\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}$$

$$\Big\downarrow \text{3140}$$

$$\frac{2\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}\text{EllipticF}\left(\frac{1}{2}\left(d+ex-\tan^{-1}(b,c)\right),\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{e\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}$$

input
```
Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]],x]
```

output
```
(2*EllipticF[(d + e*x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2
 + c^2])]*Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])
])/(e*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]])
```

3.413.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3606
```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Simp[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]   Int[1/Sqrt[a/(a
 + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x -
 ArcTan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2
, 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

3.413.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(137) = 274$.

Time = 1.35 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.73

method	result
default	$\frac{2\left(a+\sqrt{b^2+c^2}\right)\sqrt{\frac{\sqrt{b^2+c^2}\,\sin(ex+d-\arctan(-b,c))+a}{a+\sqrt{b^2+c^2}}}\,\sqrt{\frac{(\sin(ex+d-\arctan(-b,c))+1)\sqrt{b^2+c^2}}{-a+\sqrt{b^2+c^2}}}\,\sqrt{-\frac{(\sin(ex+d-\arctan(-b,c))-1)\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}}\ \mathrm{Ellip}}{\sqrt{b^2+c^2}\,\cos(ex+d-\arctan(-b,c))\sqrt{\frac{b^2\sin(ex+d-\arctan(-b,c))+c^2\sin(ex+d-\arctan(-b,}{\sqrt{b^2+c^2}}}}$

input
```
int(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(a+(b^2+c^2)^(1/2))*(((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a)/(a+(b^2
+c^2)^(1/2)))^(1/2)*((sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(-a+(b^2+
c^2)^(1/2)))^(1/2)*(-(sin(e*x+d-arctan(-b,c))-1)*(b^2+c^2)^(1/2)/(a+(b^2+c
^2)^(1/2)))^(1/2)*EllipticF((((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a)/(
a+(b^2+c^2)^(1/2)))^(1/2),(-(a+(b^2+c^2)^(1/2))/(-a+(b^2+c^2)^(1/2)))^(1/2
))/(b^2+c^2)^(1/2)/cos(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+c
^2*sin(e*x+d-arctan(-b,c))+a*(b^2+c^2)^(1/2))/(b^2+c^2)^(1/2))^(1/2)/e
```

3.413.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 506, normalized size of antiderivative = 4.69

$$\int \frac{1}{\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}\,dx$$
$$= \frac{\sqrt{2}\sqrt{b+ic}(ib+c)\text{weierstrassPInverse}\left(\frac{4\left(4a^2b^2-3b^4-4a^2c^2+6ibc^3+3c^4-2i\left(4a^2b-3b^3\right)c\right)}{3\left(b^4+2b^2c^2+c^4\right)}, -\frac{8\left(8a^3b^3-9ab^5+27abc^4-9ic\right.}{}\right.}{}$$

input
```
integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(2)*sqrt(b + I*c)*(I*b + c)*weierstrassPInverse(4/3*(4*a^2*b^2 - 3*b^
4 - 4*a^2*c^2 + 6*I*b*c^3 + 3*c^4 - 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*
c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 - 9*I*a*c^5 + 2*I*(4*a
^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 - 3*I*(8*a^3*b^2 - 9*a*b^4)*
c)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), 1/3*(2*a*b - 2*I*a*c + 3*(b^2 + c^
2)*cos(e*x + d) - 3*(I*b^2 + I*c^2)*sin(e*x + d))/(b^2 + c^2)) + sqrt(2)*s
qrt(b - I*c)*(-I*b + c)*weierstrassPInverse(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2
*c^2 - 6*I*b*c^3 + 3*c^4 + 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4
), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 + 9*I*a*c^5 - 2*I*(4*a^3 + 9*a*
b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 + 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6 +
 3*b^4*c^2 + 3*b^2*c^4 + c^6), 1/3*(2*a*b + 2*I*a*c + 3*(b^2 + c^2)*cos(e*
x + d) - 3*(-I*b^2 - I*c^2)*sin(e*x + d))/(b^2 + c^2)))/((b^2 + c^2)*e)
```

3.413.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}\,dx = \int \frac{1}{\sqrt{a+b\cos\left(d+ex\right)+c\sin\left(d+ex\right)}}\,dx$$

input
```
integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*cos(d + e*x) + c*sin(d + e*x)), x)
```

3.413.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}\,dx = \int \frac{1}{\sqrt{b\cos\left(ex+d\right)+c\sin\left(ex+d\right)+a}}\,dx$$

input
```
integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*cos(e*x + d) + c*sin(e*x + d) + a), x)
```

3.413.8 Giac [F]

$$\int \frac{1}{\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}\,dx = \int \frac{1}{\sqrt{b\cos\left(ex+d\right)+c\sin\left(ex+d\right)+a}}\,dx$$

input
```
integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*cos(e*x + d) + c*sin(e*x + d) + a), x)
```

3.413.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}\,dx=\int \frac{1}{\sqrt{a+b\,\cos\left(d+e\,x\right)+c\,\sin\left(d+e\,x\right)}}\,dx$$

input
```
int(1/(a + b*cos(d + e*x) + c*sin(d + e*x))^(1/2),x)
```

output
```
int(1/(a + b*cos(d + e*x) + c*sin(d + e*x))^(1/2), x)
```

3.414 $\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}\,dx$

3.414.1 Optimal result

Integrand size = 22, antiderivative size = 186

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}\,dx = \frac{2(c\cos(d+ex)-b\sin(d+ex))}{(a^2-b^2-c^2)\,e\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}} + \frac{2E\left(\frac{1}{2}(d+ex-\tan^{-1}(b,c))\,|\,\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}{(a^2-b^2-c^2)\,e\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}$$

output
```
2*(c*cos(e*x+d)-b*sin(e*x+d))/(a^2-b^2-c^2)/e/(a+b*cos(e*x+d)+c*sin(e*x+d)
)^(1/2)+2*(cos(1/2*d+1/2*e*x-1/2*arctan(b,c))^2)^(1/2)/cos(1/2*d+1/2*e*x-1
/2*arctan(b,c))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(b,c)),2^(1/2)*((b^2
+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2))*(a+b*cos(e*x+d)+c*sin(e*x+d))^(1/2
)/(a^2-b^2-c^2)/e/((a+b*cos(e*x+d)+c*sin(e*x+d))/(a+(b^2+c^2)^(1/2)))^(1/2
)
```

3.414.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 6.44 (sec) , antiderivative size = 1540, normalized size of antiderivative = 8.28

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-3/2),x]
```

output
```
(Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]*((-2*(b^2 + c^2))/(b*c*(-a^2 +
b^2 + c^2)) + (2*(a*c + b^2*Sin[d + e*x] + c^2*Sin[d + e*x]))/(b*(-a^2 + b
^2 + c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x]))))/e - (2*a*AppellF1[1/2,
1/2, 1/2, 3/2, -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqr
t[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1 + b^2/c^2]
*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^
2]*c))*c))]*Sec[d + e*x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*S
qrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c
^2])]*Sqrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]]]*Sqrt[(c
*Sqrt[(b^2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]
])/(-a + c*Sqrt[(b^2 + c^2)/c^2])])/(Sqrt[1 + b^2/c^2]*c*(-a^2 + b^2 + c^2
)*e) - (b^2*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((a + b*Sqrt[1 + c^2/b^
2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*(1 - a/(b*Sqrt[1 + c^2
/b^2])))), -((a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcTan[c/b]])/(b*Sqrt[
1 + c^2/b^2]*(-1 - a/(b*Sqrt[1 + c^2/b^2]))))]*Sin[d + e*x - ArcTan[c/b]])
/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[(b^2 + c^2)/b^2] - b*Sqrt[(b^2 + c^2)/b
^2]*Cos[d + e*x - ArcTan[c/b]])/(a + b*Sqrt[(b^2 + c^2)/b^2])]*Sqrt[a + b*
Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]]]*Sqrt[(b*Sqrt[(b^2 + c^2)
/b^2] + b*Sqrt[(b^2 + c^2)/b^2]*Cos[d + e*x - ArcTan[c/b]])/(-a + b*Sqrt[(
b^2 + c^2)/b^2])])) - ((2*b*(a + b*Sqrt[1 + c^2/b^2]*Cos[d + e*x - ArcT...
```

3.414.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 3607, 3042, 3598, 3042, 3132}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}dx$$

$$\Big\downarrow \text{ 3607}$$

$$\frac{\int \sqrt{a+b\cos(d+ex)+c\sin(d+ex)}dx}{a^2-b^2-c^2}+\frac{2(c\cos(d+ex)-b\sin(d+ex))}{e\left(a^2-b^2-c^2\right)\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{a+b\cos(d+ex)+c\sin(d+ex)}dx}{a^2-b^2-c^2}+\frac{2(c\cos(d+ex)-b\sin(d+ex))}{e\left(a^2-b^2-c^2\right)\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}$$

$$\Big\downarrow \text{3598}$$

$$\frac{\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}\int \sqrt{\frac{a}{a+\sqrt{b^2+c^2}}+\frac{\sqrt{b^2+c^2}\cos\left(d+ex-\tan^{-1}(b,c)\right)}{a+\sqrt{b^2+c^2}}}dx}{\left(a^2-b^2-c^2\right)\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}+\frac{2(c\cos(d+ex)-b\sin(d+ex))}{e\left(a^2-b^2-c^2\right)\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}\int \sqrt{\frac{a}{a+\sqrt{b^2+c^2}}+\frac{\sqrt{b^2+c^2}\sin\left(d+ex-\tan^{-1}(b,c)+\frac{\pi}{2}\right)}{a+\sqrt{b^2+c^2}}}dx}{\left(a^2-b^2-c^2\right)\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}+\frac{2(c\cos(d+ex)-b\sin(d+ex))}{e\left(a^2-b^2-c^2\right)\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}$$

$$\Big\downarrow \text{3132}$$

$$\frac{2\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{e\left(a^2-b^2-c^2\right)\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}+\frac{2(c\cos(d+ex)-b\sin(d+ex))}{e\left(a^2-b^2-c^2\right)\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}$$

input
```
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-3/2),x]
```

output
```
(2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/((a^2 - b^2 - c^2)*e*Sqrt[a + b*Cos[
d + e*x] + c*Sin[d + e*x]]) + (2*EllipticE[(d + e*x - ArcTan[b, c])/2, (2*
Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[a + b*Cos[d + e*x] + c*Sin[d
+ e*x]])/((a^2 - b^2 - c^2)*e*Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(
a + Sqrt[b^2 + c^2])])
```

3.414. $\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}\,dx$

3.414.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3598
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]   Int[Sqrt[a/(a + S
qrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - Arc
Tan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

rule 3607
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-3/2), x_Symbol] :> Simp[2*((c*Cos[d + e*x] - b*Sin[d + e*x])/(e*(a^2 - b^
2 - c^2)*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]])), x] + Simp[1/(a^2 - b^
2 - c^2)   Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] /; FreeQ[{
a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

3.414.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2644 vs. $2(213) = 426$.

Time = 2.44 (sec) , antiderivative size = 2645, normalized size of antiderivative = 14.22

method	result	size
default	Expression too large to display	2645

input
```
int(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(-b^2*sin(e*x+d-arctan(-b,c))-c^2*sin(e*x+d-arctan(-b,c))-a*(b^2+c^2)^(
1/2))*cos(e*x+d-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)/(b^2+c^2)^(3/2)*(b^
2*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-arctan(-b,c))+a*(b^2+c^2)^(1/2))*(
cos(e*x+d-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a)*(b^2
+c^2))^(1/2)*(sin(e*x+d-arctan(-b,c))^2*b^2+sin(e*x+d-arctan(-b,c))^2*c^2-
a^2)/(sin(e*x+d-arctan(-b,c))^2*b^2+sin(e*x+d-arctan(-b,c))^2*c^2+2*sin(e*
x+d-arctan(-b,c))*a*(b^2+c^2)^(1/2)+a^2)/(a*(((b^2+c^2)^(1/2)*sin(e*x+d-ar
ctan(-b,c))+a)*cos(e*x+d-arctan(-b,c))^2)^(1/2)-sin(e*x+d-arctan(-b,c))*(c
os(e*x+d-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a)*(b^2+
c^2))^(1/2))*(-(b^2+c^2)^(1/2)*(-b^2-c^2)*cos(e*x+d-arctan(-b,c))^2/(a^2-b
^2-c^2)/(cos(e*x+d-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c)
)+a)*(b^2+c^2))^(1/2)+a*(b^2+c^2)/(a^2-b^2-c^2)*(1/(b^2+c^2)^(1/2)*a+1)*((
(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2)*((si
n(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-si
n(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)/(cos(e
*x+d-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a)*(b^2+c^2)
)^(1/2)*EllipticF((((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a)/(a+(b^2+c^2
)^(1/2)))^(1/2),((-a-(b^2+c^2)^(1/2))/(-a+(b^2+c^2)^(1/2)))^(1/2))+2*(-(b^
2+c^2)^(3/2)+2*(b^2+c^2)^(1/2)*b^2+2*(b^2+c^2)^(1/2)*c^2)/(2*a^2-2*b^2-2*c
^2)*(1/(b^2+c^2)^(1/2)*a+1)*(((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a...
```

3.414.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 1737, normalized size of antiderivative = 9.34

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(3/2),x, algorithm="fricas")
```

output
```
-1/3*((sqrt(2)*(-I*a*b^2 - a*b*c)*cos(e*x + d) + sqrt(2)*(-I*a*b*c - a*c^2
)*sin(e*x + d) + sqrt(2)*(-I*a^2*b - a^2*c))*sqrt(b + I*c)*weierstrassPInv
erse(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2*c^2 + 6*I*b*c^3 + 3*c^4 - 2*I*(4*a^2*b
 - 3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*
c^4 - 9*I*a*c^5 + 2*I*(4*a^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 -
3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), 1/3*(2*a
*b - 2*I*a*c + 3*(b^2 + c^2)*cos(e*x + d) - 3*(I*b^2 + I*c^2)*sin(e*x + d)
)/(b^2 + c^2)) + (sqrt(2)*(I*a*b^2 - a*b*c)*cos(e*x + d) + sqrt(2)*(I*a*b*
c - a*c^2)*sin(e*x + d) + sqrt(2)*(I*a^2*b - a^2*c))*sqrt(b - I*c)*weierst
rassPInverse(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2*c^2 - 6*I*b*c^3 + 3*c^4 + 2*I*
(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 +
 27*a*b*c^4 + 9*I*a*c^5 - 2*I*(4*a^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3
)*c^2 + 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6),
1/3*(2*a*b + 2*I*a*c + 3*(b^2 + c^2)*cos(e*x + d) - 3*(-I*b^2 - I*c^2)*sin
(e*x + d))/(b^2 + c^2)) - 3*(sqrt(2)*(-I*b^3 - I*b*c^2)*cos(e*x + d) + sqr
t(2)*(-I*b^2*c - I*c^3)*sin(e*x + d) + sqrt(2)*(-I*a*b^2 - I*a*c^2))*sqrt(
b + I*c)*weierstrassZeta(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2*c^2 + 6*I*b*c^3 +
3*c^4 - 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4), -8/27*(8*a^3*b^3
 - 9*a*b^5 + 27*a*b*c^4 - 9*I*a*c^5 + 2*I*(4*a^3 + 9*a*b^2)*c^3 - 6*(4*a^3
*b - 3*a*b^3)*c^2 - 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^4*c^2 + 3*b...
```

3.414.6 Sympy [F]

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}\,dx=\int \frac{1}{\left(a+b\cos\left(d+ex\right)+c\sin\left(d+ex\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))**(3/2),x)
```

output
```
Integral((a + b*cos(d + e*x) + c*sin(d + e*x))**(-3/2), x)
```

3.414.7 Maxima [F]

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}\,dx=\int \frac{1}{(b\cos(ex+d)+c\sin(ex+d)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(e*x + d) + c*sin(e*x + d) + a)^(-3/2), x)
```

3.414.8 Giac [F]

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}\,dx=\int \frac{1}{(b\cos(ex+d)+c\sin(ex+d)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*cos(e*x + d) + c*sin(e*x + d) + a)^(-3/2), x)
```

3.414.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}\,dx=\int \frac{1}{(a+b\,\cos(d+e\,x)+c\,\sin(d+e\,x))^{3/2}}\,dx$$

input
```
int(1/(a + b*cos(d + e*x) + c*sin(d + e*x))^(3/2),x)
```

output
```
int(1/(a + b*cos(d + e*x) + c*sin(d + e*x))^(3/2), x)
```

3.415 $\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}}\,dx$

3.415.1 Optimal result

Integrand size = 22, antiderivative size = 382

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}}\,dx = \frac{2(c\cos(d+ex)-b\sin(d+ex))}{3(a^2-b^2-c^2)\,e(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}} + \frac{8(ac\cos(d+ex)-ab\sin(d+ex))}{3(a^2-b^2-c^2)^2\,e\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}} + \frac{8aE\left(\frac{1}{2}(d+ex-\tan^{-1}(b,c))\,|\,\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}{3(a^2-b^2-c^2)^2\,e\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}} - \frac{2\,\text{EllipticF}\left(\frac{1}{2}(d+ex-\tan^{-1}(b,c)),\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}{3(a^2-b^2-c^2)\,e\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}$$

output
```
2/3*(c*cos(e*x+d)-b*sin(e*x+d))/(a^2-b^2-c^2)/e/(a+b*cos(e*x+d)+c*sin(e*x+
d))^(3/2)+8/3*(a*c*cos(e*x+d)-a*b*sin(e*x+d))/(a^2-b^2-c^2)^2/e/(a+b*cos(e
*x+d)+c*sin(e*x+d))^(1/2)+8/3*a*(cos(1/2*d+1/2*e*x-1/2*arctan(b,c))^2)^(1/
2)/cos(1/2*d+1/2*e*x-1/2*arctan(b,c))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arct
an(b,c)),2^(1/2)*((b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2))*(a+b*cos(e*x
+d)+c*sin(e*x+d))^(1/2)/(a^2-b^2-c^2)^2/e/((a+b*cos(e*x+d)+c*sin(e*x+d))/(
a+(b^2+c^2)^(1/2)))^(1/2)-2/3*(cos(1/2*d+1/2*e*x-1/2*arctan(b,c))^2)^(1/2)
/cos(1/2*d+1/2*e*x-1/2*arctan(b,c))*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan
(b,c)),2^(1/2)*((b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2))*((a+b*cos(e*x+
d)+c*sin(e*x+d))/(a+(b^2+c^2)^(1/2)))^(1/2)/(a^2-b^2-c^2)/e/(a+b*cos(e*x+d
)+c*sin(e*x+d))^(1/2)
```

3.415.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 6.69 (sec) , antiderivative size = 2408, normalized size of antiderivative = 6.30

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-5/2),x]
```

output
```
(Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]*((8*a*(b^2 + c^2))/(3*b*c*(a^2
- b^2 - c^2)^2) + (2*(a*c + b^2*Sin[d + e*x] + c^2*Sin[d + e*x]))/(3*b*(-a
^2 + b^2 + c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^2) - (2*(3*a^2*c + b
^2*c + c^3 + 4*a*b^2*Sin[d + e*x] + 4*a*c^2*Sin[d + e*x]))/(3*b*(-a^2 + b^
2 + c^2)^2*(a + b*Cos[d + e*x] + c*Sin[d + e*x]))))/e + (2*a^2*AppellF1[1/
2, 1/2, 1/2, 3/2, -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(
Sqrt[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1 + b^2/c
^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2
/c^2]*c))*c))]*Sec[d + e*x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] -
c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2
)/c^2])]*Sqrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]]]*Sqrt
[(c*Sqrt[(b^2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b
/c]])/(-a + c*Sqrt[(b^2 + c^2)/c^2])])/(Sqrt[1 + b^2/c^2]*c*(-a^2 + b^2 +
c^2)^2*e) + (2*b^2*AppellF1[1/2, 1/2, 1/2, 3/2, -((a + Sqrt[1 + b^2/c^2]*c
*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*
c))*c)), -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 +
b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c))*c))]*Sec[d + e*x + ArcTan[b/c]]*Sq
rt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan
[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Si
n[d + e*x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] + c*Sqrt[(b^2 +...
```

3.415.3 Rubi [A] (verified)

Time = 1.53 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.04, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.682$, Rules used = {3042, 3608, 27, 3042, 3635, 27, 3042, 3628, 3042, 3598, 3042, 3132, 3606, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}}dx$$

$$\Big\downarrow \text{3608}$$

$$\frac{2(c\cos(d+ex)-b\sin(d+ex))}{3e\left(a^2-b^2-c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}-\frac{2\int -\frac{3a-b\cos(d+ex)-c\sin(d+ex)}{2(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}dx}{3\left(a^2-b^2-c^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{3a-b\cos(d+ex)-c\sin(d+ex)}{(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}dx}{3\left(a^2-b^2-c^2\right)}+\frac{2(c\cos(d+ex)-b\sin(d+ex))}{3e\left(a^2-b^2-c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{3a-b\cos(d+ex)-c\sin(d+ex)}{(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}dx}{3\left(a^2-b^2-c^2\right)}+\frac{2(c\cos(d+ex)-b\sin(d+ex))}{3e\left(a^2-b^2-c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}$$

$$\Big\downarrow \text{3635}$$

$$\frac{\frac{8(ac\cos(d+ex)-ab\sin(d+ex))}{e(a^2-b^2-c^2)\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}-\frac{2\int -\frac{3a^2+4b\cos(d+ex)a+4c\sin(d+ex)a+b^2+c^2}{2\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}dx}{a^2-b^2-c^2}}{3\left(a^2-b^2-c^2\right)}+\frac{2(c\cos(d+ex)-b\sin(d+ex))}{3e\left(a^2-b^2-c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{3a^2+4b\cos(d+ex)a+4c\sin(d+ex)a+b^2+c^2}{\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}dx}{a^2-b^2-c^2}+\frac{8(ac\cos(d+ex)-ab\sin(d+ex))}{e(a^2-b^2-c^2)\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}}{3\left(a^2-b^2-c^2\right)}+\frac{2(c\cos(d+ex)-b\sin(d+ex))}{3e\left(a^2-b^2-c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{3a^2+4b\cos(d+ex)a+4c\sin(d+ex)a+b^2+c^2}{\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}dx}{a^2-b^2-c^2}+\frac{8(ac\cos(d+ex)-ab\sin(d+ex))}{e(a^2-b^2-c^2)\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}}{3\left(a^2-b^2-c^2\right)}+\frac{2(c\cos(d+ex)-b\sin(d+ex))}{3e\left(a^2-b^2-c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}$$

$$\Big\downarrow \text{3628}$$

3.415. $\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}}dx$

$$\frac{\frac{4a\int \sqrt{a+b\cos(d+ex)+c\sin(d+ex)}dx-(a^2-b^2-c^2)\int \frac{1}{\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}dx}{a^2-b^2-c^2}+\frac{8(ac\cos(d+ex)-ab\sin(d+ex))}{e(a^2-b^2-c^2)\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}}{3(a^2-b^2-c^2)}+\frac{2(c\cos(d+ex)-b\sin(d+ex))}{3e(a^2-b^2-c^2)(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{4a\int \sqrt{a+b\cos(d+ex)+c\sin(d+ex)}dx-(a^2-b^2-c^2)\int \frac{1}{\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}dx}{a^2-b^2-c^2}+\frac{8(ac\cos(d+ex)-ab\sin(d+ex))}{e(a^2-b^2-c^2)\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}}{3(a^2-b^2-c^2)}+\frac{2(c\cos(d+ex)-b\sin(d+ex))}{3e(a^2-b^2-c^2)(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}$$

$\downarrow$ 3598

$$\frac{\frac{\frac{4a\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}\int\sqrt{\frac{a}{a+\sqrt{b^2+c^2}}+\frac{\sqrt{b^2+c^2}\cos\left(d+ex-\tan^{-1}(b,c)\right)}{a+\sqrt{b^2+c^2}}}dx}{\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}-(a^2-b^2-c^2)\int\frac{1}{\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}dx}{a^2-b^2-c^2}+\frac{8(ac\cos(d\cdots}{e(a^2-b^2-c^2)\sqrt{a\cdots}}}{3(a^2-b^2-c^2)}$$

$$\frac{2(c\cos(d+ex)-b\sin(d+ex))}{3e(a^2-b^2-c^2)(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{4a\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}\int\sqrt{\frac{a}{a+\sqrt{b^2+c^2}}+\frac{\sqrt{b^2+c^2}\sin\left(d+ex-\tan^{-1}(b,c)+\frac{\pi}{2}\right)}{a+\sqrt{b^2+c^2}}}dx}{\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}-(a^2-b^2-c^2)\int\frac{1}{\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}dx}{a^2-b^2-c^2}+\frac{8(ac\cos\cdots}{e(a^2-b^2-c^2)\cdots}}{3(a^2-b^2-c^2)}$$

$$\frac{2(c\cos(d+ex)-b\sin(d+ex))}{3e(a^2-b^2-c^2)(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}$$

$\downarrow$ 3132

$$\frac{\frac{\frac{8a\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{e\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}-(a^2-b^2-c^2)\int\frac{1}{\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}dx}{a^2-b^2-c^2}+\frac{8(ac\cos(d+ex)-ab\sin\cdots}{e(a^2-b^2-c^2)\sqrt{a+b\cos(d+ex\cdots}}}{3(a^2-b^2-c^2)}$$

$$\frac{2(c\cos(d+ex)-b\sin(d+ex))}{3e(a^2-b^2-c^2)(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}$$

$\downarrow$ 3606

3.415. $\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}}\,dx$

$$\frac{\frac{\frac{8a\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{e\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}-\frac{\left(a^2-b^2-c^2\right)\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}\int\frac{1}{\sqrt{\frac{a}{a+\sqrt{b^2+c^2}}+\frac{\sqrt{b^2+c^2}\cos\left(d+ex-\tan^{-1}\right.}{a+\sqrt{b^2+c^2}}}}}{\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}}{a^2-b^2-c^2}}{3\left(a^2-b^2-c^2\right)}$$

$$\frac{2(c\cos(d+ex)-b\sin(d+ex))}{3e\left(a^2-b^2-c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\frac{\frac{8a\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{e\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}-\frac{\left(a^2-b^2-c^2\right)\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}\int\frac{1}{\sqrt{\frac{a}{a+\sqrt{b^2+c^2}}+\frac{\sqrt{b^2+c^2}\sin\left(d+ex-\tan^{-}\right.}{a+\sqrt{b^2+c^2}}}}}{\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}}{a^2-b^2-c^2}}{3\left(a^2-b^2-c^2\right)}$$

$$\frac{2(c\cos(d+ex)-b\sin(d+ex))}{3e\left(a^2-b^2-c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}$$

$$\Big\downarrow \textbf{3140}$$

$$\frac{\frac{\frac{8a\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{e\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}-\frac{2\left(a^2-b^2-c^2\right)\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}\text{EllipticF}\left(\frac{1}{2}\left(d+ex-\tan^{-1}(b,c)\right),\frac{2\sqrt{b^2+}}{a+\sqrt{b^2-}}\right.}{e\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}}{a^2-b^2-c^2}}{3\left(a^2-b^2-c^2\right)}$$

$$\frac{2(c\cos(d+ex)-b\sin(d+ex))}{3e\left(a^2-b^2-c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}$$

input
```
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-5/2),x]
```

output
```
(2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(3*(a^2 - b^2 - c^2)*e*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(3/2)) + ((8*(a*c*Cos[d + e*x] - a*b*Sin[d + e*x]
))/((a^2 - b^2 - c^2)*e*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]) + ((8*a
*EllipticE[(d + e*x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2 +
 c^2])]*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*Sqrt[(a + b*Cos[d +
e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]) - (2*(a^2 - b^2 - c^2)*Elli
pticF[(d + e*x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c^2]
)]*Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])])/(e*S
qrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]))/(a^2 - b^2 - c^2))/(3*(a^2 - b^
2 - c^2))
```

3.415. $\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}}\,dx$

3.415.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3598
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]   Int[Sqrt[a/(a + S
qrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - Arc
Tan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

rule 3606
```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Simp[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]   Int[1/Sqrt[a/(a
 + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x -
 ArcTan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2
, 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

rule 3608
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Simp[
1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c
*(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x
] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] &&
NeQ[n, -3/2]
```

3.415. $\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}}\,dx$

rule 3628
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]
, x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[
A*b - a*B, 0]
```

rule 3635
```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(-(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(
a^2 - b^2 - c^2))), x] + Simp[1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a + b*Co
s[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n +
2)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x]
/; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[n, -2]
```

3.415.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 3641 vs. $2(428) = 856$.

Time = 9.28 (sec) , antiderivative size = 3642, normalized size of antiderivative = 9.53

method	result	size
default	Expression too large to display	3642

input
```
int(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(-b^2*sin(e*x+d-arctan(-b,c))-c^2*sin(e*x+d-arctan(-b,c))-a*(b^2+c^2)^(
1/2))*cos(e*x+d-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)*(b^2*sin(e*x+d-arct
an(-b,c))+c^2*sin(e*x+d-arctan(-b,c))+a*(b^2+c^2)^(1/2))*(cos(e*x+d-arctan
(-b,c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a)*(b^2+c^2))^(1/2)*(b^
4*sin(e*x+d-arctan(-b,c))^4+2*b^2*c^2*sin(e*x+d-arctan(-b,c))^4+c^4*sin(e*
x+d-arctan(-b,c))^4-2*a^2*b^2*sin(e*x+d-arctan(-b,c))^2-2*a^2*c^2*sin(e*x+
d-arctan(-b,c))^2+a^4)/(b^4*sin(e*x+d-arctan(-b,c))^3+2*b^2*c^2*sin(e*x+d-
arctan(-b,c))^3+c^4*sin(e*x+d-arctan(-b,c))^3+3*(b^2+c^2)^(1/2)*a*b^2*sin(
e*x+d-arctan(-b,c))^2+3*(b^2+c^2)^(1/2)*a*c^2*sin(e*x+d-arctan(-b,c))^2+3*
a^2*b^2*sin(e*x+d-arctan(-b,c))+3*a^2*c^2*sin(e*x+d-arctan(-b,c))+(b^2+c^2
)^(1/2)*a^3)/(2*(((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a)*cos(e*x+d-arc
tan(-b,c))^2)^(1/2)*sin(e*x+d-arctan(-b,c))*a*b^2+2*(((b^2+c^2)^(1/2)*sin(
e*x+d-arctan(-b,c))+a)*cos(e*x+d-arctan(-b,c))^2)^(1/2)*sin(e*x+d-arctan(-
b,c))*a*c^2-(cos(e*x+d-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-
b,c))+a)*(b^2+c^2))^(1/2)*sin(e*x+d-arctan(-b,c))^2*b^2-(cos(e*x+d-arctan(
-b,c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+a)*(b^2+c^2))^(1/2)*sin(
e*x+d-arctan(-b,c))^2*c^2-(cos(e*x+d-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(
e*x+d-arctan(-b,c))+a)*(b^2+c^2))^(1/2)*a^2)*(-1/4/a/(a^2-b^2-c^2)*(b^2+c^
2)^(1/2)*(cos(e*x+d-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c
))+a)*(b^2+c^2))^(1/2)/(b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-arcta...
```

3.415.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.21 (sec) , antiderivative size = 2805, normalized size of antiderivative = 7.34

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(5/2),x, algorithm="fricas")
```

output
```
1/9*((sqrt(2)*(I*a^2*b^3 + 3*I*b^5 - I*a^2*b*c^2 - a^2*c^3 - 3*I*b*c^4 - 3
*c^5 + (a^2*b^2 + 3*b^4)*c)*cos(e*x + d)^2 - 2*sqrt(2)*(-I*a^3*b^2 - 3*I*a
*b^4 - 3*I*a*b^2*c^2 - 3*a*b*c^3 - (a^3*b + 3*a*b^3)*c)*cos(e*x + d) - 2*(
sqrt(2)*(-3*I*b^2*c^3 - 3*b*c^4 - (a^2*b + 3*b^3)*c^2 - I*(a^2*b^2 + 3*b^4
)*c)*cos(e*x + d) + sqrt(2)*(-3*I*a*b*c^3 - 3*a*c^4 - (a^3 + 3*a*b^2)*c^2
- I*(a^3*b + 3*a*b^3)*c))*sin(e*x + d) + sqrt(2)*(I*a^4*b + 3*I*a^2*b^3 +
3*I*b*c^4 + 3*c^5 + (4*a^2 + 3*b^2)*c^3 + I*(4*a^2*b + 3*b^3)*c^2 + (a^4 +
 3*a^2*b^2)*c))*sqrt(b + I*c)*weierstrassPInverse(4/3*(4*a^2*b^2 - 3*b^4 -
 4*a^2*c^2 + 6*I*b*c^3 + 3*c^4 - 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2
 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 - 9*I*a*c^5 + 2*I*(4*a^3
+ 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 - 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/
(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), 1/3*(2*a*b - 2*I*a*c + 3*(b^2 + c^2)*
cos(e*x + d) - 3*(I*b^2 + I*c^2)*sin(e*x + d))/(b^2 + c^2)) + (sqrt(2)*(-I
*a^2*b^3 - 3*I*b^5 + I*a^2*b*c^2 - a^2*c^3 + 3*I*b*c^4 - 3*c^5 + (a^2*b^2
+ 3*b^4)*c)*cos(e*x + d)^2 - 2*sqrt(2)*(I*a^3*b^2 + 3*I*a*b^4 + 3*I*a*b^2*
c^2 - 3*a*b*c^3 - (a^3*b + 3*a*b^3)*c)*cos(e*x + d) - 2*(sqrt(2)*(3*I*b^2*
c^3 - 3*b*c^4 - (a^2*b + 3*b^3)*c^2 + I*(a^2*b^2 + 3*b^4)*c)*cos(e*x + d)
+ sqrt(2)*(3*I*a*b*c^3 - 3*a*c^4 - (a^3 + 3*a*b^2)*c^2 + I*(a^3*b + 3*a*b^
3)*c))*sin(e*x + d) + sqrt(2)*(-I*a^4*b - 3*I*a^2*b^3 - 3*I*b*c^4 + 3*c^5
+ (4*a^2 + 3*b^2)*c^3 - I*(4*a^2*b + 3*b^3)*c^2 + (a^4 + 3*a^2*b^2)*c))...
```

3.415.6 Sympy [F]

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}}\,dx = \int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))**(5/2),x)
```

output
```
Integral((a + b*cos(d + e*x) + c*sin(d + e*x))**(-5/2), x)
```

3.415.7 Maxima [F]

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}}\,dx = \int \frac{1}{(b\cos(ex+d)+c\sin(ex+d)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(e*x + d) + c*sin(e*x + d) + a)^(-5/2), x)
```

3.415.8 Giac [F]

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}}\,dx = \int \frac{1}{(b\cos(ex+d)+c\sin(ex+d)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*cos(e*x + d) + c*sin(e*x + d) + a)^(-5/2), x)
```

3.415.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}}\,dx = \int \frac{1}{(a+b\,\cos(d+e\,x)+c\,\sin(d+e\,x))^{5/2}}\,dx$$

input
```
int(1/(a + b*cos(d + e*x) + c*sin(d + e*x))^(5/2),x)
```

output
```
int(1/(a + b*cos(d + e*x) + c*sin(d + e*x))^(5/2), x)
```

3.416 $\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{7/2}}\,dx$

3.416.1 Optimal result

Integrand size = 22, antiderivative size = 490

$$\begin{aligned}
\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{7/2}}\,dx &= \frac{2(c\cos(d+ex)-b\sin(d+ex))}{5\,(a^2-b^2-c^2)\,e(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}} \\
&+ \frac{16(ac\cos(d+ex)-ab\sin(d+ex))}{15\,(a^2-b^2-c^2)^2\,e(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}} \\
&+ \frac{2(23a^2+9(b^2+c^2))\,E\left(\frac{1}{2}(d+ex-\tan^{-1}(b,c))\,|\,\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}{15\,(a^2-b^2-c^2)^3\,e\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}} \\
&- \frac{16a\,\mathrm{EllipticF}\left(\frac{1}{2}(d+ex-\tan^{-1}(b,c)),\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}{15\,(a^2-b^2-c^2)^2\,e\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}} \\
&+ \frac{2(c(23a^2+9(b^2+c^2))\cos(d+ex)-b(23a^2+9(b^2+c^2))\sin(d+ex))}{15\,(a^2-b^2-c^2)^3\,e\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}
\end{aligned}$$

output
```
2/5*(c*cos(e*x+d)-b*sin(e*x+d))/(a^2-b^2-c^2)/e/(a+b*cos(e*x+d)+c*sin(e*x+
d))^(5/2)+16/15*(a*c*cos(e*x+d)-a*b*sin(e*x+d))/(a^2-b^2-c^2)^2/e/(a+b*cos
(e*x+d)+c*sin(e*x+d))^(3/2)+2/15*(c*(23*a^2+9*b^2+9*c^2)*cos(e*x+d)-b*(23*
a^2+9*b^2+9*c^2)*sin(e*x+d))/(a^2-b^2-c^2)^3/e/(a+b*cos(e*x+d)+c*sin(e*x+d
))^(1/2)+2/15*(23*a^2+9*b^2+9*c^2)*(cos(1/2*d+1/2*e*x-1/2*arctan(b,c))^2)^
(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(b,c))*EllipticE(sin(1/2*d+1/2*e*x-1/2*a
rctan(b,c)),2^(1/2)*((b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2))*(a+b*cos(
e*x+d)+c*sin(e*x+d))^(1/2)/(a^2-b^2-c^2)^3/e/((a+b*cos(e*x+d)+c*sin(e*x+d)
)/(a+(b^2+c^2)^(1/2)))^(1/2)-16/15*a*(cos(1/2*d+1/2*e*x-1/2*arctan(b,c))^2
)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(b,c))*EllipticF(sin(1/2*d+1/2*e*x-1/2
*arctan(b,c)),2^(1/2)*((b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2))*((a+b*c
os(e*x+d)+c*sin(e*x+d))/(a+(b^2+c^2)^(1/2)))^(1/2)/(a^2-b^2-c^2)^2/e/(a+b*
cos(e*x+d)+c*sin(e*x+d))^(1/2)
```

3.416.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 6.57 (sec) , antiderivative size = 4116, normalized size of antiderivative = 8.40

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{7/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-7/2),x]
```

output
```
(Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]*((-2*(b^2 + c^2)*(23*a^2 + 9*b^
2 + 9*c^2))/(15*b*c*(-a^2 + b^2 + c^2)^3) + (2*(a*c + b^2*Sin[d + e*x] + c
^2*Sin[d + e*x]))/(5*b*(-a^2 + b^2 + c^2)*(a + b*Cos[d + e*x] + c*Sin[d +
e*x])^3) - (2*(5*a^2*c + 3*b^2*c + 3*c^3 + 8*a*b^2*Sin[d + e*x] + 8*a*c^2*
Sin[d + e*x]))/(15*b*(-a^2 + b^2 + c^2)^2*(a + b*Cos[d + e*x] + c*Sin[d +
e*x])^2) + (2*(15*a^3*c + 17*a*b^2*c + 17*a*c^3 + 23*a^2*b^2*Sin[d + e*x]
+ 9*b^4*Sin[d + e*x] + 23*a^2*c^2*Sin[d + e*x] + 18*b^2*c^2*Sin[d + e*x] +
 9*c^4*Sin[d + e*x]))/(15*b*(-a^2 + b^2 + c^2)^3*(a + b*Cos[d + e*x] + c*S
in[d + e*x]))))/e - (2*a^3*AppellF1[1/2, 1/2, 1/2, 3/2, -((a + Sqrt[1 + b^
2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b
^2/c^2]*c))*c)), -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(S
qrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c))*c))]*Sec[d + e*x + ArcTan[
b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqrt[(b^2 + c^2)/c^2]*Sin[d + e*x
+ ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(b^2 + c^2)
/c^2]*Sin[d + e*x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] + c*Sqrt[(
b^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[b/c]])/(-a + c*Sqrt[(b^2 + c^2)/c^2])
])/(Sqrt[1 + b^2/c^2]*c*(-a^2 + b^2 + c^2)^3*e) - (34*a*b^2*AppellF1[1/2,
1/2, 1/2, 3/2, -((a + Sqrt[1 + b^2/c^2]*c*Sin[d + e*x + ArcTan[b/c]])/(Sqr
t[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1 + b^2/c^2]
*c*Sin[d + e*x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2...
```

3.416.3 Rubi [A] (verified)

Time = 2.25 (sec) , antiderivative size = 526, normalized size of antiderivative = 1.07, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.818$, Rules used = {3042, 3608, 27, 3042, 3635, 27, 3042, 3635, 27, 3042, 3628, 3042, 3598, 3042, 3132, 3606, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{7/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{7/2}}dx$$

$$\Big\downarrow \text{3608}$$

$$\frac{2(c\cos(d+ex)-b\sin(d+ex))}{5e\left(a^2-b^2-c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}}-\frac{2\int -\frac{5a-3b\cos(d+ex)-3c\sin(d+ex)}{2(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}}dx}{5\left(a^2-b^2-c^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{5a-3b\cos(d+ex)-3c\sin(d+ex)}{(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}}dx}{5\left(a^2-b^2-c^2\right)}+\frac{2(c\cos(d+ex)-b\sin(d+ex))}{5e\left(a^2-b^2-c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{5a-3b\cos(d+ex)-3c\sin(d+ex)}{(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}}dx}{5\left(a^2-b^2-c^2\right)}+\frac{2(c\cos(d+ex)-b\sin(d+ex))}{5e\left(a^2-b^2-c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}}$$

$$\Big\downarrow \text{3635}$$

$$\frac{\frac{16(ac\cos(d+ex)-ab\sin(d+ex))}{3e(a^2-b^2-c^2)(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}-\frac{2\int -\frac{3\left(5a^2+3\left(b^2+c^2\right)\right)-8ab\cos(d+ex)-8ac\sin(d+ex)}{2(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}dx}{3(a^2-b^2-c^2)}}{5\left(a^2-b^2-c^2\right)}+\frac{2(c\cos(d+ex)-b\sin(d+ex))}{5e\left(a^2-b^2-c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{3\left(5a^2+3\left(b^2+c^2\right)\right)-8ab\cos(d+ex)-8ac\sin(d+ex)}{(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}dx}{3(a^2-b^2-c^2)}+\frac{16(ac\cos(d+ex)-ab\sin(d+ex))}{3e(a^2-b^2-c^2)(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}}{5\left(a^2-b^2-c^2\right)}+\frac{2(c\cos(d+ex)-b\sin(d+ex))}{5e\left(a^2-b^2-c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{3\left(5a^2+3\left(b^2+c^2\right)\right)-8ab\cos(d+ex)-8ac\sin(d+ex)}{(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}dx}{3(a^2-b^2-c^2)}+\frac{16(ac\cos(d+ex)-ab\sin(d+ex))}{3e(a^2-b^2-c^2)(a+b\cos(d+ex)+c\sin(d+ex))^{3/2}}}{5\left(a^2-b^2-c^2\right)}+\frac{2(c\cos(d+ex)-b\sin(d+ex))}{5e\left(a^2-b^2-c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}}$$

$$\Big\downarrow \text{3635}$$

$$\frac{\frac{\frac{2\left(c\left(23a^2+9\left(b^2+c^2\right)\right)\cos(d+ex)-b\left(23a^2+9\left(b^2+c^2\right)\right)\sin(d+ex)\right)}{e\left(a^2-b^2-c^2\right)\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}-\frac{2\int -\frac{a\left(15a^2+17\left(b^2+c^2\right)\right)+b\left(23a^2+9\left(b^2+c^2\right)\right)\cos(d+ex)+c\left(23a^2+9\left(b^2+c^2\right)\right)\sin(d+ex)}{2\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}d}{a^2-b^2-c^2}}{3(a^2-b^2-c^2)}}{5\left(a^2-b^2-c^2\right)}$$

$$\frac{2(c\cos(d+ex)-b\sin(d+ex))}{5e\left(a^2-b^2-c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}}$$

$$\Big\downarrow \text{27}$$

3.416. $\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{7/2}}\,dx$

$$\frac{\frac{\frac{\int \frac{a\left(15a^2+17\left(b^2+c^2\right)\right)+b\left(23a^2+9\left(b^2+c^2\right)\right)\cos(d+ex)+c\left(23a^2+9\left(b^2+c^2\right)\right)\sin(d+ex)}{\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}dx}{a^2-b^2-c^2}+\frac{2\left(c\left(23a^2+9\left(b^2+c^2\right)\right)\cos(d+ex)-b\left(23a^2+9\left(b^2+c^2\right)\right)\sin(d+ex)\right)}{e\left(a^2-b^2-c^2\right)\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}}{3(a^2-b^2-c^2)} \cdots}{5\left(a^2-b^2-c^2\right)}$$

$$\frac{2(c\cos(d+ex)-b\sin(d+ex))}{5e\left(a^2-b^2-c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{\int \frac{a\left(15a^2+17\left(b^2+c^2\right)\right)+b\left(23a^2+9\left(b^2+c^2\right)\right)\cos(d+ex)+c\left(23a^2+9\left(b^2+c^2\right)\right)\sin(d+ex)}{\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}dx}{a^2-b^2-c^2}+\frac{2\left(c\left(23a^2+9\left(b^2+c^2\right)\right)\cos(d+ex)-b\left(23a^2+9\left(b^2+c^2\right)\right)\sin(d+ex)\right)}{e\left(a^2-b^2-c^2\right)\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}}{3(a^2-b^2-c^2)} \cdots}{5\left(a^2-b^2-c^2\right)}$$

$$\frac{2(c\cos(d+ex)-b\sin(d+ex))}{5e\left(a^2-b^2-c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}}$$

$$\Big\downarrow \text{3628}$$

$$\frac{\frac{\frac{\left(23a^2+9\left(b^2+c^2\right)\right)\int\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}dx-8a\left(a^2-b^2-c^2\right)\int\frac{1}{\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}dx}{a^2-b^2-c^2}+\frac{2\left(c\left(23a^2+9\left(b^2+c^2\right)\right)\cos(d+ex)-b\left(23a^2+9\left(b^2+c^2\right) \cdots\right.\right.}{e\left(a^2-b^2-c^2\right)\sqrt{a+b\cos(d+ex)+c\sin(d+e \cdots}}}{3(a^2-b^2-c^2)} \cdots}{5\left(a^2-b^2-c^2\right)}$$

$$\frac{2(c\cos(d+ex)-b\sin(d+ex))}{5e\left(a^2-b^2-c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{\left(23a^2+9\left(b^2+c^2\right)\right)\int\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}dx-8a\left(a^2-b^2-c^2\right)\int\frac{1}{\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}dx}{a^2-b^2-c^2}+\frac{2\left(c\left(23a^2+9\left(b^2+c^2\right)\right)\cos(d+ex)-b\left(23a^2+9\left(b^2+c^2\right) \cdots\right.\right.}{e\left(a^2-b^2-c^2\right)\sqrt{a+b\cos(d+ex)+c\sin(d+e \cdots}}}{3(a^2-b^2-c^2)} \cdots}{5\left(a^2-b^2-c^2\right)}$$

$$\frac{2(c\cos(d+ex)-b\sin(d+ex))}{5e\left(a^2-b^2-c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}}$$

$$\Big\downarrow \text{3598}$$

$$\frac{\frac{\frac{\frac{\left(23a^2+9\left(b^2+c^2\right)\right)\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}\int\sqrt{\frac{a}{a+\sqrt{b^2+c^2}}+\frac{\sqrt{b^2+c^2}\cos\left(d+ex-\tan^{-1}(b,c)\right)}{a+\sqrt{b^2+c^2}}}dx}{\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}-8a\left(a^2-b^2-c^2\right)\int\frac{1}{\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}dx}{a^2-b^2-c^2}+\frac{2(c \cdots}{\cdots}}{3(a^2-b^2-c^2)} \cdots}{5\left(a^2-b^2-c^2\right)}$$

$$\frac{2(c\cos(d+ex)-b\sin(d+ex))}{5e\left(a^2-b^2-c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

3.416. $\displaystyle\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{7/2}}\,dx$

$$\frac{\frac{\frac{\left(23a^2+9\left(b^2+c^2\right)\right)\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}\int\sqrt{\frac{a}{a+\sqrt{b^2+c^2}}+\frac{\sqrt{b^2+c^2}\sin\left(d+ex-\tan^{-1}(b,c)+\frac{\pi}{2}\right)}{a+\sqrt{b^2+c^2}}}dx}{\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}-8a\left(a^2-b^2-c^2\right)\int\frac{1}{\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}dx}{a^2-b^2-c^2}+}{3(a^2-b^2-c^2)}}{5\left(a^2-b^2-c^2\right)}$$

$$\frac{2(c\cos(d+ex)-b\sin(d+ex))}{5e\left(a^2-b^2-c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}}$$

$$\Big\downarrow \; 3132$$

$$\frac{\frac{\frac{\frac{2\left(23a^2+9\left(b^2+c^2\right)\right)\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{e\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}-8a\left(a^2-b^2-c^2\right)\int\frac{1}{\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}dx}{a^2-b^2-c^2}+\frac{2\left(c\left(23a^2+9\left(b^2\right.\right.\right.}{e\left(a^2\right.}}{3(a^2-b^2-c^2)}}{5\left(a^2-b^2-c^2\right)}$$

$$\frac{2(c\cos(d+ex)-b\sin(d+ex))}{5e\left(a^2-b^2-c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}}$$

$$\Big\downarrow \; 3606$$

$$\frac{\frac{\frac{2\left(23a^2+9\left(b^2+c^2\right)\right)\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{e\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}-\frac{8a\left(a^2-b^2-c^2\right)\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}\int\frac{}{\sqrt{\frac{a}{a+\sqrt{b^2+c^2}}+\frac{\sqrt{b^2}\cdots}{}}}}{\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}}{a^2-b^2-c^2}}{3(a^2-b^2-c^2)} \quad 5\left(a^2-b\right.$$

$$\frac{2(c\cos(d+ex)-b\sin(d+ex))}{5e\left(a^2-b^2-c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{2\left(23a^2+9\left(b^2+c^2\right)\right)\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{e\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}-\frac{8a\left(a^2-b^2-c^2\right)\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}\int\frac{}{\sqrt{\frac{a}{a+\sqrt{b^2+c^2}}+\frac{\sqrt{b^2}\cdots}{}}}}{\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}}{a^2-b^2-c^2}}{3(a^2-b^2-c^2)} \quad 5\left(a^2-\right.$$

$$\frac{2(c\cos(d+ex)-b\sin(d+ex))}{5e\left(a^2-b^2-c^2\right)(a+b\cos(d+ex)+c\sin(d+ex))^{5/2}}$$

$$\Big\downarrow \; 3140$$

3.416. $\displaystyle\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{7/2}}\,dx$

$$\frac{\frac{\frac{\frac{2\left(23a^2+9\left(b^2+c^2\right)\right)\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{e\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}}-\frac{16a\left(a^2-b^2-c^2\right)\sqrt{\frac{a+b\cos(d+ex)+c\sin(d+ex)}{a+\sqrt{b^2+c^2}}}\text{EllipticF}\left(\frac{1}{2}\left(d+ex-t\right.\right.}{e\sqrt{a+b\cos(d+ex)+c\sin(d+ex)}}}{a^2-b^2-c^2}}{3(a^2-b^2-c^2)}}{5\left(a^2-b^2\right.}$$

$$\frac{2(c\cos(d+ex)-b\sin(d+ex))}{5e\left(a^2-b^2-c^2\right)\left(a+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}$$

input
```
Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(-7/2),x]
```

output
```
(2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(5*(a^2 - b^2 - c^2)*e*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(5/2)) + ((16*(a*c*Cos[d + e*x] - a*b*Sin[d + e*x
]))/(3*(a^2 - b^2 - c^2)*e*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(3/2)) +
((2*(c*(23*a^2 + 9*(b^2 + c^2))*Cos[d + e*x] - b*(23*a^2 + 9*(b^2 + c^2))*
Sin[d + e*x]))/((a^2 - b^2 - c^2)*e*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*
x]]) + ((2*(23*a^2 + 9*(b^2 + c^2))*EllipticE[(d + e*x - ArcTan[b, c])/2,
(2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[a + b*Cos[d + e*x] + c*Sin
[d + e*x]])/(e*Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 +
c^2])]) - (16*a*(a^2 - b^2 - c^2)*EllipticF[(d + e*x - ArcTan[b, c])/2, (2
*Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[(a + b*Cos[d + e*x] + c*Sin[
d + e*x])/(a + Sqrt[b^2 + c^2])])/(e*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e
*x]]))/(a^2 - b^2 - c^2))/(3*(a^2 - b^2 - c^2)))/(5*(a^2 - b^2 - c^2))
```

3.416.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3598
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]   Int[Sqrt[a/(a + S
qrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - Arc
Tan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

rule 3606
```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Simp[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]   Int[1/Sqrt[a/(a
 + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x -
 ArcTan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2
, 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

rule 3608
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Simp[
1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c
*(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x
] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] &&
NeQ[n, -3/2]
```

rule 3628
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]
, x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[
A*b - a*B, 0]
```

3.416. $\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{7/2}}\,dx$

rule 3635
```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(-(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(
a^2 - b^2 - c^2))), x] + Simp[1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a + b*Co
s[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n +
2)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x]
/; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[n, -2]
```

3.416.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 5027 vs. 2(535) = 1070.

Time = 30.19 (sec) , antiderivative size = 5028, normalized size of antiderivative = 10.26

method	result	size
default	Expression too large to display	5028

input
```
int(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.416.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.51 (sec) , antiderivative size = 4955, normalized size of antiderivative = 10.11

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(7/2),x, algorithm="fricas")
```

output
```
1/45*((sqrt(2)*(-I*a^3*b^4 + 33*I*a*b^6 - 99*I*a*b^2*c^4 - 99*a*b*c^5 + 3*
(a^3*b - 22*a*b^3)*c^3 + 3*I*(a^3*b^2 - 22*a*b^4)*c^2 - (a^3*b^3 - 33*a*b^
5)*c)*cos(e*x + d)^3 - 3*sqrt(2)*(I*a^4*b^3 - 33*I*a^2*b^5 - I*a^4*b*c^2 -
 a^4*c^3 + 33*I*a^2*b*c^4 + 33*a^2*c^5 + (a^4*b^2 - 33*a^2*b^4)*c)*cos(e*x
 + d)^2 - 3*sqrt(2)*(I*a^5*b^2 - 33*I*a^3*b^4 - 33*I*a*b^2*c^4 - 33*a*b*c^
5 - (32*a^3*b + 33*a*b^3)*c^3 - I*(32*a^3*b^2 + 33*a*b^4)*c^2 + (a^5*b - 3
3*a^3*b^3)*c)*cos(e*x + d) + (sqrt(2)*(-33*I*a*b*c^5 - 33*a*c^6 + (a^3 + 6
6*a*b^2)*c^4 + I*(a^3*b + 66*a*b^3)*c^3 - 3*(a^3*b^2 - 33*a*b^4)*c^2 - 3*I
*(a^3*b^3 - 33*a*b^5)*c)*cos(e*x + d)^2 - 6*sqrt(2)*(-33*I*a^2*b^2*c^3 - 3
3*a^2*b*c^4 + (a^4*b - 33*a^2*b^3)*c^2 + I*(a^4*b^2 - 33*a^2*b^4)*c)*cos(e
*x + d) + sqrt(2)*(33*I*a*b*c^5 + 33*a*c^6 + (98*a^3 + 33*a*b^2)*c^4 + I*(
98*a^3*b + 33*a*b^3)*c^3 - 3*(a^5 - 33*a^3*b^2)*c^2 - 3*I*(a^5*b - 33*a^3*
b^3)*c))*sin(e*x + d) + sqrt(2)*(-I*a^6*b + 33*I*a^4*b^3 + 99*I*a^2*b*c^4
+ 99*a^2*c^5 + 3*(10*a^4 + 33*a^2*b^2)*c^3 + 3*I*(10*a^4*b + 33*a^2*b^3)*c
^2 - (a^6 - 33*a^4*b^2)*c))*sqrt(b + I*c)*weierstrassPInverse(4/3*(4*a^2*b
^2 - 3*b^4 - 4*a^2*c^2 + 6*I*b*c^3 + 3*c^4 - 2*I*(4*a^2*b - 3*b^3)*c)/(b^4
 + 2*b^2*c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 - 9*I*a*c^5 +
 2*I*(4*a^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 - 3*I*(8*a^3*b^2 -
9*a*b^4)*c)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), 1/3*(2*a*b - 2*I*a*c + 3*
(b^2 + c^2)*cos(e*x + d) - 3*(I*b^2 + I*c^2)*sin(e*x + d))/(b^2 + c^2))...
```

3.416.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))**(7/2),x)
```

output
```
Timed out
```

3.416.7 Maxima [F]

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{7/2}}\,dx=\int \frac{1}{(b\cos(ex+d)+c\sin(ex+d)+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(e*x + d) + c*sin(e*x + d) + a)^(-7/2), x)
```

3.416.8 Giac [F]

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{7/2}}\,dx=\int \frac{1}{(b\cos(ex+d)+c\sin(ex+d)+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(e*x+d)+c*sin(e*x+d))^(7/2),x, algorithm="giac")
```

output
```
integrate((b*cos(e*x + d) + c*sin(e*x + d) + a)^(-7/2), x)
```

3.416.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(d+ex)+c\sin(d+ex))^{7/2}}\,dx=\int \frac{1}{(a+b\,\cos(d+e\,x)+c\,\sin(d+e\,x))^{7/2}}\,dx$$

input
```
int(1/(a + b*cos(d + e*x) + c*sin(d + e*x))^(7/2),x)
```

output
```
int(1/(a + b*cos(d + e*x) + c*sin(d + e*x))^(7/2), x)
```

3.417 $\int (5+4\cos(d+ex)+3\sin(d+ex))^{5/2}\,dx$

3.417.1 Optimal result

Integrand size = 22, antiderivative size = 139

$$\int (5+4\cos(d+ex)+3\sin(d+ex))^{5/2}\,dx = -\frac{320(3\cos(d+ex)-4\sin(d+ex))}{3e\sqrt{5+4\cos(d+ex)+3\sin(d+ex)}} - \frac{16(3\cos(d+ex)-4\sin(d+ex))\sqrt{5+4\cos(d+ex)+3\sin(d+ex)}}{3e} - \frac{2(3\cos(d+ex)-4\sin(d+ex))(5+4\cos(d+ex)+3\sin(d+ex))^{3/2}}{5e}$$

output
```
-2/5*(3*cos(e*x+d)-4*sin(e*x+d))*(5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2)/e-320
/3*(3*cos(e*x+d)-4*sin(e*x+d))/e/(5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2)-16/3*
(3*cos(e*x+d)-4*sin(e*x+d))*(5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2)/e
```

3.417.2 Mathematica [A] (verified)

Time = 6.25 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.94

$$\int (5+4\cos(d+ex)+3\sin(d+ex))^{5/2}\,dx = -\frac{(5+4\cos(d+ex)+3\sin(d+ex))^{5/2}\left(3750\cos\left(\frac{1}{2}(d+ex)\right)+1625\cos\left(\frac{3}{2}(d+ex)\right)+3\left(79\cos\left(\frac{5}{2}(d+ex\right)\right.\right.}{30e\left(3\cos\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{1}{2}(d+ex\right)\right.}$$

input
```
Integrate[(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(5/2),x]
```

output
```
-1/30*((5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(5/2)*(3750*Cos[(d + e*x)/2]
+ 1625*Cos[(3*(d + e*x))/2] + 3*(79*Cos[(5*(d + e*x))/2] - 3750*Sin[(d + e
*x)/2] - 375*Sin[(3*(d + e*x))/2] + 3*Sin[(5*(d + e*x))/2])))/(e*(3*Cos[(d
 + e*x)/2] + Sin[(d + e*x)/2])^5)
```

3.417.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 3592, 3042, 3592, 3042, 3591}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (3\sin(d+ex)+4\cos(d+ex)+5)^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (3\sin(d+ex)+4\cos(d+ex)+5)^{5/2}dx$$

$$\Big\downarrow \text{3592}$$

$$8\int (4\cos(d+ex)+3\sin(d+ex)+5)^{3/2}dx-\frac{2(3\cos(d+ex)-4\sin(d+ex))(3\sin(d+ex)+4\cos(d+ex)+5)^{3/2}}{5e}$$

$$\Big\downarrow \text{3042}$$

$$8\int (4\cos(d+ex)+3\sin(d+ex)+5)^{3/2}dx-\frac{2(3\cos(d+ex)-4\sin(d+ex))(3\sin(d+ex)+4\cos(d+ex)+5)^{3/2}}{5e}$$

$$\Big\downarrow \text{3592}$$

$$8\left(\frac{20}{3}\int \sqrt{4\cos(d+ex)+3\sin(d+ex)+5}dx-\frac{2(3\cos(d+ex)-4\sin(d+ex))\sqrt{3\sin(d+ex)+4\cos(d+ex)}}{3e}\right.$$
$$\frac{2(3\cos(d+ex)-4\sin(d+ex))(3\sin(d+ex)+4\cos(d+ex)+5)^{3/2}}{5e}$$

$$\Big\downarrow \text{3042}$$

$$8\left(\frac{20}{3}\int \sqrt{4\cos(d+ex)+3\sin(d+ex)+5}dx-\frac{2(3\cos(d+ex)-4\sin(d+ex))\sqrt{3\sin(d+ex)+4\cos(d+ex)+5}}{3e}\right)-\frac{2(3\cos(d+ex)-4\sin(d+ex))(3\sin(d+ex)+4\cos(d+ex)+5)^{3/2}}{5e}$$

$\Big\downarrow$ 3591

$$8\left(-\frac{2\sqrt{3\sin(d+ex)+4\cos(d+ex)+5}(3\cos(d+ex)-4\sin(d+ex))}{3e}-\frac{40(3\cos(d+ex)-4\sin(d+ex))}{3e\sqrt{3\sin(d+ex)+4\cos(d+ex)+5}}\right)-\frac{2(3\cos(d+ex)-4\sin(d+ex))(3\sin(d+ex)+4\cos(d+ex)+5)^{3/2}}{5e}$$

input
```
Int[(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(5/2),x]
```

output
```
(-2*(3*Cos[d + e*x] - 4*Sin[d + e*x])*(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x]
)^(3/2))/(5*e) + 8*((-40*(3*Cos[d + e*x] - 4*Sin[d + e*x]))/(3*e*Sqrt[5 +
4*Cos[d + e*x] + 3*Sin[d + e*x]]) - (2*(3*Cos[d + e*x] - 4*Sin[d + e*x])*S
qrt[5 + 4*Cos[d + e*x] + 3*Sin[d + e*x]])/(3*e))
```

3.417.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3591
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[-2*((c*Cos[d + e*x] - b*Sin[d + e*x])/(e*Sqrt[a + b*
Cos[d + e*x] + c*Sin[d + e*x]])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

rule 3592
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Simp[a*((2*n - 1)/n)   Int[(a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

3.417.4 Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.53

method	result	size
default	$\frac{50\left(1+\sin \left(e x+d+\arctan \left(\frac{4}{3}\right)\right)\right)\left(\sin \left(e x+d+\arctan \left(\frac{4}{3}\right)\right)-1\right)\left(3 \sin \left(e x+d+\arctan \left(\frac{4}{3}\right)\right)^{2}+14 \sin \left(e x+d+\arctan \left(\frac{4}{3}\right)\right)+43\right)}{3 \cos \left(e x+d+\arctan \left(\frac{4}{3}\right)\right) \sqrt{5+5 \sin \left(e x+d+\arctan \left(\frac{4}{3}\right)\right)}\, e}$	74

input
```
int((5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
50/3*(1+sin(e*x+d+arctan(4/3)))*(sin(e*x+d+arctan(4/3))-1)*(3*sin(e*x+d+ar
ctan(4/3))^2+14*sin(e*x+d+arctan(4/3))+43)/cos(e*x+d+arctan(4/3))/(5+5*sin
(e*x+d+arctan(4/3)))^(1/2)/e
```

3.417.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.73

$$\int (5+4\cos(d+ex)+3\sin(d+ex))^{5/2}\, dx =$$
$$-\frac{2\left(237\cos(ex+d)^3+931\cos(ex+d)^2+9\left(\cos(ex+d)^2-62\cos(ex+d)-344\right)\sin(ex+d)+1166\ c}{15\left(3\,e\cos(ex+d)+e\sin(ex+d)+3\,e\right)}$$

input
```
integrate((5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x, algorithm="fricas")
```

output
```
-2/15*(237*cos(e*x + d)^3 + 931*cos(e*x + d)^2 + 9*(cos(e*x + d)^2 - 62*co
s(e*x + d) - 344)*sin(e*x + d) + 1166*cos(e*x + d) + 472)*sqrt(4*cos(e*x +
 d) + 3*sin(e*x + d) + 5)/(3*e*cos(e*x + d) + e*sin(e*x + d) + 3*e)
```

3.417.6 Sympy [F(-1)]

Timed out.

$$\int (5+4\cos(d+ex)+3\sin(d+ex))^{5/2}\, dx = \text{Timed out}$$

input
```
integrate((5+4*cos(e*x+d)+3*sin(e*x+d))**(5/2),x)
```

output
```
Timed out
```

3.417.7 Maxima [F]

$$\int (5+4\cos(d+ex)+3\sin(d+ex))^{5/2}\,dx=\int (4\,\cos(ex+d)+3\,\sin(ex+d)+5)^{\frac{5}{2}}\,dx$$

input
```
integrate((5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x, algorithm="maxima")
```

output
```
integrate((4*cos(e*x + d) + 3*sin(e*x + d) + 5)^(5/2), x)
```

3.417.8 Giac [F]

$$\int (5+4\cos(d+ex)+3\sin(d+ex))^{5/2}\,dx=\int (4\,\cos(ex+d)+3\,\sin(ex+d)+5)^{\frac{5}{2}}\,dx$$

input
```
integrate((5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x, algorithm="giac")
```

output
```
integrate((4*cos(e*x + d) + 3*sin(e*x + d) + 5)^(5/2), x)
```

3.417.9 Mupad [F(-1)]

Timed out.

$$\int (5+4\cos(d+ex)+3\sin(d+ex))^{5/2}\,dx=\int (4\,\cos(d+e\,x)+3\,\sin(d+e\,x)+5)^{5/2}\,dx$$

input
```
int((4*cos(d + e*x) + 3*sin(d + e*x) + 5)^(5/2),x)
```

output
```
int((4*cos(d + e*x) + 3*sin(d + e*x) + 5)^(5/2), x)
```

3.418 $\int (5+4\cos(d+ex)+3\sin(d+ex))^{3/2}\,dx$

3.418.1 Optimal result

Integrand size = 22, antiderivative size = 93

$$\int (5+4\cos(d+ex)+3\sin(d+ex))^{3/2}\,dx = -\frac{40(3\cos(d+ex)-4\sin(d+ex))}{3e\sqrt{5+4\cos(d+ex)+3\sin(d+ex)}} - \frac{2(3\cos(d+ex)-4\sin(d+ex))\sqrt{5+4\cos(d+ex)+3\sin(d+ex)}}{3e}$$

output
```
-40/3*(3*cos(e*x+d)-4*sin(e*x+d))/e/(5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2)-2/
3*(3*cos(e*x+d)-4*sin(e*x+d))*(5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2)/e
```

3.418.2 Mathematica [A] (verified)

Time = 1.85 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.12

$$\int (5+4\cos(d+ex)+3\sin(d+ex))^{3/2}\,dx = \frac{(5+4\cos(d+ex)+3\sin(d+ex))^{3/2}\left(-45\cos\left(\frac{1}{2}(d+ex)\right)-13\cos\left(\frac{3}{2}(d+ex)\right)+9\left(15\sin\left(\right.\right.\right.}{3e\left(3\cos\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{1}{2}(d+ex)\right)\right)^3}$$

input
```
Integrate[(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2),x]
```

output
```
((5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2)*(-45*Cos[(d + e*x)/2] - 13*Co
s[(3*(d + e*x))/2] + 9*(15*Sin[(d + e*x)/2] + Sin[(3*(d + e*x))/2])))/(3*e
*(3*Cos[(d + e*x)/2] + Sin[(d + e*x)/2])^3)
```

3.418.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3042, 3592, 3042, 3591}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (3\sin(d+ex)+4\cos(d+ex)+5)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (3\sin(d+ex)+4\cos(d+ex)+5)^{3/2}dx$$

$$\Big\downarrow \text{3592}$$

$$\frac{20}{3}\int \sqrt{4\cos(d+ex)+3\sin(d+ex)+5}dx-\frac{2(3\cos(d+ex)-4\sin(d+ex))\sqrt{3\sin(d+ex)+4\cos(d+ex)+5}}{3e}$$

$$\Big\downarrow \text{3042}$$

$$\frac{20}{3}\int \sqrt{4\cos(d+ex)+3\sin(d+ex)+5}dx-\frac{2(3\cos(d+ex)-4\sin(d+ex))\sqrt{3\sin(d+ex)+4\cos(d+ex)+5}}{3e}$$

$$\Big\downarrow \text{3591}$$

$$-\frac{2\sqrt{3\sin(d+ex)+4\cos(d+ex)+5}(3\cos(d+ex)-4\sin(d+ex))}{3e}-\frac{40(3\cos(d+ex)-4\sin(d+ex))}{3e\sqrt{3\sin(d+ex)+4\cos(d+ex)+5}}$$

input
```
Int[(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2),x]
```

output
```
(-40*(3*Cos[d + e*x] - 4*Sin[d + e*x]))/(3*e*Sqrt[5 + 4*Cos[d + e*x] + 3*S
in[d + e*x]]) - (2*(3*Cos[d + e*x] - 4*Sin[d + e*x])*Sqrt[5 + 4*Cos[d + e*
x] + 3*Sin[d + e*x]])/(3*e)
```

3.418.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3591
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[-2*((c*Cos[d + e*x] - b*Sin[d + e*x])/(e*Sqrt[a + b*
Cos[d + e*x] + c*Sin[d + e*x]])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

rule 3592
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Simp[a*((2*n - 1)/n)   Int[(a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

3.418.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.65

method	result	size
default	$\frac{50\left(1+\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\right)\left(\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)-1\right)\left(\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)+5\right)}{3\cos\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\sqrt{5+5\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)}\, e}$	60

input
```
int((5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
50/3*(1+sin(e*x+d+arctan(4/3)))*(sin(e*x+d+arctan(4/3))-1)*(sin(e*x+d+arct
an(4/3))+5)/cos(e*x+d+arctan(4/3))/(5+5*sin(e*x+d+arctan(4/3)))^(1/2)/e
```

3.418.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.87

$$\int (5+4\cos(d+ex)+3\sin(d+ex))^{3/2}\,dx = -\frac{2\left(13\cos\left(ex+d\right)^{2}-9\left(\cos\left(ex+d\right)+8\right)\sin\left(ex+d\right)+29\cos\left(ex+d\right)+16\right)\sqrt{4\cos\left(ex+d\right)+3\sin\left(ex+d\right)+5}}{3\left(3\,e\cos\left(ex+d\right)+e\sin\left(ex+d\right)+3\,e\right)}$$

input
```
integrate((5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(13*cos(e*x + d)^2 - 9*(cos(e*x + d) + 8)*sin(e*x + d) + 29*cos(e*x +
 d) + 16)*sqrt(4*cos(e*x + d) + 3*sin(e*x + d) + 5)/(3*e*cos(e*x + d) + e*
sin(e*x + d) + 3*e)
```

3.418.6 Sympy [F]

$$\int (5+4\cos(d+ex)+3\sin(d+ex))^{3/2}\,dx = \int \left(3\sin\left(d+ex\right)+4\cos\left(d+ex\right)+5\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((5+4*cos(e*x+d)+3*sin(e*x+d))**(3/2),x)
```

output
```
Integral((3*sin(d + e*x) + 4*cos(d + e*x) + 5)**(3/2), x)
```

3.418.7 Maxima [F]

$$\int (5+4\cos(d+ex)+3\sin(d+ex))^{3/2}\,dx = \int \left(4\cos\left(ex+d\right)+3\sin\left(ex+d\right)+5\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="maxima")
```

output
```
integrate((4*cos(e*x + d) + 3*sin(e*x + d) + 5)^(3/2), x)
```

3.418.8 Giac [F]

$$\int (5+4\cos(d+ex)+3\sin(d+ex))^{3/2}\,dx=\int (4\,\cos{(ex+d)}+3\,\sin{(ex+d)}+5)^{\frac{3}{2}}\,dx$$

input
```
integrate((5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="giac")
```

output
```
integrate((4*cos(e*x + d) + 3*sin(e*x + d) + 5)^(3/2), x)
```

3.418.9 Mupad [F(-1)]

Timed out.

$$\int (5+4\cos(d+ex)+3\sin(d+ex))^{3/2}\,dx=\int (4\,\cos{(d+e\,x)}+3\,\sin{(d+e\,x)}+5)^{3/2}\,dx$$

input
```
int((4*cos(d + e*x) + 3*sin(d + e*x) + 5)^(3/2),x)
```

output
```
int((4*cos(d + e*x) + 3*sin(d + e*x) + 5)^(3/2), x)
```

3.419 $\int \sqrt{5+4\cos(d+ex)+3\sin(d+ex)}\,dx$

3.419.1 Optimal result

Integrand size = 22, antiderivative size = 44

$$\int \sqrt{5+4\cos(d+ex)+3\sin(d+ex)}\,dx=-\frac{2(3\cos(d+ex)-4\sin(d+ex))}{e\sqrt{5+4\cos(d+ex)+3\sin(d+ex)}}$$

output
```
-2*(3*cos(e*x+d)-4*sin(e*x+d))/e/(5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2)
```

3.419.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.70

$$\int \sqrt{5+4\cos(d+ex)+3\sin(d+ex)}\,dx =-\frac{2\left(\cos\left(\frac{1}{2}(d+ex)\right)-3\sin\left(\frac{1}{2}(d+ex)\right)\right)\sqrt{5+4\cos(d+ex)+3\sin(d+ex)}}{e\left(3\cos\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{1}{2}(d+ex)\right)\right)}$$

input
```
Integrate[Sqrt[5 + 4*Cos[d + e*x] + 3*Sin[d + e*x]],x]
```

output
```
(-2*(Cos[(d + e*x)/2] - 3*Sin[(d + e*x)/2])*Sqrt[5 + 4*Cos[d + e*x] + 3*Si
n[d + e*x]])/(e*(3*Cos[(d + e*x)/2] + Sin[(d + e*x)/2]))
```

3.419.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3042, 3591}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{3\sin(d+ex)+4\cos(d+ex)+5}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{3\sin(d+ex)+4\cos(d+ex)+5}dx$$

$$\Big\downarrow \text{3591}$$

$$-\frac{2(3\cos(d+ex)-4\sin(d+ex))}{e\sqrt{3\sin(d+ex)+4\cos(d+ex)+5}}$$

input
```
Int[Sqrt[5 + 4*Cos[d + e*x] + 3*Sin[d + e*x]],x]
```

output
```
(-2*(3*Cos[d + e*x] - 4*Sin[d + e*x]))/(e*Sqrt[5 + 4*Cos[d + e*x] + 3*Sin[
d + e*x]])
```

3.419.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3591
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[-2*((c*Cos[d + e*x] - b*Sin[d + e*x])/(e*Sqrt[a + b*
Cos[d + e*x] + c*Sin[d + e*x]])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

3.419.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.14

method	result
default	$\frac{10\left(\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)-1\right)\left(1+\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\right)}{\cos\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\sqrt{5+5\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)}\, e}$
risch	$-\frac{5i\sqrt{2}\,\sqrt{10+8\cos(ex+d)+6\sin(ex+d)}\,\sqrt{(4-3i)\left(25\,\mathrm{e}^{3i(ex+d)}+30i\mathrm{e}^{2i(ex+d)}+7\,\mathrm{e}^{i(ex+d)}+24i\mathrm{e}^{i(ex+d)}+40\,\mathrm{e}^{2i(ex+d)}\right)}\,\left(5\,\mathrm{e}^{i(ex+d)}-4\cdots\right.}{\left(25\,\mathrm{e}^{2i(ex+d)}+30i\mathrm{e}^{i(ex+d)}+7+24i+40\,\mathrm{e}^{i(ex+d)}\right)e\sqrt{(100-75i)\left(25\,\mathrm{e}^{3i(ex+d)}+30i\mathrm{e}^{2i(ex+d)}+7\,\mathrm{e}^{i(ex+d)}+24i\mathrm{e}^{i(ex+d)}+4\cdots\right.}}$

input
```
int((5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
10*(sin(e*x+d+arctan(4/3))-1)*(1+sin(e*x+d+arctan(4/3)))/cos(e*x+d+arctan(
4/3))/(5+5*sin(e*x+d+arctan(4/3)))^(1/2)/e
```

3.419.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.39

$$\int \sqrt{5+4\cos(d+ex)+3\sin(d+ex)}\,dx$$
$$= -\frac{2\sqrt{4\cos(ex+d)+3\sin(ex+d)+5}(\cos(ex+d)-3\sin(ex+d)+1)}{3\,e\cos(ex+d)+e\sin(ex+d)+3\,e}$$

input
```
integrate((5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(4*cos(e*x + d) + 3*sin(e*x + d) + 5)*(cos(e*x + d) - 3*sin(e*x + d
) + 1)/(3*e*cos(e*x + d) + e*sin(e*x + d) + 3*e)
```

3.419.6 Sympy [F]

$$\int \sqrt{5+4\cos(d+ex)+3\sin(d+ex)}\,dx = \int \sqrt{3\sin(d+ex)+4\cos(d+ex)+5}\,dx$$

input
```
integrate((5+4*cos(e*x+d)+3*sin(e*x+d))**(1/2),x)
```

output
```
Integral(sqrt(3*sin(d + e*x) + 4*cos(d + e*x) + 5), x)
```

3.419.7 Maxima [F]

$$\int \sqrt{5+4\cos(d+ex)+3\sin(d+ex)}\,dx = \int \sqrt{4\,\cos\left(ex+d\right)+3\,\sin\left(ex+d\right)+5}\,dx$$

input
```
integrate((5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(4*cos(e*x + d) + 3*sin(e*x + d) + 5), x)
```

3.419.8 Giac [F]

$$\int \sqrt{5+4\cos(d+ex)+3\sin(d+ex)}\,dx = \int \sqrt{4\,\cos\left(ex+d\right)+3\,\sin\left(ex+d\right)+5}\,dx$$

input
```
integrate((5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(4*cos(e*x + d) + 3*sin(e*x + d) + 5), x)
```

3.419.9 Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int \sqrt{5+4\cos(d+ex)+3\sin(d+ex)}\,dx = -\frac{2\sqrt{5}\left(3\,\cos\left(d+e\,x\right)-4\,\sin\left(d+e\,x\right)\right)}{5\,e\,\sqrt{\cos\left(d-\mathrm{atan}\left(\frac{3}{4}\right)+e\,x\right)+1}}$$

input
```
int((4*cos(d + e*x) + 3*sin(d + e*x) + 5)^(1/2),x)
```

output
```
-(2*5^(1/2)*(3*cos(d + e*x) - 4*sin(d + e*x)))/(5*e*(cos(d - atan(3/4) + e
*x) + 1)^(1/2))
```

3.420 $\int \frac{1}{\sqrt{5+4\cos(d+ex)+3\sin(d+ex)}}\,dx$

3.420.1 Optimal result

Integrand size = 22, antiderivative size = 48

$$\int \frac{1}{\sqrt{5+4\cos(d+ex)+3\sin(d+ex)}}\,dx=\frac{\sqrt{\frac{2}{5}}\operatorname{arctanh}\left(\frac{\sin\left(d+ex-\arctan\left(\frac{3}{4}\right)\right)}{\sqrt{2}\sqrt{1+\cos\left(d+ex-\arctan\left(\frac{3}{4}\right)\right)}}\right)}{e}$$

output
```
1/5*arctanh(1/2*sin(d+e*x-arctan(3/4))*2^(1/2)/(1+cos(d+e*x-arctan(3/4)))^
(1/2))*10^(1/2)/e
```

3.420.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.10

$$\int \frac{1}{\sqrt{5+4\cos(d+ex)+3\sin(d+ex)}}\,dx = -\frac{\left(\frac{2}{5}+\frac{6i}{5}\right)\sqrt{\frac{4}{5}+\frac{3i}{5}}\arctan\left(\left(\frac{1}{10}+\frac{3i}{10}\right)\sqrt{\frac{4}{5}+\frac{3i}{5}}\left(-1+3\tan\left(\frac{1}{4}(d+ex)\right)\right)\right)\left(3\cos\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{1}{2}(d+\right.\right.}{e\sqrt{5+4\cos(d+ex)+3\sin(d+ex)}}$$

input
```
Integrate[1/Sqrt[5 + 4*Cos[d + e*x] + 3*Sin[d + e*x]],x]
```

output
```
((-2/5 - (6*I)/5)*Sqrt[4/5 + (3*I)/5]*ArcTan[(1/10 + (3*I)/10)*Sqrt[4/5 +
(3*I)/5]*(-1 + 3*Tan[(d + e*x)/4])]*(3*Cos[(d + e*x)/2] + Sin[(d + e*x)/2]
))/(e*Sqrt[5 + 4*Cos[d + e*x] + 3*Sin[d + e*x]])
```

3.420.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {3042, 3594, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3\sin(d+ex)+4\cos(d+ex)+5}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{3\sin(d+ex)+4\cos(d+ex)+5}}dx$$

$$\Big\downarrow \text{ 3594}$$

$$\int \frac{1}{\sqrt{5\cos\left(-\arctan\left(\frac{3}{4}\right)+d+ex\right)+5}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{5\sin\left(-\arctan\left(\frac{3}{4}\right)+d+ex+\frac{\pi}{2}\right)+5}}dx$$

$$\Big\downarrow \text{ 3128}$$

$$-\frac{2\int \frac{1}{10-\frac{5\sin^2\left(d+ex-\arctan\left(\frac{3}{4}\right)\right)}{\cos\left(d+ex-\arctan\left(\frac{3}{4}\right)\right)+1}}d\left(-\frac{\sqrt{5}\sin\left(d+ex-\arctan\left(\frac{3}{4}\right)\right)}{\sqrt{\cos\left(d+ex-\arctan\left(\frac{3}{4}\right)\right)+1}}\right)}{e}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\sqrt{\frac{2}{5}}\text{arctanh}\left(\frac{\sin\left(-\arctan\left(\frac{3}{4}\right)+d+ex\right)}{\sqrt{2}\sqrt{\cos\left(-\arctan\left(\frac{3}{4}\right)+d+ex\right)+1}}\right)}{e}$$

input
```
Int[1/Sqrt[5 + 4*Cos[d + e*x] + 3*Sin[d + e*x]],x]
```

output
```
(Sqrt[2/5]*ArcTanh[Sin[d + e*x - ArcTan[3/4]]/(Sqrt[2]*Sqrt[1 + Cos[d + e*
x - ArcTan[3/4]]])])/e
```

3.420.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3594
```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

3.420.4 Maple [A] (verified)

Time = 2.90 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.60

method	result
default	$-\frac{\left(1+\sin \left(e x+d+\arctan \left(\frac{4}{3}\right)\right)\right) \sqrt{-5 \sin \left(e x+d+\arctan \left(\frac{4}{3}\right)\right)+5}\, \sqrt{10}\, \operatorname{arctanh}\left(\frac{\sqrt{-5 \sin \left(e x+d+\arctan \left(\frac{4}{3}\right)\right)+5}\, \sqrt{10}}{10}\right)}{5 \cos \left(e x+d+\arctan \left(\frac{4}{3}\right)\right) \sqrt{5+5 \sin \left(e x+d+\arctan \left(\frac{4}{3}\right)\right)}\, e}$
risch	$\frac{2 i \left(5 \mathrm{e}^{i (e x+d)}+4+3 i\right) \sqrt{2}\, \sqrt{(4-3 i) \left(25 \mathrm{e}^{2 i (e x+d)}+30 i \mathrm{e}^{i (e x+d)}+7+24 i+40 \mathrm{e}^{i (e x+d)}\right) \mathrm{e}^{i (e x+d)}}\, \mathrm{e}^{-i (e x+d)}}{e \sqrt{(100-75 i) \left(25 \mathrm{e}^{2 i (e x+d)}+30 i \mathrm{e}^{i (e x+d)}+7+24 i+40 \mathrm{e}^{i (e x+d)}\right) \mathrm{e}^{i (e x+d)}}\, \sqrt{-\left(3 i \mathrm{e}^{2 i (e x+d)}-4 \mathrm{e}^{2 i (e x+d)}-4-3 i-10 \mathrm{e}^{i (e x+d)}\right) \mathrm{e}^{-i (e x+d)}}}$

input
```
int(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*(1+sin(e*x+d+arctan(4/3)))*(-5*sin(e*x+d+arctan(4/3))+5)^(1/2)*10^(1/
2)*arctanh(1/10*(-5*sin(e*x+d+arctan(4/3))+5)^(1/2)*10^(1/2))/cos(e*x+d+ar
ctan(4/3))/(5+5*sin(e*x+d+arctan(4/3)))^(1/2)/e
```

3.420.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(38) = 76$.

Time = 0.25 (sec) , antiderivative size = 147, normalized size of antiderivative = 3.06

$$\int \frac{1}{\sqrt{5+4\cos(d+ex)+3\sin(d+ex)}}\,dx = \frac{\sqrt{5}\sqrt{2}\log\left(-\frac{9\cos(ex+d)^2+(13\cos(ex+d)-6)\sin(ex+d)+2\left(\sqrt{5}\sqrt{2}\cos(ex+d)-3\sqrt{5}\sqrt{2}\sin(ex+d)+\sqrt{5}\sqrt{2}\right)\sqrt{4\cos(ex+d)+3\sin(ex+d)}}{9\cos(ex+d)^2+(13\cos(ex+d)+14)\sin(ex+d)+27\cos(ex+d)+18}\right.}{10\,e}$$

input
```
integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x, algorithm="fricas")
```

output
```
1/10*sqrt(5)*sqrt(2)*log(-(9*cos(e*x + d)^2 + (13*cos(e*x + d) - 6)*sin(e*
x + d) + 2*(sqrt(5)*sqrt(2)*cos(e*x + d) - 3*sqrt(5)*sqrt(2)*sin(e*x + d)
+ sqrt(5)*sqrt(2))*sqrt(4*cos(e*x + d) + 3*sin(e*x + d) + 5) - 33*cos(e*x
+ d) - 42)/(9*cos(e*x + d)^2 + (13*cos(e*x + d) + 14)*sin(e*x + d) + 27*co
s(e*x + d) + 18))/e
```

3.420.6 Sympy [F]

$$\int \frac{1}{\sqrt{5+4\cos(d+ex)+3\sin(d+ex)}}\,dx = \int \frac{1}{\sqrt{3\sin{(d+ex)}+4\cos{(d+ex)}+5}}\,dx$$

input
```
integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))**(1/2),x)
```

output
```
Integral(1/sqrt(3*sin(d + e*x) + 4*cos(d + e*x) + 5), x)
```

3.420.7 Maxima [F]

$$\int \frac{1}{\sqrt{5+4\cos(d+ex)+3\sin(d+ex)}}\,dx = \int \frac{1}{\sqrt{4\,\cos\left(ex+d\right)+3\,\sin\left(ex+d\right)+5}}\,dx$$

input
```
integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(4*cos(e*x + d) + 3*sin(e*x + d) + 5), x)
```

3.420.8 Giac [F]

$$\int \frac{1}{\sqrt{5+4\cos(d+ex)+3\sin(d+ex)}}\,dx = \int \frac{1}{\sqrt{4\,\cos\left(ex+d\right)+3\,\sin\left(ex+d\right)+5}}\,dx$$

input
```
integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(4*cos(e*x + d) + 3*sin(e*x + d) + 5), x)
```

3.420.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{5+4\cos(d+ex)+3\sin(d+ex)}}\,dx = \int \frac{1}{\sqrt{4\,\cos\left(d+e\,x\right)+3\,\sin\left(d+e\,x\right)+5}}\,dx$$

input
```
int(1/(4*cos(d + e*x) + 3*sin(d + e*x) + 5)^(1/2),x)
```

output
```
int(1/(4*cos(d + e*x) + 3*sin(d + e*x) + 5)^(1/2), x)
```

3.421 $\int \frac{1}{(5+4\cos(d+ex)+3\sin(d+ex))^{3/2}}\,dx$

3.421.1 Optimal result

Integrand size = 22, antiderivative size = 96

$$\int \frac{1}{(5+4\cos(d+ex)+3\sin(d+ex))^{3/2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sin\left(d+ex-\arctan\left(\frac{3}{4}\right)\right)}{\sqrt{2}\sqrt{1+\cos\left(d+ex-\arctan\left(\frac{3}{4}\right)\right)}}\right)}{10\sqrt{10}e}-\frac{3\cos(d+ex)-4\sin(d+ex)}{10e(5+4\cos(d+ex)+3\sin(d+ex))^{3/2}}$$

output
```
1/10*(-3*cos(e*x+d)+4*sin(e*x+d))/e/(5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2)+1/
100*arctanh(1/2*sin(d+e*x-arctan(3/4))*2^(1/2)/(1+cos(d+e*x-arctan(3/4)))^
(1/2))*10^(1/2)/e
```

3.421.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.60

$$\int \frac{1}{(5+4\cos(d+ex)+3\sin(d+ex))^{3/2}}\,dx = -\frac{\left(\frac{1}{250}-\frac{i}{125}\right)\left(3\cos\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{1}{2}(d+ex)\right)\right)\left((5+10i)\left(\cos\left(\frac{1}{2}(d+ex)\right)-3\sin\left(\frac{1}{2}(d+ex)\right)\right)-(1-\right.}{e(5+4\cos(d+ex}$$

input
```
Integrate[(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(-3/2),x]
```

output
```
((-1/250 + I/125)*(3*Cos[(d + e*x)/2] + Sin[(d + e*x)/2])*((5 + 10*I)*(Cos
[(d + e*x)/2] - 3*Sin[(d + e*x)/2]) - (1 - I)*Sqrt[20 + 15*I]*ArcTan[(1/10
 + (3*I)/10)*Sqrt[4/5 + (3*I)/5]*(-1 + 3*Tan[(d + e*x)/4])]*(3*Cos[(d + e*
x)/2] + Sin[(d + e*x)/2])^2))/(e*(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/
2))
```

3.421.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {3042, 3595, 3042, 3594, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3\sin(d+ex)+4\cos(d+ex)+5)^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{(3\sin(d+ex)+4\cos(d+ex)+5)^{3/2}}dx$$

$$\Big\downarrow \text{ 3595}$$

$$\frac{1}{20}\int \frac{1}{\sqrt{4\cos(d+ex)+3\sin(d+ex)+5}}dx-\frac{3\cos(d+ex)-4\sin(d+ex)}{10e(3\sin(d+ex)+4\cos(d+ex)+5)^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{20}\int \frac{1}{\sqrt{4\cos(d+ex)+3\sin(d+ex)+5}}dx-\frac{3\cos(d+ex)-4\sin(d+ex)}{10e(3\sin(d+ex)+4\cos(d+ex)+5)^{3/2}}$$

$$\Big\downarrow \text{ 3594}$$

$$\frac{1}{20}\int \frac{1}{\sqrt{5\cos\left(d+ex-\arctan\left(\frac{3}{4}\right)\right)+5}}dx-\frac{3\cos(d+ex)-4\sin(d+ex)}{10e(3\sin(d+ex)+4\cos(d+ex)+5)^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{20}\int \frac{1}{\sqrt{5\sin\left(d+ex-\arctan\left(\frac{3}{4}\right)+\frac{\pi}{2}\right)+5}}dx-\frac{3\cos(d+ex)-4\sin(d+ex)}{10e(3\sin(d+ex)+4\cos(d+ex)+5)^{3/2}}$$

$$\Big\downarrow \text{ 3128}$$

$$-\frac{\int \frac{1}{10-\frac{5\sin^2\left(d+ex-\arctan\left(\frac{3}{4}\right)\right)}{\cos\left(d+ex-\arctan\left(\frac{3}{4}\right)\right)+1}}d\left(-\frac{\sqrt{5}\sin\left(d+ex-\arctan\left(\frac{3}{4}\right)\right)}{\sqrt{\cos\left(d+ex-\arctan\left(\frac{3}{4}\right)\right)+1}}\right)}{10e}-\frac{3\cos(d+ex)-4\sin(d+ex)}{10e(3\sin(d+ex)+4\cos(d+ex)+5)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{arctanh}\left(\frac{\sin\left(-\arctan\left(\frac{3}{4}\right)+d+ex\right)}{\sqrt{2}\sqrt{\cos\left(-\arctan\left(\frac{3}{4}\right)+d+ex\right)+1}}\right)}{10\sqrt{10}e}-\frac{3\cos(d+ex)-4\sin(d+ex)}{10e(3\sin(d+ex)+4\cos(d+ex)+5)^{3/2}}$$

input
```
Int[(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(-3/2),x]
```

output
```
ArcTanh[Sin[d + e*x - ArcTan[3/4]]/(Sqrt[2]*Sqrt[1 + Cos[d + e*x - ArcTan[
3/4]]])]/(10*Sqrt[10]*e) - (3*Cos[d + e*x] - 4*Sin[d + e*x])/(10*e*(5 + 4*
Cos[d + e*x] + 3*Sin[d + e*x])^(3/2))
```

3.421.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3594
```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

3.421. $\int \frac{1}{(5+4\cos(d+ex)+3\sin(d+ex))^{3/2}}\,dx$

rule 3595
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(c*Cos[d + e*x] - b*Sin[d + e*x])*((a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n/(a*e*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n + 1))
 Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b,
c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

3.421.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.22

method	result
default	$-\frac{\left(\sqrt{10}\,\operatorname{arctanh}\left(\frac{\sqrt{-5\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)+5}\,\sqrt{10}}{10}\right)\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)+\sqrt{10}\,\operatorname{arctanh}\left(\frac{\sqrt{-5\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)+5}\,\sqrt{10}}{10}\right)+2}{100\cos\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\sqrt{5+5\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)}\,e}$

input
```
int(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/100*(10^(1/2)*arctanh(1/10*(-5*sin(e*x+d+arctan(4/3))+5)^(1/2)*10^(1/2)
)*sin(e*x+d+arctan(4/3))+10^(1/2)*arctanh(1/10*(-5*sin(e*x+d+arctan(4/3))+
5)^(1/2)*10^(1/2))+2*(-5*sin(e*x+d+arctan(4/3))+5)^(1/2))*(-5*sin(e*x+d+ar
ctan(4/3))+5)^(1/2)/cos(e*x+d+arctan(4/3))/(5+5*sin(e*x+d+arctan(4/3)))^(1
/2)/e
```

3.421.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(81) = 162$.

Time = 0.26 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.79

$$\int \frac{1}{(5+4\cos(d+ex)+3\sin(d+ex))^{3/2}}\,dx = \frac{\left(9\sqrt{10}\cos\left(ex+d\right)^2+\left(13\sqrt{10}\cos\left(ex+d\right)+14\sqrt{10}\right)\sin\left(e\right.}{}$$

input
```
integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="fricas")
```

output
```
1/200*((9*sqrt(10)*cos(e*x + d)^2 + (13*sqrt(10)*cos(e*x + d) + 14*sqrt(10
))*sin(e*x + d) + 27*sqrt(10)*cos(e*x + d) + 18*sqrt(10))*log(-(9*cos(e*x
+ d)^2 + (13*cos(e*x + d) - 6)*sin(e*x + d) + 2*(sqrt(10)*cos(e*x + d) - 3
*sqrt(10)*sin(e*x + d) + sqrt(10))*sqrt(4*cos(e*x + d) + 3*sin(e*x + d) +
5) - 33*cos(e*x + d) - 42)/(9*cos(e*x + d)^2 + (13*cos(e*x + d) + 14)*sin(
e*x + d) + 27*cos(e*x + d) + 18)) - 20*sqrt(4*cos(e*x + d) + 3*sin(e*x + d
) + 5)*(cos(e*x + d) - 3*sin(e*x + d) + 1))/(9*e*cos(e*x + d)^2 + 27*e*cos
(e*x + d) + (13*e*cos(e*x + d) + 14*e)*sin(e*x + d) + 18*e)
```

3.421.6 Sympy [F]

$$\int \frac{1}{(5+4\cos(d+ex)+3\sin(d+ex))^{3/2}}\,dx=\int \frac{1}{(3\sin{(d+ex)}+4\cos{(d+ex)}+5)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))**(3/2),x)
```

output
```
Integral((3*sin(d + e*x) + 4*cos(d + e*x) + 5)**(-3/2), x)
```

3.421.7 Maxima [F]

$$\int \frac{1}{(5+4\cos(d+ex)+3\sin(d+ex))^{3/2}}\,dx=\int \frac{1}{(4\cos{(ex+d)}+3\sin{(ex+d)}+5)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="maxima")
```

output
```
integrate((4*cos(e*x + d) + 3*sin(e*x + d) + 5)^(-3/2), x)
```

3.421.8 Giac [F]

$$\int \frac{1}{(5+4\cos(d+ex)+3\sin(d+ex))^{3/2}}\,dx = \int \frac{1}{(4\,\cos\left(ex+d\right)+3\,\sin\left(ex+d\right)+5)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.421.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(5+4\cos(d+ex)+3\sin(d+ex))^{3/2}}\,dx = \int \frac{1}{(4\,\cos\left(d+e\,x\right)+3\,\sin\left(d+e\,x\right)+5)^{3/2}}\,dx$$

input
```
int(1/(4*cos(d + e*x) + 3*sin(d + e*x) + 5)^(3/2),x)
```

output
```
int(1/(4*cos(d + e*x) + 3*sin(d + e*x) + 5)^(3/2), x)
```

3.422 $\int \frac{1}{(5+4\cos(d+ex)+3\sin(d+ex))^{5/2}}\,dx$

3.422.1 Optimal result

Integrand size = 22, antiderivative size = 142

$$\int \frac{1}{(5+4\cos(d+ex)+3\sin(d+ex))^{5/2}}\,dx = \frac{3\text{arctanh}\left(\frac{\sin(d+ex-\arctan(\frac{3}{4}))}{\sqrt{2}\sqrt{1+\cos(d+ex-\arctan(\frac{3}{4}))}}\right)}{400\sqrt{10}e} - \frac{3\cos(d+ex)-4\sin(d+ex)}{20e(5+4\cos(d+ex)+3\sin(d+ex))^{5/2}} - \frac{3(3\cos(d+ex)-4\sin(d+ex))}{400e(5+4\cos(d+ex)+3\sin(d+ex))^{3/2}}$$

output
```
1/20*(-3*cos(e*x+d)+4*sin(e*x+d))/e/(5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2)-3/
400*(3*cos(e*x+d)-4*sin(e*x+d))/e/(5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2)+3/40
00*arctanh(1/2*sin(d+e*x-arctan(3/4))*2^(1/2)/(1+cos(d+e*x-arctan(3/4)))^(
1/2))*10^(1/2)/e
```

3.422.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.27

$$\int \frac{1}{(5+4\cos(d+ex)+3\sin(d+ex))^{5/2}}\,dx =$$

$$-\frac{\left(\frac{1}{20000}-\frac{i}{10000}\right)\left(3\cos\left(\frac{1}{2}(d+ex)\right)+\sin\left(\frac{1}{2}(d+ex)\right)\right)\left((-6+6i)\sqrt{20+15i}\arctan\left(\left(\frac{1}{10}+\frac{3i}{10}\right)\sqrt{\frac{4}{5}+\frac{3i}{5}}(-\right.\right.\right.}{}$$

input
```
Integrate[(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(-5/2),x]
```

output
```
((-1/20000 + I/10000)*(3*Cos[(d + e*x)/2] + Sin[(d + e*x)/2])*((-6 + 6*I)*
Sqrt[20 + 15*I]*ArcTan[(1/10 + (3*I)/10)*Sqrt[4/5 + (3*I)/5]*(-1 + 3*Tan[(
d + e*x)/4])]*(3*Cos[(d + e*x)/2] + Sin[(d + e*x)/2])^4 + (5 + 10*I)*(55*C
os[(d + e*x)/2] + 39*Cos[(3*(d + e*x))/2] - 165*Sin[(d + e*x)/2] - 27*Sin[
(3*(d + e*x))/2]))/(e*(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(5/2))
```

3.422.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {3042, 3595, 3042, 3595, 3042, 3594, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3\sin(d+ex)+4\cos(d+ex)+5)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(3\sin(d+ex)+4\cos(d+ex)+5)^{5/2}}dx$$

$$\Big\downarrow \text{3595}$$

$$\frac{3}{40}\int \frac{1}{(4\cos(d+ex)+3\sin(d+ex)+5)^{3/2}}dx-\frac{3\cos(d+ex)-4\sin(d+ex)}{20e(3\sin(d+ex)+4\cos(d+ex)+5)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{40}\int \frac{1}{(4\cos(d+ex)+3\sin(d+ex)+5)^{3/2}}dx-\frac{3\cos(d+ex)-4\sin(d+ex)}{20e(3\sin(d+ex)+4\cos(d+ex)+5)^{5/2}}$$

$$\Big\downarrow \text{3595}$$

$$\frac{3}{40}\left(\frac{1}{20}\int \frac{1}{\sqrt{4\cos(d+ex)+3\sin(d+ex)+5}}dx-\frac{3\cos(d+ex)-4\sin(d+ex)}{10e(3\sin(d+ex)+4\cos(d+ex)+5)^{3/2}}\right)-\frac{3\cos(d+ex)-4\sin(d+ex)}{20e(3\sin(d+ex)+4\cos(d+ex)+5)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{40}\left(\frac{1}{20}\int\frac{1}{\sqrt{4\cos(d+ex)+3\sin(d+ex)+5}}dx-\frac{3\cos(d+ex)-4\sin(d+ex)}{10e(3\sin(d+ex)+4\cos(d+ex)+5)^{3/2}}\right)-\frac{3\cos(d+ex)-4\sin(d+ex)}{20e(3\sin(d+ex)+4\cos(d+ex)+5)^{5/2}}$$

$$\Big\downarrow \text{3594}$$

$$\frac{3}{40}\left(\frac{1}{20}\int\frac{1}{\sqrt{5\cos\left(d+ex-\arctan\left(\frac{3}{4}\right)\right)+5}}dx-\frac{3\cos(d+ex)-4\sin(d+ex)}{10e(3\sin(d+ex)+4\cos(d+ex)+5)^{3/2}}\right)-\frac{3\cos(d+ex)-4\sin(d+ex)}{20e(3\sin(d+ex)+4\cos(d+ex)+5)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{40}\left(\frac{1}{20}\int\frac{1}{\sqrt{5\sin\left(d+ex-\arctan\left(\frac{3}{4}\right)+\frac{\pi}{2}\right)+5}}dx-\frac{3\cos(d+ex)-4\sin(d+ex)}{10e(3\sin(d+ex)+4\cos(d+ex)+5)^{3/2}}\right)-\frac{3\cos(d+ex)-4\sin(d+ex)}{20e(3\sin(d+ex)+4\cos(d+ex)+5)^{5/2}}$$

$$\Big\downarrow \text{3128}$$

$$\frac{3}{40}\left(-\frac{\int\frac{1}{10-\frac{5\sin^2\left(d+ex-\arctan\left(\frac{3}{4}\right)\right)}{\cos\left(d+ex-\arctan\left(\frac{3}{4}\right)\right)+1}}d\left(-\frac{\sqrt{5}\sin\left(d+ex-\arctan\left(\frac{3}{4}\right)\right)}{\sqrt{\cos\left(d+ex-\arctan\left(\frac{3}{4}\right)\right)+1}}\right)}{10e}-\frac{3\cos(d+ex)-4\sin(d+ex)}{10e(3\sin(d+ex)+4\cos(d+ex)+5)^{3/2}}\right)-\frac{3\cos(d+ex)-4\sin(d+ex)}{20e(3\sin(d+ex)+4\cos(d+ex)+5)^{5/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{3}{40}\left(\frac{\text{arctanh}\left(\frac{\sin\left(-\arctan\left(\frac{3}{4}\right)+d+ex\right)}{\sqrt{2}\sqrt{\cos\left(-\arctan\left(\frac{3}{4}\right)+d+ex\right)+1}}\right)}{10\sqrt{10}e}-\frac{3\cos(d+ex)-4\sin(d+ex)}{10e(3\sin(d+ex)+4\cos(d+ex)+5)^{3/2}}\right)-\frac{3\cos(d+ex)-4\sin(d+ex)}{20e(3\sin(d+ex)+4\cos(d+ex)+5)^{5/2}}$$

input
```
Int[(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(-5/2),x]
```

3.422. $\int\frac{1}{(5+4\cos(d+ex)+3\sin(d+ex))^{5/2}}\,dx$

output
```
-1/20*(3*Cos[d + e*x] - 4*Sin[d + e*x])/(e*(5 + 4*Cos[d + e*x] + 3*Sin[d +
 e*x])^(5/2)) + (3*(ArcTanh[Sin[d + e*x - ArcTan[3/4]]/(Sqrt[2]*Sqrt[1 + C
os[d + e*x - ArcTan[3/4]]])]/(10*Sqrt[10]*e) - (3*Cos[d + e*x] - 4*Sin[d +
 e*x])/(10*e*(5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2))))/40
```

3.422.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3594
```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

rule 3595
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(c*Cos[d + e*x] - b*Sin[d + e*x])*((a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n/(a*e*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n + 1))
 Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b,
c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

3.422.4 Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.34

method	result
default	$-\frac{\left(3\sqrt{10}\,\operatorname{arctanh}\left(\frac{\sqrt{-5\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)+5}\,\sqrt{10}}{10}\right)\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)^{2}+6\sqrt{10}\,\operatorname{arctanh}\left(\frac{\sqrt{-5\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)+5}\,\sqrt{10}}{10}\right.}{4000(}$

input
```
int(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4000*(3*10^(1/2)*arctanh(1/10*(-5*sin(e*x+d+arctan(4/3))+5)^(1/2)*10^(1
/2))*sin(e*x+d+arctan(4/3))^2+6*10^(1/2)*arctanh(1/10*(-5*sin(e*x+d+arctan
(4/3))+5)^(1/2)*10^(1/2))*sin(e*x+d+arctan(4/3))+3*10^(1/2)*arctanh(1/10*(
-5*sin(e*x+d+arctan(4/3))+5)^(1/2)*10^(1/2))+6*(-5*sin(e*x+d+arctan(4/3))+
5)^(1/2)*sin(e*x+d+arctan(4/3))+14*(-5*sin(e*x+d+arctan(4/3))+5)^(1/2))*(-
5*sin(e*x+d+arctan(4/3))+5)^(1/2)/(1+sin(e*x+d+arctan(4/3)))/cos(e*x+d+arc
tan(4/3))/(5+5*sin(e*x+d+arctan(4/3)))^(1/2)/e
```

3.422.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. 2(123) = 246.

Time = 0.26 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.40

$$\int \frac{1}{(5+4\cos(d+ex)+3\sin(d+ex))^{5/2}}\,dx = \frac{3\left(3\sqrt{10}\cos(ex+d)^{3}-111\sqrt{10}\cos(ex+d)^{2}-\left(79\sqrt{10}\cos\right.\right.}{}$$

input
```
integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x, algorithm="fricas")
```

output
```
1/8000*(3*(3*sqrt(10)*cos(e*x + d)^3 - 111*sqrt(10)*cos(e*x + d)^2 - (79*s
qrt(10)*cos(e*x + d)^2 + 202*sqrt(10)*cos(e*x + d) + 124*sqrt(10))*sin(e*x
 + d) - 246*sqrt(10)*cos(e*x + d) - 132*sqrt(10))*log(-(9*cos(e*x + d)^2 +
 (13*cos(e*x + d) - 6)*sin(e*x + d) + 2*(sqrt(10)*cos(e*x + d) - 3*sqrt(10
)*sin(e*x + d) + sqrt(10))*sqrt(4*cos(e*x + d) + 3*sin(e*x + d) + 5) - 33*
cos(e*x + d) - 42)/(9*cos(e*x + d)^2 + (13*cos(e*x + d) + 14)*sin(e*x + d)
 + 27*cos(e*x + d) + 18)) + 20*(39*cos(e*x + d)^2 - 3*(9*cos(e*x + d) + 32
)*sin(e*x + d) + 47*cos(e*x + d) + 8)*sqrt(4*cos(e*x + d) + 3*sin(e*x + d)
 + 5))/(3*e*cos(e*x + d)^3 - 111*e*cos(e*x + d)^2 - 246*e*cos(e*x + d) - (
79*e*cos(e*x + d)^2 + 202*e*cos(e*x + d) + 124*e)*sin(e*x + d) - 132*e)
```

3.422.6 Sympy [F]

$$\int \frac{1}{(5+4\cos(d+ex)+3\sin(d+ex))^{5/2}}\,dx = \int \frac{1}{(3\sin{(d+ex)}+4\cos{(d+ex)}+5)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))**(5/2),x)
```

output
```
Integral((3*sin(d + e*x) + 4*cos(d + e*x) + 5)**(-5/2), x)
```

3.422.7 Maxima [F]

$$\int \frac{1}{(5+4\cos(d+ex)+3\sin(d+ex))^{5/2}}\,dx = \int \frac{1}{(4\,\cos{(ex+d)}+3\,\sin{(ex+d)}+5)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x, algorithm="maxima")
```

output
```
integrate((4*cos(e*x + d) + 3*sin(e*x + d) + 5)^(-5/2), x)
```

3.422.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{(5+4\cos(d+ex)+3\sin(d+ex))^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(1/(5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.422.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(5+4\cos(d+ex)+3\sin(d+ex))^{5/2}}\,dx=\int \frac{1}{\left(4\,\cos\left(d+e\,x\right)+3\,\sin\left(d+e\,x\right)+5\right)^{5/2}}\,dx$$

input
```
int(1/(4*cos(d + e*x) + 3*sin(d + e*x) + 5)^(5/2),x)
```

output
```
int(1/(4*cos(d + e*x) + 3*sin(d + e*x) + 5)^(5/2), x)
```

3.423 $\int (-5+4\cos(d+ex)+3\sin(d+ex))^{7/2}\,dx$

3.423.1 Optimal result

Integrand size = 22, antiderivative size = 185

$$\int (-5+4\cos(d+ex)+3\sin(d+ex))^{7/2}\,dx = \frac{6400(3\cos(d+ex)-4\sin(d+ex))}{7e\sqrt{-5+4\cos(d+ex)+3\sin(d+ex)}} - \frac{320(3\cos(d+ex)-4\sin(d+ex))\sqrt{-5+4\cos(d+ex)+3\sin(d+ex)}}{7e} + \frac{24(3\cos(d+ex)-4\sin(d+ex))(-5+4\cos(d+ex)+3\sin(d+ex))^{3/2}}{7e} - \frac{2(3\cos(d+ex)-4\sin(d+ex))(-5+4\cos(d+ex)+3\sin(d+ex))^{5/2}}{7e}$$

output
```
24/7*(3*cos(e*x+d)-4*sin(e*x+d))*(-5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2)/e-2/
7*(3*cos(e*x+d)-4*sin(e*x+d))*(-5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2)/e+6400/
7*(3*cos(e*x+d)-4*sin(e*x+d))/e/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2)-320/7
*(3*cos(e*x+d)-4*sin(e*x+d))*(-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2)/e
```

3.423.2 Mathematica [A] (verified)

Time = 7.29 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.82

$$\int (-5+4\cos(d+ex)+3\sin(d+ex))^{7/2}\,dx = \frac{(-5+4\cos(d+ex)+3\sin(d+ex))^{7/2}\left(91875\cos\left(\frac{1}{2}(d+ex)\right)-11025\cos\left(\frac{3}{2}(d+ex)\right)-147\right.}{2}$$

input
```
Integrate[(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(7/2),x]
```

output
```
((-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(7/2)*(91875*Cos[(d + e*x)/2] - 11
025*Cos[(3*(d + e*x))/2] - 147*Cos[(5*(d + e*x))/2] + 249*Cos[(7*(d + e*x)
)/2] + 30625*Sin[(d + e*x)/2] - 15925*Sin[(3*(d + e*x))/2] + 3871*Sin[(5*(
d + e*x))/2] - 307*Sin[(7*(d + e*x))/2]))/(28*e*(Cos[(d + e*x)/2] - 3*Sin[
(d + e*x)/2])^7)
```

3.423.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 3592, 3042, 3592, 3042, 3592, 3042, 3591}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (3\sin(d+ex)+4\cos(d+ex)-5)^{7/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (3\sin(d+ex)+4\cos(d+ex)-5)^{7/2}dx$$

$$\Big\downarrow \; 3592$$

$$-\frac{60}{7}\int (4\cos(d+ex)+3\sin(d+ex)-5)^{5/2}dx-\frac{2(3\cos(d+ex)-4\sin(d+ex))(3\sin(d+ex)+4\cos(d+ex)-5)^{5/2}}{7e}$$

$$\Big\downarrow \; 3042$$

$$-\frac{60}{7}\int (4\cos(d+ex)+3\sin(d+ex)-5)^{5/2}dx-\frac{2(3\cos(d+ex)-4\sin(d+ex))(3\sin(d+ex)+4\cos(d+ex)-5)^{5/2}}{7e}$$

$$\Big\downarrow \; 3592$$

$$-\frac{60}{7}\left(-8\int (4\cos(d+ex)+3\sin(d+ex)-5)^{3/2}dx-\frac{2(3\cos(d+ex)-4\sin(d+ex))(3\sin(d+ex)+4\cos(d+\right.$$
$$5e$$
$$\frac{2(3\cos(d+ex)-4\sin(d+ex))(3\sin(d+ex)+4\cos(d+ex)-5)^{5/2}}{7e}$$

$\big\downarrow$ 3042

$$-\frac{60}{7}\left(-8\int (4\cos(d+ex)+3\sin(d+ex)-5)^{3/2}dx-\frac{2(3\cos(d+ex)-4\sin(d+ex))(3\sin(d+ex)+4\cos(d+\right.$$
$$\frac{2(3\cos(d+ex)-4\sin(d+ex))(3\sin(d+ex)+4\cos(d+ex)-5)^{5/2}}{7e}$$

$\big\downarrow$ 3592

$$-\frac{60}{7}\left(-8\left(-\frac{20}{3}\int \sqrt{4\cos(d+ex)+3\sin(d+ex)-5}dx-\frac{2\sqrt{3\sin(d+ex)+4\cos(d+ex)-5}(3\cos(d+ex)-\right.\right.$$
$$\frac{2(3\cos(d+ex)-4\sin(d+ex))(3\sin(d+ex)+4\cos(d+ex)-5)^{5/2}}{7e}$$

$\big\downarrow$ 3042

$$-\frac{60}{7}\left(-8\left(-\frac{20}{3}\int \sqrt{4\cos(d+ex)+3\sin(d+ex)-5}dx-\frac{2\sqrt{3\sin(d+ex)+4\cos(d+ex)-5}(3\cos(d+ex)-\right.\right.$$
$$\frac{2(3\cos(d+ex)-4\sin(d+ex))(3\sin(d+ex)+4\cos(d+ex)-5)^{5/2}}{7e}$$

$\big\downarrow$ 3591

$$-\frac{2(3\cos(d+ex)-4\sin(d+ex))(3\sin(d+ex)+4\cos(d+ex)-5)^{5/2}}{7e}-$$
$$\frac{60}{7}\left(-\frac{2(3\cos(d+ex)-4\sin(d+ex))(3\sin(d+ex)+4\cos(d+ex)-5)^{3/2}}{5e}-8\left(\frac{40(3\cos(d+ex)-4\sin(d+\right.\right.}{3e\sqrt{3\sin(d+ex)+4\cos(d+e}}$$

input
```
Int[(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(7/2),x]
```

output
```
(-2*(3*Cos[d + e*x] - 4*Sin[d + e*x])*(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x
])^(5/2))/(7*e) - (60*((-2*(3*Cos[d + e*x] - 4*Sin[d + e*x])*(-5 + 4*Cos[d
 + e*x] + 3*Sin[d + e*x])^(3/2))/(5*e) - 8*((40*(3*Cos[d + e*x] - 4*Sin[d
+ e*x]))/(3*e*Sqrt[-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x]]) - (2*(3*Cos[d +
e*x] - 4*Sin[d + e*x])*Sqrt[-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x]])/(3*e)))
)/7
```

3.423. $\int (-5+4\cos(d+ex)+3\sin(d+ex))^{7/2}\,dx$

3.423.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3591
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[-2*((c*Cos[d + e*x] - b*Sin[d + e*x])/(e*Sqrt[a + b*
Cos[d + e*x] + c*Sin[d + e*x]])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

rule 3592
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Simp[a*((2*n - 1)/n)   Int[(a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

3.423.4 Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.46

method	result
default	$\frac{250\left(\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)-1\right)\left(1+\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\right)\left(5\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)^{3}-27\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)^{2}+71\sin\left(ex+d+a\right.\right.}{7\cos\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\sqrt{-5+5\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)}\,e}$

input
```
int((-5+4*cos(e*x+d)+3*sin(e*x+d))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
250/7*(sin(e*x+d+arctan(4/3))-1)*(1+sin(e*x+d+arctan(4/3)))*(5*sin(e*x+d+a
rctan(4/3))^3-27*sin(e*x+d+arctan(4/3))^2+71*sin(e*x+d+arctan(4/3))-177)/c
os(e*x+d+arctan(4/3))/(-5+5*sin(e*x+d+arctan(4/3)))^(1/2)/e
```

3.423.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.65

$$\int (-5+4\cos(d+ex)+3\sin(d+ex))^{7/2}\,dx =$$
$$-\frac{2\left(249\cos\left(ex+d\right)^{4}+51\cos\left(ex+d\right)^{3}-3042\cos\left(ex+d\right)^{2}-\left(307\cos\left(ex+d\right)^{3}-1782\cos\left(ex+d\right)^{2}+\right.}{7\left(e\cos\left(ex+d\right)-\right.}$$

input
```
integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(7/2),x, algorithm="fricas")
```

output
```
-2/7*(249*cos(e*x + d)^4 + 51*cos(e*x + d)^3 - 3042*cos(e*x + d)^2 - (307*
cos(e*x + d)^3 - 1782*cos(e*x + d)^2 + 2860*cos(e*x + d) - 1392)*sin(e*x +
 d) + 10068*cos(e*x + d) + 12912)*sqrt(4*cos(e*x + d) + 3*sin(e*x + d) - 5
)/(e*cos(e*x + d) - 3*e*sin(e*x + d) + e)
```

3.423.6 Sympy [F(-1)]

Timed out.

$$\int (-5+4\cos(d+ex)+3\sin(d+ex))^{7/2}\,dx = \text{Timed out}$$

input
```
integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))**(7/2),x)
```

output
```
Timed out
```

3.423.7 Maxima [F]

$$\int (-5+4\cos(d+ex)+3\sin(d+ex))^{7/2}\,dx = \int \left(4\cos\left(ex+d\right)+3\sin\left(ex+d\right)-5\right)^{\frac{7}{2}}\,dx$$

input
```
integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(7/2),x, algorithm="maxima")
```

output
```
integrate((4*cos(e*x + d) + 3*sin(e*x + d) - 5)^(7/2), x)
```

3.423.8 Giac [F]

$$\int (-5+4\cos(d+ex)+3\sin(d+ex))^{7/2}\,dx=\int (4\cos(ex+d)+3\sin(ex+d)-5)^{\frac{7}{2}}\,dx$$

input
```
integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(7/2),x, algorithm="giac")
```

output
```
integrate((4*cos(e*x + d) + 3*sin(e*x + d) - 5)^(7/2), x)
```

3.423.9 Mupad [F(-1)]

Timed out.

$$\int (-5+4\cos(d+ex)+3\sin(d+ex))^{7/2}\,dx=\int (4\cos(d+e\,x)+3\sin(d+e\,x)-5)^{7/2}\,dx$$

input
```
int((4*cos(d + e*x) + 3*sin(d + e*x) - 5)^(7/2),x)
```

output
```
int((4*cos(d + e*x) + 3*sin(d + e*x) - 5)^(7/2), x)
```

3.424 $\int (-5+4\cos(d+ex)+3\sin(d+ex))^{5/2}\,dx$

3.424.1 Optimal result

Integrand size = 22, antiderivative size = 139

$$\int (-5+4\cos(d+ex)+3\sin(d+ex))^{5/2}\,dx = -\frac{320(3\cos(d+ex)-4\sin(d+ex))}{3e\sqrt{-5+4\cos(d+ex)+3\sin(d+ex)}} + \frac{16(3\cos(d+ex)-4\sin(d+ex))\sqrt{-5+4\cos(d+ex)+3\sin(d+ex)}}{3e} - \frac{2(3\cos(d+ex)-4\sin(d+ex))(-5+4\cos(d+ex)+3\sin(d+ex))^{3/2}}{5e}$$

output
```
-2/5*(3*cos(e*x+d)-4*sin(e*x+d))*(-5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2)/e-32
0/3*(3*cos(e*x+d)-4*sin(e*x+d))/e/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2)+16/
3*(3*cos(e*x+d)-4*sin(e*x+d))*(-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2)/e
```

3.424.2 Mathematica [A] (verified)

Time = 5.84 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.91

$$\int (-5+4\cos(d+ex)+3\sin(d+ex))^{5/2}\,dx = \frac{(-5+4\cos(d+ex)+3\sin(d+ex))^{5/2}\left(11250\cos\left(\frac{1}{2}(d+ex)\right)-1125\cos\left(\frac{3}{2}(d+ex)\right)-9\cos\right.}{30e\left(\cos\left(\frac{1}{2}(d+ex)\right)-3\sin\right.}$$

input
```
Integrate[(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(5/2),x]
```

output
```
((-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(5/2)*(11250*Cos[(d + e*x)/2] - 11
25*Cos[(3*(d + e*x))/2] - 9*Cos[(5*(d + e*x))/2] + 3750*Sin[(d + e*x)/2] -
 1625*Sin[(3*(d + e*x))/2] + 237*Sin[(5*(d + e*x))/2]))/(30*e*(Cos[(d + e*
x)/2] - 3*Sin[(d + e*x)/2])^5)
```

3.424.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 3592, 3042, 3592, 3042, 3591}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (3\sin(d+ex)+4\cos(d+ex)-5)^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (3\sin(d+ex)+4\cos(d+ex)-5)^{5/2}dx$$

$$\Big\downarrow \text{3592}$$

$$-8\int (4\cos(d+ex)+3\sin(d+ex)-5)^{3/2}dx-\frac{2(3\cos(d+ex)-4\sin(d+ex))(3\sin(d+ex)+4\cos(d+ex)-5)^{3/2}}{5e}$$

$$\Big\downarrow \text{3042}$$

$$-8\int (4\cos(d+ex)+3\sin(d+ex)-5)^{3/2}dx-\frac{2(3\cos(d+ex)-4\sin(d+ex))(3\sin(d+ex)+4\cos(d+ex)-5)^{3/2}}{5e}$$

$$\Big\downarrow \text{3592}$$

$$-8\left(-\frac{20}{3}\int \sqrt{4\cos(d+ex)+3\sin(d+ex)-5}dx-\frac{2\sqrt{3\sin(d+ex)+4\cos(d+ex)-5}(3\cos(d+ex)-4\sin(d\right.}{3e}$$
$$\frac{2(3\cos(d+ex)-4\sin(d+ex))(3\sin(d+ex)+4\cos(d+ex)-5)^{3/2}}{5e}$$

$$\Big\downarrow \text{3042}$$

$$-8\left(-\frac{20}{3}\int\sqrt{4\cos(d+ex)+3\sin(d+ex)-5}dx-\frac{2\sqrt{3\sin(d+ex)+4\cos(d+ex)-5}(3\cos(d+ex)-4\sin(d\right.$$
$$\frac{2(3\cos(d+ex)-4\sin(d+ex))(3\sin(d+ex)+4\cos(d+ex)-5)^{3/2}}{5e}$$
$$\Big\downarrow \text{3591}$$
$$-\frac{2(3\cos(d+ex)-4\sin(d+ex))(3\sin(d+ex)+4\cos(d+ex)-5)^{3/2}}{5e}-$$
$$8\left(\frac{40(3\cos(d+ex)-4\sin(d+ex))}{3e\sqrt{3\sin(d+ex)+4\cos(d+ex)-5}}-\frac{2(3\cos(d+ex)-4\sin(d+ex))\sqrt{3\sin(d+ex)+4\cos(d+ex)-5}}{3e}\right)$$

input
```
Int[(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(5/2),x]
```

output
```
(-2*(3*Cos[d + e*x] - 4*Sin[d + e*x])*(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x
])^(3/2))/(5*e) - 8*((40*(3*Cos[d + e*x] - 4*Sin[d + e*x]))/(3*e*Sqrt[-5 +
 4*Cos[d + e*x] + 3*Sin[d + e*x]]) - (2*(3*Cos[d + e*x] - 4*Sin[d + e*x])*
Sqrt[-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x]])/(3*e))
```

3.424.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3591
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[-2*((c*Cos[d + e*x] - b*Sin[d + e*x])/(e*Sqrt[a + b*
Cos[d + e*x] + c*Sin[d + e*x]])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

rule 3592
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Simp[a*((2*n - 1)/n)   Int[(a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

3.424.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.53

method	result	size
default	$\frac{50\left(\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)-1\right)\left(1+\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\right)\left(3\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)^{2}-14\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)+43\right)}{3\cos\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\sqrt{-5+5\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)}\, e}$	74

input
```
int((-5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
50/3*(sin(e*x+d+arctan(4/3))-1)*(1+sin(e*x+d+arctan(4/3)))*(3*sin(e*x+d+ar
ctan(4/3))^2-14*sin(e*x+d+arctan(4/3))+43)/cos(e*x+d+arctan(4/3))/(-5+5*si
n(e*x+d+arctan(4/3)))^(1/2)/e
```

3.424.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.73

$$\int(-5+4\cos(d+ex)+3\sin(d+ex))^{5/2}\,dx =$$
$$-\frac{2\left(9\cos\left(ex+d\right)^{3}+567\cos\left(ex+d\right)^{2}-\left(237\cos\left(ex+d\right)^{2}-694\cos\left(ex+d\right)+472\right)\sin\left(ex+d\right)-2538}{15\left(e\cos\left(ex+d\right)-3e\sin\left(ex+d\right)+e\right)}$$

input
```
integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x, algorithm="fricas")
```

output
```
-2/15*(9*cos(e*x + d)^3 + 567*cos(e*x + d)^2 - (237*cos(e*x + d)^2 - 694*c
os(e*x + d) + 472)*sin(e*x + d) - 2538*cos(e*x + d) - 3096)*sqrt(4*cos(e*x
 + d) + 3*sin(e*x + d) - 5)/(e*cos(e*x + d) - 3*e*sin(e*x + d) + e)
```

3.424.6 Sympy [F(-1)]

Timed out.

$$\int(-5+4\cos(d+ex)+3\sin(d+ex))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))**(5/2),x)
```

output
```
Timed out
```

3.424.7 Maxima [F]

$$\int (-5+4\cos(d+ex)+3\sin(d+ex))^{5/2}\,dx = \int (4\cos(ex+d)+3\sin(ex+d)-5)^{\frac{5}{2}}\,dx$$

input
```
integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x, algorithm="maxima")
```

output
```
integrate((4*cos(e*x + d) + 3*sin(e*x + d) - 5)^(5/2), x)
```

3.424.8 Giac [F]

$$\int (-5+4\cos(d+ex)+3\sin(d+ex))^{5/2}\,dx = \int (4\cos(ex+d)+3\sin(ex+d)-5)^{\frac{5}{2}}\,dx$$

input
```
integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x, algorithm="giac")
```

output
```
integrate((4*cos(e*x + d) + 3*sin(e*x + d) - 5)^(5/2), x)
```

3.424.9 Mupad [F(-1)]

Timed out.

$$\int (-5+4\cos(d+ex)+3\sin(d+ex))^{5/2}\,dx = \int (4\cos(d+e\,x)+3\sin(d+e\,x)-5)^{5/2}\,dx$$

input
```
int((4*cos(d + e*x) + 3*sin(d + e*x) - 5)^(5/2),x)
```

output
```
int((4*cos(d + e*x) + 3*sin(d + e*x) - 5)^(5/2), x)
```

3.425 $\int (-5+4\cos(d+ex)+3\sin(d+ex))^{3/2}\,dx$

3.425.1 Optimal result

Integrand size = 22, antiderivative size = 93

$$\int (-5+4\cos(d+ex)+3\sin(d+ex))^{3/2}\,dx=\frac{40(3\cos(d+ex)-4\sin(d+ex))}{3e\sqrt{-5+4\cos(d+ex)+3\sin(d+ex)}}-\frac{2(3\cos(d+ex)-4\sin(d+ex))\sqrt{-5+4\cos(d+ex)+3\sin(d+ex)}}{3e}$$

output
```
40/3*(3*cos(e*x+d)-4*sin(e*x+d))/e/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2)-2/
3*(3*cos(e*x+d)-4*sin(e*x+d))*(-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2)/e
```

3.425.2 Mathematica [A] (verified)

Time = 4.05 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.11

$$\int (-5+4\cos(d+ex)+3\sin(d+ex))^{3/2}\,dx=\frac{(-5+4\cos(d+ex)+3\sin(d+ex))^{3/2}\left(135\cos\left(\frac{1}{2}(d+ex)\right)-9\cos\left(\frac{3}{2}(d+ex)\right)+45\sin\left(\frac{1}{2}(d\right.\right.}{3e\left(\cos\left(\frac{1}{2}(d+ex)\right)-3\sin\left(\frac{1}{2}(d+ex)\right)\right)^3}$$

input
```
Integrate[(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2),x]
```

output
```
((-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2)*(135*Cos[(d + e*x)/2] - 9*Co
s[(3*(d + e*x))/2] + 45*Sin[(d + e*x)/2] - 13*Sin[(3*(d + e*x))/2]))/(3*e*
(Cos[(d + e*x)/2] - 3*Sin[(d + e*x)/2])^3)
```

3.425.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3042, 3592, 3042, 3591}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (3\sin(d+ex)+4\cos(d+ex)-5)^{3/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (3\sin(d+ex)+4\cos(d+ex)-5)^{3/2}dx$$

$$\Big\downarrow \; 3592$$

$$-\frac{20}{3}\int \sqrt{4\cos(d+ex)+3\sin(d+ex)-5}dx-\frac{2\sqrt{3\sin(d+ex)+4\cos(d+ex)-5}(3\cos(d+ex)-4\sin(d+ex))}{3e}$$

$$\Big\downarrow \; 3042$$

$$-\frac{20}{3}\int \sqrt{4\cos(d+ex)+3\sin(d+ex)-5}dx-\frac{2\sqrt{3\sin(d+ex)+4\cos(d+ex)-5}(3\cos(d+ex)-4\sin(d+ex))}{3e}$$

$$\Big\downarrow \; 3591$$

$$\frac{40(3\cos(d+ex)-4\sin(d+ex))}{3e\sqrt{3\sin(d+ex)+4\cos(d+ex)-5}}-\frac{2(3\cos(d+ex)-4\sin(d+ex))\sqrt{3\sin(d+ex)+4\cos(d+ex)-5}}{3e}$$

input
```
Int[(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2),x]
```

output
```
(40*(3*Cos[d + e*x] - 4*Sin[d + e*x]))/(3*e*Sqrt[-5 + 4*Cos[d + e*x] + 3*S
in[d + e*x]]) - (2*(3*Cos[d + e*x] - 4*Sin[d + e*x])*Sqrt[-5 + 4*Cos[d + e
*x] + 3*Sin[d + e*x]])/(3*e)
```

3.425.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3591
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[-2*((c*Cos[d + e*x] - b*Sin[d + e*x])/(e*Sqrt[a + b*
Cos[d + e*x] + c*Sin[d + e*x]])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

rule 3592
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Simp[a*((2*n - 1)/n)   Int[(a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

3.425.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.65

method	result	size
default	$\frac{50\left(\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)-1\right)\left(1+\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\right)\left(\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)-5\right)}{3\cos\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\sqrt{-5+5\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)}\, e}$	60

input
```
int((-5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
50/3*(sin(e*x+d+arctan(4/3))-1)*(1+sin(e*x+d+arctan(4/3)))*(sin(e*x+d+arct
an(4/3))-5)/cos(e*x+d+arctan(4/3))/(-5+5*sin(e*x+d+arctan(4/3)))^(1/2)/e
```

3.425.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.86

$$\int (-5+4\cos(d+ex)+3\sin(d+ex))^{3/2}\,dx = \frac{2\left(9\cos(ex+d)^2+(13\cos(ex+d)-16)\sin(ex+d)-63\cos(ex+d)-72\right)\sqrt{4\cos(ex+ \dots}}{3\,(e\cos(ex+d)-3\,e\sin(ex+d)+e)}$$

input
```
integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="fricas")
```

output
```
2/3*(9*cos(e*x + d)^2 + (13*cos(e*x + d) - 16)*sin(e*x + d) - 63*cos(e*x +
 d) - 72)*sqrt(4*cos(e*x + d) + 3*sin(e*x + d) - 5)/(e*cos(e*x + d) - 3*e*
sin(e*x + d) + e)
```

3.425.6 Sympy [F]

$$\int (-5+4\cos(d+ex)+3\sin(d+ex))^{3/2}\,dx = \int (3\sin(d+ex)+4\cos(d+ex)-5)^{\frac{3}{2}}\,dx$$

input
```
integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))**(3/2),x)
```

output
```
Integral((3*sin(d + e*x) + 4*cos(d + e*x) - 5)**(3/2), x)
```

3.425.7 Maxima [F]

$$\int (-5+4\cos(d+ex)+3\sin(d+ex))^{3/2}\,dx = \int (4\cos(ex+d)+3\sin(ex+d)-5)^{\frac{3}{2}}\,dx$$

input
```
integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="maxima")
```

output
```
integrate((4*cos(e*x + d) + 3*sin(e*x + d) - 5)^(3/2), x)
```

3.425.8 Giac [F]

$$\int (-5+4\cos(d+ex)+3\sin(d+ex))^{3/2}\,dx = \int (4\,\cos(ex+d)+3\,\sin(ex+d)-5)^{\frac{3}{2}}\,dx$$

input
```
integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="giac")
```

output
```
integrate((4*cos(e*x + d) + 3*sin(e*x + d) - 5)^(3/2), x)
```

3.425.9 Mupad [F(-1)]

Timed out.

$$\int (-5+4\cos(d+ex)+3\sin(d+ex))^{3/2}\,dx = \int (4\,\cos(d+e\,x)+3\,\sin(d+e\,x)-5)^{3/2}\,dx$$

input
```
int((4*cos(d + e*x) + 3*sin(d + e*x) - 5)^(3/2),x)
```

output
```
int((4*cos(d + e*x) + 3*sin(d + e*x) - 5)^(3/2), x)
```

3.426 $\int \sqrt{-5+4 \cos (d+e x)+3 \sin (d+e x)} \, dx$

3.426.1 Optimal result

Integrand size = 22, antiderivative size = 44

$$\int \sqrt{-5+4 \cos (d+e x)+3 \sin (d+e x)} \, dx=-\frac{2(3 \cos (d+e x)-4 \sin (d+e x))}{e \sqrt{-5+4 \cos (d+e x)+3 \sin (d+e x)}}$$

output
```
-2*(3*cos(e*x+d)-4*sin(e*x+d))/e/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2)
```

3.426.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.70

$$\int \sqrt{-5+4 \cos (d+e x)+3 \sin (d+e x)} \, dx =\frac{2\left(3 \cos \left(\frac{1}{2}(d+e x)\right)+\sin \left(\frac{1}{2}(d+e x)\right)\right) \sqrt{-5+4 \cos (d+e x)+3 \sin (d+e x)}}{e \left(\cos \left(\frac{1}{2}(d+e x)\right)-3 \sin \left(\frac{1}{2}(d+e x)\right)\right)}$$

input
```
Integrate[Sqrt[-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x]],x]
```

output
```
(2*(3*Cos[(d + e*x)/2] + Sin[(d + e*x)/2])*Sqrt[-5 + 4*Cos[d + e*x] + 3*Si
n[d + e*x]])/(e*(Cos[(d + e*x)/2] - 3*Sin[(d + e*x)/2]))
```

3.426.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3042, 3591}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{3\sin(d+ex)+4\cos(d+ex)-5}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{3\sin(d+ex)+4\cos(d+ex)-5}dx$$

$$\Big\downarrow \text{3591}$$

$$-\frac{2(3\cos(d+ex)-4\sin(d+ex))}{e\sqrt{3\sin(d+ex)+4\cos(d+ex)-5}}$$

input
```
Int[Sqrt[-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x]],x]
```

output
```
(-2*(3*Cos[d + e*x] - 4*Sin[d + e*x]))/(e*Sqrt[-5 + 4*Cos[d + e*x] + 3*Sin
[d + e*x]])
```

3.426.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3591
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[-2*((c*Cos[d + e*x] - b*Sin[d + e*x])/(e*Sqrt[a + b*
Cos[d + e*x] + c*Sin[d + e*x]])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

3.426.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.14

method	result
default	$\frac{10\left(\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)-1\right)\left(1+\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\right)}{\cos\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\sqrt{-5+5\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)}\, e}$
risch	$\frac{5i\sqrt{2}\,\sqrt{-10+8\cos(ex+d)+6\sin(ex+d)}\,\sqrt{(4-3i)\left(25\,e^{3i(ex+d)}-30ie^{2i(ex+d)}+7\,e^{i(ex+d)}+24ie^{i(ex+d)}-40\,e^{2i(ex+d)}\right)}\,\left(5\,e^{i(ex+d)}+4+\right.}{\left(30ie^{i(ex+d)}-25\,e^{2i(ex+d)}-7-24i+40\,e^{i(ex+d)}\right)e\sqrt{(100-75i)\left(25\,e^{3i(ex+d)}-30ie^{2i(ex+d)}+7\,e^{i(ex+d)}+24ie^{i(ex+d)}-40\right.}}$

input
```
int((-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
10*(sin(e*x+d+arctan(4/3))-1)*(1+sin(e*x+d+arctan(4/3)))/cos(e*x+d+arctan(
4/3))/(-5+5*sin(e*x+d+arctan(4/3)))^(1/2)/e
```

3.426.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.34

$$\int \sqrt{-5+4\cos(d+ex)+3\sin(d+ex)}\,dx = \frac{2\sqrt{4\cos(ex+d)+3\sin(ex+d)-5}(3\cos(ex+d)+\sin(ex+d)+3)}{e\cos(ex+d)-3\,e\sin(ex+d)+e}$$

input
```
integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(4*cos(e*x + d) + 3*sin(e*x + d) - 5)*(3*cos(e*x + d) + sin(e*x + d)
 + 3)/(e*cos(e*x + d) - 3*e*sin(e*x + d) + e)
```

3.426.6 Sympy [F]

$$\int \sqrt{-5+4\cos(d+ex)+3\sin(d+ex)}\,dx = \int \sqrt{3\sin(d+ex)+4\cos(d+ex)-5}\,dx$$

input
```
integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))**(1/2),x)
```

output
```
Integral(sqrt(3*sin(d + e*x) + 4*cos(d + e*x) - 5), x)
```

3.426.7 Maxima [F]

$$\int \sqrt{-5+4\cos(d+ex)+3\sin(d+ex)}\,dx=\int \sqrt{4\cos(ex+d)+3\sin(ex+d)-5}\,dx$$

input
```
integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(4*cos(e*x + d) + 3*sin(e*x + d) - 5), x)
```

3.426.8 Giac [F]

$$\int \sqrt{-5+4\cos(d+ex)+3\sin(d+ex)}\,dx=\int \sqrt{4\cos(ex+d)+3\sin(ex+d)-5}\,dx$$

input
```
integrate((-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(4*cos(e*x + d) + 3*sin(e*x + d) - 5), x)
```

3.426.9 Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int \sqrt{-5+4\cos(d+ex)+3\sin(d+ex)}\,dx=-\frac{2\sqrt{5}\,(3\cos(d+e\,x)-4\sin(d+e\,x))}{5\,e\,\sqrt{\cos\left(d-\operatorname{atan}\left(\frac{3}{4}\right)+e\,x\right)-1}}$$

input
```
int((4*cos(d + e*x) + 3*sin(d + e*x) - 5)^(1/2),x)
```

output
```
-(2*5^(1/2)*(3*cos(d + e*x) - 4*sin(d + e*x)))/(5*e*(cos(d - atan(3/4) + e
*x) - 1)^(1/2))
```

3.427 $\int \frac{1}{\sqrt{-5+4\cos(d+ex)+3\sin(d+ex)}}\,dx$

3.427.1 Optimal result

Integrand size = 22, antiderivative size = 49

$$\int \frac{1}{\sqrt{-5+4\cos(d+ex)+3\sin(d+ex)}}\,dx = -\frac{\sqrt{\frac{2}{5}}\arctan\left(\frac{\sin(d+ex-\arctan(\frac{3}{4}))}{\sqrt{2}\sqrt{-1+\cos(d+ex-\arctan(\frac{3}{4}))}}\right)}{e}$$

output
```
-1/5*arctan(1/2*sin(d+e*x-arctan(3/4))*2^(1/2)/(-1+cos(d+e*x-arctan(3/4)))
^(1/2))*10^(1/2)/e
```

3.427.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.02

$$\int \frac{1}{\sqrt{-5+4\cos(d+ex)+3\sin(d+ex)}}\,dx = \frac{\left(\frac{2}{5}+\frac{6i}{5}\right)\sqrt{-\frac{4}{5}-\frac{3i}{5}}\text{arctanh}\left(\left(\frac{1}{10}+\frac{3i}{10}\right)\sqrt{-\frac{4}{5}-\frac{3i}{5}}\left(3+\tan\left(\frac{1}{4}(d+ex)\right)\right)\right)\left(\cos\left(\frac{1}{2}(d+ex)\right)-3\sin\left(\frac{1}{2}(d+\right.\right.}{e\sqrt{-5+4\cos(d+ex)+3\sin(d+ex)}}$$

input
```
Integrate[1/Sqrt[-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x]],x]
```

output
```
((2/5 + (6*I)/5)*Sqrt[-4/5 - (3*I)/5]*ArcTanh[(1/10 + (3*I)/10)*Sqrt[-4/5
- (3*I)/5]*(3 + Tan[(d + e*x)/4])]*(Cos[(d + e*x)/2] - 3*Sin[(d + e*x)/2])
)/(e*Sqrt[-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x]])
```

3.427.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {3042, 3594, 3042, 3128, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3\sin(d+ex)+4\cos(d+ex)-5}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\sqrt{3\sin(d+ex)+4\cos(d+ex)-5}}dx$$

$$\Big\downarrow \ 3594$$

$$\int \frac{1}{\sqrt{5\cos\left(-\arctan\left(\frac{3}{4}\right)+d+ex\right)-5}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\sqrt{5\sin\left(-\arctan\left(\frac{3}{4}\right)+d+ex+\frac{\pi}{2}\right)-5}}dx$$

$$\Big\downarrow \ 3128$$

$$-\frac{2\int \frac{1}{-\frac{5\sin^2\left(d+ex-\arctan\left(\frac{3}{4}\right)\right)}{\cos\left(d+ex-\arctan\left(\frac{3}{4}\right)\right)-1}-10}d\left(-\frac{\sqrt{5}\sin\left(d+ex-\arctan\left(\frac{3}{4}\right)\right)}{\sqrt{\cos\left(d+ex-\arctan\left(\frac{3}{4}\right)\right)-1}}\right)}{e}$$

$$\Big\downarrow \ 217$$

$$-\frac{\sqrt{\frac{2}{5}}\arctan\left(\frac{\sin\left(-\arctan\left(\frac{3}{4}\right)+d+ex\right)}{\sqrt{2}\sqrt{\cos\left(-\arctan\left(\frac{3}{4}\right)+d+ex\right)-1}}\right)}{e}$$

input
```
Int[1/Sqrt[-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x]],x]
```

output
```
-((Sqrt[2/5]*ArcTan[Sin[d + e*x - ArcTan[3/4]]/(Sqrt[2]*Sqrt[-1 + Cos[d +
e*x - ArcTan[3/4]]])])/e)
```

3.427.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3594
```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

3.427.4 Maple [A] (verified)

Time = 2.32 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.57

method	result
default	$\frac{\left(\sin \left(e x+d+\arctan \left(\frac{4}{3}\right)\right)-1\right) \sqrt{-5 \sin \left(e x+d+\arctan \left(\frac{4}{3}\right)\right)-5}\, \sqrt{10}\, \arctan \left(\frac{\sqrt{-5 \sin \left(e x+d+\arctan \left(\frac{4}{3}\right)\right)-5}\, \sqrt{10}}{10}\right)}{5 \cos \left(e x+d+\arctan \left(\frac{4}{3}\right)\right) \sqrt{-5+5 \sin \left(e x+d+\arctan \left(\frac{4}{3}\right)\right)}\, e}$
risch	$\frac{2 i \left(5 \,{\mathrm e}^{i \left(e x+d\right)}-4-3 i\right) \sqrt{2}\, \sqrt{\left(4-3 i\right) \left(25 \,{\mathrm e}^{2 i \left(e x+d\right)}-30 i {\mathrm e}^{i \left(e x+d\right)}+7+24 i-40 \,{\mathrm e}^{i \left(e x+d\right)}\right) {\mathrm e}^{i \left(e x+d\right)}}\, {\mathrm e}^{-i \left(e x+d\right)}}{e \sqrt{\left(100-75 i\right) \left(25 \,{\mathrm e}^{2 i \left(e x+d\right)}-30 i {\mathrm e}^{i \left(e x+d\right)}+7+24 i-40 \,{\mathrm e}^{i \left(e x+d\right)}\right) {\mathrm e}^{i \left(e x+d\right)}}\, \sqrt{-\left(3 i {\mathrm e}^{2 i \left(e x+d\right)}-4 \,{\mathrm e}^{2 i \left(e x+d\right)}-4-3 i+10 \,{\mathrm e}^{i \left(e x+d\right)}\right) {\mathrm e}^{-i \left(e x+d\right)}}}$

input
```
int(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*(sin(e*x+d+arctan(4/3))-1)*(-5*sin(e*x+d+arctan(4/3))-5)^(1/2)*10^(1/2
)*arctan(1/10*(-5*sin(e*x+d+arctan(4/3))-5)^(1/2)*10^(1/2))/cos(e*x+d+arct
an(4/3))/(-5+5*sin(e*x+d+arctan(4/3)))^(1/2)/e
```

3.427.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(38) = 76$.

Time = 0.25 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.80

$$\int \frac{1}{\sqrt{-5+4\cos(d+ex)+3\sin(d+ex)}}\,dx = \frac{\sqrt{5}\sqrt{2}\arctan\left(-\frac{\left(3\sqrt{5}\sqrt{2}\cos(ex+d)+\sqrt{5}\sqrt{2}\sin(ex+d)+3\sqrt{5}\sqrt{2}\right)\sqrt{4\cos(ex+d)+3\sin(ex+d)-5}}{10\left(\cos(ex+d)-3\sin(ex+d)+1\right)}\right)}{5e}$$

input
```
integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x, algorithm="fricas")
```

output
```
1/5*sqrt(5)*sqrt(2)*arctan(-1/10*(3*sqrt(5)*sqrt(2)*cos(e*x + d) + sqrt(5)
*sqrt(2)*sin(e*x + d) + 3*sqrt(5)*sqrt(2))*sqrt(4*cos(e*x + d) + 3*sin(e*x
 + d) - 5)/(cos(e*x + d) - 3*sin(e*x + d) + 1))/e
```

3.427.6 Sympy [F]

$$\int \frac{1}{\sqrt{-5+4\cos(d+ex)+3\sin(d+ex)}}\,dx = \int \frac{1}{\sqrt{3\sin(d+ex)+4\cos(d+ex)-5}}\,dx$$

input
```
integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))**(1/2),x)
```

output
```
Integral(1/sqrt(3*sin(d + e*x) + 4*cos(d + e*x) - 5), x)
```

3.427.7 Maxima [F]

$$\int \frac{1}{\sqrt{-5+4\cos(d+ex)+3\sin(d+ex)}}\,dx = \int \frac{1}{\sqrt{4\cos(ex+d)+3\sin(ex+d)-5}}\,dx$$

input
```
integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(4*cos(e*x + d) + 3*sin(e*x + d) - 5), x)
```

3.427.8 Giac [F]

$$\int \frac{1}{\sqrt{-5+4\cos(d+ex)+3\sin(d+ex)}}\,dx = \int \frac{1}{\sqrt{4\cos(ex+d)+3\sin(ex+d)-5}}\,dx$$

input
```
integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(4*cos(e*x + d) + 3*sin(e*x + d) - 5), x)
```

3.427.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-5+4\cos(d+ex)+3\sin(d+ex)}}\,dx = \int \frac{1}{\sqrt{4\cos(d+e\,x)+3\sin(d+e\,x)-5}}\,dx$$

input
```
int(1/(4*cos(d + e*x) + 3*sin(d + e*x) - 5)^(1/2),x)
```

output
```
int(1/(4*cos(d + e*x) + 3*sin(d + e*x) - 5)^(1/2), x)
```

3.428 $\int \frac{1}{(-5+4\cos(d+ex)+3\sin(d+ex))^{3/2}}\,dx$

3.428.1 Optimal result

Integrand size = 22, antiderivative size = 96

$$\int \frac{1}{(-5+4\cos(d+ex)+3\sin(d+ex))^{3/2}}\,dx = \frac{\arctan\left(\frac{\sin\left(d+ex-\arctan\left(\frac{3}{4}\right)\right)}{\sqrt{2}\sqrt{-1+\cos\left(d+ex-\arctan\left(\frac{3}{4}\right)\right)}}\right)}{10\sqrt{10}e} + \frac{3\cos(d+ex)-4\sin(d+ex)}{10e(-5+4\cos(d+ex)+3\sin(d+ex))^{3/2}}$$

output
```
1/10*(3*cos(e*x+d)-4*sin(e*x+d))/e/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2)+1/
100*arctan(1/2*sin(d+e*x-arctan(3/4))*2^(1/2)/(-1+cos(d+e*x-arctan(3/4)))^
(1/2))*10^(1/2)/e
```

3.428.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.58

$$\int \frac{1}{(-5+4\cos(d+ex)+3\sin(d+ex))^{3/2}}\,dx = \frac{\left(\frac{1}{250}-\frac{i}{125}\right)\left(\cos\left(\frac{1}{2}(d+ex)\right)-3\sin\left(\frac{1}{2}(d+ex)\right)\right)\left((-1+i\right.}{}$$

input
```
Integrate[(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(-3/2),x]
```

output
```
((1/250 - I/125)*(Cos[(d + e*x)/2] - 3*Sin[(d + e*x)/2])*((-1 + I)*Sqrt[-2
0 - 15*I]*ArcTanh[(1/10 + (3*I)/10)*Sqrt[-4/5 - (3*I)/5]*(3 + Tan[(d + e*x
)/4])]*(Cos[(d + e*x)/2] - 3*Sin[(d + e*x)/2])^2 + (5 + 10*I)*(3*Cos[(d +
e*x)/2] + Sin[(d + e*x)/2])))/(e*(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(3
/2))
```

3.428.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {3042, 3595, 3042, 3594, 3042, 3128, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3\sin(d+ex)+4\cos(d+ex)-5)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(3\sin(d+ex)+4\cos(d+ex)-5)^{3/2}}dx$$

$$\Big\downarrow \text{3595}$$

$$\frac{3\cos(d+ex)-4\sin(d+ex)}{10e(3\sin(d+ex)+4\cos(d+ex)-5)^{3/2}}-\frac{1}{20}\int \frac{1}{\sqrt{4\cos(d+ex)+3\sin(d+ex)-5}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\cos(d+ex)-4\sin(d+ex)}{10e(3\sin(d+ex)+4\cos(d+ex)-5)^{3/2}}-\frac{1}{20}\int \frac{1}{\sqrt{4\cos(d+ex)+3\sin(d+ex)-5}}dx$$

$$\Big\downarrow \text{3594}$$

$$\frac{3\cos(d+ex)-4\sin(d+ex)}{10e(3\sin(d+ex)+4\cos(d+ex)-5)^{3/2}}-\frac{1}{20}\int \frac{1}{\sqrt{5\cos\left(d+ex-\arctan\left(\frac{3}{4}\right)\right)-5}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\cos(d+ex)-4\sin(d+ex)}{10e(3\sin(d+ex)+4\cos(d+ex)-5)^{3/2}}-\frac{1}{20}\int \frac{1}{\sqrt{5\sin\left(d+ex-\arctan\left(\frac{3}{4}\right)+\frac{\pi}{2}\right)-5}}dx$$

$$\Big\downarrow \text{3128}$$

$$\frac{\int \frac{1}{-\frac{5\sin^2\left(d+ex-\arctan\left(\frac{3}{4}\right)\right)}{\cos\left(d+ex-\arctan\left(\frac{3}{4}\right)\right)-1}-10}d\left(-\frac{\sqrt{5}\sin\left(d+ex-\arctan\left(\frac{3}{4}\right)\right)}{\sqrt{\cos\left(d+ex-\arctan\left(\frac{3}{4}\right)\right)-1}}\right)}{10e}+\frac{3\cos(d+ex)-4\sin(d+ex)}{10e(3\sin(d+ex)+4\cos(d+ex)-5)^{3/2}}$$

$$\Big\downarrow 217$$

$$\frac{\arctan\left(\frac{\sin\left(-\arctan\left(\frac{3}{4}\right)+d+ex\right)}{\sqrt{2}\sqrt{\cos\left(-\arctan\left(\frac{3}{4}\right)+d+ex\right)-1}}\right)}{10\sqrt{10}e}+\frac{3\cos(d+ex)-4\sin(d+ex)}{10e(3\sin(d+ex)+4\cos(d+ex)-5)^{3/2}}$$

input
```
Int[(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(-3/2),x]
```

output
```
ArcTan[Sin[d + e*x - ArcTan[3/4]]/(Sqrt[2]*Sqrt[-1 + Cos[d + e*x - ArcTan[
3/4]]])]/(10*Sqrt[10]*e) + (3*Cos[d + e*x] - 4*Sin[d + e*x])/(10*e*(-5 + 4
*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2))
```

3.428.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3594
```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

3.428. $\int \frac{1}{(-5+4\cos(d+ex)+3\sin(d+ex))^{3/2}}\,dx$

rule 3595
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(c*Cos[d + e*x] - b*Sin[d + e*x])*((a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n/(a*e*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n + 1))
 Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b,
c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

3.428.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.23

method	result
default	$\frac{\left(-\sqrt{10}\,\arctan\left(\frac{\sqrt{-5\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)-5}\,\sqrt{10}}{10}\right)\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)+\sqrt{10}\,\arctan\left(\frac{\sqrt{-5\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)-5}\,\sqrt{10}}{10}\right)+2\sqrt{\cdots}\right.}{100\cos\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)\sqrt{-5+5\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)}\,e}$

input
```
int(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/100*(-10^(1/2)*arctan(1/10*(-5*sin(e*x+d+arctan(4/3))-5)^(1/2)*10^(1/2))
*sin(e*x+d+arctan(4/3))+10^(1/2)*arctan(1/10*(-5*sin(e*x+d+arctan(4/3))-5)
^(1/2)*10^(1/2))+2*(-5*sin(e*x+d+arctan(4/3))-5)^(1/2))*(-5*sin(e*x+d+arct
an(4/3))-5)^(1/2)/cos(e*x+d+arctan(4/3))/(-5+5*sin(e*x+d+arctan(4/3)))^(1/
2)/e
```

3.428.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. 2(81) = 162.

Time = 0.26 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.19

$$\int \frac{1}{(-5+4\cos(d+ex)+3\sin(d+ex))^{3/2}}\,dx =$$
$$-\frac{\left(13\sqrt{10}\cos\left(ex+d\right)^2-9\left(\sqrt{10}\cos\left(ex+d\right)-2\sqrt{10}\right)\sin\left(ex+d\right)-\sqrt{10}\cos\left(ex+d\right)-14\sqrt{10}\right)\arctan\left(\cdots\right.}{100\left(13\,e\cos\left(ex+d\right)^2-e\cdots\right.}$$

input
```
integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="fricas")
```

output
```
-1/100*((13*sqrt(10)*cos(e*x + d)^2 - 9*(sqrt(10)*cos(e*x + d) - 2*sqrt(10
))*sin(e*x + d) - sqrt(10)*cos(e*x + d) - 14*sqrt(10))*arctan(-1/10*(3*sqr
t(10)*cos(e*x + d) + sqrt(10)*sin(e*x + d) + 3*sqrt(10))*sqrt(4*cos(e*x +
d) + 3*sin(e*x + d) - 5)/(cos(e*x + d) - 3*sin(e*x + d) + 1)) + 10*sqrt(4*
cos(e*x + d) + 3*sin(e*x + d) - 5)*(3*cos(e*x + d) + sin(e*x + d) + 3))/(1
3*e*cos(e*x + d)^2 - e*cos(e*x + d) - 9*(e*cos(e*x + d) - 2*e)*sin(e*x + d
) - 14*e)
```

3.428.6 Sympy [F]

$$\int \frac{1}{(-5+4\cos(d+ex)+3\sin(d+ex))^{3/2}}\,dx = \int \frac{1}{(3\sin{(d+ex)}+4\cos{(d+ex)}-5)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))**(3/2),x)
```

output
```
Integral((3*sin(d + e*x) + 4*cos(d + e*x) - 5)**(-3/2), x)
```

3.428.7 Maxima [F]

$$\int \frac{1}{(-5+4\cos(d+ex)+3\sin(d+ex))^{3/2}}\,dx = \int \frac{1}{(4\cos{(ex+d)}+3\sin{(ex+d)}-5)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="maxima")
```

output
```
integrate((4*cos(e*x + d) + 3*sin(e*x + d) - 5)^(-3/2), x)
```

3.428.8 Giac [F]

$$\int \frac{1}{(-5+4\cos(d+ex)+3\sin(d+ex))^{3/2}}\,dx = \int \frac{1}{(4\cos{(ex+d)}+3\sin{(ex+d)}-5)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.428.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-5+4\cos(d+ex)+3\sin(d+ex))^{3/2}}\,dx = \int \frac{1}{\left(4\,\cos\left(d+e\,x\right)+3\,\sin\left(d+e\,x\right)-5\right)^{3/2}}\,dx$$

input
```
int(1/(4*cos(d + e*x) + 3*sin(d + e*x) - 5)^(3/2),x)
```

output
```
int(1/(4*cos(d + e*x) + 3*sin(d + e*x) - 5)^(3/2), x)
```

3.429 $\int \frac{1}{(-5+4\cos(d+ex)+3\sin(d+ex))^{5/2}}\,dx$

3.429.1 Optimal result

Integrand size = 22, antiderivative size = 142

$$\int \frac{1}{(-5+4\cos(d+ex)+3\sin(d+ex))^{5/2}}\,dx = -\frac{3\arctan\left(\frac{\sin\left(d+ex-\arctan\left(\frac{3}{4}\right)\right)}{\sqrt{2}\sqrt{-1+\cos\left(d+ex-\arctan\left(\frac{3}{4}\right)\right)}}\right)}{400\sqrt{10}e} + \frac{3\cos(d+ex)-4\sin(d+ex)}{20e(-5+4\cos(d+ex)+3\sin(d+ex))^{5/2}} - \frac{3(3\cos(d+ex)-4\sin(d+ex))}{400e(-5+4\cos(d+ex)+3\sin(d+ex))^{3/2}}$$

output
```
1/20*(3*cos(e*x+d)-4*sin(e*x+d))/e/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2)-3/
400*(3*cos(e*x+d)-4*sin(e*x+d))/e/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(3/2)-3/4
000*arctan(1/2*sin(d+e*x-arctan(3/4))*2^(1/2)/(-1+cos(d+e*x-arctan(3/4)))^
(1/2))*10^(1/2)/e
```

3.429.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.25

$$\int \frac{1}{(-5+4\cos(d+ex)+3\sin(d+ex))^{5/2}}\,dx = \frac{\left(\frac{1}{10000}+\frac{i}{20000}\right)\left(\cos\left(\frac{1}{2}(d+ex)\right)-3\sin\left(\frac{1}{2}(d+ex)\right)\right)\left((6+\right.}{}$$

input
```
Integrate[(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(-5/2),x]
```

output
```
((1/10000 + I/20000)*(Cos[(d + e*x)/2] - 3*Sin[(d + e*x)/2])*((6 + 6*I)*Sq
rt[-20 - 15*I]*ArcTanh[(1/10 + (3*I)/10)*Sqrt[-4/5 - (3*I)/5]*(3 + Tan[(d
+ e*x)/4])]*(Cos[(d + e*x)/2] - 3*Sin[(d + e*x)/2])^4 + (10 - 5*I)*(165*Co
s[(d + e*x)/2] - 27*Cos[(3*(d + e*x))/2] + 55*Sin[(d + e*x)/2] - 39*Sin[(3
*(d + e*x))/2])))/(e*(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(5/2))
```

3.429.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {3042, 3595, 3042, 3595, 3042, 3594, 3042, 3128, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3\sin(d+ex)+4\cos(d+ex)-5)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(3\sin(d+ex)+4\cos(d+ex)-5)^{5/2}}dx$$

$$\Big\downarrow \text{3595}$$

$$\frac{3\cos(d+ex)-4\sin(d+ex)}{20e(3\sin(d+ex)+4\cos(d+ex)-5)^{5/2}}-\frac{3}{40}\int \frac{1}{(4\cos(d+ex)+3\sin(d+ex)-5)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\cos(d+ex)-4\sin(d+ex)}{20e(3\sin(d+ex)+4\cos(d+ex)-5)^{5/2}}-\frac{3}{40}\int \frac{1}{(4\cos(d+ex)+3\sin(d+ex)-5)^{3/2}}dx$$

$$\Big\downarrow \text{3595}$$

$$\frac{3\cos(d+ex)-4\sin(d+ex)}{20e(3\sin(d+ex)+4\cos(d+ex)-5)^{5/2}}-\frac{3}{40}\left(\frac{3\cos(d+ex)-4\sin(d+ex)}{10e(3\sin(d+ex)+4\cos(d+ex)-5)^{3/2}}-\frac{1}{20}\int \frac{1}{\sqrt{4\cos(d+ex)+3\sin(d+ex)-5}}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\cos(d+ex)-4\sin(d+ex)}{20e(3\sin(d+ex)+4\cos(d+ex)-5)^{5/2}}-\frac{3}{40}\left(\frac{3\cos(d+ex)-4\sin(d+ex)}{10e(3\sin(d+ex)+4\cos(d+ex)-5)^{3/2}}-\frac{1}{20}\int\frac{1}{\sqrt{4\cos(d+ex)+3\sin(d+ex)-5}}dx\right)$$

$$\Big\downarrow \text{3594}$$

$$\frac{3\cos(d+ex)-4\sin(d+ex)}{20e(3\sin(d+ex)+4\cos(d+ex)-5)^{5/2}}-\frac{3}{40}\left(\frac{3\cos(d+ex)-4\sin(d+ex)}{10e(3\sin(d+ex)+4\cos(d+ex)-5)^{3/2}}-\frac{1}{20}\int\frac{1}{\sqrt{5\cos\left(d+ex-\arctan\left(\frac{3}{4}\right)\right)-5}}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\cos(d+ex)-4\sin(d+ex)}{20e(3\sin(d+ex)+4\cos(d+ex)-5)^{5/2}}-\frac{3}{40}\left(\frac{3\cos(d+ex)-4\sin(d+ex)}{10e(3\sin(d+ex)+4\cos(d+ex)-5)^{3/2}}-\frac{1}{20}\int\frac{1}{\sqrt{5\sin\left(d+ex-\arctan\left(\frac{3}{4}\right)+\frac{\pi}{2}\right)-5}}dx\right)$$

$$\Big\downarrow \text{3128}$$

$$\frac{3\cos(d+ex)-4\sin(d+ex)}{20e(3\sin(d+ex)+4\cos(d+ex)-5)^{5/2}}-\frac{3}{40}\left(\frac{\int\frac{1}{-\frac{5\sin^2\left(d+ex-\arctan\left(\frac{3}{4}\right)\right)}{\cos\left(d+ex-\arctan\left(\frac{3}{4}\right)\right)-1}-10}d\left(-\frac{\sqrt{5}\sin\left(d+ex-\arctan\left(\frac{3}{4}\right)\right)}{\sqrt{\cos\left(d+ex-\arctan\left(\frac{3}{4}\right)\right)-1}}\right)}{10e}+\frac{3\cos(d+ex)-4\sin(d+ex)}{10e(3\sin(d+ex)+4\cos(d+ex)-5)^{3/2}}\right)$$

$$\Big\downarrow \text{217}$$

$$\frac{3\cos(d+ex)-4\sin(d+ex)}{20e(3\sin(d+ex)+4\cos(d+ex)-5)^{5/2}}-\frac{3}{40}\left(\frac{\arctan\left(\frac{\sin\left(-\arctan\left(\frac{3}{4}\right)+d+ex\right)}{\sqrt{2}\sqrt{\cos\left(-\arctan\left(\frac{3}{4}\right)+d+ex\right)-1}}\right)}{10\sqrt{10}e}+\frac{3\cos(d+ex)-4\sin(d+ex)}{10e(3\sin(d+ex)+4\cos(d+ex)-5)^{3/2}}\right)$$

input
```
Int[(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(-5/2),x]
```

3.429. $\int \frac{1}{(-5+4\cos(d+ex)+3\sin(d+ex))^{5/2}}\,dx$

output
```
(3*Cos[d + e*x] - 4*Sin[d + e*x])/(20*e*(-5 + 4*Cos[d + e*x] + 3*Sin[d + e
*x])^(5/2)) - (3*(ArcTan[Sin[d + e*x - ArcTan[3/4]]/(Sqrt[2]*Sqrt[-1 + Cos
[d + e*x - ArcTan[3/4]]])]/(10*Sqrt[10]*e) + (3*Cos[d + e*x] - 4*Sin[d + e
*x])/(10*e*(-5 + 4*Cos[d + e*x] + 3*Sin[d + e*x])^(3/2))))/40
```

3.429.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3594
```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

rule 3595
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(c*Cos[d + e*x] - b*Sin[d + e*x])*((a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n/(a*e*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n + 1))
 Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b,
c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

3.429.4 Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.34

method	result
default	$\frac{\left(3\sqrt{10}\,\arctan\left(\frac{\sqrt{-5\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)-5}\,\sqrt{10}}{10}\right)\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)^2-6\sqrt{10}\,\arctan\left(\frac{\sqrt{-5\sin\left(ex+d+\arctan\left(\frac{4}{3}\right)\right)-5}\,\sqrt{10}}{10}\right)\sin(}{4000(\sin(ex}$

input
```
int(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/4000*(3*10^(1/2)*arctan(1/10*(-5*sin(e*x+d+arctan(4/3))-5)^(1/2)*10^(1/2
))*sin(e*x+d+arctan(4/3))^2-6*10^(1/2)*arctan(1/10*(-5*sin(e*x+d+arctan(4/
3))-5)^(1/2)*10^(1/2))*sin(e*x+d+arctan(4/3))+3*10^(1/2)*arctan(1/10*(-5*s
in(e*x+d+arctan(4/3))-5)^(1/2)*10^(1/2))-6*(-5*sin(e*x+d+arctan(4/3))-5)^(
1/2)*sin(e*x+d+arctan(4/3))+14*(-5*sin(e*x+d+arctan(4/3))-5)^(1/2))*(-5*si
n(e*x+d+arctan(4/3))-5)^(1/2)/(sin(e*x+d+arctan(4/3))-1)/cos(e*x+d+arctan(
4/3))/(-5+5*sin(e*x+d+arctan(4/3)))^(1/2)/e
```

3.429.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(123) = 246$.

Time = 0.25 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.97

$$\int \frac{1}{(-5+4\cos(d+ex)+3\sin(d+ex))^{5/2}}\,dx = \frac{3\left(79\sqrt{10}\cos(ex+d)^3-123\sqrt{10}\cos(ex+d)^2+3\left(\sqrt{10}\,c\right.\right.}{}$$

input
```
integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x, algorithm="fricas")
```

output
```
1/4000*(3*(79*sqrt(10)*cos(e*x + d)^3 - 123*sqrt(10)*cos(e*x + d)^2 + 3*(s
qrt(10)*cos(e*x + d)^2 + 38*sqrt(10)*cos(e*x + d) - 44*sqrt(10))*sin(e*x +
 d) - 78*sqrt(10)*cos(e*x + d) + 124*sqrt(10))*arctan(-1/10*(3*sqrt(10)*co
s(e*x + d) + sqrt(10)*sin(e*x + d) + 3*sqrt(10))*sqrt(4*cos(e*x + d) + 3*s
in(e*x + d) - 5)/(cos(e*x + d) - 3*sin(e*x + d) + 1)) + 10*(27*cos(e*x + d
)^2 + (39*cos(e*x + d) - 8)*sin(e*x + d) - 69*cos(e*x + d) - 96)*sqrt(4*co
s(e*x + d) + 3*sin(e*x + d) - 5))/(79*e*cos(e*x + d)^3 - 123*e*cos(e*x + d
)^2 - 78*e*cos(e*x + d) + 3*(e*cos(e*x + d)^2 + 38*e*cos(e*x + d) - 44*e)*
sin(e*x + d) + 124*e)
```

3.429.6 Sympy [F]

$$\int \frac{1}{(-5+4\cos(d+ex)+3\sin(d+ex))^{5/2}}\,dx = \int \frac{1}{(3\sin{(d+ex)}+4\cos{(d+ex)}-5)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))**(5/2),x)
```

output
```
Integral((3*sin(d + e*x) + 4*cos(d + e*x) - 5)**(-5/2), x)
```

3.429.7 Maxima [F]

$$\int \frac{1}{(-5+4\cos(d+ex)+3\sin(d+ex))^{5/2}}\,dx = \int \frac{1}{(4\cos{(ex+d)}+3\sin{(ex+d)}-5)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x, algorithm="maxima")
```

output
```
integrate((4*cos(e*x + d) + 3*sin(e*x + d) - 5)^(-5/2), x)
```

3.429.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{(-5+4\cos(d+ex)+3\sin(d+ex))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(-5+4*cos(e*x+d)+3*sin(e*x+d))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.429.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(-5+4\cos(d+ex)+3\sin(d+ex))^{5/2}}\,dx = \int \frac{1}{\left(4\,\cos\left(d+e\,x\right)+3\,\sin\left(d+e\,x\right)-5\right)^{5/2}}\,dx$$

input
```
int(1/(4*cos(d + e*x) + 3*sin(d + e*x) - 5)^(5/2),x)
```

output
```
int(1/(4*cos(d + e*x) + 3*sin(d + e*x) - 5)^(5/2), x)
```

3.430 $\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{7/2}\,dx$

3.430.1 Optimal result

Integrand size = 32, antiderivative size = 258

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{7/2}dx = -\frac{256(b^2+c^2)^{3/2}\left(c\cos(d+ex)-b\sin(d+ex)\right)}{35e\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}} - \frac{64(b^2+c^2)\left(c\cos(d+ex)-b\sin(d+ex)\right)\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}{35e} - \frac{24\sqrt{b^2+c^2}(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}{35e} - \frac{2(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}{7e}$$

output
```
-24/35*(c*cos(e*x+d)-b*sin(e*x+d))*(b^2+c^2)^(1/2)*(b*cos(e*x+d)+c*sin(e*x
+d)+(b^2+c^2)^(1/2))^(3/2)/e-2/7*(c*cos(e*x+d)-b*sin(e*x+d))*(b*cos(e*x+d)
+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(5/2)/e-256/35*(b^2+c^2)^(3/2)*(c*cos(e*x+d
)-b*sin(e*x+d))/e/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(1/2)-64/35*
(b^2+c^2)*(c*cos(e*x+d)-b*sin(e*x+d))*(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)
^(1/2))^(1/2)/e
```

3.430.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 37.94 (sec) , antiderivative size = 5490, normalized size of antiderivative = 21.28

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{7/2} dx = \text{Result too large to show}$$

input
```
Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(7/2),x]
```

output
```
Result too large to show
```

3.430.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3592, 3042, 3592, 3042, 3592, 3042, 3591}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{7/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{7/2} dx$$

$$\Big\downarrow \text{3592}$$

$$\frac{12}{7}\sqrt{b^2+c^2}\int \left(b\cos(d+ex)+c\sin(d+ex)+\sqrt{b^2+c^2}\right)^{5/2} dx - \frac{2(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}{7e}$$

$$\Big\downarrow \text{3042}$$

$$\frac{12}{7}\sqrt{b^2+c^2}\int \left(b\cos(d+ex)+c\sin(d+ex)+\sqrt{b^2+c^2}\right)^{5/2} dx - \frac{2(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}{7e}$$

$$\Big\downarrow \text{3592}$$

$$\frac{12}{7}\sqrt{b^2+c^2}\left(\frac{8}{5}\sqrt{b^2+c^2}\int\left(b\cos(d+ex)+c\sin(d+ex)+\sqrt{b^2+c^2}\right)^{3/2}dx-\frac{2(c\cos(d+ex)-b\sin(d+ex))\left(\right.}{}\right.$$

$$\frac{2(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}{7e}$$

$\downarrow$ 3042

$$\frac{12}{7}\sqrt{b^2+c^2}\left(\frac{8}{5}\sqrt{b^2+c^2}\int\left(b\cos(d+ex)+c\sin(d+ex)+\sqrt{b^2+c^2}\right)^{3/2}dx-\frac{2(c\cos(d+ex)-b\sin(d+ex))\left(\right.}{}\right.$$

$$\frac{2(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}{7e}$$

$\downarrow$ 3592

$$\frac{12}{7}\sqrt{b^2+c^2}\left(\frac{8}{5}\sqrt{b^2+c^2}\left(\frac{4}{3}\sqrt{b^2+c^2}\int\sqrt{b\cos(d+ex)+c\sin(d+ex)+\sqrt{b^2+c^2}}dx-\frac{2(c\cos(d+ex)-b\sin(}{}\right.\right.$$

$$\frac{2(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}{7e}$$

$\downarrow$ 3042

$$\frac{12}{7}\sqrt{b^2+c^2}\left(\frac{8}{5}\sqrt{b^2+c^2}\left(\frac{4}{3}\sqrt{b^2+c^2}\int\sqrt{b\cos(d+ex)+c\sin(d+ex)+\sqrt{b^2+c^2}}dx-\frac{2(c\cos(d+ex)-b\sin(}{}\right.\right.$$

$$\frac{2(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}{7e}$$

$\downarrow$ 3591

$$\frac{12}{7}\sqrt{b^2+c^2}\left(\frac{8}{5}\sqrt{b^2+c^2}\left(-\frac{2\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}(c\cos(d+ex)-b\sin(d+ex))}{3e}-\frac{8\sqrt{b^2}}{3e\sqrt{\sqrt{}}}\right.\right.$$

$$\frac{2(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}{7e}$$

3.430. $\int\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{7/2}dx$

input
```
Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(7/2),x]
```

output
```
(-2*(c*Cos[d + e*x] - b*Sin[d + e*x])*(Sqrt[b^2 + c^2] + b*Cos[d + e*x] +
c*Sin[d + e*x])^(5/2))/(7*e) + (12*Sqrt[b^2 + c^2]*((-2*(c*Cos[d + e*x] -
b*Sin[d + e*x])*(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(3/2))
/(5*e) + (8*Sqrt[b^2 + c^2]*((-8*Sqrt[b^2 + c^2]*(c*Cos[d + e*x] - b*Sin[d
 + e*x]))/(3*e*Sqrt[Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]]) -
(2*(c*Cos[d + e*x] - b*Sin[d + e*x])*Sqrt[Sqrt[b^2 + c^2] + b*Cos[d + e*x]
 + c*Sin[d + e*x]])/(3*e)))/5))/7
```

3.430.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3591
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[-2*((c*Cos[d + e*x] - b*Sin[d + e*x])/(e*Sqrt[a + b*
Cos[d + e*x] + c*Sin[d + e*x]])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

rule 3592
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Simp[a*((2*n - 1)/n)   Int[(a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

3.430.4 Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.19

method	result
default	$\frac{2(\sin(ex+d-\arctan(-b,c))+1)(\sin(ex+d-\arctan(-b,c))-1)\left(5b^4\sin(ex+d-\arctan(-b,c))^3+10b^2c^2\sin(ex+d-\arctan(-b,c))^3+5c^4\right.}{}$

input
```
int((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(7/2),x,method=_RETURNVERB
OSE)
```

output
```
2/35*(sin(e*x+d-arctan(-b,c))+1)*(sin(e*x+d-arctan(-b,c))-1)*(5*b^4*sin(e*
x+d-arctan(-b,c))^3+10*b^2*c^2*sin(e*x+d-arctan(-b,c))^3+5*c^4*sin(e*x+d-a
rctan(-b,c))^3+27*b^4*sin(e*x+d-arctan(-b,c))^2+54*b^2*c^2*sin(e*x+d-arcta
n(-b,c))^2+27*c^4*sin(e*x+d-arctan(-b,c))^2+71*b^4*sin(e*x+d-arctan(-b,c))
+142*b^2*c^2*sin(e*x+d-arctan(-b,c))+71*c^4*sin(e*x+d-arctan(-b,c))+177*b^
4+354*b^2*c^2+177*c^4)/cos(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c
))+c^2*sin(e*x+d-arctan(-b,c))+b^2+c^2)/(b^2+c^2)^(1/2))^(1/2)/e
```

3.430.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.04

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{7/2}\,dx = \frac{2\left(5\left(b^4-6\,b^2c^2+c^4\right)\cos\left(ex+d\right)^4-177\,b^4-310\,b^2c^2-128\,c^4+2\left(22\,b^4+15\,b^2c^2-27\,c^4\right)c}{}$$

input
```
integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(7/2),x, algorithm="
fricas")
```

output
```
2/35*(5*(b^4 - 6*b^2*c^2 + c^4)*cos(e*x + d)^4 - 177*b^4 - 310*b^2*c^2 - 1
28*c^4 + 2*(22*b^4 + 15*b^2*c^2 - 27*c^4)*cos(e*x + d)^2 + 4*(5*(b^3*c - b
*c^3)*cos(e*x + d)^3 + (22*b^3*c + 27*b*c^3)*cos(e*x + d))*sin(e*x + d) +
2*(11*(b^3 - 3*b*c^2)*cos(e*x + d)^3 + (53*b^3 + 86*b*c^2)*cos(e*x + d) +
(53*b^2*c + 64*c^3 + 11*(3*b^2*c - c^3)*cos(e*x + d)^2)*sin(e*x + d))*sqrt
(b^2 + c^2))*sqrt(b*cos(e*x + d) + c*sin(e*x + d) + sqrt(b^2 + c^2))/(c*e*
cos(e*x + d) - b*e*sin(e*x + d))
```

3.430.6 Sympy [F(-1)]

Timed out.

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{7/2}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**(7/2),x)
```

output
```
Timed out
```

3.430.7 Maxima [F(-2)]

Exception generated.

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{7/2} dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(7/2),x, algorithm="
maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.430.8 Giac [F(-2)]

Exception generated.

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{7/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(7/2),x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.430.9 Mupad [F(-1)]

Timed out.

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{7/2} dx = \int \left(b\,\cos\left(d+e\,x\right)+c\,\sin\left(d+e\,x\right)+\sqrt{b^2+c^2}\right)^{7/2} dx$$

input
```
int((b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^(7/2),x)
```

output
```
int((b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^(7/2), x)
```

3.431 $\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2} dx$

3.431.1 Optimal result

Integrand size = 32, antiderivative size = 190

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2} dx = -\frac{64(b^2+c^2)\left(c\cos(d+ex)-b\sin(d+ex)\right)}{15e\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}} - \frac{16\sqrt{b^2+c^2}(c\cos(d+ex)-b\sin(d+ex))\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}{15e} - \frac{2(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}{5e}$$

output
```
-2/5*(c*cos(e*x+d)-b*sin(e*x+d))*(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2
))^(3/2)/e-64/15*(b^2+c^2)*(c*cos(e*x+d)-b*sin(e*x+d))/e/(b*cos(e*x+d)+c*s
in(e*x+d)+(b^2+c^2)^(1/2))^(1/2)-16/15*(c*cos(e*x+d)-b*sin(e*x+d))*(b^2+c^
2)^(1/2)*(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(1/2)/e
```

3.431.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 18.06 (sec) , antiderivative size = 5377, normalized size of antiderivative = 28.30

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2} dx = \text{Result too large to show}$$

input
```
Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(5/2),x]
```

output
```
Result too large to show
```

3.431.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3042, 3592, 3042, 3592, 3042, 3591}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2} dx$$

$$\Big\downarrow \text{3592}$$

$$\frac{8}{5}\sqrt{b^2+c^2}\int \left(b\cos(d+ex)+c\sin(d+ex)+\sqrt{b^2+c^2}\right)^{3/2} dx - \frac{2(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}{5e}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{5}\sqrt{b^2+c^2}\int \left(b\cos(d+ex)+c\sin(d+ex)+\sqrt{b^2+c^2}\right)^{3/2} dx - \frac{2(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}{5e}$$

$$\Big\downarrow \text{3592}$$

$$\frac{8}{5}\sqrt{b^2+c^2}\left(\frac{4}{3}\sqrt{b^2+c^2}\int\sqrt{b\cos(d+ex)+c\sin(d+ex)+\sqrt{b^2+c^2}}dx-\frac{2(c\cos(d+ex)-b\sin(d+ex))\sqrt{\sqrt{b^2}}}{3e}\right.$$
$$\frac{2(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}{5e}$$

$\big\downarrow$ 3042

$$\frac{8}{5}\sqrt{b^2+c^2}\left(\frac{4}{3}\sqrt{b^2+c^2}\int\sqrt{b\cos(d+ex)+c\sin(d+ex)+\sqrt{b^2+c^2}}dx-\frac{2(c\cos(d+ex)-b\sin(d+ex))\sqrt{\sqrt{b^2}}}{3e}\right.$$
$$\frac{2(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}{5e}$$

$\big\downarrow$ 3591

$$\frac{8}{5}\sqrt{b^2+c^2}\left(-\frac{2\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}(c\cos(d+ex)-b\sin(d+ex))}{3e}-\frac{8\sqrt{b^2+c^2}(c\cos(d}{3e\sqrt{\sqrt{b^2+c^2}+b\cos}}\right.$$
$$\frac{2(c\cos(d+ex)-b\sin(d+ex))\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}{5e}$$

input
```
Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(5/2),x]
```

output
```
(-2*(c*Cos[d + e*x] - b*Sin[d + e*x])*(Sqrt[b^2 + c^2] + b*Cos[d + e*x] +
c*Sin[d + e*x])^(3/2))/(5*e) + (8*Sqrt[b^2 + c^2]*((-8*Sqrt[b^2 + c^2]*(c*
Cos[d + e*x] - b*Sin[d + e*x]))/(3*e*Sqrt[Sqrt[b^2 + c^2] + b*Cos[d + e*x]
 + c*Sin[d + e*x]]) - (2*(c*Cos[d + e*x] - b*Sin[d + e*x])*Sqrt[Sqrt[b^2 +
 c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]])/(3*e)))/5
```

3.431. $\int\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}dx$

3.431.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3591
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[-2*((c*Cos[d + e*x] - b*Sin[d + e*x])/(e*Sqrt[a + b*
Cos[d + e*x] + c*Sin[d + e*x]])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

rule 3592
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Simp[a*((2*n - 1)/n)   Int[(a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

3.431.4 Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.05

method	result
default	$\frac{2(\sin(ex+d-\arctan(-b,c))+1)\sqrt{b^2+c^2}\,(\sin(ex+d-\arctan(-b,c))-1)\left(3\sin(ex+d-\arctan(-b,c))^2 b^2+3\sin(ex+d-\arctan(-b,c))^2 c^2\right.}{15\cos(ex+d-\arctan(-b,c))\sqrt{\frac{b^2\sin(ex+d-\arctan(-b,c))+c^2\sin(ex+d-\operatorname{arct}}{\sqrt{b^2+c^2}}}}$

input
```
int((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(5/2),x,method=_RETURNVERB
OSE)
```

output
```
2/15*(sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)*(sin(e*x+d-arctan(-b,c))-
1)*(3*sin(e*x+d-arctan(-b,c))^2*b^2+3*sin(e*x+d-arctan(-b,c))^2*c^2+14*b^2
*sin(e*x+d-arctan(-b,c))+14*c^2*sin(e*x+d-arctan(-b,c))+43*b^2+43*c^2)/cos
(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-arctan(-b
,c))+b^2+c^2)/(b^2+c^2)^(1/2))^(1/2)/e
```

3.431.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.99

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}\,dx = \frac{2\left(3\left(b^3-3\,bc^2\right)\cos\left(ex+d\right)^3+\left(29\,b^3+38\,bc^2\right)\cos\left(ex+d\right)+\left(29\,b^2c+32\,c^3+3\left(3\,b^2c-c^3\right)\right.}{}$$

input
```
integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(5/2),x, algorithm="
fricas")
```

output
```
2/15*(3*(b^3 - 3*b*c^2)*cos(e*x + d)^3 + (29*b^3 + 38*b*c^2)*cos(e*x + d)
+ (29*b^2*c + 32*c^3 + 3*(3*b^2*c - c^3)*cos(e*x + d)^2)*sin(e*x + d) + (2
2*b*c*cos(e*x + d)*sin(e*x + d) + 11*(b^2 - c^2)*cos(e*x + d)^2 - 43*b^2 -
 32*c^2)*sqrt(b^2 + c^2))*sqrt(b*cos(e*x + d) + c*sin(e*x + d) + sqrt(b^2
+ c^2))/(c*e*cos(e*x + d) - b*e*sin(e*x + d))
```

3.431.6 Sympy [F(-1)]

Timed out.

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**(5/2),x)
```

output
```
Timed out
```

3.431.7 Maxima [F(-2)]

Exception generated.

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(5/2),x, algorithm="
maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.431.8 Giac [F(-2)]

Exception generated.

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(5/2),x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.431.9 Mupad [F(-1)]

Timed out.

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}\,dx = \int \left(b\,\cos\left(d+e\,x\right)+c\,\sin\left(d+e\,x\right)+\sqrt{b^2+c^2}\right)^{5/2}\,dx$$

input
```
int((b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^(5/2),x)
```

output
```
int((b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^(5/2), x)
```

3.432 $\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}\,dx$

3.432.1 Optimal result

Integrand size = 32, antiderivative size = 126

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}\,dx = -\frac{8\sqrt{b^2+c^2}(c\cos(d+ex)-b\sin(d+ex))}{3e\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}} - \frac{2(c\cos(d+ex)-b\sin(d+ex))\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}{3e}$$

output
```
-8/3*(c*cos(e*x+d)-b*sin(e*x+d))*(b^2+c^2)^(1/2)/e/(b*cos(e*x+d)+c*sin(e*x
+d)+(b^2+c^2)^(1/2))^(1/2)-2/3*(c*cos(e*x+d)-b*sin(e*x+d))*(b*cos(e*x+d)+c
*sin(e*x+d)+(b^2+c^2)^(1/2))^(1/2)/e
```

3.432.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 17.98 (sec) , antiderivative size = 5279, normalized size of antiderivative = 41.90

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}\,dx = \text{Result too large to show}$$

input
```
Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(3/2),x]
```

output
```
Result too large to show
```

3.432.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3042, 3592, 3042, 3591}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}dx$$

$$\Big\downarrow \text{3592}$$

$$\frac{4}{3}\sqrt{b^2+c^2}\int \sqrt{b\cos(d+ex)+c\sin(d+ex)+\sqrt{b^2+c^2}}dx-\frac{2(c\cos(d+ex)-b\sin(d+ex))\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}{3e}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{3}\sqrt{b^2+c^2}\int \sqrt{b\cos(d+ex)+c\sin(d+ex)+\sqrt{b^2+c^2}}dx-\frac{2(c\cos(d+ex)-b\sin(d+ex))\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}{3e}$$

$$\Big\downarrow \text{3591}$$

$$-\frac{2\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}(c\cos(d+ex)-b\sin(d+ex))}{3e}-\frac{8\sqrt{b^2+c^2}(c\cos(d+ex)-b\sin(d+ex))}{3e\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}$$

input
```
Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(3/2),x]
```

output
```
(-8*Sqrt[b^2 + c^2]*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(3*e*Sqrt[Sqrt[b^2
+ c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]]) - (2*(c*Cos[d + e*x] - b*Sin[d
+ e*x])*Sqrt[Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]])/(3*e)
```

3.432.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3591
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[-2*((c*Cos[d + e*x] - b*Sin[d + e*x])/(e*Sqrt[a + b*
Cos[d + e*x] + c*Sin[d + e*x]])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

rule 3592
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Simp[a*((2*n - 1)/n)   Int[(a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

3.432.4 Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{2(\sin(ex+d-\arctan(-b,c))+1)(b^2+c^2)(\sin(ex+d-\arctan(-b,c))-1)(\sin(ex+d-\arctan(-b,c))+5)}{3\cos(ex+d-\arctan(-b,c))\sqrt{\frac{b^2\sin(ex+d-\arctan(-b,c))+c^2\sin(ex+d-\arctan(-b,c))+b^2+c^2}{\sqrt{b^2+c^2}}}\,e}$	126

input
```
int((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(3/2),x,method=_RETURNVERB
OSE)
```

output
```
2/3*(sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)*(sin(e*x+d-arctan(-b,c))-1)*(sin
(e*x+d-arctan(-b,c))+5)/cos(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,
c))+c^2*sin(e*x+d-arctan(-b,c))+b^2+c^2)/(b^2+c^2)^(1/2))^(1/2)/e
```

3.432.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.99

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}\,dx=\frac{2\left(2\,bc\cos(ex+d)\sin(ex+d)+(b^2-c^2)\cos(ex+d)^2-5\,b^2-4\,c^2+4\,\sqrt{b^2+c^2}(b\cos(ex\right.}{3\,(ce\cos(ex+d)-be\sin(ex+}$$

input
```
integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(3/2),x, algorithm="
fricas")
```

output
```
2/3*(2*b*c*cos(e*x + d)*sin(e*x + d) + (b^2 - c^2)*cos(e*x + d)^2 - 5*b^2
- 4*c^2 + 4*sqrt(b^2 + c^2)*(b*cos(e*x + d) + c*sin(e*x + d)))*sqrt(b*cos(
e*x + d) + c*sin(e*x + d) + sqrt(b^2 + c^2))/(c*e*cos(e*x + d) - b*e*sin(e
*x + d))
```

3.432.6 Sympy [F]

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}\,dx=\int \left(b\cos(d+ex)+c\sin(d+ex)+\sqrt{b^2+c^2}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**(3/2),x)
```

output
```
Integral((b*cos(d + e*x) + c*sin(d + e*x) + sqrt(b**2 + c**2))**(3/2), x)
```

3.432.7 Maxima [F(-2)]

Exception generated.

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(3/2),x, algorithm="
maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.432.8 Giac [F(-2)]

Exception generated.

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(3/2),x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.432.9 Mupad [F(-1)]

Timed out.

$$\int \left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}\,dx = \int \left(b\,\cos\left(d+e\,x\right)+c\,\sin\left(d+e\,x\right)+\sqrt{b^2+c^2}\right)^{3/2}\,dx$$

input
```
int((b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^(3/2),x)
```

output
```
int((b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^(3/2), x)
```

3.433 $\int \sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}\,dx$

3.433.1 Optimal result

Integrand size = 32, antiderivative size = 55

$$\int \sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}\,dx = -\frac{2(c\cos(d+ex)-b\sin(d+ex))}{e\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}$$

output
```
-2*(c*cos(e*x+d)-b*sin(e*x+d))/e/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2
))^(1/2)
```

3.433.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 16.60 (sec) , antiderivative size = 565, normalized size of antiderivative = 10.27

$$\int \sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}\,dx = \frac{4i(b+ic)\left(2b^4+3b^2c^2+c^4-2b^3\sqrt{b^2+c^2}-2bc^2\sqrt{b^2+c^2}+b\left(-2b^3-2bc^2+2b^2\sqrt{b^2+c^2}+c^2\sqrt{b^2+c^2}\right)\right.}{}$$

input
```
Integrate[Sqrt[Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]],x]
```

output
```
((4*I)*(b + I*c)*(2*b^4 + 3*b^2*c^2 + c^4 - 2*b^3*Sqrt[b^2 + c^2] - 2*b*c^
2*Sqrt[b^2 + c^2] + b*(-2*b^3 - 2*b*c^2 + 2*b^2*Sqrt[b^2 + c^2] + c^2*Sqrt
[b^2 + c^2])*Cos[d + e*x] + (I*b + c)*EllipticE[ArcSin[Sqrt[((-b - I*c + S
qrt[b^2 + c^2])*(Cos[d + e*x] - I*Sin[d + e*x]))/(-b + I*c + Sqrt[b^2 + c^
2])]], 1]*(Cos[(d + e*x)/2] + I*Sin[(d + e*x)/2])*(c*(2*b^2 + I*b*c + c^2
- 2*b*Sqrt[b^2 + c^2] - I*c*Sqrt[b^2 + c^2])*Cos[(d + e*x)/2] + (-4*b^3 +
b*c*(-3*c + (2*I)*Sqrt[b^2 + c^2]) + c^2*((-I)*c + Sqrt[b^2 + c^2]) + b^2*
((-2*I)*c + 4*Sqrt[b^2 + c^2]))*Sin[(d + e*x)/2])*Sqrt[((-b - I*c + Sqrt[b
^2 + c^2])*(Cos[d + e*x] - I*Sin[d + e*x]))/(-b + I*c + Sqrt[b^2 + c^2])]
- 2*b^3*c*Sin[d + e*x] - 2*b*c^3*Sin[d + e*x] + 2*b^2*c*Sqrt[b^2 + c^2]*Si
n[d + e*x] + c^3*Sqrt[b^2 + c^2]*Sin[d + e*x]))/((b + I*c - Sqrt[b^2 + c^2
])^2*(b^2 + c^2 - b*Sqrt[b^2 + c^2])*e*Sqrt[Sqrt[b^2 + c^2] + b*Cos[d + e*
x] + c*Sin[d + e*x]])
```

3.433.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {3042, 3591}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}dx$$
$$\Big\downarrow \text{ 3591}$$
$$-\frac{2(c\cos(d+ex)-b\sin(d+ex))}{e\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}$$

input
```
Int[Sqrt[Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]],x]
```

output
```
(-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[Sqrt[b^2 + c^2] + b*Cos[d +
 e*x] + c*Sin[d + e*x]])
```

3.433.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3591
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[-2*((c*Cos[d + e*x] - b*Sin[d + e*x])/(e*Sqrt[a + b*
Cos[d + e*x] + c*Sin[d + e*x]])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

3.433.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(51) = 102$.

Time = 1.28 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.05

method	result
default	$\frac{2(\sin(ex+d-\arctan(-b,c))+1)\sqrt{b^2+c^2}\,(\sin(ex+d-\arctan(-b,c))-1)}{\cos(ex+d-\arctan(-b,c))\sqrt{\frac{b^2\sin(ex+d-\arctan(-b,c))+c^2\sin(ex+d-\arctan(-b,c))+b^2+c^2}{\sqrt{b^2+c^2}}}\,e}$
risch	$-\frac{\sqrt{2\sqrt{b^2+c^2}+2b\cos(ex+d)+2c\sin(ex+d)}\,\sqrt{-ic\,e^{3i(ex+d)}+b\,e^{3i(ex+d)}+2\sqrt{b^2+c^2}\,e^{2i(ex+d)}+ic\,e^{i(ex+d)}+b\,e^{i(ex+d)}}\,\sqrt{-(ic-b)(b^2-}}{\left(-ic\,e^{2i(ex+d)}+b\,e^{2i(ex+d)}+2\sqrt{b^2+c^2}\,e^{i(ex+d)}+ic+b\right)\sqrt{-ic\,e^{3i(ex+d)}+b}}$

input
```
int((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
2*(sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2)*(sin(e*x+d-arctan(-b,c))-1)/
cos(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-arctan
(-b,c))+b^2+c^2)/(b^2+c^2)^(1/2))^(1/2)/e
```

3.433.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.45

$$\int \sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}\,dx$$
$$=\frac{2\sqrt{b\cos(ex+d)+c\sin(ex+d)+\sqrt{b^2+c^2}}\left(b\cos(ex+d)+c\sin(ex+d)-\sqrt{b^2+c^2}\right)}{ce\cos(ex+d)-be\sin(ex+d)}$$

input
```
integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(1/2),x, algorithm="
fricas")
```

output
```
2*sqrt(b*cos(e*x + d) + c*sin(e*x + d) + sqrt(b^2 + c^2))*(b*cos(e*x + d)
+ c*sin(e*x + d) - sqrt(b^2 + c^2))/(c*e*cos(e*x + d) - b*e*sin(e*x + d))
```

3.433.6 Sympy [F]

$$\int \sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}\,dx$$
$$=\int \sqrt{b\cos(d+ex)+c\sin(d+ex)+\sqrt{b^2+c^2}}\,dx$$

input
```
integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**(1/2),x)
```

output
```
Integral(sqrt(b*cos(d + e*x) + c*sin(d + e*x) + sqrt(b**2 + c**2)), x)
```

3.433.7 Maxima [F(-2)]

Exception generated.

$$\int \sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(1/2),x, algorithm="
maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.433.8 Giac [F(-2)]

Exception generated.

$$\int \sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(1/2),x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.433.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}\,dx$$
$$= \int \sqrt{b\,\cos(d+e\,x)+c\,\sin(d+e\,x)+\sqrt{b^2+c^2}}\,dx$$

input
```
int((b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^(1/2),x)
```

output
```
int((b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^(1/2), x)
```

3.434 $\int \frac{1}{\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}\,dx$

3.434.1 Optimal result

Integrand size = 32, antiderivative size = 88

$$\int \frac{1}{\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}\,dx = \frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt[4]{b^2+c^2}\sin(d+ex-\tan^{-1}(b,c))}{\sqrt{2}\sqrt{\sqrt{b^2+c^2}+\sqrt{b^2+c^2}\cos(d+ex-\tan^{-1}(b,c))}}\right)}{\sqrt[4]{b^2+c^2}e}$$

output
```
arctanh(1/2*(b^2+c^2)^(1/4)*sin(d+e*x-arctan(b,c))*2^(1/2)/((b^2+c^2)^(1/2
)+cos(d+e*x-arctan(b,c))*(b^2+c^2)^(1/2))^(1/2))*2^(1/2)/(b^2+c^2)^(1/4)/e
```

3.434.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 54.77 (sec) , antiderivative size = 63264, normalized size of antiderivative = 718.91

$$\int \frac{1}{\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/Sqrt[Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]],x]
```

output
```
Result too large to show
```

3.434.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {3042, 3594, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}dx$$

$$\Big\downarrow \text{3594}$$

$$\int \frac{1}{\sqrt{\sqrt{b^2+c^2}\cos\left(-\tan^{-1}(b,c)+d+ex\right)+\sqrt{b^2+c^2}}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\sqrt{b^2+c^2}\sin\left(-\tan^{-1}(b,c)+d+ex+\frac{\pi}{2}\right)+\sqrt{b^2+c^2}}}dx$$

$$\Big\downarrow \text{3128}$$

$$-\frac{2\int \frac{1}{2\sqrt{b^2+c^2}-\frac{\left(b^2+c^2\right)\sin^2\left(d+ex-\tan^{-1}(b,c)\right)}{\sqrt{b^2+c^2}\cos\left(d+ex-\tan^{-1}(b,c)\right)+\sqrt{b^2+c^2}}}d\left(-\frac{\sqrt{b^2+c^2}\sin\left(d+ex-\tan^{-1}(b,c)\right)}{\sqrt{\sqrt{b^2+c^2}\cos(d+ex-\tan^{-1}(b,c))+\sqrt{b^2+c^2}}}\right)}{e}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt[4]{b^2+c^2}\sin(-\tan^{-1}(b,c)+d+ex)}{\sqrt{2}\sqrt{\sqrt{b^2+c^2}\cos(-\tan^{-1}(b,c)+d+ex)+\sqrt{b^2+c^2}}}\right)}{e\sqrt[4]{b^2+c^2}}$$

input
```
Int[1/Sqrt[Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]],x]
```

output
```
(Sqrt[2]*ArcTanh[((b^2 + c^2)^(1/4)*Sin[d + e*x - ArcTan[b, c]])/(Sqrt[2]*
Sqrt[Sqrt[b^2 + c^2] + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]])])/((b
^2 + c^2)^(1/4)*e)
```

3.434.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3594
```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

3.434.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(75) = 150.

Time = 5.57 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.95

method	result
default	$-\frac{(\sin(ex+d-\arctan(-b,c))+1)\sqrt{-\sqrt{b^2+c^2}\,(\sin(ex+d-\arctan(-b,c))-1)}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-\sqrt{b^2+c^2}\,(\sin(ex+d-\arctan(-b,c))-1)}\,\sqrt{2}}{2\left(b^2+c^2\right)^{\frac{1}{4}}}\right)}{(b^2+c^2)^{\frac{1}{4}}\cos(ex+d-\arctan(-b,c))\sqrt{\frac{b^2\sin(ex+d-\arctan(-b,c))+c^2\sin(ex+d-\arctan(-b,c))+b^2+c^2}{\sqrt{b^2+c^2}}}\,e}$

input
```
int(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(1/2),x,method=_RETURNVE
RBOSE)
```

output
```
-(sin(e*x+d-arctan(-b,c))+1)*(-(b^2+c^2)^(1/2)*(sin(e*x+d-arctan(-b,c))-1)
)^(1/2)*2^(1/2)/(b^2+c^2)^(1/4)*arctanh(1/2*(-(b^2+c^2)^(1/2)*(sin(e*x+d-a
rctan(-b,c))-1))^(1/2)*2^(1/2)/(b^2+c^2)^(1/4))/cos(e*x+d-arctan(-b,c))/((
b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-arctan(-b,c))+b^2+c^2)/(b^2+c^2)
^(1/2))^(1/2)/e
```

3.434.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. $2(81) = 162$.

Time = 0.42 (sec) , antiderivative size = 349, normalized size of antiderivative = 3.97

$$\int \frac{1}{\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}\,dx$$
$$= \frac{\sqrt{2}\log\left(\frac{(3\,b^2c-c^3)\cos(ex+d)^3+(b^2c+4\,c^3)\cos(ex+d)-\left(3\,b^3+4\,bc^2+(b^3-3\,bc^2)\cos(ex+d)^2\right)\sin(ex+d)+\frac{2\sqrt{2}\left(2\left(b^3+bc^2\right)\cos(ex+d)+2\left(b^2c+\cdots\right.\right.}{\cdots}}{3\,b^2c\cos(ex+d)-(3\cdots}\right.}{\cdots}$$

input
```
integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(1/2),x, algorithm
="fricas")
```

output
```
1/2*sqrt(2)*log(((3*b^2*c - c^3)*cos(e*x + d)^3 + (b^2*c + 4*c^3)*cos(e*x
+ d) - (3*b^3 + 4*b*c^2 + (b^3 - 3*b*c^2)*cos(e*x + d)^2)*sin(e*x + d) + 2
*sqrt(2)*(2*(b^3 + b*c^2)*cos(e*x + d) + 2*(b^2*c + c^3)*sin(e*x + d) - (2
*b*c*cos(e*x + d)*sin(e*x + d) + (b^2 - c^2)*cos(e*x + d)^2 + b^2 + 2*c^2)
*sqrt(b^2 + c^2))*sqrt(b*cos(e*x + d) + c*sin(e*x + d) + sqrt(b^2 + c^2))/
(b^2 + c^2)^(1/4) - 4*(2*b*c*cos(e*x + d)^2 - (b^2 - c^2)*cos(e*x + d)*sin
(e*x + d) - b*c)*sqrt(b^2 + c^2))/(3*b^2*c*cos(e*x + d) - (3*b^2*c - c^3)*
cos(e*x + d)^3 - (b^3 - (b^3 - 3*b*c^2)*cos(e*x + d)^2)*sin(e*x + d)))/((b
^2 + c^2)^(1/4)*e)
```

3.434.6 Sympy [F]

$$\int \frac{1}{\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}\,dx$$
$$= \int \frac{1}{\sqrt{b\cos(d+ex)+c\sin(d+ex)+\sqrt{b^2+c^2}}}\,dx$$

input
```
integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**(1/2),x)
```

output
```
Integral(1/sqrt(b*cos(d + e*x) + c*sin(d + e*x) + sqrt(b**2 + c**2)), x)
```

3.434.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(1/2),x, algorithm
="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.434.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(1/2),x, algorithm
="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.434.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}\,dx$$
$$= \int \frac{1}{\sqrt{b\,\cos\left(d+e\,x\right)+c\,\sin\left(d+e\,x\right)+\sqrt{b^2+c^2}}}\,dx$$

input
```
int(1/(b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^(1/2),x)
```

output
```
int(1/(b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^(1/2), x)
```

3.434. $\int \frac{1}{\sqrt{\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}\,dx$

3.435 $\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}\,dx$

3.435.1 Optimal result

Integrand size = 32, antiderivative size = 160

$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b^2+c^2}\sin\left(d+ex-\tan^{-1}(b,c)\right)}{\sqrt{2}\sqrt{\sqrt{b^2+c^2}+\sqrt{b^2+c^2}\cos\left(d+ex-\tan^{-1}(b,c)\right)}}\right)}{2\sqrt{2}\left(b^2+c^2\right)^{3/4}e} - \frac{c\cos(d+ex)-b\sin(d+ex)}{2\sqrt{b^2+c^2}e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}$$

output
```
1/4*arctanh(1/2*(b^2+c^2)^(1/4)*sin(d+e*x-arctan(b,c))*2^(1/2)/((b^2+c^2)^
(1/2)+cos(d+e*x-arctan(b,c))*(b^2+c^2)^(1/2))^(1/2))/(b^2+c^2)^(3/4)/e*2^(
1/2)+1/2*(-c*cos(e*x+d)+b*sin(e*x+d))/e/(b^2+c^2)^(1/2)/(b*cos(e*x+d)+c*si
n(e*x+d)+(b^2+c^2)^(1/2))^(3/2)
```

3.435.2 Mathematica [F(-1)]

Timed out.

$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}\,dx = \$\text{Aborted}$$

input
```
Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-3/2),x]
```

output
```
$Aborted
```

3.435.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {3042, 3595, 3042, 3594, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3595}$$

$$\frac{\int \frac{1}{\sqrt{b\cos(d+ex)+c\sin(d+ex)+\sqrt{b^2+c^2}}}dx}{4\sqrt{b^2+c^2}}-\frac{c\cos(d+ex)-b\sin(d+ex)}{2e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\sqrt{b\cos(d+ex)+c\sin(d+ex)+\sqrt{b^2+c^2}}}dx}{4\sqrt{b^2+c^2}}-\frac{c\cos(d+ex)-b\sin(d+ex)}{2e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}$$

$$\Big\downarrow \text{3594}$$

$$\frac{\int \frac{1}{\sqrt{\sqrt{b^2+c^2}\cos(d+ex-\tan^{-1}(b,c))+\sqrt{b^2+c^2}}}dx}{4\sqrt{b^2+c^2}}-\frac{c\cos(d+ex)-b\sin(d+ex)}{2e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\sqrt{\sqrt{b^2+c^2}\sin\left(d+ex-\tan^{-1}(b,c)+\frac{\pi}{2}\right)+\sqrt{b^2+c^2}}}dx}{4\sqrt{b^2+c^2}}-\frac{c\cos(d+ex)-b\sin(d+ex)}{2e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}$$

$$\Big\downarrow \text{3128}$$

$$-\frac{\int \frac{1}{2\sqrt{b^2+c^2}-\frac{(b^2+c^2)\sin^2\left(d+ex-\tan^{-1}(b,c)\right)}{\sqrt{b^2+c^2}\cos\left(d+ex-\tan^{-1}(b,c)\right)+\sqrt{b^2+c^2}}}d\left(-\frac{\sqrt{b^2+c^2}\sin(d+ex-\tan^{-1}(b,c))}{\sqrt{\sqrt{b^2+c^2}\cos(d+ex-\tan^{-1}(b,c))+\sqrt{b^2+c^2}}}\right)}{2e\sqrt{b^2+c^2}}-\frac{c\cos(d+ex)-b\sin(d+ex)}{2e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}$$

$$\Big\downarrow \; 219$$

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{b^2+c^2}\sin(-\tan^{-1}(b,c)+d+ex)}{\sqrt{2}\sqrt{\sqrt{b^2+c^2}\cos(-\tan^{-1}(b,c)+d+ex)+\sqrt{b^2+c^2}}}\right)}{2\sqrt{2}e\,(b^2+c^2)^{3/4}}-\frac{c\cos(d+ex)-b\sin(d+ex)}{2e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}$$

input
```
Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-3/2),x]
```

output
```
ArcTanh[((b^2 + c^2)^(1/4)*Sin[d + e*x - ArcTan[b, c]])/(Sqrt[2]*Sqrt[Sqrt
[b^2 + c^2] + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]])]/(2*Sqrt[2]*(b
^2 + c^2)^(3/4)*e) - (c*Cos[d + e*x] - b*Sin[d + e*x])/(2*Sqrt[b^2 + c^2]*
e*(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(3/2))
```

3.435.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

3.435. $\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}\,dx$

rule 3594
```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

rule 3595
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(c*Cos[d + e*x] - b*Sin[d + e*x])*((a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n/(a*e*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n + 1))
 Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b,
c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

3.435.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. $2(137) = 274$.

Time = 1.54 (sec) , antiderivative size = 350, normalized size of antiderivative = 2.19

method	result
default	$-\frac{\left(\sin(ex+d-\arctan(-b,c))\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-\sqrt{b^2+c^2}\,\sin(ex+d-\arctan(-b,c))+\sqrt{b^2+c^2}}\,\sqrt{2}}{2\left(b^2+c^2\right)^{\frac{1}{4}}}\right)\left(b^2+c^2\right)+\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-\sqrt{b^2+c^2}\,\sin(e\cdots}}{\cdots}\right)\cdots}{4\left(b^2+c^2\right)^{\frac{7}{4}}\cdots}$

input
```
int(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(3/2),x,method=_RETURNVE
RBOSE)
```

output
```
-1/4/(b^2+c^2)^(7/4)*(sin(e*x+d-arctan(-b,c))*2^(1/2)*arctanh(1/2*(-(b^2+c
^2)^(1/2)*sin(e*x+d-arctan(-b,c))+(b^2+c^2)^(1/2))^(1/2)*2^(1/2)/(b^2+c^2)
^(1/4))*(b^2+c^2)+2^(1/2)*arctanh(1/2*(-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-
b,c))+(b^2+c^2)^(1/2))^(1/2)*2^(1/2)/(b^2+c^2)^(1/4))*b^2+2^(1/2)*arctanh(
1/2*(-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+(b^2+c^2)^(1/2))^(1/2)*2^(1/
2)/(b^2+c^2)^(1/4))*c^2+2*(-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+(b^2+c
^2)^(1/2))^(1/2)*(b^2+c^2)^(3/4))*(-(b^2+c^2)^(1/2)*(sin(e*x+d-arctan(-b,c
))-1))^(1/2)/cos(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+c^2*sin
(e*x+d-arctan(-b,c))+b^2+c^2)/(b^2+c^2)^(1/2))^(1/2)/e
```

3.435.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 646 vs. $2(143) = 286$.

Time = 0.51 (sec) , antiderivative size = 646, normalized size of antiderivative = 4.04

$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}\,dx =$$

$$-\frac{\left(3\sqrt{2}b^2c\cos\left(ex+d\right)-\sqrt{2}(3\,b^2c-c^3)\cos\left(ex+d\right)^3-\left(\sqrt{2}b^3-\sqrt{2}(b^3-3\,bc^2)\cos\left(ex+d\right)^2\right)\sin\left(ex+d\right)\right.}{}$$

input
```
integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(3/2),x, algorithm
="fricas")
```

output
```
-1/8*((3*sqrt(2)*b^2*c*cos(e*x + d) - sqrt(2)*(3*b^2*c - c^3)*cos(e*x + d)
^3 - (sqrt(2)*b^3 - sqrt(2)*(b^3 - 3*b*c^2)*cos(e*x + d)^2)*sin(e*x + d))*
(b^2 + c^2)^(1/4)*log(((3*b^2*c - c^3)*cos(e*x + d)^3 + (b^2*c + 4*c^3)*co
s(e*x + d) - (3*b^3 + 4*b*c^2 + (b^3 - 3*b*c^2)*cos(e*x + d)^2)*sin(e*x +
d) - 2*(2*sqrt(2)*b*c*cos(e*x + d)*sin(e*x + d) + sqrt(2)*(b^2 - c^2)*cos(
e*x + d)^2 + sqrt(2)*(b^2 + 2*c^2) - 2*(sqrt(2)*b*cos(e*x + d) + sqrt(2)*c
*sin(e*x + d))*sqrt(b^2 + c^2))*(b^2 + c^2)^(1/4)*sqrt(b*cos(e*x + d) + c*
sin(e*x + d) + sqrt(b^2 + c^2)) - 4*(2*b*c*cos(e*x + d)^2 - (b^2 - c^2)*co
s(e*x + d)*sin(e*x + d) - b*c)*sqrt(b^2 + c^2))/(3*b^2*c*cos(e*x + d) - (3
*b^2*c - c^3)*cos(e*x + d)^3 - (b^3 - (b^3 - 3*b*c^2)*cos(e*x + d)^2)*sin(
e*x + d))) + 4*(2*(b^3 + b*c^2)*cos(e*x + d) + 2*(b^2*c + c^3)*sin(e*x + d
) - (2*b*c*cos(e*x + d)*sin(e*x + d) + (b^2 - c^2)*cos(e*x + d)^2 + b^2 +
2*c^2)*sqrt(b^2 + c^2))*sqrt(b*cos(e*x + d) + c*sin(e*x + d) + sqrt(b^2 +
c^2)))/((3*b^4*c + 2*b^2*c^3 - c^5)*e*cos(e*x + d)^3 - 3*(b^4*c + b^2*c^3)
*e*cos(e*x + d) - ((b^5 - 2*b^3*c^2 - 3*b*c^4)*e*cos(e*x + d)^2 - (b^5 + b
^3*c^2)*e)*sin(e*x + d))
```

3.435.6 Sympy [F]

$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}\,dx = \int \frac{1}{\left(b\cos\left(d+ex\right)+c\sin\left(d+ex\right)+\sqrt{b^2+c^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**(3/2),x)
```

output
```
Integral((b*cos(d + e*x) + c*sin(d + e*x) + sqrt(b**2 + c**2))**(-3/2), x)
```

3.435.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(3/2),x, algorithm
="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.435.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(3/2),x, algorithm
="giac")
```

output
```
Timed out
```

3.435.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}\,dx = \int \frac{1}{\left(b\,\cos\left(d+e\,x\right)+c\,\sin\left(d+e\,x\right)+\sqrt{b^2+c^2}\right)^{3/2}}\,dx$$

input
```
int(1/(b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^(3/2),x)
```

output
```
int(1/(b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^(3/2), x)
```

3.436 $\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}\,dx$

3.436.1 Optimal result

Integrand size = 32, antiderivative size = 226

$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}\,dx = \frac{3\text{arctanh}\left(\frac{\sqrt[4]{b^2+c^2}\sin(d+ex-\tan^{-1}(b,c))}{\sqrt{2}\sqrt{\sqrt{b^2+c^2}+\sqrt{b^2+c^2}\cos(d+ex-\tan^{-1}(b,c))}}\right)}{16\sqrt{2}\,(b^2+c^2)^{5/4}\,e} - \frac{c\cos(d+ex)-b\sin(d+ex)}{4\sqrt{b^2+c^2}e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}} - \frac{3(c\cos(d+ex)-b\sin(d+ex))}{16\,(b^2+c^2)\,e\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}$$

output
```
3/32*arctanh(1/2*(b^2+c^2)^(1/4)*sin(d+e*x-arctan(b,c))*2^(1/2)/((b^2+c^2)
^(1/2)+cos(d+e*x-arctan(b,c))*(b^2+c^2)^(1/2))^(1/2))/(b^2+c^2)^(5/4)/e*2^
(1/2)+1/4*(-c*cos(e*x+d)+b*sin(e*x+d))/e/(b^2+c^2)^(1/2)/(b*cos(e*x+d)+c*s
in(e*x+d)+(b^2+c^2)^(1/2))^(5/2)-3/16*(c*cos(e*x+d)-b*sin(e*x+d))/(b^2+c^2
)/e/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(3/2)
```

3.436.2 Mathematica [F(-1)]

Timed out.

$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}\,dx=\$\text{Aborted}$$

input
```
Integrate[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-5/2),x]
```

output
```
$Aborted
```

3.436.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281, Rules used = {3042, 3595, 3042, 3595, 3042, 3594, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3595}$$

$$\frac{3\int \frac{1}{\left(b\cos(d+ex)+c\sin(d+ex)+\sqrt{b^2+c^2}\right)^{3/2}}dx}{8\sqrt{b^2+c^2}}-\frac{c\cos(d+ex)-b\sin(d+ex)}{4e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{1}{\left(b\cos(d+ex)+c\sin(d+ex)+\sqrt{b^2+c^2}\right)^{3/2}}dx}{8\sqrt{b^2+c^2}}-\frac{c\cos(d+ex)-b\sin(d+ex)}{4e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}$$

$$\Big\downarrow \text{3595}$$

$$\frac{3\left(\frac{\int \frac{1}{\sqrt{b\cos(d+ex)+c\sin(d+ex)+\sqrt{b^2+c^2}}}dx}{4\sqrt{b^2+c^2}}-\frac{c\cos(d+ex)-b\sin(d+ex)}{2e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}\right)}{8\sqrt{b^2+c^2}}-\frac{c\cos(d+ex)-b\sin(d+ex)}{4e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3\left(\frac{\int \frac{1}{\sqrt{b\cos(d+ex)+c\sin(d+ex)+\sqrt{b^2+c^2}}}dx}{4\sqrt{b^2+c^2}}-\frac{c\cos(d+ex)-b\sin(d+ex)}{2e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}\right)}{8\sqrt{b^2+c^2}}-\frac{c\cos(d+ex)-b\sin(d+ex)}{4e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}$$

$$\Big\downarrow \text{ 3594}$$

$$\frac{3\left(\frac{\int \frac{1}{\sqrt{\sqrt{b^2+c^2}\cos\left(d+ex-\tan^{-1}(b,c)\right)+\sqrt{b^2+c^2}}}dx}{4\sqrt{b^2+c^2}}-\frac{c\cos(d+ex)-b\sin(d+ex)}{2e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}\right)}{8\sqrt{b^2+c^2}}-\frac{c\cos(d+ex)-b\sin(d+ex)}{4e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3\left(\frac{\int \frac{1}{\sqrt{\sqrt{b^2+c^2}\sin\left(d+ex-\tan^{-1}(b,c)+\frac{\pi}{2}\right)+\sqrt{b^2+c^2}}}dx}{4\sqrt{b^2+c^2}}-\frac{c\cos(d+ex)-b\sin(d+ex)}{2e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}\right)}{8\sqrt{b^2+c^2}}-\frac{c\cos(d+ex)-b\sin(d+ex)}{4e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}$$

$$\Big\downarrow \text{ 3128}$$

$$\frac{3\left(-\frac{\int \frac{1}{2\sqrt{b^2+c^2}-\frac{\left(b^2+c^2\right)\sin^2\left(d+ex-\tan^{-1}(b,c)\right)}{\sqrt{b^2+c^2}\cos\left(d+ex-\tan^{-1}(b,c)\right)+\sqrt{b^2+c^2}}}d\left(-\frac{\sqrt{b^2+c^2}\sin\left(d+ex-\tan^{-1}(b,c)\right)}{\sqrt{\sqrt{b^2+c^2}\cos\left(d+ex-\tan^{-1}(b,c)\right)+\sqrt{b^2+c^2}}}\right)}{2e\sqrt{b^2+c^2}}-\frac{c\cos(d+ex)-b\sin(d+\cdots}{2e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+\cdots\right.}\cdots\right)}{8\sqrt{b^2+c^2}}-\frac{c\cos(d+ex)-b\sin(d+ex)}{4e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}$$

3.436. $\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}\,dx$

$$\Big\downarrow 219$$

$$\frac{3\left(\frac{\text{arctanh}\left(\frac{\sqrt[4]{b^2+c^2}\sin\left(-\tan^{-1}(b,c)+d+ex\right)}{\sqrt{2}\sqrt{\sqrt{b^2+c^2}\cos\left(-\tan^{-1}(b,c)+d+ex\right)+\sqrt{b^2+c^2}}}\right)}{2\sqrt{2}e(b^2+c^2)^{3/4}}-\frac{c\cos(d+ex)-b\sin(d+ex)}{2e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}\right)}{8\sqrt{b^2+c^2}}-\frac{c\cos(d+ex)-b\sin(d+ex)}{4e\sqrt{b^2+c^2}\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}$$

input
```
Int[(Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-5/2),x]
```

output
```
-1/4*(c*Cos[d + e*x] - b*Sin[d + e*x])/(Sqrt[b^2 + c^2]*e*(Sqrt[b^2 + c^2]
 + b*Cos[d + e*x] + c*Sin[d + e*x])^(5/2)) + (3*(ArcTanh[((b^2 + c^2)^(1/4
)*Sin[d + e*x - ArcTan[b, c]])/(Sqrt[2]*Sqrt[Sqrt[b^2 + c^2] + Sqrt[b^2 +
c^2]*Cos[d + e*x - ArcTan[b, c]]])]/(2*Sqrt[2]*(b^2 + c^2)^(3/4)*e) - (c*C
os[d + e*x] - b*Sin[d + e*x])/(2*Sqrt[b^2 + c^2]*e*(Sqrt[b^2 + c^2] + b*Co
s[d + e*x] + c*Sin[d + e*x])^(3/2))))/(8*Sqrt[b^2 + c^2])
```

3.436.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3594
```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

3.436. $\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}\,dx$

rule 3595
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(c*Cos[d + e*x] - b*Sin[d + e*x])*((a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n/(a*e*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n + 1))
 Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b,
c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

3.436.4 Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.55

method	result
default	$\frac{\left(\sin(ex+d-\arctan(-b,c))\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-\sqrt{b^2+c^2}\,\sin(ex+d-\arctan(-b,c))+\sqrt{b^2+c^2}}\,\sqrt{2}}{2\left(b^2+c^2\right)^{\frac{1}{4}}}\right)(b^2+c^2)+\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-\sqrt{b^2+c^2}\,\sin(ex+}}{2}\right.\right.}{4(b^2+c^2)^{\frac{5}{4}}\,\mathrm{co}}$

input
```
int(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(5/2),x,method=_RETURNVE
RBOSE)
```

output
```
1/4*(sin(e*x+d-arctan(-b,c))*2^(1/2)*arctanh(1/2*(-(b^2+c^2)^(1/2)*sin(e*x
+d-arctan(-b,c))+(b^2+c^2)^(1/2))^(1/2)*2^(1/2)/(b^2+c^2)^(1/4))*(b^2+c^2)
+2^(1/2)*arctanh(1/2*(-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+(b^2+c^2)^(
1/2))^(1/2)*2^(1/2)/(b^2+c^2)^(1/4))*b^2+2^(1/2)*arctanh(1/2*(-(b^2+c^2)^(
1/2)*sin(e*x+d-arctan(-b,c))+(b^2+c^2)^(1/2))^(1/2)*2^(1/2)/(b^2+c^2)^(1/4
))*c^2+2*(-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))+(b^2+c^2)^(1/2))^(1/2)*
(b^2+c^2)^(3/4))*(-(b^2+c^2)^(1/2)*(sin(e*x+d-arctan(-b,c))-1))^(1/2)/(b^2
+c^2)^(5/4)/cos(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(
e*x+d-arctan(-b,c))+b^2+c^2)/(b^2+c^2)^(1/2))^(1/2)/e
```

3.436.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 895 vs. $2(203) = 406$.

Time = 0.50 (sec) , antiderivative size = 895, normalized size of antiderivative = 3.96

$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(5/2),x, algorithm
="fricas")
```

output
```
1/32*(3*sqrt(1/2)*(5*b^4*c*cos(e*x + d) + (5*b^4*c - 10*b^2*c^3 + c^5)*cos
(e*x + d)^5 - 10*(b^4*c - b^2*c^3)*cos(e*x + d)^3 - (b^5 + (b^5 - 10*b^3*c
^2 + 5*b*c^4)*cos(e*x + d)^4 - 2*(b^5 - 5*b^3*c^2)*cos(e*x + d)^2)*sin(e*x
 + d))*log(((3*b^2*c - c^3)*cos(e*x + d)^3 + (b^2*c + 4*c^3)*cos(e*x + d)
- (3*b^3 + 4*b*c^2 + (b^3 - 3*b*c^2)*cos(e*x + d)^2)*sin(e*x + d) + 4*sqrt
(1/2)*(2*(b^3 + b*c^2)*cos(e*x + d) + 2*(b^2*c + c^3)*sin(e*x + d) - (2*b*
c*cos(e*x + d)*sin(e*x + d) + (b^2 - c^2)*cos(e*x + d)^2 + b^2 + 2*c^2)*sq
rt(b^2 + c^2))*sqrt(b*cos(e*x + d) + c*sin(e*x + d) + sqrt(b^2 + c^2))/(b^
2 + c^2)^(1/4) - 4*(2*b*c*cos(e*x + d)^2 - (b^2 - c^2)*cos(e*x + d)*sin(e*
x + d) - b*c)*sqrt(b^2 + c^2))/(3*b^2*c*cos(e*x + d) - (3*b^2*c - c^3)*cos
(e*x + d)^3 - (b^3 - (b^3 - 3*b*c^2)*cos(e*x + d)^2)*sin(e*x + d)))/(b^2 +
 c^2)^(1/4) + 2*(3*(b^4 - 6*b^2*c^2 + c^4)*cos(e*x + d)^4 - 7*b^4 - 26*b^2
*c^2 - 16*c^4 - 6*(2*b^4 - 3*b^2*c^2 - c^4)*cos(e*x + d)^2 + 12*((b^3*c -
b*c^3)*cos(e*x + d)^3 - (2*b^3*c + b*c^3)*cos(e*x + d))*sin(e*x + d) - 2*(
(b^3 - 3*b*c^2)*cos(e*x + d)^3 - 3*(3*b^3 + 2*b*c^2)*cos(e*x + d) - (9*b^2
*c + 8*c^3 - (3*b^2*c - c^3)*cos(e*x + d)^2)*sin(e*x + d))*sqrt(b^2 + c^2)
)*sqrt(b*cos(e*x + d) + c*sin(e*x + d) + sqrt(b^2 + c^2)))/((5*b^6*c - 5*b
^4*c^3 - 9*b^2*c^5 + c^7)*e*cos(e*x + d)^5 - 10*(b^6*c - b^2*c^5)*e*cos(e*
x + d)^3 + 5*(b^6*c + b^4*c^3)*e*cos(e*x + d) - ((b^7 - 9*b^5*c^2 - 5*b^3*
c^4 + 5*b*c^6)*e*cos(e*x + d)^4 - 2*(b^7 - 4*b^5*c^2 - 5*b^3*c^4)*e*cos...
```

3.436.6 Sympy [F]

$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}\,dx = \int \frac{1}{\left(b\cos\left(d+ex\right)+c\sin\left(d+ex\right)+\sqrt{b^2+c^2}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b**2+c**2)**(1/2))**(5/2),x)
```

output
```
Integral((b*cos(d + e*x) + c*sin(d + e*x) + sqrt(b**2 + c**2))**(-5/2), x)
```

3.436.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(5/2),x, algorithm
="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.436.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)+(b^2+c^2)^(1/2))^(5/2),x, algorithm
="giac")
```

output
```
Timed out
```

3.436.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}\,dx = \int \frac{1}{\left(b\,\cos\left(d+e\,x\right)+c\,\sin\left(d+e\,x\right)+\sqrt{b^2+c^2}\right)^{5/2}}\,dx$$

input
```
int(1/(b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^(5/2),x)
```

output
```
int(1/(b*cos(d + e*x) + c*sin(d + e*x) + (b^2 + c^2)^(1/2))^(5/2), x)
```

3.437 $\int \left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2} dx$

3.437.1 Optimal result

Integrand size = 34, antiderivative size = 196

$$\int \left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2} dx = -\frac{64(b^2+c^2)\,(c\cos(d+ex)-b\sin(d+ex))}{15e\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}} + \frac{16\sqrt{b^2+c^2}(c\cos(d+ex)-b\sin(d+ex))\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}{15e} - \frac{2(c\cos(d+ex)-b\sin(d+ex))\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}{5e}$$

output
```
-2/5*(c*cos(e*x+d)-b*sin(e*x+d))*(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2
))^(3/2)/e-64/15*(b^2+c^2)*(c*cos(e*x+d)-b*sin(e*x+d))/e/(b*cos(e*x+d)+c*s
in(e*x+d)-(b^2+c^2)^(1/2))^(1/2)+16/15*(c*cos(e*x+d)-b*sin(e*x+d))*(b^2+c^
2)^(1/2)*(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(1/2)/e
```

3.437.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 39.43 (sec) , antiderivative size = 5238, normalized size of antiderivative = 26.72

$$\int \left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2} dx = \text{Result too large to show}$$

input
```
Integrate[(-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(5/2),x]
```

output
```
Result too large to show
```

3.437.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3042, 3592, 3042, 3592, 3042, 3591}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2} dx$$

$$\Big\downarrow \text{3592}$$

$$-\frac{8}{5}\sqrt{b^2+c^2}\int \left(b\cos(d+ex)+c\sin(d+ex)-\sqrt{b^2+c^2}\right)^{3/2} dx - \frac{2(c\cos(d+ex)-b\sin(d+ex))\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}{5e}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{8}{5}\sqrt{b^2+c^2}\int \left(b\cos(d+ex)+c\sin(d+ex)-\sqrt{b^2+c^2}\right)^{3/2} dx - \frac{2(c\cos(d+ex)-b\sin(d+ex))\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}{5e}$$

$$\Big\downarrow \text{3592}$$

$$-\frac{8}{5}\sqrt{b^2+c^2}\left(-\frac{4}{3}\sqrt{b^2+c^2}\int\sqrt{b\cos(d+ex)+c\sin(d+ex)-\sqrt{b^2+c^2}}dx-\frac{2\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c}}{}\right.$$
$$\frac{2(c\cos(d+ex)-b\sin(d+ex))\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}{5e}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{8}{5}\sqrt{b^2+c^2}\left(-\frac{4}{3}\sqrt{b^2+c^2}\int\sqrt{b\cos(d+ex)+c\sin(d+ex)-\sqrt{b^2+c^2}}dx-\frac{2\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c}}{}\right.$$
$$\frac{2(c\cos(d+ex)-b\sin(d+ex))\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}{5e}$$

$$\Big\downarrow \text{3591}$$

$$-\frac{2(c\cos(d+ex)-b\sin(d+ex))\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}{5e}-$$
$$\frac{8}{5}\sqrt{b^2+c^2}\left(\frac{8\sqrt{b^2+c^2}(c\cos(d+ex)-b\sin(d+ex))}{3e\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}-\frac{2(c\cos(d+ex)-b\sin(d+ex))\sqrt{-\sqrt{b^2+c^2}+b\cos}}{3e}\right.$$

input
```
Int[(-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(5/2),x]
```

output
```
(-2*(c*Cos[d + e*x] - b*Sin[d + e*x])*(-Sqrt[b^2 + c^2] + b*Cos[d + e*x] +
 c*Sin[d + e*x])^(3/2))/(5*e) - (8*Sqrt[b^2 + c^2]*((8*Sqrt[b^2 + c^2]*(c*
Cos[d + e*x] - b*Sin[d + e*x]))/(3*e*Sqrt[-Sqrt[b^2 + c^2] + b*Cos[d + e*x
] + c*Sin[d + e*x]]) - (2*(c*Cos[d + e*x] - b*Sin[d + e*x])*Sqrt[-Sqrt[b^2
 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]])/(3*e)))/5
```

3.437. $\int\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}\,dx$

3.437.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3591
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[-2*((c*Cos[d + e*x] - b*Sin[d + e*x])/(e*Sqrt[a + b*
Cos[d + e*x] + c*Sin[d + e*x]])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

rule 3592
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Simp[a*((2*n - 1)/n)   Int[(a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

3.437.4 Maple [A] (verified)

Time = 1.45 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.04

method	result
default	$\frac{2(\sin(ex+d-\arctan(-b,c))-1)\sqrt{b^2+c^2}\,(\sin(ex+d-\arctan(-b,c))+1)\left(3\sin(ex+d-\arctan(-b,c))^2b^2+3\sin(ex+d-\arctan(-b,c))^2c^2\right.}{15\cos(ex+d-\arctan(-b,c))\sqrt{\frac{b^2\sin(ex+d-\arctan(-b,c))+c^2\sin(ex+d-\arct}{\sqrt{b^2+c^2}}}}$

input
```
int((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(5/2),x,method=_RETURNVERB
OSE)
```

output
```
2/15*(sin(e*x+d-arctan(-b,c))-1)*(b^2+c^2)^(1/2)*(sin(e*x+d-arctan(-b,c))+
1)*(3*sin(e*x+d-arctan(-b,c))^2*b^2+3*sin(e*x+d-arctan(-b,c))^2*c^2-14*b^2
*sin(e*x+d-arctan(-b,c))-14*c^2*sin(e*x+d-arctan(-b,c))+43*b^2+43*c^2)/cos
(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-arctan(-b
,c))-b^2-c^2)/(b^2+c^2)^(1/2))^(1/2)/e
```

3.437.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.98

$$\int \left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}\,dx = \frac{2\left(3\left(b^3-3bc^2\right)\cos(ex+d)^3+\left(29b^3+38bc^2\right)\cos(ex+d)+\left(29b^2c+32c^3+3\left(3b^2c-c^3\right)\right.\right.}{}$$

input
```
integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(5/2),x, algorithm="
fricas")
```

output
```
2/15*(3*(b^3 - 3*b*c^2)*cos(e*x + d)^3 + (29*b^3 + 38*b*c^2)*cos(e*x + d)
+ (29*b^2*c + 32*c^3 + 3*(3*b^2*c - c^3)*cos(e*x + d)^2)*sin(e*x + d) - (2
2*b*c*cos(e*x + d)*sin(e*x + d) + 11*(b^2 - c^2)*cos(e*x + d)^2 - 43*b^2 -
 32*c^2)*sqrt(b^2 + c^2))*sqrt(b*cos(e*x + d) + c*sin(e*x + d) - sqrt(b^2
+ c^2))/(c*e*cos(e*x + d) - b*e*sin(e*x + d))
```

3.437.6 Sympy [F(-1)]

Timed out.

$$\int \left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b**2+c**2)**(1/2))**(5/2),x)
```

output
```
Timed out
```

3.437.7 Maxima [F(-2)]

Exception generated.

$$\int \left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(5/2),x, algorithm="
maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.437.8 Giac [F(-2)]

Exception generated.

$$\int \left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(5/2),x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.437.9 Mupad [F(-1)]

Timed out.

$$\int \left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}\,dx = \int \left(b\,\cos\left(d+e\,x\right)+c\,\sin\left(d+e\,x\right)-\sqrt{b^2+c^2}\right)^{5/2}\,dx$$

input
```
int((b*cos(d + e*x) + c*sin(d + e*x) - (b^2 + c^2)^(1/2))^(5/2),x)
```

output
```
int((b*cos(d + e*x) + c*sin(d + e*x) - (b^2 + c^2)^(1/2))^(5/2), x)
```

3.438 $\int \left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2} dx$

3.438.1 Optimal result

Integrand size = 34, antiderivative size = 130

$$\int \left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2} dx = \frac{8\sqrt{b^2+c^2}(c\cos(d+ex)-b\sin(d+ex))}{3e\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}} - \frac{2(c\cos(d+ex)-b\sin(d+ex))\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}{3e}$$

output
```
8/3*(c*cos(e*x+d)-b*sin(e*x+d))*(b^2+c^2)^(1/2)/e/(b*cos(e*x+d)+c*sin(e*x+
d)-(b^2+c^2)^(1/2))^(1/2)-2/3*(c*cos(e*x+d)-b*sin(e*x+d))*(b*cos(e*x+d)+c*
sin(e*x+d)-(b^2+c^2)^(1/2))^(1/2)/e
```

3.438.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 13.03 (sec) , antiderivative size = 5142, normalized size of antiderivative = 39.55

$$\int \left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2} dx = \text{Result too large to show}$$

input
```
Integrate[(-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(3/2),x]
```

output
```
Result too large to show
```

3.438.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {3042, 3592, 3042, 3591}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}dx$$

$$\Big\downarrow \text{3592}$$

$$-\frac{4}{3}\sqrt{b^2+c^2}\int \sqrt{b\cos(d+ex)+c\sin(d+ex)-\sqrt{b^2+c^2}}dx-\frac{2\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}(c\cos(d+ex)-b\sin(d+ex))}{3e}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{4}{3}\sqrt{b^2+c^2}\int \sqrt{b\cos(d+ex)+c\sin(d+ex)-\sqrt{b^2+c^2}}dx-\frac{2\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}(c\cos(d+ex)-b\sin(d+ex))}{3e}$$

$$\Big\downarrow \text{3591}$$

$$\frac{8\sqrt{b^2+c^2}(c\cos(d+ex)-b\sin(d+ex))}{3e\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}-\frac{2(c\cos(d+ex)-b\sin(d+ex))\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}{3e}$$

input
```
Int[(-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(3/2),x]
```

output
```
(8*Sqrt[b^2 + c^2]*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(3*e*Sqrt[-Sqrt[b^2
+ c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]]) - (2*(c*Cos[d + e*x] - b*Sin[d
+ e*x])*Sqrt[-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]])/(3*e)
```

3.438.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3591
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[-2*((c*Cos[d + e*x] - b*Sin[d + e*x])/(e*Sqrt[a + b*
Cos[d + e*x] + c*Sin[d + e*x]])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

rule 3592
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Simp[a*((2*n - 1)/n)   Int[(a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e
}, x] && EqQ[a^2 - b^2 - c^2, 0] && GtQ[n, 0]
```

3.438.4 Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{2(\sin(ex+d-\arctan(-b,c))-1)\left(b^2+c^2\right)(\sin(ex+d-\arctan(-b,c))+1)(\sin(ex+d-\arctan(-b,c))-5)}{3\cos(ex+d-\arctan(-b,c))\sqrt{\frac{b^2\sin(ex+d-\arctan(-b,c))+c^2\sin(ex+d-\arctan(-b,c))-b^2-c^2}{\sqrt{b^2+c^2}}}\,e}$	130

input
```
int((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(3/2),x,method=_RETURNVERB
OSE)
```

output
```
2/3*(sin(e*x+d-arctan(-b,c))-1)*(b^2+c^2)*(sin(e*x+d-arctan(-b,c))+1)*(sin
(e*x+d-arctan(-b,c))-5)/cos(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,
c))+c^2*sin(e*x+d-arctan(-b,c))-b^2-c^2)/(b^2+c^2)^(1/2))^(1/2)/e
```

3.438.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.98

$$\int \left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}\,dx = \frac{2\left(2\,bc\cos\left(ex+d\right)\sin\left(ex+d\right)+\left(b^2-c^2\right)\cos\left(ex+d\right)^2-5\,b^2-4\,c^2-4\,\sqrt{b^2+c^2}(b\cos\left(ex+\right.\right.}{3\left(ce\cos\left(ex+d\right)-be\sin\left(ex+\right.\right.}$$

input
```
integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(3/2),x, algorithm="
fricas")
```

output
```
2/3*(2*b*c*cos(e*x + d)*sin(e*x + d) + (b^2 - c^2)*cos(e*x + d)^2 - 5*b^2
- 4*c^2 - 4*sqrt(b^2 + c^2)*(b*cos(e*x + d) + c*sin(e*x + d)))*sqrt(b*cos(
e*x + d) + c*sin(e*x + d) - sqrt(b^2 + c^2))/(c*e*cos(e*x + d) - b*e*sin(e
*x + d))
```

3.438.6 Sympy [F]

$$\int \left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}\,dx = \int \left(b\cos\left(d+ex\right)+c\sin\left(d+ex\right)-\sqrt{b^2+c^2}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b**2+c**2)**(1/2))**(3/2),x)
```

output
```
Integral((b*cos(d + e*x) + c*sin(d + e*x) - sqrt(b**2 + c**2))**(3/2), x)
```

3.438.7 Maxima [F(-2)]

Exception generated.

$$\int \left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(3/2),x, algorithm="
maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.438.8 Giac [F(-2)]

Exception generated.

$$\int \left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(3/2),x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.438.9 Mupad [F(-1)]

Timed out.

$$\int \left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}\,dx = \int \left(b\,\cos\left(d+e\,x\right)+c\,\sin\left(d+e\,x\right)-\sqrt{b^2+c^2}\right)^{3/2}\,dx$$

input
```
int((b*cos(d + e*x) + c*sin(d + e*x) - (b^2 + c^2)^(1/2))^(3/2),x)
```

output
```
int((b*cos(d + e*x) + c*sin(d + e*x) - (b^2 + c^2)^(1/2))^(3/2), x)
```

3.439 $\int \sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}\,dx$

3.439.1 Optimal result

Integrand size = 34, antiderivative size = 57

$$\int \sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}\,dx = -\frac{2(c\cos(d+ex)-b\sin(d+ex))}{e\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}$$

output
```
-2*(c*cos(e*x+d)-b*sin(e*x+d))/e/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2
))^(1/2)
```

3.439.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 18.30 (sec) , antiderivative size = 5053, normalized size of antiderivative = 88.65

$$\int \sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sqrt[-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]],x]
```

output
```
Result too large to show
```

3.439.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3042, 3591}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}dx$$
$$\Big\downarrow \text{3591}$$
$$-\frac{2(c\cos(d+ex)-b\sin(d+ex))}{e\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}$$

input
```
Int[Sqrt[-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]],x]
```

output
```
(-2*(c*Cos[d + e*x] - b*Sin[d + e*x]))/(e*Sqrt[-Sqrt[b^2 + c^2] + b*Cos[d
+ e*x] + c*Sin[d + e*x]])
```

3.439.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3591
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[-2*((c*Cos[d + e*x] - b*Sin[d + e*x])/(e*Sqrt[a + b*
Cos[d + e*x] + c*Sin[d + e*x]])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^
2 - b^2 - c^2, 0]
```

3.439.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(53) = 106$.

Time = 1.34 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.05

method	result
default	$\frac{2(\sin(ex+d-\arctan(-b,c))-1)\sqrt{b^2+c^2}\,(\sin(ex+d-\arctan(-b,c))+1)}{\cos(ex+d-\arctan(-b,c))\sqrt{\frac{b^2\sin(ex+d-\arctan(-b,c))+c^2\sin(ex+d-\arctan(-b,c))-b^2-c^2}{\sqrt{b^2+c^2}}}\,e}$
risch	$\frac{\sqrt{2}\,\sqrt{-2\sqrt{b^2+c^2}+2b\cos(ex+d)+2c\sin(ex+d)}\,(ib+c)\left(i\sqrt{b^2+c^2}\,c+b^2\mathrm{e}^{i(ex+d)}+c^2\mathrm{e}^{i(ex+d)}+\sqrt{b^2+c^2}\,b\right)\left(-i\sqrt{b^2+c^2}\,c+b^2\mathrm{e}^{i(ex+d)}+c^2\right.}{\left(ic\,\mathrm{e}^{2i(ex+d)}-b\,\mathrm{e}^{2i(ex+d)}+2\sqrt{b^2+c^2}\,\mathrm{e}^{i(ex+d)}-ic-b\right)(b^2+c^2)^2 e}$

input
```
int((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
2*(sin(e*x+d-arctan(-b,c))-1)*(b^2+c^2)^(1/2)*(sin(e*x+d-arctan(-b,c))+1)/
cos(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-arctan
(-b,c))-b^2-c^2)/(b^2+c^2)^(1/2))^(1/2)/e
```

3.439.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.40

$$\int \sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}\,dx$$
$$= \frac{2\left(b\cos(ex+d)+c\sin(ex+d)+\sqrt{b^2+c^2}\right)\sqrt{b\cos(ex+d)+c\sin(ex+d)-\sqrt{b^2+c^2}}}{ce\cos(ex+d)-be\sin(ex+d)}$$

input
```
integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(1/2),x, algorithm="
fricas")
```

output
```
2*(b*cos(e*x + d) + c*sin(e*x + d) + sqrt(b^2 + c^2))*sqrt(b*cos(e*x + d)
+ c*sin(e*x + d) - sqrt(b^2 + c^2))/(c*e*cos(e*x + d) - b*e*sin(e*x + d))
```

3.439.6 Sympy [F]

$$\int \sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}\,dx = \int \sqrt{b\cos(d+ex)+c\sin(d+ex)-\sqrt{b^2+c^2}}\,dx$$

input
```
integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b**2+c**2)**(1/2))**(1/2),x)
```

output
```
Integral(sqrt(b*cos(d + e*x) + c*sin(d + e*x) - sqrt(b**2 + c**2)), x)
```

3.439.7 Maxima [F(-2)]

Exception generated.

$$\int \sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(1/2),x, algorithm="
maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.439.8 Giac [F(-2)]

Exception generated.

$$\int \sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(1/2),x, algorithm="
giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.439.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}\,dx = \int \sqrt{b\,\cos\left(d+e\,x\right)+c\,\sin\left(d+e\,x\right)-\sqrt{b^2+c^2}}\,dx$$

input
```
int((b*cos(d + e*x) + c*sin(d + e*x) - (b^2 + c^2)^(1/2))^(1/2),x)
```

output
```
int((b*cos(d + e*x) + c*sin(d + e*x) - (b^2 + c^2)^(1/2))^(1/2), x)
```

3.440 $\int \frac{1}{\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}\,dx$

3.440.1 Optimal result

Integrand size = 34, antiderivative size = 91

$$\int \frac{1}{\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}\,dx = -\frac{\sqrt{2}\arctan\left(\frac{\sqrt[4]{b^2+c^2}\sin\left(d+ex-\tan^{-1}(b,c)\right)}{\sqrt{2}\sqrt{-\sqrt{b^2+c^2}+\sqrt{b^2+c^2}\cos\left(d+ex-\tan^{-1}(b,c)\right)}}\right)}{\sqrt[4]{b^2+c^2}\,e}$$

output
```
-arctan(1/2*(b^2+c^2)^(1/4)*sin(d+e*x-arctan(b,c))*2^(1/2)/(-(b^2+c^2)^(1/
2)+cos(d+e*x-arctan(b,c))*(b^2+c^2)^(1/2))^(1/2))*2^(1/2)/(b^2+c^2)^(1/4)/
e
```

3.440.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 61.11 (sec) , antiderivative size = 61904, normalized size of antiderivative = 680.26

$$\int \frac{1}{\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/Sqrt[-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]],x]
```

output
```
Result too large to show
```

3.440.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {3042, 3594, 3042, 3128, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}dx$$

$$\Big\downarrow \text{3594}$$

$$\int \frac{1}{\sqrt{\sqrt{b^2+c^2}\cos\left(-\tan^{-1}(b,c)+d+ex\right)-\sqrt{b^2+c^2}}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\sqrt{b^2+c^2}\sin\left(-\tan^{-1}(b,c)+d+ex+\frac{\pi}{2}\right)-\sqrt{b^2+c^2}}}dx$$

$$\Big\downarrow \text{3128}$$

$$-\frac{2\int \frac{1}{-\frac{\left(b^2+c^2\right)\sin^2\left(d+ex-\tan^{-1}(b,c)\right)}{\sqrt{b^2+c^2}\cos\left(d+ex-\tan^{-1}(b,c)\right)-\sqrt{b^2+c^2}}-2\sqrt{b^2+c^2}}d\left(-\frac{\sqrt{b^2+c^2}\sin(d+ex-\tan^{-1}(b,c))}{\sqrt{\sqrt{b^2+c^2}\cos(d+ex-\tan^{-1}(b,c))-\sqrt{b^2+c^2}}}\right)}{e}$$

$$\Big\downarrow \text{217}$$

$$-\frac{\sqrt{2}\arctan\left(\frac{\sqrt[4]{b^2+c^2}\sin(-\tan^{-1}(b,c)+d+ex)}{\sqrt{2}\sqrt{\sqrt{b^2+c^2}\cos(-\tan^{-1}(b,c)+d+ex)-\sqrt{b^2+c^2}}}\right)}{e\sqrt[4]{b^2+c^2}}$$

input
```
Int[1/Sqrt[-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x]],x]
```

output
```
-((Sqrt[2]*ArcTan[((b^2 + c^2)^(1/4)*Sin[d + e*x - ArcTan[b, c]])/(Sqrt[2]
*Sqrt[-Sqrt[b^2 + c^2] + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]])])/(
(b^2 + c^2)^(1/4)*e))
```

3.440.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3594
```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

3.440.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(78) = 156$.

Time = 4.73 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.92

method	result	s
default	$\frac{(\sin(ex+d-\arctan(-b,c))-1)\sqrt{-(\sin(ex+d-\arctan(-b,c))+1)\sqrt{b^2+c^2}}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{-(\sin(ex+d-\arctan(-b,c))+1)\sqrt{b^2+c^2}}\,\sqrt{2}}{2\left(b^2+c^2\right)^{\frac{1}{4}}}\right)}{\left(b^2+c^2\right)^{\frac{1}{4}}\cos(ex+d-\arctan(-b,c))\sqrt{\frac{b^2\sin(ex+d-\arctan(-b,c))+c^2\sin(ex+d-\arctan(-b,c))-b^2-c^2}{\sqrt{b^2+c^2}}}\,e}$	

input
```
int(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(1/2),x,method=_RETURNVE
RBOSE)
```

output
```
(sin(e*x+d-arctan(-b,c))-1)*(-(sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2))
^(1/2)*2^(1/2)/(b^2+c^2)^(1/4)*arctan(1/2*(-(sin(e*x+d-arctan(-b,c))+1)*(b
^2+c^2)^(1/2))^(1/2)*2^(1/2)/(b^2+c^2)^(1/4))/cos(e*x+d-arctan(-b,c))/((b^
2*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x+d-arctan(-b,c))-b^2-c^2)/(b^2+c^2)^(
1/2))^(1/2)/e
```

3.440.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.18

$$\int \frac{1}{\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}\,dx = \frac{\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(b\cos(ex+d)+c\sin(ex+d)+\sqrt{b^2+c^2}\right)\sqrt{b\cos(ex+d)+c\sin(ex+d)-\sqrt{b^2+c^2}}}{2\,(b^2+c^2)^{\frac{1}{4}}(c\cos(ex+d)-b\sin(ex+d))}\right)}{(b^2+c^2)^{\frac{1}{4}}e}$$

input
```
integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(1/2),x, algorithm
="fricas")
```

output
```
sqrt(2)*arctan(-1/2*sqrt(2)*(b*cos(e*x + d) + c*sin(e*x + d) + sqrt(b^2 +
c^2))*sqrt(b*cos(e*x + d) + c*sin(e*x + d) - sqrt(b^2 + c^2))/((b^2 + c^2)
^(1/4)*(c*cos(e*x + d) - b*sin(e*x + d))))/((b^2 + c^2)^(1/4)*e)
```

3.440.6 Sympy [F]

$$\int \frac{1}{\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}\,dx = \int \frac{1}{\sqrt{b\cos{(d+ex)}+c\sin{(d+ex)}-\sqrt{b^2+c^2}}}\,dx$$

input
```
integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b**2+c**2)**(1/2))**(1/2),x)
```

output
```
Integral(1/sqrt(b*cos(d + e*x) + c*sin(d + e*x) - sqrt(b**2 + c**2)), x)
```

3.440.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(1/2),x, algorithm
="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.440.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(1/2),x, algorithm
="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.440.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}\,dx = \int \frac{1}{\sqrt{b\,\cos\left(d+e\,x\right)+c\,\sin\left(d+e\,x\right)-\sqrt{b^2+c^2}}}\,dx$$

input
```
int(1/(b*cos(d + e*x) + c*sin(d + e*x) - (b^2 + c^2)^(1/2))^(1/2),x)
```

output
```
int(1/(b*cos(d + e*x) + c*sin(d + e*x) - (b^2 + c^2)^(1/2))^(1/2), x)
```

3.440. $\int \frac{1}{\sqrt{-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)}}\,dx$

3.441 $\int \frac{1}{\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}\,dx$

3.441.1 Optimal result

Integrand size = 34, antiderivative size = 164

$$\int \frac{1}{\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}\,dx=\frac{\arctan\left(\frac{\sqrt[4]{b^2+c^2}\sin\left(d+ex-\tan^{-1}(b,c)\right)}{\sqrt{2}\sqrt{-\sqrt{b^2+c^2}+\sqrt{b^2+c^2}\cos\left(d+ex-\tan^{-1}(b,c)\right)}}\right)}{2\sqrt{2}\left(b^2+c^2\right)^{3/4}e}+\frac{c\cos(d+ex)-b\sin(d+ex)}{2\sqrt{b^2+c^2}e\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}$$

output
```
1/4*arctan(1/2*(b^2+c^2)^(1/4)*sin(d+e*x-arctan(b,c))*2^(1/2)/(-(b^2+c^2)^
(1/2)+cos(d+e*x-arctan(b,c))*(b^2+c^2)^(1/2))^(1/2))/(b^2+c^2)^(3/4)/e*2^(
1/2)+1/2*(c*cos(e*x+d)-b*sin(e*x+d))/e/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2
)^(1/2))^(3/2)/(b^2+c^2)^(1/2)
```

3.441.2 Mathematica [F(-1)]

Timed out.

$$\int \frac{1}{\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}\,dx=\$\text{Aborted}$$

input
```
Integrate[(-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-3/2),x]
```

output
```
$Aborted
```

3.441.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$, Rules used = {3042, 3595, 3042, 3594, 3042, 3128, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3595}$$

$$\frac{c\cos(d+ex)-b\sin(d+ex)}{2e\sqrt{b^2+c^2}\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}-\frac{\int \frac{1}{\sqrt{b\cos(d+ex)+c\sin(d+ex)-\sqrt{b^2+c^2}}}dx}{4\sqrt{b^2+c^2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{c\cos(d+ex)-b\sin(d+ex)}{2e\sqrt{b^2+c^2}\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}-\frac{\int \frac{1}{\sqrt{b\cos(d+ex)+c\sin(d+ex)-\sqrt{b^2+c^2}}}dx}{4\sqrt{b^2+c^2}}$$

$$\Big\downarrow \text{3594}$$

$$\frac{c\cos(d+ex)-b\sin(d+ex)}{2e\sqrt{b^2+c^2}\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}-\frac{\int \frac{1}{\sqrt{\sqrt{b^2+c^2}\cos(d+ex-\tan^{-1}(b,c))-\sqrt{b^2+c^2}}}dx}{4\sqrt{b^2+c^2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{c\cos(d+ex)-b\sin(d+ex)}{2e\sqrt{b^2+c^2}\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}-\frac{\int \frac{1}{\sqrt{\sqrt{b^2+c^2}\sin\left(d+ex-\tan^{-1}(b,c)+\frac{\pi}{2}\right)-\sqrt{b^2+c^2}}}dx}{4\sqrt{b^2+c^2}}$$

$$\Big\downarrow \text{3128}$$

$$\frac{\int \frac{1}{-\frac{(b^2+c^2)\sin^2\left(d+ex-\tan^{-1}(b,c)\right)}{\sqrt{b^2+c^2}\cos\left(d+ex-\tan^{-1}(b,c)\right)-\sqrt{b^2+c^2}}-2\sqrt{b^2+c^2}}d\left(-\frac{\sqrt{b^2+c^2}\sin(d+ex-\tan^{-1}(b,c))}{\sqrt{\sqrt{b^2+c^2}\cos(d+ex-\tan^{-1}(b,c))-\sqrt{b^2+c^2}}}\right)}{2e\sqrt{b^2+c^2}}+\frac{c\cos(d+ex)-b\sin(d+ex)}{2e\sqrt{b^2+c^2}\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}$$

$$\Big\downarrow \text{217}$$

$$\frac{\arctan\left(\frac{\sqrt[4]{b^2+c^2}\sin(-\tan^{-1}(b,c)+d+ex)}{\sqrt{2}\sqrt{\sqrt{b^2+c^2}\cos(-\tan^{-1}(b,c)+d+ex)-\sqrt{b^2+c^2}}}\right)}{2\sqrt{2}e\left(b^2+c^2\right)^{3/4}}+\frac{c\cos(d+ex)-b\sin(d+ex)}{2e\sqrt{b^2+c^2}\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}$$

input
```
Int[(-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-3/2),x]
```

output
```
ArcTan[((b^2 + c^2)^(1/4)*Sin[d + e*x - ArcTan[b, c]])/(Sqrt[2]*Sqrt[-Sqrt
[b^2 + c^2] + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b, c]]])]/(2*Sqrt[2]*(b
^2 + c^2)^(3/4)*e) + (c*Cos[d + e*x] - b*Sin[d + e*x])/(2*Sqrt[b^2 + c^2]*
e*(-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(3/2))
```

3.441.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

3.441. $\int \frac{1}{\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}\,dx$

rule 3594
```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

rule 3595
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(c*Cos[d + e*x] - b*Sin[d + e*x])*((a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n/(a*e*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n + 1))
 Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b,
c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

3.441.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 363 vs. $2(141) = 282$.

Time = 1.58 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.22

method	result
default	$-\frac{\left(\sin(ex+d-\arctan(-b,c))\sqrt{2}\arctan\left(\frac{\sqrt{-\sqrt{b^2+c^2}\sin(ex+d-\arctan(-b,c))-\sqrt{b^2+c^2}}\sqrt{2}}{2\left(b^2+c^2\right)^{\frac{1}{4}}}\right)(b^2+c^2)-\sqrt{2}\arctan\left(\frac{\sqrt{-\sqrt{b^2+c^2}\sin(ex+}}{2}\right.\right.}{4(b^2+c^2)^{\frac{7}{4}}c}$

input
```
int(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(3/2),x,method=_RETURNVE
RBOSE)
```

output
```
-1/4/(b^2+c^2)^(7/4)*(sin(e*x+d-arctan(-b,c))*2^(1/2)*arctan(1/2*(-(b^2+c^
2)^(1/2)*sin(e*x+d-arctan(-b,c))-(b^2+c^2)^(1/2))^(1/2)*2^(1/2)/(b^2+c^2)^
(1/4))*(b^2+c^2)-2^(1/2)*arctan(1/2*(-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,
c))-(b^2+c^2)^(1/2))^(1/2)*2^(1/2)/(b^2+c^2)^(1/4))*b^2-2^(1/2)*arctan(1/2
*(-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-(b^2+c^2)^(1/2))^(1/2)*2^(1/2)/
(b^2+c^2)^(1/4))*c^2-2*(-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-(b^2+c^2)
^(1/2))^(1/2)*(b^2+c^2)^(3/4))*(-(sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/
2))^(1/2)/cos(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(e*
x+d-arctan(-b,c))-b^2-c^2)/(b^2+c^2)^(1/2))^(1/2)/e
```

3.441.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 442 vs. $2(147) = 294$.

Time = 0.31 (sec) , antiderivative size = 442, normalized size of antiderivative = 2.70

$$\int \frac{1}{\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}\,dx = \frac{\left(3\sqrt{2}b^2c\cos\left(ex+d\right)-\sqrt{2}(3b^2c-c^3)\cos\left(ex+d\right)^{}\right.}{}$$

input
```
integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(3/2),x, algorithm
="fricas")
```

output
```
1/4*((3*sqrt(2)*b^2*c*cos(e*x + d) - sqrt(2)*(3*b^2*c - c^3)*cos(e*x + d)^
3 - (sqrt(2)*b^3 - sqrt(2)*(b^3 - 3*b*c^2)*cos(e*x + d)^2)*sin(e*x + d))*(
b^2 + c^2)^(1/4)*arctan(-1/2*(b^2 + c^2)^(1/4)*sqrt(b*cos(e*x + d) + c*sin
(e*x + d) - sqrt(b^2 + c^2))*((sqrt(2)*b*cos(e*x + d) + sqrt(2)*c*sin(e*x
+ d))*sqrt(b^2 + c^2) + sqrt(2)*(b^2 + c^2))/((b^2*c + c^3)*cos(e*x + d) -
 (b^3 + b*c^2)*sin(e*x + d))) - 2*(2*(b^3 + b*c^2)*cos(e*x + d) + 2*(b^2*c
 + c^3)*sin(e*x + d) + (2*b*c*cos(e*x + d)*sin(e*x + d) + (b^2 - c^2)*cos(
e*x + d)^2 + b^2 + 2*c^2)*sqrt(b^2 + c^2))*sqrt(b*cos(e*x + d) + c*sin(e*x
 + d) - sqrt(b^2 + c^2)))/((3*b^4*c + 2*b^2*c^3 - c^5)*e*cos(e*x + d)^3 -
3*(b^4*c + b^2*c^3)*e*cos(e*x + d) - ((b^5 - 2*b^3*c^2 - 3*b*c^4)*e*cos(e*
x + d)^2 - (b^5 + b^3*c^2)*e)*sin(e*x + d))
```

3.441.6 Sympy [F]

$$\int \frac{1}{\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}\,dx = \int \frac{1}{\left(b\cos\left(d+ex\right)+c\sin\left(d+ex\right)-\sqrt{b^2+c^2}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b**2+c**2)**(1/2))**(3/2),x)
```

output
```
Integral((b*cos(d + e*x) + c*sin(d + e*x) - sqrt(b**2 + c**2))**(-3/2), x)
```

3.441.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(3/2),x, algorithm
="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.441.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(3/2),x, algorithm
="giac")
```

output
```
Timed out
```

3.441.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}\,dx = \int \frac{1}{\left(b\,\cos\left(d+e\,x\right)+c\,\sin\left(d+e\,x\right)-\sqrt{b^2+c^2}\right)^{3/2}}$$

input
```
int(1/(b*cos(d + e*x) + c*sin(d + e*x) - (b^2 + c^2)^(1/2))^(3/2),x)
```

output
```
int(1/(b*cos(d + e*x) + c*sin(d + e*x) - (b^2 + c^2)^(1/2))^(3/2), x)
```

3.442 $\int \frac{1}{\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}\,dx$

3.442.1 Optimal result

Integrand size = 34, antiderivative size = 232

$$\int \frac{1}{\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}\,dx = -\frac{3\arctan\left(\frac{\sqrt[4]{b^2+c^2}\sin(d+ex-\tan^{-1}(b,c))}{\sqrt{2}\sqrt{-\sqrt{b^2+c^2}+\sqrt{b^2+c^2}\cos(d+ex-\tan^{-1}(b,c))}}\right)}{16\sqrt{2}\left(b^2+c^2\right)^{5/4}e} + \frac{c\cos(d+ex)-b\sin(d+ex)}{4\sqrt{b^2+c^2}e\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}} - \frac{3(c\cos(d+ex)-b\sin(d+ex))}{16\left(b^2+c^2\right)e\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}$$

output
```
-3/32*arctan(1/2*(b^2+c^2)^(1/4)*sin(d+e*x-arctan(b,c))*2^(1/2)/(-(b^2+c^2
)^(1/2)+cos(d+e*x-arctan(b,c))*(b^2+c^2)^(1/2))^(1/2))/(b^2+c^2)^(5/4)/e*2
^(1/2)-3/16*(c*cos(e*x+d)-b*sin(e*x+d))/(b^2+c^2)/e/(b*cos(e*x+d)+c*sin(e*
x+d)-(b^2+c^2)^(1/2))^(3/2)+1/4*(c*cos(e*x+d)-b*sin(e*x+d))/e/(b*cos(e*x+d
)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(5/2)/(b^2+c^2)^(1/2)
```

3.442.2 Mathematica [F(-1)]

Timed out.

$$\int \frac{1}{\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}\,dx = \$\text{Aborted}$$

input
```
Integrate[(-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-5/2),x]
```

output
```
$Aborted
```

3.442.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.265, Rules used = {3042, 3595, 3042, 3595, 3042, 3594, 3042, 3128, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 3595}$$

$$\frac{c\cos(d+ex)-b\sin(d+ex)}{4e\sqrt{b^2+c^2}\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}-\frac{3\int \frac{1}{\left(b\cos(d+ex)+c\sin(d+ex)-\sqrt{b^2+c^2}\right)^{3/2}}dx}{8\sqrt{b^2+c^2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{c\cos(d+ex)-b\sin(d+ex)}{4e\sqrt{b^2+c^2}\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}-\frac{3\int\frac{1}{\left(b\cos(d+ex)+c\sin(d+ex)-\sqrt{b^2+c^2}\right)^{3/2}}dx}{8\sqrt{b^2+c^2}}$$

$$\Big\downarrow \text{3595}$$

$$\frac{c\cos(d+ex)-b\sin(d+ex)}{4e\sqrt{b^2+c^2}\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}-\frac{3\left(\frac{c\cos(d+ex)-b\sin(d+ex)}{2e\sqrt{b^2+c^2}\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}-\frac{\int\frac{1}{\sqrt{b\cos(d+ex)+c\sin(d+ex)-\sqrt{b^2+c^2}}}dx}{4\sqrt{b^2+c^2}}\right)}{8\sqrt{b^2+c^2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{c\cos(d+ex)-b\sin(d+ex)}{4e\sqrt{b^2+c^2}\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}-\frac{3\left(\frac{c\cos(d+ex)-b\sin(d+ex)}{2e\sqrt{b^2+c^2}\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}-\frac{\int\frac{1}{\sqrt{b\cos(d+ex)+c\sin(d+ex)-\sqrt{b^2+c^2}}}dx}{4\sqrt{b^2+c^2}}\right)}{8\sqrt{b^2+c^2}}$$

$$\Big\downarrow \text{3594}$$

$$\frac{c\cos(d+ex)-b\sin(d+ex)}{4e\sqrt{b^2+c^2}\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}-\frac{3\left(\frac{c\cos(d+ex)-b\sin(d+ex)}{2e\sqrt{b^2+c^2}\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}-\frac{\int\frac{1}{\sqrt{\sqrt{b^2+c^2}\cos\left(d+ex-\tan^{-1}(b,c)\right)-\sqrt{b^2+c^2}}}dx}{4\sqrt{b^2+c^2}}\right)}{8\sqrt{b^2+c^2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{c\cos(d+ex)-b\sin(d+ex)}{4e\sqrt{b^2+c^2}\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}-\frac{3\left(\frac{c\cos(d+ex)-b\sin(d+ex)}{2e\sqrt{b^2+c^2}\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}-\frac{\int\frac{1}{\sqrt{\sqrt{b^2+c^2}\sin\left(d+ex-\tan^{-1}(b,c)+\frac{\pi}{2}\right)-\sqrt{b^2+c^2}}}dx}{4\sqrt{b^2+c^2}}\right)}{8\sqrt{b^2+c^2}}$$

$$\Big\downarrow \text{3128}$$

3.442. $\int\frac{1}{\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}dx$

$$\frac{c\cos(d+ex)-b\sin(d+ex)}{4e\sqrt{b^2+c^2}\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}-\frac{3\left(\frac{\int\frac{1}{-\frac{(b^2+c^2)\sin^2(d+ex-\tan^{-1}(b,c))}{\sqrt{b^2+c^2}\cos(d+ex-\tan^{-1}(b,c))-\sqrt{b^2+c^2}}-2\sqrt{b^2+c^2}}d\left(-\frac{\sqrt{b^2+c^2}\sin(d+ex-\tan^{-1}(b,c))}{\sqrt{\sqrt{b^2+c^2}\cos(d+ex-\tan^{-1}(b,c))-\sqrt{b^2+c^2}}}\right)}{2e\sqrt{b^2+c^2}}+\frac{c\cos(d+ex)-b\sin(d+}{2e\sqrt{b^2+c^2}\left(-\sqrt{b^2+c^2}+b\cos(d+ex)-\right.}\right.}{8\sqrt{b^2+c^2}}$$

$$\Big\downarrow \text{217}$$

$$\frac{c\cos(d+ex)-b\sin(d+ex)}{4e\sqrt{b^2+c^2}\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}-\frac{3\left(\frac{\arctan\left(\frac{\sqrt[4]{b^2+c^2}\sin(-\tan^{-1}(b,c)+d+ex)}{\sqrt{2}\sqrt{\sqrt{b^2+c^2}\cos(-\tan^{-1}(b,c)+d+ex)-\sqrt{b^2+c^2}}}\right)}{2\sqrt{2}e(b^2+c^2)^{3/4}}+\frac{c\cos(d+ex)-b\sin(d+ex)}{2e\sqrt{b^2+c^2}\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{3/2}}\right)}{8\sqrt{b^2+c^2}}$$

input
```
Int[(-Sqrt[b^2 + c^2] + b*Cos[d + e*x] + c*Sin[d + e*x])^(-5/2),x]
```

output
```
(c*Cos[d + e*x] - b*Sin[d + e*x])/(4*Sqrt[b^2 + c^2]*e*(-Sqrt[b^2 + c^2] +
 b*Cos[d + e*x] + c*Sin[d + e*x])^(5/2)) - (3*(ArcTan[((b^2 + c^2)^(1/4)*S
in[d + e*x - ArcTan[b, c]])/(Sqrt[2]*Sqrt[-Sqrt[b^2 + c^2] + Sqrt[b^2 + c^
2]*Cos[d + e*x - ArcTan[b, c]]])]/(2*Sqrt[2]*(b^2 + c^2)^(3/4)*e) + (c*Cos
[d + e*x] - b*Sin[d + e*x])/(2*Sqrt[b^2 + c^2]*e*(-Sqrt[b^2 + c^2] + b*Cos
[d + e*x] + c*Sin[d + e*x])^(3/2))))/(8*Sqrt[b^2 + c^2])
```

3.442.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.442. $\int \frac{1}{\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}\,dx$

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3594
```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Int[1/Sqrt[a + Sqrt[b^2 + c^2]*Cos[d + e*x - ArcTan[b,
c]]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

rule 3595
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(c*Cos[d + e*x] - b*Sin[d + e*x])*((a + b*Cos[d + e
*x] + c*Sin[d + e*x])^n/(a*e*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n + 1))
 Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x] /; FreeQ[{a, b,
c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1]
```

3.442.4 Maple [A] (verified)

Time = 1.63 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.57

method	result
default	$\frac{\left(\sin(ex+d-\arctan(-b,c))\sqrt{2}\arctan\left(\frac{\sqrt{-\sqrt{b^2+c^2}\sin(ex+d-\arctan(-b,c))-\sqrt{b^2+c^2}}\sqrt{2}}{2\left(b^2+c^2\right)^{\frac{1}{4}}}\right)(b^2+c^2)-\sqrt{2}\arctan\left(\frac{\sqrt{-\sqrt{b^2+c^2}\sin(ex+d-}}{2(b^}\right.}{4(b^2+c^2)^{\frac{5}{4}}\cos(}$

input
```
int(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(5/2),x,method=_RETURNVE
RBOSE)
```

output
```
1/4*(sin(e*x+d-arctan(-b,c))*2^(1/2)*arctan(1/2*(-(b^2+c^2)^(1/2)*sin(e*x+
d-arctan(-b,c))-(b^2+c^2)^(1/2))^(1/2)*2^(1/2)/(b^2+c^2)^(1/4))*(b^2+c^2)-
2^(1/2)*arctan(1/2*(-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-(b^2+c^2)^(1/
2))^(1/2)*2^(1/2)/(b^2+c^2)^(1/4))*b^2-2^(1/2)*arctan(1/2*(-(b^2+c^2)^(1/2
)*sin(e*x+d-arctan(-b,c))-(b^2+c^2)^(1/2))^(1/2)*2^(1/2)/(b^2+c^2)^(1/4))*
c^2-2*(-(b^2+c^2)^(1/2)*sin(e*x+d-arctan(-b,c))-(b^2+c^2)^(1/2))^(1/2)*(b^
2+c^2)^(3/4))*(-(sin(e*x+d-arctan(-b,c))+1)*(b^2+c^2)^(1/2))^(1/2)/(b^2+c^
2)^(5/4)/cos(e*x+d-arctan(-b,c))/((b^2*sin(e*x+d-arctan(-b,c))+c^2*sin(e*x
+d-arctan(-b,c))-b^2-c^2)/(b^2+c^2)^(1/2))^(1/2)/e
```

3.442.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 655 vs. $2(209) = 418$.

Time = 0.35 (sec) , antiderivative size = 655, normalized size of antiderivative = 2.82

$$\int \frac{1}{\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}\,dx = \frac{\frac{3\sqrt{\frac{1}{2}}\left(5\,b^4c\cos(ex+d)+(5\,b^4c-10\,b^2c^3+c^5)\cos(ex+d)^5-10\,(b^4c-b^2c}{}}{}$$

input
```
integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(5/2),x, algorithm
="fricas")
```

output
```
1/16*(3*sqrt(1/2)*(5*b^4*c*cos(e*x + d) + (5*b^4*c - 10*b^2*c^3 + c^5)*cos
(e*x + d)^5 - 10*(b^4*c - b^2*c^3)*cos(e*x + d)^3 - (b^5 + (b^5 - 10*b^3*c
^2 + 5*b*c^4)*cos(e*x + d)^4 - 2*(b^5 - 5*b^3*c^2)*cos(e*x + d)^2)*sin(e*x
 + d))*arctan(-sqrt(1/2)*(b*cos(e*x + d) + c*sin(e*x + d) + sqrt(b^2 + c^2
))*sqrt(b*cos(e*x + d) + c*sin(e*x + d) - sqrt(b^2 + c^2))/((b^2 + c^2)^(1
/4)*(c*cos(e*x + d) - b*sin(e*x + d))))/(b^2 + c^2)^(1/4) + (3*(b^4 - 6*b^
2*c^2 + c^4)*cos(e*x + d)^4 - 7*b^4 - 26*b^2*c^2 - 16*c^4 - 6*(2*b^4 - 3*b
^2*c^2 - c^4)*cos(e*x + d)^2 + 12*((b^3*c - b*c^3)*cos(e*x + d)^3 - (2*b^3
*c + b*c^3)*cos(e*x + d))*sin(e*x + d) + 2*((b^3 - 3*b*c^2)*cos(e*x + d)^3
 - 3*(3*b^3 + 2*b*c^2)*cos(e*x + d) - (9*b^2*c + 8*c^3 - (3*b^2*c - c^3)*c
os(e*x + d)^2)*sin(e*x + d))*sqrt(b^2 + c^2))*sqrt(b*cos(e*x + d) + c*sin(
e*x + d) - sqrt(b^2 + c^2)))/((5*b^6*c - 5*b^4*c^3 - 9*b^2*c^5 + c^7)*e*co
s(e*x + d)^5 - 10*(b^6*c - b^2*c^5)*e*cos(e*x + d)^3 + 5*(b^6*c + b^4*c^3)
*e*cos(e*x + d) - ((b^7 - 9*b^5*c^2 - 5*b^3*c^4 + 5*b*c^6)*e*cos(e*x + d)^
4 - 2*(b^7 - 4*b^5*c^2 - 5*b^3*c^4)*e*cos(e*x + d)^2 + (b^7 + b^5*c^2)*e)*
sin(e*x + d))
```

3.442.6 Sympy [F]

$$\int \frac{1}{\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}\,dx = \int \frac{1}{\left(b\cos\left(d+ex\right)+c\sin\left(d+ex\right)-\sqrt{b^2+c^2}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b**2+c**2)**(1/2))**(5/2),x)
```

output
```
Integral((b*cos(d + e*x) + c*sin(d + e*x) - sqrt(b**2 + c**2))**(-5/2), x)
```

3.442.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(5/2),x, algorithm
="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.442.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*cos(e*x+d)+c*sin(e*x+d)-(b^2+c^2)^(1/2))^(5/2),x, algorithm
="giac")
```

output
```
Timed out
```

3.442.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(-\sqrt{b^2+c^2}+b\cos(d+ex)+c\sin(d+ex)\right)^{5/2}}\,dx = \int \frac{1}{\left(b\cos(d+e\,x)+c\sin(d+e\,x)-\sqrt{b^2+c^2}\right)^{5/2}}$$

input
```
int(1/(b*cos(d + e*x) + c*sin(d + e*x) - (b^2 + c^2)^(1/2))^(5/2),x)
```

output
```
int(1/(b*cos(d + e*x) + c*sin(d + e*x) - (b^2 + c^2)^(1/2))^(5/2), x)
```

3.443 $\int \frac{\sin(x)}{a+b\cos(x)+c\sin(x)}\,dx$

3.443.1 Optimal result

Integrand size = 15, antiderivative size = 101

$$\int \frac{\sin(x)}{a+b\cos(x)+c\sin(x)}\,dx = \frac{cx}{b^2+c^2} - \frac{2ac\arctan\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}\,(b^2+c^2)} - \frac{b\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}$$

output
```
c*x/(b^2+c^2)-b*ln(a+b*cos(x)+c*sin(x))/(b^2+c^2)-2*a*c*arctan((c+(a-b)*ta
n(1/2*x))/(a^2-b^2-c^2)^(1/2))/(b^2+c^2)/(a^2-b^2-c^2)^(1/2)
```

3.443.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.79

$$\int \frac{\sin(x)}{a+b\cos(x)+c\sin(x)}\,dx = \frac{cx + \frac{2ac\,\text{arctanh}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2+c^2}}\right)}{\sqrt{-a^2+b^2+c^2}} - b\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}$$

input
```
Integrate[Sin[x]/(a + b*Cos[x] + c*Sin[x]),x]
```

output
```
(c*x + (2*a*c*ArcTanh[(c + (a - b)*Tan[x/2])/Sqrt[-a^2 + b^2 + c^2]])/Sqrt
[-a^2 + b^2 + c^2] - b*Log[a + b*Cos[x] + c*Sin[x]])/(b^2 + c^2)
```

3.443.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3616, 3042, 3603, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)}{a+b\cos(x)+c\sin(x)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin(x)}{a+b\cos(x)+c\sin(x)}dx$$

$$\Big\downarrow \; 3616$$

$$-\frac{ac\int \frac{1}{a+b\cos(x)+c\sin(x)}dx}{b^2+c^2}-\frac{b\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\frac{cx}{b^2+c^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{ac\int \frac{1}{a+b\cos(x)+c\sin(x)}dx}{b^2+c^2}-\frac{b\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\frac{cx}{b^2+c^2}$$

$$\Big\downarrow \; 3603$$

$$-\frac{2ac\int \frac{1}{(a-b)\tan^2\left(\frac{x}{2}\right)+2c\tan\left(\frac{x}{2}\right)+a+b}d\tan\left(\frac{x}{2}\right)}{b^2+c^2}-\frac{b\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\frac{cx}{b^2+c^2}$$

$$\Big\downarrow \; 1083$$

$$\frac{4ac\int \frac{1}{-\left(2c+2(a-b)\tan\left(\frac{x}{2}\right)\right)^2-4(a^2-b^2-c^2)}d\left(2c+2(a-b)\tan\left(\frac{x}{2}\right)\right)}{b^2+c^2}-\frac{b\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\frac{cx}{b^2+c^2}$$

$$\Big\downarrow \; 217$$

$$-\frac{2ac\arctan\left(\frac{2(a-b)\tan\left(\frac{x}{2}\right)+2c}{2\sqrt{a^2-b^2-c^2}}\right)}{(b^2+c^2)\sqrt{a^2-b^2-c^2}}-\frac{b\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\frac{cx}{b^2+c^2}$$

input
```
Int[Sin[x]/(a + b*Cos[x] + c*Sin[x]),x]
```

3.443. $\int \frac{\sin(x)}{a+b\cos(x)+c\sin(x)}\,dx$

output
```
(c*x)/(b^2 + c^2) - (2*a*c*ArcTan[(2*c + 2*(a - b)*Tan[x/2])/(2*Sqrt[a^2 -
 b^2 - c^2])])/(Sqrt[a^2 - b^2 - c^2]*(b^2 + c^2)) - (b*Log[a + b*Cos[x] +
 c*Sin[x]])/(b^2 + c^2)
```

3.443.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

rule 3616
```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[c*C*((d + e*x)/
(e*(b^2 + c^2))), x] + (-Simp[b*C*(Log[a + b*Cos[d + e*x] + c*Sin[d + e*x]]
/(e*(b^2 + c^2))), x] + Simp[(A*(b^2 + c^2) - a*c*C)/(b^2 + c^2)   Int[1/(a
 + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x]) /; FreeQ[{a, b, c, d, e, A, C}
, x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*c*C, 0]
```

3.443. $\int \frac{\sin(x)}{a+b\cos(x)+c\sin(x)}\,dx$

3.443.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.74

method	result
default	$\frac{2b\ln\left(1+\tan\left(\frac{x}{2}\right)^2\right)+4c\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{2b^2+2c^2}+\frac{\frac{4\left(-ab+b^2\right)\ln\left(\tan\left(\frac{x}{2}\right)^2a-\tan\left(\frac{x}{2}\right)^2b+2c\tan\left(\frac{x}{2}\right)+a+b\right)}{2a-2b}+\frac{4\left(-ac-cb-\frac{\left(-ab+b^2\right)c}{a-b}\right)\arctan\left(2\right.}{\sqrt{a^2-b^2-c^2}}}{2b^2+2c^2}$
risch	Expression too large to display

input
```
int(sin(x)/(a+b*cos(x)+c*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
4/(2*b^2+2*c^2)*(1/2*b*ln(1+tan(1/2*x)^2)+c*arctan(tan(1/2*x)))+4/(2*b^2+2
*c^2)*(1/2*(-a*b+b^2)/(a-b)*ln(tan(1/2*x)^2*a-tan(1/2*x)^2*b+2*c*tan(1/2*x
)+a+b)+(-a*c-c*b-(-a*b+b^2)*c/(a-b))/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-
b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2)))
```

3.443.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. 2(95) = 190.

Time = 0.31 (sec) , antiderivative size = 579, normalized size of antiderivative = 5.73

$$\int \frac{\sin(x)}{a+b\cos(x)+c\sin(x)}\,dx$$
$$= \left[-\frac{\sqrt{-a^2+b^2+c^2}ac\log\left(\frac{a^2b^2-2b^4-c^4-(a^2+3b^2)c^2-(2a^2b^2-b^4-2a^2c^2+c^4)\cos(x)^2-2(ab^3+abc^2)\cos(x)-2(ab^2c+ac^3-(bc^3-(}{2ab\cos(x)+(b^2-c^2)\cos}\right.}{}\right.$$
$$-\frac{2\sqrt{a^2-b^2-c^2}ac\arctan\left(-\frac{(ab\cos(x)+ac\sin(x)+b^2+c^2)\sqrt{a^2-b^2-c^2}}{(c^3-(a^2-b^2)c)\cos(x)+(a^2b-b^3-bc^2)\sin(x)}\right)+2(c^3-(a^2-b^2)c)x+(a^2b-b^3-bc}{2(a^2b^2-b^4-c^4+(a^2-2b^2)}$$

input
```
integrate(sin(x)/(a+b*cos(x)+c*sin(x)),x, algorithm="fricas")
```

output
```
[-1/2*(sqrt(-a^2 + b^2 + c^2)*a*c*log((a^2*b^2 - 2*b^4 - c^4 - (a^2 + 3*b^
2)*c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(x)^2 - 2*(a*b^3 + a*b*c^2
)*cos(x) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*cos(x))*sin(x)
 - 2*(2*a*b*c*cos(x)^2 - a*b*c + (b^2*c + c^3)*cos(x) - (b^3 + b*c^2 + (a*
b^2 - a*c^2)*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2))/(2*a*b*cos(x) + (b^2
- c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x))) + 2*(c^3 - (a^
2 - b^2)*c)*x + (a^2*b - b^3 - b*c^2)*log(2*a*b*cos(x) + (b^2 - c^2)*cos(x
)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x)))/(a^2*b^2 - b^4 - c^4 + (a^
2 - 2*b^2)*c^2), -1/2*(2*sqrt(a^2 - b^2 - c^2)*a*c*arctan(-(a*b*cos(x) + a
*c*sin(x) + b^2 + c^2)*sqrt(a^2 - b^2 - c^2)/((c^3 - (a^2 - b^2)*c)*cos(x)
 + (a^2*b - b^3 - b*c^2)*sin(x))) + 2*(c^3 - (a^2 - b^2)*c)*x + (a^2*b - b
^3 - b*c^2)*log(2*a*b*cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*c
os(x) + a*c)*sin(x)))/(a^2*b^2 - b^4 - c^4 + (a^2 - 2*b^2)*c^2)]
```

3.443.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin(x)}{a + b\cos(x) + c\sin(x)}\, dx = \text{Timed out}$$

input
```
integrate(sin(x)/(a+b*cos(x)+c*sin(x)),x)
```

output
```
Timed out
```

3.443.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sin(x)}{a + b\cos(x) + c\sin(x)}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate(sin(x)/(a+b*cos(x)+c*sin(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` f
or more de
```

3.443.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.58

$$\int \frac{\sin(x)}{a+b\cos(x)+c\sin(x)}\,dx = \frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}x\right)-b\tan\left(\frac{1}{2}x\right)+c}{\sqrt{a^2-b^2-c^2}}\right)\right)ac}{\sqrt{a^2-b^2-c^2}(b^2+c^2)}+\frac{cx}{b^2+c^2} - \frac{b\log\left(-a\tan\left(\frac{1}{2}x\right)^2+b\tan\left(\frac{1}{2}x\right)^2-2c\tan\left(\frac{1}{2}x\right)-a-b\right)}{b^2+c^2}+\frac{b\log\left(\tan\left(\frac{1}{2}x\right)^2+1\right)}{b^2+c^2}$$

input
```
integrate(sin(x)/(a+b*cos(x)+c*sin(x)),x, algorithm="giac")
```

output
```
2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*ta
n(1/2*x) + c)/sqrt(a^2 - b^2 - c^2)))*a*c/(sqrt(a^2 - b^2 - c^2)*(b^2 + c^
2)) + c*x/(b^2 + c^2) - b*log(-a*tan(1/2*x)^2 + b*tan(1/2*x)^2 - 2*c*tan(1
/2*x) - a - b)/(b^2 + c^2) + b*log(tan(1/2*x)^2 + 1)/(b^2 + c^2)
```

3.443.9 Mupad [B] (verification not implemented)

Time = 40.87 (sec) , antiderivative size = 950, normalized size of antiderivative = 9.41

$$\int \frac{\sin(x)}{a+b\cos(x)+c\sin(x)}\,dx = \frac{\ln\left(\tan\left(\frac{x}{2}\right)+1\mathrm{i}\right)}{b-c\,1\mathrm{i}} + \frac{\ln\left(64\tan\left(\frac{x}{2}\right)(a-b)^2 - \frac{\left(a^2b-bc^2-b^3+ac\sqrt{-a^2+b^2+c^2}\right)\left(32a^2c+32b^2c-64abc+64\tan\left(\frac{x}{2}\right)(a-b)(-a^2+ba+c^2)+\frac{\left(a^2b-bc\cdots\right.}{\cdots}\right.}{\cdots}\right)}{\cdots} - \frac{\ln\left(64\tan\left(\frac{x}{2}\right)(a-b)^2 + \frac{\left(bc^2-a^2b+b^3+ac\sqrt{-a^2+b^2+c^2}\right)\left(32a^2c+32b^2c-64abc+64\tan\left(\frac{x}{2}\right)(a-b)(-a^2+ba+c^2)+\frac{\left(bc^2-a^2\cdots\right.}{\cdots}\right.}{\cdots}\right)}{\cdots} + \frac{\ln\left(\tan\left(\frac{x}{2}\right)-\mathrm{i}\right)1\mathrm{i}}{-c+b\,1\mathrm{i}}$$

input
```
int(sin(x)/(a + b*cos(x) + c*sin(x)),x)
```

output
```
log(tan(x/2) + 1i)/(b - c*1i) + (log(tan(x/2) - 1i)*1i)/(b*1i - c) + (log(
64*tan(x/2)*(a - b)^2 - ((a^2*b - b*c^2 - b^3 + a*c*(b^2 - a^2 + c^2)^(1/2
))*(32*a^2*c + 32*b^2*c - 64*a*b*c + 64*tan(x/2)*(a - b)*(a*b - a^2 + c^2)
 + ((a^2*b - b*c^2 - b^3 + a*c*(b^2 - a^2 + c^2)^(1/2))*(32*b*c^3 - 32*a*c
^3 - 64*b^3*c + 32*tan(x/2)*(a - b)*(2*a*b^2 - 2*a*c^2 + b*c^2 - 2*b^3) +
128*a*b^2*c - 64*a^2*b*c + (32*(a - b)*(a^2*b - b*c^2 - b^3 + a*c*(b^2 - a
^2 + c^2)^(1/2))*(3*c^4*tan(x/2) + a*c^3 + 3*b*c^3 + 3*b^3*c + 2*a^2*b^2*t
an(x/2) - 2*a^2*c^2*tan(x/2) + 3*b^2*c^2*tan(x/2) - 2*a*b^3*tan(x/2) + a*b
^2*c - 4*a^2*b*c - 2*a*b*c^2*tan(x/2)))/((b^2 + c^2)*(b^2 - a^2 + c^2))))/
((b^2 + c^2)*(b^2 - a^2 + c^2))))/((b^2 + c^2)*(b^2 - a^2 + c^2)))*(b*(a^2
 - c^2) - b^3 + a*c*(b^2 - a^2 + c^2)^(1/2)))/((b^2 + c^2)*(b^2 - a^2 + c^
2)) - (log(64*tan(x/2)*(a - b)^2 + ((b*c^2 - a^2*b + b^3 + a*c*(b^2 - a^2
+ c^2)^(1/2))*(32*a^2*c + 32*b^2*c - 64*a*b*c + 64*tan(x/2)*(a - b)*(a*b -
 a^2 + c^2) + ((b*c^2 - a^2*b + b^3 + a*c*(b^2 - a^2 + c^2)^(1/2))*(32*a*c
^3 - 32*b*c^3 + 64*b^3*c - 32*tan(x/2)*(a - b)*(2*a*b^2 - 2*a*c^2 + b*c^2
- 2*b^3) - 128*a*b^2*c + 64*a^2*b*c + (32*(a - b)*(b*c^2 - a^2*b + b^3 + a
*c*(b^2 - a^2 + c^2)^(1/2))*(3*c^4*tan(x/2) + a*c^3 + 3*b*c^3 + 3*b^3*c +
2*a^2*b^2*tan(x/2) - 2*a^2*c^2*tan(x/2) + 3*b^2*c^2*tan(x/2) - 2*a*b^3*tan
(x/2) + a*b^2*c - 4*a^2*b*c - 2*a*b*c^2*tan(x/2)))/((b^2 + c^2)*(b^2 - a^2
 + c^2))))/((b^2 + c^2)*(b^2 - a^2 + c^2))))/((b^2 + c^2)*(b^2 - a^2 + ...
```

3.443. $\int \frac{\sin(x)}{a+b\cos(x)+c\sin(x)}\,dx$

3.444 $\int \frac{\sin(x)}{1+\cos(x)+\sin(x)}\,dx$

3.444.1 Optimal result

Integrand size = 11, antiderivative size = 22

$$\int \frac{\sin(x)}{1+\cos(x)+\sin(x)}\,dx = \frac{x}{2}-\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)$$

output
```
1/2*x-ln(cos(1/2*x)+sin(1/2*x))
```

3.444.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{1+\cos(x)+\sin(x)}\,dx = \frac{x}{2}-\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[Sin[x]/(1 + Cos[x] + Sin[x]),x]
```

output
```
x/2 - Log[Cos[x/2] + Sin[x/2]]
```

3.444.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.36, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {3042, 3616, 3042, 3603, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)}{\sin(x)+\cos(x)+1}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sin(x)}{\sin(x)+\cos(x)+1}dx$$

$$\Big\downarrow \ 3616$$

$$-\frac{1}{2}\int \frac{1}{\cos(x)+\sin(x)+1}dx+\frac{x}{2}-\frac{1}{2}\log(\sin(x)+\cos(x)+1)$$

$$\Big\downarrow \ 3042$$

$$-\frac{1}{2}\int \frac{1}{\cos(x)+\sin(x)+1}dx+\frac{x}{2}-\frac{1}{2}\log(\sin(x)+\cos(x)+1)$$

$$\Big\downarrow \ 3603$$

$$-\int \frac{1}{2\tan\left(\frac{x}{2}\right)+2}d\tan\left(\frac{x}{2}\right)+\frac{x}{2}-\frac{1}{2}\log(\sin(x)+\cos(x)+1)$$

$$\Big\downarrow \ 16$$

$$\frac{x}{2}-\frac{1}{2}\log\left(\tan\left(\frac{x}{2}\right)+1\right)-\frac{1}{2}\log(\sin(x)+\cos(x)+1)$$

input
```
Int[Sin[x]/(1 + Cos[x] + Sin[x]),x]
```

output
```
x/2 - Log[1 + Cos[x] + Sin[x]]/2 - Log[1 + Tan[x/2]]/2
```

3.444.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + 
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

rule 3616
```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[c*C*((d + e*x)/
(e*(b^2 + c^2))), x] + (-Simp[b*C*(Log[a + b*Cos[d + e*x] + c*Sin[d + e*x]]
/(e*(b^2 + c^2))), x] + Simp[(A*(b^2 + c^2) - a*c*C)/(b^2 + c^2)   Int[1/(a
 + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x]) /; FreeQ[{a, b, c, d, e, A, C}
, x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*c*C, 0]
```

3.444.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.54 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

method	result	size
risch	$\frac{x}{2}+\frac{ix}{2}-\ln\left(i+{\mathrm e}^{ix}\right)$	20
default	$\frac{\ln\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)}{2}+\arctan\left(\tan\left(\frac{x}{2}\right)\right)-\ln\left(\tan\left(\frac{x}{2}\right)+1\right)$	27
parallelrisch	$\frac{x}{2}-\ln\left(-\frac{\left(-\csc\left(x\right)+\cot\left(x\right)-1\right)\sqrt{2}}{2}\right)+\ln\left(\sqrt{\frac{1}{\cos\left(x\right)+1}}\right)$	30
norman	$\frac{\frac{x}{2}+\frac{x\tan\left(\frac{x}{2}\right)^{2}}{2}}{1+\tan\left(\frac{x}{2}\right)^{2}}-\ln\left(\tan\left(\frac{x}{2}\right)+1\right)+\frac{\ln\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)}{2}$	46

input
```
int(sin(x)/(1+cos(x)+sin(x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x+1/2*I*x-ln(I+exp(I*x))
```

3.444.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.50

$$\int \frac{\sin(x)}{1+\cos(x)+\sin(x)}\,dx = \frac{1}{2}\,x - \frac{1}{2}\,\log\left(\sin\left(x\right)+1\right)$$

input
```
integrate(sin(x)/(1+cos(x)+sin(x)),x, algorithm="fricas")
```

output
```
1/2*x - 1/2*log(sin(x) + 1)
```

3.444.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{1+\cos(x)+\sin(x)}\,dx = \frac{x}{2} - \log\left(\tan\left(\frac{x}{2}\right)+1\right) + \frac{\log\left(\tan^2\left(\frac{x}{2}\right)+1\right)}{2}$$

input
```
integrate(sin(x)/(1+cos(x)+sin(x)),x)
```

output
```
x/2 - log(tan(x/2) + 1) + log(tan(x/2)**2 + 1)/2
```

3.444.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(16) = 32.

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.86

$$\int \frac{\sin(x)}{1+\cos(x)+\sin(x)}\,dx = \arctan\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}\right) - \log\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}+1\right) + \frac{1}{2}\,\log\left(\frac{\sin\left(x\right)^2}{\left(\cos\left(x\right)+1\right)^2}+1\right)$$

input
```
integrate(sin(x)/(1+cos(x)+sin(x)),x, algorithm="maxima")
```

output
```
arctan(sin(x)/(cos(x) + 1)) - log(sin(x)/(cos(x) + 1) + 1) + 1/2*log(sin(x
)^2/(cos(x) + 1)^2 + 1)
```

3.444.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{\sin(x)}{1+\cos(x)+\sin(x)}\,dx = \frac{1}{2}\,x + \frac{1}{2}\,\log\left(\tan\left(\frac{1}{2}\,x\right)^2 + 1\right) - \log\left(\left|\tan\left(\frac{1}{2}\,x\right) + 1\right|\right)$$

input
```
integrate(sin(x)/(1+cos(x)+sin(x)),x, algorithm="giac")
```

output
```
1/2*x + 1/2*log(tan(1/2*x)^2 + 1) - log(abs(tan(1/2*x) + 1))
```

3.444.9 Mupad [B] (verification not implemented)

Time = 26.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.55

$$\int \frac{\sin(x)}{1+\cos(x)+\sin(x)}\,dx = -\ln\left(\tan\left(\frac{x}{2}\right)+1\right) + \ln\left(\tan\left(\frac{x}{2}\right)-\mathrm{i}\right)\left(\frac{1}{2}-\frac{1}{2}\mathrm{i}\right) + \ln\left(\tan\left(\frac{x}{2}\right)+1\mathrm{i}\right)\left(\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)$$

input
```
int(sin(x)/(cos(x) + sin(x) + 1),x)
```

output
```
log(tan(x/2) - 1i)*(1/2 - 1i/2) - log(tan(x/2) + 1) + log(tan(x/2) + 1i)*(
1/2 + 1i/2)
```

3.445 $\int \frac{1}{a+c\sec(x)+b\tan(x)}\,dx$

3.445.1 Optimal result

Integrand size = 12, antiderivative size = 97

$$\int \frac{1}{a+c\sec(x)+b\tan(x)}\,dx = \frac{ax}{a^2+b^2}+\frac{2ac\,\text{arctanh}\left(\frac{b-(a-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{(a^2+b^2)\sqrt{a^2+b^2-c^2}}+\frac{b\log(c+a\cos(x)+b\sin(x))}{a^2+b^2}$$

output
```
a*x/(a^2+b^2)+b*ln(c+a*cos(x)+b*sin(x))/(a^2+b^2)+2*a*c*arctanh((b-(a-c)*t
an(1/2*x))/(a^2+b^2-c^2)^(1/2))/(a^2+b^2)/(a^2+b^2-c^2)^(1/2)
```

3.445.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.81

$$\int \frac{1}{a+c\sec(x)+b\tan(x)}\,dx = \frac{ax+\frac{2ac\,\text{arctanh}\left(\frac{b+(-a+c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}}+b\log(c+a\cos(x)+b\sin(x))}{a^2+b^2}$$

input
```
Integrate[(a + c*Sec[x] + b*Tan[x])^(-1),x]
```

output
```
(a*x + (2*a*c*ArcTanh[(b + (-a + c)*Tan[x/2])/Sqrt[a^2 + b^2 - c^2]])/Sqrt
[a^2 + b^2 - c^2] + b*Log[c + a*Cos[x] + b*Sin[x]])/(a^2 + b^2)
```

3.445.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 3638, 3042, 3617, 3042, 3603, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\tan(x)+c\sec(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a+b\tan(x)+c\sec(x)}dx$$

$$\Big\downarrow \text{3638}$$

$$\int \frac{\cos(x)}{a\cos(x)+b\sin(x)+c}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(x)}{a\cos(x)+b\sin(x)+c}dx$$

$$\Big\downarrow \text{3617}$$

$$-\frac{ac\int \frac{1}{c+a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\log(a\cos(x)+b\sin(x)+c)}{a^2+b^2}+\frac{ax}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{ac\int \frac{1}{c+a\cos(x)+b\sin(x)}dx}{a^2+b^2}+\frac{b\log(a\cos(x)+b\sin(x)+c)}{a^2+b^2}+\frac{ax}{a^2+b^2}$$

$$\Big\downarrow \text{3603}$$

$$-\frac{2ac\int \frac{1}{-((a-c)\tan^2(\frac{x}{2}))+2b\tan(\frac{x}{2})+a+c}d\tan\left(\frac{x}{2}\right)}{a^2+b^2}+\frac{b\log(a\cos(x)+b\sin(x)+c)}{a^2+b^2}+\frac{ax}{a^2+b^2}$$

$$\Big\downarrow \text{1083}$$

$$\frac{4ac\int \frac{1}{4(a^2+b^2-c^2)-(2b-2(a-c)\tan(\frac{x}{2}))^2}d\left(2b-2(a-c)\tan\left(\frac{x}{2}\right)\right)}{a^2+b^2}+\frac{b\log(a\cos(x)+b\sin(x)+c)}{a^2+b^2}+\frac{ax}{a^2+b^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{2ac\operatorname{arctanh}\left(\frac{2b-2(a-c)\tan\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2-c^2}}\right)}{(a^2+b^2)\sqrt{a^2+b^2-c^2}}+\frac{b\log(a\cos(x)+b\sin(x)+c)}{a^2+b^2}+\frac{ax}{a^2+b^2}$$

input
```
Int[(a + c*Sec[x] + b*Tan[x])^(-1),x]
```

output
```
(a*x)/(a^2 + b^2) + (2*a*c*ArcTanh[(2*b - 2*(a - c)*Tan[x/2])/(2*Sqrt[a^2
+ b^2 - c^2])])/((a^2 + b^2)*Sqrt[a^2 + b^2 - c^2]) + (b*Log[c + a*Cos[x]
+ b*Sin[x]])/(a^2 + b^2)
```

3.445.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

rule 3617
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[b*B*((d + e*x)/
(e*(b^2 + c^2))), x] + (Simp[c*B*(Log[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/
(e*(b^2 + c^2))), x] + Simp[(A*(b^2 + c^2) - a*b*B)/(b^2 + c^2)   Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x]) /; FreeQ[{a, b, c, d, e, A, B},
 x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*b*B, 0]
```

3.445. $\int \frac{1}{a+c\sec(x)+b\tan(x)}\,dx$

rule 3638
```
Int[((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)])
^(-1), x_Symbol] :> Int[Cos[d + e*x]/(b + a*Cos[d + e*x] + c*Sin[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

3.445.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.76

method	result
default	$\frac{\frac{2(ab-cb)\ln\left(\tan\left(\frac{x}{2}\right)^{2}a-c\tan\left(\frac{x}{2}\right)^{2}-2b\tan\left(\frac{x}{2}\right)-a-c\right)}{2a-2c}+\frac{2\left(ac-b^{2}+\frac{(ab-cb)b}{a-c}\right)\arctan\left(\frac{2(a-c)\tan\left(\frac{x}{2}\right)-2b}{2\sqrt{-a^{2}-b^{2}+c^{2}}}\right)}{\sqrt{-a^{2}-b^{2}+c^{2}}}}{a^{2}+b^{2}}+\frac{-b\ln\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)+2a\,\mathrm{arc}}{a^{2}+b^{2}}$
risch	Expression too large to display

input
```
int(1/(a+c*sec(x)+b*tan(x)),x,method=_RETURNVERBOSE)
```

output
```
2/(a^2+b^2)*(1/2*(a*b-b*c)/(a-c)*ln(tan(1/2*x)^2*a-c*tan(1/2*x)^2-2*b*tan(
1/2*x)-a-c)+(a*c-b^2+(a*b-b*c)*b/(a-c))/(-a^2-b^2+c^2)^(1/2)*arctan(1/2*(2
*(a-c)*tan(1/2*x)-2*b)/(-a^2-b^2+c^2)^(1/2)))+2/(a^2+b^2)*(-1/2*b*ln(1+tan
(1/2*x)^2)+a*arctan(tan(1/2*x)))
```

3.445.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. 2(93) = 186.

Time = 0.32 (sec) , antiderivative size = 553, normalized size of antiderivative = 5.70

$$\int\frac{1}{a+c\sec(x)+b\tan(x)}\,dx$$

$$=\left[\frac{\sqrt{a^2+b^2-c^2}ac\log\left(\frac{2a^4+3a^2b^2+b^4-(a^2-b^2)c^2+2(a^3+ab^2)c\cos(x)-(a^4-b^4-2(a^2-b^2)c^2)\cos(x)^2+2((a^2b+b^3)c-(a^3b+ab^3-2}{2ac\cos(x)+(a^2-b^2)\cos(x)^2+b}\right.}{}\right.$$

$$-\frac{2\sqrt{-a^2-b^2+c^2}ac\arctan\left(\frac{(ac\cos(x)+bc\sin(x)+a^2+b^2)\sqrt{-a^2-b^2+c^2}}{(a^2b+b^3-bc^2)\cos(x)-(a^3+ab^2-ac^2)\sin(x)}\right)-2(a^3+ab^2-ac^2)x-(a^2b+b^3-bc^2}{2(a^4+2a^2b^2+b^4-(a^2+b^2)}$$

input
```
integrate(1/(a+c*sec(x)+b*tan(x)),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a^2 + b^2 - c^2)*a*c*log((2*a^4 + 3*a^2*b^2 + b^4 - (a^2 - b^2)
*c^2 + 2*(a^3 + a*b^2)*c*cos(x) - (a^4 - b^4 - 2*(a^2 - b^2)*c^2)*cos(x)^2
 + 2*((a^2*b + b^3)*c - (a^3*b + a*b^3 - 2*a*b*c^2)*cos(x))*sin(x) + 2*(2*
a*b*c*cos(x)^2 - a*b*c + (a^2*b + b^3)*cos(x) - (a^3 + a*b^2 + (a^2 - b^2)
*c*cos(x))*sin(x))*sqrt(a^2 + b^2 - c^2))/(2*a*c*cos(x) + (a^2 - b^2)*cos(
x)^2 + b^2 + c^2 + 2*(a*b*cos(x) + b*c)*sin(x))) + 2*(a^3 + a*b^2 - a*c^2)
*x + (a^2*b + b^3 - b*c^2)*log(2*a*c*cos(x) + (a^2 - b^2)*cos(x)^2 + b^2 +
 c^2 + 2*(a*b*cos(x) + b*c)*sin(x)))/(a^4 + 2*a^2*b^2 + b^4 - (a^2 + b^2)*
c^2), -1/2*(2*sqrt(-a^2 - b^2 + c^2)*a*c*arctan((a*c*cos(x) + b*c*sin(x) +
 a^2 + b^2)*sqrt(-a^2 - b^2 + c^2)/((a^2*b + b^3 - b*c^2)*cos(x) - (a^3 +
a*b^2 - a*c^2)*sin(x))) - 2*(a^3 + a*b^2 - a*c^2)*x - (a^2*b + b^3 - b*c^2
)*log(2*a*c*cos(x) + (a^2 - b^2)*cos(x)^2 + b^2 + c^2 + 2*(a*b*cos(x) + b*
c)*sin(x)))/(a^4 + 2*a^2*b^2 + b^4 - (a^2 + b^2)*c^2)]
```

3.445.6 Sympy [F]

$$\int \frac{1}{a+c\sec(x)+b\tan(x)}\,dx=\int \frac{1}{a+b\tan{(x)}+c\sec{(x)}}\,dx$$

input
```
integrate(1/(a+c*sec(x)+b*tan(x)),x)
```

output
```
Integral(1/(a + b*tan(x) + c*sec(x)), x)
```

3.445.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a+c\sec(x)+b\tan(x)}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(1/(a+c*sec(x)+b*tan(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c^2-b^2-a^2>0)', see `assume?` f
or more de
```

3.445.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.63

$$\int \frac{1}{a+c\sec(x)+b\tan(x)}\,dx$$
$$= -\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2\,a+2\,c)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,x\right)-c\tan\left(\frac{1}{2}\,x\right)-b}{\sqrt{-a^2-b^2+c^2}}\right)\right)ac}{(a^2+b^2)\sqrt{-a^2-b^2+c^2}}+\frac{ax}{a^2+b^2}$$
$$+\frac{b\log\left(-a\tan\left(\frac{1}{2}\,x\right)^2+c\tan\left(\frac{1}{2}\,x\right)^2+2\,b\tan\left(\frac{1}{2}\,x\right)+a+c\right)}{a^2+b^2}-\frac{b\log\left(\tan\left(\frac{1}{2}\,x\right)^2+1\right)}{a^2+b^2}$$

input
```
integrate(1/(a+c*sec(x)+b*tan(x)),x, algorithm="giac")
```

output
```
-2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*c) + arctan(-(a*tan(1/2*x) - c*t
an(1/2*x) - b)/sqrt(-a^2 - b^2 + c^2)))*a*c/((a^2 + b^2)*sqrt(-a^2 - b^2 +
 c^2)) + a*x/(a^2 + b^2) + b*log(-a*tan(1/2*x)^2 + c*tan(1/2*x)^2 + 2*b*ta
n(1/2*x) + a + c)/(a^2 + b^2) - b*log(tan(1/2*x)^2 + 1)/(a^2 + b^2)
```

3.445.9 Mupad [B] (verification not implemented)

Time = 42.59 (sec) , antiderivative size = 988, normalized size of antiderivative = 10.19

$$\int \frac{1}{a+c\sec(x)+b\tan(x)}\,dx$$

$$= \frac{\ln\left(32\,a\,c-32\,c^2+32\,b\tan\left(\frac{x}{2}\right)\,(a-c)+\frac{\left(32\,a^2\,b-32\,b\,c^2+32\tan\left(\frac{x}{2}\right)(a-c)\left(-a^2+2\,a\,c+3\,b^2-2\,c^2\right)-\frac{\left(a^2\,b-b\,c^2+b^3+a\,c\,\sqrt{a^2+b^2}\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

$$-\frac{\ln\left(\tan\left(\frac{x}{2}\right)+1\mathrm{i}\right)}{b+a\,1\mathrm{i}}$$

$$+\frac{\ln\left(32\,a\,c-32\,c^2+32\,b\tan\left(\frac{x}{2}\right)\,(a-c)+\frac{\left(32\,a^2\,b-32\,b\,c^2+32\tan\left(\frac{x}{2}\right)(a-c)\left(-a^2+2\,a\,c+3\,b^2-2\,c^2\right)-\frac{\left(a^2\,b-b\,c^2+b^3-a\,c\,\sqrt{a^2\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

$$-\frac{\ln\left(\tan\left(\frac{x}{2}\right)-\mathrm{i}\right)\,1\mathrm{i}}{a+b\,1\mathrm{i}}$$

input
```
int(1/(a + b*tan(x) + c/cos(x)),x)
```

output

```
(log(32*a*c - 32*c^2 + 32*b*tan(x/2)*(a - c) + ((32*a^2*b - 32*b*c^2 + 32*
tan(x/2)*(a - c)*(2*a*c - a^2 + 3*b^2 - 2*c^2) - ((a^2*b - b*c^2 + b^3 + a
*c*(a^2 + b^2 - c^2)^(1/2))*(32*a^4 - 64*a^3*c - 64*a^2*b^2 + 32*a^2*c^2 -
 32*b^2*c^2 + 96*a*b^2*c + 32*b*tan(x/2)*(a - c)*(4*a^2 - 4*a*c + b^2) + (
32*(a - c)*(a^2*b - b*c^2 + b^3 + a*c*(a^2 + b^2 - c^2)^(1/2))*(3*b^4*tan(
x/2) + 3*a*b^3 + 3*a^3*b + b^3*c + 3*a^2*b^2*tan(x/2) + 2*a^2*c^2*tan(x/2)
 - 2*b^2*c^2*tan(x/2) - 2*a^3*c*tan(x/2) - 4*a*b*c^2 + a^2*b*c - 2*a*b^2*c
*tan(x/2)))/((a^2 + b^2)*(a^2 + b^2 - c^2))))/((a^2 + b^2)*(a^2 + b^2 - c^
2)))*(a^2*b - b*c^2 + b^3 + a*c*(a^2 + b^2 - c^2)^(1/2)))/((a^2 + b^2)*(a^
2 + b^2 - c^2)))*(b*(a^2 - c^2) + b^3 + a*c*(a^2 + b^2 - c^2)^(1/2)))/(c^2
*(a^2 + b^2 - c^2) + (a^2 + b^2 - c^2)^2) - log(tan(x/2) + 1i)/(a*1i + b)
 - (log(tan(x/2) - 1i)*1i)/(a + b*1i) + (log(32*a*c - 32*c^2 + 32*b*tan(x/2
)*(a - c) + ((32*a^2*b - 32*b*c^2 + 32*tan(x/2)*(a - c)*(2*a*c - a^2 + 3*b
^2 - 2*c^2) - ((a^2*b - b*c^2 + b^3 - a*c*(a^2 + b^2 - c^2)^(1/2))*(32*a^4
 - 64*a^3*c - 64*a^2*b^2 + 32*a^2*c^2 - 32*b^2*c^2 + 96*a*b^2*c + 32*b*tan
(x/2)*(a - c)*(4*a^2 - 4*a*c + b^2) + (32*(a - c)*(a^2*b - b*c^2 + b^3 - a
*c*(a^2 + b^2 - c^2)^(1/2))*(3*b^4*tan(x/2) + 3*a*b^3 + 3*a^3*b + b^3*c +
3*a^2*b^2*tan(x/2) + 2*a^2*c^2*tan(x/2) - 2*b^2*c^2*tan(x/2) - 2*a^3*c*tan
(x/2) - 4*a*b*c^2 + a^2*b*c - 2*a*b^2*c*tan(x/2)))/((a^2 + b^2)*(a^2 + b^2
 - c^2))))/((a^2 + b^2)*(a^2 + b^2 - c^2)))*(a^2*b - b*c^2 + b^3 - a*c*...
```

3.445. $\int \frac{1}{a+c\sec(x)+b\tan(x)}\,dx$

3.446 $\int \frac{\sec(x)}{a+c\sec(x)+b\tan(x)}\,dx$

3.446.1 Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{\sec(x)}{a+c\sec(x)+b\tan(x)}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{b-(a-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}}$$

output
```
-2*arctanh((b-(a-c)*tan(1/2*x))/(a^2+b^2-c^2)^(1/2))/(a^2+b^2-c^2)^(1/2)
```

3.446.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.98

$$\int \frac{\sec(x)}{a+c\sec(x)+b\tan(x)}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{b+(-a+c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}}$$

input
```
Integrate[Sec[x]/(a + c*Sec[x] + b*Tan[x]),x]
```

output
```
(-2*ArcTanh[(b + (-a + c)*Tan[x/2])/Sqrt[a^2 + b^2 - c^2]])/Sqrt[a^2 + b^2
 - c^2]
```

3.446.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3644, 3042, 3603, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(x)}{a+b\tan(x)+c\sec(x)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sec(x)}{a+b\tan(x)+c\sec(x)}dx$$

$$\Big\downarrow \text{ 3644}$$

$$\int \frac{1}{a\cos(x)+b\sin(x)+c}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{a\cos(x)+b\sin(x)+c}dx$$

$$\Big\downarrow \text{ 3603}$$

$$2\int \frac{1}{-\left((a-c)\tan^2\left(\frac{x}{2}\right)\right)+2b\tan\left(\frac{x}{2}\right)+a+c}d\tan\left(\frac{x}{2}\right)$$

$$\Big\downarrow \text{ 1083}$$

$$-4\int \frac{1}{4\left(a^2+b^2-c^2\right)-\left(2b-2(a-c)\tan\left(\frac{x}{2}\right)\right)^2}d\left(2b-2(a-c)\tan\left(\frac{x}{2}\right)\right)$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{2\text{arctanh}\left(\frac{2b-2(a-c)\tan\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}}$$

input
```
Int[Sec[x]/(a + c*Sec[x] + b*Tan[x]),x]
```

output
```
(-2*ArcTanh[(2*b - 2*(a - c)*Tan[x/2])/(2*Sqrt[a^2 + b^2 - c^2])])/Sqrt[a^
2 + b^2 - c^2]
```

3.446.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

rule 3644
```
Int[sec[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] +
(c_.)*tan[(d_.) + (e_.)*(x_)])^(m_), x_Symbol] :> Int[1/(b + a*Cos[d + e*x]
 + c*Sin[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && I
ntegerQ[n]
```

3.446.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

method	result
default	$-\frac{2 \arctan\left(\frac{2(a-c)\tan\left(\frac{x}{2}\right)-2b}{2\sqrt{-a^2-b^2+c^2}}\right)}{\sqrt{-a^2-b^2+c^2}}$
risch	$\frac{\ln\left(e^{ix}+\frac{icb\sqrt{a^2+b^2-c^2}+ia^3+ia\,b^2-ia\,c^2+ac\sqrt{a^2+b^2-c^2}-a^2b-b^3+c^2b}{\left(a^2+b^2\right)\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}} - \frac{\ln\left(e^{ix}+\frac{icb\sqrt{a^2+b^2-c^2}-ia^3-ia\,b^2+ia\,c^2+ac\sqrt{a^2+b^2-c^2}+a^2}{\left(a^2+b^2\right)\sqrt{a^2+b^2-c^2}}\right.}{\sqrt{a^2+b^2-c^2}}$

input
```
int(sec(x)/(a+c*sec(x)+b*tan(x)),x,method=_RETURNVERBOSE)
```

output
```
-2/(-a^2-b^2+c^2)^(1/2)*arctan(1/2*(2*(a-c)*tan(1/2*x)-2*b)/(-a^2-b^2+c^2)
^(1/2))
```

3.446.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(47) = 94$.

Time = 0.32 (sec) , antiderivative size = 349, normalized size of antiderivative = 6.84

$$\int \frac{\sec(x)}{a + c\sec(x) + b\tan(x)}\,dx$$
$$= \left[\frac{\log\left(-\frac{2a^4+3a^2b^2+b^4-(a^2-b^2)c^2+2(a^3+ab^2)c\cos(x)-(a^4-b^4-2(a^2-b^2)c^2)\cos(x)^2+2((a^2b+b^3)c-(a^3b+ab^3-2abc^2)\cos(x))\sin(x)-}{2ac\cos(x)+(a^2-b^2)\cos(x)^2+b^2+c^2+2(ab\cos(x)+}\right.}{2\sqrt{a^2+b^2-c^2}} \right.$$

input
```
integrate(sec(x)/(a+c*sec(x)+b*tan(x)),x, algorithm="fricas")
```

output
```
[1/2*log(-(2*a^4 + 3*a^2*b^2 + b^4 - (a^2 - b^2)*c^2 + 2*(a^3 + a*b^2)*c*c
os(x) - (a^4 - b^4 - 2*(a^2 - b^2)*c^2)*cos(x)^2 + 2*((a^2*b + b^3)*c - (a
^3*b + a*b^3 - 2*a*b*c^2)*cos(x))*sin(x) - 2*(2*a*b*c*cos(x)^2 - a*b*c + (
a^2*b + b^3)*cos(x) - (a^3 + a*b^2 + (a^2 - b^2)*c*cos(x))*sin(x))*sqrt(a^
2 + b^2 - c^2))/(2*a*c*cos(x) + (a^2 - b^2)*cos(x)^2 + b^2 + c^2 + 2*(a*b*
cos(x) + b*c)*sin(x)))/sqrt(a^2 + b^2 - c^2), sqrt(-a^2 - b^2 + c^2)*arcta
n((a*c*cos(x) + b*c*sin(x) + a^2 + b^2)*sqrt(-a^2 - b^2 + c^2)/((a^2*b + b
^3 - b*c^2)*cos(x) - (a^3 + a*b^2 - a*c^2)*sin(x)))/(a^2 + b^2 - c^2)]
```

3.446.6 Sympy [F]

$$\int \frac{\sec(x)}{a + c\sec(x) + b\tan(x)}\,dx = \int \frac{\sec(x)}{a + b\tan(x) + c\sec(x)}\,dx$$

input
```
integrate(sec(x)/(a+c*sec(x)+b*tan(x)),x)
```

output
```
Integral(sec(x)/(a + b*tan(x) + c*sec(x)), x)
```

3.446.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec(x)}{a + c\sec(x) + b\tan(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sec(x)/(a+c*sec(x)+b*tan(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c^2-b^2-a^2>0)', see `assume?` f
or more de
```

3.446.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.43

$$\int \frac{\sec(x)}{a + c\sec(x) + b\tan(x)}\,dx = -\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2c)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)-c\tan\left(\frac{1}{2}x\right)-b}{\sqrt{-a^2-b^2+c^2}}\right)\right)}{\sqrt{-a^2-b^2+c^2}}$$

input
```
integrate(sec(x)/(a+c*sec(x)+b*tan(x)),x, algorithm="giac")
```

output
```
-2*(pi*floor(1/2*x/pi + 1/2)*sgn(2*a - 2*c) + arctan((a*tan(1/2*x) - c*tan
(1/2*x) - b)/sqrt(-a^2 - b^2 + c^2)))/sqrt(-a^2 - b^2 + c^2)
```

3.446.9 Mupad [B] (verification not implemented)

Time = 28.00 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

$$\int \frac{\sec(x)}{a + c\sec(x) + b\tan(x)}\,dx = -\frac{2\,\operatorname{atanh}\left(\frac{b-\frac{\tan\left(\frac{x}{2}\right)(2a-2c)}{2}}{\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}}$$

input
```
int(1/(cos(x)*(a + b*tan(x) + c/cos(x))),x)
```

output
```
-(2*atanh((b - (tan(x/2)*(2*a - 2*c))/2)/(a^2 + b^2 - c^2)^(1/2)))/(a^2 +
b^2 - c^2)^(1/2)
```

3.446. $\int \frac{\sec(x)}{a+c\sec(x)+b\tan(x)}\,dx$

3.447 $\int \frac{\sec^2(x)}{a+c\sec(x)+b\tan(x)}\,dx$

3.447.1 Optimal result

Integrand size = 17, antiderivative size = 142

$$\int \frac{\sec^2(x)}{a+c\sec(x)+b\tan(x)}\,dx = -\frac{2ac\,\text{arctanh}\left(\frac{b-(a-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{(b^2-c^2)\sqrt{a^2+b^2-c^2}} - \frac{\log\left(1-\tan\left(\frac{x}{2}\right)\right)}{b+c} - \frac{\log\left(1+\tan\left(\frac{x}{2}\right)\right)}{b-c} + \frac{b\log\left(a+c+2b\tan\left(\frac{x}{2}\right)-(a-c)\tan^2\left(\frac{x}{2}\right)\right)}{b^2-c^2}$$

output
```
-ln(1-tan(1/2*x))/(b+c)-ln(1+tan(1/2*x))/(b-c)+b*ln(a+c+2*b*tan(1/2*x)-(a-
c)*tan(1/2*x)^2)/(b^2-c^2)-2*a*c*arctanh((b-(a-c)*tan(1/2*x))/(a^2+b^2-c^2
)^(1/2))/(b^2-c^2)/(a^2+b^2-c^2)^(1/2)
```

3.447.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.85

$$\int \frac{\sec^2(x)}{a+c\sec(x)+b\tan(x)}\,dx = \frac{\frac{2ac\,\text{arctanh}\left(\frac{b+(-a+c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}} + (b-c)\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right) + (b+c)\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right) - b\log(c+a}{(-b+c)(b+c)}$$

input
```
Integrate[Sec[x]^2/(a + c*Sec[x] + b*Tan[x]),x]
```

output
```
((2*a*c*ArcTanh[(b + (-a + c)*Tan[x/2])/Sqrt[a^2 + b^2 - c^2]])/Sqrt[a^2 +
 b^2 - c^2] + (b - c)*Log[Cos[x/2] - Sin[x/2]] + (b + c)*Log[Cos[x/2] + Si
n[x/2]] - b*Log[c + a*Cos[x] + b*Sin[x]])/((-b + c)*(b + c))
```

3.447.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.04, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.824$, Rules used = {3042, 4897, 3042, 4902, 2142, 27, 452, 219, 240, 1142, 27, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(x)}{a+b\tan(x)+c\sec(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(x)^2}{a+b\tan(x)+c\sec(x)}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \frac{\sec(x)}{a\cos(x)+b\sin(x)+c}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(x)}{a\cos(x)+b\sin(x)+c}dx$$

$$\Big\downarrow \text{4902}$$

$$2\int \frac{\tan^2\left(\frac{x}{2}\right)+1}{\left(1-\tan^2\left(\frac{x}{2}\right)\right)\left(-\left((a-c)\tan^2\left(\frac{x}{2}\right)\right)+2b\tan\left(\frac{x}{2}\right)+a+c\right)}d\tan\left(\frac{x}{2}\right)$$

$$\Big\downarrow \text{2142}$$

$$2\left(-\frac{\int -\frac{4\left(b^2-(a-c)\tan\left(\frac{x}{2}\right)b+ac\right)}{-\left((a-c)\tan^2\left(\frac{x}{2}\right)\right)+2b\tan\left(\frac{x}{2}\right)+a+c}d\tan\left(\frac{x}{2}\right)}{4\left(b^2-c^2\right)}-\frac{\int \frac{4\left(c-b\tan\left(\frac{x}{2}\right)\right)}{1-\tan^2\left(\frac{x}{2}\right)}d\tan\left(\frac{x}{2}\right)}{4\left(b^2-c^2\right)}\right)$$

$$\Big\downarrow \text{27}$$

$$2\left(\frac{\int \frac{b^2-(a-c)\tan\left(\frac{x}{2}\right)b+ac}{-\left((a-c)\tan^2\left(\frac{x}{2}\right)\right)+2b\tan\left(\frac{x}{2}\right)+a+c}d\tan\left(\frac{x}{2}\right)}{b^2-c^2}-\frac{\int \frac{c-b\tan\left(\frac{x}{2}\right)}{1-\tan^2\left(\frac{x}{2}\right)}d\tan\left(\frac{x}{2}\right)}{b^2-c^2}\right)$$

$$\Big\downarrow \text{ 452}$$

$$2\left(\frac{\int \frac{b^2-(a-c)\tan\left(\frac{x}{2}\right)b+ac}{-\left((a-c)\tan^2\left(\frac{x}{2}\right)\right)+2b\tan\left(\frac{x}{2}\right)+a+c}d\tan\left(\frac{x}{2}\right)}{b^2-c^2}-\frac{c\int \frac{1}{1-\tan^2\left(\frac{x}{2}\right)}d\tan\left(\frac{x}{2}\right)-b\int \frac{\tan\left(\frac{x}{2}\right)}{1-\tan^2\left(\frac{x}{2}\right)}d\tan\left(\frac{x}{2}\right)}{b^2-c^2}\right)$$

$$\Big\downarrow \text{ 219}$$

$$2\left(\frac{\int \frac{b^2-(a-c)\tan\left(\frac{x}{2}\right)b+ac}{-\left((a-c)\tan^2\left(\frac{x}{2}\right)\right)+2b\tan\left(\frac{x}{2}\right)+a+c}d\tan\left(\frac{x}{2}\right)}{b^2-c^2}-\frac{c\,\text{arctanh}\left(\tan\left(\frac{x}{2}\right)\right)-b\int \frac{\tan\left(\frac{x}{2}\right)}{1-\tan^2\left(\frac{x}{2}\right)}d\tan\left(\frac{x}{2}\right)}{b^2-c^2}\right)$$

$$\Big\downarrow \text{ 240}$$

$$2\left(\frac{\int \frac{b^2-(a-c)\tan\left(\frac{x}{2}\right)b+ac}{-\left((a-c)\tan^2\left(\frac{x}{2}\right)\right)+2b\tan\left(\frac{x}{2}\right)+a+c}d\tan\left(\frac{x}{2}\right)}{b^2-c^2}-\frac{c\,\text{arctanh}\left(\tan\left(\frac{x}{2}\right)\right)+\frac{1}{2}b\log\left(1-\tan^2\left(\frac{x}{2}\right)\right)}{b^2-c^2}\right)$$

$$\Big\downarrow \text{ 1142}$$

$$2\left(\frac{ac\int \frac{1}{-\left((a-c)\tan^2\left(\frac{x}{2}\right)\right)+2b\tan\left(\frac{x}{2}\right)+a+c}d\tan\left(\frac{x}{2}\right)+\frac{1}{2}b\int \frac{2\left(b-(a-c)\tan\left(\frac{x}{2}\right)\right)}{-\left((a-c)\tan^2\left(\frac{x}{2}\right)\right)+2b\tan\left(\frac{x}{2}\right)+a+c}d\tan\left(\frac{x}{2}\right)}{b^2-c^2}-\frac{c\,\text{arctanh}\left(\tan\left(\frac{x}{2}\right)\right)\cdots}{b\cdots}\right.$$

$$\Big\downarrow \text{ 27}$$

$$2\left(\frac{ac\int \frac{1}{-\left((a-c)\tan^2\left(\frac{x}{2}\right)\right)+2b\tan\left(\frac{x}{2}\right)+a+c}d\tan\left(\frac{x}{2}\right)+b\int \frac{b-(a-c)\tan\left(\frac{x}{2}\right)}{-\left((a-c)\tan^2\left(\frac{x}{2}\right)\right)+2b\tan\left(\frac{x}{2}\right)+a+c}d\tan\left(\frac{x}{2}\right)}{b^2-c^2}-\frac{c\,\text{arctanh}\left(\tan\left(\frac{x}{2}\right)\right)+\cdots}{b^2\cdots}\right.$$

$$\Big\downarrow \text{ 1083}$$

$$2\left(\frac{b\int \frac{b-(a-c)\tan\left(\frac{x}{2}\right)}{-\left((a-c)\tan^2\left(\frac{x}{2}\right)\right)+2b\tan\left(\frac{x}{2}\right)+a+c}d\tan\left(\frac{x}{2}\right)-2ac\int \frac{1}{4(a^2+b^2-c^2)-\left(2b-2(a-c)\tan\left(\frac{x}{2}\right)\right)^2}d\left(2b-2(a-c)\tan\left(\frac{x}{2}\right)\right)}{b^2-c^2}-\frac{\text{ca}\cdots}{\cdots}\right.$$

$$\Big\downarrow \text{ 219}$$

3.447. $\int \frac{\sec^2(x)}{a+c\sec(x)+b\tan(x)}\,dx$

$$2\left(\frac{b\int \frac{b-(a-c)\tan\left(\frac{x}{2}\right)}{-\left((a-c)\tan^2\left(\frac{x}{2}\right)\right)+2b\tan\left(\frac{x}{2}\right)+a+c}d\tan\left(\frac{x}{2}\right)-\frac{ac\text{arctanh}\left(\frac{2b-2(a-c)\tan\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}}}{b^2-c^2}-\frac{c\text{arctanh}\left(\tan\left(\frac{x}{2}\right)\right)+\frac{1}{2}b\log\left(1-\text{tan}\right.}{b^2-c^2}\right.$$

$$\Big\downarrow \text{1103}$$

$$2\left(\frac{\frac{1}{2}b\log\left(-\left((a-c)\tan^2\left(\frac{x}{2}\right)\right)+a+2b\tan\left(\frac{x}{2}\right)+c\right)-\frac{ac\text{arctanh}\left(\frac{2b-2(a-c)\tan\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}}}{b^2-c^2}-\frac{c\text{arctanh}\left(\tan\left(\frac{x}{2}\right)\right)+\frac{1}{2}b\log}{b^2-c^2}\right.$$

input
```
Int[Sec[x]^2/(a + c*Sec[x] + b*Tan[x]),x]
```

output
```
2*(-((c*ArcTanh[Tan[x/2]] + (b*Log[1 - Tan[x/2]^2])/2)/(b^2 - c^2)) + (-((
a*c*ArcTanh[(2*b - 2*(a - c)*Tan[x/2])/(2*Sqrt[a^2 + b^2 - c^2])])/Sqrt[a^
2 + b^2 - c^2]) + (b*Log[a + c + 2*b*Tan[x/2] - (a - c)*Tan[x/2]^2])/2)/(b
^2 - c^2))
```

3.447.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

3.447. $\int \frac{\sec^2(x)}{a+c\sec(x)+b\tan(x)}\,dx$

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2142
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (f_.)*(x_)^2)), x_Sym
bol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x, 2]
, q = c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2}, Simp[1/q   Int[(A*c^2*d - a
*c*C*d + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f + c*(B*c*d - b*C*d + A*b*f -
 a*B*f)*x)/(a + b*x + c*x^2), x], x] + Simp[1/q   Int[(c*C*d^2 + b*B*d*f -
A*c*d*f - a*C*d*f + a*A*f^2 - f*(B*c*d - b*C*d + A*b*f - a*B*f)*x)/(d + f*x
^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, f}, x] && PolyQ[Px, x, 2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

rule 4902
```
Int[u_, x_Symbol] :> With[{w = Block[{$ShowSteps = False, $StepCounter = Nu
ll}, Int[SubstFor[1/(1 + FreeFactors[Tan[FunctionOfTrig[u, x]/2], x]^2*x^2)
, Tan[FunctionOfTrig[u, x]/2]/FreeFactors[Tan[FunctionOfTrig[u, x]/2], x],
u, x], x]]}, Module[{v = FunctionOfTrig[u, x], d}, Simp[d = FreeFactors[Tan
[v/2], x]; 2*(d/Coefficient[v, x, 1])   Subst[Int[SubstFor[1/(1 + d^2*x^2),
 Tan[v/2]/d, u, x], x], x, Tan[v/2]/d], x]] /; CalculusFreeQ[w, x]] /; Inve
rseFunctionFreeQ[u, x] &&  !FalseQ[FunctionOfTrig[u, x]]
```

3.447. $\int \frac{\sec^2(x)}{a+c\sec(x)+b\tan(x)}\,dx$

3.447.4 Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.27

method	result
default	$-\frac{2\ln\left(\tan\left(\frac{x}{2}\right)+1\right)}{-2c+2b}-\frac{2\ln\left(\tan\left(\frac{x}{2}\right)-1\right)}{2b+2c}+\frac{\frac{2(ab-cb)\ln\left(\tan\left(\frac{x}{2}\right)^2a-c\tan\left(\frac{x}{2}\right)^2-2b\tan\left(\frac{x}{2}\right)-a-c\right)}{2a-2c}+\frac{2\left(-ac-b^2+\frac{(ab-cb)b}{a-c}\right)\arctan\left(\frac{2(a-c)}{2\sqrt{-}}\right.}{\sqrt{-a^2-b^2+c^2}}}{(b-c)(b+c)}$
risch	Expression too large to display

input
```
int(sec(x)^2/(a+c*sec(x)+b*tan(x)),x,method=_RETURNVERBOSE)
```

output
```
-2/(-2*c+2*b)*ln(tan(1/2*x)+1)-2/(2*b+2*c)*ln(tan(1/2*x)-1)+2/(b-c)/(b+c)*
(1/2*(a*b-b*c)/(a-c)*ln(tan(1/2*x)^2*a-c*tan(1/2*x)^2-2*b*tan(1/2*x)-a-c)+
(-a*c-b^2+(a*b-b*c)*b/(a-c))/(-a^2-b^2+c^2)^(1/2)*arctan(1/2*(2*(a-c)*tan(
1/2*x)-2*b)/(-a^2-b^2+c^2)^(1/2)))
```

3.447.5 Fricas [A] (verification not implemented)

Time = 1.62 (sec) , antiderivative size = 663, normalized size of antiderivative = 4.67

$$\int \frac{\sec^2(x)}{a+c\sec(x)+b\tan(x)}\,dx = \left[-\frac{\sqrt{a^2+b^2-c^2}ac\log\left(\frac{2a^4+3a^2b^2+b^4-(a^2-b^2)c^2+2(a^3+ab^2)c\cos(x)-(a^4-b^4-2(a^2-b^2)c^2)\cos(x)^2+2((a^2b+b^3)c-(a^3b+ab^3-}{2ac\cos(x)+(a^2-b^2)\cos(x)^2}\right.}{}\right.$$

input
```
integrate(sec(x)^2/(a+c*sec(x)+b*tan(x)),x, algorithm="fricas")
```

output
```
[-1/2*(sqrt(a^2 + b^2 - c^2)*a*c*log((2*a^4 + 3*a^2*b^2 + b^4 - (a^2 - b^2
)*c^2 + 2*(a^3 + a*b^2)*c*cos(x) - (a^4 - b^4 - 2*(a^2 - b^2)*c^2)*cos(x)^
2 + 2*((a^2*b + b^3)*c - (a^3*b + a*b^3 - 2*a*b*c^2)*cos(x))*sin(x) + 2*(2
*a*b*c*cos(x)^2 - a*b*c + (a^2*b + b^3)*cos(x) - (a^3 + a*b^2 + (a^2 - b^2
)*c*cos(x))*sin(x))*sqrt(a^2 + b^2 - c^2))/(2*a*c*cos(x) + (a^2 - b^2)*cos
(x)^2 + b^2 + c^2 + 2*(a*b*cos(x) + b*c)*sin(x))) - (a^2*b + b^3 - b*c^2)*
log(2*a*c*cos(x) + (a^2 - b^2)*cos(x)^2 + b^2 + c^2 + 2*(a*b*cos(x) + b*c)
*sin(x)) + (a^2*b + b^3 - b*c^2 - c^3 + (a^2 + b^2)*c)*log(sin(x) + 1) + (
a^2*b + b^3 - b*c^2 + c^3 - (a^2 + b^2)*c)*log(-sin(x) + 1))/(a^2*b^2 + b^
4 + c^4 - (a^2 + 2*b^2)*c^2), 1/2*(2*sqrt(-a^2 - b^2 + c^2)*a*c*arctan((a*
c*cos(x) + b*c*sin(x) + a^2 + b^2)*sqrt(-a^2 - b^2 + c^2)/((a^2*b + b^3 -
b*c^2)*cos(x) - (a^3 + a*b^2 - a*c^2)*sin(x))) + (a^2*b + b^3 - b*c^2)*log
(2*a*c*cos(x) + (a^2 - b^2)*cos(x)^2 + b^2 + c^2 + 2*(a*b*cos(x) + b*c)*si
n(x)) - (a^2*b + b^3 - b*c^2 - c^3 + (a^2 + b^2)*c)*log(sin(x) + 1) - (a^2
*b + b^3 - b*c^2 + c^3 - (a^2 + b^2)*c)*log(-sin(x) + 1))/(a^2*b^2 + b^4 +
 c^4 - (a^2 + 2*b^2)*c^2)]
```

3.447.6 Sympy [F]

$$\int \frac{\sec^2(x)}{a+c\sec(x)+b\tan(x)}\,dx=\int \frac{\sec^2(x)}{a+b\tan(x)+c\sec(x)}\,dx$$

input
```
integrate(sec(x)**2/(a+c*sec(x)+b*tan(x)),x)
```

output
```
Integral(sec(x)**2/(a + b*tan(x) + c*sec(x)), x)
```

3.447.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sec^2(x)}{a+c\sec(x)+b\tan(x)}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(sec(x)^2/(a+c*sec(x)+b*tan(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c^2-b^2-a^2>0)', see `assume?` f
or more de
```

3.447.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.13

$$\int \frac{\sec^2(x)}{a+c\sec(x)+b\tan(x)}\,dx$$
$$= \frac{2\left(\pi\left\lfloor \frac{x}{2\pi}+\frac{1}{2}\right\rfloor \operatorname{sgn}(-2\,a+2\,c)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,x\right)-c\tan\left(\frac{1}{2}\,x\right)-b}{\sqrt{-a^2-b^2+c^2}}\right)\right)ac}{\sqrt{-a^2-b^2+c^2}(b^2-c^2)} + \frac{b\log\left(-a\tan\left(\frac{1}{2}\,x\right)^2+c\tan\left(\frac{1}{2}\,x\right)^2+2\,b\tan\left(\frac{1}{2}\,x\right)+a+c\right)}{b^2-c^2} - \frac{\log\left(\left|\tan\left(\frac{1}{2}\,x\right)+1\right|\right)}{b-c} - \frac{\log\left(\left|\tan\left(\frac{1}{2}\,x\right)-1\right|\right)}{b+c}$$

input
```
integrate(sec(x)^2/(a+c*sec(x)+b*tan(x)),x, algorithm="giac")
```

output
```
2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*c) + arctan(-(a*tan(1/2*x) - c*ta
n(1/2*x) - b)/sqrt(-a^2 - b^2 + c^2)))*a*c/(sqrt(-a^2 - b^2 + c^2)*(b^2 -
c^2)) + b*log(-a*tan(1/2*x)^2 + c*tan(1/2*x)^2 + 2*b*tan(1/2*x) + a + c)/(
b^2 - c^2) - log(abs(tan(1/2*x) + 1))/(b - c) - log(abs(tan(1/2*x) - 1))/(
b + c)
```

3.447.9 Mupad [B] (verification not implemented)

Time = 39.08 (sec) , antiderivative size = 977, normalized size of antiderivative = 6.88

$$\int \frac{\sec^2(x)}{a + c\sec(x) + b\tan(x)}\,dx$$

$$= \frac{\ln\left(32\,a\,c - 32\,a^2 - 32\,b\tan\left(\frac{x}{2}\right)\,(a-c) - \frac{\left(a^2\,b - b\,c^2 + b^3 + a\,c\,\sqrt{a^2+b^2-c^2}\right)\left(32\,a^2\,b - 32\,b\,c^2 + 32\tan\left(\frac{x}{2}\right)(a-c)\left(2\,a^2 - 2\,a\,c + 3\,b^2\right.\right.}{}\right.}{}$$

$$- \frac{\ln\left(\tan\left(\frac{x}{2}\right) - 1\right)}{b+c} - \frac{\ln\left(\tan\left(\frac{x}{2}\right) + 1\right)}{b-c}$$

$$+ \frac{\ln\left(32\,a\,c - 32\,a^2 - 32\,b\tan\left(\frac{x}{2}\right)\,(a-c) - \frac{\left(a^2\,b - b\,c^2 + b^3 - a\,c\,\sqrt{a^2+b^2-c^2}\right)\left(32\,a^2\,b - 32\,b\,c^2 + 32\tan\left(\frac{x}{2}\right)(a-c)\left(2\,a^2 - 2\,a\,c + \right.\right.}{}\right.}{}$$

input
```
int(1/(cos(x)^2*(a + b*tan(x) + c/cos(x))),x)
```

output
```
(log(32*a*c - 32*a^2 - 32*b*tan(x/2)*(a - c) - ((a^2*b - b*c^2 + b^3 + a*c
*(a^2 + b^2 - c^2)^(1/2))*(32*a^2*b - 32*b*c^2 + 32*tan(x/2)*(a - c)*(2*a^
2 - 2*a*c + 3*b^2 + c^2) - ((a^2*b - b*c^2 + b^3 + a*c*(a^2 + b^2 - c^2)^(
1/2))*(32*c^4 - 64*a*c^3 + 32*a^2*b^2 + 32*a^2*c^2 + 64*b^2*c^2 - 96*a*b^2
*c + 32*b*tan(x/2)*(a - c)*(4*a*c + b^2 - 4*c^2) + (32*(a - c)*(a^2*b - b*
c^2 + b^3 + a*c*(a^2 + b^2 - c^2)^(1/2))*(3*b^4*tan(x/2) + a*b^3 - 3*b*c^3
 + 3*b^3*c + 2*a^2*b^2*tan(x/2) + 2*a^2*c^2*tan(x/2) - 3*b^2*c^2*tan(x/2)
- 2*a*c^3*tan(x/2) - a*b*c^2 + 4*a^2*b*c + 2*a*b^2*c*tan(x/2)))/((b^2 - c^
2)*(a^2 + b^2 - c^2))))/((b^2 - c^2)*(a^2 + b^2 - c^2))))/((b^2 - c^2)*(a^
2 + b^2 - c^2)))*(b*(a^2 - c^2) + b^3 + a*c*(a^2 + b^2 - c^2)^(1/2)))/((b^
2 - c^2)*(a^2 + b^2 - c^2)) - log(tan(x/2) - 1)/(b + c) - log(tan(x/2) + 1
)/(b - c) + (log(32*a*c - 32*a^2 - 32*b*tan(x/2)*(a - c) - ((a^2*b - b*c^2
 + b^3 - a*c*(a^2 + b^2 - c^2)^(1/2))*(32*a^2*b - 32*b*c^2 + 32*tan(x/2)*(
a - c)*(2*a^2 - 2*a*c + 3*b^2 + c^2) - ((a^2*b - b*c^2 + b^3 - a*c*(a^2 +
b^2 - c^2)^(1/2))*(32*c^4 - 64*a*c^3 + 32*a^2*b^2 + 32*a^2*c^2 + 64*b^2*c^
2 - 96*a*b^2*c + 32*b*tan(x/2)*(a - c)*(4*a*c + b^2 - 4*c^2) + (32*(a - c)
*(a^2*b - b*c^2 + b^3 - a*c*(a^2 + b^2 - c^2)^(1/2))*(3*b^4*tan(x/2) + a*b
^3 - 3*b*c^3 + 3*b^3*c + 2*a^2*b^2*tan(x/2) + 2*a^2*c^2*tan(x/2) - 3*b^2*c
^2*tan(x/2) - 2*a*c^3*tan(x/2) - a*b*c^2 + 4*a^2*b*c + 2*a*b^2*c*tan(x/2))
)/((b^2 - c^2)*(a^2 + b^2 - c^2))))/((b^2 - c^2)*(a^2 + b^2 - c^2))))/(...
```

3.447. $\int \frac{\sec^2(x)}{a+c\sec(x)+b\tan(x)}\,dx$

3.448 $\int \frac{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}{\sec^{\frac{3}{2}}(d+ex)}\,dx$

3.448.1 Optimal result

Integrand size = 33, antiderivative size = 371

$$\int \frac{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}{\sec^{\frac{3}{2}}(d+ex)}\,dx =$$

$$-\frac{2(c\cos(d+ex)-a\sin(d+ex))(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}{3e\sec^{\frac{3}{2}}(d+ex)(b+a\cos(d+ex)+c\sin(d+ex))}$$

$$+\frac{8bE\left(\frac{1}{2}(d+ex-\tan^{-1}(a,c))\,|\,\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}{3e\sec^{\frac{3}{2}}(d+ex)(b+a\cos(d+ex)+c\sin(d+ex))\sqrt{\frac{b+a\cos(d+ex)+c\sin(d+ex)}{b+\sqrt{a^2+c^2}}}}$$

$$+\frac{2(a^2-b^2+c^2)\,\text{EllipticF}\left(\frac{1}{2}(d+ex-\tan^{-1}(a,c)),\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)\sqrt{\frac{b+a\cos(d+ex)+c\sin(d+ex)}{b+\sqrt{a^2+c^2}}}(a+b\sec(d+ex)+}{3e\sec^{\frac{3}{2}}(d+ex)(b+a\cos(d+ex)+c\sin(d+ex))^2}$$

output
```
-2/3*(c*cos(e*x+d)-a*sin(e*x+d))*(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2)/e/sec
(e*x+d)^(3/2)/(b+a*cos(e*x+d)+c*sin(e*x+d))+8/3*b*(cos(1/2*d+1/2*e*x-1/2*a
rctan(a,c))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(a,c))*EllipticE(sin(1/2*
d+1/2*e*x-1/2*arctan(a,c)),2^(1/2)*((a^2+c^2)^(1/2)/(b+(a^2+c^2)^(1/2)))^(
1/2))*(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2)/e/sec(e*x+d)^(3/2)/(b+a*cos(e*x+
d)+c*sin(e*x+d))/((b+a*cos(e*x+d)+c*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(1/2)
+2/3*(a^2-b^2+c^2)*(cos(1/2*d+1/2*e*x-1/2*arctan(a,c))^2)^(1/2)/cos(1/2*d+
1/2*e*x-1/2*arctan(a,c))*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan(a,c)),2^(1
/2)*((a^2+c^2)^(1/2)/(b+(a^2+c^2)^(1/2)))^(1/2))*((b+a*cos(e*x+d)+c*sin(e*
x+d))/(b+(a^2+c^2)^(1/2)))^(1/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2)/e/sec
(e*x+d)^(3/2)/(b+a*cos(e*x+d)+c*sin(e*x+d))^2
```

3.448.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 6.82 (sec) , antiderivative size = 2490, normalized size of antiderivative = 6.71

$$\int \frac{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}{\sec^{\frac{3}{2}}(d+ex)}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2)/Sec[d + e*x]^(3/2),x
]
```

output
```
(((8*a*b)/(3*c) - (2*c*Cos[d + e*x])/3 + (2*a*Sin[d + e*x])/3)*(a + b*Sec[
d + e*x] + c*Tan[d + e*x])^(3/2))/(e*Sec[d + e*x]^(3/2)*(b + a*Cos[d + e*x
] + c*Sin[d + e*x])) + (2*a^2*AppellF1[1/2, 1/2, 1/2, 3/2, -((b + Sqrt[1 +
 a^2/c^2]*c*Sin[d + e*x + ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(1 - b/(Sqrt[1
+ a^2/c^2]*c))*c)), -((b + Sqrt[1 + a^2/c^2]*c*Sin[d + e*x + ArcTan[a/c]])
/(Sqrt[1 + a^2/c^2]*(-1 - b/(Sqrt[1 + a^2/c^2]*c))*c))]*Sec[d + e*x + ArcT
an[a/c]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e
*x + ArcTan[a/c]])/(b + c*Sqrt[(a^2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a^2 + c
^2)/c^2]*Sin[d + e*x + ArcTan[a/c]]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] + c*Sqr
t[(a^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[a/c]])/(-b + c*Sqrt[(a^2 + c^2)/c^
2])]*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2))/(3*Sqrt[1 + a^2/c^2]*c*e
*Sec[d + e*x]^(3/2)*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^(3/2)) + (2*b^2*
AppellF1[1/2, 1/2, 1/2, 3/2, -((b + Sqrt[1 + a^2/c^2]*c*Sin[d + e*x + ArcT
an[a/c]])/(Sqrt[1 + a^2/c^2]*(1 - b/(Sqrt[1 + a^2/c^2]*c))*c)), -((b + Sqr
t[1 + a^2/c^2]*c*Sin[d + e*x + ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b/(S
qrt[1 + a^2/c^2]*c))*c))]*Sec[d + e*x + ArcTan[a/c]]*Sqrt[(c*Sqrt[(a^2 + c
^2)/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[a/c]])/(b + c*Sqrt
[(a^2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[
a/c]]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] + c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*x
 + ArcTan[a/c]])/(-b + c*Sqrt[(a^2 + c^2)/c^2])]*(a + b*Sec[d + e*x] + ...
```

3.448.3 Rubi [A] (verified)

Time = 1.45 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.91, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.424$, Rules used = {3042, 3646, 3042, 3599, 27, 3042, 3628, 3042, 3598, 3042, 3132, 3606, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

3.448. $\int \frac{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}{\sec^{\frac{3}{2}}(d+ex)}\,dx$

$$\int \frac{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}{\sec^{\frac{3}{2}}(d+ex)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}{\sec(d+ex)^{3/2}}dx$$

$$\Big\downarrow \text{ 3646}$$

$$\frac{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}\int(b+a\cos(d+ex)+c\sin(d+ex))^{3/2}dx}{\sec^{\frac{3}{2}}(d+ex)(a\cos(d+ex)+b+c\sin(d+ex))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}\int(b+a\cos(d+ex)+c\sin(d+ex))^{3/2}dx}{\sec^{\frac{3}{2}}(d+ex)(a\cos(d+ex)+b+c\sin(d+ex))^{3/2}}$$

$$\Big\downarrow \text{ 3599}$$

$$\frac{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}\left(\frac{2}{3}\int\frac{a^2+4b\cos(d+ex)a+3b^2+c^2+4bc\sin(d+ex)}{2\sqrt{b+a\cos(d+ex)+c\sin(d+ex)}}dx-\frac{2(c\cos(d+ex)-a\sin(d+ex))\sqrt{a\cos(d+}}{3e}\right.}{\sec^{\frac{3}{2}}(d+ex)(a\cos(d+ex)+b+c\sin(d+ex))^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}\left(\frac{1}{3}\int\frac{a^2+4b\cos(d+ex)a+3b^2+c^2+4bc\sin(d+ex)}{\sqrt{b+a\cos(d+ex)+c\sin(d+ex)}}dx-\frac{2(c\cos(d+ex)-a\sin(d+ex))\sqrt{a\cos(d+}}{3e}\right.}{\sec^{\frac{3}{2}}(d+ex)(a\cos(d+ex)+b+c\sin(d+ex))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}\left(\frac{1}{3}\int\frac{a^2+4b\cos(d+ex)a+3b^2+c^2+4bc\sin(d+ex)}{\sqrt{b+a\cos(d+ex)+c\sin(d+ex)}}dx-\frac{2(c\cos(d+ex)-a\sin(d+ex))\sqrt{a\cos(d+}}{3e}\right.}{\sec^{\frac{3}{2}}(d+ex)(a\cos(d+ex)+b+c\sin(d+ex))^{3/2}}$$

$$\Big\downarrow \text{ 3628}$$

$$\frac{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}\left(\frac{1}{3}\left((a^2-b^2+c^2)\int\frac{1}{\sqrt{b+a\cos(d+ex)+c\sin(d+ex)}}dx+4b\int\sqrt{b+a\cos(d+ex)}\right.\right.}{\sec^{\frac{3}{2}}(d+ex)(a\cos(d+ex)+b+c\sin(d+ex))}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}\left(\frac{1}{3}\left((a^2-b^2+c^2)\int\frac{1}{\sqrt{b+a\cos(d+ex)+c\sin(d+ex)}}dx+4b\int\sqrt{b+a\cos(d+ex)}\right.\right.}{\sec^{\frac{3}{2}}(d+ex)(a\cos(d+ex)+b+c\sin(d+ex))}$$

3.448. $\int \frac{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}{\sec^{\frac{3}{2}}(d+ex)}\,dx$

$\downarrow$ 3598

$$\frac{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}\left(\frac{1}{3}\left(\left(a^2-b^2+c^2\right)\int\frac{1}{\sqrt{b+a\cos(d+ex)+c\sin(d+ex)}}dx+\frac{4b\sqrt{a\cos(d+ex)+b+c\sin(d+}}{\sqrt{}}\right.\right.}{\sec^{\frac{3}{2}}(d+ex)(a\cos(d+ex)+b+c}$$

$\downarrow$ 3042

$$\frac{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}\left(\frac{1}{3}\left(\left(a^2-b^2+c^2\right)\int\frac{1}{\sqrt{b+a\cos(d+ex)+c\sin(d+ex)}}dx+\frac{4b\sqrt{a\cos(d+ex)+b+c\sin(d+}}{}\right.\right.}{\sec^{\frac{3}{2}}(d+ex)(a\cos(d+ex)+b+}$$

$\downarrow$ 3132

$$\frac{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}\left(\frac{1}{3}\left(\left(a^2-b^2+c^2\right)\int\frac{1}{\sqrt{b+a\cos(d+ex)+c\sin(d+ex)}}dx+\frac{8b\sqrt{a\cos(d+ex)+b+c\sin(d+}}{e\sqrt{\frac{a\cos(}{}}}\right.\right.}{\sec^{\frac{3}{2}}(d+ex)(a\cos(d+ex)+b+c\sin(}$$

$\downarrow$ 3606

$$\frac{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}\left(\frac{1}{3}\left(\frac{\left(a^2-b^2+c^2\right)\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}\int\frac{1}{\sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+\frac{\sqrt{a^2+c^2}\cos\left(d+ex-\tan^{-1}(a,c)\right)}{b+\sqrt{a^2+c^2}}}}dx}{\sqrt{a\cos(d+ex)+b+c\sin(d+ex)}}\right.\right.}{\sec^{\frac{3}{2}}(d+ex)(a\cos(d+e}$$

$\downarrow$ 3042

$$\frac{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}\left(\frac{1}{3}\left(\frac{\left(a^2-b^2+c^2\right)\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}\int\frac{1}{\sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+\frac{\sqrt{a^2+c^2}\sin\left(d+ex-\tan^{-1}(a,c)+\frac{\pi}{2}\right)}{b+\sqrt{a^2+c^2}}}}}{\sqrt{a\cos(d+ex)+b+c\sin(d+ex)}}\right.\right.}{\sec^{\frac{3}{2}}(d+ex)(a\cos(d+}$$

$\downarrow$ 3140

3.448. $\int\frac{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}{\sec^{\frac{3}{2}}(d+ex)}dx$

$$\frac{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}\left(\frac{1}{3}\left(\frac{2(a^2-b^2+c^2)\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}\operatorname{EllipticF}\left(\frac{1}{2}\left(d+ex-\tan^{-1}(a,c)\right),\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{a\cos(d+ex)+b+c\sin(d+ex)}}\right.\right.}{\sec^{\frac{3}{2}}(d+ex)(a\cos(d+ex}$$

input
```
Int[(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2)/Sec[d + e*x]^(3/2),x]
```

output
```
(((-2*(c*Cos[d + e*x] - a*Sin[d + e*x])*Sqrt[b + a*Cos[d + e*x] + c*Sin[d
+ e*x]])/(3*e) + ((8*b*EllipticE[(d + e*x - ArcTan[a, c])/2, (2*Sqrt[a^2 +
 c^2])/(b + Sqrt[a^2 + c^2])]*Sqrt[b + a*Cos[d + e*x] + c*Sin[d + e*x]])/(
e*Sqrt[(b + a*Cos[d + e*x] + c*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])]) + (2*
(a^2 - b^2 + c^2)*EllipticF[(d + e*x - ArcTan[a, c])/2, (2*Sqrt[a^2 + c^2]
)/(b + Sqrt[a^2 + c^2])]*Sqrt[(b + a*Cos[d + e*x] + c*Sin[d + e*x])/(b + S
qrt[a^2 + c^2])])/(e*Sqrt[b + a*Cos[d + e*x] + c*Sin[d + e*x]]))/3)*(a + b
*Sec[d + e*x] + c*Tan[d + e*x])^(3/2))/(Sec[d + e*x]^(3/2)*(b + a*Cos[d +
e*x] + c*Sin[d + e*x])^(3/2))
```

3.448.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

3.448. $\int \frac{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}{\sec^{\frac{3}{2}}(d+ex)}\,dx$

rule 3598
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]   Int[Sqrt[a/(a + S
qrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - Arc
Tan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

rule 3599
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Simp[1/n   Int[Simp[n*a^2 + (
n - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x
], x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

rule 3606
```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Simp[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]   Int[1/Sqrt[a/(a
 + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x -
 ArcTan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2
, 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

rule 3628
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]
, x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[
A*b - a*B, 0]
```

rule 3646
```
Int[sec[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] +
(c_.)*tan[(d_.) + (e_.)*(x_)])^(m_), x_Symbol] :> Simp[Sec[d + e*x]^n*((b +
 a*Cos[d + e*x] + c*Sin[d + e*x])^n/(a + b*Sec[d + e*x] + c*Tan[d + e*x])^n
)   Int[1/(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n, x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[m + n, 0] &&  !IntegerQ[n]
```

3.448. $\int \frac{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}{\sec^{\frac{3}{2}}(d+ex)}\,dx$

3.448.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 29.33 (sec) , antiderivative size = 64069, normalized size of antiderivative = 172.69

method	result	size
default	Expression too large to display	64069

input
```
int((a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2)/sec(e*x+d)^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
result too large to display
```

3.448.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.16 (sec) , antiderivative size = 1504, normalized size of antiderivative = 4.05

$$\int \frac{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}{\sec^{\frac{3}{2}}(d+ex)}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2)/sec(e*x+d)^(3/2),x, algorith
m="fricas")
```

output
```
1/9*((-3*I*a^3 - I*a*b^2 - 3*I*a*c^2 + 3*c^3 + (3*a^2 + b^2)*c)*sqrt(2*a -
 2*I*c)*weierstrassPInverse(-4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 + 6*I*a*c^
3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), 8/27*(9*a^5*
b - 8*a^3*b^3 - 27*a*b*c^4 - 9*I*b*c^5 + 2*I*(9*a^2*b + 4*b^3)*c^3 - 6*(3*
a^3*b - 4*a*b^3)*c^2 + 3*I*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a
^2*c^4 + c^6), 1/3*(2*a*b + 2*I*b*c + 3*(a^2 + c^2)*cos(e*x + d) - 3*(-I*a
^2 - I*c^2)*sin(e*x + d))/(a^2 + c^2)) + (3*I*a^3 + I*a*b^2 + 3*I*a*c^2 +
3*c^3 + (3*a^2 + b^2)*c)*sqrt(2*a + 2*I*c)*weierstrassPInverse(-4/3*(3*a^4
 - 4*a^2*b^2 + 4*b^2*c^2 - 6*I*a*c^3 - 3*c^4 - 2*I*(3*a^3 - 4*a*b^2)*c)/(a
^4 + 2*a^2*c^2 + c^4), 8/27*(9*a^5*b - 8*a^3*b^3 - 27*a*b*c^4 + 9*I*b*c^5
- 2*I*(9*a^2*b + 4*b^3)*c^3 - 6*(3*a^3*b - 4*a*b^3)*c^2 - 3*I*(9*a^4*b - 8
*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(2*a*b - 2*I*b*c + 3
*(a^2 + c^2)*cos(e*x + d) - 3*(I*a^2 + I*c^2)*sin(e*x + d))/(a^2 + c^2)) -
 12*(-I*a^2*b - I*b*c^2)*sqrt(2*a - 2*I*c)*weierstrassZeta(-4/3*(3*a^4 - 4
*a^2*b^2 + 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 +
 2*a^2*c^2 + c^4), 8/27*(9*a^5*b - 8*a^3*b^3 - 27*a*b*c^4 - 9*I*b*c^5 + 2*
I*(9*a^2*b + 4*b^3)*c^3 - 6*(3*a^3*b - 4*a*b^3)*c^2 + 3*I*(9*a^4*b - 8*a^2
*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), weierstrassPInverse(-4/3*(3*
a^4 - 4*a^2*b^2 + 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)
/(a^4 + 2*a^2*c^2 + c^4), 8/27*(9*a^5*b - 8*a^3*b^3 - 27*a*b*c^4 - 9*I*...
```

3.448.6 Sympy [F]

$$\int \frac{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}{\sec^{\frac{3}{2}}(d+ex)}\,dx=\int \frac{(a+b\sec(d+ex)+c\tan(d+ex))^{\frac{3}{2}}}{\sec^{\frac{3}{2}}(d+ex)}\,dx$$

input
```
integrate((a+b*sec(e*x+d)+c*tan(e*x+d))**(3/2)/sec(e*x+d)**(3/2),x)
```

output
```
Integral((a + b*sec(d + e*x) + c*tan(d + e*x))**(3/2)/sec(d + e*x)**(3/2),
 x)
```

3.448.7 Maxima [F]

$$\int \frac{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}{\sec^{\frac{3}{2}}(d+ex)}\,dx=\int \frac{(b\sec{(ex+d)}+c\tan{(ex+d)}+a)^{\frac{3}{2}}}{\sec{(ex+d)}^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2)/sec(e*x+d)^(3/2),x, algorith
m="maxima")
```

output
```
integrate((b*sec(e*x + d) + c*tan(e*x + d) + a)^(3/2)/sec(e*x + d)^(3/2),
x)
```

3.448.8 Giac [F]

$$\int \frac{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}{\sec^{\frac{3}{2}}(d+ex)}\,dx=\int \frac{(b\sec{(ex+d)}+c\tan{(ex+d)}+a)^{\frac{3}{2}}}{\sec{(ex+d)}^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2)/sec(e*x+d)^(3/2),x, algorith
m="giac")
```

output
```
integrate((b*sec(e*x + d) + c*tan(e*x + d) + a)^(3/2)/sec(e*x + d)^(3/2),
x)
```

3.448.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}{\sec^{\frac{3}{2}}(d+ex)}\,dx=\int \frac{\left(a+c\tan(d+e\,x)+\frac{b}{\cos(d+e\,x)}\right)^{3/2}}{\left(\frac{1}{\cos(d+e\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + c*tan(d + e*x) + b/cos(d + e*x))^(3/2)/(1/cos(d + e*x))^(3/2),x)
```

output
```
int((a + c*tan(d + e*x) + b/cos(d + e*x))^(3/2)/(1/cos(d + e*x))^(3/2), x)
```

3.449 $\int \frac{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}{\sqrt{\sec(d+ex)}}\,dx$

3.449.1 Optimal result

Integrand size = 33, antiderivative size = 118

$$\int \frac{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}{\sqrt{\sec(d+ex)}}\,dx = \frac{2E\left(\frac{1}{2}(d+ex-\tan^{-1}(a,c))\,|\,\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}{e\sqrt{\sec(d+ex)}\sqrt{\frac{b+a\cos(d+ex)+c\sin(d+ex)}{b+\sqrt{a^2+c^2}}}}$$

output
```
2*(cos(1/2*d+1/2*e*x-1/2*arctan(a,c))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arcta
n(a,c))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(a,c)),2^(1/2)*((a^2+c^2)^(1
/2)/(b+(a^2+c^2)^(1/2)))^(1/2))*(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2)/e/sec(
e*x+d)^(1/2)/((b+a*cos(e*x+d)+c*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(1/2)
```

3.449.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 6.80 (sec) , antiderivative size = 1580, normalized size of antiderivative = 13.39

$$\int \frac{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}{\sqrt{\sec(d+ex)}}\,dx = \text{Too large to display}$$

input
```
Integrate[Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]]/Sqrt[Sec[d + e*x]],x]
```

output
```
(2*a*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]])/(c*e*Sqrt[Sec[d + e*x]]) +
 (2*b*AppellF1[1/2, 1/2, 1/2, 3/2, -((b + Sqrt[1 + a^2/c^2]*c*Sin[d + e*x
+ ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(1 - b/(Sqrt[1 + a^2/c^2]*c))*c)), -((b
 + Sqrt[1 + a^2/c^2]*c*Sin[d + e*x + ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(-1
- b/(Sqrt[1 + a^2/c^2]*c))*c))]*Sec[d + e*x + ArcTan[a/c]]*Sqrt[(c*Sqrt[(a
^2 + c^2)/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[a/c]])/(b +
c*Sqrt[(a^2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*x + A
rcTan[a/c]]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] + c*Sqrt[(a^2 + c^2)/c^2]*Sin[d
 + e*x + ArcTan[a/c]])/(-b + c*Sqrt[(a^2 + c^2)/c^2])]*Sqrt[a + b*Sec[d +
e*x] + c*Tan[d + e*x]])/(Sqrt[1 + a^2/c^2]*c*e*Sqrt[Sec[d + e*x]]*Sqrt[b +
 a*Cos[d + e*x] + c*Sin[d + e*x]]) + (a^2*(-((c*AppellF1[-1/2, -1/2, -1/2,
 1/2, -((b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 + c
^2/a^2]*(1 - b/(a*Sqrt[1 + c^2/a^2])))), -((b + a*Sqrt[1 + c^2/a^2]*Cos[d
+ e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(-1 - b/(a*Sqrt[1 + c^2/a^2])))
)]*Sin[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*Sqrt[(a*Sqrt[(a^2 + c^
2)/a^2] - a*Sqrt[(a^2 + c^2)/a^2]*Cos[d + e*x - ArcTan[c/a]])/(b + a*Sqrt[
(a^2 + c^2)/a^2])]*Sqrt[b + a*Sqrt[(a^2 + c^2)/a^2]*Cos[d + e*x - ArcTan[c
/a]]]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] + a*Sqrt[(a^2 + c^2)/a^2]*Cos[d + e*x
- ArcTan[c/a]])/(-b + a*Sqrt[(a^2 + c^2)/a^2])])) - ((2*a*(b + a*Sqrt[1 +
c^2/a^2]*Cos[d + e*x - ArcTan[c/a]]))/(a^2 + c^2) - (c*Sin[d + e*x - Ar...
```

3.449.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3042, 3646, 3042, 3598, 3042, 3132}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}{\sqrt{\sec(d+ex)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}{\sqrt{\sec(d+ex)}}dx$$

$$\Big\downarrow \text{ 3646}$$

$$\frac{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}\int \sqrt{b+a\cos(d+ex)+c\sin(d+ex)}dx}{\sqrt{\sec(d+ex)}\sqrt{a\cos(d+ex)+b+c\sin(d+ex)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}\int\sqrt{b+a\cos(d+ex)+c\sin(d+ex)}dx}{\sqrt{\sec(d+ex)}\sqrt{a\cos(d+ex)+b+c\sin(d+ex)}}$$

$$\Big\downarrow \text{3598}$$

$$\frac{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}\int\sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+\frac{\sqrt{a^2+c^2}\cos(d+ex-\tan^{-1}(a,c))}{b+\sqrt{a^2+c^2}}}dx}{\sqrt{\sec(d+ex)}\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}\int\sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+\frac{\sqrt{a^2+c^2}\sin\left(d+ex-\tan^{-1}(a,c)+\frac{\pi}{2}\right)}{b+\sqrt{a^2+c^2}}}dx}{\sqrt{\sec(d+ex)}\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}}$$

$$\Big\downarrow \text{3132}$$

$$\frac{2\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(a,c)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{\sec(d+ex)}\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}}$$

input
```
Int[Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]]/Sqrt[Sec[d + e*x]],x]
```

output
```
(2*EllipticE[(d + e*x - ArcTan[a, c])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2
 + c^2])]*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]])/(e*Sqrt[Sec[d + e*x]]
*Sqrt[(b + a*Cos[d + e*x] + c*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])])
```

3.449.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

3.449. $\int \frac{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}{\sqrt{\sec(d+ex)}}\,dx$

rule 3598
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]   Int[Sqrt[a/(a + S
qrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - Arc
Tan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

rule 3646
```
Int[sec[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] +
(c_.)*tan[(d_.) + (e_.)*(x_)])^(m_), x_Symbol] :> Simp[Sec[d + e*x]^n*((b +
 a*Cos[d + e*x] + c*Sin[d + e*x])^n/(a + b*Sec[d + e*x] + c*Tan[d + e*x])^n
)   Int[1/(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n, x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[m + n, 0] &&  !IntegerQ[n]
```

3.449.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 26.93 (sec) , antiderivative size = 2660, normalized size of antiderivative = 22.54

method	result	size
risch	Expression too large to display	2660
default	Expression too large to display	12922

input
```
int((a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2)/sec(e*x+d)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
I*(-I*exp(I*(e*x+d))^2*c+a*exp(I*(e*x+d))^2+2*b*exp(I*(e*x+d))+I*c+a)/e/(e
xp(I*(e*x+d))*(-I*exp(I*(e*x+d))^2*c+a*exp(I*(e*x+d))^2+2*b*exp(I*(e*x+d))
+I*c+a))^(1/2)/(exp(RootOf(_Z^2+1,index=1)*(e*x+d))/(exp(RootOf(_Z^2+1,ind
ex=1)*(e*x+d))^2+1))^(1/2)*(-(RootOf(_Z^2+1,index=1)*exp(RootOf(_Z^2+1,ind
ex=1)*(e*x+d))^2*c-exp(RootOf(_Z^2+1,index=1)*(e*x+d))^2*a-2*b*exp(RootOf(
_Z^2+1,index=1)*(e*x+d))-RootOf(_Z^2+1,index=1)*c-a)/(exp(RootOf(_Z^2+1,in
dex=1)*(e*x+d))^2+1))^(1/2)*(-(RootOf(_Z^2+1,index=1)*exp(RootOf(_Z^2+1,in
dex=1)*(e*x+d))^2*c-exp(RootOf(_Z^2+1,index=1)*(e*x+d))^2*a-2*b*exp(RootOf
(_Z^2+1,index=1)*(e*x+d))-RootOf(_Z^2+1,index=1)*c-a)*(exp(RootOf(_Z^2+1,i
ndex=1)*(e*x+d))^2+1))^(1/2)/(RootOf(_Z^2+1,index=1)*exp(RootOf(_Z^2+1,ind
ex=1)*(e*x+d))^2*c-exp(RootOf(_Z^2+1,index=1)*(e*x+d))^2*a-2*b*exp(RootOf(
_Z^2+1,index=1)*(e*x+d))-RootOf(_Z^2+1,index=1)*c-a)*(exp(RootOf(_Z^2+1,in
dex=1)*(e*x+d))*(-RootOf(_Z^2+1,index=1)*exp(RootOf(_Z^2+1,index=1)*(e*x+d
))^2*c+exp(RootOf(_Z^2+1,index=1)*(e*x+d))^2*a+2*b*exp(RootOf(_Z^2+1,index
=1)*(e*x+d))+RootOf(_Z^2+1,index=1)*c+a))^(1/2)/((-RootOf(_Z^2+1,index=1)*
exp(RootOf(_Z^2+1,index=1)*(e*x+d))^2*c+exp(RootOf(_Z^2+1,index=1)*(e*x+d)
)^2*a+2*b*exp(RootOf(_Z^2+1,index=1)*(e*x+d))+RootOf(_Z^2+1,index=1)*c+a)*
(exp(RootOf(_Z^2+1,index=1)*(e*x+d))^2+1))^(1/2)*2^(1/2)-I/e*(-2*b*(-b+(-a
^2+b^2-c^2)^(1/2))/(I*c-a)*((exp(I*(e*x+d))+(-b+(-a^2+b^2-c^2)^(1/2))/(I*c
-a))/(-b+(-a^2+b^2-c^2)^(1/2))*(I*c-a))^(1/2)*((exp(I*(e*x+d))-(b+(-a^2...
```

3.449.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 1367, normalized size of antiderivative = 11.58

$$\int \frac{\sqrt{a + b\sec(d + ex) + c\tan(d + ex)}}{\sqrt{\sec(d + ex)}}\, dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2)/sec(e*x+d)^(1/2),x, algorith
m="fricas")
```

output
```
1/3*((-I*a*b + b*c)*sqrt(2*a - 2*I*c)*weierstrassPInverse(-4/3*(3*a^4 - 4*
a^2*b^2 + 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 +
2*a^2*c^2 + c^4), 8/27*(9*a^5*b - 8*a^3*b^3 - 27*a*b*c^4 - 9*I*b*c^5 + 2*I
*(9*a^2*b + 4*b^3)*c^3 - 6*(3*a^3*b - 4*a*b^3)*c^2 + 3*I*(9*a^4*b - 8*a^2*
b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(2*a*b + 2*I*b*c + 3*(a^2
 + c^2)*cos(e*x + d) - 3*(-I*a^2 - I*c^2)*sin(e*x + d))/(a^2 + c^2)) + (I*
a*b + b*c)*sqrt(2*a + 2*I*c)*weierstrassPInverse(-4/3*(3*a^4 - 4*a^2*b^2 +
 4*b^2*c^2 - 6*I*a*c^3 - 3*c^4 - 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2
 + c^4), 8/27*(9*a^5*b - 8*a^3*b^3 - 27*a*b*c^4 + 9*I*b*c^5 - 2*I*(9*a^2*b
 + 4*b^3)*c^3 - 6*(3*a^3*b - 4*a*b^3)*c^2 - 3*I*(9*a^4*b - 8*a^2*b^3)*c)/(
a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(2*a*b - 2*I*b*c + 3*(a^2 + c^2)*c
os(e*x + d) - 3*(I*a^2 + I*c^2)*sin(e*x + d))/(a^2 + c^2)) - 3*(-I*a^2 - I
*c^2)*sqrt(2*a - 2*I*c)*weierstrassZeta(-4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^
2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4),
8/27*(9*a^5*b - 8*a^3*b^3 - 27*a*b*c^4 - 9*I*b*c^5 + 2*I*(9*a^2*b + 4*b^3)
*c^3 - 6*(3*a^3*b - 4*a*b^3)*c^2 + 3*I*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a
^4*c^2 + 3*a^2*c^4 + c^6), weierstrassPInverse(-4/3*(3*a^4 - 4*a^2*b^2 + 4
*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 +
 c^4), 8/27*(9*a^5*b - 8*a^3*b^3 - 27*a*b*c^4 - 9*I*b*c^5 + 2*I*(9*a^2*b +
 4*b^3)*c^3 - 6*(3*a^3*b - 4*a*b^3)*c^2 + 3*I*(9*a^4*b - 8*a^2*b^3)*c)/...
```

3.449.6 Sympy [F]

$$\int \frac{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}{\sqrt{\sec(d+ex)}}\,dx=\int \frac{\sqrt{a+b\sec\left(d+ex\right)+c\tan\left(d+ex\right)}}{\sqrt{\sec\left(d+ex\right)}}\,dx$$

input
```
integrate((a+b*sec(e*x+d)+c*tan(e*x+d))**(1/2)/sec(e*x+d)**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(d + e*x) + c*tan(d + e*x))/sqrt(sec(d + e*x)), x)
```

3.449.7 Maxima [F]

$$\int \frac{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}{\sqrt{\sec(d+ex)}}\,dx = \int \frac{\sqrt{b\sec(ex+d)+c\tan(ex+d)+a}}{\sqrt{\sec(ex+d)}}\,dx$$

input
```
integrate((a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2)/sec(e*x+d)^(1/2),x, algorith
m="maxima")
```

output
```
integrate(sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a)/sqrt(sec(e*x + d)), x)
```

3.449.8 Giac [F]

$$\int \frac{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}{\sqrt{\sec(d+ex)}}\,dx = \int \frac{\sqrt{b\sec(ex+d)+c\tan(ex+d)+a}}{\sqrt{\sec(ex+d)}}\,dx$$

input
```
integrate((a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2)/sec(e*x+d)^(1/2),x, algorith
m="giac")
```

output
```
integrate(sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a)/sqrt(sec(e*x + d)), x)
```

3.449.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}{\sqrt{\sec(d+ex)}}\,dx = \int \frac{\sqrt{a+c\tan(d+e\,x)+\frac{b}{\cos(d+e\,x)}}}{\sqrt{\frac{1}{\cos(d+e\,x)}}}\,dx$$

input
```
int((a + c*tan(d + e*x) + b/cos(d + e*x))^(1/2)/(1/cos(d + e*x))^(1/2),x)
```

output
```
int((a + c*tan(d + e*x) + b/cos(d + e*x))^(1/2)/(1/cos(d + e*x))^(1/2), x)
```

3.450 $\int \frac{\sqrt{\sec(d+ex)}}{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}\,dx$

3.450.1 Optimal result

Integrand size = 33, antiderivative size = 118

$$\int \frac{\sqrt{\sec(d+ex)}}{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}\,dx = \frac{2\,\text{EllipticF}\left(\frac{1}{2}(d+ex-\tan^{-1}(a,c)),\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)\sqrt{\sec(d+ex)}\sqrt{\frac{b+a\cos(d+ex)+c\sin(d+ex)}{b+\sqrt{a^2+c^2}}}}{e\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}$$

output
```
2*(cos(1/2*d+1/2*e*x-1/2*arctan(a,c))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arcta
n(a,c))*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan(a,c)),2^(1/2)*((a^2+c^2)^(1
/2)/(b+(a^2+c^2)^(1/2)))^(1/2))*sec(e*x+d)^(1/2)*((b+a*cos(e*x+d)+c*sin(e*
x+d))/(b+(a^2+c^2)^(1/2)))^(1/2)/e/(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2)
```

3.450.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 1.26 (sec) , antiderivative size = 339, normalized size of antiderivative = 2.87

$$\int \frac{\sqrt{\sec(d+ex)}}{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}\,dx = \frac{2\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{3}{2},\frac{b+\sqrt{1+\frac{a^2}{c^2}}c\sin\left(d+ex+\arctan\left(\frac{a}{c}\right)\right)}{b-\sqrt{1+\frac{a^2}{c^2}}c},\frac{b+\sqrt{1+\frac{a^2}{c^2}}c\sin\left(d+ex+\arctan\left(\frac{a}{c}\right)\right)}{b+\sqrt{1+\frac{a^2}{c^2}}c}\right)\sqrt{\sec(d+ex)}\sec\left(d+ex-\right.}{}$$

input
```
Integrate[Sqrt[Sec[d + e*x]]/Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]],x]
```

output
```
(2*AppellF1[1/2, 1/2, 1/2, 3/2, (b + Sqrt[1 + a^2/c^2]*c*Sin[d + e*x + Arc
Tan[a/c]])/(b - Sqrt[1 + a^2/c^2]*c), (b + Sqrt[1 + a^2/c^2]*c*Sin[d + e*x
 + ArcTan[a/c]])/(b + Sqrt[1 + a^2/c^2]*c)]*Sqrt[Sec[d + e*x]]*Sec[d + e*x
 + ArcTan[a/c]]*Sqrt[b + a*Cos[d + e*x] + c*Sin[d + e*x]]*Sqrt[-((Sqrt[1 +
 a^2/c^2]*c*(-1 + Sin[d + e*x + ArcTan[a/c]]))/(b + Sqrt[1 + a^2/c^2]*c))]
*Sqrt[(Sqrt[1 + a^2/c^2]*c*(1 + Sin[d + e*x + ArcTan[a/c]]))/(-b + Sqrt[1
+ a^2/c^2]*c)]*Sqrt[b + Sqrt[1 + a^2/c^2]*c*Sin[d + e*x + ArcTan[a/c]]])/(
Sqrt[1 + a^2/c^2]*c*e*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]])
```

3.450.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3042, 3646, 3042, 3606, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\sec(d+ex)}}{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\sec(d+ex)}}{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}dx$$

$$\Big\downarrow \text{3646}$$

$$\frac{\sqrt{\sec(d+ex)}\sqrt{a\cos(d+ex)+b+c\sin(d+ex)}\int \frac{1}{\sqrt{b+a\cos(d+ex)+c\sin(d+ex)}}dx}{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sec(d+ex)}\sqrt{a\cos(d+ex)+b+c\sin(d+ex)}\int \frac{1}{\sqrt{b+a\cos(d+ex)+c\sin(d+ex)}}dx}{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}$$

$$\Big\downarrow \text{3606}$$

$$\frac{\sqrt{\sec(d+ex)}\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}\int \frac{1}{\sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+\frac{\sqrt{a^2+c^2}\cos\left(d+ex-\tan^{-1}(a,c)\right)}{b+\sqrt{a^2+c^2}}}}dx}{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sec(d+ex)}\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}\int \frac{1}{\sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+\frac{\sqrt{a^2+c^2}\sin\left(d+ex-\tan^{-1}(a,c)+\frac{\pi}{2}\right)}{b+\sqrt{a^2+c^2}}}}dx}{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}$$

$$\Big\downarrow \text{3140}$$

$$\frac{2\sqrt{\sec(d+ex)}\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}\text{EllipticF}\left(\frac{1}{2}\left(d+ex-\tan^{-1}(a,c)\right),\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}$$

input
```
Int[Sqrt[Sec[d + e*x]]/Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]],x]
```

output
```
(2*EllipticF[(d + e*x - ArcTan[a, c])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2
 + c^2])]*Sqrt[Sec[d + e*x]]*Sqrt[(b + a*Cos[d + e*x] + c*Sin[d + e*x])/(b
 + Sqrt[a^2 + c^2])])/(e*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]])
```

3.450.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3606
```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Simp[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]   Int[1/Sqrt[a/(a
 + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x -
 ArcTan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2
, 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

3.450. $\int \frac{\sqrt{\sec(d+ex)}}{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}\,dx$

rule 3646
```
Int[sec[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] +
(c_.)*tan[(d_.) + (e_.)*(x_)])^(m_), x_Symbol] :> Simp[Sec[d + e*x]^n*((b +
 a*Cos[d + e*x] + c*Sin[d + e*x])^n/(a + b*Sec[d + e*x] + c*Tan[d + e*x])^n
)   Int[1/(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n, x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[m + n, 0] &&  !IntegerQ[n]
```

3.450.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 25.33 (sec) , antiderivative size = 1018, normalized size of antiderivative = 8.63

method	result
default	$-\frac{4\left(-i\cos(ex+d)ab-ia\sqrt{a^2-b^2+c^2}\,\sin(ex+d)+i\sqrt{a^2-b^2+c^2}\,\cos(ex+d)c-ia^2-i\cos(ex+d)c^2+i\cos(ex+d)b^2+iac\sin(ex+d)-ibc}{}$

input
```
int(sec(e*x+d)^(1/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
-4/e*(-I*cos(e*x+d)*a*b-I*a*(a^2-b^2+c^2)^(1/2)*sin(e*x+d)+I*(a^2-b^2+c^2)
^(1/2)*cos(e*x+d)*c-I*a^2-I*cos(e*x+d)*c^2+I*cos(e*x+d)*b^2+I*a*c*sin(e*x+
d)-I*b*c*sin(e*x+d)+I*a*b-I*c^2+I*b*(a^2-b^2+c^2)^(1/2)*sin(e*x+d)+I*c*(a^
2-b^2+c^2)^(1/2)-(a^2-b^2+c^2)^(1/2)*cos(e*x+d)*b+a*c*cos(e*x+d)-a^2*sin(e
*x+d)+b^2*sin(e*x+d)-a*(a^2-b^2+c^2)^(1/2)+c*b)*cos(e*x+d)*(-((a^2-b^2+c^2
)^(1/2)*sin(e*x+d)+a*cos(e*x+d)-b*cos(e*x+d)+c*sin(e*x+d)-a+b)/((a^2-b^2+c
^2)^(1/2)*sin(e*x+d)-a*cos(e*x+d)+b*cos(e*x+d)-c*sin(e*x+d)+a-b)*(I*a-I*b-
(a^2-b^2+c^2)^(1/2)+c)/(I*a-I*b+(a^2-b^2+c^2)^(1/2)+c))^(1/2)*((I*sin(e*x+
d)+cos(e*x+d)-1)/((a^2-b^2+c^2)^(1/2)*sin(e*x+d)-a*cos(e*x+d)+b*cos(e*x+d)
-c*sin(e*x+d)+a-b)*(a-b)*(a^2-b^2+c^2)^(1/2)/(I*a-I*b-(a^2-b^2+c^2)^(1/2)-
c))^(1/2)*((I*sin(e*x+d)-cos(e*x+d)+1)/((a^2-b^2+c^2)^(1/2)*sin(e*x+d)-a*c
os(e*x+d)+b*cos(e*x+d)-c*sin(e*x+d)+a-b)*(a-b)*(a^2-b^2+c^2)^(1/2)/(I*a-I*
b+(a^2-b^2+c^2)^(1/2)+c))^(1/2)*EllipticF((-((a^2-b^2+c^2)^(1/2)*sin(e*x+d
)+a*cos(e*x+d)-b*cos(e*x+d)+c*sin(e*x+d)-a+b)/((a^2-b^2+c^2)^(1/2)*sin(e*x
+d)-a*cos(e*x+d)+b*cos(e*x+d)-c*sin(e*x+d)+a-b)*(I*a-I*b-(a^2-b^2+c^2)^(1/
2)+c)/(I*a-I*b+(a^2-b^2+c^2)^(1/2)+c))^(1/2),((I*a-I*b+(a^2-b^2+c^2)^(1/2)
+c)*(I*a-I*b+(a^2-b^2+c^2)^(1/2)-c)/(I*a-I*b-(a^2-b^2+c^2)^(1/2)+c)/(I*a-I
*b-(a^2-b^2+c^2)^(1/2)-c))^(1/2))*(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2)*sec(
e*x+d)^(1/2)/(b+a*cos(e*x+d)+c*sin(e*x+d))/(I*a-I*b-(a^2-b^2+c^2)^(1/2)+c)
/(a^2-b^2+c^2)^(1/2)
```

3.450. $\int \frac{\sqrt{\sec(d+ex)}}{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}\,dx$

3.450.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 504, normalized size of antiderivative = 4.27

$$\int \frac{\sqrt{\sec(d+ex)}}{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}\,dx$$
$$= \frac{\sqrt{2a-2ic}(-ia+c)\text{weierstrassPInverse}\left(-\frac{4\left(3a^4-4a^2b^2+4b^2c^2+6iac^3-3c^4+2i(3a^3-4ab^2)c\right)}{3(a^4+2a^2c^2+c^4)}, \frac{8\left(9a^5b-8a^3b^3-27abc^4-\right.}{}\right.}{}$$

input
```
integrate(sec(e*x+d)^(1/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2),x, algorith
m="fricas")
```

output
```
(sqrt(2*a - 2*I*c)*(-I*a + c)*weierstrassPInverse(-4/3*(3*a^4 - 4*a^2*b^2
+ 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^
2 + c^4), 8/27*(9*a^5*b - 8*a^3*b^3 - 27*a*b*c^4 - 9*I*b*c^5 + 2*I*(9*a^2*
b + 4*b^3)*c^3 - 6*(3*a^3*b - 4*a*b^3)*c^2 + 3*I*(9*a^4*b - 8*a^2*b^3)*c)/
(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(2*a*b + 2*I*b*c + 3*(a^2 + c^2)*
cos(e*x + d) - 3*(-I*a^2 - I*c^2)*sin(e*x + d))/(a^2 + c^2)) + sqrt(2*a +
2*I*c)*(I*a + c)*weierstrassPInverse(-4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 -
 6*I*a*c^3 - 3*c^4 - 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), 8/2
7*(9*a^5*b - 8*a^3*b^3 - 27*a*b*c^4 + 9*I*b*c^5 - 2*I*(9*a^2*b + 4*b^3)*c^
3 - 6*(3*a^3*b - 4*a*b^3)*c^2 - 3*I*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*
c^2 + 3*a^2*c^4 + c^6), 1/3*(2*a*b - 2*I*b*c + 3*(a^2 + c^2)*cos(e*x + d)
- 3*(I*a^2 + I*c^2)*sin(e*x + d))/(a^2 + c^2)))/((a^2 + c^2)*e)
```

3.450.6 Sympy [F]

$$\int \frac{\sqrt{\sec(d+ex)}}{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}\,dx = \int \frac{\sqrt{\sec(d+ex)}}{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}\,dx$$

input
```
integrate(sec(e*x+d)**(1/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))**(1/2),x)
```

output
```
Integral(sqrt(sec(d + e*x))/sqrt(a + b*sec(d + e*x) + c*tan(d + e*x)), x)
```

3.450.7 Maxima [F]

$$\int \frac{\sqrt{\sec(d+ex)}}{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}\,dx = \int \frac{\sqrt{\sec\left(ex+d\right)}}{\sqrt{b\sec\left(ex+d\right)+c\tan\left(ex+d\right)+a}}\,dx$$

input
```
integrate(sec(e*x+d)^(1/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2),x, algorith
m="maxima")
```

output
```
integrate(sqrt(sec(e*x + d))/sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a), x)
```

3.450.8 Giac [F]

$$\int \frac{\sqrt{\sec(d+ex)}}{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}\,dx = \int \frac{\sqrt{\sec\left(ex+d\right)}}{\sqrt{b\sec\left(ex+d\right)+c\tan\left(ex+d\right)+a}}\,dx$$

input
```
integrate(sec(e*x+d)^(1/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2),x, algorith
m="giac")
```

output
```
integrate(sqrt(sec(e*x + d))/sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a), x)
```

3.450.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\sec(d+ex)}}{\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}\,dx = \int \frac{\sqrt{\frac{1}{\cos(d+e\,x)}}}{\sqrt{a+c\tan\left(d+e\,x\right)+\frac{b}{\cos(d+e\,x)}}}\,dx$$

input
```
int((1/cos(d + e*x))^(1/2)/(a + c*tan(d + e*x) + b/cos(d + e*x))^(1/2),x)
```

output
```
int((1/cos(d + e*x))^(1/2)/(a + c*tan(d + e*x) + b/cos(d + e*x))^(1/2), x)
```

3.451 $\int \frac{\sec^{\frac{3}{2}}(d+ex)}{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}\,dx$

3.451.1 Optimal result

Integrand size = 33, antiderivative size = 240

$$\int \frac{\sec^{\frac{3}{2}}(d+ex)}{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}\,dx = -\frac{2\sec^{\frac{3}{2}}(d+ex)(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+ex)+c\sin(d+ex))}{(a^2-b^2+c^2)\,e(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}} - \frac{2E\left(\frac{1}{2}(d+ex-\tan^{-1}(a,c))\,|\,\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)\sec^{\frac{3}{2}}(d+ex)(b+a\cos(d+ex)+c\sin(d+ex))^2}{(a^2-b^2+c^2)\,e\sqrt{\frac{b+a\cos(d+ex)+c\sin(d+ex)}{b+\sqrt{a^2+c^2}}}(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}$$

output
```
-2*sec(e*x+d)^(3/2)*(c*cos(e*x+d)-a*sin(e*x+d))*(b+a*cos(e*x+d)+c*sin(e*x+
d))/(a^2-b^2+c^2)/e/(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2)-2*(cos(1/2*d+1/2*e
*x-1/2*arctan(a,c))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(a,c))*EllipticE(
sin(1/2*d+1/2*e*x-1/2*arctan(a,c)),2^(1/2)*((a^2+c^2)^(1/2)/(b+(a^2+c^2)^(
1/2)))^(1/2))*sec(e*x+d)^(3/2)*(b+a*cos(e*x+d)+c*sin(e*x+d))^2/(a^2-b^2+c^
2)/e/((b+a*cos(e*x+d)+c*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(1/2)/(a+b*sec(e*
x+d)+c*tan(e*x+d))^(3/2)
```

3.451.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 6.94 (sec) , antiderivative size = 1732, normalized size of antiderivative = 7.22

$$\int \frac{\sec^{\frac{3}{2}}(d+ex)}{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[Sec[d + e*x]^(3/2)/(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2),x
]
```

output
```
(Sec[d + e*x]^(3/2)*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^2*((-2*(a^2 + c^
2))/(a*c*(a^2 - b^2 + c^2)) + (2*(b*c + a^2*Sin[d + e*x] + c^2*Sin[d + e*x
]))/(a*(a^2 - b^2 + c^2)*(b + a*Cos[d + e*x] + c*Sin[d + e*x]))))/(e*(a +
b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2)) - (2*b*AppellF1[1/2, 1/2, 1/2, 3/2
, -((b + Sqrt[1 + a^2/c^2]*c*Sin[d + e*x + ArcTan[a/c]])/(Sqrt[1 + a^2/c^2
]*(1 - b/(Sqrt[1 + a^2/c^2]*c))*c)), -((b + Sqrt[1 + a^2/c^2]*c*Sin[d + e*
x + ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b/(Sqrt[1 + a^2/c^2]*c))*c))]*S
ec[d + e*x]^(3/2)*Sec[d + e*x + ArcTan[a/c]]*(b + a*Cos[d + e*x] + c*Sin[d
 + e*x])^(3/2)*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Sin
[d + e*x + ArcTan[a/c]])/(b + c*Sqrt[(a^2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a
^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[a/c]]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] +
 c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[a/c]])/(-b + c*Sqrt[(a^2 + c
^2)/c^2])])/(Sqrt[1 + a^2/c^2]*c*(a^2 - b^2 + c^2)*e*(a + b*Sec[d + e*x] +
 c*Tan[d + e*x])^(3/2)) - (a^2*Sec[d + e*x]^(3/2)*(b + a*Cos[d + e*x] + c*
Sin[d + e*x])^(3/2)*(-((c*AppellF1[-1/2, -1/2, -1/2, 1/2, -((b + a*Sqrt[1
+ c^2/a^2]*Cos[d + e*x - ArcTan[c/a]])/(a*Sqrt[1 + c^2/a^2]*(1 - b/(a*Sqrt
[1 + c^2/a^2])))), -((b + a*Sqrt[1 + c^2/a^2]*Cos[d + e*x - ArcTan[c/a]])/
(a*Sqrt[1 + c^2/a^2]*(-1 - b/(a*Sqrt[1 + c^2/a^2]))))]*Sin[d + e*x - ArcTa
n[c/a]])/(a*Sqrt[1 + c^2/a^2]*Sqrt[(a*Sqrt[(a^2 + c^2)/a^2] - a*Sqrt[(a^2
+ c^2)/a^2]*Cos[d + e*x - ArcTan[c/a]])/(b + a*Sqrt[(a^2 + c^2)/a^2])]*...
```

3.451.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3042, 3646, 3042, 3607, 3042, 3598, 3042, 3132}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

3.451. $\int \frac{\sec^{\frac{3}{2}}(d+ex)}{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}\,dx$

$$\int \frac{\sec^{\frac{3}{2}}(d+ex)}{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}\,dx$$

$\Big\downarrow$ 3042

$$\int \frac{\sec(d+ex)^{3/2}}{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}dx$$

$\Big\downarrow$ 3646

$$\frac{\sec^{\frac{3}{2}}(d+ex)(a\cos(d+ex)+b+c\sin(d+ex))^{3/2}\int \frac{1}{(b+a\cos(d+ex)+c\sin(d+ex))^{3/2}}dx}{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{\sec^{\frac{3}{2}}(d+ex)(a\cos(d+ex)+b+c\sin(d+ex))^{3/2}\int \frac{1}{(b+a\cos(d+ex)+c\sin(d+ex))^{3/2}}dx}{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}$$

$\Big\downarrow$ 3607

$$\frac{\sec^{\frac{3}{2}}(d+ex)(a\cos(d+ex)+b+c\sin(d+ex))^{3/2}\left(-\frac{\int \sqrt{b+a\cos(d+ex)+c\sin(d+ex)}dx}{a^2-b^2+c^2}-\frac{2(c\cos(d+ex)-a\sin(d+ex))}{e(a^2-b^2+c^2)\sqrt{a\cos(d+ex)+b+c\sin}}\right.}{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{\sec^{\frac{3}{2}}(d+ex)(a\cos(d+ex)+b+c\sin(d+ex))^{3/2}\left(-\frac{\int \sqrt{b+a\cos(d+ex)+c\sin(d+ex)}dx}{a^2-b^2+c^2}-\frac{2(c\cos(d+ex)-a\sin(d+ex))}{e(a^2-b^2+c^2)\sqrt{a\cos(d+ex)+b+c\sin}}\right.}{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}$$

$\Big\downarrow$ 3598

$$\frac{\sec^{\frac{3}{2}}(d+ex)(a\cos(d+ex)+b+c\sin(d+ex))^{3/2}\left(-\frac{\sqrt{a\cos(d+ex)+b+c\sin(d+ex)}\int \sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+\frac{\sqrt{a^2+c^2}\cos\left(d+ex-\tan^{-1}(a,c\right.}{b+\sqrt{a^2+c^2}}}}{(a^2-b^2+c^2)\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}}\right.}{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{\sec^{\frac{3}{2}}(d+ex)(a\cos(d+ex)+b+c\sin(d+ex))^{3/2}\left(-\frac{\sqrt{a\cos(d+ex)+b+c\sin(d+ex)}\int \sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+\frac{\sqrt{a^2+c^2}\sin\left(d+ex-\tan^{-1}(a,c\right.}{b+\sqrt{a^2+c^2}}}}{(a^2-b^2+c^2)\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}}\right.}{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}$$

$\Big\downarrow$ 3132

3.451. $\int \frac{\sec^{\frac{3}{2}}(d+ex)}{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}\,dx$

$$\frac{\sec^{\frac{3}{2}}(d+ex)(a\cos(d+ex)+b+c\sin(d+ex))^{3/2}\left(-\frac{2\sqrt{a\cos(d+ex)+b+c\sin(d+ex)}E\left(\frac{1}{2}(d+ex-\tan^{-1}(a,c))|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e(a^2-b^2+c^2)\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}}-\right.}{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}$$

input
```
Int[Sec[d + e*x]^(3/2)/(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2),x]
```

output
```
(Sec[d + e*x]^(3/2)*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^(3/2)*((-2*(c*Co
s[d + e*x] - a*Sin[d + e*x]))/((a^2 - b^2 + c^2)*e*Sqrt[b + a*Cos[d + e*x]
 + c*Sin[d + e*x]]) - (2*EllipticE[(d + e*x - ArcTan[a, c])/2, (2*Sqrt[a^2
 + c^2])/(b + Sqrt[a^2 + c^2])]*Sqrt[b + a*Cos[d + e*x] + c*Sin[d + e*x]])
/((a^2 - b^2 + c^2)*e*Sqrt[(b + a*Cos[d + e*x] + c*Sin[d + e*x])/(b + Sqrt
[a^2 + c^2])])))/(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2)
```

3.451.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3598
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]   Int[Sqrt[a/(a + S
qrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - Arc
Tan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

rule 3607
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-3/2), x_Symbol] :> Simp[2*((c*Cos[d + e*x] - b*Sin[d + e*x])/(e*(a^2 - b^
2 - c^2)*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]])), x] + Simp[1/(a^2 - b^
2 - c^2)   Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] /; FreeQ[{
a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

3.451. $\int \frac{\sec^{\frac{3}{2}}(d+ex)}{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}\,dx$

rule 3646
```
Int[sec[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] +
(c_.)*tan[(d_.) + (e_.)*(x_)])^(m_), x_Symbol] :> Simp[Sec[d + e*x]^n*((b +
 a*Cos[d + e*x] + c*Sin[d + e*x])^n/(a + b*Sec[d + e*x] + c*Tan[d + e*x])^n
)   Int[1/(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n, x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[m + n, 0] &&  !IntegerQ[n]
```

3.451.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 20.45 (sec) , antiderivative size = 40064, normalized size of antiderivative = 166.93

method	result	size
default	Expression too large to display	40064

input
```
int(sec(e*x+d)^(3/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
result too large to display
```

3.451.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.16 (sec) , antiderivative size = 1724, normalized size of antiderivative = 7.18

$$\int \frac{\sec^{\frac{3}{2}}(d+ex)}{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(e*x+d)^(3/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x, algorith
m="fricas")
```

output
```
1/3*((I*a*b^2 - b^2*c + (I*a^2*b - a*b*c)*cos(e*x + d) + (I*a*b*c - b*c^2)
*sin(e*x + d))*sqrt(2*a - 2*I*c)*weierstrassPInverse(-4/3*(3*a^4 - 4*a^2*b
^2 + 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2
*c^2 + c^4), 8/27*(9*a^5*b - 8*a^3*b^3 - 27*a*b*c^4 - 9*I*b*c^5 + 2*I*(9*a
^2*b + 4*b^3)*c^3 - 6*(3*a^3*b - 4*a*b^3)*c^2 + 3*I*(9*a^4*b - 8*a^2*b^3)*
c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(2*a*b + 2*I*b*c + 3*(a^2 + c^
2)*cos(e*x + d) - 3*(-I*a^2 - I*c^2)*sin(e*x + d))/(a^2 + c^2)) + (-I*a*b^
2 - b^2*c + (-I*a^2*b - a*b*c)*cos(e*x + d) + (-I*a*b*c - b*c^2)*sin(e*x +
 d))*sqrt(2*a + 2*I*c)*weierstrassPInverse(-4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2
*c^2 - 6*I*a*c^3 - 3*c^4 - 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4
), 8/27*(9*a^5*b - 8*a^3*b^3 - 27*a*b*c^4 + 9*I*b*c^5 - 2*I*(9*a^2*b + 4*b
^3)*c^3 - 6*(3*a^3*b - 4*a*b^3)*c^2 - 3*I*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 +
3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(2*a*b - 2*I*b*c + 3*(a^2 + c^2)*cos(e*x
 + d) - 3*(I*a^2 + I*c^2)*sin(e*x + d))/(a^2 + c^2)) - 3*(I*a^2*b + I*b*c^
2 + (I*a^3 + I*a*c^2)*cos(e*x + d) + (I*a^2*c + I*c^3)*sin(e*x + d))*sqrt(
2*a - 2*I*c)*weierstrassZeta(-4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 + 6*I*a*c
^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), 8/27*(9*a^5
*b - 8*a^3*b^3 - 27*a*b*c^4 - 9*I*b*c^5 + 2*I*(9*a^2*b + 4*b^3)*c^3 - 6*(3
*a^3*b - 4*a*b^3)*c^2 + 3*I*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*
a^2*c^4 + c^6), weierstrassPInverse(-4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2...
```

3.451.6 Sympy [F]

$$\int \frac{\sec^{\frac{3}{2}}(d+ex)}{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}\,dx=\int \frac{\sec^{\frac{3}{2}}(d+ex)}{(a+b\sec(d+ex)+c\tan(d+ex))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(e*x+d)**(3/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))**(3/2),x)
```

output
```
Integral(sec(d + e*x)**(3/2)/(a + b*sec(d + e*x) + c*tan(d + e*x))**(3/2),
 x)
```

3.451.7 Maxima [F]

$$\int \frac{\sec^{\frac{3}{2}}(d+ex)}{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}\,dx = \int \frac{\sec\left(ex+d\right)^{\frac{3}{2}}}{\left(b\sec\left(ex+d\right)+c\tan\left(ex+d\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(e*x+d)^(3/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x, algorith
m="maxima")
```

output
```
integrate(sec(e*x + d)^(3/2)/(b*sec(e*x + d) + c*tan(e*x + d) + a)^(3/2),
x)
```

3.451.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(d+ex)}{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(e*x+d)^(3/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x, algorith
m="giac")
```

output
```
Timed out
```

3.451.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{3}{2}}(d+ex)}{(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}\,dx = \int \frac{\left(\frac{1}{\cos(d+e\,x)}\right)^{3/2}}{\left(a+c\tan\left(d+e\,x\right)+\frac{b}{\cos(d+e\,x)}\right)^{3/2}}\,dx$$

input
```
int((1/cos(d + e*x))^(3/2)/(a + c*tan(d + e*x) + b/cos(d + e*x))^(3/2),x)
```

output
```
int((1/cos(d + e*x))^(3/2)/(a + c*tan(d + e*x) + b/cos(d + e*x))^(3/2), x)
```

3.452 $\int \frac{\sec^{\frac{5}{2}}(d+ex)}{(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}\,dx$

3.452.1 Optimal result

Integrand size = 33, antiderivative size = 492

$$
\begin{aligned}
&\int \frac{\sec^{\frac{5}{2}}(d+ex)}{(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}\,dx = \\
&-\frac{2\sec^{\frac{5}{2}}(d+ex)(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+ex)+c\sin(d+ex))}{3\,(a^2-b^2+c^2)\,e(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}} \\
&+\frac{8\sec^{\frac{5}{2}}(d+ex)(bc\cos(d+ex)-ab\sin(d+ex))(b+a\cos(d+ex)+c\sin(d+ex))^2}{3\,(a^2-b^2+c^2)^2\,e(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}} \\
&+\frac{8bE\left(\frac{1}{2}(d+ex-\tan^{-1}(a,c))\,|\,\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)\sec^{\frac{5}{2}}(d+ex)(b+a\cos(d+ex)+c\sin(d+ex))^3}{3\,(a^2-b^2+c^2)^2\,e\sqrt{\frac{b+a\cos(d+ex)+c\sin(d+ex)}{b+\sqrt{a^2+c^2}}}(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}} \\
&+\frac{2\,\text{EllipticF}\left(\frac{1}{2}(d+ex-\tan^{-1}(a,c)),\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)\sec^{\frac{5}{2}}(d+ex)(b+a\cos(d+ex)+c\sin(d+ex))^2\sqrt{\frac{b+a\cos(d}{b}}}{3\,(a^2-b^2+c^2)\,e(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}
\end{aligned}
$$

output
```
-2/3*sec(e*x+d)^(5/2)*(c*cos(e*x+d)-a*sin(e*x+d))*(b+a*cos(e*x+d)+c*sin(e*
x+d))/(a^2-b^2+c^2)/e/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2)+8/3*sec(e*x+d)^(
5/2)*(b*c*cos(e*x+d)-a*b*sin(e*x+d))*(b+a*cos(e*x+d)+c*sin(e*x+d))^2/(a^2-
b^2+c^2)^2/e/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2)+8/3*b*(cos(1/2*d+1/2*e*x-
1/2*arctan(a,c))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(a,c))*EllipticE(sin
(1/2*d+1/2*e*x-1/2*arctan(a,c)),2^(1/2)*((a^2+c^2)^(1/2)/(b+(a^2+c^2)^(1/2
)))^(1/2))*sec(e*x+d)^(5/2)*(b+a*cos(e*x+d)+c*sin(e*x+d))^3/(a^2-b^2+c^2)^
2/e/((b+a*cos(e*x+d)+c*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(1/2)/(a+b*sec(e*x
+d)+c*tan(e*x+d))^(5/2)+2/3*(cos(1/2*d+1/2*e*x-1/2*arctan(a,c))^2)^(1/2)/c
os(1/2*d+1/2*e*x-1/2*arctan(a,c))*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan(a
,c)),2^(1/2)*((a^2+c^2)^(1/2)/(b+(a^2+c^2)^(1/2)))^(1/2))*sec(e*x+d)^(5/2)
*(b+a*cos(e*x+d)+c*sin(e*x+d))^2*((b+a*cos(e*x+d)+c*sin(e*x+d))/(b+(a^2+c^
2)^(1/2)))^(1/2)/(a^2-b^2+c^2)/e/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2)
```

3.452.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 7.01 (sec) , antiderivative size = 2708, normalized size of antiderivative = 5.50

$$\int \frac{\sec^{\frac{5}{2}}(d+ex)}{(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sec[d + e*x]^(5/2)/(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(5/2),x
]
```

output
```
(Sec[d + e*x]^(5/2)*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^3*((8*b*(a^2 + c
^2))/(3*a*c*(-a^2 + b^2 - c^2)^2) + (2*(b*c + a^2*Sin[d + e*x] + c^2*Sin[d
 + e*x]))/(3*a*(a^2 - b^2 + c^2)*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^2)
- (2*(a^2*c + 3*b^2*c + c^3 + 4*a^2*b*Sin[d + e*x] + 4*b*c^2*Sin[d + e*x])
)/(3*a*(a^2 - b^2 + c^2)^2*(b + a*Cos[d + e*x] + c*Sin[d + e*x]))))/(e*(a
+ b*Sec[d + e*x] + c*Tan[d + e*x])^(5/2)) + (2*a^2*AppellF1[1/2, 1/2, 1/2,
 3/2, -((b + Sqrt[1 + a^2/c^2]*c*Sin[d + e*x + ArcTan[a/c]])/(Sqrt[1 + a^2
/c^2]*(1 - b/(Sqrt[1 + a^2/c^2]*c))*c)), -((b + Sqrt[1 + a^2/c^2]*c*Sin[d
+ e*x + ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b/(Sqrt[1 + a^2/c^2]*c))*c)
)]*Sec[d + e*x]^(5/2)*Sec[d + e*x + ArcTan[a/c]]*(b + a*Cos[d + e*x] + c*S
in[d + e*x])^(5/2)*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] - c*Sqrt[(a^2 + c^2)/c^2]
*Sin[d + e*x + ArcTan[a/c]])/(b + c*Sqrt[(a^2 + c^2)/c^2])]*Sqrt[b + c*Sqr
t[(a^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[a/c]]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^
2] + c*Sqrt[(a^2 + c^2)/c^2]*Sin[d + e*x + ArcTan[a/c]])/(-b + c*Sqrt[(a^2
 + c^2)/c^2])])/(3*Sqrt[1 + a^2/c^2]*c*(a^2 - b^2 + c^2)^2*e*(a + b*Sec[d
+ e*x] + c*Tan[d + e*x])^(5/2)) + (2*b^2*AppellF1[1/2, 1/2, 1/2, 3/2, -((b
 + Sqrt[1 + a^2/c^2]*c*Sin[d + e*x + ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(1 -
 b/(Sqrt[1 + a^2/c^2]*c))*c)), -((b + Sqrt[1 + a^2/c^2]*c*Sin[d + e*x + Ar
cTan[a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b/(Sqrt[1 + a^2/c^2]*c))*c))]*Sec[d +
 e*x]^(5/2)*Sec[d + e*x + ArcTan[a/c]]*(b + a*Cos[d + e*x] + c*Sin[d + ...
```

3.452.3 Rubi [A] (verified)

Time = 1.85 (sec) , antiderivative size = 443, normalized size of antiderivative = 0.90, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.515$, Rules used = {3042, 3646, 3042, 3608, 27, 3042, 3635, 27, 3042, 3628, 3042, 3598, 3042, 3132, 3606, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^{\frac{5}{2}}(d+ex)}{(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(d+ex)^{5/2}}{(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}dx$$

$$\Big\downarrow \text{3646}$$

$$\frac{\sec^{\frac{5}{2}}(d+ex)(a\cos(d+ex)+b+c\sin(d+ex))^{5/2}\int \frac{1}{(b+a\cos(d+ex)+c\sin(d+ex))^{5/2}}dx}{(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}$$

$\Big\downarrow$ 3042

$$\frac{\sec^{\frac{5}{2}}(d+ex)(a\cos(d+ex)+b+c\sin(d+ex))^{5/2}\int \frac{1}{(b+a\cos(d+ex)+c\sin(d+ex))^{5/2}}dx}{(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}$$

$\Big\downarrow$ 3608

$$\frac{\sec^{\frac{5}{2}}(d+ex)(a\cos(d+ex)+b+c\sin(d+ex))^{5/2}\left(\frac{2\int -\frac{3b-a\cos(d+ex)-c\sin(d+ex)}{2(b+a\cos(d+ex)+c\sin(d+ex))^{3/2}}dx}{3(a^2-b^2+c^2)}-\frac{2(c\cos(d+ex)-a\sin(d+e}{3e(a^2-b^2+c^2)(a\cos(d+ex)+b+c\,\mathrm{si}}\right.}{(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}$$

$\Big\downarrow$ 27

$$\frac{\sec^{\frac{5}{2}}(d+ex)(a\cos(d+ex)+b+c\sin(d+ex))^{5/2}\left(-\frac{\int \frac{3b-a\cos(d+ex)-c\sin(d+ex)}{(b+a\cos(d+ex)+c\sin(d+ex))^{3/2}}dx}{3(a^2-b^2+c^2)}-\frac{2(c\cos(d+ex)-a\sin(d+ex)}{3e(a^2-b^2+c^2)(a\cos(d+ex)+b+c\sin(}\right.}{(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}$$

$\Big\downarrow$ 3042

$$\frac{\sec^{\frac{5}{2}}(d+ex)(a\cos(d+ex)+b+c\sin(d+ex))^{5/2}\left(-\frac{\int \frac{3b-a\cos(d+ex)-c\sin(d+ex)}{(b+a\cos(d+ex)+c\sin(d+ex))^{3/2}}dx}{3(a^2-b^2+c^2)}-\frac{2(c\cos(d+ex)-a\sin(d+ex)}{3e(a^2-b^2+c^2)(a\cos(d+ex)+b+c\sin(}\right.}{(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}$$

$\Big\downarrow$ 3635

$$\frac{\sec^{\frac{5}{2}}(d+ex)(a\cos(d+ex)+b+c\sin(d+ex))^{5/2}\left(-\frac{\frac{2\int -\frac{a^2+4b\cos(d+ex)a+3b^2+c^2+4bc\sin(d+ex)}{2\sqrt{b+a\cos(d+ex)+c\sin(d+ex)}}dx}{a^2-b^2+c^2}-\frac{8(bc\cos(d+ex)-ab\sin(}{e\left(a^2-b^2+c^2\right)\sqrt{a\cos(d+ex)+}}}{3(a^2-b^2+c^2)}\right.}{(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}$$

$\Big\downarrow$ 27

$$\frac{\sec^{\frac{5}{2}}(d+ex)(a\cos(d+ex)+b+c\sin(d+ex))^{5/2}\left(-\frac{-\frac{\int \frac{a^2+4b\cos(d+ex)a+3b^2+c^2+4bc\sin(d+ex)}{\sqrt{b+a\cos(d+ex)+c\sin(d+ex)}}dx}{a^2-b^2+c^2}-\frac{8(bc\cos(d+ex)-ab\sin(d}{e\left(a^2-b^2+c^2\right)\sqrt{a\cos(d+ex)+b}}}{3(a^2-b^2+c^2)}\right.}{(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}$$

$\Big\downarrow$ 3042

3.452. $\displaystyle\int \frac{\sec^{\frac{5}{2}}(d+ex)}{(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}\,dx$

$$\frac{\sec^{\frac{5}{2}}(d+ex)(a\cos(d+ex)+b+c\sin(d+ex))^{5/2}\left(-\frac{-\frac{\int \frac{a^2+4b\cos(d+ex)a+3b^2+c^2+4bc\sin(d+ex)}{\sqrt{b+a\cos(d+ex)+c\sin(d+ex)}}dx}{a^2-b^2+c^2}-\frac{8(bc\cos(d+ex)-ab\sin(d\ldots}{e\left(a^2-b^2+c^2\right)\sqrt{a\cos(d+ex)+b\ldots}}}{3(a^2-b^2+c^2)}\ldots\right.}{(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}$$

$\Big\downarrow$ 3628

$$\frac{\sec^{\frac{5}{2}}(d+ex)(a\cos(d+ex)+b+c\sin(d+ex))^{5/2}\left(-\frac{-\frac{\left(a^2-b^2+c^2\right)\int\frac{1}{\sqrt{b+a\cos(d+ex)+c\sin(d+ex)}}dx+4b\int\sqrt{b+a\cos(d+ex)+c\sin(d+e\ldots}}{a^2-b^2+c^2}\ldots}{3(a^2-b^2+c^2)}\ldots\right.}{(a+b\sec(d+ex)+c\tan(d+ex))^{5\ldots}}$$

$\Big\downarrow$ 3042

$$\frac{\sec^{\frac{5}{2}}(d+ex)(a\cos(d+ex)+b+c\sin(d+ex))^{5/2}\left(-\frac{-\frac{\left(a^2-b^2+c^2\right)\int\frac{1}{\sqrt{b+a\cos(d+ex)+c\sin(d+ex)}}dx+4b\int\sqrt{b+a\cos(d+ex)+c\sin(d+e\ldots}}{a^2-b^2+c^2}\ldots}{3(a^2-b^2+c^2)}\ldots\right.}{(a+b\sec(d+ex)+c\tan(d+ex))^{5\ldots}}$$

$\Big\downarrow$ 3598

$$\frac{\sec^{\frac{5}{2}}(d+ex)(a\cos(d+ex)+b+c\sin(d+ex))^{5/2}\left(-\frac{-\frac{\left(a^2-b^2+c^2\right)\int\frac{1}{\sqrt{b+a\cos(d+ex)+c\sin(d+ex)}}dx+\frac{4b\sqrt{a\cos(d+ex)+b+c\sin(d+ex)}\ldots}{\sqrt{a\ldots}}}{a^2-b^2+c^2}\ldots}{\ldots}\right.}{(a+b\sec(d+ex)\ldots}$$

$\Big\downarrow$ 3042

$$\frac{\sec^{\frac{5}{2}}(d+ex)(a\cos(d+ex)+b+c\sin(d+ex))^{5/2}\left(-\frac{-\frac{\left(a^2-b^2+c^2\right)\int\frac{1}{\sqrt{b+a\cos(d+ex)+c\sin(d+ex)}}dx+\frac{4b\sqrt{a\cos(d+ex)+b+c\sin(d+ex)}\ldots}{\sqrt{\ldots}}}{a^2-b^2+c^2}\ldots}{\ldots}\right.}{(a+b\sec(d+ex\ldots}$$

3.452. $\displaystyle\int \frac{\sec^{\frac{5}{2}}(d+ex)}{(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}\,dx$

$\Big\downarrow$ 3132

$$\frac{\sec^{\frac{5}{2}}(d+ex)(a\cos(d+ex)+b+c\sin(d+ex))^{5/2}\left(-\frac{-\frac{\left(a^2-b^2+c^2\right)\int\frac{1}{\sqrt{b+a\cos(d+ex)+c\sin(d+ex)}}dx+\frac{8b\sqrt{a\cos(d+ex)+b+c\sin(d+ex)}}{e\sqrt{\frac{a\cos(d+}{}}}}{a^2-b^2+c^2}}{3(a^2-}\right.}{(a+b\sec(d+ex)+c\,\mathrm{ta}}$$

$\Big\downarrow$ 3606

$$\frac{\sec^{\frac{5}{2}}(d+ex)(a\cos(d+ex)+b+c\sin(d+ex))^{5/2}\left(-\frac{-\frac{\frac{\left(a^2-b^2+c^2\right)\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}\int\frac{1}{\sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+\frac{\sqrt{a^2+c^2}\cos\left(d+ex\right.}{b+\sqrt{a^2+}}}}}{\sqrt{a\cos(d+ex)+b+c\sin(d+ex)}}}{}}{}\right.}{(a+b}$$

$\Big\downarrow$ 3042

$$\frac{\sec^{\frac{5}{2}}(d+ex)(a\cos(d+ex)+b+c\sin(d+ex))^{5/2}\left(-\frac{-\frac{\frac{\left(a^2-b^2+c^2\right)\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}\int\frac{1}{\sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+\frac{\sqrt{a^2+c^2}\sin\left(d+ex\right.}{b+\sqrt{a^2}}}}}{\sqrt{a\cos(d+ex)+b+c\sin(d+ex)}}}{}}{}\right.}{(a+}$$

$\Big\downarrow$ 3140

3.452. $\displaystyle\int \frac{\sec^{\frac{5}{2}}(d+ex)}{(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}\,dx$

$$\frac{\sec^{\frac{5}{2}}(d+ex)(a\cos(d+ex)+b+c\sin(d+ex))^{5/2}\left(-\frac{-\frac{\frac{2(a^2-b^2+c^2)\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}\text{EllipticF}\left(\frac{1}{2}\left(d+ex-\tan^{-1}(a,c)\right),\frac{2\cdots}{b+\cdots}\right)}{e\sqrt{a\cos(d+ex)+b+c\sin(d+ex)}}}{a^2\cdots}}{\cdots}\right.}{(a+b\,\text{se}\cdots}$$

input
```
Int[Sec[d + e*x]^(5/2)/(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(5/2),x]
```

output
```
(Sec[d + e*x]^(5/2)*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^(5/2)*((-2*(c*Co
s[d + e*x] - a*Sin[d + e*x]))/(3*(a^2 - b^2 + c^2)*e*(b + a*Cos[d + e*x] +
 c*Sin[d + e*x])^(3/2)) - ((-8*(b*c*Cos[d + e*x] - a*b*Sin[d + e*x]))/((a^
2 - b^2 + c^2)*e*Sqrt[b + a*Cos[d + e*x] + c*Sin[d + e*x]]) - ((8*b*Ellipt
icE[(d + e*x - ArcTan[a, c])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2 + c^2])]
*Sqrt[b + a*Cos[d + e*x] + c*Sin[d + e*x]])/(e*Sqrt[(b + a*Cos[d + e*x] +
c*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])]) + (2*(a^2 - b^2 + c^2)*EllipticF[(
d + e*x - ArcTan[a, c])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2 + c^2])]*Sqrt
[(b + a*Cos[d + e*x] + c*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])])/(e*Sqrt[b +
 a*Cos[d + e*x] + c*Sin[d + e*x]]))/(a^2 - b^2 + c^2))/(3*(a^2 - b^2 + c^2
))))/(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(5/2)
```

3.452.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

3.452. $\int \frac{\sec^{\frac{5}{2}}(d+ex)}{(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}\,dx$

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3598
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]   Int[Sqrt[a/(a + S
qrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - Arc
Tan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

rule 3606
```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Simp[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]   Int[1/Sqrt[a/(a
 + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x -
 ArcTan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2
, 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

rule 3608
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Simp[
1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c
*(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x
] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] &&
NeQ[n, -3/2]
```

rule 3628
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]
, x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[
A*b - a*B, 0]
```

3.452. $\int \frac{\sec^{\frac{5}{2}}(d+ex)}{(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}\,dx$

rule 3635
```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(-(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(
a^2 - b^2 - c^2))), x] + Simp[1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a + b*Co
s[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n +
2)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x]
/; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[n, -2]
```

rule 3646
```
Int[sec[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] +
(c_.)*tan[(d_.) + (e_.)*(x_)])^(m_), x_Symbol] :> Simp[Sec[d + e*x]^n*((b +
 a*Cos[d + e*x] + c*Sin[d + e*x])^n/(a + b*Sec[d + e*x] + c*Tan[d + e*x])^n
)   Int[1/(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n, x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[m + n, 0] &&  !IntegerQ[n]
```

3.452.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 25.62 (sec) , antiderivative size = 155460, normalized size of antiderivative = 315.98

method	result	size
default	Expression too large to display	155460

input
```
int(sec(e*x+d)^(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2),x,method=_RETURNV
ERBOSE)
```

output
```
result too large to display
```

3.452.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.29 (sec) , antiderivative size = 2769, normalized size of antiderivative = 5.63

$$\int \frac{\sec^{\frac{5}{2}}(d+ex)}{(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(sec(e*x+d)^(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2),x, algorith
m="fricas")
```

output
```
1/9*((-3*I*a^3*b^2 - I*a*b^4 - 3*I*a*c^4 + 3*c^5 + (3*a^2 + 4*b^2)*c^3 - I
*(3*a^3 + 4*a*b^2)*c^2 + (-3*I*a^5 - I*a^3*b^2 + I*a*b^2*c^2 - b^2*c^3 + 3
*I*a*c^4 - 3*c^5 + (3*a^4 + a^2*b^2)*c)*cos(e*x + d)^2 + (3*a^2*b^2 + b^4)
*c - 2*(3*I*a^4*b + I*a^2*b^3 + 3*I*a^2*b*c^2 - 3*a*b*c^3 - (3*a^3*b + a*b
^3)*c)*cos(e*x + d) - 2*(3*I*a*b*c^3 - 3*b*c^4 - (3*a^2*b + b^3)*c^2 + I*(
3*a^3*b + a*b^3)*c + (3*I*a^2*c^3 - 3*a*c^4 - (3*a^3 + a*b^2)*c^2 + I*(3*a
^4 + a^2*b^2)*c)*cos(e*x + d))*sin(e*x + d))*sqrt(2*a - 2*I*c)*weierstrass
PInverse(-4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*
a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), 8/27*(9*a^5*b - 8*a^3*b^3 - 27*
a*b*c^4 - 9*I*b*c^5 + 2*I*(9*a^2*b + 4*b^3)*c^3 - 6*(3*a^3*b - 4*a*b^3)*c^
2 + 3*I*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*
(2*a*b + 2*I*b*c + 3*(a^2 + c^2)*cos(e*x + d) - 3*(-I*a^2 - I*c^2)*sin(e*x
 + d))/(a^2 + c^2)) + (3*I*a^3*b^2 + I*a*b^4 + 3*I*a*c^4 + 3*c^5 + (3*a^2
+ 4*b^2)*c^3 + I*(3*a^3 + 4*a*b^2)*c^2 + (3*I*a^5 + I*a^3*b^2 - I*a*b^2*c^
2 - b^2*c^3 - 3*I*a*c^4 - 3*c^5 + (3*a^4 + a^2*b^2)*c)*cos(e*x + d)^2 + (3
*a^2*b^2 + b^4)*c - 2*(-3*I*a^4*b - I*a^2*b^3 - 3*I*a^2*b*c^2 - 3*a*b*c^3
- (3*a^3*b + a*b^3)*c)*cos(e*x + d) - 2*(-3*I*a*b*c^3 - 3*b*c^4 - (3*a^2*b
 + b^3)*c^2 - I*(3*a^3*b + a*b^3)*c + (-3*I*a^2*c^3 - 3*a*c^4 - (3*a^3 + a
*b^2)*c^2 - I*(3*a^4 + a^2*b^2)*c)*cos(e*x + d))*sin(e*x + d))*sqrt(2*a +
2*I*c)*weierstrassPInverse(-4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 - 6*I*a*...
```

3.452.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(d+ex)}{(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(e*x+d)**(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))**(5/2),x)
```

output
```
Timed out
```

3.452.7 Maxima [F]

$$\int \frac{\sec^{\frac{5}{2}}(d+ex)}{(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}\,dx = \int \frac{\sec\left(ex+d\right)^{\frac{5}{2}}}{\left(b\sec\left(ex+d\right)+c\tan\left(ex+d\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(sec(e*x+d)^(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2),x, algorith
m="maxima")
```

output
```
integrate(sec(e*x + d)^(5/2)/(b*sec(e*x + d) + c*tan(e*x + d) + a)^(5/2),
x)
```

3.452.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(d+ex)}{(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(e*x+d)^(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2),x, algorith
m="giac")
```

output
```
Timed out
```

3.452.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^{\frac{5}{2}}(d+ex)}{(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}\,dx = \int \frac{\left(\frac{1}{\cos(d+e\,x)}\right)^{5/2}}{\left(a+c\tan\left(d+e\,x\right)+\frac{b}{\cos(d+e\,x)}\right)^{5/2}}\,dx$$

input
```
int((1/cos(d + e*x))^(5/2)/(a + c*tan(d + e*x) + b/cos(d + e*x))^(5/2),x)
```

output
```
int((1/cos(d + e*x))^(5/2)/(a + c*tan(d + e*x) + b/cos(d + e*x))^(5/2), x)
```

3.453 $\int \cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}\,dx$

3.453.1 Optimal result

Integrand size = 33, antiderivative size = 371

$$\int \cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}\,dx = -\frac{2\cos^{\frac{3}{2}}(d+ex)(c\cos(d+ex)-a\sin(d+ex))(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}{3e(b+a\cos(d+ex)+c\sin(d+ex))} + \frac{8b\cos^{\frac{3}{2}}(d+ex)E\left(\frac{1}{2}(d+ex-\tan^{-1}(a,c))\,|\,\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}{3e(b+a\cos(d+ex)+c\sin(d+ex))\sqrt{\frac{b+a\cos(d+ex)+c\sin(d+ex)}{b+\sqrt{a^2+c^2}}}} + \frac{2(a^2-b^2+c^2)\cos^{\frac{3}{2}}(d+ex)\operatorname{EllipticF}\left(\frac{1}{2}(d+ex-\tan^{-1}(a,c)),\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)\sqrt{\frac{b+a\cos(d+ex)+c\sin(d+ex)}{b+\sqrt{a^2+c^2}}}(a+b\,\mathrm{s}}{3e(b+a\cos(d+ex)+c\sin(d+ex))^2}$$

output
```
-2/3*cos(e*x+d)^(3/2)*(c*cos(e*x+d)-a*sin(e*x+d))*(a+b*sec(e*x+d)+c*tan(e*
x+d))^(3/2)/e/(b+a*cos(e*x+d)+c*sin(e*x+d))+8/3*b*cos(e*x+d)^(3/2)*(cos(1/
2*d+1/2*e*x-1/2*arctan(a,c))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(a,c))*E
llipticE(sin(1/2*d+1/2*e*x-1/2*arctan(a,c)),2^(1/2)*((a^2+c^2)^(1/2)/(b+(a
^2+c^2)^(1/2)))^(1/2))*(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2)/e/(b+a*cos(e*x+
d)+c*sin(e*x+d))/((b+a*cos(e*x+d)+c*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(1/2)
+2/3*(a^2-b^2+c^2)*cos(e*x+d)^(3/2)*(cos(1/2*d+1/2*e*x-1/2*arctan(a,c))^2)
^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(a,c))*EllipticF(sin(1/2*d+1/2*e*x-1/2*
arctan(a,c)),2^(1/2)*((a^2+c^2)^(1/2)/(b+(a^2+c^2)^(1/2)))^(1/2))*((b+a*co
s(e*x+d)+c*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(1/2)*(a+b*sec(e*x+d)+c*tan(e*
x+d))^(3/2)/e/(b+a*cos(e*x+d)+c*sin(e*x+d))^2
```

3.453.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 36.99 (sec) , antiderivative size = 81485, normalized size of antiderivative = 219.64

$$\int \cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}\,dx = \text{Result too large to show}$$

input
```
Integrate[Cos[d + e*x]^(3/2)*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2),x
]
```

output
```
Result too large to show
```

3.453.3 Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.91, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.424$, Rules used = {3042, 3642, 3042, 3599, 27, 3042, 3628, 3042, 3598, 3042, 3132, 3606, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(d+ex)^{3/2}(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}dx$$

$$\Big\downarrow \text{3642}$$

$$\frac{\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}\int(b+a\cos(d+ex)+c\sin(d+ex))^{3/2}dx}{(a\cos(d+ex)+b+c\sin(d+ex))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}\int(b+a\cos(d+ex)+c\sin(d+ex))^{3/2}dx}{(a\cos(d+ex)+b+c\sin(d+ex))^{3/2}}$$

$$\Big\downarrow \text{3599}$$

$$\frac{\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}\left(\frac{2}{3}\int\frac{a^2+4b\cos(d+ex)a+3b^2+c^2+4bc\sin(d+ex)}{2\sqrt{b+a\cos(d+ex)+c\sin(d+ex)}}dx-\frac{2(c\cos(d+ex)-a\sin(d}{\ldots}\right.}{(a\cos(d+ex)+b+c\sin(d+ex))^{3/2}}$$

$\big\downarrow$ 27

$$\frac{\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}\left(\frac{1}{3}\int\frac{a^2+4b\cos(d+ex)a+3b^2+c^2+4bc\sin(d+ex)}{\sqrt{b+a\cos(d+ex)+c\sin(d+ex)}}dx-\frac{2(c\cos(d+ex)-a\sin(d\right.}{(a\cos(d+ex)+b+c\sin(d+ex))^{3/2}}$$

$\big\downarrow$ 3042

$$\frac{\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}\left(\frac{1}{3}\int\frac{a^2+4b\cos(d+ex)a+3b^2+c^2+4bc\sin(d+ex)}{\sqrt{b+a\cos(d+ex)+c\sin(d+ex)}}dx-\frac{2(c\cos(d+ex)-a\sin(d\right.}{(a\cos(d+ex)+b+c\sin(d+ex))^{3/2}}$$

$\big\downarrow$ 3628

$$\frac{\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}\left(\frac{1}{3}\left(\left(a^2-b^2+c^2\right)\int\frac{1}{\sqrt{b+a\cos(d+ex)+c\sin(d+ex)}}dx+4b\int\sqrt{b+a\right.\right.}{(a\cos(d+ex)+b+c\sin(d+ex))}$$

$\big\downarrow$ 3042

$$\frac{\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}\left(\frac{1}{3}\left(\left(a^2-b^2+c^2\right)\int\frac{1}{\sqrt{b+a\cos(d+ex)+c\sin(d+ex)}}dx+4b\int\sqrt{b+a\right.\right.}{(a\cos(d+ex)+b+c\sin(d+ex))}$$

$\big\downarrow$ 3598

$$\frac{\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}\left(\frac{1}{3}\left(\left(a^2-b^2+c^2\right)\int\frac{1}{\sqrt{b+a\cos(d+ex)+c\sin(d+ex)}}dx+\frac{4b\sqrt{a\cos(d+}}{}\right.\right.}{(a\cos(d+ex)+b+c}$$

$\big\downarrow$ 3042

$$\frac{\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}\left(\frac{1}{3}\left(\left(a^2-b^2+c^2\right)\int\frac{1}{\sqrt{b+a\cos(d+ex)+c\sin(d+ex)}}dx+\frac{4b\sqrt{a\cos(d+}}{}\right.\right.}{(a\cos(d+ex)+b+}$$

$\big\downarrow$ 3132

$$\frac{\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}\left(\frac{1}{3}\left(\left(a^2-b^2+c^2\right)\int\frac{1}{\sqrt{b+a\cos(d+ex)+c\sin(d+ex)}}dx+\frac{8b\sqrt{a\cos(d+}}{}\right.\right.}{(a\cos(d+ex)+b+c\sin(}$$

3.453. $\int\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}\,dx$

$\downarrow$ 3606

$$\frac{\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}\left(\frac{1}{3}\left(\frac{(a^2-b^2+c^2)\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}\int\frac{1}{\sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+\frac{\sqrt{a^2+c^2}\cos\left(d+e\right.}{b+\sqrt{a}}}}}{\sqrt{a\cos(d+ex)+b+c\sin(d+ex)}}\right.\right.}{(a\cos(d+e}$$

$\downarrow$ 3042

$$\frac{\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}\left(\frac{1}{3}\left(\frac{(a^2-b^2+c^2)\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}\int\frac{1}{\sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+\frac{\sqrt{a^2+c^2}\sin\left(d+e\right.}{b+\sqrt{}}}}}{\sqrt{a\cos(d+ex)+b+c\sin(d+ex)}}\right.\right.}{(a\cos(d+}$$

$\downarrow$ 3140

$$\frac{\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}\left(\frac{1}{3}\left(\frac{2(a^2-b^2+c^2)\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}\text{EllipticF}\left(\frac{1}{2}(d+ex-\tan^{-1}(a,c\right.}{e\sqrt{a\cos(d+ex)+b+c\sin(d+ex)}}\right.\right.}{(a\cos(d+ex}$$

input
```
Int[Cos[d + e*x]^(3/2)*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2),x]
```

output
```
(Cos[d + e*x]^(3/2)*((-2*(c*Cos[d + e*x] - a*Sin[d + e*x])*Sqrt[b + a*Cos[
d + e*x] + c*Sin[d + e*x]])/(3*e) + ((8*b*EllipticE[(d + e*x - ArcTan[a, c
])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2 + c^2])]*Sqrt[b + a*Cos[d + e*x] +
 c*Sin[d + e*x]])/(e*Sqrt[(b + a*Cos[d + e*x] + c*Sin[d + e*x])/(b + Sqrt[
a^2 + c^2])]) + (2*(a^2 - b^2 + c^2)*EllipticF[(d + e*x - ArcTan[a, c])/2,
 (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2 + c^2])]*Sqrt[(b + a*Cos[d + e*x] + c*S
in[d + e*x])/(b + Sqrt[a^2 + c^2])])/(e*Sqrt[b + a*Cos[d + e*x] + c*Sin[d
+ e*x]]))/3)*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2))/(b + a*Cos[d + e
*x] + c*Sin[d + e*x])^(3/2)
```

3.453. $\int\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}\,dx$

3.453.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3598
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]   Int[Sqrt[a/(a + S
qrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - Arc
Tan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

rule 3599
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Simp[1/n   Int[Simp[n*a^2 + (
n - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x
], x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

rule 3606
```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Simp[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]   Int[1/Sqrt[a/(a
 + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x -
 ArcTan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2
, 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

3.453. $\int \cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}\,dx$

rule 3628
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]
, x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[
A*b - a*B, 0]
```

rule 3642
```
Int[cos[(d_.) + (e_.)*(x_)]^(n_)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (
c_.)*tan[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[d + e*x]^n*((a +
b*Sec[d + e*x] + c*Tan[d + e*x])^n/(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n)
   Int[(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d
, e}, x] &&  !IntegerQ[n]
```

3.453.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 26.22 (sec) , antiderivative size = 64099, normalized size of antiderivative = 172.77

method	result	size
default	Expression too large to display	64099

input
```
int(cos(e*x+d)^(3/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
result too large to display
```

3.453.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 1500, normalized size of antiderivative = 4.04

$$\int \cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(e*x+d)^(3/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x, algorith
m="fricas")
```

output
```
1/9*(sqrt(2)*(-3*I*a^3 - I*a*b^2 - 3*I*a*c^2 + 3*c^3 + (3*a^2 + b^2)*c)*sq
rt(a - I*c)*weierstrassPInverse(-4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 + 6*I*
a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), 8/27*(9*
a^5*b - 8*a^3*b^3 - 27*a*b*c^4 - 9*I*b*c^5 + 2*I*(9*a^2*b + 4*b^3)*c^3 - 6
*(3*a^3*b - 4*a*b^3)*c^2 + 3*I*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 +
 3*a^2*c^4 + c^6), 1/3*(2*a*b + 2*I*b*c + 3*(a^2 + c^2)*cos(e*x + d) - 3*(
-I*a^2 - I*c^2)*sin(e*x + d))/(a^2 + c^2)) + sqrt(2)*(3*I*a^3 + I*a*b^2 +
3*I*a*c^2 + 3*c^3 + (3*a^2 + b^2)*c)*sqrt(a + I*c)*weierstrassPInverse(-4/
3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 - 6*I*a*c^3 - 3*c^4 - 2*I*(3*a^3 - 4*a*b^
2)*c)/(a^4 + 2*a^2*c^2 + c^4), 8/27*(9*a^5*b - 8*a^3*b^3 - 27*a*b*c^4 + 9*
I*b*c^5 - 2*I*(9*a^2*b + 4*b^3)*c^3 - 6*(3*a^3*b - 4*a*b^3)*c^2 - 3*I*(9*a
^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(2*a*b - 2*I
*b*c + 3*(a^2 + c^2)*cos(e*x + d) - 3*(I*a^2 + I*c^2)*sin(e*x + d))/(a^2 +
 c^2)) - 12*sqrt(2)*(-I*a^2*b - I*b*c^2)*sqrt(a - I*c)*weierstrassZeta(-4/
3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^
2)*c)/(a^4 + 2*a^2*c^2 + c^4), 8/27*(9*a^5*b - 8*a^3*b^3 - 27*a*b*c^4 - 9*
I*b*c^5 + 2*I*(9*a^2*b + 4*b^3)*c^3 - 6*(3*a^3*b - 4*a*b^3)*c^2 + 3*I*(9*a
^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), weierstrassPInve
rse(-4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 -
 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), 8/27*(9*a^5*b - 8*a^3*b^3 - 27*a*...
```

3.453.6 Sympy [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(e*x+d)**(3/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))**(3/2),x)
```

output
```
Timed out
```

3.453.7 Maxima [F]

$$\int \cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}\,dx=\int (b\sec{(ex+d)}+c\tan{(ex+d)}+a)^{\frac{3}{2}}\cos{(ex+d)}^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(e*x+d)^(3/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x, algorith
m="maxima")
```

output
```
integrate((b*sec(e*x + d) + c*tan(e*x + d) + a)^(3/2)*cos(e*x + d)^(3/2),
x)
```

3.453.8 Giac [F]

$$\int \cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}\,dx=\int (b\sec{(ex+d)}+c\tan{(ex+d)}+a)^{\frac{3}{2}}\cos{(ex+d)}^{\frac{3}{2}}\,dx$$

input
```
integrate(cos(e*x+d)^(3/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x, algorith
m="giac")
```

output
```
integrate((b*sec(e*x + d) + c*tan(e*x + d) + a)^(3/2)*cos(e*x + d)^(3/2),
x)
```

3.453.9 Mupad [F(-1)]

Timed out.

$$\int \cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}\,dx=\int \cos{(d+e\,x)}^{3/2}\left(a+c\tan(d+e\,x)+\frac{b}{\cos{(d+e\,x)}}\right)^{3/2}\,dx$$

input
```
int(cos(d + e*x)^(3/2)*(a + c*tan(d + e*x) + b/cos(d + e*x))^(3/2),x)
```

output
```
int(cos(d + e*x)^(3/2)*(a + c*tan(d + e*x) + b/cos(d + e*x))^(3/2), x)
```

3.454 $\int \sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}$

3.454.1 Optimal result

Integrand size = 33, antiderivative size = 118

$$\int \sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}\,dx = \frac{2\sqrt{\cos(d+ex)}E\left(\frac{1}{2}(d+ex-\tan^{-1}(a,c))\,|\,\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}{e\sqrt{\frac{b+a\cos(d+ex)+c\sin(d+ex)}{b+\sqrt{a^2+c^2}}}}$$

output
```
2*(cos(1/2*d+1/2*e*x-1/2*arctan(a,c))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arcta
n(a,c))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(a,c)),2^(1/2)*((a^2+c^2)^(1
/2)/(b+(a^2+c^2)^(1/2)))^(1/2))*cos(e*x+d)^(1/2)*(a+b*sec(e*x+d)+c*tan(e*x
+d))^(1/2)/e/((b+a*cos(e*x+d)+c*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(1/2)
```

3.454.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 32.80 (sec) , antiderivative size = 54676, normalized size of antiderivative = 463.36

$$\int \sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sqrt[Cos[d + e*x]]*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]],x]
```

output
```
Result too large to show
```

3.454.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3042, 3642, 3042, 3598, 3042, 3132}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}dx$$

$$\Big\downarrow \text{3642}$$

$$\frac{\sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}\int \sqrt{b+a\cos(d+ex)+c\sin(d+ex)}dx}{\sqrt{a\cos(d+ex)+b+c\sin(d+ex)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}\int \sqrt{b+a\cos(d+ex)+c\sin(d+ex)}dx}{\sqrt{a\cos(d+ex)+b+c\sin(d+ex)}}$$

$$\Big\downarrow \text{3598}$$

$$\frac{\sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}\int \sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+\frac{\sqrt{a^2+c^2}\cos(d+ex-\tan^{-1}(a,c))}{b+\sqrt{a^2+c^2}}}dx}{\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}\int \sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+\frac{\sqrt{a^2+c^2}\sin\left(d+ex-\tan^{-1}(a,c)+\frac{\pi}{2}\right)}{b+\sqrt{a^2+c^2}}}dx}{\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}}$$

$$\Big\downarrow \text{3132}$$

$$\frac{2\sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(a,c)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}}$$

input
```
Int[Sqrt[Cos[d + e*x]]*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]],x]
```

output
```
(2*Sqrt[Cos[d + e*x]]*EllipticE[(d + e*x - ArcTan[a, c])/2, (2*Sqrt[a^2 +
c^2])/(b + Sqrt[a^2 + c^2])]*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]])/(e
*Sqrt[(b + a*Cos[d + e*x] + c*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])])
```

3.454.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3598
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]   Int[Sqrt[a/(a + S
qrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - Arc
Tan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

rule 3642
```
Int[cos[(d_.) + (e_.)*(x_)]^(n_)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (
c_.)*tan[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[d + e*x]^n*((a +
b*Sec[d + e*x] + c*Tan[d + e*x])^n/(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n)
   Int[(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d
, e}, x] &&  !IntegerQ[n]
```

3.454.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 24.27 (sec) , antiderivative size = 2660, normalized size of antiderivative = 22.54

method	result	size
risch	Expression too large to display	2660
default	Expression too large to display	12921

input
```
int(cos(e*x+d)^(1/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
I*(-I*exp(I*(e*x+d))^2*c+a*exp(I*(e*x+d))^2+2*b*exp(I*(e*x+d))+I*c+a)/e/(e
xp(I*(e*x+d))*(-I*exp(I*(e*x+d))^2*c+a*exp(I*(e*x+d))^2+2*b*exp(I*(e*x+d))
+I*c+a))^(1/2)*((exp(RootOf(_Z^2+1,index=1)*(e*x+d))^2+1)/exp(RootOf(_Z^2+
1,index=1)*(e*x+d)))^(1/2)*(-(RootOf(_Z^2+1,index=1)*exp(RootOf(_Z^2+1,ind
ex=1)*(e*x+d))^2*c-exp(RootOf(_Z^2+1,index=1)*(e*x+d))^2*a-2*b*exp(RootOf(
_Z^2+1,index=1)*(e*x+d))-RootOf(_Z^2+1,index=1)*c-a)/(exp(RootOf(_Z^2+1,in
dex=1)*(e*x+d))^2+1))^(1/2)*(-(RootOf(_Z^2+1,index=1)*exp(RootOf(_Z^2+1,in
dex=1)*(e*x+d))^2*c-exp(RootOf(_Z^2+1,index=1)*(e*x+d))^2*a-2*b*exp(RootOf
(_Z^2+1,index=1)*(e*x+d))-RootOf(_Z^2+1,index=1)*c-a)*(exp(RootOf(_Z^2+1,i
ndex=1)*(e*x+d))^2+1))^(1/2)/(RootOf(_Z^2+1,index=1)*exp(RootOf(_Z^2+1,ind
ex=1)*(e*x+d))^2*c-exp(RootOf(_Z^2+1,index=1)*(e*x+d))^2*a-2*b*exp(RootOf(
_Z^2+1,index=1)*(e*x+d))-RootOf(_Z^2+1,index=1)*c-a)*(exp(RootOf(_Z^2+1,in
dex=1)*(e*x+d))*(-RootOf(_Z^2+1,index=1)*exp(RootOf(_Z^2+1,index=1)*(e*x+d
))^2*c+exp(RootOf(_Z^2+1,index=1)*(e*x+d))^2*a+2*b*exp(RootOf(_Z^2+1,index
=1)*(e*x+d))+RootOf(_Z^2+1,index=1)*c+a))^(1/2)/((-RootOf(_Z^2+1,index=1)*
exp(RootOf(_Z^2+1,index=1)*(e*x+d))^2*c+exp(RootOf(_Z^2+1,index=1)*(e*x+d)
)^2*a+2*b*exp(RootOf(_Z^2+1,index=1)*(e*x+d))+RootOf(_Z^2+1,index=1)*c+a)*
(exp(RootOf(_Z^2+1,index=1)*(e*x+d))^2+1))^(1/2)*2^(1/2)-I/e*(-2*b*(-b+(-a
^2+b^2-c^2)^(1/2))/(I*c-a)*((exp(I*(e*x+d))+(-b+(-a^2+b^2-c^2)^(1/2))/(I*c
-a))/(-b+(-a^2+b^2-c^2)^(1/2))*(I*c-a))^(1/2)*((exp(I*(e*x+d))-(b+(-a^2...
```

3.454.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 1371, normalized size of antiderivative = 11.62

$$\int \sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}\,dx = \text{Too large to display}$$

input
```
integrate(cos(e*x+d)^(1/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2),x, algorith
m="fricas")
```

output
```
1/3*(sqrt(2)*(-I*a*b + b*c)*sqrt(a - I*c)*weierstrassPInverse(-4/3*(3*a^4
- 4*a^2*b^2 + 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^
4 + 2*a^2*c^2 + c^4), 8/27*(9*a^5*b - 8*a^3*b^3 - 27*a*b*c^4 - 9*I*b*c^5 +
 2*I*(9*a^2*b + 4*b^3)*c^3 - 6*(3*a^3*b - 4*a*b^3)*c^2 + 3*I*(9*a^4*b - 8*
a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(2*a*b + 2*I*b*c + 3*
(a^2 + c^2)*cos(e*x + d) - 3*(-I*a^2 - I*c^2)*sin(e*x + d))/(a^2 + c^2)) +
 sqrt(2)*(I*a*b + b*c)*sqrt(a + I*c)*weierstrassPInverse(-4/3*(3*a^4 - 4*a
^2*b^2 + 4*b^2*c^2 - 6*I*a*c^3 - 3*c^4 - 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2
*a^2*c^2 + c^4), 8/27*(9*a^5*b - 8*a^3*b^3 - 27*a*b*c^4 + 9*I*b*c^5 - 2*I*
(9*a^2*b + 4*b^3)*c^3 - 6*(3*a^3*b - 4*a*b^3)*c^2 - 3*I*(9*a^4*b - 8*a^2*b
^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(2*a*b - 2*I*b*c + 3*(a^2
+ c^2)*cos(e*x + d) - 3*(I*a^2 + I*c^2)*sin(e*x + d))/(a^2 + c^2)) - 3*sqr
t(2)*(-I*a^2 - I*c^2)*sqrt(a - I*c)*weierstrassZeta(-4/3*(3*a^4 - 4*a^2*b^
2 + 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*
c^2 + c^4), 8/27*(9*a^5*b - 8*a^3*b^3 - 27*a*b*c^4 - 9*I*b*c^5 + 2*I*(9*a^
2*b + 4*b^3)*c^3 - 6*(3*a^3*b - 4*a*b^3)*c^2 + 3*I*(9*a^4*b - 8*a^2*b^3)*c
)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), weierstrassPInverse(-4/3*(3*a^4 - 4
*a^2*b^2 + 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 +
 2*a^2*c^2 + c^4), 8/27*(9*a^5*b - 8*a^3*b^3 - 27*a*b*c^4 - 9*I*b*c^5 + 2*
I*(9*a^2*b + 4*b^3)*c^3 - 6*(3*a^3*b - 4*a*b^3)*c^2 + 3*I*(9*a^4*b - 8*...
```

3.454.6 Sympy [F]

$$\int \sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}\,dx$$
$$= \int \sqrt{a+b\sec(d+ex)+c\tan(d+ex)}\sqrt{\cos(d+ex)}\,dx$$

input
```
integrate(cos(e*x+d)**(1/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))**(1/2),x)
```

output
```
Integral(sqrt(a + b*sec(d + e*x) + c*tan(d + e*x))*sqrt(cos(d + e*x)), x)
```

3.454.7 Maxima [F]

$$\int \sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}\,dx = \int \sqrt{b\sec\left(ex+d\right)+c\tan\left(ex+d\right)+a}\sqrt{\cos\left(ex+d\right)}\,dx$$

input
```
integrate(cos(e*x+d)^(1/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2),x, algorith
m="maxima")
```

output
```
integrate(sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a)*sqrt(cos(e*x + d)), x)
```

3.454.8 Giac [F]

$$\int \sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}\,dx = \int \sqrt{b\sec\left(ex+d\right)+c\tan\left(ex+d\right)+a}\sqrt{\cos\left(ex+d\right)}\,dx$$

input
```
integrate(cos(e*x+d)^(1/2)*(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2),x, algorith
m="giac")
```

output
```
integrate(sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a)*sqrt(cos(e*x + d)), x)
```

3.454.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}\,dx = \int \sqrt{\cos\left(d+e\,x\right)}\,\sqrt{a+c\tan\left(d+e\,x\right)+\frac{b}{\cos\left(d+e\,x\right)}}\,dx$$

input
```
int(cos(d + e*x)^(1/2)*(a + c*tan(d + e*x) + b/cos(d + e*x))^(1/2),x)
```

output
```
int(cos(d + e*x)^(1/2)*(a + c*tan(d + e*x) + b/cos(d + e*x))^(1/2), x)
```

3.455 $\int \frac{1}{\sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}\,dx$

3.455.1 Optimal result

Integrand size = 33, antiderivative size = 118

$$\int \frac{1}{\sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}\,dx = \frac{2\,\text{EllipticF}\left(\frac{1}{2}(d+ex-\tan^{-1}(a,c)),\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)\sqrt{\frac{b+a\cos(d+ex)+c\sin(d+ex)}{b+\sqrt{a^2+c^2}}}}{e\sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}$$

output
```
2*(cos(1/2*d+1/2*e*x-1/2*arctan(a,c))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arcta
n(a,c))*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan(a,c)),2^(1/2)*((a^2+c^2)^(1
/2)/(b+(a^2+c^2)^(1/2)))^(1/2))*((b+a*cos(e*x+d)+c*sin(e*x+d))/(b+(a^2+c^2
)^(1/2)))^(1/2)/e/cos(e*x+d)^(1/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2)
```

3.455.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 16.02 (sec) , antiderivative size = 506, normalized size of antiderivative = 4.29

$$\int \frac{1}{\sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}\,dx$$
$$= \frac{4\left(ia-ib+c+\sqrt{a^2-b^2+c^2}\right)\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{\left(-ia+ib+c+\sqrt{a^2-b^2+c^2}\right)(-\cos(d+ex)+i\sin(d+ex))}{ia-ib+c+\sqrt{a^2-b^2+c^2}}}\right),\frac{b+i\sqrt{a^2-\cdots}}{b-i\sqrt{a^2-\cdots}}\cdots\right.}{(a+i\,(ib+c+\cdots}$$

input
```
Integrate[1/(Sqrt[Cos[d + e*x]]*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]])
,x]
```

output
```
(4*(I*a - I*b + c + Sqrt[a^2 - b^2 + c^2])*EllipticF[ArcSin[Sqrt[(((-I)*a
+ I*b + c + Sqrt[a^2 - b^2 + c^2])*(-Cos[d + e*x] + I*Sin[d + e*x]))/(I*a
- I*b + c + Sqrt[a^2 - b^2 + c^2])]], (b + I*Sqrt[a^2 - b^2 + c^2])/(b - I
*Sqrt[a^2 - b^2 + c^2])]*Sqrt[(((-I)*a + I*b + c + Sqrt[a^2 - b^2 + c^2])*
(-Cos[d + e*x] + I*Sin[d + e*x]))/(I*a - I*b + c + Sqrt[a^2 - b^2 + c^2])]
*(Cos[d + e*x] + I*Sin[d + e*x])*Sqrt[((-I)*(-c + Sqrt[a^2 - b^2 + c^2] +
(a - b)*Tan[(d + e*x)/2]))/(((-I)*a + I*b - c + Sqrt[a^2 - b^2 + c^2])*(-I
 + Tan[(d + e*x)/2]))]*Sqrt[((-I)*(c + Sqrt[a^2 - b^2 + c^2] + (-a + b)*Ta
n[(d + e*x)/2]))/((I*a - I*b + c + Sqrt[a^2 - b^2 + c^2])*(-I + Tan[(d + e
*x)/2]))])/((a + I*(I*b + c + Sqrt[a^2 - b^2 + c^2]))*e*Sqrt[Cos[d + e*x]]
*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]])
```

3.455.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3042, 3642, 3042, 3606, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}dx$$

$$\Big\downarrow \text{3642}$$

$$\frac{\sqrt{a\cos(d+ex)+b+c\sin(d+ex)}\int \frac{1}{\sqrt{b+a\cos(d+ex)+c\sin(d+ex)}}dx}{\sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a\cos(d+ex)+b+c\sin(d+ex)}\int \frac{1}{\sqrt{b+a\cos(d+ex)+c\sin(d+ex)}}dx}{\sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}$$

$$\Big\downarrow \text{3606}$$

$$\frac{\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}\int \frac{1}{\sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+\frac{\sqrt{a^2+c^2}\cos\left(d+ex-\tan^{-1}(a,c)\right)}{b+\sqrt{a^2+c^2}}}}dx}{\sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}\int \frac{1}{\sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+\frac{\sqrt{a^2+c^2}\sin\left(d+ex-\tan^{-1}(a,c)+\frac{\pi}{2}\right)}{b+\sqrt{a^2+c^2}}}}dx}{\sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}$$

$$\Big\downarrow \text{3140}$$

$$\frac{2\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}\text{EllipticF}\left(\frac{1}{2}\left(d+ex-\tan^{-1}(a,c)\right),\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}$$

input
```
Int[1/(Sqrt[Cos[d + e*x]]*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]]),x]
```

output
```
(2*EllipticF[(d + e*x - ArcTan[a, c])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2
 + c^2])]*Sqrt[(b + a*Cos[d + e*x] + c*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])
])/(e*Sqrt[Cos[d + e*x]]*Sqrt[a + b*Sec[d + e*x] + c*Tan[d + e*x]])
```

3.455.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3606
```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Simp[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]   Int[1/Sqrt[a/(a
 + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x -
 ArcTan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2
, 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

3.455. $\int \frac{1}{\sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}dx$

rule 3642
```
Int[cos[(d_.) + (e_.)*(x_)]^(n_)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (
c_.)*tan[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[d + e*x]^n*((a +
b*Sec[d + e*x] + c*Tan[d + e*x])^n/(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n)
   Int[(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d
, e}, x] && !IntegerQ[n]
```

3.455.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 25.26 (sec) , antiderivative size = 1012, normalized size of antiderivative = 8.58

method	result
default	$\frac{4\left(i\cos(ex+d)ab+ia\sqrt{a^2-b^2+c^2}\,\sin(ex+d)-i\sqrt{a^2-b^2+c^2}\,\cos(ex+d)c-ic\sqrt{a^2-b^2+c^2}+ia^2+ibc\sin(ex+d)-i\cos(ex+d)b^2-iac\sin(\right.}{}$

input
```
int(1/cos(e*x+d)^(1/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2),x,method=_RETUR
NVERBOSE)
```

output
```
4/e*(I*cos(e*x+d)*a*b+I*a*(a^2-b^2+c^2)^(1/2)*sin(e*x+d)-I*(a^2-b^2+c^2)^(
1/2)*cos(e*x+d)*c-I*c*(a^2-b^2+c^2)^(1/2)+I*a^2+I*b*c*sin(e*x+d)-I*cos(e*x
+d)*b^2-I*a*c*sin(e*x+d)-I*a*b-I*b*(a^2-b^2+c^2)^(1/2)*sin(e*x+d)+I*c^2+I*
cos(e*x+d)*c^2+(a^2-b^2+c^2)^(1/2)*cos(e*x+d)*b+a^2*sin(e*x+d)-a*c*cos(e*x
+d)-b^2*sin(e*x+d)+a*(a^2-b^2+c^2)^(1/2)-c*b)*cos(e*x+d)^(1/2)*(-((a^2-b^2
+c^2)^(1/2)*sin(e*x+d)+a*cos(e*x+d)-b*cos(e*x+d)+c*sin(e*x+d)-a+b)/((a^2-b
^2+c^2)^(1/2)*sin(e*x+d)-a*cos(e*x+d)+b*cos(e*x+d)-c*sin(e*x+d)+a-b)*(I*a-
I*b-(a^2-b^2+c^2)^(1/2)+c)/(I*a-I*b+(a^2-b^2+c^2)^(1/2)+c))^(1/2)*((I*sin(
e*x+d)+cos(e*x+d)-1)/((a^2-b^2+c^2)^(1/2)*sin(e*x+d)-a*cos(e*x+d)+b*cos(e*
x+d)-c*sin(e*x+d)+a-b)*(a-b)*(a^2-b^2+c^2)^(1/2)/(I*a-I*b-(a^2-b^2+c^2)^(1
/2)-c))^(1/2)*((I*sin(e*x+d)-cos(e*x+d)+1)/((a^2-b^2+c^2)^(1/2)*sin(e*x+d)
-a*cos(e*x+d)+b*cos(e*x+d)-c*sin(e*x+d)+a-b)*(a-b)*(a^2-b^2+c^2)^(1/2)/(I*
a-I*b+(a^2-b^2+c^2)^(1/2)+c))^(1/2)*EllipticF((-((a^2-b^2+c^2)^(1/2)*sin(e
*x+d)+a*cos(e*x+d)-b*cos(e*x+d)+c*sin(e*x+d)-a+b)/((a^2-b^2+c^2)^(1/2)*sin
(e*x+d)-a*cos(e*x+d)+b*cos(e*x+d)-c*sin(e*x+d)+a-b)*(I*a-I*b-(a^2-b^2+c^2)
^(1/2)+c)/(I*a-I*b+(a^2-b^2+c^2)^(1/2)+c))^(1/2),((I*a-I*b+(a^2-b^2+c^2)^(
1/2)+c)*(I*a-I*b+(a^2-b^2+c^2)^(1/2)-c)/(I*a-I*b-(a^2-b^2+c^2)^(1/2)+c)/(I
*a-I*b-(a^2-b^2+c^2)^(1/2)-c))^(1/2))*(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2)/
(b+a*cos(e*x+d)+c*sin(e*x+d))/(I*a-I*b-(a^2-b^2+c^2)^(1/2)+c)/(a^2-b^2+c^2
)^(1/2)
```

3.455. $\int \frac{1}{\sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}\,dx$

3.455.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 506, normalized size of antiderivative = 4.29

$$\int \frac{1}{\sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}\,dx$$
$$= \frac{\sqrt{2}\sqrt{a-ic}(-ia+c)\text{weierstrassPInverse}\left(-\frac{4\left(3a^4-4a^2b^2+4b^2c^2+6iac^3-3c^4+2i\left(3a^3-4ab^2\right)c\right)}{3\left(a^4+2a^2c^2+c^4\right)},\frac{8\left(9a^5b-8a^3b^3-27abc^4-\right.}{}\right.}{}$$

input
```
integrate(1/cos(e*x+d)^(1/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2),x, algori
thm="fricas")
```

output
```
(sqrt(2)*sqrt(a - I*c)*(-I*a + c)*weierstrassPInverse(-4/3*(3*a^4 - 4*a^2*
b^2 + 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^
2*c^2 + c^4), 8/27*(9*a^5*b - 8*a^3*b^3 - 27*a*b*c^4 - 9*I*b*c^5 + 2*I*(9*
a^2*b + 4*b^3)*c^3 - 6*(3*a^3*b - 4*a*b^3)*c^2 + 3*I*(9*a^4*b - 8*a^2*b^3)
*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(2*a*b + 2*I*b*c + 3*(a^2 + c
^2)*cos(e*x + d) - 3*(-I*a^2 - I*c^2)*sin(e*x + d))/(a^2 + c^2)) + sqrt(2)
*sqrt(a + I*c)*(I*a + c)*weierstrassPInverse(-4/3*(3*a^4 - 4*a^2*b^2 + 4*b
^2*c^2 - 6*I*a*c^3 - 3*c^4 - 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c
^4), 8/27*(9*a^5*b - 8*a^3*b^3 - 27*a*b*c^4 + 9*I*b*c^5 - 2*I*(9*a^2*b + 4
*b^3)*c^3 - 6*(3*a^3*b - 4*a*b^3)*c^2 - 3*I*(9*a^4*b - 8*a^2*b^3)*c)/(a^6
+ 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(2*a*b - 2*I*b*c + 3*(a^2 + c^2)*cos(e
*x + d) - 3*(I*a^2 + I*c^2)*sin(e*x + d))/(a^2 + c^2)))/((a^2 + c^2)*e)
```

3.455.6 Sympy [F]

$$\int \frac{1}{\sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}\,dx$$
$$= \int \frac{1}{\sqrt{a+b\sec{(d+ex)}+c\tan{(d+ex)}}\sqrt{\cos{(d+ex)}}}\,dx$$

input
```
integrate(1/cos(e*x+d)**(1/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*sec(d + e*x) + c*tan(d + e*x))*sqrt(cos(d + e*x))),
 x)
```

3.455.7 Maxima [F]

$$\int \frac{1}{\sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}\,dx = \int \frac{1}{\sqrt{b\sec(ex+d)+c\tan(ex+d)+a}\sqrt{\cos(ex+d)}}\,dx$$

input
```
integrate(1/cos(e*x+d)^(1/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2),x, algori
thm="maxima")
```

output
```
integrate(1/(sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a)*sqrt(cos(e*x + d)))
, x)
```

3.455.8 Giac [F]

$$\int \frac{1}{\sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}\,dx = \int \frac{1}{\sqrt{b\sec(ex+d)+c\tan(ex+d)+a}\sqrt{\cos(ex+d)}}\,dx$$

input
```
integrate(1/cos(e*x+d)^(1/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(1/2),x, algori
thm="giac")
```

output
```
integrate(1/(sqrt(b*sec(e*x + d) + c*tan(e*x + d) + a)*sqrt(cos(e*x + d)))
, x)
```

3.455.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}\,dx = \int \frac{1}{\sqrt{\cos(d+e\,x)}\,\sqrt{a+c\tan(d+e\,x)+\frac{b}{\cos(d+e\,x)}}}\,dx$$

input
```
int(1/(cos(d + e*x)^(1/2)*(a + c*tan(d + e*x) + b/cos(d + e*x))^(1/2)),x)
```

output
```
int(1/(cos(d + e*x)^(1/2)*(a + c*tan(d + e*x) + b/cos(d + e*x))^(1/2)), x)
```

3.455. $\int \frac{1}{\sqrt{\cos(d+ex)}\sqrt{a+b\sec(d+ex)+c\tan(d+ex)}}\,dx$

3.456 $\int \frac{1}{\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}\,dx$

3.456.1 Optimal result

Integrand size = 33, antiderivative size = 240

$$\int \frac{1}{\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}\,dx = -\frac{2(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+ex)+c\sin(d+ex))}{(a^2-b^2+c^2)\,e\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}} - \frac{2E\left(\frac{1}{2}(d+ex-\tan^{-1}(a,c))\,|\,\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)(b+a\cos(d+ex)+c\sin(d+ex))^2}{(a^2-b^2+c^2)\,e\cos^{\frac{3}{2}}(d+ex)\sqrt{\frac{b+a\cos(d+ex)+c\sin(d+ex)}{b+\sqrt{a^2+c^2}}}(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}$$

output
```
-2*(c*cos(e*x+d)-a*sin(e*x+d))*(b+a*cos(e*x+d)+c*sin(e*x+d))/(a^2-b^2+c^2)
/e/cos(e*x+d)^(3/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2)-2*(cos(1/2*d+1/2*e
*x-1/2*arctan(a,c))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(a,c))*EllipticE(
sin(1/2*d+1/2*e*x-1/2*arctan(a,c)),2^(1/2)*((a^2+c^2)^(1/2)/(b+(a^2+c^2)^(
1/2)))^(1/2))*(b+a*cos(e*x+d)+c*sin(e*x+d))^2/(a^2-b^2+c^2)/e/cos(e*x+d)^(
3/2)/((b+a*cos(e*x+d)+c*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(1/2)/(a+b*sec(e*
x+d)+c*tan(e*x+d))^(3/2)
```

3.456.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 32.54 (sec) , antiderivative size = 54829, normalized size of antiderivative = 228.45

$$\int \frac{1}{\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/(Cos[d + e*x]^(3/2)*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2
)),x]
```

output
```
Result too large to show
```

3.456.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.242, Rules used = {3042, 3642, 3042, 3607, 3042, 3598, 3042, 3132}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(d+ex)^{3/2}(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}dx$$

$$\Big\downarrow \text{3642}$$

$$\frac{(a\cos(d+ex)+b+c\sin(d+ex))^{3/2}\int \frac{1}{(b+a\cos(d+ex)+c\sin(d+ex))^{3/2}}dx}{\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a\cos(d+ex)+b+c\sin(d+ex))^{3/2}\int \frac{1}{(b+a\cos(d+ex)+c\sin(d+ex))^{3/2}}dx}{\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}$$

$$\Big\downarrow \text{3607}$$

$$\frac{(a\cos(d+ex)+b+c\sin(d+ex))^{3/2}\left(-\frac{\int\sqrt{b+a\cos(d+ex)+c\sin(d+ex)}dx}{a^2-b^2+c^2}-\frac{2(c\cos(d+ex)-a\sin(d+ex))}{e(a^2-b^2+c^2)\sqrt{a\cos(d+ex)+b+c\sin(d+ex)}}\right)}{\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{(a\cos(d+ex)+b+c\sin(d+ex))^{3/2}\left(-\frac{\int\sqrt{b+a\cos(d+ex)+c\sin(d+ex)}dx}{a^2-b^2+c^2}-\frac{2(c\cos(d+ex)-a\sin(d+ex))}{e(a^2-b^2+c^2)\sqrt{a\cos(d+ex)+b+c\sin(d+ex)}}\right)}{\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}$$

$\Big\downarrow$ 3598

$$\frac{(a\cos(d+ex)+b+c\sin(d+ex))^{3/2}\left(-\frac{\sqrt{a\cos(d+ex)+b+c\sin(d+ex)}\int\sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+\frac{\sqrt{a^2+c^2}\cos\left(d+ex-\tan^{-1}(a,c)\right)}{b+\sqrt{a^2+c^2}}}dx}{(a^2-b^2+c^2)\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}}-\frac{}{e(a^2-b}\right.}{\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{(a\cos(d+ex)+b+c\sin(d+ex))^{3/2}\left(-\frac{\sqrt{a\cos(d+ex)+b+c\sin(d+ex)}\int\sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+\frac{\sqrt{a^2+c^2}\sin\left(d+ex-\tan^{-1}(a,c)+\frac{\pi}{2}\right)}{b+\sqrt{a^2+c^2}}}dx}{(a^2-b^2+c^2)\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}}-\frac{}{e(a}\right.}{\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}$$

$\Big\downarrow$ 3132

$$\frac{(a\cos(d+ex)+b+c\sin(d+ex))^{3/2}\left(-\frac{2\sqrt{a\cos(d+ex)+b+c\sin(d+ex)}E\left(\frac{1}{2}(d+ex-\tan^{-1}(a,c))|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e(a^2-b^2+c^2)\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}}-\frac{2(c\cos(d}{e(a^2-b^2+c^2)\sqrt{a}}\right.}{\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}$$

input
```
Int[1/(Cos[d + e*x]^(3/2)*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2)),x]
```

output
```
((b + a*Cos[d + e*x] + c*Sin[d + e*x])^(3/2)*((-2*(c*Cos[d + e*x] - a*Sin[
d + e*x]))/((a^2 - b^2 + c^2)*e*Sqrt[b + a*Cos[d + e*x] + c*Sin[d + e*x]])
 - (2*EllipticE[(d + e*x - ArcTan[a, c])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[
a^2 + c^2])]*Sqrt[b + a*Cos[d + e*x] + c*Sin[d + e*x]])/((a^2 - b^2 + c^2)
*e*Sqrt[(b + a*Cos[d + e*x] + c*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])])))/(C
os[d + e*x]^(3/2)*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(3/2))
```

3.456. $\int \frac{1}{\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}\,dx$

3.456.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3598
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]   Int[Sqrt[a/(a + S
qrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - Arc
Tan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

rule 3607
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-3/2), x_Symbol] :> Simp[2*((c*Cos[d + e*x] - b*Sin[d + e*x])/(e*(a^2 - b^
2 - c^2)*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]])), x] + Simp[1/(a^2 - b^
2 - c^2)   Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] /; FreeQ[{
a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

rule 3642
```
Int[cos[(d_.) + (e_.)*(x_)]^(n_)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (
c_.)*tan[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[d + e*x]^n*((a +
b*Sec[d + e*x] + c*Tan[d + e*x])^n/(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n)
   Int[(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d
, e}, x] &&  !IntegerQ[n]
```

3.456.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 21.59 (sec) , antiderivative size = 40403, normalized size of antiderivative = 168.35

method	result	size
default	Expression too large to display	40403

input
```
int(1/cos(e*x+d)^(3/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x,method=_RETUR
NVERBOSE)
```

output
```
result too large to display
```

3.456.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 1752, normalized size of antiderivative = 7.30

$$\int \frac{1}{\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/cos(e*x+d)^(3/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x, algori
thm="fricas")
```

output
```
1/3*((sqrt(2)*(I*a^2*b - a*b*c)*cos(e*x + d) + sqrt(2)*(I*a*b*c - b*c^2)*s
in(e*x + d) + sqrt(2)*(I*a*b^2 - b^2*c))*sqrt(a - I*c)*weierstrassPInverse
(-4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*
a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), 8/27*(9*a^5*b - 8*a^3*b^3 - 27*a*b*c^4
- 9*I*b*c^5 + 2*I*(9*a^2*b + 4*b^3)*c^3 - 6*(3*a^3*b - 4*a*b^3)*c^2 + 3*I*
(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(2*a*b +
 2*I*b*c + 3*(a^2 + c^2)*cos(e*x + d) - 3*(-I*a^2 - I*c^2)*sin(e*x + d))/(
a^2 + c^2)) + (sqrt(2)*(-I*a^2*b - a*b*c)*cos(e*x + d) + sqrt(2)*(-I*a*b*c
 - b*c^2)*sin(e*x + d) + sqrt(2)*(-I*a*b^2 - b^2*c))*sqrt(a + I*c)*weierst
rassPInverse(-4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 - 6*I*a*c^3 - 3*c^4 - 2*I
*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), 8/27*(9*a^5*b - 8*a^3*b^3 -
 27*a*b*c^4 + 9*I*b*c^5 - 2*I*(9*a^2*b + 4*b^3)*c^3 - 6*(3*a^3*b - 4*a*b^3
)*c^2 - 3*I*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6),
1/3*(2*a*b - 2*I*b*c + 3*(a^2 + c^2)*cos(e*x + d) - 3*(I*a^2 + I*c^2)*sin(
e*x + d))/(a^2 + c^2)) - 3*(sqrt(2)*(I*a^3 + I*a*c^2)*cos(e*x + d) + sqrt(
2)*(I*a^2*c + I*c^3)*sin(e*x + d) + sqrt(2)*(I*a^2*b + I*b*c^2))*sqrt(a -
I*c)*weierstrassZeta(-4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 + 6*I*a*c^3 - 3*c
^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), 8/27*(9*a^5*b - 8*a
^3*b^3 - 27*a*b*c^4 - 9*I*b*c^5 + 2*I*(9*a^2*b + 4*b^3)*c^3 - 6*(3*a^3*b -
 4*a*b^3)*c^2 + 3*I*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c...
```

3.456.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(e*x+d)**(3/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))**(3/2),x)
```

output
```
Timed out
```

3.456.7 Maxima [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}\,dx = \int \frac{1}{(b\sec{(ex+d)}+c\tan{(ex+d)}+a)^{\frac{3}{2}}\cos{(ex}$$

input
```
integrate(1/cos(e*x+d)^(3/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x, algori
thm="maxima")
```

output
```
integrate(1/((b*sec(e*x + d) + c*tan(e*x + d) + a)^(3/2)*cos(e*x + d)^(3/2
)), x)
```

3.456.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(e*x+d)^(3/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(3/2),x, algori
thm="giac")
```

output
```
Timed out
```

3.456.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{3}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{3/2}}\,dx = \int \frac{1}{\cos(d+e\,x)^{3/2}\left(a+c\tan(d+e\,x)+\frac{b}{\cos(d+e\,x)}\right)^{3/2}}$$

input
```
int(1/(cos(d + e*x)^(3/2)*(a + c*tan(d + e*x) + b/cos(d + e*x))^(3/2)),x)
```

output
```
int(1/(cos(d + e*x)^(3/2)*(a + c*tan(d + e*x) + b/cos(d + e*x))^(3/2)), x)
```

3.457 $\int \frac{1}{\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}\,dx$

3.457.1 Optimal result

Integrand size = 33, antiderivative size = 492

$$\int \frac{1}{\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}\,dx = -\frac{2(c\cos(d+ex)-a\sin(d+ex))(b+a\cos(d+ex)+c\sin(d+ex))}{3\left(a^2-b^2+c^2\right)e\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}} + \frac{8(bc\cos(d+ex)-ab\sin(d+ex))(b+a\cos(d+ex)+c\sin(d+ex))^2}{3\left(a^2-b^2+c^2\right)^2e\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}} + \frac{8bE\left(\frac{1}{2}\left(d+ex-\tan^{-1}(a,c)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)(b+a\cos(d+ex)+c\sin(d+ex))^3}{3\left(a^2-b^2+c^2\right)^2e\cos^{\frac{5}{2}}(d+ex)\sqrt{\frac{b+a\cos(d+ex)+c\sin(d+ex)}{b+\sqrt{a^2+c^2}}}(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}} + \frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(d+ex-\tan^{-1}(a,c)\right),\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)(b+a\cos(d+ex)+c\sin(d+ex))^2\sqrt{\frac{b+a\cos(d+ex)+c\sin(d+ea}{b+\sqrt{a^2+c^2}}}}{3\left(a^2-b^2+c^2\right)e\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}$$

output
```
-2/3*(c*cos(e*x+d)-a*sin(e*x+d))*(b+a*cos(e*x+d)+c*sin(e*x+d))/(a^2-b^2+c^
2)/e/cos(e*x+d)^(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2)+8/3*(b*c*cos(e*x
+d)-a*b*sin(e*x+d))*(b+a*cos(e*x+d)+c*sin(e*x+d))^2/(a^2-b^2+c^2)^2/e/cos(
e*x+d)^(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2)+8/3*b*(cos(1/2*d+1/2*e*x-
1/2*arctan(a,c))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(a,c))*EllipticE(sin
(1/2*d+1/2*e*x-1/2*arctan(a,c)),2^(1/2)*((a^2+c^2)^(1/2)/(b+(a^2+c^2)^(1/2
)))^(1/2))*(b+a*cos(e*x+d)+c*sin(e*x+d))^3/(a^2-b^2+c^2)^2/e/cos(e*x+d)^(5
/2)/((b+a*cos(e*x+d)+c*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(1/2)/(a+b*sec(e*x
+d)+c*tan(e*x+d))^(5/2)+2/3*(cos(1/2*d+1/2*e*x-1/2*arctan(a,c))^2)^(1/2)/c
os(1/2*d+1/2*e*x-1/2*arctan(a,c))*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan(a
,c)),2^(1/2)*((a^2+c^2)^(1/2)/(b+(a^2+c^2)^(1/2)))^(1/2))*(b+a*cos(e*x+d)+
c*sin(e*x+d))^2*((b+a*cos(e*x+d)+c*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(1/2)/
(a^2-b^2+c^2)/e/cos(e*x+d)^(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2)
```

3.457.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 33.65 (sec) , antiderivative size = 81703, normalized size of antiderivative = 166.06

$$\int \frac{1}{\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/(Cos[d + e*x]^(5/2)*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(5/2
)),x]
```

output
```
Result too large to show
```

3.457.3 Rubi [A] (verified)

Time = 1.79 (sec) , antiderivative size = 443, normalized size of antiderivative = 0.90, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.515$, Rules used = {3042, 3642, 3042, 3608, 27, 3042, 3635, 27, 3042, 3628, 3042, 3598, 3042, 3132, 3606, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}dx$$

$\big\downarrow$ 3042

$$\int \frac{1}{\cos(d+ex)^{5/2}(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}dx$$

$\big\downarrow$ 3642

$$\frac{(a\cos(d+ex)+b+c\sin(d+ex))^{5/2}\int \frac{1}{(b+a\cos(d+ex)+c\sin(d+ex))^{5/2}}dx}{\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}$$

$\big\downarrow$ 3042

$$\frac{(a\cos(d+ex)+b+c\sin(d+ex))^{5/2}\int \frac{1}{(b+a\cos(d+ex)+c\sin(d+ex))^{5/2}}dx}{\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}$$

$\big\downarrow$ 3608

$$\frac{(a\cos(d+ex)+b+c\sin(d+ex))^{5/2}\left(\frac{2\int -\frac{3b-a\cos(d+ex)-c\sin(d+ex)}{2(b+a\cos(d+ex)+c\sin(d+ex))^{3/2}}dx}{3(a^2-b^2+c^2)}-\frac{2(c\cos(d+ex)-a\sin(d+ex))}{3e(a^2-b^2+c^2)(a\cos(d+ex)+b+c\sin(d+ex))^{3/2}}\right)}{\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}$$

$\big\downarrow$ 27

$$\frac{(a\cos(d+ex)+b+c\sin(d+ex))^{5/2}\left(-\frac{\int \frac{3b-a\cos(d+ex)-c\sin(d+ex)}{(b+a\cos(d+ex)+c\sin(d+ex))^{3/2}}dx}{3(a^2-b^2+c^2)}-\frac{2(c\cos(d+ex)-a\sin(d+ex))}{3e(a^2-b^2+c^2)(a\cos(d+ex)+b+c\sin(d+ex))^{3/2}}\right)}{\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}$$

$\big\downarrow$ 3042

$$\frac{(a\cos(d+ex)+b+c\sin(d+ex))^{5/2}\left(-\frac{\int \frac{3b-a\cos(d+ex)-c\sin(d+ex)}{(b+a\cos(d+ex)+c\sin(d+ex))^{3/2}}dx}{3(a^2-b^2+c^2)}-\frac{2(c\cos(d+ex)-a\sin(d+ex))}{3e(a^2-b^2+c^2)(a\cos(d+ex)+b+c\sin(d+ex))^{3/2}}\right)}{\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}$$

$\big\downarrow$ 3635

$$\frac{(a\cos(d+ex)+b+c\sin(d+ex))^{5/2}\left(-\frac{\frac{2\int -\frac{a^2+4b\cos(d+ex)a+3b^2+c^2+4bc\sin(d+ex)}{2\sqrt{b+a\cos(d+ex)+c\sin(d+ex)}}dx}{a^2-b^2+c^2}-\frac{8(bc\cos(d+ex)-ab\sin(d+ex))}{e\left(a^2-b^2+c^2\right)\sqrt{a\cos(d+ex)+b+c\sin(d+ex)}}}{3(a^2-b^2+c^2)}-\right.}{\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}$$

$\big\downarrow$ 27

3.457. $\int \frac{1}{\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}dx$

$$\frac{(a\cos(d+ex)+b+c\sin(d+ex))^{5/2}\left(-\frac{-\frac{\int\frac{a^2+4b\cos(d+ex)a+3b^2+c^2+4bc\sin(d+ex)}{\sqrt{b+a\cos(d+ex)+c\sin(d+ex)}}dx}{a^2-b^2+c^2}-\frac{8(bc\cos(d+ex)-ab\sin(d+ex))}{e\left(a^2-b^2+c^2\right)\sqrt{a\cos(d+ex)+b+c\sin(d+ex)}}}{3(a^2-b^2+c^2)}-\right.}{\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}$$

$\downarrow$ 3042

$$\frac{(a\cos(d+ex)+b+c\sin(d+ex))^{5/2}\left(-\frac{-\frac{\int\frac{a^2+4b\cos(d+ex)a+3b^2+c^2+4bc\sin(d+ex)}{\sqrt{b+a\cos(d+ex)+c\sin(d+ex)}}dx}{a^2-b^2+c^2}-\frac{8(bc\cos(d+ex)-ab\sin(d+ex))}{e\left(a^2-b^2+c^2\right)\sqrt{a\cos(d+ex)+b+c\sin(d+ex)}}}{3(a^2-b^2+c^2)}-\right.}{\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}$$

$\downarrow$ 3628

$$\frac{(a\cos(d+ex)+b+c\sin(d+ex))^{5/2}\left(-\frac{-\frac{\left(a^2-b^2+c^2\right)\int\frac{1}{\sqrt{b+a\cos(d+ex)+c\sin(d+ex)}}dx+4b\int\sqrt{b+a\cos(d+ex)+c\sin(d+ex)}dx}{a^2-b^2+c^2}-\frac{8(bc}{e\left(a^2-b^2+\right.}}{3(a^2-b^2+c^2)}\right.}{\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{5}}$$

$\downarrow$ 3042

$$\frac{(a\cos(d+ex)+b+c\sin(d+ex))^{5/2}\left(-\frac{-\frac{\left(a^2-b^2+c^2\right)\int\frac{1}{\sqrt{b+a\cos(d+ex)+c\sin(d+ex)}}dx+4b\int\sqrt{b+a\cos(d+ex)+c\sin(d+ex)}dx}{a^2-b^2+c^2}-\frac{8(bc}{e\left(a^2-b^2+\right.}}{3(a^2-b^2+c^2)}\right.}{\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{5}}$$

$\downarrow$ 3598

$$\frac{(a\cos(d+ex)+b+c\sin(d+ex))^{5/2}\left(-\frac{-\frac{\left(a^2-b^2+c^2\right)\int\frac{1}{\sqrt{b+a\cos(d+ex)+c\sin(d+ex)}}dx+\frac{4b\sqrt{a\cos(d+ex)+b+c\sin(d+ex)}\int\sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+}}{\sqrt{\frac{a\cos(d+ex)+b+c\sin}{\sqrt{a^2+c^2}+b}}}}{a^2-b^2+c^2}}{3(a^2-b^2+c^2)}\right.}{\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)}$$

$\downarrow$ 3042

3.457. $\int\frac{1}{\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}dx$

$$\frac{(a\cos(d+ex)+b+c\sin(d+ex))^{5/2}\left(-\frac{-\frac{\left(a^2-b^2+c^2\right)\int\frac{1}{\sqrt{b+a\cos(d+ex)+c\sin(d+ex)}}dx+\frac{4b\sqrt{a\cos(d+ex)+b+c\sin(d+ex)}\int\sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+\cdots}}{\sqrt{\frac{a\cos(d+ex)+b+c\cdots}{\sqrt{a^2+c^2}+\cdots}}}}{a^2-b^2+c^2}}{3(a^2-b^2+c^2)\cdots}\cdots\right.}{\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)\cdots}$$

↓ 3132

$$\frac{(a\cos(d+ex)+b+c\sin(d+ex))^{5/2}\left(-\frac{-\frac{\left(a^2-b^2+c^2\right)\int\frac{1}{\sqrt{b+a\cos(d+ex)+c\sin(d+ex)}}dx+\frac{8b\sqrt{a\cos(d+ex)+b+c\sin(d+ex)}E\left(\frac{1}{2}\left(d+ex-\tan^{-1}\cdots\right.\right.}{e\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+e\cdots}{\sqrt{a^2+c^2}+b}}}}{a^2-b^2+c^2}}{3(a^2-b^2+c^2)}\cdots\right.}{\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\,\mathrm{t}\cdots}$$

↓ 3606

$$\frac{(a\cos(d+ex)+b+c\sin(d+ex))^{5/2}\left(-\frac{-\frac{\frac{\left(a^2-b^2+c^2\right)\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}\int\frac{1}{\sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+\frac{\sqrt{a^2+c^2}\cos\left(d+ex-\tan^{-1}(a,c)\right)}{b+\sqrt{a^2+c^2}}}}dx}{\sqrt{a\cos(d+ex)+b+c\sin(d+ex)}}+\cdots}{a^2-b^2+c^2}}{\cdots}\cdots\right.}{\cos^{\frac{5}{2}}(d+ex)(a+b\cdots}$$

↓ 3042

3.457. $\displaystyle\int\frac{1}{\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}\,dx$

$$\frac{(a\cos(d+ex)+b+c\sin(d+ex))^{5/2}\left(-\frac{-\frac{\frac{\left(a^2-b^2+c^2\right)\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}\int\frac{1}{\sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+\frac{\sqrt{a^2+c^2}\sin\left(d+ex-\tan^{-1}(a,c)+\frac{\pi}{2}\right)}{b+\sqrt{a^2+c^2}}}}}{\sqrt{a\cos(d+ex)+b+c\sin(d+ex)}}}{a^2-b^2+c^2}}{}\right.}{\cos^{\frac{5}{2}}(d+ex)(a+}$$

$\Big\downarrow$ 3140

$$\frac{(a\cos(d+ex)+b+c\sin(d+ex))^{5/2}\left(-\frac{-\frac{\frac{2\left(a^2-b^2+c^2\right)\sqrt{\frac{a\cos(d+ex)+b+c\sin(d+ex)}{\sqrt{a^2+c^2}+b}}\text{EllipticF}\left(\frac{1}{2}\left(d+ex-\tan^{-1}(a,c)\right),\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{a\cos(d+ex)+b+c\sin(d+ex)}}+\frac{8b\sqrt{a}}{}}{a^2-b^2+c^2}}{3}\right.}{\cos^{\frac{5}{2}}(d+ex)(a+b\,\mathrm{se}}$$

input
```
Int[1/(Cos[d + e*x]^(5/2)*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(5/2)),x]
```

output
```
((b + a*Cos[d + e*x] + c*Sin[d + e*x])^(5/2)*((-2*(c*Cos[d + e*x] - a*Sin[
d + e*x]))/(3*(a^2 - b^2 + c^2)*e*(b + a*Cos[d + e*x] + c*Sin[d + e*x])^(3
/2)) - ((-8*(b*c*Cos[d + e*x] - a*b*Sin[d + e*x]))/((a^2 - b^2 + c^2)*e*Sq
rt[b + a*Cos[d + e*x] + c*Sin[d + e*x]]) - ((8*b*EllipticE[(d + e*x - ArcT
an[a, c])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2 + c^2])]*Sqrt[b + a*Cos[d +
 e*x] + c*Sin[d + e*x]])/(e*Sqrt[(b + a*Cos[d + e*x] + c*Sin[d + e*x])/(b
+ Sqrt[a^2 + c^2])]) + (2*(a^2 - b^2 + c^2)*EllipticF[(d + e*x - ArcTan[a,
 c])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2 + c^2])]*Sqrt[(b + a*Cos[d + e*x
] + c*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])])/(e*Sqrt[b + a*Cos[d + e*x] + c
*Sin[d + e*x]]))/(a^2 - b^2 + c^2))/(3*(a^2 - b^2 + c^2))))/(Cos[d + e*x]^
(5/2)*(a + b*Sec[d + e*x] + c*Tan[d + e*x])^(5/2))
```

3.457. $\int \frac{1}{\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}\,dx$

3.457.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3598
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]   Int[Sqrt[a/(a + S
qrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - Arc
Tan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

rule 3606
```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Simp[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]   Int[1/Sqrt[a/(a
 + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x -
 ArcTan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2
, 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

rule 3608
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Simp[
1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c
*(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x
] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] &&
NeQ[n, -3/2]
```

3.457. $\int \frac{1}{\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}\,dx$

rule 3628
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]
, x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[
A*b - a*B, 0]
```

rule 3635
```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(-(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(
a^2 - b^2 - c^2))), x] + Simp[1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a + b*Co
s[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n +
2)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x]
/; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[n, -2]
```

rule 3642
```
Int[cos[(d_.) + (e_.)*(x_)]^(n_)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (
c_.)*tan[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[d + e*x]^n*((a +
b*Sec[d + e*x] + c*Tan[d + e*x])^n/(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n)
   Int[(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d
, e}, x] &&  !IntegerQ[n]
```

3.457.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 25.23 (sec) , antiderivative size = 155452, normalized size of antiderivative = 315.96

method	result	size
default	Expression too large to display	155452

input
```
int(1/cos(e*x+d)^(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2),x,method=_RETUR
NVERBOSE)
```

output
```
result too large to display
```

3.457.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.25 (sec) , antiderivative size = 2816, normalized size of antiderivative = 5.72

$$\int \frac{1}{\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/cos(e*x+d)^(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2),x, algori
thm="fricas")
```

output
```
1/9*((sqrt(2)*(-3*I*a^5 - I*a^3*b^2 + I*a*b^2*c^2 - b^2*c^3 + 3*I*a*c^4 -
3*c^5 + (3*a^4 + a^2*b^2)*c)*cos(e*x + d)^2 - 2*sqrt(2)*(3*I*a^4*b + I*a^2
*b^3 + 3*I*a^2*b*c^2 - 3*a*b*c^3 - (3*a^3*b + a*b^3)*c)*cos(e*x + d) - 2*(
sqrt(2)*(3*I*a^2*c^3 - 3*a*c^4 - (3*a^3 + a*b^2)*c^2 + I*(3*a^4 + a^2*b^2)
*c)*cos(e*x + d) + sqrt(2)*(3*I*a*b*c^3 - 3*b*c^4 - (3*a^2*b + b^3)*c^2 +
I*(3*a^3*b + a*b^3)*c))*sin(e*x + d) + sqrt(2)*(-3*I*a^3*b^2 - I*a*b^4 - 3
*I*a*c^4 + 3*c^5 + (3*a^2 + 4*b^2)*c^3 - I*(3*a^3 + 4*a*b^2)*c^2 + (3*a^2*
b^2 + b^4)*c))*sqrt(a - I*c)*weierstrassPInverse(-4/3*(3*a^4 - 4*a^2*b^2 +
 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2
 + c^4), 8/27*(9*a^5*b - 8*a^3*b^3 - 27*a*b*c^4 - 9*I*b*c^5 + 2*I*(9*a^2*b
 + 4*b^3)*c^3 - 6*(3*a^3*b - 4*a*b^3)*c^2 + 3*I*(9*a^4*b - 8*a^2*b^3)*c)/(
a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(2*a*b + 2*I*b*c + 3*(a^2 + c^2)*c
os(e*x + d) - 3*(-I*a^2 - I*c^2)*sin(e*x + d))/(a^2 + c^2)) + (sqrt(2)*(3*
I*a^5 + I*a^3*b^2 - I*a*b^2*c^2 - b^2*c^3 - 3*I*a*c^4 - 3*c^5 + (3*a^4 + a
^2*b^2)*c)*cos(e*x + d)^2 - 2*sqrt(2)*(-3*I*a^4*b - I*a^2*b^3 - 3*I*a^2*b*
c^2 - 3*a*b*c^3 - (3*a^3*b + a*b^3)*c)*cos(e*x + d) - 2*(sqrt(2)*(-3*I*a^2
*c^3 - 3*a*c^4 - (3*a^3 + a*b^2)*c^2 - I*(3*a^4 + a^2*b^2)*c)*cos(e*x + d)
 + sqrt(2)*(-3*I*a*b*c^3 - 3*b*c^4 - (3*a^2*b + b^3)*c^2 - I*(3*a^3*b + a*
b^3)*c))*sin(e*x + d) + sqrt(2)*(3*I*a^3*b^2 + I*a*b^4 + 3*I*a*c^4 + 3*c^5
 + (3*a^2 + 4*b^2)*c^3 + I*(3*a^3 + 4*a*b^2)*c^2 + (3*a^2*b^2 + b^4)*c)...
```

3.457.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(e*x+d)**(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))**(5/2),x)
```

output
```
Timed out
```

3.457.7 Maxima [F]

$$\int \frac{1}{\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}\,dx = \int \frac{1}{(b\sec(ex+d)+c\tan(ex+d)+a)^{\frac{5}{2}}\cos(ex$$

input
```
integrate(1/cos(e*x+d)^(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2),x, algori
thm="maxima")
```

output
```
integrate(1/((b*sec(e*x + d) + c*tan(e*x + d) + a)^(5/2)*cos(e*x + d)^(5/2
)), x)
```

3.457.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/cos(e*x+d)^(5/2)/(a+b*sec(e*x+d)+c*tan(e*x+d))^(5/2),x, algori
thm="giac")
```

output
```
Timed out
```

3.457.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cos^{\frac{5}{2}}(d+ex)(a+b\sec(d+ex)+c\tan(d+ex))^{5/2}}\,dx = \int \frac{1}{\cos(d+e\,x)^{5/2}\left(a+c\tan(d+e\,x)+\frac{b}{\cos(d+e\,x}\right.}$$

input
```
int(1/(cos(d + e*x)^(5/2)*(a + c*tan(d + e*x) + b/cos(d + e*x))^(5/2)),x)
```

output
```
int(1/(cos(d + e*x)^(5/2)*(a + c*tan(d + e*x) + b/cos(d + e*x))^(5/2)), x)
```

3.458 $\int \frac{1}{a+b\cot(x)+c\csc(x)}\,dx$

3.458.1 Optimal result

Integrand size = 12, antiderivative size = 98

$$\int \frac{1}{a+b\cot(x)+c\csc(x)}\,dx = \frac{ax}{a^2+b^2}+\frac{2ac\,\text{arctanh}\left(\frac{a-(b-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{(a^2+b^2)\sqrt{a^2+b^2-c^2}} - \frac{b\log(c+b\cos(x)+a\sin(x))}{a^2+b^2}$$

output
```
a*x/(a^2+b^2)-b*ln(c+b*cos(x)+a*sin(x))/(a^2+b^2)+2*a*c*arctanh((a-(b-c)*t
an(1/2*x))/(a^2+b^2-c^2)^(1/2))/(a^2+b^2)/(a^2+b^2-c^2)^(1/2)
```

3.458.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int \frac{1}{a+b\cot(x)+c\csc(x)}\,dx = \frac{ax+\frac{2ac\,\text{arctanh}\left(\frac{a+(-b+c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}}-b\log(c+b\cos(x)+a\sin(x))}{a^2+b^2}$$

input
```
Integrate[(a + b*Cot[x] + c*Csc[x])^(-1),x]
```

output
```
(a*x + (2*a*c*ArcTanh[(a + (-b + c)*Tan[x/2])/Sqrt[a^2 + b^2 - c^2]])/Sqrt
[a^2 + b^2 - c^2] - b*Log[c + b*Cos[x] + a*Sin[x]])/(a^2 + b^2)
```

3.458.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 3639, 3042, 3616, 3042, 3603, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\cot(x)+c\csc(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a+b\cot(x)+c\csc(x)}dx$$

$$\Big\downarrow \text{3639}$$

$$\int \frac{\sin(x)}{a\sin(x)+b\cos(x)+c}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)}{a\sin(x)+b\cos(x)+c}dx$$

$$\Big\downarrow \text{3616}$$

$$-\frac{ac\int \frac{1}{c+b\cos(x)+a\sin(x)}dx}{a^2+b^2}-\frac{b\log(a\sin(x)+b\cos(x)+c)}{a^2+b^2}+\frac{ax}{a^2+b^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{ac\int \frac{1}{c+b\cos(x)+a\sin(x)}dx}{a^2+b^2}-\frac{b\log(a\sin(x)+b\cos(x)+c)}{a^2+b^2}+\frac{ax}{a^2+b^2}$$

$$\Big\downarrow \text{3603}$$

$$-\frac{2ac\int \frac{1}{-((b-c)\tan^2(\frac{x}{2}))+2a\tan(\frac{x}{2})+b+c}d\tan\left(\frac{x}{2}\right)}{a^2+b^2}-\frac{b\log(a\sin(x)+b\cos(x)+c)}{a^2+b^2}+\frac{ax}{a^2+b^2}$$

$$\Big\downarrow \text{1083}$$

$$\frac{4ac\int \frac{1}{4(a^2+b^2-c^2)-(2a-2(b-c)\tan(\frac{x}{2}))^2}d\left(2a-2(b-c)\tan\left(\frac{x}{2}\right)\right)}{a^2+b^2}-\frac{b\log(a\sin(x)+b\cos(x)+c)}{a^2+b^2}+\frac{ax}{a^2+b^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{2ac\operatorname{arctanh}\left(\frac{2a-2(b-c)\tan\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2-c^2}}\right)}{(a^2+b^2)\sqrt{a^2+b^2-c^2}}-\frac{b\log(a\sin(x)+b\cos(x)+c)}{a^2+b^2}+\frac{ax}{a^2+b^2}$$

input
```
Int[(a + b*Cot[x] + c*Csc[x])^(-1),x]
```

output
```
(a*x)/(a^2 + b^2) + (2*a*c*ArcTanh[(2*a - 2*(b - c)*Tan[x/2])/(2*Sqrt[a^2
+ b^2 - c^2])])/((a^2 + b^2)*Sqrt[a^2 + b^2 - c^2]) - (b*Log[c + b*Cos[x]
+ a*Sin[x]])/(a^2 + b^2)
```

3.458.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

rule 3616
```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[c*C*((d + e*x)/
(e*(b^2 + c^2))), x] + (-Simp[b*C*(Log[a + b*Cos[d + e*x] + c*Sin[d + e*x]]
/(e*(b^2 + c^2))), x] + Simp[(A*(b^2 + c^2) - a*c*C)/(b^2 + c^2)   Int[1/(a
 + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x]) /; FreeQ[{a, b, c, d, e, A, C}
, x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*c*C, 0]
```

3.458. $\int \frac{1}{a+b\cot(x)+c\csc(x)}\,dx$

rule 3639
```
Int[((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) + cot[(d_.) + (e_.)*(x_)]*(c_.))
^(-1), x_Symbol] :> Int[Sin[d + e*x]/(b + a*Sin[d + e*x] + c*Cos[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

3.458.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.80

method	result
default	$\frac{\frac{4(b^2-cb)\ln\left(-\tan\left(\frac{x}{2}\right)^2 b+c\tan\left(\frac{x}{2}\right)^2+2a\tan\left(\frac{x}{2}\right)+b+c\right)}{-2b+2c}+\frac{4\left(-ab-ac-\frac{(b^2-cb)a}{-b+c}\right)\arctan\left(\frac{2(-b+c)\tan\left(\frac{x}{2}\right)+2a}{2\sqrt{-a^2-b^2+c^2}}\right)}{\sqrt{-a^2-b^2+c^2}}}{2a^2+2b^2}+\frac{2b\ln\left(1+\tan\left(\frac{x}{2}\right)^2\right)+}{2a^2+}$
risch	Expression too large to display

input
```
int(1/(a+b*cot(x)+c*csc(x)),x,method=_RETURNVERBOSE)
```

output
```
4/(2*a^2+2*b^2)*(1/2*(b^2-b*c)/(-b+c)*ln(-tan(1/2*x)^2*b+c*tan(1/2*x)^2+2*
a*tan(1/2*x)+b+c)+(-a*b-a*c-(b^2-b*c)*a/(-b+c))/(-a^2-b^2+c^2)^(1/2)*arcta
n(1/2*(2*(-b+c)*tan(1/2*x)+2*a)/(-a^2-b^2+c^2)^(1/2)))+4/(2*a^2+2*b^2)*(1/
2*b*ln(1+tan(1/2*x)^2)+a*arctan(tan(1/2*x)))
```

3.458.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. 2(94) = 188.

Time = 0.31 (sec) , antiderivative size = 555, normalized size of antiderivative = 5.66

$$\int \frac{1}{a+b\cot(x)+c\csc(x)}\,dx$$
$$= \left[\frac{\sqrt{a^2+b^2-c^2}ac\log\left(\frac{a^4+3a^2b^2+2b^4+(a^2-b^2)c^2+2(a^2b+b^3)c\cos(x)+(a^4-b^4-2(a^2-b^2)c^2)\cos(x)^2+2((a^3+ab^2)c-(a^3b+ab^3-2}{2bc\cos(x)-(a^2-b^2)\cos(x)^2+a}\right.}{}\right.$$
$$-\frac{2\sqrt{-a^2-b^2+c^2}ac\arctan\left(\frac{(bc\cos(x)+ac\sin(x)+a^2+b^2)\sqrt{-a^2-b^2+c^2}}{(a^3+ab^2-ac^2)\cos(x)-(a^2b+b^3-bc^2)\sin(x)}\right)-2(a^3+ab^2-ac^2)x+(a^2b+b^3-bc^2}{2(a^4+2a^2b^2+b^4-(a^2+b^2)}$$

input
```
integrate(1/(a+b*cot(x)+c*csc(x)),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(a^2 + b^2 - c^2)*a*c*log((a^4 + 3*a^2*b^2 + 2*b^4 + (a^2 - b^2)
*c^2 + 2*(a^2*b + b^3)*c*cos(x) + (a^4 - b^4 - 2*(a^2 - b^2)*c^2)*cos(x)^2
 + 2*((a^3 + a*b^2)*c - (a^3*b + a*b^3 - 2*a*b*c^2)*cos(x))*sin(x) + 2*(2*
a*b*c*cos(x)^2 - a*b*c + (a^3 + a*b^2)*cos(x) - (a^2*b + b^3 - (a^2 - b^2)
*c*cos(x))*sin(x))*sqrt(a^2 + b^2 - c^2))/(2*b*c*cos(x) - (a^2 - b^2)*cos(
x)^2 + a^2 + c^2 + 2*(a*b*cos(x) + a*c)*sin(x))) + 2*(a^3 + a*b^2 - a*c^2)
*x - (a^2*b + b^3 - b*c^2)*log(2*b*c*cos(x) - (a^2 - b^2)*cos(x)^2 + a^2 +
 c^2 + 2*(a*b*cos(x) + a*c)*sin(x)))/(a^4 + 2*a^2*b^2 + b^4 - (a^2 + b^2)*
c^2), -1/2*(2*sqrt(-a^2 - b^2 + c^2)*a*c*arctan((b*c*cos(x) + a*c*sin(x) +
 a^2 + b^2)*sqrt(-a^2 - b^2 + c^2)/((a^3 + a*b^2 - a*c^2)*cos(x) - (a^2*b
+ b^3 - b*c^2)*sin(x))) - 2*(a^3 + a*b^2 - a*c^2)*x + (a^2*b + b^3 - b*c^2
)*log(2*b*c*cos(x) - (a^2 - b^2)*cos(x)^2 + a^2 + c^2 + 2*(a*b*cos(x) + a*
c)*sin(x)))/(a^4 + 2*a^2*b^2 + b^4 - (a^2 + b^2)*c^2)]
```

3.458.6 Sympy [F]

$$\int \frac{1}{a+b\cot(x)+c\csc(x)}\,dx = \int \frac{1}{a+b\cot{(x)}+c\csc{(x)}}\,dx$$

input
```
integrate(1/(a+b*cot(x)+c*csc(x)),x)
```

output
```
Integral(1/(a + b*cot(x) + c*csc(x)), x)
```

3.458.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a+b\cot(x)+c\csc(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*cot(x)+c*csc(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c^2-b^2-a^2>0)', see `assume?` f
or more de
```

3.458.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.61

$$\int \frac{1}{a+b\cot(x)+c\csc(x)}\,dx = -\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2b+2c)+\arctan\left(-\frac{b\tan\left(\frac{1}{2}x\right)-c\tan\left(\frac{1}{2}x\right)-a}{\sqrt{-a^2-b^2+c^2}}\right)\right)ac}{(a^2+b^2)\sqrt{-a^2-b^2+c^2}}+\frac{ax}{a^2+b^2} - \frac{b\log\left(-b\tan\left(\frac{1}{2}x\right)^2+c\tan\left(\frac{1}{2}x\right)^2+2a\tan\left(\frac{1}{2}x\right)+b+c\right)}{a^2+b^2}+\frac{b\log\left(\tan\left(\frac{1}{2}x\right)^2+1\right)}{a^2+b^2}$$

input
```
integrate(1/(a+b*cot(x)+c*csc(x)),x, algorithm="giac")
```

output
```
-2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*b + 2*c) + arctan(-(b*tan(1/2*x) - c*t
an(1/2*x) - a)/sqrt(-a^2 - b^2 + c^2)))*a*c/((a^2 + b^2)*sqrt(-a^2 - b^2 +
 c^2)) + a*x/(a^2 + b^2) - b*log(-b*tan(1/2*x)^2 + c*tan(1/2*x)^2 + 2*a*ta
n(1/2*x) + b + c)/(a^2 + b^2) + b*log(tan(1/2*x)^2 + 1)/(a^2 + b^2)
```

3.458.9 Mupad [B] (verification not implemented)

Time = 41.77 (sec) , antiderivative size = 965, normalized size of antiderivative = 9.85

$$\int \frac{1}{a+b\cot(x)+c\csc(x)}\,dx = \frac{\ln\left(\tan\left(\frac{x}{2}\right)-\mathrm{i}\right)}{b+a\,1\mathrm{i}}$$

$$-\frac{\ln\left(-64\tan\left(\frac{x}{2}\right)(b-c)^2-\frac{\left(a^2b-bc^2+b^3+ac\sqrt{a^2+b^2-c^2}\right)\left(32ab^2+32ac^2-64abc-64\tan\left(\frac{x}{2}\right)(b-c)\left(a^2-c^2+bc\right)+\frac{\left(a^2b-bc^2-\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

$$-\frac{\ln\left(-64\tan\left(\frac{x}{2}\right)(b-c)^2-\frac{\left(a^2b-bc^2+b^3-ac\sqrt{a^2+b^2-c^2}\right)\left(32ab^2+32ac^2-64abc-64\tan\left(\frac{x}{2}\right)(b-c)\left(a^2-c^2+bc\right)+\frac{\left(a^2b-bc^2-\cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$$

$$+\frac{\ln\left(\tan\left(\frac{x}{2}\right)+1\mathrm{i}\right)\,1\mathrm{i}}{a+b\,1\mathrm{i}}$$

input
```
int(1/(a + c/sin(x) + b*cot(x)),x)
```

output
```
log(tan(x/2) - 1i)/(a*1i + b) + (log(tan(x/2) + 1i)*1i)/(a + b*1i) - (log(
- 64*tan(x/2)*(b - c)^2 - ((a^2*b - b*c^2 + b^3 + a*c*(a^2 + b^2 - c^2)^(1
/2))*(32*a*b^2 + 32*a*c^2 - 64*a*b*c - 64*tan(x/2)*(b - c)*(b*c + a^2 - c^
2) + ((a^2*b - b*c^2 + b^3 + a*c*(a^2 + b^2 - c^2)^(1/2))*(64*a*b^3 - 32*a
^3*b + 32*a^3*c + 32*tan(x/2)*(b - c)*(a^2*b - 2*a^2*c + 2*b^2*c - 2*b^3)
+ 64*a*b*c^2 - 128*a*b^2*c - (32*(b - c)*(a^2*b - b*c^2 + b^3 + a*c*(a^2 +
 b^2 - c^2)^(1/2))*(3*a^4*tan(x/2) + 3*a*b^3 + 3*a^3*b + a^3*c + 3*a^2*b^2
*tan(x/2) - 2*a^2*c^2*tan(x/2) + 2*b^2*c^2*tan(x/2) - 2*b^3*c*tan(x/2) - 4
*a*b*c^2 + a*b^2*c - 2*a^2*b*c*tan(x/2)))/((a^2 + b^2)*(a^2 + b^2 - c^2)))
)/((a^2 + b^2)*(a^2 + b^2 - c^2))))/((a^2 + b^2)*(a^2 + b^2 - c^2)))*(b*(a
^2 - c^2) + b^3 + a*c*(a^2 + b^2 - c^2)^(1/2)))/(c^2*(a^2 + b^2 - c^2) + (
a^2 + b^2 - c^2)^2) - (log(- 64*tan(x/2)*(b - c)^2 - ((a^2*b - b*c^2 + b^3
 - a*c*(a^2 + b^2 - c^2)^(1/2))*(32*a*b^2 + 32*a*c^2 - 64*a*b*c - 64*tan(x
/2)*(b - c)*(b*c + a^2 - c^2) + ((a^2*b - b*c^2 + b^3 - a*c*(a^2 + b^2 - c
^2)^(1/2))*(64*a*b^3 - 32*a^3*b + 32*a^3*c + 32*tan(x/2)*(b - c)*(a^2*b -
2*a^2*c + 2*b^2*c - 2*b^3) + 64*a*b*c^2 - 128*a*b^2*c - (32*(b - c)*(a^2*b
 - b*c^2 + b^3 - a*c*(a^2 + b^2 - c^2)^(1/2))*(3*a^4*tan(x/2) + 3*a*b^3 +
3*a^3*b + a^3*c + 3*a^2*b^2*tan(x/2) - 2*a^2*c^2*tan(x/2) + 2*b^2*c^2*tan(
x/2) - 2*b^3*c*tan(x/2) - 4*a*b*c^2 + a*b^2*c - 2*a^2*b*c*tan(x/2)))/((a^2
 + b^2)*(a^2 + b^2 - c^2))))/((a^2 + b^2)*(a^2 + b^2 - c^2))))/((a^2 + ...
```

3.458. $\int \frac{1}{a+b\cot(x)+c\csc(x)}\,dx$

3.459 $\int \frac{\csc(x)}{a+b\cot(x)+c\csc(x)}\,dx$

3.459.1 Optimal result

Integrand size = 15, antiderivative size = 51

$$\int \frac{\csc(x)}{a+b\cot(x)+c\csc(x)}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{a-(b-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}}$$

output
```
-2*arctanh((a-(b-c)*tan(1/2*x))/(a^2+b^2-c^2)^(1/2))/(a^2+b^2-c^2)^(1/2)
```

3.459.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.98

$$\int \frac{\csc(x)}{a+b\cot(x)+c\csc(x)}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{a+(-b+c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}}$$

input
```
Integrate[Csc[x]/(a + b*Cot[x] + c*Csc[x]),x]
```

output
```
(-2*ArcTanh[(a + (-b + c)*Tan[x/2])/Sqrt[a^2 + b^2 - c^2]])/Sqrt[a^2 + b^2
 - c^2]
```

3.459.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3645, 3042, 3603, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(x)}{a+b\cot(x)+c\csc(x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\csc(x)}{a+b\cot(x)+c\csc(x)}dx$$
$$\Big\downarrow \text{3645}$$
$$\int \frac{1}{a\sin(x)+b\cos(x)+c}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{a\sin(x)+b\cos(x)+c}dx$$
$$\Big\downarrow \text{3603}$$
$$2\int \frac{1}{-\left((b-c)\tan^2\left(\frac{x}{2}\right)\right)+2a\tan\left(\frac{x}{2}\right)+b+c}d\tan\left(\frac{x}{2}\right)$$
$$\Big\downarrow \text{1083}$$
$$-4\int \frac{1}{4\left(a^2+b^2-c^2\right)-\left(2a-2(b-c)\tan\left(\frac{x}{2}\right)\right)^2}d\left(2a-2(b-c)\tan\left(\frac{x}{2}\right)\right)$$
$$\Big\downarrow \text{219}$$
$$-\frac{2\text{arctanh}\left(\frac{2a-2(b-c)\tan\left(\frac{x}{2}\right)}{2\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}}$$

input
```
Int[Csc[x]/(a + b*Cot[x] + c*Csc[x]),x]
```

output
```
(-2*ArcTanh[(2*a - 2*(b - c)*Tan[x/2])/(2*Sqrt[a^2 + b^2 - c^2])])/Sqrt[a^
2 + b^2 - c^2]
```

3.459.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

rule 3645
```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Int[1/(b + a*Sin[d + e*x]
 + c*Cos[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && I
ntegerQ[n]
```

3.459.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

method	result
default	$\frac{2\arctan\left(\frac{2(-b+c)\tan\left(\frac{x}{2}\right)+2a}{2\sqrt{-a^{2}-b^{2}+c^{2}}}\right)}{\sqrt{-a^{2}-b^{2}+c^{2}}}$
risch	$-\frac{i\ln\left(e^{ix}+\frac{ica\sqrt{-a^{2}-b^{2}+c^{2}}+ia^{3}+ia\,b^{2}-ia\,c^{2}+cb\sqrt{-a^{2}-b^{2}+c^{2}}+a^{2}b+b^{3}-c^{2}b}{\left(a^{2}+b^{2}\right)\sqrt{-a^{2}-b^{2}+c^{2}}}\right)}{\sqrt{-a^{2}-b^{2}+c^{2}}}+\frac{i\ln\left(e^{ix}+\frac{ica\sqrt{-a^{2}-b^{2}+c^{2}}-ia^{3}-ia\,b^{2}+ia\,c^{2}+cb\sqrt{-a \cdots}}{\left(a^{2}+b^{2}\right)\sqrt{-a^{2}-b^{2}+c \cdots}}\right.}{\sqrt{-a^{2}-b^{2}+c^{2}}}$

input
```
int(csc(x)/(a+b*cot(x)+c*csc(x)),x,method=_RETURNVERBOSE)
```

output
```
2/(-a^2-b^2+c^2)^(1/2)*arctan(1/2*(2*(-b+c)*tan(1/2*x)+2*a)/(-a^2-b^2+c^2)
^(1/2))
```

3.459.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(47) = 94$.

Time = 0.31 (sec) , antiderivative size = 349, normalized size of antiderivative = 6.84

$$\int \frac{\csc(x)}{a + b\cot(x) + c\csc(x)}\,dx$$
$$= \left[\frac{\log\left(-\frac{a^4+3\,a^2b^2+2\,b^4+(a^2-b^2)c^2+2\,(a^2b+b^3)c\cos(x)+(a^4-b^4-2\,(a^2-b^2)c^2)\cos(x)^2+2\,((a^3+ab^2)c-(a^3b+ab^3-2\,abc^2)\cos(x))\sin(x)-}{2\,bc\cos(x)-(a^2-b^2)\cos(x)^2+a^2+c^2+2\,(ab\cos(x)+}\right.}{2\,\sqrt{a^2+b^2-c^2}} \right.$$

input
```
integrate(csc(x)/(a+b*cot(x)+c*csc(x)),x, algorithm="fricas")
```

output
```
[1/2*log(-(a^4 + 3*a^2*b^2 + 2*b^4 + (a^2 - b^2)*c^2 + 2*(a^2*b + b^3)*c*c
os(x) + (a^4 - b^4 - 2*(a^2 - b^2)*c^2)*cos(x)^2 + 2*((a^3 + a*b^2)*c - (a
^3*b + a*b^3 - 2*a*b*c^2)*cos(x))*sin(x) - 2*(2*a*b*c*cos(x)^2 - a*b*c + (
a^3 + a*b^2)*cos(x) - (a^2*b + b^3 - (a^2 - b^2)*c*cos(x))*sin(x))*sqrt(a^
2 + b^2 - c^2))/(2*b*c*cos(x) - (a^2 - b^2)*cos(x)^2 + a^2 + c^2 + 2*(a*b*
cos(x) + a*c)*sin(x)))/sqrt(a^2 + b^2 - c^2), sqrt(-a^2 - b^2 + c^2)*arcta
n((b*c*cos(x) + a*c*sin(x) + a^2 + b^2)*sqrt(-a^2 - b^2 + c^2)/((a^3 + a*b
^2 - a*c^2)*cos(x) - (a^2*b + b^3 - b*c^2)*sin(x)))/(a^2 + b^2 - c^2)]
```

3.459.6 Sympy [F]

$$\int \frac{\csc(x)}{a + b\cot(x) + c\csc(x)}\,dx = \int \frac{\csc(x)}{a + b\cot(x) + c\csc(x)}\,dx$$

input
```
integrate(csc(x)/(a+b*cot(x)+c*csc(x)),x)
```

output
```
Integral(csc(x)/(a + b*cot(x) + c*csc(x)), x)
```

3.459.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\csc(x)}{a + b\cot(x) + c\csc(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(csc(x)/(a+b*cot(x)+c*csc(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c^2-b^2-a^2>0)', see `assume?` f
or more de
```

3.459.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.43

$$\int \frac{\csc(x)}{a + b\cot(x) + c\csc(x)}\,dx = -\frac{2\left(\pi\left\lfloor \frac{x}{2\pi} + \frac{1}{2}\right\rfloor \operatorname{sgn}(2b - 2c) + \arctan\left(\frac{b\tan\left(\frac{1}{2}x\right) - c\tan\left(\frac{1}{2}x\right) - a}{\sqrt{-a^2 - b^2 + c^2}}\right)\right)}{\sqrt{-a^2 - b^2 + c^2}}$$

input
```
integrate(csc(x)/(a+b*cot(x)+c*csc(x)),x, algorithm="giac")
```

output
```
-2*(pi*floor(1/2*x/pi + 1/2)*sgn(2*b - 2*c) + arctan((b*tan(1/2*x) - c*tan
(1/2*x) - a)/sqrt(-a^2 - b^2 + c^2)))/sqrt(-a^2 - b^2 + c^2)
```

3.459.9 Mupad [B] (verification not implemented)

Time = 27.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

$$\int \frac{\csc(x)}{a + b\cot(x) + c\csc(x)}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{a - \frac{\tan\left(\frac{x}{2}\right)(2b - 2c)}{2}}{\sqrt{a^2 + b^2 - c^2}}\right)}{\sqrt{a^2 + b^2 - c^2}}$$

input
```
int(1/(sin(x)*(a + c/sin(x) + b*cot(x))),x)
```

output
```
-(2*atanh((a - (tan(x/2)*(2*b - 2*c))/2)/(a^2 + b^2 - c^2)^(1/2)))/(a^2 +
b^2 - c^2)^(1/2)
```

3.459. $\int \frac{\csc(x)}{a+b\cot(x)+c\csc(x)}\,dx$

3.460 $\int \frac{\csc^2(x)}{a+b\cot(x)+c\csc(x)}\,dx$

3.460.1 Optimal result

Integrand size = 17, antiderivative size = 120

$$\int \frac{\csc^2(x)}{a+b\cot(x)+c\csc(x)}\,dx = -\frac{2ac\,\mathrm{arctanh}\left(\frac{a-(b-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{(b^2-c^2)\sqrt{a^2+b^2-c^2}}+\frac{\log\left(\tan\left(\frac{x}{2}\right)\right)}{b+c} - \frac{b\log\left(b+c+2a\tan\left(\frac{x}{2}\right)-(b-c)\tan^2\left(\frac{x}{2}\right)\right)}{b^2-c^2}$$

output
```
ln(tan(1/2*x))/(b+c)-b*ln(b+c+2*a*tan(1/2*x)-(b-c)*tan(1/2*x)^2)/(b^2-c^2)
-2*a*c*arctanh((a-(b-c)*tan(1/2*x))/(a^2+b^2-c^2)^(1/2))/(b^2-c^2)/(a^2+b^
2-c^2)^(1/2)
```

3.460.2 Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.87

$$\int \frac{\csc^2(x)}{a+b\cot(x)+c\csc(x)}\,dx = \frac{\frac{2ac\,\mathrm{arctanh}\left(\frac{a+(-b+c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{\sqrt{a^2+b^2-c^2}}-(b+c)\log\left(\cos\left(\frac{x}{2}\right)\right)+(-b+c)\log\left(\sin\left(\frac{x}{2}\right)\right)+b\log(c+b\cos(x)+a\sin(x))}{(-b+c)(b+c)}$$

input
```
Integrate[Csc[x]^2/(a + b*Cot[x] + c*Csc[x]),x]
```

output
```
((2*a*c*ArcTanh[(a + (-b + c)*Tan[x/2])/Sqrt[a^2 + b^2 - c^2]])/Sqrt[a^2 +
 b^2 - c^2] - (b + c)*Log[Cos[x/2]] + (-b + c)*Log[Sin[x/2]] + b*Log[c + b
*Cos[x] + a*Sin[x]])/((-b + c)*(b + c))
```

3.460.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {3042, 4897, 3042, 4902, 27, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(x)}{a+b\cot(x)+c\csc(x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\csc(x)^2}{a+b\cot(x)+c\csc(x)}dx$$
$$\Big\downarrow \text{4897}$$
$$\int \frac{\csc(x)}{a\sin(x)+b\cos(x)+c}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\csc(x)}{a\sin(x)+b\cos(x)+c}dx$$
$$\Big\downarrow \text{4902}$$
$$2\int \frac{\cot\left(\frac{x}{2}\right)\left(\tan^2\left(\frac{x}{2}\right)+1\right)}{2\left(-\left((b-c)\tan^2\left(\frac{x}{2}\right)\right)+2a\tan\left(\frac{x}{2}\right)+b+c\right)}d\tan\left(\frac{x}{2}\right)$$
$$\Big\downarrow \text{27}$$
$$\int \frac{\left(\tan^2\left(\frac{x}{2}\right)+1\right)\cot\left(\frac{x}{2}\right)}{2a\tan\left(\frac{x}{2}\right)-\left((b-c)\tan^2\left(\frac{x}{2}\right)\right)+b+c}d\tan\left(\frac{x}{2}\right)$$
$$\Big\downarrow \text{2159}$$
$$\int \left(\frac{2\left(b\tan\left(\frac{x}{2}\right)-a\right)}{(b+c)\left(2a\tan\left(\frac{x}{2}\right)-\left((b-c)\tan^2\left(\frac{x}{2}\right)\right)+b+c\right)}+\frac{\cot\left(\frac{x}{2}\right)}{b+c}\right)d\tan\left(\frac{x}{2}\right)$$
$$\Big\downarrow \text{2009}$$

$$-\frac{2ac\operatorname{arctanh}\left(\frac{a-(b-c)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2-c^2}}\right)}{(b^2-c^2)\sqrt{a^2+b^2-c^2}}-\frac{b\log\left(2a\tan\left(\frac{x}{2}\right)-\left((b-c)\tan^2\left(\frac{x}{2}\right)\right)+b+c\right)}{b^2-c^2}+\frac{\log\left(\tan\left(\frac{x}{2}\right)\right)}{b+c}$$

input
```
Int[Csc[x]^2/(a + b*Cot[x] + c*Csc[x]),x]
```

output
```
(-2*a*c*ArcTanh[(a - (b - c)*Tan[x/2])/Sqrt[a^2 + b^2 - c^2]])/((b^2 - c^2
)*Sqrt[a^2 + b^2 - c^2]) + Log[Tan[x/2]]/(b + c) - (b*Log[b + c + 2*a*Tan[
x/2] - (b - c)*Tan[x/2]^2])/(b^2 - c^2)
```

3.460.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

rule 4902
```
Int[u_, x_Symbol] :> With[{w = Block[{$ShowSteps = False, $StepCounter = Nu
ll}, Int[SubstFor[1/(1 + FreeFactors[Tan[FunctionOfTrig[u, x]/2], x]^2*x^2)
, Tan[FunctionOfTrig[u, x]/2]/FreeFactors[Tan[FunctionOfTrig[u, x]/2], x],
u, x], x]]}, Module[{v = FunctionOfTrig[u, x], d}, Simp[d = FreeFactors[Tan
[v/2], x]; 2*(d/Coefficient[v, x, 1])   Subst[Int[SubstFor[1/(1 + d^2*x^2),
 Tan[v/2]/d, u, x], x], x, Tan[v/2]/d], x]] /; CalculusFreeQ[w, x]] /; Inve
rseFunctionFreeQ[u, x] &&  !FalseQ[FunctionOfTrig[u, x]]
```

3.460. $\int \frac{\csc^2(x)}{a+b\cot(x)+c\csc(x)}\,dx$

3.460.4 Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.02

method	result	size
default	$\frac{\frac{b \ln\left(-\tan\left(\frac{x}{2}\right)^{2} b+c \tan\left(\frac{x}{2}\right)^{2}+2 a \tan\left(\frac{x}{2}\right)+b+c\right)}{-b+c}+\frac{\left(-2 a-\frac{2 b a}{-b+c}\right) \arctan\left(\frac{2(-b+c) \tan\left(\frac{x}{2}\right)+2 a}{2 \sqrt{-a^{2}-b^{2}+c^{2}}}\right)}{\sqrt{-a^{2}-b^{2}+c^{2}}}}{b+c}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{b+c}$	123
risch	Expression too large to display	1342

input
```
int(csc(x)^2/(a+b*cot(x)+c*csc(x)),x,method=_RETURNVERBOSE)
```

output
```
1/(b+c)*(b/(-b+c)*ln(-tan(1/2*x)^2*b+c*tan(1/2*x)^2+2*a*tan(1/2*x)+b+c)+(-
2*a-2*b*a/(-b+c))/(-a^2-b^2+c^2)^(1/2)*arctan(1/2*(2*(-b+c)*tan(1/2*x)+2*a
)/(-a^2-b^2+c^2)^(1/2)))+ln(tan(1/2*x))/(b+c)
```

3.460.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 260 vs. 2(110) = 220.

Time = 1.67 (sec) , antiderivative size = 669, normalized size of antiderivative = 5.58

$$\int \frac{\csc^2(x)}{a+b\cot(x)+c\csc(x)}\,dx$$
$$= \left[-\frac{\sqrt{a^2+b^2-c^2}ac\log\left(\frac{a^4+3\,a^2b^2+2\,b^4+(a^2-b^2)c^2+2\,(a^2b+b^3)c\cos(x)+(a^4-b^4-2\,(a^2-b^2)c^2)\cos(x)^2+2\,((a^3+ab^2)c-(a^3b+ab^3-}{2\,bc\cos(x)-(a^2-b^2)\cos(x)^2-}\right.}{}\right.$$

input
```
integrate(csc(x)^2/(a+b*cot(x)+c*csc(x)),x, algorithm="fricas")
```

output
```
[-1/2*(sqrt(a^2 + b^2 - c^2)*a*c*log((a^4 + 3*a^2*b^2 + 2*b^4 + (a^2 - b^2
)*c^2 + 2*(a^2*b + b^3)*c*cos(x) + (a^4 - b^4 - 2*(a^2 - b^2)*c^2)*cos(x)^
2 + 2*((a^3 + a*b^2)*c - (a^3*b + a*b^3 - 2*a*b*c^2)*cos(x))*sin(x) + 2*(2
*a*b*c*cos(x)^2 - a*b*c + (a^3 + a*b^2)*cos(x) - (a^2*b + b^3 - (a^2 - b^2
)*c*cos(x))*sin(x))*sqrt(a^2 + b^2 - c^2))/(2*b*c*cos(x) - (a^2 - b^2)*cos
(x)^2 + a^2 + c^2 + 2*(a*b*cos(x) + a*c)*sin(x))) + (a^2*b + b^3 - b*c^2)*
log(2*b*c*cos(x) - (a^2 - b^2)*cos(x)^2 + a^2 + c^2 + 2*(a*b*cos(x) + a*c)
*sin(x)) - (a^2*b + b^3 - b*c^2 - c^3 + (a^2 + b^2)*c)*log(1/2*cos(x) + 1/
2) - (a^2*b + b^3 - b*c^2 + c^3 - (a^2 + b^2)*c)*log(-1/2*cos(x) + 1/2))/(
a^2*b^2 + b^4 + c^4 - (a^2 + 2*b^2)*c^2), 1/2*(2*sqrt(-a^2 - b^2 + c^2)*a*
c*arctan((b*c*cos(x) + a*c*sin(x) + a^2 + b^2)*sqrt(-a^2 - b^2 + c^2)/((a^
3 + a*b^2 - a*c^2)*cos(x) - (a^2*b + b^3 - b*c^2)*sin(x))) - (a^2*b + b^3
- b*c^2)*log(2*b*c*cos(x) - (a^2 - b^2)*cos(x)^2 + a^2 + c^2 + 2*(a*b*cos(
x) + a*c)*sin(x)) + (a^2*b + b^3 - b*c^2 - c^3 + (a^2 + b^2)*c)*log(1/2*co
s(x) + 1/2) + (a^2*b + b^3 - b*c^2 + c^3 - (a^2 + b^2)*c)*log(-1/2*cos(x)
+ 1/2))/(a^2*b^2 + b^4 + c^4 - (a^2 + 2*b^2)*c^2)]
```

3.460.6 Sympy [F]

$$\int \frac{\csc^2(x)}{a + b\cot(x) + c\csc(x)}\,dx = \int \frac{\csc^2{(x)}}{a + b\cot{(x)} + c\csc{(x)}}\,dx$$

input
```
integrate(csc(x)**2/(a+b*cot(x)+c*csc(x)),x)
```

output
```
Integral(csc(x)**2/(a + b*cot(x) + c*csc(x)), x)
```

3.460.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\csc^2(x)}{a + b\cot(x) + c\csc(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(csc(x)^2/(a+b*cot(x)+c*csc(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c^2-b^2-a^2>0)', see `assume?` f
or more de
```

3.460.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.18

$$\int \frac{\csc^2(x)}{a+b\cot(x)+c\csc(x)}\,dx = \frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor \operatorname{sgn}(-2\,b+2\,c)+\arctan\left(-\frac{b\tan\left(\frac{1}{2}\,x\right)-c\tan\left(\frac{1}{2}\,x\right)-a}{\sqrt{-a^2-b^2+c^2}}\right)\right)ac}{\sqrt{-a^2-b^2+c^2}(b^2-c^2)} - \frac{b\log\left(-b\tan\left(\frac{1}{2}\,x\right)^2+c\tan\left(\frac{1}{2}\,x\right)^2+2\,a\tan\left(\frac{1}{2}\,x\right)+b+c\right)}{b^2-c^2} + \frac{\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)}{b+c}$$

input
```
integrate(csc(x)^2/(a+b*cot(x)+c*csc(x)),x, algorithm="giac")
```

output
```
2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*b + 2*c) + arctan(-(b*tan(1/2*x) - c*ta
n(1/2*x) - a)/sqrt(-a^2 - b^2 + c^2)))*a*c/(sqrt(-a^2 - b^2 + c^2)*(b^2 -
c^2)) - b*log(-b*tan(1/2*x)^2 + c*tan(1/2*x)^2 + 2*a*tan(1/2*x) + b + c)/(
b^2 - c^2) + log(abs(tan(1/2*x)))/(b + c)
```

3.460.9 Mupad [B] (verification not implemented)

Time = 35.16 (sec) , antiderivative size = 531, normalized size of antiderivative = 4.42

$$\int \frac{\csc^2(x)}{a+b\cot(x)+c\csc(x)}\,dx = \frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{b+c} - \frac{\ln\left(2\,a-2\,b\tan\left(\frac{x}{2}\right)-\frac{\left(\tan\left(\frac{x}{2}\right)\left(-8\,a^2-8\,b^2+6\,b\,c+2\,c^2\right)-4\,a\,c+\frac{2\,(b-c)\left(a^2\,b-b\,c^2+b^3+a\,c\,\sqrt{a^2+b^2-c^2}\right)\left(4\tan\left(\frac{x}{2}\right)a^2+a\,b+a\,c+3\tan\left(\frac{x}{2}\right.\cdots}{\left(b^2-c^2\right)\left(a^2+b^2-c^2\right)}\cdots}{(b^2-c^2)\left(a^2+b^2-c^2\right)}\cdots\right)}{\left(b^2-c^2\right)\left(a^2+b^2-c^2\right)\cdots} - \frac{\ln\left(2\,a-2\,b\tan\left(\frac{x}{2}\right)-\frac{\left(\tan\left(\frac{x}{2}\right)\left(-8\,a^2-8\,b^2+6\,b\,c+2\,c^2\right)-4\,a\,c+\frac{2\,(b-c)\left(a^2\,b-b\,c^2+b^3-a\,c\,\sqrt{a^2+b^2-c^2}\right)\left(4\tan\left(\frac{x}{2}\right)a^2+a\,b+a\,c+3\tan\left(\frac{x}{2}\right.\cdots}{\left(b^2-c^2\right)\left(a^2+b^2-c^2\right)}\cdots}{(b^2-c^2)\left(a^2+b^2-c^2\right)}\cdots\right)}{\left(b^2-c^2\right)\left(a^2+b^2-c^2\right)\cdots}$$

input
```
int(1/(sin(x)^2*(a + c/sin(x) + b*cot(x))),x)
```

output
```
log(tan(x/2))/(b + c) - (log(2*a - 2*b*tan(x/2) - ((tan(x/2)*(6*b*c - 8*a^
2 - 8*b^2 + 2*c^2) - 4*a*c + (2*(b - c)*(a^2*b - b*c^2 + b^3 + a*c*(a^2 +
b^2 - c^2)^(1/2))*(a*b + a*c + 4*a^2*tan(x/2) + 3*b^2*tan(x/2) - 3*c^2*tan
(x/2)))/((b^2 - c^2)*(a^2 + b^2 - c^2)))*(a^2*b - b*c^2 + b^3 + a*c*(a^2 +
 b^2 - c^2)^(1/2)))/((b^2 - c^2)*(a^2 + b^2 - c^2)))*(b*(a^2 - c^2) + b^3
+ a*c*(a^2 + b^2 - c^2)^(1/2)))/((b^2 - c^2)*(a^2 + b^2 - c^2)) - (log(2*a
 - 2*b*tan(x/2) - ((tan(x/2)*(6*b*c - 8*a^2 - 8*b^2 + 2*c^2) - 4*a*c + (2*
(b - c)*(a^2*b - b*c^2 + b^3 - a*c*(a^2 + b^2 - c^2)^(1/2))*(a*b + a*c + 4
*a^2*tan(x/2) + 3*b^2*tan(x/2) - 3*c^2*tan(x/2)))/((b^2 - c^2)*(a^2 + b^2
- c^2)))*(a^2*b - b*c^2 + b^3 - a*c*(a^2 + b^2 - c^2)^(1/2)))/((b^2 - c^2)
*(a^2 + b^2 - c^2)))*(b*(a^2 - c^2) + b^3 - a*c*(a^2 + b^2 - c^2)^(1/2)))/
((b^2 - c^2)*(a^2 + b^2 - c^2))
```

3.460. $\int \frac{\csc^2(x)}{a+b\cot(x)+c\csc(x)}\,dx$

3.461 $\int \frac{\csc(x)}{2+2\cot(x)+3\csc(x)}\,dx$

3.461.1 Optimal result

Integrand size = 15, antiderivative size = 21

$$\int \frac{\csc(x)}{2+2\cot(x)+3\csc(x)}\,dx = x+2\arctan\left(\frac{\cos(x)-\sin(x)}{2+\cos(x)+\sin(x)}\right)$$

output
```
x+2*arctan((cos(x)-sin(x))/(2+cos(x)+sin(x)))
```

3.461.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 51 vs. 2(21) = 42.

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.43

$$\int \frac{\csc(x)}{2+2\cot(x)+3\csc(x)}\,dx = -\arctan\left(\frac{\cos\left(\frac{x}{2}\right)}{2\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}\right) + \arctan\left(\sec\left(\frac{x}{2}\right)\left(2\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\right)$$

input
```
Integrate[Csc[x]/(2 + 2*Cot[x] + 3*Csc[x]),x]
```

output
```
-ArcTan[Cos[x/2]/(2*Cos[x/2] + Sin[x/2])] + ArcTan[Sec[x/2]*(2*Cos[x/2] +
Sin[x/2])]
```

3.461.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3645, 3042, 3603, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(x)}{2\cot(x)+3\csc(x)+2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\csc(x)}{2\cot(x)+3\csc(x)+2}dx$$
$$\Big\downarrow \text{3645}$$
$$\int \frac{1}{2\sin(x)+2\cos(x)+3}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{2\sin(x)+2\cos(x)+3}dx$$
$$\Big\downarrow \text{3603}$$
$$2\int \frac{1}{\tan^2\left(\frac{x}{2}\right)+4\tan\left(\frac{x}{2}\right)+5}d\tan\left(\frac{x}{2}\right)$$
$$\Big\downarrow \text{1083}$$
$$-4\int \frac{1}{-\left(2\tan\left(\frac{x}{2}\right)+4\right)^2-4}d\left(2\tan\left(\frac{x}{2}\right)+4\right)$$
$$\Big\downarrow \text{217}$$
$$2\arctan\left(\frac{1}{2}\left(2\tan\left(\frac{x}{2}\right)+4\right)\right)$$

input
```
Int[Csc[x]/(2 + 2*Cot[x] + 3*Csc[x]),x]
```

output
```
2*ArcTan[(4 + 2*Tan[x/2])/2]
```

3.461.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

rule 3645
```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Int[1/(b + a*Sin[d + e*x]
 + c*Cos[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && I
ntegerQ[n]
```

3.461.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.48

method	result	size
default	$2 \arctan \left(2+\tan \left(\frac{x}{2}\right)\right)$	10
risch	$-i \ln \left({\mathrm e}^{i x}+\frac{1}{2}+\frac{i}{2}\right)+i \ln \left({\mathrm e}^{i x}+1+i\right)$	28

input
```
int(csc(x)/(2+2*cot(x)+3*csc(x)),x,method=_RETURNVERBOSE)
```

output
```
2*arctan(2+tan(1/2*x))
```

3.461.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int \frac{\csc(x)}{2+2\cot(x)+3\csc(x)}\,dx = -\arctan\left(-\frac{3\,\cos(x)+3\,\sin(x)+4}{\cos(x)-\sin(x)}\right)$$

input
```
integrate(csc(x)/(2+2*cot(x)+3*csc(x)),x, algorithm="fricas")
```

output
```
-arctan(-(3*cos(x) + 3*sin(x) + 4)/(cos(x) - sin(x)))
```

3.461.6 Sympy [F]

$$\int \frac{\csc(x)}{2+2\cot(x)+3\csc(x)}\,dx = \int \frac{\csc(x)}{2\cot(x)+3\csc(x)+2}\,dx$$

input
```
integrate(csc(x)/(2+2*cot(x)+3*csc(x)),x)
```

output
```
Integral(csc(x)/(2*cot(x) + 3*csc(x) + 2), x)
```

3.461.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

$$\int \frac{\csc(x)}{2+2\cot(x)+3\csc(x)}\,dx = 2\,\arctan\left(\frac{\sin(x)}{\cos(x)+1}+2\right)$$

input
```
integrate(csc(x)/(2+2*cot(x)+3*csc(x)),x, algorithm="maxima")
```

output
```
2*arctan(sin(x)/(cos(x) + 1) + 2)
```

3.461.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{\csc(x)}{2+2\cot(x)+3\csc(x)}\,dx = 2\,\pi\left\lfloor\frac{x}{2\,\pi}+\frac{1}{2}\right\rfloor + 2\,\arctan\left(\tan\left(\frac{1}{2}\,x\right)+2\right)$$

input
```
integrate(csc(x)/(2+2*cot(x)+3*csc(x)),x, algorithm="giac")
```

output
```
2*pi*floor(1/2*x/pi + 1/2) + 2*arctan(tan(1/2*x) + 2)
```

3.461.9 Mupad [B] (verification not implemented)

Time = 27.33 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.43

$$\int \frac{\csc(x)}{2+2\cot(x)+3\csc(x)}\,dx = 2\,\mathrm{atan}\left(\tan\left(\frac{x}{2}\right)+2\right)$$

input
```
int(1/(sin(x)*(2*cot(x) + 3/sin(x) + 2)),x)
```

output
```
2*atan(tan(x/2) + 2)
```

3.462 $\int \frac{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}}{\csc^{\frac{3}{2}}(d+ex)}\,dx$

3.462.1 Optimal result

Integrand size = 33, antiderivative size = 371

$$\int \frac{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}}{\csc^{\frac{3}{2}}(d+ex)}\,dx = \frac{8b(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}E\left(\frac{1}{2}(d+ex-\tan^{-1}(\right.}{3e\csc^{\frac{3}{2}}(d+ex)(b+c\cos(d+ex)+a\sin(d+ex))\sqrt{\frac{b+c\cos(}{}}}$$
$$+\frac{2(a^2-b^2+c^2)\,(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\,\text{EllipticF}\left(\frac{1}{2}(d+ex-\tan^{-1}(c,a)),\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)\sqrt{\frac{b+c}{}}}{3e\csc^{\frac{3}{2}}(d+ex)(b+c\cos(d+ex)+a\sin(d+ex))^2}$$
$$-\frac{2(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}(a\cos(d+ex)-c\sin(d+ex))}{3e\csc^{\frac{3}{2}}(d+ex)(b+c\cos(d+ex)+a\sin(d+ex))}$$

output
```
-2/3*(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)*(a*cos(e*x+d)-c*sin(e*x+d))/e/csc
(e*x+d)^(3/2)/(b+c*cos(e*x+d)+a*sin(e*x+d))+8/3*b*(a+c*cot(e*x+d)+b*csc(e*
x+d))^(3/2)*(cos(1/2*d+1/2*e*x-1/2*arctan(c,a))^2)^(1/2)/cos(1/2*d+1/2*e*x
-1/2*arctan(c,a))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(c,a)),2^(1/2)*((a
^2+c^2)^(1/2)/(b+(a^2+c^2)^(1/2)))^(1/2))/e/csc(e*x+d)^(3/2)/(b+c*cos(e*x+
d)+a*sin(e*x+d))/((b+c*cos(e*x+d)+a*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(1/2)
+2/3*(a^2-b^2+c^2)*(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)*(cos(1/2*d+1/2*e*x-
1/2*arctan(c,a))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(c,a))*EllipticF(sin
(1/2*d+1/2*e*x-1/2*arctan(c,a)),2^(1/2)*((a^2+c^2)^(1/2)/(b+(a^2+c^2)^(1/2
)))^(1/2))*((b+c*cos(e*x+d)+a*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(1/2)/e/csc
(e*x+d)^(3/2)/(b+c*cos(e*x+d)+a*sin(e*x+d))^2
```

3.462.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 6.81 (sec) , antiderivative size = 2490, normalized size of antiderivative = 6.71

$$\int \frac{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}}{\csc^{\frac{3}{2}}(d+ex)}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)/Csc[d + e*x]^(3/2),x
]
```

output
```
((a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*((8*b*c)/(3*a) - (2*a*Cos[d +
 e*x])/3 + (2*c*Sin[d + e*x])/3))/(e*Csc[d + e*x]^(3/2)*(b + c*Cos[d + e*x
] + a*Sin[d + e*x])) + (4*a*b*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*
(-((a*AppellF1[-1/2, -1/2, -1/2, 1/2, -((b + Sqrt[1 + a^2/c^2]*c*Cos[d + e
*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(1 - b/(Sqrt[1 + a^2/c^2]*c))*c)), -
((b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(
-1 - b/(Sqrt[1 + a^2/c^2]*c))*c))]*Sin[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a
^2/c^2]*c*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Cos[d +
e*x - ArcTan[a/c]])/(b + c*Sqrt[(a^2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a^2 +
c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] + c*Sq
rt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]])/(-b + c*Sqrt[(a^2 + c^2)/c
^2])])) - ((2*c*(b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]]))/(a^2
 + c^2) - (a*Sin[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*c))/Sqrt[b + S
qrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]]]))/(3*e*Csc[d + e*x]^(3/2)*(
b + c*Cos[d + e*x] + a*Sin[d + e*x])^(3/2)) + (4*b*c^2*(a + c*Cot[d + e*x]
 + b*Csc[d + e*x])^(3/2)*(-((a*AppellF1[-1/2, -1/2, -1/2, 1/2, -((b + Sqrt
[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(1 - b/(Sqr
t[1 + a^2/c^2]*c))*c)), -((b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/
c]])/(Sqrt[1 + a^2/c^2]*(-1 - b/(Sqrt[1 + a^2/c^2]*c))*c))]*Sin[d + e*x -
ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*c*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] - c*Sq...
```

3.462.3 Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.91, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.424$, Rules used = {3042, 3647, 3042, 3599, 27, 3042, 3628, 3042, 3598, 3042, 3132, 3606, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

3.462. $\int \frac{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}}{\csc^{\frac{3}{2}}(d+ex)}\,dx$

$$\int \frac{(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}}{\csc^{\frac{3}{2}}(d+ex)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}}{\csc(d+ex)^{3/2}}dx$$

$$\Big\downarrow \text{ 3647}$$

$$\frac{(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}\int(b+c\cos(d+ex)+a\sin(d+ex))^{3/2}dx}{\csc^{\frac{3}{2}}(d+ex)(a\sin(d+ex)+b+c\cos(d+ex))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}\int(b+c\cos(d+ex)+a\sin(d+ex))^{3/2}dx}{\csc^{\frac{3}{2}}(d+ex)(a\sin(d+ex)+b+c\cos(d+ex))^{3/2}}$$

$$\Big\downarrow \text{ 3599}$$

$$\frac{(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}\left(\frac{2}{3}\int\frac{a^2+4b\sin(d+ex)a+3b^2+c^2+4bc\cos(d+ex)}{2\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}}dx-\frac{2(a\cos(d+ex)-c\sin(d+ex))\sqrt{a\sin(d+}}{3e}\right.}{\csc^{\frac{3}{2}}(d+ex)(a\sin(d+ex)+b+c\cos(d+ex))^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}\left(\frac{1}{3}\int\frac{a^2+4b\sin(d+ex)a+3b^2+c^2+4bc\cos(d+ex)}{\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}}dx-\frac{2(a\cos(d+ex)-c\sin(d+ex))\sqrt{a\sin(d+}}{3e}\right.}{\csc^{\frac{3}{2}}(d+ex)(a\sin(d+ex)+b+c\cos(d+ex))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}\left(\frac{1}{3}\int\frac{a^2+4b\sin(d+ex)a+3b^2+c^2+4bc\cos(d+ex)}{\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}}dx-\frac{2(a\cos(d+ex)-c\sin(d+ex))\sqrt{a\sin(d+}}{3e}\right.}{\csc^{\frac{3}{2}}(d+ex)(a\sin(d+ex)+b+c\cos(d+ex))^{3/2}}$$

$$\Big\downarrow \text{ 3628}$$

$$\frac{(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}\left(\frac{1}{3}\left((a^2-b^2+c^2)\int\frac{1}{\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}}dx+4b\int\sqrt{b+c\cos(d+ex)-}\right.\right.}{\csc^{\frac{3}{2}}(d+ex)(a\sin(d+ex)+b+c\cos(d+ex))}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}\left(\frac{1}{3}\left((a^2-b^2+c^2)\int\frac{1}{\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}}dx+4b\int\sqrt{b+c\cos(d+ex)-}\right.\right.}{\csc^{\frac{3}{2}}(d+ex)(a\sin(d+ex)+b+c\cos(d+ex))}$$

3.462. $\int \frac{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}}{\csc^{\frac{3}{2}}(d+ex)}dx$

$\downarrow$ 3598

$$\frac{(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}\left(\frac{1}{3}\left(\left(a^2-b^2+c^2\right)\int\frac{1}{\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}}dx+\frac{4b\sqrt{a\sin(d+ex)+b+c\cos(d+e}}{\sqrt{}}\right.\right.}{\csc^{\frac{3}{2}}(d+ex)(a\sin(d+ex)+b+c}$$

$\downarrow$ 3042

$$\frac{(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}\left(\frac{1}{3}\left(\left(a^2-b^2+c^2\right)\int\frac{1}{\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}}dx+\frac{4b\sqrt{a\sin(d+ex)+b+c\cos(d+e}}{\sqrt{}}\right.\right.}{\csc^{\frac{3}{2}}(d+ex)(a\sin(d+ex)+b+}$$

$\downarrow$ 3132

$$\frac{(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}\left(\frac{1}{3}\left(\left(a^2-b^2+c^2\right)\int\frac{1}{\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}}dx+\frac{8b\sqrt{a\sin(d+ex)+b+c\cos(d+e}}{e\sqrt{\frac{a\sin(}{}}}\right.\right.}{\csc^{\frac{3}{2}}(d+ex)(a\sin(d+ex)+b+c\cos(}$$

$\downarrow$ 3606

$$\frac{(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}\left(\frac{1}{3}\left(\frac{\left(a^2-b^2+c^2\right)\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}\int\frac{1}{\sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+\frac{\sqrt{a^2+c^2}\cos\left(d+ex-\tan^{-1}(c,a)\right)}{b+\sqrt{a^2+c^2}}}}dx}{\sqrt{a\sin(d+ex)+b+c\cos(d+ex)}}\right.\right.}{\csc^{\frac{3}{2}}(d+ex)(a\sin(d+ex}$$

$\downarrow$ 3042

$$\frac{(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}\left(\frac{1}{3}\left(\frac{\left(a^2-b^2+c^2\right)\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}\int\frac{1}{\sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+\frac{\sqrt{a^2+c^2}\sin\left(d+ex-\tan^{-1}(c,a)+\frac{\pi}{2}\right)}{b+\sqrt{a^2+c^2}}}}}{\sqrt{a\sin(d+ex)+b+c\cos(d+ex)}}\right.\right.}{\csc^{\frac{3}{2}}(d+e}$$

$\downarrow$ 3140

3.462. $\int\frac{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}}{\csc^{\frac{3}{2}}(d+ex)}\,dx$

$$\frac{(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}\left(\frac{1}{3}\left(\frac{2(a^2-b^2+c^2)\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}\,\text{EllipticF}\left(\frac{1}{2}(d+ex-\tan^{-1}(c,a)),\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{a\sin(d+ex)+b+c\cos(d+ex)}}\right.\right.}{\csc^{\frac{3}{2}}(d+ex)(a\sin(d+ex)}$$

input
```
Int[(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)/Csc[d + e*x]^(3/2),x]
```

output
```
((a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*((-2*Sqrt[b + c*Cos[d + e*x]
+ a*Sin[d + e*x]]*(a*Cos[d + e*x] - c*Sin[d + e*x]))/(3*e) + ((8*b*Ellipti
cE[(d + e*x - ArcTan[c, a])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2 + c^2])]*
Sqrt[b + c*Cos[d + e*x] + a*Sin[d + e*x]])/(e*Sqrt[(b + c*Cos[d + e*x] + a
*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])]) + (2*(a^2 - b^2 + c^2)*EllipticF[(d
 + e*x - ArcTan[c, a])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2 + c^2])]*Sqrt[
(b + c*Cos[d + e*x] + a*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])])/(e*Sqrt[b +
c*Cos[d + e*x] + a*Sin[d + e*x]]))/3))/(Csc[d + e*x]^(3/2)*(b + c*Cos[d +
e*x] + a*Sin[d + e*x])^(3/2))
```

3.462.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3598
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]   Int[Sqrt[a/(a + S
qrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - Arc
Tan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

rule 3599
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Simp[1/n   Int[Simp[n*a^2 + (
n - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x
], x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

rule 3606
```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Simp[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]   Int[1/Sqrt[a/(a
 + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x -
 ArcTan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2
, 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

rule 3628
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]
, x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[
A*b - a*B, 0]
```

rule 3647
```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Simp[Csc[d + e*x]^n*((b +
 a*Sin[d + e*x] + c*Cos[d + e*x])^n/(a + b*Csc[d + e*x] + c*Cot[d + e*x])^n
)   Int[1/(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n, x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[m + n, 0] &&  !IntegerQ[n]
```

3.462. $\int \frac{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}}{\csc^{\frac{3}{2}}(d+ex)}\,dx$

3.462.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 29.78 (sec) , antiderivative size = 21493, normalized size of antiderivative = 57.93

method	result	size
default	Expression too large to display	21493

input
```
int((a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)/csc(e*x+d)^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
result too large to display
```

3.462.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 1511, normalized size of antiderivative = 4.07

$$\int \frac{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}}{\csc^{\frac{3}{2}}(d+ex)}\,dx = \text{Too large to display}$$

input
```
integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)/csc(e*x+d)^(3/2),x, algorith
m="fricas")
```

output
```
1/9*((3*I*a^3 + I*a*b^2 + 3*I*a*c^2 - 3*c^3 - (3*a^2 + b^2)*c)*sqrt(-2*I*a
 - 2*c)*weierstrassPInverse(4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 + 6*I*a*c^3
 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), -8/27*(-9*I*a
^5*b + 8*I*a^3*b^3 + 27*I*a*b*c^4 - 9*b*c^5 + 2*(9*a^2*b + 4*b^3)*c^3 + 6*
I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 +
3*a^2*c^4 + c^6), 1/3*(-2*I*a*b + 2*b*c + 3*(a^2 + c^2)*cos(e*x + d) - 3*(
I*a^2 + I*c^2)*sin(e*x + d))/(a^2 + c^2)) + (-3*I*a^3 - I*a*b^2 - 3*I*a*c^
2 - 3*c^3 - (3*a^2 + b^2)*c)*sqrt(2*I*a - 2*c)*weierstrassPInverse(4/3*(3*
a^4 - 4*a^2*b^2 + 4*b^2*c^2 - 6*I*a*c^3 - 3*c^4 - 2*I*(3*a^3 - 4*a*b^2)*c)
/(a^4 + 2*a^2*c^2 + c^4), -8/27*(9*I*a^5*b - 8*I*a^3*b^3 - 27*I*a*b*c^4 -
9*b*c^5 + 2*(9*a^2*b + 4*b^3)*c^3 - 6*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4
*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(2*I*a*b + 2*b
*c + 3*(a^2 + c^2)*cos(e*x + d) - 3*(-I*a^2 - I*c^2)*sin(e*x + d))/(a^2 +
c^2)) + 12*(a^2*b + b*c^2)*sqrt(-2*I*a - 2*c)*weierstrassZeta(4/3*(3*a^4 -
 4*a^2*b^2 + 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4
 + 2*a^2*c^2 + c^4), -8/27*(-9*I*a^5*b + 8*I*a^3*b^3 + 27*I*a*b*c^4 - 9*b*
c^5 + 2*(9*a^2*b + 4*b^3)*c^3 + 6*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b -
 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), weierstrassPInverse(4/
3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^
2)*c)/(a^4 + 2*a^2*c^2 + c^4), -8/27*(-9*I*a^5*b + 8*I*a^3*b^3 + 27*I*a...
```

3.462.6 Sympy [F]

$$\int \frac{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}}{\csc^{\frac{3}{2}}(d+ex)}\,dx=\int \frac{(a+b\csc{(d+ex)}+c\cot{(d+ex)})^{\frac{3}{2}}}{\csc^{\frac{3}{2}}{(d+ex)}}\,dx$$

input
```
integrate((a+c*cot(e*x+d)+b*csc(e*x+d))**(3/2)/csc(e*x+d)**(3/2),x)
```

output
```
Integral((a + b*csc(d + e*x) + c*cot(d + e*x))**(3/2)/csc(d + e*x)**(3/2),
 x)
```

3.462.7 Maxima [F]

$$\int \frac{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}}{\csc^{\frac{3}{2}}(d+ex)}\,dx=\int \frac{(c\cot(ex+d)+b\csc(ex+d)+a)^{\frac{3}{2}}}{\csc(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)/csc(e*x+d)^(3/2),x, algorith
m="maxima")
```

output
```
integrate((c*cot(e*x + d) + b*csc(e*x + d) + a)^(3/2)/csc(e*x + d)^(3/2),
x)
```

3.462.8 Giac [F]

$$\int \frac{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}}{\csc^{\frac{3}{2}}(d+ex)}\,dx=\int \frac{(c\cot(ex+d)+b\csc(ex+d)+a)^{\frac{3}{2}}}{\csc(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)/csc(e*x+d)^(3/2),x, algorith
m="giac")
```

output
```
integrate((c*cot(e*x + d) + b*csc(e*x + d) + a)^(3/2)/csc(e*x + d)^(3/2),
x)
```

3.462.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}}{\csc^{\frac{3}{2}}(d+ex)}\,dx=\int \frac{\left(a+c\cot(d+e\,x)+\frac{b}{\sin(d+e\,x)}\right)^{3/2}}{\left(\frac{1}{\sin(d+e\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + c*cot(d + e*x) + b/sin(d + e*x))^(3/2)/(1/sin(d + e*x))^(3/2),x)
```

output
```
int((a + c*cot(d + e*x) + b/sin(d + e*x))^(3/2)/(1/sin(d + e*x))^(3/2), x)
```

3.463 $\int \frac{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}}{\sqrt{\csc(d+ex)}}\,dx$

3.463.1 Optimal result

Integrand size = 33, antiderivative size = 118

$$\int \frac{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}}{\sqrt{\csc(d+ex)}}\,dx = \frac{2\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\,E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right)\middle|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{\csc(d+ex)}\sqrt{\frac{b+c\cos(d+ex)+a\sin(d+ex)}{b+\sqrt{a^2+c^2}}}}$$

output
```
2*(cos(1/2*d+1/2*e*x-1/2*arctan(c,a))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arcta
n(c,a))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(c,a)),2^(1/2)*((a^2+c^2)^(1
/2)/(b+(a^2+c^2)^(1/2)))^(1/2))*(a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)/e/csc(
e*x+d)^(1/2)/((b+c*cos(e*x+d)+a*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(1/2)
```

3.463.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 6.68 (sec) , antiderivative size = 1580, normalized size of antiderivative = 13.39

$$\int \frac{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}}{\sqrt{\csc(d+ex)}}\,dx = \text{Too large to display}$$

input
```
Integrate[Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]/Sqrt[Csc[d + e*x]],x]
```

output
```
(2*c*Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]])/(a*e*Sqrt[Csc[d + e*x]]) +
 (a*Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]*(-((a*AppellF1[-1/2, -1/2, -
1/2, 1/2, -((b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]])/(Sqrt[1 +
 a^2/c^2]*(1 - b/(Sqrt[1 + a^2/c^2]*c))*c)), -((b + Sqrt[1 + a^2/c^2]*c*Co
s[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b/(Sqrt[1 + a^2/c^2]*c)
)*c))]*Sin[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*c*Sqrt[(c*Sqrt[(a^2
+ c^2)/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]])/(b + c*S
qrt[(a^2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcT
an[a/c]]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] + c*Sqrt[(a^2 + c^2)/c^2]*Cos[d +
e*x - ArcTan[a/c]])/(-b + c*Sqrt[(a^2 + c^2)/c^2])])) - ((2*c*(b + Sqrt[1
+ a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]]))/(a^2 + c^2) - (a*Sin[d + e*x - A
rcTan[a/c]])/(Sqrt[1 + a^2/c^2]*c))/Sqrt[b + Sqrt[1 + a^2/c^2]*c*Cos[d + e
*x - ArcTan[a/c]]]))/(e*Sqrt[Csc[d + e*x]]*Sqrt[b + c*Cos[d + e*x] + a*Sin
[d + e*x]]) + (c^2*Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]*(-((a*AppellF
1[-1/2, -1/2, -1/2, 1/2, -((b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a
/c]])/(Sqrt[1 + a^2/c^2]*(1 - b/(Sqrt[1 + a^2/c^2]*c))*c)), -((b + Sqrt[1
+ a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b/(Sqrt[
1 + a^2/c^2]*c))*c))]*Sin[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*c*Sqr
t[(c*Sqrt[(a^2 + c^2)/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[
a/c]])/(b + c*Sqrt[(a^2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]*...
```

3.463.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3042, 3647, 3042, 3598, 3042, 3132}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+b\csc(d+ex)+c\cot(d+ex)}}{\sqrt{\csc(d+ex)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+b\csc(d+ex)+c\cot(d+ex)}}{\sqrt{\csc(d+ex)}}dx$$

$$\Big\downarrow \text{3647}$$

$$\frac{\sqrt{a+b\csc(d+ex)+c\cot(d+ex)}\int\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}dx}{\sqrt{\csc(d+ex)}\sqrt{a\sin(d+ex)+b+c\cos(d+ex)}}$$

$$\downarrow 3042$$

$$\frac{\sqrt{a+b\csc(d+ex)+c\cot(d+ex)}\int\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}dx}{\sqrt{\csc(d+ex)}\sqrt{a\sin(d+ex)+b+c\cos(d+ex)}}$$

$$\downarrow 3598$$

$$\frac{\sqrt{a+b\csc(d+ex)+c\cot(d+ex)}\int\sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+\frac{\sqrt{a^2+c^2}\cos(d+ex-\tan^{-1}(c,a))}{b+\sqrt{a^2+c^2}}}dx}{\sqrt{\csc(d+ex)}\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}}$$

$$\downarrow 3042$$

$$\frac{\sqrt{a+b\csc(d+ex)+c\cot(d+ex)}\int\sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+\frac{\sqrt{a^2+c^2}\sin\left(d+ex-\tan^{-1}(c,a)+\frac{\pi}{2}\right)}{b+\sqrt{a^2+c^2}}}dx}{\sqrt{\csc(d+ex)}\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}}$$

$$\downarrow 3132$$

$$\frac{2\sqrt{a+b\csc(d+ex)+c\cot(d+ex)}E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{\csc(d+ex)}\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}}$$

input
```
Int[Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]/Sqrt[Csc[d + e*x]],x]
```

output
```
(2*Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]*EllipticE[(d + e*x - ArcTan[c
, a])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2 + c^2])])/(e*Sqrt[Csc[d + e*x]]
*Sqrt[(b + c*Cos[d + e*x] + a*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])])
```

3.463.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

3.463. $\int \frac{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}}{\sqrt{\csc(d+ex)}}\,dx$

rule 3598
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]   Int[Sqrt[a/(a + S
qrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - Arc
Tan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

rule 3647
```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Simp[Csc[d + e*x]^n*((b +
 a*Sin[d + e*x] + c*Cos[d + e*x])^n/(a + b*Csc[d + e*x] + c*Cot[d + e*x])^n
)   Int[1/(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n, x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[m + n, 0] &&  !IntegerQ[n]
```

3.463.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 28.20 (sec) , antiderivative size = 1862, normalized size of antiderivative = 15.78

method	result	size
risch	Expression too large to display	1862
default	Expression too large to display	12826

input
```
int((a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)/csc(e*x+d)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
(c*exp(I*(e*x+d))^2+2*b*exp(I*(e*x+d))-I*a*exp(I*(e*x+d))^2+c+I*a)/e*2^(1/
2)*((I*exp(I*(e*x+d))^2*c+2*I*b*exp(I*(e*x+d))+a*exp(I*(e*x+d))^2+I*c-a)/(
exp(I*(e*x+d))^2-1))^(1/2)/(I*exp(I*(e*x+d))^2*c+2*I*b*exp(I*(e*x+d))+a*ex
p(I*(e*x+d))^2+I*c-a)/(I*exp(I*(e*x+d))/(exp(I*(e*x+d))^2-1))^(1/2)-I/e*(2
*I*b*(-b+(-a^2+b^2-c^2)^(1/2))/(I*a-c)*((exp(I*(e*x+d))+(-b+(-a^2+b^2-c^2)
^(1/2))/(I*a-c))/(-b+(-a^2+b^2-c^2)^(1/2))*(I*a-c))^(1/2)*((exp(I*(e*x+d))
-(b+(-a^2+b^2-c^2)^(1/2))/(I*a-c))/(-(-b+(-a^2+b^2-c^2)^(1/2))/(I*a-c)-(b+
(-a^2+b^2-c^2)^(1/2))/(I*a-c)))^(1/2)*(-exp(I*(e*x+d))/(-b+(-a^2+b^2-c^2)^
(1/2))*(I*a-c))^(1/2)/(-exp(I*(e*x+d))^3*c-2*b*exp(I*(e*x+d))^2+I*exp(I*(e
*x+d))^3*a-c*exp(I*(e*x+d))-I*a*exp(I*(e*x+d)))^(1/2)*EllipticF(((exp(I*(e
*x+d))+(-b+(-a^2+b^2-c^2)^(1/2))/(I*a-c))/(-b+(-a^2+b^2-c^2)^(1/2))*(I*a-c
))^(1/2),(-(-b+(-a^2+b^2-c^2)^(1/2))/(I*a-c)/(-(-b+(-a^2+b^2-c^2)^(1/2))/(
I*a-c)-(b+(-a^2+b^2-c^2)^(1/2))/(I*a-c)))^(1/2))+(I*c-a)*(2*(I*a*exp(I*(e*
x+d))^2-c*exp(I*(e*x+d))^2-I*a-2*b*exp(I*(e*x+d))-c)/(I*a+c)/(exp(I*(e*x+d
))*(I*a*exp(I*(e*x+d))^2-c*exp(I*(e*x+d))^2-I*a-2*b*exp(I*(e*x+d))-c))^(1/
2)+2*(1/(I*a+c)*(I*a-c)-(2*I*a-2*c)/(I*a+c))*(-b+(-a^2+b^2-c^2)^(1/2))/(I*
a-c)*((exp(I*(e*x+d))+(-b+(-a^2+b^2-c^2)^(1/2))/(I*a-c))/(-b+(-a^2+b^2-c^2
)^(1/2))*(I*a-c))^(1/2)*((exp(I*(e*x+d))-(b+(-a^2+b^2-c^2)^(1/2))/(I*a-c))
/(-(-b+(-a^2+b^2-c^2)^(1/2))/(I*a-c)-(b+(-a^2+b^2-c^2)^(1/2))/(I*a-c)))^(1
/2)*(-exp(I*(e*x+d))/(-b+(-a^2+b^2-c^2)^(1/2))*(I*a-c))^(1/2)/(-exp(I*(...
```

3.463.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 1361, normalized size of antiderivative = 11.53

$$\int \frac{\sqrt{a + c\cot(d+ex) + b\csc(d+ex)}}{\sqrt{\csc(d+ex)}}\,dx = \text{Too large to display}$$

input
```
integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)/csc(e*x+d)^(1/2),x, algorith
m="fricas")
```

output
```
1/3*((I*a*b - b*c)*sqrt(-2*I*a - 2*c)*weierstrassPInverse(4/3*(3*a^4 - 4*a
^2*b^2 + 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2
*a^2*c^2 + c^4), -8/27*(-9*I*a^5*b + 8*I*a^3*b^3 + 27*I*a*b*c^4 - 9*b*c^5
+ 2*(9*a^2*b + 4*b^3)*c^3 + 6*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*a
^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(-2*I*a*b + 2*b*c + 3*
(a^2 + c^2)*cos(e*x + d) - 3*(I*a^2 + I*c^2)*sin(e*x + d))/(a^2 + c^2)) +
(-I*a*b - b*c)*sqrt(2*I*a - 2*c)*weierstrassPInverse(4/3*(3*a^4 - 4*a^2*b^
2 + 4*b^2*c^2 - 6*I*a*c^3 - 3*c^4 - 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*
c^2 + c^4), -8/27*(9*I*a^5*b - 8*I*a^3*b^3 - 27*I*a*b*c^4 - 9*b*c^5 + 2*(9
*a^2*b + 4*b^3)*c^3 - 6*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*a^2*b^3
)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(2*I*a*b + 2*b*c + 3*(a^2 +
c^2)*cos(e*x + d) - 3*(-I*a^2 - I*c^2)*sin(e*x + d))/(a^2 + c^2)) + 3*(a^2
 + c^2)*sqrt(-2*I*a - 2*c)*weierstrassZeta(4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*
c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4)
, -8/27*(-9*I*a^5*b + 8*I*a^3*b^3 + 27*I*a*b*c^4 - 9*b*c^5 + 2*(9*a^2*b +
4*b^3)*c^3 + 6*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*a^2*b^3)*c)/(a^6
 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), weierstrassPInverse(4/3*(3*a^4 - 4*a^2*b^
2 + 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*
c^2 + c^4), -8/27*(-9*I*a^5*b + 8*I*a^3*b^3 + 27*I*a*b*c^4 - 9*b*c^5 + 2*(
9*a^2*b + 4*b^3)*c^3 + 6*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*a^2...
```

3.463.6 Sympy [F]

$$\int \frac{\sqrt{a + c\cot(d + ex) + b\csc(d + ex)}}{\sqrt{\csc(d + ex)}}\,dx = \int \frac{\sqrt{a + b\csc\left(d + ex\right) + c\cot\left(d + ex\right)}}{\sqrt{\csc\left(d + ex\right)}}\,dx$$

input
```
integrate((a+c*cot(e*x+d)+b*csc(e*x+d))**(1/2)/csc(e*x+d)**(1/2),x)
```

output
```
Integral(sqrt(a + b*csc(d + e*x) + c*cot(d + e*x))/sqrt(csc(d + e*x)), x)
```

3.463.7 Maxima [F]

$$\int \frac{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}}{\sqrt{\csc(d+ex)}}\,dx=\int \frac{\sqrt{c\cot(ex+d)+b\csc(ex+d)+a}}{\sqrt{\csc(ex+d)}}\,dx$$

input
```
integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)/csc(e*x+d)^(1/2),x, algorith
m="maxima")
```

output
```
integrate(sqrt(c*cot(e*x + d) + b*csc(e*x + d) + a)/sqrt(csc(e*x + d)), x)
```

3.463.8 Giac [F]

$$\int \frac{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}}{\sqrt{\csc(d+ex)}}\,dx=\int \frac{\sqrt{c\cot(ex+d)+b\csc(ex+d)+a}}{\sqrt{\csc(ex+d)}}\,dx$$

input
```
integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)/csc(e*x+d)^(1/2),x, algorith
m="giac")
```

output
```
integrate(sqrt(c*cot(e*x + d) + b*csc(e*x + d) + a)/sqrt(csc(e*x + d)), x)
```

3.463.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}}{\sqrt{\csc(d+ex)}}\,dx=\int \frac{\sqrt{a+c\cot(d+e\,x)+\frac{b}{\sin(d+e\,x)}}}{\sqrt{\frac{1}{\sin(d+e\,x)}}}\,dx$$

input
```
int((a + c*cot(d + e*x) + b/sin(d + e*x))^(1/2)/(1/sin(d + e*x))^(1/2),x)
```

output
```
int((a + c*cot(d + e*x) + b/sin(d + e*x))^(1/2)/(1/sin(d + e*x))^(1/2), x)
```

3.464 $\int \frac{\sqrt{\csc(d+ex)}}{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}}\,dx$

3.464.1 Optimal result

Integrand size = 33, antiderivative size = 118

$$\int \frac{\sqrt{\csc(d+ex)}}{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}}\,dx = \frac{2\sqrt{\csc(d+ex)}\,\text{EllipticF}\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right),\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)\sqrt{\frac{b+c\cos(d+ex)+a\sin(d+ex)}{b+\sqrt{a^2+c^2}}}}{e\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}}$$

output
```
2*(cos(1/2*d+1/2*e*x-1/2*arctan(c,a))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arcta
n(c,a))*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan(c,a)),2^(1/2)*((a^2+c^2)^(1
/2)/(b+(a^2+c^2)^(1/2)))^(1/2))*csc(e*x+d)^(1/2)*((b+c*cos(e*x+d)+a*sin(e*
x+d))/(b+(a^2+c^2)^(1/2)))^(1/2)/e/(a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)
```

3.464.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 1.08 (sec) , antiderivative size = 339, normalized size of antiderivative = 2.87

$$\int \frac{\sqrt{\csc(d+ex)}}{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}}\,dx = \frac{2\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{3}{2},\frac{b+a\sqrt{1+\frac{c^2}{a^2}}\sin\left(d+ex+\arctan\left(\frac{c}{a}\right)\right)}{b-a\sqrt{1+\frac{c^2}{a^2}}},\frac{b+a\sqrt{1+\frac{c^2}{a^2}}\sin\left(d+ex+\arctan\left(\frac{c}{a}\right)\right)}{b+a\sqrt{1+\frac{c^2}{a^2}}}\right)\sqrt{\csc(d+ex)}\sec\left(d+ex\right.}{}$$

input
```
Integrate[Sqrt[Csc[d + e*x]]/Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]],x]
```

output
```
(2*AppellF1[1/2, 1/2, 1/2, 3/2, (b + a*Sqrt[1 + c^2/a^2]*Sin[d + e*x + Arc
Tan[c/a]])/(b - a*Sqrt[1 + c^2/a^2]), (b + a*Sqrt[1 + c^2/a^2]*Sin[d + e*x
 + ArcTan[c/a]])/(b + a*Sqrt[1 + c^2/a^2])]*Sqrt[Csc[d + e*x]]*Sec[d + e*x
 + ArcTan[c/a]]*Sqrt[b + c*Cos[d + e*x] + a*Sin[d + e*x]]*Sqrt[-((a*Sqrt[1
 + c^2/a^2]*(-1 + Sin[d + e*x + ArcTan[c/a]]))/(b + a*Sqrt[1 + c^2/a^2]))]
*Sqrt[(a*Sqrt[1 + c^2/a^2]*(1 + Sin[d + e*x + ArcTan[c/a]]))/(-b + a*Sqrt[
1 + c^2/a^2])]*Sqrt[b + a*Sqrt[1 + c^2/a^2]*Sin[d + e*x + ArcTan[c/a]]])/(
a*Sqrt[1 + c^2/a^2]*e*Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]])
```

3.464.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3042, 3647, 3042, 3606, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\csc(d+ex)}}{\sqrt{a+b\csc(d+ex)+c\cot(d+ex)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{\csc(d+ex)}}{\sqrt{a+b\csc(d+ex)+c\cot(d+ex)}}dx$$

$$\Big\downarrow \text{3647}$$

$$\frac{\sqrt{\csc(d+ex)}\sqrt{a\sin(d+ex)+b+c\cos(d+ex)}\int \frac{1}{\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}}dx}{\sqrt{a+b\csc(d+ex)+c\cot(d+ex)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\csc(d+ex)}\sqrt{a\sin(d+ex)+b+c\cos(d+ex)}\int \frac{1}{\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}}dx}{\sqrt{a+b\csc(d+ex)+c\cot(d+ex)}}$$

$$\Big\downarrow \text{3606}$$

$$\frac{\sqrt{\csc(d+ex)}\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}\int \frac{1}{\sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+\frac{\sqrt{a^2+c^2}\cos\left(d+ex-\tan^{-1}(c,a)\right)}{b+\sqrt{a^2+c^2}}}}dx}{\sqrt{a+b\csc(d+ex)+c\cot(d+ex)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\csc(d+ex)}\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}\int \frac{1}{\sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+\frac{\sqrt{a^2+c^2}\sin\left(d+ex-\tan^{-1}(c,a)+\frac{\pi}{2}\right)}{b+\sqrt{a^2+c^2}}}}dx}{\sqrt{a+b\csc(d+ex)+c\cot(d+ex)}}$$

$$\Big\downarrow \text{3140}$$

$$\frac{2\sqrt{\csc(d+ex)}\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}\,\text{EllipticF}\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right),\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{a+b\csc(d+ex)+c\cot(d+ex)}}$$

input
```
Int[Sqrt[Csc[d + e*x]]/Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]],x]
```

output
```
(2*Sqrt[Csc[d + e*x]]*EllipticF[(d + e*x - ArcTan[c, a])/2, (2*Sqrt[a^2 +
c^2])/(b + Sqrt[a^2 + c^2])]*Sqrt[(b + c*Cos[d + e*x] + a*Sin[d + e*x])/(b
 + Sqrt[a^2 + c^2])])/(e*Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]])
```

3.464.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3606
```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Simp[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]   Int[1/Sqrt[a/(a
 + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x -
 ArcTan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2
, 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

3.464. $\int \frac{\sqrt{\csc(d+ex)}}{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}}\,dx$

rule 3647
```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Simp[Csc[d + e*x]^n*((b +
 a*Sin[d + e*x] + c*Cos[d + e*x])^n/(a + b*Csc[d + e*x] + c*Cot[d + e*x])^n
)   Int[1/(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n, x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[m + n, 0] &&  !IntegerQ[n]
```

3.464.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 31.07 (sec) , antiderivative size = 644, normalized size of antiderivative = 5.46

method	result
default	$\frac{2i\sqrt{2}\left(ib-ic+\sqrt{a^2-b^2+c^2}+a\right)\sqrt{\csc(ex+d)}\,\sqrt{a+c\cot(ex+d)+b\csc(ex+d)}\,\text{EllipticF}\left(\frac{\sqrt{2}\sqrt{-\frac{(i\cos(ex+d)+i-\sin(ex+d))^2\left(ib-ic-\sqrt{a^2}\right.}{(1+\cos(ex+d))\left(ib-ic+\sqrt{a^2-b^2+c^2}\right.}}}{2}\right.}{}$

input
```
int(csc(e*x+d)^(1/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
2*I/e*2^(1/2)*(I*b-I*c+(a^2-b^2+c^2)^(1/2)+a)*csc(e*x+d)^(1/2)*(a+c*cot(e*
x+d)+b*csc(e*x+d))^(1/2)*EllipticF(1/2*2^(1/2)*(-(I*cos(e*x+d)+I-sin(e*x+d
))^2/(1+cos(e*x+d))*(I*b-I*c-(a^2-b^2+c^2)^(1/2)-a)/(I*b-I*c+(a^2-b^2+c^2)
^(1/2)+a))^(1/2),((I*b-I*c+(a^2-b^2+c^2)^(1/2)+a)*(I*b-I*c+(a^2-b^2+c^2)^(
1/2)-a)/(I*b-I*c-(a^2-b^2+c^2)^(1/2)-a)/(I*b-I*c-(a^2-b^2+c^2)^(1/2)+a))^(
1/2))*(-(I*cos(e*x+d)+I-sin(e*x+d))^2/(1+cos(e*x+d))*(I*b-I*c-(a^2-b^2+c^2
)^(1/2)-a)/(I*b-I*c+(a^2-b^2+c^2)^(1/2)+a))^(1/2)*(I*((a^2-b^2+c^2)^(1/2)*
sin(e*x+d)-b*cos(e*x+d)+c*cos(e*x+d)+a*sin(e*x+d)+b-c)/(I*sin(e*x+d)-cos(e
*x+d)+1)/(I*b-I*c+(a^2-b^2+c^2)^(1/2)+a))^(1/2)*(-I*((a^2-b^2+c^2)^(1/2)*s
in(e*x+d)+b*cos(e*x+d)-c*cos(e*x+d)-a*sin(e*x+d)-b+c)/(I*sin(e*x+d)-cos(e*
x+d)+1)/(I*b-I*c-(a^2-b^2+c^2)^(1/2)+a))^(1/2)/(b+c*cos(e*x+d)+a*sin(e*x+d
))*(I*sin(e*x+d)^2-cos(e*x+d)*sin(e*x+d))/(I*b-I*c-(a^2-b^2+c^2)^(1/2)-a)
```

3.464.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 508, normalized size of antiderivative = 4.31

$$\int \frac{\sqrt{\csc(d+ex)}}{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}}\,dx = \frac{(i\,a-c)\sqrt{-2i\,a-2\,c}\text{weierstrassPInverse}\left(\frac{4\left(3\,a^4-4\,a^2b^2+4\,b^2c^2+6i\,ac^3-3\,c^4+2i\,(3\,a^3-4\,ab^2)\,c\right)}{3\,(a^4+2\,a^2c^2+c^4)},-\frac{8\,(-9i\,a^5b+8i\,a^3b^3+27i\,ab\cdots}{\cdots}\right.}{\cdots}$$

input
```
integrate(csc(e*x+d)^(1/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2),x, algorith
m="fricas")
```

output
```
((I*a - c)*sqrt(-2*I*a - 2*c)*weierstrassPInverse(4/3*(3*a^4 - 4*a^2*b^2 +
 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2
 + c^4), -8/27*(-9*I*a^5*b + 8*I*a^3*b^3 + 27*I*a*b*c^4 - 9*b*c^5 + 2*(9*a
^2*b + 4*b^3)*c^3 + 6*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*a^2*b^3)*
c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(-2*I*a*b + 2*b*c + 3*(a^2 + c
^2)*cos(e*x + d) - 3*(I*a^2 + I*c^2)*sin(e*x + d))/(a^2 + c^2)) + sqrt(2*I
*a - 2*c)*(-I*a - c)*weierstrassPInverse(4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^
2 - 6*I*a*c^3 - 3*c^4 - 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4),
-8/27*(9*I*a^5*b - 8*I*a^3*b^3 - 27*I*a*b*c^4 - 9*b*c^5 + 2*(9*a^2*b + 4*b
^3)*c^3 - 6*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 +
3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(2*I*a*b + 2*b*c + 3*(a^2 + c^2)*cos(e*x
 + d) - 3*(-I*a^2 - I*c^2)*sin(e*x + d))/(a^2 + c^2)))/((a^2 + c^2)*e)
```

3.464.6 Sympy [F]

$$\int \frac{\sqrt{\csc(d+ex)}}{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}}\,dx = \int \frac{\sqrt{\csc(d+ex)}}{\sqrt{a+b\csc(d+ex)+c\cot(d+ex)}}\,dx$$

input
```
integrate(csc(e*x+d)**(1/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))**(1/2),x)
```

output
```
Integral(sqrt(csc(d + e*x))/sqrt(a + b*csc(d + e*x) + c*cot(d + e*x)), x)
```

3.464.7 Maxima [F]

$$\int \frac{\sqrt{\csc(d+ex)}}{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}}\,dx=\int \frac{\sqrt{\csc(ex+d)}}{\sqrt{c\cot(ex+d)+b\csc(ex+d)+a}}\,dx$$

input
```
integrate(csc(e*x+d)^(1/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2),x, algorith
m="maxima")
```

output
```
integrate(sqrt(csc(e*x + d))/sqrt(c*cot(e*x + d) + b*csc(e*x + d) + a), x)
```

3.464.8 Giac [F]

$$\int \frac{\sqrt{\csc(d+ex)}}{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}}\,dx=\int \frac{\sqrt{\csc(ex+d)}}{\sqrt{c\cot(ex+d)+b\csc(ex+d)+a}}\,dx$$

input
```
integrate(csc(e*x+d)^(1/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2),x, algorith
m="giac")
```

output
```
integrate(sqrt(csc(e*x + d))/sqrt(c*cot(e*x + d) + b*csc(e*x + d) + a), x)
```

3.464.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\csc(d+ex)}}{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}}\,dx=\int \frac{\sqrt{\frac{1}{\sin(d+e\,x)}}}{\sqrt{a+c\cot(d+e\,x)+\frac{b}{\sin(d+e\,x)}}}\,dx$$

input
```
int((1/sin(d + e*x))^(1/2)/(a + c*cot(d + e*x) + b/sin(d + e*x))^(1/2),x)
```

output
```
int((1/sin(d + e*x))^(1/2)/(a + c*cot(d + e*x) + b/sin(d + e*x))^(1/2), x)
```

3.465 $\int \frac{\csc^{\frac{3}{2}}(d+ex)}{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}}\,dx$

3.465.1 Optimal result

Integrand size = 33, antiderivative size = 240

$$\int \frac{\csc^{\frac{3}{2}}(d+ex)}{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}}\,dx = -\frac{2\csc^{\frac{3}{2}}(d+ex)E\left(\frac{1}{2}(d+ex-\tan^{-1}(c,a))\,|\,\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)(b+c\cos(d+ex)+a\sin(d+ex))^2}{(a^2-b^2+c^2)\,e(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\sqrt{\frac{b+c\cos(d+ex)+a\sin(d+ex)}{b+\sqrt{a^2+c^2}}}} - \frac{2\csc^{\frac{3}{2}}(d+ex)(b+c\cos(d+ex)+a\sin(d+ex))(a\cos(d+ex)-c\sin(d+ex))}{(a^2-b^2+c^2)\,e(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}}$$

output
```
-2*csc(e*x+d)^(3/2)*(b+c*cos(e*x+d)+a*sin(e*x+d))*(a*cos(e*x+d)-c*sin(e*x+
d))/(a^2-b^2+c^2)/e/(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)-2*csc(e*x+d)^(3/2)
*(cos(1/2*d+1/2*e*x-1/2*arctan(c,a))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan
(c,a))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(c,a)),2^(1/2)*((a^2+c^2)^(1/
2)/(b+(a^2+c^2)^(1/2)))^(1/2))*(b+c*cos(e*x+d)+a*sin(e*x+d))^2/(a^2-b^2+c^
2)/e/(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)/((b+c*cos(e*x+d)+a*sin(e*x+d))/(b
+(a^2+c^2)^(1/2)))^(1/2)
```

3.465.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 6.88 (sec) , antiderivative size = 1732, normalized size of antiderivative = 7.22

$$\int \frac{\csc^{\frac{3}{2}}(d+ex)}{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[Csc[d + e*x]^(3/2)/(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2),x
]
```

output
```
(Csc[d + e*x]^(3/2)*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^2*((-2*(a^2 + c^
2))/(a*c*(a^2 - b^2 + c^2)) + (2*(a*b + a^2*Sin[d + e*x] + c^2*Sin[d + e*x
]))/(c*(a^2 - b^2 + c^2)*(b + c*Cos[d + e*x] + a*Sin[d + e*x]))))/(e*(a +
c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)) - (a*Csc[d + e*x]^(3/2)*(b + c*Cos
[d + e*x] + a*Sin[d + e*x])^(3/2)*(-((a*AppellF1[-1/2, -1/2, -1/2, 1/2, -(
(b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(1
 - b/(Sqrt[1 + a^2/c^2]*c))*c)), -((b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x -
ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b/(Sqrt[1 + a^2/c^2]*c))*c))]*Sin[d
 + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*c*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2]
- c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]])/(b + c*Sqrt[(a^2 + c
^2)/c^2])]*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]]]*Sq
rt[(c*Sqrt[(a^2 + c^2)/c^2] + c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan
[a/c]])/(-b + c*Sqrt[(a^2 + c^2)/c^2])])) - ((2*c*(b + Sqrt[1 + a^2/c^2]*c
*Cos[d + e*x - ArcTan[a/c]]))/(a^2 + c^2) - (a*Sin[d + e*x - ArcTan[a/c]])
/(Sqrt[1 + a^2/c^2]*c))/Sqrt[b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[
a/c]]]))/((a^2 - b^2 + c^2)*e*(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2))
 - (c^2*Csc[d + e*x]^(3/2)*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^(3/2)*(-(
(a*AppellF1[-1/2, -1/2, -1/2, 1/2, -((b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x
- ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(1 - b/(Sqrt[1 + a^2/c^2]*c))*c)), -((b
 + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(...
```

3.465.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3042, 3647, 3042, 3607, 3042, 3598, 3042, 3132}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

3.465. $\int \frac{\csc^{\frac{3}{2}}(d+ex)}{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}}\,dx$

$$\int \frac{\csc^{\frac{3}{2}}(d+ex)}{(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}}\,dx$$

$\big\downarrow$ 3042

$$\int \frac{\csc(d+ex)^{3/2}}{(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}}dx$$

$\big\downarrow$ 3647

$$\frac{\csc^{\frac{3}{2}}(d+ex)(a\sin(d+ex)+b+c\cos(d+ex))^{3/2}\int \frac{1}{(b+c\cos(d+ex)+a\sin(d+ex))^{3/2}}dx}{(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}}$$

$\big\downarrow$ 3042

$$\frac{\csc^{\frac{3}{2}}(d+ex)(a\sin(d+ex)+b+c\cos(d+ex))^{3/2}\int \frac{1}{(b+c\cos(d+ex)+a\sin(d+ex))^{3/2}}dx}{(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}}$$

$\big\downarrow$ 3607

$$\frac{\csc^{\frac{3}{2}}(d+ex)(a\sin(d+ex)+b+c\cos(d+ex))^{3/2}\left(-\frac{\int\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}dx}{a^2-b^2+c^2}-\frac{2(a\cos(d+ex)-c\sin(d+ex))}{e(a^2-b^2+c^2)\sqrt{a\sin(d+ex)+b+c\cos}}\right.}{(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}}$$

$\big\downarrow$ 3042

$$\frac{\csc^{\frac{3}{2}}(d+ex)(a\sin(d+ex)+b+c\cos(d+ex))^{3/2}\left(-\frac{\int\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}dx}{a^2-b^2+c^2}-\frac{2(a\cos(d+ex)-c\sin(d+ex))}{e(a^2-b^2+c^2)\sqrt{a\sin(d+ex)+b+c\cos}}\right.}{(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}}$$

$\big\downarrow$ 3598

$$\frac{\csc^{\frac{3}{2}}(d+ex)(a\sin(d+ex)+b+c\cos(d+ex))^{3/2}\left(-\frac{\sqrt{a\sin(d+ex)+b+c\cos(d+ex)}\int\sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+\frac{\sqrt{a^2+c^2}\cos\left(d+ex-\tan^{-1}(c,a\right.}{b+\sqrt{a^2+c^2}}}}{(a^2-b^2+c^2)\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}}\right.}{(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}}$$

$\big\downarrow$ 3042

$$\frac{\csc^{\frac{3}{2}}(d+ex)(a\sin(d+ex)+b+c\cos(d+ex))^{3/2}\left(-\frac{\sqrt{a\sin(d+ex)+b+c\cos(d+ex)}\int\sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+\frac{\sqrt{a^2+c^2}\sin\left(d+ex-\tan^{-1}(c,a\right.}{b+\sqrt{a^2+c^2}}}}{(a^2-b^2+c^2)\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}}\right.}{(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}}$$

$\big\downarrow$ 3132

3.465. $\int \frac{\csc^{\frac{3}{2}}(d+ex)}{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}}\,dx$

$$\frac{\csc^{\frac{3}{2}}(d+ex)(a\sin(d+ex)+b+c\cos(d+ex))^{3/2}\left(-\frac{2\sqrt{a\sin(d+ex)+b+c\cos(d+ex)}E\left(\frac{1}{2}(d+ex-\tan^{-1}(c,a))|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e(a^2-b^2+c^2)\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}}-\right.}{(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}}$$

input
```
Int[Csc[d + e*x]^(3/2)/(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2),x]
```

output
```
(Csc[d + e*x]^(3/2)*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^(3/2)*((-2*Ellip
ticE[(d + e*x - ArcTan[c, a])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2 + c^2])
]*Sqrt[b + c*Cos[d + e*x] + a*Sin[d + e*x]])/((a^2 - b^2 + c^2)*e*Sqrt[(b
+ c*Cos[d + e*x] + a*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])]) - (2*(a*Cos[d +
 e*x] - c*Sin[d + e*x]))/((a^2 - b^2 + c^2)*e*Sqrt[b + c*Cos[d + e*x] + a*
Sin[d + e*x]])))/(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)
```

3.465.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3598
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]   Int[Sqrt[a/(a + S
qrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - Arc
Tan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

rule 3607
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-3/2), x_Symbol] :> Simp[2*((c*Cos[d + e*x] - b*Sin[d + e*x])/(e*(a^2 - b^
2 - c^2)*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]])), x] + Simp[1/(a^2 - b^
2 - c^2)   Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] /; FreeQ[{
a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

3.465. $\int \frac{\csc^{\frac{3}{2}}(d+ex)}{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}}\,dx$

rule 3647
```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Simp[Csc[d + e*x]^n*((b +
 a*Sin[d + e*x] + c*Cos[d + e*x])^n/(a + b*Csc[d + e*x] + c*Cot[d + e*x])^n
)   Int[1/(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n, x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[m + n, 0] &&  !IntegerQ[n]
```

3.465.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 25.72 (sec) , antiderivative size = 12702, normalized size of antiderivative = 52.92

method	result	size
default	Expression too large to display	12702

input
```
int(csc(e*x+d)^(3/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
result too large to display
```

3.465.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.16 (sec) , antiderivative size = 1711, normalized size of antiderivative = 7.13

$$\int \frac{\csc^{\frac{3}{2}}(d+ex)}{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(csc(e*x+d)^(3/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2),x, algorith
m="fricas")
```

output
```
1/3*((-I*a*b^2 + b^2*c + (-I*a*b*c + b*c^2)*cos(e*x + d) + (-I*a^2*b + a*b
*c)*sin(e*x + d))*sqrt(-2*I*a - 2*c)*weierstrassPInverse(4/3*(3*a^4 - 4*a^
2*b^2 + 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*
a^2*c^2 + c^4), -8/27*(-9*I*a^5*b + 8*I*a^3*b^3 + 27*I*a*b*c^4 - 9*b*c^5 +
 2*(9*a^2*b + 4*b^3)*c^3 + 6*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*a^
2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(-2*I*a*b + 2*b*c + 3*(
a^2 + c^2)*cos(e*x + d) - 3*(I*a^2 + I*c^2)*sin(e*x + d))/(a^2 + c^2)) + (
I*a*b^2 + b^2*c + (I*a*b*c + b*c^2)*cos(e*x + d) + (I*a^2*b + a*b*c)*sin(e
*x + d))*sqrt(2*I*a - 2*c)*weierstrassPInverse(4/3*(3*a^4 - 4*a^2*b^2 + 4*
b^2*c^2 - 6*I*a*c^3 - 3*c^4 - 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 +
c^4), -8/27*(9*I*a^5*b - 8*I*a^3*b^3 - 27*I*a*b*c^4 - 9*b*c^5 + 2*(9*a^2*b
 + 4*b^3)*c^3 - 6*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*a^2*b^3)*c)/(
a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(2*I*a*b + 2*b*c + 3*(a^2 + c^2)*c
os(e*x + d) - 3*(-I*a^2 - I*c^2)*sin(e*x + d))/(a^2 + c^2)) - 3*(a^2*b + b
*c^2 + (a^2*c + c^3)*cos(e*x + d) + (a^3 + a*c^2)*sin(e*x + d))*sqrt(-2*I*
a - 2*c)*weierstrassZeta(4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 + 6*I*a*c^3 -
3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), -8/27*(-9*I*a^5*
b + 8*I*a^3*b^3 + 27*I*a*b*c^4 - 9*b*c^5 + 2*(9*a^2*b + 4*b^3)*c^3 + 6*I*(
3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a
^2*c^4 + c^6), weierstrassPInverse(4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 +...
```

3.465.6 Sympy [F]

$$\int \frac{\csc^{\frac{3}{2}}(d+ex)}{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}}\,dx=\int \frac{\csc^{\frac{3}{2}}(d+ex)}{(a+b\csc(d+ex)+c\cot(d+ex))^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(e*x+d)**(3/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))**(3/2),x)
```

output
```
Integral(csc(d + e*x)**(3/2)/(a + b*csc(d + e*x) + c*cot(d + e*x))**(3/2),
 x)
```

3.465.7 Maxima [F]

$$\int \frac{\csc^{\frac{3}{2}}(d+ex)}{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}}\,dx=\int \frac{\csc(ex+d)^{\frac{3}{2}}}{(c\cot(ex+d)+b\csc(ex+d)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(e*x+d)^(3/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2),x, algorith
m="maxima")
```

output
```
integrate(csc(e*x + d)^(3/2)/(c*cot(e*x + d) + b*csc(e*x + d) + a)^(3/2),
x)
```

3.465.8 Giac [F]

$$\int \frac{\csc^{\frac{3}{2}}(d+ex)}{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}}\,dx=\int \frac{\csc(ex+d)^{\frac{3}{2}}}{(c\cot(ex+d)+b\csc(ex+d)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(e*x+d)^(3/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2),x, algorith
m="giac")
```

output
```
integrate(csc(e*x + d)^(3/2)/(c*cot(e*x + d) + b*csc(e*x + d) + a)^(3/2),
x)
```

3.465.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^{\frac{3}{2}}(d+ex)}{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}}\,dx=\int \frac{\left(\frac{1}{\sin(d+e\,x)}\right)^{3/2}}{\left(a+c\cot(d+e\,x)+\frac{b}{\sin(d+e\,x)}\right)^{3/2}}\,dx$$

input
```
int((1/sin(d + e*x))^(3/2)/(a + c*cot(d + e*x) + b/sin(d + e*x))^(3/2),x)
```

output
```
int((1/sin(d + e*x))^(3/2)/(a + c*cot(d + e*x) + b/sin(d + e*x))^(3/2), x)
```

3.466 $\int \frac{\csc^{\frac{5}{2}}(d+ex)}{(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}}\,dx$

3.466.1 Optimal result

Integrand size = 33, antiderivative size = 492

$$\int \frac{\csc^{\frac{5}{2}}(d+ex)}{(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}}\,dx = \frac{8b\csc^{\frac{5}{2}}(d+ex)E\left(\frac{1}{2}(d+ex-\tan^{-1}(c,a))\,|\,\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)(b+c\,c}{3\,(a^2-b^2+c^2)^2\,e(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}\,}$$

$$+\frac{2\csc^{\frac{5}{2}}(d+ex)\,\text{EllipticF}\left(\frac{1}{2}(d+ex-\tan^{-1}(c,a)),\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)(b+c\cos(d+ex)+a\sin(d+ex))^2\sqrt{\frac{b+c\cos(}{}}}{3\,(a^2-b^2+c^2)\,e(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}}$$

$$-\frac{2\csc^{\frac{5}{2}}(d+ex)(b+c\cos(d+ex)+a\sin(d+ex))(a\cos(d+ex)-c\sin(d+ex))}{3\,(a^2-b^2+c^2)\,e(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}}$$

$$+\frac{8\csc^{\frac{5}{2}}(d+ex)(b+c\cos(d+ex)+a\sin(d+ex))^2(ab\cos(d+ex)-bc\sin(d+ex))}{3\,(a^2-b^2+c^2)^2\,e(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}}$$

output
```
-2/3*csc(e*x+d)^(5/2)*(b+c*cos(e*x+d)+a*sin(e*x+d))*(a*cos(e*x+d)-c*sin(e*
x+d))/(a^2-b^2+c^2)/e/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2)+8/3*csc(e*x+d)^(
5/2)*(b+c*cos(e*x+d)+a*sin(e*x+d))^2*(a*b*cos(e*x+d)-b*c*sin(e*x+d))/(a^2-
b^2+c^2)^2/e/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2)+8/3*b*csc(e*x+d)^(5/2)*(c
os(1/2*d+1/2*e*x-1/2*arctan(c,a))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(c,
a))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(c,a)),2^(1/2)*((a^2+c^2)^(1/2)/
(b+(a^2+c^2)^(1/2)))^(1/2))*(b+c*cos(e*x+d)+a*sin(e*x+d))^3/(a^2-b^2+c^2)^
2/e/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2)/((b+c*cos(e*x+d)+a*sin(e*x+d))/(b+
(a^2+c^2)^(1/2)))^(1/2)+2/3*csc(e*x+d)^(5/2)*(cos(1/2*d+1/2*e*x-1/2*arctan
(c,a))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(c,a))*EllipticF(sin(1/2*d+1/2
*e*x-1/2*arctan(c,a)),2^(1/2)*((a^2+c^2)^(1/2)/(b+(a^2+c^2)^(1/2)))^(1/2))
*(b+c*cos(e*x+d)+a*sin(e*x+d))^2*((b+c*cos(e*x+d)+a*sin(e*x+d))/(b+(a^2+c^
2)^(1/2)))^(1/2)/(a^2-b^2+c^2)/e/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2)
```

3.466.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 7.19 (sec) , antiderivative size = 2708, normalized size of antiderivative = 5.50

$$\int \frac{\csc^{\frac{5}{2}}(d+ex)}{(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[Csc[d + e*x]^(5/2)/(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2),x
]
```

output
```
(Csc[d + e*x]^(5/2)*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^3*((8*b*(a^2 + c
^2))/(3*a*c*(-a^2 + b^2 - c^2)^2) + (2*(a*b + a^2*Sin[d + e*x] + c^2*Sin[d
 + e*x]))/(3*c*(a^2 - b^2 + c^2)*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^2)
- (2*(a^3 + 3*a*b^2 + a*c^2 + 4*a^2*b*Sin[d + e*x] + 4*b*c^2*Sin[d + e*x])
)/(3*c*(a^2 - b^2 + c^2)^2*(b + c*Cos[d + e*x] + a*Sin[d + e*x]))))/(e*(a
+ c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2)) + (4*a*b*Csc[d + e*x]^(5/2)*(b +
 c*Cos[d + e*x] + a*Sin[d + e*x])^(5/2)*(-((a*AppellF1[-1/2, -1/2, -1/2, 1
/2, -((b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c
^2]*(1 - b/(Sqrt[1 + a^2/c^2]*c))*c)), -((b + Sqrt[1 + a^2/c^2]*c*Cos[d +
e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*(-1 - b/(Sqrt[1 + a^2/c^2]*c))*c))]
*Sin[d + e*x - ArcTan[a/c]])/(Sqrt[1 + a^2/c^2]*c*Sqrt[(c*Sqrt[(a^2 + c^2)
/c^2] - c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c]])/(b + c*Sqrt[(a
^2 + c^2)/c^2])]*Sqrt[b + c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x - ArcTan[a/c
]]]*Sqrt[(c*Sqrt[(a^2 + c^2)/c^2] + c*Sqrt[(a^2 + c^2)/c^2]*Cos[d + e*x -
ArcTan[a/c]])/(-b + c*Sqrt[(a^2 + c^2)/c^2])])) - ((2*c*(b + Sqrt[1 + a^2/
c^2]*c*Cos[d + e*x - ArcTan[a/c]]))/(a^2 + c^2) - (a*Sin[d + e*x - ArcTan[
a/c]])/(Sqrt[1 + a^2/c^2]*c))/Sqrt[b + Sqrt[1 + a^2/c^2]*c*Cos[d + e*x - A
rcTan[a/c]]]))/(3*(a^2 - b^2 + c^2)^2*e*(a + c*Cot[d + e*x] + b*Csc[d + e*
x])^(5/2)) + (4*b*c^2*Csc[d + e*x]^(5/2)*(b + c*Cos[d + e*x] + a*Sin[d + e
*x])^(5/2)*(-((a*AppellF1[-1/2, -1/2, -1/2, 1/2, -((b + Sqrt[1 + a^2/c^...
```

3.466.3 Rubi [A] (verified)

Time = 1.83 (sec) , antiderivative size = 443, normalized size of antiderivative = 0.90, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.515$, Rules used = {3042, 3647, 3042, 3608, 27, 3042, 3635, 27, 3042, 3628, 3042, 3598, 3042, 3132, 3606, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^{\frac{5}{2}}(d+ex)}{(a+b\csc(d+ex)+c\cot(d+ex))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc(d+ex)^{5/2}}{(a+b\csc(d+ex)+c\cot(d+ex))^{5/2}}dx$$

$$\Big\downarrow \text{3647}$$

$$\frac{\csc^{\frac{5}{2}}(d+ex)(a\sin(d+ex)+b+c\cos(d+ex))^{5/2}\int \frac{1}{(b+c\cos(d+ex)+a\sin(d+ex))^{5/2}}dx}{(a+b\csc(d+ex)+c\cot(d+ex))^{5/2}}$$

$\downarrow$ 3042

$$\frac{\csc^{\frac{5}{2}}(d+ex)(a\sin(d+ex)+b+c\cos(d+ex))^{5/2}\int\frac{1}{(b+c\cos(d+ex)+a\sin(d+ex))^{5/2}}dx}{(a+b\csc(d+ex)+c\cot(d+ex))^{5/2}}$$

$\downarrow$ 3608

$$\frac{\csc^{\frac{5}{2}}(d+ex)(a\sin(d+ex)+b+c\cos(d+ex))^{5/2}\left(\frac{2\int-\frac{3b-c\cos(d+ex)-a\sin(d+ex)}{2(b+c\cos(d+ex)+a\sin(d+ex))^{3/2}}dx}{3(a^2-b^2+c^2)}-\frac{2(a\cos(d+ex)-c\sin(d+e}{3e(a^2-b^2+c^2)(a\sin(d+ex)+b+c\cos}\right.}{(a+b\csc(d+ex)+c\cot(d+ex))^{5/2}}$$

$\downarrow$ 27

$$\frac{\csc^{\frac{5}{2}}(d+ex)(a\sin(d+ex)+b+c\cos(d+ex))^{5/2}\left(-\frac{\int\frac{3b-c\cos(d+ex)-a\sin(d+ex)}{(b+c\cos(d+ex)+a\sin(d+ex))^{3/2}}dx}{3(a^2-b^2+c^2)}-\frac{2(a\cos(d+ex)-c\sin(d+ex)}{3e(a^2-b^2+c^2)(a\sin(d+ex)+b+c\cos(}\right.}{(a+b\csc(d+ex)+c\cot(d+ex))^{5/2}}$$

$\downarrow$ 3042

$$\frac{\csc^{\frac{5}{2}}(d+ex)(a\sin(d+ex)+b+c\cos(d+ex))^{5/2}\left(-\frac{\int\frac{3b-c\cos(d+ex)-a\sin(d+ex)}{(b+c\cos(d+ex)+a\sin(d+ex))^{3/2}}dx}{3(a^2-b^2+c^2)}-\frac{2(a\cos(d+ex)-c\sin(d+ex)}{3e(a^2-b^2+c^2)(a\sin(d+ex)+b+c\cos(}\right.}{(a+b\csc(d+ex)+c\cot(d+ex))^{5/2}}$$

$\downarrow$ 3635

$$\frac{\csc^{\frac{5}{2}}(d+ex)(a\sin(d+ex)+b+c\cos(d+ex))^{5/2}\left(-\frac{\frac{2\int-\frac{a^2+4b\sin(d+ex)a+3b^2+c^2+4bc\cos(d+ex)}{2\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}}dx}{a^2-b^2+c^2}-\frac{8(ab\cos(d+ex)-bc\sin(}{e\left(a^2-b^2+c^2\right)\sqrt{a\sin(d+ex)+b}}}{3(a^2-b^2+c^2)}\right.}{(a+b\csc(d+ex)+c\cot(d+ex))^{5/2}}$$

$\downarrow$ 27

$$\frac{\csc^{\frac{5}{2}}(d+ex)(a\sin(d+ex)+b+c\cos(d+ex))^{5/2}\left(-\frac{-\frac{\int\frac{a^2+4b\sin(d+ex)a+3b^2+c^2+4bc\cos(d+ex)}{\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}}dx}{a^2-b^2+c^2}-\frac{8(ab\cos(d+ex)-bc\sin(d}{e\left(a^2-b^2+c^2\right)\sqrt{a\sin(d+ex)+b+}}}{3(a^2-b^2+c^2)}\right.}{(a+b\csc(d+ex)+c\cot(d+ex))^{5/2}}$$

$\downarrow$ 3042

3.466. $\displaystyle\int\frac{\csc^{\frac{5}{2}}(d+ex)}{(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}}\,dx$

$$\frac{\csc^{\frac{5}{2}}(d+ex)(a\sin(d+ex)+b+c\cos(d+ex))^{5/2}\left(-\frac{-\frac{\int\frac{a^2+4b\sin(d+ex)a+3b^2+c^2+4bc\cos(d+ex)}{\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}}dx}{a^2-b^2+c^2}-\frac{8(ab\cos(d+ex)-bc\sin(d\cdots}{e\left(a^2-b^2+c^2\right)\sqrt{a\sin(d+ex)+b+\cdots}}}{3(a^2-b^2+c^2)}\cdots\right.}{(a+b\csc(d+ex)+c\cot(d+ex))^{5/2}}$$

$\downarrow$ 3628

$$\frac{\csc^{\frac{5}{2}}(d+ex)(a\sin(d+ex)+b+c\cos(d+ex))^{5/2}\left(-\frac{-\frac{\left(a^2-b^2+c^2\right)\int\frac{1}{\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}}dx+4b\int\sqrt{b+c\cos(d+ex)+a\sin(d+e\cdots}}{a^2-b^2+c^2}\cdots}{3(a^2-b^2+c^2)}\cdots\right.}{(a+b\csc(d+ex)+c\cot(d+ex))^{5/\cdots}}$$

$\downarrow$ 3042

$$\frac{\csc^{\frac{5}{2}}(d+ex)(a\sin(d+ex)+b+c\cos(d+ex))^{5/2}\left(-\frac{-\frac{\left(a^2-b^2+c^2\right)\int\frac{1}{\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}}dx+4b\int\sqrt{b+c\cos(d+ex)+a\sin(d+e\cdots}}{a^2-b^2+c^2}\cdots}{3(a^2-b^2+c^2)}\cdots\right.}{(a+b\csc(d+ex)+c\cot(d+ex))^{5/\cdots}}$$

$\downarrow$ 3598

$$\frac{\csc^{\frac{5}{2}}(d+ex)(a\sin(d+ex)+b+c\cos(d+ex))^{5/2}\left(-\frac{-\frac{\left(a^2-b^2+c^2\right)\int\frac{1}{\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}}dx+\frac{4b\sqrt{a\sin(d+ex)+b+c\cos(d+ex)}\cdots}{\sqrt{a\cdots}}}{a^2-b^2+c^2}\cdots}{\cdots}\cdots\right.}{(a+b\csc(d+ex)\cdots}$$

$\downarrow$ 3042

$$\frac{\csc^{\frac{5}{2}}(d+ex)(a\sin(d+ex)+b+c\cos(d+ex))^{5/2}\left(-\frac{-\frac{\left(a^2-b^2+c^2\right)\int\frac{1}{\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}}dx+\frac{4b\sqrt{a\sin(d+ex)+b+c\cos(d+ex)}\cdots}{\sqrt{\cdots}}}{a^2-b^2+c^2}\cdots}{\cdots}\cdots\right.}{(a+b\csc(d+ex\cdots}$$

3.466. $\int\frac{\csc^{\frac{5}{2}}(d+ex)}{(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}}\,dx$

$$\Big\downarrow \; 3132$$

$$\frac{\csc^{\frac{5}{2}}(d+ex)(a\sin(d+ex)+b+c\cos(d+ex))^{5/2}\left(-\frac{-\frac{\left(a^2-b^2+c^2\right)\int\frac{1}{\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}}dx+\frac{8b\sqrt{a\sin(d+ex)+b+c\cos(d+ex)}}{e\sqrt{\frac{a\sin(d+}{}}}}{a^2-b^2+c^2}}{3(a^2-}\right.}{(a+b\csc(d+ex)+c\,c}$$

$$\Big\downarrow \; 3606$$

$$\frac{\csc^{\frac{5}{2}}(d+ex)(a\sin(d+ex)+b+c\cos(d+ex))^{5/2}\left(-\frac{-\frac{\frac{\left(a^2-b^2+c^2\right)\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}\int\frac{1}{\sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+\frac{\sqrt{a^2+c^2}\cos\left(d+ex\right.}{b+\sqrt{a^2+}}}}}{\sqrt{a\sin(d+ex)+b+c\cos(d+ex)}}}{}}{}\right.}{(a+b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\csc^{\frac{5}{2}}(d+ex)(a\sin(d+ex)+b+c\cos(d+ex))^{5/2}\left(-\frac{-\frac{\frac{\left(a^2-b^2+c^2\right)\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}\int\frac{1}{\sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+\frac{\sqrt{a^2+c^2}\sin\left(d+ex\right.}{b+\sqrt{a^2}}}}}{\sqrt{a\sin(d+ex)+b+c\cos(d+ex)}}}{}}{}\right.}{(a+}$$

$$\Big\downarrow \; 3140$$

3.466. $\displaystyle\int \frac{\csc^{\frac{5}{2}}(d+ex)}{(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}}\,dx$

$$\frac{\csc^{\frac{5}{2}}(d+ex)(a\sin(d+ex)+b+c\cos(d+ex))^{5/2}\left(-\frac{-\frac{\frac{2(a^2-b^2+c^2)\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}\text{EllipticF}\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right),\frac{2\cdots}{b+\cdots}\right.}{e\sqrt{a\sin(d+ex)+b+c\cos(d+ex)}}}{a^2-\cdots}}{\cdots}\right.}{(a+b\,c\cdots}$$

input
```
Int[Csc[d + e*x]^(5/2)/(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2),x]
```

output
```
(Csc[d + e*x]^(5/2)*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^(5/2)*((-2*(a*Co
s[d + e*x] - c*Sin[d + e*x]))/(3*(a^2 - b^2 + c^2)*e*(b + c*Cos[d + e*x] +
 a*Sin[d + e*x])^(3/2)) - ((-8*(a*b*Cos[d + e*x] - b*c*Sin[d + e*x]))/((a^
2 - b^2 + c^2)*e*Sqrt[b + c*Cos[d + e*x] + a*Sin[d + e*x]]) - ((8*b*Ellipt
icE[(d + e*x - ArcTan[c, a])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2 + c^2])]
*Sqrt[b + c*Cos[d + e*x] + a*Sin[d + e*x]])/(e*Sqrt[(b + c*Cos[d + e*x] +
a*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])]) + (2*(a^2 - b^2 + c^2)*EllipticF[(
d + e*x - ArcTan[c, a])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2 + c^2])]*Sqrt
[(b + c*Cos[d + e*x] + a*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])])/(e*Sqrt[b +
 c*Cos[d + e*x] + a*Sin[d + e*x]]))/(a^2 - b^2 + c^2))/(3*(a^2 - b^2 + c^2
))))/(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2)
```

3.466.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

3.466. $\int \frac{\csc^{\frac{5}{2}}(d+ex)}{(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}}\,dx$

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3598
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]   Int[Sqrt[a/(a + S
qrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - Arc
Tan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

rule 3606
```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Simp[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]   Int[1/Sqrt[a/(a
 + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x -
 ArcTan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2
, 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

rule 3608
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Simp[
1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c
*(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x
] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] &&
NeQ[n, -3/2]
```

rule 3628
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]
, x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[
A*b - a*B, 0]
```

3.466. $\int \frac{\csc^{\frac{5}{2}}(d+ex)}{(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}}\,dx$

rule 3635
```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(-(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(
a^2 - b^2 - c^2))), x] + Simp[1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a + b*Co
s[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n +
2)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x]
/; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[n, -2]
```

rule 3647
```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Simp[Csc[d + e*x]^n*((b +
 a*Sin[d + e*x] + c*Cos[d + e*x])^n/(a + b*Csc[d + e*x] + c*Cot[d + e*x])^n
)   Int[1/(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n, x], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[m + n, 0] &&  !IntegerQ[n]
```

3.466.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 29.43 (sec) , antiderivative size = 59351, normalized size of antiderivative = 120.63

method	result	size
default	Expression too large to display	59351

input
```
int(csc(e*x+d)^(5/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2),x,method=_RETURNV
ERBOSE)
```

output
```
result too large to display
```

3.466.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.30 (sec) , antiderivative size = 2777, normalized size of antiderivative = 5.64

$$\int \frac{\csc^{\frac{5}{2}}(d+ex)}{(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(csc(e*x+d)^(5/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2),x, algorith
m="fricas")
```

output
```
-1/9*((3*I*a^5 + 4*I*a^3*b^2 + I*a*b^4 - 3*(a^2 + b^2)*c^3 + 3*I*(a^3 + a*
b^2)*c^2 + (-3*I*a^5 - I*a^3*b^2 + I*a*b^2*c^2 - b^2*c^3 + 3*I*a*c^4 - 3*c
^5 + (3*a^4 + a^2*b^2)*c)*cos(e*x + d)^2 - (3*a^4 + 4*a^2*b^2 + b^4)*c - 2
*(-3*I*a*b*c^3 + 3*b*c^4 + (3*a^2*b + b^3)*c^2 - I*(3*a^3*b + a*b^3)*c)*co
s(e*x + d) - 2*(-3*I*a^4*b - I*a^2*b^3 - 3*I*a^2*b*c^2 + 3*a*b*c^3 + (3*a^
3*b + a*b^3)*c + (-3*I*a^2*c^3 + 3*a*c^4 + (3*a^3 + a*b^2)*c^2 - I*(3*a^4
+ a^2*b^2)*c)*cos(e*x + d))*sin(e*x + d))*sqrt(-2*I*a - 2*c)*weierstrassPI
nverse(4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3
 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), -8/27*(-9*I*a^5*b + 8*I*a^3*b^3 +
27*I*a*b*c^4 - 9*b*c^5 + 2*(9*a^2*b + 4*b^3)*c^3 + 6*I*(3*a^3*b - 4*a*b^3)
*c^2 + 3*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3
*(-2*I*a*b + 2*b*c + 3*(a^2 + c^2)*cos(e*x + d) - 3*(I*a^2 + I*c^2)*sin(e*
x + d))/(a^2 + c^2)) + (-3*I*a^5 - 4*I*a^3*b^2 - I*a*b^4 - 3*(a^2 + b^2)*c
^3 - 3*I*(a^3 + a*b^2)*c^2 + (3*I*a^5 + I*a^3*b^2 - I*a*b^2*c^2 - b^2*c^3
- 3*I*a*c^4 - 3*c^5 + (3*a^4 + a^2*b^2)*c)*cos(e*x + d)^2 - (3*a^4 + 4*a^2
*b^2 + b^4)*c - 2*(3*I*a*b*c^3 + 3*b*c^4 + (3*a^2*b + b^3)*c^2 + I*(3*a^3*
b + a*b^3)*c)*cos(e*x + d) - 2*(3*I*a^4*b + I*a^2*b^3 + 3*I*a^2*b*c^2 + 3*
a*b*c^3 + (3*a^3*b + a*b^3)*c + (3*I*a^2*c^3 + 3*a*c^4 + (3*a^3 + a*b^2)*c
^2 + I*(3*a^4 + a^2*b^2)*c)*cos(e*x + d))*sin(e*x + d))*sqrt(2*I*a - 2*c)*
weierstrassPInverse(4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 - 6*I*a*c^3 - 3*...
```

3.466.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^{\frac{5}{2}}(d+ex)}{(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(csc(e*x+d)**(5/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))**(5/2),x)
```

output
```
Timed out
```

3.466.7 Maxima [F]

$$\int \frac{\csc^{\frac{5}{2}}(d+ex)}{(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}}\,dx=\int \frac{\csc\left(ex+d\right)^{\frac{5}{2}}}{\left(c\cot\left(ex+d\right)+b\csc\left(ex+d\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(e*x+d)^(5/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2),x, algorith
m="maxima")
```

output
```
integrate(csc(e*x + d)^(5/2)/(c*cot(e*x + d) + b*csc(e*x + d) + a)^(5/2),
x)
```

3.466.8 Giac [F]

$$\int \frac{\csc^{\frac{5}{2}}(d+ex)}{(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}}\,dx=\int \frac{\csc\left(ex+d\right)^{\frac{5}{2}}}{\left(c\cot\left(ex+d\right)+b\csc\left(ex+d\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(csc(e*x+d)^(5/2)/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2),x, algorith
m="giac")
```

output
```
integrate(csc(e*x + d)^(5/2)/(c*cot(e*x + d) + b*csc(e*x + d) + a)^(5/2),
x)
```

3.466.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^{\frac{5}{2}}(d+ex)}{(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}}\,dx=\int \frac{\left(\frac{1}{\sin(d+e\,x)}\right)^{5/2}}{\left(a+c\cot\left(d+e\,x\right)+\frac{b}{\sin(d+e\,x)}\right)^{5/2}}\,dx$$

input
```
int((1/sin(d + e*x))^(5/2)/(a + c*cot(d + e*x) + b/sin(d + e*x))^(5/2),x)
```

output
```
int((1/sin(d + e*x))^(5/2)/(a + c*cot(d + e*x) + b/sin(d + e*x))^(5/2), x)
```

3.466. $\int \frac{\csc^{\frac{5}{2}}(d+ex)}{(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}}\,dx$

3.467 $\int (a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\sin^{\frac{3}{2}}(d+ex)\,dx$

3.467.1 Optimal result

Integrand size = 33, antiderivative size = 371

$$\int (a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\sin^{\frac{3}{2}}(d+ex)\,dx = \frac{8b(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}E\left(\frac{1}{2}(d+ex-\tan^{-1}(c,a))\,|\,\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)\sin^{\frac{3}{2}}(d+ex)}{3e(b+c\cos(d+ex)+a\sin(d+ex))\sqrt{\frac{b+c\cos(d+ex)+a\sin(d+ex)}{b+\sqrt{a^2+c^2}}}} + \frac{2(a^2-b^2+c^2)(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\,\text{EllipticF}\left(\frac{1}{2}(d+ex-\tan^{-1}(c,a)),\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)\sin^{\frac{3}{2}}(d}{3e(b+c\cos(d+ex)+a\sin(d+ex))^2} - \frac{2(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\sin^{\frac{3}{2}}(d+ex)(a\cos(d+ex)-c\sin(d+ex))}{3e(b+c\cos(d+ex)+a\sin(d+ex))}$$

output
```
-2/3*(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)*sin(e*x+d)^(3/2)*(a*cos(e*x+d)-c*
sin(e*x+d))/e/(b+c*cos(e*x+d)+a*sin(e*x+d))+8/3*b*(a+c*cot(e*x+d)+b*csc(e*
x+d))^(3/2)*(cos(1/2*d+1/2*e*x-1/2*arctan(c,a))^2)^(1/2)/cos(1/2*d+1/2*e*x
-1/2*arctan(c,a))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(c,a)),2^(1/2)*((a
^2+c^2)^(1/2)/(b+(a^2+c^2)^(1/2)))^(1/2))*sin(e*x+d)^(3/2)/e/(b+c*cos(e*x+
d)+a*sin(e*x+d))/((b+c*cos(e*x+d)+a*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(1/2)
+2/3*(a^2-b^2+c^2)*(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)*(cos(1/2*d+1/2*e*x-
1/2*arctan(c,a))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(c,a))*EllipticF(sin
(1/2*d+1/2*e*x-1/2*arctan(c,a)),2^(1/2)*((a^2+c^2)^(1/2)/(b+(a^2+c^2)^(1/2
)))^(1/2))*sin(e*x+d)^(3/2)*((b+c*cos(e*x+d)+a*sin(e*x+d))/(b+(a^2+c^2)^(1
/2)))^(1/2)/e/(b+c*cos(e*x+d)+a*sin(e*x+d))^2
```

3.467.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 26.31 (sec) , antiderivative size = 5904, normalized size of antiderivative = 15.91

$$\int (a + c\cot(d + ex) + b\csc(d + ex))^{3/2}\sin^{\frac{3}{2}}(d + ex)\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*Sin[d + e*x]^(3/2),x
]
```

output
```
Result too large to show
```

3.467.3 Rubi [A] (verified)

Time = 1.36 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.91, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.424$, Rules used = {3042, 3643, 3042, 3599, 27, 3042, 3628, 3042, 3598, 3042, 3132, 3606, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{\frac{3}{2}}(d + ex)(a + b\csc(d + ex) + c\cot(d + ex))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(d + ex)^{3/2}(a + b\csc(d + ex) + c\cot(d + ex))^{3/2}dx$$

$$\Big\downarrow \text{3643}$$

$$\frac{\sin^{\frac{3}{2}}(d + ex)(a + b\csc(d + ex) + c\cot(d + ex))^{3/2}\int (b + c\cos(d + ex) + a\sin(d + ex))^{3/2}dx}{(a\sin(d + ex) + b + c\cos(d + ex))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin^{\frac{3}{2}}(d + ex)(a + b\csc(d + ex) + c\cot(d + ex))^{3/2}\int (b + c\cos(d + ex) + a\sin(d + ex))^{3/2}dx}{(a\sin(d + ex) + b + c\cos(d + ex))^{3/2}}$$

$$\Big\downarrow \text{3599}$$

$$\frac{\sin^{\frac{3}{2}}(d + ex)(a + b\csc(d + ex) + c\cot(d + ex))^{3/2}\left(\frac{2}{3}\int \frac{a^2+4b\sin(d+ex)a+3b^2+c^2+4bc\cos(d+ex)}{2\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}}dx - \frac{2(a\cos(d+ex)-c\sin(d-\right.}{(a\sin(d + ex) + b + c\cos(d + ex))^{3/2}}$$

$\downarrow$ 27

$$\frac{\sin^{\frac{3}{2}}(d+ex)(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}\left(\frac{1}{3}\int\frac{a^2+4b\sin(d+ex)a+3b^2+c^2+4bc\cos(d+ex)}{\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}}dx-\frac{2(a\cos(d+ex)-c\sin(d-\right.}{(a\sin(d+ex)+b+c\cos(d+ex))^{3/2}}$$

$\downarrow$ 3042

$$\frac{\sin^{\frac{3}{2}}(d+ex)(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}\left(\frac{1}{3}\int\frac{a^2+4b\sin(d+ex)a+3b^2+c^2+4bc\cos(d+ex)}{\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}}dx-\frac{2(a\cos(d+ex)-c\sin(d-\right.}{(a\sin(d+ex)+b+c\cos(d+ex))^{3/2}}$$

$\downarrow$ 3628

$$\frac{\sin^{\frac{3}{2}}(d+ex)(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}\left(\frac{1}{3}\left(\left(a^2-b^2+c^2\right)\int\frac{1}{\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}}dx+4b\int\sqrt{b+c\right.\right.}{(a\sin(d+ex)+b+c\cos(d+ex))}$$

$\downarrow$ 3042

$$\frac{\sin^{\frac{3}{2}}(d+ex)(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}\left(\frac{1}{3}\left(\left(a^2-b^2+c^2\right)\int\frac{1}{\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}}dx+4b\int\sqrt{b+c\right.\right.}{(a\sin(d+ex)+b+c\cos(d+ex))}$$

$\downarrow$ 3598

$$\frac{\sin^{\frac{3}{2}}(d+ex)(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}\left(\frac{1}{3}\left(\left(a^2-b^2+c^2\right)\int\frac{1}{\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}}dx+\frac{4b\sqrt{a\sin(d+e}}{}\right.\right.}{(a\sin(d+ex)+b+c}$$

$\downarrow$ 3042

$$\frac{\sin^{\frac{3}{2}}(d+ex)(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}\left(\frac{1}{3}\left(\left(a^2-b^2+c^2\right)\int\frac{1}{\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}}dx+\frac{4b\sqrt{a\sin(d+e}}{}\right.\right.}{(a\sin(d+ex)+b+}$$

$\downarrow$ 3132

$$\frac{\sin^{\frac{3}{2}}(d+ex)(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}\left(\frac{1}{3}\left(\left(a^2-b^2+c^2\right)\int\frac{1}{\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}}dx+\frac{8b\sqrt{a\sin(d+e}}{}\right.\right.}{(a\sin(d+ex)+b+c\cos(}$$

3.467. $\int(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\sin^{\frac{3}{2}}(d+ex)\,dx$

$\Big\downarrow$ 3606

$$\frac{\sin^{\frac{3}{2}}(d+ex)(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}\left(\frac{1}{3}\left(\frac{(a^2-b^2+c^2)\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}\int\frac{1}{\sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+\frac{\sqrt{a^2+c^2}\cos(d+e}{b+\sqrt{a^2}}}}}{\sqrt{a\sin(d+ex)+b+c\cos(d+ex)}}\right.\right.}{(a\sin(d+ex}$$

$\Big\downarrow$ 3042

$$\frac{\sin^{\frac{3}{2}}(d+ex)(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}\left(\frac{1}{3}\left(\frac{(a^2-b^2+c^2)\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}\int\frac{1}{\sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+\frac{\sqrt{a^2+c^2}\sin(d+e}{b+\sqrt{}}}}}{\sqrt{a\sin(d+ex)+b+c\cos(d+ex)}}\right.\right.}{(a\sin(d+e}$$

$\Big\downarrow$ 3140

$$\frac{\sin^{\frac{3}{2}}(d+ex)(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}\left(\frac{1}{3}\left(\frac{2(a^2-b^2+c^2)\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}\text{EllipticF}\left(\frac{1}{2}(d+ex-\tan^{-1}(c,a)\right.}{e\sqrt{a\sin(d+ex)+b+c\cos(d+ex)}}\right.\right.}{(a\sin(d+ex)}$$

input
```
Int[(a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*Sin[d + e*x]^(3/2),x]
```

output
```
((a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*Sin[d + e*x]^(3/2)*((-2*Sqrt[
b + c*Cos[d + e*x] + a*Sin[d + e*x]]*(a*Cos[d + e*x] - c*Sin[d + e*x]))/(3
*e) + ((8*b*EllipticE[(d + e*x - ArcTan[c, a])/2, (2*Sqrt[a^2 + c^2])/(b +
 Sqrt[a^2 + c^2])]*Sqrt[b + c*Cos[d + e*x] + a*Sin[d + e*x]])/(e*Sqrt[(b +
 c*Cos[d + e*x] + a*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])]) + (2*(a^2 - b^2
+ c^2)*EllipticF[(d + e*x - ArcTan[c, a])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt
[a^2 + c^2])]*Sqrt[(b + c*Cos[d + e*x] + a*Sin[d + e*x])/(b + Sqrt[a^2 + c
^2])])/(e*Sqrt[b + c*Cos[d + e*x] + a*Sin[d + e*x]]))/3))/(b + c*Cos[d + e
*x] + a*Sin[d + e*x])^(3/2)
```

3.467. $\int (a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\sin^{\frac{3}{2}}(d+ex)\,dx$

3.467.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3598
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]   Int[Sqrt[a/(a + S
qrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - Arc
Tan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

rule 3599
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[(-(c*Cos[d + e*x] - b*Sin[d + e*x]))*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n - 1)/(e*n)), x] + Simp[1/n   Int[Simp[n*a^2 + (
n - 1)*(b^2 + c^2) + a*b*(2*n - 1)*Cos[d + e*x] + a*c*(2*n - 1)*Sin[d + e*x
], x]*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 2), x], x] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && GtQ[n, 1]
```

rule 3606
```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Simp[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]   Int[1/Sqrt[a/(a
 + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x -
 ArcTan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2
, 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

3.467. $\int (a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\sin^{\frac{3}{2}}(d+ex)\,dx$

rule 3628
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]
, x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[
A*b - a*B, 0]
```

rule 3643
```
Int[((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) + cot[(d_.) + (e_.)*(x_)]*(c_.))
^(n_)*sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbol] :> Simp[Sin[d + e*x]^n*((a +
b*Csc[d + e*x] + c*Cot[d + e*x])^n/(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n)
   Int[(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d
, e}, x] &&  !IntegerQ[n]
```

3.467.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 28.39 (sec) , antiderivative size = 21719, normalized size of antiderivative = 58.54

method	result	size
default	Expression too large to display	21719

input
```
int((a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)*sin(e*x+d)^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
result too large to display
```

3.467.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 1500, normalized size of antiderivative = 4.04

$$\int (a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\sin^{\frac{3}{2}}(d+ex)\,dx=\text{Too large to display}$$

input
```
integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)*sin(e*x+d)^(3/2),x, algorith
m="fricas")
```

output
```
1/9*(sqrt(2)*(3*a^3 + a*b^2 + 3*a*c^2 + 3*I*c^3 + I*(3*a^2 + b^2)*c)*sqrt(
I*a + c)*weierstrassPInverse(4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 + 6*I*a*c^
3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), -8/27*(-9*I*
a^5*b + 8*I*a^3*b^3 + 27*I*a*b*c^4 - 9*b*c^5 + 2*(9*a^2*b + 4*b^3)*c^3 + 6
*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 +
 3*a^2*c^4 + c^6), 1/3*(-2*I*a*b + 2*b*c + 3*(a^2 + c^2)*cos(e*x + d) - 3*
(I*a^2 + I*c^2)*sin(e*x + d))/(a^2 + c^2)) + sqrt(2)*(3*a^3 + a*b^2 + 3*a*
c^2 - 3*I*c^3 - I*(3*a^2 + b^2)*c)*sqrt(-I*a + c)*weierstrassPInverse(4/3*
(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 - 6*I*a*c^3 - 3*c^4 - 2*I*(3*a^3 - 4*a*b^2)
*c)/(a^4 + 2*a^2*c^2 + c^4), -8/27*(9*I*a^5*b - 8*I*a^3*b^3 - 27*I*a*b*c^4
 - 9*b*c^5 + 2*(9*a^2*b + 4*b^3)*c^3 - 6*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*
a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(2*I*a*b +
2*b*c + 3*(a^2 + c^2)*cos(e*x + d) - 3*(-I*a^2 - I*c^2)*sin(e*x + d))/(a^2
 + c^2)) - 12*sqrt(2)*(I*a^2*b + I*b*c^2)*sqrt(I*a + c)*weierstrassZeta(4/
3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^
2)*c)/(a^4 + 2*a^2*c^2 + c^4), -8/27*(-9*I*a^5*b + 8*I*a^3*b^3 + 27*I*a*b*
c^4 - 9*b*c^5 + 2*(9*a^2*b + 4*b^3)*c^3 + 6*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*
(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), weierstrassP
Inverse(4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^
3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), -8/27*(-9*I*a^5*b + 8*I*a^3*b^...
```

3.467.6 Sympy [F(-1)]

Timed out.

$$\int (a + c\cot(d + ex) + b\csc(d + ex))^{3/2} \sin^{\frac{3}{2}}(d + ex)\, dx = \text{Timed out}$$

input
```
integrate((a+c*cot(e*x+d)+b*csc(e*x+d))**(3/2)*sin(e*x+d)**(3/2),x)
```

output
```
Timed out
```

3.467.7 Maxima [F]

$$\int (a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\sin^{\frac{3}{2}}(d+ex)\,dx=\int (c\cot(ex+d)+b\csc(ex+d)+a)^{\frac{3}{2}}\sin(ex+d)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)*sin(e*x+d)^(3/2),x, algorith
m="maxima")
```

output
```
integrate((c*cot(e*x + d) + b*csc(e*x + d) + a)^(3/2)*sin(e*x + d)^(3/2),
x)
```

3.467.8 Giac [F]

$$\int (a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\sin^{\frac{3}{2}}(d+ex)\,dx=\int (c\cot(ex+d)+b\csc(ex+d)+a)^{\frac{3}{2}}\sin(ex+d)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)*sin(e*x+d)^(3/2),x, algorith
m="giac")
```

output
```
integrate((c*cot(e*x + d) + b*csc(e*x + d) + a)^(3/2)*sin(e*x + d)^(3/2),
x)
```

3.467.9 Mupad [F(-1)]

Timed out.

$$\int (a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\sin^{\frac{3}{2}}(d+ex)\,dx=\int \sin(d+e\,x)^{3/2}\left(a+c\cot(d+e\,x)+\frac{b}{\sin(d+e\,x)}\right)^{3/2}\,dx$$

input
```
int(sin(d + e*x)^(3/2)*(a + c*cot(d + e*x) + b/sin(d + e*x))^(3/2),x)
```

output
```
int(sin(d + e*x)^(3/2)*(a + c*cot(d + e*x) + b/sin(d + e*x))^(3/2), x)
```

3.468 $\int \sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\sqrt{\sin(d+ex)}\,d$

3.468.1 Optimal result

Integrand size = 33, antiderivative size = 118

$$\int \sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\sqrt{\sin(d+ex)}\,dx = \frac{2\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}E\left(\frac{1}{2}(d+ex-\tan^{-1}(c,a))\,|\,\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)\sqrt{\sin(d+ex)}}{e\sqrt{\frac{b+c\cos(d+ex)+a\sin(d+ex)}{b+\sqrt{a^2+c^2}}}}$$

output
```
2*(cos(1/2*d+1/2*e*x-1/2*arctan(c,a))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arcta
n(c,a))*EllipticE(sin(1/2*d+1/2*e*x-1/2*arctan(c,a)),2^(1/2)*((a^2+c^2)^(1
/2)/(b+(a^2+c^2)^(1/2)))^(1/2))*(a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)*sin(e*
x+d)^(1/2)/e/((b+c*cos(e*x+d)+a*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(1/2)
```

3.468.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 16.94 (sec) , antiderivative size = 5835, normalized size of antiderivative = 49.45

$$\int \sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\sqrt{\sin(d+ex)}\,dx = \text{Result too large to show}$$

input
```
Integrate[Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]*Sqrt[Sin[d + e*x]],x]
```

output
```
Result too large to show
```

3.468.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3042, 3643, 3042, 3598, 3042, 3132}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sin(d+ex)}\sqrt{a+b\csc(d+ex)+c\cot(d+ex)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin(d+ex)}\sqrt{a+b\csc(d+ex)+c\cot(d+ex)}dx$$

$$\Big\downarrow \text{3643}$$

$$\frac{\sqrt{\sin(d+ex)}\sqrt{a+b\csc(d+ex)+c\cot(d+ex)}\int \sqrt{b+c\cos(d+ex)+a\sin(d+ex)}dx}{\sqrt{a\sin(d+ex)+b+c\cos(d+ex)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sin(d+ex)}\sqrt{a+b\csc(d+ex)+c\cot(d+ex)}\int \sqrt{b+c\cos(d+ex)+a\sin(d+ex)}dx}{\sqrt{a\sin(d+ex)+b+c\cos(d+ex)}}$$

$$\Big\downarrow \text{3598}$$

$$\frac{\sqrt{\sin(d+ex)}\sqrt{a+b\csc(d+ex)+c\cot(d+ex)}\int \sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+\frac{\sqrt{a^2+c^2}\cos(d+ex-\tan^{-1}(c,a))}{b+\sqrt{a^2+c^2}}}dx}{\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\sin(d+ex)}\sqrt{a+b\csc(d+ex)+c\cot(d+ex)}\int \sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+\frac{\sqrt{a^2+c^2}\sin\left(d+ex-\tan^{-1}(c,a)+\frac{\pi}{2}\right)}{b+\sqrt{a^2+c^2}}}dx}{\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}}$$

$$\Big\downarrow \text{3132}$$

$$\frac{2\sqrt{\sin(d+ex)}\sqrt{a+b\csc(d+ex)+c\cot(d+ex)}E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}}$$

input
```
Int[Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]*Sqrt[Sin[d + e*x]],x]
```

3.468. $\int \sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\sqrt{\sin(d+ex)}\,dx$

output
```
(2*Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]*EllipticE[(d + e*x - ArcTan[c
, a])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2 + c^2])]*Sqrt[Sin[d + e*x]])/(e
*Sqrt[(b + c*Cos[d + e*x] + a*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])])
```

3.468.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3598
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]   Int[Sqrt[a/(a + S
qrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - Arc
Tan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

rule 3643
```
Int[((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) + cot[(d_.) + (e_.)*(x_)]*(c_.))
^(n_)*sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbol] :> Simp[Sin[d + e*x]^n*((a +
b*Csc[d + e*x] + c*Cot[d + e*x])^n/(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n)
   Int[(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d
, e}, x] &&  !IntegerQ[n]
```

3.468.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 23.30 (sec) , antiderivative size = 1851, normalized size of antiderivative = 15.69

method	result	size
risch	Expression too large to display	1851
default	Expression too large to display	12826

input
```
int((a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)*sin(e*x+d)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
(c*exp(I*(e*x+d))^2+2*b*exp(I*(e*x+d))-I*a*exp(I*(e*x+d))^2+c+I*a)/e*2^(1/
2)*((I*exp(I*(e*x+d))^2*c+2*I*b*exp(I*(e*x+d))+a*exp(I*(e*x+d))^2+I*c-a)/(
exp(I*(e*x+d))^2-1))^(1/2)/(I*exp(I*(e*x+d))^2*c+2*I*b*exp(I*(e*x+d))+a*ex
p(I*(e*x+d))^2+I*c-a)*(-I*(exp(I*(e*x+d))^2-1)/exp(I*(e*x+d)))^(1/2)-I/e*(
2*I*b*(-b+(-a^2+b^2-c^2)^(1/2))/(I*a-c)*((exp(I*(e*x+d))+(-b+(-a^2+b^2-c^2
)^(1/2))/(I*a-c))/(-b+(-a^2+b^2-c^2)^(1/2))*(I*a-c))^(1/2)*((exp(I*(e*x+d)
)-(b+(-a^2+b^2-c^2)^(1/2))/(I*a-c))/(-(-b+(-a^2+b^2-c^2)^(1/2))/(I*a-c)-(b
+(-a^2+b^2-c^2)^(1/2))/(I*a-c)))^(1/2)*(-exp(I*(e*x+d))/(-b+(-a^2+b^2-c^2)
^(1/2))*(I*a-c))^(1/2)/(exp(I*(e*x+d))^3*c+2*b*exp(I*(e*x+d))^2-I*exp(I*(e
*x+d))^3*a+c*exp(I*(e*x+d))+I*a*exp(I*(e*x+d)))^(1/2)*EllipticF(((exp(I*(e
*x+d))+(-b+(-a^2+b^2-c^2)^(1/2))/(I*a-c))/(-b+(-a^2+b^2-c^2)^(1/2))*(I*a-c
))^(1/2),(-(-b+(-a^2+b^2-c^2)^(1/2))/(I*a-c)/(-(-b+(-a^2+b^2-c^2)^(1/2))/(
I*a-c)-(b+(-a^2+b^2-c^2)^(1/2))/(I*a-c)))^(1/2))+(I*c-a)*(-2*(c*exp(I*(e*x
+d))^2+2*b*exp(I*(e*x+d))-I*a*exp(I*(e*x+d))^2+c+I*a)/(I*a+c)/(exp(I*(e*x+
d))*(c*exp(I*(e*x+d))^2+2*b*exp(I*(e*x+d))-I*a*exp(I*(e*x+d))^2+c+I*a))^(1
/2)+2*(1/(I*a+c)*(I*a-c)+(-2*I*a+2*c)/(I*a+c))*(-b+(-a^2+b^2-c^2)^(1/2))/(
I*a-c)*((exp(I*(e*x+d))+(-b+(-a^2+b^2-c^2)^(1/2))/(I*a-c))/(-b+(-a^2+b^2-c
^2)^(1/2))*(I*a-c))^(1/2)*((exp(I*(e*x+d))-(b+(-a^2+b^2-c^2)^(1/2))/(I*a-c
))/(-(-b+(-a^2+b^2-c^2)^(1/2))/(I*a-c)-(b+(-a^2+b^2-c^2)^(1/2))/(I*a-c)))^
(1/2)*(-exp(I*(e*x+d))/(-b+(-a^2+b^2-c^2)^(1/2))*(I*a-c))^(1/2)/(exp(I*...
```

3.468.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 1371, normalized size of antiderivative = 11.62

$$\int \sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\sqrt{\sin(d+ex)}\,dx = \text{Too large to display}$$

input
```
integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)*sin(e*x+d)^(1/2),x, algorith
m="fricas")
```

output
```
1/3*(sqrt(2)*(a*b + I*b*c)*sqrt(I*a + c)*weierstrassPInverse(4/3*(3*a^4 -
4*a^2*b^2 + 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4
+ 2*a^2*c^2 + c^4), -8/27*(-9*I*a^5*b + 8*I*a^3*b^3 + 27*I*a*b*c^4 - 9*b*c
^5 + 2*(9*a^2*b + 4*b^3)*c^3 + 6*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b -
8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(-2*I*a*b + 2*b*c +
 3*(a^2 + c^2)*cos(e*x + d) - 3*(I*a^2 + I*c^2)*sin(e*x + d))/(a^2 + c^2))
 + sqrt(2)*(a*b - I*b*c)*sqrt(-I*a + c)*weierstrassPInverse(4/3*(3*a^4 - 4
*a^2*b^2 + 4*b^2*c^2 - 6*I*a*c^3 - 3*c^4 - 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 +
 2*a^2*c^2 + c^4), -8/27*(9*I*a^5*b - 8*I*a^3*b^3 - 27*I*a*b*c^4 - 9*b*c^5
 + 2*(9*a^2*b + 4*b^3)*c^3 - 6*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*
a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(2*I*a*b + 2*b*c + 3*
(a^2 + c^2)*cos(e*x + d) - 3*(-I*a^2 - I*c^2)*sin(e*x + d))/(a^2 + c^2)) -
 3*sqrt(2)*(I*a^2 + I*c^2)*sqrt(I*a + c)*weierstrassZeta(4/3*(3*a^4 - 4*a^
2*b^2 + 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*
a^2*c^2 + c^4), -8/27*(-9*I*a^5*b + 8*I*a^3*b^3 + 27*I*a*b*c^4 - 9*b*c^5 +
 2*(9*a^2*b + 4*b^3)*c^3 + 6*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*a^
2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), weierstrassPInverse(4/3*(3*
a^4 - 4*a^2*b^2 + 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)
/(a^4 + 2*a^2*c^2 + c^4), -8/27*(-9*I*a^5*b + 8*I*a^3*b^3 + 27*I*a*b*c^4 -
 9*b*c^5 + 2*(9*a^2*b + 4*b^3)*c^3 + 6*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9...
```

3.468.6 Sympy [F]

$$\int \sqrt{a + c\cot(d + ex) + b\csc(d + ex)}\sqrt{\sin(d + ex)}\,dx$$
$$= \int \sqrt{a + b\csc(d + ex) + c\cot(d + ex)}\sqrt{\sin(d + ex)}\,dx$$

input
```
integrate((a+c*cot(e*x+d)+b*csc(e*x+d))**(1/2)*sin(e*x+d)**(1/2),x)
```

output
```
Integral(sqrt(a + b*csc(d + e*x) + c*cot(d + e*x))*sqrt(sin(d + e*x)), x)
```

3.468.7 Maxima [F]

$$\int \sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\sqrt{\sin(d+ex)}\,dx = \int \sqrt{c\cot(ex+d)+b\csc(ex+d)+a}\sqrt{\sin(ex+d)}\,dx$$

input
```
integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)*sin(e*x+d)^(1/2),x, algorith
m="maxima")
```

output
```
integrate(sqrt(c*cot(e*x + d) + b*csc(e*x + d) + a)*sqrt(sin(e*x + d)), x)
```

3.468.8 Giac [F]

$$\int \sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\sqrt{\sin(d+ex)}\,dx = \int \sqrt{c\cot(ex+d)+b\csc(ex+d)+a}\sqrt{\sin(ex+d)}\,dx$$

input
```
integrate((a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)*sin(e*x+d)^(1/2),x, algorith
m="giac")
```

output
```
integrate(sqrt(c*cot(e*x + d) + b*csc(e*x + d) + a)*sqrt(sin(e*x + d)), x)
```

3.468.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\sqrt{\sin(d+ex)}\,dx = \int \sqrt{\sin(d+e\,x)}\,\sqrt{a+c\cot(d+e\,x)+\frac{b}{\sin(d+e\,x)}}\,dx$$

input
```
int(sin(d + e*x)^(1/2)*(a + c*cot(d + e*x) + b/sin(d + e*x))^(1/2),x)
```

output
```
int(sin(d + e*x)^(1/2)*(a + c*cot(d + e*x) + b/sin(d + e*x))^(1/2), x)
```

3.469 $\int \frac{1}{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\sqrt{\sin(d+ex)}}\,dx$

3.469.1 Optimal result

Integrand size = 33, antiderivative size = 118

$$\int \frac{1}{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\sqrt{\sin(d+ex)}}\,dx = \frac{2\,\text{EllipticF}\left(\frac{1}{2}(d+ex-\tan^{-1}(c,a)),\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)\sqrt{\frac{b+c\cos(d+ex)+a\sin(d+ex)}{b+\sqrt{a^2+c^2}}}}{e\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\sqrt{\sin(d+ex)}}$$

output
```
2*(cos(1/2*d+1/2*e*x-1/2*arctan(c,a))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arcta
n(c,a))*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan(c,a)),2^(1/2)*((a^2+c^2)^(1
/2)/(b+(a^2+c^2)^(1/2)))^(1/2))*((b+c*cos(e*x+d)+a*sin(e*x+d))/(b+(a^2+c^2
)^(1/2)))^(1/2)/e/(a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)/sin(e*x+d)^(1/2)
```

3.469.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 18.77 (sec) , antiderivative size = 519, normalized size of antiderivative = 4.40

$$\int \frac{1}{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\sqrt{\sin(d+ex)}}\,dx = \frac{4\left(-ia-b+c+i\sqrt{a^2-b^2+c^2}\right)\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{\left(-a-ib+ic+\sqrt{a^2-b^2+c^2}\right)(-\cos(d+ex)+i\sin(d+ex))}{-a+ib-ic+\sqrt{a^2-b^2+c^2}}}\right),\frac{ib+\sqrt{a^2}\cdots}{ib-\sqrt{a^2}\cdots}\right.}{(a+ib-ic\cdots}$$

input
```
Integrate[1/(Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]*Sqrt[Sin[d + e*x]])
,x]
```

output
```
(4*((-I)*a - b + c + I*Sqrt[a^2 - b^2 + c^2])*EllipticF[ArcSin[Sqrt[((-a -
 I*b + I*c + Sqrt[a^2 - b^2 + c^2])*(-Cos[d + e*x] + I*Sin[d + e*x]))/(-a
+ I*b - I*c + Sqrt[a^2 - b^2 + c^2])]], (I*b + Sqrt[a^2 - b^2 + c^2])/(I*b
 - Sqrt[a^2 - b^2 + c^2])]*Sqrt[((-a - I*b + I*c + Sqrt[a^2 - b^2 + c^2])*
(-Cos[d + e*x] + I*Sin[d + e*x]))/(-a + I*b - I*c + Sqrt[a^2 - b^2 + c^2])
]*(Cos[d + e*x] + I*Sin[d + e*x])*Sqrt[((-I)*(a + Sqrt[a^2 - b^2 + c^2] +
(b - c)*Tan[(d + e*x)/2]))/((a - I*b + I*c + Sqrt[a^2 - b^2 + c^2])*(-I +
Tan[(d + e*x)/2]))]*Sqrt[((-I)*(-a + Sqrt[a^2 - b^2 + c^2] + (-b + c)*Tan[
(d + e*x)/2]))/((-a + I*b - I*c + Sqrt[a^2 - b^2 + c^2])*(-I + Tan[(d + e*
x)/2]))])/((a + I*b - I*c - Sqrt[a^2 - b^2 + c^2])*e*Sqrt[a + c*Cot[d + e*
x] + b*Csc[d + e*x]]*Sqrt[Sin[d + e*x]])
```

3.469.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3042, 3643, 3042, 3606, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\sin(d+ex)}\sqrt{a+b\csc(d+ex)+c\cot(d+ex)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{\sin(d+ex)}\sqrt{a+b\csc(d+ex)+c\cot(d+ex)}}dx$$

$$\Big\downarrow \text{ 3643}$$

$$\frac{\sqrt{a\sin(d+ex)+b+c\cos(d+ex)}\int \frac{1}{\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}}dx}{\sqrt{\sin(d+ex)}\sqrt{a+b\csc(d+ex)+c\cot(d+ex)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{a\sin(d+ex)+b+c\cos(d+ex)}\int \frac{1}{\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}}dx}{\sqrt{\sin(d+ex)}\sqrt{a+b\csc(d+ex)+c\cot(d+ex)}}$$

$$\Big\downarrow \text{ 3606}$$

$$\frac{\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}\int \frac{1}{\sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+\frac{\sqrt{a^2+c^2}\cos\left(d+ex-\tan^{-1}(c,a)\right)}{b+\sqrt{a^2+c^2}}}}dx}{\sqrt{\sin(d+ex)}\sqrt{a+b\csc(d+ex)+c\cot(d+ex)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}\int \frac{1}{\sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+\frac{\sqrt{a^2+c^2}\sin\left(d+ex-\tan^{-1}(c,a)+\frac{\pi}{2}\right)}{b+\sqrt{a^2+c^2}}}}dx}{\sqrt{\sin(d+ex)}\sqrt{a+b\csc(d+ex)+c\cot(d+ex)}}$$

$$\Big\downarrow \text{3140}$$

$$\frac{2\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}\operatorname{EllipticF}\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right),\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{\sin(d+ex)}\sqrt{a+b\csc(d+ex)+c\cot(d+ex)}}$$

input
```
Int[1/(Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]*Sqrt[Sin[d + e*x]]),x]
```

output
```
(2*EllipticF[(d + e*x - ArcTan[c, a])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2
 + c^2])]*Sqrt[(b + c*Cos[d + e*x] + a*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])
])/(e*Sqrt[a + c*Cot[d + e*x] + b*Csc[d + e*x]]*Sqrt[Sin[d + e*x]])
```

3.469.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3606
```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Simp[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]   Int[1/Sqrt[a/(a
 + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x -
 ArcTan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2
, 0] && NeQ[b^2 + c^2, 0] && !GtQ[a + Sqrt[b^2 + c^2], 0]
```

3.469. $\int \frac{1}{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\sqrt{\sin(d+ex)}}\,dx$

rule 3643
```
Int[((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) + cot[(d_.) + (e_.)*(x_)]*(c_.))
^(n_)*sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbol] :> Simp[Sin[d + e*x]^n*((a +
b*Csc[d + e*x] + c*Cot[d + e*x])^n/(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n)
   Int[(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d
, e}, x] &&  !IntegerQ[n]
```

3.469.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 28.57 (sec) , antiderivative size = 716, normalized size of antiderivative = 6.07

method	result
default	$-\frac{2i\,\mathrm{EllipticF}\left(\frac{\sqrt{2}\sqrt{-\frac{(i\cos(ex+d)+i-\sin(ex+d))^2\left(ib-ic-\sqrt{a^2-b^2+c^2}-a\right)}{(1+\cos(ex+d))\left(ib-ic+\sqrt{a^2-b^2+c^2}+a\right)}}}{2},\sqrt{\frac{\left(ib-ic+\sqrt{a^2-b^2+c^2}+a\right)\left(ib-ic+\sqrt{a^2-b^2+c^2}-a\right)}{\left(ib-ic-\sqrt{a^2-b^2+c^2}-a\right)\left(ib-ic-\sqrt{a^2-b^2+c^2}+a\right)}}\right)\sqrt{-\frac{i(b}{}}}{}$

input
```
int(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)/sin(e*x+d)^(1/2),x,method=_RETUR
NVERBOSE)
```

output
```
-2*I/e*EllipticF(1/2*2^(1/2)*(-(I*cos(e*x+d)+I-sin(e*x+d))^2/(1+cos(e*x+d)
)*(I*b-I*c-(a^2-b^2+c^2)^(1/2)-a)/(I*b-I*c+(a^2-b^2+c^2)^(1/2)+a))^(1/2),(
(I*b-I*c+(a^2-b^2+c^2)^(1/2)+a)*(I*b-I*c+(a^2-b^2+c^2)^(1/2)-a)/(I*b-I*c-(
a^2-b^2+c^2)^(1/2)-a)/(I*b-I*c-(a^2-b^2+c^2)^(1/2)+a))^(1/2))*(-I*(b*cos(e
*x+d)-c*cos(e*x+d)-(a^2-b^2+c^2)^(1/2)*sin(e*x+d)-a*sin(e*x+d)-b+c)/(I*sin
(e*x+d)-cos(e*x+d)+1)/(I*b-I*c+(a^2-b^2+c^2)^(1/2)+a))^(1/2)*(-I*((a^2-b^2
+c^2)^(1/2)*sin(e*x+d)+b*cos(e*x+d)-c*cos(e*x+d)-a*sin(e*x+d)-b+c)/(I*sin(
e*x+d)-cos(e*x+d)+1)/(I*b-I*c-(a^2-b^2+c^2)^(1/2)+a))^(1/2)*(-(I*cos(e*x+d
)+I-sin(e*x+d))^2/(1+cos(e*x+d))*(I*b-I*c-(a^2-b^2+c^2)^(1/2)-a)/(I*b-I*c+
(a^2-b^2+c^2)^(1/2)+a))^(1/2)*(1+cos(e*x+d))^2*(cos(e*x+d)-1)^2*(I*b*cos(e
*x+d)-I*c*cos(e*x+d)-I*(a^2-b^2+c^2)^(1/2)*sin(e*x+d)-I*sin(e*x+d)*a+cos(e
*x+d)*(a^2-b^2+c^2)^(1/2)+a*cos(e*x+d)+b*sin(e*x+d)-c*sin(e*x+d))*2^(1/2)*
(a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)/sin(e*x+d)^(7/2)/(b+c*cos(e*x+d)+a*sin
(e*x+d))/(I*b-I*c-(a^2-b^2+c^2)^(1/2)-a)
```

3.469.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 506, normalized size of antiderivative = 4.29

$$\int \frac{1}{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\sqrt{\sin(d+ex)}}\,dx$$
$$= \frac{\sqrt{2}(a+ic)\sqrt{ia+c}\text{weierstrassPInverse}\left(\frac{4\left(3a^4-4a^2b^2+4b^2c^2+6iac^3-3c^4+2i\left(3a^3-4ab^2\right)c\right)}{3\left(a^4+2a^2c^2+c^4\right)}, -\frac{8\left(-9ia^5b+8ia^3b^3+27iabc^4\right.}{}\right.}{}$$

input
```
integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)/sin(e*x+d)^(1/2),x, algori
thm="fricas")
```

output
```
(sqrt(2)*(a + I*c)*sqrt(I*a + c)*weierstrassPInverse(4/3*(3*a^4 - 4*a^2*b^
2 + 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*
c^2 + c^4), -8/27*(-9*I*a^5*b + 8*I*a^3*b^3 + 27*I*a*b*c^4 - 9*b*c^5 + 2*(
9*a^2*b + 4*b^3)*c^3 + 6*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*a^2*b^
3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(-2*I*a*b + 2*b*c + 3*(a^2
+ c^2)*cos(e*x + d) - 3*(I*a^2 + I*c^2)*sin(e*x + d))/(a^2 + c^2)) + sqrt(
2)*(a - I*c)*sqrt(-I*a + c)*weierstrassPInverse(4/3*(3*a^4 - 4*a^2*b^2 + 4
*b^2*c^2 - 6*I*a*c^3 - 3*c^4 - 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 +
 c^4), -8/27*(9*I*a^5*b - 8*I*a^3*b^3 - 27*I*a*b*c^4 - 9*b*c^5 + 2*(9*a^2*
b + 4*b^3)*c^3 - 6*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*a^2*b^3)*c)/
(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(2*I*a*b + 2*b*c + 3*(a^2 + c^2)*
cos(e*x + d) - 3*(-I*a^2 - I*c^2)*sin(e*x + d))/(a^2 + c^2)))/((a^2 + c^2)
*e)
```

3.469.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\sqrt{\sin(d+ex)}}\,dx$$
$$= \int \frac{1}{\sqrt{a+b\csc(d+ex)+c\cot(d+ex)}\sqrt{\sin(d+ex)}}\,dx$$

input
```
integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))**(1/2)/sin(e*x+d)**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*csc(d + e*x) + c*cot(d + e*x))*sqrt(sin(d + e*x))),
 x)
```

3.469.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\sqrt{\sin(d+ex)}}\,dx = \int \frac{1}{\sqrt{c\cot(ex+d)+b\csc(ex+d)+a}\sqrt{\sin(ex+d)}}\,dx$$

input
```
integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)/sin(e*x+d)^(1/2),x, algori
thm="maxima")
```

output
```
integrate(1/(sqrt(c*cot(e*x + d) + b*csc(e*x + d) + a)*sqrt(sin(e*x + d)))
, x)
```

3.469.8 Giac [F]

$$\int \frac{1}{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\sqrt{\sin(d+ex)}}\,dx = \int \frac{1}{\sqrt{c\cot(ex+d)+b\csc(ex+d)+a}\sqrt{\sin(ex+d)}}\,dx$$

input
```
integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(1/2)/sin(e*x+d)^(1/2),x, algori
thm="giac")
```

output
```
integrate(1/(sqrt(c*cot(e*x + d) + b*csc(e*x + d) + a)*sqrt(sin(e*x + d)))
, x)
```

3.469.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\sqrt{\sin(d+ex)}}\,dx = \int \frac{1}{\sqrt{\sin(d+e\,x)}\,\sqrt{a+c\cot(d+e\,x)+\frac{b}{\sin(d+e\,x)}}}\,dx$$

input
```
int(1/(sin(d + e*x)^(1/2)*(a + c*cot(d + e*x) + b/sin(d + e*x))^(1/2)),x)
```

output
```
int(1/(sin(d + e*x)^(1/2)*(a + c*cot(d + e*x) + b/sin(d + e*x))^(1/2)), x)
```

3.469. $\int \frac{1}{\sqrt{a+c\cot(d+ex)+b\csc(d+ex)}\sqrt{\sin(d+ex)}}\,dx$

3.470 $\int \frac{1}{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\sin^{\frac{3}{2}}(d+ex)}\,dx$

3.470.1 Optimal result

Integrand size = 33, antiderivative size = 240

$$\int \frac{1}{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\sin^{\frac{3}{2}}(d+ex)}\,dx = -\frac{2E\left(\frac{1}{2}(d+ex-\tan^{-1}(c,a))\,|\,\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)(b+c\cos(d+ex)+a\sin(d+ex))^2}{(a^2-b^2+c^2)\,e(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\sin^{\frac{3}{2}}(d+ex)\sqrt{\frac{b+c\cos(d+ex)+a\sin(d+ex)}{b+\sqrt{a^2+c^2}}}} - \frac{2(b+c\cos(d+ex)+a\sin(d+ex))(a\cos(d+ex)-c\sin(d+ex))}{(a^2-b^2+c^2)\,e(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\sin^{\frac{3}{2}}(d+ex)}$$

output
```
-2*(b+c*cos(e*x+d)+a*sin(e*x+d))*(a*cos(e*x+d)-c*sin(e*x+d))/(a^2-b^2+c^2)
/e/(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)/sin(e*x+d)^(3/2)-2*(cos(1/2*d+1/2*e
*x-1/2*arctan(c,a))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(c,a))*EllipticE(
sin(1/2*d+1/2*e*x-1/2*arctan(c,a)),2^(1/2)*((a^2+c^2)^(1/2)/(b+(a^2+c^2)^(
1/2)))^(1/2))*(b+c*cos(e*x+d)+a*sin(e*x+d))^2/(a^2-b^2+c^2)/e/(a+c*cot(e*x
+d)+b*csc(e*x+d))^(3/2)/sin(e*x+d)^(3/2)/((b+c*cos(e*x+d)+a*sin(e*x+d))/(b
+(a^2+c^2)^(1/2)))^(1/2)
```

3.470.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 17.46 (sec) , antiderivative size = 5959, normalized size of antiderivative = 24.83

$$\int \frac{1}{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\sin^{\frac{3}{2}}(d+ex)}\,dx = \text{Result too large to show}$$

input
```
Integrate[1/((a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*Sin[d + e*x]^(3/2
)),x]
```

output
```
Result too large to show
```

3.470.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3042, 3643, 3042, 3607, 3042, 3598, 3042, 3132}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sin^{\frac{3}{2}}(d+ex)(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin(d+ex)^{3/2}(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}}dx$$

$$\Big\downarrow \text{ 3643}$$

$$\frac{(a\sin(d+ex)+b+c\cos(d+ex))^{3/2}\int \frac{1}{(b+c\cos(d+ex)+a\sin(d+ex))^{3/2}}dx}{\sin^{\frac{3}{2}}(d+ex)(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(a\sin(d+ex)+b+c\cos(d+ex))^{3/2}\int \frac{1}{(b+c\cos(d+ex)+a\sin(d+ex))^{3/2}}dx}{\sin^{\frac{3}{2}}(d+ex)(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}}$$

$$\Big\downarrow \text{ 3607}$$

$$\frac{(a\sin(d+ex)+b+c\cos(d+ex))^{3/2}\left(-\frac{\int\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}dx}{a^2-b^2+c^2}-\frac{2(a\cos(d+ex)-c\sin(d+ex))}{e(a^2-b^2+c^2)\sqrt{a\sin(d+ex)+b+c\cos(d+ex)}}\right)}{\sin^{\frac{3}{2}}(d+ex)(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{(a\sin(d+ex)+b+c\cos(d+ex))^{3/2}\left(-\frac{\int\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}dx}{a^2-b^2+c^2}-\frac{2(a\cos(d+ex)-c\sin(d+ex))}{e(a^2-b^2+c^2)\sqrt{a\sin(d+ex)+b+c\cos(d+ex)}}\right)}{\sin^{\frac{3}{2}}(d+ex)(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}}$$

$\Big\downarrow$ 3598

$$\frac{(a\sin(d+ex)+b+c\cos(d+ex))^{3/2}\left(-\frac{\sqrt{a\sin(d+ex)+b+c\cos(d+ex)}\int\sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+\frac{\sqrt{a^2+c^2}\cos\left(d+ex-\tan^{-1}(c,a)\right)}{b+\sqrt{a^2+c^2}}}dx}{(a^2-b^2+c^2)\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}}-\frac{}{e(a^2-b}\right.}{\sin^{\frac{3}{2}}(d+ex)(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{(a\sin(d+ex)+b+c\cos(d+ex))^{3/2}\left(-\frac{\sqrt{a\sin(d+ex)+b+c\cos(d+ex)}\int\sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+\frac{\sqrt{a^2+c^2}\sin\left(d+ex-\tan^{-1}(c,a)+\frac{\pi}{2}\right)}{b+\sqrt{a^2+c^2}}}dx}{(a^2-b^2+c^2)\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}}-\frac{}{e(a^2}\right.}{\sin^{\frac{3}{2}}(d+ex)(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}}$$

$\Big\downarrow$ 3132

$$\frac{(a\sin(d+ex)+b+c\cos(d+ex))^{3/2}\left(-\frac{2\sqrt{a\sin(d+ex)+b+c\cos(d+ex)}E\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right)|\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e(a^2-b^2+c^2)\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}}-\frac{2(a\cos(d}{e(a^2-b^2+c^2)\sqrt{a}}\right.}{\sin^{\frac{3}{2}}(d+ex)(a+b\csc(d+ex)+c\cot(d+ex))^{3/2}}$$

input
```
Int[1/((a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*Sin[d + e*x]^(3/2)),x]
```

output
```
((b + c*Cos[d + e*x] + a*Sin[d + e*x])^(3/2)*((-2*EllipticE[(d + e*x - Arc
Tan[c, a])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2 + c^2])]*Sqrt[b + c*Cos[d
+ e*x] + a*Sin[d + e*x]])/((a^2 - b^2 + c^2)*e*Sqrt[(b + c*Cos[d + e*x] +
a*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])]) - (2*(a*Cos[d + e*x] - c*Sin[d + e
*x]))/((a^2 - b^2 + c^2)*e*Sqrt[b + c*Cos[d + e*x] + a*Sin[d + e*x]])))/((
a + c*Cot[d + e*x] + b*Csc[d + e*x])^(3/2)*Sin[d + e*x]^(3/2))
```

3.470. $\int \frac{1}{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\sin^{\frac{3}{2}}(d+ex)}\,dx$

3.470.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3598
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]   Int[Sqrt[a/(a + S
qrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - Arc
Tan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

rule 3607
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-3/2), x_Symbol] :> Simp[2*((c*Cos[d + e*x] - b*Sin[d + e*x])/(e*(a^2 - b^
2 - c^2)*Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]])), x] + Simp[1/(a^2 - b^
2 - c^2)   Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x], x] /; FreeQ[{
a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

rule 3643
```
Int[((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) + cot[(d_.) + (e_.)*(x_)]*(c_.))
^(n_)*sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbol] :> Simp[Sin[d + e*x]^n*((a +
b*Csc[d + e*x] + c*Cot[d + e*x])^n/(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n)
   Int[(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d
, e}, x] &&  !IntegerQ[n]
```

3.470.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 26.13 (sec) , antiderivative size = 12838, normalized size of antiderivative = 53.49

method	result	size
default	Expression too large to display	12838

3.470. $\int \frac{1}{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\sin^{\frac{3}{2}}(d+ex)}\,dx$

input
```
int(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)/sin(e*x+d)^(3/2),x,method=_RETUR
NVERBOSE)
```

output
```
result too large to display
```

3.470.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.15 (sec) , antiderivative size = 1746, normalized size of antiderivative = 7.28

$$\int \frac{1}{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\sin^{\frac{3}{2}}(d+ex)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)/sin(e*x+d)^(3/2),x, algori
thm="fricas")
```

output
```
-1/3*((sqrt(2)*(a*b*c + I*b*c^2)*cos(e*x + d) + sqrt(2)*(a^2*b + I*a*b*c)*
sin(e*x + d) + sqrt(2)*(a*b^2 + I*b^2*c))*sqrt(I*a + c)*weierstrassPInvers
e(4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 + 6*I*a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*
a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), -8/27*(-9*I*a^5*b + 8*I*a^3*b^3 + 27*I*
a*b*c^4 - 9*b*c^5 + 2*(9*a^2*b + 4*b^3)*c^3 + 6*I*(3*a^3*b - 4*a*b^3)*c^2
+ 3*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^6), 1/3*(-2*
I*a*b + 2*b*c + 3*(a^2 + c^2)*cos(e*x + d) - 3*(I*a^2 + I*c^2)*sin(e*x + d
))/(a^2 + c^2)) + (sqrt(2)*(a*b*c - I*b*c^2)*cos(e*x + d) + sqrt(2)*(a^2*b
 - I*a*b*c)*sin(e*x + d) + sqrt(2)*(a*b^2 - I*b^2*c))*sqrt(-I*a + c)*weier
strassPInverse(4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 - 6*I*a*c^3 - 3*c^4 - 2*
I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), -8/27*(9*I*a^5*b - 8*I*a^3
*b^3 - 27*I*a*b*c^4 - 9*b*c^5 + 2*(9*a^2*b + 4*b^3)*c^3 - 6*I*(3*a^3*b - 4
*a*b^3)*c^2 + 3*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^2 + 3*a^2*c^4 + c^
6), 1/3*(2*I*a*b + 2*b*c + 3*(a^2 + c^2)*cos(e*x + d) - 3*(-I*a^2 - I*c^2)
*sin(e*x + d))/(a^2 + c^2)) + 3*(sqrt(2)*(-I*a^2*c - I*c^3)*cos(e*x + d) +
 sqrt(2)*(-I*a^3 - I*a*c^2)*sin(e*x + d) + sqrt(2)*(-I*a^2*b - I*b*c^2))*s
qrt(I*a + c)*weierstrassZeta(4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 + 6*I*a*c^
3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), -8/27*(-9*I*
a^5*b + 8*I*a^3*b^3 + 27*I*a*b*c^4 - 9*b*c^5 + 2*(9*a^2*b + 4*b^3)*c^3 + 6
*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c^...
```

3.470.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\sin^{\frac{3}{2}}(d+ex)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))**(3/2)/sin(e*x+d)**(3/2),x)
```

output
```
Timed out
```

3.470.7 Maxima [F]

$$\int \frac{1}{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\sin^{\frac{3}{2}}(d+ex)}\,dx = \int \frac{1}{(c\cot(ex+d)+b\csc(ex+d)+a)^{\frac{3}{2}}\sin(ex -}$$

input
```
integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)/sin(e*x+d)^(3/2),x, algori
thm="maxima")
```

output
```
integrate(1/((c*cot(e*x + d) + b*csc(e*x + d) + a)^(3/2)*sin(e*x + d)^(3/2
)), x)
```

3.470.8 Giac [F]

$$\int \frac{1}{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\sin^{\frac{3}{2}}(d+ex)}\,dx = \int \frac{1}{(c\cot(ex+d)+b\csc(ex+d)+a)^{\frac{3}{2}}\sin(ex -}$$

input
```
integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(3/2)/sin(e*x+d)^(3/2),x, algori
thm="giac")
```

output
```
integrate(1/((c*cot(e*x + d) + b*csc(e*x + d) + a)^(3/2)*sin(e*x + d)^(3/2
)), x)
```

3.470.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+c\cot(d+ex)+b\csc(d+ex))^{3/2}\sin^{\frac{3}{2}}(d+ex)}\,dx=\int \frac{1}{\sin\left(d+e\,x\right)^{3/2}\,\left(a+c\cot\left(d+e\,x\right)+\frac{b}{\sin(d+e\,x)}\right.}$$

input
```
int(1/(sin(d + e*x)^(3/2)*(a + c*cot(d + e*x) + b/sin(d + e*x))^(3/2)),x)
```

output
```
int(1/(sin(d + e*x)^(3/2)*(a + c*cot(d + e*x) + b/sin(d + e*x))^(3/2)), x)
```

3.471 $\int \frac{1}{(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}\sin^{\frac{5}{2}}(d+ex)}\,dx$

3.471.1 Optimal result

Integrand size = 33, antiderivative size = 492

$$\int \frac{1}{(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}\sin^{\frac{5}{2}}(d+ex)}\,dx = \frac{8bE\left(\frac{1}{2}(d+ex-\tan^{-1}(c,a))\,|\,\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{3\left(a^2-b^2+c^2\right)^2 e(a+c\cot(d+ex)+b\csc(d+}$$

$$+\frac{2\,\text{EllipticF}\left(\frac{1}{2}(d+ex-\tan^{-1}(c,a)),\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)(b+c\cos(d+ex)+a\sin(d+ex))^2\sqrt{\frac{b+c\cos(d+ex)+a\sin(d+ex}{b+\sqrt{a^2+c^2}}}}{3\left(a^2-b^2+c^2\right)e(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}\sin^{\frac{5}{2}}(d+ex)}$$

$$-\frac{2(b+c\cos(d+ex)+a\sin(d+ex))(a\cos(d+ex)-c\sin(d+ex))}{3\left(a^2-b^2+c^2\right)e(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}\sin^{\frac{5}{2}}(d+ex)}$$

$$+\frac{8(b+c\cos(d+ex)+a\sin(d+ex))^2(ab\cos(d+ex)-bc\sin(d+ex))}{3\left(a^2-b^2+c^2\right)^2 e(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}\sin^{\frac{5}{2}}(d+ex)}$$

output
```
-2/3*(b+c*cos(e*x+d)+a*sin(e*x+d))*(a*cos(e*x+d)-c*sin(e*x+d))/(a^2-b^2+c^
2)/e/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2)/sin(e*x+d)^(5/2)+8/3*(b+c*cos(e*x
+d)+a*sin(e*x+d))^2*(a*b*cos(e*x+d)-b*c*sin(e*x+d))/(a^2-b^2+c^2)^2/e/(a+c
*cot(e*x+d)+b*csc(e*x+d))^(5/2)/sin(e*x+d)^(5/2)+8/3*b*(cos(1/2*d+1/2*e*x-
1/2*arctan(c,a))^2)^(1/2)/cos(1/2*d+1/2*e*x-1/2*arctan(c,a))*EllipticE(sin
(1/2*d+1/2*e*x-1/2*arctan(c,a)),2^(1/2)*((a^2+c^2)^(1/2)/(b+(a^2+c^2)^(1/2
)))^(1/2))*(b+c*cos(e*x+d)+a*sin(e*x+d))^3/(a^2-b^2+c^2)^2/e/(a+c*cot(e*x+
d)+b*csc(e*x+d))^(5/2)/sin(e*x+d)^(5/2)/((b+c*cos(e*x+d)+a*sin(e*x+d))/(b+
(a^2+c^2)^(1/2)))^(1/2)+2/3*(cos(1/2*d+1/2*e*x-1/2*arctan(c,a))^2)^(1/2)/c
os(1/2*d+1/2*e*x-1/2*arctan(c,a))*EllipticF(sin(1/2*d+1/2*e*x-1/2*arctan(c
,a)),2^(1/2)*((a^2+c^2)^(1/2)/(b+(a^2+c^2)^(1/2)))^(1/2))*(b+c*cos(e*x+d)+
a*sin(e*x+d))^2*((b+c*cos(e*x+d)+a*sin(e*x+d))/(b+(a^2+c^2)^(1/2)))^(1/2)/
(a^2-b^2+c^2)/e/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2)/sin(e*x+d)^(5/2)
```

3.471.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 18.63 (sec) , antiderivative size = 6066, normalized size of antiderivative = 12.33

$$\int \frac{1}{(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}\sin^{\frac{5}{2}}(d+ex)}\,dx=\text{Result too large to show}$$

input
```
Integrate[1/((a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2)*Sin[d + e*x]^(5/2
)),x]
```

output
```
Result too large to show
```

3.471.3 Rubi [A] (verified)

Time = 1.83 (sec) , antiderivative size = 443, normalized size of antiderivative = 0.90, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.515$, Rules used = {3042, 3643, 3042, 3608, 27, 3042, 3635, 27, 3042, 3628, 3042, 3598, 3042, 3132, 3606, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sin^{\frac{5}{2}}(d+ex)(a+b\csc(d+ex)+c\cot(d+ex))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(d+ex)^{5/2}(a+b\csc(d+ex)+c\cot(d+ex))^{5/2}}dx$$

$$\Big\downarrow \text{3643}$$

$$\frac{(a\sin(d+ex)+b+c\cos(d+ex))^{5/2}\int \frac{1}{(b+c\cos(d+ex)+a\sin(d+ex))^{5/2}}dx}{\sin^{\frac{5}{2}}(d+ex)(a+b\csc(d+ex)+c\cot(d+ex))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a\sin(d+ex)+b+c\cos(d+ex))^{5/2}\int \frac{1}{(b+c\cos(d+ex)+a\sin(d+ex))^{5/2}}dx}{\sin^{\frac{5}{2}}(d+ex)(a+b\csc(d+ex)+c\cot(d+ex))^{5/2}}$$

$$\Big\downarrow \text{3608}$$

$$\frac{(a\sin(d+ex)+b+c\cos(d+ex))^{5/2}\left(\frac{2\int -\frac{3b-c\cos(d+ex)-a\sin(d+ex)}{2(b+c\cos(d+ex)+a\sin(d+ex))^{3/2}}dx}{3(a^2-b^2+c^2)}-\frac{2(a\cos(d+ex)-c\sin(d+ex))}{3e(a^2-b^2+c^2)(a\sin(d+ex)+b+c\cos(d+ex))^{3/2}}\right)}{\sin^{\frac{5}{2}}(d+ex)(a+b\csc(d+ex)+c\cot(d+ex))^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(a\sin(d+ex)+b+c\cos(d+ex))^{5/2}\left(-\frac{\int \frac{3b-c\cos(d+ex)-a\sin(d+ex)}{(b+c\cos(d+ex)+a\sin(d+ex))^{3/2}}dx}{3(a^2-b^2+c^2)}-\frac{2(a\cos(d+ex)-c\sin(d+ex))}{3e(a^2-b^2+c^2)(a\sin(d+ex)+b+c\cos(d+ex))^{3/2}}\right)}{\sin^{\frac{5}{2}}(d+ex)(a+b\csc(d+ex)+c\cot(d+ex))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a\sin(d+ex)+b+c\cos(d+ex))^{5/2}\left(-\frac{\int \frac{3b-c\cos(d+ex)-a\sin(d+ex)}{(b+c\cos(d+ex)+a\sin(d+ex))^{3/2}}dx}{3(a^2-b^2+c^2)}-\frac{2(a\cos(d+ex)-c\sin(d+ex))}{3e(a^2-b^2+c^2)(a\sin(d+ex)+b+c\cos(d+ex))^{3/2}}\right)}{\sin^{\frac{5}{2}}(d+ex)(a+b\csc(d+ex)+c\cot(d+ex))^{5/2}}$$

$$\Big\downarrow \text{3635}$$

$$\frac{(a\sin(d+ex)+b+c\cos(d+ex))^{5/2}\left(-\frac{\frac{2\int -\frac{a^2+4b\sin(d+ex)a+3b^2+c^2+4bc\cos(d+ex)}{2\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}}dx}{a^2-b^2+c^2}-\frac{8(ab\cos(d+ex)-bc\sin(d+ex))}{e\left(a^2-b^2+c^2\right)\sqrt{a\sin(d+ex)+b+c\cos(d+ex)}}}{3(a^2-b^2+c^2)}-\right.}{\sin^{\frac{5}{2}}(d+ex)(a+b\csc(d+ex)+c\cot(d+ex))^{5/2}}$$

$$\Big\downarrow \text{27}$$

3.471. $\int \frac{1}{(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}\sin^{\frac{5}{2}}(d+ex)}\,dx$

$$\frac{(a\sin(d+ex)+b+c\cos(d+ex))^{5/2}\left(-\frac{-\frac{\int\frac{a^2+4b\sin(d+ex)a+3b^2+c^2+4bc\cos(d+ex)}{\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}}dx}{a^2-b^2+c^2}-\frac{8(ab\cos(d+ex)-bc\sin(d+ex))}{e\left(a^2-b^2+c^2\right)\sqrt{a\sin(d+ex)+b+c\cos(d+ex)}}}{3(a^2-b^2+c^2)}-\right.}{\sin^{\frac{5}{2}}(d+ex)(a+b\csc(d+ex)+c\cot(d+ex))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a\sin(d+ex)+b+c\cos(d+ex))^{5/2}\left(-\frac{-\frac{\int\frac{a^2+4b\sin(d+ex)a+3b^2+c^2+4bc\cos(d+ex)}{\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}}dx}{a^2-b^2+c^2}-\frac{8(ab\cos(d+ex)-bc\sin(d+ex))}{e\left(a^2-b^2+c^2\right)\sqrt{a\sin(d+ex)+b+c\cos(d+ex)}}}{3(a^2-b^2+c^2)}-\right.}{\sin^{\frac{5}{2}}(d+ex)(a+b\csc(d+ex)+c\cot(d+ex))^{5/2}}$$

$$\Big\downarrow \text{3628}$$

$$\frac{(a\sin(d+ex)+b+c\cos(d+ex))^{5/2}\left(-\frac{-\frac{\left(a^2-b^2+c^2\right)\int\frac{1}{\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}}dx+4b\int\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}dx}{a^2-b^2+c^2}-\frac{8(ab}{e\left(a^2-b^2+\right.}}{3(a^2-b^2+c^2)}\right.}{\sin^{\frac{5}{2}}(d+ex)(a+b\csc(d+ex)+c\cot(d+ex))^{5/}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a\sin(d+ex)+b+c\cos(d+ex))^{5/2}\left(-\frac{-\frac{\left(a^2-b^2+c^2\right)\int\frac{1}{\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}}dx+4b\int\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}dx}{a^2-b^2+c^2}-\frac{8(ab}{e\left(a^2-b^2+\right.}}{3(a^2-b^2+c^2)}\right.}{\sin^{\frac{5}{2}}(d+ex)(a+b\csc(d+ex)+c\cot(d+ex))^{5/}}$$

$$\Big\downarrow \text{3598}$$

$$\frac{(a\sin(d+ex)+b+c\cos(d+ex))^{5/2}\left(-\frac{-\frac{\left(a^2-b^2+c^2\right)\int\frac{1}{\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}}dx+\frac{4b\sqrt{a\sin(d+ex)+b+c\cos(d+ex)}\int\sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+}}{\sqrt{\frac{a\sin(d+ex)+b+c\,\mathrm{co}}{\sqrt{a^2+c^2}+b}}}}{a^2-b^2+c^2}}{3(a^2-b^2+c^2)}\right.}{\sin^{\frac{5}{2}}(d+ex)(a+b\csc(d+ex)}$$

$$\Big\downarrow \text{3042}$$

3.471. $\displaystyle\int\frac{1}{(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}\sin^{\frac{5}{2}}(d+ex)}\,dx$

$$\frac{(a\sin(d+ex)+b+c\cos(d+ex))^{5/2}\left(-\frac{-\frac{\left(a^2-b^2+c^2\right)\int\frac{1}{\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}}dx+\frac{4b\sqrt{a\sin(d+ex)+b+c\cos(d+ex)}\int\sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+}}{\sqrt{\frac{a\sin(d+ex)+b+c}{\sqrt{a^2+c^2}+}}}}{a^2-b^2+c^2}}{3(a^2-b^2+c^2)}\right.}{\sin^{\frac{5}{2}}(d+ex)(a+b\csc(d+ex)}$$

$\downarrow$ 3132

$$\frac{(a\sin(d+ex)+b+c\cos(d+ex))^{5/2}\left(-\frac{-\frac{\left(a^2-b^2+c^2\right)\int\frac{1}{\sqrt{b+c\cos(d+ex)+a\sin(d+ex)}}dx+\frac{8b\sqrt{a\sin(d+ex)+b+c\cos(d+ex)}E\left(\frac{1}{2}\left(d+ex-\tan^{-}\right.\right.}{e\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+e}{\sqrt{a^2+c^2}+b}}}}{a^2-b^2+c^2}}{3(a^2-b^2+c^2)}\right.}{\sin^{\frac{5}{2}}(d+ex)(a+b\csc(d+ex)+c\,c}$$

$\downarrow$ 3606

$$\frac{(a\sin(d+ex)+b+c\cos(d+ex))^{5/2}\left(-\frac{-\frac{\frac{\left(a^2-b^2+c^2\right)\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}\int\frac{1}{\sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+\frac{\sqrt{a^2+c^2}\cos\left(d+ex-\tan^{-1}(c,a)\right)}{b+\sqrt{a^2+c^2}}}}dx}{\sqrt{a\sin(d+ex)+b+c\cos(d+ex)}}+}{a^2-b^2+c^2}}{}\right.}{\sin^{\frac{5}{2}}(d+ex)(a+b}$$

$\downarrow$ 3042

3.471. $\displaystyle\int\frac{1}{(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}\sin^{\frac{5}{2}}(d+ex)}\,dx$

$$\frac{(a\sin(d+ex)+b+c\cos(d+ex))^{5/2}\left(-\frac{-\frac{\frac{\left(a^2-b^2+c^2\right)\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}\int\frac{1}{\sqrt{\frac{b}{b+\sqrt{a^2+c^2}}+\frac{\sqrt{a^2+c^2}\sin\left(d+ex-\tan^{-1}(c,a)+\frac{\pi}{2}\right)}{b+\sqrt{a^2+c^2}}}}}{\sqrt{a\sin(d+ex)+b+c\cos(d+ex)}}}{a^2-b^2+c^2}}{}\right.}{\sin^{\frac{5}{2}}(d+ex)(a+}$$

$\downarrow$ 3140

$$\frac{(a\sin(d+ex)+b+c\cos(d+ex))^{5/2}\left(-\frac{-\frac{\frac{2\left(a^2-b^2+c^2\right)\sqrt{\frac{a\sin(d+ex)+b+c\cos(d+ex)}{\sqrt{a^2+c^2}+b}}\operatorname{EllipticF}\left(\frac{1}{2}\left(d+ex-\tan^{-1}(c,a)\right),\frac{2\sqrt{a^2+c^2}}{b+\sqrt{a^2+c^2}}\right)}{e\sqrt{a\sin(d+ex)+b+c\cos(d+ex)}}+\frac{8b\sqrt{a}}{}}{a^2-b^2+c^2}}{3}\right.}{\sin^{\frac{5}{2}}(d+ex)(a+b\,c}$$

input
```
Int[1/((a + c*Cot[d + e*x] + b*Csc[d + e*x])^(5/2)*Sin[d + e*x]^(5/2)),x]
```

output
```
((b + c*Cos[d + e*x] + a*Sin[d + e*x])^(5/2)*((-2*(a*Cos[d + e*x] - c*Sin[
d + e*x]))/(3*(a^2 - b^2 + c^2)*e*(b + c*Cos[d + e*x] + a*Sin[d + e*x])^(3
/2)) - ((-8*(a*b*Cos[d + e*x] - b*c*Sin[d + e*x]))/((a^2 - b^2 + c^2)*e*Sq
rt[b + c*Cos[d + e*x] + a*Sin[d + e*x]]) - ((8*b*EllipticE[(d + e*x - ArcT
an[c, a])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2 + c^2])]*Sqrt[b + c*Cos[d +
 e*x] + a*Sin[d + e*x]])/(e*Sqrt[(b + c*Cos[d + e*x] + a*Sin[d + e*x])/(b
+ Sqrt[a^2 + c^2])]) + (2*(a^2 - b^2 + c^2)*EllipticF[(d + e*x - ArcTan[c,
 a])/2, (2*Sqrt[a^2 + c^2])/(b + Sqrt[a^2 + c^2])]*Sqrt[(b + c*Cos[d + e*x
] + a*Sin[d + e*x])/(b + Sqrt[a^2 + c^2])])/(e*Sqrt[b + c*Cos[d + e*x] + a
*Sin[d + e*x]]))/(a^2 - b^2 + c^2))/(3*(a^2 - b^2 + c^2))))/((a + c*Cot[d
+ e*x] + b*Csc[d + e*x])^(5/2)*Sin[d + e*x]^(5/2))
```

3.471. $\int \frac{1}{(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}\sin^{\frac{5}{2}}(d+ex)}\,dx$

3.471.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3598
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]   Int[Sqrt[a/(a + S
qrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - Arc
Tan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

rule 3606
```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Simp[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]   Int[1/Sqrt[a/(a
 + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x -
 ArcTan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2
, 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

rule 3608
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-c)*Cos[d + e*x] + b*Sin[d + e*x])*((a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Simp[
1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c
*(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x
] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] &&
NeQ[n, -3/2]
```

3.471. $\int \frac{1}{(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}\sin^{\frac{5}{2}}(d+ex)}\,dx$

rule 3628
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]
, x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[
A*b - a*B, 0]
```

rule 3635
```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(-(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(
a^2 - b^2 - c^2))), x] + Simp[1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a + b*Co
s[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n +
2)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x]
/; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[n, -2]
```

rule 3643
```
Int[((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) + cot[(d_.) + (e_.)*(x_)]*(c_.))
^(n_)*sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbol] :> Simp[Sin[d + e*x]^n*((a +
b*Csc[d + e*x] + c*Cot[d + e*x])^n/(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n)
   Int[(b + a*Sin[d + e*x] + c*Cos[d + e*x])^n, x], x] /; FreeQ[{a, b, c, d
, e}, x] &&  !IntegerQ[n]
```

3.471.4 Maple [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 28.20 (sec) , antiderivative size = 60004, normalized size of antiderivative = 121.96

method	result	size
default	Expression too large to display	60004

input
```
int(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2)/sin(e*x+d)^(5/2),x,method=_RETUR
NVERBOSE)
```

output
```
result too large to display
```

3.471.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.25 (sec) , antiderivative size = 2798, normalized size of antiderivative = 5.69

$$\int \frac{1}{(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}\sin^{\frac{5}{2}}(d+ex)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2)/sin(e*x+d)^(5/2),x, algori
thm="fricas")
```

output
```
1/9*((sqrt(2)*(3*a^5 + a^3*b^2 - a*b^2*c^2 - I*b^2*c^3 - 3*a*c^4 - 3*I*c^5
 + I*(3*a^4 + a^2*b^2)*c)*cos(e*x + d)^2 - 2*sqrt(2)*(3*a*b*c^3 + 3*I*b*c^
4 + I*(3*a^2*b + b^3)*c^2 + (3*a^3*b + a*b^3)*c)*cos(e*x + d) - 2*(sqrt(2)
*(3*a^2*c^3 + 3*I*a*c^4 + I*(3*a^3 + a*b^2)*c^2 + (3*a^4 + a^2*b^2)*c)*cos
(e*x + d) + sqrt(2)*(3*a^4*b + a^2*b^3 + 3*a^2*b*c^2 + 3*I*a*b*c^3 + I*(3*
a^3*b + a*b^3)*c))*sin(e*x + d) - sqrt(2)*(3*a^5 + 4*a^3*b^2 + a*b^4 + 3*I
*(a^2 + b^2)*c^3 + 3*(a^3 + a*b^2)*c^2 + I*(3*a^4 + 4*a^2*b^2 + b^4)*c))*s
qrt(I*a + c)*weierstrassPInverse(4/3*(3*a^4 - 4*a^2*b^2 + 4*b^2*c^2 + 6*I*
a*c^3 - 3*c^4 + 2*I*(3*a^3 - 4*a*b^2)*c)/(a^4 + 2*a^2*c^2 + c^4), -8/27*(-
9*I*a^5*b + 8*I*a^3*b^3 + 27*I*a*b*c^4 - 9*b*c^5 + 2*(9*a^2*b + 4*b^3)*c^3
 + 6*I*(3*a^3*b - 4*a*b^3)*c^2 + 3*(9*a^4*b - 8*a^2*b^3)*c)/(a^6 + 3*a^4*c
^2 + 3*a^2*c^4 + c^6), 1/3*(-2*I*a*b + 2*b*c + 3*(a^2 + c^2)*cos(e*x + d)
- 3*(I*a^2 + I*c^2)*sin(e*x + d))/(a^2 + c^2)) + (sqrt(2)*(3*a^5 + a^3*b^2
 - a*b^2*c^2 + I*b^2*c^3 - 3*a*c^4 + 3*I*c^5 - I*(3*a^4 + a^2*b^2)*c)*cos(
e*x + d)^2 - 2*sqrt(2)*(3*a*b*c^3 - 3*I*b*c^4 - I*(3*a^2*b + b^3)*c^2 + (3
*a^3*b + a*b^3)*c)*cos(e*x + d) - 2*(sqrt(2)*(3*a^2*c^3 - 3*I*a*c^4 - I*(3
*a^3 + a*b^2)*c^2 + (3*a^4 + a^2*b^2)*c)*cos(e*x + d) + sqrt(2)*(3*a^4*b +
 a^2*b^3 + 3*a^2*b*c^2 - 3*I*a*b*c^3 - I*(3*a^3*b + a*b^3)*c))*sin(e*x + d
) - sqrt(2)*(3*a^5 + 4*a^3*b^2 + a*b^4 - 3*I*(a^2 + b^2)*c^3 + 3*(a^3 + a*
b^2)*c^2 - I*(3*a^4 + 4*a^2*b^2 + b^4)*c))*sqrt(-I*a + c)*weierstrassPI...
```

3.471.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}\sin^{\frac{5}{2}}(d+ex)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))**(5/2)/sin(e*x+d)**(5/2),x)
```

output
```
Timed out
```

3.471.7 Maxima [F]

$$\int \frac{1}{(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}\sin^{\frac{5}{2}}(d+ex)}\,dx = \int \frac{1}{(c\cot(ex+d)+b\csc(ex+d)+a)^{\frac{5}{2}}\sin(ex+$$

input
```
integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2)/sin(e*x+d)^(5/2),x, algori
thm="maxima")
```

output
```
integrate(1/((c*cot(e*x + d) + b*csc(e*x + d) + a)^(5/2)*sin(e*x + d)^(5/2
)), x)
```

3.471.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}\sin^{\frac{5}{2}}(d+ex)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+c*cot(e*x+d)+b*csc(e*x+d))^(5/2)/sin(e*x+d)^(5/2),x, algori
thm="giac")
```

output
```
Timed out
```

3.471.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+c\cot(d+ex)+b\csc(d+ex))^{5/2}\sin^{\frac{5}{2}}(d+ex)}\,dx = \int \frac{1}{\sin(d+e\,x)^{5/2}\left(a+c\cot(d+e\,x)+\frac{b}{\sin(d+e\,x)}\right.$$

input
```
int(1/(sin(d + e*x)^(5/2)*(a + c*cot(d + e*x) + b/sin(d + e*x))^(5/2)),x)
```

output
```
int(1/(sin(d + e*x)^(5/2)*(a + c*cot(d + e*x) + b/sin(d + e*x))^(5/2)), x)
```

3.472 $\int \frac{1}{\cos^2(x)+\sin^2(x)}\,dx$

3.472.1 Optimal result

Integrand size = 11, antiderivative size = 1

$$\int \frac{1}{\cos^2(x)+\sin^2(x)}\,dx = x$$

output
```
x
```

3.472.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\cos^2(x)+\sin^2(x)}\,dx = x$$

input
```
Integrate[(Cos[x]^2 + Sin[x]^2)^(-1),x]
```

output
```
x
```

3.472.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 4880, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sin^2(x)+\cos^2(x)} \, dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{\sin(x)^2+\cos(x)^2}dx$$
$$\Big\downarrow \text{ 4880}$$
$$\int 1dx$$
$$\Big\downarrow \text{ 24}$$
$$x$$

input
```
Int[(Cos[x]^2 + Sin[x]^2)^(-1),x]
```

output
```
x
```

3.472.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4880
```
Int[(u_.)*((a_.) + cos[(d_.) + (e_.)*(x_)]^2*(b_.) + (c_.)*sin[(d_.) + (e_.
)*(x_)]^2)^(p_.), x_Symbol] :> Simp[(a + c)^p   Int[ActivateTrig[u], x], x]
 /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b - c, 0]
```

3.472.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 2, normalized size of antiderivative = 2.00

method	result	size
default	x	2
risch	x	2
norman	$\frac{x+x\tan\left(\frac{x}{2}\right)^{4}+2x\tan\left(\frac{x}{2}\right)^{2}}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)^{2}}$	31

input
```
int(1/(sin(x)^2+cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
x
```

3.472.5 Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\cos^2(x)+\sin^2(x)}\,dx = x$$

input
```
integrate(1/(cos(x)^2+sin(x)^2),x, algorithm="fricas")
```

output
```
x
```

3.472.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10 vs. $2(0) = 0$.

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 10.00

$$\int \frac{1}{\cos^2(x)+\sin^2(x)}\,dx = \frac{x}{\sin^2(x)+\cos^2(x)}$$

input
```
integrate(1/(cos(x)**2+sin(x)**2),x)
```

output
```
x/(sin(x)**2 + cos(x)**2)
```

3.472.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\cos^2(x) + \sin^2(x)}\, dx = x$$

input
```
integrate(1/(cos(x)^2+sin(x)^2),x, algorithm="maxima")
```

output
```
x
```

3.472.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\cos^2(x) + \sin^2(x)}\, dx = x$$

input
```
integrate(1/(cos(x)^2+sin(x)^2),x, algorithm="giac")
```

output
```
x
```

3.472.9 Mupad [B] (verification not implemented)

Time = 28.22 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\cos^2(x) + \sin^2(x)}\, dx = x$$

input
```
int(1/(cos(x)^2 + sin(x)^2),x)
```

output
```
x
```

3.473 $\int \frac{1}{\left(\cos^2(x)+\sin^2(x)\right)^2} \, dx$

3.473.1 Optimal result

Integrand size = 11, antiderivative size = 1

$$\int \frac{1}{\left(\cos^2(x)+\sin^2(x)\right)^2} \, dx = x$$

output
```
x
```

3.473.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\cos^2(x)+\sin^2(x)\right)^2} \, dx = x$$

input
```
Integrate[(Cos[x]^2 + Sin[x]^2)^(-2),x]
```

output
```
x
```

3.473.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 4880, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\sin^2(x)+\cos^2(x)\right)^2}dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{1}{\left(\sin(x)^2+\cos(x)^2\right)^2}dx$$
$$\Big\downarrow \ 4880$$
$$\int 1dx$$
$$\Big\downarrow \ 24$$
$$x$$

input
```
Int[(Cos[x]^2 + Sin[x]^2)^(-2),x]
```

output
```
x
```

3.473.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4880
```
Int[(u_.)*((a_.) + cos[(d_.) + (e_.)*(x_)]^2*(b_.) + (c_.)*sin[(d_.) + (e_.
)*(x_)]^2)^(p_.), x_Symbol] :> Simp[(a + c)^p   Int[ActivateTrig[u], x], x]
 /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b - c, 0]
```

3.473.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 2, normalized size of antiderivative = 2.00

method	result	size
default	x	2
risch	x	2
norman	$\frac{x+x\tan\left(\frac{x}{2}\right)^{8}+4x\tan\left(\frac{x}{2}\right)^{2}+6x\tan\left(\frac{x}{2}\right)^{4}+4x\tan\left(\frac{x}{2}\right)^{6}}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)^{4}}$	49

input
```
int(1/(sin(x)^2+cos(x)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
x
```

3.473.5 Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\cos^2(x)+\sin^2(x)\right)^2}\,dx = x$$

input
```
integrate(1/(cos(x)^2+sin(x)^2)^2,x, algorithm="fricas")
```

output
```
x
```

3.473.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(0) = 0.

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 22.00

$$\int \frac{1}{\left(\cos^2(x)+\sin^2(x)\right)^2}\,dx = \frac{x}{\sin^4(x)+2\sin^2(x)\cos^2(x)+\cos^4(x)}$$

input
```
integrate(1/(cos(x)**2+sin(x)**2)**2,x)
```

output
```
x/(sin(x)**4 + 2*sin(x)**2*cos(x)**2 + cos(x)**4)
```

3.473.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\cos^2(x)+\sin^2(x)\right)^2}\,dx=x$$

input
```
integrate(1/(cos(x)^2+sin(x)^2)^2,x, algorithm="maxima")
```

output
```
x
```

3.473.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\cos^2(x)+\sin^2(x)\right)^2}\,dx=x$$

input
```
integrate(1/(cos(x)^2+sin(x)^2)^2,x, algorithm="giac")
```

output
```
x
```

3.473.9 Mupad [B] (verification not implemented)

Time = 27.81 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\cos^2(x)+\sin^2(x)\right)^2}\,dx=x$$

input
```
int(1/(cos(x)^2 + sin(x)^2)^2,x)
```

output
```
x
```

3.474 $\int \frac{1}{\left(\cos^2(x)+\sin^2(x)\right)^3}\,dx$

3.474.1 Optimal result

Integrand size = 11, antiderivative size = 1

$$\int \frac{1}{\left(\cos^2(x)+\sin^2(x)\right)^3}\,dx = x$$

output
```
x
```

3.474.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\cos^2(x)+\sin^2(x)\right)^3}\,dx = x$$

input
```
Integrate[(Cos[x]^2 + Sin[x]^2)^(-3),x]
```

output
```
x
```

3.474.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 4880, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\sin^2(x)+\cos^2(x)\right)^3} \, dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\left(\sin(x)^2+\cos(x)^2\right)^3}dx$$
$$\Big\downarrow \; 4880$$
$$\int 1dx$$
$$\Big\downarrow \; 24$$
$$x$$

input
```
Int[(Cos[x]^2 + Sin[x]^2)^(-3),x]
```

output
```
x
```

3.474.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4880
```
Int[(u_.)*((a_.) + cos[(d_.) + (e_.)*(x_)]^2*(b_.) + (c_.)*sin[(d_.) + (e_.
)*(x_)]^2)^(p_.), x_Symbol] :> Simp[(a + c)^p   Int[ActivateTrig[u], x], x]
 /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b - c, 0]
```

3.474.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 2, normalized size of antiderivative = 2.00

method	result	size
default	x	2
risch	x	2
norman	$\frac{x+x\tan\left(\frac{x}{2}\right)^{12}+6x\tan\left(\frac{x}{2}\right)^{2}+15x\tan\left(\frac{x}{2}\right)^{4}+20x\tan\left(\frac{x}{2}\right)^{6}+15x\tan\left(\frac{x}{2}\right)^{8}+6x\tan\left(\frac{x}{2}\right)^{10}}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)^{6}}$	67

input
```
int(1/(sin(x)^2+cos(x)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
x
```

3.474.5 Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\cos^2(x)+\sin^2(x)\right)^3}\,dx = x$$

input
```
integrate(1/(cos(x)^2+sin(x)^2)^3,x, algorithm="fricas")
```

output
```
x
```

3.474.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(0) = 0$.

Time = 0.60 (sec) , antiderivative size = 34, normalized size of antiderivative = 34.00

$$\int \frac{1}{\left(\cos^2(x)+\sin^2(x)\right)^3}\,dx = \frac{x}{\sin^6(x)+3\sin^4(x)\cos^2(x)+3\sin^2(x)\cos^4(x)+\cos^6(x)}$$

input
```
integrate(1/(cos(x)**2+sin(x)**2)**3,x)
```

output
```
x/(sin(x)**6 + 3*sin(x)**4*cos(x)**2 + 3*sin(x)**2*cos(x)**4 + cos(x)**6)
```

3.474.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{(\cos^2(x) + \sin^2(x))^3} dx = x$$

input
```
integrate(1/(cos(x)^2+sin(x)^2)^3,x, algorithm="maxima")
```

output
```
x
```

3.474.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{(\cos^2(x) + \sin^2(x))^3} dx = x$$

input
```
integrate(1/(cos(x)^2+sin(x)^2)^3,x, algorithm="giac")
```

output
```
x
```

3.474.9 Mupad [B] (verification not implemented)

Time = 27.17 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{(\cos^2(x) + \sin^2(x))^3} dx = x$$

input
```
int(1/(cos(x)^2 + sin(x)^2)^3,x)
```

output
```
x
```

3.475 $\int \frac{1}{\cos^2(x)-\sin^2(x)} dx$

3.475.1 Optimal result

Integrand size = 13, antiderivative size = 11

$$\int \frac{1}{\cos^2(x)-\sin^2(x)} dx = \frac{1}{2}\operatorname{arctanh}(2\cos(x)\sin(x))$$

output
```
1/2*arctanh(2*cos(x)*sin(x))
```

3.475.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{1}{\cos^2(x)-\sin^2(x)} dx = \frac{1}{2}\operatorname{arctanh}(\sin(2x))$$

input
```
Integrate[(Cos[x]^2 - Sin[x]^2)^(-1),x]
```

output
```
ArcTanh[Sin[2*x]]/2
```

3.475.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.27, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 4889, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^2(x)-\sin^2(x)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\cos(x)^2-\sin(x)^2}dx$$
$$\Big\downarrow \; 4889$$
$$\int \frac{1}{1-\tan^2(x)}d\tan(x)$$
$$\Big\downarrow \; 219$$
$$\text{arctanh}(\tan(x))$$

input
```
Int[(Cos[x]^2 - Sin[x]^2)^(-1),x]
```

output
```
ArcTanh[Tan[x]]
```

3.475.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.475.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.36

method	result	size
default	$\operatorname{arctanh}(\tan(x))$	4
risch	$-\frac{\ln(e^{2ix}-i)}{2}+\frac{\ln(i+e^{2ix})}{2}$	24
norman	$\frac{\ln\left(\tan\left(\frac{x}{2}\right)^2-2\tan\left(\frac{x}{2}\right)-1\right)}{2}-\frac{\ln\left(\tan\left(\frac{x}{2}\right)^2+2\tan\left(\frac{x}{2}\right)-1\right)}{2}$	36
parallelrisch	$\ln\left(\sqrt{\tan\left(\frac{x}{2}\right)^2-2\tan\left(\frac{x}{2}\right)-1}\right)+\ln\left(\frac{1}{\sqrt{\tan\left(\frac{x}{2}\right)^2+2\tan\left(\frac{x}{2}\right)-1}}\right)$	36

input
```
int(1/(cos(x)^2-sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
arctanh(tan(x))
```

3.475.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. 2(9) = 18.

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.09

$$\int \frac{1}{\cos^2(x)-\sin^2(x)}\,dx = \frac{1}{4}\log\left(2\cos(x)\sin(x)+1\right)-\frac{1}{4}\log\left(-2\cos(x)\sin(x)+1\right)$$

input
```
integrate(1/(cos(x)^2-sin(x)^2),x, algorithm="fricas")
```

output
```
1/4*log(2*cos(x)*sin(x) + 1) - 1/4*log(-2*cos(x)*sin(x) + 1)
```

3.475.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(10) = 20$.

Time = 0.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 3.27

$$\int \frac{1}{\cos^2(x) - \sin^2(x)}\,dx = \frac{\log\left(\tan^2\left(\frac{x}{2}\right) - 2\tan\left(\frac{x}{2}\right) - 1\right)}{2} - \frac{\log\left(\tan^2\left(\frac{x}{2}\right) + 2\tan\left(\frac{x}{2}\right) - 1\right)}{2}$$

input
```
integrate(1/(cos(x)**2-sin(x)**2),x)
```

output
```
log(tan(x/2)**2 - 2*tan(x/2) - 1)/2 - log(tan(x/2)**2 + 2*tan(x/2) - 1)/2
```

3.475.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \frac{1}{\cos^2(x) - \sin^2(x)}\,dx = \frac{1}{2}\log(\tan(x) + 1) - \frac{1}{2}\log(\tan(x) - 1)$$

input
```
integrate(1/(cos(x)^2-sin(x)^2),x, algorithm="maxima")
```

output
```
1/2*log(tan(x) + 1) - 1/2*log(tan(x) - 1)
```

3.475.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(9) = 18$.

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 3.00

$$\int \frac{1}{\cos^2(x) - \sin^2(x)}\,dx = \frac{1}{8}\log\left(\left|\frac{1}{\sin(2x)} + \sin(2x) + 2\right|\right) - \frac{1}{8}\log\left(\left|\frac{1}{\sin(2x)} + \sin(2x) - 2\right|\right)$$

input
```
integrate(1/(cos(x)^2-sin(x)^2),x, algorithm="giac")
```

output
```
1/8*log(abs(1/sin(2*x) + sin(2*x) + 2)) - 1/8*log(abs(1/sin(2*x) + sin(2*x
) - 2))
```

3.475.9 Mupad [B] (verification not implemented)

Time = 27.89 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.27

$$\int \frac{1}{\cos^2(x) - \sin^2(x)} \, dx = \operatorname{atanh}(\tan(x))$$

input
```
int(1/(cos(x)^2 - sin(x)^2),x)
```

output
```
atanh(tan(x))
```

3.476 $\int \frac{1}{(\cos^2(x)-\sin^2(x))^2} dx$

3.476.1 Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{1}{(\cos^2(x)-\sin^2(x))^2} dx = \frac{\tan(x)}{1-\tan^2(x)}$$

output
```
tan(x)/(1-tan(x)^2)
```

3.476.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.62

$$\int \frac{1}{(\cos^2(x)-\sin^2(x))^2} dx = \frac{1}{2}\tan(2x)$$

input
```
Integrate[(Cos[x]^2 - Sin[x]^2)^(-2),x]
```

output
```
Tan[2*x]/2
```

3.476.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 4889, 297}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\cos^2(x)-\sin^2(x))^2} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(\cos(x)^2-\sin(x)^2)^2}dx$$

$$\Big\downarrow \text{4889}$$

$$\int \frac{\tan^2(x)+1}{(1-\tan^2(x))^2}d\tan(x)$$

$$\Big\downarrow \text{297}$$

$$\frac{\tan(x)}{1-\tan^2(x)}$$

input
```
Int[(Cos[x]^2 - Sin[x]^2)^(-2),x]
```

output
```
Tan[x]/(1 - Tan[x]^2)
```

3.476.3.1 Defintions of rubi rules used

rule 297
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[c*
x*((a + b*x^2)^(p + 1)/a), x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d
, 0] && EqQ[a*d - b*c*(2*p + 3), 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.476.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

method	result	size
risch	$\frac{i}{e^{4ix}+1}$	13
parallelrisch	$\frac{\sin(2x)}{2\cos(2x)}$	13
default	$-\frac{1}{2(\tan(x)-1)}-\frac{1}{2(1+\tan(x))}$	18
norman	$\frac{-2\tan\left(\frac{x}{2}\right)^{3}+2\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^{4}-6\tan\left(\frac{x}{2}\right)^{2}+1}$	35

input
```
int(1/(cos(x)^2-sin(x)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
I/(exp(4*I*x)+1)
```

3.476.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{1}{\left(\cos^2(x)-\sin^2(x)\right)^2}\,dx=\frac{\cos\left(x\right)\sin\left(x\right)}{2\,\cos\left(x\right)^2-1}$$

input
```
integrate(1/(cos(x)^2-sin(x)^2)^2,x, algorithm="fricas")
```

output
```
cos(x)*sin(x)/(2*cos(x)^2 - 1)
```

3.476.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(8) = 16.

Time = 0.63 (sec) , antiderivative size = 48, normalized size of antiderivative = 3.69

$$\int \frac{1}{\left(\cos^2(x)-\sin^2(x)\right)^2}\,dx = -\frac{2\tan^3\left(\frac{x}{2}\right)}{\tan^4\left(\frac{x}{2}\right)-6\tan^2\left(\frac{x}{2}\right)+1}+\frac{2\tan\left(\frac{x}{2}\right)}{\tan^4\left(\frac{x}{2}\right)-6\tan^2\left(\frac{x}{2}\right)+1}$$

input
```
integrate(1/(cos(x)**2-sin(x)**2)**2,x)
```

output
```
-2*tan(x/2)**3/(tan(x/2)**4 - 6*tan(x/2)**2 + 1) + 2*tan(x/2)/(tan(x/2)**4
 - 6*tan(x/2)**2 + 1)
```

3.476.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{1}{\left(\cos^2(x)-\sin^2(x)\right)^2}\,dx = -\frac{\tan(x)}{\tan(x)^2-1}$$

input
```
integrate(1/(cos(x)^2-sin(x)^2)^2,x, algorithm="maxima")
```

output
```
-tan(x)/(tan(x)^2 - 1)
```

3.476.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.46

$$\int \frac{1}{\left(\cos^2(x)-\sin^2(x)\right)^2}\,dx = \frac{1}{2}\tan(2x)$$

input
```
integrate(1/(cos(x)^2-sin(x)^2)^2,x, algorithm="giac")
```

output
```
1/2*tan(2*x)
```

3.476.9 Mupad [B] (verification not implemented)

Time = 27.18 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.46

$$\int \frac{1}{\left(\cos^2(x) - \sin^2(x)\right)^2}\, dx = \frac{\tan(2\,x)}{2}$$

input
```
int(1/(cos(x)^2 - sin(x)^2)^2,x)
```

output
```
tan(2*x)/2
```

3.477 $\int \frac{1}{(\cos^2(x)-\sin^2(x))^3} dx$

3.477.1 Optimal result

Integrand size = 13, antiderivative size = 32

$$\int \frac{1}{(\cos^2(x)-\sin^2(x))^3} dx = \frac{1}{4}\operatorname{arctanh}(2\cos(x)\sin(x)) + \frac{\sec^2(x)\tan(x)}{2(1-\tan^2(x))^2}$$

output
```
1/4*arctanh(2*cos(x)*sin(x))+1/2*sec(x)^2*tan(x)/(1-tan(x)^2)^2
```

3.477.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.69

$$\int \frac{1}{(\cos^2(x)-\sin^2(x))^3} dx = \frac{1}{4}\operatorname{arctanh}(\sin(2x)) + \frac{1}{4}\sec(2x)\tan(2x)$$

input
```
Integrate[(Cos[x]^2 - Sin[x]^2)^(-3),x]
```

output
```
ArcTanh[Sin[2*x]]/4 + (Sec[2*x]*Tan[2*x])/4
```

3.477.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3042, 4889, 315, 27, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\cos^2(x)-\sin^2(x)\right)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(\cos(x)^2-\sin(x)^2\right)^3}dx$$

$$\Big\downarrow \text{4889}$$

$$\int \frac{\left(\tan^2(x)+1\right)^2}{\left(1-\tan^2(x)\right)^3}d\tan(x)$$

$$\Big\downarrow \text{315}$$

$$\frac{\tan(x)\left(\tan^2(x)+1\right)}{2\left(1-\tan^2(x)\right)^2}-\frac{1}{4}\int -\frac{2}{1-\tan^2(x)}d\tan(x)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{1}{1-\tan^2(x)}d\tan(x)+\frac{\tan(x)\left(\tan^2(x)+1\right)}{2\left(1-\tan^2(x)\right)^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\text{arctanh}(\tan(x))+\frac{\tan(x)\left(\tan^2(x)+1\right)}{2\left(1-\tan^2(x)\right)^2}$$

input
```
Int[(Cos[x]^2 - Sin[x]^2)^(-3),x]
```

output
```
ArcTanh[Tan[x]]/2 + (Tan[x]*(1 + Tan[x]^2))/(2*(1 - Tan[x]^2)^2)
```

3.477.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.477. $\int \frac{1}{(\cos^2(x)-\sin^2(x))^3} \, dx$

3.477.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.50

method	result	size
default	$-\frac{1}{4(1+\tan(x))^2}+\frac{1}{4+4\tan(x)}+\frac{\ln(1+\tan(x))}{4}+\frac{1}{4(\tan(x)-1)^2}+\frac{1}{4\tan(x)-4}-\frac{\ln(\tan(x)-1)}{4}$	48
risch	$-\frac{i(e^{6ix}-e^{2ix})}{2(e^{4ix}+1)^2}-\frac{\ln(e^{2ix}-i)}{4}+\frac{\ln(i+e^{2ix})}{4}$	49
parallelrisch	$\frac{(1+\cos(4x))\ln\left(\frac{-\cos(x)-\sin(x)}{\cos(x)+1}\right)+(-\cos(4x)-1)\ln\left(\frac{-\cos(x)+\sin(x)}{\cos(x)+1}\right)+2\sin(2x)}{4\cos(4x)+4}$	67
norman	$\frac{\tan(\frac{x}{2})^3-\tan(\frac{x}{2})^5-\tan(\frac{x}{2})^7+\tan(\frac{x}{2})}{\left(\tan(\frac{x}{2})^4-6\tan(\frac{x}{2})^2+1\right)^2}+\frac{\ln\left(\tan(\frac{x}{2})^2-2\tan(\frac{x}{2})-1\right)}{4}-\frac{\ln\left(\tan(\frac{x}{2})^2+2\tan(\frac{x}{2})-1\right)}{4}$	82

input
```
int(1/(cos(x)^2-sin(x)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4/(1+tan(x))^2+1/4/(1+tan(x))+1/4*ln(1+tan(x))+1/4/(tan(x)-1)^2+1/4/(ta
n(x)-1)-1/4*ln(tan(x)-1)
```

3.477.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(26) = 52.

Time = 0.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.31

$$\int \frac{1}{(\cos^2(x)-\sin^2(x))^3}\,dx = \frac{(4\cos(x)^4-4\cos(x)^2+1)\log(2\cos(x)\sin(x)+1)-(4\cos(x)^4-4\cos(x)^2+1)\log(-2\cos(x)\sin}{8(4\cos(x)^4-4\cos(x)^2+1)}$$

input
```
integrate(1/(cos(x)^2-sin(x)^2)^3,x, algorithm="fricas")
```

output
```
1/8*((4*cos(x)^4 - 4*cos(x)^2 + 1)*log(2*cos(x)*sin(x) + 1) - (4*cos(x)^4
- 4*cos(x)^2 + 1)*log(-2*cos(x)*sin(x) + 1) + 4*cos(x)*sin(x))/(4*cos(x)^4
 - 4*cos(x)^2 + 1)
```

3.477.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 765 vs. 2(29) = 58.

Time = 1.61 (sec) , antiderivative size = 765, normalized size of antiderivative = 23.91

$$\int \frac{1}{(\cos^2(x)-\sin^2(x))^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(cos(x)**2-sin(x)**2)**3,x)
```

output
```
log(tan(x/2)**2 - 2*tan(x/2) - 1)*tan(x/2)**8/(4*tan(x/2)**8 - 48*tan(x/2)
**6 + 152*tan(x/2)**4 - 48*tan(x/2)**2 + 4) - 12*log(tan(x/2)**2 - 2*tan(x
/2) - 1)*tan(x/2)**6/(4*tan(x/2)**8 - 48*tan(x/2)**6 + 152*tan(x/2)**4 - 4
8*tan(x/2)**2 + 4) + 38*log(tan(x/2)**2 - 2*tan(x/2) - 1)*tan(x/2)**4/(4*t
an(x/2)**8 - 48*tan(x/2)**6 + 152*tan(x/2)**4 - 48*tan(x/2)**2 + 4) - 12*l
og(tan(x/2)**2 - 2*tan(x/2) - 1)*tan(x/2)**2/(4*tan(x/2)**8 - 48*tan(x/2)*
*6 + 152*tan(x/2)**4 - 48*tan(x/2)**2 + 4) + log(tan(x/2)**2 - 2*tan(x/2)
- 1)/(4*tan(x/2)**8 - 48*tan(x/2)**6 + 152*tan(x/2)**4 - 48*tan(x/2)**2 +
4) - log(tan(x/2)**2 + 2*tan(x/2) - 1)*tan(x/2)**8/(4*tan(x/2)**8 - 48*tan
(x/2)**6 + 152*tan(x/2)**4 - 48*tan(x/2)**2 + 4) + 12*log(tan(x/2)**2 + 2*
tan(x/2) - 1)*tan(x/2)**6/(4*tan(x/2)**8 - 48*tan(x/2)**6 + 152*tan(x/2)**
4 - 48*tan(x/2)**2 + 4) - 38*log(tan(x/2)**2 + 2*tan(x/2) - 1)*tan(x/2)**4
/(4*tan(x/2)**8 - 48*tan(x/2)**6 + 152*tan(x/2)**4 - 48*tan(x/2)**2 + 4) +
 12*log(tan(x/2)**2 + 2*tan(x/2) - 1)*tan(x/2)**2/(4*tan(x/2)**8 - 48*tan(
x/2)**6 + 152*tan(x/2)**4 - 48*tan(x/2)**2 + 4) - log(tan(x/2)**2 + 2*tan(
x/2) - 1)/(4*tan(x/2)**8 - 48*tan(x/2)**6 + 152*tan(x/2)**4 - 48*tan(x/2)*
*2 + 4) - 4*tan(x/2)**7/(4*tan(x/2)**8 - 48*tan(x/2)**6 + 152*tan(x/2)**4
- 48*tan(x/2)**2 + 4) - 4*tan(x/2)**5/(4*tan(x/2)**8 - 48*tan(x/2)**6 + 15
2*tan(x/2)**4 - 48*tan(x/2)**2 + 4) + 4*tan(x/2)**3/(4*tan(x/2)**8 - 48*ta
n(x/2)**6 + 152*tan(x/2)**4 - 48*tan(x/2)**2 + 4) + 4*tan(x/2)/(4*tan(x...
```

3.477.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.19

$$\int \frac{1}{(\cos^2(x)-\sin^2(x))^3}\,dx = \frac{\tan(x)^3+\tan(x)}{2\left(\tan(x)^4-2\tan(x)^2+1\right)} + \frac{1}{4}\log(\tan(x)+1) - \frac{1}{4}\log(\tan(x)-1)$$

input
```
integrate(1/(cos(x)^2-sin(x)^2)^3,x, algorithm="maxima")
```

output
```
1/2*(tan(x)^3 + tan(x))/(tan(x)^4 - 2*tan(x)^2 + 1) + 1/4*log(tan(x) + 1)
- 1/4*log(tan(x) - 1)
```

3.477.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

$$\int \frac{1}{\left(\cos^2(x)-\sin^2(x)\right)^3}\,dx = -\frac{\sin(2x)}{4\left(\sin(2x)^2-1\right)}+\frac{1}{8}\log(\sin(2x)+1)-\frac{1}{8}\log(-\sin(2x)+1)$$

input
```
integrate(1/(cos(x)^2-sin(x)^2)^3,x, algorithm="giac")
```

output
```
-1/4*sin(2*x)/(sin(2*x)^2 - 1) + 1/8*log(sin(2*x) + 1) - 1/8*log(-sin(2*x)
 + 1)
```

3.477.9 Mupad [B] (verification not implemented)

Time = 27.57 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\cos^2(x)-\sin^2(x)\right)^3}\,dx = \frac{\operatorname{atanh}(\tan(x))}{2}+\frac{\frac{\tan(x)^3}{2}+\frac{\tan(x)}{2}}{\tan(x)^4-2\tan(x)^2+1}$$

input
```
int(1/(cos(x)^2 - sin(x)^2)^3,x)
```

output
```
atanh(tan(x))/2 + (tan(x)/2 + tan(x)^3/2)/(tan(x)^4 - 2*tan(x)^2 + 1)
```

3.478 $\int \frac{1}{\cos^2(x)+a^2\sin^2(x)}\,dx$

3.478.1 Optimal result

Integrand size = 15, antiderivative size = 9

$$\int \frac{1}{\cos^2(x)+a^2\sin^2(x)}\,dx=\frac{\arctan(a\tan(x))}{a}$$

output
```
arctan(a*tan(x))/a
```

3.478.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{\cos^2(x)+a^2\sin^2(x)}\,dx=\frac{\arctan(a\tan(x))}{a}$$

input
```
Integrate[(Cos[x]^2 + a^2*Sin[x]^2)^(-1),x]
```

output
```
ArcTan[a*Tan[x]]/a
```

3.478.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4889, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a^2 \sin^2(x) + \cos^2(x)} \, dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{a^2 \sin(x)^2 + \cos(x)^2} dx$$
$$\Big\downarrow \text{ 4889}$$
$$\int \frac{1}{a^2 \tan^2(x) + 1} d\tan(x)$$
$$\Big\downarrow \text{ 216}$$
$$\frac{\arctan(a \tan(x))}{a}$$

input
```
Int[(Cos[x]^2 + a^2*Sin[x]^2)^(-1),x]
```

output
```
ArcTan[a*Tan[x]]/a
```

3.478.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.478.4 Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

method	result	size
default	$\frac{\arctan(a\tan(x))}{a}$	10
parallelrisch	$\frac{i\left(\ln\left(\tan\left(\frac{x}{2}\right)^2+2ia\tan\left(\frac{x}{2}\right)-1\right)-\ln\left(\tan\left(\frac{x}{2}\right)^2-2ia\tan\left(\frac{x}{2}\right)-1\right)\right)}{2a}$	44
risch	$-\frac{i\ln\left(\mathrm{e}^{2ix}-\frac{a-1}{a+1}\right)}{2a}+\frac{i\ln\left(\mathrm{e}^{2ix}-\frac{a+1}{a-1}\right)}{2a}$	48

input
```
int(1/(cos(x)^2+a^2*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
arctan(a*tan(x))/a
```

3.478.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(9) = 18.

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 3.89

$$\int \frac{1}{\cos^2(x)+a^2\sin^2(x)}\,dx=-\frac{\arctan\left(\frac{(a^2+1)\cos(x)^2-a^2}{2\,a\cos(x)\sin(x)}\right)}{2\,a}$$

input
```
integrate(1/(cos(x)^2+a^2*sin(x)^2),x, algorithm="fricas")
```

output
```
-1/2*arctan(1/2*((a^2 + 1)*cos(x)^2 - a^2)/(a*cos(x)*sin(x)))/a
```

3.478.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12007 vs. $2(7) = 14$.

Time = 8.29 (sec) , antiderivative size = 12007, normalized size of antiderivative = 1334.11

$$\int \frac{1}{\cos^2(x) + a^2\sin^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(cos(x)**2+a**2*sin(x)**2),x)
```

output
```
Piecewise((64*a**7*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*log(-sqrt(-2*a**
2 + 2*a*sqrt(a**2 - 1) + 1) + tan(x/2))/(64*a**7*sqrt(-2*a**2 - 2*a*sqrt(a
**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 64*a**6*sqrt(a**2 -
 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 -
1) + 1) - 96*a**5*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*
a*sqrt(a**2 - 1) + 1) + 64*a**4*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**
2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + 36*a**3*sqrt(-2*a**2
- 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 12*a**2
*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*
sqrt(a**2 - 1) + 1) - 2*a*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a
**2 + 2*a*sqrt(a**2 - 1) + 1)) - 64*a**7*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1)
 + 1)*log(sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + tan(x/2))/(64*a**7*sqrt
(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1)
- 64*a**6*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a*
*2 + 2*a*sqrt(a**2 - 1) + 1) - 96*a**5*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) +
 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) + 64*a**4*sqrt(a**2 - 1)*sqrt(-
2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) +
36*a**3*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1)*sqrt(-2*a**2 + 2*a*sqrt(a**
2 - 1) + 1) - 12*a**2*sqrt(a**2 - 1)*sqrt(-2*a**2 - 2*a*sqrt(a**2 - 1) + 1
)*sqrt(-2*a**2 + 2*a*sqrt(a**2 - 1) + 1) - 2*a*sqrt(-2*a**2 - 2*a*sqrt(...
```

3.478.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{\cos^2(x) + a^2\sin^2(x)}\,dx = \frac{\arctan(a\tan(x))}{a}$$

input
```
integrate(1/(cos(x)^2+a^2*sin(x)^2),x, algorithm="maxima")
```

output
```
arctan(a*tan(x))/a
```

3.478.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. 2(9) = 18.

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.22

$$\int \frac{1}{\cos^2(x)+a^2\sin^2(x)}\,dx=\frac{\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(a\tan\left(x\right)\right)}{a}$$

input
```
integrate(1/(cos(x)^2+a^2*sin(x)^2),x, algorithm="giac")
```

output
```
(pi*floor(x/pi + 1/2) + arctan(a*tan(x)))/a
```

3.478.9 Mupad [B] (verification not implemented)

Time = 29.70 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{1}{\cos^2(x)+a^2\sin^2(x)}\,dx=\frac{\mathrm{atan}(a\tan(x))}{a}$$

input
```
int(1/(cos(x)^2 + a^2*sin(x)^2),x)
```

output
```
atan(a*tan(x))/a
```

3.479 $\int \frac{1}{b^2 \cos^2(x)+\sin^2(x)}\,dx$

3.479.1 Optimal result

Integrand size = 15, antiderivative size = 11

$$\int \frac{1}{b^2 \cos^2(x)+\sin^2(x)}\,dx=\frac{\arctan\left(\frac{\tan(x)}{b}\right)}{b}$$

output
```
arctan(tan(x)/b)/b
```

3.479.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{b^2 \cos^2(x)+\sin^2(x)}\,dx=\frac{\arctan\left(\frac{\tan(x)}{b}\right)}{b}$$

input
```
Integrate[(b^2*Cos[x]^2 + Sin[x]^2)^(-1),x]
```

output
```
ArcTan[Tan[x]/b]/b
```

3.479.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 4889, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{b^2\cos^2(x)+\sin^2(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{b^2\cos(x)^2+\sin(x)^2}dx$$

$$\Big\downarrow \text{4889}$$

$$\int \frac{1}{b^2+\tan^2(x)}d\tan(x)$$

$$\Big\downarrow \text{216}$$

$$\frac{\arctan\left(\frac{\tan(x)}{b}\right)}{b}$$

input
```
Int[(b^2*Cos[x]^2 + Sin[x]^2)^(-1),x]
```

output
```
ArcTan[Tan[x]/b]/b
```

3.479.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.479.4 Maple [A] (verified)

Time = 3.57 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
default	$\frac{\arctan\left(\frac{\tan(x)}{b}\right)}{b}$	12
risch	$\frac{i\ln\left(\mathrm{e}^{2ix}+\frac{b+1}{b-1}\right)}{2b}-\frac{i\ln\left(\mathrm{e}^{2ix}+\frac{b-1}{b+1}\right)}{2b}$	46
parallelrisch	$-\frac{i\left(\ln\left(\frac{-b\cos(x)-i\sin(x)}{\cos(x)+1}\right)-\ln\left(\frac{i\sin(x)-b\cos(x)}{\cos(x)+1}\right)\right)}{2b}$	48

input
```
int(1/(b^2*cos(x)^2+sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
arctan(tan(x)/b)/b
```

3.479.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(11) = 22.

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.82

$$\int \frac{1}{b^2\cos^2(x)+\sin^2(x)}\,dx = -\frac{\arctan\left(\frac{(b^2+1)\cos(x)^2-1}{2\,b\cos(x)\sin(x)}\right)}{2\,b}$$

input
```
integrate(1/(b^2*cos(x)^2+sin(x)^2),x, algorithm="fricas")
```

output
```
-1/2*arctan(1/2*((b^2 + 1)*cos(x)^2 - 1)/(b*cos(x)*sin(x)))/b
```

3.479.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30189 vs. $2(7) = 14$.

Time = 10.34 (sec) , antiderivative size = 30189, normalized size of antiderivative = 2744.45

$$\int \frac{1}{b^2\cos^2(x)+\sin^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(b**2*cos(x)**2+sin(x)**2),x)
```

output
```
Piecewise((tan(x/2)/2 - 1/(2*tan(x/2)), Eq(b, 0)), (x/(sin(x)**2 + cos(x)*
*2), Eq(b, -1) | Eq(b, 1)), (-b**10*sqrt(1 - b**2)*sqrt(1 - 2*sqrt(1 - b**
2)/b**2 - 2/b**2)*log(-sqrt(1 + 2*sqrt(1 - b**2)/b**2 - 2/b**2) + tan(x/2)
)/(2*b**12*sqrt(1 - 2*sqrt(1 - b**2)/b**2 - 2/b**2)*sqrt(1 + 2*sqrt(1 - b*
*2)/b**2 - 2/b**2) + 18*b**10*sqrt(1 - b**2)*sqrt(1 - 2*sqrt(1 - b**2)/b**
2 - 2/b**2)*sqrt(1 + 2*sqrt(1 - b**2)/b**2 - 2/b**2) - 82*b**10*sqrt(1 - 2
*sqrt(1 - b**2)/b**2 - 2/b**2)*sqrt(1 + 2*sqrt(1 - b**2)/b**2 - 2/b**2) -
240*b**8*sqrt(1 - b**2)*sqrt(1 - 2*sqrt(1 - b**2)/b**2 - 2/b**2)*sqrt(1 +
2*sqrt(1 - b**2)/b**2 - 2/b**2) + 560*b**8*sqrt(1 - 2*sqrt(1 - b**2)/b**2
- 2/b**2)*sqrt(1 + 2*sqrt(1 - b**2)/b**2 - 2/b**2) + 864*b**6*sqrt(1 - b**
2)*sqrt(1 - 2*sqrt(1 - b**2)/b**2 - 2/b**2)*sqrt(1 + 2*sqrt(1 - b**2)/b**2
 - 2/b**2) - 1376*b**6*sqrt(1 - 2*sqrt(1 - b**2)/b**2 - 2/b**2)*sqrt(1 + 2
*sqrt(1 - b**2)/b**2 - 2/b**2) - 1152*b**4*sqrt(1 - b**2)*sqrt(1 - 2*sqrt(
1 - b**2)/b**2 - 2/b**2)*sqrt(1 + 2*sqrt(1 - b**2)/b**2 - 2/b**2) + 1408*b
**4*sqrt(1 - 2*sqrt(1 - b**2)/b**2 - 2/b**2)*sqrt(1 + 2*sqrt(1 - b**2)/b**
2 - 2/b**2) + 512*b**2*sqrt(1 - b**2)*sqrt(1 - 2*sqrt(1 - b**2)/b**2 - 2/b
**2)*sqrt(1 + 2*sqrt(1 - b**2)/b**2 - 2/b**2) - 512*b**2*sqrt(1 - 2*sqrt(1
 - b**2)/b**2 - 2/b**2)*sqrt(1 + 2*sqrt(1 - b**2)/b**2 - 2/b**2)) + b**10*
sqrt(1 - b**2)*sqrt(1 - 2*sqrt(1 - b**2)/b**2 - 2/b**2)*log(sqrt(1 + 2*sqr
t(1 - b**2)/b**2 - 2/b**2) + tan(x/2))/(2*b**12*sqrt(1 - 2*sqrt(1 - b**...
```

3.479.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{b^2\cos^2(x)+\sin^2(x)}\,dx = \frac{\arctan\left(\frac{\tan(x)}{b}\right)}{b}$$

input
```
integrate(1/(b^2*cos(x)^2+sin(x)^2),x, algorithm="maxima")
```

output
```
arctan(tan(x)/b)/b
```

3.479.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.00

$$\int \frac{1}{b^2\cos^2(x)+\sin^2(x)}\,dx=\frac{\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{\tan(x)}{b}\right)}{b}$$

input
```
integrate(1/(b^2*cos(x)^2+sin(x)^2),x, algorithm="giac")
```

output
```
(pi*floor(x/pi + 1/2) + arctan(tan(x)/b))/b
```

3.479.9 Mupad [B] (verification not implemented)

Time = 27.90 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{b^2\cos^2(x)+\sin^2(x)}\,dx=\frac{\mathrm{atan}\left(\frac{\tan(x)}{b}\right)}{b}$$

input
```
int(1/(sin(x)^2 + b^2*cos(x)^2),x)
```

output
```
atan(tan(x)/b)/b
```

3.480 $\int \frac{1}{b^2\cos^2(x)+a^2\sin^2(x)}\,dx$

3.480.1 Optimal result

Integrand size = 19, antiderivative size = 15

$$\int \frac{1}{b^2\cos^2(x)+a^2\sin^2(x)}\,dx=\frac{\arctan\left(\frac{a\tan(x)}{b}\right)}{ab}$$

output
```
arctan(a*tan(x)/b)/a/b
```

3.480.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{b^2\cos^2(x)+a^2\sin^2(x)}\,dx=\frac{\arctan\left(\frac{a\tan(x)}{b}\right)}{ab}$$

input
```
Integrate[(b^2*Cos[x]^2 + a^2*Sin[x]^2)^(-1),x]
```

output
```
ArcTan[(a*Tan[x])/b]/(a*b)
```

3.480.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 4889, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a^2\sin^2(x)+b^2\cos^2(x)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{a^2\sin(x)^2+b^2\cos(x)^2}dx$$

$$\Big\downarrow \; 4889$$

$$\int \frac{1}{a^2\tan^2(x)+b^2}d\tan(x)$$

$$\Big\downarrow \; 218$$

$$\frac{\arctan\left(\frac{a\tan(x)}{b}\right)}{ab}$$

input
```
Int[(b^2*Cos[x]^2 + a^2*Sin[x]^2)^(-1),x]
```

output
```
ArcTan[(a*Tan[x])/b]/(a*b)
```

3.480.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.480.4 Maple [A] (verified)

Time = 3.51 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

method	result	size
default	$\frac{\arctan\left(\frac{a\tan(x)}{b}\right)}{ab}$	16
parallelrisch	$\frac{i\left(\ln\left(\frac{ia\sin(x)-b\cos(x)}{\cos(x)+1}\right)-\ln\left(\frac{-b\cos(x)-ia\sin(x)}{\cos(x)+1}\right)\right)}{2ab}$	53
risch	$-\frac{i\ln\left(\mathrm{e}^{2ix}-\frac{a-b}{a+b}\right)}{2ab}+\frac{i\ln\left(\mathrm{e}^{2ix}-\frac{a+b}{a-b}\right)}{2ab}$	58

input
```
int(1/(b^2*cos(x)^2+a^2*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
arctan(a*tan(x)/b)/a/b
```

3.480.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. 2(15) = 30.

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.87

$$\int \frac{1}{b^2\cos^2(x)+a^2\sin^2(x)}\,dx = -\frac{\arctan\left(\frac{(a^2+b^2)\cos(x)^2-a^2}{2\,ab\cos(x)\sin(x)}\right)}{2\,ab}$$

input
```
integrate(1/(b^2*cos(x)^2+a^2*sin(x)^2),x, algorithm="fricas")
```

output
```
-1/2*arctan(1/2*((a^2 + b^2)*cos(x)^2 - a^2)/(a*b*cos(x)*sin(x)))/(a*b)
```

3.480.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71839 vs. $2(10) = 20$.

Time = 15.53 (sec) , antiderivative size = 71839, normalized size of antiderivative = 4789.27

$$\int \frac{1}{b^2 \cos^2(x) + a^2 \sin^2(x)} \, dx = \text{Too large to display}$$

input
```
integrate(1/(b**2*cos(x)**2+a**2*sin(x)**2),x)
```

output
```
Piecewise((zoo*tan(x/2)/(tan(x/2)**2 - 1), Eq(a, 0) & Eq(b, 0)), ((tan(x/2
)/2 - 1/(2*tan(x/2)))/a**2, Eq(b, 0)), (-2*tan(x/2)/(b**2*(tan(x/2)**2 - 1
)), Eq(a, 0)), (x/(b**2*sin(x)**2 + b**2*cos(x)**2), Eq(a, b) | Eq(a, -b))
, (8192*a**15*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(-sqr
t(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(8192*a**15*b
**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2
+ 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 8192*a**14*b**2*sqrt(a**2 - b**2)*sqrt
(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sq
rt(a**2 - b**2)/b**2 + 1) - 30720*a**13*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(
a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)
 + 26624*a**12*b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 -
b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + 4556
8*a**11*b**6*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a
**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33280*a**10*b**6*sqrt(a**2 -
b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**
2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) - 33792*a**9*b**8*sqrt(-2*a**2/b**2 -
2*a*sqrt(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/
b**2 + 1) + 19968*a**8*b**8*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt
(a**2 - b**2)/b**2 + 1)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1
) + 12992*a**7*b**10*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1...
```

3.480.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{b^2 \cos^2(x) + a^2 \sin^2(x)} \, dx = \frac{\arctan\left(\frac{a \tan(x)}{b}\right)}{ab}$$

input
```
integrate(1/(b^2*cos(x)^2+a^2*sin(x)^2),x, algorithm="maxima")
```

output
```
arctan(a*tan(x)/b)/(a*b)
```

3.480.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.73

$$\int \frac{1}{b^2\cos^2(x)+a^2\sin^2(x)}\,dx = \frac{\pi\left\lfloor \frac{x}{\pi}+\frac{1}{2}\right\rfloor + \arctan\left(\frac{a\tan(x)}{b}\right)}{ab}$$

input
```
integrate(1/(b^2*cos(x)^2+a^2*sin(x)^2),x, algorithm="giac")
```

output
```
(pi*floor(x/pi + 1/2) + arctan(a*tan(x)/b))/(a*b)
```

3.480.9 Mupad [B] (verification not implemented)

Time = 28.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{b^2\cos^2(x)+a^2\sin^2(x)}\,dx = \frac{\operatorname{atan}\left(\frac{a\tan(x)}{b}\right)}{a\,b}$$

input
```
int(1/(b^2*cos(x)^2 + a^2*sin(x)^2),x)
```

output
```
atan((a*tan(x))/b)/(a*b)
```

3.481 $\int \frac{1}{4\cos^2(1+2x)+3\sin^2(1+2x)}\,dx$

3.481.1 Optimal result

Integrand size = 23, antiderivative size = 53

$$\int \frac{1}{4\cos^2(1+2x)+3\sin^2(1+2x)}\,dx = \frac{x}{2\sqrt{3}} - \frac{\arctan\left(\frac{\cos(1+2x)\sin(1+2x)}{3+2\sqrt{3}+\cos^2(1+2x)}\right)}{4\sqrt{3}}$$

output
```
1/6*x*3^(1/2)-1/12*arctan(cos(1+2*x)*sin(1+2*x)/(3+cos(1+2*x)^2+2*3^(1/2))
)*3^(1/2)
```

3.481.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.47

$$\int \frac{1}{4\cos^2(1+2x)+3\sin^2(1+2x)}\,dx = \frac{\arctan\left(\frac{1}{2}\sqrt{3}\tan(1+2x)\right)}{4\sqrt{3}}$$

input
```
Integrate[(4*Cos[1 + 2*x]^2 + 3*Sin[1 + 2*x]^2)^(-1),x]
```

output
```
ArcTan[(Sqrt[3]*Tan[1 + 2*x])/2]/(4*Sqrt[3])
```

3.481.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.47, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3042, 4889, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{3\sin^2(2x+1)+4\cos^2(2x+1)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{3\sin(2x+1)^2+4\cos(2x+1)^2}dx$$

$$\Big\downarrow \text{ 4889}$$

$$\frac{1}{2}\int \frac{1}{3\tan^2(2x+1)+4}d\tan(2x+1)$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\arctan\left(\frac{1}{2}\sqrt{3}\tan(2x+1)\right)}{4\sqrt{3}}$$

input
```
Int[(4*Cos[1 + 2*x]^2 + 3*Sin[1 + 2*x]^2)^(-1),x]
```

output
```
ArcTan[(Sqrt[3]*Tan[1 + 2*x])/2]/(4*Sqrt[3])
```

3.481.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.481.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.34

method	result	size
derivativedivides	$\frac{\sqrt{3}\,\arctan\left(\frac{\tan(1+2x)\sqrt{3}}{2}\right)}{12}$	18
default	$\frac{\sqrt{3}\,\arctan\left(\frac{\tan(1+2x)\sqrt{3}}{2}\right)}{12}$	18
risch	$\frac{i\sqrt{3}\,\ln\left(\mathrm{e}^{2i(1+2x)}+4\sqrt{3}+7\right)}{24}-\frac{i\sqrt{3}\,\ln\left(\mathrm{e}^{2i(1+2x)}-4\sqrt{3}+7\right)}{24}$	48

input
```
int(1/(4*cos(1+2*x)^2+3*sin(1+2*x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/12*3^(1/2)*arctan(1/2*tan(1+2*x)*3^(1/2))
```

3.481.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.81

$$\int \frac{1}{4\cos^2(1+2x)+3\sin^2(1+2x)}\,dx = -\frac{1}{24}\,\sqrt{3}\arctan\left(\frac{7\,\sqrt{3}\cos\left(2\,x+1\right)^2-3\,\sqrt{3}}{12\,\cos\left(2\,x+1\right)\sin\left(2\,x+1\right)}\right)$$

input
```
integrate(1/(4*cos(1+2*x)^2+3*sin(1+2*x)^2),x, algorithm="fricas")
```

output
```
-1/24*sqrt(3)*arctan(1/12*(7*sqrt(3)*cos(2*x + 1)^2 - 3*sqrt(3))/(cos(2*x
+ 1)*sin(2*x + 1)))
```

3.481.6 Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.64

$$\int \frac{1}{4\cos^2(1+2x)+3\sin^2(1+2x)}\,dx = \frac{\sqrt{3}\left(\operatorname{atan}\left(\frac{2\sqrt{3}\tan\left(x+\frac{1}{2}\right)}{3}-\frac{\sqrt{3}}{3}\right)+\pi\left\lfloor\frac{x-\frac{\pi}{2}+\frac{1}{2}}{\pi}\right\rfloor\right)}{12} + \frac{\sqrt{3}\left(\operatorname{atan}\left(\frac{2\sqrt{3}\tan\left(x+\frac{1}{2}\right)}{3}+\frac{\sqrt{3}}{3}\right)+\pi\left\lfloor\frac{x-\frac{\pi}{2}+\frac{1}{2}}{\pi}\right\rfloor\right)}{12}$$

input
```
integrate(1/(4*cos(1+2*x)**2+3*sin(1+2*x)**2),x)
```

output
```
sqrt(3)*(atan(2*sqrt(3)*tan(x + 1/2)/3 - sqrt(3)/3) + pi*floor((x - pi/2 +
 1/2)/pi))/12 + sqrt(3)*(atan(2*sqrt(3)*tan(x + 1/2)/3 + sqrt(3)/3) + pi*f
loor((x - pi/2 + 1/2)/pi))/12
```

3.481.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.32

$$\int \frac{1}{4\cos^2(1+2x)+3\sin^2(1+2x)}\,dx = \frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{2}\sqrt{3}\tan(2x+1)\right)$$

input
```
integrate(1/(4*cos(1+2*x)^2+3*sin(1+2*x)^2),x, algorithm="maxima")
```

output
```
1/12*sqrt(3)*arctan(1/2*sqrt(3)*tan(2*x + 1))
```

3.481.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.15

$$\int \frac{1}{4\cos^2(1+2x)+3\sin^2(1+2x)}\,dx = \frac{1}{12}\sqrt{3}\left(2x+\arctan\left(-\frac{2\sqrt{3}\sin(4x+2)-3\sin(4x+2)}{2\sqrt{3}\cos(4x+2)+2\sqrt{3}-3\cos(4x+2)+3}\right)+1\right)$$

input
```
integrate(1/(4*cos(1+2*x)^2+3*sin(1+2*x)^2),x, algorithm="giac")
```

output
```
1/12*sqrt(3)*(2*x + arctan(-(2*sqrt(3)*sin(4*x + 2) - 3*sin(4*x + 2))/(2*s
qrt(3)*cos(4*x + 2) + 2*sqrt(3) - 3*cos(4*x + 2) + 3)) + 1)
```

3.481.9 Mupad [B] (verification not implemented)

Time = 28.91 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.68

$$\int \frac{1}{4\cos^2(1+2x)+3\sin^2(1+2x)}\,dx = \frac{\sqrt{3}\,(2\,x-\mathrm{atan}(\tan(2\,x+1)))}{12}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\tan(2\,x+1)}{2}\right)}{12}$$

input
```
int(1/(3*sin(2*x + 1)^2 + 4*cos(2*x + 1)^2),x)
```

output
```
(3^(1/2)*(2*x - atan(tan(2*x + 1))))/12 + (3^(1/2)*atan((3^(1/2)*tan(2*x +
 1))/2))/12
```

3.482 $\int \frac{\sin^2(x)}{a\cos^2(x)+b\sin^2(x)}\,dx$

3.482.1 Optimal result

Integrand size = 20, antiderivative size = 43

$$\int \frac{\sin^2(x)}{a\cos^2(x)+b\sin^2(x)}\,dx = -\frac{x}{a-b}+\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}\tan(x)}{\sqrt{a}}\right)}{(a-b)\sqrt{b}}$$

output
```
-x/(a-b)+arctan(b^(1/2)*tan(x)/a^(1/2))*a^(1/2)/(a-b)/b^(1/2)
```

3.482.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{\sin^2(x)}{a\cos^2(x)+b\sin^2(x)}\,dx = \frac{x-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{b}}}{-a+b}$$

input
```
Integrate[Sin[x]^2/(a*Cos[x]^2 + b*Sin[x]^2),x]
```

output
```
(x - (Sqrt[a]*ArcTan[(Sqrt[b]*Tan[x])/Sqrt[a]])/Sqrt[b])/(-a + b)
```

3.482.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 4889, 383, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(x)}{a\cos^2(x)+b\sin^2(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)^2}{a\cos(x)^2+b\sin(x)^2}dx$$

$$\Big\downarrow \text{4889}$$

$$\int \frac{\tan^2(x)}{(\tan^2(x)+1)\,(a+b\tan^2(x))}d\tan(x)$$

$$\Big\downarrow \text{383}$$

$$\frac{a\int \frac{1}{b\tan^2(x)+a}d\tan(x)}{a-b}-\frac{\int \frac{1}{\tan^2(x)+1}d\tan(x)}{a-b}$$

$$\Big\downarrow \text{216}$$

$$\frac{a\int \frac{1}{b\tan^2(x)+a}d\tan(x)}{a-b}-\frac{\arctan(\tan(x))}{a-b}$$

$$\Big\downarrow \text{218}$$

$$\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{b}(a-b)}-\frac{\arctan(\tan(x))}{a-b}$$

input
```
Int[Sin[x]^2/(a*Cos[x]^2 + b*Sin[x]^2),x]
```

output
```
-(ArcTan[Tan[x]]/(a - b)) + (Sqrt[a]*ArcTan[(Sqrt[b]*Tan[x])/Sqrt[a]])/((a
 - b)*Sqrt[b])
```

3.482.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 383
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Sym
bol] :> Simp[(-a)*(e^2/(b*c - a*d))   Int[(e*x)^(m - 2)/(a + b*x^2), x], x]
 + Simp[c*(e^2/(b*c - a*d))   Int[(e*x)^(m - 2)/(c + d*x^2), x], x] /; Free
Q[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LeQ[2, m, 3]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.482.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{a\arctan\left(\frac{b\tan(x)}{\sqrt{ab}}\right)}{(a-b)\sqrt{ab}}-\frac{\arctan(\tan(x))}{a-b}$	38
risch	$-\frac{x}{a-b}-\frac{\sqrt{-ab}\,\ln\left(\mathrm{e}^{2ix}+\frac{2i\sqrt{-ab}+a+b}{a-b}\right)}{2b(a-b)}+\frac{\sqrt{-ab}\,\ln\left(\mathrm{e}^{2ix}-\frac{2i\sqrt{-ab}-a-b}{a-b}\right)}{2b(a-b)}$	107

input
```
int(sin(x)^2/(a*cos(x)^2+sin(x)^2*b),x,method=_RETURNVERBOSE)
```

output
```
a/(a-b)/(a*b)^(1/2)*arctan(b*tan(x)/(a*b)^(1/2))-1/(a-b)*arctan(tan(x))
```

3.482.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 182, normalized size of antiderivative = 4.23

$$\int \frac{\sin^2(x)}{a\cos^2(x)+b\sin^2(x)}\,dx = \left[-\frac{\sqrt{-\frac{a}{b}}\log\left(\frac{(a^2+6\,ab+b^2)\cos(x)^4-2\,(3\,ab+b^2)\cos(x)^2+4\,\left((ab+b^2)\cos(x)^3-b^2\cos(x)\right)\sqrt{-\frac{a}{b}}\sin(x)+b^2}{(a^2-2\,ab+b^2)\cos(x)^4+2\,(ab-b^2)\cos(x)^2+b^2}\right)+4\,x}{4\,(a-b)}, -\frac{\sqrt{\frac{a}{b}}\arctan\left(\frac{\left((a+b)\cos(x)^2-b\right)\sqrt{\frac{a}{b}}}{2\,a\cos(x)\sin(x)}\right)+2\,x}{2\,(a-b)}\right]$$

input
```
integrate(sin(x)^2/(a*cos(x)^2+b*sin(x)^2),x, algorithm="fricas")
```

output
```
[-1/4*(sqrt(-a/b)*log(((a^2 + 6*a*b + b^2)*cos(x)^4 - 2*(3*a*b + b^2)*cos(
x)^2 + 4*((a*b + b^2)*cos(x)^3 - b^2*cos(x))*sqrt(-a/b)*sin(x) + b^2)/((a^
2 - 2*a*b + b^2)*cos(x)^4 + 2*(a*b - b^2)*cos(x)^2 + b^2)) + 4*x)/(a - b),
 -1/2*(sqrt(a/b)*arctan(1/2*((a + b)*cos(x)^2 - b)*sqrt(a/b)/(a*cos(x)*sin
(x))) + 2*x)/(a - b)]
```

3.482.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(32) = 64$.

Time = 0.58 (sec) , antiderivative size = 216, normalized size of antiderivative = 5.02

$$\int \frac{\sin^2(x)}{a\cos^2(x) + b\sin^2(x)}\,dx = \begin{cases} \tilde{\infty}x & \text{for } a = 0 \wedge b = 0 \\ \frac{-x + \frac{\sin(x)}{\cos(x)}}{a} & \text{for } b = 0 \\ \frac{x}{b} & \text{for } a = 0 \\ \frac{x\sin^2(x)}{2b\sin^2(x) + 2b\cos^2(x)} + \frac{x\cos^2(x)}{2b\sin^2(x) + 2b\cos^2(x)} - \frac{\sin(x)\cos(x)}{2b\sin^2(x) + 2b\cos^2(x)} & \text{for } a = b \\ \frac{a\log\left(-\sqrt{-\frac{a}{b}}\cos(x) + \sin(x)\right)}{2ab\sqrt{-\frac{a}{b}} - 2b^2\sqrt{-\frac{a}{b}}} - \frac{a\log\left(\sqrt{-\frac{a}{b}}\cos(x) + \sin(x)\right)}{2ab\sqrt{-\frac{a}{b}} - 2b^2\sqrt{-\frac{a}{b}}} - \frac{2bx\sqrt{-\frac{a}{b}}}{2ab\sqrt{-\frac{a}{b}} - 2b^2\sqrt{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

input
```
integrate(sin(x)**2/(a*cos(x)**2+b*sin(x)**2),x)
```

output
```
Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), ((-x + sin(x)/cos(x))/a, Eq(b, 0))
, (x/b, Eq(a, 0)), (x*sin(x)**2/(2*b*sin(x)**2 + 2*b*cos(x)**2) + x*cos(x)
**2/(2*b*sin(x)**2 + 2*b*cos(x)**2) - sin(x)*cos(x)/(2*b*sin(x)**2 + 2*b*c
os(x)**2), Eq(a, b)), (a*log(-sqrt(-a/b)*cos(x) + sin(x))/(2*a*b*sqrt(-a/b
) - 2*b**2*sqrt(-a/b)) - a*log(sqrt(-a/b)*cos(x) + sin(x))/(2*a*b*sqrt(-a/
b) - 2*b**2*sqrt(-a/b)) - 2*b*x*sqrt(-a/b)/(2*a*b*sqrt(-a/b) - 2*b**2*sqrt
(-a/b)), True))
```

3.482.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{\sin^2(x)}{a\cos^2(x) + b\sin^2(x)}\,dx = \frac{a\arctan\left(\frac{b\tan(x)}{\sqrt{ab}}\right)}{\sqrt{ab}(a-b)} - \frac{x}{a-b}$$

input
```
integrate(sin(x)^2/(a*cos(x)^2+b*sin(x)^2),x, algorithm="maxima")
```

output
```
a*arctan(b*tan(x)/sqrt(a*b))/(sqrt(a*b)*(a - b)) - x/(a - b)
```

3.482.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12

$$\int \frac{\sin^2(x)}{a\cos^2(x)+b\sin^2(x)}\,dx = \frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(x)}{\sqrt{ab}}\right)\right)a}{\sqrt{ab}(a-b)} - \frac{x}{a-b}$$

input
```
integrate(sin(x)^2/(a*cos(x)^2+b*sin(x)^2),x, algorithm="giac")
```

output
```
(pi*floor(x/pi + 1/2)*sgn(b) + arctan(b*tan(x)/sqrt(a*b)))*a/(sqrt(a*b)*(a
 - b)) - x/(a - b)
```

3.482.9 Mupad [B] (verification not implemented)

Time = 27.92 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.19

$$\int \frac{\sin^2(x)}{a\cos^2(x)+b\sin^2(x)}\,dx = \begin{cases} \frac{2x-\sin(2x)}{4b} & \text{if} \quad a=b \\ -\frac{x-\frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{b}}}{a-b} & \text{if} \quad a\neq b \end{cases}$$

input
```
int(sin(x)^2/(b*sin(x)^2 + a*cos(x)^2),x)
```

output
```
piecewise(a == b, (2*x - sin(2*x))/(4*b), a ~= b, -(x - (a^(1/2)*atan((b^(
1/2)*tan(x))/a^(1/2)))/b^(1/2))/(a - b))
```

3.483 $\int \frac{\cos^2(x)}{a\cos^2(x)+b\sin^2(x)}\,dx$

3.483.1 Optimal result

Integrand size = 20, antiderivative size = 43

$$\int \frac{\cos^2(x)}{a\cos^2(x)+b\sin^2(x)}\,dx = \frac{x}{a-b} - \frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)}$$

output
```
x/(a-b)-arctan(b^(1/2)*tan(x)/a^(1/2))*b^(1/2)/(a-b)/a^(1/2)
```

3.483.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{\cos^2(x)}{a\cos^2(x)+b\sin^2(x)}\,dx = \frac{x - \frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}}}{a-b}$$

input
```
Integrate[Cos[x]^2/(a*Cos[x]^2 + b*Sin[x]^2),x]
```

output
```
(x - (Sqrt[b]*ArcTan[(Sqrt[b]*Tan[x])/Sqrt[a]])/Sqrt[a])/(a - b)
```

3.483.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 4889, 303, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(x)}{a\cos^2(x)+b\sin^2(x)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\cos(x)^2}{a\cos(x)^2+b\sin(x)^2}dx$$
$$\Big\downarrow \; 4889$$
$$\int \frac{1}{\left(\tan^2(x)+1\right)\left(a+b\tan^2(x)\right)}d\tan(x)$$
$$\Big\downarrow \; 303$$
$$\frac{\int \frac{1}{\tan^2(x)+1}d\tan(x)}{a-b}-\frac{b\int \frac{1}{b\tan^2(x)+a}d\tan(x)}{a-b}$$
$$\Big\downarrow \; 216$$
$$\frac{\arctan(\tan(x))}{a-b}-\frac{b\int \frac{1}{b\tan^2(x)+a}d\tan(x)}{a-b}$$
$$\Big\downarrow \; 218$$
$$\frac{\arctan(\tan(x))}{a-b}-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)}$$

input
```
Int[Cos[x]^2/(a*Cos[x]^2 + b*Sin[x]^2),x]
```

output
```
ArcTan[Tan[x]]/(a - b) - (Sqrt[b]*ArcTan[(Sqrt[b]*Tan[x])/Sqrt[a]])/(Sqrt[
a]*(a - b))
```

3.483.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 303
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Simp[b/(b
*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x
^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.483.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{b \arctan\left(\frac{b \tan(x)}{\sqrt{ab}}\right)}{(a-b)\sqrt{ab}}+\frac{\arctan(\tan(x))}{a-b}$	38
risch	$\frac{x}{a-b}+\frac{\sqrt{-ab}\,\ln\left(\mathrm{e}^{2ix}+\frac{2i\sqrt{-ab}+a+b}{a-b}\right)}{2a(a-b)}-\frac{\sqrt{-ab}\,\ln\left(\mathrm{e}^{2ix}-\frac{2i\sqrt{-ab}-a-b}{a-b}\right)}{2a(a-b)}$	106

input
```
int(cos(x)^2/(a*cos(x)^2+sin(x)^2*b),x,method=_RETURNVERBOSE)
```

output
```
-b/(a-b)/(a*b)^(1/2)*arctan(b*tan(x)/(a*b)^(1/2))+1/(a-b)*arctan(tan(x))
```

3.483.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 181, normalized size of antiderivative = 4.21

$$\int \frac{\cos^2(x)}{a\cos^2(x)+b\sin^2(x)}\,dx$$
$$= \left[-\frac{\sqrt{-\frac{b}{a}}\log\left(\frac{(a^2+6\,ab+b^2)\cos(x)^4-2\,(3\,ab+b^2)\cos(x)^2-4\left((a^2+ab)\cos(x)^3-ab\cos(x)\right)\sqrt{-\frac{b}{a}}\sin(x)+b^2}{(a^2-2\,ab+b^2)\cos(x)^4+2\,(ab-b^2)\cos(x)^2+b^2}\right)-4\,x}{4\,(a-b)}, \frac{\sqrt{\frac{b}{a}}\arctan\left(\right.}{}\right.$$

input
```
integrate(cos(x)^2/(a*cos(x)^2+b*sin(x)^2),x, algorithm="fricas")
```

output
```
[-1/4*(sqrt(-b/a)*log(((a^2 + 6*a*b + b^2)*cos(x)^4 - 2*(3*a*b + b^2)*cos(
x)^2 - 4*((a^2 + a*b)*cos(x)^3 - a*b*cos(x))*sqrt(-b/a)*sin(x) + b^2)/((a^
2 - 2*a*b + b^2)*cos(x)^4 + 2*(a*b - b^2)*cos(x)^2 + b^2)) - 4*x)/(a - b),
 1/2*(sqrt(b/a)*arctan(1/2*((a + b)*cos(x)^2 - b)*sqrt(b/a)/(b*cos(x)*sin(
x))) + 2*x)/(a - b)]
```

3.483.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. 2(32) = 64.

Time = 0.59 (sec) , antiderivative size = 211, normalized size of antiderivative = 4.91

$$\int \frac{\cos^2(x)}{a\cos^2(x)+b\sin^2(x)}\,dx$$
$$= \begin{cases} \tilde{\infty}\left(-x-\frac{\cos(x)}{\sin(x)}\right) & \text{for } a=0 \wedge b=0 \\ \frac{x}{a} & \text{for } b=0 \\ \frac{-x-\frac{\cos(x)}{\sin(x)}}{b} & \text{for } a=0 \\ \frac{x\sin^2(x)}{2b\sin^2(x)+2b\cos^2(x)}+\frac{x\cos^2(x)}{2b\sin^2(x)+2b\cos^2(x)}+\frac{\sin(x)\cos(x)}{2b\sin^2(x)+2b\cos^2(x)} & \text{for } a=b \\ \frac{2x\sqrt{-\frac{a}{b}}}{2a\sqrt{-\frac{a}{b}}-2b\sqrt{-\frac{a}{b}}}-\frac{\log\left(-\sqrt{-\frac{a}{b}}\cos(x)+\sin(x)\right)}{2a\sqrt{-\frac{a}{b}}-2b\sqrt{-\frac{a}{b}}}+\frac{\log\left(\sqrt{-\frac{a}{b}}\cos(x)+\sin(x)\right)}{2a\sqrt{-\frac{a}{b}}-2b\sqrt{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(x)**2/(a*cos(x)**2+b*sin(x)**2),x)
```

output
```
Piecewise((zoo*(-x - cos(x)/sin(x)), Eq(a, 0) & Eq(b, 0)), (x/a, Eq(b, 0))
, ((-x - cos(x)/sin(x))/b, Eq(a, 0)), (x*sin(x)**2/(2*b*sin(x)**2 + 2*b*co
s(x)**2) + x*cos(x)**2/(2*b*sin(x)**2 + 2*b*cos(x)**2) + sin(x)*cos(x)/(2*
b*sin(x)**2 + 2*b*cos(x)**2), Eq(a, b)), (2*x*sqrt(-a/b)/(2*a*sqrt(-a/b) -
 2*b*sqrt(-a/b)) - log(-sqrt(-a/b)*cos(x) + sin(x))/(2*a*sqrt(-a/b) - 2*b*
sqrt(-a/b)) + log(sqrt(-a/b)*cos(x) + sin(x))/(2*a*sqrt(-a/b) - 2*b*sqrt(-
a/b)), True))
```

3.483.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \frac{\cos^2(x)}{a\cos^2(x)+b\sin^2(x)}\,dx = -\frac{b\arctan\left(\frac{b\tan(x)}{\sqrt{ab}}\right)}{\sqrt{ab}(a-b)}+\frac{x}{a-b}$$

input
```
integrate(cos(x)^2/(a*cos(x)^2+b*sin(x)^2),x, algorithm="maxima")
```

output
```
-b*arctan(b*tan(x)/sqrt(a*b))/(sqrt(a*b)*(a - b)) + x/(a - b)
```

3.483.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12

$$\int \frac{\cos^2(x)}{a\cos^2(x)+b\sin^2(x)}\,dx = -\frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(b)+\arctan\left(\frac{b\tan(x)}{\sqrt{ab}}\right)\right)b}{\sqrt{ab}(a-b)}+\frac{x}{a-b}$$

input
```
integrate(cos(x)^2/(a*cos(x)^2+b*sin(x)^2),x, algorithm="giac")
```

output
```
-(pi*floor(x/pi + 1/2)*sgn(b) + arctan(b*tan(x)/sqrt(a*b)))*b/(sqrt(a*b)*(
a - b)) + x/(a - b)
```

3.483.9 Mupad [B] (verification not implemented)

Time = 27.64 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12

$$\int \frac{\cos^2(x)}{a\cos^2(x)+b\sin^2(x)}\,dx = \begin{cases} \frac{2x+\sin(2x)}{4b} & \text{if } a=b \\ \frac{x-\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}}}{a-b} & \text{if } a\neq b \end{cases}$$

input
```
int(cos(x)^2/(b*sin(x)^2 + a*cos(x)^2),x)
```

output
```
piecewise(a == b, (2*x + sin(2*x))/(4*b), a ~= b, (x - (b^(1/2)*atan((b^(1
/2)*tan(x))/a^(1/2)))/a^(1/2))/(a - b))
```

3.484 $\int \frac{1}{\sec^2(x)+\tan^2(x)}\,dx$

3.484.1 Optimal result

Integrand size = 11, antiderivative size = 36

$$\int \frac{1}{\sec^2(x)+\tan^2(x)}\,dx=-x+\sqrt{2}x+\sqrt{2}\arctan\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\sin^2(x)}\right)$$

output
```
-x+x*2^(1/2)+arctan(cos(x)*sin(x)/(1+sin(x)^2+2^(1/2)))*2^(1/2)
```

3.484.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.53

$$\int \frac{1}{\sec^2(x)+\tan^2(x)}\,dx=-x+\sqrt{2}\arctan\left(\sqrt{2}\tan(x)\right)$$

input
```
Integrate[(Sec[x]^2 + Tan[x]^2)^(-1),x]
```

output
```
-x + Sqrt[2]*ArcTan[Sqrt[2]*Tan[x]]
```

3.484.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.58, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 4889, 1406, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\tan^2(x)+\sec^2(x)}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{\tan(x)^2+\sec(x)^2}dx$$
$$\Big\downarrow \text{ 4889}$$
$$\int \frac{1}{2\tan^4(x)+3\tan^2(x)+1}d\tan(x)$$
$$\Big\downarrow \text{ 1406}$$
$$2\int \frac{1}{2\tan^2(x)+1}d\tan(x)-2\int \frac{1}{2\tan^2(x)+2}d\tan(x)$$
$$\Big\downarrow \text{ 216}$$
$$\sqrt{2}\arctan\left(\sqrt{2}\tan(x)\right)-\arctan(\tan(x))$$

input
```
Int[(Sec[x]^2 + Tan[x]^2)^(-1),x]
```

output
```
-ArcTan[Tan[x]] + Sqrt[2]*ArcTan[Sqrt[2]*Tan[x]]
```

3.484.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.484.4 Maple [A] (verified)

Time = 3.81 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.50

method	result	size
default	$-\arctan\left(\tan\left(x\right)\right)+\sqrt{2}\,\arctan\left(\sqrt{2}\,\tan\left(x\right)\right)$	18
risch	$-x+\frac{i\sqrt{2}\,\ln\left({\mathrm e}^{2 i x}-2\sqrt{2}-3\right)}{2}-\frac{i\sqrt{2}\,\ln\left({\mathrm e}^{2 i x}+2\sqrt{2}-3\right)}{2}$	43

input
```
int(1/(sec(x)^2+tan(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-arctan(tan(x))+2^(1/2)*arctan(2^(1/2)*tan(x))
```

3.484.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int \frac{1}{\sec^2(x)+\tan^2(x)}\,dx=-\frac{1}{2}\,\sqrt{2}\arctan\left(\frac{3\,\sqrt{2}\cos\left(x\right)^{2}-2\,\sqrt{2}}{4\,\cos\left(x\right)\sin\left(x\right)}\right)-x$$

input
```
integrate(1/(sec(x)^2+tan(x)^2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - 2*sqrt(2))/(cos(x)*sin(x)))
- x
```

3.484.6 Sympy [F]

$$\int \frac{1}{\sec^2(x) + \tan^2(x)}\, dx = \int \frac{1}{\tan^2(x) + \sec^2(x)}\, dx$$

input
```
integrate(1/(sec(x)**2+tan(x)**2),x)
```

output
```
Integral(1/(tan(x)**2 + sec(x)**2), x)
```

3.484.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.42

$$\int \frac{1}{\sec^2(x) + \tan^2(x)}\, dx = \sqrt{2}\arctan\left(\sqrt{2}\tan(x)\right) - x$$

input
```
integrate(1/(sec(x)^2+tan(x)^2),x, algorithm="maxima")
```

output
```
sqrt(2)*arctan(sqrt(2)*tan(x)) - x
```

3.484.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.42

$$\int \frac{1}{\sec^2(x) + \tan^2(x)}\, dx = \sqrt{2}\arctan\left(\sqrt{2}\tan(x)\right) - x$$

input
```
integrate(1/(sec(x)^2+tan(x)^2),x, algorithm="giac")
```

output
```
sqrt(2)*arctan(sqrt(2)*tan(x)) - x
```

3.484.9 Mupad [B] (verification not implemented)

Time = 27.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.42

$$\int \frac{1}{\sec^2(x) + \tan^2(x)}\, dx = \sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\tan(x)\right) - x$$

input
```
int(1/(1/cos(x)^2 + tan(x)^2),x)
```

output
```
2^(1/2)*atan(2^(1/2)*tan(x)) - x
```

3.485 $\int \frac{1}{\left(\sec^2(x)+\tan^2(x)\right)^2}\,dx$

3.485.1 Optimal result

Integrand size = 11, antiderivative size = 49

$$\int \frac{1}{\left(\sec^2(x)+\tan^2(x)\right)^2}\,dx = x-\frac{x}{\sqrt{2}}-\frac{\arctan\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\sin^2(x)}\right)}{\sqrt{2}}+\frac{\tan(x)}{1+2\tan^2(x)}$$

output
```
x-1/2*x*2^(1/2)-1/2*arctan(cos(x)*sin(x)/(1+sin(x)^2+2^(1/2)))*2^(1/2)+tan
(x)/(1+2*tan(x)^2)
```

3.485.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(\sec^2(x)+\tan^2(x)\right)^2}\,dx = -\frac{\arctan\left(\sqrt{2}\tan(x)\right)}{\sqrt{2}}+\frac{-3x+x\cos(2x)-\sin(2x)}{-3+\cos(2x)}$$

input
```
Integrate[(Sec[x]^2 + Tan[x]^2)^(-2),x]
```

output
```
-(ArcTan[Sqrt[2]*Tan[x]]/Sqrt[2]) + (-3*x + x*Cos[2*x] - Sin[2*x])/(-3 + C
os[2*x])
```

3.485.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.67, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {3042, 4889, 316, 27, 383, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\tan^2(x)+\sec^2(x)\right)^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{(\tan(x)^2+\sec(x)^2)^2}dx$$

$$\Big\downarrow \text{ 4889}$$

$$\int \frac{1}{\left(\tan^2(x)+1\right)\left(2\tan^2(x)+1\right)^2}d\tan(x)$$

$$\Big\downarrow \text{ 316}$$

$$\frac{\tan(x)}{2\tan^2(x)+1}-\frac{1}{2}\int -\frac{2\tan^2(x)}{\left(\tan^2(x)+1\right)\left(2\tan^2(x)+1\right)}d\tan(x)$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{\tan^2(x)}{\left(\tan^2(x)+1\right)\left(2\tan^2(x)+1\right)}d\tan(x)+\frac{\tan(x)}{2\tan^2(x)+1}$$

$$\Big\downarrow \text{ 383}$$

$$\int \frac{1}{\tan^2(x)+1}d\tan(x)-\int \frac{1}{2\tan^2(x)+1}d\tan(x)+\frac{\tan(x)}{2\tan^2(x)+1}$$

$$\Big\downarrow \text{ 216}$$

$$\arctan(\tan(x))-\frac{\arctan\left(\sqrt{2}\tan(x)\right)}{\sqrt{2}}+\frac{\tan(x)}{2\tan^2(x)+1}$$

input
```
Int[(Sec[x]^2 + Tan[x]^2)^(-2),x]
```

output
```
ArcTan[Tan[x]] - ArcTan[Sqrt[2]*Tan[x]]/Sqrt[2] + Tan[x]/(1 + 2*Tan[x]^2)
```

3.485.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 383
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Sym
bol] :> Simp[(-a)*(e^2/(b*c - a*d))   Int[(e*x)^(m - 2)/(a + b*x^2), x], x]
 + Simp[c*(e^2/(b*c - a*d))   Int[(e*x)^(m - 2)/(c + d*x^2), x], x] /; Free
Q[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LeQ[2, m, 3]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.485. $\int \frac{1}{(\sec^2(x)+\tan^2(x))^2}\,dx$

3.485.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.55

$$\frac{\tan(x)}{1+2\tan(x)^2}-\frac{\sqrt{2}\arctan\left(\sqrt{2}\tan(x)\right)}{2}+x$$

input
```
int(1/(sec(x)^2+tan(x)^2)^2,x)
```

output
```
1/2*tan(x)/(tan(x)^2+1/2)-1/2*2^(1/2)*arctan(2^(1/2)*tan(x))+x
```

3.485.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.39

$$\int\frac{1}{\left(\sec^2(x)+\tan^2(x)\right)^2}\,dx=\frac{4\,x\cos(x)^2+\left(\sqrt{2}\cos(x)^2-2\sqrt{2}\right)\arctan\left(\frac{3\sqrt{2}\cos(x)^2-2\sqrt{2}}{4\cos(x)\sin(x)}\right)-4\cos(x)\sin(x)-8\,x}{4\left(\cos(x)^2-2\right)}$$

input
```
integrate(1/(sec(x)^2+tan(x)^2)^2,x, algorithm="fricas")
```

output
```
1/4*(4*x*cos(x)^2 + (sqrt(2)*cos(x)^2 - 2*sqrt(2))*arctan(1/4*(3*sqrt(2)*c
os(x)^2 - 2*sqrt(2))/(cos(x)*sin(x))) - 4*cos(x)*sin(x) - 8*x)/(cos(x)^2 -
 2)
```

3.485.6 Sympy [F]

$$\int\frac{1}{\left(\sec^2(x)+\tan^2(x)\right)^2}\,dx=\int\frac{1}{\left(\tan^2(x)+\sec^2(x)\right)^2}\,dx$$

input
```
integrate(1/(sec(x)**2+tan(x)**2)**2,x)
```

output
```
Integral((tan(x)**2 + sec(x)**2)**(-2), x)
```

3.485.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.55

$$\int \frac{1}{\left(\sec^2(x)+\tan^2(x)\right)^2}\,dx=-\frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}\tan\left(x\right)\right)+x+\frac{\tan\left(x\right)}{2\,\tan\left(x\right)^2+1}$$

input
```
integrate(1/(sec(x)^2+tan(x)^2)^2,x, algorithm="maxima")
```

output
```
-1/2*sqrt(2)*arctan(sqrt(2)*tan(x)) + x + tan(x)/(2*tan(x)^2 + 1)
```

3.485.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.55

$$\int \frac{1}{\left(\sec^2(x)+\tan^2(x)\right)^2}\,dx=-\frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}\tan\left(x\right)\right)+x+\frac{\tan\left(x\right)}{2\,\tan\left(x\right)^2+1}$$

input
```
integrate(1/(sec(x)^2+tan(x)^2)^2,x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*arctan(sqrt(2)*tan(x)) + x + tan(x)/(2*tan(x)^2 + 1)
```

3.485.9 Mupad [B] (verification not implemented)

Time = 27.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.55

$$\int \frac{1}{\left(\sec^2(x)+\tan^2(x)\right)^2}\,dx=x+\frac{\tan(x)}{2\,\left(\tan\left(x\right)^2+\frac{1}{2}\right)}-\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\tan(x)\right)}{2}$$

input
```
int(1/(1/cos(x)^2 + tan(x)^2)^2,x)
```

output
```
x + tan(x)/(2*(tan(x)^2 + 1/2)) - (2^(1/2)*atan(2^(1/2)*tan(x)))/2
```

3.486 $\int \frac{1}{(\sec^2(x)+\tan^2(x))^3}\,dx$

3.486.1 Optimal result

Integrand size = 11, antiderivative size = 74

$$\int \frac{1}{(\sec^2(x)+\tan^2(x))^3}\,dx = -x+\frac{7x}{4\sqrt{2}}+\frac{7\arctan\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\sin^2(x)}\right)}{4\sqrt{2}} + \frac{\tan(x)}{2\left(1+2\tan^2(x)\right)^2}-\frac{\tan(x)}{4\left(1+2\tan^2(x)\right)}$$

output
```
-x+7/8*x*2^(1/2)+7/8*arctan(cos(x)*sin(x)/(1+sin(x)^2+2^(1/2)))*2^(1/2)+1/
2*tan(x)/(1+2*tan(x)^2)^2-1/4*tan(x)/(1+2*tan(x)^2)
```

3.486.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.07

$$\int \frac{1}{(\sec^2(x)+\tan^2(x))^3}\,dx = -\frac{(-3+\cos(2x))\sec^6(x)\left(-76x+7\sqrt{2}\arctan\left(\sqrt{2}\tan(x)\right)(-3+\cos(2x))^2+48x\cos(2x)-4x\cos(4x)\right.}{64\left(\sec^2(x)+\tan^2(x)\right)^3}$$

input
```
Integrate[(Sec[x]^2 + Tan[x]^2)^(-3),x]
```

output
```
-1/64*((-3 + Cos[2*x])*Sec[x]^6*(-76*x + 7*Sqrt[2]*ArcTan[Sqrt[2]*Tan[x]]*
(-3 + Cos[2*x])^2 + 48*x*Cos[2*x] - 4*x*Cos[4*x] - 2*Sin[2*x] + 3*Sin[4*x]
))/(Sec[x]^2 + Tan[x]^2)^3
```

3.486.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {3042, 4889, 316, 27, 402, 25, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\tan^2(x)+\sec^2(x)\right)^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(\tan(x)^2+\sec(x)^2\right)^3}dx$$

$$\Big\downarrow \text{ 4889}$$

$$\int \frac{1}{\left(\tan^2(x)+1\right)\left(2\tan^2(x)+1\right)^3}d\tan(x)$$

$$\Big\downarrow \text{ 316}$$

$$\frac{\tan(x)}{2\left(2\tan^2(x)+1\right)^2}-\frac{1}{4}\int -\frac{2\left(3\tan^2(x)+1\right)}{\left(\tan^2(x)+1\right)\left(2\tan^2(x)+1\right)^2}d\tan(x)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\int \frac{3\tan^2(x)+1}{\left(\tan^2(x)+1\right)\left(2\tan^2(x)+1\right)^2}d\tan(x)+\frac{\tan(x)}{2\left(2\tan^2(x)+1\right)^2}$$

$$\Big\downarrow \text{ 402}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int -\frac{3-\tan^2(x)}{\left(\tan^2(x)+1\right)\left(2\tan^2(x)+1\right)}d\tan(x)-\frac{\tan(x)}{2\left(2\tan^2(x)+1\right)}\right)+\frac{\tan(x)}{2\left(2\tan^2(x)+1\right)^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{3-\tan^2(x)}{\left(\tan^2(x)+1\right)\left(2\tan^2(x)+1\right)}d\tan(x)-\frac{\tan(x)}{2\left(2\tan^2(x)+1\right)}\right)+\frac{\tan(x)}{2\left(2\tan^2(x)+1\right)^2}$$

$$\Big\downarrow \text{ 397}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(7\int \frac{1}{2\tan^2(x)+1}d\tan(x)-4\int \frac{1}{\tan^2(x)+1}d\tan(x)\right)-\frac{\tan(x)}{2\left(2\tan^2(x)+1\right)}\right)+\frac{\tan(x)}{2\left(2\tan^2(x)+1\right)^2}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{7\arctan\left(\sqrt{2}\tan(x)\right)}{\sqrt{2}}-4\arctan(\tan(x))\right)-\frac{\tan(x)}{2\left(2\tan^2(x)+1\right)}\right)+\frac{\tan(x)}{2\left(2\tan^2(x)+1\right)^2}$$

input
```
Int[(Sec[x]^2 + Tan[x]^2)^(-3),x]
```

output
```
Tan[x]/(2*(1 + 2*Tan[x]^2)^2) + ((-4*ArcTan[Tan[x]] + (7*ArcTan[Sqrt[2]*Ta
n[x]])/Sqrt[2])/2 - Tan[x]/(2*(1 + 2*Tan[x]^2)))/2
```

3.486.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.486.4 Maple [A] (verified)

Time = 14.44 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.03

method	result	size
parallelrisch	0	2
default	$-\arctan\left(\tan\left(x\right)\right)+\frac{-\frac{\tan\left(x\right)^{3}}{2}+\frac{\tan\left(x\right)}{4}}{\left(1+2\tan\left(x\right)^{2}\right)^{2}}+\frac{7\sqrt{2}\,\arctan\left(\sqrt{2}\,\tan\left(x\right)\right)}{8}$	42
risch	$-x-\frac{i\left(17\,{\mathrm e}^{6 i x}-57\,{\mathrm e}^{4 i x}+19\,{\mathrm e}^{2 i x}-3\right)}{2\left({\mathrm e}^{4 i x}-6\,{\mathrm e}^{2 i x}+1\right)^{2}}+\frac{7 i \sqrt{2}\,\ln\left({\mathrm e}^{2 i x}-2\sqrt{2}-3\right)}{16}-\frac{7 i \sqrt{2}\,\ln\left({\mathrm e}^{2 i x}+2\sqrt{2}-3\right)}{16}$	85

input
```
int(1/(sec(x)^2+tan(x)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
0
```

3.486.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.35

$$\int \frac{1}{\left(\sec^2(x)+\tan^2(x)\right)^3}\,dx = -\frac{16\,x\cos(x)^4-64\,x\cos(x)^2+7\left(\sqrt{2}\cos(x)^4-4\sqrt{2}\cos(x)^2+4\sqrt{2}\right)\arctan\left(\frac{3\sqrt{2}\cos(x)^2-2\sqrt{2}}{4\cos(x)\sin(x)}\right)-4\,(3\cos}{16\left(\cos(x)^4-4\cos(x)^2+4\right)}$$

input
```
integrate(1/(sec(x)^2+tan(x)^2)^3,x, algorithm="fricas")
```

output
```
-1/16*(16*x*cos(x)^4 - 64*x*cos(x)^2 + 7*(sqrt(2)*cos(x)^4 - 4*sqrt(2)*cos
(x)^2 + 4*sqrt(2))*arctan(1/4*(3*sqrt(2)*cos(x)^2 - 2*sqrt(2))/(cos(x)*sin
(x))) - 4*(3*cos(x)^3 - 2*cos(x))*sin(x) + 64*x)/(cos(x)^4 - 4*cos(x)^2 +
4)
```

3.486.6 Sympy [F]

$$\int \frac{1}{\left(\sec^2(x)+\tan^2(x)\right)^3}\,dx = \int \frac{1}{\left(\tan^2(x)+\sec^2(x)\right)^3}\,dx$$

input
```
integrate(1/(sec(x)**2+tan(x)**2)**3,x)
```

output
```
Integral((tan(x)**2 + sec(x)**2)**(-3), x)
```

3.486.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.61

$$\int \frac{1}{\left(\sec^2(x)+\tan^2(x)\right)^3}\,dx = \frac{7}{8}\sqrt{2}\arctan\left(\sqrt{2}\tan(x)\right)-x-\frac{2\tan(x)^3-\tan(x)}{4\left(4\tan(x)^4+4\tan(x)^2+1\right)}$$

input
```
integrate(1/(sec(x)^2+tan(x)^2)^3,x, algorithm="maxima")
```

output
```
7/8*sqrt(2)*arctan(sqrt(2)*tan(x)) - x - 1/4*(2*tan(x)^3 - tan(x))/(4*tan(
x)^4 + 4*tan(x)^2 + 1)
```

3.486.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.53

$$\int \frac{1}{(\sec^2(x)+\tan^2(x))^3}\,dx=\frac{7}{8}\sqrt{2}\arctan\left(\sqrt{2}\tan\left(x\right)\right)-x-\frac{2\,\tan\left(x\right)^3-\tan\left(x\right)}{4\left(2\,\tan\left(x\right)^2+1\right)^2}$$

input
```
integrate(1/(sec(x)^2+tan(x)^2)^3,x, algorithm="giac")
```

output
```
7/8*sqrt(2)*arctan(sqrt(2)*tan(x)) - x - 1/4*(2*tan(x)^3 - tan(x))/(2*tan(
x)^2 + 1)^2
```

3.486.9 Mupad [B] (verification not implemented)

Time = 28.56 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.54

$$\int \frac{1}{(\sec^2(x)+\tan^2(x))^3}\,dx=\frac{\frac{\tan(x)}{16}-\frac{\tan(x)^3}{8}}{\tan\left(x\right)^4+\tan\left(x\right)^2+\frac{1}{4}}-x+\frac{7\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\tan(x)\right)}{8}$$

input
```
int(1/(1/cos(x)^2 + tan(x)^2)^3,x)
```

output
```
(tan(x)/16 - tan(x)^3/8)/(tan(x)^2 + tan(x)^4 + 1/4) - x + (7*2^(1/2)*atan
(2^(1/2)*tan(x)))/8
```

3.487 $\int \frac{1}{\sec^2(x)-\tan^2(x)}\,dx$

3.487.1 Optimal result

Integrand size = 13, antiderivative size = 1

$$\int \frac{1}{\sec^2(x)-\tan^2(x)}\,dx = x$$

output
```
x
```

3.487.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sec^2(x)-\tan^2(x)}\,dx = x$$

input
```
Integrate[(Sec[x]^2 - Tan[x]^2)^(-1),x]
```

output
```
x
```

3.487.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 4881, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec^2(x)-\tan^2(x)} \, dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{\sec(x)^2-\tan(x)^2}dx$$
$$\Big\downarrow \text{ 4881}$$
$$\int 1dx$$
$$\Big\downarrow \text{ 24}$$
$$x$$

input
```
Int[(Sec[x]^2 - Tan[x]^2)^(-1),x]
```

output
```
x
```

3.487.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4881
```
Int[(u_.)*((a_.) + (c_.)*sec[(d_.) + (e_.)*(x_)]^2 + (b_.)*tan[(d_.) + (e_.
)*(x_)]^2)^(p_.), x_Symbol] :> Simp[(a + c)^p   Int[ActivateTrig[u], x], x]
 /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b + c, 0]
```

3.487.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 2, normalized size of antiderivative = 2.00

method	result	size
risch	x	2
default	$\arctan(\tan(x))$	4

input
```
int(1/(sec(x)^2-tan(x)^2),x,method=_RETURNVERBOSE)
```

output
```
x
```

3.487.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sec^2(x) - \tan^2(x)}\,dx = x$$

input
```
integrate(1/(sec(x)^2-tan(x)^2),x, algorithm="fricas")
```

output
```
x
```

3.487.6 Sympy [F]

$$\int \frac{1}{\sec^2(x) - \tan^2(x)}\,dx = \int \frac{1}{(-\tan(x) + \sec(x))(\tan(x) + \sec(x))}\,dx$$

input
```
integrate(1/(sec(x)**2-tan(x)**2),x)
```

output
```
Integral(1/((-tan(x) + sec(x))*(tan(x) + sec(x))), x)
```

3.487.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sec^2(x) - \tan^2(x)}\,dx = x$$

input
```
integrate(1/(sec(x)^2-tan(x)^2),x, algorithm="maxima")
```

output
```
x
```

3.487.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sec^2(x) - \tan^2(x)}\,dx = x$$

input
```
integrate(1/(sec(x)^2-tan(x)^2),x, algorithm="giac")
```

output
```
x
```

3.487.9 Mupad [B] (verification not implemented)

Time = 27.84 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sec^2(x) - \tan^2(x)}\,dx = x$$

input
```
int(1/(1/cos(x)^2 - tan(x)^2),x)
```

output
```
x
```

3.488 $\int \frac{1}{(\sec^2(x)-\tan^2(x))^2}\,dx$

3.488.1 Optimal result

Integrand size = 13, antiderivative size = 1

$$\int \frac{1}{(\sec^2(x)-\tan^2(x))^2}\,dx = x$$

output
```
x
```

3.488.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{(\sec^2(x)-\tan^2(x))^2}\,dx = x$$

input
```
Integrate[(Sec[x]^2 - Tan[x]^2)^(-2),x]
```

output
```
x
```

3.488.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3042, 4881, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\sec^2(x)-\tan^2(x)\right)^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(\sec(x)^2-\tan(x)^2)^2}dx$$
$$\Big\downarrow \text{4881}$$
$$\int 1dx$$
$$\Big\downarrow \text{24}$$
$$x$$

input
```
Int[(Sec[x]^2 - Tan[x]^2)^(-2),x]
```

output
```
x
```

3.488.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4881
```
Int[(u_.)*((a_.) + (c_.)*sec[(d_.) + (e_.)*(x_)]^2 + (b_.)*tan[(d_.) + (e_.
)*(x_)]^2)^(p_.), x_Symbol] :> Simp[(a + c)^p   Int[ActivateTrig[u], x], x]
 /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b + c, 0]
```

3.488.4 Maple [A] (verified)

Time = 5.35 (sec) , antiderivative size = 2, normalized size of antiderivative = 2.00

method	result	size
risch	x	2
default	$\arctan(\tan(x))$	4

input
```
int(1/(sec(x)^2-tan(x)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
x
```

3.488.5 Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\sec^2(x)-\tan^2(x)\right)^2}\,dx = x$$

input
```
integrate(1/(sec(x)^2-tan(x)^2)^2,x, algorithm="fricas")
```

output
```
x
```

3.488.6 Sympy [F]

$$\int \frac{1}{\left(\sec^2(x)-\tan^2(x)\right)^2}\,dx = \int \frac{1}{\left(-\tan(x)+\sec(x)\right)^2\left(\tan(x)+\sec(x)\right)^2}\,dx$$

input
```
integrate(1/(sec(x)**2-tan(x)**2)**2,x)
```

output
```
Integral(1/((-tan(x) + sec(x))**2*(tan(x) + sec(x))**2), x)
```

3.488.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\sec^2(x)-\tan^2(x)\right)^2}\,dx=x$$

input
```
integrate(1/(sec(x)^2-tan(x)^2)^2,x, algorithm="maxima")
```

output
```
x
```

3.488.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\sec^2(x)-\tan^2(x)\right)^2}\,dx=x$$

input
```
integrate(1/(sec(x)^2-tan(x)^2)^2,x, algorithm="giac")
```

output
```
x
```

3.488.9 Mupad [B] (verification not implemented)

Time = 27.11 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\sec^2(x)-\tan^2(x)\right)^2}\,dx=x$$

input
```
int(1/(1/cos(x)^2 - tan(x)^2)^2,x)
```

output
```
x
```

3.489 $\int \frac{1}{\left(\sec^2(x)-\tan^2(x)\right)^3} \, dx$

3.489.1 Optimal result

Integrand size = 13, antiderivative size = 1

$$\int \frac{1}{\left(\sec^2(x)-\tan^2(x)\right)^3} \, dx = x$$

output
```
x
```

3.489.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\sec^2(x)-\tan^2(x)\right)^3} \, dx = x$$

input
```
Integrate[(Sec[x]^2 - Tan[x]^2)^(-3),x]
```

output
```
x
```

3.489.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 4881, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\sec^2(x)-\tan^2(x)\right)^3} \, dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\left(\sec(x)^2-\tan(x)^2\right)^3}dx$$
$$\Big\downarrow \text{4881}$$
$$\int 1dx$$
$$\Big\downarrow \text{24}$$
$$x$$

input
```
Int[(Sec[x]^2 - Tan[x]^2)^(-3),x]
```

output
```
x
```

3.489.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4881
```
Int[(u_.)*((a_.) + (c_.)*sec[(d_.) + (e_.)*(x_)]^2 + (b_.)*tan[(d_.) + (e_.
)*(x_)]^2)^(p_.), x_Symbol] :> Simp[(a + c)^p   Int[ActivateTrig[u], x], x]
 /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b + c, 0]
```

3.489.4 Maple [A] (verified)

Time = 59.68 (sec) , antiderivative size = 2, normalized size of antiderivative = 2.00

method	result	size
risch	x	2
parallelrisch	0	2
default	$\arctan(\tan(x))$	4

input
```
int(1/(sec(x)^2-tan(x)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
x
```

3.489.5 Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{(\sec^2(x)-\tan^2(x))^3}\,dx = x$$

input
```
integrate(1/(sec(x)^2-tan(x)^2)^3,x, algorithm="fricas")
```

output
```
x
```

3.489.6 Sympy [F]

$$\int \frac{1}{(\sec^2(x)-\tan^2(x))^3}\,dx = \int \frac{1}{(-\tan(x)+\sec(x))^3(\tan(x)+\sec(x))^3}\,dx$$

input
```
integrate(1/(sec(x)**2-tan(x)**2)**3,x)
```

output
```
Integral(1/((-tan(x) + sec(x))**3*(tan(x) + sec(x))**3), x)
```

3.489.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{(\sec^2(x) - \tan^2(x))^3} \, dx = x$$

input
```
integrate(1/(sec(x)^2-tan(x)^2)^3,x, algorithm="maxima")
```

output
```
x
```

3.489.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{(\sec^2(x) - \tan^2(x))^3} \, dx = x$$

input
```
integrate(1/(sec(x)^2-tan(x)^2)^3,x, algorithm="giac")
```

output
```
x
```

3.489.9 Mupad [B] (verification not implemented)

Time = 28.38 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{(\sec^2(x) - \tan^2(x))^3} \, dx = x$$

input
```
int(1/(1/cos(x)^2 - tan(x)^2)^3,x)
```

output
```
x
```

3.490 $\int \frac{1}{\cot^2(x)+\csc^2(x)}\,dx$

3.490.1 Optimal result

Integrand size = 11, antiderivative size = 37

$$\int \frac{1}{\cot^2(x)+\csc^2(x)}\,dx = -x+\sqrt{2}x-\sqrt{2}\arctan\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)$$

output
```
-x+x*2^(1/2)-arctan(cos(x)*sin(x)/(1+cos(x)^2+2^(1/2)))*2^(1/2)
```

3.490.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.51

$$\int \frac{1}{\cot^2(x)+\csc^2(x)}\,dx = -x+\sqrt{2}\arctan\left(\frac{\tan(x)}{\sqrt{2}}\right)$$

input
```
Integrate[(Cot[x]^2 + Csc[x]^2)^(-1),x]
```

output
```
-x + Sqrt[2]*ArcTan[Tan[x]/Sqrt[2]]
```

3.490.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.57, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3042, 4889, 1450, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cot^2(x)+\csc^2(x)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\cot(x)^2+\csc(x)^2}dx$$

$$\Big\downarrow \; 4889$$

$$\int \frac{\tan^2(x)}{\tan^4(x)+3\tan^2(x)+2}d\tan(x)$$

$$\Big\downarrow \; 1450$$

$$2\int \frac{1}{\tan^2(x)+2}d\tan(x)-\int \frac{1}{\tan^2(x)+1}d\tan(x)$$

$$\Big\downarrow \; 216$$

$$\sqrt{2}\arctan\left(\frac{\tan(x)}{\sqrt{2}}\right)-\arctan(\tan(x))$$

input
```
Int[(Cot[x]^2 + Csc[x]^2)^(-1),x]
```

output
```
-ArcTan[Tan[x]] + Sqrt[2]*ArcTan[Tan[x]/Sqrt[2]]
```

3.490.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1450
```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^2/2)*(b/q + 1)   Int[(d*x)^(m - 2)/(b/
2 + q/2 + c*x^2), x], x] - Simp[(d^2/2)*(b/q - 1)   Int[(d*x)^(m - 2)/(b/2
- q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] &&
 GeQ[m, 2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.490.4 Maple [A] (verified)

Time = 3.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.51

method	result	size
default	$-\arctan\left(\tan\left(x\right)\right)+\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\tan\left(x\right)}{2}\right)$	19
risch	$-x+\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}+2\sqrt{2}+3\right)}{2}-\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}-2\sqrt{2}+3\right)}{2}$	43

input
```
int(1/(cot(x)^2+csc(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-arctan(tan(x))+2^(1/2)*arctan(1/2*2^(1/2)*tan(x))
```

3.490.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{1}{\cot^2(x)+\csc^2(x)}\,dx = -\frac{1}{2}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(x)^2-\sqrt{2}}{4\cos(x)\sin(x)}\right)-x$$

input
```
integrate(1/(cot(x)^2+csc(x)^2),x, algorithm="fricas")
```

output
```
-1/2*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - sqrt(2))/(cos(x)*sin(x))) -
x
```

3.490.6 Sympy [F]

$$\int \frac{1}{\cot^2(x)+\csc^2(x)}\,dx = \int \frac{1}{\cot^2(x)+\csc^2(x)}\,dx$$

input
```
integrate(1/(cot(x)**2+csc(x)**2),x)
```

output
```
Integral(1/(cot(x)**2 + csc(x)**2), x)
```

3.490.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.43

$$\int \frac{1}{\cot^2(x)+\csc^2(x)}\,dx = \sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\tan(x)\right)-x$$

input
```
integrate(1/(cot(x)^2+csc(x)^2),x, algorithm="maxima")
```

output
```
sqrt(2)*arctan(1/2*sqrt(2)*tan(x)) - x
```

3.490.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.32

$$\int \frac{1}{\cot^2(x)+\csc^2(x)}\,dx = \sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\sin(2x)-\sin(2x)}{\sqrt{2}\cos(2x)+\sqrt{2}-\cos(2x)+1}\right)\right)-x$$

input
```
integrate(1/(cot(x)^2+csc(x)^2),x, algorithm="giac")
```

output
```
sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - sin(2*x))/(sqrt(2)*cos(2*x) + sqr
t(2) - cos(2*x) + 1))) - x
```

3.490.9 Mupad [B] (verification not implemented)

Time = 27.62 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.43

$$\int \frac{1}{\cot^2(x)+\csc^2(x)}\,dx = \sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\tan(x)}{2}\right)-x$$

input
```
int(1/(cot(x)^2 + 1/sin(x)^2),x)
```

output
```
2^(1/2)*atan((2^(1/2)*tan(x))/2) - x
```

3.491 $\int \frac{1}{\left(\cot^2(x)+\csc^2(x)\right)^2}\,dx$

3.491.1 Optimal result

Integrand size = 11, antiderivative size = 47

$$\int \frac{1}{\left(\cot^2(x)+\csc^2(x)\right)^2}\,dx = x-\frac{x}{\sqrt{2}}+\frac{\arctan\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)}{\sqrt{2}}-\frac{\tan(x)}{2+\tan^2(x)}$$

output
```
x-1/2*x*2^(1/2)+1/2*arctan(cos(x)*sin(x)/(1+cos(x)^2+2^(1/2)))*2^(1/2)-tan
(x)/(2+tan(x)^2)
```

3.491.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.36

$$\int \frac{1}{\left(\cot^2(x)+\csc^2(x)\right)^2}\,dx = \frac{(3+\cos(2x))\csc^4(x)\left(6x+2x\cos(2x)-\sqrt{2}\arctan\left(\frac{\tan(x)}{\sqrt{2}}\right)(3+\cos(2x))-2\sin(2x)\right)}{8\left(\cot^2(x)+\csc^2(x)\right)^2}$$

input
```
Integrate[(Cot[x]^2 + Csc[x]^2)^(-2),x]
```

output
```
((3 + Cos[2*x])*Csc[x]^4*(6*x + 2*x*Cos[2*x] - Sqrt[2]*ArcTan[Tan[x]/Sqrt[
2]]*(3 + Cos[2*x]) - 2*Sin[2*x]))/(8*(Cot[x]^2 + Csc[x]^2)^2)
```

3.491.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.68, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {3042, 4889, 372, 27, 303, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\cot^2(x)+\csc^2(x)\right)^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(\cot(x)^2+\csc(x)^2\right)^2}dx$$

$$\Big\downarrow \text{ 4889}$$

$$\int \frac{\tan^4(x)}{\left(\tan^2(x)+1\right)\left(\tan^2(x)+2\right)^2}d\tan(x)$$

$$\Big\downarrow \text{ 372}$$

$$\frac{1}{2}\int \frac{2}{\left(\tan^2(x)+1\right)\left(\tan^2(x)+2\right)}d\tan(x)-\frac{\tan(x)}{\tan^2(x)+2}$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{1}{\left(\tan^2(x)+1\right)\left(\tan^2(x)+2\right)}d\tan(x)-\frac{\tan(x)}{\tan^2(x)+2}$$

$$\Big\downarrow \text{ 303}$$

$$\int \frac{1}{\tan^2(x)+1}d\tan(x)-\int \frac{1}{\tan^2(x)+2}d\tan(x)-\frac{\tan(x)}{\tan^2(x)+2}$$

$$\Big\downarrow \text{ 216}$$

$$\arctan(\tan(x))-\frac{\arctan\left(\frac{\tan(x)}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{\tan(x)}{\tan^2(x)+2}$$

input
```
Int[(Cot[x]^2 + Csc[x]^2)^(-2),x]
```

output
```
ArcTan[Tan[x]] - ArcTan[Tan[x]/Sqrt[2]]/Sqrt[2] - Tan[x]/(2 + Tan[x]^2)
```

3.491.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 303
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Simp[b/(b
*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x
^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.491. $\int \frac{1}{(\cot^2(x)+\csc^2(x))^2}\,dx$

3.491.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.60

$$-\frac{\tan(x)}{2+\tan(x)^2}-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\tan(x)}{2}\right)}{2}+x$$

input
```
int(1/(cot(x)^2+csc(x)^2)^2,x)
```

output
```
-tan(x)/(2+tan(x)^2)-1/2*2^(1/2)*arctan(1/2*2^(1/2)*tan(x))+x
```

3.491.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.40

$$\int \frac{1}{(\cot^2(x)+\csc^2(x))^2}\,dx = \frac{4\,x\cos(x)^2+\left(\sqrt{2}\cos(x)^2+\sqrt{2}\right)\arctan\left(\frac{3\sqrt{2}\cos(x)^2-\sqrt{2}}{4\cos(x)\sin(x)}\right)-4\cos(x)\sin(x)+4\,x}{4\left(\cos(x)^2+1\right)}$$

input
```
integrate(1/(cot(x)^2+csc(x)^2)^2,x, algorithm="fricas")
```

output
```
1/4*(4*x*cos(x)^2 + (sqrt(2)*cos(x)^2 + sqrt(2))*arctan(1/4*(3*sqrt(2)*cos
(x)^2 - sqrt(2))/(cos(x)*sin(x))) - 4*cos(x)*sin(x) + 4*x)/(cos(x)^2 + 1)
```

3.491.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(\cot^2(x)+\csc^2(x))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(cot(x)**2+csc(x)**2)**2,x)
```

output
```
Timed out
```

3.491.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.57

$$\int \frac{1}{\left(\cot^2(x)+\csc^2(x)\right)^2}\,dx = -\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\tan(x)\right)+x-\frac{\tan(x)}{\tan(x)^2+2}$$

input
```
integrate(1/(cot(x)^2+csc(x)^2)^2,x, algorithm="maxima")
```

output
```
-1/2*sqrt(2)*arctan(1/2*sqrt(2)*tan(x)) + x - tan(x)/(tan(x)^2 + 2)
```

3.491.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.28

$$\int \frac{1}{\left(\cot^2(x)+\csc^2(x)\right)^2}\,dx = -\frac{1}{2}\sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\sin(2x)-\sin(2x)}{\sqrt{2}\cos(2x)+\sqrt{2}-\cos(2x)+1}\right)\right)+x-\frac{\tan(x)}{\tan(x)^2+2}$$

input
```
integrate(1/(cot(x)^2+csc(x)^2)^2,x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - sin(2*x))/(sqrt(2)*cos(2*x)
+ sqrt(2) - cos(2*x) + 1))) + x - tan(x)/(tan(x)^2 + 2)
```

3.491.9 Mupad [B] (verification not implemented)

Time = 27.91 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.57

$$\int \frac{1}{\left(\cot^2(x)+\csc^2(x)\right)^2}\,dx = x-\frac{\tan(x)}{\tan(x)^2+2}-\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\tan(x)}{2}\right)}{2}$$

input
```
int(1/(cot(x)^2 + 1/sin(x)^2)^2,x)
```

output
```
x - tan(x)/(tan(x)^2 + 2) - (2^(1/2)*atan((2^(1/2)*tan(x))/2))/2
```

3.492 $\int \frac{1}{(\cot^2(x)+\csc^2(x))^3}\,dx$

3.492.1 Optimal result

Integrand size = 11, antiderivative size = 72

$$\int \frac{1}{(\cot^2(x)+\csc^2(x))^3}\,dx = -x+\frac{7x}{4\sqrt{2}}-\frac{7\arctan\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\cos^2(x)}\right)}{4\sqrt{2}} - \frac{\tan^3(x)}{2(2+\tan^2(x))^2}+\frac{\tan(x)}{4(2+\tan^2(x))}$$

output
```
-x+7/8*x*2^(1/2)-7/8*arctan(cos(x)*sin(x)/(1+cos(x)^2+2^(1/2)))*2^(1/2)-1/
2*tan(x)^3/(2+tan(x)^2)^2+1/4*tan(x)/(2+tan(x)^2)
```

3.492.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.92

$$\int \frac{1}{(\cot^2(x)+\csc^2(x))^3}\,dx = \frac{-76x-48x\cos(2x)+7\sqrt{2}\arctan\left(\frac{\tan(x)}{\sqrt{2}}\right)(3+\cos(2x))^2-4x\cos(4x)+2\sin(2x)+3\sin(4x)}{8(3+\cos(2x))^2}$$

input
```
Integrate[(Cot[x]^2 + Csc[x]^2)^(-3),x]
```

output
```
(-76*x - 48*x*Cos[2*x] + 7*Sqrt[2]*ArcTan[Tan[x]/Sqrt[2]]*(3 + Cos[2*x])^2
 - 4*x*Cos[4*x] + 2*Sin[2*x] + 3*Sin[4*x])/(8*(3 + Cos[2*x])^2)
```

3.492.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.86, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {3042, 4889, 372, 27, 440, 397, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\cot^2(x)+\csc^2(x)\right)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(\cot(x)^2+\csc(x)^2\right)^3}dx$$

$$\Big\downarrow \text{4889}$$

$$\int \frac{\tan^6(x)}{\left(\tan^2(x)+1\right)\left(\tan^2(x)+2\right)^3}d\tan(x)$$

$$\Big\downarrow \text{372}$$

$$\frac{1}{4}\int \frac{2\tan^2(x)\left(\tan^2(x)+3\right)}{\left(\tan^2(x)+1\right)\left(\tan^2(x)+2\right)^2}d\tan(x)-\frac{\tan^3(x)}{2\left(\tan^2(x)+2\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{\tan^2(x)\left(\tan^2(x)+3\right)}{\left(\tan^2(x)+1\right)\left(\tan^2(x)+2\right)^2}d\tan(x)-\frac{\tan^3(x)}{2\left(\tan^2(x)+2\right)^2}$$

$$\Big\downarrow \text{440}$$

$$\frac{1}{2}\left(\frac{\tan(x)}{2\left(\tan^2(x)+2\right)}-\frac{1}{2}\int \frac{1-3\tan^2(x)}{\left(\tan^2(x)+1\right)\left(\tan^2(x)+2\right)}d\tan(x)\right)-\frac{\tan^3(x)}{2\left(\tan^2(x)+2\right)^2}$$

$$\Big\downarrow \text{397}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(7\int \frac{1}{\tan^2(x)+2}d\tan(x)-4\int \frac{1}{\tan^2(x)+1}d\tan(x)\right)+\frac{\tan(x)}{2\left(\tan^2(x)+2\right)}\right)-\frac{\tan^3(x)}{2\left(\tan^2(x)+2\right)^2}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{7\arctan\left(\frac{\tan(x)}{\sqrt{2}}\right)}{\sqrt{2}}-4\arctan(\tan(x))\right)+\frac{\tan(x)}{2\left(\tan^2(x)+2\right)}\right)-\frac{\tan^3(x)}{2\left(\tan^2(x)+2\right)^2}$$

input
```
Int[(Cot[x]^2 + Csc[x]^2)^(-3),x]
```

output
```
-1/2*Tan[x]^3/(2 + Tan[x]^2)^2 + ((-4*ArcTan[Tan[x]] + (7*ArcTan[Tan[x]/Sq
rt[2]])/Sqrt[2])/2 + Tan[x]/(2*(2 + Tan[x]^2)))/2
```

3.492.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

3.492. $\int \frac{1}{(\cot^2(x)+\csc^2(x))^3} \, dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.492.4 Maple [A] (verified)

Time = 11.54 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.03

method	result	size
parallelrisch	0	2
default	$\frac{-\frac{\tan(x)^3}{4}+\frac{\tan(x)}{2}}{\left(2+\tan(x)^2\right)^2}+\frac{7\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,\tan(x)}{2}\right)}{8}-\arctan\left(\tan\left(x\right)\right)$	41
risch	$-x+\frac{i\left(17\,\mathrm{e}^{6ix}+57\,\mathrm{e}^{4ix}+19\,\mathrm{e}^{2ix}+3\right)}{2\left(\mathrm{e}^{4ix}+6\,\mathrm{e}^{2ix}+1\right)^2}+\frac{7i\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}+2\sqrt{2}+3\right)}{16}-\frac{7i\sqrt{2}\,\ln\left(\mathrm{e}^{2ix}-2\sqrt{2}+3\right)}{16}$	85

input
```
int(1/(cot(x)^2+csc(x)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
0
```

3.492.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.36

$$\int \frac{1}{\left(\cot^2(x)+\csc^2(x)\right)^3}\,dx =$$
$$-\frac{16\,x\cos\left(x\right)^4+32\,x\cos\left(x\right)^2+7\left(\sqrt{2}\cos\left(x\right)^4+2\,\sqrt{2}\cos\left(x\right)^2+\sqrt{2}\right)\arctan\left(\frac{3\,\sqrt{2}\cos(x)^2-\sqrt{2}}{4\,\cos(x)\sin(x)}\right)-4\left(3\,\cos\left(\right.\right.}{16\left(\cos\left(x\right)^4+2\,\cos\left(x\right)^2+1\right)}$$

input
```
integrate(1/(cot(x)^2+csc(x)^2)^3,x, algorithm="fricas")
```

output
```
-1/16*(16*x*cos(x)^4 + 32*x*cos(x)^2 + 7*(sqrt(2)*cos(x)^4 + 2*sqrt(2)*cos
(x)^2 + sqrt(2))*arctan(1/4*(3*sqrt(2)*cos(x)^2 - sqrt(2))/(cos(x)*sin(x))
) - 4*(3*cos(x)^3 - cos(x))*sin(x) + 16*x)/(cos(x)^4 + 2*cos(x)^2 + 1)
```

3.492.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(\cot^2(x)+\csc^2(x))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(cot(x)**2+csc(x)**2)**3,x)
```

output
```
Timed out
```

3.492.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.58

$$\int \frac{1}{(\cot^2(x)+\csc^2(x))^3}\,dx = \frac{7}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\tan(x)\right)-x-\frac{\tan(x)^3-2\tan(x)}{4\left(\tan(x)^4+4\tan(x)^2+4\right)}$$

input
```
integrate(1/(cot(x)^2+csc(x)^2)^3,x, algorithm="maxima")
```

output
```
7/8*sqrt(2)*arctan(1/2*sqrt(2)*tan(x)) - x - 1/4*(tan(x)^3 - 2*tan(x))/(ta
n(x)^4 + 4*tan(x)^2 + 4)
```

3.492.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.96

$$\int \frac{1}{(\cot^2(x)+\csc^2(x))^3}\,dx = \frac{7}{8}\sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\sin(2x)-\sin(2x)}{\sqrt{2}\cos(2x)+\sqrt{2}-\cos(2x)+1}\right)\right) - x - \frac{\tan(x)^3-2\tan(x)}{4\left(\tan(x)^2+2\right)^2}$$

input
```
integrate(1/(cot(x)^2+csc(x)^2)^3,x, algorithm="giac")
```

output
```
7/8*sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - sin(2*x))/(sqrt(2)*cos(2*x) +
 sqrt(2) - cos(2*x) + 1))) - x - 1/4*(tan(x)^3 - 2*tan(x))/(tan(x)^2 + 2)^
2
```

3.492.9 Mupad [B] (verification not implemented)

Time = 29.59 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.60

$$\int \frac{1}{\left(\cot^2(x)+\csc^2(x)\right)^3} \, dx = \frac{\frac{\tan(x)}{2}-\frac{\tan(x)^3}{4}}{\tan\left(x\right)^4+4\tan\left(x\right)^2+4}-x+\frac{7\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\tan(x)}{2}\right)}{8}$$

input
```
int(1/(cot(x)^2 + 1/sin(x)^2)^3,x)
```

output
```
(tan(x)/2 - tan(x)^3/4)/(4*tan(x)^2 + tan(x)^4 + 4) - x + (7*2^(1/2)*atan(
(2^(1/2)*tan(x))/2))/8
```

3.493 $\int \frac{1}{\cot^2(x)-\csc^2(x)}\,dx$

3.493.1 Optimal result

Integrand size = 13, antiderivative size = 3

$$\int \frac{1}{\cot^2(x)-\csc^2(x)}\,dx = -x$$

output
```
-x
```

3.493.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{\cot^2(x)-\csc^2(x)}\,dx = -x$$

input
```
Integrate[(Cot[x]^2 - Csc[x]^2)^(-1),x]
```

output
```
-x
```

3.493.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 4882, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cot^2(x)-\csc^2(x)} \, dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\cot(x)^2-\csc(x)^2}dx$$
$$\Big\downarrow \text{4882}$$
$$-\int 1dx$$
$$\Big\downarrow \text{24}$$
$$-x$$

input
```
Int[(Cot[x]^2 - Csc[x]^2)^(-1),x]
```

output
```
-x
```

3.493.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4882
```
Int[((a_.) + cot[(d_.) + (e_.)*(x_)]^2*(b_.) + csc[(d_.) + (e_.)*(x_)]^2*(c
_.))^(p_.)*(u_.), x_Symbol] :> Simp[(a + c)^p   Int[ActivateTrig[u], x], x]
 /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b + c, 0]
```

3.493.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
risch	$-x$	4
default	$-\arctan\left(\tan\left(x\right)\right)$	6

input
```
int(1/(cot(x)^2-csc(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-x
```

3.493.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{\cot^2(x) - \csc^2(x)}\, dx = -x$$

input
```
integrate(1/(cot(x)^2-csc(x)^2),x, algorithm="fricas")
```

output
```
-x
```

3.493.6 Sympy [F]

$$\int \frac{1}{\cot^2(x) - \csc^2(x)}\, dx = \int \frac{1}{(\cot(x) - \csc(x))\,(\cot(x) + \csc(x))}\, dx$$

input
```
integrate(1/(cot(x)**2-csc(x)**2),x)
```

output
```
Integral(1/((cot(x) - csc(x))*(cot(x) + csc(x))), x)
```

3.493.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{\cot^2(x) - \csc^2(x)}\,dx = -x$$

input
```
integrate(1/(cot(x)^2-csc(x)^2),x, algorithm="maxima")
```

output
```
-x
```

3.493.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{\cot^2(x) - \csc^2(x)}\,dx = -x$$

input
```
integrate(1/(cot(x)^2-csc(x)^2),x, algorithm="giac")
```

output
```
-x
```

3.493.9 Mupad [B] (verification not implemented)

Time = 26.31 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{\cot^2(x) - \csc^2(x)}\,dx = -x$$

input
```
int(1/(cot(x)^2 - 1/sin(x)^2),x)
```

output
```
-x
```

3.494 $\int \frac{1}{\left(\cot^2(x)-\csc^2(x)\right)^2} \, dx$

3.494.1 Optimal result

Integrand size = 13, antiderivative size = 1

$$\int \frac{1}{\left(\cot^2(x)-\csc^2(x)\right)^2} \, dx = x$$

output
```
x
```

3.494.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\cot^2(x)-\csc^2(x)\right)^2} \, dx = x$$

input
```
Integrate[(Cot[x]^2 - Csc[x]^2)^(-2),x]
```

output
```
x
```

3.494.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3042, 4882, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\cot^2(x)-\csc^2(x)\right)^2}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{\left(\cot(x)^2-\csc(x)^2\right)^2}dx$$
$$\Big\downarrow \text{ 4882}$$
$$\int 1dx$$
$$\Big\downarrow \text{ 24}$$
$$x$$

input
```
Int[(Cot[x]^2 - Csc[x]^2)^(-2),x]
```

output
```
x
```

3.494.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4882
```
Int[((a_.) + cot[(d_.) + (e_.)*(x_)]^2*(b_.) + csc[(d_.) + (e_.)*(x_)]^2*(c
_.))^(p_.)*(u_.), x_Symbol] :> Simp[(a + c)^p   Int[ActivateTrig[u], x], x]
 /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b + c, 0]
```

3.494.4 Maple [A] (verified)

Time = 3.67 (sec) , antiderivative size = 2, normalized size of antiderivative = 2.00

method	result	size
risch	x	2
default	$\arctan(\tan(x))$	4

input
```
int(1/(cot(x)^2-csc(x)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
x
```

3.494.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\cot^2(x)-\csc^2(x)\right)^2}\,dx = x$$

input
```
integrate(1/(cot(x)^2-csc(x)^2)^2,x, algorithm="fricas")
```

output
```
x
```

3.494.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(\cot^2(x)-\csc^2(x)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(cot(x)**2-csc(x)**2)**2,x)
```

output
```
Timed out
```

3.494.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\cot^2(x) - \csc^2(x)\right)^2}\,dx = x$$

input
```
integrate(1/(cot(x)^2-csc(x)^2)^2,x, algorithm="maxima")
```

output
```
x
```

3.494.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\cot^2(x) - \csc^2(x)\right)^2}\,dx = x$$

input
```
integrate(1/(cot(x)^2-csc(x)^2)^2,x, algorithm="giac")
```

output
```
x
```

3.494.9 Mupad [B] (verification not implemented)

Time = 27.01 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\cot^2(x) - \csc^2(x)\right)^2}\,dx = x$$

input
```
int(1/(cot(x)^2 - 1/sin(x)^2)^2,x)
```

output
```
x
```

3.495 $\int \frac{1}{(\cot^2(x)-\csc^2(x))^3} \, dx$

3.495.1 Optimal result

Integrand size = 13, antiderivative size = 3

$$\int \frac{1}{(\cot^2(x)-\csc^2(x))^3} \, dx = -x$$

output
```
-x
```

3.495.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{(\cot^2(x)-\csc^2(x))^3} \, dx = -x$$

input
```
Integrate[(Cot[x]^2 - Csc[x]^2)^(-3),x]
```

output
```
-x
```

3.495.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3042, 4882, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\cot^2(x)-\csc^2(x)\right)^3}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\left(\cot(x)^2-\csc(x)^2\right)^3}dx$$

$$\Big\downarrow \ 4882$$

$$-\int 1dx$$

$$\Big\downarrow \ 24$$

$$-x$$

input
```
Int[(Cot[x]^2 - Csc[x]^2)^(-3),x]
```

output
```
-x
```

3.495.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4882
```
Int[((a_.) + cot[(d_.) + (e_.)*(x_)]^2*(b_.) + csc[(d_.) + (e_.)*(x_)]^2*(c
_.))^(p_.)*(u_.), x_Symbol] :> Simp[(a + c)^p   Int[ActivateTrig[u], x], x]
 /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b + c, 0]
```

3.495.4 Maple [A] (verified)

Time = 37.71 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
risch	$-x$	4
default	$-\arctan(\tan(x))$	6

input
```
int(1/(cot(x)^2-csc(x)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-x
```

3.495.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{(\cot^2(x) - \csc^2(x))^3} \, dx = -x$$

input
```
integrate(1/(cot(x)^2-csc(x)^2)^3,x, algorithm="fricas")
```

output
```
-x
```

3.495.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(\cot^2(x) - \csc^2(x))^3} \, dx = \text{Timed out}$$

input
```
integrate(1/(cot(x)**2-csc(x)**2)**3,x)
```

output
```
Timed out
```

3.495.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\cot^2(x)-\csc^2(x)\right)^3}\,dx=-x$$

input
```
integrate(1/(cot(x)^2-csc(x)^2)^3,x, algorithm="maxima")
```

output
```
-x
```

3.495.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\cot^2(x)-\csc^2(x)\right)^3}\,dx=-x$$

input
```
integrate(1/(cot(x)^2-csc(x)^2)^3,x, algorithm="giac")
```

output
```
-x
```

3.495.9 Mupad [B] (verification not implemented)

Time = 26.51 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\cot^2(x)-\csc^2(x)\right)^3}\,dx=-x$$

input
```
int(1/(cot(x)^2 - 1/sin(x)^2)^3,x)
```

output
```
-x
```

3.496 $\int \frac{1}{a+b\cos^2(x)+c\sin^2(x)}\,dx$

3.496.1 Optimal result

Integrand size = 16, antiderivative size = 33

$$\int \frac{1}{a+b\cos^2(x)+c\sin^2(x)}\,dx=\frac{\arctan\left(\frac{\sqrt{a+c}\tan(x)}{\sqrt{a+b}}\right)}{\sqrt{a+b}\sqrt{a+c}}$$

output
```
arctan((a+c)^(1/2)*tan(x)/(a+b)^(1/2))/(a+b)^(1/2)/(a+c)^(1/2)
```

3.496.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\cos^2(x)+c\sin^2(x)}\,dx=\frac{\arctan\left(\frac{\sqrt{a+c}\tan(x)}{\sqrt{a+b}}\right)}{\sqrt{a+b}\sqrt{a+c}}$$

input
```
Integrate[(a + b*Cos[x]^2 + c*Sin[x]^2)^(-1),x]
```

output
```
ArcTan[(Sqrt[a + c]*Tan[x])/Sqrt[a + b]]/(Sqrt[a + b]*Sqrt[a + c])
```

3.496.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3042, 4889, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\cos^2(x)+c\sin^2(x)}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{a+b\cos(x)^2+c\sin(x)^2}dx$$
$$\Big\downarrow \text{ 4889}$$
$$\int \frac{1}{(a+c)\tan^2(x)+a+b}d\tan(x)$$
$$\Big\downarrow \text{ 218}$$
$$\frac{\arctan\left(\frac{\sqrt{a+c}\tan(x)}{\sqrt{a+b}}\right)}{\sqrt{a+b}\sqrt{a+c}}$$

input
```
Int[(a + b*Cos[x]^2 + c*Sin[x]^2)^(-1),x]
```

output
```
ArcTan[(Sqrt[a + c]*Tan[x])/Sqrt[a + b]]/(Sqrt[a + b]*Sqrt[a + c])
```

3.496.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.496.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

method	result
default	$\frac{\arctan\left(\frac{(a+c)\tan(x)}{\sqrt{(a+b)(a+c)}}\right)}{\sqrt{(a+b)(a+c)}}$
risch	$-\frac{\ln\left(e^{2ix}+\frac{2ia^2+2iab+2iac+2icb+2a\sqrt{-a^2-ab-ac-cb}+b\sqrt{-a^2-ab-ac-cb}+c\sqrt{-a^2-ab-ac-cb}}{\sqrt{-a^2-ab-ac-cb}\,(b-c)}\right)}{2\sqrt{-a^2-ab-ac-cb}}+\frac{\ln\left(e^{2ix}-\frac{2ia^2+2iab+2iac+2icb-2a\sqrt{}}{}\right.}{}$

input
```
int(1/(a+b*cos(x)^2+c*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/((a+b)*(a+c))^(1/2)*arctan((a+c)*tan(x)/((a+b)*(a+c))^(1/2))
```

3.496.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(25) = 50.

Time = 0.28 (sec) , antiderivative size = 259, normalized size of antiderivative = 7.85

$$\int \frac{1}{a+b\cos^2(x)+c\sin^2(x)}\,dx$$
$$= \left[-\frac{\sqrt{-a^2-ab-(a+b)c}\log\left(\frac{(8a^2+8ab+b^2+2(4a+3b)c+c^2)\cos(x)^4-2(4a^2+3ab+(5a+3b)c+c^2)\cos(x)^2+4\left((2a+b+c)\cos(x\right.}{(b^2-2bc+c^2)\cos(x)^4+2(ab-(a-b)c-c^2)\cos(x)^2+a^2}\right.}{4(a^2+ab+(a+b)c)}\right.$$
$$\left.-\frac{\arctan\left(\frac{(2a+b+c)\cos(x)^2-a-c}{2\sqrt{a^2+ab+(a+b)c}\cos(x)\sin(x)}\right)}{2\sqrt{a^2+ab+(a+b)c}}\right]$$

input
```
integrate(1/(a+b*cos(x)^2+c*sin(x)^2),x, algorithm="fricas")
```

output
```
[-1/4*sqrt(-a^2 - a*b - (a + b)*c)*log(((8*a^2 + 8*a*b + b^2 + 2*(4*a + 3*
b)*c + c^2)*cos(x)^4 - 2*(4*a^2 + 3*a*b + (5*a + 3*b)*c + c^2)*cos(x)^2 +
4*((2*a + b + c)*cos(x)^3 - (a + c)*cos(x))*sqrt(-a^2 - a*b - (a + b)*c)*s
in(x) + a^2 + 2*a*c + c^2)/((b^2 - 2*b*c + c^2)*cos(x)^4 + 2*(a*b - (a - b
)*c - c^2)*cos(x)^2 + a^2 + 2*a*c + c^2))/(a^2 + a*b + (a + b)*c), -1/2*ar
ctan(1/2*((2*a + b + c)*cos(x)^2 - a - c)/(sqrt(a^2 + a*b + (a + b)*c)*cos
(x)*sin(x)))/sqrt(a^2 + a*b + (a + b)*c)]
```

3.496.6 Sympy [F]

$$\int \frac{1}{a+b\cos^2(x)+c\sin^2(x)}\,dx = \int \frac{1}{a+b\cos^2{(x)}+c\sin^2{(x)}}\,dx$$

input
```
integrate(1/(a+b*cos(x)**2+c*sin(x)**2),x)
```

output
```
Integral(1/(a + b*cos(x)**2 + c*sin(x)**2), x)
```

3.496.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int \frac{1}{a+b\cos^2(x)+c\sin^2(x)}\,dx = \frac{\arctan\left(\frac{(a+c)\tan(x)}{\sqrt{(a+b)(a+c)}}\right)}{\sqrt{(a+b)(a+c)}}$$

input
```
integrate(1/(a+b*cos(x)^2+c*sin(x)^2),x, algorithm="maxima")
```

output
```
arctan((a + c)*tan(x)/sqrt((a + b)*(a + c)))/sqrt((a + b)*(a + c))
```

3.496.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(25) = 50$.

Time = 0.26 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.85

$$\int \frac{1}{a + b\cos^2(x) + c\sin^2(x)}\,dx = \frac{\pi \left\lfloor \frac{x}{\pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}(2\,a + 2\,c) + \arctan\left(\frac{a\tan(x) + c\tan(x)}{\sqrt{a^2 + ab + ac + bc}}\right)}{\sqrt{a^2 + ab + ac + bc}}$$

input
```
integrate(1/(a+b*cos(x)^2+c*sin(x)^2),x, algorithm="giac")
```

output
```
(pi*floor(x/pi + 1/2)*sgn(2*a + 2*c) + arctan((a*tan(x) + c*tan(x))/sqrt(a
^2 + a*b + a*c + b*c)))/sqrt(a^2 + a*b + a*c + b*c)
```

3.496.9 Mupad [B] (verification not implemented)

Time = 27.19 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.30

$$\int \frac{1}{a + b\cos^2(x) + c\sin^2(x)}\,dx = \frac{\operatorname{atan}\left(\frac{\tan(x)\,(2\,a+2\,c)}{2\,\sqrt{a\,b+a\,c+b\,c+a^2}}\right)}{\sqrt{a\,b + a\,c + b\,c + a^2}}$$

input
```
int(1/(a + c*sin(x)^2 + b*cos(x)^2),x)
```

output
```
atan((tan(x)*(2*a + 2*c))/(2*(a*b + a*c + b*c + a^2)^(1/2)))/(a*b + a*c +
b*c + a^2)^(1/2)
```

3.497 $\int \frac{x}{a+b\cos^2(x)+c\sin^2(x)}\,dx$

3.497.1 Optimal result

Integrand size = 18, antiderivative size = 239

$$\int \frac{x}{a+b\cos^2(x)+c\sin^2(x)}\,dx = -\frac{ix\log\left(1+\frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\sqrt{a+c}}\right)}{2\sqrt{a+b}\sqrt{a+c}} + \frac{ix\log\left(1+\frac{(b-c)e^{2ix}}{2a+b+c+2\sqrt{a+b}\sqrt{a+c}}\right)}{2\sqrt{a+b}\sqrt{a+c}} - \frac{\operatorname{PolyLog}\left(2,-\frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\sqrt{a+c}}\right)}{4\sqrt{a+b}\sqrt{a+c}} + \frac{\operatorname{PolyLog}\left(2,-\frac{(b-c)e^{2ix}}{2a+b+c+2\sqrt{a+b}\sqrt{a+c}}\right)}{4\sqrt{a+b}\sqrt{a+c}}$$

output
```
-1/2*I*x*ln(1+(b-c)*exp(2*I*x)/(2*a+b+c-2*(a+b)^(1/2)*(a+c)^(1/2)))/(a+b)^
(1/2)/(a+c)^(1/2)+1/2*I*x*ln(1+(b-c)*exp(2*I*x)/(2*a+b+c+2*(a+b)^(1/2)*(a+
c)^(1/2)))/(a+b)^(1/2)/(a+c)^(1/2)-1/4*polylog(2,-(b-c)*exp(2*I*x)/(2*a+b+
c-2*(a+b)^(1/2)*(a+c)^(1/2)))/(a+b)^(1/2)/(a+c)^(1/2)+1/4*polylog(2,-(b-c)
*exp(2*I*x)/(2*a+b+c+2*(a+b)^(1/2)*(a+c)^(1/2)))/(a+b)^(1/2)/(a+c)^(1/2)
```

3.497.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1022 vs. 2(239) = 478.

Time = 1.23 (sec) , antiderivative size = 1022, normalized size of antiderivative = 4.28

$$\int \frac{x}{a+b\cos^2(x)+c\sin^2(x)}\,dx$$
$$= \frac{4x\text{arctanh}\left(\frac{(a+b)\cot(x)}{\sqrt{-((a+b)(a+c))}}\right)-2\arccos\left(-\frac{2a+b+c}{b-c}\right)\text{arctanh}\left(\frac{\sqrt{-((a+b)(a+c))}\tan(x)}{a+b}\right)+\arccos\left(-\frac{2a+b+c}{b-c}\right)\log\left(\frac{}{\sqrt{}}\right.}{}$$

input
```
Integrate[x/(a + b*Cos[x]^2 + c*Sin[x]^2),x]
```

output
```
(4*x*ArcTanh[((a + b)*Cot[x])/Sqrt[-((a + b)*(a + c))]] - 2*ArcCos[-((2*a
+ b + c)/(b - c))]*ArcTanh[(Sqrt[-((a + b)*(a + c))]*Tan[x])/(a + b)] + Ar
cCos[-((2*a + b + c)/(b - c))]*Log[(Sqrt[2]*Sqrt[-((a + b)*(a + c))])/(Sqr
t[b - c]*E^(I*x)*Sqrt[2*a + b + c + (b - c)*Cos[2*x]])] - (2*I)*ArcTanh[((
a + b)*Cot[x])/Sqrt[-((a + b)*(a + c))]]*Log[(Sqrt[2]*Sqrt[-((a + b)*(a +
c))])/(Sqrt[b - c]*E^(I*x)*Sqrt[2*a + b + c + (b - c)*Cos[2*x]])] + (2*I)*
ArcTanh[(Sqrt[-((a + b)*(a + c))]*Tan[x])/(a + b)]*Log[(Sqrt[2]*Sqrt[-((a
+ b)*(a + c))])/(Sqrt[b - c]*E^(I*x)*Sqrt[2*a + b + c + (b - c)*Cos[2*x]])
] + ArcCos[-((2*a + b + c)/(b - c))]*Log[(Sqrt[2]*Sqrt[-((a + b)*(a + c))]
*E^(I*x))/(Sqrt[b - c]*Sqrt[2*a + b + c + (b - c)*Cos[2*x]])] + (2*I)*ArcT
anh[((a + b)*Cot[x])/Sqrt[-((a + b)*(a + c))]]*Log[(Sqrt[2]*Sqrt[-((a + b)
*(a + c))]*E^(I*x))/(Sqrt[b - c]*Sqrt[2*a + b + c + (b - c)*Cos[2*x]])] -
(2*I)*ArcTanh[(Sqrt[-((a + b)*(a + c))]*Tan[x])/(a + b)]*Log[(Sqrt[2]*Sqrt
[-((a + b)*(a + c))]*E^(I*x))/(Sqrt[b - c]*Sqrt[2*a + b + c + (b - c)*Cos[
2*x]])] - ArcCos[-((2*a + b + c)/(b - c))]*Log[(2*(a + b)*((-I)*a - I*c +
Sqrt[-((a + b)*(a + c))])*(-I + Tan[x]))/((b - c)*(a + b + Sqrt[-((a + b)*
(a + c))]*Tan[x]))] + (2*I)*ArcTanh[(Sqrt[-((a + b)*(a + c))]*Tan[x])/(a +
 b)]*Log[(2*(a + b)*((-I)*a - I*c + Sqrt[-((a + b)*(a + c))])*(-I + Tan[x]
))/((b - c)*(a + b + Sqrt[-((a + b)*(a + c))]*Tan[x]))] - ArcCos[-((2*a +
b + c)/(b - c))]*Log[(2*(a + b)*(I*a + I*c + Sqrt[-((a + b)*(a + c))])*...
```

3.497.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {5098, 3042, 3802, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a+b\cos^2(x)+c\sin^2(x)}dx$$

$$\Big\downarrow \text{5098}$$

$$2\int \frac{x}{2a+b+c+(b-c)\cos(2x)}dx$$

$$\Big\downarrow \text{3042}$$

$$2\int \frac{x}{2a+b+c+(b-c)\sin\left(2x+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3802}$$

$$4\int \frac{e^{2ix}x}{b+2(2a+b+c)e^{2ix}+(b-c)e^{4ix}-c}dx$$

$$\Big\downarrow \text{2694}$$

$$4\left(\frac{(b-c)\int \frac{e^{2ix}x}{2\left(2a+(b-c)e^{2ix}+b+c-2\sqrt{a+b}\sqrt{a+c}\right)}dx}{2\sqrt{a+b}\sqrt{a+c}}-\frac{(b-c)\int \frac{e^{2ix}x}{2\left(2a+(b-c)e^{2ix}+b+c+2\sqrt{a+b}\sqrt{a+c}\right)}dx}{2\sqrt{a+b}\sqrt{a+c}}\right)$$

$$\Big\downarrow \text{27}$$

$$4\left(\frac{(b-c)\int \frac{e^{2ix}x}{2a+(b-c)e^{2ix}+b+c-2\sqrt{a+b}\sqrt{a+c}}dx}{4\sqrt{a+b}\sqrt{a+c}}-\frac{(b-c)\int \frac{e^{2ix}x}{2a+(b-c)e^{2ix}+b+c+2\sqrt{a+b}\sqrt{a+c}}dx}{4\sqrt{a+b}\sqrt{a+c}}\right)$$

$$\Big\downarrow \text{2620}$$

$$4\left(\frac{(b-c)\left(\frac{i\int \log\left(\frac{e^{2ix}(b-c)}{2a+b+c-2\sqrt{a+b}\sqrt{a+c}}+1\right)dx}{2(b-c)}-\frac{ix\log\left(1+\frac{e^{2ix}(b-c)}{-2\sqrt{a+b}\sqrt{a+c}+2a+b+c}\right)}{2(b-c)}\right)}{4\sqrt{a+b}\sqrt{a+c}}-\frac{(b-c)\left(\frac{i\int \log\left(\frac{e^{2ix}(b-c)}{2a+b+c+2\sqrt{a+b}\sqrt{a+c}}+1\right)dx}{2(b-c)} \right.}{4\sqrt{a+b}\sqrt{}}\right.$$

$$\Big\downarrow \text{2715}$$

$$4\left(\frac{(b-c)\left(\frac{\int e^{-2ix}\log\left(\frac{e^{2ix}(b-c)}{2a+b+c-2\sqrt{a+b}\sqrt{a+c}}+1\right)de^{2ix}}{4(b-c)}-\frac{ix\log\left(1+\frac{e^{2ix}(b-c)}{-2\sqrt{a+b}\sqrt{a+c}+2a+b+c}\right)}{2(b-c)}\right)}{4\sqrt{a+b}\sqrt{a+c}}-\frac{(b-c)\left(\frac{\int e^{-2ix}\log\left(\frac{e^{2ix}(b-c)}{2a+b+c+2\sqrt{a+b}\cdots}\right.}{4(b-c)}\cdots\right.}{4\cdots}\cdots\right.$$

$$\Big\downarrow \text{2838}$$

$$4\left(\frac{(b-c)\left(-\frac{\text{PolyLog}\left(2,-\frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\sqrt{a+c}}\right)}{4(b-c)}-\frac{ix\log\left(1+\frac{e^{2ix}(b-c)}{-2\sqrt{a+b}\sqrt{a+c}+2a+b+c}\right)}{2(b-c)}\right)}{4\sqrt{a+b}\sqrt{a+c}}-\frac{(b-c)\left(-\frac{\text{PolyLog}\left(2,-\frac{(b-c)e^{2ix}}{2a+b+c+2\sqrt{a+b}\cdots}\right.}{4(b-c)}\cdots\right.}{4\sqrt{a+\cdots}}\cdots\right.$$

input
```
Int[x/(a + b*Cos[x]^2 + c*Sin[x]^2),x]
```

output
```
4*(((b - c)*(((-1/2*I)*x*Log[1 + ((b - c)*E^((2*I)*x))/(2*a + b + c - 2*Sq
rt[a + b]*Sqrt[a + c])])/(b - c) - PolyLog[2, -(((b - c)*E^((2*I)*x))/(2*a
 + b + c - 2*Sqrt[a + b]*Sqrt[a + c]))]/(4*(b - c))))/(4*Sqrt[a + b]*Sqrt[
a + c]) - ((b - c)*(((-1/2*I)*x*Log[1 + ((b - c)*E^((2*I)*x))/(2*a + b + c
 + 2*Sqrt[a + b]*Sqrt[a + c])])/(b - c) - PolyLog[2, -(((b - c)*E^((2*I)*x
))/(2*a + b + c + 2*Sqrt[a + b]*Sqrt[a + c]))]/(4*(b - c))))/(4*Sqrt[a + b
]*Sqrt[a + c]))
```

3.497.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3802
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*E^(I*Pi*(k - 1/2))*(E^(I*(e +
f*x))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(
e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ
[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 5098
```
Int[((f_.) + (g_.)*(x_))^(m_.)/((a_.) + Cos[(d_.) + (e_.)*(x_)]^2*(b_.) + (
c_.)*Sin[(d_.) + (e_.)*(x_)]^2), x_Symbol] :> Simp[2   Int[(f + g*x)^m/(2*a
 + b + c + (b - c)*Cos[2*d + 2*e*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}
, x] && IGtQ[m, 0] && NeQ[a + b, 0] && NeQ[a + c, 0]
```

3.497.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 819 vs. $2(189) = 378$.

Time $= 0.91$ (sec) , antiderivative size $= 820$, normalized size of antiderivative $= 3.43$

method	result
risch	$-\frac{i\ln\left(1-\frac{(b-c)\mathrm{e}^{2ix}}{-2\sqrt{(a+b)(a+c)}-2a-b-c}\right)x}{-2\sqrt{(a+b)(a+c)}-2a-b-c}-\frac{i\ln\left(1-\frac{(b-c)\mathrm{e}^{2ix}}{-2\sqrt{(a+b)(a+c)}-2a-b-c}\right)ax}{\sqrt{(a+b)(a+c)}\left(-2\sqrt{(a+b)(a+c)}-2a-b-c\right)}-\frac{i\ln\left(1-\frac{(b-c)\mathrm{e}^{2ix}}{-2\sqrt{(a+b)(a+c)}-2a-b-c}\right)b}{2\sqrt{(a+b)(a+c)}\left(-2\sqrt{(a+b)(a+c)}-2a-\right.}$

input
```
int(x/(a+b*cos(x)^2+c*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-I/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*ln(1-(b-c)*exp(2*I*x)/(-2*((a+b)*(a+c)
)^(1/2)-2*a-b-c))*x-I/((a+b)*(a+c))^(1/2)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)
*ln(1-(b-c)*exp(2*I*x)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c))*a*x-1/2*I/((a+b)*
(a+c))^(1/2)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*ln(1-(b-c)*exp(2*I*x)/(-2*((
a+b)*(a+c))^(1/2)-2*a-b-c))*b*x-1/2*I/((a+b)*(a+c))^(1/2)/(-2*((a+b)*(a+c)
)^(1/2)-2*a-b-c)*ln(1-(b-c)*exp(2*I*x)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c))*c
*x-1/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*x^2-1/((a+b)*(a+c))^(1/2)/(-2*((a+b)
*(a+c))^(1/2)-2*a-b-c)*a*x^2-1/2/((a+b)*(a+c))^(1/2)/(-2*((a+b)*(a+c))^(1/
2)-2*a-b-c)*b*x^2-1/2/((a+b)*(a+c))^(1/2)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)
*c*x^2-1/2/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*polylog(2,(b-c)*exp(2*I*x)/(-2
*((a+b)*(a+c))^(1/2)-2*a-b-c))-1/2/((a+b)*(a+c))^(1/2)/(-2*((a+b)*(a+c))^(
1/2)-2*a-b-c)*polylog(2,(b-c)*exp(2*I*x)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c))
*a-1/4/((a+b)*(a+c))^(1/2)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*polylog(2,(b-c
)*exp(2*I*x)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c))*b-1/4/((a+b)*(a+c))^(1/2)/(
-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*polylog(2,(b-c)*exp(2*I*x)/(-2*((a+b)*(a+c
))^(1/2)-2*a-b-c))*c-1/2*I/((a+b)*(a+c))^(1/2)*x*ln(1-(b-c)*exp(2*I*x)/(2*
((a+b)*(a+c))^(1/2)-2*a-b-c))-1/2/((a+b)*(a+c))^(1/2)*x^2-1/4/((a+b)*(a+c)
)^(1/2)*polylog(2,(b-c)*exp(2*I*x)/(2*((a+b)*(a+c))^(1/2)-2*a-b-c))
```

3.497.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2869 vs. 2(189) = 378.

Time = 4.76 (sec) , antiderivative size = 2869, normalized size of antiderivative = 12.00

$$\int \frac{x}{a+b\cos^2(x)+c\sin^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(x/(a+b*cos(x)^2+c*sin(x)^2),x, algorithm="fricas")
```

output
```
-1/4*(-I*(b - c)*x*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*log((
((2*a + b + c)*cos(x) + (2*I*a + I*b + I*c)*sin(x) - 2*((b - c)*cos(x) + (
I*b - I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt
(-(2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) + 2*a + b +
 c)/(b - c)) + b - c)/(b - c)) + I*(b - c)*x*sqrt((a^2 + a*b + (a + b)*c)/
(b^2 - 2*b*c + c^2))*log(-(((2*a + b + c)*cos(x) - (2*I*a + I*b + I*c)*sin
(x) - 2*((b - c)*cos(x) - (I*b - I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)
/(b^2 - 2*b*c + c^2)))*sqrt(-(2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2
- 2*b*c + c^2)) + 2*a + b + c)/(b - c)) - b + c)/(b - c)) + I*(b - c)*x*sq
rt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*log((((2*a + b + c)*cos(x)
 + (-2*I*a - I*b - I*c)*sin(x) - 2*((b - c)*cos(x) + (-I*b + I*c)*sin(x))*
sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt(-(2*(b - c)*sqrt((
a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) + 2*a + b + c)/(b - c)) + b -
c)/(b - c)) - I*(b - c)*x*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)
)*log(-(((2*a + b + c)*cos(x) - (-2*I*a - I*b - I*c)*sin(x) - 2*((b - c)*c
os(x) - (-I*b + I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c
^2)))*sqrt(-(2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) +
 2*a + b + c)/(b - c)) - b + c)/(b - c)) + I*(b - c)*x*sqrt((a^2 + a*b + (
a + b)*c)/(b^2 - 2*b*c + c^2))*log((((2*a + b + c)*cos(x) + (2*I*a + I*b +
 I*c)*sin(x) + 2*((b - c)*cos(x) - (-I*b + I*c)*sin(x))*sqrt((a^2 + a*b...
```

3.497.6 Sympy [F]

$$\int \frac{x}{a+b\cos^2(x)+c\sin^2(x)}\,dx=\int \frac{x}{a+b\cos^2{(x)}+c\sin^2{(x)}}\,dx$$

input
```
integrate(x/(a+b*cos(x)**2+c*sin(x)**2),x)
```

output
```
Integral(x/(a + b*cos(x)**2 + c*sin(x)**2), x)
```

3.497.7 Maxima [F]

$$\int \frac{x}{a + b\cos^2(x) + c\sin^2(x)}\,dx = \int \frac{x}{b\cos{(x)}^2 + c\sin{(x)}^2 + a}\,dx$$

input
```
integrate(x/(a+b*cos(x)^2+c*sin(x)^2),x, algorithm="maxima")
```

output
```
integrate(x/(b*cos(x)^2 + c*sin(x)^2 + a), x)
```

3.497.8 Giac [F]

$$\int \frac{x}{a + b\cos^2(x) + c\sin^2(x)}\,dx = \int \frac{x}{b\cos{(x)}^2 + c\sin{(x)}^2 + a}\,dx$$

input
```
integrate(x/(a+b*cos(x)^2+c*sin(x)^2),x, algorithm="giac")
```

output
```
integrate(x/(b*cos(x)^2 + c*sin(x)^2 + a), x)
```

3.497.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{a + b\cos^2(x) + c\sin^2(x)}\,dx = \int \frac{x}{b\cos{(x)}^2 + c\sin{(x)}^2 + a}\,dx$$

input
```
int(x/(a + c*sin(x)^2 + b*cos(x)^2),x)
```

output
```
int(x/(a + c*sin(x)^2 + b*cos(x)^2), x)
```

3.498 $\int \frac{x^2}{a+b\cos^2(x)+c\sin^2(x)}\,dx$

3.498.1 Optimal result

Integrand size = 20, antiderivative size = 365

$$\begin{aligned}\int \frac{x^2}{a+b\cos^2(x)+c\sin^2(x)}\,dx = & -\frac{ix^2\log\left(1+\frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\sqrt{a+c}}\right)}{2\sqrt{a+b}\sqrt{a+c}} \\ & +\frac{ix^2\log\left(1+\frac{(b-c)e^{2ix}}{2a+b+c+2\sqrt{a+b}\sqrt{a+c}}\right)}{2\sqrt{a+b}\sqrt{a+c}} \\ & -\frac{x\,\text{PolyLog}\left(2,-\frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\sqrt{a+c}}\right)}{2\sqrt{a+b}\sqrt{a+c}} \\ & +\frac{x\,\text{PolyLog}\left(2,-\frac{(b-c)e^{2ix}}{2a+b+c+2\sqrt{a+b}\sqrt{a+c}}\right)}{2\sqrt{a+b}\sqrt{a+c}} \\ & -\frac{i\,\text{PolyLog}\left(3,-\frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\sqrt{a+c}}\right)}{4\sqrt{a+b}\sqrt{a+c}} \\ & +\frac{i\,\text{PolyLog}\left(3,-\frac{(b-c)e^{2ix}}{2a+b+c+2\sqrt{a+b}\sqrt{a+c}}\right)}{4\sqrt{a+b}\sqrt{a+c}}\end{aligned}$$

output
```
-1/2*I*x^2*ln(1+(b-c)*exp(2*I*x)/(2*a+b+c-2*(a+b)^(1/2)*(a+c)^(1/2)))/(a+b
)^(1/2)/(a+c)^(1/2)+1/2*I*x^2*ln(1+(b-c)*exp(2*I*x)/(2*a+b+c+2*(a+b)^(1/2)
*(a+c)^(1/2)))/(a+b)^(1/2)/(a+c)^(1/2)-1/2*x*polylog(2,-(b-c)*exp(2*I*x)/(
2*a+b+c-2*(a+b)^(1/2)*(a+c)^(1/2)))/(a+b)^(1/2)/(a+c)^(1/2)+1/2*x*polylog(
2,-(b-c)*exp(2*I*x)/(2*a+b+c+2*(a+b)^(1/2)*(a+c)^(1/2)))/(a+b)^(1/2)/(a+c)
^(1/2)-1/4*I*polylog(3,-(b-c)*exp(2*I*x)/(2*a+b+c-2*(a+b)^(1/2)*(a+c)^(1/2
)))/(a+b)^(1/2)/(a+c)^(1/2)+1/4*I*polylog(3,-(b-c)*exp(2*I*x)/(2*a+b+c+2*(
a+b)^(1/2)*(a+c)^(1/2)))/(a+b)^(1/2)/(a+c)^(1/2)
```

3.498.2 Mathematica [A] (verified)

Time = 5.66 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.76

$$\int \frac{x^2}{a+b\cos^2(x)+c\sin^2(x)}\,dx =$$
$$-\frac{i\left(2x^2\log\left(1+\frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\sqrt{a+c}}\right)-2x^2\log\left(1+\frac{(b-c)e^{2ix}}{2a+b+c+2\sqrt{a+b}\sqrt{a+c}}\right)-2ix\,\text{PolyLog}\left(2,\frac{(-b+c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\sqrt{a}}\right.\right.}{4\sqrt{}}$$

input
```
Integrate[x^2/(a + b*Cos[x]^2 + c*Sin[x]^2),x]
```

output
```
((-1/4*I)*(2*x^2*Log[1 + ((b - c)*E^((2*I)*x))/(2*a + b + c - 2*Sqrt[a + b
]*Sqrt[a + c])] - 2*x^2*Log[1 + ((b - c)*E^((2*I)*x))/(2*a + b + c + 2*Sqr
t[a + b]*Sqrt[a + c])] - (2*I)*x*PolyLog[2, ((-b + c)*E^((2*I)*x))/(2*a +
b + c - 2*Sqrt[a + b]*Sqrt[a + c])] + (2*I)*x*PolyLog[2, ((-b + c)*E^((2*I
)*x))/(2*a + b + c + 2*Sqrt[a + b]*Sqrt[a + c])] + PolyLog[3, ((-b + c)*E^
((2*I)*x))/(2*a + b + c - 2*Sqrt[a + b]*Sqrt[a + c])] - PolyLog[3, ((-b +
c)*E^((2*I)*x))/(2*a + b + c + 2*Sqrt[a + b]*Sqrt[a + c])]))/(Sqrt[a + b]*
Sqrt[a + c])
```

3.498.3 Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {5098, 3042, 3802, 2694, 27, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a+b\cos^2(x)+c\sin^2(x)}\,dx$$

$$\Big\downarrow \; 5098$$

$$2\int \frac{x^2}{2a+b+c+(b-c)\cos(2x)}dx$$

$$\Big\downarrow \; 3042$$

$$2\int \frac{x^2}{2a+b+c+(b-c)\sin\left(2x+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 3802$$

$$4\int \frac{e^{2ix}x^2}{b+2(2a+b+c)e^{2ix}+(b-c)e^{4ix}-c}dx$$

$$\Big\downarrow \; 2694$$

$$4\left(\frac{(b-c)\int \frac{e^{2ix}x^2}{2\left(2a+(b-c)e^{2ix}+b+c-2\sqrt{a+b}\sqrt{a+c}\right)}dx}{2\sqrt{a+b}\sqrt{a+c}}-\frac{(b-c)\int \frac{e^{2ix}x^2}{2\left(2a+(b-c)e^{2ix}+b+c+2\sqrt{a+b}\sqrt{a+c}\right)}dx}{2\sqrt{a+b}\sqrt{a+c}}\right)$$

$$\Big\downarrow \; 27$$

$$4\left(\frac{(b-c)\int \frac{e^{2ix}x^2}{2a+(b-c)e^{2ix}+b+c-2\sqrt{a+b}\sqrt{a+c}}dx}{4\sqrt{a+b}\sqrt{a+c}}-\frac{(b-c)\int \frac{e^{2ix}x^2}{2a+(b-c)e^{2ix}+b+c+2\sqrt{a+b}\sqrt{a+c}}dx}{4\sqrt{a+b}\sqrt{a+c}}\right)$$

$$\Big\downarrow \; 2620$$

$$4\left(\frac{(b-c)\left(\frac{i\int x\log\left(\frac{e^{2ix}(b-c)}{2a+b+c-2\sqrt{a+b}\sqrt{a+c}}+1\right)dx}{b-c}-\frac{ix^2\log\left(1+\frac{e^{2ix}(b-c)}{-2\sqrt{a+b}\sqrt{a+c}+2a+b+c}\right)}{2(b-c)}\right)}{4\sqrt{a+b}\sqrt{a+c}}-\frac{(b-c)\left(\frac{i\int x\log\left(\frac{e^{2ix}(b-c)}{2a+b+c+2\sqrt{a+b}\sqrt{a+c}}+\right.}{b-c}\right.}{4\sqrt{a+}}\right.$$

$$\Big\downarrow \; 3011$$

$$4\left(\frac{(b-c)\left(\frac{i\left(\frac{1}{2}ix\,\text{PolyLog}\left(2,-\frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\sqrt{a+c}}\right)-\frac{1}{2}i\int \text{PolyLog}\left(2,-\frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\sqrt{a+c}}\right)dx\right)}{b-c}-\frac{ix^2\log\left(1+\frac{e^{2ix}(b-c)}{-2\sqrt{a+b}\sqrt{a+c}+2a+b+c}\right)}{2(b-c)}\right.}{4\sqrt{a+b}\sqrt{a+c}}\right.$$

$$\Big\downarrow \; 2720$$

3.498. $\int \frac{x^2}{a+b\cos^2(x)+c\sin^2(x)}\,dx$

$$4\left(\frac{(b-c)\left(\frac{i\left(\frac{1}{2}ix\,\text{PolyLog}\left(2,-\frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\sqrt{a+c}}\right)-\frac{1}{4}\int e^{-2ix}\,\text{PolyLog}\left(2,-\frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\sqrt{a+c}}\right)de^{2ix}\right)}{b-c}-\frac{ix^2\log\left(1+\frac{e^{2ix}(b-c)}{-2\sqrt{a+b}\sqrt{a+c}+\cdots}\right.}{2(b-c)}\right.}{4\sqrt{a+b}\sqrt{a+c}}\right.$$

$\downarrow$ 7143

$$4\left(\frac{(b-c)\left(\frac{i\left(\frac{1}{2}ix\,\text{PolyLog}\left(2,-\frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\sqrt{a+c}}\right)-\frac{1}{4}\,\text{PolyLog}\left(3,-\frac{(b-c)e^{2ix}}{2a+b+c-2\sqrt{a+b}\sqrt{a+c}}\right)\right)}{b-c}-\frac{ix^2\log\left(1+\frac{e^{2ix}(b-c)}{-2\sqrt{a+b}\sqrt{a+c}+2a+b+c}\right)}{2(b-c)}\right)}{4\sqrt{a+b}\sqrt{a+c}}\right.$$

input
```
Int[x^2/(a + b*Cos[x]^2 + c*Sin[x]^2),x]
```

output
```
4*(((b - c)*(((-1/2*I)*x^2*Log[1 + ((b - c)*E^((2*I)*x))/(2*a + b + c - 2*
Sqrt[a + b]*Sqrt[a + c])])/(b - c) + (I*((I/2)*x*PolyLog[2, -(((b - c)*E^(
(2*I)*x))/(2*a + b + c - 2*Sqrt[a + b]*Sqrt[a + c]))] - PolyLog[3, -(((b -
 c)*E^((2*I)*x))/(2*a + b + c - 2*Sqrt[a + b]*Sqrt[a + c]))]/4))/(b - c)))
/(4*Sqrt[a + b]*Sqrt[a + c]) - ((b - c)*(((-1/2*I)*x^2*Log[1 + ((b - c)*E^
((2*I)*x))/(2*a + b + c + 2*Sqrt[a + b]*Sqrt[a + c])])/(b - c) + (I*((I/2)
*x*PolyLog[2, -(((b - c)*E^((2*I)*x))/(2*a + b + c + 2*Sqrt[a + b]*Sqrt[a
+ c]))] - PolyLog[3, -(((b - c)*E^((2*I)*x))/(2*a + b + c + 2*Sqrt[a + b]*
Sqrt[a + c]))]/4))/(b - c)))/(4*Sqrt[a + b]*Sqrt[a + c]))
```

3.498.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

3.498. $\int \frac{x^2}{a+b\cos^2(x)+c\sin^2(x)}\,dx$

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3802
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*E^(I*Pi*(k - 1/2))*(E^(I*(e +
f*x))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(
e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ
[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 5098
```
Int[((f_.) + (g_.)*(x_))^(m_.)/((a_.) + Cos[(d_.) + (e_.)*(x_)]^2*(b_.) + (
c_.)*Sin[(d_.) + (e_.)*(x_)]^2), x_Symbol] :> Simp[2   Int[(f + g*x)^m/(2*a
 + b + c + (b - c)*Cos[2*d + 2*e*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}
, x] && IGtQ[m, 0] && NeQ[a + b, 0] && NeQ[a + c, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.498. $\int \frac{x^2}{a+b\cos^2(x)+c\sin^2(x)}\,dx$

3.498.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1160 vs. 2(289) = 578.

Time = 0.91 (sec) , antiderivative size = 1161, normalized size of antiderivative = 3.18

method	result	size
risch	Expression too large to display	1161

input
```
int(x^2/(a+b*cos(x)^2+c*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/((a+b)*(a+c))^(1/2)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*b*x^3-I/((a+b)*(
a+c))^(1/2)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*a*x^2*ln(1-(b-c)*exp(2*I*x)/(
-2*((a+b)*(a+c))^(1/2)-2*a-b-c))-1/((a+b)*(a+c))^(1/2)/(-2*((a+b)*(a+c))^(
1/2)-2*a-b-c)*a*x*polylog(2,(b-c)*exp(2*I*x)/(-2*((a+b)*(a+c))^(1/2)-2*a-b
-c))-1/2*I/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*polylog(3,(b-c)*exp(2*I*x)/(-2
*((a+b)*(a+c))^(1/2)-2*a-b-c))-2/3/((a+b)*(a+c))^(1/2)/(-2*((a+b)*(a+c))^(
1/2)-2*a-b-c)*a*x^3-1/2*I/((a+b)*(a+c))^(1/2)/(-2*((a+b)*(a+c))^(1/2)-2*a-
b-c)*a*polylog(3,(b-c)*exp(2*I*x)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c))-1/2/((
a+b)*(a+c))^(1/2)*x*polylog(2,(b-c)*exp(2*I*x)/(2*((a+b)*(a+c))^(1/2)-2*a-
b-c))-1/4*I/((a+b)*(a+c))^(1/2)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*c*polylog
(3,(b-c)*exp(2*I*x)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c))-1/(-2*((a+b)*(a+c))^
(1/2)-2*a-b-c)*x*polylog(2,(b-c)*exp(2*I*x)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-
c))-1/4*I/((a+b)*(a+c))^(1/2)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*b*polylog(3
,(b-c)*exp(2*I*x)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c))-2/3/(-2*((a+b)*(a+c))^
(1/2)-2*a-b-c)*x^3-1/2*I/((a+b)*(a+c))^(1/2)/(-2*((a+b)*(a+c))^(1/2)-2*a-b
-c)*c*x^2*ln(1-(b-c)*exp(2*I*x)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c))-1/2/((a+
b)*(a+c))^(1/2)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*c*x*polylog(2,(b-c)*exp(2
*I*x)/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c))-1/2*I/((a+b)*(a+c))^(1/2)*x^2*ln(1
-(b-c)*exp(2*I*x)/(2*((a+b)*(a+c))^(1/2)-2*a-b-c))-1/2/((a+b)*(a+c))^(1/2)
/(-2*((a+b)*(a+c))^(1/2)-2*a-b-c)*b*x*polylog(2,(b-c)*exp(2*I*x)/(-2*((...
```

3.498.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4289 vs. 2(288) = 576.

Time = 3.36 (sec) , antiderivative size = 4289, normalized size of antiderivative = 11.75

$$\int \frac{x^2}{a + b\cos^2(x) + c\sin^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(a+b*cos(x)^2+c*sin(x)^2),x, algorithm="fricas")
```

output
```
-1/4*(-I*(b - c)*x^2*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*log
((((2*a + b + c)*cos(x) + (2*I*a + I*b + I*c)*sin(x) - 2*((b - c)*cos(x) +
 (I*b - I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sq
rt(-(2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) + 2*a + b
 + c)/(b - c)) + b - c)/(b - c)) + I*(b - c)*x^2*sqrt((a^2 + a*b + (a + b)
*c)/(b^2 - 2*b*c + c^2))*log(-(((2*a + b + c)*cos(x) - (2*I*a + I*b + I*c)
*sin(x) - 2*((b - c)*cos(x) - (I*b - I*c)*sin(x))*sqrt((a^2 + a*b + (a + b
)*c)/(b^2 - 2*b*c + c^2)))*sqrt(-(2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(
b^2 - 2*b*c + c^2)) + 2*a + b + c)/(b - c)) - b + c)/(b - c)) + I*(b - c)*
x^2*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*log((((2*a + b + c)*
cos(x) + (-2*I*a - I*b - I*c)*sin(x) - 2*((b - c)*cos(x) + (-I*b + I*c)*si
n(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)))*sqrt(-(2*(b - c)*
sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2)) + 2*a + b + c)/(b - c))
+ b - c)/(b - c)) - I*(b - c)*x^2*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*
c + c^2))*log(-(((2*a + b + c)*cos(x) - (-2*I*a - I*b - I*c)*sin(x) - 2*((
b - c)*cos(x) - (-I*b + I*c)*sin(x))*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2
*b*c + c^2)))*sqrt(-(2*(b - c)*sqrt((a^2 + a*b + (a + b)*c)/(b^2 - 2*b*c +
 c^2)) + 2*a + b + c)/(b - c)) - b + c)/(b - c)) + I*(b - c)*x^2*sqrt((a^2
 + a*b + (a + b)*c)/(b^2 - 2*b*c + c^2))*log((((2*a + b + c)*cos(x) + (2*I
*a + I*b + I*c)*sin(x) + 2*((b - c)*cos(x) - (-I*b + I*c)*sin(x))*sqrt(...
```

3.498.6 Sympy [F]

$$\int \frac{x^2}{a + b\cos^2(x) + c\sin^2(x)}\,dx = \int \frac{x^2}{a + b\cos^2{(x)} + c\sin^2{(x)}}\,dx$$

input
```
integrate(x**2/(a+b*cos(x)**2+c*sin(x)**2),x)
```

output
```
Integral(x**2/(a + b*cos(x)**2 + c*sin(x)**2), x)
```

3.498.7 Maxima [F]

$$\int \frac{x^2}{a + b\cos^2(x) + c\sin^2(x)}\,dx = \int \frac{x^2}{b\cos{(x)}^2 + c\sin{(x)}^2 + a}\,dx$$

input
```
integrate(x^2/(a+b*cos(x)^2+c*sin(x)^2),x, algorithm="maxima")
```

output
```
integrate(x^2/(b*cos(x)^2 + c*sin(x)^2 + a), x)
```

3.498.8 Giac [F]

$$\int \frac{x^2}{a + b\cos^2(x) + c\sin^2(x)}\,dx = \int \frac{x^2}{b\cos{(x)}^2 + c\sin{(x)}^2 + a}\,dx$$

input
```
integrate(x^2/(a+b*cos(x)^2+c*sin(x)^2),x, algorithm="giac")
```

output
```
integrate(x^2/(b*cos(x)^2 + c*sin(x)^2 + a), x)
```

3.498.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{a + b\cos^2(x) + c\sin^2(x)}\,dx = \int \frac{x^2}{b\cos{(x)}^2 + c\sin{(x)}^2 + a}\,dx$$

input
```
int(x^2/(a + c*sin(x)^2 + b*cos(x)^2),x)
```

output
```
int(x^2/(a + c*sin(x)^2 + b*cos(x)^2), x)
```

3.499 $\int (a+b\sin(d+ex)) \left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+$

3.499.1 Optimal result

Integrand size = 39, antiderivative size = 195

$$\begin{aligned}&\int (a+b\sin(d+ex)) \left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^2 dx\\&=\frac{3}{8}a\left(a^4+12a^2b^2+8b^4\right)x-\frac{b(32a^4+69a^2b^2+4b^4)\cos(d+ex)}{10e}\\&\quad-\frac{a(15a^4+82a^2b^2+8b^4)\cos(d+ex)\sin(d+ex)}{40e}\\&\quad-\frac{b(17a^2+4b^2)\cos(d+ex)(b+a\sin(d+ex))^2}{20e}\\&\quad-\frac{(5a^2+4b^2)\cos(d+ex)(b+a\sin(d+ex))^3}{20e}-\frac{b\cos(d+ex)(b+a\sin(d+ex))^4}{5e}\end{aligned}$$

output
```
3/8*a*(a^4+12*a^2*b^2+8*b^4)*x-1/10*b*(32*a^4+69*a^2*b^2+4*b^4)*cos(e*x+d)
/e-1/40*a*(15*a^4+82*a^2*b^2+8*b^4)*cos(e*x+d)*sin(e*x+d)/e-1/20*b*(17*a^2
+4*b^2)*cos(e*x+d)*(b+a*sin(e*x+d))^2/e-1/20*(5*a^2+4*b^2)*cos(e*x+d)*(b+a
*sin(e*x+d))^3/e-1/5*b*cos(e*x+d)*(b+a*sin(e*x+d))^4/e
```

3.499.2 Mathematica [A] (verified)

Time = 2.40 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.76

$$\int (a+b\sin(d+ex)) \left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^2 dx$$
$$= \frac{-20b(29a^4+68a^2b^2+8b^4)\cos(d+ex)+a(60(a^4+12a^2b^2+8b^4)(d+ex)+10(7a^3b+8ab^3)\cos(3(d+e}{160e}$$

input
```
Integrate[(a + b*Sin[d + e*x])*(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x
]^2)^2,x]
```

output
```
(-20*b*(29*a^4 + 68*a^2*b^2 + 8*b^4)*Cos[d + e*x] + a*(60*(a^4 + 12*a^2*b^
2 + 8*b^4)*(d + e*x) + 10*(7*a^3*b + 8*a*b^3)*Cos[3*(d + e*x)] - 2*a^3*b*C
os[5*(d + e*x)] - 40*(a^4 + 10*a^2*b^2 + 4*b^4)*Sin[2*(d + e*x)] + 5*(a^4
+ 4*a^2*b^2)*Sin[4*(d + e*x)]))/(160*e)
```

3.499.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.20, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 3769, 27, 3042, 3232, 3042, 3232, 27, 3042, 3232, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sin(d+ex)) \left(a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2\right)^2 dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (a+b\sin(d+ex)) \left(a^2\sin(d+ex)^2+2ab\sin(d+ex)+b^2\right)^2 dx$$

$$\Big\downarrow \text{ 3769}$$

$$\frac{\int 16\left(\sin(d+ex)a^2+ba\right)^4 (a+b\sin(d+ex))dx}{16a^4}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \left(\sin(d+ex)a^2+ba\right)^4 (a+b\sin(d+ex))dx}{a^4}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \left(\sin(d+ex)a^2+ba\right)^4 (a+b\sin(d+ex))dx}{a^4}$$

$$\Big\downarrow \text{3232}$$

$$\frac{\frac{1}{5}\int \left(\sin(d+ex)a^2+ba\right)^3 \left(9ba^2+\left(5a^2+4b^2\right)\sin(d+ex)a\right)dx-\frac{b\cos(d+ex)\left(a^2\sin(d+ex)+ab\right)^4}{5e}}{a^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{5}\int \left(\sin(d+ex)a^2+ba\right)^3 \left(9ba^2+\left(5a^2+4b^2\right)\sin(d+ex)a\right)dx-\frac{b\cos(d+ex)\left(a^2\sin(d+ex)+ab\right)^4}{5e}}{a^4}$$

$$\Big\downarrow \text{3232}$$

$$\frac{\frac{1}{5}\Bigg(\frac{1}{4}\int 3\left(\sin(d+ex)a^2+ba\right)^2 \left(\left(5a^2+16b^2\right)a^3+b\left(17a^2+4b^2\right)\sin(d+ex)a^2\right)dx-\frac{a\left(5a^2+4b^2\right)\cos(d+ex)\left(a^2\sin(d+ex)\right.}{4e}}{a^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{5}\Bigg(\frac{3}{4}\int \left(\sin(d+ex)a^2+ba\right)^2 \left(\left(5a^2+16b^2\right)a^3+b\left(17a^2+4b^2\right)\sin(d+ex)a^2\right)dx-\frac{a\left(5a^2+4b^2\right)\cos(d+ex)\left(a^2\sin(d+ex)+\right.}{4e}}{a^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{5}\Bigg(\frac{3}{4}\int \left(\sin(d+ex)a^2+ba\right)^2 \left(\left(5a^2+16b^2\right)a^3+b\left(17a^2+4b^2\right)\sin(d+ex)a^2\right)dx-\frac{a\left(5a^2+4b^2\right)\cos(d+ex)\left(a^2\sin(d+ex)+\right.}{4e}}{a^4}$$

$$\Big\downarrow \text{3232}$$

$$\frac{\frac{1}{5}\Bigg(\frac{3}{4}\Bigg(\frac{1}{3}\int \left(\sin(d+ex)a^2+ba\right)\left(7b\left(7a^2+8b^2\right)a^4+\left(15a^4+82b^2a^2+8b^4\right)\sin(d+ex)a^3\right)dx-\frac{b\left(17a^2+4b^2\right)\cos(d+ex)}{3e}}{a^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{5}\Bigg(\frac{3}{4}\Bigg(\frac{1}{3}\int \left(\sin(d+ex)a^2+ba\right)\left(7b\left(7a^2+8b^2\right)a^4+\left(15a^4+82b^2a^2+8b^4\right)\sin(d+ex)a^3\right)dx-\frac{b\left(17a^2+4b^2\right)\cos(d+ex)}{3e}}{a^4}$$

$$\Big\downarrow \text{3213}$$

$$\frac{\frac{1}{5}\Bigg(\frac{3}{4}\Bigg(\frac{1}{3}\Bigg(-\frac{2a^4b\left(32a^4+69a^2b^2+4b^4\right)\cos(d+ex)}{e}-\frac{a^5\left(15a^4+82a^2b^2+8b^4\right)\sin(d+ex)\cos(d+ex)}{2e}+\frac{15}{2}a^5x\left(a^4+12a^2b^2+8b^4\right)\Bigg)-\frac{b(1}{}}{a^4}$$

3.499. $\int (a+b\sin(d+ex))\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^2\,dx$

input
```
Int[(a + b*Sin[d + e*x])*(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2)^2
,x]
```

output
```
(-1/5*(b*Cos[d + e*x]*(a*b + a^2*Sin[d + e*x])^4)/e + (-1/4*(a*(5*a^2 + 4*
b^2)*Cos[d + e*x]*(a*b + a^2*Sin[d + e*x])^3)/e + (3*(-1/3*(b*(17*a^2 + 4*
b^2)*Cos[d + e*x]*(a^2*b + a^3*Sin[d + e*x])^2)/e + ((15*a^5*(a^4 + 12*a^2
*b^2 + 8*b^4)*x)/2 - (2*a^4*b*(32*a^4 + 69*a^2*b^2 + 4*b^4)*Cos[d + e*x])/
e - (a^5*(15*a^4 + 82*a^2*b^2 + 8*b^4)*Cos[d + e*x]*Sin[d + e*x])/(2*e))/3
))/4)/5)/a^4
```

3.499.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

rule 3769
```
Int[((A_) + (B_.)*sin[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sin[(d_.) + (e_.)*
(x_)] + (c_.)*sin[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Simp[1/(4^n*c^n
)   Int[(A + B*Sin[d + e*x])*(b + 2*c*Sin[d + e*x])^(2*n), x], x] /; FreeQ[
{a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[n]
```

3.499. $\int (a+b\sin(d+ex))\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^2\,dx$

3.499.4 Maple [A] (verified)

Time = 2.35 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.88

method	result
parallelrisch	$\frac{(-40a^5-400a^3b^2-160a\,b^4)\sin(2ex+2d)+(70a^4b+80a^2b^3)\cos(3ex+3d)+(5a^5+20a^3b^2)\sin(4ex+4d)-2a^4b\cos(5ex+5}{160e}$
parts	$a\,b^4x-\frac{(4a^2b^3+b^5)\cos(ex+d)}{e}+\frac{(6a^3b^2+4a\,b^4)\left(-\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)}{e}-\frac{(4a^4b+6a^2b^3)\left(2+\sin(ex+d)^2\right.}{3e}$
risch	$\frac{3a^5x}{8}+\frac{9a^3b^2x}{2}+3a\,b^4x-\frac{29b\cos(ex+d)a^4}{8e}-\frac{17b^3\cos(ex+d)a^2}{2e}-\frac{b^5\cos(ex+d)}{e}-\frac{a^4b\cos(5ex+5d)}{80e}+\sin$
derivativedivides	$\frac{a\,b^4(ex+d)-4a^2b^3\cos(ex+d)+6a^3b^2\left(-\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)-\frac{4a^4b\left(2+\sin(ex+d)^2\right)\cos(ex+d)}{3}+a^5\left(-\frac{\left(\sin(ex+d)\right.}{}\right.}{}$
default	$\frac{a\,b^4(ex+d)-4a^2b^3\cos(ex+d)+6a^3b^2\left(-\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)-\frac{4a^4b\left(2+\sin(ex+d)^2\right)\cos(ex+d)}{3}+a^5\left(-\frac{\left(\sin(ex+d)\right.}{}\right.}{}$
norman	$\frac{(3a\,b^4+\frac{3}{8}a^5+\frac{9}{2}a^3b^2)x+(3a\,b^4+\frac{3}{8}a^5+\frac{9}{2}a^3b^2)x\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{10}+(15a\,b^4+\frac{15}{8}a^5+\frac{45}{2}a^3b^2)x\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2+(15a\,b^4+\frac{15}{8}a^5}{}$

input
```
int((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^2,x,method=_R
ETURNVERBOSE)
```

output
```
1/160*((-40*a^5-400*a^3*b^2-160*a*b^4)*sin(2*e*x+2*d)+(70*a^4*b+80*a^2*b^3
)*cos(3*e*x+3*d)+(5*a^5+20*a^3*b^2)*sin(4*e*x+4*d)-2*a^4*b*cos(5*e*x+5*d)+
(-580*a^4*b-1360*a^2*b^3-160*b^5)*cos(e*x+d)+60*a^5*e*x+720*a^3*b^2*e*x+48
0*a*b^4*e*x-512*a^4*b-1280*a^2*b^3-160*b^5)/e
```

3.499.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.77

$$\int (a+b\sin(d+ex))\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^2\,dx =$$

$$-\frac{8\,a^4b\cos{(ex+d)}^5-80\,(a^4b+a^2b^3)\cos{(ex+d)}^3-15\,(a^5+12\,a^3b^2+8\,ab^4)ex+40\,(5\,a^4b+10\,a^2b^3+}{40}$$

input
```
integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^2,x, al
gorithm="fricas")
```

output
```
-1/40*(8*a^4*b*cos(e*x + d)^5 - 80*(a^4*b + a^2*b^3)*cos(e*x + d)^3 - 15*(
a^5 + 12*a^3*b^2 + 8*a*b^4)*e*x + 40*(5*a^4*b + 10*a^2*b^3 + b^5)*cos(e*x
+ d) - 5*(2*(a^5 + 4*a^3*b^2)*cos(e*x + d)^3 - (5*a^5 + 44*a^3*b^2 + 16*a*
b^4)*cos(e*x + d))*sin(e*x + d))/e
```

3.499.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 566 vs. $2(178) = 356$.

Time = 0.33 (sec) , antiderivative size = 566, normalized size of antiderivative = 2.90

$$\int (a+b\sin(d+ex))\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^2\,dx$$
$$=\begin{cases}\frac{3a^5x\sin^4(d+ex)}{8}+\frac{3a^5x\sin^2(d+ex)\cos^2(d+ex)}{4}+\frac{3a^5x\cos^4(d+ex)}{8}-\frac{5a^5\sin^3(d+ex)\cos(d+ex)}{8e}-\frac{3a^5\sin(d+ex)\cos^3(d+ex)}{8e}-\frac{a}{} \\ x(a+b\sin(d))\left(a^2\sin^2(d)+2ab\sin(d)+b^2\right)^2\end{cases}$$

input
```
integrate((a+b*sin(e*x+d))*(b**2+2*a*b*sin(e*x+d)+a**2*sin(e*x+d)**2)**2,x
)
```

output
```
Piecewise((3*a**5*x*sin(d + e*x)**4/8 + 3*a**5*x*sin(d + e*x)**2*cos(d + e
*x)**2/4 + 3*a**5*x*cos(d + e*x)**4/8 - 5*a**5*sin(d + e*x)**3*cos(d + e*x
)/(8*e) - 3*a**5*sin(d + e*x)*cos(d + e*x)**3/(8*e) - a**4*b*sin(d + e*x)*
*4*cos(d + e*x)/e - 4*a**4*b*sin(d + e*x)**2*cos(d + e*x)**3/(3*e) - 4*a**
4*b*sin(d + e*x)**2*cos(d + e*x)/e - 8*a**4*b*cos(d + e*x)**5/(15*e) - 8*a
**4*b*cos(d + e*x)**3/(3*e) + 3*a**3*b**2*x*sin(d + e*x)**4/2 + 3*a**3*b**
2*x*sin(d + e*x)**2*cos(d + e*x)**2 + 3*a**3*b**2*x*sin(d + e*x)**2 + 3*a*
*3*b**2*x*cos(d + e*x)**4/2 + 3*a**3*b**2*x*cos(d + e*x)**2 - 5*a**3*b**2*
sin(d + e*x)**3*cos(d + e*x)/(2*e) - 3*a**3*b**2*sin(d + e*x)*cos(d + e*x)
**3/(2*e) - 3*a**3*b**2*sin(d + e*x)*cos(d + e*x)/e - 6*a**2*b**3*sin(d +
e*x)**2*cos(d + e*x)/e - 4*a**2*b**3*cos(d + e*x)**3/e - 4*a**2*b**3*cos(d
 + e*x)/e + 2*a*b**4*x*sin(d + e*x)**2 + 2*a*b**4*x*cos(d + e*x)**2 + a*b*
*4*x - 2*a*b**4*sin(d + e*x)*cos(d + e*x)/e - b**5*cos(d + e*x)/e, Ne(e, 0
)), (x*(a + b*sin(d))*(a**2*sin(d)**2 + 2*a*b*sin(d) + b**2)**2, True))
```

3.499.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.26

$$\int (a+b\sin(d+ex))\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^2\,dx$$
$$=\frac{15\,(12\,ex+12\,d+\sin\left(4\,ex+4\,d\right)-8\,\sin\left(2\,ex+2\,d\right))a^5-32\,\left(3\,\cos\left(ex+d\right)^5-10\,\cos\left(ex+d\right)^3+15\,\cos\right.}{}$$

input
```
integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^2,x, al
gorithm="maxima")
```

output
```
1/480*(15*(12*e*x + 12*d + sin(4*e*x + 4*d) - 8*sin(2*e*x + 2*d))*a^5 - 32
*(3*cos(e*x + d)^5 - 10*cos(e*x + d)^3 + 15*cos(e*x + d))*a^4*b + 640*(cos
(e*x + d)^3 - 3*cos(e*x + d))*a^4*b + 60*(12*e*x + 12*d + sin(4*e*x + 4*d)
 - 8*sin(2*e*x + 2*d))*a^3*b^2 + 720*(2*e*x + 2*d - sin(2*e*x + 2*d))*a^3*
b^2 + 960*(cos(e*x + d)^3 - 3*cos(e*x + d))*a^2*b^3 + 480*(2*e*x + 2*d - s
in(2*e*x + 2*d))*a*b^4 + 480*(e*x + d)*a*b^4 - 1920*a^2*b^3*cos(e*x + d) -
 480*b^5*cos(e*x + d))/e
```

3.499.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.81

$$\int (a+b\sin(d+ex))\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^2\,dx$$
$$=-\frac{a^4b\cos\left(5\,ex+5\,d\right)}{80\,e}+\frac{3}{8}\,\left(a^5+12\,a^3b^2+8\,ab^4\right)x+\frac{\left(7\,a^4b+8\,a^2b^3\right)\cos\left(3\,ex+3\,d\right)}{16\,e}-\frac{\left(29\,a^4b+68\,a^2b^3+8\,b^5\right)\cos\left(ex+d\right)}{8\,e}+\frac{\left(a^5+4\,a^3b^2\right)\sin\left(4\,ex+4\,d\right)}{32\,e}-\frac{\left(a^5+10\,a^3b^2+4\,ab^4\right)\sin\left(2\,ex+2\,d\right)}{4\,e}$$

input
```
integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^2,x, al
gorithm="giac")
```

output
```
-1/80*a^4*b*cos(5*e*x + 5*d)/e + 3/8*(a^5 + 12*a^3*b^2 + 8*a*b^4)*x + 1/16
*(7*a^4*b + 8*a^2*b^3)*cos(3*e*x + 3*d)/e - 1/8*(29*a^4*b + 68*a^2*b^3 + 8
*b^5)*cos(e*x + d)/e + 1/32*(a^5 + 4*a^3*b^2)*sin(4*e*x + 4*d)/e - 1/4*(a^
5 + 10*a^3*b^2 + 4*a*b^4)*sin(2*e*x + 2*d)/e
```

3.499.9 Mupad [B] (verification not implemented)

Time = 29.60 (sec) , antiderivative size = 456, normalized size of antiderivative = 2.34

$$\int (a+b\sin(d+ex))\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^2\,dx$$

$$=\frac{3\,a\,\mathrm{atan}\left(\frac{3\,a\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\left(a^4+12\,a^2\,b^2+8\,b^4\right)}{4\,\left(\frac{3\,a^5}{4}+9\,a^3\,b^2+6\,a\,b^4\right)}\right)\,\left(a^4+12\,a^2\,b^2+8\,b^4\right)}{4\,e}$$

$$-\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,\left(\frac{3\,a^5}{4}+9\,a^3\,b^2+4\,a\,b^4\right)-\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^9\,\left(\frac{3\,a^5}{4}+9\,a^3\,b^2+4\,a\,b^4\right)+\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^3\,\left(\frac{7\,a^5}{2}+26\right.}{}$$

$$-\frac{3\,a\,\left(\mathrm{atan}\left(\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\right)-\frac{e\,x}{2}\right)\,\left(a^4+12\,a^2\,b^2+8\,b^4\right)}{4\,e}$$

input
```
int((a + b*sin(d + e*x))*(b^2 + a^2*sin(d + e*x)^2 + 2*a*b*sin(d + e*x))^2
,x)
```

output
```
(3*a*atan((3*a*tan(d/2 + (e*x)/2)*(a^4 + 8*b^4 + 12*a^2*b^2))/(4*(6*a*b^4
+ (3*a^5)/4 + 9*a^3*b^2)))*(a^4 + 8*b^4 + 12*a^2*b^2))/(4*e) - (tan(d/2 +
(e*x)/2)*(4*a*b^4 + (3*a^5)/4 + 9*a^3*b^2) - tan(d/2 + (e*x)/2)^9*(4*a*b^4
 + (3*a^5)/4 + 9*a^3*b^2) + tan(d/2 + (e*x)/2)^3*(8*a*b^4 + (7*a^5)/2 + 26
*a^3*b^2) - tan(d/2 + (e*x)/2)^7*(8*a*b^4 + (7*a^5)/2 + 26*a^3*b^2) + tan(
d/2 + (e*x)/2)^6*(16*a^4*b + 8*b^5 + 56*a^2*b^3) + tan(d/2 + (e*x)/2)^2*(3
2*a^4*b + 8*b^5 + 72*a^2*b^3) + tan(d/2 + (e*x)/2)^4*(48*a^4*b + 12*b^5 +
104*a^2*b^3) + (32*a^4*b)/5 + 2*b^5 + 16*a^2*b^3 + tan(d/2 + (e*x)/2)^8*(2
*b^5 + 8*a^2*b^3))/(e*(5*tan(d/2 + (e*x)/2)^2 + 10*tan(d/2 + (e*x)/2)^4 +
10*tan(d/2 + (e*x)/2)^6 + 5*tan(d/2 + (e*x)/2)^8 + tan(d/2 + (e*x)/2)^10 +
 1)) - (3*a*(atan(tan(d/2 + (e*x)/2)) - (e*x)/2)*(a^4 + 8*b^4 + 12*a^2*b^2
))/(4*e)
```

3.500 $\int (a+b\sin(d+ex))\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+$

3.500.1 Optimal result

Integrand size = 37, antiderivative size = 109

$$\int (a+b\sin(d+ex))\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)\,dx = \frac{1}{2}a\left(a^2+4b^2\right)x+\frac{\left(a^4-8a^2b^2-3b^4\right)\cos(d+ex)}{3be} + \frac{a(a^2-6b^2)\cos(d+ex)\sin(d+ex)}{6e}-\frac{a^2\cos(d+ex)(a+b\sin(d+ex))^2}{3be}$$

output
```
1/2*a*(a^2+4*b^2)*x+1/3*(a^4-8*a^2*b^2-3*b^4)*cos(e*x+d)/b/e+1/6*a*(a^2-6*
b^2)*cos(e*x+d)*sin(e*x+d)/e-1/3*a^2*cos(e*x+d)*(a+b*sin(e*x+d))^2/b/e
```

3.500.2 Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.71

$$\int (a+b\sin(d+ex))\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)\,dx = \frac{-3b(11a^2+4b^2)\cos(d+ex)+a\left(6\left(a^2+4b^2\right)(d+ex)+ab\cos(3(d+ex))-3\left(a^2+2b^2\right)\sin(2(d+ex))\right)}{12e}$$

input
```
Integrate[(a + b*Sin[d + e*x])*(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x
]^2),x]
```

output
```
(-3*b*(11*a^2 + 4*b^2)*Cos[d + e*x] + a*(6*(a^2 + 4*b^2)*(d + e*x) + a*b*C
os[3*(d + e*x)] - 3*(a^2 + 2*b^2)*Sin[2*(d + e*x)]))/(12*e)
```

3.500.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.108, Rules used = {3042, 3502, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sin(d+ex))\left(a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\sin(d+ex))\left(a^2\sin(d+ex)^2+2ab\sin(d+ex)+b^2\right)dx$$

$$\Big\downarrow \text{3502}$$

$$\frac{\int (a+b\sin(d+ex))\left(b\left(2a^2+3b^2\right)-a\left(a^2-6b^2\right)\sin(d+ex)\right)dx}{3b}-\frac{a^2\cos(d+ex)(a+b\sin(d+ex))^2}{3be}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (a+b\sin(d+ex))\left(b\left(2a^2+3b^2\right)-a\left(a^2-6b^2\right)\sin(d+ex)\right)dx}{3b}-\frac{a^2\cos(d+ex)(a+b\sin(d+ex))^2}{3be}$$

$$\Big\downarrow \text{3213}$$

$$\frac{\frac{ab\left(a^2-6b^2\right)\sin(d+ex)\cos(d+ex)}{2e}+\frac{3}{2}abx\left(a^2+4b^2\right)+\frac{\left(a^4-8a^2b^2-3b^4\right)\cos(d+ex)}{e}}{3b}-\frac{a^2\cos(d+ex)(a+b\sin(d+ex))^2}{3be}$$

input
```
Int[(a + b*Sin[d + e*x])*(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2),x
]
```

output
```
-1/3*(a^2*Cos[d + e*x]*(a + b*Sin[d + e*x])^2)/(b*e) + ((3*a*b*(a^2 + 4*b^
2)*x)/2 + ((a^4 - 8*a^2*b^2 - 3*b^4)*Cos[d + e*x])/e + (a*b*(a^2 - 6*b^2)*
Cos[d + e*x]*Sin[d + e*x])/(2*e))/(3*b)
```

3.500.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

3.500.4 Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.83

method	result
parts	$a\,b^{2}x-\frac{\left(2a^{2}b+b^{3}\right)\cos\left(ex+d\right)}{e}+\frac{\left(a^{3}+2a\,b^{2}\right)\left(-\frac{\cos\left(ex+d\right)\sin\left(ex+d\right)}{2}+\frac{ex}{2}+\frac{d}{2}\right)}{e}-\frac{a^{2}b\left(2+\sin\left(ex+d\right)^{2}\right)\cos\left(ex+d\right)}{3e}$
risch	$\frac{a^{3}x}{2}+2a\,b^{2}x-\frac{11b\cos\left(ex+d\right)a^{2}}{4e}-\frac{b^{3}\cos\left(ex+d\right)}{e}+\frac{b\,a^{2}\cos\left(3ex+3d\right)}{12e}-\frac{a^{3}\sin\left(2ex+2d\right)}{4e}-\frac{a\sin\left(2ex+2d\right)b^{2}}{2e}$
parallelrisch	$\frac{6a^{3}ex+24a\,b^{2}ex+a^{2}b\cos\left(3ex+3d\right)-3\sin\left(2ex+2d\right)a^{3}-6\sin\left(2ex+2d\right)a\,b^{2}-33\cos\left(ex+d\right)a^{2}b-12b^{3}\cos\left(ex+d\right)-32a^{2}b-1}{12e}$
derivativedivides	$\frac{-\frac{a^{2}b\left(2+\sin\left(ex+d\right)^{2}\right)\cos\left(ex+d\right)}{3}+a^{3}\left(-\frac{\cos\left(ex+d\right)\sin\left(ex+d\right)}{2}+\frac{ex}{2}+\frac{d}{2}\right)+2a\,b^{2}\left(-\frac{\cos\left(ex+d\right)\sin\left(ex+d\right)}{2}+\frac{ex}{2}+\frac{d}{2}\right)-2\cos\left(ex+d\right)}{e}$
default	$\frac{-\frac{a^{2}b\left(2+\sin\left(ex+d\right)^{2}\right)\cos\left(ex+d\right)}{3}+a^{3}\left(-\frac{\cos\left(ex+d\right)\sin\left(ex+d\right)}{2}+\frac{ex}{2}+\frac{d}{2}\right)+2a\,b^{2}\left(-\frac{\cos\left(ex+d\right)\sin\left(ex+d\right)}{2}+\frac{ex}{2}+\frac{d}{2}\right)-2\cos\left(ex+d\right)}{e}$
norman	$\frac{\left(2a\,b^{2}+\frac{1}{2}a^{3}\right)x+\left(\frac{3}{2}a^{3}+6a\,b^{2}\right)x\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}+\left(\frac{3}{2}a^{3}+6a\,b^{2}\right)x\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{4}+\left(2a\,b^{2}+\frac{1}{2}a^{3}\right)x\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{6}+\frac{a\left(a^{2}+2b^{2}\right)}{}}{\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}\right.}$

input
```
int((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2),x,method=_RET
URNVERBOSE)
```

output
```
a*b^2*x-(2*a^2*b+b^3)/e*cos(e*x+d)+(a^3+2*a*b^2)/e*(-1/2*cos(e*x+d)*sin(e*
x+d)+1/2*e*x+1/2*d)-1/3*a^2*b/e*(2+sin(e*x+d)^2)*cos(e*x+d)
```

3.500.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.70

$$\int (a+b\sin(d+ex))\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)\,dx$$
$$=\frac{2\,a^2b\cos{(ex+d)}^3+3\,(a^3+4\,ab^2)ex-3\,(a^3+2\,ab^2)\cos{(ex+d)}\sin{(ex+d)}-6\,(3\,a^2b+b^3)\cos{(ex+d}}{6\,e}$$

input
```
integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2),x, algo
rithm="fricas")
```

output
```
1/6*(2*a^2*b*cos(e*x + d)^3 + 3*(a^3 + 4*a*b^2)*e*x - 3*(a^3 + 2*a*b^2)*co
s(e*x + d)*sin(e*x + d) - 6*(3*a^2*b + b^3)*cos(e*x + d))/e
```

3.500.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. 2(95) = 190.

Time = 0.14 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.87

$$\int (a+b\sin(d+ex))\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)\,dx$$
$$=\begin{cases}\frac{a^3x\sin^2{(d+ex)}}{2}+\frac{a^3x\cos^2{(d+ex)}}{2}-\frac{a^3\sin{(d+ex)}\cos{(d+ex)}}{2e}-\frac{a^2b\sin^2{(d+ex)}\cos{(d+ex)}}{e}-\frac{2a^2b\cos^3{(d+ex)}}{3e}-\frac{2a^2b\cos{(d+ex)}}{e}- \\ x(a+b\sin{(d)})\left(a^2\sin^2{(d)}+2ab\sin{(d)}+b^2\right)\end{cases}$$

input
```
integrate((a+b*sin(e*x+d))*(b**2+2*a*b*sin(e*x+d)+a**2*sin(e*x+d)**2),x)
```

output
```
Piecewise((a**3*x*sin(d + e*x)**2/2 + a**3*x*cos(d + e*x)**2/2 - a**3*sin(
d + e*x)*cos(d + e*x)/(2*e) - a**2*b*sin(d + e*x)**2*cos(d + e*x)/e - 2*a*
*2*b*cos(d + e*x)**3/(3*e) - 2*a**2*b*cos(d + e*x)/e + a*b**2*x*sin(d + e*
x)**2 + a*b**2*x*cos(d + e*x)**2 + a*b**2*x - a*b**2*sin(d + e*x)*cos(d +
e*x)/e - b**3*cos(d + e*x)/e, Ne(e, 0)), (x*(a + b*sin(d))*(a**2*sin(d)**2
 + 2*a*b*sin(d) + b**2), True))
```

3.500.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.03

$$\int (a+b\sin(d+ex))\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)\,dx$$
$$=\frac{3\,(2\,ex+2\,d-\sin\left(2\,ex+2\,d\right))a^3+4\,\left(\cos\left(ex+d\right)^3-3\,\cos\left(ex+d\right)\right)a^2b+6\,(2\,ex+2\,d-\sin\left(2\,ex+2\,d\right.}{12\,e}$$

input
```
integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2),x, algo
rithm="maxima")
```

output
```
1/12*(3*(2*e*x + 2*d - sin(2*e*x + 2*d))*a^3 + 4*(cos(e*x + d)^3 - 3*cos(e
*x + d))*a^2*b + 6*(2*e*x + 2*d - sin(2*e*x + 2*d))*a*b^2 + 12*(e*x + d)*a
*b^2 - 24*a^2*b*cos(e*x + d) - 12*b^3*cos(e*x + d))/e
```

3.500.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.72

$$\int (a+b\sin(d+ex))\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)\,dx$$
$$=\frac{a^2b\cos\left(3\,ex+3\,d\right)}{12\,e}+\frac{1}{2}\,(a^3+4\,ab^2)x$$
$$-\frac{(11\,a^2b+4\,b^3)\cos\left(ex+d\right)}{4\,e}-\frac{(a^3+2\,ab^2)\sin\left(2\,ex+2\,d\right)}{4\,e}$$

input
```
integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2),x, algo
rithm="giac")
```

output
```
1/12*a^2*b*cos(3*e*x + 3*d)/e + 1/2*(a^3 + 4*a*b^2)*x - 1/4*(11*a^2*b + 4*
b^3)*cos(e*x + d)/e - 1/4*(a^3 + 2*a*b^2)*sin(2*e*x + 2*d)/e
```

3.500.9 Mupad [B] (verification not implemented)

Time = 27.24 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.81

$$\int (a+b\sin(d+ex))\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)\,dx =$$
$$-\frac{6\,b^3\,\cos\left(d+e\,x\right)+\frac{3\,a^3\,\sin(2\,d+2\,e\,x)}{2}-\frac{a^2\,b\,\cos(3\,d+3\,e\,x)}{2}+3\,a\,b^2\,\sin\left(2\,d+2\,e\,x\right)+\frac{33\,a^2\,b\,\cos(d+e\,x)}{2}-3\,a^3\,e\,x-}{6\,e}$$

input
```
int((a + b*sin(d + e*x))*(b^2 + a^2*sin(d + e*x)^2 + 2*a*b*sin(d + e*x)),x
)
```

output
```
-(6*b^3*cos(d + e*x) + (3*a^3*sin(2*d + 2*e*x))/2 - (a^2*b*cos(3*d + 3*e*x
))/2 + 3*a*b^2*sin(2*d + 2*e*x) + (33*a^2*b*cos(d + e*x))/2 - 3*a^3*e*x -
12*a*b^2*e*x)/(6*e)
```

3.501 $\int \frac{a+b\sin(d+ex)}{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}\,dx$

3.501.1 Optimal result

Integrand size = 39, antiderivative size = 23

$$\int \frac{a+b\sin(d+ex)}{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}\,dx=-\frac{\cos(d+ex)}{e(b+a\sin(d+ex))}$$

output
```
-cos(e*x+d)/e/(b+a*sin(e*x+d))
```

3.501.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sin(d+ex)}{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}\,dx=-\frac{\cos(d+ex)}{e(b+a\sin(d+ex))}$$

input
```
Integrate[(a + b*Sin[d + e*x])/(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x
]^2),x]
```

output
```
-(Cos[d + e*x]/(e*(b + a*Sin[d + e*x])))
```

3.501.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.39, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3042, 3769, 27, 3042, 3233, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sin(d+ex)}{a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\sin(d+ex)}{a^2\sin(d+ex)^2+2ab\sin(d+ex)+b^2}dx$$

$$\Big\downarrow \text{3769}$$

$$4a^2\int \frac{a+b\sin(d+ex)}{4\left(\sin(d+ex)a^2+ba\right)^2}dx$$

$$\Big\downarrow \text{27}$$

$$a^2\int \frac{a+b\sin(d+ex)}{\left(\sin(d+ex)a^2+ba\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$a^2\int \frac{a+b\sin(d+ex)}{\left(\sin(d+ex)a^2+ba\right)^2}dx$$

$$\Big\downarrow \text{3233}$$

$$a^2\left(\frac{\int 0dx}{a^2\left(a^2-b^2\right)}-\frac{\cos(d+ex)}{e\left(a^3\sin(d+ex)+a^2b\right)}\right)$$

$$\Big\downarrow \text{24}$$

$$-\frac{a^2\cos(d+ex)}{e\left(a^3\sin(d+ex)+a^2b\right)}$$

input
```
Int[(a + b*Sin[d + e*x])/(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2),x
]
```

output
```
-((a^2*Cos[d + e*x])/(e*(a^2*b + a^3*Sin[d + e*x])))
```

3.501.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 3769
```
Int[((A_) + (B_.)*sin[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sin[(d_.) + (e_.)*
(x_)] + (c_.)*sin[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Simp[1/(4^n*c^n
)   Int[(A + B*Sin[d + e*x])*(b + 2*c*Sin[d + e*x])^(2*n), x], x] /; FreeQ[
{a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[n]
```

3.501.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

method	result	size
parallelrisch	$-\frac{\cos(ex+d)}{e(b+a\sin(ex+d))}$	24
derivativedivides	$\frac{-\frac{a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{b}-1}{e\left(\frac{\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}b}{2}+a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)+\frac{b}{2}\right)}$	54
default	$\frac{-\frac{a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{b}-1}{e\left(\frac{\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}b}{2}+a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)+\frac{b}{2}\right)}$	54
risch	$-\frac{2\left(ia+b\,\mathrm{e}^{i(ex+d)}\right)}{ae\left(a\,\mathrm{e}^{2i(ex+d)}+2ib\,\mathrm{e}^{i(ex+d)}-a\right)}$	55
norman	$\frac{\frac{2\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{4}}{e}+\frac{2\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}}{e}+\frac{2a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{be}+\frac{2a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{3}}{be}}{\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}\right)\left(\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}b+2a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)+b\right)}$	117

input
```
int((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2),x,method=_RET
URNVERBOSE)
```

output
```
-cos(e*x+d)/e/(b+a*sin(e*x+d))
```

3.501.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\sin(d+ex)}{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}\,dx=-\frac{\cos\left(ex+d\right)}{ae\sin\left(ex+d\right)+be}$$

input
```
integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2),x, algo
rithm="fricas")
```

output
```
-cos(e*x + d)/(a*e*sin(e*x + d) + b*e)
```

3.501.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\sin(d+ex)}{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sin(e*x+d))/(b**2+2*a*b*sin(e*x+d)+a**2*sin(e*x+d)**2),x)
```

output
```
Timed out
```

3.501.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sin(d+ex)}{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2),x, algo
rithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.501.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. 2(23) = 46.

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.17

$$\int \frac{a+b\sin(d+ex)}{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}\,dx \\ = -\frac{2\left(a\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+b\right)}{\left(b\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^2+2\,a\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+b\right)be}$$

input
```
integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2),x, algo
rithm="giac")
```

output
```
-2*(a*tan(1/2*e*x + 1/2*d) + b)/((b*tan(1/2*e*x + 1/2*d)^2 + 2*a*tan(1/2*e
*x + 1/2*d) + b)*b*e)
```

3.501.9 Mupad [B] (verification not implemented)

Time = 27.39 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.70

$$\int \frac{a+b\sin(d+ex)}{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}\,dx = -\frac{a\,\sin\left(d+e\,x\right)+b\left(\cos\left(d+e\,x\right)+1\right)}{b\,e\,\left(b+a\,\sin\left(d+e\,x\right)\right)}$$

input
```
int((a + b*sin(d + e*x))/(b^2 + a^2*sin(d + e*x)^2 + 2*a*b*sin(d + e*x)),x
)
```

output
```
-(a*sin(d + e*x) + b*(cos(d + e*x) + 1))/(b*e*(b + a*sin(d + e*x)))
```

3.502 $\int \frac{a+b\sin(d+ex)}{\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^2}\,dx$

3.502.1 Optimal result

Integrand size = 39, antiderivative size = 157

$$\begin{aligned}\int \frac{a+b\sin(d+ex)}{\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^2}\,dx &= \frac{2ab\,\mathrm{arctanh}\left(\frac{a+b\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}e}-\frac{\cos(d+ex)}{3e(b+a\sin(d+ex))^3}\\&\quad+\frac{b\cos(d+ex)}{3\left(a^2-b^2\right)e(b+a\sin(d+ex))^2}-\frac{\left(2a^2+b^2\right)\cos(d+ex)}{3\left(a^2-b^2\right)^2e(b+a\sin(d+ex))}\end{aligned}$$

output
```
2*a*b*arctanh((a+b*tan(1/2*e*x+1/2*d))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)/e-
1/3*cos(e*x+d)/e/(b+a*sin(e*x+d))^3+1/3*b*cos(e*x+d)/(a^2-b^2)/e/(b+a*sin(
e*x+d))^2-1/3*(2*a^2+b^2)*cos(e*x+d)/(a^2-b^2)^2/e/(b+a*sin(e*x+d))
```

3.502.2 Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.89

$$\int \frac{a+b\sin(d+ex)}{\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^2}\,dx = -\frac{\frac{6ab\arctan\left(\frac{a+b\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{-a^2+b^2}}\right)}{\left(-a^2+b^2\right)^{5/2}}+\frac{\cos(d+ex)\left(a^4-a^2b^2+3b^4+3ab\left(a^2+b^2\right)\sin(d+ex)+a^2\left(2a^2+b^2\right)\sin^2(d+ex)\right)}{(a-b)^2(a+b)^2(b+a\sin(d+ex))^3}}{3e}$$

input
```
Integrate[(a + b*Sin[d + e*x])/(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x
]^2)^2,x]
```

output
```
-1/3*((6*a*b*ArcTan[(a + b*Tan[(d + e*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^
2)^(5/2) + (Cos[d + e*x]*(a^4 - a^2*b^2 + 3*b^4 + 3*a*b*(a^2 + b^2)*Sin[d
+ e*x] + a^2*(2*a^2 + b^2)*Sin[d + e*x]^2))/((a - b)^2*(a + b)^2*(b + a*Si
n[d + e*x])^3))/e
```

3.502.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.37, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3042, 3769, 27, 3042, 3233, 27, 3042, 3233, 3042, 3233, 27, 3042, 3139, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sin(d+ex)}{\left(a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\sin(d+ex)}{\left(a^2\sin(d+ex)^2+2ab\sin(d+ex)+b^2\right)^2}dx$$

$$\Big\downarrow \text{3769}$$

$$16a^4\int \frac{a+b\sin(d+ex)}{16\left(\sin(d+ex)a^2+ba\right)^4}dx$$

$$\Big\downarrow \text{27}$$

$$a^4\int \frac{a+b\sin(d+ex)}{\left(\sin(d+ex)a^2+ba\right)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$a^4\int \frac{a+b\sin(d+ex)}{\left(\sin(d+ex)a^2+ba\right)^4}dx$$

$$\Big\downarrow \text{3233}$$

$$a^4\left(\frac{\int \frac{2a\left(a^2-b^2\right)\sin(d+ex)}{\left(\sin(d+ex)a^2+ba\right)^3}dx}{3a^2\left(a^2-b^2\right)}-\frac{\cos(d+ex)}{3ae\left(a^2\sin(d+ex)+ab\right)^3}\right)$$

$$\Big\downarrow \text{27}$$

$$a^4\left(\frac{2\int \frac{\sin(d+ex)}{(\sin(d+ex)a^2+ba)^3}dx}{3a}-\frac{\cos(d+ex)}{3ae\left(a^2\sin(d+ex)+ab\right)^3}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^4\left(\frac{2\int \frac{\sin(d+ex)}{(\sin(d+ex)a^2+ba)^3}dx}{3a}-\frac{\cos(d+ex)}{3ae\left(a^2\sin(d+ex)+ab\right)^3}\right)$$

$$\Big\downarrow \text{3233}$$

$$a^4\left(\frac{2\left(\frac{\int \frac{2a^2-ab\sin(d+ex)}{\left(\sin(d+ex)a^2+ba\right)^2}dx}{2a^2(a^2-b^2)}+\frac{b\cos(d+ex)}{2ae(a^2-b^2)(a^2\sin(d+ex)+ab)^2}\right)}{3a}-\frac{\cos(d+ex)}{3ae\left(a^2\sin(d+ex)+ab\right)^3}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^4\left(\frac{2\left(\frac{\int \frac{2a^2-ab\sin(d+ex)}{\left(\sin(d+ex)a^2+ba\right)^2}dx}{2a^2(a^2-b^2)}+\frac{b\cos(d+ex)}{2ae(a^2-b^2)(a^2\sin(d+ex)+ab)^2}\right)}{3a}-\frac{\cos(d+ex)}{3ae\left(a^2\sin(d+ex)+ab\right)^3}\right)$$

$$\Big\downarrow \text{3233}$$

$$a^4\left(\frac{2\left(\frac{\frac{\int -\frac{3a^3b}{\sin(d+ex)a^2+ba}dx}{a^2\left(a^2-b^2\right)}-\frac{\left(2a^2+b^2\right)\cos(d+ex)}{e\left(a^2-b^2\right)\left(a^2\sin(d+ex)+ab\right)}}{2a^2(a^2-b^2)}+\frac{b\cos(d+ex)}{2ae(a^2-b^2)(a^2\sin(d+ex)+ab)^2}\right)}{3a}-\frac{\cos(d+ex)}{3ae\left(a^2\sin(d+ex)+ab\right)^3}\right)$$

$$\Big\downarrow \text{27}$$

3.502. $\int \frac{a+b\sin(d+ex)}{\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^2}\,dx$

$$a^4\left(\frac{2\left(\frac{-\frac{3ab\int \frac{1}{\sin(d+ex)a^2+ba}dx}{a^2-b^2}-\frac{\left(2a^2+b^2\right)\cos(d+ex)}{e\left(a^2-b^2\right)\left(a^2\sin(d+ex)+ab\right)}}{2a^2(a^2-b^2)}+\frac{b\cos(d+ex)}{2ae(a^2-b^2)(a^2\sin(d+ex)+ab)^2}\right)}{3a}-\frac{\cos(d+ex)}{3ae\left(a^2\sin(d+ex)+ab\right)^3}\right)$$

$\downarrow$ 3042

$$a^4\left(\frac{2\left(\frac{-\frac{3ab\int \frac{1}{\sin(d+ex)a^2+ba}dx}{a^2-b^2}-\frac{\left(2a^2+b^2\right)\cos(d+ex)}{e\left(a^2-b^2\right)\left(a^2\sin(d+ex)+ab\right)}}{2a^2(a^2-b^2)}+\frac{b\cos(d+ex)}{2ae(a^2-b^2)(a^2\sin(d+ex)+ab)^2}\right)}{3a}-\frac{\cos(d+ex)}{3ae\left(a^2\sin(d+ex)+ab\right)^3}\right)$$

$\downarrow$ 3139

$$a^4\left(\frac{2\left(\frac{-\frac{6ab\int \frac{1}{2\tan\left(\frac{1}{2}(d+ex)\right)a^2+b\tan^2\left(\frac{1}{2}(d+ex)\right)a+ba}d\tan\left(\frac{1}{2}(d+ex)\right)}{e\left(a^2-b^2\right)}-\frac{\left(2a^2+b^2\right)\cos(d+ex)}{e\left(a^2-b^2\right)\left(a^2\sin(d+ex)+ab\right)}}{2a^2(a^2-b^2)}+\frac{b\cos(d+ex)}{2ae(a^2-b^2)(a^2\sin(d+ex)+ab)^2}\right)}{3a}-\frac{}{3}\right.$$

$\downarrow$ 1083

$$a^4\left(\frac{2\left(\frac{\frac{12ab\int \frac{1}{4a^2\left(a^2-b^2\right)-\left(2a^2+2b\tan\left(\frac{1}{2}(d+ex)\right)a\right)^2}d\left(2a^2+2b\tan\left(\frac{1}{2}(d+ex)\right)a\right)}{e\left(a^2-b^2\right)}-\frac{\left(2a^2+b^2\right)\cos(d+ex)}{e\left(a^2-b^2\right)\left(a^2\sin(d+ex)+ab\right)}}{2a^2(a^2-b^2)}+\frac{b\cos(d+ex)}{2ae(a^2-b^2)(a^2\sin(d+ex)+ab)^2}\right.}{3a}\right.$$

$\downarrow$ 219

3.502. $\displaystyle\int \frac{a+b\sin(d+ex)}{\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^2}\,dx$

$$a^4\left(\frac{2\left(\frac{\frac{6b\,\text{arctanh}\left(\frac{2a^2+2ab\tan\left(\frac{1}{2}(d+ex)\right)}{2a\sqrt{a^2-b^2}}\right)}{e\left(a^2-b^2\right)^{3/2}}-\frac{\left(2a^2+b^2\right)\cos(d+ex)}{e\left(a^2-b^2\right)\left(a^2\sin(d+ex)+ab\right)}}{2a^2(a^2-b^2)}+\frac{b\cos(d+ex)}{2ae(a^2-b^2)(a^2\sin(d+ex)+ab)^2}\right)}{3a}-\frac{\cos(d+ex)}{3ae\left(a^2\sin(d+ex)\right.}\right.$$

input
```
Int[(a + b*Sin[d + e*x])/(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2)^2
,x]
```

output
```
a^4*(-1/3*Cos[d + e*x]/(a*e*(a*b + a^2*Sin[d + e*x])^3) + (2*((b*Cos[d + e
*x])/(2*a*(a^2 - b^2)*e*(a*b + a^2*Sin[d + e*x])^2) + ((6*b*ArcTanh[(2*a^2
 + 2*a*b*Tan[(d + e*x)/2])/(2*a*Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*e) -
 ((2*a^2 + b^2)*Cos[d + e*x])/((a^2 - b^2)*e*(a*b + a^2*Sin[d + e*x])))/(2
*a^2*(a^2 - b^2))))/(3*a))
```

3.502.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 3769
```
Int[((A_) + (B_.)*sin[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sin[(d_.) + (e_.)*
(x_)] + (c_.)*sin[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Simp[1/(4^n*c^n
)   Int[(A + B*Sin[d + e*x])*(b + 2*c*Sin[d + e*x])^(2*n), x], x] /; FreeQ[
{a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[n]
```

3.502.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.94 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.46

method	result
risch	$-\frac{2\left(2ia^5+ia^3b^2-6ia^5e^{2i(ex+d)}-12ia^3b^2e^{2i(ex+d)}+15ia^3b^2e^{4i(ex+d)}-12a^4b\,e^{3i(ex+d)}-14a^2b^3e^{3i(ex+d)}-4\,e^{3i(ex+d)}b\right.}{3\left(a\,e^{2i(ex+d)}+2ib\,e^{i(ex+d)}-a\right)^3\left(-a^2+b^2\right)^2ea}$
derivativedivides	$\frac{\frac{-\frac{2a\left(a^4-2a^2b^2+2b^4\right)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^5}{b\left(a^4-2a^2b^2+b^4\right)}-\frac{2\left(2a^6-3a^4b^2+5a^2b^4+b^6\right)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^4}{b^2\left(a^4-2a^2b^2+b^4\right)}-\frac{4a\left(2a^6+a^4b^2+3a^2b^4+9b^6\right)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^3}{3b^3\left(a^4-2a^2b^2+b^4\right)}-\frac{4\left(a^6\right.}{}}{\left(\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2b+2a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)+b\right)^3}}{e}$
default	$\frac{\frac{-\frac{2a\left(a^4-2a^2b^2+2b^4\right)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^5}{b\left(a^4-2a^2b^2+b^4\right)}-\frac{2\left(2a^6-3a^4b^2+5a^2b^4+b^6\right)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^4}{b^2\left(a^4-2a^2b^2+b^4\right)}-\frac{4a\left(2a^6+a^4b^2+3a^2b^4+9b^6\right)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^3}{3b^3\left(a^4-2a^2b^2+b^4\right)}-\frac{4\left(a^6\right.}{}}{\left(\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2b+2a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)+b\right)^3}}{e}$

input
```
int((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^2,x,method=_R
ETURNVERBOSE)
```

output
```
-2/3*(2*I*a^5+I*a^3*b^2-6*I*a^5*exp(2*I*(e*x+d))-12*I*a^3*b^2*exp(2*I*(e*x
+d))+15*I*a^3*b^2*exp(4*I*(e*x+d))-12*a^4*b*exp(3*I*(e*x+d))-14*a^2*b^3*ex
p(3*I*(e*x+d))-4*exp(3*I*(e*x+d))*b^5+3*a^4*b*exp(5*I*(e*x+d))+9*a^4*b*exp
(I*(e*x+d))-12*I*a*b^4*exp(2*I*(e*x+d))+6*a^2*b^3*exp(I*(e*x+d)))/(a*exp(2
*I*(e*x+d))+2*I*b*exp(I*(e*x+d))-a)^3/(-a^2+b^2)^2/e/a+1/(a^2-b^2)^(1/2)*b
*a/(a+b)^2/(a-b)^2/e*ln(exp(I*(e*x+d))+(I*b*(a^2-b^2)^(1/2)+a^2-b^2)/(a^2-
b^2)^(1/2)/a)-1/(a^2-b^2)^(1/2)*b*a/(a+b)^2/(a-b)^2/e*ln(exp(I*(e*x+d))+(I
*b*(a^2-b^2)^(1/2)-a^2+b^2)/(a^2-b^2)^(1/2)/a)
```

3.502.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(146) = 292$.

Time = 0.29 (sec) , antiderivative size = 795, normalized size of antiderivative = 5.06

$$\int \frac{a + b\sin(d + ex)}{\left(b^2 + 2ab\sin(d + ex) + a^2\sin^2(d + ex)\right)^2}\,dx$$

$$= \left[-\frac{2\left(2a^6 - a^4b^2 - a^2b^4\right)\cos(ex+d)^3 - 6\left(a^5b - ab^5\right)\cos(ex+d)\sin(ex+d) - 3\left(3a^3b^2\cos(ex+d)^2 -\right.}{6\left(3\left(a^8b - 3a^6b^3 + 3a^4b^5 - a^2b^7\right)e\cos(ex+\right.}\right.$$

$$-\frac{\left(2a^6 - a^4b^2 - a^2b^4\right)\cos(ex+d)^3 - 3\left(a^5b - ab^5\right)\cos(ex+d)\sin(ex+d) - 3\left(3a^3b^2\cos(ex+d)^2 - 3\right.}{3\left(3\left(a^8b - 3a^6b^3 + 3a^4b^5 - a^2b^7\right)e\cos(ex+d)^2 - \left(3a^8b - 8a^6b^3 +\right.\right.}$$

input
```
integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^2,x, al
gorithm="fricas")
```

output
```
[-1/6*(2*(2*a^6 - a^4*b^2 - a^2*b^4)*cos(e*x + d)^3 - 6*(a^5*b - a*b^5)*co
s(e*x + d)*sin(e*x + d) - 3*(3*a^3*b^2*cos(e*x + d)^2 - 3*a^3*b^2 - a*b^4
+ (a^4*b*cos(e*x + d)^2 - a^4*b - 3*a^2*b^3)*sin(e*x + d))*sqrt(a^2 - b^2)
*log(((a^2 - 2*b^2)*cos(e*x + d)^2 + 2*a*b*sin(e*x + d) + a^2 + b^2 + 2*(b
*cos(e*x + d)*sin(e*x + d) + a*cos(e*x + d))*sqrt(a^2 - b^2))/(a^2*cos(e*x
 + d)^2 - 2*a*b*sin(e*x + d) - a^2 - b^2)) - 6*(a^6 - a^4*b^2 + a^2*b^4 -
b^6)*cos(e*x + d))/(3*(a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*e*cos(e*x
+ d)^2 - (3*a^8*b - 8*a^6*b^3 + 6*a^4*b^5 - b^9)*e + ((a^9 - 3*a^7*b^2 + 3
*a^5*b^4 - a^3*b^6)*e*cos(e*x + d)^2 - (a^9 - 6*a^5*b^4 + 8*a^3*b^6 - 3*a*
b^8)*e)*sin(e*x + d)), -1/3*((2*a^6 - a^4*b^2 - a^2*b^4)*cos(e*x + d)^3 -
3*(a^5*b - a*b^5)*cos(e*x + d)*sin(e*x + d) - 3*(3*a^3*b^2*cos(e*x + d)^2
- 3*a^3*b^2 - a*b^4 + (a^4*b*cos(e*x + d)^2 - a^4*b - 3*a^2*b^3)*sin(e*x +
 d))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*sin(e*x + d) + a)/((a^2
- b^2)*cos(e*x + d))) - 3*(a^6 - a^4*b^2 + a^2*b^4 - b^6)*cos(e*x + d))/(3
*(a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*e*cos(e*x + d)^2 - (3*a^8*b - 8
*a^6*b^3 + 6*a^4*b^5 - b^9)*e + ((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*e
*cos(e*x + d)^2 - (a^9 - 6*a^5*b^4 + 8*a^3*b^6 - 3*a*b^8)*e)*sin(e*x + d))
]
```

3.502.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\sin(d+ex)}{\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((a+b*sin(e*x+d))/(b**2+2*a*b*sin(e*x+d)+a**2*sin(e*x+d)**2)**2,x
)
```

output
```
Timed out
```

3.502.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sin(d+ex)}{\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^2,x, al
gorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.502.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 435 vs. 2(146) = 292.

Time = 0.33 (sec) , antiderivative size = 435, normalized size of antiderivative = 2.77

$$\int \frac{a+b\sin(d+ex)}{\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^2}\,dx =$$

$$-\frac{2\left(\frac{3\left(\pi\left\lfloor\frac{ex+d}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+a}{\sqrt{-a^2+b^2}}\right)\right)ab}{(a^4-2a^2b^2+b^4)\sqrt{-a^2+b^2}}+\frac{3a^5b^2\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^5-6a^3b^4\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^5+6ab^6\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^5+6\cdots}{\cdots}\right)}{\cdots}$$

input
```
integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^2,x, al
gorithm="giac")
```

output
```
-2/3*(3*(pi*floor(1/2*(e*x + d)/pi + 1/2)*sgn(b) + arctan((b*tan(1/2*e*x +
 1/2*d) + a)/sqrt(-a^2 + b^2)))*a*b/((a^4 - 2*a^2*b^2 + b^4)*sqrt(-a^2 + b
^2)) + (3*a^5*b^2*tan(1/2*e*x + 1/2*d)^5 - 6*a^3*b^4*tan(1/2*e*x + 1/2*d)^
5 + 6*a*b^6*tan(1/2*e*x + 1/2*d)^5 + 6*a^6*b*tan(1/2*e*x + 1/2*d)^4 - 9*a^
4*b^3*tan(1/2*e*x + 1/2*d)^4 + 15*a^2*b^5*tan(1/2*e*x + 1/2*d)^4 + 3*b^7*t
an(1/2*e*x + 1/2*d)^4 + 4*a^7*tan(1/2*e*x + 1/2*d)^3 + 2*a^5*b^2*tan(1/2*e
*x + 1/2*d)^3 + 6*a^3*b^4*tan(1/2*e*x + 1/2*d)^3 + 18*a*b^6*tan(1/2*e*x +
1/2*d)^3 + 6*a^6*b*tan(1/2*e*x + 1/2*d)^2 + 18*a^2*b^5*tan(1/2*e*x + 1/2*d
)^2 + 6*b^7*tan(1/2*e*x + 1/2*d)^2 + 3*a^5*b^2*tan(1/2*e*x + 1/2*d) + 12*a
*b^6*tan(1/2*e*x + 1/2*d) + a^4*b^3 - a^2*b^5 + 3*b^7)/((a^4*b^3 - 2*a^2*b
^5 + b^7)*(b*tan(1/2*e*x + 1/2*d)^2 + 2*a*tan(1/2*e*x + 1/2*d) + b)^3))/e
```

3.502.9 Mupad [B] (verification not implemented)

Time = 30.19 (sec) , antiderivative size = 497, normalized size of antiderivative = 3.17

$$\int \frac{a+b\sin(d+ex)}{\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^2}\,dx$$
$$= \frac{2\,a\,b\,\mathrm{atanh}\left(\frac{\left(2\,a+2\,b\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\right)\left(a^4-2\,a^2\,b^2+b^4\right)}{2\,(a+b)^{5/2}\,(a-b)^{5/2}}\right)}{e\,(a+b)^{5/2}\,(a-b)^{5/2}}$$
$$-\frac{\frac{2\,(a^4-a^2\,b^2+3\,b^4)}{3\,(a^4-2\,a^2\,b^2+b^4)}+\frac{4\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2\,(a^6+3\,a^2\,b^4+b^6)}{b^2\,(a^4-2\,a^2\,b^2+b^4)}+\frac{2\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^4\,(2\,a^6-3\,a^4\,b^2+5\,a^2\,b^4+b^6)}{b^2\,(a^4-2\,a^2\,b^2+b^4)}+\frac{2\,a\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,(a^4+4\,b^4)}{b\,(a^4-2\,a^2\,b^2+b^4)}+\cdots}{e\left(b^3\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^6+\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^3\,(8\,a^3+12\,a\,b^2)+\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2\,(12\,a^2\,b+3\,b^3)+\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^4\,(\cdots\right.}$$

input
```
int((a + b*sin(d + e*x))/(b^2 + a^2*sin(d + e*x)^2 + 2*a*b*sin(d + e*x))^2
,x)
```

output
```
(2*a*b*atanh(((2*a + 2*b*tan(d/2 + (e*x)/2))*(a^4 + b^4 - 2*a^2*b^2))/(2*(
a + b)^(5/2)*(a - b)^(5/2))))/(e*(a + b)^(5/2)*(a - b)^(5/2)) - ((2*(a^4 +
 3*b^4 - a^2*b^2))/(3*(a^4 + b^4 - 2*a^2*b^2)) + (4*tan(d/2 + (e*x)/2)^2*(
a^6 + b^6 + 3*a^2*b^4))/(b^2*(a^4 + b^4 - 2*a^2*b^2)) + (2*tan(d/2 + (e*x)
/2)^4*(2*a^6 + b^6 + 5*a^2*b^4 - 3*a^4*b^2))/(b^2*(a^4 + b^4 - 2*a^2*b^2))
 + (2*a*tan(d/2 + (e*x)/2)*(a^4 + 4*b^4))/(b*(a^4 + b^4 - 2*a^2*b^2)) + (2
*a*tan(d/2 + (e*x)/2)^5*(a^4 + 2*b^4 - 2*a^2*b^2))/(b*(a^4 + b^4 - 2*a^2*b
^2)) + (4*a*tan(d/2 + (e*x)/2)^3*(2*a^2 + 3*b^2)*(a^4 + 3*b^4 - a^2*b^2))/
(3*b^3*(a^4 + b^4 - 2*a^2*b^2)))/(e*(b^3*tan(d/2 + (e*x)/2)^6 + tan(d/2 +
(e*x)/2)^3*(12*a*b^2 + 8*a^3) + tan(d/2 + (e*x)/2)^2*(12*a^2*b + 3*b^3) +
tan(d/2 + (e*x)/2)^4*(12*a^2*b + 3*b^3) + b^3 + 6*a*b^2*tan(d/2 + (e*x)/2)
 + 6*a*b^2*tan(d/2 + (e*x)/2)^5))
```

3.502. $\int \frac{a+b\sin(d+ex)}{\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^2}\,dx$

3.503 $\int \frac{d+e\sin(x)}{a+b\sin(x)+c\sin^2(x)}\,dx$

3.503.1 Optimal result

Integrand size = 21, antiderivative size = 242

$$\int \frac{d+e\sin(x)}{a+b\sin(x)+c\sin^2(x)}\,dx = \frac{\sqrt{2}\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{2c+\left(b-\sqrt{b^2-4ac}\right)\tan\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{b^2-2c(a+c)-b\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-2c(a+c)-b\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{2c+\left(b+\sqrt{b^2-4ac}\right)\tan\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{b^2-2c(a+c)+b\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-2c(a+c)+b\sqrt{b^2-4ac}}}$$

output
```
arctan(1/2*(2*c+(b-(-4*a*c+b^2)^(1/2))*tan(1/2*x))*2^(1/2)/(b^2-2*c*(a+c)-
b*(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))/(
b^2-2*c*(a+c)-b*(-4*a*c+b^2)^(1/2))^(1/2)+arctan(1/2*(2*c+(b+(-4*a*c+b^2)^
(1/2))*tan(1/2*x))*2^(1/2)/(b^2-2*c*(a+c)+b*(-4*a*c+b^2)^(1/2))^(1/2))*2^(
1/2)*(e+(b*e-2*c*d)/(-4*a*c+b^2)^(1/2))/(b^2-2*c*(a+c)+b*(-4*a*c+b^2)^(1/2
))^(1/2)
```

3.503.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.39 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.18

$$\int \frac{d+e\sin(x)}{a+b\sin(x)+c\sin^2(x)}\,dx = \frac{\frac{\left(-2icd+\left(ib+\sqrt{-b^2+4ac}\right)e\right)\arctan\left(\frac{2c+\left(b-i\sqrt{-b^2+4ac}\right)\tan\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{b^2-2c(a+c)-ib\sqrt{-b^2+4ac}}}\right)}{\sqrt{b^2-2c(a+c)-ib\sqrt{-b^2+4ac}}}+\frac{\left(2icd+\left(-ib+\sqrt{-b^2+4ac}\right)e\right)\arctan\left(\frac{2c+\left(b+i\sqrt{-b^2+4ac}\right)\tan\left(\frac{x}{2}\right)}{\sqrt{2}\sqrt{b^2-2c(a+c)+ib\sqrt{-b^2+4ac}}}\right)}{\sqrt{b^2-2c(a+c)+ib\sqrt{-b^2+4ac}}}}{\sqrt{-\frac{b^2}{2}+2ac}}$$

input
```
Integrate[(d + e*Sin[x])/(a + b*Sin[x] + c*Sin[x]^2),x]
```

output
```
((((-2*I)*c*d + (I*b + Sqrt[-b^2 + 4*a*c])*e)*ArcTan[(2*c + (b - I*Sqrt[-b
^2 + 4*a*c])*Tan[x/2])/(Sqrt[2]*Sqrt[b^2 - 2*c*(a + c) - I*b*Sqrt[-b^2 + 4
*a*c]])])/Sqrt[b^2 - 2*c*(a + c) - I*b*Sqrt[-b^2 + 4*a*c]] + (((2*I)*c*d +
 ((-I)*b + Sqrt[-b^2 + 4*a*c])*e)*ArcTan[(2*c + (b + I*Sqrt[-b^2 + 4*a*c])
*Tan[x/2])/(Sqrt[2]*Sqrt[b^2 - 2*c*(a + c) + I*b*Sqrt[-b^2 + 4*a*c]])])/Sq
rt[b^2 - 2*c*(a + c) + I*b*Sqrt[-b^2 + 4*a*c]])/Sqrt[-1/2*b^2 + 2*a*c]
```

3.503.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3773, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+e\sin(x)}{a+b\sin(x)+c\sin^2(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{d+e\sin(x)}{a+b\sin(x)+c\sin(x)^2}dx$$

$$\Big\downarrow \text{3773}$$

$$\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\int \frac{1}{b+2c\sin(x)-\sqrt{b^2-4ac}}dx+\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{b+2c\sin(x)+\sqrt{b^2-4ac}}dx$$

$$\Big\downarrow \text{3042}$$

$$\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\int \frac{1}{b+2c\sin(x)-\sqrt{b^2-4ac}}dx+\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{b+2c\sin(x)+\sqrt{b^2-4ac}}dx$$

$$\Big\downarrow \text{3139}$$

$$2\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\int \frac{1}{\left(b-\sqrt{b^2-4ac}\right)\tan^2\left(\frac{x}{2}\right)+4c\tan\left(\frac{x}{2}\right)+b-\sqrt{b^2-4ac}}d\tan\left(\frac{x}{2}\right)+2\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\left(b+\sqrt{b^2-4ac}\right)\tan^2\left(\frac{x}{2}\right)+4c\tan\left(\frac{x}{2}\right)+b+\sqrt{b^2-4ac}}d\tan\left(\frac{x}{2}\right)$$

$$\Big\downarrow \text{1083}$$

$$-4\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{4\left(4c^2-\left(b+\sqrt{b^2-4ac}\right)^2\right)-\left(4c+2\left(b+\sqrt{b^2-4ac}\right)\tan\left(\frac{x}{2}\right)\right)^2}d\Big(4c+2\Big(b+\sqrt{b^2-$$

$$4\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\int \frac{1}{-\left(4c+2\left(b-\sqrt{b^2-4ac}\right)\tan\left(\frac{x}{2}\right)\right)^2-8\left(b^2-\sqrt{b^2-4ac}b-2c(a+c)\right)}d\Big(4c+2\Big(b-\sqrt{b$$

$$\Big\downarrow \text{217}$$

$$\frac{\sqrt{2}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\arctan\left(\frac{2\tan\left(\frac{x}{2}\right)\left(b-\sqrt{b^2-4ac}\right)+4c}{2\sqrt{2}\sqrt{-b\sqrt{b^2-4ac}-2c(a+c)+b^2}}\right)}{\sqrt{-b\sqrt{b^2-4ac}-2c(a+c)+b^2}}+\frac{\sqrt{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{2\tan\left(\frac{x}{2}\right)\left(\sqrt{b^2-4ac}+b\right)+4c}{2\sqrt{2}\sqrt{b\sqrt{b^2-4ac}-2c(a+c)+b^2}}\right)}{\sqrt{b\sqrt{b^2-4ac}-2c(a+c)+b^2}}$$

input
```
Int[(d + e*Sin[x])/(a + b*Sin[x] + c*Sin[x]^2),x]
```

3.503. $\int \frac{d+e\sin(x)}{a+b\sin(x)+c\sin^2(x)}\,dx$

output
```
(Sqrt[2]*(e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(4*c + 2*(b - Sqrt[b
^2 - 4*a*c])*Tan[x/2])/(2*Sqrt[2]*Sqrt[b^2 - 2*c*(a + c) - b*Sqrt[b^2 - 4*
a*c]])])/Sqrt[b^2 - 2*c*(a + c) - b*Sqrt[b^2 - 4*a*c]] + (Sqrt[2]*(e - (2*
c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(4*c + 2*(b + Sqrt[b^2 - 4*a*c])*Tan[
x/2])/(2*Sqrt[2]*Sqrt[b^2 - 2*c*(a + c) + b*Sqrt[b^2 - 4*a*c]])])/Sqrt[b^2
 - 2*c*(a + c) + b*Sqrt[b^2 - 4*a*c]]
```

3.503.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3773
```
Int[((A_) + (B_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + (b_.)*sin[(d_.) + (e_.)
*(x_)] + (c_.)*sin[(d_.) + (e_.)*(x_)]^2), x_Symbol] :> Module[{q = Rt[b^2
- 4*a*c, 2]}, Simp[(B + (b*B - 2*A*c)/q)   Int[1/(b + q + 2*c*Sin[d + e*x])
, x], x] + Simp[(B - (b*B - 2*A*c)/q)   Int[1/(b - q + 2*c*Sin[d + e*x]), x
], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0]
```

3.503. $\int \frac{d+e\sin(x)}{a+b\sin(x)+c\sin^2(x)}\,dx$

3.503.4 Maple [A] (verified)

Time = 5.27 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.09

method	result
default	$2a\left(-\frac{\sqrt{-4ac+b^2}\left(\sqrt{-4ac+b^2}\,d-2ae+bd\right)\arctan\left(\frac{2a\tan\left(\frac{x}{2}\right)+b+\sqrt{-4ac+b^2}}{\sqrt{4ac-2b^2-2b\sqrt{-4ac+b^2}+4a^2}}\right)}{(4ac-b^2)a\sqrt{4ac-2b^2-2b\sqrt{-4ac+b^2}+4a^2}}+\frac{\left(\sqrt{-4ac+b^2}\,d+2ae-bd\right)\sqrt{-4ac+b^2}\,\text{arcta}}{(4ac-b^2)a\sqrt{4ac-2b^2+2b}}\right.$
risch	Expression too large to display

input
```
int((d+e*sin(x))/(a+b*sin(x)+c*sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
2*a*(-(-4*a*c+b^2)^(1/2)*((-4*a*c+b^2)^(1/2)*d-2*a*e+b*d)/(4*a*c-b^2)/a/(4
*a*c-2*b^2-2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2)*arctan((2*a*tan(1/2*x)+b+(-
4*a*c+b^2)^(1/2))/(4*a*c-2*b^2-2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2))+((-4*a
*c+b^2)^(1/2)*d+2*a*e-b*d)*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)/a/(4*a*c-2*b^2+2
*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2)*arctan((-2*a*tan(1/2*x)+(-4*a*c+b^2)^(1
/2)-b)/(4*a*c-2*b^2+2*b*(-4*a*c+b^2)^(1/2)+4*a^2)^(1/2)))
```

3.503.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6695 vs. 2(208) = 416.

Time = 6.01 (sec) , antiderivative size = 6695, normalized size of antiderivative = 27.67

$$\int \frac{d+e\sin(x)}{a+b\sin(x)+c\sin^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate((d+e*sin(x))/(a+b*sin(x)+c*sin(x)^2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.503.6 Sympy [F(-1)]

Timed out.

$$\int \frac{d+e\sin(x)}{a+b\sin(x)+c\sin^2(x)}\,dx = \text{Timed out}$$

input
```
integrate((d+e*sin(x))/(a+b*sin(x)+c*sin(x)**2),x)
```

output
```
Timed out
```

3.503.7 Maxima [F]

$$\int \frac{d+e\sin(x)}{a+b\sin(x)+c\sin^2(x)}\,dx = \int \frac{e\sin(x)+d}{c\sin(x)^2+b\sin(x)+a}\,dx$$

input
```
integrate((d+e*sin(x))/(a+b*sin(x)+c*sin(x)^2),x, algorithm="maxima")
```

output
```
integrate((e*sin(x) + d)/(c*sin(x)^2 + b*sin(x) + a), x)
```

3.503.8 Giac [F(-1)]

Timed out.

$$\int \frac{d+e\sin(x)}{a+b\sin(x)+c\sin^2(x)}\,dx = \text{Timed out}$$

input
```
integrate((d+e*sin(x))/(a+b*sin(x)+c*sin(x)^2),x, algorithm="giac")
```

output
```
Timed out
```

3.503.9 Mupad [B] (verification not implemented)

Time = 42.93 (sec) , antiderivative size = 10465, normalized size of antiderivative = 43.24

$$\int \frac{d+e\sin(x)}{a+b\sin(x)+c\sin^2(x)}\,dx = \text{Too large to display}$$

input
```
int((d + e*sin(x))/(a + c*sin(x)^2 + b*sin(x)),x)
```

output
```
atan(((-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2)
- 8*a^3*c*e^2 + b*e^2*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b^2*e^2 + 8*a^2*c^2
*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e - 2*a*d*e*(-(4*a*c - b^
2)^3)^(1/2) + 2*b^3*c*d*e - 2*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d
^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16
*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c -
 32*a^2*b^2*c^2 + 10*a*b^4*c)))^(1/2)*((-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2
+ b*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*c*e^2 + b*e^2*(-(4*a*c - b^2)^3)^
(1/2) + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*
a*b^3*d*e - 2*a*d*e*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c*d*e - 2*c*d*e*(-(4*
a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^
2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*
c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c)))^(1/2)*((-
(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*
c*e^2 + b*e^2*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b^2*e^2 + 8*a^2*c^2*d^2 - 8
*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e - 2*a*d*e*(-(4*a*c - b^2)^3)^(1
/2) + 2*b^3*c*d*e - 2*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d^2 + 6*a
*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 + 16*a^2*c^4
 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*c - 32*a^2*
b^2*c^2 + 10*a*b^4*c)))^(1/2)*(tan(x/2)*(96*a*b^4 + 256*a^4*c - 64*a^3*...
```

3.504 $\int (a+b\sin(d+ex)) \left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2} dx$

3.504.1 Optimal result

Integrand size = 41, antiderivative size = 331

$$\begin{aligned}
&\int (a+b\sin(d+ex)) \left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2} dx = \\
&-\frac{b\cos(d+ex)\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}}{4e} \\
&-\frac{(4a^4+28a^2b^2+3b^4)\cos(d+ex)\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}}{6e(b+a\sin(d+ex))^3} \\
&-\frac{(4a^2+3b^2)\cos(d+ex)\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}}{12e(b+a\sin(d+ex))} \\
&+\frac{5a^4b(3a^2+4b^2)x\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}}{8\left(ab+a^2\sin(d+ex)\right)^3} \\
&-\frac{a^4b(29a^2+6b^2)\cos(d+ex)\sin(d+ex)\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}}{24e\left(ab+a^2\sin(d+ex)\right)^3}
\end{aligned}$$

output
```
-1/4*b*cos(e*x+d)*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2)/e-1/6*(4*a
^4+28*a^2*b^2+3*b^4)*cos(e*x+d)*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3
/2)/e/(b+a*sin(e*x+d))^3-1/12*(4*a^2+3*b^2)*cos(e*x+d)*(b^2+2*a*b*sin(e*x+
d)+a^2*sin(e*x+d)^2)^(3/2)/e/(b+a*sin(e*x+d))+5/8*a^4*b*(3*a^2+4*b^2)*x*(b
^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2)/(a*b+a^2*sin(e*x+d))^3-1/24*a^
4*b*(29*a^2+6*b^2)*cos(e*x+d)*sin(e*x+d)*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x
+d)^2)^(3/2)/e/(a*b+a^2*sin(e*x+d))^3
```

3.504.2 Mathematica [A] (verified)

Time = 2.55 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.42

$$\int (a+b\sin(d+ex))\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}\,dx=\frac{\sqrt{(b+a\sin(d+ex))^2}(-24(3a^4+21a^2b^2+4b^4)\cos(d+ex)+8a(a^3+3ab^2)\cos(3(d+ex))}{96e(b+a\sin(d+}$$

input
```
Integrate[(a + b*Sin[d + e*x])*(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x
]^2)^(3/2),x]
```

output
```
(Sqrt[(b + a*Sin[d + e*x])^2]*(-24*(3*a^4 + 21*a^2*b^2 + 4*b^4)*Cos[d + e*
x] + 8*a*(a^3 + 3*a*b^2)*Cos[3*(d + e*x)] + 3*a*b*(20*(3*a^2 + 4*b^2)*(d +
 e*x) - 8*(4*a^2 + 3*b^2)*Sin[2*(d + e*x)] + a^2*Sin[4*(d + e*x)])))/(96*e
*(b + a*Sin[d + e*x]))
```

3.504.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.65, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.220, Rules used = {3042, 3771, 27, 3042, 3232, 3042, 3232, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sin(d+ex))\left(a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\sin(d+ex))\left(a^2\sin(d+ex)^2+2ab\sin(d+ex)+b^2\right)^{3/2}dx$$

$$\Big\downarrow \text{3771}$$

$$\frac{\left(a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2\right)^{3/2}\int 8\left(\sin(d+ex)a^2+ba\right)^3(a+b\sin(d+ex))dx}{8\left(a^2\sin(d+ex)+ab\right)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2\right)^{3/2}\int \left(\sin(d+ex)a^2+ba\right)^3(a+b\sin(d+ex))dx}{\left(a^2\sin(d+ex)+ab\right)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2\right)^{3/2}\int\left(\sin(d+ex)a^2+ba\right)^3(a+b\sin(d+ex))dx}{\left(a^2\sin(d+ex)+ab\right)^3}$$

$$\Big\downarrow \text{3232}$$

$$\frac{\left(a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2\right)^{3/2}\left(\frac{1}{4}\int\left(\sin(d+ex)a^2+ba\right)^2\left(7ba^2+\left(4a^2+3b^2\right)\sin(d+ex)a\right)dx-\frac{b\,\mathrm{co}}{}\right.}{\left(a^2\sin(d+ex)+ab\right)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2\right)^{3/2}\left(\frac{1}{4}\int\left(\sin(d+ex)a^2+ba\right)^2\left(7ba^2+\left(4a^2+3b^2\right)\sin(d+ex)a\right)dx-\frac{b\,\mathrm{co}}{}\right.}{\left(a^2\sin(d+ex)+ab\right)^3}$$

$$\Big\downarrow \text{3232}$$

$$\frac{\left(a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2\right)^{3/2}\left(\frac{1}{4}\left(\frac{1}{3}\int\left(\sin(d+ex)a^2+ba\right)\left(\left(8a^2+27b^2\right)a^3+b\left(29a^2+6b^2\right)\sin(d+\right.\right.\right.}{\left(a^2\sin(d+ex)+ab\right)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2\right)^{3/2}\left(\frac{1}{4}\left(\frac{1}{3}\int\left(\sin(d+ex)a^2+ba\right)\left(\left(8a^2+27b^2\right)a^3+b\left(29a^2+6b^2\right)\sin(d+\right.\right.\right.}{\left(a^2\sin(d+ex)+ab\right)^3}$$

$$\Big\downarrow \text{3213}$$

$$\frac{\left(a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2\right)^{3/2}\left(\frac{1}{4}\left(\frac{1}{3}\left(-\frac{a^4b\left(29a^2+6b^2\right)\sin(d+ex)\cos(d+ex)}{2e}+\frac{15}{2}a^4bx\left(3a^2+4b^2\right)-\frac{2a^3\left(4a^4\right.}{}\right.\right.\right.}{\left(a^2\sin(d+ex)+ab\right)^3}$$

input
```
Int[(a + b*Sin[d + e*x])*(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2)^(
3/2),x]
```

3.504. $\int(a+b\sin(d+ex))\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}\,dx$

output
```
((b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2)^(3/2)*(-1/4*(b*Cos[d + e*
x]*(a*b + a^2*Sin[d + e*x])^3)/e + (-1/3*(a*(4*a^2 + 3*b^2)*Cos[d + e*x]*(
a*b + a^2*Sin[d + e*x])^2)/e + ((15*a^4*b*(3*a^2 + 4*b^2)*x)/2 - (2*a^3*(4
*a^4 + 28*a^2*b^2 + 3*b^4)*Cos[d + e*x])/e - (a^4*b*(29*a^2 + 6*b^2)*Cos[d
 + e*x]*Sin[d + e*x])/(2*e))/3)/4))/(a*b + a^2*Sin[d + e*x])^3
```

3.504.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

rule 3771
```
Int[((A_) + (B_.)*sin[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sin[(d_.) + (e_.)*
(x_)] + (c_.)*sin[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Simp[(a + b*Sin
[d + e*x] + c*Sin[d + e*x]^2)^n/(b + 2*c*Sin[d + e*x])^(2*n)   Int[(A + B*S
in[d + e*x])*(b + 2*c*Sin[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e,
A, B}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[n]
```

3.504. $\int (a+b\sin(d+ex))\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}\,dx$

3.504.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.70 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.58

method	result
default	$\frac{\operatorname{csgn}(b+a\sin(ex+d))\left(a\,b^3(ex+d)-3\cos(ex+d)a^2b^2+3a^3b\left(-\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)-\frac{a^4\left(2+\sin(ex+d)^2\right)\cos(ex+d)}{3}-\cos(ex+d)\right.}{}$
parts	$\frac{a\left(2\cos(ex+d)^3a^3-9\cos(ex+d)\sin(ex+d)a^2b-6a^3\cos(ex+d)-18\cos(ex+d)a\,b^2+9a^2b(ex+d)+6(ex+d)b^3-4a^3-18a\,b^2\right)\operatorname{csgn}(b+a}{6e}$

input
```
int((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2),x,metho
d=_RETURNVERBOSE)
```

output
```
csgn(b+a*sin(e*x+d))/e*(a*b^3*(e*x+d)-3*cos(e*x+d)*a^2*b^2+3*a^3*b*(-1/2*c
os(e*x+d)*sin(e*x+d)+1/2*e*x+1/2*d)-1/3*a^4*(2+sin(e*x+d)^2)*cos(e*x+d)-co
s(e*x+d)*b^4+3*a*b^3*(-1/2*cos(e*x+d)*sin(e*x+d)+1/2*e*x+1/2*d)-a^2*b^2*(2
+sin(e*x+d)^2)*cos(e*x+d)+a^3*b*(-1/4*(sin(e*x+d)^3+3/2*sin(e*x+d))*cos(e*
x+d)+3/8*e*x+3/8*d))
```

3.504.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.34

$$\int (a+b\sin(d+ex))\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}\,dx=\frac{8\left(a^4+3\,a^2b^2\right)\cos\left(ex+d\right)^3+15\left(3\,a^3b+4\,ab^3\right)ex-24\left(a^4+6\,a^2b^2+b^4\right)\cos\left(ex+d\right)+3\left(2\right.}{24\,e}$$

input
```
integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2),x
, algorithm="fricas")
```

output
```
1/24*(8*(a^4 + 3*a^2*b^2)*cos(e*x + d)^3 + 15*(3*a^3*b + 4*a*b^3)*e*x - 24
*(a^4 + 6*a^2*b^2 + b^4)*cos(e*x + d) + 3*(2*a^3*b*cos(e*x + d)^3 - (17*a^
3*b + 12*a*b^3)*cos(e*x + d))*sin(e*x + d))/e
```

3.504.6 Sympy [F(-1)]

Timed out.

$$\int (a+b\sin(d+ex))\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate((a+b*sin(e*x+d))*(b**2+2*a*b*sin(e*x+d)+a**2*sin(e*x+d)**2)**(3/
2),x)
```

output
```
Timed out
```

3.504.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 556, normalized size of antiderivative = 1.68

$$\int (a+b\sin(d+ex))\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}\,dx = \frac{4\left(3\left(3a^2b+2b^3\right)\arctan\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}\right)-\frac{4a^3+18ab^2+\frac{9a^2b\sin(ex+d)}{\cos(ex+d)+1}+\frac{18ab^2\sin(ex+d)^4}{(\cos(ex+d)+1)^4}-\frac{9a^2b\sin(ex+d)^5}{(\cos(ex+d)+1)^5}+\frac{12(}{}}{\frac{3\sin(ex+d)^2}{(\cos(ex+d)+1)^2}+\frac{3\sin(ex+d)^4}{(\cos(ex+d)+1)^4}+\frac{\sin(ex+d)^6}{(\cos(ex+d)+1)^6}+}\right.}{}$$

input
```
integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2),x
, algorithm="maxima")
```

output
```
1/12*(4*(3*(3*a^2*b + 2*b^3)*arctan(sin(e*x + d)/(cos(e*x + d) + 1)) - (4*
a^3 + 18*a*b^2 + 9*a^2*b*sin(e*x + d)/(cos(e*x + d) + 1) + 18*a*b^2*sin(e*
x + d)^4/(cos(e*x + d) + 1)^4 - 9*a^2*b*sin(e*x + d)^5/(cos(e*x + d) + 1)^
5 + 12*(a^3 + 3*a*b^2)*sin(e*x + d)^2/(cos(e*x + d) + 1)^2)/(3*sin(e*x + d
)^2/(cos(e*x + d) + 1)^2 + 3*sin(e*x + d)^4/(cos(e*x + d) + 1)^4 + sin(e*x
 + d)^6/(cos(e*x + d) + 1)^6 + 1))*a + 3*(3*(a^3 + 4*a*b^2)*arctan(sin(e*x
 + d)/(cos(e*x + d) + 1)) - (16*a^2*b + 8*b^3 + 8*b^3*sin(e*x + d)^6/(cos(
e*x + d) + 1)^6 + 3*(a^3 + 4*a*b^2)*sin(e*x + d)/(cos(e*x + d) + 1) + 8*(8
*a^2*b + 3*b^3)*sin(e*x + d)^2/(cos(e*x + d) + 1)^2 + (11*a^3 + 12*a*b^2)*
sin(e*x + d)^3/(cos(e*x + d) + 1)^3 + 24*(2*a^2*b + b^3)*sin(e*x + d)^4/(c
os(e*x + d) + 1)^4 - (11*a^3 + 12*a*b^2)*sin(e*x + d)^5/(cos(e*x + d) + 1)
^5 - 3*(a^3 + 4*a*b^2)*sin(e*x + d)^7/(cos(e*x + d) + 1)^7)/(4*sin(e*x + d
)^2/(cos(e*x + d) + 1)^2 + 6*sin(e*x + d)^4/(cos(e*x + d) + 1)^4 + 4*sin(e
*x + d)^6/(cos(e*x + d) + 1)^6 + sin(e*x + d)^8/(cos(e*x + d) + 1)^8 + 1))
*b)/e
```

3.504.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.69

$$\int (a+b\sin(d+ex))\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}\,dx=\frac{a^3b\text{sgn}(a\sin(ex+d)+b)\sin(4ex+4d)}{32e}+\frac{5}{8}\left(3a^3b\text{sgn}(a\sin(ex+d)+b)+4ab^3\text{sgn}(a\sin(ex+d)+b)\right)x+\frac{\left(a^4\text{sgn}(a\sin(ex+d)+b)+3a^2b^2\text{sgn}(a\sin(ex+d)+b)\right)\cos(3ex+3d)}{12e}-\frac{\left(3a^4\text{sgn}(a\sin(ex+d)+b)+21a^2b^2\text{sgn}(a\sin(ex+d)+b)+4b^4\text{sgn}(a\sin(ex+d)+b)\right)\cos(ex+d)}{4e}-\frac{\left(4a^3b\text{sgn}(a\sin(ex+d)+b)+3ab^3\text{sgn}(a\sin(ex+d)+b)\right)\sin(2ex+2d)}{4e}$$

input
```
integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2),x
, algorithm="giac")
```

output
```
1/32*a^3*b*sgn(a*sin(e*x + d) + b)*sin(4*e*x + 4*d)/e + 5/8*(3*a^3*b*sgn(a
*sin(e*x + d) + b) + 4*a*b^3*sgn(a*sin(e*x + d) + b))*x + 1/12*(a^4*sgn(a*
sin(e*x + d) + b) + 3*a^2*b^2*sgn(a*sin(e*x + d) + b))*cos(3*e*x + 3*d)/e
- 1/4*(3*a^4*sgn(a*sin(e*x + d) + b) + 21*a^2*b^2*sgn(a*sin(e*x + d) + b)
+ 4*b^4*sgn(a*sin(e*x + d) + b))*cos(e*x + d)/e - 1/4*(4*a^3*b*sgn(a*sin(e
*x + d) + b) + 3*a*b^3*sgn(a*sin(e*x + d) + b))*sin(2*e*x + 2*d)/e
```

3.504.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\sin(d+ex))\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}\,dx=\int (a+b\sin(d+e\,x))\,\left(a^2\sin(d+e\,x)^2+2\,a\,b\,\sin(d+e\,x)+b^2\right)^{3/2}\,dx$$

input
```
int((a + b*sin(d + e*x))*(b^2 + a^2*sin(d + e*x)^2 + 2*a*b*sin(d + e*x))^(
3/2),x)
```

output
```
int((a + b*sin(d + e*x))*(b^2 + a^2*sin(d + e*x)^2 + 2*a*b*sin(d + e*x))^(
3/2), x)
```

3.505 $\int (a+b\sin(d+ex))\sqrt{b^2+2ab\sin(d+ex)+a^2\sin^2(d$

3.505.1 Optimal result

Integrand size = 41, antiderivative size = 185

$$\begin{aligned}
&\int (a+b\sin(d+ex))\sqrt{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}\,dx \\
&= -\frac{(a^2+b^2)\cos(d+ex)\sqrt{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}}{e(b+a\sin(d+ex))} \\
&\quad+\frac{3a^2bx\sqrt{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}}{2\left(ab+a^2\sin(d+ex)\right)} \\
&\quad-\frac{a^2b\cos(d+ex)\sin(d+ex)\sqrt{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}}{2e\left(ab+a^2\sin(d+ex)\right)}
\end{aligned}$$

output
```
-(a^2+b^2)*cos(e*x+d)*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(1/2)/e/(b+a
*sin(e*x+d))+3/2*a^2*b*x*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(1/2)/(a*
b+a^2*sin(e*x+d))-1/2*a^2*b*cos(e*x+d)*sin(e*x+d)*(b^2+2*a*b*sin(e*x+d)+a^
2*sin(e*x+d)^2)^(1/2)/e/(a*b+a^2*sin(e*x+d))
```

3.505.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.38

$$\int (a+b\sin(d+ex))\sqrt{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}\,dx$$
$$=-\frac{\sqrt{(b+a\sin(d+ex))^2}(4(a^2+b^2)\cos(d+ex)+ab(-6(d+ex)+\sin(2(d+ex))))}{4e(b+a\sin(d+ex))}$$

input
```
Integrate[(a + b*Sin[d + e*x])*Sqrt[b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d +
 e*x]^2],x]
```

output
```
-1/4*(Sqrt[(b + a*Sin[d + e*x])^2]*(4*(a^2 + b^2)*Cos[d + e*x] + a*b*(-6*(
d + e*x) + Sin[2*(d + e*x)])))/(e*(b + a*Sin[d + e*x]))
```

3.505.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.54, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {3042, 3771, 27, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sin(d+ex))\sqrt{a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int (a+b\sin(d+ex))\sqrt{a^2\sin(d+ex)^2+2ab\sin(d+ex)+b^2}dx$$
$$\Big\downarrow \text{ 3771}$$
$$\frac{\sqrt{a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2}\int 2\left(\sin(d+ex)a^2+ba\right)(a+b\sin(d+ex))dx}{2\left(a^2\sin(d+ex)+ab\right)}$$
$$\Big\downarrow \text{ 27}$$
$$\frac{\sqrt{a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2}\int \left(\sin(d+ex)a^2+ba\right)(a+b\sin(d+ex))dx}{a^2\sin(d+ex)+ab}$$
$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2}\int\left(\sin(d+ex)a^2+ba\right)(a+b\sin(d+ex))dx}{a^2\sin(d+ex)+ab}$$

$$\Big\downarrow \text{3213}$$

$$\frac{\sqrt{a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2}\left(-\frac{a(a^2+b^2)\cos(d+ex)}{e}-\frac{a^2b\sin(d+ex)\cos(d+ex)}{2e}+\frac{3}{2}a^2bx\right)}{a^2\sin(d+ex)+ab}$$

input
```
Int[(a + b*Sin[d + e*x])*Sqrt[b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^
2],x]
```

output
```
(((3*a^2*b*x)/2 - (a*(a^2 + b^2)*Cos[d + e*x])/e - (a^2*b*Cos[d + e*x]*Sin
[d + e*x])/(2*e))*Sqrt[b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2])/(a*
b + a^2*Sin[d + e*x])
```

3.505.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3771
```
Int[((A_) + (B_.)*sin[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sin[(d_.) + (e_.)*
(x_)] + (c_.)*sin[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Simp[(a + b*Sin
[d + e*x] + c*Sin[d + e*x]^2)^n/(b + 2*c*Sin[d + e*x])^(2*n)   Int[(A + B*S
in[d + e*x])*(b + 2*c*Sin[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e,
A, B}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[n]
```

3.505.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.81 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.39

method	result
default	$\frac{\operatorname{csgn}(b+a\sin(ex+d))\left(ab\left(-\frac{\cos(ex+d)\sin(ex+d)}{2}+\frac{ex}{2}+\frac{d}{2}\right)-a^2\cos(ex+d)-b^2\cos(ex+d)+ab(ex+d)\right)}{e}$
parts	$-\frac{a(a\cos(ex+d)-(ex+d)b+a)\operatorname{csgn}(b+a\sin(ex+d))}{e}-\frac{b\operatorname{csgn}(b+a\sin(ex+d))(\cos(ex+d)\sin(ex+d)a+2b\cos(ex+d)-(ex+d)a+2b}{2e}$

input
```
int((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(1/2),x,metho
d=_RETURNVERBOSE)
```

output
```
csgn(b+a*sin(e*x+d))/e*(a*b*(-1/2*cos(e*x+d)*sin(e*x+d)+1/2*e*x+1/2*d)-a^2
*cos(e*x+d)-b^2*cos(e*x+d)+a*b*(e*x+d))
```

3.505.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.23

$$\int (a+b\sin(d+ex))\sqrt{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}\,dx = \frac{3\,abex-ab\cos(ex+d)\sin(ex+d)-2\,(a^2+b^2)\cos(ex+d)}{2\,e}$$

input
```
integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(1/2),x
, algorithm="fricas")
```

output
```
1/2*(3*a*b*e*x - a*b*cos(e*x + d)*sin(e*x + d) - 2*(a^2 + b^2)*cos(e*x + d
))/e
```

3.505.6 Sympy [F]

$$\int (a+b\sin(d+ex))\sqrt{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}\,dx$$
$$=\int (a+b\sin{(d+ex)})\sqrt{(a\sin{(d+ex)}+b)^2}\,dx$$

input
```
integrate((a+b*sin(e*x+d))*(b**2+2*a*b*sin(e*x+d)+a**2*sin(e*x+d)**2)**(1/
2),x)
```

output
```
Integral((a + b*sin(d + e*x))*sqrt((a*sin(d + e*x) + b)**2), x)
```

3.505.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.01

$$\int (a+b\sin(d+ex))\sqrt{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}\,dx$$
$$=\frac{2\left(b\arctan\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}\right)-\frac{a}{\frac{\sin(ex+d)^2}{(\cos(ex+d)+1)^2}+1}\right)a+\left(a\arctan\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}\right)-\frac{2\,b+\frac{a\sin(ex+d)}{\cos(ex+d)+1}+\frac{2\,b\sin(ex+d)^2}{(\cos(ex+d)+1)^2}-\frac{a\sin(}{(\cos(ex}}{\frac{2\,\sin(ex+d)^2}{(\cos(ex+d)+1)^2}+\frac{\sin(ex+d)^4}{(\cos(ex+d)+1)^4}+1}}{e}$$

input
```
integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(1/2),x
, algorithm="maxima")
```

output
```
(2*(b*arctan(sin(e*x + d)/(cos(e*x + d) + 1)) - a/(sin(e*x + d)^2/(cos(e*x
 + d) + 1)^2 + 1))*a + (a*arctan(sin(e*x + d)/(cos(e*x + d) + 1)) - (2*b +
 a*sin(e*x + d)/(cos(e*x + d) + 1) + 2*b*sin(e*x + d)^2/(cos(e*x + d) + 1)
^2 - a*sin(e*x + d)^3/(cos(e*x + d) + 1)^3)/(2*sin(e*x + d)^2/(cos(e*x + d
) + 1)^2 + sin(e*x + d)^4/(cos(e*x + d) + 1)^4 + 1))*b)/e
```

3.505.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.51

$$\int (a+b\sin(d+ex))\sqrt{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}\,dx$$
$$=\frac{3}{2}\,abx\mathrm{sgn}(a\sin{(ex+d)}+b)-\frac{a^2\cos{(ex+d)}\,\mathrm{sgn}(a\sin{(ex+d)}+b)}{e}$$
$$-\frac{b^2\cos{(ex+d)}\,\mathrm{sgn}(a\sin{(ex+d)}+b)}{e}-\frac{ab\mathrm{sgn}(a\sin{(ex+d)}+b)\sin{(2\,ex+2\,d)}}{4\,e}$$

input
```
integrate((a+b*sin(e*x+d))*(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(1/2),x
, algorithm="giac")
```

output
```
3/2*a*b*x*sgn(a*sin(e*x + d) + b) - a^2*cos(e*x + d)*sgn(a*sin(e*x + d) +
b)/e - b^2*cos(e*x + d)*sgn(a*sin(e*x + d) + b)/e - 1/4*a*b*sgn(a*sin(e*x
+ d) + b)*sin(2*e*x + 2*d)/e
```

3.505.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\sin(d+ex))\sqrt{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}\,dx$$
$$=\int (a+b\,\sin{(d+e\,x)})\,\sqrt{a^2\sin{(d+e\,x)}^2+2\,a\,b\,\sin{(d+e\,x)}+b^2}\,dx$$

input
```
int((a + b*sin(d + e*x))*(b^2 + a^2*sin(d + e*x)^2 + 2*a*b*sin(d + e*x))^(
1/2),x)
```

output
```
int((a + b*sin(d + e*x))*(b^2 + a^2*sin(d + e*x)^2 + 2*a*b*sin(d + e*x))^(
1/2), x)
```

3.506 $\int \frac{a+b\sin(d+ex)}{\sqrt{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}}\,dx$

3.506.1 Optimal result

Integrand size = 41, antiderivative size = 137

$$\int \frac{a+b\sin(d+ex)}{\sqrt{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}}\,dx = \frac{bx(b+a\sin(d+ex))}{a\sqrt{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}} - \frac{2\sqrt{a^2-b^2}\operatorname{arctanh}\left(\frac{a+b\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-b^2}}\right)(b+a\sin(d+ex))}{ae\sqrt{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}}$$

output
```
b*x*(b+a*sin(e*x+d))/a/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(1/2)-2*arc
tanh((a+b*tan(1/2*e*x+1/2*d))/(a^2-b^2)^(1/2))*(b+a*sin(e*x+d))*(a^2-b^2)^
(1/2)/a/e/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(1/2)
```

3.506.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.62

$$\int \frac{a+b\sin(d+ex)}{\sqrt{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}}\,dx = \frac{\left(b(d+ex)-2\sqrt{-a^2+b^2}\arctan\left(\frac{a+b\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{-a^2+b^2}}\right)\right)(b+a\sin(d+ex))}{ae\sqrt{(b+a\sin(d+ex))^2}}$$

input
```
Integrate[(a + b*Sin[d + e*x])/Sqrt[b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d +
 e*x]^2],x]
```

output
```
((b*(d + e*x) - 2*Sqrt[-a^2 + b^2]*ArcTan[(a + b*Tan[(d + e*x)/2])/Sqrt[-a
^2 + b^2]])*(b + a*Sin[d + e*x]))/(a*e*Sqrt[(b + a*Sin[d + e*x])^2])
```

3.506.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.83, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.220, Rules used = {3042, 3771, 27, 3042, 3214, 3042, 3139, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sin(d+ex)}{\sqrt{a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\sin(d+ex)}{\sqrt{a^2\sin(d+ex)^2+2ab\sin(d+ex)+b^2}}dx$$

$$\Big\downarrow \text{3771}$$

$$\frac{2\left(a^2\sin(d+ex)+ab\right)\int \frac{a+b\sin(d+ex)}{2(\sin(d+ex)a^2+ba)}dx}{\sqrt{a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(a^2\sin(d+ex)+ab\right)\int \frac{a+b\sin(d+ex)}{\sin(d+ex)a^2+ba}dx}{\sqrt{a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(a^2\sin(d+ex)+ab\right)\int \frac{a+b\sin(d+ex)}{\sin(d+ex)a^2+ba}dx}{\sqrt{a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2}}$$

$$\Big\downarrow \text{3214}$$

$$\frac{\left(a^2\sin(d+ex)+ab\right)\left(\frac{(a^2-b^2)\int \frac{1}{\sin(d+ex)a^2+ba}dx}{a}+\frac{bx}{a^2}\right)}{\sqrt{a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(a^2\sin(d+ex)+ab\right)\left(\frac{(a^2-b^2)\int\frac{1}{\sin(d+ex)a^2+ba}dx}{a}+\frac{bx}{a^2}\right)}{\sqrt{a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2}}$$

$$\Big\downarrow \text{3139}$$

$$\frac{\left(a^2\sin(d+ex)+ab\right)\left(\frac{2(a^2-b^2)\int\frac{1}{2\tan\left(\frac{1}{2}(d+ex)\right)a^2+b\tan^2\left(\frac{1}{2}(d+ex)\right)a+ba}d\tan\left(\frac{1}{2}(d+ex)\right)}{ae}+\frac{bx}{a^2}\right)}{\sqrt{a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2}}$$

$$\Big\downarrow \text{1083}$$

$$\frac{\left(a^2\sin(d+ex)+ab\right)\left(\frac{bx}{a^2}-\frac{4(a^2-b^2)\int\frac{1}{4a^2(a^2-b^2)-\left(2a^2+2b\tan\left(\frac{1}{2}(d+ex)\right)a\right)^2}d\left(2a^2+2b\tan\left(\frac{1}{2}(d+ex)\right)a\right)}{ae}\right)}{\sqrt{a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\left(a^2\sin(d+ex)+ab\right)\left(\frac{bx}{a^2}-\frac{2\sqrt{a^2-b^2}\text{arctanh}\left(\frac{2a^2+2ab\tan\left(\frac{1}{2}(d+ex)\right)}{2a\sqrt{a^2-b^2}}\right)}{a^2e}\right)}{\sqrt{a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2}}$$

input
```
Int[(a + b*Sin[d + e*x])/Sqrt[b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^
2],x]
```

output
```
(((b*x)/a^2 - (2*Sqrt[a^2 - b^2]*ArcTanh[(2*a^2 + 2*a*b*Tan[(d + e*x)/2])/
(2*a*Sqrt[a^2 - b^2])])/(a^2*e))*(a*b + a^2*Sin[d + e*x]))/Sqrt[b^2 + 2*a*
b*Sin[d + e*x] + a^2*Sin[d + e*x]^2]
```

3.506.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3771
```
Int[((A_) + (B_.)*sin[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sin[(d_.) + (e_.)*
(x_)] + (c_.)*sin[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Simp[(a + b*Sin
[d + e*x] + c*Sin[d + e*x]^2)^n/(b + 2*c*Sin[d + e*x])^(2*n)   Int[(A + B*S
in[d + e*x])*(b + 2*c*Sin[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e,
A, B}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[n]
```

3.506.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.64

method	result
default	$\frac{\operatorname{csgn}(b+a\sin(ex+d))\left(\frac{2b\arctan\left(\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}{a}+\frac{2\left(a^2-b^2\right)\arctan\left(\frac{2b\tan\left(\frac{ex}{2}+\frac{d}{2}\right)+2a}{2\sqrt{-a^2+b^2}}\right)}{a\sqrt{-a^2+b^2}}\right)}{e}$
parts	$-\frac{2a\operatorname{csgn}(b+a\sin(ex+d))\arctan\left(\frac{\cot(ex+d)b-b\csc(ex+d)-a}{\sqrt{-a^2+b^2}}\right)}{e\sqrt{-a^2+b^2}}+\frac{b\operatorname{csgn}(b+a\sin(ex+d))\left(2b\arctan\left(\frac{\cot(ex+d)b-b\csc(ex+d)-a}{\sqrt{-a^2+b^2}}\right)+(\right.}{ea\sqrt{-a^2+b^2}}$

input
```
int((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(1/2),x,metho
d=_RETURNVERBOSE)
```

output
```
csgn(b+a*sin(e*x+d))/e*(2*b/a*arctan(tan(1/2*e*x+1/2*d))+2*(a^2-b^2)/a/(-a
^2+b^2)^(1/2)*arctan(1/2*(2*b*tan(1/2*e*x+1/2*d)+2*a)/(-a^2+b^2)^(1/2)))
```

3.506.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.49

$$\int \frac{a+b\sin(d+ex)}{\sqrt{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}}\,dx$$
$$= \left[\frac{2\,bex+\sqrt{a^2-b^2}\log\left(-\frac{(a^2-2\,b^2)\cos(ex+d)^2+2\,ab\sin(ex+d)+a^2+b^2-2\,(b\cos(ex+d)\sin(ex+d)+a\cos(ex+d))\sqrt{a^2-b^2}}{a^2\cos(ex+d)^2-2\,ab\sin(ex+d)-a^2-b^2}\right)}{2\,ae}, \frac{bex-$$

input
```
integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(1/2),x
, algorithm="fricas")
```

output
```
[1/2*(2*b*e*x + sqrt(a^2 - b^2)*log(-((a^2 - 2*b^2)*cos(e*x + d)^2 + 2*a*b
*sin(e*x + d) + a^2 + b^2 - 2*(b*cos(e*x + d)*sin(e*x + d) + a*cos(e*x + d
))*sqrt(a^2 - b^2))/(a^2*cos(e*x + d)^2 - 2*a*b*sin(e*x + d) - a^2 - b^2))
)/(a*e), (b*e*x - sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*sin(e*x + d
) + a)/((a^2 - b^2)*cos(e*x + d))))/(a*e)]
```

3.506.6 Sympy [F]

$$\int \frac{a+b\sin(d+ex)}{\sqrt{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}}\,dx = \int \frac{a+b\sin(d+ex)}{\sqrt{(a\sin(d+ex)+b)^2}}\,dx$$

input
```
integrate((a+b*sin(e*x+d))/(b**2+2*a*b*sin(e*x+d)+a**2*sin(e*x+d)**2)**(1/
2),x)
```

output
```
Integral((a + b*sin(d + e*x))/sqrt((a*sin(d + e*x) + b)**2), x)
```

3.506.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\sin(d + ex)}{\sqrt{b^2 + 2ab\sin(d + ex) + a^2\sin^2(d + ex)}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(1/2),x
, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.506.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.80

$$\int \frac{a + b\sin(d + ex)}{\sqrt{b^2 + 2ab\sin(d + ex) + a^2\sin^2(d + ex)}}\,dx = \frac{\frac{(ex+d)b}{a\text{sgn}(a\sin(ex+d)+b)} + \frac{2\left(\pi\left\lfloor\frac{ex+d}{2\pi}+\frac{1}{2}\right\rfloor\text{sgn}(b)+\arctan\left(\frac{b\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+a}{\sqrt{-a^2+b^2}}\right)\right)(a^2-b^2)}{\sqrt{-a^2+b^2}\,a\text{sgn}(a\sin(ex+d)+b)}}{e}$$

input
```
integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(1/2),x
, algorithm="giac")
```

output
```
((e*x + d)*b/(a*sgn(a*sin(e*x + d) + b)) + 2*(pi*floor(1/2*(e*x + d)/pi +
1/2)*sgn(b) + arctan((b*tan(1/2*e*x + 1/2*d) + a)/sqrt(-a^2 + b^2)))*(a^2
- b^2)/(sqrt(-a^2 + b^2)*a*sgn(a*sin(e*x + d) + b)))/e
```

3.506.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sin(d+ex)}{\sqrt{b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)}}\,dx$$
$$=\int \frac{a+b\,\sin\left(d+e\,x\right)}{\sqrt{a^2\,{\sin\left(d+e\,x\right)}^2+2\,a\,b\,\sin\left(d+e\,x\right)+b^2}}\,dx$$

input
```
int((a + b*sin(d + e*x))/(b^2 + a^2*sin(d + e*x)^2 + 2*a*b*sin(d + e*x))^(
1/2),x)
```

output
```
int((a + b*sin(d + e*x))/(b^2 + a^2*sin(d + e*x)^2 + 2*a*b*sin(d + e*x))^(
1/2), x)
```

3.507 $\int \frac{a+b\sin(d+ex)}{\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}}\,dx$

3.507.1 Optimal result

Integrand size = 41, antiderivative size = 239

$$\begin{aligned}\int \frac{a+b\sin(d+ex)}{\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}}\,dx =& -\frac{\cos(d+ex)(b+a\sin(d+ex))}{2e\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}}\\ &-\frac{\operatorname{arctanh}\left(\frac{a+b\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-b^2}}\right)\left(ab+a^2\sin(d+ex)\right)^3}{a^2\left(a^2-b^2\right)^{3/2}e\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}}\\ &+\frac{b\cos(d+ex)\left(ab+a^2\sin(d+ex)\right)^3}{2\left(a^2-b^2\right)e\left(a^3b+a^4\sin(d+ex)\right)\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}}\end{aligned}$$

output
```
-1/2*cos(e*x+d)*(b+a*sin(e*x+d))/e/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)
^(3/2)-arctanh((a+b*tan(1/2*e*x+1/2*d))/(a^2-b^2)^(1/2))*(a*b+a^2*sin(e*x+
d))^3/a^2/(a^2-b^2)^(3/2)/e/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2)+
1/2*b*cos(e*x+d)*(a*b+a^2*sin(e*x+d))^3/(a^2-b^2)/e/(a^3*b+a^4*sin(e*x+d))
/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2)
```

3.507.2 Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.60

$$\int \frac{a+b\sin(d+ex)}{\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}}\,dx = \frac{-2a\arctan\left(\frac{a+b\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{-a^2+b^2}}\right)(b+a\sin(d+ex))^2+\sqrt{-c}}{2(-a+b)(a+b)\sqrt{-a^2+b^2}e(b+a\sin(c}$$

input
```
Integrate[(a + b*Sin[d + e*x])/(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x
]^2)^(3/2),x]
```

output
```
(-2*a*ArcTan[(a + b*Tan[(d + e*x)/2])/Sqrt[-a^2 + b^2]]*(b + a*Sin[d + e*x
])^2 + Sqrt[-a^2 + b^2]*Cos[d + e*x]*(a^2 - 2*b^2 - a*b*Sin[d + e*x]))/(2*
(-a + b)*(a + b)*Sqrt[-a^2 + b^2]*e*(b + a*Sin[d + e*x])*Sqrt[(b + a*Sin[d
 + e*x])^2])
```

3.507.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.79, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.317$, Rules used = {3042, 3771, 27, 3042, 3233, 27, 3042, 3233, 27, 3042, 3139, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sin(d+ex)}{\left(a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\sin(d+ex)}{\left(a^2\sin(d+ex)^2+2ab\sin(d+ex)+b^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{3771}$$

$$\frac{8\left(a^2\sin(d+ex)+ab\right)^3\int \frac{a+b\sin(d+ex)}{8(\sin(d+ex)a^2+ba)^3}dx}{\left(a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(a^2\sin(d+ex)+ab\right)^3\int \frac{a+b\sin(d+ex)}{(\sin(d+ex)a^2+ba)^3}dx}{\left(a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(a^2\sin(d+ex)+ab\right)^3\int\frac{a+b\sin(d+ex)}{(\sin(d+ex)a^2+ba)^3}dx}{\left(a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2\right)^{3/2}}$$

$$\Big\downarrow \text{3233}$$

$$\frac{\left(a^2\sin(d+ex)+ab\right)^3\left(\frac{\int\frac{a\left(a^2-b^2\right)\sin(d+ex)}{\left(\sin(d+ex)a^2+ba\right)^2}dx}{2a^2(a^2-b^2)}-\frac{\cos(d+ex)}{2ae(a^2\sin(d+ex)+ab)^2}\right)}{\left(a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(a^2\sin(d+ex)+ab\right)^3\left(\frac{\int\frac{\sin(d+ex)}{\left(\sin(d+ex)a^2+ba\right)^2}dx}{2a}-\frac{\cos(d+ex)}{2ae(a^2\sin(d+ex)+ab)^2}\right)}{\left(a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(a^2\sin(d+ex)+ab\right)^3\left(\frac{\int\frac{\sin(d+ex)}{\left(\sin(d+ex)a^2+ba\right)^2}dx}{2a}-\frac{\cos(d+ex)}{2ae(a^2\sin(d+ex)+ab)^2}\right)}{\left(a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2\right)^{3/2}}$$

$$\Big\downarrow \text{3233}$$

$$\frac{\left(a^2\sin(d+ex)+ab\right)^3\left(\frac{\frac{\int\frac{a^2}{\sin(d+ex)a^2+ba}dx}{a^2\left(a^2-b^2\right)}+\frac{b\cos(d+ex)}{e\left(a^2-b^2\right)\left(a^3\sin(d+ex)+a^2b\right)}}{2a}-\frac{\cos(d+ex)}{2ae(a^2\sin(d+ex)+ab)^2}\right)}{\left(a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(a^2\sin(d+ex)+ab\right)^3\left(\frac{\frac{\int\frac{1}{\sin(d+ex)a^2+ba}dx}{a^2-b^2}+\frac{b\cos(d+ex)}{e\left(a^2-b^2\right)\left(a^3\sin(d+ex)+a^2b\right)}}{2a}-\frac{\cos(d+ex)}{2ae(a^2\sin(d+ex)+ab)^2}\right)}{\left(a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

3.507. $\int\frac{a+b\sin(d+ex)}{\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}}dx$

$$\frac{\left(a^2\sin(d+ex)+ab\right)^3\left(\frac{\frac{\int\frac{1}{\sin(d+ex)a^2+ba}dx}{a^2-b^2}+\frac{b\cos(d+ex)}{e\left(a^2-b^2\right)\left(a^3\sin(d+ex)+a^2b\right)}}{2a}-\frac{\cos(d+ex)}{2ae(a^2\sin(d+ex)+ab)^2}\right)}{\left(a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2\right)^{3/2}}$$

$$\Big\downarrow \; 3139$$

$$\frac{\left(a^2\sin(d+ex)+ab\right)^3\left(\frac{\frac{2\int\frac{1}{2\tan\left(\frac{1}{2}(d+ex)\right)a^2+b\tan^2\left(\frac{1}{2}(d+ex)\right)a+ba}d\tan\left(\frac{1}{2}(d+ex)\right)}{e\left(a^2-b^2\right)}+\frac{b\cos(d+ex)}{e\left(a^2-b^2\right)\left(a^3\sin(d+ex)+a^2b\right)}}{2a}-\frac{\cos(d+ex)}{2ae(a^2\sin(d+ex)+ab}\right.}{\left(a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2\right)^{3/2}}$$

$$\Big\downarrow \; 1083$$

$$\frac{\left(a^2\sin(d+ex)+ab\right)^3\left(\frac{\frac{b\cos(d+ex)}{e\left(a^2-b^2\right)\left(a^3\sin(d+ex)+a^2b\right)}-\frac{4\int\frac{1}{4a^2\left(a^2-b^2\right)-\left(2a^2+2b\tan\left(\frac{1}{2}(d+ex)\right)a\right)^2}d\left(2a^2+2b\tan\left(\frac{1}{2}(d+ex)\right)a\right)}{e\left(a^2-b^2\right)}}{2a}-\frac{\cos(d}{2ae(a^2\sin(d}\right.}{\left(a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2\right)^{3/2}}$$

$$\Big\downarrow \; 219$$

$$\frac{\left(a^2\sin(d+ex)+ab\right)^3\left(\frac{\frac{b\cos(d+ex)}{e\left(a^2-b^2\right)\left(a^3\sin(d+ex)+a^2b\right)}-\frac{2\text{arctanh}\left(\frac{2a^2+2ab\tan\left(\frac{1}{2}(d+ex)\right)}{2a\sqrt{a^2-b^2}}\right)}{ae\left(a^2-b^2\right)^{3/2}}}{2a}-\frac{\cos(d+ex)}{2ae(a^2\sin(d+ex)+ab)^2}\right)}{\left(a^2\sin^2(d+ex)+2ab\sin(d+ex)+b^2\right)^{3/2}}$$

input
```
Int[(a + b*Sin[d + e*x])/(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2)^(
3/2),x]
```

output
```
((a*b + a^2*Sin[d + e*x])^3*(-1/2*Cos[d + e*x]/(a*e*(a*b + a^2*Sin[d + e*x
])^2) + ((-2*ArcTanh[(2*a^2 + 2*a*b*Tan[(d + e*x)/2])/(2*a*Sqrt[a^2 - b^2]
)])/(a*(a^2 - b^2)^(3/2)*e) + (b*Cos[d + e*x])/((a^2 - b^2)*e*(a^2*b + a^3
*Sin[d + e*x])))/(2*a)))/(b^2 + 2*a*b*Sin[d + e*x] + a^2*Sin[d + e*x]^2)^(
3/2)
```

3.507. $\int \frac{a+b\sin(d+ex)}{\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}}\,dx$

3.507.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 3771
```
Int[((A_) + (B_.)*sin[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sin[(d_.) + (e_.)*
(x_)] + (c_.)*sin[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Simp[(a + b*Sin
[d + e*x] + c*Sin[d + e*x]^2)^n/(b + 2*c*Sin[d + e*x])^(2*n)   Int[(A + B*S
in[d + e*x])*(b + 2*c*Sin[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e,
A, B}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[n]
```

3.507. $\int \frac{a+b\sin(d+ex)}{\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}}\,dx$

3.507.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.09 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.05

method	result
default	$\frac{\operatorname{csgn}(b+a\sin(ex+d))\left(\frac{-\frac{a(2a^2-3b^2)\tan(\frac{ex}{2}+\frac{d}{2})^3}{b(a^2-b^2)}-\frac{(2a^4-3a^2b^2-2b^4)\tan(\frac{ex}{2}+\frac{d}{2})^2}{b^2(a^2-b^2)}-\frac{a(2a^2-5b^2)\tan(\frac{ex}{2}+\frac{d}{2})}{b(a^2-b^2)}-\frac{a^2-2b^2}{a^2-b^2}}{\left(\tan(\frac{ex}{2}+\frac{d}{2})^2b+2a\tan(\frac{ex}{2}+\frac{d}{2})+b\right)^2}+\frac{4a\arctan\left(\frac{2b\cdots}{\cdots}\right)}{(4a^2-4b\cdots}\right.}{e}$
parts	Expression too large to display

input
```
int((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2),x,metho
d=_RETURNVERBOSE)
```

output
```
csgn(b+a*sin(e*x+d))/e*(8*(-1/8*a*(2*a^2-3*b^2)/b/(a^2-b^2)*tan(1/2*e*x+1/
2*d)^3-1/8*(2*a^4-3*a^2*b^2-2*b^4)/b^2/(a^2-b^2)*tan(1/2*e*x+1/2*d)^2-1/8*
a*(2*a^2-5*b^2)/b/(a^2-b^2)*tan(1/2*e*x+1/2*d)-1/8*(a^2-2*b^2)/(a^2-b^2))/
(tan(1/2*e*x+1/2*d)^2*b+2*a*tan(1/2*e*x+1/2*d)+b)^2+4*a/(4*a^2-4*b^2)/(-a^
2+b^2)^(1/2)*arctan(1/2*(2*b*tan(1/2*e*x+1/2*d)+2*a)/(-a^2+b^2)^(1/2)))
```

3.507.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 527, normalized size of antiderivative = 2.21

$$\int \frac{a+b\sin(d+ex)}{(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex))^{3/2}}\,dx = \left[-\frac{2\left(a^3b-ab^3\right)\cos\left(ex+d\right)\sin\left(ex+d\right)+\left(a^3\cos\left(ex+\cdots\right.\right.}{\cdots}\right.$$

$$-\frac{\left(a^3b-ab^3\right)\cos\left(ex+d\right)\sin\left(ex+d\right)+\left(a^3\cos\left(ex+d\right)^2-2\,a^2b\sin\left(ex+d\right)-a^3-ab^2\right)\sqrt{-a^2+b^2}\arctan\cdots}{2\left(\left(a^6-2\,a^4b^2+a^2b^4\right)e\cos\left(ex+d\right)^2-2\left(a^5b-2\,a^3b^3+ab^5\right)e\sin\left(ex+d\right)\cdots\right.}$$

input
```
integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2),x
, algorithm="fricas")
```

output
```
[-1/4*(2*(a^3*b - a*b^3)*cos(e*x + d)*sin(e*x + d) + (a^3*cos(e*x + d)^2 -
 2*a^2*b*sin(e*x + d) - a^3 - a*b^2)*sqrt(a^2 - b^2)*log(((a^2 - 2*b^2)*co
s(e*x + d)^2 + 2*a*b*sin(e*x + d) + a^2 + b^2 + 2*(b*cos(e*x + d)*sin(e*x
+ d) + a*cos(e*x + d))*sqrt(a^2 - b^2))/(a^2*cos(e*x + d)^2 - 2*a*b*sin(e*
x + d) - a^2 - b^2)) - 2*(a^4 - 3*a^2*b^2 + 2*b^4)*cos(e*x + d))/((a^6 - 2
*a^4*b^2 + a^2*b^4)*e*cos(e*x + d)^2 - 2*(a^5*b - 2*a^3*b^3 + a*b^5)*e*sin
(e*x + d) - (a^6 - a^4*b^2 - a^2*b^4 + b^6)*e), -1/2*((a^3*b - a*b^3)*cos(
e*x + d)*sin(e*x + d) + (a^3*cos(e*x + d)^2 - 2*a^2*b*sin(e*x + d) - a^3 -
 a*b^2)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*sin(e*x + d) + a)/((a
^2 - b^2)*cos(e*x + d))) - (a^4 - 3*a^2*b^2 + 2*b^4)*cos(e*x + d))/((a^6 -
 2*a^4*b^2 + a^2*b^4)*e*cos(e*x + d)^2 - 2*(a^5*b - 2*a^3*b^3 + a*b^5)*e*s
in(e*x + d) - (a^6 - a^4*b^2 - a^2*b^4 + b^6)*e)]
```

3.507.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\sin(d+ex)}{\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*sin(e*x+d))/(b**2+2*a*b*sin(e*x+d)+a**2*sin(e*x+d)**2)**(3/
2),x)
```

output
```
Timed out
```

3.507.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sin(d+ex)}{\left(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex)\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2),x
, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.507.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.22

$$\int \frac{a+b\sin(d+ex)}{(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex))^{3/2}}\,dx = \frac{\frac{\left(\pi\left\lfloor\frac{ex+d}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+a}{\sqrt{-a^2+b^2}}\right)\right)a}{(a^2\mathrm{sgn}(a\sin(ex+d)+b)-b^2\mathrm{sgn}(a\sin(ex+d)+b))\sqrt{-a^2+b^2}} - \frac{2a^3b\tan\left(\frac{1}{2}ex\right.}{}}{}$$

input
```
integrate((a+b*sin(e*x+d))/(b^2+2*a*b*sin(e*x+d)+a^2*sin(e*x+d)^2)^(3/2),x
, algorithm="giac")
```

output
```
((pi*floor(1/2*(e*x + d)/pi + 1/2)*sgn(b) + arctan((b*tan(1/2*e*x + 1/2*d)
 + a)/sqrt(-a^2 + b^2)))*a/((a^2*sgn(a*sin(e*x + d) + b) - b^2*sgn(a*sin(e
*x + d) + b))*sqrt(-a^2 + b^2)) - (2*a^3*b*tan(1/2*e*x + 1/2*d)^3 - 3*a*b^
3*tan(1/2*e*x + 1/2*d)^3 + 2*a^4*tan(1/2*e*x + 1/2*d)^2 - 3*a^2*b^2*tan(1/
2*e*x + 1/2*d)^2 - 2*b^4*tan(1/2*e*x + 1/2*d)^2 + 2*a^3*b*tan(1/2*e*x + 1/
2*d) - 5*a*b^3*tan(1/2*e*x + 1/2*d) + a^2*b^2 - 2*b^4)/((a^2*b^2*sgn(a*sin
(e*x + d) + b) - b^4*sgn(a*sin(e*x + d) + b))*(b*tan(1/2*e*x + 1/2*d)^2 +
2*a*tan(1/2*e*x + 1/2*d) + b)^2))/e
```

3.507.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\sin(d+ex)}{(b^2+2ab\sin(d+ex)+a^2\sin^2(d+ex))^{3/2}}\,dx = \int \frac{a+b\,\sin(d+e\,x)}{\left(a^2\sin(d+e\,x)^2+2\,a\,b\,\sin(d+e\,x)+b^2\right)^{3/2}}\,dx$$

input
```
int((a + b*sin(d + e*x))/(b^2 + a^2*sin(d + e*x)^2 + 2*a*b*sin(d + e*x))^(
3/2),x)
```

output
```
int((a + b*sin(d + e*x))/(b^2 + a^2*sin(d + e*x)^2 + 2*a*b*sin(d + e*x))^(
3/2), x)
```

3.508 $\int \frac{a+b\cos(x)}{b^2+2ab\cos(x)+a^2\cos^2(x)}\,dx$

3.508.1 Optimal result

Integrand size = 27, antiderivative size = 11

$$\int \frac{a+b\cos(x)}{b^2+2ab\cos(x)+a^2\cos^2(x)}\,dx=\frac{\sin(x)}{b+a\cos(x)}$$

output
```
sin(x)/(b+a*cos(x))
```

3.508.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{a+b\cos(x)}{b^2+2ab\cos(x)+a^2\cos^2(x)}\,dx=\frac{\sin(x)}{b+a\cos(x)}$$

input
```
Integrate[(a + b*Cos[x])/(b^2 + 2*a*b*Cos[x] + a^2*Cos[x]^2),x]
```

output
```
Sin[x]/(b + a*Cos[x])
```

3.508.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.82, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3042, 3770, 27, 3042, 3233, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\cos(x)}{a^2\cos^2(x)+2ab\cos(x)+b^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\cos(x)}{a^2\cos(x)^2+2ab\cos(x)+b^2}dx$$

$$\Big\downarrow \text{3770}$$

$$4a^2\int \frac{a+b\cos(x)}{4\left(\cos(x)a^2+ba\right)^2}dx$$

$$\Big\downarrow \text{27}$$

$$a^2\int \frac{a+b\cos(x)}{\left(\cos(x)a^2+ba\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$a^2\int \frac{a+b\sin\left(x+\frac{\pi}{2}\right)}{\left(\sin\left(x+\frac{\pi}{2}\right)a^2+ba\right)^2}dx$$

$$\Big\downarrow \text{3233}$$

$$a^2\left(\frac{\int 0dx}{a^2(a^2-b^2)}+\frac{\sin(x)}{a^3\cos(x)+a^2b}\right)$$

$$\Big\downarrow \text{24}$$

$$\frac{a^2\sin(x)}{a^3\cos(x)+a^2b}$$

input
```
Int[(a + b*Cos[x])/(b^2 + 2*a*b*Cos[x] + a^2*Cos[x]^2),x]
```

output
```
(a^2*Sin[x])/(a^2*b + a^3*Cos[x])
```

3.508.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 3770
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + cos[(d_.) + (e_.)*(x_)]^2*(c_.) + (a_)
)^(n_)*(cos[(d_.) + (e_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[1/(4^n*c^n
)   Int[(A + B*Cos[d + e*x])*(b + 2*c*Cos[d + e*x])^(2*n), x], x] /; FreeQ[
{a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[n]
```

3.508.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
parallelrisch	$\frac{\sin(x)}{b+a\cos(x)}$	12
default	$-\frac{2\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^2 a-\tan\left(\frac{x}{2}\right)^2 b-a-b}$	33
risch	$\frac{2i\left(b\,e^{ix}+a\right)}{a\left(a\,e^{2ix}+2b\,e^{ix}+a\right)}$	35
norman	$\frac{-2\tan\left(\frac{x}{2}\right)^3-2\tan\left(\frac{x}{2}\right)}{\left(1+\tan\left(\frac{x}{2}\right)^2\right)\left(\tan\left(\frac{x}{2}\right)^2 a-\tan\left(\frac{x}{2}\right)^2 b-a-b\right)}$	53

input
```
int((a+b*cos(x))/(b^2+2*a*b*cos(x)+cos(x)^2*a^2),x,method=_RETURNVERBOSE)
```

output
```
sin(x)/(b+a*cos(x))
```

3.508.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{a + b\cos(x)}{b^2 + 2ab\cos(x) + a^2\cos^2(x)}\,dx = \frac{\sin(x)}{a\cos(x) + b}$$

input
```
integrate((a+b*cos(x))/(b^2+2*a*b*cos(x)+a^2*cos(x)^2),x, algorithm="frica
s")
```

output
```
sin(x)/(a*cos(x) + b)
```

3.508.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\cos(x)}{b^2 + 2ab\cos(x) + a^2\cos^2(x)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(x))/(b**2+2*a*b*cos(x)+a**2*cos(x)**2),x)
```

output
```
Timed out
```

3.508.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\cos(x)}{b^2 + 2ab\cos(x) + a^2\cos^2(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*cos(x))/(b^2+2*a*b*cos(x)+a^2*cos(x)^2),x, algorithm="maxim
a")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.508.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(11) = 22.

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.91

$$\int \frac{a+b\cos(x)}{b^2+2ab\cos(x)+a^2\cos^2(x)}\,dx = -\frac{2\,\tan\left(\frac{1}{2}\,x\right)}{a\tan\left(\frac{1}{2}\,x\right)^2-b\tan\left(\frac{1}{2}\,x\right)^2-a-b}$$

input
```
integrate((a+b*cos(x))/(b^2+2*a*b*cos(x)+a^2*cos(x)^2),x, algorithm="giac"
)
```

output
```
-2*tan(1/2*x)/(a*tan(1/2*x)^2 - b*tan(1/2*x)^2 - a - b)
```

3.508.9 Mupad [B] (verification not implemented)

Time = 26.94 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.18

$$\int \frac{a+b\cos(x)}{b^2+2ab\cos(x)+a^2\cos^2(x)}\,dx = \frac{2\tan\left(\frac{x}{2}\right)}{(b-a)\,\tan\left(\frac{x}{2}\right)^2+a+b}$$

input
```
int((a + b*cos(x))/(a^2*cos(x)^2 + b^2 + 2*a*b*cos(x)),x)
```

output
```
(2*tan(x/2))/(a + b - tan(x/2)^2*(a - b))
```

3.509 $\int \frac{d+e\cos(x)}{a+b\cos(x)+c\cos^2(x)}\,dx$

3.509.1 Optimal result

Integrand size = 21, antiderivative size = 246

$$\int \frac{d+e\cos(x)}{a+b\cos(x)+c\cos^2(x)}\,dx = \frac{2\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{b-2c-\sqrt{b^2-4ac}}\tan\left(\frac{x}{2}\right)}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-2c-\sqrt{b^2-4ac}}\sqrt{b+2c-\sqrt{b^2-4ac}}} + \frac{2\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\sqrt{b-2c+\sqrt{b^2-4ac}}\tan\left(\frac{x}{2}\right)}{\sqrt{b+2c+\sqrt{b^2-4ac}}}\right)}{\sqrt{b-2c+\sqrt{b^2-4ac}}\sqrt{b+2c+\sqrt{b^2-4ac}}}$$

output
```
2*arctan((b-2*c-(-4*a*c+b^2)^(1/2))^(1/2)*tan(1/2*x)/(b+2*c-(-4*a*c+b^2)^(
1/2))^(1/2))*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))/(b-2*c-(-4*a*c+b^2)^(1/2)
)^(1/2)/(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2)+2*arctan((b-2*c+(-4*a*c+b^2)^(1/2
))^(1/2)*tan(1/2*x)/(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2))*(e+(b*e-2*c*d)/(-4*a
*c+b^2)^(1/2))/(b-2*c+(-4*a*c+b^2)^(1/2))^(1/2)/(b+2*c+(-4*a*c+b^2)^(1/2))
^(1/2)
```

3.509.2 Mathematica [A] (verified)

Time = 1.14 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.98

$$\int \frac{d+e\cos(x)}{a+b\cos(x)+c\cos^2(x)}\,dx$$
$$= \frac{\sqrt{2}\left(-\frac{\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)e\right)\operatorname{arctanh}\left(\frac{\left(b-2c+\sqrt{b^2-4ac}\right)\tan\left(\frac{x}{2}\right)}{\sqrt{-2b^2+4c(a+c)-2b\sqrt{b^2-4ac}}}\right)}{\sqrt{-b^2+2c(a+c)-b\sqrt{b^2-4ac}}}+\frac{\left(2cd+\left(-b+\sqrt{b^2-4ac}\right)e\right)\operatorname{arctanh}\left(\frac{\left(-b+2c+\sqrt{b^2-4ac}\right)\tan}{\sqrt{-2b^2+4c(a+c)+2b\sqrt{b^2}}}\right.}{\sqrt{-b^2+2c(a+c)+b\sqrt{b^2-4ac}}}\right.}{\sqrt{b^2-4ac}}$$

input
```
Integrate[(d + e*Cos[x])/(a + b*Cos[x] + c*Cos[x]^2),x]
```

output
```
(Sqrt[2]*(-(((-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*ArcTanh[((b - 2*c + Sqrt
[b^2 - 4*a*c])*Tan[x/2])/Sqrt[-2*b^2 + 4*c*(a + c) - 2*b*Sqrt[b^2 - 4*a*c]
]])/Sqrt[-b^2 + 2*c*(a + c) - b*Sqrt[b^2 - 4*a*c]]) + ((2*c*d + (-b + Sqrt
[b^2 - 4*a*c])*e)*ArcTanh[((-b + 2*c + Sqrt[b^2 - 4*a*c])*Tan[x/2])/Sqrt[-
2*b^2 + 4*c*(a + c) + 2*b*Sqrt[b^2 - 4*a*c]]])/Sqrt[-b^2 + 2*c*(a + c) + b
*Sqrt[b^2 - 4*a*c]]))/Sqrt[b^2 - 4*a*c]
```

3.509.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 3774, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+e\cos(x)}{a+b\cos(x)+c\cos^2(x)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{d+e\cos(x)}{a+b\cos(x)+c\cos(x)^2}dx$$
$$\Big\downarrow \text{3774}$$
$$\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\int \frac{1}{b+2c\cos(x)-\sqrt{b^2-4ac}}dx+\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{b+2c\cos(x)+\sqrt{b^2-4ac}}dx$$

$$\Big\downarrow \text{3042}$$

$$\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\int \frac{1}{b+2c\sin\left(x+\frac{\pi}{2}\right)-\sqrt{b^2-4ac}}dx+\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{b+2c\sin\left(x+\frac{\pi}{2}\right)+\sqrt{b^2-4ac}}dx$$

$$\Big\downarrow \text{3138}$$

$$2\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\int \frac{1}{\left(b-2c-\sqrt{b^2-4ac}\right)\tan^2\left(\frac{x}{2}\right)+b+2c-\sqrt{b^2-4ac}}d\tan\left(\frac{x}{2}\right)+2\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\left(b-2c+\sqrt{b^2-4ac}\right)\tan^2\left(\frac{x}{2}\right)+b+2c+\sqrt{b^2-4ac}}d\tan\left(\frac{x}{2}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{2\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\arctan\left(\frac{\tan\left(\frac{x}{2}\right)\sqrt{-\sqrt{b^2-4ac}+b-2c}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{-\sqrt{b^2-4ac}+b-2c}\sqrt{-\sqrt{b^2-4ac}+b+2c}}+\frac{2\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\arctan\left(\frac{\tan\left(\frac{x}{2}\right)\sqrt{\sqrt{b^2-4ac}+b-2c}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{\sqrt{b^2-4ac}+b-2c}\sqrt{\sqrt{b^2-4ac}+b+2c}}$$

input
```
Int[(d + e*Cos[x])/(a + b*Cos[x] + c*Cos[x]^2),x]
```

output
```
(2*(e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[b - 2*c - Sqrt[b^2 -
 4*a*c]]*Tan[x/2])/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b - 2*c - Sqr
t[b^2 - 4*a*c]]*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) + (2*(e - (2*c*d - b*e)
/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[b - 2*c + Sqrt[b^2 - 4*a*c]]*Tan[x/2])/Sq
rt[b + 2*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b - 2*c + Sqrt[b^2 - 4*a*c]]*Sqrt[
b + 2*c + Sqrt[b^2 - 4*a*c]])
```

3.509.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3774
```
Int[(cos[(d_.) + (e_.)*(x_)]*(B_.) + (A_))/((a_.) + cos[(d_.) + (e_.)*(x_)]
*(b_.) + cos[(d_.) + (e_.)*(x_)]^2*(c_.)), x_Symbol] :> Module[{q = Rt[b^2
- 4*a*c, 2]}, Simp[(B + (b*B - 2*A*c)/q)   Int[1/(b + q + 2*c*Cos[d + e*x])
, x], x] + Simp[(B - (b*B - 2*A*c)/q)   Int[1/(b - q + 2*c*Cos[d + e*x]), x
], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0]
```

3.509.4 Maple [A] (verified)

Time = 5.29 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.03

method	result
default	$2(a-b+c)\left(\frac{\left(\sqrt{-4ac+b^2}\,d-e\sqrt{-4ac+b^2}-2ae+bd+be-2cd\right)\arctan\left(\frac{(a-b+c)\tan\left(\frac{x}{2}\right)}{\sqrt{\left(\sqrt{-4ac+b^2}+a-c\right)(a-b+c)}}\right)}{2\sqrt{-4ac+b^2}\,(a-b+c)\sqrt{\left(\sqrt{-4ac+b^2}+a-c\right)(a-b+c)}}+\frac{\left(\sqrt{-4ac+b^2}\,d-e\sqrt{-\right.}{2\cdots}\right.$
risch	Expression too large to display

input
```
int((d+e*cos(x))/(a+b*cos(x)+c*cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
2*(a-b+c)*(1/2*((-4*a*c+b^2)^(1/2)*d-e*(-4*a*c+b^2)^(1/2)-2*a*e+b*d+b*e-2*
c*d)/(-4*a*c+b^2)^(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2)*a
rctan((a-b+c)*tan(1/2*x)/(((-4*a*c+b^2)^(1/2)+a-c)*(a-b+c))^(1/2))+1/2*((-
4*a*c+b^2)^(1/2)*d-e*(-4*a*c+b^2)^(1/2)+2*a*e-b*d-b*e+2*c*d)/(-4*a*c+b^2)^
(1/2)/(a-b+c)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)*arctanh((-a+b-c)*ta
n(1/2*x)/(((-4*a*c+b^2)^(1/2)-a+c)*(a-b+c))^(1/2)))
```

3.509.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6697 vs. 2(206) = 412.

Time = 6.40 (sec) , antiderivative size = 6697, normalized size of antiderivative = 27.22

$$\int \frac{d+e\cos(x)}{a+b\cos(x)+c\cos^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate((d+e*cos(x))/(a+b*cos(x)+c*cos(x)^2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.509.6 Sympy [F(-1)]

Timed out.

$$\int \frac{d+e\cos(x)}{a+b\cos(x)+c\cos^2(x)}\,dx = \text{Timed out}$$

input
```
integrate((d+e*cos(x))/(a+b*cos(x)+c*cos(x)**2),x)
```

output
```
Timed out
```

3.509.7 Maxima [F]

$$\int \frac{d+e\cos(x)}{a+b\cos(x)+c\cos^2(x)}\,dx = \int \frac{e\cos{(x)}+d}{c\cos{(x)}^2+b\cos{(x)}+a}\,dx$$

input
```
integrate((d+e*cos(x))/(a+b*cos(x)+c*cos(x)^2),x, algorithm="maxima")
```

output
```
integrate((e*cos(x) + d)/(c*cos(x)^2 + b*cos(x) + a), x)
```

3.509.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5300 vs. 2(206) = 412.

Time = 2.04 (sec) , antiderivative size = 5300, normalized size of antiderivative = 21.54

$$\int \frac{d+e\cos(x)}{a+b\cos(x)+c\cos^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate((d+e*cos(x))/(a+b*cos(x)+c*cos(x)^2),x, algorithm="giac")
```

output
```
((2*a^2*b^3 - 2*b^5 - 8*a^3*b*c - 12*a^2*b^2*c + 20*a*b^3*c + 4*b^4*c + 48
*a^3*c^2 - 48*a^2*b*c^2 - 24*a*b^2*c^2 - 6*b^3*c^2 + 32*a^2*c^3 + 24*a*b*c
^3 + 4*b^2*c^3 - 16*a*c^4 + 3*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*
c)*(a - b + c))*a^2*b^2 - 2*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)
*(a - b + c))*a*b^3 - 5*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a
- b + c))*b^4 - 12*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b +
 c))*a^3*c + 8*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))
*a^2*b*c + 34*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*
a*b^2*c + 6*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*b^
3*c - 56*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*a^2*c
^2 - 24*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*a*b*c^
2 - 5*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*b^2*c^2
+ 20*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*a*c^3 + 3
*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*sqrt(b^2 - 4*
a*c)*a^2*b - 2*(b^2 - 4*a*c)*a^2*b - 2*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b
^2 - 4*a*c)*(a - b + c))*sqrt(b^2 - 4*a*c)*a*b^2 - 5*sqrt(a^2 - a*b + b*c
- c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*sqrt(b^2 - 4*a*c)*b^3 + 2*(b^2 - 4*
a*c)*b^3 + 6*sqrt(a^2 - a*b + b*c - c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*s
qrt(b^2 - 4*a*c)*a^2*c + 12*(b^2 - 4*a*c)*a^2*c + 10*sqrt(a^2 - a*b + b*c
- c^2 + sqrt(b^2 - 4*a*c)*(a - b + c))*sqrt(b^2 - 4*a*c)*a*b*c - 12*(b^...
```

3.509.9 Mupad [B] (verification not implemented)

Time = 44.54 (sec) , antiderivative size = 11781, normalized size of antiderivative = 47.89

$$\int \frac{d+e\cos(x)}{a+b\cos(x)+c\cos^2(x)}\,dx = \text{Too large to display}$$

input
```
int((d + e*cos(x))/(a + b*cos(x) + c*cos(x)^2),x)
```

output
```
- atan((((-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 + b*d^2*(-(4*a*c - b^2)^3)^(1/
2) - 8*a^3*c*e^2 + b*e^2*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b^2*e^2 + 8*a^2*
c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e - 2*a*d*e*(-(4*a*c -
 b^2)^3)^(1/2) + 2*b^3*c*d*e - 2*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*
c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4 - b^6 +
 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*a^3*b^2*
c - 32*a^2*b^2*c^2 + 10*a*b^4*c)))^(1/2)*(256*a^2*c^2*d - 32*b^4*e - 32*a^
2*b^2*d - 32*a^2*b^2*e - 32*b^4*d + 256*a^2*c^2*e - 32*b^2*c^2*d - 32*b^2*
c^2*e + tan(x/2)*(-(b^4*d^2 - b^4*e^2 + 8*a*c^3*d^2 + b*d^2*(-(4*a*c - b^2
)^3)^(1/2) - 8*a^3*c*e^2 + b*e^2*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*b^2*e^2
+ 8*a^2*c^2*d^2 - 8*a^2*c^2*e^2 - 2*b^2*c^2*d^2 - 2*a*b^3*d*e - 2*a*d*e*(-
(4*a*c - b^2)^3)^(1/2) + 2*b^3*c*d*e - 2*c*d*e*(-(4*a*c - b^2)^3)^(1/2) -
6*a*b^2*c*d^2 + 6*a*b^2*c*e^2 - 8*a*b*c^2*d*e + 8*a^2*b*c*d*e)/(2*(a^2*b^4
 - b^6 + 16*a^2*c^4 + 32*a^3*c^3 + 16*a^4*c^2 + b^4*c^2 - 8*a*b^2*c^3 - 8*
a^3*b^2*c - 32*a^2*b^2*c^2 + 10*a*b^4*c)))^(1/2)*(64*a*b^4 + 256*a*c^4 - 2
56*a^4*c - 64*b^4*c - 128*a^2*b^3 + 64*a^3*b^2 + 256*a^2*c^3 - 256*a^3*c^2
 - 64*b^2*c^3 + 128*b^3*c^2 + 192*a*b^2*c^2 - 192*a^2*b^2*c - 512*a*b*c^3
+ 512*a^3*b*c) + 64*a*b^3*d + 64*a*b^3*e + 128*a*c^3*d + 128*a^3*c*d + 128
*a*c^3*e + 128*a^3*c*e + 64*b^3*c*d + 64*b^3*c*e - 256*a*b*c^2*d + 64*a*b^
2*c*d - 256*a^2*b*c*d - 256*a*b*c^2*e + 64*a*b^2*c*e - 256*a^2*b*c*e) +...
```

3.509. $\int \frac{d+e\cos(x)}{a+b\cos(x)+c\cos^2(x)}\,dx$

3.510 $\int (a+b\tan(d+ex)) (b^2+2ab\tan(d+ex)+a^2\tan^2(d$

3.510.1 Optimal result

Integrand size = 39, antiderivative size = 144

$$\int (a+b\tan(d+ex)) \left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^2 dx$$
$$= a(a^2-3b^2)(a^2+b^2)x + \frac{b(3a^2-b^2)(a^2+b^2)\log(\cos(d+ex))}{e} - \frac{a(a^4-b^4)\tan(d+ex)}{e} + \frac{b(a^2+b^2)(b+a\tan(d+ex))^2}{2e} + \frac{(a^2+b^2)(b+a\tan(d+ex))^3}{3e} + \frac{b(b+a\tan(d+ex))^4}{4e}$$

output
```
a*(a^2-3*b^2)*(a^2+b^2)*x+b*(3*a^2-b^2)*(a^2+b^2)*ln(cos(e*x+d))/e-a*(a^4-
b^4)*tan(e*x+d)/e+1/2*b*(a^2+b^2)*(b+a*tan(e*x+d))^2/e+1/3*(a^2+b^2)*(b+a*
tan(e*x+d))^3/e+1/4*b*(b+a*tan(e*x+d))^4/e
```

3.510.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.72 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.06

$$\int (a+b\tan(d+ex)) \left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^2 dx$$
$$= \frac{6(a^2+b^2)\left(-i(a-ib)^3\log(i-\tan(d+ex))+i(a+ib)^3\log(i+\tan(d+ex))\right)-12a(a^4-2a^2b^2-4b^4)\text{ta}}{12e}$$

input
```
Integrate[(a + b*Tan[d + e*x])*(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x
]^2)^2,x]
```

output
```
(6*(a^2 + b^2)*((-I)*(a - I*b)^3*Log[I - Tan[d + e*x]] + I*(a + I*b)^3*Log
[I + Tan[d + e*x]]) - 12*a*(a^4 - 2*a^2*b^2 - 4*b^4)*Tan[d + e*x] + 18*a^2
*b*(a^2 + 2*b^2)*Tan[d + e*x]^2 + 4*a^3*(a^2 + 4*b^2)*Tan[d + e*x]^3 + 3*a
^4*b*Tan[d + e*x]^4)/(12*e)
```

3.510.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.04, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.359$, Rules used = {3042, 4191, 27, 3042, 4011, 27, 3042, 4011, 3042, 4011, 3042, 4008, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\tan(d+ex))\left(a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2\right)^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(d+ex))\left(a^2\tan(d+ex)^2+2ab\tan(d+ex)+b^2\right)^2dx$$

$$\Big\downarrow \text{4191}$$

$$\frac{\int 16\left(\tan(d+ex)a^2+ba\right)^4(a+b\tan(d+ex))dx}{16a^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \left(\tan(d+ex)a^2+ba\right)^4(a+b\tan(d+ex))dx}{a^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(\tan(d+ex)a^2+ba\right)^4(a+b\tan(d+ex))dx}{a^4}$$

$$\Big\downarrow \text{4011}$$

$$\frac{\int a\left(a^2+b^2\right)\tan(d+ex)\left(\tan(d+ex)a^2+ba\right)^3dx+\frac{b\left(a^2\tan(d+ex)+ab\right)^4}{4e}}{a^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{a(a^2+b^2)\int \tan(d+ex)\left(\tan(d+ex)a^2+ba\right)^3 dx+\frac{b(a^2\tan(d+ex)+ab)^4}{4e}}{a^4}$$

$\Big\downarrow$ 3042

$$\frac{a(a^2+b^2)\int \tan(d+ex)\left(\tan(d+ex)a^2+ba\right)^3 dx+\frac{b(a^2\tan(d+ex)+ab)^4}{4e}}{a^4}$$

$\Big\downarrow$ 4011

$$\frac{a(a^2+b^2)\left(\int \left(\tan(d+ex)a^2+ba\right)^2\left(ab\tan(d+ex)-a^2\right)dx+\frac{(a^2\tan(d+ex)+ab)^3}{3e}\right)+\frac{b(a^2\tan(d+ex)+ab)^4}{4e}}{a^4}$$

$\Big\downarrow$ 3042

$$\frac{a(a^2+b^2)\left(\int \left(\tan(d+ex)a^2+ba\right)^2\left(ab\tan(d+ex)-a^2\right)dx+\frac{(a^2\tan(d+ex)+ab)^3}{3e}\right)+\frac{b(a^2\tan(d+ex)+ab)^4}{4e}}{a^4}$$

$\Big\downarrow$ 4011

$$\frac{a(a^2+b^2)\left(\int \left(\tan(d+ex)a^2+ba\right)\left(-2ba^3-(a^2-b^2)\tan(d+ex)a^2\right)dx+\frac{(a^2\tan(d+ex)+ab)^3}{3e}+\frac{ab(a^2\tan(d+ex)+ab}{2e}\right.}{a^4}$$

$\Big\downarrow$ 3042

$$\frac{a(a^2+b^2)\left(\int \left(\tan(d+ex)a^2+ba\right)\left(-2ba^3-(a^2-b^2)\tan(d+ex)a^2\right)dx+\frac{(a^2\tan(d+ex)+ab)^3}{3e}+\frac{ab(a^2\tan(d+ex)+ab}{2e}\right.}{a^4}$$

$\Big\downarrow$ 4008

$$\frac{a(a^2+b^2)\left(-a^3b(3a^2-b^2)\int \tan(d+ex)dx+\frac{ab(a^2\tan(d+ex)+ab)^2}{2e}+\frac{(a^2\tan(d+ex)+ab)^3}{3e}-\frac{a^4(a^2-b^2)\tan(d+ex)}{e}+a^4x\right.}{a^4}$$

$\Big\downarrow$ 3042

$$\frac{a(a^2+b^2)\left(-a^3b(3a^2-b^2)\int \tan(d+ex)dx+\frac{ab(a^2\tan(d+ex)+ab)^2}{2e}+\frac{(a^2\tan(d+ex)+ab)^3}{3e}-\frac{a^4(a^2-b^2)\tan(d+ex)}{e}+a^4x\right.}{a^4}$$

$\Big\downarrow$ 3956

3.510. $\int (a+b\tan(d+ex))\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^2 dx$

$$\frac{\frac{b(a^2\tan(d+ex)+ab)^4}{4e}+a(a^2+b^2)\left(\frac{ab(a^2\tan(d+ex)+ab)^2}{2e}+\frac{(a^2\tan(d+ex)+ab)^3}{3e}-\frac{a^4(a^2-b^2)\tan(d+ex)}{e}+a^4x(a^2-3b^2)+\frac{a^3}{}\right.}{a^4}$$

input
```
Int[(a + b*Tan[d + e*x])*(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2)^2
,x]
```

output
```
((b*(a*b + a^2*Tan[d + e*x])^4)/(4*e) + a*(a^2 + b^2)*(a^4*(a^2 - 3*b^2)*x
 + (a^3*b*(3*a^2 - b^2)*Log[Cos[d + e*x]])/e - (a^4*(a^2 - b^2)*Tan[d + e*
x])/e + (a*b*(a*b + a^2*Tan[d + e*x])^2)/(2*e) + (a*b + a^2*Tan[d + e*x])^
3/(3*e)))/a^4
```

3.510.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4008
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Simp[b*d*(Tan[e + f*x]/f),
x] + Simp[(b*c + a*d)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4191
```
Int[((A_) + (B_.)*tan[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*tan[(d_.) + (e_.)*
(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Simp[1/(4^n*c^n
)   Int[(A + B*Tan[d + e*x])*(b + 2*c*Tan[d + e*x])^(2*n), x], x] /; FreeQ[
{a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[n]
```

3.510.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.08

method	result
norman	$\left(a^{5}-2a^{3}b^{2}-3a\,b^{4}\right)x-\frac{a\left(a^{4}-2a^{2}b^{2}-4b^{4}\right)\tan(ex+d)}{e}+\frac{a^{3}\left(a^{2}+4b^{2}\right)\tan(ex+d)^{3}}{3e}+\frac{a^{4}b\tan(ex+d)^{4}}{4e}+ 3$
derivativedivides	$\frac{\frac{a^{4}b\tan(ex+d)^{4}}{4}+\frac{a^{5}\tan(ex+d)^{3}}{3}+\frac{4a^{3}b^{2}\tan(ex+d)^{3}}{3}+\frac{3a^{4}b\tan(ex+d)^{2}}{2}+3a^{2}b^{3}\tan(ex+d)^{2}-\tan(ex+d)a^{5}+2\tan(ex+d)a^{3}b^{2}}{e}$
default	$\frac{\frac{a^{4}b\tan(ex+d)^{4}}{4}+\frac{a^{5}\tan(ex+d)^{3}}{3}+\frac{4a^{3}b^{2}\tan(ex+d)^{3}}{3}+\frac{3a^{4}b\tan(ex+d)^{2}}{2}+3a^{2}b^{3}\tan(ex+d)^{2}-\tan(ex+d)a^{5}+2\tan(ex+d)a^{3}b^{2}}{e}$
parallelrisch	$-\frac{-3a^{4}b\tan(ex+d)^{4}-4a^{5}\tan(ex+d)^{3}-16a^{3}b^{2}\tan(ex+d)^{3}-12a^{5}ex+24a^{3}b^{2}ex+36a\,b^{4}ex-18a^{4}b\tan(ex+d)^{2}-36a^{2}b^{3}}{}$
parts	$a\,b^{4}x+\frac{\left(4a^{2}b^{3}+b^{5}\right)\ln\left(1+\tan(ex+d)^{2}\right)}{2e}+\frac{\left(6a^{3}b^{2}+4a\,b^{4}\right)\left(\tan(ex+d)-\arctan(\tan(ex+d))\right)}{e}+\frac{\left(4a^{4}b+6a^{2}b^{3}\right)\left(\tan\right.}{}$
risch	$-\frac{4ia\left(3a^{4}e^{6i(ex+d)}+3a^{2}b^{2}e^{6i(ex+d)}-6b^{4}e^{6i(ex+d)}+9ia\,b^{3}e^{6i(ex+d)}+3ia^{3}b\,e^{2i(ex+d)}+6a^{4}e^{4i(ex+d)}-3a^{2}b^{2}e^{4i(ex+d)}-\right.}{}$

input
```
int((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^2,x,method=_R
ETURNVERBOSE)
```

output
```
(a^5-2*a^3*b^2-3*a*b^4)*x-a*(a^4-2*a^2*b^2-4*b^4)/e*tan(e*x+d)+1/3*a^3*(a^
2+4*b^2)/e*tan(e*x+d)^3+1/4*a^4*b/e*tan(e*x+d)^4+3/2*a^2*b*(a^2+2*b^2)/e*t
an(e*x+d)^2-1/2*b*(3*a^4+2*a^2*b^2-b^4)/e*ln(1+tan(e*x+d)^2)
```

3.510.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.03

$$\int(a+b\tan(d+ex))\left(b^{2}+2ab\tan(d+ex)+a^{2}\tan^{2}(d+ex)\right)^{2}\,dx$$
$$=\frac{3\,a^{4}b\tan\left(ex+d\right)^{4}+4\left(a^{5}+4\,a^{3}b^{2}\right)\tan\left(ex+d\right)^{3}+12\left(a^{5}-2\,a^{3}b^{2}-3\,ab^{4}\right)ex+18\left(a^{4}b+2\,a^{2}b^{3}\right)\tan\left(ex\right.}{12\,e}$$

input
```
integrate((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^2,x, al
gorithm="fricas")
```

output
```
1/12*(3*a^4*b*tan(e*x + d)^4 + 4*(a^5 + 4*a^3*b^2)*tan(e*x + d)^3 + 12*(a^
5 - 2*a^3*b^2 - 3*a*b^4)*e*x + 18*(a^4*b + 2*a^2*b^3)*tan(e*x + d)^2 + 6*(
3*a^4*b + 2*a^2*b^3 - b^5)*log(1/(tan(e*x + d)^2 + 1)) - 12*(a^5 - 2*a^3*b
^2 - 4*a*b^4)*tan(e*x + d))/e
```

3.510.6 Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.72

$$\int (a+b\tan(d+ex))\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^2\,dx$$
$$=\begin{cases} a^5x+\frac{a^5\tan^3(d+ex)}{3e}-\frac{a^5\tan(d+ex)}{e}-\frac{3a^4b\log(\tan^2(d+ex)+1)}{2e}+\frac{a^4b\tan^4(d+ex)}{4e}+\frac{3a^4b\tan^2(d+ex)}{2e}-2a^3b^2x+\frac{4a^3b^2\tan}{} \\ x(a+b\tan(d))\left(a^2\tan^2(d)+2ab\tan(d)+b^2\right)^2 \end{cases}$$

input
```
integrate((a+b*tan(e*x+d))*(b**2+2*a*b*tan(e*x+d)+a**2*tan(e*x+d)**2)**2,x
)
```

output
```
Piecewise((a**5*x + a**5*tan(d + e*x)**3/(3*e) - a**5*tan(d + e*x)/e - 3*a
**4*b*log(tan(d + e*x)**2 + 1)/(2*e) + a**4*b*tan(d + e*x)**4/(4*e) + 3*a*
*4*b*tan(d + e*x)**2/(2*e) - 2*a**3*b**2*x + 4*a**3*b**2*tan(d + e*x)**3/(
3*e) + 2*a**3*b**2*tan(d + e*x)/e - a**2*b**3*log(tan(d + e*x)**2 + 1)/e +
 3*a**2*b**3*tan(d + e*x)**2/e - 3*a*b**4*x + 4*a*b**4*tan(d + e*x)/e + b*
*5*log(tan(d + e*x)**2 + 1)/(2*e), Ne(e, 0)), (x*(a + b*tan(d))*(a**2*tan(
d)**2 + 2*a*b*tan(d) + b**2)**2, True))
```

3.510.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.04

$$\int (a+b\tan(d+ex))\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^2\,dx$$
$$=\frac{3\,a^4b\tan(ex+d)^4+4\,(a^5+4\,a^3b^2)\tan(ex+d)^3+18\,(a^4b+2\,a^2b^3)\tan(ex+d)^2+12\,(a^5-2\,a^3b^2-3\,a}{12\,e}$$

input
```
integrate((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^2,x, al
gorithm="maxima")
```

output
```
1/12*(3*a^4*b*tan(e*x + d)^4 + 4*(a^5 + 4*a^3*b^2)*tan(e*x + d)^3 + 18*(a^
4*b + 2*a^2*b^3)*tan(e*x + d)^2 + 12*(a^5 - 2*a^3*b^2 - 3*a*b^4)*(e*x + d)
 - 6*(3*a^4*b + 2*a^2*b^3 - b^5)*log(tan(e*x + d)^2 + 1) - 12*(a^5 - 2*a^3
*b^2 - 4*a*b^4)*tan(e*x + d))/e
```

3.510.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2007 vs. 2(138) = 276.

Time = 2.43 (sec) , antiderivative size = 2007, normalized size of antiderivative = 13.94

$$\int (a+b\tan(d+ex))\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^2,x, al
gorithm="giac")
```

output
```
1/12*(12*a^5*e*x*tan(e*x)^4*tan(d)^4 - 24*a^3*b^2*e*x*tan(e*x)^4*tan(d)^4
- 36*a*b^4*e*x*tan(e*x)^4*tan(d)^4 + 18*a^4*b*log(4*(tan(e*x)^2*tan(d)^2 -
 2*tan(e*x)*tan(d) + 1)/(tan(e*x)^2*tan(d)^2 + tan(e*x)^2 + tan(d)^2 + 1))
*tan(e*x)^4*tan(d)^4 + 12*a^2*b^3*log(4*(tan(e*x)^2*tan(d)^2 - 2*tan(e*x)*
tan(d) + 1)/(tan(e*x)^2*tan(d)^2 + tan(e*x)^2 + tan(d)^2 + 1))*tan(e*x)^4*
tan(d)^4 - 6*b^5*log(4*(tan(e*x)^2*tan(d)^2 - 2*tan(e*x)*tan(d) + 1)/(tan(
e*x)^2*tan(d)^2 + tan(e*x)^2 + tan(d)^2 + 1))*tan(e*x)^4*tan(d)^4 - 48*a^5
*e*x*tan(e*x)^3*tan(d)^3 + 96*a^3*b^2*e*x*tan(e*x)^3*tan(d)^3 + 144*a*b^4*
e*x*tan(e*x)^3*tan(d)^3 + 15*a^4*b*tan(e*x)^4*tan(d)^4 + 36*a^2*b^3*tan(e*
x)^4*tan(d)^4 - 72*a^4*b*log(4*(tan(e*x)^2*tan(d)^2 - 2*tan(e*x)*tan(d) +
1)/(tan(e*x)^2*tan(d)^2 + tan(e*x)^2 + tan(d)^2 + 1))*tan(e*x)^3*tan(d)^3
- 48*a^2*b^3*log(4*(tan(e*x)^2*tan(d)^2 - 2*tan(e*x)*tan(d) + 1)/(tan(e*x)
^2*tan(d)^2 + tan(e*x)^2 + tan(d)^2 + 1))*tan(e*x)^3*tan(d)^3 + 24*b^5*log
(4*(tan(e*x)^2*tan(d)^2 - 2*tan(e*x)*tan(d) + 1)/(tan(e*x)^2*tan(d)^2 + ta
n(e*x)^2 + tan(d)^2 + 1))*tan(e*x)^3*tan(d)^3 + 12*a^5*tan(e*x)^4*tan(d)^3
 - 24*a^3*b^2*tan(e*x)^4*tan(d)^3 - 48*a*b^4*tan(e*x)^4*tan(d)^3 + 12*a^5*
tan(e*x)^3*tan(d)^4 - 24*a^3*b^2*tan(e*x)^3*tan(d)^4 - 48*a*b^4*tan(e*x)^3
*tan(d)^4 + 72*a^5*e*x*tan(e*x)^2*tan(d)^2 - 144*a^3*b^2*e*x*tan(e*x)^2*ta
n(d)^2 - 216*a*b^4*e*x*tan(e*x)^2*tan(d)^2 + 18*a^4*b*tan(e*x)^4*tan(d)^2
+ 36*a^2*b^3*tan(e*x)^4*tan(d)^2 - 24*a^4*b*tan(e*x)^3*tan(d)^3 - 72*a^...
```

3.510.9 Mupad [B] (verification not implemented)

Time = 27.08 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.42

$$\int (a+b\tan(d+ex)) \left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^2 dx$$
$$= \frac{\tan(d+e\,x)^3 \left(\frac{a^5}{3}+\frac{4\,a^3\,b^2}{3}\right)}{e} + \frac{\tan(d+e\,x)\,\left(-a^5+2\,a^3\,b^2+4\,a\,b^4\right)}{e} - \frac{\ln\left(\tan(d+e\,x)^2+1\right)\,\left(\frac{3\,a^4\,b}{2}+a^2\,b^3-\frac{b^5}{2}\right)}{e} + \frac{\tan(d+e\,x)^2\,\left(\frac{3\,a^4\,b}{2}+3\,a^2\,b^3\right)}{e} + \frac{a^4\,b\tan(d+e\,x)^4}{4\,e} - \frac{a\,\mathrm{atan}\left(\frac{a\tan(d+e\,x)\,\left(a^2-3\,b^2\right)\,\left(a^2+b^2\right)}{-a^5+2\,a^3\,b^2+3\,a\,b^4}\right)\,\left(a^2-3\,b^2\right)\,\left(a^2+b^2\right)}{e}$$

input
```
int((a + b*tan(d + e*x))*(b^2 + a^2*tan(d + e*x)^2 + 2*a*b*tan(d + e*x))^2
,x)
```

output
```
(tan(d + e*x)^3*(a^5/3 + (4*a^3*b^2)/3))/e + (tan(d + e*x)*(4*a*b^4 - a^5
+ 2*a^3*b^2))/e - (log(tan(d + e*x)^2 + 1)*((3*a^4*b)/2 - b^5/2 + a^2*b^3)
)/e + (tan(d + e*x)^2*((3*a^4*b)/2 + 3*a^2*b^3))/e + (a^4*b*tan(d + e*x)^4
)/(4*e) - (a*atan((a*tan(d + e*x)*(a^2 - 3*b^2)*(a^2 + b^2))/(3*a*b^4 - a^
5 + 2*a^3*b^2))*(a^2 - 3*b^2)*(a^2 + b^2))/e
```

3.511 $\int (a+b\tan(d+ex))\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d\right.$

3.511.1 Optimal result

Integrand size = 37, antiderivative size = 72

$$\int (a+b\tan(d+ex))\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)\,dx$$
$$=-a\left(a^2+b^2\right)x-\frac{b(a^2+b^2)\log(\cos(d+ex))}{e}+\frac{2ab^2\tan(d+ex)}{e}+\frac{a^2(a+b\tan(d+ex))^2}{2be}$$

output
```
-a*(a^2+b^2)*x-b*(a^2+b^2)*ln(cos(e*x+d))/e+2*a*b^2*tan(e*x+d)/e+1/2*a^2*(
a+b*tan(e*x+d))^2/b/e
```

3.511.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.22

$$\int (a+b\tan(d+ex))\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)\,dx$$
$$=\frac{(a^2+b^2)\left((ia+b)\log(i-\tan(d+ex))+(-ia+b)\log(i+\tan(d+ex))\right)+2a(a^2+2b^2)\tan(d+ex)+a}{2e}$$

input
```
Integrate[(a + b*Tan[d + e*x])*(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x
]^2),x]
```

output
```
((a^2 + b^2)*((I*a + b)*Log[I - Tan[d + e*x]] + ((-I)*a + b)*Log[I + Tan[d
 + e*x]]) + 2*a*(a^2 + 2*b^2)*Tan[d + e*x] + a^2*b*Tan[d + e*x]^2)/(2*e)
```

3.511.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {3042, 4113, 3042, 4008, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\tan(d+ex))\left(a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(d+ex))\left(a^2\tan(d+ex)^2+2ab\tan(d+ex)+b^2\right)dx$$

$$\Big\downarrow \text{4113}$$

$$\int (a+b\tan(d+ex))\left(-a^2+2b\tan(d+ex)a+b^2\right)dx+\frac{a^2(a+b\tan(d+ex))^2}{2be}$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\tan(d+ex))\left(-a^2+2b\tan(d+ex)a+b^2\right)dx+\frac{a^2(a+b\tan(d+ex))^2}{2be}$$

$$\Big\downarrow \text{4008}$$

$$b\left(a^2+b^2\right)\int \tan(d+ex)dx-ax\left(a^2+b^2\right)+\frac{a^2(a+b\tan(d+ex))^2}{2be}+\frac{2ab^2\tan(d+ex)}{e}$$

$$\Big\downarrow \text{3042}$$

$$b\left(a^2+b^2\right)\int \tan(d+ex)dx-ax\left(a^2+b^2\right)+\frac{a^2(a+b\tan(d+ex))^2}{2be}+\frac{2ab^2\tan(d+ex)}{e}$$

$$\Big\downarrow \text{3956}$$

$$-\frac{b\left(a^2+b^2\right)\log(\cos(d+ex))}{e}-ax\left(a^2+b^2\right)+\frac{a^2(a+b\tan(d+ex))^2}{2be}+\frac{2ab^2\tan(d+ex)}{e}$$

input
```
Int[(a + b*Tan[d + e*x])*(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2),x
]
```

output
```
-(a*(a^2 + b^2)*x) - (b*(a^2 + b^2)*Log[Cos[d + e*x]])/e + (2*a*b^2*Tan[d
+ e*x])/e + (a^2*(a + b*Tan[d + e*x])^2)/(2*b*e)
```

3.511.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4008
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Simp[b*d*(Tan[e + f*x]/f),
x] + Simp[(b*c + a*d)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

rule 4113
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*tan[(e_.)
+ (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a +
 b*Tan[e + f*x])^(m + 1)/(b*f*(m + 1))), x] + Int[(a + b*Tan[e + f*x])^m*Si
mp[A - C + B*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
NeQ[A*b^2 - a*b*B + a^2*C, 0] &&  !LeQ[m, -1]
```

3.511.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.07

method	result
norman	$\left(-a^{3}-a\,b^{2}\right) x+\frac{a\left(a^{2}+2 b^{2}\right) \tan \left(e x+d\right)}{e}+\frac{a^{2} b \tan \left(e x+d\right)^{2}}{2 e}+\frac{b\left(a^{2}+b^{2}\right) \ln \left(1+\tan \left(e x+d\right)^{2}\right)}{2 e}$
derivativedivides	$\frac{\frac{a^{2} b \tan \left(e x+d\right)^{2}}{2}+\tan \left(e x+d\right) a^{3}+2 \tan \left(e x+d\right) a\,b^{2}+\frac{\left(a^{2} b+b^{3}\right) \ln \left(1+\tan \left(e x+d\right)^{2}\right)}{2}+\left(-a^{3}-a\,b^{2}\right) \arctan \left(\tan \left(e x+d\right)\right)}{e}$
default	$\frac{\frac{a^{2} b \tan \left(e x+d\right)^{2}}{2}+\tan \left(e x+d\right) a^{3}+2 \tan \left(e x+d\right) a\,b^{2}+\frac{\left(a^{2} b+b^{3}\right) \ln \left(1+\tan \left(e x+d\right)^{2}\right)}{2}+\left(-a^{3}-a\,b^{2}\right) \arctan \left(\tan \left(e x+d\right)\right)}{e}$
parallelrisch	$\frac{-2 a^{3} e x-2 a\,b^{2} e x+a^{2} b \tan \left(e x+d\right)^{2}+\ln \left(1+\tan \left(e x+d\right)^{2}\right) a^{2} b+\ln \left(1+\tan \left(e x+d\right)^{2}\right) b^{3}+2 \tan \left(e x+d\right) a^{3}+4 \tan \left(e x+d\right) a\,b^{2}}{2 e}$
parts	$a\,b^{2} x+\frac{\left(2 a^{2} b+b^{3}\right) \ln \left(1+\tan \left(e x+d\right)^{2}\right)}{2 e}+\frac{\left(a^{3}+2 a\,b^{2}\right) \left(\tan \left(e x+d\right)-\arctan \left(\tan \left(e x+d\right)\right)\right)}{e}+\frac{a^{2} b \left(\frac{\tan \left(e x+d\right)^{2}}{2}-\frac{\ln \left(1+\right.}{}\right.}{e}$
risch	$i a^{2} b x+i b^{3} x-a^{3} x-a\,b^{2} x+\frac{2 i b\,a^{2} d}{e}+\frac{2 i b^{3} d}{e}+\frac{2 i a \left({\mathrm e}^{2 i \left(e x+d\right)} a^{2}+2 \,{\mathrm e}^{2 i \left(e x+d\right)} b^{2}-i a b\,{\mathrm e}^{2 i \left(e x+d\right)}+a^{2}+2 b^{2}\right)}{e \left(1+{\mathrm e}^{2 i \left(e x+d\right)}\right)^{2}}$

input
```
int((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2),x,method=_RET
URNVERBOSE)
```

output
```
(-a^3-a*b^2)*x+a*(a^2+2*b^2)*tan(e*x+d)/e+1/2*a^2*b/e*tan(e*x+d)^2+1/2*b*(
a^2+b^2)/e*ln(1+tan(e*x+d)^2)
```

3.511.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.03

$$\int (a+b\tan(d+ex))\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)\,dx$$
$$=\frac{a^2b\tan\left(ex+d\right)^2-2\left(a^3+ab^2\right)ex-\left(a^2b+b^3\right)\log\left(\frac{1}{\tan(ex+d)^2+1}\right)+2\left(a^3+2\,ab^2\right)\tan\left(ex+d\right)}{2\,e}$$

input
```
integrate((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2),x, algo
rithm="fricas")
```

output
```
1/2*(a^2*b*tan(e*x + d)^2 - 2*(a^3 + a*b^2)*e*x - (a^2*b + b^3)*log(1/(tan
(e*x + d)^2 + 1)) + 2*(a^3 + 2*a*b^2)*tan(e*x + d))/e
```

3.511.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.69

$$\int (a+b\tan(d+ex))\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)\,dx$$
$$=\begin{cases}-a^3x+\frac{a^3\tan\left(d+ex\right)}{e}+\frac{a^2b\log\left(\tan^2\left(d+ex\right)+1\right)}{2e}+\frac{a^2b\tan^2\left(d+ex\right)}{2e}-ab^2x+\frac{2ab^2\tan\left(d+ex\right)}{e}+\frac{b^3\log\left(\tan^2\left(d+ex\right)+1\right)}{2e} & \text{for} \\ x(a+b\tan\left(d\right))\left(a^2\tan^2\left(d\right)+2ab\tan\left(d\right)+b^2\right) & \text{oth}\end{cases}$$

input
```
integrate((a+b*tan(e*x+d))*(b**2+2*a*b*tan(e*x+d)+a**2*tan(e*x+d)**2),x)
```

output
```
Piecewise((-a**3*x + a**3*tan(d + e*x)/e + a**2*b*log(tan(d + e*x)**2 + 1)
/(2*e) + a**2*b*tan(d + e*x)**2/(2*e) - a*b**2*x + 2*a*b**2*tan(d + e*x)/e
 + b**3*log(tan(d + e*x)**2 + 1)/(2*e), Ne(e, 0)), (x*(a + b*tan(d))*(a**2
*tan(d)**2 + 2*a*b*tan(d) + b**2), True))
```

3.511.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.03

$$\int (a+b\tan(d+ex))\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)\,dx$$
$$=\frac{a^2b\tan\left(ex+d\right)^2-2\left(a^3+ab^2\right)(ex+d)+\left(a^2b+b^3\right)\log\left(\tan\left(ex+d\right)^2+1\right)+2\left(a^3+2\,ab^2\right)\tan\left(ex+d\right)}{2\,e}$$

input
```
integrate((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2),x, algo
rithm="maxima")
```

output
```
1/2*(a^2*b*tan(e*x + d)^2 - 2*(a^3 + a*b^2)*(e*x + d) + (a^2*b + b^3)*log(
tan(e*x + d)^2 + 1) + 2*(a^3 + 2*a*b^2)*tan(e*x + d))/e
```

3.511.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 592 vs. 2(70) = 140.

Time = 0.64 (sec) , antiderivative size = 592, normalized size of antiderivative = 8.22

$$\int (a+b\tan(d+ex))\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)\,dx=$$
$$-\frac{2\,a^3ex\tan\left(ex\right)^2\tan\left(d\right)^2+2\,ab^2ex\tan\left(ex\right)^2\tan\left(d\right)^2+a^2b\log\left(\frac{4\left(\tan(ex)^2\tan(d)^2-2\,\tan(ex)\tan(d)+1\right)}{\tan(ex)^2\tan(d)^2+\tan(ex)^2+\tan(d)^2+1}\right)\tan\left(e\right.}{}$$

input
```
integrate((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2),x, algo
rithm="giac")
```

output
```
-1/2*(2*a^3*e*x*tan(e*x)^2*tan(d)^2 + 2*a*b^2*e*x*tan(e*x)^2*tan(d)^2 + a^
2*b*log(4*(tan(e*x)^2*tan(d)^2 - 2*tan(e*x)*tan(d) + 1)/(tan(e*x)^2*tan(d)
^2 + tan(e*x)^2 + tan(d)^2 + 1))*tan(e*x)^2*tan(d)^2 + b^3*log(4*(tan(e*x)
^2*tan(d)^2 - 2*tan(e*x)*tan(d) + 1)/(tan(e*x)^2*tan(d)^2 + tan(e*x)^2 + t
an(d)^2 + 1))*tan(e*x)^2*tan(d)^2 - 4*a^3*e*x*tan(e*x)*tan(d) - 4*a*b^2*e*
x*tan(e*x)*tan(d) - a^2*b*tan(e*x)^2*tan(d)^2 - 2*a^2*b*log(4*(tan(e*x)^2*
tan(d)^2 - 2*tan(e*x)*tan(d) + 1)/(tan(e*x)^2*tan(d)^2 + tan(e*x)^2 + tan(
d)^2 + 1))*tan(e*x)*tan(d) - 2*b^3*log(4*(tan(e*x)^2*tan(d)^2 - 2*tan(e*x)
*tan(d) + 1)/(tan(e*x)^2*tan(d)^2 + tan(e*x)^2 + tan(d)^2 + 1))*tan(e*x)*t
an(d) + 2*a^3*tan(e*x)^2*tan(d) + 4*a*b^2*tan(e*x)^2*tan(d) + 2*a^3*tan(e*
x)*tan(d)^2 + 4*a*b^2*tan(e*x)*tan(d)^2 + 2*a^3*e*x + 2*a*b^2*e*x - a^2*b*
tan(e*x)^2 - a^2*b*tan(d)^2 + a^2*b*log(4*(tan(e*x)^2*tan(d)^2 - 2*tan(e*x
)*tan(d) + 1)/(tan(e*x)^2*tan(d)^2 + tan(e*x)^2 + tan(d)^2 + 1)) + b^3*log
(4*(tan(e*x)^2*tan(d)^2 - 2*tan(e*x)*tan(d) + 1)/(tan(e*x)^2*tan(d)^2 + ta
n(e*x)^2 + tan(d)^2 + 1)) - 2*a^3*tan(e*x) - 4*a*b^2*tan(e*x) - 2*a^3*tan(
d) - 4*a*b^2*tan(d) - a^2*b)/(e*tan(e*x)^2*tan(d)^2 - 2*e*tan(e*x)*tan(d)
+ e)
```

3.511.9 Mupad [B] (verification not implemented)

Time = 26.97 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.46

$$\int (a+b\tan(d+ex))\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)\,dx = \frac{\tan(d+e\,x)\,\left(a^3+2\,a\,b^2\right)}{e}+\frac{\ln\left(\tan(d+e\,x)^2+1\right)\,\left(\frac{a^2\,b}{2}+\frac{b^3}{2}\right)}{e} + \frac{a^2\,b\tan(d+e\,x)^2}{2\,e}-\frac{a\,\mathrm{atan}\left(\frac{a\tan(d+e\,x)\,\left(a^2+b^2\right)}{a^3+a\,b^2}\right)\,\left(a^2+b^2\right)}{e}$$

input
```
int((a + b*tan(d + e*x))*(b^2 + a^2*tan(d + e*x)^2 + 2*a*b*tan(d + e*x)),x
)
```

output
```
(tan(d + e*x)*(2*a*b^2 + a^3))/e + (log(tan(d + e*x)^2 + 1)*((a^2*b)/2 + b
^3/2))/e + (a^2*b*tan(d + e*x)^2)/(2*e) - (a*atan((a*tan(d + e*x)*(a^2 + b
^2))/(a*b^2 + a^3))*(a^2 + b^2))/e
```

3.512 $\int \frac{a+b\tan(d+ex)}{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}\,dx$

3.512.1 Optimal result

Integrand size = 39, antiderivative size = 101

$$\int \frac{a+b\tan(d+ex)}{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}\,dx = -\frac{a(a^2-3b^2)\,x}{(a^2+b^2)^2}+\frac{b(3a^2-b^2)\log(b\cos(d+ex)+a\sin(d+ex))}{(a^2+b^2)^2 e} - \frac{a^2-b^2}{(a^2+b^2)\,e(b+a\tan(d+ex))}$$

output
```
-a*(a^2-3*b^2)*x/(a^2+b^2)^2+b*(3*a^2-b^2)*ln(b*cos(e*x+d)+a*sin(e*x+d))/(
a^2+b^2)^2/e+(-a^2+b^2)/(a^2+b^2)/e/(b+a*tan(e*x+d))
```

3.512.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.91 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.85

$$\int \frac{a+b\tan(d+ex)}{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}\,dx = \frac{\frac{b(-((a+ib)\log(i-\tan(d+ex)))-(a-ib)\log(i+\tan(d+ex))+2a\log(b+a\tan(d+ex)))}{a^2+b^2}+(a-b)(a+b)\left(\frac{i\log(i-\tan(d+ex))}{(a-ib)^2}-\frac{i\log(i+\mathrm{ta}}{(a+}\right.}{2ae}$$

input
```
Integrate[(a + b*Tan[d + e*x])/(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x
]^2),x]
```

output
```
((b*(-((a + I*b)*Log[I - Tan[d + e*x]]) - (a - I*b)*Log[I + Tan[d + e*x]]
+ 2*a*Log[b + a*Tan[d + e*x]]))/(a^2 + b^2) + (a - b)*(a + b)*((I*Log[I -
Tan[d + e*x]])/(a - I*b)^2 - (I*Log[I + Tan[d + e*x]])/(a + I*b)^2 + (2*a*
(2*b*Log[b + a*Tan[d + e*x]] - (a^2 + b^2)/(b + a*Tan[d + e*x])))/(a^2 + b
^2)^2))/(2*a*e)
```

3.512.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.24, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 4191, 27, 3042, 4012, 3042, 4014, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\tan(d+ex)}{a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\tan(d+ex)}{a^2\tan(d+ex)^2+2ab\tan(d+ex)+b^2}dx$$

$$\Big\downarrow \text{4191}$$

$$4a^2\int \frac{a+b\tan(d+ex)}{4\left(\tan(d+ex)a^2+ba\right)^2}dx$$

$$\Big\downarrow \text{27}$$

$$a^2\int \frac{a+b\tan(d+ex)}{\left(\tan(d+ex)a^2+ba\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$a^2\int \frac{a+b\tan(d+ex)}{\left(\tan(d+ex)a^2+ba\right)^2}dx$$

$$\Big\downarrow \text{4012}$$

$$a^2\left(\frac{\int \frac{2a^2b-a(a^2-b^2)\tan(d+ex)}{\tan(d+ex)a^2+ba}dx}{a^2\left(a^2+b^2\right)}-\frac{a^2-b^2}{e\left(a^2+b^2\right)\left(a^3\tan(d+ex)+a^2b\right)}\right)$$

$$\Big\downarrow \; 3042$$

$$a^2\left(\frac{\int \frac{2a^2b-a(a^2-b^2)\tan(d+ex)}{\tan(d+ex)a^2+ba}dx}{a^2(a^2+b^2)}-\frac{a^2-b^2}{e(a^2+b^2)(a^3\tan(d+ex)+a^2b)}\right)$$

$$\Big\downarrow \; 4014$$

$$a^2\left(\frac{\frac{b(3a^2-b^2)\int \frac{a^2-ab\tan(d+ex)}{\tan(d+ex)a^2+ba}dx}{a^2+b^2}-\frac{ax(a^2-3b^2)}{a^2+b^2}}{a^2(a^2+b^2)}-\frac{a^2-b^2}{e(a^2+b^2)(a^3\tan(d+ex)+a^2b)}\right)$$

$$\Big\downarrow \; 3042$$

$$a^2\left(\frac{\frac{b(3a^2-b^2)\int \frac{a^2-ab\tan(d+ex)}{\tan(d+ex)a^2+ba}dx}{a^2+b^2}-\frac{ax(a^2-3b^2)}{a^2+b^2}}{a^2(a^2+b^2)}-\frac{a^2-b^2}{e(a^2+b^2)(a^3\tan(d+ex)+a^2b)}\right)$$

$$\Big\downarrow \; 4013$$

$$a^2\left(\frac{\frac{b(3a^2-b^2)\log(a\sin(d+ex)+b\cos(d+ex))}{e(a^2+b^2)}-\frac{ax(a^2-3b^2)}{a^2+b^2}}{a^2(a^2+b^2)}-\frac{a^2-b^2}{e(a^2+b^2)(a^3\tan(d+ex)+a^2b)}\right)$$

input
```
Int[(a + b*Tan[d + e*x])/(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2),x
]
```

output
```
a^2*((-((a*(a^2 - 3*b^2)*x)/(a^2 + b^2)) + (b*(3*a^2 - b^2)*Log[b*Cos[d +
e*x] + a*Sin[d + e*x]])/((a^2 + b^2)*e))/(a^2*(a^2 + b^2)) - (a^2 - b^2)/(
(a^2 + b^2)*e*(a^2*b + a^3*Tan[d + e*x])))
```

3.512.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.512. $\int \frac{a+b\tan(d+ex)}{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}\,dx$

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4191
```
Int[((A_) + (B_.)*tan[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*tan[(d_.) + (e_.)*
(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Simp[1/(4^n*c^n
)   Int[(A + B*Tan[d + e*x])*(b + 2*c*Tan[d + e*x])^(2*n), x], x] /; FreeQ[
{a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[n]
```

3.512.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.24

method	result
derivativedivides	$\frac{-\frac{a^2-b^2}{(a^2+b^2)(b+a\tan(ex+d))}+\frac{b(3a^2-b^2)\ln(b+a\tan(ex+d))}{(a^2+b^2)^2}+\frac{\frac{(-3a^2b+b^3)\ln(1+\tan(ex+d)^2)}{2}+(-a^3+3ab^2)\arctan(\tan(ex+d))}{(a^2+b^2)^2}}{e}$
default	$\frac{-\frac{a^2-b^2}{(a^2+b^2)(b+a\tan(ex+d))}+\frac{b(3a^2-b^2)\ln(b+a\tan(ex+d))}{(a^2+b^2)^2}+\frac{\frac{(-3a^2b+b^3)\ln(1+\tan(ex+d)^2)}{2}+(-a^3+3ab^2)\arctan(\tan(ex+d))}{(a^2+b^2)^2}}{e}$
norman	$\frac{\frac{(a^2-b^2)a\tan(ex+d)}{be(a^2+b^2)}-\frac{a^2(a^2-3b^2)x\tan(ex+d)}{(a^2+b^2)^2}-\frac{(a^2-3b^2)abx}{(a^2+b^2)^2}}{b+a\tan(ex+d)}+\frac{b(3a^2-b^2)\ln(b+a\tan(ex+d))}{e(a^4+2a^2b^2+b^4)}-\frac{b(3a^2-b^2)\ln(1+\tan}{2e(a^4+2a^2b^2}$
parallelrisch	$-\frac{2x\tan(ex+d)a^4be-6x\tan(ex+d)a^2b^3e+3\ln(1+\tan(ex+d)^2)\tan(ex+d)a^3b^2-\ln(1+\tan(ex+d)^2)\tan(ex+d)ab^4-}{}$
risch	$\frac{ixb}{2iab+a^2-b^2}-\frac{xa}{2iab+a^2-b^2}-\frac{6iba^2x}{a^4+2a^2b^2+b^4}+\frac{2ib^3x}{a^4+2a^2b^2+b^4}-\frac{6iba^2d}{e(a^4+2a^2b^2+b^4)}+\frac{2ib^3d}{e(a^4+2a^2b^2+b^4)}-$

input
```
int((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2),x,method=_RET
URNVERBOSE)
```

output
```
1/e*(-(a^2-b^2)/(a^2+b^2)/(b+a*tan(e*x+d))+b*(3*a^2-b^2)/(a^2+b^2)^2*ln(b+
a*tan(e*x+d))+1/(a^2+b^2)^2*(1/2*(-3*a^2*b+b^3)*ln(1+tan(e*x+d)^2)+(-a^3+3
*a*b^2)*arctan(tan(e*x+d))))
```

3.512.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.89

$$\int \frac{a+b\tan(d+ex)}{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}\,dx =$$

$$-\frac{2a^4-2a^2b^2+2(a^3b-3ab^3)ex-(3a^2b^2-b^4+(3a^3b-ab^3)\tan(ex+d))\log\left(\frac{a^2\tan(ex+d)^2+2ab\tan(ex+}{\tan(ex+d)^2+1}\right.}{2((a^5+2a^3b^2+ab^4)e\tan(ex+d)+(a^4b+2a^2b^3+b}$$

input
```
integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2),x, algo
rithm="fricas")
```

output
```
-1/2*(2*a^4 - 2*a^2*b^2 + 2*(a^3*b - 3*a*b^3)*e*x - (3*a^2*b^2 - b^4 + (3*
a^3*b - a*b^3)*tan(e*x + d))*log((a^2*tan(e*x + d)^2 + 2*a*b*tan(e*x + d)
+ b^2)/(tan(e*x + d)^2 + 1)) - 2*(a^3*b - a*b^3 - (a^4 - 3*a^2*b^2)*e*x)*t
an(e*x + d))/((a^5 + 2*a^3*b^2 + a*b^4)*e*tan(e*x + d) + (a^4*b + 2*a^2*b^
3 + b^5)*e)
```

3.512.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 1360, normalized size of antiderivative = 13.47

$$\int \frac{a+b\tan(d+ex)}{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(e*x+d))/(b**2+2*a*b*tan(e*x+d)+a**2*tan(e*x+d)**2),x)
```

output
```
Piecewise((zoo*x*tan(d), Eq(a, 0) & Eq(b, 0) & Eq(e, 0)), (log(tan(d + e*x
)**2 + 1)/(2*b*e), Eq(a, 0)), (I/(2*a*e*tan(d + e*x)**2 - 4*I*a*e*tan(d +
e*x) - 2*a*e), Eq(b, -I*a)), (-I/(2*a*e*tan(d + e*x)**2 + 4*I*a*e*tan(d +
e*x) - 2*a*e), Eq(b, I*a)), (x*(a + b*tan(d))/(a**2*tan(d)**2 + 2*a*b*tan(
d) + b**2), Eq(e, 0)), (-2*a**4*e*x*tan(d + e*x)/(2*a**5*e*tan(d + e*x) +
2*a**4*b*e + 4*a**3*b**2*e*tan(d + e*x) + 4*a**2*b**3*e + 2*a*b**4*e*tan(d
 + e*x) + 2*b**5*e) - 2*a**4/(2*a**5*e*tan(d + e*x) + 2*a**4*b*e + 4*a**3*
b**2*e*tan(d + e*x) + 4*a**2*b**3*e + 2*a*b**4*e*tan(d + e*x) + 2*b**5*e)
- 2*a**3*b*e*x/(2*a**5*e*tan(d + e*x) + 2*a**4*b*e + 4*a**3*b**2*e*tan(d +
 e*x) + 4*a**2*b**3*e + 2*a*b**4*e*tan(d + e*x) + 2*b**5*e) + 6*a**3*b*log
(tan(d + e*x) + b/a)*tan(d + e*x)/(2*a**5*e*tan(d + e*x) + 2*a**4*b*e + 4*
a**3*b**2*e*tan(d + e*x) + 4*a**2*b**3*e + 2*a*b**4*e*tan(d + e*x) + 2*b**
5*e) - 3*a**3*b*log(tan(d + e*x)**2 + 1)*tan(d + e*x)/(2*a**5*e*tan(d + e*
x) + 2*a**4*b*e + 4*a**3*b**2*e*tan(d + e*x) + 4*a**2*b**3*e + 2*a*b**4*e*
tan(d + e*x) + 2*b**5*e) + 6*a**2*b**2*e*x*tan(d + e*x)/(2*a**5*e*tan(d +
e*x) + 2*a**4*b*e + 4*a**3*b**2*e*tan(d + e*x) + 4*a**2*b**3*e + 2*a*b**4*
e*tan(d + e*x) + 2*b**5*e) + 6*a**2*b**2*log(tan(d + e*x) + b/a)/(2*a**5*e
*tan(d + e*x) + 2*a**4*b*e + 4*a**3*b**2*e*tan(d + e*x) + 4*a**2*b**3*e +
2*a*b**4*e*tan(d + e*x) + 2*b**5*e) - 3*a**2*b**2*log(tan(d + e*x)**2 + 1)
/(2*a**5*e*tan(d + e*x) + 2*a**4*b*e + 4*a**3*b**2*e*tan(d + e*x) + 4*a...
```

3.512.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.59

$$\int \frac{a+b\tan(d+ex)}{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}\,dx =$$
$$-\frac{\frac{2\,(a^3-3\,ab^2)(ex+d)}{a^4+2\,a^2b^2+b^4}-\frac{2\,(3\,a^2b-b^3)\log(a\tan(ex+d)+b)}{a^4+2\,a^2b^2+b^4}+\frac{(3\,a^2b-b^3)\log\left(\tan(ex+d)^2+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{2\,(a^2-b^2)}{a^2b+b^3+(a^3+ab^2)\tan(ex+d)}}{2\,e}$$

input
```
integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2),x, algo
rithm="maxima")
```

output
```
-1/2*(2*(a^3 - 3*a*b^2)*(e*x + d)/(a^4 + 2*a^2*b^2 + b^4) - 2*(3*a^2*b - b
^3)*log(a*tan(e*x + d) + b)/(a^4 + 2*a^2*b^2 + b^4) + (3*a^2*b - b^3)*log(
tan(e*x + d)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + 2*(a^2 - b^2)/(a^2*b + b^3 +
 (a^3 + a*b^2)*tan(e*x + d)))/e
```

3.512.8 Giac [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.97

$$\int \frac{a+b\tan(d+ex)}{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}\,dx =$$
$$-\frac{\frac{2\,(a^3-3\,ab^2)(ex+d)}{a^4+2\,a^2b^2+b^4}+\frac{(3\,a^2b-b^3)\log\left(\tan(ex+d)^2+1\right)}{a^4+2\,a^2b^2+b^4}-\frac{2\,(3\,a^3b-ab^3)\log(|a\tan(ex+d)+b|)}{a^5+2\,a^3b^2+ab^4}+\frac{2\,(3\,a^3b\tan(ex+d)-ab^3\tan(ex+d)+a^4\cdots}{(a^4+2\,a^2b^2+b^4)(a\tan(ex+d\cdots}}{2\,e}$$

input
```
integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2),x, algo
rithm="giac")
```

output
```
-1/2*(2*(a^3 - 3*a*b^2)*(e*x + d)/(a^4 + 2*a^2*b^2 + b^4) + (3*a^2*b - b^3
)*log(tan(e*x + d)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) - 2*(3*a^3*b - a*b^3)*lo
g(abs(a*tan(e*x + d) + b))/(a^5 + 2*a^3*b^2 + a*b^4) + 2*(3*a^3*b*tan(e*x
+ d) - a*b^3*tan(e*x + d) + a^4 + 3*a^2*b^2 - 2*b^4)/((a^4 + 2*a^2*b^2 + b
^4)*(a*tan(e*x + d) + b)))/e
```

3.512.9 Mupad [B] (verification not implemented)

Time = 28.05 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.50

$$\int \frac{a+b\tan(d+ex)}{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}\,dx = \frac{b\,\ln\left(b+a\tan(d+e\,x)\right)\,\left(3\,a^2-b^2\right)}{e\,{\left(a^2+b^2\right)}^2} - \frac{\ln\left(\tan(d+e\,x)+1{}\mathrm{i}\right)\,\left(a-b\,1{}\mathrm{i}\right)}{2\,e\,\left(-a^2\,1{}\mathrm{i}+2\,a\,b+b^2\,1{}\mathrm{i}\right)} - \frac{a^2-b^2}{e\,\left(a^2+b^2\right)\,\left(b+a\tan\left(d+e\,x\right)\right)} - \frac{\ln\left(\tan(d+e\,x)-\mathrm{i}\right)\,\left(a+b\,1{}\mathrm{i}\right)}{2\,e\,\left(a^2\,1{}\mathrm{i}+2\,a\,b-b^2\,1{}\mathrm{i}\right)}$$

input
```
int((a + b*tan(d + e*x))/(b^2 + a^2*tan(d + e*x)^2 + 2*a*b*tan(d + e*x)),x
)
```

output
```
(b*log(b + a*tan(d + e*x))*(3*a^2 - b^2))/(e*(a^2 + b^2)^2) - (log(tan(d +
 e*x) + 1i)*(a - b*1i))/(2*e*(2*a*b - a^2*1i + b^2*1i)) - (a^2 - b^2)/(e*(
a^2 + b^2)*(b + a*tan(d + e*x))) - (log(tan(d + e*x) - 1i)*(a + b*1i))/(2*
e*(2*a*b + a^2*1i - b^2*1i))
```

3.512. $\int \frac{a+b\tan(d+ex)}{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}\,dx$

3.513 $\int \frac{a+b\tan(d+ex)}{\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^2}\,dx$

3.513.1 Optimal result

Integrand size = 39, antiderivative size = 197

$$\int \frac{a+b\tan(d+ex)}{\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^2}\,dx = \frac{a(a^4-10a^2b^2+5b^4)\,x}{(a^2+b^2)^4} - \frac{b(5a^4-10a^2b^2+b^4)\log(b\cos(d+ex)+a\sin(d+ex))}{(a^2+b^2)^4\,e} - \frac{a^2-b^2}{3\,(a^2+b^2)\,e(b+a\tan(d+ex))^3} - \frac{b(3a^2-b^2)}{2\,(a^2+b^2)^2\,e(b+a\tan(d+ex))^2} + \frac{a^4-6a^2b^2+b^4}{(a^2+b^2)^3\,e(b+a\tan(d+ex))}$$

output
```
a*(a^4-10*a^2*b^2+5*b^4)*x/(a^2+b^2)^4-b*(5*a^4-10*a^2*b^2+b^4)*ln(b*cos(e
*x+d)+a*sin(e*x+d))/(a^2+b^2)^4/e+1/3*(-a^2+b^2)/(a^2+b^2)/e/(b+a*tan(e*x+
d))^3-1/2*b*(3*a^2-b^2)/(a^2+b^2)^2/e/(b+a*tan(e*x+d))^2+(a^4-6*a^2*b^2+b^
4)/(a^2+b^2)^3/e/(b+a*tan(e*x+d))
```

3.513.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.04 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.56

$$\int \frac{a + b\tan(d+ex)}{\left(b^2 + 2ab\tan(d+ex) + a^2\tan^2(d+ex)\right)^2}\,dx$$

$$= \frac{-\left((a-b)(a+b)\left(\frac{3i\log(i-\tan(d+ex))}{(a-ib)^4} - \frac{3i\log(i+\tan(d+ex))}{(a+ib)^4} + \frac{24a(a-b)b(a+b)\log(b+a\tan(d+ex))}{(a^2+b^2)^4} + \frac{2a}{(a^2+b^2)(b+a\tan(d+ex))^3}$$

input
```
Integrate[(a + b*Tan[d + e*x])/(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x
]^2)^2,x]
```

output
```
(-((a - b)*(a + b)*(((3*I)*Log[I - Tan[d + e*x]])/(a - I*b)^4 - ((3*I)*Log
[I + Tan[d + e*x]])/(a + I*b)^4 + (24*a*(a - b)*b*(a + b)*Log[b + a*Tan[d
+ e*x]])/(a^2 + b^2)^4 + (2*a)/((a^2 + b^2)*(b + a*Tan[d + e*x])^3) + (6*a
*b)/((a^2 + b^2)^2*(b + a*Tan[d + e*x])^2) - (6*a*(a^2 - 3*b^2))/((a^2 + b
^2)^3*(b + a*Tan[d + e*x])))) + 3*b*(Log[I - Tan[d + e*x]]/(a - I*b)^3 + L
og[I + Tan[d + e*x]]/(a + I*b)^3 + (a*(-2*(a^2 - 3*b^2)*Log[b + a*Tan[d +
e*x]] - ((a^2 + b^2)*(a^2 + 5*b^2 + 4*a*b*Tan[d + e*x]))/(b + a*Tan[d + e*
x])^2))/(a^2 + b^2)^3))/(6*a*e)
```

3.513.3 Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.32, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.359$, Rules used = {3042, 4191, 27, 3042, 4012, 3042, 4012, 25, 3042, 4012, 3042, 4014, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b\tan(d+ex)}{\left(a^2\tan^2(d+ex) + 2ab\tan(d+ex) + b^2\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a + b\tan(d+ex)}{\left(a^2\tan(d+ex)^2 + 2ab\tan(d+ex) + b^2\right)^2}dx$$

$$\Big\downarrow \text{4191}$$

$$16a^4 \int \frac{a+b\tan(d+ex)}{16\left(\tan(d+ex)a^2+ba\right)^4}dx$$

$$\Big\downarrow \text{27}$$

$$a^4 \int \frac{a+b\tan(d+ex)}{\left(\tan(d+ex)a^2+ba\right)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$a^4 \int \frac{a+b\tan(d+ex)}{\left(\tan(d+ex)a^2+ba\right)^4}dx$$

$$\Big\downarrow \text{4012}$$

$$a^4\left(\frac{\int \frac{2a^2b-a\left(a^2-b^2\right)\tan(d+ex)}{\left(\tan(d+ex)a^2+ba\right)^3}dx}{a^2\left(a^2+b^2\right)}-\frac{a^2-b^2}{3ae\left(a^2+b^2\right)\left(a^2\tan(d+ex)+ab\right)^3}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^4\left(\frac{\int \frac{2a^2b-a\left(a^2-b^2\right)\tan(d+ex)}{\left(\tan(d+ex)a^2+ba\right)^3}dx}{a^2\left(a^2+b^2\right)}-\frac{a^2-b^2}{3ae\left(a^2+b^2\right)\left(a^2\tan(d+ex)+ab\right)^3}\right)$$

$$\Big\downarrow \text{4012}$$

$$a^4\left(\frac{\frac{\int -\frac{\left(a^2-3b^2\right)a^3+b\left(3a^2-b^2\right)\tan(d+ex)a^2}{\left(\tan(d+ex)a^2+ba\right)^2}dx}{a^2(a^2+b^2)}-\frac{b(3a^2-b^2)}{2e(a^2+b^2)(a^2\tan(d+ex)+ab)^2}}{a^2\left(a^2+b^2\right)}-\frac{a^2-b^2}{3ae\left(a^2+b^2\right)\left(a^2\tan(d+ex)+ab\right)^3}\right)$$

$$\Big\downarrow \text{25}$$

$$a^4\left(\frac{-\frac{\int \frac{\left(a^2-3b^2\right)a^3+b\left(3a^2-b^2\right)\tan(d+ex)a^2}{\left(\tan(d+ex)a^2+ba\right)^2}dx}{a^2(a^2+b^2)}-\frac{b(3a^2-b^2)}{2e(a^2+b^2)(a^2\tan(d+ex)+ab)^2}}{a^2\left(a^2+b^2\right)}-\frac{a^2-b^2}{3ae\left(a^2+b^2\right)\left(a^2\tan(d+ex)+ab\right)^3}\right)$$

$$\Big\downarrow \text{3042}$$

3.513. $\int \frac{a+b\tan(d+ex)}{\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^2}\,dx$

$$a^4\left(\frac{-\frac{\int \frac{\left(a^2-3b^2\right)a^3+b\left(3a^2-b^2\right)\tan(d+ex)a^2}{\left(\tan(d+ex)a^2+ba\right)^2}dx}{a^2(a^2+b^2)}-\frac{b(3a^2-b^2)}{2e(a^2+b^2)(a^2\tan(d+ex)+ab)^2}}{a^2\left(a^2+b^2\right)}-\frac{a^2-b^2}{3ae\left(a^2+b^2\right)\left(a^2\tan(d+ex)+ab\right)^3}\right)$$

$\downarrow$ 4012

$$a^4\left(\frac{-\frac{\frac{\int\frac{4a^4b\left(a^2-b^2\right)-a^3\left(a^4-6b^2a^2+b^4\right)\tan(d+ex)}{\tan(d+ex)a^2+ba}dx}{a^2\left(a^2+b^2\right)}-\frac{a^4-6a^2b^2+b^4}{e\left(a^2+b^2\right)(a\tan(d+ex)+b)}}{a^2(a^2+b^2)}-\frac{b(3a^2-b^2)}{2e(a^2+b^2)(a^2\tan(d+ex)+ab)^2}}{a^2\left(a^2+b^2\right)}-\frac{a^2-\cdots}{3ae\left(a^2+b^2\right)\left(a^2\tan\cdots\right.}\right.$$

$\downarrow$ 3042

$$a^4\left(\frac{-\frac{\frac{\int\frac{4a^4b\left(a^2-b^2\right)-a^3\left(a^4-6b^2a^2+b^4\right)\tan(d+ex)}{\tan(d+ex)a^2+ba}dx}{a^2\left(a^2+b^2\right)}-\frac{a^4-6a^2b^2+b^4}{e\left(a^2+b^2\right)(a\tan(d+ex)+b)}}{a^2(a^2+b^2)}-\frac{b(3a^2-b^2)}{2e(a^2+b^2)(a^2\tan(d+ex)+ab)^2}}{a^2\left(a^2+b^2\right)}-\frac{a^2-\cdots}{3ae\left(a^2+b^2\right)\left(a^2\tan\cdots\right.}\right.$$

$\downarrow$ 4014

$$a^4\left(\frac{-\frac{\frac{\frac{a^2b\left(5a^4-10a^2b^2+b^4\right)\int\frac{a^2-ab\tan(d+ex)}{\tan(d+ex)a^2+ba}dx}{a^2+b^2}-\frac{a^3x\left(a^4-10a^2b^2+5b^4\right)}{a^2+b^2}}{a^2\left(a^2+b^2\right)}-\frac{a^4-6a^2b^2+b^4}{e\left(a^2+b^2\right)(a\tan(d+ex)+b)}}{a^2(a^2+b^2)}-\frac{b(3a^2-b^2)}{2e(a^2+b^2)(a^2\tan(d+ex)+ab)^2}}{a^2\left(a^2+b^2\right)}-\frac{\cdots}{3ae\left(\cdots\right.}\right.$$

$\downarrow$ 3042

$$a^4\left(\frac{-\frac{\frac{\frac{a^2b\left(5a^4-10a^2b^2+b^4\right)\int\frac{a^2-ab\tan(d+ex)}{\tan(d+ex)a^2+ba}dx}{a^2+b^2}-\frac{a^3x\left(a^4-10a^2b^2+5b^4\right)}{a^2+b^2}}{a^2\left(a^2+b^2\right)}-\frac{a^4-6a^2b^2+b^4}{e\left(a^2+b^2\right)(a\tan(d+ex)+b)}}{a^2(a^2+b^2)}-\frac{b(3a^2-b^2)}{2e(a^2+b^2)(a^2\tan(d+ex)+ab)^2}}{a^2\left(a^2+b^2\right)}-\frac{\cdots}{3ae\left(\cdots\right.}\right.$$

$\downarrow$ 4013

3.513. $\int \frac{a+b\tan(d+ex)}{(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex))^2}\,dx$

$$a^4 \left(\frac{-\frac{b(3a^2-b^2)}{2e(a^2+b^2)(a^2\tan(d+ex)+ab)^2} - \frac{\frac{\frac{a^2b\left(5a^4-10a^2b^2+b^4\right)\log(a\sin(d+ex)+b\cos(d+ex))}{e\left(a^2+b^2\right)} - \frac{a^3x\left(a^4-10a^2b^2+5b^4\right)}{a^2+b^2}}{a^2\left(a^2+b^2\right)} - \frac{a^4-6a^2b^2+b^4}{e\left(a^2+b^2\right)(a\tan(d+ex)+b)}}{a^2(a^2+b^2)}}{a^2\left(a^2+b^2\right)} - \right.$$

input
```
Int[(a + b*Tan[d + e*x])/(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2)^2
,x]
```

output
```
a^4*(-1/3*(a^2 - b^2)/(a*(a^2 + b^2)*e*(a*b + a^2*Tan[d + e*x])^3) + (-1/2
*(b*(3*a^2 - b^2))/((a^2 + b^2)*e*(a*b + a^2*Tan[d + e*x])^2) - ((-((a^3*(
a^4 - 10*a^2*b^2 + 5*b^4)*x)/(a^2 + b^2)) + (a^2*b*(5*a^4 - 10*a^2*b^2 + b
^4)*Log[b*Cos[d + e*x] + a*Sin[d + e*x]])/((a^2 + b^2)*e))/(a^2*(a^2 + b^2
)) - (a^4 - 6*a^2*b^2 + b^4)/((a^2 + b^2)*e*(b + a*Tan[d + e*x])))/(a^2*(a
^2 + b^2)))/(a^2*(a^2 + b^2)))
```

3.513.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

3.513. $\int \frac{a+b\tan(d+ex)}{\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^2}\,dx$

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

rule 4191
```
Int[((A_) + (B_.)*tan[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*tan[(d_.) + (e_.)*
(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Simp[1/(4^n*c^n
)   Int[(A + B*Tan[d + e*x])*(b + 2*c*Tan[d + e*x])^(2*n), x], x] /; FreeQ[
{a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[n]
```

3.513.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.11

method	result
derivativedivides	$\frac{-\frac{a^2-b^2}{3\left(a^2+b^2\right)(b+a\tan(ex+d))^3}-\frac{b\left(3a^2-b^2\right)}{2\left(a^2+b^2\right)^2(b+a\tan(ex+d))^2}+\frac{a^4-6a^2b^2+b^4}{\left(a^2+b^2\right)^3(b+a\tan(ex+d))}-\frac{b\left(5a^4-10a^2b^2+b^4\right)\ln(b+a\tan(ex+d}{\left(a^2+b^2\right)^4}}{e}$
default	$\frac{-\frac{a^2-b^2}{3\left(a^2+b^2\right)(b+a\tan(ex+d))^3}-\frac{b\left(3a^2-b^2\right)}{2\left(a^2+b^2\right)^2(b+a\tan(ex+d))^2}+\frac{a^4-6a^2b^2+b^4}{\left(a^2+b^2\right)^3(b+a\tan(ex+d))}-\frac{b\left(5a^4-10a^2b^2+b^4\right)\ln(b+a\tan(ex+d}{\left(a^2+b^2\right)^4}}{e}$
norman	$\frac{\frac{a^4\left(a^4-10a^2b^2+5b^4\right)x\tan(ex+d)^3}{\left(a^6+3a^4b^2+3a^2b^4+b^6\right)\left(a^2+b^2\right)}+\frac{\left(a^4-10a^2b^2+5b^4\right)a\,b^3x}{\left(a^6+3a^4b^2+3a^2b^4+b^6\right)\left(a^2+b^2\right)}-\frac{2a^8+7a^6b^2+28a^4b^4-9a^2b^6}{6a^2e\left(a^6+3a^4b^2+3a^2b^4+b^6\right)}+\frac{b\left(-a^6-14a^4b^2+3a^2b^4\right)t}{2ae\left(a^6+3a^4b^2+3a^2b^4\right.}}{(b+a\tan(ex}$
risch	$-\frac{ixb}{4ia^3b-4ia\,b^3+a^4-6a^2b^2+b^4}+\frac{xa}{4ia^3b-4ia\,b^3+a^4-6a^2b^2+b^4}+\frac{10ib\,a^4x}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8}-\frac{20ib^3}{a^8+4a^6b^2+6a^4}$
parallelrisch	Expression too large to display

input
```
int((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^2,x,method=_R
ETURNVERBOSE)
```

3.513. $\int \frac{a+b\tan(d+ex)}{\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^2}\,dx$

output
```
1/e*(-1/3*(a^2-b^2)/(a^2+b^2)/(b+a*tan(e*x+d))^3-1/2*b*(3*a^2-b^2)/(a^2+b^
2)^2/(b+a*tan(e*x+d))^2+(a^4-6*a^2*b^2+b^4)/(a^2+b^2)^3/(b+a*tan(e*x+d))-b
*(5*a^4-10*a^2*b^2+b^4)/(a^2+b^2)^4*ln(b+a*tan(e*x+d))+1/(a^2+b^2)^4*(1/2*
(5*a^4*b-10*a^2*b^3+b^5)*ln(1+tan(e*x+d)^2)+(a^5-10*a^3*b^2+5*a*b^4)*arcta
n(tan(e*x+d)))
```

3.513.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 580 vs. $2(193) = 386$.

Time = 0.28 (sec) , antiderivative size = 580, normalized size of antiderivative = 2.94

$$\int \frac{a+b\tan(d+ex)}{(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex))^2}\,dx =$$
$$-\frac{2\,a^8+7\,a^6b^2+66\,a^4b^4-27\,a^2b^6+(21\,a^7b-56\,a^5b^3+11\,a^3b^5-6\,(a^8-10\,a^6b^2+5\,a^4b^4)ex)\tan{(ex+}}{}$$

input
```
integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^2,x, al
gorithm="fricas")
```

output
```
-1/6*(2*a^8 + 7*a^6*b^2 + 66*a^4*b^4 - 27*a^2*b^6 + (21*a^7*b - 56*a^5*b^3
 + 11*a^3*b^5 - 6*(a^8 - 10*a^6*b^2 + 5*a^4*b^4)*e*x)*tan(e*x + d)^3 - 6*(
a^5*b^3 - 10*a^3*b^5 + 5*a*b^7)*e*x - 3*(2*a^8 - 31*a^6*b^2 + 46*a^4*b^4 -
 9*a^2*b^6 + 6*(a^7*b - 10*a^5*b^3 + 5*a^3*b^5)*e*x)*tan(e*x + d)^2 + 3*(5
*a^4*b^4 - 10*a^2*b^6 + b^8 + (5*a^7*b - 10*a^5*b^3 + a^3*b^5)*tan(e*x + d
)^3 + 3*(5*a^6*b^2 - 10*a^4*b^4 + a^2*b^6)*tan(e*x + d)^2 + 3*(5*a^5*b^3 -
 10*a^3*b^5 + a*b^7)*tan(e*x + d))*log((a^2*tan(e*x + d)^2 + 2*a*b*tan(e*x
 + d) + b^2)/(tan(e*x + d)^2 + 1)) - 3*(a^7*b - 46*a^5*b^3 + 35*a^3*b^5 -
6*a*b^7 + 6*(a^6*b^2 - 10*a^4*b^4 + 5*a^2*b^6)*e*x)*tan(e*x + d))/((a^11 +
 4*a^9*b^2 + 6*a^7*b^4 + 4*a^5*b^6 + a^3*b^8)*e*tan(e*x + d)^3 + 3*(a^10*b
 + 4*a^8*b^3 + 6*a^6*b^5 + 4*a^4*b^7 + a^2*b^9)*e*tan(e*x + d)^2 + 3*(a^9*
b^2 + 4*a^7*b^4 + 6*a^5*b^6 + 4*a^3*b^8 + a*b^10)*e*tan(e*x + d) + (a^8*b^
3 + 4*a^6*b^5 + 6*a^4*b^7 + 4*a^2*b^9 + b^11)*e)
```

3.513.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{a+b\tan(d+ex)}{\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^2}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate((a+b*tan(e*x+d))/(b**2+2*a*b*tan(e*x+d)+a**2*tan(e*x+d)**2)**2,x
)
```

output
```
Exception raised: AttributeError >> 'NoneType' object has no attribute 'pr
imitive'
```

3.513.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 419 vs. $2(193) = 386$.

Time = 0.31 (sec) , antiderivative size = 419, normalized size of antiderivative = 2.13

$$\int \frac{a+b\tan(d+ex)}{\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^2}\,dx$$
$$= \frac{\frac{6\,(a^5-10\,a^3b^2+5\,ab^4)(ex+d)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8} - \frac{6\,(5\,a^4b-10\,a^2b^3+b^5)\log(a\tan(ex+d)+b)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8} + \frac{3\,(5\,a^4b-10\,a^2b^3+b^5)\log\left(\tan(ex+d)^2+1\right)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8} - \frac{}{a^6b^3+3\,a^4b^5+3}}{6\,e}$$

input
```
integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^2,x, al
gorithm="maxima")
```

output
```
1/6*(6*(a^5 - 10*a^3*b^2 + 5*a*b^4)*(e*x + d)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4
 + 4*a^2*b^6 + b^8) - 6*(5*a^4*b - 10*a^2*b^3 + b^5)*log(a*tan(e*x + d) +
b)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) + 3*(5*a^4*b - 10*a^2*b
^3 + b^5)*log(tan(e*x + d)^2 + 1)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6
 + b^8) - (2*a^6 + 5*a^4*b^2 + 40*a^2*b^4 - 11*b^6 - 6*(a^6 - 6*a^4*b^2 +
a^2*b^4)*tan(e*x + d)^2 - 3*(a^5*b - 26*a^3*b^3 + 5*a*b^5)*tan(e*x + d))/(
a^6*b^3 + 3*a^4*b^5 + 3*a^2*b^7 + b^9 + (a^9 + 3*a^7*b^2 + 3*a^5*b^4 + a^3
*b^6)*tan(e*x + d)^3 + 3*(a^8*b + 3*a^6*b^3 + 3*a^4*b^5 + a^2*b^7)*tan(e*x
 + d)^2 + 3*(a^7*b^2 + 3*a^5*b^4 + 3*a^3*b^6 + a*b^8)*tan(e*x + d)))/e
```

3.513.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 437 vs. $2(193) = 386$.

Time = 0.87 (sec) , antiderivative size = 437, normalized size of antiderivative = 2.22

$$\int \frac{a+b\tan(d+ex)}{(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex))^2}\,dx$$
$$=\frac{\frac{6\,(a^5-10\,a^3b^2+5\,ab^4)(ex+d)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}+\frac{3\,(5\,a^4b-10\,a^2b^3+b^5)\log\left(\tan(ex+d)^2+1\right)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}-\frac{6\,(5\,a^5b-10\,a^3b^3+ab^5)\log(|a\tan(ex+d)+b|)}{a^9+4\,a^7b^2+6\,a^5b^4+4\,a^3b^6+ab^8}+\frac{55\,a^7b\tan(e\ldots}{}}{}$$

input
```
integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^2,x, al
gorithm="giac")
```

output
```
1/6*(6*(a^5 - 10*a^3*b^2 + 5*a*b^4)*(e*x + d)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4
 + 4*a^2*b^6 + b^8) + 3*(5*a^4*b - 10*a^2*b^3 + b^5)*log(tan(e*x + d)^2 +
1)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - 6*(5*a^5*b - 10*a^3*b
^3 + a*b^5)*log(abs(a*tan(e*x + d) + b))/(a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*
a^3*b^6 + a*b^8) + (55*a^7*b*tan(e*x + d)^3 - 110*a^5*b^3*tan(e*x + d)^3 +
 11*a^3*b^5*tan(e*x + d)^3 + 6*a^8*tan(e*x + d)^2 + 135*a^6*b^2*tan(e*x +
d)^2 - 360*a^4*b^4*tan(e*x + d)^2 + 39*a^2*b^6*tan(e*x + d)^2 + 3*a^7*b*ta
n(e*x + d) + 90*a^5*b^3*tan(e*x + d) - 393*a^3*b^5*tan(e*x + d) + 48*a*b^7
*tan(e*x + d) - 2*a^8 - 7*a^6*b^2 + 10*a^4*b^4 - 139*a^2*b^6 + 22*b^8)/((a
^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*(a*tan(e*x + d) + b)^3))/e
```

3.513.9 Mupad [B] (verification not implemented)

Time = 28.77 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.97

$$\int \frac{a+b\tan(d+ex)}{(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex))^2}\,dx$$
$$=\frac{\frac{\tan(d+e\,x)^2\,(a^6-6\,a^4\,b^2+a^2\,b^4)}{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6}-\frac{2\,a^6+5\,a^4\,b^2+40\,a^2\,b^4-11\,b^6}{6\,(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6)}+\frac{\tan(d+e\,x)\,(a^5\,b-26\,a^3\,b^3+5\,a\,b^5)}{2\,(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6)}}{e\,\left(a^3\tan\left(d+e\,x\right)^3+3\,a^2\,b\tan\left(d+e\,x\right)^2+3\,a\,b^2\tan\left(d+e\,x\right)+b^3\right)}$$
$$-\frac{\ln\left(b+a\tan(d+e\,x)\right)\,\left(\frac{5\,b}{(a^2+b^2)^2}-\frac{20\,b^3}{(a^2+b^2)^3}+\frac{16\,b^5}{(a^2+b^2)^4}\right)}{e}$$
$$+\frac{\ln\left(\tan(d+e\,x)-\mathrm{i}\right)\,(a+b\,1\mathrm{i})}{2\,e\,\left(a^4\,1\mathrm{i}+4\,a^3\,b-a^2\,b^2\,6\mathrm{i}-4\,a\,b^3+b^4\,1\mathrm{i}\right)}$$
$$-\frac{\ln\left(\tan(d+e\,x)+1\mathrm{i}\right)\,(a-b\,1\mathrm{i})}{2\,e\,\left(a^4\,1\mathrm{i}-4\,a^3\,b-a^2\,b^2\,6\mathrm{i}+4\,a\,b^3+b^4\,1\mathrm{i}\right)}$$

input
```
int((a + b*tan(d + e*x))/(b^2 + a^2*tan(d + e*x)^2 + 2*a*b*tan(d + e*x))^2
,x)
```

output
```
((tan(d + e*x)^2*(a^6 + a^2*b^4 - 6*a^4*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a
^4*b^2) - (2*a^6 - 11*b^6 + 40*a^2*b^4 + 5*a^4*b^2)/(6*(a^6 + b^6 + 3*a^2*
b^4 + 3*a^4*b^2)) + (tan(d + e*x)*(5*a*b^5 + a^5*b - 26*a^3*b^3))/(2*(a^6
+ b^6 + 3*a^2*b^4 + 3*a^4*b^2)))/(e*(b^3 + a^3*tan(d + e*x)^3 + 3*a^2*b*ta
n(d + e*x)^2 + 3*a*b^2*tan(d + e*x))) - (log(b + a*tan(d + e*x))*((5*b)/(a
^2 + b^2)^2 - (20*b^3)/(a^2 + b^2)^3 + (16*b^5)/(a^2 + b^2)^4))/e + (log(t
an(d + e*x) - 1i)*(a + b*1i))/(2*e*(4*a^3*b - 4*a*b^3 + a^4*1i + b^4*1i -
a^2*b^2*6i)) - (log(tan(d + e*x) + 1i)*(a - b*1i))/(2*e*(4*a*b^3 - 4*a^3*b
 + a^4*1i + b^4*1i - a^2*b^2*6i))
```

3.513. $\int \frac{a+b\tan(d+ex)}{\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^2}\,dx$

3.514 $\int (a+b\tan(d+ex))\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d$

3.514.1 Optimal result

Integrand size = 41, antiderivative size = 284

$$\int (a+b\tan(d+ex))\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^{3/2}\,dx=\frac{b\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^{3/2}}{3e}+\frac{(a^4-b^4)\log(\cos(d+ex))\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^{3/2}}{e(b+a\tan(d+ex))^3}+\frac{(a^2+b^2)\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^{3/2}}{2e(b+a\tan(d+ex))}-\frac{2a^4b(a^2+b^2)x\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^{3/2}}{\left(ab+a^2\tan(d+ex)\right)^3}+\frac{a^4b(a^2+b^2)\tan(d+ex)\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^{3/2}}{e\left(ab+a^2\tan(d+ex)\right)^3}$$

output
```
1/3*b*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(3/2)/e+(a^4-b^4)*ln(cos(e*x
+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(3/2)/e/(b+a*tan(e*x+d))^3+1/
2*(a^2+b^2)*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(3/2)/e/(b+a*tan(e*x+d
))-2*a^4*b*(a^2+b^2)*x*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(3/2)/(a*b+
a^2*tan(e*x+d))^3+a^4*b*(a^2+b^2)*tan(e*x+d)*(b^2+2*a*b*tan(e*x+d)+a^2*tan
(e*x+d)^2)^(3/2)/e/(a*b+a^2*tan(e*x+d))^3
```

3.514.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.15 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.52

$$\int (a + b\tan(d + ex)) \left(b^2 + 2ab\tan(d + ex) + a^2\tan^2(d + ex)\right)^{3/2} dx = \frac{\sqrt{(b + a\tan(d + ex))^2}\left(-3(a^2 + b^2)\left((a - ib)^2\log(i - \tan(d + ex)) + (a + ib)^2\log(i + \tan(d +\right.\right.}{6e(b + a\tan(}$$

input
```
Integrate[(a + b*Tan[d + e*x])*(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x
]^2)^(3/2),x]
```

output
```
(Sqrt[(b + a*Tan[d + e*x])^2]*(-3*(a^2 + b^2)*((a - I*b)^2*Log[I - Tan[d +
 e*x]] + (a + I*b)^2*Log[I + Tan[d + e*x]]) + 6*a*b*(2*a^2 + 3*b^2)*Tan[d
+ e*x] + 3*a^2*(a^2 + 3*b^2)*Tan[d + e*x]^2 + 2*a^3*b*Tan[d + e*x]^3))/(6*
e*(b + a*Tan[d + e*x]))
```

3.514.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.52, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.293, Rules used = {3042, 4193, 27, 3042, 4011, 27, 3042, 4011, 3042, 4008, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + b\tan(d + ex)) \left(a^2\tan^2(d + ex) + 2ab\tan(d + ex) + b^2\right)^{3/2} dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (a + b\tan(d + ex)) \left(a^2\tan(d + ex)^2 + 2ab\tan(d + ex) + b^2\right)^{3/2} dx$$

$$\Big\downarrow \text{ 4193}$$

$$\frac{\left(a^2\tan^2(d + ex) + 2ab\tan(d + ex) + b^2\right)^{3/2}\int 8\left(\tan(d + ex)a^2 + ba\right)^3(a + b\tan(d + ex))dx}{8\left(a^2\tan(d + ex) + ab\right)^3}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\left(a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2\right)^{3/2}\int\left(\tan(d+ex)a^2+ba\right)^3(a+b\tan(d+ex))dx}{\left(a^2\tan(d+ex)+ab\right)^3}$$

$\downarrow$ 3042

$$\frac{\left(a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2\right)^{3/2}\int\left(\tan(d+ex)a^2+ba\right)^3(a+b\tan(d+ex))dx}{\left(a^2\tan(d+ex)+ab\right)^3}$$

$\downarrow$ 4011

$$\frac{\left(a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2\right)^{3/2}\left(\int a\left(a^2+b^2\right)\tan(d+ex)\left(\tan(d+ex)a^2+ba\right)^2dx+\frac{b(a^2\tan(d+ex)+}{3e}\right.}{\left(a^2\tan(d+ex)+ab\right)^3}$$

$\downarrow$ 27

$$\frac{\left(a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2\right)^{3/2}\left(a\left(a^2+b^2\right)\int\tan(d+ex)\left(\tan(d+ex)a^2+ba\right)^2dx+\frac{b(a^2\tan(d+ex)+}{3e}\right.}{\left(a^2\tan(d+ex)+ab\right)^3}$$

$\downarrow$ 3042

$$\frac{\left(a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2\right)^{3/2}\left(a\left(a^2+b^2\right)\int\tan(d+ex)\left(\tan(d+ex)a^2+ba\right)^2dx+\frac{b(a^2\tan(d+ex)+}{3e}\right.}{\left(a^2\tan(d+ex)+ab\right)^3}$$

$\downarrow$ 4011

$$\frac{\left(a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2\right)^{3/2}\left(a\left(a^2+b^2\right)\left(\int\left(\tan(d+ex)a^2+ba\right)\left(ab\tan(d+ex)-a^2\right)dx+\frac{(a^2}{}\right.\right.}{\left(a^2\tan(d+ex)+ab\right)^3}$$

$\downarrow$ 3042

$$\frac{\left(a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2\right)^{3/2}\left(a\left(a^2+b^2\right)\left(\int\left(\tan(d+ex)a^2+ba\right)\left(ab\tan(d+ex)-a^2\right)dx+\frac{(a^2}{}\right.\right.}{\left(a^2\tan(d+ex)+ab\right)^3}$$

$\downarrow$ 4008

$$\frac{\left(a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2\right)^{3/2}\left(a\left(a^2+b^2\right)\left(-a^2\left(a^2-b^2\right)\int\tan(d+ex)dx+\frac{a^3b\tan(d+ex)}{e}-2a^3bx\right.\right.}{\left(a^2\tan(d+ex)+ab\right)^3}$$

$\downarrow$ 3042

3.514. $\int(a+b\tan(d+ex))\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^{3/2}\,dx$

$$\frac{\left(a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2\right)^{3/2}\left(a\left(a^2+b^2\right)\left(-a^2\left(a^2-b^2\right)\int\tan(d+ex)dx+\frac{a^3b\tan(d+ex)}{e}-2a^3bx-\right.\right.}{\left(a^2\tan(d+ex)+ab\right)^3}$$

$\Big\downarrow$ 3956

$$\frac{\left(a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2\right)^{3/2}\left(\frac{b\left(a^2\tan(d+ex)+ab\right)^3}{3e}+a\left(a^2+b^2\right)\left(\frac{a^3b\tan(d+ex)}{e}-2a^3bx+\frac{a^2\left(a^2-b^2\right)\log}{e}\right.\right.}{\left(a^2\tan(d+ex)+ab\right)^3}$$

input
```
Int[(a + b*Tan[d + e*x])*(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2)^(
3/2),x]
```

output
```
((b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2)^(3/2)*((b*(a*b + a^2*Tan[
d + e*x])^3)/(3*e) + a*(a^2 + b^2)*(-2*a^3*b*x + (a^2*(a^2 - b^2)*Log[Cos[
d + e*x]])/e + (a^3*b*Tan[d + e*x])/e + (a*b + a^2*Tan[d + e*x])^2/(2*e)))
)/(a*b + a^2*Tan[d + e*x])^3
```

3.514.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4008
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Simp[b*d*(Tan[e + f*x]/f),
x] + Simp[(b*c + a*d)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

rule 4011
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

rule 4193
```
Int[((A_) + (B_.)*tan[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*tan[(d_.) + (e_.)*
(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Simp[(a + b*Tan
[d + e*x] + c*Tan[d + e*x]^2)^n/(b + 2*c*Tan[d + e*x])^(2*n)   Int[(A + B*T
an[d + e*x])*(b + 2*c*Tan[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e,
A, B}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[n]
```

3.514.4 Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.56

method	result
derivativedivides	$-\frac{\left(b^{2}+2ab\tan(ex+d)+a^{2}\tan(ex+d)^{2}\right)^{\frac{3}{2}}\left(-2\tan(ex+d)^{3}a^{3}b-3a^{4}\tan(ex+d)^{2}-9\tan(ex+d)^{2}a^{2}b^{2}+3\ln\left(1+\tan(ex+d\right.\right.}{6e(b+}$
default	$-\frac{\left(b^{2}+2ab\tan(ex+d)+a^{2}\tan(ex+d)^{2}\right)^{\frac{3}{2}}\left(-2\tan(ex+d)^{3}a^{3}b-3a^{4}\tan(ex+d)^{2}-9\tan(ex+d)^{2}a^{2}b^{2}+3\ln\left(1+\tan(ex+d\right.\right.}{6e(b+}$
parts	$-\frac{a\left(b^{2}+2ab\tan(ex+d)+a^{2}\tan(ex+d)^{2}\right)^{\frac{3}{2}}\left(-\tan(ex+d)^{2}a^{3}+\ln\left(1+\tan(ex+d)^{2}\right)a^{3}-3\ln\left(1+\tan(ex+d)^{2}\right)a\,b^{2}+6\,\mathrm{arct}\right.}{2e(b+a\tan(ex+d))^{3}}$

input
```
int((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(3/2),x,metho
d=_RETURNVERBOSE)
```

output
```
-1/6/e*((b+a*tan(e*x+d))^2)^(3/2)*(-2*tan(e*x+d)^3*a^3*b-3*a^4*tan(e*x+d)^
2-9*tan(e*x+d)^2*a^2*b^2+3*ln(1+tan(e*x+d)^2)*a^4-3*ln(1+tan(e*x+d)^2)*b^4
+12*arctan(tan(e*x+d))*a^3*b+12*arctan(tan(e*x+d))*a*b^3-12*tan(e*x+d)*a^3
*b-18*tan(e*x+d)*a*b^3)/(b+a*tan(e*x+d))^3
```

3.514.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.36

$$\int (a+b\tan(d+ex)) \left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^{3/2} dx = \frac{2\,a^3b\tan\left(ex+d\right)^3-12\left(a^3b+ab^3\right)ex+3\left(a^4+3\,a^2b^2\right)\tan\left(ex+d\right)^2+3\left(a^4-b^4\right)\log\left(\frac{1}{\tan(ex+d)^2+1}\right)+\ldots}{6\,e}$$

input
```
integrate((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(3/2),x
, algorithm="fricas")
```

output
```
1/6*(2*a^3*b*tan(e*x + d)^3 - 12*(a^3*b + a*b^3)*e*x + 3*(a^4 + 3*a^2*b^2)
*tan(e*x + d)^2 + 3*(a^4 - b^4)*log(1/(tan(e*x + d)^2 + 1)) + 6*(2*a^3*b +
 3*a*b^3)*tan(e*x + d))/e
```

3.514.6 Sympy [F]

$$\int (a+b\tan(d+ex)) \left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^{3/2} dx = \int \left(a+b\tan\left(d+ex\right)\right)\left(\left(a\tan\left(d+ex\right)+b\right)^2\right)^{\frac{3}{2}} dx$$

input
```
integrate((a+b*tan(e*x+d))*(b**2+2*a*b*tan(e*x+d)+a**2*tan(e*x+d)**2)**(3/
2),x)
```

output
```
Integral((a + b*tan(d + e*x))*((a*tan(d + e*x) + b)**2)**(3/2), x)
```

3.514.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.58

$$\int (a+b\tan(d+ex)) \left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^{3/2} dx = \frac{3\left(a^3\tan\left(ex+d\right)^2+6\,a^2b\tan\left(ex+d\right)-2\left(3\,a^2b-b^3\right)(ex+d)-\left(a^3-3\,ab^2\right)\log\left(\tan\left(ex+\ldots\right.\right.}{}$$

input
```
integrate((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(3/2),x
, algorithm="maxima")
```

output
```
1/6*(3*(a^3*tan(e*x + d)^2 + 6*a^2*b*tan(e*x + d) - 2*(3*a^2*b - b^3)*(e*x
 + d) - (a^3 - 3*a*b^2)*log(tan(e*x + d)^2 + 1))*a + (2*a^3*tan(e*x + d)^3
 + 9*a^2*b*tan(e*x + d)^2 + 6*(a^3 - 3*a*b^2)*(e*x + d) - 3*(3*a^2*b - b^3
)*log(tan(e*x + d)^2 + 1) - 6*(a^3 - 3*a*b^2)*tan(e*x + d))*b)/e
```

3.514.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1535 vs. $2(270) = 540$.

Time = 1.16 (sec) , antiderivative size = 1535, normalized size of antiderivative = 5.40

$$\int (a + b\tan(d + ex)) \left(b^2 + 2ab\tan(d + ex) + a^2\tan^2(d + ex)\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate((a+b*tan(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(3/2),x
, algorithm="giac")
```

output
```
-1/6*(12*a^3*b*e*x*sgn(a*tan(e*x + d) + b)*tan(e*x)^3*tan(d)^3 + 12*a*b^3*
e*x*sgn(a*tan(e*x + d) + b)*tan(e*x)^3*tan(d)^3 - 3*a^4*log(4*(tan(e*x)^2*
tan(d)^2 - 2*tan(e*x)*tan(d) + 1)/(tan(e*x)^2*tan(d)^2 + tan(e*x)^2 + tan(
d)^2 + 1))*sgn(a*tan(e*x + d) + b)*tan(e*x)^3*tan(d)^3 + 3*b^4*log(4*(tan(
e*x)^2*tan(d)^2 - 2*tan(e*x)*tan(d) + 1)/(tan(e*x)^2*tan(d)^2 + tan(e*x)^2
 + tan(d)^2 + 1))*sgn(a*tan(e*x + d) + b)*tan(e*x)^3*tan(d)^3 - 36*a^3*b*e
*x*sgn(a*tan(e*x + d) + b)*tan(e*x)^2*tan(d)^2 - 36*a*b^3*e*x*sgn(a*tan(e*
x + d) + b)*tan(e*x)^2*tan(d)^2 - 3*a^4*sgn(a*tan(e*x + d) + b)*tan(e*x)^3
*tan(d)^3 - 9*a^2*b^2*sgn(a*tan(e*x + d) + b)*tan(e*x)^3*tan(d)^3 + 9*a^4*
log(4*(tan(e*x)^2*tan(d)^2 - 2*tan(e*x)*tan(d) + 1)/(tan(e*x)^2*tan(d)^2 +
 tan(e*x)^2 + tan(d)^2 + 1))*sgn(a*tan(e*x + d) + b)*tan(e*x)^2*tan(d)^2 -
 9*b^4*log(4*(tan(e*x)^2*tan(d)^2 - 2*tan(e*x)*tan(d) + 1)/(tan(e*x)^2*tan
(d)^2 + tan(e*x)^2 + tan(d)^2 + 1))*sgn(a*tan(e*x + d) + b)*tan(e*x)^2*tan
(d)^2 + 12*a^3*b*sgn(a*tan(e*x + d) + b)*tan(e*x)^3*tan(d)^2 + 18*a*b^3*sg
n(a*tan(e*x + d) + b)*tan(e*x)^3*tan(d)^2 + 12*a^3*b*sgn(a*tan(e*x + d) +
b)*tan(e*x)^2*tan(d)^3 + 18*a*b^3*sgn(a*tan(e*x + d) + b)*tan(e*x)^2*tan(d
)^3 + 36*a^3*b*e*x*sgn(a*tan(e*x + d) + b)*tan(e*x)*tan(d) + 36*a*b^3*e*x*
sgn(a*tan(e*x + d) + b)*tan(e*x)*tan(d) - 3*a^4*sgn(a*tan(e*x + d) + b)*ta
n(e*x)^3*tan(d) - 9*a^2*b^2*sgn(a*tan(e*x + d) + b)*tan(e*x)^3*tan(d) + 3*
a^4*sgn(a*tan(e*x + d) + b)*tan(e*x)^2*tan(d)^2 + 9*a^2*b^2*sgn(a*tan(e...
```

3.514.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\tan(d+ex)) \left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^{3/2} dx = \int (a + b\tan(d+e\,x)) \left(a^2\tan(d+e\,x)^2+2\,a\,b\tan(d+e\,x)+b^2\right)^{3/2} dx$$

input
```
int((a + b*tan(d + e*x))*(b^2 + a^2*tan(d + e*x)^2 + 2*a*b*tan(d + e*x))^(
3/2),x)
```

output
```
int((a + b*tan(d + e*x))*(b^2 + a^2*tan(d + e*x)^2 + 2*a*b*tan(d + e*x))^(
3/2), x)
```

3.515 $\int (a+b\tan(d+ex))\sqrt{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}\,dx$

3.515.1 Optimal result

Integrand size = 41, antiderivative size = 122

$$\int (a+b\tan(d+ex))\sqrt{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}\,dx = -\frac{(a^2+b^2)\log(\cos(d+ex))\sqrt{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}}{e(b+a\tan(d+ex))} + \frac{a^2b\tan(d+ex)\sqrt{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}}{e\,(ab+a^2\tan(d+ex))}$$

output
```
-(a^2+b^2)*ln(cos(e*x+d))*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(1/2)/e/
(b+a*tan(e*x+d))+a^2*b*(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(1/2)*tan(e
*x+d)/e/(a*b+a^2*tan(e*x+d))
```

3.515.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.48

$$\int (a+b\tan(d+ex))\sqrt{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}\,dx = \frac{\sqrt{(b+a\tan(d+ex))^2}(-((a^2+b^2)\log(\cos(d+ex)))+ab\tan(d+ex))}{e(b+a\tan(d+ex))}$$

input
```
Integrate[(a + b*Tan[d + e*x])*Sqrt[b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d +
 e*x]^2],x]
```

output
```
(Sqrt[(b + a*Tan[d + e*x])^2]*(-((a^2 + b^2)*Log[Cos[d + e*x]]) + a*b*Tan[
d + e*x]))/(e*(b + a*Tan[d + e*x]))
```

3.515.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.67, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {3042, 4193, 27, 3042, 4008, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\tan(d+ex))\sqrt{a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int (a+b\tan(d+ex))\sqrt{a^2\tan(d+ex)^2+2ab\tan(d+ex)+b^2}dx$$

$$\Big\downarrow \ 4193$$

$$\frac{\sqrt{a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2}\int 2\left(\tan(d+ex)a^2+ba\right)(a+b\tan(d+ex))dx}{2\left(a^2\tan(d+ex)+ab\right)}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2}\int \left(\tan(d+ex)a^2+ba\right)(a+b\tan(d+ex))dx}{a^2\tan(d+ex)+ab}$$

$$\Big\downarrow \ 3042$$

$$\frac{\sqrt{a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2}\int \left(\tan(d+ex)a^2+ba\right)(a+b\tan(d+ex))dx}{a^2\tan(d+ex)+ab}$$

$$\Big\downarrow \ 4008$$

$$\frac{\sqrt{a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2}\left(a\left(a^2+b^2\right)\int \tan(d+ex)dx+\frac{a^2b\tan(d+ex)}{e}\right)}{a^2\tan(d+ex)+ab}$$

$$\Big\downarrow \ 3042$$

$$\frac{\sqrt{a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2}\left(a\left(a^2+b^2\right)\int \tan(d+ex)dx+\frac{a^2b\tan(d+ex)}{e}\right)}{a^2\tan(d+ex)+ab}$$

$$\Big\downarrow \ 3956$$

$$\frac{\sqrt{a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2}\left(\frac{a^2b\tan(d+ex)}{e}-\frac{a(a^2+b^2)\log(\cos(d+ex))}{e}\right)}{a^2\tan(d+ex)+ab}$$

input
```
Int[(a + b*Tan[d + e*x])*Sqrt[b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^
2],x]
```

output
```
((-((a*(a^2 + b^2)*Log[Cos[d + e*x]])/e) + (a^2*b*Tan[d + e*x])/e)*Sqrt[b^
2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2])/(a*b + a^2*Tan[d + e*x])
```

3.515.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4008
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Simp[b*d*(Tan[e + f*x]/f),
x] + Simp[(b*c + a*d)   Int[Tan[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

rule 4193
```
Int[((A_) + (B_.)*tan[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*tan[(d_.) + (e_.)*
(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Simp[(a + b*Tan
[d + e*x] + c*Tan[d + e*x]^2)^n/(b + 2*c*Tan[d + e*x])^(2*n)   Int[(A + B*T
an[d + e*x])*(b + 2*c*Tan[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e,
A, B}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[n]
```

3.515. $\int(a+b\tan(d+ex))\sqrt{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}\,dx$

3.515.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.17 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.61

method	result
derivativedivides	$\frac{\operatorname{csgn}(b+a\tan(ex+d))\left(\ln\left(a^2\tan(ex+d)^2+a^2\right)a^2+\ln\left(a^2\tan(ex+d)^2+a^2\right)b^2+2ab\tan(ex+d)+2b^2\right)}{2e}$
default	$\frac{\operatorname{csgn}(b+a\tan(ex+d))\left(\ln\left(a^2\tan(ex+d)^2+a^2\right)a^2+\ln\left(a^2\tan(ex+d)^2+a^2\right)b^2+2ab\tan(ex+d)+2b^2\right)}{2e}$
parts	$\frac{a\operatorname{csgn}(b+a\tan(ex+d))\left(\ln\left(a^2\tan(ex+d)^2+a^2\right)a+2b\arctan(\tan(ex+d))\right)}{2e}+\frac{b\operatorname{csgn}(b+a\tan(ex+d))\left(b\ln\left(a^2\tan(ex-\right.\right.}{}$

input
```
int((b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(1/2)*(a+b*tan(e*x+d)),x,metho
d=_RETURNVERBOSE)
```

output
```
1/2/e*csgn(b+a*tan(e*x+d))*(ln(a^2*tan(e*x+d)^2+a^2)*a^2+ln(a^2*tan(e*x+d)
^2+a^2)*b^2+2*a*b*tan(e*x+d)+2*b^2)
```

3.515.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.31

$$\int (a+b\tan(d+ex))\sqrt{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}\,dx$$
$$=\frac{2\,ab\tan\left(ex+d\right)-\left(a^2+b^2\right)\log\left(\frac{1}{\tan(ex+d)^2+1}\right)}{2\,e}$$

input
```
integrate((b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(1/2)*(a+b*tan(e*x+d)),x
, algorithm="fricas")
```

output
```
1/2*(2*a*b*tan(e*x + d) - (a^2 + b^2)*log(1/(tan(e*x + d)^2 + 1)))/e
```

3.515.6 Sympy [F]

$$\int (a+b\tan(d+ex))\sqrt{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}\,dx$$
$$=\int (a+b\tan(d+ex))\sqrt{(a\tan(d+ex)+b)^2}\,dx$$

input
```
integrate((b**2+2*a*b*tan(e*x+d)+a**2*tan(e*x+d)**2)**(1/2)*(a+b*tan(e*x+d
)),x)
```

output
```
Integral((a + b*tan(d + e*x))*sqrt((a*tan(d + e*x) + b)**2), x)
```

3.515.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.53

$$\int (a+b\tan(d+ex))\sqrt{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}\,dx$$
$$=\frac{(2(ex+d)b+a\log(\tan(ex+d)^2+1))a-(2(ex+d)a-b\log(\tan(ex+d)^2+1)-2a\tan(ex+d))b}{2e}$$

input
```
integrate((b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(1/2)*(a+b*tan(e*x+d)),x
, algorithm="maxima")
```

output
```
1/2*((2*(e*x + d)*b + a*log(tan(e*x + d)^2 + 1))*a - (2*(e*x + d)*a - b*lo
g(tan(e*x + d)^2 + 1) - 2*a*tan(e*x + d))*b)/e
```

3.515.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. 2(118) = 236.

Time = 0.40 (sec) , antiderivative size = 323, normalized size of antiderivative = 2.65

$$\int (a+b\tan(d+ex))\sqrt{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}\,dx =$$
$$-\frac{a^2\log\left(\frac{4\left(\tan(ex)^2\tan(d)^2-2\tan(ex)\tan(d)+1\right)}{\tan(ex)^2\tan(d)^2+\tan(ex)^2+\tan(d)^2+1}\right)\mathrm{sgn}(a\tan(ex+d)+b)\tan(ex)\tan(d)+b^2\log\left(\frac{4\left(\tan(ex)^2\tan\right.}{\tan(ex)^2\tan(d}\right.}{}$$

input
```
integrate((b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(1/2)*(a+b*tan(e*x+d)),x
, algorithm="giac")
```

output
```
-1/2*(a^2*log(4*(tan(e*x)^2*tan(d)^2 - 2*tan(e*x)*tan(d) + 1)/(tan(e*x)^2*
tan(d)^2 + tan(e*x)^2 + tan(d)^2 + 1))*sgn(a*tan(e*x + d) + b)*tan(e*x)*ta
n(d) + b^2*log(4*(tan(e*x)^2*tan(d)^2 - 2*tan(e*x)*tan(d) + 1)/(tan(e*x)^2
*tan(d)^2 + tan(e*x)^2 + tan(d)^2 + 1))*sgn(a*tan(e*x + d) + b)*tan(e*x)*t
an(d) - a^2*log(4*(tan(e*x)^2*tan(d)^2 - 2*tan(e*x)*tan(d) + 1)/(tan(e*x)^
2*tan(d)^2 + tan(e*x)^2 + tan(d)^2 + 1))*sgn(a*tan(e*x + d) + b) - b^2*log
(4*(tan(e*x)^2*tan(d)^2 - 2*tan(e*x)*tan(d) + 1)/(tan(e*x)^2*tan(d)^2 + ta
n(e*x)^2 + tan(d)^2 + 1))*sgn(a*tan(e*x + d) + b) + 2*a*b*sgn(a*tan(e*x +
d) + b)*tan(e*x) + 2*a*b*sgn(a*tan(e*x + d) + b)*tan(d))/(e*tan(e*x)*tan(d
) - e)
```

3.515.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\tan(d+ex))\sqrt{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}\,dx$$
$$=\int (a+b\tan(d+e\,x))\ \sqrt{a^2\tan\left(d+e\,x\right)^2+2\,a\,b\tan\left(d+e\,x\right)+b^2}\,dx$$

input
```
int((a + b*tan(d + e*x))*(b^2 + a^2*tan(d + e*x)^2 + 2*a*b*tan(d + e*x))^(
1/2),x)
```

output
```
int((a + b*tan(d + e*x))*(b^2 + a^2*tan(d + e*x)^2 + 2*a*b*tan(d + e*x))^(
1/2), x)
```

3.516 $\int \frac{a+b\tan(d+ex)}{\sqrt{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}}\,dx$

3.516.1 Optimal result

Integrand size = 41, antiderivative size = 138

$$\int \frac{a+b\tan(d+ex)}{\sqrt{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}}\,dx = \frac{(a^2-b^2)\log(b\cos(d+ex)+a\sin(d+ex))(b+a\tan(d+ex))}{(a^2+b^2)\,e\sqrt{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}} + \frac{2bx(ab+a^2\tan(d+ex))}{(a^2+b^2)\sqrt{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}}$$

output
```
(a^2-b^2)*ln(b*cos(e*x+d)+a*sin(e*x+d))*(b+a*tan(e*x+d))/(a^2+b^2)/e/(b^2+
2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(1/2)+2*b*x*(a*b+a^2*tan(e*x+d))/(a^2+b
^2)/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(1/2)
```

3.516.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.84 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.80

$$\int \frac{a+b\tan(d+ex)}{\sqrt{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}}\,dx = \frac{(-(a+ib)^2\log(i-\tan(d+ex))-(a-ib)^2\log(i+\tan(d+ex))+2(a^2-b^2)\log(b+a\tan(d+ex)))\,(b}{2\,(a^2+b^2)\,e\sqrt{(b+a\tan(d+ex))^2}}$$

input
```
Integrate[(a + b*Tan[d + e*x])/Sqrt[b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d +
 e*x]^2],x]
```

output
```
((-((a + I*b)^2*Log[I - Tan[d + e*x]]) - (a - I*b)^2*Log[I + Tan[d + e*x]]
 + 2*(a^2 - b^2)*Log[b + a*Tan[d + e*x]])*(b + a*Tan[d + e*x]))/(2*(a^2 +
b^2)*e*Sqrt[(b + a*Tan[d + e*x])^2])
```

3.516.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.74, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3042, 4193, 27, 3042, 4014, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\tan(d+ex)}{\sqrt{a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\tan(d+ex)}{\sqrt{a^2\tan(d+ex)^2+2ab\tan(d+ex)+b^2}}dx$$

$$\Big\downarrow \text{4193}$$

$$\frac{2\left(a^2\tan(d+ex)+ab\right)\int \frac{a+b\tan(d+ex)}{2\left(\tan(d+ex)a^2+ba\right)}dx}{\sqrt{a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(a^2\tan(d+ex)+ab\right)\int \frac{a+b\tan(d+ex)}{\tan(d+ex)a^2+ba}dx}{\sqrt{a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(a^2\tan(d+ex)+ab\right)\int \frac{a+b\tan(d+ex)}{\tan(d+ex)a^2+ba}dx}{\sqrt{a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2}}$$

$$\Big\downarrow \text{4014}$$

$$\frac{\left(a^2\tan(d+ex)+ab\right)\left(\frac{(a^2-b^2)\int \frac{a^2-ab\tan(d+ex)}{\tan(d+ex)a^2+ba}dx}{a(a^2+b^2)}+\frac{2bx}{a^2+b^2}\right)}{\sqrt{a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(a^2\tan(d+ex)+ab\right)\left(\frac{(a^2-b^2)\int\frac{a^2-ab\tan(d+ex)}{\tan(d+ex)a^2+ba}dx}{a(a^2+b^2)}+\frac{2bx}{a^2+b^2}\right)}{\sqrt{a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2}}$$

$$\Big\downarrow \text{4013}$$

$$\frac{\left(a^2\tan(d+ex)+ab\right)\left(\frac{(a^2-b^2)\log(a\sin(d+ex)+b\cos(d+ex))}{ae(a^2+b^2)}+\frac{2bx}{a^2+b^2}\right)}{\sqrt{a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2}}$$

input
```
Int[(a + b*Tan[d + e*x])/Sqrt[b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^
2],x]
```

output
```
(((2*b*x)/(a^2 + b^2) + ((a^2 - b^2)*Log[b*Cos[d + e*x] + a*Sin[d + e*x]])
/(a*(a^2 + b^2)*e))*(a*b + a^2*Tan[d + e*x]))/Sqrt[b^2 + 2*a*b*Tan[d + e*x
] + a^2*Tan[d + e*x]^2]
```

3.516.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

3.516. $\int \frac{a+b\tan(d+ex)}{\sqrt{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}}\,dx$

rule 4193
```
Int[((A_) + (B_.)*tan[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*tan[(d_.) + (e_.)*
(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Simp[(a + b*Tan
[d + e*x] + c*Tan[d + e*x]^2)^n/(b + 2*c*Tan[d + e*x])^(2*n)   Int[(A + B*T
an[d + e*x])*(b + 2*c*Tan[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e,
A, B}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[n]
```

3.516.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{(b+a\tan(ex+d))\left(2\ln(b+a\tan(ex+d))a^2-2\ln(b+a\tan(ex+d))b^2-\ln\left(1+\tan(ex+d)^2\right)a^2+\ln\left(1+\tan(ex+d)^2\right)b^2+4ab\right.}{2e\sqrt{b^2+2ab\tan(ex+d)+a^2\tan(ex+d)^2}\,(a^2+b^2)}$
default	$\frac{(b+a\tan(ex+d))\left(2\ln(b+a\tan(ex+d))a^2-2\ln(b+a\tan(ex+d))b^2-\ln\left(1+\tan(ex+d)^2\right)a^2+\ln\left(1+\tan(ex+d)^2\right)b^2+4ab\right.}{2e\sqrt{b^2+2ab\tan(ex+d)+a^2\tan(ex+d)^2}\,(a^2+b^2)}$
parts	$\frac{a(b+a\tan(ex+d))\left(2a\ln(b+a\tan(ex+d))-a\ln\left(1+\tan(ex+d)^2\right)+2b\arctan(\tan(ex+d))\right)}{2e\sqrt{b^2+2ab\tan(ex+d)+a^2\tan(ex+d)^2}\,(a^2+b^2)}-\frac{b(b+a\tan(ex+d))\left(2b\ln(\right.}{2e\sqrt{}}$

input
```
int((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(1/2),x,metho
d=_RETURNVERBOSE)
```

output
```
1/2/e*(b+a*tan(e*x+d))*(2*ln(b+a*tan(e*x+d))*a^2-2*ln(b+a*tan(e*x+d))*b^2-
ln(1+tan(e*x+d)^2)*a^2+ln(1+tan(e*x+d)^2)*b^2+4*a*b*arctan(tan(e*x+d)))/((
b+a*tan(e*x+d))^2)^(1/2)/(a^2+b^2)
```

3.516.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.51

$$\int \frac{a+b\tan(d+ex)}{\sqrt{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}}\,dx = \frac{4\,abex+(a^2-b^2)\log\left(\frac{a^2\tan(ex+d)^2+2\,ab\tan(ex+d)+b^2}{\tan(ex+d)^2+1}\right)}{2\,(a^2+b^2)e}$$

input
```
integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(1/2),x
, algorithm="fricas")
```

output
```
1/2*(4*a*b*e*x + (a^2 - b^2)*log((a^2*tan(e*x + d)^2 + 2*a*b*tan(e*x + d)
+ b^2)/(tan(e*x + d)^2 + 1)))/((a^2 + b^2)*e)
```

3.516.6 Sympy [F]

$$\int \frac{a+b\tan(d+ex)}{\sqrt{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}}\,dx=\int \frac{a+b\tan\left(d+ex\right)}{\sqrt{\left(a\tan\left(d+ex\right)+b\right)^2}}\,dx$$

input
```
integrate((a+b*tan(e*x+d))/(b**2+2*a*b*tan(e*x+d)+a**2*tan(e*x+d)**2)**(1/
2),x)
```

output
```
Integral((a + b*tan(d + e*x))/sqrt((a*tan(d + e*x) + b)**2), x)
```

3.516.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.99

$$\int \frac{a+b\tan(d+ex)}{\sqrt{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}}\,dx$$
$$=\frac{a\left(\frac{2\,(ex+d)b}{a^2+b^2}+\frac{2\,a\log(a\tan(ex+d)+b)}{a^2+b^2}-\frac{a\log\left(\tan(ex+d)^2+1\right)}{a^2+b^2}\right)+\left(\frac{2\,(ex+d)a}{a^2+b^2}-\frac{2\,b\log(a\tan(ex+d)+b)}{a^2+b^2}+\frac{b\log\left(\tan(ex+d)^2+1\right)}{a^2+b^2}\right)}{2\,e}$$

input
```
integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(1/2),x
, algorithm="maxima")
```

output
```
1/2*(a*(2*(e*x + d)*b/(a^2 + b^2) + 2*a*log(a*tan(e*x + d) + b)/(a^2 + b^2
) - a*log(tan(e*x + d)^2 + 1)/(a^2 + b^2)) + (2*(e*x + d)*a/(a^2 + b^2) -
2*b*log(a*tan(e*x + d) + b)/(a^2 + b^2) + b*log(tan(e*x + d)^2 + 1)/(a^2 +
 b^2))*b)/e
```

3.516.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.17

$$\int \frac{a+b\tan(d+ex)}{\sqrt{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}}\,dx$$
$$=\frac{\frac{4(ex+d)ab}{a^2\mathrm{sgn}(a\tan(ex+d)+b)+b^2\mathrm{sgn}(a\tan(ex+d)+b)}-\frac{(a^2-b^2)\log\left(\tan(ex+d)^2+1\right)}{a^2\mathrm{sgn}(a\tan(ex+d)+b)+b^2\mathrm{sgn}(a\tan(ex+d)+b)}+\frac{2(a^3-ab^2)\log(|a\tan(ex+d)+b|)}{a^3\mathrm{sgn}(a\tan(ex+d)+b)+ab^2\mathrm{sgn}(a\tan(ex}}{2e}$$

input
```
integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(1/2),x
, algorithm="giac")
```

output
```
1/2*(4*(e*x + d)*a*b/(a^2*sgn(a*tan(e*x + d) + b) + b^2*sgn(a*tan(e*x + d)
 + b)) - (a^2 - b^2)*log(tan(e*x + d)^2 + 1)/(a^2*sgn(a*tan(e*x + d) + b)
+ b^2*sgn(a*tan(e*x + d) + b)) + 2*(a^3 - a*b^2)*log(abs(a*tan(e*x + d) +
b))/(a^3*sgn(a*tan(e*x + d) + b) + a*b^2*sgn(a*tan(e*x + d) + b)))/e
```

3.516.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\tan(d+ex)}{\sqrt{b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)}}\,dx$$
$$=\int \frac{a+b\tan(d+e\,x)}{\sqrt{a^2\tan\left(d+e\,x\right)^2+2\,a\,b\tan\left(d+e\,x\right)+b^2}}\,dx$$

input
```
int((a + b*tan(d + e*x))/(b^2 + a^2*tan(d + e*x)^2 + 2*a*b*tan(d + e*x))^(
1/2),x)
```

output
```
int((a + b*tan(d + e*x))/(b^2 + a^2*tan(d + e*x)^2 + 2*a*b*tan(d + e*x))^(
1/2), x)
```

3.517 $\int \frac{a+b\tan(d+ex)}{\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^{3/2}}\,dx$

3.517.1 Optimal result

Integrand size = 41, antiderivative size = 316

$$\begin{aligned}\int \frac{a+b\tan(d+ex)}{\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^{3/2}}\,dx =& \\ &-\frac{(a^2-b^2)\,(b+a\tan(d+ex))}{2\,(a^2+b^2)\,e\,\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^{3/2}} \\ &-\frac{(a^4-6a^2b^2+b^4)\log(b\cos(d+ex)+a\sin(d+ex))(b+a\tan(d+ex))^3}{(a^2+b^2)^3\,e\,\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^{3/2}} \\ &-\frac{4b(a^2-b^2)\,x(ab+a^2\tan(d+ex))^3}{a^2\,(a^2+b^2)^3\,\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^{3/2}} \\ &-\frac{b(3a^2-b^2)\,(ab+a^2\tan(d+ex))^3}{(a^2+b^2)^2\,e\,(a^3b+a^4\tan(d+ex))\,\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^{3/2}}\end{aligned}$$

output
```
-1/2*(a^2-b^2)*(b+a*tan(e*x+d))/(a^2+b^2)/e/(b^2+2*a*b*tan(e*x+d)+a^2*tan(
e*x+d)^2)^(3/2)-(a^4-6*a^2*b^2+b^4)*ln(b*cos(e*x+d)+a*sin(e*x+d))*(b+a*tan
(e*x+d))^3/(a^2+b^2)^3/e/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(3/2)-4*b
*(a^2-b^2)*x*(a*b+a^2*tan(e*x+d))^3/a^2/(a^2+b^2)^3/(b^2+2*a*b*tan(e*x+d)+
a^2*tan(e*x+d)^2)^(3/2)-b*(3*a^2-b^2)*(a*b+a^2*tan(e*x+d))^3/(a^2+b^2)^2/e
/(a^3*b+a^4*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(3/2)
```

3.517.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.60 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.85

$$\int \frac{a+b\tan(d+ex)}{\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^{3/2}}\,dx = \frac{(b+a\tan(d+ex))^3\left(b\left(\frac{i\log(i-\tan(d+ex))}{(a-ib)^2}-\frac{i\log(i+\tan(d+e}{(a+ib)^2}\right.\right.}{}$$

input
```
Integrate[(a + b*Tan[d + e*x])/(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x
]^2)^(3/2),x]
```

output
```
((b + a*Tan[d + e*x])^3*(b*((I*Log[I - Tan[d + e*x]])/(a - I*b)^2 - (I*Log
[I + Tan[d + e*x]])/(a + I*b)^2 + (2*a*(2*b*Log[b + a*Tan[d + e*x]] - (a^2
 + b^2)/(b + a*Tan[d + e*x])))/(a^2 + b^2)^2) + (a - b)*(a + b)*(Log[I - T
an[d + e*x]]/(a - I*b)^3 + Log[I + Tan[d + e*x]]/(a + I*b)^3 + (a*(-2*(a^2
 - 3*b^2)*Log[b + a*Tan[d + e*x]] - ((a^2 + b^2)*(a^2 + 5*b^2 + 4*a*b*Tan[
d + e*x]))/(b + a*Tan[d + e*x])^2))/(a^2 + b^2)^3)))/(2*a*e*((b + a*Tan[d
+ e*x])^2)^(3/2))
```

3.517.3 Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.74, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.293$, Rules used = {3042, 4193, 27, 3042, 4012, 3042, 4012, 25, 3042, 4014, 3042, 4013}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\tan(d+ex)}{\left(a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\tan(d+ex)}{\left(a^2\tan(d+ex)^2+2ab\tan(d+ex)+b^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{4193}$$

$$\frac{8\left(a^2\tan(d+ex)+ab\right)^3\int\frac{a+b\tan(d+ex)}{8(\tan(d+ex)a^2+ba)^3}dx}{\left(a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(a^2\tan(d+ex)+ab\right)^3\int\frac{a+b\tan(d+ex)}{(\tan(d+ex)a^2+ba)^3}dx}{\left(a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(a^2\tan(d+ex)+ab\right)^3\int\frac{a+b\tan(d+ex)}{(\tan(d+ex)a^2+ba)^3}dx}{\left(a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2\right)^{3/2}}$$

$$\Big\downarrow \text{4012}$$

$$\frac{\left(a^2\tan(d+ex)+ab\right)^3\left(\frac{\int\frac{2a^2b-a\left(a^2-b^2\right)\tan(d+ex)}{\left(\tan(d+ex)a^2+ba\right)^2}dx}{a^2(a^2+b^2)}-\frac{a^2-b^2}{2ae(a^2+b^2)(a^2\tan(d+ex)+ab)^2}\right)}{\left(a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(a^2\tan(d+ex)+ab\right)^3\left(\frac{\int\frac{2a^2b-a\left(a^2-b^2\right)\tan(d+ex)}{\left(\tan(d+ex)a^2+ba\right)^2}dx}{a^2(a^2+b^2)}-\frac{a^2-b^2}{2ae(a^2+b^2)(a^2\tan(d+ex)+ab)^2}\right)}{\left(a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2\right)^{3/2}}$$

$$\Big\downarrow \text{4012}$$

$$\frac{\left(a^2\tan(d+ex)+ab\right)^3\left(\frac{\frac{\int-\frac{\left(a^2-3b^2\right)a^3+b\left(3a^2-b^2\right)\tan(d+ex)a^2}{\tan(d+ex)a^2+ba}dx}{a^2\left(a^2+b^2\right)}-\frac{b\left(3a^2-b^2\right)}{e\left(a^2+b^2\right)\left(a^2\tan(d+ex)+ab\right)}}{a^2(a^2+b^2)}-\frac{a^2-b^2}{2ae(a^2+b^2)(a^2\tan(d+ex)+ab)^2}\right)}{\left(a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2\right)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\left(a^2\tan(d+ex)+ab\right)^3\left(\frac{-\frac{\int\frac{\left(a^2-3b^2\right)a^3+b\left(3a^2-b^2\right)\tan(d+ex)a^2}{\tan(d+ex)a^2+ba}dx}{a^2\left(a^2+b^2\right)}-\frac{b\left(3a^2-b^2\right)}{e\left(a^2+b^2\right)\left(a^2\tan(d+ex)+ab\right)}}{a^2(a^2+b^2)}-\frac{a^2-b^2}{2ae(a^2+b^2)(a^2\tan(d+ex)+ab)^2}\right)}{\left(a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2\right)^{3/2}}$$

3.517. $\int\frac{a+b\tan(d+ex)}{(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex))^{3/2}}\,dx$

$$\Big\downarrow \; 3042$$

$$\frac{(a^2\tan(d+ex)+ab)^3\left(\frac{-\frac{\int\frac{(a^2-3b^2)a^3+b(3a^2-b^2)\tan(d+ex)a^2}{\tan(d+ex)a^2+ba}dx}{a^2(a^2+b^2)}-\frac{b(3a^2-b^2)}{e(a^2+b^2)(a^2\tan(d+ex)+ab)}}{a^2(a^2+b^2)}-\frac{a^2-b^2}{2ae(a^2+b^2)(a^2\tan(d+ex)+ab)^2}\right)}{(a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2)^{3/2}}$$

$$\Big\downarrow \; 4014$$

$$\frac{(a^2\tan(d+ex)+ab)^3\left(\frac{-\frac{\frac{a(a^4-6a^2b^2+b^4)\int\frac{a^2-ab\tan(d+ex)}{\tan(d+ex)a^2+ba}dx}{a^2+b^2}+\frac{4a^2bx(a^2-b^2)}{a^2+b^2}}{a^2(a^2+b^2)}-\frac{b(3a^2-b^2)}{e(a^2+b^2)(a^2\tan(d+ex)+ab)}}{a^2(a^2+b^2)}-\frac{a^2-b^2}{2ae(a^2+b^2)(a^2\tan(d+e}\right.}{(a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2)^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{(a^2\tan(d+ex)+ab)^3\left(\frac{-\frac{\frac{a(a^4-6a^2b^2+b^4)\int\frac{a^2-ab\tan(d+ex)}{\tan(d+ex)a^2+ba}dx}{a^2+b^2}+\frac{4a^2bx(a^2-b^2)}{a^2+b^2}}{a^2(a^2+b^2)}-\frac{b(3a^2-b^2)}{e(a^2+b^2)(a^2\tan(d+ex)+ab)}}{a^2(a^2+b^2)}-\frac{a^2-b^2}{2ae(a^2+b^2)(a^2\tan(d+e}\right.}{(a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2)^{3/2}}$$

$$\Big\downarrow \; 4013$$

$$\frac{(a^2\tan(d+ex)+ab)^3\left(\frac{-\frac{b(3a^2-b^2)}{e(a^2+b^2)(a^2\tan(d+ex)+ab)}-\frac{\frac{4a^2bx(a^2-b^2)}{a^2+b^2}+\frac{a(a^4-6a^2b^2+b^4)\log(a\sin(d+ex)+b\cos(d+ex))}{e(a^2+b^2)}}{a^2(a^2+b^2)}}{a^2(a^2+b^2)}-\frac{a^2-}{2ae(a^2+b^2)(a^2\tan}\right.}{(a^2\tan^2(d+ex)+2ab\tan(d+ex)+b^2)^{3/2}}$$

input
```
Int[(a + b*Tan[d + e*x])/(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2)^(
3/2),x]
```

3.517. $\int \frac{a+b\tan(d+ex)}{(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex))^{3/2}}\,dx$

output
```
((a*b + a^2*Tan[d + e*x])^3*(-1/2*(a^2 - b^2)/(a*(a^2 + b^2)*e*(a*b + a^2*
Tan[d + e*x])^2) + (-(((4*a^2*b*(a^2 - b^2)*x)/(a^2 + b^2) + (a*(a^4 - 6*a
^2*b^2 + b^4)*Log[b*Cos[d + e*x] + a*Sin[d + e*x]])/((a^2 + b^2)*e))/(a^2*
(a^2 + b^2))) - (b*(3*a^2 - b^2))/((a^2 + b^2)*e*(a*b + a^2*Tan[d + e*x]))
)/(a^2*(a^2 + b^2))))/(b^2 + 2*a*b*Tan[d + e*x] + a^2*Tan[d + e*x]^2)^(3/2
)
```

3.517.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4012
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[e + f*x]
)^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a
, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1
]
```

rule 4013
```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

rule 4014
```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Simp[(b*c - a
*d)/(a^2 + b^2)   Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && N
eQ[a*c + b*d, 0]
```

3.517. $\int \frac{a+b\tan(d+ex)}{(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex))^{3/2}}\,dx$

rule 4193
```
Int[((A_) + (B_.)*tan[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*tan[(d_.) + (e_.)*
(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Simp[(a + b*Tan
[d + e*x] + c*Tan[d + e*x]^2)^n/(b + 2*c*Tan[d + e*x])^(2*n)   Int[(A + B*T
an[d + e*x])*(b + 2*c*Tan[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e,
A, B}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[n]
```

3.517.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 621 vs. $2(306) = 612$.

Time = 0.70 (sec) , antiderivative size = 622, normalized size of antiderivative = 1.97

method	result
derivativedivides	$-\frac{\left(-2\ln\left(1+\tan(ex+d)^2\right)a^5b\tan(ex+d)+12\ln\left(1+\tan(ex+d)^2\right)a^3b^3\tan(ex+d)-2\ln\left(1+\tan(ex+d)^2\right)a\,b^5\tan(ex+d)\right.}{}$
default	$-\frac{\left(-2\ln\left(1+\tan(ex+d)^2\right)a^5b\tan(ex+d)+12\ln\left(1+\tan(ex+d)^2\right)a^3b^3\tan(ex+d)-2\ln\left(1+\tan(ex+d)^2\right)a\,b^5\tan(ex+d)\right.}{}$
parts	Expression too large to display

input
```
int((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(3/2),x,metho
d=_RETURNVERBOSE)
```

output
```
-1/2/e*(-2*ln(1+tan(e*x+d)^2)*a^5*b*tan(e*x+d)+12*ln(1+tan(e*x+d)^2)*a^3*b
^3*tan(e*x+d)-2*ln(1+tan(e*x+d)^2)*a*b^5*tan(e*x+d)+16*arctan(tan(e*x+d))*
a^4*b^2*tan(e*x+d)-16*arctan(tan(e*x+d))*a^2*b^4*tan(e*x+d)-12*ln(b+a*tan(
e*x+d))*a^4*b^2*tan(e*x+d)^2+2*ln(b+a*tan(e*x+d))*a^2*b^4*tan(e*x+d)^2+6*l
n(1+tan(e*x+d)^2)*a^4*b^2*tan(e*x+d)^2-ln(1+tan(e*x+d)^2)*a^2*b^4*tan(e*x+
d)^2+8*arctan(tan(e*x+d))*a^5*b*tan(e*x+d)^2-8*arctan(tan(e*x+d))*a^3*b^3*
tan(e*x+d)^2+4*ln(b+a*tan(e*x+d))*a^5*b*tan(e*x+d)-24*ln(b+a*tan(e*x+d))*a
^3*b^3*tan(e*x+d)+a^6-3*b^6+6*a^5*b*tan(e*x+d)+4*a^3*b^3*tan(e*x+d)-2*a*b^
5*tan(e*x+d)-ln(1+tan(e*x+d)^2)*a^6*tan(e*x+d)^2+2*ln(b+a*tan(e*x+d))*a^4*
b^2-12*ln(b+a*tan(e*x+d))*a^2*b^4-ln(1+tan(e*x+d)^2)*a^4*b^2+6*ln(1+tan(e*
x+d)^2)*a^2*b^4+8*arctan(tan(e*x+d))*a^3*b^3-8*arctan(tan(e*x+d))*a*b^5+2*
ln(b+a*tan(e*x+d))*a^6*tan(e*x+d)^2+7*a^4*b^2+4*ln(b+a*tan(e*x+d))*a*b^5*t
an(e*x+d)+3*a^2*b^4+2*ln(b+a*tan(e*x+d))*b^6-ln(1+tan(e*x+d)^2)*b^6)*(b+a*
tan(e*x+d))/(a^2+b^2)^3/((b+a*tan(e*x+d))^2)^(3/2)
```

3.517.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.12

$$\int \frac{a+b\tan(d+ex)}{(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex))^{3/2}}\,dx =$$

$$-\frac{a^6+8\,a^4b^2-5\,a^2b^4+8\,(a^3b^3-ab^5)ex+(a^6-8\,a^4b^2+3\,a^2b^4+8\,(a^5b-a^3b^3)ex)\tan{(ex+d)}^2+(a^4b^2-}{2\,((a^8+3\,a^6b^2+3\,a^4b^4+a}$$

input
```
integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(3/2),x
, algorithm="fricas")
```

output
```
-1/2*(a^6 + 8*a^4*b^2 - 5*a^2*b^4 + 8*(a^3*b^3 - a*b^5)*e*x + (a^6 - 8*a^4
*b^2 + 3*a^2*b^4 + 8*(a^5*b - a^3*b^3)*e*x)*tan(e*x + d)^2 + (a^4*b^2 - 6*
a^2*b^4 + b^6 + (a^6 - 6*a^4*b^2 + a^2*b^4)*tan(e*x + d)^2 + 2*(a^5*b - 6*
a^3*b^3 + a*b^5)*tan(e*x + d))*log((a^2*tan(e*x + d)^2 + 2*a*b*tan(e*x + d
) + b^2)/(tan(e*x + d)^2 + 1)) + 4*(2*a^5*b - 3*a^3*b^3 + a*b^5 + 4*(a^4*b
^2 - a^2*b^4)*e*x)*tan(e*x + d))/((a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*
e*tan(e*x + d)^2 + 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*e*tan(e*x + d
) + (a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*e)
```

3.517.6 Sympy [F]

$$\int \frac{a+b\tan(d+ex)}{(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex))^{3/2}}\,dx = \int \frac{a+b\tan{(d+ex)}}{\left((a\tan{(d+ex)}+b)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*tan(e*x+d))/(b**2+2*a*b*tan(e*x+d)+a**2*tan(e*x+d)**2)**(3/
2),x)
```

output
```
Integral((a + b*tan(d + e*x))/((a*tan(d + e*x) + b)**2)**(3/2), x)
```

3.517.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.58

$$\int \frac{a+b\tan(d+ex)}{\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^{3/2}}\,dx =$$

$$-\frac{\left(\frac{2(3a^2b-b^3)(ex+d)}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{2(a^3-3ab^2)\log(a\tan(ex+d)+b)}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{(a^3-3ab^2)\log\left(\tan(ex+d)^2+1\right)}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{4a^2b\tan(ex\cdots}{a^4b^2+2a^2b^4+b^6+(a^6+2a^4b^2+a^2b^4)\tan\cdots}\right.}{}$$

input
```
integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(3/2),x
, algorithm="maxima")
```

output
```
-1/2*((2*(3*a^2*b - b^3)*(e*x + d)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 2
*(a^3 - 3*a*b^2)*log(a*tan(e*x + d) + b)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^
6) - (a^3 - 3*a*b^2)*log(tan(e*x + d)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4
+ b^6) + (4*a^2*b*tan(e*x + d) + a^3 + 5*a*b^2)/(a^4*b^2 + 2*a^2*b^4 + b^6
 + (a^6 + 2*a^4*b^2 + a^2*b^4)*tan(e*x + d)^2 + 2*(a^5*b + 2*a^3*b^3 + a*b
^5)*tan(e*x + d)))*a + (2*(a^3 - 3*a*b^2)*(e*x + d)/(a^6 + 3*a^4*b^2 + 3*a
^2*b^4 + b^6) - 2*(3*a^2*b - b^3)*log(a*tan(e*x + d) + b)/(a^6 + 3*a^4*b^2
 + 3*a^2*b^4 + b^6) + (3*a^2*b - b^3)*log(tan(e*x + d)^2 + 1)/(a^6 + 3*a^4
*b^2 + 3*a^2*b^4 + b^6) + (a^2*b - 3*b^3 + 2*(a^3 - a*b^2)*tan(e*x + d))/(
a^4*b^2 + 2*a^2*b^4 + b^6 + (a^6 + 2*a^4*b^2 + a^2*b^4)*tan(e*x + d)^2 + 2
*(a^5*b + 2*a^3*b^3 + a*b^5)*tan(e*x + d)))*b)/e
```

3.517.8 Giac [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.52

$$\int \frac{a+b\tan(d+ex)}{\left(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex)\right)^{3/2}}\,dx =$$

$$-\frac{\frac{8(a^3b-ab^3)(ex+d)}{a^6\mathrm{sgn}(a\tan(ex+d)+b)+3a^4b^2\mathrm{sgn}(a\tan(ex+d)+b)+3a^2b^4\mathrm{sgn}(a\tan(ex+d)+b)+b^6\mathrm{sgn}(a\tan(ex+d)+b)}-\frac{(a^4\cdots}{a^6\mathrm{sgn}(a\tan(ex+d)+b)+3a^4b^2\mathrm{sgn}(a\cdots}}{}$$

input
```
integrate((a+b*tan(e*x+d))/(b^2+2*a*b*tan(e*x+d)+a^2*tan(e*x+d)^2)^(3/2),x
, algorithm="giac")
```

output
```
-1/2*(8*(a^3*b - a*b^3)*(e*x + d)/(a^6*sgn(a*tan(e*x + d) + b) + 3*a^4*b^2
*sgn(a*tan(e*x + d) + b) + 3*a^2*b^4*sgn(a*tan(e*x + d) + b) + b^6*sgn(a*t
an(e*x + d) + b)) - (a^4 - 6*a^2*b^2 + b^4)*log(tan(e*x + d)^2 + 1)/(a^6*s
gn(a*tan(e*x + d) + b) + 3*a^4*b^2*sgn(a*tan(e*x + d) + b) + 3*a^2*b^4*sgn
(a*tan(e*x + d) + b) + b^6*sgn(a*tan(e*x + d) + b)) + 2*(a^5 - 6*a^3*b^2 +
 a*b^4)*log(abs(a*tan(e*x + d) + b))/(a^7*sgn(a*tan(e*x + d) + b) + 3*a^5*
b^2*sgn(a*tan(e*x + d) + b) + 3*a^3*b^4*sgn(a*tan(e*x + d) + b) + a*b^6*sg
n(a*tan(e*x + d) + b)) - (3*a^6*tan(e*x + d)^2 - 18*a^4*b^2*tan(e*x + d)^2
 + 3*a^2*b^4*tan(e*x + d)^2 - 40*a^3*b^3*tan(e*x + d) + 8*a*b^5*tan(e*x +
d) - a^6 - 4*a^4*b^2 - 21*a^2*b^4 + 6*b^6)/((a^6*sgn(a*tan(e*x + d) + b) +
 3*a^4*b^2*sgn(a*tan(e*x + d) + b) + 3*a^2*b^4*sgn(a*tan(e*x + d) + b) + b
^6*sgn(a*tan(e*x + d) + b))*(a*tan(e*x + d) + b)^2))/e
```

3.517.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\tan(d+ex)}{(b^2+2ab\tan(d+ex)+a^2\tan^2(d+ex))^{3/2}}\,dx=\int \frac{a+b\tan(d+e\,x)}{\left(a^2\tan\left(d+e\,x\right)^2+2\,a\,b\tan\left(d+e\,x\right)+b^2\right)^{3/2}}\,dx$$

input
```
int((a + b*tan(d + e*x))/(b^2 + a^2*tan(d + e*x)^2 + 2*a*b*tan(d + e*x))^(
3/2),x)
```

output
```
int((a + b*tan(d + e*x))/(b^2 + a^2*tan(d + e*x)^2 + 2*a*b*tan(d + e*x))^(
3/2), x)
```

3.518 $\int (a+b\sec(d+ex))\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d-$

3.518.1 Optimal result

Integrand size = 39, antiderivative size = 184

$$\begin{aligned}
&\int (a+b\sec(d+ex))\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^2\,dx \\
&= ab^4x+\frac{b(19a^4+56a^2b^2+8b^4)\operatorname{arctanh}(\sin(d+ex))}{8e} \\
&\quad+\frac{a(4a^4+50a^2b^2+19b^4)\tan(d+ex)}{6e}+\frac{a^2b(41a^2+26b^2)\sec(d+ex)\tan(d+ex)}{24e} \\
&\quad+\frac{(4a^2+7b^2)\left(ab+a^2\sec(d+ex)\right)^2\tan(d+ex)}{12ae}+\frac{b\left(ab+a^2\sec(d+ex)\right)^3\tan(d+ex)}{4a^2e}
\end{aligned}$$

output
```
a*b^4*x+1/8*b*(19*a^4+56*a^2*b^2+8*b^4)*arctanh(sin(e*x+d))/e+1/6*a*(4*a^4
+50*a^2*b^2+19*b^4)*tan(e*x+d)/e+1/24*a^2*b*(41*a^2+26*b^2)*sec(e*x+d)*tan
(e*x+d)/e+1/12*(4*a^2+7*b^2)*(a*b+a^2*sec(e*x+d))^2*tan(e*x+d)/a/e+1/4*b*(
a*b+a^2*sec(e*x+d))^3*tan(e*x+d)/a^2/e
```

3.518.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.15

$$\begin{aligned}
&\int (a+b\sec(d+ex))\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^2\,dx \\
&= ab^4x+\frac{19a^4b\text{arctanh}(\sin(d+ex))}{8e}+\frac{7a^2b^3\text{arctanh}(\sin(d+ex))}{e} \\
&\quad+\frac{b^5\text{arctanh}(\sin(d+ex))}{e}+\frac{a^5\tan(d+ex)}{e}+\frac{10a^3b^2\tan(d+ex)}{e} \\
&\quad+\frac{4ab^4\tan(d+ex)}{e}+\frac{19a^4b\sec(d+ex)\tan(d+ex)}{8e}+\frac{3a^2b^3\sec(d+ex)\tan(d+ex)}{e} \\
&\quad+\frac{a^4b\sec^3(d+ex)\tan(d+ex)}{4e}+\frac{a^5\tan^3(d+ex)}{3e}+\frac{4a^3b^2\tan^3(d+ex)}{3e}
\end{aligned}$$

input
```
Integrate[(a + b*Sec[d + e*x])*(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x
]^2)^2,x]
```

output
```
a*b^4*x + (19*a^4*b*ArcTanh[Sin[d + e*x]])/(8*e) + (7*a^2*b^3*ArcTanh[Sin[
d + e*x]])/e + (b^5*ArcTanh[Sin[d + e*x]])/e + (a^5*Tan[d + e*x])/e + (10*
a^3*b^2*Tan[d + e*x])/e + (4*a*b^4*Tan[d + e*x])/e + (19*a^4*b*Sec[d + e*x
]*Tan[d + e*x])/(8*e) + (3*a^2*b^3*Sec[d + e*x]*Tan[d + e*x])/e + (a^4*b*S
ec[d + e*x]^3*Tan[d + e*x])/(4*e) + (a^5*Tan[d + e*x]^3)/(3*e) + (4*a^3*b^
2*Tan[d + e*x]^3)/(3*e)
```

3.518.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.256, Rules used = {3042, 4660, 27, 3042, 4406, 3042, 4544, 3042, 4536, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(d+ex))\left(a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2\right)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\sec(d+ex))\left(a^2\sec(d+ex)^2+2ab\sec(d+ex)+b^2\right)^2dx$$

$$\Big\downarrow \text{4660}$$

$$\frac{\int 16\left(\sec(d+ex)a^2+ba\right)^4(a+b\sec(d+ex))dx}{16a^4}$$

$\downarrow$ 27

$$\frac{\int \left(\sec(d+ex)a^2+ba\right)^4(a+b\sec(d+ex))dx}{a^4}$$

$\downarrow$ 3042

$$\frac{\int \left(\csc\left(d+ex+\frac{\pi}{2}\right)a^2+ba\right)^4\left(a+b\csc\left(d+ex+\frac{\pi}{2}\right)\right)dx}{a^4}$$

$\downarrow$ 4406

$$\frac{\frac{1}{4}\int \left(\sec(d+ex)a^2+ba\right)^2\left(4b^2a^3+\left(4a^2+7b^2\right)\sec^2(d+ex)a^3+b\left(11a^2+4b^2\right)\sec(d+ex)a^2\right)dx+\frac{a^2b\tan(d+ex)(a}{4}}{a^4}$$

$\downarrow$ 3042

$$\frac{\frac{1}{4}\int \left(\csc\left(d+ex+\frac{\pi}{2}\right)a^2+ba\right)^2\left(4b^2a^3+\left(4a^2+7b^2\right)\csc\left(d+ex+\frac{\pi}{2}\right)^2a^3+b\left(11a^2+4b^2\right)\csc\left(d+ex+\frac{\pi}{2}\right)a^2\right)d}{a^4}$$

$\downarrow$ 4544

$$\frac{\frac{1}{4}\left(\frac{1}{3}\int \left(\sec(d+ex)a^2+ba\right)\left(12b^3a^4+b\left(41a^2+26b^2\right)\sec^2(d+ex)a^4+\left(8a^4+59b^2a^2+12b^4\right)\sec(d+ex)a^3\right)dx\right.}{a^4}$$

$\downarrow$ 3042

$$\frac{\frac{1}{4}\left(\frac{1}{3}\int \left(\csc\left(d+ex+\frac{\pi}{2}\right)a^2+ba\right)\left(12b^3a^4+b\left(41a^2+26b^2\right)\csc\left(d+ex+\frac{\pi}{2}\right)^2a^4+\left(8a^4+59b^2a^2+12b^4\right)\csc\left(d+\right.\right.\right.}{a^4}$$

$\downarrow$ 4536

$$\frac{\frac{1}{4}\left(\frac{1}{3}\left(\frac{1}{2}\int \left(24b^4a^5+4\left(4a^4+50b^2a^2+19b^4\right)\sec^2(d+ex)a^5+3b\left(19a^4+56b^2a^2+8b^4\right)\sec(d+ex)a^4\right)dx+\frac{a^6b(41a}{}\right.\right.}{a^4}$$

$\downarrow$ 2009

$$\frac{\frac{a^2b\tan(d+ex)\left(a^2\sec(d+ex)+ab\right)^3}{4e}+\frac{1}{4}\left(\frac{a^3\left(4a^2+7b^2\right)\tan(d+ex)\left(a^2\sec(d+ex)+ab\right)^2}{3e}+\frac{1}{3}\left(\frac{a^6b\left(41a^2+26b^2\right)\tan(d+ex)\sec(d+ex)}{2e}+\frac{1}{2}\left(24\right.\right.\right.}{a^4}$$

3.518. $\int (a+b\sec(d+ex))\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^2\,dx$

input
```
Int[(a + b*Sec[d + e*x])*(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2)^2
,x]
```

output
```
((a^2*b*(a*b + a^2*Sec[d + e*x])^3*Tan[d + e*x])/(4*e) + ((a^3*(4*a^2 + 7*
b^2)*(a*b + a^2*Sec[d + e*x])^2*Tan[d + e*x])/(3*e) + ((a^6*b*(41*a^2 + 26
*b^2)*Sec[d + e*x]*Tan[d + e*x])/(2*e) + (24*a^5*b^4*x + (3*a^4*b*(19*a^4
+ 56*a^2*b^2 + 8*b^4)*ArcTanh[Sin[d + e*x]])/e + (4*a^5*(4*a^4 + 50*a^2*b^
2 + 19*b^4)*Tan[d + e*x])/e)/2)/3)/4)/a^4
```

3.518.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4406
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m
 - 1)/(f*m)), x] + Simp[1/m   Int[(a + b*Csc[e + f*x])^(m - 2)*Simp[a^2*c*m
 + (b^2*d*(m - 1) + 2*a*b*c*m + a^2*d*m)*Csc[e + f*x] + b*(b*c*m + a*d*(2*m
 - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*
c - a*d, 0] && GtQ[m, 1] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

rule 4536
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[(-b)*C*Csc[e +
 f*x]*(Cot[e + f*x]/(2*f)), x] + Simp[1/2   Int[Simp[2*A*a + (2*B*a + b*(2*
A + C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a
, b, e, f, A, B, C}, x]
```

rule 4544
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> Simp[(-C)*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[1/(m + 1)   Int[(
a + b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m
)*Csc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[
{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

rule 4660
```
Int[((A_) + (B_.)*sec[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sec[(d_.) + (e_.)*
(x_)] + (c_.)*sec[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Simp[1/(4^n*c^n
)   Int[(A + B*Sec[d + e*x])*(b + 2*c*Sec[d + e*x])^(2*n), x], x] /; FreeQ[
{a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[n]
```

3.518.4 Maple [A] (verified)

Time = 2.88 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.09

method	result
parts	$a\,b^4x+\frac{(4a^2b^3+b^5)\ln(\sec(ex+d)+\tan(ex+d))}{e}+\frac{(6a^3b^2+4a\,b^4)\tan(ex+d)}{e}+\frac{(4a^4b+6a^2b^3)\left(\frac{\sec(ex+d)\tan(ex+d}{2}\right.}{e}$
derivativedivides	$\frac{a\,b^4(ex+d)+4a^2b^3\ln(\sec(ex+d)+\tan(ex+d))+6\tan(ex+d)a^3b^2+4a^4b\left(\frac{\sec(ex+d)\tan(ex+d)}{2}+\frac{\ln(\sec(ex+d)+\tan(ex+d))}{2}\right)}{}$
default	$\frac{a\,b^4(ex+d)+4a^2b^3\ln(\sec(ex+d)+\tan(ex+d))+6\tan(ex+d)a^3b^2+4a^4b\left(\frac{\sec(ex+d)\tan(ex+d)}{2}+\frac{\ln(\sec(ex+d)+\tan(ex+d))}{2}\right)}{}$
parallelrisch	$\frac{228b\left(\frac{3}{4}+\frac{\cos(4ex+4d)}{4}+\cos(2ex+2d)\right)\left(a^4+\frac{56}{19}a^2b^2+\frac{8}{19}b^4\right)\ln\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)-228b\left(\frac{3}{4}+\frac{\cos(4ex+4d)}{4}+\cos(2ex+2d)\right)(}{}$
norman	$\frac{a\,b^4x+a\,b^4x\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^8-4a\,b^4x\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2+6a\,b^4x\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^4-4a\,b^4x\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^6-\frac{a\left(8a^4-21a^3b+80a^2b^2-24\right.}{4}}{}$
risch	$a\,b^4x+\frac{ia\left(-57a^3b\,e^{7i(ex+d)}-72a\,b^3e^{7i(ex+d)}+144a^2b^2e^{6i(ex+d)}+96b^4e^{6i(ex+d)}-81a^3b\,e^{5i(ex+d)}-72a\,b^3e^{5i(ex+d)}+\right.}{}$

input
```
int((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^2,x,method=_R
ETURNVERBOSE)
```

output
```
a*b^4*x+(4*a^2*b^3+b^5)/e*ln(sec(e*x+d)+tan(e*x+d))+(6*a^3*b^2+4*a*b^4)/e*
tan(e*x+d)+(4*a^4*b+6*a^2*b^3)/e*(1/2*sec(e*x+d)*tan(e*x+d)+1/2*ln(sec(e*x
+d)+tan(e*x+d)))-(a^5+4*a^3*b^2)/e*(-2/3-1/3*sec(e*x+d)^2)*tan(e*x+d)+a^4*
b/e*(-(-1/4*sec(e*x+d)^3-3/8*sec(e*x+d))*tan(e*x+d)+3/8*ln(sec(e*x+d)+tan(
e*x+d)))
```

3.518.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.08

$$\int (a+b\sec(d+ex))\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^2\,dx$$
$$=\frac{48\,ab^4ex\cos\left(ex+d\right)^4+3\left(19\,a^4b+56\,a^2b^3+8\,b^5\right)\cos\left(ex+d\right)^4\log\left(\sin\left(ex+d\right)+1\right)-3\left(19\,a^4b+56\,a^2b\right.}{}$$

input
```
integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^2,x, al
gorithm="fricas")
```

output
```
1/48*(48*a*b^4*e*x*cos(e*x + d)^4 + 3*(19*a^4*b + 56*a^2*b^3 + 8*b^5)*cos(
e*x + d)^4*log(sin(e*x + d) + 1) - 3*(19*a^4*b + 56*a^2*b^3 + 8*b^5)*cos(e
*x + d)^4*log(-sin(e*x + d) + 1) + 2*(6*a^4*b + 16*(a^5 + 13*a^3*b^2 + 6*a
*b^4)*cos(e*x + d)^3 + 3*(19*a^4*b + 24*a^2*b^3)*cos(e*x + d)^2 + 8*(a^5 +
 4*a^3*b^2)*cos(e*x + d))*sin(e*x + d))/(e*cos(e*x + d)^4)
```

3.518.6 Sympy [F]

$$\int (a+b\sec(d+ex))\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^2\,dx$$
$$=\int (a+b\sec\left(d+ex\right))\left(a\sec\left(d+ex\right)+b\right)^4\,dx$$

input
```
integrate((a+b*sec(e*x+d))*(b**2+2*a*b*sec(e*x+d)+a**2*sec(e*x+d)**2)**2,x
)
```

output
```
Integral((a + b*sec(d + e*x))*(a*sec(d + e*x) + b)**4, x)
```

3.518.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.62

$$\int (a+b\sec(d+ex))\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^2\,dx$$

$$= \frac{16\left(\tan\left(ex+d\right)^3+3\,\tan\left(ex+d\right)\right)a^5+64\left(\tan\left(ex+d\right)^3+3\,\tan\left(ex+d\right)\right)a^3b^2+48\left(ex+d\right)ab^4-3\,a^4b}{}$$

input
```
integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^2,x, al
gorithm="maxima")
```

output
```
1/48*(16*(tan(e*x + d)^3 + 3*tan(e*x + d))*a^5 + 64*(tan(e*x + d)^3 + 3*ta
n(e*x + d))*a^3*b^2 + 48*(e*x + d)*a*b^4 - 3*a^4*b*(2*(3*sin(e*x + d)^3 -
5*sin(e*x + d))/(sin(e*x + d)^4 - 2*sin(e*x + d)^2 + 1) - 3*log(sin(e*x +
d) + 1) + 3*log(sin(e*x + d) - 1)) - 48*a^4*b*(2*sin(e*x + d)/(sin(e*x + d
)^2 - 1) - log(sin(e*x + d) + 1) + log(sin(e*x + d) - 1)) - 72*a^2*b^3*(2*
sin(e*x + d)/(sin(e*x + d)^2 - 1) - log(sin(e*x + d) + 1) + log(sin(e*x +
d) - 1)) + 192*a^2*b^3*log(sec(e*x + d) + tan(e*x + d)) + 48*b^5*log(sec(e
*x + d) + tan(e*x + d)) + 288*a^3*b^2*tan(e*x + d) + 192*a*b^4*tan(e*x + d
))/e
```

3.518.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 447 vs. 2(174) = 348.

Time = 0.39 (sec) , antiderivative size = 447, normalized size of antiderivative = 2.43

$$\int (a+b\sec(d+ex))\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^2\,dx$$

$$= \frac{24\left(ex+d\right)ab^4+3\left(19\,a^4b+56\,a^2b^3+8\,b^5\right)\log\left(\left|\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)+1\right|\right)-3\left(19\,a^4b+56\,a^2b^3+8\,b^5\right)\log\left(\left|\right.\right.}{}$$

input
```
integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^2,x, al
gorithm="giac")
```

output
```
1/24*(24*(e*x + d)*a*b^4 + 3*(19*a^4*b + 56*a^2*b^3 + 8*b^5)*log(abs(tan(1
/2*e*x + 1/2*d) + 1)) - 3*(19*a^4*b + 56*a^2*b^3 + 8*b^5)*log(abs(tan(1/2*
e*x + 1/2*d) - 1)) - 2*(24*a^5*tan(1/2*e*x + 1/2*d)^7 - 63*a^4*b*tan(1/2*e
*x + 1/2*d)^7 + 240*a^3*b^2*tan(1/2*e*x + 1/2*d)^7 - 72*a^2*b^3*tan(1/2*e*
x + 1/2*d)^7 + 96*a*b^4*tan(1/2*e*x + 1/2*d)^7 - 40*a^5*tan(1/2*e*x + 1/2*
d)^5 + 39*a^4*b*tan(1/2*e*x + 1/2*d)^5 - 592*a^3*b^2*tan(1/2*e*x + 1/2*d)^
5 + 72*a^2*b^3*tan(1/2*e*x + 1/2*d)^5 - 288*a*b^4*tan(1/2*e*x + 1/2*d)^5 +
 40*a^5*tan(1/2*e*x + 1/2*d)^3 + 39*a^4*b*tan(1/2*e*x + 1/2*d)^3 + 592*a^3
*b^2*tan(1/2*e*x + 1/2*d)^3 + 72*a^2*b^3*tan(1/2*e*x + 1/2*d)^3 + 288*a*b^
4*tan(1/2*e*x + 1/2*d)^3 - 24*a^5*tan(1/2*e*x + 1/2*d) - 63*a^4*b*tan(1/2*
e*x + 1/2*d) - 240*a^3*b^2*tan(1/2*e*x + 1/2*d) - 72*a^2*b^3*tan(1/2*e*x +
 1/2*d) - 96*a*b^4*tan(1/2*e*x + 1/2*d))/(tan(1/2*e*x + 1/2*d)^2 - 1)^4)/e
```

3.518.9 Mupad [B] (verification not implemented)

Time = 26.60 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.76

$$\int (a+b\sec(d+ex))\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^2\,dx$$

$$= \frac{2\,a^5\,\sin\left(d+e\,x\right)}{3\,e\,\cos\left(d+e\,x\right)}+\frac{a^5\,\sin\left(d+e\,x\right)}{3\,e\cos\left(d+e\,x\right)^3}+\frac{2\,a\,b^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{\cos\left(\frac{d}{2}+\frac{e\,x}{2}\right)}\right)}{e}+\frac{4\,a\,b^4\,\sin\left(d+e\,x\right)}{e\,\cos\left(d+e\,x\right)}+\frac{19\,a^4\,b\,\sin\left(d+e\,x\right)}{8\,e\cos\left(d+e\,x\right)^2}+\frac{a^4\,b\,\sin\left(d+e\,x\right)}{4\,e\cos\left(d+e\,x\right)^4}+\frac{26\,a^3\,b^2\,\sin\left(d+e\,x\right)}{3\,e\,\cos\left(d+e\,x\right)}+\frac{3\,a^2\,b^3\,\sin\left(d+e\,x\right)}{e\cos\left(d+e\,x\right)^2}+\frac{4\,a^3\,b^2\,\sin\left(d+e\,x\right)}{3\,e\cos\left(d+e\,x\right)^3}-\frac{b^5\,\mathrm{atan}\left(\frac{\sin\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,1{}\mathrm{i}}{\cos\left(\frac{d}{2}+\frac{e\,x}{2}\right)}\right)\,2{}\mathrm{i}}{e}-\frac{a^2\,b^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,1{}\mathrm{i}}{\cos\left(\frac{d}{2}+\frac{e\,x}{2}\right)}\right)\,14{}\mathrm{i}}{e}-\frac{a^4\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,1{}\mathrm{i}}{\cos\left(\frac{d}{2}+\frac{e\,x}{2}\right)}\right)\,19{}\mathrm{i}}{4\,e}$$

input
```
int((a + b/cos(d + e*x))*(b^2 + a^2/cos(d + e*x)^2 + (2*a*b)/cos(d + e*x))
^2,x)
```

output
```
(2*a^5*sin(d + e*x))/(3*e*cos(d + e*x)) - (b^5*atan((sin(d/2 + (e*x)/2)*1i
)/cos(d/2 + (e*x)/2))*2i)/e + (a^5*sin(d + e*x))/(3*e*cos(d + e*x)^3) - (a
^2*b^3*atan((sin(d/2 + (e*x)/2)*1i)/cos(d/2 + (e*x)/2))*14i)/e + (2*a*b^4*
atan(sin(d/2 + (e*x)/2)/cos(d/2 + (e*x)/2)))/e - (a^4*b*atan((sin(d/2 + (e
*x)/2)*1i)/cos(d/2 + (e*x)/2))*19i)/(4*e) + (4*a*b^4*sin(d + e*x))/(e*cos(
d + e*x)) + (19*a^4*b*sin(d + e*x))/(8*e*cos(d + e*x)^2) + (a^4*b*sin(d +
e*x))/(4*e*cos(d + e*x)^4) + (26*a^3*b^2*sin(d + e*x))/(3*e*cos(d + e*x))
+ (3*a^2*b^3*sin(d + e*x))/(e*cos(d + e*x)^2) + (4*a^3*b^2*sin(d + e*x))/(
3*e*cos(d + e*x)^3)
```

3.518. $\int (a+b\sec(d+ex))\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^2\,dx$

3.519 $\int (a+b\sec(d+ex)) \left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right) dx$

3.519.1 Optimal result

Integrand size = 37, antiderivative size = 76

$$\int (a+b\sec(d+ex)) \left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right) dx = ab^2x+\frac{b(5a^2+2b^2)\operatorname{arctanh}(\sin(d+ex))}{2e}+\frac{a(a^2+2b^2)\tan(d+ex)}{e}+\frac{a^2b\sec(d+ex)\tan(d+ex)}{2e}$$

output
```
a*b^2*x+1/2*b*(5*a^2+2*b^2)*arctanh(sin(e*x+d))/e+a*(a^2+2*b^2)*tan(e*x+d)
/e+1/2*a^2*b*sec(e*x+d)*tan(e*x+d)/e
```

3.519.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.84

$$\int (a+b\sec(d+ex)) \left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right) dx = \frac{2ab^2ex+b(5a^2+2b^2)\operatorname{arctanh}(\sin(d+ex))+a(2a^2+4b^2+ab\sec(d+ex))\tan(d+ex)}{2e}$$

input
```
Integrate[(a + b*Sec[d + e*x])*(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x
]^2),x]
```

output
```
(2*a*b^2*e*x + b*(5*a^2 + 2*b^2)*ArcTanh[Sin[d + e*x]] + a*(2*a^2 + 4*b^2
+ a*b*Sec[d + e*x])*Tan[d + e*x])/(2*e)
```

3.519.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.081, Rules used = {3042, 4536, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(d+ex))\left(a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\csc\left(d+ex+\frac{\pi}{2}\right)\right)\left(a^2\csc\left(d+ex+\frac{\pi}{2}\right)^2+2ab\csc\left(d+ex+\frac{\pi}{2}\right)+b^2\right)dx$$

$$\Big\downarrow \text{4536}$$

$$\frac{1}{2}\int \left(2ab^2+\left(5a^2+2b^2\right)\sec(d+ex)b+2a\left(a^2+2b^2\right)\sec^2(d+ex)\right)dx+\frac{a^2b\tan(d+ex)\sec(d+ex)}{2e}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{b\left(5a^2+2b^2\right)\text{arctanh}(\sin(d+ex))}{e}+\frac{2a\left(a^2+2b^2\right)\tan(d+ex)}{e}+2ab^2x\right)+\frac{a^2b\tan(d+ex)\sec(d+ex)}{2e}$$

input
```
Int[(a + b*Sec[d + e*x])*(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2),x
]
```

output
```
(a^2*b*Sec[d + e*x]*Tan[d + e*x])/(2*e) + (2*a*b^2*x + (b*(5*a^2 + 2*b^2)*
ArcTanh[Sin[d + e*x]])/e + (2*a*(a^2 + 2*b^2)*Tan[d + e*x])/e)/2
```

3.519.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4536
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[(-b)*C*Csc[e +
 f*x]*(Cot[e + f*x]/(2*f)), x] + Simp[1/2   Int[Simp[2*A*a + (2*B*a + b*(2*
A + C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a
, b, e, f, A, B, C}, x]
```

3.519.4 Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.25

method	result
parts	$a\,b^{2} x +\frac{\left(2 a^{2} b +b^{3}\right) \ln \left(\sec \left(e x +d \right)+\tan \left(e x +d \right)\right)}{e}+\frac{\left(a^{3}+2 a\,b^{2}\right) \tan \left(e x +d \right)}{e}+\frac{a^{2} b \left(\frac{\sec \left(e x +d \right) \tan \left(e x +d \right)}{2}+\frac{\ln \left(\sec \left(e x +d \right)+ \ldots\right.}{2}\right.}{e}$
derivativedivides	$\frac{a\,b^{2} \left(e x +d \right)+2 a^{2} b \ln \left(\sec \left(e x +d \right)+\tan \left(e x +d \right)\right)+\tan \left(e x +d \right) a^{3}+b^{3} \ln \left(\sec \left(e x +d \right)+\tan \left(e x +d \right)\right)+2 \tan \left(e x +d \right) a\,b^{2}+a^{2} b \left(\frac{\sec (\ldots}{}\right.}{e}$
default	$\frac{a\,b^{2} \left(e x +d \right)+2 a^{2} b \ln \left(\sec \left(e x +d \right)+\tan \left(e x +d \right)\right)+\tan \left(e x +d \right) a^{3}+b^{3} \ln \left(\sec \left(e x +d \right)+\tan \left(e x +d \right)\right)+2 \tan \left(e x +d \right) a\,b^{2}+a^{2} b \left(\frac{\sec (\ldots}{}\right.}{e}$
parallelrisch	$\frac{5 \left(\cos \left(2 e x +2 d \right)+1\right) b \left(a^{2}+\frac{2 b^{2}}{5}\right) \ln \left(1+\tan \left(\frac{e x}{2}+\frac{d}{2}\right)\right)-5 \left(\cos \left(2 e x +2 d \right)+1\right) b \left(a^{2}+\frac{2 b^{2}}{5}\right) \ln \left(\tan \left(\frac{e x}{2}+\frac{d}{2}\right)-1\right)+2 a \left(\cos (2 e x+\ldots\right.}{2 e \left(\cos \left(2 e x +2 d \right)+1\right)}$
risch	$a\,b^{2} x -\frac{i a \left(b a \,{\mathrm e}^{3 i \left(e x +d \right)}-2 \,{\mathrm e}^{2 i \left(e x +d \right)} a^{2}-4 \,{\mathrm e}^{2 i \left(e x +d \right)} b^{2}-a b \,{\mathrm e}^{i \left(e x +d \right)}-2 a^{2}-4 b^{2}\right)}{e \left(1+{\mathrm e}^{2 i \left(e x +d \right)}\right)^{2}}-\frac{5 b \ln \left({\mathrm e}^{i \left(e x +d \right)}-i\right) a^{2}}{2 e}-\frac{b^{3} \ln \left({\mathrm e}^{i (e\ldots}\right.}{e}$
norman	$\frac{a\,b^{2} x +a\,b^{2} x \tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{4}+\frac{a \left(2 a^{2}+a b +4 b^{2}\right) \tan \left(\frac{e x}{2}+\frac{d}{2}\right)}{e}-2 a\,b^{2} x \tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{2}-\frac{a \left(2 a^{2}-a b +4 b^{2}\right) \tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{3}}{e}}{\left(\tan \left(\frac{e x}{2}+\frac{d}{2}\right)^{2}-1\right)^{2}}-\frac{b \left(5 a^{2}\ldots\right.}{}$

input
```
int((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2),x,method=_RET
URNVERBOSE)
```

output
```
a*b^2*x+(2*a^2*b+b^3)/e*ln(sec(e*x+d)+tan(e*x+d))+(a^3+2*a*b^2)/e*tan(e*x+
d)+a^2*b/e*(1/2*sec(e*x+d)*tan(e*x+d)+1/2*ln(sec(e*x+d)+tan(e*x+d)))
```

3.519.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.64

$$\int (a + b\sec(d + ex)) \left(b^2 + 2ab\sec(d + ex) + a^2\sec^2(d + ex)\right) dx$$
$$= \frac{4\,ab^2ex\cos\left(ex + d\right)^2 + \left(5\,a^2b + 2\,b^3\right)\cos\left(ex + d\right)^2\log\left(\sin\left(ex + d\right) + 1\right) - \left(5\,a^2b + 2\,b^3\right)\cos\left(ex + d\right)^2\log\left(\right.}{4\,e\cos\left(ex + d\right)^2}$$

input
```
integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2),x, algo
rithm="fricas")
```

output
```
1/4*(4*a*b^2*e*x*cos(e*x + d)^2 + (5*a^2*b + 2*b^3)*cos(e*x + d)^2*log(sin
(e*x + d) + 1) - (5*a^2*b + 2*b^3)*cos(e*x + d)^2*log(-sin(e*x + d) + 1) +
 2*(a^2*b + 2*(a^3 + 2*a*b^2)*cos(e*x + d))*sin(e*x + d))/(e*cos(e*x + d)^
2)
```

3.519.6 Sympy [F]

$$\int (a + b\sec(d + ex)) \left(b^2 + 2ab\sec(d + ex) + a^2\sec^2(d + ex)\right) dx$$
$$= \int \left(a + b\sec\left(d + ex\right)\right)\left(a\sec\left(d + ex\right) + b\right)^2\,dx$$

input
```
integrate((a+b*sec(e*x+d))*(b**2+2*a*b*sec(e*x+d)+a**2*sec(e*x+d)**2),x)
```

output
```
Integral((a + b*sec(d + e*x))*(a*sec(d + e*x) + b)**2, x)
```

3.519.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.66

$$\int (a + b\sec(d + ex)) \left(b^2 + 2ab\sec(d + ex) + a^2\sec^2(d + ex)\right) dx$$
$$= \frac{4\,(ex + d)ab^2 - a^2b\left(\frac{2\,\sin(ex+d)}{\sin(ex+d)^2-1} - \log\left(\sin\left(ex + d\right) + 1\right) + \log\left(\sin\left(ex + d\right) - 1\right)\right) + 8\,a^2b\log\left(\sec\left(ex + d\right)\right.}{4\,e}$$

input
```
integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2),x, algo
rithm="maxima")
```

output
```
1/4*(4*(e*x + d)*a*b^2 - a^2*b*(2*sin(e*x + d)/(sin(e*x + d)^2 - 1) - log(
sin(e*x + d) + 1) + log(sin(e*x + d) - 1)) + 8*a^2*b*log(sec(e*x + d) + ta
n(e*x + d)) + 4*b^3*log(sec(e*x + d) + tan(e*x + d)) + 4*a^3*tan(e*x + d)
+ 8*a*b^2*tan(e*x + d))/e
```

3.519.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(72) = 144$.

Time = 0.30 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.39

$$\int (a + b\sec(d+ex)) \left(b^2 + 2ab\sec(d+ex) + a^2\sec^2(d+ex)\right) \, dx$$

$$= \frac{2\,(ex+d)ab^2 + (5\,a^2b + 2\,b^3)\log\left(\left|\tan\left(\frac{1}{2}\,ex + \frac{1}{2}\,d\right) + 1\right|\right) - (5\,a^2b + 2\,b^3)\log\left(\left|\tan\left(\frac{1}{2}\,ex + \frac{1}{2}\,d\right) - 1\right|\right) - \frac{2}{}}{2\,e}$$

input
```
integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2),x, algo
rithm="giac")
```

output
```
1/2*(2*(e*x + d)*a*b^2 + (5*a^2*b + 2*b^3)*log(abs(tan(1/2*e*x + 1/2*d) +
1)) - (5*a^2*b + 2*b^3)*log(abs(tan(1/2*e*x + 1/2*d) - 1)) - 2*(2*a^3*tan(
1/2*e*x + 1/2*d)^3 - a^2*b*tan(1/2*e*x + 1/2*d)^3 + 4*a*b^2*tan(1/2*e*x +
1/2*d)^3 - 2*a^3*tan(1/2*e*x + 1/2*d) - a^2*b*tan(1/2*e*x + 1/2*d) - 4*a*b
^2*tan(1/2*e*x + 1/2*d))/(tan(1/2*e*x + 1/2*d)^2 - 1)^2)/e
```

3.519.9 Mupad [B] (verification not implemented)

Time = 26.34 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.11

$$\int (a+b\sec(d+ex)) \left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)\,dx$$
$$=\frac{2\,b^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{\cos\left(\frac{d}{2}+\frac{e\,x}{2}\right)}\right)}{e}+\frac{a^3\,\sin\left(d+e\,x\right)}{e\,\cos\left(d+e\,x\right)}+\frac{2\,a\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{\cos\left(\frac{d}{2}+\frac{e\,x}{2}\right)}\right)}{e}$$
$$+\frac{5\,a^2\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{\cos\left(\frac{d}{2}+\frac{e\,x}{2}\right)}\right)}{e}+\frac{2\,a\,b^2\,\sin\left(d+e\,x\right)}{e\,\cos\left(d+e\,x\right)}+\frac{a^2\,b\,\sin\left(d+e\,x\right)}{2\,e\,{\cos\left(d+e\,x\right)}^2}$$

input
```
int((a + b/cos(d + e*x))*(b^2 + a^2/cos(d + e*x)^2 + (2*a*b)/cos(d + e*x))
,x)
```

output
```
(2*b^3*atanh(sin(d/2 + (e*x)/2)/cos(d/2 + (e*x)/2)))/e + (a^3*sin(d + e*x)
)/(e*cos(d + e*x)) + (2*a*b^2*atan(sin(d/2 + (e*x)/2)/cos(d/2 + (e*x)/2)))
/e + (5*a^2*b*atanh(sin(d/2 + (e*x)/2)/cos(d/2 + (e*x)/2)))/e + (2*a*b^2*s
in(d + e*x))/(e*cos(d + e*x)) + (a^2*b*sin(d + e*x))/(2*e*cos(d + e*x)^2)
```

3.520 $\int \frac{a+b\sec(d+ex)}{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}\,dx$

3.520.1 Optimal result

Integrand size = 39, antiderivative size = 92

$$\int \frac{a+b\sec(d+ex)}{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}\,dx = \frac{ax}{b^2}-\frac{2\sqrt{a-b}\sqrt{a+b}\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{b^2e}-\frac{a^2\tan(d+ex)}{be\left(ab+a^2\sec(d+ex)\right)}$$

output
```
a*x/b^2-2*arctan((a-b)^(1/2)*tan(1/2*e*x+1/2*d)/(a+b)^(1/2))*(a-b)^(1/2)*(
a+b)^(1/2)/b^2/e-a^2*tan(e*x+d)/b/e/(a*b+a^2*sec(e*x+d))
```

3.520.2 Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.05

$$\int \frac{a+b\sec(d+ex)}{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}\,dx = \frac{2\sqrt{-a^2+b^2}\operatorname{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{-a^2+b^2}}\right)+\frac{a(ad+aex+b(d+ex)\cos(d+ex)-b\sin(d+ex))}{a+b\cos(d+ex)}}{b^2e}$$

input
```
Integrate[(a + b*Sec[d + e*x])/(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x
]^2),x]
```

output
```
(2*Sqrt[-a^2 + b^2]*ArcTanh[((-a + b)*Tan[(d + e*x)/2])/Sqrt[-a^2 + b^2]]
+ (a*(a*d + a*e*x + b*(d + e*x)*Cos[d + e*x] - b*Sin[d + e*x]))/(a + b*Cos
[d + e*x]))/(b^2*e)
```

3.520.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.47, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 4660, 27, 3042, 4411, 3042, 4407, 3042, 4318, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec(d+ex)}{a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\sec(d+ex)}{a^2\sec(d+ex)^2+2ab\sec(d+ex)+b^2}dx$$

$$\Big\downarrow \text{4660}$$

$$4a^2\int \frac{a+b\sec(d+ex)}{4\left(\sec(d+ex)a^2+ba\right)^2}dx$$

$$\Big\downarrow \text{27}$$

$$a^2\int \frac{a+b\sec(d+ex)}{\left(\sec(d+ex)a^2+ba\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$a^2\int \frac{a+b\csc\left(d+ex+\frac{\pi}{2}\right)}{\left(\csc\left(d+ex+\frac{\pi}{2}\right)a^2+ba\right)^2}dx$$

$$\Big\downarrow \text{4411}$$

$$a^2\left(\frac{\int \frac{(a^2-b^2)a^3+b(a^2-b^2)\sec(d+ex)a^2}{\sec(d+ex)a^2+ba}dx}{a^3b\left(a^2-b^2\right)}-\frac{\tan(d+ex)}{be\left(a^2\sec(d+ex)+ab\right)}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2\left(\frac{\int \frac{(a^2-b^2)a^3+b(a^2-b^2)\csc\left(d+ex+\frac{\pi}{2}\right)a^2}{\csc\left(d+ex+\frac{\pi}{2}\right)a^2+ba}dx}{a^3b\left(a^2-b^2\right)}-\frac{\tan(d+ex)}{be\left(a^2\sec(d+ex)+ab\right)}\right)$$

$$\Big\downarrow \text{4407}$$

$$a^2\left(\frac{\frac{a^2x(a^2-b^2)}{b}-\frac{a^2(a^2-b^2)^2\int \frac{\sec(d+ex)}{\sec(d+ex)a^2+ba}dx}{b}}{a^3b\,(a^2-b^2)}-\frac{\tan(d+ex)}{be\,(a^2\sec(d+ex)+ab)}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2\left(\frac{\frac{a^2x(a^2-b^2)}{b}-\frac{a^2(a^2-b^2)^2\int \frac{\csc\left(d+ex+\frac{\pi}{2}\right)}{\csc\left(d+ex+\frac{\pi}{2}\right)a^2+ba}dx}{b}}{a^3b\,(a^2-b^2)}-\frac{\tan(d+ex)}{be\,(a^2\sec(d+ex)+ab)}\right)$$

$$\Big\downarrow \text{4318}$$

$$a^2\left(\frac{\frac{a^2x(a^2-b^2)}{b}-\frac{(a^2-b^2)^2\int \frac{1}{\frac{b\cos(d+ex)}{a}+1}dx}{b}}{a^3b\,(a^2-b^2)}-\frac{\tan(d+ex)}{be\,(a^2\sec(d+ex)+ab)}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2\left(\frac{\frac{a^2x(a^2-b^2)}{b}-\frac{(a^2-b^2)^2\int \frac{1}{\frac{b\sin\left(d+ex+\frac{\pi}{2}\right)}{a}+1}dx}{b}}{a^3b\,(a^2-b^2)}-\frac{\tan(d+ex)}{be\,(a^2\sec(d+ex)+ab)}\right)$$

$$\Big\downarrow \text{3138}$$

$$a^2\left(\frac{\frac{a^2x(a^2-b^2)}{b}-\frac{2(a^2-b^2)^2\int \frac{1}{\frac{(a-b)\tan^2\left(\frac{1}{2}(d+ex)\right)}{a}+\frac{a+b}{a}}d\tan\left(\frac{1}{2}(d+ex)\right)}{be}}{a^3b\,(a^2-b^2)}-\frac{\tan(d+ex)}{be\,(a^2\sec(d+ex)+ab)}\right)$$

$$\Big\downarrow \text{218}$$

$$a^2\left(\frac{\frac{a^2x(a^2-b^2)}{b}-\frac{2a(a^2-b^2)^2\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{be\sqrt{a-b}\sqrt{a+b}}}{a^3b\,(a^2-b^2)}-\frac{\tan(d+ex)}{be\,(a^2\sec(d+ex)+ab)}\right)$$

input
```
Int[(a + b*Sec[d + e*x])/(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2),x
]
```

3.520. $\int \frac{a+b\sec(d+ex)}{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}\,dx$

output
```
a^2*(((a^2*(a^2 - b^2)*x)/b - (2*a*(a^2 - b^2)^2*ArcTan[(Sqrt[a - b]*Tan[(
d + e*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b*Sqrt[a + b]*e))/(a^3*b*(a^2 - b^
2)) - Tan[d + e*x]/(b*e*(a*b + a^2*Sec[d + e*x])))
```

3.520.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

3.520. $\int \frac{a+b\sec(d+ex)}{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}\,dx$

rule 4411
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[b*(b*c - a*d)*Cot[e + f*x]*((a + b*Csc[e + f
*x])^(m + 1)/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2 - b^2)
)   Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
 a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && N
eQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

rule 4660
```
Int[((A_) + (B_.)*sec[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sec[(d_.) + (e_.)*
(x_)] + (c_.)*sec[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Simp[1/(4^n*c^n
)   Int[(A + B*Sec[d + e*x])*(b + 2*c*Sec[d + e*x])^(2*n), x], x] /; FreeQ[
{a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[n]
```

3.520.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.30

method	result
derivativedivides	$\frac{-\frac{2\left(\frac{ab\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2 a-\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2 b+a+b}+\frac{\left(a^2-b^2\right)\arctan\left(\frac{\tan\left(\frac{ex}{2}+\frac{d}{2}\right)(a-b)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b^2}+\frac{2a\arctan\left(\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}{b^2}}{e}$
default	$\frac{-\frac{2\left(\frac{ab\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2 a-\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2 b+a+b}+\frac{\left(a^2-b^2\right)\arctan\left(\frac{\tan\left(\frac{ex}{2}+\frac{d}{2}\right)(a-b)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b^2}+\frac{2a\arctan\left(\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}{b^2}}{e}$
risch	$\frac{ax}{b^2}-\frac{2ia\left(a\,e^{i(ex+d)}+b\right)}{b^2 e\left(b\,e^{2i(ex+d)}+2a\,e^{i(ex+d)}+b\right)}+\frac{\sqrt{-a^2+b^2}\,\ln\left(e^{i(ex+d)}+\frac{i\sqrt{-a^2+b^2}+a}{b}\right)}{e\,b^2}-\frac{\sqrt{-a^2+b^2}\,\ln\left(e^{i(ex+d)}-\frac{i\sqrt{-a}}{}\right.}{e\,b^2}$

input
```
int((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2),x,method=_RET
URNVERBOSE)
```

output
```
1/e*(-2/b^2*(a*b*tan(1/2*e*x+1/2*d)/(tan(1/2*e*x+1/2*d)^2*a-tan(1/2*e*x+1/
2*d)^2*b+a+b)+(a^2-b^2)/((a+b)*(a-b))^(1/2)*arctan(tan(1/2*e*x+1/2*d)*(a-b
)/((a+b)*(a-b))^(1/2)))+2*a/b^2*arctan(tan(1/2*e*x+1/2*d)))
```

3.520.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 279, normalized size of antiderivative = 3.03

$$\int \frac{a+b\sec(d+ex)}{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}\,dx$$
$$= \left[\frac{2\,abex\cos{(ex+d)}+2\,a^2ex-2\,ab\sin{(ex+d)}+\sqrt{-a^2+b^2}(b\cos{(ex+d)}+a)\log\left(\frac{2\,ab\cos(ex+d)+(2\,a^2-b^2}{}\right.}{2\,(b^3e\cos{(ex+d)}+ab^2e)}\right.$$

input
```
integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2),x, algo
rithm="fricas")
```

output
```
[1/2*(2*a*b*e*x*cos(e*x + d) + 2*a^2*e*x - 2*a*b*sin(e*x + d) + sqrt(-a^2
+ b^2)*(b*cos(e*x + d) + a)*log((2*a*b*cos(e*x + d) + (2*a^2 - b^2)*cos(e*
x + d)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(e*x + d) + b)*sin(e*x + d) - a^2 + 2*
b^2)/(b^2*cos(e*x + d)^2 + 2*a*b*cos(e*x + d) + a^2)))/(b^3*e*cos(e*x + d)
 + a*b^2*e), (a*b*e*x*cos(e*x + d) + a^2*e*x - a*b*sin(e*x + d) - sqrt(a^2
 - b^2)*(b*cos(e*x + d) + a)*arctan(-(a*cos(e*x + d) + b)/(sqrt(a^2 - b^2)
*sin(e*x + d))))/(b^3*e*cos(e*x + d) + a*b^2*e)]
```

3.520.6 Sympy [F]

$$\int \frac{a+b\sec(d+ex)}{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}\,dx = \int \frac{a+b\sec{(d+ex)}}{(a\sec{(d+ex)}+b)^2}\,dx$$

input
```
integrate((a+b*sec(e*x+d))/(b**2+2*a*b*sec(e*x+d)+a**2*sec(e*x+d)**2),x)
```

output
```
Integral((a + b*sec(d + e*x))/(a*sec(d + e*x) + b)**2, x)
```

3.520.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\sec(d + ex)}{b^2 + 2ab\sec(d + ex) + a^2\sec^2(d + ex)}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2),x, algo
rithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.520.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.51

$$\int \frac{a + b\sec(d + ex)}{b^2 + 2ab\sec(d + ex) + a^2\sec^2(d + ex)}\, dx$$
$$= \frac{\frac{(ex+d)a}{b^2} - \frac{2\,a\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)}{\left(a\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^2 - b\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^2 + a + b\right)b} - \frac{2\left(\pi\left\lfloor\frac{ex+d}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b) + \arctan\left(\frac{a\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right) - b\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)}{\sqrt{a^2-b^2}}\right)\right)\sqrt{a}}{b^2}}{e}$$

input
```
integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2),x, algo
rithm="giac")
```

output
```
((e*x + d)*a/b^2 - 2*a*tan(1/2*e*x + 1/2*d)/((a*tan(1/2*e*x + 1/2*d)^2 - b
*tan(1/2*e*x + 1/2*d)^2 + a + b)*b) - 2*(pi*floor(1/2*(e*x + d)/pi + 1/2)*
sgn(2*a - 2*b) + arctan((a*tan(1/2*e*x + 1/2*d) - b*tan(1/2*e*x + 1/2*d))/
sqrt(a^2 - b^2)))*sqrt(a^2 - b^2)/b^2)/e
```

3.520.9 Mupad [B] (verification not implemented)

Time = 26.38 (sec) , antiderivative size = 444, normalized size of antiderivative = 4.83

$$\int \frac{a+b\sec(d+ex)}{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}\,dx$$
$$= \frac{2\,\mathrm{atanh}\left(\frac{64\,a^3\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\sqrt{b^2-a^2}}{64\,a^4-128\,a^3\,b+128\,a\,b^3-64\,b^4}-\frac{192\,a^2\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\sqrt{b^2-a^2}}{128\,a\,b^2-128\,a^3-64\,b^3+\frac{64\,a^4}{b}}+\frac{192\,a\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\sqrt{b^2-a^2}}{128\,a\,b-64\,b^2-\frac{128\,a^3}{b}+\frac{64\,a^4}{b^2}}-\frac{64\,b\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\sqrt{b^2-a^2}}{128\,a\,b-64\,b^2-\frac{128\,a^3}{b}+\frac{64\,a^4}{b^2}}\right.}{b^2\,e}$$
$$-\frac{2\,a\,\mathrm{atan}\left(\frac{64\,a^2\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{64\,a\,b-64\,a^2-\frac{64\,a^3}{b}+\frac{64\,a^4}{b^2}}+\frac{64\,a^3\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{64\,a\,b^2-64\,a^2\,b-64\,a^3+\frac{64\,a^4}{b}}-\frac{64\,a^4\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{64\,a^4-64\,a^3\,b-64\,a^2\,b^2+64\,a\,b^3}-\frac{64\,a\,b\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{64\,a\,b-64\,a^2-\frac{64\,a^3}{b}+\frac{64\,a^4}{b^2}}\right.}{b^2\,e}$$
$$-\frac{2\,a\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)}{b\,e\,\left(\left(a-b\right)\,\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2+a+b\right)}$$

input
```
int((a + b/cos(d + e*x))/(b^2 + a^2/cos(d + e*x)^2 + (2*a*b)/cos(d + e*x))
,x)
```

output
```
(2*atanh((64*a^3*tan(d/2 + (e*x)/2)*(b^2 - a^2)^(1/2))/(128*a*b^3 - 128*a^
3*b + 64*a^4 - 64*b^4) - (192*a^2*tan(d/2 + (e*x)/2)*(b^2 - a^2)^(1/2))/(1
28*a*b^2 - 128*a^3 - 64*b^3 + (64*a^4)/b) + (192*a*tan(d/2 + (e*x)/2)*(b^2
 - a^2)^(1/2))/(128*a*b - 64*b^2 - (128*a^3)/b + (64*a^4)/b^2) - (64*b*tan
(d/2 + (e*x)/2)*(b^2 - a^2)^(1/2))/(128*a*b - 64*b^2 - (128*a^3)/b + (64*a
^4)/b^2))*(b^2 - a^2)^(1/2))/(b^2*e) - (2*a*atan((64*a^2*tan(d/2 + (e*x)/2
))/(64*a*b - 64*a^2 - (64*a^3)/b + (64*a^4)/b^2) + (64*a^3*tan(d/2 + (e*x)
/2))/(64*a*b^2 - 64*a^2*b - 64*a^3 + (64*a^4)/b) - (64*a^4*tan(d/2 + (e*x)
/2))/(64*a*b^3 - 64*a^3*b + 64*a^4 - 64*a^2*b^2) - (64*a*b*tan(d/2 + (e*x)
/2))/(64*a*b - 64*a^2 - (64*a^3)/b + (64*a^4)/b^2)))/(b^2*e) - (2*a*tan(d/
2 + (e*x)/2))/(b*e*(a + b + tan(d/2 + (e*x)/2)^2*(a - b)))
```

3.521 $\int \frac{a+b\sec(d+ex)}{\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^2}\,dx$

3.521.1 Optimal result

Integrand size = 39, antiderivative size = 230

$$\int \frac{a+b\sec(d+ex)}{\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^2}\,dx = \frac{ax}{b^4}-\frac{(a^2-2b^2)\left(2a^4-a^2b^2+b^4\right)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}b^4(a+b)^{5/2}e} - \frac{a(3a^2-5b^2)\tan(d+ex)}{6b^2\left(a^2-b^2\right)e(b+a\sec(d+ex))^2} - \frac{a(6a^4-11a^2b^2+11b^4)\tan(d+ex)}{6b^3\left(a^2-b^2\right)^2e(b+a\sec(d+ex))} - \frac{a^4\tan(d+ex)}{3be\left(ab+a^2\sec(d+ex)\right)^3}$$

output
```
a*x/b^4-(a^2-2*b^2)*(2*a^4-a^2*b^2+b^4)*arctan((a-b)^(1/2)*tan(1/2*e*x+1/2
*d)/(a+b)^(1/2))/(a-b)^(5/2)/b^4/(a+b)^(5/2)/e-1/6*a*(3*a^2-5*b^2)*tan(e*x
+d)/b^2/(a^2-b^2)/e/(b+a*sec(e*x+d))^2-1/6*a*(6*a^4-11*a^2*b^2+11*b^4)*tan
(e*x+d)/b^3/(a^2-b^2)^2/e/(b+a*sec(e*x+d))-1/3*a^4*tan(e*x+d)/b/e/(a*b+a^2
*sec(e*x+d))^3
```

3.521.2 Mathematica [A] (verified)

Time = 2.39 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.20

$$\int \frac{a+b\sec(d+ex)}{\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^2}\,dx$$

$$= \frac{(a+b\cos(d+ex))\sec^3(d+ex)(a+b\sec(d+ex))\left(6a(d+ex)(a+b\cos(d+ex))^3+\frac{6(-2a^6+5a^4b^2-3a^2b^4+\cdots}{\cdots}\cdots\right.}{6b^4e(\cdots}$$

input
```
Integrate[(a + b*Sec[d + e*x])/(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x
]^2)^2,x]
```

output
```
((a + b*Cos[d + e*x])*Sec[d + e*x]^3*(a + b*Sec[d + e*x])*(6*a*(d + e*x)*(
a + b*Cos[d + e*x])^3 + (6*(-2*a^6 + 5*a^4*b^2 - 3*a^2*b^4 + 2*b^6)*ArcTan
h[((-a + b)*Tan[(d + e*x)/2])/Sqrt[-a^2 + b^2]]*(a + b*Cos[d + e*x])^3)/(-
a^2 + b^2)^(5/2) - 2*a^3*b*Sin[d + e*x] + (a^2*b*(7*a^2 - 9*b^2)*(a + b*Co
s[d + e*x])*Sin[d + e*x])/((a - b)*(a + b)) - (a*b*(11*a^4 - 23*a^2*b^2 +
18*b^4)*(a + b*Cos[d + e*x])^2*Sin[d + e*x])/((a - b)^2*(a + b)^2))/(6*b^
4*e*(b + a*Cos[d + e*x])*(b + a*Sec[d + e*x])^4)
```

3.521.3 Rubi [A] (verified)

Time = 1.64 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.31, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.436, Rules used = {3042, 4660, 27, 3042, 4411, 3042, 4548, 3042, 4548, 27, 3042, 4407, 3042, 4318, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec(d+ex)}{\left(a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\sec(d+ex)}{\left(a^2\sec(d+ex)^2+2ab\sec(d+ex)+b^2\right)^2}dx$$

$$\Big\downarrow \text{4660}$$

$$16a^4\int \frac{a+b\sec(d+ex)}{16\left(\sec(d+ex)a^2+ba\right)^4}dx$$

$$\Big\downarrow \text{27}$$

$$a^4\int \frac{a+b\sec(d+ex)}{\left(\sec(d+ex)a^2+ba\right)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$a^4\int \frac{a+b\csc\left(d+ex+\frac{\pi}{2}\right)}{\left(\csc\left(d+ex+\frac{\pi}{2}\right)a^2+ba\right)^4}dx$$

$$\Big\downarrow \text{4411}$$

$$a^4\left(\frac{\int \frac{-2\left(a^2-b^2\right)\sec^2(d+ex)a^3+3\left(a^2-b^2\right)a^3+3b\left(a^2-b^2\right)\sec(d+ex)a^2}{\left(\sec(d+ex)a^2+ba\right)^3}dx}{3a^3b\left(a^2-b^2\right)}-\frac{\tan(d+ex)}{3be\left(a^2\sec(d+ex)+ab\right)^3}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^4\left(\frac{\int \frac{-2\left(a^2-b^2\right)\csc\left(d+ex+\frac{\pi}{2}\right)^2a^3+3\left(a^2-b^2\right)a^3+3b\left(a^2-b^2\right)\csc\left(d+ex+\frac{\pi}{2}\right)a^2}{\left(\csc\left(d+ex+\frac{\pi}{2}\right)a^2+ba\right)^3}dx}{3a^3b\left(a^2-b^2\right)}-\frac{\tan(d+ex)}{3be\left(a^2\sec(d+ex)+ab\right)^3}\right)$$

$$\Big\downarrow \text{4548}$$

$$a^4\left(\frac{\frac{\int \frac{6\left(a^2-b^2\right)^2a^5-\left(3a^2-5b^2\right)\left(a^2-b^2\right)\sec^2(d+ex)a^5+2b\left(a^2-3b^2\right)\left(a^2-b^2\right)\sec(d+ex)a^4}{\left(\sec(d+ex)a^2+ba\right)^2}dx}{2a^3b(a^2-b^2)}-\frac{\left(3a^2-5b^2\right)\tan(d+ex)}{2be(a\sec(d+ex)+b)^2}}{3a^3b\left(a^2-b^2\right)}-\frac{\tan(d+ex)}{3be\left(a^2\sec(d+ex)+\right.}\right.$$

$$\Big\downarrow \text{3042}$$

$$a^4\left(\frac{\frac{\int \frac{6\left(a^2-b^2\right)^2a^5-\left(3a^2-5b^2\right)\left(a^2-b^2\right)\csc\left(d+ex+\frac{\pi}{2}\right)^2a^5+2b\left(a^2-3b^2\right)\left(a^2-b^2\right)\csc\left(d+ex+\frac{\pi}{2}\right)a^4}{\left(\csc\left(d+ex+\frac{\pi}{2}\right)a^2+ba\right)^2}dx}{2a^3b(a^2-b^2)}-\frac{\left(3a^2-5b^2\right)\tan(d+ex)}{2be(a\sec(d+ex)+b)^2}}{3a^3b\left(a^2-b^2\right)}-\frac{\tan(d-}{3be\left(a^2\sec(d-\right.}\right.$$

$$\Big\downarrow \text{4548}$$

3.521. $\int \frac{a+b\sec(d+ex)}{\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^2}\,dx$

$$a^4\left(\frac{\frac{\frac{\int \frac{3\left(2\left(a^2-b^2\right)^3a^7+b\left(a^6-2b^2a^4+3b^4a^2-2b^6\right)\sec(d+ex)a^6\right)}{\sec(d+ex)a^2+ba}dx}{a^3b\left(a^2-b^2\right)}-\frac{a^4\left(6a^4-11a^2b^2+11b^4\right)\tan(d+ex)}{be\left(a^2\sec(d+ex)+ab\right)}}{2a^3b(a^2-b^2)}-\frac{(3a^2-5b^2)\tan(d+ex)}{2be(a\sec(d+ex)+b)^2}}{3a^3b\left(a^2-b^2\right)}-\frac{\tan(d}{3be\left(a^2\sec(d\right.}\right.$$

$$\Big\downarrow \text{27}$$

$$a^4\left(\frac{\frac{\frac{3\int \frac{2\left(a^2-b^2\right)^3a^7+b\left(a^6-2b^2a^4+3b^4a^2-2b^6\right)\sec(d+ex)a^6}{\sec(d+ex)a^2+ba}dx}{a^3b\left(a^2-b^2\right)}-\frac{a^4\left(6a^4-11a^2b^2+11b^4\right)\tan(d+ex)}{be\left(a^2\sec(d+ex)+ab\right)}}{2a^3b(a^2-b^2)}-\frac{(3a^2-5b^2)\tan(d+ex)}{2be(a\sec(d+ex)+b)^2}}{3a^3b\left(a^2-b^2\right)}-\frac{\tan(d+}{3be\left(a^2\sec(d+\right.}\right.$$

$$\Big\downarrow \text{3042}$$

$$a^4\left(\frac{\frac{\frac{3\int \frac{2\left(a^2-b^2\right)^3a^7+b\left(a^6-2b^2a^4+3b^4a^2-2b^6\right)\csc\left(d+ex+\frac{\pi}{2}\right)a^6}{\csc\left(d+ex+\frac{\pi}{2}\right)a^2+ba}dx}{a^3b\left(a^2-b^2\right)}-\frac{a^4\left(6a^4-11a^2b^2+11b^4\right)\tan(d+ex)}{be\left(a^2\sec(d+ex)+ab\right)}}{2a^3b(a^2-b^2)}-\frac{(3a^2-5b^2)\tan(d+ex)}{2be(a\sec(d+ex)+b)^2}}{3a^3b\left(a^2-b^2\right)}-\frac{\tan(}{3be\left(a^2\sec(\right.}\right.$$

$$\Big\downarrow \text{4407}$$

$$a^4\left(\frac{\frac{\frac{3\left(\frac{2a^6x\left(a^2-b^2\right)^3}{b}-\frac{a^6\left(a^2-2b^2\right)\left(a^2-b^2\right)\left(2a^4-a^2b^2+b^4\right)\int \frac{\sec(d+ex)}{\sec(d+ex)a^2+ba}dx}{b}\right)}{a^3b\left(a^2-b^2\right)}-\frac{a^4\left(6a^4-11a^2b^2+11b^4\right)\tan(d+ex)}{be\left(a^2\sec(d+ex)+ab\right)}}{2a^3b(a^2-b^2)}-\frac{(3a^2-5b^2)\tan(d+ex)}{2be(a\sec(d+ex)+b)^2}}{3a^3b\left(a^2-b^2\right)}-\right.$$

$$\Big\downarrow \text{3042}$$

3.521. $\int \frac{a+b\sec(d+ex)}{(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex))^2}\,dx$

$$a^4\left(\frac{\frac{\frac{3\left(\frac{2a^6x\left(a^2-b^2\right)^3}{b}-\frac{a^6\left(a^2-2b^2\right)\left(a^2-b^2\right)\left(2a^4-a^2b^2+b^4\right)\int \frac{\csc\left(d+ex+\frac{\pi}{2}\right)}{\csc\left(d+ex+\frac{\pi}{2}\right)a^2+ba}dx}{b}\right)}{a^3b\left(a^2-b^2\right)}-\frac{a^4\left(6a^4-11a^2b^2+11b^4\right)\tan(d+ex)}{be\left(a^2\sec(d+ex)+ab\right)}}{2a^3b(a^2-b^2)}-\frac{\left(3a^2-5b^2\right)\tan(d+ex)}{2be(a\sec(d+ex)+b)^2}}{3a^3b\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \ 4318$$

$$a^4\left(\frac{\frac{\frac{3\left(\frac{2a^6x\left(a^2-b^2\right)^3}{b}-\frac{a^4\left(a^2-2b^2\right)\left(a^2-b^2\right)\left(2a^4-a^2b^2+b^4\right)\int \frac{1}{\frac{b\cos(d+ex)}{a}+1}dx}{b}\right)}{a^3b\left(a^2-b^2\right)}-\frac{a^4\left(6a^4-11a^2b^2+11b^4\right)\tan(d+ex)}{be\left(a^2\sec(d+ex)+ab\right)}}{2a^3b(a^2-b^2)}-\frac{\left(3a^2-5b^2\right)\tan(d+ex)}{2be(a\sec(d+ex)+b)^2}}{3a^3b\left(a^2-b^2\right)}-\right.$$

$$\Big\downarrow \ 3042$$

$$a^4\left(\frac{\frac{\frac{3\left(\frac{2a^6x\left(a^2-b^2\right)^3}{b}-\frac{a^4\left(a^2-2b^2\right)\left(a^2-b^2\right)\left(2a^4-a^2b^2+b^4\right)\int \frac{1}{\frac{b\sin\left(d+ex+\frac{\pi}{2}\right)}{a}+1}dx}{b}\right)}{a^3b\left(a^2-b^2\right)}-\frac{a^4\left(6a^4-11a^2b^2+11b^4\right)\tan(d+ex)}{be\left(a^2\sec(d+ex)+ab\right)}}{2a^3b(a^2-b^2)}-\frac{\left(3a^2-5b^2\right)\tan(d+ex)}{2be(a\sec(d+ex)+b)^2}}{3a^3b\left(a^2-b^2\right)}\right.$$

$$\Big\downarrow \ 3138$$

$$a^4\left(\frac{\frac{\frac{3\left(\frac{2a^6x\left(a^2-b^2\right)^3}{b}-\frac{2a^4\left(a^2-2b^2\right)\left(a^2-b^2\right)\left(2a^4-a^2b^2+b^4\right)\int \frac{1}{\frac{(a-b)\tan^2\left(\frac{1}{2}(d+ex)\right)}{a}+\frac{a+b}{a}}d\tan\left(\frac{1}{2}(d+ex)\right)}{be}\right)}{a^3b\left(a^2-b^2\right)}-\frac{a^4\left(6a^4-11a^2b^2+11b^4\right)\tan(d+ex)}{be\left(a^2\sec(d+ex)+ab\right)}}{2a^3b(a^2-b^2)}-}{3a^3b\left(a^2-b^2\right)}\right.$$

3.521. $\displaystyle\int \frac{a+b\sec(d+ex)}{\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^2}\,dx$

$\downarrow$ 218

$$a^4\left(\frac{\frac{\frac{3\left(\frac{2a^6x\left(a^2-b^2\right)^3}{b}-\frac{2a^5\left(a^2-2b^2\right)\left(a^2-b^2\right)\left(2a^4-a^2b^2+b^4\right)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{be\sqrt{a-b}\sqrt{a+b}}\right)}{a^3b\left(a^2-b^2\right)}-\frac{a^4\left(6a^4-11a^2b^2+11b^4\right)\tan(d+ex)}{be\left(a^2\sec(d+ex)+ab\right)}}{2a^3b(a^2-b^2)}-\frac{(3a^2-5b^2)\tan(}{2be(a\sec(d+ex}}{3a^3b\left(a^2-b^2\right)}\right.$$

input
```
Int[(a + b*Sec[d + e*x])/(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2)^2
,x]
```

output
```
a^4*(-1/3*Tan[d + e*x]/(b*e*(a*b + a^2*Sec[d + e*x])^3) + (-1/2*((3*a^2 -
5*b^2)*Tan[d + e*x])/(b*e*(b + a*Sec[d + e*x])^2) + ((3*((2*a^6*(a^2 - b^2
)^3*x)/b - (2*a^5*(a^2 - 2*b^2)*(a^2 - b^2)*(2*a^4 - a^2*b^2 + b^4)*ArcTan
[(Sqrt[a - b]*Tan[(d + e*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b*Sqrt[a + b]*e
)))/(a^3*b*(a^2 - b^2)) - (a^4*(6*a^4 - 11*a^2*b^2 + 11*b^4)*Tan[d + e*x])
/(b*e*(a*b + a^2*Sec[d + e*x])))/(2*a^3*b*(a^2 - b^2)))/(3*a^3*b*(a^2 - b^
2)))
```

3.521.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.521. $\int \frac{a+b\sec(d+ex)}{\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^2}\,dx$

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 4411
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[b*(b*c - a*d)*Cot[e + f*x]*((a + b*Csc[e + f
*x])^(m + 1)/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2 - b^2)
)   Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
 a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && N
eQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

rule 4548
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(
m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x
] + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

rule 4660
```
Int[((A_) + (B_.)*sec[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sec[(d_.) + (e_.)*
(x_)] + (c_.)*sec[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Simp[1/(4^n*c^n
)   Int[(A + B*Sec[d + e*x])*(b + 2*c*Sec[d + e*x])^(2*n), x], x] /; FreeQ[
{a, b, c, d, e, A, B}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[n]
```

3.521. $\int \frac{a+b\sec(d+ex)}{(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex))^2}\,dx$

3.521.4 Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.33

method	result
derivativedivides	$-\frac{\frac{2\left(\frac{\frac{\left(2a^4-a^3b-4a^2b^2+3a\,b^3+6b^4\right)ab\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^5}{2a^2+4ab+2b^2}+\frac{2\left(3a^4-8a^2b^2+9b^4\right)ab\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^3}{3(a+b)(a-b)}+\frac{\left(2a^4+a^3b-4a^2b^2-3a\,b^3+6b^4\right)ab\tan}{2a^2-4ab+2b^2}}{\left(\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2a-\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2b+a+b\right)^3}}{b^4}}{e}$
default	$-\frac{\frac{2\left(\frac{\frac{\left(2a^4-a^3b-4a^2b^2+3a\,b^3+6b^4\right)ab\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^5}{2a^2+4ab+2b^2}+\frac{2\left(3a^4-8a^2b^2+9b^4\right)ab\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^3}{3(a+b)(a-b)}+\frac{\left(2a^4+a^3b-4a^2b^2-3a\,b^3+6b^4\right)ab\tan}{2a^2-4ab+2b^2}}{\left(\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2a-\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2b+a+b\right)^3}}{b^4}}{e}$
risch	$\frac{ax}{b^4}-\frac{ia\left(18a^5b^2\mathrm{e}^{5i(ex+d)}-39a^3b^4\mathrm{e}^{5i(ex+d)}+27a\,b^6\mathrm{e}^{5i(ex+d)}+54a^6b\,\mathrm{e}^{4i(ex+d)}-105a^4b^3\mathrm{e}^{4i(ex+d)}+63a^2b^5\mathrm{e}^{4i(ex+d)}+\right.}{}$

input
```
int((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^2,x,method=_R
ETURNVERBOSE)
```

output
```
1/e*(-2/b^4*((1/2*(2*a^4-a^3*b-4*a^2*b^2+3*a*b^3+6*b^4)*a*b/(a^2+2*a*b+b^2
)*tan(1/2*e*x+1/2*d)^5+2/3*(3*a^4-8*a^2*b^2+9*b^4)*a*b/(a+b)/(a-b)*tan(1/2
*e*x+1/2*d)^3+1/2*(2*a^4+a^3*b-4*a^2*b^2-3*a*b^3+6*b^4)*a*b/(a^2-2*a*b+b^2
)*tan(1/2*e*x+1/2*d))/(tan(1/2*e*x+1/2*d)^2*a-tan(1/2*e*x+1/2*d)^2*b+a+b)^
3+1/2*(2*a^6-5*a^4*b^2+3*a^2*b^4-2*b^6)/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^
(1/2)*arctan(tan(1/2*e*x+1/2*d)*(a-b)/((a+b)*(a-b))^(1/2)))+2*a/b^4*arctan
(tan(1/2*e*x+1/2*d)))
```

3.521.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 633 vs. 2(215) = 430.

Time = 0.33 (sec) , antiderivative size = 1335, normalized size of antiderivative = 5.80

$$\int \frac{a+b\sec(d+ex)}{\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^2,x, al
gorithm="fricas")
```

output
```
[1/12*(12*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 - a*b^9)*e*x*cos(e*x + d)^3 + 3
6*(a^8*b^2 - 3*a^6*b^4 + 3*a^4*b^6 - a^2*b^8)*e*x*cos(e*x + d)^2 + 36*(a^9
*b - 3*a^7*b^3 + 3*a^5*b^5 - a^3*b^7)*e*x*cos(e*x + d) + 12*(a^10 - 3*a^8*
b^2 + 3*a^6*b^4 - a^4*b^6)*e*x + 3*(2*a^9 - 5*a^7*b^2 + 3*a^5*b^4 - 2*a^3*
b^6 + (2*a^6*b^3 - 5*a^4*b^5 + 3*a^2*b^7 - 2*b^9)*cos(e*x + d)^3 + 3*(2*a^
7*b^2 - 5*a^5*b^4 + 3*a^3*b^6 - 2*a*b^8)*cos(e*x + d)^2 + 3*(2*a^8*b - 5*a
^6*b^3 + 3*a^4*b^5 - 2*a^2*b^7)*cos(e*x + d))*sqrt(-a^2 + b^2)*log((2*a*b*
cos(e*x + d) + (2*a^2 - b^2)*cos(e*x + d)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(e*
x + d) + b)*sin(e*x + d) - a^2 + 2*b^2)/(b^2*cos(e*x + d)^2 + 2*a*b*cos(e*
x + d) + a^2)) - 2*(6*a^9*b - 17*a^7*b^3 + 22*a^5*b^5 - 11*a^3*b^7 + (11*a
^7*b^3 - 34*a^5*b^5 + 41*a^3*b^7 - 18*a*b^9)*cos(e*x + d)^2 + 3*(5*a^8*b^2
 - 15*a^6*b^4 + 19*a^4*b^6 - 9*a^2*b^8)*cos(e*x + d))*sin(e*x + d))/((a^6*
b^7 - 3*a^4*b^9 + 3*a^2*b^11 - b^13)*e*cos(e*x + d)^3 + 3*(a^7*b^6 - 3*a^5
*b^8 + 3*a^3*b^10 - a*b^12)*e*cos(e*x + d)^2 + 3*(a^8*b^5 - 3*a^6*b^7 + 3*
a^4*b^9 - a^2*b^11)*e*cos(e*x + d) + (a^9*b^4 - 3*a^7*b^6 + 3*a^5*b^8 - a^
3*b^10)*e), 1/6*(6*(a^7*b^3 - 3*a^5*b^5 + 3*a^3*b^7 - a*b^9)*e*x*cos(e*x +
 d)^3 + 18*(a^8*b^2 - 3*a^6*b^4 + 3*a^4*b^6 - a^2*b^8)*e*x*cos(e*x + d)^2
+ 18*(a^9*b - 3*a^7*b^3 + 3*a^5*b^5 - a^3*b^7)*e*x*cos(e*x + d) + 6*(a^10
- 3*a^8*b^2 + 3*a^6*b^4 - a^4*b^6)*e*x - 3*(2*a^9 - 5*a^7*b^2 + 3*a^5*b^4
- 2*a^3*b^6 + (2*a^6*b^3 - 5*a^4*b^5 + 3*a^2*b^7 - 2*b^9)*cos(e*x + d)^...
```

3.521.6 Sympy [F]

$$\int \frac{a+b\sec(d+ex)}{\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^2}\,dx=\int \frac{a+b\sec\left(d+ex\right)}{\left(a\sec\left(d+ex\right)+b\right)^4}\,dx$$

input
```
integrate((a+b*sec(e*x+d))/(b**2+2*a*b*sec(e*x+d)+a**2*sec(e*x+d)**2)**2,x
)
```

output
```
Integral((a + b*sec(d + e*x))/(a*sec(d + e*x) + b)**4, x)
```

3.521.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec(d+ex)}{\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^2,x, al
gorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.521.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 469 vs. 2(215) = 430.

Time = 0.39 (sec) , antiderivative size = 469, normalized size of antiderivative = 2.04

$$\int \frac{a+b\sec(d+ex)}{\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^2}\,dx$$
$$=\frac{\frac{3\left(2a^6-5a^4b^2+3a^2b^4-2b^6\right)\left(\pi\left\lfloor\frac{ex+d}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)-b\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^4b^4-2a^2b^6+b^8\right)\sqrt{a^2-b^2}}+\frac{3(ex+d)a}{b^4}-\frac{6a^7\tan\left(\frac{1}{2}ex+\frac{1}{2}\right.}{}}{}$$

input
```
integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^2,x, al
gorithm="giac")
```

output
```
1/3*(3*(2*a^6 - 5*a^4*b^2 + 3*a^2*b^4 - 2*b^6)*(pi*floor(1/2*(e*x + d)/pi
+ 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*e*x + 1/2*d) - b*tan(1/2*e*x +
 1/2*d))/sqrt(a^2 - b^2)))/((a^4*b^4 - 2*a^2*b^6 + b^8)*sqrt(a^2 - b^2)) +
 3*(e*x + d)*a/b^4 - (6*a^7*tan(1/2*e*x + 1/2*d)^5 - 15*a^6*b*tan(1/2*e*x
+ 1/2*d)^5 + 30*a^4*b^3*tan(1/2*e*x + 1/2*d)^5 - 12*a^3*b^4*tan(1/2*e*x +
1/2*d)^5 - 27*a^2*b^5*tan(1/2*e*x + 1/2*d)^5 + 18*a*b^6*tan(1/2*e*x + 1/2*
d)^5 + 12*a^7*tan(1/2*e*x + 1/2*d)^3 - 44*a^5*b^2*tan(1/2*e*x + 1/2*d)^3 +
 68*a^3*b^4*tan(1/2*e*x + 1/2*d)^3 - 36*a*b^6*tan(1/2*e*x + 1/2*d)^3 + 6*a
^7*tan(1/2*e*x + 1/2*d) + 15*a^6*b*tan(1/2*e*x + 1/2*d) - 30*a^4*b^3*tan(1
/2*e*x + 1/2*d) - 12*a^3*b^4*tan(1/2*e*x + 1/2*d) + 27*a^2*b^5*tan(1/2*e*x
 + 1/2*d) + 18*a*b^6*tan(1/2*e*x + 1/2*d))/((a^4*b^3 - 2*a^2*b^5 + b^7)*(a
*tan(1/2*e*x + 1/2*d)^2 - b*tan(1/2*e*x + 1/2*d)^2 + a + b)^3))/e
```

3.521.9 Mupad [B] (verification not implemented)

Time = 37.13 (sec) , antiderivative size = 5469, normalized size of antiderivative = 23.78

$$\int \frac{a + b\sec(d + ex)}{(b^2 + 2ab\sec(d + ex) + a^2\sec^2(d + ex))^2}\,dx = \text{Too large to display}$$

input
```
int((a + b/cos(d + e*x))/(b^2 + a^2/cos(d + e*x)^2 + (2*a*b)/cos(d + e*x))
^2,x)
```

output
```
- ((tan(d/2 + (e*x)/2)*(6*a*b^4 + a^4*b + 2*a^5 - 3*a^2*b^3 - 4*a^3*b^2))/
(b^5 - 2*a*b^4 + a^2*b^3) + (tan(d/2 + (e*x)/2)^5*(6*a*b^4 - a^4*b + 2*a^5
 + 3*a^2*b^3 - 4*a^3*b^2))/(b^3*(a + b)^2) + (4*tan(d/2 + (e*x)/2)^3*(9*a*
b^4 + 3*a^5 - 8*a^3*b^2))/(3*(a*b^3 - b^4)*(a + b)))/(e*(3*a*b^2 - tan(d/2
 + (e*x)/2)^4*(3*a*b^2 + 3*a^2*b - 3*a^3 - 3*b^3) - tan(d/2 + (e*x)/2)^2*(
3*a*b^2 - 3*a^2*b - 3*a^3 + 3*b^3) + 3*a^2*b + a^3 + b^3 + tan(d/2 + (e*x)
/2)^6*(3*a*b^2 - 3*a^2*b + a^3 - b^3))) - (2*a*atan(((a*((a*((8*(4*b^18 -
14*a^2*b^16 - 6*a^3*b^15 + 26*a^4*b^14 + 14*a^5*b^13 - 30*a^6*b^12 - 10*a^
7*b^11 + 18*a^8*b^10 + 2*a^9*b^9 - 4*a^10*b^8))/(a*b^15 + b^16 - 3*a^2*b^1
4 - 3*a^3*b^13 + 3*a^4*b^12 + 3*a^5*b^11 - a^6*b^10 - a^7*b^9) - (a*tan(d/
2 + (e*x)/2)*(8*a*b^17 - 8*a^2*b^16 - 32*a^3*b^15 + 32*a^4*b^14 + 48*a^5*b
^13 - 48*a^6*b^12 - 32*a^7*b^11 + 32*a^8*b^10 + 8*a^9*b^9 - 8*a^10*b^8)*8i
)/(b^4*(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 -
a^6*b^7 - a^7*b^6)))*1i)/b^4 - (8*tan(d/2 + (e*x)/2)*(8*a^11*b - 8*a^12 -
4*b^12 + 8*a^2*b^10 + 8*a^3*b^9 - 17*a^4*b^8 - 32*a^5*b^7 + 30*a^6*b^6 + 4
8*a^7*b^5 - 45*a^8*b^4 - 32*a^9*b^3 + 32*a^10*b^2))/(a*b^12 + b^13 - 3*a^2
*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6)))/b^4 - (a
*((a*((8*(4*b^18 - 14*a^2*b^16 - 6*a^3*b^15 + 26*a^4*b^14 + 14*a^5*b^13 -
30*a^6*b^12 - 10*a^7*b^11 + 18*a^8*b^10 + 2*a^9*b^9 - 4*a^10*b^8))/(a*b^15
 + b^16 - 3*a^2*b^14 - 3*a^3*b^13 + 3*a^4*b^12 + 3*a^5*b^11 - a^6*b^10 ...
```

3.521. $\int \frac{a+b\sec(d+ex)}{(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex))^2}\,dx$

3.522 $\int (a+b\sec(d+ex)) \left(b^2+2ab\sec(d+ex)+a^2\sec^2(d$

3.522.1 Optimal result

Integrand size = 41, antiderivative size = 359

$$\int (a+b\sec(d+ex)) \left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^{3/2} dx = \frac{(a^4+9a^2b^2+2b^4)\operatorname{arctanh}(\sin(d+ex)) \left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^{3/2}}{2e(b+a\sec(d+ex))^3} + \frac{a^4b^3x(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex))^{3/2}}{(ab+a^2\sec(d+ex))^3} + \frac{a^4b(11a^2+8b^2)\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^{3/2}\tan(d+ex)}{3e\left(ab+a^2\sec(d+ex)\right)^3} + \frac{a^5(3a^2+5b^2)\sec(d+ex)\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^{3/2}\tan(d+ex)}{6e\left(ab+a^2\sec(d+ex)\right)^3} + \frac{b\left(a^2b+a^3\sec(d+ex)\right)^2\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^{3/2}\tan(d+ex)}{3e\left(ab+a^2\sec(d+ex)\right)^3}$$

output
```
1/2*(a^4+9*a^2*b^2+2*b^4)*arctanh(sin(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*se
c(e*x+d)^2)^(3/2)/e/(b+a*sec(e*x+d))^3+a^4*b^3*x*(b^2+2*a*b*sec(e*x+d)+a^2
*sec(e*x+d)^2)^(3/2)/(a*b+a^2*sec(e*x+d))^3+1/3*a^4*b*(11*a^2+8*b^2)*(b^2+
2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(3/2)*tan(e*x+d)/e/(a*b+a^2*sec(e*x+d))
^3+1/6*a^5*(3*a^2+5*b^2)*sec(e*x+d)*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2
)^(3/2)*tan(e*x+d)/e/(a*b+a^2*sec(e*x+d))^3+1/3*b*(a^2*b+a^3*sec(e*x+d))^2
*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(3/2)*tan(e*x+d)/e/(a*b+a^2*sec(e
*x+d))^3
```

3.522.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.36

$$\int (a+b\sec(d+ex))\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^{3/2}\,dx=\frac{\cos(d+ex)\sqrt{(b+a\sec(d+ex))^2}(6ab^3ex+3(a^4+9a^2b^2+2b^4)\operatorname{arctanh}(\sin(d+ex))+3a(8}{6e(a+b\cos(d+ex))}$$

input
```
Integrate[(a + b*Sec[d + e*x])*(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x
]^2)^(3/2),x]
```

output
```
(Cos[d + e*x]*Sqrt[(b + a*Sec[d + e*x])^2]*(6*a*b^3*e*x + 3*(a^4 + 9*a^2*b
^2 + 2*b^4)*ArcTanh[Sin[d + e*x]] + 3*a*(8*a^2*b + 6*b^3 + a*(a^2 + 3*b^2)
*Sec[d + e*x])*Tan[d + e*x] + 2*a^3*b*Tan[d + e*x]^3))/(6*e*(a + b*Cos[d +
 e*x]))
```

3.522.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.53, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.195, Rules used = {3042, 4662, 27, 3042, 4406, 3042, 4536, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(d+ex))\left(a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2\right)^{3/2}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int (a+b\sec(d+ex))\left(a^2\sec(d+ex)^2+2ab\sec(d+ex)+b^2\right)^{3/2}dx$$

$$\Big\downarrow \ 4662$$

$$\frac{\left(a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2\right)^{3/2}\int 8\left(\sec(d+ex)a^2+ba\right)^3(a+b\sec(d+ex))dx}{8\left(a^2\sec(d+ex)+ab\right)^3}$$

$$\Big\downarrow \ 27$$

$$\frac{\left(a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2\right)^{3/2}\int \left(\sec(d+ex)a^2+ba\right)^3(a+b\sec(d+ex))dx}{\left(a^2\sec(d+ex)+ab\right)^3}$$

$\Big\downarrow$ 3042

$$\frac{\left(a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2\right)^{3/2}\int\left(\csc\left(d+ex+\frac{\pi}{2}\right)a^2+ba\right)^3\left(a+b\csc\left(d+ex+\frac{\pi}{2}\right)\right)dx}{\left(a^2\sec(d+ex)+ab\right)^3}$$

$\Big\downarrow$ 4406

$$\frac{\left(a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2\right)^{3/2}\left(\frac{1}{3}\int\left(\sec(d+ex)a^2+ba\right)\left(3b^2a^3+\left(3a^2+5b^2\right)\sec^2(d+ex)a^3+b(8a\right.\right.}{\left(a^2\sec(d+ex)+ab\right)^3}$$

$\Big\downarrow$ 3042

$$\frac{\left(a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2\right)^{3/2}\left(\frac{1}{3}\int\left(\csc\left(d+ex+\frac{\pi}{2}\right)a^2+ba\right)\left(3b^2a^3+\left(3a^2+5b^2\right)\csc\left(d+ex+\frac{\pi}{2}\right.\right.\right.}{\left(a^2\sec(d+ex)+ab\right)^3}$$

$\Big\downarrow$ 4536

$$\frac{\left(a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2\right)^{3/2}\left(\frac{1}{3}\left(\frac{1}{2}\int\left(6b^3a^4+2b\left(11a^2+8b^2\right)\sec^2(d+ex)a^4+3\left(a^4+9b^2a^2+2b^4\right.\right.\right.\right.}{\left(a^2\sec(d+ex)+ab\right)^3}$$

$\Big\downarrow$ 2009

$$\frac{\left(a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2\right)^{3/2}\left(\frac{b\tan(d+ex)\left(a^3\sec(d+ex)+a^2b\right)^2}{3e}+\frac{1}{3}\left(\frac{a^5\left(3a^2+5b^2\right)\tan(d+ex)\sec(d+ex)}{2e}+\frac{1}{2}\left(6a\right.\right.\right.}{\left(a^2\sec(d+ex)+ab\right)^3}$$

input
```
Int[(a + b*Sec[d + e*x])*(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2)^(
3/2),x]
```

output
```
((b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2)^(3/2)*((b*(a^2*b + a^3*Se
c[d + e*x])^2*Tan[d + e*x])/(3*e) + ((a^5*(3*a^2 + 5*b^2)*Sec[d + e*x]*Tan
[d + e*x])/(2*e) + (6*a^4*b^3*x + (3*a^3*(a^4 + 9*a^2*b^2 + 2*b^4)*ArcTanh
[Sin[d + e*x]])/e + (2*a^4*b*(11*a^2 + 8*b^2)*Tan[d + e*x])/e)/2)/3))/(a*b
 + a^2*Sec[d + e*x])^3
```

3.522. $\int (a+b\sec(d+ex))\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^{3/2}\,dx$

3.522.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4406
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m
 - 1)/(f*m)), x] + Simp[1/m   Int[(a + b*Csc[e + f*x])^(m - 2)*Simp[a^2*c*m
 + (b^2*d*(m - 1) + 2*a*b*c*m + a^2*d*m)*Csc[e + f*x] + b*(b*c*m + a*d*(2*m
 - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*
c - a*d, 0] && GtQ[m, 1] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

rule 4536
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[(-b)*C*Csc[e +
 f*x]*(Cot[e + f*x]/(2*f)), x] + Simp[1/2   Int[Simp[2*A*a + (2*B*a + b*(2*
A + C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a
, b, e, f, A, B, C}, x]
```

rule 4662
```
Int[((A_) + (B_.)*sec[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sec[(d_.) + (e_.)*
(x_)] + (c_.)*sec[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Simp[(a + b*Sec
[d + e*x] + c*Sec[d + e*x]^2)^n/(b + 2*c*Sec[d + e*x])^(2*n)   Int[(A + B*S
ec[d + e*x])*(b + 2*c*Sec[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e,
A, B}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[n]
```

3.522. $\int (a+b\sec(d+ex)) \left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^{3/2} \, dx$

3.522.4 Maple [A] (verified)

Time = 5.18 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.84

method	result
default	$-\frac{\sqrt{\sec(ex+d)^2(b\cos(ex+d)+a)^2}\,(3\cos(ex+d)\ln(-\cot(ex+d)+\csc(ex+d)-1)a^4+27\cos(ex+d)\ln(-\cot(ex+d)+\csc(ex+d)-1)a^2}{}$
parts	$\frac{a\sqrt{\sec(ex+d)^2(b\cos(ex+d)+a)^2}\,(-\ln(-\cot(ex+d)+\csc(ex+d)-1)\cos(ex+d)a^3-6\ln(-\cot(ex+d)+\csc(ex+d)-1)\cos(ex+d)a\,b^2}{}$
risch	$\frac{(1+e^{2i(ex+d)})\sqrt{\frac{(b\,e^{2i(ex+d)}+2a\,e^{i(ex+d)}+b)^2}{(1+e^{2i(ex+d)})^2}}\,xa\,b^3}{b\,e^{2i(ex+d)}+2a\,e^{i(ex+d)}+b}-\frac{i\sqrt{\frac{(b\,e^{2i(ex+d)}+2a\,e^{i(ex+d)}+b)^2}{(1+e^{2i(ex+d)})^2}}\,a(3a^3e^{5i(ex+d)}+9a\,b^2e^{5i(ex+d)}-18a^2b\,e^{4i}}{3(b\,e^{2i(ex+}}$

input
```
int((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(3/2),x,metho
d=_RETURNVERBOSE)
```

output
```
-1/6/e*(sec(e*x+d)^2*(b*cos(e*x+d)+a)^2)^(1/2)/(b*cos(e*x+d)+a)*(3*cos(e*x
+d)*ln(-cot(e*x+d)+csc(e*x+d)-1)*a^4+27*cos(e*x+d)*ln(-cot(e*x+d)+csc(e*x+
d)-1)*a^2*b^2+6*cos(e*x+d)*ln(-cot(e*x+d)+csc(e*x+d)-1)*b^4-3*cos(e*x+d)*l
n(-cot(e*x+d)+csc(e*x+d)+1)*a^4-27*cos(e*x+d)*ln(-cot(e*x+d)+csc(e*x+d)+1)
*a^2*b^2-6*cos(e*x+d)*ln(-cot(e*x+d)+csc(e*x+d)+1)*b^4-6*cos(e*x+d)*a*b^3*
(e*x+d)-22*a^3*b*sin(e*x+d)-18*sin(e*x+d)*a*b^3-3*a^4*tan(e*x+d)-9*tan(e*x
+d)*a^2*b^2-2*tan(e*x+d)*sec(e*x+d)*a^3*b)
```

3.522.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.45

$$\int (a+b\sec(d+ex))\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^{3/2}\,dx=\frac{12\,ab^3ex\cos(ex+d)^3+3\,(a^4+9\,a^2b^2+2\,b^4)\cos(ex+d)^3\log(\sin(ex+d)+1)-3\,(a^4+9\,a}{}$$

input
```
integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(3/2),x
, algorithm="fricas")
```

output
```
1/12*(12*a*b^3*e*x*cos(e*x + d)^3 + 3*(a^4 + 9*a^2*b^2 + 2*b^4)*cos(e*x +
d)^3*log(sin(e*x + d) + 1) - 3*(a^4 + 9*a^2*b^2 + 2*b^4)*cos(e*x + d)^3*lo
g(-sin(e*x + d) + 1) + 2*(2*a^3*b + 2*(11*a^3*b + 9*a*b^3)*cos(e*x + d)^2
+ 3*(a^4 + 3*a^2*b^2)*cos(e*x + d))*sin(e*x + d))/(e*cos(e*x + d)^3)
```

3.522.6 Sympy [F]

$$\int (a+b\sec(d+ex)) \left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^{3/2} dx = \int (a+b\sec(d+ex)) \left((a\sec(d+ex)+b)^2\right)^{\frac{3}{2}} dx$$

input
```
integrate((a+b*sec(e*x+d))*(b**2+2*a*b*sec(e*x+d)+a**2*sec(e*x+d)**2)**(3/
2),x)
```

output
```
Integral((a + b*sec(d + e*x))*((a*sec(d + e*x) + b)**2)**(3/2), x)
```

3.522.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 440, normalized size of antiderivative = 1.23

$$\int (a+b\sec(d+ex)) \left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^{3/2} dx = \frac{3\left(4b^3\arctan\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}\right)+(a^3+6ab^2)\log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}+1\right)-(a^3+6ab^2)\log\left(\frac{\sin(ex+d)}{\cos(ex+d)+\cdots}\right.\right.}{}$$

input
```
integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(3/2),x
, algorithm="maxima")
```

output
```
1/6*(3*(4*b^3*arctan(sin(e*x + d)/(cos(e*x + d) + 1)) + (a^3 + 6*a*b^2)*lo
g(sin(e*x + d)/(cos(e*x + d) + 1) + 1) - (a^3 + 6*a*b^2)*log(sin(e*x + d)/
(cos(e*x + d) + 1) - 1) - 2*((a^3 + 6*a^2*b)*sin(e*x + d)/(cos(e*x + d) +
1) + (a^3 - 6*a^2*b)*sin(e*x + d)^3/(cos(e*x + d) + 1)^3)/(2*sin(e*x + d)^
2/(cos(e*x + d) + 1)^2 - sin(e*x + d)^4/(cos(e*x + d) + 1)^4 - 1))*a + (3*
(3*a^2*b + 2*b^3)*log(sin(e*x + d)/(cos(e*x + d) + 1) + 1) - 3*(3*a^2*b +
2*b^3)*log(sin(e*x + d)/(cos(e*x + d) + 1) - 1) - 2*(3*(2*a^3 + 3*a^2*b +
6*a*b^2)*sin(e*x + d)/(cos(e*x + d) + 1) - 4*(a^3 + 9*a*b^2)*sin(e*x + d)^
3/(cos(e*x + d) + 1)^3 + 3*(2*a^3 - 3*a^2*b + 6*a*b^2)*sin(e*x + d)^5/(cos
(e*x + d) + 1)^5)/(3*sin(e*x + d)^2/(cos(e*x + d) + 1)^2 - 3*sin(e*x + d)^
4/(cos(e*x + d) + 1)^4 + sin(e*x + d)^6/(cos(e*x + d) + 1)^6 - 1))*b)/e
```

3.522.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 605, normalized size of antiderivative = 1.69

$$\int (a+b\sec(d+ex)) \left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^{3/2} dx = \frac{6\,(ex+d)ab^3\mathrm{sgn}\left(b\cos{(ex+d)}^2+a\cos{(ex+d)}\right)+3\,\left(a^4\mathrm{sgn}\left(b\cos{(ex+d)}^2+a\cos{(ex+d)}\right.\right.}{}$$

input
```
integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(3/2),x
, algorithm="giac")
```

output
```
1/6*(6*(e*x + d)*a*b^3*sgn(b*cos(e*x + d)^2 + a*cos(e*x + d)) + 3*(a^4*sgn
(b*cos(e*x + d)^2 + a*cos(e*x + d)) + 9*a^2*b^2*sgn(b*cos(e*x + d)^2 + a*c
os(e*x + d)) + 2*b^4*sgn(b*cos(e*x + d)^2 + a*cos(e*x + d)))*log(abs(tan(1
/2*e*x + 1/2*d) + 1)) - 3*(a^4*sgn(b*cos(e*x + d)^2 + a*cos(e*x + d)) + 9*
a^2*b^2*sgn(b*cos(e*x + d)^2 + a*cos(e*x + d)) + 2*b^4*sgn(b*cos(e*x + d)^
2 + a*cos(e*x + d)))*log(abs(tan(1/2*e*x + 1/2*d) - 1)) + 2*(3*a^4*sgn(b*c
os(e*x + d)^2 + a*cos(e*x + d))*tan(1/2*e*x + 1/2*d)^5 - 24*a^3*b*sgn(b*co
s(e*x + d)^2 + a*cos(e*x + d))*tan(1/2*e*x + 1/2*d)^5 + 9*a^2*b^2*sgn(b*co
s(e*x + d)^2 + a*cos(e*x + d))*tan(1/2*e*x + 1/2*d)^5 - 18*a*b^3*sgn(b*cos
(e*x + d)^2 + a*cos(e*x + d))*tan(1/2*e*x + 1/2*d)^5 + 40*a^3*b*sgn(b*cos(
e*x + d)^2 + a*cos(e*x + d))*tan(1/2*e*x + 1/2*d)^3 + 36*a*b^3*sgn(b*cos(e
*x + d)^2 + a*cos(e*x + d))*tan(1/2*e*x + 1/2*d)^3 - 3*a^4*sgn(b*cos(e*x +
 d)^2 + a*cos(e*x + d))*tan(1/2*e*x + 1/2*d) - 24*a^3*b*sgn(b*cos(e*x + d)
^2 + a*cos(e*x + d))*tan(1/2*e*x + 1/2*d) - 9*a^2*b^2*sgn(b*cos(e*x + d)^2
 + a*cos(e*x + d))*tan(1/2*e*x + 1/2*d) - 18*a*b^3*sgn(b*cos(e*x + d)^2 +
a*cos(e*x + d))*tan(1/2*e*x + 1/2*d))/(tan(1/2*e*x + 1/2*d)^2 - 1)^3)/e
```

3.522.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\sec(d+ex)) \left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^{3/2} dx = \int \left(a+\frac{b}{\cos{(d+e\,x)}}\right)\left(b^2+\frac{a^2}{\cos{(d+e\,x)}^2}+\frac{2\,a\,b}{\cos{(d+e\,x)}}\right)^{3/2} dx$$

input
```
int((a + b/cos(d + e*x))*(b^2 + a^2/cos(d + e*x)^2 + (2*a*b)/cos(d + e*x))
^(3/2),x)
```

output
```
int((a + b/cos(d + e*x))*(b^2 + a^2/cos(d + e*x)^2 + (2*a*b)/cos(d + e*x))
^(3/2), x)
```

3.522. $\int (a + b\sec(d + ex)) \left(b^2 + 2ab\sec(d + ex) + a^2\sec^2(d + ex)\right)^{3/2} dx$

3.523 $\int (a+b\sec(d+ex))\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d$

3.523.1 Optimal result

Integrand size = 41, antiderivative size = 173

$$\int (a+b\sec(d+ex))\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}\,dx = \frac{(a^2+b^2)\operatorname{arctanh}(\sin(d+ex))\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}}{e(b+a\sec(d+ex))} + \frac{a^2bx\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}}{ab+a^2\sec(d+ex)} + \frac{a^2b\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}\tan(d+ex)}{e\,(ab+a^2\sec(d+ex))}$$

output
```
(a^2+b^2)*arctanh(sin(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(1/2
)/e/(b+a*sec(e*x+d))+a^2*b*x*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(1/2)
/(a*b+a^2*sec(e*x+d))+a^2*b*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(1/2)*
tan(e*x+d)/e/(a*b+a^2*sec(e*x+d))
```

3.523.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.39

$$\int (a+b\sec(d+ex))\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}\,dx$$
$$=\frac{\cos(d+ex)\sqrt{(b+a\sec(d+ex))^2}\left((a^2+b^2)\operatorname{arctanh}(\sin(d+ex))+ab(ex+\tan(d+ex))\right)}{e(a+b\cos(d+ex))}$$

input
```
Integrate[(a + b*Sec[d + e*x])*Sqrt[b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d +
 e*x]^2],x]
```

output
```
(Cos[d + e*x]*Sqrt[(b + a*Sec[d + e*x])^2]*((a^2 + b^2)*ArcTanh[Sin[d + e*
x]] + a*b*(e*x + Tan[d + e*x])))/(e*(a + b*Cos[d + e*x]))
```

3.523.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.50, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.220, Rules used = {3042, 4662, 27, 3042, 4402, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sec(d+ex))\sqrt{a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (a+b\sec(d+ex))\sqrt{a^2\sec(d+ex)^2+2ab\sec(d+ex)+b^2}dx$$

$$\Big\downarrow \text{ 4662}$$

$$\frac{\sqrt{a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2}\int 2\left(\sec(d+ex)a^2+ba\right)(a+b\sec(d+ex))dx}{2\left(a^2\sec(d+ex)+ab\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2}\int \left(\sec(d+ex)a^2+ba\right)(a+b\sec(d+ex))dx}{a^2\sec(d+ex)+ab}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2}\int\left(\csc\left(d+ex+\frac{\pi}{2}\right)a^2+ba\right)\left(a+b\csc\left(d+ex+\frac{\pi}{2}\right)\right)dx}{a^2\sec(d+ex)+ab}$$

$$\Big\downarrow \text{4402}$$

$$\frac{\sqrt{a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2}\left(a\left(a^2+b^2\right)\int\sec(d+ex)dx+a^2b\int\sec^2(d+ex)dx+a^2bx\right)}{a^2\sec(d+ex)+ab}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2}\left(a\left(a^2+b^2\right)\int\csc\left(d+ex+\frac{\pi}{2}\right)dx+a^2b\int\csc\left(d+ex+\frac{\pi}{2}\right)^2dx+a^2bx\right)}{a^2\sec(d+ex)+ab}$$

$$\Big\downarrow \text{4254}$$

$$\frac{\sqrt{a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2}\left(a\left(a^2+b^2\right)\int\csc\left(d+ex+\frac{\pi}{2}\right)dx-\frac{a^2b\int 1d(-\tan(d+ex))}{e}+a^2bx\right)}{a^2\sec(d+ex)+ab}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sqrt{a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2}\left(a\left(a^2+b^2\right)\int\csc\left(d+ex+\frac{\pi}{2}\right)dx+\frac{a^2b\tan(d+ex)}{e}+a^2bx\right)}{a^2\sec(d+ex)+ab}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\sqrt{a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2}\left(\frac{a\left(a^2+b^2\right)\text{arctanh}(\sin(d+ex))}{e}+\frac{a^2b\tan(d+ex)}{e}+a^2bx\right)}{a^2\sec(d+ex)+ab}$$

input
```
Int[(a + b*Sec[d + e*x])*Sqrt[b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^
2],x]
```

output
```
(Sqrt[b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2]*(a^2*b*x + (a*(a^2 +
b^2)*ArcTanh[Sin[d + e*x]])/e + (a^2*b*Tan[d + e*x])/e))/(a*b + a^2*Sec[d
+ e*x])
```

3.523. $\int(a+b\sec(d+ex))\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}\,dx$

3.523.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4402
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))*(csc[(e_.) + (f_.)*(x_)]*(d_.) +
 (c_)), x_Symbol] :> Simp[a*c*x, x] + (Simp[b*d   Int[Csc[e + f*x]^2, x], x
] + Simp[(b*c + a*d)   Int[Csc[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e, f
}, x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

rule 4662
```
Int[((A_) + (B_.)*sec[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sec[(d_.) + (e_.)*
(x_)] + (c_.)*sec[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Simp[(a + b*Sec
[d + e*x] + c*Sec[d + e*x]^2)^n/(b + 2*c*Sec[d + e*x])^(2*n)   Int[(A + B*S
ec[d + e*x])*(b + 2*c*Sec[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e,
A, B}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[n]
```

3.523. $\int (a+b\sec(d+ex))\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}\,dx$

3.523.4 Maple [A] (verified)

Time = 4.93 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.02

method	result
default	$-\frac{(\ln(-\cot(ex+d)+\csc(ex+d)-1)\cos(ex+d)a^2+\ln(-\cot(ex+d)+\csc(ex+d)-1)\cos(ex+d)b^2-\ln(-\cot(ex+d)+\csc(ex+d)+1)\cos}{e(b\cos(ex+}$
parts	$-\frac{a\cos(ex+d)(a\ln(-\cot(ex+d)+\csc(ex+d)-1)-a\ln(-\cot(ex+d)+\csc(ex+d)+1)-(ex+d)b)\sqrt{\sec(ex+d)^2(b\cos(ex+d)+a)^2}}{e(b\cos(ex+d)+a)} -$
risch	$\frac{\sqrt{\frac{\left(b\,e^{2i(ex+d)}+2a\,e^{i(ex+d)}+b\right)^2}{\left(1+e^{2i(ex+d)}\right)^2}}\,(1+e^{2i(ex+d)})xab}{b\,e^{2i(ex+d)}+2a\,e^{i(ex+d)}+b}+\frac{2i\sqrt{\frac{\left(b\,e^{2i(ex+d)}+2a\,e^{i(ex+d)}+b\right)^2}{\left(1+e^{2i(ex+d)}\right)^2}}\,ab}{\left(b\,e^{2i(ex+d)}+2a\,e^{i(ex+d)}+b\right)e}-\frac{\sqrt{\frac{\left(b\,e^{2i(ex+d)}+2a\,e^{i(ex+d)}+b\right)^2}{\left(1+e^{2i(ex+d)}\right)^2}}\,(1+}{\left(b\,e^{2i(ex+d)}+2a\right.}$

input
```
int((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(1/2),x,metho
d=_RETURNVERBOSE)
```

output
```
-1/e*(ln(-cot(e*x+d)+csc(e*x+d)-1)*cos(e*x+d)*a^2+ln(-cot(e*x+d)+csc(e*x+d
)-1)*cos(e*x+d)*b^2-ln(-cot(e*x+d)+csc(e*x+d)+1)*cos(e*x+d)*a^2-ln(-cot(e*
x+d)+csc(e*x+d)+1)*cos(e*x+d)*b^2-cos(e*x+d)*a*b*(e*x+d)-a*b*sin(e*x+d))*(
sec(e*x+d)^2*(b*cos(e*x+d)+a)^2)^(1/2)/(b*cos(e*x+d)+a)
```

3.523.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.49

$$\int (a+b\sec(d+ex))\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}\,dx$$
$$=\frac{2\,abex\cos{(ex+d)}+(a^2+b^2)\cos{(ex+d)}\log{(\sin{(ex+d)}+1)}-(a^2+b^2)\cos{(ex+d)}\log{(-\sin{(ex+}}{2\,e\cos{(ex+d)}}$$

input
```
integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(1/2),x
, algorithm="fricas")
```

output
```
1/2*(2*a*b*e*x*cos(e*x + d) + (a^2 + b^2)*cos(e*x + d)*log(sin(e*x + d) +
1) - (a^2 + b^2)*cos(e*x + d)*log(-sin(e*x + d) + 1) + 2*a*b*sin(e*x + d))
/(e*cos(e*x + d))
```

3.523.6 Sympy [F]

$$\int (a+b\sec(d+ex))\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}\,dx = \int (a+b\sec(d+ex))\sqrt{(a\sec(d+ex)+b)^2}\,dx$$

input
```
integrate((a+b*sec(e*x+d))*(b**2+2*a*b*sec(e*x+d)+a**2*sec(e*x+d)**2)**(1/
2),x)
```

output
```
Integral((a + b*sec(d + e*x))*sqrt((a*sec(d + e*x) + b)**2), x)
```

3.523.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.95

$$\int (a+b\sec(d+ex))\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}\,dx = \frac{\left(2\,b\arctan\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}\right)+a\log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}+1\right)-a\log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}-1\right)\right)a+\left(b\log\left(\frac{\sin(ex+d)}{\cos(ex+d)+1}+1\right)\right.}{e}$$

input
```
integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(1/2),x
, algorithm="maxima")
```

output
```
((2*b*arctan(sin(e*x + d)/(cos(e*x + d) + 1)) + a*log(sin(e*x + d)/(cos(e*
x + d) + 1) + 1) - a*log(sin(e*x + d)/(cos(e*x + d) + 1) - 1))*a + (b*log(
sin(e*x + d)/(cos(e*x + d) + 1) + 1) - b*log(sin(e*x + d)/(cos(e*x + d) +
1) - 1) - 2*a*sin(e*x + d)/((sin(e*x + d)^2/(cos(e*x + d) + 1)^2 - 1)*(cos
(e*x + d) + 1)))*b)/e
```

3.523.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.20

$$\int (a+b\sec(d+ex))\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}\,dx$$
$$= \frac{(ex+d)ab\mathrm{sgn}\left(b\cos(ex+d)^2+a\cos(ex+d)\right) - \frac{2\,ab\mathrm{sgn}\left(b\cos(ex+d)^2+a\cos(ex+d)\right)\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)}{\tan\left(\frac{1}{2}\,ex+\frac{1}{2}\,d\right)^2-1} + \left(a^2\mathrm{sgn}\left(b\cos\left(\right.\right.\right.}{}$$

input
```
integrate((a+b*sec(e*x+d))*(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(1/2),x
, algorithm="giac")
```

output
```
((e*x + d)*a*b*sgn(b*cos(e*x + d)^2 + a*cos(e*x + d)) - 2*a*b*sgn(b*cos(e*
x + d)^2 + a*cos(e*x + d))*tan(1/2*e*x + 1/2*d)/(tan(1/2*e*x + 1/2*d)^2 -
1) + (a^2*sgn(b*cos(e*x + d)^2 + a*cos(e*x + d)) + b^2*sgn(b*cos(e*x + d)^
2 + a*cos(e*x + d)))*log(abs(tan(1/2*e*x + 1/2*d) + 1)) - (a^2*sgn(b*cos(e
*x + d)^2 + a*cos(e*x + d)) + b^2*sgn(b*cos(e*x + d)^2 + a*cos(e*x + d)))*
log(abs(tan(1/2*e*x + 1/2*d) - 1)))/e
```

3.523.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\sec(d+ex))\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}\,dx$$
$$= \int \left(a+\frac{b}{\cos(d+e\,x)}\right)\sqrt{b^2+\frac{a^2}{\cos(d+e\,x)^2}+\frac{2\,a\,b}{\cos(d+e\,x)}}\,dx$$

input
```
int((a + b/cos(d + e*x))*(b^2 + a^2/cos(d + e*x)^2 + (2*a*b)/cos(d + e*x))
^(1/2),x)
```

output
```
int((a + b/cos(d + e*x))*(b^2 + a^2/cos(d + e*x)^2 + (2*a*b)/cos(d + e*x))
^(1/2), x)
```

3.524 $\int \frac{a+b\sec(d+ex)}{\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}}\,dx$

3.524.1 Optimal result

Integrand size = 41, antiderivative size = 142

$$\int \frac{a+b\sec(d+ex)}{\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}}\,dx = -\frac{2\sqrt{a-b}\sqrt{a+b}\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)(b+a\sec(d+ex))}{be\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}} + \frac{x(ab+a^2\sec(d+ex))}{b\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}}$$

output
```
x*(a*b+a^2*sec(e*x+d))/b/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(1/2)-2*a
rctan((a-b)^(1/2)*tan(1/2*e*x+1/2*d)/(a+b)^(1/2))*(b+a*sec(e*x+d))*(a-b)^(
1/2)*(a+b)^(1/2)/b/e/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(1/2)
```

3.524.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.65

$$\int \frac{a+b\sec(d+ex)}{\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}}\,dx = \frac{\left(a(d+ex)+2\sqrt{-a^2+b^2}\,\text{arctanh}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{-a^2+b^2}}\right)\right)(a+b\cos(d+ex))\sec(d+ex)}{be\sqrt{(b+a\sec(d+ex))^2}}$$

input
```
Integrate[(a + b*Sec[d + e*x])/Sqrt[b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d +
 e*x]^2],x]
```

output
```
((a*(d + e*x) + 2*Sqrt[-a^2 + b^2]*ArcTanh[((-a + b)*Tan[(d + e*x)/2])/Sqr
t[-a^2 + b^2]])*(a + b*Cos[d + e*x])*Sec[d + e*x])/(b*e*Sqrt[(b + a*Sec[d
+ e*x])^2])
```

3.524.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.81, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.244, Rules used = {3042, 4662, 27, 3042, 4407, 3042, 4318, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec(d+ex)}{\sqrt{a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\sec(d+ex)}{\sqrt{a^2\sec(d+ex)^2+2ab\sec(d+ex)+b^2}}dx$$

$$\Big\downarrow \text{4662}$$

$$\frac{2\left(a^2\sec(d+ex)+ab\right)\int \frac{a+b\sec(d+ex)}{2\left(\sec(d+ex)a^2+ba\right)}dx}{\sqrt{a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(a^2\sec(d+ex)+ab\right)\int \frac{a+b\sec(d+ex)}{\sec(d+ex)a^2+ba}dx}{\sqrt{a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(a^2\sec(d+ex)+ab\right)\int \frac{a+b\csc\left(d+ex+\frac{\pi}{2}\right)}{\csc\left(d+ex+\frac{\pi}{2}\right)a^2+ba}dx}{\sqrt{a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2}}$$

$$\Big\downarrow \text{4407}$$

$$\frac{\left(a^2\sec(d+ex)+ab\right)\left(\frac{x}{b}-\frac{\left(a^2-b^2\right)\int \frac{\sec(d+ex)}{\sec(d+ex)a^2+ba}dx}{b}\right)}{\sqrt{a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\left(a^2\sec(d+ex)+ab\right)\left(\frac{x}{b}-\frac{(a^2-b^2)\int\frac{\csc\left(d+ex+\frac{\pi}{2}\right)}{\csc\left(d+ex+\frac{\pi}{2}\right)a^2+ba}dx}{b}\right)}{\sqrt{a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2}}$$

$$\Big\downarrow \ 4318$$

$$\frac{\left(a^2\sec(d+ex)+ab\right)\left(\frac{x}{b}-\frac{(a^2-b^2)\int\frac{1}{\frac{b\cos(d+ex)}{a}+1}dx}{a^2b}\right)}{\sqrt{a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\left(a^2\sec(d+ex)+ab\right)\left(\frac{x}{b}-\frac{(a^2-b^2)\int\frac{1}{\frac{b\sin\left(d+ex+\frac{\pi}{2}\right)}{a}+1}dx}{a^2b}\right)}{\sqrt{a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2}}$$

$$\Big\downarrow \ 3138$$

$$\frac{\left(a^2\sec(d+ex)+ab\right)\left(\frac{x}{b}-\frac{2(a^2-b^2)\int\frac{1}{\frac{(a-b)\tan^2\left(\frac{1}{2}(d+ex)\right)}{a}+\frac{a+b}{a}}d\tan\left(\frac{1}{2}(d+ex)\right)}{a^2be}\right)}{\sqrt{a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2}}$$

$$\Big\downarrow \ 218$$

$$\frac{\left(a^2\sec(d+ex)+ab\right)\left(\frac{x}{b}-\frac{2(a^2-b^2)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{abe\sqrt{a-b}\sqrt{a+b}}\right)}{\sqrt{a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2}}$$

input
```
Int[(a + b*Sec[d + e*x])/Sqrt[b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^
2],x]
```

output
```
((x/b - (2*(a^2 - b^2)*ArcTan[(Sqrt[a - b]*Tan[(d + e*x)/2])/Sqrt[a + b]])
/(a*Sqrt[a - b]*b*Sqrt[a + b]*e))*(a*b + a^2*Sec[d + e*x]))/Sqrt[b^2 + 2*a
*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2]
```

3.524. $\int\frac{a+b\sec(d+ex)}{\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}}\,dx$

3.524.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 4662
```
Int[((A_) + (B_.)*sec[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sec[(d_.) + (e_.)*
(x_)] + (c_.)*sec[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Simp[(a + b*Sec
[d + e*x] + c*Sec[d + e*x]^2)^n/(b + 2*c*Sec[d + e*x])^(2*n)   Int[(A + B*S
ec[d + e*x])*(b + 2*c*Sec[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e,
A, B}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[n]
```

3.524. $\int \frac{a+b\sec(d+ex)}{\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}}\,dx$

3.524.4 Maple [A] (verified)

Time = 5.08 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.04

method	result
default	$\frac{\left(a(ex+d)\sqrt{(a+b)(a-b)}+2\arctan\left(\frac{(\cot(ex+d)-\csc(ex+d))(a-b)}{\sqrt{(a+b)(a-b)}}\right)a^2-2\arctan\left(\frac{(\cot(ex+d)-\csc(ex+d))(a-b)}{\sqrt{(a+b)(a-b)}}\right)b^2\right)(b+a\sec(ex+d))}{e\sqrt{\sec(ex+d)^2(b\cos(ex+d)+a)^2}\,b\sqrt{(a+b)(a-b)}}$
parts	$-\frac{a\left(2a\arctan\left(\frac{(-\cot(ex+d)+\csc(ex+d))(a-b)}{\sqrt{(a+b)(a-b)}}\right)-(ex+d)\sqrt{(a+b)(a-b)}\right)(b+a\sec(ex+d))}{e\sqrt{\sec(ex+d)^2(b\cos(ex+d)+a)^2}\,b\sqrt{(a+b)(a-b)}}+\frac{2b\arctan\left(\frac{(-\cot(ex+d)+\csc(ex+d))(a-}{\sqrt{(a+b)(a-b)}}\right.}{e\sqrt{\sec(ex+d)^2(b\cos(ex+d)+a)^2}}$
risch	$\frac{(b\,e^{2i(ex+d)}+2a\,e^{i(ex+d)}+b)ax}{\sqrt{\frac{(b\,e^{2i(ex+d)}+2a\,e^{i(ex+d)}+b)^2}{(1+e^{2i(ex+d)})^2}}(1+e^{2i(ex+d)})b}+\frac{(b\,e^{2i(ex+d)}+2a\,e^{i(ex+d)}+b)\sqrt{-a^2+b^2}\,\ln\left(e^{i(ex+d)}+\frac{i\sqrt{-a^2+b^2}+a}{b}\right)}{\sqrt{\frac{(b\,e^{2i(ex+d)}+2a\,e^{i(ex+d)}+b)^2}{(1+e^{2i(ex+d)})^2}}(1+e^{2i(ex+d)})eb}-\frac{(b\,e^{2i(}}{}$

input
```
int((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(1/2),x,metho
d=_RETURNVERBOSE)
```

output
```
1/e*(a*(e*x+d)*((a+b)*(a-b))^(1/2)+2*arctan((cot(e*x+d)-csc(e*x+d))*(a-b)/
((a+b)*(a-b))^(1/2))*a^2-2*arctan((cot(e*x+d)-csc(e*x+d))*(a-b)/((a+b)*(a-
b))^(1/2))*b^2)/(sec(e*x+d)^2*(b*cos(e*x+d)+a)^2)^(1/2)*(b+a*sec(e*x+d))/b
/((a+b)*(a-b))^(1/2)
```

3.524.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.30

$$\int \frac{a+b\sec(d+ex)}{\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}}\,dx$$
$$=\left[\frac{2\,aex+\sqrt{-a^2+b^2}\log\left(\frac{2\,ab\cos(ex+d)+(2\,a^2-b^2)\cos(ex+d)^2+2\,\sqrt{-a^2+b^2}(a\cos(ex+d)+b)\sin(ex+d)-a^2+2\,b^2}{b^2\cos(ex+d)^2+2\,ab\cos(ex+d)+a^2}\right)}{2\,be},\ \frac{aex-\sqrt{a^2}}{}\right.$$

input
```
integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(1/2),x
, algorithm="fricas")
```

output
```
[1/2*(2*a*e*x + sqrt(-a^2 + b^2)*log((2*a*b*cos(e*x + d) + (2*a^2 - b^2)*c
os(e*x + d)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(e*x + d) + b)*sin(e*x + d) - a^2
 + 2*b^2)/(b^2*cos(e*x + d)^2 + 2*a*b*cos(e*x + d) + a^2)))/(b*e), (a*e*x
- sqrt(a^2 - b^2)*arctan(-(a*cos(e*x + d) + b)/(sqrt(a^2 - b^2)*sin(e*x +
d))))/(b*e)]
```

3.524.6 Sympy [F]

$$\int \frac{a+b\sec(d+ex)}{\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}}\,dx=\int \frac{a+b\sec\left(d+ex\right)}{\sqrt{\left(a\sec\left(d+ex\right)+b\right)^2}}\,dx$$

input
```
integrate((a+b*sec(e*x+d))/(b**2+2*a*b*sec(e*x+d)+a**2*sec(e*x+d)**2)**(1/
2),x)
```

output
```
Integral((a + b*sec(d + e*x))/sqrt((a*sec(d + e*x) + b)**2), x)
```

3.524.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec(d+ex)}{\sqrt{b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(1/2),x
, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.524.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 383 vs. $2(129) = 258$.

Time = 0.55 (sec) , antiderivative size = 383, normalized size of antiderivative = 2.70

$$\int \frac{a + b\sec(d + ex)}{\sqrt{b^2 + 2ab\sec(d + ex) + a^2\sec^2(d + ex)}}\,dx =$$

$$-\frac{\frac{\left(a|b|\left|\operatorname{sgn}\left(b\cos(ex+d)^2+a\cos(ex+d)\right)\right|-b|b|\left|\operatorname{sgn}\left(b\cos(ex+d)^2+a\cos(ex+d)\right)\right|-2\,a^2\operatorname{sgn}\left(b\cos(ex+d)^2+a\cos(ex+d)\right)+ab\operatorname{sgn}\left(b\cos(ex+d)^2-\right.\right.}{a|b|\left|\operatorname{sgn}\left(b\cos(ex+d)^2+a\cos(ex+d)\right)\right|\operatorname{sgn}\left(b\cos(ex+d)^2+a\cos(ex+d)\right)-b^2}}{}$$

input
```
integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(1/2),x
, algorithm="giac")
```

output
```
-1/2*((a*abs(b)*abs(sgn(b*cos(e*x + d)^2 + a*cos(e*x + d))) - b*abs(b)*abs
(sgn(b*cos(e*x + d)^2 + a*cos(e*x + d))) - 2*a^2*sgn(b*cos(e*x + d)^2 + a*
cos(e*x + d)) + a*b*sgn(b*cos(e*x + d)^2 + a*cos(e*x + d)) + b^2*sgn(b*cos
(e*x + d)^2 + a*cos(e*x + d)))*(e*x + d)/(a*abs(b)*abs(sgn(b*cos(e*x + d)^
2 + a*cos(e*x + d)))*sgn(b*cos(e*x + d)^2 + a*cos(e*x + d)) - b^2) + 2*(sq
rt(a^2 - b^2)*abs(a - b)*abs(b)*abs(sgn(b*cos(e*x + d)^2 + a*cos(e*x + d))
) + sqrt(a^2 - b^2)*(2*a + b)*abs(a - b)*sgn(b*cos(e*x + d)^2 + a*cos(e*x
+ d)))*(pi*floor(1/2*(e*x + d)/pi + 1/2) + arctan(sqrt(a^2 - b^2)*tan(1/2*
e*x + 1/2*d)/(a + b)))/((a^2 - a*b)*abs(b)*abs(sgn(b*cos(e*x + d)^2 + a*co
s(e*x + d)))*sgn(b*cos(e*x + d)^2 + a*cos(e*x + d)) + (a - b)*b^2))/e
```

3.524.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\sec(d + ex)}{\sqrt{b^2 + 2ab\sec(d + ex) + a^2\sec^2(d + ex)}}\,dx = \int \frac{a + \frac{b}{\cos(d+e\,x)}}{\sqrt{b^2 + \frac{a^2}{\cos(d+e\,x)^2} + \frac{2\,a\,b}{\cos(d+e\,x)}}}\,dx$$

input
```
int((a + b/cos(d + e*x))/(b^2 + a^2/cos(d + e*x)^2 + (2*a*b)/cos(d + e*x))
^(1/2),x)
```

output
```
int((a + b/cos(d + e*x))/(b^2 + a^2/cos(d + e*x)^2 + (2*a*b)/cos(d + e*x))
^(1/2), x)
```

3.525 $\int \frac{a+b\sec(d+ex)}{(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex))^{3/2}}\,dx$

3.525.1 Optimal result

Integrand size = 41, antiderivative size = 330

$$\begin{aligned}
&\int \frac{a+b\sec(d+ex)}{(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex))^{3/2}}\,dx = \\
&-\frac{(2a^4-3a^2b^2+2b^4)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)(b+a\sec(d+ex))^3}{(a-b)^{3/2}b^3(a+b)^{3/2}e\,(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex))^{3/2}} \\
&+\frac{x(ab+a^2\sec(d+ex))^3}{a^2b^3\,(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex))^{3/2}} \\
&-\frac{(ab+a^2\sec(d+ex))\tan(d+ex)}{2be\,(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex))^{3/2}} \\
&-\frac{(2a^2-3b^2)\,(ab+a^2\sec(d+ex))^3\tan(d+ex)}{2b^2\,(a^2-b^2)\,e\,(a^2b+a^3\sec(d+ex))\,(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex))^{3/2}}
\end{aligned}$$

output
```
-(2*a^4-3*a^2*b^2+2*b^4)*arctan((a-b)^(1/2)*tan(1/2*e*x+1/2*d)/(a+b)^(1/2)
)*(b+a*sec(e*x+d))^3/(a-b)^(3/2)/b^3/(a+b)^(3/2)/e/(b^2+2*a*b*sec(e*x+d)+a
^2*sec(e*x+d)^2)^(3/2)+x*(a*b+a^2*sec(e*x+d))^3/a^2/b^3/(b^2+2*a*b*sec(e*x
+d)+a^2*sec(e*x+d)^2)^(3/2)-1/2*(a*b+a^2*sec(e*x+d))*tan(e*x+d)/b/e/(b^2+2
*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(3/2)-1/2*(2*a^2-3*b^2)*(a*b+a^2*sec(e*x
+d))^3*tan(e*x+d)/b^2/(a^2-b^2)/e/(a^2*b+a^3*sec(e*x+d))/(b^2+2*a*b*sec(e*
x+d)+a^2*sec(e*x+d)^2)^(3/2)
```

3.525.2 Mathematica [A] (verified)

Time = 2.50 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.65

$$\int \frac{a+b\sec(d+ex)}{(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex))^{3/2}}\,dx = \frac{(a+b\cos(d+ex))\sec^2(d+ex)(a+b\sec(d+ex))\left(2a\right.}{}$$

input
```
Integrate[(a + b*Sec[d + e*x])/(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x
]^2)^(3/2),x]
```

output
```
((a + b*Cos[d + e*x])*Sec[d + e*x]^2*(a + b*Sec[d + e*x])*(2*a*(d + e*x)*(
a + b*Cos[d + e*x])^2 + (2*(2*a^4 - 3*a^2*b^2 + 2*b^4)*ArcTanh[((-a + b)*T
an[(d + e*x)/2])/Sqrt[-a^2 + b^2]]*(a + b*Cos[d + e*x])^2)/(-a^2 + b^2)^(3
/2) + a^2*b*Sin[d + e*x] + (a*b*(3*a^2 - 4*b^2)*(a + b*Cos[d + e*x])*Sin[d
 + e*x])/((-a + b)*(a + b))))/(2*b^3*e*(b + a*Cos[d + e*x])*((b + a*Sec[d
+ e*x])^2)^(3/2))
```

3.525.3 Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.81, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.341, Rules used = {3042, 4662, 27, 3042, 4411, 3042, 4548, 3042, 4407, 3042, 4318, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\sec(d+ex)}{(a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\sec(d+ex)}{(a^2\sec(d+ex)^2+2ab\sec(d+ex)+b^2)^{3/2}}dx$$

$$\Big\downarrow \text{4662}$$

$$\frac{8\left(a^2\sec(d+ex)+ab\right)^3\int \frac{a+b\sec(d+ex)}{8(\sec(d+ex)a^2+ba)^3}dx}{(a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(a^2\sec(d+ex)+ab\right)^3\int\frac{a+b\sec(d+ex)}{\left(\sec(d+ex)a^2+ba\right)^3}dx}{\left(a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(a^2\sec(d+ex)+ab\right)^3\int\frac{a+b\csc\left(d+ex+\frac{\pi}{2}\right)}{\left(\csc\left(d+ex+\frac{\pi}{2}\right)a^2+ba\right)^3}dx}{\left(a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2\right)^{3/2}}$$

$$\Big\downarrow \text{4411}$$

$$\frac{\left(a^2\sec(d+ex)+ab\right)^3\left(\frac{\int\frac{-\left(\left(a^2-b^2\right)\sec^2(d+ex)a^3\right)+2\left(a^2-b^2\right)a^3+2b\left(a^2-b^2\right)\sec(d+ex)a^2}{\left(\sec(d+ex)a^2+ba\right)^2}dx}{2a^3b(a^2-b^2)}-\frac{\tan(d+ex)}{2be(a^2\sec(d+ex)+ab)^2}\right)}{\left(a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(a^2\sec(d+ex)+ab\right)^3\left(\frac{\int\frac{-\left(\left(a^2-b^2\right)\csc\left(d+ex+\frac{\pi}{2}\right)^2a^3\right)+2\left(a^2-b^2\right)a^3+2b\left(a^2-b^2\right)\csc\left(d+ex+\frac{\pi}{2}\right)a^2}{\left(\csc\left(d+ex+\frac{\pi}{2}\right)a^2+ba\right)^2}dx}{2a^3b(a^2-b^2)}-\frac{\tan(d+ex)}{2be(a^2\sec(d+ex)+ab)^2}\right)}{\left(a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2\right)^{3/2}}$$

$$\Big\downarrow \text{4548}$$

$$\frac{\left(a^2\sec(d+ex)+ab\right)^3\left(\frac{\frac{\int\frac{2\left(a^2-b^2\right)^2a^5+b\left(a^2-2b^2\right)\left(a^2-b^2\right)\sec(d+ex)a^4}{\sec(d+ex)a^2+ba}dx}{a^3b\left(a^2-b^2\right)}-\frac{a^2\left(2a^2-3b^2\right)\tan(d+ex)}{be\left(a^2\sec(d+ex)+ab\right)}}{2a^3b(a^2-b^2)}-\frac{\tan(d+ex)}{2be(a^2\sec(d+ex)+ab)^2}\right)}{\left(a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(a^2\sec(d+ex)+ab\right)^3\left(\frac{\frac{\int\frac{2\left(a^2-b^2\right)^2a^5+b\left(a^2-2b^2\right)\left(a^2-b^2\right)\csc\left(d+ex+\frac{\pi}{2}\right)a^4}{\csc\left(d+ex+\frac{\pi}{2}\right)a^2+ba}dx}{a^3b\left(a^2-b^2\right)}-\frac{a^2\left(2a^2-3b^2\right)\tan(d+ex)}{be\left(a^2\sec(d+ex)+ab\right)}}{2a^3b(a^2-b^2)}-\frac{\tan(d+ex)}{2be(a^2\sec(d+ex)+ab)^2}\right)}{\left(a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2\right)^{3/2}}$$

$$\Big\downarrow \text{4407}$$

3.525. $\int\frac{a+b\sec(d+ex)}{\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^{3/2}}dx$

$$\frac{\left(a^2\sec(d+ex)+ab\right)^3\left(\frac{\frac{\frac{2a^4x\left(a^2-b^2\right)^2}{b}-\frac{a^4\left(a^2-b^2\right)\left(2a^4-3a^2b^2+2b^4\right)\int\frac{\sec(d+ex)}{\sec(d+ex)a^2+ba}dx}{b}}{a^3b\left(a^2-b^2\right)}-\frac{a^2\left(2a^2-3b^2\right)\tan(d+ex)}{be\left(a^2\sec(d+ex)+ab\right)}}{2a^3b(a^2-b^2)}-\frac{\tan(d+ex)}{2be(a^2\sec(d+ex)+a}\right.}{(a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(a^2\sec(d+ex)+ab\right)^3\left(\frac{\frac{\frac{2a^4x\left(a^2-b^2\right)^2}{b}-\frac{a^4\left(a^2-b^2\right)\left(2a^4-3a^2b^2+2b^4\right)\int\frac{\csc\left(d+ex+\frac{\pi}{2}\right)}{\csc\left(d+ex+\frac{\pi}{2}\right)a^2+ba}dx}{b}}{a^3b\left(a^2-b^2\right)}-\frac{a^2\left(2a^2-3b^2\right)\tan(d+ex)}{be\left(a^2\sec(d+ex)+ab\right)}}{2a^3b(a^2-b^2)}-\frac{\tan(d+ex}{2be(a^2\sec(d+ex}\right.}{(a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2)^{3/2}}$$

$$\Big\downarrow \text{4318}$$

$$\frac{\left(a^2\sec(d+ex)+ab\right)^3\left(\frac{\frac{\frac{2a^4x\left(a^2-b^2\right)^2}{b}-\frac{a^2\left(a^2-b^2\right)\left(2a^4-3a^2b^2+2b^4\right)\int\frac{1}{\frac{b\cos(d+ex)}{a}+1}dx}{b}}{a^3b\left(a^2-b^2\right)}-\frac{a^2\left(2a^2-3b^2\right)\tan(d+ex)}{be\left(a^2\sec(d+ex)+ab\right)}}{2a^3b(a^2-b^2)}-\frac{\tan(d+ex}{2be(a^2\sec(d+ex)+ab}\right.}{(a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(a^2\sec(d+ex)+ab\right)^3\left(\frac{\frac{\frac{2a^4x\left(a^2-b^2\right)^2}{b}-\frac{a^2\left(a^2-b^2\right)\left(2a^4-3a^2b^2+2b^4\right)\int\frac{1}{\frac{b\sin\left(d+ex+\frac{\pi}{2}\right)}{a}+1}dx}{b}}{a^3b\left(a^2-b^2\right)}-\frac{a^2\left(2a^2-3b^2\right)\tan(d+ex)}{be\left(a^2\sec(d+ex)+ab\right)}}{2a^3b(a^2-b^2)}-\frac{\tan(d+ex)}{2be(a^2\sec(d+ex)}\right.}{(a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2)^{3/2}}$$

$$\Big\downarrow \text{3138}$$

$$\frac{\left(a^2\sec(d+ex)+ab\right)^3\left(\frac{\frac{\frac{2a^4x\left(a^2-b^2\right)^2}{b}-\frac{2a^2\left(a^2-b^2\right)\left(2a^4-3a^2b^2+2b^4\right)\int\frac{1}{\frac{(a-b)\tan^2\left(\frac{1}{2}(d+ex)\right)}{a}+\frac{a+b}{a}}d\tan\left(\frac{1}{2}(d+ex)\right)}{be}}{a^3b\left(a^2-b^2\right)}-\frac{a^2\left(2a^2-3b^2\right)\tan(d+ex}{be\left(a^2\sec(d+ex)+ab\right)}}{2a^3b(a^2-b^2)}\right.}{(a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2)^{3/2}}$$

3.525. $\displaystyle\int\frac{a+b\sec(d+ex)}{\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^{3/2}}\,dx$

$$\Big\downarrow \text{218}$$

$$\frac{\left(a^2\sec(d+ex)+ab\right)^3\left(\frac{\frac{\frac{2a^4x\left(a^2-b^2\right)^2}{b}-\frac{2a^3\left(a^2-b^2\right)\left(2a^4-3a^2b^2+2b^4\right)\arctan\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{be\sqrt{a-b}\sqrt{a+b}}}{a^3b\left(a^2-b^2\right)}-\frac{a^2\left(2a^2-3b^2\right)\tan(d+ex)}{be\left(a^2\sec(d+ex)+ab\right)}}{2a^3b(a^2-b^2)}-\frac{\tan}{2be(a^2\sec}\right.}{(a^2\sec^2(d+ex)+2ab\sec(d+ex)+b^2)^{3/2}}$$

input
```
Int[(a + b*Sec[d + e*x])/(b^2 + 2*a*b*Sec[d + e*x] + a^2*Sec[d + e*x]^2)^(
3/2),x]
```

output
```
((a*b + a^2*Sec[d + e*x])^3*(-1/2*Tan[d + e*x]/(b*e*(a*b + a^2*Sec[d + e*x
])^2) + (((2*a^4*(a^2 - b^2)^2*x)/b - (2*a^3*(a^2 - b^2)*(2*a^4 - 3*a^2*b^
2 + 2*b^4)*ArcTan[(Sqrt[a - b]*Tan[(d + e*x)/2])/Sqrt[a + b]])/(Sqrt[a - b
]*b*Sqrt[a + b]*e))/(a^3*b*(a^2 - b^2)) - (a^2*(2*a^2 - 3*b^2)*Tan[d + e*x
])/(b*e*(a*b + a^2*Sec[d + e*x])))/(2*a^3*b*(a^2 - b^2))))/(b^2 + 2*a*b*Se
c[d + e*x] + a^2*Sec[d + e*x]^2)^(3/2)
```

3.525.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

3.525. $\int \frac{a+b\sec(d+ex)}{(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex))^{3/2}}\,dx$

rule 4318
```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[1/b   Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

rule 4407
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Simp[(b*c - a*d)/a   Int[Csc[e + f*
x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0]
```

rule 4411
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[b*(b*c - a*d)*Cot[e + f*x]*((a + b*Csc[e + f
*x])^(m + 1)/(a*f*(m + 1)*(a^2 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2 - b^2)
)   Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
 a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && N
eQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

rule 4548
```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Simp[1/(a*(m + 1)*(a^2 - b^2))   Int[(a + b*Csc[e + f*x])^(
m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x
] + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

rule 4662
```
Int[((A_) + (B_.)*sec[(d_.) + (e_.)*(x_)])*((a_) + (b_.)*sec[(d_.) + (e_.)*
(x_)] + (c_.)*sec[(d_.) + (e_.)*(x_)]^2)^(n_), x_Symbol] :> Simp[(a + b*Sec
[d + e*x] + c*Sec[d + e*x]^2)^n/(b + 2*c*Sec[d + e*x])^(2*n)   Int[(A + B*S
ec[d + e*x])*(b + 2*c*Sec[d + e*x])^(2*n), x], x] /; FreeQ[{a, b, c, d, e,
A, B}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[n]
```

3.525. $\int \frac{a+b\sec(d+ex)}{\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^{3/2}}\,dx$

3.525.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.60 (sec) , antiderivative size = 678, normalized size of antiderivative = 2.05

method	result
risch	$\frac{\left(b\,\mathrm{e}^{2i(ex+d)}+2a\,\mathrm{e}^{i(ex+d)}+b\right)ax}{\left(1+\mathrm{e}^{2i(ex+d)}\right)\sqrt{\frac{\left(b\,\mathrm{e}^{2i(ex+d)}+2a\,\mathrm{e}^{i(ex+d)}+b\right)^2}{\left(1+\mathrm{e}^{2i(ex+d)}\right)^2}}\,b^3} - \frac{ia\left(4a^3b\,\mathrm{e}^{3i(ex+d)}-5a\,b^3\mathrm{e}^{3i(ex+d)}+6a^4\mathrm{e}^{2i(ex+d)}-5a^2b^2\mathrm{e}^{2i(ex+d)}-4b^4\mathrm{e}^{2i(ex+d)}\right.}{\left(1+\mathrm{e}^{2i(ex+d)}\right)\left(b\,\mathrm{e}^{2i(ex+d)}+2a\,\mathrm{e}^{i(ex+d)}+b\right)\sqrt{\frac{\left(b\,\mathrm{e}^{2i(ex+d)}+\right.}{\left(1+\mathrm{e}^{2i}\right.}}}$
default	$-\frac{2\sin(ex+d)\tan(ex+d)\sqrt{(a+b)(a-b)}\,a^3b^2(ex+d)+2\sqrt{(a+b)(a-b)}\,\cos(ex+d)a\,b^4(ex+d)+4\arctan\left(\frac{(-\cot(ex+d)+\csc(ex+d))(a-b)}{\sqrt{(a+b)(a-b)}}\right.}{}$
parts	Expression too large to display

input
```
int((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(3/2),x,metho
d=_RETURNVERBOSE)
```

output
```
1/(1+exp(2*I*(e*x+d)))*(b*exp(2*I*(e*x+d))+2*a*exp(I*(e*x+d))+b)/((b*exp(2
*I*(e*x+d))+2*a*exp(I*(e*x+d))+b)^2/(1+exp(2*I*(e*x+d)))^2)^(1/2)*a*x/b^3-
I/(1+exp(2*I*(e*x+d)))/(b*exp(2*I*(e*x+d))+2*a*exp(I*(e*x+d))+b)/((b*exp(2
*I*(e*x+d))+2*a*exp(I*(e*x+d))+b)^2/(1+exp(2*I*(e*x+d)))^2)^(1/2)*a/b^3*(4
*a^3*b*exp(3*I*(e*x+d))-5*a*b^3*exp(3*I*(e*x+d))+6*a^4*exp(2*I*(e*x+d))-5*
a^2*b^2*exp(2*I*(e*x+d))-4*b^4*exp(2*I*(e*x+d))+8*a^3*b*exp(I*(e*x+d))-11*
a*b^3*exp(I*(e*x+d))+3*a^2*b^2-4*b^4)/(a^2-b^2)/e-1/2/(1+exp(2*I*(e*x+d)))
*(b*exp(2*I*(e*x+d))+2*a*exp(I*(e*x+d))+b)/((b*exp(2*I*(e*x+d))+2*a*exp(I*
(e*x+d))+b)^2/(1+exp(2*I*(e*x+d)))^2)^(1/2)/(-a^2+b^2)^(1/2)*(2*a^4-3*a^2*
b^2+2*b^4)/(a+b)/(a-b)/e/b^3*ln(exp(I*(e*x+d))+(-I*a^2+I*b^2+(-a^2+b^2)^(1
/2)*a)/(-a^2+b^2)^(1/2)/b)+1/2/(1+exp(2*I*(e*x+d)))*(b*exp(2*I*(e*x+d))+2*
a*exp(I*(e*x+d))+b)/((b*exp(2*I*(e*x+d))+2*a*exp(I*(e*x+d))+b)^2/(1+exp(2*
I*(e*x+d)))^2)^(1/2)/(-a^2+b^2)^(1/2)*(2*a^4-3*a^2*b^2+2*b^4)/(a+b)/(a-b)/
e/b^3*ln(exp(I*(e*x+d))+(I*a^2-I*b^2+(-a^2+b^2)^(1/2)*a)/(-a^2+b^2)^(1/2)/
b)
```

3.525.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 798, normalized size of antiderivative = 2.42

$$\int \frac{a+b\sec(d+ex)}{(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex))^{3/2}}\,dx = \left[\frac{4\,(a^5b^2-2\,a^3b^4+ab^6)ex\cos{(ex+d)}^2+8\,(a^6b-2\,a^4b\right.$$

input
```
integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(3/2),x
, algorithm="fricas")
```

output
```
[1/4*(4*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*e*x*cos(e*x + d)^2 + 8*(a^6*b - 2*a^
4*b^3 + a^2*b^5)*e*x*cos(e*x + d) + 4*(a^7 - 2*a^5*b^2 + a^3*b^4)*e*x + (2
*a^6 - 3*a^4*b^2 + 2*a^2*b^4 + (2*a^4*b^2 - 3*a^2*b^4 + 2*b^6)*cos(e*x + d
)^2 + 2*(2*a^5*b - 3*a^3*b^3 + 2*a*b^5)*cos(e*x + d))*sqrt(-a^2 + b^2)*log
((2*a*b*cos(e*x + d) + (2*a^2 - b^2)*cos(e*x + d)^2 + 2*sqrt(-a^2 + b^2)*(
a*cos(e*x + d) + b)*sin(e*x + d) - a^2 + 2*b^2)/(b^2*cos(e*x + d)^2 + 2*a*
b*cos(e*x + d) + a^2)) - 2*(2*a^6*b - 5*a^4*b^3 + 3*a^2*b^5 + (3*a^5*b^2 -
 7*a^3*b^4 + 4*a*b^6)*cos(e*x + d))*sin(e*x + d))/((a^4*b^5 - 2*a^2*b^7 +
b^9)*e*cos(e*x + d)^2 + 2*(a^5*b^4 - 2*a^3*b^6 + a*b^8)*e*cos(e*x + d) + (
a^6*b^3 - 2*a^4*b^5 + a^2*b^7)*e), 1/2*(2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*e*
x*cos(e*x + d)^2 + 4*(a^6*b - 2*a^4*b^3 + a^2*b^5)*e*x*cos(e*x + d) + 2*(a
^7 - 2*a^5*b^2 + a^3*b^4)*e*x - (2*a^6 - 3*a^4*b^2 + 2*a^2*b^4 + (2*a^4*b^
2 - 3*a^2*b^4 + 2*b^6)*cos(e*x + d)^2 + 2*(2*a^5*b - 3*a^3*b^3 + 2*a*b^5)*
cos(e*x + d))*sqrt(a^2 - b^2)*arctan(-(a*cos(e*x + d) + b)/(sqrt(a^2 - b^2
)*sin(e*x + d))) - (2*a^6*b - 5*a^4*b^3 + 3*a^2*b^5 + (3*a^5*b^2 - 7*a^3*b
^4 + 4*a*b^6)*cos(e*x + d))*sin(e*x + d))/((a^4*b^5 - 2*a^2*b^7 + b^9)*e*c
os(e*x + d)^2 + 2*(a^5*b^4 - 2*a^3*b^6 + a*b^8)*e*cos(e*x + d) + (a^6*b^3
- 2*a^4*b^5 + a^2*b^7)*e)]
```

3.525.6 Sympy [F]

$$\int \frac{a+b\sec(d+ex)}{(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex))^{3/2}}\,dx = \int \frac{a+b\sec{(d+ex)}}{\left((a\sec{(d+ex)}+b)^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*sec(e*x+d))/(b**2+2*a*b*sec(e*x+d)+a**2*sec(e*x+d)**2)**(3/
2),x)
```

output
```
Integral((a + b*sec(d + e*x))/((a*sec(d + e*x) + b)**2)**(3/2), x)
```

3.525.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\sec(d+ex)}{\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(3/2),x
, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.525.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.23

$$\int \frac{a+b\sec(d+ex)}{\left(b^2+2ab\sec(d+ex)+a^2\sec^2(d+ex)\right)^{3/2}}\,dx =$$

$$-\frac{\frac{(2a^4-3a^2b^2+2b^4)\left(\pi\left\lfloor\frac{ex+d}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)-b\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)}{\sqrt{a^2-b^2}}\right)\right)}{\left(a^2b^3\mathrm{sgn}\left(b\cos(ex+d)^2+a\cos(ex+d)\right)-b^5\mathrm{sgn}\left(b\cos(ex+d)^2+a\cos(ex+d)\right)\right)\sqrt{a^2-b^2}}+\frac{2a^4\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^3-3a^3b\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^3-\cdots}{\left(a^2b^2\mathrm{sgn}\left(b\cos(e\cdots\right.\right.}}{}$$

input
```
integrate((a+b*sec(e*x+d))/(b^2+2*a*b*sec(e*x+d)+a^2*sec(e*x+d)^2)^(3/2),x
, algorithm="giac")
```

output
```
-((2*a^4 - 3*a^2*b^2 + 2*b^4)*(pi*floor(1/2*(e*x + d)/pi + 1/2)*sgn(2*a -
2*b) + arctan((a*tan(1/2*e*x + 1/2*d) - b*tan(1/2*e*x + 1/2*d))/sqrt(a^2 -
 b^2)))/((a^2*b^3*sgn(b*cos(e*x + d)^2 + a*cos(e*x + d)) - b^5*sgn(b*cos(e
*x + d)^2 + a*cos(e*x + d)))*sqrt(a^2 - b^2)) + (2*a^4*tan(1/2*e*x + 1/2*d
)^3 - 3*a^3*b*tan(1/2*e*x + 1/2*d)^3 - 3*a^2*b^2*tan(1/2*e*x + 1/2*d)^3 +
4*a*b^3*tan(1/2*e*x + 1/2*d)^3 + 2*a^4*tan(1/2*e*x + 1/2*d) + 3*a^3*b*tan(
1/2*e*x + 1/2*d) - 3*a^2*b^2*tan(1/2*e*x + 1/2*d) - 4*a*b^3*tan(1/2*e*x +
1/2*d))/((a^2*b^2*sgn(b*cos(e*x + d)^2 + a*cos(e*x + d)) - b^4*sgn(b*cos(e
*x + d)^2 + a*cos(e*x + d)))*(a*tan(1/2*e*x + 1/2*d)^2 - b*tan(1/2*e*x + 1
/2*d)^2 + a + b)^2) - (e*x + d)*a/(b^3*sgn(b*cos(e*x + d)^2 + a*cos(e*x +
d))))/e
```

3.525.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\sec(d + ex)}{(b^2 + 2ab\sec(d + ex) + a^2\sec^2(d + ex))^{3/2}}\,dx = \int \frac{a + \frac{b}{\cos(d+e\,x)}}{\left(b^2 + \frac{a^2}{\cos(d+e\,x)^2} + \frac{2\,a\,b}{\cos(d+e\,x)}\right)^{3/2}}\,dx$$

input
```
int((a + b/cos(d + e*x))/(b^2 + a^2/cos(d + e*x)^2 + (2*a*b)/cos(d + e*x))
^(3/2),x)
```

output
```
int((a + b/cos(d + e*x))/(b^2 + a^2/cos(d + e*x)^2 + (2*a*b)/cos(d + e*x))
^(3/2), x)
```

3.526 $\int \frac{\cos(x)-i\sin(x)}{\cos(x)+i\sin(x)}\,dx$

3.526.1 Optimal result

Integrand size = 21, antiderivative size = 17

$$\int \frac{\cos(x)-i\sin(x)}{\cos(x)+i\sin(x)}\,dx = \frac{1}{2}i(\cos(x)-i\sin(x))^2$$

output
```
1/2*I*(cos(x)-I*sin(x))^2
```

3.526.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{\cos(x)-i\sin(x)}{\cos(x)+i\sin(x)}\,dx = \frac{1}{2}i\cos(2x)+\frac{1}{2}\sin(2x)$$

input
```
Integrate[(Cos[x] - I*Sin[x])/(Cos[x] + I*Sin[x]),x]
```

output
```
(I/2)*Cos[2*x] + Sin[2*x]/2
```

3.526.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3042, 4885}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(x)-i\sin(x)}{\cos(x)+i\sin(x)}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\cos(x)-i\sin(x)}{\cos(x)+i\sin(x)}dx$$
$$\Big\downarrow \text{ 4885}$$
$$\frac{1}{2}i(\cos(x)-i\sin(x))^2$$

input
```
Int[(Cos[x] - I*Sin[x])/(Cos[x] + I*Sin[x]),x]
```

output
```
(I/2)*(Cos[x] - I*Sin[x])^2
```

3.526.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4885
```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[ActivateTrig[
y], ActivateTrig[u], x]}, Simp[q*(ActivateTrig[y^(m + 1)]/(m + 1)), x] /;
!FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1] &&  !InertTrigFreeQ[u]
```

3.526.4 Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.47

method	result	size
default	$\frac{1}{\tan(x)-i}$	8
risch	$\frac{i\mathrm{e}^{-2ix}}{2}$	9
norman	$\frac{-4i\tan\left(\frac{x}{2}\right)^2-2\tan\left(\frac{x}{2}\right)^3+2\tan\left(\frac{x}{2}\right)}{\left(1+\tan\left(\frac{x}{2}\right)^2\right)^2}$	36

input
```
int((cos(x)-I*sin(x))/(cos(x)+I*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
1/(tan(x)-I)
```

3.526.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.35

$$\int \frac{\cos(x)-i\sin(x)}{\cos(x)+i\sin(x)}\,dx = \frac{1}{2}i\,e^{(-2i\,x)}$$

input
```
integrate((cos(x)-I*sin(x))/(cos(x)+I*sin(x)),x, algorithm="fricas")
```

output
```
1/2*I*e^(-2*I*x)
```

3.526.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.47

$$\int \frac{\cos(x)-i\sin(x)}{\cos(x)+i\sin(x)}\,dx = \frac{ie^{-2ix}}{2}$$

input
```
integrate((cos(x)-I*sin(x))/(cos(x)+I*sin(x)),x)
```

output
```
I*exp(-2*I*x)/2
```

3.526.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos(x) - i\sin(x)}{\cos(x) + i\sin(x)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((cos(x)-I*sin(x))/(cos(x)+I*sin(x)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.526.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{\cos(x) - i\sin(x)}{\cos(x) + i\sin(x)}\,dx = -\frac{2\tan\left(\frac{1}{2}x\right)}{\left(\tan\left(\frac{1}{2}x\right) - i\right)^2}$$

input
```
integrate((cos(x)-I*sin(x))/(cos(x)+I*sin(x)),x, algorithm="giac")
```

output
```
-2*tan(1/2*x)/(tan(1/2*x) - I)^2
```

3.526.9 Mupad [B] (verification not implemented)

Time = 26.94 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{\cos(x) - i\sin(x)}{\cos(x) + i\sin(x)}\,dx = -\frac{\cos(x)}{-\sin(x) + \cos(x)\ 1\mathrm{i}}$$

input
```
int((cos(x) - sin(x)*1i)/(cos(x) + sin(x)*1i),x)
```

output
```
-cos(x)/(cos(x)*1i - sin(x))
```

3.527 $\int \frac{\cos(x)+i\sin(x)}{\cos(x)-i\sin(x)}\,dx$

3.527.1 Optimal result

Integrand size = 21, antiderivative size = 17

$$\int \frac{\cos(x)+i\sin(x)}{\cos(x)-i\sin(x)}\,dx=-\frac{i}{2(\cos(x)-i\sin(x))^2}$$

output
```
-1/2*I/(cos(x)-I*sin(x))^2
```

3.527.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{\cos(x)+i\sin(x)}{\cos(x)-i\sin(x)}\,dx=-\frac{1}{2}i\cos(2x)+\frac{1}{2}\sin(2x)$$

input
```
Integrate[(Cos[x] + I*Sin[x])/(Cos[x] - I*Sin[x]),x]
```

output
```
(-1/2*I)*Cos[2*x] + Sin[2*x]/2
```

3.527.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3042, 4885}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(x)+i\sin(x)}{\cos(x)-i\sin(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(x)+i\sin(x)}{\cos(x)-i\sin(x)}dx$$

$$\Big\downarrow \text{4885}$$

$$-\frac{i}{2(\cos(x)-i\sin(x))^2}$$

input
```
Int[(Cos[x] + I*Sin[x])/(Cos[x] - I*Sin[x]),x]
```

output
```
(-1/2*I)/(Cos[x] - I*Sin[x])^2
```

3.527.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4885
```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[ActivateTrig[
y], ActivateTrig[u], x]}, Simp[q*(ActivateTrig[y^(m + 1)]/(m + 1)), x] /;
!FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1] && !InertTrigFreeQ[u]
```

3.527.4 Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.47

method	result	size
default	$\frac{1}{\tan(x)+i}$	8
risch	$-\frac{i e^{2ix}}{2}$	9
norman	$\frac{4i\tan\left(\frac{x}{2}\right)^{2}-2\tan\left(\frac{x}{2}\right)^{3}+2\tan\left(\frac{x}{2}\right)}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)^{2}}$	36

input
```
int((cos(x)+I*sin(x))/(cos(x)-I*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
1/(tan(x)+I)
```

3.527.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.35

$$\int \frac{\cos(x)+i\sin(x)}{\cos(x)-i\sin(x)}\,dx = -\frac{1}{2} i\, e^{(2i\,x)}$$

input
```
integrate((cos(x)+I*sin(x))/(cos(x)-I*sin(x)),x, algorithm="fricas")
```

output
```
-1/2*I*e^(2*I*x)
```

3.527.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.59

$$\int \frac{\cos(x)+i\sin(x)}{\cos(x)-i\sin(x)}\,dx = -\frac{ie^{2ix}}{2}$$

input
```
integrate((cos(x)+I*sin(x))/(cos(x)-I*sin(x)),x)
```

output
```
-I*exp(2*I*x)/2
```

3.527.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos(x) + i\sin(x)}{\cos(x) - i\sin(x)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((cos(x)+I*sin(x))/(cos(x)-I*sin(x)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.527.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{\cos(x) + i\sin(x)}{\cos(x) - i\sin(x)}\,dx = -\frac{2\tan\left(\frac{1}{2}x\right)}{\left(\tan\left(\frac{1}{2}x\right) + i\right)^2}$$

input
```
integrate((cos(x)+I*sin(x))/(cos(x)-I*sin(x)),x, algorithm="giac")
```

output
```
-2*tan(1/2*x)/(tan(1/2*x) + I)^2
```

3.527.9 Mupad [B] (verification not implemented)

Time = 26.71 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \frac{\cos(x) + i\sin(x)}{\cos(x) - i\sin(x)}\,dx = \frac{\sin(x)}{\cos(x) - \sin(x)\ 1\mathrm{i}}$$

input
```
int((cos(x) + sin(x)*1i)/(cos(x) - sin(x)*1i),x)
```

output
```
sin(x)/(cos(x) - sin(x)*1i)
```

3.528 $\int \frac{\cos(x)-\sin(x)}{\cos(x)+\sin(x)}\,dx$

3.528.1 Optimal result

Integrand size = 15, antiderivative size = 6

$$\int \frac{\cos(x)-\sin(x)}{\cos(x)+\sin(x)}\,dx = \log(\cos(x)+\sin(x))$$

output
```
ln(cos(x)+sin(x))
```

3.528.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)-\sin(x)}{\cos(x)+\sin(x)}\,dx = \log(\cos(x)+\sin(x))$$

input
```
Integrate[(Cos[x] - Sin[x])/(Cos[x] + Sin[x]),x]
```

output
```
Log[Cos[x] + Sin[x]]
```

3.528.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3042, 3612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(x)-\sin(x)}{\sin(x)+\cos(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(x)-\sin(x)}{\sin(x)+\cos(x)}dx$$

$$\Big\downarrow \text{3612}$$

$$\log(\sin(x)+\cos(x))$$

input
```
Int[(Cos[x] - Sin[x])/(Cos[x] + Sin[x]),x]
```

output
```
Log[Cos[x] + Sin[x]]
```

3.528.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3612
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

3.528.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

method	result	size
derivativedivides	$\ln(\sin(x)+\cos(x))$	7
default	$\ln(\sin(x)+\cos(x))$	7
risch	$-ix+\ln(i+e^{2ix})$	15
norman	$-\ln\left(1+\tan\left(\frac{x}{2}\right)^2\right)+\ln\left(\tan\left(\frac{x}{2}\right)^2-2\tan\left(\frac{x}{2}\right)-1\right)$	28
parallelrisch	$-\ln\left(\frac{1}{\cos(x)+1}\right)+\ln\left(\frac{-\cos(x)-\sin(x)}{\cos(x)+1}\right)$	28

input
```
int((cos(x)-sin(x))/(sin(x)+cos(x)),x,method=_RETURNVERBOSE)
```

output
```
ln(sin(x)+cos(x))
```

3.528.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.83

$$\int \frac{\cos(x)-\sin(x)}{\cos(x)+\sin(x)}\,dx = \frac{1}{2}\log(2\cos(x)\sin(x)+1)$$

input
```
integrate((cos(x)-sin(x))/(cos(x)+sin(x)),x, algorithm="fricas")
```

output
```
1/2*log(2*cos(x)*sin(x) + 1)
```

3.528.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

$$\int \frac{\cos(x)-\sin(x)}{\cos(x)+\sin(x)}\,dx = \log(\sin(x)+\cos(x))$$

input
```
integrate((cos(x)-sin(x))/(cos(x)+sin(x)),x)
```

output
```
log(sin(x) + cos(x))
```

3.528.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x) - \sin(x)}{\cos(x) + \sin(x)}\, dx = \log(\cos(x) + \sin(x))$$

input
```
integrate((cos(x)-sin(x))/(cos(x)+sin(x)),x, algorithm="maxima")
```

output
```
log(cos(x) + sin(x))
```

3.528.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. 2(6) = 12.

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.67

$$\int \frac{\cos(x) - \sin(x)}{\cos(x) + \sin(x)}\, dx = -\frac{1}{2}\log\left(\tan(x)^2 + 1\right) + \log(|\tan(x) + 1|)$$

input
```
integrate((cos(x)-sin(x))/(cos(x)+sin(x)),x, algorithm="giac")
```

output
```
-1/2*log(tan(x)^2 + 1) + log(abs(tan(x) + 1))
```

3.528.9 Mupad [B] (verification not implemented)

Time = 26.93 (sec) , antiderivative size = 32, normalized size of antiderivative = 5.33

$$\int \frac{\cos(x) - \sin(x)}{\cos(x) + \sin(x)}\, dx = 2\,\mathrm{atanh}\left(\frac{128\tan\left(\frac{x}{2}\right) + 128}{16\tan\left(\frac{x}{2}\right)^2 + 32\tan\left(\frac{x}{2}\right) + 48} - 3\right)$$

input
```
int((cos(x) - sin(x))/(cos(x) + sin(x)),x)
```

output
```
2*atanh((128*tan(x/2) + 128)/(32*tan(x/2) + 16*tan(x/2)^2 + 48) - 3)
```

3.529 $\int \frac{B\cos(x)+C\sin(x)}{b\cos(x)+c\sin(x)}\,dx$

3.529.1 Optimal result

Integrand size = 21, antiderivative size = 47

$$\int \frac{B\cos(x)+C\sin(x)}{b\cos(x)+c\sin(x)}\,dx=\frac{(bB+cC)x}{b^2+c^2}+\frac{(Bc-bC)\log(b\cos(x)+c\sin(x))}{b^2+c^2}$$

output
```
(B*b+C*c)*x/(b^2+c^2)+(B*c-C*b)*ln(b*cos(x)+c*sin(x))/(b^2+c^2)
```

3.529.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \frac{B\cos(x)+C\sin(x)}{b\cos(x)+c\sin(x)}\,dx=\frac{(bB+cC)x+(Bc-bC)\log(b\cos(x)+c\sin(x))}{b^2+c^2}$$

input
```
Integrate[(B*Cos[x] + C*Sin[x])/(b*Cos[x] + c*Sin[x]),x]
```

output
```
((b*B + c*C)*x + (B*c - b*C)*Log[b*Cos[x] + c*Sin[x]])/(b^2 + c^2)
```

3.529.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3042, 3612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\cos(x)+C\sin(x)}{b\cos(x)+c\sin(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{B\cos(x)+C\sin(x)}{b\cos(x)+c\sin(x)}dx$$

$$\Big\downarrow \text{3612}$$

$$\frac{x(bB+cC)}{b^2+c^2}+\frac{(Bc-bC)\log(b\cos(x)+c\sin(x))}{b^2+c^2}$$

input
```
Int[(B*Cos[x] + C*Sin[x])/(b*Cos[x] + c*Sin[x]),x]
```

output
```
((b*B + c*C)*x)/(b^2 + c^2) + ((B*c - b*C)*Log[b*Cos[x] + c*Sin[x]])/(b^2
+ c^2)
```

3.529.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3612
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

3.529.4 Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.40

method	result	size
default	$\frac{\frac{(-Bc+bC)\ln\left(1+\tan(x)^2\right)}{2}+(Bb+Cc)\arctan(\tan(x))}{b^2+c^2}+\frac{(Bc-bC)\ln(c\tan(x)+b)}{b^2+c^2}$	66
parallelrisch	$\frac{-Bc\ln\left(\frac{1}{\cos(x)+1}\right)+Bc\ln\left(\frac{-b\cos(x)-c\sin(x)}{\cos(x)+1}\right)+Bxb+bC\ln\left(\frac{1}{\cos(x)+1}\right)-bC\ln\left(\frac{-b\cos(x)-c\sin(x)}{\cos(x)+1}\right)+Cxc}{b^2+c^2}$	86
norman	$\frac{\frac{(Bb+Cc)x}{b^2+c^2}+\frac{(Bb+Cc)x\tan\left(\frac{x}{2}\right)^2}{b^2+c^2}}{1+\tan\left(\frac{x}{2}\right)^2}+\frac{(Bc-bC)\ln\left(\tan\left(\frac{x}{2}\right)^2b-2c\tan\left(\frac{x}{2}\right)-b\right)}{b^2+c^2}-\frac{(Bc-bC)\ln\left(1+\tan\left(\frac{x}{2}\right)^2\right)}{b^2+c^2}$	122
risch	$\frac{ixC}{ic-b}-\frac{Bx}{ic-b}-\frac{2ixBc}{b^2+c^2}+\frac{2ixbC}{b^2+c^2}+\frac{\ln\left(e^{2ix}-\frac{ic+b}{ic-b}\right)Bc}{b^2+c^2}-\frac{\ln\left(e^{2ix}-\frac{ic+b}{ic-b}\right)bC}{b^2+c^2}$	136

input
```
int((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
1/(b^2+c^2)*(1/2*(-B*c+C*b)*ln(1+tan(x)^2)+(B*b+C*c)*arctan(tan(x)))+(B*c-
C*b)/(b^2+c^2)*ln(c*tan(x)+b)
```

3.529.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.26

$$\int \frac{B\cos(x)+C\sin(x)}{b\cos(x)+c\sin(x)}\,dx = \frac{2\,(Bb+Cc)x-(Cb-Bc)\log\left(2\,bc\cos{(x)}\sin{(x)}+(b^2-c^2)\cos{(x)}^2+c^2\right)}{2\,(b^2+c^2)}$$

input
```
integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x)),x, algorithm="fricas")
```

output
```
1/2*(2*(B*b + C*c)*x - (C*b - B*c)*log(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*c
os(x)^2 + c^2))/(b^2 + c^2)
```

3.529.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 360, normalized size of antiderivative = 7.66

$$\int \frac{B\cos(x)+C\sin(x)}{b\cos(x)+c\sin(x)}\,dx$$

$$= \begin{cases} \tilde{\infty}(B\log(\sin(x))+Cx) \\ \frac{Bx-C\log(\cos(x))}{b} \\ -\frac{Bx\sin(x)}{2ic\sin(x)+2c\cos(x)}+\frac{iBx\cos(x)}{2ic\sin(x)+2c\cos(x)}+\frac{iB\sin(x)}{2ic\sin(x)+2c\cos(x)}+\frac{iCx\sin(x)}{2ic\sin(x)+2c\cos(x)}+\frac{Cx\cos(x)}{2ic\sin(x)+2c\cos(x)}-\frac{C\sin}{2ic\sin(x)+} \\ -\frac{Bx\sin(x)}{-2ic\sin(x)+2c\cos(x)}-\frac{iBx\cos(x)}{-2ic\sin(x)+2c\cos(x)}-\frac{iB\sin(x)}{-2ic\sin(x)+2c\cos(x)}-\frac{iCx\sin(x)}{-2ic\sin(x)+2c\cos(x)}+\frac{Cx\cos(x)}{-2ic\sin(x)+2c\cos(x)}- \\ \frac{Bbx}{b^2+c^2}+\frac{Bc\log\left(\frac{b\cos(x)}{c}+\sin(x)\right)}{b^2+c^2}-\frac{Cb\log\left(\frac{b\cos(x)}{c}+\sin(x)\right)}{b^2+c^2}+\frac{Ccx}{b^2+c^2} \end{cases}$$

input
```
integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x)),x)
```

output
```
Piecewise((zoo*(B*log(sin(x)) + C*x), Eq(b, 0) & Eq(c, 0)), ((B*x - C*log(
cos(x)))/b, Eq(c, 0)), (-B*x*sin(x)/(2*I*c*sin(x) + 2*c*cos(x)) + I*B*x*co
s(x)/(2*I*c*sin(x) + 2*c*cos(x)) + I*B*sin(x)/(2*I*c*sin(x) + 2*c*cos(x))
+ I*C*x*sin(x)/(2*I*c*sin(x) + 2*c*cos(x)) + C*x*cos(x)/(2*I*c*sin(x) + 2*
c*cos(x)) - C*sin(x)/(2*I*c*sin(x) + 2*c*cos(x)), Eq(b, -I*c)), (-B*x*sin(
x)/(-2*I*c*sin(x) + 2*c*cos(x)) - I*B*x*cos(x)/(-2*I*c*sin(x) + 2*c*cos(x)
) - I*B*sin(x)/(-2*I*c*sin(x) + 2*c*cos(x)) - I*C*x*sin(x)/(-2*I*c*sin(x)
+ 2*c*cos(x)) + C*x*cos(x)/(-2*I*c*sin(x) + 2*c*cos(x)) - C*sin(x)/(-2*I*c
*sin(x) + 2*c*cos(x)), Eq(b, I*c)), (B*b*x/(b**2 + c**2) + B*c*log(b*cos(x
)/c + sin(x))/(b**2 + c**2) - C*b*log(b*cos(x)/c + sin(x))/(b**2 + c**2) +
 C*c*x/(b**2 + c**2), True))
```

3.529.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(48) = 96$.

Time = 0.32 (sec) , antiderivative size = 181, normalized size of antiderivative = 3.85

$$\int \frac{B\cos(x) + C\sin(x)}{b\cos(x) + c\sin(x)}\,dx$$
$$= B\left(\frac{2\,b\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{b^2+c^2} + \frac{c\log\left(-b - \frac{2\,c\sin(x)}{\cos(x)+1} + \frac{b\sin(x)^2}{(\cos(x)+1)^2}\right)}{b^2+c^2} - \frac{c\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{b^2+c^2}\right)$$
$$+ C\left(\frac{2\,c\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{b^2+c^2} - \frac{b\log\left(-b - \frac{2\,c\sin(x)}{\cos(x)+1} + \frac{b\sin(x)^2}{(\cos(x)+1)^2}\right)}{b^2+c^2} + \frac{b\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{b^2+c^2}\right)$$

input
```
integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x)),x, algorithm="maxima")
```

output
```
B*(2*b*arctan(sin(x)/(cos(x) + 1))/(b^2 + c^2) + c*log(-b - 2*c*sin(x)/(co
s(x) + 1) + b*sin(x)^2/(cos(x) + 1)^2)/(b^2 + c^2) - c*log(sin(x)^2/(cos(x
) + 1)^2 + 1)/(b^2 + c^2)) + C*(2*c*arctan(sin(x)/(cos(x) + 1))/(b^2 + c^2
) - b*log(-b - 2*c*sin(x)/(cos(x) + 1) + b*sin(x)^2/(cos(x) + 1)^2)/(b^2 +
 c^2) + b*log(sin(x)^2/(cos(x) + 1)^2 + 1)/(b^2 + c^2))
```

3.529.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.64

$$\int \frac{B\cos(x) + C\sin(x)}{b\cos(x) + c\sin(x)}\,dx = \frac{(Bb+Cc)x}{b^2+c^2} + \frac{(Cb-Bc)\log\left(\tan(x)^2+1\right)}{2\,(b^2+c^2)} - \frac{(Cbc-Bc^2)\log\left(|c\tan(x)+b|\right)}{b^2c+c^3}$$

input
```
integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x)),x, algorithm="giac")
```

output
```
(B*b + C*c)*x/(b^2 + c^2) + 1/2*(C*b - B*c)*log(tan(x)^2 + 1)/(b^2 + c^2)
- (C*b*c - B*c^2)*log(abs(c*tan(x) + b))/(b^2*c + c^3)
```

3.529.9 Mupad [B] (verification not implemented)

Time = 38.64 (sec) , antiderivative size = 1976, normalized size of antiderivative = 42.04

$$\int \frac{B\cos(x) + C\sin(x)}{b\cos(x) + c\sin(x)}\, dx = \text{Too large to display}$$

input
```
int((B*cos(x) + C*sin(x))/(b*cos(x) + c*sin(x)),x)
```

output
```
(log(b + 2*c*tan(x/2) - b*tan(x/2)^2)*(B*c - C*b))/(b^2 + c^2) - (log(1/(c
os(x) + 1))*(2*B*c - 2*C*b))/(2*(b^2 + c^2)) + (2*atan(((((32*B*C^2*b^2 -
((2*B*c - 2*C*b)*(((2*B*c - 2*C*b)*(64*B*b^2*c^2 - 32*B*b^4 + 32*C*b*c^3 -
 64*C*b^3*c + ((2*B*c - 2*C*b)*(96*b^4*c + 96*b^2*c^3))/(2*(b^2 + c^2))))/
(2*(b^2 + c^2)) - 32*C^2*b^2*c - 32*B^2*b^2*c + 64*B*C*b^3 + 64*B*C*b*c^2)
)/(2*(b^2 + c^2)) + ((B*b + C*c)*(((B*b + C*c)*(64*B*b^2*c^2 - 32*B*b^4 +
32*C*b*c^3 - 64*C*b^3*c + ((2*B*c - 2*C*b)*(96*b^4*c + 96*b^2*c^3))/(2*(b^
2 + c^2))))/(b^2 + c^2) + ((B*b + C*c)*(2*B*c - 2*C*b)*(96*b^4*c + 96*b^2*
c^3))/(2*(b^2 + c^2)^2)))/(b^2 + c^2) - 32*B^2*C*b*c + ((B*b + C*c)^2*(2*B
*c - 2*C*b)*(96*b^4*c + 96*b^2*c^3))/(2*(b^2 + c^2)^3))*(12*B^2*b*c^3 - 6*
B^2*b^3*c - 6*C^2*b*c^3 + 12*C^2*b^3*c + 4*B*C*b^4 + 4*B*C*c^4 - 28*B*C*b^
2*c^2))/((b^2 + c^2)^2*(B^2*b^2 + 4*B^2*c^2 + 4*C^2*b^2 + C^2*c^2 - 6*B*C*
b*c)^2) - tan(x/2)*(((32*B^3*b*c - 32*B^2*C*b^2 - 64*C^3*b^2 + ((2*B*c - 2
*C*b)*(32*B^2*b^3 - 96*B^2*b*c^2 + 64*C^2*b*c^2 - ((2*B*c - 2*C*b)*(32*C*b
^2*c^2 - 64*C*b^4 + 32*B*b*c^3 + 128*B*b^3*c - ((2*B*c - 2*C*b)*(96*b*c^4
+ 96*b^3*c^2))/(2*(b^2 + c^2))))/(2*(b^2 + c^2)) + 192*B*C*b^2*c))/(2*(b^2
 + c^2)) + ((B*b + C*c)*(((B*b + C*c)*(32*C*b^2*c^2 - 64*C*b^4 + 32*B*b*c^
3 + 128*B*b^3*c - ((2*B*c - 2*C*b)*(96*b*c^4 + 96*b^3*c^2))/(2*(b^2 + c^2)
)))/(b^2 + c^2) - ((B*b + C*c)*(2*B*c - 2*C*b)*(96*b*c^4 + 96*b^3*c^2))/(2
*(b^2 + c^2)^2)))/(b^2 + c^2) + 64*B*C^2*b*c - ((B*b + C*c)^2*(2*B*c - ...
```

3.530 $\int \frac{B\cos(x)+C\sin(x)}{(b\cos(x)+c\sin(x))^2}\,dx$

3.530.1 Optimal result

Integrand size = 21, antiderivative size = 74

$$\int \frac{B\cos(x)+C\sin(x)}{(b\cos(x)+c\sin(x))^2}\,dx = -\frac{(bB+cC)\text{arctanh}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{3/2}} - \frac{Bc-bC}{(b^2+c^2)\,(b\cos(x)+c\sin(x))}$$

output
```
-(B*b+C*c)*arctanh((c*cos(x)-b*sin(x))/(b^2+c^2)^(1/2))/(b^2+c^2)^(3/2)+(-
B*c+C*b)/(b^2+c^2)/(b*cos(x)+c*sin(x))
```

3.530.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.01

$$\int \frac{B\cos(x)+C\sin(x)}{(b\cos(x)+c\sin(x))^2}\,dx = \frac{2(bB+cC)\text{arctanh}\left(\frac{-c+b\tan\left(\frac{x}{2}\right)}{\sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{3/2}} + \frac{-Bc+bC}{(b^2+c^2)\,(b\cos(x)+c\sin(x))}$$

input
```
Integrate[(B*Cos[x] + C*Sin[x])/(b*Cos[x] + c*Sin[x])^2,x]
```

output
```
(2*(b*B + c*C)*ArcTanh[(-c + b*Tan[x/2])/Sqrt[b^2 + c^2]])/(b^2 + c^2)^(3/
2) + (-(B*c) + b*C)/((b^2 + c^2)*(b*Cos[x] + c*Sin[x]))
```

3.530.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 3632, 3042, 3553, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\cos(x)+C\sin(x)}{(b\cos(x)+c\sin(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{B\cos(x)+C\sin(x)}{(b\cos(x)+c\sin(x))^2}dx$$

$$\Big\downarrow \text{3632}$$

$$\frac{(bB+cC)\int \frac{1}{b\cos(x)+c\sin(x)}dx}{b^2+c^2}-\frac{Bc-bC}{(b^2+c^2)(b\cos(x)+c\sin(x))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(bB+cC)\int \frac{1}{b\cos(x)+c\sin(x)}dx}{b^2+c^2}-\frac{Bc-bC}{(b^2+c^2)(b\cos(x)+c\sin(x))}$$

$$\Big\downarrow \text{3553}$$

$$-\frac{(bB+cC)\int \frac{1}{b^2+c^2-(c\cos(x)-b\sin(x))^2}d(c\cos(x)-b\sin(x))}{b^2+c^2}-\frac{Bc-bC}{(b^2+c^2)(b\cos(x)+c\sin(x))}$$

$$\Big\downarrow \text{219}$$

$$-\frac{(bB+cC)\text{arctanh}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{3/2}}-\frac{Bc-bC}{(b^2+c^2)(b\cos(x)+c\sin(x))}$$

input
```
Int[(B*Cos[x] + C*Sin[x])/(b*Cos[x] + c*Sin[x])^2,x]
```

output
```
-(((b*B + c*C)*ArcTanh[(c*Cos[x] - b*Sin[x])/Sqrt[b^2 + c^2]])/(b^2 + c^2)
^(3/2)) - (B*c - b*C)/((b^2 + c^2)*(b*Cos[x] + c*Sin[x]))
```

3.530.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3632
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Simp[(a*A - b*B - c*C)/(a^2 - b^2 - c^2)   Int[1/(a + b*Cos[d + e*x] + c*S
in[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
 - c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

3.530.4 Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.53

method	result
default	$-\frac{2\left(-\frac{c(Bc-bC)\tan\left(\frac{x}{2}\right)}{b\left(b^{2}+c^{2}\right)}-\frac{Bc-bC}{b^{2}+c^{2}}\right)}{\tan\left(\frac{x}{2}\right)^{2}b-2c\tan\left(\frac{x}{2}\right)-b}+\frac{2(Bb+Cc)\,\mathrm{arctanh}\left(\frac{2b\tan\left(\frac{x}{2}\right)-2c}{2\sqrt{b^{2}+c^{2}}}\right)}{\left(b^{2}+c^{2}\right)^{\frac{3}{2}}}$
risch	$-\frac{2\,\mathrm{e}^{ix}(Bc-bC)}{(ic+b)(-ic+b)(-ic\,\mathrm{e}^{2ix}+b\,\mathrm{e}^{2ix}+ic+b)}+\frac{bB\ln\left(\mathrm{e}^{ix}+\frac{ib^{3}+ib\,c^{2}-b^{2}c-c^{3}}{\left(b^{2}+c^{2}\right)^{\frac{3}{2}}}\right)}{\left(b^{2}+c^{2}\right)^{\frac{3}{2}}}+\frac{cC\ln\left(\mathrm{e}^{ix}+\frac{ib^{3}+ib\,c^{2}-b^{2}c-c^{3}}{\left(b^{2}+c^{2}\right)^{\frac{3}{2}}}\right)}{\left(b^{2}+c^{2}\right)^{\frac{3}{2}}}-\frac{bB\ln\left(\mathrm{e}^{ix}-i\cdots\right)}{\left(b^{2}\cdots\right.}$

input
```
int((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-2*(-c*(B*c-C*b)/b/(b^2+c^2)*tan(1/2*x)-(B*c-C*b)/(b^2+c^2))/(tan(1/2*x)^2
*b-2*c*tan(1/2*x)-b)+2*(B*b+C*c)/(b^2+c^2)^(3/2)*arctanh(1/2*(2*b*tan(1/2*
x)-2*c)/(b^2+c^2)^(1/2))
```

3.530.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. 2(69) = 138.

Time = 0.25 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.62

$$\int \frac{B\cos(x)+C\sin(x)}{(b\cos(x)+c\sin(x))^2}\,dx = \frac{2\,Cb^3-2\,Bb^2c+2\,Cbc^2-2\,Bc^3+\sqrt{b^2+c^2}((Bb^2+Cbc)\cos(x)+(Bbc+Cc^2)\sin(x))\log\left(-\frac{2\,bc\cos(x)\sin\cdots}{\cdots}\right.}{2\,((b^5+2\,b^3c^2+bc^4)\cos(x)+(b^4c+2\,b^2c^3+c^5)\sin(x))}$$

input
```
integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="fricas")
```

output
```
1/2*(2*C*b^3 - 2*B*b^2*c + 2*C*b*c^2 - 2*B*c^3 + sqrt(b^2 + c^2)*((B*b^2 +
 C*b*c)*cos(x) + (B*b*c + C*c^2)*sin(x))*log(-(2*b*c*cos(x)*sin(x) + (b^2
- c^2)*cos(x)^2 - 2*b^2 - c^2 + 2*sqrt(b^2 + c^2)*(c*cos(x) - b*sin(x)))/(
2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 + c^2)))/((b^5 + 2*b^3*c^2 + b*
c^4)*cos(x) + (b^4*c + 2*b^2*c^3 + c^5)*sin(x))
```

3.530.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{B\cos(x)+C\sin(x)}{(b\cos(x)+c\sin(x))^2}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))**2,x)
```

output
```
Exception raised: AttributeError >> 'NoneType' object has no attribute 'pr
imitive'
```

3.530.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(69) = 138$.

Time = 0.32 (sec) , antiderivative size = 271, normalized size of antiderivative = 3.66

$$\int \frac{B\cos(x) + C\sin(x)}{(b\cos(x) + c\sin(x))^2}\,dx = -B\left(\frac{b\log\left(\frac{c-\frac{b\sin(x)}{\cos(x)+1}+\sqrt{b^2+c^2}}{c-\frac{b\sin(x)}{\cos(x)+1}-\sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{\frac{3}{2}}} + \frac{2\left(bc+\frac{c^2\sin(x)}{\cos(x)+1}\right)}{b^4+b^2c^2+\frac{2(b^3c+bc^3)\sin(x)}{\cos(x)+1}-\frac{(b^4+b^2c^2)\sin(x)^2}{(\cos(x)+1)^2}}\right) - C\left(\frac{c\log\left(\frac{c-\frac{b\sin(x)}{\cos(x)+1}+\sqrt{b^2+c^2}}{c-\frac{b\sin(x)}{\cos(x)+1}-\sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{\frac{3}{2}}} - \frac{2\left(b+\frac{c\sin(x)}{\cos(x)+1}\right)}{b^3+bc^2+\frac{2(b^2c+c^3)\sin(x)}{\cos(x)+1}-\frac{(b^3+bc^2)\sin(x)^2}{(\cos(x)+1)^2}}\right)$$

input
```
integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="maxima")
```

output
```
-B*(b*log((c - b*sin(x)/(cos(x) + 1) + sqrt(b^2 + c^2))/(c - b*sin(x)/(cos
(x) + 1) - sqrt(b^2 + c^2)))/(b^2 + c^2)^(3/2) + 2*(b*c + c^2*sin(x)/(cos(
x) + 1))/(b^4 + b^2*c^2 + 2*(b^3*c + b*c^3)*sin(x)/(cos(x) + 1) - (b^4 + b
^2*c^2)*sin(x)^2/(cos(x) + 1)^2)) - C*(c*log((c - b*sin(x)/(cos(x) + 1) +
sqrt(b^2 + c^2))/(c - b*sin(x)/(cos(x) + 1) - sqrt(b^2 + c^2)))/(b^2 + c^2
)^(3/2) - 2*(b + c*sin(x)/(cos(x) + 1))/(b^3 + b*c^2 + 2*(b^2*c + c^3)*sin
(x)/(cos(x) + 1) - (b^3 + b*c^2)*sin(x)^2/(cos(x) + 1)^2))
```

3.530.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.78

$$\int \frac{B\cos(x) + C\sin(x)}{(b\cos(x) + c\sin(x))^2}\,dx = -\frac{(Bb+Cc)\log\left(\frac{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,c-2\,\sqrt{b^2+c^2}\right|}{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,c+2\,\sqrt{b^2+c^2}\right|}\right)}{(b^2+c^2)^{\frac{3}{2}}} - \frac{2\left(Cbc\tan\left(\frac{1}{2}\,x\right)-Bc^2\tan\left(\frac{1}{2}\,x\right)+Cb^2-Bbc\right)}{(b^3+bc^2)\left(b\tan\left(\frac{1}{2}\,x\right)^2-2\,c\tan\left(\frac{1}{2}\,x\right)-b\right)}$$

input
```
integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="giac")
```

output
```
-(B*b + C*c)*log(abs(2*b*tan(1/2*x) - 2*c - 2*sqrt(b^2 + c^2))/abs(2*b*tan
(1/2*x) - 2*c + 2*sqrt(b^2 + c^2)))/(b^2 + c^2)^(3/2) - 2*(C*b*c*tan(1/2*x
) - B*c^2*tan(1/2*x) + C*b^2 - B*b*c)/((b^3 + b*c^2)*(b*tan(1/2*x)^2 - 2*c
*tan(1/2*x) - b))
```

3.530.9 Mupad [B] (verification not implemented)

Time = 26.14 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.74

$$\int \frac{B\cos(x)+C\sin(x)}{(b\cos(x)+c\sin(x))^2}\,dx = -\frac{\frac{2\,(B\,c-C\,b)}{b^2+c^2}+\frac{2\,c\tan\left(\frac{x}{2}\right)\,(B\,c-C\,b)}{b\,(b^2+c^2)}}{-b\tan\left(\frac{x}{2}\right)^2+2\,c\tan\left(\frac{x}{2}\right)+b} + \frac{\mathrm{atan}\left(\frac{b^2\,c\,1\mathrm{i}+c^3\,1\mathrm{i}-b\tan\left(\frac{x}{2}\right)\,(b^2+c^2)\,1\mathrm{i}}{(b^2+c^2)^{3/2}}\right)\,(B\,b+C\,c)\,2\mathrm{i}}{(b^2+c^2)^{3/2}}$$

input
```
int((B*cos(x) + C*sin(x))/(b*cos(x) + c*sin(x))^2,x)
```

output
```
(atan((b^2*c*1i + c^3*1i - b*tan(x/2)*(b^2 + c^2)*1i)/(b^2 + c^2)^(3/2))*(
B*b + C*c)*2i)/(b^2 + c^2)^(3/2) - ((2*(B*c - C*b))/(b^2 + c^2) + (2*c*tan
(x/2)*(B*c - C*b))/(b*(b^2 + c^2)))/(b + 2*c*tan(x/2) - b*tan(x/2)^2)
```

3.531 $\int \frac{B\cos(x)+C\sin(x)}{(b\cos(x)+c\sin(x))^3}\,dx$

3.531.1 Optimal result

Integrand size = 21, antiderivative size = 66

$$\int \frac{B\cos(x)+C\sin(x)}{(b\cos(x)+c\sin(x))^3}\,dx = -\frac{Bc-bC}{2(b^2+c^2)(b\cos(x)+c\sin(x))^2} + \frac{(bB+cC)\sin(x)}{b(b^2+c^2)(b\cos(x)+c\sin(x))}$$

output
```
1/2*(-B*c+C*b)/(b^2+c^2)/(b*cos(x)+c*sin(x))^2+(B*b+C*c)*sin(x)/b/(b^2+c^2
)/(b*cos(x)+c*sin(x))
```

3.531.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.97

$$\int \frac{B\cos(x)+C\sin(x)}{(b\cos(x)+c\sin(x))^3}\,dx = \frac{(b^2+c^2)\,C-c(bB+cC)\cos(2x)+b(bB+cC)\sin(2x)}{2b(b^2+c^2)(b\cos(x)+c\sin(x))^2}$$

input
```
Integrate[(B*Cos[x] + C*Sin[x])/(b*Cos[x] + c*Sin[x])^3,x]
```

output
```
((b^2 + c^2)*C - c*(b*B + c*C)*Cos[2*x] + b*(b*B + c*C)*Sin[2*x])/(2*b*(b^
2 + c^2)*(b*Cos[x] + c*Sin[x])^2)
```

3.531.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 3635, 27, 3042, 3554}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\cos(x)+C\sin(x)}{(b\cos(x)+c\sin(x))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{B\cos(x)+C\sin(x)}{(b\cos(x)+c\sin(x))^3}dx$$

$$\Big\downarrow \text{3635}$$

$$\frac{\int \frac{2(bB+cC)}{(b\cos(x)+c\sin(x))^2}dx}{2(b^2+c^2)}-\frac{Bc-bC}{2(b^2+c^2)(b\cos(x)+c\sin(x))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{(bB+cC)\int \frac{1}{(b\cos(x)+c\sin(x))^2}dx}{b^2+c^2}-\frac{Bc-bC}{2(b^2+c^2)(b\cos(x)+c\sin(x))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(bB+cC)\int \frac{1}{(b\cos(x)+c\sin(x))^2}dx}{b^2+c^2}-\frac{Bc-bC}{2(b^2+c^2)(b\cos(x)+c\sin(x))^2}$$

$$\Big\downarrow \text{3554}$$

$$\frac{\sin(x)(bB+cC)}{b(b^2+c^2)(b\cos(x)+c\sin(x))}-\frac{Bc-bC}{2(b^2+c^2)(b\cos(x)+c\sin(x))^2}$$

input
```
Int[(B*Cos[x] + C*Sin[x])/(b*Cos[x] + c*Sin[x])^3,x]
```

output
```
-1/2*(B*c - b*C)/((b^2 + c^2)*(b*Cos[x] + c*Sin[x])^2) + ((b*B + c*C)*Sin[
x])/(b*(b^2 + c^2)*(b*Cos[x] + c*Sin[x]))
```

3.531.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3554
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3635
```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(-(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(
a^2 - b^2 - c^2))), x] + Simp[1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a + b*Co
s[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n +
2)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x]
/; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[n, -2]
```

3.531.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.56

method	result	size
default	$-\frac{C}{c^{2}\left(c\tan\left(x\right)+b\right)}-\frac{Bc-bC}{2c^{2}\left(c\tan\left(x\right)+b\right)^{2}}$	37
parallelrisch	$\frac{2Bb\sin\left(2x\right)-Bc\cos\left(2x\right)+Bc-bC\cos\left(2x\right)+bC}{2b^{2}\left(2\sin\left(2x\right)bc+\cos\left(2x\right)b^{2}-c^{2}\cos\left(2x\right)+b^{2}+c^{2}\right)}$	71
risch	$\frac{2i\left(Bb\,\mathrm{e}^{2ix}-Cc\,\mathrm{e}^{2ix}-iBc\,\mathrm{e}^{2ix}-iCb\,\mathrm{e}^{2ix}+Bb+Cc\right)}{\left(-ic+b\right)^{2}\left(-ic\,\mathrm{e}^{2ix}+b\,\mathrm{e}^{2ix}+ic+b\right)^{2}}$	80
norman	$\frac{\frac{\left(2Bc+2bC\right)\tan\left(\frac{x}{2}\right)^{2}}{b^{2}}+\frac{\left(2Bc+2bC\right)\tan\left(\frac{x}{2}\right)^{4}}{b^{2}}+\frac{2B\tan\left(\frac{x}{2}\right)}{b}-\frac{2B\tan\left(\frac{x}{2}\right)^{5}}{b}}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)\left(\tan\left(\frac{x}{2}\right)^{2}b-2c\tan\left(\frac{x}{2}\right)-b\right)^{2}}$	94

input
```
int((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^3,x,method=_RETURNVERBOSE)
```

output
```
-C/c^2/(c*tan(x)+b)-1/2*(B*c-C*b)/c^2/(c*tan(x)+b)^2
```

3.531.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(64) = 128$.

Time = 0.25 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.30

$$\int \frac{B\cos(x)+C\sin(x)}{(b\cos(x)+c\sin(x))^3}\,dx = \frac{Cb^3+Bb^2c+3\,Cbc^2-Bc^3-4\,(Bb^2c+Cbc^2)\cos{(x)}^2+2\,(Bb^3+Cb^2c-Bbc^2-Cc^3)\cos{(x)}\sin{(x)}}{2\,\left(b^4c^2+2\,b^2c^4+c^6+(b^6+b^4c^2-b^2c^4-c^6)\cos{(x)}^2+2\,(b^5c+2\,b^3c^3+bc^5)\cos{(x)}\sin{(x)}\right)}$$

input
```
integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="fricas")
```

output
```
1/2*(C*b^3 + B*b^2*c + 3*C*b*c^2 - B*c^3 - 4*(B*b^2*c + C*b*c^2)*cos(x)^2
+ 2*(B*b^3 + C*b^2*c - B*b*c^2 - C*c^3)*cos(x)*sin(x))/(b^4*c^2 + 2*b^2*c^
4 + c^6 + (b^6 + b^4*c^2 - b^2*c^4 - c^6)*cos(x)^2 + 2*(b^5*c + 2*b^3*c^3
+ b*c^5)*cos(x)*sin(x))
```

3.531.6 Sympy [F(-1)]

Timed out.

$$\int \frac{B\cos(x)+C\sin(x)}{(b\cos(x)+c\sin(x))^3}\,dx = \text{Timed out}$$

input
```
integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))**3,x)
```

output
```
Timed out
```

3.531.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(64) = 128.

Time = 0.27 (sec) , antiderivative size = 199, normalized size of antiderivative = 3.02

$$\int \frac{B\cos(x)+C\sin(x)}{(b\cos(x)+c\sin(x))^3}\,dx = \frac{2\,B\left(\frac{b\sin(x)}{\cos(x)+1}+\frac{c\sin(x)^2}{(\cos(x)+1)^2}-\frac{b\sin(x)^3}{(\cos(x)+1)^3}\right)}{b^4+\frac{4\,b^3c\sin(x)}{\cos(x)+1}-\frac{4\,b^3c\sin(x)^3}{(\cos(x)+1)^3}+\frac{b^4\sin(x)^4}{(\cos(x)+1)^4}-\frac{2\,(b^4-2\,b^2c^2)\sin(x)^2}{(\cos(x)+1)^2}} + \frac{2\,C\sin{(x)}^2}{\left(b^3+\frac{4\,b^2c\sin(x)}{\cos(x)+1}-\frac{4\,b^2c\sin(x)^3}{(\cos(x)+1)^3}+\frac{b^3\sin(x)^4}{(\cos(x)+1)^4}-\frac{2\,(b^3-2\,bc^2)\sin(x)^2}{(\cos(x)+1)^2}\right)(\cos{(x)}+1)^2}$$

input
```
integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="maxima")
```

output
```
2*B*(b*sin(x)/(cos(x) + 1) + c*sin(x)^2/(cos(x) + 1)^2 - b*sin(x)^3/(cos(x
) + 1)^3)/(b^4 + 4*b^3*c*sin(x)/(cos(x) + 1) - 4*b^3*c*sin(x)^3/(cos(x) +
1)^3 + b^4*sin(x)^4/(cos(x) + 1)^4 - 2*(b^4 - 2*b^2*c^2)*sin(x)^2/(cos(x)
+ 1)^2) + 2*C*sin(x)^2/((b^3 + 4*b^2*c*sin(x)/(cos(x) + 1) - 4*b^2*c*sin(x
)^3/(cos(x) + 1)^3 + b^3*sin(x)^4/(cos(x) + 1)^4 - 2*(b^3 - 2*b*c^2)*sin(x
)^2/(cos(x) + 1)^2)*(cos(x) + 1)^2)
```

3.531.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.39

$$\int \frac{B\cos(x)+C\sin(x)}{(b\cos(x)+c\sin(x))^3}\,dx = -\frac{2\,Cc\tan{(x)}+Cb+Bc}{2\,(c\tan{(x)}+b)^2c^2}$$

input
```
integrate((B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="giac")
```

output
```
-1/2*(2*C*c*tan(x) + C*b + B*c)/((c*tan(x) + b)^2*c^2)
```

3.531.9 Mupad [B] (verification not implemented)

Time = 26.24 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.44

$$\int \frac{B\cos(x) + C\sin(x)}{(b\cos(x) + c\sin(x))^3}\,dx = \frac{\frac{2\tan\left(\frac{x}{2}\right)^2 (B\,c+C\,b)}{b^2} - \frac{2\,B\tan\left(\frac{x}{2}\right)^3}{b} + \frac{2\,B\tan\left(\frac{x}{2}\right)}{b}}{b^2 - \tan\left(\frac{x}{2}\right)^2\,(2\,b^2 - 4\,c^2) + b^2\tan\left(\frac{x}{2}\right)^4 + 4\,b\,c\tan\left(\frac{x}{2}\right) - 4\,b\,c\tan\left(\frac{x}{2}\right)^3}$$

input
```
int((B*cos(x) + C*sin(x))/(b*cos(x) + c*sin(x))^3,x)
```

output
```
((2*tan(x/2)^2*(B*c + C*b))/b^2 - (2*B*tan(x/2)^3)/b + (2*B*tan(x/2))/b)/(
b^2 - tan(x/2)^2*(2*b^2 - 4*c^2) + b^2*tan(x/2)^4 + 4*b*c*tan(x/2) - 4*b*c
*tan(x/2)^3)
```

3.532 $\int \frac{A+B\cos(x)+C\sin(x)}{b\cos(x)+c\sin(x)}\,dx$

3.532.1 Optimal result

Integrand size = 22, antiderivative size = 84

$$\int \frac{A+B\cos(x)+C\sin(x)}{b\cos(x)+c\sin(x)}\,dx = \frac{(bB+cC)x}{b^2+c^2} - \frac{A\,\mathrm{arctanh}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}} + \frac{(Bc-bC)\log(b\cos(x)+c\sin(x))}{b^2+c^2}$$

output
```
(B*b+C*c)*x/(b^2+c^2)+(B*c-C*b)*ln(b*cos(x)+c*sin(x))/(b^2+c^2)-A*arctanh(
(c*cos(x)-b*sin(x))/(b^2+c^2)^(1/2))/(b^2+c^2)^(1/2)
```

3.532.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.93

$$\int \frac{A+B\cos(x)+C\sin(x)}{b\cos(x)+c\sin(x)}\,dx = \frac{(bB+cC)x+2A\sqrt{b^2+c^2}\,\mathrm{arctanh}\left(\frac{-c+b\tan\left(\frac{x}{2}\right)}{\sqrt{b^2+c^2}}\right)+(Bc-bC)\log(b\cos(x)+c\sin(x))}{b^2+c^2}$$

input
```
Integrate[(A + B*Cos[x] + C*Sin[x])/(b*Cos[x] + c*Sin[x]),x]
```

output
```
((b*B + c*C)*x + 2*A*Sqrt[b^2 + c^2]*ArcTanh[(-c + b*Tan[x/2])/Sqrt[b^2 +
c^2]] + (B*c - b*C)*Log[b*Cos[x] + c*Sin[x]])/(b^2 + c^2)
```

3.532.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {3042, 3615, 3042, 3553, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(x)+C\sin(x)}{b\cos(x)+c\sin(x)} \, dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{A+B\cos(x)+C\sin(x)}{b\cos(x)+c\sin(x)}dx$$

$$\Big\downarrow \; 3615$$

$$A\int \frac{1}{b\cos(x)+c\sin(x)}dx+\frac{x(bB+cC)}{b^2+c^2}+\frac{(Bc-bC)\log(b\cos(x)+c\sin(x))}{b^2+c^2}$$

$$\Big\downarrow \; 3042$$

$$A\int \frac{1}{b\cos(x)+c\sin(x)}dx+\frac{x(bB+cC)}{b^2+c^2}+\frac{(Bc-bC)\log(b\cos(x)+c\sin(x))}{b^2+c^2}$$

$$\Big\downarrow \; 3553$$

$$-A\int \frac{1}{b^2+c^2-(c\cos(x)-b\sin(x))^2}d(c\cos(x)-b\sin(x))+\frac{x(bB+cC)}{b^2+c^2}+\frac{(Bc-bC)\log(b\cos(x)+c\sin(x))}{b^2+c^2}$$

$$\Big\downarrow \; 219$$

$$-\frac{A\text{arctanh}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}}+\frac{x(bB+cC)}{b^2+c^2}+\frac{(Bc-bC)\log(b\cos(x)+c\sin(x))}{b^2+c^2}$$

input
```
Int[(A + B*Cos[x] + C*Sin[x])/(b*Cos[x] + c*Sin[x]),x]
```

output
```
((b*B + c*C)*x)/(b^2 + c^2) - (A*ArcTanh[(c*Cos[x] - b*Sin[x])/Sqrt[b^2 +
c^2]])/Sqrt[b^2 + c^2] + ((B*c - b*C)*Log[b*Cos[x] + c*Sin[x]])/(b^2 + c^2
)
```

3.532.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3615
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + (Simp[(c*B - b*C)*(Log[a
 + b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] + Simp[(A*(b^2 + c
^2) - a*(b*B + c*C))/(b^2 + c^2)   Int[1/(a + b*Cos[d + e*x] + c*Sin[d + e*
x]), x], x]) /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] &&
NeQ[A*(b^2 + c^2) - a*(b*B + c*C), 0]
```

3.532.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(80) = 160$.

Time = 0.89 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.99

method	result
default	$\frac{(-Bc+bC)\ln\left(1+\tan\left(\frac{x}{2}\right)^2\right)+2(Bb+Cc)\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{b^2+c^2}+\frac{\frac{\left(bBc-b^2C\right)\ln\left(\tan\left(\frac{x}{2}\right)^2 b-2c\tan\left(\frac{x}{2}\right)-b\right)}{b}-\frac{2\left(-A\,b^2-A\,c^2-B\,c^2+Cbc+\frac{(b}{}\right.}{}}{b^2+c^2}$
risch	$\frac{ixC}{ic-b}-\frac{Bx}{ic-b}-\frac{2ixB\,b^2c}{b^4+2b^2c^2+c^4}-\frac{2ixB\,c^3}{b^4+2b^2c^2+c^4}+\frac{2iCx\,b^3}{b^4+2b^2c^2+c^4}+\frac{2iCx\,c^2b}{b^4+2b^2c^2+c^4}+\frac{\ln\left(e^{ix}+\frac{(ib-c)\sqrt{A^2b^2+A^2c^2}}{A\left(b^2+c^2\right)}\right)Bc}{b^2+c^2}-\ln($

input
```
int((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
2/(b^2+c^2)*(1/2*(-B*c+C*b)*ln(1+tan(1/2*x)^2)+(B*b+C*c)*arctan(tan(1/2*x)
))+2/(b^2+c^2)*(1/2*(B*b*c-C*b^2)/b*ln(tan(1/2*x)^2*b-2*c*tan(1/2*x)-b)-(-
A*b^2-A*c^2-B*c^2+C*b*c+(B*b*c-C*b^2)*c/b)/(b^2+c^2)^(1/2)*arctanh(1/2*(2*
b*tan(1/2*x)-2*c)/(b^2+c^2)^(1/2)))
```

3.532.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.85

$$\int \frac{A+B\cos(x)+C\sin(x)}{b\cos(x)+c\sin(x)}\,dx = \frac{\sqrt{b^2+c^2}A\log\left(-\frac{2\,bc\cos(x)\sin(x)+(b^2-c^2)\cos(x)^2-2\,b^2-c^2+2\,\sqrt{b^2+c^2}(c\cos(x)-b\sin(x))}{2\,bc\cos(x)\sin(x)+(b^2-c^2)\cos(x)^2+c^2}\right)+2\,(Bb+Cc)x-(Cb-Bc)\,\mathrm{l}}{2\,(b^2+c^2)}$$

input
```
integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x)),x, algorithm="fricas")
```

output
```
1/2*(sqrt(b^2 + c^2)*A*log(-(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 -
2*b^2 - c^2 + 2*sqrt(b^2 + c^2)*(c*cos(x) - b*sin(x)))/(2*b*c*cos(x)*sin(x
) + (b^2 - c^2)*cos(x)^2 + c^2)) + 2*(B*b + C*c)*x - (C*b - B*c)*log(2*b*c
*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 + c^2))/(b^2 + c^2)
```

3.532.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.95 (sec) , antiderivative size = 1030, normalized size of antiderivative = 12.26

$$\int \frac{A+B\cos(x)+C\sin(x)}{b\cos(x)+c\sin(x)}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x)),x)
```

output
```
Piecewise((zoo*(A*log(tan(x/2)) - B*log(tan(x/2)**2 + 1) + B*log(tan(x/2))
 + C*x), Eq(b, 0) & Eq(c, 0)), ((A*log(tan(x/2)) - B*log(tan(x/2)**2 + 1)
+ B*log(tan(x/2)) + C*x)/c, Eq(b, 0)), (-2*A/(2*I*c*sin(x) + 2*c*cos(x)) -
 B*x*sin(x)/(2*I*c*sin(x) + 2*c*cos(x)) + I*B*x*cos(x)/(2*I*c*sin(x) + 2*c
*cos(x)) + I*B*sin(x)/(2*I*c*sin(x) + 2*c*cos(x)) + I*C*x*sin(x)/(2*I*c*si
n(x) + 2*c*cos(x)) + C*x*cos(x)/(2*I*c*sin(x) + 2*c*cos(x)) - C*sin(x)/(2*
I*c*sin(x) + 2*c*cos(x)), Eq(b, -I*c)), (-2*A/(-2*I*c*sin(x) + 2*c*cos(x))
 - B*x*sin(x)/(-2*I*c*sin(x) + 2*c*cos(x)) - I*B*x*cos(x)/(-2*I*c*sin(x) +
 2*c*cos(x)) - I*B*sin(x)/(-2*I*c*sin(x) + 2*c*cos(x)) - I*C*x*sin(x)/(-2*
I*c*sin(x) + 2*c*cos(x)) + C*x*cos(x)/(-2*I*c*sin(x) + 2*c*cos(x)) - C*sin
(x)/(-2*I*c*sin(x) + 2*c*cos(x)), Eq(b, I*c)), (-A*b**2*log(tan(x/2) - c/b
 - sqrt(b**2 + c**2)/b)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2))
+ A*b**2*log(tan(x/2) - c/b + sqrt(b**2 + c**2)/b)/(b**2*sqrt(b**2 + c**2)
 + c**2*sqrt(b**2 + c**2)) - A*c**2*log(tan(x/2) - c/b - sqrt(b**2 + c**2)
/b)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2)) + A*c**2*log(tan(x/2
) - c/b + sqrt(b**2 + c**2)/b)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 +
c**2)) + B*b*x*sqrt(b**2 + c**2)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2
+ c**2)) - B*c*sqrt(b**2 + c**2)*log(tan(x/2)**2 + 1)/(b**2*sqrt(b**2 + c*
*2) + c**2*sqrt(b**2 + c**2)) + B*c*sqrt(b**2 + c**2)*log(tan(x/2) - c/b -
 sqrt(b**2 + c**2)/b)/(b**2*sqrt(b**2 + c**2) + c**2*sqrt(b**2 + c**2))...
```

3.532.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(81) = 162$.

Time = 0.35 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.89

$$\begin{aligned}\int \frac{A+B\cos(x)+C\sin(x)}{b\cos(x)+c\sin(x)}\,dx &= B\left(\frac{2\,b\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{b^2+c^2}+\frac{c\log\left(-b-\frac{2\,c\sin(x)}{\cos(x)+1}+\frac{b\sin(x)^2}{(\cos(x)+1)^2}\right)}{b^2+c^2}-\frac{c\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{b^2+c^2}\right)\\ &\quad+C\left(\frac{2\,c\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{b^2+c^2}-\frac{b\log\left(-b-\frac{2\,c\sin(x)}{\cos(x)+1}+\frac{b\sin(x)^2}{(\cos(x)+1)^2}\right)}{b^2+c^2}+\frac{b\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{b^2+c^2}\right)\\ &\quad-\frac{A\log\left(\frac{c-\frac{b\sin(x)}{\cos(x)+1}+\sqrt{b^2+c^2}}{c-\frac{b\sin(x)}{\cos(x)+1}-\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}}\end{aligned}$$

input
```
integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x)),x, algorithm="maxima")
```

output
```
B*(2*b*arctan(sin(x)/(cos(x) + 1))/(b^2 + c^2) + c*log(-b - 2*c*sin(x)/(co
s(x) + 1) + b*sin(x)^2/(cos(x) + 1)^2)/(b^2 + c^2) - c*log(sin(x)^2/(cos(x
) + 1)^2 + 1)/(b^2 + c^2)) + C*(2*c*arctan(sin(x)/(cos(x) + 1))/(b^2 + c^2
) - b*log(-b - 2*c*sin(x)/(cos(x) + 1) + b*sin(x)^2/(cos(x) + 1)^2)/(b^2 +
 c^2) + b*log(sin(x)^2/(cos(x) + 1)^2 + 1)/(b^2 + c^2)) - A*log((c - b*sin
(x)/(cos(x) + 1) + sqrt(b^2 + c^2))/(c - b*sin(x)/(cos(x) + 1) - sqrt(b^2
+ c^2)))/sqrt(b^2 + c^2)
```

3.532.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.76

$$\int \frac{A+B\cos(x)+C\sin(x)}{b\cos(x)+c\sin(x)}\,dx = -\frac{A\log\left(\frac{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,c-2\,\sqrt{b^2+c^2}\right|}{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,c+2\,\sqrt{b^2+c^2}\right|}\right)}{\sqrt{b^2+c^2}} + \frac{(Bb+Cc)x}{b^2+c^2} + \frac{(Cb-Bc)\log\left(\tan\left(\frac{1}{2}\,x\right)^2+1\right)}{b^2+c^2} - \frac{(Cb-Bc)\log\left(\left|b\tan\left(\frac{1}{2}\,x\right)^2-2\,c\tan\left(\frac{1}{2}\,x\right)-b\right|\right)}{b^2+c^2}$$

input
```
integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x)),x, algorithm="giac")
```

output
```
-A*log(abs(2*b*tan(1/2*x) - 2*c - 2*sqrt(b^2 + c^2))/abs(2*b*tan(1/2*x) -
2*c + 2*sqrt(b^2 + c^2)))/sqrt(b^2 + c^2) + (B*b + C*c)*x/(b^2 + c^2) + (C
*b - B*c)*log(tan(1/2*x)^2 + 1)/(b^2 + c^2) - (C*b - B*c)*log(abs(b*tan(1/
2*x)^2 - 2*c*tan(1/2*x) - b))/(b^2 + c^2)
```

3.532.9 Mupad [B] (verification not implemented)

Time = 34.61 (sec) , antiderivative size = 1099, normalized size of antiderivative = 13.08

$$\int \frac{A+B\cos(x)+C\sin(x)}{b\cos(x)+c\sin(x)}\,dx = \ln\left(32\,A^2\,B\,b^2-32\,A\,B^2\,b^2-32\,A\,C^2\,b^2-32\,B\,C^2\,b^2\right.$$

$$+32\,b\tan\left(\frac{x}{2}\right)\,\left(c\,A^2\,B+b\,A^2\,C-2\,c\,A\,B^2-2\,c\,A\,C^2+c\,B^3-b\,B^2\,C+2\,c\,B\,C^2-2\,b\,C^3\right)$$

$$-\frac{\left(A\,\sqrt{{\left(b^2+c^2\right)}^3}+B\,c^3-C\,b^3+B\,b^2\,c-C\,b\,c^2\right)\,\left(32\,b\tan\left(\frac{x}{2}\right)\,\left(A^2\,b^2-A^2\,c^2+4\,A\,B\,c^2-4\,A\,C\,b\,c+\right.\right.}{}$$

$$\left.-32\,A^2\,C\,b\,c+32\,B^2\,C\,b\,c\right)\left(\frac{B\,c-C\,b}{b^2+c^2}+\frac{A\,\sqrt{{\left(b^2+c^2\right)}^3}}{{\left(b^2+c^2\right)}^2}\right)$$

$$+\ln\left(32\,A^2\,B\,b^2-32\,A\,B^2\,b^2-32\,A\,C^2\,b^2-32\,B\,C^2\,b^2\right.$$

$$+32\,b\tan\left(\frac{x}{2}\right)\,\left(c\,A^2\,B+b\,A^2\,C-2\,c\,A\,B^2-2\,c\,A\,C^2+c\,B^3-b\,B^2\,C+2\,c\,B\,C^2-2\,b\,C^3\right)$$

$$\left(A\,\sqrt{{\left(b^2+c^2\right)}^3}-B\,c^3+C\,b^3-B\,b^2\,c+C\,b\,c^2\right)\,\left(64\,A^2\,b^2\,c+32\,B^2\,b^2\,c+32\,C^2\,b^2\,c-32\,b\tan\left(\frac{x}{2}\right)\,\left(\right.\right.$$

3.532. $\int \frac{A+B\cos(x)+C\sin(x)}{b\cos(x)+c\sin(x)}\,dx$

input
```
int((A + B*cos(x) + C*sin(x))/(b*cos(x) + c*sin(x)),x)
```

output
```
log(32*A^2*B*b^2 - 32*A*B^2*b^2 - 32*A*C^2*b^2 - 32*B*C^2*b^2 + 32*b*tan(x
/2)*(B^3*c - 2*C^3*b - 2*A*B^2*c + A^2*B*c + A^2*C*b - 2*A*C^2*c - B^2*C*b
 + 2*B*C^2*c) - ((A*((b^2 + c^2)^3)^(1/2) + B*c^3 - C*b^3 + B*b^2*c - C*b*
c^2)*(32*b*tan(x/2)*(A^2*b^2 - A^2*c^2 + B^2*b^2 - 3*B^2*c^2 + 2*C^2*c^2 +
 4*A*B*c^2 - 4*A*C*b*c + 6*B*C*b*c) - 32*B^2*b^2*c - 32*C^2*b^2*c - 64*A^2
*b^2*c - 64*A*C*b^3 + 64*B*C*b^3 + ((A*((b^2 + c^2)^3)^(1/2) + B*c^3 - C*b
^3 + B*b^2*c - C*b*c^2)*(32*A*b^4 + 32*B*b^4 + 32*A*b^2*c^2 - 64*B*b^2*c^2
 - 32*C*b*c^3 + 64*C*b^3*c + 32*b*tan(x/2)*(2*A*c^3 + B*c^3 - 2*C*b^3 + 2*
A*b^2*c + 4*B*b^2*c + C*b*c^2) + (96*b*c*(b + c*tan(x/2))*(A*((b^2 + c^2)^
3)^(1/2) + B*c^3 - C*b^3 + B*b^2*c - C*b*c^2))/(b^2 + c^2)))/(b^2 + c^2)^2
 + 64*A*B*b^2*c + 64*B*C*b*c^2))/(b^2 + c^2)^2 - 32*A^2*C*b*c + 32*B^2*C*b
*c)*((B*c - C*b)/(b^2 + c^2) + (A*((b^2 + c^2)^3)^(1/2))/(b^2 + c^2)^2) +
log(32*A^2*B*b^2 - 32*A*B^2*b^2 - 32*A*C^2*b^2 - 32*B*C^2*b^2 + 32*b*tan(x
/2)*(B^3*c - 2*C^3*b - 2*A*B^2*c + A^2*B*c + A^2*C*b - 2*A*C^2*c - B^2*C*b
 + 2*B*C^2*c) - ((A*((b^2 + c^2)^3)^(1/2) - B*c^3 + C*b^3 - B*b^2*c + C*b*
c^2)*(64*A^2*b^2*c + 32*B^2*b^2*c + 32*C^2*b^2*c - 32*b*tan(x/2)*(A^2*b^2
- A^2*c^2 + B^2*b^2 - 3*B^2*c^2 + 2*C^2*c^2 + 4*A*B*c^2 - 4*A*C*b*c + 6*B*
C*b*c) + 64*A*C*b^3 - 64*B*C*b^3 + ((A*((b^2 + c^2)^3)^(1/2) - B*c^3 + C*b
^3 - B*b^2*c + C*b*c^2)*(32*A*b^4 + 32*B*b^4 + 32*A*b^2*c^2 - 64*B*b^2*c^2
 - 32*C*b*c^3 + 64*C*b^3*c + 32*b*tan(x/2)*(2*A*c^3 + B*c^3 - 2*C*b^3 +...
```

3.532. $\int \frac{A+B\cos(x)+C\sin(x)}{b\cos(x)+c\sin(x)}\,dx$

3.533 $\int \frac{A+B\cos(x)+C\sin(x)}{(b\cos(x)+c\sin(x))^2}\,dx$

3.533.1 Optimal result

Integrand size = 22, antiderivative size = 85

$$\int \frac{A+B\cos(x)+C\sin(x)}{(b\cos(x)+c\sin(x))^2}\,dx = -\frac{(bB+cC)\text{arctanh}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{3/2}} - \frac{Bc-bC+Ac\cos(x)-Ab\sin(x)}{(b^2+c^2)\,(b\cos(x)+c\sin(x))}$$

output
```
-(B*b+C*c)*arctanh((c*cos(x)-b*sin(x))/(b^2+c^2)^(1/2))/(b^2+c^2)^(3/2)+(-
B*c+b*C-A*c*cos(x)+A*b*sin(x))/(b^2+c^2)/(b*cos(x)+c*sin(x))
```

3.533.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.08

$$\int \frac{A+B\cos(x)+C\sin(x)}{(b\cos(x)+c\sin(x))^2}\,dx = \frac{2(bB+cC)\text{arctanh}\left(\frac{-c+b\tan\left(\frac{x}{2}\right)}{\sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{3/2}} + \frac{b(-Bc+bC)+A(b^2+c^2)\sin(x)}{b\,(b^2+c^2)\,(b\cos(x)+c\sin(x))}$$

input
```
Integrate[(A + B*Cos[x] + C*Sin[x])/(b*Cos[x] + c*Sin[x])^2,x]
```

output
```
(2*(b*B + c*C)*ArcTanh[(-c + b*Tan[x/2])/Sqrt[b^2 + c^2]])/(b^2 + c^2)^(3/
2) + (b*(-(B*c) + b*C) + A*(b^2 + c^2)*Sin[x])/(b*(b^2 + c^2)*(b*Cos[x] +
c*Sin[x]))
```

3.533.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {3042, 3632, 3042, 3553, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(x)+C\sin(x)}{(b\cos(x)+c\sin(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\cos(x)+C\sin(x)}{(b\cos(x)+c\sin(x))^2}dx$$

$$\Big\downarrow \text{3632}$$

$$\frac{(bB+cC)\int \frac{1}{b\cos(x)+c\sin(x)}dx}{b^2+c^2}-\frac{-Ab\sin(x)+Ac\cos(x)-bC+Bc}{(b^2+c^2)(b\cos(x)+c\sin(x))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(bB+cC)\int \frac{1}{b\cos(x)+c\sin(x)}dx}{b^2+c^2}-\frac{-Ab\sin(x)+Ac\cos(x)-bC+Bc}{(b^2+c^2)(b\cos(x)+c\sin(x))}$$

$$\Big\downarrow \text{3553}$$

$$-\frac{(bB+cC)\int \frac{1}{b^2+c^2-(c\cos(x)-b\sin(x))^2}d(c\cos(x)-b\sin(x))}{b^2+c^2}-\frac{-Ab\sin(x)+Ac\cos(x)-bC+Bc}{(b^2+c^2)(b\cos(x)+c\sin(x))}$$

$$\Big\downarrow \text{219}$$

$$-\frac{-Ab\sin(x)+Ac\cos(x)-bC+Bc}{(b^2+c^2)(b\cos(x)+c\sin(x))}-\frac{(bB+cC)\text{arctanh}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{3/2}}$$

input
```
Int[(A + B*Cos[x] + C*Sin[x])/(b*Cos[x] + c*Sin[x])^2,x]
```

output
```
-(((b*B + c*C)*ArcTanh[(c*Cos[x] - b*Sin[x])/Sqrt[b^2 + c^2]])/(b^2 + c^2)
^(3/2)) - (B*c - b*C + A*c*Cos[x] - A*b*Sin[x])/((b^2 + c^2)*(b*Cos[x] + c
*Sin[x]))
```

3.533.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3632
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Simp[(a*A - b*B - c*C)/(a^2 - b^2 - c^2)   Int[1/(a + b*Cos[d + e*x] + c*S
in[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
 - c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

3.533.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.46

method	result
default	$\frac{-\frac{2\left(A\,b^{2}+A\,c^{2}-B\,c^{2}+Cbc\right)\tan\left(\frac{x}{2}\right)}{b\left(b^{2}+c^{2}\right)}+\frac{2(Bc-bC)}{b^{2}+c^{2}}}{\tan\left(\frac{x}{2}\right)^{2}b-2c\tan\left(\frac{x}{2}\right)-b}+\frac{2(Bb+Cc)\,\text{arctanh}\left(\frac{2b\tan\left(\frac{x}{2}\right)-2c}{2\sqrt{b^{2}+c^{2}}}\right)}{(b^{2}+c^{2})^{\frac{3}{2}}}$
risch	$-\frac{2i\left(cA-iAb+Bc\,\mathrm{e}^{ix}-Cb\,\mathrm{e}^{ix}\right)}{(-ib+c)(ib+c)(c\,\mathrm{e}^{2ix}+ib\,\mathrm{e}^{2ix}-c+ib)}+\frac{bB\ln\left(\mathrm{e}^{ix}+\frac{ib^{3}+ib\,c^{2}-b^{2}c-c^{3}}{\left(b^{2}+c^{2}\right)^{\frac{3}{2}}}\right)}{(b^{2}+c^{2})^{\frac{3}{2}}}+\frac{cC\ln\left(\mathrm{e}^{ix}+\frac{ib^{3}+ib\,c^{2}-b^{2}c-c^{3}}{\left(b^{2}+c^{2}\right)^{\frac{3}{2}}}\right)}{(b^{2}+c^{2})^{\frac{3}{2}}}-\frac{bB\ln\left(\mathrm{e}^{ix}-\frac{ib^{3}}{}\right.}{(b^{2}+}$

input
```
int((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^2,x,method=_RETURNVERBOSE)
```

output
```
2*(-(A*b^2+A*c^2-B*c^2+C*b*c)/b/(b^2+c^2)*tan(1/2*x)+(B*c-C*b)/(b^2+c^2))/
(tan(1/2*x)^2*b-2*c*tan(1/2*x)-b)+2*(B*b+C*c)/(b^2+c^2)^(3/2)*arctanh(1/2*
(2*b*tan(1/2*x)-2*c)/(b^2+c^2)^(1/2))
```

3.533.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(81) = 162$.

Time = 0.27 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.66

$$\int \frac{A + B\cos(x) + C\sin(x)}{(b\cos(x) + c\sin(x))^2}\,dx = \frac{2\,Cb^3 - 2\,Bb^2c + 2\,Cbc^2 - 2\,Bc^3 + \sqrt{b^2+c^2}((Bb^2 + Cbc)\cos(x) + (Bbc + Cc^2)\sin(x))\log\left(-\frac{2\,bc\cos(x)\sin}{}\right.}{2\,((b^5 + 2\,b^3c^2 + bc^4)\cos(x) + (b}$$

input
```
integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="fricas
")
```

output
```
1/2*(2*C*b^3 - 2*B*b^2*c + 2*C*b*c^2 - 2*B*c^3 + sqrt(b^2 + c^2)*((B*b^2 +
 C*b*c)*cos(x) + (B*b*c + C*c^2)*sin(x))*log(-(2*b*c*cos(x)*sin(x) + (b^2
- c^2)*cos(x)^2 - 2*b^2 - c^2 + 2*sqrt(b^2 + c^2)*(c*cos(x) - b*sin(x)))/(
2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 + c^2)) - 2*(A*b^2*c + A*c^3)*c
os(x) + 2*(A*b^3 + A*b*c^2)*sin(x))/((b^5 + 2*b^3*c^2 + b*c^4)*cos(x) + (b
^4*c + 2*b^2*c^3 + c^5)*sin(x))
```

3.533.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{A + B\cos(x) + C\sin(x)}{(b\cos(x) + c\sin(x))^2}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))**2,x)
```

output
```
Exception raised: AttributeError >> 'NoneType' object has no attribute 'pr
imitive'
```

3.533.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. 2(81) = 162.

Time = 0.32 (sec) , antiderivative size = 286, normalized size of antiderivative = 3.36

$$\int \frac{A+B\cos(x)+C\sin(x)}{(b\cos(x)+c\sin(x))^2}\,dx$$
$$= -B\left(\frac{b\log\left(\frac{c-\frac{b\sin(x)}{\cos(x)+1}+\sqrt{b^2+c^2}}{c-\frac{b\sin(x)}{\cos(x)+1}-\sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{\frac{3}{2}}}+\frac{2\left(bc+\frac{c^2\sin(x)}{\cos(x)+1}\right)}{b^4+b^2c^2+\frac{2(b^3c+bc^3)\sin(x)}{\cos(x)+1}-\frac{(b^4+b^2c^2)\sin(x)^2}{(\cos(x)+1)^2}}\right)$$
$$- C\left(\frac{c\log\left(\frac{c-\frac{b\sin(x)}{\cos(x)+1}+\sqrt{b^2+c^2}}{c-\frac{b\sin(x)}{\cos(x)+1}-\sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{\frac{3}{2}}}-\frac{2\left(b+\frac{c\sin(x)}{\cos(x)+1}\right)}{b^3+bc^2+\frac{2(b^2c+c^3)\sin(x)}{\cos(x)+1}-\frac{(b^3+bc^2)\sin(x)^2}{(\cos(x)+1)^2}}\right)$$
$$-\frac{A}{c^2\tan(x)+bc}$$

input
```
integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="maxima
")
```

output
```
-B*(b*log((c - b*sin(x)/(cos(x) + 1) + sqrt(b^2 + c^2))/(c - b*sin(x)/(cos
(x) + 1) - sqrt(b^2 + c^2)))/(b^2 + c^2)^(3/2) + 2*(b*c + c^2*sin(x)/(cos(
x) + 1))/(b^4 + b^2*c^2 + 2*(b^3*c + b*c^3)*sin(x)/(cos(x) + 1) - (b^4 + b
^2*c^2)*sin(x)^2/(cos(x) + 1)^2)) - C*(c*log((c - b*sin(x)/(cos(x) + 1) +
sqrt(b^2 + c^2))/(c - b*sin(x)/(cos(x) + 1) - sqrt(b^2 + c^2)))/(b^2 + c^2
)^(3/2) - 2*(b + c*sin(x)/(cos(x) + 1))/(b^3 + b*c^2 + 2*(b^2*c + c^3)*sin
(x)/(cos(x) + 1) - (b^3 + b*c^2)*sin(x)^2/(cos(x) + 1)^2)) - A/(c^2*tan(x)
 + b*c)
```

3.533.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.76

$$\int \frac{A+B\cos(x)+C\sin(x)}{(b\cos(x)+c\sin(x))^2}\,dx$$
$$= -\frac{(Bb+Cc)\log\left(\frac{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,c-2\,\sqrt{b^2+c^2}\right|}{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,c+2\,\sqrt{b^2+c^2}\right|}\right)}{(b^2+c^2)^{\frac{3}{2}}}$$
$$-\frac{2\left(Ab^2\tan\left(\frac{1}{2}\,x\right)+Cbc\tan\left(\frac{1}{2}\,x\right)+Ac^2\tan\left(\frac{1}{2}\,x\right)-Bc^2\tan\left(\frac{1}{2}\,x\right)+Cb^2-Bbc\right)}{(b^3+bc^2)\left(b\tan\left(\frac{1}{2}\,x\right)^2-2\,c\tan\left(\frac{1}{2}\,x\right)-b\right)}$$

input
```
integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^2,x, algorithm="giac")
```

output
```
-(B*b + C*c)*log(abs(2*b*tan(1/2*x) - 2*c - 2*sqrt(b^2 + c^2))/abs(2*b*tan
(1/2*x) - 2*c + 2*sqrt(b^2 + c^2)))/(b^2 + c^2)^(3/2) - 2*(A*b^2*tan(1/2*x
) + C*b*c*tan(1/2*x) + A*c^2*tan(1/2*x) - B*c^2*tan(1/2*x) + C*b^2 - B*b*c
)/((b^3 + b*c^2)*(b*tan(1/2*x)^2 - 2*c*tan(1/2*x) - b))
```

3.533.9 Mupad [B] (verification not implemented)

Time = 27.60 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.66

$$\int \frac{A+B\cos(x)+C\sin(x)}{(b\cos(x)+c\sin(x))^2}\,dx = -\frac{\frac{2\,(B\,c-C\,b)}{b^2+c^2}-\frac{2\tan\left(\frac{x}{2}\right)\,\left(A\,b^2+A\,c^2-B\,c^2+C\,b\,c\right)}{b\,(b^2+c^2)}}{-b\tan\left(\frac{x}{2}\right)^2+2\,c\tan\left(\frac{x}{2}\right)+b} + \frac{\mathrm{atan}\left(\frac{b^2\,c\,1\mathrm{i}+c^3\,1\mathrm{i}-b\tan\left(\frac{x}{2}\right)\,\left(b^2+c^2\right)\,1\mathrm{i}}{(b^2+c^2)^{3/2}}\right)\,\left(B\,b+C\,c\right)\,2\mathrm{i}}{\left(b^2+c^2\right)^{3/2}}$$

input
```
int((A + B*cos(x) + C*sin(x))/(b*cos(x) + c*sin(x))^2,x)
```

output
```
(atan((b^2*c*1i + c^3*1i - b*tan(x/2)*(b^2 + c^2)*1i)/(b^2 + c^2)^(3/2))*(
B*b + C*c)*2i)/(b^2 + c^2)^(3/2) - ((2*(B*c - C*b))/(b^2 + c^2) - (2*tan(x
/2)*(A*b^2 + A*c^2 - B*c^2 + C*b*c))/(b*(b^2 + c^2)))/(b + 2*c*tan(x/2) -
b*tan(x/2)^2)
```

3.534 $\int \frac{A+B\cos(x)+C\sin(x)}{(b\cos(x)+c\sin(x))^3}\,dx$

3.534.1 Optimal result

Integrand size = 22, antiderivative size = 129

$$\int \frac{A+B\cos(x)+C\sin(x)}{(b\cos(x)+c\sin(x))^3}\,dx = -\frac{A\operatorname{arctanh}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{2(b^2+c^2)^{3/2}} - \frac{Bc-bC+Ac\cos(x)-Ab\sin(x)}{2(b^2+c^2)(b\cos(x)+c\sin(x))^2} - \frac{c(bB+cC)\cos(x)-b(bB+cC)\sin(x)}{(b^2+c^2)^2(b\cos(x)+c\sin(x))}$$

output
```
-1/2*A*arctanh((c*cos(x)-b*sin(x))/(b^2+c^2)^(1/2))/(b^2+c^2)^(3/2)+1/2*(-
B*c+b*C-A*c*cos(x)+A*b*sin(x))/(b^2+c^2)/(b*cos(x)+c*sin(x))^2+(-c*(B*b+C*
c)*cos(x)+b*(B*b+C*c)*sin(x))/(b^2+c^2)^2/(b*cos(x)+c*sin(x))
```

3.534.2 Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.95

$$\int \frac{A+B\cos(x)+C\sin(x)}{(b\cos(x)+c\sin(x))^3}\,dx = \frac{A\operatorname{arctanh}\left(\frac{-c+b\tan\left(\frac{x}{2}\right)}{\sqrt{b^2+c^2}}\right)}{(b^2+c^2)^{3/2}} + \frac{b^2C+c^2C-Abc\cos(x)-c(bB+cC)\cos(2x)+Ab^2\sin(x)+b^2B\sin(2x)+bcC\sin(2x)}{2b(b^2+c^2)(b\cos(x)+c\sin(x))^2}$$

input
```
Integrate[(A + B*Cos[x] + C*Sin[x])/(b*Cos[x] + c*Sin[x])^3,x]
```

output
```
(A*ArcTanh[(-c + b*Tan[x/2])/Sqrt[b^2 + c^2]])/(b^2 + c^2)^(3/2) + (b^2*C
+ c^2*C - A*b*c*Cos[x] - c*(b*B + c*C)*Cos[2*x] + A*b^2*Sin[x] + b^2*B*Sin
[2*x] + b*c*C*Sin[2*x])/(2*b*(b^2 + c^2)*(b*Cos[x] + c*Sin[x])^2)
```

3.534.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {3042, 3635, 3042, 3632, 3042, 3553, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(x)+C\sin(x)}{(b\cos(x)+c\sin(x))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\cos(x)+C\sin(x)}{(b\cos(x)+c\sin(x))^3}dx$$

$$\Big\downarrow \text{3635}$$

$$\frac{\int \frac{2(bB+cC)+Ab\cos(x)+Ac\sin(x)}{(b\cos(x)+c\sin(x))^2}dx}{2(b^2+c^2)}-\frac{-Ab\sin(x)+Ac\cos(x)-bC+Bc}{2(b^2+c^2)(b\cos(x)+c\sin(x))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{2(bB+cC)+Ab\cos(x)+Ac\sin(x)}{(b\cos(x)+c\sin(x))^2}dx}{2(b^2+c^2)}-\frac{-Ab\sin(x)+Ac\cos(x)-bC+Bc}{2(b^2+c^2)(b\cos(x)+c\sin(x))^2}$$

$$\Big\downarrow \text{3632}$$

$$\frac{A\int \frac{1}{b\cos(x)+c\sin(x)}dx-\frac{2(c\cos(x)(bB+cC)-b\sin(x)(bB+cC))}{(b^2+c^2)(b\cos(x)+c\sin(x))}}{2(b^2+c^2)}-\frac{-Ab\sin(x)+Ac\cos(x)-bC+Bc}{2(b^2+c^2)(b\cos(x)+c\sin(x))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{A\int \frac{1}{b\cos(x)+c\sin(x)}dx-\frac{2(c\cos(x)(bB+cC)-b\sin(x)(bB+cC))}{(b^2+c^2)(b\cos(x)+c\sin(x))}}{2(b^2+c^2)}-\frac{-Ab\sin(x)+Ac\cos(x)-bC+Bc}{2(b^2+c^2)(b\cos(x)+c\sin(x))^2}$$

$$\Big\downarrow \text{3553}$$

$$\frac{-A\int \frac{1}{b^2+c^2-(c\cos(x)-b\sin(x))^2}d(c\cos(x)-b\sin(x))-\frac{2(c\cos(x)(bB+cC)-b\sin(x)(bB+cC))}{(b^2+c^2)(b\cos(x)+c\sin(x))}}{2\left(b^2+c^2\right)}-\frac{-Ab\sin(x)+Ac\cos(x)-bC+Bc}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{-\frac{A\text{arctanh}\left(\frac{c\cos(x)-b\sin(x)}{\sqrt{b^2+c^2}}\right)}{\sqrt{b^2+c^2}}-\frac{2(c\cos(x)(bB+cC)-b\sin(x)(bB+cC))}{(b^2+c^2)(b\cos(x)+c\sin(x))}}{2\left(b^2+c^2\right)}-\frac{-Ab\sin(x)+Ac\cos(x)-bC+Bc}{2\left(b^2+c^2\right)(b\cos(x)+c\sin(x))^2}$$

input
```
Int[(A + B*Cos[x] + C*Sin[x])/(b*Cos[x] + c*Sin[x])^3,x]
```

output
```
-1/2*(B*c - b*C + A*c*Cos[x] - A*b*Sin[x])/((b^2 + c^2)*(b*Cos[x] + c*Sin[
x])^2) + (-((A*ArcTanh[(c*Cos[x] - b*Sin[x])/Sqrt[b^2 + c^2]])/Sqrt[b^2 +
c^2]) - (2*(c*(b*B + c*C)*Cos[x] - b*(b*B + c*C)*Sin[x]))/((b^2 + c^2)*(b*
Cos[x] + c*Sin[x])))/(2*(b^2 + c^2))
```

3.534.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3553
```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Simp[-d^(-1)   Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c +
d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

rule 3632
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Simp[(a*A - b*B - c*C)/(a^2 - b^2 - c^2)   Int[1/(a + b*Cos[d + e*x] + c*S
in[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
 - c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

rule 3635
```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(-(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(
a^2 - b^2 - c^2))), x] + Simp[1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a + b*Co
s[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n +
2)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x]
/; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[n, -2]
```

3.534.4 Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.69

method	result
default	$-\frac{2\left(-\frac{\left(A\,b^{2}+2A\,c^{2}-2B\,b^{2}-2B\,c^{2}\right)\tan\left(\frac{x}{2}\right)^{3}}{2b\left(b^{2}+c^{2}\right)}-\frac{\left(A\,b^{2}c-2A\,c^{3}+2B\,b^{2}c+2B\,c^{3}+2C\,b^{3}+2Cb\,c^{2}\right)\tan\left(\frac{x}{2}\right)^{2}}{2\left(b^{2}+c^{2}\right)b^{2}}-\frac{\left(A\,b^{2}-2A\,c^{2}+2B\,b^{2}+2B\,c^{2}\right)\tan\left(\frac{x}{2}\right)}{2\left(b^{2}+c^{2}\right)b}\right.}{\left(\tan\left(\frac{x}{2}\right)^{2}b-2c\tan\left(\frac{x}{2}\right)-b\right)^{2}}$
risch	$-\frac{i\left(2iB\,b^{2}{\mathrm e}^{2ix}-2bBc+2C\,b^{2}{\mathrm e}^{2ix}+iA\,b^{2}{\mathrm e}^{ix}+iA\,c^{2}{\mathrm e}^{ix}+2iB\,b^{2}+2C\,c^{2}{\mathrm e}^{2ix}-2C\,c^{2}+2iCbc-2Abc\,{\mathrm e}^{3ix}-iA\,b^{2}{\mathrm e}^{3ix}+iA\,c^{2}{\mathrm e}^{3ix}+2iB\,c^{2}{\mathrm e}^{2}\right.}{\left(c\,{\mathrm e}^{2ix}+ib\,{\mathrm e}^{2ix}-c+ib\right)^{2}\left(-ib+c\right)\left(ib+c\right)^{2}}$

input
```
int((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^3,x,method=_RETURNVERBOSE)
```

output
```
-2*(-1/2*(A*b^2+2*A*c^2-2*B*b^2-2*B*c^2)/b/(b^2+c^2)*tan(1/2*x)^3-1/2*(A*b
^2*c-2*A*c^3+2*B*b^2*c+2*B*c^3+2*C*b^3+2*C*b*c^2)/(b^2+c^2)/b^2*tan(1/2*x)
^2-1/2*(A*b^2-2*A*c^2+2*B*b^2+2*B*c^2)/(b^2+c^2)/b*tan(1/2*x)+1/2*c*A/(b^2
+c^2))/(tan(1/2*x)^2*b-2*c*tan(1/2*x)-b)^2+A/(b^2+c^2)^(3/2)*arctanh(1/2*(
2*b*tan(1/2*x)-2*c)/(b^2+c^2)^(1/2))
```

3.534.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(121) = 242.

Time = 0.26 (sec) , antiderivative size = 311, normalized size of antiderivative = 2.41

$$\int \frac{A+B\cos(x)+C\sin(x)}{(b\cos(x)+c\sin(x))^3}\,dx$$

$$=\frac{2\,Cb^{3}+2\,Bb^{2}c+6\,Cbc^{2}-2\,Bc^{3}-8\left(Bb^{2}c+Cbc^{2}\right)\cos\left(x\right)^{2}+\left(2\,Abc\cos\left(x\right)\sin\left(x\right)+Ac^{2}+\left(Ab^{2}-Ac^{2}\right)\right.}{4\left(b^{4}c^{2}+2\,b^{2}c\right.}$$

input
```
integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="fricas
")
```

output
```
1/4*(2*C*b^3 + 2*B*b^2*c + 6*C*b*c^2 - 2*B*c^3 - 8*(B*b^2*c + C*b*c^2)*cos
(x)^2 + (2*A*b*c*cos(x)*sin(x) + A*c^2 + (A*b^2 - A*c^2)*cos(x)^2)*sqrt(b^
2 + c^2)*log(-(2*b*c*cos(x)*sin(x) + (b^2 - c^2)*cos(x)^2 - 2*b^2 - c^2 +
2*sqrt(b^2 + c^2)*(c*cos(x) - b*sin(x)))/(2*b*c*cos(x)*sin(x) + (b^2 - c^2
)*cos(x)^2 + c^2)) - 2*(A*b^2*c + A*c^3)*cos(x) + 2*(A*b^3 + A*b*c^2 + 2*(
B*b^3 + C*b^2*c - B*b*c^2 - C*c^3)*cos(x))*sin(x))/(b^4*c^2 + 2*b^2*c^4 +
c^6 + (b^6 + b^4*c^2 - b^2*c^4 - c^6)*cos(x)^2 + 2*(b^5*c + 2*b^3*c^3 + b*
c^5)*cos(x)*sin(x))
```

3.534.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(x)+C\sin(x)}{(b\cos(x)+c\sin(x))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))**3,x)
```

output
```
Timed out
```

3.534.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 451 vs. $2(121) = 242$.

Time = 0.38 (sec) , antiderivative size = 451, normalized size of antiderivative = 3.50

$$\int \frac{A+B\cos(x)+C\sin(x)}{(b\cos(x)+c\sin(x))^3}\,dx =$$

$$-\frac{1}{2}A\left(\frac{2\left(b^2c-\frac{(b^3-2bc^2)\sin(x)}{\cos(x)+1}-\frac{(b^2c-2c^3)\sin(x)^2}{(\cos(x)+1)^2}-\frac{(b^3+2bc^2)\sin(x)^3}{(\cos(x)+1)^3}\right)}{b^6+b^4c^2+\frac{4(b^5c+b^3c^3)\sin(x)}{\cos(x)+1}-\frac{2(b^6-b^4c^2-2b^2c^4)\sin(x)^2}{(\cos(x)+1)^2}-\frac{4(b^5c+b^3c^3)\sin(x)^3}{(\cos(x)+1)^3}+\frac{(b^6+b^4c^2)\sin(x)^4}{(\cos(x)+1)^4}}+\frac{\log\left(\frac{c-\frac{b\,s\cdots}{\cos\cdots}}{c-\frac{b\,s\cdots}{\cos\cdots}}\right)}{(b^2\cdots}\right.$$

$$+\frac{2B\left(\frac{b\sin(x)}{\cos(x)+1}+\frac{c\sin(x)^2}{(\cos(x)+1)^2}-\frac{b\sin(x)^3}{(\cos(x)+1)^3}\right)}{b^4+\frac{4b^3c\sin(x)}{\cos(x)+1}-\frac{4b^3c\sin(x)^3}{(\cos(x)+1)^3}+\frac{b^4\sin(x)^4}{(\cos(x)+1)^4}-\frac{2(b^4-2b^2c^2)\sin(x)^2}{(\cos(x)+1)^2}}$$

$$+\frac{2C\sin(x)^2}{\left(b^3+\frac{4b^2c\sin(x)}{\cos(x)+1}-\frac{4b^2c\sin(x)^3}{(\cos(x)+1)^3}+\frac{b^3\sin(x)^4}{(\cos(x)+1)^4}-\frac{2(b^3-2bc^2)\sin(x)^2}{(\cos(x)+1)^2}\right)(\cos(x)+1)^2}$$

input
```
integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="maxima
")
```

output
```
-1/2*A*(2*(b^2*c - (b^3 - 2*b*c^2)*sin(x)/(cos(x) + 1) - (b^2*c - 2*c^3)*s
in(x)^2/(cos(x) + 1)^2 - (b^3 + 2*b*c^2)*sin(x)^3/(cos(x) + 1)^3)/(b^6 + b
^4*c^2 + 4*(b^5*c + b^3*c^3)*sin(x)/(cos(x) + 1) - 2*(b^6 - b^4*c^2 - 2*b^
2*c^4)*sin(x)^2/(cos(x) + 1)^2 - 4*(b^5*c + b^3*c^3)*sin(x)^3/(cos(x) + 1)
^3 + (b^6 + b^4*c^2)*sin(x)^4/(cos(x) + 1)^4) + log((c - b*sin(x)/(cos(x)
+ 1) + sqrt(b^2 + c^2))/(c - b*sin(x)/(cos(x) + 1) - sqrt(b^2 + c^2)))/(b^
2 + c^2)^(3/2)) + 2*B*(b*sin(x)/(cos(x) + 1) + c*sin(x)^2/(cos(x) + 1)^2 -
 b*sin(x)^3/(cos(x) + 1)^3)/(b^4 + 4*b^3*c*sin(x)/(cos(x) + 1) - 4*b^3*c*s
in(x)^3/(cos(x) + 1)^3 + b^4*sin(x)^4/(cos(x) + 1)^4 - 2*(b^4 - 2*b^2*c^2)
*sin(x)^2/(cos(x) + 1)^2) + 2*C*sin(x)^2/((b^3 + 4*b^2*c*sin(x)/(cos(x) +
1) - 4*b^2*c*sin(x)^3/(cos(x) + 1)^3 + b^3*sin(x)^4/(cos(x) + 1)^4 - 2*(b^
3 - 2*b*c^2)*sin(x)^2/(cos(x) + 1)^2)*(cos(x) + 1)^2)
```

3.534.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(121) = 242$.

Time = 0.32 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.09

$$\int \frac{A + B\cos(x) + C\sin(x)}{(b\cos(x) + c\sin(x))^3}\,dx = \frac{A\log\left(\frac{\left|-2\,b\tan\left(\frac{1}{2}\,x\right)+2\,c-2\,\sqrt{b^2+c^2}\right|}{\left|-2\,b\tan\left(\frac{1}{2}\,x\right)+2\,c+2\,\sqrt{b^2+c^2}\right|}\right)}{2\,(b^2+c^2)^{\frac{3}{2}}} + \frac{Ab^3\tan\left(\frac{1}{2}\,x\right)^3 - 2\,Bb^3\tan\left(\frac{1}{2}\,x\right)^3 + 2\,Abc^2\tan\left(\frac{1}{2}\,x\right)^3 - 2\,Bbc^2\tan\left(\frac{1}{2}\,x\right)^3 + 2\,Cb^3\tan\left(\frac{1}{2}\,x\right)^2 + Ab^2c\tan}{}$$

input
```
integrate((A+B*cos(x)+C*sin(x))/(b*cos(x)+c*sin(x))^3,x, algorithm="giac")
```

output
```
1/2*A*log(abs(-2*b*tan(1/2*x) + 2*c - 2*sqrt(b^2 + c^2))/abs(-2*b*tan(1/2*
x) + 2*c + 2*sqrt(b^2 + c^2)))/(b^2 + c^2)^(3/2) + (A*b^3*tan(1/2*x)^3 - 2
*B*b^3*tan(1/2*x)^3 + 2*A*b*c^2*tan(1/2*x)^3 - 2*B*b*c^2*tan(1/2*x)^3 + 2*
C*b^3*tan(1/2*x)^2 + A*b^2*c*tan(1/2*x)^2 + 2*B*b^2*c*tan(1/2*x)^2 + 2*C*b
*c^2*tan(1/2*x)^2 - 2*A*c^3*tan(1/2*x)^2 + 2*B*c^3*tan(1/2*x)^2 + A*b^3*ta
n(1/2*x) + 2*B*b^3*tan(1/2*x) - 2*A*b*c^2*tan(1/2*x) + 2*B*b*c^2*tan(1/2*x
) - A*b^2*c)/((b^4 + b^2*c^2)*(b*tan(1/2*x)^2 - 2*c*tan(1/2*x) - b)^2)
```

3.534.9 Mupad [B] (verification not implemented)

Time = 27.37 (sec) , antiderivative size = 264, normalized size of antiderivative = 2.05

$$\int \frac{A+B\cos(x)+C\sin(x)}{(b\cos(x)+c\sin(x))^3}\,dx$$
$$= \frac{\frac{\tan\left(\frac{x}{2}\right)\,\left(A\,b^2-2\,A\,c^2+2\,B\,b^2+2\,B\,c^2\right)}{b\,\left(b^2+c^2\right)}-\frac{A\,c}{b^2+c^2}+\frac{{\tan\left(\frac{x}{2}\right)}^2\,\left(2\,B\,c^3-2\,A\,c^3+2\,C\,b^3+A\,b^2\,c+2\,B\,b^2\,c+2\,C\,b\,c^2\right)}{b^2\,\left(b^2+c^2\right)}+\frac{{\tan\left(\frac{x}{2}\right)}^3\,\left(A\,b^2+2\,A\,c^2-2\,B\right.}{b\,\left(b^2+c^2\right)}}{b^2-{\tan\left(\frac{x}{2}\right)}^2\,\left(2\,b^2-4\,c^2\right)+b^2\,{\tan\left(\frac{x}{2}\right)}^4+4\,b\,c\,\tan\left(\frac{x}{2}\right)-4\,b\,c\,{\tan\left(\frac{x}{2}\right)}^3}$$
$$+\frac{A\,\mathrm{atan}\left(\frac{b^2\,c\,1\mathrm{i}+c^3\,1\mathrm{i}-b\,\tan\left(\frac{x}{2}\right)\,\left(b^2+c^2\right)\,1\mathrm{i}}{{\left(b^2+c^2\right)}^{3/2}}\right)\,1\mathrm{i}}{{\left(b^2+c^2\right)}^{3/2}}$$

input
```
int((A + B*cos(x) + C*sin(x))/(b*cos(x) + c*sin(x))^3,x)
```

output
```
((tan(x/2)*(A*b^2 - 2*A*c^2 + 2*B*b^2 + 2*B*c^2))/(b*(b^2 + c^2)) - (A*c)/
(b^2 + c^2) + (tan(x/2)^2*(2*B*c^3 - 2*A*c^3 + 2*C*b^3 + A*b^2*c + 2*B*b^2
*c + 2*C*b*c^2))/(b^2*(b^2 + c^2)) + (tan(x/2)^3*(A*b^2 + 2*A*c^2 - 2*B*b^
2 - 2*B*c^2))/(b*(b^2 + c^2)))/(b^2 - tan(x/2)^2*(2*b^2 - 4*c^2) + b^2*tan
(x/2)^4 + 4*b*c*tan(x/2) - 4*b*c*tan(x/2)^3) + (A*atan((b^2*c*1i + c^3*1i
- b*tan(x/2)*(b^2 + c^2)*1i)/(b^2 + c^2)^(3/2))*1i)/(b^2 + c^2)^(3/2)
```

3.535 $\int \frac{A+B\cos(x)}{a+b\cos(x)+c\sin(x)}\,dx$

3.535.1 Optimal result

Integrand size = 19, antiderivative size = 115

$$\int \frac{A+B\cos(x)}{a+b\cos(x)+c\sin(x)}\,dx = \frac{bBx}{b^2+c^2} - \frac{2(abB-A(b^2+c^2))\arctan\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}\,(b^2+c^2)} + \frac{Bc\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}$$

output
```
b*B*x/(b^2+c^2)+B*c*ln(a+b*cos(x)+c*sin(x))/(b^2+c^2)-2*(a*b*B-A*(b^2+c^2)
)*arctan((c+(a-b)*tan(1/2*x))/(a^2-b^2-c^2)^(1/2))/(b^2+c^2)/(a^2-b^2-c^2)
^(1/2)
```

3.535.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.83

$$\int \frac{A+B\cos(x)}{a+b\cos(x)+c\sin(x)}\,dx = \frac{-\frac{2(-abB+A(b^2+c^2))\operatorname{arctanh}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2+c^2}}\right)}{\sqrt{-a^2+b^2+c^2}} + B(bx+c\log(a+b\cos(x)+c\sin(x)))}{b^2+c^2}$$

input
```
Integrate[(A + B*Cos[x])/(a + b*Cos[x] + c*Sin[x]),x]
```

output
```
((-2*(-(a*b*B) + A*(b^2 + c^2))*ArcTanh[(c + (a - b)*Tan[x/2])/Sqrt[-a^2 +
 b^2 + c^2]])/Sqrt[-a^2 + b^2 + c^2] + B*(b*x + c*Log[a + b*Cos[x] + c*Sin
[x]]))/(b^2 + c^2)
```

3.535.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 3617, 3042, 3603, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(x)}{a+b\cos(x)+c\sin(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\cos(x)}{a+b\cos(x)+c\sin(x)}dx$$

$$\Big\downarrow \text{3617}$$

$$\left(A-\frac{abB}{b^2+c^2}\right)\int \frac{1}{a+b\cos(x)+c\sin(x)}dx+\frac{Bc\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\frac{bBx}{b^2+c^2}$$

$$\Big\downarrow \text{3042}$$

$$\left(A-\frac{abB}{b^2+c^2}\right)\int \frac{1}{a+b\cos(x)+c\sin(x)}dx+\frac{Bc\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\frac{bBx}{b^2+c^2}$$

$$\Big\downarrow \text{3603}$$

$$2\left(A-\frac{abB}{b^2+c^2}\right)\int \frac{1}{(a-b)\tan^2\left(\frac{x}{2}\right)+2c\tan\left(\frac{x}{2}\right)+a+b}d\tan\left(\frac{x}{2}\right)+\frac{Bc\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\frac{bBx}{b^2+c^2}$$

$$\Big\downarrow \text{1083}$$

$$-4\left(A-\frac{abB}{b^2+c^2}\right)\int \frac{1}{-\left(2c+2(a-b)\tan\left(\frac{x}{2}\right)\right)^2-4\left(a^2-b^2-c^2\right)}d\left(2c+2(a-b)\tan\left(\frac{x}{2}\right)\right)+\frac{Bc\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\frac{bBx}{b^2+c^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{2\left(A-\frac{abB}{b^2+c^2}\right)\arctan\left(\frac{2(a-b)\tan\left(\frac{x}{2}\right)+2c}{2\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}}+\frac{Bc\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\frac{bBx}{b^2+c^2}$$

input
```
Int[(A + B*Cos[x])/(a + b*Cos[x] + c*Sin[x]),x]
```

output
```
(b*B*x)/(b^2 + c^2) + (2*(A - (a*b*B)/(b^2 + c^2))*ArcTan[(2*c + 2*(a - b)
*Tan[x/2])/(2*Sqrt[a^2 - b^2 - c^2])])/Sqrt[a^2 - b^2 - c^2] + (B*c*Log[a
+ b*Cos[x] + c*Sin[x]])/(b^2 + c^2)
```

3.535.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

rule 3617
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[b*B*((d + e*x)/
(e*(b^2 + c^2))), x] + (Simp[c*B*(Log[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/
(e*(b^2 + c^2))), x] + Simp[(A*(b^2 + c^2) - a*b*B)/(b^2 + c^2)   Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x]) /; FreeQ[{a, b, c, d, e, A, B},
 x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*b*B, 0]
```

3.535. $\int \frac{A+B\cos(x)}{a+b\cos(x)+c\sin(x)}\,dx$

3.535.4 Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.61

method	result
default	$\frac{2B\left(-\frac{c\ln\left(1+\tan\left(\frac{x}{2}\right)^2\right)}{2}+b\arctan\left(\tan\left(\frac{x}{2}\right)\right)\right)}{b^2+c^2}+\frac{\frac{2(aBc-bBc)\ln\left(\tan\left(\frac{x}{2}\right)^2a-\tan\left(\frac{x}{2}\right)^2b+2c\tan\left(\frac{x}{2}\right)+a+b\right)}{2a-2b}+\frac{2\left(Ab^2+Ac^2-abB+Bc^2-(a\right.}{}}{b^2+c^2}$
risch	Expression too large to display

input
```
int((A+B*cos(x))/(a+b*cos(x)+c*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
2*B/(b^2+c^2)*(-1/2*c*ln(1+tan(1/2*x)^2)+b*arctan(tan(1/2*x)))+2/(b^2+c^2)
*(1/2*(B*a*c-B*b*c)/(a-b)*ln(tan(1/2*x)^2*a-tan(1/2*x)^2*b+2*c*tan(1/2*x)+
a+b)+(A*b^2+A*c^2-a*b*B+B*c^2-(B*a*c-B*b*c)*c/(a-b))/(a^2-b^2-c^2)^(1/2)*a
rctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2)))
```

3.535.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(109) = 218$.

Time = 0.32 (sec) , antiderivative size = 625, normalized size of antiderivative = 5.43

$$\int \frac{A+B\cos(x)}{a+b\cos(x)+c\sin(x)}\,dx$$
$$= \left[-\frac{(Bab-Ab^2-Ac^2)\sqrt{-a^2+b^2+c^2}\log\left(\frac{a^2b^2-2\,b^4-c^4-(a^2+3\,b^2)c^2-(2\,a^2b^2-b^4-2\,a^2c^2+c^4)\cos(x)^2-2\,(ab^3+abc^2)\cos(x\right.}{}}{}\right.$$
$$-\frac{2\,(Bab-Ab^2-Ac^2)\sqrt{a^2-b^2-c^2}\arctan\left(-\frac{(ab\cos(x)+ac\sin(x)+b^2+c^2)\sqrt{a^2-b^2-c^2}}{(c^3-(a^2-b^2)c)\cos(x)+(a^2b-b^3-bc^2)\sin(x)}\right)-2\,(Ba^2b-Bb^3-Bb}{2\,(a^2b^2-b^4-c}$$

input
```
integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="fricas")
```

output
```
[-1/2*((B*a*b - A*b^2 - A*c^2)*sqrt(-a^2 + b^2 + c^2)*log((a^2*b^2 - 2*b^4
 - c^4 - (a^2 + 3*b^2)*c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(x)^2
- 2*(a*b^3 + a*b*c^2)*cos(x) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^
3)*c)*cos(x))*sin(x) - 2*(2*a*b*c*cos(x)^2 - a*b*c + (b^2*c + c^3)*cos(x)
- (b^3 + b*c^2 + (a*b^2 - a*c^2)*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2))/(
2*a*b*cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin
(x))) - 2*(B*a^2*b - B*b^3 - B*b*c^2)*x + (B*c^3 - (B*a^2 - B*b^2)*c)*log(
2*a*b*cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin
(x)))/(a^2*b^2 - b^4 - c^4 + (a^2 - 2*b^2)*c^2), -1/2*(2*(B*a*b - A*b^2 -
A*c^2)*sqrt(a^2 - b^2 - c^2)*arctan(-(a*b*cos(x) + a*c*sin(x) + b^2 + c^2)
*sqrt(a^2 - b^2 - c^2)/((c^3 - (a^2 - b^2)*c)*cos(x) + (a^2*b - b^3 - b*c^
2)*sin(x))) - 2*(B*a^2*b - B*b^3 - B*b*c^2)*x + (B*c^3 - (B*a^2 - B*b^2)*c
)*log(2*a*b*cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*
c)*sin(x)))/(a^2*b^2 - b^4 - c^4 + (a^2 - 2*b^2)*c^2)]
```

3.535.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cos(x)}{a + b\cos(x) + c\sin(x)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x)),x)
```

output
```
Timed out
```

3.535.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\cos(x)}{a + b\cos(x) + c\sin(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` f
or more de
```

3.535.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.55

$$\int \frac{A+B\cos(x)}{a+b\cos(x)+c\sin(x)}\,dx = \frac{Bbx}{b^2+c^2} + \frac{Bc\log\left(-a\tan\left(\frac{1}{2}x\right)^2+b\tan\left(\frac{1}{2}x\right)^2-2c\tan\left(\frac{1}{2}x\right)-a-b\right)}{b^2+c^2} - \frac{Bc\log\left(\tan\left(\frac{1}{2}x\right)^2+1\right)}{b^2+c^2} + \frac{2\left(Bab-Ab^2-Ac^2\right)\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}x\right)-b\tan\left(\frac{1}{2}x\right)+c}{\sqrt{a^2-b^2-c^2}}\right)\right)}{\sqrt{a^2-b^2-c^2}\left(b^2+c^2\right)}$$

input
```
integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="giac")
```

output
```
B*b*x/(b^2 + c^2) + B*c*log(-a*tan(1/2*x)^2 + b*tan(1/2*x)^2 - 2*c*tan(1/2
*x) - a - b)/(b^2 + c^2) - B*c*log(tan(1/2*x)^2 + 1)/(b^2 + c^2) + 2*(B*a*
b - A*b^2 - A*c^2)*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*
tan(1/2*x) - b*tan(1/2*x) + c)/sqrt(a^2 - b^2 - c^2)))/(sqrt(a^2 - b^2 - c
^2)*(b^2 + c^2))
```

3.535.9 Mupad [B] (verification not implemented)

Time = 62.05 (sec) , antiderivative size = 1709, normalized size of antiderivative = 14.86

$$\int \frac{A+B\cos(x)}{a+b\cos(x)+c\sin(x)}\,dx = \text{Too large to display}$$

input
```
int((A + B*cos(x))/(a + b*cos(x) + c*sin(x)),x)
```

output
```
(log(32*B^3*a^2 - 32*A*B^2*a^2 + 32*A*B^2*b^2 - 32*A^2*B*b^2 - 32*B^3*a*b
+ 32*A^2*B*a*b - ((B*c^3 + A*b^2*(b^2 - a^2 + c^2)^(1/2) - B*a^2*c + A*c^2
*(b^2 - a^2 + c^2)^(1/2) + B*b^2*c - B*a*b*(b^2 - a^2 + c^2)^(1/2))*(64*A^
2*b^2*c - 32*B^2*a^2*c + 32*B^2*b^2*c + 32*tan(x/2)*(a - b)*(A^2*b^2 + 2*B
^2*a^2 - A^2*c^2 + B^2*b^2 - 3*B^2*c^2 + 4*A*B*c^2 - 2*B^2*a*b - 2*A*B*a*b
) + 64*A*B*a^2*c - 64*A*B*b^2*c - 64*A^2*a*b*c + ((B*c^3 + A*b^2*(b^2 - a^
2 + c^2)^(1/2) - B*a^2*c + A*c^2*(b^2 - a^2 + c^2)^(1/2) + B*b^2*c - B*a*b
*(b^2 - a^2 + c^2)^(1/2))*(32*A*a^2*c^2 - 32*B*b^4 - 32*A*a^2*b^2 - 32*A*b
^4 - 32*B*a^2*b^2 - 32*A*b^2*c^2 + 32*B*a^2*c^2 + 64*B*b^2*c^2 + 64*A*a*b^
3 + 64*B*a*b^3 - 96*B*a*b*c^2 + 32*c*tan(x/2)*(a - b)*(2*A*b^2 + 2*A*c^2 +
 4*B*b^2 + B*c^2 - 2*A*a*b - 4*B*a*b) + (32*(a - b)*(B*c^3 + A*b^2*(b^2 -
a^2 + c^2)^(1/2) - B*a^2*c + A*c^2*(b^2 - a^2 + c^2)^(1/2) + B*b^2*c - B*a
*b*(b^2 - a^2 + c^2)^(1/2))*(3*c^4*tan(x/2) + a*c^3 + 3*b*c^3 + 3*b^3*c +
2*a^2*b^2*tan(x/2) - 2*a^2*c^2*tan(x/2) + 3*b^2*c^2*tan(x/2) - 2*a*b^3*tan
(x/2) + a*b^2*c - 4*a^2*b*c - 2*a*b*c^2*tan(x/2)))/((b^2 + c^2)*(b^2 - a^2
 + c^2))))/((b^2 + c^2)*(b^2 - a^2 + c^2))))/((b^2 + c^2)*(b^2 - a^2 + c^2
)) + 32*B*c*tan(x/2)*(A - B)^2*(a - b))*(B*c^3 + b^2*(A*(b^2 - a^2 + c^2)^
(1/2) + B*c) - B*a^2*c + A*c^2*(b^2 - a^2 + c^2)^(1/2) - B*a*b*(b^2 - a^2
+ c^2)^(1/2)))/((b^2 + c^2)*(b^2 - a^2 + c^2)) - (B*log(tan(x/2) + 1i))/(b
*1i + c) - (B*log(tan(x/2) - 1i)*1i)/(b + c*1i) - (log(32*B^3*a^2 - 32*...
```

3.535. $\int \frac{A+B\cos(x)}{a+b\cos(x)+c\sin(x)}\,dx$

3.536 $\int \frac{A+B\cos(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx$

3.536.1 Optimal result

Integrand size = 19, antiderivative size = 113

$$\int \frac{A+B\cos(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx = \frac{2(aA-bB)\arctan\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{3/2}} + \frac{Bc+Ac\cos(x)-(Ab-aB)\sin(x)}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}$$

output
```
2*(A*a-B*b)*arctan((c+(a-b)*tan(1/2*x))/(a^2-b^2-c^2)^(1/2))/(a^2-b^2-c^2)
^(3/2)+(B*c+A*c*cos(x)-(A*b-B*a)*sin(x))/(a^2-b^2-c^2)/(a+b*cos(x)+c*sin(x
))
```

3.536.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.04

$$\int \frac{A+B\cos(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx = \frac{2(aA-bB)\text{arctanh}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2+c^2}}\right)}{\left(-a^2+b^2+c^2\right)^{3/2}} + \frac{(aA-bB)c+\left(-abB+A\left(b^2+c^2\right)\right)\sin(x)}{b\left(-a^2+b^2+c^2\right)(a+b\cos(x)+c\sin(x))}$$

input
```
Integrate[(A + B*Cos[x])/(a + b*Cos[x] + c*Sin[x])^2,x]
```

output
```
(2*(a*A - b*B)*ArcTanh[(c + (a - b)*Tan[x/2])/Sqrt[-a^2 + b^2 + c^2]])/(-a
^2 + b^2 + c^2)^(3/2) + ((a*A - b*B)*c + (-(a*b*B) + A*(b^2 + c^2))*Sin[x]
)/(b*(-a^2 + b^2 + c^2)*(a + b*Cos[x] + c*Sin[x]))
```

3.536.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3042, 3634, 3042, 3603, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(x)}{(a+b\cos(x)+c\sin(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\cos(x)}{(a+b\cos(x)+c\sin(x))^2}dx$$

$$\Big\downarrow \text{3634}$$

$$\frac{(aA-bB)\int \frac{1}{a+b\cos(x)+c\sin(x)}dx}{a^2-b^2-c^2}+\frac{-\sin(x)(Ab-aB)+Ac\cos(x)+Bc}{(a^2-b^2-c^2)(a+b\cos(x)+c\sin(x))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(aA-bB)\int \frac{1}{a+b\cos(x)+c\sin(x)}dx}{a^2-b^2-c^2}+\frac{-\sin(x)(Ab-aB)+Ac\cos(x)+Bc}{(a^2-b^2-c^2)(a+b\cos(x)+c\sin(x))}$$

$$\Big\downarrow \text{3603}$$

$$\frac{2(aA-bB)\int \frac{1}{(a-b)\tan^2\left(\frac{x}{2}\right)+2c\tan\left(\frac{x}{2}\right)+a+b}d\tan\left(\frac{x}{2}\right)}{a^2-b^2-c^2}+\frac{-\sin(x)(Ab-aB)+Ac\cos(x)+Bc}{(a^2-b^2-c^2)(a+b\cos(x)+c\sin(x))}$$

$$\Big\downarrow \text{1083}$$

$$\frac{-\sin(x)(Ab-aB)+Ac\cos(x)+Bc}{(a^2-b^2-c^2)(a+b\cos(x)+c\sin(x))}-\frac{4(aA-bB)\int \frac{1}{-\left(2c+2(a-b)\tan\left(\frac{x}{2}\right)\right)^2-4(a^2-b^2-c^2)}d\left(2c+2(a-b)\tan\left(\frac{x}{2}\right)\right)}{a^2-b^2-c^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{2(aA-bB)\arctan\left(\frac{2(a-b)\tan\left(\frac{x}{2}\right)+2c}{2\sqrt{a^2-b^2-c^2}}\right)}{(a^2-b^2-c^2)^{3/2}}+\frac{-\sin(x)(Ab-aB)+Ac\cos(x)+Bc}{(a^2-b^2-c^2)(a+b\cos(x)+c\sin(x))}$$

input
```
Int[(A + B*Cos[x])/(a + b*Cos[x] + c*Sin[x])^2,x]
```

output
```
(2*(a*A - b*B)*ArcTan[(2*c + 2*(a - b)*Tan[x/2])/(2*Sqrt[a^2 - b^2 - c^2])
])/(a^2 - b^2 - c^2)^(3/2) + (B*c + A*c*Cos[x] - (A*b - a*B)*Sin[x])/((a^2
 - b^2 - c^2)*(a + b*Cos[x] + c*Sin[x]))
```

3.536.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

rule 3634
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[(c*B + c*A*Co
s[d + e*x] + (a*B - b*A)*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])), x] + Simp[(a*A - b*B)/(a^2 - b^2 - c^2)   Int[1/(a
 + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B},
 x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - b*B, 0]
```

3.536.4 Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.82

method	result
default	$\frac{-\frac{2\left(A a b-A\,b^{2}-A\,c^{2}-B\,a^{2}+a b B+B\,c^{2}\right)\tan\left(\frac{x}{2}\right)}{a^{3}-a^{2} b-a\,b^{2}-a\,c^{2}+b^{3}+c^{2} b}+\frac{2(A a-B b) c}{a^{3}-a^{2} b-a\,b^{2}-a\,c^{2}+b^{3}+c^{2} b}}{\tan\left(\frac{x}{2}\right)^{2} a-\tan\left(\frac{x}{2}\right)^{2} b+2 c\tan\left(\frac{x}{2}\right)+a+b}+\frac{2(A a-B b)\arctan\left(\frac{2(a-b)\tan\left(\frac{x}{2}\right)+2 c}{2\sqrt{a^{2}-b^{2}-c^{2}}}\right)}{\left(a^{2}-b^{2}-c^{2}\right)^{\frac{3}{2}}}$
risch	$\frac{2 i\left(-i A a c\,\mathrm{e}^{i x}+i B b c\,\mathrm{e}^{i x}+A a b\,\mathrm{e}^{i x}-B\,a^{2} \mathrm{e}^{i x}+B\,c^{2} \mathrm{e}^{i x}+A\,b^{2}+A\,c^{2}-a b B\right)}{\left(-a^{2}+b^{2}+c^{2}\right)\left(-i c+b\right)\left(-i c\,\mathrm{e}^{2 i x}+b\,\mathrm{e}^{2 i x}+i c+2 a\,\mathrm{e}^{i x}+b\right)}-\frac{\ln\left(\mathrm{e}^{i x}+\frac{i a c\sqrt{-a^{2}+b^{2}+c^{2}}+i a^{2} b-i b^{3}-i b\,c^{2}+a b\sqrt{-a^{2}+b^{2}+c^{2}}}{\left(b^{2}+c^{2}\right)\sqrt{-a^{2}+b^{2}+c^{2}}}\right.}{\sqrt{-a^{2}+b^{2}+c^{2}}\,\left(a^{2}-b^{2}-c^{2}\right)}$

input
```
int((A+B*cos(x))/(a+b*cos(x)+c*sin(x))^2,x,method=_RETURNVERBOSE)
```

output
```
2*(-(A*a*b-A*b^2-A*c^2-B*a^2+B*a*b+B*c^2)/(a^3-a^2*b-a*b^2-a*c^2+b^3+b*c^2
)*tan(1/2*x)+(A*a-B*b)*c/(a^3-a^2*b-a*b^2-a*c^2+b^3+b*c^2))/(tan(1/2*x)^2*
a-tan(1/2*x)^2*b+2*c*tan(1/2*x)+a+b)+2*(A*a-B*b)/(a^2-b^2-c^2)^(3/2)*arcta
n(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))
```

3.536.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 559 vs. 2(106) = 212.

Time = 0.32 (sec) , antiderivative size = 1277, normalized size of antiderivative = 11.30

$$\int \frac{A+B\cos(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx=\text{Too large to display}$$

input
```
integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="fricas")
```

output
```
[-1/2*(2*B*c^5 - 4*(B*a^2 - B*b^2)*c^3 + (A*a^2*b^2 - B*a*b^3 + (A*a^2 - B
*a*b)*c^2 + (A*a*b^3 - B*b^4 + (A*a*b - B*b^2)*c^2)*cos(x) + ((A*a - B*b)*
c^3 + (A*a*b^2 - B*b^3)*c)*sin(x))*sqrt(-a^2 + b^2 + c^2)*log(-(a^2*b^2 -
2*b^4 - c^4 - (a^2 + 3*b^2)*c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(
x)^2 - 2*(a*b^3 + a*b*c^2)*cos(x) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b
 - b^3)*c)*cos(x))*sin(x) + 2*(2*a*b*c*cos(x)^2 - a*b*c + (b^2*c + c^3)*co
s(x) - (b^3 + b*c^2 + (a*b^2 - a*c^2)*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^
2))/(2*a*b*cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c
)*sin(x))) + 2*(B*a^4 - 2*B*a^2*b^2 + B*b^4)*c + 2*(A*c^5 - (A*a^2 + B*a*b
 - 2*A*b^2)*c^3 + (B*a^3*b - A*a^2*b^2 - B*a*b^3 + A*b^4)*c)*cos(x) - 2*(B
*a^3*b^2 - A*a^2*b^3 - B*a*b^4 + A*b^5 + A*b*c^4 - (A*a^2*b + B*a*b^2 - 2*
A*b^3)*c^2)*sin(x))/(a^5*b^2 - 2*a^3*b^4 + a*b^6 + a*c^6 - (2*a^3 - 3*a*b^
2)*c^4 + (a^5 - 4*a^3*b^2 + 3*a*b^4)*c^2 + (a^4*b^3 - 2*a^2*b^5 + b^7 + b*
c^6 - (2*a^2*b - 3*b^3)*c^4 + (a^4*b - 4*a^2*b^3 + 3*b^5)*c^2)*cos(x) + (c
^7 - (2*a^2 - 3*b^2)*c^5 + (a^4 - 4*a^2*b^2 + 3*b^4)*c^3 + (a^4*b^2 - 2*a^
2*b^4 + b^6)*c)*sin(x)), -(B*c^5 - 2*(B*a^2 - B*b^2)*c^3 - (A*a^2*b^2 - B*
a*b^3 + (A*a^2 - B*a*b)*c^2 + (A*a*b^3 - B*b^4 + (A*a*b - B*b^2)*c^2)*cos(
x) + ((A*a - B*b)*c^3 + (A*a*b^2 - B*b^3)*c)*sin(x))*sqrt(a^2 - b^2 - c^2)
*arctan(-(a*b*cos(x) + a*c*sin(x) + b^2 + c^2)*sqrt(a^2 - b^2 - c^2)/((c^3
 - (a^2 - b^2)*c)*cos(x) + (a^2*b - b^3 - b*c^2)*sin(x))) + (B*a^4 - 2*...
```

3.536.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cos(x)}{(a + b\cos(x) + c\sin(x))^2} \, dx = \text{Timed out}$$

input
```
integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x))**2,x)
```

output
```
Timed out
```

3.536.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\cos(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` f
or more de
```

3.536.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.85

$$\int \frac{A+B\cos(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx$$
$$= -\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}x\right)-b\tan\left(\frac{1}{2}x\right)+c}{\sqrt{a^2-b^2-c^2}}\right)\right)(Aa-Bb)}{(a^2-b^2-c^2)^{\frac{3}{2}}}$$
$$+\frac{2\left(Ba^2\tan\left(\frac{1}{2}x\right)-Aab\tan\left(\frac{1}{2}x\right)-Bab\tan\left(\frac{1}{2}x\right)+Ab^2\tan\left(\frac{1}{2}x\right)+Ac^2\tan\left(\frac{1}{2}x\right)-Bc^2\tan\left(\frac{1}{2}x\right)+A\right.}{(a^3-a^2b-ab^2+b^3-ac^2+bc^2)\left(a\tan\left(\frac{1}{2}x\right)^2-b\tan\left(\frac{1}{2}x\right)^2+2c\tan\left(\frac{1}{2}x\right)+a+b\right)}$$

input
```
integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="giac")
```

output
```
-2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*t
an(1/2*x) + c)/sqrt(a^2 - b^2 - c^2)))*(A*a - B*b)/(a^2 - b^2 - c^2)^(3/2)
 + 2*(B*a^2*tan(1/2*x) - A*a*b*tan(1/2*x) - B*a*b*tan(1/2*x) + A*b^2*tan(1
/2*x) + A*c^2*tan(1/2*x) - B*c^2*tan(1/2*x) + A*a*c - B*b*c)/((a^3 - a^2*b
 - a*b^2 + b^3 - a*c^2 + b*c^2)*(a*tan(1/2*x)^2 - b*tan(1/2*x)^2 + 2*c*tan
(1/2*x) + a + b))
```

3.536.9 Mupad [B] (verification not implemented)

Time = 26.22 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.81

$$\int \frac{A+B\cos(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx = \frac{2\,\mathrm{atanh}\left(\frac{\tan\left(\frac{x}{2}\right)\,(2\,a-2\,b)+\frac{2\,\left(-a^2\,c+b^2\,c+c^3\right)}{-a^2+b^2+c^2}}{2\,\sqrt{-a^2+b^2+c^2}}\right)\,(A\,a-B\,b)}{\left(-a^2+b^2+c^2\right)^{3/2}} - \frac{\frac{2\,(A\,a\,c-B\,b\,c)}{(a-b)\,\left(-a^2+b^2+c^2\right)}+\frac{2\tan\left(\frac{x}{2}\right)\,\left(A\,b^2+B\,a^2+A\,c^2-B\,c^2-A\,a\,b-B\,a\,b\right)}{(a-b)\,\left(-a^2+b^2+c^2\right)}}{(a-b)\,\tan\left(\frac{x}{2}\right)^2+2\,c\tan\left(\frac{x}{2}\right)+a+b}$$

input
```
int((A + B*cos(x))/(a + b*cos(x) + c*sin(x))^2,x)
```

output
```
(2*atanh((tan(x/2)*(2*a - 2*b) + (2*(b^2*c - a^2*c + c^3))/(b^2 - a^2 + c^
2))/(2*(b^2 - a^2 + c^2)^(1/2)))*(A*a - B*b))/(b^2 - a^2 + c^2)^(3/2) - ((
2*(A*a*c - B*b*c))/((a - b)*(b^2 - a^2 + c^2)) + (2*tan(x/2)*(A*b^2 + B*a^
2 + A*c^2 - B*c^2 - A*a*b - B*a*b))/((a - b)*(b^2 - a^2 + c^2)))/(a + b +
2*c*tan(x/2) + tan(x/2)^2*(a - b))
```

3.537 $\int \frac{A+B\cos(x)}{(a+b\cos(x)+c\sin(x))^3}\,dx$

3.537.1 Optimal result

Integrand size = 19, antiderivative size = 200

$$\int \frac{A+B\cos(x)}{(a+b\cos(x)+c\sin(x))^3}\,dx = \frac{(2a^2A-3abB+A(b^2+c^2))\arctan\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{(a^2-b^2-c^2)^{5/2}} + \frac{Bc+Ac\cos(x)-(Ab-aB)\sin(x)}{2(a^2-b^2-c^2)(a+b\cos(x)+c\sin(x))^2} + \frac{aBc+(3aA-2bB)c\cos(x)-(3aAb-a^2B-2b^2B)\sin(x)}{2(a^2-b^2-c^2)^2(a+b\cos(x)+c\sin(x))}$$

output
```
(2*a^2*A-3*a*b*B+A*(b^2+c^2))*arctan((c+(a-b)*tan(1/2*x))/(a^2-b^2-c^2)^(1
/2))/(a^2-b^2-c^2)^(5/2)+1/2*(B*c+A*c*cos(x)-(A*b-B*a)*sin(x))/(a^2-b^2-c^
2)/(a+b*cos(x)+c*sin(x))^2+1/2*(a*B*c+(3*A*a-2*B*b)*c*cos(x)-(3*A*a*b-B*a^
2-2*B*b^2)*sin(x))/(a^2-b^2-c^2)^2/(a+b*cos(x)+c*sin(x))
```

3.537.2 Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.63

$$\int \frac{A+B\cos(x)}{(a+b\cos(x)+c\sin(x))^3}\,dx = -\frac{(2a^2A-3abB+A(b^2+c^2))\operatorname{arctanh}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2+c^2}}\right)}{(-a^2+b^2+c^2)^{5/2}} + \frac{-6a^3Ac-3aAb^2c+9a^2bBc-3aAc^3-2bc(2a^2A-3abB+A(b^2+c^2))\cos(x)+c(-a^2bB+3aA(b^2+$$

input
```
Integrate[(A + B*Cos[x])/(a + b*Cos[x] + c*Sin[x])^3,x]
```

output
```
-(((2*a^2*A - 3*a*b*B + A*(b^2 + c^2))*ArcTanh[(c + (a - b)*Tan[x/2])/Sqrt
[-a^2 + b^2 + c^2]])/(-a^2 + b^2 + c^2)^(5/2)) + (-6*a^3*A*c - 3*a*A*b^2*c
 + 9*a^2*b*B*c - 3*a*A*c^3 - 2*b*c*(2*a^2*A - 3*a*b*B + A*(b^2 + c^2))*Cos
[x] + c*(-(a^2*b*B) + 3*a*A*(b^2 + c^2) - 2*b*B*(b^2 + c^2))*Cos[2*x] - 8*
a^2*A*b^2*Sin[x] + 2*A*b^4*Sin[x] + 4*a^3*b*B*Sin[x] + 2*a*b^3*B*Sin[x] -
12*a^2*A*c^2*Sin[x] + 2*A*b^2*c^2*Sin[x] + 8*a*b*B*c^2*Sin[x] - 3*a*A*b^3*
Sin[2*x] + a^2*b^2*B*Sin[2*x] + 2*b^4*B*Sin[2*x] - 3*a*A*b*c^2*Sin[2*x] +
2*b^2*B*c^2*Sin[2*x])/(4*b*(-a^2 + b^2 + c^2)^2*(a + b*Cos[x] + c*Sin[x])^
2)
```

3.537.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {3042, 3637, 25, 3042, 3632, 3042, 3603, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(x)}{(a+b\cos(x)+c\sin(x))^3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{A+B\cos(x)}{(a+b\cos(x)+c\sin(x))^3}dx$$

$$\Big\downarrow \; 3637$$

$$\frac{-\sin(x)(Ab-aB)+Ac\cos(x)+Bc}{2(a^2-b^2-c^2)(a+b\cos(x)+c\sin(x))^2} - \frac{\int -\frac{2(aA-bB)-(Ab-aB)\cos(x)-Ac\sin(x)}{(a+b\cos(x)+c\sin(x))^2}dx}{2(a^2-b^2-c^2)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{2(aA-bB)-(Ab-aB)\cos(x)-Ac\sin(x)}{(a+b\cos(x)+c\sin(x))^2}dx}{2\left(a^2-b^2-c^2\right)}+\frac{-\sin(x)(Ab-aB)+Ac\cos(x)+Bc}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{2(aA-bB)-(Ab-aB)\cos(x)-Ac\sin(x)}{(a+b\cos(x)+c\sin(x))^2}dx}{2\left(a^2-b^2-c^2\right)}+\frac{-\sin(x)(Ab-aB)+Ac\cos(x)+Bc}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2}$$

$$\Big\downarrow \; 3632$$

$$\frac{\frac{\left(2a^2A-3abB+A\left(b^2+c^2\right)\right)\int \frac{1}{a+b\cos(x)+c\sin(x)}dx}{a^2-b^2-c^2}+\frac{-\sin(x)\left(a^2(-B)+3aAb-2b^2B\right)+c\cos(x)(3aA-2bB)+aBc}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}}{2\left(a^2-b^2-c^2\right)}+\frac{-\sin(x)(Ab-aB)+Ac\cos(x)+Bc}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\left(2a^2A-3abB+A\left(b^2+c^2\right)\right)\int \frac{1}{a+b\cos(x)+c\sin(x)}dx}{a^2-b^2-c^2}+\frac{-\sin(x)\left(a^2(-B)+3aAb-2b^2B\right)+c\cos(x)(3aA-2bB)+aBc}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}}{2\left(a^2-b^2-c^2\right)}+\frac{-\sin(x)(Ab-aB)+Ac\cos(x)+Bc}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2}$$

$$\Big\downarrow \; 3603$$

$$\frac{\frac{2\left(2a^2A-3abB+A\left(b^2+c^2\right)\right)\int \frac{1}{(a-b)\tan^2\left(\frac{x}{2}\right)+2c\tan\left(\frac{x}{2}\right)+a+b}d\tan\left(\frac{x}{2}\right)}{a^2-b^2-c^2}+\frac{-\sin(x)\left(a^2(-B)+3aAb-2b^2B\right)+c\cos(x)(3aA-2bB)+aBc}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}}{2\left(a^2-b^2-c^2\right)}+\frac{-\sin(x)(Ab-aB)+Ac\cos(x)+Bc}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2}$$

$$\Big\downarrow \; 1083$$

$$\frac{\frac{-\sin(x)\left(a^2(-B)+3aAb-2b^2B\right)+c\cos(x)(3aA-2bB)+aBc}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}-\frac{4\left(2a^2A-3abB+A\left(b^2+c^2\right)\right)\int \frac{1}{-\left(2c+2(a-b)\tan\left(\frac{x}{2}\right)\right)^2-4\left(a^2-b^2-c^2\right)}d\left(2c+2(a-b)\right.}{a^2-b^2-c^2}}{2\left(a^2-b^2-c^2\right)}+\frac{-\sin(x)(Ab-aB)+Ac\cos(x)+Bc}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{2\left(2a^2A-3abB+A\left(b^2+c^2\right)\right)\arctan\left(\frac{2(a-b)\tan\left(\frac{x}{2}\right)+2c}{2\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{3/2}}+\frac{-\sin(x)\left(a^2(-B)+3aAb-2b^2B\right)+c\cos(x)(3aA-2bB)+aBc}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}}{2\left(a^2-b^2-c^2\right)}+\frac{-\sin(x)(Ab-aB)+Ac\cos(x)+Bc}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2}$$

3.537. $\int \frac{A+B\cos(x)}{(a+b\cos(x)+c\sin(x))^3}\,dx$

input
```
Int[(A + B*Cos[x])/(a + b*Cos[x] + c*Sin[x])^3,x]
```

output
```
(B*c + A*c*Cos[x] - (A*b - a*B)*Sin[x])/(2*(a^2 - b^2 - c^2)*(a + b*Cos[x]
 + c*Sin[x])^2) + ((2*(2*a^2*A - 3*a*b*B + A*(b^2 + c^2))*ArcTan[(2*c + 2*
(a - b)*Tan[x/2])/(2*Sqrt[a^2 - b^2 - c^2])])/(a^2 - b^2 - c^2)^(3/2) + (a
*B*c + (3*a*A - 2*b*B)*c*Cos[x] - (3*a*A*b - a^2*B - 2*b^2*B)*Sin[x])/((a^
2 - b^2 - c^2)*(a + b*Cos[x] + c*Sin[x])))/(2*(a^2 - b^2 - c^2))
```

3.537.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

rule 3632
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Simp[(a*A - b*B - c*C)/(a^2 - b^2 - c^2)   Int[1/(a + b*Cos[d + e*x] + c*S
in[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
 - c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

rule 3637
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))*((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(c*B + c
*A*Cos[d + e*x] + (a*B - b*A)*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d
 + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Simp[1/((n + 1)*(a^2
- b^2 - c^2))   Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n +
 1)*(a*A - b*B) + (n + 2)*(a*B - b*A)*Cos[d + e*x] - (n + 2)*c*A*Sin[d + e*
x], x], x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && LtQ[n, -1] && NeQ[a^2
- b^2 - c^2, 0] && NeQ[n, -2]
```

3.537.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 852 vs. $2(190) = 380$.

Time = 1.73 (sec) , antiderivative size = 853, normalized size of antiderivative = 4.26

method	result
default	$-\frac{\frac{\left(4A\,a^{3}b-7A\,a^{2}b^{2}-5A\,a^{2}c^{2}+2Aa\,b^{3}+2Aab\,c^{2}+A\,b^{4}+3A\,b^{2}c^{2}+2A\,c^{4}-2B\,a^{4}+3B\,a^{3}b-2B\,a^{2}b^{2}+4B\,a^{2}c^{2}+3Ba\,b^{3}-2B\,b^{4}-4B\,b^{2}c^{2}-2B\,c^{4}\right)\tan\left(\right.}{(a-b)\left(a^{4}-2a^{2}b^{2}-2a^{2}c^{2}+b^{4}+2b^{2}c^{2}+c^{4}\right)}}{}$
risch	Expression too large to display

input
```
int((A+B*cos(x))/(a+b*cos(x)+c*sin(x))^3,x,method=_RETURNVERBOSE)
```

output
```
2*(-1/2*(4*A*a^3*b-7*A*a^2*b^2-5*A*a^2*c^2+2*A*a*b^3+2*A*a*b*c^2+A*b^4+3*A
*b^2*c^2+2*A*c^4-2*B*a^4+3*B*a^3*b-2*B*a^2*b^2+4*B*a^2*c^2+3*B*a*b^3-2*B*b
^4-4*B*b^2*c^2-2*B*c^4)/(a-b)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)*
tan(1/2*x)^3+1/2*c*(4*A*a^4-12*A*a^3*b+13*A*a^2*b^2+7*A*a^2*c^2-6*A*a*b^3-
6*A*a*b*c^2+A*b^4-A*b^2*c^2-2*A*c^4+2*B*a^4-9*B*a^3*b+14*B*a^2*b^2-4*B*a^2
*c^2-9*B*a*b^3+2*B*b^4+4*B*b^2*c^2+2*B*c^4)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2
*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tan(1/2*x)^2-1/2*(4*A*a^4*b-5*A*a^3*b^2-11*A
*a^3*c^2-3*A*a^2*b^3+3*A*a^2*b*c^2+5*A*a*b^4+7*A*a*b^2*c^2+2*A*a*c^4-A*b^5
+A*b^3*c^2+2*A*b*c^4-2*B*a^5+3*B*a^4*b-B*a^3*b^2+4*B*a^3*c^2-B*a^2*b^3+8*B
*a^2*b*c^2+3*B*a*b^4-8*B*a*b^2*c^2-2*B*a*c^4-2*B*b^5-4*B*b^3*c^2-2*B*b*c^4
)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tan(1/2*x)+1
/2*c*(4*A*a^4-3*A*a^2*b^2-A*a^2*c^2-A*b^4-A*b^2*c^2-5*B*a^3*b+5*B*a*b^3+2*
B*a*b*c^2)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2))/(t
an(1/2*x)^2*a-tan(1/2*x)^2*b+2*c*tan(1/2*x)+a+b)^2+(2*A*a^2+A*b^2+A*c^2-3*
B*a*b)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-b^2-c^2)^(1/2)*arc
tan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))
```

3.537.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1620 vs. $2(187) = 374$.

Time = 0.45 (sec) , antiderivative size = 3402, normalized size of antiderivative = 17.01

$$\int \frac{A + B\cos(x)}{(a + b\cos(x) + c\sin(x))^3} \, dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="fricas")
```

output
```
[1/4*(2*B*c^7 - 2*(3*B*a^2 - 3*A*a*b - B*b^2)*c^5 + 2*(3*B*a^4 - 3*A*a^3*b
 - 5*B*a^2*b^2 + 6*A*a*b^3 - B*b^4)*c^3 - 4*((3*A*a*b - 2*B*b^2)*c^5 - (3*
A*a^3*b - B*a^2*b^2 - 6*A*a*b^3 + 4*B*b^4)*c^3 + (B*a^4*b^2 - 3*A*a^3*b^3
+ B*a^2*b^4 + 3*A*a*b^5 - 2*B*b^6)*c)*cos(x)^2 - (2*A*a^4*b^2 - 3*B*a^3*b^
3 + A*a^2*b^4 + A*c^6 + (3*A*a^2 - 3*B*a*b + 2*A*b^2)*c^4 + (2*A*a^4 - 3*B
*a^3*b + 4*A*a^2*b^2 - 3*B*a*b^3 + A*b^4)*c^2 + (2*A*a^2*b^4 - 3*B*a*b^5 +
 A*b^6 + A*b^4*c^2 - A*c^6 - (2*A*a^2 - 3*B*a*b + A*b^2)*c^4)*cos(x)^2 + 2
*(2*A*a^3*b^3 - 3*B*a^2*b^4 + A*a*b^5 + A*a*b*c^4 + (2*A*a^3*b - 3*B*a^2*b
^2 + 2*A*a*b^3)*c^2)*cos(x) + 2*(A*a*c^5 + (2*A*a^3 - 3*B*a^2*b + 2*A*a*b^
2)*c^3 + (2*A*a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4)*c + (A*b*c^5 + (2*A*a^2*b -
 3*B*a*b^2 + 2*A*b^3)*c^3 + (2*A*a^2*b^3 - 3*B*a*b^4 + A*b^5)*c)*cos(x))*s
in(x))*sqrt(-a^2 + b^2 + c^2)*log(-(a^2*b^2 - 2*b^4 - c^4 - (a^2 + 3*b^2)*
c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(x)^2 - 2*(a*b^3 + a*b*c^2)*c
os(x) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*cos(x))*sin(x) +
2*(2*a*b*c*cos(x)^2 - a*b*c + (b^2*c + c^3)*cos(x) - (b^3 + b*c^2 + (a*b^2
 - a*c^2)*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2))/(2*a*b*cos(x) + (b^2 - c
^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x))) - 2*(B*a^6 - 4*B*
a^4*b^2 + 3*A*a^3*b^3 + 2*B*a^2*b^4 - 3*A*a*b^5 + B*b^6)*c + 2*(A*c^7 - (5
*A*a^2 - B*a*b - 3*A*b^2)*c^5 + (4*A*a^4 + B*a^3*b - 10*A*a^2*b^2 + 2*B*a*
b^3 + 3*A*b^4)*c^3 - (2*B*a^5*b - 4*A*a^4*b^2 - B*a^3*b^3 + 5*A*a^2*b^4...
```

3.537.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cos(x)}{(a + b\cos(x) + c\sin(x))^3} \, dx = \text{Timed out}$$

input
```
integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x))**3,x)
```

output
```
Timed out
```

3.537.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\cos(x)}{(a+b\cos(x)+c\sin(x))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` f
or more de
```

3.537.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1162 vs. $2(187) = 374$.

Time = 0.37 (sec) , antiderivative size = 1162, normalized size of antiderivative = 5.81

$$\int \frac{A+B\cos(x)}{(a+b\cos(x)+c\sin(x))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="giac")
```

output
```
-(2*A*a^2 - 3*B*a*b + A*b^2 + A*c^2)*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a +
2*b) + arctan(-(a*tan(1/2*x) - b*tan(1/2*x) + c)/sqrt(a^2 - b^2 - c^2)))/(
(a^4 - 2*a^2*b^2 + b^4 - 2*a^2*c^2 + 2*b^2*c^2 + c^4)*sqrt(a^2 - b^2 - c^2
)) + (2*B*a^5*tan(1/2*x)^3 - 4*A*a^4*b*tan(1/2*x)^3 - 5*B*a^4*b*tan(1/2*x)
^3 + 11*A*a^3*b^2*tan(1/2*x)^3 + 5*B*a^3*b^2*tan(1/2*x)^3 - 9*A*a^2*b^3*ta
n(1/2*x)^3 - 5*B*a^2*b^3*tan(1/2*x)^3 + A*a*b^4*tan(1/2*x)^3 + 5*B*a*b^4*t
an(1/2*x)^3 + A*b^5*tan(1/2*x)^3 - 2*B*b^5*tan(1/2*x)^3 + 5*A*a^3*c^2*tan(
1/2*x)^3 - 4*B*a^3*c^2*tan(1/2*x)^3 - 7*A*a^2*b*c^2*tan(1/2*x)^3 + 4*B*a^2
*b*c^2*tan(1/2*x)^3 - A*a*b^2*c^2*tan(1/2*x)^3 + 4*B*a*b^2*c^2*tan(1/2*x)^
3 + 3*A*b^3*c^2*tan(1/2*x)^3 - 4*B*b^3*c^2*tan(1/2*x)^3 - 2*A*a*c^4*tan(1/
2*x)^3 + 2*B*a*c^4*tan(1/2*x)^3 + 2*A*b*c^4*tan(1/2*x)^3 - 2*B*b*c^4*tan(1
/2*x)^3 + 4*A*a^4*c*tan(1/2*x)^2 + 2*B*a^4*c*tan(1/2*x)^2 - 12*A*a^3*b*c*t
an(1/2*x)^2 - 9*B*a^3*b*c*tan(1/2*x)^2 + 13*A*a^2*b^2*c*tan(1/2*x)^2 + 14*
B*a^2*b^2*c*tan(1/2*x)^2 - 6*A*a*b^3*c*tan(1/2*x)^2 - 9*B*a*b^3*c*tan(1/2*
x)^2 + A*b^4*c*tan(1/2*x)^2 + 2*B*b^4*c*tan(1/2*x)^2 + 7*A*a^2*c^3*tan(1/2
*x)^2 - 4*B*a^2*c^3*tan(1/2*x)^2 - 6*A*a*b*c^3*tan(1/2*x)^2 - A*b^2*c^3*ta
n(1/2*x)^2 + 4*B*b^2*c^3*tan(1/2*x)^2 - 2*A*c^5*tan(1/2*x)^2 + 2*B*c^5*tan
(1/2*x)^2 + 2*B*a^5*tan(1/2*x) - 4*A*a^4*b*tan(1/2*x) - 3*B*a^4*b*tan(1/2*
x) + 5*A*a^3*b^2*tan(1/2*x) + B*a^3*b^2*tan(1/2*x) + 3*A*a^2*b^3*tan(1/2*x
) + B*a^2*b^3*tan(1/2*x) - 5*A*a*b^4*tan(1/2*x) - 3*B*a*b^4*tan(1/2*x) ...
```

3.537.9 Mupad [B] (verification not implemented)

Time = 31.45 (sec) , antiderivative size = 946, normalized size of antiderivative = 4.73

$$\int \frac{A+B\cos(x)}{(a+b\cos(x)+c\sin(x))^3}\,dx =$$

$$-\frac{\frac{-4\,A\,a^4\,c+5\,B\,a^3\,b\,c+3\,A\,a^2\,b^2\,c+A\,a^2\,c^3-5\,B\,a\,b^3\,c-2\,B\,a\,b\,c^3+A\,b^4\,c+A\,b^2\,c^3}{(a-b)^2\,(a^4-2\,a^2\,b^2-2\,a^2\,c^2+b^4+2\,b^2\,c^2+c^4)}-\frac{\tan\left(\frac{x}{2}\right)\,(A\,b^5+2\,B\,a^5+2\,B\,b^5+3\,A\,a^2\,b^3+5\,A\,a^3\,b^2+11\,A\cdots}{\cdots}}{\cdots}$$

$$-\frac{\mathrm{atanh}\left(\frac{2\,a^4\,c-4\,a^2\,b^2\,c-4\,a^2\,c^3+2\,b^4\,c+4\,b^2\,c^3+2\,c^5}{2\,(-a^2+b^2+c^2)^{5/2}}+\frac{\tan\left(\frac{x}{2}\right)\,(2\,a-2\,b)\,(a^4-2\,a^2\,b^2-2\,a^2\,c^2+b^4+2\,b^2\,c^2+c^4)}{2\,(-a^2+b^2+c^2)^{5/2}}\right)\,(2\,A\,a^2-3\,B\,a\,b\cdots}{\left(-a^2+b^2+c^2\right)^{5/2}}$$

input
```
int((A + B*cos(x))/(a + b*cos(x) + c*sin(x))^3,x)
```

output

```
- ((A*a^2*c^3 + A*b^2*c^3 - 4*A*a^4*c + A*b^4*c - 2*B*a*b*c^3 - 5*B*a*b^3*
c + 5*B*a^3*b*c + 3*A*a^2*b^2*c)/((a - b)^2*(a^4 + b^4 + c^4 - 2*a^2*b^2 -
 2*a^2*c^2 + 2*b^2*c^2)) - (tan(x/2)*(A*b^5 + 2*B*a^5 + 2*B*b^5 + 3*A*a^2*
b^3 + 5*A*a^3*b^2 + 11*A*a^3*c^2 + B*a^2*b^3 + B*a^3*b^2 - A*b^3*c^2 - 4*B
*a^3*c^2 + 4*B*b^3*c^2 - 5*A*a*b^4 - 4*A*a^4*b - 2*A*a*c^4 - 3*B*a*b^4 - 3
*B*a^4*b - 2*A*b*c^4 + 2*B*a*c^4 + 2*B*b*c^4 - 7*A*a*b^2*c^2 - 3*A*a^2*b*c
^2 + 8*B*a*b^2*c^2 - 8*B*a^2*b*c^2))/((a - b)^2*(a^4 + b^4 + c^4 - 2*a^2*b
^2 - 2*a^2*c^2 + 2*b^2*c^2)) - (tan(x/2)^2*(2*B*c^5 - 2*A*c^5 + 7*A*a^2*c^
3 - A*b^2*c^3 - 4*B*a^2*c^3 + 4*B*b^2*c^3 + 4*A*a^4*c + A*b^4*c + 2*B*a^4*
c + 2*B*b^4*c - 6*A*a*b*c^3 - 6*A*a*b^3*c - 12*A*a^3*b*c - 9*B*a*b^3*c - 9
*B*a^3*b*c + 13*A*a^2*b^2*c + 14*B*a^2*b^2*c))/((a - b)^2*(a^4 + b^4 + c^4
 - 2*a^2*b^2 - 2*a^2*c^2 + 2*b^2*c^2)) + (tan(x/2)^3*(A*b^4 - 2*B*a^4 + 2*
A*c^4 - 2*B*b^4 - 2*B*c^4 - 7*A*a^2*b^2 - 5*A*a^2*c^2 - 2*B*a^2*b^2 + 3*A*
b^2*c^2 + 4*B*a^2*c^2 - 4*B*b^2*c^2 + 2*A*a*b^3 + 4*A*a^3*b + 3*B*a*b^3 +
3*B*a^3*b + 2*A*a*b*c^2))/((a - b)*(a^4 + b^4 + c^4 - 2*a^2*b^2 - 2*a^2*c^
2 + 2*b^2*c^2)))/(tan(x/2)^4*(a^2 - 2*a*b + b^2) + 2*a*b + tan(x/2)*(4*a*c
 + 4*b*c) + tan(x/2)^3*(4*a*c - 4*b*c) + a^2 + b^2 + tan(x/2)^2*(2*a^2 - 2
*b^2 + 4*c^2)) - (atanh((2*a^4*c + 2*b^4*c + 2*c^5 - 4*a^2*c^3 + 4*b^2*c^3
 - 4*a^2*b^2*c)/(2*(b^2 - a^2 + c^2)^(5/2)) + (tan(x/2)*(2*a - 2*b)*(a^4 +
 b^4 + c^4 - 2*a^2*b^2 - 2*a^2*c^2 + 2*b^2*c^2))/(2*(b^2 - a^2 + c^2)^(...
```

3.537. $\int \frac{A+B\cos(x)}{(a+b\cos(x)+c\sin(x))^3}\,dx$

3.538 $\int \frac{A+B\cos(x)}{a+b\cos(x)+ib\sin(x)}\,dx$

3.538.1 Optimal result

Integrand size = 22, antiderivative size = 84

$$\int \frac{A+B\cos(x)}{a+b\cos(x)+ib\sin(x)}\,dx = \frac{(2aA-bB)x}{2a^2}+\frac{iB\cos(x)}{2a} + \frac{i(2aAb-a^2B-b^2B)\log(a+b\cos(x)+ib\sin(x))}{2a^2b} + \frac{B\sin(x)}{2a}$$

output
```
1/2*(2*A*a-B*b)*x/a^2+1/2*I*B*cos(x)/a+1/2*I*(2*A*a*b-B*a^2-B*b^2)*ln(a+b*
cos(x)+I*b*sin(x))/a^2/b+1/2*B*sin(x)/a
```

3.538.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.75

$$\int \frac{A+B\cos(x)}{a+b\cos(x)+ib\sin(x)}\,dx = \frac{2aAbx+a^2Bx-b^2Bx+2(-2aAb+a^2B+b^2B)\arctan\left(\frac{(a+b)\cot\left(\frac{x}{2}\right)}{a-b}\right)+2iabB\cos(x)+2iaAb\log\left(a^2+\right.}{4a^2b}$$

input
```
Integrate[(A + B*Cos[x])/(a + b*Cos[x] + I*b*Sin[x]),x]
```

output
```
(2*a*A*b*x + a^2*B*x - b^2*B*x + 2*(-2*a*A*b + a^2*B + b^2*B)*ArcTan[((a +
 b)*Cot[x/2])/(a - b)] + (2*I)*a*b*B*Cos[x] + (2*I)*a*A*b*Log[a^2 + b^2 +
2*a*b*Cos[x]] - I*a^2*B*Log[a^2 + b^2 + 2*a*b*Cos[x]] - I*b^2*B*Log[a^2 +
b^2 + 2*a*b*Cos[x]] + 2*a*b*B*Sin[x])/(4*a^2*b)
```

3.538.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3042, 3611}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(x)}{a+ib\sin(x)+b\cos(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\cos(x)}{a+ib\sin(x)+b\cos(x)}dx$$

$$\Big\downarrow \text{3611}$$

$$\frac{i(a^2(-B)+2aAb-b^2B)\log(a+ib\sin(x)+b\cos(x))}{2a^2b}+\frac{x(2aA-bB)}{2a^2}+\frac{B\sin(x)}{2a}+\frac{iB\cos(x)}{2a}$$

input
```
Int[(A + B*Cos[x])/(a + b*Cos[x] + I*b*Sin[x]),x]
```

output
```
((2*a*A - b*B)*x)/(2*a^2) + ((I/2)*B*Cos[x])/a + ((I/2)*(2*a*A*b - a^2*B -
 b^2*B)*Log[a + b*Cos[x] + I*b*Sin[x]])/(a^2*b) + (B*Sin[x])/(2*a)
```

3.538.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3611
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/(cos[(d_.) + (e_.)*(x_)]*(b_.)
+ (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(2*a*A - b*B)*(x
/(2*a^2)), x] + (Simp[B*(Sin[d + e*x]/(2*a*e)), x] - Simp[b*B*(Cos[d + e*x]
/(2*a*c*e)), x] + Simp[(a^2*B - 2*a*b*A + b^2*B)*(Log[RemoveContent[a + b*C
os[d + e*x] + c*Sin[d + e*x], x]]/(2*a^2*c*e)), x]) /; FreeQ[{a, b, c, d, e
, A, B}, x] && EqQ[b^2 + c^2, 0]
```

3.538.4 Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.02

method	result
risch	$\frac{iB\,\mathrm{e}^{-ix}}{2a}+\frac{xA}{a}-\frac{bxB}{2a^2}+\frac{i\ln\left(\mathrm{e}^{ix}+\frac{a}{b}\right)A}{a}-\frac{i\ln\left(\mathrm{e}^{ix}+\frac{a}{b}\right)B}{2b}-\frac{ib\ln\left(\mathrm{e}^{ix}+\frac{a}{b}\right)B}{2a^2}$
default	$-\frac{i(2Aa-Bb)\ln\left(-i+\tan\left(\frac{x}{2}\right)\right)}{2a^2}+\frac{B}{a\left(-i+\tan\left(\frac{x}{2}\right)\right)}+\frac{i\left(2Aab-B\,a^2-B\,b^2\right)\ln\left(ia+ib+a\tan\left(\frac{x}{2}\right)-b\tan\left(\frac{x}{2}\right)\right)}{2a^2b}+\frac{iB\ln\left(\tan\left(\frac{x}{2}\right)+i\right)}{2b}$

input
```
int((A+B*cos(x))/(a+b*cos(x)+I*b*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*I*B/a*exp(-I*x)+1/a*x*A-1/2/a^2*b*x*B+I/a*ln(exp(I*x)+a/b)*A-1/2*I/b*l
n(exp(I*x)+a/b)*B-1/2*I/a^2*b*ln(exp(I*x)+a/b)*B
```

3.538.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.86

$$\int \frac{A+B\cos(x)}{a+b\cos(x)+ib\sin(x)}\,dx = \frac{\left(i\,Bab+(2\,Aab-Bb^2)xe^{(i\,x)}+(-i\,Ba^2+2i\,Aab-i\,Bb^2)e^{(i\,x)}\log\left(\frac{be^{(i\,x)}+a}{b}\right)\right)e^{(-i\,x)}}{2\,a^2b}$$

input
```
integrate((A+B*cos(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="fricas")
```

output
```
1/2*(I*B*a*b + (2*A*a*b - B*b^2)*x*e^(I*x) + (-I*B*a^2 + 2*I*A*a*b - I*B*b
^2)*e^(I*x)*log((b*e^(I*x) + a)/b))*e^(-I*x)/(a^2*b)
```

3.538.6 Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.12

$$\int \frac{A+B\cos(x)}{a+b\cos(x)+ib\sin(x)}\,dx = \begin{cases} \frac{iBe^{-ix}}{2a} & \text{for } a\neq 0 \\ x\left(-\frac{2Aa-Bb}{2a^2}+\frac{2Aa+Ba-Bb}{2a^2}\right) & \text{otherwise} \end{cases} + \frac{x(2Aa-Bb)}{2a^2} - \frac{i(-2Aab+Ba^2+Bb^2)\log\left(\frac{a}{b}+e^{ix}\right)}{2a^2b}$$

input
```
integrate((A+B*cos(x))/(a+b*cos(x)+I*b*sin(x)),x)
```

output
```
Piecewise((I*B*exp(-I*x)/(2*a), Ne(a, 0)), (x*(-(2*A*a - B*b)/(2*a**2) + (
2*A*a + B*a - B*b)/(2*a**2)), True)) + x*(2*A*a - B*b)/(2*a**2) - I*(-2*A*
a*b + B*a**2 + B*b**2)*log(a/b + exp(I*x))/(2*a**2*b)
```

3.538.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\cos(x)}{a+b\cos(x)+ib\sin(x)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*cos(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.538.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(70) = 140$.

Time = 0.27 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.00

$$\int \frac{A+B\cos(x)}{a+b\cos(x)+ib\sin(x)}\,dx = -\frac{(-2iAa+iBb)\log\left(-a\tan\left(\frac{1}{2}x\right)^2+b\tan\left(\frac{1}{2}x\right)^2-2ia\tan\left(\frac{1}{2}x\right)+a+b\right)}{4a^2} - \frac{(2iAa-iBb)\log\left(\tan\left(\frac{1}{2}x\right)-i\right)}{2a^2} + \frac{(2Ba^2-2Aab+Bb^2)\left(x+2\arctan\left(\frac{-ia\cos(x)-a\sin(x)-ia}{a\cos(x)-ia\sin(x)-a+2b}\right)\right)}{4a^2b} - \frac{-2iAa\tan\left(\frac{1}{2}x\right)+iBb\tan\left(\frac{1}{2}x\right)-2Aa-2Ba+Bb}{2a^2\left(\tan\left(\frac{1}{2}x\right)-i\right)}$$

input
```
integrate((A+B*cos(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="giac")
```

output
```
-1/4*(-2*I*A*a + I*B*b)*log(-a*tan(1/2*x)^2 + b*tan(1/2*x)^2 - 2*I*a*tan(1
/2*x) + a + b)/a^2 - 1/2*(2*I*A*a - I*B*b)*log(tan(1/2*x) - I)/a^2 + 1/4*(
2*B*a^2 - 2*A*a*b + B*b^2)*(x + 2*arctan((-I*a*cos(x) - a*sin(x) - I*a)/(a
*cos(x) - I*a*sin(x) - a + 2*b)))/(a^2*b) - 1/2*(-2*I*A*a*tan(1/2*x) + I*B
*b*tan(1/2*x) - 2*A*a - 2*B*a + B*b)/(a^2*(tan(1/2*x) - I))
```

3.538.9 Mupad [B] (verification not implemented)

Time = 28.31 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.18

$$\int \frac{A+B\cos(x)}{a+b\cos(x)+ib\sin(x)}\,dx = \frac{B}{a\left(\tan\left(\frac{x}{2}\right)-\mathrm{i}\right)} + \frac{B\ln\left(\tan\left(\frac{x}{2}\right)+1\mathrm{i}\right)\,1\mathrm{i}}{2b} - \frac{\ln\left(\tan\left(\frac{x}{2}\right)-\mathrm{i}\right)\left(A\,a\,1\mathrm{i}-\frac{B\,b\,1\mathrm{i}}{2}\right)}{a^2} - \frac{\ln\left(a+b-a\tan\left(\frac{x}{2}\right)\,1\mathrm{i}+b\tan\left(\frac{x}{2}\right)\,1\mathrm{i}\right)\left(B\,a^2-2\,A\,a\,b+B\,b^2\right)\,1\mathrm{i}}{2\,a^2\,b}$$

input
```
int((A + B*cos(x))/(a + b*cos(x) + b*sin(x)*1i),x)
```

output
```
B/(a*(tan(x/2) - 1i)) + (B*log(tan(x/2) + 1i)*1i)/(2*b) - (log(tan(x/2) -
1i)*(A*a*1i - (B*b*1i)/2))/a^2 - (log(a + b - a*tan(x/2)*1i + b*tan(x/2)*1
i)*(B*a^2 + B*b^2 - 2*A*a*b)*1i)/(2*a^2*b)
```

3.539 $\int \frac{A+B\cos(x)}{a+b\cos(x)-ib\sin(x)}\,dx$

3.539.1 Optimal result

Integrand size = 22, antiderivative size = 84

$$\int \frac{A+B\cos(x)}{a+b\cos(x)-ib\sin(x)}\,dx = \frac{(2aA-bB)x}{2a^2}-\frac{iB\cos(x)}{2a}-\frac{i(2aAb-a^2B-b^2B)\log(a+b\cos(x)-ib\sin(x))}{2a^2b}+\frac{B\sin(x)}{2a}$$

output
```
1/2*(2*A*a-B*b)*x/a^2-1/2*I*B*cos(x)/a-1/2*I*(2*A*a*b-B*a^2-B*b^2)*ln(a+b*
cos(x)-I*b*sin(x))/a^2/b+1/2*B*sin(x)/a
```

3.539.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.75

$$\int \frac{A+B\cos(x)}{a+b\cos(x)-ib\sin(x)}\,dx = \frac{2aAbx+a^2Bx-b^2Bx+2(-2aAb+a^2B+b^2B)\arctan\left(\frac{(a+b)\cot\left(\frac{x}{2}\right)}{a-b}\right)-2iabB\cos(x)-2iaAb\log\left(a^2+\right.}{4a^2b}$$

input
```
Integrate[(A + B*Cos[x])/(a + b*Cos[x] - I*b*Sin[x]),x]
```

output
```
(2*a*A*b*x + a^2*B*x - b^2*B*x + 2*(-2*a*A*b + a^2*B + b^2*B)*ArcTan[((a +
 b)*Cot[x/2])/(a - b)] - (2*I)*a*b*B*Cos[x] - (2*I)*a*A*b*Log[a^2 + b^2 +
2*a*b*Cos[x]] + I*a^2*B*Log[a^2 + b^2 + 2*a*b*Cos[x]] + I*b^2*B*Log[a^2 +
b^2 + 2*a*b*Cos[x]] + 2*a*b*B*Sin[x])/(4*a^2*b)
```

3.539.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3042, 3611}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(x)}{a-ib\sin(x)+b\cos(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\cos(x)}{a-ib\sin(x)+b\cos(x)}dx$$

$$\Big\downarrow \text{3611}$$

$$-\frac{i(a^2(-B)+2aAb-b^2B)\log(a-ib\sin(x)+b\cos(x))}{2a^2b}+\frac{x(2aA-bB)}{2a^2}+\frac{B\sin(x)}{2a}-\frac{iB\cos(x)}{2a}$$

input
```
Int[(A + B*Cos[x])/(a + b*Cos[x] - I*b*Sin[x]),x]
```

output
```
((2*a*A - b*B)*x)/(2*a^2) - ((I/2)*B*Cos[x])/a - ((I/2)*(2*a*A*b - a^2*B -
 b^2*B)*Log[a + b*Cos[x] - I*b*Sin[x]])/(a^2*b) + (B*Sin[x])/(2*a)
```

3.539.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3611
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/(cos[(d_.) + (e_.)*(x_)]*(b_.)
+ (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(2*a*A - b*B)*(x
/(2*a^2)), x] + (Simp[B*(Sin[d + e*x]/(2*a*e)), x] - Simp[b*B*(Cos[d + e*x]
/(2*a*c*e)), x] + Simp[(a^2*B - 2*a*b*A + b^2*B)*(Log[RemoveContent[a + b*C
os[d + e*x] + c*Sin[d + e*x], x]]/(2*a^2*c*e)), x]) /; FreeQ[{a, b, c, d, e
, A, B}, x] && EqQ[b^2 + c^2, 0]
```

3.539.4 Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.94

method	result
risch	$-\frac{iB\,\mathrm{e}^{ix}}{2a}+\frac{Bx}{2b}-\frac{i\ln\left(\mathrm{e}^{ix}+\frac{b}{a}\right)A}{a}+\frac{i\ln\left(\mathrm{e}^{ix}+\frac{b}{a}\right)B}{2b}+\frac{ib\ln\left(\mathrm{e}^{ix}+\frac{b}{a}\right)B}{2a^2}$
default	$-\frac{iB\ln\left(-i+\tan\left(\frac{x}{2}\right)\right)}{2b}+\frac{i(2Aa-Bb)\ln\left(\tan\left(\frac{x}{2}\right)+i\right)}{2a^2}+\frac{B}{a\left(\tan\left(\frac{x}{2}\right)+i\right)}+\frac{i\left(2Aab-B\,a^2-B\,b^2\right)(a-b)\ln\left(ia+ib-a\tan\left(\frac{x}{2}\right)+b\tan\left(\frac{x}{2}\right)\right)}{2a^2b(-a+b)}$

input
```
int((A+B*cos(x))/(a+b*cos(x)-I*b*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*B/a*exp(I*x)+1/2*B*x/b-I/a*ln(exp(I*x)+b/a)*A+1/2*I/b*ln(exp(I*x)+b
/a)*B+1/2*I/a^2*b*ln(exp(I*x)+b/a)*B
```

3.539.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.67

$$\int \frac{A+B\cos(x)}{a+b\cos(x)-ib\sin(x)}\,dx=\frac{Ba^2x-i\,Babe^{(i\,x)}+\left(i\,Ba^2-2i\,Aab+i\,Bb^2\right)\log\left(\frac{ae^{(i\,x)}+b}{a}\right)}{2\,a^2b}$$

input
```
integrate((A+B*cos(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="fricas")
```

output
```
1/2*(B*a^2*x - I*B*a*b*e^(I*x) + (I*B*a^2 - 2*I*A*a*b + I*B*b^2)*log((a*e^
(I*x) + b)/a))/(a^2*b)
```

3.539.6 Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.87

$$\int \frac{A+B\cos(x)}{a+b\cos(x)-ib\sin(x)}\,dx = \frac{Bx}{2b} + \begin{cases} -\frac{iBe^{ix}}{2a} & \text{for } a \neq 0 \\ x\left(-\frac{B}{2b}+\frac{Ba+Bb}{2ab}\right) & \text{otherwise} \end{cases} + \frac{i(-2Aab+Ba^2+Bb^2)\log\left(e^{ix}+\frac{b}{a}\right)}{2a^2b}$$

input
```
integrate((A+B*cos(x))/(a+b*cos(x)-I*b*sin(x)),x)
```

output
```
B*x/(2*b) + Piecewise((-I*B*exp(I*x)/(2*a), Ne(a, 0)), (x*(-B/(2*b) + (B*a
 + B*b)/(2*a*b)), True)) + I*(-2*A*a*b + B*a**2 + B*b**2)*log(exp(I*x) + b
/a)/(2*a**2*b)
```

3.539.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\cos(x)}{a+b\cos(x)-ib\sin(x)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*cos(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.539.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(70) = 140$.

Time = 0.28 (sec) , antiderivative size = 168, normalized size of antiderivative = 2.00

$$\int \frac{A+B\cos(x)}{a+b\cos(x)-ib\sin(x)}\,dx = -\frac{(2iAa-iBb)\log\left(-a\tan\left(\frac{1}{2}x\right)^2+b\tan\left(\frac{1}{2}x\right)^2+2ia\tan\left(\frac{1}{2}x\right)+a+b\right)}{4a^2} - \frac{(-2iAa+iBb)\log\left(\tan\left(\frac{1}{2}x\right)+i\right)}{2a^2} + \frac{(2Ba^2-2Aab+Bb^2)\left(x+2\arctan\left(\frac{ia\cos(x)-a\sin(x)+ia}{a\cos(x)+ia\sin(x)-a+2b}\right)\right)}{4a^2b} - \frac{2iAa\tan\left(\frac{1}{2}x\right)-iBb\tan\left(\frac{1}{2}x\right)-2Aa-2Ba+Bb}{2a^2\left(\tan\left(\frac{1}{2}x\right)+i\right)}$$

input
```
integrate((A+B*cos(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="giac")
```

output
```
-1/4*(2*I*A*a - I*B*b)*log(-a*tan(1/2*x)^2 + b*tan(1/2*x)^2 + 2*I*a*tan(1/
2*x) + a + b)/a^2 - 1/2*(-2*I*A*a + I*B*b)*log(tan(1/2*x) + I)/a^2 + 1/4*(
2*B*a^2 - 2*A*a*b + B*b^2)*(x + 2*arctan((I*a*cos(x) - a*sin(x) + I*a)/(a*
cos(x) + I*a*sin(x) - a + 2*b)))/(a^2*b) - 1/2*(2*I*A*a*tan(1/2*x) - I*B*b
*tan(1/2*x) - 2*A*a - 2*B*a + B*b)/(a^2*(tan(1/2*x) + I))
```

3.539.9 Mupad [B] (verification not implemented)

Time = 33.71 (sec) , antiderivative size = 584, normalized size of antiderivative = 6.95

$$\int \frac{A+B\cos(x)}{a+b\cos(x)-ib\sin(x)}\,dx = \left(\sum_{k=1}^{3}\ln\left(-(a-b)^2\left(4A^2a^2-4ABab-B^2a^2+B^2b^2\right)1\mathrm{i} - \mathrm{root}\left(a^4b^2d^3\,64\mathrm{i}-ABab^3d\,64\mathrm{i}-ABa^3bd\,32\mathrm{i}+B^2a^2b^2d\,16\mathrm{i}+A^2a^2b^2d\,64\mathrm{i}+B^2b^4d\,16\mathrm{i}+B^2a^4d\,16\mathrm{i}- \right.\right.\right.$$
$$\left.+\tan\left(\frac{x}{2}\right)(a-b)^2\left(Ba-2Aa+Bb\right)^2\right)\mathrm{root}\left(a^4b^2d^3\,64\mathrm{i}-ABab^3d\,64\mathrm{i}-ABa^3bd\,32\mathrm{i}+B^2a^2b^2d\,16\mathrm{i}+A^2a^2b^2d\,64\mathrm{i}+B^2b^4d\,16\mathrm{i}+B^2a^4d\,16\mathrm{i}-32A^2Ba^2b\right.$$
$$\left.\left.+32AB^2ab^2-8B^3a^2b+16AB^2a^3-8B^3b^3,d,k\right)\right)+\frac{B}{a\left(\tan\left(\frac{x}{2}\right)+1\mathrm{i}\right)}$$

input
```
int((A + B*cos(x))/(a + b*cos(x) - b*sin(x)*1i),x)
```

output
```
symsum(log(tan(x/2)*(a - b)^2*(B*a - 2*A*a + B*b)^2 - root(a^4*b^2*d^3*64i
 - A*B*a*b^3*d*64i - A*B*a^3*b*d*32i + B^2*a^2*b^2*d*16i + A^2*a^2*b^2*d*6
4i + B^2*b^4*d*16i + B^2*a^4*d*16i - 32*A^2*B*a^2*b + 32*A*B^2*a*b^2 - 8*B
^3*a^2*b + 16*A*B^2*a^3 - 8*B^3*b^3, d, k)*(4*A*a^3*(a - b)^2 - 8*root(a^4
*b^2*d^3*64i - A*B*a*b^3*d*64i - A*B*a^3*b*d*32i + B^2*a^2*b^2*d*16i + A^2
*a^2*b^2*d*64i + B^2*b^4*d*16i + B^2*a^4*d*16i - 32*A^2*B*a^2*b + 32*A*B^2
*a*b^2 - 8*B^3*a^2*b + 16*A*B^2*a^3 - 8*B^3*b^3, d, k)*a^2*(a - b)^2*(a^2*
tan(x/2) + b^2*tan(x/2) - a^2*1i + b^2*1i - a*b*tan(x/2)) + 4*a*tan(x/2)*(
a - b)^2*(A*a^2*1i + B*a^2*1i + B*b^2*1i - A*a*b*2i - B*a*b*1i)) - (a - b)
^2*(4*A^2*a^2 - B^2*a^2 + B^2*b^2 - 4*A*B*a*b)*1i)*root(a^4*b^2*d^3*64i -
A*B*a*b^3*d*64i - A*B*a^3*b*d*32i + B^2*a^2*b^2*d*16i + A^2*a^2*b^2*d*64i
+ B^2*b^4*d*16i + B^2*a^4*d*16i - 32*A^2*B*a^2*b + 32*A*B^2*a*b^2 - 8*B^3*
a^2*b + 16*A*B^2*a^3 - 8*B^3*b^3, d, k), k, 1, 3) + B/(a*(tan(x/2) + 1i))
```

3.539. $\int \frac{A+B\cos(x)}{a+b\cos(x)-ib\sin(x)}\,dx$

3.540 $\int \frac{A+C\sin(x)}{a+b\cos(x)+c\sin(x)}\,dx$

3.540.1 Optimal result

Integrand size = 19, antiderivative size = 116

$$\int \frac{A+C\sin(x)}{a+b\cos(x)+c\sin(x)}\,dx = \frac{cCx}{b^2+c^2}+\frac{2(A(b^2+c^2)-acC)\arctan\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}\,(b^2+c^2)} - \frac{bC\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}$$

output
```
c*C*x/(b^2+c^2)-b*C*ln(a+b*cos(x)+c*sin(x))/(b^2+c^2)+2*(A*(b^2+c^2)-a*c*C
)*arctan((c+(a-b)*tan(1/2*x))/(a^2-b^2-c^2)^(1/2))/(b^2+c^2)/(a^2-b^2-c^2)
^(1/2)
```

3.540.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.83

$$\int \frac{A+C\sin(x)}{a+b\cos(x)+c\sin(x)}\,dx = \frac{-\frac{2(A(b^2+c^2)-acC)\,\text{arctanh}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2+c^2}}\right)}{\sqrt{-a^2+b^2+c^2}}+C(cx-b\log(a+b\cos(x)+c\sin(x)))}{b^2+c^2}$$

input
```
Integrate[(A + C*Sin[x])/(a + b*Cos[x] + c*Sin[x]),x]
```

output
```
((-2*(A*(b^2 + c^2) - a*c*C)*ArcTanh[(c + (a - b)*Tan[x/2])/Sqrt[-a^2 + b^
2 + c^2]])/Sqrt[-a^2 + b^2 + c^2] + C*(c*x - b*Log[a + b*Cos[x] + c*Sin[x]
]))/(b^2 + c^2)
```

3.540.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3042, 3616, 3042, 3603, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+C\sin(x)}{a+b\cos(x)+c\sin(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+C\sin(x)}{a+b\cos(x)+c\sin(x)}dx$$

$$\Big\downarrow \text{3616}$$

$$\left(A-\frac{acC}{b^2+c^2}\right)\int \frac{1}{a+b\cos(x)+c\sin(x)}dx-\frac{bC\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\frac{cCx}{b^2+c^2}$$

$$\Big\downarrow \text{3042}$$

$$\left(A-\frac{acC}{b^2+c^2}\right)\int \frac{1}{a+b\cos(x)+c\sin(x)}dx-\frac{bC\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\frac{cCx}{b^2+c^2}$$

$$\Big\downarrow \text{3603}$$

$$2\left(A-\frac{acC}{b^2+c^2}\right)\int \frac{1}{(a-b)\tan^2\left(\frac{x}{2}\right)+2c\tan\left(\frac{x}{2}\right)+a+b}d\tan\left(\frac{x}{2}\right)-\frac{bC\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\frac{cCx}{b^2+c^2}$$

$$\Big\downarrow \text{1083}$$

$$-4\left(A-\frac{acC}{b^2+c^2}\right)\int \frac{1}{-\left(2c+2(a-b)\tan\left(\frac{x}{2}\right)\right)^2-4\left(a^2-b^2-c^2\right)}d\left(2c+2(a-b)\tan\left(\frac{x}{2}\right)\right)-\frac{bC\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\frac{cCx}{b^2+c^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{2\left(A-\frac{acC}{b^2+c^2}\right)\arctan\left(\frac{2(a-b)\tan\left(\frac{x}{2}\right)+2c}{2\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}}-\frac{bC\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\frac{cCx}{b^2+c^2}$$

input
```
Int[(A + C*Sin[x])/(a + b*Cos[x] + c*Sin[x]),x]
```

output
```
(c*C*x)/(b^2 + c^2) + (2*(A - (a*c*C)/(b^2 + c^2))*ArcTan[(2*c + 2*(a - b)
*Tan[x/2])/(2*Sqrt[a^2 - b^2 - c^2])])/Sqrt[a^2 - b^2 - c^2] - (b*C*Log[a
+ b*Cos[x] + c*Sin[x]])/(b^2 + c^2)
```

3.540.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

rule 3616
```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[c*C*((d + e*x)/
(e*(b^2 + c^2))), x] + (-Simp[b*C*(Log[a + b*Cos[d + e*x] + c*Sin[d + e*x]]
/(e*(b^2 + c^2))), x] + Simp[(A*(b^2 + c^2) - a*c*C)/(b^2 + c^2)   Int[1/(a
 + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x]) /; FreeQ[{a, b, c, d, e, A, C}
, x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*c*C, 0]
```

3.540. $\int \frac{A+C\sin(x)}{a+b\cos(x)+c\sin(x)}\,dx$

3.540.4 Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.61

method	result
default	$\frac{\frac{2\left(-abC+b^2C\right)\ln\left(\tan\left(\frac{x}{2}\right)^2a-\tan\left(\frac{x}{2}\right)^2b+2c\tan\left(\frac{x}{2}\right)+a+b\right)}{2a-2b}+\frac{2\left(A\,b^2+A\,c^2-acC-Cbc-\frac{\left(-abC+b^2C\right)c}{a-b}\right)\arctan\left(\frac{2(a-b)\tan\left(\frac{x}{2}\right)+2c}{2\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}}}{b^2+c^2}+ -$ 2C
risch	Expression too large to display

input
```
int((A+C*sin(x))/(a+b*cos(x)+c*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
2/(b^2+c^2)*(1/2*(-C*a*b+C*b^2)/(a-b)*ln(tan(1/2*x)^2*a-tan(1/2*x)^2*b+2*c
*tan(1/2*x)+a+b)+(A*b^2+A*c^2-a*c*C-C*b*c-(-C*a*b+C*b^2)*c/(a-b))/(a^2-b^2
-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2)))+2*C/
(b^2+c^2)*(1/2*b*ln(1+tan(1/2*x)^2)+c*arctan(tan(1/2*x)))
```

3.540.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. 2(110) = 220.

Time = 0.32 (sec) , antiderivative size = 625, normalized size of antiderivative = 5.39

$$\int \frac{A+C\sin(x)}{a+b\cos(x)+c\sin(x)}\,dx$$
$$= \left[\frac{(Ab^2-Cac+Ac^2)\sqrt{-a^2+b^2+c^2}\log\left(\frac{a^2b^2-2\,b^4-c^4-(a^2+3\,b^2)c^2-(2\,a^2b^2-b^4-2\,a^2c^2+c^4)\cos(x)^2-2\,(ab^3+abc^2)\cos(x)-}{2\,a}\right.}{}\right.$$

input
```
integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="fricas")
```

output
```
[1/2*((A*b^2 - C*a*c + A*c^2)*sqrt(-a^2 + b^2 + c^2)*log((a^2*b^2 - 2*b^4
- c^4 - (a^2 + 3*b^2)*c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(x)^2 -
 2*(a*b^3 + a*b*c^2)*cos(x) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3
)*c)*cos(x))*sin(x) - 2*(2*a*b*c*cos(x)^2 - a*b*c + (b^2*c + c^3)*cos(x) -
 (b^3 + b*c^2 + (a*b^2 - a*c^2)*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2))/(2
*a*b*cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(
x))) - 2*(C*c^3 - (C*a^2 - C*b^2)*c)*x - (C*a^2*b - C*b^3 - C*b*c^2)*log(2
*a*b*cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(
x)))/(a^2*b^2 - b^4 - c^4 + (a^2 - 2*b^2)*c^2), 1/2*(2*(A*b^2 - C*a*c + A*
c^2)*sqrt(a^2 - b^2 - c^2)*arctan(-(a*b*cos(x) + a*c*sin(x) + b^2 + c^2)*s
qrt(a^2 - b^2 - c^2)/((c^3 - (a^2 - b^2)*c)*cos(x) + (a^2*b - b^3 - b*c^2)
*sin(x))) - 2*(C*c^3 - (C*a^2 - C*b^2)*c)*x - (C*a^2*b - C*b^3 - C*b*c^2)*
log(2*a*b*cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)
*sin(x)))/(a^2*b^2 - b^4 - c^4 + (a^2 - 2*b^2)*c^2)]
```

3.540.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + C\sin(x)}{a + b\cos(x) + c\sin(x)}\,dx = \text{Timed out}$$

input
```
integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x)),x)
```

output
```
Timed out
```

3.540.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A + C\sin(x)}{a + b\cos(x) + c\sin(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` f
or more de
```

3.540.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.53

$$\int \frac{A+C\sin(x)}{a+b\cos(x)+c\sin(x)}\,dx$$
$$= \frac{Ccx}{b^2+c^2} - \frac{Cb\log\left(-a\tan\left(\frac{1}{2}x\right)^2+b\tan\left(\frac{1}{2}x\right)^2-2c\tan\left(\frac{1}{2}x\right)-a-b\right)}{b^2+c^2}$$
$$+ \frac{Cb\log\left(\tan\left(\frac{1}{2}x\right)^2+1\right)}{b^2+c^2}$$
$$- \frac{2\left(Ab^2-Cac+Ac^2\right)\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}x\right)-b\tan\left(\frac{1}{2}x\right)+c}{\sqrt{a^2-b^2-c^2}}\right)\right)}{\sqrt{a^2-b^2-c^2}\left(b^2+c^2\right)}$$

input
```
integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="giac")
```

output
```
C*c*x/(b^2 + c^2) - C*b*log(-a*tan(1/2*x)^2 + b*tan(1/2*x)^2 - 2*c*tan(1/2
*x) - a - b)/(b^2 + c^2) + C*b*log(tan(1/2*x)^2 + 1)/(b^2 + c^2) - 2*(A*b^
2 - C*a*c + A*c^2)*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*
tan(1/2*x) - b*tan(1/2*x) + c)/sqrt(a^2 - b^2 - c^2)))/(sqrt(a^2 - b^2 - c
^2)*(b^2 + c^2))
```

3.540.9 Mupad [B] (verification not implemented)

Time = 62.21 (sec) , antiderivative size = 1741, normalized size of antiderivative = 15.01

$$\int \frac{A+C\sin(x)}{a+b\cos(x)+c\sin(x)}\,dx = \text{Too large to display}$$

input
```
int((A + C*sin(x))/(a + b*cos(x) + c*sin(x)),x)
```

output
```
(C*log(tan(x/2) + 1i))/(b - c*1i) + (C*log(tan(x/2) - 1i)*1i)/(b*1i - c) -
 (log(32*A*C^2*a^2 + 32*A*C^2*b^2 - 64*A*C^2*a*b - 32*A^2*C*a*c + 32*A^2*C
*b*c + 32*C*tan(x/2)*(a - b)*(A^2*b + 2*C^2*a - 2*C^2*b - 2*A*C*c) + ((C*b
^3 + A*b^2*(b^2 - a^2 + c^2)^(1/2) - C*a^2*b + A*c^2*(b^2 - a^2 + c^2)^(1/
2) + C*b*c^2 - C*a*c*(b^2 - a^2 + c^2)^(1/2))*(64*A^2*b^2*c + 32*C^2*a^2*c
 + 32*C^2*b^2*c + 64*A*C*b^3 + 32*tan(x/2)*(a - b)*(A^2*b^2 - A^2*c^2 - 2*
C^2*a^2 + 2*C^2*c^2 + 2*C^2*a*b + 2*A*C*a*c - 4*A*C*b*c) - 128*A*C*a*b^2 +
 64*A*C*a^2*b - 64*A^2*a*b*c - 64*C^2*a*b*c + ((C*b^3 + A*b^2*(b^2 - a^2 +
 c^2)^(1/2) - C*a^2*b + A*c^2*(b^2 - a^2 + c^2)^(1/2) + C*b*c^2 - C*a*c*(b
^2 - a^2 + c^2)^(1/2))*(32*A*b^4 + 32*A*a^2*b^2 - 32*A*a^2*c^2 + 32*A*b^2*
c^2 - 32*tan(x/2)*(a - b)*(2*A*c^3 - 2*C*b^3 + 2*A*b^2*c + 2*C*a*b^2 - 2*C
*a*c^2 + C*b*c^2 - 2*A*a*b*c) - 64*A*a*b^3 + 32*C*a*c^3 - 32*C*b*c^3 + 64*
C*b^3*c - 128*C*a*b^2*c + 64*C*a^2*b*c + (32*(a - b)*(C*b^3 + A*b^2*(b^2 -
 a^2 + c^2)^(1/2) - C*a^2*b + A*c^2*(b^2 - a^2 + c^2)^(1/2) + C*b*c^2 - C*
a*c*(b^2 - a^2 + c^2)^(1/2))*(3*c^4*tan(x/2) + a*c^3 + 3*b*c^3 + 3*b^3*c +
 2*a^2*b^2*tan(x/2) - 2*a^2*c^2*tan(x/2) + 3*b^2*c^2*tan(x/2) - 2*a*b^3*ta
n(x/2) + a*b^2*c - 4*a^2*b*c - 2*a*b*c^2*tan(x/2)))/((b^2 + c^2)*(b^2 - a^
2 + c^2))))/((b^2 + c^2)*(b^2 - a^2 + c^2))))/((b^2 + c^2)*(b^2 - a^2 + c^
2)))*(C*b^3 - b*(C*a^2 - C*c^2) + A*b^2*(b^2 - a^2 + c^2)^(1/2) + A*c^2*(b
^2 - a^2 + c^2)^(1/2) - C*a*c*(b^2 - a^2 + c^2)^(1/2)))/((b^2 + c^2)*(b...
```

3.540. $\int \frac{A+C\sin(x)}{a+b\cos(x)+c\sin(x)}\,dx$

3.541 $\int \frac{A+C\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx$

3.541.1 Optimal result

Integrand size = 19, antiderivative size = 114

$$\int \frac{A+C\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx = \frac{2(aA-cC)\arctan\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{(a^2-b^2-c^2)^{3/2}} - \frac{bC-(Ac-aC)\cos(x)+Ab\sin(x)}{(a^2-b^2-c^2)\,(a+b\cos(x)+c\sin(x))}$$

output
```
2*(A*a-C*c)*arctan((c+(a-b)*tan(1/2*x))/(a^2-b^2-c^2)^(1/2))/(a^2-b^2-c^2)
^(3/2)+(-b*C+(A*c-C*a)*cos(x)-A*b*sin(x))/(a^2-b^2-c^2)/(a+b*cos(x)+c*sin(
x))
```

3.541.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.08

$$\int \frac{A+C\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx = \frac{2(aA-cC)\text{arctanh}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2+c^2}}\right)}{(-a^2+b^2+c^2)^{3/2}} + \frac{aAc-a^2C+b^2C+(A(b^2+c^2)-acC)\sin(x)}{b\,(-a^2+b^2+c^2)\,(a+b\cos(x)+c\sin(x))}$$

input
```
Integrate[(A + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^2,x]
```

output
```
(2*(a*A - c*C)*ArcTanh[(c + (a - b)*Tan[x/2])/Sqrt[-a^2 + b^2 + c^2]])/(-a
^2 + b^2 + c^2)^(3/2) + (a*A*c - a^2*C + b^2*C + (A*(b^2 + c^2) - a*c*C)*S
in[x])/(b*(-a^2 + b^2 + c^2)*(a + b*Cos[x] + c*Sin[x]))
```

3.541.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3042, 3633, 3042, 3603, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+C\sin(x)}{(a+b\cos(x)+c\sin(x))^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+C\sin(x)}{(a+b\cos(x)+c\sin(x))^2}dx$$

$$\Big\downarrow \text{ 3633}$$

$$\frac{(aA-cC)\int \frac{1}{a+b\cos(x)+c\sin(x)}dx}{a^2-b^2-c^2}-\frac{-\cos(x)(Ac-aC)+Ab\sin(x)+bC}{(a^2-b^2-c^2)(a+b\cos(x)+c\sin(x))}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(aA-cC)\int \frac{1}{a+b\cos(x)+c\sin(x)}dx}{a^2-b^2-c^2}-\frac{-\cos(x)(Ac-aC)+Ab\sin(x)+bC}{(a^2-b^2-c^2)(a+b\cos(x)+c\sin(x))}$$

$$\Big\downarrow \text{ 3603}$$

$$\frac{2(aA-cC)\int \frac{1}{(a-b)\tan^2\left(\frac{x}{2}\right)+2c\tan\left(\frac{x}{2}\right)+a+b}d\tan\left(\frac{x}{2}\right)}{a^2-b^2-c^2}-\frac{-\cos(x)(Ac-aC)+Ab\sin(x)+bC}{(a^2-b^2-c^2)(a+b\cos(x)+c\sin(x))}$$

$$\Big\downarrow \text{ 1083}$$

$$-\frac{4(aA-cC)\int \frac{1}{-\left(2c+2(a-b)\tan\left(\frac{x}{2}\right)\right)^2-4(a^2-b^2-c^2)}d\left(2c+2(a-b)\tan\left(\frac{x}{2}\right)\right)}{a^2-b^2-c^2}-\frac{-\cos(x)(Ac-aC)+Ab\sin(x)+bC}{(a^2-b^2-c^2)(a+b\cos(x)+c\sin(x))}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{2(aA-cC)\arctan\left(\frac{2(a-b)\tan\left(\frac{x}{2}\right)+2c}{2\sqrt{a^2-b^2-c^2}}\right)}{(a^2-b^2-c^2)^{3/2}}-\frac{-\cos(x)(Ac-aC)+Ab\sin(x)+bC}{(a^2-b^2-c^2)(a+b\cos(x)+c\sin(x))}$$

input
```
Int[(A + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^2,x]
```

output
```
(2*(a*A - c*C)*ArcTan[(2*c + 2*(a - b)*Tan[x/2])/(2*Sqrt[a^2 - b^2 - c^2])
])/(a^2 - b^2 - c^2)^(3/2) - (b*C - (A*c - a*C)*Cos[x] + A*b*Sin[x])/((a^2
 - b^2 - c^2)*(a + b*Cos[x] + c*Sin[x]))
```

3.541.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

rule 3633
```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[-(b*C + (a*C
- c*A)*Cos[d + e*x] + b*A*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
 e*x] + c*Sin[d + e*x])), x] + Simp[(a*A - c*C)/(a^2 - b^2 - c^2)   Int[1/(
a + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, C}
, x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - c*C, 0]
```

3.541. $\int \frac{A+C\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx$

3.541.4 Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.82

method	result
default	$\frac{-\frac{2\left(Aab-A\,b^{2}-A\,c^{2}+acC-Cbc\right)\tan\left(\frac{x}{2}\right)}{a^{3}-a^{2}b-a\,b^{2}-a\,c^{2}+b^{3}+c^{2}b}+\frac{2\left(Aac-C\,a^{2}+b^{2}C\right)}{a^{3}-a^{2}b-a\,b^{2}-a\,c^{2}+b^{3}+c^{2}b}}{\tan\left(\frac{x}{2}\right)^{2}a-\tan\left(\frac{x}{2}\right)^{2}b+2c\tan\left(\frac{x}{2}\right)+a+b}+\frac{2(Aa-Cc)\arctan\left(\frac{2(a-b)\tan\left(\frac{x}{2}\right)+2c}{2\sqrt{a^{2}-b^{2}-c^{2}}}\right)}{\left(a^{2}-b^{2}-c^{2}\right)^{\frac{3}{2}}}$
risch	$\frac{2i\left(-iA\,b^{2}-iA\,c^{2}+iacC-iAab\,\mathrm{e}^{ix}+iCbc\,\mathrm{e}^{ix}-Aac\,\mathrm{e}^{ix}+C\,a^{2}\mathrm{e}^{ix}-C\,b^{2}\mathrm{e}^{ix}\right)}{\left(a^{2}-b^{2}-c^{2}\right)(ib+c)\left(-ic\,\mathrm{e}^{2ix}+b\,\mathrm{e}^{2ix}+ic+2a\,\mathrm{e}^{ix}+b\right)}-\frac{\ln\left(\mathrm{e}^{ix}+\frac{iac\sqrt{-a^{2}+b^{2}+c^{2}}+ia^{2}b-ib^{3}-ib\,c^{2}+ab\sqrt{-a^{2}+b^{2}-}}{\left(b^{2}+c^{2}\right)\sqrt{-a^{2}+b^{2}+c^{2}}}\right.}{\sqrt{-a^{2}+b^{2}+c^{2}}\,\left(a^{2}-b^{2}-c^{2}\right)}$

input
```
int((A+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x,method=_RETURNVERBOSE)
```

output
```
2*(-(A*a*b-A*b^2-A*c^2+C*a*c-C*b*c)/(a^3-a^2*b-a*b^2-a*c^2+b^3+b*c^2)*tan(
1/2*x)+(A*a*c-C*a^2+C*b^2)/(a^3-a^2*b-a*b^2-a*c^2+b^3+b*c^2))/(tan(1/2*x)^
2*a-tan(1/2*x)^2*b+2*c*tan(1/2*x)+a+b)+2*(A*a-C*c)/(a^2-b^2-c^2)^(3/2)*arc
tan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))
```

3.541.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 569 vs. 2(107) = 214.

Time = 0.32 (sec) , antiderivative size = 1301, normalized size of antiderivative = 11.41

$$\int \frac{A+C\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="fricas")
```

output
```
[1/2*(2*C*a^4*b - 4*C*a^2*b^3 + 2*C*b^5 + 2*C*b*c^4 - 4*(C*a^2*b - C*b^3)*
c^2 - (A*a^2*b^2 - C*a*b^2*c + A*a^2*c^2 - C*a*c^3 + (A*a*b^3 - C*b^3*c +
A*a*b*c^2 - C*b*c^3)*cos(x) + (A*a*b^2*c - C*b^2*c^2 + A*a*c^3 - C*c^4)*si
n(x))*sqrt(-a^2 + b^2 + c^2)*log(-(a^2*b^2 - 2*b^4 - c^4 - (a^2 + 3*b^2)*c
^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(x)^2 - 2*(a*b^3 + a*b*c^2)*co
s(x) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*cos(x))*sin(x) + 2
*(2*a*b*c*cos(x)^2 - a*b*c + (b^2*c + c^3)*cos(x) - (b^3 + b*c^2 + (a*b^2
- a*c^2)*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2))/(2*a*b*cos(x) + (b^2 - c^
2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x))) + 2*(C*a*c^4 - A*c
^5 + (A*a^2 - 2*A*b^2)*c^3 - (C*a^3 - C*a*b^2)*c^2 + (A*a^2*b^2 - A*b^4)*c
)*cos(x) - 2*(A*a^2*b^3 - A*b^5 + C*a*b*c^3 - A*b*c^4 + (A*a^2*b - 2*A*b^3
)*c^2 - (C*a^3*b - C*a*b^3)*c)*sin(x))/(a^5*b^2 - 2*a^3*b^4 + a*b^6 + a*c^
6 - (2*a^3 - 3*a*b^2)*c^4 + (a^5 - 4*a^3*b^2 + 3*a*b^4)*c^2 + (a^4*b^3 - 2
*a^2*b^5 + b^7 + b*c^6 - (2*a^2*b - 3*b^3)*c^4 + (a^4*b - 4*a^2*b^3 + 3*b^
5)*c^2)*cos(x) + (c^7 - (2*a^2 - 3*b^2)*c^5 + (a^4 - 4*a^2*b^2 + 3*b^4)*c^
3 + (a^4*b^2 - 2*a^2*b^4 + b^6)*c)*sin(x)), (C*a^4*b - 2*C*a^2*b^3 + C*b^5
 + C*b*c^4 - 2*(C*a^2*b - C*b^3)*c^2 + (A*a^2*b^2 - C*a*b^2*c + A*a^2*c^2
- C*a*c^3 + (A*a*b^3 - C*b^3*c + A*a*b*c^2 - C*b*c^3)*cos(x) + (A*a*b^2*c
- C*b^2*c^2 + A*a*c^3 - C*c^4)*sin(x))*sqrt(a^2 - b^2 - c^2)*arctan(-(a*b*
cos(x) + a*c*sin(x) + b^2 + c^2)*sqrt(a^2 - b^2 - c^2)/((c^3 - (a^2 - b...
```

3.541.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + C\sin(x)}{(a + b\cos(x) + c\sin(x))^2} \, dx = \text{Timed out}$$

input
```
integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x))**2,x)
```

output
```
Timed out
```

3.541.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+C\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` f
or more de
```

3.541.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.81

$$\int \frac{A+C\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx$$
$$= -\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}x\right)-b\tan\left(\frac{1}{2}x\right)+c}{\sqrt{a^2-b^2-c^2}}\right)\right)(Aa-Cc)}{(a^2-b^2-c^2)^{\frac{3}{2}}}$$
$$-\frac{2\left(Aab\tan\left(\frac{1}{2}x\right)-Ab^2\tan\left(\frac{1}{2}x\right)+Cac\tan\left(\frac{1}{2}x\right)-Cbc\tan\left(\frac{1}{2}x\right)-Ac^2\tan\left(\frac{1}{2}x\right)+Ca^2-Cb^2-Aac\right)}{(a^3-a^2b-ab^2+b^3-ac^2+bc^2)\left(a\tan\left(\frac{1}{2}x\right)^2-b\tan\left(\frac{1}{2}x\right)^2+2c\tan\left(\frac{1}{2}x\right)+a+b\right)}$$

input
```
integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="giac")
```

output
```
-2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*t
an(1/2*x) + c)/sqrt(a^2 - b^2 - c^2)))*(A*a - C*c)/(a^2 - b^2 - c^2)^(3/2)
 - 2*(A*a*b*tan(1/2*x) - A*b^2*tan(1/2*x) + C*a*c*tan(1/2*x) - C*b*c*tan(1
/2*x) - A*c^2*tan(1/2*x) + C*a^2 - C*b^2 - A*a*c)/((a^3 - a^2*b - a*b^2 +
b^3 - a*c^2 + b*c^2)*(a*tan(1/2*x)^2 - b*tan(1/2*x)^2 + 2*c*tan(1/2*x) + a
 + b))
```

3.541.9 Mupad [B] (verification not implemented)

Time = 28.45 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.79

$$\int \frac{A+C\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx = \frac{2\,\mathrm{atanh}\left(\frac{\tan\left(\frac{x}{2}\right)(2\,a-2\,b)+\frac{2\left(-a^2\,c+b^2\,c+c^3\right)}{-a^2+b^2+c^2}}{2\,\sqrt{-a^2+b^2+c^2}}\right)(A\,a-C\,c)}{\left(-a^2+b^2+c^2\right)^{3/2}} - \frac{\frac{2\left(-C\,a^2+A\,c\,a+C\,b^2\right)}{(a-b)\left(-a^2+b^2+c^2\right)}+\frac{2\tan\left(\frac{x}{2}\right)\left(A\,b^2+C\,b\,c-A\,a\,b+A\,c^2-C\,a\,c\right)}{(a-b)\left(-a^2+b^2+c^2\right)}}{(a-b)\,\tan\left(\frac{x}{2}\right)^2+2\,c\tan\left(\frac{x}{2}\right)+a+b}$$

input
```
int((A + C*sin(x))/(a + b*cos(x) + c*sin(x))^2,x)
```

output
```
(2*atanh((tan(x/2)*(2*a - 2*b) + (2*(b^2*c - a^2*c + c^3))/(b^2 - a^2 + c^
2))/(2*(b^2 - a^2 + c^2)^(1/2)))*(A*a - C*c))/(b^2 - a^2 + c^2)^(3/2) - ((
2*(C*b^2 - C*a^2 + A*a*c))/((a - b)*(b^2 - a^2 + c^2)) + (2*tan(x/2)*(A*b^
2 + A*c^2 - A*a*b - C*a*c + C*b*c))/((a - b)*(b^2 - a^2 + c^2)))/(a + b +
2*c*tan(x/2) + tan(x/2)^2*(a - b))
```

3.542 $\int \frac{A+C\sin(x)}{(a+b\cos(x)+c\sin(x))^3}\,dx$

3.542.1 Optimal result

Integrand size = 19, antiderivative size = 200

$$\begin{aligned}\int \frac{A+C\sin(x)}{(a+b\cos(x)+c\sin(x))^3}\,dx &= \frac{(2a^2A+A(b^2+c^2)-3acC)\arctan\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{(a^2-b^2-c^2)^{5/2}}\\ &\quad-\frac{bC-(Ac-aC)\cos(x)+Ab\sin(x)}{2(a^2-b^2-c^2)(a+b\cos(x)+c\sin(x))^2}\\ &\quad-\frac{abC-(3aAc-a^2C-2c^2C)\cos(x)+b(3aA-2cC)\sin(x)}{2(a^2-b^2-c^2)^2(a+b\cos(x)+c\sin(x))}\end{aligned}$$

output
```
(2*a^2*A+A*(b^2+c^2)-3*a*c*C)*arctan((c+(a-b)*tan(1/2*x))/(a^2-b^2-c^2)^(1
/2))/(a^2-b^2-c^2)^(5/2)+1/2*(-b*C+(A*c-C*a)*cos(x)-A*b*sin(x))/(a^2-b^2-c
^2)/(a+b*cos(x)+c*sin(x))^2+1/2*(-a*b*C+(3*A*a*c-C*a^2-2*C*c^2)*cos(x)-b*(
3*A*a-2*C*c)*sin(x))/(a^2-b^2-c^2)^2/(a+b*cos(x)+c*sin(x))
```

3.542.2 Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.80

$$\int \frac{A+C\sin(x)}{(a+b\cos(x)+c\sin(x))^3}\,dx = -\frac{(2a^2A+A(b^2+c^2)-3acC)\,\text{arctanh}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2+c^2}}\right)}{(-a^2+b^2+c^2)^{5/2}} + \frac{-6a^3Ac-3aAb^2c-3aAc^3+2a^4C-4a^2b^2C+2b^4C+5a^2c^2C+4b^2c^2C+2c^4C-2bc(2a^2A+A(b^2+\cdots}{\cdots}$$

input
```
Integrate[(A + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^3,x]
```

output
```
-(((2*a^2*A + A*(b^2 + c^2) - 3*a*c*C)*ArcTanh[(c + (a - b)*Tan[x/2])/Sqrt
[-a^2 + b^2 + c^2]])/(-a^2 + b^2 + c^2)^(5/2)) + (-6*a^3*A*c - 3*a*A*b^2*c
 - 3*a*A*c^3 + 2*a^4*C - 4*a^2*b^2*C + 2*b^4*C + 5*a^2*c^2*C + 4*b^2*c^2*C
 + 2*c^4*C - 2*b*c*(2*a^2*A + A*(b^2 + c^2) - 3*a*c*C)*Cos[x] - c*(-3*a*A*
(b^2 + c^2) + a^2*c*C + 2*c*(b^2 + c^2)*C)*Cos[2*x] - 8*a^2*A*b^2*Sin[x] +
 2*A*b^4*Sin[x] - 12*a^2*A*c^2*Sin[x] + 2*A*b^2*c^2*Sin[x] + 4*a^3*c*C*Sin
[x] + 2*a*b^2*c*C*Sin[x] + 8*a*c^3*C*Sin[x] - 3*a*A*b^3*Sin[2*x] - 3*a*A*b
*c^2*Sin[2*x] + a^2*b*c*C*Sin[2*x] + 2*b^3*c*C*Sin[2*x] + 2*b*c^3*C*Sin[2*
x])/(4*b*(-a^2 + b^2 + c^2)^2*(a + b*Cos[x] + c*Sin[x])^2)
```

3.542.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.13, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {3042, 3636, 25, 3042, 3632, 3042, 3603, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+C\sin(x)}{(a+b\cos(x)+c\sin(x))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+C\sin(x)}{(a+b\cos(x)+c\sin(x))^3}dx$$

$$\Big\downarrow \text{3636}$$

$$-\frac{\int -\frac{2(aA-cC)-Ab\cos(x)-(Ac-aC)\sin(x)}{(a+b\cos(x)+c\sin(x))^2}dx}{2(a^2-b^2-c^2)} - \frac{-\cos(x)(Ac-aC)+Ab\sin(x)+bC}{2(a^2-b^2-c^2)(a+b\cos(x)+c\sin(x))^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{2(aA-cC)-Ab\cos(x)-(Ac-aC)\sin(x)}{(a+b\cos(x)+c\sin(x))^2}dx}{2\left(a^2-b^2-c^2\right)}-\frac{-\cos(x)(Ac-aC)+Ab\sin(x)+bC}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{2(aA-cC)-Ab\cos(x)-(Ac-aC)\sin(x)}{(a+b\cos(x)+c\sin(x))^2}dx}{2\left(a^2-b^2-c^2\right)}-\frac{-\cos(x)(Ac-aC)+Ab\sin(x)+bC}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2}$$

$$\Big\downarrow \; 3632$$

$$\frac{\frac{\left(2a^2A-3acC+A\left(b^2+c^2\right)\right)\int \frac{1}{a+b\cos(x)+c\sin(x)}dx}{a^2-b^2-c^2}-\frac{-\cos(x)\left(a^2(-C)+3aAc-2c^2C\right)+b\sin(x)(3aA-2cC)+abC}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}}{2\left(a^2-b^2-c^2\right)}-\frac{-\cos(x)(Ac-aC)+Ab\sin(x)+bC}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\left(2a^2A-3acC+A\left(b^2+c^2\right)\right)\int \frac{1}{a+b\cos(x)+c\sin(x)}dx}{a^2-b^2-c^2}-\frac{-\cos(x)\left(a^2(-C)+3aAc-2c^2C\right)+b\sin(x)(3aA-2cC)+abC}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}}{2\left(a^2-b^2-c^2\right)}-\frac{-\cos(x)(Ac-aC)+Ab\sin(x)+bC}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2}$$

$$\Big\downarrow \; 3603$$

$$\frac{\frac{2\left(2a^2A-3acC+A\left(b^2+c^2\right)\right)\int \frac{1}{(a-b)\tan^2\left(\frac{x}{2}\right)+2c\tan\left(\frac{x}{2}\right)+a+b}d\tan\left(\frac{x}{2}\right)}{a^2-b^2-c^2}-\frac{-\cos(x)\left(a^2(-C)+3aAc-2c^2C\right)+b\sin(x)(3aA-2cC)+abC}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}}{2\left(a^2-b^2-c^2\right)}-\frac{-\cos(x)(Ac-aC)+Ab\sin(x)+bC}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2}$$

$$\Big\downarrow \; 1083$$

$$\frac{-\frac{4\left(2a^2A-3acC+A\left(b^2+c^2\right)\right)\int \frac{1}{-\left(2c+2(a-b)\tan\left(\frac{x}{2}\right)\right)^2-4\left(a^2-b^2-c^2\right)}d\left(2c+2(a-b)\tan\left(\frac{x}{2}\right)\right)}{a^2-b^2-c^2}-\frac{-\cos(x)\left(a^2(-C)+3aAc-2c^2C\right)+b\sin(x)(3aA-}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}}{2\left(a^2-b^2-c^2\right)}$$
$$\frac{-\cos(x)(Ac-aC)+Ab\sin(x)+bC}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{2\left(2a^2A-3acC+A\left(b^2+c^2\right)\right)\arctan\left(\frac{2(a-b)\tan\left(\frac{x}{2}\right)+2c}{2\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{3/2}}-\frac{-\cos(x)\left(a^2(-C)+3aAc-2c^2C\right)+b\sin(x)(3aA-2cC)+abC}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}}{2\left(a^2-b^2-c^2\right)}-\frac{-\cos(x)(Ac-aC)+Ab\sin(x)+bC}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2}$$

3.542. $\int \frac{A+C\sin(x)}{(a+b\cos(x)+c\sin(x))^3}\,dx$

input
```
Int[(A + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^3,x]
```

output
```
-1/2*(b*C - (A*c - a*C)*Cos[x] + A*b*Sin[x])/((a^2 - b^2 - c^2)*(a + b*Cos
[x] + c*Sin[x])^2) + ((2*(2*a^2*A + A*(b^2 + c^2) - 3*a*c*C)*ArcTan[(2*c +
 2*(a - b)*Tan[x/2])/(2*Sqrt[a^2 - b^2 - c^2])])/(a^2 - b^2 - c^2)^(3/2) -
 (a*b*C - (3*a*A*c - a^2*C - 2*c^2*C)*Cos[x] + b*(3*a*A - 2*c*C)*Sin[x])/(
(a^2 - b^2 - c^2)*(a + b*Cos[x] + c*Sin[x])))/(2*(a^2 - b^2 - c^2))
```

3.542.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

rule 3632
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Simp[(a*A - b*B - c*C)/(a^2 - b^2 - c^2)   Int[1/(a + b*Cos[d + e*x] + c*S
in[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
 - c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

rule 3636
```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(b*C + (a*
C - c*A)*Cos[d + e*x] + b*A*Sin[d + e*x])*((a + b*Cos[d + e*x] + c*Sin[d +
e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Simp[1/((n + 1)*(a^2 - b
^2 - c^2))   Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)
*(a*A - c*C) - (n + 2)*b*A*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, A, C}, x] && LtQ[n, -1] && NeQ[a^2 - b
^2 - c^2, 0] && NeQ[n, -2]
```

3.542.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 831 vs. $2(192) = 384$.

Time = 1.74 (sec) , antiderivative size = 832, normalized size of antiderivative = 4.16

method	result
default	$-\frac{\left(4A\,a^{3}b-7A\,a^{2}b^{2}-5A\,a^{2}c^{2}+2Aa\,b^{3}+2Aab\,c^{2}+A\,b^{4}+3A\,b^{2}c^{2}+2A\,c^{4}+3C\,a^{3}c-6C\,a^{2}bc+3Ca\,b^{2}c\right)\tan\left(\frac{x}{2}\right)^{3}}{\left(a^{4}-2a^{2}b^{2}-2a^{2}c^{2}+b^{4}+2b^{2}c^{2}+c^{4}\right)(a-b)}+\frac{\left(4A\,a^{4}c-12A\,a^{3}bc+13A\,a^{2}b^{2}c+7\dots\right.}{}$
risch	Expression too large to display

input
```
int((A+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x,method=_RETURNVERBOSE)
```

output
```
2*(-1/2*(4*A*a^3*b-7*A*a^2*b^2-5*A*a^2*c^2+2*A*a*b^3+2*A*a*b*c^2+A*b^4+3*A
*b^2*c^2+2*A*c^4+3*C*a^3*c-6*C*a^2*b*c+3*C*a*b^2*c)/(a^4-2*a^2*b^2-2*a^2*c
^2+b^4+2*b^2*c^2+c^4)/(a-b)*tan(1/2*x)^3+1/2*(4*A*a^4*c-12*A*a^3*b*c+13*A*
a^2*b^2*c+7*A*a^2*c^3-6*A*a*b^3*c-6*A*a*b*c^3+A*b^4*c-A*b^2*c^3-2*A*c^5-2*
C*a^5+2*C*a^4*b+4*C*a^3*b^2-5*C*a^3*c^2-4*C*a^2*b^3+14*C*a^2*b*c^2-2*C*a*b
^4-13*C*a*b^2*c^2-2*C*a*c^4+2*C*b^5+4*C*b^3*c^2+2*C*b*c^4)/(a^4-2*a^2*b^2-
2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tan(1/2*x)^2-1/2*(4*A*a^4*b-5
*A*a^3*b^2-11*A*a^3*c^2-3*A*a^2*b^3+3*A*a^2*b*c^2+5*A*a*b^4+7*A*a*b^2*c^2+
2*A*a*c^4-A*b^5+A*b^3*c^2+2*A*b*c^4+5*C*a^4*c-5*C*a^3*b*c-5*C*a^2*b^2*c+4*
C*a^2*c^3+5*C*a*b^3*c-4*C*a*b*c^3)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+
c^4)/(a^2-2*a*b+b^2)*tan(1/2*x)+1/2*(4*A*a^4*c-3*A*a^2*b^2*c-A*a^2*c^3-A*b
^4*c-A*b^2*c^3-2*C*a^5+4*C*a^3*b^2-C*a^3*c^2-2*C*a*b^4+C*a*b^2*c^2)/(a^4-2
*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2))/(tan(1/2*x)^2*a-tan
(1/2*x)^2*b+2*c*tan(1/2*x)+a+b)^2+(2*A*a^2+A*b^2+A*c^2-3*C*a*c)/(a^4-2*a^2
*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*
tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))
```

3.542.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1676 vs. $2(187) = 374$.

Time = 0.47 (sec) , antiderivative size = 3513, normalized size of antiderivative = 17.56

$$\int \frac{A + C\sin(x)}{(a + b\cos(x) + c\sin(x))^3}\, dx = \text{Too large to display}$$

input
```
integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="fricas")
```

output
```
[1/4*(2*C*a^6*b - 6*C*a^4*b^3 + 6*C*a^2*b^5 - 2*C*b^7 + 6*A*a*b*c^5 - 6*C*
b*c^6 + 2*(4*C*a^2*b - 7*C*b^3)*c^4 - 6*(A*a^3*b - 2*A*a*b^3)*c^3 - 2*(2*C
*a^4*b - 7*C*a^2*b^3 + 5*C*b^5)*c^2 - 4*(3*A*a*b*c^5 - 2*C*b*c^6 + (C*a^2*
b - 4*C*b^3)*c^4 - 3*(A*a^3*b - 2*A*a*b^3)*c^3 + (C*a^4*b + C*a^2*b^3 - 2*
C*b^5)*c^2 - 3*(A*a^3*b^3 - A*a*b^5)*c)*cos(x)^2 - (2*A*a^4*b^2 + A*a^2*b^
4 - 3*C*a^3*b^2*c - 3*C*a*c^5 + A*c^6 + (3*A*a^2 + 2*A*b^2)*c^4 - 3*(C*a^3
 + C*a*b^2)*c^3 + (2*A*a^4 + 4*A*a^2*b^2 + A*b^4)*c^2 + (2*A*a^2*b^4 + A*b
^6 - 3*C*a*b^4*c + A*b^4*c^2 + 3*C*a*c^5 - A*c^6 - (2*A*a^2 + A*b^2)*c^4)*
cos(x)^2 + 2*(2*A*a^3*b^3 + A*a*b^5 - 3*C*a^2*b^3*c - 3*C*a^2*b*c^3 + A*a*
b*c^4 + 2*(A*a^3*b + A*a*b^3)*c^2)*cos(x) - 2*(3*C*a^2*b^2*c^2 + 3*C*a^2*c
^4 - A*a*c^5 - 2*(A*a^3 + A*a*b^2)*c^3 - (2*A*a^3*b^2 + A*a*b^4)*c + (3*C*
a*b^3*c^2 + 3*C*a*b*c^4 - A*b*c^5 - 2*(A*a^2*b + A*b^3)*c^3 - (2*A*a^2*b^3
 + A*b^5)*c)*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2)*log(-(a^2*b^2 - 2*b^4
- c^4 - (a^2 + 3*b^2)*c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(x)^2 -
 2*(a*b^3 + a*b*c^2)*cos(x) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3
)*c)*cos(x))*sin(x) + 2*(2*a*b*c*cos(x)^2 - a*b*c + (b^2*c + c^3)*cos(x) -
 (b^3 + b*c^2 + (a*b^2 - a*c^2)*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2))/(2
*a*b*cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(
x))) - 6*(A*a^3*b^3 - A*a*b^5)*c + 2*(C*a*c^6 + A*c^7 - (5*A*a^2 - 3*A*b^2
)*c^5 + (C*a^3 + 2*C*a*b^2)*c^4 + (4*A*a^4 - 10*A*a^2*b^2 + 3*A*b^4)*c^...
```

3.542.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + C\sin(x)}{(a + b\cos(x) + c\sin(x))^3}\, dx = \text{Timed out}$$

input
```
integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x))**3,x)
```

output
```
Timed out
```

3.542.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A + C\sin(x)}{(a + b\cos(x) + c\sin(x))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` f
or more de
```

3.542.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1054 vs. $2(187) = 374$.

Time = 0.38 (sec) , antiderivative size = 1054, normalized size of antiderivative = 5.27

$$\int \frac{A + C\sin(x)}{(a + b\cos(x) + c\sin(x))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="giac")
```

output
```
-(2*A*a^2 + A*b^2 - 3*C*a*c + A*c^2)*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a +
2*b) + arctan(-(a*tan(1/2*x) - b*tan(1/2*x) + c)/sqrt(a^2 - b^2 - c^2)))/(
(a^4 - 2*a^2*b^2 + b^4 - 2*a^2*c^2 + 2*b^2*c^2 + c^4)*sqrt(a^2 - b^2 - c^2
)) - (4*A*a^4*b*tan(1/2*x)^3 - 11*A*a^3*b^2*tan(1/2*x)^3 + 9*A*a^2*b^3*tan
(1/2*x)^3 - A*a*b^4*tan(1/2*x)^3 - A*b^5*tan(1/2*x)^3 + 3*C*a^4*c*tan(1/2*
x)^3 - 9*C*a^3*b*c*tan(1/2*x)^3 + 9*C*a^2*b^2*c*tan(1/2*x)^3 - 3*C*a*b^3*c
*tan(1/2*x)^3 - 5*A*a^3*c^2*tan(1/2*x)^3 + 7*A*a^2*b*c^2*tan(1/2*x)^3 + A*
a*b^2*c^2*tan(1/2*x)^3 - 3*A*b^3*c^2*tan(1/2*x)^3 + 2*A*a*c^4*tan(1/2*x)^3
 - 2*A*b*c^4*tan(1/2*x)^3 + 2*C*a^5*tan(1/2*x)^2 - 2*C*a^4*b*tan(1/2*x)^2
- 4*C*a^3*b^2*tan(1/2*x)^2 + 4*C*a^2*b^3*tan(1/2*x)^2 + 2*C*a*b^4*tan(1/2*
x)^2 - 2*C*b^5*tan(1/2*x)^2 - 4*A*a^4*c*tan(1/2*x)^2 + 12*A*a^3*b*c*tan(1/
2*x)^2 - 13*A*a^2*b^2*c*tan(1/2*x)^2 + 6*A*a*b^3*c*tan(1/2*x)^2 - A*b^4*c*
tan(1/2*x)^2 + 5*C*a^3*c^2*tan(1/2*x)^2 - 14*C*a^2*b*c^2*tan(1/2*x)^2 + 13
*C*a*b^2*c^2*tan(1/2*x)^2 - 4*C*b^3*c^2*tan(1/2*x)^2 - 7*A*a^2*c^3*tan(1/2
*x)^2 + 6*A*a*b*c^3*tan(1/2*x)^2 + A*b^2*c^3*tan(1/2*x)^2 + 2*C*a*c^4*tan(
1/2*x)^2 - 2*C*b*c^4*tan(1/2*x)^2 + 2*A*c^5*tan(1/2*x)^2 + 4*A*a^4*b*tan(1
/2*x) - 5*A*a^3*b^2*tan(1/2*x) - 3*A*a^2*b^3*tan(1/2*x) + 5*A*a*b^4*tan(1/
2*x) - A*b^5*tan(1/2*x) + 5*C*a^4*c*tan(1/2*x) - 5*C*a^3*b*c*tan(1/2*x) -
5*C*a^2*b^2*c*tan(1/2*x) + 5*C*a*b^3*c*tan(1/2*x) - 11*A*a^3*c^2*tan(1/2*x
) + 3*A*a^2*b*c^2*tan(1/2*x) + 7*A*a*b^2*c^2*tan(1/2*x) + A*b^3*c^2*tan...
```

3.542.9 Mupad [B] (verification not implemented)

Time = 33.69 (sec) , antiderivative size = 912, normalized size of antiderivative = 4.56

$$\int \frac{A+C\sin(x)}{(a+b\cos(x)+c\sin(x))^3}\,dx =$$

$$-\frac{\frac{2\,C\,a^5-4\,A\,a^4\,c-4\,C\,a^3\,b^2+C\,a^3\,c^2+3\,A\,a^2\,b^2\,c+A\,a^2\,c^3+2\,C\,a\,b^4-C\,a\,b^2\,c^2+A\,b^4\,c+A\,b^2\,c^3}{(a-b)^2\,(a^4-2\,a^2\,b^2-2\,a^2\,c^2+b^4+2\,b^2\,c^2+c^4)}+\frac{\tan\left(\frac{x}{2}\right)\,(4\,A\,a^4\,b+5\,C\,a^4\,c-5\,A\,a^3\,b^2-5\,C\,a^3}{}}{}$$

$$-\frac{\operatorname{atanh}\left(\frac{2\,a^4\,c-4\,a^2\,b^2\,c-4\,a^2\,c^3+2\,b^4\,c+4\,b^2\,c^3+2\,c^5}{2\,(-a^2+b^2+c^2)^{5/2}}+\frac{\tan\left(\frac{x}{2}\right)\,(2\,a-2\,b)\,(a^4-2\,a^2\,b^2-2\,a^2\,c^2+b^4+2\,b^2\,c^2+c^4)}{2\,(-a^2+b^2+c^2)^{5/2}}\right)\,(2\,A\,a^2-3\,C\,a\,c}{(-a^2+b^2+c^2)^{5/2}}$$

input
```
int((A + C*sin(x))/(a + b*cos(x) + c*sin(x))^3,x)
```

output

```
- ((2*C*a^5 + A*a^2*c^3 + A*b^2*c^3 - 4*C*a^3*b^2 + C*a^3*c^2 - 4*A*a^4*c
+ A*b^4*c + 2*C*a*b^4 + 3*A*a^2*b^2*c - C*a*b^2*c^2)/((a - b)^2*(a^4 + b^4
 + c^4 - 2*a^2*b^2 - 2*a^2*c^2 + 2*b^2*c^2)) + (tan(x/2)*(A*b^3*c^2 - 3*A*
a^2*b^3 - 5*A*a^3*b^2 - 11*A*a^3*c^2 - A*b^5 + 4*C*a^2*c^3 + 5*A*a*b^4 + 4
*A*a^4*b + 2*A*a*c^4 + 2*A*b*c^4 + 5*C*a^4*c - 4*C*a*b*c^3 + 5*C*a*b^3*c -
 5*C*a^3*b*c + 7*A*a*b^2*c^2 + 3*A*a^2*b*c^2 - 5*C*a^2*b^2*c))/((a - b)^2*
(a^4 + b^4 + c^4 - 2*a^2*b^2 - 2*a^2*c^2 + 2*b^2*c^2)) + (tan(x/2)^2*(2*A*
c^5 + 2*C*a^5 - 2*C*b^5 - 7*A*a^2*c^3 + A*b^2*c^3 + 4*C*a^2*b^3 - 4*C*a^3*
b^2 + 5*C*a^3*c^2 - 4*C*b^3*c^2 - 4*A*a^4*c - A*b^4*c + 2*C*a*b^4 - 2*C*a^
4*b + 2*C*a*c^4 - 2*C*b*c^4 + 6*A*a*b*c^3 + 6*A*a*b^3*c + 12*A*a^3*b*c - 1
3*A*a^2*b^2*c + 13*C*a*b^2*c^2 - 14*C*a^2*b*c^2))/((a - b)^2*(a^4 + b^4 +
c^4 - 2*a^2*b^2 - 2*a^2*c^2 + 2*b^2*c^2)) + (tan(x/2)^3*(A*b^4 + 2*A*c^4 -
 7*A*a^2*b^2 - 5*A*a^2*c^2 + 3*A*b^2*c^2 + 2*A*a*b^3 + 4*A*a^3*b + 3*C*a^3
*c + 2*A*a*b*c^2 + 3*C*a*b^2*c - 6*C*a^2*b*c))/((a - b)*(a^4 + b^4 + c^4 -
 2*a^2*b^2 - 2*a^2*c^2 + 2*b^2*c^2)))/(tan(x/2)^4*(a^2 - 2*a*b + b^2) + 2*
a*b + tan(x/2)*(4*a*c + 4*b*c) + tan(x/2)^3*(4*a*c - 4*b*c) + a^2 + b^2 +
tan(x/2)^2*(2*a^2 - 2*b^2 + 4*c^2)) - (atanh((2*a^4*c + 2*b^4*c + 2*c^5 -
4*a^2*c^3 + 4*b^2*c^3 - 4*a^2*b^2*c)/(2*(b^2 - a^2 + c^2)^(5/2)) + (tan(x/
2)*(2*a - 2*b)*(a^4 + b^4 + c^4 - 2*a^2*b^2 - 2*a^2*c^2 + 2*b^2*c^2))/(2*(
b^2 - a^2 + c^2)^(5/2)))*(2*A*a^2 + A*b^2 + A*c^2 - 3*C*a*c))/(b^2 - a^...
```

3.542. $\int \frac{A+C\sin(x)}{(a+b\cos(x)+c\sin(x))^3}\,dx$

3.543 $\int \frac{A+C\sin(x)}{a+b\cos(x)+ib\sin(x)}\,dx$

3.543.1 Optimal result

Integrand size = 22, antiderivative size = 85

$$\int \frac{A+C\sin(x)}{a+b\cos(x)+ib\sin(x)}\,dx = \frac{(2aA-ibC)x}{2a^2}-\frac{C\cos(x)}{2a}+\frac{(2iaAb-a^2C+b^2C)\log(a+b\cos(x)+ib\sin(x))}{2a^2b}+\frac{iC\sin(x)}{2a}$$

output

```
1/2*(2*A*a-I*b*C)*x/a^2-1/2*C*cos(x)/a+1/2*(2*I*a*A*b-C*a^2+b^2*C)*ln(a+b*
cos(x)+I*b*sin(x))/a^2/b+1/2*I*C*sin(x)/a
```

3.543.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.79

$$\int \frac{A+C\sin(x)}{a+b\cos(x)+ib\sin(x)}\,dx = \frac{2aAbx-ia^2Cx-ib^2Cx+(-4aAb-2ia^2C+2ib^2C)\arctan\left(\frac{(a+b)\cot\left(\frac{x}{2}\right)}{a-b}\right)-2abC\cos(x)+2iaAb\log\left(a^2 \cdots\right.}{4a^2b}$$

input

```
Integrate[(A + C*Sin[x])/(a + b*Cos[x] + I*b*Sin[x]),x]
```

output
```
(2*a*A*b*x - I*a^2*C*x - I*b^2*C*x + (-4*a*A*b - (2*I)*a^2*C + (2*I)*b^2*C
)*ArcTan[((a + b)*Cot[x/2])/(a - b)] - 2*a*b*C*Cos[x] + (2*I)*a*A*b*Log[a^
2 + b^2 + 2*a*b*Cos[x]] - a^2*C*Log[a^2 + b^2 + 2*a*b*Cos[x]] + b^2*C*Log[
a^2 + b^2 + 2*a*b*Cos[x]] + (2*I)*a*b*C*Sin[x])/(4*a^2*b)
```

3.543.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3042, 3610}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+C\sin(x)}{a+ib\sin(x)+b\cos(x)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{A+C\sin(x)}{a+ib\sin(x)+b\cos(x)}dx$$
$$\Big\downarrow \text{3610}$$
$$\frac{(a^2(-C)+2iaAb+b^2C)\log(a+ib\sin(x)+b\cos(x))}{2a^2b}+\frac{x(2aA-ibC)}{2a^2}+\frac{iC\sin(x)}{2a}-\frac{C\cos(x)}{2a}$$

input
```
Int[(A + C*Sin[x])/(a + b*Cos[x] + I*b*Sin[x]),x]
```

output
```
((2*a*A - I*b*C)*x)/(2*a^2) - (C*Cos[x])/(2*a) + (((2*I)*a*A*b - a^2*C + b
^2*C)*Log[a + b*Cos[x] + I*b*Sin[x]])/(2*a^2*b) + ((I/2)*C*Sin[x])/a
```

3.543.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3610
```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/(cos[(d_.) + (e_.)*(x_)]*(b_.)
+ (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(2*a*A - c*C)*(x
/(2*a^2)), x] + (-Simp[C*(Cos[d + e*x]/(2*a*e)), x] + Simp[c*C*(Sin[d + e*x
]/(2*a*b*e)), x] + Simp[((-a^2)*C + 2*a*c*A + b^2*C)*(Log[RemoveContent[a +
 b*Cos[d + e*x] + c*Sin[d + e*x], x]]/(2*a^2*b*e)), x]) /; FreeQ[{a, b, c,
d, e, A, C}, x] && EqQ[b^2 + c^2, 0]
```

3.543.4 Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.99

method	result
risch	$-\frac{C\,\mathrm{e}^{-ix}}{2a}-\frac{ibxC}{2a^{2}}+\frac{xA}{a}-\frac{\ln\left(\mathrm{e}^{ix}+\frac{a}{b}\right)C}{2b}+\frac{b\ln\left(\mathrm{e}^{ix}+\frac{a}{b}\right)C}{2a^{2}}+\frac{i\ln\left(\mathrm{e}^{ix}+\frac{a}{b}\right)A}{a}$
default	$\frac{i\left(iC\,a^{2}-iC\,b^{2}+2Aab\right)\ln\left(ia+ib+a\tan\left(\frac{x}{2}\right)-b\tan\left(\frac{x}{2}\right)\right)}{2a^{2}b}+\frac{iC}{a\left(-i+\tan\left(\frac{x}{2}\right)\right)}+\frac{\left(-2iAa-bC\right)\ln\left(-i+\tan\left(\frac{x}{2}\right)\right)}{2a^{2}}+\frac{C\ln\left(\tan\left(\frac{x}{2}\right)+i\right)}{2b}$

input
```
int((A+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*C/a*exp(-I*x)-1/2*I*b/a^2*x*C+1/a*x*A-1/2/b*ln(exp(I*x)+a/b)*C+1/2*b/
a^2*ln(exp(I*x)+a/b)*C+I/a*ln(exp(I*x)+a/b)*A
```

3.543.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.84

$$\int \frac{A+C\sin(x)}{a+b\cos(x)+ib\sin(x)}\,dx$$
$$= -\frac{\left(Cab-(2\,Aab-i\,Cb^{2})xe^{(i\,x)}+(Ca^{2}-2i\,Aab-Cb^{2})e^{(i\,x)}\log\left(\frac{be^{(i\,x)}+a}{b}\right)\right)e^{(-i\,x)}}{2\,a^{2}b}$$

input
```
integrate((A+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="fricas")
```

output
```
-1/2*(C*a*b - (2*A*a*b - I*C*b^2)*x*e^(I*x) + (C*a^2 - 2*I*A*a*b - C*b^2)*
e^(I*x)*log((b*e^(I*x) + a)/b))*e^(-I*x)/(a^2*b)
```

3.543.6 Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.18

$$\int \frac{A+C\sin(x)}{a+b\cos(x)+ib\sin(x)}\,dx = \begin{cases} -\frac{Ce^{-ix}}{2a} & \text{for } a\neq 0 \\ x\left(-\frac{2Aa-iCb}{2a^2}+\frac{2Aa+iCa-iCb}{2a^2}\right) & \text{otherwise} \end{cases} + \frac{x(2Aa-iCb)}{2a^2} - \frac{(-2iAab+Ca^2-Cb^2)\log\left(\frac{a}{b}+e^{ix}\right)}{2a^2b}$$

input
```
integrate((A+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x)
```

output
```
Piecewise((-C*exp(-I*x)/(2*a), Ne(a, 0)), (x*(-(2*A*a - I*C*b)/(2*a**2) +
(2*A*a + I*C*a - I*C*b)/(2*a**2)), True)) + x*(2*A*a - I*C*b)/(2*a**2) - (
-2*I*A*a*b + C*a**2 - C*b**2)*log(a/b + exp(I*x))/(2*a**2*b)
```

3.543.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+C\sin(x)}{a+b\cos(x)+ib\sin(x)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.543.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 169 vs. 2(69) = 138.

Time = 0.27 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.99

$$\int \frac{A+C\sin(x)}{a+b\cos(x)+ib\sin(x)}\,dx = -\frac{(-2iAa-Cb)\log\left(-a\tan\left(\frac{1}{2}x\right)^2+b\tan\left(\frac{1}{2}x\right)^2-2ia\tan\left(\frac{1}{2}x\right)+a+b\right)}{4a^2} - \frac{(2iAa+Cb)\log\left(\tan\left(\frac{1}{2}x\right)-i\right)}{2a^2} - \frac{(2iCa^2+2Aab-iCb^2)\left(x+2\arctan\left(\frac{-ia\cos(x)-a\sin(x)-ia}{a\cos(x)-ia\sin(x)-a+2b}\right)\right)}{4a^2b} - \frac{-2iAa\tan\left(\frac{1}{2}x\right)-Cb\tan\left(\frac{1}{2}x\right)-2Aa-2iCa+iCb}{2a^2\left(\tan\left(\frac{1}{2}x\right)-i\right)}$$

input
```
integrate((A+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="giac")
```

output
```
-1/4*(-2*I*A*a - C*b)*log(-a*tan(1/2*x)^2 + b*tan(1/2*x)^2 - 2*I*a*tan(1/2
*x) + a + b)/a^2 - 1/2*(2*I*A*a + C*b)*log(tan(1/2*x) - I)/a^2 - 1/4*(2*I*
C*a^2 + 2*A*a*b - I*C*b^2)*(x + 2*arctan((-I*a*cos(x) - a*sin(x) - I*a)/(a
*cos(x) - I*a*sin(x) - a + 2*b)))/(a^2*b) - 1/2*(-2*I*A*a*tan(1/2*x) - C*b
*tan(1/2*x) - 2*A*a - 2*I*C*a + I*C*b)/(a^2*(tan(1/2*x) - I))
```

3.543.9 Mupad [B] (verification not implemented)

Time = 29.90 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.13

$$\int \frac{A+C\sin(x)}{a+b\cos(x)+ib\sin(x)}\,dx = \ln\left(a+b-a\tan\left(\frac{x}{2}\right)\,1\mathrm{i}+b\tan\left(\frac{x}{2}\right)\,1\mathrm{i}\right)\left(\frac{C\,b}{2\,a^2}-\frac{C}{2\,b}+\frac{A\,1\mathrm{i}}{a}\right) + \frac{C\,1\mathrm{i}}{a\,\left(\tan\left(\frac{x}{2}\right)-\mathrm{i}\right)} + \frac{C\,\ln\left(\tan\left(\frac{x}{2}\right)+1\mathrm{i}\right)}{2\,b} - \frac{\ln\left(\tan\left(\frac{x}{2}\right)-\mathrm{i}\right)\,\left(\frac{C\,b}{2}+A\,a\,1\mathrm{i}\right)}{a^2}$$

input
```
int((A + C*sin(x))/(a + b*cos(x) + b*sin(x)*1i),x)
```

output
```
log(a + b - a*tan(x/2)*1i + b*tan(x/2)*1i)*((A*1i)/a - C/(2*b) + (C*b)/(2*
a^2)) + (C*1i)/(a*(tan(x/2) - 1i)) + (C*log(tan(x/2) + 1i))/(2*b) - (log(t
an(x/2) - 1i)*(A*a*1i + (C*b)/2))/a^2
```

3.544 $\int \frac{A+C\sin(x)}{a+b\cos(x)-ib\sin(x)}\,dx$

3.544.1 Optimal result

Integrand size = 22, antiderivative size = 85

$$\int \frac{A+C\sin(x)}{a+b\cos(x)-ib\sin(x)}\,dx=\frac{(2aA+ibC)x}{2a^2}-\frac{C\cos(x)}{2a}-\frac{(2iaAb+a^2C-b^2C)\log(a+b\cos(x)-ib\sin(x))}{2a^2b}-\frac{iC\sin(x)}{2a}$$

output
```
1/2*(2*A*a+I*b*C)*x/a^2-1/2*C*cos(x)/a-1/2*(2*I*a*A*b+C*a^2-b^2*C)*ln(a+b*
cos(x)-I*b*sin(x))/a^2/b-1/2*I*C*sin(x)/a
```

3.544.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.79

$$\int \frac{A+C\sin(x)}{a+b\cos(x)-ib\sin(x)}\,dx=\frac{2aAbx+ia^2Cx+ib^2Cx+2i(2iaAb+a^2C-b^2C)\arctan\left(\frac{(a+b)\cot\left(\frac{x}{2}\right)}{a-b}\right)-2abC\cos(x)-2iaAb\log\left(a^2+\right.}{4a^2b}$$

input
```
Integrate[(A + C*Sin[x])/(a + b*Cos[x] - I*b*Sin[x]),x]
```

output
```
(2*a*A*b*x + I*a^2*C*x + I*b^2*C*x + (2*I)*((2*I)*a*A*b + a^2*C - b^2*C)*A
rcTan[((a + b)*Cot[x/2])/(a - b)] - 2*a*b*C*Cos[x] - (2*I)*a*A*b*Log[a^2 +
 b^2 + 2*a*b*Cos[x]] - a^2*C*Log[a^2 + b^2 + 2*a*b*Cos[x]] + b^2*C*Log[a^2
 + b^2 + 2*a*b*Cos[x]] - (2*I)*a*b*C*Sin[x])/(4*a^2*b)
```

3.544.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3042, 3610}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+C\sin(x)}{a-ib\sin(x)+b\cos(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+C\sin(x)}{a-ib\sin(x)+b\cos(x)}dx$$

$$\Big\downarrow \text{3610}$$

$$-\frac{(a^2C+2iaAb-b^2C)\log(a-ib\sin(x)+b\cos(x))}{2a^2b}+\frac{x(2aA+ibC)}{2a^2}-\frac{iC\sin(x)}{2a}-\frac{C\cos(x)}{2a}$$

input
```
Int[(A + C*Sin[x])/(a + b*Cos[x] - I*b*Sin[x]),x]
```

output
```
((2*a*A + I*b*C)*x)/(2*a^2) - (C*Cos[x])/(2*a) - (((2*I)*a*A*b + a^2*C - b
^2*C)*Log[a + b*Cos[x] - I*b*Sin[x]])/(2*a^2*b) - ((I/2)*C*Sin[x])/a
```

3.544.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3610
```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/(cos[(d_.) + (e_.)*(x_)]*(b_.)
+ (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(2*a*A - c*C)*(x
/(2*a^2)), x] + (-Simp[C*(Cos[d + e*x]/(2*a*e)), x] + Simp[c*C*(Sin[d + e*x
]/(2*a*b*e)), x] + Simp[((-a^2)*C + 2*a*c*A + b^2*C)*(Log[RemoveContent[a +
 b*Cos[d + e*x] + c*Sin[d + e*x], x]]/(2*a^2*b*e)), x]) /; FreeQ[{a, b, c,
d, e, A, C}, x] && EqQ[b^2 + c^2, 0]
```

3.544.4 Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

method	result
risch	$-\frac{C\,\mathrm{e}^{ix}}{2a}+\frac{ixC}{2b}-\frac{\ln\left(\mathrm{e}^{ix}+\frac{b}{a}\right)C}{2b}+\frac{b\ln\left(\mathrm{e}^{ix}+\frac{b}{a}\right)C}{2a^2}-\frac{i\ln\left(\mathrm{e}^{ix}+\frac{b}{a}\right)A}{a}$
default	$\frac{C\ln(-i+\tan(\frac{x}{2}))}{2b}+\frac{i(-iC\,a^2+iC\,b^2+2Aab)(a-b)\ln(ia+ib-a\tan(\frac{x}{2})+b\tan(\frac{x}{2}))}{2a^2b(-a+b)}-\frac{iC}{a(\tan(\frac{x}{2})+i)}+\frac{(2iAa-bC)\ln(\tan(\frac{x}{2})+$

input
```
int((A+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*C/a*exp(I*x)+1/2*I/b*x*C-1/2/b*ln(exp(I*x)+b/a)*C+1/2*b/a^2*ln(exp(I*
x)+b/a)*C-I/a*ln(exp(I*x)+b/a)*A
```

3.544.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.67

$$\int \frac{A+C\sin(x)}{a+b\cos(x)-ib\sin(x)}\,dx=\frac{i\,Ca^2x-Cabe^{(i\,x)}-(Ca^2+2i\,Aab-Cb^2)\log\left(\frac{ae^{(i\,x)}+b}{a}\right)}{2\,a^2b}$$

input
```
integrate((A+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="fricas")
```

output
```
1/2*(I*C*a^2*x - C*a*b*e^(I*x) - (C*a^2 + 2*I*A*a*b - C*b^2)*log((a*e^(I*x
) + b)/a))/(a^2*b)
```

3.544.6 Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int \frac{A+C\sin(x)}{a+b\cos(x)-ib\sin(x)}\,dx = \frac{iCx}{2b} + \begin{cases} -\frac{Ce^{ix}}{2a} & \text{for } a\neq 0 \\ x\left(-\frac{iC}{2b}+\frac{iCa-iCb}{2ab}\right) & \text{otherwise} \end{cases} - \frac{(2iAab+Ca^2-Cb^2)\log\left(e^{ix}+\frac{b}{a}\right)}{2a^2b}$$

input
```
integrate((A+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x)
```

output
```
I*C*x/(2*b) + Piecewise((-C*exp(I*x)/(2*a), Ne(a, 0)), (x*(-I*C/(2*b) + (I
*C*a - I*C*b)/(2*a*b)), True)) - (2*I*A*a*b + C*a**2 - C*b**2)*log(exp(I*x
) + b/a)/(2*a**2*b)
```

3.544.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+C\sin(x)}{a+b\cos(x)-ib\sin(x)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.544.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 169 vs. 2(69) = 138.

Time = 0.27 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.99

$$\int \frac{A+C\sin(x)}{a+b\cos(x)-ib\sin(x)}\,dx$$
$$= -\frac{(2iAa-Cb)\log\left(-a\tan\left(\frac{1}{2}x\right)^2+b\tan\left(\frac{1}{2}x\right)^2+2ia\tan\left(\frac{1}{2}x\right)+a+b\right)}{4a^2}$$
$$-\frac{(-2iAa+Cb)\log\left(\tan\left(\frac{1}{2}x\right)+i\right)}{2a^2}$$
$$-\frac{(-2iCa^2+2Aab+iCb^2)\left(x+2\arctan\left(\frac{ia\cos(x)-a\sin(x)+ia}{a\cos(x)+ia\sin(x)-a+2b}\right)\right)}{4a^2b}$$
$$-\frac{2iAa\tan\left(\frac{1}{2}x\right)-Cb\tan\left(\frac{1}{2}x\right)-2Aa+2iCa-iCb}{2a^2\left(\tan\left(\frac{1}{2}x\right)+i\right)}$$

input
```
integrate((A+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="giac")
```

output
```
-1/4*(2*I*A*a - C*b)*log(-a*tan(1/2*x)^2 + b*tan(1/2*x)^2 + 2*I*a*tan(1/2*
x) + a + b)/a^2 - 1/2*(-2*I*A*a + C*b)*log(tan(1/2*x) + I)/a^2 - 1/4*(-2*I
*C*a^2 + 2*A*a*b + I*C*b^2)*(x + 2*arctan((I*a*cos(x) - a*sin(x) + I*a)/(a
*cos(x) + I*a*sin(x) - a + 2*b)))/(a^2*b) - 1/2*(2*I*A*a*tan(1/2*x) - C*b*
tan(1/2*x) - 2*A*a + 2*I*C*a - I*C*b)/(a^2*(tan(1/2*x) + I))
```

3.544.9 Mupad [B] (verification not implemented)

Time = 28.45 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.13

$$\int \frac{A+C\sin(x)}{a+b\cos(x)-ib\sin(x)}\,dx = -\ln\left(a+b+a\tan\left(\frac{x}{2}\right)\,1\mathrm{i}-b\tan\left(\frac{x}{2}\right)\,1\mathrm{i}\right)\left(\frac{C}{2b}-\frac{Cb}{2a^2}+\frac{A\,1\mathrm{i}}{a}\right)-\frac{C\,1\mathrm{i}}{a\left(\tan\left(\frac{x}{2}\right)+1\mathrm{i}\right)}+\frac{C\ln\left(\tan\left(\frac{x}{2}\right)-\mathrm{i}\right)}{2b}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)+1\mathrm{i}\right)\left(-\frac{Cb}{2}+Aa\,1\mathrm{i}\right)}{a^2}$$

input
```
int((A + C*sin(x))/(a + b*cos(x) - b*sin(x)*1i),x)
```

output
```
(C*log(tan(x/2) - 1i))/(2*b) - (C*1i)/(a*(tan(x/2) + 1i)) - log(a + b + a*
tan(x/2)*1i - b*tan(x/2)*1i)*((A*1i)/a + C/(2*b) - (C*b)/(2*a^2)) + (log(t
an(x/2) + 1i)*(A*a*1i - (C*b)/2))/a^2
```

3.545 $\int \frac{B\cos(x)+C\sin(x)}{a+b\cos(x)+c\sin(x)}\,dx$

3.545.1 Optimal result

Integrand size = 22, antiderivative size = 119

$$\int \frac{B\cos(x)+C\sin(x)}{a+b\cos(x)+c\sin(x)}\,dx = \frac{(bB+cC)x}{b^2+c^2} - \frac{2a(bB+cC)\arctan\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}\,(b^2+c^2)} + \frac{(Bc-bC)\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}$$

output
```
(B*b+C*c)*x/(b^2+c^2)+(B*c-C*b)*ln(a+b*cos(x)+c*sin(x))/(b^2+c^2)-2*a*(B*b
+C*c)*arctan((c+(a-b)*tan(1/2*x))/(a^2-b^2-c^2)^(1/2))/(b^2+c^2)/(a^2-b^2-
c^2)^(1/2)
```

3.545.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.82

$$\int \frac{B\cos(x)+C\sin(x)}{a+b\cos(x)+c\sin(x)}\,dx = \frac{(bB+cC)x + \frac{2a(bB+cC)\operatorname{arctanh}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2+c^2}}\right)}{\sqrt{-a^2+b^2+c^2}} + (Bc-bC)\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}$$

input
```
Integrate[(B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x]),x]
```

output
```
((b*B + c*C)*x + (2*a*(b*B + c*C)*ArcTanh[(c + (a - b)*Tan[x/2])/Sqrt[-a^2
 + b^2 + c^2]])/Sqrt[-a^2 + b^2 + c^2] + (B*c - b*C)*Log[a + b*Cos[x] + c*
Sin[x]])/(b^2 + c^2)
```

3.545.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 3615, 3042, 3603, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\cos(x)+C\sin(x)}{a+b\cos(x)+c\sin(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{B\cos(x)+C\sin(x)}{a+b\cos(x)+c\sin(x)}dx$$

$$\Big\downarrow \text{3615}$$

$$-\frac{a(bB+cC)\int \frac{1}{a+b\cos(x)+c\sin(x)}dx}{b^2+c^2}+\frac{(Bc-bC)\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\frac{x(bB+cC)}{b^2+c^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a(bB+cC)\int \frac{1}{a+b\cos(x)+c\sin(x)}dx}{b^2+c^2}+\frac{(Bc-bC)\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\frac{x(bB+cC)}{b^2+c^2}$$

$$\Big\downarrow \text{3603}$$

$$-\frac{2a(bB+cC)\int \frac{1}{(a-b)\tan^2\left(\frac{x}{2}\right)+2c\tan\left(\frac{x}{2}\right)+a+b}d\tan\left(\frac{x}{2}\right)}{b^2+c^2}+\frac{(Bc-bC)\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\frac{x(bB+cC)}{b^2+c^2}$$

$$\Big\downarrow \text{1083}$$

$$\frac{4a(bB+cC)\int \frac{1}{-\left(2c+2(a-b)\tan\left(\frac{x}{2}\right)\right)^2-4(a^2-b^2-c^2)}d\left(2c+2(a-b)\tan\left(\frac{x}{2}\right)\right)}{b^2+c^2}+\frac{(Bc-bC)\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\frac{x(bB+cC)}{b^2+c^2}$$

$$\Big\downarrow \text{217}$$

$$-\frac{2a(bB+cC)\arctan\left(\frac{2(a-b)\tan\left(\frac{x}{2}\right)+2c}{2\sqrt{a^2-b^2-c^2}}\right)}{(b^2+c^2)\sqrt{a^2-b^2-c^2}}+\frac{(Bc-bC)\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\frac{x(bB+cC)}{b^2+c^2}$$

input
```
Int[(B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x]),x]
```

output
```
((b*B + c*C)*x)/(b^2 + c^2) - (2*a*(b*B + c*C)*ArcTan[(2*c + 2*(a - b)*Tan
[x/2])/(2*Sqrt[a^2 - b^2 - c^2])])/(Sqrt[a^2 - b^2 - c^2]*(b^2 + c^2)) + (
(B*c - b*C)*Log[a + b*Cos[x] + c*Sin[x]])/(b^2 + c^2)
```

3.545.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

rule 3615
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + (Simp[(c*B - b*C)*(Log[a
 + b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] + Simp[(A*(b^2 + c
^2) - a*(b*B + c*C))/(b^2 + c^2)   Int[1/(a + b*Cos[d + e*x] + c*Sin[d + e*
x]), x], x]) /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] &&
NeQ[A*(b^2 + c^2) - a*(b*B + c*C), 0]
```

3.545. $\int \frac{B\cos(x)+C\sin(x)}{a+b\cos(x)+c\sin(x)}\,dx$

3.545.4 Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.82

method	result
default	$\frac{\frac{2\left(aBc-bBc-abC+b^{2}C\right)\ln\left(\tan\left(\frac{x}{2}\right)^{2}a-\tan\left(\frac{x}{2}\right)^{2}b+2c\tan\left(\frac{x}{2}\right)+a+b\right)}{2a-2b}+\frac{2\left(-abB+B\,c^{2}-acC-Cbc-\frac{\left(aBc-bBc-abC+b^{2}C\right)c}{a-b}\right)\arctan\left(\frac{2(a-b)\,\mathrm{t}}{2\sqrt{a^{2}}}\right.}{\sqrt{a^{2}-b^{2}-c^{2}}}}{b^{2}+c^{2}}$
risch	Expression too large to display

input
```
int((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
2/(b^2+c^2)*(1/2*(B*a*c-B*b*c-C*a*b+C*b^2)/(a-b)*ln(tan(1/2*x)^2*a-tan(1/2
*x)^2*b+2*c*tan(1/2*x)+a+b)+(-a*b*B+B*c^2-a*c*C-C*b*c-(B*a*c-B*b*c-C*a*b+C
*b^2)*c/(a-b))/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^
2-b^2-c^2)^(1/2)))+2/(b^2+c^2)*(1/2*(-B*c+C*b)*ln(1+tan(1/2*x)^2)+(B*b+C*c
)*arctan(tan(1/2*x)))
```

3.545.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. 2(114) = 228.

Time = 0.33 (sec) , antiderivative size = 687, normalized size of antiderivative = 5.77

$$\int \frac{B\cos(x)+C\sin(x)}{a+b\cos(x)+c\sin(x)}\,dx$$

$$= \left[-\frac{(Bab+Cac)\sqrt{-a^2+b^2+c^2}\log\left(\frac{a^2b^2-2\,b^4-c^4-(a^2+3\,b^2)c^2-(2\,a^2b^2-b^4-2\,a^2c^2+c^4)\cos(x)^2-2\,(ab^3+abc^2)\cos(x)-2\,(ab^2}{2\,ab\cos(x}\right.}{}\right.$$

$$-\frac{2\,(Bab+Cac)\sqrt{a^2-b^2-c^2}\arctan\left(-\frac{(ab\cos(x)+ac\sin(x)+b^2+c^2)\sqrt{a^2-b^2-c^2}}{(c^3-(a^2-b^2)c)\cos(x)+(a^2b-b^3-bc^2)\sin(x)}\right)-2\,(Ba^2b-Bb^3-Bbc^2-C}{}$$

input
```
integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="fricas")
```

output
```
[-1/2*((B*a*b + C*a*c)*sqrt(-a^2 + b^2 + c^2)*log((a^2*b^2 - 2*b^4 - c^4 -
 (a^2 + 3*b^2)*c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(x)^2 - 2*(a*b
^3 + a*b*c^2)*cos(x) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*co
s(x))*sin(x) - 2*(2*a*b*c*cos(x)^2 - a*b*c + (b^2*c + c^3)*cos(x) - (b^3 +
 b*c^2 + (a*b^2 - a*c^2)*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2))/(2*a*b*co
s(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x))) -
2*(B*a^2*b - B*b^3 - B*b*c^2 - C*c^3 + (C*a^2 - C*b^2)*c)*x + (C*a^2*b - C
*b^3 - C*b*c^2 + B*c^3 - (B*a^2 - B*b^2)*c)*log(2*a*b*cos(x) + (b^2 - c^2)
*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x)))/(a^2*b^2 - b^4 - c^4
 + (a^2 - 2*b^2)*c^2), -1/2*(2*(B*a*b + C*a*c)*sqrt(a^2 - b^2 - c^2)*arcta
n(-(a*b*cos(x) + a*c*sin(x) + b^2 + c^2)*sqrt(a^2 - b^2 - c^2)/((c^3 - (a^
2 - b^2)*c)*cos(x) + (a^2*b - b^3 - b*c^2)*sin(x))) - 2*(B*a^2*b - B*b^3 -
 B*b*c^2 - C*c^3 + (C*a^2 - C*b^2)*c)*x + (C*a^2*b - C*b^3 - C*b*c^2 + B*c
^3 - (B*a^2 - B*b^2)*c)*log(2*a*b*cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^
2 + 2*(b*c*cos(x) + a*c)*sin(x)))/(a^2*b^2 - b^4 - c^4 + (a^2 - 2*b^2)*c^2
)]
```

3.545.6 Sympy [F(-1)]

Timed out.

$$\int \frac{B\cos(x) + C\sin(x)}{a + b\cos(x) + c\sin(x)}\,dx = \text{Timed out}$$

input
```
integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x)),x)
```

output
```
Timed out
```

3.545.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{B\cos(x) + C\sin(x)}{a + b\cos(x) + c\sin(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` f
or more de
```

3.545.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.57

$$\int \frac{B\cos(x)+C\sin(x)}{a+b\cos(x)+c\sin(x)}\,dx = \frac{(Bb+Cc)x}{b^2+c^2} - \frac{(Cb-Bc)\log\left(-a\tan\left(\frac{1}{2}x\right)^2+b\tan\left(\frac{1}{2}x\right)^2-2c\tan\left(\frac{1}{2}x\right)-a-b\right)}{b^2+c^2} + \frac{(Cb-Bc)\log\left(\tan\left(\frac{1}{2}x\right)^2+1\right)}{b^2+c^2} + \frac{2\left(Bab+Cac\right)\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}x\right)-b\tan\left(\frac{1}{2}x\right)+c}{\sqrt{a^2-b^2-c^2}}\right)\right)}{\sqrt{a^2-b^2-c^2}(b^2+c^2)}$$

input
```
integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="giac")
```

output
```
(B*b + C*c)*x/(b^2 + c^2) - (C*b - B*c)*log(-a*tan(1/2*x)^2 + b*tan(1/2*x)
^2 - 2*c*tan(1/2*x) - a - b)/(b^2 + c^2) + (C*b - B*c)*log(tan(1/2*x)^2 +
1)/(b^2 + c^2) + 2*(B*a*b + C*a*c)*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*
b) + arctan(-(a*tan(1/2*x) - b*tan(1/2*x) + c)/sqrt(a^2 - b^2 - c^2)))/(sq
rt(a^2 - b^2 - c^2)*(b^2 + c^2))
```

3.545.9 Mupad [B] (verification not implemented)

Time = 67.96 (sec) , antiderivative size = 1864, normalized size of antiderivative = 15.66

$$\int \frac{B\cos(x)+C\sin(x)}{a+b\cos(x)+c\sin(x)}\,dx = \text{Too large to display}$$

input
```
int((B*cos(x) + C*sin(x))/(a + b*cos(x) + c*sin(x)),x)
```

output
```
(log(tan(x/2) - 1i)*(B + C*1i))/(b*1i - c) - (log(tan(x/2) + 1i)*(B - C*1i
))/(b*1i + c) - (log(32*B^3*a^2 + 32*B*C^2*a^2 + 32*B*C^2*b^2 + 32*tan(x/2
)*(a - b)*(2*C^3*a + B^3*c - 2*C^3*b + 2*B^2*C*a - B^2*C*b + 2*B*C^2*c) -
32*B^3*a*b - 64*B*C^2*a*b + 32*B^2*C*a*c - 32*B^2*C*b*c + ((C*b^3 - B*c^3
+ B*a^2*c - C*a^2*b - B*b^2*c + C*b*c^2 + B*a*b*(b^2 - a^2 + c^2)^(1/2) +
C*a*c*(b^2 - a^2 + c^2)^(1/2))*(32*B^2*b^2*c - 32*B^2*a^2*c + 32*C^2*a^2*c
 + 32*C^2*b^2*c + 32*tan(x/2)*(a - b)*(2*B^2*a^2 + B^2*b^2 - 2*C^2*a^2 - 3
*B^2*c^2 + 2*C^2*c^2 - 2*B^2*a*b + 2*C^2*a*b - 4*B*C*a*c + 6*B*C*b*c) - 12
8*B*C*a^3 - 64*B*C*b^3 + 192*B*C*a^2*b + 64*B*C*a*c^2 - 64*B*C*b*c^2 - 64*
C^2*a*b*c + ((C*b^3 - B*c^3 + B*a^2*c - C*a^2*b - B*b^2*c + C*b*c^2 + B*a*
b*(b^2 - a^2 + c^2)^(1/2) + C*a*c*(b^2 - a^2 + c^2)^(1/2))*(32*B*b^4 + 32*
B*a^2*b^2 - 32*B*a^2*c^2 - 64*B*b^2*c^2 - 32*tan(x/2)*(a - b)*(B*c^3 - 2*C
*b^3 + 2*C*a*b^2 + 4*B*b^2*c - 2*C*a*c^2 + C*b*c^2 - 4*B*a*b*c) - 64*B*a*b
^3 + 32*C*a*c^3 - 32*C*b*c^3 + 64*C*b^3*c + 96*B*a*b*c^2 - 128*C*a*b^2*c +
 64*C*a^2*b*c + (32*(a - b)*(C*b^3 - B*c^3 + B*a^2*c - C*a^2*b - B*b^2*c +
 C*b*c^2 + B*a*b*(b^2 - a^2 + c^2)^(1/2) + C*a*c*(b^2 - a^2 + c^2)^(1/2))*
(3*c^4*tan(x/2) + a*c^3 + 3*b*c^3 + 3*b^3*c + 2*a^2*b^2*tan(x/2) - 2*a^2*c
^2*tan(x/2) + 3*b^2*c^2*tan(x/2) - 2*a*b^3*tan(x/2) + a*b^2*c - 4*a^2*b*c
- 2*a*b*c^2*tan(x/2)))/((b^2 + c^2)*(b^2 - a^2 + c^2))))/((b^2 + c^2)*(b^2
 - a^2 + c^2))))/((b^2 + c^2)*(b^2 - a^2 + c^2)))*(C*b^3 - B*c^3 + B*a^...
```

3.545. $\int \frac{B\cos(x)+C\sin(x)}{a+b\cos(x)+c\sin(x)}\,dx$

3.546 $\int \frac{B\cos(x)+C\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx$

3.546.1 Optimal result

Integrand size = 22, antiderivative size = 110

$$\int \frac{B\cos(x)+C\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx = -\frac{2(bB+cC)\arctan\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{3/2}} + \frac{Bc-bC-aC\cos(x)+aB\sin(x)}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}$$

output
```
-2*(B*b+C*c)*arctan((c+(a-b)*tan(1/2*x))/(a^2-b^2-c^2)^(1/2))/(a^2-b^2-c^2
)^(3/2)+(B*c-b*C-a*C*cos(x)+a*B*sin(x))/(a^2-b^2-c^2)/(a+b*cos(x)+c*sin(x)
)
```

3.546.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.05

$$\int \frac{B\cos(x)+C\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx = -\frac{2(bB+cC)\text{arctanh}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2+c^2}}\right)}{\left(-a^2+b^2+c^2\right)^{3/2}} - \frac{bBc+a^2C-b^2C+a(bB+cC)\sin(x)}{b\left(-a^2+b^2+c^2\right)(a+b\cos(x)+c\sin(x))}$$

input
```
Integrate[(B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^2,x]
```

output
```
(-2*(b*B + c*C)*ArcTanh[(c + (a - b)*Tan[x/2])/Sqrt[-a^2 + b^2 + c^2]])/(-
a^2 + b^2 + c^2)^(3/2) - (b*B*c + a^2*C - b^2*C + a*(b*B + c*C)*Sin[x])/(b
*(-a^2 + b^2 + c^2)*(a + b*Cos[x] + c*Sin[x]))
```

3.546.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 3632, 3042, 3603, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\cos(x)+C\sin(x)}{(a+b\cos(x)+c\sin(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{B\cos(x)+C\sin(x)}{(a+b\cos(x)+c\sin(x))^2}dx$$

$$\Big\downarrow \text{3632}$$

$$\frac{aB\sin(x)-aC\cos(x)-bC+Bc}{(a^2-b^2-c^2)(a+b\cos(x)+c\sin(x))}-\frac{(bB+cC)\int \frac{1}{a+b\cos(x)+c\sin(x)}dx}{a^2-b^2-c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{aB\sin(x)-aC\cos(x)-bC+Bc}{(a^2-b^2-c^2)(a+b\cos(x)+c\sin(x))}-\frac{(bB+cC)\int \frac{1}{a+b\cos(x)+c\sin(x)}dx}{a^2-b^2-c^2}$$

$$\Big\downarrow \text{3603}$$

$$\frac{aB\sin(x)-aC\cos(x)-bC+Bc}{(a^2-b^2-c^2)(a+b\cos(x)+c\sin(x))}-\frac{2(bB+cC)\int \frac{1}{(a-b)\tan^2\left(\frac{x}{2}\right)+2c\tan\left(\frac{x}{2}\right)+a+b}d\tan\left(\frac{x}{2}\right)}{a^2-b^2-c^2}$$

$$\Big\downarrow \text{1083}$$

$$\frac{4(bB+cC)\int \frac{1}{-\left(2c+2(a-b)\tan\left(\frac{x}{2}\right)\right)^2-4(a^2-b^2-c^2)}d\left(2c+2(a-b)\tan\left(\frac{x}{2}\right)\right)}{a^2-b^2-c^2}+\frac{aB\sin(x)-aC\cos(x)-bC+Bc}{(a^2-b^2-c^2)(a+b\cos(x)+c\sin(x))}$$

$$\Big\downarrow \text{217}$$

$$\frac{aB\sin(x)-aC\cos(x)-bC+Bc}{(a^2-b^2-c^2)(a+b\cos(x)+c\sin(x))}-\frac{2(bB+cC)\arctan\left(\frac{2(a-b)\tan\left(\frac{x}{2}\right)+2c}{2\sqrt{a^2-b^2-c^2}}\right)}{(a^2-b^2-c^2)^{3/2}}$$

input
```
Int[(B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^2,x]
```

output
```
(-2*(b*B + c*C)*ArcTan[(2*c + 2*(a - b)*Tan[x/2])/(2*Sqrt[a^2 - b^2 - c^2]
)])/(a^2 - b^2 - c^2)^(3/2) + (B*c - b*C - a*C*Cos[x] + a*B*Sin[x])/((a^2
- b^2 - c^2)*(a + b*Cos[x] + c*Sin[x]))
```

3.546.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

rule 3632
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Simp[(a*A - b*B - c*C)/(a^2 - b^2 - c^2)   Int[1/(a + b*Cos[d + e*x] + c*S
in[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
 - c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

3.546.4 Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.87

method	result
default	$-\frac{2\left(-\frac{\left(B\,a^{2}-abB-B\,c^{2}-acC+Cbc\right)\tan\left(\frac{x}{2}\right)}{a^{3}-a^{2}b-a\,b^{2}-a\,c^{2}+b^{3}+c^{2}b}+\frac{bBc+C\,a^{2}-b^{2}C}{a^{3}-a^{2}b-a\,b^{2}-a\,c^{2}+b^{3}+c^{2}b}\right)}{\tan\left(\frac{x}{2}\right)^{2}a-\tan\left(\frac{x}{2}\right)^{2}b+2c\tan\left(\frac{x}{2}\right)+a+b}-\frac{2(Bb+Cc)\arctan\left(\frac{2(a-b)\tan\left(\frac{x}{2}\right)+2c}{2\sqrt{a^{2}-b^{2}-c^{2}}}\right)}{\left(a^{2}-b^{2}-c^{2}\right)^{\frac{3}{2}}}$
risch	$\frac{-2iBab-2iacC-2iB\,e^{ix}a^{2}+2iB\,c^{2}e^{ix}-2iCbc\,e^{ix}-2Bbc\,e^{ix}-2C\,a^{2}e^{ix}+2C\,b^{2}e^{ix}}{\left(-a^{2}+b^{2}+c^{2}\right)(-ic+b)\left(-ic\,e^{2ix}+b\,e^{2ix}+ic+2a\,e^{ix}+b\right)}-\frac{\ln\left(e^{ix}+\frac{iac\sqrt{-a^{2}+b^{2}+c^{2}}-ia^{2}b+ib^{3}+ib\,c^{2}+ab\sqrt{}}{\left(b^{2}+c^{2}\right)\sqrt{-a^{2}+b^{2}}}\right.}{\sqrt{-a^{2}+b^{2}+c^{2}}\,\left(a^{2}-b^{2}\right.}$

input
```
int((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-2*(-(B*a^2-B*a*b-B*c^2-C*a*c+C*b*c)/(a^3-a^2*b-a*b^2-a*c^2+b^3+b*c^2)*tan
(1/2*x)+(B*b*c+C*a^2-C*b^2)/(a^3-a^2*b-a*b^2-a*c^2+b^3+b*c^2))/(tan(1/2*x)
^2*a-tan(1/2*x)^2*b+2*c*tan(1/2*x)+a+b)-2*(B*b+C*c)/(a^2-b^2-c^2)^(3/2)*ar
ctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))
```

3.546.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 577 vs. $2(105) = 210$.

Time = 0.32 (sec) , antiderivative size = 1316, normalized size of antiderivative = 11.96

$$\int \frac{B\cos(x)+C\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx = \text{Too large to display}$$

input
```
integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="fricas
")
```

output
```
[1/2*(2*C*a^4*b - 4*C*a^2*b^3 + 2*C*b^5 + 2*C*b*c^4 - 2*B*c^5 + 4*(B*a^2 -
 B*b^2)*c^3 - 4*(C*a^2*b - C*b^3)*c^2 + (B*a*b^3 + C*a*b^2*c + B*a*b*c^2 +
 C*a*c^3 + (B*b^4 + C*b^3*c + B*b^2*c^2 + C*b*c^3)*cos(x) + (B*b^3*c + C*b
^2*c^2 + B*b*c^3 + C*c^4)*sin(x))*sqrt(-a^2 + b^2 + c^2)*log(-(a^2*b^2 - 2
*b^4 - c^4 - (a^2 + 3*b^2)*c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(x
)^2 - 2*(a*b^3 + a*b*c^2)*cos(x) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b
- b^3)*c)*cos(x))*sin(x) + 2*(2*a*b*c*cos(x)^2 - a*b*c + (b^2*c + c^3)*cos
(x) - (b^3 + b*c^2 + (a*b^2 - a*c^2)*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2
))/(2*a*b*cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)
*sin(x))) - 2*(B*a^4 - 2*B*a^2*b^2 + B*b^4)*c + 2*(B*a*b*c^3 + C*a*c^4 - (
C*a^3 - C*a*b^2)*c^2 - (B*a^3*b - B*a*b^3)*c)*cos(x) + 2*(B*a^3*b^2 - B*a*
b^4 - B*a*b^2*c^2 - C*a*b*c^3 + (C*a^3*b - C*a*b^3)*c)*sin(x))/(a^5*b^2 -
2*a^3*b^4 + a*b^6 + a*c^6 - (2*a^3 - 3*a*b^2)*c^4 + (a^5 - 4*a^3*b^2 + 3*a
*b^4)*c^2 + (a^4*b^3 - 2*a^2*b^5 + b^7 + b*c^6 - (2*a^2*b - 3*b^3)*c^4 + (
a^4*b - 4*a^2*b^3 + 3*b^5)*c^2)*cos(x) + (c^7 - (2*a^2 - 3*b^2)*c^5 + (a^4
 - 4*a^2*b^2 + 3*b^4)*c^3 + (a^4*b^2 - 2*a^2*b^4 + b^6)*c)*sin(x)), (C*a^4
*b - 2*C*a^2*b^3 + C*b^5 + C*b*c^4 - B*c^5 + 2*(B*a^2 - B*b^2)*c^3 - 2*(C*
a^2*b - C*b^3)*c^2 - (B*a*b^3 + C*a*b^2*c + B*a*b*c^2 + C*a*c^3 + (B*b^4 +
 C*b^3*c + B*b^2*c^2 + C*b*c^3)*cos(x) + (B*b^3*c + C*b^2*c^2 + B*b*c^3 +
C*c^4)*sin(x))*sqrt(a^2 - b^2 - c^2)*arctan(-(a*b*cos(x) + a*c*sin(x) +...
```

3.546.6 Sympy [F(-1)]

Timed out.

$$\int \frac{B\cos(x) + C\sin(x)}{(a + b\cos(x) + c\sin(x))^2} \, dx = \text{Timed out}$$

input
```
integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))**2,x)
```

output
```
Timed out
```

3.546.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{B\cos(x) + C\sin(x)}{(a + b\cos(x) + c\sin(x))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` f
or more de
```

3.546.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.86

$$\int \frac{B\cos(x) + C\sin(x)}{(a + b\cos(x) + c\sin(x))^2}\,dx = \frac{2\left(\pi\left\lfloor\frac{x}{2\pi} + \frac{1}{2}\right\rfloor\operatorname{sgn}(-2a + 2b) + \arctan\left(-\frac{a\tan\left(\frac{1}{2}x\right) - b\tan\left(\frac{1}{2}x\right) + c}{\sqrt{a^2 - b^2 - c^2}}\right)\right)(Bb + Cc)}{(a^2 - b^2 - c^2)^{\frac{3}{2}}} + \frac{2\left(Ba^2\tan\left(\frac{1}{2}x\right) - Bab\tan\left(\frac{1}{2}x\right) - Cac\tan\left(\frac{1}{2}x\right) + Cbc\tan\left(\frac{1}{2}x\right) - Bc^2\tan\left(\frac{1}{2}x\right) - Ca^2 + Cb^2 - Bbc}{(a^3 - a^2b - ab^2 + b^3 - ac^2 + bc^2)\left(a\tan\left(\frac{1}{2}x\right)^2 - b\tan\left(\frac{1}{2}x\right)^2 + 2c\tan\left(\frac{1}{2}x\right) + a + b\right)}$$

input
```
integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="giac")
```

output
```
2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*ta
n(1/2*x) + c)/sqrt(a^2 - b^2 - c^2)))*(B*b + C*c)/(a^2 - b^2 - c^2)^(3/2)
+ 2*(B*a^2*tan(1/2*x) - B*a*b*tan(1/2*x) - C*a*c*tan(1/2*x) + C*b*c*tan(1/
2*x) - B*c^2*tan(1/2*x) - C*a^2 + C*b^2 - B*b*c)/((a^3 - a^2*b - a*b^2 + b
^3 - a*c^2 + b*c^2)*(a*tan(1/2*x)^2 - b*tan(1/2*x)^2 + 2*c*tan(1/2*x) + a
+ b))
```

3.546.9 Mupad [B] (verification not implemented)

Time = 27.41 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.84

$$\int \frac{B\cos(x)+C\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx=\frac{\frac{2\,(C\,a^2-C\,b^2+B\,c\,b)}{(a-b)\,(-a^2+b^2+c^2)}+\frac{2\tan\left(\frac{x}{2}\right)\,(-B\,a^2+C\,a\,c+B\,b\,a+B\,c^2-C\,b\,c)}{(a-b)\,(-a^2+b^2+c^2)}}{(a-b)\,\tan\left(\frac{x}{2}\right)^2+2\,c\tan\left(\frac{x}{2}\right)+a+b}-\frac{2\,\mathrm{atanh}\left(\frac{\tan\left(\frac{x}{2}\right)\,(2\,a-2\,b)+\frac{2\,\left(-a^2\,c+b^2\,c+c^3\right)}{-a^2+b^2+c^2}}{2\,\sqrt{-a^2+b^2+c^2}}\right)\,(B\,b+C\,c)}{\left(-a^2+b^2+c^2\right)^{3/2}}$$

input
```
int((B*cos(x) + C*sin(x))/(a + b*cos(x) + c*sin(x))^2,x)
```

output
```
((2*(C*a^2 - C*b^2 + B*b*c))/((a - b)*(b^2 - a^2 + c^2)) + (2*tan(x/2)*(B*
c^2 - B*a^2 + B*a*b + C*a*c - C*b*c))/((a - b)*(b^2 - a^2 + c^2)))/(a + b
+ 2*c*tan(x/2) + tan(x/2)^2*(a - b)) - (2*atanh((tan(x/2)*(2*a - 2*b) + (2
*(b^2*c - a^2*c + c^3))/(b^2 - a^2 + c^2))/(2*(b^2 - a^2 + c^2)^(1/2)))*(B
*b + C*c))/(b^2 - a^2 + c^2)^(3/2)
```

3.547 $\int \frac{B\cos(x)+C\sin(x)}{(a+b\cos(x)+c\sin(x))^3}\,dx$

3.547.1 Optimal result

Integrand size = 22, antiderivative size = 197

$$\int \frac{B\cos(x)+C\sin(x)}{(a+b\cos(x)+c\sin(x))^3}\,dx = -\frac{3a(bB+cC)\arctan\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{(a^2-b^2-c^2)^{5/2}}+\frac{Bc-bC-aC\cos(x)+aB\sin(x)}{2(a^2-b^2-c^2)(a+b\cos(x)+c\sin(x))^2}+\frac{a(Bc-bC)-(2bBc+(a^2+2c^2)C)\cos(x)+(a^2B+2b(bB+cC))\sin(x)}{2(a^2-b^2-c^2)^2(a+b\cos(x)+c\sin(x))}$$

output
```
-3*a*(B*b+C*c)*arctan((c+(a-b)*tan(1/2*x))/(a^2-b^2-c^2)^(1/2))/(a^2-b^2-c
^2)^(5/2)+1/2*(B*c-b*C-a*C*cos(x)+a*B*sin(x))/(a^2-b^2-c^2)/(a+b*cos(x)+c*
sin(x))^2+1/2*(a*(B*c-C*b)-(2*b*B*c+(a^2+2*c^2)*C)*cos(x)+(B*a^2+2*b*(B*b+
C*c))*sin(x))/(a^2-b^2-c^2)^2/(a+b*cos(x)+c*sin(x))
```

3.547.2 Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.58

$$\int \frac{B\cos(x)+C\sin(x)}{(a+b\cos(x)+c\sin(x))^3}\,dx = \frac{3a(bB+cC)\text{arctanh}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2+c^2}}\right)}{(-a^2+b^2+c^2)^{5/2}} + \frac{9a^2bBc+2a^4C-4a^2b^2C+2b^4C+5a^2c^2C+4b^2c^2C+2c^4C+6abc(bB+cC)\cos(x)-c(a^2+2(b^2+$$

input
```
Integrate[(B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^3,x]
```

output
```
(3*a*(b*B + c*C)*ArcTanh[(c + (a - b)*Tan[x/2])/Sqrt[-a^2 + b^2 + c^2]])/(
-a^2 + b^2 + c^2)^(5/2) + (9*a^2*b*B*c + 2*a^4*C - 4*a^2*b^2*C + 2*b^4*C +
 5*a^2*c^2*C + 4*b^2*c^2*C + 2*c^4*C + 6*a*b*c*(b*B + c*C)*Cos[x] - c*(a^2
 + 2*(b^2 + c^2))*(b*B + c*C)*Cos[2*x] + 4*a^3*b*B*Sin[x] + 2*a*b^3*B*Sin[
x] + 8*a*b*B*c^2*Sin[x] + 4*a^3*c*C*Sin[x] + 2*a*b^2*c*C*Sin[x] + 8*a*c^3*
C*Sin[x] + a^2*b^2*B*Sin[2*x] + 2*b^4*B*Sin[2*x] + 2*b^2*B*c^2*Sin[2*x] +
a^2*b*c*C*Sin[2*x] + 2*b^3*c*C*Sin[2*x] + 2*b*c^3*C*Sin[2*x])/(4*b*(-a^2 +
 b^2 + c^2)^2*(a + b*Cos[x] + c*Sin[x])^2)
```

3.547.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 3635, 3042, 3632, 3042, 3603, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\cos(x)+C\sin(x)}{(a+b\cos(x)+c\sin(x))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{B\cos(x)+C\sin(x)}{(a+b\cos(x)+c\sin(x))^3}dx$$

$$\Big\downarrow \text{3635}$$

$$\frac{aB\sin(x)-aC\cos(x)-bC+Bc}{2(a^2-b^2-c^2)(a+b\cos(x)+c\sin(x))^2}-\frac{\int \frac{2(bB+cC)-aB\cos(x)-aC\sin(x)}{(a+b\cos(x)+c\sin(x))^2}dx}{2(a^2-b^2-c^2)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{aB\sin(x)-aC\cos(x)-bC+Bc}{2(a^2-b^2-c^2)(a+b\cos(x)+c\sin(x))^2}-\frac{\int \frac{2(bB+cC)-aB\cos(x)-aC\sin(x)}{(a+b\cos(x)+c\sin(x))^2}dx}{2(a^2-b^2-c^2)}$$

$$\Big\downarrow \text{3632}$$

$$\frac{aB\sin(x)-aC\cos(x)-bC+Bc}{2(a^2-b^2-c^2)(a+b\cos(x)+c\sin(x))^2}-\frac{\frac{3a(bB+cC)\int \frac{1}{a+b\cos(x)+c\sin(x)}dx}{a^2-b^2-c^2}-\frac{\sin(x)(a^2B+2b(bB+cC))-\cos(x)(a^2C+2c(bB+cC))+a(Bc-bC)}{(a^2-b^2-c^2)(a+b\cos(x)+c\sin(x))}}{2(a^2-b^2-c^2)}$$

$$\Big\downarrow \; \text{3042}$$

$$\frac{aB\sin(x)-aC\cos(x)-bC+Bc}{2\left(a^2-b^2-c^2\right)\left(a+b\cos(x)+c\sin(x)\right)^2}-\frac{\frac{3a(bB+cC)\int\frac{1}{a+b\cos(x)+c\sin(x)}dx}{a^2-b^2-c^2}-\frac{\sin(x)\left(a^2B+2b(bB+cC)\right)-\cos(x)\left(a^2C+2c(bB+cC)\right)+a(Bc-bC)}{\left(a^2-b^2-c^2\right)\left(a+b\cos(x)+c\sin(x)\right)}}{2\left(a^2-b^2-c^2\right)}$$

$$\Big\downarrow \; \text{3603}$$

$$\frac{aB\sin(x)-aC\cos(x)-bC+Bc}{2\left(a^2-b^2-c^2\right)\left(a+b\cos(x)+c\sin(x)\right)^2}-\frac{\frac{6a(bB+cC)\int\frac{1}{(a-b)\tan^2\left(\frac{x}{2}\right)+2c\tan\left(\frac{x}{2}\right)+a+b}d\tan\left(\frac{x}{2}\right)}{a^2-b^2-c^2}-\frac{\sin(x)\left(a^2B+2b(bB+cC)\right)-\cos(x)\left(a^2C+2c(bB+cC)\right)+a(Bc-bC)}{\left(a^2-b^2-c^2\right)\left(a+b\cos(x)+c\sin(x)\right)}}{2\left(a^2-b^2-c^2\right)}$$

$$\Big\downarrow \; \text{1083}$$

$$\frac{aB\sin(x)-aC\cos(x)-bC+Bc}{2\left(a^2-b^2-c^2\right)\left(a+b\cos(x)+c\sin(x)\right)^2}-\frac{-\frac{12a(bB+cC)\int\frac{1}{-\left(2c+2(a-b)\tan\left(\frac{x}{2}\right)\right)^2-4\left(a^2-b^2-c^2\right)}d\left(2c+2(a-b)\tan\left(\frac{x}{2}\right)\right)}{a^2-b^2-c^2}-\frac{\sin(x)\left(a^2B+2b(bB+cC)\right)-\cos(x)\left(a^2C+2c(bB+cC)\right)+a(Bc-b}{\left(a^2-b^2-c^2\right)\left(a+b\cos(x)+c\sin(x)\right)}}{2\left(a^2-b^2-c^2\right)}$$

$$\Big\downarrow \; \text{217}$$

$$\frac{aB\sin(x)-aC\cos(x)-bC+Bc}{2\left(a^2-b^2-c^2\right)\left(a+b\cos(x)+c\sin(x)\right)^2}-\frac{\frac{6a(bB+cC)\arctan\left(\frac{2(a-b)\tan\left(\frac{x}{2}\right)+2c}{2\sqrt{a^2-b^2-c^2}}\right)}{\left(a^2-b^2-c^2\right)^{3/2}}-\frac{\sin(x)\left(a^2B+2b(bB+cC)\right)-\cos(x)\left(a^2C+2c(bB+cC)\right)+a(Bc-bC)}{\left(a^2-b^2-c^2\right)\left(a+b\cos(x)+c\sin(x)\right)}}{2\left(a^2-b^2-c^2\right)}$$

input
```
Int[(B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^3,x]
```

output
```
(B*c - b*C - a*C*Cos[x] + a*B*Sin[x])/(2*(a^2 - b^2 - c^2)*(a + b*Cos[x] +
 c*Sin[x])^2) - ((6*a*(b*B + c*C)*ArcTan[(2*c + 2*(a - b)*Tan[x/2])/(2*Sqr
t[a^2 - b^2 - c^2])])/(a^2 - b^2 - c^2)^(3/2) - (a*(B*c - b*C) - (a^2*C +
2*c*(b*B + c*C))*Cos[x] + (a^2*B + 2*b*(b*B + c*C))*Sin[x])/((a^2 - b^2 -
c^2)*(a + b*Cos[x] + c*Sin[x])))/(2*(a^2 - b^2 - c^2))
```

3.547. $\int \frac{B\cos(x)+C\sin(x)}{(a+b\cos(x)+c\sin(x))^3}\,dx$

3.547.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

rule 3632
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Simp[(a*A - b*B - c*C)/(a^2 - b^2 - c^2)   Int[1/(a + b*Cos[d + e*x] + c*S
in[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
 - c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

rule 3635
```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(-(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(
a^2 - b^2 - c^2))), x] + Simp[1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a + b*Co
s[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n +
2)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x]
/; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[n, -2]
```

3.547. $\int \frac{B\cos(x)+C\sin(x)}{(a+b\cos(x)+c\sin(x))^3}\,dx$

3.547.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 794 vs. $2(187) = 374$.

Time = 1.61 (sec) , antiderivative size = 795, normalized size of antiderivative = 4.04

method	result
default	$-\frac{2\left(-\frac{\left(2B\,a^{4}-3B\,a^{3}b+2B\,a^{2}b^{2}-4B\,a^{2}c^{2}-3Ba\,b^{3}+2B\,b^{4}+4B\,b^{2}c^{2}+2B\,c^{4}-3C\,a^{3}c+6C\,a^{2}bc-3Ca\,b^{2}c\right)\tan\left(\frac{x}{2}\right)^{3}}{2\left(a^{4}-2a^{2}b^{2}-2a^{2}c^{2}+b^{4}+2b^{2}c^{2}+c^{4}\right)(a-b)}-\frac{\left(2B\,a^{4}c-9B\,a^{3}bc+14B\,a^{2}b\right.}{}\right.}{}$
risch	$\frac{i\left(5iBab\,c^{2}\mathrm{e}^{ix}-3iBab\,c^{2}\mathrm{e}^{3ix}+9iC\,a^{2}bc\,\mathrm{e}^{2ix}+3iCa\,b^{2}c\,\mathrm{e}^{3ix}+5iCa\,b^{2}c\,\mathrm{e}^{ix}+2\,\mathrm{e}^{2ix}a^{4}C+2C\,b^{4}\mathrm{e}^{2ix}+2iB\,b^{4}+2C\,c^{4}\mathrm{e}^{2ix}-2C\,c^{4}+5C\,a^{2}c^{2}\right.}{}$

input
```
int((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x,method=_RETURNVERBOSE)
```

output
```
-2*(-1/2*(2*B*a^4-3*B*a^3*b+2*B*a^2*b^2-4*B*a^2*c^2-3*B*a*b^3+2*B*b^4+4*B*
b^2*c^2+2*B*c^4-3*C*a^3*c+6*C*a^2*b*c-3*C*a*b^2*c)/(a^4-2*a^2*b^2-2*a^2*c^
2+b^4+2*b^2*c^2+c^4)/(a-b)*tan(1/2*x)^3-1/2*(2*B*a^4*c-9*B*a^3*b*c+14*B*a^
2*b^2*c-4*B*a^2*c^3-9*B*a*b^3*c+2*B*b^4*c+4*B*b^2*c^3+2*B*c^5-2*C*a^5+2*C*
a^4*b+4*C*a^3*b^2-5*C*a^3*c^2-4*C*a^2*b^3+14*C*a^2*b*c^2-2*C*a*b^4-13*C*a*
b^2*c^2-2*C*a*c^4+2*C*b^5+4*C*b^3*c^2+2*C*b*c^4)/(a^4-2*a^2*b^2-2*a^2*c^2+
b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tan(1/2*x)^2-1/2*(2*B*a^5-3*B*a^4*b+B*a
^3*b^2-4*B*a^3*c^2+B*a^2*b^3-8*B*a^2*b*c^2-3*B*a*b^4+8*B*a*b^2*c^2+2*B*a*c
^4+2*B*b^5+4*B*b^3*c^2+2*B*b*c^4-5*C*a^4*c+5*C*a^3*b*c+5*C*a^2*b^2*c-4*C*a
^2*c^3-5*C*a*b^3*c+4*C*a*b*c^3)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4
)/(a^2-2*a*b+b^2)*tan(1/2*x)+1/2*a*(5*B*a^2*b*c-5*B*b^3*c-2*B*b*c^3+2*C*a^
4-4*C*a^2*b^2+C*a^2*c^2+2*C*b^4-C*b^2*c^2)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*
b^2*c^2+c^4)/(a^2-2*a*b+b^2))/(tan(1/2*x)^2*a-tan(1/2*x)^2*b+2*c*tan(1/2*x
)+a+b)^2-3*a*(B*b+C*c)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-b^
2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)^(1/2))
```

3.547.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1551 vs. $2(187) = 374$.

Time = 0.47 (sec) , antiderivative size = 3264, normalized size of antiderivative = 16.57

$$\int \frac{B\cos(x) + C\sin(x)}{(a + b\cos(x) + c\sin(x))^3}\,dx = \text{Too large to display}$$

input
```
integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="fricas
")
```

output
```
[1/4*(2*C*a^6*b - 6*C*a^4*b^3 + 6*C*a^2*b^5 - 2*C*b^7 - 6*C*b*c^6 + 2*B*c^
7 - 2*(3*B*a^2 - B*b^2)*c^5 + 2*(4*C*a^2*b - 7*C*b^3)*c^4 + 2*(3*B*a^4 - 5
*B*a^2*b^2 - B*b^4)*c^3 - 2*(2*C*a^4*b - 7*C*a^2*b^3 + 5*C*b^5)*c^2 + 4*(2
*B*b^2*c^5 + 2*C*b*c^6 - (C*a^2*b - 4*C*b^3)*c^4 - (B*a^2*b^2 - 4*B*b^4)*c
^3 - (C*a^4*b + C*a^2*b^3 - 2*C*b^5)*c^2 - (B*a^4*b^2 + B*a^2*b^4 - 2*B*b^
6)*c)*cos(x)^2 - 3*(B*a^3*b^3 + C*a^3*b^2*c + B*a*b*c^4 + C*a*c^5 + (C*a^3
 + C*a*b^2)*c^3 + (B*a^3*b + B*a*b^3)*c^2 + (B*a*b^5 + C*a*b^4*c - B*a*b*c
^4 - C*a*c^5)*cos(x)^2 + 2*(B*a^2*b^4 + C*a^2*b^3*c + B*a^2*b^2*c^2 + C*a^
2*b*c^3)*cos(x) + 2*(B*a^2*b^3*c + C*a^2*b^2*c^2 + B*a^2*b*c^3 + C*a^2*c^4
 + (B*a*b^4*c + C*a*b^3*c^2 + B*a*b^2*c^3 + C*a*b*c^4)*cos(x))*sin(x))*sqr
t(-a^2 + b^2 + c^2)*log((a^2*b^2 - 2*b^4 - c^4 - (a^2 + 3*b^2)*c^2 - (2*a^
2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(x)^2 - 2*(a*b^3 + a*b*c^2)*cos(x) - 2*(
a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*cos(x))*sin(x) - 2*(2*a*b*c*
cos(x)^2 - a*b*c + (b^2*c + c^3)*cos(x) - (b^3 + b*c^2 + (a*b^2 - a*c^2)*c
os(x))*sin(x))*sqrt(-a^2 + b^2 + c^2))/(2*a*b*cos(x) + (b^2 - c^2)*cos(x)^
2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x))) - 2*(B*a^6 - 4*B*a^4*b^2 + 2
*B*a^2*b^4 + B*b^6)*c + 2*(B*a*b*c^5 + C*a*c^6 + (C*a^3 + 2*C*a*b^2)*c^4 +
 (B*a^3*b + 2*B*a*b^3)*c^3 - (2*C*a^5 - C*a^3*b^2 - C*a*b^4)*c^2 - (2*B*a^
5*b - B*a^3*b^3 - B*a*b^5)*c)*cos(x) + 2*(2*B*a^5*b^2 - B*a^3*b^4 - B*a*b^
6 - B*a*b^2*c^4 - C*a*b*c^5 - (C*a^3*b + 2*C*a*b^3)*c^3 - (B*a^3*b^2 + ...
```

3.547.6 Sympy [F(-1)]

Timed out.

$$\int \frac{B\cos(x) + C\sin(x)}{(a + b\cos(x) + c\sin(x))^3}\,dx = \text{Timed out}$$

input
```
integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))**3,x)
```

output
```
Timed out
```

3.547.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{B\cos(x) + C\sin(x)}{(a + b\cos(x) + c\sin(x))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` f
or more de
```

3.547.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1034 vs. 2(187) = 374.

Time = 0.36 (sec) , antiderivative size = 1034, normalized size of antiderivative = 5.25

$$\int \frac{B\cos(x) + C\sin(x)}{(a + b\cos(x) + c\sin(x))^3}\,dx = \text{Too large to display}$$

input
```
integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="giac")
```

output
```
3*(B*a*b + C*a*c)*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*t
an(1/2*x) - b*tan(1/2*x) + c)/sqrt(a^2 - b^2 - c^2)))/((a^4 - 2*a^2*b^2 +
b^4 - 2*a^2*c^2 + 2*b^2*c^2 + c^4)*sqrt(a^2 - b^2 - c^2)) + (2*B*a^5*tan(1
/2*x)^3 - 5*B*a^4*b*tan(1/2*x)^3 + 5*B*a^3*b^2*tan(1/2*x)^3 - 5*B*a^2*b^3*
tan(1/2*x)^3 + 5*B*a*b^4*tan(1/2*x)^3 - 2*B*b^5*tan(1/2*x)^3 - 3*C*a^4*c*t
an(1/2*x)^3 + 9*C*a^3*b*c*tan(1/2*x)^3 - 9*C*a^2*b^2*c*tan(1/2*x)^3 + 3*C*
a*b^3*c*tan(1/2*x)^3 - 4*B*a^3*c^2*tan(1/2*x)^3 + 4*B*a^2*b*c^2*tan(1/2*x)
^3 + 4*B*a*b^2*c^2*tan(1/2*x)^3 - 4*B*b^3*c^2*tan(1/2*x)^3 + 2*B*a*c^4*tan
(1/2*x)^3 - 2*B*b*c^4*tan(1/2*x)^3 - 2*C*a^5*tan(1/2*x)^2 + 2*C*a^4*b*tan(
1/2*x)^2 + 4*C*a^3*b^2*tan(1/2*x)^2 - 4*C*a^2*b^3*tan(1/2*x)^2 - 2*C*a*b^4
*tan(1/2*x)^2 + 2*C*b^5*tan(1/2*x)^2 + 2*B*a^4*c*tan(1/2*x)^2 - 9*B*a^3*b*
c*tan(1/2*x)^2 + 14*B*a^2*b^2*c*tan(1/2*x)^2 - 9*B*a*b^3*c*tan(1/2*x)^2 +
2*B*b^4*c*tan(1/2*x)^2 - 5*C*a^3*c^2*tan(1/2*x)^2 + 14*C*a^2*b*c^2*tan(1/2
*x)^2 - 13*C*a*b^2*c^2*tan(1/2*x)^2 + 4*C*b^3*c^2*tan(1/2*x)^2 - 4*B*a^2*c
^3*tan(1/2*x)^2 + 4*B*b^2*c^3*tan(1/2*x)^2 - 2*C*a*c^4*tan(1/2*x)^2 + 2*C*
b*c^4*tan(1/2*x)^2 + 2*B*c^5*tan(1/2*x)^2 + 2*B*a^5*tan(1/2*x) - 3*B*a^4*b
*tan(1/2*x) + B*a^3*b^2*tan(1/2*x) + B*a^2*b^3*tan(1/2*x) - 3*B*a*b^4*tan(
1/2*x) + 2*B*b^5*tan(1/2*x) - 5*C*a^4*c*tan(1/2*x) + 5*C*a^3*b*c*tan(1/2*x
) + 5*C*a^2*b^2*c*tan(1/2*x) - 5*C*a*b^3*c*tan(1/2*x) - 4*B*a^3*c^2*tan(1/
2*x) - 8*B*a^2*b*c^2*tan(1/2*x) + 8*B*a*b^2*c^2*tan(1/2*x) + 4*B*b^3*c^...
```

3.547.9 Mupad [B] (verification not implemented)

Time = 30.83 (sec) , antiderivative size = 923, normalized size of antiderivative = 4.69

$$\int \frac{B\cos(x) + C\sin(x)}{(a + b\cos(x) + c\sin(x))^3}\,dx$$
$$= \frac{\frac{\tan\left(\frac{x}{2}\right)^3 \left(2\,B\,a^4-3\,B\,a^3\,b-3\,C\,a^3\,c+2\,B\,a^2\,b^2+6\,C\,a^2\,b\,c-4\,B\,a^2\,c^2-3\,B\,a\,b^3-3\,C\,a\,b^2\,c+2\,B\,b^4+4\,B\,b^2\,c^2+2\,B\,c^4\right)}{(a-b)\left(a^4-2\,a^2\,b^2-2\,a^2\,c^2+b^4+2\,b^2\,c^2+c^4\right)} - \frac{2\,C\,a^5-4\,C\,a^3\,b^2+5\,B\,a^3 \cdots}{(a-b)^2 \cdots}}{\cdots}$$
$$+ \frac{3\,a\,\mathrm{atanh}\left(\frac{3\,a\,(B\,b+C\,c)\left(\tan\left(\frac{x}{2}\right)(2\,a-2\,b)+\frac{2\,a^4\,c-4\,a^2\,b^2\,c-4\,a^2\,c^3+2\,b^4\,c+4\,b^2\,c^3+2\,c^5}{a^4-2\,a^2\,b^2-2\,a^2\,c^2+b^4+2\,b^2\,c^2+c^4}\right)\left(a^4-2\,a^2\,b^2-2\,a^2\,c^2+b^4+2\,b^2\,c^2+c^4\right)}{2\,(3\,B\,a\,b+3\,C\,a\,c)\left(-a^2+b^2+c^2\right)^{5/2}}\right)(B\,b+ \cdots}{\left(-a^2+b^2+c^2\right)^{5/2}}$$

input
```
int((B*cos(x) + C*sin(x))/(a + b*cos(x) + c*sin(x))^3,x)
```

output
```
((tan(x/2)^3*(2*B*a^4 + 2*B*b^4 + 2*B*c^4 + 2*B*a^2*b^2 - 4*B*a^2*c^2 + 4*
B*b^2*c^2 - 3*B*a*b^3 - 3*B*a^3*b - 3*C*a^3*c - 3*C*a*b^2*c + 6*C*a^2*b*c)
)/((a - b)*(a^4 + b^4 + c^4 - 2*a^2*b^2 - 2*a^2*c^2 + 2*b^2*c^2)) - (2*C*a
^5 - 4*C*a^3*b^2 + C*a^3*c^2 + 2*C*a*b^4 - 2*B*a*b*c^3 - 5*B*a*b^3*c + 5*B
*a^3*b*c - C*a*b^2*c^2)/((a - b)^2*(a^4 + b^4 + c^4 - 2*a^2*b^2 - 2*a^2*c^
2 + 2*b^2*c^2)) + (tan(x/2)^2*(2*B*c^5 - 2*C*a^5 + 2*C*b^5 - 4*B*a^2*c^3 -
 4*C*a^2*b^3 + 4*C*a^3*b^2 + 4*B*b^2*c^3 - 5*C*a^3*c^2 + 4*C*b^3*c^2 + 2*B
*a^4*c - 2*C*a*b^4 + 2*C*a^4*b + 2*B*b^4*c - 2*C*a*c^4 + 2*C*b*c^4 - 9*B*a
*b^3*c - 9*B*a^3*b*c + 14*B*a^2*b^2*c - 13*C*a*b^2*c^2 + 14*C*a^2*b*c^2))/
((a - b)^2*(a^4 + b^4 + c^4 - 2*a^2*b^2 - 2*a^2*c^2 + 2*b^2*c^2)) + (tan(x
/2)*(2*B*a^5 + 2*B*b^5 + B*a^2*b^3 + B*a^3*b^2 - 4*B*a^3*c^2 + 4*B*b^3*c^2
 - 4*C*a^2*c^3 - 3*B*a*b^4 - 3*B*a^4*b + 2*B*a*c^4 + 2*B*b*c^4 - 5*C*a^4*c
 + 4*C*a*b*c^3 - 5*C*a*b^3*c + 5*C*a^3*b*c + 8*B*a*b^2*c^2 - 8*B*a^2*b*c^2
 + 5*C*a^2*b^2*c))/((a - b)^2*(a^4 + b^4 + c^4 - 2*a^2*b^2 - 2*a^2*c^2 + 2
*b^2*c^2)))/(tan(x/2)^4*(a^2 - 2*a*b + b^2) + 2*a*b + tan(x/2)*(4*a*c + 4*
b*c) + tan(x/2)^3*(4*a*c - 4*b*c) + a^2 + b^2 + tan(x/2)^2*(2*a^2 - 2*b^2
+ 4*c^2)) + (3*a*atanh((3*a*(B*b + C*c)*(tan(x/2)*(2*a - 2*b) + (2*a^4*c +
 2*b^4*c + 2*c^5 - 4*a^2*c^3 + 4*b^2*c^3 - 4*a^2*b^2*c)/(a^4 + b^4 + c^4 -
 2*a^2*b^2 - 2*a^2*c^2 + 2*b^2*c^2))*(a^4 + b^4 + c^4 - 2*a^2*b^2 - 2*a^2*
c^2 + 2*b^2*c^2))/(2*(3*B*a*b + 3*C*a*c)*(b^2 - a^2 + c^2)^(5/2)))*(B*b...
```

3.547. $\int \frac{B\cos(x)+C\sin(x)}{(a+b\cos(x)+c\sin(x))^3}\,dx$

3.548 $\int \frac{B\cos(x)+C\sin(x)}{a+b\cos(x)+ib\sin(x)}\,dx$

3.548.1 Optimal result

Integrand size = 25, antiderivative size = 92

$$\int \frac{B\cos(x)+C\sin(x)}{a+b\cos(x)+ib\sin(x)}\,dx = -\frac{b(B+iC)x}{2a^2} - \frac{(ib^2(B+iC)+a^2(iB+C))\log(a+b\cos(x)+ib\sin(x))}{2a^2b} + \frac{(iB-C)(\cos(x)-i\sin(x))}{2a}$$

output
```
-1/2*b*(B+I*C)*x/a^2-1/2*(I*b^2*(B+I*C)+a^2*(I*B+C))*ln(a+b*cos(x)+I*b*sin
(x))/a^2/b+1/2*(I*B-C)*(cos(x)-I*sin(x))/a
```

3.548.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 195 vs. 2(92) = 184.

Time = 0.55 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.12

$$\int \frac{B\cos(x)+C\sin(x)}{a+b\cos(x)+ib\sin(x)}\,dx=\frac{(a^2B-b^2B-ia^2C-ib^2C)\,x}{4a^2b}-\frac{(a^2B+b^2B-ia^2C+ib^2C)\arctan\left(\frac{(a+b)\cos\left(\frac{x}{2}\right)}{-a\sin\left(\frac{x}{2}\right)+b\sin\left(\frac{x}{2}\right)}\right)}{2a^2b}+\frac{i(B+iC)\cos(x)}{2a}-\frac{i(a^2B+b^2B-ia^2C+ib^2C)\log\left(a^2+b^2+2ab\cos(x)\right)}{4a^2b}+\frac{(B+iC)\sin(x)}{2a}$$

input
```
Integrate[(B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + I*b*Sin[x]),x]
```

output
```
((a^2*B - b^2*B - I*a^2*C - I*b^2*C)*x)/(4*a^2*b) - ((a^2*B + b^2*B - I*a^
2*C + I*b^2*C)*ArcTan[((a + b)*Cos[x/2])/(-(a*Sin[x/2]) + b*Sin[x/2])])/(2
*a^2*b) + ((I/2)*(B + I*C)*Cos[x])/a - ((I/4)*(a^2*B + b^2*B - I*a^2*C + I
*b^2*C)*Log[a^2 + b^2 + 2*a*b*Cos[x]])/(a^2*b) + ((B + I*C)*Sin[x])/(2*a)
```

3.548.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.95, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {3042, 3609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\cos(x)+C\sin(x)}{a+ib\sin(x)+b\cos(x)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{B\cos(x)+C\sin(x)}{a+ib\sin(x)+b\cos(x)}dx$$

$$\Big\downarrow \; 3609$$

$$-\frac{\left(\frac{ib^2(B+iC)}{a^2}+iB+C\right)\log(a+ib\sin(x)+b\cos(x))}{2b}-\frac{bx(B+iC)}{2a^2}+\frac{(-C+iB)(\cos(x)-i\sin(x))}{2a}$$

input
```
Int[(B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + I*b*Sin[x]),x]
```

output
```
-1/2*(b*(B + I*C)*x)/a^2 - ((I*B + (I*b^2*(B + I*C))/a^2 + C)*Log[a + b*Co
s[x] + I*b*Sin[x]])/(2*b) + ((I*B - C)*(Cos[x] - I*Sin[x]))/(2*a)
```

3.548.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3609
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_
Symbol] :> Simp[(2*a*A - b*B - c*C)*(x/(2*a^2)), x] + (-Simp[(b*B + c*C)*((
b*Cos[d + e*x] - c*Sin[d + e*x])/(2*a*b*c*e)), x] + Simp[(a^2*(b*B - c*C) -
 2*a*A*b^2 + b^2*(b*B + c*C))*(Log[RemoveContent[a + b*Cos[d + e*x] + c*Sin
[d + e*x], x]]/(2*a^2*b*c*e)), x]) /; FreeQ[{a, b, c, d, e, A, B, C}, x] &&
 EqQ[b^2 + c^2, 0]
```

3.548.4 Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.28

method	result
risch	$-\frac{C\,\mathrm{e}^{-ix}}{2a}+\frac{iB\,\mathrm{e}^{-ix}}{2a}-\frac{ibxC}{2a^2}-\frac{bxB}{2a^2}-\frac{\ln\left(\mathrm{e}^{ix}+\frac{a}{b}\right)C}{2b}+\frac{b\ln\left(\mathrm{e}^{ix}+\frac{a}{b}\right)C}{2a^2}-\frac{i\ln\left(\mathrm{e}^{ix}+\frac{a}{b}\right)B}{2b}-\frac{ib\ln\left(\mathrm{e}^{ix}+\frac{a}{b}\right)B}{2a^2}$
default	$-\frac{-iC-B}{a\left(-i+\tan\left(\frac{x}{2}\right)\right)}+\frac{b(iB-C)\ln\left(-i+\tan\left(\frac{x}{2}\right)\right)}{2a^2}-\frac{i\left(-iC\,a^2+iC\,b^2+B\,a^2+B\,b^2\right)\ln\left(ia+ib+a\tan\left(\frac{x}{2}\right)-b\tan\left(\frac{x}{2}\right)\right)}{2a^2b}+\frac{i(-iC+B)\ln}{2}$

input
```
int((B*cos(x)+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*C/a*exp(-I*x)+1/2*I*B/a*exp(-I*x)-1/2*I*b/a^2*x*C-1/2/a^2*b*x*B-1/2/b
*ln(exp(I*x)+a/b)*C+1/2*b/a^2*ln(exp(I*x)+a/b)*C-1/2*I/b*ln(exp(I*x)+a/b)*
B-1/2*I/a^2*b*ln(exp(I*x)+a/b)*B
```

3.548.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.85

$$\int \frac{B\cos(x)+C\sin(x)}{a+b\cos(x)+ib\sin(x)}\,dx = -\frac{\left((B+iC)b^2xe^{(ix)}-(iB-C)ab-((-iB-C)a^2+(-iB+C)b^2)e^{(ix)}\log\left(\frac{be^{(ix)}+a}{b}\right)\right)e^{(-ix)}}{2a^2b}$$

input
```
integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="fricas
")
```

output
```
-1/2*((B + I*C)*b^2*x*e^(I*x) - (I*B - C)*a*b - ((-I*B - C)*a^2 + (-I*B +
C)*b^2)*e^(I*x)*log((b*e^(I*x) + a)/b))*e^(-I*x)/(a^2*b)
```

3.548.6 Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.20

$$\int \frac{B\cos(x)+C\sin(x)}{a+b\cos(x)+ib\sin(x)}\,dx = \begin{cases} \frac{(iB-C)e^{-ix}}{2a} & \text{for } a\neq 0 \\ x\left(-\frac{-Bb-iCb}{2a^2}+\frac{Ba-Bb+iCa-iCb}{2a^2}\right) & \text{otherwise} \end{cases} + \frac{x(-Bb-iCb)}{2a^2} - \frac{i(Ba^2+Bb^2-iCa^2+iCb^2)\log\left(\frac{a}{b}+e^{ix}\right)}{2a^2b}$$

input
```
integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x)
```

output
```
Piecewise(((I*B - C)*exp(-I*x)/(2*a), Ne(a, 0)), (x*(-(-B*b - I*C*b)/(2*a*
*2) + (B*a - B*b + I*C*a - I*C*b)/(2*a**2)), True)) + x*(-B*b - I*C*b)/(2*
a**2) - I*(B*a**2 + B*b**2 - I*C*a**2 + I*C*b**2)*log(a/b + exp(I*x))/(2*a
**2*b)
```

3.548.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{B\cos(x) + C\sin(x)}{a + b\cos(x) + ib\sin(x)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.548.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(70) = 140.

Time = 0.27 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.93

$$\begin{aligned}\int \frac{B\cos(x) + C\sin(x)}{a + b\cos(x) + ib\sin(x)}\,dx &= -\frac{(iBb - Cb)\log\left(-a\tan\left(\frac{1}{2}x\right)^2 + b\tan\left(\frac{1}{2}x\right)^2 - 2i\,a\tan\left(\frac{1}{2}x\right) + a + b\right)}{4\,a^2}\\ &\quad - \frac{(-iBb + Cb)\log\left(\tan\left(\frac{1}{2}x\right) - i\right)}{2\,a^2}\\ &\quad + \frac{(2\,Ba^2 - 2i\,Ca^2 + Bb^2 + i\,Cb^2)\left(x + 2\arctan\left(\frac{-i\,a\cos(x) - a\sin(x) - i\,a}{a\cos(x) - i\,a\sin(x) - a + 2\,b}\right)\right)}{4\,a^2b}\\ &\quad - \frac{i\,Bb\tan\left(\frac{1}{2}x\right) - Cb\tan\left(\frac{1}{2}x\right) - 2\,Ba - 2i\,Ca + Bb + i\,Cb}{2\,a^2\left(\tan\left(\frac{1}{2}x\right) - i\right)}\end{aligned}$$

input
```
integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="giac")
```

output
```
-1/4*(I*B*b - C*b)*log(-a*tan(1/2*x)^2 + b*tan(1/2*x)^2 - 2*I*a*tan(1/2*x)
 + a + b)/a^2 - 1/2*(-I*B*b + C*b)*log(tan(1/2*x) - I)/a^2 + 1/4*(2*B*a^2
- 2*I*C*a^2 + B*b^2 + I*C*b^2)*(x + 2*arctan((-I*a*cos(x) - a*sin(x) - I*a
)/(a*cos(x) - I*a*sin(x) - a + 2*b)))/(a^2*b) - 1/2*(I*B*b*tan(1/2*x) - C*
b*tan(1/2*x) - 2*B*a - 2*I*C*a + B*b + I*C*b)/(a^2*(tan(1/2*x) - I))
```

3.548.9 Mupad [B] (verification not implemented)

Time = 29.10 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.28

$$\int \frac{B\cos(x)+C\sin(x)}{a+b\cos(x)+ib\sin(x)}\,dx = -\ln\left(a+b-a\tan\left(\frac{x}{2}\right)\,1\mathrm{i}+b\tan\left(\frac{x}{2}\right)\,1\mathrm{i}\right)\left(\frac{\frac{C}{2}+\frac{B\,1\mathrm{i}}{2}}{b}+\frac{-\frac{C\,b^2}{2}+\frac{B\,b^2\,1\mathrm{i}}{2}}{a^2\,b}\right)+\frac{B+C\,1\mathrm{i}}{a\,\left(\tan\left(\frac{x}{2}\right)-\mathrm{i}\right)}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)+1\mathrm{i}\right)\,(C+B\,1\mathrm{i})}{2\,b}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)-\mathrm{i}\right)\,(-C\,b+B\,b\,1\mathrm{i})}{2\,a^2}$$

input
```
int((B*cos(x) + C*sin(x))/(a + b*cos(x) + b*sin(x)*1i),x)
```

output
```
(B + C*1i)/(a*(tan(x/2) - 1i)) - log(a + b - a*tan(x/2)*1i + b*tan(x/2)*1i
)*(((B*1i)/2 + C/2)/b + ((B*b^2*1i)/2 - (C*b^2)/2)/(a^2*b)) + (log(tan(x/2
) + 1i)*(B*1i + C))/(2*b) + (log(tan(x/2) - 1i)*(B*b*1i - C*b))/(2*a^2)
```

3.549 $\int \frac{B\cos(x)+C\sin(x)}{a+b\cos(x)-ib\sin(x)}\,dx$

3.549.1 Optimal result

Integrand size = 25, antiderivative size = 90

$$\int \frac{B\cos(x)+C\sin(x)}{a+b\cos(x)-ib\sin(x)}\,dx = -\frac{b(B-iC)x}{2a^2} + \frac{(ia^2(B+iC)+b^2(iB+C))\log(a+b\cos(x)-ib\sin(x))}{2a^2b} - \frac{(iB+C)(\cos(x)+i\sin(x))}{2a}$$

output
```
-1/2*b*(B-I*C)*x/a^2+1/2*(I*a^2*(B+I*C)+b^2*(I*B+C))*ln(a+b*cos(x)-I*b*sin
(x))/a^2/b-1/2*(I*B+C)*(cos(x)+I*sin(x))/a
```

3.549.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 195 vs. $2(90) = 180$.

Time = 0.56 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.17

$$\int \frac{B\cos(x)+C\sin(x)}{a+b\cos(x)-ib\sin(x)}\,dx = \frac{(a^2B-b^2B+ia^2C+ib^2C)\,x}{4a^2b} + \frac{(a^2B+b^2B+ia^2C-ib^2C)\arctan\left(\frac{(a+b)\cos\left(\frac{x}{2}\right)}{a\sin\left(\frac{x}{2}\right)-b\sin\left(\frac{x}{2}\right)}\right)}{2a^2b} - \frac{i(B-iC)\cos(x)}{2a} + \frac{i(a^2B+b^2B+ia^2C-ib^2C)\log\left(a^2+b^2+2ab\cos(x)\right)}{4a^2b} + \frac{(B-iC)\sin(x)}{2a}$$

input
```
Integrate[(B*Cos[x] + C*Sin[x])/(a + b*Cos[x] - I*b*Sin[x]),x]
```

output
```
((a^2*B - b^2*B + I*a^2*C + I*b^2*C)*x)/(4*a^2*b) + ((a^2*B + b^2*B + I*a^
2*C - I*b^2*C)*ArcTan[((a + b)*Cos[x/2])/(a*Sin[x/2] - b*Sin[x/2])])/(2*a^
2*b) - ((I/2)*(B - I*C)*Cos[x])/a + ((I/4)*(a^2*B + b^2*B + I*a^2*C - I*b^
2*C)*Log[a^2 + b^2 + 2*a*b*Cos[x]])/(a^2*b) + ((B - I*C)*Sin[x])/(2*a)
```

3.549.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.94, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3042, 3609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\cos(x)+C\sin(x)}{a-ib\sin(x)+b\cos(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{B\cos(x)+C\sin(x)}{a-ib\sin(x)+b\cos(x)}dx$$

$$\Big\downarrow \text{3609}$$

$$-\frac{bx(B-iC)}{2a^2}+\frac{1}{2}\left(\frac{b(C+iB)}{a^2}+\frac{i(B+iC)}{b}\right)\log(a-ib\sin(x)+b\cos(x))-\frac{(C+iB)(\cos(x)+i\sin(x))}{2a}$$

input
```
Int[(B*Cos[x] + C*Sin[x])/(a + b*Cos[x] - I*b*Sin[x]),x]
```

output
```
-1/2*(b*(B - I*C)*x)/a^2 + (((I*(B + I*C))/b + (b*(I*B + C))/a^2)*Log[a + 
b*Cos[x] - I*b*Sin[x]])/2 - ((I*B + C)*(Cos[x] + I*Sin[x]))/(2*a)
```

3.549.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3609
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_
Symbol] :> Simp[(2*a*A - b*B - c*C)*(x/(2*a^2)), x] + (-Simp[(b*B + c*C)*((
b*Cos[d + e*x] - c*Sin[d + e*x])/(2*a*b*c*e)), x] + Simp[(a^2*(b*B - c*C) -
 2*a*A*b^2 + b^2*(b*B + c*C))*(Log[RemoveContent[a + b*Cos[d + e*x] + c*Sin
[d + e*x], x]]/(2*a^2*b*c*e)), x]) /; FreeQ[{a, b, c, d, e, A, B, C}, x] &&
 EqQ[b^2 + c^2, 0]
```

3.549.4 Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.29

method	result
risch	$-\frac{C\,\mathrm{e}^{ix}}{2a}-\frac{iB\,\mathrm{e}^{ix}}{2a}+\frac{ixC}{2b}+\frac{Bx}{2b}-\frac{\ln\left(\mathrm{e}^{ix}+\frac{b}{a}\right)C}{2b}+\frac{b\ln\left(\mathrm{e}^{ix}+\frac{b}{a}\right)C}{2a^2}+\frac{i\ln\left(\mathrm{e}^{ix}+\frac{b}{a}\right)B}{2b}+\frac{ib\ln\left(\mathrm{e}^{ix}+\frac{b}{a}\right)B}{2a^2}$
default	$-\frac{i(iC+B)\ln\left(-i+\tan\left(\frac{x}{2}\right)\right)}{2b}-\frac{i\left(iC\,a^2-iC\,b^2+B\,a^2+B\,b^2\right)(a-b)\ln\left(ia+ib-a\tan\left(\frac{x}{2}\right)+b\tan\left(\frac{x}{2}\right)\right)}{2a^2b(-a+b)}-\frac{iC-B}{a\left(\tan\left(\frac{x}{2}\right)+i\right)}-\frac{b(iB+C)\ln}{2}$

input
```
int((B*cos(x)+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*C/a*exp(I*x)-1/2*I*B/a*exp(I*x)+1/2*I/b*x*C+1/2*B*x/b-1/2/b*ln(exp(I*
x)+b/a)*C+1/2*b/a^2*ln(exp(I*x)+b/a)*C+1/2*I/b*ln(exp(I*x)+b/a)*B+1/2*I*b/
a^2*ln(exp(I*x)+b/a)*B
```

3.549.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

$$\int \frac{B\cos(x)+C\sin(x)}{a+b\cos(x)-ib\sin(x)}\,dx = \frac{(B+i\,C)a^2x+(-i\,B-C)abe^{(i\,x)}+((i\,B-C)a^2+(i\,B+C)b^2)\log\left(\frac{ae^{(i\,x)}+b}{a}\right)}{2\,a^2b}$$

input
```
integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="fricas
")
```

output
```
1/2*((B + I*C)*a^2*x + (-I*B - C)*a*b*e^(I*x) + ((I*B - C)*a^2 + (I*B + C)
*b^2)*log((a*e^(I*x) + b)/a))/(a^2*b)
```

3.549.6 Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.10

$$\int \frac{B\cos(x)+C\sin(x)}{a+b\cos(x)-ib\sin(x)}\,dx = \begin{cases} \frac{(-iB-C)e^{ix}}{2a} & \text{for } a\neq 0 \\ x\left(-\frac{B+iC}{2b}+\frac{Ba+Bb+iCa-iCb}{2ab}\right) & \text{otherwise} \end{cases} + \frac{x(B+iC)}{2b} + \frac{i(Ba^2+Bb^2+iCa^2-iCb^2)\log\left(e^{ix}+\frac{b}{a}\right)}{2a^2b}$$

input
```
integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x)
```

output
```
Piecewise(((-I*B - C)*exp(I*x)/(2*a), Ne(a, 0)), (x*(-(B + I*C)/(2*b) + (B
*a + B*b + I*C*a - I*C*b)/(2*a*b)), True)) + x*(B + I*C)/(2*b) + I*(B*a**2
 + B*b**2 + I*C*a**2 - I*C*b**2)*log(exp(I*x) + b/a)/(2*a**2*b)
```

3.549.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{B\cos(x) + C\sin(x)}{a + b\cos(x) - ib\sin(x)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.549.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(73) = 146.

Time = 0.27 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.98

$$\int \frac{B\cos(x) + C\sin(x)}{a + b\cos(x) - ib\sin(x)}\,dx$$
$$= -\frac{(-iBb - Cb)\log\left(-a\tan\left(\frac{1}{2}x\right)^2 + b\tan\left(\frac{1}{2}x\right)^2 + 2ia\tan\left(\frac{1}{2}x\right) + a + b\right)}{4a^2}$$
$$- \frac{(iBb + Cb)\log\left(\tan\left(\frac{1}{2}x\right) + i\right)}{2a^2}$$
$$+ \frac{(2Ba^2 + 2iCa^2 + Bb^2 - iCb^2)\left(x + 2\arctan\left(\frac{ia\cos(x) - a\sin(x) + ia}{a\cos(x) + ia\sin(x) - a + 2b}\right)\right)}{4a^2b}$$
$$- \frac{-iBb\tan\left(\frac{1}{2}x\right) - Cb\tan\left(\frac{1}{2}x\right) - 2Ba + 2iCa + Bb - iCb}{2a^2\left(\tan\left(\frac{1}{2}x\right) + i\right)}$$

input
```
integrate((B*cos(x)+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="giac")
```

output
```
-1/4*(-I*B*b - C*b)*log(-a*tan(1/2*x)^2 + b*tan(1/2*x)^2 + 2*I*a*tan(1/2*x
) + a + b)/a^2 - 1/2*(I*B*b + C*b)*log(tan(1/2*x) + I)/a^2 + 1/4*(2*B*a^2
+ 2*I*C*a^2 + B*b^2 - I*C*b^2)*(x + 2*arctan((I*a*cos(x) - a*sin(x) + I*a)
/(a*cos(x) + I*a*sin(x) - a + 2*b)))/(a^2*b) - 1/2*(-I*B*b*tan(1/2*x) - C*
b*tan(1/2*x) - 2*B*a + 2*I*C*a + B*b - I*C*b)/(a^2*(tan(1/2*x) + I))
```

3.549.9 Mupad [B] (verification not implemented)

Time = 28.00 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.31

$$\int \frac{B\cos(x)+C\sin(x)}{a+b\cos(x)-ib\sin(x)}\,dx = \ln\left(a+b+a\tan\left(\frac{x}{2}\right)\,1\mathrm{i}-b\tan\left(\frac{x}{2}\right)\,1\mathrm{i}\right)\left(\frac{-\frac{C}{2}+\frac{B\,1\mathrm{i}}{2}}{b}+\frac{\frac{C\,b^2}{2}+\frac{B\,b^2\,1\mathrm{i}}{2}}{a^2\,b}\right)+\frac{B-C\,1\mathrm{i}}{a\,\left(\tan\left(\frac{x}{2}\right)+1\mathrm{i}\right)}-\frac{\ln\left(\tan\left(\frac{x}{2}\right)+1\mathrm{i}\right)\,\left(C\,b+B\,b\,1\mathrm{i}\right)}{2\,a^2}-\frac{\ln\left(\tan\left(\frac{x}{2}\right)-\mathrm{i}\right)\,\left(-C+B\,1\mathrm{i}\right)}{2\,b}$$

input
```
int((B*cos(x) + C*sin(x))/(a + b*cos(x) - b*sin(x)*1i),x)
```

output
```
log(a + b + a*tan(x/2)*1i - b*tan(x/2)*1i)*(((B*1i)/2 - C/2)/b + ((B*b^2*1
i)/2 + (C*b^2)/2)/(a^2*b)) + (B - C*1i)/(a*(tan(x/2) + 1i)) - (log(tan(x/2
) + 1i)*(B*b*1i + C*b))/(2*a^2) - (log(tan(x/2) - 1i)*(B*1i - C))/(2*b)
```

3.550 $\int \frac{A+B\cos(x)+C\sin(x)}{a+b\cos(x)+c\sin(x)}\,dx$

3.550.1 Optimal result

Integrand size = 23, antiderivative size = 131

$$\int \frac{A+B\cos(x)+C\sin(x)}{a+b\cos(x)+c\sin(x)}\,dx=\frac{(bB+cC)x}{b^2+c^2}+\frac{2(A(b^2+c^2)-a(bB+cC))\arctan\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}\,(b^2+c^2)}+\frac{(Bc-bC)\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}$$

output
```
(B*b+C*c)*x/(b^2+c^2)+(B*c-C*b)*ln(a+b*cos(x)+c*sin(x))/(b^2+c^2)+2*(A*(b^
2+c^2)-a*(B*b+C*c))*arctan((c+(a-b)*tan(1/2*x))/(a^2-b^2-c^2)^(1/2))/(b^2+
c^2)/(a^2-b^2-c^2)^(1/2)
```

3.550.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.84

$$\int \frac{A+B\cos(x)+C\sin(x)}{a+b\cos(x)+c\sin(x)}\,dx=\frac{(bB+cC)x+\frac{2(-A(b^2+c^2)+a(bB+cC))\text{arctanh}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2+c^2}}\right)}{\sqrt{-a^2+b^2+c^2}}+(Bc-bC)\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}$$

input
```
Integrate[(A + B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x]),x]
```

output
```
((b*B + c*C)*x + (2*(-(A*(b^2 + c^2)) + a*(b*B + c*C))*ArcTanh[(c + (a - b
)*Tan[x/2])/Sqrt[-a^2 + b^2 + c^2]])/Sqrt[-a^2 + b^2 + c^2] + (B*c - b*C)*
Log[a + b*Cos[x] + c*Sin[x]])/(b^2 + c^2)
```

3.550.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 3615, 3042, 3603, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(x)+C\sin(x)}{a+b\cos(x)+c\sin(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\cos(x)+C\sin(x)}{a+b\cos(x)+c\sin(x)}dx$$

$$\Big\downarrow \text{3615}$$

$$\left(A-\frac{a(bB+cC)}{b^2+c^2}\right)\int \frac{1}{a+b\cos(x)+c\sin(x)}dx+\frac{(Bc-bC)\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\frac{x(bB+cC)}{b^2+c^2}$$

$$\Big\downarrow \text{3042}$$

$$\left(A-\frac{a(bB+cC)}{b^2+c^2}\right)\int \frac{1}{a+b\cos(x)+c\sin(x)}dx+\frac{(Bc-bC)\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\frac{x(bB+cC)}{b^2+c^2}$$

$$\Big\downarrow \text{3603}$$

$$2\left(A-\frac{a(bB+cC)}{b^2+c^2}\right)\int \frac{1}{(a-b)\tan^2\left(\frac{x}{2}\right)+2c\tan\left(\frac{x}{2}\right)+a+b}d\tan\left(\frac{x}{2}\right)+\frac{(Bc-bC)\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\frac{x(bB+cC)}{b^2+c^2}$$

$$\Big\downarrow \text{1083}$$

$$-4\left(A-\frac{a(bB+cC)}{b^2+c^2}\right)\int \frac{1}{-\left(2c+2(a-b)\tan\left(\frac{x}{2}\right)\right)^2-4\left(a^2-b^2-c^2\right)}d\left(2c+2(a-b)\tan\left(\frac{x}{2}\right)\right)+\frac{(Bc-bC)\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\frac{x(bB+cC)}{b^2+c^2}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{2\left(A-\frac{a(bB+cC)}{b^2+c^2}\right)\arctan\left(\frac{2(a-b)\tan\left(\frac{x}{2}\right)+2c}{2\sqrt{a^2-b^2-c^2}}\right)}{\sqrt{a^2-b^2-c^2}}+\frac{(Bc-bC)\log(a+b\cos(x)+c\sin(x))}{b^2+c^2}+\frac{x(bB+cC)}{b^2+c^2}$$

input
```
Int[(A + B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x]),x]
```

output
```
((b*B + c*C)*x)/(b^2 + c^2) + (2*(A - (a*(b*B + c*C))/(b^2 + c^2))*ArcTan[
(2*c + 2*(a - b)*Tan[x/2])/(2*Sqrt[a^2 - b^2 - c^2])])/Sqrt[a^2 - b^2 - c^
2] + ((B*c - b*C)*Log[a + b*Cos[x] + c*Sin[x]])/(b^2 + c^2)
```

3.550.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

rule 3615
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + (Simp[(c*B - b*C)*(Log[a
 + b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] + Simp[(A*(b^2 + c
^2) - a*(b*B + c*C))/(b^2 + c^2)   Int[1/(a + b*Cos[d + e*x] + c*Sin[d + e*
x]), x], x]) /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] &&
NeQ[A*(b^2 + c^2) - a*(b*B + c*C), 0]
```

3.550.4 Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.73

method	result
default	$\frac{\frac{2\left(aBc-bBc-abC+b^2C\right)\ln\left(\tan\left(\frac{x}{2}\right)^2a-\tan\left(\frac{x}{2}\right)^2b+2c\tan\left(\frac{x}{2}\right)+a+b\right)}{2a-2b}+\frac{2\left(A b^2+A c^2-abB+B c^2-acC-Cbc-\frac{\left(aBc-bBc-abC+b^2C\right)c}{a-b}\right)\arctan}{\sqrt{a^2-b^2-c^2}}}{b^2+c^2}$
risch	Expression too large to display

input
```
int((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
2/(b^2+c^2)*(1/2*(B*a*c-B*b*c-C*a*b+C*b^2)/(a-b)*ln(tan(1/2*x)^2*a-tan(1/2
*x)^2*b+2*c*tan(1/2*x)+a+b)+(A*b^2+A*c^2-a*b*B+B*c^2-a*c*C-C*b*c-(B*a*c-B*
b*c-C*a*b+C*b^2)*c/(a-b))/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*
x)+2*c)/(a^2-b^2-c^2)^(1/2)))+2/(b^2+c^2)*(1/2*(-B*c+C*b)*ln(1+tan(1/2*x)^
2)+(B*b+C*c)*arctan(tan(1/2*x)))
```

3.550.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. 2(126) = 252.

Time = 0.32 (sec) , antiderivative size = 711, normalized size of antiderivative = 5.43

$$\int \frac{A + B\cos(x) + C\sin(x)}{a + b\cos(x) + c\sin(x)}\,dx$$
$$= \left[-\frac{(Bab - Ab^2 + Cac - Ac^2)\sqrt{-a^2 + b^2 + c^2}\log\left(\frac{a^2b^2 - 2\,b^4 - c^4 - (a^2 + 3\,b^2)c^2 - (2\,a^2b^2 - b^4 - 2\,a^2c^2 + c^4)\cos(x)^2 - 2\,(ab^3 + ab\right.}{}\right.$$

$$-\frac{2\,(Bab - Ab^2 + Cac - Ac^2)\sqrt{a^2 - b^2 - c^2}\arctan\left(-\frac{(ab\cos(x) + ac\sin(x) + b^2 + c^2)\sqrt{a^2 - b^2 - c^2}}{(c^3 - (a^2 - b^2)c)\cos(x) + (a^2b - b^3 - bc^2)\sin(x)}\right) - 2\,(Ba^2b - B}{}$$

input
```
integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="fricas
")
```

output
```
[-1/2*((B*a*b - A*b^2 + C*a*c - A*c^2)*sqrt(-a^2 + b^2 + c^2)*log((a^2*b^2
 - 2*b^4 - c^4 - (a^2 + 3*b^2)*c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*c
os(x)^2 - 2*(a*b^3 + a*b*c^2)*cos(x) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^
2*b - b^3)*c)*cos(x))*sin(x) - 2*(2*a*b*c*cos(x)^2 - a*b*c + (b^2*c + c^3)
*cos(x) - (b^3 + b*c^2 + (a*b^2 - a*c^2)*cos(x))*sin(x))*sqrt(-a^2 + b^2 +
 c^2))/(2*a*b*cos(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) +
a*c)*sin(x))) - 2*(B*a^2*b - B*b^3 - B*b*c^2 - C*c^3 + (C*a^2 - C*b^2)*c)*
x + (C*a^2*b - C*b^3 - C*b*c^2 + B*c^3 - (B*a^2 - B*b^2)*c)*log(2*a*b*cos(
x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x)))/(a^2
*b^2 - b^4 - c^4 + (a^2 - 2*b^2)*c^2), -1/2*(2*(B*a*b - A*b^2 + C*a*c - A*
c^2)*sqrt(a^2 - b^2 - c^2)*arctan(-(a*b*cos(x) + a*c*sin(x) + b^2 + c^2)*s
qrt(a^2 - b^2 - c^2)/((c^3 - (a^2 - b^2)*c)*cos(x) + (a^2*b - b^3 - b*c^2)
*sin(x))) - 2*(B*a^2*b - B*b^3 - B*b*c^2 - C*c^3 + (C*a^2 - C*b^2)*c)*x +
(C*a^2*b - C*b^3 - C*b*c^2 + B*c^3 - (B*a^2 - B*b^2)*c)*log(2*a*b*cos(x) +
 (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x)))/(a^2*b^2
 - b^4 - c^4 + (a^2 - 2*b^2)*c^2)]
```

3.550. $\int \frac{A+B\cos(x)+C\sin(x)}{a+b\cos(x)+c\sin(x)}\,dx$

3.550.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(x)+C\sin(x)}{a+b\cos(x)+c\sin(x)}\,dx=\text{Timed out}$$

input
```
integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x)),x)
```

output
```
Timed out
```

3.550.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\cos(x)+C\sin(x)}{a+b\cos(x)+c\sin(x)}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` f
or more de
```

3.550.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.52

$$\int \frac{A+B\cos(x)+C\sin(x)}{a+b\cos(x)+c\sin(x)}\,dx$$
$$=\frac{(Bb+Cc)x}{b^2+c^2}-\frac{(Cb-Bc)\log\left(-a\tan\left(\frac{1}{2}x\right)^2+b\tan\left(\frac{1}{2}x\right)^2-2c\tan\left(\frac{1}{2}x\right)-a-b\right)}{b^2+c^2}$$
$$+\frac{(Cb-Bc)\log\left(\tan\left(\frac{1}{2}x\right)^2+1\right)}{b^2+c^2}$$
$$+\frac{2\left(Bab-Ab^2+Cac-Ac^2\right)\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}x\right)-b\tan\left(\frac{1}{2}x\right)+c}{\sqrt{a^2-b^2-c^2}}\right)\right)}{\sqrt{a^2-b^2-c^2}(b^2+c^2)}$$

input
```
integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x)),x, algorithm="giac")
```

output
```
(B*b + C*c)*x/(b^2 + c^2) - (C*b - B*c)*log(-a*tan(1/2*x)^2 + b*tan(1/2*x)
^2 - 2*c*tan(1/2*x) - a - b)/(b^2 + c^2) + (C*b - B*c)*log(tan(1/2*x)^2 +
1)/(b^2 + c^2) + 2*(B*a*b - A*b^2 + C*a*c - A*c^2)*(pi*floor(1/2*x/pi + 1/
2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*tan(1/2*x) + c)/sqrt(a^2 -
b^2 - c^2)))/(sqrt(a^2 - b^2 - c^2)*(b^2 + c^2))
```

3.550.9 Mupad [B] (verification not implemented)

Time = 119.49 (sec) , antiderivative size = 2711, normalized size of antiderivative = 20.69

$$\int \frac{A + B\cos(x) + C\sin(x)}{a + b\cos(x) + c\sin(x)}\,dx = \text{Too large to display}$$

input
```
int((A + B*cos(x) + C*sin(x))/(a + b*cos(x) + c*sin(x)),x)
```

output
```
(log(tan(x/2) - 1i)*(B + C*1i))/(b*1i - c) - (log(tan(x/2) + 1i)*(B - C*1i
))/(b*1i + c) + (log(32*B^3*a^2 - 32*A*B^2*a^2 + 32*A*B^2*b^2 + 32*A*C^2*a
^2 - 32*A^2*B*b^2 + 32*A*C^2*b^2 + 32*B*C^2*a^2 + 32*B*C^2*b^2 + 32*tan(x/
2)*(a - b)*(2*C^3*a + B^3*c - 2*C^3*b - 2*A*B^2*c + A^2*B*c + A^2*C*b + 2*
B^2*C*a - 2*A*C^2*c - B^2*C*b + 2*B*C^2*c - 2*A*B*C*a) - 32*B^3*a*b + 32*A
^2*B*a*b - 64*A*C^2*a*b - 64*B*C^2*a*b - 32*A^2*C*a*c + 32*A^2*C*b*c + 32*
B^2*C*a*c - 32*B^2*C*b*c - ((B*c^3 - C*b^3 - A*b^2*(b^2 - a^2 + c^2)^(1/2)
 - B*a^2*c + C*a^2*b - A*c^2*(b^2 - a^2 + c^2)^(1/2) + B*b^2*c - C*b*c^2 +
 B*a*b*(b^2 - a^2 + c^2)^(1/2) + C*a*c*(b^2 - a^2 + c^2)^(1/2))*(64*A^2*b^
2*c - 32*B^2*a^2*c + 32*B^2*b^2*c + 32*C^2*a^2*c + 32*C^2*b^2*c + 32*tan(x
/2)*(a - b)*(A^2*b^2 + 2*B^2*a^2 - A^2*c^2 + B^2*b^2 - 2*C^2*a^2 - 3*B^2*c
^2 + 2*C^2*c^2 + 4*A*B*c^2 - 2*B^2*a*b + 2*C^2*a*b - 2*A*B*a*b + 2*A*C*a*c
 - 4*A*C*b*c - 4*B*C*a*c + 6*B*C*b*c) + 64*A*C*b^3 - 128*B*C*a^3 - 64*B*C*
b^3 + 64*A*B*a^2*c - 128*A*C*a*b^2 + 64*A*C*a^2*b - 64*A*B*b^2*c + 192*B*C
*a^2*b + 64*B*C*a*c^2 - 64*B*C*b*c^2 - 64*A^2*a*b*c - 64*C^2*a*b*c - ((B*c
^3 - C*b^3 - A*b^2*(b^2 - a^2 + c^2)^(1/2) - B*a^2*c + C*a^2*b - A*c^2*(b^
2 - a^2 + c^2)^(1/2) + B*b^2*c - C*b*c^2 + B*a*b*(b^2 - a^2 + c^2)^(1/2) +
 C*a*c*(b^2 - a^2 + c^2)^(1/2))*(32*A*b^4 + 32*B*b^4 - 32*tan(x/2)*(a - b)
*(2*A*c^3 + B*c^3 - 2*C*b^3 + 2*A*b^2*c + 2*C*a*b^2 + 4*B*b^2*c - 2*C*a*c^
2 + C*b*c^2 - 2*A*a*b*c - 4*B*a*b*c) + 32*A*a^2*b^2 - 32*A*a^2*c^2 + 32...
```

3.551 $\int \frac{A+B\cos(x)+C\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx$

3.551.1 Optimal result

Integrand size = 23, antiderivative size = 127

$$\int \frac{A+B\cos(x)+C\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx = \frac{2(aA-bB-cC)\arctan\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{(a^2-b^2-c^2)^{3/2}} + \frac{Bc-bC+(Ac-aC)\cos(x)-(Ab-aB)\sin(x)}{(a^2-b^2-c^2)\,(a+b\cos(x)+c\sin(x))}$$

output
```
2*(A*a-B*b-C*c)*arctan((c+(a-b)*tan(1/2*x))/(a^2-b^2-c^2)^(1/2))/(a^2-b^2-
c^2)^(3/2)+(B*c-b*C+(A*c-C*a)*cos(x)-(A*b-B*a)*sin(x))/(a^2-b^2-c^2)/(a+b*
cos(x)+c*sin(x))
```

3.551.2 Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.08

$$\begin{aligned}&\int \frac{A+B\cos(x)+C\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx\\ &= \frac{2(aA-bB-cC)\text{arctanh}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2+c^2}}\right)}{(-a^2+b^2+c^2)^{3/2}}\\ &\quad+\frac{aAc-a^2C+b(-Bc+bC)+(A(b^2+c^2)-a(bB+cC))\sin(x)}{b\,(-a^2+b^2+c^2)\,(a+b\cos(x)+c\sin(x))}\end{aligned}$$

input
```
Integrate[(A + B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^2,x]
```

output
```
(2*(a*A - b*B - c*C)*ArcTanh[(c + (a - b)*Tan[x/2])/Sqrt[-a^2 + b^2 + c^2]
])/(-a^2 + b^2 + c^2)^(3/2) + (a*A*c - a^2*C + b*(-(B*c) + b*C) + (A*(b^2
+ c^2) - a*(b*B + c*C))*Sin[x])/(b*(-a^2 + b^2 + c^2)*(a + b*Cos[x] + c*Si
n[x]))
```

3.551.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 3632, 3042, 3603, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(x)+C\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\cos(x)+C\sin(x)}{(a+b\cos(x)+c\sin(x))^2}dx$$

$$\Big\downarrow \text{3632}$$

$$\frac{(aA-bB-cC)\int \frac{1}{a+b\cos(x)+c\sin(x)}dx}{a^2-b^2-c^2}+\frac{-\sin(x)(Ab-aB)+\cos(x)(Ac-aC)-bC+Bc}{(a^2-b^2-c^2)\,(a+b\cos(x)+c\sin(x))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(aA-bB-cC)\int \frac{1}{a+b\cos(x)+c\sin(x)}dx}{a^2-b^2-c^2}+\frac{-\sin(x)(Ab-aB)+\cos(x)(Ac-aC)-bC+Bc}{(a^2-b^2-c^2)\,(a+b\cos(x)+c\sin(x))}$$

$$\Big\downarrow \text{3603}$$

$$\frac{2(aA-bB-cC)\int \frac{1}{(a-b)\tan^2\left(\frac{x}{2}\right)+2c\tan\left(\frac{x}{2}\right)+a+b}d\tan\left(\frac{x}{2}\right)}{a^2-b^2-c^2}+\frac{-\sin(x)(Ab-aB)+\cos(x)(Ac-aC)-bC+Bc}{(a^2-b^2-c^2)\,(a+b\cos(x)+c\sin(x))}$$

$$\Big\downarrow \text{1083}$$

$$\frac{-\sin(x)(Ab-aB)+\cos(x)(Ac-aC)-bC+Bc}{(a^2-b^2-c^2)\,(a+b\cos(x)+c\sin(x))}-\frac{4(aA-bB-cC)\int\frac{1}{-(2c+2(a-b)\tan(\frac{x}{2}))^2-4(a^2-b^2-c^2)}d\left(2c+2(a-b)\tan\left(\frac{x}{2}\right)\right)}{a^2-b^2-c^2}$$

$$\Big\downarrow \; 217$$

$$\frac{2(aA-bB-cC)\arctan\left(\frac{2(a-b)\tan(\frac{x}{2})+2c}{2\sqrt{a^2-b^2-c^2}}\right)}{(a^2-b^2-c^2)^{3/2}}+\frac{-\sin(x)(Ab-aB)+\cos(x)(Ac-aC)-bC+Bc}{(a^2-b^2-c^2)\,(a+b\cos(x)+c\sin(x))}$$

input
```
Int[(A + B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^2,x]
```

output
```
(2*(a*A - b*B - c*C)*ArcTan[(2*c + 2*(a - b)*Tan[x/2])/(2*Sqrt[a^2 - b^2 -
 c^2])])/(a^2 - b^2 - c^2)^(3/2) + (B*c - b*C + (A*c - a*C)*Cos[x] - (A*b
- a*B)*Sin[x])/((a^2 - b^2 - c^2)*(a + b*Cos[x] + c*Sin[x]))
```

3.551.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

3.551. $\int \frac{A+B\cos(x)+C\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx$

rule 3632
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Simp[(a*A - b*B - c*C)/(a^2 - b^2 - c^2)   Int[1/(a + b*Cos[d + e*x] + c*S
in[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
 - c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

3.551.4 Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.82

method	result
default	$\frac{-\frac{2\left(Aab-A\,b^{2}-A\,c^{2}-B\,a^{2}+abB+B\,c^{2}+acC-Cbc\right)\tan\left(\frac{x}{2}\right)}{a^{3}-a^{2}b-a\,b^{2}-a\,c^{2}+b^{3}+c^{2}b}+\frac{2\left(Aac-bBc-C\,a^{2}+b^{2}C\right)}{a^{3}-a^{2}b-a\,b^{2}-a\,c^{2}+b^{3}+c^{2}b}}{\tan\left(\frac{x}{2}\right)^{2}a-\tan\left(\frac{x}{2}\right)^{2}b+2c\tan\left(\frac{x}{2}\right)+a+b}+\frac{2(Aa-Bb-Cc)\arctan\left(\frac{2(a-b)\tan\left(\frac{x}{2}\right)+2c}{2\sqrt{a^{2}-b^{2}-c^{2}}}\right)}{\left(a^{2}-b^{2}-c^{2}\right)^{\frac{3}{2}}}$
risch	$\frac{2iA\,b^{2}+2iA\,c^{2}-2iBab-2iacC+2iAab\,e^{ix}-2iB\,e^{ix}a^{2}+2iB\,c^{2}e^{ix}-2iCbc\,e^{ix}+2Aac\,e^{ix}-2Bbc\,e^{ix}-2C\,a^{2}e^{ix}+2C\,b^{2}e^{ix}}{\left(-a^{2}+b^{2}+c^{2}\right)\left(-ic+b\right)\left(-ic\,e^{2ix}+b\,e^{2ix}+ic+2a\,e^{ix}+b\right)}-\ln\left(e^{ix}+\right.$

input
```
int((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x,method=_RETURNVERBOSE)
```

output
```
2*(-(A*a*b-A*b^2-A*c^2-B*a^2+B*a*b+B*c^2+C*a*c-C*b*c)/(a^3-a^2*b-a*b^2-a*c
^2+b^3+b*c^2)*tan(1/2*x)+(A*a*c-B*b*c-C*a^2+C*b^2)/(a^3-a^2*b-a*b^2-a*c^2+
b^3+b*c^2))/(tan(1/2*x)^2*a-tan(1/2*x)^2*b+2*c*tan(1/2*x)+a+b)+2*(A*a-B*b-
C*c)/(a^2-b^2-c^2)^(3/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2-c^2)
^(1/2))
```

3.551.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 696 vs. $2(122) = 244$.

Time = 0.33 (sec) , antiderivative size = 1556, normalized size of antiderivative = 12.25

$$\int \frac{A+B\cos(x)+C\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="fric
as")
```

output
```
[1/2*(2*C*a^4*b - 4*C*a^2*b^3 + 2*C*b^5 + 2*C*b*c^4 - 2*B*c^5 + 4*(B*a^2 -
 B*b^2)*c^3 - 4*(C*a^2*b - C*b^3)*c^2 - (A*a^2*b^2 - B*a*b^3 - C*a*b^2*c -
 C*a*c^3 + (A*a^2 - B*a*b)*c^2 + (A*a*b^3 - B*b^4 - C*b^3*c - C*b*c^3 + (A
*a*b - B*b^2)*c^2)*cos(x) - (C*b^2*c^2 + C*c^4 - (A*a - B*b)*c^3 - (A*a*b^
2 - B*b^3)*c)*sin(x))*sqrt(-a^2 + b^2 + c^2)*log(-(a^2*b^2 - 2*b^4 - c^4 -
 (a^2 + 3*b^2)*c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(x)^2 - 2*(a*b
^3 + a*b*c^2)*cos(x) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*co
s(x))*sin(x) + 2*(2*a*b*c*cos(x)^2 - a*b*c + (b^2*c + c^3)*cos(x) - (b^3 +
 b*c^2 + (a*b^2 - a*c^2)*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2))/(2*a*b*co
s(x) + (b^2 - c^2)*cos(x)^2 + a^2 + c^2 + 2*(b*c*cos(x) + a*c)*sin(x))) -
2*(B*a^4 - 2*B*a^2*b^2 + B*b^4)*c + 2*(C*a*c^4 - A*c^5 + (A*a^2 + B*a*b -
2*A*b^2)*c^3 - (C*a^3 - C*a*b^2)*c^2 - (B*a^3*b - A*a^2*b^2 - B*a*b^3 + A*
b^4)*c)*cos(x) + 2*(B*a^3*b^2 - A*a^2*b^3 - B*a*b^4 + A*b^5 - C*a*b*c^3 +
A*b*c^4 - (A*a^2*b + B*a*b^2 - 2*A*b^3)*c^2 + (C*a^3*b - C*a*b^3)*c)*sin(x
))/(a^5*b^2 - 2*a^3*b^4 + a*b^6 + a*c^6 - (2*a^3 - 3*a*b^2)*c^4 + (a^5 - 4
*a^3*b^2 + 3*a*b^4)*c^2 + (a^4*b^3 - 2*a^2*b^5 + b^7 + b*c^6 - (2*a^2*b -
3*b^3)*c^4 + (a^4*b - 4*a^2*b^3 + 3*b^5)*c^2)*cos(x) + (c^7 - (2*a^2 - 3*b
^2)*c^5 + (a^4 - 4*a^2*b^2 + 3*b^4)*c^3 + (a^4*b^2 - 2*a^2*b^4 + b^6)*c)*s
in(x)), (C*a^4*b - 2*C*a^2*b^3 + C*b^5 + C*b*c^4 - B*c^5 + 2*(B*a^2 - B*b^
2)*c^3 - 2*(C*a^2*b - C*b^3)*c^2 + (A*a^2*b^2 - B*a*b^3 - C*a*b^2*c - C...
```

3.551.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cos(x) + C\sin(x)}{(a + b\cos(x) + c\sin(x))^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))**2,x)
```

output
```
Timed out
```

3.551.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\cos(x)+C\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="maxi
ma")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` f
or more de
```

3.551.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.90

$$\int \frac{A+B\cos(x)+C\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx$$
$$= -\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}x\right)-b\tan\left(\frac{1}{2}x\right)+c}{\sqrt{a^2-b^2-c^2}}\right)\right)(Aa-Bb-Cc)}{(a^2-b^2-c^2)^{\frac{3}{2}}}$$
$$+\frac{2\left(Ba^2\tan\left(\frac{1}{2}x\right)-Aab\tan\left(\frac{1}{2}x\right)-Bab\tan\left(\frac{1}{2}x\right)+Ab^2\tan\left(\frac{1}{2}x\right)-Cac\tan\left(\frac{1}{2}x\right)+Cbc\tan\left(\frac{1}{2}x\right)+A\right.}{(a^3-a^2b-ab^2+b^3-ac^2+bc^2)\left(a\tan\left(\frac{1}{2}x\right)^2-b\tan\left(\frac{1}{2}x\right)^2+2\right.}$$

input
```
integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algorithm="giac
")
```

output
```
-2*(pi*floor(1/2*x/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*t
an(1/2*x) + c)/sqrt(a^2 - b^2 - c^2)))*(A*a - B*b - C*c)/(a^2 - b^2 - c^2)
^(3/2) + 2*(B*a^2*tan(1/2*x) - A*a*b*tan(1/2*x) - B*a*b*tan(1/2*x) + A*b^2
*tan(1/2*x) - C*a*c*tan(1/2*x) + C*b*c*tan(1/2*x) + A*c^2*tan(1/2*x) - B*c
^2*tan(1/2*x) - C*a^2 + C*b^2 + A*a*c - B*b*c)/((a^3 - a^2*b - a*b^2 + b^3
 - a*c^2 + b*c^2)*(a*tan(1/2*x)^2 - b*tan(1/2*x)^2 + 2*c*tan(1/2*x) + a +
b))
```

3.551.9 Mupad [B] (verification not implemented)

Time = 26.59 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.79

$$\int \frac{A+B\cos(x)+C\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx$$
$$=\frac{\frac{2\,(C\,a^2-A\,c\,a-C\,b^2+B\,c\,b)}{(a-b)\,(-a^2+b^2+c^2)}-\frac{2\tan\left(\frac{x}{2}\right)\,(A\,b^2+B\,a^2+A\,c^2-B\,c^2-A\,a\,b-B\,a\,b-C\,a\,c+C\,b\,c)}{(a-b)\,(-a^2+b^2+c^2)}}{(a-b)\,\tan\left(\frac{x}{2}\right)^2+2\,c\tan\left(\frac{x}{2}\right)+a+b}$$
$$-\frac{2\,\mathrm{atanh}\left(\frac{\tan\left(\frac{x}{2}\right)\,(2\,a-2\,b)+\frac{2\,(-a^2\,c+b^2\,c+c^3)}{-a^2+b^2+c^2}}{2\,\sqrt{-a^2+b^2+c^2}}\right)\,(B\,b-A\,a+C\,c)}{(-a^2+b^2+c^2)^{3/2}}$$

input
```
int((A + B*cos(x) + C*sin(x))/(a + b*cos(x) + c*sin(x))^2,x)
```

output
```
((2*(C*a^2 - C*b^2 - A*a*c + B*b*c))/((a - b)*(b^2 - a^2 + c^2)) - (2*tan(
x/2)*(A*b^2 + B*a^2 + A*c^2 - B*c^2 - A*a*b - B*a*b - C*a*c + C*b*c))/((a
- b)*(b^2 - a^2 + c^2)))/(a + b + 2*c*tan(x/2) + tan(x/2)^2*(a - b)) - (2*
atanh((tan(x/2)*(2*a - 2*b) + (2*(b^2*c - a^2*c + c^3))/(b^2 - a^2 + c^2))
/(2*(b^2 - a^2 + c^2)^(1/2)))*(B*b - A*a + C*c))/(b^2 - a^2 + c^2)^(3/2)
```

3.552 $\int \frac{A+B\cos(x)+C\sin(x)}{(a+b\cos(x)+c\sin(x))^3}\,dx$

3.552.1 Optimal result

Integrand size = 23, antiderivative size = 237

$$\int \frac{A+B\cos(x)+C\sin(x)}{(a+b\cos(x)+c\sin(x))^3}\,dx = \frac{(2a^2A+A(b^2+c^2)-3a(bB+cC))\arctan\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2-c^2}}\right)}{(a^2-b^2-c^2)^{5/2}} + \frac{Bc-bC+(Ac-aC)\cos(x)-(Ab-aB)\sin(x)}{2(a^2-b^2-c^2)(a+b\cos(x)+c\sin(x))^2} + \frac{a(Bc-bC)+(3aAc-a^2C-2c(bB+cC))\cos(x)-(3aAb-a^2B-2b(bB+cC))\sin(x)}{2(a^2-b^2-c^2)^2(a+b\cos(x)+c\sin(x))}$$

output
```
(2*a^2*A+A*(b^2+c^2)-3*a*(B*b+C*c))*arctan((c+(a-b)*tan(1/2*x))/(a^2-b^2-c
^2)^(1/2))/(a^2-b^2-c^2)^(5/2)+1/2*(B*c-b*C+(A*c-C*a)*cos(x)-(A*b-B*a)*sin
(x))/(a^2-b^2-c^2)/(a+b*cos(x)+c*sin(x))^2+1/2*(a*(B*c-C*b)+(3*A*a*c-C*a^2
-2*c*(B*b+C*c))*cos(x)-(3*A*a*b-B*a^2-2*b*(B*b+C*c))*sin(x))/(a^2-b^2-c^2)
^2/(a+b*cos(x)+c*sin(x))
```

3.552.2 Mathematica [A] (verified)

Time = 1.26 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.91

$$\int \frac{A+B\cos(x)+C\sin(x)}{(a+b\cos(x)+c\sin(x))^3}\,dx$$
$$= -\frac{(2a^2A+A(b^2+c^2)-3a(bB+cC))\operatorname{arctanh}\left(\frac{c+(a-b)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2+c^2}}\right)}{(-a^2+b^2+c^2)^{5/2}}$$
$$+\frac{-6a^3Ac-3aAb^2c+9a^2bBc-3aAc^3+2a^4C-4a^2b^2C+2b^4C+5a^2c^2C+4b^2c^2C+2c^4C-2bc(2a^2A}{}$$

input
```
Integrate[(A + B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^3,x]
```

output
```
-(((2*a^2*A + A*(b^2 + c^2) - 3*a*(b*B + c*C))*ArcTanh[(c + (a - b)*Tan[x/
2])/Sqrt[-a^2 + b^2 + c^2]])/(-a^2 + b^2 + c^2)^(5/2)) + (-6*a^3*A*c - 3*a
*A*b^2*c + 9*a^2*b*B*c - 3*a*A*c^3 + 2*a^4*C - 4*a^2*b^2*C + 2*b^4*C + 5*a
^2*c^2*C + 4*b^2*c^2*C + 2*c^4*C - 2*b*c*(2*a^2*A + A*(b^2 + c^2) - 3*a*(b
*B + c*C))*Cos[x] - c*(-3*a*A*(b^2 + c^2) + a^2*(b*B + c*C) + 2*(b^2 + c^2
)*(b*B + c*C))*Cos[2*x] - 8*a^2*A*b^2*Sin[x] + 2*A*b^4*Sin[x] + 4*a^3*b*B*
Sin[x] + 2*a*b^3*B*Sin[x] - 12*a^2*A*c^2*Sin[x] + 2*A*b^2*c^2*Sin[x] + 8*a
*b*B*c^2*Sin[x] + 4*a^3*c*C*Sin[x] + 2*a*b^2*c*C*Sin[x] + 8*a*c^3*C*Sin[x]
 - 3*a*A*b^3*Sin[2*x] + a^2*b^2*B*Sin[2*x] + 2*b^4*B*Sin[2*x] - 3*a*A*b*c^
2*Sin[2*x] + 2*b^2*B*c^2*Sin[2*x] + a^2*b*c*C*Sin[2*x] + 2*b^3*c*C*Sin[2*x
] + 2*b*c^3*C*Sin[2*x])/(4*b*(-a^2 + b^2 + c^2)^2*(a + b*Cos[x] + c*Sin[x]
)^2)
```

3.552.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 3635, 25, 3042, 3632, 3042, 3603, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(x)+C\sin(x)}{(a+b\cos(x)+c\sin(x))^3}\,dx$$
$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\cos(x)+C\sin(x)}{(a+b\cos(x)+c\sin(x))^3}dx$$

$$\Big\downarrow \text{3635}$$

$$\frac{-\sin(x)(Ab-aB)+\cos(x)(Ac-aC)-bC+Bc}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2}-\frac{\int -\frac{2(aA-bB-cC)-(Ab-aB)\cos(x)-(Ac-aC)\sin(x)}{(a+b\cos(x)+c\sin(x))^2}dx}{2\left(a^2-b^2-c^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{2(aA-bB-cC)-(Ab-aB)\cos(x)-(Ac-aC)\sin(x)}{(a+b\cos(x)+c\sin(x))^2}dx}{2\left(a^2-b^2-c^2\right)}+\frac{-\sin(x)(Ab-aB)+\cos(x)(Ac-aC)-bC+Bc}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{2(aA-bB-cC)-(Ab-aB)\cos(x)-(Ac-aC)\sin(x)}{(a+b\cos(x)+c\sin(x))^2}dx}{2\left(a^2-b^2-c^2\right)}+\frac{-\sin(x)(Ab-aB)+\cos(x)(Ac-aC)-bC+Bc}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2}$$

$$\Big\downarrow \text{3632}$$

$$\frac{\frac{\left(2a^2A-3a(bB+cC)+A\left(b^2+c^2\right)\right)\int \frac{1}{a+b\cos(x)+c\sin(x)}dx}{a^2-b^2-c^2}+\frac{-\sin(x)\left(a^2(-B)+3aAb-2b(bB+cC)\right)+\cos(x)\left(a^2(-C)+3aAc-2c(bB+cC)\right)+a(Bc-}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}}{2\left(a^2-b^2-c^2\right)}$$

$$\frac{-\sin(x)(Ab-aB)+\cos(x)(Ac-aC)-bC+Bc}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\left(2a^2A-3a(bB+cC)+A\left(b^2+c^2\right)\right)\int \frac{1}{a+b\cos(x)+c\sin(x)}dx}{a^2-b^2-c^2}+\frac{-\sin(x)\left(a^2(-B)+3aAb-2b(bB+cC)\right)+\cos(x)\left(a^2(-C)+3aAc-2c(bB+cC)\right)+a(Bc-}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}}{2\left(a^2-b^2-c^2\right)}$$

$$\frac{-\sin(x)(Ab-aB)+\cos(x)(Ac-aC)-bC+Bc}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2}$$

$$\Big\downarrow \text{3603}$$

$$\frac{\frac{2\left(2a^2A-3a(bB+cC)+A\left(b^2+c^2\right)\right)\int \frac{1}{(a-b)\tan^2\left(\frac{x}{2}\right)+2c\tan\left(\frac{x}{2}\right)+a+b}d\tan\left(\frac{x}{2}\right)}{a^2-b^2-c^2}+\frac{-\sin(x)\left(a^2(-B)+3aAb-2b(bB+cC)\right)+\cos(x)\left(a^2(-C)+3aAc-2\right.}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}}{2\left(a^2-b^2-c^2\right)}$$

$$\frac{-\sin(x)(Ab-aB)+\cos(x)(Ac-aC)-bC+Bc}{2\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^2}$$

$$\Big\downarrow \text{1083}$$

3.552. $\int \frac{A+B\cos(x)+C\sin(x)}{(a+b\cos(x)+c\sin(x))^3}\,dx$

$$\frac{\frac{-\sin(x)\left(a^2(-B)+3aAb-2b(bB+cC)\right)+\cos(x)\left(a^2(-C)+3aAc-2c(bB+cC)\right)+a(Bc-bC)}{(a^2-b^2-c^2)(a+b\cos(x)+c\sin(x))}-\frac{4\left(2a^2A-3a(bB+cC)+A(b^2+c^2)\right)\int\frac{}{-\left(2c+2(a-b)\tan\ldots\right.}}{a^2-b^2-\ldots}}{2\left(a^2-b^2-c^2\right)}$$
$$\frac{-\sin(x)(Ab-aB)+\cos(x)(Ac-aC)-bC+Bc}{2\left(a^2-b^2-c^2\right)\left(a+b\cos(x)+c\sin(x)\right)^2}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{2\left(2a^2A-3a(bB+cC)+A(b^2+c^2)\right)\arctan\left(\frac{2(a-b)\tan\left(\frac{x}{2}\right)+2c}{2\sqrt{a^2-b^2-c^2}}\right)}{(a^2-b^2-c^2)^{3/2}}+\frac{-\sin(x)\left(a^2(-B)+3aAb-2b(bB+cC)\right)+\cos(x)\left(a^2(-C)+3aAc-2c(bB+cC)\right)+\ldots}{(a^2-b^2-c^2)(a+b\cos(x)+c\sin(x))}}{2\left(a^2-b^2-c^2\right)}$$
$$\frac{-\sin(x)(Ab-aB)+\cos(x)(Ac-aC)-bC+Bc}{2\left(a^2-b^2-c^2\right)\left(a+b\cos(x)+c\sin(x)\right)^2}$$

input
```
Int[(A + B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + c*Sin[x])^3,x]
```

output
```
(B*c - b*C + (A*c - a*C)*Cos[x] - (A*b - a*B)*Sin[x])/(2*(a^2 - b^2 - c^2)
*(a + b*Cos[x] + c*Sin[x])^2) + ((2*(2*a^2*A + A*(b^2 + c^2) - 3*a*(b*B +
c*C))*ArcTan[(2*c + 2*(a - b)*Tan[x/2])/(2*Sqrt[a^2 - b^2 - c^2])])/(a^2 -
 b^2 - c^2)^(3/2) + (a*(B*c - b*C) + (3*a*A*c - a^2*C - 2*c*(b*B + c*C))*C
os[x] - (3*a*A*b - a^2*B - 2*b*(b*B + c*C))*Sin[x])/((a^2 - b^2 - c^2)*(a
+ b*Cos[x] + c*Sin[x])))/(2*(a^2 - b^2 - c^2))
```

3.552.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3603
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Simp[2*(f
/e)   Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)
/2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

rule 3632
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] +
 Simp[(a*A - b*B - c*C)/(a^2 - b^2 - c^2)   Int[1/(a + b*Cos[d + e*x] + c*S
in[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2
 - c^2, 0] && NeQ[a*A - b*B - c*C, 0]
```

rule 3635
```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(-(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(
a^2 - b^2 - c^2))), x] + Simp[1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a + b*Co
s[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n +
2)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x]
/; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[n, -2]
```

3.552.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1079 vs. $2(227) = 454$.

Time = 1.87 (sec) , antiderivative size = 1080, normalized size of antiderivative = 4.56

method	result	size
default	Expression too large to display	1080
risch	Expression too large to display	2157

input
```
int((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x,method=_RETURNVERBOSE)
```

output
```
2*(-1/2*(4*A*a^3*b-7*A*a^2*b^2-5*A*a^2*c^2+2*A*a*b^3+2*A*a*b*c^2+A*b^4+3*A
*b^2*c^2+2*A*c^4-2*B*a^4+3*B*a^3*b-2*B*a^2*b^2+4*B*a^2*c^2+3*B*a*b^3-2*B*b
^4-4*B*b^2*c^2-2*B*c^4+3*C*a^3*c-6*C*a^2*b*c+3*C*a*b^2*c)/(a^4-2*a^2*b^2-2
*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a-b)*tan(1/2*x)^3+1/2*(4*A*a^4*c-12*A*a^3*b*c
+13*A*a^2*b^2*c+7*A*a^2*c^3-6*A*a*b^3*c-6*A*a*b*c^3+A*b^4*c-A*b^2*c^3-2*A*
c^5+2*B*a^4*c-9*B*a^3*b*c+14*B*a^2*b^2*c-4*B*a^2*c^3-9*B*a*b^3*c+2*B*b^4*c
+4*B*b^2*c^3+2*B*c^5-2*C*a^5+2*C*a^4*b+4*C*a^3*b^2-5*C*a^3*c^2-4*C*a^2*b^3
+14*C*a^2*b*c^2-2*C*a*b^4-13*C*a*b^2*c^2-2*C*a*c^4+2*C*b^5+4*C*b^3*c^2+2*C
*b*c^4)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^4)/(a^2-2*a*b+b^2)*tan(1/
2*x)^2-1/2*(4*A*a^4*b-5*A*a^3*b^2-11*A*a^3*c^2-3*A*a^2*b^3+3*A*a^2*b*c^2+5
*A*a*b^4+7*A*a*b^2*c^2+2*A*a*c^4-A*b^5+A*b^3*c^2+2*A*b*c^4-2*B*a^5+3*B*a^4
*b-B*a^3*b^2+4*B*a^3*c^2-B*a^2*b^3+8*B*a^2*b*c^2+3*B*a*b^4-8*B*a*b^2*c^2-2
*B*a*c^4-2*B*b^5-4*B*b^3*c^2-2*B*b*c^4+5*C*a^4*c-5*C*a^3*b*c-5*C*a^2*b^2*c
+4*C*a^2*c^3+5*C*a*b^3*c-4*C*a*b*c^3)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c
^2+c^4)/(a^2-2*a*b+b^2)*tan(1/2*x)+1/2*(4*A*a^4*c-3*A*a^2*b^2*c-A*a^2*c^3-
A*b^4*c-A*b^2*c^3-5*B*a^3*b*c+5*B*a*b^3*c+2*B*a*b*c^3-2*C*a^5+4*C*a^3*b^2-
C*a^3*c^2-2*C*a*b^4+C*a*b^2*c^2)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^2+c^
4)/(a^2-2*a*b+b^2))/(tan(1/2*x)^2*a-tan(1/2*x)^2*b+2*c*tan(1/2*x)+a+b)^2+(
2*A*a^2+A*b^2+A*c^2-3*B*a*b-3*C*a*c)/(a^4-2*a^2*b^2-2*a^2*c^2+b^4+2*b^2*c^
2+c^4)/(a^2-b^2-c^2)^(1/2)*arctan(1/2*(2*(a-b)*tan(1/2*x)+2*c)/(a^2-b^2...
```

3.552.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2038 vs. $2(225) = 450$.

Time = 0.51 (sec) , antiderivative size = 4240, normalized size of antiderivative = 17.89

$$\int \frac{A + B\cos(x) + C\sin(x)}{(a + b\cos(x) + c\sin(x))^3} \, dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="fric
as")
```

output
```
[1/4*(2*C*a^6*b - 6*C*a^4*b^3 + 6*C*a^2*b^5 - 2*C*b^7 - 6*C*b*c^6 + 2*B*c^
7 - 2*(3*B*a^2 - 3*A*a*b - B*b^2)*c^5 + 2*(4*C*a^2*b - 7*C*b^3)*c^4 + 2*(3
*B*a^4 - 3*A*a^3*b - 5*B*a^2*b^2 + 6*A*a*b^3 - B*b^4)*c^3 - 2*(2*C*a^4*b -
 7*C*a^2*b^3 + 5*C*b^5)*c^2 + 4*(2*C*b*c^6 - (3*A*a*b - 2*B*b^2)*c^5 - (C*
a^2*b - 4*C*b^3)*c^4 + (3*A*a^3*b - B*a^2*b^2 - 6*A*a*b^3 + 4*B*b^4)*c^3 -
 (C*a^4*b + C*a^2*b^3 - 2*C*b^5)*c^2 - (B*a^4*b^2 - 3*A*a^3*b^3 + B*a^2*b^
4 + 3*A*a*b^5 - 2*B*b^6)*c)*cos(x)^2 - (2*A*a^4*b^2 - 3*B*a^3*b^3 + A*a^2*
b^4 - 3*C*a^3*b^2*c - 3*C*a*c^5 + A*c^6 + (3*A*a^2 - 3*B*a*b + 2*A*b^2)*c^
4 - 3*(C*a^3 + C*a*b^2)*c^3 + (2*A*a^4 - 3*B*a^3*b + 4*A*a^2*b^2 - 3*B*a*b
^3 + A*b^4)*c^2 + (2*A*a^2*b^4 - 3*B*a*b^5 + A*b^6 - 3*C*a*b^4*c + A*b^4*c
^2 + 3*C*a*c^5 - A*c^6 - (2*A*a^2 - 3*B*a*b + A*b^2)*c^4)*cos(x)^2 + 2*(2*
A*a^3*b^3 - 3*B*a^2*b^4 + A*a*b^5 - 3*C*a^2*b^3*c - 3*C*a^2*b*c^3 + A*a*b*
c^4 + (2*A*a^3*b - 3*B*a^2*b^2 + 2*A*a*b^3)*c^2)*cos(x) - 2*(3*C*a^2*b^2*c
^2 + 3*C*a^2*c^4 - A*a*c^5 - (2*A*a^3 - 3*B*a^2*b + 2*A*a*b^2)*c^3 - (2*A*
a^3*b^2 - 3*B*a^2*b^3 + A*a*b^4)*c + (3*C*a*b^3*c^2 + 3*C*a*b*c^4 - A*b*c^
5 - (2*A*a^2*b - 3*B*a*b^2 + 2*A*b^3)*c^3 - (2*A*a^2*b^3 - 3*B*a*b^4 + A*b
^5)*c)*cos(x))*sin(x))*sqrt(-a^2 + b^2 + c^2)*log(-(a^2*b^2 - 2*b^4 - c^4
- (a^2 + 3*b^2)*c^2 - (2*a^2*b^2 - b^4 - 2*a^2*c^2 + c^4)*cos(x)^2 - 2*(a*
b^3 + a*b*c^2)*cos(x) - 2*(a*b^2*c + a*c^3 - (b*c^3 - (2*a^2*b - b^3)*c)*c
os(x))*sin(x) + 2*(2*a*b*c*cos(x)^2 - a*b*c + (b^2*c + c^3)*cos(x) - (b...
```

3.552.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cos(x) + C\sin(x)}{(a + b\cos(x) + c\sin(x))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))**3,x)
```

output
```
Timed out
```

3.552.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\cos(x) + C\sin(x)}{(a + b\cos(x) + c\sin(x))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="maxi
ma")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` f
or more de
```

3.552.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1506 vs. 2(225) = 450.

Time = 0.37 (sec) , antiderivative size = 1506, normalized size of antiderivative = 6.35

$$\int \frac{A + B\cos(x) + C\sin(x)}{(a + b\cos(x) + c\sin(x))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+c*sin(x))^3,x, algorithm="giac
")
```

output
```
-(2*A*a^2 - 3*B*a*b + A*b^2 - 3*C*a*c + A*c^2)*(pi*floor(1/2*x/pi + 1/2)*s
gn(-2*a + 2*b) + arctan(-(a*tan(1/2*x) - b*tan(1/2*x) + c)/sqrt(a^2 - b^2
- c^2)))/((a^4 - 2*a^2*b^2 + b^4 - 2*a^2*c^2 + 2*b^2*c^2 + c^4)*sqrt(a^2 -
 b^2 - c^2)) + (2*B*a^5*tan(1/2*x)^3 - 4*A*a^4*b*tan(1/2*x)^3 - 5*B*a^4*b*
tan(1/2*x)^3 + 11*A*a^3*b^2*tan(1/2*x)^3 + 5*B*a^3*b^2*tan(1/2*x)^3 - 9*A*
a^2*b^3*tan(1/2*x)^3 - 5*B*a^2*b^3*tan(1/2*x)^3 + A*a*b^4*tan(1/2*x)^3 + 5
*B*a*b^4*tan(1/2*x)^3 + A*b^5*tan(1/2*x)^3 - 2*B*b^5*tan(1/2*x)^3 - 3*C*a^
4*c*tan(1/2*x)^3 + 9*C*a^3*b*c*tan(1/2*x)^3 - 9*C*a^2*b^2*c*tan(1/2*x)^3 +
 3*C*a*b^3*c*tan(1/2*x)^3 + 5*A*a^3*c^2*tan(1/2*x)^3 - 4*B*a^3*c^2*tan(1/2
*x)^3 - 7*A*a^2*b*c^2*tan(1/2*x)^3 + 4*B*a^2*b*c^2*tan(1/2*x)^3 - A*a*b^2*
c^2*tan(1/2*x)^3 + 4*B*a*b^2*c^2*tan(1/2*x)^3 + 3*A*b^3*c^2*tan(1/2*x)^3 -
 4*B*b^3*c^2*tan(1/2*x)^3 - 2*A*a*c^4*tan(1/2*x)^3 + 2*B*a*c^4*tan(1/2*x)^
3 + 2*A*b*c^4*tan(1/2*x)^3 - 2*B*b*c^4*tan(1/2*x)^3 - 2*C*a^5*tan(1/2*x)^2
 + 2*C*a^4*b*tan(1/2*x)^2 + 4*C*a^3*b^2*tan(1/2*x)^2 - 4*C*a^2*b^3*tan(1/2
*x)^2 - 2*C*a*b^4*tan(1/2*x)^2 + 2*C*b^5*tan(1/2*x)^2 + 4*A*a^4*c*tan(1/2*
x)^2 + 2*B*a^4*c*tan(1/2*x)^2 - 12*A*a^3*b*c*tan(1/2*x)^2 - 9*B*a^3*b*c*ta
n(1/2*x)^2 + 13*A*a^2*b^2*c*tan(1/2*x)^2 + 14*B*a^2*b^2*c*tan(1/2*x)^2 - 6
*A*a*b^3*c*tan(1/2*x)^2 - 9*B*a*b^3*c*tan(1/2*x)^2 + A*b^4*c*tan(1/2*x)^2
+ 2*B*b^4*c*tan(1/2*x)^2 - 5*C*a^3*c^2*tan(1/2*x)^2 + 14*C*a^2*b*c^2*tan(1
/2*x)^2 - 13*C*a*b^2*c^2*tan(1/2*x)^2 + 4*C*b^3*c^2*tan(1/2*x)^2 + 7*A*...
```

3.552.9 Mupad [B] (verification not implemented)

Time = 33.52 (sec) , antiderivative size = 1160, normalized size of antiderivative = 4.89

$$\int \frac{A+B\cos(x)+C\sin(x)}{(a+b\cos(x)+c\sin(x))^3}\,dx =$$

$$-\frac{\frac{2\,C\,a^5-4\,A\,a^4\,c-4\,C\,a^3\,b^2+5\,B\,a^3\,b\,c+C\,a^3\,c^2+3\,A\,a^2\,b^2\,c+A\,a^2\,c^3+2\,C\,a\,b^4-5\,B\,a\,b^3\,c-C\,a\,b^2\,c^2-2\,B\,a\,b\,c^3+A\,b^4\,c+A\,b^2\,c^3}{(a-b)^2\,(a^4-2\,a^2\,b^2-2\,a^2\,c^2+b^4+2\,b^2\,c^2+c^4)}+\frac{\tan\left(\frac{x}{2}\right)^3\,(A}{}}{}$$

$$-\frac{\mathrm{atanh}\left(\frac{2\,a^4\,c-4\,a^2\,b^2\,c-4\,a^2\,c^3+2\,b^4\,c+4\,b^2\,c^3+2\,c^5}{2\,(-a^2+b^2+c^2)^{5/2}}+\frac{\tan\left(\frac{x}{2}\right)\,(2\,a-2\,b)\,(a^4-2\,a^2\,b^2-2\,a^2\,c^2+b^4+2\,b^2\,c^2+c^4)}{2\,(-a^2+b^2+c^2)^{5/2}}\right)\,(2\,A\,a^2-3\,B\,a\,b}{(-a^2+b^2+c^2)^{5/2}}$$

input
```
int((A + B*cos(x) + C*sin(x))/(a + b*cos(x) + c*sin(x))^3,x)
```

output
```
- ((2*C*a^5 + A*a^2*c^3 + A*b^2*c^3 - 4*C*a^3*b^2 + C*a^3*c^2 - 4*A*a^4*c
+ A*b^4*c + 2*C*a*b^4 - 2*B*a*b*c^3 - 5*B*a*b^3*c + 5*B*a^3*b*c + 3*A*a^2*
b^2*c - C*a*b^2*c^2)/((a - b)^2*(a^4 + b^4 + c^4 - 2*a^2*b^2 - 2*a^2*c^2 +
 2*b^2*c^2)) + (tan(x/2)^3*(A*b^4 - 2*B*a^4 + 2*A*c^4 - 2*B*b^4 - 2*B*c^4
- 7*A*a^2*b^2 - 5*A*a^2*c^2 - 2*B*a^2*b^2 + 3*A*b^2*c^2 + 4*B*a^2*c^2 - 4*
B*b^2*c^2 + 2*A*a*b^3 + 4*A*a^3*b + 3*B*a*b^3 + 3*B*a^3*b + 3*C*a^3*c + 2*
A*a*b*c^2 + 3*C*a*b^2*c - 6*C*a^2*b*c))/((a - b)*(a^4 + b^4 + c^4 - 2*a^2*
b^2 - 2*a^2*c^2 + 2*b^2*c^2)) - (tan(x/2)^2*(2*B*c^5 - 2*C*a^5 - 2*A*c^5 +
 2*C*b^5 + 7*A*a^2*c^3 - A*b^2*c^3 - 4*B*a^2*c^3 - 4*C*a^2*b^3 + 4*C*a^3*b
^2 + 4*B*b^2*c^3 - 5*C*a^3*c^2 + 4*C*b^3*c^2 + 4*A*a^4*c + A*b^4*c + 2*B*a
^4*c - 2*C*a*b^4 + 2*C*a^4*b + 2*B*b^4*c - 2*C*a*c^4 + 2*C*b*c^4 - 6*A*a*b
*c^3 - 6*A*a*b^3*c - 12*A*a^3*b*c - 9*B*a*b^3*c - 9*B*a^3*b*c + 13*A*a^2*b
^2*c + 14*B*a^2*b^2*c - 13*C*a*b^2*c^2 + 14*C*a^2*b*c^2))/((a - b)^2*(a^4
+ b^4 + c^4 - 2*a^2*b^2 - 2*a^2*c^2 + 2*b^2*c^2)) - (tan(x/2)*(A*b^5 + 2*B
*a^5 + 2*B*b^5 + 3*A*a^2*b^3 + 5*A*a^3*b^2 + 11*A*a^3*c^2 + B*a^2*b^3 + B*
a^3*b^2 - A*b^3*c^2 - 4*B*a^3*c^2 + 4*B*b^3*c^2 - 4*C*a^2*c^3 - 5*A*a*b^4
- 4*A*a^4*b - 2*A*a*c^4 - 3*B*a*b^4 - 3*B*a^4*b - 2*A*b*c^4 + 2*B*a*c^4 +
2*B*b*c^4 - 5*C*a^4*c + 4*C*a*b*c^3 - 5*C*a*b^3*c + 5*C*a^3*b*c - 7*A*a*b^
2*c^2 - 3*A*a^2*b*c^2 + 8*B*a*b^2*c^2 - 8*B*a^2*b*c^2 + 5*C*a^2*b^2*c))/((
a - b)^2*(a^4 + b^4 + c^4 - 2*a^2*b^2 - 2*a^2*c^2 + 2*b^2*c^2)))/(tan(x...
```

3.552. $\int \frac{A+B\cos(x)+C\sin(x)}{(a+b\cos(x)+c\sin(x))^3}\,dx$

3.553 $\int \frac{A+B\cos(x)+C\sin(x)}{a+b\cos(x)+ib\sin(x)}\,dx$

3.553.1 Optimal result

Integrand size = 26, antiderivative size = 105

$$\int \frac{A+B\cos(x)+C\sin(x)}{a+b\cos(x)+ib\sin(x)}\,dx = \frac{(2aA-b(B+iC))x}{2a^2}+\frac{i(2aAb-a^2(B-iC)-b^2(B+iC))\log(a+b\cos(x)+ib\sin(x))}{2a^2b} + \frac{(iB-C)(\cos(x)-i\sin(x))}{2a}$$

output
```
1/2*(2*A*a-b*(B+I*C))*x/a^2+1/2*I*(2*A*a*b-a^2*(B-I*C)-b^2*(B+I*C))*ln(a+b
*cos(x)+I*b*sin(x))/a^2/b+1/2*(I*B-C)*(cos(x)-I*sin(x))/a
```

3.553.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.57

$$\int \frac{A+B\cos(x)+C\sin(x)}{a+b\cos(x)+ib\sin(x)}\,dx = \frac{(2aAb+a^2(B-iC)-b^2(B+iC))\,x+2(-2aAb+a^2(B-iC)+b^2(B+iC))\arctan\left(\frac{(a+b)\cot\left(\frac{x}{2}\right)}{a-b}\right)+2ia}{}$$

input
```
Integrate[(A + B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + I*b*Sin[x]),x]
```

output
```
((2*a*A*b + a^2*(B - I*C) - b^2*(B + I*C))*x + 2*(-2*a*A*b + a^2*(B - I*C)
 + b^2*(B + I*C))*ArcTan[((a + b)*Cot[x/2])/(a - b)] + (2*I)*a*b*(B + I*C)
*Cos[x] + ((2*I)*a*A*b + a^2*((-I)*B - C) + b^2*((-I)*B + C))*Log[a^2 + b^
2 + 2*a*b*Cos[x]] + 2*a*b*(B + I*C)*Sin[x])/(4*a^2*b)
```

3.553.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3042, 3609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(x)+C\sin(x)}{a+ib\sin(x)+b\cos(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\cos(x)+C\sin(x)}{a+ib\sin(x)+b\cos(x)}dx$$

$$\Big\downarrow \text{3609}$$

$$\frac{i\left(-\left(a^2(B-iC)\right)+2aAb-b^2(B+iC)\right)\log(a+ib\sin(x)+b\cos(x))}{2a^2b}+\frac{x(2aA-b(B+iC))}{2a^2}+\frac{(-C+iB)(\cos(x)-i\sin(x))}{2a}$$

input
```
Int[(A + B*Cos[x] + C*Sin[x])/(a + b*Cos[x] + I*b*Sin[x]),x]
```

output
```
((2*a*A - b*(B + I*C))*x)/(2*a^2) + ((I/2)*(2*a*A*b - a^2*(B - I*C) - b^2*
(B + I*C))*Log[a + b*Cos[x] + I*b*Sin[x]])/(a^2*b) + ((I*B - C)*(Cos[x] -
I*Sin[x]))/(2*a)
```

3.553.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3609
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_
Symbol] :> Simp[(2*a*A - b*B - c*C)*(x/(2*a^2)), x] + (-Simp[(b*B + c*C)*((
b*Cos[d + e*x] - c*Sin[d + e*x])/(2*a*b*c*e)), x] + Simp[(a^2*(b*B - c*C) -
 2*a*A*b^2 + b^2*(b*B + c*C))*(Log[RemoveContent[a + b*Cos[d + e*x] + c*Sin
[d + e*x], x]]/(2*a^2*b*c*e)), x]) /; FreeQ[{a, b, c, d, e, A, B, C}, x] &&
 EqQ[b^2 + c^2, 0]
```

3.553.4 Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.30

method	result
default	$\frac{(-2iAa+iBb-bC)\ln(-i+\tan(\frac{x}{2}))}{2a^2}-\frac{-iC-B}{a(-i+\tan(\frac{x}{2}))}+\frac{i(-iC+B)\ln(\tan(\frac{x}{2})+i)}{2b}+\frac{i(iCa^2-iCb^2+2Aab-Ba^2-Bb^2)\ln(ia+$
risch	$-\frac{C\,\mathrm{e}^{-ix}}{2a}+\frac{iB\,\mathrm{e}^{-ix}}{2a}-\frac{ibxC}{2a^2}+\frac{xA}{a}-\frac{bxB}{2a^2}-\frac{\ln(\mathrm{e}^{ix}+\frac{a}{b})C}{2b}+\frac{b\ln(\mathrm{e}^{ix}+\frac{a}{b})C}{2a^2}+\frac{i\ln(\mathrm{e}^{ix}+\frac{a}{b})A}{a}-\frac{i\ln(\mathrm{e}^{ix}+\frac{a}{b})B}{2b}-\frac{ib\ln(\mathrm{e}$

input
```
int((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
1/2/a^2*(-2*I*A*a+I*B*b-b*C)*ln(-I+tan(1/2*x))-(-I*C-B)/a/(-I+tan(1/2*x))+
1/2*I*(B-I*C)/b*ln(tan(1/2*x)+I)+1/2*I*(I*C*a^2-I*C*b^2+2*A*a*b-B*a^2-B*b^
2)/a^2/b*ln(I*a+I*b+a*tan(1/2*x)-b*tan(1/2*x))
```

3.553.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.85

$$\int \frac{A+B\cos(x)+C\sin(x)}{a+b\cos(x)+ib\sin(x)}\,dx$$
$$=\frac{\left((iB-C)ab+(2Aab-(B+iC)b^2)xe^{(ix)}+((-iB-C)a^2+2iAab+(-iB+C)b^2)e^{(ix)}\log\left(\frac{be^{(ix)}+a}{b}\right.\right.}{2a^2b}$$

input
```
integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="fric
as")
```

output
```
1/2*((I*B - C)*a*b + (2*A*a*b - (B + I*C)*b^2)*x*e^(I*x) + ((-I*B - C)*a^2
 + 2*I*A*a*b + (-I*B + C)*b^2)*e^(I*x)*log((b*e^(I*x) + a)/b))*e^(-I*x)/(a
^2*b)
```

3.553.6 Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.23

$$\int \frac{A+B\cos(x)+C\sin(x)}{a+b\cos(x)+ib\sin(x)}\,dx = \begin{cases} \frac{(iB-C)e^{-ix}}{2a} & \text{for } a\neq 0 \\ x\left(-\frac{2Aa-Bb-iCb}{2a^2}+\frac{2Aa+Ba-Bb+iCa-iCb}{2a^2}\right) & \text{otherwise} \end{cases} + \frac{x(2Aa-Bb-iCb)}{2a^2} - \frac{i(-2Aab+Ba^2+Bb^2-iCa^2+iCb^2)\log\left(\frac{a}{b}+e^{ix}\right)}{2a^2b}$$

input
```
integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x)
```

output
```
Piecewise(((I*B - C)*exp(-I*x)/(2*a), Ne(a, 0)), (x*(-(2*A*a - B*b - I*C*b
)/(2*a**2) + (2*A*a + B*a - B*b + I*C*a - I*C*b)/(2*a**2)), True)) + x*(2*
A*a - B*b - I*C*b)/(2*a**2) - I*(-2*A*a*b + B*a**2 + B*b**2 - I*C*a**2 + I
*C*b**2)*log(a/b + exp(I*x))/(2*a**2*b)
```

3.553.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\cos(x)+C\sin(x)}{a+b\cos(x)+ib\sin(x)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="maxi
ma")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.553.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 203 vs. 2(83) = 166.

Time = 0.27 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.93

$$\int \frac{A+B\cos(x)+C\sin(x)}{a+b\cos(x)+ib\sin(x)}\,dx$$
$$= -\frac{(-2iAa+iBb-Cb)\log\left(-a\tan\left(\frac{1}{2}x\right)^2+b\tan\left(\frac{1}{2}x\right)^2-2ia\tan\left(\frac{1}{2}x\right)+a+b\right)}{4a^2}$$
$$-\frac{(2iAa-iBb+Cb)\log\left(\tan\left(\frac{1}{2}x\right)-i\right)}{2a^2}$$
$$+\frac{(2Ba^2-2iCa^2-2Aab+Bb^2+iCb^2)\left(x+2\arctan\left(\frac{-ia\cos(x)-a\sin(x)-ia}{a\cos(x)-ia\sin(x)-a+2b}\right)\right)}{4a^2b}$$
$$-\frac{-2iAa\tan\left(\frac{1}{2}x\right)+iBb\tan\left(\frac{1}{2}x\right)-Cb\tan\left(\frac{1}{2}x\right)-2Aa-2Ba-2iCa+Bb+iCb}{2a^2\left(\tan\left(\frac{1}{2}x\right)-i\right)}$$

input
```
integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)+I*b*sin(x)),x, algorithm="giac
")
```

output
```
-1/4*(-2*I*A*a + I*B*b - C*b)*log(-a*tan(1/2*x)^2 + b*tan(1/2*x)^2 - 2*I*a
*tan(1/2*x) + a + b)/a^2 - 1/2*(2*I*A*a - I*B*b + C*b)*log(tan(1/2*x) - I)
/a^2 + 1/4*(2*B*a^2 - 2*I*C*a^2 - 2*A*a*b + B*b^2 + I*C*b^2)*(x + 2*arctan
((-I*a*cos(x) - a*sin(x) - I*a)/(a*cos(x) - I*a*sin(x) - a + 2*b)))/(a^2*b
) - 1/2*(-2*I*A*a*tan(1/2*x) + I*B*b*tan(1/2*x) - C*b*tan(1/2*x) - 2*A*a -
 2*B*a - 2*I*C*a + B*b + I*C*b)/(a^2*(tan(1/2*x) - I))
```

3.553.9 Mupad [B] (verification not implemented)

Time = 31.68 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.26

$$\int \frac{A+B\cos(x)+C\sin(x)}{a+b\cos(x)+ib\sin(x)}\,dx = -\ln\left(a+b-a\tan\left(\frac{x}{2}\right)\,1\mathrm{i}+b\tan\left(\frac{x}{2}\right)\,1\mathrm{i}\right)\left(\frac{\frac{C}{2}+\frac{B\,1\mathrm{i}}{2}}{b}-\frac{\frac{C\,b^2}{2}-\frac{B\,b^2\,1\mathrm{i}}{2}+A\,a\,b\,1\mathrm{i}}{a^2\,b}\right)+\frac{\ln\left(\tan\left(\frac{x}{2}\right)+1\mathrm{i}\right)\,(C+B\,1\mathrm{i})}{2\,b}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)-\mathrm{i}\right)\,(B\,b-2\,A\,a+C\,b\,1\mathrm{i})\,1\mathrm{i}}{2\,a^2}+\frac{5\,B+C\,5\mathrm{i}}{5\,a\,\left(\tan\left(\frac{x}{2}\right)-\mathrm{i}\right)}$$

input
```
int((A + B*cos(x) + C*sin(x))/(a + b*cos(x) + b*sin(x)*1i),x)
```

output
```
(log(tan(x/2) + 1i)*(B*1i + C))/(2*b) - log(a + b - a*tan(x/2)*1i + b*tan(
x/2)*1i)*(((B*1i)/2 + C/2)/b - ((C*b^2)/2 - (B*b^2*1i)/2 + A*a*b*1i)/(a^2*
b)) + (log(tan(x/2) - 1i)*(B*b - 2*A*a + C*b*1i)*1i)/(2*a^2) + (5*B + C*5i
)/(5*a*(tan(x/2) - 1i))
```

3.553. $\int \frac{A+B\cos(x)+C\sin(x)}{a+b\cos(x)+ib\sin(x)}\,dx$

3.554 $\int \frac{A+B\cos(x)+C\sin(x)}{a+b\cos(x)-ib\sin(x)}\,dx$

3.554.1 Optimal result

Integrand size = 26, antiderivative size = 103

$$\int \frac{A+B\cos(x)+C\sin(x)}{a+b\cos(x)-ib\sin(x)}\,dx = \frac{(2aA-bB+ibC)x}{2a^2} - \frac{i(2aAb-b^2(B-iC)-a^2(B+iC))\log(a+b\cos(x)-ib\sin(x))}{2a^2b} - \frac{(iB+C)(\cos(x)+i\sin(x))}{2a}$$

output
```
1/2*(2*A*a-B*b+I*b*C)*x/a^2-1/2*I*(2*A*a*b-b^2*(B-I*C)-a^2*(B+I*C))*ln(a+b
*cos(x)-I*b*sin(x))/a^2/b-1/2*(I*B+C)*(cos(x)+I*sin(x))/a
```

3.554.2 Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.62

$$\int \frac{A+B\cos(x)+C\sin(x)}{a+b\cos(x)-ib\sin(x)}\,dx = \frac{\left(2aA-b(B-iC)+\frac{a^2(B+iC)}{b}\right)x+\frac{2\left(-2aAb+b^2(B-iC)+a^2(B+iC)\right)\arctan\left(\frac{(a+b)\cot\left(\frac{x}{2}\right)}{a-b}\right)}{b}-2ia(B-iC)\cos(x)+\cdots}{4a^2}$$

input
```
Integrate[(A + B*Cos[x] + C*Sin[x])/(a + b*Cos[x] - I*b*Sin[x]),x]
```

output
```
((2*a*A - b*(B - I*C) + (a^2*(B + I*C))/b)*x + (2*(-2*a*A*b + b^2*(B - I*C
) + a^2*(B + I*C))*ArcTan[((a + b)*Cot[x/2])/(a - b)])/b - (2*I)*a*(B - I*
C)*Cos[x] + (((-2*I)*a*A*b + I*a^2*(B + I*C) + b^2*(I*B + C))*Log[a^2 + b^
2 + 2*a*b*Cos[x]])/b + 2*a*(B - I*C)*Sin[x])/(4*a^2)
```

3.554.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3042, 3609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(x)+C\sin(x)}{a-ib\sin(x)+b\cos(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\cos(x)+C\sin(x)}{a-ib\sin(x)+b\cos(x)}dx$$

$$\Big\downarrow \text{3609}$$

$$-\frac{i\left(-\left(a^2(B+iC)\right)+2aAb-b^2(B-iC)\right)\log(a-ib\sin(x)+b\cos(x))}{2a^2b}+\frac{x(2aA-bB+ibC)}{2a^2}-\frac{(C+iB)(\cos(x)+i\sin(x))}{2a}$$

input
```
Int[(A + B*Cos[x] + C*Sin[x])/(a + b*Cos[x] - I*b*Sin[x]),x]
```

output
```
((2*a*A - b*B + I*b*C)*x)/(2*a^2) - ((I/2)*(2*a*A*b - b^2*(B - I*C) - a^2*
(B + I*C))*Log[a + b*Cos[x] - I*b*Sin[x]])/(a^2*b) - ((I*B + C)*(Cos[x] +
I*Sin[x]))/(2*a)
```

3.554.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3609
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_
Symbol] :> Simp[(2*a*A - b*B - c*C)*(x/(2*a^2)), x] + (-Simp[(b*B + c*C)*((
b*Cos[d + e*x] - c*Sin[d + e*x])/(2*a*b*c*e)), x] + Simp[(a^2*(b*B - c*C) -
 2*a*A*b^2 + b^2*(b*B + c*C))*(Log[RemoveContent[a + b*Cos[d + e*x] + c*Sin
[d + e*x], x]]/(2*a^2*b*c*e)), x]) /; FreeQ[{a, b, c, d, e, A, B, C}, x] &&
 EqQ[b^2 + c^2, 0]
```

3.554.4 Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.31

method	result
risch	$-\frac{C\,\mathrm{e}^{ix}}{2a}-\frac{iB\,\mathrm{e}^{ix}}{2a}+\frac{ixC}{2b}+\frac{Bx}{2b}-\frac{\ln\left(\mathrm{e}^{ix}+\frac{b}{a}\right)C}{2b}+\frac{b\ln\left(\mathrm{e}^{ix}+\frac{b}{a}\right)C}{2a^2}-\frac{i\ln\left(\mathrm{e}^{ix}+\frac{b}{a}\right)A}{a}+\frac{i\ln\left(\mathrm{e}^{ix}+\frac{b}{a}\right)B}{2b}+\frac{ib\ln\left(\mathrm{e}^{ix}+\frac{b}{a}\right)B}{2a^2}$
default	$\frac{i(-iC\,a^2+iC\,b^2+2Aab-B\,a^2-B\,b^2)(a-b)\ln(ia+ib-a\tan(\frac{x}{2})+b\tan(\frac{x}{2}))}{2a^2b(-a+b)}-\frac{i(iC+B)\ln(-i+\tan(\frac{x}{2}))}{2b}-\frac{iC-B}{a(\tan(\frac{x}{2})+i)}+\frac{(2iA$

input
```
int((A+B*cos(x)+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*C/a*exp(I*x)-1/2*I*B/a*exp(I*x)+1/2*I/b*x*C+1/2*B*x/b-1/2/b*ln(exp(I*
x)+b/a)*C+1/2*b/a^2*ln(exp(I*x)+b/a)*C-I/a*ln(exp(I*x)+b/a)*A+1/2*I/b*ln(e
xp(I*x)+b/a)*B+1/2*I*b/a^2*ln(exp(I*x)+b/a)*B
```

3.554.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.71

$$\int \frac{A+B\cos(x)+C\sin(x)}{a+b\cos(x)-ib\sin(x)}\,dx$$
$$=\frac{(B+i\,C)a^2x+(-i\,B-C)abe^{(i\,x)}+((i\,B-C)a^2-2i\,Aab+(i\,B+C)b^2)\log\left(\frac{ae^{(i\,x)}+b}{a}\right)}{2\,a^2b}$$

input
```
integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="fric
as")
```

output
```
1/2*((B + I*C)*a^2*x + (-I*B - C)*a*b*e^(I*x) + ((I*B - C)*a^2 - 2*I*A*a*b
 + (I*B + C)*b^2)*log((a*e^(I*x) + b)/a))/(a^2*b)
```

3.554.6 Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.02

$$\int \frac{A+B\cos(x)+C\sin(x)}{a+b\cos(x)-ib\sin(x)}\,dx = \begin{cases} \frac{(-iB-C)e^{ix}}{2a} & \text{for } a\neq 0 \\ x\left(-\frac{B+iC}{2b}+\frac{Ba+Bb+iCa-iCb}{2ab}\right) & \text{otherwise} \end{cases} + \frac{x(B+iC)}{2b} + \frac{i(-2Aab+Ba^2+Bb^2+iCa^2-iCb^2)\log\left(e^{ix}+\frac{b}{a}\right)}{2a^2b}$$

input
```
integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x)
```

output
```
Piecewise(((-I*B - C)*exp(I*x)/(2*a), Ne(a, 0)), (x*(-(B + I*C)/(2*b) + (B
*a + B*b + I*C*a - I*C*b)/(2*a*b)), True)) + x*(B + I*C)/(2*b) + I*(-2*A*a
*b + B*a**2 + B*b**2 + I*C*a**2 - I*C*b**2)*log(exp(I*x) + b/a)/(2*a**2*b)
```

3.554.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\cos(x)+C\sin(x)}{a+b\cos(x)-ib\sin(x)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="maxi
ma")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.554.8 Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 203 vs. 2(85) = 170.

Time = 0.27 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.97

$$\int \frac{A+B\cos(x)+C\sin(x)}{a+b\cos(x)-ib\sin(x)}\,dx = -\frac{(2iAa-iBb-Cb)\log\left(-a\tan\left(\frac{1}{2}x\right)^2+b\tan\left(\frac{1}{2}x\right)^2+2ia\tan\left(\frac{1}{2}x\right)+a+b\right)}{4a^2} - \frac{(-2iAa+iBb+Cb)\log\left(\tan\left(\frac{1}{2}x\right)+i\right)}{2a^2} + \frac{(2Ba^2+2iCa^2-2Aab+Bb^2-iCb^2)\left(x+2\arctan\left(\frac{ia\cos(x)-a\sin(x)+ia}{a\cos(x)+ia\sin(x)-a+2b}\right)\right)}{4a^2b} - \frac{2iAa\tan\left(\frac{1}{2}x\right)-iBb\tan\left(\frac{1}{2}x\right)-Cb\tan\left(\frac{1}{2}x\right)-2Aa-2Ba+2iCa+Bb-iCb}{2a^2\left(\tan\left(\frac{1}{2}x\right)+i\right)}$$

input
```
integrate((A+B*cos(x)+C*sin(x))/(a+b*cos(x)-I*b*sin(x)),x, algorithm="giac
")
```

output
```
-1/4*(2*I*A*a - I*B*b - C*b)*log(-a*tan(1/2*x)^2 + b*tan(1/2*x)^2 + 2*I*a*
tan(1/2*x) + a + b)/a^2 - 1/2*(-2*I*A*a + I*B*b + C*b)*log(tan(1/2*x) + I)
/a^2 + 1/4*(2*B*a^2 + 2*I*C*a^2 - 2*A*a*b + B*b^2 - I*C*b^2)*(x + 2*arctan
((I*a*cos(x) - a*sin(x) + I*a)/(a*cos(x) + I*a*sin(x) - a + 2*b)))/(a^2*b)
 - 1/2*(2*I*A*a*tan(1/2*x) - I*B*b*tan(1/2*x) - C*b*tan(1/2*x) - 2*A*a - 2
*B*a + 2*I*C*a + B*b - I*C*b)/(a^2*(tan(1/2*x) + I))
```

3.554.9 Mupad [B] (verification not implemented)

Time = 31.92 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.29

$$\int \frac{A+B\cos(x)+C\sin(x)}{a+b\cos(x)-ib\sin(x)}\,dx = \ln\left(a+b+a\tan\left(\frac{x}{2}\right)\,1\mathrm{i}-b\tan\left(\frac{x}{2}\right)\,1\mathrm{i}\right)\left(\frac{-\frac{C}{2}+\frac{B\,1\mathrm{i}}{2}}{b}+\frac{\frac{B\,b^2\,1\mathrm{i}}{2}+\frac{C\,b^2}{2}-A\,a\,b\,1\mathrm{i}}{a^2\,b}\right)+\frac{\ln\left(\tan\left(\frac{x}{2}\right)+1\mathrm{i}\right)\,\left(2\,A\,a-B\,b+C\,b\,1\mathrm{i}\right)\,1\mathrm{i}}{2\,a^2}+\frac{5\,B-C\,5\mathrm{i}}{5\,a\,\left(\tan\left(\frac{x}{2}\right)+1\mathrm{i}\right)}-\frac{\ln\left(\tan\left(\frac{x}{2}\right)-\mathrm{i}\right)\,\left(-C+B\,1\mathrm{i}\right)}{2\,b}$$

input
```
int((A + B*cos(x) + C*sin(x))/(a + b*cos(x) - b*sin(x)*1i),x)
```

output
```
log(a + b + a*tan(x/2)*1i - b*tan(x/2)*1i)*(((B*1i)/2 - C/2)/b + ((B*b^2*1
i)/2 + (C*b^2)/2 - A*a*b*1i)/(a^2*b)) + (log(tan(x/2) + 1i)*(2*A*a - B*b +
 C*b*1i)*1i)/(2*a^2) + (5*B - C*5i)/(5*a*(tan(x/2) + 1i)) - (log(tan(x/2)
- 1i)*(B*1i - C))/(2*b)
```

3.554. $\int \frac{A+B\cos(x)+C\sin(x)}{a+b\cos(x)-ib\sin(x)}\,dx$

3.555 $\int \frac{b^2+c^2+ab\cos(x)+ac\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx$

3.555.1 Optimal result

Integrand size = 30, antiderivative size = 24

$$\int \frac{b^2+c^2+ab\cos(x)+ac\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx=-\frac{c\cos(x)-b\sin(x)}{a+b\cos(x)+c\sin(x)}$$

output
```
(-c*cos(x)+b*sin(x))/(a+b*cos(x)+c*sin(x))
```

3.555.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

$$\int \frac{b^2+c^2+ab\cos(x)+ac\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx=\frac{ac+b^2\sin(x)+c^2\sin(x)}{b(a+b\cos(x)+c\sin(x))}$$

input
```
Integrate[(b^2 + c^2 + a*b*Cos[x] + a*c*Sin[x])/(a + b*Cos[x] + c*Sin[x])^
2,x]
```

output
```
(a*c + b^2*Sin[x] + c^2*Sin[x])/(b*(a + b*Cos[x] + c*Sin[x]))
```

3.555.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 68 vs. 2(24) = 48.

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.83, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3042, 3629}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{ab\cos(x)+ac\sin(x)+b^2+c^2}{(a+b\cos(x)+c\sin(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{ab\cos(x)+ac\sin(x)+b^2+c^2}{(a+b\cos(x)+c\sin(x))^2}dx$$

$$\Big\downarrow \text{3629}$$

$$-\frac{c\cos(x)\left(a^2-b^2-c^2\right)-b\sin(x)\left(a^2-b^2-c^2\right)}{\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))}$$

input
```
Int[(b^2 + c^2 + a*b*Cos[x] + a*c*Sin[x])/(a + b*Cos[x] + c*Sin[x])^2,x]
```

output
```
-((c*(a^2 - b^2 - c^2)*Cos[x] - b*(a^2 - b^2 - c^2)*Sin[x])/((a^2 - b^2 -
c^2)*(a + b*Cos[x] + c*Sin[x])))
```

3.555.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3629
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2,
 x_Symbol] :> Simp[(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)*Sin[
d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d + e*x] + c*Sin[d + e*x])), x] /
; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[a^2 - b^2 - c^2, 0] && EqQ[a*A
- b*B - c*C, 0]
```

3.555.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.50

method	result	size
parallelrisch	$\frac{-b^2\cos(x)-c^2\cos(x)-ab}{(a+b\cos(x)+c\sin(x))c}$	36
risch	$-\frac{2i(-ib+c-ia\,e^{ix})}{c\,e^{2ix}+ib\,e^{2ix}-c+2ia\,e^{ix}+ib}$	54
default	$-\frac{2\left(-\frac{(ab-b^2-c^2)\tan(\frac{x}{2})}{a-b}+\frac{ac}{a-b}\right)}{\tan(\frac{x}{2})^2a-\tan(\frac{x}{2})^2b+2c\tan(\frac{x}{2})+a+b}$	70
norman	$\frac{-\frac{2ab+2b^2+2c^2}{2c}-\frac{(2ab-2b^2-2c^2)\tan(\frac{x}{2})^4}{2c}-\frac{2ab\tan(\frac{x}{2})^2}{c}}{\left(1+\tan(\frac{x}{2})^2\right)\left(\tan(\frac{x}{2})^2a-\tan(\frac{x}{2})^2b+2c\tan(\frac{x}{2})+a+b\right)}$	101

input
```
int((b^2+c^2+a*b*cos(x)+a*c*sin(x))/(a+b*cos(x)+c*sin(x))^2,x,method=_RETU
RNVERBOSE)
```

output
```
(-b^2*cos(x)-c^2*cos(x)-a*b)/(a+b*cos(x)+c*sin(x))/c
```

3.555.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{b^2+c^2+ab\cos(x)+ac\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx = -\frac{c\cos(x)-b\sin(x)}{b\cos(x)+c\sin(x)+a}$$

input
```
integrate((b^2+c^2+a*b*cos(x)+a*c*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algor
ithm="fricas")
```

output
```
-(c*cos(x) - b*sin(x))/(b*cos(x) + c*sin(x) + a)
```

3.555.6 Sympy [F(-1)]

Timed out.

$$\int \frac{b^2+c^2+ab\cos(x)+ac\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx = \text{Timed out}$$

input
```
integrate((b**2+c**2+a*b*cos(x)+a*c*sin(x))/(a+b*cos(x)+c*sin(x))**2,x)
```

output
```
Timed out
```

3.555.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{b^2+c^2+ab\cos(x)+ac\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b^2+c^2+a*b*cos(x)+a*c*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algor
ithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c^2+b^2-a^2>0)', see `assume?` f
or more de
```

3.555.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(24) = 48$.

Time = 0.30 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.83

$$\int \frac{b^2+c^2+ab\cos(x)+ac\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx = \frac{2\left(ab\tan\left(\frac{1}{2}x\right)-b^2\tan\left(\frac{1}{2}x\right)-c^2\tan\left(\frac{1}{2}x\right)-ac\right)}{\left(a\tan\left(\frac{1}{2}x\right)^2-b\tan\left(\frac{1}{2}x\right)^2+2\,c\tan\left(\frac{1}{2}x\right)+a+b\right)(a-b)}$$

input
```
integrate((b^2+c^2+a*b*cos(x)+a*c*sin(x))/(a+b*cos(x)+c*sin(x))^2,x, algor
ithm="giac")
```

output
```
2*(a*b*tan(1/2*x) - b^2*tan(1/2*x) - c^2*tan(1/2*x) - a*c)/((a*tan(1/2*x)^
2 - b*tan(1/2*x)^2 + 2*c*tan(1/2*x) + a + b)*(a - b))
```

3.555.9 Mupad [B] (verification not implemented)

Time = 26.05 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.58

$$\int \frac{b^2+c^2+ab\cos(x)+ac\sin(x)}{(a+b\cos(x)+c\sin(x))^2}\,dx = -\frac{\frac{2\,a\,c}{a-b}+\frac{2\tan\left(\frac{x}{2}\right)\,\left(b^2-a\,b+c^2\right)}{a-b}}{(a-b)\,\tan\left(\frac{x}{2}\right)^2+2\,c\tan\left(\frac{x}{2}\right)+a+b}$$

input
```
int((b^2 + c^2 + a*c*sin(x) + a*b*cos(x))/(a + b*cos(x) + c*sin(x))^2,x)
```

output
```
-((2*a*c)/(a - b) + (2*tan(x/2)*(b^2 - a*b + c^2))/(a - b))/(a + b + 2*c*t
an(x/2) + tan(x/2)^2*(a - b))
```

3.556 $\int (a+b\cos(x)+c\sin(x))^{5/2}(d+be\cos(x)+ce\sin(x))\,dx$

3.556.1 Optimal result

Integrand size = 27, antiderivative size = 390

$$\int (a+b\cos(x)+c\sin(x))^{5/2}(d+be\cos(x)+ce\sin(x))\,dx = \frac{2(161a^2d+63(b^2+c^2)\,d+15a^3e+145a(b^2+c^2)\,e)\,E\left(\frac{1}{2}(x-\tan^{-1}(b,c))\,|\,\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)\sqrt{a+}}{105\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}$$
$$-\frac{2(a^2-b^2-c^2)\,(56ad+15a^2e+25(b^2+c^2)\,e)\,\text{EllipticF}\left(\frac{1}{2}(x-\tan^{-1}(b,c)),\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}{105\sqrt{a+b\cos(x)+c\sin(x)}}$$
$$-\frac{2}{7}(a+b\cos(x)+c\sin(x))^{5/2}(ce\cos(x)-be\sin(x))-\frac{2}{35}(a+b\cos(x)+c\sin(x))^{3/2}(c(7d+5ae)\cos(x)-b(7d+$$

output
```
-2/7*(a+b*cos(x)+c*sin(x))^(5/2)*(c*e*cos(x)-b*e*sin(x))-2/35*(a+b*cos(x)+
c*sin(x))^(3/2)*(c*(5*a*e+7*d)*cos(x)-b*(5*a*e+7*d)*sin(x))-2/105*(c*(56*a
*d+15*a^2*e+25*(b^2+c^2)*e)*cos(x)-b*(56*a*d+15*a^2*e+25*(b^2+c^2)*e)*sin(
x))*(a+b*cos(x)+c*sin(x))^(1/2)+2/105*(161*a^2*d+63*(b^2+c^2)*d+15*a^3*e+1
45*a*(b^2+c^2)*e)*(cos(1/2*x-1/2*arctan(b,c))^2)^(1/2)/cos(1/2*x-1/2*arcta
n(b,c))*EllipticE(sin(1/2*x-1/2*arctan(b,c)),2^(1/2)*((b^2+c^2)^(1/2)/(a+(
b^2+c^2)^(1/2)))^(1/2))*(a+b*cos(x)+c*sin(x))^(1/2)/((a+b*cos(x)+c*sin(x))
/(a+(b^2+c^2)^(1/2)))^(1/2)-2/105*(a^2-b^2-c^2)*(56*a*d+15*a^2*e+25*(b^2+c
^2)*e)*(cos(1/2*x-1/2*arctan(b,c))^2)^(1/2)/cos(1/2*x-1/2*arctan(b,c))*Ell
ipticF(sin(1/2*x-1/2*arctan(b,c)),2^(1/2)*((b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1
/2)))^(1/2))*((a+b*cos(x)+c*sin(x))/(a+(b^2+c^2)^(1/2)))^(1/2)/(a+b*cos(x)
+c*sin(x))^(1/2)
```

3.556.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 6.78 (sec) , antiderivative size = 7823, normalized size of antiderivative = 20.06

$$\int (a+b\cos(x)+c\sin(x))^{5/2}(d+be\cos(x)+ce\sin(x))\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Cos[x] + c*Sin[x])^(5/2)*(d + b*e*Cos[x] + c*e*Sin[x]),x]
```

output
```
Result too large to show
```

3.556.3 Rubi [A] (verified)

Time = 2.23 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.09, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 3625, 27, 3042, 3625, 27, 3042, 3625, 27, 3042, 3628, 3042, 3598, 3042, 3132, 3606, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cos(x)+c\sin(x))^{5/2}(be\cos(x)+ce\sin(x)+d)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\cos(x)+c\sin(x))^{5/2}(be\cos(x)+ce\sin(x)+d)dx$$

$$\Big\downarrow \; 3625$$

$$\frac{2\int \frac{1}{2}(a+b\cos(x)+c\sin(x))^{3/2}\left(a\left(7ad+5\left(b^2+c^2\right)e\right)+ab(7d+5ae)\cos(x)+ac(7d+5ae)\sin(x)\right)dx}{7a}-\frac{2}{7}(a+b\cos(x)+c\sin(x))^{5/2}(ce\cos(x)-be\sin(x))$$

$$\Big\downarrow \; 27$$

$$\frac{\int (a+b\cos(x)+c\sin(x))^{3/2}\left(a\left(7ad+5\left(b^2+c^2\right)e\right)+ab(7d+5ae)\cos(x)+ac(7d+5ae)\sin(x)\right)dx}{7a}-\frac{2}{7}(a+b\cos(x)+c\sin(x))^{5/2}(ce\cos(x)-be\sin(x))$$

$$\Big\downarrow \; 3042$$

$$\frac{\int (a+b\cos(x)+c\sin(x))^{3/2}\left(a\left(7ad+5\left(b^2+c^2\right)e\right)+ab(7d+5ae)\cos(x)+ac(7d+5ae)\sin(x)\right)dx}{7a}-\frac{2}{7}(a+b\cos(x)+c\sin(x))^{5/2}(ce\cos(x)-be\sin(x))$$

$$\Big\downarrow \; 3625$$

$$\frac{\frac{2\int \frac{1}{2}\sqrt{a+b\cos(x)+c\sin(x)}\left(\left(35da^2+40\left(b^2+c^2\right)ea+21\left(b^2+c^2\right)d\right)a^2+b\left(15ea^2+56da+25\left(b^2+c^2\right)e\right)\cos(x)a^2+c\left(15ea^2+56da+25\left(b^2+c^2\right)e\right)\sin(x)}{5a}}{7a}$$

$$\frac{2}{7}(a+b\cos(x)+c\sin(x))^{5/2}(ce\cos(x)-be\sin(x))$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \sqrt{a+b\cos(x)+c\sin(x)}\left(\left(35da^2+40\left(b^2+c^2\right)ea+21\left(b^2+c^2\right)d\right)a^2+b\left(15ea^2+56da+25\left(b^2+c^2\right)e\right)\cos(x)a^2+c\left(15ea^2+56da+25\left(b^2+c^2\right)e\right)\sin(x)a^2\right)}{5a}}{7a}$$

$$\frac{2}{7}(a+b\cos(x)+c\sin(x))^{5/2}(ce\cos(x)-be\sin(x))$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \sqrt{a+b\cos(x)+c\sin(x)}\left(\left(35da^2+40\left(b^2+c^2\right)ea+21\left(b^2+c^2\right)d\right)a^2+b\left(15ea^2+56da+25\left(b^2+c^2\right)e\right)\cos(x)a^2+c\left(15ea^2+56da+25\left(b^2+c^2\right)e\right)\sin(x)a^2\right)}{5a}}{7a}$$

$$\frac{2}{7}(a+b\cos(x)+c\sin(x))^{5/2}(ce\cos(x)-be\sin(x))$$

$$\Big\downarrow \; 3625$$

3.556. $\int (a+b\cos(x)+c\sin(x))^{5/2}(d+be\cos(x)+ce\sin(x))\,dx$

$$\frac{\frac{2\int \frac{\left(105da^3+135\left(b^2+c^2\right)ea^2+119\left(b^2+c^2\right)da+25\left(b^2+c^2\right)^2e\right)a^3+b\left(15ea^3+161da^2+145\left(b^2+c^2\right)ea+63\left(b^2+c^2\right)d\right)\cos(x)a^3+c\left(15ea^3+161da^2+145\left(b^2+c^2\right)ea+\right.}{2\sqrt{a+b\cos(x)+c\sin(x)}}}{3a}}{5a}$$

$$\frac{2}{7}(a+b\cos(x)+c\sin(x))^{5/2}(ce\cos(x)-be\sin(x))$$

$\downarrow$ 27

$$\frac{\frac{\int \frac{\left(105da^3+135\left(b^2+c^2\right)ea^2+119\left(b^2+c^2\right)da+25\left(b^2+c^2\right)^2e\right)a^3+b\left(15ea^3+161da^2+145\left(b^2+c^2\right)ea+63\left(b^2+c^2\right)d\right)\cos(x)a^3+c\left(15ea^3+161da^2+145\left(b^2+c^2\right)ea+6\right.}{\sqrt{a+b\cos(x)+c\sin(x)}}}{3a}}{5a}$$

$$\frac{2}{7}(a+b\cos(x)+c\sin(x))^{5/2}(ce\cos(x)-be\sin(x))$$

$\downarrow$ 3042

$$\frac{\frac{\int \frac{\left(105da^3+135\left(b^2+c^2\right)ea^2+119\left(b^2+c^2\right)da+25\left(b^2+c^2\right)^2e\right)a^3+b\left(15ea^3+161da^2+145\left(b^2+c^2\right)ea+63\left(b^2+c^2\right)d\right)\cos(x)a^3+c\left(15ea^3+161da^2+145\left(b^2+c^2\right)ea+6\right.}{\sqrt{a+b\cos(x)+c\sin(x)}}}{3a}}{5a}$$

$$\frac{2}{7}(a+b\cos(x)+c\sin(x))^{5/2}(ce\cos(x)-be\sin(x))$$

$\downarrow$ 3628

$$\frac{\frac{a^3\left(15a^3e+161a^2d+145ae\left(b^2+c^2\right)+63d\left(b^2+c^2\right)\right)\int\sqrt{a+b\cos(x)+c\sin(x)}dx-a^3\left(a^2-b^2-c^2\right)\left(15a^2e+56ad+25e\left(b^2+c^2\right)\right)\int\frac{1}{\sqrt{a+b\cos(x)+c\sin(x)}}dx}{3a}-\frac{2}{3}\sqrt{a+b}}{5a}$$

$$\frac{2}{7}(a+b\cos(x)+c\sin(x))^{5/2}(ce\cos(x)-be\sin(x))$$

$\downarrow$ 3042

$$\frac{\frac{a^3\left(15a^3e+161a^2d+145ae\left(b^2+c^2\right)+63d\left(b^2+c^2\right)\right)\int\sqrt{a+b\cos(x)+c\sin(x)}dx-a^3\left(a^2-b^2-c^2\right)\left(15a^2e+56ad+25e\left(b^2+c^2\right)\right)\int\frac{1}{\sqrt{a+b\cos(x)+c\sin(x)}}dx}{3a}-\frac{2}{3}\sqrt{a+b}}{5a}$$

$$\frac{2}{7}(a+b\cos(x)+c\sin(x))^{5/2}(ce\cos(x)-be\sin(x))$$

$\downarrow$ 3598

3.556. $\int (a+b\cos(x)+c\sin(x))^{5/2}(d+be\cos(x)+ce\sin(x))\,dx$

$$\frac{\frac{\frac{a^3\left(15a^3e+161a^2d+145ae\left(b^2+c^2\right)+63d\left(b^2+c^2\right)\right)\sqrt{a+b\cos(x)+c\sin(x)}\int\sqrt{\frac{a}{a+\sqrt{b^2+c^2}}+\frac{\sqrt{b^2+c^2}\cos\left(x-\tan^{-1}(b,c)\right)}{a+\sqrt{b^2+c^2}}}dx}{\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}-a^3\left(a^2-b^2-c^2\right)\left(15a^2e+56ad+25e\left(b^2\cdots\right.\right.}{3a}}{5a}$$

$$\frac{2}{7}(a+b\cos(x)+c\sin(x))^{5/2}(ce\cos(x)-be\sin(x))$$

$\downarrow$ 3042

$$\frac{\frac{\frac{a^3\left(15a^3e+161a^2d+145ae\left(b^2+c^2\right)+63d\left(b^2+c^2\right)\right)\sqrt{a+b\cos(x)+c\sin(x)}\int\sqrt{\frac{a}{a+\sqrt{b^2+c^2}}+\frac{\sqrt{b^2+c^2}\sin\left(x-\tan^{-1}(b,c)+\frac{\pi}{2}\right)}{a+\sqrt{b^2+c^2}}}dx}{\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}-a^3\left(a^2-b^2-c^2\right)\left(15a^2e+56ad+25e\cdots\right.}{3a}}{5a}$$

$$\frac{2}{7}(a+b\cos(x)+c\sin(x))^{5/2}(ce\cos(x)-be\sin(x))$$

$\downarrow$ 3132

$$\frac{\frac{\frac{2a^3\left(15a^3e+161a^2d+145ae\left(b^2+c^2\right)+63d\left(b^2+c^2\right)\right)\sqrt{a+b\cos(x)+c\sin(x)}E\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}-a^3\left(a^2-b^2-c^2\right)\left(15a^2e+56ad+25e\left(b^2+c^2\right)\right)\int\frac{\cdots}{\sqrt{a\cdots}}}{3a}}{5a}$$

$$\frac{2}{7}(a+b\cos(x)+c\sin(x))^{5/2}(ce\cos(x)-be\sin(x))$$

$\downarrow$ 3606

$$\frac{\frac{2a^3\left(15a^3e+161a^2d+145ae\left(b^2+c^2\right)+63d\left(b^2+c^2\right)\right)\sqrt{a+b\cos(x)+c\sin(x)}E\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}-\frac{a^3\left(a^2-b^2-c^2\right)\left(15a^2e+56ad+25e\left(b^2+c^2\right)\right)\sqrt{\frac{a+\cdots}{\cdots}}}{\sqrt{\cdots}}}{3a}$$

$$\frac{2}{7}(a+b\cos(x)+c\sin(x))^{5/2}(ce\cos(x)-be\sin(x))$$

$\downarrow$ 3042

$$\frac{\frac{2a^3\left(15a^3e+161a^2d+145ae\left(b^2+c^2\right)+63d\left(b^2+c^2\right)\right)\sqrt{a+b\cos(x)+c\sin(x)}E\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}-\frac{a^3\left(a^2-b^2-c^2\right)\left(15a^2e+56ad+25e\left(b^2+c^2\right)\right)\sqrt{\frac{a+\cdots}{\cdots}}}{\cdots}}{3a}$$

$$\frac{2}{7}(a+b\cos(x)+c\sin(x))^{5/2}(ce\cos(x)-be\sin(x))$$

3.556. $\int(a+b\cos(x)+c\sin(x))^{5/2}(d+be\cos(x)+ce\sin(x))\,dx$

$\downarrow$ 3140

$$\frac{\frac{2a^3\left(15a^3e+161a^2d+145ae\left(b^2+c^2\right)+63d\left(b^2+c^2\right)\right)\sqrt{a+b\cos(x)+c\sin(x)}E\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}-\frac{2a^3\left(a^2-b^2-c^2\right)\left(15a^2e+56ad+25e\left(b^2+c^2\right)\right)\sqrt{a\cdots}}{\sqrt{a+\cdots}}}{3a}$$

$$\frac{2}{7}(a+b\cos(x)+c\sin(x))^{5/2}(ce\cos(x)-be\sin(x))$$

input
```
Int[(a + b*Cos[x] + c*Sin[x])^(5/2)*(d + b*e*Cos[x] + c*e*Sin[x]),x]
```

output
```
(-2*(a + b*Cos[x] + c*Sin[x])^(5/2)*(c*e*Cos[x] - b*e*Sin[x]))/7 + ((-2*(a
 + b*Cos[x] + c*Sin[x])^(3/2)*(a*c*(7*d + 5*a*e)*Cos[x] - a*b*(7*d + 5*a*e
)*Sin[x]))/5 + ((-2*Sqrt[a + b*Cos[x] + c*Sin[x]]*(a^2*c*(56*a*d + 15*a^2*
e + 25*(b^2 + c^2)*e)*Cos[x] - a^2*b*(56*a*d + 15*a^2*e + 25*(b^2 + c^2)*e
)*Sin[x]))/3 + ((2*a^3*(161*a^2*d + 63*(b^2 + c^2)*d + 15*a^3*e + 145*a*(b
^2 + c^2)*e)*EllipticE[(x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt
[b^2 + c^2])]*Sqrt[a + b*Cos[x] + c*Sin[x]])/Sqrt[(a + b*Cos[x] + c*Sin[x]
)/(a + Sqrt[b^2 + c^2])] - (2*a^3*(a^2 - b^2 - c^2)*(56*a*d + 15*a^2*e + 2
5*(b^2 + c^2)*e)*EllipticF[(x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a +
Sqrt[b^2 + c^2])]*Sqrt[(a + b*Cos[x] + c*Sin[x])/(a + Sqrt[b^2 + c^2])])/S
qrt[a + b*Cos[x] + c*Sin[x]])/(3*a))/(5*a))/(7*a)
```

3.556.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3598
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]   Int[Sqrt[a/(a + S
qrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - Arc
Tan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

rule 3606
```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Simp[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]   Int[1/Sqrt[a/(a
 + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x -
 ArcTan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2
, 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

rule 3625
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*((a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^n/(a*e*(n + 1))), x] + Simp[1/(a*(n + 1
))   Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
 + a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
 + (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

rule 3628
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]
, x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[
A*b - a*B, 0]
```

3.556. $\int (a+b\cos(x)+c\sin(x))^{5/2}(d+be\cos(x)+ce\sin(x))\,dx$

3.556.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 3451 vs. $2(414) = 828$.

Time = 12.96 (sec) , antiderivative size = 3452, normalized size of antiderivative = 8.85

method	result	size
default	Expression too large to display	3452
parts	Expression too large to display	38062

input
```
int((a+b*cos(x)+c*sin(x))^(5/2)*(d+b*e*cos(x)+c*e*sin(x)),x,method=_RETURN
VERBOSE)
```

output
```
(-(-b^2*sin(x-arctan(-b,c))-c^2*sin(x-arctan(-b,c))-a*(b^2+c^2)^(1/2))*cos
(x-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)/(b^2+c^2)*(2*a^3*b^2*d*(1/(b^2+c
^2)^(1/2)*a+1)*(((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)
))^(1/2)*((sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1
/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)/(
-(-b^2*sin(x-arctan(-b,c))-c^2*sin(x-arctan(-b,c))-a*(b^2+c^2)^(1/2))*cos(
x-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)*EllipticF((((b^2+c^2)^(1/2)*sin(x
-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2),((-a-(b^2+c^2)^(1/2))/(-a+(b^
2+c^2)^(1/2)))^(1/2))+2*a^3*c^2*d*(1/(b^2+c^2)^(1/2)*a+1)*(((b^2+c^2)^(1/2
)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2)*((sin(x-arctan(-b,c))+
1)*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(
b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)/(-(-b^2*sin(x-arctan(-b,c))-c^2*
sin(x-arctan(-b,c))-a*(b^2+c^2)^(1/2))*cos(x-arctan(-b,c))^2/(b^2+c^2)^(1/
2))^(1/2)*EllipticF((((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^
(1/2)))^(1/2),((-a-(b^2+c^2)^(1/2))/(-a+(b^2+c^2)^(1/2)))^(1/2))+(b^6*e+3*
b^4*c^2*e+3*b^2*c^4*e+c^6*e)*(-2/7/(b^2+c^2)^(1/2)*sin(x-arctan(-b,c))^2*(
((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)*cos(x-arctan(-b,c))^2)^(1/2)+12/35
/(b^2+c^2)*a*sin(x-arctan(-b,c))*(((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)*
cos(x-arctan(-b,c))^2)^(1/2)-2/3*(5/7+24/35/(b^2+c^2)*a^2)/(b^2+c^2)^(1/2)
*(((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)*cos(x-arctan(-b,c))^2)^(1/2)+...
```

3.556.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.21 (sec) , antiderivative size = 2014, normalized size of antiderivative = 5.16

$$\int (a+b\cos(x)+c\sin(x))^{5/2}(d+be\cos(x)+ce\sin(x))\,dx = \text{Too large to display}$$

input
```
integrate((a+b*cos(x)+c*sin(x))^(5/2)*(d+b*e*cos(x)+c*e*sin(x)),x, algorit
hm="fricas")
```

output
```
1/315*(sqrt(2)*(-7*I*(a^3*b - 33*a*b^3 - 33*a*b*c^2)*d + 7*(33*a*c^3 - (a^
3 - 33*a*b^2)*c)*d - 5*I*(6*a^4*b - 23*a^2*b^3 - 15*b^5 - 15*b*c^4 - (23*a
^2*b + 30*b^3)*c^2)*e + 5*(15*c^5 + (23*a^2 + 30*b^2)*c^3 - (6*a^4 - 23*a^
2*b^2 - 15*b^4)*c)*e)*sqrt(b + I*c)*weierstrassPInverse(4/3*(4*a^2*b^2 - 3
*b^4 - 4*a^2*c^2 + 6*I*b*c^3 + 3*c^4 - 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b
^2*c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 - 9*I*a*c^5 + 2*I*(
4*a^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 - 3*I*(8*a^3*b^2 - 9*a*b^
4)*c)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), 1/3*(2*a*b - 2*I*a*c + 3*(b^2 +
 c^2)*cos(x) - 3*(I*b^2 + I*c^2)*sin(x))/(b^2 + c^2)) + sqrt(2)*(7*I*(a^3*
b - 33*a*b^3 - 33*a*b*c^2)*d + 7*(33*a*c^3 - (a^3 - 33*a*b^2)*c)*d + 5*I*(
6*a^4*b - 23*a^2*b^3 - 15*b^5 - 15*b*c^4 - (23*a^2*b + 30*b^3)*c^2)*e + 5*
(15*c^5 + (23*a^2 + 30*b^2)*c^3 - (6*a^4 - 23*a^2*b^2 - 15*b^4)*c)*e)*sqrt
(b - I*c)*weierstrassPInverse(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2*c^2 - 6*I*b*c
^3 + 3*c^4 + 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4), -8/27*(8*a^
3*b^3 - 9*a*b^5 + 27*a*b*c^4 + 9*I*a*c^5 - 2*I*(4*a^3 + 9*a*b^2)*c^3 - 6*(
4*a^3*b - 3*a*b^3)*c^2 + 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^4*c^2 + 3
*b^2*c^4 + c^6), 1/3*(2*a*b + 2*I*a*c + 3*(b^2 + c^2)*cos(x) - 3*(-I*b^2 -
 I*c^2)*sin(x))/(b^2 + c^2)) - 3*sqrt(2)*(7*I*(23*a^2*b^2 + 9*b^4 + 9*c^4
+ (23*a^2 + 18*b^2)*c^2)*d + 5*I*(3*a^3*b^2 + 29*a*b^4 + 29*a*c^4 + (3*a^3
 + 58*a*b^2)*c^2)*e)*sqrt(b + I*c)*weierstrassZeta(4/3*(4*a^2*b^2 - 3*b...
```

3.556.6 Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(x)+c\sin(x))^{5/2}(d+be\cos(x)+ce\sin(x))\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(x)+c*sin(x))**(5/2)*(d+b*e*cos(x)+c*e*sin(x)),x)
```

output
```
Timed out
```

3.556.7 Maxima [F]

$$\int (a+b\cos(x)+c\sin(x))^{5/2}(d+be\cos(x)+ce\sin(x))\,dx = \int (be\cos(x)+ce\sin(x)+d)(b\cos(x)+c\sin(x)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cos(x)+c*sin(x))^(5/2)*(d+b*e*cos(x)+c*e*sin(x)),x, algorit
hm="maxima")
```

output
```
integrate((b*e*cos(x) + c*e*sin(x) + d)*(b*cos(x) + c*sin(x) + a)^(5/2), x
)
```

3.556.8 Giac [F]

$$\int (a+b\cos(x)+c\sin(x))^{5/2}(d+be\cos(x)+ce\sin(x))\,dx = \int (be\cos(x)+ce\sin(x)+d)(b\cos(x)+c\sin(x)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cos(x)+c*sin(x))^(5/2)*(d+b*e*cos(x)+c*e*sin(x)),x, algorit
hm="giac")
```

output
```
integrate((b*e*cos(x) + c*e*sin(x) + d)*(b*cos(x) + c*sin(x) + a)^(5/2), x
)
```

3.556.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(x)+c\sin(x))^{5/2}(d+be\cos(x)+ce\sin(x))\,dx = \int \left(a+b\,\cos\left(x\right)+c\,\sin\left(x\right)\right)^{5/2}\,\left(d+b\,e\,\cos\left(x\right)+c\,e\,\sin\left(x\right)\right)\,dx$$

input
```
int((a + b*cos(x) + c*sin(x))^(5/2)*(d + b*e*cos(x) + c*e*sin(x)),x)
```

output
```
int((a + b*cos(x) + c*sin(x))^(5/2)*(d + b*e*cos(x) + c*e*sin(x)), x)
```

3.557 $\int (a+b\cos(x)+c\sin(x))^{3/2}(d+be\cos(x)+ce\sin(x))\,dx$

3.557.1 Optimal result

Integrand size = 27, antiderivative size = 294

$$\int (a+b\cos(x)+c\sin(x))^{3/2}(d+be\cos(x)+ce\sin(x))\,dx=\frac{2(20ad+3a^2e+9(b^2+c^2)e)\,E\left(\frac{1}{2}(x-\tan^{-1}(b,c))\,|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)\sqrt{a+b\cos(x)+c\sin(x)}}{15\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}-\frac{2(a^2-b^2-c^2)(5d+3ae)\operatorname{EllipticF}\left(\frac{1}{2}(x-\tan^{-1}(b,c)),\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}{15\sqrt{a+b\cos(x)+c\sin(x)}}-\frac{2}{5}(a+b\cos(x)+c\sin(x))^{3/2}(ce\cos(x)-be\sin(x))-\frac{2}{15}\sqrt{a+b\cos(x)+c\sin(x)}(c(5d+3ae)\cos(x)-b(5d+3$$

output
```
-2/5*(a+b*cos(x)+c*sin(x))^(3/2)*(c*e*cos(x)-b*e*sin(x))-2/15*(c*(3*a*e+5*
d)*cos(x)-b*(3*a*e+5*d)*sin(x))*(a+b*cos(x)+c*sin(x))^(1/2)+2/15*(20*a*d+3
*a^2*e+9*(b^2+c^2)*e)*(cos(1/2*x-1/2*arctan(b,c))^2)^(1/2)/cos(1/2*x-1/2*a
rctan(b,c))*EllipticE(sin(1/2*x-1/2*arctan(b,c)),2^(1/2)*((b^2+c^2)^(1/2)/
(a+(b^2+c^2)^(1/2)))^(1/2))*(a+b*cos(x)+c*sin(x))^(1/2)/((a+b*cos(x)+c*sin
(x))/(a+(b^2+c^2)^(1/2)))^(1/2)-2/15*(a^2-b^2-c^2)*(3*a*e+5*d)*(cos(1/2*x-
1/2*arctan(b,c))^2)^(1/2)/cos(1/2*x-1/2*arctan(b,c))*EllipticF(sin(1/2*x-1
/2*arctan(b,c)),2^(1/2)*((b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2))*((a+b
*cos(x)+c*sin(x))/(a+(b^2+c^2)^(1/2)))^(1/2)/(a+b*cos(x)+c*sin(x))^(1/2)
```

3.557.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 6.59 (sec) , antiderivative size = 5218, normalized size of antiderivative = 17.75

$$\int (a+b\cos(x)+c\sin(x))^{3/2}(d+be\cos(x)+ce\sin(x))\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Cos[x] + c*Sin[x])^(3/2)*(d + b*e*Cos[x] + c*e*Sin[x]),x]
```

output
```
Result too large to show
```

3.557.3 Rubi [A] (verified)

Time = 1.48 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.07, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {3042, 3625, 27, 3042, 3625, 27, 3042, 3628, 3042, 3598, 3042, 3132, 3606, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cos(x)+c\sin(x))^{3/2}(be\cos(x)+ce\sin(x)+d)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\cos(x)+c\sin(x))^{3/2}(be\cos(x)+ce\sin(x)+d)dx$$

$$\Big\downarrow \text{3625}$$

$$\frac{2\int \frac{1}{2}\sqrt{a+b\cos(x)+c\sin(x)}\left(a\left(5ad+3\left(b^2+c^2\right)e\right)+ab(5d+3ae)\cos(x)+ac(5d+3ae)\sin(x)\right)dx}{5a}-\frac{2}{5}(a+b\cos(x)+c\sin(x))^{3/2}(ce\cos(x)-be\sin(x))$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \sqrt{a+b\cos(x)+c\sin(x)}\left(a\left(5ad+3\left(b^2+c^2\right)e\right)+ab(5d+3ae)\cos(x)+ac(5d+3ae)\sin(x)\right)dx}{5a}-\frac{2}{5}(a+b\cos(x)+c\sin(x))^{3/2}(ce\cos(x)-be\sin(x))$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{a+b\cos(x)+c\sin(x)}\left(a\left(5ad+3\left(b^2+c^2\right)e\right)+ab(5d+3ae)\cos(x)+ac(5d+3ae)\sin(x)\right)dx}{5a}-\frac{2}{5}(a+b\cos(x)+c\sin(x))^{3/2}(ce\cos(x)-be\sin(x))$$

$\Big\downarrow$ 3625

$$\frac{\frac{2\int \frac{\left(15da^2+12\left(b^2+c^2\right)ea+5\left(b^2+c^2\right)d\right)a^2+b\left(3ea^2+20da+9\left(b^2+c^2\right)e\right)\cos(x)a^2+c\left(3ea^2+20da+9\left(b^2+c^2\right)e\right)\sin(x)a^2}{2\sqrt{a+b\cos(x)+c\sin(x)}}dx}{3a}-\frac{2}{3}\sqrt{a+b\cos(x)+c\sin(}{5a}\frac{2}{5}(a+b\cos(x)+c\sin(x))^{3/2}(ce\cos(x)-be\sin(x))$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{\left(15da^2+12\left(b^2+c^2\right)ea+5\left(b^2+c^2\right)d\right)a^2+b\left(3ea^2+20da+9\left(b^2+c^2\right)e\right)\cos(x)a^2+c\left(3ea^2+20da+9\left(b^2+c^2\right)e\right)\sin(x)a^2}{\sqrt{a+b\cos(x)+c\sin(x)}}dx}{3a}-\frac{2}{3}\sqrt{a+b\cos(x)+c\sin(x}{5a}\frac{2}{5}(a+b\cos(x)+c\sin(x))^{3/2}(ce\cos(x)-be\sin(x))$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{\left(15da^2+12\left(b^2+c^2\right)ea+5\left(b^2+c^2\right)d\right)a^2+b\left(3ea^2+20da+9\left(b^2+c^2\right)e\right)\cos(x)a^2+c\left(3ea^2+20da+9\left(b^2+c^2\right)e\right)\sin(x)a^2}{\sqrt{a+b\cos(x)+c\sin(x)}}dx}{3a}-\frac{2}{3}\sqrt{a+b\cos(x)+c\sin(x}{5a}\frac{2}{5}(a+b\cos(x)+c\sin(x))^{3/2}(ce\cos(x)-be\sin(x))$$

$\Big\downarrow$ 3628

$$\frac{\frac{a^2\left(3a^2e+20ad+9e\left(b^2+c^2\right)\right)\int\sqrt{a+b\cos(x)+c\sin(x)}dx-a^2\left(a^2-b^2-c^2\right)(3ae+5d)\int\frac{1}{\sqrt{a+b\cos(x)+c\sin(x)}}dx}{3a}-\frac{2}{3}\sqrt{a+b\cos(x)+c\sin(x)}(}{5a}\frac{2}{5}(a+b\cos(x)+c\sin(x))^{3/2}(ce\cos(x)-be\sin(x))$$

$\Big\downarrow$ 3042

$$\frac{\frac{a^2\left(3a^2e+20ad+9e\left(b^2+c^2\right)\right)\int\sqrt{a+b\cos(x)+c\sin(x)}dx-a^2\left(a^2-b^2-c^2\right)(3ae+5d)\int\frac{1}{\sqrt{a+b\cos(x)+c\sin(x)}}dx}{3a}-\frac{2}{3}\sqrt{a+b\cos(x)+c\sin(x)}(}{5a}\frac{2}{5}(a+b\cos(x)+c\sin(x))^{3/2}(ce\cos(x)-be\sin(x))$$

$\Big\downarrow$ 3598

3.557. $\int (a+b\cos(x)+c\sin(x))^{3/2}(d+be\cos(x)+ce\sin(x))\,dx$

$$\frac{\frac{\frac{a^2\left(3a^2e+20ad+9e\left(b^2+c^2\right)\right)\sqrt{a+b\cos(x)+c\sin(x)}\int\sqrt{\frac{a}{a+\sqrt{b^2+c^2}}+\frac{\sqrt{b^2+c^2}\cos\left(x-\tan^{-1}(b,c)\right)}{a+\sqrt{b^2+c^2}}}dx}{\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}-a^2(a^2-b^2-c^2)(3ae+5d)\int\frac{1}{\sqrt{a+b\cos(x)+c\sin(x)}}dx}{3a}-}{5a}$$

$$\frac{2}{5}(a+b\cos(x)+c\sin(x))^{3/2}(ce\cos(x)-be\sin(x))$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{a^2\left(3a^2e+20ad+9e\left(b^2+c^2\right)\right)\sqrt{a+b\cos(x)+c\sin(x)}\int\sqrt{\frac{a}{a+\sqrt{b^2+c^2}}+\frac{\sqrt{b^2+c^2}\sin\left(x-\tan^{-1}(b,c)+\frac{\pi}{2}\right)}{a+\sqrt{b^2+c^2}}}dx}{\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}-a^2(a^2-b^2-c^2)(3ae+5d)\int\frac{1}{\sqrt{a+b\cos(x)+c\sin(x)}}dx}{3a}}{5a}$$

$$\frac{2}{5}(a+b\cos(x)+c\sin(x))^{3/2}(ce\cos(x)-be\sin(x))$$

$\Big\downarrow$ 3132

$$\frac{\frac{\frac{2a^2\left(3a^2e+20ad+9e\left(b^2+c^2\right)\right)\sqrt{a+b\cos(x)+c\sin(x)}E\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}-a^2(a^2-b^2-c^2)(3ae+5d)\int\frac{1}{\sqrt{a+b\cos(x)+c\sin(x)}}dx}{3a}-\frac{2}{3}\sqrt{a+b}}{5a}$$

$$\frac{2}{5}(a+b\cos(x)+c\sin(x))^{3/2}(ce\cos(x)-be\sin(x))$$

$\Big\downarrow$ 3606

$$\frac{\frac{\frac{2a^2\left(3a^2e+20ad+9e\left(b^2+c^2\right)\right)\sqrt{a+b\cos(x)+c\sin(x)}E\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}-\frac{a^2\left(a^2-b^2-c^2\right)(3ae+5d)\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}\int\frac{}{\sqrt{\frac{a}{a+\sqrt{b^2+c^2}}+\frac{\sqrt{b^2}}{}}}}{\sqrt{a+b\cos(x)+c\sin(x)}}}{3a}}{5a}$$

$$\frac{2}{5}(a+b\cos(x)+c\sin(x))^{3/2}(ce\cos(x)-be\sin(x))$$

$\Big\downarrow$ 3042

$$\frac{\frac{\frac{2a^2\left(3a^2e+20ad+9e\left(b^2+c^2\right)\right)\sqrt{a+b\cos(x)+c\sin(x)}E\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}-\frac{a^2\left(a^2-b^2-c^2\right)(3ae+5d)\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}\int\frac{}{\sqrt{\frac{a}{a+\sqrt{b^2+c^2}}+\frac{\sqrt{b^2}}{}}}}{\sqrt{a+b\cos(x)+c\sin(x)}}}{3a}}{5a}$$

$$\frac{2}{5}(a+b\cos(x)+c\sin(x))^{3/2}(ce\cos(x)-be\sin(x))$$

$\Big\downarrow$ 3140

3.557. $\int(a+b\cos(x)+c\sin(x))^{3/2}(d+be\cos(x)+ce\sin(x))\,dx$

$$\frac{\frac{2a^2\left(3a^2e+20ad+9e\left(b^2+c^2\right)\right)\sqrt{a+b\cos(x)+c\sin(x)}E\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}-\frac{2a^2\left(a^2-b^2-c^2\right)(3ae+5d)\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}\text{EllipticF}\left(\frac{1}{2}\left(x-\tan^{-1}\right.\right.}{\sqrt{a+b\cos(x)+c\sin(x)}}}{3a}$$

$$5a$$

$$\frac{2}{5}(a+b\cos(x)+c\sin(x))^{3/2}(ce\cos(x)-be\sin(x))$$

input
```
Int[(a + b*Cos[x] + c*Sin[x])^(3/2)*(d + b*e*Cos[x] + c*e*Sin[x]),x]
```

output
```
(-2*(a + b*Cos[x] + c*Sin[x])^(3/2)*(c*e*Cos[x] - b*e*Sin[x]))/5 + ((-2*Sq
rt[a + b*Cos[x] + c*Sin[x]]*(a*c*(5*d + 3*a*e)*Cos[x] - a*b*(5*d + 3*a*e)*
Sin[x]))/3 + ((2*a^2*(20*a*d + 3*a^2*e + 9*(b^2 + c^2)*e)*EllipticE[(x - A
rcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[a + b*Cos[
x] + c*Sin[x]])/Sqrt[(a + b*Cos[x] + c*Sin[x])/(a + Sqrt[b^2 + c^2])] - (2
*a^2*(a^2 - b^2 - c^2)*(5*d + 3*a*e)*EllipticF[(x - ArcTan[b, c])/2, (2*Sq
rt[b^2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[(a + b*Cos[x] + c*Sin[x])/(a +
Sqrt[b^2 + c^2])])/Sqrt[a + b*Cos[x] + c*Sin[x]])/(3*a))/(5*a)
```

3.557.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3598
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]   Int[Sqrt[a/(a + S
qrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - Arc
Tan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

rule 3606
```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Simp[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]   Int[1/Sqrt[a/(a
 + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x -
 ArcTan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2
, 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

rule 3625
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*((a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^n/(a*e*(n + 1))), x] + Simp[1/(a*(n + 1
))   Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
 + a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
 + (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

rule 3628
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]
, x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[
A*b - a*B, 0]
```

3.557. $\int (a+b\cos(x)+c\sin(x))^{3/2}(d+be\cos(x)+ce\sin(x))\,dx$

3.557.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2204 vs. $2(322) = 644$.

Time = 8.22 (sec) , antiderivative size = 2205, normalized size of antiderivative = 7.50

method	result	size
default	Expression too large to display	2205
parts	Expression too large to display	13403

input
```
int((a+b*cos(x)+c*sin(x))^(3/2)*(d+b*e*cos(x)+c*e*sin(x)),x,method=_RETURN
VERBOSE)
```

output
```
(-(-b^2*sin(x-arctan(-b,c))-c^2*sin(x-arctan(-b,c))-a*(b^2+c^2)^(1/2))*cos
(x-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)/(b^2+c^2)^(1/2)*(2*a^2*d*(b^2+c^
2)^(1/2)*(1/(b^2+c^2)^(1/2)*a+1)*(((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/
(a+(b^2+c^2)^(1/2)))^(1/2)*((sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(-a+(b
^2+c^2)^(1/2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^
2)^(1/2)))^(1/2)/(-(-b^2*sin(x-arctan(-b,c))-c^2*sin(x-arctan(-b,c))-a*(b^
2+c^2)^(1/2))*cos(x-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)*EllipticF((((b^
2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2),((-a-(b^2+c
^2)^(1/2))/(-a+(b^2+c^2)^(1/2)))^(1/2))+(b^4*e+2*b^2*c^2*e+c^4*e)*(-2/5/(b
^2+c^2)^(1/2)*sin(x-arctan(-b,c))*(((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)
*cos(x-arctan(-b,c))^2)^(1/2)+8/15/(b^2+c^2)*a*(((b^2+c^2)^(1/2)*sin(x-arc
tan(-b,c))+a)*cos(x-arctan(-b,c))^2)^(1/2)+4/15/(b^2+c^2)^(1/2)*a*(1/(b^2+
c^2)^(1/2)*a+1)*(((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2
)))^(1/2)*((sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(
1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)/
(((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)*cos(x-arctan(-b,c))^2)^(1/2)*Elli
pticF((((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2),
((-a-(b^2+c^2)^(1/2))/(-a+(b^2+c^2)^(1/2)))^(1/2))+2*(3/5+8/15/(b^2+c^2)*a
^2)*(1/(b^2+c^2)^(1/2)*a+1)*(((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b
^2+c^2)^(1/2)))^(1/2)*((sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(-a+(b^2...
```

3.557.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.16 (sec) , antiderivative size = 1692, normalized size of antiderivative = 5.76

$$\int (a+b\cos(x)+c\sin(x))^{3/2}(d+be\cos(x)+ce\sin(x))\,dx = \text{Too large to display}$$

input
```
integrate((a+b*cos(x)+c*sin(x))^(3/2)*(d+b*e*cos(x)+c*e*sin(x)),x, algorit
hm="fricas")
```

output
```
1/45*(sqrt(2)*(5*I*(a^2*b + 3*b^3 + 3*b*c^2)*d + 5*(3*c^3 + (a^2 + 3*b^2)*
c)*d - 6*I*(a^3*b - 3*a*b^3 - 3*a*b*c^2)*e + 6*(3*a*c^3 - (a^3 - 3*a*b^2)*
c)*e)*sqrt(b + I*c)*weierstrassPInverse(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2*c^2
 + 6*I*b*c^3 + 3*c^4 - 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4), -
8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 - 9*I*a*c^5 + 2*I*(4*a^3 + 9*a*b^2)
*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 - 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b
^4*c^2 + 3*b^2*c^4 + c^6), 1/3*(2*a*b - 2*I*a*c + 3*(b^2 + c^2)*cos(x) - 3
*(I*b^2 + I*c^2)*sin(x))/(b^2 + c^2)) + sqrt(2)*(-5*I*(a^2*b + 3*b^3 + 3*b
*c^2)*d + 5*(3*c^3 + (a^2 + 3*b^2)*c)*d + 6*I*(a^3*b - 3*a*b^3 - 3*a*b*c^2
)*e + 6*(3*a*c^3 - (a^3 - 3*a*b^2)*c)*e)*sqrt(b - I*c)*weierstrassPInverse
(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2*c^2 - 6*I*b*c^3 + 3*c^4 + 2*I*(4*a^2*b - 3
*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4
+ 9*I*a*c^5 - 2*I*(4*a^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 + 3*I*
(8*a^3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), 1/3*(2*a*b +
 2*I*a*c + 3*(b^2 + c^2)*cos(x) - 3*(-I*b^2 - I*c^2)*sin(x))/(b^2 + c^2))
- 3*sqrt(2)*(20*I*(a*b^2 + a*c^2)*d + 3*I*(a^2*b^2 + 3*b^4 + 3*c^4 + (a^2
+ 6*b^2)*c^2)*e)*sqrt(b + I*c)*weierstrassZeta(4/3*(4*a^2*b^2 - 3*b^4 - 4*
a^2*c^2 + 6*I*b*c^3 + 3*c^4 - 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2 +
c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 - 9*I*a*c^5 + 2*I*(4*a^3 + 9
*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 - 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/...
```

3.557.6 Sympy [F]

$$\int (a+b\cos(x)+c\sin(x))^{3/2}(d+be\cos(x)+ce\sin(x))\,dx = \int (a + b\cos{(x)} + c\sin{(x)})^{\frac{3}{2}}\left(be\cos{(x)} + ce\sin{(x)} + d\right)\,dx$$

input
```
integrate((a+b*cos(x)+c*sin(x))**(3/2)*(d+b*e*cos(x)+c*e*sin(x)),x)
```

output
```
Integral((a + b*cos(x) + c*sin(x))**(3/2)*(b*e*cos(x) + c*e*sin(x) + d), x
)
```

3.557.7 Maxima [F]

$$\int (a+b\cos(x)+c\sin(x))^{3/2}(d+be\cos(x)+ce\sin(x))\,dx=\int (be\cos(x)+ce\sin(x)+d)(b\cos(x)+c\sin(x)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cos(x)+c*sin(x))^(3/2)*(d+b*e*cos(x)+c*e*sin(x)),x, algorit
hm="maxima")
```

output
```
integrate((b*e*cos(x) + c*e*sin(x) + d)*(b*cos(x) + c*sin(x) + a)^(3/2), x
)
```

3.557.8 Giac [F]

$$\int (a+b\cos(x)+c\sin(x))^{3/2}(d+be\cos(x)+ce\sin(x))\,dx=\int (be\cos(x)+ce\sin(x)+d)(b\cos(x)+c\sin(x)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cos(x)+c*sin(x))^(3/2)*(d+b*e*cos(x)+c*e*sin(x)),x, algorit
hm="giac")
```

output
```
integrate((b*e*cos(x) + c*e*sin(x) + d)*(b*cos(x) + c*sin(x) + a)^(3/2), x
)
```

3.557.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(x)+c\sin(x))^{3/2}(d+be\cos(x)+ce\sin(x))\,dx=\int \left(a+b\,\cos\left(x\right)+c\,\sin\left(x\right)\right)^{3/2}\,\left(d+b\,e\,\cos\left(x\right)+c\,e\,\sin\left(x\right)\right)\,dx$$

input
```
int((a + b*cos(x) + c*sin(x))^(3/2)*(d + b*e*cos(x) + c*e*sin(x)),x)
```

output
```
int((a + b*cos(x) + c*sin(x))^(3/2)*(d + b*e*cos(x) + c*e*sin(x)), x)
```

3.558 $\int \sqrt{a+b\cos(x)+c\sin(x)}(d+be\cos(x)+ce\sin(x))\,d$

3.558.1 Optimal result

Integrand size = 27, antiderivative size = 229

$$\begin{aligned}\int \sqrt{a+b\cos(x)+c\sin(x)}(d+be\cos(x)+ce\sin(x))\,dx &= \frac{2(3d+ae)E\left(\frac{1}{2}(x-\tan^{-1}(b,c))\,|\,\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)\sqrt{a+b\cos(x)+c\sin(x)}}{3\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}\\ &\quad-\frac{2(a^2-b^2-c^2)\,e\,\text{EllipticF}\left(\frac{1}{2}(x-\tan^{-1}(b,c)),\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}{3\sqrt{a+b\cos(x)+c\sin(x)}}\\ &\quad-\frac{2}{3}\sqrt{a+b\cos(x)+c\sin(x)}(ce\cos(x)-be\sin(x))\end{aligned}$$

output
```
-2/3*(c*e*cos(x)-b*e*sin(x))*(a+b*cos(x)+c*sin(x))^(1/2)+2/3*(a*e+3*d)*(co
s(1/2*x-1/2*arctan(b,c))^2)^(1/2)/cos(1/2*x-1/2*arctan(b,c))*EllipticE(sin
(1/2*x-1/2*arctan(b,c)),2^(1/2)*((b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2
))*(a+b*cos(x)+c*sin(x))^(1/2)/((a+b*cos(x)+c*sin(x))/(a+(b^2+c^2)^(1/2)))
^(1/2)-2/3*(a^2-b^2-c^2)*e*(cos(1/2*x-1/2*arctan(b,c))^2)^(1/2)/cos(1/2*x-
1/2*arctan(b,c))*EllipticF(sin(1/2*x-1/2*arctan(b,c)),2^(1/2)*((b^2+c^2)^(
1/2)/(a+(b^2+c^2)^(1/2)))^(1/2))*((a+b*cos(x)+c*sin(x))/(a+(b^2+c^2)^(1/2)
))^(1/2)/(a+b*cos(x)+c*sin(x))^(1/2)
```

3.558.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 6.27 (sec) , antiderivative size = 3006, normalized size of antiderivative = 13.13

$$\int \sqrt{a+b\cos(x)+c\sin(x)}(d+be\cos(x)+ce\sin(x))\,dx = \text{Result too large to show}$$

input
```
Integrate[Sqrt[a + b*Cos[x] + c*Sin[x]]*(d + b*e*Cos[x] + c*e*Sin[x]),x]
```

output
```
Sqrt[a + b*Cos[x] + c*Sin[x]]*((2*b*(3*d + a*e))/(3*c) - (2*c*e*Cos[x])/3
+ (2*b*e*Sin[x])/3) + (2*a*d*AppellF1[1/2, 1/2, 1/2, 3/2, -((a + Sqrt[1 +
b^2/c^2]*c*Sin[x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c
^2]*c))*c)), -((a + Sqrt[1 + b^2/c^2]*c*Sin[x + ArcTan[b/c]])/(Sqrt[1 + b^
2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c))*c))]*Sec[x + ArcTan[b/c]]*Sqrt[(c*Sq
rt[(b^2 + c^2)/c^2] - c*Sqrt[(b^2 + c^2)/c^2]*Sin[x + ArcTan[b/c]])/(a + c
*Sqrt[(b^2 + c^2)/c^2])]*Sqrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[x + ArcTan[b
/c]]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*Sin[x + ArcT
an[b/c]])/(-a + c*Sqrt[(b^2 + c^2)/c^2])])/(Sqrt[1 + b^2/c^2]*c) + (2*b^2*
e*AppellF1[1/2, 1/2, 1/2, 3/2, -((a + Sqrt[1 + b^2/c^2]*c*Sin[x + ArcTan[b
/c]])/(Sqrt[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1
+ b^2/c^2]*c*Sin[x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^
2/c^2]*c))*c))]*Sec[x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sqr
t[(b^2 + c^2)/c^2]*Sin[x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*Sq
rt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[(b^2 + c
^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*Sin[x + ArcTan[b/c]])/(-a + c*Sqrt[(b^2
 + c^2)/c^2])])/(3*Sqrt[1 + b^2/c^2]*c) + (2*c*e*AppellF1[1/2, 1/2, 1/2, 3
/2, -((a + Sqrt[1 + b^2/c^2]*c*Sin[x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(1
 - a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1 + b^2/c^2]*c*Sin[x + ArcTan
[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b^2/c^2]*c))*c))]*Sec[x + ...
```

3.558.3 Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3042, 3625, 27, 3042, 3628, 3042, 3598, 3042, 3132, 3606, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\cos(x)+c\sin(x)}(be\cos(x)+ce\sin(x)+d)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt{a+b\cos(x)+c\sin(x)}(be\cos(x)+ce\sin(x)+d)dx$$

$$\Big\downarrow \; 3625$$

$$\frac{2\int \frac{a(3ad+(b^2+c^2)e)+ab(3d+ae)\cos(x)+ac(3d+ae)\sin(x)}{2\sqrt{a+b\cos(x)+c\sin(x)}}dx}{3a}-\frac{2}{3}\sqrt{a+b\cos(x)+c\sin(x)}(ce\cos(x)-be\sin(x))$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{a(3ad+(b^2+c^2)e)+ab(3d+ae)\cos(x)+ac(3d+ae)\sin(x)}{\sqrt{a+b\cos(x)+c\sin(x)}}dx}{3a}-\frac{2}{3}\sqrt{a+b\cos(x)+c\sin(x)}(ce\cos(x)-be\sin(x))$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{a(3ad+(b^2+c^2)e)+ab(3d+ae)\cos(x)+ac(3d+ae)\sin(x)}{\sqrt{a+b\cos(x)+c\sin(x)}}dx}{3a}-\frac{2}{3}\sqrt{a+b\cos(x)+c\sin(x)}(ce\cos(x)-be\sin(x))$$

$$\Big\downarrow \; 3628$$

$$\frac{a(ae+3d)\int \sqrt{a+b\cos(x)+c\sin(x)}dx-ae(a^2-b^2-c^2)\int \frac{1}{\sqrt{a+b\cos(x)+c\sin(x)}}dx}{3a}-\frac{2}{3}\sqrt{a+b\cos(x)+c\sin(x)}(ce\cos(x)-be\sin(x))$$

$$\Big\downarrow \; 3042$$

$$\frac{a(ae+3d)\int \sqrt{a+b\cos(x)+c\sin(x)}dx-ae(a^2-b^2-c^2)\int \frac{1}{\sqrt{a+b\cos(x)+c\sin(x)}}dx}{3a}-\frac{2}{3}\sqrt{a+b\cos(x)+c\sin(x)}(ce\cos(x)-be\sin(x))$$

$$\Big\downarrow \; 3598$$

$$\frac{\frac{a(ae+3d)\sqrt{a+b\cos(x)+c\sin(x)}\int \sqrt{\frac{a}{a+\sqrt{b^2+c^2}}+\frac{\sqrt{b^2+c^2}\cos\left(x-\tan^{-1}(b,c)\right)}{a+\sqrt{b^2+c^2}}}dx}{\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}-ae(a^2-b^2-c^2)\int \frac{1}{\sqrt{a+b\cos(x)+c\sin(x)}}dx}{3a}-\frac{2}{3}\sqrt{a+b\cos(x)+c\sin(x)}(ce\cos(x)-be\sin(x))$$

$$\Big\downarrow \; 3042$$

3.558. $\int \sqrt{a+b\cos(x)+c\sin(x)}(d+be\cos(x)+ce\sin(x))\,dx$

$$\frac{\frac{a(ae+3d)\sqrt{a+b\cos(x)+c\sin(x)}\int \sqrt{\frac{a}{a+\sqrt{b^2+c^2}}+\frac{\sqrt{b^2+c^2}\sin\left(x-\tan^{-1}(b,c)+\frac{\pi}{2}\right)}{a+\sqrt{b^2+c^2}}}dx}{\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}-ae\left(a^2-b^2-c^2\right)\int \frac{1}{\sqrt{a+b\cos(x)+c\sin(x)}}dx}{3a}-$$
$$\frac{2}{3}\sqrt{a+b\cos(x)+c\sin(x)}(ce\cos(x)-be\sin(x))$$

$\downarrow$ 3132

$$\frac{\frac{2a(ae+3d)\sqrt{a+b\cos(x)+c\sin(x)}E\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}-ae\left(a^2-b^2-c^2\right)\int \frac{1}{\sqrt{a+b\cos(x)+c\sin(x)}}dx}{3a}-$$
$$\frac{2}{3}\sqrt{a+b\cos(x)+c\sin(x)}(ce\cos(x)-be\sin(x))$$

$\downarrow$ 3606

$$\frac{\frac{2a(ae+3d)\sqrt{a+b\cos(x)+c\sin(x)}E\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}-\frac{ae\left(a^2-b^2-c^2\right)\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}\int \frac{1}{\sqrt{\frac{a}{a+\sqrt{b^2+c^2}}+\frac{\sqrt{b^2+c^2}\cos\left(x-\tan^{-1}(\right.}{a+\sqrt{b^2+c^2}}}}}{\sqrt{a+b\cos(x)+c\sin(x)}}}{3a}$$
$$\frac{2}{3}\sqrt{a+b\cos(x)+c\sin(x)}(ce\cos(x)-be\sin(x))$$

$\downarrow$ 3042

$$\frac{\frac{2a(ae+3d)\sqrt{a+b\cos(x)+c\sin(x)}E\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}-\frac{ae\left(a^2-b^2-c^2\right)\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}\int \frac{1}{\sqrt{\frac{a}{a+\sqrt{b^2+c^2}}+\frac{\sqrt{b^2+c^2}\sin\left(x-\tan^{-1}(\right.}{a+\sqrt{b^2+c^2}}}}}{\sqrt{a+b\cos(x)+c\sin(x)}}}{3a}$$
$$\frac{2}{3}\sqrt{a+b\cos(x)+c\sin(x)}(ce\cos(x)-be\sin(x))$$

$\downarrow$ 3140

$$\frac{\frac{2a(ae+3d)\sqrt{a+b\cos(x)+c\sin(x)}E\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}-\frac{2ae\left(a^2-b^2-c^2\right)\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}\text{EllipticF}\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right),\frac{2\sqrt{b^2+}}{a+\sqrt{b^2-}}\right.}{\sqrt{a+b\cos(x)+c\sin(x)}}}{3a}$$
$$\frac{2}{3}\sqrt{a+b\cos(x)+c\sin(x)}(ce\cos(x)-be\sin(x))$$

input
```
Int[Sqrt[a + b*Cos[x] + c*Sin[x]]*(d + b*e*Cos[x] + c*e*Sin[x]),x]
```

3.558. $\int \sqrt{a+b\cos(x)+c\sin(x)}(d+be\cos(x)+ce\sin(x))\,dx$

output
```
(-2*Sqrt[a + b*Cos[x] + c*Sin[x]]*(c*e*Cos[x] - b*e*Sin[x]))/3 + ((2*a*(3*
d + a*e)*EllipticE[(x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2
 + c^2])]*Sqrt[a + b*Cos[x] + c*Sin[x]])/Sqrt[(a + b*Cos[x] + c*Sin[x])/(a
 + Sqrt[b^2 + c^2])] - (2*a*(a^2 - b^2 - c^2)*e*EllipticF[(x - ArcTan[b, c
])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[(a + b*Cos[x] + c*Si
n[x])/(a + Sqrt[b^2 + c^2])])/Sqrt[a + b*Cos[x] + c*Sin[x]])/(3*a)
```

3.558.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3598
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]   Int[Sqrt[a/(a + S
qrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - Arc
Tan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

rule 3606
```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Simp[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]   Int[1/Sqrt[a/(a
 + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x -
 ArcTan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2
, 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

rule 3625
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_.)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(B*c - b*C - a*C*Cos[d + e*x] + a*B*Sin[d + e*x])*((a
 + b*Cos[d + e*x] + c*Sin[d + e*x])^n/(a*e*(n + 1))), x] + Simp[1/(a*(n + 1
))   Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n - 1)*Simp[a*(b*B + c*C)*n
 + a^2*A*(n + 1) + (n*(a^2*B - B*c^2 + b*c*C) + a*b*A*(n + 1))*Cos[d + e*x]
 + (n*(b*B*c + a^2*C - b^2*C) + a*c*A*(n + 1))*Sin[d + e*x], x], x], x] /;
FreeQ[{a, b, c, d, e, A, B, C}, x] && GtQ[n, 0] && NeQ[a^2 - b^2 - c^2, 0]
```

rule 3628
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]
, x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[
A*b - a*B, 0]
```

3.558.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 1437 vs. 2(261) = 522.

Time = 5.77 (sec) , antiderivative size = 1438, normalized size of antiderivative = 6.28

method	result	size
default	Expression too large to display	1438
parts	Expression too large to display	6201

input
```
int((a+b*cos(x)+c*sin(x))^(1/2)*(d+b*e*cos(x)+c*e*sin(x)),x,method=_RETURN
VERBOSE)
```

output
```
(-(-b^2*sin(x-arctan(-b,c))-c^2*sin(x-arctan(-b,c))-a*(b^2+c^2)^(1/2))*cos
(x-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)/(b^2+c^2)^(1/2)*(2*a*d*(b^2+c^2)
^(1/2)*(1/(b^2+c^2)^(1/2)*a+1)*(((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a
+(b^2+c^2)^(1/2)))^(1/2)*((sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(-a+(b^2
+c^2)^(1/2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)
^(1/2)))^(1/2)/(-(-b^2*sin(x-arctan(-b,c))-c^2*sin(x-arctan(-b,c))-a*(b^2+
c^2)^(1/2))*cos(x-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)*EllipticF((((b^2+
c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2),((-a-(b^2+c^2
)^(1/2))/(-a+(b^2+c^2)^(1/2)))^(1/2))+2*(a*b^2*e+a*c^2*e+b^2*d+c^2*d)*(1/(
b^2+c^2)^(1/2)*a+1)*(((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^
(1/2)))^(1/2)*((sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)
))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1
/2)/(((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)*cos(x-arctan(-b,c))^2)^(1/2)*
((-1/(b^2+c^2)^(1/2)*a+1)*EllipticE((((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+
a)/(a+(b^2+c^2)^(1/2)))^(1/2),((-a-(b^2+c^2)^(1/2))/(-a+(b^2+c^2)^(1/2)))^
(1/2))-EllipticF((((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/
2)))^(1/2),((-a-(b^2+c^2)^(1/2))/(-a+(b^2+c^2)^(1/2)))^(1/2)))+((b^2+c^2)^
(1/2)*b^2*e+(b^2+c^2)^(1/2)*c^2*e)*(-2/3/(b^2+c^2)^(1/2)*(((b^2+c^2)^(1/2)
*sin(x-arctan(-b,c))+a)*cos(x-arctan(-b,c))^2)^(1/2)+2/3*(1/(b^2+c^2)^(1/2
)*a+1)*(((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1...
```

3.558.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.14 (sec) , antiderivative size = 1498, normalized size of antiderivative = 6.54

$$\int \sqrt{a+b\cos(x)+c\sin(x)}(d+be\cos(x)+ce\sin(x))\,dx = \text{Too large to display}$$

input
```
integrate((a+b*cos(x)+c*sin(x))^(1/2)*(d+b*e*cos(x)+c*e*sin(x)),x, algorit
hm="fricas")
```

output
```
1/9*(sqrt(2)*(3*I*a*b*d + 3*a*c*d - I*(2*a^2*b - 3*b^3 - 3*b*c^2)*e + (3*c
^3 - (2*a^2 - 3*b^2)*c)*e)*sqrt(b + I*c)*weierstrassPInverse(4/3*(4*a^2*b^
2 - 3*b^4 - 4*a^2*c^2 + 6*I*b*c^3 + 3*c^4 - 2*I*(4*a^2*b - 3*b^3)*c)/(b^4
+ 2*b^2*c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 - 9*I*a*c^5 +
2*I*(4*a^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 - 3*I*(8*a^3*b^2 - 9
*a*b^4)*c)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), 1/3*(2*a*b - 2*I*a*c + 3*(
b^2 + c^2)*cos(x) - 3*(I*b^2 + I*c^2)*sin(x))/(b^2 + c^2)) + sqrt(2)*(-3*I
*a*b*d + 3*a*c*d + I*(2*a^2*b - 3*b^3 - 3*b*c^2)*e + (3*c^3 - (2*a^2 - 3*b
^2)*c)*e)*sqrt(b - I*c)*weierstrassPInverse(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2
*c^2 - 6*I*b*c^3 + 3*c^4 + 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4
), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 + 9*I*a*c^5 - 2*I*(4*a^3 + 9*a*
b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 + 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6 +
 3*b^4*c^2 + 3*b^2*c^4 + c^6), 1/3*(2*a*b + 2*I*a*c + 3*(b^2 + c^2)*cos(x)
 - 3*(-I*b^2 - I*c^2)*sin(x))/(b^2 + c^2)) - 3*sqrt(2)*(3*I*(b^2 + c^2)*d
+ I*(a*b^2 + a*c^2)*e)*sqrt(b + I*c)*weierstrassZeta(4/3*(4*a^2*b^2 - 3*b^
4 - 4*a^2*c^2 + 6*I*b*c^3 + 3*c^4 - 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*
c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 - 9*I*a*c^5 + 2*I*(4*a
^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 - 3*I*(8*a^3*b^2 - 9*a*b^4)*
c)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), weierstrassPInverse(4/3*(4*a^2*b^2
 - 3*b^4 - 4*a^2*c^2 + 6*I*b*c^3 + 3*c^4 - 2*I*(4*a^2*b - 3*b^3)*c)/(b^...
```

3.558.6 Sympy [F]

$$\int \sqrt{a+b\cos(x)+c\sin(x)}(d+be\cos(x)+ce\sin(x))\,dx$$
$$= \int \sqrt{a+b\cos{(x)}+c\sin{(x)}}(be\cos{(x)}+ce\sin{(x)}+d)\,dx$$

input
```
integrate((a+b*cos(x)+c*sin(x))**(1/2)*(d+b*e*cos(x)+c*e*sin(x)),x)
```

output
```
Integral(sqrt(a + b*cos(x) + c*sin(x))*(b*e*cos(x) + c*e*sin(x) + d), x)
```

3.558.7 Maxima [F]

$$\int \sqrt{a+b\cos(x)+c\sin(x)}(d+be\cos(x)+ce\sin(x))\,dx = \int (be\cos(x)+ce\sin(x)+d)\sqrt{b\cos(x)+c\sin(x)+a}\,dx$$

input
```
integrate((a+b*cos(x)+c*sin(x))^(1/2)*(d+b*e*cos(x)+c*e*sin(x)),x, algorit
hm="maxima")
```

output
```
integrate((b*e*cos(x) + c*e*sin(x) + d)*sqrt(b*cos(x) + c*sin(x) + a), x)
```

3.558.8 Giac [F]

$$\int \sqrt{a+b\cos(x)+c\sin(x)}(d+be\cos(x)+ce\sin(x))\,dx = \int (be\cos(x)+ce\sin(x)+d)\sqrt{b\cos(x)+c\sin(x)+a}\,dx$$

input
```
integrate((a+b*cos(x)+c*sin(x))^(1/2)*(d+b*e*cos(x)+c*e*sin(x)),x, algorit
hm="giac")
```

output
```
integrate((b*e*cos(x) + c*e*sin(x) + d)*sqrt(b*cos(x) + c*sin(x) + a), x)
```

3.558.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\cos(x)+c\sin(x)}(d+be\cos(x)+ce\sin(x))\,dx = \int \sqrt{a+b\,\cos(x)+c\,\sin(x)}\,\left(d+b\,e\,\cos(x)+c\,e\,\sin(x)\right)\,dx$$

input
```
int((a + b*cos(x) + c*sin(x))^(1/2)*(d + b*e*cos(x) + c*e*sin(x)),x)
```

output
```
int((a + b*cos(x) + c*sin(x))^(1/2)*(d + b*e*cos(x) + c*e*sin(x)), x)
```

3.559 $\int \frac{d+be\cos(x)+ce\sin(x)}{\sqrt{a+b\cos(x)+c\sin(x)}}\,dx$

3.559.1 Optimal result

Integrand size = 27, antiderivative size = 180

$$\int \frac{d+be\cos(x)+ce\sin(x)}{\sqrt{a+b\cos(x)+c\sin(x)}}\,dx = \frac{2eE\left(\frac{1}{2}(x-\tan^{-1}(b,c))\,|\,\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)\sqrt{a+b\cos(x)+c\sin(x)}}{\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}} + \frac{2(d-ae)\,\text{EllipticF}\left(\frac{1}{2}(x-\tan^{-1}(b,c)),\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}{\sqrt{a+b\cos(x)+c\sin(x)}}$$

output
```
2*e*(cos(1/2*x-1/2*arctan(b,c))^2)^(1/2)/cos(1/2*x-1/2*arctan(b,c))*Ellipt
icE(sin(1/2*x-1/2*arctan(b,c)),2^(1/2)*((b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)
))^(1/2))*(a+b*cos(x)+c*sin(x))^(1/2)/((a+b*cos(x)+c*sin(x))/(a+(b^2+c^2)^
(1/2)))^(1/2)+2*(-a*e+d)*(cos(1/2*x-1/2*arctan(b,c))^2)^(1/2)/cos(1/2*x-1/
2*arctan(b,c))*EllipticF(sin(1/2*x-1/2*arctan(b,c)),2^(1/2)*((b^2+c^2)^(1/
2)/(a+(b^2+c^2)^(1/2)))^(1/2))*((a+b*cos(x)+c*sin(x))/(a+(b^2+c^2)^(1/2)))
^(1/2)/(a+b*cos(x)+c*sin(x))^(1/2)
```

3.559.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 5.97 (sec) , antiderivative size = 570, normalized size of antiderivative = 3.17

$$\int \frac{d+be\cos(x)+ce\sin(x)}{\sqrt{a+b\cos(x)+c\sin(x)}}\,dx$$

$$= \frac{2d\,\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{3}{2},\frac{a+b\cos(x)+c\sin(x)}{a-\sqrt{1+\frac{b^2}{c^2}}c},\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{1+\frac{b^2}{c^2}}c}\right)\sec\left(x+\arctan\left(\frac{b}{c}\right)\right)\sqrt{\frac{\sqrt{1+\frac{b^2}{c^2}}c-b\cos(x)-c\sin(x)}{a+\sqrt{1+\frac{b^2}{c^2}}c}}\sqrt{a}}{\sqrt{1+\frac{b^2}{c^2}}c}$$
$$-\frac{(b^2+c^2)\,e\,\text{AppellF1}\left(-\frac{1}{2},-\frac{1}{2},-\frac{1}{2},\frac{1}{2},\frac{a+b\cos(x)+c\sin(x)}{a-b\sqrt{1+\frac{c^2}{b^2}}},\frac{a+b\cos(x)+c\sin(x)}{a+b\sqrt{1+\frac{c^2}{b^2}}}\right)\sin\left(x-\arctan\left(\frac{c}{b}\right)\right)}{b\sqrt{1+\frac{c^2}{b^2}}\sqrt{\frac{b\sqrt{1+\frac{c^2}{b^2}}-b\cos(x)-c\sin(x)}{a+b\sqrt{1+\frac{c^2}{b^2}}}}\sqrt{a+b\cos(x)+c\sin(x)}\sqrt{\frac{b\sqrt{1+\frac{c^2}{b^2}}+b\cos(x)+c\sin(x)}{-a+b\sqrt{1+\frac{c^2}{b^2}}}}}$$
$$+\frac{e\left(2b^3\sqrt{1+\frac{c^2}{b^2}}\cos(x)-2b(b^2+c^2)\cos\left(x-\arctan\left(\frac{c}{b}\right)\right)+2b^2c\sqrt{1+\frac{c^2}{b^2}}\sin(x)+b^2c\sin\left(x-\arctan\left(\frac{c}{b}\right)\right)\right.}{bc\sqrt{1+\frac{c^2}{b^2}}\sqrt{a+b\cos(x)+c\sin(x)}}$$

input
```
Integrate[(d + b*e*Cos[x] + c*e*Sin[x])/Sqrt[a + b*Cos[x] + c*Sin[x]],x]
```

output
```
(2*d*AppellF1[1/2, 1/2, 1/2, 3/2, (a + b*Cos[x] + c*Sin[x])/(a - Sqrt[1 +
b^2/c^2]*c), (a + b*Cos[x] + c*Sin[x])/(a + Sqrt[1 + b^2/c^2]*c)]*Sec[x +
ArcTan[b/c]]*Sqrt[(Sqrt[1 + b^2/c^2]*c - b*Cos[x] - c*Sin[x])/(a + Sqrt[1
+ b^2/c^2]*c)]*Sqrt[a + b*Cos[x] + c*Sin[x]]*Sqrt[(b*Cos[x] + c*(Sqrt[1 +
b^2/c^2] + Sin[x]))/(-a + Sqrt[1 + b^2/c^2]*c)])/(Sqrt[1 + b^2/c^2]*c) - (
(b^2 + c^2)*e*AppellF1[-1/2, -1/2, -1/2, 1/2, (a + b*Cos[x] + c*Sin[x])/(a
 - b*Sqrt[1 + c^2/b^2]), (a + b*Cos[x] + c*Sin[x])/(a + b*Sqrt[1 + c^2/b^2
])]*Sin[x - ArcTan[c/b]])/(b*Sqrt[1 + c^2/b^2]*Sqrt[(b*Sqrt[1 + c^2/b^2] -
 b*Cos[x] - c*Sin[x])/(a + b*Sqrt[1 + c^2/b^2])]*Sqrt[a + b*Cos[x] + c*Sin
[x]]*Sqrt[(b*Sqrt[1 + c^2/b^2] + b*Cos[x] + c*Sin[x])/(-a + b*Sqrt[1 + c^2
/b^2])]) + (e*(2*b^3*Sqrt[1 + c^2/b^2]*Cos[x] - 2*b*(b^2 + c^2)*Cos[x - Ar
cTan[c/b]] + 2*b^2*c*Sqrt[1 + c^2/b^2]*Sin[x] + b^2*c*Sin[x - ArcTan[c/b]]
 + c^3*Sin[x - ArcTan[c/b]]))/(b*c*Sqrt[1 + c^2/b^2]*Sqrt[a + b*Cos[x] + c
*Sin[x]])
```

3.559.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3628, 3042, 3598, 3042, 3132, 3606, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{be\cos(x)+ce\sin(x)+d}{\sqrt{a+b\cos(x)+c\sin(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{be\cos(x)+ce\sin(x)+d}{\sqrt{a+b\cos(x)+c\sin(x)}}dx$$

$$\Big\downarrow \text{3628}$$

$$(d-ae)\int \frac{1}{\sqrt{a+b\cos(x)+c\sin(x)}}dx+e\int \sqrt{a+b\cos(x)+c\sin(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$(d-ae)\int \frac{1}{\sqrt{a+b\cos(x)+c\sin(x)}}dx+e\int \sqrt{a+b\cos(x)+c\sin(x)}dx$$

$$\Big\downarrow \text{3598}$$

$$\frac{e\sqrt{a+b\cos(x)+c\sin(x)}\int \sqrt{\frac{a}{a+\sqrt{b^2+c^2}}+\frac{\sqrt{b^2+c^2}\cos(x-\tan^{-1}(b,c))}{a+\sqrt{b^2+c^2}}}dx}{\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}+(d-ae)\int \frac{1}{\sqrt{a+b\cos(x)+c\sin(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{e\sqrt{a+b\cos(x)+c\sin(x)}\int \sqrt{\frac{a}{a+\sqrt{b^2+c^2}}+\frac{\sqrt{b^2+c^2}\sin\left(x-\tan^{-1}(b,c)+\frac{\pi}{2}\right)}{a+\sqrt{b^2+c^2}}}dx}{\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}+(d-ae)\int \frac{1}{\sqrt{a+b\cos(x)+c\sin(x)}}dx$$

$$\Big\downarrow \text{3132}$$

$$(d-ae)\int \frac{1}{\sqrt{a+b\cos(x)+c\sin(x)}}dx+\frac{2e\sqrt{a+b\cos(x)+c\sin(x)}E\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}$$

$$\Big\downarrow \text{3606}$$

$$\frac{(d-ae)\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}\int \frac{1}{\sqrt{\frac{a}{a+\sqrt{b^2+c^2}}+\frac{\sqrt{b^2+c^2}\cos\left(x-\tan^{-1}(b,c)\right)}{a+\sqrt{b^2+c^2}}}}dx}{\sqrt{a+b\cos(x)+c\sin(x)}}+\frac{2e\sqrt{a+b\cos(x)+c\sin(x)}E\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(d-ae)\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}\int \frac{1}{\sqrt{\frac{a}{a+\sqrt{b^2+c^2}}+\frac{\sqrt{b^2+c^2}\sin\left(x-\tan^{-1}(b,c)+\frac{\pi}{2}\right)}{a+\sqrt{b^2+c^2}}}}dx}{\sqrt{a+b\cos(x)+c\sin(x)}}+\frac{2e\sqrt{a+b\cos(x)+c\sin(x)}E\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}$$

$$\Big\downarrow \text{3140}$$

$$\frac{2(d-ae)\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}\,\text{EllipticF}\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right),\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{\sqrt{a+b\cos(x)+c\sin(x)}}+\frac{2e\sqrt{a+b\cos(x)+c\sin(x)}E\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}$$

input
```
Int[(d + b*e*Cos[x] + c*e*Sin[x])/Sqrt[a + b*Cos[x] + c*Sin[x]],x]
```

output
```
(2*e*EllipticE[(x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c
^2])]*Sqrt[a + b*Cos[x] + c*Sin[x]])/Sqrt[(a + b*Cos[x] + c*Sin[x])/(a + S
qrt[b^2 + c^2])] + (2*(d - a*e)*EllipticF[(x - ArcTan[b, c])/2, (2*Sqrt[b^
2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[(a + b*Cos[x] + c*Sin[x])/(a + Sqrt[
b^2 + c^2])])/Sqrt[a + b*Cos[x] + c*Sin[x]]
```

3.559. $\int \frac{d+be\cos(x)+ce\sin(x)}{\sqrt{a+b\cos(x)+c\sin(x)}}\,dx$

3.559.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3598
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]   Int[Sqrt[a/(a + S
qrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - Arc
Tan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

rule 3606
```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Simp[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]   Int[1/Sqrt[a/(a
 + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x -
 ArcTan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2
, 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

rule 3628
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]
, x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[
A*b - a*B, 0]
```

3.559. $\int \frac{d+be\cos(x)+ce\sin(x)}{\sqrt{a+b\cos(x)+c\sin(x)}}\,dx$

3.559.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 765 vs. $2(220) = 440$.

Time = 8.37 (sec) , antiderivative size = 766, normalized size of antiderivative = 4.26

method	result
default	$\frac{\sqrt{-\frac{\left(-b^{2} \sin(x-\arctan(-b,c))-c^{2} \sin(x-\arctan(-b,c))-a\sqrt{b^{2}+c^{2}}\right) \cos(x-\arctan(-b,c))^{2}}{\sqrt{b^{2}+c^{2}}}}\left(\frac{2d\sqrt{b^{2}+c^{2}}\left(\frac{a}{\sqrt{b^{2}+c^{2}}}+1\right)\sqrt{\frac{\sqrt{b^{2}+c^{2}}\sin(x-\arctan(-}{a+\sqrt{b^{2}+c^{2}}}}}{}\right.}{}$
parts	Expression too large to display
risch	Expression too large to display

input
```
int((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(1/2),x,method=_RETURN
VERBOSE)
```

output
```
(-(-b^2*sin(x-arctan(-b,c))-c^2*sin(x-arctan(-b,c))-a*(b^2+c^2)^(1/2))*cos
(x-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)/(b^2+c^2)^(1/2)*(2*d*(b^2+c^2)^(
1/2)*(1/(b^2+c^2)^(1/2)*a+1)*(((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(
b^2+c^2)^(1/2)))^(1/2)*((sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(-a+(b^2+c
^2)^(1/2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(
1/2)))^(1/2)/(-(-b^2*sin(x-arctan(-b,c))-c^2*sin(x-arctan(-b,c))-a*(b^2+c^
2)^(1/2))*cos(x-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)*EllipticF((((b^2+c^
2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2),((-a-(b^2+c^2)^
(1/2))/(-a+(b^2+c^2)^(1/2)))^(1/2))+2*(b^2*e+c^2*e)*(1/(b^2+c^2)^(1/2)*a+1
)*(((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2)*((si
n(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(-a+(b^2+c^2)^(1/2)))^(1/2)*((-sin(x-
arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2)/(((b^2+c^2)^(1
/2)*sin(x-arctan(-b,c))+a)*cos(x-arctan(-b,c))^2)^(1/2)*((-1/(b^2+c^2)^(1/
2)*a+1)*EllipticE((((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^(1
/2)))^(1/2),((-a-(b^2+c^2)^(1/2))/(-a+(b^2+c^2)^(1/2)))^(1/2))-EllipticF((
((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2),((-a-(b
^2+c^2)^(1/2))/(-a+(b^2+c^2)^(1/2)))^(1/2))))/cos(x-arctan(-b,c))/((b^2*si
n(x-arctan(-b,c))+c^2*sin(x-arctan(-b,c))+a*(b^2+c^2)^(1/2))/(b^2+c^2)^(1/
2))^(1/2)
```

3.559.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 1352, normalized size of antiderivative = 7.51

$$\int \frac{d+be\cos(x)+ce\sin(x)}{\sqrt{a+b\cos(x)+c\sin(x)}}\,dx=\text{Too large to display}$$

input
```
integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(1/2),x, algorit
hm="fricas")
```

output
```
1/3*(-3*I*sqrt(2)*(b^2 + c^2)*sqrt(b + I*c)*e*weierstrassZeta(4/3*(4*a^2*b
^2 - 3*b^4 - 4*a^2*c^2 + 6*I*b*c^3 + 3*c^4 - 2*I*(4*a^2*b - 3*b^3)*c)/(b^4
 + 2*b^2*c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 - 9*I*a*c^5 +
 2*I*(4*a^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 - 3*I*(8*a^3*b^2 -
9*a*b^4)*c)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), weierstrassPInverse(4/3*(
4*a^2*b^2 - 3*b^4 - 4*a^2*c^2 + 6*I*b*c^3 + 3*c^4 - 2*I*(4*a^2*b - 3*b^3)*
c)/(b^4 + 2*b^2*c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 - 9*I*
a*c^5 + 2*I*(4*a^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 - 3*I*(8*a^3
*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), 1/3*(2*a*b - 2*I*a
*c + 3*(b^2 + c^2)*cos(x) - 3*(I*b^2 + I*c^2)*sin(x))/(b^2 + c^2))) + 3*I*
sqrt(2)*(b^2 + c^2)*sqrt(b - I*c)*e*weierstrassZeta(4/3*(4*a^2*b^2 - 3*b^4
 - 4*a^2*c^2 - 6*I*b*c^3 + 3*c^4 + 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c
^2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 + 9*I*a*c^5 - 2*I*(4*a^
3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 + 3*I*(8*a^3*b^2 - 9*a*b^4)*c
)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), weierstrassPInverse(4/3*(4*a^2*b^2
- 3*b^4 - 4*a^2*c^2 - 6*I*b*c^3 + 3*c^4 + 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 +
2*b^2*c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 + 9*I*a*c^5 - 2*
I*(4*a^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 + 3*I*(8*a^3*b^2 - 9*a
*b^4)*c)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), 1/3*(2*a*b + 2*I*a*c + 3*(b^
2 + c^2)*cos(x) - 3*(-I*b^2 - I*c^2)*sin(x))/(b^2 + c^2))) + sqrt(2)*(-...
```

3.559.6 Sympy [F]

$$\int \frac{d+be\cos(x)+ce\sin(x)}{\sqrt{a+b\cos(x)+c\sin(x)}}\,dx=\int \frac{be\cos{(x)}+ce\sin{(x)}+d}{\sqrt{a+b\cos{(x)}+c\sin{(x)}}}\,dx$$

input
```
integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))**(1/2),x)
```

output
```
Integral((b*e*cos(x) + c*e*sin(x) + d)/sqrt(a + b*cos(x) + c*sin(x)), x)
```

3.559.7 Maxima [F]

$$\int \frac{d+be\cos(x)+ce\sin(x)}{\sqrt{a+b\cos(x)+c\sin(x)}}\,dx=\int \frac{be\cos\left(x\right)+ce\sin\left(x\right)+d}{\sqrt{b\cos\left(x\right)+c\sin\left(x\right)+a}}\,dx$$

input
```
integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(1/2),x, algorit
hm="maxima")
```

output
```
integrate((b*e*cos(x) + c*e*sin(x) + d)/sqrt(b*cos(x) + c*sin(x) + a), x)
```

3.559.8 Giac [F]

$$\int \frac{d+be\cos(x)+ce\sin(x)}{\sqrt{a+b\cos(x)+c\sin(x)}}\,dx=\int \frac{be\cos\left(x\right)+ce\sin\left(x\right)+d}{\sqrt{b\cos\left(x\right)+c\sin\left(x\right)+a}}\,dx$$

input
```
integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(1/2),x, algorit
hm="giac")
```

output
```
integrate((b*e*cos(x) + c*e*sin(x) + d)/sqrt(b*cos(x) + c*sin(x) + a), x)
```

3.559.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+be\cos(x)+ce\sin(x)}{\sqrt{a+b\cos(x)+c\sin(x)}}\,dx=\int \frac{d+b\,e\,\cos\left(x\right)+c\,e\,\sin\left(x\right)}{\sqrt{a+b\,\cos\left(x\right)+c\,\sin\left(x\right)}}\,dx$$

input
```
int((d + b*e*cos(x) + c*e*sin(x))/(a + b*cos(x) + c*sin(x))^(1/2),x)
```

output
```
int((d + b*e*cos(x) + c*e*sin(x))/(a + b*cos(x) + c*sin(x))^(1/2), x)
```

3.560 $\int \frac{d+be\cos(x)+ce\sin(x)}{(a+b\cos(x)+c\sin(x))^{3/2}}\,dx$

3.560.1 Optimal result

Integrand size = 27, antiderivative size = 250

$$\int \frac{d+be\cos(x)+ce\sin(x)}{(a+b\cos(x)+c\sin(x))^{3/2}}\,dx=\frac{2(d-ae)E\left(\frac{1}{2}(x-\tan^{-1}(b,c))\,|\,\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)\sqrt{a+b\cos(x)+c\sin(x)}}{(a^2-b^2-c^2)\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}$$
$$+\frac{2e\,\text{EllipticF}\left(\frac{1}{2}(x-\tan^{-1}(b,c)),\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}{\sqrt{a+b\cos(x)+c\sin(x)}}$$
$$+\frac{2(c(d-ae)\cos(x)-b(d-ae)\sin(x))}{(a^2-b^2-c^2)\sqrt{a+b\cos(x)+c\sin(x)}}$$

output
```
2*(c*(-a*e+d)*cos(x)-b*(-a*e+d)*sin(x))/(a^2-b^2-c^2)/(a+b*cos(x)+c*sin(x)
)^(1/2)+2*(-a*e+d)*(cos(1/2*x-1/2*arctan(b,c))^2)^(1/2)/cos(1/2*x-1/2*arct
an(b,c))*EllipticE(sin(1/2*x-1/2*arctan(b,c)),2^(1/2)*((b^2+c^2)^(1/2)/(a+
(b^2+c^2)^(1/2)))^(1/2))*(a+b*cos(x)+c*sin(x))^(1/2)/(a^2-b^2-c^2)/((a+b*c
os(x)+c*sin(x))/(a+(b^2+c^2)^(1/2)))^(1/2)+2*e*(cos(1/2*x-1/2*arctan(b,c))
^2)^(1/2)/cos(1/2*x-1/2*arctan(b,c))*EllipticF(sin(1/2*x-1/2*arctan(b,c)),
2^(1/2)*((b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2))*((a+b*cos(x)+c*sin(x)
)/(a+(b^2+c^2)^(1/2)))^(1/2)/(a+b*cos(x)+c*sin(x))^(1/2)
```

3.560.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 6.45 (sec) , antiderivative size = 3176, normalized size of antiderivative = 12.70

$$\int \frac{d+be\cos(x)+ce\sin(x)}{(a+b\cos(x)+c\sin(x))^{3/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(d + b*e*Cos[x] + c*e*Sin[x])/(a + b*Cos[x] + c*Sin[x])^(3/2),x]
```

output
```
Sqrt[a + b*Cos[x] + c*Sin[x]]*((2*(b^2 + c^2)*(-d + a*e))/(b*c*(-a^2 + b^2
 + c^2)) - (2*(-(a*c*d) + a^2*c*e - b^2*d*Sin[x] - c^2*d*Sin[x] + a*b^2*e*
Sin[x] + a*c^2*e*Sin[x]))/(b*(-a^2 + b^2 + c^2)*(a + b*Cos[x] + c*Sin[x]))
) - (2*a*d*AppellF1[1/2, 1/2, 1/2, 3/2, -((a + Sqrt[1 + b^2/c^2]*c*Sin[x +
 ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a
+ Sqrt[1 + b^2/c^2]*c*Sin[x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sq
rt[1 + b^2/c^2]*c))*c))]*Sec[x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2
] - c*Sqrt[(b^2 + c^2)/c^2]*Sin[x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/
c^2])]*Sqrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[x + ArcTan[b/c]]]*Sqrt[(c*Sqrt
[(b^2 + c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*Sin[x + ArcTan[b/c]])/(-a + c*
Sqrt[(b^2 + c^2)/c^2])])/(Sqrt[1 + b^2/c^2]*c*(-a^2 + b^2 + c^2)) + (2*b^2
*e*AppellF1[1/2, 1/2, 1/2, 3/2, -((a + Sqrt[1 + b^2/c^2]*c*Sin[x + ArcTan[
b/c]])/(Sqrt[1 + b^2/c^2]*(1 - a/(Sqrt[1 + b^2/c^2]*c))*c)), -((a + Sqrt[1
 + b^2/c^2]*c*Sin[x + ArcTan[b/c]])/(Sqrt[1 + b^2/c^2]*(-1 - a/(Sqrt[1 + b
^2/c^2]*c))*c))]*Sec[x + ArcTan[b/c]]*Sqrt[(c*Sqrt[(b^2 + c^2)/c^2] - c*Sq
rt[(b^2 + c^2)/c^2]*Sin[x + ArcTan[b/c]])/(a + c*Sqrt[(b^2 + c^2)/c^2])]*S
qrt[a + c*Sqrt[(b^2 + c^2)/c^2]*Sin[x + ArcTan[b/c]]]*Sqrt[(c*Sqrt[(b^2 +
c^2)/c^2] + c*Sqrt[(b^2 + c^2)/c^2]*Sin[x + ArcTan[b/c]])/(-a + c*Sqrt[(b^
2 + c^2)/c^2])])/(Sqrt[1 + b^2/c^2]*c*(-a^2 + b^2 + c^2)) + (2*c*e*AppellF
1[1/2, 1/2, 1/2, 3/2, -((a + Sqrt[1 + b^2/c^2]*c*Sin[x + ArcTan[b/c]])/...
```

3.560.3 Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3042, 3635, 27, 3042, 3628, 3042, 3598, 3042, 3132, 3606, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{be\cos(x)+ce\sin(x)+d}{(a+b\cos(x)+c\sin(x))^{3/2}}dx$$

$\big\downarrow$ 3042

$$\int \frac{be\cos(x)+ce\sin(x)+d}{(a+b\cos(x)+c\sin(x))^{3/2}}dx$$

$\big\downarrow$ 3635

$$\frac{2(c\cos(x)(d-ae)-b\sin(x)(d-ae))}{(a^2-b^2-c^2)\sqrt{a+b\cos(x)+c\sin(x)}}-\frac{2\int -\frac{ad-(b^2+c^2)e+b(d-ae)\cos(x)+c(d-ae)\sin(x)}{2\sqrt{a+b\cos(x)+c\sin(x)}}dx}{a^2-b^2-c^2}$$

$\big\downarrow$ 27

$$\frac{\int \frac{ad-(b^2+c^2)e+b(d-ae)\cos(x)+c(d-ae)\sin(x)}{\sqrt{a+b\cos(x)+c\sin(x)}}dx}{a^2-b^2-c^2}+\frac{2(c\cos(x)(d-ae)-b\sin(x)(d-ae))}{(a^2-b^2-c^2)\sqrt{a+b\cos(x)+c\sin(x)}}$$

$\big\downarrow$ 3042

$$\frac{\int \frac{ad-(b^2+c^2)e+b(d-ae)\cos(x)+c(d-ae)\sin(x)}{\sqrt{a+b\cos(x)+c\sin(x)}}dx}{a^2-b^2-c^2}+\frac{2(c\cos(x)(d-ae)-b\sin(x)(d-ae))}{(a^2-b^2-c^2)\sqrt{a+b\cos(x)+c\sin(x)}}$$

$\big\downarrow$ 3628

$$\frac{e(a^2-b^2-c^2)\int \frac{1}{\sqrt{a+b\cos(x)+c\sin(x)}}dx+(d-ae)\int \sqrt{a+b\cos(x)+c\sin(x)}dx}{a^2-b^2-c^2}+\frac{2(c\cos(x)(d-ae)-b\sin(x)(d-ae))}{(a^2-b^2-c^2)\sqrt{a+b\cos(x)+c\sin(x)}}$$

$\big\downarrow$ 3042

$$\frac{e(a^2-b^2-c^2)\int \frac{1}{\sqrt{a+b\cos(x)+c\sin(x)}}dx+(d-ae)\int \sqrt{a+b\cos(x)+c\sin(x)}dx}{a^2-b^2-c^2}+\frac{2(c\cos(x)(d-ae)-b\sin(x)(d-ae))}{(a^2-b^2-c^2)\sqrt{a+b\cos(x)+c\sin(x)}}$$

$\big\downarrow$ 3598

$$\frac{e(a^2-b^2-c^2)\int \frac{1}{\sqrt{a+b\cos(x)+c\sin(x)}}dx+\frac{(d-ae)\sqrt{a+b\cos(x)+c\sin(x)}\int \sqrt{\frac{a}{a+\sqrt{b^2+c^2}}+\frac{\sqrt{b^2+c^2}\cos\left(x-\tan^{-1}(b,c)\right)}{a+\sqrt{b^2+c^2}}}dx}{\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}}{a^2-b^2-c^2}+\frac{2(c\cos(x)(d-ae)-b\sin(x)(d-ae))}{(a^2-b^2-c^2)\sqrt{a+b\cos(x)+c\sin(x)}}$$

$\big\downarrow$ 3042

3.560. $\int \frac{d+be\cos(x)+ce\sin(x)}{(a+b\cos(x)+c\sin(x))^{3/2}}\,dx$

$$\frac{e\left(a^2-b^2-c^2\right)\int\frac{1}{\sqrt{a+b\cos(x)+c\sin(x)}}dx+\frac{(d-ae)\sqrt{a+b\cos(x)+c\sin(x)}\int\sqrt{\frac{a}{a+\sqrt{b^2+c^2}}+\frac{\sqrt{b^2+c^2}\sin\left(x-\tan^{-1}(b,c)+\frac{\pi}{2}\right)}{a+\sqrt{b^2+c^2}}}dx}{\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}}{a^2-b^2-c^2}+\frac{2(c\cos(x)(d-ae)-b\sin(x)(d-ae))}{\left(a^2-b^2-c^2\right)\sqrt{a+b\cos(x)+c\sin(x)}}$$

$$\Big\downarrow \; 3132$$

$$\frac{e\left(a^2-b^2-c^2\right)\int\frac{1}{\sqrt{a+b\cos(x)+c\sin(x)}}dx+\frac{2(d-ae)\sqrt{a+b\cos(x)+c\sin(x)}E\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}}{a^2-b^2-c^2}+\frac{2(c\cos(x)(d-ae)-b\sin(x)(d-ae))}{\left(a^2-b^2-c^2\right)\sqrt{a+b\cos(x)+c\sin(x)}}$$

$$\Big\downarrow \; 3606$$

$$\frac{\frac{e\left(a^2-b^2-c^2\right)\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}\int\frac{1}{\sqrt{\frac{a}{a+\sqrt{b^2+c^2}}+\frac{\sqrt{b^2+c^2}\cos\left(x-\tan^{-1}(b,c)\right)}{a+\sqrt{b^2+c^2}}}}dx}{\sqrt{a+b\cos(x)+c\sin(x)}}+\frac{2(d-ae)\sqrt{a+b\cos(x)+c\sin(x)}E\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}}{a^2-b^2-c^2}+\frac{2(c\cos(x)(d-ae)-b\sin(x)(d-ae))}{\left(a^2-b^2-c^2\right)\sqrt{a+b\cos(x)+c\sin(x)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{e\left(a^2-b^2-c^2\right)\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}\int\frac{1}{\sqrt{\frac{a}{a+\sqrt{b^2+c^2}}+\frac{\sqrt{b^2+c^2}\sin\left(x-\tan^{-1}(b,c)+\frac{\pi}{2}\right)}{a+\sqrt{b^2+c^2}}}}dx}{\sqrt{a+b\cos(x)+c\sin(x)}}+\frac{2(d-ae)\sqrt{a+b\cos(x)+c\sin(x)}E\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}}{a^2-b^2-c^2}+\frac{2(c\cos(x)(d-ae)-b\sin(x)(d-ae))}{\left(a^2-b^2-c^2\right)\sqrt{a+b\cos(x)+c\sin(x)}}$$

$$\Big\downarrow \; 3140$$

$$\frac{\frac{2e\left(a^2-b^2-c^2\right)\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}\operatorname{EllipticF}\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right),\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{\sqrt{a+b\cos(x)+c\sin(x)}}+\frac{2(d-ae)\sqrt{a+b\cos(x)+c\sin(x)}E\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}}{a^2-b^2-c^2}+\frac{2(c\cos(x)(d-ae)-b\sin(x)(d-ae))}{\left(a^2-b^2-c^2\right)\sqrt{a+b\cos(x)+c\sin(x)}}$$

3.560. $\int\frac{d+be\cos(x)+ce\sin(x)}{(a+b\cos(x)+c\sin(x))^{3/2}}\,dx$

input
```
Int[(d + b*e*Cos[x] + c*e*Sin[x])/(a + b*Cos[x] + c*Sin[x])^(3/2),x]
```

output
```
(2*(c*(d - a*e)*Cos[x] - b*(d - a*e)*Sin[x]))/((a^2 - b^2 - c^2)*Sqrt[a +
b*Cos[x] + c*Sin[x]]) + ((2*(d - a*e)*EllipticE[(x - ArcTan[b, c])/2, (2*S
qrt[b^2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[a + b*Cos[x] + c*Sin[x]])/Sqrt
[(a + b*Cos[x] + c*Sin[x])/(a + Sqrt[b^2 + c^2])] + (2*(a^2 - b^2 - c^2)*e
*EllipticF[(x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c^2])
]*Sqrt[(a + b*Cos[x] + c*Sin[x])/(a + Sqrt[b^2 + c^2])])/Sqrt[a + b*Cos[x]
 + c*Sin[x]])/(a^2 - b^2 - c^2)
```

3.560.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3598
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]   Int[Sqrt[a/(a + S
qrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - Arc
Tan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

rule 3606
```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Simp[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]   Int[1/Sqrt[a/(a
 + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x -
 ArcTan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2
, 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

rule 3628
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]
, x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[
A*b - a*B, 0]
```

rule 3635
```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(-(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(
a^2 - b^2 - c^2))), x] + Simp[1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a + b*Co
s[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n +
2)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x]
/; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[n, -2]
```

3.560.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2910 vs. $2(288) = 576$.

Time = 28.98 (sec) , antiderivative size = 2911, normalized size of antiderivative = 11.64

method	result	size
default	Expression too large to display	2911
parts	Expression too large to display	113315

input
```
int((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(3/2),x,method=_RETURN
VERBOSE)
```

output
```
(-(-b^2*sin(x-arctan(-b,c))-c^2*sin(x-arctan(-b,c))-a*(b^2+c^2)^(1/2))*cos
(x-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)/(b^2+c^2)^(1/2)*(2*(b^2+c^2)^(1/
2)*e*(1/(b^2+c^2)^(1/2)*a+1)*(((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(
b^2+c^2)^(1/2)))^(1/2)*((sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(-a+(b^2+c
^2)^(1/2)))^(1/2)*((-sin(x-arctan(-b,c))+1)*(b^2+c^2)^(1/2)/(a+(b^2+c^2)^(
1/2)))^(1/2)/(-(-b^2*sin(x-arctan(-b,c))-c^2*sin(x-arctan(-b,c))-a*(b^2+c^
2)^(1/2))*cos(x-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)*EllipticF((((b^2+c^
2)^(1/2)*sin(x-arctan(-b,c))+a)/(a+(b^2+c^2)^(1/2)))^(1/2),((-a-(b^2+c^2)^
(1/2))/(-a+(b^2+c^2)^(1/2)))^(1/2))+(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2
)*sin(x-arctan(-b,c))+a)*(b^2+c^2))^(1/2)*(sin(x-arctan(-b,c))^2*b^2+sin(x
-arctan(-b,c))^2*c^2-a^2)*(b^2*sin(x-arctan(-b,c))+c^2*sin(x-arctan(-b,c))
+a*(b^2+c^2)^(1/2))/((((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)*cos(x-arctan
(-b,c))^2)^(1/2)*a*b^2+(((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)*cos(x-arct
an(-b,c))^2)^(1/2)*a*c^2-(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arc
tan(-b,c))+a)*(b^2+c^2))^(1/2)*sin(x-arctan(-b,c))*b^2-(cos(x-arctan(-b,c)
)^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)*(b^2+c^2))^(1/2)*sin(x-arctan(
-b,c))*c^2)/(sin(x-arctan(-b,c))^2*b^2+sin(x-arctan(-b,c))^2*c^2+2*(b^2+c^
2)^(1/2)*sin(x-arctan(-b,c))*a+a^2)*((b^2+c^2)*cos(x-arctan(-b,c))^2*(a*e-
d)/(a^2-b^2-c^2)/(((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)*cos(x-arctan(-b,
c))^2)^(1/2)+a*(b^2+c^2)^(1/2)*(a*e-d)/(a^2-b^2-c^2)*(1/(b^2+c^2)^(1/2)...
```

3.560.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.17 (sec) , antiderivative size = 2086, normalized size of antiderivative = 8.34

$$\int \frac{d + be\cos(x) + ce\sin(x)}{(a + b\cos(x) + c\sin(x))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(3/2),x, algorit
hm="fricas")
```

output
```
-1/3*((sqrt(2)*(-I*a*b^2*d - a*b*c*d - I*(2*a^2*b^2 - 3*b^4 - 3*b^2*c^2)*e
 + (3*b*c^3 - (2*a^2*b - 3*b^3)*c)*e)*cos(x) + sqrt(2)*(-I*a*b*c*d - a*c^2
*d + I*(3*b*c^3 - (2*a^2*b - 3*b^3)*c)*e + (3*c^4 - (2*a^2 - 3*b^2)*c^2)*e
)*sin(x) + sqrt(2)*(-I*a^2*b*d - a^2*c*d - I*(2*a^3*b - 3*a*b^3 - 3*a*b*c^
2)*e + (3*a*c^3 - (2*a^3 - 3*a*b^2)*c)*e))*sqrt(b + I*c)*weierstrassPInver
se(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2*c^2 + 6*I*b*c^3 + 3*c^4 - 2*I*(4*a^2*b -
 3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^
4 - 9*I*a*c^5 + 2*I*(4*a^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 - 3*
I*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), 1/3*(2*a*b
 - 2*I*a*c + 3*(b^2 + c^2)*cos(x) - 3*(I*b^2 + I*c^2)*sin(x))/(b^2 + c^2))
 + (sqrt(2)*(I*a*b^2*d - a*b*c*d + I*(2*a^2*b^2 - 3*b^4 - 3*b^2*c^2)*e + (
3*b*c^3 - (2*a^2*b - 3*b^3)*c)*e)*cos(x) + sqrt(2)*(I*a*b*c*d - a*c^2*d -
I*(3*b*c^3 - (2*a^2*b - 3*b^3)*c)*e + (3*c^4 - (2*a^2 - 3*b^2)*c^2)*e)*sin
(x) + sqrt(2)*(I*a^2*b*d - a^2*c*d + I*(2*a^3*b - 3*a*b^3 - 3*a*b*c^2)*e +
 (3*a*c^3 - (2*a^3 - 3*a*b^2)*c)*e))*sqrt(b - I*c)*weierstrassPInverse(4/3
*(4*a^2*b^2 - 3*b^4 - 4*a^2*c^2 - 6*I*b*c^3 + 3*c^4 + 2*I*(4*a^2*b - 3*b^3
)*c)/(b^4 + 2*b^2*c^2 + c^4), -8/27*(8*a^3*b^3 - 9*a*b^5 + 27*a*b*c^4 + 9*
I*a*c^5 - 2*I*(4*a^3 + 9*a*b^2)*c^3 - 6*(4*a^3*b - 3*a*b^3)*c^2 + 3*I*(8*a
^3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^4*c^2 + 3*b^2*c^4 + c^6), 1/3*(2*a*b + 2*I
*a*c + 3*(b^2 + c^2)*cos(x) - 3*(-I*b^2 - I*c^2)*sin(x))/(b^2 + c^2)) -...
```

3.560.6 Sympy [F(-1)]

Timed out.

$$\int \frac{d + be\cos(x) + ce\sin(x)}{(a + b\cos(x) + c\sin(x))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))**(3/2),x)
```

output
```
Timed out
```

3.560.7 Maxima [F]

$$\int \frac{d+be\cos(x)+ce\sin(x)}{(a+b\cos(x)+c\sin(x))^{3/2}}\,dx=\int \frac{be\cos{(x)}+ce\sin{(x)}+d}{(b\cos{(x)}+c\sin{(x)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(3/2),x, algorit
hm="maxima")
```

output
```
integrate((b*e*cos(x) + c*e*sin(x) + d)/(b*cos(x) + c*sin(x) + a)^(3/2), x
)
```

3.560.8 Giac [F]

$$\int \frac{d+be\cos(x)+ce\sin(x)}{(a+b\cos(x)+c\sin(x))^{3/2}}\,dx=\int \frac{be\cos{(x)}+ce\sin{(x)}+d}{(b\cos{(x)}+c\sin{(x)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(3/2),x, algorit
hm="giac")
```

output
```
integrate((b*e*cos(x) + c*e*sin(x) + d)/(b*cos(x) + c*sin(x) + a)^(3/2), x
)
```

3.560.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d+be\cos(x)+ce\sin(x)}{(a+b\cos(x)+c\sin(x))^{3/2}}\,dx=\int \frac{d+b\,e\,\cos{(x)}+c\,e\,\sin{(x)}}{(a+b\,\cos{(x)}+c\,\sin{(x)})^{3/2}}\,dx$$

input
```
int((d + b*e*cos(x) + c*e*sin(x))/(a + b*cos(x) + c*sin(x))^(3/2),x)
```

output
```
int((d + b*e*cos(x) + c*e*sin(x))/(a + b*cos(x) + c*sin(x))^(3/2), x)
```

3.561 $\int \frac{d+be\cos(x)+ce\sin(x)}{(a+b\cos(x)+c\sin(x))^{5/2}}\,dx$

3.561.1 Optimal result

Integrand size = 27, antiderivative size = 378

$$\int \frac{d+be\cos(x)+ce\sin(x)}{(a+b\cos(x)+c\sin(x))^{5/2}}\,dx = \frac{2(4ad-a^2e-3(b^2+c^2)e)\,E\left(\frac{1}{2}(x-\tan^{-1}(b,c))\,|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)\sqrt{a+b\cos}}{3(a^2-b^2-c^2)^2\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}$$
$$-\frac{2(d-ae)\operatorname{EllipticF}\left(\frac{1}{2}(x-\tan^{-1}(b,c)),\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}{3(a^2-b^2-c^2)\sqrt{a+b\cos(x)+c\sin(x)}}$$
$$+\frac{2(c(d-ae)\cos(x)-b(d-ae)\sin(x))}{3(a^2-b^2-c^2)(a+b\cos(x)+c\sin(x))^{3/2}}$$
$$+\frac{2(c(4ad-a^2e-3(b^2+c^2)e)\cos(x)-b(4ad-a^2e-3(b^2+c^2)e)\sin(x))}{3(a^2-b^2-c^2)^2\sqrt{a+b\cos(x)+c\sin(x)}}$$

output
```
2/3*(c*(-a*e+d)*cos(x)-b*(-a*e+d)*sin(x))/(a^2-b^2-c^2)/(a+b*cos(x)+c*sin(
x))^(3/2)+2/3*(c*(4*a*d-a^2*e-3*(b^2+c^2)*e)*cos(x)-b*(4*a*d-a^2*e-3*(b^2+
c^2)*e)*sin(x))/(a^2-b^2-c^2)^2/(a+b*cos(x)+c*sin(x))^(1/2)+2/3*(4*a*d-a^2
*e-3*(b^2+c^2)*e)*(cos(1/2*x-1/2*arctan(b,c))^2)^(1/2)/cos(1/2*x-1/2*arcta
n(b,c))*EllipticE(sin(1/2*x-1/2*arctan(b,c)),2^(1/2)*((b^2+c^2)^(1/2)/(a+(
b^2+c^2)^(1/2)))^(1/2))*(a+b*cos(x)+c*sin(x))^(1/2)/(a^2-b^2-c^2)^2/((a+b*
cos(x)+c*sin(x))/(a+(b^2+c^2)^(1/2)))^(1/2)-2/3*(-a*e+d)*(cos(1/2*x-1/2*ar
ctan(b,c))^2)^(1/2)/cos(1/2*x-1/2*arctan(b,c))*EllipticF(sin(1/2*x-1/2*arc
tan(b,c)),2^(1/2)*((b^2+c^2)^(1/2)/(a+(b^2+c^2)^(1/2)))^(1/2))*((a+b*cos(x
)+c*sin(x))/(a+(b^2+c^2)^(1/2)))^(1/2)/(a^2-b^2-c^2)/(a+b*cos(x)+c*sin(x))
^(1/2)
```

3.561.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 4 in optimal.

Time = 6.73 (sec) , antiderivative size = 5554, normalized size of antiderivative = 14.69

$$\int \frac{d+be\cos(x)+ce\sin(x)}{(a+b\cos(x)+c\sin(x))^{5/2}}\,dx = \text{Result too large to show}$$

input
```
Integrate[(d + b*e*Cos[x] + c*e*Sin[x])/(a + b*Cos[x] + c*Sin[x])^(5/2),x]
```

output
```
Result too large to show
```

3.561.3 Rubi [A] (verified)

Time = 1.74 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.04, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {3042, 3635, 27, 3042, 3635, 27, 3042, 3628, 3042, 3598, 3042, 3132, 3606, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{be\cos(x)+ce\sin(x)+d}{(a+b\cos(x)+c\sin(x))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{be\cos(x)+ce\sin(x)+d}{(a+b\cos(x)+c\sin(x))^{5/2}}dx$$

$$\Big\downarrow \text{3635}$$

$$\frac{2(c\cos(x)(d-ae)-b\sin(x)(d-ae))}{3(a^2-b^2-c^2)(a+b\cos(x)+c\sin(x))^{3/2}}-\frac{2\int -\frac{3(ad-(b^2+c^2)e)-b(d-ae)\cos(x)-c(d-ae)\sin(x)}{2(a+b\cos(x)+c\sin(x))^{3/2}}dx}{3(a^2-b^2-c^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{3(ad-(b^2+c^2)e)-b(d-ae)\cos(x)-c(d-ae)\sin(x)}{(a+b\cos(x)+c\sin(x))^{3/2}}dx}{3(a^2-b^2-c^2)}+\frac{2(c\cos(x)(d-ae)-b\sin(x)(d-ae))}{3(a^2-b^2-c^2)(a+b\cos(x)+c\sin(x))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{3(ad-(b^2+c^2)e)-b(d-ae)\cos(x)-c(d-ae)\sin(x)}{(a+b\cos(x)+c\sin(x))^{3/2}}dx}{3(a^2-b^2-c^2)}+\frac{2(c\cos(x)(d-ae)-b\sin(x)(d-ae))}{3(a^2-b^2-c^2)(a+b\cos(x)+c\sin(x))^{3/2}}$$

$$\Big\downarrow \text{3635}$$

$$\frac{\frac{2\left(c\cos(x)\left(a^2(-e)+4ad-3e\left(b^2+c^2\right)\right)-b\sin(x)\left(a^2(-e)+4ad-3e\left(b^2+c^2\right)\right)\right)}{\left(a^2-b^2-c^2\right)\sqrt{a+b\cos(x)+c\sin(x)}}-\frac{2\int -\frac{3da^2-4\left(b^2+c^2\right)ea+\left(b^2+c^2\right)d+b\left(-ea^2+4da-3\left(b^2+c^2\right)e\right)\cos(x)+}{2\sqrt{a+b\cos(x)+c\sin(x)}}}{a^2-b^2-c^2}}{3\left(a^2-b^2-c^2\right)}$$
$$\frac{2(c\cos(x)(d-ae)-b\sin(x)(d-ae))}{3\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{3da^2-4\left(b^2+c^2\right)ea+\left(b^2+c^2\right)d+b\left(-ea^2+4da-3\left(b^2+c^2\right)e\right)\cos(x)+c\left(-ea^2+4da-3\left(b^2+c^2\right)e\right)\sin(x)}{\sqrt{a+b\cos(x)+c\sin(x)}}dx}{a^2-b^2-c^2}+\frac{2\left(c\cos(x)\left(a^2(-e)+4ad-3e\left(b^2+c^2\right)\right)-b\sin(a}{\left(a^2-b^2-c^2\right)\sqrt{a+b\cos(x)}}}{3\left(a^2-b^2-c^2\right)}$$
$$\frac{2(c\cos(x)(d-ae)-b\sin(x)(d-ae))}{3\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{3da^2-4\left(b^2+c^2\right)ea+\left(b^2+c^2\right)d+b\left(-ea^2+4da-3\left(b^2+c^2\right)e\right)\cos(x)+c\left(-ea^2+4da-3\left(b^2+c^2\right)e\right)\sin(x)}{\sqrt{a+b\cos(x)+c\sin(x)}}dx}{a^2-b^2-c^2}+\frac{2\left(c\cos(x)\left(a^2(-e)+4ad-3e\left(b^2+c^2\right)\right)-b\sin(a}{\left(a^2-b^2-c^2\right)\sqrt{a+b\cos(x)}}}{3\left(a^2-b^2-c^2\right)}$$
$$\frac{2(c\cos(x)(d-ae)-b\sin(x)(d-ae))}{3\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^{3/2}}$$

$$\Big\downarrow \text{3628}$$

$$\frac{\frac{\left(a^2(-e)+4ad-3e\left(b^2+c^2\right)\right)\int \sqrt{a+b\cos(x)+c\sin(x)}dx-\left(a^2-b^2-c^2\right)(d-ae)\int \frac{1}{\sqrt{a+b\cos(x)+c\sin(x)}}dx}{a^2-b^2-c^2}+\frac{2\left(c\cos(x)\left(a^2(-e)+4ad-3e\left(b^2+c^2\right)\right)-b}{\left(a^2-b^2-c^2\right)\sqrt{a+b c}}}{3\left(a^2-b^2-c^2\right)}$$
$$\frac{2(c\cos(x)(d-ae)-b\sin(x)(d-ae))}{3\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\left(a^2(-e)+4ad-3e\left(b^2+c^2\right)\right)\int \sqrt{a+b\cos(x)+c\sin(x)}dx-\left(a^2-b^2-c^2\right)(d-ae)\int \frac{1}{\sqrt{a+b\cos(x)+c\sin(x)}}dx}{a^2-b^2-c^2}+\frac{2\left(c\cos(x)\left(a^2(-e)+4ad-3e\left(b^2+c^2\right)\right)-b}{\left(a^2-b^2-c^2\right)\sqrt{a+b c}}}{3\left(a^2-b^2-c^2\right)}$$
$$\frac{2(c\cos(x)(d-ae)-b\sin(x)(d-ae))}{3\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^{3/2}}$$

$$\Big\downarrow \text{3598}$$

3.561. $\int \frac{d+be\cos(x)+ce\sin(x)}{(a+b\cos(x)+c\sin(x))^{5/2}}\,dx$

$$\frac{\frac{\frac{\left(a^2(-e)+4ad-3e\left(b^2+c^2\right)\right)\sqrt{a+b\cos(x)+c\sin(x)}\int\sqrt{\frac{a}{a+\sqrt{b^2+c^2}}+\frac{\sqrt{b^2+c^2}\cos\left(x-\tan^{-1}(b,c)\right)}{a+\sqrt{b^2+c^2}}}dx}{\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}-\left(a^2-b^2-c^2\right)(d-ae)\int\frac{1}{\sqrt{a+b\cos(x)+c\sin(x)}}dx}{a^2-b^2-c^2}+\frac{2(c c\cdots}{}}{3\left(a^2-b^2-c^2\right)}$$

$$\frac{2(c\cos(x)(d-ae)-b\sin(x)(d-ae))}{3\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{\left(a^2(-e)+4ad-3e\left(b^2+c^2\right)\right)\sqrt{a+b\cos(x)+c\sin(x)}\int\sqrt{\frac{a}{a+\sqrt{b^2+c^2}}+\frac{\sqrt{b^2+c^2}\sin\left(x-\tan^{-1}(b,c)+\frac{\pi}{2}\right)}{a+\sqrt{b^2+c^2}}}dx}{\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}-\left(a^2-b^2-c^2\right)(d-ae)\int\frac{1}{\sqrt{a+b\cos(x)+c\sin(x)}}dx}{a^2-b^2-c^2}+\frac{2\cdots}{}}{3\left(a^2-b^2-c^2\right)}$$

$$\frac{2(c\cos(x)(d-ae)-b\sin(x)(d-ae))}{3\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^{3/2}}$$

$$\Big\downarrow \; 3132$$

$$\frac{\frac{\frac{2\left(a^2(-e)+4ad-3e\left(b^2+c^2\right)\right)\sqrt{a+b\cos(x)+c\sin(x)}E\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}-\left(a^2-b^2-c^2\right)(d-ae)\int\frac{1}{\sqrt{a+b\cos(x)+c\sin(x)}}dx}{a^2-b^2-c^2}+\frac{2(c\cos(x)(a^2(-\cdots}{}}{3\left(a^2-b^2-c^2\right)}$$

$$\frac{2(c\cos(x)(d-ae)-b\sin(x)(d-ae))}{3\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^{3/2}}$$

$$\Big\downarrow \; 3606$$

$$\frac{\frac{\frac{2\left(a^2(-e)+4ad-3e\left(b^2+c^2\right)\right)\sqrt{a+b\cos(x)+c\sin(x)}E\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}-\frac{\left(a^2-b^2-c^2\right)(d-ae)\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}\int\frac{1}{\sqrt{\frac{a}{a+\sqrt{b^2+c^2}}+\frac{\sqrt{b^2+c^2}\cos\cdots}{a+\cdots}}}}{\sqrt{a+b\cos(x)+c\sin(x)}}}{a^2-b^2-c^2}\cdots}{3\left(a^2-b^2-c^2\right)}$$

$$\frac{2(c\cos(x)(d-ae)-b\sin(x)(d-ae))}{3\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^{3/2}}$$

$$\Big\downarrow \; 3042$$

3.561. $\int\frac{d+be\cos(x)+ce\sin(x)}{(a+b\cos(x)+c\sin(x))^{5/2}}\,dx$

$$\frac{\frac{\frac{2\left(a^2(-e)+4ad-3e\left(b^2+c^2\right)\right)\sqrt{a+b\cos(x)+c\sin(x)}E\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}-\frac{\left(a^2-b^2-c^2\right)(d-ae)\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}\int\frac{1}{\sqrt{\frac{a}{a+\sqrt{b^2+c^2}}+\frac{\sqrt{b^2+c^2}\sin\dots}{\dots}}}}{\sqrt{a+b\cos(x)+c\sin(x)}}}{a^2-b^2-c^2}}{3\left(a^2-b^2-c^2\right)}$$

$$\frac{2(c\cos(x)(d-ae)-b\sin(x)(d-ae))}{3\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^{3/2}}$$

$\downarrow$ 3140

$$\frac{\frac{\frac{2\left(a^2(-e)+4ad-3e\left(b^2+c^2\right)\right)\sqrt{a+b\cos(x)+c\sin(x)}E\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)|\frac{2\sqrt{b^2+c^2}}{a+\sqrt{b^2+c^2}}\right)}{\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}}-\frac{2\left(a^2-b^2-c^2\right)(d-ae)\sqrt{\frac{a+b\cos(x)+c\sin(x)}{a+\sqrt{b^2+c^2}}}\text{EllipticF}\left(\frac{1}{2}\left(x-\tan^{-1}(b,c)\right)\dots\right)}{\sqrt{a+b\cos(x)+c\sin(x)}}}{a^2-b^2-c^2}}{3\left(a^2-b^2-c^2\right)}$$

$$\frac{2(c\cos(x)(d-ae)-b\sin(x)(d-ae))}{3\left(a^2-b^2-c^2\right)(a+b\cos(x)+c\sin(x))^{3/2}}$$

input
```
Int[(d + b*e*Cos[x] + c*e*Sin[x])/(a + b*Cos[x] + c*Sin[x])^(5/2),x]
```

output
```
(2*(c*(d - a*e)*Cos[x] - b*(d - a*e)*Sin[x]))/(3*(a^2 - b^2 - c^2)*(a + b*
Cos[x] + c*Sin[x])^(3/2)) + ((2*(c*(4*a*d - a^2*e - 3*(b^2 + c^2)*e)*Cos[x
] - b*(4*a*d - a^2*e - 3*(b^2 + c^2)*e)*Sin[x]))/((a^2 - b^2 - c^2)*Sqrt[a
 + b*Cos[x] + c*Sin[x]]) + ((2*(4*a*d - a^2*e - 3*(b^2 + c^2)*e)*EllipticE
[(x - ArcTan[b, c])/2, (2*Sqrt[b^2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[a +
 b*Cos[x] + c*Sin[x]])/Sqrt[(a + b*Cos[x] + c*Sin[x])/(a + Sqrt[b^2 + c^2]
)] - (2*(a^2 - b^2 - c^2)*(d - a*e)*EllipticF[(x - ArcTan[b, c])/2, (2*Sqr
t[b^2 + c^2])/(a + Sqrt[b^2 + c^2])]*Sqrt[(a + b*Cos[x] + c*Sin[x])/(a + S
qrt[b^2 + c^2])])/Sqrt[a + b*Cos[x] + c*Sin[x]])/(a^2 - b^2 - c^2))/(3*(a^
2 - b^2 - c^2))
```

3.561.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.561. $\int \frac{d+be\cos(x)+ce\sin(x)}{(a+b\cos(x)+c\sin(x))^{5/2}}\,dx$

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3598
```
Int[Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_
)]], x_Symbol] :> Simp[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]/Sqrt[(a +
b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sqrt[b^2 + c^2])]   Int[Sqrt[a/(a + S
qrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x - Arc
Tan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
 && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

rule 3606
```
Int[1/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(
x_)]], x_Symbol] :> Simp[Sqrt[(a + b*Cos[d + e*x] + c*Sin[d + e*x])/(a + Sq
rt[b^2 + c^2])]/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]   Int[1/Sqrt[a/(a
 + Sqrt[b^2 + c^2]) + (Sqrt[b^2 + c^2]/(a + Sqrt[b^2 + c^2]))*Cos[d + e*x -
 ArcTan[b, c]]], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2
, 0] && NeQ[b^2 + c^2, 0] &&  !GtQ[a + Sqrt[b^2 + c^2], 0]
```

rule 3628
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/Sqrt[cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)]]
, x_Symbol] :> Simp[B/b   Int[Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]], x]
, x] + Simp[(A*b - a*B)/b   Int[1/Sqrt[a + b*Cos[d + e*x] + c*Sin[d + e*x]]
, x], x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && EqQ[B*c - b*C, 0] && NeQ[
A*b - a*B, 0]
```

3.561. $\int \frac{d+be\cos(x)+ce\sin(x)}{(a+b\cos(x)+c\sin(x))^{5/2}}\,dx$

rule 3635
```
Int[((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])
^(n_)*((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)
]), x_Symbol] :> Simp[(-(c*B - b*C - (a*C - c*A)*Cos[d + e*x] + (a*B - b*A)
*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(
a^2 - b^2 - c^2))), x] + Simp[1/((n + 1)*(a^2 - b^2 - c^2))   Int[(a + b*Co
s[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)*(a*A - b*B - c*C) + (n +
2)*(a*B - b*A)*Cos[d + e*x] + (n + 2)*(a*C - c*A)*Sin[d + e*x], x], x], x]
/; FreeQ[{a, b, c, d, e, A, B, C}, x] && LtQ[n, -1] && NeQ[a^2 - b^2 - c^2,
 0] && NeQ[n, -2]
```

3.561.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 4019 vs. $2(406) = 812$.

Time = 142.43 (sec) , antiderivative size = 4020, normalized size of antiderivative = 10.63

method	result	size
default	Expression too large to display	4020
parts	Expression too large to display	1032654

input
```
int((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(5/2),x,method=_RETURN
VERBOSE)
```

output
```
-(-(-b^2*sin(x-arctan(-b,c))-c^2*sin(x-arctan(-b,c))-a*(b^2+c^2)^(1/2))*co
s(x-arctan(-b,c))^2/(b^2+c^2)^(1/2))^(1/2)/(b^2+c^2)^(1/2)*(b^4*sin(x-arct
an(-b,c))^4+2*b^2*c^2*sin(x-arctan(-b,c))^4+c^4*sin(x-arctan(-b,c))^4-2*a^
2*b^2*sin(x-arctan(-b,c))^2-2*a^2*c^2*sin(x-arctan(-b,c))^2+a^4)*(cos(x-ar
ctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)*(b^2+c^2))^(1/2)*(si
n(x-arctan(-b,c))^2*b^2*e+sin(x-arctan(-b,c))^2*c^2*e+e*a*sin(x-arctan(-b,
c))*(b^2+c^2)^(1/2)+d*sin(x-arctan(-b,c))*(b^2+c^2)^(1/2)+a*d)/(2*(((b^2+c
^2)^(1/2)*sin(x-arctan(-b,c))+a)*cos(x-arctan(-b,c))^2)^(1/2)*sin(x-arctan
(-b,c))^2*a*b^2*e+2*(((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)*cos(x-arctan(
-b,c))^2)^(1/2)*sin(x-arctan(-b,c))^2*a*c^2*e-(cos(x-arctan(-b,c))^2*((b^2
+c^2)^(1/2)*sin(x-arctan(-b,c))+a)*(b^2+c^2))^(1/2)*sin(x-arctan(-b,c))^3*
b^2*e-(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)*(b^2+
c^2))^(1/2)*sin(x-arctan(-b,c))^3*c^2*e-(((b^2+c^2)^(1/2)*sin(x-arctan(-b,
c))+a)*cos(x-arctan(-b,c))^2)^(1/2)*sin(x-arctan(-b,c))^2*b^2*d-(((b^2+c^2
)^(1/2)*sin(x-arctan(-b,c))+a)*cos(x-arctan(-b,c))^2)^(1/2)*sin(x-arctan(-
b,c))^2*c^2*d-e*a^2*sin(x-arctan(-b,c))*(cos(x-arctan(-b,c))^2*((b^2+c^2)^
(1/2)*sin(x-arctan(-b,c))+a)*(b^2+c^2))^(1/2)-d*a^2*(((b^2+c^2)^(1/2)*sin(
x-arctan(-b,c))+a)*cos(x-arctan(-b,c))^2)^(1/2)+2*d*a*sin(x-arctan(-b,c))*
(cos(x-arctan(-b,c))^2*((b^2+c^2)^(1/2)*sin(x-arctan(-b,c))+a)*(b^2+c^2))^
(1/2))/(b^4*sin(x-arctan(-b,c))^3+2*b^2*c^2*sin(x-arctan(-b,c))^3+c^4*s...
```

3.561.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.37 (sec) , antiderivative size = 4057, normalized size of antiderivative = 10.73

$$\int \frac{d + be\cos(x) + ce\sin(x)}{(a + b\cos(x) + c\sin(x))^{5/2}}\, dx = \text{Too large to display}$$

input
```
integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(5/2),x, algorit
hm="fricas")
```

output
```
1/9*((sqrt(2)*(I*(a^2*b^3 + 3*b^5 - a^2*b*c^2 - 3*b*c^4)*d - (a^2*c^3 + 3*
c^5 - (a^2*b^2 + 3*b^4)*c)*d + 2*I*(a^3*b^3 - 3*a*b^5 - a^3*b*c^2 + 3*a*b*
c^4)*e - 2*(a^3*c^3 - 3*a*c^5 - (a^3*b^2 - 3*a*b^4)*c)*e)*cos(x)^2 - 2*sqr
t(2)*(-I*(a^3*b^2 + 3*a*b^4 + 3*a*b^2*c^2)*d - (3*a*b*c^3 + (a^3*b + 3*a*b
^3)*c)*d - 2*I*(a^4*b^2 - 3*a^2*b^4 - 3*a^2*b^2*c^2)*e + 2*(3*a^2*b*c^3 -
(a^4*b - 3*a^2*b^3)*c)*e)*cos(x) - 2*(sqrt(2)*(-I*(3*b^2*c^3 + (a^2*b^2 +
3*b^4)*c)*d - (3*b*c^4 + (a^2*b + 3*b^3)*c^2)*d + 2*I*(3*a*b^2*c^3 - (a^3*
b^2 - 3*a*b^4)*c)*e + 2*(3*a*b*c^4 - (a^3*b - 3*a*b^3)*c^2)*e)*cos(x) + sq
rt(2)*(-I*(3*a*b*c^3 + (a^3*b + 3*a*b^3)*c)*d - (3*a*c^4 + (a^3 + 3*a*b^2)
*c^2)*d + 2*I*(3*a^2*b*c^3 - (a^4*b - 3*a^2*b^3)*c)*e + 2*(3*a^2*c^4 - (a^
4 - 3*a^2*b^2)*c^2)*e))*sin(x) + sqrt(2)*(I*(a^4*b + 3*a^2*b^3 + 3*b*c^4 +
 (4*a^2*b + 3*b^3)*c^2)*d + (3*c^5 + (4*a^2 + 3*b^2)*c^3 + (a^4 + 3*a^2*b^
2)*c)*d + 2*I*(a^5*b - 3*a^3*b^3 - 3*a*b*c^4 - (2*a^3*b + 3*a*b^3)*c^2)*e
- 2*(3*a*c^5 + (2*a^3 + 3*a*b^2)*c^3 - (a^5 - 3*a^3*b^2)*c)*e))*sqrt(b + I
*c)*weierstrassPInverse(4/3*(4*a^2*b^2 - 3*b^4 - 4*a^2*c^2 + 6*I*b*c^3 + 3
*c^4 - 2*I*(4*a^2*b - 3*b^3)*c)/(b^4 + 2*b^2*c^2 + c^4), -8/27*(8*a^3*b^3
- 9*a*b^5 + 27*a*b*c^4 - 9*I*a*c^5 + 2*I*(4*a^3 + 9*a*b^2)*c^3 - 6*(4*a^3*
b - 3*a*b^3)*c^2 - 3*I*(8*a^3*b^2 - 9*a*b^4)*c)/(b^6 + 3*b^4*c^2 + 3*b^2*c
^4 + c^6), 1/3*(2*a*b - 2*I*a*c + 3*(b^2 + c^2)*cos(x) - 3*(I*b^2 + I*c^2)
*sin(x))/(b^2 + c^2)) + (sqrt(2)*(-I*(a^2*b^3 + 3*b^5 - a^2*b*c^2 - 3*b...
```

3.561.6 Sympy [F(-1)]

Timed out.

$$\int \frac{d+be\cos(x)+ce\sin(x)}{(a+b\cos(x)+c\sin(x))^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))**(5/2),x)
```

output
```
Timed out
```

3.561.7 Maxima [F]

$$\int \frac{d + be\cos(x) + ce\sin(x)}{(a + b\cos(x) + c\sin(x))^{5/2}} \, dx = \int \frac{be\cos(x) + ce\sin(x) + d}{(b\cos(x) + c\sin(x) + a)^{\frac{5}{2}}} \, dx$$

input
```
integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(5/2),x, algorit
hm="maxima")
```

output
```
integrate((b*e*cos(x) + c*e*sin(x) + d)/(b*cos(x) + c*sin(x) + a)^(5/2), x
)
```

3.561.8 Giac [F]

$$\int \frac{d + be\cos(x) + ce\sin(x)}{(a + b\cos(x) + c\sin(x))^{5/2}} \, dx = \int \frac{be\cos(x) + ce\sin(x) + d}{(b\cos(x) + c\sin(x) + a)^{\frac{5}{2}}} \, dx$$

input
```
integrate((d+b*e*cos(x)+c*e*sin(x))/(a+b*cos(x)+c*sin(x))^(5/2),x, algorit
hm="giac")
```

output
```
integrate((b*e*cos(x) + c*e*sin(x) + d)/(b*cos(x) + c*sin(x) + a)^(5/2), x
)
```

3.561.9 Mupad [F(-1)]

Timed out.

$$\int \frac{d + be\cos(x) + ce\sin(x)}{(a + b\cos(x) + c\sin(x))^{5/2}} \, dx = \int \frac{d + b\,e\,\cos(x) + c\,e\,\sin(x)}{(a + b\,\cos(x) + c\,\sin(x))^{5/2}} \, dx$$

input
```
int((d + b*e*cos(x) + c*e*sin(x))/(a + b*cos(x) + c*sin(x))^(5/2),x)
```

output
```
int((d + b*e*cos(x) + c*e*sin(x))/(a + b*cos(x) + c*sin(x))^(5/2), x)
```

3.562 $\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{a+c\sin(d+ex)}\,dx$

3.562.1 Optimal result

Integrand size = 31, antiderivative size = 84

$$\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{a+c\sin(d+ex)}\,dx = \frac{Cx}{c}+\frac{2(Ac-aC)\arctan\left(\frac{c+a\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-c^2}}\right)}{c\sqrt{a^2-c^2}e}+\frac{B\log(a+c\sin(d+ex))}{ce}$$

output
```
C*x/c+B*ln(a+c*sin(e*x+d))/c/e+2*(A*c-C*a)*arctan((c+a*tan(1/2*e*x+1/2*d))
/(a^2-c^2)^(1/2))/c/e/(a^2-c^2)^(1/2)
```

3.562.2 Mathematica [A] (verified)

Time = 3.67 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.95

$$\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{a+c\sin(d+ex)}\,dx = \frac{C(d+ex)+\frac{2(Ac-aC)\arctan\left(\frac{c+a\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-c^2}}\right)}{\sqrt{a^2-c^2}}+B\log(a+c\sin(d+ex))}{ce}$$

input
```
Integrate[(A + B*Cos[d + e*x] + C*Sin[d + e*x])/(a + c*Sin[d + e*x]),x]
```

output
```
(C*(d + e*x) + (2*(A*c - a*C)*ArcTan[(c + a*Tan[(d + e*x)/2])/Sqrt[a^2 - c
^2]])/Sqrt[a^2 - c^2] + B*Log[a + c*Sin[d + e*x]])/(c*e)
```

3.562.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {3042, 4876, 3042, 3147, 16, 3214, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{a+c\sin(d+ex)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{a+c\sin(d+ex)}dx$$

$$\Big\downarrow \text{4876}$$

$$\int \frac{A+C\sin(d+ex)}{a+c\sin(d+ex)}dx+B\int \frac{\cos(d+ex)}{a+c\sin(d+ex)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+C\sin(d+ex)}{a+c\sin(d+ex)}dx+B\int \frac{\cos(d+ex)}{a+c\sin(d+ex)}dx$$

$$\Big\downarrow \text{3147}$$

$$\int \frac{A+C\sin(d+ex)}{a+c\sin(d+ex)}dx+\frac{B\int \frac{1}{a+c\sin(d+ex)}d(c\sin(d+ex))}{ce}$$

$$\Big\downarrow \text{16}$$

$$\int \frac{A+C\sin(d+ex)}{a+c\sin(d+ex)}dx+\frac{B\log(a+c\sin(d+ex))}{ce}$$

$$\Big\downarrow \text{3214}$$

$$\frac{(Ac-aC)\int \frac{1}{a+c\sin(d+ex)}dx}{c}+\frac{B\log(a+c\sin(d+ex))}{ce}+\frac{Cx}{c}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(Ac-aC)\int \frac{1}{a+c\sin(d+ex)}dx}{c}+\frac{B\log(a+c\sin(d+ex))}{ce}+\frac{Cx}{c}$$

$$\Big\downarrow \text{3139}$$

$$\frac{2(Ac-aC)\int \frac{1}{a\tan^2\left(\frac{1}{2}(d+ex)\right)+2c\tan\left(\frac{1}{2}(d+ex)\right)+a}d\tan\left(\frac{1}{2}(d+ex)\right)}{ce}+\frac{B\log(a+c\sin(d+ex))}{ce}+\frac{Cx}{c}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{4(Ac-aC)\int \frac{1}{-\left(2c+2a\tan\left(\frac{1}{2}(d+ex)\right)\right)^2-4(a^2-c^2)}d\left(2c+2a\tan\left(\frac{1}{2}(d+ex)\right)\right)}{ce}+\frac{B\log(a+c\sin(d+ex))}{ce}+\frac{Cx}{c}$$

$$\Big\downarrow \text{217}$$

$$\frac{2(Ac-aC)\arctan\left(\frac{2a\tan\left(\frac{1}{2}(d+ex)\right)+2c}{2\sqrt{a^2-c^2}}\right)}{ce\sqrt{a^2-c^2}}+\frac{B\log(a+c\sin(d+ex))}{ce}+\frac{Cx}{c}$$

input
```
Int[(A + B*Cos[d + e*x] + C*Sin[d + e*x])/(a + c*Sin[d + e*x]),x]
```

output
```
(C*x)/c + (2*(A*c - a*C)*ArcTan[(2*c + 2*a*Tan[(d + e*x)/2])/(2*Sqrt[a^2 -
 c^2])])/(c*Sqrt[a^2 - c^2]*e) + (B*Log[a + c*Sin[d + e*x]])/(c*e)
```

3.562.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

3.562. $\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{a+c\sin(d+ex)}\,dx$

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 4876
```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Sin[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Simp[d   Int[ActivateTrig[u]*Cos[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Sin[
c*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Intege
rQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

3.562.4 Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.14

method	result
parts	$\frac{\frac{2(cA-Ca)\arctan\left(\frac{2a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)+2c}{2\sqrt{a^2-c^2}}\right)}{c\sqrt{a^2-c^2}}+\frac{2C\arctan\left(\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}{c}}{e}+\frac{B\ln(a+c\sin(ex+d))}{ce}$
derivativedivides	$\frac{\frac{-B\ln\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2\right)+2C\arctan\left(\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}{c}+\frac{B\ln\left(\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2a+2c\tan\left(\frac{ex}{2}+\frac{d}{2}\right)+a\right)+\frac{2(cA-Ca)\arctan\left(\frac{2a\tan\left(\frac{ex}{2}+\frac{c}{2}\right.}{2\sqrt{a^2-c}}\right.}{\sqrt{a^2-c^2}}}{c}}{e}$
default	$\frac{\frac{-B\ln\left(1+\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2\right)+2C\arctan\left(\tan\left(\frac{ex}{2}+\frac{d}{2}\right)\right)}{c}+\frac{B\ln\left(\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^2a+2c\tan\left(\frac{ex}{2}+\frac{d}{2}\right)+a\right)+\frac{2(cA-Ca)\arctan\left(\frac{2a\tan\left(\frac{ex}{2}+\frac{c}{2}\right.}{2\sqrt{a^2-c}}\right.}{\sqrt{a^2-c^2}}}{c}}{e}$
risch	$\frac{ixB}{c}+\frac{Cx}{c}-\frac{2iB\,a^2ce^2x}{a^2c^2e^2-c^4e^2}+\frac{2iB\,c^3e^2x}{a^2c^2e^2-c^4e^2}-\frac{2iB\,a^2cde}{a^2c^2e^2-c^4e^2}+\frac{2iB\,c^3de}{a^2c^2e^2-c^4e^2}+\frac{\ln\left(e^{i(ex+d)}+\frac{iaAc-iC\,a^2+\sqrt{-A}}{}\right.}{}$

input
```
int((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d)),x,method=_RETURNVERBOSE
)
```

output
```
1/e*(2*(A*c-C*a)/c/(a^2-c^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*e*x+1/2*d)+2*c)
/(a^2-c^2)^(1/2))+2*C/c*arctan(tan(1/2*e*x+1/2*d)))+B*ln(a+c*sin(e*x+d))/c
/e
```

3.562.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 346, normalized size of antiderivative = 4.12

$$\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{a+c\sin(d+ex)}\,dx$$
$$= \left[\frac{2\left(Ca^2-Cc^2\right)ex+(Ca-Ac)\sqrt{-a^2+c^2}\log\left(\frac{(2\,a^2-c^2)\cos(ex+d)^2-2\,ac\sin(ex+d)-a^2-c^2+2\,(a\cos(ex+d)\sin(ex+d)+c\,c}{c^2\cos(ex+d)^2-2\,ac\sin(ex+d)-a^2-c^2}\right.}{2\left(a^2c-c^3\right)e}\right.$$

input
```
integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d)),x, algorithm="fri
cas")
```

output
```
[1/2*(2*(C*a^2 - C*c^2)*e*x + (C*a - A*c)*sqrt(-a^2 + c^2)*log(((2*a^2 - c
^2)*cos(e*x + d)^2 - 2*a*c*sin(e*x + d) - a^2 - c^2 + 2*(a*cos(e*x + d)*si
n(e*x + d) + c*cos(e*x + d))*sqrt(-a^2 + c^2))/(c^2*cos(e*x + d)^2 - 2*a*c
*sin(e*x + d) - a^2 - c^2)) + (B*a^2 - B*c^2)*log(-c^2*cos(e*x + d)^2 + 2*
a*c*sin(e*x + d) + a^2 + c^2))/((a^2*c - c^3)*e), 1/2*(2*(C*a^2 - C*c^2)*e
*x + 2*(C*a - A*c)*sqrt(a^2 - c^2)*arctan(-(a*sin(e*x + d) + c)/(sqrt(a^2
- c^2)*cos(e*x + d))) + (B*a^2 - B*c^2)*log(-c^2*cos(e*x + d)^2 + 2*a*c*si
n(e*x + d) + a^2 + c^2))/((a^2*c - c^3)*e)]
```

3.562.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1110 vs. 2(70) = 140.

Time = 13.61 (sec) , antiderivative size = 1110, normalized size of antiderivative = 13.21

$$\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{a+c\sin(d+ex)}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d)),x)
```

output
```
Piecewise((zoo*x*(A + B*cos(d) + C*sin(d))/sin(d), Eq(a, 0) & Eq(c, 0) & E
q(e, 0)), ((A*log(tan(d/2 + e*x/2))/e - B*log(tan(d/2 + e*x/2)**2 + 1)/e +
 B*log(tan(d/2 + e*x/2))/e + C*x)/c, Eq(a, 0)), (2*A/(c*e*tan(d/2 + e*x/2)
 - c*e) + 2*B*log(tan(d/2 + e*x/2) - 1)*tan(d/2 + e*x/2)/(c*e*tan(d/2 + e*
x/2) - c*e) - 2*B*log(tan(d/2 + e*x/2) - 1)/(c*e*tan(d/2 + e*x/2) - c*e) -
 B*log(tan(d/2 + e*x/2)**2 + 1)*tan(d/2 + e*x/2)/(c*e*tan(d/2 + e*x/2) - c
*e) + B*log(tan(d/2 + e*x/2)**2 + 1)/(c*e*tan(d/2 + e*x/2) - c*e) + C*e*x*
tan(d/2 + e*x/2)/(c*e*tan(d/2 + e*x/2) - c*e) - C*e*x/(c*e*tan(d/2 + e*x/2
) - c*e) + 2*C/(c*e*tan(d/2 + e*x/2) - c*e), Eq(a, -c)), (-2*A/(c*e*tan(d/
2 + e*x/2) + c*e) + 2*B*log(tan(d/2 + e*x/2) + 1)*tan(d/2 + e*x/2)/(c*e*ta
n(d/2 + e*x/2) + c*e) + 2*B*log(tan(d/2 + e*x/2) + 1)/(c*e*tan(d/2 + e*x/2
) + c*e) - B*log(tan(d/2 + e*x/2)**2 + 1)*tan(d/2 + e*x/2)/(c*e*tan(d/2 +
e*x/2) + c*e) - B*log(tan(d/2 + e*x/2)**2 + 1)/(c*e*tan(d/2 + e*x/2) + c*e
) + C*e*x*tan(d/2 + e*x/2)/(c*e*tan(d/2 + e*x/2) + c*e) + C*e*x/(c*e*tan(d
/2 + e*x/2) + c*e) + 2*C/(c*e*tan(d/2 + e*x/2) + c*e), Eq(a, c)), ((A*x +
B*sin(d + e*x)/e - C*cos(d + e*x)/e)/a, Eq(c, 0)), (x*(A + B*cos(d) + C*si
n(d))/(a + c*sin(d)), Eq(e, 0)), (-A*c*sqrt(-a**2 + c**2)*log(tan(d/2 + e*
x/2) + c/a - sqrt(-a**2 + c**2)/a)/(a**2*c*e - c**3*e) + A*c*sqrt(-a**2 +
c**2)*log(tan(d/2 + e*x/2) + c/a + sqrt(-a**2 + c**2)/a)/(a**2*c*e - c**3*
e) - B*a**2*log(tan(d/2 + e*x/2)**2 + 1)/(a**2*c*e - c**3*e) + B*a**2*l...
```

3.562.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\cos(d + ex) + C\sin(d + ex)}{a + c\sin(d + ex)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d)),x, algorithm="max
ima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*a^2>0)', see `assume?` f
or more de
```

3.562.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.62

$$\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{a+c\sin(d+ex)}\,dx$$
$$= \frac{\frac{(ex+d)C}{c} + \frac{B\log\left(a\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^2+2c\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+a\right)}{c} - \frac{B\log\left(\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^2+1\right)}{c} - \frac{2\left(\pi\left\lfloor\frac{ex+d}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}ex+\right.}{\sqrt{a^2-\right.}}\right.}{\sqrt{a^2-c^2}c}}{e}$$

input
```
integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d)),x, algorithm="gia
c")
```

output
```
((e*x + d)*C/c + B*log(a*tan(1/2*e*x + 1/2*d)^2 + 2*c*tan(1/2*e*x + 1/2*d)
 + a)/c - B*log(tan(1/2*e*x + 1/2*d)^2 + 1)/c - 2*(pi*floor(1/2*(e*x + d)/
pi + 1/2)*sgn(a) + arctan((a*tan(1/2*e*x + 1/2*d) + c)/sqrt(a^2 - c^2)))*(
C*a - A*c)/(sqrt(a^2 - c^2)*c))/e
```

3.562.9 Mupad [B] (verification not implemented)

Time = 34.85 (sec) , antiderivative size = 1143, normalized size of antiderivative = 13.61

$$\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{a+c\sin(d+ex)}\,dx$$
$$= \frac{\ln\left(32B^3a^2-32AB^2a^2+32AC^2a^2+32BC^2a^2+32a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\left(cA^2B-2cAB^2-2aABC\right.\right.}{}$$
$$-\frac{\ln\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)+1\mathrm{i}\right)(B-C\,1\mathrm{i})}{ce} - \frac{\ln\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)-\mathrm{i}\right)(B+C\,1\mathrm{i})}{ce}$$
$$+\frac{\ln\left(32B^3a^2-32AB^2a^2+32AC^2a^2+32BC^2a^2+32a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)\left(cA^2B-2cAB^2-2aABC\right.\right.}{}$$

input
```
int((A + B*cos(d + e*x) + C*sin(d + e*x))/(a + c*sin(d + e*x)),x)
```

output
```
(log(32*B^3*a^2 - 32*A*B^2*a^2 + 32*A*C^2*a^2 + 32*B*C^2*a^2 + 32*a*tan(d/
2 + (e*x)/2)*(2*C^3*a + B^3*c - 2*A*B^2*c + A^2*B*c + 2*B^2*C*a - 2*A*C^2*
c + 2*B*C^2*c - 2*A*B*C*a) - 32*A^2*C*a*c + 32*B^2*C*a*c - ((B*a^2 - B*c^2
 + A*c*(c^2 - a^2)^(1/2) - C*a*(c^2 - a^2)^(1/2))*(32*C^2*a^2*c - 32*B^2*a
^2*c - 128*B*C*a^3 + 32*a*tan(d/2 + (e*x)/2)*(2*B^2*a^2 - A^2*c^2 - 2*C^2*
a^2 - 3*B^2*c^2 + 2*C^2*c^2 + 4*A*B*c^2 + 2*A*C*a*c - 4*B*C*a*c) + 64*A*B*
a^2*c + 64*B*C*a*c^2 + ((B*a^2 - B*c^2 + A*c*(c^2 - a^2)^(1/2) - C*a*(c^2
- a^2)^(1/2))*(32*A*a^2*c^2 + 32*B*a^2*c^2 - 32*C*a*c^3 + 32*a*c^2*tan(d/2
 + (e*x)/2)*(2*A*c - 2*C*a + B*c) + (32*a*c*(a*c - 2*a^2*tan(d/2 + (e*x)/2
) + 3*c^2*tan(d/2 + (e*x)/2))*(B*a^2 - B*c^2 + A*c*(c^2 - a^2)^(1/2) - C*a
*(c^2 - a^2)^(1/2)))/(a^2 - c^2)))/(c*(a^2 - c^2))))/(c*(a^2 - c^2)))*(B*a
^2 - B*c^2 + A*c*(c^2 - a^2)^(1/2) - C*a*(c^2 - a^2)^(1/2)))/(c*e*(a^2 - c
^2)) - (log(tan(d/2 + (e*x)/2) + 1i)*(B - C*1i))/(c*e) - (log(tan(d/2 + (e
*x)/2) - 1i)*(B + C*1i))/(c*e) + (log(32*B^3*a^2 - 32*A*B^2*a^2 + 32*A*C^2
*a^2 + 32*B*C^2*a^2 + 32*a*tan(d/2 + (e*x)/2)*(2*C^3*a + B^3*c - 2*A*B^2*c
 + A^2*B*c + 2*B^2*C*a - 2*A*C^2*c + 2*B*C^2*c - 2*A*B*C*a) - 32*A^2*C*a*c
 + 32*B^2*C*a*c - ((B*a^2 - B*c^2 - A*c*(c^2 - a^2)^(1/2) + C*a*(c^2 - a^2
)^(1/2))*(32*C^2*a^2*c - 32*B^2*a^2*c - 128*B*C*a^3 + 32*a*tan(d/2 + (e*x)
/2)*(2*B^2*a^2 - A^2*c^2 - 2*C^2*a^2 - 3*B^2*c^2 + 2*C^2*c^2 + 4*A*B*c^2 +
 2*A*C*a*c - 4*B*C*a*c) + 64*A*B*a^2*c + 64*B*C*a*c^2 + ((B*a^2 - B*c^2...
```

3.562. $\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{a+c\sin(d+ex)}\,dx$

3.563 $\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{(a+c\sin(d+ex))^2}\,dx$

3.563.1 Optimal result

Integrand size = 31, antiderivative size = 118

$$\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{(a+c\sin(d+ex))^2}\,dx=\frac{2(aA-cC)\arctan\left(\frac{c+a\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-c^2}}\right)}{(a^2-c^2)^{3/2}\,e}-\frac{B}{ce(a+c\sin(d+ex))}+\frac{(Ac-aC)\cos(d+ex)}{(a^2-c^2)\,e(a+c\sin(d+ex))}$$

output
```
2*(A*a-C*c)*arctan((c+a*tan(1/2*e*x+1/2*d))/(a^2-c^2)^(1/2))/(a^2-c^2)^(3/
2)/e-B/c/e/(a+c*sin(e*x+d))+(A*c-C*a)*cos(e*x+d)/(a^2-c^2)/e/(a+c*sin(e*x+
d))
```

3.563.2 Mathematica [A] (verified)

Time = 2.59 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.97

$$\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{(a+c\sin(d+ex))^2}\,dx=\frac{\frac{2(aA-cC)\arctan\left(\frac{c+a\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-c^2}}\right)}{(a^2-c^2)^{3/2}}+\frac{B(a^2-c^2)-c(Ac-aC)\cos(d+ex)}{c(-a+c)(a+c)(a+c\sin(d+ex))}}{e}$$

input
```
Integrate[(A + B*Cos[d + e*x] + C*Sin[d + e*x])/(a + c*Sin[d + e*x])^2,x]
```

output
```
((2*(a*A - c*C)*ArcTan[(c + a*Tan[(d + e*x)/2])/Sqrt[a^2 - c^2]])/(a^2 - c
^2)^(3/2) + (B*(a^2 - c^2) - c*(A*c - a*C)*Cos[d + e*x])/(c*(-a + c)*(a +
c)*(a + c*Sin[d + e*x])))/e
```

3.563.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.05, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.387, Rules used = {3042, 4876, 3042, 3147, 17, 3233, 25, 27, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{(a+c\sin(d+ex))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{(a+c\sin(d+ex))^2}dx$$

$$\Big\downarrow \text{4876}$$

$$\int \frac{A+C\sin(d+ex)}{(a+c\sin(d+ex))^2}dx+B\int \frac{\cos(d+ex)}{(a+c\sin(d+ex))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+C\sin(d+ex)}{(a+c\sin(d+ex))^2}dx+B\int \frac{\cos(d+ex)}{(a+c\sin(d+ex))^2}dx$$

$$\Big\downarrow \text{3147}$$

$$\int \frac{A+C\sin(d+ex)}{(a+c\sin(d+ex))^2}dx+\frac{B\int \frac{1}{(a+c\sin(d+ex))^2}d(c\sin(d+ex))}{ce}$$

$$\Big\downarrow \text{17}$$

$$\int \frac{A+C\sin(d+ex)}{(a+c\sin(d+ex))^2}dx-\frac{B}{ce(a+c\sin(d+ex))}$$

$$\Big\downarrow \text{3233}$$

$$-\frac{\int -\frac{aA-cC}{a+c\sin(d+ex)}dx}{a^2-c^2}+\frac{(Ac-aC)\cos(d+ex)}{e(a^2-c^2)(a+c\sin(d+ex))}-\frac{B}{ce(a+c\sin(d+ex))}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{aA-cC}{a+c\sin(d+ex)}dx}{a^2-c^2}+\frac{(Ac-aC)\cos(d+ex)}{e(a^2-c^2)(a+c\sin(d+ex))}-\frac{B}{ce(a+c\sin(d+ex))}$$

$$\Big\downarrow \; 27$$

$$\frac{(aA-cC)\int \frac{1}{a+c\sin(d+ex)}dx}{a^2-c^2}+\frac{(Ac-aC)\cos(d+ex)}{e(a^2-c^2)(a+c\sin(d+ex))}-\frac{B}{ce(a+c\sin(d+ex))}$$

$$\Big\downarrow \; 3042$$

$$\frac{(aA-cC)\int \frac{1}{a+c\sin(d+ex)}dx}{a^2-c^2}+\frac{(Ac-aC)\cos(d+ex)}{e(a^2-c^2)(a+c\sin(d+ex))}-\frac{B}{ce(a+c\sin(d+ex))}$$

$$\Big\downarrow \; 3139$$

$$\frac{2(aA-cC)\int \frac{1}{a\tan^2\left(\frac{1}{2}(d+ex)\right)+2c\tan\left(\frac{1}{2}(d+ex)\right)+a}d\tan\left(\frac{1}{2}(d+ex)\right)}{e(a^2-c^2)}+\frac{(Ac-aC)\cos(d+ex)}{e(a^2-c^2)(a+c\sin(d+ex))}-\frac{B}{ce(a+c\sin(d+ex))}$$

$$\Big\downarrow \; 1083$$

$$-\frac{4(aA-cC)\int \frac{1}{-\left(2c+2a\tan\left(\frac{1}{2}(d+ex)\right)\right)^2-4(a^2-c^2)}d\left(2c+2a\tan\left(\frac{1}{2}(d+ex)\right)\right)}{e(a^2-c^2)}+\frac{(Ac-aC)\cos(d+ex)}{e(a^2-c^2)(a+c\sin(d+ex))}-\frac{B}{ce(a+c\sin(d+ex))}$$

$$\Big\downarrow \; 217$$

$$\frac{2(aA-cC)\arctan\left(\frac{2a\tan\left(\frac{1}{2}(d+ex)\right)+2c}{2\sqrt{a^2-c^2}}\right)}{e(a^2-c^2)^{3/2}}+\frac{(Ac-aC)\cos(d+ex)}{e(a^2-c^2)(a+c\sin(d+ex))}-\frac{B}{ce(a+c\sin(d+ex))}$$

input
```
Int[(A + B*Cos[d + e*x] + C*Sin[d + e*x])/(a + c*Sin[d + e*x])^2,x]
```

output
```
(2*(a*A - c*C)*ArcTan[(2*c + 2*a*Tan[(d + e*x)/2])/(2*Sqrt[a^2 - c^2])])/(
(a^2 - c^2)^(3/2)*e) - B/(c*e*(a + c*Sin[d + e*x])) + ((A*c - a*C)*Cos[d +
 e*x])/((a^2 - c^2)*e*(a + c*Sin[d + e*x]))
```

3.563. $\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{(a+c\sin(d+ex))^2}\,dx$

3.563.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.563. $\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{(a+c\sin(d+ex))^2}\,dx$

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 4876
```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Sin[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Simp[d   Int[ActivateTrig[u]*Cos[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Sin[
c*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Intege
rQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

3.563.4 Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.31

method	result
derivativedivides	$\frac{\frac{\frac{2\left(A\,c^{2}+B\,a^{2}-B\,c^{2}-acC\right)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{a\left(a^{2}-c^{2}\right)}+\frac{2(cA-Ca)}{a^{2}-c^{2}}}{\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}a+2c\tan\left(\frac{ex}{2}+\frac{d}{2}\right)+a}+\frac{2(Aa-Cc)\arctan\left(\frac{2a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)+2c}{2\sqrt{a^{2}-c^{2}}}\right)}{\left(a^{2}-c^{2}\right)^{\frac{3}{2}}}}{e}$
default	$\frac{\frac{\frac{2\left(A\,c^{2}+B\,a^{2}-B\,c^{2}-acC\right)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{a\left(a^{2}-c^{2}\right)}+\frac{2(cA-Ca)}{a^{2}-c^{2}}}{\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}a+2c\tan\left(\frac{ex}{2}+\frac{d}{2}\right)+a}+\frac{2(Aa-Cc)\arctan\left(\frac{2a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)+2c}{2\sqrt{a^{2}-c^{2}}}\right)}{\left(a^{2}-c^{2}\right)^{\frac{3}{2}}}}{e}$
parts	$\frac{\frac{\frac{2c(cA-Ca)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)}{\left(a^{2}-c^{2}\right)a}+\frac{2(cA-Ca)}{a^{2}-c^{2}}}{\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}a+2c\tan\left(\frac{ex}{2}+\frac{d}{2}\right)+a}+\frac{2(Aa-Cc)\arctan\left(\frac{2a\tan\left(\frac{ex}{2}+\frac{d}{2}\right)+2c}{2\sqrt{a^{2}-c^{2}}}\right)}{\left(a^{2}-c^{2}\right)^{\frac{3}{2}}}}{e}-\frac{B}{ce(a+c\sin(ex+d))}$
risch	$-\frac{2i\left(-iB\,a^{2}e^{i(ex+d)}+iB\,c^{2}e^{i(ex+d)}+iA\,c^{2}+Aac\,e^{i(ex+d)}-iacC-C\,a^{2}e^{i(ex+d)}\right)}{ce(a^{2}-c^{2})\left(-ic\,e^{2i(ex+d)}+ic+2a\,e^{i(ex+d)}\right)}-\frac{\ln\left(e^{i(ex+d)}+\frac{i\sqrt{-a^{2}+c^{2}}\,a-a^{2}+c^{2}}{\sqrt{-a^{2}+c^{2}}\,c}\right.}{\sqrt{-a^{2}+c^{2}}\,(a+c)(a-c)e}$

input
```
int((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^2,x,method=_RETURNVERBO
SE)
```

output
```
1/e*(2*((A*c^2+B*a^2-B*c^2-C*a*c)/a/(a^2-c^2)*tan(1/2*e*x+1/2*d)+(A*c-C*a)
/(a^2-c^2))/(tan(1/2*e*x+1/2*d)^2*a+2*c*tan(1/2*e*x+1/2*d)+a)+2*(A*a-C*c)/
(a^2-c^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*e*x+1/2*d)+2*c)/(a^2-c^2)^(1/2)))
```

3.563.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 458, normalized size of antiderivative = 3.88

$$\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{(a+c\sin(d+ex))^2}\,dx$$
$$= \left[-\frac{2\,Ba^4-4\,Ba^2c^2+2\,Bc^4+(Aa^2c-Cac^2+(Aac^2-Cc^3)\sin(ex+d))\sqrt{-a^2+c^2}\log\left(\frac{(2\,a^2-c^2)\cos(ex+c}{}\right.}{2\,((a^4c^2-2\,a^2c^4+c^6)e\sin(}\right.$$
$$-\frac{Ba^4-2\,Ba^2c^2+Bc^4+(Aa^2c-Cac^2+(Aac^2-Cc^3)\sin(ex+d))\sqrt{a^2-c^2}\arctan\left(-\frac{a\sin(ex+d)+c}{\sqrt{a^2-c^2}\cos(ex+d)}\right.}{(a^4c^2-2\,a^2c^4+c^6)e\sin(ex+d)+(a^5c-2\,a^3c^3+ac^5)e}$$

input
```
integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^2,x, algorithm="f
ricas")
```

output
```
[-1/2*(2*B*a^4 - 4*B*a^2*c^2 + 2*B*c^4 + (A*a^2*c - C*a*c^2 + (A*a*c^2 - C
*c^3)*sin(e*x + d))*sqrt(-a^2 + c^2)*log(((2*a^2 - c^2)*cos(e*x + d)^2 - 2
*a*c*sin(e*x + d) - a^2 - c^2 + 2*(a*cos(e*x + d)*sin(e*x + d) + c*cos(e*x
 + d))*sqrt(-a^2 + c^2))/(c^2*cos(e*x + d)^2 - 2*a*c*sin(e*x + d) - a^2 -
c^2)) + 2*(C*a^3*c - A*a^2*c^2 - C*a*c^3 + A*c^4)*cos(e*x + d))/((a^4*c^2
- 2*a^2*c^4 + c^6)*e*sin(e*x + d) + (a^5*c - 2*a^3*c^3 + a*c^5)*e), -(B*a^
4 - 2*B*a^2*c^2 + B*c^4 + (A*a^2*c - C*a*c^2 + (A*a*c^2 - C*c^3)*sin(e*x +
 d))*sqrt(a^2 - c^2)*arctan(-(a*sin(e*x + d) + c)/(sqrt(a^2 - c^2)*cos(e*x
 + d))) + (C*a^3*c - A*a^2*c^2 - C*a*c^3 + A*c^4)*cos(e*x + d))/((a^4*c^2
- 2*a^2*c^4 + c^6)*e*sin(e*x + d) + (a^5*c - 2*a^3*c^3 + a*c^5)*e)]
```

3.563.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{(a+c\sin(d+ex))^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))**2,x)
```

output
```
Timed out
```

3.563.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{(a+c\sin(d+ex))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^2,x, algorithm="m
axima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*a^2>0)', see `assume?` f
or more de
```

3.563.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.53

$$\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{(a+c\sin(d+ex))^2}\,dx$$
$$= \frac{2\left(\frac{\left(\pi\left\lfloor\frac{ex+d}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+c}{\sqrt{a^2-c^2}}\right)\right)(Aa-Cc)}{(a^2-c^2)^{\frac{3}{2}}}+\frac{Ba^2\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)-Cac\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+Ac^2\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)-Bc^2\tan}{(a^3-ac^2)\left(a\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)^2+2c\tan\left(\frac{1}{2}ex+\frac{1}{2}d\right)+\right.}}{e}$$

input
```
integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^2,x, algorithm="g
iac")
```

output
```
2*((pi*floor(1/2*(e*x + d)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*e*x + 1/2*
d) + c)/sqrt(a^2 - c^2)))*(A*a - C*c)/(a^2 - c^2)^(3/2) + (B*a^2*tan(1/2*e
*x + 1/2*d) - C*a*c*tan(1/2*e*x + 1/2*d) + A*c^2*tan(1/2*e*x + 1/2*d) - B*
c^2*tan(1/2*e*x + 1/2*d) - C*a^2 + A*a*c)/((a^3 - a*c^2)*(a*tan(1/2*e*x +
1/2*d)^2 + 2*c*tan(1/2*e*x + 1/2*d) + a)))/e
```

3.563.9 Mupad [B] (verification not implemented)

Time = 26.95 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.92

$$\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{(a+c\sin(d+ex))^2}\,dx$$
$$=\frac{\frac{2\,(A\,c-C\,a)}{a^2-c^2}+\frac{2\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,(B\,a^2+A\,c^2-B\,c^2-C\,a\,c)}{a\,(a^2-c^2)}}{e\,\left(a\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)^2+2\,c\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)+a\right)}$$
$$+\frac{2\,\mathrm{atan}\left(\frac{(a^2-c^2)\,\left(\frac{2\,a\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\,(A\,a-C\,c)}{(a+c)^{3/2}\,(a-c)^{3/2}}+\frac{2\,\left(a^2\,c-c^3\right)\,(A\,a-C\,c)}{(a+c)^{3/2}\,\left(a^2-c^2\right)\,(a-c)^{3/2}}\right)}{2\,(A\,a-C\,c)}\right)\,(A\,a-C\,c)}{e\,(a+c)^{3/2}\,(a-c)^{3/2}}$$

input
```
int((A + B*cos(d + e*x) + C*sin(d + e*x))/(a + c*sin(d + e*x))^2,x)
```

output
```
((2*(A*c - C*a))/(a^2 - c^2) + (2*tan(d/2 + (e*x)/2)*(B*a^2 + A*c^2 - B*c^
2 - C*a*c))/(a*(a^2 - c^2)))/(e*(a + 2*c*tan(d/2 + (e*x)/2) + a*tan(d/2 +
(e*x)/2)^2)) + (2*atan(((a^2 - c^2)*((2*a*tan(d/2 + (e*x)/2)*(A*a - C*c))/
((a + c)^(3/2)*(a - c)^(3/2)) + (2*(a^2*c - c^3)*(A*a - C*c))/((a + c)^(3/
2)*(a^2 - c^2)*(a - c)^(3/2))))/(2*(A*a - C*c)))*(A*a - C*c))/(e*(a + c)^(
3/2)*(a - c)^(3/2))
```

3.564 $\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{(a+c\sin(d+ex))^3}\,dx$

3.564.1 Optimal result

Integrand size = 31, antiderivative size = 185

$$\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{(a+c\sin(d+ex))^3}\,dx = \frac{(2a^2A+Ac^2-3acC)\arctan\left(\frac{c+a\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-c^2}}\right)}{(a^2-c^2)^{5/2}e} - \frac{B}{2ce(a+c\sin(d+ex))^2} + \frac{(Ac-aC)\cos(d+ex)}{2(a^2-c^2)e(a+c\sin(d+ex))^2} + \frac{(3aAc-a^2C-2c^2C)\cos(d+ex)}{2(a^2-c^2)^2e(a+c\sin(d+ex))}$$

output
```
(2*A*a^2+A*c^2-3*C*a*c)*arctan((c+a*tan(1/2*e*x+1/2*d))/(a^2-c^2)^(1/2))/(
a^2-c^2)^(5/2)/e-1/2*B/c/e/(a+c*sin(e*x+d))^2+1/2*(A*c-C*a)*cos(e*x+d)/(a^
2-c^2)/e/(a+c*sin(e*x+d))^2+1/2*(3*A*a*c-C*a^2-2*C*c^2)*cos(e*x+d)/(a^2-c^
2)^2/e/(a+c*sin(e*x+d))
```

3.564.2 Mathematica [A] (verified)

Time = 1.77 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.94

$$\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{(a+c\sin(d+ex))^3}\,dx = \frac{\frac{2(2a^2A+Ac^2-3acC)\arctan\left(\frac{c+a\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-c^2}}\right)}{(a^2-c^2)^{5/2}}+\frac{B(-a^2+c^2)+c(Ac-aC)\cos(d+ex)}{(a-c)c(a+c)(a+c\sin(d+ex))^2}-\frac{(-3aAc+a^2C+2c^2C)\cos(d+ex)}{(a-c)^2(a+c)^2(a+c\sin(d+ex))}}{2e}$$

input
```
Integrate[(A + B*Cos[d + e*x] + C*Sin[d + e*x])/(a + c*Sin[d + e*x])^3,x]
```

output
```
((2*(2*a^2*A + A*c^2 - 3*a*c*C)*ArcTan[(c + a*Tan[(d + e*x)/2])/Sqrt[a^2 -
 c^2]])/(a^2 - c^2)^(5/2) + (B*(-a^2 + c^2) + c*(A*c - a*C)*Cos[d + e*x])/
((a - c)*c*(a + c)*(a + c*Sin[d + e*x])^2) - ((-3*a*A*c + a^2*C + 2*c^2*C)
*Cos[d + e*x])/((a - c)^2*(a + c)^2*(a + c*Sin[d + e*x])))/(2*e)
```

3.564.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.11, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.484$, Rules used = {3042, 4876, 3042, 3147, 17, 3233, 25, 3042, 3233, 25, 27, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{(a+c\sin(d+ex))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{(a+c\sin(d+ex))^3}dx$$

$$\Big\downarrow \text{4876}$$

$$\int \frac{A+C\sin(d+ex)}{(a+c\sin(d+ex))^3}dx+B\int \frac{\cos(d+ex)}{(a+c\sin(d+ex))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+C\sin(d+ex)}{(a+c\sin(d+ex))^3}dx+B\int \frac{\cos(d+ex)}{(a+c\sin(d+ex))^3}dx$$

$$\Big\downarrow \; 3147$$

$$\int \frac{A+C\sin(d+ex)}{(a+c\sin(d+ex))^3}dx+\frac{B\int \frac{1}{(a+c\sin(d+ex))^3}d(c\sin(d+ex))}{ce}$$

$$\Big\downarrow \; 17$$

$$\int \frac{A+C\sin(d+ex)}{(a+c\sin(d+ex))^3}dx-\frac{B}{2ce(a+c\sin(d+ex))^2}$$

$$\Big\downarrow \; 3233$$

$$-\frac{\int -\frac{2(aA-cC)-(Ac-aC)\sin(d+ex)}{(a+c\sin(d+ex))^2}dx}{2(a^2-c^2)}+\frac{(Ac-aC)\cos(d+ex)}{2e(a^2-c^2)(a+c\sin(d+ex))^2}-\frac{B}{2ce(a+c\sin(d+ex))^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{2(aA-cC)-(Ac-aC)\sin(d+ex)}{(a+c\sin(d+ex))^2}dx}{2(a^2-c^2)}+\frac{(Ac-aC)\cos(d+ex)}{2e(a^2-c^2)(a+c\sin(d+ex))^2}-\frac{B}{2ce(a+c\sin(d+ex))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{2(aA-cC)-(Ac-aC)\sin(d+ex)}{(a+c\sin(d+ex))^2}dx}{2(a^2-c^2)}+\frac{(Ac-aC)\cos(d+ex)}{2e(a^2-c^2)(a+c\sin(d+ex))^2}-\frac{B}{2ce(a+c\sin(d+ex))^2}$$

$$\Big\downarrow \; 3233$$

$$\frac{\frac{(a^2(-C)+3aAc-2c^2C)\cos(d+ex)}{e(a^2-c^2)(a+c\sin(d+ex))}-\frac{\int -\frac{2Aa^2-3cCa+Ac^2}{a+c\sin(d+ex)}dx}{a^2-c^2}}{2(a^2-c^2)}+\frac{(Ac-aC)\cos(d+ex)}{2e(a^2-c^2)(a+c\sin(d+ex))^2}-\frac{B}{2ce(a+c\sin(d+ex))^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{2Aa^2-3cCa+Ac^2}{a+c\sin(d+ex)}dx}{a^2-c^2}+\frac{(a^2(-C)+3aAc-2c^2C)\cos(d+ex)}{e(a^2-c^2)(a+c\sin(d+ex))}}{2(a^2-c^2)}+\frac{(Ac-aC)\cos(d+ex)}{2e(a^2-c^2)(a+c\sin(d+ex))^2}-\frac{B}{2ce(a+c\sin(d+ex))^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{(2a^2A-3acC+Ac^2)\int \frac{1}{a+c\sin(d+ex)}dx}{a^2-c^2}+\frac{(a^2(-C)+3aAc-2c^2C)\cos(d+ex)}{e(a^2-c^2)(a+c\sin(d+ex))}}{2(a^2-c^2)}+\frac{(Ac-aC)\cos(d+ex)}{2e(a^2-c^2)(a+c\sin(d+ex))^2}-\frac{B}{2ce(a+c\sin(d+ex))^2}$$

$$\Big\downarrow \; 3042$$

3.564. $\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{(a+c\sin(d+ex))^3}\,dx$

$$\frac{\frac{(2a^2A-3acC+Ac^2)\int \frac{1}{a+c\sin(d+ex)}dx}{a^2-c^2}+\frac{(a^2(-C)+3aAc-2c^2C)\cos(d+ex)}{e(a^2-c^2)(a+c\sin(d+ex))}}{2(a^2-c^2)}+\frac{(Ac-aC)\cos(d+ex)}{2e(a^2-c^2)(a+c\sin(d+ex))^2}-\frac{B}{2ce(a+c\sin(d+ex))^2}$$

$$\Big\downarrow \text{3139}$$

$$\frac{\frac{2(2a^2A-3acC+Ac^2)\int \frac{1}{a\tan^2\left(\frac{1}{2}(d+ex)\right)+2c\tan\left(\frac{1}{2}(d+ex)\right)+a}d\tan\left(\frac{1}{2}(d+ex)\right)}{e(a^2-c^2)}+\frac{(a^2(-C)+3aAc-2c^2C)\cos(d+ex)}{e(a^2-c^2)(a+c\sin(d+ex))}}{2(a^2-c^2)}+\frac{(Ac-aC)\cos(d+ex)}{2e(a^2-c^2)(a+c\sin(d+ex))^2}-\frac{B}{2ce(a+c\sin(d+ex))^2}$$

$$\Big\downarrow \text{1083}$$

$$\frac{\frac{(a^2(-C)+3aAc-2c^2C)\cos(d+ex)}{e(a^2-c^2)(a+c\sin(d+ex))}-\frac{4(2a^2A-3acC+Ac^2)\int \frac{1}{-\left(2c+2a\tan\left(\frac{1}{2}(d+ex)\right)\right)^2-4(a^2-c^2)}d\left(2c+2a\tan\left(\frac{1}{2}(d+ex)\right)\right)}{e(a^2-c^2)}}{2(a^2-c^2)}+\frac{(Ac-aC)\cos(d+ex)}{2e(a^2-c^2)(a+c\sin(d+ex))^2}-\frac{B}{2ce(a+c\sin(d+ex))^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{\frac{2(2a^2A-3acC+Ac^2)\arctan\left(\frac{2a\tan\left(\frac{1}{2}(d+ex)\right)+2c}{2\sqrt{a^2-c^2}}\right)}{e(a^2-c^2)^{3/2}}+\frac{(a^2(-C)+3aAc-2c^2C)\cos(d+ex)}{e(a^2-c^2)(a+c\sin(d+ex))}}{2(a^2-c^2)}+\frac{(Ac-aC)\cos(d+ex)}{2e(a^2-c^2)(a+c\sin(d+ex))^2}-\frac{B}{2ce(a+c\sin(d+ex))^2}$$

input
```
Int[(A + B*Cos[d + e*x] + C*Sin[d + e*x])/(a + c*Sin[d + e*x])^3,x]
```

output
```
-1/2*B/(c*e*(a + c*Sin[d + e*x])^2) + ((A*c - a*C)*Cos[d + e*x])/(2*(a^2 -
 c^2)*e*(a + c*Sin[d + e*x])^2) + ((2*(2*a^2*A + A*c^2 - 3*a*c*C)*ArcTan[(
2*c + 2*a*Tan[(d + e*x)/2])/(2*Sqrt[a^2 - c^2])])/((a^2 - c^2)^(3/2)*e) +
((3*a*A*c - a^2*C - 2*c^2*C)*Cos[d + e*x])/((a^2 - c^2)*e*(a + c*Sin[d + e
*x])))/(2*(a^2 - c^2))
```

3.564. $\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{(a+c\sin(d+ex))^3}\,dx$

3.564.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.564. $\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{(a+c\sin(d+ex))^3}\,dx$

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 4876
```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Sin[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Simp[d   Int[ActivateTrig[u]*Cos[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Sin[
c*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Intege
rQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

3.564.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 372 vs. $2(174) = 348$.

Time = 1.91 (sec) , antiderivative size = 373, normalized size of antiderivative = 2.02

method	result
parts	$\frac{c\left(5A\,a^{2}c-2A\,c^{3}-3C\,a^{3}\right)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{3}}{a\left(a^{4}-2a^{2}c^{2}+c^{4}\right)}+\frac{\left(4A\,a^{4}c+7A\,a^{2}c^{3}-2A\,c^{5}-2C\,a^{5}-5C\,a^{3}c^{2}-2Ca\,c^{4}\right)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}}{\left(a^{4}-2a^{2}c^{2}+c^{4}\right)a^{2}}+\frac{c\left(11A\,a^{2}c-2A\,c^{3}-5\right.}{a\left(a^{4}\right.}$ / $\left(\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}a+2c\tan\left(\frac{ex}{2}+\frac{d}{2}\right)+a\right)^{2}$ / e
derivativedivides	$\frac{\left(5A\,a^{2}c^{2}-2A\,c^{4}+2B\,a^{4}-4B\,a^{2}c^{2}+2B\,c^{4}-3C\,a^{3}c\right)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{3}}{\left(a^{4}-2a^{2}c^{2}+c^{4}\right)a}+\frac{\left(4A\,a^{4}c+7A\,a^{2}c^{3}-2A\,c^{5}+2B\,a^{4}c-4B\,a^{2}c^{3}+2B\,c^{5}-2C\,a^{5}-5C\right.}{\left(a^{4}-2a^{2}c^{2}+c^{4}\right)a^{2}}$ / $\left(\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}\right.$
default	$\frac{\left(5A\,a^{2}c^{2}-2A\,c^{4}+2B\,a^{4}-4B\,a^{2}c^{2}+2B\,c^{4}-3C\,a^{3}c\right)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{3}}{\left(a^{4}-2a^{2}c^{2}+c^{4}\right)a}+\frac{\left(4A\,a^{4}c+7A\,a^{2}c^{3}-2A\,c^{5}+2B\,a^{4}c-4B\,a^{2}c^{3}+2B\,c^{5}-2C\,a^{5}-5C\right.}{\left(a^{4}-2a^{2}c^{2}+c^{4}\right)a^{2}}$ / $\left(\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}\right.$
risch	$-\frac{iC\,a^{2}c^{2}-2ic^{4}C\,\mathrm{e}^{2i(ex+d)}+2c^{2}A\,a^{2}\mathrm{e}^{3i(ex+d)}+c^{4}A\,\mathrm{e}^{3i(ex+d)}-2iC\,a^{4}\mathrm{e}^{2i(ex+d)}+6icA\,a^{3}\mathrm{e}^{2i(ex+d)}-5ic^{2}C\,a^{2}\mathrm{e}^{2i(ex+d)}}{}$

input
```
int((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^3,x,method=_RETURNVERBO
SE)
```

output
```
1/e*(2*(1/2*c*(5*A*a^2*c-2*A*c^3-3*C*a^3)/a/(a^4-2*a^2*c^2+c^4)*tan(1/2*e*
x+1/2*d)^3+1/2*(4*A*a^4*c+7*A*a^2*c^3-2*A*c^5-2*C*a^5-5*C*a^3*c^2-2*C*a*c^
4)/(a^4-2*a^2*c^2+c^4)/a^2*tan(1/2*e*x+1/2*d)^2+1/2*c*(11*A*a^2*c-2*A*c^3-
5*C*a^3-4*C*a*c^2)/a/(a^4-2*a^2*c^2+c^4)*tan(1/2*e*x+1/2*d)+1/2*(4*A*a^2*c
-A*c^3-2*C*a^3-C*a*c^2)/(a^4-2*a^2*c^2+c^4))/(tan(1/2*e*x+1/2*d)^2*a+2*c*t
an(1/2*e*x+1/2*d)+a)^2+(2*A*a^2+A*c^2-3*C*a*c)/(a^4-2*a^2*c^2+c^4)/(a^2-c^
2)^(1/2)*arctan(1/2*(2*a*tan(1/2*e*x+1/2*d)+2*c)/(a^2-c^2)^(1/2)))-1/2*B/c
/e/(a+c*sin(e*x+d))^2
```

3.564.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 397 vs. $2(173) = 346$.

Time = 0.29 (sec) , antiderivative size = 880, normalized size of antiderivative = 4.76

$$\int \frac{A + B\cos(d+ex) + C\sin(d+ex)}{(a+c\sin(d+ex))^3}\,dx$$
$$= \left[\frac{2\,Ba^6 - 6\,Ba^4c^2 + 6\,Ba^2c^4 - 2\,Bc^6 + 2\,(Ca^4c^2 - 3\,Aa^3c^3 + Ca^2c^4 + 3\,Aac^5 - 2\,Cc^6)\cos(ex+d)\sin(e\right.$$

input
```
integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^3,x, algorithm="f
ricas")
```

output
```
[1/4*(2*B*a^6 - 6*B*a^4*c^2 + 6*B*a^2*c^4 - 2*B*c^6 + 2*(C*a^4*c^2 - 3*A*a
^3*c^3 + C*a^2*c^4 + 3*A*a*c^5 - 2*C*c^6)*cos(e*x + d)*sin(e*x + d) + (2*A
*a^4*c - 3*C*a^3*c^2 + 3*A*a^2*c^3 - 3*C*a*c^4 + A*c^5 - (2*A*a^2*c^3 - 3*
C*a*c^4 + A*c^5)*cos(e*x + d)^2 + 2*(2*A*a^3*c^2 - 3*C*a^2*c^3 + A*a*c^4)*
sin(e*x + d))*sqrt(-a^2 + c^2)*log(((2*a^2 - c^2)*cos(e*x + d)^2 - 2*a*c*s
in(e*x + d) - a^2 - c^2 + 2*(a*cos(e*x + d)*sin(e*x + d) + c*cos(e*x + d))
*sqrt(-a^2 + c^2))/(c^2*cos(e*x + d)^2 - 2*a*c*sin(e*x + d) - a^2 - c^2))
+ 2*(2*C*a^5*c - 4*A*a^4*c^2 - C*a^3*c^3 + 5*A*a^2*c^4 - C*a*c^5 - A*c^6)*
cos(e*x + d))/((a^6*c^3 - 3*a^4*c^5 + 3*a^2*c^7 - c^9)*e*cos(e*x + d)^2 -
2*(a^7*c^2 - 3*a^5*c^4 + 3*a^3*c^6 - a*c^8)*e*sin(e*x + d) - (a^8*c - 2*a^
6*c^3 + 2*a^2*c^7 - c^9)*e), 1/2*(B*a^6 - 3*B*a^4*c^2 + 3*B*a^2*c^4 - B*c^
6 + (C*a^4*c^2 - 3*A*a^3*c^3 + C*a^2*c^4 + 3*A*a*c^5 - 2*C*c^6)*cos(e*x +
d)*sin(e*x + d) + (2*A*a^4*c - 3*C*a^3*c^2 + 3*A*a^2*c^3 - 3*C*a*c^4 + A*c
^5 - (2*A*a^2*c^3 - 3*C*a*c^4 + A*c^5)*cos(e*x + d)^2 + 2*(2*A*a^3*c^2 - 3
*C*a^2*c^3 + A*a*c^4)*sin(e*x + d))*sqrt(a^2 - c^2)*arctan(-(a*sin(e*x + d
) + c)/(sqrt(a^2 - c^2)*cos(e*x + d))) + (2*C*a^5*c - 4*A*a^4*c^2 - C*a^3*
c^3 + 5*A*a^2*c^4 - C*a*c^5 - A*c^6)*cos(e*x + d))/((a^6*c^3 - 3*a^4*c^5 +
 3*a^2*c^7 - c^9)*e*cos(e*x + d)^2 - 2*(a^7*c^2 - 3*a^5*c^4 + 3*a^3*c^6 -
a*c^8)*e*sin(e*x + d) - (a^8*c - 2*a^6*c^3 + 2*a^2*c^7 - c^9)*e)]
```

3.564.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cos(d + ex) + C\sin(d + ex)}{(a + c\sin(d + ex))^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))**3,x)
```

output
```
Timed out
```

3.564.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\cos(d + ex) + C\sin(d + ex)}{(a + c\sin(d + ex))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^3,x, algorithm="m
axima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*a^2>0)', see `assume?` f
or more de
```

3.564.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 571 vs. $2(173) = 346$.

Time = 0.33 (sec) , antiderivative size = 571, normalized size of antiderivative = 3.09

$$\int \frac{A + B\cos(d + ex) + C\sin(d + ex)}{(a + c\sin(d + ex))^3} dx$$

$$= \frac{\frac{(2\,Aa^2 - 3\,Cac + Ac^2)\left(\pi\left\lfloor\frac{ex+d}{2\,\pi} + \frac{1}{2}\right\rfloor\mathrm{sgn}(a) + \arctan\left(\frac{a\tan\left(\frac{1}{2}\,ex + \frac{1}{2}\,d\right) + c}{\sqrt{a^2 - c^2}}\right)\right)}{(a^4 - 2\,a^2c^2 + c^4)\sqrt{a^2 - c^2}} + \frac{2\,Ba^5\tan\left(\frac{1}{2}\,ex + \frac{1}{2}\,d\right)^3 - 3\,Ca^4c\tan\left(\frac{1}{2}\,ex + \frac{1}{2}\,d\right)^3 + 5\,Aa^3c^2\tan\left(\frac{1}{2}\right.}{}}{}$$

input
```
integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^3,x, algorithm="g
iac")
```

output
```
((2*A*a^2 - 3*C*a*c + A*c^2)*(pi*floor(1/2*(e*x + d)/pi + 1/2)*sgn(a) + ar
ctan((a*tan(1/2*e*x + 1/2*d) + c)/sqrt(a^2 - c^2)))/((a^4 - 2*a^2*c^2 + c^
4)*sqrt(a^2 - c^2)) + (2*B*a^5*tan(1/2*e*x + 1/2*d)^3 - 3*C*a^4*c*tan(1/2*
e*x + 1/2*d)^3 + 5*A*a^3*c^2*tan(1/2*e*x + 1/2*d)^3 - 4*B*a^3*c^2*tan(1/2*
e*x + 1/2*d)^3 - 2*A*a*c^4*tan(1/2*e*x + 1/2*d)^3 + 2*B*a*c^4*tan(1/2*e*x
+ 1/2*d)^3 - 2*C*a^5*tan(1/2*e*x + 1/2*d)^2 + 4*A*a^4*c*tan(1/2*e*x + 1/2*
d)^2 + 2*B*a^4*c*tan(1/2*e*x + 1/2*d)^2 - 5*C*a^3*c^2*tan(1/2*e*x + 1/2*d)
^2 + 7*A*a^2*c^3*tan(1/2*e*x + 1/2*d)^2 - 4*B*a^2*c^3*tan(1/2*e*x + 1/2*d)
^2 - 2*C*a*c^4*tan(1/2*e*x + 1/2*d)^2 - 2*A*c^5*tan(1/2*e*x + 1/2*d)^2 + 2
*B*c^5*tan(1/2*e*x + 1/2*d)^2 + 2*B*a^5*tan(1/2*e*x + 1/2*d) - 5*C*a^4*c*t
an(1/2*e*x + 1/2*d) + 11*A*a^3*c^2*tan(1/2*e*x + 1/2*d) - 4*B*a^3*c^2*tan(
1/2*e*x + 1/2*d) - 4*C*a^2*c^3*tan(1/2*e*x + 1/2*d) - 2*A*a*c^4*tan(1/2*e*
x + 1/2*d) + 2*B*a*c^4*tan(1/2*e*x + 1/2*d) - 2*C*a^5 + 4*A*a^4*c - C*a^3*
c^2 - A*a^2*c^3)/((a^6 - 2*a^4*c^2 + a^2*c^4)*(a*tan(1/2*e*x + 1/2*d)^2 +
2*c*tan(1/2*e*x + 1/2*d) + a)^2))/e
```

3.564.9 Mupad [B] (verification not implemented)

Time = 28.88 (sec) , antiderivative size = 557, normalized size of antiderivative = 3.01

$$\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{(a+c\sin(d+ex))^3}\,dx$$
$$= \frac{\operatorname{atan}\left(\frac{\left(\frac{(2Aa^2-3Cac+Ac^2)(2a^4c-4a^2c^3+2c^5)}{2(a+c)^{5/2}(a-c)^{5/2}(a^4-2a^2c^2+c^4)}+\frac{a\tan\left(\frac{d}{2}+\frac{ex}{2}\right)(2Aa^2-3Cac+Ac^2)}{(a+c)^{5/2}(a-c)^{5/2}}\right)(a^4-2a^2c^2+c^4)}{2Aa^2-3Cac+Ac^2}\right)(2Aa^2-3Cac+Ac^2}{e(a+c)^{5/2}(a-c)^{5/2}}$$
$$-\frac{\frac{2Ca^3-4Aa^2c+Cac^2+Ac^3}{a^4-2a^2c^2+c^4}-\frac{\tan\left(\frac{d}{2}+\frac{ex}{2}\right)^3(2Ba^4-2Ac^4+2Bc^4+5Aa^2c^2-4Ba^2c^2-3Ca^3c)}{a(a^4-2a^2c^2+c^4)}+\frac{\tan\left(\frac{d}{2}+\frac{ex}{2}\right)(2Ac^4-2Ba^4-2B}{a(}}{e\left(\tan\left(\frac{d}{2}+\frac{ex}{2}\right)^2(2a^2+4c^2)+a^2\tan\left(\frac{d}{2}+\frac{e}{2}\right.\right.}$$

input
```
int((A + B*cos(d + e*x) + C*sin(d + e*x))/(a + c*sin(d + e*x))^3,x)
```

output
```
(atan(((((2*A*a^2 + A*c^2 - 3*C*a*c)*(2*a^4*c + 2*c^5 - 4*a^2*c^3))/(2*(a
+ c)^(5/2)*(a - c)^(5/2)*(a^4 + c^4 - 2*a^2*c^2)) + (a*tan(d/2 + (e*x)/2)*
(2*A*a^2 + A*c^2 - 3*C*a*c))/((a + c)^(5/2)*(a - c)^(5/2)))*(a^4 + c^4 - 2
*a^2*c^2))/(2*A*a^2 + A*c^2 - 3*C*a*c))*(2*A*a^2 + A*c^2 - 3*C*a*c))/(e*(a
 + c)^(5/2)*(a - c)^(5/2)) - ((A*c^3 + 2*C*a^3 - 4*A*a^2*c + C*a*c^2)/(a^4
 + c^4 - 2*a^2*c^2) - (tan(d/2 + (e*x)/2)^3*(2*B*a^4 - 2*A*c^4 + 2*B*c^4 +
 5*A*a^2*c^2 - 4*B*a^2*c^2 - 3*C*a^3*c))/(a*(a^4 + c^4 - 2*a^2*c^2)) + (ta
n(d/2 + (e*x)/2)*(2*A*c^4 - 2*B*a^4 - 2*B*c^4 - 11*A*a^2*c^2 + 4*B*a^2*c^2
 + 4*C*a*c^3 + 5*C*a^3*c))/(a*(a^4 + c^4 - 2*a^2*c^2)) + (tan(d/2 + (e*x)/
2)^2*(2*A*c^5 + 2*C*a^5 - 2*B*c^5 - 7*A*a^2*c^3 + 4*B*a^2*c^3 + 5*C*a^3*c^
2 - 4*A*a^4*c - 2*B*a^4*c + 2*C*a*c^4))/(a^2*(a^4 + c^4 - 2*a^2*c^2)))/(e*
(tan(d/2 + (e*x)/2)^2*(2*a^2 + 4*c^2) + a^2*tan(d/2 + (e*x)/2)^4 + a^2 + 4
*a*c*tan(d/2 + (e*x)/2)^3 + 4*a*c*tan(d/2 + (e*x)/2)))
```

3.565 $\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{(a+c\sin(d+ex))^4}\,dx$

3.565.1 Optimal result

Integrand size = 31, antiderivative size = 258

$$\begin{aligned}\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{(a+c\sin(d+ex))^4}\,dx &= \frac{(2a^3A+3aAc^2-4a^2cC-c^3C)\arctan\left(\frac{c+a\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-c^2}}\right)}{(a^2-c^2)^{7/2}\,e} \\ &\quad - \frac{B}{3ce(a+c\sin(d+ex))^3} + \frac{(Ac-aC)\cos(d+ex)}{3\,(a^2-c^2)\,e(a+c\sin(d+ex))^3} \\ &\quad + \frac{(5aAc-2a^2C-3c^2C)\cos(d+ex)}{6\,(a^2-c^2)^2\,e(a+c\sin(d+ex))^2} + \frac{(11a^2Ac+4Ac^3-2a^3C-13ac^2C)\cos(d+ex)}{6\,(a^2-c^2)^3\,e(a+c\sin(d+ex))}\end{aligned}$$

output
```
(2*A*a^3+3*A*a*c^2-4*C*a^2*c-C*c^3)*arctan((c+a*tan(1/2*e*x+1/2*d))/(a^2-c
^2)^(1/2))/(a^2-c^2)^(7/2)/e-1/3*B/c/e/(a+c*sin(e*x+d))^3+1/3*(A*c-C*a)*co
s(e*x+d)/(a^2-c^2)/e/(a+c*sin(e*x+d))^3+1/6*(5*A*a*c-2*C*a^2-3*C*c^2)*cos(
e*x+d)/(a^2-c^2)^2/e/(a+c*sin(e*x+d))^2+1/6*(11*A*a^2*c+4*A*c^3-2*C*a^3-13
*C*a*c^2)*cos(e*x+d)/(a^2-c^2)^3/e/(a+c*sin(e*x+d))
```

3.565.2 Mathematica [A] (verified)

Time = 3.14 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.95

$$\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{(a+c\sin(d+ex))^4}\,dx$$
$$= \frac{\frac{6(2a^3A+3aAc^2-4a^2cC-c^3C)\arctan\left(\frac{c+a\tan\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a^2-c^2}}\right)}{(a^2-c^2)^{7/2}}+\frac{2B(-a^2+c^2)+2c(Ac-aC)\cos(d+ex)}{(a-c)c(a+c)(a+c\sin(d+ex))^3}+\frac{(5aAc-2a^2C-3c^2C)\cos(d+ex)}{(a-c)^2(a+c)^2(a+c\sin(d+ex))^2}+ \cdots}{6e}$$

input
```
Integrate[(A + B*Cos[d + e*x] + C*Sin[d + e*x])/(a + c*Sin[d + e*x])^4,x]
```

output
```
((6*(2*a^3*A + 3*a*A*c^2 - 4*a^2*c*C - c^3*C)*ArcTan[(c + a*Tan[(d + e*x)/
2])/Sqrt[a^2 - c^2]])/(a^2 - c^2)^(7/2) + (2*B*(-a^2 + c^2) + 2*c*(A*c - a
*C)*Cos[d + e*x])/((a - c)*c*(a + c)*(a + c*Sin[d + e*x])^3) + ((5*a*A*c -
 2*a^2*C - 3*c^2*C)*Cos[d + e*x])/((a - c)^2*(a + c)^2*(a + c*Sin[d + e*x]
)^2) + ((11*a^2*A*c + 4*A*c^3 - 2*a^3*C - 13*a*c^2*C)*Cos[d + e*x])/((a -
c)^3*(a + c)^3*(a + c*Sin[d + e*x])))/(6*e)
```

3.565.3 Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.14, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.548$, Rules used = {3042, 4876, 3042, 3147, 17, 3233, 25, 3042, 3233, 25, 3042, 3233, 27, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{(a+c\sin(d+ex))^4}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{(a+c\sin(d+ex))^4}dx$$
$$\Big\downarrow \text{4876}$$
$$\int \frac{A+C\sin(d+ex)}{(a+c\sin(d+ex))^4}dx+B\int \frac{\cos(d+ex)}{(a+c\sin(d+ex))^4}dx$$
$$\Big\downarrow \text{3042}$$

$$\int \frac{A+C\sin(d+ex)}{(a+c\sin(d+ex))^4}dx+B\int \frac{\cos(d+ex)}{(a+c\sin(d+ex))^4}dx$$

$$\Big\downarrow \textbf{3147}$$

$$\int \frac{A+C\sin(d+ex)}{(a+c\sin(d+ex))^4}dx+\frac{B\int \frac{1}{(a+c\sin(d+ex))^4}d(c\sin(d+ex))}{ce}$$

$$\Big\downarrow \textbf{17}$$

$$\int \frac{A+C\sin(d+ex)}{(a+c\sin(d+ex))^4}dx-\frac{B}{3ce(a+c\sin(d+ex))^3}$$

$$\Big\downarrow \textbf{3233}$$

$$-\frac{\int -\frac{3(aA-cC)-2(Ac-aC)\sin(d+ex)}{(a+c\sin(d+ex))^3}dx}{3(a^2-c^2)}+\frac{(Ac-aC)\cos(d+ex)}{3e(a^2-c^2)(a+c\sin(d+ex))^3}-\frac{B}{3ce(a+c\sin(d+ex))^3}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{\int \frac{3(aA-cC)-2(Ac-aC)\sin(d+ex)}{(a+c\sin(d+ex))^3}dx}{3(a^2-c^2)}+\frac{(Ac-aC)\cos(d+ex)}{3e(a^2-c^2)(a+c\sin(d+ex))^3}-\frac{B}{3ce(a+c\sin(d+ex))^3}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\int \frac{3(aA-cC)-2(Ac-aC)\sin(d+ex)}{(a+c\sin(d+ex))^3}dx}{3(a^2-c^2)}+\frac{(Ac-aC)\cos(d+ex)}{3e(a^2-c^2)(a+c\sin(d+ex))^3}-\frac{B}{3ce(a+c\sin(d+ex))^3}$$

$$\Big\downarrow \textbf{3233}$$

$$\frac{\frac{(-2a^2C+5aAc-3c^2C)\cos(d+ex)}{2e(a^2-c^2)(a+c\sin(d+ex))^2}-\frac{\int -\frac{2\left(3Aa^2-5cCa+2Ac^2\right)-\left(-2Ca^2+5Aca-3c^2C\right)\sin(d+ex)}{(a+c\sin(d+ex))^2}dx}{2(a^2-c^2)}}{3(a^2-c^2)}+\frac{(Ac-aC)\cos(d+ex)}{3e(a^2-c^2)(a+c\sin(d+ex))^3}-\frac{B}{3ce(a+c\sin(d+ex))^3}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{\frac{\int \frac{2\left(3Aa^2-5cCa+2Ac^2\right)-\left(-2Ca^2+5Aca-3c^2C\right)\sin(d+ex)}{(a+c\sin(d+ex))^2}dx}{2(a^2-c^2)}+\frac{(-2a^2C+5aAc-3c^2C)\cos(d+ex)}{2e(a^2-c^2)(a+c\sin(d+ex))^2}}{3(a^2-c^2)}+\frac{(Ac-aC)\cos(d+ex)}{3e(a^2-c^2)(a+c\sin(d+ex))^3}-\frac{B}{3ce(a+c\sin(d+ex))^3}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\frac{\int \frac{2\left(3Aa^2-5cCa+2Ac^2\right)-\left(-2Ca^2+5Aca-3c^2C\right)\sin(d+ex)}{(a+c\sin(d+ex))^2}dx}{2(a^2-c^2)}+\frac{(-2a^2C+5aAc-3c^2C)\cos(d+ex)}{2e(a^2-c^2)(a+c\sin(d+ex))^2}}{3(a^2-c^2)}+\frac{(Ac-aC)\cos(d+ex)}{3e(a^2-c^2)(a+c\sin(d+ex))^3}-\frac{B}{3ce(a+c\sin(d+ex))^3}$$

3.565. $\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{(a+c\sin(d+ex))^4}\,dx$

$$\Big\downarrow \; 3233$$

$$\frac{\frac{\frac{\left(-2a^3C+11a^2Ac-13ac^2C+4Ac^3\right)\cos(d+ex)}{e\left(a^2-c^2\right)(a+c\sin(d+ex))}-\frac{\int -\frac{3\left(2Aa^3-4cCa^2+3Ac^2a-c^3C\right)}{a+c\sin(d+ex)}dx}{a^2-c^2}}{2(a^2-c^2)}+\frac{(-2a^2C+5aAc-3c^2C)\cos(d+ex)}{2e(a^2-c^2)(a+c\sin(d+ex))^2}}{3\left(a^2-c^2\right)}+\frac{(Ac-aC)\cos(d+ex)}{3e\left(a^2-c^2\right)(a+c\sin(d+ex))^3}-\frac{B}{3ce(a+c\sin(d+ex))^3}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{3\left(2a^3A-4a^2cC+3aAc^2-c^3C\right)\int\frac{1}{a+c\sin(d+ex)}dx}{a^2-c^2}+\frac{\left(-2a^3C+11a^2Ac-13ac^2C+4Ac^3\right)\cos(d+ex)}{e\left(a^2-c^2\right)(a+c\sin(d+ex))}}{2(a^2-c^2)}+\frac{(-2a^2C+5aAc-3c^2C)\cos(d+ex)}{2e(a^2-c^2)(a+c\sin(d+ex))^2}}{3\left(a^2-c^2\right)}+\frac{(Ac-aC)\cos(d+ex)}{3e\left(a^2-c^2\right)(a+c\sin(d+ex))^3}-\frac{B}{3ce(a+c\sin(d+ex))^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\frac{3\left(2a^3A-4a^2cC+3aAc^2-c^3C\right)\int\frac{1}{a+c\sin(d+ex)}dx}{a^2-c^2}+\frac{\left(-2a^3C+11a^2Ac-13ac^2C+4Ac^3\right)\cos(d+ex)}{e\left(a^2-c^2\right)(a+c\sin(d+ex))}}{2(a^2-c^2)}+\frac{(-2a^2C+5aAc-3c^2C)\cos(d+ex)}{2e(a^2-c^2)(a+c\sin(d+ex))^2}}{3\left(a^2-c^2\right)}+\frac{(Ac-aC)\cos(d+ex)}{3e\left(a^2-c^2\right)(a+c\sin(d+ex))^3}-\frac{B}{3ce(a+c\sin(d+ex))^3}$$

$$\Big\downarrow \; 3139$$

$$\frac{\frac{\frac{6\left(2a^3A-4a^2cC+3aAc^2-c^3C\right)\int\frac{1}{a\tan^2\left(\frac{1}{2}(d+ex)\right)+2c\tan\left(\frac{1}{2}(d+ex)\right)+a}d\tan\left(\frac{1}{2}(d+ex)\right)}{e\left(a^2-c^2\right)}+\frac{\left(-2a^3C+11a^2Ac-13ac^2C+4Ac^3\right)\cos(d+ex)}{e\left(a^2-c^2\right)(a+c\sin(d+ex))}}{2(a^2-c^2)}+\frac{(-2a^2C+5aAc-3c^2C)\cos(d+ex)}{2e(a^2-c^2)(a+c\sin(d+ex))^2}}{3\left(a^2-c^2\right)}+\frac{(Ac-aC)\cos(d+ex)}{3e\left(a^2-c^2\right)(a+c\sin(d+ex))^3}-\frac{B}{3ce(a+c\sin(d+ex))^3}$$

$$\Big\downarrow \; 1083$$

$$\frac{\frac{\frac{\left(-2a^3C+11a^2Ac-13ac^2C+4Ac^3\right)\cos(d+ex)}{e\left(a^2-c^2\right)(a+c\sin(d+ex))}-\frac{12\left(2a^3A-4a^2cC+3aAc^2-c^3C\right)\int\frac{1}{-\left(2c+2a\tan\left(\frac{1}{2}(d+ex)\right)\right)^2-4\left(a^2-c^2\right)}d\left(2c+2a\tan\left(\frac{1}{2}(d+ex)\right)\right)}{e\left(a^2-c^2\right)}}{2(a^2-c^2)}+\frac{(-2a^2C+5aAc-3c^2C)\cos(d+ex)}{2e(a^2-c^2)(a+c\sin(d+ex))^2}}{3\left(a^2-c^2\right)}+\frac{(Ac-aC)\cos(d+ex)}{3e\left(a^2-c^2\right)(a+c\sin(d+ex))^3}-\frac{B}{3ce(a+c\sin(d+ex))^3}$$

$$\Big\downarrow \; 217$$

3.565. $\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{(a+c\sin(d+ex))^4}\,dx$

$$\frac{\frac{(Ac-aC)\cos(d+ex)}{3e(a^2-c^2)(a+c\sin(d+ex))^3}+\frac{\frac{(-2a^2C+5aAc-3c^2C)\cos(d+ex)}{2e(a^2-c^2)(a+c\sin(d+ex))^2}+\frac{\frac{6\left(2a^3A-4a^2cC+3aAc^2-c^3C\right)\arctan\left(\frac{2a\tan\left(\frac{1}{2}(d+ex)\right)+2c}{2\sqrt{a^2-c^2}}\right)}{e\left(a^2-c^2\right)^{3/2}}+\frac{\left(-2a^3C+11a^2Ac-13ac^2C+4Ac^3\right)\cos(d+ex)}{e\left(a^2-c^2\right)(a+c\sin(d+ex))}}{2(a^2-c^2)}}{3(a^2-c^2)}-\frac{B}{3ce(a+c\sin(d+ex))^3}$$

input
```
Int[(A + B*Cos[d + e*x] + C*Sin[d + e*x])/(a + c*Sin[d + e*x])^4,x]
```

output
```
-1/3*B/(c*e*(a + c*Sin[d + e*x])^3) + ((A*c - a*C)*Cos[d + e*x])/(3*(a^2 -
 c^2)*e*(a + c*Sin[d + e*x])^3) + (((5*a*A*c - 2*a^2*C - 3*c^2*C)*Cos[d +
e*x])/(2*(a^2 - c^2)*e*(a + c*Sin[d + e*x])^2) + ((6*(2*a^3*A + 3*a*A*c^2
- 4*a^2*c*C - c^3*C)*ArcTan[(2*c + 2*a*Tan[(d + e*x)/2])/(2*Sqrt[a^2 - c^2
])])/((a^2 - c^2)^(3/2)*e) + ((11*a^2*A*c + 4*A*c^3 - 2*a^3*C - 13*a*c^2*C
)*Cos[d + e*x])/((a^2 - c^2)*e*(a + c*Sin[d + e*x])))/(2*(a^2 - c^2)))/(3*
(a^2 - c^2))
```

3.565.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 4876
```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Sin[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Simp[d   Int[ActivateTrig[u]*Cos[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Sin[
c*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Intege
rQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

3.565. $\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{(a+c\sin(d+ex))^4}\,dx$

3.565.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 714 vs. $2(245) = 490$.

Time = 2.57 (sec) , antiderivative size = 715, normalized size of antiderivative = 2.77

method	result
parts	$\frac{\frac{c\left(9A\,a^4c-6A\,a^2c^3+2A\,c^5-4C\,a^5-C\,a^3c^2\right)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^5}{a\left(a^6-3a^4c^2+3a^2c^4-c^6\right)}+\frac{\left(6A\,a^6c+27A\,a^4c^3-12A\,a^2c^5+4A\,c^7-2C\,a^7-14C\,a^5c^2-11C\,a^3c^4+2C}{\left(a^6-3a^4c^2+3a^2c^4-c^6\right)a^2}}{}$
derivativedivides	$\frac{\frac{\left(9A\,a^4c^2-6A\,a^2c^4+2A\,c^6+2B\,a^6-6B\,a^4c^2+6B\,a^2c^4-2B\,c^6-4C\,a^5c-C\,a^3c^3\right)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^5}{a\left(a^6-3a^4c^2+3a^2c^4-c^6\right)}+\frac{\left(6A\,a^6c+27A\,a^4c^3-12A\,a^2c^5+4A}{}}{}$
default	$\frac{\frac{\left(9A\,a^4c^2-6A\,a^2c^4+2A\,c^6+2B\,a^6-6B\,a^4c^2+6B\,a^2c^4-2B\,c^6-4C\,a^5c-C\,a^3c^3\right)\tan\left(\frac{ex}{2}+\frac{d}{2}\right)^5}{a\left(a^6-3a^4c^2+3a^2c^4-c^6\right)}+\frac{\left(6A\,a^6c+27A\,a^4c^3-12A\,a^2c^5+4A}{}}{}$
risch	Expression too large to display

input
```
int((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^4,x,method=_RETURNVERBO
SE)
```

output
```
1/e*(2*(1/2*c*(9*A*a^4*c-6*A*a^2*c^3+2*A*c^5-4*C*a^5-C*a^3*c^2)/a/(a^6-3*a
^4*c^2+3*a^2*c^4-c^6)*tan(1/2*e*x+1/2*d)^5+1/2*(6*A*a^6*c+27*A*a^4*c^3-12*
A*a^2*c^5+4*A*c^7-2*C*a^7-14*C*a^5*c^2-11*C*a^3*c^4+2*C*a*c^6)/(a^6-3*a^4*
c^2+3*a^2*c^4-c^6)/a^2*tan(1/2*e*x+1/2*d)^4+1/3/a^3*c*(54*A*a^6*c+21*A*a^4
*c^3-4*A*a^2*c^5+4*A*c^7-18*C*a^7-42*C*a^5*c^2-17*C*a^3*c^4+2*C*a*c^6)/(a^
6-3*a^4*c^2+3*a^2*c^4-c^6)*tan(1/2*e*x+1/2*d)^3+1/a^2*(6*A*a^6*c+20*A*a^4*
c^3-3*A*a^2*c^5+2*A*c^7-2*C*a^7-10*C*a^5*c^2-14*C*a^3*c^4+C*a*c^6)/(a^6-3*
a^4*c^2+3*a^2*c^4-c^6)*tan(1/2*e*x+1/2*d)^2+1/2*c*(27*A*a^4*c-4*A*a^2*c^3+
2*A*c^5-8*C*a^5-19*C*a^3*c^2+2*C*a*c^4)/a/(a^6-3*a^4*c^2+3*a^2*c^4-c^6)*ta
n(1/2*e*x+1/2*d)+1/6*(18*A*a^4*c-5*A*a^2*c^3+2*A*c^5-6*C*a^5-10*C*a^3*c^2+
C*a*c^4)/(a^6-3*a^4*c^2+3*a^2*c^4-c^6))/(tan(1/2*e*x+1/2*d)^2*a+2*c*tan(1/
2*e*x+1/2*d)+a)^3+(2*A*a^3+3*A*a*c^2-4*C*a^2*c-C*c^3)/(a^6-3*a^4*c^2+3*a^2
*c^4-c^6)/(a^2-c^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*e*x+1/2*d)+2*c)/(a^2-c^2
)^(1/2)))-1/3*B/c/e/(a+c*sin(e*x+d))^3
```

3.565.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 664 vs. 2(245) = 490.

Time = 0.34 (sec) , antiderivative size = 1411, normalized size of antiderivative = 5.47

$$\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{(a+c\sin(d+ex))^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^4,x, algorithm="f
ricas")
```

output
```
[1/12*(4*B*a^8 - 16*B*a^6*c^2 + 24*B*a^4*c^4 - 16*B*a^2*c^6 + 4*B*c^8 - 2*
(2*C*a^5*c^3 - 11*A*a^4*c^4 + 11*C*a^3*c^5 + 7*A*a^2*c^6 - 13*C*a*c^7 + 4*
A*c^8)*cos(e*x + d)^3 + 6*(2*C*a^6*c^2 - 9*A*a^5*c^3 + 7*C*a^4*c^4 + 8*A*a
^3*c^5 - 10*C*a^2*c^6 + A*a*c^7 + C*c^8)*cos(e*x + d)*sin(e*x + d) + 3*(2*
A*a^6*c - 4*C*a^5*c^2 + 9*A*a^4*c^3 - 13*C*a^3*c^4 + 9*A*a^2*c^5 - 3*C*a*c
^6 - 3*(2*A*a^4*c^3 - 4*C*a^3*c^4 + 3*A*a^2*c^5 - C*a*c^6)*cos(e*x + d)^2
+ (6*A*a^5*c^2 - 12*C*a^4*c^3 + 11*A*a^3*c^4 - 7*C*a^2*c^5 + 3*A*a*c^6 - C
*c^7 - (2*A*a^3*c^4 - 4*C*a^2*c^5 + 3*A*a*c^6 - C*c^7)*cos(e*x + d)^2)*sin
(e*x + d))*sqrt(-a^2 + c^2)*log(((2*a^2 - c^2)*cos(e*x + d)^2 - 2*a*c*sin(
e*x + d) - a^2 - c^2 + 2*(a*cos(e*x + d)*sin(e*x + d) + c*cos(e*x + d))*sq
rt(-a^2 + c^2))/(c^2*cos(e*x + d)^2 - 2*a*c*sin(e*x + d) - a^2 - c^2)) + 1
2*(C*a^7*c - 3*A*a^6*c^2 + C*a^5*c^3 + 2*A*a^4*c^4 - 2*C*a*c^7 + A*c^8)*co
s(e*x + d))/(3*(a^9*c^3 - 4*a^7*c^5 + 6*a^5*c^7 - 4*a^3*c^9 + a*c^11)*e*co
s(e*x + d)^2 - (a^11*c - a^9*c^3 - 6*a^7*c^5 + 14*a^5*c^7 - 11*a^3*c^9 + 3
*a*c^11)*e + ((a^8*c^4 - 4*a^6*c^6 + 6*a^4*c^8 - 4*a^2*c^10 + c^12)*e*cos(
e*x + d)^2 - (3*a^10*c^2 - 11*a^8*c^4 + 14*a^6*c^6 - 6*a^4*c^8 - a^2*c^10
+ c^12)*e)*sin(e*x + d)), 1/6*(2*B*a^8 - 8*B*a^6*c^2 + 12*B*a^4*c^4 - 8*B*
a^2*c^6 + 2*B*c^8 - (2*C*a^5*c^3 - 11*A*a^4*c^4 + 11*C*a^3*c^5 + 7*A*a^2*c
^6 - 13*C*a*c^7 + 4*A*c^8)*cos(e*x + d)^3 + 3*(2*C*a^6*c^2 - 9*A*a^5*c^3 +
 7*C*a^4*c^4 + 8*A*a^3*c^5 - 10*C*a^2*c^6 + A*a*c^7 + C*c^8)*cos(e*x + ...
```

3.565.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{(a+c\sin(d+ex))^4}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))**4,x)
```

output
```
Timed out
```

3.565.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\cos(d + ex) + C\sin(d + ex)}{(a + c\sin(d + ex))^4} dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^4,x, algorithm="m
axima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c^2-4*a^2>0)', see `assume?` f
or more de
```

3.565.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1281 vs. $2(245) = 490$.

Time = 0.34 (sec) , antiderivative size = 1281, normalized size of antiderivative = 4.97

$$\int \frac{A + B\cos(d + ex) + C\sin(d + ex)}{(a + c\sin(d + ex))^4} dx = \text{Too large to display}$$

input
```
integrate((A+B*cos(e*x+d)+C*sin(e*x+d))/(a+c*sin(e*x+d))^4,x, algorithm="g
iac")
```

output
```
1/3*(3*(2*A*a^3 - 4*C*a^2*c + 3*A*a*c^2 - C*c^3)*(pi*floor(1/2*(e*x + d)/p
i + 1/2)*sgn(a) + arctan((a*tan(1/2*e*x + 1/2*d) + c)/sqrt(a^2 - c^2)))/((
a^6 - 3*a^4*c^2 + 3*a^2*c^4 - c^6)*sqrt(a^2 - c^2)) + (6*B*a^8*tan(1/2*e*x
 + 1/2*d)^5 - 12*C*a^7*c*tan(1/2*e*x + 1/2*d)^5 + 27*A*a^6*c^2*tan(1/2*e*x
 + 1/2*d)^5 - 18*B*a^6*c^2*tan(1/2*e*x + 1/2*d)^5 - 3*C*a^5*c^3*tan(1/2*e*
x + 1/2*d)^5 - 18*A*a^4*c^4*tan(1/2*e*x + 1/2*d)^5 + 18*B*a^4*c^4*tan(1/2*
e*x + 1/2*d)^5 + 6*A*a^2*c^6*tan(1/2*e*x + 1/2*d)^5 - 6*B*a^2*c^6*tan(1/2*
e*x + 1/2*d)^5 - 6*C*a^8*tan(1/2*e*x + 1/2*d)^4 + 18*A*a^7*c*tan(1/2*e*x +
 1/2*d)^4 + 12*B*a^7*c*tan(1/2*e*x + 1/2*d)^4 - 42*C*a^6*c^2*tan(1/2*e*x +
 1/2*d)^4 + 81*A*a^5*c^3*tan(1/2*e*x + 1/2*d)^4 - 36*B*a^5*c^3*tan(1/2*e*x
 + 1/2*d)^4 - 33*C*a^4*c^4*tan(1/2*e*x + 1/2*d)^4 - 36*A*a^3*c^5*tan(1/2*e
*x + 1/2*d)^4 + 36*B*a^3*c^5*tan(1/2*e*x + 1/2*d)^4 + 6*C*a^2*c^6*tan(1/2*
e*x + 1/2*d)^4 + 12*A*a*c^7*tan(1/2*e*x + 1/2*d)^4 - 12*B*a*c^7*tan(1/2*e*
x + 1/2*d)^4 + 12*B*a^8*tan(1/2*e*x + 1/2*d)^3 - 36*C*a^7*c*tan(1/2*e*x +
1/2*d)^3 + 108*A*a^6*c^2*tan(1/2*e*x + 1/2*d)^3 - 28*B*a^6*c^2*tan(1/2*e*x
 + 1/2*d)^3 - 84*C*a^5*c^3*tan(1/2*e*x + 1/2*d)^3 + 42*A*a^4*c^4*tan(1/2*e
*x + 1/2*d)^3 + 12*B*a^4*c^4*tan(1/2*e*x + 1/2*d)^3 - 34*C*a^3*c^5*tan(1/2
*e*x + 1/2*d)^3 - 8*A*a^2*c^6*tan(1/2*e*x + 1/2*d)^3 + 12*B*a^2*c^6*tan(1/
2*e*x + 1/2*d)^3 + 4*C*a*c^7*tan(1/2*e*x + 1/2*d)^3 + 8*A*c^8*tan(1/2*e*x
+ 1/2*d)^3 - 8*B*c^8*tan(1/2*e*x + 1/2*d)^3 - 12*C*a^8*tan(1/2*e*x + 1/...
```

3.565.9 Mupad [B] (verification not implemented)

Time = 30.50 (sec) , antiderivative size = 1085, normalized size of antiderivative = 4.21

$$\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{(a+c\sin(d+ex))^4}\,dx$$

$$= \frac{\frac{-6\,C\,a^5+18\,A\,a^4\,c-10\,C\,a^3\,c^2-5\,A\,a^2\,c^3+C\,a\,c^4+2\,A\,c^5}{3\,(a^6-3\,a^4\,c^2+3\,a^2\,c^4-c^6)}+\frac{\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\left(2\,B\,a^6+2\,A\,c^6-2\,B\,c^6-4\,A\,a^2\,c^4+27\,A\,a^4\,c^2+6\,B\,a^2\,c^4-6\,B\,a^4\,c^2-19\right.}{a\,(a^6-3\,a^4\,c^2+3\,a^2\,c^4-c^6)}}{}$$

$$+\frac{\mathrm{atan}\left(\frac{\left(\frac{\left(2\,A\,a^3-4\,C\,a^2\,c+3\,A\,a\,c^2-C\,c^3\right)\left(2\,a^6\,c-6\,a^4\,c^3+6\,a^2\,c^5-2\,c^7\right)}{2\,{(a+c)}^{7/2}\,{(a-c)}^{7/2}\,\left(a^6-3\,a^4\,c^2+3\,a^2\,c^4-c^6\right)}+\frac{a\tan\left(\frac{d}{2}+\frac{e\,x}{2}\right)\left(2\,A\,a^3-4\,C\,a^2\,c+3\,A\,a\,c^2-C\,c^3\right)}{{(a+c)}^{7/2}\,{(a-c)}^{7/2}}\right)\left(a^6-3\,a^4\,c^2+3\,a^2\,c^4-c\right.}{2\,A\,a^3-4\,C\,a^2\,c+3\,A\,a\,c^2-C\,c^3}\right.}{e\,{(a+c)}^{7/2}\,{(a-c)}^{7/2}}$$

input
```
int((A + B*cos(d + e*x) + C*sin(d + e*x))/(a + c*sin(d + e*x))^4,x)
```

output
```
((2*A*c^5 - 6*C*a^5 - 5*A*a^2*c^3 - 10*C*a^3*c^2 + 18*A*a^4*c + C*a*c^4)/(
3*(a^6 - c^6 + 3*a^2*c^4 - 3*a^4*c^2)) + (tan(d/2 + (e*x)/2)*(2*B*a^6 + 2*
A*c^6 - 2*B*c^6 - 4*A*a^2*c^4 + 27*A*a^4*c^2 + 6*B*a^2*c^4 - 6*B*a^4*c^2 -
 19*C*a^3*c^3 + 2*C*a*c^5 - 8*C*a^5*c))/(a*(a^6 - c^6 + 3*a^2*c^4 - 3*a^4*
c^2)) + (2*tan(d/2 + (e*x)/2)^2*(2*A*c^7 - 2*C*a^7 - 2*B*c^7 - 3*A*a^2*c^5
 + 20*A*a^4*c^3 + 6*B*a^2*c^5 - 6*B*a^4*c^3 - 14*C*a^3*c^4 - 10*C*a^5*c^2
+ 6*A*a^6*c + 2*B*a^6*c + C*a*c^6))/(a^2*(a^6 - c^6 + 3*a^2*c^4 - 3*a^4*c^
2)) + (tan(d/2 + (e*x)/2)^4*(4*A*c^7 - 2*C*a^7 - 4*B*c^7 - 12*A*a^2*c^5 +
27*A*a^4*c^3 + 12*B*a^2*c^5 - 12*B*a^4*c^3 - 11*C*a^3*c^4 - 14*C*a^5*c^2 +
 6*A*a^6*c + 4*B*a^6*c + 2*C*a*c^6))/(a^2*(a^6 - c^6 + 3*a^2*c^4 - 3*a^4*c
^2)) - (tan(d/2 + (e*x)/2)^5*(2*B*c^6 - 2*A*c^6 - 2*B*a^6 + 6*A*a^2*c^4 -
9*A*a^4*c^2 - 6*B*a^2*c^4 + 6*B*a^4*c^2 + C*a^3*c^3 + 4*C*a^5*c))/(a*(a^6
- c^6 + 3*a^2*c^4 - 3*a^4*c^2)) + (2*tan(d/2 + (e*x)/2)^3*(3*a^2 + 2*c^2)*
(2*B*a^6 + 2*A*c^6 - 2*B*c^6 - 5*A*a^2*c^4 + 18*A*a^4*c^2 + 6*B*a^2*c^4 -
6*B*a^4*c^2 - 10*C*a^3*c^3 + C*a*c^5 - 6*C*a^5*c))/(3*a^3*(a^6 - c^6 + 3*a
^2*c^4 - 3*a^4*c^2)))/(e*(a^3*tan(d/2 + (e*x)/2)^6 + tan(d/2 + (e*x)/2)^2*
(12*a*c^2 + 3*a^3) + tan(d/2 + (e*x)/2)^4*(12*a*c^2 + 3*a^3) + tan(d/2 + (
e*x)/2)^3*(12*a^2*c + 8*c^3) + a^3 + 6*a^2*c*tan(d/2 + (e*x)/2) + 6*a^2*c*
tan(d/2 + (e*x)/2)^5)) + (atan(((((2*A*a^3 - C*c^3 + 3*A*a*c^2 - 4*C*a^2*c
)*(2*a^6*c - 2*c^7 + 6*a^2*c^5 - 6*a^4*c^3))/(2*(a + c)^(7/2)*(a - c)^(...
```

3.565. $\int \frac{A+B\cos(d+ex)+C\sin(d+ex)}{(a+c\sin(d+ex))^4}\,dx$

3.566 $\int (a+b\cos(c+dx)\sin(c+dx))^m\,dx$

3.566.1 Optimal result

Integrand size = 18, antiderivative size = 131

$$\int (a+b\cos(c+dx)\sin(c+dx))^m\,dx = -\frac{\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-m,\frac{3}{2},\frac{1}{2}(1-\sin(2c+2dx)),\frac{b(1-\sin(2c+2dx))}{2a+b}\right)\cos(2c+2dx)\left(a+\frac{1}{2}b\sin(2c+2dx)\right)^m\left(2\right.}{\sqrt{2}d\sqrt{1+\sin(2c+2dx)}}$$

output
```
-1/2*AppellF1(1/2,-m,1/2,3/2,b*(1-sin(2*d*x+2*c))/(2*a+b),1/2-1/2*sin(2*d*
x+2*c))*cos(2*d*x+2*c)*(a+1/2*b*sin(2*d*x+2*c))^m/d/(((2*a+b*sin(2*d*x+2*c
))/(2*a+b))^m)*2^(1/2)/(1+sin(2*d*x+2*c))^(1/2)
```

3.566.2 Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.11

$$\int (a+b\cos(c+dx)\sin(c+dx))^m\,dx = \frac{\text{AppellF1}\left(1+m,\frac{1}{2},\frac{1}{2},2+m,\frac{2a+b\sin(2(c+dx))}{2a-b},\frac{2a+b\sin(2(c+dx))}{2a+b}\right)\sec(2(c+dx))\sqrt{-\frac{b(-1+\sin(2(c+dx)))}{2a+b}}\sqrt{\frac{b(1+\sin(\cdots}{-2\cdots}}}{bd(1+m)}$$

input
```
Integrate[(a + b*Cos[c + d*x]*Sin[c + d*x])^m,x]
```

output
```
(AppellF1[1 + m, 1/2, 1/2, 2 + m, (2*a + b*Sin[2*(c + d*x)])/(2*a - b), (2
*a + b*Sin[2*(c + d*x)])/(2*a + b)]*Sec[2*(c + d*x)]*Sqrt[-((b*(-1 + Sin[2
*(c + d*x)]))/(2*a + b))]*Sqrt[(b*(1 + Sin[2*(c + d*x)]))/(-2*a + b)]*(a +
 (b*Sin[2*(c + d*x)])/2)^(1 + m))/(b*d*(1 + m))
```

3.566.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3145, 3042, 3144, 156, 155}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sin(c+dx)\cos(c+dx))^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\sin(c+dx)\cos(c+dx))^m dx$$

$$\Big\downarrow \text{3145}$$

$$\int \left(a+\frac{1}{2}b\sin(2c+2dx)\right)^m dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+\frac{1}{2}b\sin(2c+2dx)\right)^m dx$$

$$\Big\downarrow \text{3144}$$

$$\frac{\cos(2c+2dx)\int \frac{\left(a+\frac{1}{2}b\sin(2c+2dx)\right)^m}{\sqrt{1-\sin(2c+2dx)}\sqrt{\sin(2c+2dx)+1}}d\sin(2c+2dx)}{2d\sqrt{1-\sin(2c+2dx)}\sqrt{\sin(2c+2dx)+1}}$$

$$\Big\downarrow \text{156}$$

$$\frac{\cos(2c+2dx)\left(a+\frac{1}{2}b\sin(2c+2dx)\right)^m\left(\frac{2a+b\sin(2c+2dx)}{2a+b}\right)^{-m}\int \frac{\left(\frac{2a}{2a+b}+\frac{b\sin(2c+2dx)}{2a+b}\right)^m}{\sqrt{1-\sin(2c+2dx)}\sqrt{\sin(2c+2dx)+1}}d\sin(2c+2dx)}{2d\sqrt{1-\sin(2c+2dx)}\sqrt{\sin(2c+2dx)+1}}$$

$$\Big\downarrow \text{155}$$

$$-\frac{\cos(2c+2dx)\left(a+\frac{1}{2}b\sin(2c+2dx)\right)^m\left(\frac{2a+b\sin(2c+2dx)}{2a+b}\right)^{-m}\text{AppellF1}\left(\frac{1}{2},\frac{1}{2},-m,\frac{3}{2},\frac{1}{2}(1-\sin(2c+2dx)),\frac{b(1-\sin}{2}\right.}{\sqrt{2}d\sqrt{\sin(2c+2dx)+1}}$$

input
```
Int[(a + b*Cos[c + d*x]*Sin[c + d*x])^m,x]
```

output
```
-((AppellF1[1/2, 1/2, -m, 3/2, (1 - Sin[2*c + 2*d*x])/2, (b*(1 - Sin[2*c +
 2*d*x]))/(2*a + b)]*Cos[2*c + 2*d*x]*(a + (b*Sin[2*c + 2*d*x])/2)^m)/(Sqr
t[2]*d*Sqrt[1 + Sin[2*c + 2*d*x]]*((2*a + b*Sin[2*c + 2*d*x])/(2*a + b))^m
))
```

3.566.3.1 Defintions of rubi rules used

rule 155
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[((a + b*x)^(m + 1)/(b*(m + 1)*Simplify[b/(b*c - a*d)]^n*
Simplify[b/(b*e - a*f)]^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/
(b*c - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f,
 m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[Sim
plify[b/(b*c - a*d)], 0] && GtQ[Simplify[b/(b*e - a*f)], 0] && !(GtQ[Simpl
ify[d/(d*a - c*b)], 0] && GtQ[Simplify[d/(d*e - c*f)], 0] && SimplerQ[c + d
*x, a + b*x]) && !(GtQ[Simplify[f/(f*a - e*b)], 0] && GtQ[Simplify[f/(f*c
- e*d)], 0] && SimplerQ[e + f*x, a + b*x])
```

rule 156
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(e + f*x)^FracPart[p]/(Simplify[b/(b*e - a*f)]^IntPart[p
]*(b*((e + f*x)/(b*e - a*f)))^FracPart[p])   Int[(a + b*x)^m*(c + d*x)^n*Si
mp[b*(e/(b*e - a*f)) + b*f*(x/(b*e - a*f)), x]^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] &
& GtQ[Simplify[b/(b*c - a*d)], 0] && !GtQ[Simplify[b/(b*e - a*f)], 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3144
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]])   Subst[Int[(a + b*x
)^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c,
d, n}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*n]
```

rule 3145
```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + b*(Sin[2*c + 2*d*x]/2))^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

3.566.4 Maple [F]

$$\int (a + \cos(dx + c)\sin(dx + c)\, b)^m\, dx$$

input
```
int((a+cos(d*x+c)*sin(d*x+c)*b)^m,x)
```

output
```
int((a+cos(d*x+c)*sin(d*x+c)*b)^m,x)
```

3.566.5 Fricas [F]

$$\int (a + b\cos(c + dx)\sin(c + dx))^m\, dx = \int (b\cos(dx + c)\sin(dx + c) + a)^m\, dx$$

input
```
integrate((a+b*cos(d*x+c)*sin(d*x+c))^m,x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c)*sin(d*x + c) + a)^m, x)
```

3.566.6 Sympy [F(-1)]

Timed out.

$$\int (a + b\cos(c + dx)\sin(c + dx))^m\, dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c)*sin(d*x+c))**m,x)
```

output
```
Timed out
```

3.566.7 Maxima [F]

$$\int (a+b\cos(c+dx)\sin(c+dx))^m\,dx = \int (b\cos(dx+c)\sin(dx+c)+a)^m\,dx$$

input
```
integrate((a+b*cos(d*x+c)*sin(d*x+c))^m,x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c)*sin(d*x + c) + a)^m, x)
```

3.566.8 Giac [F]

$$\int (a+b\cos(c+dx)\sin(c+dx))^m\,dx = \int (b\cos(dx+c)\sin(dx+c)+a)^m\,dx$$

input
```
integrate((a+b*cos(d*x+c)*sin(d*x+c))^m,x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c)*sin(d*x + c) + a)^m, x)
```

3.566.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx)\sin(c+dx))^m\,dx = \int (a+b\,\cos(c+d\,x)\,\sin(c+d\,x))^m\,dx$$

input
```
int((a + b*cos(c + d*x)*sin(c + d*x))^m,x)
```

output
```
int((a + b*cos(c + d*x)*sin(c + d*x))^m, x)
```

3.567 $\int (a+b\cos(c+dx)\sin(c+dx))^3\,dx$

3.567.1 Optimal result

Integrand size = 18, antiderivative size = 107

$$\int (a+b\cos(c+dx)\sin(c+dx))^3\,dx=\frac{1}{8}a\left(8a^2+3b^2\right)x-\frac{b(16a^2+b^2)\cos(2c+2dx)}{24d}-\frac{5ab^2\cos(2c+2dx)\sin(2c+2dx)}{48d}-\frac{b\cos(2c+2dx)(2a+b\sin(2c+2dx))^2}{48d}$$

output
```
1/8*a*(8*a^2+3*b^2)*x-1/24*b*(16*a^2+b^2)*cos(2*d*x+2*c)/d-5/48*a*b^2*cos(
2*d*x+2*c)*sin(2*d*x+2*c)/d-1/48*b*cos(2*d*x+2*c)*(2*a+b*sin(2*d*x+2*c))^2
/d
```

3.567.2 Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.70

$$\int (a+b\cos(c+dx)\sin(c+dx))^3\,dx =\frac{-9(16a^2b+b^3)\cos(2(c+dx))+b^3\cos(6(c+dx))+6a\left(4\left(8a^2+3b^2\right)(c+dx)-3b^2\sin(4(c+dx))\right)}{192d}$$

input
```
Integrate[(a + b*Cos[c + d*x]*Sin[c + d*x])^3,x]
```

output
```
(-9*(16*a^2*b + b^3)*Cos[2*(c + d*x)] + b^3*Cos[6*(c + d*x)] + 6*a*(4*(8*a
^2 + 3*b^2)*(c + d*x) - 3*b^2*Sin[4*(c + d*x)]))/(192*d)
```

3.567.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3042, 3145, 3042, 3135, 27, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sin(c+dx)\cos(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\sin(c+dx)\cos(c+dx))^3dx$$

$$\Big\downarrow \text{3145}$$

$$\int \left(a+\frac{1}{2}b\sin(2c+2dx)\right)^3 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+\frac{1}{2}b\sin(2c+2dx)\right)^3 dx$$

$$\Big\downarrow \text{3135}$$

$$\frac{1}{3}\int \frac{1}{4}(2a+b\sin(2c+2dx))\left(6a^2+5b\sin(2c+2dx)a+b^2\right)dx-\frac{b\cos(2c+2dx)(2a+b\sin(2c+2dx))^2}{48d}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{12}\int (2a+b\sin(2c+2dx))\left(6a^2+5b\sin(2c+2dx)a+b^2\right)dx-\frac{b\cos(2c+2dx)(2a+b\sin(2c+2dx))^2}{48d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{12}\int (2a+b\sin(2c+2dx))\left(6a^2+5b\sin(2c+2dx)a+b^2\right)dx-\frac{b\cos(2c+2dx)(2a+b\sin(2c+2dx))^2}{48d}$$

$$\Big\downarrow \text{3213}$$

$$\frac{1}{12}\left(-\frac{b(16a^2+b^2)\cos(2c+2dx)}{2d}+\frac{3}{2}ax(8a^2+3b^2)-\frac{5ab^2\sin(2c+2dx)\cos(2c+2dx)}{4d}\right)-\frac{b\cos(2c+2dx)(2a+b\sin(2c+2dx))^2}{48d}$$

input
```
Int[(a + b*Cos[c + d*x]*Sin[c + d*x])^3,x]
```

output
```
-1/48*(b*Cos[2*c + 2*d*x]*(2*a + b*Sin[2*c + 2*d*x])^2)/d + ((3*a*(8*a^2 +
 3*b^2)*x)/2 - (b*(16*a^2 + b^2)*Cos[2*c + 2*d*x])/(2*d) - (5*a*b^2*Cos[2*
c + 2*d*x]*Sin[2*c + 2*d*x])/(4*d))/12
```

3.567.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3135
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[1/n   Int[(a + b*
Sin[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x]
, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] &&
 IntegerQ[2*n]
```

rule 3145
```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + b*(Sin[2*c + 2*d*x]/2))^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

3.567.4 Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.79

method	result
risch	$a^3x+\frac{3a\,b^2x}{8}+\frac{b^3\cos(6dx+6c)}{192d}-\frac{3a\,b^2\sin(4dx+4c)}{32d}-\frac{3b\cos(2dx+2c)a^2}{4d}-\frac{3b^3\cos(2dx+2c)}{64d}$
parallelrisch	$\frac{192a^3xd+72a\,b^2dx+b^3\cos(6dx+6c)-18a\,b^2\sin(4dx+4c)-144\cos(2dx+2c)a^2b-9\cos(2dx+2c)b^3+144a^2b+8b^3}{192d}$
parts	$a^3x+\frac{b^3\left(-\frac{\sin(dx+c)^6}{6}+\frac{\sin(dx+c)^4}{4}\right)}{d}+\frac{3a^2b\sin(dx+c)^2}{2d}+\frac{3a\,b^2\left(-\frac{\sin(dx+c)\cos(dx+c)^3}{4}+\frac{\sin(dx+c)\cos(dx+c)}{8}+\frac{dx}{8}\right.}{d}$
derivativedivides	$\frac{a^3(dx+c)-\frac{3\cos(dx+c)^2a^2b}{2}+3a\,b^2\left(-\frac{\sin(dx+c)\cos(dx+c)^3}{4}+\frac{\sin(dx+c)\cos(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)+b^3\left(-\frac{\sin(dx+c)^2\cos(dx+c)^4}{6}-\right.}{d}$
default	$\frac{a^3(dx+c)-\frac{3\cos(dx+c)^2a^2b}{2}+3a\,b^2\left(-\frac{\sin(dx+c)\cos(dx+c)^3}{4}+\frac{\sin(dx+c)\cos(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)+b^3\left(-\frac{\sin(dx+c)^2\cos(dx+c)^4}{6}-\right.}{d}$
norman	$\frac{(a^3+\frac{3}{8}a\,b^2)x+(a^3+\frac{3}{8}a\,b^2)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{12}+(6a^3+\frac{9}{4}a\,b^2)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+(6a^3+\frac{9}{4}a\,b^2)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}+(15a^3+\frac{45}{8}a}{}$

input
```
int((a+cos(d*x+c)*sin(d*x+c)*b)^3,x,method=_RETURNVERBOSE)
```

output
```
a^3*x+3/8*a*b^2*x+1/192*b^3/d*cos(6*d*x+6*c)-3/32*a*b^2/d*sin(4*d*x+4*c)-3
/4*b*cos(2*d*x+2*c)/d*a^2-3/64*b^3*cos(2*d*x+2*c)/d
```

3.567.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.91

$$\int (a+b\cos(c+dx)\sin(c+dx))^3\,dx$$
$$=\frac{4\,b^3\cos{(dx+c)}^6-6\,b^3\cos{(dx+c)}^4-36\,a^2b\cos{(dx+c)}^2+3\,(8\,a^3+3\,ab^2)dx-9\,\left(2\,ab^2\cos{(dx+c)}^3-\right.}{24\,d}$$

input
```
integrate((a+b*cos(d*x+c)*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/24*(4*b^3*cos(d*x + c)^6 - 6*b^3*cos(d*x + c)^4 - 36*a^2*b*cos(d*x + c)^
2 + 3*(8*a^3 + 3*a*b^2)*d*x - 9*(2*a*b^2*cos(d*x + c)^3 - a*b^2*cos(d*x +
c))*sin(d*x + c))/d
```

3.567.6 Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.78

$$\int (a+b\cos(c+dx)\sin(c+dx))^3\,dx$$
$$=\begin{cases} a^3x-\frac{3a^2b\cos^2(c+dx)}{2d}+\frac{3ab^2x\sin^4(c+dx)}{8}+\frac{3ab^2x\sin^2(c+dx)\cos^2(c+dx)}{4}+\frac{3ab^2x\cos^4(c+dx)}{8}+\frac{3ab^2\sin^3(c+dx)\cos(c+dx)}{8d}- \\ x(a+b\sin(c)\cos(c))^3 \end{cases}$$

input
```
integrate((a+b*cos(d*x+c)*sin(d*x+c))**3,x)
```

output
```
Piecewise((a**3*x - 3*a**2*b*cos(c + d*x)**2/(2*d) + 3*a*b**2*x*sin(c + d*
x)**4/8 + 3*a*b**2*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*a*b**2*x*cos(c
+ d*x)**4/8 + 3*a*b**2*sin(c + d*x)**3*cos(c + d*x)/(8*d) - 3*a*b**2*sin(c
 + d*x)*cos(c + d*x)**3/(8*d) + b**3*sin(c + d*x)**6/(12*d) + b**3*sin(c +
 d*x)**4*cos(c + d*x)**2/(4*d), Ne(d, 0)), (x*(a + b*sin(c)*cos(c))**3, Tr
ue))
```

3.567.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.75

$$\int (a+b\cos(c+dx)\sin(c+dx))^3\,dx = a^3x-\frac{3\,a^2b\cos(dx+c)^2}{2\,d}+\frac{3\,(4\,dx+4\,c-\sin(4\,dx+4\,c))ab^2}{32\,d}-\frac{(2\,\sin(dx+c)^6-3\,\sin(dx+c)^4)b^3}{12\,d}$$

input
```
integrate((a+b*cos(d*x+c)*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
a^3*x - 3/2*a^2*b*cos(d*x + c)^2/d + 3/32*(4*d*x + 4*c - sin(4*d*x + 4*c))
*a*b^2/d - 1/12*(2*sin(d*x + c)^6 - 3*sin(d*x + c)^4)*b^3/d
```

3.567.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.70

$$\int (a+b\cos(c+dx)\sin(c+dx))^3\,dx = \frac{b^3\cos(6\,dx+6\,c)}{192\,d} - \frac{3\,ab^2\sin(4\,dx+4\,c)}{32\,d} + \frac{1}{8}\left(8\,a^3+3\,ab^2\right)x - \frac{3\,(16\,a^2b+b^3)\cos(2\,dx+2\,c)}{64\,d}$$

input
```
integrate((a+b*cos(d*x+c)*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
1/192*b^3*cos(6*d*x + 6*c)/d - 3/32*a*b^2*sin(4*d*x + 4*c)/d + 1/8*(8*a^3
+ 3*a*b^2)*x - 3/64*(16*a^2*b + b^3)*cos(2*d*x + 2*c)/d
```

3.567.9 Mupad [B] (verification not implemented)

Time = 27.19 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.17

$$\int (a+b\cos(c+dx)\sin(c+dx))^3\,dx = a^3\,x - \frac{\tan(c+d\,x)^2\,(72\,a^2\,b+6\,b^3)+36\,a^2\,b+2\,b^3+36\,a^2\,b\tan(c+d\,x)^4-9\,a\,b^2\tan(c+d\,x)^5+9\,a\,b^2\tan(c}{d\,\left(24\tan(c+d\,x)^6+72\tan(c+d\,x)^4+72\tan(c+d\,x)^2+24\right)} + \frac{3\,a\,b^2\,x}{8}$$

input
```
int((a + b*cos(c + d*x)*sin(c + d*x))^3,x)
```

output
```
a^3*x - (tan(c + d*x)^2*(72*a^2*b + 6*b^3) + 36*a^2*b + 2*b^3 + 36*a^2*b*t
an(c + d*x)^4 - 9*a*b^2*tan(c + d*x)^5 + 9*a*b^2*tan(c + d*x))/(d*(72*tan(
c + d*x)^2 + 72*tan(c + d*x)^4 + 24*tan(c + d*x)^6 + 24)) + (3*a*b^2*x)/8
```

3.568 $\int (a+b\cos(c+dx)\sin(c+dx))^2\,dx$

3.568.1 Optimal result

Integrand size = 18, antiderivative size = 61

$$\int (a+b\cos(c+dx)\sin(c+dx))^2\,dx=\frac{1}{8}\left(8a^2+b^2\right)x-\frac{ab\cos(2c+2dx)}{2d}-\frac{b^2\cos(2c+2dx)\sin(2c+2dx)}{16d}$$

output
```
1/8*(8*a^2+b^2)*x-1/2*a*b*cos(2*d*x+2*c)/d-1/16*b^2*cos(2*d*x+2*c)*sin(2*d
*x+2*c)/d
```

3.568.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.79

$$\int (a+b\cos(c+dx)\sin(c+dx))^2\,dx=-\frac{-4(8a^2+b^2)(c+dx)+16ab\cos(2(c+dx))+b^2\sin(4(c+dx))}{32d}$$

input
```
Integrate[(a + b*Cos[c + d*x]*Sin[c + d*x])^2,x]
```

output
```
-1/32*(-4*(8*a^2 + b^2)*(c + d*x) + 16*a*b*Cos[2*(c + d*x)] + b^2*Sin[4*(c
 + d*x)])/d
```

3.568.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3042, 3145, 3042, 3123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sin(c+dx)\cos(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\sin(c+dx)\cos(c+dx))^2dx$$

$$\Big\downarrow \text{3145}$$

$$\int \left(a+\frac{1}{2}b\sin(2c+2dx)\right)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+\frac{1}{2}b\sin(2c+2dx)\right)^2\,dx$$

$$\Big\downarrow \text{3123}$$

$$\frac{1}{8}x\left(8a^2+b^2\right)-\frac{ab\cos(2c+2dx)}{2d}-\frac{b^2\sin(2c+2dx)\cos(2c+2dx)}{16d}$$

input
```
Int[(a + b*Cos[c + d*x]*Sin[c + d*x])^2,x]
```

output
```
((8*a^2 + b^2)*x)/8 - (a*b*Cos[2*c + 2*d*x])/(2*d) - (b^2*Cos[2*c + 2*d*x]
*Sin[2*c + 2*d*x])/(16*d)
```

3.568.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3123
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(2*a^2 + b^
2)*(x/2), x] + (-Simp[2*a*b*(Cos[c + d*x]/d), x] - Simp[b^2*Cos[c + d*x]*(S
in[c + d*x]/(2*d)), x]) /; FreeQ[{a, b, c, d}, x]
```

rule 3145
```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + b*(Sin[2*c + 2*d*x]/2))^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

3.568.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

method	result
risch	$a^2x+\frac{x\,b^2}{8}-\frac{b^2\sin(4dx+4c)}{32d}-\frac{ab\cos(2dx+2c)}{2d}$
parallelrisch	$\frac{32a^2dx+4b^2dx-b^2\sin(4dx+4c)-16ab\cos(2dx+2c)+16ab}{32d}$
parts	$a^2x+\frac{b^2\left(-\frac{\sin(dx+c)\cos(dx+c)^3}{4}+\frac{\sin(dx+c)\cos(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)}{d}+\frac{ab\sin(dx+c)^2}{d}$
derivativedivides	$\frac{a^2(dx+c)-\cos(dx+c)^2ab+b^2\left(-\frac{\sin(dx+c)\cos(dx+c)^3}{4}+\frac{\sin(dx+c)\cos(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)}{d}$
default	$\frac{a^2(dx+c)-\cos(dx+c)^2ab+b^2\left(-\frac{\sin(dx+c)\cos(dx+c)^3}{4}+\frac{\sin(dx+c)\cos(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)}{d}$
norman	$\left(a^2+\frac{b^2}{8}\right)x+\left(a^2+\frac{b^2}{8}\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8+\left(4a^2+\frac{b^2}{2}\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\left(4a^2+\frac{b^2}{2}\right)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+\left(6a^2+\frac{3b^2}{4}\right)x\tan\left(\frac{dx}{2}+\right.$ ... $\left(1+\right.$

input
```
int((a+cos(d*x+c)*sin(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*x+1/8*x*b^2-1/32*b^2/d*sin(4*d*x+4*c)-1/2*a*b*cos(2*d*x+2*c)/d
```

3.568.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.03

$$\int (a+b\cos(c+dx)\sin(c+dx))^2\,dx$$
$$= -\frac{8\,ab\cos\left(dx+c\right)^2 - (8\,a^2+b^2)dx + \left(2\,b^2\cos\left(dx+c\right)^3 - b^2\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{8\,d}$$

input
```
integrate((a+b*cos(d*x+c)*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/8*(8*a*b*cos(d*x + c)^2 - (8*a^2 + b^2)*d*x + (2*b^2*cos(d*x + c)^3 - b
^2*cos(d*x + c))*sin(d*x + c))/d
```

3.568.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(53) = 106.

Time = 0.18 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.11

$$\int (a+b\cos(c+dx)\sin(c+dx))^2\,dx$$
$$= \begin{cases} a^2x - \frac{ab\cos^2(c+dx)}{d} + \frac{b^2x\sin^4(c+dx)}{8} + \frac{b^2x\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{b^2x\cos^4(c+dx)}{8} + \frac{b^2\sin^3(c+dx)\cos(c+dx)}{8d} - \frac{b^2\sin(c+d}{} \\ x(a+b\sin(c)\cos(c))^2 \end{cases}$$

input
```
integrate((a+b*cos(d*x+c)*sin(d*x+c))**2,x)
```

output
```
Piecewise((a**2*x - a*b*cos(c + d*x)**2/d + b**2*x*sin(c + d*x)**4/8 + b**
2*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + b**2*x*cos(c + d*x)**4/8 + b**2*si
n(c + d*x)**3*cos(c + d*x)/(8*d) - b**2*sin(c + d*x)*cos(c + d*x)**3/(8*d)
, Ne(d, 0)), (x*(a + b*sin(c)*cos(c))**2, True))
```

3.568.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.79

$$\int (a+b\cos(c+dx)\sin(c+dx))^2\,dx = a^2x - \frac{ab\cos(dx+c)^2}{d} + \frac{(4\,dx+4\,c-\sin(4\,dx+4\,c))b^2}{32\,d}$$

input
```
integrate((a+b*cos(d*x+c)*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
a^2*x - a*b*cos(d*x + c)^2/d + 1/32*(4*d*x + 4*c - sin(4*d*x + 4*c))*b^2/d
```

3.568.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int (a+b\cos(c+dx)\sin(c+dx))^2\,dx = \frac{1}{8}\left(8\,a^2+b^2\right)x - \frac{ab\cos(2\,dx+2\,c)}{2\,d} - \frac{b^2\sin(4\,dx+4\,c)}{32\,d}$$

input
```
integrate((a+b*cos(d*x+c)*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
1/8*(8*a^2 + b^2)*x - 1/2*a*b*cos(2*d*x + 2*c)/d - 1/32*b^2*sin(4*d*x + 4*
c)/d
```

3.568.9 Mupad [B] (verification not implemented)

Time = 27.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.28

$$\int (a+b\cos(c+dx)\sin(c+dx))^2\,dx = x\left(a^2+\frac{b^2}{8}\right) - \frac{-\frac{b^2\tan(c+d\,x)^3}{8}+\frac{b^2\tan(c+d\,x)}{8}+a\,b\tan(c+d\,x)^2+a\,b}{d\,\left(\tan\left(c+d\,x\right)^4+2\tan\left(c+d\,x\right)^2+1\right)}$$

input
```
int((a + b*cos(c + d*x)*sin(c + d*x))^2,x)
```

output
```
x*(a^2 + b^2/8) - (a*b + (b^2*tan(c + d*x))/8 - (b^2*tan(c + d*x)^3)/8 + a
*b*tan(c + d*x)^2)/(d*(2*tan(c + d*x)^2 + tan(c + d*x)^4 + 1))
```

3.569 $\int (a + b\cos(c + dx)\sin(c + dx))\,dx$

3.569.1 Optimal result

Integrand size = 16, antiderivative size = 20

$$\int (a + b\cos(c + dx)\sin(c + dx))\,dx = ax + \frac{b\sin^2(c + dx)}{2d}$$

output
```
a*x+1/2*b*sin(d*x+c)^2/d
```

3.569.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.90

$$\int (a + b\cos(c + dx)\sin(c + dx))\,dx = ax - \frac{b\cos(2c)\cos(2dx)}{4d} + \frac{b\sin(2c)\sin(2dx)}{4d}$$

input
```
Integrate[a + b*Cos[c + d*x]*Sin[c + d*x],x]
```

output
```
a*x - (b*Cos[2*c]*Cos[2*d*x])/(4*d) + (b*Sin[2*c]*Sin[2*d*x])/(4*d)
```

3.569.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sin(c+dx)\cos(c+dx))\,dx$$

$$\Big\downarrow \text{2009}$$

$$ax+\frac{b\sin^2(c+dx)}{2d}$$

input
```
Int[a + b*Cos[c + d*x]*Sin[c + d*x],x]
```

output
```
a*x + (b*Sin[c + d*x]^2)/(2*d)
```

3.569.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.569.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
default	$ax+\frac{b\sin(dx+c)^2}{2d}$	19
parts	$ax+\frac{b\sin(dx+c)^2}{2d}$	19
risch	$ax-\frac{b\cos(2dx+2c)}{4d}$	20
derivativedivides	$\frac{(dx+c)a+\frac{b\sin(dx+c)^2}{2}}{d}$	24
parallelrisch	$\frac{b(1-\cos(2dx+2c))}{4d}+ax$	24
norman	$\frac{ax+ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+2ax\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+\frac{2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{d}}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^2}$	67

input
```
int(a+cos(d*x+c)*sin(d*x+c)*b,x,method=_RETURNVERBOSE)
```

output
```
a*x+1/2*b*sin(d*x+c)^2/d
```

3.569.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (a+b\cos(c+dx)\sin(c+dx))\,dx=\frac{2\,adx-b\cos{(dx+c)}^2}{2\,d}$$

input
```
integrate(a+b*cos(d*x+c)*sin(d*x+c),x, algorithm="fricas")
```

output
```
1/2*(2*a*d*x - b*cos(d*x + c)^2)/d
```

3.569.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int (a+b\cos(c+dx)\sin(c+dx))\,dx=ax+b\left(\begin{cases}-\frac{\cos^2{(c+dx)}}{2d} & \text{for } d\neq 0\\ x\sin{(c)}\cos{(c)} & \text{otherwise}\end{cases}\right)$$

input
```
integrate(a+b*cos(d*x+c)*sin(d*x+c),x)
```

output
```
a*x + b*Piecewise((-cos(c + d*x)**2/(2*d), Ne(d, 0)), (x*sin(c)*cos(c), Tr
ue))
```

3.569.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int (a+b\cos(c+dx)\sin(c+dx))\,dx=ax-\frac{b\cos{(dx+c)}^2}{2\,d}$$

input
```
integrate(a+b*cos(d*x+c)*sin(d*x+c),x, algorithm="maxima")
```

output
```
a*x - 1/2*b*cos(d*x + c)^2/d
```

3.569.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int (a + b\cos(c + dx)\sin(c + dx))\,dx = ax + \frac{b\sin{(dx + c)}^2}{2\,d}$$

input
```
integrate(a+b*cos(d*x+c)*sin(d*x+c),x, algorithm="giac")
```

output
```
a*x + 1/2*b*sin(d*x + c)^2/d
```

3.569.9 Mupad [B] (verification not implemented)

Time = 26.77 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (a + b\cos(c + dx)\sin(c + dx))\,dx = -\frac{\frac{b\cos(c+d\,x)^2}{2} - a\,d\,x}{d}$$

input
```
int(a + b*cos(c + d*x)*sin(c + d*x),x)
```

output
```
-((b*cos(c + d*x)^2)/2 - a*d*x)/d
```

3.570 $\int \frac{1}{a+b\cos(c+dx)\sin(c+dx)}\,dx$

3.570.1 Optimal result

Integrand size = 18, antiderivative size = 48

$$\int \frac{1}{a+b\cos(c+dx)\sin(c+dx)}\,dx = \frac{2\arctan\left(\frac{b+2a\tan(c+dx)}{\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}d}$$

output
```
2*arctan((b+2*a*tan(d*x+c))/(4*a^2-b^2)^(1/2))/d/(4*a^2-b^2)^(1/2)
```

3.570.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{1}{a+b\cos(c+dx)\sin(c+dx)}\,dx = \frac{2\arctan\left(\frac{b+2a\tan(c+dx)}{\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}d}$$

input
```
Integrate[(a + b*Cos[c + d*x]*Sin[c + d*x])^(-1),x]
```

output
```
(2*ArcTan[(b + 2*a*Tan[c + d*x])/Sqrt[4*a^2 - b^2]])/(Sqrt[4*a^2 - b^2]*d)
```

3.570.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3145, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\sin(c+dx)\cos(c+dx)}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{a+b\sin(c+dx)\cos(c+dx)}dx$$

$$\Big\downarrow \ 3145$$

$$\int \frac{1}{a+\frac{1}{2}b\sin(2c+2dx)}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{a+\frac{1}{2}b\sin(2c+2dx)}dx$$

$$\Big\downarrow \ 3139$$

$$\frac{\int \frac{1}{a\tan^2(c+dx)+b\tan(c+dx)+a}d\tan(c+dx)}{d}$$

$$\Big\downarrow \ 1083$$

$$-\frac{2\int \frac{1}{-4a^2+b^2-(b+2a\tan(c+dx))^2}d(b+2a\tan(c+dx))}{d}$$

$$\Big\downarrow \ 217$$

$$\frac{2\arctan\left(\frac{2a\tan(c+dx)+b}{\sqrt{4a^2-b^2}}\right)}{d\sqrt{4a^2-b^2}}$$

input
```
Int[(a + b*Cos[c + d*x]*Sin[c + d*x])^(-1),x]
```

output
```
(2*ArcTan[(b + 2*a*Tan[c + d*x])/Sqrt[4*a^2 - b^2]])/(Sqrt[4*a^2 - b^2]*d)
```

3.570.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3145
```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + b*(Sin[2*c + 2*d*x]/2))^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

3.570.4 Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$\frac{2\arctan\left(\frac{b+2a\tan(dx+c)}{\sqrt{4a^2-b^2}}\right)}{d\sqrt{4a^2-b^2}}$	45
default	$\frac{2\arctan\left(\frac{b+2a\tan(dx+c)}{\sqrt{4a^2-b^2}}\right)}{d\sqrt{4a^2-b^2}}$	45
risch	$-\frac{\ln\left(\mathrm{e}^{2i(dx+c)}+\frac{2ia\sqrt{-4a^2+b^2}-4a^2+b^2}{b\sqrt{-4a^2+b^2}}\right)}{\sqrt{-4a^2+b^2}\,d}+\frac{\ln\left(\mathrm{e}^{2i(dx+c)}+\frac{2ia\sqrt{-4a^2+b^2}+4a^2-b^2}{b\sqrt{-4a^2+b^2}}\right)}{\sqrt{-4a^2+b^2}\,d}$	135

input
```
int(1/(a+cos(d*x+c)*sin(d*x+c)*b),x,method=_RETURNVERBOSE)
```

output
```
2*arctan((b+2*a*tan(d*x+c))/(4*a^2-b^2)^(1/2))/d/(4*a^2-b^2)^(1/2)
```

3.570.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 290, normalized size of antiderivative = 6.04

$$\int \frac{1}{a+b\cos(c+dx)\sin(c+dx)}\,dx$$
$$= \left[-\frac{\sqrt{-4\,a^2+b^2}\log\left(-\frac{2\,(8\,a^2-b^2)\cos(dx+c)^4-4\,ab\cos(dx+c)\sin(dx+c)-2\,(8\,a^2-b^2)\cos(dx+c)^2+2\,a^2-b^2+\left(2\,b\cos(dx+c)^2+4\,\left(2\,a\right.\right.}{b^2\cos(dx+c)^4-b^2\cos(dx+c)^2-2\,ab\cos(dx+c)\sin(dx+c)-a}\right.}{2\,(4\,a^2-b^2)d}\right.$$
$$\left.-\frac{\arctan\left(-\frac{(4\,a\cos(dx+c)\sin(dx+c)+b)\sqrt{4\,a^2-b^2}}{2\,(4\,a^2-b^2)\cos(dx+c)^2-4\,a^2+b^2}\right)}{\sqrt{4\,a^2-b^2}d}\right]$$

input
```
integrate(1/(a+b*cos(d*x+c)*sin(d*x+c)),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-4*a^2 + b^2)*log(-(2*(8*a^2 - b^2)*cos(d*x + c)^4 - 4*a*b*cos(
d*x + c)*sin(d*x + c) - 2*(8*a^2 - b^2)*cos(d*x + c)^2 + 2*a^2 - b^2 + (2*
b*cos(d*x + c)^2 + 4*(2*a*cos(d*x + c)^3 - a*cos(d*x + c))*sin(d*x + c) -
b)*sqrt(-4*a^2 + b^2))/(b^2*cos(d*x + c)^4 - b^2*cos(d*x + c)^2 - 2*a*b*co
s(d*x + c)*sin(d*x + c) - a^2))/((4*a^2 - b^2)*d), -arctan(-(4*a*cos(d*x +
 c)*sin(d*x + c) + b)*sqrt(4*a^2 - b^2)/(2*(4*a^2 - b^2)*cos(d*x + c)^2 -
4*a^2 + b^2))/(sqrt(4*a^2 - b^2)*d)]
```

3.570.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{a+b\cos(c+dx)\sin(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c)*sin(d*x+c)),x)
```

output
```
Timed out
```

3.570.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a+b\cos(c+dx)\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*cos(d*x+c)*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b^2-4*a^2>0)', see `assume?` for
 more deta
```

3.570.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.27

$$\int \frac{1}{a+b\cos(c+dx)\sin(c+dx)}\,dx = \frac{2\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{2\,a\tan(dx+c)+b}{\sqrt{4\,a^2-b^2}}\right)\right)}{\sqrt{4\,a^2-b^2}d}$$

input
```
integrate(1/(a+b*cos(d*x+c)*sin(d*x+c)),x, algorithm="giac")
```

output
```
2*(pi*floor((d*x + c)/pi + 1/2)*sgn(a) + arctan((2*a*tan(d*x + c) + b)/sqr
t(4*a^2 - b^2)))/(sqrt(4*a^2 - b^2)*d)
```

3.570.9 Mupad [B] (verification not implemented)

Time = 27.35 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

$$\int \frac{1}{a+b\cos(c+dx)\sin(c+dx)}\,dx = \frac{2\,\mathrm{atan}\left(\frac{b+2\,a\tan(c+d\,x)}{\sqrt{4\,a^2-b^2}}\right)}{d\,\sqrt{4\,a^2-b^2}}$$

input
```
int(1/(a + b*cos(c + d*x)*sin(c + d*x)),x)
```

output
```
(2*atan((b + 2*a*tan(c + d*x))/(4*a^2 - b^2)^(1/2)))/(d*(4*a^2 - b^2)^(1/2
))
```

3.571 $\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^2}\,dx$

3.571.1 Optimal result

Integrand size = 18, antiderivative size = 95

$$\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^2}\,dx = \frac{8a\arctan\left(\frac{b+2a\tan(c+dx)}{\sqrt{4a^2-b^2}}\right)}{(4a^2-b^2)^{3/2}\,d} + \frac{2b\cos(2c+2dx)}{(4a^2-b^2)\,d(2a+b\sin(2c+2dx))}$$

output
```
8*a*arctan((b+2*a*tan(d*x+c))/(4*a^2-b^2)^(1/2))/(4*a^2-b^2)^(3/2)/d+2*b*c
os(2*d*x+2*c)/(4*a^2-b^2)/d/(2*a+b*sin(2*d*x+2*c))
```

3.571.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.99

$$\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^2}\,dx = \frac{2\left(\frac{4a\arctan\left(\frac{b+2a\tan(c+dx)}{\sqrt{4a^2-b^2}}\right)}{(4a^2-b^2)^{3/2}}+\frac{b\cos(2(c+dx))}{(2a-b)(2a+b)(2a+b\sin(2(c+dx)))}\right)}{d}$$

input
```
Integrate[(a + b*Cos[c + d*x]*Sin[c + d*x])^(-2),x]
```

output
```
(2*((4*a*ArcTan[(b + 2*a*Tan[c + d*x])/Sqrt[4*a^2 - b^2]])/(4*a^2 - b^2)^(
3/2) + (b*Cos[2*(c + d*x)])/((2*a - b)*(2*a + b)*(2*a + b*Sin[2*(c + d*x)]
))))/d
```

3.571.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3145, 3042, 3143, 27, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sin(c+dx)\cos(c+dx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+b\sin(c+dx)\cos(c+dx))^2}dx$$

$$\Big\downarrow \text{3145}$$

$$\int \frac{1}{\left(a+\frac{1}{2}b\sin(2c+2dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a+\frac{1}{2}b\sin(2c+2dx)\right)^2}dx$$

$$\Big\downarrow \text{3143}$$

$$\frac{2b\cos(2c+2dx)}{d\,(4a^2-b^2)\,(2a+b\sin(2c+2dx))}-\frac{4\int -\frac{2a}{2a+b\sin(2c+2dx)}dx}{4a^2-b^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{8a\int \frac{1}{2a+b\sin(2c+2dx)}dx}{4a^2-b^2}+\frac{2b\cos(2c+2dx)}{d\,(4a^2-b^2)\,(2a+b\sin(2c+2dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8a\int \frac{1}{2a+b\sin(2c+2dx)}dx}{4a^2-b^2}+\frac{2b\cos(2c+2dx)}{d\,(4a^2-b^2)\,(2a+b\sin(2c+2dx))}$$

$$\Big\downarrow \text{3139}$$

$$\frac{8a\int \frac{1}{2a\tan^2(c+dx)+2b\tan(c+dx)+2a}d\tan(c+dx)}{d\left(4a^2-b^2\right)}+\frac{2b\cos(2c+2dx)}{d\left(4a^2-b^2\right)(2a+b\sin(2c+2dx))}$$

$$\Big\downarrow \text{1083}$$

$$\frac{2b\cos(2c+2dx)}{d\left(4a^2-b^2\right)(2a+b\sin(2c+2dx))}-\frac{16a\int \frac{1}{-(2b+4a\tan(c+dx))^2-4(4a^2-b^2)}d(2b+4a\tan(c+dx))}{d\left(4a^2-b^2\right)}$$

$$\Big\downarrow \text{217}$$

$$\frac{8a\arctan\left(\frac{4a\tan(c+dx)+2b}{2\sqrt{4a^2-b^2}}\right)}{d\left(4a^2-b^2\right)^{3/2}}+\frac{2b\cos(2c+2dx)}{d\left(4a^2-b^2\right)(2a+b\sin(2c+2dx))}$$

input
```
Int[(a + b*Cos[c + d*x]*Sin[c + d*x])^(-2),x]
```

output
```
(8*a*ArcTan[(2*b + 4*a*Tan[c + d*x])/(2*Sqrt[4*a^2 - b^2])])/((4*a^2 - b^2
)^(3/2)*d) + (2*b*Cos[2*c + 2*d*x])/((4*a^2 - b^2)*d*(2*a + b*Sin[2*c + 2*
d*x]))
```

3.571.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.571. $\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^2}\,dx$

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3145
```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + b*(Sin[2*c + 2*d*x]/2))^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

3.571.4 Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.20

method	result
derivativedivides	$\frac{\frac{\frac{b^{2} \tan(dx+c)}{a\left(4a^{2}-b^{2}\right)}+\frac{2b}{4a^{2}-b^{2}}}{a\tan(dx+c)^{2}+b\tan(dx+c)+a}+\frac{8a\arctan\left(\frac{b+2a\tan(dx+c)}{\sqrt{4a^{2}-b^{2}}}\right)}{\left(4a^{2}-b^{2}\right)^{\frac{3}{2}}}}{d}$
default	$\frac{\frac{\frac{b^{2} \tan(dx+c)}{a\left(4a^{2}-b^{2}\right)}+\frac{2b}{4a^{2}-b^{2}}}{a\tan(dx+c)^{2}+b\tan(dx+c)+a}+\frac{8a\arctan\left(\frac{b+2a\tan(dx+c)}{\sqrt{4a^{2}-b^{2}}}\right)}{\left(4a^{2}-b^{2}\right)^{\frac{3}{2}}}}{d}$
risch	$\frac{8\,\mathrm{e}^{2i(dx+c)}a+4ib}{(4a^{2}-b^{2})d\left(b\,\mathrm{e}^{4i(dx+c)}+4ia\,\mathrm{e}^{2i(dx+c)}-b\right)}-\frac{4a\ln\left(\mathrm{e}^{2i(dx+c)}+\frac{2ia\sqrt{-4a^{2}+b^{2}}-4a^{2}+b^{2}}{b\sqrt{-4a^{2}+b^{2}}}\right)}{\sqrt{-4a^{2}+b^{2}}\,(2a+b)(2a-b)d}+\frac{4a\ln\left(\mathrm{e}^{2i(dx+c)}+\frac{2ia\sqrt{-4a^{2}}}{b\sqrt{-}}\right.}{\sqrt{-4a^{2}+b^{2}}\,(2a+b)(}$

input
```
int(1/(a+cos(d*x+c)*sin(d*x+c)*b)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*((b^2/a/(4*a^2-b^2)*tan(d*x+c)+2*b/(4*a^2-b^2))/(a*tan(d*x+c)^2+b*tan(
d*x+c)+a)+8*a/(4*a^2-b^2)^(3/2)*arctan((b+2*a*tan(d*x+c))/(4*a^2-b^2)^(1/2
)))
```

3.571.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(91) = 182$.

Time = 0.28 (sec) , antiderivative size = 493, normalized size of antiderivative = 5.19

$$\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^2}\,dx$$

$$= \left[-\frac{4\,a^2b-b^3-2\,(4\,a^2b-b^3)\cos{(dx+c)}^2-2\,(ab\cos{(dx+c)}\sin{(dx+c)}+a^2)\sqrt{-4\,a^2+b^2}\log\left(\frac{2\,(8\,a^2-\cdots}{\cdots}\right.}{(16\,a^4b-8\,a^2b^3+b^5)d\cos{(dx\cdots}}\right.$$

$$-\frac{4\,a^2b-b^3-2\,(4\,a^2b-b^3)\cos{(dx+c)}^2+4\,(ab\cos{(dx+c)}\sin{(dx+c)}+a^2)\sqrt{4\,a^2-b^2}\arctan\left(-\frac{(4\,a\,c\cdots}{2\,(\cdots}\right.}{(16\,a^4b-8\,a^2b^3+b^5)d\cos{(dx+c)}\sin{(dx+c)}+(16\,a^5-8\,a^3b^2+ab^4)d}$$

input
```
integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
[-(4*a^2*b - b^3 - 2*(4*a^2*b - b^3)*cos(d*x + c)^2 - 2*(a*b*cos(d*x + c)*
sin(d*x + c) + a^2)*sqrt(-4*a^2 + b^2)*log((2*(8*a^2 - b^2)*cos(d*x + c)^4
 - 4*a*b*cos(d*x + c)*sin(d*x + c) - 2*(8*a^2 - b^2)*cos(d*x + c)^2 + 2*a^
2 - b^2 - (2*b*cos(d*x + c)^2 + 4*(2*a*cos(d*x + c)^3 - a*cos(d*x + c))*si
n(d*x + c) - b)*sqrt(-4*a^2 + b^2))/(b^2*cos(d*x + c)^4 - b^2*cos(d*x + c)
^2 - 2*a*b*cos(d*x + c)*sin(d*x + c) - a^2)))/((16*a^4*b - 8*a^2*b^3 + b^5
)*d*cos(d*x + c)*sin(d*x + c) + (16*a^5 - 8*a^3*b^2 + a*b^4)*d), -(4*a^2*b
 - b^3 - 2*(4*a^2*b - b^3)*cos(d*x + c)^2 + 4*(a*b*cos(d*x + c)*sin(d*x +
c) + a^2)*sqrt(4*a^2 - b^2)*arctan(-(4*a*cos(d*x + c)*sin(d*x + c) + b)*sq
rt(4*a^2 - b^2)/(2*(4*a^2 - b^2)*cos(d*x + c)^2 - 4*a^2 + b^2)))/((16*a^4*
b - 8*a^2*b^3 + b^5)*d*cos(d*x + c)*sin(d*x + c) + (16*a^5 - 8*a^3*b^2 + a
*b^4)*d)]
```

3.571.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))**2,x)
```

output
```
Timed out
```

3.571.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b^2-4*a^2>0)', see `assume?` for
 more deta
```

3.571.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.22

$$\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^2}\,dx = \frac{\frac{8\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{2a\tan(dx+c)+b}{\sqrt{4a^2-b^2}}\right)\right)a}{(4a^2-b^2)^{\frac{3}{2}}}+\frac{b^2\tan(dx+c)+2ab}{(4a^3-ab^2)\left(a\tan(dx+c)^2+b\tan(dx+c)+a\right)}}{d}$$

input
```
integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
(8*(pi*floor((d*x + c)/pi + 1/2)*sgn(a) + arctan((2*a*tan(d*x + c) + b)/sq
rt(4*a^2 - b^2)))*a/(4*a^2 - b^2)^(3/2) + (b^2*tan(d*x + c) + 2*a*b)/((4*a
^3 - a*b^2)*(a*tan(d*x + c)^2 + b*tan(d*x + c) + a)))/d
```

3.571.9 Mupad [B] (verification not implemented)

Time = 27.40 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.91

$$\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^2}\,dx = \frac{\frac{2\,b}{4\,a^2-b^2}+\frac{b^2\tan(c+d\,x)}{a\,(4\,a^2-b^2)}}{d\,\left(a\tan\left(c+d\,x\right)^2+b\tan\left(c+d\,x\right)+a\right)} + \frac{8\,a\,\mathrm{atan}\left(\frac{(4\,a^2-b^2)\,\left(\frac{8\,a^2\tan(c+d\,x)}{(2\,a+b)^{3/2}\,(2\,a-b)^{3/2}}+\frac{4\,a\,\left(4\,a^2\,b-b^3\right)}{(2\,a+b)^{3/2}\,\left(4\,a^2-b^2\right)\,(2\,a-b)^{3/2}}\right)}{4\,a}\right)}{d\,(2\,a+b)^{3/2}\,(2\,a-b)^{3/2}}$$

input
```
int(1/(a + b*cos(c + d*x)*sin(c + d*x))^2,x)
```

output
```
((2*b)/(4*a^2 - b^2) + (b^2*tan(c + d*x))/(a*(4*a^2 - b^2)))/(d*(a + b*tan
(c + d*x) + a*tan(c + d*x)^2)) + (8*a*atan(((4*a^2 - b^2)*((8*a^2*tan(c +
d*x))/((2*a + b)^(3/2)*(2*a - b)^(3/2)) + (4*a*(4*a^2*b - b^3))/((2*a + b)
^(3/2)*(4*a^2 - b^2)*(2*a - b)^(3/2))))/(4*a)))/(d*(2*a + b)^(3/2)*(2*a -
b)^(3/2))
```

3.572 $\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^3}\,dx$

3.572.1 Optimal result

Integrand size = 18, antiderivative size = 149

$$\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^3}\,dx = \frac{4(8a^2+b^2)\arctan\left(\frac{b+2a\tan(c+dx)}{\sqrt{4a^2-b^2}}\right)}{(4a^2-b^2)^{5/2}d} + \frac{2b\cos(2c+2dx)}{(4a^2-b^2)d(2a+b\sin(2c+2dx))^2} + \frac{12ab\cos(2c+2dx)}{(4a^2-b^2)^2d(2a+b\sin(2c+2dx))}$$

output
```
4*(8*a^2+b^2)*arctan((b+2*a*tan(d*x+c))/(4*a^2-b^2)^(1/2))/(4*a^2-b^2)^(5/
2)/d+2*b*cos(2*d*x+2*c)/(4*a^2-b^2)/d/(2*a+b*sin(2*d*x+2*c))^2+12*a*b*cos(
2*d*x+2*c)/(4*a^2-b^2)^2/d/(2*a+b*sin(2*d*x+2*c))
```

3.572.2 Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.81

$$\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^3}\,dx = \frac{2\left(\frac{2(8a^2+b^2)\arctan\left(\frac{b+2a\tan(c+dx)}{\sqrt{4a^2-b^2}}\right)}{(4a^2-b^2)^{5/2}} + \frac{b\cos(2(c+dx))(16a^2-b^2+6ab\sin(2(c+dx)))}{(-4a^2+b^2)^2(2a+b\sin(2(c+dx)))^2}\right)}{d}$$

input
```
Integrate[(a + b*Cos[c + d*x]*Sin[c + d*x])^(-3),x]
```

output
```
(2*((2*(8*a^2 + b^2)*ArcTan[(b + 2*a*Tan[c + d*x])/Sqrt[4*a^2 - b^2]])/(4*
a^2 - b^2)^(5/2) + (b*Cos[2*(c + d*x)]*(16*a^2 - b^2 + 6*a*b*Sin[2*(c + d*
x)]))/((-4*a^2 + b^2)^2*(2*a + b*Sin[2*(c + d*x)])^2)))/d
```

3.572.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.13, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.722$, Rules used = {3042, 3145, 3042, 3143, 27, 3042, 3233, 25, 27, 3042, 3139, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sin(c+dx)\cos(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+b\sin(c+dx)\cos(c+dx))^3}dx$$

$$\Big\downarrow \text{3145}$$

$$\int \frac{1}{\left(a+\frac{1}{2}b\sin(2c+2dx)\right)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a+\frac{1}{2}b\sin(2c+2dx)\right)^3}dx$$

$$\Big\downarrow \text{3143}$$

$$\frac{2b\cos(2c+2dx)}{d\left(4a^2-b^2\right)(2a+b\sin(2c+2dx))^2}-\frac{2\int -\frac{2(4a-b\sin(2c+2dx))}{(2a+b\sin(2c+2dx))^2}dx}{4a^2-b^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{4\int \frac{4a-b\sin(2c+2dx)}{(2a+b\sin(2c+2dx))^2}dx}{4a^2-b^2}+\frac{2b\cos(2c+2dx)}{d\left(4a^2-b^2\right)(2a+b\sin(2c+2dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\int \frac{4a-b\sin(2c+2dx)}{(2a+b\sin(2c+2dx))^2}dx}{4a^2-b^2}+\frac{2b\cos(2c+2dx)}{d\left(4a^2-b^2\right)(2a+b\sin(2c+2dx))^2}$$

$$\Big\downarrow \text{3233}$$

$$\frac{4\left(\frac{3ab\cos(2c+2dx)}{d(4a^2-b^2)(2a+b\sin(2c+2dx))}-\frac{\int -\frac{8a^2+b^2}{2a+b\sin(2c+2dx)}dx}{4a^2-b^2}\right)}{4a^2-b^2}+\frac{2b\cos(2c+2dx)}{d\left(4a^2-b^2\right)(2a+b\sin(2c+2dx))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{4\left(\frac{\int \frac{8a^2+b^2}{2a+b\sin(2c+2dx)}dx}{4a^2-b^2}+\frac{3ab\cos(2c+2dx)}{d(4a^2-b^2)(2a+b\sin(2c+2dx))}\right)}{4a^2-b^2}+\frac{2b\cos(2c+2dx)}{d\left(4a^2-b^2\right)(2a+b\sin(2c+2dx))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{4\left(\frac{(8a^2+b^2)\int \frac{1}{2a+b\sin(2c+2dx)}dx}{4a^2-b^2}+\frac{3ab\cos(2c+2dx)}{d(4a^2-b^2)(2a+b\sin(2c+2dx))}\right)}{4a^2-b^2}+\frac{2b\cos(2c+2dx)}{d\left(4a^2-b^2\right)(2a+b\sin(2c+2dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\left(\frac{(8a^2+b^2)\int \frac{1}{2a+b\sin(2c+2dx)}dx}{4a^2-b^2}+\frac{3ab\cos(2c+2dx)}{d(4a^2-b^2)(2a+b\sin(2c+2dx))}\right)}{4a^2-b^2}+\frac{2b\cos(2c+2dx)}{d\left(4a^2-b^2\right)(2a+b\sin(2c+2dx))^2}$$

$$\Big\downarrow \text{3139}$$

$$\frac{4\left(\frac{(8a^2+b^2)\int \frac{1}{2a\tan^2(c+dx)+2b\tan(c+dx)+2a}d\tan(c+dx)}{d(4a^2-b^2)}+\frac{3ab\cos(2c+2dx)}{d(4a^2-b^2)(2a+b\sin(2c+2dx))}\right)}{4a^2-b^2}+\frac{2b\cos(2c+2dx)}{d\left(4a^2-b^2\right)(2a+b\sin(2c+2dx))^2}$$

$$\Big\downarrow \text{1083}$$

$$\frac{4\left(\frac{3ab\cos(2c+2dx)}{d(4a^2-b^2)(2a+b\sin(2c+2dx))}-\frac{2(8a^2+b^2)\int \frac{1}{-(2b+4a\tan(c+dx))^2-4\left(4a^2-b^2\right)}d(2b+4a\tan(c+dx))}{d(4a^2-b^2)}\right)}{4a^2-b^2}+\frac{2b\cos(2c+2dx)}{d\left(4a^2-b^2\right)(2a+b\sin(2c+2dx))^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{4\left(\frac{(8a^2+b^2)\arctan\left(\frac{4a\tan(c+dx)+2b}{2\sqrt{4a^2-b^2}}\right)}{d(4a^2-b^2)^{3/2}}+\frac{3ab\cos(2c+2dx)}{d(4a^2-b^2)(2a+b\sin(2c+2dx))}\right)}{4a^2-b^2}+\frac{2b\cos(2c+2dx)}{d\left(4a^2-b^2\right)(2a+b\sin(2c+2dx))^2}$$

3.572. $\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^3}\,dx$

input
```
Int[(a + b*Cos[c + d*x]*Sin[c + d*x])^(-3),x]
```

output
```
(2*b*Cos[2*c + 2*d*x])/((4*a^2 - b^2)*d*(2*a + b*Sin[2*c + 2*d*x])^2) + (4
*(((8*a^2 + b^2)*ArcTan[(2*b + 4*a*Tan[c + d*x])/(2*Sqrt[4*a^2 - b^2])])/(
(4*a^2 - b^2)^(3/2)*d) + (3*a*b*Cos[2*c + 2*d*x])/((4*a^2 - b^2)*d*(2*a +
b*Sin[2*c + 2*d*x]))))/(4*a^2 - b^2)
```

3.572.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3145
```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + b*(Sin[2*c + 2*d*x]/2))^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

3.572.4 Maple [A] (verified)

Time = 2.63 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.82

method	result
derivativedivides	$\frac{\frac{\frac{b^{2}\left(10a^{2}-b^{2}\right)\tan(dx+c)^{3}}{\left(16a^{4}-8a^{2}b^{2}+b^{4}\right)a}+\frac{b\left(32a^{4}+14a^{2}b^{2}-b^{4}\right)\tan(dx+c)^{2}}{2\left(16a^{4}-8a^{2}b^{2}+b^{4}\right)a^{2}}+\frac{b^{2}\left(22a^{2}-b^{2}\right)\tan(dx+c)}{a\left(16a^{4}-8a^{2}b^{2}+b^{4}\right)}+\frac{b\left(16a^{2}-b^{2}\right)}{16a^{4}-8a^{2}b^{2}+b^{4}}}{\left(a\tan(dx+c)^{2}+b\tan(dx+c)+a\right)^{2}}+\frac{4\left(8a^{2}+b^{2}\right)\arctan\left(\frac{b}{}\right.}{\left(16a^{4}-8a^{2}b^{2}+b\right.}}{d}$
default	$\frac{\frac{\frac{b^{2}\left(10a^{2}-b^{2}\right)\tan(dx+c)^{3}}{\left(16a^{4}-8a^{2}b^{2}+b^{4}\right)a}+\frac{b\left(32a^{4}+14a^{2}b^{2}-b^{4}\right)\tan(dx+c)^{2}}{2\left(16a^{4}-8a^{2}b^{2}+b^{4}\right)a^{2}}+\frac{b^{2}\left(22a^{2}-b^{2}\right)\tan(dx+c)}{a\left(16a^{4}-8a^{2}b^{2}+b^{4}\right)}+\frac{b\left(16a^{2}-b^{2}\right)}{16a^{4}-8a^{2}b^{2}+b^{4}}}{\left(a\tan(dx+c)^{2}+b\tan(dx+c)+a\right)^{2}}+\frac{4\left(8a^{2}+b^{2}\right)\arctan\left(\frac{b}{}\right.}{\left(16a^{4}-8a^{2}b^{2}+b\right.}}{d}$
risch	$-\frac{4i\left(-8ia^{2}b\,e^{6i(dx+c)}-ie^{6i(dx+c)}b^{3}+48a^{3}e^{4i(dx+c)}+6a\,b^{2}e^{4i(dx+c)}+40ia^{2}b\,e^{2i(dx+c)}-ib^{3}e^{2i(dx+c)}-6a\,b^{2}\right)}{\left(ib-ib\,e^{4i(dx+c)}+4\,e^{2i(dx+c)}a\right)^{2}\left(4a^{2}-b^{2}\right)^{2}d}-\frac{16\ln\left(\right.}{}$

input
```
int(1/(a+cos(d*x+c)*sin(d*x+c)*b)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*((b^2*(10*a^2-b^2)/(16*a^4-8*a^2*b^2+b^4)/a*tan(d*x+c)^3+1/2*b*(32*a^4
+14*a^2*b^2-b^4)/(16*a^4-8*a^2*b^2+b^4)/a^2*tan(d*x+c)^2+b^2*(22*a^2-b^2)/
a/(16*a^4-8*a^2*b^2+b^4)*tan(d*x+c)+b*(16*a^2-b^2)/(16*a^4-8*a^2*b^2+b^4))
/(a*tan(d*x+c)^2+b*tan(d*x+c)+a)^2+4*(8*a^2+b^2)/(16*a^4-8*a^2*b^2+b^4)/(4
*a^2-b^2)^(1/2)*arctan((b+2*a*tan(d*x+c))/(4*a^2-b^2)^(1/2)))
```

3.572.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 429 vs. $2(145) = 290$.

Time $= 0.31$ (sec) , antiderivative size $= 969$, normalized size of antiderivative $= 6.50$

$$\int \frac{1}{(a + b\cos(c + dx)\sin(c + dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
[1/2*(64*a^4*b - 20*a^2*b^3 + b^5 - 2*(64*a^4*b - 20*a^2*b^3 + b^5)*cos(d*
x + c)^2 - 2*((8*a^2*b^2 + b^4)*cos(d*x + c)^4 - 8*a^4 - a^2*b^2 - (8*a^2*
b^2 + b^4)*cos(d*x + c)^2 - 2*(8*a^3*b + a*b^3)*cos(d*x + c)*sin(d*x + c))
*sqrt(-4*a^2 + b^2)*log(-(2*(8*a^2 - b^2)*cos(d*x + c)^4 - 4*a*b*cos(d*x +
 c)*sin(d*x + c) - 2*(8*a^2 - b^2)*cos(d*x + c)^2 + 2*a^2 - b^2 + (2*b*cos
(d*x + c)^2 + 4*(2*a*cos(d*x + c)^3 - a*cos(d*x + c))*sin(d*x + c) - b)*sq
rt(-4*a^2 + b^2))/(b^2*cos(d*x + c)^4 - b^2*cos(d*x + c)^2 - 2*a*b*cos(d*x
 + c)*sin(d*x + c) - a^2)) - 12*(2*(4*a^3*b^2 - a*b^4)*cos(d*x + c)^3 - (4
*a^3*b^2 - a*b^4)*cos(d*x + c))*sin(d*x + c))/((64*a^6*b^2 - 48*a^4*b^4 +
12*a^2*b^6 - b^8)*d*cos(d*x + c)^4 - (64*a^6*b^2 - 48*a^4*b^4 + 12*a^2*b^6
 - b^8)*d*cos(d*x + c)^2 - 2*(64*a^7*b - 48*a^5*b^3 + 12*a^3*b^5 - a*b^7)*
d*cos(d*x + c)*sin(d*x + c) - (64*a^8 - 48*a^6*b^2 + 12*a^4*b^4 - a^2*b^6)
*d), 1/2*(64*a^4*b - 20*a^2*b^3 + b^5 - 2*(64*a^4*b - 20*a^2*b^3 + b^5)*co
s(d*x + c)^2 - 4*((8*a^2*b^2 + b^4)*cos(d*x + c)^4 - 8*a^4 - a^2*b^2 - (8*
a^2*b^2 + b^4)*cos(d*x + c)^2 - 2*(8*a^3*b + a*b^3)*cos(d*x + c)*sin(d*x +
 c))*sqrt(4*a^2 - b^2)*arctan(-(4*a*cos(d*x + c)*sin(d*x + c) + b)*sqrt(4*
a^2 - b^2)/(2*(4*a^2 - b^2)*cos(d*x + c)^2 - 4*a^2 + b^2)) - 12*(2*(4*a^3*
b^2 - a*b^4)*cos(d*x + c)^3 - (4*a^3*b^2 - a*b^4)*cos(d*x + c))*sin(d*x +
c))/((64*a^6*b^2 - 48*a^4*b^4 + 12*a^2*b^6 - b^8)*d*cos(d*x + c)^4 - (64*a
^6*b^2 - 48*a^4*b^4 + 12*a^2*b^6 - b^8)*d*cos(d*x + c)^2 - 2*(64*a^7*b ...
```

3.572.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))**3,x)
```

output
```
Timed out
```

3.572.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b^2-4*a^2>0)', see `assume?` for
 more deta
```

3.572.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.69

$$\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^3}\,dx = \frac{\frac{8\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{2a\tan(dx+c)+b}{\sqrt{4a^2-b^2}}\right)\right)(8a^2+b^2)}{(16a^4-8a^2b^2+b^4)\sqrt{4a^2-b^2}} + \frac{20a^3b^2\tan(dx+c)^3-2ab^4\tan(dx+c)^3+32a^4b\tan(dx+c)^2+14a^2b^3\tan(dx+c)}{(16a^6-8a^4b^2+a^2b^4)\left(a\tan(dx\right.}}{2d}$$

input
```
integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
1/2*(8*(pi*floor((d*x + c)/pi + 1/2)*sgn(a) + arctan((2*a*tan(d*x + c) + b
)/sqrt(4*a^2 - b^2)))*(8*a^2 + b^2)/((16*a^4 - 8*a^2*b^2 + b^4)*sqrt(4*a^2
 - b^2)) + (20*a^3*b^2*tan(d*x + c)^3 - 2*a*b^4*tan(d*x + c)^3 + 32*a^4*b*
tan(d*x + c)^2 + 14*a^2*b^3*tan(d*x + c)^2 - b^5*tan(d*x + c)^2 + 44*a^3*b
^2*tan(d*x + c) - 2*a*b^4*tan(d*x + c) + 32*a^4*b - 2*a^2*b^3)/((16*a^6 -
8*a^4*b^2 + a^2*b^4)*(a*tan(d*x + c)^2 + b*tan(d*x + c) + a)^2))/d
```

3.572.9 Mupad [B] (verification not implemented)

Time = 27.39 (sec) , antiderivative size = 396, normalized size of antiderivative = 2.66

$$\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^3}\,dx$$
$$= \frac{\frac{16\,a^2\,b-b^3}{16\,a^4-8\,a^2\,b^2+b^4}+\frac{b\tan(c+d\,x)\,\left(22\,a^2\,b-b^3\right)}{a\,\left(16\,a^4-8\,a^2\,b^2+b^4\right)}+\frac{\tan(c+d\,x)^2\,\left(16\,a^2\,b-b^3\right)\,\left(2\,a^2+b^2\right)}{2\,a^2\,\left(16\,a^4-8\,a^2\,b^2+b^4\right)}+\frac{b\tan(c+d\,x)^3\,\left(10\,a^2\,b-b^3\right)}{a\,\left(16\,a^4-8\,a^2\,b^2+b^4\right)}}{d\,\left(\tan\left(c+d\,x\right)^2\,\left(2\,a^2+b^2\right)+a^2+a^2\tan\left(c+d\,x\right)^4+2\,a\,b\tan\left(c+d\,x\right)+2\,a\,b\tan\left(c+d\,x\right)^3\right)}$$
$$+\frac{4\,\mathrm{atan}\left(\frac{\left(\frac{4\,a\tan(c+d\,x)\,\left(8\,a^2+b^2\right)}{(2\,a+b)^{5/2}\,(2\,a-b)^{5/2}}+\frac{2\,\left(8\,a^2+b^2\right)\,\left(16\,a^4\,b-8\,a^2\,b^3+b^5\right)}{(2\,a+b)^{5/2}\,(2\,a-b)^{5/2}\,\left(16\,a^4-8\,a^2\,b^2+b^4\right)}\right)\,\left(16\,a^4-8\,a^2\,b^2+b^4\right)}{16\,a^2+2\,b^2}\right)\,\left(8\,a^2+b^2\right)}{d\,\left(2\,a+b\right)^{5/2}\,\left(2\,a-b\right)^{5/2}}$$

input
```
int(1/(a + b*cos(c + d*x)*sin(c + d*x))^3,x)
```

output
```
((16*a^2*b - b^3)/(16*a^4 + b^4 - 8*a^2*b^2) + (b*tan(c + d*x)*(22*a^2*b -
 b^3))/(a*(16*a^4 + b^4 - 8*a^2*b^2)) + (tan(c + d*x)^2*(16*a^2*b - b^3)*(
2*a^2 + b^2))/(2*a^2*(16*a^4 + b^4 - 8*a^2*b^2)) + (b*tan(c + d*x)^3*(10*a
^2*b - b^3))/(a*(16*a^4 + b^4 - 8*a^2*b^2)))/(d*(tan(c + d*x)^2*(2*a^2 + b
^2) + a^2 + a^2*tan(c + d*x)^4 + 2*a*b*tan(c + d*x) + 2*a*b*tan(c + d*x)^3
)) + (4*atan((((4*a*tan(c + d*x)*(8*a^2 + b^2))/((2*a + b)^(5/2)*(2*a - b)
^(5/2)) + (2*(8*a^2 + b^2)*(16*a^4*b + b^5 - 8*a^2*b^3))/((2*a + b)^(5/2)*
(2*a - b)^(5/2)*(16*a^4 + b^4 - 8*a^2*b^2)))*(16*a^4 + b^4 - 8*a^2*b^2))/(
16*a^2 + 2*b^2))*(8*a^2 + b^2))/(d*(2*a + b)^(5/2)*(2*a - b)^(5/2))
```

3.573 $\int (a+b\cos(c+dx)\sin(c+dx))^{5/2}\,dx$

3.573.1 Optimal result

Integrand size = 20, antiderivative size = 265

$$\int (a+b\cos(c+dx)\sin(c+dx))^{5/2}\,dx = -\frac{2\sqrt{2}ab\cos(2c+2dx)\sqrt{2a+b\sin(2c+2dx)}}{15d} - \frac{b\cos(2c+2dx)(2a+b\sin(2c+2dx))^{3/2}}{20\sqrt{2}d} + \frac{(92a^2+9b^2)\,E\left(c-\frac{\pi}{4}+dx\middle|\frac{2b}{2a+b}\right)\sqrt{2a+b\sin(2c+2dx)}}{60\sqrt{2}d\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}} - \frac{2\sqrt{2}a(4a^2-b^2)\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,\frac{2b}{2a+b}\right)\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}{15d\sqrt{2a+b\sin(2c+2dx)}}$$

output
```
-1/40*b*cos(2*d*x+2*c)*(2*a+b*sin(2*d*x+2*c))^(3/2)/d*2^(1/2)-2/15*a*b*cos
(2*d*x+2*c)*2^(1/2)*(2*a+b*sin(2*d*x+2*c))^(1/2)/d-1/120*(92*a^2+9*b^2)*(s
in(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^
(1/2)*(b/(2*a+b))^(1/2))*(2*a+b*sin(2*d*x+2*c))^(1/2)/d*2^(1/2)/((2*a+b*si
n(2*d*x+2*c))/(2*a+b))^(1/2)+2/15*a*(4*a^2-b^2)*(sin(c+1/4*Pi+d*x)^2)^(1/2
)/sin(c+1/4*Pi+d*x)*EllipticF(cos(c+1/4*Pi+d*x),2^(1/2)*(b/(2*a+b))^(1/2))
*2^(1/2)*((2*a+b*sin(2*d*x+2*c))/(2*a+b))^(1/2)/d/(2*a+b*sin(2*d*x+2*c))^(
1/2)
```

3.573.2 Mathematica [A] (verified)

Time = 2.05 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.76

$$\int (a+b\cos(c+dx)\sin(c+dx))^{5/2}\,dx = \frac{2(184a^3+92a^2b+18ab^2+9b^3)\,E\left(c-\frac{\pi}{4}+dx\middle|\frac{2b}{2a+b}\right)\sqrt{\frac{2a+b\sin(2(c+dx))}{2a+b}} - 32a(4a^2-b^2)\,\text{Elliptic}}{120d\sqrt{4}}$$

input
```
Integrate[(a + b*Cos[c + d*x]*Sin[c + d*x])^(5/2),x]
```

output
```
(2*(184*a^3 + 92*a^2*b + 18*a*b^2 + 9*b^3)*EllipticE[c - Pi/4 + d*x, (2*b)
/(2*a + b)]*Sqrt[(2*a + b*Sin[2*(c + d*x)])/(2*a + b)] - 32*a*(4*a^2 - b^2
)*EllipticF[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[(2*a + b*Sin[2*(c + d*x)
])/(2*a + b)] - b*(88*a^2*Cos[2*(c + d*x)] + b*(28*a + 3*b*Sin[2*(c + d*x)
])*Sin[4*(c + d*x)]))/(120*d*Sqrt[4*a + 2*b*Sin[2*(c + d*x)]])
```

3.573.3 Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.98, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.850$, Rules used = {3042, 3145, 3042, 3135, 27, 3042, 3232, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sin(c+dx)\cos(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (a+b\sin(c+dx)\cos(c+dx))^{5/2}dx$$

$$\Big\downarrow \; 3145$$

$$\int \left(a+\frac{1}{2}b\sin(2c+2dx)\right)^{5/2}dx$$

$$\Big\downarrow \; 3042$$

$$\int \left(a+\frac{1}{2}b\sin(2c+2dx)\right)^{5/2}dx$$

$$\Big\downarrow \; 3135$$

$$\frac{2}{5}\int \frac{\sqrt{2a+b\sin(2c+2dx)}\left(20a^2+16b\sin(2c+2dx)a+3b^2\right)}{8\sqrt{2}}dx-\frac{b\cos(2c+2dx)(2a+b\sin(2c+2dx))^{3/2}}{20\sqrt{2}d}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \sqrt{2a+b\sin(2c+2dx)}\left(20a^2+16b\sin(2c+2dx)a+3b^2\right)dx}{20\sqrt{2}}-\frac{b\cos(2c+2dx)(2a+b\sin(2c+2dx))^{3/2}}{20\sqrt{2}d}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \sqrt{2a+b\sin(2c+2dx)}\left(20a^2+16b\sin(2c+2dx)a+3b^2\right)dx}{20\sqrt{2}}-\frac{b\cos(2c+2dx)(2a+b\sin(2c+2dx))^{3/2}}{20\sqrt{2}d}$$

$$\Big\downarrow \; 3232$$

$$\frac{\frac{2}{3}\int \frac{2a(60a^2+17b^2)+b(92a^2+9b^2)\sin(2c+2dx)}{2\sqrt{2a+b\sin(2c+2dx)}}dx-\frac{16ab\cos(2c+2dx)\sqrt{2a+b\sin(2c+2dx)}}{3d}}{20\sqrt{2}}-\frac{b\cos(2c+2dx)(2a+b\sin(2c+2dx))^{3/2}}{20\sqrt{2}d}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{3}\int \frac{2a(60a^2+17b^2)+b(92a^2+9b^2)\sin(2c+2dx)}{\sqrt{2a+b\sin(2c+2dx)}}dx-\frac{16ab\cos(2c+2dx)\sqrt{2a+b\sin(2c+2dx)}}{3d}}{20\sqrt{2}}-\frac{b\cos(2c+2dx)(2a+b\sin(2c+2dx))^{3/2}}{20\sqrt{2}d}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{1}{3}\int \frac{2a(60a^2+17b^2)+b(92a^2+9b^2)\sin(2c+2dx)}{\sqrt{2a+b\sin(2c+2dx)}}dx-\frac{16ab\cos(2c+2dx)\sqrt{2a+b\sin(2c+2dx)}}{3d}}{20\sqrt{2}}-\frac{b\cos(2c+2dx)(2a+b\sin(2c+2dx))^{3/2}}{20\sqrt{2}d}$$

$$\Big\downarrow \; 3231$$

3.573. $\int (a+b\cos(c+dx)\sin(c+dx))^{5/2}\,dx$

$$\frac{\frac{1}{3}\left(\left(92a^2+9b^2\right)\int\sqrt{2a+b\sin(2c+2dx)}dx-16a\left(4a^2-b^2\right)\int\frac{1}{\sqrt{2a+b\sin(2c+2dx)}}dx\right)-\frac{16ab\cos(2c+2dx)\sqrt{2a+b\sin(2c+2d}}{3d}}{20\sqrt{2}}$$
$$\frac{b\cos(2c+2dx)(2a+b\sin(2c+2dx))^{3/2}}{20\sqrt{2}d}$$

$\downarrow$ 3042

$$\frac{\frac{1}{3}\left(\left(92a^2+9b^2\right)\int\sqrt{2a+b\sin(2c+2dx)}dx-16a\left(4a^2-b^2\right)\int\frac{1}{\sqrt{2a+b\sin(2c+2dx)}}dx\right)-\frac{16ab\cos(2c+2dx)\sqrt{2a+b\sin(2c+2d}}{3d}}{20\sqrt{2}}$$
$$\frac{b\cos(2c+2dx)(2a+b\sin(2c+2dx))^{3/2}}{20\sqrt{2}d}$$

$\downarrow$ 3134

$$\frac{\frac{1}{3}\left(\frac{\left(92a^2+9b^2\right)\sqrt{2a+b\sin(2c+2dx)}\int\sqrt{\frac{2a}{2a+b}+\frac{b\sin(2c+2dx)}{2a+b}}dx}{\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}-16a\left(4a^2-b^2\right)\int\frac{1}{\sqrt{2a+b\sin(2c+2dx)}}dx\right)-\frac{16ab\cos(2c+2dx)\sqrt{2a+b}}{3d}}{20\sqrt{2}}$$
$$\frac{b\cos(2c+2dx)(2a+b\sin(2c+2dx))^{3/2}}{20\sqrt{2}d}$$

$\downarrow$ 3042

$$\frac{\frac{1}{3}\left(\frac{\left(92a^2+9b^2\right)\sqrt{2a+b\sin(2c+2dx)}\int\sqrt{\frac{2a}{2a+b}+\frac{b\sin(2c+2dx)}{2a+b}}dx}{\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}-16a\left(4a^2-b^2\right)\int\frac{1}{\sqrt{2a+b\sin(2c+2dx)}}dx\right)-\frac{16ab\cos(2c+2dx)\sqrt{2a+b}}{3d}}{20\sqrt{2}}$$
$$\frac{b\cos(2c+2dx)(2a+b\sin(2c+2dx))^{3/2}}{20\sqrt{2}d}$$

$\downarrow$ 3132

$$\frac{\frac{1}{3}\left(\frac{\left(92a^2+9b^2\right)\sqrt{2a+b\sin(2c+2dx)}E\left(c+dx-\frac{\pi}{4}|\frac{2b}{2a+b}\right)}{d\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}-16a\left(4a^2-b^2\right)\int\frac{1}{\sqrt{2a+b\sin(2c+2dx)}}dx\right)-\frac{16ab\cos(2c+2dx)\sqrt{2a+b\sin(2c+}}{3d}}{20\sqrt{2}}$$
$$\frac{b\cos(2c+2dx)(2a+b\sin(2c+2dx))^{3/2}}{20\sqrt{2}d}$$

$\downarrow$ 3142

3.573. $\int(a+b\cos(c+dx)\sin(c+dx))^{5/2}\,dx$

$$\frac{\frac{1}{3}\left(\frac{(92a^2+9b^2)\sqrt{2a+b\sin(2c+2dx)}E\left(c+dx-\frac{\pi}{4}|\frac{2b}{2a+b}\right)}{d\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}-\frac{16a(4a^2-b^2)\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}\int\frac{1}{\sqrt{\frac{2a}{2a+b}+\frac{b\sin(2c+2dx)}{2a+b}}}dx}{\sqrt{2a+b\sin(2c+2dx)}}\right)-\frac{16ab\cos(2c+2dx)\sqrt{\ldots}}{3\ldots}}{20\sqrt{2}}$$
$$\frac{b\cos(2c+2dx)(2a+b\sin(2c+2dx))^{3/2}}{20\sqrt{2}d}$$

$\downarrow$ 3042

$$\frac{\frac{1}{3}\left(\frac{(92a^2+9b^2)\sqrt{2a+b\sin(2c+2dx)}E\left(c+dx-\frac{\pi}{4}|\frac{2b}{2a+b}\right)}{d\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}-\frac{16a(4a^2-b^2)\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}\int\frac{1}{\sqrt{\frac{2a}{2a+b}+\frac{b\sin(2c+2dx)}{2a+b}}}dx}{\sqrt{2a+b\sin(2c+2dx)}}\right)-\frac{16ab\cos(2c+2dx)\sqrt{\ldots}}{3\ldots}}{20\sqrt{2}}$$
$$\frac{b\cos(2c+2dx)(2a+b\sin(2c+2dx))^{3/2}}{20\sqrt{2}d}$$

$\downarrow$ 3140

$$\frac{\frac{1}{3}\left(\frac{(92a^2+9b^2)\sqrt{2a+b\sin(2c+2dx)}E\left(c+dx-\frac{\pi}{4}|\frac{2b}{2a+b}\right)}{d\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}-\frac{16a(4a^2-b^2)\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}\text{EllipticF}\left(c+dx-\frac{\pi}{4},\frac{2b}{2a+b}\right)}{d\sqrt{2a+b\sin(2c+2dx)}}\right)-\frac{16ab\cos(2c+2dx)\sqrt{\ldots}}{3\ldots}}{20\sqrt{2}}$$
$$\frac{b\cos(2c+2dx)(2a+b\sin(2c+2dx))^{3/2}}{20\sqrt{2}d}$$

input
```
Int[(a + b*Cos[c + d*x]*Sin[c + d*x])^(5/2),x]
```

output
```
-1/20*(b*Cos[2*c + 2*d*x]*(2*a + b*Sin[2*c + 2*d*x])^(3/2))/(Sqrt[2]*d) +
((-16*a*b*Cos[2*c + 2*d*x]*Sqrt[2*a + b*Sin[2*c + 2*d*x]])/(3*d) + (((92*a
^2 + 9*b^2)*EllipticE[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[2*a + b*Sin[2*
c + 2*d*x]])/(d*Sqrt[(2*a + b*Sin[2*c + 2*d*x])/(2*a + b)]) - (16*a*(4*a^2
 - b^2)*EllipticF[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[(2*a + b*Sin[2*c +
 2*d*x])/(2*a + b)])/(d*Sqrt[2*a + b*Sin[2*c + 2*d*x]]))/3)/(20*Sqrt[2])
```

3.573. $\int (a+b\cos(c+dx)\sin(c+dx))^{5/2}\,dx$

3.573.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3135
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[1/n   Int[(a + b*
Sin[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x]
, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] &&
 IntegerQ[2*n]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3145
```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + b*(Sin[2*c + 2*d*x]/2))^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

3.573.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1137 vs. $2(293) = 586$.

Time = 2.58 (sec) , antiderivative size = 1138, normalized size of antiderivative = 4.29

method	result	size
default	Expression too large to display	1138

input
```
int((a+cos(d*x+c)*sin(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/60*(240*a^4*((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*(-(sin(2*d*x+2*c)-1)*
b/(2*a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*EllipticF(((2*a+b*s
in(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))+64*EllipticF(((2*a+
b*sin(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*((2*a+b*sin(2*d*
x+2*c))/(2*a-b))^(1/2)*(-(sin(2*d*x+2*c)-1)*b/(2*a+b))^(1/2)*(-(1+sin(2*d*
x+2*c))*b/(2*a-b))^(1/2)*a^3*b-24*a^2*((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/
2)*(-(sin(2*d*x+2*c)-1)*b/(2*a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(
1/2)*EllipticF(((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1
/2))*b^2-16*a*((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*(-(sin(2*d*x+2*c)-1)*
b/(2*a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*EllipticF(((2*a+b*s
in(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*b^3-9*EllipticF(((2
*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*((2*a+b*sin(2
*d*x+2*c))/(2*a-b))^(1/2)*(-(sin(2*d*x+2*c)-1)*b/(2*a+b))^(1/2)*(-(1+sin(2
*d*x+2*c))*b/(2*a-b))^(1/2)*b^4-368*EllipticE(((2*a+b*sin(2*d*x+2*c))/(2*a
-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)
*(-(sin(2*d*x+2*c)-1)*b/(2*a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/
2)*a^4+56*EllipticE(((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b
))^(1/2))*((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*(-(sin(2*d*x+2*c)-1)*b/(2
*a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*a^2*b^2+9*EllipticE(((2
*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*((2*a+b*si...
```

3.573.5 Fricas [F]

$$\int (a+b\cos(c+dx)\sin(c+dx))^{5/2}\,dx=\int (b\cos(dx+c)\sin(dx+c)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c)*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
integral(-(b^2*cos(d*x + c)^4 - b^2*cos(d*x + c)^2 - 2*a*b*cos(d*x + c)*si
n(d*x + c) - a^2)*sqrt(b*cos(d*x + c)*sin(d*x + c) + a), x)
```

3.573.6 Sympy [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx)\sin(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c)*sin(d*x+c))**(5/2),x)
```

output
```
Timed out
```

3.573.7 Maxima [F]

$$\int (a+b\cos(c+dx)\sin(c+dx))^{5/2}\,dx = \int (b\cos(dx+c)\sin(dx+c)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c)*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c)*sin(d*x + c) + a)^(5/2), x)
```

3.573.8 Giac [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx)\sin(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c)*sin(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.573.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx)\sin(c+dx))^{5/2}\,dx = \int \left(a+b\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)\right)^{5/2}\,dx$$

input
```
int((a + b*cos(c + d*x)*sin(c + d*x))^(5/2),x)
```

output
```
int((a + b*cos(c + d*x)*sin(c + d*x))^(5/2), x)
```

3.574 $\int (a+b\cos(c+dx)\sin(c+dx))^{3/2}\,dx$

3.574.1 Optimal result

Integrand size = 20, antiderivative size = 212

$$\int (a+b\cos(c+dx)\sin(c+dx))^{3/2}\,dx = -\frac{b\cos(2c+2dx)\sqrt{2a+b\sin(2c+2dx)}}{6\sqrt{2}d} + \frac{2\sqrt{2}aE\left(c-\frac{\pi}{4}+dx\middle|\frac{2b}{2a+b}\right)\sqrt{2a+b\sin(2c+2dx)}}{3d\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}} - \frac{(4a^2-b^2)\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,\frac{2b}{2a+b}\right)\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}{6\sqrt{2}d\sqrt{2a+b\sin(2c+2dx)}}$$

output
```
-1/12*b*cos(2*d*x+2*c)*(2*a+b*sin(2*d*x+2*c))^(1/2)/d*2^(1/2)-2/3*a*(sin(c
+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^(1/2
)*(b/(2*a+b))^(1/2))*2^(1/2)*(2*a+b*sin(2*d*x+2*c))^(1/2)/d/((2*a+b*sin(2*
d*x+2*c))/(2*a+b))^(1/2)+1/12*(4*a^2-b^2)*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(
c+1/4*Pi+d*x)*EllipticF(cos(c+1/4*Pi+d*x),2^(1/2)*(b/(2*a+b))^(1/2))*((2*a
+b*sin(2*d*x+2*c))/(2*a+b))^(1/2)/d*2^(1/2)/(2*a+b*sin(2*d*x+2*c))^(1/2)
```

3.574.2 Mathematica [A] (verified)

Time = 2.03 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.79

$$\int (a+b\cos(c+dx)\sin(c+dx))^{3/2}\,dx = \frac{-b\cos(2(c+dx))(2a+b\sin(2(c+dx)))+8a(2a+b)E\left(c-\frac{\pi}{4}+dx|\frac{2b}{2a+b}\right)\sqrt{\frac{2a+b\sin(2(c+dx))}{2a+b}} -}{6d\sqrt{4a+2b\sin(2(c+dx))}}$$

input
```
Integrate[(a + b*Cos[c + d*x]*Sin[c + d*x])^(3/2),x]
```

output
```
(-(b*Cos[2*(c + d*x)]*(2*a + b*Sin[2*(c + d*x)])) + 8*a*(2*a + b)*Elliptic
E[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[(2*a + b*Sin[2*(c + d*x)])/(2*a +
b)] - (4*a^2 - b^2)*EllipticF[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[(2*a +
 b*Sin[2*(c + d*x)])/(2*a + b)])/(6*d*Sqrt[4*a + 2*b*Sin[2*(c + d*x)]])
```

3.574.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.98, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {3042, 3145, 3042, 3135, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sin(c+dx)\cos(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\sin(c+dx)\cos(c+dx))^{3/2}dx$$

$$\Big\downarrow \text{3145}$$

$$\int \left(a+\frac{1}{2}b\sin(2c+2dx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+\frac{1}{2}b\sin(2c+2dx)\right)^{3/2}dx$$

$$\Big\downarrow \text{3135}$$

$$\frac{2}{3}\int \frac{12a^2+8b\sin(2c+2dx)a+b^2}{4\sqrt{2}\sqrt{2a+b\sin(2c+2dx)}}dx-\frac{b\cos(2c+2dx)\sqrt{2a+b\sin(2c+2dx)}}{6\sqrt{2}d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{12a^2+8b\sin(2c+2dx)a+b^2}{\sqrt{2a+b\sin(2c+2dx)}}dx}{6\sqrt{2}}-\frac{b\cos(2c+2dx)\sqrt{2a+b\sin(2c+2dx)}}{6\sqrt{2}d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{12a^2+8b\sin(2c+2dx)a+b^2}{\sqrt{2a+b\sin(2c+2dx)}}dx}{6\sqrt{2}}-\frac{b\cos(2c+2dx)\sqrt{2a+b\sin(2c+2dx)}}{6\sqrt{2}d}$$

$$\Big\downarrow \text{ 3231}$$

$$\frac{8a\int \sqrt{2a+b\sin(2c+2dx)}dx-\left(4a^2-b^2\right)\int \frac{1}{\sqrt{2a+b\sin(2c+2dx)}}dx}{6\sqrt{2}}-\frac{b\cos(2c+2dx)\sqrt{2a+b\sin(2c+2dx)}}{6\sqrt{2}d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{8a\int \sqrt{2a+b\sin(2c+2dx)}dx-\left(4a^2-b^2\right)\int \frac{1}{\sqrt{2a+b\sin(2c+2dx)}}dx}{6\sqrt{2}}-\frac{b\cos(2c+2dx)\sqrt{2a+b\sin(2c+2dx)}}{6\sqrt{2}d}$$

$$\Big\downarrow \text{ 3134}$$

$$\frac{\frac{8a\sqrt{2a+b\sin(2c+2dx)}\int \sqrt{\frac{2a}{2a+b}+\frac{b\sin(2c+2dx)}{2a+b}}dx}{\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}-\left(4a^2-b^2\right)\int \frac{1}{\sqrt{2a+b\sin(2c+2dx)}}dx}{6\sqrt{2}}-\frac{b\cos(2c+2dx)\sqrt{2a+b\sin(2c+2dx)}}{6\sqrt{2}d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{8a\sqrt{2a+b\sin(2c+2dx)}\int \sqrt{\frac{2a}{2a+b}+\frac{b\sin(2c+2dx)}{2a+b}}dx}{\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}-\left(4a^2-b^2\right)\int \frac{1}{\sqrt{2a+b\sin(2c+2dx)}}dx}{6\sqrt{2}}-\frac{b\cos(2c+2dx)\sqrt{2a+b\sin(2c+2dx)}}{6\sqrt{2}d}$$

$$\Big\downarrow \text{ 3132}$$

3.574. $\int (a+b\cos(c+dx)\sin(c+dx))^{3/2}\,dx$

$$\frac{\frac{8a\sqrt{2a+b\sin(2c+2dx)}E\left(c+dx-\frac{\pi}{4}|\frac{2b}{2a+b}\right)}{d\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}-(4a^2-b^2)\int\frac{1}{\sqrt{2a+b\sin(2c+2dx)}}dx}{6\sqrt{2}}-\frac{b\cos(2c+2dx)\sqrt{2a+b\sin(2c+2dx)}}{6\sqrt{2}d}$$

$$\Big\downarrow \text{3142}$$

$$\frac{\frac{8a\sqrt{2a+b\sin(2c+2dx)}E\left(c+dx-\frac{\pi}{4}|\frac{2b}{2a+b}\right)}{d\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}-\frac{(4a^2-b^2)\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}\int\frac{1}{\sqrt{\frac{2a}{2a+b}+\frac{b\sin(2c+2dx)}{2a+b}}}dx}{\sqrt{2a+b\sin(2c+2dx)}}}{6\sqrt{2}}-\frac{b\cos(2c+2dx)\sqrt{2a+b\sin(2c+2dx)}}{6\sqrt{2}d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{8a\sqrt{2a+b\sin(2c+2dx)}E\left(c+dx-\frac{\pi}{4}|\frac{2b}{2a+b}\right)}{d\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}-\frac{(4a^2-b^2)\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}\int\frac{1}{\sqrt{\frac{2a}{2a+b}+\frac{b\sin(2c+2dx)}{2a+b}}}dx}{\sqrt{2a+b\sin(2c+2dx)}}}{6\sqrt{2}}-\frac{b\cos(2c+2dx)\sqrt{2a+b\sin(2c+2dx)}}{6\sqrt{2}d}$$

$$\Big\downarrow \text{3140}$$

$$\frac{\frac{8a\sqrt{2a+b\sin(2c+2dx)}E\left(c+dx-\frac{\pi}{4}|\frac{2b}{2a+b}\right)}{d\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}-\frac{(4a^2-b^2)\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}\,\text{EllipticF}\left(c+dx-\frac{\pi}{4},\frac{2b}{2a+b}\right)}{d\sqrt{2a+b\sin(2c+2dx)}}}{6\sqrt{2}}-\frac{b\cos(2c+2dx)\sqrt{2a+b\sin(2c+2dx)}}{6\sqrt{2}d}$$

input
```
Int[(a + b*Cos[c + d*x]*Sin[c + d*x])^(3/2),x]
```

output
```
-1/6*(b*Cos[2*c + 2*d*x]*Sqrt[2*a + b*Sin[2*c + 2*d*x]])/(Sqrt[2]*d) + ((8
*a*EllipticE[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[2*a + b*Sin[2*c + 2*d*x
]])/(d*Sqrt[(2*a + b*Sin[2*c + 2*d*x])/(2*a + b)]) - ((4*a^2 - b^2)*Ellipt
icF[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[(2*a + b*Sin[2*c + 2*d*x])/(2*a
+ b)])/(d*Sqrt[2*a + b*Sin[2*c + 2*d*x]]))/(6*Sqrt[2])
```

3.574. $\int (a+b\cos(c+dx)\sin(c+dx))^{3/2}\,dx$

3.574.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3135
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[1/n   Int[(a + b*
Sin[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x]
, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] &&
 IntegerQ[2*n]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3145
```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + b*(Sin[2*c + 2*d*x]/2))^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

3.574.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 843 vs. $2(246) = 492$.

Time = 1.57 (sec) , antiderivative size = 844, normalized size of antiderivative = 3.98

method	result
default	$\frac{24a^3\sqrt{\frac{2a+b\sin(2dx+2c)}{2a-b}}\sqrt{-\frac{(\sin(2dx+2c)-1)b}{2a+b}}\sqrt{-\frac{(1+\sin(2dx+2c))b}{2a-b}}\,\text{EllipticF}\left(\sqrt{\frac{2a+b\sin(2dx+2c)}{2a-b}},\sqrt{\frac{2a-b}{2a+b}}\right)+4\,\text{EllipticF}\left(\sqrt{\frac{2a+b\sin(}{2a-}}\right.}{}$

input
```
int((a+cos(d*x+c)*sin(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(24*a^3*((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*(-(sin(2*d*x+2*c)-1)*b/
(2*a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*EllipticF(((2*a+b*sin
(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))+4*EllipticF(((2*a+b*s
in(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*((2*a+b*sin(2*d*x+2
*c))/(2*a-b))^(1/2)*(-(sin(2*d*x+2*c)-1)*b/(2*a+b))^(1/2)*(-(1+sin(2*d*x+2
*c))*b/(2*a-b))^(1/2)*a^2*b-6*((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*(-(si
n(2*d*x+2*c)-1)*b/(2*a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*Ell
ipticF(((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*b^2
*a-((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*(-(sin(2*d*x+2*c)-1)*b/(2*a+b))^
(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*EllipticF(((2*a+b*sin(2*d*x+2*
c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*b^3-32*EllipticE(((2*a+b*sin(2
*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*((2*a+b*sin(2*d*x+2*c))
/(2*a-b))^(1/2)*(-(sin(2*d*x+2*c)-1)*b/(2*a+b))^(1/2)*(-(1+sin(2*d*x+2*c))
*b/(2*a-b))^(1/2)*a^3+8*EllipticE(((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2),(
(2*a-b)/(2*a+b))^(1/2))*((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*(-(sin(2*d*
x+2*c)-1)*b/(2*a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*a*b^2+sin
(2*d*x+2*c)^3*b^3+2*sin(2*d*x+2*c)^2*a*b^2-sin(2*d*x+2*c)*b^3-2*a*b^2)/b/c
os(2*d*x+2*c)/(4*a+2*b*sin(2*d*x+2*c))^(1/2)/d
```

3.574.5 Fricas [F]

$$\int (a+b\cos(c+dx)\sin(c+dx))^{3/2}\,dx = \int (b\cos(dx+c)\sin(dx+c)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c)*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
integral((b*cos(d*x + c)*sin(d*x + c) + a)^(3/2), x)
```

3.574.6 Sympy [F]

$$\int (a+b\cos(c+dx)\sin(c+dx))^{3/2}\,dx = \int (a+b\sin(c+dx)\cos(c+dx))^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c)*sin(d*x+c))**(3/2),x)
```

output
```
Integral((a + b*sin(c + d*x)*cos(c + d*x))**(3/2), x)
```

3.574.7 Maxima [F]

$$\int (a+b\cos(c+dx)\sin(c+dx))^{3/2}\,dx = \int (b\cos(dx+c)\sin(dx+c)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cos(d*x+c)*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c)*sin(d*x + c) + a)^(3/2), x)
```

3.574.8 Giac [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx)\sin(c+dx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cos(d*x+c)*sin(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.574.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\cos(c+dx)\sin(c+dx))^{3/2}\,dx = \int (a+b\,\cos(c+d\,x)\,\sin(c+d\,x))^{3/2}\,dx$$

input
```
int((a + b*cos(c + d*x)*sin(c + d*x))^(3/2),x)
```

output
```
int((a + b*cos(c + d*x)*sin(c + d*x))^(3/2), x)
```

3.575 $\int \sqrt{a+b\cos(c+dx)\sin(c+dx)}\,dx$

3.575.1 Optimal result

Integrand size = 20, antiderivative size = 76

$$\int \sqrt{a+b\cos(c+dx)\sin(c+dx)}\,dx=\frac{E\left(c-\frac{\pi}{4}+dx\middle|\frac{2b}{2a+b}\right)\sqrt{2a+b\sin(2c+2dx)}}{\sqrt{2}d\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}$$

output
```
-1/2*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+
d*x),2^(1/2)*(b/(2*a+b))^(1/2))*(2*a+b*sin(2*d*x+2*c))^(1/2)/d*2^(1/2)/((2
*a+b*sin(2*d*x+2*c))/(2*a+b))^(1/2)
```

3.575.2 Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.99

$$\int \sqrt{a+b\cos(c+dx)\sin(c+dx)}\,dx=\frac{(2a+b)E\left(c-\frac{\pi}{4}+dx\middle|\frac{2b}{2a+b}\right)\sqrt{\frac{2a+b\sin(2(c+dx))}{2a+b}}}{d\sqrt{4a+2b\sin(2(c+dx))}}$$

input
```
Integrate[Sqrt[a + b*Cos[c + d*x]*Sin[c + d*x]],x]
```

output
```
((2*a + b)*EllipticE[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[(2*a + b*Sin[2*
(c + d*x)])/(2*a + b)])/(d*Sqrt[4*a + 2*b*Sin[2*(c + d*x)]])
```

3.575.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 3145, 3042, 3134, 3042, 3132}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\sin(c+dx)\cos(c+dx)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sqrt{a+b\sin(c+dx)\cos(c+dx)}dx$$

$$\Big\downarrow \ 3145$$

$$\int \sqrt{a+\frac{1}{2}b\sin(2c+2dx)}dx$$

$$\Big\downarrow \ 3042$$

$$\int \sqrt{a+\frac{1}{2}b\sin(2c+2dx)}dx$$

$$\Big\downarrow \ 3134$$

$$\frac{\sqrt{2a+b\sin(2c+2dx)}\int \sqrt{\frac{2a}{2a+b}+\frac{b\sin(2c+2dx)}{2a+b}}dx}{\sqrt{2}\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\sqrt{2a+b\sin(2c+2dx)}\int \sqrt{\frac{2a}{2a+b}+\frac{b\sin(2c+2dx)}{2a+b}}dx}{\sqrt{2}\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}$$

$$\Big\downarrow \ 3132$$

$$\frac{\sqrt{2a+b\sin(2c+2dx)}E\left(c+dx-\frac{\pi}{4}|\frac{2b}{2a+b}\right)}{\sqrt{2}d\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}$$

input
```
Int[Sqrt[a + b*Cos[c + d*x]*Sin[c + d*x]],x]
```

output
```
(EllipticE[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[2*a + b*Sin[2*c + 2*d*x]]
)/(Sqrt[2]*d*Sqrt[(2*a + b*Sin[2*c + 2*d*x])/(2*a + b)])
```

3.575.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3145
```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + b*(Sin[2*c + 2*d*x]/2))^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

3.575.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(99) = 198$.

Time = 1.23 (sec) , antiderivative size = 311, normalized size of antiderivative = 4.09

method	result
default	$\frac{\sqrt{\frac{2a+b\sin(2dx+2c)}{2a-b}}\sqrt{-\frac{(\sin(2dx+2c)-1)b}{2a+b}}\sqrt{-\frac{(1+\sin(2dx+2c))b}{2a-b}}\,(2a-b)\left(2a\operatorname{EllipticF}\left(\sqrt{\frac{2a+b\sin(2dx+2c)}{2a-b}},\sqrt{\frac{2a-b}{2a+b}}\right)+\operatorname{EllipticF}\left(\sqrt{\frac{2a+}{}}\right.\right.}{b\cos(2dx+2c)\sqrt{4a+2b\sin(2}}$

input
```
int((a+cos(d*x+c)*sin(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*(-(sin(2*d*x+2*c)-1)*b/(2*a+b))^(1/
2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*(2*a-b)/b*(2*a*EllipticF(((2*a+b*
sin(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))+EllipticF(((2*a+b*
sin(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*b-2*EllipticE(((2*
a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*a-EllipticE(((
2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*b)/cos(2*d*x
+2*c)/(4*a+2*b*sin(2*d*x+2*c))^(1/2)/d
```

3.575.5 Fricas [F]

$$\int \sqrt{a+b\cos(c+dx)\sin(c+dx)}\,dx=\int \sqrt{b\cos\left(dx+c\right)\sin\left(dx+c\right)+a}\,dx$$

input
```
integrate((a+b*cos(d*x+c)*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(b*cos(d*x + c)*sin(d*x + c) + a), x)
```

3.575.6 Sympy [F]

$$\int \sqrt{a+b\cos(c+dx)\sin(c+dx)}\,dx=\int \sqrt{a+b\sin\left(c+dx\right)\cos\left(c+dx\right)}\,dx$$

input
```
integrate((a+b*cos(d*x+c)*sin(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a + b*sin(c + d*x)*cos(c + d*x)), x)
```

3.575.7 Maxima [F]

$$\int \sqrt{a+b\cos(c+dx)\sin(c+dx)}\,dx=\int \sqrt{b\cos\left(dx+c\right)\sin\left(dx+c\right)+a}\,dx$$

input
```
integrate((a+b*cos(d*x+c)*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*cos(d*x + c)*sin(d*x + c) + a), x)
```

3.575.8 Giac [F]

$$\int \sqrt{a+b\cos(c+dx)\sin(c+dx)}\,dx = \int \sqrt{b\cos\left(dx+c\right)\sin\left(dx+c\right)+a}\,dx$$

input
```
integrate((a+b*cos(d*x+c)*sin(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*cos(d*x + c)*sin(d*x + c) + a), x)
```

3.575.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\cos(c+dx)\sin(c+dx)}\,dx = \int \sqrt{a+b\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}\,dx$$

input
```
int((a + b*cos(c + d*x)*sin(c + d*x))^(1/2),x)
```

output
```
int((a + b*cos(c + d*x)*sin(c + d*x))^(1/2), x)
```

3.576 $\int \frac{1}{\sqrt{a+b\cos(c+dx)\sin(c+dx)}}\,dx$

3.576.1 Optimal result

Integrand size = 20, antiderivative size = 76

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)\sin(c+dx)}}\,dx=\frac{\sqrt{2}\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,\frac{2b}{2a+b}\right)\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}{d\sqrt{2a+b\sin(2c+2dx)}}$$

output
```
-(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(c+1/4*Pi+d*x)
,2^(1/2)*(b/(2*a+b))^(1/2))*2^(1/2)*((2*a+b*sin(2*d*x+2*c))/(2*a+b))^(1/2)
/d/(2*a+b*sin(2*d*x+2*c))^(1/2)
```

3.576.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)\sin(c+dx)}}\,dx=\frac{\text{EllipticF}\left(c-\frac{\pi}{4}+dx,\frac{2b}{2a+b}\right)\sqrt{\frac{2a+b\sin(2(c+dx))}{2a+b}}}{d\sqrt{a+\frac{1}{2}b\sin(2(c+dx))}}$$

input
```
Integrate[1/Sqrt[a + b*Cos[c + d*x]*Sin[c + d*x]],x]
```

output
```
(EllipticF[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[(2*a + b*Sin[2*(c + d*x)]
)/(2*a + b)])/(d*Sqrt[a + (b*Sin[2*(c + d*x)])/2])
```

3.576.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 3145, 3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\sin(c+dx)\cos(c+dx)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sqrt{a+b\sin(c+dx)\cos(c+dx)}}dx$$
$$\Big\downarrow \text{3145}$$
$$\int \frac{1}{\sqrt{a+\frac{1}{2}b\sin(2c+2dx)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sqrt{a+\frac{1}{2}b\sin(2c+2dx)}}dx$$
$$\Big\downarrow \text{3142}$$
$$\frac{\sqrt{2}\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}\int \frac{1}{\sqrt{\frac{2a}{2a+b}+\frac{b\sin(2c+2dx)}{2a+b}}}dx}{\sqrt{2a+b\sin(2c+2dx)}}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\sqrt{2}\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}\int \frac{1}{\sqrt{\frac{2a}{2a+b}+\frac{b\sin(2c+2dx)}{2a+b}}}dx}{\sqrt{2a+b\sin(2c+2dx)}}$$
$$\Big\downarrow \text{3140}$$
$$\frac{\sqrt{2}\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}\text{EllipticF}\left(c+dx-\frac{\pi}{4},\frac{2b}{2a+b}\right)}{d\sqrt{2a+b\sin(2c+2dx)}}$$

input
```
Int[1/Sqrt[a + b*Cos[c + d*x]*Sin[c + d*x]],x]
```

output
```
(Sqrt[2]*EllipticF[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[(2*a + b*Sin[2*c
+ 2*d*x])/(2*a + b)])/(d*Sqrt[2*a + b*Sin[2*c + 2*d*x]])
```

3.576.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3145
```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + b*(Sin[2*c + 2*d*x]/2))^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

3.576.4 Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.17

method	result	size
default	$\frac{2(2a-b)\sqrt{\frac{2a+b\sin(2dx+2c)}{2a-b}}\,\sqrt{-\frac{(\sin(2dx+2c)-1)b}{2a+b}}\,\sqrt{-\frac{(1+\sin(2dx+2c))b}{2a-b}}\,\operatorname{EllipticF}\left(\sqrt{\frac{2a+b\sin(2dx+2c)}{2a-b}},\sqrt{\frac{2a-b}{2a+b}}\right)}{b\cos(2dx+2c)\sqrt{4a+2b\sin(2dx+2c)}\,d}$	165

input
```
int(1/(a+cos(d*x+c)*sin(d*x+c)*b)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(2*a-b)*((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*(-(sin(2*d*x+2*c)-1)*b/(2
*a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*EllipticF(((2*a+b*sin(2
*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))/b/cos(2*d*x+2*c)/(4*a+2
*b*sin(2*d*x+2*c))^(1/2)/d
```

3.576.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 304, normalized size of antiderivative = 4.00

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)\sin(c+dx)}}\,dx = -\frac{\left(b\sqrt{-\frac{4a^2-b^2}{b^2}}-2i\,a\right)\sqrt{4i\,b}\sqrt{\frac{b\sqrt{-\frac{4a^2-b^2}{b^2}}+2i\,a}{b}}F\left(\arcsin\left(\sqrt{\frac{b\sqrt{-\frac{4a^2-b^2}{b^2}}+2i\,a}{b}}(\cos(dx+c)+i\sin(dx+c))\right)\right.}{}$$

input
```
integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
-1/2*((b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)*sqrt(4*I*b)*sqrt((b*sqrt(-(4*a^
2 - b^2)/b^2) + 2*I*a)/b)*elliptic_f(arcsin(sqrt((b*sqrt(-(4*a^2 - b^2)/b^
2) + 2*I*a)/b)*(cos(d*x + c) + I*sin(d*x + c))), (4*I*a*b*sqrt(-(4*a^2 - b
^2)/b^2) + 8*a^2 - b^2)/b^2) + (sqrt(-4*I*b)*b*sqrt(-(4*a^2 - b^2)/b^2) +
2*I*a*sqrt(-4*I*b))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b)*elliptic_
f(arcsin(sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b)*(cos(d*x + c) - I*si
n(d*x + c))), (-4*I*a*b*sqrt(-(4*a^2 - b^2)/b^2) + 8*a^2 - b^2)/b^2))/(b^2
*d)
```

3.576.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)\sin(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+b\sin(c+dx)\cos(c+dx)}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*sin(c + d*x)*cos(c + d*x)), x)
```

3.576.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)\sin(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\cos\left(dx+c\right)\sin\left(dx+c\right)+a}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*cos(d*x + c)*sin(d*x + c) + a), x)
```

3.576.8 Giac [F]

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)\sin(c+dx)}}\,dx = \int \frac{1}{\sqrt{b\cos\left(dx+c\right)\sin\left(dx+c\right)+a}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*cos(d*x + c)*sin(d*x + c) + a), x)
```

3.576.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\cos(c+dx)\sin(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+b\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}}\,dx$$

input
```
int(1/(a + b*cos(c + d*x)*sin(c + d*x))^(1/2),x)
```

output
```
int(1/(a + b*cos(c + d*x)*sin(c + d*x))^(1/2), x)
```

3.577 $\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^{3/2}}\,dx$

3.577.1 Optimal result

Integrand size = 20, antiderivative size = 143

$$\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^{3/2}}\,dx = \frac{2\sqrt{2}b\cos(2c+2dx)}{(4a^2-b^2)\,d\sqrt{2a+b\sin(2c+2dx)}} + \frac{2\sqrt{2}E\left(c-\frac{\pi}{4}+dx\middle|\frac{2b}{2a+b}\right)\sqrt{2a+b\sin(2c+2dx)}}{(4a^2-b^2)\,d\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}$$

output
```
2*b*cos(2*d*x+2*c)*2^(1/2)/(4*a^2-b^2)/d/(2*a+b*sin(2*d*x+2*c))^(1/2)-2*(s
in(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*Pi+d*x),2^
(1/2)*(b/(2*a+b))^(1/2))*2^(1/2)*(2*a+b*sin(2*d*x+2*c))^(1/2)/(4*a^2-b^2)/
d/((2*a+b*sin(2*d*x+2*c))/(2*a+b))^(1/2)
```

3.577.2 Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.71

$$\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^{3/2}}\,dx = \frac{2\left(b\cos(2(c+dx))+(2a+b)E\left(c-\frac{\pi}{4}+dx\middle|\frac{2b}{2a+b}\right)\sqrt{\frac{2a+b\sin(2(c+dx}{2a+b}}\right.}{(4a^2-b^2)\,d\sqrt{a+\frac{1}{2}b\sin(2(c+dx))}}$$

input
```
Integrate[(a + b*Cos[c + d*x]*Sin[c + d*x])^(-3/2),x]
```

output
```
(2*(b*Cos[2*(c + d*x)] + (2*a + b)*EllipticE[c - Pi/4 + d*x, (2*b)/(2*a +
b)]*Sqrt[(2*a + b*Sin[2*(c + d*x)])/(2*a + b)]))/((4*a^2 - b^2)*d*Sqrt[a +
 (b*Sin[2*(c + d*x)])/2])
```

3.577.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {3042, 3145, 3042, 3143, 27, 3042, 3134, 3042, 3132}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sin(c+dx)\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(a+b\sin(c+dx)\cos(c+dx))^{3/2}}dx$$

$$\Big\downarrow \text{3145}$$

$$\int \frac{1}{\left(a+\frac{1}{2}b\sin(2c+2dx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a+\frac{1}{2}b\sin(2c+2dx)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3143}$$

$$\frac{2\sqrt{2}b\cos(2c+2dx)}{d(4a^2-b^2)\sqrt{2a+b\sin(2c+2dx)}}-\frac{8\int -\frac{\sqrt{2a+b\sin(2c+2dx)}}{2\sqrt{2}}dx}{4a^2-b^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\sqrt{2}\int \sqrt{2a+b\sin(2c+2dx)}dx}{4a^2-b^2}+\frac{2\sqrt{2}b\cos(2c+2dx)}{d(4a^2-b^2)\sqrt{2a+b\sin(2c+2dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sqrt{2}\int \sqrt{2a+b\sin(2c+2dx)}dx}{4a^2-b^2}+\frac{2\sqrt{2}b\cos(2c+2dx)}{d(4a^2-b^2)\sqrt{2a+b\sin(2c+2dx)}}$$

$$\Big\downarrow \text{3134}$$

$$\frac{2\sqrt{2}\sqrt{2a+b\sin(2c+2dx)}\int\sqrt{\frac{2a}{2a+b}+\frac{b\sin(2c+2dx)}{2a+b}}dx}{(4a^2-b^2)\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}+\frac{2\sqrt{2}b\cos(2c+2dx)}{d(4a^2-b^2)\sqrt{2a+b\sin(2c+2dx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sqrt{2}\sqrt{2a+b\sin(2c+2dx)}\int\sqrt{\frac{2a}{2a+b}+\frac{b\sin(2c+2dx)}{2a+b}}dx}{(4a^2-b^2)\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}+\frac{2\sqrt{2}b\cos(2c+2dx)}{d(4a^2-b^2)\sqrt{2a+b\sin(2c+2dx)}}$$

$$\Big\downarrow \text{3132}$$

$$\frac{2\sqrt{2}b\cos(2c+2dx)}{d(4a^2-b^2)\sqrt{2a+b\sin(2c+2dx)}}+\frac{2\sqrt{2}\sqrt{2a+b\sin(2c+2dx)}E\left(c+dx-\frac{\pi}{4}|\frac{2b}{2a+b}\right)}{d(4a^2-b^2)\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}$$

input
```
Int[(a + b*Cos[c + d*x]*Sin[c + d*x])^(-3/2),x]
```

output
```
(2*Sqrt[2]*b*Cos[2*c + 2*d*x])/((4*a^2 - b^2)*d*Sqrt[2*a + b*Sin[2*c + 2*d
*x]]) + (2*Sqrt[2]*EllipticE[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[2*a + b
*Sin[2*c + 2*d*x]])/((4*a^2 - b^2)*d*Sqrt[(2*a + b*Sin[2*c + 2*d*x])/(2*a
+ b)])
```

3.577.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3145
```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + b*(Sin[2*c + 2*d*x]/2))^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

3.577.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 569 vs. $2(161) = 322$.

Time = 1.20 (sec) , antiderivative size = 570, normalized size of antiderivative = 3.99

method	result
default	$\frac{16a^2\sqrt{\frac{2a+b\sin(2dx+2c)}{2a-b}}\sqrt{-\frac{(\sin(2dx+2c)-1)b}{2a+b}}\sqrt{-\frac{(1+\sin(2dx+2c))b}{2a-b}}\text{ EllipticF}\left(\sqrt{\frac{2a+b\sin(2dx+2c)}{2a-b}},\sqrt{\frac{2a-b}{2a+b}}\right)-4\text{ EllipticF}\left(\sqrt{\frac{2a+b\sin(2}{2a-}}\right.}{}$

input
```
int(1/(a+cos(d*x+c)*sin(d*x+c)*b)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
4/b*(4*a^2*((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*(-(sin(2*d*x+2*c)-1)*b/(
2*a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*EllipticF(((2*a+b*sin(
2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))-EllipticF(((2*a+b*sin(
2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*((2*a+b*sin(2*d*x+2*c)
)/(2*a-b))^(1/2)*(-(sin(2*d*x+2*c)-1)*b/(2*a+b))^(1/2)*(-(1+sin(2*d*x+2*c)
)*b/(2*a-b))^(1/2)*b^2-4*EllipticE(((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2),
((2*a-b)/(2*a+b))^(1/2))*((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*(-(sin(2*d
*x+2*c)-1)*b/(2*a+b))^(1/2)*(-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*a^2+Elli
pticE(((2*a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*((2*
a+b*sin(2*d*x+2*c))/(2*a-b))^(1/2)*(-(sin(2*d*x+2*c)-1)*b/(2*a+b))^(1/2)*(
-(1+sin(2*d*x+2*c))*b/(2*a-b))^(1/2)*b^2-sin(2*d*x+2*c)^2*b^2+b^2)/(4*a^2-
b^2)/cos(2*d*x+2*c)/(4*a+2*b*sin(2*d*x+2*c))^(1/2)/d
```

3.577.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 914, normalized size of antiderivative = 6.39

$$\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^{3/2}}\,dx =$$

$$-\frac{\left(2\,ab^{2}\cos\left(dx+c\right)\sin\left(dx+c\right)+2\,a^{2}b+\left(-i\,b^{3}\cos\left(dx+c\right)\sin\left(dx+c\right)-i\,ab^{2}\right)\sqrt{-\frac{4\,a^{2}-b^{2}}{b^{2}}}\right)\sqrt{4i\,b}\sqrt{\frac{b\sqrt{-4}}{}}}{}$$

input
```
integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-((2*a*b^2*cos(d*x + c)*sin(d*x + c) + 2*a^2*b + (-I*b^3*cos(d*x + c)*sin(
d*x + c) - I*a*b^2)*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(4*I*b)*sqrt((b*sqrt(-(4
*a^2 - b^2)/b^2) + 2*I*a)/b)*elliptic_e(arcsin(sqrt((b*sqrt(-(4*a^2 - b^2)
/b^2) + 2*I*a)/b)*(cos(d*x + c) + I*sin(d*x + c))), (4*I*a*b*sqrt(-(4*a^2
- b^2)/b^2) + 8*a^2 - b^2)/b^2) - (4*I*a^3 + 2*a^2*b + 2*(2*I*a^2*b + a*b^
2)*cos(d*x + c)*sin(d*x + c) - (2*a^2*b + I*a*b^2 + (2*a*b^2 + I*b^3)*cos(
d*x + c)*sin(d*x + c))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(4*I*b)*sqrt((b*sqrt(
-(4*a^2 - b^2)/b^2) + 2*I*a)/b)*elliptic_f(arcsin(sqrt((b*sqrt(-(4*a^2 - b
^2)/b^2) + 2*I*a)/b)*(cos(d*x + c) + I*sin(d*x + c))), (4*I*a*b*sqrt(-(4*a
^2 - b^2)/b^2) + 8*a^2 - b^2)/b^2) + ((I*b^3*cos(d*x + c)*sin(d*x + c) + I
*a*b^2)*sqrt(-4*I*b)*sqrt(-(4*a^2 - b^2)/b^2) + 2*(a*b^2*cos(d*x + c)*sin(
d*x + c) + a^2*b)*sqrt(-4*I*b))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/
b)*elliptic_e(arcsin(sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b)*(cos(d*x
 + c) - I*sin(d*x + c))), (-4*I*a*b*sqrt(-(4*a^2 - b^2)/b^2) + 8*a^2 - b^2
)/b^2) + ((2*a^2*b - I*a*b^2 + (2*a*b^2 - I*b^3)*cos(d*x + c)*sin(d*x + c)
)*sqrt(-4*I*b)*sqrt(-(4*a^2 - b^2)/b^2) - 2*(-2*I*a^3 + a^2*b + (-2*I*a^2*
b + a*b^2)*cos(d*x + c)*sin(d*x + c))*sqrt(-4*I*b))*sqrt((b*sqrt(-(4*a^2 -
 b^2)/b^2) - 2*I*a)/b)*elliptic_f(arcsin(sqrt((b*sqrt(-(4*a^2 - b^2)/b^2)
- 2*I*a)/b)*(cos(d*x + c) - I*sin(d*x + c))), (-4*I*a*b*sqrt(-(4*a^2 - b^2
)/b^2) + 8*a^2 - b^2)/b^2) - 2*(2*b^3*cos(d*x + c)^2 - b^3)*sqrt(b*cos(...
```

3.577.6 Sympy [F]

$$\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^{3/2}}\,dx = \int \frac{1}{(a+b\sin{(c+dx)}\cos{(c+dx)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))**(3/2),x)
```

output
```
Integral((a + b*sin(c + d*x)*cos(c + d*x))**(-3/2), x)
```

3.577.7 Maxima [F]

$$\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\cos{(dx+c)}\sin{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c)*sin(d*x + c) + a)^(-3/2), x)
```

3.577.8 Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^{3/2}}\,dx = \int \frac{1}{(b\cos{(dx+c)}\sin{(dx+c)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c)*sin(d*x + c) + a)^(-3/2), x)
```

3.577.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a+b\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int(1/(a + b*cos(c + d*x)*sin(c + d*x))^(3/2),x)
```

output
```
int(1/(a + b*cos(c + d*x)*sin(c + d*x))^(3/2), x)
```

3.578 $\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^{5/2}}\,dx$

3.578.1 Optimal result

Integrand size = 20, antiderivative size = 295

$$\begin{aligned}\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^{5/2}}\,dx &= \frac{4\sqrt{2}b\cos(2c+2dx)}{3(4a^2-b^2)\,d(2a+b\sin(2c+2dx))^{3/2}} \\ &+ \frac{32\sqrt{2}ab\cos(2c+2dx)}{3(4a^2-b^2)^2\,d\sqrt{2a+b\sin(2c+2dx)}} \\ &+ \frac{32\sqrt{2}aE\left(c-\frac{\pi}{4}+dx\middle|\frac{2b}{2a+b}\right)\sqrt{2a+b\sin(2c+2dx)}}{3(4a^2-b^2)^2\,d\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}} \\ &- \frac{4\sqrt{2}\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,\frac{2b}{2a+b}\right)\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}{3(4a^2-b^2)\,d\sqrt{2a+b\sin(2c+2dx)}}\end{aligned}$$

output
```
4/3*b*cos(2*d*x+2*c)*2^(1/2)/(4*a^2-b^2)/d/(2*a+b*sin(2*d*x+2*c))^(3/2)+32
/3*a*b*cos(2*d*x+2*c)*2^(1/2)/(4*a^2-b^2)^2/d/(2*a+b*sin(2*d*x+2*c))^(1/2)
-32/3*a*(sin(c+1/4*Pi+d*x)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticE(cos(c+1/4*
Pi+d*x),2^(1/2)*(b/(2*a+b))^(1/2))*2^(1/2)*(2*a+b*sin(2*d*x+2*c))^(1/2)/(4
*a^2-b^2)^2/d/((2*a+b*sin(2*d*x+2*c))/(2*a+b))^(1/2)+4/3*(sin(c+1/4*Pi+d*x
)^2)^(1/2)/sin(c+1/4*Pi+d*x)*EllipticF(cos(c+1/4*Pi+d*x),2^(1/2)*(b/(2*a+b
))^(1/2))*2^(1/2)*((2*a+b*sin(2*d*x+2*c))/(2*a+b))^(1/2)/(4*a^2-b^2)/d/(2*
a+b*sin(2*d*x+2*c))^(1/2)
```

3.578.2 Mathematica [A] (verified)

Time = 1.62 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.68

$$\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^{5/2}}\,dx =$$
$$-\frac{4\sqrt{2}\left(-\frac{8aE\left(c-\frac{\pi}{4}+dx|\frac{2b}{2a+b}\right)(2a+b\sin(2(c+dx)))^2}{\sqrt{\frac{2a+b\sin(2(c+dx))}{2a+b}}}+(2a-b)(2a+b)^2\,\text{EllipticF}\left(c-\frac{\pi}{4}+dx,\frac{2b}{2a+b}\right)\left(\frac{2a+b\sin(2(c+dx))}{2a+b}\right)\right.}{3\left(-4a^2+b^2\right)^2d(2a+b\sin(2(c+dx)))^{3/2}}$$

input
```
Integrate[(a + b*Cos[c + d*x]*Sin[c + d*x])^(-5/2),x]
```

output
```
(-4*Sqrt[2]*((-8*a*EllipticE[c - Pi/4 + d*x, (2*b)/(2*a + b)]*(2*a + b*Sin
[2*(c + d*x)])^2)/Sqrt[(2*a + b*Sin[2*(c + d*x)])/(2*a + b)] + (2*a - b)*(
2*a + b)^2*EllipticF[c - Pi/4 + d*x, (2*b)/(2*a + b)]*((2*a + b*Sin[2*(c +
 d*x)])/(2*a + b))^(3/2) + b*Cos[2*(c + d*x)]*(-20*a^2 + b^2 - 8*a*b*Sin[2
*(c + d*x)])))/(3*(-4*a^2 + b^2)^2*d*(2*a + b*Sin[2*(c + d*x)])^(3/2))
```

3.578.3 Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.01, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {3042, 3145, 3042, 3143, 25, 27, 3042, 3233, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\sin(c+dx)\cos(c+dx))^{5/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(a+b\sin(c+dx)\cos(c+dx))^{5/2}}dx$$
$$\Big\downarrow \text{3145}$$
$$\int \frac{1}{\left(a+\frac{1}{2}b\sin(2c+2dx)\right)^{5/2}}dx$$
$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a+\frac{1}{2} b \sin(2c+2dx)\right)^{5/2}}dx$$

$\Big\downarrow$ 3143

$$\frac{4\sqrt{2}b\cos(2c+2dx)}{3d\left(4a^2-b^2\right)(2a+b\sin(2c+2dx))^{3/2}}-\frac{8\int -\frac{6a-b\sin(2c+2dx)}{\sqrt{2}(2a+b\sin(2c+2dx))^{3/2}}dx}{3\left(4a^2-b^2\right)}$$

$\Big\downarrow$ 25

$$\frac{8\int \frac{6a-b\sin(2c+2dx)}{\sqrt{2}(2a+b\sin(2c+2dx))^{3/2}}dx}{3\left(4a^2-b^2\right)}+\frac{4\sqrt{2}b\cos(2c+2dx)}{3d\left(4a^2-b^2\right)(2a+b\sin(2c+2dx))^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{4\sqrt{2}\int \frac{6a-b\sin(2c+2dx)}{(2a+b\sin(2c+2dx))^{3/2}}dx}{3\left(4a^2-b^2\right)}+\frac{4\sqrt{2}b\cos(2c+2dx)}{3d\left(4a^2-b^2\right)(2a+b\sin(2c+2dx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{4\sqrt{2}\int \frac{6a-b\sin(2c+2dx)}{(2a+b\sin(2c+2dx))^{3/2}}dx}{3\left(4a^2-b^2\right)}+\frac{4\sqrt{2}b\cos(2c+2dx)}{3d\left(4a^2-b^2\right)(2a+b\sin(2c+2dx))^{3/2}}$$

$\Big\downarrow$ 3233

$$\frac{4\sqrt{2}\left(\frac{8ab\cos(2c+2dx)}{d(4a^2-b^2)\sqrt{2a+b\sin(2c+2dx)}}-\frac{2\int -\frac{12a^2+8b\sin(2c+2dx)a+b^2}{2\sqrt{2a+b\sin(2c+2dx)}}dx}{4a^2-b^2}\right)}{3\left(4a^2-b^2\right)}+\frac{4\sqrt{2}b\cos(2c+2dx)}{3d\left(4a^2-b^2\right)(2a+b\sin(2c+2dx))^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{4\sqrt{2}\left(\frac{\int \frac{12a^2+8b\sin(2c+2dx)a+b^2}{\sqrt{2a+b\sin(2c+2dx)}}dx}{4a^2-b^2}+\frac{8ab\cos(2c+2dx)}{d(4a^2-b^2)\sqrt{2a+b\sin(2c+2dx)}}\right)}{3\left(4a^2-b^2\right)}+\frac{4\sqrt{2}b\cos(2c+2dx)}{3d\left(4a^2-b^2\right)(2a+b\sin(2c+2dx))^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{4\sqrt{2}\left(\frac{\int \frac{12a^2+8b\sin(2c+2dx)a+b^2}{\sqrt{2a+b\sin(2c+2dx)}}dx}{4a^2-b^2}+\frac{8ab\cos(2c+2dx)}{d(4a^2-b^2)\sqrt{2a+b\sin(2c+2dx)}}\right)}{3\left(4a^2-b^2\right)}+\frac{4\sqrt{2}b\cos(2c+2dx)}{3d\left(4a^2-b^2\right)(2a+b\sin(2c+2dx))^{3/2}}$$

$\Big\downarrow$ 3231

3.578. $\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^{5/2}}\,dx$

$$\frac{4\sqrt{2}\left(\frac{8a\int\sqrt{2a+b\sin(2c+2dx)}dx-(4a^2-b^2)\int\frac{1}{\sqrt{2a+b\sin(2c+2dx)}}dx}{4a^2-b^2}+\frac{8ab\cos(2c+2dx)}{d(4a^2-b^2)\sqrt{2a+b\sin(2c+2dx)}}\right)}{3(4a^2-b^2)}+\frac{4\sqrt{2}b\cos(2c+2dx)}{3d(4a^2-b^2)(2a+b\sin(2c+2dx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{4\sqrt{2}\left(\frac{8a\int\sqrt{2a+b\sin(2c+2dx)}dx-(4a^2-b^2)\int\frac{1}{\sqrt{2a+b\sin(2c+2dx)}}dx}{4a^2-b^2}+\frac{8ab\cos(2c+2dx)}{d(4a^2-b^2)\sqrt{2a+b\sin(2c+2dx)}}\right)}{3(4a^2-b^2)}+\frac{4\sqrt{2}b\cos(2c+2dx)}{3d(4a^2-b^2)(2a+b\sin(2c+2dx))^{3/2}}$$

$\downarrow$ 3134

$$\frac{4\sqrt{2}\left(\frac{\frac{8a\sqrt{2a+b\sin(2c+2dx)}\int\sqrt{\frac{2a}{2a+b}+\frac{b\sin(2c+2dx)}{2a+b}}dx}{\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}-(4a^2-b^2)\int\frac{1}{\sqrt{2a+b\sin(2c+2dx)}}dx}{4a^2-b^2}+\frac{8ab\cos(2c+2dx)}{d(4a^2-b^2)\sqrt{2a+b\sin(2c+2dx)}}\right)}{3(4a^2-b^2)}+\frac{4\sqrt{2}b\cos(2c+2dx)}{3d(4a^2-b^2)(2a+b\sin(2c+2dx))^{3/2}}$$

$\downarrow$ 3042

$$\frac{4\sqrt{2}\left(\frac{\frac{8a\sqrt{2a+b\sin(2c+2dx)}\int\sqrt{\frac{2a}{2a+b}+\frac{b\sin(2c+2dx)}{2a+b}}dx}{\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}-(4a^2-b^2)\int\frac{1}{\sqrt{2a+b\sin(2c+2dx)}}dx}{4a^2-b^2}+\frac{8ab\cos(2c+2dx)}{d(4a^2-b^2)\sqrt{2a+b\sin(2c+2dx)}}\right)}{3(4a^2-b^2)}+\frac{4\sqrt{2}b\cos(2c+2dx)}{3d(4a^2-b^2)(2a+b\sin(2c+2dx))^{3/2}}$$

$\downarrow$ 3132

$$\frac{4\sqrt{2}\left(\frac{\frac{8a\sqrt{2a+b\sin(2c+2dx)}E\left(c+dx-\frac{\pi}{4}|\frac{2b}{2a+b}\right)}{d\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}-(4a^2-b^2)\int\frac{1}{\sqrt{2a+b\sin(2c+2dx)}}dx}{4a^2-b^2}+\frac{8ab\cos(2c+2dx)}{d(4a^2-b^2)\sqrt{2a+b\sin(2c+2dx)}}\right)}{3(4a^2-b^2)}+\frac{4\sqrt{2}b\cos(2c+2dx)}{3d(4a^2-b^2)(2a+b\sin(2c+2dx))^{3/2}}$$

$\downarrow$ 3142

3.578. $\int\frac{1}{(a+b\cos(c+dx)\sin(c+dx))^{5/2}}\,dx$

$$\frac{4\sqrt{2}\left(\frac{\frac{8a\sqrt{2a+b\sin(2c+2dx)}E\left(c+dx-\frac{\pi}{4}|\frac{2b}{2a+b}\right)}{d\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}-\frac{\left(4a^2-b^2\right)\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}\int\frac{1}{\sqrt{\frac{2a}{2a+b}+\frac{b\sin(2c+2dx)}{2a+b}}}dx}{\sqrt{2a+b\sin(2c+2dx)}}}{4a^2-b^2}+\frac{8ab\cos(2c+2dx)}{d(4a^2-b^2)\sqrt{2a+b\sin(2c+2dx)}}\right)}{3\left(4a^2-b^2\right)}+\frac{4\sqrt{2}b\cos(2c+2dx)}{3d\left(4a^2-b^2\right)(2a+b\sin(2c+2dx))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\sqrt{2}\left(\frac{\frac{8a\sqrt{2a+b\sin(2c+2dx)}E\left(c+dx-\frac{\pi}{4}|\frac{2b}{2a+b}\right)}{d\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}-\frac{\left(4a^2-b^2\right)\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}\int\frac{1}{\sqrt{\frac{2a}{2a+b}+\frac{b\sin(2c+2dx)}{2a+b}}}dx}{\sqrt{2a+b\sin(2c+2dx)}}}{4a^2-b^2}+\frac{8ab\cos(2c+2dx)}{d(4a^2-b^2)\sqrt{2a+b\sin(2c+2dx)}}\right)}{3\left(4a^2-b^2\right)}+\frac{4\sqrt{2}b\cos(2c+2dx)}{3d\left(4a^2-b^2\right)(2a+b\sin(2c+2dx))^{3/2}}$$

$$\Big\downarrow \text{3140}$$

$$\frac{4\sqrt{2}b\cos(2c+2dx)}{3d\left(4a^2-b^2\right)(2a+b\sin(2c+2dx))^{3/2}}+\frac{4\sqrt{2}\left(\frac{8ab\cos(2c+2dx)}{d(4a^2-b^2)\sqrt{2a+b\sin(2c+2dx)}}+\frac{\frac{8a\sqrt{2a+b\sin(2c+2dx)}E\left(c+dx-\frac{\pi}{4}|\frac{2b}{2a+b}\right)}{d\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}}-\frac{\left(4a^2-b^2\right)\sqrt{\frac{2a+b\sin(2c+2dx)}{2a+b}}\text{EllipticF}\left(c+dx-\frac{\pi}{4},\frac{2b}{2a+b}\right)}{d\sqrt{2a+b\sin(2c+2dx)}}}{4a^2-b^2}\right)}{3\left(4a^2-b^2\right)}$$

input
```
Int[(a + b*Cos[c + d*x]*Sin[c + d*x])^(-5/2),x]
```

output
```
(4*Sqrt[2]*b*Cos[2*c + 2*d*x])/(3*(4*a^2 - b^2)*d*(2*a + b*Sin[2*c + 2*d*x
])^(3/2)) + (4*Sqrt[2]*((8*a*b*Cos[2*c + 2*d*x])/((4*a^2 - b^2)*d*Sqrt[2*a
 + b*Sin[2*c + 2*d*x]]) + ((8*a*EllipticE[c - Pi/4 + d*x, (2*b)/(2*a + b)]
*Sqrt[2*a + b*Sin[2*c + 2*d*x]])/(d*Sqrt[(2*a + b*Sin[2*c + 2*d*x])/(2*a +
 b)]) - ((4*a^2 - b^2)*EllipticF[c - Pi/4 + d*x, (2*b)/(2*a + b)]*Sqrt[(2*
a + b*Sin[2*c + 2*d*x])/(2*a + b)])/(d*Sqrt[2*a + b*Sin[2*c + 2*d*x]]))/(4
*a^2 - b^2)))/(3*(4*a^2 - b^2))
```

3.578. $\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^{5/2}}\,dx$

3.578.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

3.578. $\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^{5/2}}\,dx$

rule 3145
```
Int[((a_) + cos[(c_.) + (d_.)*(x_)]*(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_
Symbol] :> Int[(a + b*(Sin[2*c + 2*d*x]/2))^n, x] /; FreeQ[{a, b, c, d, n},
 x]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

3.578.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1553 vs. $2(323) = 646$.

Time = 1.43 (sec) , antiderivative size = 1554, normalized size of antiderivative = 5.27

method	result	size
default	Expression too large to display	1554

input
```
int(1/(a+cos(d*x+c)*sin(d*x+c)*b)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
8/3*(8*a*b^3*cos(2*d*x+2*c)^2*sin(2*d*x+2*c)+(20*a^2*b^2-b^4)*cos(2*d*x+2*
c)^2-(-b/(2*a-b)*sin(2*d*x+2*c)-b/(2*a-b))^(1/2)*(-b/(2*a+b)*sin(2*d*x+2*c
)+b/(2*a+b))^(1/2)*(b/(2*a-b)*sin(2*d*x+2*c)+2*a/(2*a-b))^(1/2)*b*(32*Elli
pticE((b/(2*a-b)*sin(2*d*x+2*c)+2*a/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2)
)*a^3-8*EllipticE((b/(2*a-b)*sin(2*d*x+2*c)+2*a/(2*a-b))^(1/2),((2*a-b)/(2
*a+b))^(1/2))*a*b^2-24*EllipticF((b/(2*a-b)*sin(2*d*x+2*c)+2*a/(2*a-b))^(1
/2),((2*a-b)/(2*a+b))^(1/2))*a^3-4*EllipticF((b/(2*a-b)*sin(2*d*x+2*c)+2*a
/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*a^2*b+6*EllipticF((b/(2*a-b)*sin(
2*d*x+2*c)+2*a/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*a*b^2+EllipticF((b/
(2*a-b)*sin(2*d*x+2*c)+2*a/(2*a-b))^(1/2),((2*a-b)/(2*a+b))^(1/2))*b^3)*si
n(2*d*x+2*c)+48*EllipticF((b/(2*a-b)*sin(2*d*x+2*c)+2*a/(2*a-b))^(1/2),((2
*a-b)/(2*a+b))^(1/2))*(b/(2*a-b)*sin(2*d*x+2*c)+2*a/(2*a-b))^(1/2)*(-b/(2*
a+b)*sin(2*d*x+2*c)+b/(2*a+b))^(1/2)*(-b/(2*a-b)*sin(2*d*x+2*c)-b/(2*a-b))
^(1/2)*a^4+8*EllipticF((b/(2*a-b)*sin(2*d*x+2*c)+2*a/(2*a-b))^(1/2),((2*a-
b)/(2*a+b))^(1/2))*(b/(2*a-b)*sin(2*d*x+2*c)+2*a/(2*a-b))^(1/2)*(-b/(2*a+b
)*sin(2*d*x+2*c)+b/(2*a+b))^(1/2)*(-b/(2*a-b)*sin(2*d*x+2*c)-b/(2*a-b))^(1
/2)*a^3*b-12*EllipticF((b/(2*a-b)*sin(2*d*x+2*c)+2*a/(2*a-b))^(1/2),((2*a-
b)/(2*a+b))^(1/2))*(b/(2*a-b)*sin(2*d*x+2*c)+2*a/(2*a-b))^(1/2)*(-b/(2*a+b
)*sin(2*d*x+2*c)+b/(2*a+b))^(1/2)*(-b/(2*a-b)*sin(2*d*x+2*c)-b/(2*a-b))^(1
/2)*a^2*b^2-2*EllipticF((b/(2*a-b)*sin(2*d*x+2*c)+2*a/(2*a-b))^(1/2),((...
```

3.578.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 1506, normalized size of antiderivative = 5.11

$$\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(8*(2*a^2*b^3*cos(d*x + c)^4 - 2*a^2*b^3*cos(d*x + c)^2 - 4*a^3*b^2*c
os(d*x + c)*sin(d*x + c) - 2*a^4*b - (I*a*b^4*cos(d*x + c)^4 - I*a*b^4*cos
(d*x + c)^2 - 2*I*a^2*b^3*cos(d*x + c)*sin(d*x + c) - I*a^3*b^2)*sqrt(-(4*
a^2 - b^2)/b^2))*sqrt(4*I*b)*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b)*
elliptic_e(arcsin(sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b)*(cos(d*x +
c) + I*sin(d*x + c))), (4*I*a*b*sqrt(-(4*a^2 - b^2)/b^2) + 8*a^2 - b^2)/b^
2) - (-24*I*a^5 - 16*a^4*b - 2*I*a^3*b^2 + 2*(12*I*a^3*b^2 + 8*a^2*b^3 + I
*a*b^4)*cos(d*x + c)^4 + 2*(-12*I*a^3*b^2 - 8*a^2*b^3 - I*a*b^4)*cos(d*x +
 c)^2 + 4*(-12*I*a^4*b - 8*a^3*b^2 - I*a^2*b^3)*cos(d*x + c)*sin(d*x + c)
+ (12*a^4*b + 8*I*a^3*b^2 + a^2*b^3 - (12*a^2*b^3 + 8*I*a*b^4 + b^5)*cos(d
*x + c)^4 + (12*a^2*b^3 + 8*I*a*b^4 + b^5)*cos(d*x + c)^2 + 2*(12*a^3*b^2
+ 8*I*a^2*b^3 + a*b^4)*cos(d*x + c)*sin(d*x + c))*sqrt(-(4*a^2 - b^2)/b^2)
)*sqrt(4*I*b)*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b)*elliptic_f(arcs
in(sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b)*(cos(d*x + c) + I*sin(d*x
+ c))), (4*I*a*b*sqrt(-(4*a^2 - b^2)/b^2) + 8*a^2 - b^2)/b^2) - 8*((-I*a*b
^4*cos(d*x + c)^4 + I*a*b^4*cos(d*x + c)^2 + 2*I*a^2*b^3*cos(d*x + c)*sin(
d*x + c) + I*a^3*b^2)*sqrt(-4*I*b)*sqrt(-(4*a^2 - b^2)/b^2) - 2*(a^2*b^3*c
os(d*x + c)^4 - a^2*b^3*cos(d*x + c)^2 - 2*a^3*b^2*cos(d*x + c)*sin(d*x +
c) - a^4*b)*sqrt(-4*I*b))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b)*ell
iptic_e(arcsin(sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b)*(cos(d*x + ...
```

3.578.6 Sympy [F]

$$\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^{5/2}}\,dx=\int \frac{1}{(a+b\sin{(c+dx)}\cos{(c+dx)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))**(5/2),x)
```

output
```
Integral((a + b*sin(c + d*x)*cos(c + d*x))**(-5/2), x)
```

3.578.7 Maxima [F]

$$\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\cos(dx+c)\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*cos(d*x + c)*sin(d*x + c) + a)^(-5/2), x)
```

3.578.8 Giac [F]

$$\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^{5/2}}\,dx = \int \frac{1}{(b\cos(dx+c)\sin(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*cos(d*x+c)*sin(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*cos(d*x + c)*sin(d*x + c) + a)^(-5/2), x)
```

3.578.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cos(c+dx)\sin(c+dx))^{5/2}}\,dx = \int \frac{1}{(a+b\cos(c+d\,x)\,\sin(c+d\,x))^{5/2}}\,dx$$

input
```
int(1/(a + b*cos(c + d*x)*sin(c + d*x))^(5/2),x)
```

output
```
int(1/(a + b*cos(c + d*x)*sin(c + d*x))^(5/2), x)
```

3.579 $\int \frac{x^3}{a+b\cos(x)\sin(x)}\,dx$

3.579.1 Optimal result

Integrand size = 14, antiderivative size = 461

$$\begin{aligned}\int \frac{x^3}{a+b\cos(x)\sin(x)}\,dx = &-\frac{ix^3\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}}+\frac{ix^3\log\left(1-\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}}\\&-\frac{3x^2\,\text{PolyLog}\left(2,\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}}+\frac{3x^2\,\text{PolyLog}\left(2,\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}}\\&-\frac{3ix\,\text{PolyLog}\left(3,\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}}+\frac{3ix\,\text{PolyLog}\left(3,\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}}\\&+\frac{3\,\text{PolyLog}\left(4,\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{4\sqrt{4a^2-b^2}}-\frac{3\,\text{PolyLog}\left(4,\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{4\sqrt{4a^2-b^2}}\end{aligned}$$

output
```
-I*x^3*ln(1-I*b*exp(2*I*x)/(2*a-(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)+I*x^
3*ln(1-I*b*exp(2*I*x)/(2*a+(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)-3/2*x^2*p
olylog(2,I*b*exp(2*I*x)/(2*a-(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)+3/2*x^2
*polylog(2,I*b*exp(2*I*x)/(2*a+(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)-3/2*I
*x*polylog(3,I*b*exp(2*I*x)/(2*a-(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)+3/2
*I*x*polylog(3,I*b*exp(2*I*x)/(2*a+(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)+3
/4*polylog(4,I*b*exp(2*I*x)/(2*a-(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)-3/4
*polylog(4,I*b*exp(2*I*x)/(2*a+(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)
```

3.579.2 Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 340, normalized size of antiderivative = 0.74

$$\int \frac{x^3}{a+b\cos(x)\sin(x)}\,dx$$
$$= \frac{-4ix^3\log\left(1+\frac{ibe^{2ix}}{-2a+\sqrt{4a^2-b^2}}\right)+4ix^3\log\left(1-\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)-6x^2\operatorname{PolyLog}\left(2,-\frac{ibe^{2ix}}{-2a+\sqrt{4a^2-b^2}}\right)+6x^2\operatorname{PolyL}}{}$$

input
```
Integrate[x^3/(a + b*Cos[x]*Sin[x]),x]
```

output
```
((-4*I)*x^3*Log[1 + (I*b*E^((2*I)*x))/(-2*a + Sqrt[4*a^2 - b^2])] + (4*I)*
x^3*Log[1 - (I*b*E^((2*I)*x))/(2*a + Sqrt[4*a^2 - b^2])] - 6*x^2*PolyLog[2
, ((-I)*b*E^((2*I)*x))/(-2*a + Sqrt[4*a^2 - b^2])] + 6*x^2*PolyLog[2, (I*b
*E^((2*I)*x))/(2*a + Sqrt[4*a^2 - b^2])] - (6*I)*x*PolyLog[3, ((-I)*b*E^((
2*I)*x))/(-2*a + Sqrt[4*a^2 - b^2])] + (6*I)*x*PolyLog[3, (I*b*E^((2*I)*x)
)/(2*a + Sqrt[4*a^2 - b^2])] + 3*PolyLog[4, ((-I)*b*E^((2*I)*x))/(-2*a + S
qrt[4*a^2 - b^2])] - 3*PolyLog[4, (I*b*E^((2*I)*x))/(2*a + Sqrt[4*a^2 - b^
2])])/(4*Sqrt[4*a^2 - b^2])
```

3.579.3 Rubi [A] (verified)

Time = 1.41 (sec) , antiderivative size = 425, normalized size of antiderivative = 0.92, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {5095, 3042, 3804, 27, 2694, 27, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{a+b\sin(x)\cos(x)}\,dx$$
$$\Big\downarrow \; 5095$$
$$\int \frac{x^3}{a+\frac{1}{2}b\sin(2x)}dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{x^3}{a+\frac{1}{2}b\sin(2x)}dx$$

$$\Big\downarrow \text{3804}$$

$$2\int \frac{2e^{2ix}x^3}{4e^{2ix}a-ibe^{4ix}+ib}dx$$

$$\Big\downarrow \text{27}$$

$$4\int \frac{e^{2ix}x^3}{4e^{2ix}a-ibe^{4ix}+ib}dx$$

$$\Big\downarrow \text{2694}$$

$$4\left(\frac{ib\int \frac{e^{2ix}x^3}{2\left(2a-ibe^{2ix}+\sqrt{4a^2-b^2}\right)}dx}{\sqrt{4a^2-b^2}}-\frac{ib\int \frac{e^{2ix}x^3}{2\left(2a-ibe^{2ix}-\sqrt{4a^2-b^2}\right)}dx}{\sqrt{4a^2-b^2}}\right)$$

$$\Big\downarrow \text{27}$$

$$4\left(\frac{ib\int \frac{e^{2ix}x^3}{2a-ibe^{2ix}+\sqrt{4a^2-b^2}}dx}{2\sqrt{4a^2-b^2}}-\frac{ib\int \frac{e^{2ix}x^3}{2a-ibe^{2ix}-\sqrt{4a^2-b^2}}dx}{2\sqrt{4a^2-b^2}}\right)$$

$$\Big\downarrow \text{2620}$$

$$4\left(\frac{ib\left(\frac{x^3\log\left(1-\frac{ibe^{2ix}}{\sqrt{4a^2-b^2}+2a}\right)}{2b}-\frac{3\int x^2\log\left(1-\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)dx}{2b}\right)}{2\sqrt{4a^2-b^2}}-\frac{ib\left(\frac{x^3\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{2b}-\frac{3\int x^2\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)dx}{2b}\right.}{2\sqrt{4a^2-b^2}}\right.$$

$$\Big\downarrow \text{3011}$$

$$4\left(\frac{ib\left(\frac{x^3\log\left(1-\frac{ibe^{2ix}}{\sqrt{4a^2-b^2}+2a}\right)}{2b}-\frac{3\left(\frac{1}{2}ix^2\text{PolyLog}\left(2,\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)-i\int x\,\text{PolyLog}\left(2,\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)dx\right)}{2b}\right)}{2\sqrt{4a^2-b^2}}-\frac{ib\left(\frac{x^3\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-}}\right.}{2b}\right.}{}\right.$$

$$\Big\downarrow \text{7163}$$

$$4\left(\frac{ib\left(\frac{x^3\log\left(1-\frac{ibe^{2ix}}{\sqrt{4a^2-b^2}+2a}\right)}{2b}-\frac{3\left(\frac{1}{2}ix^2\text{PolyLog}\left(2,\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)-i\left(\frac{1}{2}i\int\text{PolyLog}\left(3,\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)dx-\frac{1}{2}ix\,\text{PolyLog}\left(3,\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)\right.\right.}{2b}\right.}{2\sqrt{4a^2-b^2}}\right.$$

3.579. $\int \frac{x^3}{a+b\cos(x)\sin(x)}\,dx$

$$\Big\downarrow \text{2720}$$

$$4\left(\frac{ib\left(\frac{x^3\log\left(1-\frac{ibe^{2ix}}{\sqrt{4a^2-b^2}+2a}\right)}{2b}-\frac{3\left(\frac{1}{2}ix^2\,\text{PolyLog}\left(2,\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)-i\left(\frac{1}{4}\int e^{-2ix}\,\text{PolyLog}\left(3,\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)de^{2ix}-\frac{1}{2}ix\,\text{PolyLog}\left(3,\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)\right)\right)}{2b}\right)}{2\sqrt{4a^2-b^2}}\right.$$

$$\Big\downarrow \text{7143}$$

$$4\left(\frac{ib\left(\frac{x^3\log\left(1-\frac{ibe^{2ix}}{\sqrt{4a^2-b^2}+2a}\right)}{2b}-\frac{3\left(\frac{1}{2}ix^2\,\text{PolyLog}\left(2,\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)-i\left(\frac{1}{4}\,\text{PolyLog}\left(4,\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)-\frac{1}{2}ix\,\text{PolyLog}\left(3,\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)\right)\right)}{2b}\right)}{2\sqrt{4a^2-b^2}}\right.$$

input
```
Int[x^3/(a + b*Cos[x]*Sin[x]),x]
```

output
```
4*(((-1/2*I)*b*((x^3*Log[1 - (I*b*E^((2*I)*x))/(2*a - Sqrt[4*a^2 - b^2])])
/(2*b) - (3*((I/2)*x^2*PolyLog[2, (I*b*E^((2*I)*x))/(2*a - Sqrt[4*a^2 - b^
2])] - I*((-1/2*I)*x*PolyLog[3, (I*b*E^((2*I)*x))/(2*a - Sqrt[4*a^2 - b^2]
)] + PolyLog[4, (I*b*E^((2*I)*x))/(2*a - Sqrt[4*a^2 - b^2])]/4)))/(2*b)))/
Sqrt[4*a^2 - b^2] + ((I/2)*b*((x^3*Log[1 - (I*b*E^((2*I)*x))/(2*a + Sqrt[4
*a^2 - b^2])])/(2*b) - (3*((I/2)*x^2*PolyLog[2, (I*b*E^((2*I)*x))/(2*a + S
qrt[4*a^2 - b^2])] - I*((-1/2*I)*x*PolyLog[3, (I*b*E^((2*I)*x))/(2*a + Sqr
t[4*a^2 - b^2])] + PolyLog[4, (I*b*E^((2*I)*x))/(2*a + Sqrt[4*a^2 - b^2])]
/4)))/(2*b)))/Sqrt[4*a^2 - b^2])
```

3.579. $\int \frac{x^3}{a+b\cos(x)\sin(x)}\,dx$

3.579.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3804
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[2   Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[a^2 - b^2, 0] && IGtQ[m, 0]
```

3.579. $\int \frac{x^3}{a+b\cos(x)\sin(x)}\,dx$

rule 5095
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + Cos[(c_.) + (d_.)*(x_)]*(b_.)*Sin[(c
_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(e + f*x)^m*(a + b*(Sin[2*c + 2*
d*x]/2))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.579.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2281 vs. $2(389) = 778$.

Time = 1.02 (sec) , antiderivative size = 2282, normalized size of antiderivative = 4.95

method	result	size
risch	Expression too large to display	2282

input
```
int(x^3/(a+b*cos(x)*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
-4*I/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*a^2*x^4+6*I/(8*a^2-2*
b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*polylog(4,b*exp(2*I*x)/(-2*I*a+(-(2
*a-b)*(2*a+b))^(1/2)))*a^2-3/2*I/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^
(1/2))*polylog(4,b*exp(2*I*x)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2)))*b^2+12/(8
*a^2-2*b^2)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))*polylog(3,b*exp(2*I*x)/(-2*I
*a-(-(2*a-b)*(2*a+b))^(1/2)))*a^2*x-3/(8*a^2-2*b^2)/(-2*I*a-(-(2*a-b)*(2*a
+b))^(1/2))*polylog(3,b*exp(2*I*x)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2)))*b^2*
x+3/(8*a^2-2*b^2)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))*(-(2*a-b)*(2*a+b))^(1/
2)*polylog(4,b*exp(2*I*x)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2)))*a+6*I/(8*a^2-
2*b^2)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))*polylog(4,b*exp(2*I*x)/(-2*I*a-(-
(2*a-b)*(2*a+b))^(1/2)))*a^2+6*I/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^
(1/2))*(-(2*a-b)*(2*a+b))^(1/2)*polylog(3,b*exp(2*I*x)/(-2*I*a+(-(2*a-b)*(
2*a+b))^(1/2)))*a*x+4*I/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*(-
(2*a-b)*(2*a+b))^(1/2)*ln(1-b*exp(2*I*x)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))
)*a*x^3-3/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*polylog(3,b*exp(
2*I*x)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2)))*b^2*x-3/(8*a^2-2*b^2)/(-2*I*a+(-
(2*a-b)*(2*a+b))^(1/2))*(-(2*a-b)*(2*a+b))^(1/2)*polylog(4,b*exp(2*I*x)/(-
2*I*a+(-(2*a-b)*(2*a+b))^(1/2)))*a+I/(8*a^2-2*b^2)/(-2*I*a-(-(2*a-b)*(2*a+
b))^(1/2))*b^2*x^4+I/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*b^2*x
^4-6*I/(8*a^2-2*b^2)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))*(-(2*a-b)*(2*a+b...
```

3.579.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3308 vs. $2(367) = 734$.

Time = 0.98 (sec) , antiderivative size = 3308, normalized size of antiderivative = 7.18

$$\int \frac{x^3}{a + b\cos(x)\sin(x)}\,dx = \text{Too large to display}$$

input
```
integrate(x^3/(a+b*cos(x)*sin(x)),x, algorithm="fricas")
```

output
```
-1/2*(b*x^3*sqrt(-(4*a^2 - b^2)/b^2)*log(-((2*I*a*cos(x) + 2*a*sin(x) - (b
*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2
)/b^2) + 2*I*a)/b) - b)/b) + b*x^3*sqrt(-(4*a^2 - b^2)/b^2)*log(-((-2*I*a*
cos(x) - 2*a*sin(x) + (b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sq
rt((b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b) - b)/b) - b*x^3*sqrt(-(4*a^2 -
b^2)/b^2)*log(-((2*I*a*cos(x) - 2*a*sin(x) - (b*cos(x) + I*b*sin(x))*sqrt(
-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b) - b)/b)
 - b*x^3*sqrt(-(4*a^2 - b^2)/b^2)*log(-((-2*I*a*cos(x) + 2*a*sin(x) + (b*c
os(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)
/b^2) + 2*I*a)/b) - b)/b) + b*x^3*sqrt(-(4*a^2 - b^2)/b^2)*log(-((2*I*a*co
s(x) - 2*a*sin(x) + (b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt
((b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b) - b)/b) + b*x^3*sqrt(-(4*a^2 - b^
2)/b^2)*log(-((-2*I*a*cos(x) + 2*a*sin(x) - (b*cos(x) + I*b*sin(x))*sqrt(-
(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b) - b)/b) -
 b*x^3*sqrt(-(4*a^2 - b^2)/b^2)*log(-((2*I*a*cos(x) + 2*a*sin(x) + (b*cos(
x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^
2) - 2*I*a)/b) - b)/b) - b*x^3*sqrt(-(4*a^2 - b^2)/b^2)*log(-((-2*I*a*cos(
x) - 2*a*sin(x) - (b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-
(b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b) - b)/b) + 3*I*b*x^2*sqrt(-(4*a^2 -
 b^2)/b^2)*dilog(((2*I*a*cos(x) + 2*a*sin(x) - (b*cos(x) - I*b*sin(x))*...
```

3.579.6 Sympy [F]

$$\int \frac{x^3}{a+b\cos(x)\sin(x)}\,dx=\int \frac{x^3}{a+b\sin{(x)}\cos{(x)}}\,dx$$

input
```
integrate(x**3/(a+b*cos(x)*sin(x)),x)
```

output
```
Integral(x**3/(a + b*sin(x)*cos(x)), x)
```

3.579.7 Maxima [F]

$$\int \frac{x^3}{a+b\cos(x)\sin(x)}\,dx = \int \frac{x^3}{b\cos(x)\sin(x)+a}\,dx$$

input
```
integrate(x^3/(a+b*cos(x)*sin(x)),x, algorithm="maxima")
```

output
```
integrate(x^3/(b*cos(x)*sin(x) + a), x)
```

3.579.8 Giac [F]

$$\int \frac{x^3}{a+b\cos(x)\sin(x)}\,dx = \int \frac{x^3}{b\cos(x)\sin(x)+a}\,dx$$

input
```
integrate(x^3/(a+b*cos(x)*sin(x)),x, algorithm="giac")
```

output
```
integrate(x^3/(b*cos(x)*sin(x) + a), x)
```

3.579.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{a+b\cos(x)\sin(x)}\,dx = \int \frac{x^3}{a+b\,\cos(x)\,\sin(x)}\,dx$$

input
```
int(x^3/(a + b*cos(x)*sin(x)),x)
```

output
```
int(x^3/(a + b*cos(x)*sin(x)), x)
```

3.580 $\int \frac{x^2}{a+b\cos(x)\sin(x)}\,dx$

3.580.1 Optimal result

Integrand size = 14, antiderivative size = 340

$$\int \frac{x^2}{a+b\cos(x)\sin(x)}\,dx = -\frac{ix^2\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}}+\frac{ix^2\log\left(1-\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} - \frac{x\,\text{PolyLog}\left(2,\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}}+\frac{x\,\text{PolyLog}\left(2,\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} - \frac{i\,\text{PolyLog}\left(3,\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}}+\frac{i\,\text{PolyLog}\left(3,\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}}$$

output
```
-I*x^2*ln(1-I*b*exp(2*I*x)/(2*a-(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)+I*x^
2*ln(1-I*b*exp(2*I*x)/(2*a+(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)-x*polylog
(2,I*b*exp(2*I*x)/(2*a-(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)+x*polylog(2,I
*b*exp(2*I*x)/(2*a+(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)-1/2*I*polylog(3,I
*b*exp(2*I*x)/(2*a-(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)+1/2*I*polylog(3,I
*b*exp(2*I*x)/(2*a+(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)
```

3.580.2 Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.75

$$\int \frac{x^2}{a+b\cos(x)\sin(x)}\,dx =$$
$$-\frac{i\left(2x^2\log\left(1+\frac{ibe^{2ix}}{-2a+\sqrt{4a^2-b^2}}\right)-2x^2\log\left(1-\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)-2ix\,\text{PolyLog}\left(2,-\frac{ibe^{2ix}}{-2a+\sqrt{4a^2-b^2}}\right)+2ix\,\text{PolyL}\right.}{2\sqrt{4a^2-b^2}}$$

input
```
Integrate[x^2/(a + b*Cos[x]*Sin[x]),x]
```

output
```
((-1/2*I)*(2*x^2*Log[1 + (I*b*E^((2*I)*x))/(-2*a + Sqrt[4*a^2 - b^2])] - 2
*x^2*Log[1 - (I*b*E^((2*I)*x))/(2*a + Sqrt[4*a^2 - b^2])] - (2*I)*x*PolyLo
g[2, ((-I)*b*E^((2*I)*x))/(-2*a + Sqrt[4*a^2 - b^2])] + (2*I)*x*PolyLog[2,
 (I*b*E^((2*I)*x))/(2*a + Sqrt[4*a^2 - b^2])] + PolyLog[3, ((-I)*b*E^((2*I
)*x))/(-2*a + Sqrt[4*a^2 - b^2])] - PolyLog[3, (I*b*E^((2*I)*x))/(2*a + Sq
rt[4*a^2 - b^2])]))/Sqrt[4*a^2 - b^2]
```

3.580.3 Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 321, normalized size of antiderivative = 0.94, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {5095, 3042, 3804, 27, 2694, 27, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a+b\sin(x)\cos(x)}\,dx$$
$$\Big\downarrow \text{5095}$$
$$\int \frac{x^2}{a+\frac{1}{2}b\sin(2x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{x^2}{a+\frac{1}{2}b\sin(2x)}dx$$
$$\Big\downarrow \text{3804}$$

$$2\int \frac{2e^{2ix}x^2}{4e^{2ix}a-ibe^{4ix}+ib}dx$$

$\downarrow$ 27

$$4\int \frac{e^{2ix}x^2}{4e^{2ix}a-ibe^{4ix}+ib}dx$$

$\downarrow$ 2694

$$4\left(\frac{ib\int \frac{e^{2ix}x^2}{2\left(2a-ibe^{2ix}+\sqrt{4a^2-b^2}\right)}dx}{\sqrt{4a^2-b^2}}-\frac{ib\int \frac{e^{2ix}x^2}{2\left(2a-ibe^{2ix}-\sqrt{4a^2-b^2}\right)}dx}{\sqrt{4a^2-b^2}}\right)$$

$\downarrow$ 27

$$4\left(\frac{ib\int \frac{e^{2ix}x^2}{2a-ibe^{2ix}+\sqrt{4a^2-b^2}}dx}{2\sqrt{4a^2-b^2}}-\frac{ib\int \frac{e^{2ix}x^2}{2a-ibe^{2ix}-\sqrt{4a^2-b^2}}dx}{2\sqrt{4a^2-b^2}}\right)$$

$\downarrow$ 2620

$$4\left(\frac{ib\left(\frac{x^2\log\left(1-\frac{ibe^{2ix}}{\sqrt{4a^2-b^2}+2a}\right)}{2b}-\frac{\int x\log\left(1-\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)dx}{b}\right)}{2\sqrt{4a^2-b^2}}-\frac{ib\left(\frac{x^2\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{2b}-\frac{\int x\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)dx}{b}\right)}{2\sqrt{4a^2-b^2}}\right)$$

$\downarrow$ 3011

$$4\Bigg(\frac{ib\left(\frac{x^2\log\left(1-\frac{ibe^{2ix}}{\sqrt{4a^2-b^2}+2a}\right)}{2b}-\frac{\frac{1}{2}ix\,\text{PolyLog}\left(2,\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)-\frac{1}{2}i\int \text{PolyLog}\left(2,\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)dx}{b}\right)}{2\sqrt{4a^2-b^2}}-ib\Bigg(\frac{x^2\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{2b}$$

$\downarrow$ 2720

$$4\Bigg(\frac{ib\left(\frac{x^2\log\left(1-\frac{ibe^{2ix}}{\sqrt{4a^2-b^2}+2a}\right)}{2b}-\frac{\frac{1}{2}ix\,\text{PolyLog}\left(2,\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)-\frac{1}{4}\int e^{-2ix}\,\text{PolyLog}\left(2,\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)de^{2ix}}{b}\right)}{2\sqrt{4a^2-b^2}}-ib\Bigg(\frac{x^2\log\Big(1-\frac{ibe^{2i}}{2a-\sqrt{4a}}}{2b}$$

$\downarrow$ 7143

3.580. $\int \frac{x^2}{a+b\cos(x)\sin(x)}\,dx$

$$4\left(\frac{ib\left(\frac{x^2\log\left(1-\frac{ibe^{2ix}}{\sqrt{4a^2-b^2}+2a}\right)}{2b}-\frac{\frac{1}{2}ix\operatorname{PolyLog}\left(2,\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)-\frac{1}{4}\operatorname{PolyLog}\left(3,\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{b}\right)}{2\sqrt{4a^2-b^2}}-\frac{ib\left(\frac{x^2\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{2b}-\frac{\frac{1}{2}i}{}\right.}{}\right.$$

input
```
Int[x^2/(a + b*Cos[x]*Sin[x]),x]
```

output
```
4*(((-1/2*I)*b*((x^2*Log[1 - (I*b*E^((2*I)*x))/(2*a - Sqrt[4*a^2 - b^2])])
/(2*b) - ((I/2)*x*PolyLog[2, (I*b*E^((2*I)*x))/(2*a - Sqrt[4*a^2 - b^2])]
- PolyLog[3, (I*b*E^((2*I)*x))/(2*a - Sqrt[4*a^2 - b^2])]/4)/b))/Sqrt[4*a^
2 - b^2] + ((I/2)*b*((x^2*Log[1 - (I*b*E^((2*I)*x))/(2*a + Sqrt[4*a^2 - b^
2])])/(2*b) - ((I/2)*x*PolyLog[2, (I*b*E^((2*I)*x))/(2*a + Sqrt[4*a^2 - b^
2])] - PolyLog[3, (I*b*E^((2*I)*x))/(2*a + Sqrt[4*a^2 - b^2])]/4)/b))/Sqrt
[4*a^2 - b^2])
```

3.580.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3804
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[2   Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 5095
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + Cos[(c_.) + (d_.)*(x_)]*(b_.)*Sin[(c
_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(e + f*x)^m*(a + b*(Sin[2*c + 2*
d*x]/2))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.580.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1781 vs. 2(290) = 580.

Time = 0.96 (sec) , antiderivative size = 1782, normalized size of antiderivative = 5.24

method	result	size
risch	Expression too large to display	1782

input
```
int(x^2/(a+b*cos(x)*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
-8/3/(8*a^2-2*b^2)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))*(-(2*a-b)*(2*a+b))^(1
/2)*a*x^3-16/3*I/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*a^2*x^3+4
/3*I/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*b^2*x^3-2*I/(8*a^2-2*
b^2)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))*polylog(3,b*exp(2*I*x)/(-2*I*a-(-(2
*a-b)*(2*a+b))^(1/2)))*(-(2*a-b)*(2*a+b))^(1/2)*a+8/(8*a^2-2*b^2)/(-2*I*a-
(-(2*a-b)*(2*a+b))^(1/2))*ln(1-b*exp(2*I*x)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/
2)))*a^2*x^2-2/(8*a^2-2*b^2)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))*ln(1-b*exp(
2*I*x)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2)))*b^2*x^2-4/(8*a^2-2*b^2)/(-2*I*a-
(-(2*a-b)*(2*a+b))^(1/2))*polylog(2,b*exp(2*I*x)/(-2*I*a-(-(2*a-b)*(2*a+b)
)^(1/2)))*(-(2*a-b)*(2*a+b))^(1/2)*a*x-4*I/(8*a^2-2*b^2)/(-2*I*a-(-(2*a-b)
*(2*a+b))^(1/2))*ln(1-b*exp(2*I*x)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2)))*(-(2
*a-b)*(2*a+b))^(1/2)*a*x^2-8*I/(8*a^2-2*b^2)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1
/2))*polylog(2,b*exp(2*I*x)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2)))*a^2*x+2*I/(
8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*polylog(3,b*exp(2*I*x)/(-2*
I*a+(-(2*a-b)*(2*a+b))^(1/2)))*(-(2*a-b)*(2*a+b))^(1/2)*a+4/(8*a^2-2*b^2)/
(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))*polylog(3,b*exp(2*I*x)/(-2*I*a-(-(2*a-b)
*(2*a+b))^(1/2)))*a^2-1/(8*a^2-2*b^2)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))*po
lylog(3,b*exp(2*I*x)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2)))*b^2+8/3/(8*a^2-2*b
^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*(-(2*a-b)*(2*a+b))^(1/2)*a*x^3+2*I/(
8*a^2-2*b^2)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))*polylog(2,b*exp(2*I*x)/(...
```

3.580.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2492 vs. $2(272) = 544$.

Time = 0.89 (sec) , antiderivative size = 2492, normalized size of antiderivative = 7.33

$$\int \frac{x^2}{a + b\cos(x)\sin(x)}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(a+b*cos(x)*sin(x)),x, algorithm="fricas")
```

output
```
-1/2*(b*x^2*sqrt(-(4*a^2 - b^2)/b^2)*log(-((2*I*a*cos(x) + 2*a*sin(x) - (b
*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2
)/b^2) + 2*I*a)/b) - b)/b) + b*x^2*sqrt(-(4*a^2 - b^2)/b^2)*log(-((-2*I*a*
cos(x) - 2*a*sin(x) + (b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sq
rt((b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b) - b)/b) - b*x^2*sqrt(-(4*a^2 -
b^2)/b^2)*log(-((2*I*a*cos(x) - 2*a*sin(x) - (b*cos(x) + I*b*sin(x))*sqrt(
-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b) - b)/b)
 - b*x^2*sqrt(-(4*a^2 - b^2)/b^2)*log(-((-2*I*a*cos(x) + 2*a*sin(x) + (b*c
os(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)
/b^2) + 2*I*a)/b) - b)/b) + b*x^2*sqrt(-(4*a^2 - b^2)/b^2)*log(-((2*I*a*co
s(x) - 2*a*sin(x) + (b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt
((b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b) - b)/b) + b*x^2*sqrt(-(4*a^2 - b^
2)/b^2)*log(-((-2*I*a*cos(x) + 2*a*sin(x) - (b*cos(x) + I*b*sin(x))*sqrt(-
(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b) - b)/b) -
 b*x^2*sqrt(-(4*a^2 - b^2)/b^2)*log(-((2*I*a*cos(x) + 2*a*sin(x) + (b*cos(
x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^
2) - 2*I*a)/b) - b)/b) - b*x^2*sqrt(-(4*a^2 - b^2)/b^2)*log(-((-2*I*a*cos(
x) - 2*a*sin(x) - (b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-
(b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b) - b)/b) + 2*I*b*x*sqrt(-(4*a^2 - b
^2)/b^2)*dilog(((2*I*a*cos(x) + 2*a*sin(x) - (b*cos(x) - I*b*sin(x))*sq...
```

3.580.6 Sympy [F]

$$\int \frac{x^2}{a+b\cos(x)\sin(x)}\,dx=\int \frac{x^2}{a+b\sin{(x)}\cos{(x)}}\,dx$$

input
```
integrate(x**2/(a+b*cos(x)*sin(x)),x)
```

output
```
Integral(x**2/(a + b*sin(x)*cos(x)), x)
```

3.580.7 Maxima [F]

$$\int \frac{x^2}{a + b\cos(x)\sin(x)}\,dx = \int \frac{x^2}{b\cos(x)\sin(x) + a}\,dx$$

input
```
integrate(x^2/(a+b*cos(x)*sin(x)),x, algorithm="maxima")
```

output
```
integrate(x^2/(b*cos(x)*sin(x) + a), x)
```

3.580.8 Giac [F]

$$\int \frac{x^2}{a + b\cos(x)\sin(x)}\,dx = \int \frac{x^2}{b\cos(x)\sin(x) + a}\,dx$$

input
```
integrate(x^2/(a+b*cos(x)*sin(x)),x, algorithm="giac")
```

output
```
integrate(x^2/(b*cos(x)*sin(x) + a), x)
```

3.580.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{a + b\cos(x)\sin(x)}\,dx = \int \frac{x^2}{a + b\,\cos(x)\,\sin(x)}\,dx$$

input
```
int(x^2/(a + b*cos(x)*sin(x)),x)
```

output
```
int(x^2/(a + b*cos(x)*sin(x)), x)
```

3.581 $\int \frac{x}{a+b\cos(x)\sin(x)}\,dx$

3.581.1 Optimal result

Integrand size = 12, antiderivative size = 225

$$\int \frac{x}{a+b\cos(x)\sin(x)}\,dx = -\frac{ix\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}}+\frac{ix\log\left(1-\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} - \frac{\operatorname{PolyLog}\left(2,\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}}+\frac{\operatorname{PolyLog}\left(2,\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{2\sqrt{4a^2-b^2}}$$

output

```
-I*x*ln(1-I*b*exp(2*I*x)/(2*a-(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)+I*x*ln
(1-I*b*exp(2*I*x)/(2*a+(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)-1/2*polylog(2
,I*b*exp(2*I*x)/(2*a-(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)+1/2*polylog(2,I
*b*exp(2*I*x)/(2*a+(4*a^2-b^2)^(1/2)))/(4*a^2-b^2)^(1/2)
```

3.581.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 788 vs. 2(225) = 450.

Time = 1.25 (sec) , antiderivative size = 788, normalized size of antiderivative = 3.50

$$\int \frac{x}{a+b\cos(x)\sin(x)}\,dx = \frac{1}{2}\left(\frac{\pi\arctan\left(\frac{b+2a\tan(x)}{\sqrt{4a^2-b^2}}\right)}{\sqrt{4a^2-b^2}} + \frac{2\arccos\left(-\frac{2a}{b}\right)\operatorname{arctanh}\left(\frac{(2a-b)\cot\left(\frac{\pi}{4}+x\right)}{\sqrt{-4a^2+b^2}}\right)+(\pi-4x)\operatorname{arctanh}\left(\frac{(2a+b)\tan\left(\frac{\pi}{4}+x\right)}{\sqrt{-4a^2+b^2}}\right)-\left(\arccos\left(-\frac{2a}{b}\right)+2i\operatorname{arcta}\right.}{}\right.$$

input
```
Integrate[x/(a + b*Cos[x]*Sin[x]),x]
```

output
```
((Pi*ArcTan[(b + 2*a*Tan[x])/Sqrt[4*a^2 - b^2]])/Sqrt[4*a^2 - b^2] + (2*Ar
cCos[(-2*a)/b]*ArcTanh[((2*a - b)*Cot[Pi/4 + x])/Sqrt[-4*a^2 + b^2]] + (Pi
 - 4*x)*ArcTanh[((2*a + b)*Tan[Pi/4 + x])/Sqrt[-4*a^2 + b^2]] - (ArcCos[(-
2*a)/b] + (2*I)*ArcTanh[((2*a - b)*Cot[Pi/4 + x])/Sqrt[-4*a^2 + b^2]])*Log
[((2*a + b)*(-2*a + b - I*Sqrt[-4*a^2 + b^2])*(1 + I*Cot[Pi/4 + x]))/(b*(2
*a + b + Sqrt[-4*a^2 + b^2]*Cot[Pi/4 + x]))] - (ArcCos[(-2*a)/b] - (2*I)*A
rcTanh[((2*a - b)*Cot[Pi/4 + x])/Sqrt[-4*a^2 + b^2]])*Log[((2*a + b)*((2*I
)*a - I*b + Sqrt[-4*a^2 + b^2])*(I + Cot[Pi/4 + x]))/(b*(2*a + b + Sqrt[-4
*a^2 + b^2]*Cot[Pi/4 + x]))] + (ArcCos[(-2*a)/b] + (2*I)*(ArcTanh[((2*a -
b)*Cot[Pi/4 + x])/Sqrt[-4*a^2 + b^2]] + ArcTanh[((2*a + b)*Tan[Pi/4 + x])/
Sqrt[-4*a^2 + b^2]]))*Log[((-1)^(1/4)*Sqrt[-4*a^2 + b^2])/(2*Sqrt[b]*E^(I*
x)*Sqrt[a + b*Cos[x]*Sin[x]])] + (ArcCos[(-2*a)/b] - (2*I)*ArcTanh[((2*a -
 b)*Cot[Pi/4 + x])/Sqrt[-4*a^2 + b^2]] - (2*I)*ArcTanh[((2*a + b)*Tan[Pi/4
 + x])/Sqrt[-4*a^2 + b^2]])*Log[-1/2*((-1)^(3/4)*Sqrt[-4*a^2 + b^2]*E^(I*x
))/(Sqrt[b]*Sqrt[a + b*Cos[x]*Sin[x]])] + I*(PolyLog[2, ((2*a - I*Sqrt[-4*
a^2 + b^2])*(2*a + b - Sqrt[-4*a^2 + b^2]*Cot[Pi/4 + x]))/(b*(2*a + b + Sq
rt[-4*a^2 + b^2]*Cot[Pi/4 + x]))] - PolyLog[2, ((2*a + I*Sqrt[-4*a^2 + b^2
])*(2*a + b - Sqrt[-4*a^2 + b^2]*Cot[Pi/4 + x]))/(b*(2*a + b + Sqrt[-4*a^2
 + b^2]*Cot[Pi/4 + x]))]))/Sqrt[-4*a^2 + b^2])/2
```

3.581.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5095, 3042, 3804, 27, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a+b\sin(x)\cos(x)}\,dx$$

$$\Big\downarrow \text{5095}$$

$$\int \frac{x}{a+\frac{1}{2}b\sin(2x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{x}{a+\frac{1}{2}b\sin(2x)}dx$$

$$\Big\downarrow \text{3804}$$

$$2\int \frac{2e^{2ix}x}{4e^{2ix}a-ibe^{4ix}+ib}dx$$

$$\Big\downarrow \text{27}$$

$$4\int \frac{e^{2ix}x}{4e^{2ix}a-ibe^{4ix}+ib}dx$$

$$\Big\downarrow \text{2694}$$

$$4\left(\frac{ib\int \frac{e^{2ix}x}{2\left(2a-ibe^{2ix}+\sqrt{4a^2-b^2}\right)}dx}{\sqrt{4a^2-b^2}}-\frac{ib\int \frac{e^{2ix}x}{2\left(2a-ibe^{2ix}-\sqrt{4a^2-b^2}\right)}dx}{\sqrt{4a^2-b^2}}\right)$$

$$\Big\downarrow \text{27}$$

$$4\left(\frac{ib\int \frac{e^{2ix}x}{2a-ibe^{2ix}+\sqrt{4a^2-b^2}}dx}{2\sqrt{4a^2-b^2}}-\frac{ib\int \frac{e^{2ix}x}{2a-ibe^{2ix}-\sqrt{4a^2-b^2}}dx}{2\sqrt{4a^2-b^2}}\right)$$

$$\Big\downarrow \text{2620}$$

$$4\left(\frac{ib\left(\frac{x\log\left(1-\frac{ibe^{2ix}}{\sqrt{4a^2-b^2}+2a}\right)}{2b}-\frac{\int\log\left(1-\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)dx}{2b}\right)}{2\sqrt{4a^2-b^2}}-\frac{ib\left(\frac{x\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{2b}-\frac{\int\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)dx}{2b}\right)}{2\sqrt{4a^2-b^2}}\right)$$

$\Big\downarrow$ 2715

$$4\left(\frac{ib\left(\frac{i\int e^{-2ix}\log\left(1-\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)de^{2ix}}{4b}+\frac{x\log\left(1-\frac{ibe^{2ix}}{\sqrt{4a^2-b^2}+2a}\right)}{2b}\right)}{2\sqrt{4a^2-b^2}}-\frac{ib\left(\frac{i\int e^{-2ix}\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)de^{2ix}}{4b}+\frac{x\log\left(1-\frac{ibe^{\cdots}}{2a-\sqrt{4\cdots}}\right.}{2b}\right.}{2\sqrt{4a^2-b^2}}\right.$$

$\Big\downarrow$ 2838

$$4\left(\frac{ib\left(\frac{x\log\left(1-\frac{ibe^{2ix}}{\sqrt{4a^2-b^2}+2a}\right)}{2b}-\frac{i\,\mathrm{PolyLog}\left(2,\frac{ibe^{2ix}}{2a+\sqrt{4a^2-b^2}}\right)}{4b}\right)}{2\sqrt{4a^2-b^2}}-\frac{ib\left(\frac{x\log\left(1-\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{2b}-\frac{i\,\mathrm{PolyLog}\left(2,\frac{ibe^{2ix}}{2a-\sqrt{4a^2-b^2}}\right)}{4b}\right)}{2\sqrt{4a^2-b^2}}\right)$$

input

```
Int[x/(a + b*Cos[x]*Sin[x]),x]
```

output

```
4*(((-1/2*I)*b*((x*Log[1 - (I*b*E^((2*I)*x))/(2*a - Sqrt[4*a^2 - b^2])])/(
2*b) - ((I/4)*PolyLog[2, (I*b*E^((2*I)*x))/(2*a - Sqrt[4*a^2 - b^2])])/b))
/Sqrt[4*a^2 - b^2] + ((I/2)*b*((x*Log[1 - (I*b*E^((2*I)*x))/(2*a + Sqrt[4*
a^2 - b^2])])/(2*b) - ((I/4)*PolyLog[2, (I*b*E^((2*I)*x))/(2*a + Sqrt[4*a^
2 - b^2])])/b))/Sqrt[4*a^2 - b^2])
```

3.581. $\int \frac{x}{a+b\cos(x)\sin(x)}\,dx$

3.581.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3804
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[2   Int[(c + d*x)^m*(E^(I*(e + f*x))/(I*b + 2*a*E^(I*(e + f*x
)) - I*b*E^(2*I*(e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 5095
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + Cos[(c_.) + (d_.)*(x_)]*(b_.)*Sin[(c
_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(e + f*x)^m*(a + b*(Sin[2*c + 2*
d*x]/2))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.581. $\int \frac{x}{a+b\cos(x)\sin(x)}\,dx$

3.581.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1283 vs. 2(191) = 382.

Time = 0.91 (sec) , antiderivative size = 1284, normalized size of antiderivative = 5.71

method	result	size
risch	Expression too large to display	1284

input
```
int(x/(a+b*cos(x)*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
4*I/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*ln(1-b*exp(2*I*x)/(-2*
I*a+(-(2*a-b)*(2*a+b))^(1/2)))*(-(2*a-b)*(2*a+b))^(1/2)*a*x+4/(8*a^2-2*b^2
)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*(-(2*a-b)*(2*a+b))^(1/2)*a*x^2-2/(8*a^
2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*ln(1-b*exp(2*I*x)/(-2*I*a+(-(2*
a-b)*(2*a+b))^(1/2)))*b^2*x+2/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/
2))*polylog(2,b*exp(2*I*x)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2)))*(-(2*a-b)*(2
*a+b))^(1/2)*a+8/(8*a^2-2*b^2)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))*ln(1-b*ex
p(2*I*x)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2)))*a^2*x-4*I/(8*a^2-2*b^2)/(-2*I*
a+(-(2*a-b)*(2*a+b))^(1/2))*polylog(2,b*exp(2*I*x)/(-2*I*a+(-(2*a-b)*(2*a+
b))^(1/2)))*a^2+I/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*polylog(
2,b*exp(2*I*x)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2)))*b^2+2*I/(8*a^2-2*b^2)/(-
2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*x^2*b^2-4*I/(8*a^2-2*b^2)/(-2*I*a-(-(2*a-b
)*(2*a+b))^(1/2))*(-(2*a-b)*(2*a+b))^(1/2)*ln(1-b*exp(2*I*x)/(-2*I*a-(-(2*
a-b)*(2*a+b))^(1/2)))*a*x-8*I/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/
2))*a^2*x^2+8/(8*a^2-2*b^2)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2))*ln(1-b*exp(2
*I*x)/(-2*I*a+(-(2*a-b)*(2*a+b))^(1/2)))*a^2*x-4/(8*a^2-2*b^2)/(-2*I*a-(-(
2*a-b)*(2*a+b))^(1/2))*(-(2*a-b)*(2*a+b))^(1/2)*a*x^2-2/(8*a^2-2*b^2)/(-2*
I*a-(-(2*a-b)*(2*a+b))^(1/2))*ln(1-b*exp(2*I*x)/(-2*I*a-(-(2*a-b)*(2*a+b))
^(1/2)))*b^2*x-2/(8*a^2-2*b^2)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2))*(-(2*a-b)
*(2*a+b))^(1/2)*polylog(2,b*exp(2*I*x)/(-2*I*a-(-(2*a-b)*(2*a+b))^(1/2)...
```

3.581.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1676 vs. 2(179) = 358.

Time = 1.19 (sec) , antiderivative size = 1676, normalized size of antiderivative = 7.45

$$\int \frac{x}{a+b\cos(x)\sin(x)}\,dx = \text{Too large to display}$$

input
```
integrate(x/(a+b*cos(x)*sin(x)),x, algorithm="fricas")
```

output
```
-1/2*(b*x*sqrt(-(4*a^2 - b^2)/b^2)*log(-((2*I*a*cos(x) + 2*a*sin(x) - (b*c
os(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/
b^2) + 2*I*a)/b) - b)/b) + b*x*sqrt(-(4*a^2 - b^2)/b^2)*log(-((-2*I*a*cos(
x) - 2*a*sin(x) + (b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((
b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b) - b)/b) - b*x*sqrt(-(4*a^2 - b^2)/b
^2)*log(-((2*I*a*cos(x) - 2*a*sin(x) - (b*cos(x) + I*b*sin(x))*sqrt(-(4*a^
2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) + 2*I*a)/b) - b)/b) - b*x
*sqrt(-(4*a^2 - b^2)/b^2)*log(-((-2*I*a*cos(x) + 2*a*sin(x) + (b*cos(x) +
I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) +
2*I*a)/b) - b)/b) + b*x*sqrt(-(4*a^2 - b^2)/b^2)*log(-((2*I*a*cos(x) - 2*a
*sin(x) + (b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt((b*sqrt(-
(4*a^2 - b^2)/b^2) - 2*I*a)/b) - b)/b) + b*x*sqrt(-(4*a^2 - b^2)/b^2)*log(
-((-2*I*a*cos(x) + 2*a*sin(x) - (b*cos(x) + I*b*sin(x))*sqrt(-(4*a^2 - b^2
)/b^2))*sqrt((b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b) - b)/b) - b*x*sqrt(-(
4*a^2 - b^2)/b^2)*log(-((2*I*a*cos(x) + 2*a*sin(x) + (b*cos(x) - I*b*sin(x
))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2 - b^2)/b^2) - 2*I*a)/b)
 - b)/b) - b*x*sqrt(-(4*a^2 - b^2)/b^2)*log(-((-2*I*a*cos(x) - 2*a*sin(x)
- (b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^2))*sqrt(-(b*sqrt(-(4*a^2
- b^2)/b^2) - 2*I*a)/b) - b)/b) + I*b*sqrt(-(4*a^2 - b^2)/b^2)*dilog(((2*I
*a*cos(x) + 2*a*sin(x) - (b*cos(x) - I*b*sin(x))*sqrt(-(4*a^2 - b^2)/b^...
```

3.581.6 Sympy [F]

$$\int \frac{x}{a+b\cos(x)\sin(x)}\,dx = \int \frac{x}{a+b\sin(x)\cos(x)}\,dx$$

input
```
integrate(x/(a+b*cos(x)*sin(x)),x)
```

output
```
Integral(x/(a + b*sin(x)*cos(x)), x)
```

3.581.7 Maxima [F]

$$\int \frac{x}{a+b\cos(x)\sin(x)}\,dx = \int \frac{x}{b\cos\left(x\right)\sin\left(x\right)+a}\,dx$$

input
```
integrate(x/(a+b*cos(x)*sin(x)),x, algorithm="maxima")
```

output
```
integrate(x/(b*cos(x)*sin(x) + a), x)
```

3.581.8 Giac [F]

$$\int \frac{x}{a+b\cos(x)\sin(x)}\,dx = \int \frac{x}{b\cos\left(x\right)\sin\left(x\right)+a}\,dx$$

input
```
integrate(x/(a+b*cos(x)*sin(x)),x, algorithm="giac")
```

output
```
integrate(x/(b*cos(x)*sin(x) + a), x)
```

3.581.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{a+b\cos(x)\sin(x)}\,dx = \int \frac{x}{a+b\,\cos\left(x\right)\,\sin\left(x\right)}\,dx$$

input
```
int(x/(a + b*cos(x)*sin(x)),x)
```

output
```
int(x/(a + b*cos(x)*sin(x)), x)
```

3.582 $\int \frac{1}{x(a+b\cos(x)\sin(x))}\,dx$

3.582.1 Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{x(a+b\cos(x)\sin(x))}\,dx = \operatorname{Int}\left(\frac{1}{x\left(a+\frac{1}{2}b\sin(2x)\right)},x\right)$$

output
```
Unintegrable(1/x/(a+1/2*b*sin(2*x)),x)
```

3.582.2 Mathematica [N/A]

Not integrable

Time = 2.52 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x(a+b\cos(x)\sin(x))}\,dx = \int \frac{1}{x(a+b\cos(x)\sin(x))}\,dx$$

input
```
Integrate[1/(x*(a + b*Cos[x]*Sin[x])),x]
```

output
```
Integrate[1/(x*(a + b*Cos[x]*Sin[x])), x]
```

3.582.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5095, 3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a+b\sin(x)\cos(x))}dx$$
$$\Big\downarrow \text{5095}$$
$$\int \frac{1}{x\left(a+\frac{1}{2}b\sin(2x)\right)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{x\left(a+\frac{1}{2}b\sin(2x)\right)}dx$$
$$\Big\downarrow \text{3807}$$
$$\int \frac{1}{x\left(a+\frac{1}{2}b\sin(2x)\right)}dx$$

input
```
Int[1/(x*(a + b*Cos[x]*Sin[x])),x]
```

output
```
$Aborted
```

3.582.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

rule 5095
```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + Cos[(c_.) + (d_.)*(x_)]*(b_.)*Sin[(c
_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(e + f*x)^m*(a + b*(Sin[2*c + 2*
d*x]/2))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.582.4 Maple [N/A] (verified)

Not integrable

Time = 0.64 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\cos\left(x\right)\sin\left(x\right)\right)}dx$$

input
```
int(1/x/(a+b*cos(x)*sin(x)),x)
```

output
```
int(1/x/(a+b*cos(x)*sin(x)),x)
```

3.582.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{x(a+b\cos(x)\sin(x))}\,dx=\int \frac{1}{(b\cos\left(x\right)\sin\left(x\right)+a)x}\,dx$$

input
```
integrate(1/x/(a+b*cos(x)*sin(x)),x, algorithm="fricas")
```

output
```
integral(1/(b*x*cos(x)*sin(x) + a*x), x)
```

3.582.6 Sympy [N/A]

Not integrable

Time = 24.99 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+b\cos(x)\sin(x))}\,dx=\int \frac{1}{x\left(a+b\sin\left(x\right)\cos\left(x\right)\right)}\,dx$$

input
```
integrate(1/x/(a+b*cos(x)*sin(x)),x)
```

output
```
Integral(1/(x*(a + b*sin(x)*cos(x))), x)
```

3.582.7 Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x(a+b\cos(x)\sin(x))}\,dx = \int \frac{1}{(b\cos{(x)}\sin{(x)}+a)x}\,dx$$

input
```
integrate(1/x/(a+b*cos(x)*sin(x)),x, algorithm="maxima")
```

output
```
integrate(1/((b*cos(x)*sin(x) + a)*x), x)
```

3.582.8 Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x(a+b\cos(x)\sin(x))}\,dx = \int \frac{1}{(b\cos{(x)}\sin{(x)}+a)x}\,dx$$

input
```
integrate(1/x/(a+b*cos(x)*sin(x)),x, algorithm="giac")
```

output
```
integrate(1/((b*cos(x)*sin(x) + a)*x), x)
```

3.582.9 Mupad [N/A]

Not integrable

Time = 26.47 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x(a+b\cos(x)\sin(x))}\,dx = \int \frac{1}{x\ (a+b\ \cos\left(x\right)\ \sin\left(x\right))}\,dx$$

input
```
int(1/(x*(a + b*cos(x)*sin(x))),x)
```

output
```
int(1/(x*(a + b*cos(x)*sin(x))), x)
```

3.583 $\int \frac{(bx)^{2-n}\sin^n(ax)}{(acx\cos(ax)-c\sin(ax))^2}\,dx$

3.583.1 Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{(bx)^{2-n}\sin^n(ax)}{(acx\cos(ax)-c\sin(ax))^2}\,dx = \frac{b(bx)^{1-n}\sin^{-1+n}(ax)}{a^2\left(ac^2x\cos(ax)-c^2\sin(ax)\right)} + \frac{b^2(1-n)\text{Int}\left((bx)^{-n}\sin^{-2+n}(ax),x\right)}{a^2c^2}$$

output
```
b*(b*x)^(1-n)*sin(a*x)^(-1+n)/a^2/(a*c^2*x*cos(a*x)-c^2*sin(a*x))+b^2*(1-n
)*Unintegrable(sin(a*x)^(-2+n)/((b*x)^n),x)/a^2/c^2
```

3.583.2 Mathematica [N/A]

Not integrable

Time = 8.96 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(bx)^{2-n}\sin^n(ax)}{(acx\cos(ax)-c\sin(ax))^2}\,dx = \int \frac{(bx)^{2-n}\sin^n(ax)}{(acx\cos(ax)-c\sin(ax))^2}\,dx$$

input
```
Integrate[((b*x)^(2 - n)*Sin[a*x]^n)/(a*c*x*Cos[a*x] - c*Sin[a*x])^2,x]
```

output
```
Integrate[((b*x)^(2 - n)*Sin[a*x]^n)/(a*c*x*Cos[a*x] - c*Sin[a*x])^2, x]
```

3.583.3 Rubi [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5109, 3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx)^{2-n}\sin^n(ax)}{(acx\cos(ax)-c\sin(ax))^2}dx$$

$$\Big\downarrow \text{5109}$$

$$\frac{b^2(1-n)\int (bx)^{-n}\sin^{n-2}(ax)dx}{a^2c^2}+\frac{b(bx)^{1-n}\sin^{n-1}(ax)}{a^2\left(ac^2x\cos(ax)-c^2\sin(ax)\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2(1-n)\int (bx)^{-n}\sin(ax)^{n-2}dx}{a^2c^2}+\frac{b(bx)^{1-n}\sin^{n-1}(ax)}{a^2\left(ac^2x\cos(ax)-c^2\sin(ax)\right)}$$

$$\Big\downarrow \text{3807}$$

$$\frac{b^2(1-n)\int (bx)^{-n}\sin^{n-2}(ax)dx}{a^2c^2}+\frac{b(bx)^{1-n}\sin^{n-1}(ax)}{a^2\left(ac^2x\cos(ax)-c^2\sin(ax)\right)}$$

input
```
Int[((b*x)^(2 - n)*Sin[a*x]^n)/(a*c*x*Cos[a*x] - c*Sin[a*x])^2,x]
```

output
```
$Aborted
```

3.583.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

rule 5109
```
Int[(((b_.)*(x_))^(m_)*Sin[(a_.)*(x_)]^(n_))/(Cos[(a_.)*(x_)]*(d_.)*(x_) +
(c_.)*Sin[(a_.)*(x_)])^2, x_Symbol] :> Simp[b*(b*x)^(m - 1)*(Sin[a*x]^(n -
1)/(a*d*(c*Sin[a*x] + d*x*Cos[a*x]))), x] - Simp[b^2*((n - 1)/d^2)   Int[(b
*x)^(m - 2)*Sin[a*x]^(n - 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ
[a*c + d, 0] && EqQ[m, 2 - n]
```

3.583.4 Maple [N/A] (verified)

Not integrable

Time = 0.96 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{(xb)^{2-n} \sin(ax)^n}{(acx\cos(ax) - c\sin(ax))^2} dx$$

input
```
int((x*b)^(2-n)*sin(a*x)^n/(a*c*x*cos(a*x)-c*sin(a*x))^2,x)
```

output
```
int((x*b)^(2-n)*sin(a*x)^n/(a*c*x*cos(a*x)-c*sin(a*x))^2,x)
```

3.583.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.94

$$\int \frac{(bx)^{2-n}\sin^n(ax)}{(acx\cos(ax) - c\sin(ax))^2} dx = \int \frac{(bx)^{-n+2}\sin(ax)^n}{(acx\cos(ax) - c\sin(ax))^2} dx$$

input
```
integrate((b*x)^(2-n)*sin(a*x)^n/(a*c*x*cos(a*x)-c*sin(a*x))^2,x, algorith
m="fricas")
```

output
```
integral(-(b*x)^(-n + 2)*sin(a*x)^n/(2*a*c^2*x*cos(a*x)*sin(a*x) - (a^2*c^
2*x^2 - c^2)*cos(a*x)^2 - c^2), x)
```

3.583.6 Sympy [N/A]

Not integrable

Time = 82.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.56

$$\int \frac{(bx)^{2-n}\sin^n(ax)}{(acx\cos(ax)-c\sin(ax))^2}\,dx = \frac{\int \frac{(bx)^{2-n}\sin^n(ax)}{a^2x^2\cos^2(ax)-2ax\sin(ax)\cos(ax)+\sin^2(ax)}\,dx}{c^2}$$

input
```
integrate((b*x)**(2-n)*sin(a*x)**n/(a*c*x*cos(a*x)-c*sin(a*x))**2,x)
```

output
```
Integral((b*x)**(2 - n)*sin(a*x)**n/(a**2*x**2*cos(a*x)**2 - 2*a*x*sin(a*x
)*cos(a*x) + sin(a*x)**2), x)/c**2
```

3.583.7 Maxima [N/A]

Not integrable

Time = 2.80 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(bx)^{2-n}\sin^n(ax)}{(acx\cos(ax)-c\sin(ax))^2}\,dx = \int \frac{(bx)^{-n+2}\sin(ax)^n}{(acx\cos(ax)-c\sin(ax))^2}\,dx$$

input
```
integrate((b*x)^(2-n)*sin(a*x)^n/(a*c*x*cos(a*x)-c*sin(a*x))^2,x, algorith
m="maxima")
```

output
```
integrate((b*x)^(-n + 2)*sin(a*x)^n/(a*c*x*cos(a*x) - c*sin(a*x))^2, x)
```

3.583.8 Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(bx)^{2-n}\sin^n(ax)}{(acx\cos(ax)-c\sin(ax))^2}\,dx = \int \frac{(bx)^{-n+2}\sin(ax)^n}{(acx\cos(ax)-c\sin(ax))^2}\,dx$$

input
```
integrate((b*x)^(2-n)*sin(a*x)^n/(a*c*x*cos(a*x)-c*sin(a*x))^2,x, algorith
m="giac")
```

output
```
integrate((b*x)^(-n + 2)*sin(a*x)^n/(a*c*x*cos(a*x) - c*sin(a*x))^2, x)
```

3.583.9 Mupad [N/A]

Not integrable

Time = 28.27 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(bx)^{2-n}\sin^n(ax)}{(acx\cos(ax)-c\sin(ax))^2}\,dx=\int \frac{\sin\left(a\,x\right)^n\,{\left(b\,x\right)}^{2-n}}{{\left(c\,\sin\left(a\,x\right)-a\,c\,x\,\cos\left(a\,x\right)\right)}^2}\,dx$$

input
```
int((sin(a*x)^n*(b*x)^(2 - n))/(c*sin(a*x) - a*c*x*cos(a*x))^2,x)
```

output
```
int((sin(a*x)^n*(b*x)^(2 - n))/(c*sin(a*x) - a*c*x*cos(a*x))^2, x)
```

3.584 $\int \frac{(bx)^{2-n}\cos^n(ax)}{(c\cos(ax)+acx\sin(ax))^2}\,dx$

3.584.1 Optimal result

Integrand size = 33, antiderivative size = 33

$$\int \frac{(bx)^{2-n}\cos^n(ax)}{(c\cos(ax)+acx\sin(ax))^2}\,dx = -\frac{b(bx)^{1-n}\cos^{-1+n}(ax)}{a^2\left(c^2\cos(ax)+ac^2x\sin(ax)\right)} + \frac{b^2(1-n)\text{Int}\left((bx)^{-n}\cos^{-2+n}(ax),x\right)}{a^2c^2}$$

output
```
-b*(b*x)^(1-n)*cos(a*x)^(-1+n)/a^2/(c^2*cos(a*x)+a*c^2*x*sin(a*x))+b^2*(1-
n)*Unintegrable(cos(a*x)^(-2+n)/((b*x)^n),x)/a^2/c^2
```

3.584.2 Mathematica [N/A]

Not integrable

Time = 7.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{(bx)^{2-n}\cos^n(ax)}{(c\cos(ax)+acx\sin(ax))^2}\,dx = \int \frac{(bx)^{2-n}\cos^n(ax)}{(c\cos(ax)+acx\sin(ax))^2}\,dx$$

input
```
Integrate[((b*x)^(2 - n)*Cos[a*x]^n)/(c*Cos[a*x] + a*c*x*Sin[a*x])^2,x]
```

output
```
Integrate[((b*x)^(2 - n)*Cos[a*x]^n)/(c*Cos[a*x] + a*c*x*Sin[a*x])^2, x]
```

3.584.3 Rubi [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5110, 3042, 3807}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx)^{2-n}\cos^n(ax)}{(acx\sin(ax)+c\cos(ax))^2}dx$$

$$\Big\downarrow \text{5110}$$

$$\frac{b^2(1-n)\int (bx)^{-n}\cos^{n-2}(ax)dx}{a^2c^2}-\frac{b(bx)^{1-n}\cos^{n-1}(ax)}{a^2\left(ac^2x\sin(ax)+c^2\cos(ax)\right)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b^2(1-n)\int (bx)^{-n}\sin\left(ax+\frac{\pi}{2}\right)^{n-2}dx}{a^2c^2}-\frac{b(bx)^{1-n}\cos^{n-1}(ax)}{a^2\left(ac^2x\sin(ax)+c^2\cos(ax)\right)}$$

$$\Big\downarrow \text{3807}$$

$$\frac{b^2(1-n)\int (bx)^{-n}\cos^{n-2}(ax)dx}{a^2c^2}-\frac{b(bx)^{1-n}\cos^{n-1}(ax)}{a^2\left(ac^2x\sin(ax)+c^2\cos(ax)\right)}$$

input
```
Int[((b*x)^(2 - n)*Cos[a*x]^n)/(c*Cos[a*x] + a*c*x*Sin[a*x])^2,x]
```

output
```
$Aborted
```

3.584.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3807
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.
), x_Symbol] :> Unintegrable[(c + d*x)^m*(a + b*Sin[e + f*x])^n, x] /; Free
Q[{a, b, c, d, e, f, m, n}, x]
```

rule 5110
```
Int[(Cos[(a_.)*(x_)]^(n_)*((b_.)*(x_))^(m_))/(Cos[(a_.)*(x_)]*(c_.) + (d_.)
*(x_)*Sin[(a_.)*(x_)])^2, x_Symbol] :> Simp[(-b)*(b*x)^(m - 1)*(Cos[a*x]^(n
 - 1)/(a*d*(c*Cos[a*x] + d*x*Sin[a*x]))), x] - Simp[b^2*((n - 1)/d^2)   Int
[(b*x)^(m - 2)*Cos[a*x]^(n - 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&
EqQ[a*c - d, 0] && EqQ[m, 2 - n]
```

3.584.4 Maple [N/A] (verified)

Not integrable

Time = 0.99 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{(xb)^{2-n} \cos(ax)^n}{(c\cos(ax) + acx\sin(ax))^2} dx$$

input
```
int((x*b)^(2-n)*cos(a*x)^n/(c*cos(a*x)+a*c*x*sin(a*x))^2,x)
```

output
```
int((x*b)^(2-n)*cos(a*x)^n/(c*cos(a*x)+a*c*x*sin(a*x))^2,x)
```

3.584.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.12

$$\int \frac{(bx)^{2-n} \cos^n(ax)}{(c\cos(ax) + acx\sin(ax))^2} dx = \int \frac{(bx)^{-n+2} \cos(ax)^n}{(acx\sin(ax) + c\cos(ax))^2} dx$$

input
```
integrate((b*x)^(2-n)*cos(a*x)^n/(c*cos(a*x)+a*c*x*sin(a*x))^2,x, algorith
m="fricas")
```

output
```
integral((b*x)^(-n + 2)*cos(a*x)^n/(a^2*c^2*x^2 + 2*a*c^2*x*cos(a*x)*sin(a
*x) - (a^2*c^2*x^2 - c^2)*cos(a*x)^2), x)
```

3.584.6 Sympy [N/A]

Not integrable

Time = 82.93 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.61

$$\int \frac{(bx)^{2-n}\cos^n(ax)}{(c\cos(ax)+acx\sin(ax))^2}\,dx = \frac{\int \frac{(bx)^{2-n}\cos^n(ax)}{a^2x^2\sin^2(ax)+2ax\sin(ax)\cos(ax)+\cos^2(ax)}\,dx}{c^2}$$

input
```
integrate((b*x)**(2-n)*cos(a*x)**n/(c*cos(a*x)+a*c*x*sin(a*x))**2,x)
```

output
```
Integral((b*x)**(2 - n)*cos(a*x)**n/(a**2*x**2*sin(a*x)**2 + 2*a*x*sin(a*x
)*cos(a*x) + cos(a*x)**2), x)/c**2
```

3.584.7 Maxima [N/A]

Not integrable

Time = 2.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{(bx)^{2-n}\cos^n(ax)}{(c\cos(ax)+acx\sin(ax))^2}\,dx = \int \frac{(bx)^{-n+2}\cos(ax)^n}{(acx\sin(ax)+c\cos(ax))^2}\,dx$$

input
```
integrate((b*x)^(2-n)*cos(a*x)^n/(c*cos(a*x)+a*c*x*sin(a*x))^2,x, algorith
m="maxima")
```

output
```
integrate((b*x)^(-n + 2)*cos(a*x)^n/(a*c*x*sin(a*x) + c*cos(a*x))^2, x)
```

3.584.8 Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{(bx)^{2-n}\cos^n(ax)}{(c\cos(ax)+acx\sin(ax))^2}\,dx = \int \frac{(bx)^{-n+2}\cos(ax)^n}{(acx\sin(ax)+c\cos(ax))^2}\,dx$$

input
```
integrate((b*x)^(2-n)*cos(a*x)^n/(c*cos(a*x)+a*c*x*sin(a*x))^2,x, algorith
m="giac")
```

output
```
integrate((b*x)^(-n + 2)*cos(a*x)^n/(a*c*x*sin(a*x) + c*cos(a*x))^2, x)
```

3.584.9 Mupad [N/A]

Not integrable

Time = 27.86 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{(bx)^{2-n}\cos^n(ax)}{(c\cos(ax)+acx\sin(ax))^2}\,dx=\int \frac{{\cos\left(a\,x\right)}^n\,{\left(b\,x\right)}^{2-n}}{{\left(c\,\cos\left(a\,x\right)+a\,c\,x\,\sin\left(a\,x\right)\right)}^2}\,dx$$

input
```
int((cos(a*x)^n*(b*x)^(2 - n))/(c*cos(a*x) + a*c*x*sin(a*x))^2,x)
```

output
```
int((cos(a*x)^n*(b*x)^(2 - n))/(c*cos(a*x) + a*c*x*sin(a*x))^2, x)
```

3.585 $\int \frac{\sin^6(ax)}{x^4(ax\cos(ax)-\sin(ax))^2}\,dx$

3.585.1 Optimal result

Integrand size = 26, antiderivative size = 175

$$\begin{aligned}\int \frac{\sin^6(ax)}{x^4(ax\cos(ax)-\sin(ax))^2}\,dx &= \frac{a^2}{x}+\frac{a\cos(ax)\sin(ax)}{x^2}+\frac{\sin^2(ax)}{x^3}\\ &-\frac{10a^2\sin^2(ax)}{x}+\frac{\cos(ax)\sin^3(ax)}{ax^4}\\ &-\frac{8a\cos(ax)\sin^3(ax)}{3x^2}+\frac{\sin^4(ax)}{a^2x^5}-\frac{4\sin^4(ax)}{3x^3}\\ &+\frac{32a^2\sin^4(ax)}{3x}+\frac{\sin^5(ax)}{a^2x^5(ax\cos(ax)-\sin(ax))}\\ &-\frac{2}{3}a^3\mathrm{Si}(2ax)+\frac{16}{3}a^3\mathrm{Si}(4ax)\end{aligned}$$

output
```
a^2/x-2/3*a^3*Si(2*a*x)+16/3*a^3*Si(4*a*x)+a*cos(a*x)*sin(a*x)/x^2+sin(a*x
)^2/x^3-10*a^2*sin(a*x)^2/x+cos(a*x)*sin(a*x)^3/a/x^4-8/3*a*cos(a*x)*sin(a
*x)^3/x^2+sin(a*x)^4/a^2/x^5-4/3*sin(a*x)^4/x^3+32/3*a^2*sin(a*x)^4/x+sin(
a*x)^5/a^2/x^5/(a*x*cos(a*x)-sin(a*x))
```

3.585.2 Mathematica [A] (verified)

Time = 1.63 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.13

$$\int \frac{\sin^6(ax)}{x^4(ax\cos(ax)-\sin(ax))^2}\,dx$$
$$= \frac{8ax\cos(ax)-8a^3x^3\cos(ax)-12ax\cos(3ax)+24a^3x^3\cos(3ax)+4ax\cos(5ax)+32a^3x^3\cos(5ax)+10\,\mathrm{s}}{}$$

input
```
Integrate[Sin[a*x]^6/(x^4*(a*x*Cos[a*x] - Sin[a*x])^2),x]
```

output
```
(8*a*x*Cos[a*x] - 8*a^3*x^3*Cos[a*x] - 12*a*x*Cos[3*a*x] + 24*a^3*x^3*Cos[
3*a*x] + 4*a*x*Cos[5*a*x] + 32*a^3*x^3*Cos[5*a*x] + 10*Sin[a*x] - 12*a^2*x
^2*Sin[a*x] - 5*Sin[3*a*x] + 44*a^2*x^2*Sin[3*a*x] + Sin[5*a*x] - 24*a^2*x
^2*Sin[5*a*x] - 32*a^3*x^3*(a*x*Cos[a*x] - Sin[a*x])*SinIntegral[2*a*x] +
256*a^3*x^3*(a*x*Cos[a*x] - Sin[a*x])*SinIntegral[4*a*x])/(48*x^3*(a*x*Cos
[a*x] - Sin[a*x]))
```

3.585.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.39, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {5109, 3042, 3795, 3042, 3795, 15, 3042, 3794, 27, 2009, 3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^6(ax)}{x^4(ax\cos(ax)-\sin(ax))^2}\,dx$$
$$\Big\downarrow \ 5109$$
$$\frac{\sin^5(ax)}{a^2x^5(ax\cos(ax)-\sin(ax))}-\frac{5\int \frac{\sin^4(ax)}{x^6}dx}{a^2}$$
$$\Big\downarrow \ 3042$$
$$\frac{\sin^5(ax)}{a^2x^5(ax\cos(ax)-\sin(ax))}-\frac{5\int \frac{\sin(ax)^4}{x^6}dx}{a^2}$$
$$\Big\downarrow \ 3795$$

$$\frac{\sin^5(ax)}{a^2x^5(ax\cos(ax)-\sin(ax))}-\frac{5\left(-\frac{4}{5}a^2\int\frac{\sin^4(ax)}{x^4}dx+\frac{3}{5}a^2\int\frac{\sin^2(ax)}{x^4}dx-\frac{\sin^4(ax)}{5x^5}-\frac{a\sin^3(ax)\cos(ax)}{5x^4}\right)}{a^2}$$

$\downarrow$ 3042

$$\frac{\sin^5(ax)}{a^2x^5(ax\cos(ax)-\sin(ax))}-\frac{5\left(\frac{3}{5}a^2\int\frac{\sin(ax)^2}{x^4}dx-\frac{4}{5}a^2\int\frac{\sin(ax)^4}{x^4}dx-\frac{\sin^4(ax)}{5x^5}-\frac{a\sin^3(ax)\cos(ax)}{5x^4}\right)}{a^2}$$

$\downarrow$ 3795

$$\frac{\sin^5(ax)}{a^2x^5(ax\cos(ax)-\sin(ax))}-$$
$$\frac{5\left(\frac{3}{5}a^2\left(\frac{1}{3}a^2\int\frac{1}{x^2}dx-\frac{2}{3}a^2\int\frac{\sin^2(ax)}{x^2}dx-\frac{\sin^2(ax)}{3x^3}-\frac{a\sin(ax)\cos(ax)}{3x^2}\right)-\frac{4}{5}a^2\left(-\frac{8}{3}a^2\int\frac{\sin^4(ax)}{x^2}dx+2a^2\int\frac{\sin^2(ax)}{x^2}dx-\text{si}\right.}{a^2}$$

$\downarrow$ 15

$$\frac{\sin^5(ax)}{a^2x^5(ax\cos(ax)-\sin(ax))}-$$
$$\frac{5\left(\frac{3}{5}a^2\left(-\frac{2}{3}a^2\int\frac{\sin^2(ax)}{x^2}dx-\frac{a^2}{3x}-\frac{\sin^2(ax)}{3x^3}-\frac{a\sin(ax)\cos(ax)}{3x^2}\right)-\frac{4}{5}a^2\left(-\frac{8}{3}a^2\int\frac{\sin^4(ax)}{x^2}dx+2a^2\int\frac{\sin^2(ax)}{x^2}dx-\frac{\sin^4(ax)}{3x^3}-\right.}{a^2}$$

$\downarrow$ 3042

$$\frac{\sin^5(ax)}{a^2x^5(ax\cos(ax)-\sin(ax))}-$$
$$\frac{5\left(\frac{3}{5}a^2\left(-\frac{2}{3}a^2\int\frac{\sin(ax)^2}{x^2}dx-\frac{a^2}{3x}-\frac{\sin^2(ax)}{3x^3}-\frac{a\sin(ax)\cos(ax)}{3x^2}\right)-\frac{4}{5}a^2\left(2a^2\int\frac{\sin(ax)^2}{x^2}dx-\frac{8}{3}a^2\int\frac{\sin(ax)^4}{x^2}dx-\frac{\sin^4(ax)}{3x^3}-\right.}{a^2}$$

$\downarrow$ 3794

$$\frac{\sin^5(ax)}{a^2x^5(ax\cos(ax)-\sin(ax))}-$$
$$\frac{5\left(\frac{3}{5}a^2\left(-\frac{2}{3}a^2\left(2a\int\frac{\sin(2ax)}{2x}dx-\frac{\sin^2(ax)}{x}\right)-\frac{a^2}{3x}-\frac{\sin^2(ax)}{3x^3}-\frac{a\sin(ax)\cos(ax)}{3x^2}\right)-\frac{4}{5}a^2\left(-\frac{8}{3}a^2\left(4a\int\left(\frac{\sin(2ax)}{4x}-\frac{\sin(4ax)}{8x}\right)\right.\right.\right.}{a^2}$$

$\downarrow$ 27

$$\frac{\sin^5(ax)}{a^2x^5(ax\cos(ax)-\sin(ax))}-$$
$$\frac{5\left(\frac{3}{5}a^2\left(-\frac{2}{3}a^2\left(a\int\frac{\sin(2ax)}{x}dx-\frac{\sin^2(ax)}{x}\right)-\frac{a^2}{3x}-\frac{\sin^2(ax)}{3x^3}-\frac{a\sin(ax)\cos(ax)}{3x^2}\right)-\frac{4}{5}a^2\left(-\frac{8}{3}a^2\left(4a\int\left(\frac{\sin(2ax)}{4x}-\frac{\sin(4ax)}{8x}\right)d\right.\right.\right.}{a^2}$$

$\downarrow$ 2009

3.585. $\int\frac{\sin^6(ax)}{x^4(ax\cos(ax)-\sin(ax))^2}\,dx$

$$\frac{\sin^5(ax)}{a^2x^5(ax\cos(ax)-\sin(ax))}-\frac{5\left(-\frac{4}{5}a^2\left(2a^2\left(a\int\frac{\sin(2ax)}{x}dx-\frac{\sin^2(ax)}{x}\right)-\frac{8}{3}a^2\left(4a\left(\frac{\text{Si}(2ax)}{4}-\frac{\text{Si}(4ax)}{8}\right)-\frac{\sin^4(ax)}{x}\right)-\frac{\sin^4(ax)}{3x^3}-\frac{2a\sin^3(ax)\cos(ax)}{3x^2}\right)+\cdots\right)}{a^2}$$

$\downarrow$ 3042

$$\frac{\sin^5(ax)}{a^2x^5(ax\cos(ax)-\sin(ax))}-\frac{5\left(-\frac{4}{5}a^2\left(2a^2\left(a\int\frac{\sin(2ax)}{x}dx-\frac{\sin^2(ax)}{x}\right)-\frac{8}{3}a^2\left(4a\left(\frac{\text{Si}(2ax)}{4}-\frac{\text{Si}(4ax)}{8}\right)-\frac{\sin^4(ax)}{x}\right)-\frac{\sin^4(ax)}{3x^3}-\frac{2a\sin^3(ax)\cos(ax)}{3x^2}\right)+\cdots\right)}{a^2}$$

$\downarrow$ 3780

$$\frac{\sin^5(ax)}{a^2x^5(ax\cos(ax)-\sin(ax))}-\frac{5\left(\frac{3}{5}a^2\left(-\frac{2}{3}a^2\left(a\text{Si}(2ax)-\frac{\sin^2(ax)}{x}\right)-\frac{a^2}{3x}-\frac{\sin^2(ax)}{3x^3}-\frac{a\sin(ax)\cos(ax)}{3x^2}\right)-\frac{4}{5}a^2\left(-\frac{8}{3}a^2\left(4a\left(\frac{\text{Si}(2ax)}{4}-\frac{\text{Si}(4ax)}{8}\right)-\frac{\sin^4(ax)}{x}\cdots\right)\right)\right)}{a^2}$$

input
```
Int[Sin[a*x]^6/(x^4*(a*x*Cos[a*x] - Sin[a*x])^2),x]
```

output
```
Sin[a*x]^5/(a^2*x^5*(a*x*Cos[a*x] - Sin[a*x])) - (5*(-1/5*(a*Cos[a*x]*Sin[
a*x]^3)/x^4 - Sin[a*x]^4/(5*x^5) + (3*a^2*(-1/3*a^2/x - (a*Cos[a*x]*Sin[a*
x])/(3*x^2) - Sin[a*x]^2/(3*x^3) - (2*a^2*(-(Sin[a*x]^2/x) + a*SinIntegral
[2*a*x]))/3))/5 - (4*a^2*((-2*a*Cos[a*x]*Sin[a*x]^3)/(3*x^2) - Sin[a*x]^4/
(3*x^3) + 2*a^2*(-(Sin[a*x]^2/x) + a*SinIntegral[2*a*x]) - (8*a^2*(-(Sin[a
*x]^4/x) + 4*a*(SinIntegral[2*a*x]/4 - SinIntegral[4*a*x]/8)))/3))/5))/a^2
```

3.585.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.585. $\int\frac{\sin^6(ax)}{x^4(ax\cos(ax)-\sin(ax))^2}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

rule 5109
```
Int[(((b_.)*(x_))^(m_)*Sin[(a_.)*(x_)]^(n_))/(Cos[(a_.)*(x_)]*(d_.)*(x_) +
(c_.)*Sin[(a_.)*(x_)])^2, x_Symbol] :> Simp[b*(b*x)^(m - 1)*(Sin[a*x]^(n -
1)/(a*d*(c*Sin[a*x] + d*x*Cos[a*x]))), x] - Simp[b^2*((n - 1)/d^2)   Int[(b
*x)^(m - 2)*Sin[a*x]^(n - 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ
[a*c + d, 0] && EqQ[m, 2 - n]
```

3.585.4 Maple [F(-1)]

Timed out.

$$\int \frac{\sin(ax)^6}{x^4 (ax\cos(ax) - \sin(ax))^2} dx$$

input
```
int(sin(a*x)^6/x^4/(a*x*cos(a*x)-sin(a*x))^2,x)
```

output
```
int(sin(a*x)^6/x^4/(a*x*cos(a*x)-sin(a*x))^2,x)
```

3.585.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.06

$$\int \frac{\sin^6(ax)}{x^4(ax\cos(ax)-\sin(ax))^2}\,dx$$
$$= \frac{4\left(8\,a^3x^3+ax\right)\cos\left(ax\right)^5-2\left(17\,a^3x^3+4\,ax\right)\cos\left(ax\right)^3+\left(16\,a^4x^4\,\mathrm{Si}\left(4\,ax\right)-2\,a^4x^4\,\mathrm{Si}\left(2\,ax\right)+5\,a^3x^3+4\right.}{3\left(ax\right.}$$

input
```
integrate(sin(a*x)^6/x^4/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="fricas")
```

output
```
1/3*(4*(8*a^3*x^3 + a*x)*cos(a*x)^5 - 2*(17*a^3*x^3 + 4*a*x)*cos(a*x)^3 +
(16*a^4*x^4*sin_integral(4*a*x) - 2*a^4*x^4*sin_integral(2*a*x) + 5*a^3*x^
3 + 4*a*x)*cos(a*x) - (16*a^3*x^3*sin_integral(4*a*x) - 2*a^3*x^3*sin_inte
gral(2*a*x) + (24*a^2*x^2 - 1)*cos(a*x)^4 + 5*a^2*x^2 - (29*a^2*x^2 - 2)*c
os(a*x)^2 - 1)*sin(a*x))/(a*x^4*cos(a*x) - x^3*sin(a*x))
```

3.585.6 Sympy [F]

$$\int \frac{\sin^6(ax)}{x^4(ax\cos(ax)-\sin(ax))^2}\,dx = \int \frac{\sin^6\left(ax\right)}{x^4\left(ax\cos\left(ax\right)-\sin\left(ax\right)\right)^2}\,dx$$

input
```
integrate(sin(a*x)**6/x**4/(a*x*cos(a*x)-sin(a*x))**2,x)
```

output
```
Integral(sin(a*x)**6/(x**4*(a*x*cos(a*x) - sin(a*x))**2), x)
```

3.585.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sin^6(ax)}{x^4(ax\cos(ax)-\sin(ax))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(sin(a*x)^6/x^4/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.585.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.72 (sec) , antiderivative size = 7347, normalized size of antiderivative = 41.98

$$\int \frac{\sin^6(ax)}{x^4(ax\cos(ax)-\sin(ax))^2}\,dx = \text{Too large to display}$$

input
```
integrate(sin(a*x)^6/x^4/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="giac")
```

output
```
1/12*(32*a^8*x^8*imag_part(cos_integral(4*a*x))*tan(2*a*x)^2*tan(a*x)^2*ta
n(1/2*a*x)^2 - 4*a^8*x^8*imag_part(cos_integral(2*a*x))*tan(2*a*x)^2*tan(a
*x)^2*tan(1/2*a*x)^2 + 4*a^8*x^8*imag_part(cos_integral(-2*a*x))*tan(2*a*x
)^2*tan(a*x)^2*tan(1/2*a*x)^2 - 32*a^8*x^8*imag_part(cos_integral(-4*a*x))
*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 + 64*a^8*x^8*sin_integral(4*a*x)*t
an(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 - 8*a^8*x^8*sin_integral(2*a*x)*tan(
2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 - 32*a^8*x^8*imag_part(cos_integral(4*a
*x))*tan(2*a*x)^2*tan(a*x)^2 + 4*a^8*x^8*imag_part(cos_integral(2*a*x))*ta
n(2*a*x)^2*tan(a*x)^2 - 4*a^8*x^8*imag_part(cos_integral(-2*a*x))*tan(2*a*
x)^2*tan(a*x)^2 + 32*a^8*x^8*imag_part(cos_integral(-4*a*x))*tan(2*a*x)^2*
tan(a*x)^2 - 64*a^8*x^8*sin_integral(4*a*x)*tan(2*a*x)^2*tan(a*x)^2 + 8*a^
8*x^8*sin_integral(2*a*x)*tan(2*a*x)^2*tan(a*x)^2 + 32*a^8*x^8*imag_part(c
os_integral(4*a*x))*tan(2*a*x)^2*tan(1/2*a*x)^2 - 4*a^8*x^8*imag_part(cos_
integral(2*a*x))*tan(2*a*x)^2*tan(1/2*a*x)^2 + 4*a^8*x^8*imag_part(cos_int
egral(-2*a*x))*tan(2*a*x)^2*tan(1/2*a*x)^2 - 32*a^8*x^8*imag_part(cos_inte
gral(-4*a*x))*tan(2*a*x)^2*tan(1/2*a*x)^2 + 64*a^8*x^8*sin_integral(4*a*x)
*tan(2*a*x)^2*tan(1/2*a*x)^2 - 8*a^8*x^8*sin_integral(2*a*x)*tan(2*a*x)^2*
tan(1/2*a*x)^2 + 32*a^8*x^8*imag_part(cos_integral(4*a*x))*tan(a*x)^2*tan(
1/2*a*x)^2 - 4*a^8*x^8*imag_part(cos_integral(2*a*x))*tan(a*x)^2*tan(1/2*a
*x)^2 + 4*a^8*x^8*imag_part(cos_integral(-2*a*x))*tan(a*x)^2*tan(1/2*a*...
```

3.585.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^6(ax)}{x^4(ax\cos(ax)-\sin(ax))^2}\,dx = \int \frac{\sin(a\,x)^6}{x^4\,\left(\sin(a\,x)-a\,x\,\cos(a\,x)\right)^2}\,dx$$

input
```
int(sin(a*x)^6/(x^4*(sin(a*x) - a*x*cos(a*x))^2),x)
```

output
```
int(sin(a*x)^6/(x^4*(sin(a*x) - a*x*cos(a*x))^2), x)
```

3.585. $\int \frac{\sin^6(ax)}{x^4(ax\cos(ax)-\sin(ax))^2}\,dx$

3.586 $\int \frac{\sin^5(ax)}{x^3(ax\cos(ax)-\sin(ax))^2}\,dx$

3.586.1 Optimal result

Integrand size = 26, antiderivative size = 131

$$\int \frac{\sin^5(ax)}{x^3(ax\cos(ax)-\sin(ax))^2}\,dx = \frac{a\cos(ax)}{x}+\frac{\sin(ax)}{x^2}+\frac{\cos(ax)\sin^2(ax)}{ax^3} - \frac{9a\cos(ax)\sin^2(ax)}{2x}+\frac{\sin^3(ax)}{a^2x^4}-\frac{3\sin^3(ax)}{2x^2} + \frac{\sin^4(ax)}{a^2x^4(ax\cos(ax)-\sin(ax))}-\frac{1}{8}a^2\text{Si}(ax)+\frac{27}{8}a^2\text{Si}(3ax)$$

output
```
a*cos(a*x)/x-1/8*a^2*Si(a*x)+27/8*a^2*Si(3*a*x)+sin(a*x)/x^2+cos(a*x)*sin(
a*x)^2/a/x^3-9/2*a*cos(a*x)*sin(a*x)^2/x+sin(a*x)^3/a^2/x^4-3/2*sin(a*x)^3
/x^2+sin(a*x)^4/a^2/x^4/(a*x*cos(a*x)-sin(a*x))
```

3.586.2 Mathematica [A] (verified)

Time = 1.45 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.08

$$\int \frac{\sin^5(ax)}{x^3(ax\cos(ax)-\sin(ax))^2}\,dx = \frac{3-a^2x^2-4\cos(2ax)+8a^2x^2\cos(2ax)+\cos(4ax)+9a^2x^2\cos(4ax)+12ax\sin(2ax)-6ax\sin(4ax)-2}{16x^2(ax\cos(ax)-\sin(ax))}$$

input
```
Integrate[Sin[a*x]^5/(x^3*(a*x*Cos[a*x] - Sin[a*x])^2),x]
```

output
```
(3 - a^2*x^2 - 4*Cos[2*a*x] + 8*a^2*x^2*Cos[2*a*x] + Cos[4*a*x] + 9*a^2*x^
2*Cos[4*a*x] + 12*a*x*Sin[2*a*x] - 6*a*x*Sin[4*a*x] - 2*a^2*x^2*(a*x*Cos[a
*x] - Sin[a*x])*SinIntegral[a*x] + 54*a^2*x^2*(a*x*Cos[a*x] - Sin[a*x])*Si
nIntegral[3*a*x])/(16*x^2*(a*x*Cos[a*x] - Sin[a*x]))
```

3.586.3 Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.38, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.577, Rules used = {5109, 3042, 3795, 3042, 3778, 3042, 3778, 25, 3042, 3780, 3795, 3042, 3780, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^5(ax)}{x^3(ax\cos(ax)-\sin(ax))^2}dx$$

$$\Big\downarrow \text{5109}$$

$$\frac{\sin^4(ax)}{a^2x^4(ax\cos(ax)-\sin(ax))}-\frac{4\int \frac{\sin^3(ax)}{x^5}dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin^4(ax)}{a^2x^4(ax\cos(ax)-\sin(ax))}-\frac{4\int \frac{\sin(ax)^3}{x^5}dx}{a^2}$$

$$\Big\downarrow \text{3795}$$

$$\frac{\sin^4(ax)}{a^2x^4(ax\cos(ax)-\sin(ax))}-\frac{4\left(-\frac{3}{4}a^2\int \frac{\sin^3(ax)}{x^3}dx+\frac{1}{2}a^2\int \frac{\sin(ax)}{x^3}dx-\frac{\sin^3(ax)}{4x^4}-\frac{a\sin^2(ax)\cos(ax)}{4x^3}\right)}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin^4(ax)}{a^2x^4(ax\cos(ax)-\sin(ax))}-\frac{4\left(\frac{1}{2}a^2\int \frac{\sin(ax)}{x^3}dx-\frac{3}{4}a^2\int \frac{\sin(ax)^3}{x^3}dx-\frac{\sin^3(ax)}{4x^4}-\frac{a\sin^2(ax)\cos(ax)}{4x^3}\right)}{a^2}$$

$$\Big\downarrow \text{3778}$$

$$\frac{\sin^4(ax)}{a^2x^4(ax\cos(ax)-\sin(ax))}-\frac{4\left(-\frac{3}{4}a^2\int \frac{\sin(ax)^3}{x^3}dx+\frac{1}{2}a^2\left(\frac{1}{2}a\int \frac{\cos(ax)}{x^2}dx-\frac{\sin(ax)}{2x^2}\right)-\frac{\sin^3(ax)}{4x^4}-\frac{a\sin^2(ax)\cos(ax)}{4x^3}\right)}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin^4(ax)}{a^2x^4(ax\cos(ax)-\sin(ax))}-\frac{4\left(-\frac{3}{4}a^2\int\frac{\sin(ax)^3}{x^3}dx+\frac{1}{2}a^2\left(\frac{1}{2}a\int\frac{\sin\left(ax+\frac{\pi}{2}\right)}{x^2}dx-\frac{\sin(ax)}{2x^2}\right)-\frac{\sin^3(ax)}{4x^4}-\frac{a\sin^2(ax)\cos(ax)}{4x^3}\right)}{a^2}$$

$$\Big\downarrow \text{ 3778}$$

$$\frac{\sin^4(ax)}{a^2x^4(ax\cos(ax)-\sin(ax))}-\frac{4\left(-\frac{3}{4}a^2\int\frac{\sin(ax)^3}{x^3}dx+\frac{1}{2}a^2\left(\frac{1}{2}a\left(a\int-\frac{\sin(ax)}{x}dx-\frac{\cos(ax)}{x}\right)-\frac{\sin(ax)}{2x^2}\right)-\frac{\sin^3(ax)}{4x^4}-\frac{a\sin^2(ax)\cos(ax)}{4x^3}\right)}{a^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\sin^4(ax)}{a^2x^4(ax\cos(ax)-\sin(ax))}-\frac{4\left(-\frac{3}{4}a^2\int\frac{\sin(ax)^3}{x^3}dx+\frac{1}{2}a^2\left(\frac{1}{2}a\left(-a\int\frac{\sin(ax)}{x}dx-\frac{\cos(ax)}{x}\right)-\frac{\sin(ax)}{2x^2}\right)-\frac{\sin^3(ax)}{4x^4}-\frac{a\sin^2(ax)\cos(ax)}{4x^3}\right)}{a^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sin^4(ax)}{a^2x^4(ax\cos(ax)-\sin(ax))}-\frac{4\left(-\frac{3}{4}a^2\int\frac{\sin(ax)^3}{x^3}dx+\frac{1}{2}a^2\left(\frac{1}{2}a\left(-a\int\frac{\sin(ax)}{x}dx-\frac{\cos(ax)}{x}\right)-\frac{\sin(ax)}{2x^2}\right)-\frac{\sin^3(ax)}{4x^4}-\frac{a\sin^2(ax)\cos(ax)}{4x^3}\right)}{a^2}$$

$$\Big\downarrow \text{ 3780}$$

$$\frac{\sin^4(ax)}{a^2x^4(ax\cos(ax)-\sin(ax))}-\frac{4\left(-\frac{3}{4}a^2\int\frac{\sin(ax)^3}{x^3}dx+\frac{1}{2}a^2\left(\frac{1}{2}a\left(-a\text{Si}(ax)-\frac{\cos(ax)}{x}\right)-\frac{\sin(ax)}{2x^2}\right)-\frac{\sin^3(ax)}{4x^4}-\frac{a\sin^2(ax)\cos(ax)}{4x^3}\right)}{a^2}$$

$$\Big\downarrow \text{ 3795}$$

$$\frac{\sin^4(ax)}{a^2x^4(ax\cos(ax)-\sin(ax))}-\frac{4\left(-\frac{3}{4}a^2\left(-\frac{9}{2}a^2\int\frac{\sin^3(ax)}{x}dx+3a^2\int\frac{\sin(ax)}{x}dx-\frac{\sin^3(ax)}{2x^2}-\frac{3a\sin^2(ax)\cos(ax)}{2x}\right)+\frac{1}{2}a^2\left(\frac{1}{2}a\left(-a\text{Si}(ax)-\frac{\cos(ax)}{x}\right)-\frac{\sin(a}{2x^2}\right.\right.}{a^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sin^4(ax)}{a^2x^4(ax\cos(ax)-\sin(ax))}-\frac{4\left(-\frac{3}{4}a^2\left(3a^2\int\frac{\sin(ax)}{x}dx-\frac{9}{2}a^2\int\frac{\sin(ax)^3}{x}dx-\frac{\sin^3(ax)}{2x^2}-\frac{3a\sin^2(ax)\cos(ax)}{2x}\right)+\frac{1}{2}a^2\left(\frac{1}{2}a\left(-a\text{Si}(ax)-\frac{\cos(ax)}{x}\right)-\frac{\sin(ax)}{2x^2}\right.\right.}{a^2}$$

$$\Big\downarrow \text{ 3780}$$

3.586. $\int\frac{\sin^5(ax)}{x^3(ax\cos(ax)-\sin(ax))^2}\,dx$

$$\frac{\sin^4(ax)}{a^2x^4(ax\cos(ax)-\sin(ax))} - \frac{4\left(-\frac{3}{4}a^2\left(-\frac{9}{2}a^2\int\frac{\sin(ax)^3}{x}dx+3a^2\text{Si}(ax)-\frac{\sin^3(ax)}{2x^2}-\frac{3a\sin^2(ax)\cos(ax)}{2x}\right)+\frac{1}{2}a^2\left(\frac{1}{2}a\left(-a\text{Si}(ax)-\frac{\cos(ax)}{x}\right)-\frac{\sin(ax)}{2x^2}\right)-\right.}{a^2}$$

$\downarrow$ 3793

$$\frac{\sin^4(ax)}{a^2x^4(ax\cos(ax)-\sin(ax))} - \frac{4\left(-\frac{3}{4}a^2\left(-\frac{9}{2}a^2\int\left(\frac{3\sin(ax)}{4x}-\frac{\sin(3ax)}{4x}\right)dx+3a^2\text{Si}(ax)-\frac{\sin^3(ax)}{2x^2}-\frac{3a\sin^2(ax)\cos(ax)}{2x}\right)+\frac{1}{2}a^2\left(\frac{1}{2}a\left(-a\text{Si}(ax)-\frac{\cos(ax)}{x}\right.\right.\right.}{a^2}$$

$\downarrow$ 2009

$$\frac{\sin^4(ax)}{a^2x^4(ax\cos(ax)-\sin(ax))} - \frac{4\left(-\frac{3}{4}a^2\left(3a^2\text{Si}(ax)-\frac{9}{2}a^2\left(\frac{3\text{Si}(ax)}{4}-\frac{\text{Si}(3ax)}{4}\right)-\frac{\sin^3(ax)}{2x^2}-\frac{3a\sin^2(ax)\cos(ax)}{2x}\right)+\frac{1}{2}a^2\left(\frac{1}{2}a\left(-a\text{Si}(ax)-\frac{\cos(ax)}{x}\right)-\frac{\sin(\cdot}{2x}\right.\right.}{a^2}$$

input
```
Int[Sin[a*x]^5/(x^3*(a*x*Cos[a*x] - Sin[a*x])^2),x]
```

output
```
Sin[a*x]^4/(a^2*x^4*(a*x*Cos[a*x] - Sin[a*x])) - (4*(-1/4*(a*Cos[a*x]*Sin[
a*x]^2)/x^3 - Sin[a*x]^3/(4*x^4) + (a^2*(-1/2*Sin[a*x]/x^2 + (a*(-(Cos[a*x
]/x) - a*SinIntegral[a*x]))/2))/2 - (3*a^2*((-3*a*Cos[a*x]*Sin[a*x]^2)/(2*
x) - Sin[a*x]^3/(2*x^2) + 3*a^2*SinIntegral[a*x] - (9*a^2*((3*SinIntegral[
a*x])/4 - SinIntegral[3*a*x]/4))/2))/4))/a^2
```

3.586.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.586. $\int \frac{\sin^5(ax)}{x^3(ax\cos(ax)-\sin(ax))^2}\,dx$

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

rule 5109
```
Int[(((b_.)*(x_))^(m_)*Sin[(a_.)*(x_)]^(n_))/(Cos[(a_.)*(x_)]*(d_.)*(x_) +
(c_.)*Sin[(a_.)*(x_)])^2, x_Symbol] :> Simp[b*(b*x)^(m - 1)*(Sin[a*x]^(n -
1)/(a*d*(c*Sin[a*x] + d*x*Cos[a*x]))), x] - Simp[b^2*((n - 1)/d^2)   Int[(b
*x)^(m - 2)*Sin[a*x]^(n - 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ
[a*c + d, 0] && EqQ[m, 2 - n]
```

3.586.4 Maple [F(-1)]

Timed out.

$$\int \frac{\sin{(ax)}^5}{x^3 (ax\cos{(ax)} - \sin{(ax)})^2} dx$$

input
```
int(sin(a*x)^5/x^3/(a*x*cos(a*x)-sin(a*x))^2,x)
```

output
```
int(sin(a*x)^5/x^3/(a*x*cos(a*x)-sin(a*x))^2,x)
```

3.586.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.08

$$\int \frac{\sin^5(ax)}{x^3(ax\cos(ax)-\sin(ax))^2}\,dx$$
$$= \frac{4\left(9\,a^2x^2+1\right)\cos\left(ax\right)^4-4\left(7\,a^2x^2+2\right)\cos\left(ax\right)^2+\left(27\,a^3x^3\,\mathrm{Si}\left(3\,ax\right)-a^3x^3\,\mathrm{Si}\left(ax\right)\right)\cos\left(ax\right)-\left(24\,ax\cos\right.}{8\left(ax^3\cos\left(ax\right)-x^2\sin\left(ax\right)\right)}$$

input
```
integrate(sin(a*x)^5/x^3/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="fricas")
```

output
```
1/8*(4*(9*a^2*x^2 + 1)*cos(a*x)^4 - 4*(7*a^2*x^2 + 2)*cos(a*x)^2 + (27*a^3
*x^3*sin_integral(3*a*x) - a^3*x^3*sin_integral(a*x))*cos(a*x) - (24*a*x*c
os(a*x)^3 + 27*a^2*x^2*sin_integral(3*a*x) - a^2*x^2*sin_integral(a*x) - 2
4*a*x*cos(a*x))*sin(a*x) + 4)/(a*x^3*cos(a*x) - x^2*sin(a*x))
```

3.586.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(ax)}{x^3(ax\cos(ax)-\sin(ax))^2}\,dx = \text{Timed out}$$

input
```
integrate(sin(a*x)**5/x**3/(a*x*cos(a*x)-sin(a*x))**2,x)
```

output
```
Timed out
```

3.586.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sin^5(ax)}{x^3(ax\cos(ax)-\sin(ax))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(sin(a*x)^5/x^3/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.586.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.55 (sec) , antiderivative size = 4175, normalized size of antiderivative = 31.87

$$\int \frac{\sin^5(ax)}{x^3(ax\cos(ax)-\sin(ax))^2}\,dx = \text{Too large to display}$$

input
```
integrate(sin(a*x)^5/x^3/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="giac")
```

output
```
1/16*(27*a^7*x^7*imag_part(cos_integral(3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x
)^4 - a^7*x^7*imag_part(cos_integral(a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^4 +
 a^7*x^7*imag_part(cos_integral(-a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^4 - 27*
a^7*x^7*imag_part(cos_integral(-3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^4 + 54
*a^7*x^7*sin_integral(3*a*x)*tan(3/2*a*x)^2*tan(1/2*a*x)^4 - 2*a^7*x^7*sin
_integral(a*x)*tan(3/2*a*x)^2*tan(1/2*a*x)^4 + 27*a^7*x^7*imag_part(cos_in
tegral(3*a*x))*tan(1/2*a*x)^4 - a^7*x^7*imag_part(cos_integral(a*x))*tan(1
/2*a*x)^4 + a^7*x^7*imag_part(cos_integral(-a*x))*tan(1/2*a*x)^4 - 27*a^7*
x^7*imag_part(cos_integral(-3*a*x))*tan(1/2*a*x)^4 + 54*a^7*x^7*sin_integr
al(3*a*x)*tan(1/2*a*x)^4 - 2*a^7*x^7*sin_integral(a*x)*tan(1/2*a*x)^4 + 54
*a^6*x^6*imag_part(cos_integral(3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 - 2*
a^6*x^6*imag_part(cos_integral(a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 + 2*a^6
*x^6*imag_part(cos_integral(-a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 - 54*a^6*
x^6*imag_part(cos_integral(-3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3 + 108*a^
6*x^6*sin_integral(3*a*x)*tan(3/2*a*x)^2*tan(1/2*a*x)^3 - 4*a^6*x^6*sin_in
tegral(a*x)*tan(3/2*a*x)^2*tan(1/2*a*x)^3 - 16*a^6*x^6*tan(3/2*a*x)^2*tan(
1/2*a*x)^4 - 27*a^7*x^7*imag_part(cos_integral(3*a*x))*tan(3/2*a*x)^2 + a^
7*x^7*imag_part(cos_integral(a*x))*tan(3/2*a*x)^2 - a^7*x^7*imag_part(cos_
integral(-a*x))*tan(3/2*a*x)^2 + 27*a^7*x^7*imag_part(cos_integral(-3*a*x)
)*tan(3/2*a*x)^2 - 54*a^7*x^7*sin_integral(3*a*x)*tan(3/2*a*x)^2 + 2*a^...
```

3.586.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^5(ax)}{x^3(ax\cos(ax)-\sin(ax))^2}\,dx = \int \frac{\sin{(a\,x)}^5}{x^3\,{\left(\sin{(a\,x)}-a\,x\,\cos{(a\,x)}\right)}^2}\,dx$$

input
```
int(sin(a*x)^5/(x^3*(sin(a*x) - a*x*cos(a*x))^2),x)
```

output
```
int(sin(a*x)^5/(x^3*(sin(a*x) - a*x*cos(a*x))^2), x)
```

3.586. $\int \frac{\sin^5(ax)}{x^3(ax\cos(ax)-\sin(ax))^2}\,dx$

3.587 $\int \frac{\sin^4(ax)}{x^2(ax\cos(ax)-\sin(ax))^2}\,dx$

3.587.1 Optimal result

Integrand size = 26, antiderivative size = 80

$$\int \frac{\sin^4(ax)}{x^2(ax\cos(ax)-\sin(ax))^2}\,dx = \frac{1}{x}+\frac{\cos(ax)\sin(ax)}{ax^2}+\frac{\sin^2(ax)}{a^2x^3}-\frac{2\sin^2(ax)}{x} + \frac{\sin^3(ax)}{a^2x^3(ax\cos(ax)-\sin(ax))}+2a\text{Si}(2ax)$$

output
```
1/x+2*a*Si(2*a*x)+cos(a*x)*sin(a*x)/a/x^2+sin(a*x)^2/a^2/x^3-2*sin(a*x)^2/
x+sin(a*x)^3/a^2/x^3/(a*x*cos(a*x)-sin(a*x))
```

3.587.2 Mathematica [A] (verified)

Time = 1.36 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.96

$$\int \frac{\sin^4(ax)}{x^2(ax\cos(ax)-\sin(ax))^2}\,dx = \frac{2ax\cos(ax)+2ax\cos(3ax)+3\sin(ax)-\sin(3ax)+8ax(ax\cos(ax)-\sin(ax))\text{Si}(2ax)}{4x(ax\cos(ax)-\sin(ax))}$$

input
```
Integrate[Sin[a*x]^4/(x^2*(a*x*Cos[a*x] - Sin[a*x])^2),x]
```

output
```
(2*a*x*Cos[a*x] + 2*a*x*Cos[3*a*x] + 3*Sin[a*x] - Sin[3*a*x] + 8*a*x*(a*x*
Cos[a*x] - Sin[a*x])*SinIntegral[2*a*x])/(4*x*(a*x*Cos[a*x] - Sin[a*x]))
```

3.587.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.26, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {5109, 3042, 3795, 15, 3042, 3794, 27, 3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^4(ax)}{x^2(ax\cos(ax)-\sin(ax))^2}dx$$

$$\Big\downarrow \text{5109}$$

$$\frac{\sin^3(ax)}{a^2x^3(ax\cos(ax)-\sin(ax))}-\frac{3\int \frac{\sin^2(ax)}{x^4}dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin^3(ax)}{a^2x^3(ax\cos(ax)-\sin(ax))}-\frac{3\int \frac{\sin(ax)^2}{x^4}dx}{a^2}$$

$$\Big\downarrow \text{3795}$$

$$\frac{\sin^3(ax)}{a^2x^3(ax\cos(ax)-\sin(ax))}-\frac{3\left(\frac{1}{3}a^2\int \frac{1}{x^2}dx-\frac{2}{3}a^2\int \frac{\sin^2(ax)}{x^2}dx-\frac{\sin^2(ax)}{3x^3}-\frac{a\sin(ax)\cos(ax)}{3x^2}\right)}{a^2}$$

$$\Big\downarrow \text{15}$$

$$\frac{\sin^3(ax)}{a^2x^3(ax\cos(ax)-\sin(ax))}-\frac{3\left(-\frac{2}{3}a^2\int \frac{\sin^2(ax)}{x^2}dx-\frac{a^2}{3x}-\frac{\sin^2(ax)}{3x^3}-\frac{a\sin(ax)\cos(ax)}{3x^2}\right)}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin^3(ax)}{a^2x^3(ax\cos(ax)-\sin(ax))}-\frac{3\left(-\frac{2}{3}a^2\int \frac{\sin(ax)^2}{x^2}dx-\frac{a^2}{3x}-\frac{\sin^2(ax)}{3x^3}-\frac{a\sin(ax)\cos(ax)}{3x^2}\right)}{a^2}$$

$$\Big\downarrow \text{3794}$$

$$\frac{\sin^3(ax)}{a^2x^3(ax\cos(ax)-\sin(ax))}-\frac{3\left(-\frac{2}{3}a^2\left(2a\int \frac{\sin(2ax)}{2x}dx-\frac{\sin^2(ax)}{x}\right)-\frac{a^2}{3x}-\frac{\sin^2(ax)}{3x^3}-\frac{a\sin(ax)\cos(ax)}{3x^2}\right)}{a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sin^3(ax)}{a^2x^3(ax\cos(ax)-\sin(ax))}-\frac{3\left(-\frac{2}{3}a^2\left(a\int \frac{\sin(2ax)}{x}dx-\frac{\sin^2(ax)}{x}\right)-\frac{a^2}{3x}-\frac{\sin^2(ax)}{3x^3}-\frac{a\sin(ax)\cos(ax)}{3x^2}\right)}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin^3(ax)}{a^2x^3(ax\cos(ax)-\sin(ax))}-\frac{3\left(-\frac{2}{3}a^2\left(a\int\frac{\sin(2ax)}{x}dx-\frac{\sin^2(ax)}{x}\right)-\frac{a^2}{3x}-\frac{\sin^2(ax)}{3x^3}-\frac{a\sin(ax)\cos(ax)}{3x^2}\right)}{a^2}$$

$$\Big\downarrow \text{3780}$$

$$\frac{\sin^3(ax)}{a^2x^3(ax\cos(ax)-\sin(ax))}-\frac{3\left(-\frac{2}{3}a^2\left(a\text{Si}(2ax)-\frac{\sin^2(ax)}{x}\right)-\frac{a^2}{3x}-\frac{\sin^2(ax)}{3x^3}-\frac{a\sin(ax)\cos(ax)}{3x^2}\right)}{a^2}$$

input
```
Int[Sin[a*x]^4/(x^2*(a*x*Cos[a*x] - Sin[a*x])^2),x]
```

output
```
Sin[a*x]^3/(a^2*x^3*(a*x*Cos[a*x] - Sin[a*x])) - (3*(-1/3*a^2/x - (a*Cos[a
*x]*Sin[a*x])/(3*x^2) - Sin[a*x]^2/(3*x^3) - (2*a^2*(-(Sin[a*x]^2/x) + a*S
inIntegral[2*a*x]))/3))/a^2
```

3.587.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

3.587. $\int \frac{\sin^4(ax)}{x^2(ax\cos(ax)-\sin(ax))^2}\,dx$

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

rule 5109
```
Int[(((b_.)*(x_))^(m_)*Sin[(a_.)*(x_)]^(n_))/(Cos[(a_.)*(x_)]*(d_.)*(x_) +
(c_.)*Sin[(a_.)*(x_)])^2, x_Symbol] :> Simp[b*(b*x)^(m - 1)*(Sin[a*x]^(n -
1)/(a*d*(c*Sin[a*x] + d*x*Cos[a*x]))), x] - Simp[b^2*((n - 1)/d^2)   Int[(b
*x)^(m - 2)*Sin[a*x]^(n - 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ
[a*c + d, 0] && EqQ[m, 2 - n]
```

3.587.4 Maple [F(-1)]

Timed out.

$$\int \frac{\sin(ax)^4}{x^2(ax\cos(ax)-\sin(ax))^2}dx$$

input
```
int(sin(a*x)^4/x^2/(a*x*cos(a*x)-sin(a*x))^2,x)
```

output
```
int(sin(a*x)^4/x^2/(a*x*cos(a*x)-sin(a*x))^2,x)
```

3.587.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.96

$$\int \frac{\sin^4(ax)}{x^2(ax\cos(ax)-\sin(ax))^2}\,dx = \frac{2\,ax\cos(ax)^3+(2\,a^2x^2\,\mathrm{Si}\,(2\,ax)-ax)\cos(ax)-\left(2\,ax\,\mathrm{Si}\,(2\,ax)+\cos(ax)^2-1\right)\sin(ax)}{ax^2\cos(ax)-x\sin(ax)}$$

input
```
integrate(sin(a*x)^4/x^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="fricas")
```

output
```
(2*a*x*cos(a*x)^3 + (2*a^2*x^2*sin_integral(2*a*x) - a*x)*cos(a*x) - (2*a*
x*sin_integral(2*a*x) + cos(a*x)^2 - 1)*sin(a*x))/(a*x^2*cos(a*x) - x*sin(
a*x))
```

3.587.6 Sympy [F]

$$\int \frac{\sin^4(ax)}{x^2(ax\cos(ax)-\sin(ax))^2}\,dx = \int \frac{\sin^4(ax)}{x^2\left(ax\cos(ax)-\sin(ax)\right)^2}\,dx$$

input
```
integrate(sin(a*x)**4/x**2/(a*x*cos(a*x)-sin(a*x))**2,x)
```

output
```
Integral(sin(a*x)**4/(x**2*(a*x*cos(a*x) - sin(a*x))**2), x)
```

3.587.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sin^4(ax)}{x^2(ax\cos(ax)-\sin(ax))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(sin(a*x)^4/x^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.587.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.39 (sec) , antiderivative size = 1033, normalized size of antiderivative = 12.91

$$\int \frac{\sin^4(ax)}{x^2(ax\cos(ax)-\sin(ax))^2}\,dx = \text{Too large to display}$$

input
```
integrate(sin(a*x)^4/x^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="giac")
```

output
```
(a^4*x^4*imag_part(cos_integral(2*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 - a^4*x^
4*imag_part(cos_integral(-2*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 + 2*a^4*x^4*si
n_integral(2*a*x)*tan(a*x)^2*tan(1/2*a*x)^2 - a^4*x^4*imag_part(cos_integr
al(2*a*x))*tan(a*x)^2 + a^4*x^4*imag_part(cos_integral(-2*a*x))*tan(a*x)^2
 - 2*a^4*x^4*sin_integral(2*a*x)*tan(a*x)^2 + a^4*x^4*imag_part(cos_integr
al(2*a*x))*tan(1/2*a*x)^2 - a^4*x^4*imag_part(cos_integral(-2*a*x))*tan(1/
2*a*x)^2 + 2*a^4*x^4*sin_integral(2*a*x)*tan(1/2*a*x)^2 + 2*a^3*x^3*imag_p
art(cos_integral(2*a*x))*tan(a*x)^2*tan(1/2*a*x) - 2*a^3*x^3*imag_part(cos
_integral(-2*a*x))*tan(a*x)^2*tan(1/2*a*x) + 4*a^3*x^3*sin_integral(2*a*x)
*tan(a*x)^2*tan(1/2*a*x) - a^3*x^3*tan(a*x)^2*tan(1/2*a*x)^2 - a^4*x^4*ima
g_part(cos_integral(2*a*x)) + a^4*x^4*imag_part(cos_integral(-2*a*x)) - 2*
a^4*x^4*sin_integral(2*a*x) + a^2*x^2*imag_part(cos_integral(2*a*x))*tan(a
*x)^2*tan(1/2*a*x)^2 - a^2*x^2*imag_part(cos_integral(-2*a*x))*tan(a*x)^2*
tan(1/2*a*x)^2 + 2*a^2*x^2*sin_integral(2*a*x)*tan(a*x)^2*tan(1/2*a*x)^2 +
 a^3*x^3*tan(a*x)^2 + 2*a^3*x^3*imag_part(cos_integral(2*a*x))*tan(1/2*a*x
) - 2*a^3*x^3*imag_part(cos_integral(-2*a*x))*tan(1/2*a*x) + 4*a^3*x^3*sin
_integral(2*a*x)*tan(1/2*a*x) + a^3*x^3*tan(1/2*a*x)^2 - a^2*x^2*imag_part
(cos_integral(2*a*x))*tan(a*x)^2 + a^2*x^2*imag_part(cos_integral(-2*a*x))
*tan(a*x)^2 - 2*a^2*x^2*sin_integral(2*a*x)*tan(a*x)^2 - 2*a^2*x^2*tan(a*x
)^2*tan(1/2*a*x) + a^2*x^2*imag_part(cos_integral(2*a*x))*tan(1/2*a*x)^...
```

3.587.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^4(ax)}{x^2(ax\cos(ax)-\sin(ax))^2}\,dx=\int \frac{{\sin\left(a\,x\right)}^4}{x^2\,{\left(\sin\left(a\,x\right)-a\,x\,\cos\left(a\,x\right)\right)}^2}\,dx$$

input
```
int(sin(a*x)^4/(x^2*(sin(a*x) - a*x*cos(a*x))^2),x)
```

output
```
int(sin(a*x)^4/(x^2*(sin(a*x) - a*x*cos(a*x))^2), x)
```

3.588 $\int \frac{\sin^3(ax)}{x(ax\cos(ax)-\sin(ax))^2}\,dx$

3.588.1 Optimal result

Integrand size = 26, antiderivative size = 56

$$\int \frac{\sin^3(ax)}{x(ax\cos(ax)-\sin(ax))^2}\,dx=\frac{\cos(ax)}{ax}+\frac{\sin(ax)}{a^2x^2}+\frac{\sin^2(ax)}{a^2x^2(ax\cos(ax)-\sin(ax))}+\mathrm{Si}(ax)$$

output
```
cos(a*x)/a/x+Si(a*x)+sin(a*x)/a^2/x^2+sin(a*x)^2/a^2/x^2/(a*x*cos(a*x)-sin
(a*x))
```

3.588.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.05 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.86

$$\int \frac{\sin^3(ax)}{x(ax\cos(ax)-\sin(ax))^2}\,dx=\frac{1}{2}\left(\frac{e^{-iax}}{-i+ax}+\frac{e^{iax}}{i+ax}-ie\,\mathrm{CosIntegral}(i-ax)\right.$$
$$+ie\,\mathrm{CosIntegral}(i+ax)+ie\,\mathrm{ExpIntegralEi}(-1-iax)$$
$$-ie\,\mathrm{ExpIntegralEi}(-1+iax)$$
$$+\frac{2}{(-i+ax)(i+ax)(ax\cos(ax)-\sin(ax))}+2\mathrm{Si}(ax)$$
$$\left.+e\mathrm{Si}(i-ax)-e\mathrm{Si}(i+ax)\right)$$

input
```
Integrate[Sin[a*x]^3/(x*(a*x*Cos[a*x] - Sin[a*x])^2),x]
```

output
```
(1/(E^(I*a*x)*(-I + a*x)) + E^(I*a*x)/(I + a*x) - I*E*CosIntegral[I - a*x]
 + I*E*CosIntegral[I + a*x] + I*E*ExpIntegralEi[-1 - I*a*x] - I*E*ExpInteg
ralEi[-1 + I*a*x] + 2/((-I + a*x)*(I + a*x)*(a*x*Cos[a*x] - Sin[a*x])) + 2
*SinIntegral[a*x] + E*SinIntegral[I - a*x] - E*SinIntegral[I + a*x])/2
```

3.588.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.23, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5109, 3042, 3778, 3042, 3778, 25, 3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(ax)}{x(ax\cos(ax)-\sin(ax))^2}dx$$

$$\Big\downarrow \text{5109}$$

$$\frac{\sin^2(ax)}{a^2x^2(ax\cos(ax)-\sin(ax))}-\frac{2\int \frac{\sin(ax)}{x^3}dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin^2(ax)}{a^2x^2(ax\cos(ax)-\sin(ax))}-\frac{2\int \frac{\sin(ax)}{x^3}dx}{a^2}$$

$$\Big\downarrow \text{3778}$$

$$\frac{\sin^2(ax)}{a^2x^2(ax\cos(ax)-\sin(ax))}-\frac{2\left(\frac{1}{2}a\int \frac{\cos(ax)}{x^2}dx-\frac{\sin(ax)}{2x^2}\right)}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin^2(ax)}{a^2x^2(ax\cos(ax)-\sin(ax))}-\frac{2\left(\frac{1}{2}a\int \frac{\sin\left(ax+\frac{\pi}{2}\right)}{x^2}dx-\frac{\sin(ax)}{2x^2}\right)}{a^2}$$

$$\Big\downarrow \text{3778}$$

$$\frac{\sin^2(ax)}{a^2x^2(ax\cos(ax)-\sin(ax))}-\frac{2\left(\frac{1}{2}a\left(a\int -\frac{\sin(ax)}{x}dx-\frac{\cos(ax)}{x}\right)-\frac{\sin(ax)}{2x^2}\right)}{a^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sin^2(ax)}{a^2x^2(ax\cos(ax)-\sin(ax))}-\frac{2\left(\frac{1}{2}a\left(-a\int\frac{\sin(ax)}{x}dx-\frac{\cos(ax)}{x}\right)-\frac{\sin(ax)}{2x^2}\right)}{a^2}$$

$\Big\downarrow$ 3042

$$\frac{\sin^2(ax)}{a^2x^2(ax\cos(ax)-\sin(ax))}-\frac{2\left(\frac{1}{2}a\left(-a\int\frac{\sin(ax)}{x}dx-\frac{\cos(ax)}{x}\right)-\frac{\sin(ax)}{2x^2}\right)}{a^2}$$

$\Big\downarrow$ 3780

$$\frac{\sin^2(ax)}{a^2x^2(ax\cos(ax)-\sin(ax))}-\frac{2\left(\frac{1}{2}a\left(-a\text{Si}(ax)-\frac{\cos(ax)}{x}\right)-\frac{\sin(ax)}{2x^2}\right)}{a^2}$$

input
```
Int[Sin[a*x]^3/(x*(a*x*Cos[a*x] - Sin[a*x])^2),x]
```

output
```
Sin[a*x]^2/(a^2*x^2*(a*x*Cos[a*x] - Sin[a*x])) - (2*(-1/2*Sin[a*x]/x^2 + (
a*(-(Cos[a*x]/x) - a*SinIntegral[a*x]))/2))/a^2
```

3.588.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

3.588. $\int \frac{\sin^3(ax)}{x(ax\cos(ax)-\sin(ax))^2}\,dx$

rule 5109
```
Int[(((b_.)*(x_))^(m_)*Sin[(a_.)*(x_)]^(n_))/(Cos[(a_.)*(x_)]*(d_.)*(x_) +
(c_.)*Sin[(a_.)*(x_)])^2, x_Symbol] :> Simp[b*(b*x)^(m - 1)*(Sin[a*x]^(n -
1)/(a*d*(c*Sin[a*x] + d*x*Cos[a*x]))), x] - Simp[b^2*((n - 1)/d^2)   Int[(b
*x)^(m - 2)*Sin[a*x]^(n - 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ
[a*c + d, 0] && EqQ[m, 2 - n]
```

3.588.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.61 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.14

method	result	size
risch	$\frac{i\left(i\,\mathrm{Ei}_1(-iax)ax-\mathrm{Ei}_1(-iax)+\mathrm{e}^{iax}\right)}{2iax-2}+\frac{i\mathrm{e}^{-iax}}{2iax+2}-\frac{i\,\mathrm{Ei}_1(iax)}{2}+\frac{2\,\mathrm{e}^{iax}}{(ax+i)(ax-i)(ax\,\mathrm{e}^{2iax}+i\mathrm{e}^{2iax}+ax-i)}$	120

input
```
int(sin(a*x)^3/x/(a*x*cos(a*x)-sin(a*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*I*(I*Ei(1,-I*a*x)*a*x-Ei(1,-I*a*x)+exp(I*a*x))/(-1+I*a*x)+1/2*I*exp(-I
*a*x)/(I*a*x+1)-1/2*I*Ei(1,I*a*x)+2*exp(I*a*x)/(a*x+I)/(a*x-I)/(a*x*exp(2*
I*a*x)+I*exp(2*I*a*x)+a*x-I)
```

3.588.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.80

$$\int \frac{\sin^3(ax)}{x(ax\cos(ax)-\sin(ax))^2}\,dx=\frac{ax\cos(ax)\,\mathrm{Si}\,(ax)+\cos(ax)^2-\sin(ax)\,\mathrm{Si}\,(ax)}{ax\cos(ax)-\sin(ax)}$$

input
```
integrate(sin(a*x)^3/x/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="fricas")
```

output
```
(a*x*cos(a*x)*sin_integral(a*x) + cos(a*x)^2 - sin(a*x)*sin_integral(a*x))
/(a*x*cos(a*x) - sin(a*x))
```

3.588.6 Sympy [F]

$$\int \frac{\sin^3(ax)}{x(ax\cos(ax)-\sin(ax))^2}\,dx = \int \frac{\sin^3{(ax)}}{x\left(ax\cos{(ax)}-\sin{(ax)}\right)^2}\,dx$$

input
```
integrate(sin(a*x)**3/x/(a*x*cos(a*x)-sin(a*x))**2,x)
```

output
```
Integral(sin(a*x)**3/(x*(a*x*cos(a*x) - sin(a*x))**2), x)
```

3.588.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sin^3(ax)}{x(ax\cos(ax)-\sin(ax))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(sin(a*x)^3/x/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.588.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.36 (sec) , antiderivative size = 496, normalized size of antiderivative = 8.86

$$\int \frac{\sin^3(ax)}{x(ax\cos(ax)-\sin(ax))^2}\,dx$$
$$= \frac{a^3x^3\Im(\mathrm{Ci}\,(ax))\tan\left(\frac{1}{2}\,ax\right)^4 - a^3x^3\Im(\mathrm{Ci}\,(-ax))\tan\left(\frac{1}{2}\,ax\right)^4 + 2\,a^3x^3\,\mathrm{Si}\,(ax)\tan\left(\frac{1}{2}\,ax\right)^4 + 2\,a^2x^2\Im(\mathrm{Ci}\,(ax)}{}$$

input
```
integrate(sin(a*x)^3/x/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="giac")
```

output
```
1/2*(a^3*x^3*imag_part(cos_integral(a*x))*tan(1/2*a*x)^4 - a^3*x^3*imag_pa
rt(cos_integral(-a*x))*tan(1/2*a*x)^4 + 2*a^3*x^3*sin_integral(a*x)*tan(1/
2*a*x)^4 + 2*a^2*x^2*imag_part(cos_integral(a*x))*tan(1/2*a*x)^3 - 2*a^2*x
^2*imag_part(cos_integral(-a*x))*tan(1/2*a*x)^3 + 4*a^2*x^2*sin_integral(a
*x)*tan(1/2*a*x)^3 - 2*a^2*x^2*tan(1/2*a*x)^4 - a^3*x^3*imag_part(cos_inte
gral(a*x)) + a^3*x^3*imag_part(cos_integral(-a*x)) - 2*a^3*x^3*sin_integra
l(a*x) + a*x*imag_part(cos_integral(a*x))*tan(1/2*a*x)^4 - a*x*imag_part(c
os_integral(-a*x))*tan(1/2*a*x)^4 + 2*a*x*sin_integral(a*x)*tan(1/2*a*x)^4
 + 2*a^2*x^2*imag_part(cos_integral(a*x))*tan(1/2*a*x) - 2*a^2*x^2*imag_pa
rt(cos_integral(-a*x))*tan(1/2*a*x) + 4*a^2*x^2*sin_integral(a*x)*tan(1/2*
a*x) + 4*a^2*x^2*tan(1/2*a*x)^2 - 2*a^2*x^2 + 2*imag_part(cos_integral(a*x
))*tan(1/2*a*x)^3 - 2*imag_part(cos_integral(-a*x))*tan(1/2*a*x)^3 + 4*sin
_integral(a*x)*tan(1/2*a*x)^3 - 4*tan(1/2*a*x)^4 - a*x*imag_part(cos_integ
ral(a*x)) + a*x*imag_part(cos_integral(-a*x)) - 2*a*x*sin_integral(a*x) +
2*imag_part(cos_integral(a*x))*tan(1/2*a*x) - 2*imag_part(cos_integral(-a*
x))*tan(1/2*a*x) + 4*sin_integral(a*x)*tan(1/2*a*x) - 4)/(a^3*x^3*tan(1/2*
a*x)^4 + 2*a^2*x^2*tan(1/2*a*x)^3 - a^3*x^3 + a*x*tan(1/2*a*x)^4 + 2*a^2*x
^2*tan(1/2*a*x) + 2*tan(1/2*a*x)^3 - a*x + 2*tan(1/2*a*x))
```

3.588.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^3(ax)}{x(ax\cos(ax)-\sin(ax))^2}\,dx = \int \frac{\sin\left(a\,x\right)^3}{x\,\left(\sin\left(a\,x\right)-a\,x\,\cos\left(a\,x\right)\right)^2}\,dx$$

input
```
int(sin(a*x)^3/(x*(sin(a*x) - a*x*cos(a*x))^2),x)
```

output
```
int(sin(a*x)^3/(x*(sin(a*x) - a*x*cos(a*x))^2), x)
```

3.589 $\int \frac{\sin^2(ax)}{(ax\cos(ax)-\sin(ax))^2} dx$

3.589.1 Optimal result

Integrand size = 23, antiderivative size = 35

$$\int \frac{\sin^2(ax)}{(ax\cos(ax)-\sin(ax))^2} dx = \frac{1}{a^2x} + \frac{\sin(ax)}{a^2x(ax\cos(ax)-\sin(ax))}$$

output
```
1/a^2/x+sin(a*x)/a^2/x/(a*x*cos(a*x)-sin(a*x))
```

3.589.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.69

$$\int \frac{\sin^2(ax)}{(ax\cos(ax)-\sin(ax))^2} dx = \frac{\cos(ax)}{a^2x\cos(ax)-a\sin(ax)}$$

input
```
Integrate[Sin[a*x]^2/(a*x*Cos[a*x] - Sin[a*x])^2,x]
```

output
```
Cos[a*x]/(a^2*x*Cos[a*x] - a*Sin[a*x])
```

3.589.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {5107}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(ax)}{(ax\cos(ax)-\sin(ax))^2}\,dx$$

$$\Big\downarrow \; 5107$$

$$\frac{1}{a^2x}+\frac{\sin(ax)}{a^2x(ax\cos(ax)-\sin(ax))}$$

input
```
Int[Sin[a*x]^2/(a*x*Cos[a*x] - Sin[a*x])^2,x]
```

output
```
1/(a^2*x) + Sin[a*x]/(a^2*x*(a*x*Cos[a*x] - Sin[a*x]))
```

3.589.3.1 Defintions of rubi rules used

rule 5107
```
Int[Sin[(a_.)*(x_)]^2/(Cos[(a_.)*(x_)]*(d_.)*(x_) + (c_.)*Sin[(a_.)*(x_)])^
2, x_Symbol] :> Simp[1/(d^2*x), x] + Simp[Sin[a*x]/(a*d*x*(d*x*Cos[a*x] + c
*Sin[a*x])), x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0]
```

3.589.4 Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

method	result	size
parallelrisch	$\frac{\tan\left(\frac{ax}{2}\right)^{2}-1}{a\left(ax\tan\left(\frac{ax}{2}\right)^{2}-ax+2\tan\left(\frac{ax}{2}\right)\right)}$	38
risch	$\frac{1}{a(ax+i)}+\frac{2i}{(ax+i)(ax\,{\mathrm e}^{2iax}+i{\mathrm e}^{2iax}+ax-i)a}$	54
norman	$\frac{\frac{\tan\left(\frac{ax}{2}\right)^{4}}{a}+\frac{\tan\left(\frac{ax}{2}\right)^{6}}{a}-\frac{1}{a}-\frac{\tan\left(\frac{ax}{2}\right)^{2}}{a}}{\left(1+\tan\left(\frac{ax}{2}\right)^{2}\right)^{2}\left(ax\tan\left(\frac{ax}{2}\right)^{2}-ax+2\tan\left(\frac{ax}{2}\right)\right)}$	77

input
```
int(sin(a*x)^2/(a*x*cos(a*x)-sin(a*x))^2,x,method=_RETURNVERBOSE)
```

output
```
(tan(1/2*a*x)^2-1)/a/(a*x*tan(1/2*a*x)^2-a*x+2*tan(1/2*a*x))
```

3.589.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.69

$$\int \frac{\sin^2(ax)}{(ax\cos(ax)-\sin(ax))^2}\,dx = \frac{\cos(ax)}{a^2x\cos(ax)-a\sin(ax)}$$

input
```
integrate(sin(a*x)^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="fricas")
```

output
```
cos(a*x)/(a^2*x*cos(a*x) - a*sin(a*x))
```

3.589.6 Sympy [A] (verification not implemented)

Time = 2.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.57

$$\int \frac{\sin^2(ax)}{(ax\cos(ax)-\sin(ax))^2}\,dx = \frac{\cos(ax)}{a^2x\cos(ax)-a\sin(ax)}$$

input
```
integrate(sin(a*x)**2/(a*x*cos(a*x)-sin(a*x))**2,x)
```

output
```
cos(a*x)/(a**2*x*cos(a*x) - a*sin(a*x))
```

3.589.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(35) = 70.

Time = 0.20 (sec) , antiderivative size = 114, normalized size of antiderivative = 3.26

$$\int \frac{\sin^2(ax)}{(ax\cos(ax)-\sin(ax))^2}\,dx$$
$$= \frac{ax\cos(2ax)^2+ax\sin(2ax)^2+2ax\cos(2ax)+ax-2\sin(2ax)}{(a^2x^2+(a^2x^2+1)\cos(2ax)^2-4ax\sin(2ax)+(a^2x^2+1)\sin(2ax)^2+2(a^2x^2-1)\cos(2ax)+1)a}$$

input
```
integrate(sin(a*x)^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="maxima")
```

output
```
(a*x*cos(2*a*x)^2 + a*x*sin(2*a*x)^2 + 2*a*x*cos(2*a*x) + a*x - 2*sin(2*a*
x))/((a^2*x^2 + (a^2*x^2 + 1)*cos(2*a*x)^2 - 4*a*x*sin(2*a*x) + (a^2*x^2 +
 1)*sin(2*a*x)^2 + 2*(a^2*x^2 - 1)*cos(2*a*x) + 1)*a)
```

3.589.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int \frac{\sin^2(ax)}{(ax\cos(ax)-\sin(ax))^2}\,dx = \frac{\tan\left(\frac{1}{2}\,ax\right)^2-1}{a^2x\tan\left(\frac{1}{2}\,ax\right)^2-a^2x+2\,a\tan\left(\frac{1}{2}\,ax\right)}$$

input
```
integrate(sin(a*x)^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="giac")
```

output
```
(tan(1/2*a*x)^2 - 1)/(a^2*x*tan(1/2*a*x)^2 - a^2*x + 2*a*tan(1/2*a*x))
```

3.589.9 Mupad [B] (verification not implemented)

Time = 27.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.69

$$\int \frac{\sin^2(ax)}{(ax\cos(ax)-\sin(ax))^2}\,dx = -\frac{\cos(a\,x)}{a\,\left(\sin(a\,x)-a\,x\,\cos(a\,x)\right)}$$

input
```
int(sin(a*x)^2/(sin(a*x) - a*x*cos(a*x))^2,x)
```

output
```
-cos(a*x)/(a*(sin(a*x) - a*x*cos(a*x)))
```

3.590 $\int \frac{x\sin(ax)}{(ax\cos(ax)-\sin(ax))^2}\,dx$

3.590.1 Optimal result

Integrand size = 22, antiderivative size = 20

$$\int \frac{x\sin(ax)}{(ax\cos(ax)-\sin(ax))^2}\,dx = \frac{1}{a^2(ax\cos(ax)-\sin(ax))}$$

output
```
1/a^2/(a*x*cos(a*x)-sin(a*x))
```

3.590.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x\sin(ax)}{(ax\cos(ax)-\sin(ax))^2}\,dx = -\frac{1}{a^2(-ax\cos(ax)+\sin(ax))}$$

input
```
Integrate[(x*Sin[a*x])/(a*x*Cos[a*x] - Sin[a*x])^2,x]
```

output
```
-(1/(a^2*(-(a*x*Cos[a*x]) + Sin[a*x])))
```

3.590.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {7237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sin(ax)}{(ax\cos(ax)-\sin(ax))^2}\,dx$$
$$\Big\downarrow \; 7237$$
$$\frac{1}{a^2(ax\cos(ax)-\sin(ax))}$$

input
```
Int[(x*Sin[a*x])/(a*x*Cos[a*x] - Sin[a*x])^2,x]
```

output
```
1/(a^2*(a*x*Cos[a*x] - Sin[a*x]))
```

3.590.3.1 Defintions of rubi rules used

rule 7237
```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[q*(y^(m + 1)/(m + 1)), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

3.590.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$\frac{1}{a^2(ax\cos(ax)-\sin(ax))}$	21
default	$\frac{1}{a^2(ax\cos(ax)-\sin(ax))}$	21
parallelrisch	$\frac{1}{a^2(ax\cos(ax)-\sin(ax))}$	21
risch	$\frac{2\,\mathrm{e}^{iax}}{a^2(ax\,\mathrm{e}^{2iax}+i\mathrm{e}^{2iax}+ax-i)}$	38
norman	$\frac{-\frac{1}{a^2}-\frac{2\tan\left(\frac{ax}{2}\right)^2}{a^2}-\frac{\tan\left(\frac{ax}{2}\right)^4}{a^2}}{\left(1+\tan\left(\frac{ax}{2}\right)^2\right)\left(ax\tan\left(\frac{ax}{2}\right)^2-ax+2\tan\left(\frac{ax}{2}\right)\right)}$	67

input
```
int(x*sin(a*x)/(a*x*cos(a*x)-sin(a*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^2/(a*x*cos(a*x)-sin(a*x))
```

3.590.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \frac{x\sin(ax)}{(ax\cos(ax)-\sin(ax))^2}\,dx = \frac{1}{a^3x\cos(ax)-a^2\sin(ax)}$$

input
```
integrate(x*sin(a*x)/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="fricas")
```

output
```
1/(a^3*x*cos(a*x) - a^2*sin(a*x))
```

3.590.6 Sympy [A] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{x\sin(ax)}{(ax\cos(ax)-\sin(ax))^2}\,dx = \frac{1}{a^3x\cos(ax)-a^2\sin(ax)}$$

input
```
integrate(x*sin(a*x)/(a*x*cos(a*x)-sin(a*x))**2,x)
```

output
```
1/(a**3*x*cos(a*x) - a**2*sin(a*x))
```

3.590.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x\sin(ax)}{(ax\cos(ax)-\sin(ax))^2}\,dx = \frac{1}{(ax\cos(ax)-\sin(ax))a^2}$$

input
```
integrate(x*sin(a*x)/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="maxima")
```

output
```
1/((a*x*cos(a*x) - sin(a*x))*a^2)
```

3.590.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(20) = 40$.

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.10

$$\int \frac{x\sin(ax)}{(ax\cos(ax)-\sin(ax))^2}\,dx = -\frac{2\left(\tan\left(\frac{1}{2}\,ax\right)^2+1\right)}{a^3x\tan\left(\frac{1}{2}\,ax\right)^2-a^3x+2\,a^2\tan\left(\frac{1}{2}\,ax\right)}$$

input
```
integrate(x*sin(a*x)/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="giac")
```

output
```
-2*(tan(1/2*a*x)^2 + 1)/(a^3*x*tan(1/2*a*x)^2 - a^3*x + 2*a^2*tan(1/2*a*x)
)
```

3.590.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{x\sin(ax)}{(ax\cos(ax)-\sin(ax))^2}\,dx = -\frac{1}{a^2\,\sin\left(a\,x\right)-a^3\,x\,\cos\left(a\,x\right)}$$

input
```
int((x*sin(a*x))/(sin(a*x) - a*x*cos(a*x))^2,x)
```

output
```
-1/(a^2*sin(a*x) - a^3*x*cos(a*x))
```

3.591 $\int \frac{x^2}{(ax\cos(ax)-\sin(ax))^2}\,dx$

3.591.1 Optimal result

Integrand size = 20, antiderivative size = 35

$$\int \frac{x^2}{(ax\cos(ax)-\sin(ax))^2}\,dx=-\frac{\cot(ax)}{a^3}+\frac{x\csc(ax)}{a^2(ax\cos(ax)-\sin(ax))}$$

output
```
-cot(a*x)/a^3+x*csc(a*x)/a^2/(a*x*cos(a*x)-sin(a*x))
```

3.591.2 Mathematica [A] (verified)

Time = 1.47 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \frac{x^2}{(ax\cos(ax)-\sin(ax))^2}\,dx=\frac{\cos(ax)+ax\sin(ax)}{a^3(ax\cos(ax)-\sin(ax))}$$

input
```
Integrate[x^2/(a*x*Cos[a*x] - Sin[a*x])^2,x]
```

output
```
(Cos[a*x] + a*x*Sin[a*x])/(a^3*(a*x*Cos[a*x] - Sin[a*x]))
```

3.591.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5105, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(ax\cos(ax)-\sin(ax))^2}dx$$

$$\Big\downarrow \text{5105}$$

$$\frac{\int \csc^2(ax)dx}{a^2}+\frac{x\csc(ax)}{a^2(ax\cos(ax)-\sin(ax))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc(ax)^2dx}{a^2}+\frac{x\csc(ax)}{a^2(ax\cos(ax)-\sin(ax))}$$

$$\Big\downarrow \text{4254}$$

$$\frac{x\csc(ax)}{a^2(ax\cos(ax)-\sin(ax))}-\frac{\int 1d\cot(ax)}{a^3}$$

$$\Big\downarrow \text{24}$$

$$\frac{x\csc(ax)}{a^2(ax\cos(ax)-\sin(ax))}-\frac{\cot(ax)}{a^3}$$

input
```
Int[x^2/(a*x*Cos[a*x] - Sin[a*x])^2,x]
```

output
```
-(Cot[a*x]/a^3) + (x*Csc[a*x])/(a^2*(a*x*Cos[a*x] - Sin[a*x]))
```

3.591.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 5105
```
Int[(x_)^2/(Cos[(a_.)*(x_)]*(d_.)*(x_) + (c_.)*Sin[(a_.)*(x_)])^2, x_Symbol
] :> Simp[x/(a*d*Sin[a*x]*(c*Sin[a*x] + d*x*Cos[a*x])), x] + Simp[1/d^2   I
nt[1/Sin[a*x]^2, x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0]
```

3.591.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

method	result	size
risch	$\frac{2i(ax-i)}{a^3(ax\,\mathrm{e}^{2iax}+i\mathrm{e}^{2iax}+ax-i)}$	39
parallelrisch	$\frac{-1-2\tan\left(\frac{ax}{2}\right)ax+\tan\left(\frac{ax}{2}\right)^2}{a^3\left(ax\tan\left(\frac{ax}{2}\right)^2-ax+2\tan\left(\frac{ax}{2}\right)\right)}$	47
norman	$\frac{\frac{\tan\left(\frac{ax}{2}\right)^2}{a^3}-\frac{1}{a^3}-\frac{2x\tan\left(\frac{ax}{2}\right)}{a^2}}{ax\tan\left(\frac{ax}{2}\right)^2-ax+2\tan\left(\frac{ax}{2}\right)}$	54

input
```
int(x^2/(a*x*cos(a*x)-sin(a*x))^2,x,method=_RETURNVERBOSE)
```

output
```
2*I*(a*x-I)/a^3/(a*x*exp(2*I*a*x)+I*exp(2*I*a*x)+a*x-I)
```

3.591.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int \frac{x^2}{(ax\cos(ax)-\sin(ax))^2}\,dx = \frac{ax\sin(ax)+\cos(ax)}{a^4x\cos(ax)-a^3\sin(ax)}$$

input
```
integrate(x^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="fricas")
```

output
```
(a*x*sin(a*x) + cos(a*x))/(a^4*x*cos(a*x) - a^3*sin(a*x))
```

3.591.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(31) = 62.

Time = 2.92 (sec) , antiderivative size = 112, normalized size of antiderivative = 3.20

$$\int \frac{x^2}{(ax\cos(ax)-\sin(ax))^2}\,dx = -\frac{2ax\tan\left(\frac{ax}{2}\right)}{a^4x\tan^2\left(\frac{ax}{2}\right)-a^4x+2a^3\tan\left(\frac{ax}{2}\right)} + \frac{\tan^2\left(\frac{ax}{2}\right)}{a^4x\tan^2\left(\frac{ax}{2}\right)-a^4x+2a^3\tan\left(\frac{ax}{2}\right)} - \frac{1}{a^4x\tan^2\left(\frac{ax}{2}\right)-a^4x+2a^3\tan\left(\frac{ax}{2}\right)}$$

input
```
integrate(x**2/(a*x*cos(a*x)-sin(a*x))**2,x)
```

output
```
-2*a*x*tan(a*x/2)/(a**4*x*tan(a*x/2)**2 - a**4*x + 2*a**3*tan(a*x/2)) + ta
n(a*x/2)**2/(a**4*x*tan(a*x/2)**2 - a**4*x + 2*a**3*tan(a*x/2)) - 1/(a**4*
x*tan(a*x/2)**2 - a**4*x + 2*a**3*tan(a*x/2))
```

3.591.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(35) = 70.

Time = 0.22 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.86

$$\int \frac{x^2}{(ax\cos(ax)-\sin(ax))^2}\,dx = \frac{2\left(2\,ax\cos(2\,ax)+(a^2x^2-1)\sin(2\,ax)\right)}{\left(a^2x^2+(a^2x^2+1)\cos(2\,ax)^2-4\,ax\sin(2\,ax)+(a^2x^2+1)\sin(2\,ax)^2+2\,(a^2x^2-1)\cos(2\,ax)+1\right)a^3}$$

input
```
integrate(x^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="maxima")
```

output
```
2*(2*a*x*cos(2*a*x) + (a^2*x^2 - 1)*sin(2*a*x))/((a^2*x^2 + (a^2*x^2 + 1)*
cos(2*a*x)^2 - 4*a*x*sin(2*a*x) + (a^2*x^2 + 1)*sin(2*a*x)^2 + 2*(a^2*x^2
- 1)*cos(2*a*x) + 1)*a^3)
```

3.591.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.51

$$\int \frac{x^2}{(ax\cos(ax)-\sin(ax))^2}\,dx = -\frac{2\,ax\tan\left(\frac{1}{2}\,ax\right)-\tan\left(\frac{1}{2}\,ax\right)^2+1}{a^4x\tan\left(\frac{1}{2}\,ax\right)^2-a^4x+2\,a^3\tan\left(\frac{1}{2}\,ax\right)}$$

input
```
integrate(x^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="giac")
```

output
```
-(2*a*x*tan(1/2*a*x) - tan(1/2*a*x)^2 + 1)/(a^4*x*tan(1/2*a*x)^2 - a^4*x +
 2*a^3*tan(1/2*a*x))
```

3.591.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(ax\cos(ax)-\sin(ax))^2}\,dx = \int \frac{x^2}{\left(\sin\left(a\,x\right)-a\,x\,\cos\left(a\,x\right)\right)^2}\,dx$$

input
```
int(x^2/(sin(a*x) - a*x*cos(a*x))^2,x)
```

output
```
int(x^2/(sin(a*x) - a*x*cos(a*x))^2, x)
```

3.592 $\int \frac{x^3 \csc(ax)}{(ax\cos(ax)-\sin(ax))^2}\,dx$

3.592.1 Optimal result

Integrand size = 24, antiderivative size = 104

$$\int \frac{x^3 \csc(ax)}{(ax\cos(ax)-\sin(ax))^2}\,dx = -\frac{2x\text{arctanh}(e^{iax})}{a^3} - \frac{\csc(ax)}{a^4} - \frac{x\cot(ax)\csc(ax)}{a^3} + \frac{i\,\text{PolyLog}(2,-e^{iax})}{a^4} - \frac{i\,\text{PolyLog}(2,e^{iax})}{a^4} + \frac{x^2\csc^2(ax)}{a^2(ax\cos(ax)-\sin(ax))}$$

output
```
-2*x*arctanh(exp(I*a*x))/a^3-csc(a*x)/a^4-x*cot(a*x)*csc(a*x)/a^3+I*polylo
g(2,-exp(I*a*x))/a^4-I*polylog(2,exp(I*a*x))/a^4+x^2*csc(a*x)^2/a^2/(a*x*c
os(a*x)-sin(a*x))
```

3.592.2 Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.51

$$\int \frac{x^3 \csc(ax)}{(ax\cos(ax)-\sin(ax))^2}\,dx = \frac{\csc(ax) + a^2x^2\csc(ax) - ax\log(1-e^{iax}) + a^2x^2\cot(ax)\log(1-e^{iax}) + ax\log(1+e^{iax}) - a^2x^2\cot(ax)}{a^4(-1+ax\cot(}$$

input
```
Integrate[(x^3*Csc[a*x])/(a*x*Cos[a*x] - Sin[a*x])^2,x]
```

output
```
(Csc[a*x] + a^2*x^2*Csc[a*x] - a*x*Log[1 - E^(I*a*x)] + a^2*x^2*Cot[a*x]*L
og[1 - E^(I*a*x)] + a*x*Log[1 + E^(I*a*x)] - a^2*x^2*Cot[a*x]*Log[1 + E^(I
*a*x)] + I*(-1 + a*x*Cot[a*x])*PolyLog[2, -E^(I*a*x)] - I*(-1 + a*x*Cot[a*
x])*PolyLog[2, E^(I*a*x)])/(a^4*(-1 + a*x*Cot[a*x]))
```

3.592.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {5111, 3042, 4673, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \csc(ax)}{(ax\cos(ax)-\sin(ax))^2}dx$$

$$\Big\downarrow \text{5111}$$

$$\frac{2\int x\csc^3(ax)dx}{a^2}+\frac{x^2\csc^2(ax)}{a^2(ax\cos(ax)-\sin(ax))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int x\csc(ax)^3dx}{a^2}+\frac{x^2\csc^2(ax)}{a^2(ax\cos(ax)-\sin(ax))}$$

$$\Big\downarrow \text{4673}$$

$$\frac{2\left(\frac{1}{2}\int x\csc(ax)dx-\frac{\csc(ax)}{2a^2}-\frac{x\cot(ax)\csc(ax)}{2a}\right)}{a^2}+\frac{x^2\csc^2(ax)}{a^2(ax\cos(ax)-\sin(ax))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(\frac{1}{2}\int x\csc(ax)dx-\frac{\csc(ax)}{2a^2}-\frac{x\cot(ax)\csc(ax)}{2a}\right)}{a^2}+\frac{x^2\csc^2(ax)}{a^2(ax\cos(ax)-\sin(ax))}$$

$$\Big\downarrow \text{4671}$$

$$\frac{x^2\csc^2(ax)}{a^2(ax\cos(ax)-\sin(ax))}+\frac{2\left(\frac{1}{2}\left(-\frac{\int\log(1-e^{iax})dx}{a}+\frac{\int\log(1+e^{iax})dx}{a}-\frac{2x\text{arctanh}(e^{iax})}{a}\right)-\frac{\csc(ax)}{2a^2}-\frac{x\cot(ax)\csc(ax)}{2a}\right)}{a^2}$$

$$\Big\downarrow \text{2715}$$

$$\frac{x^2\csc^2(ax)}{a^2(ax\cos(ax)-\sin(ax))}+\frac{2\left(\frac{1}{2}\left(\frac{i\int e^{-iax}\log(1-e^{iax})de^{iax}}{a^2}-\frac{i\int e^{-iax}\log(1+e^{iax})de^{iax}}{a^2}-\frac{2x\text{arctanh}(e^{iax})}{a}\right)-\frac{\csc(ax)}{2a^2}-\frac{x\cot(ax)\csc(ax)}{2a}\right)}{a^2}$$

$$\Big\downarrow \; 2838$$

$$\frac{x^2\csc^2(ax)}{a^2(ax\cos(ax)-\sin(ax))}+\frac{2\left(\frac{1}{2}\left(\frac{i\,\text{PolyLog}(2,-e^{iax})}{a^2}-\frac{i\,\text{PolyLog}(2,e^{iax})}{a^2}-\frac{2x\text{arctanh}(e^{iax})}{a}\right)-\frac{\csc(ax)}{2a^2}-\frac{x\cot(ax)\csc(ax)}{2a}\right)}{a^2}$$

input
```
Int[(x^3*Csc[a*x])/(a*x*Cos[a*x] - Sin[a*x])^2,x]
```

output
```
(2*(-1/2*Csc[a*x]/a^2 - (x*Cot[a*x]*Csc[a*x])/(2*a) + ((-2*x*ArcTanh[E^(I*
a*x)])/a + (I*PolyLog[2, -E^(I*a*x)])/a^2 - (I*PolyLog[2, E^(I*a*x)])/a^2)
/2))/a^2 + (x^2*Csc[a*x]^2)/(a^2*(a*x*Cos[a*x] - Sin[a*x]))
```

3.592.3.1 Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

3.592. $\int \frac{x^3\csc(ax)}{(ax\cos(ax)-\sin(ax))^2}\,dx$

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

rule 5111
```
Int[(Csc[(a_.)*(x_)]^(n_.)*((b_.)*(x_))^(m_.))/(Cos[(a_.)*(x_)]*(d_.)*(x_)
+ (c_.)*Sin[(a_.)*(x_)])^2, x_Symbol] :> Simp[b*(b*x)^(m - 1)*(Csc[a*x]^(n
+ 1)/(a*d*(c*Sin[a*x] + d*x*Cos[a*x]))), x] + Simp[b^2*((n + 1)/d^2)   Int[
(b*x)^(m - 2)*Csc[a*x]^(n + 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && E
qQ[a*c + d, 0] && EqQ[m, n + 2]
```

3.592.4 Maple [F]

$$\int \frac{x^3 \csc(ax)}{(ax\cos(ax) - \sin(ax))^2} dx$$

input
```
int(x^3*csc(a*x)/(a*x*cos(a*x)-sin(a*x))^2,x)
```

output
```
int(x^3*csc(a*x)/(a*x*cos(a*x)-sin(a*x))^2,x)
```

3.592.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 295 vs. 2(89) = 178.

Time = 0.28 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.84

$$\int \frac{x^3 \csc(ax)}{(ax\cos(ax) - \sin(ax))^2} dx = \frac{2a^2x^2 - (i\,ax\cos(ax) - i\,\sin(ax))\mathrm{Li}_2(\cos(ax) + i\,\sin(ax)) - (-i\,ax\cos(ax) + i\,\sin(ax))\mathrm{Li}_2(\cos(ax) -$$

input
```
integrate(x^3*csc(a*x)/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="fricas")
```

output
```
1/2*(2*a^2*x^2 - (I*a*x*cos(a*x) - I*sin(a*x))*dilog(cos(a*x) + I*sin(a*x)
) - (-I*a*x*cos(a*x) + I*sin(a*x))*dilog(cos(a*x) - I*sin(a*x)) - (I*a*x*c
os(a*x) - I*sin(a*x))*dilog(-cos(a*x) + I*sin(a*x)) - (-I*a*x*cos(a*x) + I
*sin(a*x))*dilog(-cos(a*x) - I*sin(a*x)) - (a^2*x^2*cos(a*x) - a*x*sin(a*x
))*log(cos(a*x) + I*sin(a*x) + 1) - (a^2*x^2*cos(a*x) - a*x*sin(a*x))*log(
cos(a*x) - I*sin(a*x) + 1) + (a^2*x^2*cos(a*x) - a*x*sin(a*x))*log(-cos(a*
x) + I*sin(a*x) + 1) + (a^2*x^2*cos(a*x) - a*x*sin(a*x))*log(-cos(a*x) - I
*sin(a*x) + 1) + 2)/(a^5*x*cos(a*x) - a^4*sin(a*x))
```

3.592.6 Sympy [F]

$$\int \frac{x^3 \csc(ax)}{(ax\cos(ax) - \sin(ax))^2}\,dx = \int \frac{x^3 \csc{(ax)}}{(ax\cos{(ax)} - \sin{(ax)})^2}\,dx$$

input
```
integrate(x**3*csc(a*x)/(a*x*cos(a*x)-sin(a*x))**2,x)
```

output
```
Integral(x**3*csc(a*x)/(a*x*cos(a*x) - sin(a*x))**2, x)
```

3.592.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3 \csc(ax)}{(ax\cos(ax) - \sin(ax))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*csc(a*x)/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.592.8 Giac [F]

$$\int \frac{x^3 \csc(ax)}{(ax\cos(ax)-\sin(ax))^2}\,dx = \int \frac{x^3 \csc(ax)}{(ax\cos(ax)-\sin(ax))^2}\,dx$$

input
```
integrate(x^3*csc(a*x)/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="giac")
```

output
```
integrate(x^3*csc(a*x)/(a*x*cos(a*x) - sin(a*x))^2, x)
```

3.592.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \csc(ax)}{(ax\cos(ax)-\sin(ax))^2}\,dx = \int \frac{x^3}{\sin(a\,x)\,(\sin(a\,x)-a\,x\,\cos(a\,x))^2}\,dx$$

input
```
int(x^3/(sin(a*x)*(sin(a*x) - a*x*cos(a*x))^2),x)
```

output
```
int(x^3/(sin(a*x)*(sin(a*x) - a*x*cos(a*x))^2), x)
```

3.593 $\int \frac{x^4 \csc^2(ax)}{(ax\cos(ax)-\sin(ax))^2}\,dx$

3.593.1 Optimal result

Integrand size = 26, antiderivative size = 127

$$\int \frac{x^4 \csc^2(ax)}{(ax\cos(ax)-\sin(ax))^2}\,dx = -\frac{2ix^2}{a^3}-\frac{\cot(ax)}{a^5}-\frac{2x^2\cot(ax)}{a^3}-\frac{x\csc^2(ax)}{a^4} - \frac{x^2\cot(ax)\csc^2(ax)}{a^3}+\frac{4x\log\left(1-e^{2iax}\right)}{a^4} - \frac{2i\,\text{PolyLog}\left(2,e^{2iax}\right)}{a^5}+\frac{x^3\csc^3(ax)}{a^2(ax\cos(ax)-\sin(ax))}$$

output
```
-2*I*x^2/a^3-cot(a*x)/a^5-2*x^2*cot(a*x)/a^3-x*csc(a*x)^2/a^4-x^2*cot(a*x)
*csc(a*x)^2/a^3+4*x*ln(1-exp(2*I*a*x))/a^4-2*I*polylog(2,exp(2*I*a*x))/a^5
+x^3*csc(a*x)^3/a^2/(a*x*cos(a*x)-sin(a*x))
```

3.593.2 Mathematica [A] (verified)

Time = 1.30 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.80

$$\int \frac{x^4 \csc^2(ax)}{(ax\cos(ax)-\sin(ax))^2}\,dx = \frac{\frac{1}{x}+a^2x-a^3x^2\cot(ax)+4a^2x\log\left(1-e^{2iax}\right)-2ia\left(a^2x^2+\text{PolyLog}\left(2,e^{2iax}\right)\right)+\frac{(1+a^2x^2)^2\sin(ax)}{x(ax\cos(ax)-\sin(ax))}}{a^6}$$

input
```
Integrate[(x^4*Csc[a*x]^2)/(a*x*Cos[a*x] - Sin[a*x])^2,x]
```

output
```
(x^(-1) + a^2*x - a^3*x^2*Cot[a*x] + 4*a^2*x*Log[1 - E^((2*I)*a*x)] - (2*I
)*a*(a^2*x^2 + PolyLog[2, E^((2*I)*a*x)]) + ((1 + a^2*x^2)^2*Sin[a*x])/(x*
(a*x*Cos[a*x] - Sin[a*x])))/a^6
```

3.593.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.24, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {5111, 3042, 4674, 3042, 4254, 24, 4672, 3042, 25, 4200, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \csc^2(ax)}{(ax\cos(ax)-\sin(ax))^2}dx$$

$$\Big\downarrow \text{5111}$$

$$\frac{3\int x^2\csc^4(ax)dx}{a^2}+\frac{x^3\csc^3(ax)}{a^2(ax\cos(ax)-\sin(ax))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int x^2\csc(ax)^4dx}{a^2}+\frac{x^3\csc^3(ax)}{a^2(ax\cos(ax)-\sin(ax))}$$

$$\Big\downarrow \text{4674}$$

$$\frac{3\left(\frac{\int \csc^2(ax)dx}{3a^2}+\frac{2}{3}\int x^2\csc^2(ax)dx-\frac{x\csc^2(ax)}{3a^2}-\frac{x^2\cot(ax)\csc^2(ax)}{3a}\right)}{a^2}+\frac{x^3\csc^3(ax)}{a^2(ax\cos(ax)-\sin(ax))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{\int \csc(ax)^2dx}{3a^2}+\frac{2}{3}\int x^2\csc(ax)^2dx-\frac{x\csc^2(ax)}{3a^2}-\frac{x^2\cot(ax)\csc^2(ax)}{3a}\right)}{a^2}+\frac{x^3\csc^3(ax)}{a^2(ax\cos(ax)-\sin(ax))}$$

$$\Big\downarrow \text{4254}$$

$$\frac{3\left(-\frac{\int 1d\cot(ax)}{3a^3}+\frac{2}{3}\int x^2\csc(ax)^2dx-\frac{x\csc^2(ax)}{3a^2}-\frac{x^2\cot(ax)\csc^2(ax)}{3a}\right)}{a^2}+\frac{x^3\csc^3(ax)}{a^2(ax\cos(ax)-\sin(ax))}$$

$$\Big\downarrow \text{24}$$

$$\frac{3\left(\frac{2}{3}\int x^2\csc(ax)^2dx-\frac{\cot(ax)}{3a^3}-\frac{x\csc^2(ax)}{3a^2}-\frac{x^2\cot(ax)\csc^2(ax)}{3a}\right)}{a^2}+\frac{x^3\csc^3(ax)}{a^2(ax\cos(ax)-\sin(ax))}$$

$$\Big\downarrow \text{4672}$$

$$\frac{3\left(\frac{2}{3}\left(\frac{2\int x\cot(ax)dx}{a}-\frac{x^2\cot(ax)}{a}\right)-\frac{\cot(ax)}{3a^3}-\frac{x\csc^2(ax)}{3a^2}-\frac{x^2\cot(ax)\csc^2(ax)}{3a}\right)}{a^2}+\frac{x^3\csc^3(ax)}{a^2(ax\cos(ax)-\sin(ax))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{2}{3}\left(\frac{2\int -x\tan\left(ax+\frac{\pi}{2}\right)dx}{a}-\frac{x^2\cot(ax)}{a}\right)-\frac{\cot(ax)}{3a^3}-\frac{x\csc^2(ax)}{3a^2}-\frac{x^2\cot(ax)\csc^2(ax)}{3a}\right)}{a^2}+\frac{x^3\csc^3(ax)}{a^2(ax\cos(ax)-\sin(ax))}$$

$$\Big\downarrow \text{25}$$

$$\frac{3\left(\frac{2}{3}\left(-\frac{2\int x\tan\left(ax+\frac{\pi}{2}\right)dx}{a}-\frac{x^2\cot(ax)}{a}\right)-\frac{\cot(ax)}{3a^3}-\frac{x\csc^2(ax)}{3a^2}-\frac{x^2\cot(ax)\csc^2(ax)}{3a}\right)}{a^2}+\frac{x^3\csc^3(ax)}{a^2(ax\cos(ax)-\sin(ax))}$$

$$\Big\downarrow \text{4200}$$

$$\frac{x^3\csc^3(ax)}{a^2(ax\cos(ax)-\sin(ax))}+\frac{3\left(\frac{2}{3}\left(-\frac{x^2\cot(ax)}{a}-\frac{2\left(\frac{ix^2}{2}-2i\int -\frac{e^{2iax}x}{1-e^{2iax}}dx\right)}{a}\right)-\frac{\cot(ax)}{3a^3}-\frac{x\csc^2(ax)}{3a^2}-\frac{x^2\cot(ax)\csc^2(ax)}{3a}\right)}{a^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{x^3\csc^3(ax)}{a^2(ax\cos(ax)-\sin(ax))}+\frac{3\left(\frac{2}{3}\left(-\frac{x^2\cot(ax)}{a}-\frac{2\left(2i\int \frac{e^{2iax}x}{1-e^{2iax}}dx+\frac{ix^2}{2}\right)}{a}\right)-\frac{\cot(ax)}{3a^3}-\frac{x\csc^2(ax)}{3a^2}-\frac{x^2\cot(ax)\csc^2(ax)}{3a}\right)}{a^2}$$

$$\Big\downarrow \text{2620}$$

$$\frac{x^3\csc^3(ax)}{a^2(ax\cos(ax)-\sin(ax))}+\frac{3\left(\frac{2}{3}\left(-\frac{x^2\cot(ax)}{a}-\frac{2\left(2i\left(\frac{ix\log\left(1-e^{2iax}\right)}{2a}-\frac{i\int\log\left(1-e^{2iax}\right)dx}{2a}\right)+\frac{ix^2}{2}\right)}{a}\right)-\frac{\cot(ax)}{3a^3}-\frac{x\csc^2(ax)}{3a^2}-\frac{x^2\cot(ax)\csc^2(ax)}{3a}\right)}{a^2}$$

$$\Big\downarrow \text{2715}$$

3.593. $\int \frac{x^4\csc^2(ax)}{(ax\cos(ax)-\sin(ax))^2}\,dx$

$$\frac{\frac{x^3\csc^3(ax)}{a^2(ax\cos(ax)-\sin(ax))}+3\left(\frac{2}{3}\left(-\frac{x^2\cot(ax)}{a}-\frac{2\left(2i\left(\frac{ix\log\left(1-e^{2iax}\right)}{2a}-\frac{\int e^{-2iax}\log\left(1-e^{2iax}\right)de^{2iax}}{4a^2}\right)+\frac{ix^2}{2}\right)}{a}\right)-\frac{\cot(ax)}{3a^3}-\frac{x\csc^2(ax)}{3a^2}-\frac{x^2\cot(ax)\csc^2(ax)}{3a}\right)}{a^2}$$

$$\Big\downarrow \; 2838$$

$$\frac{\frac{x^3\csc^3(ax)}{a^2(ax\cos(ax)-\sin(ax))}+3\left(-\frac{\cot(ax)}{3a^3}+\frac{2}{3}\left(-\frac{x^2\cot(ax)}{a}-\frac{2\left(2i\left(\frac{\text{PolyLog}\left(2,e^{2iax}\right)}{4a^2}+\frac{ix\log\left(1-e^{2iax}\right)}{2a}\right)+\frac{ix^2}{2}\right)}{a}\right)-\frac{x\csc^2(ax)}{3a^2}-\frac{x^2\cot(ax)\csc^2(ax)}{3a}\right)}{a^2}$$

input
```
Int[(x^4*Csc[a*x]^2)/(a*x*Cos[a*x] - Sin[a*x])^2,x]
```

output
```
(3*(-1/3*Cot[a*x]/a^3 - (x*Csc[a*x]^2)/(3*a^2) - (x^2*Cot[a*x]*Csc[a*x]^2)
/(3*a) + (2*(-((x^2*Cot[a*x])/a) - (2*((I/2)*x^2 + (2*I)*(((I/2)*x*Log[1 -
 E^((2*I)*a*x)])/a + PolyLog[2, E^((2*I)*a*x)]/(4*a^2))))/a))/3))/a^2 + (x
^3*Csc[a*x]^3)/(a^2*(a*x*Cos[a*x] - Sin[a*x]))
```

3.593.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

3.593. $\int \frac{x^4\csc^2(ax)}{(ax\cos(ax)-\sin(ax))^2}\,dx$

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4200
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 5111
```
Int[(Csc[(a_.)*(x_)]^(n_.)*((b_.)*(x_))^(m_.))/(Cos[(a_.)*(x_)]*(d_.)*(x_)
+ (c_.)*Sin[(a_.)*(x_)])^2, x_Symbol] :> Simp[b*(b*x)^(m - 1)*(Csc[a*x]^(n
+ 1)/(a*d*(c*Sin[a*x] + d*x*Cos[a*x]))), x] + Simp[b^2*((n + 1)/d^2)   Int[
(b*x)^(m - 2)*Csc[a*x]^(n + 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && E
qQ[a*c + d, 0] && EqQ[m, n + 2]
```

3.593. $\int \frac{x^4 \csc^2(ax)}{(ax\cos(ax)-\sin(ax))^2} \, dx$

3.593.4 Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.35

method	result
risch	$-\frac{2i\left(2ia^2x^2 e^{2iax}+2a^3x^3-2ia^2x^2-ax\,e^{2iax}+ie^{2iax}+ax-i\right)}{\left(e^{2iax}-1\right)\left(ax\,e^{2iax}+ie^{2iax}+ax-i\right)a^5}-\frac{4ix^2}{a^3}+\frac{4x\ln\left(1-e^{iax}\right)}{a^4}-\frac{4i\,\mathrm{polylog}\left(2,e^{iax}\right)}{a^5}+\frac{4x\ln\left(e^{iax}+1\right)}{a^4}$

input
```
int(x^4*csc(a*x)^2/(a*x*cos(a*x)-sin(a*x))^2,x,method=_RETURNVERBOSE)
```

output
```
-2*I*(2*I*a^2*x^2*exp(2*I*a*x)+2*a^3*x^3-2*I*a^2*x^2-a*x*exp(2*I*a*x)+I*ex
p(2*I*a*x)+a*x-I)/(exp(2*I*a*x)-1)/(a*x*exp(2*I*a*x)+I*exp(2*I*a*x)+a*x-I)
/a^5-4*I/a^3*x^2+4/a^4*x*ln(1-exp(I*a*x))-4*I/a^5*polylog(2,exp(I*a*x))+4/
a^4*x*ln(exp(I*a*x)+1)-4*I/a^5*polylog(2,-exp(I*a*x))
```

3.593.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 410 vs. 2(116) = 232.

Time = 0.29 (sec) , antiderivative size = 410, normalized size of antiderivative = 3.23

$$\int \frac{x^4\csc^2(ax)}{(ax\cos(ax)-\sin(ax))^2}\,dx = \frac{a^3x^3-\left(2\,a^3x^3+ax\right)\cos\left(ax\right)^2+\left(2\,a^2x^2+1\right)\cos\left(ax\right)\sin\left(ax\right)+ax-2\left(i\,ax\cos\left(ax\right)\sin\left(ax\right)+i\,\cos\left(ax\right)\right.}{}$$

input
```
integrate(x^4*csc(a*x)^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="fricas")
```

output
```
(a^3*x^3 - (2*a^3*x^3 + a*x)*cos(a*x)^2 + (2*a^2*x^2 + 1)*cos(a*x)*sin(a*x
) + a*x - 2*(I*a*x*cos(a*x)*sin(a*x) + I*cos(a*x)^2 - I)*dilog(cos(a*x) +
I*sin(a*x)) - 2*(-I*a*x*cos(a*x)*sin(a*x) - I*cos(a*x)^2 + I)*dilog(cos(a*
x) - I*sin(a*x)) - 2*(-I*a*x*cos(a*x)*sin(a*x) - I*cos(a*x)^2 + I)*dilog(-
cos(a*x) + I*sin(a*x)) - 2*(I*a*x*cos(a*x)*sin(a*x) + I*cos(a*x)^2 - I)*di
log(-cos(a*x) - I*sin(a*x)) + 2*(a^2*x^2*cos(a*x)*sin(a*x) + a*x*cos(a*x)^
2 - a*x)*log(cos(a*x) + I*sin(a*x) + 1) + 2*(a^2*x^2*cos(a*x)*sin(a*x) + a
*x*cos(a*x)^2 - a*x)*log(cos(a*x) - I*sin(a*x) + 1) + 2*(a^2*x^2*cos(a*x)*
sin(a*x) + a*x*cos(a*x)^2 - a*x)*log(-cos(a*x) + I*sin(a*x) + 1) + 2*(a^2*
x^2*cos(a*x)*sin(a*x) + a*x*cos(a*x)^2 - a*x)*log(-cos(a*x) - I*sin(a*x) +
 1))/(a^6*x*cos(a*x)*sin(a*x) + a^5*cos(a*x)^2 - a^5)
```

3.593.6 Sympy [F]

$$\int \frac{x^4 \csc^2(ax)}{(ax\cos(ax)-\sin(ax))^2}\,dx = \int \frac{x^4 \csc^2(ax)}{(ax\cos(ax)-\sin(ax))^2}\,dx$$

input
```
integrate(x**4*csc(a*x)**2/(a*x*cos(a*x)-sin(a*x))**2,x)
```

output
```
Integral(x**4*csc(a*x)**2/(a*x*cos(a*x) - sin(a*x))**2, x)
```

3.593.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 594 vs. 2(116) = 232.

Time = 0.26 (sec) , antiderivative size = 594, normalized size of antiderivative = 4.68

$$\int \frac{x^4 \csc^2(ax)}{(ax\cos(ax)-\sin(ax))^2}\,dx = -\frac{2\left(ax+2\left(a^2x^2+2i\,ax\cos(2\,ax)-2\,ax\sin(2\,ax)-i\,ax-(a^2x^2+i\,ax)\cos(4\,ax)+(-i\,a^2x^2+ax)\sin\right.}{}$$

input
```
integrate(x^4*csc(a*x)^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="maxima")
```

output
```
-2*(a*x + 2*(a^2*x^2 + 2*I*a*x*cos(2*a*x) - 2*a*x*sin(2*a*x) - I*a*x - (a^
2*x^2 + I*a*x)*cos(4*a*x) + (-I*a^2*x^2 + a*x)*sin(4*a*x))*arctan2(sin(a*x
), cos(a*x) + 1) - 2*(a^2*x^2 + 2*I*a*x*cos(2*a*x) - 2*a*x*sin(2*a*x) - I*
a*x - (a^2*x^2 + I*a*x)*cos(4*a*x) - (I*a^2*x^2 - a*x)*sin(4*a*x))*arctan2
(sin(a*x), -cos(a*x) + 1) + 2*(a^3*x^3 + I*a^2*x^2)*cos(4*a*x) + (-2*I*a^2
*x^2 - a*x + I)*cos(2*a*x) - 2*(a*x - (a*x + I)*cos(4*a*x) - (I*a*x - 1)*s
in(4*a*x) + 2*I*cos(2*a*x) - 2*sin(2*a*x) - I)*dilog(-e^(I*a*x)) - 2*(a*x
- (a*x + I)*cos(4*a*x) - (I*a*x - 1)*sin(4*a*x) + 2*I*cos(2*a*x) - 2*sin(2
*a*x) - I)*dilog(e^(I*a*x)) + (-I*a^2*x^2 + 2*a*x*cos(2*a*x) + 2*I*a*x*sin
(2*a*x) - a*x + (I*a^2*x^2 - a*x)*cos(4*a*x) - (a^2*x^2 + I*a*x)*sin(4*a*x
))*log(cos(a*x)^2 + sin(a*x)^2 + 2*cos(a*x) + 1) + (-I*a^2*x^2 + 2*a*x*cos
(2*a*x) + 2*I*a*x*sin(2*a*x) - a*x + (I*a^2*x^2 - a*x)*cos(4*a*x) - (a^2*x
^2 + I*a*x)*sin(4*a*x))*log(cos(a*x)^2 + sin(a*x)^2 - 2*cos(a*x) + 1) + 2*
(I*a^3*x^3 - a^2*x^2)*sin(4*a*x) + (2*a^2*x^2 - I*a*x - 1)*sin(2*a*x) - I)
/((I*a*x + (-I*a*x + 1)*cos(4*a*x) + (a*x + I)*sin(4*a*x) - 2*cos(2*a*x) -
 2*I*sin(2*a*x) + 1)*a^5)
```

3.593.8 Giac [F]

$$\int \frac{x^4 \csc^2(ax)}{(ax\cos(ax)-\sin(ax))^2}\,dx = \int \frac{x^4 \csc{(ax)}^2}{(ax\cos{(ax)}-\sin{(ax)})^2}\,dx$$

input
```
integrate(x^4*csc(a*x)^2/(a*x*cos(a*x)-sin(a*x))^2,x, algorithm="giac")
```

output
```
integrate(x^4*csc(a*x)^2/(a*x*cos(a*x) - sin(a*x))^2, x)
```

3.593.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \csc^2(ax)}{(ax\cos(ax)-\sin(ax))^2}\,dx = \int \frac{x^4}{\sin{(a\,x)}^2\,(\sin{(a\,x)}-a\,x\,\cos{(a\,x)})^2}\,dx$$

input
```
int(x^4/(sin(a*x)^2*(sin(a*x) - a*x*cos(a*x))^2),x)
```

output
```
int(x^4/(sin(a*x)^2*(sin(a*x) - a*x*cos(a*x))^2), x)
```

3.594 $\int \frac{\cos^6(ax)}{x^4(\cos(ax)+ax\sin(ax))^2}\,dx$

3.594.1 Optimal result

Integrand size = 24, antiderivative size = 176

$$\int \frac{\cos^6(ax)}{x^4(\cos(ax)+ax\sin(ax))^2}\,dx = \frac{a^2}{x}+\frac{\cos^2(ax)}{x^3}-\frac{10a^2\cos^2(ax)}{x}+\frac{\cos^4(ax)}{a^2x^5}-\frac{4\cos^4(ax)}{3x^3}+\frac{32a^2\cos^4(ax)}{3x}-\frac{a\cos(ax)\sin(ax)}{x^2}-\frac{\cos^3(ax)\sin(ax)}{ax^4}+\frac{8a\cos^3(ax)\sin(ax)}{3x^2}-\frac{\cos^5(ax)}{a^2x^5(\cos(ax)+ax\sin(ax))}+\frac{2}{3}a^3\text{Si}(2ax)+\frac{16}{3}a^3\text{Si}(4ax)$$

output
```
a^2/x+cos(a*x)^2/x^3-10*a^2*cos(a*x)^2/x+cos(a*x)^4/a^2/x^5-4/3*cos(a*x)^4
/x^3+32/3*a^2*cos(a*x)^4/x+2/3*a^3*Si(2*a*x)+16/3*a^3*Si(4*a*x)-a*cos(a*x)
*sin(a*x)/x^2-cos(a*x)^3*sin(a*x)/a/x^4+8/3*a*cos(a*x)^3*sin(a*x)/x^2-cos(
a*x)^5/a^2/x^5/(cos(a*x)+a*x*sin(a*x))
```

3.594.2 Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.10

$$\int \frac{\cos^6(ax)}{x^4(\cos(ax)+ax\sin(ax))^2}\,dx = \frac{-10\cos(ax)+12a^2x^2\cos(ax)-5\cos(3ax)+44a^2x^2\cos(3ax)-\cos(5ax)+24a^2x^2\cos(5ax)+8ax\sin(a}{}$$

input
```
Integrate[Cos[a*x]^6/(x^4*(Cos[a*x] + a*x*Sin[a*x])^2),x]
```

output
```
(-10*Cos[a*x] + 12*a^2*x^2*Cos[a*x] - 5*Cos[3*a*x] + 44*a^2*x^2*Cos[3*a*x]
 - Cos[5*a*x] + 24*a^2*x^2*Cos[5*a*x] + 8*a*x*Sin[a*x] - 8*a^3*x^3*Sin[a*x
] + 12*a*x*Sin[3*a*x] - 24*a^3*x^3*Sin[3*a*x] + 4*a*x*Sin[5*a*x] + 32*a^3*
x^3*Sin[5*a*x] + 32*a^3*x^3*(Cos[a*x] + a*x*Sin[a*x])*SinIntegral[2*a*x] +
 256*a^3*x^3*(Cos[a*x] + a*x*Sin[a*x])*SinIntegral[4*a*x])/(48*x^3*(Cos[a*
x] + a*x*Sin[a*x]))
```

3.594.3 Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.39, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5110, 3042, 3795, 3042, 3795, 15, 3042, 3794, 27, 2009, 3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^6(ax)}{x^4(ax\sin(ax)+\cos(ax))^2}dx$$

$$\Big\downarrow \text{5110}$$

$$-\frac{5\int \frac{\cos^4(ax)}{x^6}dx}{a^2}-\frac{\cos^5(ax)}{a^2x^5(ax\sin(ax)+\cos(ax))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{5\int \frac{\sin\left(ax+\frac{\pi}{2}\right)^4}{x^6}dx}{a^2}-\frac{\cos^5(ax)}{a^2x^5(ax\sin(ax)+\cos(ax))}$$

$$\Big\downarrow \text{3795}$$

$$-\frac{5\left(-\frac{4}{5}a^2\int \frac{\cos^4(ax)}{x^4}dx+\frac{3}{5}a^2\int \frac{\cos^2(ax)}{x^4}dx-\frac{\cos^4(ax)}{5x^5}+\frac{a\sin(ax)\cos^3(ax)}{5x^4}\right)}{a^2}-\frac{\cos^5(ax)}{a^2x^5(ax\sin(ax)+\cos(ax))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{5\left(\frac{3}{5}a^2\int \frac{\sin\left(ax+\frac{\pi}{2}\right)^2}{x^4}dx-\frac{4}{5}a^2\int \frac{\sin\left(ax+\frac{\pi}{2}\right)^4}{x^4}dx-\frac{\cos^4(ax)}{5x^5}+\frac{a\sin(ax)\cos^3(ax)}{5x^4}\right)}{a^2}-\frac{\cos^5(ax)}{a^2x^5(ax\sin(ax)+\cos(ax))}$$

$$\Big\downarrow \text{3795}$$

$$-\frac{\frac{5\left(\frac{3}{5}a^2\left(\frac{1}{3}a^2\int\frac{1}{x^2}dx-\frac{2}{3}a^2\int\frac{\cos^2(ax)}{x^2}dx-\frac{\cos^2(ax)}{3x^3}+\frac{a\sin(ax)\cos(ax)}{3x^2}\right)-\frac{4}{5}a^2\left(-\frac{8}{3}a^2\int\frac{\cos^4(ax)}{x^2}dx+2a^2\int\frac{\cos^2(ax)}{x^2}dx-\right.}{a^2}}{\frac{\cos^5(ax)}{a^2x^5(ax\sin(ax)+\cos(ax))}}$$

$$\Big\downarrow \text{15}$$

$$-\frac{\frac{5\left(\frac{3}{5}a^2\left(-\frac{2}{3}a^2\int\frac{\cos^2(ax)}{x^2}dx-\frac{a^2}{3x}-\frac{\cos^2(ax)}{3x^3}+\frac{a\sin(ax)\cos(ax)}{3x^2}\right)-\frac{4}{5}a^2\left(-\frac{8}{3}a^2\int\frac{\cos^4(ax)}{x^2}dx+2a^2\int\frac{\cos^2(ax)}{x^2}dx-\frac{\cos^4(a}{3x^3}\right.}{a^2}}{\frac{\cos^5(ax)}{a^2x^5(ax\sin(ax)+\cos(ax))}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{5\left(\frac{3}{5}a^2\left(-\frac{2}{3}a^2\int\frac{\sin\left(ax+\frac{\pi}{2}\right)^2}{x^2}dx-\frac{a^2}{3x}-\frac{\cos^2(ax)}{3x^3}+\frac{a\sin(ax)\cos(ax)}{3x^2}\right)-\frac{4}{5}a^2\left(2a^2\int\frac{\sin\left(ax+\frac{\pi}{2}\right)^2}{x^2}dx-\frac{8}{3}a^2\int\frac{\sin\left(ax+\frac{\pi}{2}\right)^4}{x^2}dx\right.}{a^2}}{\frac{\cos^5(ax)}{a^2x^5(ax\sin(ax)+\cos(ax))}}$$

$$\Big\downarrow \text{3794}$$

$$-\frac{5\left(\frac{3}{5}a^2\left(-\frac{2}{3}a^2\left(2a\int-\frac{\sin(2ax)}{2x}dx-\frac{\cos^2(ax)}{x}\right)-\frac{a^2}{3x}-\frac{\cos^2(ax)}{3x^3}+\frac{a\sin(ax)\cos(ax)}{3x^2}\right)-\frac{4}{5}a^2\left(-\frac{8}{3}a^2\left(4a\int\left(-\frac{\sin(2ax)}{4x}-\frac{\sin(}{8}\right.\right.\right.}{\frac{\cos^5(ax)}{a^2x^5(ax\sin(ax)+\cos(ax))}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{5\left(\frac{3}{5}a^2\left(-\frac{2}{3}a^2\left(-a\int\frac{\sin(2ax)}{x}dx-\frac{\cos^2(ax)}{x}\right)-\frac{a^2}{3x}-\frac{\cos^2(ax)}{3x^3}+\frac{a\sin(ax)\cos(ax)}{3x^2}\right)-\frac{4}{5}a^2\left(-\frac{8}{3}a^2\left(4a\int\left(-\frac{\sin(2ax)}{4x}-\frac{\sin(4}{8x}\right.\right.\right.}{\frac{\cos^5(ax)}{a^2x^5(ax\sin(ax)+\cos(ax))}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{5\left(-\frac{4}{5}a^2\left(2a^2\left(-a\int\frac{\sin(2ax)}{x}dx-\frac{\cos^2(ax)}{x}\right)-\frac{8}{3}a^2\left(4a\left(-\frac{1}{4}\text{Si}(2ax)-\frac{\text{Si}(4ax)}{8}\right)-\frac{\cos^4(ax)}{x}\right)-\frac{\cos^4(ax)}{3x^3}+\frac{2a\sin(ax)\cos^3(}{3x^2}\right.}{a^2}}{\frac{\cos^5(ax)}{a^2x^5(ax\sin(ax)+\cos(ax))}}$$

$$\Big\downarrow \text{3042}$$

3.594. $\int\frac{\cos^6(ax)}{x^4(\cos(ax)+ax\sin(ax))^2}\,dx$

$$-\frac{5\left(-\frac{4}{5}a^2\left(2a^2\left(-a\int\frac{\sin(2ax)}{x}dx-\frac{\cos^2(ax)}{x}\right)-\frac{8}{3}a^2\left(4a\left(-\frac{1}{4}\text{Si}(2ax)-\frac{\text{Si}(4ax)}{8}\right)-\frac{\cos^4(ax)}{x}\right)-\frac{\cos^4(ax)}{3x^3}+\frac{2a\sin(ax)\cos^3(\ldots}{3x^2}\right.}{a^2}$$
$$\frac{\cos^5(ax)}{a^2x^5(ax\sin(ax)+\cos(ax))}$$

$\downarrow$ 3780

$$-\frac{5\left(\frac{3}{5}a^2\left(-\frac{2}{3}a^2\left(-a\text{Si}(2ax)-\frac{\cos^2(ax)}{x}\right)-\frac{a^2}{3x}-\frac{\cos^2(ax)}{3x^3}+\frac{a\sin(ax)\cos(ax)}{3x^2}\right)-\frac{4}{5}a^2\left(-\frac{8}{3}a^2\left(4a\left(-\frac{1}{4}\text{Si}(2ax)-\frac{\text{Si}(4ax)}{8}\right)\ldots\right.\right.}{a^2}$$
$$\frac{\cos^5(ax)}{a^2x^5(ax\sin(ax)+\cos(ax))}$$

input
```
Int[Cos[a*x]^6/(x^4*(Cos[a*x] + a*x*Sin[a*x])^2),x]
```

output
```
-(Cos[a*x]^5/(a^2*x^5*(Cos[a*x] + a*x*Sin[a*x]))) - (5*(-1/5*Cos[a*x]^4/x^
5 + (a*Cos[a*x]^3*Sin[a*x])/(5*x^4) + (3*a^2*(-1/3*a^2/x - Cos[a*x]^2/(3*x
^3) + (a*Cos[a*x]*Sin[a*x])/(3*x^2) - (2*a^2*(-(Cos[a*x]^2/x) - a*SinInteg
ral[2*a*x]))/3))/5 - (4*a^2*(-1/3*Cos[a*x]^4/x^3 + (2*a*Cos[a*x]^3*Sin[a*x
])/(3*x^2) + 2*a^2*(-(Cos[a*x]^2/x) - a*SinIntegral[2*a*x]) - (8*a^2*(-(Co
s[a*x]^4/x) + 4*a*(-1/4*SinIntegral[2*a*x] - SinIntegral[4*a*x]/8)))/3))/5
))/a^2
```

3.594.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.594. $\int\frac{\cos^6(ax)}{x^4(\cos(ax)+ax\sin(ax))^2}\,dx$

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

rule 5110
```
Int[(Cos[(a_.)*(x_)]^(n_)*((b_.)*(x_))^(m_))/(Cos[(a_.)*(x_)]*(c_.) + (d_.)
*(x_)*Sin[(a_.)*(x_)])^2, x_Symbol] :> Simp[(-b)*(b*x)^(m - 1)*(Cos[a*x]^(n
 - 1)/(a*d*(c*Cos[a*x] + d*x*Sin[a*x]))), x] - Simp[b^2*((n - 1)/d^2)   Int
[(b*x)^(m - 2)*Cos[a*x]^(n - 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&
EqQ[a*c - d, 0] && EqQ[m, 2 - n]
```

3.594.4 Maple [F(-1)]

Timed out.

$$\int \frac{\cos{(ax)}^6}{x^4 (\cos{(ax)} + ax \sin{(ax)})^2} dx$$

input
```
int(cos(a*x)^6/x^4/(cos(a*x)+a*x*sin(a*x))^2,x)
```

output
```
int(cos(a*x)^6/x^4/(cos(a*x)+a*x*sin(a*x))^2,x)
```

3.594.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.92

$$\int \frac{\cos^6(ax)}{x^4(\cos(ax)+ax\sin(ax))^2}\,dx =$$
$$-\frac{19\,a^2x^2\cos{(ax)}^3-(24\,a^2x^2-1)\cos{(ax)}^5-2\,(8\,a^3x^3\,\mathrm{Si}\,(4\,ax)+a^3x^3\,\mathrm{Si}\,(2\,ax))\cos{(ax)}-(16\,a^4x^4\,\mathrm{Si}\,(}{3\,(ax^4\sin{(ax)}+x^3\,\mathrm{co}}$$

input
```
integrate(cos(a*x)^6/x^4/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="fricas")
```

output
```
-1/3*(19*a^2*x^2*cos(a*x)^3 - (24*a^2*x^2 - 1)*cos(a*x)^5 - 2*(8*a^3*x^3*s
in_integral(4*a*x) + a^3*x^3*sin_integral(2*a*x))*cos(a*x) - (16*a^4*x^4*s
in_integral(4*a*x) + 2*a^4*x^4*sin_integral(2*a*x) - 30*a^3*x^3*cos(a*x)^2
 + 3*a^3*x^3 + 4*(8*a^3*x^3 + a*x)*cos(a*x)^4)*sin(a*x))/(a*x^4*sin(a*x) +
 x^3*cos(a*x))
```

3.594.6 Sympy [F]

$$\int \frac{\cos^6(ax)}{x^4(\cos(ax)+ax\sin(ax))^2}\,dx = \int \frac{\cos^6{(ax)}}{x^4\,(ax\sin{(ax)}+\cos{(ax)})^2}\,dx$$

input
```
integrate(cos(a*x)**6/x**4/(cos(a*x)+a*x*sin(a*x))**2,x)
```

output
```
Integral(cos(a*x)**6/(x**4*(a*x*sin(a*x) + cos(a*x))**2), x)
```

3.594.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^6(ax)}{x^4(\cos(ax)+ax\sin(ax))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(a*x)^6/x^4/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.594.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.71 (sec) , antiderivative size = 7279, normalized size of antiderivative = 41.36

$$\int \frac{\cos^6(ax)}{x^4(\cos(ax) + ax\sin(ax))^2}\, dx = \text{Too large to display}$$

input
```
integrate(cos(a*x)^6/x^4/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="giac")
```

output
```
1/12*(64*a^8*x^8*imag_part(cos_integral(4*a*x))*tan(2*a*x)^2*tan(a*x)^2*ta
n(1/2*a*x) + 8*a^8*x^8*imag_part(cos_integral(2*a*x))*tan(2*a*x)^2*tan(a*x
)^2*tan(1/2*a*x) - 8*a^8*x^8*imag_part(cos_integral(-2*a*x))*tan(2*a*x)^2*
tan(a*x)^2*tan(1/2*a*x) - 64*a^8*x^8*imag_part(cos_integral(-4*a*x))*tan(2
*a*x)^2*tan(a*x)^2*tan(1/2*a*x) + 128*a^8*x^8*sin_integral(4*a*x)*tan(2*a*
x)^2*tan(a*x)^2*tan(1/2*a*x) + 16*a^8*x^8*sin_integral(2*a*x)*tan(2*a*x)^2
*tan(a*x)^2*tan(1/2*a*x) - 32*a^7*x^7*imag_part(cos_integral(4*a*x))*tan(2
*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 - 4*a^7*x^7*imag_part(cos_integral(2*a*x
))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 + 4*a^7*x^7*imag_part(cos_integr
al(-2*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 + 32*a^7*x^7*imag_part(
cos_integral(-4*a*x))*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 - 64*a^7*x^7*
sin_integral(4*a*x)*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 - 8*a^7*x^7*sin
_integral(2*a*x)*tan(2*a*x)^2*tan(a*x)^2*tan(1/2*a*x)^2 + 64*a^8*x^8*imag_
part(cos_integral(4*a*x))*tan(2*a*x)^2*tan(1/2*a*x) + 8*a^8*x^8*imag_part(
cos_integral(2*a*x))*tan(2*a*x)^2*tan(1/2*a*x) - 8*a^8*x^8*imag_part(cos_i
ntegral(-2*a*x))*tan(2*a*x)^2*tan(1/2*a*x) - 64*a^8*x^8*imag_part(cos_inte
gral(-4*a*x))*tan(2*a*x)^2*tan(1/2*a*x) + 128*a^8*x^8*sin_integral(4*a*x)*
tan(2*a*x)^2*tan(1/2*a*x) + 16*a^8*x^8*sin_integral(2*a*x)*tan(2*a*x)^2*ta
n(1/2*a*x) + 64*a^8*x^8*imag_part(cos_integral(4*a*x))*tan(a*x)^2*tan(1/2*
a*x) + 8*a^8*x^8*imag_part(cos_integral(2*a*x))*tan(a*x)^2*tan(1/2*a*x)...
```

3.594.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^6(ax)}{x^4(\cos(ax) + ax\sin(ax))^2}\, dx = \int \frac{\cos\left(a\,x\right)^6}{x^4\,\left(\cos\left(a\,x\right)+a\,x\,\sin\left(a\,x\right)\right)^2}\, dx$$

input
```
int(cos(a*x)^6/(x^4*(cos(a*x) + a*x*sin(a*x))^2),x)
```

output
```
int(cos(a*x)^6/(x^4*(cos(a*x) + a*x*sin(a*x))^2), x)
```

3.594. $\int \frac{\cos^6(ax)}{x^4(\cos(ax)+ax\sin(ax))^2}\,dx$

3.595 $\int \frac{\cos^5(ax)}{x^3(\cos(ax)+ax\sin(ax))^2}\,dx$

3.595.1 Optimal result

Integrand size = 24, antiderivative size = 132

$$\int \frac{\cos^5(ax)}{x^3(\cos(ax)+ax\sin(ax))^2}\,dx = \frac{\cos(ax)}{x^2}+\frac{\cos^3(ax)}{a^2x^4}-\frac{3\cos^3(ax)}{2x^2}-\frac{1}{8}a^2\,\text{CosIntegral}(ax) -\frac{27}{8}a^2\,\text{CosIntegral}(3ax)-\frac{a\sin(ax)}{x}-\frac{\cos^2(ax)\sin(ax)}{ax^3} +\frac{9a\cos^2(ax)\sin(ax)}{2x}-\frac{\cos^4(ax)}{a^2x^4(\cos(ax)+ax\sin(ax))}$$

output
```
-1/8*a^2*Ci(a*x)-27/8*a^2*Ci(3*a*x)+cos(a*x)/x^2+cos(a*x)^3/a^2/x^4-3/2*co
s(a*x)^3/x^2-a*sin(a*x)/x-cos(a*x)^2*sin(a*x)/a/x^3+9/2*a*cos(a*x)^2*sin(a
*x)/x-cos(a*x)^4/a^2/x^4/(cos(a*x)+a*x*sin(a*x))
```

3.595.2 Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.03

$$\int \frac{\cos^5(ax)}{x^3(\cos(ax)+ax\sin(ax))^2}\,dx = -\frac{3-a^2x^2+4\cos(2ax)-8a^2x^2\cos(2ax)+\cos(4ax)+9a^2x^2\cos(4ax)+2a^2x^2\,\text{CosIntegral}(ax)(\cos(ax)}{16x^2(\cos(ax)+}$$

input
```
Integrate[Cos[a*x]^5/(x^3*(Cos[a*x] + a*x*Sin[a*x])^2),x]
```

output
```
-1/16*(3 - a^2*x^2 + 4*Cos[2*a*x] - 8*a^2*x^2*Cos[2*a*x] + Cos[4*a*x] + 9*
a^2*x^2*Cos[4*a*x] + 2*a^2*x^2*CosIntegral[a*x]*(Cos[a*x] + a*x*Sin[a*x])
+ 54*a^2*x^2*CosIntegral[3*a*x]*(Cos[a*x] + a*x*Sin[a*x]) - 12*a*x*Sin[2*a
*x] - 6*a*x*Sin[4*a*x])/(x^2*(Cos[a*x] + a*x*Sin[a*x]))
```

3.595.3 Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.36, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5110, 3042, 3795, 3042, 3778, 25, 3042, 3778, 3042, 3783, 3795, 3042, 3783, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^5(ax)}{x^3(ax\sin(ax)+\cos(ax))^2}dx$$

$$\Big\downarrow \text{5110}$$

$$-\frac{4\int \frac{\cos^3(ax)}{x^5}dx}{a^2}-\frac{\cos^4(ax)}{a^2x^4(ax\sin(ax)+\cos(ax))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{4\int \frac{\sin\left(ax+\frac{\pi}{2}\right)^3}{x^5}dx}{a^2}-\frac{\cos^4(ax)}{a^2x^4(ax\sin(ax)+\cos(ax))}$$

$$\Big\downarrow \text{3795}$$

$$-\frac{4\left(-\frac{3}{4}a^2\int \frac{\cos^3(ax)}{x^3}dx+\frac{1}{2}a^2\int \frac{\cos(ax)}{x^3}dx-\frac{\cos^3(ax)}{4x^4}+\frac{a\sin(ax)\cos^2(ax)}{4x^3}\right)}{a^2}-\frac{\cos^4(ax)}{a^2x^4(ax\sin(ax)+\cos(ax))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{4\left(\frac{1}{2}a^2\int \frac{\sin\left(ax+\frac{\pi}{2}\right)}{x^3}dx-\frac{3}{4}a^2\int \frac{\sin\left(ax+\frac{\pi}{2}\right)^3}{x^3}dx-\frac{\cos^3(ax)}{4x^4}+\frac{a\sin(ax)\cos^2(ax)}{4x^3}\right)}{a^2}-\frac{\cos^4(ax)}{a^2x^4(ax\sin(ax)+\cos(ax))}$$

$$\Big\downarrow \text{3778}$$

$$-\frac{4\left(-\frac{3}{4}a^2\int\frac{\sin\left(ax+\frac{\pi}{2}\right)^3}{x^3}dx+\frac{1}{2}a^2\left(\frac{1}{2}a\int-\frac{\sin(ax)}{x^2}dx-\frac{\cos(ax)}{2x^2}\right)-\frac{\cos^3(ax)}{4x^4}+\frac{a\sin(ax)\cos^2(ax)}{4x^3}\right)}{a^2}-\frac{\cos^4(ax)}{a^2x^4(ax\sin(ax)+\cos(ax))}$$

$$\Big\downarrow \ 25$$

$$-\frac{4\left(-\frac{3}{4}a^2\int\frac{\sin\left(ax+\frac{\pi}{2}\right)^3}{x^3}dx+\frac{1}{2}a^2\left(-\frac{1}{2}a\int\frac{\sin(ax)}{x^2}dx-\frac{\cos(ax)}{2x^2}\right)-\frac{\cos^3(ax)}{4x^4}+\frac{a\sin(ax)\cos^2(ax)}{4x^3}\right)}{a^2}-\frac{\cos^4(ax)}{a^2x^4(ax\sin(ax)+\cos(ax))}$$

$$\Big\downarrow \ 3042$$

$$-\frac{4\left(-\frac{3}{4}a^2\int\frac{\sin\left(ax+\frac{\pi}{2}\right)^3}{x^3}dx+\frac{1}{2}a^2\left(-\frac{1}{2}a\int\frac{\sin(ax)}{x^2}dx-\frac{\cos(ax)}{2x^2}\right)-\frac{\cos^3(ax)}{4x^4}+\frac{a\sin(ax)\cos^2(ax)}{4x^3}\right)}{a^2}-\frac{\cos^4(ax)}{a^2x^4(ax\sin(ax)+\cos(ax))}$$

$$\Big\downarrow \ 3778$$

$$-\frac{4\left(-\frac{3}{4}a^2\int\frac{\sin\left(ax+\frac{\pi}{2}\right)^3}{x^3}dx+\frac{1}{2}a^2\left(-\frac{1}{2}a\left(a\int\frac{\cos(ax)}{x}dx-\frac{\sin(ax)}{x}\right)-\frac{\cos(ax)}{2x^2}\right)-\frac{\cos^3(ax)}{4x^4}+\frac{a\sin(ax)\cos^2(ax)}{4x^3}\right)}{a^2}-\frac{\cos^4(ax)}{a^2x^4(ax\sin(ax)+\cos(ax))}$$

$$\Big\downarrow \ 3042$$

$$-\frac{4\left(-\frac{3}{4}a^2\int\frac{\sin\left(ax+\frac{\pi}{2}\right)^3}{x^3}dx+\frac{1}{2}a^2\left(-\frac{1}{2}a\left(a\int\frac{\sin\left(ax+\frac{\pi}{2}\right)}{x}dx-\frac{\sin(ax)}{x}\right)-\frac{\cos(ax)}{2x^2}\right)-\frac{\cos^3(ax)}{4x^4}+\frac{a\sin(ax)\cos^2(ax)}{4x^3}\right)}{a^2}-\frac{\cos^4(ax)}{a^2x^4(ax\sin(ax)+\cos(ax))}$$

$$\Big\downarrow \ 3783$$

$$-\frac{4\left(-\frac{3}{4}a^2\int\frac{\sin\left(ax+\frac{\pi}{2}\right)^3}{x^3}dx+\frac{1}{2}a^2\left(-\frac{1}{2}a\left(a\,\text{CosIntegral}(ax)-\frac{\sin(ax)}{x}\right)-\frac{\cos(ax)}{2x^2}\right)-\frac{\cos^3(ax)}{4x^4}+\frac{a\sin(ax)\cos^2(ax)}{4x^3}\right)}{a^2}-\frac{\cos^4(ax)}{a^2x^4(ax\sin(ax)+\cos(ax))}$$

$$\Big\downarrow \ 3795$$

3.595. $\int\frac{\cos^5(ax)}{x^3(\cos(ax)+ax\sin(ax))^2}\,dx$

$$-\frac{4\left(-\frac{3}{4}a^2\left(-\frac{9}{2}a^2\int\frac{\cos^3(ax)}{x}dx+3a^2\int\frac{\cos(ax)}{x}dx-\frac{\cos^3(ax)}{2x^2}+\frac{3a\sin(ax)\cos^2(ax)}{2x}\right)+\frac{1}{2}a^2\left(-\frac{1}{2}a\left(a\,\text{CosIntegral}(ax)-\frac{\sin}{}\right.\right.\right.}{a^2}$$

$$\frac{\cos^4(ax)}{a^2x^4(ax\sin(ax)+\cos(ax))}$$

$$\Big\downarrow \; 3042$$

$$-\frac{4\left(-\frac{3}{4}a^2\left(3a^2\int\frac{\sin\left(ax+\frac{\pi}{2}\right)}{x}dx-\frac{9}{2}a^2\int\frac{\sin\left(ax+\frac{\pi}{2}\right)^3}{x}dx-\frac{\cos^3(ax)}{2x^2}+\frac{3a\sin(ax)\cos^2(ax)}{2x}\right)+\frac{1}{2}a^2\left(-\frac{1}{2}a\left(a\,\text{CosIntegral}(ax)\right.\right.\right.}{a^2}$$

$$\frac{\cos^4(ax)}{a^2x^4(ax\sin(ax)+\cos(ax))}$$

$$\Big\downarrow \; 3783$$

$$-\frac{4\left(-\frac{3}{4}a^2\left(-\frac{9}{2}a^2\int\frac{\sin\left(ax+\frac{\pi}{2}\right)^3}{x}dx+3a^2\,\text{CosIntegral}(ax)-\frac{\cos^3(ax)}{2x^2}+\frac{3a\sin(ax)\cos^2(ax)}{2x}\right)+\frac{1}{2}a^2\left(-\frac{1}{2}a\left(a\,\text{CosIntegral}\right.\right.\right.}{a^2}$$

$$\frac{\cos^4(ax)}{a^2x^4(ax\sin(ax)+\cos(ax))}$$

$$\Big\downarrow \; 3793$$

$$-\frac{4\left(-\frac{3}{4}a^2\left(-\frac{9}{2}a^2\int\left(\frac{3\cos(ax)}{4x}+\frac{\cos(3ax)}{4x}\right)dx+3a^2\,\text{CosIntegral}(ax)-\frac{\cos^3(ax)}{2x^2}+\frac{3a\sin(ax)\cos^2(ax)}{2x}\right)+\frac{1}{2}a^2\left(-\frac{1}{2}a\left(a\,\text{Co}\right.\right.\right.}{a^2}$$

$$\frac{\cos^4(ax)}{a^2x^4(ax\sin(ax)+\cos(ax))}$$

$$\Big\downarrow \; 2009$$

$$-\frac{4\left(-\frac{3}{4}a^2\left(3a^2\,\text{CosIntegral}(ax)-\frac{9}{2}a^2\left(\frac{3\,\text{CosIntegral}(ax)}{4}+\frac{\text{CosIntegral}(3ax)}{4}\right)-\frac{\cos^3(ax)}{2x^2}+\frac{3a\sin(ax)\cos^2(ax)}{2x}\right)+\frac{1}{2}a^2\left(-\frac{1}{2}\right.\right.}{a^2}$$

$$\frac{\cos^4(ax)}{a^2x^4(ax\sin(ax)+\cos(ax))}$$

input
```
Int[Cos[a*x]^5/(x^3*(Cos[a*x] + a*x*Sin[a*x])^2),x]
```

output
```
-(Cos[a*x]^4/(a^2*x^4*(Cos[a*x] + a*x*Sin[a*x]))) - (4*(-1/4*Cos[a*x]^3/x^
4 + (a*Cos[a*x]^2*Sin[a*x])/(4*x^3) - (3*a^2*(-1/2*Cos[a*x]^3/x^2 + 3*a^2*
CosIntegral[a*x] - (9*a^2*((3*CosIntegral[a*x])/4 + CosIntegral[3*a*x]/4))
/2 + (3*a*Cos[a*x]^2*Sin[a*x])/(2*x)))/4 + (a^2*(-1/2*Cos[a*x]/x^2 - (a*(a
*CosIntegral[a*x] - Sin[a*x]/x))/2))/2))/a^2
```

3.595. $\int\frac{\cos^5(ax)}{x^3(\cos(ax)+ax\sin(ax))^2}\,dx$

3.595.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

rule 5110
```
Int[(Cos[(a_.)*(x_)]^(n_)*((b_.)*(x_))^(m_))/(Cos[(a_.)*(x_)]*(c_.) + (d_.)
*(x_)*Sin[(a_.)*(x_)])^2, x_Symbol] :> Simp[(-b)*(b*x)^(m - 1)*(Cos[a*x]^(n
 - 1)/(a*d*(c*Cos[a*x] + d*x*Sin[a*x]))), x] - Simp[b^2*((n - 1)/d^2)   Int
[(b*x)^(m - 2)*Cos[a*x]^(n - 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&
EqQ[a*c - d, 0] && EqQ[m, 2 - n]
```

3.595. $\int \frac{\cos^5(ax)}{x^3(\cos(ax)+ax\sin(ax))^2} dx$

3.595.4 Maple [F(-1)]

Timed out.

$$\int \frac{\cos(ax)^5}{x^3(\cos(ax) + ax\sin(ax))^2}dx$$

input
```
int(cos(a*x)^5/x^3/(cos(a*x)+a*x*sin(a*x))^2,x)
```

output
```
int(cos(a*x)^5/x^3/(cos(a*x)+a*x*sin(a*x))^2,x)
```

3.595.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.02

$$\int \frac{\cos^5(ax)}{x^3(\cos(ax) + ax\sin(ax))^2}\,dx$$
$$= \frac{44\,a^2x^2\cos(ax)^2 - 4\,(9\,a^2x^2 + 1)\cos(ax)^4 - 8\,a^2x^2 - (27\,a^2x^2\,\mathrm{Ci}(3\,ax) + a^2x^2\,\mathrm{Ci}(ax))\cos(ax) - (27\,a^3}{8\,(ax^3\sin(ax) + x^2\cos(ax))}$$

input
```
integrate(cos(a*x)^5/x^3/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="fricas")
```

output
```
1/8*(44*a^2*x^2*cos(a*x)^2 - 4*(9*a^2*x^2 + 1)*cos(a*x)^4 - 8*a^2*x^2 - (2
7*a^2*x^2*cos_integral(3*a*x) + a^2*x^2*cos_integral(a*x))*cos(a*x) - (27*
a^3*x^3*cos_integral(3*a*x) + a^3*x^3*cos_integral(a*x) - 24*a*x*cos(a*x)^
3)*sin(a*x))/(a*x^3*sin(a*x) + x^2*cos(a*x))
```

3.595.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^5(ax)}{x^3(\cos(ax) + ax\sin(ax))^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(a*x)**5/x**3/(cos(a*x)+a*x*sin(a*x))**2,x)
```

output
```
Timed out
```

3.595.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^5(ax)}{x^3(\cos(ax) + ax\sin(ax))^2} dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(a*x)^5/x^3/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.595.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.48 (sec) , antiderivative size = 3130, normalized size of antiderivative = 23.71

$$\int \frac{\cos^5(ax)}{x^3(\cos(ax) + ax\sin(ax))^2} dx = \text{Too large to display}$$

input
```
integrate(cos(a*x)^5/x^3/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="giac")
```

output
```
-1/16*(54*a^7*x^7*real_part(cos_integral(3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*
x)^3 + 2*a^7*x^7*real_part(cos_integral(a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^
3 + 2*a^7*x^7*real_part(cos_integral(-a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3
+ 54*a^7*x^7*real_part(cos_integral(-3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^3
 - 27*a^6*x^6*real_part(cos_integral(3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^4
 - a^6*x^6*real_part(cos_integral(a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^4 - a^
6*x^6*real_part(cos_integral(-a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^4 - 27*a^6
*x^6*real_part(cos_integral(-3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x)^4 + 54*a^
7*x^7*real_part(cos_integral(3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x) + 2*a^7*x
^7*real_part(cos_integral(a*x))*tan(3/2*a*x)^2*tan(1/2*a*x) + 2*a^7*x^7*re
al_part(cos_integral(-a*x))*tan(3/2*a*x)^2*tan(1/2*a*x) + 54*a^7*x^7*real_
part(cos_integral(-3*a*x))*tan(3/2*a*x)^2*tan(1/2*a*x) + 54*a^7*x^7*real_p
art(cos_integral(3*a*x))*tan(1/2*a*x)^3 + 2*a^7*x^7*real_part(cos_integral
(a*x))*tan(1/2*a*x)^3 + 2*a^7*x^7*real_part(cos_integral(-a*x))*tan(1/2*a*
x)^3 + 54*a^7*x^7*real_part(cos_integral(-3*a*x))*tan(1/2*a*x)^3 - 27*a^6*
x^6*real_part(cos_integral(3*a*x))*tan(1/2*a*x)^4 - a^6*x^6*real_part(cos_
integral(a*x))*tan(1/2*a*x)^4 - a^6*x^6*real_part(cos_integral(-a*x))*tan(
1/2*a*x)^4 - 27*a^6*x^6*real_part(cos_integral(-3*a*x))*tan(1/2*a*x)^4 + 5
4*a^7*x^7*real_part(cos_integral(3*a*x))*tan(1/2*a*x) + 2*a^7*x^7*real_par
t(cos_integral(a*x))*tan(1/2*a*x) + 2*a^7*x^7*real_part(cos_integral(-a...
```

3.595.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^5(ax)}{x^3(\cos(ax)+ax\sin(ax))^2}\,dx=\int \frac{\cos\left(a\,x\right)^5}{x^3\,\left(\cos\left(a\,x\right)+a\,x\,\sin\left(a\,x\right)\right)^2}\,dx$$

input
```
int(cos(a*x)^5/(x^3*(cos(a*x) + a*x*sin(a*x))^2),x)
```

output
```
int(cos(a*x)^5/(x^3*(cos(a*x) + a*x*sin(a*x))^2), x)
```

3.596 $\int \frac{\cos^4(ax)}{x^2(\cos(ax)+ax\sin(ax))^2}\,dx$

3.596.1 Optimal result

Integrand size = 24, antiderivative size = 80

$$\int \frac{\cos^4(ax)}{x^2(\cos(ax)+ax\sin(ax))^2}\,dx = \frac{1}{x}+\frac{\cos^2(ax)}{a^2x^3}-\frac{2\cos^2(ax)}{x}-\frac{\cos(ax)\sin(ax)}{ax^2}-\frac{\cos^3(ax)}{a^2x^3(\cos(ax)+ax\sin(ax))}-2a\text{Si}(2ax)$$

output
```
1/x+cos(a*x)^2/a^2/x^3-2*cos(a*x)^2/x-2*a*Si(2*a*x)-cos(a*x)*sin(a*x)/a/x^
2-cos(a*x)^3/a^2/x^3/(cos(a*x)+a*x*sin(a*x))
```

3.596.2 Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.89

$$\int \frac{\cos^4(ax)}{x^2(\cos(ax)+ax\sin(ax))^2}\,dx = -\frac{3\cos(ax)+\cos(3ax)-2ax\sin(ax)+2ax\sin(3ax)+8ax(\cos(ax)+ax\sin(ax))\text{Si}(2ax)}{4x(\cos(ax)+ax\sin(ax))}$$

input
```
Integrate[Cos[a*x]^4/(x^2*(Cos[a*x] + a*x*Sin[a*x])^2),x]
```

output
```
-1/4*(3*Cos[a*x] + Cos[3*a*x] - 2*a*x*Sin[a*x] + 2*a*x*Sin[3*a*x] + 8*a*x*
(Cos[a*x] + a*x*Sin[a*x])*SinIntegral[2*a*x])/(x*(Cos[a*x] + a*x*Sin[a*x])
)
```

3.596.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.26, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5110, 3042, 3795, 15, 3042, 3794, 27, 3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(ax)}{x^2(ax\sin(ax)+\cos(ax))^2}dx$$

$$\Big\downarrow \text{5110}$$

$$-\frac{3\int \frac{\cos^2(ax)}{x^4}dx}{a^2}-\frac{\cos^3(ax)}{a^2x^3(ax\sin(ax)+\cos(ax))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\int \frac{\sin\left(ax+\frac{\pi}{2}\right)^2}{x^4}dx}{a^2}-\frac{\cos^3(ax)}{a^2x^3(ax\sin(ax)+\cos(ax))}$$

$$\Big\downarrow \text{3795}$$

$$-\frac{3\left(\frac{1}{3}a^2\int \frac{1}{x^2}dx-\frac{2}{3}a^2\int \frac{\cos^2(ax)}{x^2}dx-\frac{\cos^2(ax)}{3x^3}+\frac{a\sin(ax)\cos(ax)}{3x^2}\right)}{a^2}-\frac{\cos^3(ax)}{a^2x^3(ax\sin(ax)+\cos(ax))}$$

$$\Big\downarrow \text{15}$$

$$-\frac{3\left(-\frac{2}{3}a^2\int \frac{\cos^2(ax)}{x^2}dx-\frac{a^2}{3x}-\frac{\cos^2(ax)}{3x^3}+\frac{a\sin(ax)\cos(ax)}{3x^2}\right)}{a^2}-\frac{\cos^3(ax)}{a^2x^3(ax\sin(ax)+\cos(ax))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\left(-\frac{2}{3}a^2\int \frac{\sin\left(ax+\frac{\pi}{2}\right)^2}{x^2}dx-\frac{a^2}{3x}-\frac{\cos^2(ax)}{3x^3}+\frac{a\sin(ax)\cos(ax)}{3x^2}\right)}{a^2}-\frac{\cos^3(ax)}{a^2x^3(ax\sin(ax)+\cos(ax))}$$

$$\Big\downarrow \text{3794}$$

$$-\frac{3\left(-\frac{2}{3}a^2\left(2a\int -\frac{\sin(2ax)}{2x}dx-\frac{\cos^2(ax)}{x}\right)-\frac{a^2}{3x}-\frac{\cos^2(ax)}{3x^3}+\frac{a\sin(ax)\cos(ax)}{3x^2}\right)}{a^2}-\frac{\cos^3(ax)}{a^2x^3(ax\sin(ax)+\cos(ax))}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3\left(-\frac{2}{3}a^2\left(-a\int\frac{\sin(2ax)}{x}dx-\frac{\cos^2(ax)}{x}\right)-\frac{a^2}{3x}-\frac{\cos^2(ax)}{3x^3}+\frac{a\sin(ax)\cos(ax)}{3x^2}\right)}{a^2}-\frac{\cos^3(ax)}{a^2x^3(ax\sin(ax)+\cos(ax))}$$

$$\Big\downarrow \; 3042$$

$$-\frac{3\left(-\frac{2}{3}a^2\left(-a\int\frac{\sin(2ax)}{x}dx-\frac{\cos^2(ax)}{x}\right)-\frac{a^2}{3x}-\frac{\cos^2(ax)}{3x^3}+\frac{a\sin(ax)\cos(ax)}{3x^2}\right)}{a^2}-\frac{\cos^3(ax)}{a^2x^3(ax\sin(ax)+\cos(ax))}$$

$$\Big\downarrow \; 3780$$

$$-\frac{3\left(-\frac{2}{3}a^2\left(-a\text{Si}(2ax)-\frac{\cos^2(ax)}{x}\right)-\frac{a^2}{3x}-\frac{\cos^2(ax)}{3x^3}+\frac{a\sin(ax)\cos(ax)}{3x^2}\right)}{a^2}-\frac{\cos^3(ax)}{a^2x^3(ax\sin(ax)+\cos(ax))}$$

input
```
Int[Cos[a*x]^4/(x^2*(Cos[a*x] + a*x*Sin[a*x])^2),x]
```

output
```
-(Cos[a*x]^3/(a^2*x^3*(Cos[a*x] + a*x*Sin[a*x]))) - (3*(-1/3*a^2/x - Cos[a
*x]^2/(3*x^3) + (a*Cos[a*x]*Sin[a*x])/(3*x^2) - (2*a^2*(-(Cos[a*x]^2/x) -
a*SinIntegral[2*a*x]))/3))/a^2
```

3.596.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

3.596. $\int\frac{\cos^4(ax)}{x^2(\cos(ax)+ax\sin(ax))^2}\,dx$

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

rule 5110
```
Int[(Cos[(a_.)*(x_)]^(n_)*((b_.)*(x_))^(m_))/(Cos[(a_.)*(x_)]*(c_.) + (d_.)
*(x_)*Sin[(a_.)*(x_)])^2, x_Symbol] :> Simp[(-b)*(b*x)^(m - 1)*(Cos[a*x]^(n
 - 1)/(a*d*(c*Cos[a*x] + d*x*Sin[a*x]))), x] - Simp[b^2*((n - 1)/d^2)   Int
[(b*x)^(m - 2)*Cos[a*x]^(n - 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&
EqQ[a*c - d, 0] && EqQ[m, 2 - n]
```

3.596.4 Maple [F(-1)]

Timed out.

$$\int \frac{\cos (ax)^4}{x^2 (\cos (ax) + ax \sin (ax))^2} dx$$

input
```
int(cos(a*x)^4/x^2/(cos(a*x)+a*x*sin(a*x))^2,x)
```

output
```
int(cos(a*x)^4/x^2/(cos(a*x)+a*x*sin(a*x))^2,x)
```

3.596.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.91

$$\int \frac{\cos^4(ax)}{x^2(\cos(ax)+ax\sin(ax))^2}\,dx$$
$$= -\frac{2\,ax\cos\left(ax\right)\operatorname{Si}\left(2\,ax\right)+\cos\left(ax\right)^3+\left(2\,a^2x^2\operatorname{Si}\left(2\,ax\right)+2\,ax\cos\left(ax\right)^2-ax\right)\sin\left(ax\right)}{ax^2\sin\left(ax\right)+x\cos\left(ax\right)}$$

input
```
integrate(cos(a*x)^4/x^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="fricas")
```

output
```
-(2*a*x*cos(a*x)*sin_integral(2*a*x) + cos(a*x)^3 + (2*a^2*x^2*sin_integra
l(2*a*x) + 2*a*x*cos(a*x)^2 - a*x)*sin(a*x))/(a*x^2*sin(a*x) + x*cos(a*x))
```

3.596.6 Sympy [F]

$$\int \frac{\cos^4(ax)}{x^2(\cos(ax)+ax\sin(ax))^2}\,dx = \int \frac{\cos^4\left(ax\right)}{x^2\left(ax\sin\left(ax\right)+\cos\left(ax\right)\right)^2}\,dx$$

input
```
integrate(cos(a*x)**4/x**2/(cos(a*x)+a*x*sin(a*x))**2,x)
```

output
```
Integral(cos(a*x)**4/(x**2*(a*x*sin(a*x) + cos(a*x))**2), x)
```

3.596.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^4(ax)}{x^2(\cos(ax)+ax\sin(ax))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(a*x)^4/x^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.596.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.40 (sec) , antiderivative size = 997, normalized size of antiderivative = 12.46

$$\int \frac{\cos^4(ax)}{x^2(\cos(ax)+ax\sin(ax))^2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(a*x)^4/x^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="giac")
```

output
```
-(2*a^4*x^4*imag_part(cos_integral(2*a*x))*tan(a*x)^2*tan(1/2*a*x) - 2*a^4
*x^4*imag_part(cos_integral(-2*a*x))*tan(a*x)^2*tan(1/2*a*x) + 4*a^4*x^4*s
in_integral(2*a*x)*tan(a*x)^2*tan(1/2*a*x) - a^3*x^3*imag_part(cos_integra
l(2*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 + a^3*x^3*imag_part(cos_integral(-2*a*
x))*tan(a*x)^2*tan(1/2*a*x)^2 - 2*a^3*x^3*sin_integral(2*a*x)*tan(a*x)^2*t
an(1/2*a*x)^2 + 2*a^4*x^4*imag_part(cos_integral(2*a*x))*tan(1/2*a*x) - 2*
a^4*x^4*imag_part(cos_integral(-2*a*x))*tan(1/2*a*x) + 4*a^4*x^4*sin_integ
ral(2*a*x)*tan(1/2*a*x) + a^3*x^3*imag_part(cos_integral(2*a*x))*tan(a*x)^
2 - a^3*x^3*imag_part(cos_integral(-2*a*x))*tan(a*x)^2 + 2*a^3*x^3*sin_int
egral(2*a*x)*tan(a*x)^2 - 2*a^3*x^3*tan(a*x)^2*tan(1/2*a*x) - a^3*x^3*imag
_part(cos_integral(2*a*x))*tan(1/2*a*x)^2 + a^3*x^3*imag_part(cos_integral
(-2*a*x))*tan(1/2*a*x)^2 - 2*a^3*x^3*sin_integral(2*a*x)*tan(1/2*a*x)^2 +
2*a^2*x^2*imag_part(cos_integral(2*a*x))*tan(a*x)^2*tan(1/2*a*x) - 2*a^2*x
^2*imag_part(cos_integral(-2*a*x))*tan(a*x)^2*tan(1/2*a*x) + 4*a^2*x^2*sin
_integral(2*a*x)*tan(a*x)^2*tan(1/2*a*x) + a^2*x^2*tan(a*x)^2*tan(1/2*a*x)
^2 + a^3*x^3*imag_part(cos_integral(2*a*x)) - a^3*x^3*imag_part(cos_integr
al(-2*a*x)) + 2*a^3*x^3*sin_integral(2*a*x) + 2*a^3*x^3*tan(1/2*a*x) - a*x
*imag_part(cos_integral(2*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 + a*x*imag_part(
cos_integral(-2*a*x))*tan(a*x)^2*tan(1/2*a*x)^2 - 2*a*x*sin_integral(2*a*x
)*tan(a*x)^2*tan(1/2*a*x)^2 - a^2*x^2*tan(a*x)^2 + 2*a^2*x^2*imag_part(...
```

3.596.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^4(ax)}{x^2(\cos(ax)+ax\sin(ax))^2}\,dx = \int \frac{{\cos\left(a\,x\right)}^4}{x^2\,{\left(\cos\left(a\,x\right)+a\,x\,\sin\left(a\,x\right)\right)}^2}\,dx$$

input
```
int(cos(a*x)^4/(x^2*(cos(a*x) + a*x*sin(a*x))^2),x)
```

output
```
int(cos(a*x)^4/(x^2*(cos(a*x) + a*x*sin(a*x))^2), x)
```

3.596. $\int \frac{\cos^4(ax)}{x^2(\cos(ax)+ax\sin(ax))^2}\,dx$

3.597 $\int \frac{\cos^3(ax)}{x(\cos(ax)+ax\sin(ax))^2}\,dx$

3.597.1 Optimal result

Integrand size = 24, antiderivative size = 56

$$\int \frac{\cos^3(ax)}{x(\cos(ax)+ax\sin(ax))^2}\,dx = \frac{\cos(ax)}{a^2x^2} + \text{CosIntegral}(ax) - \frac{\sin(ax)}{ax} - \frac{\cos^2(ax)}{a^2x^2(\cos(ax)+ax\sin(ax))}$$

output
```
Ci(a*x)+cos(a*x)/a^2/x^2-sin(a*x)/a/x-cos(a*x)^2/a^2/x^2/(cos(a*x)+a*x*sin
(a*x))
```

3.597.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.86 (sec) , antiderivative size = 181, normalized size of antiderivative = 3.23

$$\int \frac{\cos^3(ax)}{x(\cos(ax)+ax\sin(ax))^2}\,dx = \frac{1}{2}\left(2\,\text{CosIntegral}(ax) - e\,\text{CosIntegral}(i-ax) - e\,\text{CosIntegral}(i+ax) + \frac{i\cos(ax)+e(i+ax)\,\text{ExpIntegralEi}(-1+iax)-\sin(ax)}{i+ax} - \frac{i\cos(ax)+e(i-ax)\,\text{ExpIntegralEi}(-1-iax)+\sin(ax)}{-i+ax} - \frac{2}{(-i+ax)(i+ax)(\cos(ax)+ax\sin(ax))} - ie\text{Si}(i-ax) - ie\text{Si}(i+ax)\right)$$

input
```
Integrate[Cos[a*x]^3/(x*(Cos[a*x] + a*x*Sin[a*x])^2),x]
```

output
```
(2*CosIntegral[a*x] - E*CosIntegral[I - a*x] - E*CosIntegral[I + a*x] + (I
*Cos[a*x] + E*(I + a*x)*ExpIntegralEi[-1 + I*a*x] - Sin[a*x])/(I + a*x) -
(I*Cos[a*x] + E*(I - a*x)*ExpIntegralEi[-1 - I*a*x] + Sin[a*x])/(-I + a*x)
 - 2/((-I + a*x)*(I + a*x)*(Cos[a*x] + a*x*Sin[a*x])) - I*E*SinIntegral[I
- a*x] - I*E*SinIntegral[I + a*x])/2
```

3.597.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.20, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5110, 3042, 3778, 25, 3042, 3778, 3042, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(ax)}{x(ax\sin(ax)+\cos(ax))^2}dx$$

$$\Big\downarrow \text{5110}$$

$$-\frac{2\int \frac{\cos(ax)}{x^3}dx}{a^2}-\frac{\cos^2(ax)}{a^2x^2(ax\sin(ax)+\cos(ax))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\int \frac{\sin\left(ax+\frac{\pi}{2}\right)}{x^3}dx}{a^2}-\frac{\cos^2(ax)}{a^2x^2(ax\sin(ax)+\cos(ax))}$$

$$\Big\downarrow \text{3778}$$

$$-\frac{2\left(\frac{1}{2}a\int -\frac{\sin(ax)}{x^2}dx-\frac{\cos(ax)}{2x^2}\right)}{a^2}-\frac{\cos^2(ax)}{a^2x^2(ax\sin(ax)+\cos(ax))}$$

$$\Big\downarrow \text{25}$$

$$-\frac{2\left(-\frac{1}{2}a\int \frac{\sin(ax)}{x^2}dx-\frac{\cos(ax)}{2x^2}\right)}{a^2}-\frac{\cos^2(ax)}{a^2x^2(ax\sin(ax)+\cos(ax))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\left(-\frac{1}{2}a\int \frac{\sin(ax)}{x^2}dx-\frac{\cos(ax)}{2x^2}\right)}{a^2}-\frac{\cos^2(ax)}{a^2x^2(ax\sin(ax)+\cos(ax))}$$

$$\Big\downarrow \text{3778}$$

$$-\frac{2\left(-\frac{1}{2}a\left(a\int\frac{\cos(ax)}{x}dx-\frac{\sin(ax)}{x}\right)-\frac{\cos(ax)}{2x^2}\right)}{a^2}-\frac{\cos^2(ax)}{a^2x^2(ax\sin(ax)+\cos(ax))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\left(-\frac{1}{2}a\left(a\int\frac{\sin\left(ax+\frac{\pi}{2}\right)}{x}dx-\frac{\sin(ax)}{x}\right)-\frac{\cos(ax)}{2x^2}\right)}{a^2}-\frac{\cos^2(ax)}{a^2x^2(ax\sin(ax)+\cos(ax))}$$

$$\Big\downarrow \text{3783}$$

$$-\frac{2\left(-\frac{1}{2}a\left(a\,\text{CosIntegral}(ax)-\frac{\sin(ax)}{x}\right)-\frac{\cos(ax)}{2x^2}\right)}{a^2}-\frac{\cos^2(ax)}{a^2x^2(ax\sin(ax)+\cos(ax))}$$

input
```
Int[Cos[a*x]^3/(x*(Cos[a*x] + a*x*Sin[a*x])^2),x]
```

output
```
-(Cos[a*x]^2/(a^2*x^2*(Cos[a*x] + a*x*Sin[a*x]))) - (2*(-1/2*Cos[a*x]/x^2
- (a*(a*CosIntegral[a*x] - Sin[a*x]/x))/2))/a^2
```

3.597.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

3.597. $\int \frac{\cos^3(ax)}{x(\cos(ax)+ax\sin(ax))^2}\,dx$

rule 5110
```
Int[(Cos[(a_.)*(x_)]^(n_)*((b_.)*(x_))^(m_))/(Cos[(a_.)*(x_)]*(c_.) + (d_.)
*(x_)*Sin[(a_.)*(x_)])^2, x_Symbol] :> Simp[(-b)*(b*x)^(m - 1)*(Cos[a*x]^(n
 - 1)/(a*d*(c*Cos[a*x] + d*x*Sin[a*x]))), x] - Simp[b^2*((n - 1)/d^2)   Int
[(b*x)^(m - 2)*Cos[a*x]^(n - 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&
EqQ[a*c - d, 0] && EqQ[m, 2 - n]
```

3.597.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.38 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.12

method	result	size
risch	$-\frac{i\,\mathrm{Ei}_1(-iax)ax-\mathrm{Ei}_1(-iax)+\mathrm{e}^{iax}}{2(iax-1)}+\frac{\mathrm{e}^{-iax}}{2iax+2}-\frac{\mathrm{Ei}_1(iax)}{2}-\frac{2i\mathrm{e}^{iax}}{(ax+i)(ax-i)(ax\,\mathrm{e}^{2iax}-ax+i\mathrm{e}^{2iax}+i)}$	119

input
```
int(cos(a*x)^3/x/(cos(a*x)+a*x*sin(a*x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(I*Ei(1,-I*a*x)*a*x-Ei(1,-I*a*x)+exp(I*a*x))/(-1+I*a*x)+1/2*exp(-I*a*
x)/(I*a*x+1)-1/2*Ei(1,I*a*x)-2*I*exp(I*a*x)/(a*x+I)/(a*x-I)/(a*x*exp(2*I*a
*x)-a*x+I*exp(2*I*a*x)+I)
```

3.597.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int \frac{\cos^3(ax)}{x(\cos(ax)+ax\sin(ax))^2}\,dx=\frac{ax\,\mathrm{Ci}\,(ax)\sin\,(ax)+\cos\,(ax)^2+\cos\,(ax)\,\mathrm{Ci}\,(ax)-1}{ax\sin\,(ax)+\cos\,(ax)}$$

input
```
integrate(cos(a*x)^3/x/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="fricas")
```

output
```
(a*x*cos_integral(a*x)*sin(a*x) + cos(a*x)^2 + cos(a*x)*cos_integral(a*x)
- 1)/(a*x*sin(a*x) + cos(a*x))
```

3.597.6 Sympy [F]

$$\int \frac{\cos^3(ax)}{x(\cos(ax) + ax\sin(ax))^2}\,dx = \int \frac{\cos^3(ax)}{x(ax\sin(ax) + \cos(ax))^2}\,dx$$

input
```
integrate(cos(a*x)**3/x/(cos(a*x)+a*x*sin(a*x))**2,x)
```

output
```
Integral(cos(a*x)**3/(x*(a*x*sin(a*x) + cos(a*x))**2), x)
```

3.597.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^3(ax)}{x(\cos(ax) + ax\sin(ax))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(cos(a*x)^3/x/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.597.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.34 (sec) , antiderivative size = 366, normalized size of antiderivative = 6.54

$$\int \frac{\cos^3(ax)}{x(\cos(ax) + ax\sin(ax))^2}\,dx$$
$$= \frac{2a^3x^3\Re(\text{Ci}(ax))\tan\left(\frac{1}{2}ax\right)^3 + 2a^3x^3\Re(\text{Ci}(-ax))\tan\left(\frac{1}{2}ax\right)^3 - a^2x^2\Re(\text{Ci}(ax))\tan\left(\frac{1}{2}ax\right)^4 - a^2x^2\Re(\text{Ci}(\ldots}{}$$

input
```
integrate(cos(a*x)^3/x/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="giac")
```

output
```
1/2*(2*a^3*x^3*real_part(cos_integral(a*x))*tan(1/2*a*x)^3 + 2*a^3*x^3*rea
l_part(cos_integral(-a*x))*tan(1/2*a*x)^3 - a^2*x^2*real_part(cos_integral
(a*x))*tan(1/2*a*x)^4 - a^2*x^2*real_part(cos_integral(-a*x))*tan(1/2*a*x)
^4 + 2*a^3*x^3*real_part(cos_integral(a*x))*tan(1/2*a*x) + 2*a^3*x^3*real_
part(cos_integral(-a*x))*tan(1/2*a*x) - 8*a^2*x^2*tan(1/2*a*x)^2 + 2*a*x*r
eal_part(cos_integral(a*x))*tan(1/2*a*x)^3 + 2*a*x*real_part(cos_integral(
-a*x))*tan(1/2*a*x)^3 + a^2*x^2*real_part(cos_integral(a*x)) + a^2*x^2*rea
l_part(cos_integral(-a*x)) - real_part(cos_integral(a*x))*tan(1/2*a*x)^4 -
 real_part(cos_integral(-a*x))*tan(1/2*a*x)^4 + 2*a*x*real_part(cos_integr
al(a*x))*tan(1/2*a*x) + 2*a*x*real_part(cos_integral(-a*x))*tan(1/2*a*x) -
 2*tan(1/2*a*x)^4 - 12*tan(1/2*a*x)^2 + real_part(cos_integral(a*x)) + rea
l_part(cos_integral(-a*x)) - 2)/(2*a^3*x^3*tan(1/2*a*x)^3 - a^2*x^2*tan(1/
2*a*x)^4 + 2*a^3*x^3*tan(1/2*a*x) + 2*a*x*tan(1/2*a*x)^3 + a^2*x^2 - tan(1
/2*a*x)^4 + 2*a*x*tan(1/2*a*x) + 1)
```

3.597.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(ax)}{x(\cos(ax)+ax\sin(ax))^2}\,dx = \int \frac{\cos\left(a\,x\right)^3}{x\,\left(\cos\left(a\,x\right)+a\,x\,\sin\left(a\,x\right)\right)^2}\,dx$$

input
```
int(cos(a*x)^3/(x*(cos(a*x) + a*x*sin(a*x))^2),x)
```

output
```
int(cos(a*x)^3/(x*(cos(a*x) + a*x*sin(a*x))^2), x)
```

3.598 $\int \frac{\cos^2(ax)}{(\cos(ax)+ax\sin(ax))^2}\,dx$

3.598.1 Optimal result

Integrand size = 21, antiderivative size = 34

$$\int \frac{\cos^2(ax)}{(\cos(ax)+ax\sin(ax))^2}\,dx = \frac{1}{a^2x} - \frac{\cos(ax)}{a^2x(\cos(ax)+ax\sin(ax))}$$

output
```
1/a^2/x-cos(a*x)/a^2/x/(cos(a*x)+a*x*sin(a*x))
```

3.598.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int \frac{\cos^2(ax)}{(\cos(ax)+ax\sin(ax))^2}\,dx = \frac{\sin(ax)}{a(\cos(ax)+ax\sin(ax))}$$

input
```
Integrate[Cos[a*x]^2/(Cos[a*x] + a*x*Sin[a*x])^2,x]
```

output
```
Sin[a*x]/(a*(Cos[a*x] + a*x*Sin[a*x]))
```

3.598.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {5108}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(ax)}{(ax\sin(ax)+\cos(ax))^2}\,dx$$
$$\Big\downarrow \text{ 5108}$$
$$\frac{1}{a^2x}-\frac{\cos(ax)}{a^2x(ax\sin(ax)+\cos(ax))}$$

input
```
Int[Cos[a*x]^2/(Cos[a*x] + a*x*Sin[a*x])^2,x]
```

output
```
1/(a^2*x) - Cos[a*x]/(a^2*x*(Cos[a*x] + a*x*Sin[a*x]))
```

3.598.3.1 Defintions of rubi rules used

rule 5108
```
Int[Cos[(a_.)*(x_)]^2/(Cos[(a_.)*(x_)]*(c_.) + (d_.)*(x_)*Sin[(a_.)*(x_)])^
2, x_Symbol] :> Simp[1/(d^2*x), x] - Simp[Cos[a*x]/(a*d*x*(d*x*Sin[a*x] + c
*Cos[a*x])), x] /; FreeQ[{a, c, d}, x] && EqQ[a*c - d, 0]
```

3.598.4 Maple [A] (verified)

Time = 2.40 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

method	result	size
parallelrisch	$-\frac{2\tan\left(\frac{ax}{2}\right)}{\left(-1-2\tan\left(\frac{ax}{2}\right)ax+\tan\left(\frac{ax}{2}\right)^2\right)a}$	31
risch	$\frac{1}{a(ax+i)}-\frac{2i}{(ax+i)(ax\,e^{2iax}-ax+ie^{2iax}+i)a}$	55
norman	$\frac{\frac{2\tan\left(\frac{ax}{2}\right)}{a}+\frac{4\tan\left(\frac{ax}{2}\right)^3}{a}+\frac{2\tan\left(\frac{ax}{2}\right)^5}{a}}{\left(1+\tan\left(\frac{ax}{2}\right)^2\right)^2\left(1+2\tan\left(\frac{ax}{2}\right)ax-\tan\left(\frac{ax}{2}\right)^2\right)}$	70

input
```
int(cos(a*x)^2/(cos(a*x)+a*x*sin(a*x))^2,x,method=_RETURNVERBOSE)
```

output
```
-2*tan(1/2*a*x)/(-1-2*tan(1/2*a*x)*a*x+tan(1/2*a*x)^2)/a
```

3.598.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

$$\int \frac{\cos^2(ax)}{(\cos(ax)+ax\sin(ax))^2}\,dx=\frac{\sin{(ax)}}{a^2x\sin{(ax)}+a\cos{(ax)}}$$

input
```
integrate(cos(a*x)^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="fricas")
```

output
```
sin(a*x)/(a^2*x*sin(a*x) + a*cos(a*x))
```

3.598.6 Sympy [A] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.59

$$\int \frac{\cos^2(ax)}{(\cos(ax)+ax\sin(ax))^2}\,dx=\frac{\sin{(ax)}}{a^2x\sin{(ax)}+a\cos{(ax)}}$$

input
```
integrate(cos(a*x)**2/(cos(a*x)+a*x*sin(a*x))**2,x)
```

output
```
sin(a*x)/(a**2*x*sin(a*x) + a*cos(a*x))
```

3.598.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(34) = 68.

Time = 0.21 (sec) , antiderivative size = 114, normalized size of antiderivative = 3.35

$$\int \frac{\cos^2(ax)}{(\cos(ax)+ax\sin(ax))^2}\,dx$$
$$=\frac{ax\cos{(2\,ax)}^2+ax\sin{(2\,ax)}^2-2\,ax\cos{(2\,ax)}+ax+2\,\sin{(2\,ax)}}{(a^2x^2+(a^2x^2+1)\cos{(2\,ax)}^2+4\,ax\sin{(2\,ax)}+(a^2x^2+1)\sin{(2\,ax)}^2-2\,(a^2x^2-1)\cos{(2\,ax)}+1)a}$$

input
```
integrate(cos(a*x)^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="maxima")
```

output
```
(a*x*cos(2*a*x)^2 + a*x*sin(2*a*x)^2 - 2*a*x*cos(2*a*x) + a*x + 2*sin(2*a*
x))/((a^2*x^2 + (a^2*x^2 + 1)*cos(2*a*x)^2 + 4*a*x*sin(2*a*x) + (a^2*x^2 +
 1)*sin(2*a*x)^2 - 2*(a^2*x^2 - 1)*cos(2*a*x) + 1)*a)
```

3.598.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{\cos^2(ax)}{(\cos(ax) + ax\sin(ax))^2} dx = \frac{2\tan\left(\frac{1}{2}ax\right)}{2a^2x\tan\left(\frac{1}{2}ax\right) - a\tan\left(\frac{1}{2}ax\right)^2 + a}$$

input
```
integrate(cos(a*x)^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="giac")
```

output
```
2*tan(1/2*a*x)/(2*a^2*x*tan(1/2*a*x) - a*tan(1/2*a*x)^2 + a)
```

3.598.9 Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int \frac{\cos^2(ax)}{(\cos(ax) + ax\sin(ax))^2} dx = \frac{\sin(a\,x)}{a\,(\cos(a\,x) + a\,x\,\sin(a\,x))}$$

input
```
int(cos(a*x)^2/(cos(a*x) + a*x*sin(a*x))^2,x)
```

output
```
sin(a*x)/(a*(cos(a*x) + a*x*sin(a*x)))
```

3.599 $\int \frac{x\cos(ax)}{(\cos(ax)+ax\sin(ax))^2}\,dx$

3.599.1 Optimal result

Integrand size = 20, antiderivative size = 19

$$\int \frac{x\cos(ax)}{(\cos(ax)+ax\sin(ax))^2}\,dx=-\frac{1}{a^2(\cos(ax)+ax\sin(ax))}$$

output
```
-1/a^2/(cos(a*x)+a*x*sin(a*x))
```

3.599.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{x\cos(ax)}{(\cos(ax)+ax\sin(ax))^2}\,dx=-\frac{1}{a^2(\cos(ax)+ax\sin(ax))}$$

input
```
Integrate[(x*Cos[a*x])/(Cos[a*x] + a*x*Sin[a*x])^2,x]
```

output
```
-(1/(a^2*(Cos[a*x] + a*x*Sin[a*x])))
```

3.599.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {7237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\cos(ax)}{(ax\sin(ax)+\cos(ax))^2}\,dx$$
$$\Big\downarrow \text{7237}$$
$$-\frac{1}{a^2(ax\sin(ax)+\cos(ax))}$$

input
```
Int[(x*Cos[a*x])/(Cos[a*x] + a*x*Sin[a*x])^2,x]
```

output
```
-(1/(a^2*(Cos[a*x] + a*x*Sin[a*x])))
```

3.599.3.1 Defintions of rubi rules used

rule 7237
```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[q*(y^(m + 1)/(m + 1)), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

3.599.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$-\frac{1}{a^{2}(\cos(ax)+ax\sin(ax))}$	20
default	$-\frac{1}{a^{2}(\cos(ax)+ax\sin(ax))}$	20
parallelrisch	$\frac{2+2\tan\left(\frac{ax}{2}\right)ax}{a^{2}\left(-1-2\tan\left(\frac{ax}{2}\right)ax+\tan\left(\frac{ax}{2}\right)^{2}\right)}$	36
risch	$-\frac{2ie^{iax}}{a^{2}(ax\,e^{2iax}-ax+ie^{2iax}+i)}$	40
norman	$\frac{-\frac{2\tan\left(\frac{ax}{2}\right)^{2}}{a^{2}}-\frac{2}{a^{2}}-\frac{2x\tan\left(\frac{ax}{2}\right)}{a}-\frac{2x\tan\left(\frac{ax}{2}\right)^{3}}{a}}{\left(1+\tan\left(\frac{ax}{2}\right)^{2}\right)\left(1+2\tan\left(\frac{ax}{2}\right)ax-\tan\left(\frac{ax}{2}\right)^{2}\right)}$	77

input
```
int(x*cos(a*x)/(cos(a*x)+a*x*sin(a*x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/a^2/(cos(a*x)+a*x*sin(a*x))
```

3.599.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \frac{x\cos(ax)}{(\cos(ax)+ax\sin(ax))^2}\,dx = -\frac{1}{a^3x\sin{(ax)}+a^2\cos{(ax)}}$$

input
```
integrate(x*cos(a*x)/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="fricas")
```

output
```
-1/(a^3*x*sin(a*x) + a^2*cos(a*x))
```

3.599.6 Sympy [A] (verification not implemented)

Time = 2.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{x\cos(ax)}{(\cos(ax)+ax\sin(ax))^2}\,dx = -\frac{1}{a^3x\sin{(ax)}+a^2\cos{(ax)}}$$

input
```
integrate(x*cos(a*x)/(cos(a*x)+a*x*sin(a*x))**2,x)
```

output
```
-1/(a**3*x*sin(a*x) + a**2*cos(a*x))
```

3.599.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{x\cos(ax)}{(\cos(ax)+ax\sin(ax))^2}\,dx = -\frac{1}{(ax\sin{(ax)}+\cos{(ax)})a^2}$$

input
```
integrate(x*cos(a*x)/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="maxima")
```

output
```
-1/((a*x*sin(a*x) + cos(a*x))*a^2)
```

3.599.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(19) = 38$.

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.11

$$\int \frac{x\cos(ax)}{(\cos(ax)+ax\sin(ax))^2}\,dx = -\frac{2\left(\tan\left(\frac{1}{2}\,ax\right)^2+1\right)}{2\,a^3x\tan\left(\frac{1}{2}\,ax\right)-a^2\tan\left(\frac{1}{2}\,ax\right)^2+a^2}$$

input
```
integrate(x*cos(a*x)/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="giac")
```

output
```
-2*(tan(1/2*a*x)^2 + 1)/(2*a^3*x*tan(1/2*a*x) - a^2*tan(1/2*a*x)^2 + a^2)
```

3.599.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \frac{x\cos(ax)}{(\cos(ax)+ax\sin(ax))^2}\,dx = -\frac{1}{a^2\,\cos\left(a\,x\right)+a^3\,x\,\sin\left(a\,x\right)}$$

input
```
int((x*cos(a*x))/(cos(a*x) + a*x*sin(a*x))^2,x)
```

output
```
-1/(a^2*cos(a*x) + a^3*x*sin(a*x))
```

3.600 $\int \frac{x^2}{(\cos(ax)+ax\sin(ax))^2}\,dx$

3.600.1 Optimal result

Integrand size = 18, antiderivative size = 33

$$\int \frac{x^2}{(\cos(ax)+ax\sin(ax))^2}\,dx = -\frac{x\sec(ax)}{a^2(\cos(ax)+ax\sin(ax))}+\frac{\tan(ax)}{a^3}$$

output
```
-x*sec(a*x)/a^2/(cos(a*x)+a*x*sin(a*x))+tan(a*x)/a^3
```

3.600.2 Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{(\cos(ax)+ax\sin(ax))^2}\,dx = \frac{-ax\cos(ax)+\sin(ax)}{a^3(\cos(ax)+ax\sin(ax))}$$

input
```
Integrate[x^2/(Cos[a*x] + a*x*Sin[a*x])^2,x]
```

output
```
(-(a*x*Cos[a*x]) + Sin[a*x])/(a^3*(Cos[a*x] + a*x*Sin[a*x]))
```

3.600.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5106, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(ax\sin(ax)+\cos(ax))^2}dx$$

$$\Big\downarrow \text{ 5106}$$

$$\frac{\int \sec^2(ax)dx}{a^2}-\frac{x\sec(ax)}{a^2(ax\sin(ax)+\cos(ax))}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \csc\left(ax+\frac{\pi}{2}\right)^2dx}{a^2}-\frac{x\sec(ax)}{a^2(ax\sin(ax)+\cos(ax))}$$

$$\Big\downarrow \text{ 4254}$$

$$-\frac{\int 1d(-\tan(ax))}{a^3}-\frac{x\sec(ax)}{a^2(ax\sin(ax)+\cos(ax))}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{\tan(ax)}{a^3}-\frac{x\sec(ax)}{a^2(ax\sin(ax)+\cos(ax))}$$

input
```
Int[x^2/(Cos[a*x] + a*x*Sin[a*x])^2,x]
```

output
```
-((x*Sec[a*x])/(a^2*(Cos[a*x] + a*x*Sin[a*x]))) + Tan[a*x]/a^3
```

3.600.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 5106
```
Int[(x_)^2/(Cos[(a_.)*(x_)]*(c_.) + (d_.)*(x_)*Sin[(a_.)*(x_)])^2, x_Symbol
] :> Simp[-x/(a*d*Cos[a*x]*(c*Cos[a*x] + d*x*Sin[a*x])), x] + Simp[1/d^2
Int[1/Cos[a*x]^2, x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c - d, 0]
```

3.600.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.89 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.21

method	result	size
risch	$-\frac{2i(ax-i)}{a^3(ax\,\mathrm{e}^{2iax}-ax+i\mathrm{e}^{2iax}+i)}$	40
parallelrisch	$\frac{-ax\tan\left(\frac{ax}{2}\right)^2+ax-2\tan\left(\frac{ax}{2}\right)}{a^3\left(-1-2\tan\left(\frac{ax}{2}\right)ax+\tan\left(\frac{ax}{2}\right)^2\right)}$	47
norman	$\frac{\frac{x\tan\left(\frac{ax}{2}\right)^2}{a^2}+\frac{2\tan\left(\frac{ax}{2}\right)}{a^3}-\frac{x}{a^2}}{1+2\tan\left(\frac{ax}{2}\right)ax-\tan\left(\frac{ax}{2}\right)^2}$	53

input
```
int(x^2/(cos(a*x)+a*x*sin(a*x))^2,x,method=_RETURNVERBOSE)
```

output
```
-2*I*(a*x-I)/a^3/(a*x*exp(2*I*a*x)-a*x+I*exp(2*I*a*x)+I)
```

3.600.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{(\cos(ax)+ax\sin(ax))^2}\,dx = -\frac{ax\cos(ax)-\sin(ax)}{a^4x\sin(ax)+a^3\cos(ax)}$$

input
```
integrate(x^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="fricas")
```

output
```
-(a*x*cos(a*x) - sin(a*x))/(a^4*x*sin(a*x) + a^3*cos(a*x))
```

3.600.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(31) = 62$.

Time = 2.41 (sec) , antiderivative size = 109, normalized size of antiderivative = 3.30

$$\int \frac{x^2}{(\cos(ax)+ax\sin(ax))^2}\,dx = \frac{ax\tan^2\left(\frac{ax}{2}\right)}{2a^4x\tan\left(\frac{ax}{2}\right)-a^3\tan^2\left(\frac{ax}{2}\right)+a^3} - \frac{ax}{2a^4x\tan\left(\frac{ax}{2}\right)-a^3\tan^2\left(\frac{ax}{2}\right)+a^3} + \frac{2\tan\left(\frac{ax}{2}\right)}{2a^4x\tan\left(\frac{ax}{2}\right)-a^3\tan^2\left(\frac{ax}{2}\right)+a^3}$$

input
```
integrate(x**2/(cos(a*x)+a*x*sin(a*x))**2,x)
```

output
```
a*x*tan(a*x/2)**2/(2*a**4*x*tan(a*x/2) - a**3*tan(a*x/2)**2 + a**3) - a*x/
(2*a**4*x*tan(a*x/2) - a**3*tan(a*x/2)**2 + a**3) + 2*tan(a*x/2)/(2*a**4*x
*tan(a*x/2) - a**3*tan(a*x/2)**2 + a**3)
```

3.600.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(33) = 66$.

Time = 0.22 (sec) , antiderivative size = 100, normalized size of antiderivative = 3.03

$$\int \frac{x^2}{(\cos(ax)+ax\sin(ax))^2}\,dx = -\frac{2\left(2\,ax\cos(2\,ax)+(a^2x^2-1)\sin(2\,ax)\right)}{\left(a^2x^2+(a^2x^2+1)\cos(2\,ax)^2+4\,ax\sin(2\,ax)+(a^2x^2+1)\sin(2\,ax)^2-2\,(a^2x^2-1)\cos(2\,ax)+1\right)a^3}$$

input
```
integrate(x^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="maxima")
```

output
```
-2*(2*a*x*cos(2*a*x) + (a^2*x^2 - 1)*sin(2*a*x))/((a^2*x^2 + (a^2*x^2 + 1)
*cos(2*a*x)^2 + 4*a*x*sin(2*a*x) + (a^2*x^2 + 1)*sin(2*a*x)^2 - 2*(a^2*x^2
 - 1)*cos(2*a*x) + 1)*a^3)
```

3.600.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.58

$$\int \frac{x^2}{(\cos(ax) + ax\sin(ax))^2} \, dx = \frac{ax\tan\left(\frac{1}{2}\,ax\right)^2 - ax + 2\,\tan\left(\frac{1}{2}\,ax\right)}{2\,a^4x\tan\left(\frac{1}{2}\,ax\right) - a^3\tan\left(\frac{1}{2}\,ax\right)^2 + a^3}$$

input
```
integrate(x^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="giac")
```

output
```
(a*x*tan(1/2*a*x)^2 - a*x + 2*tan(1/2*a*x))/(2*a^4*x*tan(1/2*a*x) - a^3*ta
n(1/2*a*x)^2 + a^3)
```

3.600.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(\cos(ax) + ax\sin(ax))^2} \, dx = \int \frac{x^2}{\left(\cos\left(a\,x\right) + a\,x\,\sin\left(a\,x\right)\right)^2} \, dx$$

input
```
int(x^2/(cos(a*x) + a*x*sin(a*x))^2,x)
```

output
```
int(x^2/(cos(a*x) + a*x*sin(a*x))^2, x)
```

3.601 $\int \frac{x^3 \sec(ax)}{(\cos(ax)+ax\sin(ax))^2} \, dx$

3.601.1 Optimal result

Integrand size = 22, antiderivative size = 110

$$\int \frac{x^3 \sec(ax)}{(\cos(ax)+ax\sin(ax))^2} \, dx = -\frac{2ix\arctan\left(e^{iax}\right)}{a^3} + \frac{i\,\text{PolyLog}\left(2,-ie^{iax}\right)}{a^4} - \frac{i\,\text{PolyLog}\left(2,ie^{iax}\right)}{a^4} - \frac{\sec(ax)}{a^4} - \frac{x^2\sec^2(ax)}{a^2(\cos(ax)+ax\sin(ax))} + \frac{x\sec(ax)\tan(ax)}{a^3}$$

output
```
-2*I*x*arctan(exp(I*a*x))/a^3+I*polylog(2,-I*exp(I*a*x))/a^4-I*polylog(2,I
*exp(I*a*x))/a^4-sec(a*x)/a^4-x^2*sec(a*x)^2/a^2/(cos(a*x)+a*x*sin(a*x))+x
*sec(a*x)*tan(a*x)/a^3
```

3.601.2 Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.60

$$\int \frac{x^3 \sec(ax)}{(\cos(ax)+ax\sin(ax))^2} \, dx = -\frac{-ax\log\left(1-ie^{iax}\right)+ax\log\left(1+ie^{iax}\right)+\sec(ax)+a^2x^2\sec(ax)-a^2x^2\log\left(1-ie^{iax}\right)\tan(ax)+a^2x^2\,\text{l}}{a^4(1+ax}$$

input
```
Integrate[(x^3*Sec[a*x])/(Cos[a*x] + a*x*Sin[a*x])^2,x]
```

output
```
-((-(a*x*Log[1 - I*E^(I*a*x)]) + a*x*Log[1 + I*E^(I*a*x)] + Sec[a*x] + a^2
*x^2*Sec[a*x] - a^2*x^2*Log[1 - I*E^(I*a*x)]*Tan[a*x] + a^2*x^2*Log[1 + I*
E^(I*a*x)]*Tan[a*x] - I*PolyLog[2, (-I)*E^(I*a*x)]*(1 + a*x*Tan[a*x]) + I*
PolyLog[2, I*E^(I*a*x)]*(1 + a*x*Tan[a*x]))/(a^4*(1 + a*x*Tan[a*x])))
```

3.601.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {5112, 3042, 4673, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \sec(ax)}{(ax\sin(ax)+\cos(ax))^2}dx$$

$$\Big\downarrow \text{5112}$$

$$\frac{2\int x\sec^3(ax)dx}{a^2}-\frac{x^2\sec^2(ax)}{a^2(ax\sin(ax)+\cos(ax))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int x\csc\left(ax+\frac{\pi}{2}\right)^3dx}{a^2}-\frac{x^2\sec^2(ax)}{a^2(ax\sin(ax)+\cos(ax))}$$

$$\Big\downarrow \text{4673}$$

$$\frac{2\left(\frac{1}{2}\int x\sec(ax)dx-\frac{\sec(ax)}{2a^2}+\frac{x\tan(ax)\sec(ax)}{2a}\right)}{a^2}-\frac{x^2\sec^2(ax)}{a^2(ax\sin(ax)+\cos(ax))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(\frac{1}{2}\int x\csc\left(ax+\frac{\pi}{2}\right)dx-\frac{\sec(ax)}{2a^2}+\frac{x\tan(ax)\sec(ax)}{2a}\right)}{a^2}-\frac{x^2\sec^2(ax)}{a^2(ax\sin(ax)+\cos(ax))}$$

$$\Big\downarrow \text{4669}$$

$$-\frac{x^2\sec^2(ax)}{a^2(ax\sin(ax)+\cos(ax))}+\frac{2\left(\frac{1}{2}\left(-\frac{\int\log(1-ie^{iax})dx}{a}+\frac{\int\log(1+ie^{iax})dx}{a}-\frac{2ix\arctan(e^{iax})}{a}\right)-\frac{\sec(ax)}{2a^2}+\frac{x\tan(ax)\sec(ax)}{2a}\right)}{a^2}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{x^2\sec^2(ax)}{a^2(ax\sin(ax)+\cos(ax))}+\frac{2\left(\frac{1}{2}\left(\frac{i\int e^{-iax}\log(1-ie^{iax})de^{iax}}{a^2}-\frac{i\int e^{-iax}\log(1+ie^{iax})de^{iax}}{a^2}-\frac{2ix\arctan(e^{iax})}{a}\right)-\frac{\sec(ax)}{2a^2}+\frac{x\tan(ax)\sec(ax)}{2a}\right)}{a^2}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{x^2\sec^2(ax)}{a^2(ax\sin(ax)+\cos(ax))}+\frac{2\left(\frac{1}{2}\left(\frac{i\,\text{PolyLog}(2,-ie^{iax})}{a^2}-\frac{i\,\text{PolyLog}(2,ie^{iax})}{a^2}-\frac{2ix\arctan(e^{iax})}{a}\right)-\frac{\sec(ax)}{2a^2}+\frac{x\tan(ax)\sec(ax)}{2a}\right)}{a^2}$$

input
```
Int[(x^3*Sec[a*x])/(Cos[a*x] + a*x*Sin[a*x])^2,x]
```

output
```
-((x^2*Sec[a*x]^2)/(a^2*(Cos[a*x] + a*x*Sin[a*x]))) + (2*((((-2*I)*x*ArcTa
n[E^(I*a*x)])/a + (I*PolyLog[2, (-I)*E^(I*a*x)])/a^2 - (I*PolyLog[2, I*E^(
I*a*x)])/a^2)/2 - Sec[a*x]/(2*a^2) + (x*Sec[a*x]*Tan[a*x])/(2*a)))/a^2
```

3.601.3.1 Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

3.601. $\int \frac{x^3\sec(ax)}{(\cos(ax)+ax\sin(ax))^2}\,dx$

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

rule 5112
```
Int[(((b_.)*(x_))^(m_.)*Sec[(a_.)*(x_)]^(n_.))/(Cos[(a_.)*(x_)]*(c_.) + (d_
.)*(x_)*Sin[(a_.)*(x_)])^2, x_Symbol] :> Simp[(-b)*(b*x)^(m - 1)*(Sec[a*x]^
(n + 1)/(a*d*(c*Cos[a*x] + d*x*Sin[a*x]))), x] + Simp[b^2*((n + 1)/d^2)   I
nt[(b*x)^(m - 2)*Sec[a*x]^(n + 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] &
& EqQ[a*c - d, 0] && EqQ[m, n + 2]
```

3.601.4 Maple [F]

$$\int \frac{x^3 \sec(ax)}{(\cos(ax) + ax\sin(ax))^2} dx$$

input
```
int(x^3*sec(a*x)/(cos(a*x)+a*x*sin(a*x))^2,x)
```

output
```
int(x^3*sec(a*x)/(cos(a*x)+a*x*sin(a*x))^2,x)
```

3.601.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 290 vs. 2(89) = 178.

Time = 0.27 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.64

$$\int \frac{x^3 \sec(ax)}{(\cos(ax) + ax\sin(ax))^2} dx = -\frac{2a^2x^2 - (-i\,ax\sin(ax) - i\,\cos(ax))\mathrm{Li}_2(i\,\cos(ax) + \sin(ax)) - (-i\,ax\sin(ax) - i\,\cos(ax))\mathrm{Li}_2(i\,\cos$$

input
```
integrate(x^3*sec(a*x)/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="fricas")
```

output
```
-1/2*(2*a^2*x^2 - (-I*a*x*sin(a*x) - I*cos(a*x))*dilog(I*cos(a*x) + sin(a*
x)) - (-I*a*x*sin(a*x) - I*cos(a*x))*dilog(I*cos(a*x) - sin(a*x)) - (I*a*x
*sin(a*x) + I*cos(a*x))*dilog(-I*cos(a*x) + sin(a*x)) - (I*a*x*sin(a*x) +
I*cos(a*x))*dilog(-I*cos(a*x) - sin(a*x)) - (a^2*x^2*sin(a*x) + a*x*cos(a*
x))*log(I*cos(a*x) + sin(a*x) + 1) + (a^2*x^2*sin(a*x) + a*x*cos(a*x))*log
(I*cos(a*x) - sin(a*x) + 1) - (a^2*x^2*sin(a*x) + a*x*cos(a*x))*log(-I*cos
(a*x) + sin(a*x) + 1) + (a^2*x^2*sin(a*x) + a*x*cos(a*x))*log(-I*cos(a*x)
- sin(a*x) + 1) + 2)/(a^5*x*sin(a*x) + a^4*cos(a*x))
```

3.601.6 Sympy [F]

$$\int \frac{x^3 \sec(ax)}{(\cos(ax) + ax\sin(ax))^2}\,dx = \int \frac{x^3 \sec{(ax)}}{(ax\sin{(ax)} + \cos{(ax)})^2}\,dx$$

input
```
integrate(x**3*sec(a*x)/(cos(a*x)+a*x*sin(a*x))**2,x)
```

output
```
Integral(x**3*sec(a*x)/(a*x*sin(a*x) + cos(a*x))**2, x)
```

3.601.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3 \sec(ax)}{(\cos(ax) + ax\sin(ax))^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*sec(a*x)/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.601.8 Giac [F]

$$\int \frac{x^3 \sec(ax)}{(\cos(ax) + ax\sin(ax))^2}\,dx = \int \frac{x^3 \sec(ax)}{(ax\sin(ax) + \cos(ax))^2}\,dx$$

input
```
integrate(x^3*sec(a*x)/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="giac")
```

output
```
integrate(x^3*sec(a*x)/(a*x*sin(a*x) + cos(a*x))^2, x)
```

3.601.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \sec(ax)}{(\cos(ax) + ax\sin(ax))^2}\,dx = \int \frac{x^3}{\cos(a\,x)\,\left(\cos(a\,x) + a\,x\,\sin(a\,x)\right)^2}\,dx$$

input
```
int(x^3/(cos(a*x)*(cos(a*x) + a*x*sin(a*x))^2),x)
```

output
```
int(x^3/(cos(a*x)*(cos(a*x) + a*x*sin(a*x))^2), x)
```

3.602 $\int \frac{x^4 \sec^2(ax)}{(\cos(ax)+ax\sin(ax))^2}\,dx$

3.602.1 Optimal result

Integrand size = 24, antiderivative size = 124

$$\int \frac{x^4 \sec^2(ax)}{(\cos(ax)+ax\sin(ax))^2}\,dx = -\frac{2ix^2}{a^3}+\frac{4x\log\left(1+e^{2iax}\right)}{a^4}-\frac{2i\,\mathrm{PolyLog}\left(2,-e^{2iax}\right)}{a^5} -\frac{x\sec^2(ax)}{a^4}-\frac{x^3\sec^3(ax)}{a^2(\cos(ax)+ax\sin(ax))} +\frac{\tan(ax)}{a^5}+\frac{2x^2\tan(ax)}{a^3}+\frac{x^2\sec^2(ax)\tan(ax)}{a^3}$$

output
```
-2*I*x^2/a^3+4*x*ln(1+exp(2*I*a*x))/a^4-2*I*polylog(2,-exp(2*I*a*x))/a^5-x
*sec(a*x)^2/a^4-x^3*sec(a*x)^3/a^2/(cos(a*x)+a*x*sin(a*x))+tan(a*x)/a^5+2*
x^2*tan(a*x)/a^3+x^2*sec(a*x)^2*tan(a*x)/a^3
```

3.602.2 Mathematica [A] (verified)

Time = 1.83 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.05

$$\int \frac{x^4 \sec^2(ax)}{(\cos(ax)+ax\sin(ax))^2}\,dx = \frac{-ax\left(1+2iax+a^2x^2-4\log\left(1+e^{2iax}\right)\right)+\left(1+2a^2x^2-2ia^3x^3+4a^2x^2\log\left(1+e^{2iax}\right)\right)\tan(ax)+a^3x^3\,\mathrm{ta}}{a^5(1+ax\tan(ax))}$$

input
```
Integrate[(x^4*Sec[a*x]^2)/(Cos[a*x] + a*x*Sin[a*x])^2,x]
```

output
```
(-(a*x*(1 + (2*I)*a*x + a^2*x^2 - 4*Log[1 + E^((2*I)*a*x)])) + (1 + 2*a^2*
x^2 - (2*I)*a^3*x^3 + 4*a^2*x^2*Log[1 + E^((2*I)*a*x)])*Tan[a*x] + a^3*x^3
*Tan[a*x]^2 - (2*I)*PolyLog[2, -E^((2*I)*a*x)]*(1 + a*x*Tan[a*x]))/(a^5*(1
 + a*x*Tan[a*x]))
```

3.602.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.26, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {5112, 3042, 4674, 3042, 4254, 24, 4672, 25, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \sec^2(ax)}{(ax\sin(ax)+\cos(ax))^2}dx$$

$$\Big\downarrow \text{5112}$$

$$\frac{3\int x^2\sec^4(ax)dx}{a^2}-\frac{x^3\sec^3(ax)}{a^2(ax\sin(ax)+\cos(ax))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int x^2\csc\left(ax+\frac{\pi}{2}\right)^4dx}{a^2}-\frac{x^3\sec^3(ax)}{a^2(ax\sin(ax)+\cos(ax))}$$

$$\Big\downarrow \text{4674}$$

$$\frac{3\left(\frac{\int \sec^2(ax)dx}{3a^2}+\frac{2}{3}\int x^2\sec^2(ax)dx-\frac{x\sec^2(ax)}{3a^2}+\frac{x^2\tan(ax)\sec^2(ax)}{3a}\right)}{a^2}-\frac{x^3\sec^3(ax)}{a^2(ax\sin(ax)+\cos(ax))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{\int \csc\left(ax+\frac{\pi}{2}\right)^2dx}{3a^2}+\frac{2}{3}\int x^2\csc\left(ax+\frac{\pi}{2}\right)^2dx-\frac{x\sec^2(ax)}{3a^2}+\frac{x^2\tan(ax)\sec^2(ax)}{3a}\right)}{a^2}-\frac{x^3\sec^3(ax)}{a^2(ax\sin(ax)+\cos(ax))}$$

$$\Big\downarrow \text{4254}$$

$$\frac{3\left(-\frac{\int 1d(-\tan(ax))}{3a^3}+\frac{2}{3}\int x^2\csc\left(ax+\frac{\pi}{2}\right)^2dx-\frac{x\sec^2(ax)}{3a^2}+\frac{x^2\tan(ax)\sec^2(ax)}{3a}\right)}{a^2}-\frac{x^3\sec^3(ax)}{a^2(ax\sin(ax)+\cos(ax))}$$

$$\Big\downarrow \text{24}$$

$$\frac{3\left(\frac{2}{3}\int x^2\csc\left(ax+\frac{\pi}{2}\right)^2dx+\frac{\tan(ax)}{3a^3}-\frac{x\sec^2(ax)}{3a^2}+\frac{x^2\tan(ax)\sec^2(ax)}{3a}\right)}{a^2}-\frac{x^3\sec^3(ax)}{a^2(ax\sin(ax)+\cos(ax))}$$

$$\Big\downarrow \text{4672}$$

$$\frac{3\left(\frac{2}{3}\left(\frac{2\int -x\tan(ax)dx}{a}+\frac{x^2\tan(ax)}{a}\right)+\frac{\tan(ax)}{3a^3}-\frac{x\sec^2(ax)}{3a^2}+\frac{x^2\tan(ax)\sec^2(ax)}{3a}\right)}{a^2}-\frac{x^3\sec^3(ax)}{a^2(ax\sin(ax)+\cos(ax))}$$

$$\Big\downarrow \text{25}$$

$$\frac{3\left(\frac{2}{3}\left(\frac{x^2\tan(ax)}{a}-\frac{2\int x\tan(ax)dx}{a}\right)+\frac{\tan(ax)}{3a^3}-\frac{x\sec^2(ax)}{3a^2}+\frac{x^2\tan(ax)\sec^2(ax)}{3a}\right)}{a^2}-\frac{x^3\sec^3(ax)}{a^2(ax\sin(ax)+\cos(ax))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{2}{3}\left(\frac{x^2\tan(ax)}{a}-\frac{2\int x\tan(ax)dx}{a}\right)+\frac{\tan(ax)}{3a^3}-\frac{x\sec^2(ax)}{3a^2}+\frac{x^2\tan(ax)\sec^2(ax)}{3a}\right)}{a^2}-\frac{x^3\sec^3(ax)}{a^2(ax\sin(ax)+\cos(ax))}$$

$$\Big\downarrow \text{4202}$$

$$-\frac{x^3\sec^3(ax)}{a^2(ax\sin(ax)+\cos(ax))}+\frac{3\left(\frac{2}{3}\left(\frac{x^2\tan(ax)}{a}-\frac{2\left(\frac{ix^2}{2}-2i\int\frac{e^{2iax}x}{1+e^{2iax}}dx\right)}{a}\right)+\frac{\tan(ax)}{3a^3}-\frac{x\sec^2(ax)}{3a^2}+\frac{x^2\tan(ax)\sec^2(ax)}{3a}\right)}{a^2}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{x^3\sec^3(ax)}{a^2(ax\sin(ax)+\cos(ax))}+\frac{3\left(\frac{2}{3}\left(\frac{x^2\tan(ax)}{a}-\frac{2\left(\frac{ix^2}{2}-2i\left(\frac{i\int\log\left(1+e^{2iax}\right)dx}{2a}-\frac{ix\log\left(1+e^{2iax}\right)}{2a}\right)\right)}{a}\right)+\frac{\tan(ax)}{3a^3}-\frac{x\sec^2(ax)}{3a^2}+\frac{x^2\tan(ax)\sec^2(ax)}{3a}\right)}{a^2}$$

$$\Big\downarrow \text{2715}$$

3.602. $\int \frac{x^4\sec^2(ax)}{(\cos(ax)+ax\sin(ax))^2}\,dx$

$$-\frac{x^3\sec^3(ax)}{a^2(ax\sin(ax)+\cos(ax))}+\frac{3\left(\frac{2}{3}\left(\frac{x^2\tan(ax)}{a}-\frac{2\left(\frac{ix^2}{2}-2i\left(\frac{\int e^{-2iax}\log\left(1+e^{2iax}\right)de^{2iax}}{4a^2}-\frac{ix\log\left(1+e^{2iax}\right)}{2a}\right)\right)}{a}\right)+\frac{\tan(ax)}{3a^3}-\frac{x\sec^2(ax)}{3a^2}+\frac{x^2\tan(ax)\sec^2(ax)}{3a}\right)}{a^2}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{x^3\sec^3(ax)}{a^2(ax\sin(ax)+\cos(ax))}+\frac{3\left(\frac{\tan(ax)}{3a^3}+\frac{2}{3}\left(\frac{x^2\tan(ax)}{a}-\frac{2\left(\frac{ix^2}{2}-2i\left(-\frac{\text{PolyLog}\left(2,-e^{2iax}\right)}{4a^2}-\frac{ix\log\left(1+e^{2iax}\right)}{2a}\right)\right)}{a}\right)-\frac{x\sec^2(ax)}{3a^2}+\frac{x^2\tan(ax)\sec^2(ax)}{3a}\right)}{a^2}$$

input
```
Int[(x^4*Sec[a*x]^2)/(Cos[a*x] + a*x*Sin[a*x])^2,x]
```

output
```
-((x^3*Sec[a*x]^3)/(a^2*(Cos[a*x] + a*x*Sin[a*x]))) + (3*(-1/3*(x*Sec[a*x]
^2)/a^2 + Tan[a*x]/(3*a^3) + (x^2*Sec[a*x]^2*Tan[a*x])/(3*a) + (2*((-2*((I
/2)*x^2 - (2*I)*(((-1/2*I)*x*Log[1 + E^((2*I)*a*x)])/a - PolyLog[2, -E^((2
*I)*a*x)]/(4*a^2))))/a + (x^2*Tan[a*x])/a))/3))/a^2
```

3.602.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

3.602. $\int \frac{x^4\sec^2(ax)}{(\cos(ax)+ax\sin(ax))^2}\,dx$

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 5112
```
Int[(((b_.)*(x_))^(m_.)*Sec[(a_.)*(x_)]^(n_.))/(Cos[(a_.)*(x_)]*(c_.) + (d_
.)*(x_)*Sin[(a_.)*(x_)])^2, x_Symbol] :> Simp[(-b)*(b*x)^(m - 1)*(Sec[a*x]^
(n + 1)/(a*d*(c*Cos[a*x] + d*x*Sin[a*x]))), x] + Simp[b^2*((n + 1)/d^2)   I
nt[(b*x)^(m - 2)*Sec[a*x]^(n + 2), x], x] /; FreeQ[{a, b, c, d, m, n}, x] &
& EqQ[a*c - d, 0] && EqQ[m, n + 2]
```

3.602. $\int \frac{x^4 \sec^2(ax)}{(\cos(ax)+ax\sin(ax))^2} \, dx$

3.602.4 Maple [A] (verified)

Time = 3.54 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.14

method	result	size
risch	$-\frac{2i\left(-2ia^{2}x^{2}\mathrm{e}^{2iax}+2a^{3}x^{3}-2ia^{2}x^{2}+ax\,\mathrm{e}^{2iax}-i\,\mathrm{e}^{2iax}+ax-i\right)}{\left(1+\mathrm{e}^{2iax}\right)\left(ax\,\mathrm{e}^{2iax}-ax+i\,\mathrm{e}^{2iax}+i\right)a^{5}}-\frac{4ix^{2}}{a^{3}}+\frac{4x\ln\left(1+\mathrm{e}^{2iax}\right)}{a^{4}}-\frac{2i\,\mathrm{polylog}\left(2,-\mathrm{e}^{2iax}\right)}{a^{5}}$	141

input
```
int(x^4*sec(a*x)^2/(cos(a*x)+a*x*sin(a*x))^2,x,method=_RETURNVERBOSE)
```

output
```
-2*I*(-2*I*a^2*x^2*exp(2*I*a*x)+2*a^3*x^3-2*I*a^2*x^2+a*x*exp(2*I*a*x)-I*e
xp(2*I*a*x)+a*x-I)/(1+exp(2*I*a*x))/(a*x*exp(2*I*a*x)-a*x+I*exp(2*I*a*x)+I
)/a^5-4*I/a^3*x^2+4*x*ln(1+exp(2*I*a*x))/a^4-2*I*polylog(2,-exp(2*I*a*x))/
a^5
```

3.602.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 382 vs. 2(113) = 226.

Time = 0.28 (sec) , antiderivative size = 382, normalized size of antiderivative = 3.08

$$\int \frac{x^4\sec^2(ax)}{(\cos(ax)+ax\sin(ax))^2}\,dx = \frac{a^3x^3-\left(2\,a^3x^3+ax\right)\cos\left(ax\right)^2+\left(2\,a^2x^2+1\right)\cos\left(ax\right)\sin\left(ax\right)-2\left(-i\,ax\cos\left(ax\right)\sin\left(ax\right)-i\,\cos\left(ax\right)^2\right)\mathrm{I}}{}$$

input
```
integrate(x^4*sec(a*x)^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="fricas")
```

output
```
(a^3*x^3 - (2*a^3*x^3 + a*x)*cos(a*x)^2 + (2*a^2*x^2 + 1)*cos(a*x)*sin(a*x
) - 2*(-I*a*x*cos(a*x)*sin(a*x) - I*cos(a*x)^2)*dilog(I*cos(a*x) + sin(a*x
)) - 2*(I*a*x*cos(a*x)*sin(a*x) + I*cos(a*x)^2)*dilog(I*cos(a*x) - sin(a*x
)) - 2*(I*a*x*cos(a*x)*sin(a*x) + I*cos(a*x)^2)*dilog(-I*cos(a*x) + sin(a*
x)) - 2*(-I*a*x*cos(a*x)*sin(a*x) - I*cos(a*x)^2)*dilog(-I*cos(a*x) - sin(
a*x)) + 2*(a^2*x^2*cos(a*x)*sin(a*x) + a*x*cos(a*x)^2)*log(I*cos(a*x) + si
n(a*x) + 1) + 2*(a^2*x^2*cos(a*x)*sin(a*x) + a*x*cos(a*x)^2)*log(I*cos(a*x
) - sin(a*x) + 1) + 2*(a^2*x^2*cos(a*x)*sin(a*x) + a*x*cos(a*x)^2)*log(-I*
cos(a*x) + sin(a*x) + 1) + 2*(a^2*x^2*cos(a*x)*sin(a*x) + a*x*cos(a*x)^2)*
log(-I*cos(a*x) - sin(a*x) + 1))/(a^6*x*cos(a*x)*sin(a*x) + a^5*cos(a*x)^2
)
```

3.602.6 Sympy [F]

$$\int \frac{x^4 \sec^2(ax)}{(\cos(ax)+ax\sin(ax))^2}\,dx = \int \frac{x^4 \sec^2(ax)}{(ax\sin(ax)+\cos(ax))^2}\,dx$$

input
```
integrate(x**4*sec(a*x)**2/(cos(a*x)+a*x*sin(a*x))**2,x)
```

output
```
Integral(x**4*sec(a*x)**2/(a*x*sin(a*x) + cos(a*x))**2, x)
```

3.602.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 372 vs. 2(113) = 226.

Time = 0.32 (sec) , antiderivative size = 372, normalized size of antiderivative = 3.00

$$\int \frac{x^4 \sec^2(ax)}{(\cos(ax)+ax\sin(ax))^2}\,dx = -\frac{2\left(ax+2\left(a^2x^2-2i\,ax\cos(2\,ax)+2\,ax\sin(2\,ax)-i\,ax-(a^2x^2+i\,ax)\cos(4\,ax)+(-i\,a^2x^2+ax)\sin\right.}{}$$

input
```
integrate(x^4*sec(a*x)^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="maxima")
```

output
```
-2*(a*x + 2*(a^2*x^2 - 2*I*a*x*cos(2*a*x) + 2*a*x*sin(2*a*x) - I*a*x - (a^
2*x^2 + I*a*x)*cos(4*a*x) + (-I*a^2*x^2 + a*x)*sin(4*a*x))*arctan2(sin(2*a
*x), cos(2*a*x) + 1) + 2*(a^3*x^3 + I*a^2*x^2)*cos(4*a*x) + (2*I*a^2*x^2 +
 a*x - I)*cos(2*a*x) - (a*x - (a*x + I)*cos(4*a*x) - (I*a*x - 1)*sin(4*a*x
) - 2*I*cos(2*a*x) + 2*sin(2*a*x) - I)*dilog(-e^(2*I*a*x)) + (-I*a^2*x^2 -
 2*a*x*cos(2*a*x) - 2*I*a*x*sin(2*a*x) - a*x + (I*a^2*x^2 - a*x)*cos(4*a*x
) - (a^2*x^2 + I*a*x)*sin(4*a*x))*log(cos(2*a*x)^2 + sin(2*a*x)^2 + 2*cos(
2*a*x) + 1) + 2*(I*a^3*x^3 - a^2*x^2)*sin(4*a*x) - (2*a^2*x^2 - I*a*x - 1)
*sin(2*a*x) - I)/((I*a*x + (-I*a*x + 1)*cos(4*a*x) + (a*x + I)*sin(4*a*x)
+ 2*cos(2*a*x) + 2*I*sin(2*a*x) + 1)*a^5)
```

3.602.8 Giac [F]

$$\int \frac{x^4 \sec^2(ax)}{(\cos(ax) + ax\sin(ax))^2}\, dx = \int \frac{x^4 \sec{(ax)}^2}{(ax\sin{(ax)} + \cos{(ax)})^2}\, dx$$

input
```
integrate(x^4*sec(a*x)^2/(cos(a*x)+a*x*sin(a*x))^2,x, algorithm="giac")
```

output
```
integrate(x^4*sec(a*x)^2/(a*x*sin(a*x) + cos(a*x))^2, x)
```

3.602.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \sec^2(ax)}{(\cos(ax) + ax\sin(ax))^2}\, dx = \int \frac{x^4}{\cos{(a\,x)}^2\,(\cos{(a\,x)} + a\,x\,\sin{(a\,x)})^2}\, dx$$

input
```
int(x^4/(cos(a*x)^2*(cos(a*x) + a*x*sin(a*x))^2),x)
```

output
```
int(x^4/(cos(a*x)^2*(cos(a*x) + a*x*sin(a*x))^2), x)
```

3.603 $\int \sec^4(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$

3.603.1 Optimal result

Integrand size = 31, antiderivative size = 157

$$\int \sec^4(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = -\frac{2c\tan(2a+2bx)}{5b\sqrt{-c+c\sec(2a+2bx)}}+\frac{c\sec^3(2a+2bx)\tan(2a+2bx)}{7b\sqrt{-c+c\sec(2a+2bx)}} - \frac{4\sqrt{-c+c\sec(2a+2bx)}\tan(2a+2bx)}{35b}-\frac{6(-c+c\sec(2a+2bx))^{3/2}\tan(2a+2bx)}{35bc}$$

output
```
-6/35*(-c+c*sec(2*b*x+2*a))^(3/2)*tan(2*b*x+2*a)/b/c-2/5*c*tan(2*b*x+2*a)/
b/(-c+c*sec(2*b*x+2*a))^(1/2)+1/7*c*sec(2*b*x+2*a)^3*tan(2*b*x+2*a)/b/(-c+
c*sec(2*b*x+2*a))^(1/2)-4/35*(-c+c*sec(2*b*x+2*a))^(1/2)*tan(2*b*x+2*a)/b
```

3.603.2 Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.41

$$\int \sec^4(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = -\frac{(7\cos(3(a+bx))+2\cos(7(a+bx)))\csc(a+bx)\sec^3(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}}{35b}$$

input
```
Integrate[Sec[2*(a + b*x)]^4*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

output
```
-1/35*((7*Cos[3*(a + b*x)] + 2*Cos[7*(a + b*x)])*Csc[a + b*x]*Sec[2*(a + b
*x)]^3*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]])/b
```

3.603.3 Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.355, Rules used = {3042, 4897, 3042, 4290, 3042, 4287, 27, 3042, 4489, 3042, 4279}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(2a+2bx)^4\sqrt{c\tan(a+bx)\tan(2a+2bx)}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \sec^4(2a+2bx)\sqrt{c\sec(2a+2bx)-c}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(2a+2bx+\frac{\pi}{2}\right)^4\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}dx$$

$$\Big\downarrow \text{4290}$$

$$\frac{c\tan(2a+2bx)\sec^3(2a+2bx)}{7b\sqrt{c\sec(2a+2bx)-c}}-\frac{6}{7}\int \sec^3(2a+2bx)\sqrt{c\sec(2a+2bx)-c}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{c\tan(2a+2bx)\sec^3(2a+2bx)}{7b\sqrt{c\sec(2a+2bx)-c}}-\frac{6}{7}\int \csc\left(2a+2bx+\frac{\pi}{2}\right)^3\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}dx$$

$$\Big\downarrow \text{4287}$$

$$\frac{c\tan(2a+2bx)\sec^3(2a+2bx)}{7b\sqrt{c\sec(2a+2bx)-c}}-\frac{6}{7}\left(\frac{2\int \frac{1}{2}\sec(2a+2bx)\sqrt{c\sec(2a+2bx)-c}(2\sec(2a+2bx)c+3c)dx}{5c}+\frac{\tan(2a+2bx)(c\sec(2a+2bx)-c)^{3/2}}{5bc}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{c\tan(2a+2bx)\sec^3(2a+2bx)}{7b\sqrt{c\sec(2a+2bx)-c}}-\frac{6}{7}\left(\frac{\int\sec(2a+2bx)\sqrt{c\sec(2a+2bx)-c}(2\sec(2a+2bx)c+3c)dx}{5c}+\frac{\tan(2a+2bx)(c\sec(2a+2bx)-c)^{3/2}}{5bc}\right)$$

$\Big\downarrow$ 3042

$$\frac{c\tan(2a+2bx)\sec^3(2a+2bx)}{7b\sqrt{c\sec(2a+2bx)-c}}-\frac{6}{7}\left(\frac{\int\csc\left(2a+2bx+\frac{\pi}{2}\right)\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}\left(2\csc\left(2a+2bx+\frac{\pi}{2}\right)c+3c\right)dx}{5c}+\frac{\tan(2a+2bx)(c\sec(2a+2}{5bc}\right.$$

$\Big\downarrow$ 4489

$$\frac{c\tan(2a+2bx)\sec^3(2a+2bx)}{7b\sqrt{c\sec(2a+2bx)-c}}-\frac{6}{7}\left(\frac{\frac{7}{3}c\int\sec(2a+2bx)\sqrt{c\sec(2a+2bx)-c}dx+\frac{2c\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{3b}}{5c}+\frac{\tan(2a+2bx)(c\sec(2a+2bx)-c}{5bc}\right.$$

$\Big\downarrow$ 3042

$$\frac{c\tan(2a+2bx)\sec^3(2a+2bx)}{7b\sqrt{c\sec(2a+2bx)-c}}-\frac{6}{7}\left(\frac{\frac{7}{3}c\int\csc\left(2a+2bx+\frac{\pi}{2}\right)\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}dx+\frac{2c\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{3b}}{5c}+\frac{\tan(2a+2bx)(c\sec(2a}{5bc}\right.$$

$\Big\downarrow$ 4279

$$\frac{c\tan(2a+2bx)\sec^3(2a+2bx)}{7b\sqrt{c\sec(2a+2bx)-c}}-\frac{6}{7}\left(\frac{\frac{7c^2\tan(2a+2bx)}{3b\sqrt{c\sec(2a+2bx)-c}}+\frac{2c\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{3b}}{5c}+\frac{\tan(2a+2bx)(c\sec(2a+2bx)-c)^{3/2}}{5bc}\right)$$

input
```
Int[Sec[2*(a + b*x)]^4*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

output
```
(c*Sec[2*a + 2*b*x]^3*Tan[2*a + 2*b*x])/(7*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]
) - (6*(((-c + c*Sec[2*a + 2*b*x])^(3/2)*Tan[2*a + 2*b*x])/(5*b*c) + ((7*c
^2*Tan[2*a + 2*b*x])/(3*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]) + (2*c*Sqrt[-c +
c*Sec[2*a + 2*b*x]]*Tan[2*a + 2*b*x])/(3*b))/(5*c)))/7
```

3.603. $\int\sec^4(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$

3.603.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4279
```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4287
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(
m + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 -
b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 4290
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[2*a*d*((n - 1)/(b*(2*n -
1)))   Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Fre
eQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

rule 4489
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[(a*B*m + A*b*(m + 1))/(b*(m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B
, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b
*(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.603. $\int \sec^4(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$

3.603.4 Maple [A] (verified)

Time = 7.24 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.57

method	result	size
default	$-\frac{\sqrt{2}\,\cot(xb+a)\sqrt{\frac{c\sin(xb+a)^2}{2\cos(xb+a)^2-1}}\left(128\cos(xb+a)^6-224\cos(xb+a)^4+140\cos(xb+a)^2-35\right)\sqrt{4}}{70b\left(2\cos(xb+a)^2-1\right)^3}$	90

input
```
int(sec(2*b*x+2*a)^4*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
-1/70*2^(1/2)/b*cot(b*x+a)*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(1/2)*(128*
cos(b*x+a)^6-224*cos(b*x+a)^4+140*cos(b*x+a)^2-35)/(2*cos(b*x+a)^2-1)^3*4^
(1/2)
```

3.603.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.68

$$\int \sec^4(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$$
$$= -\frac{\sqrt{2}\left(35\tan(bx+a)^6-35\tan(bx+a)^4+49\tan(bx+a)^2-9\right)\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}}{35\left(b\tan(bx+a)^7-3\,b\tan(bx+a)^5+3\,b\tan(bx+a)^3-b\tan(bx+a)\right)}$$

input
```
integrate(sec(2*b*x+2*a)^4*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorith
m="fricas")
```

output
```
-1/35*sqrt(2)*(35*tan(b*x + a)^6 - 35*tan(b*x + a)^4 + 49*tan(b*x + a)^2 -
 9)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))/(b*tan(b*x + a)^7 - 3*b*t
an(b*x + a)^5 + 3*b*tan(b*x + a)^3 - b*tan(b*x + a))
```

3.603.6 Sympy [F(-1)]

Timed out.

$$\int \sec^4(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = \text{Timed out}$$

input
```
integrate(sec(2*b*x+2*a)**4*(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
```

output
```
Timed out
```

3.603.7 Maxima [F]

$$\int \sec^4(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$$
$$= \int \sqrt{c\tan\left(2\,bx+2\,a\right)\tan\left(bx+a\right)}\sec\left(2\,bx+2\,a\right)^4\,dx$$

input
```
integrate(sec(2*b*x+2*a)^4*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorith
m="maxima")
```

output
```
-8/35*(70*(b*cos(4*b*x + 4*a)^2 + b*sin(4*b*x + 4*a)^2 + 2*b*cos(4*b*x + 4
*a) + b)*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1
)^(3/4)*sqrt(c)*integrate(-(((cos(20*b*x + 20*a)*cos(4*b*x + 4*a) + 4*cos(
16*b*x + 16*a)*cos(4*b*x + 4*a) + 6*cos(12*b*x + 12*a)*cos(4*b*x + 4*a) +
4*cos(8*b*x + 8*a)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + sin(20*b*x + 20
*a)*sin(4*b*x + 4*a) + 4*sin(16*b*x + 16*a)*sin(4*b*x + 4*a) + 6*sin(12*b*
x + 12*a)*sin(4*b*x + 4*a) + 4*sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + sin(4*b
*x + 4*a)^2)*cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) + (
cos(4*b*x + 4*a)*sin(20*b*x + 20*a) + 4*cos(4*b*x + 4*a)*sin(16*b*x + 16*a
) + 6*cos(4*b*x + 4*a)*sin(12*b*x + 12*a) + 4*cos(4*b*x + 4*a)*sin(8*b*x +
 8*a) - cos(20*b*x + 20*a)*sin(4*b*x + 4*a) - 4*cos(16*b*x + 16*a)*sin(4*b
*x + 4*a) - 6*cos(12*b*x + 12*a)*sin(4*b*x + 4*a) - 4*cos(8*b*x + 8*a)*sin
(4*b*x + 4*a))*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)))*
cos(5/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a))) + ((cos(4*b*x + 4*a)*
sin(20*b*x + 20*a) + 4*cos(4*b*x + 4*a)*sin(16*b*x + 16*a) + 6*cos(4*b*x +
 4*a)*sin(12*b*x + 12*a) + 4*cos(4*b*x + 4*a)*sin(8*b*x + 8*a) - cos(20*b*
x + 20*a)*sin(4*b*x + 4*a) - 4*cos(16*b*x + 16*a)*sin(4*b*x + 4*a) - 6*cos
(12*b*x + 12*a)*sin(4*b*x + 4*a) - 4*cos(8*b*x + 8*a)*sin(4*b*x + 4*a))*co
s(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) - (cos(20*b*x + 20
*a)*cos(4*b*x + 4*a) + 4*cos(16*b*x + 16*a)*cos(4*b*x + 4*a) + 6*cos(12...
```

3.603.8 Giac [F(-1)]

Timed out.

$$\int \sec^4(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = \text{Timed out}$$

input
```
integrate(sec(2*b*x+2*a)^4*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorith
m="giac")
```

output
```
Timed out
```

3.603.9 Mupad [B] (verification not implemented)

Time = 34.13 (sec) , antiderivative size = 463, normalized size of antiderivative = 2.95

$$\int \sec^4(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$$
$$= -\frac{e^{a\,2i+b\,x\,2i}\sqrt{\frac{c\,(e^{a\,2i+b\,x\,2i}\,1i-i)\,(e^{a\,4i+b\,x\,4i}\,1i-i)}{(e^{a\,2i+b\,x\,2i}+1)\,(e^{a\,4i+b\,x\,4i}+1)}}\,16i}{35\,b\,(e^{a\,2i+b\,x\,2i}-1)}$$
$$+\frac{\left(\frac{8i}{7\,b}-\frac{e^{a\,2i+b\,x\,2i}\,8i}{7\,b}\right)\sqrt{\frac{c\,(e^{a\,2i+b\,x\,2i}\,1i-i)\,(e^{a\,4i+b\,x\,4i}\,1i-i)}{(e^{a\,2i+b\,x\,2i}+1)\,(e^{a\,4i+b\,x\,4i}+1)}}}{(e^{a\,2i+b\,x\,2i}-1)\,(e^{a\,4i+b\,x\,4i}+1)^3}$$
$$-\frac{\left(\frac{8i}{5\,b}-\frac{e^{a\,2i+b\,x\,2i}\,64i}{35\,b}\right)\sqrt{\frac{c\,(e^{a\,2i+b\,x\,2i}\,1i-i)\,(e^{a\,4i+b\,x\,4i}\,1i-i)}{(e^{a\,2i+b\,x\,2i}+1)\,(e^{a\,4i+b\,x\,4i}+1)}}}{(e^{a\,2i+b\,x\,2i}-1)\,(e^{a\,4i+b\,x\,4i}+1)^2}$$
$$-\frac{e^{a\,2i+b\,x\,2i}\sqrt{\frac{c\,(e^{a\,2i+b\,x\,2i}\,1i-i)\,(e^{a\,4i+b\,x\,4i}\,1i-i)}{(e^{a\,2i+b\,x\,2i}+1)\,(e^{a\,4i+b\,x\,4i}+1)}}\,8i}{35\,b\,(e^{a\,2i+b\,x\,2i}-1)\,(e^{a\,4i+b\,x\,4i}+1)}$$

input
```
int((c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2)/cos(2*a + 2*b*x)^4,x)
```

output
```
((8i/(7*b) - (exp(a*2i + b*x*2i)*8i)/(7*b))*((c*(exp(a*2i + b*x*2i)*1i - 1
i)*(exp(a*4i + b*x*4i)*1i - 1i))/((exp(a*2i + b*x*2i) + 1)*(exp(a*4i + b*x
*4i) + 1)))^(1/2))/((exp(a*2i + b*x*2i) - 1)*(exp(a*4i + b*x*4i) + 1)^3) -
 (exp(a*2i + b*x*2i)*((c*(exp(a*2i + b*x*2i)*1i - 1i)*(exp(a*4i + b*x*4i)*
1i - 1i))/((exp(a*2i + b*x*2i) + 1)*(exp(a*4i + b*x*4i) + 1)))^(1/2)*16i)/
(35*b*(exp(a*2i + b*x*2i) - 1)) - ((8i/(5*b) - (exp(a*2i + b*x*2i)*64i)/(3
5*b))*((c*(exp(a*2i + b*x*2i)*1i - 1i)*(exp(a*4i + b*x*4i)*1i - 1i))/((exp
(a*2i + b*x*2i) + 1)*(exp(a*4i + b*x*4i) + 1)))^(1/2))/((exp(a*2i + b*x*2i
) - 1)*(exp(a*4i + b*x*4i) + 1)^2) - (exp(a*2i + b*x*2i)*((c*(exp(a*2i + b
*x*2i)*1i - 1i)*(exp(a*4i + b*x*4i)*1i - 1i))/((exp(a*2i + b*x*2i) + 1)*(e
xp(a*4i + b*x*4i) + 1)))^(1/2)*8i)/(35*b*(exp(a*2i + b*x*2i) - 1)*(exp(a*4
i + b*x*4i) + 1))
```

3.604 $\int \sec^3(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$

3.604.1 Optimal result

Integrand size = 31, antiderivative size = 110

$$\int \sec^3(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = \frac{7c\tan(2a+2bx)}{15b\sqrt{-c+c\sec(2a+2bx)}}+\frac{2\sqrt{-c+c\sec(2a+2bx)}\tan(2a+2bx)}{15b}+\frac{(-c+c\sec(2a+2bx))^{3/2}\tan(2a+2bx)}{5bc}$$

output
```
1/5*(-c+c*sec(2*b*x+2*a))^(3/2)*tan(2*b*x+2*a)/b/c+7/15*c*tan(2*b*x+2*a)/b
/(-c+c*sec(2*b*x+2*a))^(1/2)+2/15*(-c+c*sec(2*b*x+2*a))^(1/2)*tan(2*b*x+2*
a)/b
```

3.604.2 Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.56

$$\int \sec^3(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = \frac{(5\cos(a+bx)+2\cos(5(a+bx)))\csc(a+bx)\sec^2(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}}{15b}$$

input
```
Integrate[Sec[2*(a + b*x)]^3*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

output
```
((5*Cos[a + b*x] + 2*Cos[5*(a + b*x)])*Csc[a + b*x]*Sec[2*(a + b*x)]^2*Sqr
t[c*Tan[a + b*x]*Tan[2*(a + b*x)]])/(15*b)
```

3.604.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.290, Rules used = {3042, 4897, 3042, 4287, 27, 3042, 4489, 3042, 4279}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(2a+2bx)^3\sqrt{c\tan(a+bx)\tan(2a+2bx)}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \sec^3(2a+2bx)\sqrt{c\sec(2a+2bx)-c}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(2a+2bx+\frac{\pi}{2}\right)^3\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}dx$$

$$\Big\downarrow \text{4287}$$

$$\frac{2\int \frac{1}{2}\sec(2a+2bx)\sqrt{c\sec(2a+2bx)-c}(2\sec(2a+2bx)c+3c)dx}{5c}+\frac{\tan(2a+2bx)(c\sec(2a+2bx)-c)^{3/2}}{5bc}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \sec(2a+2bx)\sqrt{c\sec(2a+2bx)-c}(2\sec(2a+2bx)c+3c)dx}{5c}+\frac{\tan(2a+2bx)(c\sec(2a+2bx)-c)^{3/2}}{5bc}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc\left(2a+2bx+\frac{\pi}{2}\right)\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}\left(2\csc\left(2a+2bx+\frac{\pi}{2}\right)c+3c\right)dx}{5c}+\frac{\tan(2a+2bx)(c\sec(2a+2bx)-c)^{3/2}}{5bc}$$

$$\Big\downarrow \text{4489}$$

$$\frac{\frac{7}{3}c\int \sec(2a+2bx)\sqrt{c\sec(2a+2bx)-c}dx+\frac{2c\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{3b}}{5c}+\frac{\tan(2a+2bx)(c\sec(2a+2bx)-c)^{3/2}}{5bc}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{7}{3}c\int \csc\left(2a+2bx+\frac{\pi}{2}\right)\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}dx+\frac{2c\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{3b}}{5c}+\frac{\tan(2a+2bx)(c\sec(2a+2bx)-c)^{3/2}}{5bc}$$

$$\Big\downarrow \text{4279}$$

$$\frac{\frac{7c^2\tan(2a+2bx)}{3b\sqrt{c\sec(2a+2bx)-c}}+\frac{2c\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{3b}}{5c}+\frac{\tan(2a+2bx)(c\sec(2a+2bx)-c)^{3/2}}{5bc}$$

input
```
Int[Sec[2*(a + b*x)]^3*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

output
```
((-c + c*Sec[2*a + 2*b*x])^(3/2)*Tan[2*a + 2*b*x])/(5*b*c) + ((7*c^2*Tan[2
*a + 2*b*x])/(3*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]) + (2*c*Sqrt[-c + c*Sec[2*
a + 2*b*x]]*Tan[2*a + 2*b*x])/(3*b))/(5*c)
```

3.604.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4279
```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

3.604. $\int \sec^3(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$

rule 4287
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(
m + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 -
b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 4489
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[(a*B*m + A*b*(m + 1))/(b*(m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B
, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b
*(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.604.4 Maple [A] (verified)

Time = 6.55 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.73

method	result	size
default	$\frac{\sqrt{2}\,\cot(xb+a)\sqrt{\frac{c\sin(xb+a)^2}{2\cos(xb+a)^2-1}}\left(32\cos(xb+a)^4-40\cos(xb+a)^2+15\right)\sqrt{4}}{30b\left(2\cos(xb+a)^2-1\right)^2}$	80

input
```
int(sec(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
1/30*2^(1/2)/b*cot(b*x+a)*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(1/2)*(32*co
s(b*x+a)^4-40*cos(b*x+a)^2+15)/(2*cos(b*x+a)^2-1)^2*4^(1/2)
```

3.604.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.76

$$\int \sec^3(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = \frac{\sqrt{2}\left(15\,\tan\left(bx+a\right)^4-10\,\tan\left(bx+a\right)^2+7\right)\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}}{15\left(b\tan\left(bx+a\right)^5-2\,b\tan\left(bx+a\right)^3+b\tan\left(bx+a\right)\right)}$$

input
```
integrate(sec(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorith
m="fricas")
```

output
```
1/15*sqrt(2)*(15*tan(b*x + a)^4 - 10*tan(b*x + a)^2 + 7)*sqrt(-c*tan(b*x +
 a)^2/(tan(b*x + a)^2 - 1))/(b*tan(b*x + a)^5 - 2*b*tan(b*x + a)^3 + b*tan
(b*x + a))
```

3.604.6 Sympy [F(-1)]

Timed out.

$$\int \sec^3(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = \text{Timed out}$$

input
```
integrate(sec(2*b*x+2*a)**3*(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
```

output
```
Timed out
```

3.604.7 Maxima [F]

$$\int \sec^3(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = \int \sqrt{c\tan\left(2\,bx+2\,a\right)\tan\left(bx+a\right)}\sec\left(2\,bx+2\,a\right)^3\,dx$$

input
```
integrate(sec(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorith
m="maxima")
```

output
```
4/15*(30*(b*cos(4*b*x + 4*a)^2 + b*sin(4*b*x + 4*a)^2 + 2*b*cos(4*b*x + 4*
a) + b)*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)
^(1/4)*sqrt(c)*integrate(-(((cos(16*b*x + 16*a)*cos(4*b*x + 4*a) + 3*cos(1
2*b*x + 12*a)*cos(4*b*x + 4*a) + 3*cos(8*b*x + 8*a)*cos(4*b*x + 4*a) + cos
(4*b*x + 4*a)^2 + sin(16*b*x + 16*a)*sin(4*b*x + 4*a) + 3*sin(12*b*x + 12*
a)*sin(4*b*x + 4*a) + 3*sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + sin(4*b*x + 4*
a)^2)*cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) + (cos(4*b
*x + 4*a)*sin(16*b*x + 16*a) + 3*cos(4*b*x + 4*a)*sin(12*b*x + 12*a) + 3*c
os(4*b*x + 4*a)*sin(8*b*x + 8*a) - cos(16*b*x + 16*a)*sin(4*b*x + 4*a) - 3
*cos(12*b*x + 12*a)*sin(4*b*x + 4*a) - 3*cos(8*b*x + 8*a)*sin(4*b*x + 4*a)
)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)))*cos(3/2*arcta
n2(sin(4*b*x + 4*a), cos(4*b*x + 4*a))) + ((cos(4*b*x + 4*a)*sin(16*b*x +
16*a) + 3*cos(4*b*x + 4*a)*sin(12*b*x + 12*a) + 3*cos(4*b*x + 4*a)*sin(8*b
*x + 8*a) - cos(16*b*x + 16*a)*sin(4*b*x + 4*a) - 3*cos(12*b*x + 12*a)*sin
(4*b*x + 4*a) - 3*cos(8*b*x + 8*a)*sin(4*b*x + 4*a))*cos(1/2*arctan2(sin(4
*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) - (cos(16*b*x + 16*a)*cos(4*b*x + 4*a
) + 3*cos(12*b*x + 12*a)*cos(4*b*x + 4*a) + 3*cos(8*b*x + 8*a)*cos(4*b*x +
 4*a) + cos(4*b*x + 4*a)^2 + sin(16*b*x + 16*a)*sin(4*b*x + 4*a) + 3*sin(1
2*b*x + 12*a)*sin(4*b*x + 4*a) + 3*sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + sin
(4*b*x + 4*a)^2)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - ...
```

3.604.8 Giac [F(-1)]

Timed out.

$$\int \sec^3(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = \text{Timed out}$$

input
```
integrate(sec(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorith
m="giac")
```

output
```
Timed out
```

3.604.9 Mupad [B] (verification not implemented)

Time = 43.08 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.35

$$\int \sec^3(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$$
$$=\frac{4\left(e^{a\,4i+b\,x\,4i}\,5i+e^{a\,6i+b\,x\,6i}\,5i+e^{a\,10i+b\,x\,10i}\,2i+2i\right)\sqrt{\frac{c\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)\left(e^{a\,4i+b\,x\,4i}\,1i-i\right)}{\left(e^{a\,2i+b\,x\,2i}+1\right)\left(e^{a\,4i+b\,x\,4i}+1\right)}}}{15\,b\,\left(e^{a\,2i+b\,x\,2i}-1\right)\left(e^{a\,4i+b\,x\,4i}+1\right)^2}$$

input
```
int((c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2)/cos(2*a + 2*b*x)^3,x)
```

output
```
(4*(exp(a*4i + b*x*4i)*5i + exp(a*6i + b*x*6i)*5i + exp(a*10i + b*x*10i)*2
i + 2i)*((c*(exp(a*2i + b*x*2i)*1i - 1i)*(exp(a*4i + b*x*4i)*1i - 1i))/((e
xp(a*2i + b*x*2i) + 1)*(exp(a*4i + b*x*4i) + 1)))^(1/2))/(15*b*(exp(a*2i +
 b*x*2i) - 1)*(exp(a*4i + b*x*4i) + 1)^2)
```

3.605 $\int \sec^2(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$

3.605.1 Optimal result

Integrand size = 31, antiderivative size = 72

$$\int \sec^2(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = -\frac{c\tan(2a+2bx)}{3b\sqrt{-c+c\sec(2a+2bx)}}+\frac{\sqrt{-c+c\sec(2a+2bx)}\tan(2a+2bx)}{3b}$$

output
```
-1/3*c*tan(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)+1/3*(-c+c*sec(2*b*x+2*
a))^(1/2)*tan(2*b*x+2*a)/b
```

3.605.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.61

$$\int \sec^2(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = \frac{\sqrt{c\tan(a+bx)\tan(2(a+bx))}(-\cot(a+bx)+\tan(2(a+bx)))}{3b}$$

input
```
Integrate[Sec[2*(a + b*x)]^2*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

output
```
(Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]]*(-Cot[a + b*x] + Tan[2*(a + b*x)]))
/(3*b)
```

3.605.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3042, 4897, 3042, 4285, 3042, 4279}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(2a+2bx)^2\sqrt{c\tan(a+bx)\tan(2a+2bx)}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \sec^2(2a+2bx)\sqrt{c\sec(2a+2bx)-c}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(2a+2bx+\frac{\pi}{2}\right)^2\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}dx$$

$$\Big\downarrow \text{4285}$$

$$\frac{\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{3b}-\frac{1}{3}\int \sec(2a+2bx)\sqrt{c\sec(2a+2bx)-c}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{3b}-\frac{1}{3}\int \csc\left(2a+2bx+\frac{\pi}{2}\right)\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}dx$$

$$\Big\downarrow \text{4279}$$

$$\frac{\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{3b}-\frac{c\tan(2a+2bx)}{3b\sqrt{c\sec(2a+2bx)-c}}$$

input
```
Int[Sec[2*(a + b*x)]^2*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

output
```
-1/3*(c*Tan[2*a + 2*b*x])/(b*Sqrt[-c + c*Sec[2*a + 2*b*x]]) + (Sqrt[-c + c
*Sec[2*a + 2*b*x]]*Tan[2*a + 2*b*x])/(3*b)
```

3.605.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4279
```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4285
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x]
+ Simp[a*(m/(b*(m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x]
/; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.605.4 Maple [A] (verified)

Time = 6.75 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.97

method	result	size
default	$-\frac{\sqrt{2}\,\cot(xb+a)\sqrt{\frac{c\sin(xb+a)^2}{2\cos(xb+a)^2-1}}\left(4\cos(xb+a)^2-3\right)\sqrt{4}}{6b\left(2\cos(xb+a)^2-1\right)}$	70

input
```
int(sec(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
-1/6*2^(1/2)/b*cot(b*x+a)*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(1/2)*(4*cos
(b*x+a)^2-3)/(2*cos(b*x+a)^2-1)*4^(1/2)
```

3.605.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.89

$$\int \sec^2(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = -\frac{\sqrt{2}\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(3\tan\left(bx+a\right)^2-1\right)}{3\left(b\tan\left(bx+a\right)^3-b\tan\left(bx+a\right)\right)}$$

input
```
integrate(sec(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorith
m="fricas")
```

output
```
-1/3*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(3*tan(b*x + a)^
2 - 1)/(b*tan(b*x + a)^3 - b*tan(b*x + a))
```

3.605.6 Sympy [F(-1)]

Timed out.

$$\int \sec^2(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = \text{Timed out}$$

input
```
integrate(sec(2*b*x+2*a)**2*(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
```

output
```
Timed out
```

3.605.7 Maxima [F]

$$\int \sec^2(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = \int \sqrt{c\tan\left(2\,bx+2\,a\right)\tan\left(bx+a\right)}\sec\left(2\,bx+2\,a\right)^2\,dx$$

input
```
integrate(sec(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorith
m="maxima")
```

output
```
-2/3*(6*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)
^(3/4)*b*sqrt(c)*integrate(-(((cos(12*b*x + 12*a)*cos(4*b*x + 4*a) + 2*cos
(8*b*x + 8*a)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + sin(12*b*x + 12*a)*s
in(4*b*x + 4*a) + 2*sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + sin(4*b*x + 4*a)^2
)*cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) + (cos(4*b*x +
 4*a)*sin(12*b*x + 12*a) + 2*cos(4*b*x + 4*a)*sin(8*b*x + 8*a) - cos(12*b*
x + 12*a)*sin(4*b*x + 4*a) - 2*cos(8*b*x + 8*a)*sin(4*b*x + 4*a))*sin(1/2*
arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)))*cos(3/2*arctan2(sin(4*b
*x + 4*a), cos(4*b*x + 4*a))) + ((cos(4*b*x + 4*a)*sin(12*b*x + 12*a) + 2*
cos(4*b*x + 4*a)*sin(8*b*x + 8*a) - cos(12*b*x + 12*a)*sin(4*b*x + 4*a) -
2*cos(8*b*x + 8*a)*sin(4*b*x + 4*a))*cos(1/2*arctan2(sin(4*b*x + 4*a), -co
s(4*b*x + 4*a) - 1)) - (cos(12*b*x + 12*a)*cos(4*b*x + 4*a) + 2*cos(8*b*x
+ 8*a)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + sin(12*b*x + 12*a)*sin(4*b*
x + 4*a) + 2*sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + sin(4*b*x + 4*a)^2)*sin(1
/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)))*sin(3/2*arctan2(sin(
4*b*x + 4*a), cos(4*b*x + 4*a))))/(((2*(2*cos(8*b*x + 8*a) + cos(4*b*x + 4
*a))*cos(12*b*x + 12*a) + cos(12*b*x + 12*a)^2 + 4*cos(8*b*x + 8*a)^2 + 4*
cos(8*b*x + 8*a)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 2*(2*sin(8*b*x +
8*a) + sin(4*b*x + 4*a))*sin(12*b*x + 12*a) + sin(12*b*x + 12*a)^2 + 4*sin
(8*b*x + 8*a)^2 + 4*sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + sin(4*b*x + 4*a...
```

3.605.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 413066 vs. $2(64) = 128$.

Time = 142.29 (sec) , antiderivative size = 413066, normalized size of antiderivative = 5737.03

$$\int \sec^2(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = \text{Too large to display}$$

input
```
integrate(sec(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorith
m="giac")
```

output
```
1/3*sqrt(2)*(((((((c^5*sgn(tan(1/2*b*x + 2*a)^4*tan(1/2*a)^12 - 66*tan(1/2
*b*x + 2*a)^4*tan(1/2*a)^10 + 48*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^11 - 6*ta
n(1/2*b*x + 2*a)^2*tan(1/2*a)^12 + 495*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^8 -
 880*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^9 + 396*tan(1/2*b*x + 2*a)^2*tan(1/2*
a)^10 - 48*tan(1/2*b*x + 2*a)*tan(1/2*a)^11 + tan(1/2*a)^12 - 924*tan(1/2*
b*x + 2*a)^4*tan(1/2*a)^6 + 3168*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^7 - 2970*
tan(1/2*b*x + 2*a)^2*tan(1/2*a)^8 + 880*tan(1/2*b*x + 2*a)*tan(1/2*a)^9 -
66*tan(1/2*a)^10 + 495*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^4 - 3168*tan(1/2*b*
x + 2*a)^3*tan(1/2*a)^5 + 5544*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^6 - 3168*ta
n(1/2*b*x + 2*a)*tan(1/2*a)^7 + 495*tan(1/2*a)^8 - 66*tan(1/2*b*x + 2*a)^4
*tan(1/2*a)^2 + 880*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^3 - 2970*tan(1/2*b*x +
 2*a)^2*tan(1/2*a)^4 + 3168*tan(1/2*b*x + 2*a)*tan(1/2*a)^5 - 924*tan(1/2*
a)^6 + tan(1/2*b*x + 2*a)^4 - 48*tan(1/2*b*x + 2*a)^3*tan(1/2*a) + 396*tan
(1/2*b*x + 2*a)^2*tan(1/2*a)^2 - 880*tan(1/2*b*x + 2*a)*tan(1/2*a)^3 + 495
*tan(1/2*a)^4 - 6*tan(1/2*b*x + 2*a)^2 + 48*tan(1/2*b*x + 2*a)*tan(1/2*a)
- 66*tan(1/2*a)^2 + 1)*sgn(-3*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^5 + tan(1/2*
b*x + 2*a)*tan(1/2*a)^6 + 10*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^3 - 15*tan(1/
2*b*x + 2*a)*tan(1/2*a)^4 + 3*tan(1/2*a)^5 - 3*tan(1/2*b*x + 2*a)^2*tan(1/
2*a) + 15*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 - 10*tan(1/2*a)^3 - tan(1/2*b*x
+ 2*a) + 3*tan(1/2*a))*tan(1/2*a)^174 - 147*c^5*sgn(tan(1/2*b*x + 2*a)^...
```

3.605.9 Mupad [B] (verification not implemented)

Time = 32.89 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.79

$$\int \sec^2(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$$
$$= -\frac{2\left(e^{a\,6i+b\,x\,6i}\,1i+1i\right)\sqrt{\frac{c\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)\left(e^{a\,4i+b\,x\,4i}\,1i-i\right)}{\left(e^{a\,2i+b\,x\,2i}+1\right)\left(e^{a\,4i+b\,x\,4i}+1\right)}}}{3\,b\,\left(e^{a\,2i+b\,x\,2i}-e^{a\,4i+b\,x\,4i}+e^{a\,6i+b\,x\,6i}-1\right)}$$

input
```
int((c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2)/cos(2*a + 2*b*x)^2,x)
```

output
```
-(2*(exp(a*6i + b*x*6i)*1i + 1i)*((c*(exp(a*2i + b*x*2i)*1i - 1i)*(exp(a*4
i + b*x*4i)*1i - 1i))/((exp(a*2i + b*x*2i) + 1)*(exp(a*4i + b*x*4i) + 1)))
^(1/2))/(3*b*(exp(a*2i + b*x*2i) - exp(a*4i + b*x*4i) + exp(a*6i + b*x*6i)
 - 1))
```

3.606 $\int \sec(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$

3.606.1 Optimal result

Integrand size = 29, antiderivative size = 33

$$\int \sec(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = \frac{c\tan(2a+2bx)}{b\sqrt{-c+c\sec(2a+2bx)}}$$

output
```
c*tan(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)
```

3.606.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

$$\int \sec(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = \frac{\cot(a+bx)\sqrt{c\tan(a+bx)\tan(2(a+bx))}}{b}$$

input
```
Integrate[Sec[2*(a + b*x)]*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

output
```
(Cot[a + b*x]*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]])/b
```

3.606.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {3042, 4897, 3042, 4279}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sec(2a+2bx)\sqrt{c\tan(a+bx)\tan(2a+2bx)}dx$$

$$\Big\downarrow \; 4897$$

$$\int \sec(2a+2bx)\sqrt{c\sec(2a+2bx)-c}dx$$

$$\Big\downarrow \; 3042$$

$$\int \csc\left(2a+2bx+\frac{\pi}{2}\right)\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}dx$$

$$\Big\downarrow \; 4279$$

$$\frac{c\tan(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}$$

input
```
Int[Sec[2*(a + b*x)]*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

output
```
(c*Tan[2*a + 2*b*x])/(b*Sqrt[-c + c*Sec[2*a + 2*b*x]])
```

3.606.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4279
```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.606.4 Maple [A] (verified)

Time = 6.32 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.33

method	result	size
default	$\frac{\sqrt{2}\,\cot(xb+a)\sqrt{\frac{c\sin(xb+a)^2}{2\cos(xb+a)^2-1}}\,\sqrt{4}}{2b}$	44

input
```
int(sec(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
1/2*2^(1/2)/b*cot(b*x+a)*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(1/2)*4^(1/2)
```

3.606.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.21

$$\int \sec(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = \frac{\sqrt{2}\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}}{b\tan(bx+a)}$$

input
```
integrate(sec(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm=
"fricas")
```

output
```
sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))/(b*tan(b*x + a))
```

3.606.6 Sympy [F(-1)]

Timed out.

$$\int \sec(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = \text{Timed out}$$

input
```
integrate(sec(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
```

output
```
Timed out
```

3.606.7 Maxima [F]

$$\int \sec(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$$
$$=\int \sqrt{c\tan\left(2\,bx+2\,a\right)\tan\left(bx+a\right)}\sec\left(2\,bx+2\,a\right)\,dx$$

input
```
integrate(sec(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm=
"maxima")
```

output
```
(2*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4
)*b*sqrt(c)*integrate(-(((cos(8*b*x + 8*a)*cos(4*b*x + 4*a) + cos(4*b*x +
4*a)^2 + sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + sin(4*b*x + 4*a)^2)*cos(1/2*a
rctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) + (cos(4*b*x + 4*a)*sin(8
*b*x + 8*a) - cos(8*b*x + 8*a)*sin(4*b*x + 4*a))*sin(1/2*arctan2(sin(4*b*x
 + 4*a), -cos(4*b*x + 4*a) - 1)))*cos(1/2*arctan2(sin(4*b*x + 4*a), cos(4*
b*x + 4*a))) + ((cos(4*b*x + 4*a)*sin(8*b*x + 8*a) - cos(8*b*x + 8*a)*sin(
4*b*x + 4*a))*cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) -
(cos(8*b*x + 8*a)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + sin(8*b*x + 8*a)
*sin(4*b*x + 4*a) + sin(4*b*x + 4*a)^2)*sin(1/2*arctan2(sin(4*b*x + 4*a),
-cos(4*b*x + 4*a) - 1)))*sin(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a
))))/(((cos(8*b*x + 8*a)^2 + 2*cos(8*b*x + 8*a)*cos(4*b*x + 4*a) + cos(4*b
*x + 4*a)^2 + sin(8*b*x + 8*a)^2 + 2*sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + s
in(4*b*x + 4*a)^2)*cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1
))^2 + (cos(8*b*x + 8*a)^2 + 2*cos(8*b*x + 8*a)*cos(4*b*x + 4*a) + cos(4*b
*x + 4*a)^2 + sin(8*b*x + 8*a)^2 + 2*sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + s
in(4*b*x + 4*a)^2)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1
))^2)*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(
1/4)), x) - sqrt(c)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) -
1)))/((cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1...
```

3.606.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80097 vs. $2(31) = 62$.

Time = 37.40 (sec) , antiderivative size = 80097, normalized size of antiderivative = 2427.18

$$\int \sec(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = \text{Too large to display}$$

input
```
integrate(sec(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm=
"giac")
```

output
```
-sqrt(2)*(((c*sgn(tan(1/2*b*x + 2*a)^4*tan(1/2*a)^12 - 66*tan(1/2*b*x + 2*
a)^4*tan(1/2*a)^10 + 48*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^11 - 6*tan(1/2*b*x
 + 2*a)^2*tan(1/2*a)^12 + 495*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^8 - 880*tan(
1/2*b*x + 2*a)^3*tan(1/2*a)^9 + 396*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^10 - 4
8*tan(1/2*b*x + 2*a)*tan(1/2*a)^11 + tan(1/2*a)^12 - 924*tan(1/2*b*x + 2*a
)^4*tan(1/2*a)^6 + 3168*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^7 - 2970*tan(1/2*b
*x + 2*a)^2*tan(1/2*a)^8 + 880*tan(1/2*b*x + 2*a)*tan(1/2*a)^9 - 66*tan(1/
2*a)^10 + 495*tan(1/2*b*x + 2*a)^4*tan(1/2*a)^4 - 3168*tan(1/2*b*x + 2*a)^
3*tan(1/2*a)^5 + 5544*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^6 - 3168*tan(1/2*b*x
 + 2*a)*tan(1/2*a)^7 + 495*tan(1/2*a)^8 - 66*tan(1/2*b*x + 2*a)^4*tan(1/2*
a)^2 + 880*tan(1/2*b*x + 2*a)^3*tan(1/2*a)^3 - 2970*tan(1/2*b*x + 2*a)^2*t
an(1/2*a)^4 + 3168*tan(1/2*b*x + 2*a)*tan(1/2*a)^5 - 924*tan(1/2*a)^6 + ta
n(1/2*b*x + 2*a)^4 - 48*tan(1/2*b*x + 2*a)^3*tan(1/2*a) + 396*tan(1/2*b*x
+ 2*a)^2*tan(1/2*a)^2 - 880*tan(1/2*b*x + 2*a)*tan(1/2*a)^3 + 495*tan(1/2*
a)^4 - 6*tan(1/2*b*x + 2*a)^2 + 48*tan(1/2*b*x + 2*a)*tan(1/2*a) - 66*tan(
1/2*a)^2 + 1)*sgn(-3*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^5 + tan(1/2*b*x + 2*a
)*tan(1/2*a)^6 + 10*tan(1/2*b*x + 2*a)^2*tan(1/2*a)^3 - 15*tan(1/2*b*x + 2
*a)*tan(1/2*a)^4 + 3*tan(1/2*a)^5 - 3*tan(1/2*b*x + 2*a)^2*tan(1/2*a) + 15
*tan(1/2*b*x + 2*a)*tan(1/2*a)^2 - 10*tan(1/2*a)^3 - tan(1/2*b*x + 2*a) +
3*tan(1/2*a))*tan(1/2*a)^78 + 21*c*sgn(tan(1/2*b*x + 2*a)^4*tan(1/2*a)^...
```

3.606.9 Mupad [B] (verification not implemented)

Time = 27.42 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.64

$$\int \sec(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = -\frac{\sin\left(2\,a+2\,b\,x\right)\,\sqrt{\frac{c\,\left(\cos(2\,a+2\,b\,x)-\cos(6\,a+6\,b\,x)\right)}{3\,\cos(2\,a+2\,b\,x)+2\,\cos(4\,a+4\,b\,x)+\cos(6\,a+6\,b\,x)+2}}}{b\,\left(\cos\left(2\,a+2\,b\,x\right)-1\right)}$$

input
```
int((c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2)/cos(2*a + 2*b*x),x)
```

output
```
-(sin(2*a + 2*b*x)*((c*(cos(2*a + 2*b*x) - cos(6*a + 6*b*x)))/(3*cos(2*a +
 2*b*x) + 2*cos(4*a + 4*b*x) + cos(6*a + 6*b*x) + 2))^(1/2))/(b*(cos(2*a +
 2*b*x) - 1))
```

3.607 $\int \sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$

3.607.1 Optimal result

Integrand size = 20, antiderivative size = 45

$$\int \sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = -\frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{b}$$

output
```
-arctanh(c^(1/2)*tan(2*b*x+2*a)/(-c+c*sec(2*b*x+2*a))^(1/2))*c^(1/2)/b
```

3.607.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.62

$$\int \sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = -\frac{\text{arctanh}\left(\frac{\sqrt{2}\cos(a+bx)}{\sqrt{\cos(2(a+bx))}}\right)\sqrt{\cos(2(a+bx))}\csc(a+bx)\sqrt{c\tan(a+bx)\tan(2(a+bx))}}{\sqrt{2}b}$$

input
```
Integrate[Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

output
```
-((ArcTanh[(Sqrt[2]*Cos[a + b*x])/Sqrt[Cos[2*(a + b*x)]]]*Sqrt[Cos[2*(a +
b*x)]]*Csc[a + b*x]*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]])/(Sqrt[2]*b))
```

3.607.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 4897, 3042, 4261, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{c\tan(a+bx)\tan(2a+2bx)}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \sqrt{c\sec(2a+2bx)-c}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}dx$$

$$\Big\downarrow \text{4261}$$

$$-\frac{c\int \frac{1}{\frac{c^2\tan^2(2a+2bx)}{c\sec(2a+2bx)-c}-c}d\left(-\frac{c\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{b}$$

$$\Big\downarrow \text{220}$$

$$-\frac{\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{b}$$

input
```
Int[Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

output
```
-((Sqrt[c]*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]
]])/b)
```

3.607.3.1 Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.607.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(39) = 78$.

Time = 7.24 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.71

method	result	size
default	$\frac{\sqrt{\frac{c\sin(xb+a)^2}{2\cos(xb+a)^2-1}}\,\sin(xb+a)\sqrt{\frac{2\cos(xb+a)^2-1}{(1+\cos(xb+a))^2}}\,\operatorname{arctanh}\left(\frac{\cos(xb+a)\sqrt{2}}{(1+\cos(xb+a))\sqrt{\frac{2\cos(xb+a)^2-1}{(1+\cos(xb+a))^2}}}\right)\sqrt{4}}{2b(\cos(xb+a)-1)}$	122

input
```
int((c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/b*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(1/2)*sin(b*x+a)*((2*cos(b*x+a)^
2-1)/(1+cos(b*x+a))^2)^(1/2)*arctanh(cos(b*x+a)/(1+cos(b*x+a))/((2*cos(b*x
+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*2^(1/2))/(cos(b*x+a)-1)*4^(1/2)
```

3.607.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 201, normalized size of antiderivative = 4.47

$$\int \sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$$
$$= \left[\frac{\sqrt{c}\log\left(-\frac{c\tan(bx+a)^5-14\,c\tan(bx+a)^3-4\sqrt{2}\left(\tan(bx+a)^4-4\tan(bx+a)^2+3\right)\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\sqrt{c}+17\,c\tan(bx+a)}{\tan(bx+a)^5+2\tan(bx+a)^3+\tan(bx+a)}\right)}{4\,b}, \sqrt{-c}\,\mathrm{arcta}\ldots\right.$$

input
```
integrate((c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="fricas")
```

output
```
[1/4*sqrt(c)*log(-(c*tan(b*x + a)^5 - 14*c*tan(b*x + a)^3 - 4*sqrt(2)*(tan
(b*x + a)^4 - 4*tan(b*x + a)^2 + 3)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2
 - 1))*sqrt(c) + 17*c*tan(b*x + a))/(tan(b*x + a)^5 + 2*tan(b*x + a)^3 + t
an(b*x + a)))/b, 1/2*sqrt(-c)*arctan(2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan
(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a)^3 - 3*c*ta
n(b*x + a)))/b]
```

3.607.6 Sympy [F(-2)]

Exception generated.

$$\int \sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.607.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(39) = 78$.

Time = 0.35 (sec) , antiderivative size = 430, normalized size of antiderivative = 9.56

$$\int \sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$$
$$= \frac{\sqrt{c}\left(\log\left(4\sqrt{\cos\left(4\,bx+4\,a\right)^2+\sin\left(4\,bx+4\,a\right)^2+2\,\cos\left(4\,bx+4\,a\right)+1}\cos\left(\frac{1}{2}\arctan\left(\sin\left(4\,bx+4\,a\right),c\right.\right.\right.\right.}{}$$

input
```
integrate((c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="maxima")
```

output
```
1/4*sqrt(c)*(log(4*sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*
b*x + 4*a) + 1)*cos(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a) + 1))^2
 + 4*sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1
)*sin(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a) + 1))^2 + 8*(cos(4*b*
x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)*cos(1/2*ar
ctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a) + 1)) + 4) - log(cos(2*b*x + 2*a)
^2 + sin(2*b*x + 2*a)^2 + sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2
*cos(4*b*x + 4*a) + 1)*(cos(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a)
 + 1))^2 + sin(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a) + 1))^2) + 2
*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)*
(cos(2*b*x + 2*a)*cos(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a) + 1))
 + sin(2*b*x + 2*a)*sin(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a) + 1
)))))/b
```

3.607.8 Giac [F]

$$\int \sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = \int \sqrt{c\tan\left(2\,bx+2\,a\right)\tan\left(bx+a\right)}\,dx$$

input
```
integrate((c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*tan(2*b*x + 2*a)*tan(b*x + a)), x)
```

3.607.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = \int \sqrt{c\tan\left(a+b\,x\right)\,\tan\left(2\,a+2\,b\,x\right)}\,dx$$

input
```
int((c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2),x)
```

output
```
int((c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2), x)
```

3.608 $\int \cos(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$

3.608.1 Optimal result

Integrand size = 29, antiderivative size = 84

$$\int \cos(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = \frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{2b} - \frac{c\sin(2a+2bx)}{2b\sqrt{-c+c\sec(2a+2bx)}}$$

output
```
1/2*arctanh(c^(1/2)*tan(2*b*x+2*a)/(-c+c*sec(2*b*x+2*a))^(1/2))*c^(1/2)/b-
1/2*c*sin(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)
```

3.608.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.10

$$\int \cos(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = -\frac{\left(\cos(a+bx)-\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\cos(a+bx)}{\sqrt{\cos(2(a+bx))}}\right)\sqrt{\cos(2(a+bx))}+\cos(3(a+bx))\right)\csc(a+bx)\sqrt{c\tan(a+b}}{4b}$$

input
```
Integrate[Cos[2*(a + b*x)]*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

output
```
-1/4*((Cos[a + b*x] - Sqrt[2]*ArcTanh[(Sqrt[2]*Cos[a + b*x])/Sqrt[Cos[2*(a
 + b*x)]]]*Sqrt[Cos[2*(a + b*x)]] + Cos[3*(a + b*x)])*Csc[a + b*x]*Sqrt[c*
Tan[a + b*x]*Tan[2*(a + b*x)]])/b
```

3.608.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {3042, 4897, 3042, 4292, 3042, 4261, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(2a+2bx)\sqrt{c\tan(a+bx)\tan(2a+2bx)}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \cos(2a+2bx)\sqrt{c\sec(2a+2bx)-c}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}}{\csc\left(2a+2bx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4292}$$

$$-\frac{1}{2}\int \sqrt{c\sec(2a+2bx)-c}dx-\frac{c\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{2}\int \sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}dx-\frac{c\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}$$

$$\Big\downarrow \text{4261}$$

$$\frac{c\int \frac{1}{\frac{c^2\tan^2(2a+2bx)}{c\sec(2a+2bx)-c}-c}d\left(-\frac{c\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{2b}-\frac{c\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}$$

$$\Big\downarrow \text{220}$$

$$\frac{\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{2b}-\frac{c\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}$$

input
```
Int[Cos[2*(a + b*x)]*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

output
```
(Sqrt[c]*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]]]
)/(2*b) - (c*Sin[2*a + 2*b*x])/(2*b*Sqrt[-c + c*Sec[2*a + 2*b*x]])
```

3.608.3.1 Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4292
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Simp[a*((2*n + 1)/(2*b*d*n))   Int[Sqrt[a + b*Csc
[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.608.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(72) = 144$.

Time = 6.33 (sec) , antiderivative size = 341, normalized size of antiderivative = 4.06

method	result
default	$-\frac{\sqrt{\frac{c\sin(xb+a)^2}{2\cos(xb+a)^2-1}}\,\sin(xb+a)\sqrt{\frac{2\cos(xb+a)^2-1}{(1+\cos(xb+a))^2}}\,\operatorname{arctanh}\left(\frac{\cos(xb+a)\sqrt{2}}{(1+\cos(xb+a))\sqrt{\frac{2\cos(xb+a)^2-1}{(1+\cos(xb+a))^2}}}\right)\sqrt{4}}{2b(\cos(xb+a)-1)}-\frac{\sqrt{2}\,\csc(xb+a)\left(\sqrt{2}\,\operatorname{arctanh}\left(\right.\right.}{}$

input
```
int(cos(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
-1/2/b*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(1/2)*sin(b*x+a)*((2*cos(b*x+a)
^2-1)/(1+cos(b*x+a))^2)^(1/2)*arctanh(cos(b*x+a)/(1+cos(b*x+a))/((2*cos(b*
x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*2^(1/2))/(cos(b*x+a)-1)*4^(1/2)-1/8*2^(1
/2)/b*csc(b*x+a)*(2^(1/2)*arctanh(cos(b*x+a)/(1+cos(b*x+a))/((2*cos(b*x+a)
^2-1)/(1+cos(b*x+a))^2)^(1/2)*2^(1/2))*((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^
2)^(1/2)*cos(b*x+a)+2^(1/2)*arctanh(cos(b*x+a)/(1+cos(b*x+a))/((2*cos(b*x+
a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*2^(1/2))*((2*cos(b*x+a)^2-1)/(1+cos(b*x+a)
)^2)^(1/2)+4*cos(b*x+a)^3-2*cos(b*x+a))*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1)
)^(1/2)*4^(1/2)
```

3.608.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(72) = 144$.

Time = 0.29 (sec) , antiderivative size = 351, normalized size of antiderivative = 4.18

$$\int \cos(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$$

$$= \left[\frac{\left(\tan(bx+a)^3+\tan(bx+a)\right)\sqrt{c}\log\left(-\frac{c\tan(bx+a)^5-14\,c\tan(bx+a)^3+4\sqrt{2}\left(\tan(bx+a)^4-4\tan(bx+a)^2+3\right)\sqrt{-\frac{c\tan(bx+a}{\tan(bx+a)^2}}}{\tan(bx+a)^5+2\tan(bx+a)^3+\tan(bx+a)}\right.}{8\left(b\tan(bx+a)^3+b\tan(bx+a)\right)}\right.$$

$$-\frac{\left(\tan(bx+a)^3+\tan(bx+a)\right)\sqrt{-c}\arctan\left(\frac{2\sqrt{2}\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{-c}}{c\tan(bx+a)^3-3\,c\tan(bx+a)}\right)-2\sqrt{2}\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-}}}{4\left(b\tan(bx+a)^3+b\tan(bx+a)\right)}$$

input
```
integrate(cos(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm=
"fricas")
```

output
```
[1/8*((tan(b*x + a)^3 + tan(b*x + a))*sqrt(c)*log(-(c*tan(b*x + a)^5 - 14*
c*tan(b*x + a)^3 + 4*sqrt(2)*(tan(b*x + a)^4 - 4*tan(b*x + a)^2 + 3)*sqrt(
-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*sqrt(c) + 17*c*tan(b*x + a))/(tan(
b*x + a)^5 + 2*tan(b*x + a)^3 + tan(b*x + a))) + 4*sqrt(2)*sqrt(-c*tan(b*x
 + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1))/(b*tan(b*x + a)^3 + b*
tan(b*x + a)), -1/4*((tan(b*x + a)^3 + tan(b*x + a))*sqrt(-c)*arctan(2*sqr
t(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqr
t(-c)/(c*tan(b*x + a)^3 - 3*c*tan(b*x + a))) - 2*sqrt(2)*sqrt(-c*tan(b*x +
 a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1))/(b*tan(b*x + a)^3 + b*ta
n(b*x + a))]
```

3.608.6 Sympy [F(-1)]

Timed out.

$$\int \cos(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = \text{Timed out}$$

input
```
integrate(cos(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
```

output
```
Timed out
```

3.608.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1049 vs. $2(72) = 144$.

Time $= 0.45$ (sec) , antiderivative size $= 1049$, normalized size of antiderivative $= 12.49$

$$\int \cos(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = \text{Too large to display}$$

input
```
integrate(cos(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm=
"maxima")
```

output
```
1/16*(4*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)
^(1/4)*(cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))*sin(2*b*
x + 2*a) + (cos(2*b*x + 2*a) + 1)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4
*b*x + 4*a) - 1)))*sqrt(c) - sqrt(c)*(log(sqrt(cos(4*b*x + 4*a)^2 + sin(4*
b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*cos(1/2*arctan2(sin(4*b*x + 4*a), -
cos(4*b*x + 4*a) - 1))^2 + sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 +
2*cos(4*b*x + 4*a) + 1)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a
) - 1))^2 + 2*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a
) + 1)^(1/4)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) + 1
) - log(sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a)
+ 1)*cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + sqrt(co
s(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(1/2*ar
ctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 - 2*(cos(4*b*x + 4*a)^2
+ sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)*sin(1/2*arctan2(sin(4
*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) + 1) + log(((cos(2*b*x + 2*a)^2 + sin
(2*b*x + 2*a)^2)*cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))
^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2)*sin(1/2*arctan2(sin(4*b*x +
 4*a), -cos(4*b*x + 4*a) - 1))^2)*sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*
a)^2 + 2*cos(4*b*x + 4*a) + 1) + 2*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^
2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)*(cos(1/2*arctan2(sin(4*b*x + 4*a), -c...
```

3.608.8 Giac [F]

$$\int \cos(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = \int \sqrt{c\tan\left(2\,bx+2\,a\right)\tan\left(bx+a\right)}\cos\left(2\,bx+2\,a\right)\,dx$$

input
```
integrate(cos(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm=
"giac")
```

output
```
integrate(sqrt(c*tan(2*b*x + 2*a)*tan(b*x + a))*cos(2*b*x + 2*a), x)
```

3.608.9 Mupad [F(-1)]

Timed out.

$$\int \cos(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$$
$$=\int \cos(2a+2bx)\ \sqrt{c\tan(a+bx)\ \tan(2a+2bx)}\,dx$$

input
```
int(cos(2*a + 2*b*x)*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2),x)
```

output
```
int(cos(2*a + 2*b*x)*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2), x)
```

3.609 $\int \cos^2(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$

3.609.1 Optimal result

Integrand size = 31, antiderivative size = 129

$$\int \cos^2(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = -\frac{3\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{8b} + \frac{3c\sin(2a+2bx)}{8b\sqrt{-c+c\sec(2a+2bx)}} - \frac{c\cos(2a+2bx)\sin(2a+2bx)}{4b\sqrt{-c+c\sec(2a+2bx)}}$$

output
```
-3/8*arctanh(c^(1/2)*tan(2*b*x+2*a)/(-c+c*sec(2*b*x+2*a))^(1/2))*c^(1/2)/b
+3/8*c*sin(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)-1/4*c*cos(2*b*x+2*a)*s
in(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)
```

3.609.2 Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.81

$$\int \cos^2(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = \frac{\left(-3\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\cos(a+bx)}{\sqrt{\cos(2(a+bx))}}\right)\sqrt{\cos(2(a+bx))}\csc(a+bx)+2(\cot(a+bx)-\sin(2(a+bx))+\sin(4(a+}{16b}$$

input
```
Integrate[Cos[2*(a + b*x)]^2*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

output
```
((-3*Sqrt[2]*ArcTanh[(Sqrt[2]*Cos[a + b*x])/Sqrt[Cos[2*(a + b*x)]]]*Sqrt[C
os[2*(a + b*x)]]*Csc[a + b*x] + 2*(Cot[a + b*x] - Sin[2*(a + b*x)] + Sin[4
*(a + b*x)]))*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]])/(16*b)
```

3.609.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.290, Rules used = {3042, 4897, 3042, 4292, 3042, 4292, 3042, 4261, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(2a+2bx)^2\sqrt{c\tan(a+bx)\tan(2a+2bx)}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \cos^2(2a+2bx)\sqrt{c\sec(2a+2bx)-c}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}}{\csc\left(2a+2bx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{4292}$$

$$-\frac{3}{4}\int \cos(2a+2bx)\sqrt{c\sec(2a+2bx)-c}dx-\frac{c\sin(2a+2bx)\cos(2a+2bx)}{4b\sqrt{c\sec(2a+2bx)-c}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3}{4}\int \frac{\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}}{\csc\left(2a+2bx+\frac{\pi}{2}\right)}dx-\frac{c\sin(2a+2bx)\cos(2a+2bx)}{4b\sqrt{c\sec(2a+2bx)-c}}$$

$$\Big\downarrow \text{4292}$$

$$-\frac{3}{4}\left(-\frac{1}{2}\int \sqrt{c\sec(2a+2bx)-c}dx-\frac{c\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}\right)-\frac{c\sin(2a+2bx)\cos(2a+2bx)}{4b\sqrt{c\sec(2a+2bx)-c}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3}{4}\left(-\frac{1}{2}\int\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}dx-\frac{c\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}\right)-\frac{c\sin(2a+2bx)\cos(2a+2bx)}{4b\sqrt{c\sec(2a+2bx)-c}}$$

$$\Big\downarrow \text{4261}$$

$$-\frac{3}{4}\left(\frac{c\int\frac{1}{\frac{c^2\tan^2(2a+2bx)}{c\sec(2a+2bx)-c}-c}d\left(-\frac{c\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{2b}-\frac{c\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}\right)-\frac{c\sin(2a+2bx)\cos(2a+2bx)}{4b\sqrt{c\sec(2a+2bx)-c}}$$

$$\Big\downarrow \text{220}$$

$$-\frac{3}{4}\left(\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{2b}-\frac{c\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}\right)-\frac{c\sin(2a+2bx)\cos(2a+2bx)}{4b\sqrt{c\sec(2a+2bx)-c}}$$

input
```
Int[Cos[2*(a + b*x)]^2*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

output
```
-1/4*(c*Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x])/(b*Sqrt[-c + c*Sec[2*a + 2*b*x]
]) - (3*((Sqrt[c]*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a +
 2*b*x]]])/(2*b) - (c*Sin[2*a + 2*b*x])/(2*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]
)))/4
```

3.609.3.1 Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

3.609. $\int\cos^2(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$

rule 4292
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Simp[a*((2*n + 1)/(2*b*d*n))   Int[Sqrt[a + b*Csc
[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.609.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 570 vs. $2(113) = 226$.

Time = 6.14 (sec) , antiderivative size = 571, normalized size of antiderivative = 4.43

method	result
default	$\frac{\sqrt{\frac{c\sin(xb+a)^2}{2\cos(xb+a)^2-1}}\,\sin(xb+a)\sqrt{\frac{2\cos(xb+a)^2-1}{(1+\cos(xb+a))^2}}\,\operatorname{arctanh}\left(\frac{\cos(xb+a)\sqrt{2}}{(1+\cos(xb+a))\sqrt{\frac{2\cos(xb+a)^2-1}{(1+\cos(xb+a))^2}}}\right)\sqrt{4}}{2b(\cos(xb+a)-1)} + \frac{\sqrt{2}\csc(xb+a)\left(\sqrt{2}\operatorname{arctanh}\left(\frac{}{(1+\mathrm{c}}\right.\right.}{}$

input
```
int(cos(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
1/2/b*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(1/2)*sin(b*x+a)*((2*cos(b*x+a)^
2-1)/(1+cos(b*x+a))^2)^(1/2)*arctanh(cos(b*x+a)/(1+cos(b*x+a))/((2*cos(b*x
+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*2^(1/2))/(cos(b*x+a)-1)*4^(1/2)+1/4*2^(1/
2)/b*csc(b*x+a)*(2^(1/2)*arctanh(cos(b*x+a)/(1+cos(b*x+a))/((2*cos(b*x+a)^
2-1)/(1+cos(b*x+a))^2)^(1/2)*2^(1/2))*((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2
)^(1/2)*cos(b*x+a)+2^(1/2)*arctanh(cos(b*x+a)/(1+cos(b*x+a))/((2*cos(b*x+a
)^2-1)/(1+cos(b*x+a))^2)^(1/2)*2^(1/2))*((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))
^2)^(1/2)+4*cos(b*x+a)^3-2*cos(b*x+a))*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))
^(1/2)*4^(1/2)-1/32*2^(1/2)/b*csc(b*x+a)*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1
))^(1/2)*(16*cos(b*x+a)^5+3*2^(1/2)*arctanh(cos(b*x+a)/(1+cos(b*x+a))/((2*
cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*2^(1/2))*((2*cos(b*x+a)^2-1)/(1+co
s(b*x+a))^2)^(1/2)*cos(b*x+a)+3*2^(1/2)*arctanh(cos(b*x+a)/(1+cos(b*x+a))/
((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*2^(1/2))*((2*cos(b*x+a)^2-1)/(
1+cos(b*x+a))^2)^(1/2)+4*cos(b*x+a)^3-6*cos(b*x+a))*4^(1/2)
```

3.609.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 419, normalized size of antiderivative = 3.25

$$\int \cos^2(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$$
$$= \left[\frac{3\left(\tan(bx+a)^5+2\tan(bx+a)^3+\tan(bx+a)\right)\sqrt{c}\log\left(-\frac{c\tan(bx+a)^5-14c\tan(bx+a)^3-4\sqrt{2}\left(\tan(bx+a)^4-4\tan\right.}{\tan(bx+a)^5+2\tan(bx+}\right.}{32\left(b\tan(bx+a)^5+2b\tan\right.}\right.$$

input
```
integrate(cos(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorith
m="fricas")
```

output
```
[1/32*(3*(tan(b*x + a)^5 + 2*tan(b*x + a)^3 + tan(b*x + a))*sqrt(c)*log(-(
c*tan(b*x + a)^5 - 14*c*tan(b*x + a)^3 - 4*sqrt(2)*(tan(b*x + a)^4 - 4*tan
(b*x + a)^2 + 3)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*sqrt(c) + 17
*c*tan(b*x + a))/(tan(b*x + a)^5 + 2*tan(b*x + a)^3 + tan(b*x + a))) - 4*s
qrt(2)*(5*tan(b*x + a)^4 - 4*tan(b*x + a)^2 - 1)*sqrt(-c*tan(b*x + a)^2/(t
an(b*x + a)^2 - 1)))/(b*tan(b*x + a)^5 + 2*b*tan(b*x + a)^3 + b*tan(b*x +
a)), 1/16*(3*(tan(b*x + a)^5 + 2*tan(b*x + a)^3 + tan(b*x + a))*sqrt(-c)*a
rctan(2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)
^2 - 1)*sqrt(-c)/(c*tan(b*x + a)^3 - 3*c*tan(b*x + a))) - 2*sqrt(2)*(5*tan
(b*x + a)^4 - 4*tan(b*x + a)^2 - 1)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2
 - 1)))/(b*tan(b*x + a)^5 + 2*b*tan(b*x + a)^3 + b*tan(b*x + a))]
```

3.609.6 Sympy [F(-1)]

Timed out.

$$\int \cos^2(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = \text{Timed out}$$

input
```
integrate(cos(2*b*x+2*a)**2*(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
```

output
```
Timed out
```

3.609.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1421 vs. 2(113) = 226.

Time = 0.51 (sec) , antiderivative size = 1421, normalized size of antiderivative = 11.02

$$\int \cos^2(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = \text{Too large to display}$$

input
```
integrate(cos(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorith
m="maxima")
```

output
```
-1/64*(4*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1
)^(1/4)*((cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))*sin(4*
b*x + 4*a) - (cos(4*b*x + 4*a) - 2)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos
(4*b*x + 4*a) - 1)))*cos(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a)))
- cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))*sin(4*b*x + 4*
a) - (cos(4*b*x + 4*a) - 2)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x +
 4*a) - 1)) - ((cos(4*b*x + 4*a) - 2)*cos(1/2*arctan2(sin(4*b*x + 4*a), -c
os(4*b*x + 4*a) - 1)) + sin(4*b*x + 4*a)*sin(1/2*arctan2(sin(4*b*x + 4*a),
 -cos(4*b*x + 4*a) - 1)))*sin(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*
a))))*sqrt(c) - 3*sqrt(c)*(log(sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^
2 + 2*cos(4*b*x + 4*a) + 1)*cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x +
 4*a) - 1))^2 + sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x
 + 4*a) + 1)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 +
 2*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4
)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) + 1) - log(sqr
t(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*cos(1/
2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + sqrt(cos(4*b*x + 4
*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(1/2*arctan2(sin(4
*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 - 2*(cos(4*b*x + 4*a)^2 + sin(4*b*x
 + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)*sin(1/2*arctan2(sin(4*b*x + 4...
```

3.609.8 Giac [F]

$$\int \cos^2(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$$
$$= \int \sqrt{c\tan\left(2\,bx+2\,a\right)\tan\left(bx+a\right)}\cos\left(2\,bx+2\,a\right)^2\,dx$$

input
```
integrate(cos(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorith
m="giac")
```

output
```
integrate(sqrt(c*tan(2*b*x + 2*a)*tan(b*x + a))*cos(2*b*x + 2*a)^2, x)
```

3.609.9 Mupad [F(-1)]

Timed out.

$$\int \cos^2(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$$
$$= \int \cos\left(2\,a+2\,b\,x\right)^2\,\sqrt{c\tan\left(a+b\,x\right)\,\tan\left(2\,a+2\,b\,x\right)}\,dx$$

input
```
int(cos(2*a + 2*b*x)^2*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2),x)
```

output
```
int(cos(2*a + 2*b*x)^2*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2), x)
```

3.610 $\int \cos^3(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$

3.610.1 Optimal result

Integrand size = 31, antiderivative size = 176

$$\int \cos^3(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = \frac{5\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{16b} - \frac{5c\sin(2a+2bx)}{16b\sqrt{-c+c\sec(2a+2bx)}} + \frac{5c\cos(2a+2bx)\sin(2a+2bx)}{24b\sqrt{-c+c\sec(2a+2bx)}} - \frac{c\cos^2(2a+2bx)\sin(2a+2bx)}{6b\sqrt{-c+c\sec(2a+2bx)}}$$

output
```
5/16*arctanh(c^(1/2)*tan(2*b*x+2*a)/(-c+c*sec(2*b*x+2*a))^(1/2))*c^(1/2)/b
-5/16*c*sin(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)+5/24*c*cos(2*b*x+2*a)
*sin(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)-1/6*c*cos(2*b*x+2*a)^2*sin(2
*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)
```

3.610.2 Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.66

$$\int \cos^3(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = \frac{\left(-26\cot(a+bx)+15\sqrt{2}\,\text{arctanh}\left(\frac{\sqrt{2}\cos(a+bx)}{\sqrt{\cos(2(a+bx))}}\right)\sqrt{\cos(2(a+bx))}\csc(a+bx)+30\sin(2(a+bx))-2\sin\right.}{96b}$$

input
```
Integrate[Cos[2*(a + b*x)]^3*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

output
```
((-26*Cot[a + b*x] + 15*Sqrt[2]*ArcTanh[(Sqrt[2]*Cos[a + b*x])/Sqrt[Cos[2*
(a + b*x)]]]*Sqrt[Cos[2*(a + b*x)]]*Csc[a + b*x] + 30*Sin[2*(a + b*x)] - 2
*Sin[4*(a + b*x)] + 4*Sin[6*(a + b*x)])*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x
)]])/(96*b)
```

3.610.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules used = {3042, 4897, 3042, 4292, 3042, 4292, 3042, 4292, 3042, 4261, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(2a+2bx)^3\sqrt{c\tan(a+bx)\tan(2a+2bx)}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \cos^3(2a+2bx)\sqrt{c\sec(2a+2bx)-c}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}}{\csc\left(2a+2bx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{4292}$$

$$-\frac{5}{6}\int \cos^2(2a+2bx)\sqrt{c\sec(2a+2bx)-c}dx-\frac{c\sin(2a+2bx)\cos^2(2a+2bx)}{6b\sqrt{c\sec(2a+2bx)-c}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{5}{6}\int \frac{\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}}{\csc\left(2a+2bx+\frac{\pi}{2}\right)^2}dx-\frac{c\sin(2a+2bx)\cos^2(2a+2bx)}{6b\sqrt{c\sec(2a+2bx)-c}}$$

$$\Big\downarrow \text{4292}$$

$$-\frac{5}{6}\left(-\frac{3}{4}\int \cos(2a+2bx)\sqrt{c\sec(2a+2bx)-c}dx-\frac{c\sin(2a+2bx)\cos(2a+2bx)}{4b\sqrt{c\sec(2a+2bx)-c}}\right)-\frac{c\sin(2a+2bx)\cos^2(2a+2bx)}{6b\sqrt{c\sec(2a+2bx)-c}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{5}{6}\left(-\frac{3}{4}\int \frac{\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}}{\csc\left(2a+2bx+\frac{\pi}{2}\right)}dx-\frac{c\sin(2a+2bx)\cos(2a+2bx)}{4b\sqrt{c\sec(2a+2bx)-c}}\right)-\frac{c\sin(2a+2bx)\cos^2(2a+2bx)}{6b\sqrt{c\sec(2a+2bx)-c}}$$

$$\Big\downarrow \text{4292}$$

$$-\frac{5}{6}\left(-\frac{3}{4}\left(-\frac{1}{2}\int \sqrt{c\sec(2a+2bx)-c}dx-\frac{c\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}\right)-\frac{c\sin(2a+2bx)\cos(2a+2bx)}{4b\sqrt{c\sec(2a+2bx)-c}}\right)-\frac{c\sin(2a+2bx)\cos^2(2a+2bx)}{6b\sqrt{c\sec(2a+2bx)-c}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{5}{6}\left(-\frac{3}{4}\left(-\frac{1}{2}\int \sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}dx-\frac{c\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}\right)-\frac{c\sin(2a+2bx)\cos(2a+2bx)}{4b\sqrt{c\sec(2a+2bx)-c}}\right)-\frac{c\sin(2a+2bx)\cos^2(2a+2bx)}{6b\sqrt{c\sec(2a+2bx)-c}}$$

$$\Big\downarrow \text{4261}$$

$$-\frac{5}{6}\left(-\frac{3}{4}\left(\frac{c\int \frac{1}{\frac{c^2\tan^2(2a+2bx)}{c\sec(2a+2bx)-c}-c}d\left(-\frac{c\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{2b}-\frac{c\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}\right)-\frac{c\sin(2a+2bx)\cos(2a+2bx)}{4b\sqrt{c\sec(2a+2bx)-c}}\right)-\frac{c\sin(2a+2bx)\cos^2(2a+2bx)}{6b\sqrt{c\sec(2a+2bx)-c}}$$

$$\Big\downarrow \text{220}$$

$$-\frac{5}{6}\left(-\frac{3}{4}\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{2b}-\frac{c\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}\right)-\frac{c\sin(2a+2bx)\cos(2a+2bx)}{4b\sqrt{c\sec(2a+2bx)-c}}\right)-\frac{c\sin(2a+2bx)\cos^2(2a+2bx)}{6b\sqrt{c\sec(2a+2bx)-c}}$$

3.610. $\int \cos^3(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$

input
```
Int[Cos[2*(a + b*x)]^3*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

output
```
-1/6*(c*Cos[2*a + 2*b*x]^2*Sin[2*a + 2*b*x])/(b*Sqrt[-c + c*Sec[2*a + 2*b*
x]]) - (5*(-1/4*(c*Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x])/(b*Sqrt[-c + c*Sec[2
*a + 2*b*x]]) - (3*((Sqrt[c]*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c +
c*Sec[2*a + 2*b*x]]])/(2*b) - (c*Sin[2*a + 2*b*x])/(2*b*Sqrt[-c + c*Sec[2*
a + 2*b*x]])))/4))/6
```

3.610.3.1 Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4292
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Simp[a*((2*n + 1)/(2*b*d*n))   Int[Sqrt[a + b*Csc
[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.610.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 810 vs. $2(156) = 312$.

Time = 7.42 (sec) , antiderivative size = 811, normalized size of antiderivative = 4.61

method	result	size
default	Expression too large to display	811

input
```
int(cos(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
-1/2/b*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(1/2)*sin(b*x+a)*((2*cos(b*x+a)
^2-1)/(1+cos(b*x+a))^2)^(1/2)*arctanh(cos(b*x+a)/(1+cos(b*x+a))/((2*cos(b*
x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*2^(1/2))/(cos(b*x+a)-1)*4^(1/2)-3/8*2^(1
/2)/b*csc(b*x+a)*(2^(1/2)*arctanh(cos(b*x+a)/(1+cos(b*x+a))/((2*cos(b*x+a)
^2-1)/(1+cos(b*x+a))^2)^(1/2)*2^(1/2))*((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^
2)^(1/2)*cos(b*x+a)+2^(1/2)*arctanh(cos(b*x+a)/(1+cos(b*x+a))/((2*cos(b*x+
a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*2^(1/2))*((2*cos(b*x+a)^2-1)/(1+cos(b*x+a)
)^2)^(1/2)+4*cos(b*x+a)^3-2*cos(b*x+a))*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1)
)^(1/2)*4^(1/2)+3/32*2^(1/2)/b*csc(b*x+a)*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-
1))^(1/2)*(16*cos(b*x+a)^5+3*2^(1/2)*arctanh(cos(b*x+a)/(1+cos(b*x+a))/((2
*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*2^(1/2))*((2*cos(b*x+a)^2-1)/(1+c
os(b*x+a))^2)^(1/2)*cos(b*x+a)+3*2^(1/2)*arctanh(cos(b*x+a)/(1+cos(b*x+a))
/((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*2^(1/2))*((2*cos(b*x+a)^2-1)/
(1+cos(b*x+a))^2)^(1/2)+4*cos(b*x+a)^3-6*cos(b*x+a))*4^(1/2)-1/192*2^(1/2)
/b*csc(b*x+a)*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(1/2)*(128*cos(b*x+a)^7+
16*cos(b*x+a)^5+15*2^(1/2)*arctanh(cos(b*x+a)/(1+cos(b*x+a))/((2*cos(b*x+a
)^2-1)/(1+cos(b*x+a))^2)^(1/2)*2^(1/2))*((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))
^2)^(1/2)*cos(b*x+a)+15*2^(1/2)*arctanh(cos(b*x+a)/(1+cos(b*x+a))/((2*cos(
b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*2^(1/2))*((2*cos(b*x+a)^2-1)/(1+cos(b*
x+a))^2)^(1/2)+20*cos(b*x+a)^3-30*cos(b*x+a))*4^(1/2)
```

3.610.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 481, normalized size of antiderivative = 2.73

$$\int \cos^3(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$$

$$= \left[\frac{15\left(\tan(bx+a)^7+3\tan(bx+a)^5+3\tan(bx+a)^3+\tan(bx+a)\right)\sqrt{c}\log\left(-\frac{c\tan(bx+a)^5-14c\tan(bx+a)^3+\cdots}{\cdots}\right.}{192\left(b\tan(bx+a)^7\cdots\right.}\right.$$

$$-\frac{15\left(\tan(bx+a)^7+3\tan(bx+a)^5+3\tan(bx+a)^3+\tan(bx+a)\right)\sqrt{-c}\arctan\left(\frac{2\sqrt{2}\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\mathrm{ta}\cdots\right.}{c\tan(bx+a)^3-3c\cdots}\right.}{96\left(b\tan(bx+a)^7+3b\tan(bx+a)^5\cdots\right.}$$

input
```
integrate(cos(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorith
m="fricas")
```

output
```
[1/192*(15*(tan(b*x + a)^7 + 3*tan(b*x + a)^5 + 3*tan(b*x + a)^3 + tan(b*x
 + a))*sqrt(c)*log(-(c*tan(b*x + a)^5 - 14*c*tan(b*x + a)^3 + 4*sqrt(2)*(t
an(b*x + a)^4 - 4*tan(b*x + a)^2 + 3)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)
^2 - 1))*sqrt(c) + 17*c*tan(b*x + a))/(tan(b*x + a)^5 + 2*tan(b*x + a)^3 +
 tan(b*x + a))) + 4*sqrt(2)*(33*tan(b*x + a)^6 - 19*tan(b*x + a)^4 - tan(b
*x + a)^2 - 13)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*tan(b*x +
 a)^7 + 3*b*tan(b*x + a)^5 + 3*b*tan(b*x + a)^3 + b*tan(b*x + a)), -1/96*(
15*(tan(b*x + a)^7 + 3*tan(b*x + a)^5 + 3*tan(b*x + a)^3 + tan(b*x + a))*s
qrt(-c)*arctan(2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan
(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a)^3 - 3*c*tan(b*x + a))) - 2*sqrt(
2)*(33*tan(b*x + a)^6 - 19*tan(b*x + a)^4 - tan(b*x + a)^2 - 13)*sqrt(-c*t
an(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*tan(b*x + a)^7 + 3*b*tan(b*x + a)^
5 + 3*b*tan(b*x + a)^3 + b*tan(b*x + a))]
```

3.610.6 Sympy [F(-1)]

Timed out.

$$\int \cos^3(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = \text{Timed out}$$

input
```
integrate(cos(2*b*x+2*a)**3*(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
```

output
```
Timed out
```

3.610.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2333 vs. $2(156) = 312$.

Time = 0.76 (sec) , antiderivative size = 2333, normalized size of antiderivative = 13.26

$$\int \cos^3(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx = \text{Too large to display}$$

input
```
integrate(cos(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorith
m="maxima")
```

output
```
-1/384*(8*(cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 + sin(2/
3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 + 2*cos(2/3*arctan2(sin(6
*b*x + 6*a), cos(6*b*x + 6*a))) + 1)^(3/4)*(cos(3/2*arctan2(sin(2/3*arctan
2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))), -cos(2/3*arctan2(sin(6*b*x + 6*a),
 cos(6*b*x + 6*a))) - 1))*sin(6*b*x + 6*a) + (cos(6*b*x + 6*a) + 1)*sin(3/
2*arctan2(sin(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))), -cos(2/3*a
rctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))) - 1)))*sqrt(c) + 12*(cos(2/3*a
rctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 + sin(2/3*arctan2(sin(6*b*x
+ 6*a), cos(6*b*x + 6*a)))^2 + 2*cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b
*x + 6*a))) + 1)^(1/4)*((sin(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a
))) - 5*sin(1/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))))*cos(1/2*arct
an2(sin(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))), -cos(2/3*arctan2
(sin(6*b*x + 6*a), cos(6*b*x + 6*a))) - 1)) + (cos(2/3*arctan2(sin(6*b*x +
 6*a), cos(6*b*x + 6*a))) - 3*cos(1/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x
+ 6*a))) - 4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x
+ 6*a))), -cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))) - 1)))*sqr
t(c) + 15*sqrt(c)*(log(sqrt(cos(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x +
6*a)))^2 + sin(2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a)))^2 + 2*cos(
2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))) + 1)*cos(1/2*arctan2(sin(
2/3*arctan2(sin(6*b*x + 6*a), cos(6*b*x + 6*a))), -cos(2/3*arctan2(sin(...
```

3.610.8 Giac [F]

$$\int \cos^3(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$$
$$= \int \sqrt{c\tan\left(2\,bx+2\,a\right)\tan\left(bx+a\right)}\cos\left(2\,bx+2\,a\right)^3\,dx$$

input
```
integrate(cos(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorith
m="giac")
```

output
```
integrate(sqrt(c*tan(2*b*x + 2*a)*tan(b*x + a))*cos(2*b*x + 2*a)^3, x)
```

3.610.9 Mupad [F(-1)]

Timed out.

$$\int \cos^3(2(a+bx))\sqrt{c\tan(a+bx)\tan(2(a+bx))}\,dx$$
$$= \int \cos(2a+2bx)^3 \sqrt{c\tan(a+bx)\ \tan(2a+2bx)}\,dx$$

input
```
int(cos(2*a + 2*b*x)^3*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2),x)
```

output
```
int(cos(2*a + 2*b*x)^3*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2), x)
```

3.611 $\int \sec^4(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx$

3.611.1 Optimal result

Integrand size = 31, antiderivative size = 208

$$\int \sec^4(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = \frac{34c^2\tan(2a+2bx)}{45b\sqrt{-c+c\sec(2a+2bx)}} - \frac{17c^2\sec^3(2a+2bx)\tan(2a+2bx)}{63b\sqrt{-c+c\sec(2a+2bx)}} + \frac{c^2\sec^4(2a+2bx)\tan(2a+2bx)}{9b\sqrt{-c+c\sec(2a+2bx)}} + \frac{68c\sqrt{-c+c\sec(2a+2bx)}\tan(2a+2bx)}{315b} + \frac{34(-c+c\sec(2a+2bx))^{3/2}\tan(2a+2bx)}{105b}$$

output
```
34/105*(-c+c*sec(2*b*x+2*a))^(3/2)*tan(2*b*x+2*a)/b+34/45*c^2*tan(2*b*x+2*
a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)-17/63*c^2*sec(2*b*x+2*a)^3*tan(2*b*x+2*a)
/b/(-c+c*sec(2*b*x+2*a))^(1/2)+1/9*c^2*sec(2*b*x+2*a)^4*tan(2*b*x+2*a)/b/(
-c+c*sec(2*b*x+2*a))^(1/2)+68/315*c*(-c+c*sec(2*b*x+2*a))^(1/2)*tan(2*b*x+
2*a)/b
```

3.611.2 Mathematica [A] (verified)

Time = 1.22 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.41

$$\int \sec^4(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = \frac{\cot(a+bx)\,(-84+188\cot(a+bx)\cot(2(a+bx))+52\sec(2(a+bx))-50\sec$$
$$315b$$

input
```
Integrate[Sec[2*(a + b*x)]^4*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

output
```
(Cot[a + b*x]*(-84 + 188*Cot[a + b*x]*Cot[2*(a + b*x)] + 52*Sec[2*(a + b*x
)] - 50*Sec[2*(a + b*x)]^2 + 35*Sec[2*(a + b*x)]^3)*(c*Tan[a + b*x]*Tan[2*
(a + b*x)])^(3/2))/(315*b)
```

3.611.3 Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.10, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.484$, Rules used = {3042, 4897, 3042, 4301, 27, 2011, 3042, 4290, 3042, 4287, 27, 3042, 4489, 3042, 4279}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^4(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(2a+2bx)^4(c\tan(a+bx)\tan(2a+2bx))^{3/2}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \sec^4(2a+2bx)(c\sec(2a+2bx)-c)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(2a+2bx+\frac{\pi}{2}\right)^4\left(c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c\right)^{3/2}dx$$

$$\Big\downarrow \text{4301}$$

$$\frac{2}{9}c\int \frac{17\sec^4(2a+2bx)(c-c\sec(2a+2bx))}{2\sqrt{c\sec(2a+2bx)-c}}dx+\frac{c^2\tan(2a+2bx)\sec^4(2a+2bx)}{9b\sqrt{c\sec(2a+2bx)-c}}$$

$$\Big\downarrow \text{27}$$

$$\frac{17}{9}c\int \frac{\sec^4(2a+2bx)(c-c\sec(2a+2bx))}{\sqrt{c\sec(2a+2bx)-c}}dx+\frac{c^2\tan(2a+2bx)\sec^4(2a+2bx)}{9b\sqrt{c\sec(2a+2bx)-c}}$$

$$\Big\downarrow \text{2011}$$

$$\frac{c^2\tan(2a+2bx)\sec^4(2a+2bx)}{9b\sqrt{c\sec(2a+2bx)-c}}-\frac{17}{9}c\int \sec^4(2a+2bx)\sqrt{c\sec(2a+2bx)-c}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{c^2\tan(2a+2bx)\sec^4(2a+2bx)}{9b\sqrt{c\sec(2a+2bx)-c}}-\frac{17}{9}c\int \csc\left(2a+2bx+\frac{\pi}{2}\right)^4\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}dx$$

$$\Big\downarrow \text{4290}$$

$$\frac{c^2\tan(2a+2bx)\sec^4(2a+2bx)}{9b\sqrt{c\sec(2a+2bx)-c}}-\frac{17}{9}c\left(\frac{c\tan(2a+2bx)\sec^3(2a+2bx)}{7b\sqrt{c\sec(2a+2bx)-c}}-\frac{6}{7}\int \sec^3(2a+2bx)\sqrt{c\sec(2a+2bx)-c}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{c^2\tan(2a+2bx)\sec^4(2a+2bx)}{9b\sqrt{c\sec(2a+2bx)-c}}-\frac{17}{9}c\left(\frac{c\tan(2a+2bx)\sec^3(2a+2bx)}{7b\sqrt{c\sec(2a+2bx)-c}}-\frac{6}{7}\int \csc\left(2a+2bx+\frac{\pi}{2}\right)^3\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}dx\right)$$

$$\Big\downarrow \text{4287}$$

$$\frac{c^2\tan(2a+2bx)\sec^4(2a+2bx)}{9b\sqrt{c\sec(2a+2bx)-c}}-\frac{17}{9}c\left(\frac{c\tan(2a+2bx)\sec^3(2a+2bx)}{7b\sqrt{c\sec(2a+2bx)-c}}-\frac{6}{7}\left(\frac{2\int \frac{1}{2}\sec(2a+2bx)\sqrt{c\sec(2a+2bx)-c}(2\sec(2a+2bx)c+3c)dx}{5c}+\text{t}\right.\right.$$

$$\Big\downarrow \text{27}$$

$$\frac{c^2\tan(2a+2bx)\sec^4(2a+2bx)}{9b\sqrt{c\sec(2a+2bx)-c}}-\frac{17}{9}c\left(\frac{c\tan(2a+2bx)\sec^3(2a+2bx)}{7b\sqrt{c\sec(2a+2bx)-c}}-\frac{6}{7}\left(\frac{\int \sec(2a+2bx)\sqrt{c\sec(2a+2bx)-c}(2\sec(2a+2bx)c+3c)dx}{5c}+\frac{\tan(}{}\right.\right.$$

$$\Big\downarrow \text{3042}$$

3.611. $\int \sec^4(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx$

$$\frac{c^2\tan(2a+2bx)\sec^4(2a+2bx)}{9b\sqrt{c\sec(2a+2bx)-c}}-$$
$$\frac{17}{9}c\left(\frac{c\tan(2a+2bx)\sec^3(2a+2bx)}{7b\sqrt{c\sec(2a+2bx)-c}}-\frac{6}{7}\left(\frac{\int\csc\left(2a+2bx+\frac{\pi}{2}\right)\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}\left(2\csc\left(2a+2bx+\frac{\pi}{2}\right)\right.}{5c}\right.\right.$$

$\Big\downarrow$ 4489

$$\frac{c^2\tan(2a+2bx)\sec^4(2a+2bx)}{9b\sqrt{c\sec(2a+2bx)-c}}-$$
$$\frac{17}{9}c\left(\frac{c\tan(2a+2bx)\sec^3(2a+2bx)}{7b\sqrt{c\sec(2a+2bx)-c}}-\frac{6}{7}\left(\frac{\frac{7}{3}c\int\sec(2a+2bx)\sqrt{c\sec(2a+2bx)-c}dx+\frac{2c\tan(2a+2bx)\sqrt{c\sec(2a+2bx}}{3b}}{5c}\right.\right.$$

$\Big\downarrow$ 3042

$$\frac{c^2\tan(2a+2bx)\sec^4(2a+2bx)}{9b\sqrt{c\sec(2a+2bx)-c}}-$$
$$\frac{17}{9}c\left(\frac{c\tan(2a+2bx)\sec^3(2a+2bx)}{7b\sqrt{c\sec(2a+2bx)-c}}-\frac{6}{7}\left(\frac{\frac{7}{3}c\int\csc\left(2a+2bx+\frac{\pi}{2}\right)\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}dx+\frac{2c\tan(2a+2bx)\sqrt{}}{3}}{5c}\right.\right.$$

$\Big\downarrow$ 4279

$$\frac{c^2\tan(2a+2bx)\sec^4(2a+2bx)}{9b\sqrt{c\sec(2a+2bx)-c}}-$$
$$\frac{17}{9}c\left(\frac{c\tan(2a+2bx)\sec^3(2a+2bx)}{7b\sqrt{c\sec(2a+2bx)-c}}-\frac{6}{7}\left(\frac{\frac{7c^2\tan(2a+2bx)}{3b\sqrt{c\sec(2a+2bx)-c}}+\frac{2c\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{3b}}{5c}+\frac{\tan(2a+2bx)(c\sec}{5}\right.\right.$$

input
```
Int[Sec[2*(a + b*x)]^4*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

output
```
(c^2*Sec[2*a + 2*b*x]^4*Tan[2*a + 2*b*x])/(9*b*Sqrt[-c + c*Sec[2*a + 2*b*x
]]) - (17*c*((c*Sec[2*a + 2*b*x]^3*Tan[2*a + 2*b*x])/(7*b*Sqrt[-c + c*Sec[
2*a + 2*b*x]]) - (6*(((-c + c*Sec[2*a + 2*b*x])^(3/2)*Tan[2*a + 2*b*x])/(5
*b*c) + ((7*c^2*Tan[2*a + 2*b*x])/(3*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]) + (2
*c*Sqrt[-c + c*Sec[2*a + 2*b*x]]*Tan[2*a + 2*b*x])/(3*b))/(5*c)))/7))/9
```

3.611. $\int\sec^4(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx$

3.611.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4279
```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4287
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(
m + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 -
b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 4290
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[2*a*d*((n - 1)/(b*(2*n -
1)))   Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Fre
eQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

3.611. $\int \sec^4(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx$

rule 4489
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[(a*B*m + A*b*(m + 1))/(b*(m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B
, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b
*(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.611.4 Maple [A] (verified)

Time = 6.25 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.49

method	result	size
default	$\frac{\sqrt{2}\,\cot(xb+a)\sqrt{\frac{c\sin(xb+a)^2}{2\cos(xb+a)^2-1}}\left(2176\cos(xb+a)^8-4896\cos(xb+a)^6+4284\cos(xb+a)^4-1785\cos(xb+a)^2+315\right)c\sqrt{4}}{315b\left(2\cos(xb+a)^2-1\right)^4}$	101

input
```
int(sec(2*b*x+2*a)^4*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
1/315*2^(1/2)/b*cot(b*x+a)*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(1/2)*(2176
*cos(b*x+a)^8-4896*cos(b*x+a)^6+4284*cos(b*x+a)^4-1785*cos(b*x+a)^2+315)*c
/(2*cos(b*x+a)^2-1)^4*4^(1/2)
```

3.611.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.63

$$\int \sec^4(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = \frac{2\sqrt{2}(315\,c\tan(bx+a)^8 - 525\,c\tan(bx+a)^6 + 819\,c\tan(bx+a)^4 - 423\,c\tan}{315\,(b\tan(bx+a)^9 - 4\,b\tan(bx+a)^7 + 6\,b\tan(bx+a)^5 - 4\,b\tan(ba}$$

input
```
integrate(sec(2*b*x+2*a)^4*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorith
m="fricas")
```

output
```
2/315*sqrt(2)*(315*c*tan(b*x + a)^8 - 525*c*tan(b*x + a)^6 + 819*c*tan(b*x
 + a)^4 - 423*c*tan(b*x + a)^2 + 94*c)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a
)^2 - 1))/(b*tan(b*x + a)^9 - 4*b*tan(b*x + a)^7 + 6*b*tan(b*x + a)^5 - 4*
b*tan(b*x + a)^3 + b*tan(b*x + a))
```

3.611.6 Sympy [F(-1)]

Timed out.

$$\int \sec^4(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(2*b*x+2*a)**4*(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
```

output
```
Timed out
```

3.611.7 Maxima [F]

$$\int \sec^4(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = \int (c\tan(2\,bx+2\,a)\tan(bx+a))^{\frac{3}{2}}\sec(2\,bx+2\,a)^4\,dx$$

input
```
integrate(sec(2*b*x+2*a)^4*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorith
m="maxima")
```

output
```
-8/315*(630*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a)
+ 1)^(1/4)*((b*c*cos(4*b*x + 4*a)^4 + b*c*sin(4*b*x + 4*a)^4 + 4*b*c*cos(4
*b*x + 4*a)^3 + 6*b*c*cos(4*b*x + 4*a)^2 + 4*b*c*cos(4*b*x + 4*a) + 2*(b*c
*cos(4*b*x + 4*a)^2 + 2*b*c*cos(4*b*x + 4*a) + b*c)*sin(4*b*x + 4*a)^2 + b
*c)*integrate(-(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*
a) + 1)^(1/4)*(((cos(20*b*x + 20*a)*cos(4*b*x + 4*a) + 4*cos(16*b*x + 16*a
)*cos(4*b*x + 4*a) + 6*cos(12*b*x + 12*a)*cos(4*b*x + 4*a) + 4*cos(8*b*x +
 8*a)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + sin(20*b*x + 20*a)*sin(4*b*x
 + 4*a) + 4*sin(16*b*x + 16*a)*sin(4*b*x + 4*a) + 6*sin(12*b*x + 12*a)*sin
(4*b*x + 4*a) + 4*sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + sin(4*b*x + 4*a)^2)*
cos(3/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) + (cos(4*b*x + 4
*a)*sin(20*b*x + 20*a) + 4*cos(4*b*x + 4*a)*sin(16*b*x + 16*a) + 6*cos(4*b
*x + 4*a)*sin(12*b*x + 12*a) + 4*cos(4*b*x + 4*a)*sin(8*b*x + 8*a) - cos(2
0*b*x + 20*a)*sin(4*b*x + 4*a) - 4*cos(16*b*x + 16*a)*sin(4*b*x + 4*a) - 6
*cos(12*b*x + 12*a)*sin(4*b*x + 4*a) - 4*cos(8*b*x + 8*a)*sin(4*b*x + 4*a)
)*sin(3/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)))*cos(7/2*arcta
n2(sin(4*b*x + 4*a), cos(4*b*x + 4*a))) + ((cos(4*b*x + 4*a)*sin(20*b*x +
20*a) + 4*cos(4*b*x + 4*a)*sin(16*b*x + 16*a) + 6*cos(4*b*x + 4*a)*sin(12*
b*x + 12*a) + 4*cos(4*b*x + 4*a)*sin(8*b*x + 8*a) - cos(20*b*x + 20*a)*sin
(4*b*x + 4*a) - 4*cos(16*b*x + 16*a)*sin(4*b*x + 4*a) - 6*cos(12*b*x + ...
```

3.611.8 Giac [F(-1)]

Timed out.

$$\int \sec^4(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(2*b*x+2*a)^4*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorith
m="giac")
```

output
```
Timed out
```

3.611.9 Mupad [B] (verification not implemented)

Time = 34.92 (sec) , antiderivative size = 594, normalized size of antiderivative = 2.86

$$\int \sec^4(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = \frac{\left(\frac{c\,16i}{9\,b}+\frac{c\,e^{a\,2i+b\,x\,2i}\,16i}{9\,b}\right)\sqrt{\frac{c\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)\,\left(e^{a\,4i+b\,x\,4i}\,1i-i\right)}{\left(e^{a\,2i+b\,x\,2i}+1\right)\,\left(e^{a\,4i+b\,x\,4i}+1\right)}}}{\left(e^{a\,2i+b\,x\,2i}-1\right)\,\left(e^{a\,4i+b\,x\,4i}+1\right)^4} - \frac{\left(\frac{c\,40i}{7\,b}+\frac{c\,e^{a\,2i+b\,x\,2i}\,88i}{63\,b}\right)\sqrt{\frac{c\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)\,\left(e^{a\,4i+b\,x\,4i}\,1i-i\right)}{\left(e^{a\,2i+b\,x\,2i}+1\right)\,\left(e^{a\,4i+b\,x\,4i}+1\right)}}}{\left(e^{a\,2i+b\,x\,2i}-1\right)\,\left(e^{a\,4i+b\,x\,4i}+1\right)^3} + \frac{\left(\frac{c\,24i}{5\,b}-\frac{c\,e^{a\,2i+b\,x\,2i}\,176i}{105\,b}\right)\sqrt{\frac{c\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)\,\left(e^{a\,4i+b\,x\,4i}\,1i-i\right)}{\left(e^{a\,2i+b\,x\,2i}+1\right)\,\left(e^{a\,4i+b\,x\,4i}+1\right)}}}{\left(e^{a\,2i+b\,x\,2i}-1\right)\,\left(e^{a\,4i+b\,x\,4i}+1\right)^2} + \frac{c\,e^{a\,2i+b\,x\,2i}\sqrt{\frac{c\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)\,\left(e^{a\,4i+b\,x\,4i}\,1i-i\right)}{\left(e^{a\,2i+b\,x\,2i}+1\right)\,\left(e^{a\,4i+b\,x\,4i}+1\right)}}\,272i}{315\,b\,\left(e^{a\,2i+b\,x\,2i}-1\right)} + \frac{c\,e^{a\,2i+b\,x\,2i}\sqrt{\frac{c\,\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)\,\left(e^{a\,4i+b\,x\,4i}\,1i-i\right)}{\left(e^{a\,2i+b\,x\,2i}+1\right)\,\left(e^{a\,4i+b\,x\,4i}+1\right)}}\,136i}{315\,b\,\left(e^{a\,2i+b\,x\,2i}-1\right)\,\left(e^{a\,4i+b\,x\,4i}+1\right)}$$

input
```
int((c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2)/cos(2*a + 2*b*x)^4,x)
```

output
```
(((c*16i)/(9*b) + (c*exp(a*2i + b*x*2i)*16i)/(9*b))*((c*(exp(a*2i + b*x*2i
)*1i - 1i)*(exp(a*4i + b*x*4i)*1i - 1i))/((exp(a*2i + b*x*2i) + 1)*(exp(a*
4i + b*x*4i) + 1)))^(1/2))/((exp(a*2i + b*x*2i) - 1)*(exp(a*4i + b*x*4i) +
 1)^4) - (((c*40i)/(7*b) + (c*exp(a*2i + b*x*2i)*88i)/(63*b))*((c*(exp(a*2
i + b*x*2i)*1i - 1i)*(exp(a*4i + b*x*4i)*1i - 1i))/((exp(a*2i + b*x*2i) +
1)*(exp(a*4i + b*x*4i) + 1)))^(1/2))/((exp(a*2i + b*x*2i) - 1)*(exp(a*4i +
 b*x*4i) + 1)^3) + (((c*24i)/(5*b) - (c*exp(a*2i + b*x*2i)*176i)/(105*b))*
((c*(exp(a*2i + b*x*2i)*1i - 1i)*(exp(a*4i + b*x*4i)*1i - 1i))/((exp(a*2i
+ b*x*2i) + 1)*(exp(a*4i + b*x*4i) + 1)))^(1/2))/((exp(a*2i + b*x*2i) - 1)
*(exp(a*4i + b*x*4i) + 1)^2) + (c*exp(a*2i + b*x*2i)*((c*(exp(a*2i + b*x*2
i)*1i - 1i)*(exp(a*4i + b*x*4i)*1i - 1i))/((exp(a*2i + b*x*2i) + 1)*(exp(a
*4i + b*x*4i) + 1)))^(1/2)*272i)/(315*b*(exp(a*2i + b*x*2i) - 1)) + (c*exp
(a*2i + b*x*2i)*((c*(exp(a*2i + b*x*2i)*1i - 1i)*(exp(a*4i + b*x*4i)*1i -
1i))/((exp(a*2i + b*x*2i) + 1)*(exp(a*4i + b*x*4i) + 1)))^(1/2)*136i)/(315
*b*(exp(a*2i + b*x*2i) - 1)*(exp(a*4i + b*x*4i) + 1))
```

3.612 $\int \sec^3(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx$

3.612.1 Optimal result

Integrand size = 31, antiderivative size = 148

$$\int \sec^3(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = -\frac{76c^2\tan(2a+2bx)}{105b\sqrt{-c+c\sec(2a+2bx)}}+\frac{19c\sqrt{-c+c\sec(2a+2bx)}\tan(2a+2bx)}{105b}+\frac{2(-c+c\sec(2a+2bx))^{3/2}\tan(2a+2bx)}{35b}+\frac{(-c+c\sec(2a+2bx))^{5/2}\tan(2a+2bx)}{7bc}$$

output
```
2/35*(-c+c*sec(2*b*x+2*a))^(3/2)*tan(2*b*x+2*a)/b+1/7*(-c+c*sec(2*b*x+2*a)
)^(5/2)*tan(2*b*x+2*a)/b/c-76/105*c^2*tan(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a
))^(1/2)+19/105*c*(-c+c*sec(2*b*x+2*a))^(1/2)*tan(2*b*x+2*a)/b
```

3.612.2 Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.49

$$\int \sec^3(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = -\frac{\cot(a+bx)\left(-28+76\cot(a+bx)\cot(2(a+bx))+24\sec(2(a+bx))-15\sec^2(2(a+bx))\right)(c\tan(a+bx)}{105b}$$

input
```
Integrate[Sec[2*(a + b*x)]^3*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

output
```
-1/105*(Cot[a + b*x]*(-28 + 76*Cot[a + b*x]*Cot[2*(a + b*x)] + 24*Sec[2*(a
 + b*x)] - 15*Sec[2*(a + b*x)]^2)*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2))
/b
```

3.612.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$, Rules used = {3042, 4897, 3042, 4287, 27, 3042, 4489, 3042, 4280, 3042, 4279}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^3(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(2a+2bx)^3(c\tan(a+bx)\tan(2a+2bx))^{3/2}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \sec^3(2a+2bx)(c\sec(2a+2bx)-c)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(2a+2bx+\frac{\pi}{2}\right)^3\left(c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c\right)^{3/2}dx$$

$$\Big\downarrow \text{4287}$$

$$\frac{2\int \frac{1}{2}\sec(2a+2bx)(c\sec(2a+2bx)-c)^{3/2}(2\sec(2a+2bx)c+5c)dx}{7c}+\frac{\tan(2a+2bx)(c\sec(2a+2bx)-c)^{5/2}}{7bc}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \sec(2a+2bx)(c\sec(2a+2bx)-c)^{3/2}(2\sec(2a+2bx)c+5c)dx}{7c}+\frac{\tan(2a+2bx)(c\sec(2a+2bx)-c)^{5/2}}{7bc}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc\left(2a+2bx+\frac{\pi}{2}\right)\left(c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c\right)^{3/2}\left(2\csc\left(2a+2bx+\frac{\pi}{2}\right)c+5c\right)dx}{7c}+\frac{\tan(2a+2bx)(c\sec(2a+2bx)-c)^{5/2}}{7bc}$$

$$\Big\downarrow \text{4489}$$

$$\frac{\frac{19}{5}c\int \sec(2a+2bx)(c\sec(2a+2bx)-c)^{3/2}dx+\frac{2c\tan(2a+2bx)(c\sec(2a+2bx)-c)^{3/2}}{5b}}{7c}+\frac{\tan(2a+2bx)(c\sec(2a+2bx)-c)^{5/2}}{7bc}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{19}{5}c\int \csc\left(2a+2bx+\frac{\pi}{2}\right)\left(c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c\right)^{3/2}dx+\frac{2c\tan(2a+2bx)(c\sec(2a+2bx)-c)^{3/2}}{5b}}{7c}+\frac{\tan(2a+2bx)(c\sec(2a+2bx)-c)^{5/2}}{7bc}$$

$$\Big\downarrow \text{4280}$$

$$\frac{\frac{19}{5}c\left(\frac{c\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{3b}-\frac{4}{3}c\int \sec(2a+2bx)\sqrt{c\sec(2a+2bx)-c}dx\right)+\frac{2c\tan(2a+2bx)(c\sec(2a+2bx)-c)^{3/2}}{5b}}{7c}+\frac{\tan(2a+2bx)(c\sec(2a+2bx)-c)^{5/2}}{7bc}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{19}{5}c\left(\frac{c\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{3b}-\frac{4}{3}c\int \csc\left(2a+2bx+\frac{\pi}{2}\right)\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}dx\right)+\frac{2c\tan(2a+2bx)(c\sec(2a+2bx)-c)^{3/2}}{5b}}{7c}+\frac{\tan(2a+2bx)(c\sec(2a+2bx)-c)^{5/2}}{7bc}$$

$$\Big\downarrow \text{4279}$$

$$\frac{\frac{19}{5}c\left(\frac{c\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{3b}-\frac{4c^2\tan(2a+2bx)}{3b\sqrt{c\sec(2a+2bx)-c}}\right)+\frac{2c\tan(2a+2bx)(c\sec(2a+2bx)-c)^{3/2}}{5b}}{7c}+\frac{\tan(2a+2bx)(c\sec(2a+2bx)-c)^{5/2}}{7bc}$$

input
```
Int[Sec[2*(a + b*x)]^3*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

3.612. $\int \sec^3(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx$

output
```
((-c + c*Sec[2*a + 2*b*x])^(5/2)*Tan[2*a + 2*b*x])/(7*b*c) + ((2*c*(-c + c
*Sec[2*a + 2*b*x])^(3/2)*Tan[2*a + 2*b*x])/(5*b) + (19*c*((-4*c^2*Tan[2*a
+ 2*b*x])/(3*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]) + (c*Sqrt[-c + c*Sec[2*a + 2
*b*x]]*Tan[2*a + 2*b*x])/(3*b)))/5)/(7*c)
```

3.612.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4279
```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4280
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m - 1)/(f*m)), x]
+ Simp[a*((2*m - 1)/m)   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x],
 x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && Intege
rQ[2*m]
```

rule 4287
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(
m + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 -
b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 4489
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[(a*B*m + A*b*(m + 1))/(b*(m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B
, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b
*(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.612.4 Maple [A] (verified)

Time = 5.80 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.61

method	result	size
default	$-\frac{\sqrt{2}\,\cot(xb+a)\sqrt{\frac{c\sin(xb+a)^2}{2\cos(xb+a)^2-1}}\left(416\cos(xb+a)^6-728\cos(xb+a)^4+455\cos(xb+a)^2-105\right)c\sqrt{4}}{105b\left(2\cos(xb+a)^2-1\right)^3}$	91

input
```
int(sec(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
-1/105*2^(1/2)/b*cot(b*x+a)*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(1/2)*(416
*cos(b*x+a)^6-728*cos(b*x+a)^4+455*cos(b*x+a)^2-105)*c/(2*cos(b*x+a)^2-1)^
3*4^(1/2)
```

3.612.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.75

$$\int \sec^3(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx =$$
$$-\frac{2\sqrt{2}\left(105\,c\tan\left(bx+a\right)^6-140\,c\tan\left(bx+a\right)^4+133\,c\tan\left(bx+a\right)^2-38\,c\right)\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}}{105\left(b\tan\left(bx+a\right)^7-3\,b\tan\left(bx+a\right)^5+3\,b\tan\left(bx+a\right)^3-b\tan\left(bx+a\right)\right)}$$

input
```
integrate(sec(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorith
m="fricas")
```

output
```
-2/105*sqrt(2)*(105*c*tan(b*x + a)^6 - 140*c*tan(b*x + a)^4 + 133*c*tan(b*
x + a)^2 - 38*c)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))/(b*tan(b*x +
 a)^7 - 3*b*tan(b*x + a)^5 + 3*b*tan(b*x + a)^3 - b*tan(b*x + a))
```

3.612.6 Sympy [F(-1)]

Timed out.

$$\int \sec^3(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(2*b*x+2*a)**3*(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
```

output
```
Timed out
```

3.612.7 Maxima [F]

$$\int \sec^3(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = \int (c\tan(2bx+2a)\tan(bx+a))^{\frac{3}{2}}\sec(2bx+2a)^3\,dx$$

input
```
integrate(sec(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorith
m="maxima")
```

output
```
4/105*(210*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) +
 1)^(3/4)*(3*(b*c*cos(4*b*x + 4*a)^2 + b*c*sin(4*b*x + 4*a)^2 + 2*b*c*cos(
4*b*x + 4*a) + b*c)*integrate(-(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 +
2*cos(4*b*x + 4*a) + 1)^(1/4)*(((cos(16*b*x + 16*a)*cos(4*b*x + 4*a) + 3*c
os(12*b*x + 12*a)*cos(4*b*x + 4*a) + 3*cos(8*b*x + 8*a)*cos(4*b*x + 4*a) +
 cos(4*b*x + 4*a)^2 + sin(16*b*x + 16*a)*sin(4*b*x + 4*a) + 3*sin(12*b*x +
 12*a)*sin(4*b*x + 4*a) + 3*sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + sin(4*b*x
+ 4*a)^2)*cos(3/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) + (cos
(4*b*x + 4*a)*sin(16*b*x + 16*a) + 3*cos(4*b*x + 4*a)*sin(12*b*x + 12*a) +
 3*cos(4*b*x + 4*a)*sin(8*b*x + 8*a) - cos(16*b*x + 16*a)*sin(4*b*x + 4*a)
 - 3*cos(12*b*x + 12*a)*sin(4*b*x + 4*a) - 3*cos(8*b*x + 8*a)*sin(4*b*x +
4*a))*sin(3/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)))*cos(5/2*a
rctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a))) + ((cos(4*b*x + 4*a)*sin(16*b*
x + 16*a) + 3*cos(4*b*x + 4*a)*sin(12*b*x + 12*a) + 3*cos(4*b*x + 4*a)*sin
(8*b*x + 8*a) - cos(16*b*x + 16*a)*sin(4*b*x + 4*a) - 3*cos(12*b*x + 12*a)
*sin(4*b*x + 4*a) - 3*cos(8*b*x + 8*a)*sin(4*b*x + 4*a))*cos(3/2*arctan2(s
in(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) - (cos(16*b*x + 16*a)*cos(4*b*x +
 4*a) + 3*cos(12*b*x + 12*a)*cos(4*b*x + 4*a) + 3*cos(8*b*x + 8*a)*cos(4*b
*x + 4*a) + cos(4*b*x + 4*a)^2 + sin(16*b*x + 16*a)*sin(4*b*x + 4*a) + 3*s
in(12*b*x + 12*a)*sin(4*b*x + 4*a) + 3*sin(8*b*x + 8*a)*sin(4*b*x + 4*a...
```

3.612.8 Giac [F(-1)]

Timed out.

$$\int \sec^3(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorith
m="giac")
```

output
```
Timed out
```

3.612.9 Mupad [B] (verification not implemented)

Time = 36.54 (sec) , antiderivative size = 479, normalized size of antiderivative = 3.24

$$\int \sec^3(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = \frac{\left(\frac{c\,8\mathrm{i}}{7\,b}-\frac{c\,\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,8\mathrm{i}}{7\,b}\right)\sqrt{\frac{c\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,\left(\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\left(\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}+1\right)}}}{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)\,{\left(\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}+1\right)}^3} - \frac{\left(\frac{c\,4\mathrm{i}}{5\,b}-\frac{c\,\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,92\mathrm{i}}{35\,b}\right)\sqrt{\frac{c\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,\left(\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\left(\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}+1\right)}}}{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)\,{\left(\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}+1\right)}^2} - \frac{\left(\frac{c\,4\mathrm{i}}{3\,b}+\frac{c\,\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,52\mathrm{i}}{105\,b}\right)\sqrt{\frac{c\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,\left(\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\left(\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}+1\right)}}}{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)\,\left(\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}+1\right)} - \frac{c\,\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,\sqrt{\frac{c\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,\left(\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\left(\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}+1\right)}}\,104\mathrm{i}}{105\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}$$

input
```
int((c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2)/cos(2*a + 2*b*x)^3,x)
```

output
```
(((c*8i)/(7*b) - (c*exp(a*2i + b*x*2i)*8i)/(7*b))*((c*(exp(a*2i + b*x*2i)*
1i - 1i)*(exp(a*4i + b*x*4i)*1i - 1i))/((exp(a*2i + b*x*2i) + 1)*(exp(a*4i
 + b*x*4i) + 1)))^(1/2))/((exp(a*2i + b*x*2i) - 1)*(exp(a*4i + b*x*4i) + 1
)^3) - (((c*4i)/(5*b) - (c*exp(a*2i + b*x*2i)*92i)/(35*b))*((c*(exp(a*2i +
 b*x*2i)*1i - 1i)*(exp(a*4i + b*x*4i)*1i - 1i))/((exp(a*2i + b*x*2i) + 1)*
(exp(a*4i + b*x*4i) + 1)))^(1/2))/((exp(a*2i + b*x*2i) - 1)*(exp(a*4i + b*
x*4i) + 1)^2) - (((c*4i)/(3*b) + (c*exp(a*2i + b*x*2i)*52i)/(105*b))*((c*(
exp(a*2i + b*x*2i)*1i - 1i)*(exp(a*4i + b*x*4i)*1i - 1i))/((exp(a*2i + b*x
*2i) + 1)*(exp(a*4i + b*x*4i) + 1)))^(1/2))/((exp(a*2i + b*x*2i) - 1)*(exp
(a*4i + b*x*4i) + 1)) - (c*exp(a*2i + b*x*2i)*((c*(exp(a*2i + b*x*2i)*1i -
 1i)*(exp(a*4i + b*x*4i)*1i - 1i))/((exp(a*2i + b*x*2i) + 1)*(exp(a*4i + b
*x*4i) + 1)))^(1/2)*104i)/(105*b*(exp(a*2i + b*x*2i) - 1))
```

3.613 $\int \sec^2(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx$

3.613.1 Optimal result

Integrand size = 31, antiderivative size = 110

$$\int \sec^2(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx=\frac{4c^2\tan(2a+2bx)}{5b\sqrt{-c+c\sec(2a+2bx)}}-\frac{c\sqrt{-c+c\sec(2a+2bx)}\tan(2a+2bx)}{5b}+\frac{(-c+c\sec(2a+2bx))^{3/2}\tan(2a+2bx)}{5b}$$

output
```
1/5*(-c+c*sec(2*b*x+2*a))^(3/2)*tan(2*b*x+2*a)/b+4/5*c^2*tan(2*b*x+2*a)/b/
(-c+c*sec(2*b*x+2*a))^(1/2)-1/5*c*(-c+c*sec(2*b*x+2*a))^(1/2)*tan(2*b*x+2*
a)/b
```

3.613.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.54

$$\int \sec^2(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx=\frac{\cot(a+bx)(-2+4\cot(a+bx)\cot(2(a+bx))+\sec(2(a+bx)))(c\tan(a+bx)}{5b}$$

input
```
Integrate[Sec[2*(a + b*x)]^2*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

output
```
(Cot[a + b*x]*(-2 + 4*Cot[a + b*x]*Cot[2*(a + b*x)] + Sec[2*(a + b*x)])*(c
*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2))/(5*b)
```

3.613.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {3042, 4897, 3042, 4285, 3042, 4280, 3042, 4279}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sec(2a+2bx)^2(c\tan(a+bx)\tan(2a+2bx))^{3/2}dx$$

$$\Big\downarrow \text{ 4897}$$

$$\int \sec^2(2a+2bx)(c\sec(2a+2bx)-c)^{3/2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \csc\left(2a+2bx+\frac{\pi}{2}\right)^2\left(c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c\right)^{3/2}dx$$

$$\Big\downarrow \text{ 4285}$$

$$\frac{\tan(2a+2bx)(c\sec(2a+2bx)-c)^{3/2}}{5b}-\frac{3}{5}\int \sec(2a+2bx)(c\sec(2a+2bx)-c)^{3/2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\tan(2a+2bx)(c\sec(2a+2bx)-c)^{3/2}}{5b}-\frac{3}{5}\int \csc\left(2a+2bx+\frac{\pi}{2}\right)\left(c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c\right)^{3/2}dx$$

$$\Big\downarrow \text{ 4280}$$

$$\frac{\tan(2a+2bx)(c\sec(2a+2bx)-c)^{3/2}}{5b}-\frac{3}{5}\left(\frac{c\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{3b}-\frac{4}{3}c\int \sec(2a+2bx)\sqrt{c\sec(2a+2bx)-c}dx\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\tan(2a+2bx)(c\sec(2a+2bx)-c)^{3/2}}{5b}-\frac{3}{5}\left(\frac{c\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{3b}-\frac{4}{3}c\int \csc\left(2a+2bx+\frac{\pi}{2}\right)\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}dx\right)$$

$$\Big\downarrow \text{ 4279}$$

$$\frac{\tan(2a+2bx)(c\sec(2a+2bx)-c)^{3/2}}{5b} - \frac{3}{5}\left(\frac{c\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{3b} - \frac{4c^2\tan(2a+2bx)}{3b\sqrt{c\sec(2a+2bx)-c}}\right)$$

input
```
Int[Sec[2*(a + b*x)]^2*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

output
```
((-c + c*Sec[2*a + 2*b*x])^(3/2)*Tan[2*a + 2*b*x])/(5*b) - (3*((-4*c^2*Tan
[2*a + 2*b*x])/(3*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]) + (c*Sqrt[-c + c*Sec[2*
a + 2*b*x]]*Tan[2*a + 2*b*x])/(3*b)))/5
```

3.613.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4279
```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4280
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m - 1)/(f*m)), x]
+ Simp[a*((2*m - 1)/m)   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x],
 x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && Intege
rQ[2*m]
```

rule 4285
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x]
+ Simp[a*(m/(b*(m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x]
/; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.613.4 Maple [A] (verified)

Time = 5.67 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{\sqrt{2}\,\cot(xb+a)\sqrt{\frac{c\sin(xb+a)^2}{2\cos(xb+a)^2-1}}\left(12\cos(xb+a)^4-15\cos(xb+a)^2+5\right)c\sqrt{4}}{5b\left(2\cos(xb+a)^2-1\right)^2}$	81

input
```
int(sec(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
1/5*2^(1/2)/b*cot(b*x+a)*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(1/2)*(12*cos
(b*x+a)^4-15*cos(b*x+a)^2+5)*c/(2*cos(b*x+a)^2-1)^2*4^(1/2)
```

3.613.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.80

$$\int \sec^2(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = \frac{2\sqrt{2}\left(5\,c\tan(bx+a)^4-5\,c\tan(bx+a)^2+2\,c\right)\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}}{5\left(b\tan(bx+a)^5-2\,b\tan(bx+a)^3+b\tan(bx+a)\right)}$$

input
```
integrate(sec(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorith
m="fricas")
```

output
```
2/5*sqrt(2)*(5*c*tan(b*x + a)^4 - 5*c*tan(b*x + a)^2 + 2*c)*sqrt(-c*tan(b*
x + a)^2/(tan(b*x + a)^2 - 1))/(b*tan(b*x + a)^5 - 2*b*tan(b*x + a)^3 + b*
tan(b*x + a))
```

3.613.6 Sympy [F(-1)]

Timed out.

$$\int \sec^2(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(2*b*x+2*a)**2*(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
```

output
```
Timed out
```

3.613.7 Maxima [F]

$$\int \sec^2(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = \int (c\tan(2\,bx+2\,a)\tan(bx+a))^{\frac{3}{2}}\sec(2\,bx+2\,a)^2\,dx$$

input
```
integrate(sec(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorith
m="maxima")
```

output
```
-2/5*(10*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1
)^(1/4)*((b*c*cos(4*b*x + 4*a)^2 + b*c*sin(4*b*x + 4*a)^2 + 2*b*c*cos(4*b*
x + 4*a) + b*c)*integrate(-(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*co
s(4*b*x + 4*a) + 1)^(1/4)*(((cos(12*b*x + 12*a)*cos(4*b*x + 4*a) + 2*cos(8
*b*x + 8*a)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + sin(12*b*x + 12*a)*sin
(4*b*x + 4*a) + 2*sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + sin(4*b*x + 4*a)^2)*
cos(3/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) + (cos(4*b*x + 4
*a)*sin(12*b*x + 12*a) + 2*cos(4*b*x + 4*a)*sin(8*b*x + 8*a) - cos(12*b*x
+ 12*a)*sin(4*b*x + 4*a) - 2*cos(8*b*x + 8*a)*sin(4*b*x + 4*a))*sin(3/2*ar
ctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)))*cos(5/2*arctan2(sin(4*b*x
 + 4*a), cos(4*b*x + 4*a))) + ((cos(4*b*x + 4*a)*sin(12*b*x + 12*a) + 2*co
s(4*b*x + 4*a)*sin(8*b*x + 8*a) - cos(12*b*x + 12*a)*sin(4*b*x + 4*a) - 2*
cos(8*b*x + 8*a)*sin(4*b*x + 4*a))*cos(3/2*arctan2(sin(4*b*x + 4*a), -cos(
4*b*x + 4*a) - 1)) - (cos(12*b*x + 12*a)*cos(4*b*x + 4*a) + 2*cos(8*b*x +
8*a)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + sin(12*b*x + 12*a)*sin(4*b*x
+ 4*a) + 2*sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + sin(4*b*x + 4*a)^2)*sin(3/2
*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)))*sin(5/2*arctan2(sin(4*
b*x + 4*a), cos(4*b*x + 4*a))))/((cos(4*b*x + 4*a)^4 + sin(4*b*x + 4*a)^4
+ (cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*cos(1
2*b*x + 12*a)^2 + 4*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*...
```

3.613.8 Giac [F(-1)]

Timed out.

$$\int \sec^2(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorith
m="giac")
```

output
```
Timed out
```

3.613.9 Mupad [B] (verification not implemented)

Time = 45.17 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.35

$$\int \sec^2(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = \frac{2\,c\,\left(\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}\,5\mathrm{i}+\mathrm{e}^{a\,6\mathrm{i}+b\,x\,6\mathrm{i}}\,5\mathrm{i}+\mathrm{e}^{a\,10\mathrm{i}+b\,x\,10\mathrm{i}}\,3\mathrm{i}+3\mathrm{i}\right)\,\sqrt{\frac{c\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)\,\left(\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)\,\left(\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}+1\right)}}}{5\,b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)\,\left(\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}+1\right)^2}$$

input
```
int((c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2)/cos(2*a + 2*b*x)^2,x)
```

output
```
(2*c*(exp(a*4i + b*x*4i)*5i + exp(a*6i + b*x*6i)*5i + exp(a*10i + b*x*10i)
*3i + 3i)*((c*(exp(a*2i + b*x*2i)*1i - 1i)*(exp(a*4i + b*x*4i)*1i - 1i))/(
(exp(a*2i + b*x*2i) + 1)*(exp(a*4i + b*x*4i) + 1)))^(1/2))/(5*b*(exp(a*2i
+ b*x*2i) - 1)*(exp(a*4i + b*x*4i) + 1)^2)
```

3.614 $\int \sec(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx$

3.614.1 Optimal result

Integrand size = 29, antiderivative size = 75

$$\int \sec(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = -\frac{4c^2\tan(2a+2bx)}{3b\sqrt{-c+c\sec(2a+2bx)}}+\frac{c\sqrt{-c+c\sec(2a+2bx)}\tan(2a+2bx)}{3b}$$

output
```
-4/3*c^2*tan(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)+1/3*c*(-c+c*sec(2*b*
x+2*a))^(1/2)*tan(2*b*x+2*a)/b
```

3.614.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.68

$$\int \sec(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = -\frac{\cot(a+bx)(-1+4\cot(a+bx)\cot(2(a+bx)))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}{3b}$$

input
```
Integrate[Sec[2*(a + b*x)]*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

output
```
-1/3*(Cot[a + b*x]*(-1 + 4*Cot[a + b*x]*Cot[2*(a + b*x)])*(c*Tan[a + b*x]*
Tan[2*(a + b*x)])^(3/2))/b
```

3.614.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {3042, 4897, 3042, 4280, 3042, 4279}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sec(2a+2bx)(c\tan(a+bx)\tan(2a+2bx))^{3/2}dx$$

$$\Big\downarrow \text{ 4897}$$

$$\int \sec(2a+2bx)(c\sec(2a+2bx)-c)^{3/2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \csc\left(2a+2bx+\frac{\pi}{2}\right)\left(c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c\right)^{3/2}dx$$

$$\Big\downarrow \text{ 4280}$$

$$\frac{c\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{3b}-\frac{4}{3}c\int \sec(2a+2bx)\sqrt{c\sec(2a+2bx)-c}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{c\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{3b}-\frac{4}{3}c\int \csc\left(2a+2bx+\frac{\pi}{2}\right)\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}dx$$

$$\Big\downarrow \text{ 4279}$$

$$\frac{c\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{3b}-\frac{4c^2\tan(2a+2bx)}{3b\sqrt{c\sec(2a+2bx)-c}}$$

input
```
Int[Sec[2*(a + b*x)]*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

output
```
(-4*c^2*Tan[2*a + 2*b*x])/(3*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]) + (c*Sqrt[-c
 + c*Sec[2*a + 2*b*x]]*Tan[2*a + 2*b*x])/(3*b)
```

3.614.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4279
```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4280
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m - 1)/(f*m)), x]
+ Simp[a*((2*m - 1)/m)   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x],
 x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && Intege
rQ[2*m]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.614.4 Maple [A] (verified)

Time = 5.74 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.95

method	result	size
default	$-\frac{\sqrt{2}\,\cot(xb+a)c\left(5\cos(xb+a)^{2}-3\right)\sqrt{\frac{c\sin(xb+a)^{2}}{2\cos(xb+a)^{2}-1}}\,\sqrt{4}}{3b\left(2\cos(xb+a)^{2}-1\right)}$	71

input
```
int(sec(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x,method=_RETURNVER
BOSE)
```

output
```
-1/3*2^(1/2)/b*cot(b*x+a)*c*(5*cos(b*x+a)^2-3)*(c*sin(b*x+a)^2/(2*cos(b*x+
a)^2-1))^(1/2)/(2*cos(b*x+a)^2-1)*4^(1/2)
```

3.614.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.89

$$\int \sec(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = -\frac{2\sqrt{2}\left(3\,c\tan\left(bx+a\right)^{2}-2\,c\right)\sqrt{-\frac{c\tan(bx+a)^{2}}{\tan(bx+a)^{2}-1}}}{3\left(b\tan\left(bx+a\right)^{3}-b\tan\left(bx+a\right)\right)}$$

input
```
integrate(sec(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm=
"fricas")
```

output
```
-2/3*sqrt(2)*(3*c*tan(b*x + a)^2 - 2*c)*sqrt(-c*tan(b*x + a)^2/(tan(b*x +
a)^2 - 1))/(b*tan(b*x + a)^3 - b*tan(b*x + a))
```

3.614.6 Sympy [F(-1)]

Timed out.

$$\int \sec(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
```

output
```
Timed out
```

3.614.7 Maxima [F]

$$\int \sec(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = \int \left(c\tan\left(2\,bx+2\,a\right)\tan\left(bx+a\right)\right)^{\frac{3}{2}}\sec\left(2\,bx+2\,a\right)\,dx$$

input
```
integrate(sec(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm=
"maxima")
```

output
```
1/3*(6*(3*b*c*integrate(-(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(
4*b*x + 4*a) + 1)^(1/4)*(((cos(8*b*x + 8*a)*cos(4*b*x + 4*a) + cos(4*b*x +
 4*a)^2 + sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + sin(4*b*x + 4*a)^2)*cos(3/2*
arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) + (cos(4*b*x + 4*a)*sin(
8*b*x + 8*a) - cos(8*b*x + 8*a)*sin(4*b*x + 4*a))*sin(3/2*arctan2(sin(4*b*
x + 4*a), -cos(4*b*x + 4*a) - 1)))*cos(3/2*arctan2(sin(4*b*x + 4*a), cos(4
*b*x + 4*a))) + ((cos(4*b*x + 4*a)*sin(8*b*x + 8*a) - cos(8*b*x + 8*a)*sin
(4*b*x + 4*a))*cos(3/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) -
 (cos(8*b*x + 8*a)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + sin(8*b*x + 8*a
)*sin(4*b*x + 4*a) + sin(4*b*x + 4*a)^2)*sin(3/2*arctan2(sin(4*b*x + 4*a),
 -cos(4*b*x + 4*a) - 1)))*sin(3/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*
a))))/((cos(4*b*x + 4*a)^4 + sin(4*b*x + 4*a)^4 + (cos(4*b*x + 4*a)^2 + si
n(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*cos(8*b*x + 8*a)^2 + 2*cos(4*b*
x + 4*a)^3 + (cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a)
 + 1)*sin(8*b*x + 8*a)^2 + (2*cos(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)
*sin(4*b*x + 4*a)^2 + 2*(cos(4*b*x + 4*a)^3 + cos(4*b*x + 4*a)*sin(4*b*x +
 4*a)^2 + 2*cos(4*b*x + 4*a)^2 + cos(4*b*x + 4*a))*cos(8*b*x + 8*a) + cos(
4*b*x + 4*a)^2 + 2*(sin(4*b*x + 4*a)^3 + (cos(4*b*x + 4*a)^2 + 2*cos(4*b*x
 + 4*a) + 1)*sin(4*b*x + 4*a))*sin(8*b*x + 8*a))*cos(3/2*arctan2(sin(4*b*x
 + 4*a), -cos(4*b*x + 4*a) - 1))^2 + (cos(4*b*x + 4*a)^4 + sin(4*b*x + ...
```

3.614.8 Giac [F(-1)]

Timed out.

$$\int \sec(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(sec(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm=
"giac")
```

output
```
Timed out
```

3.614.9 Mupad [B] (verification not implemented)

Time = 35.04 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.11

$$\int \sec(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = -\frac{c\left(e^{a\,2i+b\,x\,2i}\,3i+e^{a\,4i+b\,x\,4i}\,3i+e^{a\,6i+b\,x\,6i}\,5i+5i\right)\sqrt{\frac{c\left(e^{a\,2i+b\,x\,2i}\,1i-i\right)\left(e^{a\,4i+b\,x\,4i}\,1i-i\right)}{\left(e^{a\,2i+b\,x\,2i}+1\right)\left(e^{a\,4i+b\,x\,4i}+1\right)}}}{3\,b\,\left(e^{a\,2i+b\,x\,2i}-e^{a\,4i+b\,x\,4i}+e^{a\,6i+b\,x\,6i}-1\right)}$$

input
```
int((c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2)/cos(2*a + 2*b*x),x)
```

output
```
-(c*(exp(a*2i + b*x*2i)*3i + exp(a*4i + b*x*4i)*3i + exp(a*6i + b*x*6i)*5i
 + 5i)*((c*(exp(a*2i + b*x*2i)*1i - 1i)*(exp(a*4i + b*x*4i)*1i - 1i))/((ex
p(a*2i + b*x*2i) + 1)*(exp(a*4i + b*x*4i) + 1)))^(1/2))/(3*b*(exp(a*2i + b
*x*2i) - exp(a*4i + b*x*4i) + exp(a*6i + b*x*6i) - 1))
```

3.615 $\int (c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx$

3.615.1 Optimal result

Integrand size = 20, antiderivative size = 80

$$\int (c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx=\frac{c^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{b}+\frac{c^2\tan(2a+2bx)}{b\sqrt{-c+c\sec(2a+2bx)}}$$

output
```
c^(3/2)*arctanh(c^(1/2)*tan(2*b*x+2*a)/(-c+c*sec(2*b*x+2*a))^(1/2))/b+c^2*
tan(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)
```

3.615.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.08

$$\int (c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx=\frac{c\left(2\cot(a+bx)+\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}\cos(a+bx)}{\sqrt{\cos(2(a+bx))}}\right)\sqrt{\cos(2(a+bx))}\csc(a+bx)\right)\sqrt{c}}{2b}$$

input
```
Integrate[(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

output
```
(c*(2*Cot[a + b*x] + Sqrt[2]*ArcTanh[(Sqrt[2]*Cos[a + b*x])/Sqrt[Cos[2*(a
+ b*x)]]]*Sqrt[Cos[2*(a + b*x)]]*Csc[a + b*x])*Sqrt[c*Tan[a + b*x]*Tan[2*(
a + b*x)]])/(2*b)
```

3.615.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 4897, 3042, 4262, 27, 3042, 4261, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (c\tan(a+bx)\tan(2a+2bx))^{3/2}dx$$

$$\Big\downarrow \; 4897$$

$$\int (c\sec(2a+2bx)-c)^{3/2}dx$$

$$\Big\downarrow \; 3042$$

$$\int \left(c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c\right)^{3/2}dx$$

$$\Big\downarrow \; 4262$$

$$\frac{c^2\tan(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}-2c\int \frac{1}{2}\sqrt{c\sec(2a+2bx)-c}dx$$

$$\Big\downarrow \; 27$$

$$\frac{c^2\tan(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}-c\int \sqrt{c\sec(2a+2bx)-c}dx$$

$$\Big\downarrow \; 3042$$

$$\frac{c^2\tan(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}-c\int \sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}dx$$

$$\Big\downarrow \; 4261$$

$$\frac{c^2\int \frac{1}{\frac{c^2\tan^2(2a+2bx)}{c\sec(2a+2bx)-c}-c}d\left(-\frac{c\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{b}+\frac{c^2\tan(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}$$

$$\Big\downarrow \; 220$$

$$\frac{c^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{b}+\frac{c^2\tan(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}$$

input
```
Int[(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

output
```
(c^(3/2)*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]]]
)/b + (c^2*Tan[2*a + 2*b*x])/(b*Sqrt[-c + c*Sec[2*a + 2*b*x]])
```

3.615.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4262
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-b^2)*C
ot[c + d*x]*((a + b*Csc[c + d*x])^(n - 2)/(d*(n - 1))), x] + Simp[a/(n - 1)
   Int[(a + b*Csc[c + d*x])^(n - 2)*(a*(n - 1) + b*(3*n - 4)*Csc[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && Inte
gerQ[2*n]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.615.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. 2(72) = 144.

Time = 17.21 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.80

method	result
default	$-\frac{\sqrt{2}\, c \sqrt{\frac{c \sin(xb+a)^2}{2\cos(xb+a)^2-1}} \left(\cot(xb+a)\sqrt{\frac{2\cos(xb+a)^2-1}{(1+\cos(xb+a))^2}}\, \operatorname{arctanh}\left(\frac{\cos(xb+a)\sqrt{2}}{(1+\cos(xb+a))\sqrt{\frac{2\cos(xb+a)^2-1}{(1+\cos(xb+a))^2}}}\right)\sqrt{2}+\csc(xb+a)\sqrt{\frac{2\cos(xb+a)^2-1}{(1+\cos(xb+a))^2}}\right.}{2b\left(2+\sqrt{2}\right)\left(-2+\sqrt{2}\right)}$

input
```
int((c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*2^(1/2)/b*c*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(1/2)*(cot(b*x+a)*((2
*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*arctanh(cos(b*x+a)/(1+cos(b*x+a))
/((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*2^(1/2))*2^(1/2)+csc(b*x+a)*(
(2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*arctanh(cos(b*x+a)/(1+cos(b*x+a
))/((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*2^(1/2))*2^(1/2)+2*cot(b*x+
a))*4^(1/2)/(2+2^(1/2))/(-2+2^(1/2))
```

3.615.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 296, normalized size of antiderivative = 3.70

$$\int (c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = \left[\frac{c^{\frac{3}{2}}\log\left(-\frac{c\tan(bx+a)^5-14\,c\tan(bx+a)^3+4\sqrt{2}\left(\tan(bx+a)^4-4\tan(bx+a)^2+3\right)\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\sqrt{c}-}{\tan(bx+a)^5+2\tan(bx+a)^3+\tan(bx+a)}\right.}{4\,b\tan(bx+a)} - \frac{\sqrt{-c}c\arctan\left(\frac{2\sqrt{2}\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{-c}}{c\tan(bx+a)^3-3\,c\tan(bx+a)}\right)\tan(bx+a)-2\sqrt{2}\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}c}{2\,b\tan(bx+a)}\right]$$

input
```
integrate((c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(c^(3/2)*log(-(c*tan(b*x + a)^5 - 14*c*tan(b*x + a)^3 + 4*sqrt(2)*(ta
n(b*x + a)^4 - 4*tan(b*x + a)^2 + 3)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^
2 - 1))*sqrt(c) + 17*c*tan(b*x + a))/(tan(b*x + a)^5 + 2*tan(b*x + a)^3 +
tan(b*x + a)))*tan(b*x + a) + 4*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x +
a)^2 - 1))*c)/(b*tan(b*x + a)), -1/2*(sqrt(-c)*c*arctan(2*sqrt(2)*sqrt(-c*
tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(
b*x + a)^3 - 3*c*tan(b*x + a)))*tan(b*x + a) - 2*sqrt(2)*sqrt(-c*tan(b*x +
 a)^2/(tan(b*x + a)^2 - 1))*c)/(b*tan(b*x + a))]
```

3.615.6 Sympy [F(-1)]

Timed out.

$$\int (c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
```

output
```
Timed out
```

3.615.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1317 vs. $2(72) = 144$.

Time = 0.47 (sec) , antiderivative size = 1317, normalized size of antiderivative = 16.46

$$\int (c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate((c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="maxima")
```

output
```
-1/8*((cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(
1/4)*(c*log(sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4
*a) + 1)*cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + sqr
t(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(1/
2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + 2*(cos(4*b*x + 4*a
)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)*sin(1/2*arctan2(s
in(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) + 1) - c*log(sqrt(cos(4*b*x + 4*a
)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*cos(1/2*arctan2(sin(4*b
*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x
 + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos
(4*b*x + 4*a) - 1))^2 - 2*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos
(4*b*x + 4*a) + 1)^(1/4)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*
a) - 1)) + 1) + c*log(((cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a
) - 1))^2 + sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2)*c
os(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a)))^2 + (cos(1/2*arctan2(s
in(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + sin(1/2*arctan2(sin(4*b*x + 4
*a), -cos(4*b*x + 4*a) - 1))^2)*sin(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*
x + 4*a)))^2)*sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x +
 4*a) + 1) + 2*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*
a) + 1)^(1/4)*(cos(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a)))*sin...
```

3.615.8 Giac [F(-1)]

Timed out.

$$\int (c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.615.9 Mupad [F(-1)]

Timed out.

$$\int (c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = \int (c\tan(a+b\,x)\,\tan(2\,a+2\,b\,x))^{3/2}\,dx$$

input
```
int((c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2),x)
```

output
```
int((c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2), x)
```

3.616 $\int \cos(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx$

3.616.1 Optimal result

Integrand size = 29, antiderivative size = 86

$$\int \cos(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = -\frac{3c^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{2b}+\frac{c^2\sin(2a+2bx)}{2b\sqrt{-c+c\sec(2a+2bx)}}$$

output
```
-3/2*c^(3/2)*arctanh(c^(1/2)*tan(2*b*x+2*a)/(-c+c*sec(2*b*x+2*a))^(1/2))/b
+1/2*c^2*sin(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)
```

3.616.2 Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.08

$$\int \cos(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = \frac{c\left(\cos(a+bx)-3\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\cos(a+bx)}{\sqrt{\cos(2(a+bx))}}\right)\sqrt{\cos(2(a+bx))}+\cos(3(a+bx))\right)}{4b}$$

input
```
Integrate[Cos[2*(a + b*x)]*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

output
```
(c*(Cos[a + b*x] - 3*Sqrt[2]*ArcTanh[(Sqrt[2]*Cos[a + b*x])/Sqrt[Cos[2*(a
+ b*x)]]]*Sqrt[Cos[2*(a + b*x)]] + Cos[3*(a + b*x)])*Csc[a + b*x]*Sqrt[c*T
an[a + b*x]*Tan[2*(a + b*x)]])/(4*b)
```

3.616.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.44, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {3042, 4897, 3042, 4301, 27, 3042, 4292, 3042, 4261, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(2a+2bx)(c\tan(a+bx)\tan(2a+2bx))^{3/2}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \cos(2a+2bx)(c\sec(2a+2bx)-c)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c\right)^{3/2}}{\csc\left(2a+2bx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{4301}$$

$$-2c\int \frac{3}{2}\cos(2a+2bx)\sqrt{c\sec(2a+2bx)-c}dx-\frac{c^2\sin(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}$$

$$\Big\downarrow \text{27}$$

$$-3c\int \cos(2a+2bx)\sqrt{c\sec(2a+2bx)-c}dx-\frac{c^2\sin(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}$$

$$\Big\downarrow \text{3042}$$

$$-3c\int \frac{\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}}{\csc\left(2a+2bx+\frac{\pi}{2}\right)}dx-\frac{c^2\sin(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}$$

$$\Big\downarrow \text{4292}$$

$$-3c\left(-\frac{1}{2}\int \sqrt{c\sec(2a+2bx)-c}dx-\frac{c\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}\right)-\frac{c^2\sin(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}$$

$$\Big\downarrow \text{3042}$$

$$-3c\left(-\frac{1}{2}\int\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}dx-\frac{c\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}\right)-\frac{c^2\sin(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}$$

$$\Big\downarrow \text{4261}$$

$$-3c\left(\frac{c\int\frac{1}{\frac{c^2\tan^2(2a+2bx)}{c\sec(2a+2bx)-c}-c}d\left(-\frac{c\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{2b}-\frac{c\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}\right)-\frac{c^2\sin(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}$$

$$\Big\downarrow \text{220}$$

$$-3c\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{2b}-\frac{c\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}\right)-\frac{c^2\sin(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}$$

input
```
Int[Cos[2*(a + b*x)]*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

output
```
-((c^2*Sin[2*a + 2*b*x])/(b*Sqrt[-c + c*Sec[2*a + 2*b*x]])) - 3*c*((Sqrt[c
]*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]]])/(2*b)
 - (c*Sin[2*a + 2*b*x])/(2*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]))
```

3.616.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4292
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Simp[a*((2*n + 1)/(2*b*d*n))   Int[Sqrt[a + b*Csc
[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

rule 4301
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-b^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m -
2)*((d*Csc[e + f*x])^n/(f*(m + n - 1))), x] + Simp[b/(m + n - 1)   Int[(a +
 b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*(b*(m + 2*n - 1) + a*(3*m + 2*n
 - 4)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^
2, 0] && GtQ[m, 1] && NeQ[m + n - 1, 0] && IntegerQ[2*m]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.616.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 456 vs. $2(74) = 148$.

Time = 4.95 (sec) , antiderivative size = 457, normalized size of antiderivative = 5.31

method	result
default	$\frac{\sqrt{2}\, c\sqrt{\frac{c\sin(xb+a)^2}{2\cos(xb+a)^2-1}}\left(\cot(xb+a)\sqrt{\frac{2\cos(xb+a)^2-1}{(1+\cos(xb+a))^2}}\ \operatorname{arctanh}\left(\frac{\cos(xb+a)\sqrt{2}}{(1+\cos(xb+a))\sqrt{\frac{2\cos(xb+a)^2-1}{(1+\cos(xb+a))^2}}}\right)\sqrt{2}+\csc(xb+a)\sqrt{\frac{2\cos(xb+a)^2-1}{(1+\cos(xb+a))^2}}\ \operatorname{arc}}{2b\left(2+\sqrt{2}\right)\left(-2+\sqrt{2}\right)}$

input
```
int(cos(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x,method=_RETURNVER
BOSE)
```

output
```
1/2*2^(1/2)/b*c*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(1/2)*(cot(b*x+a)*((2*
cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*arctanh(cos(b*x+a)/(1+cos(b*x+a))/
((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*2^(1/2))*2^(1/2)+csc(b*x+a)*((
2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*arctanh(cos(b*x+a)/(1+cos(b*x+a)
)/((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*2^(1/2))*2^(1/2)+2*cot(b*x+a
))*4^(1/2)/(2+2^(1/2))/(-2+2^(1/2))+2^(1/2)/b*csc(b*x+a)*(2^(1/2)*arctanh(
cos(b*x+a)/(1+cos(b*x+a))/((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*2^(1
/2))*((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*cos(b*x+a)+2^(1/2)*arctan
h(cos(b*x+a)/(1+cos(b*x+a))/((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*2^
(1/2))*((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)-4*cos(b*x+a)^3-2*cos(b*
x+a))*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(1/2)*c*4^(1/2)/(-2+2^(1/2))^3/(
2+2^(1/2))^3
```

3.616.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(74) = 148$.

Time = 0.29 (sec) , antiderivative size = 369, normalized size of antiderivative = 4.29

$$\int \cos(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = \left[\frac{3\left(c\tan(bx+a)^3+c\tan(bx+a)\right)\sqrt{c}\log\left(-\frac{c\tan(bx+a)^5-14\,c\tan(bx+a)^3-4\sqrt{2}\left(\tan(\right.}{\tan(bx+a}\right.}{8\left(b\tan(b\right.}\right.$$

input
```
integrate(cos(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm=
"fricas")
```

output
```
[1/8*(3*(c*tan(b*x + a)^3 + c*tan(b*x + a))*sqrt(c)*log(-(c*tan(b*x + a)^5
 - 14*c*tan(b*x + a)^3 - 4*sqrt(2)*(tan(b*x + a)^4 - 4*tan(b*x + a)^2 + 3)
*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*sqrt(c) + 17*c*tan(b*x + a))
/(tan(b*x + a)^5 + 2*tan(b*x + a)^3 + tan(b*x + a))) - 4*sqrt(2)*(c*tan(b*
x + a)^2 - c)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*tan(b*x + a
)^3 + b*tan(b*x + a)), 1/4*(3*(c*tan(b*x + a)^3 + c*tan(b*x + a))*sqrt(-c)
*arctan(2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x +
a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a)^3 - 3*c*tan(b*x + a))) - 2*sqrt(2)*(c*t
an(b*x + a)^2 - c)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*tan(b*
x + a)^3 + b*tan(b*x + a))]
```

3.616.6 Sympy [F(-1)]

Timed out.

$$\int \cos(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
```

output
```
Timed out
```

3.616.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1058 vs. $2(74) = 148$.

Time = 0.46 (sec) , antiderivative size = 1058, normalized size of antiderivative = 12.30

$$\int \cos(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm=
"maxima")
```

output
```
-1/16*(4*(c*cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))*sin(
2*b*x + 2*a) + (c*cos(2*b*x + 2*a) + c)*sin(1/2*arctan2(sin(4*b*x + 4*a),
-cos(4*b*x + 4*a) - 1)))*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(
4*b*x + 4*a) + 1)^(1/4)*sqrt(c) - 3*(c*log(sqrt(cos(4*b*x + 4*a)^2 + sin(4
*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*cos(1/2*arctan2(sin(4*b*x + 4*a),
-cos(4*b*x + 4*a) - 1))^2 + sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 +
 2*cos(4*b*x + 4*a) + 1)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*
a) - 1))^2 + 2*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*
a) + 1)^(1/4)*sin(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) +
1) - c*log(sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*
a) + 1)*cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 + sqrt
(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*sin(1/2
*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2 - 2*(cos(4*b*x + 4*a)
^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)*sin(1/2*arctan2(si
n(4*b*x + 4*a), -cos(4*b*x + 4*a) - 1)) + 1) + c*log(((cos(2*b*x + 2*a)^2
+ sin(2*b*x + 2*a)^2)*cos(1/2*arctan2(sin(4*b*x + 4*a), -cos(4*b*x + 4*a)
- 1))^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2)*sin(1/2*arctan2(sin(4*
b*x + 4*a), -cos(4*b*x + 4*a) - 1))^2)*sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x
 + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1) + 2*(cos(4*b*x + 4*a)^2 + sin(4*b*x +
4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)*(cos(1/2*arctan2(sin(4*b*x + 4*a...
```

3.616.8 Giac [F(-1)]

Timed out.

$$\int \cos(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(cos(2*b*x+2*a)*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm=
"giac")
```

output
```
Timed out
```

3.616.9 Mupad [F(-1)]

Timed out.

$$\int \cos(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = \int \cos\left(2\,a + 2\,b\,x\right)\,\left(c\tan(a+b\,x)\,\tan(2\,a+2\,b\,x)\right)^{3/2}\,dx$$

input
```
int(cos(2*a + 2*b*x)*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2),x)
```

output
```
int(cos(2*a + 2*b*x)*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2), x)
```

3.617 $\int \cos^2(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx$

3.617.1 Optimal result

Integrand size = 31, antiderivative size = 133

$$\int \cos^2(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx=\frac{7c^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{8b}-\frac{7c^2\sin(2a+2bx)}{8b\sqrt{-c+c\sec(2a+2bx)}}+\frac{c^2\cos(2a+2bx)\sin(2a+2bx)}{4b\sqrt{-c+c\sec(2a+2bx)}}$$

output
```
7/8*c^(3/2)*arctanh(c^(1/2)*tan(2*b*x+2*a)/(-c+c*sec(2*b*x+2*a))^(1/2))/b-
7/8*c^2*sin(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)+1/4*c^2*cos(2*b*x+2*a
)*sin(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)
```

3.617.2 Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.79

$$\int \cos^2(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx=\frac{c\left(-5\cos(a+bx)+7\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\cos(a+bx)}{\sqrt{\cos(2(a+bx))}}\right)\sqrt{\cos(2(a+bx))}-6\cos(3(a+\dots\right.}{16b}$$

input
```
Integrate[Cos[2*(a + b*x)]^2*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

output
```
(c*(-5*Cos[a + b*x] + 7*Sqrt[2]*ArcTanh[(Sqrt[2]*Cos[a + b*x])/Sqrt[Cos[2*
(a + b*x)]]]*Sqrt[Cos[2*(a + b*x)]] - 6*Cos[3*(a + b*x)] + Cos[5*(a + b*x)
])*Csc[a + b*x]*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]])/(16*b)
```

3.617.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.355, Rules used = {3042, 4897, 3042, 4300, 27, 2011, 3042, 4292, 3042, 4261, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(2a+2bx)^2(c\tan(a+bx)\tan(2a+2bx))^{3/2}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \cos^2(2a+2bx)(c\sec(2a+2bx)-c)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c\right)^{3/2}}{\csc\left(2a+2bx+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{4300}$$

$$\frac{c^2\sin(2a+2bx)\cos(2a+2bx)}{4b\sqrt{c\sec(2a+2bx)-c}}-\frac{1}{2}c\int \frac{7\cos(2a+2bx)(c-c\sec(2a+2bx))}{2\sqrt{c\sec(2a+2bx)-c}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{c^2\sin(2a+2bx)\cos(2a+2bx)}{4b\sqrt{c\sec(2a+2bx)-c}}-\frac{7}{4}c\int \frac{\cos(2a+2bx)(c-c\sec(2a+2bx))}{\sqrt{c\sec(2a+2bx)-c}}dx$$

$$\Big\downarrow \text{2011}$$

$$\frac{7}{4}c\int \cos(2a+2bx)\sqrt{c\sec(2a+2bx)-c}dx+\frac{c^2\sin(2a+2bx)\cos(2a+2bx)}{4b\sqrt{c\sec(2a+2bx)-c}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{4}c\int \frac{\sqrt{c\csc \left(2a+2bx+\frac{\pi }{2}\right)-c}}{\csc \left(2a+2bx+\frac{\pi }{2}\right)}dx+\frac{c^2\sin (2a+2bx)\cos (2a+2bx)}{4b\sqrt{c\sec (2a+2bx)-c}}$$

$$\Big\downarrow \text{4292}$$

$$\frac{7}{4}c\left(-\frac{1}{2}\int \sqrt{c\sec (2a+2bx)-c}dx-\frac{c\sin (2a+2bx)}{2b\sqrt{c\sec (2a+2bx)-c}}\right)+\frac{c^2\sin (2a+2bx)\cos (2a+2bx)}{4b\sqrt{c\sec (2a+2bx)-c}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7}{4}c\left(-\frac{1}{2}\int \sqrt{c\csc \left(2a+2bx+\frac{\pi }{2}\right)-c}dx-\frac{c\sin (2a+2bx)}{2b\sqrt{c\sec (2a+2bx)-c}}\right)+\frac{c^2\sin (2a+2bx)\cos (2a+2bx)}{4b\sqrt{c\sec (2a+2bx)-c}}$$

$$\Big\downarrow \text{4261}$$

$$\frac{7}{4}c\left(\frac{c\int \frac{1}{\frac{c^2\tan ^2(2a+2bx)}{c\sec (2a+2bx)-c}-c}d\left(-\frac{c\tan (2a+2bx)}{\sqrt{c\sec (2a+2bx)-c}}\right)}{2b}-\frac{c\sin (2a+2bx)}{2b\sqrt{c\sec (2a+2bx)-c}}\right)+\frac{c^2\sin (2a+2bx)\cos (2a+2bx)}{4b\sqrt{c\sec (2a+2bx)-c}}$$

$$\Big\downarrow \text{220}$$

$$\frac{7}{4}c\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}\tan (2a+2bx)}{\sqrt{c\sec (2a+2bx)-c}}\right)}{2b}-\frac{c\sin (2a+2bx)}{2b\sqrt{c\sec (2a+2bx)-c}}\right)+\frac{c^2\sin (2a+2bx)\cos (2a+2bx)}{4b\sqrt{c\sec (2a+2bx)-c}}$$

input
```
Int[Cos[2*(a + b*x)]^2*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

output
```
(c^2*Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x])/(4*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]
) + (7*c*((Sqrt[c]*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a
+ 2*b*x]]])/(2*b) - (c*Sin[2*a + 2*b*x])/(2*b*Sqrt[-c + c*Sec[2*a + 2*b*x]
])))/4
```

3.617. $\int \cos^2(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx$

3.617.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4292
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Simp[a*((2*n + 1)/(2*b*d*n))   Int[Sqrt[a + b*Csc
[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

rule 4300
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[a/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
 && (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.617. $\int \cos^2(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx$

3.617.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 701 vs. 2(117) = 234.

Time = 5.32 (sec) , antiderivative size = 702, normalized size of antiderivative = 5.28

method	result
default	$-\frac{\sqrt{2}\,c\sqrt{\frac{c\sin(xb+a)^2}{2\cos(xb+a)^2-1}}\left(\cot(xb+a)\sqrt{\frac{2\cos(xb+a)^2-1}{(1+\cos(xb+a))^2}}\,\operatorname{arctanh}\left(\frac{\cos(xb+a)\sqrt{2}}{(1+\cos(xb+a))\sqrt{\frac{2\cos(xb+a)^2-1}{(1+\cos(xb+a))^2}}}\right)\sqrt{2}+\csc(xb+a)\sqrt{\frac{2\cos(xb+a)^2-1}{(1+\cos(xb+a))^2}}}{2b\left(2+\sqrt{2}\right)\left(-2+\sqrt{2}\right)}$

input

```
int(cos(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x,method=_RETURNV
ERBOSE)
```

output

```
-1/2*2^(1/2)/b*c*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(1/2)*(cot(b*x+a)*((2
*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*arctanh(cos(b*x+a)/(1+cos(b*x+a))
/((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*2^(1/2))*2^(1/2)+csc(b*x+a)*(
(2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*arctanh(cos(b*x+a)/(1+cos(b*x+a
))/((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*2^(1/2))*2^(1/2)+2*cot(b*x+
a))*4^(1/2)/(2+2^(1/2))/(-2+2^(1/2))-2*2^(1/2)/b*csc(b*x+a)*(2^(1/2)*arcta
nh(cos(b*x+a)/(1+cos(b*x+a))/((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*2
^(1/2))*((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*cos(b*x+a)+2^(1/2)*arc
tanh(cos(b*x+a)/(1+cos(b*x+a))/((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)
*2^(1/2))*((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)-4*cos(b*x+a)^3-2*cos
(b*x+a))*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(1/2)*c*4^(1/2)/(-2+2^(1/2))^
3/(2+2^(1/2))^3+2^(1/2)/b*csc(b*x+a)*(-16*cos(b*x+a)^5+9*2^(1/2)*arctanh(c
os(b*x+a)/(1+cos(b*x+a))/((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*2^(1/
2))*((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*cos(b*x+a)+9*2^(1/2)*arcta
nh(cos(b*x+a)/(1+cos(b*x+a))/((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*2
^(1/2))*((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)+12*cos(b*x+a)^3-18*cos
(b*x+a))*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(1/2)*c*4^(1/2)/(-2+2^(1/2))^
5/(2+2^(1/2))^5
```

3.617.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 437, normalized size of antiderivative = 3.29

$$\int \cos^2(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = \left[\frac{7\left(c\tan(bx+a)^5+2\,c\tan(bx+a)^3+c\tan(bx+a)\right)\sqrt{c}\log\left(-\frac{c\tan(bx+a)^5-14\cdots}{\cdots}\right.}{\cdots}\right.$$

$$-\frac{7\left(c\tan(bx+a)^5+2\,c\tan(bx+a)^3+c\tan(bx+a)\right)\sqrt{-c}\arctan\left(\frac{2\sqrt{2}\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{-c}}{c\tan(bx+a)^3-3\,c\tan(bx+a)}\right)-\cdots}{16\left(b\tan(bx+a)^5+2\,b\tan(bx+a)^3+b\tan(b\cdots\right.}$$

input
```
integrate(cos(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorith
m="fricas")
```

output
```
[1/32*(7*(c*tan(b*x + a)^5 + 2*c*tan(b*x + a)^3 + c*tan(b*x + a))*sqrt(c)*
log(-(c*tan(b*x + a)^5 - 14*c*tan(b*x + a)^3 + 4*sqrt(2)*(tan(b*x + a)^4 -
 4*tan(b*x + a)^2 + 3)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*sqrt(c
) + 17*c*tan(b*x + a))/(tan(b*x + a)^5 + 2*tan(b*x + a)^3 + tan(b*x + a)))
 + 4*sqrt(2)*(9*c*tan(b*x + a)^4 - 4*c*tan(b*x + a)^2 - 5*c)*sqrt(-c*tan(b
*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*tan(b*x + a)^5 + 2*b*tan(b*x + a)^3 +
b*tan(b*x + a)), -1/16*(7*(c*tan(b*x + a)^5 + 2*c*tan(b*x + a)^3 + c*tan(b
*x + a))*sqrt(-c)*arctan(2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2
- 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a)^3 - 3*c*tan(b*x + a)))
 - 2*sqrt(2)*(9*c*tan(b*x + a)^4 - 4*c*tan(b*x + a)^2 - 5*c)*sqrt(-c*tan(b
*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*tan(b*x + a)^5 + 2*b*tan(b*x + a)^3 +
b*tan(b*x + a))]
```

3.617.6 Sympy [F(-1)]

Timed out.

$$\int \cos^2(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(2*b*x+2*a)**2*(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
```

output
```
Timed out
```

3.617.7 Maxima [F(-1)]

Timed out.

$$\int \cos^2(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorith
m="maxima")
```

output
```
Timed out
```

3.617.8 Giac [F(-1)]

Timed out.

$$\int \cos^2(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(2*b*x+2*a)^2*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorith
m="giac")
```

output
```
Timed out
```

3.617.9 Mupad [F(-1)]

Timed out.

$$\int \cos^2(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = \int \cos\left(2\,a+2\,b\,x\right)^2\,{\left(c\tan\left(a+b\,x\right)\,\tan\left(2\,a+2\,b\,x\right)\right)}^{3/2}\,dx$$

input
```
int(cos(2*a + 2*b*x)^2*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2),x)
```

output
```
int(cos(2*a + 2*b*x)^2*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2), x)
```

3.618 $\int \cos^3(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx$

3.618.1 Optimal result

Integrand size = 31, antiderivative size = 182

$$\int \cos^3(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = -\frac{11c^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{16b}+\frac{11c^2\sin(2a+2bx)}{16b\sqrt{-c+c\sec(2a+2bx)}}-\frac{11c^2\cos(2a+2bx)\sin(2a+2bx)}{24b\sqrt{-c+c\sec(2a+2bx)}}+\frac{c^2\cos^2(2a+2bx)\sin(2a+2bx)}{6b\sqrt{-c+c\sec(2a+2bx)}}$$

output
```
-11/16*c^(3/2)*arctanh(c^(1/2)*tan(2*b*x+2*a)/(-c+c*sec(2*b*x+2*a))^(1/2))
/b+11/16*c^2*sin(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)-11/24*c^2*cos(2*
b*x+2*a)*sin(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)+1/6*c^2*cos(2*b*x+2*
a)^2*sin(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)
```

3.618.2 Mathematica [A] (verified)

Time = 1.18 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.64

$$\int \cos^3(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = \frac{c\left(38\cot(a+bx)-33\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\cos(a+bx)}{\sqrt{\cos(2(a+bx))}}\right)\sqrt{\cos(2(a+bx))}\csc(a+bx)-\right.}{}$$

input
```
Integrate[Cos[2*(a + b*x)]^3*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

output
```
(c*(38*Cot[a + b*x] - 33*Sqrt[2]*ArcTanh[(Sqrt[2]*Cos[a + b*x])/Sqrt[Cos[2
*(a + b*x)]]]*Sqrt[Cos[2*(a + b*x)]]*Csc[a + b*x] - 42*Sin[2*(a + b*x)] +
14*Sin[4*(a + b*x)] - 4*Sin[6*(a + b*x)])*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b
*x)]])/(96*b)
```

3.618.3 Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.04, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$, Rules used = {3042, 4897, 3042, 4300, 27, 2011, 3042, 4292, 3042, 4292, 3042, 4261, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^3(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(2a+2bx)^3(c\tan(a+bx)\tan(2a+2bx))^{3/2}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \cos^3(2a+2bx)(c\sec(2a+2bx)-c)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c\right)^{3/2}}{\csc\left(2a+2bx+\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{4300}$$

$$\frac{c^2\sin(2a+2bx)\cos^2(2a+2bx)}{6b\sqrt{c\sec(2a+2bx)-c}}-\frac{1}{3}c\int \frac{11\cos^2(2a+2bx)(c-c\sec(2a+2bx))}{2\sqrt{c\sec(2a+2bx)-c}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{c^2\sin(2a+2bx)\cos^2(2a+2bx)}{6b\sqrt{c\sec(2a+2bx)-c}}-\frac{11}{6}c\int \frac{\cos^2(2a+2bx)(c-c\sec(2a+2bx))}{\sqrt{c\sec(2a+2bx)-c}}dx$$

$$\Big\downarrow \text{2011}$$

$$\frac{11}{6}c\int \cos^2(2a+2bx)\sqrt{c\sec(2a+2bx)-c}dx+\frac{c^2\sin(2a+2bx)\cos^2(2a+2bx)}{6b\sqrt{c\sec(2a+2bx)-c}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{11}{6}c\int \frac{\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}}{\csc\left(2a+2bx+\frac{\pi}{2}\right)^2}dx+\frac{c^2\sin(2a+2bx)\cos^2(2a+2bx)}{6b\sqrt{c\sec(2a+2bx)-c}}$$

$$\Big\downarrow \text{4292}$$

$$\frac{11}{6}c\left(-\frac{3}{4}\int \cos(2a+2bx)\sqrt{c\sec(2a+2bx)-c}dx-\frac{c\sin(2a+2bx)\cos(2a+2bx)}{4b\sqrt{c\sec(2a+2bx)-c}}\right)+\frac{c^2\sin(2a+2bx)\cos^2(2a+2bx)}{6b\sqrt{c\sec(2a+2bx)-c}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{11}{6}c\left(-\frac{3}{4}\int \frac{\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}}{\csc\left(2a+2bx+\frac{\pi}{2}\right)}dx-\frac{c\sin(2a+2bx)\cos(2a+2bx)}{4b\sqrt{c\sec(2a+2bx)-c}}\right)+\frac{c^2\sin(2a+2bx)\cos^2(2a+2bx)}{6b\sqrt{c\sec(2a+2bx)-c}}$$

$$\Big\downarrow \text{4292}$$

$$\frac{11}{6}c\left(-\frac{3}{4}\left(-\frac{1}{2}\int \sqrt{c\sec(2a+2bx)-c}dx-\frac{c\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}\right)-\frac{c\sin(2a+2bx)\cos(2a+2bx)}{4b\sqrt{c\sec(2a+2bx)-c}}\right)+\frac{c^2\sin(2a+2bx)\cos^2(2a+2bx)}{6b\sqrt{c\sec(2a+2bx)-c}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{11}{6}c\left(-\frac{3}{4}\left(-\frac{1}{2}\int \sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}dx-\frac{c\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}\right)-\frac{c\sin(2a+2bx)\cos(2a+2bx)}{4b\sqrt{c\sec(2a+2bx)-c}}\right)+\frac{c^2\sin(2a+2bx)\cos^2(2a+2bx)}{6b\sqrt{c\sec(2a+2bx)-c}}$$

$$\Big\downarrow \text{4261}$$

$$\frac{11}{6}c\left(-\frac{3}{4}\left(\frac{c\int \frac{1}{\frac{c^2\tan^2(2a+2bx)}{c\sec(2a+2bx)-c}-c}d\left(-\frac{c\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{2b}-\frac{c\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}\right)-\frac{c\sin(2a+2bx)\cos(2a+2bx)}{4b\sqrt{c\sec(2a+2bx)-c}}\right)+\frac{c^2\sin(2a+2bx)\cos^2(2a+2bx)}{6b\sqrt{c\sec(2a+2bx)-c}}$$

3.618. $\int \cos^3(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx$

$$\Big\downarrow 220$$

$$\frac{11}{6}c\left(-\frac{3}{4}\left(\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{2b}-\frac{c\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}\right)-\frac{c\sin(2a+2bx)\cos(2a+2bx)}{4b\sqrt{c\sec(2a+2bx)-c}}\right)+\frac{c^2\sin(2a+2bx)\cos^2(2a+2bx)}{6b\sqrt{c\sec(2a+2bx)-c}}$$

input
```
Int[Cos[2*(a + b*x)]^3*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

output
```
(c^2*Cos[2*a + 2*b*x]^2*Sin[2*a + 2*b*x])/(6*b*Sqrt[-c + c*Sec[2*a + 2*b*x
]]) + (11*c*(-1/4*(c*Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x])/(b*Sqrt[-c + c*Sec
[2*a + 2*b*x]]) - (3*((Sqrt[c]*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c
+ c*Sec[2*a + 2*b*x]]])/(2*b) - (c*Sin[2*a + 2*b*x])/(2*b*Sqrt[-c + c*Sec[
2*a + 2*b*x]])))/4))/6
```

3.618.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.618. $\int \cos^3(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx$

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4292
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Simp[a*((2*n + 1)/(2*b*d*n))   Int[Sqrt[a + b*Csc
[e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
 EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

rule 4300
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b^2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 2)*
((d*Csc[e + f*x])^n/(f*n)), x] - Simp[a/(d*n)   Int[(a + b*Csc[e + f*x])^(m
 - 2)*(d*Csc[e + f*x])^(n + 1)*(b*(m - 2*n - 2) - a*(m + 2*n - 1)*Csc[e + f
*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1]
 && (LtQ[n, -1] || (EqQ[m, 3/2] && EqQ[n, -2^(-1)])) && IntegerQ[2*m]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.618.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 957 vs. $2(162) = 324$.

Time = 6.48 (sec) , antiderivative size = 958, normalized size of antiderivative = 5.26

method	result	size
default	Expression too large to display	958

input
```
int(cos(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
1/2*2^(1/2)/b*c*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(1/2)*(cot(b*x+a)*((2*
cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*arctanh(cos(b*x+a)/(1+cos(b*x+a))/
((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*2^(1/2))*2^(1/2)+csc(b*x+a)*((
2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*arctanh(cos(b*x+a)/(1+cos(b*x+a)
)/((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*2^(1/2))*2^(1/2)+2*cot(b*x+a
))*4^(1/2)/(2+2^(1/2))/(-2+2^(1/2))+3*2^(1/2)/b*csc(b*x+a)*(2^(1/2)*arctan
h(cos(b*x+a)/(1+cos(b*x+a))/((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*2^
(1/2))*((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*cos(b*x+a)+2^(1/2)*arct
anh(cos(b*x+a)/(1+cos(b*x+a))/((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*
2^(1/2))*((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)-4*cos(b*x+a)^3-2*cos(
b*x+a))*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(1/2)*c*4^(1/2)/(-2+2^(1/2))^3
/(2+2^(1/2))^3-3*2^(1/2)/b*csc(b*x+a)*(-16*cos(b*x+a)^5+9*2^(1/2)*arctanh(
cos(b*x+a)/(1+cos(b*x+a))/((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*2^(1
/2))*((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*cos(b*x+a)+9*2^(1/2)*arct
anh(cos(b*x+a)/(1+cos(b*x+a))/((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*
2^(1/2))*((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)+12*cos(b*x+a)^3-18*co
s(b*x+a))*(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(1/2)*c*4^(1/2)/(-2+2^(1/2))
^5/(2+2^(1/2))^5+2/3*2^(1/2)/b*csc(b*x+a)*(-128*cos(b*x+a)^7+80*cos(b*x+a)
^5+75*2^(1/2)*arctanh(cos(b*x+a)/(1+cos(b*x+a))/((2*cos(b*x+a)^2-1)/(1+cos
(b*x+a))^2)^(1/2)*2^(1/2))*((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*...
```

3.618.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 503, normalized size of antiderivative = 2.76

$$\int \cos^3(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = \left[\frac{33\left(c\tan(bx+a)^7+3\,c\tan(bx+a)^5+3\,c\tan(bx+a)^3+c\tan(bx+a)\right)\sqrt{c}}{}\right.$$

input
```
integrate(cos(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorith
m="fricas")
```

output
```
[1/192*(33*(c*tan(b*x + a)^7 + 3*c*tan(b*x + a)^5 + 3*c*tan(b*x + a)^3 + c
*tan(b*x + a))*sqrt(c)*log(-(c*tan(b*x + a)^5 - 14*c*tan(b*x + a)^3 - 4*sq
rt(2)*(tan(b*x + a)^4 - 4*tan(b*x + a)^2 + 3)*sqrt(-c*tan(b*x + a)^2/(tan(
b*x + a)^2 - 1))*sqrt(c) + 17*c*tan(b*x + a))/(tan(b*x + a)^5 + 2*tan(b*x
+ a)^3 + tan(b*x + a))) - 4*sqrt(2)*(63*c*tan(b*x + a)^6 - 13*c*tan(b*x +
a)^4 - 31*c*tan(b*x + a)^2 - 19*c)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2
- 1)))/(b*tan(b*x + a)^7 + 3*b*tan(b*x + a)^5 + 3*b*tan(b*x + a)^3 + b*tan
(b*x + a)), 1/96*(33*(c*tan(b*x + a)^7 + 3*c*tan(b*x + a)^5 + 3*c*tan(b*x
+ a)^3 + c*tan(b*x + a))*sqrt(-c)*arctan(2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/
(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a)^3 - 3*
c*tan(b*x + a))) - 2*sqrt(2)*(63*c*tan(b*x + a)^6 - 13*c*tan(b*x + a)^4 -
31*c*tan(b*x + a)^2 - 19*c)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/
(b*tan(b*x + a)^7 + 3*b*tan(b*x + a)^5 + 3*b*tan(b*x + a)^3 + b*tan(b*x +
a))]
```

3.618.6 Sympy [F(-1)]

Timed out.

$$\int \cos^3(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(2*b*x+2*a)**3*(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
```

output
```
Timed out
```

3.618.7 Maxima [F(-1)]

Timed out.

$$\int \cos^3(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cos(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorith
m="maxima")
```

output
```
Timed out
```

3.618.8 Giac [F(-1)]

Timed out.

$$\int \cos^3(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(cos(2*b*x+2*a)^3*(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorith
m="giac")
```

output
```
Timed out
```

3.618.9 Mupad [F(-1)]

Timed out.

$$\int \cos^3(2(a+bx))(c\tan(a+bx)\tan(2(a+bx)))^{3/2}\,dx=\int \cos\left(2\,a+2\,b\,x\right)^3\,{\left(c\tan\left(a+b\,x\right)\,\tan\left(2\,a+2\,b\,x\right)\right)}^{3/2}\,dx$$

input
```
int(cos(2*a + 2*b*x)^3*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2),x)
```

output
```
int(cos(2*a + 2*b*x)^3*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2), x)
```

3.619 $\int \frac{\sec^4(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx$

3.619.1 Optimal result

Integrand size = 31, antiderivative size = 175

$$\int \frac{\sec^4(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{-c+c\sec(2a+2bx)}}\right)}{\sqrt{2}b\sqrt{c}} + \frac{14\tan(2a+2bx)}{15b\sqrt{-c+c\sec(2a+2bx)}} + \frac{\sec^2(2a+2bx)\tan(2a+2bx)}{5b\sqrt{-c+c\sec(2a+2bx)}} + \frac{\sqrt{-c+c\sec(2a+2bx)}\tan(2a+2bx)}{15bc}$$

output
```
-1/2*arctanh(1/2*c^(1/2)*tan(2*b*x+2*a)*2^(1/2)/(-c+c*sec(2*b*x+2*a))^(1/2
))/b*2^(1/2)/c^(1/2)+14/15*tan(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)+1/
5*sec(2*b*x+2*a)^2*tan(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)+1/15*(-c+c
*sec(2*b*x+2*a))^(1/2)*tan(2*b*x+2*a)/b/c
```

3.619.2 Mathematica [A] (verified)

Time = 1.79 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.64

$$\int \frac{\sec^4(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx = \frac{\cos(a+bx)\sec^3(2(a+bx))\sin(a+bx)\left(38+4\cos(2(a+bx))+26\cos(4(a+bx))+30\arctan\left(\sqrt{-1+\tan^2(a+bx)}\right)\cos^2(2(a+bx))\sqrt{-1+\tan^2(a+bx)}\right)}{30b\sqrt{c\tan(a+bx)\tan(2(a+bx))}}$$

input
```
Integrate[Sec[2*(a + b*x)]^4/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

output
```
(Cos[a + b*x]*Sec[2*(a + b*x)]^3*Sin[a + b*x]*(38 + 4*Cos[2*(a + b*x)] + 2
6*Cos[4*(a + b*x)] + 30*ArcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Cos[2*(a + b*x)]
^2*Sqrt[-1 + Tan[a + b*x]^2]))/(30*b*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]]
)
```

3.619.3 Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {3042, 4897, 3042, 4309, 3042, 4498, 27, 3042, 4489, 3042, 4282, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(2a+2bx)^4}{\sqrt{c\tan(a+bx)\tan(2a+2bx)}}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \frac{\sec^4(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(2a+2bx+\frac{\pi}{2}\right)^4}{\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}}dx$$

$$\Big\downarrow \text{4309}$$

$$\frac{\int \frac{\sec^2(2a+2bx)(\sec(2a+2bx)c+4c)}{\sqrt{c\sec(2a+2bx)-c}}dx}{5c}+\frac{\tan(2a+2bx)\sec^2(2a+2bx)}{5b\sqrt{c\sec(2a+2bx)-c}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(2a+2bx+\frac{\pi}{2}\right)^2\left(\csc\left(2a+2bx+\frac{\pi}{2}\right)c+4c\right)}{\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}}dx}{5c}+\frac{\tan(2a+2bx)\sec^2(2a+2bx)}{5b\sqrt{c\sec(2a+2bx)-c}}$$

$$\Big\downarrow \text{4498}$$

$$\frac{\frac{2\int \frac{\sec(2a+2bx)\left(14\sec(2a+2bx)c^2+c^2\right)}{2\sqrt{c\sec(2a+2bx)-c}}dx}{3c}+\frac{\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{3b}}{5c}+\frac{\tan(2a+2bx)\sec^2(2a+2bx)}{5b\sqrt{c\sec(2a+2bx)-c}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\sec(2a+2bx)\left(14\sec(2a+2bx)c^2+c^2\right)}{\sqrt{c\sec(2a+2bx)-c}}dx}{3c}+\frac{\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{3b}}{5c}+\frac{\tan(2a+2bx)\sec^2(2a+2bx)}{5b\sqrt{c\sec(2a+2bx)-c}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{\csc\left(2a+2bx+\frac{\pi}{2}\right)\left(14\csc\left(2a+2bx+\frac{\pi}{2}\right)c^2+c^2\right)}{\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}}dx}{3c}+\frac{\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{3b}}{5c}+\frac{\tan(2a+2bx)\sec^2(2a+2bx)}{5b\sqrt{c\sec(2a+2bx)-c}}$$

$$\Big\downarrow \text{4489}$$

$$\frac{\frac{15c^2\int \frac{\sec(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}dx+\frac{14c^2\tan(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}}{3c}+\frac{\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{3b}}{5c}+\frac{\tan(2a+2bx)\sec^2(2a+2bx)}{5b\sqrt{c\sec(2a+2bx)-c}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{15c^2\int \frac{\csc\left(2a+2bx+\frac{\pi}{2}\right)}{\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}}dx+\frac{14c^2\tan(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}}{3c}+\frac{\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{3b}}{5c}+\frac{\tan(2a+2bx)\sec^2(2a+2bx)}{5b\sqrt{c\sec(2a+2bx)-c}}$$

$$\Big\downarrow \text{4282}$$

3.619. $\int \frac{\sec^4(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx$

$$\frac{\frac{\frac{14c^2\tan(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}-\frac{15c^2\int\frac{1}{\frac{c^2\tan^2(2a+2bx)}{c\sec(2a+2bx)-c}-2c}d\left(-\frac{c\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{b}}{3c}+\frac{\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{3b}}{5c}+\frac{\tan(2a+2bx)\sec^2(2a+2bx)}{5b\sqrt{c\sec(2a+2bx)-c}}$$

$$\Big\downarrow \ 220$$

$$\frac{\frac{\frac{14c^2\tan(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}-\frac{15c^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{\sqrt{2}b}}{3c}+\frac{\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{3b}}{5c}+\frac{\tan(2a+2bx)\sec^2(2a+2bx)}{5b\sqrt{c\sec(2a+2bx)-c}}$$

input
```
Int[Sec[2*(a + b*x)]^4/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

output
```
(Sec[2*a + 2*b*x]^2*Tan[2*a + 2*b*x])/(5*b*Sqrt[-c + c*Sec[2*a + 2*b*x]])
+ ((Sqrt[-c + c*Sec[2*a + 2*b*x]]*Tan[2*a + 2*b*x])/(3*b) + ((-15*c^(3/2)*
ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a + 2*b*x]])
])/(Sqrt[2]*b) + (14*c^2*Tan[2*a + 2*b*x])/(b*Sqrt[-c + c*Sec[2*a + 2*b*x]
]))/(3*c))/(5*c)
```

3.619.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

3.619. $\int \frac{\sec^4(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx$

rule 4309
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*d^2*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 2)/(
f*(2*n - 3)*Sqrt[a + b*Csc[e + f*x]])), x] + Simp[d^2/(b*(2*n - 3))   Int[(
d*Csc[e + f*x])^(n - 2)*((2*b*(n - 2) - a*Csc[e + f*x])/Sqrt[a + b*Csc[e +
f*x]]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n,
2] && IntegerQ[2*n]
```

rule 4489
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[(a*B*m + A*b*(m + 1))/(b*(m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B
, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b
*(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

rule 4498
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*
((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m + 2))   Int
[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)
*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a
*B, 0] &&  !LtQ[m, -1]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.619.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 891 vs. $2(154) = 308$.

Time = 11.23 (sec) , antiderivative size = 892, normalized size of antiderivative = 5.10

method	result	size
default	Expression too large to display	892

input
```
int(sec(2*b*x+2*a)^4/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
-1/120*2^(1/2)/b*sin(b*x+a)*(120*cos(b*x+a)^6*ln(2*(cos(b*x+a)*((2*cos(b*x
+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)+((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/
2)-2*cos(b*x+a)-1)/(1+cos(b*x+a)))-120*cos(b*x+a)^6*arctanh((2*cos(b*x+a)-
1)/(1+cos(b*x+a))/((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2))+208*((2*cos
(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*cos(b*x+a)^6+208*((2*cos(b*x+a)^2-1)/
(1+cos(b*x+a))^2)^(1/2)*cos(b*x+a)^5-180*cos(b*x+a)^4*ln(2*(cos(b*x+a)*((2
*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)+((2*cos(b*x+a)^2-1)/(1+cos(b*x+a)
)^2)^(1/2)-2*cos(b*x+a)-1)/(1+cos(b*x+a)))+180*cos(b*x+a)^4*arctanh((2*cos
(b*x+a)-1)/(1+cos(b*x+a))/((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2))-200
*((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*cos(b*x+a)^4-200*((2*cos(b*x+
a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*cos(b*x+a)^3+90*cos(b*x+a)^2*ln(2*(cos(b*x
+a)*((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)+((2*cos(b*x+a)^2-1)/(1+cos
(b*x+a))^2)^(1/2)-2*cos(b*x+a)-1)/(1+cos(b*x+a)))-90*cos(b*x+a)^2*arctanh(
(2*cos(b*x+a)-1)/(1+cos(b*x+a))/((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2
))+60*((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*cos(b*x+a)^2+60*cos(b*x+
a)*((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)-15*ln(2*(cos(b*x+a)*((2*cos
(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)+((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)
^(1/2)-2*cos(b*x+a)-1)/(1+cos(b*x+a)))+15*arctanh((2*cos(b*x+a)-1)/(1+cos(
b*x+a))/((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)))/(2*cos(b*x+a)^2-1)^3
/(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(1/2)/((2*cos(b*x+a)^2-1)/(1+cos(b...
```

3.619. $\int \frac{\sec^4(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx$

3.619.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 380, normalized size of antiderivative = 2.17

$$\int \frac{\sec^4(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx$$

$$= \left[\frac{4\sqrt{2}\left(15\tan(bx+a)^4-20\tan(bx+a)^2+17\right)\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}+\frac{15\sqrt{2}\left(c\tan(bx+a)^5-2c\tan(bx+a)^3+c\tan(bx+a)\right)}{}}{60\left(bc\tan(bx+a)^5-2bc\tan(bx+a)^3+bc\tan(bx+a\right.}\right.$$

$$-\frac{15\sqrt{2}\left(c\tan(bx+a)^5-2c\tan(bx+a)^3+c\tan(bx+a)\right)\sqrt{-\frac{1}{c}}\arctan\left(\frac{\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{-}}{\tan(bx+a)}\right.}{30\left(bc\tan(bx+a)^5-2bc\tan(bx+a)^3+bc\,t\right.}$$

input
```
integrate(sec(2*b*x+2*a)^4/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorith
m="fricas")
```

output
```
[1/60*(4*sqrt(2)*(15*tan(b*x + a)^4 - 20*tan(b*x + a)^2 + 17)*sqrt(-c*tan(
b*x + a)^2/(tan(b*x + a)^2 - 1)) + 15*sqrt(2)*(c*tan(b*x + a)^5 - 2*c*tan(
b*x + a)^3 + c*tan(b*x + a))*log((tan(b*x + a)^3 - 2*sqrt(-c*tan(b*x + a)^
2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)/sqrt(c) - 2*tan(b*x + a))/tan
(b*x + a)^3)/sqrt(c))/(b*c*tan(b*x + a)^5 - 2*b*c*tan(b*x + a)^3 + b*c*tan
(b*x + a)), -1/30*(15*sqrt(2)*(c*tan(b*x + a)^5 - 2*c*tan(b*x + a)^3 + c*t
an(b*x + a))*sqrt(-1/c)*arctan(sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)
)*(tan(b*x + a)^2 - 1)*sqrt(-1/c)/tan(b*x + a)) - 2*sqrt(2)*(15*tan(b*x +
a)^4 - 20*tan(b*x + a)^2 + 17)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)
))/(b*c*tan(b*x + a)^5 - 2*b*c*tan(b*x + a)^3 + b*c*tan(b*x + a))]
```

3.619.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^4(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx = \text{Timed out}$$

input
```
integrate(sec(2*b*x+2*a)**4/(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
```

output
```
Timed out
```

3.619.7 Maxima [F]

$$\int \frac{\sec^4(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx = \int \frac{\sec(2\,bx+2\,a)^4}{\sqrt{c\tan(2\,bx+2\,a)\tan(bx+a)}}\,dx$$

input
```
integrate(sec(2*b*x+2*a)^4/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorith
m="maxima")
```

output
```
integrate(sec(2*b*x + 2*a)^4/sqrt(c*tan(2*b*x + 2*a)*tan(b*x + a)), x)
```

3.619.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sec^4(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx = \text{Timed out}$$

input
```
integrate(sec(2*b*x+2*a)^4/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorith
m="giac")
```

output
```
Timed out
```

3.619.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^4(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx$$
$$=\int \frac{1}{\cos\left(2\,a+2\,b\,x\right)^4\,\sqrt{c\tan\left(a+b\,x\right)\,\tan\left(2\,a+2\,b\,x\right)}}\,dx$$

input
```
int(1/(cos(2*a + 2*b*x)^4*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2)),x)
```

output
```
int(1/(cos(2*a + 2*b*x)^4*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2)), x)
```

3.620 $\int \frac{\sec^3(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx$

3.620.1 Optimal result

Integrand size = 31, antiderivative size = 129

$$\int \frac{\sec^3(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{-c+c\sec(2a+2bx)}}\right)}{\sqrt{2}b\sqrt{c}} + \frac{2\tan(2a+2bx)}{3b\sqrt{-c+c\sec(2a+2bx)}} + \frac{\sqrt{-c+c\sec(2a+2bx)}\tan(2a+2bx)}{3bc}$$

output
```
-1/2*arctanh(1/2*c^(1/2)*tan(2*b*x+2*a)*2^(1/2)/(-c+c*sec(2*b*x+2*a))^(1/2
))/b*2^(1/2)/c^(1/2)+2/3*tan(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)+1/3*
(-c+c*sec(2*b*x+2*a))^(1/2)*tan(2*b*x+2*a)/b/c
```

3.620.2 Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.69

$$\int \frac{\sec^3(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx = \frac{\cos^2(a+bx)\csc(2(a+bx))\left(2+2\sec(2(a+bx))+3\arctan\left(\sqrt{-1+\tan^2(a+bx)}\right)\sqrt{-1+\tan^2(a+bx}}{3bc}$$

input
```
Integrate[Sec[2*(a + b*x)]^3/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

output
```
(Cos[a + b*x]^2*Csc[2*(a + b*x)]*(2 + 2*Sec[2*(a + b*x)] + 3*ArcTan[Sqrt[-
1 + Tan[a + b*x]^2]]*Sqrt[-1 + Tan[a + b*x]^2])*Sqrt[c*Tan[a + b*x]*Tan[2*
(a + b*x)]])/(3*b*c)
```

3.620.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$, Rules used = {3042, 4897, 3042, 4287, 27, 3042, 4489, 3042, 4282, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sec(2a+2bx)^3}{\sqrt{c\tan(a+bx)\tan(2a+2bx)}}dx$$

$$\Big\downarrow \text{ 4897}$$

$$\int \frac{\sec^3(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(2a+2bx+\frac{\pi}{2}\right)^3}{\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}}dx$$

$$\Big\downarrow \text{ 4287}$$

$$\frac{2\int \frac{\sec(2a+2bx)(2\sec(2a+2bx)c+c)}{2\sqrt{c\sec(2a+2bx)-c}}dx}{3c}+\frac{\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{3bc}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\sec(2a+2bx)(2\sec(2a+2bx)c+c)}{\sqrt{c\sec(2a+2bx)-c}}dx}{3c}+\frac{\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{3bc}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{\csc\left(2a+2bx+\frac{\pi}{2}\right)\left(2\csc\left(2a+2bx+\frac{\pi}{2}\right)c+c\right)}{\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}}dx}{3c}+\frac{\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{3bc}$$

$$\Big\downarrow \text{ 4489}$$

$$\frac{3c\int \frac{\sec(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}dx+\frac{2c\tan(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}}{3c}+\frac{\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{3bc}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3c\int \frac{\csc\left(2a+2bx+\frac{\pi}{2}\right)}{\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}}dx+\frac{2c\tan(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}}{3c}+\frac{\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{3bc}$$

$$\Big\downarrow \text{ 4282}$$

$$\frac{\frac{2c\tan(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}-\frac{3c\int \frac{1}{\frac{c^2\tan^2(2a+2bx)}{c\sec(2a+2bx)-c}-2c}d\left(-\frac{c\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{b}}{3c}+\frac{\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{3bc}$$

$$\Big\downarrow \text{ 220}$$

$$\frac{\frac{2c\tan(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}-\frac{3\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{\sqrt{2}b}}{3c}+\frac{\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{3bc}$$

input
```
Int[Sec[2*(a + b*x)]^3/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

output
```
(Sqrt[-c + c*Sec[2*a + 2*b*x]]*Tan[2*a + 2*b*x])/(3*b*c) + ((-3*Sqrt[c]*Ar
cTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a + 2*b*x]])])
/(Sqrt[2]*b) + (2*c*Tan[2*a + 2*b*x])/(b*Sqrt[-c + c*Sec[2*a + 2*b*x]]))/(
3*c)
```

3.620.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

3.620. $\int \frac{\sec^3(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4287
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Simp[1/(b*(m + 2))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(
m + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 -
b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 4489
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[(a*B*m + A*b*(m + 1))/(b*(m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B
, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b
*(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.620.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 606 vs. $2(112) = 224$.

Time = 9.81 (sec) , antiderivative size = 607, normalized size of antiderivative = 4.71

method	result
default	$\frac{\sqrt{2}\,\sin(xb+a)\left(12\cos(xb+a)^4\ln\left(\frac{2\cos(xb+a)\sqrt{\frac{2\cos(xb+a)^2-1}{(1+\cos(xb+a))^2}}+2\sqrt{\frac{2\cos(xb+a)^2-1}{(1+\cos(xb+a))^2}}-4\cos(xb+a)-2}{1+\cos(xb+a)}\right)-12\cos(xb+a)^4\operatorname{arctanh}\left(\frac{}{(1+\cos\ldots}\right.\right.}{}$

input
```
int(sec(2*b*x+2*a)^3/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
1/24*2^(1/2)/b*sin(b*x+a)*(12*cos(b*x+a)^4*ln(2*(cos(b*x+a)*((2*cos(b*x+a)
^2-1)/(1+cos(b*x+a))^2)^(1/2)+((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)-
2*cos(b*x+a)-1)/(1+cos(b*x+a)))-12*cos(b*x+a)^4*arctanh((2*cos(b*x+a)-1)/(
1+cos(b*x+a))/((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2))+8*((2*cos(b*x+a
)^2-1)/(1+cos(b*x+a))^2)^(1/2)*cos(b*x+a)^4+8*((2*cos(b*x+a)^2-1)/(1+cos(b
*x+a))^2)^(1/2)*cos(b*x+a)^3-12*cos(b*x+a)^2*ln(2*(cos(b*x+a)*((2*cos(b*x+
a)^2-1)/(1+cos(b*x+a))^2)^(1/2)+((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2
)-2*cos(b*x+a)-1)/(1+cos(b*x+a)))+12*cos(b*x+a)^2*arctanh((2*cos(b*x+a)-1)
/(1+cos(b*x+a))/((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2))+3*ln(2*(cos(b
*x+a)*((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)+((2*cos(b*x+a)^2-1)/(1+c
os(b*x+a))^2)^(1/2)-2*cos(b*x+a)-1)/(1+cos(b*x+a)))-3*arctanh((2*cos(b*x+a
)-1)/(1+cos(b*x+a))/((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)))/(2*cos(b
*x+a)^2-1)^2/(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(1/2)/((2*cos(b*x+a)^2-1)
/(1+cos(b*x+a))^2)^(1/2)/(1+cos(b*x+a))*4^(1/2)/(-3+2*2^(1/2))^2/(3+2*2^(1
/2))^2
```

3.620.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 294, normalized size of antiderivative = 2.28

$$\int \frac{\sec^3(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx$$

$$= \left[\frac{\frac{3\sqrt{2}\left(c\tan(bx+a)^3-c\tan(bx+a)\right)\log\left(\frac{\tan(bx+a)^3-\frac{2\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)}{\sqrt{c}}-2\tan(bx+a)}{\tan(bx+a)^3}\right)}{\sqrt{c}}-8\sqrt{2}\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}}{12\left(bc\tan\left(bx+a\right)^3-bc\tan\left(bx+a\right)\right)},\right.$$

$$-\frac{3\sqrt{2}\left(c\tan\left(bx+a\right)^3-c\tan\left(bx+a\right)\right)\sqrt{-\frac{1}{c}}\arctan\left(\frac{\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{-\frac{1}{c}}}{\tan(bx+a)}\right)+4\sqrt{2}\sqrt{-\frac{c\tan(}{\tan(bx}}}{6\left(bc\tan\left(bx+a\right)^3-bc\tan\left(bx+a\right)\right)}$$

input
```
integrate(sec(2*b*x+2*a)^3/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorith
m="fricas")
```

output
```
[1/12*(3*sqrt(2)*(c*tan(b*x + a)^3 - c*tan(b*x + a))*log((tan(b*x + a)^3 -
 2*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)/sqrt(
c) - 2*tan(b*x + a))/tan(b*x + a)^3)/sqrt(c) - 8*sqrt(2)*sqrt(-c*tan(b*x +
 a)^2/(tan(b*x + a)^2 - 1)))/(b*c*tan(b*x + a)^3 - b*c*tan(b*x + a)), -1/6
*(3*sqrt(2)*(c*tan(b*x + a)^3 - c*tan(b*x + a))*sqrt(-1/c)*arctan(sqrt(-c*
tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-1/c)/tan(b
*x + a)) + 4*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*c*ta
n(b*x + a)^3 - b*c*tan(b*x + a))]
```

3.620.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^3(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx = \text{Timed out}$$

input
```
integrate(sec(2*b*x+2*a)**3/(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
```

output
```
Timed out
```

3.620.7 Maxima [F]

$$\int \frac{\sec^3(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx = \int \frac{\sec\left(2\,bx+2\,a\right)^3}{\sqrt{c\tan\left(2\,bx+2\,a\right)\tan\left(bx+a\right)}}\,dx$$

input
```
integrate(sec(2*b*x+2*a)^3/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorith
m="maxima")
```

output
```
integrate(sec(2*b*x + 2*a)^3/sqrt(c*tan(2*b*x + 2*a)*tan(b*x + a)), x)
```

3.620.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sec^3(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx = \text{Timed out}$$

input
```
integrate(sec(2*b*x+2*a)^3/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorith
m="giac")
```

output
```
Timed out
```

3.620.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx = \int \frac{1}{\cos\left(2\,a+2\,b\,x\right)^3\,\sqrt{c\tan\left(a+b\,x\right)\,\tan\left(2\,a+2\,b\,x\right)}}\,dx$$

input
```
int(1/(cos(2*a + 2*b*x)^3*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2)),x)
```

output
```
int(1/(cos(2*a + 2*b*x)^3*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2)), x)
```

3.621 $\int \frac{\sec^2(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx$

3.621.1 Optimal result

Integrand size = 31, antiderivative size = 88

$$\int \frac{\sec^2(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{-c+c\sec(2a+2bx)}}\right)}{\sqrt{2}b\sqrt{c}} + \frac{\tan(2a+2bx)}{b\sqrt{-c+c\sec(2a+2bx)}}$$

output
```
-1/2*arctanh(1/2*c^(1/2)*tan(2*b*x+2*a)*2^(1/2)/(-c+c*sec(2*b*x+2*a))^(1/2
))/b*2^(1/2)/c^(1/2)+tan(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)
```

3.621.2 Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.76

$$\int \frac{\sec^2(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx = \frac{\left(2+\arctan\left(\sqrt{-1+\tan^2(a+bx)}\right)\sqrt{-1+\tan^2(a+bx)}\right)\tan(2(a+bx))}{2b\sqrt{c\tan(a+bx)\tan(2(a+bx))}}$$

input
```
Integrate[Sec[2*(a + b*x)]^2/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

output
```
((2 + ArcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Sqrt[-1 + Tan[a + b*x]^2])*Tan[2*(
a + b*x)])/(2*b*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]])
```

3.621.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {3042, 4897, 3042, 4285, 3042, 4282, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(2a+2bx)^2}{\sqrt{c\tan(a+bx)\tan(2a+2bx)}}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \frac{\sec^2(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(2a+2bx+\frac{\pi}{2}\right)^2}{\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}}dx$$

$$\Big\downarrow \text{4285}$$

$$\int \frac{\sec(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}dx+\frac{\tan(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(2a+2bx+\frac{\pi}{2}\right)}{\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}}dx+\frac{\tan(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}$$

$$\Big\downarrow \text{4282}$$

$$\frac{\tan(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}-\frac{\int \frac{1}{\frac{c^2\tan^2(2a+2bx)}{c\sec(2a+2bx)-c}-2c}d\left(-\frac{c\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{b}$$

$$\Big\downarrow \text{220}$$

$$\frac{\tan(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{\sqrt{2}b\sqrt{c}}$$

input
```
Int[Sec[2*(a + b*x)]^2/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

output
```
-(ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a + 2*b*x]
])]/(Sqrt[2]*b*Sqrt[c])) + Tan[2*a + 2*b*x]/(b*Sqrt[-c + c*Sec[2*a + 2*b*x
]])
```

3.621.3.1 Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4285
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x]
+ Simp[a*(m/(b*(m + 1)))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x]
/; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.621. $\int \frac{\sec^2(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx$

3.621.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(77) = 154$.

Time = 4.09 (sec) , antiderivative size = 370, normalized size of antiderivative = 4.20

method	result
default	$\frac{\sqrt{2}\,(\cos(xb+a)-1)\left(\operatorname{arctanh}\left(\frac{2\cos(xb+a)-1}{(1+\cos(xb+a))\sqrt{\frac{2\cos(xb+a)^2-1}{(1+\cos(xb+a))^2}}}\right)\sqrt{\frac{2\cos(xb+a)^2-1}{(1+\cos(xb+a))^2}}\,\sin(xb+a)^2-\ln\left(\frac{2\cos(xb+a)\sqrt{\frac{2\cos(xb+a)^2-1}{(1+\cos(xb+a))^2}}+2\sqrt{\cdots}}{1+\cos(x\cdots}\cdots\right.\right.}{2b\sqrt{\frac{c\sin(xb+a)^2}{2\cos(xb+a)^2-1}}\,\left(\sin(xb+a)^4-6\cos(xb+a)^2\sin(xb+a)^2+\cos(xb+a)^4+12\cos(xb+\cdots\right.}$

input
```
int(sec(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
1/2*2^(1/2)/b*(cos(b*x+a)-1)*(arctanh((2*cos(b*x+a)-1)/(1+cos(b*x+a))/((2*
cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2))*((2*cos(b*x+a)^2-1)/(1+cos(b*x+a)
)^2)^(1/2)*sin(b*x+a)^2-ln(2*(cos(b*x+a)*((2*cos(b*x+a)^2-1)/(1+cos(b*x+a)
)^2)^(1/2)+((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)-2*cos(b*x+a)-1)/(1+
cos(b*x+a)))*((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*sin(b*x+a)^2-2*si
n(b*x+a)^2+2*cos(b*x+a)^2-4*cos(b*x+a)+2)*sin(b*x+a)/(c*sin(b*x+a)^2/(2*co
s(b*x+a)^2-1))^(1/2)/(sin(b*x+a)^4-6*cos(b*x+a)^2*sin(b*x+a)^2+cos(b*x+a)^
4+12*cos(b*x+a)*sin(b*x+a)^2-4*cos(b*x+a)^3-6*sin(b*x+a)^2+6*cos(b*x+a)^2-
4*cos(b*x+a)+1)*4^(1/2)
```

3.621.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.78

$$\int \frac{\sec^2(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx = \left[\frac{\sqrt{2}\sqrt{c}\log\left(\frac{\tan(bx+a)^3-\frac{2\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)}{\sqrt{c}}-2\tan(bx+a)}{\tan(bx+a)^3}\right)\tan(bx+a)+4\sqrt{2}\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}}{4\,bc\tan(bx+a)}, -\frac{\sqrt{2}c\sqrt{-\frac{1}{c}}\arctan\left(\frac{\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{-\frac{1}{c}}}{\tan(bx+a)}\right)\tan(bx+a)-2\sqrt{2}\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}}{2\,bc\tan(bx+a)}\right]$$

input
```
integrate(sec(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorith
m="fricas")
```

output
```
[1/4*(sqrt(2)*sqrt(c)*log((tan(b*x + a)^3 - 2*sqrt(-c*tan(b*x + a)^2/(tan(
b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)/sqrt(c) - 2*tan(b*x + a))/tan(b*x +
a)^3)*tan(b*x + a) + 4*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)
))/(b*c*tan(b*x + a)), -1/2*(sqrt(2)*c*sqrt(-1/c)*arctan(sqrt(-c*tan(b*x +
 a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-1/c)/tan(b*x + a))*
tan(b*x + a) - 2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*
c*tan(b*x + a))]
```

3.621.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^2(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx = \text{Timed out}$$

input
```
integrate(sec(2*b*x+2*a)**2/(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
```

output
```
Timed out
```

3.621.7 Maxima [F]

$$\int \frac{\sec^2(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx = \int \frac{\sec(2\,bx+2\,a)^2}{\sqrt{c\tan(2\,bx+2\,a)\tan(bx+a)}}\,dx$$

input
```
integrate(sec(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorith
m="maxima")
```

output
```
integrate(sec(2*b*x + 2*a)^2/sqrt(c*tan(2*b*x + 2*a)*tan(b*x + a)), x)
```

3.621.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sec^2(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx = \text{Timed out}$$

input
```
integrate(sec(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorith
m="giac")
```

output
```
Timed out
```

3.621.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx$$
$$=\int \frac{1}{\cos\left(2\,a+2\,b\,x\right)^2\,\sqrt{c\tan\left(a+b\,x\right)\,\tan\left(2\,a+2\,b\,x\right)}}\,dx$$

input
```
int(1/(cos(2*a + 2*b*x)^2*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2)),x)
```

output
```
int(1/(cos(2*a + 2*b*x)^2*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2)), x)
```

3.622 $\int \frac{\sec(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx$

3.622.1 Optimal result

Integrand size = 29, antiderivative size = 55

$$\int \frac{\sec(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{-c+c\sec(2a+2bx)}}\right)}{\sqrt{2}b\sqrt{c}}$$

output
```
-1/2*arctanh(1/2*c^(1/2)*tan(2*b*x+2*a)*2^(1/2)/(-c+c*sec(2*b*x+2*a))^(1/2
))/b*2^(1/2)/c^(1/2)
```

3.622.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.16

$$\int \frac{\sec(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx = \frac{\arctan\left(\sqrt{-1+\tan^2(a+bx)}\right)\sqrt{-1+\tan^2(a+bx)}\tan(2(a+bx))}{2b\sqrt{c\tan(a+bx)\tan(2(a+bx))}}$$

input
```
Integrate[Sec[2*(a + b*x)]/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

output
```
(ArcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Sqrt[-1 + Tan[a + b*x]^2]*Tan[2*(a + b*
x)])/(2*b*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]])
```

3.622.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {3042, 4897, 3042, 4282, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(2a+2bx)}{\sqrt{c\tan(a+bx)\tan(2a+2bx)}}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \frac{\sec(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(2a+2bx+\frac{\pi}{2}\right)}{\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}}dx$$

$$\Big\downarrow \text{4282}$$

$$-\frac{\int \frac{1}{\frac{c^2\tan^2(2a+2bx)}{c\sec(2a+2bx)-c}-2c}d\left(-\frac{c\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{b}$$

$$\Big\downarrow \text{220}$$

$$-\frac{\text{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{\sqrt{2}b\sqrt{c}}$$

input
```
Int[Sec[2*(a + b*x)]/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

output
```
-(ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a + 2*b*x]
])]/(Sqrt[2]*b*Sqrt[c]))
```

3.622.3.1 Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.622.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 208 vs. $2(46) = 92$.

Time = 3.51 (sec) , antiderivative size = 209, normalized size of antiderivative = 3.80

method	result
default	$\frac{\sqrt{2}\,\sin(xb+a)\left(\ln\left(\frac{2\cos(xb+a)\sqrt{\frac{2\cos(xb+a)^2-1}{(1+\cos(xb+a))^2}}+2\sqrt{\frac{2\cos(xb+a)^2-1}{(1+\cos(xb+a))^2}}-4\cos(xb+a)-2}{1+\cos(xb+a)}\right)-\operatorname{arctanh}\left(\frac{2\cos(xb+a)-1}{(1+\cos(xb+a))\sqrt{\frac{2\cos(xb+a)^2-1}{(1+\cos(xb+a))^2}}}\right)\right)}{8b(1+\cos(xb+a))\sqrt{\frac{c\sin(xb+a)^2}{2\cos(xb+a)^2-1}}\,\sqrt{\frac{2\cos(xb+a)^2-1}{(1+\cos(xb+a))^2}}}$

input
```
int(sec(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
1/8*2^(1/2)/b*sin(b*x+a)*(ln(2*(cos(b*x+a)*((2*cos(b*x+a)^2-1)/(1+cos(b*x+
a))^2)^(1/2)+((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)-2*cos(b*x+a)-1)/(
1+cos(b*x+a)))-arctanh((2*cos(b*x+a)-1)/(1+cos(b*x+a))/((2*cos(b*x+a)^2-1)
/(1+cos(b*x+a))^2)^(1/2)))/(1+cos(b*x+a))/(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-
1))^(1/2)/((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*4^(1/2)
```

3.622.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.65

$$\int \frac{\sec(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx = \left[\frac{\sqrt{2}\log\left(\frac{\tan(bx+a)^3-\frac{2\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)}{\sqrt{c}}-2\tan(bx+a)}{\tan(bx+a)^3}\right)}{4b\sqrt{c}}, -\frac{\sqrt{2}\sqrt{-\frac{1}{c}}\arctan\left(\frac{\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{-\frac{1}{c}}}{\tan(bx+a)}\right)}{2b}\right]$$

input
```
integrate(sec(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm=
"fricas")
```

output
```
[1/4*sqrt(2)*log((tan(b*x + a)^3 - 2*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^
2 - 1))*(tan(b*x + a)^2 - 1)/sqrt(c) - 2*tan(b*x + a))/tan(b*x + a)^3)/(b*
sqrt(c)), -1/2*sqrt(2)*sqrt(-1/c)*arctan(sqrt(-c*tan(b*x + a)^2/(tan(b*x +
 a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-1/c)/tan(b*x + a))/b]
```

3.622.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx = \text{Timed out}$$

input
```
integrate(sec(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
```

output
```
Timed out
```

3.622.7 Maxima [F]

$$\int \frac{\sec(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx = \int \frac{\sec(2\,bx+2\,a)}{\sqrt{c\tan(2\,bx+2\,a)\tan(bx+a)}}\,dx$$

input
```
integrate(sec(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm=
"maxima")
```

output
```
integrate(sec(2*b*x + 2*a)/sqrt(c*tan(2*b*x + 2*a)*tan(b*x + a)), x)
```

3.622.8 Giac [F]

$$\int \frac{\sec(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx = \int \frac{\sec(2\,bx+2\,a)}{\sqrt{c\tan(2\,bx+2\,a)\tan(bx+a)}}\,dx$$

input
```
integrate(sec(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm=
"giac")
```

output
```
integrate(sec(2*b*x + 2*a)/sqrt(c*tan(2*b*x + 2*a)*tan(b*x + a)), x)
```

3.622.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx = \int \frac{1}{\cos(2\,a+2\,b\,x)\,\sqrt{c\tan(a+b\,x)\,\tan(2\,a+2\,b\,x)}}\,dx$$

input
```
int(1/(cos(2*a + 2*b*x)*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2)),x)
```

output
```
int(1/(cos(2*a + 2*b*x)*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2)), x)
```

3.623 $\int \frac{1}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx$

3.623.1 Optimal result

Integrand size = 20, antiderivative size = 100

$$\int \frac{1}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{b\sqrt{c}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{-c+c\sec(2a+2bx)}}\right)}{\sqrt{2}b\sqrt{c}}$$

output
```
arctanh(c^(1/2)*tan(2*b*x+2*a)/(-c+c*sec(2*b*x+2*a))^(1/2))/b/c^(1/2)-1/2*
arctanh(1/2*c^(1/2)*tan(2*b*x+2*a)*2^(1/2)/(-c+c*sec(2*b*x+2*a))^(1/2))/b*
2^(1/2)/c^(1/2)
```

3.623.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx=-\frac{\left(\sqrt{2}\operatorname{arctanh}\left(\sqrt{1-\tan^2(a+bx)}\right)-2\operatorname{arctanh}\left(\frac{\sqrt{1-\tan^2(a+bx)}}{\sqrt{2}}\right)\right)\tan(a+bx)}{b\sqrt{2-2\tan^2(a+bx)}\sqrt{c\tan(a+bx)\tan(2(a+bx))}}$$

input
```
Integrate[1/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

output
```
-(((Sqrt[2]*ArcTanh[Sqrt[1 - Tan[a + b*x]^2]] - 2*ArcTanh[Sqrt[1 - Tan[a +
 b*x]^2]/Sqrt[2]])*Tan[a + b*x])/(b*Sqrt[2 - 2*Tan[a + b*x]^2]*Sqrt[c*Tan[
a + b*x]*Tan[2*(a + b*x)]]))
```

3.623.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {3042, 4897, 3042, 4263, 3042, 4261, 220, 4282, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{c\tan(a+bx)\tan(2a+2bx)}}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \frac{1}{\sqrt{c\sec(2a+2bx)-c}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}}dx$$

$$\Big\downarrow \text{4263}$$

$$\int \frac{\sec(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}dx-\frac{\int \sqrt{c\sec(2a+2bx)-c}dx}{c}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(2a+2bx+\frac{\pi}{2}\right)}{\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}}dx-\frac{\int \sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}dx}{c}$$

$$\Big\downarrow \text{4261}$$

$$\frac{\int \frac{1}{\frac{c^2\tan^2(2a+2bx)}{c\sec(2a+2bx)-c}-c}d\left(-\frac{c\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{b}+\int \frac{\csc\left(2a+2bx+\frac{\pi}{2}\right)}{\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}}dx$$

$$\Big\downarrow \ 220$$

$$\int \frac{\csc\left(2a+2bx+\frac{\pi}{2}\right)}{\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}}dx+\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{b\sqrt{c}}$$

$$\Big\downarrow \ 4282$$

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{b\sqrt{c}}-\frac{\int \frac{1}{\frac{c^2\tan^2(2a+2bx)}{c\sec(2a+2bx)-c}-2c}d\left(-\frac{c\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{b}$$

$$\Big\downarrow \ 220$$

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{b\sqrt{c}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{\sqrt{2}b\sqrt{c}}$$

input
```
Int[1/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

output
```
ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]]]/(b*Sqrt[
c]) - ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a + 2*
b*x]])]/(Sqrt[2]*b*Sqrt[c])
```

3.623.3.1 Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4263
```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[1/a   I
nt[Sqrt[a + b*Csc[c + d*x]], x], x] - Simp[b/a   Int[Csc[c + d*x]/Sqrt[a +
b*Csc[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

3.623. $\int \frac{1}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx$

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.623.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(85) = 170$.

Time = 4.07 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.60

method	result
default	$\frac{\sqrt{2}\sin(xb+a)\left(2\sqrt{2}\,\text{arctanh}\left(\frac{\cos(xb+a)\sqrt{2}}{(1+\cos(xb+a))\sqrt{\frac{2\cos(xb+a)^2-1}{(1+\cos(xb+a))^2}}}\right)+\ln\left(\frac{2\cos(xb+a)\sqrt{\frac{2\cos(xb+a)^2-1}{(1+\cos(xb+a))^2}}+2\sqrt{\frac{2\cos(xb+a)^2-1}{(1+\cos(xb+a))^2}}-4\cos(xb+a)-2}{1+\cos(xb+a)}\right.\right.}{8b(1+\cos(xb+a))\sqrt{\frac{c\sin(xb+a)^2}{2\cos(xb+a)^2-1}}\sqrt{\frac{2\cos(xb+a)^2-1}{(1+\cos(xb+a))^2}}}$

input
```
int(1/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*2^(1/2)/b*sin(b*x+a)*(2*2^(1/2)*arctanh(cos(b*x+a)/(1+cos(b*x+a))/((2*
cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*2^(1/2))+ln(2*(cos(b*x+a)*((2*cos(
b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)+((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^
(1/2)-2*cos(b*x+a)-1)/(1+cos(b*x+a)))-arctanh((2*cos(b*x+a)-1)/(1+cos(b*x+
a))/((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)))/(1+cos(b*x+a))/(c*sin(b*
x+a)^2/(2*cos(b*x+a)^2-1))^(1/2)/((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/
2)*4^(1/2)
```

3.623.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 309, normalized size of antiderivative = 3.09

$$\int \frac{1}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx$$
$$= \left[\frac{\sqrt{2}\sqrt{c}\log\left(\frac{c\tan(bx+a)^3-2\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{c}-2\,c\tan(bx+a)}{\tan(bx+a)^3}\right)+2\sqrt{c}\log\left(\frac{c\tan(bx+a)^3+2\sqrt{2}\sqrt{-\frac{c\tan(bx\cdots}{\tan(bx+a\cdots}}}{\tan(bx+\cdots}\right.}{4\,bc}\right.$$
$$-\frac{\sqrt{2}\sqrt{-c}\arctan\left(\frac{\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{-c}}{c\tan(bx+a)}\right)-2\sqrt{-c}\arctan\left(\frac{\sqrt{2}\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{-c}}{2\,c\tan(bx+a)}\right)}{2\,bc}$$

input
```
integrate(1/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(2)*sqrt(c)*log((c*tan(b*x + a)^3 - 2*sqrt(-c*tan(b*x + a)^2/(ta
n(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(c) - 2*c*tan(b*x + a))/tan(b*
x + a)^3) + 2*sqrt(c)*log((c*tan(b*x + a)^3 + 2*sqrt(2)*sqrt(-c*tan(b*x +
a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(c) - 3*c*tan(b*x + a)
)/(tan(b*x + a)^3 + tan(b*x + a))))/(b*c), -1/2*(sqrt(2)*sqrt(-c)*arctan(s
qrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/
(c*tan(b*x + a))) - 2*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(
tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a))))/(b*c
)]
```

3.623.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(1/(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

3.623.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 1685, normalized size of antiderivative = 16.85

$$\int \frac{1}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="maxima")
```

output
```
-1/8*sqrt(2)*(sqrt(2)*log(4*sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 +
 2*cos(4*b*x + 4*a) + 1)*cos(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a
) + 1))^2 + 4*sqrt(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x +
 4*a) + 1)*sin(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a) + 1))^2 + 8*
(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)^(1/4)*c
os(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x + 4*a) + 1)) + 4) - sqrt(2)*log
(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + sqrt(cos(4*b*x + 4*a)^2 + sin(4
*b*x + 4*a)^2 + 2*cos(4*b*x + 4*a) + 1)*(cos(1/2*arctan2(sin(4*b*x + 4*a),
 cos(4*b*x + 4*a) + 1))^2 + sin(1/2*arctan2(sin(4*b*x + 4*a), cos(4*b*x +
4*a) + 1))^2) + 2*(cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 + 2*cos(4*b*x +
 4*a) + 1)^(1/4)*(cos(2*b*x + 2*a)*cos(1/2*arctan2(sin(4*b*x + 4*a), cos(4
*b*x + 4*a) + 1)) + sin(2*b*x + 2*a)*sin(1/2*arctan2(sin(4*b*x + 4*a), cos
(4*b*x + 4*a) + 1)))) - 2*log((sqrt(abs(2*e^(2*I*b*x + 2*I*a) - 2)^4 + 16*
cos(2*b*x + 2*a)^4 + 16*sin(2*b*x + 2*a)^4 + 8*(cos(2*b*x + 2*a)^2 - sin(2
*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*abs(2*e^(2*I*b*x + 2*I*a) - 2)^2 +
 64*cos(2*b*x + 2*a)^3 + 32*(cos(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*
sin(2*b*x + 2*a)^2 + 96*cos(2*b*x + 2*a)^2 + 64*cos(2*b*x + 2*a) + 16)*cos
(1/2*arctan2(8*(cos(2*b*x + 2*a) + 1)*sin(2*b*x + 2*a)/abs(2*e^(2*I*b*x +
2*I*a) - 2)^2, (abs(2*e^(2*I*b*x + 2*I*a) - 2)^2 + 4*cos(2*b*x + 2*a)^2 -
4*sin(2*b*x + 2*a)^2 + 8*cos(2*b*x + 2*a) + 4)/abs(2*e^(2*I*b*x + 2*I*a...
```

3.623.8 Giac [F]

$$\int \frac{1}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx = \int \frac{1}{\sqrt{c\tan\left(2\,bx+2\,a\right)\tan\left(bx+a\right)}}\,dx$$

input
```
integrate(1/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(c*tan(2*b*x + 2*a)*tan(b*x + a)), x)
```

3.623.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx = \int \frac{1}{\sqrt{c\tan\left(a+b\,x\right)\,\tan\left(2\,a+2\,b\,x\right)}}\,dx$$

input
```
int(1/(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2),x)
```

output
```
int(1/(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2), x)
```

3.624 $\int \frac{\cos(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx$

3.624.1 Optimal result

Integrand size = 29, antiderivative size = 138

$$\int \frac{\cos(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{2b\sqrt{c}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{-c+c\sec(2a+2bx)}}\right)}{\sqrt{2}b\sqrt{c}} + \frac{\sin(2a+2bx)}{2b\sqrt{-c+c\sec(2a+2bx)}}$$

output
```
1/2*arctanh(c^(1/2)*tan(2*b*x+2*a)/(-c+c*sec(2*b*x+2*a))^(1/2))/b/c^(1/2)-
1/2*arctanh(1/2*c^(1/2)*tan(2*b*x+2*a)*2^(1/2)/(-c+c*sec(2*b*x+2*a))^(1/2)
)/b*2^(1/2)/c^(1/2)+1/2*sin(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)
```

3.624.2 Mathematica [A] (verified)

Time = 2.43 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.18

$$\int \frac{\cos(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx = -\frac{\tan(a+bx)\left(\operatorname{arctanh}\left(\sqrt{1-\tan^2(a+bx)}\right)-\sqrt{2}\left(\operatorname{arctanh}\left(\frac{\sqrt{1-\tan^2(a+bx)}}{\sqrt{2}}\right)+\cos^2(a+bx)\sqrt{\frac{1}{1+\sec(2(a+b}}\right.\right.}{2b\sqrt{1-\tan^2(a+bx)}\sqrt{c\tan(a+bx)\,t}}$$

input
```
Integrate[Cos[2*(a + b*x)]/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

output
```
-1/2*(Tan[a + b*x]*(ArcTanh[Sqrt[1 - Tan[a + b*x]^2]] - Sqrt[2]*(ArcTanh[S
qrt[1 - Tan[a + b*x]^2]/Sqrt[2]] + Cos[a + b*x]^2*Sqrt[(1 + Sec[2*(a + b*x
)])^(-1)]*(2 + ArcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Sec[2*(a + b*x)]*Sqrt[-1
+ Tan[a + b*x]^2]))))/(b*Sqrt[1 - Tan[a + b*x]^2]*Sqrt[c*Tan[a + b*x]*Tan[
2*(a + b*x)]])
```

3.624.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.379$, Rules used = {3042, 4897, 3042, 4310, 25, 3042, 4392, 3042, 4375, 383, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(2a+2bx)}{\sqrt{c\tan(a+bx)\tan(2a+2bx)}}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \frac{\cos(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(2a+2bx+\frac{\pi}{2}\right)\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}}dx$$

$$\Big\downarrow \text{4310}$$

$$\frac{\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}-\frac{\int -\frac{\sec(2a+2bx)c+c}{\sqrt{c\sec(2a+2bx)-c}}dx}{2c}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sec(2a+2bx)c+c}{\sqrt{c\sec(2a+2bx)-c}}dx}{2c}+\frac{\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{\csc(2a+2bx+\frac{\pi}{2})c+c}{\sqrt{c\csc(2a+2bx+\frac{\pi}{2})-c}}dx}{2c}+\frac{\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}$$

$$\Big\downarrow \text{ 4392}$$

$$\frac{1}{2}c\int \frac{\tan^2(2a+2bx)}{(c\sec(2a+2bx)-c)^{3/2}}dx+\frac{\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}c\int \frac{\cot\left(2a+2bx+\frac{\pi}{2}\right)^2}{\left(c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c\right)^{3/2}}dx+\frac{\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}$$

$$\Big\downarrow \text{ 4375}$$

$$\frac{\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}-\frac{c\int \frac{\tan^2(2a+2bx)}{(c\sec(2a+2bx)-c)\left(1-\frac{c\tan^2(2a+2bx)}{c\sec(2a+2bx)-c}\right)\left(2-\frac{c\tan^2(2a+2bx)}{c\sec(2a+2bx)-c}\right)}d\left(-\frac{\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{2b}$$

$$\Big\downarrow \text{ 383}$$

$$\frac{\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}-\frac{c\left(\frac{\int \frac{1}{1-\frac{c\tan^2(2a+2bx)}{c\sec(2a+2bx)-c}}d\left(-\frac{\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{c}-\frac{2\int \frac{1}{2-\frac{c\tan^2(2a+2bx)}{c\sec(2a+2bx)-c}}d\left(-\frac{\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{c}\right)}{2b}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}-\frac{c\left(\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{c^{3/2}}-\frac{\text{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{c^{3/2}}\right)}{2b}$$

input
```
Int[Cos[2*(a + b*x)]/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

output
```
-1/2*(c*(-(ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]
]]/c^(3/2)) + (Sqrt[2]*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c
 + c*Sec[2*a + 2*b*x]])])/c^(3/2)))/b + Sin[2*a + 2*b*x]/(2*b*Sqrt[-c + c*
Sec[2*a + 2*b*x]])
```

3.624. $\int \frac{\cos(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx$

3.624.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 383
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Sym
bol] :> Simp[(-a)*(e^2/(b*c - a*d))   Int[(e*x)^(m - 2)/(a + b*x^2), x], x]
 + Simp[c*(e^2/(b*c - a*d))   Int[(e*x)^(m - 2)/(c + d*x^2), x], x] /; Free
Q[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LeQ[2, m, 3]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4310
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a +
b*Csc[e + f*x]])), x] + Simp[1/(2*b*d*n)   Int[(d*Csc[e + f*x])^(n + 1)*((a
 + b*(2*n + 1)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0] && IntegerQ[2*n]
```

rule 4375
```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Simp[-2*(a^(m/2 + n + 1/2)/d)   Subst[Int[x^m*((2 + a*x^2
)^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]
]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && I
ntegerQ[n - 1/2]
```

rule 4392
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Simp[((-a)*c)^m   Int[Cot[e + f*x]^(2*m)*(
c + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && E
qQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(
IntegerQ[n] && GtQ[m - n, 0])
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.624. $\int \frac{\cos(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx$

3.624.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 589 vs. $2(117) = 234$.

Time = 3.28 (sec) , antiderivative size = 590, normalized size of antiderivative = 4.28

method	result
default	$\frac{\sqrt{2}\,\sin(xb+a)\left(2\sqrt{\frac{2\cos(xb+a)^2-1}{(1+\cos(xb+a))^2}}\cos(xb+a)^2+3\sqrt{2}\,\operatorname{arctanh}\left(\frac{\cos(xb+a)\sqrt{2}}{(1+\cos(xb+a))\sqrt{\frac{2\cos(xb+a)^2-1}{(1+\cos(xb+a))^2}}}\right)+2\cos(xb+a)\sqrt{\frac{2\cos(xb+a)^2-1}{(1+\cos(xb+a))^2}}-2}{8b(1+\cos(xb+a))\sqrt{\frac{2\cos(xb+a)^2-}{(1+\cos(xb+a)}}}$

input
```
int(cos(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x,method=_RETURNVER
BOSE)
```

output
```
1/8*2^(1/2)/b*sin(b*x+a)*(2*((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*co
s(b*x+a)^2+3*2^(1/2)*arctanh(cos(b*x+a)/(1+cos(b*x+a))/((2*cos(b*x+a)^2-1)
/(1+cos(b*x+a))^2)^(1/2)*2^(1/2))+2*cos(b*x+a)*((2*cos(b*x+a)^2-1)/(1+cos(
b*x+a))^2)^(1/2)-2*arctanh((2*cos(b*x+a)-1)/(1+cos(b*x+a))/((2*cos(b*x+a)^
2-1)/(1+cos(b*x+a))^2)^(1/2))+2*ln(2*(cos(b*x+a)*((2*cos(b*x+a)^2-1)/(1+co
s(b*x+a))^2)^(1/2)+((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)-2*cos(b*x+a
)-1)/(1+cos(b*x+a))))/(1+cos(b*x+a))/((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)
^(1/2)/(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(1/2)*4^(1/2)-1/8*2^(1/2)/b*sin
(b*x+a)*(2*2^(1/2)*arctanh(cos(b*x+a)/(1+cos(b*x+a))/((2*cos(b*x+a)^2-1)/(
1+cos(b*x+a))^2)^(1/2)*2^(1/2))+ln(2*(cos(b*x+a)*((2*cos(b*x+a)^2-1)/(1+co
s(b*x+a))^2)^(1/2)+((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)-2*cos(b*x+a
)-1)/(1+cos(b*x+a)))-arctanh((2*cos(b*x+a)-1)/(1+cos(b*x+a))/((2*cos(b*x+a
)^2-1)/(1+cos(b*x+a))^2)^(1/2)))/(1+cos(b*x+a))/(c*sin(b*x+a)^2/(2*cos(b*x
+a)^2-1))^(1/2)/((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*4^(1/2)
```

3.624.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 481, normalized size of antiderivative = 3.49

$$\int \frac{\cos(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx$$

$$= \left[\frac{\sqrt{2}\left(\tan(bx+a)^3+\tan(bx+a)\right)\sqrt{c}\log\left(\frac{c\tan(bx+a)^3-2\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{c}-2\,c\tan(bx+a)}{\tan(bx+a)^3}\right)+\left(\tan\right.}{4\left(b\right.}\right.$$

$$-\frac{\sqrt{2}\left(\tan(bx+a)^3+\tan(bx+a)\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{-c}}{c\tan(bx+a)}\right)-\left(\tan(bx+a)^3+\tan\right.}{2\left(bc\tan(bx+a)^3\right.}$$

input
```
integrate(cos(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm=
"fricas")
```

output
```
[1/4*(sqrt(2)*(tan(b*x + a)^3 + tan(b*x + a))*sqrt(c)*log((c*tan(b*x + a)^
3 - 2*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sq
rt(c) - 2*c*tan(b*x + a))/tan(b*x + a)^3) + (tan(b*x + a)^3 + tan(b*x + a)
)*sqrt(c)*log((c*tan(b*x + a)^3 + 2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*
x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(c) - 3*c*tan(b*x + a))/(tan(b*x +
 a)^3 + tan(b*x + a))) - 2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2
 - 1))*(tan(b*x + a)^2 - 1))/(b*c*tan(b*x + a)^3 + b*c*tan(b*x + a)), -1/2*
(sqrt(2)*(tan(b*x + a)^3 + tan(b*x + a))*sqrt(-c)*arctan(sqrt(-c*tan(b*x +
 a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a))
) - (tan(b*x + a)^3 + tan(b*x + a))*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-c*ta
n(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*
x + a))) + sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x +
 a)^2 - 1))/(b*c*tan(b*x + a)^3 + b*c*tan(b*x + a))]
```

3.624.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx = \text{Timed out}$$

input
```
integrate(cos(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
```

output
```
Timed out
```

3.624.7 Maxima [F]

$$\int \frac{\cos(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx = \int \frac{\cos(2\,bx+2\,a)}{\sqrt{c\tan(2\,bx+2\,a)\tan(bx+a)}}\,dx$$

input
```
integrate(cos(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm=
"maxima")
```

output
```
integrate(cos(2*b*x + 2*a)/sqrt(c*tan(2*b*x + 2*a)*tan(b*x + a)), x)
```

3.624.8 Giac [F(-1)]

Timed out.

$$\int \frac{\cos(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx = \text{Timed out}$$

input
```
integrate(cos(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorithm=
"giac")
```

output
```
Timed out
```

3.624.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx = \int \frac{\cos\left(2\,a+2\,b\,x\right)}{\sqrt{c\tan\left(a+b\,x\right)\,\tan\left(2\,a+2\,b\,x\right)}}\,dx$$

input
```
int(cos(2*a + 2*b*x)/(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2),x)
```

output
```
int(cos(2*a + 2*b*x)/(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2), x)
```

3.625 $\int \frac{\cos^2(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx$

3.625.1 Optimal result

Integrand size = 31, antiderivative size = 182

$$\int \frac{\cos^2(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx = \frac{7\text{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{8b\sqrt{c}} - \frac{\text{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{-c+c\sec(2a+2bx)}}\right)}{\sqrt{2}b\sqrt{c}} + \frac{\sin(2a+2bx)}{8b\sqrt{-c+c\sec(2a+2bx)}} + \frac{\cos(2a+2bx)\sin(2a+2bx)}{4b\sqrt{-c+c\sec(2a+2bx)}}$$

output
```
7/8*arctanh(c^(1/2)*tan(2*b*x+2*a)/(-c+c*sec(2*b*x+2*a))^(1/2))/b/c^(1/2)-
1/2*arctanh(1/2*c^(1/2)*tan(2*b*x+2*a)*2^(1/2)/(-c+c*sec(2*b*x+2*a))^(1/2)
)/b*2^(1/2)/c^(1/2)+1/8*sin(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)+1/4*c
os(2*b*x+2*a)*sin(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(1/2)
```

3.625.2 Mathematica [A] (verified)

Time = 3.98 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.02

$$\int \frac{\cos^2(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx =$$
$$-\frac{\tan(a+bx)\left(7\text{arctanh}\left(\sqrt{1-\tan^2(a+bx)}\right)-\sqrt{2}\left(7\text{arctanh}\left(\frac{\sqrt{1-\tan^2(a+bx)}}{\sqrt{2}}\right)+\cos^2(a+bx)\sec(2(a+\ldots}{8b\sqrt{1-\tan^2(a+\ldots}$$

input
```
Integrate[Cos[2*(a + b*x)]^2/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

output
```
-1/8*(Tan[a + b*x]*(7*ArcTanh[Sqrt[1 - Tan[a + b*x]^2]] - Sqrt[2]*(7*ArcTa
nh[Sqrt[1 - Tan[a + b*x]^2]/Sqrt[2]] + Cos[a + b*x]^2*Sec[2*(a + b*x)]*Sqr
t[(1 + Sec[2*(a + b*x)])^(-1)]*(2*(1 + Cos[2*(a + b*x)] + Cos[4*(a + b*x)]
) + ArcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Sqrt[-1 + Tan[a + b*x]^2]))))/(b*Sqr
t[1 - Tan[a + b*x]^2]*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]])
```

3.625.3 Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.08, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.484$, Rules used = {3042, 4897, 3042, 4310, 25, 3042, 4510, 27, 3042, 4408, 3042, 4261, 220, 4282, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos(2a+2bx)^2}{\sqrt{c\tan(a+bx)\tan(2a+2bx)}}dx$$
$$\Big\downarrow \text{4897}$$
$$\int \frac{\cos^2(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}dx$$
$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(2a+2bx+\frac{\pi}{2}\right)^2\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}}dx$$

$$\Big\downarrow \text{4310}$$

$$\frac{\sin(2a+2bx)\cos(2a+2bx)}{4b\sqrt{c\sec(2a+2bx)-c}}-\frac{\int -\frac{\cos(2a+2bx)(3\sec(2a+2bx)c+c)}{\sqrt{c\sec(2a+2bx)-c}}dx}{4c}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\cos(2a+2bx)(3\sec(2a+2bx)c+c)}{\sqrt{c\sec(2a+2bx)-c}}dx}{4c}+\frac{\sin(2a+2bx)\cos(2a+2bx)}{4b\sqrt{c\sec(2a+2bx)-c}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{3\csc\left(2a+2bx+\frac{\pi}{2}\right)c+c}{\csc\left(2a+2bx+\frac{\pi}{2}\right)\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}}dx}{4c}+\frac{\sin(2a+2bx)\cos(2a+2bx)}{4b\sqrt{c\sec(2a+2bx)-c}}$$

$$\Big\downarrow \text{4510}$$

$$\frac{\frac{\int \frac{\sec(2a+2bx)c^2+7c^2}{2\sqrt{c\sec(2a+2bx)-c}}dx}{c}+\frac{c\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}}{4c}+\frac{\sin(2a+2bx)\cos(2a+2bx)}{4b\sqrt{c\sec(2a+2bx)-c}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\sec(2a+2bx)c^2+7c^2}{\sqrt{c\sec(2a+2bx)-c}}dx}{2c}+\frac{c\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}}{4c}+\frac{\sin(2a+2bx)\cos(2a+2bx)}{4b\sqrt{c\sec(2a+2bx)-c}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{\csc\left(2a+2bx+\frac{\pi}{2}\right)c^2+7c^2}{\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}}dx}{2c}+\frac{c\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}}{4c}+\frac{\sin(2a+2bx)\cos(2a+2bx)}{4b\sqrt{c\sec(2a+2bx)-c}}$$

$$\Big\downarrow \text{4408}$$

$$\frac{\frac{8c^2\int \frac{\sec(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}dx-7c\int \sqrt{c\sec(2a+2bx)-c}dx}{2c}+\frac{c\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}}{4c}+\frac{\sin(2a+2bx)\cos(2a+2bx)}{4b\sqrt{c\sec(2a+2bx)-c}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{8c^2\int \frac{\csc\left(2a+2bx+\frac{\pi}{2}\right)}{\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}}dx-7c\int \sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}dx}{2c}+\frac{c\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}}{4c}+\frac{\sin(2a+2bx)\cos(2a+2bx)}{4b\sqrt{c\sec(2a+2bx)-c}}$$

3.625. $\int \frac{\cos^2(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx$

$$\Big\downarrow \ 4261$$

$$\frac{\frac{8c^2\int \frac{\csc\left(2a+2bx+\frac{\pi}{2}\right)}{\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}}dx+\frac{7c^2\int \frac{1}{\frac{c^2\tan^2(2a+2bx)}{c\sec(2a+2bx)-c}-c}d\left(-\frac{c\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{b}}{2c}+\frac{c\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}}{4c}+\frac{\sin(2a+2bx)\cos(2a+2bx)}{4b\sqrt{c\sec(2a+2bx)-c}}$$

$$\Big\downarrow \ 220$$

$$\frac{\frac{8c^2\int \frac{\csc\left(2a+2bx+\frac{\pi}{2}\right)}{\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}}dx+\frac{7c^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{b}}{2c}+\frac{c\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}}{4c}+\frac{\sin(2a+2bx)\cos(2a+2bx)}{4b\sqrt{c\sec(2a+2bx)-c}}$$

$$\Big\downarrow \ 4282$$

$$\frac{\frac{\frac{7c^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{b}-\frac{8c^2\int \frac{1}{\frac{c^2\tan^2(2a+2bx)}{c\sec(2a+2bx)-c}-2c}d\left(-\frac{c\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{b}}{2c}+\frac{c\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}}{4c}+\frac{\sin(2a+2bx)\cos(2a+2bx)}{4b\sqrt{c\sec(2a+2bx)-c}}$$

$$\Big\downarrow \ 220$$

$$\frac{\frac{\frac{7c^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{b}-\frac{4\sqrt{2}c^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{b}}{2c}+\frac{c\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}}{4c}+\frac{\sin(2a+2bx)\cos(2a+2bx)}{4b\sqrt{c\sec(2a+2bx)-c}}$$

input
```
Int[Cos[2*(a + b*x)]^2/Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]],x]
```

output
```
(Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x])/(4*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]) +
(((7*c^(3/2)*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*
x]]])/b - (4*Sqrt[2]*c^(3/2)*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*S
qrt[-c + c*Sec[2*a + 2*b*x]])])/b)/(2*c) + (c*Sin[2*a + 2*b*x])/(2*b*Sqrt[
-c + c*Sec[2*a + 2*b*x]]))/(4*c)
```

3.625. $\int \frac{\cos^2(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx$

3.625.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4310
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a +
b*Csc[e + f*x]])), x] + Simp[1/(2*b*d*n)   Int[(d*Csc[e + f*x])^(n + 1)*((a
 + b*(2*n + 1)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0] && IntegerQ[2*n]
```

rule 4408
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c/a   Int[Sqrt[a + b*Csc[e + f*x]], x], x] -
Simp[(b*c - a*d)/a   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

3.625. $\int \frac{\cos^2(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx$

rule 4510
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(b*d
*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*
n - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.625.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 988 vs. $2(157) = 314$.

Time = 3.57 (sec) , antiderivative size = 989, normalized size of antiderivative = 5.43

method	result	size
default	Expression too large to display	989

input
```
int(cos(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
1/8*2^(1/2)/b*sin(b*x+a)*(2*2^(1/2)*arctanh(cos(b*x+a)/(1+cos(b*x+a))/((2*
cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*2^(1/2))+ln(2*(cos(b*x+a)*((2*cos(
b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)+((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^
(1/2)-2*cos(b*x+a)-1)/(1+cos(b*x+a)))-arctanh((2*cos(b*x+a)-1)/(1+cos(b*x+
a))/((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)))/(1+cos(b*x+a))/(c*sin(b*
x+a)^2/(2*cos(b*x+a)^2-1))^(1/2)/((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/
2)*4^(1/2)-1/4*2^(1/2)/b*sin(b*x+a)*(2*((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^
2)^(1/2)*cos(b*x+a)^2+3*2^(1/2)*arctanh(cos(b*x+a)/(1+cos(b*x+a))/((2*cos(
b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*2^(1/2))+2*cos(b*x+a)*((2*cos(b*x+a)^2
-1)/(1+cos(b*x+a))^2)^(1/2)-2*arctanh((2*cos(b*x+a)-1)/(1+cos(b*x+a))/((2*
cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2))+2*ln(2*(cos(b*x+a)*((2*cos(b*x+a)
^2-1)/(1+cos(b*x+a))^2)^(1/2)+((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)-
2*cos(b*x+a)-1)/(1+cos(b*x+a))))/(1+cos(b*x+a))/((2*cos(b*x+a)^2-1)/(1+cos
(b*x+a))^2)^(1/2)/(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(1/2)*4^(1/2)+1/32*2
^(1/2)/b*sin(b*x+a)*(8*((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*cos(b*x
+a)^4+8*((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*cos(b*x+a)^3+14*((2*co
s(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*cos(b*x+a)^2+23*2^(1/2)*arctanh(cos(
b*x+a)/(1+cos(b*x+a))/((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*2^(1/2))
+14*cos(b*x+a)*((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)-16*arctanh((2*c
os(b*x+a)-1)/(1+cos(b*x+a))/((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)...
```

3.625.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 569, normalized size of antiderivative = 3.13

$$\int \frac{\cos^2(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx$$
$$= \left[\frac{4\sqrt{2}\left(\tan(bx+a)^5+2\tan(bx+a)^3+\tan(bx+a)\right)\sqrt{c}\log\left(\frac{c\tan(bx+a)^3-2\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{c}-}{\tan(bx+a)^3}\right.}{}\right.$$
$$-\frac{4\sqrt{2}\left(\tan(bx+a)^5+2\tan(bx+a)^3+\tan(bx+a)\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{-c}}{c\tan(bx+a)}\right)-}{8}$$

input
```
integrate(cos(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorith
m="fricas")
```

output
```
[1/16*(4*sqrt(2)*(tan(b*x + a)^5 + 2*tan(b*x + a)^3 + tan(b*x + a))*sqrt(c
)*log((c*tan(b*x + a)^3 - 2*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(
tan(b*x + a)^2 - 1)*sqrt(c) - 2*c*tan(b*x + a))/tan(b*x + a)^3) + 7*(tan(b
*x + a)^5 + 2*tan(b*x + a)^3 + tan(b*x + a))*sqrt(c)*log((c*tan(b*x + a)^3
 + 2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2
- 1)*sqrt(c) - 3*c*tan(b*x + a))/(tan(b*x + a)^3 + tan(b*x + a))) + 2*sqrt
(2)*(tan(b*x + a)^4 - 4*tan(b*x + a)^2 + 3)*sqrt(-c*tan(b*x + a)^2/(tan(b*
x + a)^2 - 1)))/(b*c*tan(b*x + a)^5 + 2*b*c*tan(b*x + a)^3 + b*c*tan(b*x +
 a)), -1/8*(4*sqrt(2)*(tan(b*x + a)^5 + 2*tan(b*x + a)^3 + tan(b*x + a))*s
qrt(-c)*arctan(sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^
2 - 1)*sqrt(-c)/(c*tan(b*x + a))) - 7*(tan(b*x + a)^5 + 2*tan(b*x + a)^3 +
 tan(b*x + a))*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x
 + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a))) - sqrt(2)*(t
an(b*x + a)^4 - 4*tan(b*x + a)^2 + 3)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)
^2 - 1)))/(b*c*tan(b*x + a)^5 + 2*b*c*tan(b*x + a)^3 + b*c*tan(b*x + a))]
```

3.625.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx = \text{Timed out}$$

input
```
integrate(cos(2*b*x+2*a)**2/(c*tan(b*x+a)*tan(2*b*x+2*a))**(1/2),x)
```

output
```
Timed out
```

3.625.7 Maxima [F]

$$\int \frac{\cos^2(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx = \int \frac{\cos(2\,bx+2\,a)^2}{\sqrt{c\tan(2\,bx+2\,a)\tan(bx+a)}}\,dx$$

input
```
integrate(cos(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorith
m="maxima")
```

output
```
integrate(cos(2*b*x + 2*a)^2/sqrt(c*tan(2*b*x + 2*a)*tan(b*x + a)), x)
```

3.625.8 Giac [F(-1)]

Timed out.

$$\int \frac{\cos^2(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx = \text{Timed out}$$

input
```
integrate(cos(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(1/2),x, algorith
m="giac")
```

output
```
Timed out
```

3.625.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(2(a+bx))}{\sqrt{c\tan(a+bx)\tan(2(a+bx))}}\,dx = \int \frac{\cos\left(2\,a+2\,b\,x\right)^2}{\sqrt{c\tan\left(a+b\,x\right)\,\tan\left(2\,a+2\,b\,x\right)}}\,dx$$

input
```
int(cos(2*a + 2*b*x)^2/(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2),x)
```

output
```
int(cos(2*a + 2*b*x)^2/(c*tan(a + b*x)*tan(2*a + 2*b*x))^(1/2), x)
```

3.626 $\int \frac{\sec^4(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx$

3.626.1 Optimal result

Integrand size = 31, antiderivative size = 180

$$\int \frac{\sec^4(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = -\frac{11\text{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{-c+c\sec(2a+2bx)}}\right)}{4\sqrt{2}bc^{3/2}} - \frac{\sec^2(2a+2bx)\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} + \frac{13\tan(2a+2bx)}{6bc\sqrt{-c+c\sec(2a+2bx)}} + \frac{7\sqrt{-c+c\sec(2a+2bx)}\tan(2a+2bx)}{12bc^2}$$

output
```
-11/8*arctanh(1/2*c^(1/2)*tan(2*b*x+2*a)*2^(1/2)/(-c+c*sec(2*b*x+2*a))^(1/
2))/b/c^(3/2)*2^(1/2)-1/4*sec(2*b*x+2*a)^2*tan(2*b*x+2*a)/b/(-c+c*sec(2*b*
x+2*a))^(3/2)+13/6*tan(2*b*x+2*a)/b/c/(-c+c*sec(2*b*x+2*a))^(1/2)+7/12*(-c
+c*sec(2*b*x+2*a))^(1/2)*tan(2*b*x+2*a)/b/c^2
```

3.626.2 Mathematica [A] (verified)

Time = 2.97 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.56

$$\int \frac{\sec^4(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = -\frac{\cot(a+bx)\left(\csc^2(a+bx)(-24+(11+19\cos(4(a+bx)))\sec(2(a+bx)))-66\arctan\left(\sqrt{-1+\tan^2(a+\right.\right.}{48bc^2}$$

input
```
Integrate[Sec[2*(a + b*x)]^4/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

output
```
-1/48*(Cot[a + b*x]*(Csc[a + b*x]^2*(-24 + (11 + 19*Cos[4*(a + b*x)])*Sec[
2*(a + b*x)]) - 66*ArcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Sqrt[-1 + Tan[a + b*x
]^2])*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]])/(b*c^2)
```

3.626.3 Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.05, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$, Rules used = {3042, 4897, 3042, 4303, 27, 3042, 4498, 27, 3042, 4489, 3042, 4282, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(2a+2bx)^4}{(c\tan(a+bx)\tan(2a+2bx))^{3/2}}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \frac{\sec^4(2a+2bx)}{(c\sec(2a+2bx)-c)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(2a+2bx+\frac{\pi}{2}\right)^4}{\left(c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c\right)^{3/2}}dx$$

$$\Big\downarrow \text{4303}$$

$$\frac{\int \frac{\sec^2(2a+2bx)(7\sec(2a+2bx)c+4c)}{2\sqrt{c\sec(2a+2bx)-c}}dx}{2c^2}-\frac{\tan(2a+2bx)\sec^2(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sec^2(2a+2bx)(7\sec(2a+2bx)c+4c)}{\sqrt{c\sec(2a+2bx)-c}}dx}{4c^2}-\frac{\tan(2a+2bx)\sec^2(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(2a+2bx+\frac{\pi}{2}\right)^2\left(7\csc\left(2a+2bx+\frac{\pi}{2}\right)c+4c\right)}{\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}}dx}{4c^2}-\frac{\tan(2a+2bx)\sec^2(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

$$\Big\downarrow \text{4498}$$

$$\frac{\frac{2\int \frac{\sec(2a+2bx)\left(26\sec(2a+2bx)c^2+7c^2\right)}{2\sqrt{c\sec(2a+2bx)-c}}dx}{3c}+\frac{7\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{3b}}{4c^2}-\frac{\tan(2a+2bx)\sec^2(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\sec(2a+2bx)\left(26\sec(2a+2bx)c^2+7c^2\right)}{\sqrt{c\sec(2a+2bx)-c}}dx}{3c}+\frac{7\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{3b}}{4c^2}-\frac{\tan(2a+2bx)\sec^2(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{\csc\left(2a+2bx+\frac{\pi}{2}\right)\left(26\csc\left(2a+2bx+\frac{\pi}{2}\right)c^2+7c^2\right)}{\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}}dx}{3c}+\frac{7\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{3b}}{4c^2}-\frac{\tan(2a+2bx)\sec^2(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

$$\Big\downarrow \text{4489}$$

$$\frac{\frac{33c^2\int \frac{\sec(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}dx+\frac{26c^2\tan(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}}{3c}+\frac{7\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{3b}}{4c^2}-\frac{\tan(2a+2bx)\sec^2(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{33c^2\int \frac{\csc\left(2a+2bx+\frac{\pi}{2}\right)}{\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}}dx+\frac{26c^2\tan(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}}{3c}+\frac{7\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{3b}}{4c^2}-\frac{\tan(2a+2bx)\sec^2(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

$$\Big\downarrow \text{4282}$$

$$\frac{\frac{\frac{26c^2\tan(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}-\frac{33c^2\int \frac{1}{\frac{c^2\tan^2(2a+2bx)}{c\sec(2a+2bx)-c}-2c}d\left(-\frac{c\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{b}}{3c}+\frac{7\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{3b}}{4c^2}-\frac{\tan(2a+2bx)\sec^2(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

$$\Big\downarrow \text{220}$$

3.626. $\displaystyle\int \frac{\sec^4(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx$

$$\frac{\frac{\frac{26c^2\tan(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}-\frac{33c^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{\sqrt{2}b}}{3c}+\frac{7\tan(2a+2bx)\sqrt{c\sec(2a+2bx)-c}}{3b}}{4c^2}-\frac{\tan(2a+2bx)\sec^2(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

input
```
Int[Sec[2*(a + b*x)]^4/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

output
```
-1/4*(Sec[2*a + 2*b*x]^2*Tan[2*a + 2*b*x])/(b*(-c + c*Sec[2*a + 2*b*x])^(3
/2)) + ((7*Sqrt[-c + c*Sec[2*a + 2*b*x]]*Tan[2*a + 2*b*x])/(3*b) + ((-33*c
^(3/2)*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a + 2
*b*x]])])/(Sqrt[2]*b) + (26*c^2*Tan[2*a + 2*b*x])/(b*Sqrt[-c + c*Sec[2*a +
 2*b*x]]))/(3*c))/(4*c^2)
```

3.626.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4303
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-d^2)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d
*Csc[e + f*x])^(n - 2)/(f*(2*m + 1))), x] + Simp[d^2/(a*b*(2*m + 1))   Int[
(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*(b*(n - 2) + a*(m - n
 + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2,
0] && LtQ[m, -1] && GtQ[n, 2] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

3.626. $\int \frac{\sec^4(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx$

rule 4489
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[(a*B*m + A*b*(m + 1))/(b*(m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B
, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b
*(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

rule 4498
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*
((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m + 2))   Int
[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)
*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a
*B, 0] &&  !LtQ[m, -1]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.626.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1069 vs. $2(157) = 314$.

Time = 9.49 (sec) , antiderivative size = 1070, normalized size of antiderivative = 5.94

method	result	size
default	Expression too large to display	1070

input
```
int(sec(2*b*x+2*a)^4/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
1/96*2^(1/2)/b*csc(b*x+a)*(152*((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)
*cos(b*x+a)^5-132*arctanh((2*cos(b*x+a)-1)/(1+cos(b*x+a))/((2*cos(b*x+a)^2
-1)/(1+cos(b*x+a))^2)^(1/2))*cos(b*x+a)^5+132*ln(2*(cos(b*x+a)*((2*cos(b*x
+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)+((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/
2)-2*cos(b*x+a)-1)/(1+cos(b*x+a)))*cos(b*x+a)^5+132*cos(b*x+a)^4*arctanh((
2*cos(b*x+a)-1)/(1+cos(b*x+a))/((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)
)-132*cos(b*x+a)^4*ln(2*(cos(b*x+a)*((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^
(1/2)+((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)-2*cos(b*x+a)-1)/(1+cos(b
*x+a)))-200*((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*cos(b*x+a)^3+132*a
rctanh((2*cos(b*x+a)-1)/(1+cos(b*x+a))/((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^
2)^(1/2))*cos(b*x+a)^3-132*ln(2*(cos(b*x+a)*((2*cos(b*x+a)^2-1)/(1+cos(b*x
+a))^2)^(1/2)+((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)-2*cos(b*x+a)-1)/
(1+cos(b*x+a)))*cos(b*x+a)^3-132*cos(b*x+a)^2*arctanh((2*cos(b*x+a)-1)/(1+
cos(b*x+a))/((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2))+132*cos(b*x+a)^2*
ln(2*(cos(b*x+a)*((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)+((2*cos(b*x+a
)^2-1)/(1+cos(b*x+a))^2)^(1/2)-2*cos(b*x+a)-1)/(1+cos(b*x+a)))+54*cos(b*x+
a)*((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)-33*arctanh((2*cos(b*x+a)-1)
/(1+cos(b*x+a))/((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2))*cos(b*x+a)+33
*ln(2*(cos(b*x+a)*((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)+((2*cos(b*x+
a)^2-1)/(1+cos(b*x+a))^2)^(1/2)-2*cos(b*x+a)-1)/(1+cos(b*x+a)))*cos(b*x...
```

3.626.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.94

$$\int \frac{\sec^4(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \left[\frac{33\sqrt{2}\left(\tan(bx+a)^5-\tan(bx+a)^3\right)\sqrt{c}\log\left(\frac{c\tan(bx+a)^3-2\sqrt{-\frac{c\,t}{\tan}}}{}\right.}{48}\right.$$

$$-\frac{33\sqrt{2}\left(\tan(bx+a)^5-\tan(bx+a)^3\right)\sqrt{-c}\arctan\left(\frac{\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{-c}}{c\tan(bx+a)}\right)-\sqrt{2}\left(27\tan(bx+a\right.}{24\left(bc^2\tan(bx+a)^5-bc^2\tan(bx+a)^3\right)}$$

input
```
integrate(sec(2*b*x+2*a)^4/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorith
m="fricas")
```

output
```
[1/48*(33*sqrt(2)*(tan(b*x + a)^5 - tan(b*x + a)^3)*sqrt(c)*log((c*tan(b*x
 + a)^3 - 2*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 -
 1)*sqrt(c) - 2*c*tan(b*x + a))/tan(b*x + a)^3) + 2*sqrt(2)*(27*tan(b*x +
a)^4 - 46*tan(b*x + a)^2 + 3)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))
)/(b*c^2*tan(b*x + a)^5 - b*c^2*tan(b*x + a)^3), -1/24*(33*sqrt(2)*(tan(b*
x + a)^5 - tan(b*x + a)^3)*sqrt(-c)*arctan(sqrt(-c*tan(b*x + a)^2/(tan(b*x
 + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a))) - sqrt(2)*(2
7*tan(b*x + a)^4 - 46*tan(b*x + a)^2 + 3)*sqrt(-c*tan(b*x + a)^2/(tan(b*x
+ a)^2 - 1)))/(b*c^2*tan(b*x + a)^5 - b*c^2*tan(b*x + a)^3)]
```

3.626.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^4(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(2*b*x+2*a)**4/(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
```

output
```
Timed out
```

3.626.7 Maxima [F]

$$\int \frac{\sec^4(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \int \frac{\sec(2\,bx+2\,a)^4}{(c\tan(2\,bx+2\,a)\tan(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(2*b*x+2*a)^4/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorith
m="maxima")
```

output
```
integrate(sec(2*b*x + 2*a)^4/(c*tan(2*b*x + 2*a)*tan(b*x + a))^(3/2), x)
```

3.626.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sec^4(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(2*b*x+2*a)^4/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorith
m="giac")
```

output
```
Timed out
```

3.626.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^4(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \int \frac{1}{\cos\left(2\,a+2\,b\,x\right)^4\,{\left(c\tan\left(a+b\,x\right)\,\tan\left(2\,a+2\,b\,x\right)\right)}^{3/2}}\,dx$$

input
```
int(1/(cos(2*a + 2*b*x)^4*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2)),x)
```

output
```
int(1/(cos(2*a + 2*b*x)^4*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2)), x)
```

3.627 $\int \frac{\sec^3(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx$

3.627.1 Optimal result

Integrand size = 31, antiderivative size = 128

$$\int \frac{\sec^3(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = -\frac{7\text{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{-c+c\sec(2a+2bx)}}\right)}{4\sqrt{2}bc^{3/2}} - \frac{\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} + \frac{\tan(2a+2bx)}{bc\sqrt{-c+c\sec(2a+2bx)}}$$

output
```
-7/8*arctanh(1/2*c^(1/2)*tan(2*b*x+2*a)*2^(1/2)/(-c+c*sec(2*b*x+2*a))^(1/2
))/b/c^(3/2)*2^(1/2)-1/4*tan(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(3/2)+tan(
2*b*x+2*a)/b/c/(-c+c*sec(2*b*x+2*a))^(1/2)
```

3.627.2 Mathematica [A] (verified)

Time = 1.66 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.73

$$\int \frac{\sec^3(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \frac{\left(-5+4\sec(2(a+bx))+7\arctan\left(\sqrt{-1+\tan^2(a+bx)}\right)\sec(2(a+bx))\right.}{4b(c\tan(a+bx)\tan(}$$

input
```
Integrate[Sec[2*(a + b*x)]^3/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

output
```
((-5 + 4*Sec[2*(a + b*x)] + 7*ArcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Sec[2*(a +
 b*x)]*Sin[a + b*x]^2*Sqrt[-1 + Tan[a + b*x]^2])*Tan[2*(a + b*x)])/(4*b*(c
*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2))
```

3.627.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {3042, 4897, 3042, 4286, 27, 3042, 4489, 3042, 4282, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(2a+2bx)^3}{(c\tan(a+bx)\tan(2a+2bx))^{3/2}}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \frac{\sec^3(2a+2bx)}{(c\sec(2a+2bx)-c)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(2a+2bx+\frac{\pi}{2}\right)^3}{\left(c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c\right)^{3/2}}dx$$

$$\Big\downarrow \text{4286}$$

$$\frac{\int \frac{\sec(2a+2bx)(4\sec(2a+2bx)c+3c)}{2\sqrt{c\sec(2a+2bx)-c}}dx}{2c^2}-\frac{\tan(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sec(2a+2bx)(4\sec(2a+2bx)c+3c)}{\sqrt{c\sec(2a+2bx)-c}}dx}{4c^2}-\frac{\tan(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\csc\left(2a+2bx+\frac{\pi}{2}\right)\left(4\csc\left(2a+2bx+\frac{\pi}{2}\right)c+3c\right)}{\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}}dx}{4c^2}-\frac{\tan(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

$$\Big\downarrow \text{4489}$$

$$\frac{7c\int \frac{\sec(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}dx+\frac{4c\tan(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}}{4c^2}-\frac{\tan(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{7c\int \frac{\csc\left(2a+2bx+\frac{\pi}{2}\right)}{\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}}dx+\frac{4c\tan(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}}{4c^2}-\frac{\tan(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

$$\Big\downarrow \text{ 4282}$$

$$\frac{\frac{4c\tan(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}-\frac{7c\int \frac{1}{\frac{c^2\tan^2(2a+2bx)}{c\sec(2a+2bx)-c}-2c}d\left(-\frac{c\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{b}}{4c^2}-\frac{\tan(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

$$\Big\downarrow \text{ 220}$$

$$\frac{\frac{4c\tan(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}-\frac{7\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{\sqrt{2}b}}{4c^2}-\frac{\tan(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

input
```
Int[Sec[2*(a + b*x)]^3/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

output
```
-1/4*Tan[2*a + 2*b*x]/(b*(-c + c*Sec[2*a + 2*b*x])^(3/2)) + ((-7*Sqrt[c]*A
rcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a + 2*b*x]])]
)/(Sqrt[2]*b) + (4*c*Tan[2*a + 2*b*x])/(b*Sqrt[-c + c*Sec[2*a + 2*b*x]]))/
(4*c^2)
```

3.627.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

3.627. $\int \frac{\sec^3(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx$

rule 4286
```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))),
x] - Simp[1/(a^2*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)
*(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[
a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 4489
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Simp[(a*B*m + A*b*(m + 1))/(b*(m +
 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B
, e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b
*(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.627.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 494 vs. $2(111) = 222$.

Time = 3.91 (sec) , antiderivative size = 495, normalized size of antiderivative = 3.87

method	result
default	$\frac{\sqrt{2}\,\csc(xb+a)(1-\cos(xb+a))\left(\csc(xb+a)^6(1-\cos(xb+a))^6+14\csc(xb+a)^2\ln\left(\csc(xb+a)^2(1-\cos(xb+a))^2-3+\sqrt{\csc(xb+a)^4(1-\mathrm{co}\right.\right.}{}$

input
```
int(sec(2*b*x+2*a)^3/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
1/64*2^(1/2)/b*csc(b*x+a)/(csc(b*x+a)^2*(1-cos(b*x+a))^2*c/(csc(b*x+a)^4*(
1-cos(b*x+a))^4-6*csc(b*x+a)^2*(1-cos(b*x+a))^2+1))^(3/2)*(1-cos(b*x+a))*(
csc(b*x+a)^6*(1-cos(b*x+a))^6+14*csc(b*x+a)^2*ln(csc(b*x+a)^2*(1-cos(b*x+a
))^2-3+(csc(b*x+a)^4*(1-cos(b*x+a))^4-6*csc(b*x+a)^2*(1-cos(b*x+a))^2+1)^(
1/2))*(1-cos(b*x+a))^2*(csc(b*x+a)^4*(1-cos(b*x+a))^4-6*csc(b*x+a)^2*(1-co
s(b*x+a))^2+1)^(1/2)+14*csc(b*x+a)^2*arctanh((3*csc(b*x+a)^2*(1-cos(b*x+a)
)^2-1)/(csc(b*x+a)^4*(1-cos(b*x+a))^4-6*csc(b*x+a)^2*(1-cos(b*x+a))^2+1)^(
1/2))*(1-cos(b*x+a))^2*(csc(b*x+a)^4*(1-cos(b*x+a))^4-6*csc(b*x+a)^2*(1-co
s(b*x+a))^2+1)^(1/2)-39*csc(b*x+a)^4*(1-cos(b*x+a))^4+39*csc(b*x+a)^2*(1-c
os(b*x+a))^2-1)/(csc(b*x+a)^4*(1-cos(b*x+a))^4-6*csc(b*x+a)^2*(1-cos(b*x+a
))^2+1)^2*4^(1/2)
```

3.627.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 276, normalized size of antiderivative = 2.16

$$\int \frac{\sec^3(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \left[\frac{7\sqrt{2}\sqrt{c}\log\left(\frac{c\tan(bx+a)^3-2\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{c}-2c\tan(bx+a)}{\tan(bx+a)^3}\right.}{16\,bc^2\tan(}\right.$$

$$-\frac{7\sqrt{2}\sqrt{-c}\arctan\left(\frac{\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{-c}}{c\tan(bx+a)}\right)\tan(bx+a)^3-\sqrt{2}\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(9\tan(bx+a)^2-\right.}{8\,bc^2\tan(bx+a)^3}$$

input
```
integrate(sec(2*b*x+2*a)^3/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorith
m="fricas")
```

output
```
[1/16*(7*sqrt(2)*sqrt(c)*log((c*tan(b*x + a)^3 - 2*sqrt(-c*tan(b*x + a)^2/
(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(c) - 2*c*tan(b*x + a))/tan
(b*x + a)^3)*tan(b*x + a)^3 + 2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x +
a)^2 - 1))*(9*tan(b*x + a)^2 - 1))/(b*c^2*tan(b*x + a)^3), -1/8*(7*sqrt(2)
*sqrt(-c)*arctan(sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a
)^2 - 1)*sqrt(-c)/(c*tan(b*x + a)))*tan(b*x + a)^3 - sqrt(2)*sqrt(-c*tan(b
*x + a)^2/(tan(b*x + a)^2 - 1))*(9*tan(b*x + a)^2 - 1))/(b*c^2*tan(b*x + a
)^3)]
```

3.627.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^3(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(2*b*x+2*a)**3/(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
```

output
```
Timed out
```

3.627.7 Maxima [F]

$$\int \frac{\sec^3(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \int \frac{\sec\left(2\,bx+2\,a\right)^3}{\left(c\tan\left(2\,bx+2\,a\right)\tan\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(2*b*x+2*a)^3/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorith
m="maxima")
```

output
```
integrate(sec(2*b*x + 2*a)^3/(c*tan(2*b*x + 2*a)*tan(b*x + a))^(3/2), x)
```

3.627.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sec^3(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(2*b*x+2*a)^3/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorith
m="giac")
```

output
```
Timed out
```

3.627.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^3(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \int \frac{1}{\cos\left(2\,a+2\,b\,x\right)^3\,{\left(c\tan\left(a+b\,x\right)\,\tan\left(2\,a+2\,b\,x\right)\right)}^{3/2}}\,dx$$

input
```
int(1/(cos(2*a + 2*b*x)^3*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2)),x)
```

output
```
int(1/(cos(2*a + 2*b*x)^3*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2)), x)
```

3.628 $\int \frac{\sec^2(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx$

3.628.1 Optimal result

Integrand size = 31, antiderivative size = 93

$$\int \frac{\sec^2(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = -\frac{3\operatorname{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{-c+c\sec(2a+2bx)}}\right)}{4\sqrt{2}bc^{3/2}} - \frac{\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}}$$

output
```
-3/8*arctanh(1/2*c^(1/2)*tan(2*b*x+2*a)*2^(1/2)/(-c+c*sec(2*b*x+2*a))^(1/2
))/b/c^(3/2)*2^(1/2)-1/4*tan(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(3/2)
```

3.628.2 Mathematica [A] (verified)

Time = 1.43 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.90

$$\int \frac{\sec^2(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \frac{\left(-1+3\arctan\left(\sqrt{-1+\tan^2(a+bx)}\right)\sec(2(a+bx))\sin^2(a+b\ldots}{4b(c\tan(a+bx)\tan(2(a+bx)\ldots}$$

input
```
Integrate[Sec[2*(a + b*x)]^2/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

output
```
((-1 + 3*ArcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Sec[2*(a + b*x)]*Sin[a + b*x]^2
*Sqrt[-1 + Tan[a + b*x]^2])*Tan[2*(a + b*x)])/(4*b*(c*Tan[a + b*x]*Tan[2*(
a + b*x)])^(3/2))
```

3.628.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {3042, 4897, 3042, 4284, 3042, 4282, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(2a+2bx)^2}{(c\tan(a+bx)\tan(2a+2bx))^{3/2}}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \frac{\sec^2(2a+2bx)}{(c\sec(2a+2bx)-c)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(2a+2bx+\frac{\pi}{2}\right)^2}{\left(c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c\right)^{3/2}}dx$$

$$\Big\downarrow \text{4284}$$

$$\frac{3\int \frac{\sec(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}dx}{4c}-\frac{\tan(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{\csc\left(2a+2bx+\frac{\pi}{2}\right)}{\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}}dx}{4c}-\frac{\tan(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

$$\Big\downarrow \text{4282}$$

$$-\frac{3\int \frac{1}{\frac{c^2\tan^2(2a+2bx)}{c\sec(2a+2bx)-c}-2c}d\left(-\frac{c\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{4bc}-\frac{\tan(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

$$\Big\downarrow \text{220}$$

$$-\frac{3\text{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{4\sqrt{2}bc^{3/2}}-\frac{\tan(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

input
```
Int[Sec[2*(a + b*x)]^2/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

output
```
(-3*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a + 2*b*
x]])])/(4*Sqrt[2]*b*c^(3/2)) - Tan[2*a + 2*b*x]/(4*b*(-c + c*Sec[2*a + 2*b
*x])^(3/2))
```

3.628.3.1 Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4284
```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^m/(f*(2*m + 1))), x
] + Simp[m/(b*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x
], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.628.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(78) = 156$.

Time = 3.89 (sec) , antiderivative size = 380, normalized size of antiderivative = 4.09

method	result
default	$\frac{\sqrt{2}\,\csc(xb+a)\left(3\,\mathrm{arctanh}\left(\frac{2\cos(xb+a)-1}{(1+\cos(xb+a))\sqrt{\frac{2\cos(xb+a)^2-1}{(1+\cos(xb+a))^2}}}\right)\cos(xb+a)-3\ln\left(\frac{2\cos(xb+a)\sqrt{\frac{2\cos(xb+a)^2-1}{(1+\cos(xb+a))^2}}+2\sqrt{\frac{2\cos(xb+a)^2-1}{(1+\cos(xb+a))^2}}-4\cos}{1+\cos(xb+a)}\right.\right.}{}$

input
```
int(sec(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
1/32*2^(1/2)/b*csc(b*x+a)*(3*arctanh((2*cos(b*x+a)-1)/(1+cos(b*x+a))/((2*c
os(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2))*cos(b*x+a)-3*ln(2*(cos(b*x+a)*((2*
cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)+((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))
^2)^(1/2)-2*cos(b*x+a)-1)/(1+cos(b*x+a)))*cos(b*x+a)-2*cos(b*x+a)*((2*cos(
b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)-3*arctanh((2*cos(b*x+a)-1)/(1+cos(b*x+
a))/((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2))+3*ln(2*(cos(b*x+a)*((2*co
s(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)+((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2
)^(1/2)-2*cos(b*x+a)-1)/(1+cos(b*x+a))))/((2*cos(b*x+a)^2-1)/(1+cos(b*x+a)
)^2)^(1/2)/(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(1/2)/c*4^(1/2)
```

3.628.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.92

$$\int \frac{\sec^2(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \left[\frac{3\sqrt{2}\sqrt{c}\log\left(\frac{c\tan(bx+a)^3-2\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{c}-2c\tan(bx+}{\tan(bx+a)^3}\right.}{16\,bc^2\tan(b}\right.$$

$$-\frac{3\sqrt{2}\sqrt{-c}\arctan\left(\frac{\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{-c}}{c\tan(bx+a)}\right)\tan(bx+a)^3-\sqrt{2}\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)}{8\,bc^2\tan(bx+a)^3}$$

input
```
integrate(sec(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorith
m="fricas")
```

output
```
[1/16*(3*sqrt(2)*sqrt(c)*log((c*tan(b*x + a)^3 - 2*sqrt(-c*tan(b*x + a)^2/
(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(c) - 2*c*tan(b*x + a))/tan
(b*x + a)^3)*tan(b*x + a)^3 + 2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x +
a)^2 - 1))*(tan(b*x + a)^2 - 1))/(b*c^2*tan(b*x + a)^3), -1/8*(3*sqrt(2)*s
qrt(-c)*arctan(sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^
2 - 1)*sqrt(-c)/(c*tan(b*x + a)))*tan(b*x + a)^3 - sqrt(2)*sqrt(-c*tan(b*x
 + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1))/(b*c^2*tan(b*x + a)^3)
]
```

3.628.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^2(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(2*b*x+2*a)**2/(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
```

output
```
Timed out
```

3.628.7 Maxima [F]

$$\int \frac{\sec^2(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \int \frac{\sec\left(2\,bx+2\,a\right)^2}{\left(c\tan\left(2\,bx+2\,a\right)\tan\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorith
m="maxima")
```

output
```
integrate(sec(2*b*x + 2*a)^2/(c*tan(2*b*x + 2*a)*tan(b*x + a))^(3/2), x)
```

3.628.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sec^2(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorith
m="giac")
```

output
```
Timed out
```

3.628.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \int \frac{1}{\cos\left(2\,a+2\,b\,x\right)^2\,\left(c\tan\left(a+b\,x\right)\,\tan\left(2\,a+2\,b\,x\right)\right)^{3/2}}\,dx$$

input
```
int(1/(cos(2*a + 2*b*x)^2*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2)),x)
```

output
```
int(1/(cos(2*a + 2*b*x)^2*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2)), x)
```

3.629 $\int \frac{\sec(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx$

3.629.1 Optimal result

Integrand size = 29, antiderivative size = 93

$$\int \frac{\sec(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{-c+c\sec(2a+2bx)}}\right)}{4\sqrt{2}bc^{3/2}} - \frac{\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}}$$

output
```
1/8*arctanh(1/2*c^(1/2)*tan(2*b*x+2*a)*2^(1/2)/(-c+c*sec(2*b*x+2*a))^(1/2)
)/b/c^(3/2)*2^(1/2)-1/4*tan(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(3/2)
```

3.629.2 Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.89

$$\int \frac{\sec(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = -\frac{\left(1+\arctan\left(\sqrt{-1+\tan^2(a+bx)}\right)\sec(2(a+bx))\sin^2(a+bx)\sqrt{-1+\tan^2(a+bx)}\right)\tan(2(a+bx))}{4b(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}$$

input
```
Integrate[Sec[2*(a + b*x)]/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

output
```
-1/4*((1 + ArcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Sec[2*(a + b*x)]*Sin[a + b*x]
^2*Sqrt[-1 + Tan[a + b*x]^2])*Tan[2*(a + b*x)])/(b*(c*Tan[a + b*x]*Tan[2*(
a + b*x)])^(3/2))
```

3.629.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {3042, 4897, 3042, 4283, 3042, 4282, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(2a+2bx)}{(c\tan(a+bx)\tan(2a+2bx))^{3/2}}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \frac{\sec(2a+2bx)}{(c\sec(2a+2bx)-c)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(2a+2bx+\frac{\pi}{2}\right)}{\left(c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c\right)^{3/2}}dx$$

$$\Big\downarrow \text{4283}$$

$$-\frac{\int \frac{\sec(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}dx}{4c}-\frac{\tan(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\csc\left(2a+2bx+\frac{\pi}{2}\right)}{\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}}dx}{4c}-\frac{\tan(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

$$\Big\downarrow \text{4282}$$

$$\frac{\int \frac{1}{\frac{c^2\tan^2(2a+2bx)}{c\sec(2a+2bx)-c}-2c}d\left(-\frac{c\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{4bc}-\frac{\tan(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

$$\Big\downarrow 220$$

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{4\sqrt{2}bc^{3/2}}-\frac{\tan(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

input
```
Int[Sec[2*(a + b*x)]/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

output
```
ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a + 2*b*x]])
]/(4*Sqrt[2]*b*c^(3/2)) - Tan[2*a + 2*b*x]/(4*b*(-c + c*Sec[2*a + 2*b*x])^
(3/2))
```

3.629.3.1 Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4283
```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Simp[(m + 1)/(a*(2*m + 1))   Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m +
1), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)
] && IntegerQ[2*m]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.629. $\int \frac{\sec(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx$

3.629.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 376 vs. 2(78) = 156.

Time = 4.18 (sec) , antiderivative size = 377, normalized size of antiderivative = 4.05

method	result
default	$-\frac{\sqrt{2}\,\csc(xb+a)\left(2\cos(xb+a)\sqrt{\frac{2\cos(xb+a)^2-1}{(1+\cos(xb+a))^2}}+\operatorname{arctanh}\left(\frac{2\cos(xb+a)-1}{(1+\cos(xb+a))\sqrt{\frac{2\cos(xb+a)^2-1}{(1+\cos(xb+a))^2}}}\right)\cos(xb+a)-\ln\left(\frac{2\cos(xb+a)\sqrt{\frac{2\cos(xb+a}{(1+\cos(xb}}}{}\right.\right.}{}$

input
```
int(sec(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x,method=_RETURNVER
BOSE)
```

output
```
-1/32*2^(1/2)/c/b*csc(b*x+a)*(2*cos(b*x+a)*((2*cos(b*x+a)^2-1)/(1+cos(b*x+
a))^2)^(1/2)+arctanh((2*cos(b*x+a)-1)/(1+cos(b*x+a))/((2*cos(b*x+a)^2-1)/(
1+cos(b*x+a))^2)^(1/2))*cos(b*x+a)-ln(2*(cos(b*x+a)*((2*cos(b*x+a)^2-1)/(1
+cos(b*x+a))^2)^(1/2)+((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)-2*cos(b*
x+a)-1)/(1+cos(b*x+a)))*cos(b*x+a)-arctanh((2*cos(b*x+a)-1)/(1+cos(b*x+a))
/((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2))+ln(2*(cos(b*x+a)*((2*cos(b*x
+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)+((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/
2)-2*cos(b*x+a)-1)/(1+cos(b*x+a))))/(c*sin(b*x+a)^2/(2*cos(b*x+a)^2-1))^(1
/2)/((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*4^(1/2)
```

3.629.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.89

$$\int \frac{\sec(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \left[\frac{\sqrt{2}\sqrt{c}\log\left(\frac{c\tan(bx+a)^3+2\sqrt{-\frac{c\tan(bx+a)^2}{\tan(bx+a)^2-1}}\left(\tan(bx+a)^2-1\right)\sqrt{c}-2\,c\tan(bx+a)}{\tan(bx+a)^3}\right.}{16\,bc^2\tan(bx}\right.$$

input
```
integrate(sec(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm=
"fricas")
```

output
```
[1/16*(sqrt(2)*sqrt(c)*log((c*tan(b*x + a)^3 + 2*sqrt(-c*tan(b*x + a)^2/(t
an(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(c) - 2*c*tan(b*x + a))/tan(b
*x + a)^3)*tan(b*x + a)^3 + 2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)
^2 - 1))*(tan(b*x + a)^2 - 1))/(b*c^2*tan(b*x + a)^3), 1/8*(sqrt(2)*sqrt(-
c)*arctan(sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1
)*sqrt(-c)/(c*tan(b*x + a)))*tan(b*x + a)^3 + sqrt(2)*sqrt(-c*tan(b*x + a)
^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1))/(b*c^2*tan(b*x + a)^3)]
```

3.629.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
```

output
```
Timed out
```

3.629.7 Maxima [F]

$$\int \frac{\sec(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \int \frac{\sec(2\,bx+2\,a)}{(c\tan(2\,bx+2\,a)\tan(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate(sec(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm=
"maxima")
```

output
```
integrate(sec(2*b*x + 2*a)/(c*tan(2*b*x + 2*a)*tan(b*x + a))^(3/2), x)
```

3.629.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sec(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(sec(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm=
"giac")
```

output
```
Timed out
```

3.629.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \int \frac{1}{\cos(2\,a+2\,b\,x)\,\left(c\tan(a+b\,x)\,\tan(2\,a+2\,b\,x)\right)^{3/2}}\,dx$$

input
```
int(1/(cos(2*a + 2*b*x)*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2)),x)
```

output
```
int(1/(cos(2*a + 2*b*x)*(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2)), x)
```

3.630 $\int \frac{1}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx$

3.630.1 Optimal result

Integrand size = 20, antiderivative size = 138

$$\int \frac{1}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{bc^{3/2}} + \frac{5\operatorname{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{-c+c\sec(2a+2bx)}}\right)}{4\sqrt{2}bc^{3/2}} - \frac{\tan(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}}$$

output
```
-arctanh(c^(1/2)*tan(2*b*x+2*a)/(-c+c*sec(2*b*x+2*a))^(1/2))/b/c^(3/2)+5/8
*arctanh(1/2*c^(1/2)*tan(2*b*x+2*a)*2^(1/2)/(-c+c*sec(2*b*x+2*a))^(1/2))/b
/c^(3/2)*2^(1/2)-1/4*tan(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(3/2)
```

3.630.2 Mathematica [A] (verified)

Time = 3.20 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.43

$$\int \frac{1}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = -\frac{\cot(a+bx)\left(-4\operatorname{arctanh}\left(\sqrt{1-\tan^2(a+bx)}\right)\cos(2(a+bx))\sec^2(a+bx) + 4\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{1-\tan^2(a+bx)}}{\sqrt{2}}\right)\right.}{}$$

input
```
Integrate[(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(-3/2),x]
```

output
```
-1/8*(Cot[a + b*x]*(-4*ArcTanh[Sqrt[1 - Tan[a + b*x]^2]]*Cos[2*(a + b*x)]*
Sec[a + b*x]^2 + 4*Sqrt[2]*ArcTanh[Sqrt[1 - Tan[a + b*x]^2]/Sqrt[2]]*Cos[2
*(a + b*x)]*Sec[a + b*x]^2 + Cot[a + b*x]^2*(Cos[2*(a + b*x)]*Sec[a + b*x]
^2)^(3/2) + ArcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Sqrt[-(-1 + Tan[a + b*x]^2)^
2])*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x)]])/(b*c^2*Sqrt[1 - Tan[a + b*x]^2]
)
```

3.630.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.04, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 4897, 3042, 4264, 27, 3042, 4408, 3042, 4261, 220, 4282, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(c\tan(a+bx)\tan(2a+2bx))^{3/2}}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \frac{1}{(c\sec(2a+2bx)-c)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c\right)^{3/2}}dx$$

$$\Big\downarrow \text{4264}$$

$$-\frac{\int \frac{\sec(2a+2bx)c+4c}{2\sqrt{c\sec(2a+2bx)-c}}dx}{2c^2}-\frac{\tan(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\sec(2a+2bx)c+4c}{\sqrt{c\sec(2a+2bx)-c}}dx}{4c^2}-\frac{\tan(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\csc(2a+2bx+\frac{\pi}{2})c+4c}{\sqrt{c\csc(2a+2bx+\frac{\pi}{2})-c}}dx}{4c^2}-\frac{\tan(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

$$\Big\downarrow \text{4408}$$

$$-\frac{5c\int \frac{\sec(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}dx-4\int \sqrt{c\sec(2a+2bx)-c}dx}{4c^2}-\frac{\tan(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{5c\int \frac{\csc(2a+2bx+\frac{\pi}{2})}{\sqrt{c\csc(2a+2bx+\frac{\pi}{2})-c}}dx-4\int \sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}dx}{4c^2}-\frac{\tan(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

$$\Big\downarrow \text{4261}$$

$$-\frac{\frac{4c\int \frac{1}{\frac{c^2\tan^2(2a+2bx)}{c\sec(2a+2bx)-c}-c}d\left(-\frac{c\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{b}+5c\int \frac{\csc(2a+2bx+\frac{\pi}{2})}{\sqrt{c\csc(2a+2bx+\frac{\pi}{2})-c}}dx}{4c^2}-\frac{\tan(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

$$\Big\downarrow \text{220}$$

$$-\frac{5c\int \frac{\csc(2a+2bx+\frac{\pi}{2})}{\sqrt{c\csc(2a+2bx+\frac{\pi}{2})-c}}dx+\frac{4\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{b}}{4c^2}-\frac{\tan(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

$$\Big\downarrow \text{4282}$$

$$-\frac{\frac{4\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{b}-\frac{5c\int \frac{1}{\frac{c^2\tan^2(2a+2bx)}{c\sec(2a+2bx)-c}-2c}d\left(-\frac{c\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{b}}{4c^2}-\frac{\tan(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

$$\Big\downarrow \text{220}$$

$$-\frac{\frac{4\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{b}-\frac{5\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{\sqrt{2}b}}{4c^2}-\frac{\tan(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

input
```
Int[(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(-3/2),x]
```

output
```
-1/4*((4*Sqrt[c]*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a +
2*b*x]]])/b - (5*Sqrt[c]*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[
-c + c*Sec[2*a + 2*b*x]])])/(Sqrt[2]*b))/c^2 - Tan[2*a + 2*b*x]/(4*b*(-c +
 c*Sec[2*a + 2*b*x])^(3/2))
```

3.630. $\int \frac{1}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx$

3.630.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4264
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(-Cot[c
+ d*x])*((a + b*Csc[c + d*x])^n/(d*(2*n + 1))), x] + Simp[1/(a^2*(2*n + 1))
   Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && Int
egerQ[2*n]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4408
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c/a   Int[Sqrt[a + b*Csc[e + f*x]], x], x] -
Simp[(b*c - a*d)/a   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.630.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 551 vs. $2(117) = 234$.

Time = 4.37 (sec) , antiderivative size = 552, normalized size of antiderivative = 4.00

method	result
default	$\frac{\sqrt{2}\,\csc(xb+a)(1-\cos(xb+a))\left(16\csc(xb+a)^{2}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\left(\csc(xb+a)^{2}(1-\cos(xb+a))^{2}-1\right)\sqrt{2}}{\sqrt{\csc(xb+a)^{4}(1-\cos(xb+a))^{4}-6\csc(xb+a)^{2}(1-\cos(xb+a))^{2}+1}}\right)(1-\cos(xb+a)\cdots}{\cdots}$

input
```
int(1/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/64*2^(1/2)/b*csc(b*x+a)/(csc(b*x+a)^2*(1-cos(b*x+a))^2*c/(csc(b*x+a)^4*(
1-cos(b*x+a))^4-6*csc(b*x+a)^2*(1-cos(b*x+a))^2+1))^(3/2)*(1-cos(b*x+a))*(
16*csc(b*x+a)^2*2^(1/2)*arctanh((csc(b*x+a)^2*(1-cos(b*x+a))^2-1)*2^(1/2)/
(csc(b*x+a)^4*(1-cos(b*x+a))^4-6*csc(b*x+a)^2*(1-cos(b*x+a))^2+1)^(1/2))*(
1-cos(b*x+a))^2-10*csc(b*x+a)^2*ln(csc(b*x+a)^2*(1-cos(b*x+a))^2-3+(csc(b*
x+a)^4*(1-cos(b*x+a))^4-6*csc(b*x+a)^2*(1-cos(b*x+a))^2+1)^(1/2))*(1-cos(b
*x+a))^2-10*csc(b*x+a)^2*arctanh((3*csc(b*x+a)^2*(1-cos(b*x+a))^2-1)/(csc(
b*x+a)^4*(1-cos(b*x+a))^4-6*csc(b*x+a)^2*(1-cos(b*x+a))^2+1)^(1/2))*(1-cos
(b*x+a))^2+csc(b*x+a)^2*(1-cos(b*x+a))^2*(csc(b*x+a)^4*(1-cos(b*x+a))^4-6*
csc(b*x+a)^2*(1-cos(b*x+a))^2+1)^(1/2)-(csc(b*x+a)^4*(1-cos(b*x+a))^4-6*cs
c(b*x+a)^2*(1-cos(b*x+a))^2+1)^(1/2))/(csc(b*x+a)^4*(1-cos(b*x+a))^4-6*csc
(b*x+a)^2*(1-cos(b*x+a))^2+1)^(3/2)*4^(1/2)
```

3.630.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 438, normalized size of antiderivative = 3.17

$$\int \frac{1}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \left[\frac{5\sqrt{2}\sqrt{c}\log\left(\frac{c\tan(bx+a)^{3}+2\sqrt{-\frac{c\tan(bx+a)^{2}}{\tan(bx+a)^{2}-1}}\left(\tan(bx+a)^{2}-1\right)\sqrt{c}-2c\tan(bx+\cdots}{\tan(bx+a)^{3}}\right.\cdots}{\cdots}\right.$$

input
```
integrate(1/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="fricas")
```

output
```
[1/16*(5*sqrt(2)*sqrt(c)*log((c*tan(b*x + a)^3 + 2*sqrt(-c*tan(b*x + a)^2/
(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(c) - 2*c*tan(b*x + a))/tan
(b*x + a)^3)*tan(b*x + a)^3 + 8*sqrt(c)*log((c*tan(b*x + a)^3 - 2*sqrt(2)*
sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(c)
- 3*c*tan(b*x + a))/(tan(b*x + a)^3 + tan(b*x + a)))*tan(b*x + a)^3 + 2*sq
rt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1))/(
b*c^2*tan(b*x + a)^3), 1/8*(5*sqrt(2)*sqrt(-c)*arctan(sqrt(-c*tan(b*x + a)
^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a)))*t
an(b*x + a)^3 - 8*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(
b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a)))*tan(b*x +
 a)^3 + sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)
^2 - 1))/(b*c^2*tan(b*x + a)^3)]
```

3.630.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
```

output
```
Timed out
```

3.630.7 Maxima [F]

$$\int \frac{1}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \int \frac{1}{(c\tan(2\,bx+2\,a)\tan(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*tan(2*b*x + 2*a)*tan(b*x + a))^(-3/2), x)
```

3.630.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm="giac")
```

output
```
Timed out
```

3.630.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \int \frac{1}{\left(c\tan\left(a+b\,x\right)\,\tan\left(2\,a+2\,b\,x\right)\right)^{3/2}}\,dx$$

input
```
int(1/(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2),x)
```

output
```
int(1/(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2), x)
```

3.631 $\int \frac{\cos(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx$

3.631.1 Optimal result

Integrand size = 29, antiderivative size = 178

$$\int \frac{\cos(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = -\frac{3\operatorname{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{2bc^{3/2}}+\frac{9\operatorname{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{-c+c\sec(2a+2bx)}}\right)}{4\sqrt{2}bc^{3/2}}-\frac{\sin(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}}-\frac{3\sin(2a+2bx)}{4bc\sqrt{-c+c\sec(2a+2bx)}}$$

output
```
-3/2*arctanh(c^(1/2)*tan(2*b*x+2*a)/(-c+c*sec(2*b*x+2*a))^(1/2))/b/c^(3/2)
-1/4*sin(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(3/2)+9/8*arctanh(1/2*c^(1/2)*
tan(2*b*x+2*a)*2^(1/2)/(-c+c*sec(2*b*x+2*a))^(1/2))/b/c^(3/2)*2^(1/2)-3/4*
sin(2*b*x+2*a)/b/c/(-c+c*sec(2*b*x+2*a))^(1/2)
```

3.631.2 Mathematica [A] (verified)

Time = 5.86 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.22

$$\int \frac{\cos(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \frac{\csc(a+bx)\sec(a+bx)\left(6\operatorname{arctanh}\left(\sqrt{1-\tan^2(a+bx)}\right)\cos(2(a-$$

input
```
Integrate[Cos[2*(a + b*x)]/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

output
```
(Csc[a + b*x]*Sec[a + b*x]*(6*ArcTanh[Sqrt[1 - Tan[a + b*x]^2]]*Cos[2*(a +
 b*x)] - 6*Sqrt[2]*ArcTanh[Sqrt[1 - Tan[a + b*x]^2]/Sqrt[2]]*Cos[2*(a + b*
x)] + (1 - 3*Cos[2*(a + b*x)] + Cos[4*(a + b*x)])*Cot[a + b*x]^2*Sqrt[Cos[
2*(a + b*x)]*Sec[a + b*x]^2] - 3*ArcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Cos[a +
 b*x]^2*Sqrt[-(-1 + Tan[a + b*x]^2)^2])*Sqrt[c*Tan[a + b*x]*Tan[2*(a + b*x
)]])/(8*b*c^2*Sqrt[1 - Tan[a + b*x]^2])
```

3.631.3 Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.02, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.483$, Rules used = {3042, 4897, 3042, 4304, 27, 3042, 4510, 3042, 4408, 3042, 4261, 220, 4282, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(2a+2bx)}{(c\tan(a+bx)\tan(2a+2bx))^{3/2}}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \frac{\cos(2a+2bx)}{(c\sec(2a+2bx)-c)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(2a+2bx+\frac{\pi}{2}\right)\left(c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c\right)^{3/2}}dx$$

$$\Big\downarrow \text{4304}$$

$$-\frac{\int \frac{3\cos(2a+2bx)(\sec(2a+2bx)c+2c)}{2\sqrt{c\sec(2a+2bx)-c}}dx}{2c^2}-\frac{\sin(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3\int \frac{\cos(2a+2bx)(\sec(2a+2bx)c+2c)}{\sqrt{c\sec(2a+2bx)-c}}dx}{4c^2}-\frac{\sin(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\int \frac{\csc\left(2a+2bx+\frac{\pi}{2}\right)c+2c}{\csc\left(2a+2bx+\frac{\pi}{2}\right)\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}}dx}{4c^2}-\frac{\sin(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

$$\Big\downarrow \text{4510}$$

$$-\frac{3\left(\frac{\int \frac{\sec(2a+2bx)c^2+2c^2}{\sqrt{c\sec(2a+2bx)-c}}dx}{c}+\frac{c\sin(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}\right)}{4c^2}-\frac{\sin(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\left(\frac{\int \frac{\csc\left(2a+2bx+\frac{\pi}{2}\right)c^2+2c^2}{\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}}dx}{c}+\frac{c\sin(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}\right)}{4c^2}-\frac{\sin(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

$$\Big\downarrow \text{4408}$$

$$-\frac{3\left(\frac{3c^2\int \frac{\sec(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}dx-2c\int \sqrt{c\sec(2a+2bx)-c}dx}{c}+\frac{c\sin(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}\right)}{4c^2}-\frac{\sin(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\left(\frac{3c^2\int \frac{\csc\left(2a+2bx+\frac{\pi}{2}\right)}{\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}}dx-2c\int \sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}dx}{c}+\frac{c\sin(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}\right)}{4c^2}-\frac{\sin(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

$$\Big\downarrow \text{4261}$$

$$-\frac{3\left(\frac{3c^2\int \frac{\csc\left(2a+2bx+\frac{\pi}{2}\right)}{\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}}dx+\frac{2c^2\int \frac{1}{\frac{c^2\tan^2(2a+2bx)}{c\sec(2a+2bx)-c}-c}d\left(-\frac{c\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{b}}{c}+\frac{c\sin(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}\right)}{4c^2}-\frac{\sin(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

$$\Big\downarrow \text{220}$$

3.631. $\int \frac{\cos(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx$

$$-\frac{3\left(\frac{3c^2\int \frac{\csc\left(2a+2bx+\frac{\pi}{2}\right)}{\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}}dx+\frac{2c^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{b}}{c}+\frac{c\sin(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}\right)}{4c^2}-\frac{\sin(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

$$\Big\downarrow \; 4282$$

$$-\frac{3\left(\frac{\frac{2c^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{b}-\frac{3c^2\int \frac{1}{\frac{c^2\tan^2(2a+2bx)}{c\sec(2a+2bx)-c}-2c}d\left(-\frac{c\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{b}}{c}+\frac{c\sin(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}\right)}{4c^2}-\frac{\sin(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

$$\Big\downarrow \; 220$$

$$-\frac{3\left(\frac{\frac{2c^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{b}-\frac{3c^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{\sqrt{2}b}}{c}+\frac{c\sin(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}\right)}{4c^2}-\frac{\sin(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

input
```
Int[Cos[2*(a + b*x)]/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

output
```
-1/4*Sin[2*a + 2*b*x]/(b*(-c + c*Sec[2*a + 2*b*x])^(3/2)) - (3*(((2*c^(3/2
)*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2*a + 2*b*x]]])/b - (
3*c^(3/2)*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/(Sqrt[2]*Sqrt[-c + c*Sec[2*a
+ 2*b*x]])])/(Sqrt[2]*b))/c + (c*Sin[2*a + 2*b*x])/(b*Sqrt[-c + c*Sec[2*a
+ 2*b*x]])))/(4*c^2)
```

3.631. $\int \frac{\cos(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx$

3.631.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4408
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c/a   Int[Sqrt[a + b*Csc[e + f*x]], x], x] -
Simp[(b*c - a*d)/a   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

3.631. $\int \frac{\cos(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx$

rule 4510
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(b*d
*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*
n - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.631.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1074 vs. 2(151) = 302.

Time = 2.96 (sec) , antiderivative size = 1075, normalized size of antiderivative = 6.04

method	result	size
default	Expression too large to display	1075

input
```
int(cos(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x,method=_RETURNVER
BOSE)
```

output
```
-1/64*2^(1/2)/b*csc(b*x+a)/(csc(b*x+a)^2*(1-cos(b*x+a))^2*c/(csc(b*x+a)^4*
(1-cos(b*x+a))^4-6*csc(b*x+a)^2*(1-cos(b*x+a))^2+1))^(3/2)*(1-cos(b*x+a))*
(16*csc(b*x+a)^2*2^(1/2)*arctanh((csc(b*x+a)^2*(1-cos(b*x+a))^2-1)*2^(1/2)
/(csc(b*x+a)^4*(1-cos(b*x+a))^4-6*csc(b*x+a)^2*(1-cos(b*x+a))^2+1)^(1/2))*
(1-cos(b*x+a))^2-10*csc(b*x+a)^2*ln(csc(b*x+a)^2*(1-cos(b*x+a))^2-3+(csc(b
*x+a)^4*(1-cos(b*x+a))^4-6*csc(b*x+a)^2*(1-cos(b*x+a))^2+1)^(1/2))*(1-cos(
b*x+a))^2-10*csc(b*x+a)^2*arctanh((3*csc(b*x+a)^2*(1-cos(b*x+a))^2-1)/(csc
(b*x+a)^4*(1-cos(b*x+a))^4-6*csc(b*x+a)^2*(1-cos(b*x+a))^2+1)^(1/2))*(1-co
s(b*x+a))^2+csc(b*x+a)^2*(1-cos(b*x+a))^2*(csc(b*x+a)^4*(1-cos(b*x+a))^4-6
*csc(b*x+a)^2*(1-cos(b*x+a))^2+1)^(1/2)-(csc(b*x+a)^4*(1-cos(b*x+a))^4-6*c
sc(b*x+a)^2*(1-cos(b*x+a))^2+1)^(1/2))/(csc(b*x+a)^4*(1-cos(b*x+a))^4-6*cs
c(b*x+a)^2*(1-cos(b*x+a))^2+1)^(3/2)*4^(1/2)+1/16*2^(1/2)/b*csc(b*x+a)*(4*
((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*cos(b*x+a)^3+10*2^(1/2)*arctan
h(cos(b*x+a)/(1+cos(b*x+a))/((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*2^
(1/2))*cos(b*x+a)-10*2^(1/2)*arctanh(cos(b*x+a)/(1+cos(b*x+a))/((2*cos(b*x
+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*2^(1/2))-6*cos(b*x+a)*((2*cos(b*x+a)^2-1)
/(1+cos(b*x+a))^2)^(1/2)-7*arctanh((2*cos(b*x+a)-1)/(1+cos(b*x+a))/((2*cos
(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2))*cos(b*x+a)+7*ln(2*(cos(b*x+a)*((2*co
s(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)+((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2
)^(1/2)-2*cos(b*x+a)-1)/(1+cos(b*x+a)))*cos(b*x+a)+7*arctanh((2*cos(b*x...
```

3.631.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 528, normalized size of antiderivative = 2.97

$$\int \frac{\cos(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \left[\frac{9\sqrt{2}\left(\tan(bx+a)^5+\tan(bx+a)^3\right)\sqrt{c}\log\left(\frac{c\tan(bx+a)^3+2\sqrt{-\frac{c\tan}{\tan(b}}}{}\right.}{}\right.$$

input
```
integrate(cos(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm=
"fricas")
```

output
```
[1/16*(9*sqrt(2)*(tan(b*x + a)^5 + tan(b*x + a)^3)*sqrt(c)*log((c*tan(b*x
+ a)^3 + 2*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 -
1)*sqrt(c) - 2*c*tan(b*x + a))/tan(b*x + a)^3) + 12*(tan(b*x + a)^5 + tan(
b*x + a)^3)*sqrt(c)*log((c*tan(b*x + a)^3 - 2*sqrt(2)*sqrt(-c*tan(b*x + a)
^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(c) - 3*c*tan(b*x + a))/
(tan(b*x + a)^3 + tan(b*x + a))) + 2*sqrt(2)*(5*tan(b*x + a)^4 - 4*tan(b*x
 + a)^2 - 1)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*c^2*tan(b*x
+ a)^5 + b*c^2*tan(b*x + a)^3), 1/8*(9*sqrt(2)*(tan(b*x + a)^5 + tan(b*x +
 a)^3)*sqrt(-c)*arctan(sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b
*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a))) - 12*(tan(b*x + a)^5 + tan(b*x +
 a)^3)*sqrt(-c)*arctan(1/2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2
- 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a))) + sqrt(2)*(5*tan(b*x
 + a)^4 - 4*tan(b*x + a)^2 - 1)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1
)))/(b*c^2*tan(b*x + a)^5 + b*c^2*tan(b*x + a)^3)]
```

3.631.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
```

output
```
Timed out
```

3.631.7 Maxima [F]

$$\int \frac{\cos(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \int \frac{\cos(2\,bx+2\,a)}{(c\tan(2\,bx+2\,a)\tan(bx+a))^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm=
"maxima")
```

output
```
integrate(cos(2*b*x + 2*a)/(c*tan(2*b*x + 2*a)*tan(b*x + a))^(3/2), x)
```

3.631.8 Giac [F(-1)]

Timed out.

$$\int \frac{\cos(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(2*b*x+2*a)/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorithm=
"giac")
```

output
```
Timed out
```

3.631.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \int \frac{\cos(2\,a+2\,b\,x)}{\left(c\tan(a+b\,x)\,\tan(2\,a+2\,b\,x)\right)^{3/2}}\,dx$$

input
```
int(cos(2*a + 2*b*x)/(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2),x)
```

output
```
int(cos(2*a + 2*b*x)/(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2), x)
```

3.632 $\int \frac{\cos^2(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx$

3.632.1 Optimal result

Integrand size = 31, antiderivative size = 234

$$\int \frac{\cos^2(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = -\frac{19\operatorname{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{-c+c\sec(2a+2bx)}}\right)}{8bc^{3/2}} + \frac{13\operatorname{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{-c+c\sec(2a+2bx)}}\right)}{4\sqrt{2}bc^{3/2}} - \frac{\cos(2a+2bx)\sin(2a+2bx)}{4b(-c+c\sec(2a+2bx))^{3/2}} - \frac{7\sin(2a+2bx)}{8bc\sqrt{-c+c\sec(2a+2bx)}} - \frac{\cos(2a+2bx)\sin(2a+2bx)}{2bc\sqrt{-c+c\sec(2a+2bx)}}$$

output

```
-19/8*arctanh(c^(1/2)*tan(2*b*x+2*a)/(-c+c*sec(2*b*x+2*a))^(1/2))/b/c^(3/2
)-1/4*cos(2*b*x+2*a)*sin(2*b*x+2*a)/b/(-c+c*sec(2*b*x+2*a))^(3/2)+13/8*arc
tanh(1/2*c^(1/2)*tan(2*b*x+2*a)*2^(1/2)/(-c+c*sec(2*b*x+2*a))^(1/2))/b/c^(
3/2)*2^(1/2)-7/8*sin(2*b*x+2*a)/b/c/(-c+c*sec(2*b*x+2*a))^(1/2)-1/2*cos(2*
b*x+2*a)*sin(2*b*x+2*a)/b/c/(-c+c*sec(2*b*x+2*a))^(1/2)
```

3.632.2 Mathematica [A] (verified)

Time = 7.59 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.53

$$\int \frac{\cos^2(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \frac{\tan^{\frac{3}{2}}(a+bx)\left(-\frac{7\arctan\left(\sqrt{-1+\tan^2(a+bx)}\right)\csc^2(a+bx)\sec^2(a+bx)\tan^{\frac{3}{2}}(a+bx)}{(1+\tan^2(a+bx))^2}\right.}{}$$
$$+\frac{\left(-\frac{5}{8}\cot(a+bx)-\frac{1}{8}\cot(a+bx)\csc^2(a+bx)+\frac{7}{8}\sin(2(a+bx))+\frac{1}{8}\sin(4(a+bx))\right)\tan^2(a+bx)\tan^2(2(a}{b(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}$$

input
```
Integrate[Cos[2*(a + b*x)]^2/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

output
```
(Tan[a + b*x]^(3/2)*((-7*ArcTan[Sqrt[-1 + Tan[a + b*x]^2]]*Csc[a + b*x]^2*
Sec[a + b*x]^2*Tan[a + b*x]^(3/2)*Sqrt[-1 + Tan[a + b*x]^2]*Sqrt[Tan[2*(a
+ b*x)]])/(1 + Tan[a + b*x]^2)^2 + (19*(Sqrt[2]*ArcTanh[Sqrt[1 - Tan[a + b
*x]^2]] - 2*ArcTanh[Sqrt[1 - Tan[a + b*x]^2]/Sqrt[2]])*Cos[2*(a + b*x)]*Cs
c[a + b*x]^2*Sec[a + b*x]^2*Tan[a + b*x]^(3/2)*Sqrt[Tan[2*(a + b*x)]])/(Sq
rt[2]*Sqrt[1 - Tan[a + b*x]^2]*(1 + Tan[a + b*x]^2)))*Tan[2*(a + b*x)]^(3/
2))/(16*b*(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2)) + (((-5*Cot[a + b*x])/8
 - (Cot[a + b*x]*Csc[a + b*x]^2)/8 + (7*Sin[2*(a + b*x)])/8 + Sin[4*(a + b
*x)]/8)*Tan[a + b*x]^2*Tan[2*(a + b*x)]^2)/(b*(c*Tan[a + b*x]*Tan[2*(a + b
*x)])^(3/2))
```

3.632.3 Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.06, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.581$, Rules used = {3042, 4897, 3042, 4304, 27, 3042, 4510, 27, 3042, 4510, 27, 3042, 4408, 3042, 4261, 220, 4282, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos(2a+2bx)^2}{(c\tan(a+bx)\tan(2a+2bx))^{3/2}}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \frac{\cos^2(2a+2bx)}{(c\sec(2a+2bx)-c)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\csc\left(2a+2bx+\frac{\pi}{2}\right)^2\left(c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c\right)^{3/2}}dx$$

$$\Big\downarrow \text{4304}$$

$$-\frac{\int \frac{\cos^2(2a+2bx)(5\sec(2a+2bx)c+8c)}{2\sqrt{c\sec(2a+2bx)-c}}dx}{2c^2}-\frac{\sin(2a+2bx)\cos(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\cos^2(2a+2bx)(5\sec(2a+2bx)c+8c)}{\sqrt{c\sec(2a+2bx)-c}}dx}{4c^2}-\frac{\sin(2a+2bx)\cos(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{5\csc\left(2a+2bx+\frac{\pi}{2}\right)c+8c}{\csc\left(2a+2bx+\frac{\pi}{2}\right)^2\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}}dx}{4c^2}-\frac{\sin(2a+2bx)\cos(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

$$\Big\downarrow \text{4510}$$

$$-\frac{\frac{\int \frac{2\cos(2a+2bx)\left(6\sec(2a+2bx)c^2+7c^2\right)}{\sqrt{c\sec(2a+2bx)-c}}dx}{2c}+\frac{2c\sin(2a+2bx)\cos(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}}{4c^2}-\frac{\sin(2a+2bx)\cos(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\int \frac{\cos(2a+2bx)\left(6\sec(2a+2bx)c^2+7c^2\right)}{\sqrt{c\sec(2a+2bx)-c}}dx}{c}+\frac{2c\sin(2a+2bx)\cos(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}}{4c^2}-\frac{\sin(2a+2bx)\cos(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\int \frac{6\csc\left(2a+2bx+\frac{\pi}{2}\right)c^2+7c^2}{\csc\left(2a+2bx+\frac{\pi}{2}\right)\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}}dx}{c}+\frac{2c\sin(2a+2bx)\cos(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}}{4c^2}-\frac{\sin(2a+2bx)\cos(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

$$\Big\downarrow \text{4510}$$

$$-\frac{\frac{\frac{\int \frac{7\sec(2a+2bx)c^3+19c^3}{2\sqrt{c\sec(2a+2bx)-c}}dx}{c}+\frac{7c^2\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}}{c}+\frac{2c\sin(2a+2bx)\cos(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}}{4c^2}-\frac{\sin(2a+2bx)\cos(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

3.632. $\int \frac{\cos^2(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}dx$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\frac{\int \frac{7\sec(2a+2bx)c^3+19c^3}{\sqrt{c\sec(2a+2bx)-c}}dx}{2c}+\frac{7c^2\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}}{c}+\frac{2c\sin(2a+2bx)\cos(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}}{4c^2}-\frac{\sin(2a+2bx)\cos(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\frac{\int \frac{7\csc\left(2a+2bx+\frac{\pi}{2}\right)c^3+19c^3}{\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}}dx}{2c}+\frac{7c^2\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}}{c}+\frac{2c\sin(2a+2bx)\cos(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}}{4c^2}-\frac{\sin(2a+2bx)\cos(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

$$\Big\downarrow \; 4408$$

$$-\frac{\frac{\frac{26c^3\int \frac{\sec(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}dx-19c^2\int \sqrt{c\sec(2a+2bx)-c}dx}{2c}+\frac{7c^2\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}}{c}+\frac{2c\sin(2a+2bx)\cos(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}}{4c^2}-\frac{\sin(2a+2bx)\cos(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{\frac{26c^3\int \frac{\csc\left(2a+2bx+\frac{\pi}{2}\right)}{\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}}dx-19c^2\int \sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}dx}{2c}+\frac{7c^2\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}}{c}+\frac{2c\sin(2a+2bx)\cos(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}}{4c^2}-\frac{\sin(2a+2bx)\cos(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

$$\Big\downarrow \; 4261$$

$$-\frac{\frac{\frac{26c^3\int \frac{\csc\left(2a+2bx+\frac{\pi}{2}\right)}{\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}}dx+\frac{19c^3\int \frac{1}{\frac{c^2\tan^2(2a+2bx)}{c\sec(2a+2bx)-c}-c}d\left(-\frac{c\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{b}}{2c}+\frac{7c^2\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}}{c}+\frac{2c\sin(2a+2bx)\cos(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}}{4c^2}-\frac{\sin(2a+2bx)\cos(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

$$\Big\downarrow \; 220$$

3.632. $\int \frac{\cos^2(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx$

$$-\frac{\frac{\frac{26c^3\int \frac{\csc\left(2a+2bx+\frac{\pi}{2}\right)}{\sqrt{c\csc\left(2a+2bx+\frac{\pi}{2}\right)-c}}dx+\frac{19c^{5/2}\text{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{b}}{2c}+\frac{7c^2\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}}{c}+\frac{2c\sin(2a+2bx)\cos(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}}{4c^2}-\frac{\sin(2a+2bx)\cos(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

$$\Big\downarrow \; 4282$$

$$-\frac{\frac{\frac{\frac{19c^{5/2}\text{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{b}-\frac{26c^3\int \frac{1}{\frac{c^2\tan^2(2a+2bx)}{c\sec(2a+2bx)-c}-2c}d\left(-\frac{c\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{b}}{2c}+\frac{7c^2\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}}{c}+\frac{2c\sin(2a+2bx)\cos(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}}{4c^2}-\frac{\sin(2a+2bx)\cos(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

$$\Big\downarrow \; 220$$

$$-\frac{\frac{\frac{\frac{19c^{5/2}\text{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{c\sec(2a+2bx)-c}}\right)}{b}-\frac{13\sqrt{2}c^{5/2}\text{arctanh}\left(\frac{\sqrt{c}\tan(2a+2bx)}{\sqrt{2}\sqrt{c\sec(2a+2bx)-c}}\right)}{b}}{2c}+\frac{7c^2\sin(2a+2bx)}{2b\sqrt{c\sec(2a+2bx)-c}}}{c}+\frac{2c\sin(2a+2bx)\cos(2a+2bx)}{b\sqrt{c\sec(2a+2bx)-c}}}{4c^2}-\frac{\sin(2a+2bx)\cos(2a+2bx)}{4b(c\sec(2a+2bx)-c)^{3/2}}$$

input
```
Int[Cos[2*(a + b*x)]^2/(c*Tan[a + b*x]*Tan[2*(a + b*x)])^(3/2),x]
```

output
```
-1/4*(Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x])/(b*(-c + c*Sec[2*a + 2*b*x])^(3/2
)) - ((2*c*Cos[2*a + 2*b*x]*Sin[2*a + 2*b*x])/(b*Sqrt[-c + c*Sec[2*a + 2*b
*x]]) + (((19*c^(5/2)*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/Sqrt[-c + c*Sec[2
*a + 2*b*x]]])/b - (13*Sqrt[2]*c^(5/2)*ArcTanh[(Sqrt[c]*Tan[2*a + 2*b*x])/
(Sqrt[2]*Sqrt[-c + c*Sec[2*a + 2*b*x]])])/b)/(2*c) + (7*c^2*Sin[2*a + 2*b*
x])/(2*b*Sqrt[-c + c*Sec[2*a + 2*b*x]]))/c)/(4*c^2)
```

3.632. $\int \frac{\cos^2(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx$

3.632.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4261
```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(b/d)
  Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 4282
```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2/f   Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[
a + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

rule 4304
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-Cot[e + f*x])*(a + b*Csc[e + f*x])^m*((d*Csc
[e + f*x])^n/(f*(2*m + 1))), x] + Simp[1/(a^2*(2*m + 1))   Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*(a*(2*m + n + 1) - b*(m + n + 1)*Csc[e
+ f*x]), x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && LtQ
[m, -1] && (IntegersQ[2*m, 2*n] || IntegerQ[m])
```

rule 4408
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Simp[c/a   Int[Sqrt[a + b*Csc[e + f*x]], x], x] -
Simp[(b*c - a*d)/a   Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

3.632. $\int \frac{\cos^2(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx$

rule 4510
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Simp[1/(b*d
*n)   Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*
n - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.632.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1631 vs. $2(203) = 406$.

Time = 3.26 (sec) , antiderivative size = 1632, normalized size of antiderivative = 6.97

method	result	size
default	Expression too large to display	1632

input
```
int(cos(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x,method=_RETURNV
ERBOSE)
```

output
```
1/32*2^(1/2)/b*csc(b*x+a)*(8*((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*c
os(b*x+a)^5+14*((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*cos(b*x+a)^3+51
*2^(1/2)*arctanh(cos(b*x+a)/(1+cos(b*x+a))/((2*cos(b*x+a)^2-1)/(1+cos(b*x+
a))^2)^(1/2)*2^(1/2))*cos(b*x+a)-51*2^(1/2)*arctanh(cos(b*x+a)/(1+cos(b*x+
a))/((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)*2^(1/2))-30*cos(b*x+a)*((2
*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)-36*arctanh((2*cos(b*x+a)-1)/(1+co
s(b*x+a))/((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2))*cos(b*x+a)+36*ln(2*
(cos(b*x+a)*((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)+((2*cos(b*x+a)^2-1
)/(1+cos(b*x+a))^2)^(1/2)-2*cos(b*x+a)-1)/(1+cos(b*x+a)))*cos(b*x+a)+36*ar
ctanh((2*cos(b*x+a)-1)/(1+cos(b*x+a))/((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2
)^(1/2))-36*ln(2*(cos(b*x+a)*((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)+(
(2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)-2*cos(b*x+a)-1)/(1+cos(b*x+a)))
)/((2*cos(b*x+a)^2-1)/(1+cos(b*x+a))^2)^(1/2)/(c*sin(b*x+a)^2/(2*cos(b*x+a
)^2-1))^(1/2)/c*4^(1/2)+1/64*2^(1/2)/b*csc(b*x+a)/(csc(b*x+a)^2*(1-cos(b*x
+a))^2*c/(csc(b*x+a)^4*(1-cos(b*x+a))^4-6*csc(b*x+a)^2*(1-cos(b*x+a))^2+1)
)^(3/2)*(1-cos(b*x+a))*(16*csc(b*x+a)^2*2^(1/2)*arctanh((csc(b*x+a)^2*(1-c
os(b*x+a))^2-1)*2^(1/2)/(csc(b*x+a)^4*(1-cos(b*x+a))^4-6*csc(b*x+a)^2*(1-c
os(b*x+a))^2+1)^(1/2))*(1-cos(b*x+a))^2-10*csc(b*x+a)^2*ln(csc(b*x+a)^2*(1
-cos(b*x+a))^2-3+(csc(b*x+a)^4*(1-cos(b*x+a))^4-6*csc(b*x+a)^2*(1-cos(b*x+
a))^2+1)^(1/2))*(1-cos(b*x+a))^2-10*csc(b*x+a)^2*arctanh((3*csc(b*x+a)^...
```

3.632.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 616, normalized size of antiderivative = 2.63

$$\int \frac{\cos^2(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \left[\frac{13\sqrt{2}\left(\tan(bx+a)^7+2\tan(bx+a)^5+\tan(bx+a)^3\right)\sqrt{c}\log\Bigg(}{}\right.$$

input
```
integrate(cos(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorith
m="fricas")
```

output
```
[1/16*(13*sqrt(2)*(tan(b*x + a)^7 + 2*tan(b*x + a)^5 + tan(b*x + a)^3)*sqr
t(c)*log((c*tan(b*x + a)^3 + 2*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)
)*(tan(b*x + a)^2 - 1)*sqrt(c) - 2*c*tan(b*x + a))/tan(b*x + a)^3) + 19*(t
an(b*x + a)^7 + 2*tan(b*x + a)^5 + tan(b*x + a)^3)*sqrt(c)*log((c*tan(b*x
+ a)^3 - 2*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x +
 a)^2 - 1)*sqrt(c) - 3*c*tan(b*x + a))/(tan(b*x + a)^3 + tan(b*x + a))) +
2*sqrt(2)*(4*tan(b*x + a)^6 + 5*tan(b*x + a)^4 - 8*tan(b*x + a)^2 - 1)*sqr
t(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*c^2*tan(b*x + a)^7 + 2*b*c^2
*tan(b*x + a)^5 + b*c^2*tan(b*x + a)^3), 1/8*(13*sqrt(2)*(tan(b*x + a)^7 +
 2*tan(b*x + a)^5 + tan(b*x + a)^3)*sqrt(-c)*arctan(sqrt(-c*tan(b*x + a)^2
/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)*sqrt(-c)/(c*tan(b*x + a))) - 1
9*(tan(b*x + a)^7 + 2*tan(b*x + a)^5 + tan(b*x + a)^3)*sqrt(-c)*arctan(1/2
*sqrt(2)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1))*(tan(b*x + a)^2 - 1)
*sqrt(-c)/(c*tan(b*x + a))) + sqrt(2)*(4*tan(b*x + a)^6 + 5*tan(b*x + a)^4
 - 8*tan(b*x + a)^2 - 1)*sqrt(-c*tan(b*x + a)^2/(tan(b*x + a)^2 - 1)))/(b*
c^2*tan(b*x + a)^7 + 2*b*c^2*tan(b*x + a)^5 + b*c^2*tan(b*x + a)^3)]
```

3.632.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(2*b*x+2*a)**2/(c*tan(b*x+a)*tan(2*b*x+2*a))**(3/2),x)
```

output
```
Timed out
```

3.632.7 Maxima [F]

$$\int \frac{\cos^2(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \int \frac{\cos\left(2\,bx+2\,a\right)^2}{\left(c\tan\left(2\,bx+2\,a\right)\tan\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cos(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorith
m="maxima")
```

output
```
integrate(cos(2*b*x + 2*a)^2/(c*tan(2*b*x + 2*a)*tan(b*x + a))^(3/2), x)
```

3.632.8 Giac [F(-1)]

Timed out.

$$\int \frac{\cos^2(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cos(2*b*x+2*a)^2/(c*tan(b*x+a)*tan(2*b*x+2*a))^(3/2),x, algorith
m="giac")
```

output
```
Timed out
```

3.632.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(2(a+bx))}{(c\tan(a+bx)\tan(2(a+bx)))^{3/2}}\,dx = \int \frac{\cos\left(2\,a+2\,b\,x\right)^2}{\left(c\tan\left(a+b\,x\right)\,\tan\left(2\,a+2\,b\,x\right)\right)^{3/2}}\,dx$$

input
```
int(cos(2*a + 2*b*x)^2/(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2),x)
```

output
```
int(cos(2*a + 2*b*x)^2/(c*tan(a + b*x)*tan(2*a + 2*b*x))^(3/2), x)
```

3.633 $\int \frac{\cot(x)\csc(x)}{\sqrt{\sin(2x)}}\,dx$

3.633.1 Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{\cot(x)\csc(x)}{\sqrt{\sin(2x)}}\,dx = -\frac{2\cos(x)\cot(x)}{3\sqrt{\sin(2x)}}$$

output
```
-2/3*cos(x)*cot(x)/sin(2*x)^(1/2)
```

3.633.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\cot(x)\csc(x)}{\sqrt{\sin(2x)}}\,dx = -\frac{1}{3}\cot(x)\csc(x)\sqrt{\sin(2x)}$$

input
```
Integrate[(Cot[x]*Csc[x])/Sqrt[Sin[2*x]],x]
```

output
```
-1/3*(Cot[x]*Csc[x]*Sqrt[Sin[2*x]])
```

3.633.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 4890, 4889, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(x)\csc(x)}{\sqrt{\sin(2x)}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cot(x)\csc(x)}{\sqrt{\sin(2x)}}dx$$

$$\Big\downarrow \text{ 4890}$$

$$\frac{\sin(x)\int \cot(x)\csc(x)\csc(x)\sqrt{\tan(x)}dx}{\sqrt{\sin(2x)}\sqrt{\tan(x)}}$$

$$\Big\downarrow \text{ 4889}$$

$$\frac{\sin(x)\int \frac{1}{\tan^{\frac{5}{2}}(x)}d\tan(x)}{\sqrt{\sin(2x)}\sqrt{\tan(x)}}$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{2\cos(x)\cot(x)}{3\sqrt{\sin(2x)}}$$

input
```
Int[(Cot[x]*Csc[x])/Sqrt[Sin[2*x]],x]
```

output
```
(-2*Cos[x]*Cot[x])/(3*Sqrt[Sin[2*x]])
```

3.633.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

rule 4890
```
Int[(u_)*((c_.)*sin[v_])^(m_), x_Symbol] :> With[{w = FunctionOfTrig[u*(Sin
[v/2]^(2*m)/(c*Tan[v/2])^m), x]}, Simp[(c*Sin[v])^m*((c*Tan[v/2])^m/Sin[v/2
]^(2*m))   Int[u*(Sin[v/2]^(2*m)/(c*Tan[v/2])^m), x], x] /;  !FalseQ[w] &&
FunctionOfQ[NonfreeFactors[Tan[w], x], u*(Sin[v/2]^(2*m)/(c*Tan[v/2])^m), x
]] /; FreeQ[c, x] && LinearQ[v, x] && IntegerQ[m + 1/2] &&  !SumQ[u] && Inv
erseFunctionFreeQ[u, x]
```

3.633.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 1.27 (sec) , antiderivative size = 119, normalized size of antiderivative = 7.44

method	result	s
default	$\frac{\sqrt{-\frac{\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^{2}-1}}\,\left(\tan\left(\frac{x}{2}\right)^{2}-1\right)\left(4\sqrt{\tan\left(\frac{x}{2}\right)+1}\,\sqrt{-2\tan\left(\frac{x}{2}\right)+2}\,\sqrt{-\tan\left(\frac{x}{2}\right)}\,\operatorname{EllipticF}\left(\sqrt{\tan\left(\frac{x}{2}\right)+1},\frac{\sqrt{2}}{2}\right)\tan\left(\frac{x}{2}\right)+\tan\left(\frac{x}{2}\right)^{4}-1\right)}{6\tan\left(\frac{x}{2}\right)\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan\left(\frac{x}{2}\right)^{2}-1\right)}\,\sqrt{\tan\left(\frac{x}{2}\right)^{3}-\tan\left(\frac{x}{2}\right)}}$	1

input
```
int(cot(x)*csc(x)/sin(2*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1)/tan(1/2*x)*(4*(t
an(1/2*x)+1)^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((
tan(1/2*x)+1)^(1/2),1/2*2^(1/2))*tan(1/2*x)+tan(1/2*x)^4-1)/(tan(1/2*x)*(t
an(1/2*x)^2-1))^(1/2)/(tan(1/2*x)^3-tan(1/2*x))^(1/2)
```

3.633.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(12) = 24$.

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{\cot(x)\csc(x)}{\sqrt{\sin(2x)}}\,dx = \frac{\sqrt{2}\sqrt{\cos(x)\sin(x)}\cos(x) + \cos(x)^2 - 1}{3\left(\cos(x)^2 - 1\right)}$$

input
```
integrate(cot(x)*csc(x)/sin(2*x)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(sqrt(2)*sqrt(cos(x)*sin(x))*cos(x) + cos(x)^2 - 1)/(cos(x)^2 - 1)
```

3.633.6 Sympy [F]

$$\int \frac{\cot(x)\csc(x)}{\sqrt{\sin(2x)}}\,dx = \int \frac{\cot(x)\csc(x)}{\sqrt{\sin(2x)}}\,dx$$

input
```
integrate(cot(x)*csc(x)/sin(2*x)**(1/2),x)
```

output
```
Integral(cot(x)*csc(x)/sqrt(sin(2*x)), x)
```

3.633.7 Maxima [F]

$$\int \frac{\cot(x)\csc(x)}{\sqrt{\sin(2x)}}\,dx = \int \frac{\cot(x)\csc(x)}{\sqrt{\sin(2\,x)}}\,dx$$

input
```
integrate(cot(x)*csc(x)/sin(2*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(cot(x)*csc(x)/sqrt(sin(2*x)), x)
```

3.633.8 Giac [F]

$$\int \frac{\cot(x)\csc(x)}{\sqrt{\sin(2x)}}\,dx = \int \frac{\cot(x)\csc(x)}{\sqrt{\sin(2x)}}\,dx$$

input
```
integrate(cot(x)*csc(x)/sin(2*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(cot(x)*csc(x)/sqrt(sin(2*x)), x)
```

3.633.9 Mupad [B] (verification not implemented)

Time = 26.37 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\cot(x)\csc(x)}{\sqrt{\sin(2x)}}\,dx = -\frac{\sqrt{\sin(2x)}\,\cos(x)}{3\sin(x)^2}$$

input
```
int(cot(x)/(sin(2*x)^(1/2)*sin(x)),x)
```

output
```
-(sin(2*x)^(1/2)*cos(x))/(3*sin(x)^2)
```

3.634 $\int \frac{\csc^2(x)\sec(x)}{\sqrt{\sin(2x)}(-2+\tan(x))}\,dx$

3.634.1 Optimal result

Integrand size = 21, antiderivative size = 69

$$\int \frac{\csc^2(x)\sec(x)}{\sqrt{\sin(2x)}(-2+\tan(x))}\,dx=\frac{\cos(x)}{2\sqrt{\sin(2x)}}+\frac{\cos(x)\cot(x)}{3\sqrt{\sin(2x)}}-\frac{5\text{arctanh}\left(\frac{\sqrt{\tan(x)}}{\sqrt{2}}\right)\sin(x)}{2\sqrt{2}\sqrt{\sin(2x)}\sqrt{\tan(x)}}$$

output
```
1/2*cos(x)/sin(2*x)^(1/2)+1/3*cos(x)*cot(x)/sin(2*x)^(1/2)-5/4*arctanh(1/2
*tan(x)^(1/2)*2^(1/2))*sin(x)*2^(1/2)/sin(2*x)^(1/2)/tan(x)^(1/2)
```

3.634.2 Mathematica [A] (verified)

Time = 1.81 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.16

$$\int \frac{\csc^2(x)\sec(x)}{\sqrt{\sin(2x)}(-2+\tan(x))}\,dx=\frac{1}{4}\sqrt{\sin(2x)}\left(\left(1+\frac{2\cot(x)}{3}\right)\csc(x)-\frac{5\arctan\left(\frac{\sqrt{\tan\left(\frac{x}{2}\right)}}{\sqrt{-1+\tan^2\left(\frac{x}{2}\right)}}\right)\sqrt{-\frac{\cos(x)}{2+2\cos(x)}}\sec(x)}{\sqrt{\tan\left(\frac{x}{2}\right)}}\right)$$

input
```
Integrate[(Csc[x]^2*Sec[x])/(Sqrt[Sin[2*x]]*(-2 + Tan[x])),x]
```

output
```
(Sqrt[Sin[2*x]]*((1 + (2*Cot[x])/3)*Csc[x] - (5*ArcTan[Sqrt[Tan[x/2]]/Sqrt
[-1 + Tan[x/2]^2]]*Sqrt[-(Cos[x]/(2 + 2*Cos[x]))]*Sec[x])/Sqrt[Tan[x/2]]))
/4
```

3.634.3 Rubi [A] (warning: unable to verify)

Time = 0.57 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.80, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4890, 25, 4889, 518, 1585, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(x)\sec(x)}{\sqrt{\sin(2x)}(\tan(x)-2)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sec(x)}{\sin(x)^2\sqrt{\sin(2x)}(\tan(x)-2)}dx$$

$$\Big\downarrow \; 4890$$

$$\frac{\sin(x)\int -\frac{\csc(x)\sec(x)\sqrt{\tan(x)}}{\sin(x)^2(2-\tan(x))}dx}{\sqrt{\sin(2x)}\sqrt{\tan(x)}}$$

$$\Big\downarrow \; 25$$

$$-\frac{\sin(x)\int \frac{\csc(x)\sec(x)\sqrt{\tan(x)}}{\sin(x)^2(2-\tan(x))}dx}{\sqrt{\sin(2x)}\sqrt{\tan(x)}}$$

$$\Big\downarrow \; 4889$$

$$-\frac{\sin(x)\int \frac{\tan^2(x)+1}{(2-\tan(x))\tan^{\frac{5}{2}}(x)}d\tan(x)}{\sqrt{\sin(2x)}\sqrt{\tan(x)}}$$

$$\Big\downarrow \; 518$$

$$-\frac{2\sin(x)\int \frac{\cot^4(x)(\tan^2(x)+1)}{2-\tan(x)}d\sqrt{\tan(x)}}{\sqrt{\sin(2x)}\sqrt{\tan(x)}}$$

$$\Big\downarrow \; 1585$$

$$-\frac{2\sin(x)\int \left(\frac{\cot^4(x)}{2}+\frac{\cot^2(x)}{4}-\frac{5}{4(\tan(x)-2)}\right)d\sqrt{\tan(x)}}{\sqrt{\sin(2x)}\sqrt{\tan(x)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\sin(x)\left(\frac{5\text{arctanh}\left(\frac{\sqrt{\tan(x)}}{\sqrt{2}}\right)}{4\sqrt{2}}-\frac{1}{6}\cot^3(x)-\frac{\cot(x)}{4}\right)}{\sqrt{\sin(2x)}\sqrt{\tan(x)}}$$

input
```
Int[(Csc[x]^2*Sec[x])/(Sqrt[Sin[2*x]]*(-2 + Tan[x])),x]
```

output
```
(-2*((5*ArcTanh[Sqrt[Tan[x]]/Sqrt[2]])/(4*Sqrt[2]) - Cot[x]/4 - Cot[x]^3/6
)*Sin[x])/(Sqrt[Sin[2*x]]*Sqrt[Tan[x]])
```

3.634.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 518
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.),
 x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(2*m + 1)*(e*c + d*x^2)^
n*(a*e^2 + b*x^4)^p, x], x, Sqrt[e*x]], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1585
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.634. $\int \frac{\csc^2(x)\sec(x)}{\sqrt{\sin(2x)}(-2+\tan(x))}\,dx$

rule 4890
```
Int[(u_)*((c_.)*sin[v_])^(m_), x_Symbol] :> With[{w = FunctionOfTrig[u*(Sin
[v/2]^(2*m)/(c*Tan[v/2])^m), x]}, Simp[(c*Sin[v])^m*((c*Tan[v/2])^m/Sin[v/2
]^(2*m))   Int[u*(Sin[v/2]^(2*m)/(c*Tan[v/2])^m), x], x] /;  !FalseQ[w] &&
FunctionOfQ[NonfreeFactors[Tan[w], x], u*(Sin[v/2]^(2*m)/(c*Tan[v/2])^m), x
]] /; FreeQ[c, x] && LinearQ[v, x] && IntegerQ[m + 1/2] &&  !SumQ[u] && Inv
erseFunctionFreeQ[u, x]
```

3.634.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.62 (sec) , antiderivative size = 396, normalized size of antiderivative = 5.74

method	result
default	$-\frac{\sqrt{-\frac{\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^{2}-1}}\left(-140\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan\left(\frac{x}{2}\right)^{2}-1\right)}\,\sqrt{-2\tan\left(\frac{x}{2}\right)+2}\,\operatorname{EllipticF}\left(\sqrt{\tan\left(\frac{x}{2}\right)+1},\frac{\sqrt{2}}{2}\right)\sqrt{\tan\left(\frac{x}{2}\right)+1}\,\sqrt{-\tan\left(\frac{x}{2}\right)}\,\tan\left(\frac{x}{2}\right)+2\right.}{}$

input
```
int(csc(x)^2*sec(x)/sin(2*x)^(1/2)/(-2+tan(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/480*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)/tan(1/2*x)^2*(-140*(tan(1/2*x)
*(tan(1/2*x)^2-1))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*EllipticF((tan(1/2*x)+1)^
(1/2),1/2*2^(1/2))*(tan(1/2*x)+1)^(1/2)*(-tan(1/2*x))^(1/2)*tan(1/2*x)+240
*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*EllipticE((ta
n(1/2*x)+1)^(1/2),1/2*2^(1/2))*(tan(1/2*x)+1)^(1/2)*(-tan(1/2*x))^(1/2)*ta
n(1/2*x)+2^(1/2)*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^3-tan(1/2
*x))^(1/2)*sum((14*_alpha^3+3*_alpha^2+14*_alpha-11)*(_alpha^3+2*_alpha-3)
*(tan(1/2*x)+1)^(1/2)*(1-tan(1/2*x))^(1/2)*(-tan(1/2*x))^(1/2)/(tan(1/2*x)
*(tan(1/2*x)^2-1))^(1/2)*EllipticPi((tan(1/2*x)+1)^(1/2),-1/4*_alpha^3-1/2
*_alpha+3/4,1/2*2^(1/2)),_alpha=RootOf(_Z^4+_Z^3+2*_Z^2-_Z+1))*tan(1/2*x)+
40*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*tan(1/2*x)^4+120*tan(1/2*x)^3*(tan(
1/2*x)^3-tan(1/2*x))^(1/2)-120*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*tan(1/2*x)-
40*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2))/(tan(1/2*x)^3-tan(1/2*x))^(1/2)
```

3.634.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(50) = 100.

Time = 0.26 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.74

$$\int \frac{\csc^2(x)\sec(x)}{\sqrt{\sin(2x)}(-2+\tan(x))}\,dx = -\frac{4\sqrt{2}\sqrt{\cos(x)\sin(x)}(2\cos(x)+3\sin(x)) - 4\cos(x)^2 - 15\left(\cos(x)^2-1\right)\log\left(-\frac{1}{2}\sqrt{2}\sqrt{\cos(x)\sin(x)}\right.}{}$$

input
```
integrate(csc(x)^2*sec(x)/sin(2*x)^(1/2)/(-2+tan(x)),x, algorithm="fricas"
)
```

output
```
-1/48*(4*sqrt(2)*sqrt(cos(x)*sin(x))*(2*cos(x) + 3*sin(x)) - 4*cos(x)^2 -
15*(cos(x)^2 - 1)*log(-1/2*sqrt(2)*sqrt(cos(x)*sin(x))*(4*cos(x) + 3*sin(x
)) + 1/2*cos(x)^2 + 7/2*cos(x)*sin(x) + 1/2) + 15*(cos(x)^2 - 1)*log(1/2*c
os(x)^2 + 1/2*sqrt(2)*sqrt(cos(x)*sin(x))*sin(x) - 1/2*cos(x)*sin(x) + 1/2
) + 4)/(cos(x)^2 - 1)
```

3.634.6 Sympy [F]

$$\int \frac{\csc^2(x)\sec(x)}{\sqrt{\sin(2x)}(-2+\tan(x))}\,dx = \int \frac{\csc^2(x)\sec(x)}{(\tan(x)-2)\sqrt{\sin(2x)}}\,dx$$

input
```
integrate(csc(x)**2*sec(x)/sin(2*x)**(1/2)/(-2+tan(x)),x)
```

output
```
Integral(csc(x)**2*sec(x)/((tan(x) - 2)*sqrt(sin(2*x))), x)
```

3.634.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\csc^2(x)\sec(x)}{\sqrt{\sin(2x)}(-2+\tan(x))}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(csc(x)^2*sec(x)/sin(2*x)^(1/2)/(-2+tan(x)),x, algorithm="maxima"
)
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

3.634.8 Giac [F]

$$\int \frac{\csc^2(x)\sec(x)}{\sqrt{\sin(2x)}(-2+\tan(x))}\,dx = \int \frac{\csc(x)^2\sec(x)}{(\tan(x)-2)\sqrt{\sin(2x)}}\,dx$$

input
```
integrate(csc(x)^2*sec(x)/sin(2*x)^(1/2)/(-2+tan(x)),x, algorithm="giac")
```

output
```
integrate(csc(x)^2*sec(x)/((tan(x) - 2)*sqrt(sin(2*x))), x)
```

3.634.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\csc^2(x)\sec(x)}{\sqrt{\sin(2x)}(-2+\tan(x))}\,dx = \int \frac{1}{\sqrt{\sin(2x)}\,\cos(x)\,\sin(x)^2\,(\tan(x)-2)}\,dx$$

input
```
int(1/(sin(2*x)^(1/2)*cos(x)*sin(x)^2*(tan(x) - 2)),x)
```

output
```
int(1/(sin(2*x)^(1/2)*cos(x)*sin(x)^2*(tan(x) - 2)), x)
```

3.635 $\int \frac{\cos^2(x)\sin(x)}{(\sin^2(x)-\sin(2x))\sin^{\frac{5}{2}}(2x)}\,dx$

3.635.1 Optimal result

Integrand size = 28, antiderivative size = 79

$$\int \frac{\cos^2(x)\sin(x)}{(\sin^2(x)-\sin(2x))\sin^{\frac{5}{2}}(2x)}\,dx = \frac{\cos^4(x)\sin(x)}{3\sin^{\frac{5}{2}}(2x)}+\frac{\cos^3(x)\sin^2(x)}{2\sin^{\frac{5}{2}}(2x)} - \frac{5\text{arctanh}\left(\frac{\sqrt{\tan(x)}}{\sqrt{2}}\right)\sin^5(x)}{2\sqrt{2}\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}$$

output
```
1/3*cos(x)^4*sin(x)/sin(2*x)^(5/2)+1/2*cos(x)^3*sin(x)^2/sin(2*x)^(5/2)-5/
4*arctanh(1/2*tan(x)^(1/2)*2^(1/2))*sin(x)^5/sin(2*x)^(5/2)*2^(1/2)/tan(x)
^(5/2)
```

3.635.2 Mathematica [A] (verified)

Time = 2.23 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.54

$$\int \frac{\cos^2(x)\sin(x)}{(\sin^2(x)-\sin(2x))\sin^{\frac{5}{2}}(2x)}\,dx$$

$$= \frac{\csc^2\left(\frac{x}{2}\right)\sqrt{\sin(2x)}\left(-15\arctan\left(\frac{\sqrt{\tan\left(\frac{x}{2}\right)}}{\sqrt{-1+\tan^2\left(\frac{x}{2}\right)}}\right)(-1+\cos(x))+\sqrt{2}\sqrt{-\frac{\cos(x)}{1+\cos(x)}}(2\cos(x)+3\sin(x))\sqrt{\tan\cdots}\right.}{96(1+\cos(x))\sqrt{\tan\left(\frac{x}{2}\right)}\sqrt{-1+\tan^2\left(\frac{x}{2}\right)}}$$

input
```
Integrate[(Cos[x]^2*Sin[x])/((Sin[x]^2 - Sin[2*x])*Sin[2*x]^(5/2)),x]
```

output
```
(Csc[x/2]^2*Sqrt[Sin[2*x]]*(-15*ArcTan[Sqrt[Tan[x/2]]/Sqrt[-1 + Tan[x/2]^2
]]*(-1 + Cos[x]) + Sqrt[2]*Sqrt[-(Cos[x]/(1 + Cos[x]))]*(2*Cos[x] + 3*Sin[
x])*Sqrt[Tan[x/2]]))/(96*(1 + Cos[x])*Sqrt[Tan[x/2]]*Sqrt[-1 + Tan[x/2]^2]
)
```

3.635.3 Rubi [A] (warning: unable to verify)

Time = 0.72 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.72, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 4890, 4889, 25, 518, 1585, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)\cos^2(x)}{\left(\sin^2(x)-\sin(2x)\right)\sin^{\frac{5}{2}}(2x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)\cos(x)^2}{\left(\sin(x)^2-\sin(2x)\right)\sin(2x)^{5/2}}dx$$

$$\Big\downarrow \text{4890}$$

$$\frac{\sin^5(x)\int \frac{\cos(x)^2\csc^5(x)\sin(x)\tan^{\frac{5}{2}}(x)}{\sin(x)^2-\sin(2x)}dx}{\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}$$

$$\Big\downarrow \text{4889}$$

$$\frac{\sin^5(x)\int -\frac{\tan^2(x)+1}{(2-\tan(x))\tan^{\frac{5}{2}}(x)}d\tan(x)}{\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sin^5(x)\int \frac{\tan^2(x)+1}{(2-\tan(x))\tan^{\frac{5}{2}}(x)}d\tan(x)}{\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}$$

$$\Big\downarrow \text{518}$$

$$-\frac{2\sin^5(x)\int \frac{\cot^4(x)\left(\tan^2(x)+1\right)}{2-\tan(x)}d\sqrt{\tan(x)}}{\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}$$

$$\Big\downarrow \; 1585$$

$$-\frac{2\sin^5(x)\int\left(\frac{\cot^4(x)}{2}+\frac{\cot^2(x)}{4}-\frac{5}{4(\tan(x)-2)}\right)d\sqrt{\tan(x)}}{\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}$$

$$\Big\downarrow \; 2009$$

$$-\frac{2\sin^5(x)\left(\frac{5\text{arctanh}\left(\frac{\sqrt{\tan(x)}}{\sqrt{2}}\right)}{4\sqrt{2}}-\frac{1}{6}\cot^3(x)-\frac{\cot(x)}{4}\right)}{\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}$$

input
```
Int[(Cos[x]^2*Sin[x])/((Sin[x]^2 - Sin[2*x])*Sin[2*x]^(5/2)),x]
```

output
```
(-2*((5*ArcTanh[Sqrt[Tan[x]]/Sqrt[2]])/(4*Sqrt[2]) - Cot[x]/4 - Cot[x]^3/6
)*Sin[x]^5)/(Sin[2*x]^(5/2)*Tan[x]^(5/2))
```

3.635.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 518
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.),
 x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(2*m + 1)*(e*c + d*x^2)^
n*(a*e^2 + b*x^4)^p, x], x, Sqrt[e*x]], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1585
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.635. $\int \frac{\cos^2(x)\sin(x)}{(\sin^2(x)-\sin(2x))\sin^{\frac{5}{2}}(2x)}\,dx$

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

rule 4890
```
Int[(u_)*((c_.)*sin[v_])^(m_), x_Symbol] :> With[{w = FunctionOfTrig[u*(Sin
[v/2]^(2*m)/(c*Tan[v/2])^m), x]}, Simp[(c*Sin[v])^m*((c*Tan[v/2])^m/Sin[v/2
]^(2*m))   Int[u*(Sin[v/2]^(2*m)/(c*Tan[v/2])^m), x], x] /;  !FalseQ[w] &&
FunctionOfQ[NonfreeFactors[Tan[w], x], u*(Sin[v/2]^(2*m)/(c*Tan[v/2])^m), x
]] /; FreeQ[c, x] && LinearQ[v, x] && IntegerQ[m + 1/2] &&  !SumQ[u] && Inv
erseFunctionFreeQ[u, x]
```

3.635.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.90 (sec) , antiderivative size = 396, normalized size of antiderivative = 5.01

method	result
default	$-\frac{\sqrt{-\frac{\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)^{2}-1}}\left(-140\sqrt{\tan\left(\frac{x}{2}\right)\left(\tan\left(\frac{x}{2}\right)^{2}-1\right)}\,\sqrt{-2\tan\left(\frac{x}{2}\right)+2}\,\text{EllipticF}\left(\sqrt{\tan\left(\frac{x}{2}\right)+1},\frac{\sqrt{2}}{2}\right)\sqrt{\tan\left(\frac{x}{2}\right)+1}\,\sqrt{-\tan\left(\frac{x}{2}\right)}\,\tan\left(\frac{x}{2}\right)+2\right.}{}$

input
```
int(cos(x)^2*sin(x)/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x,method=_RETURNVER
BOSE)
```

output
```
-1/1920*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)/tan(1/2*x)^2*(-140*(tan(1/2*x
)*(tan(1/2*x)^2-1))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*EllipticF((tan(1/2*x)+1)
^(1/2),1/2*2^(1/2))*(tan(1/2*x)+1)^(1/2)*(-tan(1/2*x))^(1/2)*tan(1/2*x)+24
0*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*EllipticE((t
an(1/2*x)+1)^(1/2),1/2*2^(1/2))*(tan(1/2*x)+1)^(1/2)*(-tan(1/2*x))^(1/2)*t
an(1/2*x)+2^(1/2)*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^3-tan(1/
2*x))^(1/2)*sum((14*_alpha^3+3*_alpha^2+14*_alpha-11)*(_alpha^3+2*_alpha-3
)*(tan(1/2*x)+1)^(1/2)*(1-tan(1/2*x))^(1/2)*(-tan(1/2*x))^(1/2)/(tan(1/2*x
)*(tan(1/2*x)^2-1))^(1/2)*EllipticPi((tan(1/2*x)+1)^(1/2),-1/4*_alpha^3-1/
2*_alpha+3/4,1/2*2^(1/2)),_alpha=RootOf(_Z^4+_Z^3+2*_Z^2-_Z+1))*tan(1/2*x)
+40*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*tan(1/2*x)^4+120*tan(1/2*x)^3*(tan
(1/2*x)^3-tan(1/2*x))^(1/2)-120*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*tan(1/2*x)
-40*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2))/(tan(1/2*x)^3-tan(1/2*x))^(1/2)
```

3.635.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.52

$$\int \frac{\cos^2(x)\sin(x)}{(\sin^2(x)-\sin(2x))\sin^{\frac{5}{2}}(2x)}\,dx =$$
$$-\frac{4\sqrt{2}\sqrt{\cos(x)\sin(x)}(2\cos(x)+3\sin(x))-4\cos(x)^2-15\left(\cos(x)^2-1\right)\log\left(-\frac{1}{2}\sqrt{2}\sqrt{\cos(x)\sin(x)}\right.}{}$$

input
```
integrate(cos(x)^2*sin(x)/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x, algorithm=
"fricas")
```

output
```
-1/192*(4*sqrt(2)*sqrt(cos(x)*sin(x))*(2*cos(x) + 3*sin(x)) - 4*cos(x)^2 -
 15*(cos(x)^2 - 1)*log(-1/2*sqrt(2)*sqrt(cos(x)*sin(x))*(4*cos(x) + 3*sin(
x)) + 1/2*cos(x)^2 + 7/2*cos(x)*sin(x) + 1/2) + 15*(cos(x)^2 - 1)*log(1/2*
cos(x)^2 + 1/2*sqrt(2)*sqrt(cos(x)*sin(x))*sin(x) - 1/2*cos(x)*sin(x) + 1/
2) + 4)/(cos(x)^2 - 1)
```

3.635.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(x)\sin(x)}{(\sin^2(x)-\sin(2x))\sin^{\frac{5}{2}}(2x)}\,dx = \text{Timed out}$$

input
```
integrate(cos(x)**2*sin(x)/(sin(x)**2-sin(2*x))/sin(2*x)**(5/2),x)
```

output
```
Timed out
```

3.635.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\cos^2(x)\sin(x)}{(\sin^2(x)-\sin(2x))\sin^{\frac{5}{2}}(2x)}\,dx = \text{Timed out}$$

input
```
integrate(cos(x)^2*sin(x)/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x, algorithm=
"maxima")
```

output
```
Timed out
```

3.635.8 Giac [F]

$$\int \frac{\cos^2(x)\sin(x)}{(\sin^2(x)-\sin(2x))\sin^{\frac{5}{2}}(2x)}\,dx = \int \frac{\cos(x)^2\sin(x)}{(\sin(x)^2-\sin(2x))\sin(2x)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(x)^2*sin(x)/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x, algorithm=
"giac")
```

output
```
integrate(cos(x)^2*sin(x)/((sin(x)^2 - sin(2*x))*sin(2*x)^(5/2)), x)
```

3.635.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(x)\sin(x)}{(\sin^2(x)-\sin(2x))\sin^{\frac{5}{2}}(2x)}\,dx = -\int \frac{\cos(x)^2\,\sin(x)}{\sin(2\,x)^{5/2}\,\left(\sin(2\,x)-\sin(x)^2\right)}\,dx$$

input
```
int(-(cos(x)^2*sin(x))/(sin(2*x)^(5/2)*(sin(2*x) - sin(x)^2)),x)
```

output
```
-int((cos(x)^2*sin(x))/(sin(2*x)^(5/2)*(sin(2*x) - sin(x)^2)), x)
```

3.636 $\int \frac{\cos^3(x)\cos(2x)}{(\sin^2(x)-\sin(2x))\sin^{\frac{5}{2}}(2x)}\,dx$

3.636.1 Optimal result

Integrand size = 30, antiderivative size = 95

$$\int \frac{\cos^3(x)\cos(2x)}{(\sin^2(x)-\sin(2x))\sin^{\frac{5}{2}}(2x)}\,dx=\frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)}+\frac{\cos^4(x)\sin(x)}{6\sin^{\frac{5}{2}}(2x)}-\frac{3\cos^3(x)\sin^2(x)}{4\sin^{\frac{5}{2}}(2x)}+\frac{3\text{arctanh}\left(\frac{\sqrt{\tan(x)}}{\sqrt{2}}\right)\sin^5(x)}{4\sqrt{2}\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}$$

output
```
1/5*cos(x)^5/sin(2*x)^(5/2)+1/6*cos(x)^4*sin(x)/sin(2*x)^(5/2)-3/4*cos(x)^
3*sin(x)^2/sin(2*x)^(5/2)+3/8*arctanh(1/2*tan(x)^(1/2)*2^(1/2))*sin(x)^5/s
in(2*x)^(5/2)*2^(1/2)/tan(x)^(5/2)
```

3.636.2 Mathematica [A] (verified)

Time = 4.51 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.97

$$\int \frac{\cos^3(x)\cos(2x)}{(\sin^2(x)-\sin(2x))\sin^{\frac{5}{2}}(2x)}\,dx=\frac{\sec(x)\sqrt{\sin(2x)}\left(4\cot(x)\csc^2(x)(-33+57\cos(2x)+10\sin(2x))+\frac{180\sqrt{2}\arctan\left(\frac{\sqrt{\tan\left(\frac{x}{2}\right)}}{\sqrt{-1+\tan^2\left(\frac{x}{2}\right)}}\right)\sqrt{-\frac{1}{1+\sec(x)}}}{\sqrt{\tan\left(\frac{x}{2}\right)}}\right)}{3840}$$

input
```
Integrate[(Cos[x]^3*Cos[2*x])/((Sin[x]^2 - Sin[2*x])*Sin[2*x]^(5/2)),x]
```

output
```
(Sec[x]*Sqrt[Sin[2*x]]*(4*Cot[x]*Csc[x]^2*(-33 + 57*Cos[2*x] + 10*Sin[2*x]
) + (180*Sqrt[2]*ArcTan[Sqrt[Tan[x/2]]/Sqrt[-1 + Tan[x/2]^2]]*Sqrt[-(1 + S
ec[x])^(-1)])/Sqrt[Tan[x/2]]))/3840
```

3.636.3 Rubi [A] (warning: unable to verify)

Time = 0.77 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.68, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {3042, 4890, 4889, 25, 518, 1585, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(x)\cos(2x)}{(\sin^2(x)-\sin(2x))\sin^{\frac{5}{2}}(2x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(x)^3\cos(2x)}{(\sin(x)^2-\sin(2x))\sin(2x)^{5/2}}dx$$

$$\Big\downarrow \text{4890}$$

$$\frac{\sin^5(x)\int \frac{\cos(x)^3\cos(2x)\csc^5(x)\tan^{\frac{5}{2}}(x)}{\sin(x)^2-\sin(2x)}dx}{\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}$$

$$\Big\downarrow \text{4889}$$

$$\frac{\sin^5(x)\int -\frac{1-\tan^2(x)}{(2-\tan(x))\tan^{\frac{7}{2}}(x)}d\tan(x)}{\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sin^5(x)\int \frac{1-\tan^2(x)}{(2-\tan(x))\tan^{\frac{7}{2}}(x)}d\tan(x)}{\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}$$

$$\Big\downarrow \text{518}$$

$$-\frac{2\sin^5(x)\int \frac{\cot^6(x)(1-\tan^2(x))}{2-\tan(x)}d\sqrt{\tan(x)}}{\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}$$

$$\Big\downarrow \text{1585}$$

$$-\frac{2\sin^5(x)\int\left(\frac{\cot^6(x)}{2}+\frac{\cot^4(x)}{4}-\frac{3\cot^2(x)}{8}+\frac{3}{8(\tan(x)-2)}\right)d\sqrt{\tan(x)}}{\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\sin^5(x)\left(-\frac{3\text{arctanh}\left(\frac{\sqrt{\tan(x)}}{\sqrt{2}}\right)}{8\sqrt{2}}-\frac{1}{10}\cot^5(x)-\frac{\cot^3(x)}{12}+\frac{3\cot(x)}{8}\right)}{\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}$$

input
```
Int[(Cos[x]^3*Cos[2*x])/((Sin[x]^2 - Sin[2*x])*Sin[2*x]^(5/2)),x]
```

output
```
(-2*((-3*ArcTanh[Sqrt[Tan[x]]/Sqrt[2]])/(8*Sqrt[2]) + (3*Cot[x])/8 - Cot[x
]^3/12 - Cot[x]^5/10)*Sin[x]^5)/(Sin[2*x]^(5/2)*Tan[x]^(5/2))
```

3.636.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 518
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.),
 x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(2*m + 1)*(e*c + d*x^2)^
n*(a*e^2 + b*x^4)^p, x], x, Sqrt[e*x]], x] /; FreeQ[{a, b, c, d, e}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1585
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.636. $\int \frac{\cos^3(x)\cos(2x)}{(\sin^2(x)-\sin(2x))\sin^{\frac{5}{2}}(2x)}\,dx$

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

rule 4890
```
Int[(u_)*((c_.)*sin[v_])^(m_), x_Symbol] :> With[{w = FunctionOfTrig[u*(Sin
[v/2]^(2*m)/(c*Tan[v/2])^m), x]}, Simp[(c*Sin[v])^m*((c*Tan[v/2])^m/Sin[v/2
]^(2*m))   Int[u*(Sin[v/2]^(2*m)/(c*Tan[v/2])^m), x], x] /;  !FalseQ[w] &&
FunctionOfQ[NonfreeFactors[Tan[w], x], u*(Sin[v/2]^(2*m)/(c*Tan[v/2])^m), x
]] /; FreeQ[c, x] && LinearQ[v, x] && IntegerQ[m + 1/2] &&  !SumQ[u] && Inv
erseFunctionFreeQ[u, x]
```

3.636.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.83 (sec) , antiderivative size = 761, normalized size of antiderivative = 8.01

method	result	size
default	Expression too large to display	761

input
```
int(cos(x)^3*cos(2*x)/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x,method=_RETURNV
ERBOSE)
```

output
```
1/3840*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)/tan(1/2*x)^3*(-4464*(tan(1/2*x
)+1)^(1/2)*(-tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*EllipticE((tan(1/2*
x)+1)^(1/2),1/2*2^(1/2))*(tan(1/2*x)*(tan(1/2*x)-1)*(tan(1/2*x)+1))^(1/2)*
(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*tan(1/2*x)^2+1772*(tan(1/2*x)+1)^(1/2)
*(-tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*EllipticF((tan(1/2*x)+1)^(1/2
),1/2*2^(1/2))*(tan(1/2*x)*(tan(1/2*x)-1)*(tan(1/2*x)+1))^(1/2)*(tan(1/2*x
)*(tan(1/2*x)^2-1))^(1/2)*tan(1/2*x)^2+24*(tan(1/2*x)*(tan(1/2*x)-1)*(tan(
1/2*x)+1))^(1/2)*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*tan(1/2*x)^6+3*2^(1/2
)*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*(tan(1/2*x)*(tan(1/2*x)-1)*(tan(1/2*x)+1
))^(1/2)*sum((6*_alpha^3+7*_alpha^2+6*_alpha+1)*(_alpha^3+2*_alpha-3)*(tan
(1/2*x)+1)^(1/2)*(1-tan(1/2*x))^(1/2)*(-tan(1/2*x))^(1/2)*EllipticPi((tan(
1/2*x)+1)^(1/2),-1/4*_alpha^3-1/2*_alpha+3/4,1/2*2^(1/2))/(tan(1/2*x)*(tan
(1/2*x)^2-1))^(1/2),_alpha=RootOf(_Z^4+_Z^3+2*_Z^2-_Z+1))*(tan(1/2*x)*(tan
(1/2*x)^2-1))^(1/2)*tan(1/2*x)^2-40*(tan(1/2*x)*(tan(1/2*x)-1)*(tan(1/2*x)
+1))^(1/2)*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*tan(1/2*x)^5-1272*tan(1/2*x
)^4*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*(tan(1/2*x)*(tan(1/2*x)-1)*(tan(1/2*x)
+1))^(1/2)-1920*tan(1/2*x)^4*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*(tan(1/2*x)*(
tan(1/2*x)^2-1))^(1/2)-24*tan(1/2*x)^4*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)
*(tan(1/2*x)*(tan(1/2*x)-1)*(tan(1/2*x)+1))^(1/2)+1272*(tan(1/2*x)^3-tan(1
/2*x))^(1/2)*(tan(1/2*x)*(tan(1/2*x)-1)*(tan(1/2*x)+1))^(1/2)*tan(1/2*x...
```

3.636.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.43

$$\int \frac{\cos^3(x)\cos(2x)}{(\sin^2(x)-\sin(2x))\sin^{\frac{5}{2}}(2x)}\,dx =$$
$$-\frac{45\left(\cos(x)^2-1\right)\log\left(-\frac{1}{2}\sqrt{2}\sqrt{\cos(x)\sin(x)}(4\cos(x)+3\sin(x))+\frac{1}{2}\cos(x)^2+\frac{7}{2}\cos(x)\sin(x)+\frac{1}{2}\right.}{}$$

input
```
integrate(cos(x)^3*cos(2*x)/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x, algorith
m="fricas")
```

output
```
-1/1920*(45*(cos(x)^2 - 1)*log(-1/2*sqrt(2)*sqrt(cos(x)*sin(x))*(4*cos(x)
+ 3*sin(x)) + 1/2*cos(x)^2 + 7/2*cos(x)*sin(x) + 1/2)*sin(x) - 45*(cos(x)^
2 - 1)*log(1/2*cos(x)^2 + 1/2*sqrt(2)*sqrt(cos(x)*sin(x))*sin(x) - 1/2*cos
(x)*sin(x) + 1/2)*sin(x) + 4*sqrt(2)*(57*cos(x)^2 + 10*cos(x)*sin(x) - 45)
*sqrt(cos(x)*sin(x)) + 268*(cos(x)^2 - 1)*sin(x))/((cos(x)^2 - 1)*sin(x))
```

3.636.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(x)\cos(2x)}{(\sin^2(x)-\sin(2x))\sin^{\frac{5}{2}}(2x)}\,dx = \text{Timed out}$$

input
```
integrate(cos(x)**3*cos(2*x)/(sin(x)**2-sin(2*x))/sin(2*x)**(5/2),x)
```

output
```
Timed out
```

3.636.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\cos^3(x)\cos(2x)}{(\sin^2(x)-\sin(2x))\sin^{\frac{5}{2}}(2x)}\,dx = \text{Timed out}$$

input
```
integrate(cos(x)^3*cos(2*x)/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x, algorith
m="maxima")
```

output
```
Timed out
```

3.636.8 Giac [F]

$$\int \frac{\cos^3(x)\cos(2x)}{(\sin^2(x)-\sin(2x))\sin^{\frac{5}{2}}(2x)}\,dx = \int \frac{\cos(2x)\cos(x)^3}{(\sin(x)^2-\sin(2x))\sin(2x)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cos(x)^3*cos(2*x)/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x, algorith
m="giac")
```

output
```
integrate(cos(2*x)*cos(x)^3/((sin(x)^2 - sin(2*x))*sin(2*x)^(5/2)), x)
```

3.636.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(x)\cos(2x)}{(\sin^2(x)-\sin(2x))\sin^{\frac{5}{2}}(2x)}\,dx = \int -\frac{\cos(2x)\cos(x)^3}{\sin(2x)^{5/2}\left(\sin(2x)-\sin(x)^2\right)}\,dx$$

input
```
int(-(cos(2*x)*cos(x)^3)/(sin(2*x)^(5/2)*(sin(2*x) - sin(x)^2)),x)
```

output
```
int(-(cos(2*x)*cos(x)^3)/(sin(2*x)^(5/2)*(sin(2*x) - sin(x)^2)), x)
```

3.637 $\int (b\sec(c+dx)+a\sin(c+dx))^n(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx$

3.637.1 Optimal result

Integrand size = 43, antiderivative size = 30

$$\int (b\sec(c+dx)+a\sin(c+dx))^n(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx = \frac{(b\sec(c+dx)+a\sin(c+dx))^{1+n}}{d(1+n)}$$

output
```
(b*sec(d*x+c)+a*sin(d*x+c))^(1+n)/d/(1+n)
```

3.637.2 Mathematica [A] (verified)

Time = 5.45 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (b\sec(c+dx)+a\sin(c+dx))^n(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx = \frac{(b\sec(c+dx)+a\sin(c+dx))^{1+n}}{d(1+n)}$$

input
```
Integrate[(b*Sec[c + d*x] + a*Sin[c + d*x])^n*(a*Cos[c + d*x] + b*Sec[c +
d*x]*Tan[c + d*x]),x]
```

output
```
(b*Sec[c + d*x] + a*Sin[c + d*x])^(1 + n)/(d*(1 + n))
```

3.637.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.047, Rules used = {3042, 4885}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(c+dx)+b\sec(c+dx))^n(a\cos(c+dx)+b\tan(c+dx)\sec(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(c+dx)+b\sec(c+dx))^n(a\cos(c+dx)+b\tan(c+dx)\sec(c+dx))dx$$

$$\Big\downarrow \text{4885}$$

$$\frac{(a\sin(c+dx)+b\sec(c+dx))^{n+1}}{d(n+1)}$$

input
```
Int[(b*Sec[c + d*x] + a*Sin[c + d*x])^n*(a*Cos[c + d*x] + b*Sec[c + d*x]*T
an[c + d*x]),x]
```

output
```
(b*Sec[c + d*x] + a*Sin[c + d*x])^(1 + n)/(d*(1 + n))
```

3.637.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4885
```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[ActivateTrig[
y], ActivateTrig[u], x]}, Simp[q*(ActivateTrig[y^(m + 1)]/(m + 1)), x] /;
!FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1] &&  !InertTrigFreeQ[u]
```

3.637.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\frac{(b\sec(dx+c)+\sin(dx+c)\,a)^{n+1}}{d\,(n+1)}$$

input
```
int((b*sec(d*x+c)+sin(d*x+c)*a)^n*(cos(d*x+c)*a+b*sec(d*x+c)*tan(d*x+c)),x
)
```

output
```
(b*sec(d*x+c)+sin(d*x+c)*a)^(n+1)/d/(n+1)
```

3.637.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.97

$$\int (b\sec(c+dx)+a\sin(c+dx))^n(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx = \frac{(a\cos(dx+c)\sin(dx+c)+b)\left(\frac{a\cos(dx+c)\sin(dx+c)+b}{\cos(dx+c)}\right)^n}{(dn+d)\cos(dx+c)}$$

input
```
integrate((b*sec(d*x+c)+a*sin(d*x+c))^n*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x
+c)),x, algorithm="fricas")
```

output
```
(a*cos(d*x + c)*sin(d*x + c) + b)*((a*cos(d*x + c)*sin(d*x + c) + b)/cos(d
*x + c))^n/((d*n + d)*cos(d*x + c))
```

3.637.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(24) = 48$.

Time = 18.29 (sec) , antiderivative size = 138, normalized size of antiderivative = 4.60

$$\int (b\sec(c+dx)+a\sin(c+dx))^n(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx = \begin{cases} \frac{x(a\cos(c)+b\tan(c)\sec(c))}{a\sin(c)+b\sec(c)} & \text{for}\ d=0\wedge n=-1 \\ x(a\sin(c)+b\sec(c))^n\,(a\cos(c)+b\tan(c)\sec(c)) & \text{for}\ d=0 \\ \frac{\log\left(\frac{a\sin(c+dx)}{b}+\sec(c+dx)\right)}{d} & \text{for}\ n=-1 \\ \frac{a(a\sin(c+dx)+b\sec(c+dx))^n\sin(c+dx)}{dn+d}+\frac{b(a\sin(c+dx)+b\sec(c+dx))^n\sec(c+dx)}{dn+d} & \text{otherwise} \end{cases}$$

input
```
integrate((b*sec(d*x+c)+a*sin(d*x+c))**n*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*
x+c)),x)
```

output
```
Piecewise((x*(a*cos(c) + b*tan(c)*sec(c))/(a*sin(c) + b*sec(c)), Eq(d, 0)
& Eq(n, -1)), (x*(a*sin(c) + b*sec(c))**n*(a*cos(c) + b*tan(c)*sec(c)), Eq
(d, 0)), (log(a*sin(c + d*x)/b + sec(c + d*x))/d, Eq(n, -1)), (a*(a*sin(c
+ d*x) + b*sec(c + d*x))**n*sin(c + d*x)/(d*n + d) + b*(a*sin(c + d*x) + b
*sec(c + d*x))**n*sec(c + d*x)/(d*n + d), True))
```

3.637.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (b\sec(c+dx)+a\sin(c+dx))^n(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx = \frac{(b\sec(dx+c)+a\sin(dx+c))^{n+1}}{d(n+1)}$$

input
```
integrate((b*sec(d*x+c)+a*sin(d*x+c))^n*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x
+c)),x, algorithm="maxima")
```

output
```
(b*sec(d*x + c) + a*sin(d*x + c))^(n + 1)/(d*(n + 1))
```

3.637.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. 2(30) = 60.

Time = 1.05 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.83

$$\int (b\sec(c+dx)+a\sin(c+dx))^n(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx = \frac{\left(-\frac{b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-1}\right)^{n+1}}{d(n+1)}$$

input
```
integrate((b*sec(d*x+c)+a*sin(d*x+c))^n*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x
+c)),x, algorithm="giac")
```

output
```
(-(b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^3 + 2*b*tan(1/2*d*x
 + 1/2*c)^2 + 2*a*tan(1/2*d*x + 1/2*c) + b)/(tan(1/2*d*x + 1/2*c)^4 - 1))^
(n + 1)/(d*(n + 1))
```

3.637.9 Mupad [B] (verification not implemented)

Time = 30.76 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.10

$$\int (b\sec(c+dx)+a\sin(c+dx))^n(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx$$
$$=\begin{cases} \frac{\ln\left(a\,\sin(c+d\,x)+\frac{b}{\cos(c+d\,x)}\right)}{d} & \text{if}\ \ n=-1 \\ \frac{\left(a\,\sin(c+d\,x)+\frac{b}{\cos(c+d\,x)}\right)^{n+1}}{d\,(n+1)} & \text{if}\ \ n\neq -1 \end{cases}$$

input
```
int((a*sin(c + d*x) + b/cos(c + d*x))^n*(a*cos(c + d*x) + (b*tan(c + d*x))
/cos(c + d*x)),x)
```

output
```
piecewise(n == -1, log(a*sin(c + d*x) + b/cos(c + d*x))/d, n ~= -1, (a*sin
(c + d*x) + b/cos(c + d*x))^(n + 1)/(d*(n + 1)))
```

3.638 $\int (b\sec(c+dx)+a\sin(c+dx))^3(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx$

3.638.1 Optimal result

Integrand size = 43, antiderivative size = 26

$$\int (b\sec(c+dx)+a\sin(c+dx))^3(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx = \frac{(b\sec(c+dx)+a\sin(c+dx))^4}{4d}$$

output
```
1/4*(b*sec(d*x+c)+a*sin(d*x+c))^4/d
```

3.638.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 938 vs. $2(26) = 52$.

Time = 12.23 (sec) , antiderivative size = 938, normalized size of antiderivative = 36.08

$$\int (b\sec(c+dx)+a\sin(c+dx))^3(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx$$
$$= \frac{8b^4\cos(c+dx)(b\sec(c+dx)+a\sin(c+dx))^3(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))}{d(3a\cos(c+dx)+a\cos(3c+3dx)+4b\sin(c+dx))(2b+a\sin(2c+2dx))^3}$$
$$+ \frac{a^4\cos(4c)\cos(4dx)\cos^5(c+dx)(b\sec(c+dx)+a\sin(c+dx))^3(a\cos(c+dx)+b\sec(c+dx)\tan(c+d}{d(3a\cos(c+dx)+a\cos(3c+3dx)+4b\sin(c+dx))(2b+a\sin(2c+2dx))^3}$$
$$+ \frac{16ab^2\cos^3(c+dx)\sec(c)(3a\cos(c)+2b\sin(c))(b\sec(c+dx)+a\sin(c+dx))^3(a\cos(c+dx)+b\sec(c+}{d(3a\cos(c+dx)+a\cos(3c+3dx)+4b\sin(c+dx))(2b+a\sin(2c+2dx))^3}$$
$$- \frac{4a^3\cos(2dx)\cos^5(c+dx)(a\cos(2c)+4b\sin(2c))(b\sec(c+dx)+a\sin(c+dx))^3(a\cos(c+dx)+b\sec(c}{d(3a\cos(c+dx)+a\cos(3c+3dx)+4b\sin(c+dx))(2b+a\sin(2c+2dx))^3}$$
$$+ \frac{32ab^3\cos^2(c+dx)\sec(c)\sin(dx)(b\sec(c+dx)+a\sin(c+dx))^3(a\cos(c+dx)+b\sec(c+dx)\tan(c+}{d(3a\cos(c+dx)+a\cos(3c+3dx)+4b\sin(c+dx))(2b+a\sin(2c+2dx))^3}$$
$$+ \frac{32a^3b\cos^4(c+dx)\sec(c)\sin(dx)(b\sec(c+dx)+a\sin(c+dx))^3(a\cos(c+dx)+b\sec(c+dx)\tan(c+}{d(3a\cos(c+dx)+a\cos(3c+3dx)+4b\sin(c+dx))(2b+a\sin(2c+2dx))^3}$$
$$+ \frac{4a^3\cos^5(c+dx)(-4b\cos(2c)+a\sin(2c))\sin(2dx)(b\sec(c+dx)+a\sin(c+dx))^3(a\cos(c+dx)+b\sec}{d(3a\cos(c+dx)+a\cos(3c+3dx)+4b\sin(c+dx))(2b+a\sin(2c+2dx))^3}$$
$$- \frac{a^4\cos^5(c+dx)\sin(4c)\sin(4dx)(b\sec(c+dx)+a\sin(c+dx))^3(a\cos(c+dx)+b\sec(c+dx)\tan(c+d}{d(3a\cos(c+dx)+a\cos(3c+3dx)+4b\sin(c+dx))(2b+a\sin(2c+2dx))^3}$$

input
```
Integrate[(b*Sec[c + d*x] + a*Sin[c + d*x])^3*(a*Cos[c + d*x] + b*Sec[c + 
d*x]*Tan[c + d*x]),x]
```

output
```
(8*b^4*Cos[c + d*x]*(b*Sec[c + d*x] + a*Sin[c + d*x])^3*(a*Cos[c + d*x] +
b*Sec[c + d*x]*Tan[c + d*x]))/(d*(3*a*Cos[c + d*x] + a*Cos[3*c + 3*d*x] +
4*b*Sin[c + d*x])*(2*b + a*Sin[2*c + 2*d*x])^3) + (a^4*Cos[4*c]*Cos[4*d*x]
*Cos[c + d*x]^5*(b*Sec[c + d*x] + a*Sin[c + d*x])^3*(a*Cos[c + d*x] + b*Se
c[c + d*x]*Tan[c + d*x]))/(d*(3*a*Cos[c + d*x] + a*Cos[3*c + 3*d*x] + 4*b*
Sin[c + d*x])*(2*b + a*Sin[2*c + 2*d*x])^3) + (16*a*b^2*Cos[c + d*x]^3*Sec
[c]*(3*a*Cos[c] + 2*b*Sin[c])*(b*Sec[c + d*x] + a*Sin[c + d*x])^3*(a*Cos[c
 + d*x] + b*Sec[c + d*x]*Tan[c + d*x]))/(d*(3*a*Cos[c + d*x] + a*Cos[3*c +
 3*d*x] + 4*b*Sin[c + d*x])*(2*b + a*Sin[2*c + 2*d*x])^3) - (4*a^3*Cos[2*d
*x]*Cos[c + d*x]^5*(a*Cos[2*c] + 4*b*Sin[2*c])*(b*Sec[c + d*x] + a*Sin[c +
 d*x])^3*(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x]))/(d*(3*a*Cos[c + d
*x] + a*Cos[3*c + 3*d*x] + 4*b*Sin[c + d*x])*(2*b + a*Sin[2*c + 2*d*x])^3)
 + (32*a*b^3*Cos[c + d*x]^2*Sec[c]*Sin[d*x]*(b*Sec[c + d*x] + a*Sin[c + d*
x])^3*(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x]))/(d*(3*a*Cos[c + d*x]
 + a*Cos[3*c + 3*d*x] + 4*b*Sin[c + d*x])*(2*b + a*Sin[2*c + 2*d*x])^3) +
(32*a^3*b*Cos[c + d*x]^4*Sec[c]*Sin[d*x]*(b*Sec[c + d*x] + a*Sin[c + d*x])
^3*(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x]))/(d*(3*a*Cos[c + d*x] +
a*Cos[3*c + 3*d*x] + 4*b*Sin[c + d*x])*(2*b + a*Sin[2*c + 2*d*x])^3) + (4*
a^3*Cos[c + d*x]^5*(-4*b*Cos[2*c] + a*Sin[2*c])*Sin[2*d*x]*(b*Sec[c + d*x]
 + a*Sin[c + d*x])^3*(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x]))/(d...
```

3.638.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.047, Rules used = {3042, 4885}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(c+dx)+b\sec(c+dx))^3(a\cos(c+dx)+b\tan(c+dx)\sec(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(c+dx)+b\sec(c+dx))^3(a\cos(c+dx)+b\tan(c+dx)\sec(c+dx))dx$$

$$\Big\downarrow \text{4885}$$

$$\frac{(a\sin(c+dx)+b\sec(c+dx))^4}{4d}$$

input
```
Int[(b*Sec[c + d*x] + a*Sin[c + d*x])^3*(a*Cos[c + d*x] + b*Sec[c + d*x]*T
an[c + d*x]),x]
```

output
```
(b*Sec[c + d*x] + a*Sin[c + d*x])^4/(4*d)
```

3.638.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4885
```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[ActivateTrig[
y], ActivateTrig[u], x]}, Simp[q*(ActivateTrig[y^(m + 1)]/(m + 1)), x] /;
!FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1] &&  !InertTrigFreeQ[u]
```

3.638.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. 2(24) = 48.

Time = 222.00 (sec) , antiderivative size = 187, normalized size of antiderivative = 7.19

method	result
derivativedivides	$\frac{\frac{a^4\sin(dx+c)^4}{4}+a^3b\left(\frac{\sin(dx+c)^5}{\cos(dx+c)}+\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)+3a^3b\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right.}{d}$
default	$\frac{\frac{a^4\sin(dx+c)^4}{4}+a^3b\left(\frac{\sin(dx+c)^5}{\cos(dx+c)}+\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)+3a^3b\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right.}{d}$
parts	$\frac{\frac{a^4\sin(dx+c)^4}{4}+3a^3b\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-3a^2b^2\ln(\cos(dx+c))+a\,b^3\tan(dx+c)}{d}+\frac{b^4\sec(dx+c)^4}{4d}+\frac{a^3b\left(\right.}{}$
risch	$\frac{a^4e^{4i(dx+c)}}{64d}+\frac{ia^3e^{2i(dx+c)}b}{4d}-\frac{a^4e^{2i(dx+c)}}{16d}-\frac{ia^3e^{-2i(dx+c)}b}{4d}-\frac{a^4e^{-2i(dx+c)}}{16d}+\frac{a^4e^{-4i(dx+c)}}{64d}+\frac{2b(ia^3e^{6i(dx}}{}$

input
```
int((b*sec(d*x+c)+sin(d*x+c)*a)^3*(cos(d*x+c)*a+b*sec(d*x+c)*tan(d*x+c)),x
,method=_RETURNVERBOSE)
```

output
```
1/d*(1/4*a^4*sin(d*x+c)^4+a^3*b*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2
*sin(d*x+c))*cos(d*x+c)-3/2*d*x-3/2*c)+3*a^3*b*(-1/2*sin(d*x+c)*cos(d*x+c)
+1/2*d*x+1/2*c)+3*a^2*b^2*(1/2*tan(d*x+c)^2+ln(cos(d*x+c)))-3*a^2*b^2*ln(c
os(d*x+c))+a*b^3*sin(d*x+c)^3/cos(d*x+c)^3+a*b^3*tan(d*x+c)+1/4*b^4/cos(d*
x+c)^4)
```

3.638.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(24) = 48$.

Time = 0.27 (sec) , antiderivative size = 122, normalized size of antiderivative = 4.69

$$\int (b\sec(c+dx)+a\sin(c+dx))^3(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx$$
$$= \frac{8\,a^4\cos\left(dx+c\right)^8-16\,a^4\cos\left(dx+c\right)^6+5\,a^4\cos\left(dx+c\right)^4+48\,a^2b^2\cos\left(dx+c\right)^2+8\,b^4-32\,\left(a^3b\cos\left(dx\right.\right.}{32\,d\cos\left(dx+c\right)^4}$$

input
```
integrate((b*sec(d*x+c)+a*sin(d*x+c))^3*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x
+c)),x, algorithm="fricas")
```

output
```
1/32*(8*a^4*cos(d*x + c)^8 - 16*a^4*cos(d*x + c)^6 + 5*a^4*cos(d*x + c)^4
+ 48*a^2*b^2*cos(d*x + c)^2 + 8*b^4 - 32*(a^3*b*cos(d*x + c)^5 - a^3*b*cos
(d*x + c)^3 - a*b^3*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4)
```

3.638.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(20) = 40$.

Time = 2.51 (sec) , antiderivative size = 129, normalized size of antiderivative = 4.96

$$\int (b\sec(c+dx)+a\sin(c+dx))^3(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx$$
$$= \begin{cases} \frac{a^4\sin^4\left(c+dx\right)}{4d}+\frac{a^3b\sin^3\left(c+dx\right)\sec\left(c+dx\right)}{d}+\frac{3a^2b^2\sin^2\left(c+dx\right)\sec^2\left(c+dx\right)}{2d}+\frac{ab^3\sin\left(c+dx\right)\sec^3\left(c+dx\right)}{d}+\frac{b^4\sec^4\left(c+dx\right)}{4d} & \text{for } d = \\ x(a\sin\left(c\right)+b\sec\left(c\right))^3\left(a\cos\left(c\right)+b\tan\left(c\right)\sec\left(c\right)\right) & \text{other} \end{cases}$$

input
```
integrate((b*sec(d*x+c)+a*sin(d*x+c))**3*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*
x+c)),x)
```

output
```
Piecewise((a**4*sin(c + d*x)**4/(4*d) + a**3*b*sin(c + d*x)**3*sec(c + d*x
)/d + 3*a**2*b**2*sin(c + d*x)**2*sec(c + d*x)**2/(2*d) + a*b**3*sin(c + d
*x)*sec(c + d*x)**3/d + b**4*sec(c + d*x)**4/(4*d), Ne(d, 0)), (x*(a*sin(c
) + b*sec(c))**3*(a*cos(c) + b*tan(c)*sec(c)), True))
```

3.638.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int (b\sec(c+dx)+a\sin(c+dx))^3(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx$$
$$=\frac{(b\sec(dx+c)+a\sin(dx+c))^4}{4\,d}$$

input
```
integrate((b*sec(d*x+c)+a*sin(d*x+c))^3*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x
+c)),x, algorithm="maxima")
```

output
```
1/4*(b*sec(d*x + c) + a*sin(d*x + c))^4/d
```

3.638.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. 2(24) = 48.

Time = 0.81 (sec) , antiderivative size = 142, normalized size of antiderivative = 5.46

$$\int (b\sec(c+dx)+a\sin(c+dx))^3(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx$$
$$=\frac{b^4\tan(dx+c)^4+4\,ab^3\tan(dx+c)^3+6\,a^2b^2\tan(dx+c)^2+2\,b^4\tan(dx+c)^2+4\,a^3b\tan(dx+c)+4\,a}{4\,d}$$

input
```
integrate((b*sec(d*x+c)+a*sin(d*x+c))^3*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x
+c)),x, algorithm="giac")
```

output
```
1/4*(b^4*tan(d*x + c)^4 + 4*a*b^3*tan(d*x + c)^3 + 6*a^2*b^2*tan(d*x + c)^
2 + 2*b^4*tan(d*x + c)^2 + 4*a^3*b*tan(d*x + c) + 4*a*b^3*tan(d*x + c) - (
4*a^3*b*tan(d*x + c)^3 + 2*a^4*tan(d*x + c)^2 + 4*a^3*b*tan(d*x + c) + a^4
)/(tan(d*x + c)^2 + 1)^2)/d
```

3.638.9 Mupad [B] (verification not implemented)

Time = 27.19 (sec) , antiderivative size = 185, normalized size of antiderivative = 7.12

$$\int (b\sec(c+dx)+a\sin(c+dx))^3(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx$$
$$=\frac{a^4\cos(2c+2dx)^4-2a^4\cos(2c+2dx)^2+a^4-8\sin(2c+2dx)\,a^3b\cos(2c+2dx)^2+8\sin(2c+2}{d\,(16\cos(2c+}$$

input
```
int((a*sin(c + d*x) + b/cos(c + d*x))^3*(a*cos(c + d*x) + (b*tan(c + d*x))
/cos(c + d*x)),x)
```

output
```
(a^4*cos(2*c + 2*d*x)^4 - 2*a^4*cos(2*c + 2*d*x)^2 - 4*b^4*cos(2*c + 2*d*x
)^2 + a^4 + 12*b^4 + 24*a^2*b^2 - 8*b^4*cos(2*c + 2*d*x) - 24*a^2*b^2*cos(
2*c + 2*d*x)^2 + 32*a*b^3*sin(2*c + 2*d*x) + 8*a^3*b*sin(2*c + 2*d*x) - 8*
a^3*b*cos(2*c + 2*d*x)^2*sin(2*c + 2*d*x))/(d*(32*cos(2*c + 2*d*x) + 16*co
s(2*c + 2*d*x)^2 + 16))
```

3.639 $\int (b\sec(c+dx)+a\sin(c+dx))^2(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx$

3.639.1 Optimal result

Integrand size = 43, antiderivative size = 26

$$\int (b\sec(c+dx)+a\sin(c+dx))^2(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx = \frac{(b\sec(c+dx)+a\sin(c+dx))^3}{3d}$$

output
```
1/3*(b*sec(d*x+c)+a*sin(d*x+c))^3/d
```

3.639.2 Mathematica [A] (verified)

Time = 5.95 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int (b\sec(c+dx)+a\sin(c+dx))^2(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx = \frac{(b\sec(c+dx)+a\sin(c+dx))^3}{3d}$$

input
```
Integrate[(b*Sec[c + d*x] + a*Sin[c + d*x])^2*(a*Cos[c + d*x] + b*Sec[c +
d*x]*Tan[c + d*x]),x]
```

output
```
(b*Sec[c + d*x] + a*Sin[c + d*x])^3/(3*d)
```

3.639.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.047$, Rules used = {3042, 4885}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(c+dx)+b\sec(c+dx))^2(a\cos(c+dx)+b\tan(c+dx)\sec(c+dx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (a\sin(c+dx)+b\sec(c+dx))^2(a\cos(c+dx)+b\tan(c+dx)\sec(c+dx))dx$$

$$\Big\downarrow \text{ 4885}$$

$$\frac{(a\sin(c+dx)+b\sec(c+dx))^3}{3d}$$

input
```
Int[(b*Sec[c + d*x] + a*Sin[c + d*x])^2*(a*Cos[c + d*x] + b*Sec[c + d*x]*T
an[c + d*x]),x]
```

output
```
(b*Sec[c + d*x] + a*Sin[c + d*x])^3/(3*d)
```

3.639.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4885
```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[ActivateTrig[
y], ActivateTrig[u], x]}, Simp[q*(ActivateTrig[y^(m + 1)]/(m + 1)), x] /;
!FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1] &&  !InertTrigFreeQ[u]
```

3.639.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(24) = 48.

Time = 48.38 (sec) , antiderivative size = 152, normalized size of antiderivative = 5.85

method	result
derivativedivides	$\frac{\frac{\sin(dx+c)^3a^3}{3}+a^2b\left(\frac{\sin(dx+c)^4}{\cos(dx+c)}+\left(2+\sin(dx+c)^2\right)\cos(dx+c)\right)-2\cos(dx+c)a^2b+2a\,b^2\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec}{}\right.}{d}$
default	$\frac{\frac{\sin(dx+c)^3a^3}{3}+a^2b\left(\frac{\sin(dx+c)^4}{\cos(dx+c)}+\left(2+\sin(dx+c)^2\right)\cos(dx+c)\right)-2\cos(dx+c)a^2b+2a\,b^2\left(\frac{\sin(dx+c)^3}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec}{}\right.}{d}$
parts	$\frac{\frac{\sin(dx+c)^3a^3}{3}-2\cos(dx+c)a^2b+a\,b^2\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{b^3\sec(dx+c)^3}{3d}+\frac{a^2b\left(\frac{\sin(dx+c)^4}{\cos(dx+c)}+\left(2+\sin(dx+c)^2\right.\right.}{d}$
risch	$\frac{-3ia^3e^{5i(dx+c)}+3ia^3e^{i(dx+c)}-12\,e^{-i(dx+c)}a^2b+24\,e^{3i(dx+c)}a^2b-ia^3e^{-3i(dx+c)}+64\,e^{3i(dx+c)}b^3+ia^3e^{9i(dx+c)}-12a^2b}{24d\left(e^{2i(dx+c)}+1\right)^3}$

input
```
int((b*sec(d*x+c)+sin(d*x+c)*a)^2*(cos(d*x+c)*a+b*sec(d*x+c)*tan(d*x+c)),x
,method=_RETURNVERBOSE)
```

output
```
1/d*(1/3*sin(d*x+c)^3*a^3+a^2*b*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*
cos(d*x+c))-2*cos(d*x+c)*a^2*b+2*a*b^2*(1/2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*
sin(d*x+c)-1/2*ln(sec(d*x+c)+tan(d*x+c)))+a*b^2*ln(sec(d*x+c)+tan(d*x+c))+
1/3*b^3/cos(d*x+c)^3)
```

3.639.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(24) = 48.

Time = 0.26 (sec) , antiderivative size = 92, normalized size of antiderivative = 3.54

$$\int (b\sec(c+dx)+a\sin(c+dx))^2(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx =$$
$$-\frac{3\,a^2b\cos\left(dx+c\right)^4-3\,a^2b\cos\left(dx+c\right)^2-b^3+\left(a^3\cos\left(dx+c\right)^5-a^3\cos\left(dx+c\right)^3-3\,ab^2\cos\left(dx+c\right)\right)}{3\,d\cos\left(dx+c\right)^3}$$

input
```
integrate((b*sec(d*x+c)+a*sin(d*x+c))^2*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x
+c)),x, algorithm="fricas")
```

output
```
-1/3*(3*a^2*b*cos(d*x + c)^4 - 3*a^2*b*cos(d*x + c)^2 - b^3 + (a^3*cos(d*x
 + c)^5 - a^3*cos(d*x + c)^3 - 3*a*b^2*cos(d*x + c))*sin(d*x + c))/(d*cos(
d*x + c)^3)
```

3.639.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(20) = 40$.

Time = 1.00 (sec) , antiderivative size = 100, normalized size of antiderivative = 3.85

$$\int (b\sec(c+dx)+a\sin(c+dx))^2(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx = \begin{cases} \frac{a^3\sin^3(c+dx)}{3d}+\frac{a^2b\sin^2(c+dx)\sec(c+dx)}{d}+\frac{ab^2\sin(c+dx)\sec^2(c+dx)}{d}+\frac{b^3\sec^3(c+dx)}{3d} & \text{for } d\neq 0 \\ x(a\sin(c)+b\sec(c))^2(a\cos(c)+b\tan(c)\sec(c)) & \text{otherwise} \end{cases}$$

input
```
integrate((b*sec(d*x+c)+a*sin(d*x+c))**2*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*
x+c)),x)
```

output
```
Piecewise((a**3*sin(c + d*x)**3/(3*d) + a**2*b*sin(c + d*x)**2*sec(c + d*x
)/d + a*b**2*sin(c + d*x)*sec(c + d*x)**2/d + b**3*sec(c + d*x)**3/(3*d),
Ne(d, 0)), (x*(a*sin(c) + b*sec(c))**2*(a*cos(c) + b*tan(c)*sec(c)), True)
)
```

3.639.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int (b\sec(c+dx)+a\sin(c+dx))^2(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx = \frac{(b\sec(dx+c)+a\sin(dx+c))^3}{3\,d}$$

input
```
integrate((b*sec(d*x+c)+a*sin(d*x+c))^2*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x
+c)),x, algorithm="maxima")
```

output
```
1/3*(b*sec(d*x + c) + a*sin(d*x + c))^3/d
```

3.639.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. 2(24) = 48.

Time = 0.70 (sec) , antiderivative size = 321, normalized size of antiderivative = 12.35

$$\int (b\sec(c+dx)+a\sin(c+dx))^2(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx$$
$$= \frac{2\left(3\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-6\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}-3\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}+4\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+9\right.}{}$$

input
```
integrate((b*sec(d*x+c)+a*sin(d*x+c))^2*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x
+c)),x, algorithm="giac")
```

output
```
2/3*(3*a*b^2*tan(1/2*d*x + 1/2*c)^11 - 6*a^2*b*tan(1/2*d*x + 1/2*c)^10 - 3
*b^3*tan(1/2*d*x + 1/2*c)^10 + 4*a^3*tan(1/2*d*x + 1/2*c)^9 + 9*a*b^2*tan(
1/2*d*x + 1/2*c)^9 - 9*b^3*tan(1/2*d*x + 1/2*c)^8 - 12*a^3*tan(1/2*d*x + 1
/2*c)^7 + 6*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 12*a^2*b*tan(1/2*d*x + 1/2*c)^6
 - 10*b^3*tan(1/2*d*x + 1/2*c)^6 + 12*a^3*tan(1/2*d*x + 1/2*c)^5 - 6*a*b^2
*tan(1/2*d*x + 1/2*c)^5 - 6*b^3*tan(1/2*d*x + 1/2*c)^4 - 4*a^3*tan(1/2*d*x
 + 1/2*c)^3 - 9*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 6*a^2*b*tan(1/2*d*x + 1/2*c
)^2 - 3*b^3*tan(1/2*d*x + 1/2*c)^2 - 3*a*b^2*tan(1/2*d*x + 1/2*c) - b^3)/(
(tan(1/2*d*x + 1/2*c)^4 - 1)^3*d)
```

3.639.9 Mupad [B] (verification not implemented)

Time = 26.84 (sec) , antiderivative size = 100, normalized size of antiderivative = 3.85

$$\int (b\sec(c+dx)+a\sin(c+dx))^2(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx$$
$$= \frac{a^3\,\sin\left(c+d\,x\right)}{3\,d}+\frac{a^2\,b\cos\left(c+d\,x\right)^2+\sin\left(c+d\,x\right)\,a\,b^2\,\cos\left(c+d\,x\right)+\frac{b^3}{3}}{d\cos\left(c+d\,x\right)^3}$$
$$-\frac{a^3\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{3\,d}-\frac{a^2\,b\,\cos\left(c+d\,x\right)}{d}$$

input
```
int((a*sin(c + d*x) + b/cos(c + d*x))^2*(a*cos(c + d*x) + (b*tan(c + d*x))
/cos(c + d*x)),x)
```

output
```
(a^3*sin(c + d*x))/(3*d) + (b^3/3 + a^2*b*cos(c + d*x)^2 + a*b^2*cos(c + d
*x)*sin(c + d*x))/(d*cos(c + d*x)^3) - (a^3*cos(c + d*x)^2*sin(c + d*x))/(
3*d) - (a^2*b*cos(c + d*x))/d
```

3.639. $\int (b\sec(c+dx)+a\sin(c+dx))^2(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx$

3.640 $\int (b\sec(c+dx)+a\sin(c+dx))(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx$

3.640.1 Optimal result

Integrand size = 41, antiderivative size = 26

$$\int (b\sec(c+dx)+a\sin(c+dx))(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx = \frac{(b\sec(c+dx)+a\sin(c+dx))^2}{2d}$$

output
```
1/2*(b*sec(d*x+c)+a*sin(d*x+c))^2/d
```

3.640.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 67 vs. 2(26) = 52.

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.58

$$\int (b\sec(c+dx)+a\sin(c+dx))(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx = abx-\frac{ab\arctan(\tan(c+dx))}{d}-\frac{a^2\cos^2(c+dx)}{2d}+\frac{b^2\sec^2(c+dx)}{2d}+\frac{ab\tan(c+dx)}{d}$$

input
```
Integrate[(b*Sec[c + d*x] + a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sec[c + d*
x]*Tan[c + d*x]),x]
```

output
```
a*b*x - (a*b*ArcTan[Tan[c + d*x]])/d - (a^2*Cos[c + d*x]^2)/(2*d) + (b^2*S
ec[c + d*x]^2)/(2*d) + (a*b*Tan[c + d*x])/d
```

3.640.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.049, Rules used = {3042, 4885}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(c+dx)+b\sec(c+dx))(a\cos(c+dx)+b\tan(c+dx)\sec(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(c+dx)+b\sec(c+dx))(a\cos(c+dx)+b\tan(c+dx)\sec(c+dx))dx$$

$$\Big\downarrow \text{4885}$$

$$\frac{(a\sin(c+dx)+b\sec(c+dx))^2}{2d}$$

input
```
Int[(b*Sec[c + d*x] + a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan
[c + d*x]),x]
```

output
```
(b*Sec[c + d*x] + a*Sin[c + d*x])^2/(2*d)
```

3.640.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4885
```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[ActivateTrig[
y], ActivateTrig[u], x]}, Simp[q*(ActivateTrig[y^(m + 1)]/(m + 1)), x] /;
!FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1] &&  !InertTrigFreeQ[u]
```

3.640.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(24) = 48$.

Time = 8.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.19

method	result	size
derivativedivides	$\frac{-\frac{\cos(dx+c)^2 a^2}{2}+ab(\tan(dx+c)-dx-c)+ab(dx+c)+\frac{b^2}{2\cos(dx+c)^2}}{d}$	57
default	$\frac{-\frac{\cos(dx+c)^2 a^2}{2}+ab(\tan(dx+c)-dx-c)+ab(dx+c)+\frac{b^2}{2\cos(dx+c)^2}}{d}$	57
parts	$\frac{-\frac{\cos(dx+c)^2 a^2}{2}+ab(dx+c)}{d}+\frac{b^2\sec(dx+c)^2}{2d}+\frac{ab(\tan(dx+c)-dx-c)}{d}$	64
risch	$-\frac{a^2 e^{2i(dx+c)}}{8d}-\frac{a^2 e^{-2i(dx+c)}}{8d}+\frac{2b(ia\,e^{2i(dx+c)}+b\,e^{2i(dx+c)}+ia)}{d(e^{2i(dx+c)}+1)^2}$	84

input
```
int((b*sec(d*x+c)+sin(d*x+c)*a)*(cos(d*x+c)*a+b*sec(d*x+c)*tan(d*x+c)),x,m
ethod=_RETURNVERBOSE)
```

output
```
1/d*(-1/2*cos(d*x+c)^2*a^2+a*b*(tan(d*x+c)-d*x-c)+a*b*(d*x+c)+1/2*b^2/cos(
d*x+c)^2)
```

3.640.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(24) = 48$.

Time = 0.24 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.35

$$\int (b\sec(c+dx)+a\sin(c+dx))(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx$$
$$= -\frac{2\,a^2\cos(dx+c)^4 - a^2\cos(dx+c)^2 - 4\,ab\cos(dx+c)\sin(dx+c) - 2\,b^2}{4\,d\cos(dx+c)^2}$$

input
```
integrate((b*sec(d*x+c)+a*sin(d*x+c))*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c
)),x, algorithm="fricas")
```

output
```
-1/4*(2*a^2*cos(d*x + c)^4 - a^2*cos(d*x + c)^2 - 4*a*b*cos(d*x + c)*sin(d
*x + c) - 2*b^2)/(d*cos(d*x + c)^2)
```

3.640.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(20) = 40$.

Time = 0.41 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.81

$$\int (b\sec(c+dx)+a\sin(c+dx))(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx$$
$$= \begin{cases} \frac{a^2\sin^2{(c+dx)}}{2d}+\frac{ab\sin{(c+dx)}\sec{(c+dx)}}{d}+\frac{b^2\sec^2{(c+dx)}}{2d} & \text{for } d\neq 0 \\ x(a\sin{(c)}+b\sec{(c)})\,(a\cos{(c)}+b\tan{(c)}\sec{(c)}) & \text{otherwise} \end{cases}$$

input
```
integrate((b*sec(d*x+c)+a*sin(d*x+c))*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c
)),x)
```

output
```
Piecewise((a**2*sin(c + d*x)**2/(2*d) + a*b*sin(c + d*x)*sec(c + d*x)/d +
b**2*sec(c + d*x)**2/(2*d), Ne(d, 0)), (x*(a*sin(c) + b*sec(c))*(a*cos(c)
+ b*tan(c)*sec(c)), True))
```

3.640.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int (b\sec(c+dx)+a\sin(c+dx))(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx$$
$$= \frac{(b\sec{(dx+c)}+a\sin{(dx+c)})^2}{2\,d}$$

input
```
integrate((b*sec(d*x+c)+a*sin(d*x+c))*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c
)),x, algorithm="maxima")
```

output
```
1/2*(b*sec(d*x + c) + a*sin(d*x + c))^2/d
```

3.640.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.73

$$\int (b\sec(c+dx)+a\sin(c+dx))(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx = \frac{b^2\tan{(dx+c)}^2+2\,ab\tan{(dx+c)}-\frac{a^2}{\tan(dx+c)^2+1}}{2\,d}$$

input
```
integrate((b*sec(d*x+c)+a*sin(d*x+c))*(a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c
)),x, algorithm="giac")
```

output
```
1/2*(b^2*tan(d*x + c)^2 + 2*a*b*tan(d*x + c) - a^2/(tan(d*x + c)^2 + 1))/d
```

3.640.9 Mupad [B] (verification not implemented)

Time = 26.37 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.35

$$\int (b\sec(c+dx)+a\sin(c+dx))(a\cos(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx = -\frac{\frac{a^2\,\left(2\sin(2\,c+2\,d\,x)^2-1\right)}{16}+\frac{a^2}{16}+\frac{b^2}{2}+\frac{a\,b\,\sin(2\,c+2\,d\,x)}{2}}{d\,\left(\sin{(c+d\,x)}^2-1\right)}$$

input
```
int((a*sin(c + d*x) + b/cos(c + d*x))*(a*cos(c + d*x) + (b*tan(c + d*x))/c
os(c + d*x)),x)
```

output
```
-((a^2*(2*sin(2*c + 2*d*x)^2 - 1))/16 + a^2/16 + b^2/2 + (a*b*sin(2*c + 2*
d*x))/2)/(d*(sin(c + d*x)^2 - 1))
```

3.641 $\int \frac{a\cos(c+dx)+b\sec(c+dx)\tan(c+dx)}{b\sec(c+dx)+a\sin(c+dx)}\,dx$

3.641.1 Optimal result

Integrand size = 43, antiderivative size = 22

$$\int \frac{a\cos(c+dx)+b\sec(c+dx)\tan(c+dx)}{b\sec(c+dx)+a\sin(c+dx)}\,dx=\frac{\log(b\sec(c+dx)+a\sin(c+dx))}{d}$$

output
```
ln(b*sec(d*x+c)+a*sin(d*x+c))/d
```

3.641.2 Mathematica [A] (verified)

Time = 3.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.32

$$\int \frac{a\cos(c+dx)+b\sec(c+dx)\tan(c+dx)}{b\sec(c+dx)+a\sin(c+dx)}\,dx = \frac{-\log(\cos(c+dx))+\log(2b+a\sin(2(c+dx)))}{d}$$

input
```
Integrate[(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x])/(b*Sec[c + d*x] +
 a*Sin[c + d*x]),x]
```

output
```
(-Log[Cos[c + d*x]] + Log[2*b + a*Sin[2*(c + d*x)]])/d
```

3.641.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.047$, Rules used = {3042, 4883}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a\cos(c+dx)+b\tan(c+dx)\sec(c+dx)}{a\sin(c+dx)+b\sec(c+dx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{a\cos(c+dx)+b\tan(c+dx)\sec(c+dx)}{a\sin(c+dx)+b\sec(c+dx)}dx$$
$$\Big\downarrow \text{4883}$$
$$\frac{\log(a\sin(c+dx)+b\sec(c+dx))}{d}$$

input
```
Int[(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x])/(b*Sec[c + d*x] + a*Sin
[c + d*x]),x]
```

output
```
Log[b*Sec[c + d*x] + a*Sin[c + d*x]]/d
```

3.641.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4883
```
Int[(u_)/(y_), x_Symbol] :> With[{q = DerivativeDivides[ActivateTrig[y], Ac
tivateTrig[u], x]}, Simp[q*Log[RemoveContent[ActivateTrig[y], x]], x] /;  !
FalseQ[q]] /;  !InertTrigFreeQ[u]
```

3.641.4 Maple [A] (verified)

Time = 2.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$\frac{\ln(b\sec(dx+c)+\sin(dx+c)a)}{d}$	23
default	$\frac{\ln(b\sec(dx+c)+\sin(dx+c)a)}{d}$	23
parts	$\frac{\ln\left(b\tan(dx+c)^2+a\tan(dx+c)+b\right)}{d}-\frac{\ln\left(1+\tan(dx+c)^2\right)}{2d}$	43
risch	$-ix-\frac{2ic}{d}-\frac{\ln(e^{2i(dx+c)}+1)}{d}+\frac{\ln\left(e^{4i(dx+c)}+\frac{4ib\,e^{2i(dx+c)}}{a}-1\right)}{d}$	62

input
```
int((cos(d*x+c)*a+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+sin(d*x+c)*a),x,m
ethod=_RETURNVERBOSE)
```

output
```
ln(b*sec(d*x+c)+sin(d*x+c)*a)/d
```

3.641.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.50

$$\int \frac{a\cos(c+dx)+b\sec(c+dx)\tan(c+dx)}{b\sec(c+dx)+a\sin(c+dx)}\,dx = \frac{\log\left(a\cos\left(dx+c\right)\sin\left(dx+c\right)+b\right)-\log\left(-\cos\left(dx+c\right)\right)}{d}$$

input
```
integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c
)),x, algorithm="fricas")
```

output
```
(log(a*cos(d*x + c)*sin(d*x + c) + b) - log(-cos(d*x + c)))/d
```

3.641.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(19) = 38$.

Time = 3.78 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.77

$$\int \frac{a\cos(c+dx)+b\sec(c+dx)\tan(c+dx)}{b\sec(c+dx)+a\sin(c+dx)}\,dx = \begin{cases} \frac{x\cos(c)}{\sin(c)} & \text{for } b=0 \wedge d=0 \\ \frac{\log(\sin(c+dx))}{d} & \text{for } b=0 \\ \frac{x(a\cos(c)+b\tan(c)\sec(c))}{a\sin(c)+b\sec(c)} & \text{for } d=0 \\ \frac{\log\left(\frac{a\sin(c+dx)}{b}+\sec(c+dx)\right)}{d} & \text{otherwise} \end{cases}$$

input
```
integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c
)),x)
```

output
```
Piecewise((x*cos(c)/sin(c), Eq(b, 0) & Eq(d, 0)), (log(sin(c + d*x))/d, Eq
(b, 0)), (x*(a*cos(c) + b*tan(c)*sec(c))/(a*sin(c) + b*sec(c)), Eq(d, 0)),
 (log(a*sin(c + d*x)/b + sec(c + d*x))/d, True))
```

3.641.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{a\cos(c+dx)+b\sec(c+dx)\tan(c+dx)}{b\sec(c+dx)+a\sin(c+dx)}\,dx = \frac{\log(b\sec(dx+c)+a\sin(dx+c))}{d}$$

input
```
integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c
)),x, algorithm="maxima")
```

output
```
log(b*sec(d*x + c) + a*sin(d*x + c))/d
```

3.641.8 Giac [A] (verification not implemented)

Time = 133.53 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.91

$$\int \frac{a\cos(c+dx)+b\sec(c+dx)\tan(c+dx)}{b\sec(c+dx)+a\sin(c+dx)}\,dx$$
$$=\frac{2\log\left(b\tan\left(dx+c\right)^2+a\tan\left(dx+c\right)+b\right)-\log\left(\tan\left(dx+c\right)^2+1\right)}{2\,d}$$

input
```
integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c
)),x, algorithm="giac")
```

output
```
1/2*(2*log(b*tan(d*x + c)^2 + a*tan(d*x + c) + b) - log(tan(d*x + c)^2 + 1
))/d
```

3.641.9 Mupad [B] (verification not implemented)

Time = 29.03 (sec) , antiderivative size = 133, normalized size of antiderivative = 6.05

$$\int \frac{a\cos(c+dx)+b\sec(c+dx)\tan(c+dx)}{b\sec(c+dx)+a\sin(c+dx)}\,dx$$
$$=\frac{\mathrm{atan}\left(\frac{-\cos(c+d\,x)\,a^6+8\,\cos(c+d\,x)\,a^4\,b^2-16\,\cos(c+d\,x)\,a^2\,b^4+\frac{\sin(2\,c+2\,d\,x)\,a\,b^5}{2}+b^6}{1\mathrm{i}\,\cos(c+d\,x)\,a^6-8\mathrm{i}\,\cos(c+d\,x)\,a^4\,b^2+16\mathrm{i}\,\cos(c+d\,x)\,a^2\,b^4+\frac{1\mathrm{i}\,\sin(2\,c+2\,d\,x)\,a\,b^5}{2}+b^6\,1\mathrm{i}}\right)\,2\mathrm{i}}{d}$$

input
```
int((a*cos(c + d*x) + (b*tan(c + d*x))/cos(c + d*x))/(a*sin(c + d*x) + b/c
os(c + d*x)),x)
```

output
```
(atan((b^6 - a^6*cos(c + d*x) - 16*a^2*b^4*cos(c + d*x) + 8*a^4*b^2*cos(c
+ d*x) + (a*b^5*sin(2*c + 2*d*x))/2)/(a^6*cos(c + d*x)*1i + b^6*1i + a^2*b
^4*cos(c + d*x)*16i - a^4*b^2*cos(c + d*x)*8i + (a*b^5*sin(2*c + 2*d*x)*1i
)/2))*2i)/d
```

3.642 $\int \frac{a\cos(c+dx)+b\sec(c+dx)\tan(c+dx)}{(b\sec(c+dx)+a\sin(c+dx))^2}\,dx$

3.642.1 Optimal result

Integrand size = 43, antiderivative size = 24

$$\int \frac{a\cos(c+dx)+b\sec(c+dx)\tan(c+dx)}{(b\sec(c+dx)+a\sin(c+dx))^2}\,dx=-\frac{1}{d(b\sec(c+dx)+a\sin(c+dx))}$$

output
```
-1/d/(b*sec(d*x+c)+a*sin(d*x+c))
```

3.642.2 Mathematica [A] (verified)

Time = 1.57 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.12

$$\int \frac{a\cos(c+dx)+b\sec(c+dx)\tan(c+dx)}{(b\sec(c+dx)+a\sin(c+dx))^2}\,dx=-\frac{2\cos(c+dx)}{d(2b+a\sin(2(c+dx)))}$$

input
```
Integrate[(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x])/(b*Sec[c + d*x] +
 a*Sin[c + d*x])^2,x]
```

output
```
(-2*Cos[c + d*x])/(d*(2*b + a*Sin[2*(c + d*x)]))
```

3.642.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.047, Rules used = {3042, 4885}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a\cos(c+dx)+b\tan(c+dx)\sec(c+dx)}{(a\sin(c+dx)+b\sec(c+dx))^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{a\cos(c+dx)+b\tan(c+dx)\sec(c+dx)}{(a\sin(c+dx)+b\sec(c+dx))^2}dx$$
$$\Big\downarrow \text{4885}$$
$$-\frac{1}{d(a\sin(c+dx)+b\sec(c+dx))}$$

input
```
Int[(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x])/(b*Sec[c + d*x] + a*Sin
[c + d*x])^2,x]
```

output
```
-(1/(d*(b*Sec[c + d*x] + a*Sin[c + d*x])))
```

3.642.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4885
```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[ActivateTrig[
y], ActivateTrig[u], x]}, Simp[q*(ActivateTrig[y^(m + 1)]/(m + 1)), x] /;
!FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1] &&  !InertTrigFreeQ[u]
```

3.642.4 Maple [A] (verified)

Time = 4.66 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

method	result
derivativedivides	$-\frac{1}{d(b\sec(dx+c)+\sin(dx+c)a)}$
default	$-\frac{1}{d(b\sec(dx+c)+\sin(dx+c)a)}$
risch	$-\frac{2i(e^{3i(dx+c)}+e^{i(dx+c)})}{d(a\,e^{4i(dx+c)}+4ib\,e^{2i(dx+c)}-a)}$
parts	$\frac{a\left(-\frac{2\left(-\frac{(a^2-3b^2)\tan(\frac{dx}{2}+\frac{c}{2})^3}{b(a^2-4b^2)}+\frac{(a^2-2b^2)\tan(\frac{dx}{2}+\frac{c}{2})^2}{a(a^2-4b^2)}+\frac{(a^2-b^2)\tan(\frac{dx}{2}+\frac{c}{2})}{b(a^2-4b^2)}+\frac{a^2-2b^2}{a(a^2-4b^2)}\right)}{\tan(\frac{dx}{2}+\frac{c}{2})^4 b-2\tan(\frac{dx}{2}+\frac{c}{2})^3 a+2\tan(\frac{dx}{2}+\frac{c}{2})^2 b+2a\tan(\frac{dx}{2}+\frac{c}{2})+b}-\frac{b\left(\_R=\mathrm{RootOf}(b\_Z^4-\right.}{}\right.}{d}$

input
```
int((cos(d*x+c)*a+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+sin(d*x+c)*a)^2,x
,method=_RETURNVERBOSE)
```

output
```
-1/d/(b*sec(d*x+c)+sin(d*x+c)*a)
```

3.642.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \frac{a\cos(c+dx)+b\sec(c+dx)\tan(c+dx)}{(b\sec(c+dx)+a\sin(c+dx))^2}\,dx = -\frac{\cos(dx+c)}{ad\cos(dx+c)\sin(dx+c)+bd}$$

input
```
integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c
))^2,x, algorithm="fricas")
```

output
```
-cos(d*x + c)/(a*d*cos(d*x + c)*sin(d*x + c) + b*d)
```

3.642.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(20) = 40$.

Time = 11.18 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.04

$$\int \frac{a\cos(c+dx)+b\sec(c+dx)\tan(c+dx)}{(b\sec(c+dx)+a\sin(c+dx))^2}\,dx = \begin{cases} -\frac{1}{ad\sin(c+dx)+bd\sec(c+dx)} & \text{for } d\neq 0 \\ \frac{x(a\cos(c)+b\tan(c)\sec(c))}{(a\sin(c)+b\sec(c))^2} & \text{otherwise} \end{cases}$$

input
```
integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c
))**2,x)
```

output
```
Piecewise((-1/(a*d*sin(c + d*x) + b*d*sec(c + d*x)), Ne(d, 0)), (x*(a*cos(
c) + b*tan(c)*sec(c))/(a*sin(c) + b*sec(c))**2, True))
```

3.642.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{a\cos(c+dx)+b\sec(c+dx)\tan(c+dx)}{(b\sec(c+dx)+a\sin(c+dx))^2}\,dx = -\frac{1}{(b\sec(dx+c)+a\sin(dx+c))d}$$

input
```
integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c
))^2,x, algorithm="maxima")
```

output
```
-1/((b*sec(d*x + c) + a*sin(d*x + c))*d)
```

3.642.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(24) = 48$.

Time = 0.62 (sec) , antiderivative size = 108, normalized size of antiderivative = 4.50

$$\int \frac{a\cos(c+dx)+b\sec(c+dx)\tan(c+dx)}{(b\sec(c+dx)+a\sin(c+dx))^2}\,dx$$
$$= \frac{2\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\right)}{\left(b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b\right)bd}$$

input
```
integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c
))^2,x, algorithm="giac")
```

output
```
2*(a*tan(1/2*d*x + 1/2*c)^3 - b*tan(1/2*d*x + 1/2*c)^2 - a*tan(1/2*d*x + 1
/2*c) - b)/((b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^3 + 2*b*t
an(1/2*d*x + 1/2*c)^2 + 2*a*tan(1/2*d*x + 1/2*c) + b)*b*d)
```

3.642.9 Mupad [B] (verification not implemented)

Time = 26.69 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.96

$$\int \frac{a\cos(c+dx)+b\sec(c+dx)\tan(c+dx)}{(b\sec(c+dx)+a\sin(c+dx))^2}\,dx = -\frac{b\,(\cos\left(c+d\,x\right)+1)+\frac{a\,\sin(2\,c+2\,d\,x)}{2}}{b\,d\,\left(b+\frac{a\,\sin(2\,c+2\,d\,x)}{2}\right)}$$

input
```
int((a*cos(c + d*x) + (b*tan(c + d*x))/cos(c + d*x))/(a*sin(c + d*x) + b/c
os(c + d*x))^2,x)
```

output
```
-(b*(cos(c + d*x) + 1) + (a*sin(2*c + 2*d*x))/2)/(b*d*(b + (a*sin(2*c + 2*
d*x))/2))
```

3.643 $\int \frac{a\cos(c+dx)+b\sec(c+dx)\tan(c+dx)}{(b\sec(c+dx)+a\sin(c+dx))^3}\,dx$

3.643.1 Optimal result

Integrand size = 43, antiderivative size = 26

$$\int \frac{a\cos(c+dx)+b\sec(c+dx)\tan(c+dx)}{(b\sec(c+dx)+a\sin(c+dx))^3}\,dx=-\frac{1}{2d(b\sec(c+dx)+a\sin(c+dx))^2}$$

output
```
-1/2/d/(b*sec(d*x+c)+a*sin(d*x+c))^2
```

3.643.2 Mathematica [A] (verified)

Time = 3.99 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.12

$$\int \frac{a\cos(c+dx)+b\sec(c+dx)\tan(c+dx)}{(b\sec(c+dx)+a\sin(c+dx))^3}\,dx=-\frac{2\cos^2(c+dx)}{d(2b+a\sin(2(c+dx)))^2}$$

input
```
Integrate[(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x])/(b*Sec[c + d*x] +
 a*Sin[c + d*x])^3,x]
```

output
```
(-2*Cos[c + d*x]^2)/(d*(2*b + a*Sin[2*(c + d*x)])^2)
```

3.643.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.047, Rules used = {3042, 4885}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a\cos(c+dx)+b\tan(c+dx)\sec(c+dx)}{(a\sin(c+dx)+b\sec(c+dx))^3}dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{a\cos(c+dx)+b\tan(c+dx)\sec(c+dx)}{(a\sin(c+dx)+b\sec(c+dx))^3}dx$$
$$\Big\downarrow \; 4885$$
$$-\frac{1}{2d(a\sin(c+dx)+b\sec(c+dx))^2}$$

input
```
Int[(a*Cos[c + d*x] + b*Sec[c + d*x]*Tan[c + d*x])/(b*Sec[c + d*x] + a*Sin
[c + d*x])^3,x]
```

output
```
-1/2*1/(d*(b*Sec[c + d*x] + a*Sin[c + d*x])^2)
```

3.643.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4885
```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[ActivateTrig[
y], ActivateTrig[u], x]}, Simp[q*(ActivateTrig[y^(m + 1)]/(m + 1)), x] /;
!FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1] && !InertTrigFreeQ[u]
```

3.643.4 Maple [A] (verified)

Time = 18.50 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

method	result
derivativedivides	$-\frac{1}{2d(b\sec(dx+c)+\sin(dx+c)a)^2}$
default	$-\frac{1}{2d(b\sec(dx+c)+\sin(dx+c)a)^2}$
risch	$\frac{2\,\mathrm{e}^{6i(dx+c)}+4\,\mathrm{e}^{4i(dx+c)}+2\,\mathrm{e}^{2i(dx+c)}}{\left(a\,\mathrm{e}^{4i(dx+c)}+4ib\,\mathrm{e}^{2i(dx+c)}-a\right)^2d}$
parts	$\frac{a\left(\frac{2b\tan(dx+c)+a}{2\left(-a^2+4b^2\right)\left(b\tan(dx+c)^2+a\tan(dx+c)+b\right)^2}+\frac{3b\left(\frac{2b\tan(dx+c)+a}{\left(-a^2+4b^2\right)\left(b\tan(dx+c)^2+a\tan(dx+c)+b\right)}+\frac{4b\arctan\left(\frac{2b\tan(dx+c)+a}{\sqrt{-a^2+4b^2}}\right)}{\left(-a^2+4b^2\right)^{\frac{3}{2}}}\right.}{-a^2+4b^2}\right.}{d}$

input
```
int((cos(d*x+c)*a+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+sin(d*x+c)*a)^3,x
,method=_RETURNVERBOSE)
```

output
```
-1/2/d/(b*sec(d*x+c)+sin(d*x+c)*a)^2
```

3.643.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. 2(24) = 48.

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.42

$$\int \frac{a\cos(c+dx)+b\sec(c+dx)\tan(c+dx)}{(b\sec(c+dx)+a\sin(c+dx))^3}\,dx = \frac{\cos(dx+c)^2}{2\left(a^2d\cos(dx+c)^4-a^2d\cos(dx+c)^2-2\,abd\cos(dx+c)\sin(dx+c)-b^2d\right)}$$

input
```
integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c
))^3,x, algorithm="fricas")
```

output
```
1/2*cos(d*x + c)^2/(a^2*d*cos(d*x + c)^4 - a^2*d*cos(d*x + c)^2 - 2*a*b*d*
cos(d*x + c)*sin(d*x + c) - b^2*d)
```

3.643.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(24) = 48$.

Time = 23.68 (sec) , antiderivative size = 80, normalized size of antiderivative = 3.08

$$\int \frac{a\cos(c+dx)+b\sec(c+dx)\tan(c+dx)}{(b\sec(c+dx)+a\sin(c+dx))^3}\,dx = \begin{cases} -\frac{1}{2a^2d\sin^2(c+dx)+4abd\sin(c+dx)\sec(c+dx)+2b^2d\sec^2(c+dx)} & \text{for } d \neq 0 \\ \frac{x(a\cos(c)+b\tan(c)\sec(c))}{(a\sin(c)+b\sec(c))^3} & \text{otherwise} \end{cases}$$

input
```
integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c
))**3,x)
```

output
```
Piecewise((-1/(2*a**2*d*sin(c + d*x)**2 + 4*a*b*d*sin(c + d*x)*sec(c + d*x
) + 2*b**2*d*sec(c + d*x)**2), Ne(d, 0)), (x*(a*cos(c) + b*tan(c)*sec(c))/
(a*sin(c) + b*sec(c))**3, True))
```

3.643.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{a\cos(c+dx)+b\sec(c+dx)\tan(c+dx)}{(b\sec(c+dx)+a\sin(c+dx))^3}\,dx = -\frac{1}{2\,(b\sec(dx+c)+a\sin(dx+c))^2d}$$

input
```
integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c
))^3,x, algorithm="maxima")
```

output
```
-1/2/((b*sec(d*x + c) + a*sin(d*x + c))^2*d)
```

3.643.8 Giac [A] (verification not implemented)

Time = 142.41 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.42

$$\int \frac{a\cos(c+dx)+b\sec(c+dx)\tan(c+dx)}{(b\sec(c+dx)+a\sin(c+dx))^3}\,dx = -\frac{\tan(dx+c)^2+1}{2\left(b\tan(dx+c)^2+a\tan(dx+c)+b\right)^2 d}$$

input
```
integrate((a*cos(d*x+c)+b*sec(d*x+c)*tan(d*x+c))/(b*sec(d*x+c)+a*sin(d*x+c
))^3,x, algorithm="giac")
```

output
```
-1/2*(tan(d*x + c)^2 + 1)/((b*tan(d*x + c)^2 + a*tan(d*x + c) + b)^2*d)
```

3.643.9 Mupad [B] (verification not implemented)

Time = 30.84 (sec) , antiderivative size = 291, normalized size of antiderivative = 11.19

$$\int \frac{a\cos(c+dx)+b\sec(c+dx)\tan(c+dx)}{(b\sec(c+dx)+a\sin(c+dx))^3}\,dx$$
$$= \frac{\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(a^2+b^2)}{b^2}+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6(a^2+b^2)}{b^2}+\frac{2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{b}-\frac{4\tan\left(\frac{c}{2}+\frac{d}{}\right.}{b^2}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(4a^2+4b^2)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6(4a^2+4b^2)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4(8a^2-6b^2)+b^2\tan\left(\frac{c}{2}+\frac{d}{}\right.\right.}$$

input
```
int((a*cos(c + d*x) + (b*tan(c + d*x))/cos(c + d*x))/(a*sin(c + d*x) + b/c
os(c + d*x))^3,x)
```

output
```
((2*tan(c/2 + (d*x)/2)^2*(a^2 + b^2))/b^2 + (2*tan(c/2 + (d*x)/2)^6*(a^2 +
 b^2))/b^2 + (2*a*tan(c/2 + (d*x)/2))/b - (4*tan(c/2 + (d*x)/2)^4*(a^2 - b
^2))/b^2 + (2*a*tan(c/2 + (d*x)/2)^3)/b - (2*a*tan(c/2 + (d*x)/2)^5)/b - (
2*a*tan(c/2 + (d*x)/2)^7)/b)/(d*(tan(c/2 + (d*x)/2)^2*(4*a^2 + 4*b^2) + ta
n(c/2 + (d*x)/2)^6*(4*a^2 + 4*b^2) - tan(c/2 + (d*x)/2)^4*(8*a^2 - 6*b^2)
+ b^2*tan(c/2 + (d*x)/2)^8 + b^2 + 4*a*b*tan(c/2 + (d*x)/2)^3 - 4*a*b*tan(
c/2 + (d*x)/2)^5 - 4*a*b*tan(c/2 + (d*x)/2)^7 + 4*a*b*tan(c/2 + (d*x)/2)))
```

3.644 $\int F(c,d,\cos(a+bx),r,s)\sin(a+bx)\,dx$

3.644.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int F(c,d,\cos(a+bx),r,s)\sin(a+bx)\,dx=\mathrm{Int}(F(c,d,\cos(a+bx),r,s)\sin(a+bx),x)$$

output
```
CannotIntegrate(F(c,d,cos(b*x+a),r,s)*sin(b*x+a),x)
```

3.644.2 Mathematica [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int F(c,d,\cos(a+bx),r,s)\sin(a+bx)\,dx=\int F(c,d,\cos(a+bx),r,s)\sin(a+bx)\,dx$$

input
```
Integrate[F[c, d, Cos[a + b*x], r, s]*Sin[a + b*x],x]
```

output
```
Integrate[F[c, d, Cos[a + b*x], r, s]*Sin[a + b*x], x]
```

3.644.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {4835, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)F(c,d,\cos(a+bx),r,s)\,dx$$
$$\Big\downarrow \text{4835}$$
$$-\frac{\int F(c,d,\cos(a+bx),r,s)d\cos(a+bx)}{b}$$
$$\Big\downarrow \text{7299}$$
$$-\frac{\int F(c,d,\cos(a+bx),r,s)d\cos(a+bx)}{b}$$

input
```
Int[F[c, d, Cos[a + b*x], r, s]*Sin[a + b*x],x]
```

output
```
$Aborted
```

3.644.3.1 Defintions of rubi rules used

rule 4835
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[1, Cos[c*(a +
b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.644.4 Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int F(c,d,\cos(xb+a),r,s)\sin(xb+a)\,dx$$

input
```
int(F(c,d,cos(b*x+a),r,s)*sin(b*x+a),x)
```

output
```
int(F(c,d,cos(b*x+a),r,s)*sin(b*x+a),x)
```

3.644.5 Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int F(c,d,\cos(a+bx),r,s)\sin(a+bx)\,dx = \int F(c,d,\cos(bx+a),r,s)\sin(bx+a)\,dx$$

input
```
integrate(F(c,d,cos(b*x+a),r,s)*sin(b*x+a),x, algorithm="fricas")
```

output
```
integral(F(c, d, cos(b*x + a), r, s)*sin(b*x + a), x)
```

3.644.6 Sympy [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int F(c,d,\cos(a+bx),r,s)\sin(a+bx)\,dx = \int F(c,d,\cos(a+bx),r,s)\sin(a+bx)\,dx$$

input
```
integrate(F(c,d,cos(b*x+a),r,s)*sin(b*x+a),x)
```

output
```
Integral(F(c, d, cos(a + b*x), r, s)*sin(a + b*x), x)
```

3.644.7 Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int F(c,d,\cos(a+bx),r,s)\sin(a+bx)\,dx = \int F(c,d,\cos{(bx+a)}\,,r,s)\sin{(bx+a)}\,\,dx$$

input
```
integrate(F(c,d,cos(b*x+a),r,s)*sin(b*x+a),x, algorithm="maxima")
```

output
```
integrate(F(c, d, cos(b*x + a), r, s)*sin(b*x + a), x)
```

3.644.8 Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int F(c,d,\cos(a+bx),r,s)\sin(a+bx)\,dx = \int F(c,d,\cos{(bx+a)}\,,r,s)\sin{(bx+a)}\,\,dx$$

input
```
integrate(F(c,d,cos(b*x+a),r,s)*sin(b*x+a),x, algorithm="giac")
```

output
```
integrate(F(c, d, cos(b*x + a), r, s)*sin(b*x + a), x)
```

3.644.9 Mupad [N/A]

Not integrable

Time = 26.71 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int F(c,d,\cos(a+bx),r,s)\sin(a+bx)\,dx = \int \sin{(a+b\,x)}\,\,F(c,d,\cos{(a+b\,x)}\,,r,s)\,\,dx$$

input
```
int(sin(a + b*x)*F(c, d, cos(a + b*x), r, s),x)
```

output
```
int(sin(a + b*x)*F(c, d, cos(a + b*x), r, s), x)
```

3.645 $\int \cos(a+bx)F(c,d,\sin(a+bx),r,s)\,dx$

3.645.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \cos(a+bx)F(c,d,\sin(a+bx),r,s)\,dx = \text{Int}(\cos(a+bx)F(c,d,\sin(a+bx),r,s),x)$$

output
```
CannotIntegrate(cos(b*x+a)*F(c,d,sin(b*x+a),r,s),x)
```

3.645.2 Mathematica [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \cos(a+bx)F(c,d,\sin(a+bx),r,s)\,dx = \int \cos(a+bx)F(c,d,\sin(a+bx),r,s)\,dx$$

input
```
Integrate[Cos[a + b*x]*F[c, d, Sin[a + b*x], r, s],x]
```

output
```
Integrate[Cos[a + b*x]*F[c, d, Sin[a + b*x], r, s], x]
```

3.645.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {4834, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)F(c,d,\sin(a+bx),r,s)\,dx$$

$$\Big\downarrow \text{4834}$$

$$\frac{\int F(c,d,\sin(a+bx),r,s)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{7299}$$

$$\frac{\int F(c,d,\sin(a+bx),r,s)d\sin(a+bx)}{b}$$

input
```
Int[Cos[a + b*x]*F[c, d, Sin[a + b*x], r, s],x]
```

output
```
$Aborted
```

3.645.3.1 Defintions of rubi rules used

rule 4834
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.645.4 Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \cos\left(xb+a\right) F\left(c,d,\sin\left(xb+a\right),r,s\right)dx$$

input
```
int(cos(b*x+a)*F(c,d,sin(b*x+a),r,s),x)
```

output
```
int(cos(b*x+a)*F(c,d,sin(b*x+a),r,s),x)
```

3.645.5 Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \cos(a+bx)F(c,d,\sin(a+bx),r,s)\,dx = \int F\left(c,d,\sin\left(bx+a\right),r,s\right)\cos\left(bx+a\right)\,dx$$

input
```
integrate(cos(b*x+a)*F(c,d,sin(b*x+a),r,s),x, algorithm="fricas")
```

output
```
integral(F(c, d, sin(b*x + a), r, s)*cos(b*x + a), x)
```

3.645.6 Sympy [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \cos(a+bx)F(c,d,\sin(a+bx),r,s)\,dx = \int F\left(c,d,\sin\left(a+bx\right),r,s\right)\cos\left(a+bx\right)dx$$

input
```
integrate(cos(b*x+a)*F(c,d,sin(b*x+a),r,s),x)
```

output
```
Integral(F(c, d, sin(a + b*x), r, s)*cos(a + b*x), x)
```

3.645.7 Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \cos(a+bx)F(c,d,\sin(a+bx),r,s)\,dx = \int F(c,d,\sin{(bx+a)},r,s)\cos{(bx+a)}\,dx$$

input
```
integrate(cos(b*x+a)*F(c,d,sin(b*x+a),r,s),x, algorithm="maxima")
```

output
```
integrate(F(c, d, sin(b*x + a), r, s)*cos(b*x + a), x)
```

3.645.8 Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \cos(a+bx)F(c,d,\sin(a+bx),r,s)\,dx = \int F(c,d,\sin{(bx+a)},r,s)\cos{(bx+a)}\,dx$$

input
```
integrate(cos(b*x+a)*F(c,d,sin(b*x+a),r,s),x, algorithm="giac")
```

output
```
integrate(F(c, d, sin(b*x + a), r, s)*cos(b*x + a), x)
```

3.645.9 Mupad [N/A]

Not integrable

Time = 27.58 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \cos(a+bx)F(c,d,\sin(a+bx),r,s)\,dx = \int \cos{(a+b\,x)}\,F(c,d,\sin{(a+b\,x)},r,s)\,dx$$

input
```
int(cos(a + b*x)*F(c, d, sin(a + b*x), r, s),x)
```

output
```
int(cos(a + b*x)*F(c, d, sin(a + b*x), r, s), x)
```

3.646 $\int F(c,d,\tan(a+bx),r,s)\sec^2(a+bx)\,dx$

3.646.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int F(c,d,\tan(a+bx),r,s)\sec^2(a+bx)\,dx = \text{Int}\big(F(c,d,\tan(a+bx),r,s)\sec^2(a+bx),x\big)$$

output
```
CannotIntegrate(F(c,d,tan(b*x+a),r,s)*sec(b*x+a)^2,x)
```

3.646.2 Mathematica [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int F(c,d,\tan(a+bx),r,s)\sec^2(a+bx)\,dx = \int F(c,d,\tan(a+bx),r,s)\sec^2(a+bx)\,dx$$

input
```
Integrate[F[c, d, Tan[a + b*x], r, s]*Sec[a + b*x]^2,x]
```

output
```
Integrate[F[c, d, Tan[a + b*x], r, s]*Sec[a + b*x]^2, x]
```

3.646.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {4842, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(a+bx)F(c,d,\tan(a+bx),r,s)\,dx$$

$$\Big\downarrow \text{ 4842}$$

$$\frac{\int F(c,d,\tan(a+bx),r,s)d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{ 7299}$$

$$\frac{\int F(c,d,\tan(a+bx),r,s)d\tan(a+bx)}{b}$$

input
```
Int[F[c, d, Tan[a + b*x], r, s]*Sec[a + b*x]^2,x]
```

output
```
$Aborted
```

3.646.3.1 Defintions of rubi rules used

rule 4842
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Tan[c*(a +
 b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] |
| EqQ[F, sec])
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.646.4 Maple [N/A] (verified)

Not integrable

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int F(c,d,\tan(xb+a),r,s)\sec(xb+a)^2\,dx$$

input
```
int(F(c,d,tan(b*x+a),r,s)*sec(b*x+a)^2,x)
```

output
```
int(F(c,d,tan(b*x+a),r,s)*sec(b*x+a)^2,x)
```

3.646.5 Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int F(c,d,\tan(a+bx),r,s)\sec^2(a+bx)\,dx = \int F(c,d,\tan(bx+a),r,s)\sec(bx+a)^2\,dx$$

input
```
integrate(F(c,d,tan(b*x+a),r,s)*sec(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(F(c, d, tan(b*x + a), r, s)*sec(b*x + a)^2, x)
```

3.646.6 Sympy [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int F(c,d,\tan(a+bx),r,s)\sec^2(a+bx)\,dx = \int F(c,d,\tan(a+bx),r,s)\sec^2(a+bx)\,dx$$

input
```
integrate(F(c,d,tan(b*x+a),r,s)*sec(b*x+a)**2,x)
```

output
```
Integral(F(c, d, tan(a + b*x), r, s)*sec(a + b*x)**2, x)
```

3.646.7 Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int F(c,d,\tan(a+bx),r,s)\sec^2(a+bx)\,dx = \int F(c,d,\tan(bx+a),r,s)\sec(bx+a)^2\,dx$$

input
```
integrate(F(c,d,tan(b*x+a),r,s)*sec(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate(F(c, d, tan(b*x + a), r, s)*sec(b*x + a)^2, x)
```

3.646.8 Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int F(c,d,\tan(a+bx),r,s)\sec^2(a+bx)\,dx = \int F(c,d,\tan(bx+a),r,s)\sec(bx+a)^2\,dx$$

input
```
integrate(F(c,d,tan(b*x+a),r,s)*sec(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(F(c, d, tan(b*x + a), r, s)*sec(b*x + a)^2, x)
```

3.646.9 Mupad [N/A]

Not integrable

Time = 27.42 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int F(c,d,\tan(a+bx),r,s)\sec^2(a+bx)\,dx = \int \frac{F(c,d,\tan(a+b\,x),r,s)}{\cos(a+b\,x)^2}\,dx$$

input
```
int(F(c, d, tan(a + b*x), r, s)/cos(a + b*x)^2,x)
```

output
```
int(F(c, d, tan(a + b*x), r, s)/cos(a + b*x)^2, x)
```

3.647 $\int \csc^2(a+bx)F(c,d,\cot(a+bx),r,s)\,dx$

3.647.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \csc^2(a+bx)F(c,d,\cot(a+bx),r,s)\,dx=\operatorname{Int}\big(\csc^2(a+bx)F(c,d,\cot(a+bx),r,s),x\big)$$

output
```
CannotIntegrate(csc(b*x+a)^2*F(c,d,cot(b*x+a),r,s),x)
```

3.647.2 Mathematica [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \csc^2(a+bx)F(c,d,\cot(a+bx),r,s)\,dx=\int \csc^2(a+bx)F(c,d,\cot(a+bx),r,s)\,dx$$

input
```
Integrate[Csc[a + b*x]^2*F[c, d, Cot[a + b*x], r, s],x]
```

output
```
Integrate[Csc[a + b*x]^2*F[c, d, Cot[a + b*x], r, s], x]
```

3.647.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {4844, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(a+bx)F(c,d,\cot(a+bx),r,s)\,dx$$

$$\Big\downarrow \text{4844}$$

$$-\frac{\int F(c,d,\cot(a+bx),r,s)d\cot(a+bx)}{b}$$

$$\Big\downarrow \text{7299}$$

$$-\frac{\int F(c,d,\cot(a+bx),r,s)d\cot(a+bx)}{b}$$

input
```
Int[Csc[a + b*x]^2*F[c, d, Cot[a + b*x], r, s],x]
```

output
```
$Aborted
```

3.647.3.1 Defintions of rubi rules used

rule 4844
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Cot[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[1, Cot[c*(a
+ b*x)]/d, u, x], x], x, Cot[c*(a + b*x)]/d], x] /; FunctionOfQ[Cot[c*(a +
b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Csc]
|| EqQ[F, csc])
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.647.4 Maple [N/A] (verified)

Not integrable

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \csc (xb+a)^2 F(c,d,\cot (xb+a),r,s)\,dx$$

input
```
int(csc(b*x+a)^2*F(c,d,cot(b*x+a),r,s),x)
```

output
```
int(csc(b*x+a)^2*F(c,d,cot(b*x+a),r,s),x)
```

3.647.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \csc^2(a+bx)F(c,d,\cot(a+bx),r,s)\,dx = \int F(c,d,\cot (bx+a),r,s)\csc (bx+a)^2\,dx$$

input
```
integrate(csc(b*x+a)^2*F(c,d,cot(b*x+a),r,s),x, algorithm="fricas")
```

output
```
integral(F(c, d, cot(b*x + a), r, s)*csc(b*x + a)^2, x)
```

3.647.6 Sympy [N/A]

Not integrable

Time = 3.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \csc^2(a+bx)F(c,d,\cot(a+bx),r,s)\,dx = \int F(c,d,\cot (a+bx),r,s)\csc^2 (a+bx)\,dx$$

input
```
integrate(csc(b*x+a)**2*F(c,d,cot(b*x+a),r,s),x)
```

output
```
Integral(F(c, d, cot(a + b*x), r, s)*csc(a + b*x)**2, x)
```

3.647.7 Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \csc^2(a+bx)F(c,d,\cot(a+bx),r,s)\,dx = \int F(c,d,\cot(bx+a),r,s)\csc(bx+a)^2\,dx$$

input
```
integrate(csc(b*x+a)^2*F(c,d,cot(b*x+a),r,s),x, algorithm="maxima")
```

output
```
integrate(F(c, d, cot(b*x + a), r, s)*csc(b*x + a)^2, x)
```

3.647.8 Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \csc^2(a+bx)F(c,d,\cot(a+bx),r,s)\,dx = \int F(c,d,\cot(bx+a),r,s)\csc(bx+a)^2\,dx$$

input
```
integrate(csc(b*x+a)^2*F(c,d,cot(b*x+a),r,s),x, algorithm="giac")
```

output
```
integrate(F(c, d, cot(b*x + a), r, s)*csc(b*x + a)^2, x)
```

3.647.9 Mupad [N/A]

Not integrable

Time = 27.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \csc^2(a+bx)F(c,d,\cot(a+bx),r,s)\,dx = \int \frac{F(c,d,\cot(a+b\,x),r,s)}{\sin(a+b\,x)^2}\,dx$$

input
```
int(F(c, d, cot(a + b*x), r, s)/sin(a + b*x)^2,x)
```

output
```
int(F(c, d, cot(a + b*x), r, s)/sin(a + b*x)^2, x)
```

3.648 $\int \frac{\sin(x)}{a+b\cos(x)}\,dx$

3.648.1 Optimal result

Integrand size = 11, antiderivative size = 12

$$\int \frac{\sin(x)}{a+b\cos(x)}\,dx=-\frac{\log(a+b\cos(x))}{b}$$

output
```
-ln(a+b*cos(x))/b
```

3.648.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{a+b\cos(x)}\,dx=-\frac{\log(a+b\cos(x))}{b}$$

input
```
Integrate[Sin[x]/(a + b*Cos[x]),x]
```

output
```
-(Log[a + b*Cos[x]]/b)
```

3.648.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 3147, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)}{a+b\cos(x)} \, dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)}{a-b\sin\left(x-\frac{\pi}{2}\right)}dx$$
$$\Big\downarrow \text{ 3147}$$
$$-\frac{\int \frac{1}{a+b\cos(x)}d(b\cos(x))}{b}$$
$$\Big\downarrow \text{ 16}$$
$$-\frac{\log(a+b\cos(x))}{b}$$

input
```
Int[Sin[x]/(a + b*Cos[x]),x]
```

output
```
-(Log[a + b*Cos[x]]/b)
```

3.648.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + 
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.648.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$-\frac{\ln(a+b\cos(x))}{b}$	13
default	$-\frac{\ln(a+b\cos(x))}{b}$	13
parallelrisch	$\frac{\ln\left(\frac{1}{\cos(x)+1}\right)-\ln\left(\frac{a+b\cos(x)}{\cos(x)+1}\right)}{b}$	29
risch	$\frac{ix}{b}-\frac{\ln\left(\mathrm{e}^{2ix}+1+\frac{2a\,\mathrm{e}^{ix}}{b}\right)}{b}$	33
norman	$\frac{\ln\left(1+\tan\left(\frac{x}{2}\right)^2\right)}{b}-\frac{\ln\left(\tan\left(\frac{x}{2}\right)^2a-\tan\left(\frac{x}{2}\right)^2b+a+b\right)}{b}$	41

input
```
int(sin(x)/(a+b*cos(x)),x,method=_RETURNVERBOSE)
```

output
```
-ln(a+b*cos(x))/b
```

3.648.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{\sin(x)}{a+b\cos(x)}\,dx=-\frac{\log\left(-b\cos\left(x\right)-a\right)}{b}$$

input
```
integrate(sin(x)/(a+b*cos(x)),x, algorithm="fricas")
```

output
```
-log(-b*cos(x) - a)/b
```

3.648.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \frac{\sin(x)}{a+b\cos(x)}\,dx = \begin{cases} -\frac{\log\left(\frac{a}{b}+\cos(x)\right)}{b} & \text{for } b \neq 0 \\ -\frac{\cos(x)}{a} & \text{otherwise} \end{cases}$$

input
```
integrate(sin(x)/(a+b*cos(x)),x)
```

output
```
Piecewise((-log(a/b + cos(x))/b, Ne(b, 0)), (-cos(x)/a, True))
```

3.648.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{a+b\cos(x)}\,dx = -\frac{\log(b\cos(x)+a)}{b}$$

input
```
integrate(sin(x)/(a+b*cos(x)),x, algorithm="maxima")
```

output
```
-log(b*cos(x) + a)/b
```

3.648.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{\sin(x)}{a+b\cos(x)}\,dx = -\frac{\log(|b\cos(x)+a|)}{b}$$

input
```
integrate(sin(x)/(a+b*cos(x)),x, algorithm="giac")
```

output
```
-log(abs(b*cos(x) + a))/b
```

3.648.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{a+b\cos(x)}\,dx = -\frac{\ln\left(a+b\,\cos\left(x\right)\right)}{b}$$

input
```
int(sin(x)/(a + b*cos(x)),x)
```

output
```
-log(a + b*cos(x))/b
```

3.649 $\int (a+b\cos(x))^n \sin(x)\,dx$

3.649.1 Optimal result

Integrand size = 11, antiderivative size = 20

$$\int (a+b\cos(x))^n \sin(x)\,dx = -\frac{(a+b\cos(x))^{1+n}}{b(1+n)}$$

output
```
-(a+b*cos(x))^(1+n)/b/(1+n)
```

3.649.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (a+b\cos(x))^n \sin(x)\,dx = -\frac{(a+b\cos(x))^{1+n}}{b(1+n)}$$

input
```
Integrate[(a + b*Cos[x])^n*Sin[x],x]
```

output
```
-((a + b*Cos[x])^(1 + n)/(b*(1 + n)))
```

3.649.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 3147, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)(a+b\cos(x))^n\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \cos\left(x-\frac{\pi}{2}\right)\left(a-b\sin\left(x-\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{ 3147}$$

$$-\frac{\int (a+b\cos(x))^n d(b\cos(x))}{b}$$

$$\Big\downarrow \text{ 17}$$

$$-\frac{(a+b\cos(x))^{n+1}}{b(n+1)}$$

input
```
Int[(a + b*Cos[x])^n*Sin[x],x]
```

output
```
-((a + b*Cos[x])^(1 + n)/(b*(1 + n)))
```

3.649.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.649.4 Maple [A] (verified)

Time = 2.42 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

method	result
derivativedivides	$-\frac{(a+b\cos(x))^{n+1}}{b(n+1)}$
default	$-\frac{(a+b\cos(x))^{n+1}}{b(n+1)}$
parallelrisch	$-\frac{(a+b\cos(x))^{n+1}}{b(n+1)}$
norman	$\frac{-\frac{(a+b)\,\mathrm{e}^{n\ln\left(a+\frac{b\left(1-\tan\left(\frac{x}{2}\right)^{2}\right)}{1+\tan\left(\frac{x}{2}\right)^{2}}\right)}}{(n+1)b}-\frac{(a-b)\tan\left(\frac{x}{2}\right)^{2}\mathrm{e}^{n\ln\left(a+\frac{b\left(1-\tan\left(\frac{x}{2}\right)^{2}\right)}{1+\tan\left(\frac{x}{2}\right)^{2}}\right)}}{(n+1)b}}{1+\tan\left(\frac{x}{2}\right)^{2}}$
risch	$-\frac{\left(a\,\mathrm{e}^{ix}+\frac{b\,\mathrm{e}^{2ix}}{2}+\frac{b}{2}\right)^{n}\left(b\,\mathrm{e}^{\frac{i\left(\pi\,\mathrm{csgn}(ia+ib\cos(x))^{2}n\,\mathrm{csgn}\left(i\mathrm{e}^{-ix}\right)-\pi\,\mathrm{csgn}(ia+ib\cos(x))n\,\mathrm{csgn}\left(\frac{ib\,\mathrm{e}^{2ix}}{2}+ia\,\mathrm{e}^{ix}+\frac{ib}{2}\right)\mathrm{csgn}\left(i\mathrm{e}^{-ix}\cdots\right.\right.}{2}}\cdots\right.}{\cdots}$

input
```
int((a+b*cos(x))^n*sin(x),x,method=_RETURNVERBOSE)
```

output
```
-(a+b*cos(x))^(n+1)/b/(n+1)
```

3.649.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int (a+b\cos(x))^n\sin(x)\,dx=-\frac{(b\cos(x)+a)(b\cos(x)+a)^n}{bn+b}$$

input
```
integrate((a+b*cos(x))^n*sin(x),x, algorithm="fricas")
```

output
```
-(b*cos(x) + a)*(b*cos(x) + a)^n/(b*n + b)
```

3.649.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(15) = 30$.

Time = 0.52 (sec) , antiderivative size = 63, normalized size of antiderivative = 3.15

$$\int (a+b\cos(x))^n \sin(x)\,dx = \begin{cases} -\frac{\cos(x)}{a} & \text{for } b=0 \wedge n=-1 \\ -a^n\cos(x) & \text{for } b=0 \\ -\frac{\log\left(\frac{a}{b}+\cos(x)\right)}{b} & \text{for } n=-1 \\ -\frac{a(a+b\cos(x))^n}{bn+b} - \frac{b(a+b\cos(x))^n\cos(x)}{bn+b} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*cos(x))**n*sin(x),x)
```

output
```
Piecewise((-cos(x)/a, Eq(b, 0) & Eq(n, -1)), (-a**n*cos(x), Eq(b, 0)), (-l
og(a/b + cos(x))/b, Eq(n, -1)), (-a*(a + b*cos(x))**n/(b*n + b) - b*(a + b
*cos(x))**n*cos(x)/(b*n + b), True))
```

3.649.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (a+b\cos(x))^n \sin(x)\,dx = -\frac{(b\cos(x)+a)^{n+1}}{b(n+1)}$$

input
```
integrate((a+b*cos(x))^n*sin(x),x, algorithm="maxima")
```

output
```
-(b*cos(x) + a)^(n + 1)/(b*(n + 1))
```

3.649.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (a+b\cos(x))^n \sin(x)\,dx = -\frac{(b\cos(x)+a)^{n+1}}{b(n+1)}$$

input
```
integrate((a+b*cos(x))^n*sin(x),x, algorithm="giac")
```

output
```
-(b*cos(x) + a)^(n + 1)/(b*(n + 1))
```

3.649.9 Mupad [B] (verification not implemented)

Time = 27.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (a+b\cos(x))^n\sin(x)\,dx=-\frac{(a+b\,\cos\left(x\right))^{n+1}}{b\,\left(n+1\right)}$$

input
```
int(sin(x)*(a + b*cos(x))^n,x)
```

output
```
-(a + b*cos(x))^(n + 1)/(b*(n + 1))
```

3.650 $\int \frac{\sin(x)}{\sqrt{1+\cos^2(x)}}\,dx$

3.650.1 Optimal result

Integrand size = 13, antiderivative size = 5

$$\int \frac{\sin(x)}{\sqrt{1+\cos^2(x)}}\,dx = -\text{arcsinh}(\cos(x))$$

output
```
-arcsinh(cos(x))
```

3.650.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 16 vs. 2(5) = 10.

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 3.20

$$\int \frac{\sin(x)}{\sqrt{1+\cos^2(x)}}\,dx = \log\left(-\cos(x)+\sqrt{1+\cos^2(x)}\right)$$

input
```
Integrate[Sin[x]/Sqrt[1 + Cos[x]^2],x]
```

output
```
Log[-Cos[x] + Sqrt[1 + Cos[x]^2]]
```

3.650.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 25, 3669, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)}{\sqrt{\cos^2(x)+1}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{\cos\left(x+\frac{\pi}{2}\right)}{\sqrt{\sin\left(x+\frac{\pi}{2}\right)^2+1}}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{\cos\left(x+\frac{\pi}{2}\right)}{\sqrt{\sin\left(x+\frac{\pi}{2}\right)^2+1}}dx$$

$$\Big\downarrow \text{ 3669}$$

$$-\int \frac{1}{\sqrt{\cos^2(x)+1}}d\cos(x)$$

$$\Big\downarrow \text{ 222}$$

$$-\text{arcsinh}(\cos(x))$$

input
```
Int[Sin[x]/Sqrt[1 + Cos[x]^2],x]
```

output
```
-ArcSinh[Cos[x]]
```

3.650.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.650.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

method	result	size
derivativedivides	$-\operatorname{arcsinh}(\cos(x))$	6
default	$-\operatorname{arcsinh}(\cos(x))$	6

input
```
int(sin(x)/(cos(x)^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-arcsinh(cos(x))
```

3.650.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(5) = 10$.

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 7.20

$$\int \frac{\sin(x)}{\sqrt{1+\cos^2(x)}}\,dx = \frac{1}{4}\log\left(8\cos(x)^4+8\cos(x)^2-4\left(2\cos(x)^3+\cos(x)\right)\sqrt{\cos(x)^2+1}+1\right)$$

input
```
integrate(sin(x)/(1+cos(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/4*log(8*cos(x)^4 + 8*cos(x)^2 - 4*(2*cos(x)^3 + cos(x))*sqrt(cos(x)^2 +
1) + 1)
```

3.650.6 Sympy [F]

$$\int \frac{\sin(x)}{\sqrt{1+\cos^2(x)}}\,dx = \int \frac{\sin(x)}{\sqrt{\cos^2(x)+1}}\,dx$$

input
```
integrate(sin(x)/(1+cos(x)**2)**(1/2),x)
```

output
```
Integral(sin(x)/sqrt(cos(x)**2 + 1), x)
```

3.650.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{\sqrt{1+\cos^2(x)}}\,dx = -\operatorname{arsinh}(\cos(x))$$

input
```
integrate(sin(x)/(1+cos(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-arcsinh(cos(x))
```

3.650.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14 vs. 2(5) = 10.

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 2.80

$$\int \frac{\sin(x)}{\sqrt{1+\cos^2(x)}}\,dx = \log\left(\sqrt{\cos(x)^2+1}-\cos(x)\right)$$

input
```
integrate(sin(x)/(1+cos(x)^2)^(1/2),x, algorithm="giac")
```

output
```
log(sqrt(cos(x)^2 + 1) - cos(x))
```

3.650.9 Mupad [B] (verification not implemented)

Time = 26.47 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x)}{\sqrt{1+\cos^2(x)}}\,dx = -\text{asinh}(\cos{(x)})$$

input
```
int(sin(x)/(cos(x)^2 + 1)^(1/2),x)
```

output
```
-asinh(cos(x))
```

3.651 $\int \cos(\cos(x))\sin(x)\,dx$

3.651.1 Optimal result

Integrand size = 6, antiderivative size = 5

$$\int \cos(\cos(x))\sin(x)\,dx = -\sin(\cos(x))$$

output
```
-sin(cos(x))
```

3.651.2 Mathematica [A] (verified)

Time = 4.39 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \cos(\cos(x))\sin(x)\,dx = -\sin(\cos(x))$$

input
```
Integrate[Cos[Cos[x]]*Sin[x],x]
```

output
```
-Sin[Cos[x]]
```

3.651.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4835, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\cos(\cos(x))\,dx$$

$$\Big\downarrow \text{ 4835}$$

$$-\int \cos(\cos(x))d\cos(x)$$

$$\Big\downarrow \text{ 3042}$$

$$-\int \sin\left(\cos(x)+\frac{\pi}{2}\right)d\cos(x)$$

$$\Big\downarrow \text{ 3117}$$

$$-\sin(\cos(x))$$

input
```
Int[Cos[Cos[x]]*Sin[x],x]
```

output
```
-Sin[Cos[x]]
```

3.651.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4835
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[1, Cos[c*(a +
b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

3.651.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

method	result	size
derivativedivides	$-\sin(\cos(x))$	6
default	$-\sin(\cos(x))$	6
risch	$-\sin(\cos(x))$	6
parallelrisch	$-\sin(\cos(x))$	6
norman	$\frac{-2\tan\left(\frac{x}{2}\right)^{2}\tan\left(\frac{1-\tan\left(\frac{x}{2}\right)^{2}}{2+2\tan\left(\frac{x}{2}\right)^{2}}\right)-2\tan\left(\frac{1-\tan\left(\frac{x}{2}\right)^{2}}{2+2\tan\left(\frac{x}{2}\right)^{2}}\right)}{\left(1+\tan\left(\frac{1-\tan\left(\frac{x}{2}\right)^{2}}{2+2\tan\left(\frac{x}{2}\right)^{2}}\right)^{2}\right)\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)}$	98

input
```
int(cos(cos(x))*sin(x),x,method=_RETURNVERBOSE)
```

output
```
-sin(cos(x))
```

3.651.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. 2(5) = 10.

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 4.00

$$\int \cos(\cos(x))\sin(x)\,dx = \sin\left(\frac{\tan\left(\frac{1}{2}\,x\right)^{2}-1}{\tan\left(\frac{1}{2}\,x\right)^{2}+1}\right)$$

input
```
integrate(cos(cos(x))*sin(x),x, algorithm="fricas")
```

output
```
sin((tan(1/2*x)^2 - 1)/(tan(1/2*x)^2 + 1))
```

3.651.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \cos(\cos(x))\sin(x)\,dx = -\sin(\cos(x))$$

input
```
integrate(cos(cos(x))*sin(x),x)
```

output
```
-sin(cos(x))
```

3.651.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \cos(\cos(x))\sin(x)\,dx = -\sin(\cos(x))$$

input
```
integrate(cos(cos(x))*sin(x),x, algorithm="maxima")
```

output
```
-sin(cos(x))
```

3.651.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \cos(\cos(x))\sin(x)\,dx = -\sin(\cos(x))$$

input
```
integrate(cos(cos(x))*sin(x),x, algorithm="giac")
```

output
```
-sin(cos(x))
```

3.651.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \cos(\cos(x))\sin(x)\,dx = -\sin\left(\cos\left(x\right)\right)$$

input
```
int(cos(cos(x))*sin(x),x)
```

output
```
-sin(cos(x))
```

3.652 $\int \cos(x)\cos(\cos(x))\sin(x)\sin(\cos(x))\,dx$

3.652.1 Optimal result

Integrand size = 11, antiderivative size = 28

$$\int \cos(x)\cos(\cos(x))\sin(x)\sin(\cos(x))\,dx = \frac{\cos(x)}{4} - \frac{1}{4}\cos(\cos(x))\sin(\cos(x)) - \frac{1}{2}\cos(x)\sin^2(\cos(x))$$

output
```
1/4*cos(x)-1/4*cos(cos(x))*sin(cos(x))-1/2*cos(x)*sin(cos(x))^2
```

3.652.2 Mathematica [A] (verified)

Time = 2.05 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

$$\int \cos(x)\cos(\cos(x))\sin(x)\sin(\cos(x))\,dx = \frac{1}{4}\cos(x)\cos(2\cos(x)) - \frac{1}{8}\sin(2\cos(x))$$

input
```
Integrate[Cos[x]*Cos[Cos[x]]*Sin[x]*Sin[Cos[x]],x]
```

output
```
(Cos[x]*Cos[2*Cos[x]])/4 - Sin[2*Cos[x]]/8
```

3.652.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.18, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {4835, 3924, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\cos(x)\cos(\cos(x))\sin(\cos(x))\,dx$$

$$\Big\downarrow \text{4835}$$

$$-\int \cos(x)\cos(\cos(x))\sin(\cos(x))d\cos(x)$$

$$\Big\downarrow \text{3924}$$

$$\frac{1}{2}\int \sin^2(\cos(x))d\cos(x)-\frac{1}{2}\cos(x)\sin^2(\cos(x))$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \sin(\cos(x))^2d\cos(x)-\frac{1}{2}\cos(x)\sin^2(\cos(x))$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{2}\left(\frac{\int 1d\cos(x)}{2}-\frac{1}{2}\cos(\cos(x))\sin(\cos(x))\right)-\frac{1}{2}\cos(x)\sin^2(\cos(x))$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{2}\left(\frac{\cos(x)}{2}-\frac{1}{2}\cos(\cos(x))\sin(\cos(x))\right)-\frac{1}{2}\cos(x)\sin^2(\cos(x))$$

input
```
Int[Cos[x]*Cos[Cos[x]]*Sin[x]*Sin[Cos[x]],x]
```

output
```
-1/2*(Cos[x]*Sin[Cos[x]]^2) + (Cos[x]/2 - (Cos[Cos[x]]*Sin[Cos[x]])/2)/2
```

3.652.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3924
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)
)), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sin[a + b*x^n]^(p +
 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

rule 4835
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[1, Cos[c*(a +
b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

3.652.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{\cos(x)\cos(\cos(x))^2}{2}-\frac{\cos(\cos(x))\sin(\cos(x))}{4}-\frac{\cos(x)}{4}$	23
default	$\frac{\cos(x)\cos(\cos(x))^2}{2}-\frac{\cos(\cos(x))\sin(\cos(x))}{4}-\frac{\cos(x)}{4}$	23
risch	$\frac{\cos(-2\cos(x)+x)}{8}+\frac{\cos(2\cos(x)+x)}{8}-\frac{\sin(2\cos(x))}{8}$	27
parallelrisch	$-\frac{1}{4}+\frac{\cos(-2\cos(x)+x)}{8}+\frac{\cos(2\cos(x)+x)}{8}-\frac{\sin(2\cos(x))}{8}$	28

input
```
int(cos(x)*cos(cos(x))*sin(x)*sin(cos(x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*cos(x)*cos(cos(x))^2-1/4*cos(cos(x))*sin(cos(x))-1/4*cos(x)
```

3.652.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(22) = 44$.

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.61

$$\int \cos(x)\cos(\cos(x))\sin(x)\sin(\cos(x))\,dx = \frac{1}{2}\cos(x)\cos\left(\frac{\tan\left(\frac{1}{2}x\right)^2-1}{\tan\left(\frac{1}{2}x\right)^2+1}\right)^2 + \frac{1}{4}\cos\left(\frac{\tan\left(\frac{1}{2}x\right)^2-1}{\tan\left(\frac{1}{2}x\right)^2+1}\right)\sin\left(\frac{\tan\left(\frac{1}{2}x\right)^2-1}{\tan\left(\frac{1}{2}x\right)^2+1}\right) - \frac{1}{4}\cos(x)$$

input
```
integrate(cos(x)*cos(cos(x))*sin(x)*sin(cos(x)),x, algorithm="fricas")
```

output
```
1/2*cos(x)*cos((tan(1/2*x)^2 - 1)/(tan(1/2*x)^2 + 1))^2 + 1/4*cos((tan(1/2
*x)^2 - 1)/(tan(1/2*x)^2 + 1))*sin((tan(1/2*x)^2 - 1)/(tan(1/2*x)^2 + 1))
- 1/4*cos(x)
```

3.652.6 Sympy [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.21

$$\int \cos(x)\cos(\cos(x))\sin(x)\sin(\cos(x))\,dx = -\frac{\sin^2(\cos(x))\cos(x)}{4} - \frac{\sin(\cos(x))\cos(\cos(x))}{4} + \frac{\cos(x)\cos^2(\cos(x))}{4}$$

input
```
integrate(cos(x)*cos(cos(x))*sin(x)*sin(cos(x)),x)
```

output
```
-sin(cos(x))**2*cos(x)/4 - sin(cos(x))*cos(cos(x))/4 + cos(x)*cos(cos(x))*
*2/4
```

3.652.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.61

$$\int \cos(x)\cos(\cos(x))\sin(x)\sin(\cos(x))\,dx = \frac{1}{4}\cos(x)\cos(2\cos(x)) - \frac{1}{8}\sin(2\cos(x))$$

input
```
integrate(cos(x)*cos(cos(x))*sin(x)*sin(cos(x)),x, algorithm="maxima")
```

output
```
1/4*cos(x)*cos(2*cos(x)) - 1/8*sin(2*cos(x))
```

3.652.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.61

$$\int \cos(x)\cos(\cos(x))\sin(x)\sin(\cos(x))\,dx = \frac{1}{4}\cos(x)\cos(2\cos(x)) - \frac{1}{8}\sin(2\cos(x))$$

input
```
integrate(cos(x)*cos(cos(x))*sin(x)*sin(cos(x)),x, algorithm="giac")
```

output
```
1/4*cos(x)*cos(2*cos(x)) - 1/8*sin(2*cos(x))
```

3.652.9 Mupad [B] (verification not implemented)

Time = 26.84 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \cos(x)\cos(\cos(x))\sin(x)\sin(\cos(x))\,dx = \frac{\cos(x)\cos(\cos(x))^2}{2} - \frac{\sin(\cos(x))\cos(\cos(x))}{4} - \frac{\cos(x)}{4}$$

input
```
int(cos(cos(x))*sin(cos(x))*cos(x)*sin(x),x)
```

output
```
(cos(cos(x))^2*cos(x))/2 - cos(x)/4 - (cos(cos(x))*sin(cos(x)))/4
```

3.653 $\int \cos(\cos(x))\sin(x)\sin^2(6\cos(x))\,dx$

3.653.1 Optimal result

Integrand size = 13, antiderivative size = 26

$$\int \cos(\cos(x))\sin(x)\sin^2(6\cos(x))\,dx = -\frac{1}{2}\sin(\cos(x)) + \frac{1}{44}\sin(11\cos(x)) + \frac{1}{52}\sin(13\cos(x))$$

output
```
-1/2*sin(cos(x))+1/44*sin(11*cos(x))+1/52*sin(13*cos(x))
```

3.653.2 Mathematica [A] (verified)

Time = 4.55 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \cos(\cos(x))\sin(x)\sin^2(6\cos(x))\,dx = -\frac{1}{2}\sin(\cos(x)) + \frac{1}{44}\sin(11\cos(x)) + \frac{1}{52}\sin(13\cos(x))$$

input
```
Integrate[Cos[Cos[x]]*Sin[x]*Sin[6*Cos[x]]^2,x]
```

output
```
-1/2*Sin[Cos[x]] + Sin[11*Cos[x]]/44 + Sin[13*Cos[x]]/52
```

3.653.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {4835, 3042, 4854, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\cos(\cos(x))\sin^2(6\cos(x))\,dx$$

$$\Big\downarrow \text{4835}$$

$$-\int \cos(\cos(x))\sin^2(6\cos(x))d\cos(x)$$

$$\Big\downarrow \text{3042}$$

$$-\int \cos(\cos(x))\sin(6\cos(x))^2d\cos(x)$$

$$\Big\downarrow \text{4854}$$

$$-\int \left(\frac{1}{2}\cos(\cos(x))-\frac{1}{4}\cos(11\cos(x))-\frac{1}{4}\cos(13\cos(x))\right)d\cos(x)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2}\sin(\cos(x))+\frac{1}{44}\sin(11\cos(x))+\frac{1}{52}\sin(13\cos(x))$$

input
```
Int[Cos[Cos[x]]*Sin[x]*Sin[6*Cos[x]]^2,x]
```

output
```
-1/2*Sin[Cos[x]] + Sin[11*Cos[x]]/44 + Sin[13*Cos[x]]/52
```

3.653.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4835
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[1, Cos[c*(a +
b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

rule 4854
```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol
] :> Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /
; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] ||
 EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]
```

3.653.4 Maple [A] (verified)

Time = 4.43 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

method	result
derivativedivides	$-\frac{\sin(\cos(x))}{2}+\frac{\sin(11\cos(x))}{44}+\frac{\sin(13\cos(x))}{52}$
default	$-\frac{\sin(\cos(x))}{2}+\frac{\sin(11\cos(x))}{44}+\frac{\sin(13\cos(x))}{52}$
risch	$-\frac{\sin(\cos(x))}{2}+\frac{\sin(11\cos(x))}{44}+\frac{\sin(13\cos(x))}{52}$
parallelrisch	$\frac{\left(\sin\left(\frac{3\cos(x)}{2}\right)-3\sin\left(\frac{\cos(x)}{2}\right)\right)\left(\cos\left(\frac{3\cos(x)}{2}\right)+3\cos\left(\frac{\cos(x)}{2}\right)\right)(213+105\cos(6\cos(x))+46\cos(8\cos(x))+11\cos(10\cos($ 143

input
```
int(cos(cos(x))*sin(x)*sin(6*cos(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*sin(cos(x))+1/44*sin(11*cos(x))+1/52*sin(13*cos(x))
```

3.653.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(20) = 40.

Time = 0.25 (sec) , antiderivative size = 168, normalized size of antiderivative = 6.46

$$\int\cos(\cos(x))\sin(x)\sin^2(6\cos(x))\,dx =$$
$$-\frac{4}{143}\left(2816\cos\left(\frac{\tan\left(\frac{1}{2}x\right)^2-1}{\tan\left(\frac{1}{2}x\right)^2+1}\right)^{12}-6912\cos\left(\frac{\tan\left(\frac{1}{2}x\right)^2-1}{\tan\left(\frac{1}{2}x\right)^2+1}\right)^{10}+6048\cos\left(\frac{\tan\left(\frac{1}{2}x\right)^2-1}{\tan\left(\frac{1}{2}x\right)^2+1}\right)^{8}-\right.$$

input
```
integrate(cos(cos(x))*sin(x)*sin(6*cos(x))^2,x, algorithm="fricas")
```

output
```
-4/143*(2816*cos((tan(1/2*x)^2 - 1)/(tan(1/2*x)^2 + 1))^12 - 6912*cos((tan
(1/2*x)^2 - 1)/(tan(1/2*x)^2 + 1))^10 + 6048*cos((tan(1/2*x)^2 - 1)/(tan(1
/2*x)^2 + 1))^8 - 2240*cos((tan(1/2*x)^2 - 1)/(tan(1/2*x)^2 + 1))^6 + 315*
cos((tan(1/2*x)^2 - 1)/(tan(1/2*x)^2 + 1))^4 - 9*cos((tan(1/2*x)^2 - 1)/(t
an(1/2*x)^2 + 1))^2 - 18)*sin((tan(1/2*x)^2 - 1)/(tan(1/2*x)^2 + 1))
```

3.653.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(22) = 44$.

Time = 3.39 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.08

$$\int \cos(\cos(x))\sin(x)\sin^2(6\cos(x))\,dx = -\frac{71\sin(\cos(x))\sin^2(6\cos(x))}{143} - \frac{72\sin(\cos(x))\cos^2(6\cos(x))}{143} + \frac{12\sin(6\cos(x))\cos(\cos(x))\cos(6\cos(x))}{143}$$

input
```
integrate(cos(cos(x))*sin(x)*sin(6*cos(x))**2,x)
```

output
```
-71*sin(cos(x))*sin(6*cos(x))**2/143 - 72*sin(cos(x))*cos(6*cos(x))**2/143
 + 12*sin(6*cos(x))*cos(cos(x))*cos(6*cos(x))/143
```

3.653.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \cos(\cos(x))\sin(x)\sin^2(6\cos(x))\,dx = \frac{1}{52}\sin(13\cos(x)) + \frac{1}{44}\sin(11\cos(x)) - \frac{1}{2}\sin(\cos(x))$$

input
```
integrate(cos(cos(x))*sin(x)*sin(6*cos(x))^2,x, algorithm="maxima")
```

output
```
1/52*sin(13*cos(x)) + 1/44*sin(11*cos(x)) - 1/2*sin(cos(x))
```

3.653.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \cos(\cos(x))\sin(x)\sin^2(6\cos(x))\,dx = \frac{1}{52}\sin(13\cos(x)) + \frac{1}{44}\sin(11\cos(x)) - \frac{1}{2}\sin(\cos(x))$$

input
```
integrate(cos(cos(x))*sin(x)*sin(6*cos(x))^2,x, algorithm="giac")
```

output
```
1/52*sin(13*cos(x)) + 1/44*sin(11*cos(x)) - 1/2*sin(cos(x))
```

3.653.9 Mupad [B] (verification not implemented)

Time = 26.75 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \cos(\cos(x))\sin(x)\sin^2(6\cos(x))\,dx = \frac{\sin(11\cos(x))}{44} - \frac{\sin(\cos(x))}{2} + \frac{\sin(13\cos(x))}{52}$$

input
```
int(cos(cos(x))*sin(6*cos(x))^2*sin(x),x)
```

output
```
sin(11*cos(x))/44 - sin(cos(x))/2 + sin(13*cos(x))/52
```

3.654 $\int \cos^3(x) \left(a+b\cos^2(x)\right)^3 \sin(x)\,dx$

3.654.1 Optimal result

Integrand size = 17, antiderivative size = 36

$$\int \cos^3(x) \left(a+b\cos^2(x)\right)^3 \sin(x)\,dx=\frac{a\left(a+b\cos^2(x)\right)^4}{8b^2}-\frac{\left(a+b\cos^2(x)\right)^5}{10b^2}$$

output
```
1/8*a*(a+b*cos(x)^2)^4/b^2-1/10*(a+b*cos(x)^2)^5/b^2
```

3.654.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 137 vs. 2(36) = 72.

Time = 0.24 (sec) , antiderivative size = 137, normalized size of antiderivative = 3.81

$$\int \cos^3(x) \left(a+b\cos^2(x)\right)^3 \sin(x)\,dx=\frac{1}{32}\Bigg(-12a^2b\cos^4(x)-8ab^2\cos^6(x)-2b^3\cos^8(x) - 4a^3\cos(2x)-4a^2b\cos^3(x)\cos(3x)-a^3\cos(4x) - \frac{1}{32}ab^2(48\cos(2x)+36\cos(4x)+16\cos(6x) + 3\cos(8x))-\frac{1}{320}b^3(140\cos(2x)+100\cos(4x) + 50\cos(6x)+15\cos(8x)+2\cos(10x))\Bigg)$$

input
```
Integrate[Cos[x]^3*(a + b*Cos[x]^2)^3*Sin[x],x]
```

output
```
(-12*a^2*b*Cos[x]^4 - 8*a*b^2*Cos[x]^6 - 2*b^3*Cos[x]^8 - 4*a^3*Cos[2*x] -
 4*a^2*b*Cos[x]^3*Cos[3*x] - a^3*Cos[4*x] - (a*b^2*(48*Cos[2*x] + 36*Cos[4
*x] + 16*Cos[6*x] + 3*Cos[8*x]))/32 - (b^3*(140*Cos[2*x] + 100*Cos[4*x] +
50*Cos[6*x] + 15*Cos[8*x] + 2*Cos[10*x]))/320)/32
```

3.654.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3042, 4835, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\cos^3(x)\left(a+b\cos^2(x)\right)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x)\cos(x)^3\left(a+b\cos(x)^2\right)^3dx$$

$$\Big\downarrow \text{4835}$$

$$-\int \cos^3(x)\left(b\cos^2(x)+a\right)^3d\cos(x)$$

$$\Big\downarrow \text{243}$$

$$-\frac{1}{2}\int \cos^2(x)\left(b\cos^2(x)+a\right)^3d\cos^2(x)$$

$$\Big\downarrow \text{49}$$

$$-\frac{1}{2}\int \left(\frac{\left(b\cos^2(x)+a\right)^4}{b}-\frac{a\left(b\cos^2(x)+a\right)^3}{b}\right)d\cos^2(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{a\left(a+b\cos^2(x)\right)^4}{4b^2}-\frac{\left(a+b\cos^2(x)\right)^5}{5b^2}\right)$$

input
```
Int[Cos[x]^3*(a + b*Cos[x]^2)^3*Sin[x],x]
```

output
```
((a*(a + b*Cos[x]^2)^4)/(4*b^2) - (a + b*Cos[x]^2)^5/(5*b^2))/2
```

3.654. $\int \cos^3(x)\left(a+b\cos^2(x)\right)^3\sin(x)\,dx$

3.654.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4835
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[1, Cos[c*(a +
b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

3.654.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{\frac{a\left(a+b\cos(x)^2\right)^4}{4}-\frac{\left(a+b\cos(x)^2\right)^5}{5}}{2b^2}$
default	$\frac{\frac{a\left(a+b\cos(x)^2\right)^4}{4}-\frac{\left(a+b\cos(x)^2\right)^5}{5}}{2b^2}$
parts	$-\frac{\cos(x)^4a^3}{4}-\frac{\cos(x)^{10}b^3}{10}-\frac{3\cos(x)^8a\,b^2}{8}-\frac{\cos(x)^6a^2b}{2}$
parallelrisch	$\frac{\left(-64a^3-120a^2b-84a\,b^2-21b^3\right)\cos(2x)}{512}+\frac{\left(-8a^3-24a^2b-21a\,b^2-6b^3\right)\cos(4x)}{256}-\frac{b\left(a+\frac{3b}{4}\right)^2\cos(6x)}{64}+\frac{\left(-3a\,b^2-2\right.}{10}$
risch	$-\frac{b^3\cos(10x)}{5120}-\frac{3\cos(8x)a\,b^2}{1024}-\frac{\cos(8x)b^3}{512}-\frac{\cos(6x)a^2b}{64}-\frac{3\cos(6x)a\,b^2}{128}-\frac{9\cos(6x)b^3}{1024}-\frac{\cos(4x)a^3}{32}-3\cos$

input
```
int(cos(x)^3*(a+b*cos(x)^2)^3*sin(x),x,method=_RETURNVERBOSE)
```

output
```
1/2/b^2*(1/4*a*(a+b*cos(x)^2)^4-1/5*(a+b*cos(x)^2)^5)
```

3.654.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int \cos^3(x)\left(a+b\cos^2(x)\right)^3\sin(x)\,dx=-\frac{1}{10}\,b^3\cos\left(x\right)^{10}-\frac{3}{8}\,ab^2\cos\left(x\right)^8-\frac{1}{2}\,a^2b\cos\left(x\right)^6-\frac{1}{4}\,a^3\cos\left(x\right)^4$$

input
```
integrate(cos(x)^3*(a+b*cos(x)^2)^3*sin(x),x, algorithm="fricas")
```

output
```
-1/10*b^3*cos(x)^10 - 3/8*a*b^2*cos(x)^8 - 1/2*a^2*b*cos(x)^6 - 1/4*a^3*co
s(x)^4
```

3.654.6 Sympy [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.28

$$\int \cos^3(x)\left(a+b\cos^2(x)\right)^3\sin(x)\,dx=-\frac{a^3\cos^4\left(x\right)}{4}-\frac{a^2b\cos^6\left(x\right)}{2}-\frac{3ab^2\cos^8\left(x\right)}{8}-\frac{b^3\cos^{10}\left(x\right)}{10}$$

input
```
integrate(cos(x)**3*(a+b*cos(x)**2)**3*sin(x),x)
```

output
```
-a**3*cos(x)**4/4 - a**2*b*cos(x)**6/2 - 3*a*b**2*cos(x)**8/8 - b**3*cos(x
)**10/10
```

3.654.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(32) = 64.

Time = 0.20 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.86

$$\int \cos^3(x)\left(a+b\cos^2(x)\right)^3\sin(x)\,dx=\frac{1}{10}\,b^3\sin\left(x\right)^{10}-\frac{1}{8}\left(3\,ab^2+4\,b^3\right)\sin\left(x\right)^8+\frac{1}{2}\left(a^2b+3\,ab^2+2\,b^3\right)\sin\left(x\right)^6-\frac{1}{4}\left(a^3+6\,a^2b+9\,ab^2+4\,b^3\right)\sin\left(x\right)^4+\frac{1}{2}\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\sin\left(x\right)^2$$

input
```
integrate(cos(x)^3*(a+b*cos(x)^2)^3*sin(x),x, algorithm="maxima")
```

output
```
1/10*b^3*sin(x)^10 - 1/8*(3*a*b^2 + 4*b^3)*sin(x)^8 + 1/2*(a^2*b + 3*a*b^2
 + 2*b^3)*sin(x)^6 - 1/4*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*sin(x)^4 + 1/2*
(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sin(x)^2
```

3.654.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int \cos^3(x) \left(a+b\cos^2(x)\right)^3 \sin(x)\,dx = -\frac{1}{10}\,b^3\cos\left(x\right)^{10} - \frac{3}{8}\,ab^2\cos\left(x\right)^8 - \frac{1}{2}\,a^2b\cos\left(x\right)^6 - \frac{1}{4}\,a^3\cos\left(x\right)^4$$

input
```
integrate(cos(x)^3*(a+b*cos(x)^2)^3*sin(x),x, algorithm="giac")
```

output
```
-1/10*b^3*cos(x)^10 - 3/8*a*b^2*cos(x)^8 - 1/2*a^2*b*cos(x)^6 - 1/4*a^3*co
s(x)^4
```

3.654.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int \cos^3(x) \left(a+b\cos^2(x)\right)^3 \sin(x)\,dx = -\frac{a^3\cos\left(x\right)^4}{4} - \frac{a^2\,b\cos\left(x\right)^6}{2} - \frac{3\,a\,b^2\cos\left(x\right)^8}{8} - \frac{b^3\cos\left(x\right)^{10}}{10}$$

input
```
int(cos(x)^3*sin(x)*(a + b*cos(x)^2)^3,x)
```

output
```
- (a^3*cos(x)^4)/4 - (b^3*cos(x)^10)/10 - (a^2*b*cos(x)^6)/2 - (3*a*b^2*co
s(x)^8)/8
```

3.655 $\int \sin(3x)\sin(\cos(3x))\,dx$

3.655.1 Optimal result

Integrand size = 10, antiderivative size = 9

$$\int \sin(3x)\sin(\cos(3x))\,dx = \frac{1}{3}\cos(\cos(3x))$$

output
```
1/3*cos(cos(3*x))
```

3.655.2 Mathematica [A] (verified)

Time = 2.31 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \sin(3x)\sin(\cos(3x))\,dx = \frac{1}{3}\cos(\cos(3x))$$

input
```
Integrate[Sin[3*x]*Sin[Cos[3*x]],x]
```

output
```
Cos[Cos[3*x]]/3
```

3.655.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4835, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(3x)\sin(\cos(3x))\,dx$$

$$\Big\downarrow \text{4835}$$

$$-\frac{1}{3}\int \sin(\cos(3x))d\cos(3x)$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{3}\int \sin(\cos(3x))d\cos(3x)$$

$$\Big\downarrow \text{3118}$$

$$\frac{1}{3}\cos(\cos(3x))$$

input
```
Int[Sin[3*x]*Sin[Cos[3*x]],x]
```

output
```
Cos[Cos[3*x]]/3
```

3.655.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 4835
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[1, Cos[c*(a +
b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

3.655.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{\cos(\cos(3x))}{3}$	8
default	$\frac{\cos(\cos(3x))}{3}$	8
risch	$\frac{\cos(\cos(3x))}{3}$	8
parallelrisch	$\frac{\cos(\cos(3x))}{3}+\frac{1}{3}$	10
norman	$\frac{\frac{2\tan\left(\frac{3x}{2}\right)^{2}}{3}+\frac{2}{3}}{\left(1+\tan\left(\frac{3x}{2}\right)^{2}\right)\left(1+\tan\left(\frac{1-\tan\left(\frac{3x}{2}\right)^{2}}{2+2\tan\left(\frac{3x}{2}\right)^{2}}\right)^{2}\right)}$	51

input
```
int(sin(3*x)*sin(cos(3*x)),x,method=_RETURNVERBOSE)
```

output
```
1/3*cos(cos(3*x))
```

3.655.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(7) = 14$.

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.44

$$\int \sin(3x)\sin(\cos(3x))\,dx = \frac{1}{3}\cos\left(\frac{\tan\left(\frac{3}{2}\,x\right)^2-1}{\tan\left(\frac{3}{2}\,x\right)^2+1}\right)$$

input
```
integrate(sin(3*x)*sin(cos(3*x)),x, algorithm="fricas")
```

output
```
1/3*cos((tan(3/2*x)^2 - 1)/(tan(3/2*x)^2 + 1))
```

3.655.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \sin(3x)\sin(\cos(3x))\,dx = \frac{\cos(\cos(3x))}{3}$$

input
```
integrate(sin(3*x)*sin(cos(3*x)),x)
```

output
```
cos(cos(3*x))/3
```

3.655.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \sin(3x)\sin(\cos(3x))\,dx = \frac{1}{3}\cos(\cos(3x))$$

input
```
integrate(sin(3*x)*sin(cos(3*x)),x, algorithm="maxima")
```

output
```
1/3*cos(cos(3*x))
```

3.655.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \sin(3x)\sin(\cos(3x))\,dx = \frac{1}{3}\cos(\cos(3x))$$

input
```
integrate(sin(3*x)*sin(cos(3*x)),x, algorithm="giac")
```

output
```
1/3*cos(cos(3*x))
```

3.655.9 Mupad [B] (verification not implemented)

Time = 26.80 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \sin(3x)\sin(\cos(3x))\,dx = \frac{\cos\left(\cos\left(3\,x\right)\right)}{3}$$

input
```
int(sin(3*x)*sin(cos(3*x)),x)
```

output
```
cos(cos(3*x))/3
```

3.656 $\int e^{\cos(1+3x)} \cos(1+3x) \sin(1+3x) \, dx$

3.656.1 Optimal result

Integrand size = 21, antiderivative size = 31

$$\int e^{\cos(1+3x)} \cos(1+3x) \sin(1+3x) \, dx = \frac{1}{3} e^{\cos(1+3x)} - \frac{1}{3} e^{\cos(1+3x)} \cos(1+3x)$$

output
```
1/3*exp(cos(1+3*x))-1/3*exp(cos(1+3*x))*cos(1+3*x)
```

3.656.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

$$\int e^{\cos(1+3x)} \cos(1+3x) \sin(1+3x) \, dx = \frac{2}{3} e^{\cos(1+3x)} \sin^2\left(\frac{1}{2}(1+3x)\right)$$

input
```
Integrate[E^Cos[1 + 3*x]*Cos[1 + 3*x]*Sin[1 + 3*x],x]
```

output
```
(2*E^Cos[1 + 3*x]*Sin[(1 + 3*x)/2]^2)/3
```

3.656.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4835, 2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(3x+1)e^{\cos(3x+1)}\cos(3x+1)\,dx$$

$$\Big\downarrow \text{ 4835}$$

$$-\frac{1}{3}\int e^{\cos(3x+1)}\cos(3x+1)d\cos(3x+1)$$

$$\Big\downarrow \text{ 2607}$$

$$\frac{1}{3}\left(\int e^{\cos(3x+1)}d\cos(3x+1)-e^{\cos(3x+1)}\cos(3x+1)\right)$$

$$\Big\downarrow \text{ 2624}$$

$$\frac{1}{3}\left(e^{\cos(3x+1)}-e^{\cos(3x+1)}\cos(3x+1)\right)$$

input
```
Int[E^Cos[1 + 3*x]*Cos[1 + 3*x]*Sin[1 + 3*x],x]
```

output
```
(E^Cos[1 + 3*x] - E^Cos[1 + 3*x]*Cos[1 + 3*x])/3
```

3.656.3.1 Defintions of rubi rules used

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 4835
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[1, Cos[c*(a +
b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

3.656.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.58

method	result
parallelrisch	$-\frac{e^{\cos(1+3x)}(-1+\cos(1+3x))}{3}$
derivativedivides	$\frac{e^{\cos(1+3x)}}{3}-\frac{e^{\cos(1+3x)}\cos(1+3x)}{3}$
default	$\frac{e^{\cos(1+3x)}}{3}-\frac{e^{\cos(1+3x)}\cos(1+3x)}{3}$
norman	$\frac{\frac{2\tan\left(\frac{1}{2}+\frac{3x}{2}\right)^{2} e^{\frac{1-\tan\left(\frac{1}{2}+\frac{3x}{2}\right)^{2}}{1+\tan\left(\frac{1}{2}+\frac{3x}{2}\right)^{2}}}}{3}+\frac{2\tan\left(\frac{1}{2}+\frac{3x}{2}\right)^{4} e^{\frac{1-\tan\left(\frac{1}{2}+\frac{3x}{2}\right)^{2}}{1+\tan\left(\frac{1}{2}+\frac{3x}{2}\right)^{2}}}}{3}}{\left(1+\tan\left(\frac{1}{2}+\frac{3x}{2}\right)^{2}\right)^{2}}$
risch	$-\frac{e^{4\cos(1)\cos(x)^{3}-3\cos(1)\cos(x)-4\sin(1)\cos(x)^{2}\sin(x)+\sin(1)\sin(x)} e^{i} e^{3ix}}{6}+\frac{e^{4\cos(1)\cos(x)^{3}-3\cos(1)\cos(x)-4\sin(1)\cos(x)^{2}}}{3}$

input
```
int(exp(cos(1+3*x))*cos(1+3*x)*sin(1+3*x),x,method=_RETURNVERBOSE)
```

output
```
-1/3*exp(cos(1+3*x))*(-1+cos(1+3*x))
```

3.656.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.55

$$\int e^{\cos(1+3x)}\cos(1+3x)\sin(1+3x)\,dx=-\frac{1}{3}\left(\cos\left(3\,x+1\right)-1\right)e^{\left(\cos\left(3\,x+1\right)\right)}$$

input
```
integrate(exp(cos(1+3*x))*cos(1+3*x)*sin(1+3*x),x, algorithm="fricas")
```

output
```
-1/3*(cos(3*x + 1) - 1)*e^(cos(3*x + 1))
```

3.656.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

$$\int e^{\cos(1+3x)}\cos(1+3x)\sin(1+3x)\,dx = -\frac{e^{\cos(3x+1)}\cos(3x+1)}{3} + \frac{e^{\cos(3x+1)}}{3}$$

input
```
integrate(exp(cos(1+3*x))*cos(1+3*x)*sin(1+3*x),x)
```

output
```
-exp(cos(3*x + 1))*cos(3*x + 1)/3 + exp(cos(3*x + 1))/3
```

3.656.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.55

$$\int e^{\cos(1+3x)}\cos(1+3x)\sin(1+3x)\,dx = -\frac{1}{3}\left(\cos\left(3\,x+1\right)-1\right)e^{\left(\cos\left(3\,x+1\right)\right)}$$

input
```
integrate(exp(cos(1+3*x))*cos(1+3*x)*sin(1+3*x),x, algorithm="maxima")
```

output
```
-1/3*(cos(3*x + 1) - 1)*e^(cos(3*x + 1))
```

3.656.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.55

$$\int e^{\cos(1+3x)}\cos(1+3x)\sin(1+3x)\,dx = -\frac{1}{3}\left(\cos\left(3\,x+1\right)-1\right)e^{\left(\cos\left(3\,x+1\right)\right)}$$

input
```
integrate(exp(cos(1+3*x))*cos(1+3*x)*sin(1+3*x),x, algorithm="giac")
```

output
```
-1/3*(cos(3*x + 1) - 1)*e^(cos(3*x + 1))
```

3.656.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.55

$$\int e^{\cos(1+3x)}\cos(1+3x)\sin(1+3x)\,dx = -\frac{e^{\cos(3\,x+1)}\,\left(\cos\left(3\,x+1\right)-1\right)}{3}$$

input
```
int(exp(cos(3*x + 1))*cos(3*x + 1)*sin(3*x + 1),x)
```

output
```
-(exp(cos(3*x + 1))*(cos(3*x + 1) - 1))/3
```

3.657 $\int \frac{\cos^2(x)\sin(x)}{\sqrt{1-\cos^6(x)}}\,dx$

3.657.1 Optimal result

Integrand size = 19, antiderivative size = 9

$$\int \frac{\cos^2(x)\sin(x)}{\sqrt{1-\cos^6(x)}}\,dx = -\frac{1}{3}\arcsin\left(\cos^3(x)\right)$$

output
```
-1/3*arcsin(cos(x)^3)
```

3.657.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 2.64 (sec) , antiderivative size = 162, normalized size of antiderivative = 18.00

$$\int \frac{\cos^2(x)\sin(x)}{\sqrt{1-\cos^6(x)}}\,dx = -\frac{i\cos^2(x)\,\text{EllipticPi}\left(\frac{3}{2}+\frac{i\sqrt{3}}{2}, i\,\text{arcsinh}\left(\sqrt{-\frac{2i}{-3i+\sqrt{3}}}\tan(x)\right), \frac{3i-\sqrt{3}}{3i+\sqrt{3}}\right)\sin(x)\sqrt{1-\frac{2i\tan^2(x)}{-3i+\sqrt{3}}}\sqrt{1+\frac{2i\tan^2(x)}{3i+\sqrt{3}}}}{\sqrt{2}\sqrt{-\frac{i}{-3i+\sqrt{3}}}\sqrt{1-\cos^6(x)}}$$

input
```
Integrate[(Cos[x]^2*Sin[x])/Sqrt[1 - Cos[x]^6],x]
```

output
```
((-I)*Cos[x]^2*EllipticPi[3/2 + (I/2)*Sqrt[3], I*ArcSinh[Sqrt[(-2*I)/(-3*I
 + Sqrt[3])]*Tan[x]], (3*I - Sqrt[3])/(3*I + Sqrt[3])]*Sin[x]*Sqrt[1 - ((2
*I)*Tan[x]^2)/(-3*I + Sqrt[3])]*Sqrt[1 + ((2*I)*Tan[x]^2)/(3*I + Sqrt[3])]
)/(Sqrt[2]*Sqrt[(-I)/(-3*I + Sqrt[3])]*Sqrt[1 - Cos[x]^6])
```

3.657.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 4835, 807, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)\cos^2(x)}{\sqrt{1-\cos^6(x)}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)\cos(x)^2}{\sqrt{1-\cos(x)^6}}dx$$

$$\Big\downarrow \text{4835}$$

$$-\int \frac{\cos^2(x)}{\sqrt{1-\cos^6(x)}}d\cos(x)$$

$$\Big\downarrow \text{807}$$

$$-\frac{1}{3}\int \frac{1}{\sqrt{1-\cos^6(x)}}d\cos^3(x)$$

$$\Big\downarrow \text{223}$$

$$-\frac{1}{3}\arcsin\left(\cos^3(x)\right)$$

input
```
Int[(Cos[x]^2*Sin[x])/Sqrt[1 - Cos[x]^6],x]
```

output
```
-1/3*ArcSin[Cos[x]^3]
```

3.657.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4835
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[1, Cos[c*(a +
b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

3.657.4 Maple [F]

$$\int \frac{\cos(x)^2 \sin(x)}{\sqrt{1-\cos(x)^6}}\,dx$$

input
```
int(cos(x)^2*sin(x)/(1-cos(x)^6)^(1/2),x)
```

output
```
int(cos(x)^2*sin(x)/(1-cos(x)^6)^(1/2),x)
```

3.657.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(7) = 14$.

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 3.22

$$\int \frac{\cos^2(x)\sin(x)}{\sqrt{1-\cos^6(x)}}\,dx = \frac{1}{6}\arctan\left(\frac{2\sqrt{-\cos(x)^6+1}\cos(x)^3}{2\cos(x)^6-1}\right)$$

input
```
integrate(cos(x)^2*sin(x)/(1-cos(x)^6)^(1/2),x, algorithm="fricas")
```

output
```
1/6*arctan(2*sqrt(-cos(x)^6 + 1)*cos(x)^3/(2*cos(x)^6 - 1))
```

3.657.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^2(x)\sin(x)}{\sqrt{1-\cos^6(x)}}\,dx = \text{Timed out}$$

input
```
integrate(cos(x)**2*sin(x)/(1-cos(x)**6)**(1/2),x)
```

output
```
Timed out
```

3.657.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. $2(7) = 14$.

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.00

$$\int \frac{\cos^2(x)\sin(x)}{\sqrt{1-\cos^6(x)}}\,dx = \frac{1}{3}\arctan\left(\frac{\sqrt{-\cos(x)^6+1}}{\cos(x)^3}\right)$$

input
```
integrate(cos(x)^2*sin(x)/(1-cos(x)^6)^(1/2),x, algorithm="maxima")
```

output
```
1/3*arctan(sqrt(-cos(x)^6 + 1)/cos(x)^3)
```

3.657.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{\cos^2(x)\sin(x)}{\sqrt{1-\cos^6(x)}}\,dx = -\frac{1}{3}\arcsin\left(\cos(x)^3\right)$$

input
```
integrate(cos(x)^2*sin(x)/(1-cos(x)^6)^(1/2),x, algorithm="giac")
```

output
```
-1/3*arcsin(cos(x)^3)
```

3.657.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(x)\sin(x)}{\sqrt{1-\cos^6(x)}}\,dx = \int \frac{\cos(x)^2\sin(x)}{\sqrt{1-\cos(x)^6}}\,dx$$

input
```
int((cos(x)^2*sin(x))/(1 - cos(x)^6)^(1/2),x)
```

output
```
int((cos(x)^2*sin(x))/(1 - cos(x)^6)^(1/2), x)
```

3.658 $\int \frac{\sin^5(x)}{\sqrt{1-5\cos(x)}}\,dx$

3.658.1 Optimal result

Integrand size = 15, antiderivative size = 71

$$\int \frac{\sin^5(x)}{\sqrt{1-5\cos(x)}}\,dx = \frac{1152\sqrt{1-5\cos(x)}}{3125}+\frac{64(1-5\cos(x))^{3/2}}{3125} -\frac{88(1-5\cos(x))^{5/2}}{15625}-\frac{8(1-5\cos(x))^{7/2}}{21875}+\frac{2(1-5\cos(x))^{9/2}}{28125}$$

output
```
64/3125*(1-5*cos(x))^(3/2)-88/15625*(1-5*cos(x))^(5/2)-8/21875*(1-5*cos(x)
)^(7/2)+2/28125*(1-5*cos(x))^(9/2)+1152/3125*(1-5*cos(x))^(1/2)
```

3.658.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.83

$$\int \frac{\sin^5(x)}{\sqrt{1-5\cos(x)}}\,dx = \frac{180607\left(-1+\sqrt{1-5\cos(x)}\right)}{562500}+\sqrt{1-5\cos(x)}\left(-\frac{6772\cos(x)}{196875} -\frac{2227\cos(2x)}{39375}+\frac{4\cos(3x)}{1575}+\frac{1}{180}\cos(4x)\right)$$

input
```
Integrate[Sin[x]^5/Sqrt[1 - 5*Cos[x]],x]
```

output
```
(180607*(-1 + Sqrt[1 - 5*Cos[x]]))/562500 + Sqrt[1 - 5*Cos[x]]*((-6772*Cos
[x])/196875 - (2227*Cos[2*x])/39375 + (4*Cos[3*x])/1575 + Cos[4*x]/180)
```

3.658.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 3147, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^5(x)}{\sqrt{1-5\cos(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos\left(x-\frac{\pi}{2}\right)^5}{\sqrt{5\sin\left(x-\frac{\pi}{2}\right)+1}}dx$$

$$\Big\downarrow \text{3147}$$

$$\frac{\int \frac{\left(25-25\cos^2(x)\right)^2}{\sqrt{1-5\cos(x)}}d(-5\cos(x))}{3125}$$

$$\Big\downarrow \text{476}$$

$$\frac{\int \left((1-5\cos(x))^{7/2}-4(1-5\cos(x))^{5/2}-44(1-5\cos(x))^{3/2}+96\sqrt{1-5\cos(x)}+\frac{576}{\sqrt{1-5\cos(x)}}\right)d(-5\cos(x))}{3125}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{2}{9}(1-5\cos(x))^{9/2}-\frac{8}{7}(1-5\cos(x))^{7/2}-\frac{88}{5}(1-5\cos(x))^{5/2}+64(1-5\cos(x))^{3/2}+1152\sqrt{1-5\cos(x)}}{3125}$$

input
```
Int[Sin[x]^5/Sqrt[1 - 5*Cos[x]],x]
```

output
```
(1152*Sqrt[1 - 5*Cos[x]] + 64*(1 - 5*Cos[x])^(3/2) - (88*(1 - 5*Cos[x])^(5
/2))/5 - (8*(1 - 5*Cos[x])^(7/2))/7 + (2*(1 - 5*Cos[x])^(9/2))/9)/3125
```

3.658.3.1 Defintions of rubi rules used

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.658.4 Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$\frac{64(1-5\cos(x))^{\frac{3}{2}}}{3125}-\frac{88(1-5\cos(x))^{\frac{5}{2}}}{15625}-\frac{8(1-5\cos(x))^{\frac{7}{2}}}{21875}+\frac{2(1-5\cos(x))^{\frac{9}{2}}}{28125}+\frac{1152\sqrt{1-5\cos(x)}}{3125}$	52
default	$\frac{64(1-5\cos(x))^{\frac{3}{2}}}{3125}-\frac{88(1-5\cos(x))^{\frac{5}{2}}}{15625}-\frac{8(1-5\cos(x))^{\frac{7}{2}}}{21875}+\frac{2(1-5\cos(x))^{\frac{9}{2}}}{28125}+\frac{1152\sqrt{1-5\cos(x)}}{3125}$	52

input
```
int(sin(x)^5/(1-5*cos(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
64/3125*(1-5*cos(x))^(3/2)-88/15625*(1-5*cos(x))^(5/2)-8/21875*(1-5*cos(x)
)^(7/2)+2/28125*(1-5*cos(x))^(9/2)+1152/3125*(1-5*cos(x))^(1/2)
```

3.658.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.48

$$\int \frac{\sin^5(x)}{\sqrt{1-5\cos(x)}}\,dx$$
$$=\frac{2}{984375}\left(21875\cos(x)^4+5000\cos(x)^3-77550\cos(x)^2-20680\cos(x)+188603\right)\sqrt{-5\cos(x)+1}$$

input
```
integrate(sin(x)^5/(1-5*cos(x))^(1/2),x, algorithm="fricas")
```

output
```
2/984375*(21875*cos(x)^4 + 5000*cos(x)^3 - 77550*cos(x)^2 - 20680*cos(x) +
 188603)*sqrt(-5*cos(x) + 1)
```

3.658.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin^5(x)}{\sqrt{1-5\cos(x)}}\,dx=\text{Timed out}$$

input
```
integrate(sin(x)**5/(1-5*cos(x))**(1/2),x)
```

output
```
Timed out
```

3.658.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.72

$$\int \frac{\sin^5(x)}{\sqrt{1-5\cos(x)}}\,dx=\frac{2}{28125}(-5\cos(x)+1)^{\frac{9}{2}}-\frac{8}{21875}(-5\cos(x)+1)^{\frac{7}{2}}-\frac{88}{15625}(-5\cos(x)+1)^{\frac{5}{2}}+\frac{64}{3125}(-5\cos(x)+1)^{\frac{3}{2}}+\frac{1152}{3125}\sqrt{-5\cos(x)+1}$$

input
```
integrate(sin(x)^5/(1-5*cos(x))^(1/2),x, algorithm="maxima")
```

output
```
2/28125*(-5*cos(x) + 1)^(9/2) - 8/21875*(-5*cos(x) + 1)^(7/2) - 88/15625*(
-5*cos(x) + 1)^(5/2) + 64/3125*(-5*cos(x) + 1)^(3/2) + 1152/3125*sqrt(-5*c
os(x) + 1)
```

3.658.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.06

$$\int \frac{\sin^5(x)}{\sqrt{1-5\cos(x)}}\,dx = \frac{2}{28125}\,(5\,\cos\left(x\right)-1)^4\sqrt{-5\,\cos\left(x\right)+1} + \frac{8}{21875}\,(5\,\cos\left(x\right)-1)^3\sqrt{-5\,\cos\left(x\right)+1} - \frac{88}{15625}\,(5\,\cos\left(x\right)-1)^2\sqrt{-5\,\cos\left(x\right)+1} + \frac{64}{3125}\,(-5\,\cos\left(x\right)+1)^{\frac{3}{2}} + \frac{1152}{3125}\,\sqrt{-5\,\cos\left(x\right)+1}$$

input
```
integrate(sin(x)^5/(1-5*cos(x))^(1/2),x, algorithm="giac")
```

output
```
2/28125*(5*cos(x) - 1)^4*sqrt(-5*cos(x) + 1) + 8/21875*(5*cos(x) - 1)^3*sq
rt(-5*cos(x) + 1) - 88/15625*(5*cos(x) - 1)^2*sqrt(-5*cos(x) + 1) + 64/312
5*(-5*cos(x) + 1)^(3/2) + 1152/3125*sqrt(-5*cos(x) + 1)
```

3.658.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin^5(x)}{\sqrt{1-5\cos(x)}}\,dx = \int \frac{\sin\left(x\right)^5}{\sqrt{1-5\,\cos\left(x\right)}}\,dx$$

input
```
int(sin(x)^5/(1 - 5*cos(x))^(1/2),x)
```

output
```
int(sin(x)^5/(1 - 5*cos(x))^(1/2), x)
```

3.659 $\int e^{n\cos(a+bx)}\sin(a+bx)\,dx$

3.659.1 Optimal result

Integrand size = 17, antiderivative size = 18

$$\int e^{n\cos(a+bx)}\sin(a+bx)\,dx=-\frac{e^{n\cos(a+bx)}}{bn}$$

output
```
-exp(n*cos(b*x+a))/b/n
```

3.659.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int e^{n\cos(a+bx)}\sin(a+bx)\,dx=-\frac{e^{n\cos(a+bx)}}{bn}$$

input
```
Integrate[E^(n*Cos[a + b*x])*Sin[a + b*x],x]
```

output
```
-(E^(n*Cos[a + b*x])/(b*n))
```

3.659.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4835, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)e^{n\cos(a+bx)}\,dx$$

$$\Big\downarrow \text{ 4835}$$

$$-\frac{\int e^{n\cos(a+bx)}d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{ 2624}$$

$$-\frac{e^{n\cos(a+bx)}}{bn}$$

input
```
Int[E^(n*Cos[a + b*x])*Sin[a + b*x],x]
```

output
```
-(E^(n*Cos[a + b*x])/(b*n))
```

3.659.3.1 Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 4835
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[1, Cos[c*(a +
b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

3.659.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$-\frac{e^{n\cos(xb+a)}}{bn}$	18
default	$-\frac{e^{n\cos(xb+a)}}{bn}$	18
risch	$-\frac{e^{n\cos(xb+a)}}{bn}$	18
parallelrisch	$-\frac{e^{n\cos(xb+a)}}{bn}$	18
norman	$\frac{-\frac{e^{\frac{n\left(1-\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}\right)}{1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}}}}{nb}-\frac{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}e^{\frac{n\left(1-\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}\right)}{1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}}}}{nb}}{1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}}$	111

input
```
int(exp(n*cos(b*x+a))*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-exp(n*cos(b*x+a))/b/n
```

3.659.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int e^{n\cos(a+bx)}\sin(a+bx)\,dx=-\frac{e^{(n\cos(bx+a))}}{bn}$$

input
```
integrate(exp(n*cos(b*x+a))*sin(b*x+a),x, algorithm="fricas")
```

output
```
-e^(n*cos(b*x + a))/(b*n)
```

3.659.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(14) = 28.

Time = 0.14 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.17

$$\int e^{n\cos(a+bx)}\sin(a+bx)\,dx = \begin{cases} x\sin(a) & \text{for } b=0\wedge n=0 \\ xe^{n\cos(a)}\sin(a) & \text{for } b=0 \\ -\frac{\cos(a+bx)}{b} & \text{for } n=0 \\ -\frac{e^{n\cos(a+bx)}}{bn} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(n*cos(b*x+a))*sin(b*x+a),x)
```

output
```
Piecewise((x*sin(a), Eq(b, 0) & Eq(n, 0)), (x*exp(n*cos(a))*sin(a), Eq(b,
0)), (-cos(a + b*x)/b, Eq(n, 0)), (-exp(n*cos(a + b*x))/(b*n), True))
```

3.659.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int e^{n\cos(a+bx)}\sin(a+bx)\,dx = -\frac{e^{(n\cos(bx+a))}}{bn}$$

input
```
integrate(exp(n*cos(b*x+a))*sin(b*x+a),x, algorithm="maxima")
```

output
```
-e^(n*cos(b*x + a))/(b*n)
```

3.659.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int e^{n\cos(a+bx)}\sin(a+bx)\,dx = -\frac{e^{(n\cos(bx+a))}}{bn}$$

input
```
integrate(exp(n*cos(b*x+a))*sin(b*x+a),x, algorithm="giac")
```

output
```
-e^(n*cos(b*x + a))/(b*n)
```

3.659.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int e^{n\cos(a+bx)}\sin(a+bx)\,dx=-\frac{\mathrm{e}^{n\,\cos(a+b\,x)}}{b\,n}$$

input
```
int(exp(n*cos(a + b*x))*sin(a + b*x),x)
```

output
```
-exp(n*cos(a + b*x))/(b*n)
```

3.660 $\int e^{n\cos(ac+bcx)}\sin(c(a+bx))\,dx$

3.660.1 Optimal result

Integrand size = 22, antiderivative size = 23

$$\int e^{n\cos(ac+bcx)}\sin(c(a+bx))\,dx=-\frac{e^{n\cos(c(a+bx))}}{bcn}$$

output
```
-exp(n*cos(c*(b*x+a)))/b/c/n
```

3.660.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int e^{n\cos(ac+bcx)}\sin(c(a+bx))\,dx=-\frac{e^{n\cos(c(a+bx))}}{bcn}$$

input
```
Integrate[E^(n*Cos[a*c + b*c*x])*Sin[c*(a + b*x)],x]
```

output
```
-(E^(n*Cos[c*(a + b*x)])/(b*c*n))
```

3.660.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4835, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(c(a+bx))e^{n\cos(ac+bcx)}\,dx$$

$$\Big\downarrow \text{ 4835}$$

$$-\frac{\int e^{n\cos(c(a+bx))}d\cos(c(a+bx))}{bc}$$

$$\Big\downarrow \text{ 2624}$$

$$-\frac{e^{n\cos(c(a+bx))}}{bcn}$$

input
```
Int[E^(n*Cos[a*c + b*c*x])*Sin[c*(a + b*x)],x]
```

output
```
-(E^(n*Cos[c*(a + b*x)])/(b*c*n))
```

3.660.3.1 Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 4835
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[1, Cos[c*(a +
b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

3.660.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

method	result	size
risch	$-\frac{e^{n\cos(c(xb+a))}}{bcn}$	23
parallelrisch	$-\frac{e^{n\cos(c(xb+a))}}{bcn}$	23
derivativedivides	$-\frac{e^{n\cos(bcx+ac)}}{bcn}$	24
default	$-\frac{e^{n\cos(bcx+ac)}}{bcn}$	24

input
```
int(exp(n*cos(b*c*x+a*c))*sin(c*(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-exp(n*cos(c*(b*x+a)))/b/c/n
```

3.660.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int e^{n\cos(ac+bcx)}\sin(c(a+bx))\,dx=-\frac{e^{(n\cos(bcx+ac))}}{bcn}$$

input
```
integrate(exp(n*cos(b*c*x+a*c))*sin(c*(b*x+a)),x, algorithm="fricas")
```

output
```
-e^(n*cos(b*c*x + a*c))/(b*c*n)
```

3.660.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(17) = 34$.

Time = 4.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.30

$$\int e^{n\cos(ac+bcx)}\sin(c(a+bx))\,dx=\begin{cases} xe^{n\cos(ac)}\sin(ac) & \text{for } b=0 \\ 0 & \text{for } c=0 \\ \begin{cases} x\sin(ac) & \text{for } b=0 \\ 0 & \text{for } c=0 \\ -\frac{\cos(c(a+bx))}{bc} & \text{otherwise} \end{cases} & \text{for } n=0 \\ -\frac{e^{n\cos(ac+bcx)}}{bcn} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(n*cos(b*c*x+a*c))*sin(c*(b*x+a)),x)
```

output
```
Piecewise((x*exp(n*cos(a*c))*sin(a*c), Eq(b, 0)), (0, Eq(c, 0)), (Piecewis
e((x*sin(a*c), Eq(b, 0)), (0, Eq(c, 0)), (-cos(c*(a + b*x))/(b*c), True)),
 Eq(n, 0)), (-exp(n*cos(a*c + b*c*x))/(b*c*n), True))
```

3.660.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int e^{n\cos(ac+bcx)}\sin(c(a+bx))\,dx=-\frac{e^{(n\cos(bcx+ac))}}{bcn}$$

input
```
integrate(exp(n*cos(b*c*x+a*c))*sin(c*(b*x+a)),x, algorithm="maxima")
```

output
```
-e^(n*cos(b*c*x + a*c))/(b*c*n)
```

3.660.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int e^{n\cos(ac+bcx)}\sin(c(a+bx))\,dx=-\frac{e^{(n\cos(bcx+ac))}}{bcn}$$

input
```
integrate(exp(n*cos(b*c*x+a*c))*sin(c*(b*x+a)),x, algorithm="giac")
```

output
```
-e^(n*cos(b*c*x + a*c))/(b*c*n)
```

3.660.9 Mupad [B] (verification not implemented)

Time = 26.56 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int e^{n\cos(ac+bcx)}\sin(c(a+bx))\,dx=-\frac{\mathrm{e}^{n\,\cos(a\,c+b\,c\,x)}}{b\,c\,n}$$

input
```
int(sin(c*(a + b*x))*exp(n*cos(a*c + b*c*x)),x)
```

output
```
-exp(n*cos(a*c + b*c*x))/(b*c*n)
```

3.660. $\int e^{n\cos(ac+bcx)}\sin(c(a+bx))\,dx$

3.661 $\int e^{n\cos(c(a+bx))}\sin(ac+bcx)\,dx$

3.661.1 Optimal result

Integrand size = 22, antiderivative size = 24

$$\int e^{n\cos(c(a+bx))}\sin(ac+bcx)\,dx=-\frac{e^{n\cos(ac+bcx)}}{bcn}$$

output
```
-exp(n*cos(b*c*x+a*c))/b/c/n
```

3.661.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int e^{n\cos(c(a+bx))}\sin(ac+bcx)\,dx=-\frac{e^{n\cos(c(a+bx))}}{bcn}$$

input
```
Integrate[E^(n*Cos[c*(a + b*x)])*Sin[a*c + b*c*x],x]
```

output
```
-(E^(n*Cos[c*(a + b*x)])/(b*c*n))
```

3.661.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4835, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(ac+bcx)e^{n\cos(c(a+bx))}\,dx$$
$$\Big\downarrow \text{4835}$$
$$-\frac{\int e^{n\cos(ac+bxc)}d\cos(ac+bxc)}{bc}$$
$$\Big\downarrow \text{2624}$$
$$-\frac{e^{n\cos(ac+bcx)}}{bcn}$$

input
```
Int[E^(n*Cos[c*(a + b*x)])*Sin[a*c + b*c*x],x]
```

output
```
-(E^(n*Cos[a*c + b*c*x])/(b*c*n))
```

3.661.3.1 Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 4835
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[1, Cos[c*(a +
b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

3.661.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$-\frac{e^{n\cos(c(xb+a))}}{bcn}$	23
default	$-\frac{e^{n\cos(c(xb+a))}}{bcn}$	23
risch	$-\frac{e^{n\cos(c(xb+a))}}{bcn}$	23
parallelrisch	$-\frac{e^{n\cos(c(xb+a))}}{bcn}$	23

input
```
int(exp(n*cos(c*(b*x+a)))*sin(b*c*x+a*c),x,method=_RETURNVERBOSE)
```

output
```
-exp(n*cos(c*(b*x+a)))/b/c/n
```

3.661.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int e^{n\cos(c(a+bx))}\sin(ac+bcx)\,dx = -\frac{e^{(n\cos(bcx+ac))}}{bcn}$$

input
```
integrate(exp(n*cos(c*(b*x+a)))*sin(b*c*x+a*c),x, algorithm="fricas")
```

output
```
-e^(n*cos(b*c*x + a*c))/(b*c*n)
```

3.661.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(19) = 38.

Time = 0.30 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.12

$$\int e^{n\cos(c(a+bx))}\sin(ac+bcx)\,dx = \begin{cases} 0 & \text{for } b=0 \wedge c=0 \wedge n=0 \\ xe^{n\cos(ac)}\sin(ac) & \text{for } b=0 \\ 0 & \text{for } c=0 \\ -\frac{\cos(ac+bcx)}{bc} & \text{for } n=0 \\ -\frac{e^{n\cos(ac+bcx)}}{bcn} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(n*cos(c*(b*x+a)))*sin(b*c*x+a*c),x)
```

output
```
Piecewise((0, Eq(b, 0) & Eq(c, 0) & Eq(n, 0)), (x*exp(n*cos(a*c))*sin(a*c)
, Eq(b, 0)), (0, Eq(c, 0)), (-cos(a*c + b*c*x)/(b*c), Eq(n, 0)), (-exp(n*c
os(a*c + b*c*x))/(b*c*n), True))
```

3.661.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int e^{n\cos(c(a+bx))}\sin(ac+bcx)\,dx=-\frac{e^{(n\cos(bcx+ac))}}{bcn}$$

input
```
integrate(exp(n*cos(c*(b*x+a)))*sin(b*c*x+a*c),x, algorithm="maxima")
```

output
```
-e^(n*cos(b*c*x + a*c))/(b*c*n)
```

3.661.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int e^{n\cos(c(a+bx))}\sin(ac+bcx)\,dx=-\frac{e^{(n\cos(bcx+ac))}}{bcn}$$

input
```
integrate(exp(n*cos(c*(b*x+a)))*sin(b*c*x+a*c),x, algorithm="giac")
```

output
```
-e^(n*cos(b*c*x + a*c))/(b*c*n)
```

3.661.9 Mupad [B] (verification not implemented)

Time = 25.87 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int e^{n\cos(c(a+bx))}\sin(ac+bcx)\,dx=-\frac{\mathrm{e}^{n\,\cos(a\,c+b\,c\,x)}}{b\,c\,n}$$

input
```
int(exp(n*cos(c*(a + b*x)))*sin(a*c + b*c*x),x)
```

output
```
-exp(n*cos(a*c + b*c*x))/(b*c*n)
```

3.661. $\int e^{n\cos(c(a+bx))}\sin(ac+bcx)\,dx$

3.662 $\int e^{n\cos(a+bx)}\tan(a+bx)\,dx$

3.662.1 Optimal result

Integrand size = 17, antiderivative size = 14

$$\int e^{n\cos(a+bx)}\tan(a+bx)\,dx = -\frac{\text{ExpIntegralEi}(n\cos(a+bx))}{b}$$

output
```
-Ei(n*cos(b*x+a))/b
```

3.662.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int e^{n\cos(a+bx)}\tan(a+bx)\,dx = -\frac{\text{ExpIntegralEi}(n\cos(a+bx))}{b}$$

input
```
Integrate[E^(n*Cos[a + b*x])*Tan[a + b*x],x]
```

output
```
-(ExpIntegralEi[n*Cos[a + b*x]]/b)
```

3.662.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4839, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(a+bx)e^{n\cos(a+bx)}\,dx$$

$$\Big\downarrow \text{4839}$$

$$-\frac{\int e^{n\cos(a+bx)}\sec(a+bx)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{2609}$$

$$-\frac{\text{ExpIntegralEi}(n\cos(a+bx))}{b}$$

input
```
Int[E^(n*Cos[a + b*x])*Tan[a + b*x],x]
```

output
```
-(ExpIntegralEi[n*Cos[a + b*x]]/b)
```

3.662.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 4839
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-(b*c)^(-1)   Subst[Int[SubstFor[1/x, Cos[c*
(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
 + b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, t
an])
```

3.662.4 Maple [F]

$$\int e^{n\cos(xb+a)}\tan(xb+a)\,dx$$

input
```
int(exp(n*cos(b*x+a))*tan(b*x+a),x)
```

output
```
int(exp(n*cos(b*x+a))*tan(b*x+a),x)
```

3.662.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int e^{n\cos(a+bx)}\tan(a+bx)\,dx = -\frac{\mathrm{Ei}(n\cos(bx+a))}{b}$$

input
```
integrate(exp(n*cos(b*x+a))*tan(b*x+a),x, algorithm="fricas")
```

output
```
-Ei(n*cos(b*x + a))/b
```

3.662.6 Sympy [F]

$$\int e^{n\cos(a+bx)}\tan(a+bx)\,dx = \int e^{n\cos(a+bx)}\tan(a+bx)\,dx$$

input
```
integrate(exp(n*cos(b*x+a))*tan(b*x+a),x)
```

output
```
Integral(exp(n*cos(a + b*x))*tan(a + b*x), x)
```

3.662.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int e^{n\cos(a+bx)}\tan(a+bx)\,dx = -\frac{\mathrm{Ei}(n\cos{(bx+a)})}{b}$$

input
```
integrate(exp(n*cos(b*x+a))*tan(b*x+a),x, algorithm="maxima")
```

output
```
-Ei(n*cos(b*x + a))/b
```

3.662.8 Giac [F]

$$\int e^{n\cos(a+bx)}\tan(a+bx)\,dx = \int e^{(n\cos(bx+a))}\tan{(bx+a)}\,dx$$

input
```
integrate(exp(n*cos(b*x+a))*tan(b*x+a),x, algorithm="giac")
```

output
```
integrate(e^(n*cos(b*x + a))*tan(b*x + a), x)
```

3.662.9 Mupad [F(-1)]

Timed out.

$$\int e^{n\cos(a+bx)}\tan(a+bx)\,dx = \int \mathrm{e}^{n\cos(a+b\,x)}\tan(a+b\,x)\,dx$$

input
```
int(exp(n*cos(a + b*x))*tan(a + b*x),x)
```

output
```
int(exp(n*cos(a + b*x))*tan(a + b*x), x)
```

3.663 $\int e^{n\cos(ac+bcx)}\tan(c(a+bx))\,dx$

3.663.1 Optimal result

Integrand size = 22, antiderivative size = 19

$$\int e^{n\cos(ac+bcx)}\tan(c(a+bx))\,dx=-\frac{\text{ExpIntegralEi}(n\cos(c(a+bx)))}{bc}$$

output
```
-Ei(n*cos(c*(b*x+a)))/b/c
```

3.663.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int e^{n\cos(ac+bcx)}\tan(c(a+bx))\,dx=-\frac{\text{ExpIntegralEi}(n\cos(c(a+bx)))}{bc}$$

input
```
Integrate[E^(n*Cos[a*c + b*c*x])*Tan[c*(a + b*x)],x]
```

output
```
-(ExpIntegralEi[n*Cos[c*(a + b*x)]]/(b*c))
```

3.663.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4839, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(c(a+bx))e^{n\cos(ac+bcx)}\,dx$$

$$\Big\downarrow \text{4839}$$

$$-\frac{\int e^{n\cos(c(a+bx))}\sec(c(a+bx))d\cos(c(a+bx))}{bc}$$

$$\Big\downarrow \text{2609}$$

$$-\frac{\text{ExpIntegralEi}(n\cos(c(a+bx)))}{bc}$$

input
```
Int[E^(n*Cos[a*c + b*c*x])*Tan[c*(a + b*x)],x]
```

output
```
-(ExpIntegralEi[n*Cos[c*(a + b*x)]]/(b*c))
```

3.663.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

rule 4839
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-(b*c)^(-1)   Subst[Int[SubstFor[1/x, Cos[c*
(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
 + b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, t
an])
```

3.663.4 Maple [F]

$$\int e^{n\cos(bcx+ac)} \tan\left(c(xb+a)\right) dx$$

input
```
int(exp(n*cos(b*c*x+a*c))*tan(c*(b*x+a)),x)
```

output
```
int(exp(n*cos(b*c*x+a*c))*tan(c*(b*x+a)),x)
```

3.663.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int e^{n\cos(ac+bcx)} \tan(c(a+bx))\, dx = -\frac{\operatorname{Ei}(n\cos\left(bcx+ac\right))}{bc}$$

input
```
integrate(exp(n*cos(b*c*x+a*c))*tan(c*(b*x+a)),x, algorithm="fricas")
```

output
```
-Ei(n*cos(b*c*x + a*c))/(b*c)
```

3.663.6 Sympy [F]

$$\int e^{n\cos(ac+bcx)} \tan(c(a+bx))\, dx = \int e^{n\cos\left(ac+bcx\right)} \tan\left(ac+bcx\right) dx$$

input
```
integrate(exp(n*cos(b*c*x+a*c))*tan(c*(b*x+a)),x)
```

output
```
Integral(exp(n*cos(a*c + b*c*x))*tan(a*c + b*c*x), x)
```

3.663.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int e^{n\cos(ac+bcx)}\tan(c(a+bx))\,dx = -\frac{\mathrm{Ei}(n\cos{(bcx+ac)})}{bc}$$

input
```
integrate(exp(n*cos(b*c*x+a*c))*tan(c*(b*x+a)),x, algorithm="maxima")
```

output
```
-Ei(n*cos(b*c*x + a*c))/(b*c)
```

3.663.8 Giac [F]

$$\int e^{n\cos(ac+bcx)}\tan(c(a+bx))\,dx = \int e^{(n\cos(bcx+ac))}\tan{((bx+a)c)}\,dx$$

input
```
integrate(exp(n*cos(b*c*x+a*c))*tan(c*(b*x+a)),x, algorithm="giac")
```

output
```
integrate(e^(n*cos(b*c*x + a*c))*tan((b*x + a)*c), x)
```

3.663.9 Mupad [F(-1)]

Timed out.

$$\int e^{n\cos(ac+bcx)}\tan(c(a+bx))\,dx = \int \tan(c\,(a+b\,x))\,\mathrm{e}^{n\,\cos(a\,c+b\,c\,x)}\,dx$$

input
```
int(tan(c*(a + b*x))*exp(n*cos(a*c + b*c*x)),x)
```

output
```
int(tan(c*(a + b*x))*exp(n*cos(a*c + b*c*x)), x)
```

3.664 $\int e^{n\cos(c(a+bx))}\tan(ac+bcx)\,dx$

3.664.1 Optimal result

Integrand size = 22, antiderivative size = 20

$$\int e^{n\cos(c(a+bx))}\tan(ac+bcx)\,dx = -\frac{\text{ExpIntegralEi}(n\cos(ac+bcx))}{bc}$$

output
```
-Ei(n*cos(b*c*x+a*c))/b/c
```

3.664.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int e^{n\cos(c(a+bx))}\tan(ac+bcx)\,dx = -\frac{\text{ExpIntegralEi}(n\cos(c(a+bx)))}{bc}$$

input
```
Integrate[E^(n*Cos[c*(a + b*x)])*Tan[a*c + b*c*x],x]
```

output
```
-(ExpIntegralEi[n*Cos[c*(a + b*x)]]/(b*c))
```

3.664.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4839, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(ac+bcx)e^{n\cos(c(a+bx))}\,dx$$

$$\Big\downarrow \text{4839}$$

$$-\frac{\int e^{n\cos(ac+bxc)}\sec(ac+bxc)d\cos(ac+bxc)}{bc}$$

$$\Big\downarrow \text{2609}$$

$$-\frac{\text{ExpIntegralEi}(n\cos(ac+bxc))}{bc}$$

input
```
Int[E^(n*Cos[c*(a + b*x)])*Tan[a*c + b*c*x],x]
```

output
```
-(ExpIntegralEi[n*Cos[a*c + b*c*x]]/(b*c))
```

3.664.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 4839
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-(b*c)^(-1)   Subst[Int[SubstFor[1/x, Cos[c*
(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
 + b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, t
an])
```

3.664.4 Maple [F]

$$\int e^{n\cos(c(xb+a))}\tan(bcx+ac)\,dx$$

input
```
int(exp(n*cos(c*(b*x+a)))*tan(b*c*x+a*c),x)
```

output
```
int(exp(n*cos(c*(b*x+a)))*tan(b*c*x+a*c),x)
```

3.664.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int e^{n\cos(c(a+bx))}\tan(ac+bcx)\,dx = -\frac{\mathrm{Ei}(n\cos(bcx+ac))}{bc}$$

input
```
integrate(exp(n*cos(c*(b*x+a)))*tan(b*c*x+a*c),x, algorithm="fricas")
```

output
```
-Ei(n*cos(b*c*x + a*c))/(b*c)
```

3.664.6 Sympy [F]

$$\int e^{n\cos(c(a+bx))}\tan(ac+bcx)\,dx = \int e^{n\cos(ac+bcx)}\tan(ac+bcx)\,dx$$

input
```
integrate(exp(n*cos(c*(b*x+a)))*tan(b*c*x+a*c),x)
```

output
```
Integral(exp(n*cos(a*c + b*c*x))*tan(a*c + b*c*x), x)
```

3.664.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int e^{n\cos(c(a+bx))}\tan(ac+bcx)\,dx = -\frac{\operatorname{Ei}(n\cos(bcx+ac))}{bc}$$

input
```
integrate(exp(n*cos(c*(b*x+a)))*tan(b*c*x+a*c),x, algorithm="maxima")
```

output
```
-Ei(n*cos(b*c*x + a*c))/(b*c)
```

3.664.8 Giac [F]

$$\int e^{n\cos(c(a+bx))}\tan(ac+bcx)\,dx = \int e^{(n\cos((bx+a)c))}\tan(bcx+ac)\,dx$$

input
```
integrate(exp(n*cos(c*(b*x+a)))*tan(b*c*x+a*c),x, algorithm="giac")
```

output
```
integrate(e^(n*cos((b*x + a)*c))*tan(b*c*x + a*c), x)
```

3.664.9 Mupad [F(-1)]

Timed out.

$$\int e^{n\cos(c(a+bx))}\tan(ac+bcx)\,dx = \int e^{n\cos(c(a+bx))}\tan(ac+bcx)\,dx$$

input
```
int(exp(n*cos(c*(a + b*x)))*tan(a*c + b*c*x),x)
```

output
```
int(exp(n*cos(c*(a + b*x)))*tan(a*c + b*c*x), x)
```

3.665 $\int \frac{\cos(x)}{a+b\sin(x)}\,dx$

3.665.1 Optimal result

Integrand size = 11, antiderivative size = 11

$$\int \frac{\cos(x)}{a+b\sin(x)}\,dx=\frac{\log(a+b\sin(x))}{b}$$

output
```
ln(a+b*sin(x))/b
```

3.665.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{a+b\sin(x)}\,dx=\frac{\log(a+b\sin(x))}{b}$$

input
```
Integrate[Cos[x]/(a + b*Sin[x]),x]
```

output
```
Log[a + b*Sin[x]]/b
```

3.665.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 3147, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(x)}{a+b\sin(x)}dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\cos(x)}{a+b\sin(x)}dx$$
$$\Big\downarrow \; 3147$$
$$\frac{\int \frac{1}{a+b\sin(x)}d(b\sin(x))}{b}$$
$$\Big\downarrow \; 16$$
$$\frac{\log(a+b\sin(x))}{b}$$

input
```
Int[Cos[x]/(a + b*Sin[x]),x]
```

output
```
Log[a + b*Sin[x]]/b
```

3.665.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.665.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$\frac{\ln(a+b\sin(x))}{b}$	12
default	$\frac{\ln(a+b\sin(x))}{b}$	12
parallelrisch	$\frac{-\ln\left(\frac{1}{\cos(x)+1}\right)+\ln\left(\frac{a+b\sin(x)}{\cos(x)+1}\right)}{b}$	29
risch	$-\frac{ix}{b}+\frac{\ln\left(\mathrm{e}^{2ix}+\frac{2ia\,\mathrm{e}^{ix}}{b}-1\right)}{b}$	33
norman	$\frac{\ln\left(\tan\left(\frac{x}{2}\right)^2a+2b\tan\left(\frac{x}{2}\right)+a\right)}{b}-\frac{\ln\left(1+\tan\left(\frac{x}{2}\right)^2\right)}{b}$	38

input
```
int(cos(x)/(a+b*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
ln(a+b*sin(x))/b
```

3.665.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{a+b\sin(x)}\,dx = \frac{\log\left(b\sin\left(x\right)+a\right)}{b}$$

input
```
integrate(cos(x)/(a+b*sin(x)),x, algorithm="fricas")
```

output
```
log(b*sin(x) + a)/b
```

3.665.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int \frac{\cos(x)}{a + b\sin(x)}\,dx = \begin{cases} \frac{\log\left(\frac{a}{b} + \sin{(x)}\right)}{b} & \text{for } b \neq 0 \\ \frac{\sin{(x)}}{a} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(x)/(a+b*sin(x)),x)
```

output
```
Piecewise((log(a/b + sin(x))/b, Ne(b, 0)), (sin(x)/a, True))
```

3.665.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{a + b\sin(x)}\,dx = \frac{\log\left(b\sin{(x)} + a\right)}{b}$$

input
```
integrate(cos(x)/(a+b*sin(x)),x, algorithm="maxima")
```

output
```
log(b*sin(x) + a)/b
```

3.665.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{\cos(x)}{a + b\sin(x)}\,dx = \frac{\log\left(|b\sin{(x)} + a|\right)}{b}$$

input
```
integrate(cos(x)/(a+b*sin(x)),x, algorithm="giac")
```

output
```
log(abs(b*sin(x) + a))/b
```

3.665.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{a + b\sin(x)}\, dx = \frac{\ln\left(a + b\, \sin\left(x\right)\right)}{b}$$

input
```
int(cos(x)/(a + b*sin(x)),x)
```

output
```
log(a + b*sin(x))/b
```

3.666 $\int \cos(x)(a + b\sin(x))^n\,dx$

3.666.1 Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \cos(x)(a + b\sin(x))^n\,dx = \frac{(a + b\sin(x))^{1+n}}{b(1+n)}$$

output
```
(a+b*sin(x))^(1+n)/b/(1+n)
```

3.666.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \cos(x)(a + b\sin(x))^n\,dx = \frac{(a + b\sin(x))^{1+n}}{b(1+n)}$$

input
```
Integrate[Cos[x]*(a + b*Sin[x])^n,x]
```

output
```
(a + b*Sin[x])^(1 + n)/(b*(1 + n))
```

3.666.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 3147, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)(a+b\sin(x))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(x)(a+b\sin(x))^n dx$$

$$\Big\downarrow \text{3147}$$

$$\frac{\int (a+b\sin(x))^n d(b\sin(x))}{b}$$

$$\Big\downarrow \text{17}$$

$$\frac{(a+b\sin(x))^{n+1}}{b(n+1)}$$

input
```
Int[Cos[x]*(a + b*Sin[x])^n,x]
```

output
```
(a + b*Sin[x])^(1 + n)/(b*(1 + n))
```

3.666.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

3.666.4 Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$\frac{(a+b\sin(x))^{n+1}}{b(n+1)}$	20
default	$\frac{(a+b\sin(x))^{n+1}}{b(n+1)}$	20
parallelrisch	$\frac{(a+b\sin(x))^{n+1}}{b(n+1)}$	20
norman	$\frac{\frac{a\,\mathrm{e}^{n\ln\left(a+\frac{2b\tan\left(\frac{x}{2}\right)}{1+\tan\left(\frac{x}{2}\right)^{2}}\right)}}{b(n+1)}+\frac{a\tan\left(\frac{x}{2}\right)^{2}\mathrm{e}^{n\ln\left(a+\frac{2b\tan\left(\frac{x}{2}\right)}{1+\tan\left(\frac{x}{2}\right)^{2}}\right)}}{b(n+1)}+\frac{2\tan\left(\frac{x}{2}\right)\mathrm{e}^{n\ln\left(a+\frac{2b\tan\left(\frac{x}{2}\right)}{1+\tan\left(\frac{x}{2}\right)^{2}}\right)}}{n+1}}{1+\tan\left(\frac{x}{2}\right)^{2}}$	119

input
```
int(cos(x)*(a+b*sin(x))^n,x,method=_RETURNVERBOSE)
```

output
```
(a+b*sin(x))^(n+1)/b/(n+1)
```

3.666.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int\cos(x)(a+b\sin(x))^n\,dx=\frac{(b\sin{(x)}+a)(b\sin{(x)}+a)^n}{bn+b}$$

input
```
integrate(cos(x)*(a+b*sin(x))^n,x, algorithm="fricas")
```

output
```
(b*sin(x) + a)*(b*sin(x) + a)^n/(b*n + b)
```

3.666.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(14) = 28.

Time = 0.50 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.95

$$\int\cos(x)(a+b\sin(x))^n\,dx=\begin{cases}\frac{\sin{(x)}}{a} & \text{for } b=0\wedge n=-1\\ a^n\sin{(x)} & \text{for } b=0\\ \frac{\log{\left(\frac{a}{b}+\sin{(x)}\right)}}{b} & \text{for } n=-1\\ \frac{a(a+b\sin{(x)})^n}{bn+b}+\frac{b(a+b\sin{(x)})^n\sin{(x)}}{bn+b} & \text{otherwise}\end{cases}$$

input
```
integrate(cos(x)*(a+b*sin(x))**n,x)
```

output
```
Piecewise((sin(x)/a, Eq(b, 0) & Eq(n, -1)), (a**n*sin(x), Eq(b, 0)), (log(
a/b + sin(x))/b, Eq(n, -1)), (a*(a + b*sin(x))**n/(b*n + b) + b*(a + b*sin
(x))**n*sin(x)/(b*n + b), True))
```

3.666.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \cos(x)(a + b\sin(x))^n\,dx = \frac{(b\sin(x) + a)^{n+1}}{b(n + 1)}$$

input
```
integrate(cos(x)*(a+b*sin(x))^n,x, algorithm="maxima")
```

output
```
(b*sin(x) + a)^(n + 1)/(b*(n + 1))
```

3.666.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \cos(x)(a + b\sin(x))^n\,dx = \frac{(b\sin(x) + a)^{n+1}}{b(n + 1)}$$

input
```
integrate(cos(x)*(a+b*sin(x))^n,x, algorithm="giac")
```

output
```
(b*sin(x) + a)^(n + 1)/(b*(n + 1))
```

3.666.9 Mupad [B] (verification not implemented)

Time = 26.72 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \cos(x)(a+b\sin(x))^n\,dx = \frac{(a+b\,\sin(x))^{n+1}}{b\,(n+1)}$$

input
```
int(cos(x)*(a + b*sin(x))^n,x)
```

output
```
(a + b*sin(x))^(n + 1)/(b*(n + 1))
```

3.667 $\int \frac{\cos(x)}{\sqrt{1+\sin^2(x)}}\,dx$

3.667.1 Optimal result

Integrand size = 13, antiderivative size = 3

$$\int \frac{\cos(x)}{\sqrt{1+\sin^2(x)}}\,dx = \text{arcsinh}(\sin(x))$$

output
```
arcsinh(sin(x))
```

3.667.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{\sqrt{1+\sin^2(x)}}\,dx = \text{arcsinh}(\sin(x))$$

input
```
Integrate[Cos[x]/Sqrt[1 + Sin[x]^2],x]
```

output
```
ArcSinh[Sin[x]]
```

3.667.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3042, 3669, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(x)}{\sqrt{\sin^2(x)+1}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(x)}{\sqrt{\sin(x)^2+1}}dx$$

$$\Big\downarrow \text{ 3669}$$

$$\int \frac{1}{\sqrt{\sin^2(x)+1}}d\sin(x)$$

$$\Big\downarrow \text{ 222}$$

$$\operatorname{arcsinh}(\sin(x))$$

input
```
Int[Cos[x]/Sqrt[1 + Sin[x]^2],x]
```

output
```
ArcSinh[Sin[x]]
```

3.667.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.667.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
derivativedivides	$\operatorname{arcsinh}(\sin(x))$	4
default	$\operatorname{arcsinh}(\sin(x))$	4

input
```
int(cos(x)/(sin(x)^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arcsinh(sin(x))
```

3.667.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(3) = 6.

Time = 0.26 (sec) , antiderivative size = 39, normalized size of antiderivative = 13.00

$$\int \frac{\cos(x)}{\sqrt{1+\sin^2(x)}}\,dx = \frac{1}{4}\log\left(8\cos(x)^4 - 4\left(2\cos(x)^2-3\right)\sqrt{-\cos(x)^2+2}\sin(x) - 24\cos(x)^2+17\right)$$

input
```
integrate(cos(x)/(1+sin(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/4*log(8*cos(x)^4 - 4*(2*cos(x)^2 - 3)*sqrt(-cos(x)^2 + 2)*sin(x) - 24*co
s(x)^2 + 17)
```

3.667.6 Sympy [F]

$$\int \frac{\cos(x)}{\sqrt{1+\sin^2(x)}}\,dx = \int \frac{\cos(x)}{\sqrt{\sin^2(x)+1}}\,dx$$

input
```
integrate(cos(x)/(1+sin(x)**2)**(1/2),x)
```

output
```
Integral(cos(x)/sqrt(sin(x)**2 + 1), x)
```

3.667.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{\sqrt{1+\sin^2(x)}}\,dx = \operatorname{arsinh}\left(\sin\left(x\right)\right)$$

input
```
integrate(cos(x)/(1+sin(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
arcsinh(sin(x))
```

3.667.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. 2(3) = 6.

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 5.33

$$\int \frac{\cos(x)}{\sqrt{1+\sin^2(x)}}\,dx = -\log\left(\sqrt{\sin\left(x\right)^2+1}-\sin\left(x\right)\right)$$

input
```
integrate(cos(x)/(1+sin(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-log(sqrt(sin(x)^2 + 1) - sin(x))
```

3.667.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 9, normalized size of antiderivative = 3.00

$$\int \frac{\cos(x)}{\sqrt{1+\sin^2(x)}}\,dx = -\mathrm{asin}(\sin\left(x\right)\ 1\mathrm{i})\ 1\mathrm{i}$$

input
```
int(cos(x)/(sin(x)^2 + 1)^(1/2),x)
```

output
```
-asin(sin(x)*1i)*1i
```

3.668 $\int \frac{\cos(x)}{\sqrt{4-\sin^2(x)}}\,dx$

3.668.1 Optimal result

Integrand size = 15, antiderivative size = 7

$$\int \frac{\cos(x)}{\sqrt{4-\sin^2(x)}}\,dx = \arcsin\left(\frac{\sin(x)}{2}\right)$$

output
```
arcsin(1/2*sin(x))
```

3.668.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{\sqrt{4-\sin^2(x)}}\,dx = \arcsin\left(\frac{\sin(x)}{2}\right)$$

input
```
Integrate[Cos[x]/Sqrt[4 - Sin[x]^2],x]
```

output
```
ArcSin[Sin[x]/2]
```

3.668.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3669, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(x)}{\sqrt{4-\sin^2(x)}}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\cos(x)}{\sqrt{4-\sin(x)^2}}dx$$
$$\Big\downarrow \text{ 3669}$$
$$\int \frac{1}{\sqrt{4-\sin^2(x)}}d\sin(x)$$
$$\Big\downarrow \text{ 223}$$
$$\arcsin\left(\frac{\sin(x)}{2}\right)$$

input
```
Int[Cos[x]/Sqrt[4 - Sin[x]^2],x]
```

output
```
ArcSin[Sin[x]/2]
```

3.668.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.668.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\arcsin\left(\frac{\sin(x)}{2}\right)$	6
default	$\arcsin\left(\frac{\sin(x)}{2}\right)$	6

input
```
int(cos(x)/(4-sin(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arcsin(1/2*sin(x))
```

3.668.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(5) = 10.

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 7.57

$$\int \frac{\cos(x)}{\sqrt{4-\sin^2(x)}}\,dx = \frac{1}{2}\arctan\left(\frac{\sqrt{\cos(x)^2+3}\left(\cos(x)^2+1\right)\sin(x)-4\cos(x)\sin(x)}{\cos(x)^4+6\cos(x)^2-3}\right) + \frac{1}{2}\arctan\left(\frac{\sin(x)}{\cos(x)}\right)$$

input
```
integrate(cos(x)/(4-sin(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*arctan((sqrt(cos(x)^2 + 3)*(cos(x)^2 + 1)*sin(x) - 4*cos(x)*sin(x))/(c
os(x)^4 + 6*cos(x)^2 - 3)) + 1/2*arctan(sin(x)/cos(x))
```

3.668.6 Sympy [F]

$$\int \frac{\cos(x)}{\sqrt{4-\sin^2(x)}}\,dx = \int \frac{\cos(x)}{\sqrt{-(\sin(x)-2)(\sin(x)+2)}}\,dx$$

input
```
integrate(cos(x)/(4-sin(x)**2)**(1/2),x)
```

output
```
Integral(cos(x)/sqrt(-(sin(x) - 2)*(sin(x) + 2)), x)
```

3.668.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \frac{\cos(x)}{\sqrt{4-\sin^2(x)}}\,dx = \arcsin\left(\frac{1}{2}\sin(x)\right)$$

input
```
integrate(cos(x)/(4-sin(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
arcsin(1/2*sin(x))
```

3.668.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \frac{\cos(x)}{\sqrt{4-\sin^2(x)}}\,dx = \arcsin\left(\frac{1}{2}\sin(x)\right)$$

input
```
integrate(cos(x)/(4-sin(x)^2)^(1/2),x, algorithm="giac")
```

output
```
arcsin(1/2*sin(x))
```

3.668.9 Mupad [B] (verification not implemented)

Time = 26.85 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \frac{\cos(x)}{\sqrt{4-\sin^2(x)}}\, dx = \mathrm{asin}\left(\frac{\sin(x)}{2}\right)$$

input
```
int(cos(x)/(4 - sin(x)^2)^(1/2),x)
```

output
```
asin(sin(x)/2)
```

3.669 $\int \frac{\cos(3x)}{\sqrt{4-\sin^2(3x)}}\,dx$

3.669.1 Optimal result

Integrand size = 19, antiderivative size = 13

$$\int \frac{\cos(3x)}{\sqrt{4-\sin^2(3x)}}\,dx = \frac{1}{3}\arcsin\left(\frac{1}{2}\sin(3x)\right)$$

output
```
1/3*arcsin(1/2*sin(3*x))
```

3.669.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\cos(3x)}{\sqrt{4-\sin^2(3x)}}\,dx = \frac{1}{3}\arcsin\left(\frac{1}{2}\sin(3x)\right)$$

input
```
Integrate[Cos[3*x]/Sqrt[4 - Sin[3*x]^2],x]
```

output
```
ArcSin[Sin[3*x]/2]/3
```

3.669.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 3669, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(3x)}{\sqrt{4-\sin^2(3x)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos(3x)}{\sqrt{4-\sin(3x)^2}}dx$$
$$\Big\downarrow \text{3669}$$
$$\frac{1}{3}\int \frac{1}{\sqrt{4-\sin^2(3x)}}d\sin(3x)$$
$$\Big\downarrow \text{223}$$
$$\frac{1}{3}\arcsin\left(\frac{1}{2}\sin(3x)\right)$$

input
```
Int[Cos[3*x]/Sqrt[4 - Sin[3*x]^2],x]
```

output
```
ArcSin[Sin[3*x]/2]/3
```

3.669.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.669.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{\arcsin\left(\frac{\sin(3x)}{2}\right)}{3}$	10
default	$\frac{\arcsin\left(\frac{\sin(3x)}{2}\right)}{3}$	10

input
```
int(cos(3*x)/(4-sin(3*x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*arcsin(1/2*sin(3*x))
```

3.669.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(9) = 18.

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 5.46

$$\int \frac{\cos(3x)}{\sqrt{4-\sin^2(3x)}}\,dx = \frac{1}{6}\arctan\left(\frac{\sqrt{\cos(3x)^2+3}(\cos(3x)^2+1)\sin(3x)-4\cos(3x)\sin(3x)}{\cos(3x)^4+6\cos(3x)^2-3}\right) + \frac{1}{6}\arctan\left(\frac{\sin(3x)}{\cos(3x)}\right)$$

input
```
integrate(cos(3*x)/(4-sin(3*x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/6*arctan((sqrt(cos(3*x)^2 + 3)*(cos(3*x)^2 + 1)*sin(3*x) - 4*cos(3*x)*si
n(3*x))/(cos(3*x)^4 + 6*cos(3*x)^2 - 3)) + 1/6*arctan(sin(3*x)/cos(3*x))
```

3.669.6 Sympy [F]

$$\int \frac{\cos(3x)}{\sqrt{4-\sin^2(3x)}}\,dx = \int \frac{\cos(3x)}{\sqrt{-(\sin(3x)-2)(\sin(3x)+2)}}\,dx$$

input
```
integrate(cos(3*x)/(4-sin(3*x)**2)**(1/2),x)
```

output
```
Integral(cos(3*x)/sqrt(-(sin(3*x) - 2)*(sin(3*x) + 2)), x)
```

3.669.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{\cos(3x)}{\sqrt{4-\sin^2(3x)}}\,dx = \frac{1}{3}\arcsin\left(\frac{1}{2}\sin(3x)\right)$$

input
```
integrate(cos(3*x)/(4-sin(3*x)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*arcsin(1/2*sin(3*x))
```

3.669.8 Giac [F]

$$\int \frac{\cos(3x)}{\sqrt{4-\sin^2(3x)}}\,dx = \int \frac{\cos(3x)}{\sqrt{-\sin(3x)^2+4}}\,dx$$

input
```
integrate(cos(3*x)/(4-sin(3*x)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(cos(3*x)/sqrt(-sin(3*x)^2 + 4), x)
```

3.669.9 Mupad [B] (verification not implemented)

Time = 26.91 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{\cos(3x)}{\sqrt{4-\sin^2(3x)}}\,dx = \frac{\operatorname{asin}\left(\frac{\sin(3\,x)}{2}\right)}{3}$$

input
```
int(cos(3*x)/(4 - sin(3*x)^2)^(1/2),x)
```

output
```
asin(sin(3*x)/2)/3
```

3.670 $\int \cos(x)\sqrt{1+\csc(x)}\,dx$

3.670.1 Optimal result

Integrand size = 11, antiderivative size = 21

$$\int \cos(x)\sqrt{1+\csc(x)}\,dx = \operatorname{arctanh}\left(\sqrt{1+\csc(x)}\right)+\sqrt{1+\csc(x)}\sin(x)$$

output
```
arctanh((1+csc(x))^(1/2))+sin(x)*(1+csc(x))^(1/2)
```

3.670.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \cos(x)\sqrt{1+\csc(x)}\,dx = \operatorname{arctanh}\left(\sqrt{1+\csc(x)}\right)+\sqrt{1+\csc(x)}\sin(x)$$

input
```
Integrate[Cos[x]*Sqrt[1 + Csc[x]],x]
```

output
```
ArcTanh[Sqrt[1 + Csc[x]]] + Sqrt[1 + Csc[x]]*Sin[x]
```

3.670.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {3042, 4361, 51, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\sqrt{\csc(x)+1}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(x)\sqrt{\csc(x)+1}dx$$

$$\Big\downarrow \text{4361}$$

$$-\int \sqrt{\csc(x)+1}\sin^2(x)d\csc(x)$$

$$\Big\downarrow \text{51}$$

$$\sin(x)\sqrt{\csc(x)+1}-\frac{1}{2}\int \frac{\sin(x)}{\sqrt{\csc(x)+1}}d\csc(x)$$

$$\Big\downarrow \text{73}$$

$$\sin(x)\sqrt{\csc(x)+1}-\int \sin(x)d\sqrt{\csc(x)+1}$$

$$\Big\downarrow \text{220}$$

$$\text{arctanh}\left(\sqrt{\csc(x)+1}\right)+\sin(x)\sqrt{\csc(x)+1}$$

input
```
Int[Cos[x]*Sqrt[1 + Csc[x]],x]
```

output
```
ArcTanh[Sqrt[1 + Csc[x]]] + Sqrt[1 + Csc[x]]*Sin[x]
```

3.670.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4361
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> Simp[-(f*b^(p - 1))^(-1)   Subst[Int[(-a + b*x)^((p - 1)/2
)*((a + b*x)^(m + (p - 1)/2)/x^(p + 1)), x], x, Csc[e + f*x]], x] /; FreeQ[
{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

3.670.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(17) = 34$.

Time = 0.64 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.29

method	result	size
derivativedivides	$\frac{1}{2\sqrt{\csc(x)+1}+2}+\frac{\ln\left(\sqrt{\csc(x)+1}+1\right)}{2}+\frac{1}{2\sqrt{\csc(x)+1}-2}-\frac{\ln\left(\sqrt{\csc(x)+1}-1\right)}{2}$	48
default	$\frac{1}{2\sqrt{\csc(x)+1}+2}+\frac{\ln\left(\sqrt{\csc(x)+1}+1\right)}{2}+\frac{1}{2\sqrt{\csc(x)+1}-2}-\frac{\ln\left(\sqrt{\csc(x)+1}-1\right)}{2}$	48

3.670. $\int \cos(x)\sqrt{1+\csc(x)}\,dx$

input
```
int(cos(x)*(csc(x)+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/((csc(x)+1)^(1/2)+1)+1/2*ln((csc(x)+1)^(1/2)+1)+1/2/((csc(x)+1)^(1/2)-
1)-1/2*ln((csc(x)+1)^(1/2)-1)
```

3.670.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(17) = 34$.

Time = 0.24 (sec) , antiderivative size = 79, normalized size of antiderivative = 3.76

$$\int \cos(x)\sqrt{1+\csc(x)}\,dx = \sqrt{\frac{\sin(x)+1}{\sin(x)}}\sin(x) + \frac{1}{2}\log\left(\frac{2\left(\sqrt{\frac{\sin(x)+1}{\sin(x)}}\sin(x)+\sin(x)+1\right)}{\cos(x)+\sin(x)+1}\right) - \frac{1}{2}\log\left(-\frac{2\left(\sqrt{\frac{\sin(x)+1}{\sin(x)}}\sin(x)-\sin(x)-1\right)}{\cos(x)+\sin(x)+1}\right)$$

input
```
integrate(cos(x)*(1+csc(x))^(1/2),x, algorithm="fricas")
```

output
```
sqrt((sin(x) + 1)/sin(x))*sin(x) + 1/2*log(2*(sqrt((sin(x) + 1)/sin(x))*si
n(x) + sin(x) + 1)/(cos(x) + sin(x) + 1)) - 1/2*log(-2*(sqrt((sin(x) + 1)/
sin(x))*sin(x) - sin(x) - 1)/(cos(x) + sin(x) + 1))
```

3.670.6 Sympy [F]

$$\int \cos(x)\sqrt{1+\csc(x)}\,dx = \int \sqrt{\csc(x)+1}\cos(x)\,dx$$

input
```
integrate(cos(x)*(1+csc(x))**(1/2),x)
```

output
```
Integral(sqrt(csc(x) + 1)*cos(x), x)
```

3.670.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(17) = 34$.

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.81

$$\int \cos(x)\sqrt{1+\csc(x)}\,dx = \sqrt{\frac{1}{\sin(x)}+1}\sin(x) + \frac{1}{2}\log\left(\sqrt{\frac{1}{\sin(x)}+1}+1\right) - \frac{1}{2}\log\left(\sqrt{\frac{1}{\sin(x)}+1}-1\right)$$

input
```
integrate(cos(x)*(1+csc(x))^(1/2),x, algorithm="maxima")
```

output
```
sqrt(1/sin(x) + 1)*sin(x) + 1/2*log(sqrt(1/sin(x) + 1) + 1) - 1/2*log(sqrt
(1/sin(x) + 1) - 1)
```

3.670.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(17) = 34$.

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.81

$$\int \cos(x)\sqrt{1+\csc(x)}\,dx = \frac{1}{2}\left(2\sqrt{\sin(x)^2+\sin(x)} - \log\left(\left|2\sqrt{\sin(x)^2+\sin(x)} - 2\sin(x) - 1\right|\right)\right)\operatorname{sgn}(\sin(x))$$

input
```
integrate(cos(x)*(1+csc(x))^(1/2),x, algorithm="giac")
```

output
```
1/2*(2*sqrt(sin(x)^2 + sin(x)) - log(abs(2*sqrt(sin(x)^2 + sin(x)) - 2*sin
(x) - 1)))*sgn(sin(x))
```

3.670.9 Mupad [B] (verification not implemented)

Time = 26.73 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.24

$$\int \cos(x)\sqrt{1+\csc(x)}\,dx = \sin(x)\,\sqrt{\frac{1}{\sin(x)}+1} + \frac{\ln\left(\sin(x)+\sqrt{\sin(x)^2+\sin(x)}+\frac{1}{2}\right)\sin(x)\,\sqrt{\frac{1}{\sin(x)}+1}}{2\sqrt{\sin(x)^2+\sin(x)}}$$

input
```
int(cos(x)*(1/sin(x) + 1)^(1/2),x)
```

output
```
sin(x)*(1/sin(x) + 1)^(1/2) + (log(sin(x) + (sin(x) + sin(x)^2)^(1/2) + 1/
2)*sin(x)*(1/sin(x) + 1)^(1/2))/(2*(sin(x) + sin(x)^2)^(1/2))
```

3.671 $\int \cos(x)\sqrt{4-\sin^2(x)}\,dx$

3.671.1 Optimal result

Integrand size = 15, antiderivative size = 28

$$\int \cos(x)\sqrt{4-\sin^2(x)}\,dx = 2\arcsin\left(\frac{\sin(x)}{2}\right)+\frac{1}{2}\sin(x)\sqrt{4-\sin^2(x)}$$

output
```
2*arcsin(1/2*sin(x))+1/2*sin(x)*(4-sin(x)^2)^(1/2)
```

3.671.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \cos(x)\sqrt{4-\sin^2(x)}\,dx = 2\arcsin\left(\frac{\sin(x)}{2}\right)+\frac{1}{2}\sin(x)\sqrt{4-\sin^2(x)}$$

input
```
Integrate[Cos[x]*Sqrt[4 - Sin[x]^2],x]
```

output
```
2*ArcSin[Sin[x]/2] + (Sin[x]*Sqrt[4 - Sin[x]^2])/2
```

3.671.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 3669, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{4-\sin^2(x)}\cos(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{4-\sin(x)^2}\cos(x)dx$$

$$\Big\downarrow \text{3669}$$

$$\int \sqrt{4-\sin^2(x)}d\sin(x)$$

$$\Big\downarrow \text{211}$$

$$2\int \frac{1}{\sqrt{4-\sin^2(x)}}d\sin(x)+\frac{1}{2}\sqrt{4-\sin^2(x)}\sin(x)$$

$$\Big\downarrow \text{223}$$

$$2\arcsin\left(\frac{\sin(x)}{2}\right)+\frac{1}{2}\sin(x)\sqrt{4-\sin^2(x)}$$

input
```
Int[Cos[x]*Sqrt[4 - Sin[x]^2],x]
```

output
```
2*ArcSin[Sin[x]/2] + (Sin[x]*Sqrt[4 - Sin[x]^2])/2
```

3.671.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.671.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$2\arcsin\left(\frac{\sin(x)}{2}\right)+\frac{\sin(x)\sqrt{4-\sin(x)^{2}}}{2}$	23
default	$2\arcsin\left(\frac{\sin(x)}{2}\right)+\frac{\sin(x)\sqrt{4-\sin(x)^{2}}}{2}$	23

input
```
int(cos(x)*(4-sin(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*arcsin(1/2*sin(x))+1/2*sin(x)*(4-sin(x)^2)^(1/2)
```

3.671.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(22) = 44.

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.18

$$\int \cos(x)\sqrt{4-\sin^2(x)}\,dx$$
$$=\frac{1}{2}\sqrt{\cos(x)^2+3}\sin(x)$$
$$+\arctan\left(\frac{\sqrt{\cos(x)^2+3}(\cos(x)^2+1)\sin(x)-4\cos(x)\sin(x)}{\cos(x)^4+6\cos(x)^2-3}\right)+\arctan\left(\frac{\sin(x)}{\cos(x)}\right)$$

input
```
integrate(cos(x)*(4-sin(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(cos(x)^2 + 3)*sin(x) + arctan((sqrt(cos(x)^2 + 3)*(cos(x)^2 + 1)*
sin(x) - 4*cos(x)*sin(x))/(cos(x)^4 + 6*cos(x)^2 - 3)) + arctan(sin(x)/cos
(x))
```

3.671.6 Sympy [F]

$$\int \cos(x)\sqrt{4-\sin^2(x)}\,dx = \int \sqrt{-\left(\sin\left(x\right)-2\right)\left(\sin\left(x\right)+2\right)}\cos\left(x\right)dx$$

input
```
integrate(cos(x)*(4-sin(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(-(sin(x) - 2)*(sin(x) + 2))*cos(x), x)
```

3.671.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \cos(x)\sqrt{4-\sin^2(x)}\,dx = \frac{1}{2}\sqrt{-\sin\left(x\right)^2+4}\sin\left(x\right)+2\arcsin\left(\frac{1}{2}\sin\left(x\right)\right)$$

input
```
integrate(cos(x)*(4-sin(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(-sin(x)^2 + 4)*sin(x) + 2*arcsin(1/2*sin(x))
```

3.671.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \cos(x)\sqrt{4-\sin^2(x)}\,dx = \frac{1}{2}\sqrt{-\sin\left(x\right)^2+4}\sin\left(x\right)+2\arcsin\left(\frac{1}{2}\sin\left(x\right)\right)$$

input
```
integrate(cos(x)*(4-sin(x)^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(-sin(x)^2 + 4)*sin(x) + 2*arcsin(1/2*sin(x))
```

3.671.9 Mupad [B] (verification not implemented)

Time = 27.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.71

$$\int \cos(x)\sqrt{4-\sin^2(x)}\,dx = 2\,\mathrm{asin}\left(\frac{\sin(x)}{2}\right)+\frac{\sin(x)\ \sqrt{\cos(x)^2+3}}{2}$$

input
```
int(cos(x)*(4 - sin(x)^2)^(1/2),x)
```

output
```
2*asin(sin(x)/2) + (sin(x)*(cos(x)^2 + 3)^(1/2))/2
```

3.672 $\int \cos(x)\sin(x)\sqrt{1+\sin^2(x)}\,dx$

3.672.1 Optimal result

Integrand size = 15, antiderivative size = 14

$$\int \cos(x)\sin(x)\sqrt{1+\sin^2(x)}\,dx = \frac{1}{3}\left(1+\sin^2(x)\right)^{3/2}$$

output
```
1/3*(1+sin(x)^2)^(3/2)
```

3.672.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \cos(x)\sin(x)\sqrt{1+\sin^2(x)}\,dx = \frac{1}{3}\left(1+\sin^2(x)\right)^{3/2}$$

input
```
Integrate[Cos[x]*Sin[x]*Sqrt[1 + Sin[x]^2],x]
```

output
```
(1 + Sin[x]^2)^(3/2)/3
```

3.672.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3677, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\sqrt{\sin^2(x)+1}\cos(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x)\sqrt{\sin(x)^2+1}\cos(x)dx$$

$$\Big\downarrow \text{3677}$$

$$\int \sin(x)\sqrt{\sin^2(x)+1}d\sin(x)$$

$$\Big\downarrow \text{241}$$

$$\frac{1}{3}\left(\sin^2(x)+1\right)^{3/2}$$

input
```
Int[Cos[x]*Sin[x]*Sqrt[1 + Sin[x]^2],x]
```

output
```
(1 + Sin[x]^2)^(3/2)/3
```

3.672.3.1 Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3677
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFa
ctors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[(d*ff*x)^n*(1 - ff^2*x^2)^((
m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
d, e, f, n, p}, x] && IntegerQ[(m - 1)/2]
```

3.672.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\frac{\left(\sin(x)^2+1\right)^{\frac{3}{2}}}{3}$	11
default	$\frac{\left(\sin(x)^2+1\right)^{\frac{3}{2}}}{3}$	11

input
```
int(cos(x)*sin(x)*(sin(x)^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(sin(x)^2+1)^(3/2)
```

3.672.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \cos(x)\sin(x)\sqrt{1+\sin^2(x)}\,dx = \frac{1}{3}\left(-\cos\left(x\right)^2+2\right)^{\frac{3}{2}}$$

input
```
integrate(cos(x)*sin(x)*(1+sin(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(-cos(x)^2 + 2)^(3/2)
```

3.672.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(10) = 20$.

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.93

$$\int \cos(x)\sin(x)\sqrt{1+\sin^2(x)}\,dx = \frac{\sqrt{\sin^2{(x)}+1}\sin^2{(x)}}{3} + \frac{\sqrt{\sin^2{(x)}+1}}{3}$$

input
```
integrate(cos(x)*sin(x)*(1+sin(x)**2)**(1/2),x)
```

output
```
sqrt(sin(x)**2 + 1)*sin(x)**2/3 + sqrt(sin(x)**2 + 1)/3
```

3.672.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos(x)\sin(x)\sqrt{1+\sin^2(x)}\,dx = \frac{1}{3}\left(\sin{(x)}^2+1\right)^{\frac{3}{2}}$$

input
```
integrate(cos(x)*sin(x)*(1+sin(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*(sin(x)^2 + 1)^(3/2)
```

3.672.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos(x)\sin(x)\sqrt{1+\sin^2(x)}\,dx = \frac{1}{3}\left(\sin{(x)}^2+1\right)^{\frac{3}{2}}$$

input
```
integrate(cos(x)*sin(x)*(1+sin(x)^2)^(1/2),x, algorithm="giac")
```

output
```
1/3*(sin(x)^2 + 1)^(3/2)
```

3.672.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \cos(x)\sin(x)\sqrt{1+\sin^2(x)}\,dx = \frac{\left(\sin\left(x\right)^2+1\right)^{3/2}}{3}$$

input
```
int(cos(x)*sin(x)*(sin(x)^2 + 1)^(1/2),x)
```

output
```
(sin(x)^2 + 1)^(3/2)/3
```

3.673 $\int \frac{\cos(x)}{\sqrt{2\sin(x)+\sin^2(x)}}\,dx$

3.673.1 Optimal result

Integrand size = 16, antiderivative size = 19

$$\int \frac{\cos(x)}{\sqrt{2\sin(x)+\sin^2(x)}}\,dx = 2\text{arctanh}\left(\frac{\sin(x)}{\sqrt{2\sin(x)+\sin^2(x)}}\right)$$

output
```
2*arctanh(sin(x)/(2*sin(x)+sin(x)^2)^(1/2))
```

3.673.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 40 vs. 2(19) = 38.

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.11

$$\int \frac{\cos(x)}{\sqrt{2\sin(x)+\sin^2(x)}}\,dx = \frac{2\text{arcsinh}\left(\frac{\sqrt{\sin(x)}}{\sqrt{2}}\right)\sqrt{\sin(x)}\sqrt{2+\sin(x)}}{\sqrt{\sin(x)(2+\sin(x))}}$$

input
```
Integrate[Cos[x]/Sqrt[2*Sin[x] + Sin[x]^2],x]
```

output
```
(2*ArcSinh[Sqrt[Sin[x]]/Sqrt[2]]*Sqrt[Sin[x]]*Sqrt[2 + Sin[x]])/Sqrt[Sin[x
]*(2 + Sin[x])]
```

3.673.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3739, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(x)}{\sqrt{\sin^2(x)+2\sin(x)}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(x)}{\sqrt{\sin(x)^2+2\sin(x)}}dx$$

$$\Big\downarrow \text{ 3739}$$

$$\int \frac{1}{\sqrt{\sin^2(x)+2\sin(x)}}d\sin(x)$$

$$\Big\downarrow \text{ 1091}$$

$$2\int \frac{1}{1-\frac{\sin^2(x)}{\sin^2(x)+2\sin(x)}}d\frac{\sin(x)}{\sqrt{\sin^2(x)+2\sin(x)}}$$

$$\Big\downarrow \text{ 219}$$

$$2\text{arctanh}\left(\frac{\sin(x)}{\sqrt{\sin^2(x)+2\sin(x)}}\right)$$

input
```
Int[Cos[x]/Sqrt[2*Sin[x] + Sin[x]^2],x]
```

output
```
2*ArcTanh[Sin[x]/Sqrt[2*Sin[x] + Sin[x]^2]]
```

3.673.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3739
```
Int[cos[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*sin[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*sin[(d_.) + (e_.)*(x_)])^(n2_.))^(p_.), x_Symbol
] :> Module[{g = FreeFactors[Sin[d + e*x], x]}, Simp[g/e   Subst[Int[(1 - g
^2*x^2)^((m - 1)/2)*(a + b*(f*g*x)^n + c*(f*g*x)^(2*n))^p, x], x, Sin[d + e
*x]/g], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[n2, 2*n] && Intege
rQ[(m - 1)/2]
```

3.673.4 Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\ln\left(1+\sin\left(x\right)+\sqrt{2\sin\left(x\right)+\sin\left(x\right)^{2}}\right)$	17
default	$\ln\left(1+\sin\left(x\right)+\sqrt{2\sin\left(x\right)+\sin\left(x\right)^{2}}\right)$	17

input
```
int(cos(x)/(2*sin(x)+sin(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
ln(1+sin(x)+(2*sin(x)+sin(x)^2)^(1/2))
```

3.673.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(17) = 34$.

Time = 0.35 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.84

$$\int \frac{\cos(x)}{\sqrt{2\sin(x)+\sin^2(x)}}\,dx = \frac{1}{2}\log\left(-2\cos(x)^2+2\sqrt{-\cos(x)^2+2\sin(x)+1}(\sin(x)+1)+4\sin(x)+3\right)$$

input
```
integrate(cos(x)/(2*sin(x)+sin(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*log(-2*cos(x)^2 + 2*sqrt(-cos(x)^2 + 2*sin(x) + 1)*(sin(x) + 1) + 4*si
n(x) + 3)
```

3.673.6 Sympy [F]

$$\int \frac{\cos(x)}{\sqrt{2\sin(x)+\sin^2(x)}}\,dx = \int \frac{\cos(x)}{\sqrt{(\sin(x)+2)\sin(x)}}\,dx$$

input
```
integrate(cos(x)/(2*sin(x)+sin(x)**2)**(1/2),x)
```

output
```
Integral(cos(x)/sqrt((sin(x) + 2)*sin(x)), x)
```

3.673.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{\cos(x)}{\sqrt{2\sin(x)+\sin^2(x)}}\,dx = \log\left(2\sqrt{\sin(x)^2+2\sin(x)}+2\sin(x)+2\right)$$

input
```
integrate(cos(x)/(2*sin(x)+sin(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
log(2*sqrt(sin(x)^2 + 2*sin(x)) + 2*sin(x) + 2)
```

3.673.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{\cos(x)}{\sqrt{2\sin(x)+\sin^2(x)}}\,dx = -\log\left(-\sqrt{\sin(x)^2+2\sin(x)}+\sin(x)+1\right)$$

input
```
integrate(cos(x)/(2*sin(x)+sin(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-log(-sqrt(sin(x)^2 + 2*sin(x)) + sin(x) + 1)
```

3.673.9 Mupad [B] (verification not implemented)

Time = 26.44 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{\cos(x)}{\sqrt{2\sin(x)+\sin^2(x)}}\,dx = \ln\left(\sin(x)+\sqrt{\sin(x)\,(\sin(x)+2)}+1\right)$$

input
```
int(cos(x)/(2*sin(x) + sin(x)^2)^(1/2),x)
```

output
```
log(sin(x) + (sin(x)*(sin(x) + 2))^(1/2) + 1)
```

3.674 $\int \cos(x)\cos(\sin(x))\,dx$

3.674.1 Optimal result

Integrand size = 6, antiderivative size = 3

$$\int \cos(x)\cos(\sin(x))\,dx = \sin(\sin(x))$$

output
```
sin(sin(x))
```

3.674.2 Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \cos(x)\cos(\sin(x))\,dx = \sin(\sin(x))$$

input
```
Integrate[Cos[x]*Cos[Sin[x]],x]
```

output
```
Sin[Sin[x]]
```

3.674.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4834, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\cos(\sin(x))\,dx$$

$$\Big\downarrow \; 4834$$

$$\int \cos(\sin(x))d\sin(x)$$

$$\Big\downarrow \; 3042$$

$$\int \sin\left(\sin(x)+\frac{\pi}{2}\right)d\sin(x)$$

$$\Big\downarrow \; 3117$$

$$\sin(\sin(x))$$

input
```
Int[Cos[x]*Cos[Sin[x]],x]
```

output
```
Sin[Sin[x]]
```

3.674.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4834
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

3.674.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
derivativedivides	$\sin \left(\sin \left(x \right)\right)$	4
default	$\sin \left(\sin \left(x \right)\right)$	4
risch	$\sin \left(\sin \left(x \right)\right)$	4
parallelrisch	$\sin \left(\sin \left(x \right)\right)$	4
norman	$\frac{2 \tan \left(\frac{x}{2}\right)^{2} \tan \left(\frac{\tan \left(\frac{x}{2}\right)}{1+\tan \left(\frac{x}{2}\right)^{2}}\right)+2 \tan \left(\frac{\tan \left(\frac{x}{2}\right)}{1+\tan \left(\frac{x}{2}\right)^{2}}\right)}{\left(1+\tan \left(\frac{x}{2}\right)^{2}\right) \left(1+\tan \left(\frac{\tan \left(\frac{x}{2}\right)}{1+\tan \left(\frac{x}{2}\right)^{2}}\right)^{2}\right)}$	77

input
```
int(cos(x)*cos(sin(x)),x,method=_RETURNVERBOSE)
```

output
```
sin(sin(x))
```

3.674.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(3) = 6.

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 5.67

$$\int \cos(x)\cos(\sin(x))\,dx = \sin\left(\frac{2\,\tan\left(\frac{1}{2}\,x\right)}{\tan\left(\frac{1}{2}\,x\right)^2+1}\right)$$

input
```
integrate(cos(x)*cos(sin(x)),x, algorithm="fricas")
```

output
```
sin(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))
```

3.674.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \cos(x)\cos(\sin(x))\,dx = \sin(\sin(x))$$

input
```
integrate(cos(x)*cos(sin(x)),x)
```

output
```
sin(sin(x))
```

3.674.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \cos(x)\cos(\sin(x))\,dx = \sin(\sin(x))$$

input
```
integrate(cos(x)*cos(sin(x)),x, algorithm="maxima")
```

output
```
sin(sin(x))
```

3.674.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \cos(x)\cos(\sin(x))\,dx = \sin(\sin(x))$$

input
```
integrate(cos(x)*cos(sin(x)),x, algorithm="giac")
```

output
```
sin(sin(x))
```

3.674.9 Mupad [B] (verification not implemented)

Time = 26.96 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \cos(x)\cos(\sin(x))\,dx = \sin(\sin(x))$$

input
```
int(cos(sin(x))*cos(x),x)
```

output
```
sin(sin(x))
```

3.675 $\int \cos(x)\cos(\sin(x))\cos(\sin(\sin(x)))\,dx$

3.675.1 Optimal result

Integrand size = 10, antiderivative size = 4

$$\int \cos(x)\cos(\sin(x))\cos(\sin(\sin(x)))\,dx = \sin(\sin(\sin(x)))$$

output
```
sin(sin(sin(x)))
```

3.675.2 Mathematica [A] (verified)

Time = 3.76 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \cos(x)\cos(\sin(x))\cos(\sin(\sin(x)))\,dx = \sin(\sin(\sin(x)))$$

input
```
Integrate[Cos[x]*Cos[Sin[x]]*Cos[Sin[Sin[x]]],x]
```

output
```
Sin[Sin[Sin[x]]]
```

3.675.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4834, 4834, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\cos(\sin(x))\cos(\sin(\sin(x)))\,dx$$

$$\Big\downarrow \text{4834}$$

$$\int \cos(\sin(x))\cos(\sin(\sin(x)))d\sin(x)$$

$$\Big\downarrow \text{4834}$$

$$\int \cos(\sin(\sin(x)))d\sin(\sin(x))$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(\sin(\sin(x))+\frac{\pi}{2}\right)d\sin(\sin(x))$$

$$\Big\downarrow \text{3117}$$

$$\sin(\sin(\sin(x)))$$

input
```
Int[Cos[x]*Cos[Sin[x]]*Cos[Sin[Sin[x]]],x]
```

output
```
Sin[Sin[Sin[x]]]
```

3.675.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4834
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

3.675.4 Maple [A] (verified)

Time = 10.41 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

method	result	size
derivativedivides	$\sin(\sin(\sin(x)))$	5
default	$\sin(\sin(\sin(x)))$	5
risch	$\sin(\sin(\sin(x)))$	5
parallelrisch	$\sin(\sin(\sin(x)))$	5

input
```
int(cos(x)*cos(sin(x))*cos(sin(sin(x))),x,method=_RETURNVERBOSE)
```

output
```
sin(sin(sin(x)))
```

3.675.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(4) = 8$.

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 10.25

$$\int \cos(x)\cos(\sin(x))\cos(\sin(\sin(x)))\,dx = \sin\left(\frac{2\tan\left(\frac{\tan\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)^2+1}\right)}{\tan\left(\frac{\tan\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)^2+1}\right)^2+1}\right)$$

input
```
integrate(cos(x)*cos(sin(x))*cos(sin(sin(x))),x, algorithm="fricas")
```

output
```
sin(2*tan(tan(1/2*x)/(tan(1/2*x)^2 + 1))/(tan(tan(1/2*x)/(tan(1/2*x)^2 + 1
))^2 + 1))
```

3.675.6 Sympy [A] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

$$\int \cos(x)\cos(\sin(x))\cos(\sin(\sin(x)))\,dx = \sin\left(\sin\left(\sin\left(x\right)\right)\right)$$

input
```
integrate(cos(x)*cos(sin(x))*cos(sin(sin(x))),x)
```

output
```
sin(sin(sin(x)))
```

3.675.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \cos(x)\cos(\sin(x))\cos(\sin(\sin(x)))\,dx = \sin\left(\sin\left(\sin\left(x\right)\right)\right)$$

input
```
integrate(cos(x)*cos(sin(x))*cos(sin(sin(x))),x, algorithm="maxima")
```

output
```
sin(sin(sin(x)))
```

3.675.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \cos(x)\cos(\sin(x))\cos(\sin(\sin(x)))\,dx = \sin\left(\sin\left(\sin\left(x\right)\right)\right)$$

input
```
integrate(cos(x)*cos(sin(x))*cos(sin(sin(x))),x, algorithm="giac")
```

output
```
sin(sin(sin(x)))
```

3.675.9 Mupad [B] (verification not implemented)

Time = 27.63 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \cos(x)\cos(\sin(x))\cos(\sin(\sin(x)))\,dx = \sin(\sin(\sin(x)))$$

input
```
int(cos(sin(x))*cos(sin(sin(x)))*cos(x),x)
```

output
```
sin(sin(sin(x)))
```

3.676 $\int \cos(x)\sec(\sin(x))\,dx$

3.676.1 Optimal result

Integrand size = 6, antiderivative size = 4

$$\int \cos(x)\sec(\sin(x))\,dx = \operatorname{arctanh}(\sin(\sin(x)))$$

output
```
arctanh(sin(sin(x)))
```

3.676.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \cos(x)\sec(\sin(x))\,dx = \operatorname{arctanh}(\sin(\sin(x)))$$

input
```
Integrate[Cos[x]*Sec[Sin[x]],x]
```

output
```
ArcTanh[Sin[Sin[x]]]
```

3.676.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4834, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\sec(\sin(x))\,dx$$

$$\Big\downarrow \text{4834}$$

$$\int \sec(\sin(x))d\sin(x)$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(\sin(x)+\frac{\pi}{2}\right)d\sin(x)$$

$$\Big\downarrow \text{4257}$$

$$\text{arctanh}(\sin(\sin(x)))$$

input
```
Int[Cos[x]*Sec[Sin[x]],x]
```

output
```
ArcTanh[Sin[Sin[x]]]
```

3.676.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4834
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

3.676.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 9, normalized size of antiderivative = 2.25

method	result	size
derivativedivides	$\ln\left(\sec\left(\sin\left(x\right)\right)+\tan\left(\sin\left(x\right)\right)\right)$	9
default	$\ln\left(\sec\left(\sin\left(x\right)\right)+\tan\left(\sin\left(x\right)\right)\right)$	9
parallelrisch	$-\ln\left(\tan\left(\frac{\sin(x)}{2}\right)-1\right)+\ln\left(\tan\left(\frac{\sin(x)}{2}\right)+1\right)$	20
risch	$\ln\left(\mathrm{e}^{i\sin(x)}+i\right)-\ln\left(\mathrm{e}^{i\sin(x)}-i\right)$	24
norman	$-\ln\left(\tan\left(\frac{\tan\left(\frac{x}{2}\right)}{1+\tan\left(\frac{x}{2}\right)^{2}}\right)-1\right)+\ln\left(\tan\left(\frac{\tan\left(\frac{x}{2}\right)}{1+\tan\left(\frac{x}{2}\right)^{2}}\right)+1\right)$	42

input
```
int(cos(x)*sec(sin(x)),x,method=_RETURNVERBOSE)
```

output
```
ln(sec(sin(x))+tan(sin(x)))
```

3.676.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(4) = 8.

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 11.75

$$\int \cos(x)\sec(\sin(x))\,dx = \frac{1}{2}\log\left(\sin\left(\frac{2\tan\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)^{2}+1}\right)+1\right) - \frac{1}{2}\log\left(-\sin\left(\frac{2\tan\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)^{2}+1}\right)+1\right)$$

input
```
integrate(cos(x)*sec(sin(x)),x, algorithm="fricas")
```

output
```
1/2*log(sin(2*tan(1/2*x)/(tan(1/2*x)^2 + 1)) + 1) - 1/2*log(-sin(2*tan(1/2
*x)/(tan(1/2*x)^2 + 1)) + 1)
```

3.676.6 Sympy [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 10, normalized size of antiderivative = 2.50

$$\int \cos(x)\sec(\sin(x))\,dx = \log\left(\tan\left(\sin\left(x\right)\right) + \sec\left(\sin\left(x\right)\right)\right)$$

input
```
integrate(cos(x)*sec(sin(x)),x)
```

output
```
log(tan(sin(x)) + sec(sin(x)))
```

3.676.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 8, normalized size of antiderivative = 2.00

$$\int \cos(x)\sec(\sin(x))\,dx = \log\left(\sec\left(\sin\left(x\right)\right) + \tan\left(\sin\left(x\right)\right)\right)$$

input
```
integrate(cos(x)*sec(sin(x)),x, algorithm="maxima")
```

output
```
log(sec(sin(x)) + tan(sin(x)))
```

3.676.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(4) = 8.

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 7.25

$$\int \cos(x)\sec(\sin(x))\,dx = \frac{1}{4}\log\left(\left|\frac{1}{\sin\left(\sin\left(x\right)\right)} + \sin\left(\sin\left(x\right)\right) + 2\right|\right) - \frac{1}{4}\log\left(\left|\frac{1}{\sin\left(\sin\left(x\right)\right)} + \sin\left(\sin\left(x\right)\right) - 2\right|\right)$$

input
```
integrate(cos(x)*sec(sin(x)),x, algorithm="giac")
```

output
```
1/4*log(abs(1/sin(sin(x)) + sin(sin(x)) + 2)) - 1/4*log(abs(1/sin(sin(x))
+ sin(sin(x)) - 2))
```

3.676.9 Mupad [B] (verification not implemented)

Time = 27.37 (sec) , antiderivative size = 21, normalized size of antiderivative = 5.25

$$\int \cos(x)\sec(\sin(x))\,dx = -\mathrm{atan}\left(\mathrm{e}^{-\frac{\mathrm{e}^{-x\,1\mathrm{i}}}{2}}\,\mathrm{e}^{\frac{\mathrm{e}^{x\,1\mathrm{i}}}{2}}\right)\,2\mathrm{i}$$

input
```
int(cos(x)/cos(sin(x)),x)
```

output
```
-atan(exp(-exp(-x*1i)/2)*exp(exp(x*1i)/2))*2i
```

3.677 $\int \cos(x)\sin^3(x)\left(a+b\sin^2(x)\right)^3\,dx$

3.677.1 Optimal result

Integrand size = 17, antiderivative size = 36

$$\int \cos(x)\sin^3(x)\left(a+b\sin^2(x)\right)^3\,dx=-\frac{a\left(a+b\sin^2(x)\right)^4}{8b^2}+\frac{\left(a+b\sin^2(x)\right)^5}{10b^2}$$

output
```
-1/8*a*(a+b*sin(x)^2)^4/b^2+1/10*(a+b*sin(x)^2)^5/b^2
```

3.677.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 128 vs. 2(36) = 72.

Time = 0.26 (sec) , antiderivative size = 128, normalized size of antiderivative = 3.56

$$\int \cos(x)\sin^3(x)\left(a+b\sin^2(x)\right)^3\,dx$$
$$=\frac{-20(64a^3+24ab^2+7b^3)\cos(2x)+20(16a^3+18ab^2+5b^3)\cos(4x)+b(-10b(16a+5b)\cos(6x)+15b(2a}{102}$$

input
```
Integrate[Cos[x]*Sin[x]^3*(a + b*Sin[x]^2)^3,x]
```

output
```
(-20*(64*a^3 + 24*a*b^2 + 7*b^3)*Cos[2*x] + 20*(16*a^3 + 18*a*b^2 + 5*b^3)
*Cos[4*x] + b*(-10*b*(16*a + 5*b)*Cos[6*x] + 15*b*(2*a + b)*Cos[8*x] - 2*b
^2*Cos[10*x] + 3840*a^2*Sin[x]^4 + 2560*a*b*Sin[x]^6 + 640*b^2*Sin[x]^8 -
1280*a^2*Sin[x]^3*Sin[3*x]))/10240
```

3.677.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3042, 3677, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(x)\cos(x)\left(a+b\sin^2(x)\right)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x)^3\cos(x)\left(a+b\sin(x)^2\right)^3dx$$

$$\Big\downarrow \text{3677}$$

$$\int \sin^3(x)\left(a+b\sin^2(x)\right)^3d\sin(x)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \sin^2(x)\left(b\sin^2(x)+a\right)^3d\sin^2(x)$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\frac{\left(b\sin^2(x)+a\right)^4}{b}-\frac{a\left(b\sin^2(x)+a\right)^3}{b}\right)d\sin^2(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{\left(a+b\sin^2(x)\right)^5}{5b^2}-\frac{a\left(a+b\sin^2(x)\right)^4}{4b^2}\right)$$

input
```
Int[Cos[x]*Sin[x]^3*(a + b*Sin[x]^2)^3,x]
```

output
```
(-1/4*(a*(a + b*Sin[x]^2)^4)/b^2 + (a + b*Sin[x]^2)^5/(5*b^2))/2
```

3.677.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3677
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFa
ctors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[(d*ff*x)^n*(1 - ff^2*x^2)^((
m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
d, e, f, n, p}, x] && IntegerQ[(m - 1)/2]
```

3.677.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

method	result
derivativedivides	$-\frac{\frac{a\left(a+\sin\left(x\right)^{2} b\right)^{4}}{4}-\frac{\left(a+\sin\left(x\right)^{2} b\right)^{5}}{5}}{2 b^{2}}$
default	$-\frac{\frac{a\left(a+\sin\left(x\right)^{2} b\right)^{4}}{4}-\frac{\left(a+\sin\left(x\right)^{2} b\right)^{5}}{5}}{2 b^{2}}$
parallelrisch	$\frac{\left(-64 a^{3}-120 a^{2} b-84 a\,b^{2}-21 b^{3}\right)\cos\left(2 x\right)}{512}+\frac{\left(8 a^{3}+24 a^{2} b+21 a\,b^{2}+6 b^{3}\right)\cos\left(4 x\right)}{256}-\frac{b\left(a+\frac{3 b}{4}\right)^{2}\cos\left(6 x\right)}{64}+\frac{\left(3 a\,b^{2}+2 b^{3}\right)}{1024}$
risch	$-\frac{b^{3}\cos\left(10 x\right)}{5120}+\frac{3\cos\left(8 x\right) a\,b^{2}}{1024}+\frac{\cos\left(8 x\right) b^{3}}{512}-\frac{\cos\left(6 x\right) a^{2} b}{64}-\frac{3\cos\left(6 x\right) a\,b^{2}}{128}-\frac{9\cos\left(6 x\right) b^{3}}{1024}+\frac{\cos\left(4 x\right) a^{3}}{32}+\frac{3\cos}{}$

input
```
int(cos(x)*sin(x)^3*(a+sin(x)^2*b)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/b^2*(1/4*a*(a+sin(x)^2*b)^4-1/5*(a+sin(x)^2*b)^5)
```

3.677.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(32) = 64.

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.86

$$\int \cos(x)\sin^3(x)\left(a+b\sin^2(x)\right)^3\,dx = -\frac{1}{10}\,b^3\cos(x)^{10}+\frac{1}{8}\left(3\,ab^2+4\,b^3\right)\cos(x)^8 - \frac{1}{2}\left(a^2b+3\,ab^2+2\,b^3\right)\cos(x)^6 + \frac{1}{4}\left(a^3+6\,a^2b+9\,ab^2+4\,b^3\right)\cos(x)^4 - \frac{1}{2}\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\cos(x)^2$$

input
```
integrate(cos(x)*sin(x)^3*(a+b*sin(x)^2)^3,x, algorithm="fricas")
```

output
```
-1/10*b^3*cos(x)^10 + 1/8*(3*a*b^2 + 4*b^3)*cos(x)^8 - 1/2*(a^2*b + 3*a*b^
2 + 2*b^3)*cos(x)^6 + 1/4*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*cos(x)^4 - 1/2
*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(x)^2
```

3.677.6 Sympy [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int \cos(x)\sin^3(x)\left(a+b\sin^2(x)\right)^3\,dx = \frac{a^3\sin^4(x)}{4}+\frac{a^2b\sin^6(x)}{2}+\frac{3ab^2\sin^8(x)}{8}+\frac{b^3\sin^{10}(x)}{10}$$

input
```
integrate(cos(x)*sin(x)**3*(a+b*sin(x)**2)**3,x)
```

output
```
a**3*sin(x)**4/4 + a**2*b*sin(x)**6/2 + 3*a*b**2*sin(x)**8/8 + b**3*sin(x)
**10/10
```

3.677.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int \cos(x)\sin^3(x)\left(a+b\sin^2(x)\right)^3\,dx=\frac{1}{10}\,b^3\sin\left(x\right)^{10}+\frac{3}{8}\,ab^2\sin\left(x\right)^8+\frac{1}{2}\,a^2b\sin\left(x\right)^6+\frac{1}{4}\,a^3\sin\left(x\right)^4$$

input
```
integrate(cos(x)*sin(x)^3*(a+b*sin(x)^2)^3,x, algorithm="maxima")
```

output
```
1/10*b^3*sin(x)^10 + 3/8*a*b^2*sin(x)^8 + 1/2*a^2*b*sin(x)^6 + 1/4*a^3*sin
(x)^4
```

3.677.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int \cos(x)\sin^3(x)\left(a+b\sin^2(x)\right)^3\,dx=\frac{1}{10}\,b^3\sin\left(x\right)^{10}+\frac{3}{8}\,ab^2\sin\left(x\right)^8+\frac{1}{2}\,a^2b\sin\left(x\right)^6+\frac{1}{4}\,a^3\sin\left(x\right)^4$$

input
```
integrate(cos(x)*sin(x)^3*(a+b*sin(x)^2)^3,x, algorithm="giac")
```

output
```
1/10*b^3*sin(x)^10 + 3/8*a*b^2*sin(x)^8 + 1/2*a^2*b*sin(x)^6 + 1/4*a^3*sin
(x)^4
```

3.677.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.03

$$\int \cos(x)\sin^3(x)\left(a+b\sin^2(x)\right)^3\,dx=\frac{b^2\cos\left(x\right)^8\left(3\,a+4\,b\right)}{8}-\frac{b^3\cos\left(x\right)^{10}}{10}-\frac{\cos\left(x\right)^2\left(a+b\right)^3}{2}-\frac{b\cos\left(x\right)^6\left(a^2+3\,a\,b+2\,b^2\right)}{2}+\frac{\cos\left(x\right)^4\left(a+b\right)^2\left(a+4\,b\right)}{4}$$

input
```
int(cos(x)*sin(x)^3*(a + b*sin(x)^2)^3,x)
```

output
```
(b^2*cos(x)^8*(3*a + 4*b))/8 - (b^3*cos(x)^10)/10 - (cos(x)^2*(a + b)^3)/2
 - (b*cos(x)^6*(3*a*b + a^2 + 2*b^2))/2 + (cos(x)^4*(a + b)^2*(a + 4*b))/4
```

3.677. $\int \cos(x)\sin^3(x)\left(a+b\sin^2(x)\right)^3\,dx$

3.678 $\int e^{\sin(x)} \cos(x) \sin(x) \, dx$

3.678.1 Optimal result

Integrand size = 9, antiderivative size = 14

$$\int e^{\sin(x)} \cos(x) \sin(x) \, dx = -e^{\sin(x)} + e^{\sin(x)} \sin(x)$$

output
```
-exp(sin(x))+exp(sin(x))*sin(x)
```

3.678.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.64

$$\int e^{\sin(x)} \cos(x) \sin(x) \, dx = e^{\sin(x)}(-1 + \sin(x))$$

input
```
Integrate[E^Sin[x]*Cos[x]*Sin[x],x]
```

output
```
E^Sin[x]*(-1 + Sin[x])
```

3.678.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4834, 2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\sin(x)}\sin(x)\cos(x)\,dx$$
$$\Big\downarrow \text{4834}$$
$$\int e^{\sin(x)}\sin(x)d\sin(x)$$
$$\Big\downarrow \text{2607}$$
$$e^{\sin(x)}\sin(x)-\int e^{\sin(x)}d\sin(x)$$
$$\Big\downarrow \text{2624}$$
$$e^{\sin(x)}\sin(x)-e^{\sin(x)}$$

input
```
Int[E^Sin[x]*Cos[x]*Sin[x],x]
```

output
```
-E^Sin[x] + E^Sin[x]*Sin[x]
```

3.678.3.1 Defintions of rubi rules used

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 4834
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

3.678.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.64

method	result	size
parallelrisch	$(\sin (x)-1) \mathrm{e}^{\sin (x)}$	9
derivativedivides	$-\mathrm{e}^{\sin (x)}+\mathrm{e}^{\sin (x)} \sin (x)$	13
default	$-\mathrm{e}^{\sin (x)}+\mathrm{e}^{\sin (x)} \sin (x)$	13
risch	$-\mathrm{e}^{\sin (x)}+\mathrm{e}^{\sin (x)} \sin (x)$	13
norman	$\frac{2 \tan \left(\frac{x}{2}\right) \mathrm{e}^{\frac{2 \tan \left(\frac{x}{2}\right)}{1+\tan \left(\frac{x}{2}\right)^{2}}}-2 \tan \left(\frac{x}{2}\right)^{2} \mathrm{e}^{\frac{2 \tan \left(\frac{x}{2}\right)}{1+\tan \left(\frac{x}{2}\right)^{2}}}+2 \tan \left(\frac{x}{2}\right)^{3} \mathrm{e}^{\frac{2 \tan \left(\frac{x}{2}\right)}{1+\tan \left(\frac{x}{2}\right)^{2}}}-\tan \left(\frac{x}{2}\right)^{4} \mathrm{e}^{\frac{2 \tan \left(\frac{x}{2}\right)}{1+\tan \left(\frac{x}{2}\right)^{2}}}-\mathrm{e}^{\frac{2 \tan \left(\frac{x}{2}\right)}{1+\tan \left(\frac{x}{2}\right)^{2}}}}{\left(1+\tan \left(\frac{x}{2}\right)^{2}\right)^{2}}$	130

input
```
int(exp(sin(x))*cos(x)*sin(x),x,method=_RETURNVERBOSE)
```

output
```
(sin(x)-1)*exp(sin(x))
```

3.678.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int e^{\sin(x)} \cos(x) \sin(x) \, dx = (\sin (x) - 1)e^{\sin (x)}$$

input
```
integrate(exp(sin(x))*cos(x)*sin(x),x, algorithm="fricas")
```

output
```
(sin(x) - 1)*e^sin(x)
```

3.678.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int e^{\sin(x)}\cos(x)\sin(x)\,dx = e^{\sin(x)}\sin(x) - e^{\sin(x)}$$

input
```
integrate(exp(sin(x))*cos(x)*sin(x),x)
```

output
```
exp(sin(x))*sin(x) - exp(sin(x))
```

3.678.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int e^{\sin(x)}\cos(x)\sin(x)\,dx = (\sin(x) - 1)e^{\sin(x)}$$

input
```
integrate(exp(sin(x))*cos(x)*sin(x),x, algorithm="maxima")
```

output
```
(sin(x) - 1)*e^sin(x)
```

3.678.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int e^{\sin(x)}\cos(x)\sin(x)\,dx = (\sin(x) - 1)e^{\sin(x)}$$

input
```
integrate(exp(sin(x))*cos(x)*sin(x),x, algorithm="giac")
```

output
```
(sin(x) - 1)*e^sin(x)
```

3.678.9 Mupad [B] (verification not implemented)

Time = 26.69 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int e^{\sin(x)}\cos(x)\sin(x)\,dx = \mathrm{e}^{\sin(x)}\left(\sin\left(x\right)-1\right)$$

input
```
int(exp(sin(x))*cos(x)*sin(x),x)
```

output
```
exp(sin(x))*(sin(x) - 1)
```

3.679 $\int \frac{\cos^3(x)}{\sqrt{\sin^3(x)}}\,dx$

3.679.1 Optimal result

Integrand size = 13, antiderivative size = 25

$$\int \frac{\cos^3(x)}{\sqrt{\sin^3(x)}}\,dx = -\frac{2\sin(x)}{\sqrt{\sin^3(x)}} - \frac{2}{3}\sqrt{\sin^3(x)}$$

output
```
-2*sin(x)/(sin(x)^3)^(1/2)-2/3*(sin(x)^3)^(1/2)
```

3.679.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{\cos^3(x)}{\sqrt{\sin^3(x)}}\,dx = \frac{(-7+\cos(2x))\sin(x)}{3\sqrt{\sin^3(x)}}$$

input
```
Integrate[Cos[x]^3/Sqrt[Sin[x]^3],x]
```

output
```
((-7 + Cos[2*x])*Sin[x])/(3*Sqrt[Sin[x]^3])
```

3.679.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.36, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3042, 3686, 3042, 3044, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(x)}{\sqrt{\sin^3(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(x)^3}{\sqrt{\sin(x)^3}}dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{\sin^{\frac{3}{2}}(x)\int \frac{\cos^3(x)}{\sin^{\frac{3}{2}}(x)}dx}{\sqrt{\sin^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sin^{\frac{3}{2}}(x)\int \frac{\cos(x)^3}{\sin(x)^{3/2}}dx}{\sqrt{\sin^3(x)}}$$

$$\Big\downarrow \text{3044}$$

$$\frac{\sin^{\frac{3}{2}}(x)\int \frac{1-\sin^2(x)}{\sin^{\frac{3}{2}}(x)}d\sin(x)}{\sqrt{\sin^3(x)}}$$

$$\Big\downarrow \text{244}$$

$$\frac{\sin^{\frac{3}{2}}(x)\int \left(\frac{1}{\sin^{\frac{3}{2}}(x)}-\sqrt{\sin(x)}\right)d\sin(x)}{\sqrt{\sin^3(x)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sin^{\frac{3}{2}}(x)\left(-\frac{2}{3}\sin^{\frac{3}{2}}(x)-\frac{2}{\sqrt{\sin(x)}}\right)}{\sqrt{\sin^3(x)}}$$

input
```
Int[Cos[x]^3/Sqrt[Sin[x]^3],x]
```

3.679. $\int \frac{\cos^3(x)}{\sqrt{\sin^3(x)}}dx$

output
```
(Sin[x]^(3/2)*(-2/Sqrt[Sin[x]] - (2*Sin[x]^(3/2))/3))/Sqrt[Sin[x]^3]
```

3.679.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

3.679.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

method	result	size
derivativedivides	$-\frac{4\sin(x)\left(\sin(x)^2+3\right)}{3\sqrt{-\sin(3x)+3\sin(x)}}$	17
default	$-\frac{4\sin(x)\left(\sin(x)^2+3\right)}{3\sqrt{-\sin(3x)+3\sin(x)}}$	17

input
```
int(cos(x)^3/(sin(x)^3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*sin(x)*(sin(x)^2+3)/(sin(x)^3)^(1/2)
```

3.679.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int \frac{\cos^3(x)}{\sqrt{\sin^3(x)}}\,dx = -\frac{2\left(\cos(x)^2-4\right)\sqrt{-\left(\cos(x)^2-1\right)\sin(x)}}{3\left(\cos(x)^2-1\right)}$$

input
```
integrate(cos(x)^3/(sin(x)^3)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(cos(x)^2 - 4)*sqrt(-(cos(x)^2 - 1)*sin(x))/(cos(x)^2 - 1)
```

3.679.6 Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.44

$$\int \frac{\cos^3(x)}{\sqrt{\sin^3(x)}}\,dx = -\frac{8\sin^3(x)}{3\sqrt{\sin^3(x)}} - \frac{2\sin(x)\cos^2(x)}{\sqrt{\sin^3(x)}}$$

input
```
integrate(cos(x)**3/(sin(x)**3)**(1/2),x)
```

output
```
-8*sin(x)**3/(3*sqrt(sin(x)**3)) - 2*sin(x)*cos(x)**2/sqrt(sin(x)**3)
```

3.679.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{\cos^3(x)}{\sqrt{\sin^3(x)}}\,dx = -\frac{2}{3}\sqrt{\sin(x)^3} - \frac{2\sin(x)}{\sqrt{\sin(x)^3}}$$

input
```
integrate(cos(x)^3/(sin(x)^3)^(1/2),x, algorithm="maxima")
```

output
```
-2/3*sqrt(sin(x)^3) - 2*sin(x)/sqrt(sin(x)^3)
```

3.679.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.52

$$\int \frac{\cos^3(x)}{\sqrt{\sin^3(x)}}\,dx = -\frac{2}{3}\sin(x)^{\frac{3}{2}} - \frac{2}{\sqrt{\sin(x)}}$$

input
```
integrate(cos(x)^3/(sin(x)^3)^(1/2),x, algorithm="giac")
```

output
```
-2/3*sin(x)^(3/2) - 2/sqrt(sin(x))
```

3.679.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(x)}{\sqrt{\sin^3(x)}}\,dx = \int \frac{\cos(x)^3}{\sqrt{\sin(x)^3}}\,dx$$

input
```
int(cos(x)^3/(sin(x)^3)^(1/2),x)
```

output
```
int(cos(x)^3/(sin(x)^3)^(1/2), x)
```

3.680 $\int \frac{e^{\sqrt{\sin(x)}}\cos(x)}{\sqrt{\sin(x)}}\,dx$

3.680.1 Optimal result

Integrand size = 17, antiderivative size = 10

$$\int \frac{e^{\sqrt{\sin(x)}}\cos(x)}{\sqrt{\sin(x)}}\,dx = 2e^{\sqrt{\sin(x)}}$$

output
```
2*exp(sin(x)^(1/2))
```

3.680.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{e^{\sqrt{\sin(x)}}\cos(x)}{\sqrt{\sin(x)}}\,dx = 2e^{\sqrt{\sin(x)}}$$

input
```
Integrate[(E^Sqrt[Sin[x]]*Cos[x])/Sqrt[Sin[x]],x]
```

output
```
2*E^Sqrt[Sin[x]]
```

3.680.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {4834, 2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\sqrt{\sin(x)}}\cos(x)}{\sqrt{\sin(x)}}\,dx$$

$$\Big\downarrow \text{ 4834}$$

$$\int \frac{e^{\sqrt{\sin(x)}}}{\sqrt{\sin(x)}}d\sin(x)$$

$$\Big\downarrow \text{ 2638}$$

$$2e^{\sqrt{\sin(x)}}$$

input
```
Int[(E^Sqrt[Sin[x]]*Cos[x])/Sqrt[Sin[x]],x]
```

output
```
2*E^Sqrt[Sin[x]]
```

3.680.3.1 Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 4834
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

3.680.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$2\,\mathrm{e}^{\sqrt{\sin(x)}}$	8
default	$2\,\mathrm{e}^{\sqrt{\sin(x)}}$	8

input
```
int(exp(sin(x)^(1/2))*cos(x)/sin(x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*exp(sin(x)^(1/2))
```

3.680.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{e^{\sqrt{\sin(x)}}\cos(x)}{\sqrt{\sin(x)}}\,dx = 2\,e^{\sqrt{\sin(x)}}$$

input
```
integrate(exp(sin(x)^(1/2))*cos(x)/sin(x)^(1/2),x, algorithm="fricas")
```

output
```
2*e^sqrt(sin(x))
```

3.680.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{e^{\sqrt{\sin(x)}}\cos(x)}{\sqrt{\sin(x)}}\,dx = 2e^{\sqrt{\sin(x)}}$$

input
```
integrate(exp(sin(x)**(1/2))*cos(x)/sin(x)**(1/2),x)
```

output
```
2*exp(sqrt(sin(x)))
```

3.680.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{e^{\sqrt{\sin(x)}}\cos(x)}{\sqrt{\sin(x)}}\,dx = 2\,e^{\sqrt{\sin(x)}}$$

input
```
integrate(exp(sin(x)^(1/2))*cos(x)/sin(x)^(1/2),x, algorithm="maxima")
```

output
```
2*e^sqrt(sin(x))
```

3.680.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{e^{\sqrt{\sin(x)}}\cos(x)}{\sqrt{\sin(x)}}\,dx = 2\,e^{\sqrt{\sin(x)}}$$

input
```
integrate(exp(sin(x)^(1/2))*cos(x)/sin(x)^(1/2),x, algorithm="giac")
```

output
```
2*e^sqrt(sin(x))
```

3.680.9 Mupad [B] (verification not implemented)

Time = 26.99 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{e^{\sqrt{\sin(x)}}\cos(x)}{\sqrt{\sin(x)}}\,dx = 2\,\mathrm{e}^{\sqrt{\sin(x)}}$$

input
```
int((exp(sin(x)^(1/2))*cos(x))/sin(x)^(1/2),x)
```

output
```
2*exp(sin(x)^(1/2))
```

3.681 $\int e^{4+\sin(x)} \cos(x) \, dx$

3.681.1 Optimal result

Integrand size = 9, antiderivative size = 6

$$\int e^{4+\sin(x)} \cos(x) \, dx = e^{4+\sin(x)}$$

output
```
exp(4+sin(x))
```

3.681.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int e^{4+\sin(x)} \cos(x) \, dx = e^{4+\sin(x)}$$

input
```
Integrate[E^(4 + Sin[x])*Cos[x],x]
```

output
```
E^(4 + Sin[x])
```

3.681.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4834, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\sin(x)+4}\cos(x)\,dx$$
$$\Big\downarrow \text{4834}$$
$$\int e^{\sin(x)+4}d\sin(x)$$
$$\Big\downarrow \text{2624}$$
$$e^{\sin(x)+4}$$

input
```
Int[E^(4 + Sin[x])*Cos[x],x]
```

output
```
E^(4 + Sin[x])
```

3.681.3.1 Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 4834
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

3.681.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$e^{4+\sin(x)}$	6
default	$e^{4+\sin(x)}$	6
risch	$e^{4+\sin(x)}$	6
parallelrisch	$e^{4+\sin(x)}$	6
norman	$\frac{\tan\left(\frac{x}{2}\right)^{2} e^{4+\frac{2\tan\left(\frac{x}{2}\right)}{1+\tan\left(\frac{x}{2}\right)^{2}}}+e^{4+\frac{2\tan\left(\frac{x}{2}\right)}{1+\tan\left(\frac{x}{2}\right)^{2}}}}{1+\tan\left(\frac{x}{2}\right)^{2}}$	58

input
```
int(exp(4+sin(x))*cos(x),x,method=_RETURNVERBOSE)
```

output
```
exp(4+sin(x))
```

3.681.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int e^{4+\sin(x)}\cos(x)\,dx = e^{(\sin(x)+4)}$$

input
```
integrate(exp(4+sin(x))*cos(x),x, algorithm="fricas")
```

output
```
e^(sin(x) + 4)
```

3.681.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

$$\int e^{4+\sin(x)}\cos(x)\,dx = e^{4}e^{\sin(x)}$$

input
```
integrate(exp(4+sin(x))*cos(x),x)
```

output
```
exp(4)*exp(sin(x))
```

3.681.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int e^{4+\sin(x)}\cos(x)\,dx = e^{(\sin(x)+4)}$$

input
```
integrate(exp(4+sin(x))*cos(x),x, algorithm="maxima")
```

output
```
e^(sin(x) + 4)
```

3.681.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int e^{4+\sin(x)}\cos(x)\,dx = e^{(\sin(x)+4)}$$

input
```
integrate(exp(4+sin(x))*cos(x),x, algorithm="giac")
```

output
```
e^(sin(x) + 4)
```

3.681.9 Mupad [B] (verification not implemented)

Time = 26.31 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int e^{4+\sin(x)}\cos(x)\,dx = \mathrm{e}^{\sin(x)}\,\mathrm{e}^4$$

input
```
int(exp(sin(x) + 4)*cos(x),x)
```

output
```
exp(sin(x))*exp(4)
```

3.682 $\int e^{\cos(x)\sin(x)}\cos(2x)\,dx$

3.682.1 Optimal result

Integrand size = 12, antiderivative size = 10

$$\int e^{\cos(x)\sin(x)}\cos(2x)\,dx = e^{\frac{1}{2}\sin(2x)}$$

output
```
exp(1/2*sin(2*x))
```

3.682.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int e^{\cos(x)\sin(x)}\cos(2x)\,dx = e^{\cos(x)\sin(x)}$$

input
```
Integrate[E^(Cos[x]*Sin[x])*Cos[2*x],x]
```

output
```
E^(Cos[x]*Sin[x])
```

3.682.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4856, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(2x)e^{\sin(x)\cos(x)}\,dx$$

$$\Big\downarrow \text{ 4856}$$

$$\frac{1}{2}\int e^{\frac{1}{2}\sin(2x)}d\sin(2x)$$

$$\Big\downarrow \text{ 2624}$$

$$e^{\frac{1}{2}\sin(2x)}$$

input
```
Int[E^(Cos[x]*Sin[x])*Cos[2*x],x]
```

output
```
E^(Sin[2*x]/2)
```

3.682.3.1 Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 4856
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

3.682.4 Maple [A] (verified)

Time = 208.58 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$e^{\cos(x)\sin(x)}$	7
default	$e^{\cos(x)\sin(x)}$	7
risch	$e^{\cos(x)\sin(x)}$	7

input
```
int(exp(cos(x)*sin(x))*cos(2*x),x,method=_RETURNVERBOSE)
```

output
```
exp(cos(x)*sin(x))
```

3.682.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int e^{\cos(x)\sin(x)}\cos(2x)\,dx = e^{(\cos(x)\sin(x))}$$

input
```
integrate(exp(cos(x)*sin(x))*cos(2*x),x, algorithm="fricas")
```

output
```
e^(cos(x)*sin(x))
```

3.682.6 Sympy [F(-1)]

Timed out.

$$\int e^{\cos(x)\sin(x)}\cos(2x)\,dx = \text{Timed out}$$

input
```
integrate(exp(cos(x)*sin(x))*cos(2*x),x)
```

output
```
Timed out
```

3.682.7 Maxima [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int e^{\cos(x)\sin(x)}\cos(2x)\,dx = e^{\left(\frac{1}{2}\sin(2x)\right)}$$

input
```
integrate(exp(cos(x)*sin(x))*cos(2*x),x, algorithm="maxima")
```

output
```
e^(1/2*sin(2*x))
```

3.682.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int e^{\cos(x)\sin(x)}\cos(2x)\,dx = e^{\left(\frac{\tan(x)}{\tan(x)^2+1}\right)}$$

input
```
integrate(exp(cos(x)*sin(x))*cos(2*x),x, algorithm="giac")
```

output
```
e^(tan(x)/(tan(x)^2 + 1))
```

3.682.9 Mupad [B] (verification not implemented)

Time = 26.28 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int e^{\cos(x)\sin(x)}\cos(2x)\,dx = \mathrm{e}^{\frac{\sin(2x)}{2}}$$

input
```
int(cos(2*x)*exp(cos(x)*sin(x)),x)
```

output
```
exp(sin(2*x)/2)
```

3.683 $\int e^{\cos\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)}\cos(x)\,dx$

3.683.1 Optimal result

Integrand size = 18, antiderivative size = 10

$$\int e^{\cos\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)}\cos(x)\,dx=2e^{\frac{\sin(x)}{2}}$$

output
```
2*exp(1/2*sin(x))
```

3.683.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int e^{\cos\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)}\cos(x)\,dx=2e^{\frac{\sin(x)}{2}}$$

input
```
Integrate[E^(Cos[x/2]*Sin[x/2])*Cos[x],x]
```

output
```
2*E^(Sin[x]/2)
```

3.683.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4856, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)e^{\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)}\,dx$$

$$\Big\downarrow \text{4856}$$

$$\int e^{\frac{\sin(x)}{2}}d\sin(x)$$

$$\Big\downarrow \text{2624}$$

$$2e^{\frac{\sin(x)}{2}}$$

input
```
Int[E^(Cos[x/2]*Sin[x/2])*Cos[x],x]
```

output
```
2*E^(Sin[x]/2)
```

3.683.3.1 Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 4856
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

3.683.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.30

$$2\,\mathrm{e}^{\cos\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)}$$

input
```
int(exp(cos(1/2*x)*sin(1/2*x))*cos(x),x)
```

output
```
2*exp(cos(1/2*x)*sin(1/2*x))
```

3.683.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int e^{\cos\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)}\cos(x)\,dx = 2\,e^{\left(\cos\left(\frac{1}{2}\,x\right)\sin\left(\frac{1}{2}\,x\right)\right)}$$

input
```
integrate(exp(cos(1/2*x)*sin(1/2*x))*cos(x),x, algorithm="fricas")
```

output
```
2*e^(cos(1/2*x)*sin(1/2*x))
```

3.683.6 Sympy [F]

$$\int e^{\cos\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)}\cos(x)\,dx = \int e^{\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)}\cos\left(x\right)dx$$

input
```
integrate(exp(cos(1/2*x)*sin(1/2*x))*cos(x),x)
```

output
```
Integral(exp(sin(x/2)*cos(x/2))*cos(x), x)
```

3.683.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int e^{\cos\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)}\cos(x)\,dx = 2\,e^{\left(\frac{1}{2}\sin(x)\right)}$$

input
```
integrate(exp(cos(1/2*x)*sin(1/2*x))*cos(x),x, algorithm="maxima")
```

output
```
2*e^(1/2*sin(x))
```

3.683.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. 2(7) = 14.

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

$$\int e^{\cos\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)}\cos(x)\,dx = 2\,e^{\left(\frac{\tan\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)^2+1}\right)}$$

input
```
integrate(exp(cos(1/2*x)*sin(1/2*x))*cos(x),x, algorithm="giac")
```

output
```
2*e^(tan(1/2*x)/(tan(1/2*x)^2 + 1))
```

3.683.9 Mupad [B] (verification not implemented)

Time = 26.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int e^{\cos\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)}\cos(x)\,dx = 2\,\mathrm{e}^{\frac{\sin(x)}{2}}$$

input
```
int(exp(cos(x/2)*sin(x/2))*cos(x),x)
```

output
```
2*exp(sin(x)/2)
```

3.684 $\int e^{n\sin(a+bx)}\cos(a+bx)\,dx$

3.684.1 Optimal result

Integrand size = 17, antiderivative size = 17

$$\int e^{n\sin(a+bx)}\cos(a+bx)\,dx=\frac{e^{n\sin(a+bx)}}{bn}$$

output
```
exp(n*sin(b*x+a))/b/n
```

3.684.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int e^{n\sin(a+bx)}\cos(a+bx)\,dx=\frac{e^{n\sin(a+bx)}}{bn}$$

input
```
Integrate[E^(n*Sin[a + b*x])*Cos[a + b*x],x]
```

output
```
E^(n*Sin[a + b*x])/(b*n)
```

3.684.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4834, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)e^{n\sin(a+bx)}\,dx$$

$$\Big\downarrow \text{4834}$$

$$\frac{\int e^{n\sin(a+bx)}d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{2624}$$

$$\frac{e^{n\sin(a+bx)}}{bn}$$

input
```
Int[E^(n*Sin[a + b*x])*Cos[a + b*x],x]
```

output
```
E^(n*Sin[a + b*x])/(b*n)
```

3.684.3.1 Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 4834
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

3.684.4 Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{e^{n\sin(xb+a)}}{bn}$	17
default	$\frac{e^{n\sin(xb+a)}}{bn}$	17
risch	$\frac{e^{n\sin(xb+a)}}{bn}$	17
parallelrisch	$\frac{e^{n\sin(xb+a)}}{bn}$	17
norman	$\frac{\frac{e^{\frac{2n\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}}}}{nb}+\frac{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}e^{\frac{2n\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}}}}{nb}}{1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}}$	99

input
```
int(exp(n*sin(b*x+a))*cos(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
exp(n*sin(b*x+a))/b/n
```

3.684.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int e^{n\sin(a+bx)}\cos(a+bx)\,dx=\frac{e^{(n\sin(bx+a))}}{bn}$$

input
```
integrate(exp(n*sin(b*x+a))*cos(b*x+a),x, algorithm="fricas")
```

output
```
e^(n*sin(b*x + a))/(b*n)
```

3.684.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(12) = 24.

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.12

$$\int e^{n\sin(a+bx)}\cos(a+bx)\,dx = \begin{cases} x\cos(a) & \text{for } b=0 \wedge n=0 \\ xe^{n\sin(a)}\cos(a) & \text{for } b=0 \\ \frac{\sin(a+bx)}{b} & \text{for } n=0 \\ \frac{e^{n\sin(a+bx)}}{bn} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(n*sin(b*x+a))*cos(b*x+a),x)
```

output
```
Piecewise((x*cos(a), Eq(b, 0) & Eq(n, 0)), (x*exp(n*sin(a))*cos(a), Eq(b,
0)), (sin(a + b*x)/b, Eq(n, 0)), (exp(n*sin(a + b*x))/(b*n), True))
```

3.684.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int e^{n\sin(a+bx)}\cos(a+bx)\,dx = \frac{e^{(n\sin(bx+a))}}{bn}$$

input
```
integrate(exp(n*sin(b*x+a))*cos(b*x+a),x, algorithm="maxima")
```

output
```
e^(n*sin(b*x + a))/(b*n)
```

3.684.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int e^{n\sin(a+bx)}\cos(a+bx)\,dx = \frac{e^{(n\sin(bx+a))}}{bn}$$

input
```
integrate(exp(n*sin(b*x+a))*cos(b*x+a),x, algorithm="giac")
```

output
```
e^(n*sin(b*x + a))/(b*n)
```

3.684.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int e^{n\sin(a+bx)}\cos(a+bx)\,dx = \frac{e^{n\,\sin(a+b\,x)}}{b\,n}$$

input
```
int(cos(a + b*x)*exp(n*sin(a + b*x)),x)
```

output
```
exp(n*sin(a + b*x))/(b*n)
```

3.685 $\int e^{n \sin(ac+bcx)} \cos(c(a+bx)) \, dx$

3.685.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int e^{n \sin(ac+bcx)} \cos(c(a+bx)) \, dx = \frac{e^{n \sin(c(a+bx))}}{bcn}$$

output
```
exp(n*sin(c*(b*x+a)))/b/c/n
```

3.685.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int e^{n \sin(ac+bcx)} \cos(c(a+bx)) \, dx = \frac{e^{n \sin(c(a+bx))}}{bcn}$$

input
```
Integrate[E^(n*Sin[a*c + b*c*x])*Cos[c*(a + b*x)],x]
```

output
```
E^(n*Sin[c*(a + b*x)])/(b*c*n)
```

3.685.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4834, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(c(a+bx))e^{n\sin(ac+bcx)}\,dx$$

$$\Big\downarrow \text{ 4834}$$

$$\frac{\int e^{n\sin(c(a+bx))}d\sin(c(a+bx))}{bc}$$

$$\Big\downarrow \text{ 2624}$$

$$\frac{e^{n\sin(c(a+bx))}}{bcn}$$

input
```
Int[E^(n*Sin[a*c + b*c*x])*Cos[c*(a + b*x)],x]
```

output
```
E^(n*Sin[c*(a + b*x)])/(b*c*n)
```

3.685.3.1 Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 4834
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

3.685.4 Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

method	result	size
risch	$\frac{e^{n \sin(c(xb+a))}}{bcn}$	22
parallelrisch	$\frac{e^{n \sin(c(xb+a))}}{bcn}$	22
derivativedivides	$\frac{e^{n \sin(bcx+ac)}}{bcn}$	23
default	$\frac{e^{n \sin(bcx+ac)}}{bcn}$	23

input
```
int(exp(n*sin(b*c*x+a*c))*cos(c*(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
exp(n*sin(c*(b*x+a)))/b/c/n
```

3.685.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int e^{n\sin(ac+bcx)}\cos(c(a+bx))\,dx = \frac{e^{(n\sin(bcx+ac))}}{bcn}$$

input
```
integrate(exp(n*sin(b*c*x+a*c))*cos(c*(b*x+a)),x, algorithm="fricas")
```

output
```
e^(n*sin(b*c*x + a*c))/(b*c*n)
```

3.685.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(15) = 30.

Time = 4.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.23

$$\int e^{n\sin(ac+bcx)}\cos(c(a+bx))\,dx = \begin{cases} xe^{n\sin(ac)}\cos(ac) & \text{for } b=0 \\ x & \text{for } c=0 \\ \begin{cases} x\cos(ac) & \text{for } b=0 \\ x & \text{for } c=0 \\ \frac{\sin(c(a+bx))}{bc} & \text{otherwise} \end{cases} & \text{for } n=0 \\ \frac{e^{n\sin(ac+bcx)}}{bcn} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(n*sin(b*c*x+a*c))*cos(c*(b*x+a)),x)
```

output
```
Piecewise((x*exp(n*sin(a*c))*cos(a*c), Eq(b, 0)), (x, Eq(c, 0)), (Piecewis
e((x*cos(a*c), Eq(b, 0)), (x, Eq(c, 0)), (sin(c*(a + b*x))/(b*c), True)),
Eq(n, 0)), (exp(n*sin(a*c + b*c*x))/(b*c*n), True))
```

3.685.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int e^{n\sin(ac+bcx)}\cos(c(a+bx))\,dx=\frac{e^{(n\sin(bcx+ac))}}{bcn}$$

input
```
integrate(exp(n*sin(b*c*x+a*c))*cos(c*(b*x+a)),x, algorithm="maxima")
```

output
```
e^(n*sin(b*c*x + a*c))/(b*c*n)
```

3.685.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int e^{n\sin(ac+bcx)}\cos(c(a+bx))\,dx=\frac{e^{(n\sin(bcx+ac))}}{bcn}$$

input
```
integrate(exp(n*sin(b*c*x+a*c))*cos(c*(b*x+a)),x, algorithm="giac")
```

output
```
e^(n*sin(b*c*x + a*c))/(b*c*n)
```

3.685.9 Mupad [B] (verification not implemented)

Time = 25.77 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int e^{n\sin(ac+bcx)}\cos(c(a+bx))\,dx=\frac{\mathrm{e}^{n\,\sin(a\,c+b\,c\,x)}}{b\,c\,n}$$

input
```
int(cos(c*(a + b*x))*exp(n*sin(a*c + b*c*x)),x)
```

output
```
exp(n*sin(a*c + b*c*x))/(b*c*n)
```

3.685. $\int e^{n\sin(ac+bcx)}\cos(c(a+bx))\,dx$

3.686 $\int e^{n \sin(c(a+bx))} \cos(ac+bcx) \, dx$

3.686.1 Optimal result

Integrand size = 22, antiderivative size = 23

$$\int e^{n \sin(c(a+bx))} \cos(ac+bcx) \, dx = \frac{e^{n \sin(ac+bcx)}}{bcn}$$

output
```
exp(n*sin(b*c*x+a*c))/b/c/n
```

3.686.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int e^{n \sin(c(a+bx))} \cos(ac+bcx) \, dx = \frac{e^{n \sin(c(a+bx))}}{bcn}$$

input
```
Integrate[E^(n*Sin[c*(a + b*x)])*Cos[a*c + b*c*x],x]
```

output
```
E^(n*Sin[c*(a + b*x)])/(b*c*n)
```

3.686.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4834, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(ac+bcx)e^{n\sin(c(a+bx))}\,dx$$

$$\Big\downarrow \text{4834}$$

$$\frac{\int e^{n\sin(ac+bxc)}d\sin(ac+bxc)}{bc}$$

$$\Big\downarrow \text{2624}$$

$$\frac{e^{n\sin(ac+bcx)}}{bcn}$$

input
```
Int[E^(n*Sin[c*(a + b*x)])*Cos[a*c + b*c*x],x]
```

output
```
E^(n*Sin[a*c + b*c*x])/(b*c*n)
```

3.686.3.1 Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 4834
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

3.686.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{e^{n\sin(c(xb+a))}}{bcn}$	22
default	$\frac{e^{n\sin(c(xb+a))}}{bcn}$	22
risch	$\frac{e^{n\sin(c(xb+a))}}{bcn}$	22
parallelrisch	$\frac{e^{n\sin(c(xb+a))}}{bcn}$	22

input
```
int(exp(n*sin(c*(b*x+a)))*cos(b*c*x+a*c),x,method=_RETURNVERBOSE)
```

output
```
exp(n*sin(c*(b*x+a)))/b/c/n
```

3.686.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int e^{n\sin(c(a+bx))}\cos(ac+bcx)\,dx=\frac{e^{(n\sin(bcx+ac))}}{bcn}$$

input
```
integrate(exp(n*sin(c*(b*x+a)))*cos(b*c*x+a*c),x, algorithm="fricas")
```

output
```
e^(n*sin(b*c*x + a*c))/(b*c*n)
```

3.686.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(17) = 34$.

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.09

$$\int e^{n\sin(c(a+bx))}\cos(ac+bcx)\,dx=\begin{cases} x & \text{for } b=0\wedge c=0\wedge n=0\\ xe^{n\sin(ac)}\cos(ac) & \text{for } b=0\\ x & \text{for } c=0\\ \frac{\sin(ac+bcx)}{bc} & \text{for } n=0\\ \frac{e^{n\sin(ac+bcx)}}{bcn} & \text{otherwise}\end{cases}$$

input
```
integrate(exp(n*sin(c*(b*x+a)))*cos(b*c*x+a*c),x)
```

output
```
Piecewise((x, Eq(b, 0) & Eq(c, 0) & Eq(n, 0)), (x*exp(n*sin(a*c))*cos(a*c)
, Eq(b, 0)), (x, Eq(c, 0)), (sin(a*c + b*c*x)/(b*c), Eq(n, 0)), (exp(n*sin
(a*c + b*c*x))/(b*c*n), True))
```

3.686.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int e^{n\sin(c(a+bx))}\cos(ac+bcx)\,dx=\frac{e^{(n\sin(bcx+ac))}}{bcn}$$

input
```
integrate(exp(n*sin(c*(b*x+a)))*cos(b*c*x+a*c),x, algorithm="maxima")
```

output
```
e^(n*sin(b*c*x + a*c))/(b*c*n)
```

3.686.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int e^{n\sin(c(a+bx))}\cos(ac+bcx)\,dx=\frac{e^{(n\sin(bcx+ac))}}{bcn}$$

input
```
integrate(exp(n*sin(c*(b*x+a)))*cos(b*c*x+a*c),x, algorithm="giac")
```

output
```
e^(n*sin(b*c*x + a*c))/(b*c*n)
```

3.686.9 Mupad [B] (verification not implemented)

Time = 26.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int e^{n\sin(c(a+bx))}\cos(ac+bcx)\,dx=\frac{\mathrm{e}^{n\,\sin(a\,c+b\,c\,x)}}{b\,c\,n}$$

input
```
int(exp(n*sin(c*(a + b*x)))*cos(a*c + b*c*x),x)
```

output
```
exp(n*sin(a*c + b*c*x))/(b*c*n)
```

3.686. $\int e^{n\sin(c(a+bx))}\cos(ac+bcx)\,dx$

3.687 $\int e^{n \sin(a+bx)} \cot(a+bx)\,dx$

3.687.1 Optimal result

Integrand size = 17, antiderivative size = 13

$$\int e^{n \sin(a+bx)} \cot(a+bx)\,dx = \frac{\text{ExpIntegralEi}(n \sin(a+bx))}{b}$$

output
```
Ei(n*sin(b*x+a))/b
```

3.687.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int e^{n \sin(a+bx)} \cot(a+bx)\,dx = \frac{\text{ExpIntegralEi}(n \sin(a+bx))}{b}$$

input
```
Integrate[E^(n*Sin[a + b*x])*Cot[a + b*x],x]
```

output
```
ExpIntegralEi[n*Sin[a + b*x]]/b
```

3.687.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {4838, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(a+bx)e^{n\sin(a+bx)}\,dx$$

$$\Big\downarrow \text{4838}$$

$$\frac{\int e^{n\sin(a+bx)}\csc(a+bx)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{2609}$$

$$\frac{\text{ExpIntegralEi}(n\sin(a+bx))}{b}$$

input
```
Int[E^(n*Sin[a + b*x])*Cot[a + b*x],x]
```

output
```
ExpIntegralEi[n*Sin[a + b*x]]/b
```

3.687.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 4838
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[1/(b*c)   Subst[Int[SubstFor[1/x, Sin[c*(a +
 b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

3.687.4 Maple [F]

$$\int e^{n\sin(xb+a)}\cot(xb+a)\,dx$$

input
```
int(exp(n*sin(b*x+a))*cot(b*x+a),x)
```

output
```
int(exp(n*sin(b*x+a))*cot(b*x+a),x)
```

3.687.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int e^{n\sin(a+bx)}\cot(a+bx)\,dx = \frac{\mathrm{Ei}(n\sin(bx+a))}{b}$$

input
```
integrate(exp(n*sin(b*x+a))*cot(b*x+a),x, algorithm="fricas")
```

output
```
Ei(n*sin(b*x + a))/b
```

3.687.6 Sympy [F]

$$\int e^{n\sin(a+bx)}\cot(a+bx)\,dx = \int e^{n\sin(a+bx)}\cot(a+bx)\,dx$$

input
```
integrate(exp(n*sin(b*x+a))*cot(b*x+a),x)
```

output
```
Integral(exp(n*sin(a + b*x))*cot(a + b*x), x)
```

3.687.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int e^{n\sin(a+bx)}\cot(a+bx)\,dx=\frac{\operatorname{Ei}(n\sin(bx+a))}{b}$$

input
```
integrate(exp(n*sin(b*x+a))*cot(b*x+a),x, algorithm="maxima")
```

output
```
Ei(n*sin(b*x + a))/b
```

3.687.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int e^{n\sin(a+bx)}\cot(a+bx)\,dx=\frac{\operatorname{Ei}(n\sin(bx+a))}{b}$$

input
```
integrate(exp(n*sin(b*x+a))*cot(b*x+a),x, algorithm="giac")
```

output
```
Ei(n*sin(b*x + a))/b
```

3.687.9 Mupad [F(-1)]

Timed out.

$$\int e^{n\sin(a+bx)}\cot(a+bx)\,dx=\int\cot(a+b\,x)\,\mathrm{e}^{n\,\sin(a+b\,x)}\,dx$$

input
```
int(cot(a + b*x)*exp(n*sin(a + b*x)),x)
```

output
```
int(cot(a + b*x)*exp(n*sin(a + b*x)), x)
```

3.688 $\int e^{n\sin(ac+bcx)}\cot(c(a+bx))\,dx$

3.688.1 Optimal result

Integrand size = 22, antiderivative size = 18

$$\int e^{n\sin(ac+bcx)}\cot(c(a+bx))\,dx=\frac{\text{ExpIntegralEi}(n\sin(c(a+bx)))}{bc}$$

output
```
Ei(n*sin(c*(b*x+a)))/b/c
```

3.688.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int e^{n\sin(ac+bcx)}\cot(c(a+bx))\,dx=\frac{\text{ExpIntegralEi}(n\sin(c(a+bx)))}{bc}$$

input
```
Integrate[E^(n*Sin[a*c + b*c*x])*Cot[c*(a + b*x)],x]
```

output
```
ExpIntegralEi[n*Sin[c*(a + b*x)]]/(b*c)
```

3.688.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4838, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(c(a+bx))e^{n\sin(ac+bcx)}\,dx$$

$$\Big\downarrow \text{4838}$$

$$\frac{\int e^{n\sin(c(a+bx))}\csc(c(a+bx))d\sin(c(a+bx))}{bc}$$

$$\Big\downarrow \text{2609}$$

$$\frac{\text{ExpIntegralEi}(n\sin(c(a+bx)))}{bc}$$

input
```
Int[E^(n*Sin[a*c + b*c*x])*Cot[c*(a + b*x)],x]
```

output
```
ExpIntegralEi[n*Sin[c*(a + b*x)]]/(b*c)
```

3.688.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 4838
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[1/(b*c)   Subst[Int[SubstFor[1/x, Sin[c*(a +
 b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

3.688.4 Maple [F]

$$\int e^{n\sin(bcx+ac)}\cot\left(c(xb+a)\right)dx$$

input
```
int(exp(n*sin(b*c*x+a*c))*cot(c*(b*x+a)),x)
```

output
```
int(exp(n*sin(b*c*x+a*c))*cot(c*(b*x+a)),x)
```

3.688.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int e^{n\sin(ac+bcx)}\cot(c(a+bx))\,dx=\frac{\text{Ei}(n\sin\left(bcx+ac\right))}{bc}$$

input
```
integrate(exp(n*sin(b*c*x+a*c))*cot(c*(b*x+a)),x, algorithm="fricas")
```

output
```
Ei(n*sin(b*c*x + a*c))/(b*c)
```

3.688.6 Sympy [F]

$$\int e^{n\sin(ac+bcx)}\cot(c(a+bx))\,dx=\int e^{n\sin\left(ac+bcx\right)}\cot\left(ac+bcx\right)dx$$

input
```
integrate(exp(n*sin(b*c*x+a*c))*cot(c*(b*x+a)),x)
```

output
```
Integral(exp(n*sin(a*c + b*c*x))*cot(a*c + b*c*x), x)
```

3.688.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int e^{n\sin(ac+bcx)}\cot(c(a+bx))\,dx = \frac{\mathrm{Ei}(n\sin{(bcx+ac)})}{bc}$$

input
```
integrate(exp(n*sin(b*c*x+a*c))*cot(c*(b*x+a)),x, algorithm="maxima")
```

output
```
Ei(n*sin(b*c*x + a*c))/(b*c)
```

3.688.8 Giac [F]

$$\int e^{n\sin(ac+bcx)}\cot(c(a+bx))\,dx = \int \cot\left((bx+a)c\right)e^{(n\sin(bcx+ac))}\,dx$$

input
```
integrate(exp(n*sin(b*c*x+a*c))*cot(c*(b*x+a)),x, algorithm="giac")
```

output
```
integrate(cot((b*x + a)*c)*e^(n*sin(b*c*x + a*c)), x)
```

3.688.9 Mupad [F(-1)]

Timed out.

$$\int e^{n\sin(ac+bcx)}\cot(c(a+bx))\,dx = \int \cot(c\,(a+b\,x))\,\mathrm{e}^{n\,\sin(a\,c+b\,c\,x)}\,dx$$

input
```
int(cot(c*(a + b*x))*exp(n*sin(a*c + b*c*x)),x)
```

output
```
int(cot(c*(a + b*x))*exp(n*sin(a*c + b*c*x)), x)
```

3.689 $\int e^{n \sin(c(a+bx))} \cot(ac + bcx) \, dx$

3.689.1 Optimal result

Integrand size = 22, antiderivative size = 19

$$\int e^{n \sin(c(a+bx))} \cot(ac + bcx) \, dx = \frac{\text{ExpIntegralEi}(n \sin(ac + bcx))}{bc}$$

output
```
Ei(n*sin(b*c*x+a*c))/b/c
```

3.689.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int e^{n \sin(c(a+bx))} \cot(ac + bcx) \, dx = \frac{\text{ExpIntegralEi}(n \sin(c(a + bx)))}{bc}$$

input
```
Integrate[E^(n*Sin[c*(a + b*x)])*Cot[a*c + b*c*x],x]
```

output
```
ExpIntegralEi[n*Sin[c*(a + b*x)]]/(b*c)
```

3.689.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4838, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(ac+bcx)e^{n\sin(c(a+bx))}\,dx$$

$$\Big\downarrow \text{4838}$$

$$\frac{\int e^{n\sin(ac+bxc)}\csc(ac+bxc)d\sin(ac+bxc)}{bc}$$

$$\Big\downarrow \text{2609}$$

$$\frac{\text{ExpIntegralEi}(n\sin(ac+bxc))}{bc}$$

input
```
Int[E^(n*Sin[c*(a + b*x)])*Cot[a*c + b*c*x],x]
```

output
```
ExpIntegralEi[n*Sin[a*c + b*c*x]]/(b*c)
```

3.689.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

rule 4838
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[1/(b*c)   Subst[Int[SubstFor[1/x, Sin[c*(a +
 b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

3.689.4 Maple [F]

$$\int e^{n\sin(c(xb+a))}\cot(bcx+ac)\,dx$$

input
```
int(exp(n*sin(c*(b*x+a)))*cot(b*c*x+a*c),x)
```

output
```
int(exp(n*sin(c*(b*x+a)))*cot(b*c*x+a*c),x)
```

3.689.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int e^{n\sin(c(a+bx))}\cot(ac+bcx)\,dx=\frac{\mathrm{Ei}(n\sin(bcx+ac))}{bc}$$

input
```
integrate(exp(n*sin(c*(b*x+a)))*cot(b*c*x+a*c),x, algorithm="fricas")
```

output
```
Ei(n*sin(b*c*x + a*c))/(b*c)
```

3.689.6 Sympy [F]

$$\int e^{n\sin(c(a+bx))}\cot(ac+bcx)\,dx=\int e^{n\sin(ac+bcx)}\cot(ac+bcx)\,dx$$

input
```
integrate(exp(n*sin(c*(b*x+a)))*cot(b*c*x+a*c),x)
```

output
```
Integral(exp(n*sin(a*c + b*c*x))*cot(a*c + b*c*x), x)
```

3.689.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int e^{n\sin(c(a+bx))}\cot(ac+bcx)\,dx=\frac{\mathrm{Ei}(n\sin{(bcx+ac)})}{bc}$$

input
```
integrate(exp(n*sin(c*(b*x+a)))*cot(b*c*x+a*c),x, algorithm="maxima")
```

output
```
Ei(n*sin(b*c*x + a*c))/(b*c)
```

3.689.8 Giac [F]

$$\int e^{n\sin(c(a+bx))}\cot(ac+bcx)\,dx=\int\cot{(bcx+ac)}\,e^{(n\sin((bx+a)c))}\,dx$$

input
```
integrate(exp(n*sin(c*(b*x+a)))*cot(b*c*x+a*c),x, algorithm="giac")
```

output
```
integrate(cot(b*c*x + a*c)*e^(n*sin((b*x + a)*c)), x)
```

3.689.9 Mupad [F(-1)]

Timed out.

$$\int e^{n\sin(c(a+bx))}\cot(ac+bcx)\,dx=\int e^{n\,\sin(c\,(a+b\,x))}\cot(a\,c+b\,c\,x)\,dx$$

input
```
int(exp(n*sin(c*(a + b*x)))*cot(a*c + b*c*x),x)
```

output
```
int(exp(n*sin(c*(a + b*x)))*cot(a*c + b*c*x), x)
```

3.690 $\int \frac{\sec^2(x)}{a+b\tan(x)}\,dx$

3.690.1 Optimal result

Integrand size = 13, antiderivative size = 11

$$\int \frac{\sec^2(x)}{a+b\tan(x)}\,dx = \frac{\log(a+b\tan(x))}{b}$$

output
```
ln(a+b*tan(x))/b
```

3.690.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(x)}{a+b\tan(x)}\,dx = \frac{\log(a+b\tan(x))}{b}$$

input
```
Integrate[Sec[x]^2/(a + b*Tan[x]),x]
```

output
```
Log[a + b*Tan[x]]/b
```

3.690.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 3987, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(x)}{a+b\tan(x)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sec(x)^2}{a+b\tan(x)}dx$$

$$\Big\downarrow \; 3987$$

$$\frac{\int \frac{1}{a+b\tan(x)}d(b\tan(x))}{b}$$

$$\Big\downarrow \; 16$$

$$\frac{\log(a+b\tan(x))}{b}$$

input
```
Int[Sec[x]^2/(a + b*Tan[x]),x]
```

output
```
Log[a + b*Tan[x]]/b
```

3.690.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + 
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

3.690.4 Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$\frac{\ln(a+b\tan(x))}{b}$	12
default	$\frac{\ln(a+b\tan(x))}{b}$	12
risch	$-\frac{\ln(e^{2ix}+1)}{b}+\frac{\ln\left(e^{2ix}-\frac{ib+a}{ib-a}\right)}{b}$	44

input
```
int(sec(x)^2/(a+b*tan(x)),x,method=_RETURNVERBOSE)
```

output
```
ln(a+b*tan(x))/b
```

3.690.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(11) = 22.

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 3.64

$$\int \frac{\sec^2(x)}{a+b\tan(x)}\,dx = \frac{\log\left(2\,ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2+b^2\right)-\log\left(\cos(x)^2\right)}{2\,b}$$

input
```
integrate(sec(x)^2/(a+b*tan(x)),x, algorithm="fricas")
```

output
```
1/2*(log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) - log(cos(x)^2)
)/b
```

3.690.6 Sympy [F]

$$\int \frac{\sec^2(x)}{a+b\tan(x)}\,dx = \int \frac{\sec^2\left(x\right)}{a+b\tan\left(x\right)}\,dx$$

input
```
integrate(sec(x)**2/(a+b*tan(x)),x)
```

output
```
Integral(sec(x)**2/(a + b*tan(x)), x)
```

3.690.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(x)}{a+b\tan(x)}\,dx = \frac{\log\left(b\tan\left(x\right)+a\right)}{b}$$

input
```
integrate(sec(x)^2/(a+b*tan(x)),x, algorithm="maxima")
```

output
```
log(b*tan(x) + a)/b
```

3.690.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{\sec^2(x)}{a+b\tan(x)}\,dx = \frac{\log\left(|b\tan\left(x\right)+a|\right)}{b}$$

input
```
integrate(sec(x)^2/(a+b*tan(x)),x, algorithm="giac")
```

output
```
log(abs(b*tan(x) + a))/b
```

3.690.9 Mupad [B] (verification not implemented)

Time = 26.48 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(x)}{a+b\tan(x)}\,dx = \frac{\ln\left(a+b\tan(x)\right)}{b}$$

input
```
int(1/(cos(x)^2*(a + b*tan(x))),x)
```

output
```
log(a + b*tan(x))/b
```

3.691 $\int \frac{\sec^2(x)}{1-\tan^2(x)}\,dx$

3.691.1 Optimal result

Integrand size = 15, antiderivative size = 11

$$\int \frac{\sec^2(x)}{1-\tan^2(x)}\,dx = \frac{1}{2}\operatorname{arctanh}(2\cos(x)\sin(x))$$

output
```
1/2*arctanh(2*cos(x)*sin(x))
```

3.691.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{\sec^2(x)}{1-\tan^2(x)}\,dx = \frac{1}{2}\operatorname{arctanh}(\sin(2x))$$

input
```
Integrate[Sec[x]^2/(1 - Tan[x]^2),x]
```

output
```
ArcTanh[Sin[2*x]]/2
```

3.691.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.27, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4158, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(x)}{1-\tan^2(x)}\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{\sec(x)^2}{1-\tan(x)^2}dx$$
$$\Big\downarrow \ 4158$$
$$\int \frac{1}{1-\tan^2(x)}d\tan(x)$$
$$\Big\downarrow \ 219$$
$$\text{arctanh}(\tan(x))$$

input
```
Int[Sec[x]^2/(1 - Tan[x]^2),x]
```

output
```
ArcTanh[Tan[x]]
```

3.691.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

3.691.4 Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.36

method	result	size
default	$\operatorname{arctanh}(\tan(x))$	4
risch	$-\frac{\ln(e^{2ix}-i)}{2}+\frac{\ln(i+e^{2ix})}{2}$	24

input
```
int(sec(x)^2/(1-tan(x)^2),x,method=_RETURNVERBOSE)
```

output
```
arctanh(tan(x))
```

3.691.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. 2(9) = 18.

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.09

$$\int \frac{\sec^2(x)}{1-\tan^2(x)}\,dx = \frac{1}{4}\log(2\cos(x)\sin(x)+1) - \frac{1}{4}\log(-2\cos(x)\sin(x)+1)$$

input
```
integrate(sec(x)^2/(1-tan(x)^2),x, algorithm="fricas")
```

output
```
1/4*log(2*cos(x)*sin(x) + 1) - 1/4*log(-2*cos(x)*sin(x) + 1)
```

3.691.6 Sympy [F]

$$\int \frac{\sec^2(x)}{1-\tan^2(x)}\,dx = -\int \frac{\sec^2{(x)}}{\tan^2{(x)}-1}\,dx$$

input
```
integrate(sec(x)**2/(1-tan(x)**2),x)
```

output
```
-Integral(sec(x)**2/(tan(x)**2 - 1), x)
```

3.691.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \frac{\sec^2(x)}{1-\tan^2(x)}\,dx = \frac{1}{2}\log\left(\tan\left(x\right)+1\right) - \frac{1}{2}\log\left(\tan\left(x\right)-1\right)$$

input
```
integrate(sec(x)^2/(1-tan(x)^2),x, algorithm="maxima")
```

output
```
1/2*log(tan(x) + 1) - 1/2*log(tan(x) - 1)
```

3.691.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \frac{\sec^2(x)}{1-\tan^2(x)}\,dx = \frac{1}{2}\log\left(\left|\tan\left(x\right)+1\right|\right) - \frac{1}{2}\log\left(\left|\tan\left(x\right)-1\right|\right)$$

input
```
integrate(sec(x)^2/(1-tan(x)^2),x, algorithm="giac")
```

output
```
1/2*log(abs(tan(x) + 1)) - 1/2*log(abs(tan(x) - 1))
```

3.691.9 Mupad [B] (verification not implemented)

Time = 26.55 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.27

$$\int \frac{\sec^2(x)}{1-\tan^2(x)}\,dx = \operatorname{atanh}(\tan(x))$$

input
```
int(-1/(cos(x)^2*(tan(x)^2 - 1)),x)
```

output
```
atanh(tan(x))
```

3.692 $\int \frac{\sec^2(x)}{9+\tan^2(x)}\,dx$

3.692.1 Optimal result

Integrand size = 13, antiderivative size = 27

$$\int \frac{\sec^2(x)}{9+\tan^2(x)}\,dx=\frac{x}{3}-\frac{1}{3}\arctan\left(\frac{2\cos(x)\sin(x)}{1+2\cos^2(x)}\right)$$

output
```
1/3*x-1/3*arctan(2*cos(x)*sin(x)/(1+2*cos(x)^2))
```

3.692.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.33

$$\int \frac{\sec^2(x)}{9+\tan^2(x)}\,dx=-\frac{1}{3}\arctan(3\cot(x))$$

input
```
Integrate[Sec[x]^2/(9 + Tan[x]^2),x]
```

output
```
-1/3*ArcTan[3*Cot[x]]
```

3.692.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.41, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3042, 4158, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(x)}{\tan^2(x)+9} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(x)^2}{\tan(x)^2+9}dx$$

$$\Big\downarrow \text{4158}$$

$$\int \frac{1}{\tan^2(x)+9}d\tan(x)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{3}\arctan\left(\frac{\tan(x)}{3}\right)$$

input
```
Int[Sec[x]^2/(9 + Tan[x]^2),x]
```

output
```
ArcTan[Tan[x]/3]/3
```

3.692.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

3.692.4 Maple [A] (verified)

Time = 6.79 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.30

method	result	size
default	$\frac{\arctan\left(\frac{\tan(x)}{3}\right)}{3}$	8
risch	$\frac{i\ln\left(e^{2ix}+2\right)}{6}-\frac{i\ln\left(e^{2ix}+\frac{1}{2}\right)}{6}$	24

input
```
int(sec(x)^2/(9+tan(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/3*arctan(1/3*tan(x))
```

3.692.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{\sec^2(x)}{9+\tan^2(x)}\,dx = -\frac{1}{6}\arctan\left(\frac{10\cos(x)^2-1}{6\cos(x)\sin(x)}\right)$$

input
```
integrate(sec(x)^2/(9+tan(x)^2),x, algorithm="fricas")
```

output
```
-1/6*arctan(1/6*(10*cos(x)^2 - 1)/(cos(x)*sin(x)))
```

3.692.6 Sympy [F]

$$\int \frac{\sec^2(x)}{9+\tan^2(x)}\,dx = \int \frac{\sec^2(x)}{\tan^2(x)+9}\,dx$$

input
```
integrate(sec(x)**2/(9+tan(x)**2),x)
```

output
```
Integral(sec(x)**2/(tan(x)**2 + 9), x)
```

3.692.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.26

$$\int \frac{\sec^2(x)}{9+\tan^2(x)}\,dx = \frac{1}{3}\arctan\left(\frac{1}{3}\tan(x)\right)$$

input
```
integrate(sec(x)^2/(9+tan(x)^2),x, algorithm="maxima")
```

output
```
1/3*arctan(1/3*tan(x))
```

3.692.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.26

$$\int \frac{\sec^2(x)}{9+\tan^2(x)}\,dx = \frac{1}{3}\arctan\left(\frac{1}{3}\tan(x)\right)$$

input
```
integrate(sec(x)^2/(9+tan(x)^2),x, algorithm="giac")
```

output
```
1/3*arctan(1/3*tan(x))
```

3.692.9 Mupad [B] (verification not implemented)

Time = 26.38 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.26

$$\int \frac{\sec^2(x)}{9+\tan^2(x)}\,dx = \frac{\operatorname{atan}\left(\frac{\tan(x)}{3}\right)}{3}$$

input
```
int(1/(cos(x)^2*(tan(x)^2 + 9)),x)
```

output
```
atan(tan(x)/3)/3
```

3.693 $\int \sec^2(x)(a+b\tan(x))^n\,dx$

3.693.1 Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \sec^2(x)(a+b\tan(x))^n\,dx=\frac{(a+b\tan(x))^{1+n}}{b(1+n)}$$

output
```
(a+b*tan(x))^(1+n)/b/(1+n)
```

3.693.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \sec^2(x)(a+b\tan(x))^n\,dx=\frac{(a+b\tan(x))^{1+n}}{b(1+n)}$$

input
```
Integrate[Sec[x]^2*(a + b*Tan[x])^n,x]
```

output
```
(a + b*Tan[x])^(1 + n)/(b*(1 + n))
```

3.693.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3042, 3987, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec^2(x)(a+b\tan(x))^n\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sec(x)^2(a+b\tan(x))^n dx$$

$$\Big\downarrow \text{ 3987}$$

$$\frac{\int (a+b\tan(x))^n d(b\tan(x))}{b}$$

$$\Big\downarrow \text{ 17}$$

$$\frac{(a+b\tan(x))^{n+1}}{b(n+1)}$$

input
```
Int[Sec[x]^2*(a + b*Tan[x])^n,x]
```

output
```
(a + b*Tan[x])^(1 + n)/(b*(1 + n))
```

3.693.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

3.693.4 Maple [A] (verified)

Time = 12.98 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$\frac{(a+b\tan(x))^{n+1}}{b(n+1)}$	20
default	$\frac{(a+b\tan(x))^{n+1}}{b(n+1)}$	20

input
```
int(sec(x)^2*(a+b*tan(x))^n,x,method=_RETURNVERBOSE)
```

output
```
(a+b*tan(x))^(n+1)/b/(n+1)
```

3.693.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.95

$$\int \sec^2(x)(a+b\tan(x))^n\,dx = \frac{(a\cos(x)+b\sin(x))\left(\frac{a\cos(x)+b\sin(x)}{\cos(x)}\right)^n}{(bn+b)\cos(x)}$$

input
```
integrate(sec(x)^2*(a+b*tan(x))^n,x, algorithm="fricas")
```

output
```
(a*cos(x) + b*sin(x))*((a*cos(x) + b*sin(x))/cos(x))^n/((b*n + b)*cos(x))
```

3.693.6 Sympy [F]

$$\int \sec^2(x)(a+b\tan(x))^n\,dx = \int (a+b\tan(x))^n \sec^2(x)\,dx$$

input
```
integrate(sec(x)**2*(a+b*tan(x))**n,x)
```

output
```
Integral((a + b*tan(x))**n*sec(x)**2, x)
```

3.693.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \sec^2(x)(a+b\tan(x))^n\,dx = \frac{(b\tan(x)+a)^{n+1}}{b(n+1)}$$

input
```
integrate(sec(x)^2*(a+b*tan(x))^n,x, algorithm="maxima")
```

output
```
(b*tan(x) + a)^(n + 1)/(b*(n + 1))
```

3.693.8 Giac [F(-2)]

Exception generated.

$$\int \sec^2(x)(a+b\tan(x))^n\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sec(x)^2*(a+b*tan(x))^n,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,1,0]%%%} / %%%{1,[0,0,1]%%%} Error: Bad Argument Valu
e
```

3.693.9 Mupad [B] (verification not implemented)

Time = 27.66 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.95

$$\int \sec^2(x)(a+b\tan(x))^n\,dx = \begin{cases} \frac{\ln(a+b\tan(x))}{b} & \text{if } n=-1 \\ \frac{(a+b\tan(x))^{n+1}}{b\,(n+1)} & \text{if } n\neq -1 \end{cases}$$

input
```
int((a + b*tan(x))^n/cos(x)^2,x)
```

output
```
piecewise(n == -1, log(a + b*tan(x))/b, n ~= -1, (a + b*tan(x))^(n + 1)/(b
*(n + 1)))
```

3.694 $\int \sec^2(x) \left(1+\frac{1}{1+\tan^2(x)}\right) \, dx$

3.694.1 Optimal result

Integrand size = 15, antiderivative size = 4

$$\int \sec^2(x) \left(1+\frac{1}{1+\tan^2(x)}\right) \, dx = x+\tan(x)$$

output
```
x+tan(x)
```

3.694.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 11 vs. 2(4) = 8.

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 2.75

$$\int \sec^2(x) \left(1+\frac{1}{1+\tan^2(x)}\right) \, dx = 2x-\arctan(\tan(x))+\tan(x)$$

input
```
Integrate[Sec[x]^2*(1 + (1 + Tan[x]^2)^(-1)),x]
```

output
```
2*x - ArcTan[Tan[x]] + Tan[x]
```

3.694.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.50, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 4889, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{1}{\tan^2(x)+1}+1\right)\sec^2(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(\frac{1}{\tan(x)^2+1}+1\right)\sec(x)^2dx$$

$$\Big\downarrow \text{4889}$$

$$\int \left(\frac{1}{\tan^2(x)+1}+1\right)d\tan(x)$$

$$\Big\downarrow \text{2009}$$

$$\arctan(\tan(x))+\tan(x)$$

input
```
Int[Sec[x]^2*(1 + (1 + Tan[x]^2)^(-1)),x]
```

output
```
ArcTan[Tan[x]] + Tan[x]
```

3.694.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.694.4 Maple [A] (verified)

Time = 5.99 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

method	result	size
default	$x+\tan\left(x\right)$	5
risch	$x+\frac{2i}{{\rm e}^{2ix}+1}$	15

input
```
int(sec(x)^2*(1+1/(1+tan(x)^2)),x,method=_RETURNVERBOSE)
```

output
```
x+tan(x)
```

3.694.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. 2(4) = 8.

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 3.00

$$\int \sec^2(x)\left(1+\frac{1}{1+\tan^2(x)}\right)\,dx=\frac{x\cos\left(x\right)+\sin\left(x\right)}{\cos\left(x\right)}$$

input
```
integrate(sec(x)^2*(1+1/(1+tan(x)^2)),x, algorithm="fricas")
```

output
```
(x*cos(x) + sin(x))/cos(x)
```

3.694.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(3) = 6$.

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 6.75

$$\int \sec^2(x)\left(1+\frac{1}{1+\tan^2(x)}\right)\,dx = \frac{x\sec^2(x)}{\tan^2(x)+1}+\frac{\tan(x)\sec^2(x)}{\tan^2(x)+1}$$

input
```
integrate(sec(x)**2*(1+1/(1+tan(x)**2)),x)
```

output
```
x*sec(x)**2/(tan(x)**2 + 1) + tan(x)*sec(x)**2/(tan(x)**2 + 1)
```

3.694.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \sec^2(x)\left(1+\frac{1}{1+\tan^2(x)}\right)\,dx = x+\tan(x)$$

input
```
integrate(sec(x)^2*(1+1/(1+tan(x)^2)),x, algorithm="maxima")
```

output
```
x + tan(x)
```

3.694.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \sec^2(x)\left(1+\frac{1}{1+\tan^2(x)}\right)\,dx = x+\tan(x)$$

input
```
integrate(sec(x)^2*(1+1/(1+tan(x)^2)),x, algorithm="giac")
```

output
```
x + tan(x)
```

3.694.9 Mupad [B] (verification not implemented)

Time = 26.03 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \sec^2(x)\left(1+\frac{1}{1+\tan^2(x)}\right)\,dx=x+\tan(x)$$

input
```
int((1/(tan(x)^2 + 1) + 1)/cos(x)^2,x)
```

output
```
x + tan(x)
```

3.695 $\int \frac{\sec^2(x)(2+\tan^2(x))}{1+\tan^2(x)} dx$

3.695.1 Optimal result

Integrand size = 19, antiderivative size = 4

$$\int \frac{\sec^2(x)\left(2+\tan^2(x)\right)}{1+\tan^2(x)}\,dx = x+\tan(x)$$

output
```
x+tan(x)
```

3.695.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 11 vs. 2(4) = 8.

Time = 0.00 (sec) , antiderivative size = 11, normalized size of antiderivative = 2.75

$$\int \frac{\sec^2(x)\left(2+\tan^2(x)\right)}{1+\tan^2(x)}\,dx = 2x-\arctan(\tan(x))+\tan(x)$$

input
```
Integrate[(Sec[x]^2*(2 + Tan[x]^2))/(1 + Tan[x]^2),x]
```

output
```
2*x - ArcTan[Tan[x]] + Tan[x]
```

3.695.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 4140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(\tan^2(x)+2)\sec^2(x)}{\tan^2(x)+1} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(\tan(x)^2+2)\sec(x)^2}{\tan(x)^2+1} dx$$

$$\Big\downarrow \text{4140}$$

$$\int (\tan^2(x)+2) \, dx$$

$$\Big\downarrow \text{2009}$$

$$x+\tan(x)$$

input
```
Int[(Sec[x]^2*(2 + Tan[x]^2))/(1 + Tan[x]^2),x]
```

output
```
x + Tan[x]
```

3.695.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

3.695.4 Maple [A] (verified)

Time = 5.95 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

method	result	size
default	$x+\tan\left(x\right)$	5
risch	$x+\frac{2i}{e^{2ix}+1}$	15

input
```
int(sec(x)^2*(2+tan(x)^2)/(1+tan(x)^2),x,method=_RETURNVERBOSE)
```

output
```
x+tan(x)
```

3.695.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. 2(4) = 8.

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 3.00

$$\int \frac{\sec^2(x)\left(2+\tan^2(x)\right)}{1+\tan^2(x)}\,dx=\frac{x\cos\left(x\right)+\sin\left(x\right)}{\cos\left(x\right)}$$

input
```
integrate(sec(x)^2*(2+tan(x)^2)/(1+tan(x)^2),x, algorithm="fricas")
```

output
```
(x*cos(x) + sin(x))/cos(x)
```

3.695.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(3) = 6.

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 6.75

$$\int \frac{\sec^2(x)\left(2+\tan^2(x)\right)}{1+\tan^2(x)}\,dx=\frac{x\sec^2\left(x\right)}{\tan^2\left(x\right)+1}+\frac{\tan\left(x\right)\sec^2\left(x\right)}{\tan^2\left(x\right)+1}$$

input
```
integrate(sec(x)**2*(2+tan(x)**2)/(1+tan(x)**2),x)
```

output
```
x*sec(x)**2/(tan(x)**2 + 1) + tan(x)*sec(x)**2/(tan(x)**2 + 1)
```

3.695.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(x) (2+\tan^2(x))}{1+\tan^2(x)} dx = x + \tan (x)$$

input
```
integrate(sec(x)^2*(2+tan(x)^2)/(1+tan(x)^2),x, algorithm="maxima")
```

output
```
x + tan(x)
```

3.695.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(x) (2+\tan^2(x))}{1+\tan^2(x)} dx = x + \tan (x)$$

input
```
integrate(sec(x)^2*(2+tan(x)^2)/(1+tan(x)^2),x, algorithm="giac")
```

output
```
x + tan(x)
```

3.695.9 Mupad [B] (verification not implemented)

Time = 25.60 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(x) (2+\tan^2(x))}{1+\tan^2(x)} dx = x + \tan(x)$$

input
```
int((tan(x)^2 + 2)/(cos(x)^2*(tan(x)^2 + 1)),x)
```

output
```
x + tan(x)
```

3.696 $\int \frac{\sec^2(x)}{2+2\tan(x)+\tan^2(x)}\,dx$

3.696.1 Optimal result

Integrand size = 17, antiderivative size = 33

$$\int \frac{\sec^2(x)}{2+2\tan(x)+\tan^2(x)}\,dx = x-\arctan\left(\frac{1-2\cos^2(x)+\cos(x)\sin(x)}{2+\cos^2(x)+2\cos(x)\sin(x)}\right)$$

output
```
x-arctan((1-2*cos(x)^2+cos(x)*sin(x))/(2+cos(x)^2+2*cos(x)*sin(x)))
```

3.696.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{\sec^2(x)}{2+2\tan(x)+\tan^2(x)}\,dx = 2\left(-\frac{1}{4}\arctan\left(\frac{\cos(x)}{\cos(x)+\sin(x)}\right) + \frac{1}{4}\arctan(\sec(x)(\cos(x)+\sin(x)))\right)$$

input
```
Integrate[Sec[x]^2/(2 + 2*Tan[x] + Tan[x]^2),x]
```

output
```
2*(-1/4*ArcTan[Cos[x]/(Cos[x] + Sin[x])] + ArcTan[Sec[x]*(Cos[x] + Sin[x])
]/4)
```

3.696.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.15, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3042, 4842, 1082, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(x)}{\tan^2(x)+2\tan(x)+2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(x)^2}{\tan(x)^2+2\tan(x)+2}dx$$

$$\Big\downarrow \text{4842}$$

$$\int \frac{1}{\tan^2(x)+2\tan(x)+2}d\tan(x)$$

$$\Big\downarrow \text{1082}$$

$$-\int \frac{1}{-(\tan(x)+1)^2-1}d(\tan(x)+1)$$

$$\Big\downarrow \text{217}$$

$$\arctan(\tan(x)+1)$$

input
```
Int[Sec[x]^2/(2 + 2*Tan[x] + Tan[x]^2),x]
```

output
```
ArcTan[1 + Tan[x]]
```

3.696.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4842
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Tan[c*(a +
 b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] |
| EqQ[F, sec])
```

3.696.4 Maple [A] (verified)

Time = 8.09 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.18

method	result	size
default	$\arctan\left(1+\tan\left(x\right)\right)$	6
risch	$-\frac{i \ln\left({\mathrm e}^{2 i x}+\frac{1}{5}+\frac{2 i}{5}\right)}{2}+\frac{i \ln\left({\mathrm e}^{2 i x}+1+2 i\right)}{2}$	28

input
```
int(sec(x)^2/(2+2*tan(x)+tan(x)^2),x,method=_RETURNVERBOSE)
```

output
```
arctan(1+tan(x))
```

3.696.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

$$\int \frac{\sec^2(x)}{2+2\tan(x)+\tan^2(x)}\,dx = -\frac{1}{2}\,\arctan\left(-\frac{3\,\cos\left(x\right)^{2}+6\,\cos\left(x\right)\sin\left(x\right)+1}{2\,\left(2\,\cos\left(x\right)^{2}-\cos\left(x\right)\sin\left(x\right)-1\right)}\right)$$

input
```
integrate(sec(x)^2/(2+2*tan(x)+tan(x)^2),x, algorithm="fricas")
```

output
```
-1/2*arctan(-1/2*(3*cos(x)^2 + 6*cos(x)*sin(x) + 1)/(2*cos(x)^2 - cos(x)*s
in(x) - 1))
```

3.696.6 Sympy [F]

$$\int \frac{\sec^2(x)}{2+2\tan(x)+\tan^2(x)}\,dx = \int \frac{\sec^2{(x)}}{\tan^2{(x)}+2\tan{(x)}+2}\,dx$$

input
```
integrate(sec(x)**2/(2+2*tan(x)+tan(x)**2),x)
```

output
```
Integral(sec(x)**2/(tan(x)**2 + 2*tan(x) + 2), x)
```

3.696.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.15

$$\int \frac{\sec^2(x)}{2+2\tan(x)+\tan^2(x)}\,dx = \arctan\left(\tan\left(x\right)+1\right)$$

input
```
integrate(sec(x)^2/(2+2*tan(x)+tan(x)^2),x, algorithm="maxima")
```

output
```
arctan(tan(x) + 1)
```

3.696.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.15

$$\int \frac{\sec^2(x)}{2+2\tan(x)+\tan^2(x)}\,dx = \arctan\left(\tan\left(x\right)+1\right)$$

input
```
integrate(sec(x)^2/(2+2*tan(x)+tan(x)^2),x, algorithm="giac")
```

output
```
arctan(tan(x) + 1)
```

3.696.9 Mupad [B] (verification not implemented)

Time = 26.79 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.15

$$\int \frac{\sec^2(x)}{2+2\tan(x)+\tan^2(x)}\,dx = \operatorname{atan}(\tan(x)+1)$$

input
```
int(1/(cos(x)^2*(2*tan(x) + tan(x)^2 + 2)),x)
```

output
```
atan(tan(x) + 1)
```

3.697 $\int \frac{\sec^2(x)}{\tan^2(x)+\tan^3(x)}\,dx$

3.697.1 Optimal result

Integrand size = 16, antiderivative size = 10

$$\int \frac{\sec^2(x)}{\tan^2(x)+\tan^3(x)}\,dx = -\cot(x)+\log(1+\cot(x))$$

output
```
-cot(x)+ln(1+cot(x))
```

3.697.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \frac{\sec^2(x)}{\tan^2(x)+\tan^3(x)}\,dx = -\cot(x)-\log(\sin(x))+\log(\cos(x)+\sin(x))$$

input
```
Integrate[Sec[x]^2/(Tan[x]^2 + Tan[x]^3),x]
```

output
```
-Cot[x] - Log[Sin[x]] + Log[Cos[x] + Sin[x]]
```

3.697.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.50, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 4842, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(x)}{\tan^3(x)+\tan^2(x)} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(x)^2}{\tan(x)^3+\tan(x)^2}dx$$

$$\Big\downarrow \text{4842}$$

$$\int \frac{\cot^2(x)}{\tan(x)+1}d\tan(x)$$

$$\Big\downarrow \text{54}$$

$$\int \left(\frac{1}{\tan(x)+1}+\cot^2(x)-\cot(x)\right)d\tan(x)$$

$$\Big\downarrow \text{2009}$$

$$-\cot(x)-\log(\tan(x))+\log(\tan(x)+1)$$

input
```
Int[Sec[x]^2/(Tan[x]^2 + Tan[x]^3),x]
```

output
```
-Cot[x] - Log[Tan[x]] + Log[1 + Tan[x]]
```

3.697.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4842
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Tan[c*(a +
 b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] |
| EqQ[F, sec])
```

3.697.4 Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

method	result	size
default	$\ln\left(1+\tan\left(x\right)\right)-\frac{1}{\tan\left(x\right)}-\ln\left(\tan\left(x\right)\right)$	18
risch	$-\frac{2i}{{\rm e}^{2ix}-1}-\ln\left({\rm e}^{2ix}-1\right)+\ln\left(i+{\rm e}^{2ix}\right)$	33

input
```
int(sec(x)^2/(tan(x)^2+tan(x)^3),x,method=_RETURNVERBOSE)
```

output
```
ln(1+tan(x))-1/tan(x)-ln(tan(x))
```

3.697.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(10) = 20$.

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 3.60

$$\int \frac{\sec^2(x)}{\tan^2(x)+\tan^3(x)}\,dx = -\frac{\log\left(-\frac{1}{4}\cos\left(x\right)^2+\frac{1}{4}\right)\sin\left(x\right)-\log\left(2\cos\left(x\right)\sin\left(x\right)+1\right)\sin\left(x\right)+2\cos\left(x\right)}{2\sin\left(x\right)}$$

input
```
integrate(sec(x)^2/(tan(x)^2+tan(x)^3),x, algorithm="fricas")
```

output
```
-1/2*(log(-1/4*cos(x)^2 + 1/4)*sin(x) - log(2*cos(x)*sin(x) + 1)*sin(x) +
2*cos(x))/sin(x)
```

3.697.6 Sympy [F]

$$\int \frac{\sec^2(x)}{\tan^2(x)+\tan^3(x)}\,dx = \int \frac{\sec^2(x)}{(\tan(x)+1)\tan^2(x)}\,dx$$

input
```
integrate(sec(x)**2/(tan(x)**2+tan(x)**3),x)
```

output
```
Integral(sec(x)**2/((tan(x) + 1)*tan(x)**2), x)
```

3.697.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.70

$$\int \frac{\sec^2(x)}{\tan^2(x)+\tan^3(x)}\,dx = -\frac{1}{\tan(x)} + \log(\tan(x)+1) - \log(\tan(x))$$

input
```
integrate(sec(x)^2/(tan(x)^2+tan(x)^3),x, algorithm="maxima")
```

output
```
-1/tan(x) + log(tan(x) + 1) - log(tan(x))
```

3.697.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.90

$$\int \frac{\sec^2(x)}{\tan^2(x)+\tan^3(x)}\,dx = -\frac{1}{\tan(x)} + \log(|\tan(x)+1|) - \log(|\tan(x)|)$$

input
```
integrate(sec(x)^2/(tan(x)^2+tan(x)^3),x, algorithm="giac")
```

output
```
-1/tan(x) + log(abs(tan(x) + 1)) - log(abs(tan(x)))
```

3.697.9 Mupad [B] (verification not implemented)

Time = 27.75 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \frac{\sec^2(x)}{\tan^2(x)+\tan^3(x)}\,dx = 2\,\mathrm{atanh}(2\tan(x)+1)-\frac{1}{\tan(x)}$$

input
```
int(1/(cos(x)^2*(tan(x)^2 + tan(x)^3)),x)
```

output
```
2*atanh(2*tan(x) + 1) - 1/tan(x)
```

3.698 $\int \frac{\sec^2(x)}{-\tan^2(x)+\tan^3(x)}\,dx$

3.698.1 Optimal result

Integrand size = 18, antiderivative size = 10

$$\int \frac{\sec^2(x)}{-\tan^2(x)+\tan^3(x)}\,dx = \cot(x)+\log(1-\cot(x))$$

output
```
cot(x)+ln(1-cot(x))
```

3.698.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \frac{\sec^2(x)}{-\tan^2(x)+\tan^3(x)}\,dx = \cot(x)+\log(\cos(x)-\sin(x))-\log(\sin(x))$$

input
```
Integrate[Sec[x]^2/(-Tan[x]^2 + Tan[x]^3),x]
```

output
```
Cot[x] + Log[Cos[x] - Sin[x]] - Log[Sin[x]]
```

3.698.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.50, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3042, 4842, 25, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(x)}{\tan^3(x)-\tan^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(x)^2}{\tan(x)^3-\tan(x)^2}dx$$

$$\Big\downarrow \text{4842}$$

$$\int -\frac{\cot^2(x)}{1-\tan(x)}d\tan(x)$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\cot^2(x)}{1-\tan(x)}d\tan(x)$$

$$\Big\downarrow \text{54}$$

$$-\int \left(\cot^2(x)+\cot(x)+\frac{1}{1-\tan(x)}\right)d\tan(x)$$

$$\Big\downarrow \text{2009}$$

$$\cot(x)+\log(1-\tan(x))-\log(\tan(x))$$

input
```
Int[Sec[x]^2/(-Tan[x]^2 + Tan[x]^3),x]
```

output
```
Cot[x] + Log[1 - Tan[x]] - Log[Tan[x]]
```

3.698. $\int \frac{\sec^2(x)}{-\tan^2(x)+\tan^3(x)}dx$

3.698.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4842
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Tan[c*(a +
 b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] |
| EqQ[F, sec])
```

3.698.4 Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

method	result	size
default	$\frac{1}{\tan(x)}-\ln\left(\tan\left(x\right)\right)+\ln\left(\tan\left(x\right)-1\right)$	16
risch	$\frac{2i}{\mathrm{e}^{2ix}-1}-\ln\left(\mathrm{e}^{2ix}-1\right)+\ln\left(\mathrm{e}^{2ix}-i\right)$	33

input
```
int(sec(x)^2/(-tan(x)^2+tan(x)^3),x,method=_RETURNVERBOSE)
```

output
```
1/tan(x)-ln(tan(x))+ln(tan(x)-1)
```

3.698.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(10) = 20.

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 3.60

$$\int \frac{\sec^2(x)}{-\tan^2(x)+\tan^3(x)}\,dx = -\frac{\log\left(-\frac{1}{4}\cos(x)^2+\frac{1}{4}\right)\sin(x)-\log\left(-2\cos(x)\sin(x)+1\right)\sin(x)-2\cos(x)}{2\sin(x)}$$

input
```
integrate(sec(x)^2/(-tan(x)^2+tan(x)^3),x, algorithm="fricas")
```

output
```
-1/2*(log(-1/4*cos(x)^2 + 1/4)*sin(x) - log(-2*cos(x)*sin(x) + 1)*sin(x) -
 2*cos(x))/sin(x)
```

3.698.6 Sympy [F]

$$\int \frac{\sec^2(x)}{-\tan^2(x)+\tan^3(x)}\,dx = \int \frac{\sec^2(x)}{(\tan(x)-1)\tan^2(x)}\,dx$$

input
```
integrate(sec(x)**2/(-tan(x)**2+tan(x)**3),x)
```

output
```
Integral(sec(x)**2/((tan(x) - 1)*tan(x)**2), x)
```

3.698.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.50

$$\int \frac{\sec^2(x)}{-\tan^2(x)+\tan^3(x)}\,dx = \frac{1}{\tan(x)}+\log(\tan(x)-1)-\log(\tan(x))$$

input
```
integrate(sec(x)^2/(-tan(x)^2+tan(x)^3),x, algorithm="maxima")
```

output
```
1/tan(x) + log(tan(x) - 1) - log(tan(x))
```

3.698.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.70

$$\int \frac{\sec^2(x)}{-\tan^2(x)+\tan^3(x)}\,dx = \frac{1}{\tan(x)} + \log(|\tan(x)-1|) - \log(|\tan(x)|)$$

input
```
integrate(sec(x)^2/(-tan(x)^2+tan(x)^3),x, algorithm="giac")
```

output
```
1/tan(x) + log(abs(tan(x) - 1)) - log(abs(tan(x)))
```

3.698.9 Mupad [B] (verification not implemented)

Time = 26.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.40

$$\int \frac{\sec^2(x)}{-\tan^2(x)+\tan^3(x)}\,dx = \frac{1}{\tan(x)} - 2\,\mathrm{atanh}(2\tan(x)-1)$$

input
```
int(-1/(cos(x)^2*(tan(x)^2 - tan(x)^3)),x)
```

output
```
1/tan(x) - 2*atanh(2*tan(x) - 1)
```

3.699 $\int \frac{\sec^2(x)}{3-4\tan^3(x)}\,dx$

3.699.1 Optimal result

Integrand size = 15, antiderivative size = 176

$$\int \frac{\sec^2(x)}{3-4\tan^3(x)}\,dx = \frac{x}{3\ 2^{2/3}\sqrt[6]{3}} - \frac{\arctan\left(\frac{6^{2/3}-2\ 6^{2/3}\cos^2(x)+2\left(3-2\sqrt[3]{6}\right)\cos(x)\sin(x)}{3\ 2^{2/3}\sqrt[6]{3}+4\sqrt[3]{6}+\left(6-4\sqrt[3]{6}\right)\cos^2(x)+2\ 6^{2/3}\cos(x)\sin(x)}\right)}{3\ 2^{2/3}\sqrt[6]{3}} - \frac{\log\left(\sqrt[3]{3}-2^{2/3}\tan(x)\right)}{3\ 6^{2/3}} + \frac{\log\left(3^{2/3}+2^{2/3}\sqrt[3]{3}\tan(x)+2\sqrt[3]{2}\tan^2(x)\right)}{6\ 6^{2/3}}$$

output
```
1/18*x*2^(1/3)*3^(5/6)-1/18*arctan((6^(2/3)-2*6^(2/3)*cos(x)^2+2*(3-2*6^(1
/3))*cos(x)*sin(x))/(3*2^(2/3)*3^(1/6)+4*6^(1/3)+(6-4*6^(1/3))*cos(x)^2+2*
6^(2/3)*cos(x)*sin(x)))*2^(1/3)*3^(5/6)-1/18*ln(3^(1/3)-2^(2/3)*tan(x))*6^
(1/3)+1/36*ln(3^(2/3)+2^(2/3)*3^(1/3)*tan(x)+2*2^(1/3)*tan(x)^2)*6^(1/3)
```

3.699.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.42

$$\int \frac{\sec^2(x)}{3-4\tan^3(x)}\,dx = \frac{2\sqrt{3}\arctan\left(\frac{3+2\ 6^{2/3}\tan(x)}{3\sqrt{3}}\right)-2\log\left(3-6^{2/3}\tan(x)\right)+\log\left(3+6^{2/3}\tan(x)+2\sqrt[3]{6}\tan^2(x)\right)}{6\ 6^{2/3}}$$

input
```
Integrate[Sec[x]^2/(3 - 4*Tan[x]^3),x]
```

output
```
(2*Sqrt[3]*ArcTan[(3 + 2*6^(2/3)*Tan[x])/(3*Sqrt[3])] - 2*Log[3 - 6^(2/3)*
Tan[x]] + Log[3 + 6^(2/3)*Tan[x] + 2*6^(1/3)*Tan[x]^2])/(6*6^(2/3))
```

3.699.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.65, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 4158, 750, 16, 27, 1142, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(x)}{3-4\tan^3(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(x)^2}{3-4\tan(x)^3}dx$$

$$\Big\downarrow \text{4158}$$

$$\int \frac{1}{3-4\tan^3(x)}d\tan(x)$$

$$\Big\downarrow \text{750}$$

$$\frac{\int \frac{2^{2/3}\left(\tan(x)+\sqrt[3]{6}\right)}{2\sqrt[3]{2}\tan^2(x)+2^{2/3}\sqrt[3]{3}\tan(x)+3^{2/3}}d\tan(x)}{3\ 3^{2/3}}+\frac{\int \frac{1}{\sqrt[3]{3}-2^{2/3}\tan(x)}d\tan(x)}{3\ 3^{2/3}}$$

$$\Big\downarrow \text{16}$$

$$\frac{\int \frac{2^{2/3}\left(\tan(x)+\sqrt[3]{6}\right)}{2\sqrt[3]{2}\tan^2(x)+2^{2/3}\sqrt[3]{3}\tan(x)+3^{2/3}}d\tan(x)}{3\ 3^{2/3}}-\frac{\log\left(\sqrt[3]{3}-2^{2/3}\tan(x)\right)}{3\ 6^{2/3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{2}{3}\right)^{2/3}\int \frac{\tan(x)+\sqrt[3]{6}}{2\sqrt[3]{2}\tan^2(x)+2^{2/3}\sqrt[3]{3}\tan(x)+3^{2/3}}d\tan(x)-\frac{\log\left(\sqrt[3]{3}-2^{2/3}\tan(x)\right)}{3\ 6^{2/3}}$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{3}\left(\frac{2}{3}\right)^{2/3}\left(\frac{3\sqrt[3]{3}\int \frac{1}{2\sqrt[3]{2}\tan^2(x)+2^{2/3}\sqrt[3]{3}\tan(x)+3^{2/3}}d\tan(x)}{2\ 2^{2/3}}+\frac{\int \frac{\sqrt[3]{2}\left(4\tan(x)+\sqrt[3]{6}\right)}{2\sqrt[3]{2}\tan^2(x)+2^{2/3}\sqrt[3]{3}\tan(x)+3^{2/3}}d\tan(x)}{4\sqrt[3]{2}}\right)-\frac{\log\left(\sqrt[3]{3}-2^{2/3}\tan(x)\right)}{3\ 6^{2/3}}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\left(\frac{2}{3}\right)^{2/3}\left(\frac{3\sqrt[3]{3}\int \frac{1}{2\sqrt[3]{2}\tan^2(x)+2^{2/3}\sqrt[3]{3}\tan(x)+3^{2/3}}d\tan(x)}{2\ 2^{2/3}}+\frac{1}{4}\int \frac{4\tan(x)+\sqrt[3]{6}}{2\sqrt[3]{2}\tan^2(x)+2^{2/3}\sqrt[3]{3}\tan(x)+3^{2/3}}d\tan(x)\right)-\frac{\log\left(\sqrt[3]{3}-2^{2/3}\tan(x)\right)}{3\ 6^{2/3}}$$

$$\Big\downarrow \; 1082$$

$$\frac{1}{3}\left(\frac{2}{3}\right)^{2/3}\left(\frac{1}{4}\int \frac{4\tan(x)+\sqrt[3]{6}}{2\sqrt[3]{2}\tan^2(x)+2^{2/3}\sqrt[3]{3}\tan(x)+3^{2/3}}d\tan(x)-\frac{3\int \frac{1}{-\left(\frac{2\ 2^{2/3}\tan(x)}{\sqrt[3]{3}}+1\right)^2-3}d\left(\frac{2\ 2^{2/3}\tan(x)}{\sqrt[3]{3}}+1\right)}{2\sqrt[3]{2}}\right)-\frac{\log\left(\sqrt[3]{3}-2^{2/3}\tan(x)\right)}{3\ 6^{2/3}}$$

$$\Big\downarrow \; 217$$

$$\frac{1}{3}\left(\frac{2}{3}\right)^{2/3}\left(\frac{1}{4}\int \frac{4\tan(x)+\sqrt[3]{6}}{2\sqrt[3]{2}\tan^2(x)+2^{2/3}\sqrt[3]{3}\tan(x)+3^{2/3}}d\tan(x)+\frac{\sqrt{3}\arctan\left(\frac{\frac{2\ 2^{2/3}\tan(x)}{\sqrt[3]{3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}}\right)-\frac{\log\left(\sqrt[3]{3}-2^{2/3}\tan(x)\right)}{3\ 6^{2/3}}$$

$$\Big\downarrow \; 1103$$

3.699. $\int \frac{\sec^2(x)}{3-4\tan^3(x)}\,dx$

$$\frac{1}{3}\left(\frac{2}{3}\right)^{2/3}\left(\frac{\sqrt{3}\arctan\left(\frac{\frac{2\ 2^{2/3}\tan(x)}{\sqrt[3]{3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{2}}+\frac{\log\left(2\sqrt[3]{2}\tan^2(x)+2^{2/3}\sqrt[3]{3}\tan(x)+3^{2/3}\right)}{4\sqrt[3]{2}}\right)-\frac{\log\left(\sqrt[3]{3}-2^{2/3}\tan(x)\right)}{3\ 6^{2/3}}$$

input
```
Int[Sec[x]^2/(3 - 4*Tan[x]^3),x]
```

output
```
-1/3*Log[3^(1/3) - 2^(2/3)*Tan[x]]/6^(2/3) + ((2/3)^(2/3)*((Sqrt[3]*ArcTan
[(1 + (2*2^(2/3)*Tan[x])/3^(1/3))/Sqrt[3]])/(2*2^(1/3)) + Log[3^(2/3) + 2^
(2/3)*3^(1/3)*Tan[x] + 2*2^(1/3)*Tan[x]^2]/(4*2^(1/3))))/3
```

3.699.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.699. $\int \frac{\sec^2(x)}{3-4\tan^3(x)}\,dx$

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

3.699.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.42 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.22

method	result
risch	$4\left(\sum_{\_R=\mathrm{RootOf}\left(62208\_Z^3+1\right)}\_R\ln\left(\mathrm{e}^{2ix}+\left(\frac{41472}{25}-\frac{31104i}{25}\right)\_R^2+\left(\frac{864}{25}+\frac{1152i}{25}\right)\_R-\frac{7}{25}+\right.\right.$
derivativedivides	$-\frac{3^{\frac{1}{3}}4^{\frac{2}{3}}\ln\left(\tan(x)-\frac{3^{\frac{1}{3}}4^{\frac{2}{3}}}{4}\right)}{36}+\frac{3^{\frac{1}{3}}4^{\frac{2}{3}}\ln\left(\tan(x)^2+\frac{3^{\frac{1}{3}}4^{\frac{2}{3}}\tan(x)}{4}+\frac{3^{\frac{2}{3}}4^{\frac{1}{3}}}{4}\right)}{72}+\frac{3^{\frac{5}{6}}4^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}4^{\frac{1}{3}}\tan(x)}{3}+1\right)}{3}\right)}{36}$
default	$-\frac{3^{\frac{1}{3}}4^{\frac{2}{3}}\ln\left(\tan(x)-\frac{3^{\frac{1}{3}}4^{\frac{2}{3}}}{4}\right)}{36}+\frac{3^{\frac{1}{3}}4^{\frac{2}{3}}\ln\left(\tan(x)^2+\frac{3^{\frac{1}{3}}4^{\frac{2}{3}}\tan(x)}{4}+\frac{3^{\frac{2}{3}}4^{\frac{1}{3}}}{4}\right)}{72}+\frac{3^{\frac{5}{6}}4^{\frac{2}{3}}\arctan\left(\frac{\sqrt{3}\left(\frac{2\,3^{\frac{2}{3}}4^{\frac{1}{3}}\tan(x)}{3}+1\right)}{3}\right)}{36}$

input
```
int(sec(x)^2/(3-4*tan(x)^3),x,method=_RETURNVERBOSE)
```

output
```
4*sum(_R*ln(exp(2*I*x)+(41472/25-31104/25*I)*_R^2+(864/25+1152/25*I)*_R-7/
25+24/25*I),_R=RootOf(62208*_Z^3+1))
```

3.699.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 441 vs. 2(131) = 262.

Time = 0.35 (sec) , antiderivative size = 441, normalized size of antiderivative = 2.51

$$\int \frac{\sec^2(x)}{3-4\tan^3(x)}\,dx = -\frac{1}{36} \cdot 36^{\frac{1}{6}}\sqrt{3}(-1)^{\frac{1}{3}}\arctan\left(-\frac{36^{\frac{1}{6}}\left(28\left(36^{\frac{2}{3}}\sqrt{3}(-1)^{\frac{2}{3}}-9\sqrt{3}(-1)^{\frac{1}{3}}\right)\cos(x)^6-4\left(14\cdot 36^{\frac{2}{3}}\sqrt{3}(-1)^{\frac{2}{3}}+36\cdot 36^{\frac{1}{3}}\right.}{}\right) - \frac{1}{432}\cdot 36^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(-3\left(2\cdot 36^{\frac{2}{3}}(-1)^{\frac{1}{3}}-8\cdot 36^{\frac{1}{3}}(-1)^{\frac{2}{3}}+25\right)\cos(x)^4 + 3\left(3\cdot 36^{\frac{2}{3}}(-1)^{\frac{1}{3}}-4\cdot 36^{\frac{1}{3}}(-1)^{\frac{2}{3}}+32\right)\cos(x)^2 - 2\left(\left(4\cdot 36^{\frac{2}{3}}(-1)^{\frac{1}{3}}+9\cdot 36^{\frac{1}{3}}(-1)^{\frac{2}{3}}\right)\cos(x)^3-4\left(36^{\frac{2}{3}}(-1)^{\frac{1}{3}}-9\right)\cos(x)\right)\sin(x)-12\cdot 36^{\frac{1}{3}}(-1)^{\frac{2}{3}}-48\right) + \frac{1}{216}\cdot 36^{\frac{2}{3}}(-1)^{\frac{1}{3}}\log\left(3\left(2\cdot 36^{\frac{2}{3}}(-1)^{\frac{1}{3}}+8\cdot 36^{\frac{1}{3}}(-1)^{\frac{2}{3}}-7\right)\cos(x)^2 + 2\left(4\cdot 36^{\frac{2}{3}}(-1)^{\frac{1}{3}}-9\cdot 36^{\frac{1}{3}}(-1)^{\frac{2}{3}}+36\right)\cos(x)\sin(x)-3\cdot 36^{\frac{2}{3}}(-1)^{\frac{1}{3}}-12\cdot 36^{\frac{1}{3}}(-1)^{\frac{2}{3}}+48\right)$$

input
```
integrate(sec(x)^2/(3-4*tan(x)^3),x, algorithm="fricas")
```

output
```
-1/36*36^(1/6)*sqrt(3)*(-1)^(1/3)*arctan(-1/108*36^(1/6)*(28*(36^(2/3)*sqr
t(3)*(-1)^(2/3) - 9*sqrt(3)*(-1)^(1/3))*cos(x)^6 - 4*(14*36^(2/3)*sqrt(3)*
(-1)^(2/3) + 36*36^(1/3)*sqrt(3) - 63*sqrt(3)*(-1)^(1/3))*cos(x)^4 + (37*3
6^(2/3)*sqrt(3)*(-1)^(2/3) + 144*36^(1/3)*sqrt(3) + 144*sqrt(3)*(-1)^(1/3)
)*cos(x)^2 - 6*(16*(36^(2/3)*sqrt(3)*(-1)^(2/3) - 9*sqrt(3)*(-1)^(1/3))*co
s(x)^5 - (24*36^(2/3)*sqrt(3)*(-1)^(2/3) - 7*36^(1/3)*sqrt(3) - 72*sqrt(3)
*(-1)^(1/3))*cos(x)^3 + 4*(36^(2/3)*sqrt(3)*(-1)^(2/3) - 4*36^(1/3)*sqrt(3
) + 9*sqrt(3)*(-1)^(1/3))*cos(x))*sin(x) - 18*36^(1/3)*sqrt(3) - 144*sqrt(
3)*(-1)^(1/3))/(48*cos(x)^6 - 72*cos(x)^4 + 18*cos(x)^2 + 14*(cos(x)^5 - c
os(x)^3)*sin(x) + 3)) - 1/432*36^(2/3)*(-1)^(1/3)*log(-3*(2*36^(2/3)*(-1)^
(1/3) - 8*36^(1/3)*(-1)^(2/3) + 25)*cos(x)^4 + 3*(3*36^(2/3)*(-1)^(1/3) -
4*36^(1/3)*(-1)^(2/3) + 32)*cos(x)^2 - 2*((4*36^(2/3)*(-1)^(1/3) + 9*36^(1
/3)*(-1)^(2/3))*cos(x)^3 - 4*(36^(2/3)*(-1)^(1/3) - 9)*cos(x))*sin(x) - 12
*36^(1/3)*(-1)^(2/3) - 48) + 1/216*36^(2/3)*(-1)^(1/3)*log(3*(2*36^(2/3)*(
-1)^(1/3) + 8*36^(1/3)*(-1)^(2/3) - 7)*cos(x)^2 + 2*(4*36^(2/3)*(-1)^(1/3)
 - 9*36^(1/3)*(-1)^(2/3) + 36)*cos(x)*sin(x) - 3*36^(2/3)*(-1)^(1/3) - 12*
36^(1/3)*(-1)^(2/3) + 48)
```

3.699.6 Sympy [F]

$$\int \frac{\sec^2(x)}{3-4\tan^3(x)}\,dx = -\int \frac{\sec^2{(x)}}{4\tan^3{(x)}-3}\,dx$$

input
```
integrate(sec(x)**2/(3-4*tan(x)**3),x)
```

output
```
-Integral(sec(x)**2/(4*tan(x)**3 - 3), x)
```

3.699.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.51

$$\begin{aligned}\int \frac{\sec^2(x)}{3-4\tan^3(x)}\,dx &= \frac{1}{36}\cdot 4^{\frac{2}{3}}3^{\frac{5}{6}}\arctan\left(\frac{1}{12}\cdot 4^{\frac{2}{3}}3^{\frac{1}{6}}\left(2\cdot 4^{\frac{2}{3}}\tan{(x)}+4^{\frac{1}{3}}3^{\frac{1}{3}}\right)\right)\\ &\quad+\frac{1}{72}\cdot 4^{\frac{2}{3}}3^{\frac{1}{3}}\log\left(4^{\frac{2}{3}}\tan{(x)}^2+4^{\frac{1}{3}}3^{\frac{1}{3}}\tan{(x)}+3^{\frac{2}{3}}\right)\\ &\quad-\frac{1}{36}\cdot 4^{\frac{2}{3}}3^{\frac{1}{3}}\log\left(\frac{1}{4}\cdot 4^{\frac{2}{3}}\left(4^{\frac{1}{3}}\tan{(x)}-3^{\frac{1}{3}}\right)\right)\end{aligned}$$

input
```
integrate(sec(x)^2/(3-4*tan(x)^3),x, algorithm="maxima")
```

output
```
1/36*4^(2/3)*3^(5/6)*arctan(1/12*4^(2/3)*3^(1/6)*(2*4^(2/3)*tan(x) + 4^(1/
3)*3^(1/3))) + 1/72*4^(2/3)*3^(1/3)*log(4^(2/3)*tan(x)^2 + 4^(1/3)*3^(1/3)
*tan(x) + 3^(2/3)) - 1/36*4^(2/3)*3^(1/3)*log(1/4*4^(2/3)*(4^(1/3)*tan(x)
- 3^(1/3)))
```

3.699.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.35

$$\int \frac{\sec^2(x)}{3-4\tan^3(x)}\,dx = \frac{1}{9}\sqrt{3}\left(\frac{3}{4}\right)^{\frac{1}{3}}\arctan\left(\frac{4}{9}\sqrt{3}\left(\frac{3}{4}\right)^{\frac{2}{3}}\left(\left(\frac{3}{4}\right)^{\frac{1}{3}}+2\tan(x)\right)\right) + \frac{1}{36}\cdot 6^{\frac{1}{3}}\log\left(\tan(x)^2+\left(\frac{3}{4}\right)^{\frac{1}{3}}\tan(x)+\left(\frac{3}{4}\right)^{\frac{2}{3}}\right) - \frac{1}{9}\left(\frac{3}{4}\right)^{\frac{1}{3}}\log\left(\left|-\left(\frac{3}{4}\right)^{\frac{1}{3}}+\tan(x)\right|\right)$$

input
```
integrate(sec(x)^2/(3-4*tan(x)^3),x, algorithm="giac")
```

output
```
1/9*sqrt(3)*(3/4)^(1/3)*arctan(4/9*sqrt(3)*(3/4)^(2/3)*((3/4)^(1/3) + 2*ta
n(x))) + 1/36*6^(1/3)*log(tan(x)^2 + (3/4)^(1/3)*tan(x) + (3/4)^(2/3)) - 1
/9*(3/4)^(1/3)*log(abs(-(3/4)^(1/3) + tan(x)))
```

3.699.9 Mupad [B] (verification not implemented)

Time = 27.29 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.43

$$\int \frac{\sec^2(x)}{3-4\tan^3(x)}\,dx = -\frac{6^{1/3}\ln\left(\tan(x)-\frac{6^{1/3}}{2}\right)}{18} - \frac{6^{1/3}\ln\left(\tan(x)-\frac{6^{1/3}\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{4}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{36} + \frac{6^{1/3}\ln\left(\tan(x)+\frac{6^{1/3}\left(1+\sqrt{3}\,1\mathrm{i}\right)}{4}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{36}$$

input
```
int(-1/(cos(x)^2*(4*tan(x)^3 - 3)),x)
```

output
```
(6^(1/3)*log(tan(x) + (6^(1/3)*(3^(1/2)*1i + 1))/4)*(3^(1/2)*1i + 1))/36 -
 (6^(1/3)*log(tan(x) - (6^(1/3)*(3^(1/2)*1i - 1))/4)*(3^(1/2)*1i - 1))/36
- (6^(1/3)*log(tan(x) - 6^(1/3)/2))/18
```

3.699. $\int \frac{\sec^2(x)}{3-4\tan^3(x)}\,dx$

3.700 $\int \frac{\sec^2(x)}{11-5\tan(x)+5\tan^2(x)}\,dx$

3.700.1 Optimal result

Integrand size = 19, antiderivative size = 53

$$\int \frac{\sec^2(x)}{11-5\tan(x)+5\tan^2(x)}\,dx=\frac{2x}{\sqrt{195}}-\frac{2\arctan\left(\frac{-5+10\cos^2(x)+12\cos(x)\sin(x)}{10+\sqrt{195}+12\cos^2(x)-10\cos(x)\sin(x)}\right)}{\sqrt{195}}$$

output
```
2/195*x*195^(1/2)-2/195*arctan((-5+10*cos(x)^2+12*cos(x)*sin(x))/(10+12*co
s(x)^2-10*cos(x)*sin(x)+195^(1/2)))*195^(1/2)
```

3.700.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.42

$$\int \frac{\sec^2(x)}{11-5\tan(x)+5\tan^2(x)}\,dx=-\frac{2\arctan\left(\sqrt{\frac{5}{39}}(1-2\tan(x))\right)}{\sqrt{195}}$$

input
```
Integrate[Sec[x]^2/(11 - 5*Tan[x] + 5*Tan[x]^2),x]
```

output
```
(-2*ArcTan[Sqrt[5/39]*(1 - 2*Tan[x])])/Sqrt[195]
```

3.700.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.38, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 4842, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(x)}{5\tan^2(x)-5\tan(x)+11}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(x)^2}{5\tan(x)^2-5\tan(x)+11}dx$$

$$\Big\downarrow \text{4842}$$

$$\int \frac{1}{5\tan^2(x)-5\tan(x)+11}d\tan(x)$$

$$\Big\downarrow \text{1083}$$

$$-2\int \frac{1}{-(10\tan(x)-5)^2-195}d(10\tan(x)-5)$$

$$\Big\downarrow \text{217}$$

$$\frac{2\arctan\left(\frac{10\tan(x)-5}{\sqrt{195}}\right)}{\sqrt{195}}$$

input
```
Int[Sec[x]^2/(11 - 5*Tan[x] + 5*Tan[x]^2),x]
```

output
```
(2*ArcTan[(-5 + 10*Tan[x])/Sqrt[195]])/Sqrt[195]
```

3.700.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4842
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Tan[c*(a +
 b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] |
| EqQ[F, sec])
```

3.700.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.34

method	result	size
default	$\frac{2\sqrt{195}\,\arctan\left(\frac{(10\tan(x)-5)\sqrt{195}}{195}\right)}{195}$	18
risch	$\frac{i\sqrt{195}\,\ln\left(e^{2ix}+\frac{6\sqrt{195}}{61}-\frac{5i\sqrt{195}}{61}+\frac{96}{61}-\frac{80i}{61}\right)}{195}-\frac{i\sqrt{195}\,\ln\left(e^{2ix}-\frac{6\sqrt{195}}{61}+\frac{5i\sqrt{195}}{61}+\frac{96}{61}-\frac{80i}{61}\right)}{195}$	56

input
```
int(sec(x)^2/(11-5*tan(x)+5*tan(x)^2),x,method=_RETURNVERBOSE)
```

output
```
2/195*195^(1/2)*arctan(1/195*(10*tan(x)-5)*195^(1/2))
```

3.700.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

$$\int \frac{\sec^2(x)}{11-5\tan(x)+5\tan^2(x)}\,dx$$
$$=\frac{1}{195}\sqrt{195}\arctan\left(-\frac{192\sqrt{195}\cos(x)^2-160\sqrt{195}\cos(x)\sin(x)-35\sqrt{195}}{195\left(10\cos(x)^2+12\cos(x)\sin(x)-5\right)}\right)$$

input
```
integrate(sec(x)^2/(11-5*tan(x)+5*tan(x)^2),x, algorithm="fricas")
```

output
```
1/195*sqrt(195)*arctan(-1/195*(192*sqrt(195)*cos(x)^2 - 160*sqrt(195)*cos(
x)*sin(x) - 35*sqrt(195))/(10*cos(x)^2 + 12*cos(x)*sin(x) - 5))
```

3.700.6 Sympy [F]

$$\int \frac{\sec^2(x)}{11-5\tan(x)+5\tan^2(x)}\,dx = \int \frac{\sec^2(x)}{5\tan^2(x)-5\tan(x)+11}\,dx$$

input
```
integrate(sec(x)**2/(11-5*tan(x)+5*tan(x)**2),x)
```

output
```
Integral(sec(x)**2/(5*tan(x)**2 - 5*tan(x) + 11), x)
```

3.700.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.32

$$\int \frac{\sec^2(x)}{11-5\tan(x)+5\tan^2(x)}\,dx = \frac{2}{195}\sqrt{195}\arctan\left(\frac{1}{39}\sqrt{195}(2\tan(x)-1)\right)$$

input
```
integrate(sec(x)^2/(11-5*tan(x)+5*tan(x)^2),x, algorithm="maxima")
```

output
```
2/195*sqrt(195)*arctan(1/39*sqrt(195)*(2*tan(x) - 1))
```

3.700.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.32

$$\int \frac{\sec^2(x)}{11-5\tan(x)+5\tan^2(x)}\,dx = \frac{2}{195}\sqrt{195}\arctan\left(\frac{1}{39}\sqrt{195}(2\tan(x)-1)\right)$$

input
```
integrate(sec(x)^2/(11-5*tan(x)+5*tan(x)^2),x, algorithm="giac")
```

output
```
2/195*sqrt(195)*arctan(1/39*sqrt(195)*(2*tan(x) - 1))
```

3.700.9 Mupad [B] (verification not implemented)

Time = 26.82 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.32

$$\int \frac{\sec^2(x)}{11-5\tan(x)+5\tan^2(x)}\,dx=\frac{2\sqrt{195}\,\mathrm{atan}\left(\frac{\sqrt{195}\,(2\tan(x)-1)}{39}\right)}{195}$$

input
```
int(1/(cos(x)^2*(5*tan(x)^2 - 5*tan(x) + 11)),x)
```

output
```
(2*195^(1/2)*atan((195^(1/2)*(2*tan(x) - 1))/39))/195
```

3.701 $\int \frac{\sec^2(x)(a+b\tan(x))}{c+d\tan(x)}\,dx$

3.701.1 Optimal result

Integrand size = 19, antiderivative size = 28

$$\int \frac{\sec^2(x)(a+b\tan(x))}{c+d\tan(x)}\,dx = -\frac{(bc-ad)\log(c+d\tan(x))}{d^2}+\frac{b\tan(x)}{d}$$

output
```
-(-a*d+b*c)*ln(c+d*tan(x))/d^2+b*tan(x)/d
```

3.701.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{\sec^2(x)(a+b\tan(x))}{c+d\tan(x)}\,dx = \frac{(-bc+ad)\log(c+d\tan(x))+bd\tan(x)}{d^2}$$

input
```
Integrate[(Sec[x]^2*(a + b*Tan[x]))/(c + d*Tan[x]),x]
```

output
```
((-(b*c) + a*d)*Log[c + d*Tan[x]] + b*d*Tan[x])/d^2
```

3.701.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 4842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(x)(a+b\tan(x))}{c+d\tan(x)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\sec(x)^2(a+b\tan(x))}{c+d\tan(x)}dx$$
$$\Big\downarrow \; 4842$$
$$\int \frac{a+b\tan(x)}{c+d\tan(x)}d\tan(x)$$
$$\Big\downarrow \; 49$$
$$\int \left(\frac{ad-bc}{d(c+d\tan(x))}+\frac{b}{d}\right)d\tan(x)$$
$$\Big\downarrow \; 2009$$
$$\frac{b\tan(x)}{d}-\frac{(bc-ad)\log(c+d\tan(x))}{d^2}$$

input
```
Int[(Sec[x]^2*(a + b*Tan[x]))/(c + d*Tan[x]),x]
```

output
```
-(((b*c - a*d)*Log[c + d*Tan[x]])/d^2) + (b*Tan[x])/d
```

3.701.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4842
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Tan[c*(a +
 b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] |
| EqQ[F, sec])
```

3.701.4 Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{b\tan(x)}{d}+\frac{(ad-cb)\ln(c+d\tan(x))}{d^2}$	28
risch	$\frac{2ib}{d(e^{2ix}+1)}-\frac{\ln(e^{2ix}+1)a}{d}+\frac{\ln(e^{2ix}+1)cb}{d^2}+\frac{\ln\left(e^{2ix}-\frac{id+c}{id-c}\right)a}{d}-\frac{\ln\left(e^{2ix}-\frac{id+c}{id-c}\right)cb}{d^2}$	108

input
```
int(sec(x)^2*(a+b*tan(x))/(c+d*tan(x)),x,method=_RETURNVERBOSE)
```

output
```
b*tan(x)/d+(a*d-b*c)/d^2*ln(c+d*tan(x))
```

3.701.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(28) = 56.

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.54

$$\int \frac{\sec^2(x)(a+b\tan(x))}{c+d\tan(x)}\,dx = -\frac{(bc-ad)\cos(x)\log\left(2\,cd\cos(x)\sin(x)+(c^2-d^2)\cos(x)^2+d^2\right)-(bc-ad)\cos(x)\log\left(\cos(x)^2\right)-2}{2\,d^2\cos(x)}$$

input
```
integrate(sec(x)^2*(a+b*tan(x))/(c+d*tan(x)),x, algorithm="fricas")
```

output
```
-1/2*((b*c - a*d)*cos(x)*log(2*c*d*cos(x)*sin(x) + (c^2 - d^2)*cos(x)^2 +
d^2) - (b*c - a*d)*cos(x)*log(cos(x)^2) - 2*b*d*sin(x))/(d^2*cos(x))
```

3.701.6 Sympy [A] (verification not implemented)

Time = 1.35 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{\sec^2(x)(a+b\tan(x))}{c+d\tan(x)}\,dx = \frac{b\tan(x)}{d} + \frac{(ad-bc)\left(\begin{cases} \frac{\tan(x)}{c} & \text{for } d=0 \\ \frac{\log(c+d\tan(x))}{d} & \text{otherwise} \end{cases}\right)}{d}$$

input
```
integrate(sec(x)**2*(a+b*tan(x))/(c+d*tan(x)),x)
```

output
```
b*tan(x)/d + (a*d - b*c)*Piecewise((tan(x)/c, Eq(d, 0)), (log(c + d*tan(x)
)/d, True))/d
```

3.701.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(x)(a+b\tan(x))}{c+d\tan(x)}\,dx = \frac{b\tan(x)}{d} - \frac{(bc-ad)\log(d\tan(x)+c)}{d^2}$$

input
```
integrate(sec(x)^2*(a+b*tan(x))/(c+d*tan(x)),x, algorithm="maxima")
```

output
```
b*tan(x)/d - (b*c - a*d)*log(d*tan(x) + c)/d^2
```

3.701.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04

$$\int \frac{\sec^2(x)(a+b\tan(x))}{c+d\tan(x)}\,dx = \frac{b\tan(x)}{d} - \frac{(bc-ad)\log(|d\tan(x)+c|)}{d^2}$$

input
```
integrate(sec(x)^2*(a+b*tan(x))/(c+d*tan(x)),x, algorithm="giac")
```

output
```
b*tan(x)/d - (b*c - a*d)*log(abs(d*tan(x) + c))/d^2
```

3.701.9 Mupad [B] (verification not implemented)

Time = 28.34 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{\sec^2(x)(a+b\tan(x))}{c+d\tan(x)}\,dx = \frac{b\tan(x)}{d} + \frac{\ln\left(c+d\tan(x)\right)\,\left(a\,d-b\,c\right)}{d^2}$$

input
```
int((a + b*tan(x))/(cos(x)^2*(c + d*tan(x))),x)
```

output
```
(b*tan(x))/d + (log(c + d*tan(x))*(a*d - b*c))/d^2
```

3.702 $\int \frac{\sec^2(x)(a+b\tan(x))^2}{c+d\tan(x)}\,dx$

3.702.1 Optimal result

Integrand size = 21, antiderivative size = 53

$$\int \frac{\sec^2(x)(a+b\tan(x))^2}{c+d\tan(x)}\,dx = \frac{(bc-ad)^2\log(c+d\tan(x))}{d^3} - \frac{b(bc-ad)\tan(x)}{d^2} + \frac{(a+b\tan(x))^2}{2d}$$

output
```
(-a*d+b*c)^2*ln(c+d*tan(x))/d^3-b*(-a*d+b*c)*tan(x)/d^2+1/2*(a+b*tan(x))^2
/d
```

3.702.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

$$\int \frac{\sec^2(x)(a+b\tan(x))^2}{c+d\tan(x)}\,dx = \frac{2(bc-ad)^2\log(c+d\tan(x)) + b^2d^2\sec^2(x) - 2bd(bc-2ad)\tan(x)}{2d^3}$$

input
```
Integrate[(Sec[x]^2*(a + b*Tan[x])^2)/(c + d*Tan[x]),x]
```

output
```
(2*(b*c - a*d)^2*Log[c + d*Tan[x]] + b^2*d^2*Sec[x]^2 - 2*b*d*(b*c - 2*a*d
)*Tan[x])/(2*d^3)
```

3.702.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 4842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(x)(a+b\tan(x))^2}{c+d\tan(x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sec(x)^2(a+b\tan(x))^2}{c+d\tan(x)}dx$$
$$\Big\downarrow \text{4842}$$
$$\int \frac{(a+b\tan(x))^2}{c+d\tan(x)}d\tan(x)$$
$$\Big\downarrow \text{49}$$
$$\int \left(\frac{(ad-bc)^2}{d^2(c+d\tan(x))}-\frac{b(bc-ad)}{d^2}+\frac{b(a+b\tan(x))}{d}\right)d\tan(x)$$
$$\Big\downarrow \text{2009}$$
$$\frac{(bc-ad)^2\log(c+d\tan(x))}{d^3}-\frac{b\tan(x)(bc-ad)}{d^2}+\frac{(a+b\tan(x))^2}{2d}$$

input
```
Int[(Sec[x]^2*(a + b*Tan[x])^2)/(c + d*Tan[x]),x]
```

output
```
((b*c - a*d)^2*Log[c + d*Tan[x]])/d^3 - (b*(b*c - a*d)*Tan[x])/d^2 + (a +
b*Tan[x])^2/(2*d)
```

3.702.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4842
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Tan[c*(a +
 b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] |
| EqQ[F, sec])
```

3.702.4 Maple [A] (verified)

Time = 1.94 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.13

method	result
derivativedivides	$\frac{b\left(\frac{b\tan(x)^2 d}{2}+2\tan(x)ad-\tan(x)bc\right)}{d^2}+\frac{(d^2a^2-2cdab+b^2c^2)\ln(c+d\tan(x))}{d^3}$
default	$\frac{b\left(\frac{b\tan(x)^2 d}{2}+2\tan(x)ad-\tan(x)bc\right)}{d^2}+\frac{(d^2a^2-2cdab+b^2c^2)\ln(c+d\tan(x))}{d^3}$
risch	$\frac{2ib(2ad\,e^{2ix}-bc\,e^{2ix}-ibd\,e^{2ix}+2ad-cb)}{(e^{2ix}+1)^2 d^2}-\frac{\ln(e^{2ix}+1)a^2}{d}+\frac{2\ln(e^{2ix}+1)cab}{d^2}-\frac{\ln(e^{2ix}+1)b^2c^2}{d^3}+\frac{\ln\left(e^{2ix}-\frac{id+c}{id-c}\right.}{d}$

input
```
int(sec(x)^2*(a+b*tan(x))^2/(c+d*tan(x)),x,method=_RETURNVERBOSE)
```

output
```
b/d^2*(1/2*b*tan(x)^2*d+2*tan(x)*a*d-tan(x)*b*c)+(a^2*d^2-2*a*b*c*d+b^2*c^
2)/d^3*ln(c+d*tan(x))
```

3.702.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. 2(51) = 102.

Time = 0.26 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.30

$$\int \frac{\sec^2(x)(a+b\tan(x))^2}{c+d\tan(x)}\,dx$$
$$=\frac{b^2d^2+(b^2c^2-2\,abcd+a^2d^2)\cos(x)^2\log\left(2\,cd\cos(x)\sin(x)+(c^2-d^2)\cos(x)^2+d^2\right)-(b^2c^2-2\,abcd+}{2\,d^3\cos(x)^2}$$

input
```
integrate(sec(x)^2*(a+b*tan(x))^2/(c+d*tan(x)),x, algorithm="fricas")
```

output
```
1/2*(b^2*d^2 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cos(x)^2*log(2*c*d*cos(x)*s
in(x) + (c^2 - d^2)*cos(x)^2 + d^2) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cos(
x)^2*log(cos(x)^2) - 2*(b^2*c*d - 2*a*b*d^2)*cos(x)*sin(x))/(d^3*cos(x)^2)
```

3.702.6 Sympy [A] (verification not implemented)

Time = 1.89 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06

$$\int \frac{\sec^2(x)(a+b\tan(x))^2}{c+d\tan(x)}\,dx = \frac{b^2\tan^2(x)}{2d} + \frac{(ad-bc)^2\left(\begin{cases}\frac{\tan(x)}{c} & \text{for } d=0\\ \frac{\log(c+d\tan(x))}{d} & \text{otherwise}\end{cases}\right)}{d^2} + \frac{(2abd-b^2c)\tan(x)}{d^2}$$

input
```
integrate(sec(x)**2*(a+b*tan(x))**2/(c+d*tan(x)),x)
```

output
```
b**2*tan(x)**2/(2*d) + (a*d - b*c)**2*Piecewise((tan(x)/c, Eq(d, 0)), (log
(c + d*tan(x))/d, True))/d**2 + (2*a*b*d - b**2*c)*tan(x)/d**2
```

3.702.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.19

$$\int \frac{\sec^2(x)(a+b\tan(x))^2}{c+d\tan(x)}\,dx = \frac{b^2d\tan(x)^2 - 2(b^2c-2abd)\tan(x)}{2d^2} + \frac{(b^2c^2-2abcd+a^2d^2)\log(d\tan(x)+c)}{d^3}$$

input
```
integrate(sec(x)^2*(a+b*tan(x))^2/(c+d*tan(x)),x, algorithm="maxima")
```

output
```
1/2*(b^2*d*tan(x)^2 - 2*(b^2*c - 2*a*b*d)*tan(x))/d^2 + (b^2*c^2 - 2*a*b*c
*d + a^2*d^2)*log(d*tan(x) + c)/d^3
```

3.702.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.21

$$\int \frac{\sec^2(x)(a+b\tan(x))^2}{c+d\tan(x)}\,dx = \frac{b^2d\tan{(x)}^2 - 2\,b^2c\tan{(x)} + 4\,abd\tan{(x)}}{2\,d^2} + \frac{(b^2c^2 - 2\,abcd + a^2d^2)\log{(|d\tan{(x)} + c|)}}{d^3}$$

input
```
integrate(sec(x)^2*(a+b*tan(x))^2/(c+d*tan(x)),x, algorithm="giac")
```

output
```
1/2*(b^2*d*tan(x)^2 - 2*b^2*c*tan(x) + 4*a*b*d*tan(x))/d^2 + (b^2*c^2 - 2*
a*b*c*d + a^2*d^2)*log(abs(d*tan(x) + c))/d^3
```

3.702.9 Mupad [B] (verification not implemented)

Time = 28.52 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.23

$$\int \frac{\sec^2(x)(a+b\tan(x))^2}{c+d\tan(x)}\,dx = \frac{\ln(c+d\tan(x))\,\left(a^2\,d^2 - 2\,a\,b\,c\,d + b^2\,c^2\right)}{d^3} - \tan(x)\,\left(\frac{b^2\,c}{d^2} - \frac{2\,a\,b}{d}\right) + \frac{b^2\tan(x)^2}{2\,d}$$

input
```
int((a + b*tan(x))^2/(cos(x)^2*(c + d*tan(x))),x)
```

output
```
(log(c + d*tan(x))*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/d^3 - tan(x)*((b^2*c)/
d^2 - (2*a*b)/d) + (b^2*tan(x)^2)/(2*d)
```

3.703 $\int \frac{\sec^2(x)(a+b\tan(x))^3}{c+d\tan(x)}\,dx$

3.703.1 Optimal result

Integrand size = 21, antiderivative size = 78

$$\int \frac{\sec^2(x)(a+b\tan(x))^3}{c+d\tan(x)}\,dx = -\frac{(bc-ad)^3\log(c+d\tan(x))}{d^4}+\frac{b(bc-ad)^2\tan(x)}{d^3} - \frac{(bc-ad)(a+b\tan(x))^2}{2d^2}+\frac{(a+b\tan(x))^3}{3d}$$

output
```
-(-a*d+b*c)^3*ln(c+d*tan(x))/d^4+b*(-a*d+b*c)^2*tan(x)/d^3-1/2*(-a*d+b*c)*
(a+b*tan(x))^2/d^2+1/3*(a+b*tan(x))^3/d
```

3.703.2 Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.56

$$\int \frac{\sec^2(x)(a+b\tan(x))^3}{c+d\tan(x)}\,dx = \frac{(c\cos(x)+d\sin(x))(a+b\tan(x))^3\left(-6(bc-ad)^3\cos^2(x)\log(c+d\tan(x))+6b^3c^2d\cos(x)\sin(x)+bd^2(9\right.}{6d^4(a\cos(x)+b\sin(x))^3(c+d\tan(x))}$$

input
```
Integrate[(Sec[x]^2*(a + b*Tan[x])^3)/(c + d*Tan[x]),x]
```

output
```
((c*Cos[x] + d*Sin[x])*(a + b*Tan[x])^3*(-6*(b*c - a*d)^3*Cos[x]^2*Log[c +
 d*Tan[x]] + 6*b^3*c^2*d*Cos[x]*Sin[x] + b*d^2*(9*a*(-(b*c) + a*d)*Sin[2*x
] + b*(-3*b*c + 9*a*d + 2*b*d*Sin[x]^2*Tan[x]))))/(6*d^4*(a*Cos[x] + b*Sin
[x])^3*(c + d*Tan[x]))
```

3.703.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 4842, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(x)(a+b\tan(x))^3}{c+d\tan(x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sec(x)^2(a+b\tan(x))^3}{c+d\tan(x)}dx$$
$$\Big\downarrow \text{4842}$$
$$\int \frac{(a+b\tan(x))^3}{c+d\tan(x)}d\tan(x)$$
$$\Big\downarrow \text{49}$$
$$\int \left(\frac{(ad-bc)^3}{d^3(c+d\tan(x))}+\frac{b(bc-ad)^2}{d^3}-\frac{b(bc-ad)(a+b\tan(x))}{d^2}+\frac{b(a+b\tan(x))^2}{d}\right)d\tan(x)$$
$$\Big\downarrow \text{2009}$$
$$-\frac{(bc-ad)^3\log(c+d\tan(x))}{d^4}+\frac{b\tan(x)(bc-ad)^2}{d^3}-\frac{(bc-ad)(a+b\tan(x))^2}{2d^2}+\frac{(a+b\tan(x))^3}{3d}$$

input
```
Int[(Sec[x]^2*(a + b*Tan[x])^3)/(c + d*Tan[x]),x]
```

output
```
-(((b*c - a*d)^3*Log[c + d*Tan[x]])/d^4) + (b*(b*c - a*d)^2*Tan[x])/d^3 -
((b*c - a*d)*(a + b*Tan[x])^2)/(2*d^2) + (a + b*Tan[x])^3/(3*d)
```

3.703.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4842
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Tan[c*(a +
 b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] |
| EqQ[F, sec])
```

3.703.4 Maple [A] (verified)

Time = 3.90 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.49

method	result
derivativedivides	$\frac{b\left(\frac{b^2\tan(x)^3d^2}{3}+\frac{3ab\,d^2\tan(x)^2}{2}-\frac{b^2cd\tan(x)^2}{2}+3\tan(x)d^2a^2-3\tan(x)cdab+\tan(x)b^2c^2\right)}{d^3}+\frac{(d^3a^3-3c\,d^2a^2b+3c^2da\,b^2}{d^4}$
default	$\frac{b\left(\frac{b^2\tan(x)^3d^2}{3}+\frac{3ab\,d^2\tan(x)^2}{2}-\frac{b^2cd\tan(x)^2}{2}+3\tan(x)d^2a^2-3\tan(x)cdab+\tan(x)b^2c^2\right)}{d^3}+\frac{(d^3a^3-3c\,d^2a^2b+3c^2da\,b^2}{d^4}$
risch	$\frac{2ib(9a^2d^2e^{4ix}-9abcd\,e^{4ix}+3b^2c^2e^{4ix}-3b^2d^2e^{4ix}-9iab\,d^2e^{4ix}+3ib^2cd\,e^{4ix}+18a^2d^2e^{2ix}-18abcd\,e^{2ix}+6b^2c^2e^{2ix}-9iab}{3d^3(e^{2ix}+1)^3}$

input
```
int(sec(x)^2*(a+b*tan(x))^3/(c+d*tan(x)),x,method=_RETURNVERBOSE)
```

output
```
b/d^3*(1/3*b^2*tan(x)^3*d^2+3/2*a*b*d^2*tan(x)^2-1/2*b^2*c*d*tan(x)^2+3*ta
n(x)*d^2*a^2-3*tan(x)*c*d*a*b+tan(x)*b^2*c^2)+(a^3*d^3-3*a^2*b*c*d^2+3*a*b
^2*c^2*d-b^3*c^3)/d^4*ln(c+d*tan(x))
```

3.703.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(74) = 148.

Time = 0.30 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.58

$$\int \frac{\sec^2(x)(a+b\tan(x))^3}{c+d\tan(x)}\,dx =$$
$$-\frac{3\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\cos(x)^3\log\left(2\,cd\cos(x)\sin(x)+\left(c^2-d^2\right)\cos(x)^2+d^2\right)-3\left(b^3c^3\right.}{}$$

input
```
integrate(sec(x)^2*(a+b*tan(x))^3/(c+d*tan(x)),x, algorithm="fricas")
```

output
```
-1/6*(3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cos(x)^3*log(2
*c*d*cos(x)*sin(x) + (c^2 - d^2)*cos(x)^2 + d^2) - 3*(b^3*c^3 - 3*a*b^2*c^
2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cos(x)^3*log(cos(x)^2) + 3*(b^3*c*d^2 - 3*a
*b^2*d^3)*cos(x) - 2*(b^3*d^3 + (3*b^3*c^2*d - 9*a*b^2*c*d^2 + (9*a^2*b -
b^3)*d^3)*cos(x)^2)*sin(x))/(d^4*cos(x)^3)
```

3.703.6 Sympy [A] (verification not implemented)

Time = 2.84 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.22

$$\int \frac{\sec^2(x)(a+b\tan(x))^3}{c+d\tan(x)}\,dx = \frac{b^3\tan^3(x)}{3d} + \frac{(3ab^2d-b^3c)\tan^2(x)}{2d^2} + \frac{(ad-bc)^3\left(\begin{cases}\frac{\tan(x)}{c} & \text{for } d=0\\ \frac{\log(c+d\tan(x))}{d} & \text{otherwise}\end{cases}\right)}{d^3} + \frac{(3a^2bd^2-3ab^2cd+b^3c^2)\tan(x)}{d^3}$$

input
```
integrate(sec(x)**2*(a+b*tan(x))**3/(c+d*tan(x)),x)
```

output
```
b**3*tan(x)**3/(3*d) + (3*a*b**2*d - b**3*c)*tan(x)**2/(2*d**2) + (a*d - b
*c)**3*Piecewise((tan(x)/c, Eq(d, 0)), (log(c + d*tan(x))/d, True))/d**3 +
 (3*a**2*b*d**2 - 3*a*b**2*c*d + b**3*c**2)*tan(x)/d**3
```

3.703.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.51

$$\int \frac{\sec^2(x)(a+b\tan(x))^3}{c+d\tan(x)}\,dx = \frac{2\,b^3d^2\tan(x)^3 - 3\,(b^3cd-3\,ab^2d^2)\tan(x)^2 + 6\,(b^3c^2-3\,ab^2cd+3\,a^2bd^2)\tan(x)}{6\,d^3} - \frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\log(d\tan(x)+c)}{d^4}$$

input
```
integrate(sec(x)^2*(a+b*tan(x))^3/(c+d*tan(x)),x, algorithm="maxima")
```

output
```
1/6*(2*b^3*d^2*tan(x)^3 - 3*(b^3*c*d - 3*a*b^2*d^2)*tan(x)^2 + 6*(b^3*c^2
- 3*a*b^2*c*d + 3*a^2*b*d^2)*tan(x))/d^3 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^
2*b*c*d^2 - a^3*d^3)*log(d*tan(x) + c)/d^4
```

3.703.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.58

$$\int \frac{\sec^2(x)(a+b\tan(x))^3}{c+d\tan(x)}\,dx = \frac{2\,b^3d^2\tan{(x)}^3 - 3\,b^3cd\tan{(x)}^2 + 9\,ab^2d^2\tan{(x)}^2 + 6\,b^3c^2\tan{(x)} - 18\,ab^2cd\tan{(x)} + 18\,a^2bd^2\tan{(x)}}{6\,d^3} - \frac{(b^3c^3 - 3\,ab^2c^2d + 3\,a^2bcd^2 - a^3d^3)\log{(|d\tan{(x)} + c|)}}{d^4}$$

input
```
integrate(sec(x)^2*(a+b*tan(x))^3/(c+d*tan(x)),x, algorithm="giac")
```

output
```
1/6*(2*b^3*d^2*tan(x)^3 - 3*b^3*c*d*tan(x)^2 + 9*a*b^2*d^2*tan(x)^2 + 6*b^
3*c^2*tan(x) - 18*a*b^2*c*d*tan(x) + 18*a^2*b*d^2*tan(x))/d^3 - (b^3*c^3 -
 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(abs(d*tan(x) + c))/d^4
```

3.703.9 Mupad [B] (verification not implemented)

Time = 28.90 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.56

$$\int \frac{\sec^2(x)(a+b\tan(x))^3}{c+d\tan(x)}\,dx = \tan(x)\left(\frac{3\,a^2\,b}{d} - \frac{c\left(\frac{3\,a\,b^2}{d} - \frac{b^3\,c}{d^2}\right)}{d}\right) + \tan(x)^2\left(\frac{3\,a\,b^2}{2\,d} - \frac{b^3\,c}{2\,d^2}\right) + \frac{b^3\tan(x)^3}{3\,d} + \frac{\ln\left(c + d\tan(x)\right)\,\left(a^3\,d^3 - 3\,a^2\,b\,c\,d^2 + 3\,a\,b^2\,c^2\,d - b^3\,c^3\right)}{d^4}$$

input
```
int((a + b*tan(x))^3/(cos(x)^2*(c + d*tan(x))),x)
```

output
```
tan(x)*((3*a^2*b)/d - (c*((3*a*b^2)/d - (b^3*c)/d^2))/d) + tan(x)^2*((3*a*
b^2)/(2*d) - (b^3*c)/(2*d^2)) + (b^3*tan(x)^3)/(3*d) + (log(c + d*tan(x))*
(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/d^4
```

3.703. $\int \frac{\sec^2(x)(a+b\tan(x))^3}{c+d\tan(x)}\,dx$

3.704 $\int \frac{\sec^2(x)\tan^2(x)}{(2+\tan^3(x))^2}\,dx$

3.704.1 Optimal result

Integrand size = 17, antiderivative size = 12

$$\int \frac{\sec^2(x)\tan^2(x)}{(2+\tan^3(x))^2}\,dx = -\frac{1}{3(2+\tan^3(x))}$$

output
```
-1/3/(2+tan(x)^3)
```

3.704.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(x)\tan^2(x)}{(2+\tan^3(x))^2}\,dx = -\frac{1}{3(2+\tan^3(x))}$$

input
```
Integrate[(Sec[x]^2*Tan[x]^2)/(2 + Tan[x]^3)^2,x]
```

output
```
-1/3*1/(2 + Tan[x]^3)
```

3.704.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3042, 4842, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(x)\sec^2(x)}{\left(\tan^3(x)+2\right)^2}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\tan(x)^2\sec(x)^2}{\left(\tan(x)^3+2\right)^2}dx$$

$$\Big\downarrow \ 4842$$

$$\int \frac{\tan^2(x)}{\left(\tan^3(x)+2\right)^2}d\tan(x)$$

$$\Big\downarrow \ 793$$

$$-\frac{1}{3\left(\tan^3(x)+2\right)}$$

input
```
Int[(Sec[x]^2*Tan[x]^2)/(2 + Tan[x]^3)^2,x]
```

output
```
-1/3*1/(2 + Tan[x]^3)
```

3.704.3.1 Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4842
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Tan[c*(a +
 b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] |
| EqQ[F, sec])
```

3.704.4 Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$-\frac{1}{3\left(2+\tan\left(x\right)^{3}\right)}$	11
default	$-\frac{1}{3\left(2+\tan\left(x\right)^{3}\right)}$	11
risch	$\frac{\left(-\frac{8}{15}-\frac{2i}{5}\right)\left(3\,{\mathrm e}^{4ix}+1\right)}{5\,{\mathrm e}^{6ix}+9\,{\mathrm e}^{4ix}-12i{\mathrm e}^{4ix}+15\,{\mathrm e}^{2ix}+3-4i}$	49

input
```
int(sec(x)^2*tan(x)^2/(2+tan(x)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3/(2+tan(x)^3)
```

3.704.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(10) = 20.

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 3.00

$$\int \frac{\sec^2(x)\tan^2(x)}{(2+\tan^3(x))^2}\,dx = -\frac{\cos\left(x\right)^{3} + 2\left(\cos\left(x\right)^{2} - 1\right)\sin\left(x\right)}{15\left(2\,\cos\left(x\right)^{3} - \left(\cos\left(x\right)^{2} - 1\right)\sin\left(x\right)\right)}$$

input
```
integrate(sec(x)^2*tan(x)^2/(2+tan(x)^3)^2,x, algorithm="fricas")
```

output
```
-1/15*(cos(x)^3 + 2*(cos(x)^2 - 1)*sin(x))/(2*cos(x)^3 - (cos(x)^2 - 1)*si
n(x))
```

3.704.6 Sympy [A] (verification not implemented)

Time = 160.23 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\sec^2(x)\tan^2(x)}{(2+\tan^3(x))^2}\,dx = -\frac{1}{3\left(\tan^3(x)+2\right)}$$

input
```
integrate(sec(x)**2*tan(x)**2/(2+tan(x)**3)**2,x)
```

output
```
-1/(3*(tan(x)**3 + 2))
```

3.704.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\sec^2(x)\tan^2(x)}{(2+\tan^3(x))^2}\,dx = -\frac{1}{3\left(\tan(x)^3+2\right)}$$

input
```
integrate(sec(x)^2*tan(x)^2/(2+tan(x)^3)^2,x, algorithm="maxima")
```

output
```
-1/3/(tan(x)^3 + 2)
```

3.704.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\sec^2(x)\tan^2(x)}{(2+\tan^3(x))^2}\,dx = -\frac{1}{3\left(\tan(x)^3+2\right)}$$

input
```
integrate(sec(x)^2*tan(x)^2/(2+tan(x)^3)^2,x, algorithm="giac")
```

output
```
-1/3/(tan(x)^3 + 2)
```

3.704.9 Mupad [B] (verification not implemented)

Time = 28.61 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(x)\tan^2(x)}{(2+\tan^3(x))^2}\,dx = -\frac{1}{3\,\left(\tan\left(x\right)^3+2\right)}$$

input
```
int(tan(x)^2/(cos(x)^2*(tan(x)^3 + 2)^2),x)
```

output
```
-1/(3*(tan(x)^3 + 2))
```

3.705 $\int \sec^2(x)\tan^6(x)\left(1+\tan^2(x)\right)^3\,dx$

3.705.1 Optimal result

Integrand size = 17, antiderivative size = 33

$$\int \sec^2(x)\tan^6(x)\left(1+\tan^2(x)\right)^3\,dx=\frac{\tan^7(x)}{7}+\frac{\tan^9(x)}{3}+\frac{3\tan^{11}(x)}{11}+\frac{\tan^{13}(x)}{13}$$

output
```
1/7*tan(x)^7+1/3*tan(x)^9+3/11*tan(x)^11+1/13*tan(x)^13
```

3.705.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 67 vs. 2(33) = 66.

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.03

$$\begin{aligned}\int \sec^2(x)\tan^6(x)\left(1+\tan^2(x)\right)^3\,dx=&-\frac{16\tan(x)}{3003}-\frac{8\sec^2(x)\tan(x)}{3003}-\frac{2\sec^4(x)\tan(x)}{1001}\\&-\frac{5\sec^6(x)\tan(x)}{3003}+\frac{53}{429}\sec^8(x)\tan(x)\\&-\frac{27}{143}\sec^{10}(x)\tan(x)+\frac{1}{13}\sec^{12}(x)\tan(x)\end{aligned}$$

input
```
Integrate[Sec[x]^2*Tan[x]^6*(1 + Tan[x]^2)^3,x]
```

output
```
(-16*Tan[x])/3003 - (8*Sec[x]^2*Tan[x])/3003 - (2*Sec[x]^4*Tan[x])/1001 -
(5*Sec[x]^6*Tan[x])/3003 + (53*Sec[x]^8*Tan[x])/429 - (27*Sec[x]^10*Tan[x]
)/143 + (Sec[x]^12*Tan[x])/13
```

3.705.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3042, 4140, 3042, 3087, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^6(x) \left(\tan^2(x)+1\right)^3 \sec^2(x) \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(x)^6 \left(\tan(x)^2+1\right)^3 \sec(x)^2 dx$$

$$\Big\downarrow \text{4140}$$

$$\int \tan^6(x) \sec^8(x) dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(x)^6 \sec(x)^8 dx$$

$$\Big\downarrow \text{3087}$$

$$\int \tan^6(x) \left(\tan^2(x)+1\right)^3 d\tan(x)$$

$$\Big\downarrow \text{244}$$

$$\int \left(\tan^{12}(x)+3\tan^{10}(x)+3\tan^8(x)+\tan^6(x)\right) d\tan(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{\tan^{13}(x)}{13}+\frac{3\tan^{11}(x)}{11}+\frac{\tan^9(x)}{3}+\frac{\tan^7(x)}{7}$$

input
```
Int[Sec[x]^2*Tan[x]^6*(1 + Tan[x]^2)^3,x]
```

output
```
Tan[x]^7/7 + Tan[x]^9/3 + (3*Tan[x]^11)/11 + Tan[x]^13/13
```

3.705.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

3.705.4 Maple [A] (verified)

Time = 5.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\frac{\tan(x)^{7}}{7}+\frac{\tan(x)^{9}}{3}+\frac{3\tan(x)^{11}}{11}+\frac{\tan(x)^{13}}{13}$	26
default	$\frac{\tan(x)^{7}}{7}+\frac{\tan(x)^{9}}{3}+\frac{3\tan(x)^{11}}{11}+\frac{\tan(x)^{13}}{13}$	26
risch	$-\frac{32i(3003\,\mathrm{e}^{18ix}-9009\,\mathrm{e}^{16ix}+18018\,\mathrm{e}^{14ix}-16302\,\mathrm{e}^{12ix}+10296\,\mathrm{e}^{10ix}-2288\,\mathrm{e}^{8ix}+286\,\mathrm{e}^{6ix}+78\,\mathrm{e}^{4ix}+13\,\mathrm{e}^{2ix}+1)}{3003(\mathrm{e}^{2ix}+1)^{13}}$	78

input
```
int(sec(x)^2*tan(x)^6*(1+tan(x)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/7*tan(x)^7+1/3*tan(x)^9+3/11*tan(x)^11+1/13*tan(x)^13
```

3.705.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.39

$$\int \sec^2(x)\tan^6(x)\left(1+\tan^2(x)\right)^3\,dx = -\frac{\left(16\,\cos(x)^{12}+8\,\cos(x)^{10}+6\,\cos(x)^{8}+5\,\cos(x)^{6}-371\,\cos(x)^{4}+567\,\cos(x)^{2}-231\right)\sin(x)}{3003\,\cos(x)^{13}}$$

input
```
integrate(sec(x)^2*tan(x)^6*(1+tan(x)^2)^3,x, algorithm="fricas")
```

output
```
-1/3003*(16*cos(x)^12 + 8*cos(x)^10 + 6*cos(x)^8 + 5*cos(x)^6 - 371*cos(x)
^4 + 567*cos(x)^2 - 231)*sin(x)/cos(x)^13
```

3.705.6 Sympy [A] (verification not implemented)

Time = 15.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \sec^2(x)\tan^6(x)\left(1+\tan^2(x)\right)^3\,dx = \frac{\tan^{13}(x)}{13}+\frac{3\tan^{11}(x)}{11}+\frac{\tan^{9}(x)}{3}+\frac{\tan^{7}(x)}{7}$$

input
```
integrate(sec(x)**2*tan(x)**6*(1+tan(x)**2)**3,x)
```

output
```
tan(x)**13/13 + 3*tan(x)**11/11 + tan(x)**9/3 + tan(x)**7/7
```

3.705.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \sec^2(x)\tan^6(x)\left(1+\tan^2(x)\right)^3\,dx = \frac{1}{13}\,\tan(x)^{13}+\frac{3}{11}\,\tan(x)^{11}+\frac{1}{3}\,\tan(x)^{9}+\frac{1}{7}\,\tan(x)^{7}$$

input
```
integrate(sec(x)^2*tan(x)^6*(1+tan(x)^2)^3,x, algorithm="maxima")
```

output
```
1/13*tan(x)^13 + 3/11*tan(x)^11 + 1/3*tan(x)^9 + 1/7*tan(x)^7
```

3.705.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \sec^2(x)\tan^6(x)\left(1+\tan^2(x)\right)^3\,dx=\frac{1}{13}\tan\left(x\right)^{13}+\frac{3}{11}\tan\left(x\right)^{11}+\frac{1}{3}\tan\left(x\right)^{9}+\frac{1}{7}\tan\left(x\right)^{7}$$

input
```
integrate(sec(x)^2*tan(x)^6*(1+tan(x)^2)^3,x, algorithm="giac")
```

output
```
1/13*tan(x)^13 + 3/11*tan(x)^11 + 1/3*tan(x)^9 + 1/7*tan(x)^7
```

3.705.9 Mupad [B] (verification not implemented)

Time = 27.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \sec^2(x)\tan^6(x)\left(1+\tan^2(x)\right)^3\,dx=\frac{\tan(x)^{13}}{13}+\frac{3\tan(x)^{11}}{11}+\frac{\tan(x)^{9}}{3}+\frac{\tan(x)^{7}}{7}$$

input
```
int((tan(x)^6*(tan(x)^2 + 1)^3)/cos(x)^2,x)
```

output
```
tan(x)^7/7 + tan(x)^9/3 + (3*tan(x)^11)/11 + tan(x)^13/13
```

3.706 $\int \frac{\sec^2(x)(2+\tan^2(x))}{1+\tan^3(x)}\,dx$

3.706.1 Optimal result

Integrand size = 19, antiderivative size = 46

$$\int \frac{\sec^2(x)\,(2+\tan^2(x))}{1+\tan^3(x)}\,dx = \frac{2x}{\sqrt{3}} + \frac{2\arctan\left(\frac{1-2\cos^2(x)}{2+\sqrt{3}-2\cos(x)\sin(x)}\right)}{\sqrt{3}} + \log(1+\tan(x))$$

output
```
ln(1+tan(x))+2/3*x*3^(1/2)+2/3*arctan((1-2*cos(x)^2)/(2-2*cos(x)*sin(x)+3^
(1/2)))*3^(1/2)
```

3.706.2 Mathematica [A] (verified)

Time = 6.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.70

$$\int \frac{\sec^2(x)\,(2+\tan^2(x))}{1+\tan^3(x)}\,dx = -\frac{2\arctan\left(\frac{1-2\tan(x)}{\sqrt{3}}\right)}{\sqrt{3}} - \log(\cos(x)) + \log(\cos(x)+\sin(x))$$

input
```
Integrate[(Sec[x]^2*(2 + Tan[x]^2))/(1 + Tan[x]^3),x]
```

output
```
(-2*ArcTan[(1 - 2*Tan[x])/Sqrt[3]])/Sqrt[3] - Log[Cos[x]] + Log[Cos[x] + S
in[x]]
```

3.706.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.57, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3042, 4842, 2402, 16, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(\tan^2(x)+2)\sec^2(x)}{\tan^3(x)+1}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(\tan(x)^2+2)\sec(x)^2}{\tan(x)^3+1}dx$$

$$\Big\downarrow \text{4842}$$

$$\int \frac{\tan^2(x)+2}{\tan^3(x)+1}d\tan(x)$$

$$\Big\downarrow \text{2402}$$

$$\int \frac{1}{\tan^2(x)-\tan(x)+1}d\tan(x)+\int \frac{1}{\tan(x)+1}d\tan(x)$$

$$\Big\downarrow \text{16}$$

$$\int \frac{1}{\tan^2(x)-\tan(x)+1}d\tan(x)+\log(\tan(x)+1)$$

$$\Big\downarrow \text{1083}$$

$$\log(\tan(x)+1)-2\int \frac{1}{-(2\tan(x)-1)^2-3}d(2\tan(x)-1)$$

$$\Big\downarrow \text{217}$$

$$\frac{2\arctan\left(\frac{2\tan(x)-1}{\sqrt{3}}\right)}{\sqrt{3}}+\log(\tan(x)+1)$$

input
```
Int[(Sec[x]^2*(2 + Tan[x]^2))/(1 + Tan[x]^3),x]
```

output
```
(2*ArcTan[(-1 + 2*Tan[x])/Sqrt[3]])/Sqrt[3] + Log[1 + Tan[x]]
```

3.706.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 2402
```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, With[{q = a^(1/3)/b^(1/3)}, Simp[C
/b   Int[1/(q + x), x], x] + Simp[(B + C*q)/b   Int[1/(q^2 - q*x + x^2), x]
, x]] /; EqQ[A*b^(2/3) - a^(1/3)*b^(1/3)*B - 2*a^(2/3)*C, 0]] /; FreeQ[{a,
b}, x] && PolyQ[P2, x, 2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4842
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Tan[c*(a +
 b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] |
| EqQ[F, sec])
```

3.706. $\int \frac{\sec^2(x)(2+\tan^2(x))}{1+\tan^3(x)}\,dx$

3.706.4 Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.52

method	result	size
derivativedivides	$\frac{2\sqrt{3}\,\arctan\left(\frac{(2\tan(x)-1)\sqrt{3}}{3}\right)}{3}+\ln\left(1+\tan\left(x\right)\right)$	24
default	$\frac{2\sqrt{3}\,\arctan\left(\frac{(2\tan(x)-1)\sqrt{3}}{3}\right)}{3}+\ln\left(1+\tan\left(x\right)\right)$	24
risch	$\ln\left(i+{\mathrm e}^{2 i x}\right)-\ln\left({\mathrm e}^{2 i x}+1\right)+\frac{i\sqrt{3}\,\ln\left({\mathrm e}^{2 i x}-2 i-i\sqrt{3}\right)}{3}-\frac{i\sqrt{3}\,\ln\left({\mathrm e}^{2 i x}-2 i+i\sqrt{3}\right)}{3}$	63

input
```
int(sec(x)^2*(2+tan(x)^2)/(1+tan(x)^3),x,method=_RETURNVERBOSE)
```

output
```
2/3*3^(1/2)*arctan(1/3*(2*tan(x)-1)*3^(1/2))+ln(1+tan(x))
```

3.706.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13

$$\int \frac{\sec^2(x)\left(2+\tan^2(x)\right)}{1+\tan^3(x)}\,dx = \frac{1}{3}\,\sqrt{3}\arctan\left(\frac{4\,\sqrt{3}\cos\left(x\right)\sin\left(x\right)-\sqrt{3}}{3\,\left(2\,\cos\left(x\right)^{2}-1\right)}\right) - \frac{1}{2}\,\log\left(\cos\left(x\right)^{2}\right)+\frac{1}{2}\,\log\left(2\,\cos\left(x\right)\sin\left(x\right)+1\right)$$

input
```
integrate(sec(x)^2*(2+tan(x)^2)/(1+tan(x)^3),x, algorithm="fricas")
```

output
```
1/3*sqrt(3)*arctan(1/3*(4*sqrt(3)*cos(x)*sin(x) - sqrt(3))/(2*cos(x)^2 - 1
)) - 1/2*log(cos(x)^2) + 1/2*log(2*cos(x)*sin(x) + 1)
```

3.706.6 Sympy [A] (verification not implemented)

Time = 2.50 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int \frac{\sec^2(x)\left(2+\tan^2(x)\right)}{1+\tan^3(x)}\,dx = \frac{2\sqrt{3}\left(\operatorname{atan}\left(\frac{2\sqrt{3}\left(\tan\left(x\right)-\frac{1}{2}\right)}{3}\right)+\pi\left\lfloor\frac{x-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{3}+\log\left(\tan\left(x\right)+1\right)$$

input
```
integrate(sec(x)**2*(2+tan(x)**2)/(1+tan(x)**3),x)
```

output
```
2*sqrt(3)*(atan(2*sqrt(3)*(tan(x) - 1/2)/3) + pi*floor((x - pi/2)/pi))/3 +
 log(tan(x) + 1)
```

3.706.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.50

$$\int \frac{\sec^2(x)\,(2+\tan^2(x))}{1+\tan^3(x)}\,dx = \frac{2}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,\tan(x)-1)\right)+\log(\tan(x)+1)$$

input
```
integrate(sec(x)^2*(2+tan(x)^2)/(1+tan(x)^3),x, algorithm="maxima")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*tan(x) - 1)) + log(tan(x) + 1)
```

3.706.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.52

$$\int \frac{\sec^2(x)\,(2+\tan^2(x))}{1+\tan^3(x)}\,dx = \frac{2}{3}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,\tan(x)-1)\right)+\log(|\tan(x)+1|)$$

input
```
integrate(sec(x)^2*(2+tan(x)^2)/(1+tan(x)^3),x, algorithm="giac")
```

output
```
2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*tan(x) - 1)) + log(abs(tan(x) + 1))
```

3.706.9 Mupad [B] (verification not implemented)

Time = 26.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.65

$$\int \frac{\sec^2(x)\,(2+\tan^2(x))}{1+\tan^3(x)}\,dx = \ln(\tan(x)+1)-\frac{2\,\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}-\sqrt{3}\,\tan(x)}{\tan(x)+1}\right)}{3}$$

input
```
int((tan(x)^2 + 2)/(cos(x)^2*(tan(x)^3 + 1)),x)
```

output
```
log(tan(x) + 1) - (2*3^(1/2)*atan((3^(1/2) - 3^(1/2)*tan(x))/(tan(x) + 1))
)/3
```

3.706. $\int \frac{\sec^2(x)(2+\tan^2(x))}{1+\tan^3(x)}\,dx$

3.707 $\int (1+\cos^2(x)) \sec^2(x) \, dx$

3.707.1 Optimal result

Integrand size = 11, antiderivative size = 4

$$\int (1+\cos^2(x)) \sec^2(x) \, dx = x + \tan(x)$$

output
```
x+tan(x)
```

3.707.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int (1+\cos^2(x)) \sec^2(x) \, dx = x + \tan(x)$$

input
```
Integrate[(1 + Cos[x]^2)*Sec[x]^2,x]
```

output
```
x + Tan[x]
```

3.707.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3042, 3491, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\cos^2(x)+1\right)\sec^2(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(x+\frac{\pi}{2}\right)^2+1}{\sin\left(x+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3491}$$

$$\int 1dx+\tan(x)$$

$$\Big\downarrow \text{24}$$

$$x+\tan(x)$$

input
```
Int[(1 + Cos[x]^2)*Sec[x]^2,x]
```

output
```
x + Tan[x]
```

3.707.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

3.707.4 Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

method	result	size
default	$x+\tan\left(x\right)$	5
parallelrisch	$x+\tan\left(x\right)$	5
parts	$x+\tan\left(x\right)$	5
risch	$x+\frac{2i}{{\mathrm e}^{2ix}+1}$	15
norman	$\frac{x\tan\left(\frac{x}{2}\right)^{4}+x\tan\left(\frac{x}{2}\right)^{6}-x-4\tan\left(\frac{x}{2}\right)^{3}-2\tan\left(\frac{x}{2}\right)^{5}-x\tan\left(\frac{x}{2}\right)^{2}-2\tan\left(\frac{x}{2}\right)}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)^{2}\left(\tan\left(\frac{x}{2}\right)^{2}-1\right)}$	73

input
```
int((cos(x)^2+1)*sec(x)^2,x,method=_RETURNVERBOSE)
```

output
```
x+tan(x)
```

3.707.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. 2(4) = 8.

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 3.00

$$\int\left(1+\cos^2(x)\right)\sec^2(x)\,dx=\frac{x\cos\left(x\right)+\sin\left(x\right)}{\cos\left(x\right)}$$

input
```
integrate((1+cos(x)^2)*sec(x)^2,x, algorithm="fricas")
```

output
```
(x*cos(x) + sin(x))/cos(x)
```

3.707.6 Sympy [A] (verification not implemented)

Time = 3.77 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int\left(1+\cos^2(x)\right)\sec^2(x)\,dx=x+\tan\left(x\right)$$

input
```
integrate((1+cos(x)**2)*sec(x)**2,x)
```

output
```
x + tan(x)
```

3.707.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int (1+\cos^2(x)) \sec^2(x)\, dx = x + \tan(x)$$

input
```
integrate((1+cos(x)^2)*sec(x)^2,x, algorithm="maxima")
```

output
```
x + tan(x)
```

3.707.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(4) = 8.

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 3.75

$$\int (1+\cos^2(x)) \sec^2(x)\, dx = -\pi \left\lfloor \frac{x}{\pi} + \frac{1}{2} \right\rfloor + x + \tan(x)$$

input
```
integrate((1+cos(x)^2)*sec(x)^2,x, algorithm="giac")
```

output
```
-pi*floor(x/pi + 1/2) + x + tan(x)
```

3.707.9 Mupad [B] (verification not implemented)

Time = 26.47 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int (1+\cos^2(x)) \sec^2(x)\, dx = x + \tan(x)$$

input
```
int((cos(x)^2 + 1)/cos(x)^2,x)
```

output
```
x + tan(x)
```

3.708 $\int \frac{\sec^2(x)}{1+\sec^2(x)-3\tan(x)}\,dx$

3.708.1 Optimal result

Integrand size = 17, antiderivative size = 21

$$\int \frac{\sec^2(x)}{1+\sec^2(x)-3\tan(x)}\,dx = -\log(\cos(x)-\sin(x))+\log(2\cos(x)-\sin(x))$$

output
```
-ln(cos(x)-sin(x))+ln(2*cos(x)-sin(x))
```

3.708.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.38

$$\int \frac{\sec^2(x)}{1+\sec^2(x)-3\tan(x)}\,dx = 2\left(-\frac{1}{2}\log(\cos(x)-\sin(x))+\frac{1}{2}\log(2\cos(x)-\sin(x))\right)$$

input
```
Integrate[Sec[x]^2/(1 + Sec[x]^2 - 3*Tan[x]),x]
```

output
```
2*(-1/2*Log[Cos[x] - Sin[x]] + Log[2*Cos[x] - Sin[x]]/2)
```

3.708.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3042, 4889, 1081, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(x)}{-3\tan(x)+\sec^2(x)+1}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(x)^2}{-3\tan(x)+\sec(x)^2+1}dx$$

$$\Big\downarrow \text{4889}$$

$$\int \frac{1}{\tan^2(x)-3\tan(x)+2}d\tan(x)$$

$$\Big\downarrow \text{1081}$$

$$\int \left(\frac{1}{\tan(x)-2}+\frac{1}{1-\tan(x)}\right)d\tan(x)$$

$$\Big\downarrow \text{2009}$$

$$\log(2-\tan(x))-\log(1-\tan(x))$$

input
```
Int[Sec[x]^2/(1 + Sec[x]^2 - 3*Tan[x]),x]
```

output
```
-Log[1 - Tan[x]] + Log[2 - Tan[x]]
```

3.708.3.1 Defintions of rubi rules used

rule 1081
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
- 4*a*c, 2]}, Simp[c   Int[ExpandIntegrand[1/((b/2 - q/2 + c*x)*(b/2 + q/2
+ c*x)), x], x], x]] /; FreeQ[{a, b, c}, x] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.708.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.67

method	result	size
default	$-\ln\left(\tan\left(x\right)-1\right)+\ln\left(-2+\tan\left(x\right)\right)$	14
risch	$\ln\left(\mathrm{e}^{2ix}+\frac{3}{5}-\frac{4i}{5}\right)-\ln\left(\mathrm{e}^{2ix}-i\right)$	23

input
```
int(sec(x)^2/(1+sec(x)^2-3*tan(x)),x,method=_RETURNVERBOSE)
```

output
```
-ln(tan(x)-1)+ln(-2+tan(x))
```

3.708.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.38

$$\int \frac{\sec^2(x)}{1+\sec^2(x)-3\tan(x)}\,dx=\frac{1}{2}\log\left(\frac{3}{4}\cos\left(x\right)^2-\cos\left(x\right)\sin\left(x\right)+\frac{1}{4}\right)-\frac{1}{2}\log\left(-2\cos\left(x\right)\sin\left(x\right)+1\right)$$

input
```
integrate(sec(x)^2/(1+sec(x)^2-3*tan(x)),x, algorithm="fricas")
```

output
```
1/2*log(3/4*cos(x)^2 - cos(x)*sin(x) + 1/4) - 1/2*log(-2*cos(x)*sin(x) + 1
)
```

3.708.6 Sympy [F]

$$\int \frac{\sec^2(x)}{1+\sec^2(x)-3\tan(x)}\,dx = \int \frac{\sec^2(x)}{-3\tan(x)+\sec^2(x)+1}\,dx$$

input
```
integrate(sec(x)**2/(1+sec(x)**2-3*tan(x)),x)
```

output
```
Integral(sec(x)**2/(-3*tan(x) + sec(x)**2 + 1), x)
```

3.708.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.62

$$\int \frac{\sec^2(x)}{1+\sec^2(x)-3\tan(x)}\,dx = -\log(\tan(x)-1)+\log(\tan(x)-2)$$

input
```
integrate(sec(x)^2/(1+sec(x)^2-3*tan(x)),x, algorithm="maxima")
```

output
```
-log(tan(x) - 1) + log(tan(x) - 2)
```

3.708.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.71

$$\int \frac{\sec^2(x)}{1+\sec^2(x)-3\tan(x)}\,dx = -\log(|\tan(x)-1|)+\log(|\tan(x)-2|)$$

input
```
integrate(sec(x)^2/(1+sec(x)^2-3*tan(x)),x, algorithm="giac")
```

output
```
-log(abs(tan(x) - 1)) + log(abs(tan(x) - 2))
```

3.708.9 Mupad [B] (verification not implemented)

Time = 27.38 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.43

$$\int \frac{\sec^2(x)}{1+\sec^2(x)-3\tan(x)}\,dx = -2\,\text{atanh}(2\tan(x)-3)$$

input
```
int(1/(cos(x)^2*(1/cos(x)^2 - 3*tan(x) + 1)),x)
```

output
```
-2*atanh(2*tan(x) - 3)
```

3.709 $\int \frac{\sec^2(x)}{\sqrt{4-\sec^2(x)}}\,dx$

3.709.1 Optimal result

Integrand size = 17, antiderivative size = 9

$$\int \frac{\sec^2(x)}{\sqrt{4-\sec^2(x)}}\,dx = \arcsin\left(\frac{\tan(x)}{\sqrt{3}}\right)$$

output
```
arcsin(1/3*tan(x)*3^(1/2))
```

3.709.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 43 vs. 2(9) = 18.

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 4.78

$$\int \frac{\sec^2(x)}{\sqrt{4-\sec^2(x)}}\,dx = \frac{\arctan\left(\frac{\sin(x)}{\sqrt{3-4\sin^2(x)}}\right)\sqrt{1+2\cos(2x)}\sec(x)}{\sqrt{4-\sec^2(x)}}$$

input
```
Integrate[Sec[x]^2/Sqrt[4 - Sec[x]^2],x]
```

output
```
(ArcTan[Sin[x]/Sqrt[3 - 4*Sin[x]^2]]*Sqrt[1 + 2*Cos[2*x]]*Sec[x])/Sqrt[4 -
 Sec[x]^2]
```

3.709.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3042, 4634, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(x)}{\sqrt{4-\sec^2(x)}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sec(x)^2}{\sqrt{4-\sec(x)^2}}dx$$

$$\Big\downarrow \text{ 4634}$$

$$\int \frac{1}{\sqrt{3-\tan^2(x)}}d\tan(x)$$

$$\Big\downarrow \text{ 223}$$

$$\arcsin\left(\frac{\tan(x)}{\sqrt{3}}\right)$$

input
```
Int[Sec[x]^2/Sqrt[4 - Sec[x]^2],x]
```

output
```
ArcSin[Tan[x]/Sqrt[3]]
```

3.709.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4634
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_)
)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[ff/f
Subst[Int[(1 + ff^2*x^2)^(m/2 - 1)*ExpandToSum[a + b*(1 + ff^2*x^2)^(n/2),
x]^p, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ
[m/2] && IntegerQ[n/2]
```

3.709.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(8) = 16$.

Time = 16.84 (sec) , antiderivative size = 109, normalized size of antiderivative = 12.11

method	result
default	$\frac{\left(\arctan\left(\frac{4\sin(x)+3}{(\cos(x)+1)\sqrt{\frac{(2\cos(x)-1)(1+2\cos(x))}{(\cos(x)+1)^2}}}\right)+\arctan\left(\frac{4\sin(x)-3}{(\cos(x)+1)\sqrt{\frac{(2\cos(x)-1)(1+2\cos(x))}{(\cos(x)+1)^2}}}\right)\right)\sqrt{\frac{(2\cos(x)-1)(1+2\cos(x))}{(\cos(x)+1)^2}}\,(1+\sec(x$

input
```
int(sec(x)^2/(4-sec(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(arctan((4*sin(x)+3)/(cos(x)+1)/((2*cos(x)-1)*(1+2*cos(x))/(cos(x)+1)^
2)^(1/2))+arctan((4*sin(x)-3)/(cos(x)+1)/((2*cos(x)-1)*(1+2*cos(x))/(cos(x
)+1)^2)^(1/2)))*((2*cos(x)-1)*(1+2*cos(x))/(cos(x)+1)^2)^(1/2)/(4-sec(x)^2
)^(1/2)*(1+sec(x))
```

3.709.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(8) = 16$.

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.78

$$\int \frac{\sec^2(x)}{\sqrt{4-\sec^2(x)}}\,dx = -\arctan\left(\frac{\sqrt{\frac{4\cos(x)^2-1}{\cos(x)^2}}\cos(x)}{\sin(x)}\right)$$

input
```
integrate(sec(x)^2/(4-sec(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-arctan(sqrt((4*cos(x)^2 - 1)/cos(x)^2)*cos(x)/sin(x))
```

3.709.6 Sympy [F]

$$\int \frac{\sec^2(x)}{\sqrt{4-\sec^2(x)}}\,dx = \int \frac{\sec^2(x)}{\sqrt{-\left(\sec(x)-2\right)\left(\sec(x)+2\right)}}\,dx$$

input
```
integrate(sec(x)**2/(4-sec(x)**2)**(1/2),x)
```

output
```
Integral(sec(x)**2/sqrt(-(sec(x) - 2)*(sec(x) + 2)), x)
```

3.709.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int \frac{\sec^2(x)}{\sqrt{4-\sec^2(x)}}\,dx = \arcsin\left(\frac{1}{3}\sqrt{3}\tan(x)\right)$$

input
```
integrate(sec(x)^2/(4-sec(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
arcsin(1/3*sqrt(3)*tan(x))
```

3.709.8 Giac [F]

$$\int \frac{\sec^2(x)}{\sqrt{4-\sec^2(x)}}\,dx = \int \frac{\sec(x)^2}{\sqrt{-\sec(x)^2+4}}\,dx$$

input
```
integrate(sec(x)^2/(4-sec(x)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(x)^2/sqrt(-sec(x)^2 + 4), x)
```

3.709.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(x)}{\sqrt{4-\sec^2(x)}}\,dx = \int \frac{1}{\cos(x)^2\,\sqrt{4-\frac{1}{\cos(x)^2}}}\,dx$$

input
```
int(1/(cos(x)^2*(4 - 1/cos(x)^2)^(1/2)),x)
```

output
```
int(1/(cos(x)^2*(4 - 1/cos(x)^2)^(1/2)), x)
```

3.710 $\int \frac{\sec^2(x)}{\sqrt{1-4\tan^2(x)}}\,dx$

3.710.1 Optimal result

Integrand size = 17, antiderivative size = 9

$$\int \frac{\sec^2(x)}{\sqrt{1-4\tan^2(x)}}\,dx = \frac{1}{2}\arcsin(2\tan(x))$$

output
```
1/2*arcsin(2*tan(x))
```

3.710.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 47 vs. 2(9) = 18.

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 5.22

$$\int \frac{\sec^2(x)}{\sqrt{1-4\tan^2(x)}}\,dx = \frac{\arctan\left(\frac{2\sin(x)}{\sqrt{1-5\sin^2(x)}}\right)\sqrt{-3+5\cos(2x)}\sec(x)}{2\sqrt{2-8\tan^2(x)}}$$

input
```
Integrate[Sec[x]^2/Sqrt[1 - 4*Tan[x]^2],x]
```

output
```
(ArcTan[(2*Sin[x])/Sqrt[1 - 5*Sin[x]^2]]*Sqrt[-3 + 5*Cos[2*x]]*Sec[x])/(2*
Sqrt[2 - 8*Tan[x]^2])
```

3.710.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {3042, 4158, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(x)}{\sqrt{1-4\tan^2(x)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sec(x)^2}{\sqrt{1-4\tan(x)^2}}dx$$

$$\Big\downarrow \text{ 4158}$$

$$\int \frac{1}{\sqrt{1-4\tan^2(x)}}d\tan(x)$$

$$\Big\downarrow \text{ 223}$$

$$\frac{1}{2}\arcsin(2\tan(x))$$

input
```
Int[Sec[x]^2/Sqrt[1 - 4*Tan[x]^2],x]
```

output
```
ArcSin[2*Tan[x]]/2
```

3.710.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

3.710.4 Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{\arcsin(2\tan(x))}{2}$	8
default	$\frac{\arcsin(2\tan(x))}{2}$	8

input
```
int(sec(x)^2/(1-4*tan(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arcsin(2*tan(x))
```

3.710.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(7) = 14$.

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 5.00

$$\int \frac{\sec^2(x)}{\sqrt{1-4\tan^2(x)}}\,dx = -\frac{1}{4}\arctan\left(\frac{\left(9\cos(x)^3-8\cos(x)\right)\sqrt{\frac{5\cos(x)^2-4}{\cos(x)^2}}}{4\left(5\cos(x)^2-4\right)\sin(x)}\right)$$

input
```
integrate(sec(x)^2/(1-4*tan(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*arctan(1/4*(9*cos(x)^3 - 8*cos(x))*sqrt((5*cos(x)^2 - 4)/cos(x)^2)/((
5*cos(x)^2 - 4)*sin(x)))
```

3.710.6 Sympy [F]

$$\int \frac{\sec^2(x)}{\sqrt{1-4\tan^2(x)}}\,dx = \int \frac{\sec^2(x)}{\sqrt{-(2\tan(x)-1)(2\tan(x)+1)}}\,dx$$

input
```
integrate(sec(x)**2/(1-4*tan(x)**2)**(1/2),x)
```

output
```
Integral(sec(x)**2/sqrt(-(2*tan(x) - 1)*(2*tan(x) + 1)), x)
```

3.710.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{\sec^2(x)}{\sqrt{1-4\tan^2(x)}}\,dx = \frac{1}{2}\arcsin(2\tan(x))$$

input
```
integrate(sec(x)^2/(1-4*tan(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*arcsin(2*tan(x))
```

3.710.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{\sec^2(x)}{\sqrt{1-4\tan^2(x)}}\,dx = \frac{1}{2}\arcsin(2\tan(x))$$

input
```
integrate(sec(x)^2/(1-4*tan(x)^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*arcsin(2*tan(x))
```

3.710.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(x)}{\sqrt{1-4\tan^2(x)}}\,dx = \int \frac{1}{\cos(x)^2\sqrt{1-4\tan(x)^2}}\,dx$$

input
```
int(1/(cos(x)^2*(1 - 4*tan(x)^2)^(1/2)),x)
```

output
```
int(1/(cos(x)^2*(1 - 4*tan(x)^2)^(1/2)), x)
```

3.711 $\int \frac{\sec^2(x)}{\sqrt{-4+\tan^2(x)}}\,dx$

3.711.1 Optimal result

Integrand size = 15, antiderivative size = 14

$$\int \frac{\sec^2(x)}{\sqrt{-4+\tan^2(x)}}\,dx = \operatorname{arctanh}\left(\frac{\tan(x)}{\sqrt{-4+\tan^2(x)}}\right)$$

output
```
arctanh(tan(x)/(-4+tan(x)^2)^(1/2))
```

3.711.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 46 vs. 2(14) = 28.

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 3.29

$$\int \frac{\sec^2(x)}{\sqrt{-4+\tan^2(x)}}\,dx = \frac{\arctan\left(\frac{\sin(x)}{\sqrt{4-5\sin^2(x)}}\right)\sqrt{3+5\cos(2x)}\sec(x)}{\sqrt{2}\sqrt{-4+\tan^2(x)}}$$

input
```
Integrate[Sec[x]^2/Sqrt[-4 + Tan[x]^2],x]
```

output
```
(ArcTan[Sin[x]/Sqrt[4 - 5*Sin[x]^2]]*Sqrt[3 + 5*Cos[2*x]]*Sec[x])/(Sqrt[2]
*Sqrt[-4 + Tan[x]^2])
```

3.711.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 4158, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(x)}{\sqrt{\tan^2(x)-4}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sec(x)^2}{\sqrt{\tan(x)^2-4}}dx$$

$$\Big\downarrow \text{ 4158}$$

$$\int \frac{1}{\sqrt{\tan^2(x)-4}}d\tan(x)$$

$$\Big\downarrow \text{ 224}$$

$$\int \frac{1}{1-\frac{\tan^2(x)}{\tan^2(x)-4}}d\frac{\tan(x)}{\sqrt{\tan^2(x)-4}}$$

$$\Big\downarrow \text{ 219}$$

$$\text{arctanh}\left(\frac{\tan(x)}{\sqrt{\tan^2(x)-4}}\right)$$

input
```
Int[Sec[x]^2/Sqrt[-4 + Tan[x]^2],x]
```

output
```
ArcTanh[Tan[x]/Sqrt[-4 + Tan[x]^2]]
```

3.711.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

3.711.4 Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\ln\left(\tan\left(x\right)+\sqrt{-4+\tan\left(x\right)^{2}}\right)$	13
default	$\ln\left(\tan\left(x\right)+\sqrt{-4+\tan\left(x\right)^{2}}\right)$	13

input
```
int(sec(x)^2/(-4+tan(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
ln(tan(x)+(-4+tan(x)^2)^(1/2))
```

3.711.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(12) = 24$.

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 4.79

$$\int \frac{\sec^2(x)}{\sqrt{-4+\tan^2(x)}}\,dx = \frac{1}{4}\log\left(\frac{1}{2}\sqrt{-\frac{5\cos(x)^2-1}{\cos(x)^2}}\cos(x)\sin(x) - \frac{3}{2}\cos(x)^2 + \frac{1}{2}\right) - \frac{1}{4}\log\left(-\frac{1}{2}\sqrt{-\frac{5\cos(x)^2-1}{\cos(x)^2}}\cos(x)\sin(x) - \frac{3}{2}\cos(x)^2 + \frac{1}{2}\right)$$

input
```
integrate(sec(x)^2/(-4+tan(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/4*log(1/2*sqrt(-(5*cos(x)^2 - 1)/cos(x)^2)*cos(x)*sin(x) - 3/2*cos(x)^2
+ 1/2) - 1/4*log(-1/2*sqrt(-(5*cos(x)^2 - 1)/cos(x)^2)*cos(x)*sin(x) - 3/2
*cos(x)^2 + 1/2)
```

3.711.6 Sympy [F]

$$\int \frac{\sec^2(x)}{\sqrt{-4+\tan^2(x)}}\,dx = \int \frac{\sec^2(x)}{\sqrt{(\tan(x)-2)(\tan(x)+2)}}\,dx$$

input
```
integrate(sec(x)**2/(-4+tan(x)**2)**(1/2),x)
```

output
```
Integral(sec(x)**2/sqrt((tan(x) - 2)*(tan(x) + 2)), x)
```

3.711.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\sec^2(x)}{\sqrt{-4+\tan^2(x)}}\,dx = \log\left(2\sqrt{\tan(x)^2-4} + 2\tan(x)\right)$$

input
```
integrate(sec(x)^2/(-4+tan(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
log(2*sqrt(tan(x)^2 - 4) + 2*tan(x))
```

3.711.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.21

$$\int \frac{\sec^2(x)}{\sqrt{-4+\tan^2(x)}}\,dx = -\log\left(\left|\sqrt{\tan(x)^2-4}-\tan(x)\right|\right)$$

input
```
integrate(sec(x)^2/(-4+tan(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-log(abs(sqrt(tan(x)^2 - 4) - tan(x)))
```

3.711.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sec^2(x)}{\sqrt{-4+\tan^2(x)}}\,dx = \int \frac{1}{\cos(x)^2\sqrt{\tan(x)^2-4}}\,dx$$

input
```
int(1/(cos(x)^2*(tan(x)^2 - 4)^(1/2)),x)
```

output
```
int(1/(cos(x)^2*(tan(x)^2 - 4)^(1/2)), x)
```

3.712 $\int \sqrt{1-\cot^2(x)}\sec^2(x)\,dx$

3.712.1 Optimal result

Integrand size = 17, antiderivative size = 19

$$\int \sqrt{1-\cot^2(x)}\sec^2(x)\,dx=\arcsin(\cot(x))+\sqrt{1-\cot^2(x)}\tan(x)$$

output
```
arcsin(cot(x))+(1-cot(x)^2)^(1/2)*tan(x)
```

3.712.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 52 vs. 2(19) = 38.

Time = 0.44 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.74

$$\int \sqrt{1-\cot^2(x)}\sec^2(x)\,dx=\left(-\arctan\left(\frac{\cos(x)}{\sqrt{-\cos(2x)}}\right)\cos(x)\sqrt{-\cos(2x)}+\cos(2x)\right)\sqrt{1-\cot^2(x)}\sec(2x)\tan(x)$$

input
```
Integrate[Sqrt[1 - Cot[x]^2]*Sec[x]^2,x]
```

output
```
(-(ArcTan[Cos[x]/Sqrt[-Cos[2*x]]]*Cos[x]*Sqrt[-Cos[2*x]]) + Cos[2*x])*Sqrt
[1 - Cot[x]^2]*Sec[2*x]*Tan[x]
```

3.712.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3042, 4146, 247, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{1-\cot^2(x)}\sec^2(x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{1-\tan\left(x+\frac{\pi}{2}\right)^2}}{\sin\left(x+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{ 4146}$$

$$-\int \sqrt{1-\cot^2(x)}\tan^2(x)d\cot(x)$$

$$\Big\downarrow \text{ 247}$$

$$\int \frac{1}{\sqrt{1-\cot^2(x)}}d\cot(x)+\tan(x)\sqrt{1-\cot^2(x)}$$

$$\Big\downarrow \text{ 223}$$

$$\arcsin(\cot(x))+\tan(x)\sqrt{1-\cot^2(x)}$$

input
```
Int[Sqrt[1 - Cot[x]^2]*Sec[x]^2,x]
```

output
```
ArcSin[Cot[x]] + Sqrt[1 - Cot[x]^2]*Tan[x]
```

3.712.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

3.712.4 Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.74

method	result	size
derivativedivides	$\frac{\left(1-\cot(x)^2\right)^{\frac{3}{2}}}{\cot(x)}+\cot\left(x\right)\sqrt{1-\cot\left(x\right)^{2}}+\arcsin\left(\cot\left(x\right)\right)$	33
default	$\frac{\left(1-\cot(x)^2\right)^{\frac{3}{2}}}{\cot(x)}+\cot\left(x\right)\sqrt{1-\cot\left(x\right)^{2}}+\arcsin\left(\cot\left(x\right)\right)$	33

input
```
int(sec(x)^2*(1-cot(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/cot(x)*(1-cot(x)^2)^(3/2)+cot(x)*(1-cot(x)^2)^(1/2)+arcsin(cot(x))
```

3.712.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(17) = 34$.

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 4.11

$$\int \sqrt{1-\cot^2(x)}\sec^2(x)\,dx = -\frac{\arctan\left(\frac{\left(3\cos(x)^2-1\right)\sqrt{\frac{2\cos(x)^2-1}{\cos(x)^2-1}}\sin(x)}{2\left(2\cos(x)^3-\cos(x)\right)}\right)\cos(x)-2\sqrt{\frac{2\cos(x)^2-1}{\cos(x)^2-1}}\sin(x)}{2\cos(x)}$$

input
```
integrate(sec(x)^2*(1-cot(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(arctan(1/2*(3*cos(x)^2 - 1)*sqrt((2*cos(x)^2 - 1)/(cos(x)^2 - 1))*si
n(x)/(2*cos(x)^3 - cos(x)))*cos(x) - 2*sqrt((2*cos(x)^2 - 1)/(cos(x)^2 - 1
))*sin(x))/cos(x)
```

3.712.6 Sympy [F]

$$\int \sqrt{1-\cot^2(x)}\sec^2(x)\,dx = \int \sqrt{-(\cot(x)-1)(\cot(x)+1)}\sec^2(x)\,dx$$

input
```
integrate(sec(x)**2*(1-cot(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(-(cot(x) - 1)*(cot(x) + 1))*sec(x)**2, x)
```

3.712.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.58

$$\int \sqrt{1-\cot^2(x)}\sec^2(x)\,dx = \sqrt{-\frac{1}{\tan(x)^2}+1}\tan(x)-\arctan\left(\sqrt{-\frac{1}{\tan(x)^2}+1}\tan(x)\right)$$

input
```
integrate(sec(x)^2*(1-cot(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(-1/tan(x)^2 + 1)*tan(x) - arctan(sqrt(-1/tan(x)^2 + 1)*tan(x))
```

3.712.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 142, normalized size of antiderivative = 7.47

$$\int \sqrt{1-\cot^2(x)}\sec^2(x)\,dx = -\frac{1}{2}\left(\pi+2\arctan\left(-i\right)+2i\right)\mathrm{sgn}(\sin\left(x\right)) + \frac{1}{4}\left(2\,\pi\mathrm{sgn}(\cos\left(x\right))+\sqrt{2}\left(\frac{\sqrt{2}\sqrt{-2\,\cos\left(x\right)^2+1}-\sqrt{2}}{\cos\left(x\right)}-\frac{4\,\cos\left(x\right)}{\sqrt{2}\sqrt{-2\,\cos\left(x\right)^2+1}-\sqrt{2}}\right)+4\arctan\left(\right.\right.$$

input
```
integrate(sec(x)^2*(1-cot(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*(pi + 2*arctan(-I) + 2*I)*sgn(sin(x)) + 1/4*(2*pi*sgn(cos(x)) + sqrt(
2)*((sqrt(2)*sqrt(-2*cos(x)^2 + 1) - sqrt(2))/cos(x) - 4*cos(x)/(sqrt(2)*s
qrt(-2*cos(x)^2 + 1) - sqrt(2))) + 4*arctan(-1/4*sqrt(2)*((sqrt(2)*sqrt(-2
*cos(x)^2 + 1) - sqrt(2))^2/cos(x)^2 - 4)*cos(x)/(sqrt(2)*sqrt(-2*cos(x)^2
 + 1) - sqrt(2))))*sgn(sin(x))
```

3.712.9 Mupad [B] (verification not implemented)

Time = 26.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \sqrt{1-\cot^2(x)}\sec^2(x)\,dx = \mathrm{asin}(\cot(x)) + \frac{\sqrt{1-\cot\left(x\right)^2}}{\cot\left(x\right)}$$

input
```
int((1 - cot(x)^2)^(1/2)/cos(x)^2,x)
```

output
```
asin(cot(x)) + (1 - cot(x)^2)^(1/2)/cot(x)
```

3.713 $\int \sec^2(x)\sqrt{1-\tan^2(x)}\,dx$

3.713.1 Optimal result

Integrand size = 17, antiderivative size = 26

$$\int \sec^2(x)\sqrt{1-\tan^2(x)}\,dx=\frac{1}{2}\arcsin(\tan(x))+\frac{1}{2}\tan(x)\sqrt{1-\tan^2(x)}$$

output
```
1/2*arcsin(tan(x))+1/2*(1-tan(x)^2)^(1/2)*tan(x)
```

3.713.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 63 vs. 2(26) = 52.

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.42

$$\int \sec^2(x)\sqrt{1-\tan^2(x)}\,dx = \frac{\cos(2x)\tan(x)+\arcsin\left(\frac{\sin(x)}{\sqrt{\cos^2(x)}}\right)\cos(x)\sqrt{\cos^2(x)}\sqrt{1-\tan^2(x)}}{2\sqrt{\cos^2(x)}\sqrt{\cos(2x)}}$$

input
```
Integrate[Sec[x]^2*Sqrt[1 - Tan[x]^2],x]
```

output
```
(Cos[2*x]*Tan[x] + ArcSin[Sin[x]/Sqrt[Cos[x]^2]]*Cos[x]*Sqrt[Cos[x]^2]*Sqr
t[1 - Tan[x]^2])/(2*Sqrt[Cos[x]^2]*Sqrt[Cos[2*x]])
```

3.713.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3042, 4158, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{1-\tan^2(x)}\sec^2(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{1-\tan(x)^2}\sec(x)^2dx$$

$$\Big\downarrow \text{4158}$$

$$\int \sqrt{1-\tan^2(x)}d\tan(x)$$

$$\Big\downarrow \text{211}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{1-\tan^2(x)}}d\tan(x)+\frac{1}{2}\sqrt{1-\tan^2(x)}\tan(x)$$

$$\Big\downarrow \text{223}$$

$$\frac{1}{2}\arcsin(\tan(x))+\frac{1}{2}\tan(x)\sqrt{1-\tan^2(x)}$$

input
```
Int[Sec[x]^2*Sqrt[1 - Tan[x]^2],x]
```

output
```
ArcSin[Tan[x]]/2 + (Tan[x]*Sqrt[1 - Tan[x]^2])/2
```

3.713.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

3.713.4 Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{\arcsin(\tan(x))}{2}+\frac{\sqrt{1-\tan(x)^2}\,\tan(x)}{2}$	21
default	$\frac{\arcsin(\tan(x))}{2}+\frac{\sqrt{1-\tan(x)^2}\,\tan(x)}{2}$	21

input
```
int(sec(x)^2*(1-tan(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arcsin(tan(x))+1/2*(1-tan(x)^2)^(1/2)*tan(x)
```

3.713.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. 2(20) = 40.

Time = 0.26 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.77

$$\int \sec^2(x)\sqrt{1-\tan^2(x)}\,dx = -\frac{\arctan\left(\frac{\left(3\cos(x)^3-2\cos(x)\right)\sqrt{\frac{2\cos(x)^2-1}{\cos(x)^2}}}{2\left(2\cos(x)^2-1\right)\sin(x)}\right)\cos(x)-2\sqrt{\frac{2\cos(x)^2-1}{\cos(x)^2}}\sin(x)}{4\cos(x)}$$

input
```
integrate(sec(x)^2*(1-tan(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*(arctan(1/2*(3*cos(x)^3 - 2*cos(x))*sqrt((2*cos(x)^2 - 1)/cos(x)^2)/(
(2*cos(x)^2 - 1)*sin(x)))*cos(x) - 2*sqrt((2*cos(x)^2 - 1)/cos(x)^2)*sin(x
))/cos(x)
```

3.713.6 Sympy [F]

$$\int \sec^2(x)\sqrt{1-\tan^2(x)}\,dx = \int \sqrt{-(\tan(x)-1)(\tan(x)+1)}\sec^2(x)\,dx$$

input
```
integrate(sec(x)**2*(1-tan(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(-(tan(x) - 1)*(tan(x) + 1))*sec(x)**2, x)
```

3.713.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \sec^2(x)\sqrt{1-\tan^2(x)}\,dx = \frac{1}{2}\sqrt{-\tan(x)^2+1}\tan(x) + \frac{1}{2}\arcsin(\tan(x))$$

input
```
integrate(sec(x)^2*(1-tan(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(-tan(x)^2 + 1)*tan(x) + 1/2*arcsin(tan(x))
```

3.713.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77

$$\int \sec^2(x)\sqrt{1-\tan^2(x)}\,dx = \frac{1}{2}\sqrt{-\tan(x)^2+1}\tan(x) + \frac{1}{2}\arcsin(\tan(x))$$

input
```
integrate(sec(x)^2*(1-tan(x)^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(-tan(x)^2 + 1)*tan(x) + 1/2*arcsin(tan(x))
```

3.713.9 Mupad [F(-1)]

Timed out.

$$\int \sec^2(x)\sqrt{1-\tan^2(x)}\,dx = \int \frac{\sqrt{1-\tan(x)^2}}{\cos(x)^2}\,dx$$

input
```
int((1 - tan(x)^2)^(1/2)/cos(x)^2,x)
```

output
```
int((1 - tan(x)^2)^(1/2)/cos(x)^2, x)
```

3.714 $\int e^{\tan(x)} \sec^2(x) \, dx$

3.714.1 Optimal result

Integrand size = 9, antiderivative size = 4

$$\int e^{\tan(x)} \sec^2(x) \, dx = e^{\tan(x)}$$

output
```
exp(tan(x))
```

3.714.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int e^{\tan(x)} \sec^2(x) \, dx = e^{\tan(x)}$$

input
```
Integrate[E^Tan[x]*Sec[x]^2,x]
```

output
```
E^Tan[x]
```

3.714.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4842, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\tan(x)}\sec^2(x)\,dx$$

$$\Big\downarrow \text{4842}$$

$$\int e^{\tan(x)}d\tan(x)$$

$$\Big\downarrow \text{2624}$$

$$e^{\tan(x)}$$

input
```
Int[E^Tan[x]*Sec[x]^2,x]
```

output
```
E^Tan[x]
```

3.714.3.1 Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 4842
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Tan[c*(a +
 b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] |
| EqQ[F, sec])
```

3.714.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$e^{\tan(x)}$	4
default	$e^{\tan(x)}$	4
risch	$e^{\frac{\sin(x)}{\cos(x)}}$	9

input
```
int(exp(tan(x))*sec(x)^2,x,method=_RETURNVERBOSE)
```

output
```
exp(tan(x))
```

3.714.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8 vs. 2(3) = 6.

Time = 0.24 (sec) , antiderivative size = 8, normalized size of antiderivative = 2.00

$$\int e^{\tan(x)} \sec^2(x)\,dx = e^{\frac{\sin(x)}{\cos(x)}}$$

input
```
integrate(exp(tan(x))*sec(x)^2,x, algorithm="fricas")
```

output
```
e^(sin(x)/cos(x))
```

3.714.6 Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int e^{\tan(x)} \sec^2(x)\,dx = e^{\tan(x)}$$

input
```
integrate(exp(tan(x))*sec(x)**2,x)
```

output
```
exp(tan(x))
```

3.714.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int e^{\tan(x)} \sec^2(x)\, dx = e^{\tan(x)}$$

input
```
integrate(exp(tan(x))*sec(x)^2,x, algorithm="maxima")
```

output
```
e^tan(x)
```

3.714.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int e^{\tan(x)} \sec^2(x)\, dx = e^{\tan(x)}$$

input
```
integrate(exp(tan(x))*sec(x)^2,x, algorithm="giac")
```

output
```
e^tan(x)
```

3.714.9 Mupad [B] (verification not implemented)

Time = 26.56 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int e^{\tan(x)} \sec^2(x)\, dx = \mathrm{e}^{\tan(x)}$$

input
```
int(exp(tan(x))/cos(x)^2,x)
```

output
```
exp(tan(x))
```

3.715 $\int \sec^4(x) \left(-1+\sec^2(x)\right)^2 \tan(x) \, dx$

3.715.1 Optimal result

Integrand size = 15, antiderivative size = 17

$$\int \sec^4(x) \left(-1+\sec^2(x)\right)^2 \tan(x) \, dx = \frac{\tan^6(x)}{6}+\frac{\tan^8(x)}{8}$$

output
```
1/6*tan(x)^6+1/8*tan(x)^8
```

3.715.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.47

$$\int \sec^4(x) \left(-1+\sec^2(x)\right)^2 \tan(x) \, dx = \frac{\sec^4(x)}{4}-\frac{\sec^6(x)}{3}+\frac{\sec^8(x)}{8}$$

input
```
Integrate[Sec[x]^4*(-1 + Sec[x]^2)^2*Tan[x],x]
```

output
```
Sec[x]^4/4 - Sec[x]^6/3 + Sec[x]^8/8
```

3.715.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 4608, 3042, 3087, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(x)\sec^4(x)\left(\sec^2(x)-1\right)^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(x)\sec(x)^4\left(\sec(x)^2-1\right)^2dx$$

$$\Big\downarrow \text{4608}$$

$$\int \tan^5(x)\sec^4(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(x)^5\sec(x)^4dx$$

$$\Big\downarrow \text{3087}$$

$$\int \tan^5(x)\left(\tan^2(x)+1\right)d\tan(x)$$

$$\Big\downarrow \text{244}$$

$$\int \left(\tan^7(x)+\tan^5(x)\right)d\tan(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{\tan^8(x)}{8}+\frac{\tan^6(x)}{6}$$

input
```
Int[Sec[x]^4*(-1 + Sec[x]^2)^2*Tan[x],x]
```

output
```
Tan[x]^6/6 + Tan[x]^8/8
```

3.715.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

rule 4608
```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
b^p   Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.715.4 Maple [A] (verified)

Time = 19.94 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{\tan(x)^6}{6}+\frac{\tan(x)^8}{8}$	14
default	$\frac{\tan(x)^6}{6}+\frac{\tan(x)^8}{8}$	14
parts	$\frac{\sec(x)^8}{8}+\frac{\sec(x)^4}{4}-\frac{\sec(x)^6}{3}$	20
risch	$\frac{4\,e^{12ix}-\frac{16\,e^{10ix}}{3}+\frac{40\,e^{8ix}}{3}-\frac{16\,e^{6ix}}{3}+4\,e^{4ix}}{\left(e^{2ix}+1\right)^8}$	48

input
```
int(sec(x)^4*(-1+sec(x)^2)^2*tan(x),x,method=_RETURNVERBOSE)
```

output
```
1/6*tan(x)^6+1/8*tan(x)^8
```

3.715.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \sec^4(x)\left(-1+\sec^2(x)\right)^2\tan(x)\,dx=\frac{6\cos(x)^4-8\cos(x)^2+3}{24\cos(x)^8}$$

input
```
integrate(sec(x)^4*(-1+sec(x)^2)^2*tan(x),x, algorithm="fricas")
```

output
```
1/24*(6*cos(x)^4 - 8*cos(x)^2 + 3)/cos(x)^8
```

3.715.6 Sympy [A] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \sec^4(x)\left(-1+\sec^2(x)\right)^2\tan(x)\,dx=\frac{\sec^8(x)}{8}-\frac{\sec^6(x)}{3}+\frac{\sec^4(x)}{4}$$

input
```
integrate(sec(x)**4*(-1+sec(x)**2)**2*tan(x),x)
```

output
```
sec(x)**8/8 - sec(x)**6/3 + sec(x)**4/4
```

3.715.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(13) = 26.

Time = 0.22 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.47

$$\int \sec^4(x)\left(-1+\sec^2(x)\right)^2\tan(x)\,dx=\frac{6\sin(x)^4-4\sin(x)^2+1}{24\left(\sin(x)^8-4\sin(x)^6+6\sin(x)^4-4\sin(x)^2+1\right)}$$

input
```
integrate(sec(x)^4*(-1+sec(x)^2)^2*tan(x),x, algorithm="maxima")
```

output
```
1/24*(6*sin(x)^4 - 4*sin(x)^2 + 1)/(sin(x)^8 - 4*sin(x)^6 + 6*sin(x)^4 - 4
*sin(x)^2 + 1)
```

3.715.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.18

$$\int \sec^4(x)\left(-1+\sec^2(x)\right)^2\tan(x)\,dx = \frac{6\cos(x)^4-8\cos(x)^2+3}{24\cos(x)^8}$$

input
```
integrate(sec(x)^4*(-1+sec(x)^2)^2*tan(x),x, algorithm="giac")
```

output
```
1/24*(6*cos(x)^4 - 8*cos(x)^2 + 3)/cos(x)^8
```

3.715.9 Mupad [B] (verification not implemented)

Time = 26.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \sec^4(x)\left(-1+\sec^2(x)\right)^2\tan(x)\,dx = \frac{\tan(x)^6\left(3\tan(x)^2+4\right)}{24}$$

input
```
int((tan(x)*(1/cos(x)^2 - 1)^2)/cos(x)^4,x)
```

output
```
(tan(x)^6*(3*tan(x)^2 + 4))/24
```

3.716 $\int \frac{\csc^2(x)}{a+b\cot(x)}\,dx$

3.716.1 Optimal result

Integrand size = 13, antiderivative size = 12

$$\int \frac{\csc^2(x)}{a+b\cot(x)}\,dx=-\frac{\log(a+b\cot(x))}{b}$$

output
```
-ln(a+b*cot(x))/b
```

3.716.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \frac{\csc^2(x)}{a+b\cot(x)}\,dx=\frac{\log(\sin(x))-\log(b\cos(x)+a\sin(x))}{b}$$

input
```
Integrate[Csc[x]^2/(a + b*Cot[x]),x]
```

output
```
(Log[Sin[x]] - Log[b*Cos[x] + a*Sin[x]])/b
```

3.716.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3042, 3987, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(x)}{a+b\cot(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec\left(x-\frac{\pi}{2}\right)^2}{a-b\tan\left(x-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3987}$$

$$-\frac{\int \frac{1}{a+b\cot(x)}d(b\cot(x))}{b}$$

$$\Big\downarrow \text{16}$$

$$-\frac{\log(a+b\cot(x))}{b}$$

input
```
Int[Csc[x]^2/(a + b*Cot[x]),x]
```

output
```
-(Log[a + b*Cot[x]]/b)
```

3.716.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + 
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

3.716.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$-\frac{\ln(a+b\cot(x))}{b}$	13
default	$-\frac{\ln(a+b\cot(x))}{b}$	13
risch	$-\frac{\ln\left(\mathrm{e}^{2ix}+\frac{ib-a}{ib+a}\right)}{b}+\frac{\ln(\mathrm{e}^{2ix}-1)}{b}$	43

input
```
int(csc(x)^2/(a+b*cot(x)),x,method=_RETURNVERBOSE)
```

output
```
-ln(a+b*cot(x))/b
```

3.716.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(12) = 24.

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 3.75

$$\int \frac{\csc^2(x)}{a+b\cot(x)}\,dx = -\frac{\log\left(2\,ab\cos\left(x\right)\sin\left(x\right)-\left(a^2-b^2\right)\cos\left(x\right)^2+a^2\right)-\log\left(-\frac{1}{4}\,\cos\left(x\right)^2+\frac{1}{4}\right)}{2\,b}$$

input
```
integrate(csc(x)^2/(a+b*cot(x)),x, algorithm="fricas")
```

output
```
-1/2*(log(2*a*b*cos(x)*sin(x) - (a^2 - b^2)*cos(x)^2 + a^2) - log(-1/4*cos
(x)^2 + 1/4))/b
```

3.716.6 Sympy [F]

$$\int \frac{\csc^2(x)}{a+b\cot(x)}\,dx = \int \frac{\csc^2{(x)}}{a+b\cot{(x)}}\,dx$$

input
```
integrate(csc(x)**2/(a+b*cot(x)),x)
```

output
```
Integral(csc(x)**2/(a + b*cot(x)), x)
```

3.716.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\csc^2(x)}{a+b\cot(x)}\,dx = -\frac{\log{(b\cot{(x)}+a)}}{b}$$

input
```
integrate(csc(x)^2/(a+b*cot(x)),x, algorithm="maxima")
```

output
```
-log(b*cot(x) + a)/b
```

3.716.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\csc^2(x)}{a+b\cot(x)}\,dx = -\frac{\log{(|a\tan{(x)}+b|)}}{b} + \frac{\log{(|\tan{(x)}|)}}{b}$$

input
```
integrate(csc(x)^2/(a+b*cot(x)),x, algorithm="giac")
```

output
```
-log(abs(a*tan(x) + b))/b + log(abs(tan(x)))/b
```

3.716.9 Mupad [B] (verification not implemented)

Time = 26.57 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \frac{\csc^2(x)}{a+b\cot(x)}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{2\,a\tan(x)}{b}+1\right)}{b}$$

input
```
int(1/(sin(x)^2*(a + b*cot(x))),x)
```

output
```
-(2*atanh((2*a*tan(x))/b + 1))/b
```

3.717 $\int (a+b\cot(x))^n \csc^2(x)\,dx$

3.717.1 Optimal result

Integrand size = 13, antiderivative size = 20

$$\int (a+b\cot(x))^n \csc^2(x)\,dx=-\frac{(a+b\cot(x))^{1+n}}{b(1+n)}$$

output
```
-(a+b*cot(x))^(1+n)/b/(1+n)
```

3.717.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (a+b\cot(x))^n \csc^2(x)\,dx=-\frac{(a+b\cot(x))^{1+n}}{b(1+n)}$$

input
```
Integrate[(a + b*Cot[x])^n*Csc[x]^2,x]
```

output
```
-((a + b*Cot[x])^(1 + n)/(b*(1 + n)))
```

3.717.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3042, 3987, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(x)(a+b\cot(x))^n\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sec\left(x-\frac{\pi}{2}\right)^2\left(a-b\tan\left(x-\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \ 3987$$

$$-\frac{\int (a+b\cot(x))^n d(b\cot(x))}{b}$$

$$\Big\downarrow \ 17$$

$$-\frac{(a+b\cot(x))^{n+1}}{b(n+1)}$$

input
```
Int[(a + b*Cot[x])^n*Csc[x]^2,x]
```

output
```
-((a + b*Cot[x])^(1 + n)/(b*(1 + n)))
```

3.717.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3987
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_
), x_Symbol] :> Simp[1/(b*f)   Subst[Int[(a + x)^n*(1 + x^2/b^2)^(m/2 - 1),
 x], x, b*Tan[e + f*x]], x] /; FreeQ[{a, b, e, f, n}, x] && NeQ[a^2 + b^2,
0] && IntegerQ[m/2]
```

3.717.4 Maple [A] (verified)

Time = 9.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$-\frac{(a+b\cot(x))^{n+1}}{b(n+1)}$	21
default	$-\frac{(a+b\cot(x))^{n+1}}{b(n+1)}$	21

input
```
int((a+b*cot(x))^n*csc(x)^2,x,method=_RETURNVERBOSE)
```

output
```
-(a+b*cot(x))^(n+1)/b/(n+1)
```

3.717.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.90

$$\int (a+b\cot(x))^n \csc^2(x)\,dx = -\frac{(b\cos(x)+a\sin(x))\left(\frac{b\cos(x)+a\sin(x)}{\sin(x)}\right)^n}{(bn+b)\sin(x)}$$

input
```
integrate((a+b*cot(x))^n*csc(x)^2,x, algorithm="fricas")
```

output
```
-(b*cos(x) + a*sin(x))*((b*cos(x) + a*sin(x))/sin(x))^n/((b*n + b)*sin(x))
```

3.717.6 Sympy [F]

$$\int (a+b\cot(x))^n \csc^2(x)\,dx = \int (a+b\cot(x))^n \csc^2(x)\,dx$$

input
```
integrate((a+b*cot(x))**n*csc(x)**2,x)
```

output
```
Integral((a + b*cot(x))**n*csc(x)**2, x)
```

3.717.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (a+b\cot(x))^n\csc^2(x)\,dx = -\frac{(b\cot(x)+a)^{n+1}}{b(n+1)}$$

input
```
integrate((a+b*cot(x))^n*csc(x)^2,x, algorithm="maxima")
```

output
```
-(b*cot(x) + a)^(n + 1)/(b*(n + 1))
```

3.717.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(20) = 40.

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.05

$$\int (a+b\cot(x))^n\csc^2(x)\,dx = -\frac{\left(-\frac{b\tan\left(\frac{1}{2}x\right)^2-2a\tan\left(\frac{1}{2}x\right)-b}{2\tan\left(\frac{1}{2}x\right)}\right)^{n+1}}{b(n+1)}$$

input
```
integrate((a+b*cot(x))^n*csc(x)^2,x, algorithm="giac")
```

output
```
-(-1/2*(b*tan(1/2*x)^2 - 2*a*tan(1/2*x) - b)/tan(1/2*x))^(n + 1)/(b*(n + 1
))
```

3.717.9 Mupad [B] (verification not implemented)

Time = 26.63 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.15

$$\int (a+b\cot(x))^n\csc^2(x)\,dx = \begin{cases} -\frac{\ln\left(a+\frac{b}{\tan(x)}\right)}{b} & \text{if } n=-1 \\ -\frac{\left(a+\frac{b}{\tan(x)}\right)^{n+1}}{b\,(n+1)} & \text{if } n\neq -1 \end{cases}$$

input
```
int((a + b*cot(x))^n/sin(x)^2,x)
```

output
```
piecewise(n == -1, -log(a + b/tan(x))/b, n ~= -1, -(a + b/tan(x))^(n + 1)/
(b*(n + 1)))
```

3.718 $\int \csc^2(x) \left(1+\sin^2(x)\right) \, dx$

3.718.1 Optimal result

Integrand size = 11, antiderivative size = 6

$$\int \csc^2(x) \left(1+\sin^2(x)\right) \, dx = x-\cot(x)$$

output
```
x-cot(x)
```

3.718.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \csc^2(x) \left(1+\sin^2(x)\right) \, dx = x-\cot(x)$$

input
```
Integrate[Csc[x]^2*(1 + Sin[x]^2),x]
```

output
```
x - Cot[x]
```

3.718.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 3491, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\sin^2(x)+1\right)\csc^2(x)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sin(x)^2+1}{\sin(x)^2}dx$$

$$\Big\downarrow \ 3491$$

$$\int 1dx-\cot(x)$$

$$\Big\downarrow \ 24$$

$$x-\cot(x)$$

input
```
Int[Csc[x]^2*(1 + Sin[x]^2),x]
```

output
```
x - Cot[x]
```

3.718.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3491
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[A*Cos[e + f*x]*((b*Sin[e + f*x])^(m + 1)/(b*f*(m
+ 1))), x] + Simp[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1))   Int[(b*Sin[e + f*
x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

3.718.4 Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

method	result	size
default	$x-\cot\left(x\right)$	7
parallelrisch	$x-\cot\left(x\right)$	7
risch	$x-\frac{2i}{{\mathrm e}^{2ix}-1}$	15
norman	$\frac{-\frac{1}{2}+x\tan\left(\frac{x}{2}\right)+x\tan\left(\frac{x}{2}\right)^{5}-\frac{\tan\left(\frac{x}{2}\right)^{2}}{2}+\frac{\tan\left(\frac{x}{2}\right)^{4}}{2}+\frac{\tan\left(\frac{x}{2}\right)^{6}}{2}+2x\tan\left(\frac{x}{2}\right)^{3}}{\tan\left(\frac{x}{2}\right)\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)^{2}}$	67

input
```
int(csc(x)^2*(sin(x)^2+1),x,method=_RETURNVERBOSE)
```

output
```
x-cot(x)
```

3.718.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14 vs. 2(6) = 12.

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 2.33

$$\int \csc^2(x)\left(1+\sin^2(x)\right)\,dx=\frac{x\sin\left(x\right)-\cos\left(x\right)}{\sin\left(x\right)}$$

input
```
integrate(csc(x)^2*(1+sin(x)^2),x, algorithm="fricas")
```

output
```
(x*sin(x) - cos(x))/sin(x)
```

3.718.6 Sympy [A] (verification not implemented)

Time = 2.47 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.50

$$\int \csc^2(x)\left(1+\sin^2(x)\right)\,dx=x-\cot\left(x\right)$$

input
```
integrate(csc(x)**2*(1+sin(x)**2),x)
```

output
```
x - cot(x)
```

3.718.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.33

$$\int \csc^2(x)\left(1+\sin^2(x)\right)\,dx = x - \frac{1}{\tan(x)}$$

input
```
integrate(csc(x)^2*(1+sin(x)^2),x, algorithm="maxima")
```

output
```
x - 1/tan(x)
```

3.718.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. 2(6) = 12.

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.67

$$\int \csc^2(x)\left(1+\sin^2(x)\right)\,dx = x - \frac{1}{2\tan\left(\frac{1}{2}x\right)} + \frac{1}{2}\tan\left(\frac{1}{2}x\right)$$

input
```
integrate(csc(x)^2*(1+sin(x)^2),x, algorithm="giac")
```

output
```
x - 1/2/tan(1/2*x) + 1/2*tan(1/2*x)
```

3.718.9 Mupad [B] (verification not implemented)

Time = 26.32 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \csc^2(x)\left(1+\sin^2(x)\right)\,dx = x - \cot(x)$$

input
```
int((sin(x)^2 + 1)/sin(x)^2,x)
```

output
```
x - cot(x)
```

3.719 $\int \left(1+\frac{1}{1+\cot^2(x)}\right) \csc^2(x)\,dx$

3.719.1 Optimal result

Integrand size = 15, antiderivative size = 6

$$\int \left(1+\frac{1}{1+\cot^2(x)}\right) \csc^2(x)\,dx = x-\cot(x)$$

output
```
x-cot(x)
```

3.719.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 3.67

$$\int \left(1+\frac{1}{1+\cot^2(x)}\right) \csc^2(x)\,dx = 2x-\cot(x)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(x)\right)$$

input
```
Integrate[(1 + (1 + Cot[x]^2)^(-1))*Csc[x]^2,x]
```

output
```
2*x - Cot[x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[x]^2]
```

3.719.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.33, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 4889, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{1}{\cot^2(x)+1}+1\right)\csc^2(x)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \left(\frac{1}{\cot(x)^2+1}+1\right)\csc(x)^2dx$$
$$\Big\downarrow \text{4889}$$
$$\int \cot^2(x)\left(\frac{1}{\cot^2(x)+1}+1\right)d\tan(x)$$
$$\Big\downarrow \text{2010}$$
$$\int \left(\frac{1}{\tan^2(x)+1}+\cot^2(x)\right)d\tan(x)$$
$$\Big\downarrow \text{2009}$$
$$\arctan(\tan(x))-\cot(x)$$

input
```
Int[(1 + (1 + Cot[x]^2)^(-1))*Csc[x]^2,x]
```

output
```
ArcTan[Tan[x]] - Cot[x]
```

3.719.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.719.4 Maple [A] (verified)

Time = 3.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

method	result	size
default	$x-\cot\left(x\right)$	7
risch	$x-\frac{2i}{{\mathrm e}^{2ix}-1}$	15

input
```
int((1+1/(1+cot(x)^2))*csc(x)^2,x,method=_RETURNVERBOSE)
```

output
```
x-cot(x)
```

3.719.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14 vs. 2(6) = 12.

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 2.33

$$\int \left(1+\frac{1}{1+\cot^2(x)}\right)\csc^2(x)\,dx=\frac{x\sin\left(x\right)-\cos\left(x\right)}{\sin\left(x\right)}$$

input
```
integrate((1+1/(1+cot(x)^2))*csc(x)^2,x, algorithm="fricas")
```

output
```
(x*sin(x) - cos(x))/sin(x)
```

3.719.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(3) = 6$.

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 4.50

$$\int \left(1+\frac{1}{1+\cot^2(x)}\right)\csc^2(x)\,dx = \frac{x\csc^2(x)}{\cot^2(x)+1} - \frac{\cot(x)\csc^2(x)}{\cot^2(x)+1}$$

input
```
integrate((1+1/(1+cot(x)**2))*csc(x)**2,x)
```

output
```
x*csc(x)**2/(cot(x)**2 + 1) - cot(x)*csc(x)**2/(cot(x)**2 + 1)
```

3.719.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.33

$$\int \left(1+\frac{1}{1+\cot^2(x)}\right)\csc^2(x)\,dx = x - \frac{1}{\tan(x)}$$

input
```
integrate((1+1/(1+cot(x)^2))*csc(x)^2,x, algorithm="maxima")
```

output
```
x - 1/tan(x)
```

3.719.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. $2(6) = 12$.

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.67

$$\int \left(1+\frac{1}{1+\cot^2(x)}\right)\csc^2(x)\,dx = x - \frac{1}{2\tan\left(\frac{1}{2}x\right)} + \frac{1}{2}\tan\left(\frac{1}{2}x\right)$$

input
```
integrate((1+1/(1+cot(x)^2))*csc(x)^2,x, algorithm="giac")
```

output
```
x - 1/2/tan(1/2*x) + 1/2*tan(1/2*x)
```

3.719.9 Mupad [B] (verification not implemented)

Time = 26.24 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \left(1+\frac{1}{1+\cot^2(x)}\right)\csc^2(x)\,dx=x-\cot(x)$$

input
```
int((1/(cot(x)^2 + 1) + 1)/sin(x)^2,x)
```

output
```
x - cot(x)
```

3.720 $\int \frac{(a+b\cot(x))\csc^2(x)}{c+d\cot(x)}\,dx$

3.720.1 Optimal result

Integrand size = 19, antiderivative size = 28

$$\int \frac{(a+b\cot(x))\csc^2(x)}{c+d\cot(x)}\,dx=-\frac{b\cot(x)}{d}+\frac{(bc-ad)\log(c+d\cot(x))}{d^2}$$

output
```
-b*cot(x)/d+(-a*d+b*c)*ln(c+d*cot(x))/d^2
```

3.720.2 Mathematica [A] (verified)

Time = 1.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.00

$$\int \frac{(a+b\cot(x))\csc^2(x)}{c+d\cot(x)}\,dx = \frac{(a+b\cot(x))(-bd\cot(x)-(bc-ad)(\log(\sin(x))-\log(d\cos(x)+c\sin(x))))\sin(x)}{d^2(b\cos(x)+a\sin(x))}$$

input
```
Integrate[((a + b*Cot[x])*Csc[x]^2)/(c + d*Cot[x]),x]
```

output
```
((a + b*Cot[x])*(-(b*d*Cot[x]) - (b*c - a*d)*(Log[Sin[x]] - Log[d*Cos[x] +
 c*Sin[x]]))*Sin[x])/(d^2*(b*Cos[x] + a*Sin[x]))
```

3.720.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 4844, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(x)(a+b\cot(x))}{c+d\cot(x)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\csc(x)^2(a+b\cot(x))}{c+d\cot(x)}dx$$
$$\Big\downarrow \; 4844$$
$$-\int \frac{a+b\cot(x)}{c+d\cot(x)}d\cot(x)$$
$$\Big\downarrow \; 49$$
$$-\int \left(\frac{b}{d}+\frac{ad-bc}{d(c+d\cot(x))}\right)d\cot(x)$$
$$\Big\downarrow \; 2009$$
$$\frac{(bc-ad)\log(c+d\cot(x))}{d^2}-\frac{b\cot(x)}{d}$$

input
```
Int[((a + b*Cot[x])*Csc[x]^2)/(c + d*Cot[x]),x]
```

output
```
-((b*Cot[x])/d) + ((b*c - a*d)*Log[c + d*Cot[x]])/d^2
```

3.720.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4844
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Cot[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[1, Cot[c*(a
+ b*x)]/d, u, x], x], x, Cot[c*(a + b*x)]/d], x] /; FunctionOfQ[Cot[c*(a +
b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Csc]
|| EqQ[F, csc])
```

3.720.4 Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.68

method	result	size
default	$-\frac{(ad-cb)\ln(c\tan(x)+d)}{d^2}-\frac{b}{d\tan(x)}+\frac{(ad-cb)\ln(\tan(x))}{d^2}$	47
risch	$-\frac{2ib}{d(\mathrm{e}^{2ix}-1)}+\frac{\ln(\mathrm{e}^{2ix}-1)a}{d}-\frac{\ln(\mathrm{e}^{2ix}-1)cb}{d^2}-\frac{\ln\left(\mathrm{e}^{2ix}+\frac{id-c}{id+c}\right)a}{d}+\frac{\ln\left(\mathrm{e}^{2ix}+\frac{id-c}{id+c}\right)cb}{d^2}$	106

input
```
int((a+b*cot(x))*csc(x)^2/(c+d*cot(x)),x,method=_RETURNVERBOSE)
```

output
```
-(a*d-b*c)/d^2*ln(c*tan(x)+d)-b/d/tan(x)+(a*d-b*c)/d^2*ln(tan(x))
```

3.720.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(28) = 56.

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.71

$$\int \frac{(a+b\cot(x))\csc^2(x)}{c+d\cot(x)}\,dx =$$
$$-\frac{2\,bd\cos{(x)}-(bc-ad)\log{\left(2\,cd\cos{(x)}\sin{(x)}-(c^2-d^2)\cos{(x)}^2+c^2\right)}\sin{(x)}+(bc-ad)\log{\left(-\frac{1}{4}\cos\right.}}{2\,d^2\sin{(x)}}$$

input
```
integrate((a+b*cot(x))*csc(x)^2/(c+d*cot(x)),x, algorithm="fricas")
```

output
```
-1/2*(2*b*d*cos(x) - (b*c - a*d)*log(2*c*d*cos(x)*sin(x) - (c^2 - d^2)*cos
(x)^2 + c^2)*sin(x) + (b*c - a*d)*log(-1/4*cos(x)^2 + 1/4)*sin(x))/(d^2*si
n(x))
```

3.720.6 Sympy [A] (verification not implemented)

Time = 9.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int \frac{(a+b\cot(x))\csc^2(x)}{c+d\cot(x)}\,dx = -\frac{b\cot{(x)}}{d} - \frac{(ad-bc)\left(\begin{cases}\frac{\cot{(x)}}{c} & \text{for}\ d=0\\ \frac{\log{(c+d\cot{(x)})}}{d} & \text{otherwise}\end{cases}\right)}{d}$$

input
```
integrate((a+b*cot(x))*csc(x)**2/(c+d*cot(x)),x)
```

output
```
-b*cot(x)/d - (a*d - b*c)*Piecewise((cot(x)/c, Eq(d, 0)), (log(c + d*cot(x
))/d, True))/d
```

3.720.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.64

$$\int \frac{(a+b\cot(x))\csc^2(x)}{c+d\cot(x)}\,dx = \frac{(bc-ad)\log{(c\tan{(x)}+d)}}{d^2} - \frac{(bc-ad)\log{(\tan{(x)})}}{d^2} - \frac{b}{d\tan{(x)}}$$

input
```
integrate((a+b*cot(x))*csc(x)^2/(c+d*cot(x)),x, algorithm="maxima")
```

output
```
(b*c - a*d)*log(c*tan(x) + d)/d^2 - (b*c - a*d)*log(tan(x))/d^2 - b/(d*tan
(x))
```

3.720.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(28) = 56$.

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.43

$$\int \frac{(a+b\cot(x))\csc^2(x)}{c+d\cot(x)}\,dx = -\frac{(bc-ad)\log(|\tan(x)|)}{d^2} + \frac{(bc^2-acd)\log(|c\tan(x)+d|)}{cd^2} + \frac{bc\tan(x)-ad\tan(x)-bd}{d^2\tan(x)}$$

input
```
integrate((a+b*cot(x))*csc(x)^2/(c+d*cot(x)),x, algorithm="giac")
```

output
```
-(b*c - a*d)*log(abs(tan(x)))/d^2 + (b*c^2 - a*c*d)*log(abs(c*tan(x) + d))
/(c*d^2) + (b*c*tan(x) - a*d*tan(x) - b*d)/(d^2*tan(x))
```

3.720.9 Mupad [B] (verification not implemented)

Time = 26.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.25

$$\int \frac{(a+b\cot(x))\csc^2(x)}{c+d\cot(x)}\,dx = -\frac{b}{d\tan(x)} - \frac{2\,\mathrm{atanh}\left(\frac{2\,c\tan(x)}{d}+1\right)\,(a\,d-b\,c)}{d^2}$$

input
```
int((a + b*cot(x))/(sin(x)^2*(c + d*cot(x))),x)
```

output
```
- b/(d*tan(x)) - (2*atanh((2*c*tan(x))/d + 1)*(a*d - b*c))/d^2
```

3.721 $\int \frac{(a+b\cot(x))^2\csc^2(x)}{c+d\cot(x)}\,dx$

3.721.1 Optimal result

Integrand size = 21, antiderivative size = 53

$$\int \frac{(a+b\cot(x))^2\csc^2(x)}{c+d\cot(x)}\,dx = \frac{b(bc-ad)\cot(x)}{d^2} - \frac{(a+b\cot(x))^2}{2d} - \frac{(bc-ad)^2\log(c+d\cot(x))}{d^3}$$

output
```
b*(-a*d+b*c)*cot(x)/d^2-1/2*(a+b*cot(x))^2/d-(-a*d+b*c)^2*ln(c+d*cot(x))/d
^3
```

3.721.2 Mathematica [A] (verified)

Time = 2.17 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.17

$$\int \frac{(a+b\cot(x))^2\csc^2(x)}{c+d\cot(x)}\,dx = \frac{2bd(bc-2ad)\cot(x) - b^2d^2\csc^2(x) + 2(bc-ad)^2(\log(\sin(x)) - \log(d\cos(x)+c\sin(x)))}{2d^3}$$

input
```
Integrate[((a + b*Cot[x])^2*Csc[x]^2)/(c + d*Cot[x]),x]
```

output
```
(2*b*d*(b*c - 2*a*d)*Cot[x] - b^2*d^2*Csc[x]^2 + 2*(b*c - a*d)^2*(Log[Sin[
x]] - Log[d*Cos[x] + c*Sin[x]]))/(2*d^3)
```

3.721.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 4844, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(x)(a+b\cot(x))^2}{c+d\cot(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc(x)^2(a+b\cot(x))^2}{c+d\cot(x)}dx$$

$$\Big\downarrow \text{4844}$$

$$-\int \frac{(a+b\cot(x))^2}{c+d\cot(x)}d\cot(x)$$

$$\Big\downarrow \text{49}$$

$$-\int \left(\frac{(ad-bc)^2}{d^2(c+d\cot(x))}-\frac{b(bc-ad)}{d^2}+\frac{b(a+b\cot(x))}{d}\right)d\cot(x)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(bc-ad)^2\log(c+d\cot(x))}{d^3}+\frac{b\cot(x)(bc-ad)}{d^2}-\frac{(a+b\cot(x))^2}{2d}$$

input
```
Int[((a + b*Cot[x])^2*Csc[x]^2)/(c + d*Cot[x]),x]
```

output
```
(b*(b*c - a*d)*Cot[x])/d^2 - (a + b*Cot[x])^2/(2*d) - ((b*c - a*d)^2*Log[c
 + d*Cot[x]])/d^3
```

3.721.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4844
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Cot[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[1, Cot[c*(a
+ b*x)]/d, u, x], x], x, Cot[c*(a + b*x)]/d], x] /; FunctionOfQ[Cot[c*(a +
b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Csc]
|| EqQ[F, csc])
```

3.721.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.17

method	result
derivativedivides	$-\frac{b\left(\frac{b\cot(x)^2 d}{2}+2\cot(x)ad-\cot(x)cb\right)}{d^2}-\frac{(d^2a^2-2cdab+b^2c^2)\ln(c+d\cot(x))}{d^3}$
default	$-\frac{b\left(\frac{b\cot(x)^2 d}{2}+2\cot(x)ad-\cot(x)cb\right)}{d^2}-\frac{(d^2a^2-2cdab+b^2c^2)\ln(c+d\cot(x))}{d^3}$
risch	$\frac{2ib(-2ad\,e^{2ix}+bc\,e^{2ix}-ibd\,e^{2ix}+2ad-cb)}{(e^{2ix}-1)^2d^2}+\frac{\ln(e^{2ix}-1)a^2}{d}-\frac{2\ln(e^{2ix}-1)cab}{d^2}+\frac{\ln(e^{2ix}-1)b^2c^2}{d^3}-\frac{\ln\left(e^{2ix}+\frac{id-}{id+}\right.}{d}$

input
```
int((a+b*cot(x))^2*csc(x)^2/(c+d*cot(x)),x,method=_RETURNVERBOSE)
```

output
```
-b/d^2*(1/2*b*cot(x)^2*d+2*cot(x)*a*d-cot(x)*c*b)-(a^2*d^2-2*a*b*c*d+b^2*c
^2)/d^3*ln(c+d*cot(x))
```

3.721.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(51) = 102.

Time = 0.28 (sec) , antiderivative size = 182, normalized size of antiderivative = 3.43

$$\int \frac{(a+b\cot(x))^2\csc^2(x)}{c+d\cot(x)}\,dx$$
$$=\frac{b^2d^2-2\left(b^2cd-2\,abd^2\right)\cos(x)\sin(x)+\left(b^2c^2-2\,abcd+a^2d^2-\left(b^2c^2-2\,abcd+a^2d^2\right)\cos(x)^2\right)\log\left(2\,cd\right.}{2}$$

input
```
integrate((a+b*cot(x))^2*csc(x)^2/(c+d*cot(x)),x, algorithm="fricas")
```

output
```
1/2*(b^2*d^2 - 2*(b^2*c*d - 2*a*b*d^2)*cos(x)*sin(x) + (b^2*c^2 - 2*a*b*c*
d + a^2*d^2 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cos(x)^2)*log(2*c*d*cos(x)*s
in(x) - (c^2 - d^2)*cos(x)^2 + c^2) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2 - (b^
2*c^2 - 2*a*b*c*d + a^2*d^2)*cos(x)^2)*log(-1/4*cos(x)^2 + 1/4))/(d^3*cos(
x)^2 - d^3)
```

3.721.6 Sympy [A] (verification not implemented)

Time = 36.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.09

$$\int \frac{(a+b\cot(x))^2\csc^2(x)}{c+d\cot(x)}\,dx = -\frac{b^2\cot^2(x)}{2d} - \frac{(ad-bc)^2\left(\begin{cases}\frac{\cot(x)}{c} & \text{for } d=0\\ \frac{\log(c+d\cot(x))}{d} & \text{otherwise}\end{cases}\right)}{d^2} - \frac{(2abd-b^2c)\cot(x)}{d^2}$$

input
```
integrate((a+b*cot(x))**2*csc(x)**2/(c+d*cot(x)),x)
```

output
```
-b**2*cot(x)**2/(2*d) - (a*d - b*c)**2*Piecewise((cot(x)/c, Eq(d, 0)), (lo
g(c + d*cot(x))/d, True))/d**2 - (2*a*b*d - b**2*c)*cot(x)/d**2
```

3.721.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.74

$$\int \frac{(a+b\cot(x))^2\csc^2(x)}{c+d\cot(x)}\,dx = -\frac{(b^2c^2-2\,abcd+a^2d^2)\log(c\tan(x)+d)}{d^3} + \frac{(b^2c^2-2\,abcd+a^2d^2)\log(\tan(x))}{d^3} - \frac{b^2d-2\,(b^2c-2\,abd)\tan(x)}{2\,d^2\tan(x)^2}$$

input
```
integrate((a+b*cot(x))^2*csc(x)^2/(c+d*cot(x)),x, algorithm="maxima")
```

output
```
-(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(c*tan(x) + d)/d^3 + (b^2*c^2 - 2*a*b*
c*d + a^2*d^2)*log(tan(x))/d^3 - 1/2*(b^2*d - 2*(b^2*c - 2*a*b*d)*tan(x))/
(d^2*tan(x)^2)
```

3.721.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(51) = 102.

Time = 0.31 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.62

$$\int \frac{(a+b\cot(x))^2\csc^2(x)}{c+d\cot(x)}\,dx = \frac{(b^2c^2-2\,abcd+a^2d^2)\log(|\tan(x)|)}{d^3} - \frac{(b^2c^3-2\,abc^2d+a^2cd^2)\log(|c\tan(x)+d|)}{cd^3} - \frac{3\,b^2c^2\tan(x)^2-6\,abcd\tan(x)^2+3\,a^2d^2\tan(x)^2-2\,b^2cd\tan(x)+4\,abd^2\tan(x)+b^2d^2}{2\,d^3\tan(x)^2}$$

input
```
integrate((a+b*cot(x))^2*csc(x)^2/(c+d*cot(x)),x, algorithm="giac")
```

output
```
(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(abs(tan(x)))/d^3 - (b^2*c^3 - 2*a*b*c^
2*d + a^2*c*d^2)*log(abs(c*tan(x) + d))/(c*d^3) - 1/2*(3*b^2*c^2*tan(x)^2
- 6*a*b*c*d*tan(x)^2 + 3*a^2*d^2*tan(x)^2 - 2*b^2*c*d*tan(x) + 4*a*b*d^2*t
an(x) + b^2*d^2)/(d^3*tan(x)^2)
```

3.721.9 Mupad [B] (verification not implemented)

Time = 27.29 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.74

$$\int \frac{(a+b\cot(x))^2\csc^2(x)}{c+d\cot(x)}\,dx = -\frac{\frac{b^2}{2\,d}+\frac{b\tan(x)\,(2\,a\,d-b\,c)}{d^2}}{\tan(x)^2} - \frac{2\,\mathrm{atanh}\left(\frac{(d+2\,c\tan(x))\,(a\,d-b\,c)^2}{d\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}\right)\,(a\,d-b\,c)^2}{d^3}$$

input
```
int((a + b*cot(x))^2/(sin(x)^2*(c + d*cot(x))),x)
```

output
```
- (b^2/(2*d) + (b*tan(x)*(2*a*d - b*c))/d^2)/tan(x)^2 - (2*atanh(((d + 2*c
*tan(x))*(a*d - b*c)^2)/(d*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))*(a*d - b*c)^2
)/d^3
```

3.722 $\int \frac{(a+b\cot(x))^3\csc^2(x)}{c+d\cot(x)}\,dx$

3.722.1 Optimal result

Integrand size = 21, antiderivative size = 78

$$\int \frac{(a+b\cot(x))^3\csc^2(x)}{c+d\cot(x)}\,dx = -\frac{b(bc-ad)^2\cot(x)}{d^3} + \frac{(bc-ad)(a+b\cot(x))^2}{2d^2} - \frac{(a+b\cot(x))^3}{3d} + \frac{(bc-ad)^3\log(c+d\cot(x))}{d^4}$$

output
```
-b*(-a*d+b*c)^2*cot(x)/d^3+1/2*(-a*d+b*c)*(a+b*cot(x))^2/d^2-1/3*(a+b*cot(
x))^3/d+(-a*d+b*c)^3*ln(c+d*cot(x))/d^4
```

3.722.2 Mathematica [A] (verified)

Time = 3.08 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.73

$$\int \frac{(a+b\cot(x))^3\csc^2(x)}{c+d\cot(x)}\,dx$$
$$= \frac{(a+b\cot(x))^3(d\cos(x)+c\sin(x))\left(-2b^3d^3\cot(x)-6(bc-ad)^3(\log(\sin(x))-\log(d\cos(x)+c\sin(x)))\,\mathrm{s}\right.}{6d^4(c+d\cot(x))(b\cos(x)+a\sin(x)}$$

input
```
Integrate[((a + b*Cot[x])^3*Csc[x]^2)/(c + d*Cot[x]),x]
```

output
```
((a + b*Cot[x])^3*(d*Cos[x] + c*Sin[x])*(-2*b^3*d^3*Cot[x] - 6*(b*c - a*d)
^3*(Log[Sin[x]] - Log[d*Cos[x] + c*Sin[x]])*Sin[x]^2 + b*d*(3*b*d*(b*c - 3
*a*d) + (9*a*b*c*d - 9*a^2*d^2 + b^2*(-3*c^2 + d^2))*Sin[2*x])))/(6*d^4*(c
 + d*Cot[x])*(b*Cos[x] + a*Sin[x])^3)
```

3.722.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 4844, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(x)(a+b\cot(x))^3}{c+d\cot(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc(x)^2(a+b\cot(x))^3}{c+d\cot(x)}dx$$

$$\Big\downarrow \text{4844}$$

$$-\int \frac{(a+b\cot(x))^3}{c+d\cot(x)}d\cot(x)$$

$$\Big\downarrow \text{49}$$

$$-\int \left(\frac{(ad-bc)^3}{d^3(c+d\cot(x))}+\frac{b(bc-ad)^2}{d^3}+\frac{b(a+b\cot(x))^2}{d}-\frac{b(bc-ad)(a+b\cot(x))}{d^2}\right)d\cot(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{(bc-ad)^3\log(c+d\cot(x))}{d^4}-\frac{b\cot(x)(bc-ad)^2}{d^3}+\frac{(bc-ad)(a+b\cot(x))^2}{2d^2}-\frac{(a+b\cot(x))^3}{3d}$$

input
```
Int[((a + b*Cot[x])^3*Csc[x]^2)/(c + d*Cot[x]),x]
```

output
```
-((b*(b*c - a*d)^2*Cot[x])/d^3) + ((b*c - a*d)*(a + b*Cot[x])^2)/(2*d^2) -
 (a + b*Cot[x])^3/(3*d) + ((b*c - a*d)^3*Log[c + d*Cot[x]])/d^4
```

3.722.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4844
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Cot[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[1, Cot[c*(a
+ b*x)]/d, u, x], x], x, Cot[c*(a + b*x)]/d], x] /; FunctionOfQ[Cot[c*(a +
b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Csc]
|| EqQ[F, csc])
```

3.722.4 Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.51

method	result
derivativedivides	$-\frac{b\left(\frac{b^2\cot(x)^3d^2}{3}+\frac{3ab\,d^2\cot(x)^2}{2}-\frac{b^2cd\cot(x)^2}{2}+3\cot(x)d^2a^2-3\cot(x)cdab+\cot(x)b^2c^2\right)}{d^3}-\frac{(d^3a^3-3c\,d^2a^2b+3c^2da\,b^2}{d^4}$
default	$-\frac{b\left(\frac{b^2\cot(x)^3d^2}{3}+\frac{3ab\,d^2\cot(x)^2}{2}-\frac{b^2cd\cot(x)^2}{2}+3\cot(x)d^2a^2-3\cot(x)cdab+\cot(x)b^2c^2\right)}{d^3}-\frac{(d^3a^3-3c\,d^2a^2b+3c^2da\,b^2}{d^4}$
risch	$-\frac{2ib(9a^2d^2e^{4ix}-9abcd\,e^{4ix}+3b^2c^2e^{4ix}-3b^2d^2e^{4ix}+9iab\,d^2e^{4ix}-3ib^2cd\,e^{4ix}-18a^2d^2e^{2ix}+18abcd\,e^{2ix}-6b^2c^2e^{2ix}-9i}{3d^3(e^{2ix}-1)^3}$

input
```
int((a+b*cot(x))^3*csc(x)^2/(c+d*cot(x)),x,method=_RETURNVERBOSE)
```

output
```
-b/d^3*(1/3*b^2*cot(x)^3*d^2+3/2*a*b*d^2*cot(x)^2-1/2*b^2*c*d*cot(x)^2+3*c
ot(x)*d^2*a^2-3*cot(x)*c*d*a*b+cot(x)*b^2*c^2)-(a^3*d^3-3*a^2*b*c*d^2+3*a*
b^2*c^2*d-b^3*c^3)/d^4*ln(c+d*cot(x))
```

3.722.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. 2(74) = 148.

Time = 0.29 (sec) , antiderivative size = 320, normalized size of antiderivative = 4.10

$$\int \frac{(a+b\cot(x))^3\csc^2(x)}{c+d\cot(x)}\,dx =$$
$$-\frac{2\left(3\,b^3c^2d-9\,ab^2cd^2+(9\,a^2b-b^3)d^3\right)\cos(x)^3+3\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3-(b^3c^3-3\,ab^2c^2d\right.}{}$$

input
```
integrate((a+b*cot(x))^3*csc(x)^2/(c+d*cot(x)),x, algorithm="fricas")
```

output
```
-1/6*(2*(3*b^3*c^2*d - 9*a*b^2*c*d^2 + (9*a^2*b - b^3)*d^3)*cos(x)^3 + 3*(
b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3 - (b^3*c^3 - 3*a*b^2*c^2
*d + 3*a^2*b*c*d^2 - a^3*d^3)*cos(x)^2)*log(2*c*d*cos(x)*sin(x) - (c^2 - d
^2)*cos(x)^2 + c^2)*sin(x) - 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
a^3*d^3 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cos(x)^2)*lo
g(-1/4*cos(x)^2 + 1/4)*sin(x) - 6*(b^3*c^2*d - 3*a*b^2*c*d^2 + 3*a^2*b*d^3
)*cos(x) + 3*(b^3*c*d^2 - 3*a*b^2*d^3)*sin(x))/((d^4*cos(x)^2 - d^4)*sin(x
))
```

3.722.6 Sympy [A] (verification not implemented)

Time = 33.51 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.24

$$\int \frac{(a+b\cot(x))^3\csc^2(x)}{c+d\cot(x)}\,dx = -\frac{b^3\cot^3(x)}{3d} - \frac{(3ab^2d-b^3c)\cot^2(x)}{2d^2} - \frac{(ad-bc)^3\left(\begin{cases}\frac{\cot(x)}{c} & \text{for } d=0\\ \frac{\log(c+d\cot(x))}{d} & \text{otherwise}\end{cases}\right)}{d^3} - \frac{(3a^2bd^2-3ab^2cd+b^3c^2)\cot(x)}{d^3}$$

input
```
integrate((a+b*cot(x))**3*csc(x)**2/(c+d*cot(x)),x)
```

output
```
-b**3*cot(x)**3/(3*d) - (3*a*b**2*d - b**3*c)*cot(x)**2/(2*d**2) - (a*d -
b*c)**3*Piecewise((cot(x)/c, Eq(d, 0)), (log(c + d*cot(x))/d, True))/d**3
- (3*a**2*b*d**2 - 3*a*b**2*c*d + b**3*c**2)*cot(x)/d**3
```

3.722.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 161 vs. $2(74) = 148$.

Time = 0.22 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.06

$$\int \frac{(a+b\cot(x))^3\csc^2(x)}{c+d\cot(x)}\,dx = \frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\log\left(c\tan\left(x\right)+d\right)}{d^4} - \frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\log\left(\tan\left(x\right)\right)}{d^4} - \frac{2\,b^3d^2+6\,(b^3c^2-3\,ab^2cd+3\,a^2bd^2)\tan\left(x\right)^2-3\,(b^3cd-3\,ab^2d^2)\tan\left(x\right)}{6\,d^3\tan\left(x\right)^3}$$

input
```
integrate((a+b*cot(x))^3*csc(x)^2/(c+d*cot(x)),x, algorithm="maxima")
```

output
```
(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(c*tan(x) + d)/d^4
- (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(tan(x))/d^4 - 1/
6*(2*b^3*d^2 + 6*(b^3*c^2 - 3*a*b^2*c*d + 3*a^2*b*d^2)*tan(x)^2 - 3*(b^3*c
*d - 3*a*b^2*d^2)*tan(x))/(d^3*tan(x)^3)
```

3.722.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. 2(74) = 148.

Time = 0.29 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.97

$$\int \frac{(a+b\cot(x))^3\csc^2(x)}{c+d\cot(x)}\,dx = -\frac{(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3)\log\left(\left|\tan\left(x\right)\right|\right)}{d^4} + \frac{(b^3c^4-3\,ab^2c^3d+3\,a^2bc^2d^2-a^3cd^3)\log\left(\left|c\tan\left(x\right)+d\right|\right)}{cd^4} + \frac{11\,b^3c^3\tan\left(x\right)^3-33\,ab^2c^2d\tan\left(x\right)^3+33\,a^2bcd^2\tan\left(x\right)^3-11\,a^3d^3\tan\left(x\right)^3-6\,b^3c^2d\tan\left(x\right)^2+18\,ab^2cd}{6\,d^4\tan\left(x\right)^3}$$

input
```
integrate((a+b*cot(x))^3*csc(x)^2/(c+d*cot(x)),x, algorithm="giac")
```

output
```
-(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(abs(tan(x)))/d^4
+ (b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2 - a^3*c*d^3)*log(abs(c*tan(x)
 + d))/(c*d^4) + 1/6*(11*b^3*c^3*tan(x)^3 - 33*a*b^2*c^2*d*tan(x)^3 + 33*a
^2*b*c*d^2*tan(x)^3 - 11*a^3*d^3*tan(x)^3 - 6*b^3*c^2*d*tan(x)^2 + 18*a*b^
2*c*d^2*tan(x)^2 - 18*a^2*b*d^3*tan(x)^2 + 3*b^3*c*d^2*tan(x) - 9*a*b^2*d^
3*tan(x) - 2*b^3*d^3)/(d^4*tan(x)^3)
```

3.722.9 Mupad [B] (verification not implemented)

Time = 26.51 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.81

$$\int \frac{(a+b\cot(x))^3\csc^2(x)}{c+d\cot(x)}\,dx = -\frac{\frac{b^3}{3\,d}+\frac{b^2\tan(x)\,(3\,a\,d-b\,c)}{2\,d^2}+\frac{b\tan(x)^2\,(3\,a^2\,d^2-3\,a\,b\,c\,d+b^2\,c^2)}{d^3}}{\tan(x)^3} - \frac{2\,\mathrm{atanh}\left(\frac{(d+2\,c\tan(x))\,(a\,d-b\,c)^3}{d\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}\right)\,(a\,d-b\,c)^3}{d^4}$$

input
```
int((a + b*cot(x))^3/(sin(x)^2*(c + d*cot(x))),x)
```

output
```
- (b^3/(3*d) + (b^2*tan(x)*(3*a*d - b*c))/(2*d^2) + (b*tan(x)^2*(3*a^2*d^2
 + b^2*c^2 - 3*a*b*c*d))/d^3)/tan(x)^3 - (2*atanh(((d + 2*c*tan(x))*(a*d -
 b*c)^3)/(d*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)))*(a*d - b
*c)^3)/d^4
```

3.723 $\int e^{-\cot(x)}\csc^2(x)\,dx$

3.723.1 Optimal result

Integrand size = 11, antiderivative size = 6

$$\int e^{-\cot(x)}\csc^2(x)\,dx = e^{-\cot(x)}$$

output
```
exp(-cot(x))
```

3.723.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int e^{-\cot(x)}\csc^2(x)\,dx = e^{-\cot(x)}$$

input
```
Integrate[Csc[x]^2/E^Cot[x],x]
```

output
```
E^(-Cot[x])
```

3.723.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4844, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-\cot(x)}\csc^2(x)\,dx$$
$$\Big\downarrow \text{4844}$$
$$-\int e^{-\cot(x)}d\cot(x)$$
$$\Big\downarrow \text{2624}$$
$$e^{-\cot(x)}$$

input
```
Int[Csc[x]^2/E^Cot[x],x]
```

output
```
E^(-Cot[x])
```

3.723.3.1 Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 4844
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Cot[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[1, Cot[c*(a
+ b*x)]/d, u, x], x], x, Cot[c*(a + b*x)]/d], x] /; FunctionOfQ[Cot[c*(a +
b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Csc]
|| EqQ[F, csc])
```

3.723.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$e^{-\cot(x)}$	6
default	$e^{-\cot(x)}$	6
risch	$e^{-\frac{\cos(x)}{\sin(x)}}$	10

input
```
int(csc(x)^2/exp(cot(x)),x,method=_RETURNVERBOSE)
```

output
```
1/exp(cot(x))
```

3.723.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.50

$$\int e^{-\cot(x)}\csc^2(x)\,dx = e^{\left(-\frac{\cos(x)}{\sin(x)}\right)}$$

input
```
integrate(csc(x)^2/exp(cot(x)),x, algorithm="fricas")
```

output
```
e^(-cos(x)/sin(x))
```

3.723.6 Sympy [A] (verification not implemented)

Time = 17.17 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int e^{-\cot(x)}\csc^2(x)\,dx = e^{-\cot(x)}$$

input
```
integrate(csc(x)**2/exp(cot(x)),x)
```

output
```
exp(-cot(x))
```

3.723.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int e^{-\cot(x)}\csc^2(x)\,dx = e^{(-\cot(x))}$$

input
```
integrate(csc(x)^2/exp(cot(x)),x, algorithm="maxima")
```

output
```
e^(-cot(x))
```

3.723.8 Giac [F]

$$\int e^{-\cot(x)}\csc^2(x)\,dx = \int \csc(x)^2 e^{(-\cot(x))}\,dx$$

input
```
integrate(csc(x)^2/exp(cot(x)),x, algorithm="giac")
```

output
```
integrate(csc(x)^2*e^(-cot(x)), x)
```

3.723.9 Mupad [B] (verification not implemented)

Time = 26.25 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

$$\int e^{-\cot(x)}\csc^2(x)\,dx = \mathrm{e}^{-\frac{1}{\tan(x)}}$$

input
```
int(exp(-cot(x))/sin(x)^2,x)
```

output
```
exp(-1/tan(x))
```

3.724 $\int \frac{\sec(x)\tan(x)}{a+b\sec(x)}\,dx$

3.724.1 Optimal result

Integrand size = 13, antiderivative size = 11

$$\int \frac{\sec(x)\tan(x)}{a+b\sec(x)}\,dx = \frac{\log(a+b\sec(x))}{b}$$

output
```
ln(a+b*sec(x))/b
```

3.724.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.82

$$\int \frac{\sec(x)\tan(x)}{a+b\sec(x)}\,dx = -\frac{\log(\cos(x))}{b} + \frac{\log(b+a\cos(x))}{b}$$

input
```
Integrate[(Sec[x]*Tan[x])/(a + b*Sec[x]),x]
```

output
```
-(Log[Cos[x]]/b) + Log[b + a*Cos[x]]/b
```

3.724.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.82, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3042, 4839, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(x)\sec(x)}{a+b\sec(x)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\tan(x)\sec(x)}{a+b\sec(x)}dx$$

$$\Big\downarrow \; 4839$$

$$-\int \frac{\sec(x)}{b+a\cos(x)}d\cos(x)$$

$$\Big\downarrow \; 47$$

$$\frac{a\int \frac{1}{b+a\cos(x)}d\cos(x)}{b}-\frac{\int \sec(x)d\cos(x)}{b}$$

$$\Big\downarrow \; 14$$

$$\frac{a\int \frac{1}{b+a\cos(x)}d\cos(x)}{b}-\frac{\log(\cos(x))}{b}$$

$$\Big\downarrow \; 16$$

$$\frac{\log(a\cos(x)+b)}{b}-\frac{\log(\cos(x))}{b}$$

input
```
Int[(Sec[x]*Tan[x])/(a + b*Sec[x]),x]
```

output
```
-(Log[Cos[x]]/b) + Log[b + a*Cos[x]]/b
```

3.724.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4839
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-(b*c)^(-1)   Subst[Int[SubstFor[1/x, Cos[c*
(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
 + b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, t
an])
```

3.724.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$\frac{\ln(a+b\sec(x))}{b}$	12
default	$\frac{\ln(a+b\sec(x))}{b}$	12
risch	$\frac{\ln\left(\mathrm{e}^{2ix}+\frac{2b\,\mathrm{e}^{ix}}{a}+1\right)}{b}-\frac{\ln(\mathrm{e}^{2ix}+1)}{b}$	38

input
```
int(sec(x)*tan(x)/(a+b*sec(x)),x,method=_RETURNVERBOSE)
```

output
```
ln(a+b*sec(x))/b
```

3.724.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int \frac{\sec(x)\tan(x)}{a+b\sec(x)}\,dx = \frac{\log\left(a\cos\left(x\right)+b\right)-\log\left(-\cos\left(x\right)\right)}{b}$$

input
```
integrate(sec(x)*tan(x)/(a+b*sec(x)),x, algorithm="fricas")
```

output
```
(log(a*cos(x) + b) - log(-cos(x)))/b
```

3.724.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int \frac{\sec(x)\tan(x)}{a+b\sec(x)}\,dx = \begin{cases} \frac{\log\left(\frac{a}{b}+\sec\left(x\right)\right)}{b} & \text{for } b \neq 0 \\ \frac{\sec\left(x\right)}{a} & \text{otherwise} \end{cases}$$

input
```
integrate(sec(x)*tan(x)/(a+b*sec(x)),x)
```

output
```
Piecewise((log(a/b + sec(x))/b, Ne(b, 0)), (sec(x)/a, True))
```

3.724.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\sec(x)\tan(x)}{a+b\sec(x)}\,dx = \frac{\log\left(b\sec\left(x\right)+a\right)}{b}$$

input
```
integrate(sec(x)*tan(x)/(a+b*sec(x)),x, algorithm="maxima")
```

output
```
log(b*sec(x) + a)/b
```

3.724.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.00

$$\int \frac{\sec(x)\tan(x)}{a+b\sec(x)}\,dx = \frac{\log\left(|a\cos\left(x\right)+b|\right)}{b} - \frac{\log\left(|\cos\left(x\right)|\right)}{b}$$

input
```
integrate(sec(x)*tan(x)/(a+b*sec(x)),x, algorithm="giac")
```

output
```
log(abs(a*cos(x) + b))/b - log(abs(cos(x)))/b
```

3.724.9 Mupad [B] (verification not implemented)

Time = 26.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 4.36

$$\int \frac{\sec(x)\tan(x)}{a+b\sec(x)}\,dx = \frac{\operatorname{atan}\left(\frac{b\sin\left(\frac{x}{2}\right)^2}{a\cos\left(\frac{x}{2}\right)^2 1\mathrm{i}+b\cos\left(\frac{x}{2}\right)^2 1\mathrm{i}-a\sin\left(\frac{x}{2}\right)^2 1\mathrm{i}}\right)\,2\mathrm{i}}{b}$$

input
```
int(tan(x)/(cos(x)*(a + b/cos(x))),x)
```

output
```
(atan((b*sin(x/2)^2)/(a*cos(x/2)^2*1i + b*cos(x/2)^2*1i - a*sin(x/2)^2*1i)
)*2i)/b
```

3.725 $\int \frac{\sec(x)\tan(x)}{1+\sec^2(x)}\,dx$

3.725.1 Optimal result

Integrand size = 13, antiderivative size = 5

$$\int \frac{\sec(x)\tan(x)}{1+\sec^2(x)}\,dx = -\arctan(\cos(x))$$

output
```
-arctan(cos(x))
```

3.725.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{\sec(x)\tan(x)}{1+\sec^2(x)}\,dx = -\arctan(\cos(x))$$

input
```
Integrate[(Sec[x]*Tan[x])/(1 + Sec[x]^2),x]
```

output
```
-ArcTan[Cos[x]]
```

3.725.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3042, 4839, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(x)\sec(x)}{\sec^2(x)+1}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\tan(x)\sec(x)}{\sec(x)^2+1}dx$$
$$\Big\downarrow \text{4839}$$
$$-\int \frac{1}{\cos^2(x)+1}d\cos(x)$$
$$\Big\downarrow \text{216}$$
$$-\arctan(\cos(x))$$

input
```
Int[(Sec[x]*Tan[x])/(1 + Sec[x]^2),x]
```

output
```
-ArcTan[Cos[x]]
```

3.725.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4839
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-(b*c)^(-1)   Subst[Int[SubstFor[1/x, Cos[c*
(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
 + b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, t
an])
```

3.725.4 Maple [A] (verified)

Time = 2.26 (sec) , antiderivative size = 4, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\arctan(\sec(x))$	4
default	$\arctan(\sec(x))$	4
risch	$-\frac{i\ln(2i e^{ix}+e^{2ix}+1)}{2}+\frac{i\ln(-2i e^{ix}+e^{2ix}+1)}{2}$	40

input
```
int(sec(x)*tan(x)/(1+sec(x)^2),x,method=_RETURNVERBOSE)
```

output
```
arctan(sec(x))
```

3.725.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{\sec(x)\tan(x)}{1+\sec^2(x)}\,dx = -\arctan(\cos(x))$$

input
```
integrate(sec(x)*tan(x)/(1+sec(x)^2),x, algorithm="fricas")
```

output
```
-arctan(cos(x))
```

3.725.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.60

$$\int \frac{\sec(x)\tan(x)}{1+\sec^2(x)}\,dx = \operatorname{atan}(\sec(x))$$

input
```
integrate(sec(x)*tan(x)/(1+sec(x)**2),x)
```

output
```
atan(sec(x))
```

3.725.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{\sec(x)\tan(x)}{1+\sec^2(x)}\,dx = -\arctan(\cos(x))$$

input
```
integrate(sec(x)*tan(x)/(1+sec(x)^2),x, algorithm="maxima")
```

output
```
-arctan(cos(x))
```

3.725.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{\sec(x)\tan(x)}{1+\sec^2(x)}\,dx = -\arctan(\cos(x))$$

input
```
integrate(sec(x)*tan(x)/(1+sec(x)^2),x, algorithm="giac")
```

output
```
-arctan(cos(x))
```

3.725.9 Mupad [B] (verification not implemented)

Time = 26.47 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.40

$$\int \frac{\sec(x)\tan(x)}{1+\sec^2(x)}\,dx = \operatorname{atan}\left(\tan\left(\frac{x}{2}\right)^2\right)$$

input
```
int(tan(x)/(cos(x)*(1/cos(x)^2 + 1)),x)
```

output
```
atan(tan(x/2)^2)
```

3.726 $\int \frac{\sec(x)\tan(x)}{9+4\sec^2(x)}\,dx$

3.726.1 Optimal result

Integrand size = 15, antiderivative size = 11

$$\int \frac{\sec(x)\tan(x)}{9+4\sec^2(x)}\,dx = -\frac{1}{6}\arctan\left(\frac{3\cos(x)}{2}\right)$$

output
```
-1/6*arctan(3/2*cos(x))
```

3.726.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\sec(x)\tan(x)}{9+4\sec^2(x)}\,dx = -\frac{1}{6}\arctan\left(\frac{3\cos(x)}{2}\right)$$

input
```
Integrate[(Sec[x]*Tan[x])/(9 + 4*Sec[x]^2),x]
```

output
```
-1/6*ArcTan[(3*Cos[x])/2]
```

3.726.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 4839, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(x)\sec(x)}{4\sec^2(x)+9}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\tan(x)\sec(x)}{4\sec(x)^2+9}dx$$
$$\Big\downarrow \; 4839$$
$$-\int \frac{1}{9\cos^2(x)+4}d\cos(x)$$
$$\Big\downarrow \; 216$$
$$-\frac{1}{6}\arctan\left(\frac{3\cos(x)}{2}\right)$$

input
```
Int[(Sec[x]*Tan[x])/(9 + 4*Sec[x]^2),x]
```

output
```
-1/6*ArcTan[(3*Cos[x])/2]
```

3.726.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4839
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-(b*c)^(-1)   Subst[Int[SubstFor[1/x, Cos[c*
(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
 + b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, t
an])
```

3.726.4 Maple [A] (verified)

Time = 2.36 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$\frac{\arctan\left(\frac{2\sec(x)}{3}\right)}{6}$	8
default	$\frac{\arctan\left(\frac{2\sec(x)}{3}\right)}{6}$	8
risch	$-\frac{i\ln\left(\frac{4i e^{ix}}{3}+e^{2ix}+1\right)}{12}+\frac{i\ln\left(-\frac{4i e^{ix}}{3}+e^{2ix}+1\right)}{12}$	40

input
```
int(sec(x)*tan(x)/(9+4*sec(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/6*arctan(2/3*sec(x))
```

3.726.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{\sec(x)\tan(x)}{9+4\sec^2(x)}\,dx = -\frac{1}{6}\arctan\left(\frac{3}{2}\cos(x)\right)$$

input
```
integrate(sec(x)*tan(x)/(9+4*sec(x)^2),x, algorithm="fricas")
```

output
```
-1/6*arctan(3/2*cos(x))
```

3.726.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int \frac{\sec(x)\tan(x)}{9+4\sec^2(x)}\,dx = \frac{\operatorname{atan}\left(\frac{2\sec(x)}{3}\right)}{6}$$

input
```
integrate(sec(x)*tan(x)/(9+4*sec(x)**2),x)
```

output
```
atan(2*sec(x)/3)/6
```

3.726.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{\sec(x)\tan(x)}{9+4\sec^2(x)}\,dx = -\frac{1}{6}\arctan\left(\frac{3}{2}\cos(x)\right)$$

input
```
integrate(sec(x)*tan(x)/(9+4*sec(x)^2),x, algorithm="maxima")
```

output
```
-1/6*arctan(3/2*cos(x))
```

3.726.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.64

$$\int \frac{\sec(x)\tan(x)}{9+4\sec^2(x)}\,dx = -\frac{1}{6}\arctan\left(\frac{3}{2}\cos(x)\right)$$

input
```
integrate(sec(x)*tan(x)/(9+4*sec(x)^2),x, algorithm="giac")
```

output
```
-1/6*arctan(3/2*cos(x))
```

3.726.9 Mupad [B] (verification not implemented)

Time = 26.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{\sec(x)\tan(x)}{9+4\sec^2(x)}\,dx = \frac{\operatorname{atan}\left(\frac{13\tan\left(\frac{x}{2}\right)^2}{12}-\frac{5}{12}\right)}{6}$$

input
```
int(tan(x)/(cos(x)*(4/cos(x)^2 + 9)),x)
```

output
```
atan((13*tan(x/2)^2)/12 - 5/12)/6
```

3.727 $\int \frac{\sec(x)\tan(x)}{\sec(x)+\sec^2(x)}\,dx$

3.727.1 Optimal result

Integrand size = 14, antiderivative size = 7

$$\int \frac{\sec(x)\tan(x)}{\sec(x)+\sec^2(x)}\,dx = -\log(1+\cos(x))$$

output
```
-ln(1+cos(x))
```

3.727.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \frac{\sec(x)\tan(x)}{\sec(x)+\sec^2(x)}\,dx = -2\log\left(\cos\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[(Sec[x]*Tan[x])/(Sec[x] + Sec[x]^2),x]
```

output
```
-2*Log[Cos[x/2]]
```

3.727.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 4839, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(x)\sec(x)}{\sec^2(x)+\sec(x)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\tan(x)\sec(x)}{\sec(x)^2+\sec(x)}dx$$

$$\Big\downarrow \; 4839$$

$$-\int \frac{1}{\cos(x)+1}d\cos(x)$$

$$\Big\downarrow \; 16$$

$$-\log(\cos(x)+1)$$

input
```
Int[(Sec[x]*Tan[x])/(Sec[x] + Sec[x]^2),x]
```

output
```
-Log[1 + Cos[x]]
```

3.727.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4839
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-(b*c)^(-1)   Subst[Int[SubstFor[1/x, Cos[c*
(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
 + b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, t
an])
```

3.727.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.71

method	result	size
derivativedivides	$\ln(\sec(x)) - \ln(1 + \sec(x))$	12
default	$\ln(\sec(x)) - \ln(1 + \sec(x))$	12
risch	$ix - 2\ln(e^{ix} + 1)$	16

input
```
int(sec(x)*tan(x)/(sec(x)+sec(x)^2),x,method=_RETURNVERBOSE)
```

output
```
ln(sec(x))-ln(1+sec(x))
```

3.727.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \frac{\sec(x)\tan(x)}{\sec(x)+\sec^2(x)}\,dx = -\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right)$$

input
```
integrate(sec(x)*tan(x)/(sec(x)+sec(x)^2),x, algorithm="fricas")
```

output
```
-log(1/2*cos(x) + 1/2)
```

3.727.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(7) = 14.

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 2.14

$$\int \frac{\sec(x)\tan(x)}{\sec(x)+\sec^2(x)}\,dx = \frac{\log\left(\tan^2(x)+1\right)}{2} - \log\left(\sec(x)+1\right)$$

input
```
integrate(sec(x)*tan(x)/(sec(x)+sec(x)**2),x)
```

output
```
log(tan(x)**2 + 1)/2 - log(sec(x) + 1)
```

3.727.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{\sec(x)\tan(x)}{\sec(x)+\sec^2(x)}\,dx = -\log\left(\cos(x)+1\right)$$

input
```
integrate(sec(x)*tan(x)/(sec(x)+sec(x)^2),x, algorithm="maxima")
```

output
```
-log(cos(x) + 1)
```

3.727.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{\sec(x)\tan(x)}{\sec(x)+\sec^2(x)}\,dx = -\log\left(\cos(x)+1\right)$$

input
```
integrate(sec(x)*tan(x)/(sec(x)+sec(x)^2),x, algorithm="giac")
```

output
```
-log(cos(x) + 1)
```

3.727.9 Mupad [B] (verification not implemented)

Time = 26.60 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \frac{\sec(x)\tan(x)}{\sec(x)+\sec^2(x)}\,dx = \ln\left(\tan\left(\frac{x}{2}\right)^2+1\right)$$

input
```
int(tan(x)/(cos(x)*(1/cos(x) + 1/cos(x)^2)),x)
```

output
```
log(tan(x/2)^2 + 1)
```

3.728 $\int \frac{\sec(x)\tan(x)}{\sqrt{4+\sec^2(x)}}\,dx$

3.728.1 Optimal result

Integrand size = 15, antiderivative size = 5

$$\int \frac{\sec(x)\tan(x)}{\sqrt{4+\sec^2(x)}}\,dx = \operatorname{csch}^{-1}(2\cos(x))$$

output
```
arccsch(2*cos(x))
```

3.728.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 38 vs. 2(5) = 10.

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 7.60

$$\int \frac{\sec(x)\tan(x)}{\sqrt{4+\sec^2(x)}}\,dx = \frac{\operatorname{arctanh}\left(\sqrt{1+4\cos^2(x)}\right)\sqrt{3+2\cos(2x)}\sec(x)}{\sqrt{4+\sec^2(x)}}$$

input
```
Integrate[(Sec[x]*Tan[x])/Sqrt[4 + Sec[x]^2],x]
```

output
```
(ArcTanh[Sqrt[1 + 4*Cos[x]^2]]*Sqrt[3 + 2*Cos[2*x]]*Sec[x])/Sqrt[4 + Sec[x
]^2]
```

3.728.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.40, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 4839, 858, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(x)\sec(x)}{\sqrt{\sec^2(x)+4}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(x)\sec(x)}{\sqrt{\sec(x)^2+4}}dx$$

$$\Big\downarrow \text{4839}$$

$$-\int \frac{\sec^2(x)}{\sqrt{\sec^2(x)+4}}d\cos(x)$$

$$\Big\downarrow \text{858}$$

$$\int \frac{1}{\sqrt{\sec^2(x)+4}}d\sec(x)$$

$$\Big\downarrow \text{222}$$

$$\text{arcsinh}\left(\frac{\sec(x)}{2}\right)$$

input
```
Int[(Sec[x]*Tan[x])/Sqrt[4 + Sec[x]^2],x]
```

output
```
ArcSinh[Sec[x]/2]
```

3.728.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4839
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-(b*c)^(-1)   Subst[Int[SubstFor[1/x, Cos[c*
(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
 + b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, t
an])
```

3.728.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

method	result	size
derivativedivides	$\operatorname{arcsinh}\left(\frac{\sec(x)}{2}\right)$	6
default	$\operatorname{arcsinh}\left(\frac{\sec(x)}{2}\right)$	6

input
```
int(sec(x)*tan(x)/(4+sec(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arcsinh(1/2*sec(x))
```

3.728.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(5) = 10.

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 5.40

$$\int \frac{\sec(x)\tan(x)}{\sqrt{4+\sec^2(x)}}\,dx = \log\left(-\frac{\sqrt{\frac{4\cos(x)^2+1}{\cos(x)^2}}\cos(x)+1}{\cos(x)}\right)$$

input
```
integrate(sec(x)*tan(x)/(4+sec(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
log(-(sqrt((4*cos(x)^2 + 1)/cos(x)^2)*cos(x) + 1)/cos(x))
```

3.728.6 Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{\sec(x)\tan(x)}{\sqrt{4+\sec^2(x)}}\,dx = \operatorname{asinh}\left(\frac{\sec(x)}{2}\right)$$

input
```
integrate(sec(x)*tan(x)/(4+sec(x)**2)**(1/2),x)
```

output
```
asinh(sec(x)/2)
```

3.728.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(5) = 10.

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 6.60

$$\int \frac{\sec(x)\tan(x)}{\sqrt{4+\sec^2(x)}}\,dx$$
$$= \frac{1}{2}\log\left(\sqrt{\frac{1}{\cos(x)^2}+4}\cos(x)+1\right) - \frac{1}{2}\log\left(\sqrt{\frac{1}{\cos(x)^2}+4}\cos(x)-1\right)$$

input
```
integrate(sec(x)*tan(x)/(4+sec(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*log(sqrt(1/cos(x)^2 + 4)*cos(x) + 1) - 1/2*log(sqrt(1/cos(x)^2 + 4)*co
s(x) - 1)
```

3.728.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(5) = 10.

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 7.20

$$\int \frac{\sec(x)\tan(x)}{\sqrt{4+\sec^2(x)}}\,dx = \frac{\log\left(\sqrt{4\cos(x)^2+1}+1\right) - \log\left(\sqrt{4\cos(x)^2+1}-1\right)}{2\operatorname{sgn}(\cos(x))}$$

input
```
integrate(sec(x)*tan(x)/(4+sec(x)^2)^(1/2),x, algorithm="giac")
```

output
```
1/2*(log(sqrt(4*cos(x)^2 + 1) + 1) - log(sqrt(4*cos(x)^2 + 1) - 1))/sgn(co
s(x))
```

3.728.9 Mupad [B] (verification not implemented)

Time = 27.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.40

$$\int \frac{\sec(x)\tan(x)}{\sqrt{4+\sec^2(x)}}\,dx = \operatorname{asinh}\left(\frac{1}{2\,\cos(x)}\right)$$

input
```
int(tan(x)/(cos(x)*(1/cos(x)^2 + 4)^(1/2)),x)
```

output
```
asinh(1/(2*cos(x)))
```

3.729 $\int \frac{\sec(x)\tan(x)}{\sqrt{1+\cos^2(x)}}\,dx$

3.729.1 Optimal result

Integrand size = 15, antiderivative size = 13

$$\int \frac{\sec(x)\tan(x)}{\sqrt{1+\cos^2(x)}}\,dx = \sqrt{1+\cos^2(x)}\sec(x)$$

output
```
sec(x)*(1+cos(x)^2)^(1/2)
```

3.729.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\sec(x)\tan(x)}{\sqrt{1+\cos^2(x)}}\,dx = \sqrt{1+\cos^2(x)}\sec(x)$$

input
```
Integrate[(Sec[x]*Tan[x])/Sqrt[1 + Cos[x]^2],x]
```

output
```
Sqrt[1 + Cos[x]^2]*Sec[x]
```

3.729.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 4879, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(x)\sec(x)}{\sqrt{\cos^2(x)+1}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\cos(x)\sqrt{\cos(x)^2+1}\cot(x)}dx$$

$$\Big\downarrow \; 4879$$

$$-\int \frac{\sec^2(x)}{\sqrt{\cos^2(x)+1}}d\cos(x)$$

$$\Big\downarrow \; 242$$

$$\sqrt{\cos^2(x)+1}\sec(x)$$

input
```
Int[(Sec[x]*Tan[x])/Sqrt[1 + Cos[x]^2],x]
```

output
```
Sqrt[1 + Cos[x]^2]*Sec[x]
```

3.729.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4879
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cos[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Cos[v]/d
, u/Sin[v], x], x], x, Cos[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[Nonfree
Factors[Cos[v], x], u/Sin[v], x]]
```

3.729.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(11) = 22$.

Time = 0.69 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.92

method	result	size
derivativedivides	$\frac{1+\sec(x)^2}{\sqrt{\frac{1+\sec(x)^2}{\sec(x)^2}}\,\sec(x)}$	25
default	$\frac{1+\sec(x)^2}{\sqrt{\frac{1+\sec(x)^2}{\sec(x)^2}}\,\sec(x)}$	25

input
```
int(sec(x)*tan(x)/(cos(x)^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/((1+sec(x)^2)/sec(x)^2)^(1/2)/sec(x)*(1+sec(x)^2)
```

3.729.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.23

$$\int \frac{\sec(x)\tan(x)}{\sqrt{1+\cos^2(x)}}\,dx = \frac{\sqrt{\cos{(x)}^2+1}+\cos{(x)}}{\cos{(x)}}$$

input
```
integrate(sec(x)*tan(x)/(1+cos(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(cos(x)^2 + 1) + cos(x))/cos(x)
```

3.729.6 Sympy [F]

$$\int \frac{\sec(x)\tan(x)}{\sqrt{1+\cos^2(x)}}\,dx = \int \frac{\tan(x)\sec(x)}{\sqrt{\cos^2(x)+1}}\,dx$$

input
```
integrate(sec(x)*tan(x)/(1+cos(x)**2)**(1/2),x)
```

output
```
Integral(tan(x)*sec(x)/sqrt(cos(x)**2 + 1), x)
```

3.729.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\sec(x)\tan(x)}{\sqrt{1+\cos^2(x)}}\,dx = \frac{\sqrt{\cos(x)^2+1}}{\cos(x)}$$

input
```
integrate(sec(x)*tan(x)/(1+cos(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(cos(x)^2 + 1)/cos(x)
```

3.729.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62

$$\int \frac{\sec(x)\tan(x)}{\sqrt{1+\cos^2(x)}}\,dx = -\frac{2}{\left(\sqrt{\cos(x)^2+1}-\cos(x)\right)^2-1}$$

input
```
integrate(sec(x)*tan(x)/(1+cos(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-2/((sqrt(cos(x)^2 + 1) - cos(x))^2 - 1)
```

3.729.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\sec(x)\tan(x)}{\sqrt{1+\cos^2(x)}}\,dx = \frac{\sqrt{\cos(x)^2+1}}{\cos(x)}$$

input
```
int(tan(x)/(cos(x)*(cos(x)^2 + 1)^(1/2)),x)
```

output
```
(cos(x)^2 + 1)^(1/2)/cos(x)
```

3.730 $\int e^{\sec(x)} \sec(x) \tan(x) \, dx$

3.730.1 Optimal result

Integrand size = 9, antiderivative size = 4

$$\int e^{\sec(x)} \sec(x) \tan(x) \, dx = e^{\sec(x)}$$

output
```
exp(sec(x))
```

3.730.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int e^{\sec(x)} \sec(x) \tan(x) \, dx = e^{\sec(x)}$$

input
```
Integrate[E^Sec[x]*Sec[x]*Tan[x],x]
```

output
```
E^Sec[x]
```

3.730.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4839, 2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(x)e^{\sec(x)}\sec(x)\,dx$$
$$\Big\downarrow \text{4839}$$
$$-\int e^{\sec(x)}\sec^2(x)d\cos(x)$$
$$\Big\downarrow \text{2638}$$
$$e^{\sec(x)}$$

input
```
Int[E^Sec[x]*Sec[x]*Tan[x],x]
```

output
```
E^Sec[x]
```

3.730.3.1 Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 4839
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-(b*c)^(-1)   Subst[Int[SubstFor[1/x, Cos[c*
(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
 + b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, t
an])
```

3.730.4 Maple [A] (verified)

Time = 2.78 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$e^{\sec(x)}$	4
default	$e^{\sec(x)}$	4
risch	$e^{\frac{1}{\cos(x)}}$	6

input
```
int(exp(sec(x))*sec(x)*tan(x),x,method=_RETURNVERBOSE)
```

output
```
exp(sec(x))
```

3.730.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

$$\int e^{\sec(x)}\sec(x)\tan(x)\,dx = e^{\frac{1}{\cos(x)}}$$

input
```
integrate(exp(sec(x))*sec(x)*tan(x),x, algorithm="fricas")
```

output
```
e^(1/cos(x))
```

3.730.6 Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int e^{\sec(x)}\sec(x)\tan(x)\,dx = e^{\sec(x)}$$

input
```
integrate(exp(sec(x))*sec(x)*tan(x),x)
```

output
```
exp(sec(x))
```

3.730.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int e^{\sec(x)}\sec(x)\tan(x)\,dx = e^{\sec(x)}$$

input
```
integrate(exp(sec(x))*sec(x)*tan(x),x, algorithm="maxima")
```

output
```
e^sec(x)
```

3.730.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

$$\int e^{\sec(x)}\sec(x)\tan(x)\,dx = e^{\frac{1}{\cos(x)}}$$

input
```
integrate(exp(sec(x))*sec(x)*tan(x),x, algorithm="giac")
```

output
```
e^(1/cos(x))
```

3.730.9 Mupad [B] (verification not implemented)

Time = 27.73 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

$$\int e^{\sec(x)}\sec(x)\tan(x)\,dx = \mathrm{e}^{\frac{1}{\cos(x)}}$$

input
```
int((exp(1/cos(x))*tan(x))/cos(x),x)
```

output
```
exp(1/cos(x))
```

3.731 $\int 2^{\sec(x)} \sec(x) \tan(x) \, dx$

3.731.1 Optimal result

Integrand size = 9, antiderivative size = 9

$$\int 2^{\sec(x)} \sec(x) \tan(x) \, dx = \frac{2^{\sec(x)}}{\log(2)}$$

output
```
2^sec(x)/ln(2)
```

3.731.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int 2^{\sec(x)} \sec(x) \tan(x) \, dx = \frac{2^{\sec(x)}}{\log(2)}$$

input
```
Integrate[2^Sec[x]*Sec[x]*Tan[x],x]
```

output
```
2^Sec[x]/Log[2]
```

3.731.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4839, 2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(x)2^{\sec(x)}\sec(x)\,dx$$

$$\Big\downarrow \text{4839}$$

$$-\int 2^{\sec(x)}\sec^2(x)d\cos(x)$$

$$\Big\downarrow \text{2638}$$

$$\frac{2^{\sec(x)}}{\log(2)}$$

input
```
Int[2^Sec[x]*Sec[x]*Tan[x],x]
```

output
```
2^Sec[x]/Log[2]
```

3.731.3.1 Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 4839
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-(b*c)^(-1)   Subst[Int[SubstFor[1/x, Cos[c*
(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
 + b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, t
an])
```

3.731.4 Maple [A] (verified)

Time = 21.10 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

method	result	size
derivativedivides	$\frac{2^{\sec(x)}}{\ln(2)}$	10
default	$\frac{2^{\sec(x)}}{\ln(2)}$	10
risch	$\frac{2^{\sec(x)}}{\ln(2)}$	10

input
```
int(2^sec(x)*sec(x)*tan(x),x,method=_RETURNVERBOSE)
```

output
```
2^sec(x)/ln(2)
```

3.731.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int 2^{\sec(x)}\sec(x)\tan(x)\,dx = \frac{2^{\left(\frac{1}{\cos(x)}\right)}}{\log(2)}$$

input
```
integrate(2^sec(x)*sec(x)*tan(x),x, algorithm="fricas")
```

output
```
2^(1/cos(x))/log(2)
```

3.731.6 Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int 2^{\sec(x)}\sec(x)\tan(x)\,dx = \frac{2^{\sec(x)}}{\log(2)}$$

input
```
integrate(2**sec(x)*sec(x)*tan(x),x)
```

output
```
2**sec(x)/log(2)
```

3.731.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int 2^{\sec(x)} \sec(x) \tan(x) \, dx = \frac{2^{\sec(x)}}{\log(2)}$$

input
```
integrate(2^sec(x)*sec(x)*tan(x),x, algorithm="maxima")
```

output
```
2^sec(x)/log(2)
```

3.731.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int 2^{\sec(x)} \sec(x) \tan(x) \, dx = \frac{2^{\left(\frac{1}{\cos(x)}\right)}}{\log(2)}$$

input
```
integrate(2^sec(x)*sec(x)*tan(x),x, algorithm="giac")
```

output
```
2^(1/cos(x))/log(2)
```

3.731.9 Mupad [B] (verification not implemented)

Time = 27.58 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int 2^{\sec(x)} \sec(x) \tan(x) \, dx = \frac{2^{\frac{1}{\cos(x)}}}{\ln(2)}$$

input
```
int((2^(1/cos(x))*tan(x))/cos(x),x)
```

output
```
2^(1/cos(x))/log(2)
```

3.732 $\int \frac{\sec(2x)\tan(2x)}{(1+\sec(2x))^{3/2}}\,dx$

3.732.1 Optimal result

Integrand size = 19, antiderivative size = 12

$$\int \frac{\sec(2x)\tan(2x)}{(1+\sec(2x))^{3/2}}\,dx = -\frac{1}{\sqrt{1+\sec(2x)}}$$

output
```
-1/(1+sec(2*x))^(1/2)
```

3.732.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sec(2x)\tan(2x)}{(1+\sec(2x))^{3/2}}\,dx = -\frac{1}{\sqrt{1+\sec(2x)}}$$

input
```
Integrate[(Sec[2*x]*Tan[2*x])/(1 + Sec[2*x])^(3/2),x]
```

output
```
-(1/Sqrt[1 + Sec[2*x]])
```

3.732.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 4839, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(2x)\sec(2x)}{(\sec(2x)+1)^{3/2}}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\tan(2x)\sec(2x)}{(\sec(2x)+1)^{3/2}}dx$$
$$\Big\downarrow \text{ 4839}$$
$$-\frac{1}{2}\int \frac{\sec^2(2x)}{(\sec(2x)+1)^{3/2}}d\cos(2x)$$
$$\Big\downarrow \text{ 793}$$
$$-\frac{1}{\sqrt{\sec(2x)+1}}$$

input
```
Int[(Sec[2*x]*Tan[2*x])/(1 + Sec[2*x])^(3/2),x]
```

output
```
-(1/Sqrt[1 + Sec[2*x]])
```

3.732.3.1 Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4839
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-(b*c)^(-1)   Subst[Int[SubstFor[1/x, Cos[c*
(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
 + b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, t
an])
```

3.732.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$-\frac{1}{\sqrt{1+\sec(2x)}}$	11
default	$-\frac{1}{\sqrt{1+\sec(2x)}}$	11

input
```
int(sec(2*x)*tan(2*x)/(1+sec(2*x))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(1+sec(2*x))^(1/2)
```

3.732.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(10) = 20.

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.42

$$\int \frac{\sec(2x)\tan(2x)}{(1+\sec(2x))^{3/2}}\,dx = -\frac{\sqrt{\frac{\cos(2\,x)+1}{\cos(2\,x)}}\cos(2\,x)}{\cos(2\,x)+1}$$

input
```
integrate(sec(2*x)*tan(2*x)/(1+sec(2*x))^(3/2),x, algorithm="fricas")
```

output
```
-sqrt((cos(2*x) + 1)/cos(2*x))*cos(2*x)/(cos(2*x) + 1)
```

3.732.6 Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sec(2x)\tan(2x)}{(1+\sec(2x))^{3/2}}\,dx = -\frac{1}{\sqrt{\sec(2x)+1}}$$

input
```
integrate(sec(2*x)*tan(2*x)/(1+sec(2*x))**(3/2),x)
```

output
```
-1/sqrt(sec(2*x) + 1)
```

3.732.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\sec(2x)\tan(2x)}{(1+\sec(2x))^{3/2}}\,dx = -\frac{1}{\sqrt{\sec(2\,x)+1}}$$

input
```
integrate(sec(2*x)*tan(2*x)/(1+sec(2*x))^(3/2),x, algorithm="maxima")
```

output
```
-1/sqrt(sec(2*x) + 1)
```

3.732.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(10) = 20.

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.58

$$\int \frac{\sec(2x)\tan(2x)}{(1+\sec(2x))^{3/2}}\,dx = \frac{1}{\left(\sqrt{\cos(2\,x)^2+\cos(2\,x)}-\cos(2\,x)-1\right)\operatorname{sgn}(\cos(2\,x))}$$

input
```
integrate(sec(2*x)*tan(2*x)/(1+sec(2*x))^(3/2),x, algorithm="giac")
```

output
```
1/((sqrt(cos(2*x)^2 + cos(2*x)) - cos(2*x) - 1)*sgn(cos(2*x)))
```

3.732.9 Mupad [B] (verification not implemented)

Time = 25.97 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \frac{\sec(2x)\tan(2x)}{(1+\sec(2x))^{3/2}}\,dx = -\frac{1}{\sqrt{\cos\left(2\,x\right)+1}\,\sqrt{\frac{1}{\cos\left(2\,x\right)}}}$$

input
```
int(tan(2*x)/(cos(2*x)*(1/cos(2*x) + 1)^(3/2)),x)
```

output
```
-1/((cos(2*x) + 1)^(1/2)*(1/cos(2*x))^(1/2))
```

3.733 $\int \sqrt{1+5\cos^2(3x)}\sec(3x)\tan(3x)\,dx$

3.733.1 Optimal result

Integrand size = 23, antiderivative size = 43

$$\int \sqrt{1+5\cos^2(3x)}\sec(3x)\tan(3x)\,dx = -\frac{1}{3}\sqrt{5}\text{arcsinh}\left(\sqrt{5}\cos(3x)\right) + \frac{1}{3}\sqrt{1+5\cos^2(3x)}\sec(3x)$$

output
```
-1/3*arcsinh(cos(3*x)*5^(1/2))*5^(1/2)+1/3*sec(3*x)*(1+5*cos(3*x)^2)^(1/2)
```

3.733.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \sqrt{1+5\cos^2(3x)}\sec(3x)\tan(3x)\,dx = \frac{1}{3}\left(-\sqrt{5}\text{arcsinh}\left(\sqrt{5}\cos(3x)\right) + \sqrt{1+5\cos^2(3x)}\sec(3x)\right)$$

input
```
Integrate[Sqrt[1 + 5*Cos[3*x]^2]*Sec[3*x]*Tan[3*x],x]
```

output
```
(-(Sqrt[5]*ArcSinh[Sqrt[5]*Cos[3*x]]) + Sqrt[1 + 5*Cos[3*x]^2]*Sec[3*x])/3
```

3.733.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4879, 247, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{5\cos^2(3x)+1}\tan(3x)\sec(3x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{5\cos(3x)^2+1}}{\cos(3x)\cot(3x)}dx$$

$$\Big\downarrow \text{4879}$$

$$-\frac{1}{3}\int \sqrt{5\cos^2(3x)+1}\sec^2(3x)d\cos(3x)$$

$$\Big\downarrow \text{247}$$

$$\frac{1}{3}\left(\sqrt{5\cos^2(3x)+1}\sec(3x)-5\int \frac{1}{\sqrt{5\cos^2(3x)+1}}d\cos(3x)\right)$$

$$\Big\downarrow \text{222}$$

$$\frac{1}{3}\left(\sqrt{5\cos^2(3x)+1}\sec(3x)-\sqrt{5}\text{arcsinh}\left(\sqrt{5}\cos(3x)\right)\right)$$

input
```
Int[Sqrt[1 + 5*Cos[3*x]^2]*Sec[3*x]*Tan[3*x],x]
```

output
```
(-(Sqrt[5]*ArcSinh[Sqrt[5]*Cos[3*x]]) + Sqrt[1 + 5*Cos[3*x]^2]*Sec[3*x])/3
```

3.733.3.1 Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4879
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cos[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Cos[v]/d
, u/Sin[v], x], x], x, Cos[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[Nonfree
Factors[Cos[v], x], u/Sin[v], x]]
```

3.733.4 Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.51

method	result	size
derivativedivides	$\frac{\sqrt{\frac{\sec(3x)^2+5}{\sec(3x)^2}}\,\sec(3x)\left(\sqrt{\sec(3x)^2+5}-\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{\sec(3x)^2+5}}\right)\right)}{3\sqrt{\sec(3x)^2+5}}$	65
default	$\frac{\sqrt{\frac{\sec(3x)^2+5}{\sec(3x)^2}}\,\sec(3x)\left(\sqrt{\sec(3x)^2+5}-\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{\sec(3x)^2+5}}\right)\right)}{3\sqrt{\sec(3x)^2+5}}$	65

input
```
int(sec(3*x)*(1+5*cos(3*x)^2)^(1/2)*tan(3*x),x,method=_RETURNVERBOSE)
```

output
```
1/3*((sec(3*x)^2+5)/sec(3*x)^2)^(1/2)*sec(3*x)/(sec(3*x)^2+5)^(1/2)*((sec(
3*x)^2+5)^(1/2)-5^(1/2)*arctanh(5^(1/2)/(sec(3*x)^2+5)^(1/2)))
```

3.733.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. 2(33) = 66.

Time = 0.26 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.84

$$\int \sqrt{1+5\cos^2(3x)}\sec(3x)\tan(3x)\,dx$$
$$= \frac{\sqrt{5}\cos(3x)\log\left(80000\cos(3x)^8 + 32000\cos(3x)^6 + 4000\cos(3x)^4 + 160\cos(3x)^2 - 8\left(2000\sqrt{5}\cos(\right.\right.}{24\cos}$$

input
```
integrate(sec(3*x)*(1+5*cos(3*x)^2)^(1/2)*tan(3*x),x, algorithm="fricas")
```

output
```
1/24*(sqrt(5)*cos(3*x)*log(80000*cos(3*x)^8 + 32000*cos(3*x)^6 + 4000*cos(
3*x)^4 + 160*cos(3*x)^2 - 8*(2000*sqrt(5)*cos(3*x)^7 + 600*sqrt(5)*cos(3*x
)^5 + 50*sqrt(5)*cos(3*x)^3 + sqrt(5)*cos(3*x))*sqrt(5*cos(3*x)^2 + 1) + 1
) + 8*sqrt(5*cos(3*x)^2 + 1))/cos(3*x)
```

3.733.6 Sympy [F]

$$\int \sqrt{1+5\cos^2(3x)}\sec(3x)\tan(3x)\,dx = \int \sqrt{5\cos^2(3x)+1}\tan(3x)\sec(3x)\,dx$$

input
```
integrate(sec(3*x)*(1+5*cos(3*x)**2)**(1/2)*tan(3*x),x)
```

output
```
Integral(sqrt(5*cos(3*x)**2 + 1)*tan(3*x)*sec(3*x), x)
```

3.733.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \sqrt{1+5\cos^2(3x)}\sec(3x)\tan(3x)\,dx = -\frac{1}{3}\sqrt{5}\,\mathrm{arsinh}\left(\sqrt{5}\cos(3x)\right) + \frac{\sqrt{5\cos(3x)^2+1}}{3\cos(3x)}$$

input
```
integrate(sec(3*x)*(1+5*cos(3*x)^2)^(1/2)*tan(3*x),x, algorithm="maxima")
```

output
```
-1/3*sqrt(5)*arcsinh(sqrt(5)*cos(3*x)) + 1/3*sqrt(5*cos(3*x)^2 + 1)/cos(3*
x)
```

3.733.8 Giac [F]

$$\int \sqrt{1+5\cos^2(3x)}\sec(3x)\tan(3x)\,dx=\int \sqrt{5\,\cos\left(3\,x\right)^{2}+1}\sec\left(3\,x\right)\tan\left(3\,x\right)\,dx$$

input
```
integrate(sec(3*x)*(1+5*cos(3*x)^2)^(1/2)*tan(3*x),x, algorithm="giac")
```

output
```
integrate(sqrt(5*cos(3*x)^2 + 1)*sec(3*x)*tan(3*x), x)
```

3.733.9 Mupad [B] (verification not implemented)

Time = 26.70 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \sqrt{1+5\cos^2(3x)}\sec(3x)\tan(3x)\,dx=\frac{\sqrt{\frac{5\,\cos(6\,x)}{2}+\frac{7}{2}}}{3\,\cos\left(3\,x\right)}+\frac{\sqrt{5}\,\mathrm{asin}\left(\sqrt{5}\,\cos\left(3\,x\right)\,1\mathrm{i}\right)\,1\mathrm{i}}{3}$$

input
```
int((tan(3*x)*(5*cos(3*x)^2 + 1)^(1/2))/cos(3*x),x)
```

output
```
(5^(1/2)*asin(5^(1/2)*cos(3*x)*1i)*1i)/3 + ((5*cos(6*x))/2 + 7/2)^(1/2)/(3
*cos(3*x))
```

3.734 $\int \frac{\sec(3x)\tan(3x)}{\sqrt{1+5\cos^2(3x)}}\,dx$

3.734.1 Optimal result

Integrand size = 23, antiderivative size = 22

$$\int \frac{\sec(3x)\tan(3x)}{\sqrt{1+5\cos^2(3x)}}\,dx = \frac{1}{3}\sqrt{1+5\cos^2(3x)}\sec(3x)$$

output
```
1/3*sec(3*x)*(1+5*cos(3*x)^2)^(1/2)
```

3.734.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sec(3x)\tan(3x)}{\sqrt{1+5\cos^2(3x)}}\,dx = \frac{1}{3}\sqrt{1+5\cos^2(3x)}\sec(3x)$$

input
```
Integrate[(Sec[3*x]*Tan[3*x])/Sqrt[1 + 5*Cos[3*x]^2],x]
```

output
```
(Sqrt[1 + 5*Cos[3*x]^2]*Sec[3*x])/3
```

3.734.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3042, 4879, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(3x)\sec(3x)}{\sqrt{5\cos^2(3x)+1}}dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\cos(3x)\sqrt{5\cos(3x)^2+1}\cot(3x)}dx$$
$$\Big\downarrow \; 4879$$
$$-\frac{1}{3}\int \frac{\sec^2(3x)}{\sqrt{5\cos^2(3x)+1}}d\cos(3x)$$
$$\Big\downarrow \; 242$$
$$\frac{1}{3}\sqrt{5\cos^2(3x)+1}\sec(3x)$$

input
```
Int[(Sec[3*x]*Tan[3*x])/Sqrt[1 + 5*Cos[3*x]^2],x]
```

output
```
(Sqrt[1 + 5*Cos[3*x]^2]*Sec[3*x])/3
```

3.734.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4879
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cos[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Cos[v]/d
, u/Sin[v], x], x], x, Cos[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[Nonfree
Factors[Cos[v], x], u/Sin[v], x]]
```

3.734.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.55

method	result	size
derivativedivides	$\frac{\sec(3x)^2+5}{3\sqrt{\frac{\sec(3x)^2+5}{\sec(3x)^2}}\,\sec(3x)}$	34
default	$\frac{\sec(3x)^2+5}{3\sqrt{\frac{\sec(3x)^2+5}{\sec(3x)^2}}\,\sec(3x)}$	34

input
```
int(sec(3*x)*tan(3*x)/(1+5*cos(3*x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/((sec(3*x)^2+5)/sec(3*x)^2)^(1/2)/sec(3*x)*(sec(3*x)^2+5)
```

3.734.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sec(3x)\tan(3x)}{\sqrt{1+5\cos^2(3x)}}\,dx = \frac{\sqrt{5\,\cos\left(3\,x\right)^2+1}}{3\,\cos\left(3\,x\right)}$$

input
```
integrate(sec(3*x)*tan(3*x)/(1+5*cos(3*x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(5*cos(3*x)^2 + 1)/cos(3*x)
```

3.734.6 Sympy [F]

$$\int \frac{\sec(3x)\tan(3x)}{\sqrt{1+5\cos^2(3x)}}\,dx = \int \frac{\tan(3x)\sec(3x)}{\sqrt{5\cos^2(3x)+1}}\,dx$$

input
```
integrate(sec(3*x)*tan(3*x)/(1+5*cos(3*x)**2)**(1/2),x)
```

output
```
Integral(tan(3*x)*sec(3*x)/sqrt(5*cos(3*x)**2 + 1), x)
```

3.734.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sec(3x)\tan(3x)}{\sqrt{1+5\cos^2(3x)}}\,dx = \frac{\sqrt{5\cos(3x)^2+1}}{3\cos(3x)}$$

input
```
integrate(sec(3*x)*tan(3*x)/(1+5*cos(3*x)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(5*cos(3*x)^2 + 1)/cos(3*x)
```

3.734.8 Giac [F]

$$\int \frac{\sec(3x)\tan(3x)}{\sqrt{1+5\cos^2(3x)}}\,dx = \int \frac{\sec(3x)\tan(3x)}{\sqrt{5\cos(3x)^2+1}}\,dx$$

input
```
integrate(sec(3*x)*tan(3*x)/(1+5*cos(3*x)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sec(3*x)*tan(3*x)/sqrt(5*cos(3*x)^2 + 1), x)
```

3.734.9 Mupad [B] (verification not implemented)

Time = 26.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{\sec(3x)\tan(3x)}{\sqrt{1+5\cos^2(3x)}}\,dx = \frac{\sqrt{\frac{5\,\cos(6\,x)}{2}+\frac{7}{2}}}{3\,\cos(3\,x)}$$

input
```
int(tan(3*x)/(cos(3*x)*(5*cos(3*x)^2 + 1)^(1/2)),x)
```

output
```
((5*cos(6*x))/2 + 7/2)^(1/2)/(3*cos(3*x))
```

3.735 $\int \frac{\cot(x)\csc(x)}{a+b\csc(x)}\,dx$

3.735.1 Optimal result

Integrand size = 13, antiderivative size = 12

$$\int \frac{\cot(x)\csc(x)}{a+b\csc(x)}\,dx = -\frac{\log(a+b\csc(x))}{b}$$

output
```
-ln(a+b*csc(x))/b
```

3.735.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \frac{\cot(x)\csc(x)}{a+b\csc(x)}\,dx = \frac{\log(\sin(x))}{b} - \frac{\log(b+a\sin(x))}{b}$$

input
```
Integrate[(Cot[x]*Csc[x])/(a + b*Csc[x]),x]
```

output
```
Log[Sin[x]]/b - Log[b + a*Sin[x]]/b
```

3.735.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3042, 4838, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(x)\csc(x)}{a+b\csc(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cot(x)\csc(x)}{a+b\csc(x)}dx$$

$$\Big\downarrow \text{4838}$$

$$\int \frac{\csc(x)}{a\sin(x)+b}d\sin(x)$$

$$\Big\downarrow \text{47}$$

$$\frac{\int \csc(x)d\sin(x)}{b}-\frac{a\int \frac{1}{b+a\sin(x)}d\sin(x)}{b}$$

$$\Big\downarrow \text{14}$$

$$\frac{\log(\sin(x))}{b}-\frac{a\int \frac{1}{b+a\sin(x)}d\sin(x)}{b}$$

$$\Big\downarrow \text{16}$$

$$\frac{\log(\sin(x))}{b}-\frac{\log(a\sin(x)+b)}{b}$$

input
```
Int[(Cot[x]*Csc[x])/(a + b*Csc[x]),x]
```

output
```
Log[Sin[x]]/b - Log[b + a*Sin[x]]/b
```

3.735.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4838
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[1/(b*c)   Subst[Int[SubstFor[1/x, Sin[c*(a +
 b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

3.735.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$-\frac{\ln(a+b\csc(x))}{b}$	13
default	$-\frac{\ln(a+b\csc(x))}{b}$	13
risch	$-\frac{\ln\left(\mathrm{e}^{2ix}-1+\frac{2ib\,\mathrm{e}^{ix}}{a}\right)}{b}+\frac{\ln(\mathrm{e}^{2ix}-1)}{b}$	39

input
```
int(cot(x)*csc(x)/(a+b*csc(x)),x,method=_RETURNVERBOSE)
```

output
```
-ln(a+b*csc(x))/b
```

3.735.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \frac{\cot(x)\csc(x)}{a + b\csc(x)}\,dx = -\frac{\log\left(a\sin\left(x\right) + b\right) - \log\left(-\frac{1}{2}\,\sin\left(x\right)\right)}{b}$$

input
```
integrate(cot(x)*csc(x)/(a+b*csc(x)),x, algorithm="fricas")
```

output
```
-(log(a*sin(x) + b) - log(-1/2*sin(x)))/b
```

3.735.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \frac{\cot(x)\csc(x)}{a + b\csc(x)}\,dx = \begin{cases} -\frac{\log\left(\frac{a}{b} + \csc\left(x\right)\right)}{b} & \text{for}\: b \neq 0 \\ -\frac{\csc\left(x\right)}{a} & \text{otherwise} \end{cases}$$

input
```
integrate(cot(x)*csc(x)/(a+b*csc(x)),x)
```

output
```
Piecewise((-log(a/b + csc(x))/b, Ne(b, 0)), (-csc(x)/a, True))
```

3.735.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\cot(x)\csc(x)}{a + b\csc(x)}\,dx = -\frac{\log\left(b\csc\left(x\right) + a\right)}{b}$$

input
```
integrate(cot(x)*csc(x)/(a+b*csc(x)),x, algorithm="maxima")
```

output
```
-log(b*csc(x) + a)/b
```

3.735.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\cot(x)\csc(x)}{a+b\csc(x)}\,dx = -\frac{\log(|a\sin(x)+b|)}{b} + \frac{\log(|\sin(x)|)}{b}$$

input
```
integrate(cot(x)*csc(x)/(a+b*csc(x)),x, algorithm="giac")
```

output
```
-log(abs(a*sin(x) + b))/b + log(abs(sin(x)))/b
```

3.735.9 Mupad [B] (verification not implemented)

Time = 26.75 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.58

$$\int \frac{\cot(x)\csc(x)}{a+b\csc(x)}\,dx = -\frac{\ln\left(b\tan\left(\frac{x}{2}\right)^2 + 2a\tan\left(\frac{x}{2}\right) + b\right) - \ln\left(\tan\left(\frac{x}{2}\right)\right)}{b}$$

input
```
int(cot(x)/(sin(x)*(a + b/sin(x))),x)
```

output
```
-(log(b + 2*a*tan(x/2) + b*tan(x/2)^2) - log(tan(x/2)))/b
```

3.736 $\int 5^{\csc(3x)} \cot(3x) \csc(3x) \, dx$

3.736.1 Optimal result

Integrand size = 15, antiderivative size = 14

$$\int 5^{\csc(3x)} \cot(3x) \csc(3x) \, dx = -\frac{5^{\csc(3x)}}{3\log(5)}$$

output
```
-1/3*5^csc(3*x)/ln(5)
```

3.736.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int 5^{\csc(3x)} \cot(3x) \csc(3x) \, dx = -\frac{5^{\csc(3x)}}{3\log(5)}$$

input
```
Integrate[5^Csc[3*x]*Cot[3*x]*Csc[3*x],x]
```

output
```
-1/3*5^Csc[3*x]/Log[5]
```

3.736.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4838, 2638}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(3x)5^{\csc(3x)}\csc(3x)\,dx$$

$$\Big\downarrow \text{4838}$$

$$\frac{1}{3}\int 5^{\csc(3x)}\csc^2(3x)d\sin(3x)$$

$$\Big\downarrow \text{2638}$$

$$-\frac{5^{\csc(3x)}}{3\log(5)}$$

input
```
Int[5^Csc[3*x]*Cot[3*x]*Csc[3*x],x]
```

output
```
-1/3*5^Csc[3*x]/Log[5]
```

3.736.3.1 Defintions of rubi rules used

rule 2638
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^n*(F^(a + b*(c + d*x)^n)/(b*f*n*(c + d*x)^n
*Log[F])), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

rule 4838
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[1/(b*c)   Subst[Int[SubstFor[1/x, Sin[c*(a +
 b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

3.736.4 Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
risch	$-\frac{5^{\csc(3x)}}{3\ln(5)}$	13
derivativedivides	$-\frac{5^{\frac{1}{4\cos(x)^2\sin(x)-\sin(x)}}}{3\ln(5)}$	24
default	$-\frac{5^{\frac{1}{4\cos(x)^2\sin(x)-\sin(x)}}}{3\ln(5)}$	24

input
```
int(5^csc(3*x)*cot(3*x)*csc(3*x),x,method=_RETURNVERBOSE)
```

output
```
-1/3*5^csc(3*x)/ln(5)
```

3.736.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int 5^{\csc(3x)}\cot(3x)\csc(3x)\,dx=-\frac{5^{\left(\frac{1}{\sin(3x)}\right)}}{3\log(5)}$$

input
```
integrate(5^csc(3*x)*cot(3*x)*csc(3*x),x, algorithm="fricas")
```

output
```
-1/3*5^(1/sin(3*x))/log(5)
```

3.736.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int 5^{\csc(3x)}\cot(3x)\csc(3x)\,dx=-\frac{5^{\csc(3x)}}{3\log(5)}$$

input
```
integrate(5**csc(3*x)*cot(3*x)*csc(3*x),x)
```

output
```
-5**csc(3*x)/(3*log(5))
```

3.736.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int 5^{\csc(3x)} \cot(3x) \csc(3x)\, dx = -\frac{5^{\csc(3\,x)}}{3\,\log(5)}$$

input
```
integrate(5^csc(3*x)*cot(3*x)*csc(3*x),x, algorithm="maxima")
```

output
```
-1/3*5^csc(3*x)/log(5)
```

3.736.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int 5^{\csc(3x)} \cot(3x) \csc(3x)\, dx = -\frac{5^{\left(\frac{1}{\sin(3\,x)}\right)}}{3\,\log(5)}$$

input
```
integrate(5^csc(3*x)*cot(3*x)*csc(3*x),x, algorithm="giac")
```

output
```
-1/3*5^(1/sin(3*x))/log(5)
```

3.736.9 Mupad [B] (verification not implemented)

Time = 26.65 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int 5^{\csc(3x)} \cot(3x) \csc(3x)\, dx = -\frac{5^{\frac{1}{\sin(3\,x)}}}{3\,\ln(5)}$$

input
```
int((5^(1/sin(3*x))*cot(3*x))/sin(3*x),x)
```

output
```
-5^(1/sin(3*x))/(3*log(5))
```

3.737 $\int \frac{\cot(x)\csc(x)}{1+\csc^2(x)}\,dx$

3.737.1 Optimal result

Integrand size = 13, antiderivative size = 3

$$\int \frac{\cot(x)\csc(x)}{1+\csc^2(x)}\,dx = \arctan(\sin(x))$$

output
```
arctan(sin(x))
```

3.737.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{\cot(x)\csc(x)}{1+\csc^2(x)}\,dx = \arctan(\sin(x))$$

input
```
Integrate[(Cot[x]*Csc[x])/(1 + Csc[x]^2),x]
```

output
```
ArcTan[Sin[x]]
```

3.737.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 4838, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(x)\csc(x)}{\csc^2(x)+1}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\cot(x)\csc(x)}{\csc(x)^2+1}dx$$

$$\Big\downarrow \ 4838$$

$$\int \frac{1}{\sin^2(x)+1}d\sin(x)$$

$$\Big\downarrow \ 216$$

$$\arctan(\sin(x))$$

input
```
Int[(Cot[x]*Csc[x])/(1 + Csc[x]^2),x]
```

output
```
ArcTan[Sin[x]]
```

3.737.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4838
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[1/(b*c)   Subst[Int[SubstFor[1/x, Sin[c*(a +
 b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

3.737.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 6, normalized size of antiderivative = 2.00

method	result	size
derivativedivides	$-\arctan\left(\csc\left(x\right)\right)$	6
default	$-\arctan\left(\csc\left(x\right)\right)$	6
risch	$\frac{i\ln\left(e^{2ix}-2\,e^{ix}-1\right)}{2}-\frac{i\ln\left(e^{2ix}+2\,e^{ix}-1\right)}{2}$	38

input
```
int(cot(x)*csc(x)/(1+csc(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-arctan(csc(x))
```

3.737.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{\cot(x)\csc(x)}{1+\csc^2(x)}\,dx = \arctan\left(\sin\left(x\right)\right)$$

input
```
integrate(cot(x)*csc(x)/(1+csc(x)^2),x, algorithm="fricas")
```

output
```
arctan(sin(x))
```

3.737.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.67

$$\int \frac{\cot(x)\csc(x)}{1+\csc^2(x)}\,dx = -\operatorname{atan}\left(\csc\left(x\right)\right)$$

input
```
integrate(cot(x)*csc(x)/(1+csc(x)**2),x)
```

output
```
-atan(csc(x))
```

3.737.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{\cot(x)\csc(x)}{1+\csc^2(x)}\,dx = \arctan\left(\sin\left(x\right)\right)$$

input
```
integrate(cot(x)*csc(x)/(1+csc(x)^2),x, algorithm="maxima")
```

output
```
arctan(sin(x))
```

3.737.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{\cot(x)\csc(x)}{1+\csc^2(x)}\,dx = \arctan\left(\sin\left(x\right)\right)$$

input
```
integrate(cot(x)*csc(x)/(1+csc(x)^2),x, algorithm="giac")
```

output
```
arctan(sin(x))
```

3.737.9 Mupad [B] (verification not implemented)

Time = 27.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 8.67

$$\int \frac{\cot(x)\csc(x)}{1+\csc^2(x)}\,dx = \operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)^3}{2}+\frac{5\tan\left(\frac{x}{2}\right)}{2}\right)-\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{2}\right)$$

input
```
int(cot(x)/(sin(x)*(1/sin(x)^2 + 1)),x)
```

output
```
atan((5*tan(x/2))/2 + tan(x/2)^3/2) - atan(tan(x/2)/2)
```

3.738 $\int \frac{\cot(6x)\csc(6x)}{(5-11\csc^2(6x))^2}\,dx$

3.738.1 Optimal result

Integrand size = 21, antiderivative size = 43

$$\int \frac{\cot(6x)\csc(6x)}{(5-11\csc^2(6x))^2}\,dx = -\frac{\operatorname{arctanh}\left(\sqrt{\frac{5}{11}}\sin(6x)\right)}{60\sqrt{55}} + \frac{\sin(6x)}{60\,(11-5\sin^2(6x))}$$

output
```
1/60*sin(6*x)/(11-5*sin(6*x)^2)-1/3300*arctanh(1/11*sin(6*x)*55^(1/2))*55^
(1/2)
```

3.738.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int \frac{\cot(6x)\csc(6x)}{(5-11\csc^2(6x))^2}\,dx = -\frac{\operatorname{arctanh}\left(\sqrt{\frac{5}{11}}\sin(6x)\right)}{60\sqrt{55}} + \frac{\sin(6x)}{30(17+5\cos(12x))}$$

input
```
Integrate[(Cot[6*x]*Csc[6*x])/(5 - 11*Csc[6*x]^2)^2,x]
```

output
```
-1/60*ArcTanh[Sqrt[5/11]*Sin[6*x]]/Sqrt[55] + Sin[6*x]/(30*(17 + 5*Cos[12*
x]))
```

3.738.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 4838, 252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(6x)\csc(6x)}{(5-11\csc^2(6x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cot(6x)\csc(6x)}{(5-11\csc(6x)^2)^2}dx$$

$$\Big\downarrow \text{4838}$$

$$\frac{1}{6}\int \frac{\sin^2(6x)}{(11-5\sin^2(6x))^2}d\sin(6x)$$

$$\Big\downarrow \text{252}$$

$$\frac{1}{6}\left(\frac{\sin(6x)}{10(11-5\sin^2(6x))}-\frac{1}{10}\int \frac{1}{11-5\sin^2(6x)}d\sin(6x)\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{6}\left(\frac{\sin(6x)}{10(11-5\sin^2(6x))}-\frac{\operatorname{arctanh}\left(\sqrt{\frac{5}{11}}\sin(6x)\right)}{10\sqrt{55}}\right)$$

input
```
Int[(Cot[6*x]*Csc[6*x])/(5 - 11*Csc[6*x]^2)^2,x]
```

output
```
(-1/10*ArcTanh[Sqrt[5/11]*Sin[6*x]]/Sqrt[55] + Sin[6*x]/(10*(11 - 5*Sin[6*
x]^2)))/6
```

3.738.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4838
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[1/(b*c)   Subst[Int[SubstFor[1/x, Sin[c*(a +
 b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b
*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

3.738.4 Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{\csc(6x)}{660\csc(6x)^2-300}-\frac{\sqrt{55}\,\operatorname{arctanh}\left(\frac{\csc(6x)\sqrt{55}}{5}\right)}{3300}$	35
default	$\frac{\csc(6x)}{660\csc(6x)^2-300}-\frac{\sqrt{55}\,\operatorname{arctanh}\left(\frac{\csc(6x)\sqrt{55}}{5}\right)}{3300}$	35
risch	$-\frac{i\left(e^{18ix}-e^{6ix}\right)}{30\left(5\,e^{24ix}+34\,e^{12ix}+5\right)}-\frac{\sqrt{55}\,\ln\left(e^{12ix}+\frac{2i\sqrt{55}\,e^{6ix}}{5}-1\right)}{6600}+\frac{\sqrt{55}\,\ln\left(e^{12ix}-\frac{2i\sqrt{55}\,e^{6ix}}{5}-1\right)}{6600}$	84

input
```
int(cot(6*x)*csc(6*x)/(5-11*csc(6*x)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/60*csc(6*x)/(11*csc(6*x)^2-5)-1/3300*55^(1/2)*arctanh(1/5*csc(6*x)*55^(1
/2))
```

3.738.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(34) = 68$.

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.70

$$\int \frac{\cot(6x)\csc(6x)}{(5-11\csc^2(6x))^2}\,dx = \frac{\left(5\sqrt{55}\cos(6x)^2+6\sqrt{55}\right)\log\left(-\frac{5\cos(6x)^2+2\sqrt{55}\sin(6x)-16}{5\cos(6x)^2+6}\right)+110\sin(6x)}{6600\left(5\cos(6x)^2+6\right)}$$

input
```
integrate(cot(6*x)*csc(6*x)/(5-11*csc(6*x)^2)^2,x, algorithm="fricas")
```

output
```
1/6600*((5*sqrt(55)*cos(6*x)^2 + 6*sqrt(55))*log(-(5*cos(6*x)^2 + 2*sqrt(5
5)*sin(6*x) - 16)/(5*cos(6*x)^2 + 6)) + 110*sin(6*x))/(5*cos(6*x)^2 + 6)
```

3.738.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(34) = 68$.

Time = 0.52 (sec) , antiderivative size = 151, normalized size of antiderivative = 3.51

$$\begin{aligned}\int \frac{\cot(6x)\csc(6x)}{(5-11\csc^2(6x))^2}\,dx &= \frac{11\sqrt{55}\log\left(\csc(6x)-\frac{\sqrt{55}}{11}\right)\csc^2(6x)}{72600\csc^2(6x)-33000} \\ &- \frac{5\sqrt{55}\log\left(\csc(6x)-\frac{\sqrt{55}}{11}\right)}{72600\csc^2(6x)-33000} \\ &- \frac{11\sqrt{55}\log\left(\csc(6x)+\frac{\sqrt{55}}{11}\right)\csc^2(6x)}{72600\csc^2(6x)-33000} \\ &+ \frac{5\sqrt{55}\log\left(\csc(6x)+\frac{\sqrt{55}}{11}\right)}{72600\csc^2(6x)-33000} + \frac{110\csc(6x)}{72600\csc^2(6x)-33000}\end{aligned}$$

input
```
integrate(cot(6*x)*csc(6*x)/(5-11*csc(6*x)**2)**2,x)
```

output
```
11*sqrt(55)*log(csc(6*x) - sqrt(55)/11)*csc(6*x)**2/(72600*csc(6*x)**2 - 3
3000) - 5*sqrt(55)*log(csc(6*x) - sqrt(55)/11)/(72600*csc(6*x)**2 - 33000)
 - 11*sqrt(55)*log(csc(6*x) + sqrt(55)/11)*csc(6*x)**2/(72600*csc(6*x)**2
- 33000) + 5*sqrt(55)*log(csc(6*x) + sqrt(55)/11)/(72600*csc(6*x)**2 - 330
00) + 110*csc(6*x)/(72600*csc(6*x)**2 - 33000)
```

3.738.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.14

$$\int \frac{\cot(6x)\csc(6x)}{(5-11\csc^2(6x))^2}\,dx = \frac{1}{6600}\sqrt{55}\log\left(-\frac{\sqrt{55}-5\,\sin(6\,x)}{\sqrt{55}+5\,\sin(6\,x)}\right) - \frac{\sin(6\,x)}{60\,\left(5\,\sin(6\,x)^2-11\right)}$$

input
```
integrate(cot(6*x)*csc(6*x)/(5-11*csc(6*x)^2)^2,x, algorithm="maxima")
```

output
```
1/6600*sqrt(55)*log(-(sqrt(55) - 5*sin(6*x))/(sqrt(55) + 5*sin(6*x))) - 1/
60*sin(6*x)/(5*sin(6*x)^2 - 11)
```

3.738.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12

$$\int \frac{\cot(6x)\csc(6x)}{(5-11\csc^2(6x))^2}\,dx = \frac{1}{6600}\sqrt{55}\log\left(\frac{\sqrt{55}-5\,\sin(6\,x)}{\sqrt{55}+5\,\sin(6\,x)}\right) - \frac{\sin(6\,x)}{60\,\left(5\,\sin(6\,x)^2-11\right)}$$

input
```
integrate(cot(6*x)*csc(6*x)/(5-11*csc(6*x)^2)^2,x, algorithm="giac")
```

output
```
1/6600*sqrt(55)*log((sqrt(55) - 5*sin(6*x))/(sqrt(55) + 5*sin(6*x))) - 1/6
0*sin(6*x)/(5*sin(6*x)^2 - 11)
```

3.738.9 Mupad [B] (verification not implemented)

Time = 27.44 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.33

$$\int \frac{\cot(6x)\csc(6x)}{(5-11\csc^2(6x))^2}\,dx = -\frac{55\,\sin(6\,x)-11\,\sqrt{55}\,\mathrm{atanh}\left(\frac{\sqrt{55}\,\sin(6\,x)}{11}\right)+5\,\sqrt{55}\,\sin(6\,x)^2\,\mathrm{atanh}\left(\frac{\sqrt{55}\,\sin(6\,x)}{11}\right)}{16500\,\sin(6\,x)^2-36300}$$

input
```
int(cot(6*x)/(sin(6*x)*(11/sin(6*x)^2 - 5)^2),x)
```

output
```
-(55*sin(6*x) - 11*55^(1/2)*atanh((55^(1/2)*sin(6*x))/11) + 5*55^(1/2)*sin
(6*x)^2*atanh((55^(1/2)*sin(6*x))/11))/(16500*sin(6*x)^2 - 36300)
```

3.739 $\int \frac{\cot(x)\csc(x)}{\sqrt{1+\sin^2(x)}}\,dx$

3.739.1 Optimal result

Integrand size = 15, antiderivative size = 14

$$\int \frac{\cot(x)\csc(x)}{\sqrt{1+\sin^2(x)}}\,dx = -\csc(x)\sqrt{1+\sin^2(x)}$$

output
```
-csc(x)*(1+sin(x)^2)^(1/2)
```

3.739.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\cot(x)\csc(x)}{\sqrt{1+\sin^2(x)}}\,dx = -\csc(x)\sqrt{1+\sin^2(x)}$$

input
```
Integrate[(Cot[x]*Csc[x])/Sqrt[1 + Sin[x]^2],x]
```

output
```
-(Csc[x]*Sqrt[1 + Sin[x]^2])
```

3.739.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4878, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(x)\csc(x)}{\sqrt{\sin^2(x)+1}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin(x)\sqrt{\sin(x)^2+1}\tan(x)}dx$$

$$\Big\downarrow \text{ 4878}$$

$$\int \frac{\csc^2(x)}{\sqrt{\sin^2(x)+1}}d\sin(x)$$

$$\Big\downarrow \text{ 242}$$

$$\sqrt{\sin^2(x)+1}(-\csc(x))$$

input
```
Int[(Cot[x]*Csc[x])/Sqrt[1 + Sin[x]^2],x]
```

output
```
-(Csc[x]*Sqrt[1 + Sin[x]^2])
```

3.739.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

3.739.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result	size
default	$-\frac{\sqrt{\sin(x)^2+1}}{\sin(x)}$	15

input
```
int(cot(x)*csc(x)/(sin(x)^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/sin(x)*(sin(x)^2+1)^(1/2)
```

3.739.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.50

$$\int \frac{\cot(x)\csc(x)}{\sqrt{1+\sin^2(x)}}\,dx = -\frac{\sqrt{-\cos(x)^2+2}-\sin(x)}{\sin(x)}$$

input
```
integrate(cot(x)*csc(x)/(1+sin(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-(sqrt(-cos(x)^2 + 2) - sin(x))/sin(x)
```

3.739.6 Sympy [F]

$$\int \frac{\cot(x)\csc(x)}{\sqrt{1+\sin^2(x)}}\,dx = \int \frac{\cot(x)\csc(x)}{\sqrt{\sin^2(x)+1}}\,dx$$

input
```
integrate(cot(x)*csc(x)/(1+sin(x)**2)**(1/2),x)
```

output
```
Integral(cot(x)*csc(x)/sqrt(sin(x)**2 + 1), x)
```

3.739.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\cot(x)\csc(x)}{\sqrt{1+\sin^2(x)}}\,dx = -\frac{\sqrt{\sin(x)^2+1}}{\sin(x)}$$

input
```
integrate(cot(x)*csc(x)/(1+sin(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(sin(x)^2 + 1)/sin(x)
```

3.739.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.50

$$\int \frac{\cot(x)\csc(x)}{\sqrt{1+\sin^2(x)}}\,dx = \frac{2}{\left(\sqrt{\sin(x)^2+1}-\sin(x)\right)^2-1}$$

input
```
integrate(cot(x)*csc(x)/(1+sin(x)^2)^(1/2),x, algorithm="giac")
```

output
```
2/((sqrt(sin(x)^2 + 1) - sin(x))^2 - 1)
```

3.739.9 Mupad [B] (verification not implemented)

Time = 26.83 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.43

$$\int \frac{\cot(x)\csc(x)}{\sqrt{1+\sin^2(x)}}\,dx = -\frac{\sqrt{\frac{1}{\sin(x)^2}+1}}{\sin(x)\,\left(\sqrt{\frac{1}{\sin(x)^2}+1}+1\right)\,\sqrt{\sin(x)^2+1}}$$

input
```
int(cot(x)/(sin(x)*(sin(x)^2 + 1)^(1/2)),x)
```

output
```
-(1/sin(x)^2 + 1)^(1/2)/(sin(x)*((1/sin(x)^2 + 1)^(1/2) + 1)*(sin(x)^2 + 1
)^(1/2))
```

3.740 $\int \frac{\cot(5x)\csc^3(5x)}{\sqrt{1+\sin^2(5x)}}\,dx$

3.740.1 Optimal result

Integrand size = 23, antiderivative size = 43

$$\int \frac{\cot(5x)\csc^3(5x)}{\sqrt{1+\sin^2(5x)}}\,dx = \frac{2}{15}\csc(5x)\sqrt{1+\sin^2(5x)} - \frac{1}{15}\csc^3(5x)\sqrt{1+\sin^2(5x)}$$

output
```
2/15*csc(5*x)*(1+sin(5*x)^2)^(1/2)-1/15*csc(5*x)^3*(1+sin(5*x)^2)^(1/2)
```

3.740.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.65

$$\int \frac{\cot(5x)\csc^3(5x)}{\sqrt{1+\sin^2(5x)}}\,dx = -\frac{1}{15}\csc(5x)\left(-2+\csc^2(5x)\right)\sqrt{1+\sin^2(5x)}$$

input
```
Integrate[(Cot[5*x]*Csc[5*x]^3)/Sqrt[1 + Sin[5*x]^2],x]
```

output
```
-1/15*(Csc[5*x]*(-2 + Csc[5*x]^2)*Sqrt[1 + Sin[5*x]^2])
```

3.740.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 4878, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(5x)\csc^3(5x)}{\sqrt{\sin^2(5x)+1}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin(5x)^3\sqrt{\sin(5x)^2+1}\tan(5x)}dx$$

$$\Big\downarrow \text{ 4878}$$

$$\frac{1}{5}\int \frac{\csc^4(5x)}{\sqrt{\sin^2(5x)+1}}d\sin(5x)$$

$$\Big\downarrow \text{ 245}$$

$$\frac{1}{5}\left(-\frac{2}{3}\int \frac{\csc^2(5x)}{\sqrt{\sin^2(5x)+1}}d\sin(5x)-\frac{1}{3}\sqrt{\sin^2(5x)+1}\csc^3(5x)\right)$$

$$\Big\downarrow \text{ 242}$$

$$\frac{1}{5}\left(\frac{2}{3}\sqrt{\sin^2(5x)+1}\csc(5x)-\frac{1}{3}\sqrt{\sin^2(5x)+1}\csc^3(5x)\right)$$

input
```
Int[(Cot[5*x]*Csc[5*x]^3)/Sqrt[1 + Sin[5*x]^2],x]
```

output
```
((2*Csc[5*x]*Sqrt[1 + Sin[5*x]^2])/3 - (Csc[5*x]^3*Sqrt[1 + Sin[5*x]^2])/3
)/5
```

3.740.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

3.740.4 Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{\sqrt{1+\sin(5x)^2}}{15\sin(5x)^3}+\frac{2\sqrt{1+\sin(5x)^2}}{15\sin(5x)}$	38

input
```
int(cot(5*x)*csc(5*x)^3/(1+sin(5*x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15/sin(5*x)^3*(1+sin(5*x)^2)^(1/2)+2/15/sin(5*x)*(1+sin(5*x)^2)^(1/2)
```

3.740.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.33

$$\int \frac{\cot(5x)\csc^3(5x)}{\sqrt{1+\sin^2(5x)}}\,dx = -\frac{2\left(\cos\left(5\,x\right)^2-1\right)\sin\left(5\,x\right)-\left(2\,\cos\left(5\,x\right)^2-1\right)\sqrt{-\cos\left(5\,x\right)^2+2}}{15\left(\cos\left(5\,x\right)^2-1\right)\sin\left(5\,x\right)}$$

input
```
integrate(cot(5*x)*csc(5*x)^3/(1+sin(5*x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/15*(2*(cos(5*x)^2 - 1)*sin(5*x) - (2*cos(5*x)^2 - 1)*sqrt(-cos(5*x)^2 +
 2))/((cos(5*x)^2 - 1)*sin(5*x))
```

3.740.6 Sympy [F]

$$\int \frac{\cot(5x)\csc^3(5x)}{\sqrt{1+\sin^2(5x)}}\,dx = \int \frac{\cot(5x)\csc^3(5x)}{\sqrt{\sin^2(5x)+1}}\,dx$$

input
```
integrate(cot(5*x)*csc(5*x)**3/(1+sin(5*x)**2)**(1/2),x)
```

output
```
Integral(cot(5*x)*csc(5*x)**3/sqrt(sin(5*x)**2 + 1), x)
```

3.740.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{\cot(5x)\csc^3(5x)}{\sqrt{1+\sin^2(5x)}}\,dx = \frac{2\sqrt{\sin\left(5\,x\right)^2+1}}{15\,\sin\left(5\,x\right)}-\frac{\sqrt{\sin\left(5\,x\right)^2+1}}{15\,\sin\left(5\,x\right)^3}$$

input
```
integrate(cot(5*x)*csc(5*x)^3/(1+sin(5*x)^2)^(1/2),x, algorithm="maxima")
```

output
```
2/15*sqrt(sin(5*x)^2 + 1)/sin(5*x) - 1/15*sqrt(sin(5*x)^2 + 1)/sin(5*x)^3
```

3.740.8 Giac [F]

$$\int \frac{\cot(5x)\csc^3(5x)}{\sqrt{1+\sin^2(5x)}}\,dx = \int \frac{\cot(5x)\csc(5x)^3}{\sqrt{\sin(5x)^2+1}}\,dx$$

input
```
integrate(cot(5*x)*csc(5*x)^3/(1+sin(5*x)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(cot(5*x)*csc(5*x)^3/sqrt(sin(5*x)^2 + 1), x)
```

3.740.9 Mupad [B] (verification not implemented)

Time = 26.34 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.65

$$\int \frac{\cot(5x)\csc^3(5x)}{\sqrt{1+\sin^2(5x)}}\,dx = \frac{\sqrt{\sin(5x)^2+1}\,\left(2\sin(5x)^2-1\right)}{15\sin(5x)^3}$$

input
```
int(cot(5*x)/(sin(5*x)^3*(sin(5*x)^2 + 1)^(1/2)),x)
```

output
```
((sin(5*x)^2 + 1)^(1/2)*(2*sin(5*x)^2 - 1))/(15*sin(5*x)^3)
```

3.741 $\int e^{n\sin(a+bx)}\sin(2a+2bx)\,dx$

3.741.1 Optimal result

Integrand size = 20, antiderivative size = 43

$$\int e^{n\sin(a+bx)}\sin(2a+2bx)\,dx=-\frac{2e^{n\sin(a+bx)}}{bn^2}+\frac{2e^{n\sin(a+bx)}\sin(a+bx)}{bn}$$

output
```
-2*exp(n*sin(b*x+a))/b/n^2+2*exp(n*sin(b*x+a))*sin(b*x+a)/b/n
```

3.741.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.65

$$\int e^{n\sin(a+bx)}\sin(2a+2bx)\,dx=\frac{2e^{n\sin(a+bx)}(-1+n\sin(a+bx))}{bn^2}$$

input
```
Integrate[E^(n*Sin[a + b*x])*Sin[2*a + 2*b*x],x]
```

output
```
(2*E^(n*Sin[a + b*x])*(-1 + n*Sin[a + b*x]))/(b*n^2)
```

3.741.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4878, 27, 2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(2a+2bx)e^{n\sin(a+bx)}\,dx$$

$$\Big\downarrow \text{4878}$$

$$\frac{\int 2e^{n\sin(a+bx)}\sin(a+bx)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int e^{n\sin(a+bx)}\sin(a+bx)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{2607}$$

$$\frac{2\left(\frac{\sin(a+bx)e^{n\sin(a+bx)}}{n}-\frac{\int e^{n\sin(a+bx)}d\sin(a+bx)}{n}\right)}{b}$$

$$\Big\downarrow \text{2624}$$

$$\frac{2\left(\frac{\sin(a+bx)e^{n\sin(a+bx)}}{n}-\frac{e^{n\sin(a+bx)}}{n^2}\right)}{b}$$

input
```
Int[E^(n*Sin[a + b*x])*Sin[2*a + 2*b*x],x]
```

output
```
(2*(-(E^(n*Sin[a + b*x])/n^2) + (E^(n*Sin[a + b*x])*Sin[a + b*x])/n))/b
```

3.741.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

3.741.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.33 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.42

method	result	size
risch	$-\frac{i \mathrm{e}^{n \sin(xb) \cos(a)+n \cos(xb) \sin(a)} \mathrm{e}^{ibx} \mathrm{e}^{ia}}{nb}+\frac{i \mathrm{e}^{n \sin(xb) \cos(a)+n \cos(xb) \sin(a)} \mathrm{e}^{-ibx} \mathrm{e}^{-ia}}{nb}-\frac{2\, \mathrm{e}^{n(\sin(xb) \cos(a)+\cos(xb) \sin(a))}}{n^{2} b}$	104

input
```
int(exp(n*sin(b*x+a))*sin(2*b*x+2*a),x,method=_RETURNVERBOSE)
```

output
```
-I/n/b*exp(n*sin(x*b)*cos(a)+n*cos(x*b)*sin(a))*exp(I*b*x)*exp(I*a)+I/n/b*
exp(n*sin(x*b)*cos(a)+n*cos(x*b)*sin(a))*exp(-I*x*b)*exp(-I*a)-2/n^2/b*exp
(n*(sin(x*b)*cos(a)+cos(x*b)*sin(a)))
```

3.741.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.63

$$\int e^{n\sin(a+bx)}\sin(2a+2bx)\,dx = \frac{2\,(n\sin{(bx+a)}-1)e^{(n\sin(bx+a))}}{bn^2}$$

input
```
integrate(exp(n*sin(b*x+a))*sin(2*b*x+2*a),x, algorithm="fricas")
```

output
```
2*(n*sin(b*x + a) - 1)*e^(n*sin(b*x + a))/(b*n^2)
```

3.741.6 Sympy [F]

$$\int e^{n\sin(a+bx)}\sin(2a+2bx)\,dx = \int e^{n\sin{(a+bx)}}\sin{(2a+2bx)}\,dx$$

input
```
integrate(exp(n*sin(b*x+a))*sin(2*b*x+2*a),x)
```

output
```
Integral(exp(n*sin(a + b*x))*sin(2*a + 2*b*x), x)
```

3.741.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int e^{n\sin(a+bx)}\sin(2a+2bx)\,dx = \frac{2\left(ne^{(n\sin(bx+a))}\sin{(bx+a)}-e^{(n\sin(bx+a))}\right)}{bn^2}$$

input
```
integrate(exp(n*sin(b*x+a))*sin(2*b*x+2*a),x, algorithm="maxima")
```

output
```
2*(n*e^(n*sin(b*x + a))*sin(b*x + a) - e^(n*sin(b*x + a)))/(b*n^2)
```

3.741.8 Giac [F]

$$\int e^{n\sin(a+bx)}\sin(2a+2bx)\,dx=\int e^{(n\sin(bx+a))}\sin\left(2\,bx+2\,a\right)\,dx$$

input
```
integrate(exp(n*sin(b*x+a))*sin(2*b*x+2*a),x, algorithm="giac")
```

output
```
integrate(e^(n*sin(b*x + a))*sin(2*b*x + 2*a), x)
```

3.741.9 Mupad [B] (verification not implemented)

Time = 27.67 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.63

$$\int e^{n\sin(a+bx)}\sin(2a+2bx)\,dx=\frac{2\,\mathrm{e}^{n\,\sin(a+b\,x)}\,\left(n\,\sin\left(a+b\,x\right)-1\right)}{b\,n^2}$$

input
```
int(exp(n*sin(a + b*x))*sin(2*a + 2*b*x),x)
```

output
```
(2*exp(n*sin(a + b*x))*(n*sin(a + b*x) - 1))/(b*n^2)
```

3.742 $\int e^{n\sin(a+bx)}\sin(2(a+bx))\,dx$

3.742.1 Optimal result

Integrand size = 19, antiderivative size = 43

$$\int e^{n\sin(a+bx)}\sin(2(a+bx))\,dx=-\frac{2e^{n\sin(a+bx)}}{bn^2}+\frac{2e^{n\sin(a+bx)}\sin(a+bx)}{bn}$$

output
```
-2*exp(n*sin(b*x+a))/b/n^2+2*exp(n*sin(b*x+a))*sin(b*x+a)/b/n
```

3.742.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.65

$$\int e^{n\sin(a+bx)}\sin(2(a+bx))\,dx=\frac{2e^{n\sin(a+bx)}(-1+n\sin(a+bx))}{bn^2}$$

input
```
Integrate[E^(n*Sin[a + b*x])*Sin[2*(a + b*x)],x]
```

output
```
(2*E^(n*Sin[a + b*x])*(-1 + n*Sin[a + b*x]))/(b*n^2)
```

3.742.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {4878, 27, 2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(2(a+bx))e^{n\sin(a+bx)}\,dx$$

$$\Big\downarrow \text{4878}$$

$$\frac{\int 2e^{n\sin(a+bx)}\sin(a+bx)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int e^{n\sin(a+bx)}\sin(a+bx)d\sin(a+bx)}{b}$$

$$\Big\downarrow \text{2607}$$

$$\frac{2\left(\frac{\sin(a+bx)e^{n\sin(a+bx)}}{n}-\frac{\int e^{n\sin(a+bx)}d\sin(a+bx)}{n}\right)}{b}$$

$$\Big\downarrow \text{2624}$$

$$\frac{2\left(\frac{\sin(a+bx)e^{n\sin(a+bx)}}{n}-\frac{e^{n\sin(a+bx)}}{n^2}\right)}{b}$$

input
```
Int[E^(n*Sin[a + b*x])*Sin[2*(a + b*x)],x]
```

output
```
(2*(-(E^(n*Sin[a + b*x])/n^2) + (E^(n*Sin[a + b*x])*Sin[a + b*x])/n))/b
```

3.742.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

3.742.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.11 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.42

method	result	size
risch	$-\frac{i e^{n \sin(xb) \cos(a)+n \cos(xb) \sin(a)} e^{ibx} e^{ia}}{nb}+\frac{i e^{n \sin(xb) \cos(a)+n \cos(xb) \sin(a)} e^{-ibx} e^{-ia}}{nb}-\frac{2 e^{n(\sin(xb) \cos(a)+\cos(xb) \sin(a))}}{n^2 b}$	104

input
```
int(exp(n*sin(b*x+a))*sin(2*b*x+2*a),x,method=_RETURNVERBOSE)
```

output
```
-I/n/b*exp(n*sin(x*b)*cos(a)+n*cos(x*b)*sin(a))*exp(I*b*x)*exp(I*a)+I/n/b*
exp(n*sin(x*b)*cos(a)+n*cos(x*b)*sin(a))*exp(-I*x*b)*exp(-I*a)-2/n^2/b*exp
(n*(sin(x*b)*cos(a)+cos(x*b)*sin(a)))
```

3.742.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.63

$$\int e^{n\sin(a+bx)}\sin(2(a+bx))\,dx = \frac{2\,(n\sin{(bx+a)}-1)e^{(n\sin(bx+a))}}{bn^2}$$

input
```
integrate(exp(n*sin(b*x+a))*sin(2*b*x+2*a),x, algorithm="fricas")
```

output
```
2*(n*sin(b*x + a) - 1)*e^(n*sin(b*x + a))/(b*n^2)
```

3.742.6 Sympy [F]

$$\int e^{n\sin(a+bx)}\sin(2(a+bx))\,dx = \int e^{n\sin{(a+bx)}}\sin{(2a+2bx)}\,dx$$

input
```
integrate(exp(n*sin(b*x+a))*sin(2*b*x+2*a),x)
```

output
```
Integral(exp(n*sin(a + b*x))*sin(2*a + 2*b*x), x)
```

3.742.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int e^{n\sin(a+bx)}\sin(2(a+bx))\,dx = \frac{2\,\left(ne^{(n\sin(bx+a))}\sin{(bx+a)}-e^{(n\sin(bx+a))}\right)}{bn^2}$$

input
```
integrate(exp(n*sin(b*x+a))*sin(2*b*x+2*a),x, algorithm="maxima")
```

output
```
2*(n*e^(n*sin(b*x + a))*sin(b*x + a) - e^(n*sin(b*x + a)))/(b*n^2)
```

3.742.8 Giac [F]

$$\int e^{n\sin(a+bx)}\sin(2(a+bx))\,dx = \int e^{(n\sin(bx+a))}\sin\left(2\,bx+2\,a\right)\,dx$$

input
```
integrate(exp(n*sin(b*x+a))*sin(2*b*x+2*a),x, algorithm="giac")
```

output
```
integrate(e^(n*sin(b*x + a))*sin(2*b*x + 2*a), x)
```

3.742.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.63

$$\int e^{n\sin(a+bx)}\sin(2(a+bx))\,dx = \frac{2\,e^{n\,\sin(a+b\,x)}\,(n\,\sin\left(a+b\,x\right)-1)}{b\,n^2}$$

input
```
int(exp(n*sin(a + b*x))*sin(2*a + 2*b*x),x)
```

output
```
(2*exp(n*sin(a + b*x))*(n*sin(a + b*x) - 1))/(b*n^2)
```

3.743 $\int e^{n \sin \left(\frac{a}{2}+\frac{bx}{2}\right)} \sin(a+bx)\,dx$

3.743.1 Optimal result

Integrand size = 24, antiderivative size = 64

$$\int e^{n \sin \left(\frac{a}{2}+\frac{bx}{2}\right)} \sin(a+bx)\,dx=-\frac{4e^{n \sin \left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}+\frac{4e^{n \sin \left(\frac{a}{2}+\frac{bx}{2}\right)} \sin \left(\frac{a}{2}+\frac{bx}{2}\right)}{bn}$$

output
```
-4*exp(n*sin(1/2*a+1/2*b*x))/b/n^2+4*exp(n*sin(1/2*a+1/2*b*x))*sin(1/2*a+1
/2*b*x)/b/n
```

3.743.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.56

$$\int e^{n \sin \left(\frac{a}{2}+\frac{bx}{2}\right)} \sin(a+bx)\,dx=\frac{4e^{n \sin \left(\frac{1}{2}(a+bx)\right)} \left(-1+n \sin \left(\frac{1}{2}(a+bx)\right)\right)}{bn^2}$$

input
```
Integrate[E^(n*Sin[a/2 + (b*x)/2])*Sin[a + b*x],x]
```

output
```
(4*E^(n*Sin[(a + b*x)/2])*(-1 + n*Sin[(a + b*x)/2]))/(b*n^2)
```

3.743.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4878, 27, 2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}dx$$

$$\Big\downarrow \text{4878}$$

$$\frac{2\int 2e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}\sin\left(\frac{a}{2}+\frac{bx}{2}\right)d\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{4\int e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}\sin\left(\frac{a}{2}+\frac{bx}{2}\right)d\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}{b}$$

$$\Big\downarrow \text{2607}$$

$$\frac{4\left(\frac{\sin\left(\frac{a}{2}+\frac{bx}{2}\right)e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}}{n}-\frac{\int e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}d\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}{n}\right)}{b}$$

$$\Big\downarrow \text{2624}$$

$$\frac{4\left(\frac{\sin\left(\frac{a}{2}+\frac{bx}{2}\right)e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}}{n}-\frac{e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}}{n^2}\right)}{b}$$

input
```
Int[E^(n*Sin[a/2 + (b*x)/2])*Sin[a + b*x],x]
```

output
```
(4*(-(E^(n*Sin[a/2 + (b*x)/2])/n^2) + (E^(n*Sin[a/2 + (b*x)/2])*Sin[a/2 +
(b*x)/2])/n))/b
```

3.743.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

3.743.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.41 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.91

method	result
risch	$-\frac{2 i e^{n \sin(\frac{a}{2}) \cos(\frac{x b}{2})+n \cos(\frac{a}{2}) \sin(\frac{x b}{2})} e^{\frac{i b x}{2}} e^{\frac{i a}{2}}}{n b}+\frac{2 i e^{n \sin(\frac{a}{2}) \cos(\frac{x b}{2})+n \cos(\frac{a}{2}) \sin(\frac{x b}{2})} e^{-\frac{i b x}{2}} e^{-\frac{i a}{2}}}{n b}-\frac{4 e^{n(\sin(\frac{a}{2}) \cos(\frac{x b}{2})+\cos(}}{n^{2} b}$

input
```
int(exp(n*sin(1/2*a+1/2*x*b))*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-2*I/n/b*exp(n*sin(1/2*a)*cos(1/2*x*b)+n*cos(1/2*a)*sin(1/2*x*b))*exp(1/2*
I*b*x)*exp(1/2*I*a)+2*I/n/b*exp(n*sin(1/2*a)*cos(1/2*x*b)+n*cos(1/2*a)*sin
(1/2*x*b))*exp(-1/2*I*b*x)*exp(-1/2*I*a)-4/n^2/b*exp(n*(sin(1/2*a)*cos(1/2
*x*b)+cos(1/2*a)*sin(1/2*x*b)))
```

3.743.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.52

$$\int e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}\sin(a+bx)\,dx=\frac{4\left(n\sin\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)-1\right)e^{\left(n\sin\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)}}{bn^2}$$

input
```
integrate(exp(n*sin(1/2*b*x+1/2*a))*sin(b*x+a),x, algorithm="fricas")
```

output
```
4*(n*sin(1/2*b*x + 1/2*a) - 1)*e^(n*sin(1/2*b*x + 1/2*a))/(b*n^2)
```

3.743.6 Sympy [F]

$$\int e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}\sin(a+bx)\,dx=\int e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}\sin\left(a+bx\right)dx$$

input
```
integrate(exp(n*sin(1/2*b*x+1/2*a))*sin(b*x+a),x)
```

output
```
Integral(exp(n*sin(a/2 + b*x/2))*sin(a + b*x), x)
```

3.743.7 Maxima [F]

$$\int e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}\sin(a+bx)\,dx=\int e^{\left(n\sin\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)}\sin\left(bx+a\right)\,dx$$

input
```
integrate(exp(n*sin(1/2*b*x+1/2*a))*sin(b*x+a),x, algorithm="maxima")
```

output
```
integrate(e^(n*sin(1/2*b*x + 1/2*a))*sin(b*x + a), x)
```

3.743.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(50) = 100$.

Time = 0.32 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.16

$$\int e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}\sin(a+bx)\,dx = \frac{4\left(2\,ne^{\left(\frac{2\,n\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)}{\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2+1}\right)}\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)-e^{\left(\frac{2\,n\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)}{\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2+1}\right)}\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2-e^{\left(\frac{2\,n\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)}{\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2+1}\right)}\right)}{bn^2\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^2+bn^2}$$

input
```
integrate(exp(n*sin(1/2*b*x+1/2*a))*sin(b*x+a),x, algorithm="giac")
```

output
```
4*(2*n*e^(2*n*tan(1/4*b*x + 1/4*a)/(tan(1/4*b*x + 1/4*a)^2 + 1))*tan(1/4*b
*x + 1/4*a) - e^(2*n*tan(1/4*b*x + 1/4*a)/(tan(1/4*b*x + 1/4*a)^2 + 1))*ta
n(1/4*b*x + 1/4*a)^2 - e^(2*n*tan(1/4*b*x + 1/4*a)/(tan(1/4*b*x + 1/4*a)^2
 + 1)))/(b*n^2*tan(1/4*b*x + 1/4*a)^2 + b*n^2)
```

3.743.9 Mupad [B] (verification not implemented)

Time = 28.51 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.52

$$\int e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}\sin(a+bx)\,dx = \frac{4\,\mathrm{e}^{n\,\sin\left(\frac{a}{2}+\frac{b\,x}{2}\right)}\,\left(n\,\sin\left(\frac{a}{2}+\frac{b\,x}{2}\right)-1\right)}{b\,n^2}$$

input
```
int(exp(n*sin(a/2 + (b*x)/2))*sin(a + b*x),x)
```

output
```
(4*exp(n*sin(a/2 + (b*x)/2))*(n*sin(a/2 + (b*x)/2) - 1))/(b*n^2)
```

3.744 $\int e^{n \sin \left(\frac{1}{2}(a+bx)\right)} \sin(a+bx)\,dx$

3.744.1 Optimal result

Integrand size = 21, antiderivative size = 64

$$\int e^{n \sin \left(\frac{1}{2}(a+bx)\right)} \sin(a+bx)\,dx = -\frac{4e^{n \sin \left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}+\frac{4e^{n \sin \left(\frac{a}{2}+\frac{bx}{2}\right)} \sin \left(\frac{a}{2}+\frac{bx}{2}\right)}{bn}$$

output
```
-4*exp(n*sin(1/2*a+1/2*b*x))/b/n^2+4*exp(n*sin(1/2*a+1/2*b*x))*sin(1/2*a+1
/2*b*x)/b/n
```

3.744.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.56

$$\int e^{n \sin \left(\frac{1}{2}(a+bx)\right)} \sin(a+bx)\,dx = \frac{4e^{n \sin \left(\frac{1}{2}(a+bx)\right)} \left(-1+n \sin \left(\frac{1}{2}(a+bx)\right)\right)}{bn^2}$$

input
```
Integrate[E^(n*Sin[(a + b*x)/2])*Sin[a + b*x],x]
```

output
```
(4*E^(n*Sin[(a + b*x)/2])*(-1 + n*Sin[(a + b*x)/2]))/(b*n^2)
```

3.744.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {4878, 27, 2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)e^{n\sin\left(\frac{1}{2}(a+bx)\right)}dx$$

$$\Big\downarrow \text{4878}$$

$$\frac{2\int 2e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}\sin\left(\frac{a}{2}+\frac{bx}{2}\right)d\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{4\int e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}\sin\left(\frac{a}{2}+\frac{bx}{2}\right)d\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}{b}$$

$$\Big\downarrow \text{2607}$$

$$\frac{4\left(\frac{\sin\left(\frac{a}{2}+\frac{bx}{2}\right)e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}}{n}-\frac{\int e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}d\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}{n}\right)}{b}$$

$$\Big\downarrow \text{2624}$$

$$\frac{4\left(\frac{\sin\left(\frac{a}{2}+\frac{bx}{2}\right)e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}}{n}-\frac{e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}}{n^2}\right)}{b}$$

input
```
Int[E^(n*Sin[(a + b*x)/2])*Sin[a + b*x],x]
```

output
```
(4*(-(E^(n*Sin[a/2 + (b*x)/2])/n^2) + (E^(n*Sin[a/2 + (b*x)/2])*Sin[a/2 +
(b*x)/2])/n))/b
```

3.744.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

3.744.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.09 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.91

method	result
risch	$-\frac{2i\,e^{n \sin\left(\frac{a}{2}\right) \cos\left(\frac{xb}{2}\right)+n \cos\left(\frac{a}{2}\right) \sin\left(\frac{xb}{2}\right)} e^{\frac{ibx}{2}} e^{\frac{ia}{2}}}{nb}+\frac{2i\,e^{n \sin\left(\frac{a}{2}\right) \cos\left(\frac{xb}{2}\right)+n \cos\left(\frac{a}{2}\right) \sin\left(\frac{xb}{2}\right)} e^{-\frac{ibx}{2}} e^{-\frac{ia}{2}}}{nb}-\frac{4\,e^{n\left(\sin\left(\frac{a}{2}\right) \cos\left(\frac{xb}{2}\right)+\cos\left(\right.\right.}}{n^{2}b}$

input
```
int(exp(n*sin(1/2*a+1/2*x*b))*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-2*I/n/b*exp(n*sin(1/2*a)*cos(1/2*x*b)+n*cos(1/2*a)*sin(1/2*x*b))*exp(1/2*
I*b*x)*exp(1/2*I*a)+2*I/n/b*exp(n*sin(1/2*a)*cos(1/2*x*b)+n*cos(1/2*a)*sin
(1/2*x*b))*exp(-1/2*I*b*x)*exp(-1/2*I*a)-4/n^2/b*exp(n*(sin(1/2*a)*cos(1/2
*x*b)+cos(1/2*a)*sin(1/2*x*b)))
```

3.744.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.52

$$\int e^{n\sin\left(\frac{1}{2}(a+bx)\right)}\sin(a+bx)\,dx=\frac{4\left(n\sin\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)-1\right)e^{\left(n\sin\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)}}{bn^2}$$

input
```
integrate(exp(n*sin(1/2*b*x+1/2*a))*sin(b*x+a),x, algorithm="fricas")
```

output
```
4*(n*sin(1/2*b*x + 1/2*a) - 1)*e^(n*sin(1/2*b*x + 1/2*a))/(b*n^2)
```

3.744.6 Sympy [F]

$$\int e^{n\sin\left(\frac{1}{2}(a+bx)\right)}\sin(a+bx)\,dx=\int e^{n\sin\left(\frac{a}{2}+\frac{bx}{2}\right)}\sin\left(a+bx\right)dx$$

input
```
integrate(exp(n*sin(1/2*b*x+1/2*a))*sin(b*x+a),x)
```

output
```
Integral(exp(n*sin(a/2 + b*x/2))*sin(a + b*x), x)
```

3.744.7 Maxima [F]

$$\int e^{n\sin\left(\frac{1}{2}(a+bx)\right)}\sin(a+bx)\,dx=\int e^{\left(n\sin\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)}\sin\left(bx+a\right)\,dx$$

input
```
integrate(exp(n*sin(1/2*b*x+1/2*a))*sin(b*x+a),x, algorithm="maxima")
```

output
```
integrate(e^(n*sin(1/2*b*x + 1/2*a))*sin(b*x + a), x)
```

3.744.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(50) = 100.

Time = 0.33 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.16

$$\int e^{n\sin\left(\frac{1}{2}(a+bx)\right)}\sin(a+bx)\,dx = \frac{4\left(2\,ne^{\left(\frac{2\,n\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)}{\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^{2}+1}\right)}\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)-e^{\left(\frac{2\,n\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)}{\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^{2}+1}\right)}\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^{2}-e^{\left(\frac{2\,n\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)}{\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^{2}+1}\right)}\right)}{bn^{2}\tan\left(\frac{1}{4}\,bx+\frac{1}{4}\,a\right)^{2}+bn^{2}}$$

input
```
integrate(exp(n*sin(1/2*b*x+1/2*a))*sin(b*x+a),x, algorithm="giac")
```

output
```
4*(2*n*e^(2*n*tan(1/4*b*x + 1/4*a)/(tan(1/4*b*x + 1/4*a)^2 + 1))*tan(1/4*b
*x + 1/4*a) - e^(2*n*tan(1/4*b*x + 1/4*a)/(tan(1/4*b*x + 1/4*a)^2 + 1))*ta
n(1/4*b*x + 1/4*a)^2 - e^(2*n*tan(1/4*b*x + 1/4*a)/(tan(1/4*b*x + 1/4*a)^2
 + 1)))/(b*n^2*tan(1/4*b*x + 1/4*a)^2 + b*n^2)
```

3.744.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.52

$$\int e^{n\sin\left(\frac{1}{2}(a+bx)\right)}\sin(a+bx)\,dx = \frac{4\,\mathrm{e}^{n\,\sin\left(\frac{a}{2}+\frac{b\,x}{2}\right)}\,\left(n\,\sin\left(\frac{a}{2}+\frac{b\,x}{2}\right)-1\right)}{b\,n^2}$$

input
```
int(exp(n*sin(a/2 + (b*x)/2))*sin(a + b*x),x)
```

output
```
(4*exp(n*sin(a/2 + (b*x)/2))*(n*sin(a/2 + (b*x)/2) - 1))/(b*n^2)
```

3.745 $\int e^{n\cos(a+bx)}\sin(2a+2bx)\,dx$

3.745.1 Optimal result

Integrand size = 20, antiderivative size = 43

$$\int e^{n\cos(a+bx)}\sin(2a+2bx)\,dx=\frac{2e^{n\cos(a+bx)}}{bn^2}-\frac{2e^{n\cos(a+bx)}\cos(a+bx)}{bn}$$

output
```
2*exp(n*cos(b*x+a))/b/n^2-2*exp(n*cos(b*x+a))*cos(b*x+a)/b/n
```

3.745.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.65

$$\int e^{n\cos(a+bx)}\sin(2a+2bx)\,dx=-\frac{2e^{n\cos(a+bx)}(-1+n\cos(a+bx))}{bn^2}$$

input
```
Integrate[E^(n*Cos[a + b*x])*Sin[2*a + 2*b*x],x]
```

output
```
(-2*E^(n*Cos[a + b*x])*(-1 + n*Cos[a + b*x]))/(b*n^2)
```

3.745.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4879, 27, 2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(2a+2bx)e^{n\cos(a+bx)}\,dx$$
$$\Big\downarrow \text{4879}$$
$$-\frac{\int 2e^{n\cos(a+bx)}\cos(a+bx)d\cos(a+bx)}{b}$$
$$\Big\downarrow \text{27}$$
$$-\frac{2\int e^{n\cos(a+bx)}\cos(a+bx)d\cos(a+bx)}{b}$$
$$\Big\downarrow \text{2607}$$
$$-\frac{2\left(\frac{\cos(a+bx)e^{n\cos(a+bx)}}{n}-\frac{\int e^{n\cos(a+bx)}d\cos(a+bx)}{n}\right)}{b}$$
$$\Big\downarrow \text{2624}$$
$$-\frac{2\left(\frac{\cos(a+bx)e^{n\cos(a+bx)}}{n}-\frac{e^{n\cos(a+bx)}}{n^2}\right)}{b}$$

input
```
Int[E^(n*Cos[a + b*x])*Sin[2*a + 2*b*x],x]
```

output
```
(-2*(-(E^(n*Cos[a + b*x])/n^2) + (E^(n*Cos[a + b*x])*Cos[a + b*x])/n))/b
```

3.745.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 4879
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cos[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Cos[v]/d
, u/Sin[v], x], x], x, Cos[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[Nonfree
Factors[Cos[v], x], u/Sin[v], x]]
```

3.745.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.25 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.44

method	result	size
risch	$-\frac{{\mathrm e}^{n \cos\left(x b \right) \cos\left(a \right)-n \sin\left(x b \right) \sin\left(a \right)} {\mathrm e}^{-i b x} {\mathrm e}^{-i a}}{b n}-\frac{{\mathrm e}^{n \cos\left(x b \right) \cos\left(a \right)-n \sin\left(x b \right) \sin\left(a \right)} {\mathrm e}^{i b x} {\mathrm e}^{i a}}{b n}+\frac{2 \,{\mathrm e}^{n \left(\cos\left(x b \right) \cos\left(a \right)-\sin\left(x b \right) \sin\left(a \right)\right)}}{b \,n^{2}}$	105

input
```
int(exp(n*cos(b*x+a))*sin(2*b*x+2*a),x,method=_RETURNVERBOSE)
```

output
```
-1/b/n*exp(n*cos(x*b)*cos(a)-n*sin(x*b)*sin(a))*exp(-I*x*b)*exp(-I*a)-1/b/
n*exp(n*cos(x*b)*cos(a)-n*sin(x*b)*sin(a))*exp(I*b*x)*exp(I*a)+2/b/n^2*exp
(n*(cos(x*b)*cos(a)-sin(x*b)*sin(a)))
```

3.745.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.63

$$\int e^{n\cos(a+bx)}\sin(2a+2bx)\,dx = -\frac{2\,(n\cos{(bx+a)}-1)e^{(n\cos{(bx+a)})}}{bn^2}$$

input
```
integrate(exp(n*cos(b*x+a))*sin(2*b*x+2*a),x, algorithm="fricas")
```

output
```
-2*(n*cos(b*x + a) - 1)*e^(n*cos(b*x + a))/(b*n^2)
```

3.745.6 Sympy [F]

$$\int e^{n\cos(a+bx)}\sin(2a+2bx)\,dx = \int e^{n\cos{(a+bx)}}\sin{(2a+2bx)}\,dx$$

input
```
integrate(exp(n*cos(b*x+a))*sin(2*b*x+2*a),x)
```

output
```
Integral(exp(n*cos(a + b*x))*sin(2*a + 2*b*x), x)
```

3.745.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int e^{n\cos(a+bx)}\sin(2a+2bx)\,dx = -\frac{2\left(n\cos{(bx+a)}\,e^{(n\cos{(bx+a)})}-e^{(n\cos{(bx+a)})}\right)}{bn^2}$$

input
```
integrate(exp(n*cos(b*x+a))*sin(2*b*x+2*a),x, algorithm="maxima")
```

output
```
-2*(n*cos(b*x + a)*e^(n*cos(b*x + a)) - e^(n*cos(b*x + a)))/(b*n^2)
```

3.745.8 Giac [F]

$$\int e^{n\cos(a+bx)}\sin(2a+2bx)\,dx=\int e^{(n\cos(bx+a))}\sin\left(2\,bx+2\,a\right)\,dx$$

input
```
integrate(exp(n*cos(b*x+a))*sin(2*b*x+2*a),x, algorithm="giac")
```

output
```
integrate(e^(n*cos(b*x + a))*sin(2*b*x + 2*a), x)
```

3.745.9 Mupad [B] (verification not implemented)

Time = 29.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.63

$$\int e^{n\cos(a+bx)}\sin(2a+2bx)\,dx=-\frac{2\,e^{n\,\cos(a+b\,x)}\,\left(n\,\cos\left(a+b\,x\right)-1\right)}{b\,n^2}$$

input
```
int(exp(n*cos(a + b*x))*sin(2*a + 2*b*x),x)
```

output
```
-(2*exp(n*cos(a + b*x))*(n*cos(a + b*x) - 1))/(b*n^2)
```

3.746 $\int e^{n\cos(a+bx)}\sin(2(a+bx))\,dx$

3.746.1 Optimal result

Integrand size = 19, antiderivative size = 43

$$\int e^{n\cos(a+bx)}\sin(2(a+bx))\,dx=\frac{2e^{n\cos(a+bx)}}{bn^2}-\frac{2e^{n\cos(a+bx)}\cos(a+bx)}{bn}$$

output
```
2*exp(n*cos(b*x+a))/b/n^2-2*exp(n*cos(b*x+a))*cos(b*x+a)/b/n
```

3.746.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.65

$$\int e^{n\cos(a+bx)}\sin(2(a+bx))\,dx=-\frac{2e^{n\cos(a+bx)}(-1+n\cos(a+bx))}{bn^2}$$

input
```
Integrate[E^(n*Cos[a + b*x])*Sin[2*(a + b*x)],x]
```

output
```
(-2*E^(n*Cos[a + b*x])*(-1 + n*Cos[a + b*x]))/(b*n^2)
```

3.746.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {4879, 27, 2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(2(a+bx))e^{n\cos(a+bx)}dx$$

$$\Big\downarrow \text{4879}$$

$$-\frac{\int 2e^{n\cos(a+bx)}\cos(a+bx)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2\int e^{n\cos(a+bx)}\cos(a+bx)d\cos(a+bx)}{b}$$

$$\Big\downarrow \text{2607}$$

$$-\frac{2\left(\frac{\cos(a+bx)e^{n\cos(a+bx)}}{n}-\frac{\int e^{n\cos(a+bx)}d\cos(a+bx)}{n}\right)}{b}$$

$$\Big\downarrow \text{2624}$$

$$-\frac{2\left(\frac{\cos(a+bx)e^{n\cos(a+bx)}}{n}-\frac{e^{n\cos(a+bx)}}{n^2}\right)}{b}$$

input
```
Int[E^(n*Cos[a + b*x])*Sin[2*(a + b*x)],x]
```

output
```
(-2*(-(E^(n*Cos[a + b*x])/n^2) + (E^(n*Cos[a + b*x])*Cos[a + b*x])/n))/b
```

3.746.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 4879
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cos[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Cos[v]/d
, u/Sin[v], x], x], x, Cos[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[Nonfree
Factors[Cos[v], x], u/Sin[v], x]]
```

3.746.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.02 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.44

method	result	size
risch	$-\frac{e^{n \cos(xb) \cos(a)-n \sin(xb) \sin(a)} e^{-ibx} e^{-ia}}{bn}-\frac{e^{n \cos(xb) \cos(a)-n \sin(xb) \sin(a)} e^{ibx} e^{ia}}{bn}+\frac{2 e^{n(\cos(xb) \cos(a)-\sin(xb) \sin(a))}}{b n^{2}}$	105

input
```
int(exp(n*cos(b*x+a))*sin(2*b*x+2*a),x,method=_RETURNVERBOSE)
```

output
```
-1/b/n*exp(n*cos(x*b)*cos(a)-n*sin(x*b)*sin(a))*exp(-I*x*b)*exp(-I*a)-1/b/
n*exp(n*cos(x*b)*cos(a)-n*sin(x*b)*sin(a))*exp(I*b*x)*exp(I*a)+2/b/n^2*exp
(n*(cos(x*b)*cos(a)-sin(x*b)*sin(a)))
```

3.746.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.63

$$\int e^{n\cos(a+bx)}\sin(2(a+bx))\,dx = -\frac{2\,(n\cos{(bx+a)}-1)e^{(n\cos{(bx+a)})}}{bn^2}$$

input
```
integrate(exp(n*cos(b*x+a))*sin(2*b*x+2*a),x, algorithm="fricas")
```

output
```
-2*(n*cos(b*x + a) - 1)*e^(n*cos(b*x + a))/(b*n^2)
```

3.746.6 Sympy [F]

$$\int e^{n\cos(a+bx)}\sin(2(a+bx))\,dx = \int e^{n\cos{(a+bx)}}\sin{(2a+2bx)}\,dx$$

input
```
integrate(exp(n*cos(b*x+a))*sin(2*b*x+2*a),x)
```

output
```
Integral(exp(n*cos(a + b*x))*sin(2*a + 2*b*x), x)
```

3.746.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int e^{n\cos(a+bx)}\sin(2(a+bx))\,dx = -\frac{2\left(n\cos{(bx+a)}\,e^{(n\cos{(bx+a)})}-e^{(n\cos{(bx+a)})}\right)}{bn^2}$$

input
```
integrate(exp(n*cos(b*x+a))*sin(2*b*x+2*a),x, algorithm="maxima")
```

output
```
-2*(n*cos(b*x + a)*e^(n*cos(b*x + a)) - e^(n*cos(b*x + a)))/(b*n^2)
```

3.746.8 Giac [F]

$$\int e^{n\cos(a+bx)}\sin(2(a+bx))\,dx = \int e^{(n\cos(bx+a))}\sin(2\,bx+2\,a)\;dx$$

input
```
integrate(exp(n*cos(b*x+a))*sin(2*b*x+2*a),x, algorithm="giac")
```

output
```
integrate(e^(n*cos(b*x + a))*sin(2*b*x + 2*a), x)
```

3.746.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.63

$$\int e^{n\cos(a+bx)}\sin(2(a+bx))\,dx = -\frac{2\,e^{n\,\cos(a+b\,x)}\,(n\,\cos(a+b\,x)-1)}{b\,n^2}$$

input
```
int(exp(n*cos(a + b*x))*sin(2*a + 2*b*x),x)
```

output
```
-(2*exp(n*cos(a + b*x))*(n*cos(a + b*x) - 1))/(b*n^2)
```

3.747 $\int e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}\sin(a+bx)\,dx$

3.747.1 Optimal result

Integrand size = 24, antiderivative size = 64

$$\int e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}\sin(a+bx)\,dx=\frac{4e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}-\frac{4e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn}$$

output
```
4*exp(n*cos(1/2*a+1/2*b*x))/b/n^2-4*exp(n*cos(1/2*a+1/2*b*x))*cos(1/2*a+1/
2*b*x)/b/n
```

3.747.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.56

$$\int e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}\sin(a+bx)\,dx=-\frac{4e^{n\cos\left(\frac{1}{2}(a+bx)\right)}\left(-1+n\cos\left(\frac{1}{2}(a+bx)\right)\right)}{bn^2}$$

input
```
Integrate[E^(n*Cos[a/2 + (b*x)/2])*Sin[a + b*x],x]
```

output
```
(-4*E^(n*Cos[(a + b*x)/2])*(-1 + n*Cos[(a + b*x)/2]))/(b*n^2)
```

3.747.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4879, 27, 2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}dx$$

$$\Big\downarrow \; 4879$$

$$-\frac{2\int 2e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}\cos\left(\frac{a}{2}+\frac{bx}{2}\right)d\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}{b}$$

$$\Big\downarrow \; 27$$

$$-\frac{4\int e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}\cos\left(\frac{a}{2}+\frac{bx}{2}\right)d\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}{b}$$

$$\Big\downarrow \; 2607$$

$$-\frac{4\left(\frac{\cos\left(\frac{a}{2}+\frac{bx}{2}\right)e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}}{n}-\frac{\int e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}d\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}{n}\right)}{b}$$

$$\Big\downarrow \; 2624$$

$$-\frac{4\left(\frac{\cos\left(\frac{a}{2}+\frac{bx}{2}\right)e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}}{n}-\frac{e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}}{n^2}\right)}{b}$$

input
```
Int[E^(n*Cos[a/2 + (b*x)/2])*Sin[a + b*x],x]
```

output
```
(-4*(-(E^(n*Cos[a/2 + (b*x)/2])/n^2) + (E^(n*Cos[a/2 + (b*x)/2])*Cos[a/2 +
 (b*x)/2])/n))/b
```

3.747.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 4879
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cos[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Cos[v]/d
, u/Sin[v], x], x], x, Cos[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[Nonfree
Factors[Cos[v], x], u/Sin[v], x]]
```

3.747.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.28 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.92

method	result
risch	$-\frac{2\,\mathrm{e}^{n\cos\left(\frac{a}{2}\right)\cos\left(\frac{xb}{2}\right)-n\sin\left(\frac{a}{2}\right)\sin\left(\frac{xb}{2}\right)}\mathrm{e}^{-\frac{ibx}{2}}\mathrm{e}^{-\frac{ia}{2}}}{bn}-\frac{2\,\mathrm{e}^{n\cos\left(\frac{a}{2}\right)\cos\left(\frac{xb}{2}\right)-n\sin\left(\frac{a}{2}\right)\sin\left(\frac{xb}{2}\right)}\mathrm{e}^{\frac{ibx}{2}}\mathrm{e}^{\frac{ia}{2}}}{bn}+\frac{4\,\mathrm{e}^{n\left(\cos\left(\frac{a}{2}\right)\cos\left(\frac{xb}{2}\right)-\sin\left(\right.\right.}}{b\,n^{2}}$

input
```
int(exp(n*cos(1/2*a+1/2*x*b))*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-2/b/n*exp(n*cos(1/2*a)*cos(1/2*x*b)-n*sin(1/2*a)*sin(1/2*x*b))*exp(-1/2*I
*b*x)*exp(-1/2*I*a)-2/b/n*exp(n*cos(1/2*a)*cos(1/2*x*b)-n*sin(1/2*a)*sin(1
/2*x*b))*exp(1/2*I*b*x)*exp(1/2*I*a)+4/b/n^2*exp(n*(cos(1/2*a)*cos(1/2*x*b
)-sin(1/2*a)*sin(1/2*x*b)))
```

3.747.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.52

$$\int e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}\sin(a+bx)\,dx=-\frac{4\left(n\cos\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)-1\right)e^{\left(n\cos\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)}}{bn^2}$$

input
```
integrate(exp(n*cos(1/2*b*x+1/2*a))*sin(b*x+a),x, algorithm="fricas")
```

output
```
-4*(n*cos(1/2*b*x + 1/2*a) - 1)*e^(n*cos(1/2*b*x + 1/2*a))/(b*n^2)
```

3.747.6 Sympy [F]

$$\int e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}\sin(a+bx)\,dx=\int e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}\sin\left(a+bx\right)dx$$

input
```
integrate(exp(n*cos(1/2*b*x+1/2*a))*sin(b*x+a),x)
```

output
```
Integral(exp(n*cos(a/2 + b*x/2))*sin(a + b*x), x)
```

3.747.7 Maxima [F]

$$\int e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}\sin(a+bx)\,dx=\int e^{\left(n\cos\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)}\sin\left(bx+a\right)\,dx$$

input
```
integrate(exp(n*cos(1/2*b*x+1/2*a))*sin(b*x+a),x, algorithm="maxima")
```

output
```
integrate(e^(n*cos(1/2*b*x + 1/2*a))*sin(b*x + a), x)
```

3.747.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(50) = 100$.

Time = 0.34 (sec) , antiderivative size = 195, normalized size of antiderivative = 3.05

$$\int e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}\sin(a+bx)\,dx$$
$$= \frac{4\left(ne^{\left(-\frac{n\tan\left(\frac{1}{4}bx+\frac{1}{4}a\right)^2-n}{\tan\left(\frac{1}{4}bx+\frac{1}{4}a\right)^2+1}\right)}\tan\left(\frac{1}{4}bx+\frac{1}{4}a\right)^2+e^{\left(-\frac{n\tan\left(\frac{1}{4}bx+\frac{1}{4}a\right)^2-n}{\tan\left(\frac{1}{4}bx+\frac{1}{4}a\right)^2+1}\right)}\tan\left(\frac{1}{4}bx+\frac{1}{4}a\right)^2-ne^{\left(-\frac{n\tan\left(\frac{1}{4}bx+\frac{1}{4}a\right)^2-n}{\tan\left(\frac{1}{4}bx+\frac{1}{4}a\right)^2+1}\right)}\right.}{bn^2\tan\left(\frac{1}{4}bx+\frac{1}{4}a\right)^2+bn^2}$$

input
```
integrate(exp(n*cos(1/2*b*x+1/2*a))*sin(b*x+a),x, algorithm="giac")
```

output
```
4*(n*e^(-(n*tan(1/4*b*x + 1/4*a)^2 - n)/(tan(1/4*b*x + 1/4*a)^2 + 1))*tan(
1/4*b*x + 1/4*a)^2 + e^(-(n*tan(1/4*b*x + 1/4*a)^2 - n)/(tan(1/4*b*x + 1/4
*a)^2 + 1))*tan(1/4*b*x + 1/4*a)^2 - n*e^(-(n*tan(1/4*b*x + 1/4*a)^2 - n)/
(tan(1/4*b*x + 1/4*a)^2 + 1)) + e^(-(n*tan(1/4*b*x + 1/4*a)^2 - n)/(tan(1/
4*b*x + 1/4*a)^2 + 1)))/(b*n^2*tan(1/4*b*x + 1/4*a)^2 + b*n^2)
```

3.747.9 Mupad [B] (verification not implemented)

Time = 26.77 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.52

$$\int e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}\sin(a+bx)\,dx = -\frac{4\,e^{n\,\cos\left(\frac{a}{2}+\frac{b\,x}{2}\right)}\,\left(n\,\cos\left(\frac{a}{2}+\frac{b\,x}{2}\right)-1\right)}{b\,n^2}$$

input
```
int(exp(n*cos(a/2 + (b*x)/2))*sin(a + b*x),x)
```

output
```
-(4*exp(n*cos(a/2 + (b*x)/2))*(n*cos(a/2 + (b*x)/2) - 1))/(b*n^2)
```

3.748 $\int e^{n\cos\left(\frac{1}{2}(a+bx)\right)}\sin(a+bx)\,dx$

3.748.1 Optimal result

Integrand size = 21, antiderivative size = 64

$$\int e^{n\cos\left(\frac{1}{2}(a+bx)\right)}\sin(a+bx)\,dx=\frac{4e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}}{bn^2}-\frac{4e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}{bn}$$

output
```
4*exp(n*cos(1/2*a+1/2*b*x))/b/n^2-4*exp(n*cos(1/2*a+1/2*b*x))*cos(1/2*a+1/
2*b*x)/b/n
```

3.748.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.56

$$\int e^{n\cos\left(\frac{1}{2}(a+bx)\right)}\sin(a+bx)\,dx=-\frac{4e^{n\cos\left(\frac{1}{2}(a+bx)\right)}\left(-1+n\cos\left(\frac{1}{2}(a+bx)\right)\right)}{bn^2}$$

input
```
Integrate[E^(n*Cos[(a + b*x)/2])*Sin[a + b*x],x]
```

output
```
(-4*E^(n*Cos[(a + b*x)/2])*(-1 + n*Cos[(a + b*x)/2]))/(b*n^2)
```

3.748.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {4879, 27, 2607, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)e^{n\cos\left(\frac{1}{2}(a+bx)\right)}dx$$

$$\Big\downarrow \text{ 4879}$$

$$-\frac{2\int 2e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}\cos\left(\frac{a}{2}+\frac{bx}{2}\right)d\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}{b}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{4\int e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}\cos\left(\frac{a}{2}+\frac{bx}{2}\right)d\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}{b}$$

$$\Big\downarrow \text{ 2607}$$

$$-\frac{4\left(\frac{\cos\left(\frac{a}{2}+\frac{bx}{2}\right)e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}}{n}-\frac{\int e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}d\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}{n}\right)}{b}$$

$$\Big\downarrow \text{ 2624}$$

$$-\frac{4\left(\frac{\cos\left(\frac{a}{2}+\frac{bx}{2}\right)e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}}{n}-\frac{e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}}{n^2}\right)}{b}$$

input
```
Int[E^(n*Cos[(a + b*x)/2])*Sin[a + b*x],x]
```

output
```
(-4*(-(E^(n*Cos[a/2 + (b*x)/2])/n^2) + (E^(n*Cos[a/2 + (b*x)/2])*Cos[a/2 +
 (b*x)/2])/n))/b
```

3.748.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2607
```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[(c + d*x)^m*((b*F^(g*(e + f*x)))^n/(f*g*n*Log[F])),
x] - Simp[d*(m/(f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^
n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*
m] &&  !TrueQ[$UseGamma]
```

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 4879
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cos[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Cos[v]/d
, u/Sin[v], x], x], x, Cos[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[Nonfree
Factors[Cos[v], x], u/Sin[v], x]]
```

3.748.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.03 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.92

method	result
risch	$-\frac{2 \mathrm{e}^{n \cos\left(\frac{a}{2}\right) \cos\left(\frac{x b}{2}\right)-n \sin\left(\frac{a}{2}\right) \sin\left(\frac{x b}{2}\right)} \mathrm{e}^{-\frac{i b x}{2}} \mathrm{e}^{-\frac{i a}{2}}}{b n}-\frac{2 \mathrm{e}^{n \cos\left(\frac{a}{2}\right) \cos\left(\frac{x b}{2}\right)-n \sin\left(\frac{a}{2}\right) \sin\left(\frac{x b}{2}\right)} \mathrm{e}^{\frac{i b x}{2}} \mathrm{e}^{\frac{i a}{2}}}{b n}+\frac{4 \mathrm{e}^{n\left(\cos\left(\frac{a}{2}\right) \cos\left(\frac{x b}{2}\right)-\sin\left(\cdots\right)\right)}}{b n^{2}}$

input
```
int(exp(n*cos(1/2*a+1/2*x*b))*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-2/b/n*exp(n*cos(1/2*a)*cos(1/2*x*b)-n*sin(1/2*a)*sin(1/2*x*b))*exp(-1/2*I
*b*x)*exp(-1/2*I*a)-2/b/n*exp(n*cos(1/2*a)*cos(1/2*x*b)-n*sin(1/2*a)*sin(1
/2*x*b))*exp(1/2*I*b*x)*exp(1/2*I*a)+4/b/n^2*exp(n*(cos(1/2*a)*cos(1/2*x*b
)-sin(1/2*a)*sin(1/2*x*b)))
```

3.748.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.52

$$\int e^{n\cos\left(\frac{1}{2}(a+bx)\right)}\sin(a+bx)\,dx = -\frac{4\left(n\cos\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)-1\right)e^{\left(n\cos\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)}}{bn^2}$$

input
```
integrate(exp(n*cos(1/2*b*x+1/2*a))*sin(b*x+a),x, algorithm="fricas")
```

output
```
-4*(n*cos(1/2*b*x + 1/2*a) - 1)*e^(n*cos(1/2*b*x + 1/2*a))/(b*n^2)
```

3.748.6 Sympy [F]

$$\int e^{n\cos\left(\frac{1}{2}(a+bx)\right)}\sin(a+bx)\,dx = \int e^{n\cos\left(\frac{a}{2}+\frac{bx}{2}\right)}\sin\left(a+bx\right)dx$$

input
```
integrate(exp(n*cos(1/2*b*x+1/2*a))*sin(b*x+a),x)
```

output
```
Integral(exp(n*cos(a/2 + b*x/2))*sin(a + b*x), x)
```

3.748.7 Maxima [F]

$$\int e^{n\cos\left(\frac{1}{2}(a+bx)\right)}\sin(a+bx)\,dx = \int e^{\left(n\cos\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)\right)}\sin\left(bx+a\right)\,dx$$

input
```
integrate(exp(n*cos(1/2*b*x+1/2*a))*sin(b*x+a),x, algorithm="maxima")
```

output
```
integrate(e^(n*cos(1/2*b*x + 1/2*a))*sin(b*x + a), x)
```

3.748.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(50) = 100$.

Time = 0.34 (sec) , antiderivative size = 195, normalized size of antiderivative = 3.05

$$\int e^{n\cos\left(\frac{1}{2}(a+bx)\right)}\sin(a+bx)\,dx$$
$$= \frac{4\left(ne^{\left(-\frac{n\tan\left(\frac{1}{4}bx+\frac{1}{4}a\right)^2-n}{\tan\left(\frac{1}{4}bx+\frac{1}{4}a\right)^2+1}\right)}\tan\left(\frac{1}{4}bx+\frac{1}{4}a\right)^2+e^{\left(-\frac{n\tan\left(\frac{1}{4}bx+\frac{1}{4}a\right)^2-n}{\tan\left(\frac{1}{4}bx+\frac{1}{4}a\right)^2+1}\right)}\tan\left(\frac{1}{4}bx+\frac{1}{4}a\right)^2-ne^{\left(-\frac{n\tan\left(\frac{1}{4}bx+\frac{1}{4}a\right)^2-n}{\tan\left(\frac{1}{4}bx+\frac{1}{4}a\right)^2+1}\right.}\right.}{bn^2\tan\left(\frac{1}{4}bx+\frac{1}{4}a\right)^2+bn^2}$$

input
```
integrate(exp(n*cos(1/2*b*x+1/2*a))*sin(b*x+a),x, algorithm="giac")
```

output
```
4*(n*e^(-(n*tan(1/4*b*x + 1/4*a)^2 - n)/(tan(1/4*b*x + 1/4*a)^2 + 1))*tan(
1/4*b*x + 1/4*a)^2 + e^(-(n*tan(1/4*b*x + 1/4*a)^2 - n)/(tan(1/4*b*x + 1/4
*a)^2 + 1))*tan(1/4*b*x + 1/4*a)^2 - n*e^(-(n*tan(1/4*b*x + 1/4*a)^2 - n)/
(tan(1/4*b*x + 1/4*a)^2 + 1)) + e^(-(n*tan(1/4*b*x + 1/4*a)^2 - n)/(tan(1/
4*b*x + 1/4*a)^2 + 1)))/(b*n^2*tan(1/4*b*x + 1/4*a)^2 + b*n^2)
```

3.748.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.52

$$\int e^{n\cos\left(\frac{1}{2}(a+bx)\right)}\sin(a+bx)\,dx = -\frac{4\,e^{n\,\cos\left(\frac{a}{2}+\frac{b\,x}{2}\right)}\,\left(n\,\cos\left(\frac{a}{2}+\frac{b\,x}{2}\right)-1\right)}{b\,n^2}$$

input
```
int(exp(n*cos(a/2 + (b*x)/2))*sin(a + b*x),x)
```

output
```
-(4*exp(n*cos(a/2 + (b*x)/2))*(n*cos(a/2 + (b*x)/2) - 1))/(b*n^2)
```

3.749 $\int \csc(x) \log(\tan(x)) \sec(x) \, dx$

3.749.1 Optimal result

Integrand size = 8, antiderivative size = 9

$$\int \csc(x) \log(\tan(x)) \sec(x) \, dx = \frac{1}{2} \log^2(\tan(x))$$

output
```
1/2*ln(tan(x))^2
```

3.749.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \csc(x) \log(\tan(x)) \sec(x) \, dx = \frac{1}{2} \log^2(\tan(x))$$

input
```
Integrate[Csc[x]*Log[Tan[x]]*Sec[x],x]
```

output
```
Log[Tan[x]]^2/2
```

3.749.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {7237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(x)\sec(x)\log(\tan(x))\,dx$$

$$\Big\downarrow \text{7237}$$

$$\frac{1}{2}\log^2(\tan(x))$$

input
```
Int[Csc[x]*Log[Tan[x]]*Sec[x],x]
```

output
```
Log[Tan[x]]^2/2
```

3.749.3.1 Defintions of rubi rules used

rule 7237
```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[q*(y^(m + 1)/(m + 1)), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

3.749.4 Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{\ln(\tan(x))^2}{2}$	8
default	$\frac{\ln(\tan(x))^2}{2}$	8
risch	Expression too large to display	764

input
```
int(csc(x)*ln(tan(x))*sec(x),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(tan(x))^2
```

3.749.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.33

$$\int \csc(x)\log(\tan(x))\sec(x)\,dx = \frac{1}{2}\log\left(\frac{\sin(x)}{\cos(x)}\right)^2$$

input
```
integrate(csc(x)*log(tan(x))*sec(x),x, algorithm="fricas")
```

output
```
1/2*log(sin(x)/cos(x))^2
```

3.749.6 Sympy [F]

$$\int \csc(x)\log(\tan(x))\sec(x)\,dx = \int \log(\tan(x))\csc(x)\sec(x)\,dx$$

input
```
integrate(csc(x)*ln(tan(x))*sec(x),x)
```

output
```
Integral(log(tan(x))*csc(x)*sec(x), x)
```

3.749.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \csc(x)\log(\tan(x))\sec(x)\,dx = \frac{1}{2}\log(\tan(x))^2$$

input
```
integrate(csc(x)*log(tan(x))*sec(x),x, algorithm="maxima")
```

output
```
1/2*log(tan(x))^2
```

3.749.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \csc(x)\log(\tan(x))\sec(x)\,dx = \frac{1}{2}\log\left(\tan\left(x\right)\right)^2$$

input
```
integrate(csc(x)*log(tan(x))*sec(x),x, algorithm="giac")
```

output
```
1/2*log(tan(x))^2
```

3.749.9 Mupad [B] (verification not implemented)

Time = 29.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 3.00

$$\int \csc(x)\log(\tan(x))\sec(x)\,dx = \frac{\ln\left(-\frac{\mathrm{e}^{x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}}{\mathrm{e}^{x\,2\mathrm{i}}+1}\right)^2}{2}$$

input
```
int(log(tan(x))/(cos(x)*sin(x)),x)
```

output
```
log(-(exp(x*2i)*1i - 1i)/(exp(x*2i) + 1))^2/2
```

3.750 $\int \csc(2x)\log(\tan(x))\,dx$

3.750.1 Optimal result

Integrand size = 8, antiderivative size = 9

$$\int \csc(2x)\log(\tan(x))\,dx = \frac{1}{4}\log^2(\tan(x))$$

output
```
1/4*ln(tan(x))^2
```

3.750.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \csc(2x)\log(\tan(x))\,dx = \frac{1}{4}\log^2(\tan(x))$$

input
```
Integrate[Csc[2*x]*Log[Tan[x]],x]
```

output
```
Log[Tan[x]]^2/4
```

3.750.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {7237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(2x)\log(\tan(x))\,dx$$

$$\Big\downarrow \text{ 7237}$$

$$\frac{1}{4}\log^2(\tan(x))$$

input
```
Int[Csc[2*x]*Log[Tan[x]],x]
```

output
```
Log[Tan[x]]^2/4
```

3.750.3.1 Defintions of rubi rules used

rule 7237
```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[q*(y^(m + 1)/(m + 1)), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

3.750.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{\ln(\tan(x))^2}{4}$	8
default	$\frac{\ln(\tan(x))^2}{4}$	8
norman	$\frac{\ln(\tan(x))^2}{4}$	8
parallelrisch	$\frac{\ln(\tan(x))^2}{4}$	8
risch	Expression too large to display	764

input
```
int(csc(2*x)*ln(tan(x)),x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(tan(x))^2
```

3.750.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \csc(2x)\log(\tan(x))\,dx = \frac{1}{4}\log{(\tan{(x)})}^2$$

input
```
integrate(csc(2*x)*log(tan(x)),x, algorithm="fricas")
```

output
```
1/4*log(tan(x))^2
```

3.750.6 Sympy [F(-1)]

Timed out.

$$\int \csc(2x)\log(\tan(x))\,dx = \text{Timed out}$$

input
```
integrate(csc(2*x)*ln(tan(x)),x)
```

output
```
Timed out
```

3.750.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(7) = 14$.

Time = 0.31 (sec) , antiderivative size = 265, normalized size of antiderivative = 29.44

$$\int \csc(2x)\log(\tan(x))\,dx$$
$$= \frac{1}{4}\left(\pi - 2\arctan\left(\sin(x), \cos(x) + 1\right) - 2\arctan\left(\sin(x), \cos(x) - 1\right)\right)\arctan\left(\sin(2x), \cos(2x) + 1\right) + \frac{1}{4}\arctan\left(\sin(2x), \cos(2x) + 1\right)^2 - \frac{1}{4}\left(\pi - 2\arctan\left(\sin(x), \cos(x) - 1\right)\right)\arctan\left(\sin(x), \cos(x) + 1\right) + \frac{1}{4}\arctan\left(\sin(x), \cos(x) + 1\right)^2 - \frac{1}{4}\pi\arctan\left(\sin(x), \cos(x) - 1\right) + \frac{1}{4}\arctan\left(\sin(x), \cos(x) - 1\right)^2 + \frac{1}{8}\left(\log\left(\cos(x)^2 + \sin(x)^2 + 2\cos(x) + 1\right) + \log\left(\cos(x)^2 + \sin(x)^2 - 2\cos(x) + 1\right)\right)\log\left(\cos(2x)^2 + \sin(2x)^2 + 2\cos(2x) + 1\right) - \frac{1}{16}\log\left(\cos(2x)^2 + \sin(2x)^2 + 2\cos(2x) + 1\right)^2 - \frac{1}{16}\log\left(\cos(x)^2 + \sin(x)^2 + 2\cos(x) + 1\right)^2 - \frac{1}{8}\log\left(\cos(x)^2 + \sin(x)^2 + 2\cos(x) + 1\right)\log\left(\cos(x)^2 + \sin(x)^2 - 2\cos(x) + 1\right) - \frac{1}{16}\log\left(\cos(x)^2 + \sin(x)^2 - 2\cos(x) + 1\right)^2 - \frac{1}{2}\log\left(\cot(2x) + \csc(2x)\right)\log\left(\tan(x)\right)$$

input
```
integrate(csc(2*x)*log(tan(x)),x, algorithm="maxima")
```

output
```
1/4*(pi - 2*arctan2(sin(x), cos(x) + 1) - 2*arctan2(sin(x), cos(x) - 1))*a
rctan2(sin(2*x), cos(2*x) + 1) + 1/4*arctan2(sin(2*x), cos(2*x) + 1)^2 - 1
/4*(pi - 2*arctan2(sin(x), cos(x) - 1))*arctan2(sin(x), cos(x) + 1) + 1/4*
arctan2(sin(x), cos(x) + 1)^2 - 1/4*pi*arctan2(sin(x), cos(x) - 1) + 1/4*a
rctan2(sin(x), cos(x) - 1)^2 + 1/8*(log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1
) + log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1))*log(cos(2*x)^2 + sin(2*x)^2 +
 2*cos(2*x) + 1) - 1/16*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)^2 -
1/16*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)^2 - 1/8*log(cos(x)^2 + sin(x)
^2 + 2*cos(x) + 1)*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) - 1/16*log(cos(
x)^2 + sin(x)^2 - 2*cos(x) + 1)^2 - 1/2*log(cot(2*x) + csc(2*x))*log(tan(x
))
```

3.750.8 Giac [F]

$$\int \csc(2x)\log(\tan(x))\,dx = \int \csc(2x)\log(\tan(x))\,dx$$

input
```
integrate(csc(2*x)*log(tan(x)),x, algorithm="giac")
```

output
```
integrate(csc(2*x)*log(tan(x)), x)
```

3.750.9 Mupad [B] (verification not implemented)

Time = 27.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 3.00

$$\int \csc(2x)\log(\tan(x))\,dx = \frac{\ln\left(-\frac{\mathrm{e}^{x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}}{\mathrm{e}^{x\,2\mathrm{i}}+1}\right)^2}{4}$$

input
```
int(log(tan(x))/sin(2*x),x)
```

output
```
log(-(exp(x*2i)*1i - 1i)/(exp(x*2i) + 1))^2/4
```

3.751 $\int e^{\cos^2(x)+\sin^2(x)}\,dx$

3.751.1 Optimal result

Integrand size = 11, antiderivative size = 3

$$\int e^{\cos^2(x)+\sin^2(x)}\,dx = ex$$

output
```
exp(1)*x
```

3.751.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int e^{\cos^2(x)+\sin^2(x)}\,dx = ex$$

input
```
Integrate[E^(Cos[x]^2 + Sin[x]^2),x]
```

output
```
E*x
```

3.751.3 Rubi [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 1 in optimal.

Time = 0.15 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.67, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4889, 27, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\sin^2(x)+\cos^2(x)}\,dx$$

$$\Big\downarrow \text{ 4889}$$

$$\int \frac{e}{\tan^2(x)+1}d\tan(x)$$

$$\Big\downarrow \text{ 27}$$

$$e\int \frac{1}{\tan^2(x)+1}d\tan(x)$$

$$\Big\downarrow \text{ 216}$$

$$e\arctan(\tan(x))$$

input
```
Int[E^(Cos[x]^2 + Sin[x]^2),x]
```

output
```
E*ArcTan[Tan[x]]
```

3.751.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.751.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.67

method	result	size
default	$\mathrm{e}x$	5
risch	$\mathrm{e}x$	5
norman	$x\,\mathrm{e}^{\frac{4\tan\left(\frac{x}{2}\right)^{2}}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)^{2}}+\frac{\left(1-\tan\left(\frac{x}{2}\right)^{2}\right)^{2}}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)^{2}}}$	46

input
```
int(exp(sin(x)^2+cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
exp(1)*x
```

3.751.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

$$\int e^{\cos^2(x)+\sin^2(x)}\,dx = xe$$

input
```
integrate(exp(cos(x)^2+sin(x)^2),x, algorithm="fricas")
```

output
```
x*e
```

3.751.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14 vs. 2(3) = 6.

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 4.67

$$\int e^{\cos^2(x)+\sin^2(x)}\,dx = xe^{\sin^2(x)}e^{\cos^2(x)}$$

input
```
integrate(exp(cos(x)**2+sin(x)**2),x)
```

output
```
x*exp(sin(x)**2)*exp(cos(x)**2)
```

3.751.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

$$\int e^{\cos^2(x)+\sin^2(x)}\,dx = xe$$

input
```
integrate(exp(cos(x)^2+sin(x)^2),x, algorithm="maxima")
```

output
```
x*e
```

3.751.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

$$\int e^{\cos^2(x)+\sin^2(x)}\,dx = xe$$

input
```
integrate(exp(cos(x)^2+sin(x)^2),x, algorithm="giac")
```

output
```
x*e
```

3.751.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

$$\int e^{\cos^2(x)+\sin^2(x)}\,dx = x\,\mathrm{e}$$

input
```
int(exp(cos(x)^2 + sin(x)^2),x)
```

output
```
x*exp(1)
```

3.752 $\int x\sec^2(x)\,dx$

3.752.1 Optimal result

Integrand size = 6, antiderivative size = 8

$$\int x\sec^2(x)\,dx = \log(\cos(x)) + x\tan(x)$$

output
```
ln(cos(x))+x*tan(x)
```

3.752.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int x\sec^2(x)\,dx = \log(\cos(x)) + x\tan(x)$$

input
```
Integrate[x*Sec[x]^2,x]
```

output
```
Log[Cos[x]] + x*Tan[x]
```

3.752.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {3042, 4672, 25, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sec^2(x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int x\csc\left(x+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{ 4672}$$

$$\int -\tan(x)dx + x\tan(x)$$

$$\Big\downarrow \text{ 25}$$

$$x\tan(x) - \int \tan(x)dx$$

$$\Big\downarrow \text{ 3042}$$

$$x\tan(x) - \int \tan(x)dx$$

$$\Big\downarrow \text{ 3956}$$

$$x\tan(x) + \log(\cos(x))$$

input
```
Int[x*Sec[x]^2,x]
```

output
```
Log[Cos[x]] + x*Tan[x]
```

3.752.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.752.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

method	result	size
default	$\ln(\cos(x)) + x\tan(x)$	9
risch	$-2ix + \frac{2ix}{e^{2ix}+1} + \ln(e^{2ix}+1)$	27
parallelrisch	$-\ln\left(\frac{2}{\cos(x)+1}\right) + \ln(-\cot(x) + \csc(x) - 1) + \ln(\csc(x) - \cot(x) + 1) + x\tan(x)$	35
norman	$-\frac{2x\tan(\frac{x}{2})}{\tan(\frac{x}{2})^2-1} - \ln\left(1+\tan\left(\frac{x}{2}\right)^2\right) + \ln\left(\tan\left(\frac{x}{2}\right) - 1\right) + \ln\left(\tan\left(\frac{x}{2}\right) + 1\right)$	44

input
```
int(x*sec(x)^2,x,method=_RETURNVERBOSE)
```

output
```
ln(cos(x))+x*tan(x)
```

3.752.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. 2(8) = 16.

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.25

$$\int x\sec^2(x)\,dx = \frac{\cos(x)\log(-\cos(x)) + x\sin(x)}{\cos(x)}$$

input
```
integrate(x*sec(x)^2,x, algorithm="fricas")
```

output
```
(cos(x)*log(-cos(x)) + x*sin(x))/cos(x)
```

3.752.6 Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int x\sec^2(x)\,dx = x\tan(x) + \log(\cos(x))$$

input
```
integrate(x*sec(x)**2,x)
```

output
```
x*tan(x) + log(cos(x))
```

3.752.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(8) = 16.

Time = 0.29 (sec) , antiderivative size = 74, normalized size of antiderivative = 9.25

$$\int x\sec^2(x)\,dx$$
$$= \frac{(\cos(2x)^2 + \sin(2x)^2 + 2\cos(2x) + 1)\log(\cos(2x)^2 + \sin(2x)^2 + 2\cos(2x) + 1) + 4x\sin(2x)}{2(\cos(2x)^2 + \sin(2x)^2 + 2\cos(2x) + 1)}$$

input
```
integrate(x*sec(x)^2,x, algorithm="maxima")
```

output
```
1/2*((cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)*log(cos(2*x)^2 + sin(2*x)^
2 + 2*cos(2*x) + 1) + 4*x*sin(2*x))/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x)
+ 1)
```

3.752.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(8) = 16.

Time = 0.29 (sec) , antiderivative size = 103, normalized size of antiderivative = 12.88

$$\int x\sec^2(x)\,dx = \frac{\log\left(\frac{4\left(\tan\left(\frac{1}{2}\,x\right)^4-2\,\tan\left(\frac{1}{2}\,x\right)^2+1\right)}{\tan\left(\frac{1}{2}\,x\right)^4+2\,\tan\left(\frac{1}{2}\,x\right)^2+1}\right)\tan\left(\frac{1}{2}\,x\right)^2-4\,x\tan\left(\frac{1}{2}\,x\right)-\log\left(\frac{4\left(\tan\left(\frac{1}{2}\,x\right)^4-2\,\tan\left(\frac{1}{2}\,x\right)^2+1\right)}{\tan\left(\frac{1}{2}\,x\right)^4+2\,\tan\left(\frac{1}{2}\,x\right)^2+1}\right)}{2\left(\tan\left(\frac{1}{2}\,x\right)^2-1\right)}$$

input
```
integrate(x*sec(x)^2,x, algorithm="giac")
```

output
```
1/2*(log(4*(tan(1/2*x)^4 - 2*tan(1/2*x)^2 + 1)/(tan(1/2*x)^4 + 2*tan(1/2*x
)^2 + 1))*tan(1/2*x)^2 - 4*x*tan(1/2*x) - log(4*(tan(1/2*x)^4 - 2*tan(1/2*
x)^2 + 1)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1)))/(tan(1/2*x)^2 - 1)
```

3.752.9 Mupad [B] (verification not implemented)

Time = 26.84 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int x\sec^2(x)\,dx = \ln\left(\cos\left(x\right)\right)+x\tan(x)$$

input
```
int(x/cos(x)^2,x)
```

output
```
log(cos(x)) + x*tan(x)
```

3.753 $\int x \cos^4(x^2)\, dx$

3.753.1 Optimal result

Integrand size = 8, antiderivative size = 34

$$\int x \cos^4(x^2)\, dx = \frac{3x^2}{16} + \frac{3}{16}\cos(x^2)\sin(x^2) + \frac{1}{8}\cos^3(x^2)\sin(x^2)$$

output
```
3/16*x^2+3/16*cos(x^2)*sin(x^2)+1/8*cos(x^2)^3*sin(x^2)
```

3.753.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int x \cos^4(x^2)\, dx = \frac{3x^2}{16} + \frac{1}{8}\sin(2x^2) + \frac{1}{64}\sin(4x^2)$$

input
```
Integrate[x*Cos[x^2]^4,x]
```

output
```
(3*x^2)/16 + Sin[2*x^2]/8 + Sin[4*x^2]/64
```

3.753.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.26, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3861, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cos^4\left(x^2\right)\,dx$$

$$\Big\downarrow \text{ 3861}$$

$$\frac{1}{2}\int \cos^4\left(x^2\right)dx^2$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}\int \sin\left(x^2+\frac{\pi}{2}\right)^4dx^2$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{1}{2}\left(\frac{3}{4}\int \cos^2\left(x^2\right)dx^2+\frac{1}{4}\sin\left(x^2\right)\cos^3\left(x^2\right)\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}\left(\frac{3}{4}\int \sin\left(x^2+\frac{\pi}{2}\right)^2dx^2+\frac{1}{4}\sin\left(x^2\right)\cos^3\left(x^2\right)\right)$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{1}{2}\left(\frac{3}{4}\left(\frac{\int 1dx^2}{2}+\frac{1}{2}\sin\left(x^2\right)\cos\left(x^2\right)\right)+\frac{1}{4}\sin\left(x^2\right)\cos^3\left(x^2\right)\right)$$

$$\Big\downarrow \text{ 24}$$

$$\frac{1}{2}\left(\frac{1}{4}\sin\left(x^2\right)\cos^3\left(x^2\right)+\frac{3}{4}\left(\frac{x^2}{2}+\frac{1}{2}\sin\left(x^2\right)\cos\left(x^2\right)\right)\right)$$

input
```
Int[x*Cos[x^2]^4,x]
```

output
```
((Cos[x^2]^3*Sin[x^2])/4 + (3*(x^2/2 + (Cos[x^2]*Sin[x^2])/2))/4)/2
```

3.753.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3861
```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

3.753.4 Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

method	result
risch	$\frac{3x^2}{16} + \frac{\sin(4x^2)}{64} + \frac{\sin(2x^2)}{8}$
parallelrisch	$\frac{3x^2}{16} + \frac{\sin(4x^2)}{64} + \frac{\sin(2x^2)}{8}$
derivativedivides	$\frac{\left(\cos(x^2)^3 + \frac{3\cos(x^2)}{2}\right)\sin(x^2)}{8} + \frac{3x^2}{16}$
default	$\frac{\left(\cos(x^2)^3 + \frac{3\cos(x^2)}{2}\right)\sin(x^2)}{8} + \frac{3x^2}{16}$
norman	$\frac{\frac{3x^2}{16} - \frac{3\tan\left(\frac{x^2}{2}\right)^3}{8} + \frac{3\tan\left(\frac{x^2}{2}\right)^5}{8} - \frac{5\tan\left(\frac{x^2}{2}\right)^7}{8} + \frac{3x^2\tan\left(\frac{x^2}{2}\right)^2}{4} + \frac{9x^2\tan\left(\frac{x^2}{2}\right)^4}{8} + \frac{3x^2\tan\left(\frac{x^2}{2}\right)^6}{4} + \frac{3x^2\tan\left(\frac{x^2}{2}\right)^8}{16} + \frac{5\tan\left(\cdots\right.}{8}\cdots}{\left(1+\tan\left(\frac{x^2}{2}\right)^2\right)^4}$

input
```
int(x*cos(x^2)^4,x,method=_RETURNVERBOSE)
```

output
```
3/16*x^2+1/64*sin(4*x^2)+1/8*sin(2*x^2)
```

3.753.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int x\cos^4\left(x^2\right)\,dx = \frac{3}{16}\,x^2 + \frac{1}{16}\left(2\,\cos\left(x^2\right)^3 + 3\,\cos\left(x^2\right)\right)\sin\left(x^2\right)$$

input
```
integrate(x*cos(x^2)^4,x, algorithm="fricas")
```

output
```
3/16*x^2 + 1/16*(2*cos(x^2)^3 + 3*cos(x^2))*sin(x^2)
```

3.753.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(32) = 64.

Time = 0.17 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.24

$$\int x\cos^4\left(x^2\right)\,dx = \frac{3x^2\sin^4\left(x^2\right)}{16} + \frac{3x^2\sin^2\left(x^2\right)\cos^2\left(x^2\right)}{8} + \frac{3x^2\cos^4\left(x^2\right)}{16} + \frac{3\sin^3\left(x^2\right)\cos\left(x^2\right)}{16} + \frac{5\sin\left(x^2\right)\cos^3\left(x^2\right)}{16}$$

input
```
integrate(x*cos(x**2)**4,x)
```

output
```
3*x**2*sin(x**2)**4/16 + 3*x**2*sin(x**2)**2*cos(x**2)**2/8 + 3*x**2*cos(x
**2)**4/16 + 3*sin(x**2)**3*cos(x**2)/16 + 5*sin(x**2)*cos(x**2)**3/16
```

3.753.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int x\cos^4\left(x^2\right)\,dx = \frac{3}{16}\,x^2 + \frac{1}{64}\,\sin\left(4\,x^2\right) + \frac{1}{8}\,\sin\left(2\,x^2\right)$$

input
```
integrate(x*cos(x^2)^4,x, algorithm="maxima")
```

output
```
3/16*x^2 + 1/64*sin(4*x^2) + 1/8*sin(2*x^2)
```

3.753.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int x\cos^4\left(x^2\right)\,dx=\frac{3}{16}\,x^2+\frac{1}{64}\,\sin\left(4\,x^2\right)+\frac{1}{8}\,\sin\left(2\,x^2\right)$$

input
```
integrate(x*cos(x^2)^4,x, algorithm="giac")
```

output
```
3/16*x^2 + 1/64*sin(4*x^2) + 1/8*sin(2*x^2)
```

3.753.9 Mupad [B] (verification not implemented)

Time = 26.93 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int x\cos^4\left(x^2\right)\,dx=\frac{\sin\left(2\,x^2\right)}{8}+\frac{\sin\left(4\,x^2\right)}{64}+\frac{3\,x^2}{16}$$

input
```
int(x*cos(x^2)^4,x)
```

output
```
sin(2*x^2)/8 + sin(4*x^2)/64 + (3*x^2)/16
```

3.754 $\int \sqrt{\cos(x)}\sin(x)\,dx$

3.754.1 Optimal result

Integrand size = 9, antiderivative size = 10

$$\int \sqrt{\cos(x)}\sin(x)\,dx = -\frac{2}{3}\cos^{\frac{3}{2}}(x)$$

output
```
-2/3*cos(x)^(3/2)
```

3.754.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \sqrt{\cos(x)}\sin(x)\,dx = -\frac{2}{3}\cos^{\frac{3}{2}}(x)$$

input
```
Integrate[Sqrt[Cos[x]]*Sin[x],x]
```

output
```
(-2*Cos[x]^(3/2))/3
```

3.754.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3045, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\sqrt{\cos(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x)\sqrt{\cos(x)}dx$$

$$\Big\downarrow \text{3045}$$

$$-\int \sqrt{\cos(x)}d\cos(x)$$

$$\Big\downarrow \text{15}$$

$$-\frac{2}{3}\cos^{\frac{3}{2}}(x)$$

input
```
Int[Sqrt[Cos[x]]*Sin[x],x]
```

output
```
(-2*Cos[x]^(3/2))/3
```

3.754.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3045
```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Simp[-(a*f)^(-1)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x],
x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
 !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

3.754.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$-\frac{2 \cos(x)^{\frac{3}{2}}}{3}$	7
default	$-\frac{2 \cos(x)^{\frac{3}{2}}}{3}$	7

input
```
int(sin(x)*cos(x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*cos(x)^(3/2)
```

3.754.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \sqrt{\cos(x)} \sin(x) \, dx = -\frac{2}{3} \cos(x)^{\frac{3}{2}}$$

input
```
integrate(sin(x)*cos(x)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*cos(x)^(3/2)
```

3.754.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \sqrt{\cos(x)} \sin(x) \, dx = -\frac{2 \cos^{\frac{3}{2}}(x)}{3}$$

input
```
integrate(sin(x)*cos(x)**(1/2),x)
```

output
```
-2*cos(x)**(3/2)/3
```

3.754.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \sqrt{\cos(x)}\sin(x)\,dx = -\frac{2}{3}\cos(x)^{\frac{3}{2}}$$

input
```
integrate(sin(x)*cos(x)^(1/2),x, algorithm="maxima")
```

output
```
-2/3*cos(x)^(3/2)
```

3.754.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \sqrt{\cos(x)}\sin(x)\,dx = -\frac{2}{3}\cos(x)^{\frac{3}{2}}$$

input
```
integrate(sin(x)*cos(x)^(1/2),x, algorithm="giac")
```

output
```
-2/3*cos(x)^(3/2)
```

3.754.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \sqrt{\cos(x)}\sin(x)\,dx = -\frac{2\cos(x)^{3/2}}{3}$$

input
```
int(cos(x)^(1/2)*sin(x),x)
```

output
```
-(2*cos(x)^(3/2))/3
```

3.755 $\int e^{-2x} \tan(e^{-2x})\, dx$

3.755.1 Optimal result

Integrand size = 12, antiderivative size = 11

$$\int e^{-2x} \tan\left(e^{-2x}\right)\, dx = \frac{1}{2}\log\left(\cos\left(e^{-2x}\right)\right)$$

output
```
1/2*ln(cos(exp(-2*x)))
```

3.755.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int e^{-2x} \tan\left(e^{-2x}\right)\, dx = \frac{1}{2}\log\left(\cos\left(e^{-2x}\right)\right)$$

input
```
Integrate[Tan[E^(-2*x)]/E^(2*x),x]
```

output
```
Log[Cos[E^(-2*x)]]/2
```

3.755.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2720, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-2x} \tan \left(e^{-2x}\right) \, dx$$

$$\Big\downarrow \text{2720}$$

$$-\frac{1}{2} \int \tan \left(e^{-2x}\right) de^{-2x}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{2} \int \tan \left(e^{-2x}\right) de^{-2x}$$

$$\Big\downarrow \text{3956}$$

$$\frac{1}{2} \log \left(\cos \left(e^{-2x}\right)\right)$$

input
```
Int[Tan[E^(-2*x)]/E^(2*x),x]
```

output
```
Log[Cos[E^(-2*x)]]/2
```

3.755.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

3.755.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{\ln(\cos(e^{-2x}))}{2}$	9
norman	$-\frac{\ln\left(1+\tan(e^{-2x})^2\right)}{4}$	13
risch	$-\frac{i e^{-2x}}{2}+\frac{\ln\left(e^{2i e^{-2x}}+1\right)}{2}$	22

input
```
int(tan(exp(-2*x))/exp(2*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(cos(exp(-2*x)))
```

3.755.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int e^{-2x}\tan\left(e^{-2x}\right)\,dx=\frac{1}{4}\log\left(\frac{1}{\tan\left(e^{(-2x)}\right)^2+1}\right)$$

input
```
integrate(tan(exp(-2*x))/exp(2*x),x, algorithm="fricas")
```

output
```
1/4*log(1/(tan(e^(-2*x))^2 + 1))
```

3.755.6 Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int e^{-2x} \tan \left(e^{-2x}\right) \, dx = -\frac{\log \left(\tan^2 \left(e^{-2x}\right) + 1\right)}{4}$$

input
```
integrate(tan(exp(-2*x))/exp(2*x),x)
```

output
```
-log(tan(exp(-2*x))**2 + 1)/4
```

3.755.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int e^{-2x} \tan \left(e^{-2x}\right) \, dx = -\frac{1}{2} \log \left(\sec \left(e^{(-2x)}\right)\right)$$

input
```
integrate(tan(exp(-2*x))/exp(2*x),x, algorithm="maxima")
```

output
```
-1/2*log(sec(e^(-2*x)))
```

3.755.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int e^{-2x} \tan \left(e^{-2x}\right) \, dx = \frac{1}{2} \log \left(\left|\cos \left(e^{(-2x)}\right)\right|\right)$$

input
```
integrate(tan(exp(-2*x))/exp(2*x),x, algorithm="giac")
```

output
```
1/2*log(abs(cos(e^(-2*x))))
```

3.755.9 Mupad [B] (verification not implemented)

Time = 27.40 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int e^{-2x}\tan\left(e^{-2x}\right)\,dx = -\frac{\ln\left(\tan(\mathrm{e}^{-2\,x})^2+1\right)}{4}$$

input
```
int(exp(-2*x)*tan(exp(-2*x)),x)
```

output
```
-log(tan(exp(-2*x))^2 + 1)/4
```

3.756 $\int \frac{\sec(x)\sin(2x)}{1+\cos(x)}\,dx$

3.756.1 Optimal result

Integrand size = 13, antiderivative size = 7

$$\int \frac{\sec(x)\sin(2x)}{1+\cos(x)}\,dx = -2\log(1+\cos(x))$$

output
```
-2*ln(1+cos(x))
```

3.756.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \frac{\sec(x)\sin(2x)}{1+\cos(x)}\,dx = -4\log\left(\cos\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[(Sec[x]*Sin[2*x])/(1 + Cos[x]),x]
```

output
```
-4*Log[Cos[x/2]]
```

3.756.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3042, 4879, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(2x)\sec(x)}{\cos(x)+1}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(2x)}{\cos(x)(\cos(x)+1)}dx$$

$$\Big\downarrow \text{4879}$$

$$-\int \frac{2}{\cos(x)+1}d\cos(x)$$

$$\Big\downarrow \text{16}$$

$$-2\log(\cos(x)+1)$$

input
```
Int[(Sec[x]*Sin[2*x])/(1 + Cos[x]),x]
```

output
```
-2*Log[1 + Cos[x]]
```

3.756.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4879
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Cos[v], x]}, -d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Cos[v]/d
, u/Sin[v], x], x], x, Cos[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[Nonfree
Factors[Cos[v], x], u/Sin[v], x]]
```

3.756.4 Maple [A] (verified)

Time = 2.50 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

method	result	size
derivativedivides	$-2\ln\left(\cos\left(x\right)+1\right)$	8
default	$-2\ln\left(\cos\left(x\right)+1\right)$	8
risch	$2ix-4\ln\left(\mathrm{e}^{ix}+1\right)$	16

input
```
int(sec(x)*sin(2*x)/(cos(x)+1),x,method=_RETURNVERBOSE)
```

output
```
-2*ln(cos(x)+1)
```

3.756.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \frac{\sec(x)\sin(2x)}{1+\cos(x)}\,dx = -2\,\log\left(\frac{1}{2}\,\cos\left(x\right)+\frac{1}{2}\right)$$

input
```
integrate(sec(x)*sin(2*x)/(1+cos(x)),x, algorithm="fricas")
```

output
```
-2*log(1/2*cos(x) + 1/2)
```

3.756.6 Sympy [A] (verification not implemented)

Time = 1.34 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

$$\int \frac{\sec(x)\sin(2x)}{1+\cos(x)}\,dx = -2\log\left(\cos\left(x\right)+1\right)$$

input
```
integrate(sec(x)*sin(2*x)/(1+cos(x)),x)
```

output
```
-2*log(cos(x) + 1)
```

3.756.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{\sec(x)\sin(2x)}{1+\cos(x)}\,dx = -2\,\log\left(\cos\left(x\right)+1\right)$$

input
```
integrate(sec(x)*sin(2*x)/(1+cos(x)),x, algorithm="maxima")
```

output
```
-2*log(cos(x) + 1)
```

3.756.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{\sec(x)\sin(2x)}{1+\cos(x)}\,dx = -2\,\log\left(\cos\left(x\right)+1\right)$$

input
```
integrate(sec(x)*sin(2*x)/(1+cos(x)),x, algorithm="giac")
```

output
```
-2*log(cos(x) + 1)
```

3.756.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{\sec(x)\sin(2x)}{1+\cos(x)}\,dx = -2\,\ln\left(\cos\left(x\right)+1\right)$$

input
```
int(sin(2*x)/(cos(x)*(cos(x) + 1)),x)
```

output
```
-2*log(cos(x) + 1)
```

3.757 $\int x \sec^2(3x)\,dx$

3.757.1 Optimal result

Integrand size = 8, antiderivative size = 19

$$\int x \sec^2(3x)\,dx = \frac{1}{9}\log(\cos(3x)) + \frac{1}{3}x\tan(3x)$$

output
```
1/9*ln(cos(3*x))+1/3*x*tan(3*x)
```

3.757.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int x \sec^2(3x)\,dx = \frac{1}{9}\log(\cos(3x)) + \frac{1}{3}x\tan(3x)$$

input
```
Integrate[x*Sec[3*x]^2,x]
```

output
```
Log[Cos[3*x]]/9 + (x*Tan[3*x])/3
```

3.757.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {3042, 4672, 25, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sec^2(3x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int x\csc\left(3x+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{ 4672}$$

$$\frac{1}{3}\int -\tan(3x)dx+\frac{1}{3}x\tan(3x)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{3}x\tan(3x)-\frac{1}{3}\int \tan(3x)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{3}x\tan(3x)-\frac{1}{3}\int \tan(3x)dx$$

$$\Big\downarrow \text{ 3956}$$

$$\frac{1}{3}x\tan(3x)+\frac{1}{9}\log(\cos(3x))$$

input
```
Int[x*Sec[3*x]^2,x]
```

output
```
Log[Cos[3*x]]/9 + (x*Tan[3*x])/3
```

3.757.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.757.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{\ln(\cos(3x))}{9}+\frac{x\tan(3x)}{3}$	16
default	$\frac{\ln(\cos(3x))}{9}+\frac{x\tan(3x)}{3}$	16
risch	$-\frac{2ix}{3}+\frac{2ix}{3(e^{6ix}+1)}+\frac{\ln(e^{6ix}+1)}{9}$	29
norman	$-\frac{2x\tan(\frac{3x}{2})}{3\left(\tan(\frac{3x}{2})^2-1\right)}+\frac{\ln(\tan(\frac{3x}{2})-1)}{9}+\frac{\ln(\tan(\frac{3x}{2})+1)}{9}-\frac{\ln\left(1+\tan(\frac{3x}{2})^2\right)}{9}$	48
parallelrisch	$\frac{\ln(\tan(\frac{3x}{2})-1)\cos(3x)+\ln(\tan(\frac{3x}{2})+1)\cos(3x)-\ln\left(\sec(\frac{3x}{2})^2\right)\cos(3x)+3x\sin(3x)}{9\cos(3x)}$	54

input
```
int(x*sec(3*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/9*ln(cos(3*x))+1/3*x*tan(3*x)
```

3.757.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.47

$$\int x\sec^2(3x)\,dx = \frac{\cos(3x)\log(-\cos(3x)) + 3x\sin(3x)}{9\cos(3x)}$$

input
```
integrate(x*sec(3*x)^2,x, algorithm="fricas")
```

output
```
1/9*(cos(3*x)*log(-cos(3*x)) + 3*x*sin(3*x))/cos(3*x)
```

3.757.6 Sympy [F]

$$\int x\sec^2(3x)\,dx = \int x\sec^2(3x)\,dx$$

input
```
integrate(x*sec(3*x)**2,x)
```

output
```
Integral(x*sec(3*x)**2, x)
```

3.757.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(15) = 30.

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 3.89

$$\int x\sec^2(3x)\,dx = \frac{(\cos(6x)^2 + \sin(6x)^2 + 2\cos(6x) + 1)\log(\cos(6x)^2 + \sin(6x)^2 + 2\cos(6x) + 1) + 12x\sin(6x)}{18(\cos(6x)^2 + \sin(6x)^2 + 2\cos(6x) + 1)}$$

input
```
integrate(x*sec(3*x)^2,x, algorithm="maxima")
```

output
```
1/18*((cos(6*x)^2 + sin(6*x)^2 + 2*cos(6*x) + 1)*log(cos(6*x)^2 + sin(6*x)
^2 + 2*cos(6*x) + 1) + 12*x*sin(6*x))/(cos(6*x)^2 + sin(6*x)^2 + 2*cos(6*x
) + 1)
```

3.757.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(15) = 30.

Time = 0.29 (sec) , antiderivative size = 103, normalized size of antiderivative = 5.42

$$\int x\sec^2(3x)\,dx$$
$$= \frac{\log\left(\frac{4\left(\tan\left(\frac{3}{2}x\right)^4-2\tan\left(\frac{3}{2}x\right)^2+1\right)}{\tan\left(\frac{3}{2}x\right)^4+2\tan\left(\frac{3}{2}x\right)^2+1}\right)\tan\left(\frac{3}{2}x\right)^2-12\,x\tan\left(\frac{3}{2}x\right)-\log\left(\frac{4\left(\tan\left(\frac{3}{2}x\right)^4-2\tan\left(\frac{3}{2}x\right)^2+1\right)}{\tan\left(\frac{3}{2}x\right)^4+2\tan\left(\frac{3}{2}x\right)^2+1}\right)}{18\left(\tan\left(\frac{3}{2}x\right)^2-1\right)}$$

input
```
integrate(x*sec(3*x)^2,x, algorithm="giac")
```

output
```
1/18*(log(4*(tan(3/2*x)^4 - 2*tan(3/2*x)^2 + 1)/(tan(3/2*x)^4 + 2*tan(3/2*
x)^2 + 1))*tan(3/2*x)^2 - 12*x*tan(3/2*x) - log(4*(tan(3/2*x)^4 - 2*tan(3/
2*x)^2 + 1)/(tan(3/2*x)^4 + 2*tan(3/2*x)^2 + 1)))/(tan(3/2*x)^2 - 1)
```

3.757.9 Mupad [B] (verification not implemented)

Time = 25.92 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int x\sec^2(3x)\,dx = \frac{\ln(\cos(3\,x))}{9} + \frac{x\tan(3\,x)}{3}$$

input
```
int(x/cos(3*x)^2,x)
```

output
```
log(cos(3*x))/9 + (x*tan(3*x))/3
```

3.758 $\int e^{-2\pi x}\cos(2\pi x)\,dx$

3.758.1 Optimal result

Integrand size = 12, antiderivative size = 37

$$\int e^{-2\pi x}\cos(2\pi x)\,dx=-\frac{e^{-2\pi x}\cos(2\pi x)}{4\pi}+\frac{e^{-2\pi x}\sin(2\pi x)}{4\pi}$$

output
```
-1/4*cos(2*Pi*x)/exp(2*Pi*x)/Pi+1/4*sin(2*Pi*x)/exp(2*Pi*x)/Pi
```

3.758.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70

$$\int e^{-2\pi x}\cos(2\pi x)\,dx=\frac{e^{-2\pi x}(-\cos(2\pi x)+\sin(2\pi x))}{4\pi}$$

input
```
Integrate[Cos[2*Pi*x]/E^(2*Pi*x),x]
```

output
```
(-Cos[2*Pi*x] + Sin[2*Pi*x])/(4*E^(2*Pi*x)*Pi)
```

3.758.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4933}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-2\pi x}\cos(2\pi x)\,dx$$

$$\Big\downarrow \text{4933}$$

$$\frac{e^{-2\pi x}\sin(2\pi x)}{4\pi}-\frac{e^{-2\pi x}\cos(2\pi x)}{4\pi}$$

input
```
Int[Cos[2*Pi*x]/E^(2*Pi*x),x]
```

output
```
-1/4*Cos[2*Pi*x]/(E^(2*Pi*x)*Pi) + Sin[2*Pi*x]/(4*E^(2*Pi*x)*Pi)
```

3.758.3.1 Defintions of rubi rules used

rule 4933
```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

3.758.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.65

method	result	size
parallelrisch	$\frac{\mathrm{e}^{-2\pi x}\left(-\cos(2\pi x)+\sin(2\pi x)\right)}{4\pi}$	24
derivativedivides	$\frac{-\frac{\mathrm{e}^{-2\pi x}\cos(2\pi x)}{2}+\frac{\mathrm{e}^{-2\pi x}\sin(2\pi x)}{2}}{2\pi}$	31
default	$\frac{-\frac{\mathrm{e}^{-2\pi x}\cos(2\pi x)}{2}+\frac{\mathrm{e}^{-2\pi x}\sin(2\pi x)}{2}}{2\pi}$	31
norman	$\frac{\left(-\frac{1}{4\pi}+\frac{\tan(\pi x)}{2\pi}+\frac{\tan(\pi x)^{2}}{4\pi}\right)\mathrm{e}^{-2\pi x}}{1+\tan(\pi x)^{2}}$	45
risch	$-\frac{\mathrm{e}^{(-2+2i)\pi x}}{8\pi}-\frac{i\mathrm{e}^{(-2+2i)\pi x}}{8\pi}-\frac{\mathrm{e}^{(-2-2i)\pi x}}{8\pi}+\frac{i\mathrm{e}^{(-2-2i)\pi x}}{8\pi}$	52

input
```
int(cos(2*Pi*x)/exp(2*Pi*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*exp(-2*Pi*x)/Pi*(-cos(2*Pi*x)+sin(2*Pi*x))
```

3.758.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int e^{-2\pi x}\cos(2\pi x)\,dx = -\frac{\cos\left(2\,\pi x\right)e^{(-2\,\pi x)} - e^{(-2\,\pi x)}\sin\left(2\,\pi x\right)}{4\,\pi}$$

input
```
integrate(cos(2*pi*x)/exp(2*pi*x),x, algorithm="fricas")
```

output
```
-1/4*(cos(2*pi*x)*e^(-2*pi*x) - e^(-2*pi*x)*sin(2*pi*x))/pi
```

3.758.6 Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int e^{-2\pi x}\cos(2\pi x)\,dx = \frac{e^{-2\pi x}\sin\left(2\pi x\right)}{4\pi} - \frac{e^{-2\pi x}\cos\left(2\pi x\right)}{4\pi}$$

input
```
integrate(cos(2*pi*x)/exp(2*pi*x),x)
```

output
```
exp(-2*pi*x)*sin(2*pi*x)/(4*pi) - exp(-2*pi*x)*cos(2*pi*x)/(4*pi)
```

3.758.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70

$$\int e^{-2\pi x}\cos(2\pi x)\,dx = -\frac{(\pi\cos\left(2\,\pi x\right) - \pi\sin\left(2\,\pi x\right))e^{(-2\,\pi x)}}{4\,\pi^2}$$

input
```
integrate(cos(2*pi*x)/exp(2*pi*x),x, algorithm="maxima")
```

output
```
-1/4*(pi*cos(2*pi*x) - pi*sin(2*pi*x))*e^(-2*pi*x)/pi^2
```

3.758.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.62

$$\int e^{-2\pi x}\cos(2\pi x)\,dx = -\frac{(\cos(2\pi x) - \sin(2\pi x))e^{(-2\pi x)}}{4\pi}$$

input
```
integrate(cos(2*pi*x)/exp(2*pi*x),x, algorithm="giac")
```

output
```
-1/4*(cos(2*pi*x) - sin(2*pi*x))*e^(-2*pi*x)/pi
```

3.758.9 Mupad [B] (verification not implemented)

Time = 26.39 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.68

$$\int e^{-2\pi x}\cos(2\pi x)\,dx = -\frac{e^{-2\Pi x}\,(2\cos(2\Pi x) - 2\sin(2\Pi x))}{8\Pi}$$

input
```
int(exp(-2*Pi*x)*cos(2*Pi*x),x)
```

output
```
-(exp(-2*Pi*x)*(2*cos(2*Pi*x) - 2*sin(2*Pi*x)))/(8*Pi)
```

3.759 $\int (\cos^{12}(x)\sin^{10}(x)-\cos^{10}(x)\sin^{12}(x))\,dx$

3.759.1 Optimal result

Integrand size = 20, antiderivative size = 12

$$\int (\cos^{12}(x)\sin^{10}(x)-\cos^{10}(x)\sin^{12}(x))\,dx=\frac{1}{11}\cos^{11}(x)\sin^{11}(x)$$

output
```
1/11*cos(x)^11*sin(x)^11
```

3.759.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 49 vs. 2(12) = 24.

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 4.08

$$\int (\cos^{12}(x)\sin^{10}(x)-\cos^{10}(x)\sin^{12}(x))\,dx=\frac{21\sin(2x)}{1048576}-\frac{15\sin(6x)}{1048576}+\frac{15\sin(10x)}{2097152}-\frac{5\sin(14x)}{2097152}+\frac{\sin(18x)}{2097152}-\frac{\sin(22x)}{23068672}$$

input
```
Integrate[Cos[x]^12*Sin[x]^10 - Cos[x]^10*Sin[x]^12,x]
```

output
```
(21*Sin[2*x])/1048576 - (15*Sin[6*x])/1048576 + (15*Sin[10*x])/2097152 - (
5*Sin[14*x])/2097152 + Sin[18*x]/2097152 - Sin[22*x]/23068672
```

3.759.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 129 vs. $2(12) = 24$.

Time = 0.43 (sec) , antiderivative size = 129, normalized size of antiderivative = 10.75, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\sin^{10}(x)\cos^{12}(x) - \sin^{12}(x)\cos^{10}(x)\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{22}\sin^9(x)\cos^{13}(x) - \frac{9}{440}\sin^7(x)\cos^{13}(x) - \frac{7}{880}\sin^5(x)\cos^{13}(x) - \frac{7\sin^3(x)\cos^{13}(x)}{2816} - \frac{3\sin(x)\cos^{13}(x)}{5632} + \frac{1}{22}\sin^{11}(x)\cos^{11}(x) + \frac{1}{40}\sin^9(x)\cos^{11}(x) + \frac{1}{80}\sin^7(x)\cos^{11}(x) + \frac{7\sin^5(x)\cos^{11}(x)}{1280} + \frac{1}{512}\sin^3(x)\cos^{11}(x) + \frac{3\sin(x)\cos^{11}(x)}{5632}$$

input

```
Int[Cos[x]^12*Sin[x]^10 - Cos[x]^10*Sin[x]^12,x]
```

output

```
(3*Cos[x]^11*Sin[x])/5632 - (3*Cos[x]^13*Sin[x])/5632 + (Cos[x]^11*Sin[x]^
3)/512 - (7*Cos[x]^13*Sin[x]^3)/2816 + (7*Cos[x]^11*Sin[x]^5)/1280 - (7*Co
s[x]^13*Sin[x]^5)/880 + (Cos[x]^11*Sin[x]^7)/80 - (9*Cos[x]^13*Sin[x]^7)/4
40 + (Cos[x]^11*Sin[x]^9)/40 - (Cos[x]^13*Sin[x]^9)/22 + (Cos[x]^11*Sin[x]
^11)/22
```

3.759.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.759.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(10) = 20.

Time = 2.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 3.17

method	result
risch	$-\frac{\sin(22x)}{23068672}+\frac{\sin(18x)}{2097152}-\frac{5\sin(14x)}{2097152}+\frac{15\sin(10x)}{2097152}-\frac{15\sin(6x)}{1048576}+\frac{21\sin(2x)}{1048576}$
parallelrisch	$-\frac{5\left(\frac{\sin(11x)}{55}-\frac{\sin(9x)}{5}+\sin(7x)-3\sin(5x)+6\sin(3x)-\frac{42\sin(x)}{5}\right)(\cos(11x)+11\cos(9x)+55\cos(7x)+165\cos(5x)+330\cos(3x)+}{1048576}$
default	$-\frac{\cos(x)^{13}\sin(x)^{9}}{22}-\frac{9\sin(x)^{7}\cos(x)^{13}}{440}-\frac{7\sin(x)^{5}\cos(x)^{13}}{880}-\frac{7\sin(x)^{3}\cos(x)^{13}}{2816}-\frac{3\sin(x)\cos(x)^{13}}{5632}+$ $\left(\cos(x)^{11}+\frac{11}{}\right.$
parts	$-\frac{\cos(x)^{13}\sin(x)^{9}}{22}-\frac{9\sin(x)^{7}\cos(x)^{13}}{440}-\frac{7\sin(x)^{5}\cos(x)^{13}}{880}-\frac{7\sin(x)^{3}\cos(x)^{13}}{2816}-\frac{3\sin(x)\cos(x)^{13}}{5632}+$ $\left(\cos(x)^{11}+\frac{11}{}\right.$

input
```
int(cos(x)^12*sin(x)^10-cos(x)^10*sin(x)^12,x,method=_RETURNVERBOSE)
```

output
```
-1/23068672*sin(22*x)+1/2097152*sin(18*x)-5/2097152*sin(14*x)+15/2097152*s
in(10*x)-15/1048576*sin(6*x)+21/1048576*sin(2*x)
```

3.759.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(10) = 20.

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 3.25

$$\int\left(\cos^{12}(x)\sin^{10}(x)-\cos^{10}(x)\sin^{12}(x)\right)\,dx$$
$$=-\frac{1}{11}\left(\cos\left(x\right)^{21}-5\,\cos\left(x\right)^{19}+10\,\cos\left(x\right)^{17}-10\,\cos\left(x\right)^{15}+5\,\cos\left(x\right)^{13}-\cos\left(x\right)^{11}\right)\sin\left(x\right)$$

input
```
integrate(cos(x)^12*sin(x)^10-cos(x)^10*sin(x)^12,x, algorithm="fricas")
```

output
```
-1/11*(cos(x)^21 - 5*cos(x)^19 + 10*cos(x)^17 - 10*cos(x)^15 + 5*cos(x)^13
 - cos(x)^11)*sin(x)
```

3.759.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(10) = 20.

Time = 0.03 (sec) , antiderivative size = 236, normalized size of antiderivative = 19.67

$$\int \left(\cos^{12}(x)\sin^{10}(x)-\cos^{10}(x)\sin^{12}(x)\right)\,dx=-\frac{\sin^{21}(x)\cos(x)}{22}+\frac{89\sin^{19}(x)\cos(x)}{440}-\frac{301\sin^{17}(x)\cos(x)}{880}+\frac{3683\sin^{15}(x)\cos(x)}{14080}-\frac{433\sin^{13}(x)\cos(x)}{5632}+\frac{\sin^{11}(x)\cos(x)}{22528}+\frac{\sin^{9}(x)\cos(x)}{20480}+\frac{9\sin^{7}(x)\cos(x)}{163840}+\frac{21\sin^{5}(x)\cos(x)}{327680}+\frac{21\sin^{3}(x)\cos(x)}{262144}-\frac{\sin(x)\cos^{21}(x)}{22}+\frac{89\sin(x)\cos^{19}(x)}{440}-\frac{301\sin(x)\cos^{17}(x)}{880}+\frac{3683\sin(x)\cos^{15}(x)}{14080}-\frac{433\sin(x)\cos^{13}(x)}{5632}+\frac{\sin(x)\cos^{11}(x)}{22528}+\frac{\sin(x)\cos^{9}(x)}{20480}+\frac{9\sin(x)\cos^{7}(x)}{163840}+\frac{21\sin(x)\cos^{5}(x)}{327680}+\frac{21\sin(x)\cos^{3}(x)}{262144}+\frac{63\sin(x)\cos(x)}{262144}$$

input
```
integrate(cos(x)**12*sin(x)**10-cos(x)**10*sin(x)**12,x)
```

output
```
-sin(x)**21*cos(x)/22 + 89*sin(x)**19*cos(x)/440 - 301*sin(x)**17*cos(x)/8
80 + 3683*sin(x)**15*cos(x)/14080 - 433*sin(x)**13*cos(x)/5632 + sin(x)**1
1*cos(x)/22528 + sin(x)**9*cos(x)/20480 + 9*sin(x)**7*cos(x)/163840 + 21*s
in(x)**5*cos(x)/327680 + 21*sin(x)**3*cos(x)/262144 - sin(x)*cos(x)**21/22
 + 89*sin(x)*cos(x)**19/440 - 301*sin(x)*cos(x)**17/880 + 3683*sin(x)*cos(
x)**15/14080 - 433*sin(x)*cos(x)**13/5632 + sin(x)*cos(x)**11/22528 + sin(
x)*cos(x)**9/20480 + 9*sin(x)*cos(x)**7/163840 + 21*sin(x)*cos(x)**5/32768
0 + 21*sin(x)*cos(x)**3/262144 + 63*sin(x)*cos(x)/262144
```

3.759.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \left(\cos^{12}(x)\sin^{10}(x)-\cos^{10}(x)\sin^{12}(x)\right)\,dx=\frac{1}{22528}\sin\left(2\,x\right)^{11}$$

input
```
integrate(cos(x)^12*sin(x)^10-cos(x)^10*sin(x)^12,x, algorithm="maxima")
```

output
```
1/22528*sin(2*x)^11
```

3.759.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(10) = 20.

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 3.08

$$\int \left(\cos^{12}(x)\sin^{10}(x)-\cos^{10}(x)\sin^{12}(x)\right)\,dx=-\frac{1}{23068672}\sin\left(22\,x\right)+\frac{1}{2097152}\sin\left(18\,x\right)-\frac{5}{2097152}\sin\left(14\,x\right)+\frac{15}{2097152}\sin\left(10\,x\right)-\frac{15}{1048576}\sin\left(6\,x\right)+\frac{21}{1048576}\sin\left(2\,x\right)$$

input
```
integrate(cos(x)^12*sin(x)^10-cos(x)^10*sin(x)^12,x, algorithm="giac")
```

output
```
-1/23068672*sin(22*x) + 1/2097152*sin(18*x) - 5/2097152*sin(14*x) + 15/209
7152*sin(10*x) - 15/1048576*sin(6*x) + 21/1048576*sin(2*x)
```

3.759.9 Mupad [B] (verification not implemented)

Time = 26.58 (sec) , antiderivative size = 49, normalized size of antiderivative = 4.08

$$\int \left(\cos^{12}(x)\sin^{10}(x)-\cos^{10}(x)\sin^{12}(x)\right)\,dx=-\frac{\sin\left(x\right)\,{\cos\left(x\right)}^{21}}{11}+\frac{5\,\sin\left(x\right)\,{\cos\left(x\right)}^{19}}{11}-\frac{10\,\sin\left(x\right)\,{\cos\left(x\right)}^{17}}{11}+\frac{10\,\sin\left(x\right)\,{\cos\left(x\right)}^{15}}{11}-\frac{5\,\sin\left(x\right)\,{\cos\left(x\right)}^{13}}{11}+\frac{\sin\left(x\right)\,{\cos\left(x\right)}^{11}}{11}$$

input
```
int(cos(x)^12*sin(x)^10 - cos(x)^10*sin(x)^12,x)
```

output
```
(cos(x)^11*sin(x))/11 - (5*cos(x)^13*sin(x))/11 + (10*cos(x)^15*sin(x))/11
 - (10*cos(x)^17*sin(x))/11 + (5*cos(x)^19*sin(x))/11 - (cos(x)^21*sin(x))
/11
```

3.759. $\int \left(\cos^{12}(x)\sin^{10}(x) - \cos^{10}(x)\sin^{12}(x)\right)\, dx$

3.760 $\int x \cot(x^2)\,dx$

3.760.1 Optimal result

Integrand size = 6, antiderivative size = 9

$$\int x \cot(x^2)\,dx = \frac{1}{2}\log(\sin(x^2))$$

output
```
1/2*ln(sin(x^2))
```

3.760.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.67

$$\int x \cot(x^2)\,dx = \frac{1}{2}(\log(\cos(x^2)) + \log(\tan(x^2)))$$

input
```
Integrate[x*Cot[x^2],x]
```

output
```
(Log[Cos[x^2]] + Log[Tan[x^2]])/2
```

3.760.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {4235, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cot\left(x^2\right)\,dx$$

$$\Big\downarrow \text{4235}$$

$$\frac{1}{2}\int \cot\left(x^2\right)dx^2$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int -\tan\left(x^2+\frac{\pi}{2}\right)dx^2$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}\int \tan\left(x^2+\frac{\pi}{2}\right)dx^2$$

$$\Big\downarrow \text{3956}$$

$$\frac{1}{2}\log\left(\sin\left(x^2\right)\right)$$

input
```
Int[x*Cot[x^2],x]
```

output
```
Log[Sin[x^2]]/2
```

3.760.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4235
```
Int[((a_.) + Cot[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cot[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
 1)/n], 0] && IntegerQ[p]
```

3.760.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{\ln(\sin(x^2))}{2}$	8
default	$\frac{\ln(\sin(x^2))}{2}$	8
parallelrisch	$\ln\left(\sqrt{\tan\left(x^2\right)}\right)+\ln\left(\frac{1}{\left(\sec(x^2)^2\right)^{\frac{1}{4}}}\right)$	18
norman	$\frac{\ln(\tan(x^2))}{2}-\frac{\ln\left(1+\tan(x^2)^2\right)}{4}$	20
risch	$-\frac{ix^2}{2}+\frac{\ln\left(e^{2ix^2}-1\right)}{2}$	20

input
```
int(x*cot(x^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(sin(x^2))
```

3.760.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.44

$$\int x\cot\left(x^2\right)\,dx=\frac{1}{4}\log\left(-\frac{1}{2}\cos\left(2\,x^2\right)+\frac{1}{2}\right)$$

input
```
integrate(x*cot(x^2),x, algorithm="fricas")
```

output
```
1/4*log(-1/2*cos(2*x^2) + 1/2)
```

3.760.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(7) = 14.

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 2.11

$$\int x\cot\left(x^2\right)\,dx = -\frac{\log\left(\tan^2\left(x^2\right)+1\right)}{4}+\frac{\log\left(\tan\left(x^2\right)\right)}{2}$$

input
```
integrate(x*cot(x**2),x)
```

output
```
-log(tan(x**2)**2 + 1)/4 + log(tan(x**2))/2
```

3.760.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int x\cot\left(x^2\right)\,dx = \frac{1}{2}\log\left(\sin\left(x^2\right)\right)$$

input
```
integrate(x*cot(x^2),x, algorithm="maxima")
```

output
```
1/2*log(sin(x^2))
```

3.760.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int x\cot\left(x^2\right)\,dx = \frac{1}{2}\log\left(\left|\sin\left(x^2\right)\right|\right)$$

input
```
integrate(x*cot(x^2),x, algorithm="giac")
```

output
```
1/2*log(abs(sin(x^2)))
```

3.760.9 Mupad [B] (verification not implemented)

Time = 26.40 (sec) , antiderivative size = 19, normalized size of antiderivative = 2.11

$$\int x\cot \left (x^2\right) \,dx=\frac {\ln \left ({\mathrm {e}}^{x^2\,2{}\mathrm {i}}-1\right)}{2}-\frac {x^2\,1{}\mathrm {i}}{2}$$

input
```
int(x*cot(x^2),x)
```

output
```
log(exp(x^2*2i) - 1)/2 - (x^2*1i)/2
```

3.761 $\int x \sec^2(x^2) \, dx$

3.761.1 Optimal result

Integrand size = 8, antiderivative size = 8

$$\int x \sec^2(x^2) \, dx = \frac{\tan(x^2)}{2}$$

output
```
1/2*tan(x^2)
```

3.761.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int x \sec^2(x^2) \, dx = \frac{\tan(x^2)}{2}$$

input
```
Integrate[x*Sec[x^2]^2,x]
```

output
```
Tan[x^2]/2
```

3.761.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4692, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sec^2\left(x^2\right) \, dx$$

$$\Big\downarrow \text{ 4692}$$

$$\frac{1}{2} \int \sec^2\left(x^2\right) dx^2$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2} \int \csc\left(x^2 + \frac{\pi}{2}\right)^2 dx^2$$

$$\Big\downarrow \text{ 4254}$$

$$-\frac{1}{2} \int 1 d\left(-\tan\left(x^2\right)\right)$$

$$\Big\downarrow \text{ 24}$$

$$\frac{\tan\left(x^2\right)}{2}$$

input
```
Int[x*Sec[x^2]^2,x]
```

output
```
Tan[x^2]/2
```

3.761.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4692
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
 1)/n], 0] && IntegerQ[p]
```

3.761.4 Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{\tan(x^2)}{2}$	7
default	$\frac{\tan(x^2)}{2}$	7
risch	$\frac{i}{e^{2ix^2}+1}$	15
norman	$-\frac{\tan\left(\frac{x^2}{2}\right)}{\tan\left(\frac{x^2}{2}\right)^2-1}$	21
parallelrisch	$-\frac{\tan\left(\frac{x^2}{2}\right)}{\tan\left(\frac{x^2}{2}\right)^2-1}$	21

input
```
int(x*sec(x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*tan(x^2)
```

3.761.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.50

$$\int x\sec^2\left(x^2\right)\,dx = \frac{\sin\left(x^2\right)}{2\,\cos\left(x^2\right)}$$

input
```
integrate(x*sec(x^2)^2,x, algorithm="fricas")
```

output
```
1/2*sin(x^2)/cos(x^2)
```

3.761.6 Sympy [F]

$$\int x\sec^2\left(x^2\right)\,dx=\int x\sec^2\left(x^2\right)dx$$

input
```
integrate(x*sec(x**2)**2,x)
```

output
```
Integral(x*sec(x**2)**2, x)
```

3.761.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(6) = 12.

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 4.38

$$\int x\sec^2\left(x^2\right)\,dx=\frac{\sin\left(2\,x^2\right)}{\cos\left(2\,x^2\right)^2+\sin\left(2\,x^2\right)^2+2\,\cos\left(2\,x^2\right)+1}$$

input
```
integrate(x*sec(x^2)^2,x, algorithm="maxima")
```

output
```
sin(2*x^2)/(cos(2*x^2)^2 + sin(2*x^2)^2 + 2*cos(2*x^2) + 1)
```

3.761.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int x\sec^2\left(x^2\right)\,dx=\frac{1}{2}\,\tan\left(x^2\right)$$

input
```
integrate(x*sec(x^2)^2,x, algorithm="giac")
```

output
```
1/2*tan(x^2)
```

3.761.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.75

$$\int x \sec^2\left(x^2\right) \, dx = \frac{1\mathrm{i}}{\mathrm{e}^{x^2\,2\mathrm{i}}+1}$$

input
```
int(x/cos(x^2)^2,x)
```

output
```
1i/(exp(x^2*2i) + 1)
```

3.762 $\int \frac{\sin(8x)}{9+\sin^4(4x)}\,dx$

3.762.1 Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{\sin(8x)}{9+\sin^4(4x)}\,dx = \frac{1}{12}\arctan\left(\frac{1}{3}\sin^2(4x)\right)$$

output
```
1/12*arctan(1/3*sin(4*x)^2)
```

3.762.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\sin(8x)}{9+\sin^4(4x)}\,dx = \frac{1}{12}\arctan\left(\frac{1}{3}\sin^2(4x)\right)$$

input
```
Integrate[Sin[8*x]/(9 + Sin[4*x]^4),x]
```

output
```
ArcTan[Sin[4*x]^2/3]/12
```

3.762.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4878, 27, 807, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(8x)}{\sin^4(4x)+9} \, dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sin(8x)}{\sin(4x)^4+9}dx$$

$$\Big\downarrow \ 4878$$

$$\frac{1}{4}\int \frac{2\sin(4x)}{\sin^4(4x)+9}d\sin(4x)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{2}\int \frac{\sin(4x)}{\sin^4(4x)+9}d\sin(4x)$$

$$\Big\downarrow \ 807$$

$$\frac{1}{4}\int \frac{1}{\sin^4(4x)+9}d\sin^2(4x)$$

$$\Big\downarrow \ 216$$

$$\frac{1}{12}\arctan\left(\frac{1}{3}\sin^2(4x)\right)$$

input
```
Int[Sin[8*x]/(9 + Sin[4*x]^4),x]
```

output
```
ArcTan[Sin[4*x]^2/3]/12
```

3.762.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

3.762.4 Maple [A] (verified)

Time = 26.90 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{\arctan\left(\frac{\sin(4x)^2}{3}\right)}{12}$	12
default	$\frac{\arctan\left(\frac{\sin(4x)^2}{3}\right)}{12}$	12
risch	$\frac{i\ln\left(e^{16ix}+(-2-12i)e^{8ix}+1\right)}{24}-\frac{i\ln\left(e^{16ix}+(-2+12i)e^{8ix}+1\right)}{24}$	42

input
```
int(sin(8*x)/(9+sin(4*x)^4),x,method=_RETURNVERBOSE)
```

output
```
1/12*arctan(1/3*sin(4*x)^2)
```

3.762.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{\sin(8x)}{9+\sin^4(4x)}\,dx = -\frac{1}{12}\arctan\left(\frac{1}{3}\cos\left(4\,x\right)^2-\frac{1}{3}\right)$$

input
```
integrate(sin(8*x)/(9+sin(4*x)^4),x, algorithm="fricas")
```

output
```
-1/12*arctan(1/3*cos(4*x)^2 - 1/3)
```

3.762.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin(8x)}{9+\sin^4(4x)}\,dx = \text{Timed out}$$

input
```
integrate(sin(8*x)/(9+sin(4*x)**4),x)
```

output
```
Timed out
```

3.762.7 Maxima [F]

$$\int \frac{\sin(8x)}{9+\sin^4(4x)}\,dx = \int \frac{\sin\left(8\,x\right)}{\sin\left(4\,x\right)^4+9}\,dx$$

input
```
integrate(sin(8*x)/(9+sin(4*x)^4),x, algorithm="maxima")
```

output
```
integrate(sin(8*x)/(sin(4*x)^4 + 9), x)
```

3.762.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{\sin(8x)}{9+\sin^4(4x)}\,dx = \frac{1}{12}\arctan\left(\frac{1}{3}\sin\left(4\,x\right)^2\right)$$

input
```
integrate(sin(8*x)/(9+sin(4*x)^4),x, algorithm="giac")
```

output
```
1/12*arctan(1/3*sin(4*x)^2)
```

3.762.9 Mupad [B] (verification not implemented)

Time = 26.98 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{\sin(8x)}{9+\sin^4(4x)}\,dx = \frac{\mathrm{atan}\left(\frac{10\tan(4\,x)^2}{3}+3\right)}{12}$$

input
```
int(sin(8*x)/(sin(4*x)^4 + 9),x)
```

output
```
atan((10*tan(4*x)^2)/3 + 3)/12
```

3.763 $\int \frac{\cos(2x)}{8+\sin^2(2x)}\,dx$

3.763.1 Optimal result

Integrand size = 15, antiderivative size = 23

$$\int \frac{\cos(2x)}{8+\sin^2(2x)}\,dx = \frac{\arctan\left(\frac{\sin(2x)}{2\sqrt{2}}\right)}{4\sqrt{2}}$$

output
```
1/8*arctan(1/4*sin(2*x)*2^(1/2))*2^(1/2)
```

3.763.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{\cos(2x)}{8+\sin^2(2x)}\,dx = \frac{\arctan\left(\frac{\cos(x)\sin(x)}{\sqrt{2}}\right)}{4\sqrt{2}}$$

input
```
Integrate[Cos[2*x]/(8 + Sin[2*x]^2),x]
```

output
```
ArcTan[(Cos[x]*Sin[x])/Sqrt[2]]/(4*Sqrt[2])
```

3.763.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3669, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(2x)}{\sin^2(2x)+8}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(2x)}{\sin(2x)^2+8}dx$$

$$\Big\downarrow \text{3669}$$

$$\frac{1}{2}\int \frac{1}{\sin^2(2x)+8}d\sin(2x)$$

$$\Big\downarrow \text{216}$$

$$\frac{\arctan\left(\frac{\sin(2x)}{2\sqrt{2}}\right)}{4\sqrt{2}}$$

input
```
Int[Cos[2*x]/(8 + Sin[2*x]^2),x]
```

output
```
ArcTan[Sin[2*x]/(2*Sqrt[2])]/(4*Sqrt[2])
```

3.763.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

3.763.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$\frac{\arctan\left(\frac{\sin(2x)\sqrt{2}}{4}\right)\sqrt{2}}{8}$	16
default	$\frac{\arctan\left(\frac{\sin(2x)\sqrt{2}}{4}\right)\sqrt{2}}{8}$	16
risch	$\frac{i\sqrt{2}\,\ln\left(e^{4ix}-4\sqrt{2}\,e^{2ix}-1\right)}{16}-\frac{i\sqrt{2}\,\ln\left(e^{4ix}+4\sqrt{2}\,e^{2ix}-1\right)}{16}$	50

input
```
int(cos(2*x)/(8+sin(2*x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/8*arctan(1/4*sin(2*x)*2^(1/2))*2^(1/2)
```

3.763.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \frac{\cos(2x)}{8+\sin^2(2x)}\,dx = \frac{1}{8}\,\sqrt{2}\arctan\left(\frac{1}{4}\,\sqrt{2}\sin\left(2\,x\right)\right)$$

input
```
integrate(cos(2*x)/(8+sin(2*x)^2),x, algorithm="fricas")
```

output
```
1/8*sqrt(2)*arctan(1/4*sqrt(2)*sin(2*x))
```

3.763.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\cos(2x)}{8+\sin^2(2x)}\,dx = \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\sin(2x)}{4}\right)}{8}$$

input
```
integrate(cos(2*x)/(8+sin(2*x)**2),x)
```

output
```
sqrt(2)*atan(sqrt(2)*sin(2*x)/4)/8
```

3.763.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \frac{\cos(2x)}{8+\sin^2(2x)}\,dx = \frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\sin(2x)\right)$$

input
```
integrate(cos(2*x)/(8+sin(2*x)^2),x, algorithm="maxima")
```

output
```
1/8*sqrt(2)*arctan(1/4*sqrt(2)*sin(2*x))
```

3.763.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \frac{\cos(2x)}{8+\sin^2(2x)}\,dx = \frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}\sin(2x)\right)$$

input
```
integrate(cos(2*x)/(8+sin(2*x)^2),x, algorithm="giac")
```

output
```
1/8*sqrt(2)*arctan(1/4*sqrt(2)*sin(2*x))
```

3.763.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \frac{\cos(2x)}{8+\sin^2(2x)}\,dx = \frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,\sin(2\,x)}{4}\right)}{8}$$

input
```
int(cos(2*x)/(sin(2*x)^2 + 8),x)
```

output
```
(2^(1/2)*atan((2^(1/2)*sin(2*x))/4))/8
```

3.764 $\int x(\cos^3(x^2) - \sin^3(x^2))\,dx$

3.764.1 Optimal result

Integrand size = 17, antiderivative size = 37

$$\int x(\cos^3(x^2) - \sin^3(x^2))\,dx = \frac{\cos(x^2)}{2} - \frac{1}{6}\cos^3(x^2) + \frac{\sin(x^2)}{2} - \frac{1}{6}\sin^3(x^2)$$

output
```
1/2*cos(x^2)-1/6*cos(x^2)^3+1/2*sin(x^2)-1/6*sin(x^2)^3
```

3.764.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int x(\cos^3(x^2) - \sin^3(x^2))\,dx = \frac{3\cos(x^2)}{8} - \frac{1}{24}\cos(3x^2) + \frac{\sin(x^2)}{2} - \frac{1}{6}\sin^3(x^2)$$

input
```
Integrate[x*(Cos[x^2]^3 - Sin[x^2]^3),x]
```

output
```
(3*Cos[x^2])/8 - Cos[3*x^2]/24 + Sin[x^2]/2 - Sin[x^2]^3/6
```

3.764.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(\cos^3(x^2) - \sin^3(x^2))\,dx$$

$$\Big\downarrow \text{2010}$$

$$\int (x\cos^3(x^2) - x\sin^3(x^2))\,dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{6}\sin^3(x^2) + \frac{\sin(x^2)}{2} - \frac{1}{6}\cos^3(x^2) + \frac{\cos(x^2)}{2}$$

input
```
Int[x*(Cos[x^2]^3 - Sin[x^2]^3),x]
```

output
```
Cos[x^2]/2 - Cos[x^2]^3/6 + Sin[x^2]/2 - Sin[x^2]^3/6
```

3.764.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

3.764.4 Maple [A] (verified)

Time = 2.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{\left(2+\cos\left(x^2\right)^2\right)\sin\left(x^2\right)}{6}+\frac{\left(2+\sin\left(x^2\right)^2\right)\cos\left(x^2\right)}{6}$	30
default	$\frac{\left(2+\cos\left(x^2\right)^2\right)\sin\left(x^2\right)}{6}+\frac{\left(2+\sin\left(x^2\right)^2\right)\cos\left(x^2\right)}{6}$	30
risch	$\frac{3\cos\left(x^2\right)}{8}+\frac{3\sin\left(x^2\right)}{8}-\frac{\cos\left(3x^2\right)}{24}+\frac{\sin\left(3x^2\right)}{24}$	30
parts	$\frac{\left(2+\cos\left(x^2\right)^2\right)\sin\left(x^2\right)}{6}+\frac{\left(2+\sin\left(x^2\right)^2\right)\cos\left(x^2\right)}{6}$	30
norman	$\frac{\tan\left(\frac{x^2}{2}\right)^5+2\tan\left(\frac{x^2}{2}\right)^2+\frac{2\tan\left(\frac{x^2}{2}\right)^3}{3}+\frac{2}{3}+\tan\left(\frac{x^2}{2}\right)}{\left(1+\tan\left(\frac{x^2}{2}\right)^2\right)^3}$	50
parallelrisch	$\frac{2+3\tan\left(\frac{x^2}{2}\right)^5+2\tan\left(\frac{x^2}{2}\right)^3+6\tan\left(\frac{x^2}{2}\right)^2+3\tan\left(\frac{x^2}{2}\right)}{3\left(1+\tan\left(\frac{x^2}{2}\right)^2\right)^3}$	55

input
```
int(x*(cos(x^2)^3-sin(x^2)^3),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2+cos(x^2)^2)*sin(x^2)+1/6*(2+sin(x^2)^2)*cos(x^2)
```

3.764.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int x\left(\cos^3\left(x^2\right)-\sin^3\left(x^2\right)\right)\,dx=-\frac{1}{6}\cos\left(x^2\right)^3+\frac{1}{6}\left(\cos\left(x^2\right)^2+2\right)\sin\left(x^2\right)+\frac{1}{2}\cos\left(x^2\right)$$

input
```
integrate(x*(cos(x^2)^3-sin(x^2)^3),x, algorithm="fricas")
```

output
```
-1/6*cos(x^2)^3 + 1/6*(cos(x^2)^2 + 2)*sin(x^2) + 1/2*cos(x^2)
```

3.764.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int x\left(\cos^3\left(x^2\right)-\sin^3\left(x^2\right)\right)\,dx=\frac{\sin^3\left(x^2\right)}{3}+\frac{\sin^2\left(x^2\right)\cos\left(x^2\right)}{2}+\frac{\sin\left(x^2\right)\cos^2\left(x^2\right)}{2}+\frac{\cos^3\left(x^2\right)}{3}$$

input
```
integrate(x*(cos(x**2)**3-sin(x**2)**3),x)
```

output
```
sin(x**2)**3/3 + sin(x**2)**2*cos(x**2)/2 + sin(x**2)*cos(x**2)**2/2 + cos
(x**2)**3/3
```

3.764.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int x\left(\cos^3\left(x^2\right)-\sin^3\left(x^2\right)\right)\,dx=-\frac{1}{24}\cos\left(3\,x^2\right)+\frac{3}{8}\cos\left(x^2\right)+\frac{1}{24}\sin\left(3\,x^2\right)+\frac{3}{8}\sin\left(x^2\right)$$

input
```
integrate(x*(cos(x^2)^3-sin(x^2)^3),x, algorithm="maxima")
```

output
```
-1/24*cos(3*x^2) + 3/8*cos(x^2) + 1/24*sin(3*x^2) + 3/8*sin(x^2)
```

3.764.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int x\left(\cos^3\left(x^2\right)-\sin^3\left(x^2\right)\right)\,dx=-\frac{1}{6}\cos\left(x^2\right)^3-\frac{1}{6}\sin\left(x^2\right)^3+\frac{1}{2}\cos\left(x^2\right)+\frac{1}{2}\sin\left(x^2\right)$$

input
```
integrate(x*(cos(x^2)^3-sin(x^2)^3),x, algorithm="giac")
```

output
```
-1/6*cos(x^2)^3 - 1/6*sin(x^2)^3 + 1/2*cos(x^2) + 1/2*sin(x^2)
```

3.764.9 Mupad [B] (verification not implemented)

Time = 26.43 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int x\left(\cos^3\left(x^2\right)-\sin^3\left(x^2\right)\right)\,dx=-\frac{\cos\left(x^2\right)^3}{6}+\frac{\sin\left(x^2\right)\,\cos\left(x^2\right)^2}{6}+\frac{\cos\left(x^2\right)}{2}+\frac{\sin\left(x^2\right)}{3}$$

input
```
int(x*(cos(x^2)^3 - sin(x^2)^3),x)
```

output
```
cos(x^2)/2 + sin(x^2)/3 + (cos(x^2)^2*sin(x^2))/6 - cos(x^2)^3/6
```

3.765 $\int \frac{\cos(x)\sin(x)}{1-\cos(x)}\,dx$

3.765.1 Optimal result

Integrand size = 13, antiderivative size = 10

$$\int \frac{\cos(x)\sin(x)}{1-\cos(x)}\,dx = \cos(x) + \log(1-\cos(x))$$

output
```
cos(x)+ln(1-cos(x))
```

3.765.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\cos(x)\sin(x)}{1-\cos(x)}\,dx = \cos(x) + 2\log\left(\sin\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[(Cos[x]*Sin[x])/(1 - Cos[x]),x]
```

output
```
Cos[x] + 2*Log[Sin[x/2]]
```

3.765.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3042, 25, 3312, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)\cos(x)}{1-\cos(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sin\left(x+\frac{\pi}{2}\right)\cos\left(x+\frac{\pi}{2}\right)}{1-\sin\left(x+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\cos\left(x+\frac{\pi}{2}\right)\sin\left(x+\frac{\pi}{2}\right)}{1-\sin\left(x+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3312}$$

$$\int \frac{\cos(x)}{1-\cos(x)}d(-\cos(x))$$

$$\Big\downarrow \text{25}$$

$$-\int -\frac{\cos(x)}{1-\cos(x)}d(-\cos(x))$$

$$\Big\downarrow \text{49}$$

$$-\int \left(1+\frac{1}{\cos(x)-1}\right)d(-\cos(x))$$

$$\Big\downarrow \text{2009}$$

$$\cos(x)+\log(1-\cos(x))$$

input
```
Int[(Cos[x]*Sin[x])/(1 - Cos[x]),x]
```

output
```
Cos[x] + Log[1 - Cos[x]]
```

3.765.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3312
```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b*f)   Su
bst[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x]
```

3.765.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$\cos(x)+\ln(\cos(x)-1)$	9
default	$\cos(x)+\ln(\cos(x)-1)$	9
parallelrisch	$\cos(x)+2\ln(\csc(x)-\cot(x))-\ln\left(\frac{2}{\cos(x)+1}\right)+1$	26
risch	$-ix+\frac{e^{ix}}{2}+\frac{e^{-ix}}{2}+2\ln(e^{ix}-1)$	30
norman	$\frac{2\tan(\frac{x}{2})+2\tan(\frac{x}{2})^{3}}{\left(1+\tan(\frac{x}{2})^{2}\right)^{2}\tan(\frac{x}{2})}+2\ln\left(\tan\left(\frac{x}{2}\right)\right)-\ln\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)$	52

input
```
int(cos(x)*sin(x)/(1-cos(x)),x,method=_RETURNVERBOSE)
```

output
```
cos(x)+ln(cos(x)-1)
```

3.765.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)\sin(x)}{1-\cos(x)}\,dx = \cos(x) + \log\left(-\frac{1}{2}\cos(x) + \frac{1}{2}\right)$$

input
```
integrate(cos(x)*sin(x)/(1-cos(x)),x, algorithm="fricas")
```

output
```
cos(x) + log(-1/2*cos(x) + 1/2)
```

3.765.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\cos(x)\sin(x)}{1-\cos(x)}\,dx = \log(\cos(x) - 1) + \cos(x)$$

input
```
integrate(cos(x)*sin(x)/(1-cos(x)),x)
```

output
```
log(cos(x) - 1) + cos(x)
```

3.765.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\cos(x)\sin(x)}{1-\cos(x)}\,dx = \cos(x) + \log(\cos(x) - 1)$$

input
```
integrate(cos(x)*sin(x)/(1-cos(x)),x, algorithm="maxima")
```

output
```
cos(x) + log(cos(x) - 1)
```

3.765.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)\sin(x)}{1-\cos(x)}\,dx = \cos\left(x\right) + \log\left(-\cos\left(x\right) + 1\right)$$

input
```
integrate(cos(x)*sin(x)/(1-cos(x)),x, algorithm="giac")
```

output
```
cos(x) + log(-cos(x) + 1)
```

3.765.9 Mupad [B] (verification not implemented)

Time = 26.60 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\cos(x)\sin(x)}{1-\cos(x)}\,dx = \ln\left(\cos\left(x\right) - 1\right) + \cos\left(x\right)$$

input
```
int(-(cos(x)*sin(x))/(cos(x) - 1),x)
```

output
```
log(cos(x) - 1) + cos(x)
```

3.766 $\int x \cos(x^2)\, dx$

3.766.1 Optimal result

Integrand size = 6, antiderivative size = 8

$$\int x \cos(x^2)\, dx = \frac{\sin(x^2)}{2}$$

output
```
1/2*sin(x^2)
```

3.766.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int x \cos(x^2)\, dx = \frac{\sin(x^2)}{2}$$

input
```
Integrate[x*Cos[x^2],x]
```

output
```
Sin[x^2]/2
```

3.766.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3861, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cos\left(x^2\right)\,dx$$

$$\Big\downarrow \text{3861}$$

$$\frac{1}{2}\int \cos\left(x^2\right)dx^2$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \sin\left(x^2+\frac{\pi}{2}\right)dx^2$$

$$\Big\downarrow \text{3117}$$

$$\frac{\sin\left(x^2\right)}{2}$$

input
```
Int[x*Cos[x^2],x]
```

output
```
Sin[x^2]/2
```

3.766.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3861
```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

3.766.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{\sin\left(x^2\right)}{2}$	7
default	$\frac{\sin\left(x^2\right)}{2}$	7
meijerg	$\frac{\sin\left(x^2\right)}{2}$	7
risch	$\frac{\sin\left(x^2\right)}{2}$	7
parallelrisch	$\frac{\sin\left(x^2\right)}{2}$	7
norman	$\frac{\tan\left(\frac{x^2}{2}\right)}{1+\tan\left(\frac{x^2}{2}\right)^2}$	20
parts	$\frac{\sqrt{2}\,\sqrt{\pi}\,\operatorname{FresnelC}\left(\frac{\sqrt{2}\,x}{\sqrt{\pi}}\right)x}{2}-\frac{\pi\left(\frac{\operatorname{FresnelC}\left(\frac{\sqrt{2}\,x}{\sqrt{\pi}}\right)\sqrt{2}\,x}{\sqrt{\pi}}-\frac{\sin\left(x^2\right)}{\pi}\right)}{2}$	50

input
```
int(x*cos(x^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*sin(x^2)
```

3.766.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int x\cos\left(x^2\right)\,dx=\frac{1}{2}\sin\left(x^2\right)$$

input
```
integrate(x*cos(x^2),x, algorithm="fricas")
```

output
```
1/2*sin(x^2)
```

3.766.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int x\cos\left(x^2\right)\,dx=\frac{\sin\left(x^2\right)}{2}$$

input
```
integrate(x*cos(x**2),x)
```

output
```
sin(x**2)/2
```

3.766.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int x\cos\left(x^2\right)\,dx=\frac{1}{2}\sin\left(x^2\right)$$

input
```
integrate(x*cos(x^2),x, algorithm="maxima")
```

output
```
1/2*sin(x^2)
```

3.766.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int x\cos\left(x^2\right)\,dx=\frac{1}{2}\sin\left(x^2\right)$$

input
```
integrate(x*cos(x^2),x, algorithm="giac")
```

output
```
1/2*sin(x^2)
```

3.766.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int x\cos\left(x^2\right)\,dx=\frac{\sin\left(x^2\right)}{2}$$

input
```
int(x*cos(x^2),x)
```

output
```
sin(x^2)/2
```

3.767 $\int x^2 \cos(4x^3)\,dx$

3.767.1 Optimal result

Integrand size = 10, antiderivative size = 10

$$\int x^2 \cos(4x^3)\,dx = \frac{1}{12}\sin(4x^3)$$

output
```
1/12*sin(4*x^3)
```

3.767.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int x^2 \cos(4x^3)\,dx = \frac{1}{12}\sin(4x^3)$$

input
```
Integrate[x^2*Cos[4*x^3],x]
```

output
```
Sin[4*x^3]/12
```

3.767.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3861, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \cos \left(4 x^3\right) \, dx$$

$$\Big\downarrow \text{ 3861}$$

$$\frac{1}{3} \int \cos \left(4 x^3\right)dx^3$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{3} \int \sin \left(4 x^3+\frac{\pi }{2}\right)dx^3$$

$$\Big\downarrow \text{ 3117}$$

$$\frac{1}{12} \sin \left(4 x^3\right)$$

input
```
Int[x^2*Cos[4*x^3],x]
```

output
```
Sin[4*x^3]/12
```

3.767.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3861
```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

3.767.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$\frac{\sin(4x^3)}{12}$	9
default	$\frac{\sin(4x^3)}{12}$	9
meijerg	$\frac{\sin(4x^3)}{12}$	9
risch	$\frac{\sin(4x^3)}{12}$	9
parallelrisch	$\frac{\sin(4x^3)}{12}$	9
norman	$\frac{\tan(2x^3)}{6+6\tan(2x^3)^2}$	21

input
```
int(x^2*cos(4*x^3),x,method=_RETURNVERBOSE)
```

output
```
1/12*sin(4*x^3)
```

3.767.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int x^2 \cos\left(4x^3\right) \,dx = \frac{1}{12} \sin\left(4\,x^3\right)$$

input
```
integrate(x^2*cos(4*x^3),x, algorithm="fricas")
```

output
```
1/12*sin(4*x^3)
```

3.767.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int x^2 \cos(4x^3) \, dx = \frac{\sin(4x^3)}{12}$$

input
```
integrate(x**2*cos(4*x**3),x)
```

output
```
sin(4*x**3)/12
```

3.767.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int x^2 \cos(4x^3) \, dx = \frac{1}{12} \sin(4x^3)$$

input
```
integrate(x^2*cos(4*x^3),x, algorithm="maxima")
```

output
```
1/12*sin(4*x^3)
```

3.767.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int x^2 \cos(4x^3) \, dx = \frac{1}{12} \sin(4x^3)$$

input
```
integrate(x^2*cos(4*x^3),x, algorithm="giac")
```

output
```
1/12*sin(4*x^3)
```

3.767.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int x^2 \cos(4x^3)\, dx = \frac{\sin(4\, x^3)}{12}$$

input
```
int(x^2*cos(4*x^3),x)
```

output
```
sin(4*x^3)/12
```

3.768 $\int x^3 \cos(x^4) \, dx$

3.768.1 Optimal result

Integrand size = 8, antiderivative size = 8

$$\int x^3 \cos(x^4) \, dx = \frac{\sin(x^4)}{4}$$

output
```
1/4*sin(x^4)
```

3.768.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int x^3 \cos(x^4) \, dx = \frac{\sin(x^4)}{4}$$

input
```
Integrate[x^3*Cos[x^4],x]
```

output
```
Sin[x^4]/4
```

3.768.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3861, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \cos\left(x^4\right) \, dx$$

$$\Big\downarrow \; 3861$$

$$\frac{1}{4} \int \cos\left(x^4\right) dx^4$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{4} \int \sin\left(x^4 + \frac{\pi}{2}\right) dx^4$$

$$\Big\downarrow \; 3117$$

$$\frac{\sin\left(x^4\right)}{4}$$

input
```
Int[x^3*Cos[x^4],x]
```

output
```
Sin[x^4]/4
```

3.768.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3861
```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

3.768.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{\sin(x^4)}{4}$	7
default	$\frac{\sin(x^4)}{4}$	7
meijerg	$\frac{\sin(x^4)}{4}$	7
risch	$\frac{\sin(x^4)}{4}$	7
parallelrisch	$\frac{\sin(x^4)}{4}$	7
norman	$\frac{\tan\left(\frac{x^4}{2}\right)}{2+2\tan\left(\frac{x^4}{2}\right)^2}$	21

input
```
int(x^3*cos(x^4),x,method=_RETURNVERBOSE)
```

output
```
1/4*sin(x^4)
```

3.768.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int x^3 \cos\left(x^4\right) \, dx = \frac{1}{4} \sin\left(x^4\right)$$

input
```
integrate(x^3*cos(x^4),x, algorithm="fricas")
```

output
```
1/4*sin(x^4)
```

3.768.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int x^3 \cos\left(x^4\right)\, dx = \frac{\sin\left(x^4\right)}{4}$$

input
```
integrate(x**3*cos(x**4),x)
```

output
```
sin(x**4)/4
```

3.768.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int x^3 \cos\left(x^4\right)\, dx = \frac{1}{4}\,\sin\left(x^4\right)$$

input
```
integrate(x^3*cos(x^4),x, algorithm="maxima")
```

output
```
1/4*sin(x^4)
```

3.768.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int x^3 \cos\left(x^4\right)\, dx = \frac{1}{4}\,\sin\left(x^4\right)$$

input
```
integrate(x^3*cos(x^4),x, algorithm="giac")
```

output
```
1/4*sin(x^4)
```

3.768.9 Mupad [B] (verification not implemented)

Time = 25.88 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int x^3 \cos\left(x^4\right) \, dx = \frac{\sin\left(x^4\right)}{4}$$

input
```
int(x^3*cos(x^4),x)
```

output
```
sin(x^4)/4
```

3.769 $\int x \sin\left(\frac{x^2}{2}\right) dx$

3.769.1 Optimal result

Integrand size = 10, antiderivative size = 10

$$\int x \sin\left(\frac{x^2}{2}\right) dx = -\cos\left(\frac{x^2}{2}\right)$$

output
```
-cos(1/2*x^2)
```

3.769.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int x \sin\left(\frac{x^2}{2}\right) dx = -\cos\left(\frac{x^2}{2}\right)$$

input
```
Integrate[x*Sin[x^2/2],x]
```

output
```
-Cos[x^2/2]
```

3.769.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3860, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sin\left(\frac{x^2}{2}\right) dx$$

$$\Big\downarrow \text{ 3860}$$

$$\frac{1}{2} \int \sin\left(\frac{x^2}{2}\right) dx^2$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2} \int \sin\left(\frac{x^2}{2}\right) dx^2$$

$$\Big\downarrow \text{ 3118}$$

$$-\cos\left(\frac{x^2}{2}\right)$$

input
```
Int[x*Sin[x^2/2],x]
```

output
```
-Cos[x^2/2]
```

3.769.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

3.769.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$-\cos\left(\frac{x^2}{2}\right)$	9
default	$-\cos\left(\frac{x^2}{2}\right)$	9
risch	$-\cos\left(\frac{x^2}{2}\right)$	9
parallelrisch	$-\cos\left(\frac{x^2}{2}\right)-1$	11
norman	$-\frac{2}{1+\tan\left(\frac{x^2}{4}\right)^2}$	15
meijerg	$\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos\left(\frac{x^2}{2}\right)}{\sqrt{\pi}}\right)$	20
parts	$\sqrt{\pi}\ \mathrm{FresnelS}\left(\frac{x}{\sqrt{\pi}}\right)x-\pi\left(\frac{\mathrm{FresnelS}\left(\frac{x}{\sqrt{\pi}}\right)x}{\sqrt{\pi}}+\frac{\cos\left(\frac{x^2}{2}\right)}{\pi}\right)$	38

input
```
int(x*sin(1/2*x^2),x,method=_RETURNVERBOSE)
```

output
```
-cos(1/2*x^2)
```

3.769.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int x\sin\left(\frac{x^2}{2}\right)\,dx=-\cos\left(\frac{1}{2}\,x^2\right)$$

input
```
integrate(x*sin(1/2*x^2),x, algorithm="fricas")
```

output
```
-cos(1/2*x^2)
```

3.769.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int x\sin\left(\frac{x^2}{2}\right)\,dx = -\cos\left(\frac{x^2}{2}\right)$$

input
```
integrate(x*sin(1/2*x**2),x)
```

output
```
-cos(x**2/2)
```

3.769.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int x\sin\left(\frac{x^2}{2}\right)\,dx = -\cos\left(\frac{1}{2}\,x^2\right)$$

input
```
integrate(x*sin(1/2*x^2),x, algorithm="maxima")
```

output
```
-cos(1/2*x^2)
```

3.769.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int x\sin\left(\frac{x^2}{2}\right)\,dx = -\cos\left(\frac{1}{2}\,x^2\right)$$

input
```
integrate(x*sin(1/2*x^2),x, algorithm="giac")
```

output
```
-cos(1/2*x^2)
```

3.769.9 Mupad [B] (verification not implemented)

Time = 26.99 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int x\sin\left(\frac{x^2}{2}\right)\,dx = -\cos\left(\frac{x^2}{2}\right)$$

input
```
int(x*sin(x^2/2),x)
```

output
```
-cos(x^2/2)
```

3.770 $\int x \sec(x^2) \tan(x^2)\, dx$

3.770.1 Optimal result

Integrand size = 10, antiderivative size = 8

$$\int x \sec(x^2) \tan(x^2)\, dx = \frac{\sec(x^2)}{2}$$

output
```
1/2*sec(x^2)
```

3.770.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int x \sec(x^2) \tan(x^2)\, dx = \frac{\sec(x^2)}{2}$$

input
```
Integrate[x*Sec[x^2]*Tan[x^2],x]
```

output
```
Sec[x^2]/2
```

3.770.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {7266, 3042, 3086, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\tan\left(x^2\right)\sec\left(x^2\right)\,dx$$

$$\Big\downarrow \; 7266$$

$$\frac{1}{2}\int \sec\left(x^2\right)\tan\left(x^2\right)dx^2$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{2}\int \sec\left(x^2\right)\tan\left(x^2\right)dx^2$$

$$\Big\downarrow \; 3086$$

$$\frac{1}{2}\int 1d\sec\left(x^2\right)$$

$$\Big\downarrow \; 24$$

$$\frac{\sec\left(x^2\right)}{2}$$

input
```
Int[x*Sec[x^2]*Tan[x^2],x]
```

output
```
Sec[x^2]/2
```

3.770.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

3.770.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{\sec(x^2)}{2}$	7
default	$\frac{\sec(x^2)}{2}$	7
risch	$\frac{e^{i x^2}}{e^{2 i x^2}+1}$	20

input
```
int(x*sec(x^2)*tan(x^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*sec(x^2)
```

3.770.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int x \sec\left(x^2\right) \tan\left(x^2\right)\, dx = \frac{1}{2 \cos\left(x^2\right)}$$

input
```
integrate(x*sec(x^2)*tan(x^2),x, algorithm="fricas")
```

output
```
1/2/cos(x^2)
```

3.770.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int x\sec\left(x^2\right)\tan\left(x^2\right)\,dx=\frac{\sec\left(x^2\right)}{2}$$

input
```
integrate(x*sec(x**2)*tan(x**2),x)
```

output
```
sec(x**2)/2
```

3.770.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(6) = 12.

Time = 0.20 (sec) , antiderivative size = 56, normalized size of antiderivative = 7.00

$$\int x\sec\left(x^2\right)\tan\left(x^2\right)\,dx=\frac{\cos\left(2\,x^2\right)\cos\left(x^2\right)+\sin\left(2\,x^2\right)\sin\left(x^2\right)+\cos\left(x^2\right)}{\cos\left(2\,x^2\right)^2+\sin\left(2\,x^2\right)^2+2\,\cos\left(2\,x^2\right)+1}$$

input
```
integrate(x*sec(x^2)*tan(x^2),x, algorithm="maxima")
```

output
```
(cos(2*x^2)*cos(x^2) + sin(2*x^2)*sin(x^2) + cos(x^2))/(cos(2*x^2)^2 + sin
(2*x^2)^2 + 2*cos(2*x^2) + 1)
```

3.770.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int x\sec\left(x^2\right)\tan\left(x^2\right)\,dx=\frac{1}{2\,\cos\left(x^2\right)}$$

input
```
integrate(x*sec(x^2)*tan(x^2),x, algorithm="giac")
```

output
```
1/2/cos(x^2)
```

3.770.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int x \sec\left(x^2\right) \tan\left(x^2\right) \, dx = \frac{1}{2 \cos\left(x^2\right)}$$

input
```
int((x*tan(x^2))/cos(x^2),x)
```

output
```
1/(2*cos(x^2))
```

3.771 $\int \frac{\tan^2\left(\frac{1}{x}\right)}{x^2} dx$

3.771.1 Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\tan^2\left(\frac{1}{x}\right)}{x^2} dx = \frac{1}{x} - \tan\left(\frac{1}{x}\right)$$

output
```
1/x-tan(1/x)
```

3.771.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\tan^2\left(\frac{1}{x}\right)}{x^2} dx = \arctan\left(\tan\left(\frac{1}{x}\right)\right) - \tan\left(\frac{1}{x}\right)$$

input
```
Integrate[Tan[x^(-1)]^2/x^2,x]
```

output
```
ArcTan[Tan[x^(-1)]] - Tan[x^(-1)]
```

3.771.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4234, 3042, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2\left(\frac{1}{x}\right)}{x^2} \, dx$$
$$\Big\downarrow \ 4234$$
$$-\int \tan^2\left(\frac{1}{x}\right)d\frac{1}{x}$$
$$\Big\downarrow \ 3042$$
$$-\int \tan\left(\frac{1}{x}\right)^2 d\frac{1}{x}$$
$$\Big\downarrow \ 3954$$
$$\int 1d\frac{1}{x}-\tan\left(\frac{1}{x}\right)$$
$$\Big\downarrow \ 24$$
$$\frac{1}{x}-\tan\left(\frac{1}{x}\right)$$

input
```
Int[Tan[x^(-1)]^2/x^2,x]
```

output
```
x^(-1) - Tan[x^(-1)]
```

3.771.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4234
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Tan[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Tan[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
 1)/n], 0] && IntegerQ[p]
```

3.771.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.30

method	result	size
derivativedivides	$-\tan\left(\frac{1}{x}\right)+\arctan\left(\tan\left(\frac{1}{x}\right)\right)$	13
default	$-\tan\left(\frac{1}{x}\right)+\arctan\left(\tan\left(\frac{1}{x}\right)\right)$	13
norman	$\frac{1-\tan\left(\frac{1}{x}\right)x}{x}$	14
parallelrisch	$-\frac{\tan\left(\frac{1}{x}\right)x-1}{x}$	14
risch	$\frac{1}{x}-\frac{2i}{e^{\frac{2i}{x}}+1}$	19

input
```
int(tan(1/x)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-tan(1/x)+arctan(tan(1/x))
```

3.771.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.30

$$\int \frac{\tan^2\left(\frac{1}{x}\right)}{x^2}\,dx=-\frac{x\tan\left(\frac{1}{x}\right)-1}{x}$$

input
```
integrate(tan(1/x)^2/x^2,x, algorithm="fricas")
```

output
```
-(x*tan(1/x) - 1)/x
```

3.771.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int \frac{\tan^2\left(\frac{1}{x}\right)}{x^2}\,dx = -\tan\left(\frac{1}{x}\right) + \frac{1}{x}$$

input
```
integrate(tan(1/x)**2/x**2,x)
```

output
```
-tan(1/x) + 1/x
```

3.771.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. 2(10) = 20.

Time = 0.19 (sec) , antiderivative size = 67, normalized size of antiderivative = 6.70

$$\int \frac{\tan^2\left(\frac{1}{x}\right)}{x^2}\,dx = \frac{\cos\left(\frac{2}{x}\right)^2 - 2\,x\sin\left(\frac{2}{x}\right) + \sin\left(\frac{2}{x}\right)^2 + 2\,\cos\left(\frac{2}{x}\right) + 1}{\left(\cos\left(\frac{2}{x}\right)^2 + \sin\left(\frac{2}{x}\right)^2 + 2\,\cos\left(\frac{2}{x}\right) + 1\right)x}$$

input
```
integrate(tan(1/x)^2/x^2,x, algorithm="maxima")
```

output
```
(cos(2/x)^2 - 2*x*sin(2/x) + sin(2/x)^2 + 2*cos(2/x) + 1)/((cos(2/x)^2 + s
in(2/x)^2 + 2*cos(2/x) + 1)*x)
```

3.771.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\tan^2\left(\frac{1}{x}\right)}{x^2}\,dx = \frac{1}{x} - \tan\left(\frac{1}{x}\right)$$

input
```
integrate(tan(1/x)^2/x^2,x, algorithm="giac")
```

output
```
1/x - tan(1/x)
```

3.771.9 Mupad [B] (verification not implemented)

Time = 26.48 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\tan^2\left(\frac{1}{x}\right)}{x^2}\,dx = \frac{1}{x} - \tan\left(\frac{1}{x}\right)$$

input
```
int(tan(1/x)^2/x^2,x)
```

output
```
1/x - tan(1/x)
```

3.772 $\int x \tan(1+x^2)\,dx$

3.772.1 Optimal result

Integrand size = 8, antiderivative size = 11

$$\int x \tan(1+x^2)\,dx = -\frac{1}{2}\log(\cos(1+x^2))$$

output
```
-1/2*ln(cos(x^2+1))
```

3.772.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int x \tan(1+x^2)\,dx = -\frac{1}{2}\log(\cos(1+x^2))$$

input
```
Integrate[x*Tan[1 + x^2],x]
```

output
```
-1/2*Log[Cos[1 + x^2]]
```

3.772.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4234, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \tan \left(x^2+1\right) \, dx$$

$$\Big\downarrow \ 4234$$

$$\frac{1}{2} \int \tan \left(x^2+1\right)dx^2$$

$$\Big\downarrow \ 3042$$

$$\frac{1}{2} \int \tan \left(x^2+1\right)dx^2$$

$$\Big\downarrow \ 3956$$

$$-\frac{1}{2} \log \left(\cos \left(x^2+1\right)\right)$$

input
```
Int[x*Tan[1 + x^2],x]
```

output
```
-1/2*Log[Cos[1 + x^2]]
```

3.772.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4234
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Tan[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Tan[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
 1)/n], 0] && IntegerQ[p]
```

3.772.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$-\frac{\ln(\cos(x^2+1))}{2}$	10
default	$-\frac{\ln(\cos(x^2+1))}{2}$	10
norman	$\frac{\ln\left(1+\tan(x^2+1)^2\right)}{4}$	14
parallelrisch	$\frac{\ln\left(1+\tan(x^2+1)^2\right)}{4}$	14
risch	$\frac{ix^2}{2}+i-\frac{\ln\left(\mathrm{e}^{2i(x^2+1)}+1\right)}{2}$	24

input
```
int(x*tan(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(cos(x^2+1))
```

3.772.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int x\tan\left(1+x^2\right)\,dx=-\frac{1}{4}\log\left(\frac{1}{\tan\left(x^2+1\right)^2+1}\right)$$

input
```
integrate(x*tan(x^2+1),x, algorithm="fricas")
```

output
```
-1/4*log(1/(tan(x^2 + 1)^2 + 1))
```

3.772.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int x\tan\left(1+x^2\right)\,dx=\frac{\log\left(\tan^2\left(x^2+1\right)+1\right)}{4}$$

input
```
integrate(x*tan(x**2+1),x)
```

output
```
log(tan(x**2 + 1)**2 + 1)/4
```

3.772.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int x\tan\left(1+x^2\right)\,dx = \frac{1}{2}\log\left(\sec\left(x^2+1\right)\right)$$

input
```
integrate(x*tan(x^2+1),x, algorithm="maxima")
```

output
```
1/2*log(sec(x^2 + 1))
```

3.772.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int x\tan\left(1+x^2\right)\,dx = -\frac{1}{2}\log\left(\left|\cos\left(x^2+1\right)\right|\right)$$

input
```
integrate(x*tan(x^2+1),x, algorithm="giac")
```

output
```
-1/2*log(abs(cos(x^2 + 1)))
```

3.772.9 Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int x\tan\left(1+x^2\right)\,dx = \frac{\ln\left(\tan(x^2+1)^2+1\right)}{4}$$

input
```
int(x*tan(x^2 + 1),x)
```

output
```
log(tan(x^2 + 1)^2 + 1)/4
```

3.773 $\int \sin(\pi(1+2x))\,dx$

3.773.1 Optimal result

Integrand size = 8, antiderivative size = 12

$$\int \sin(\pi(1+2x))\,dx = \frac{\cos(2\pi x)}{2\pi}$$

output
```
1/2*cos(2*Pi*x)/Pi
```

3.773.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \sin(\pi(1+2x))\,dx = \frac{\cos(2\pi x)}{2\pi}$$

input
```
Integrate[Sin[Pi*(1 + 2*x)],x]
```

output
```
Cos[2*Pi*x]/(2*Pi)
```

3.773.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 25, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(\pi(2x+1))\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -\sin(2\pi x)dx$$

$$\Big\downarrow \; 25$$

$$-\int \sin(2\pi x)dx$$

$$\Big\downarrow \; 3118$$

$$\frac{\cos(2\pi x)}{2\pi}$$

input
```
Int[Sin[Pi*(1 + 2*x)],x]
```

output
```
Cos[2*Pi*x]/(2*Pi)
```

3.773.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

3.773.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{\cos(2\pi x)}{2\pi}$	11
default	$\frac{\cos(2\pi x)}{2\pi}$	11
risch	$\frac{\cos(2\pi x)}{2\pi}$	11
parallelrisch	$\frac{-1+\cos(2\pi x)}{2\pi}$	13
meijerg	$-\frac{\frac{1}{\sqrt{\pi}}-\frac{\cos(2\pi x)}{\sqrt{\pi}}}{2\sqrt{\pi}}$	20
norman	$-\frac{1}{\pi\left(1+\tan\left(\frac{\pi(1+2x)}{2}\right)^2\right)}$	21

input
```
int(sin(Pi*(1+2*x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*cos(2*Pi*x)/Pi
```

3.773.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \sin(\pi(1+2x))\,dx = -\frac{\cos(\pi + 2\,\pi x)}{2\,\pi}$$

input
```
integrate(sin(pi*(1+2*x)),x, algorithm="fricas")
```

output
```
-1/2*cos(pi + 2*pi*x)/pi
```

3.773.6 Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \sin(\pi(1+2x))\,dx = -\frac{\cos(\pi(2x+1))}{2\pi}$$

input
```
integrate(sin(pi*(1+2*x)),x)
```

output
```
-cos(pi*(2*x + 1))/(2*pi)
```

3.773.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \sin(\pi(1+2x))\,dx = \frac{\cos(2\,\pi x)}{2\,\pi}$$

input
```
integrate(sin(pi*(1+2*x)),x, algorithm="maxima")
```

output
```
1/2*cos(2*pi*x)/pi
```

3.773.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \sin(\pi(1+2x))\,dx = \frac{\cos(2\,\pi x)}{2\,\pi}$$

input
```
integrate(sin(pi*(1+2*x)),x, algorithm="giac")
```

output
```
1/2*cos(2*pi*x)/pi
```

3.773.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \sin(\pi(1+2x))\,dx = -\frac{\cos\left(\Pi\left(2\,x+1\right)\right)}{2\,\Pi}$$

input
```
int(sin(Pi*(2*x + 1)),x)
```

output
```
-cos(Pi*(2*x + 1))/(2*Pi)
```

3.774 $\int \frac{\cot(x)+\csc^2(x)}{1-\cos^2(x)}\,dx$

3.774.1 Optimal result

Integrand size = 18, antiderivative size = 21

$$\int \frac{\cot(x)+\csc^2(x)}{1-\cos^2(x)}\,dx = -\cot(x)-\frac{\cot^2(x)}{2}-\frac{\cot^3(x)}{3}$$

output
```
-cot(x)-1/2*cot(x)^2-1/3*cot(x)^3
```

3.774.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{\cot(x)+\csc^2(x)}{1-\cos^2(x)}\,dx = -\frac{2\cot(x)}{3}-\frac{\csc^2(x)}{2}-\frac{1}{3}\cot(x)\csc^2(x)$$

input
```
Integrate[(Cot[x] + Csc[x]^2)/(1 - Cos[x]^2),x]
```

output
```
(-2*Cot[x])/3 - Csc[x]^2/2 - (Cot[x]*Csc[x]^2)/3
```

3.774.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3042, 4889, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(x)+\csc^2(x)}{1-\cos^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cot(x)+\csc(x)^2}{1-\cos(x)^2}dx$$

$$\Big\downarrow \text{4889}$$

$$\int \left(\tan^2(x)+\tan(x)+1\right)\cot^4(x)d\tan(x)$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\cot^4(x)+\cot^3(x)+\cot^2(x)\right)d\tan(x)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{3}\cot^3(x)-\frac{\cot^2(x)}{2}-\cot(x)$$

input
```
Int[(Cot[x] + Csc[x]^2)/(1 - Cos[x]^2),x]
```

output
```
-Cot[x] - Cot[x]^2/2 - Cot[x]^3/3
```

3.774.3.1 Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.774.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$-\cot\left(x\right)-\frac{\cot\left(x\right)^{2}}{2}-\frac{\cot\left(x\right)^{3}}{3}$	18
default	$-\cot\left(x\right)-\frac{\cot\left(x\right)^{2}}{2}-\frac{\cot\left(x\right)^{3}}{3}$	18
risch	$\frac{2\,{\mathrm e}^{4 i x}+4 i {\mathrm e}^{2 i x}-2\,{\mathrm e}^{2 i x}-\frac{4 i}{3}}{\left({\mathrm e}^{2 i x}-1\right)^{3}}$	37

input
```
int((cot(x)+csc(x)^2)/(1-cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-cot(x)-1/2*cot(x)^2-1/3*cot(x)^3
```

3.774.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.38

$$\int \frac{\cot(x)+\csc^2(x)}{1-\cos^2(x)}\,dx=-\frac{4\,\cos\left(x\right)^{3}-6\,\cos\left(x\right)-3\,\sin\left(x\right)}{6\,\left(\cos\left(x\right)^{2}-1\right)\sin\left(x\right)}$$

input
```
integrate((cot(x)+csc(x)^2)/(1-cos(x)^2),x, algorithm="fricas")
```

output
```
-1/6*(4*cos(x)^3 - 6*cos(x) - 3*sin(x))/((cos(x)^2 - 1)*sin(x))
```

3.774.6 Sympy [F]

$$\int \frac{\cot(x)+\csc^2(x)}{1-\cos^2(x)}\,dx = -\int \frac{\cot(x)}{\cos^2(x)-1}\,dx - \int \frac{\csc^2(x)}{\cos^2(x)-1}\,dx$$

input
```
integrate((cot(x)+csc(x)**2)/(1-cos(x)**2),x)
```

output
```
-Integral(cot(x)/(cos(x)**2 - 1), x) - Integral(csc(x)**2/(cos(x)**2 - 1),
 x)
```

3.774.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int \frac{\cot(x)+\csc^2(x)}{1-\cos^2(x)}\,dx = -\frac{6\tan(x)^2+3\tan(x)+2}{6\tan(x)^3}$$

input
```
integrate((cot(x)+csc(x)^2)/(1-cos(x)^2),x, algorithm="maxima")
```

output
```
-1/6*(6*tan(x)^2 + 3*tan(x) + 2)/tan(x)^3
```

3.774.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int \frac{\cot(x)+\csc^2(x)}{1-\cos^2(x)}\,dx = -\frac{6\tan(x)^2+3\tan(x)+2}{6\tan(x)^3}$$

input
```
integrate((cot(x)+csc(x)^2)/(1-cos(x)^2),x, algorithm="giac")
```

output
```
-1/6*(6*tan(x)^2 + 3*tan(x) + 2)/tan(x)^3
```

3.774.9 Mupad [B] (verification not implemented)

Time = 26.58 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.76

$$\int \frac{\cot(x) + \csc^2(x)}{1 - \cos^2(x)} \, dx = -\frac{\cot(x) \left(2\cot(x)^2 + 3\cot(x) + 6\right)}{6}$$

input
```
int(-(cot(x) + 1/sin(x)^2)/(cos(x)^2 - 1),x)
```

output
```
-(cot(x)*(3*cot(x) + 2*cot(x)^2 + 6))/6
```

3.775 $\int x^2 \cos(4x^3) \cos(5x^3)\,dx$

3.775.1 Optimal result

Integrand size = 16, antiderivative size = 19

$$\int x^2 \cos(4x^3) \cos(5x^3)\,dx = \frac{\sin(x^3)}{6} + \frac{1}{54}\sin(9x^3)$$

output
```
1/6*sin(x^3)+1/54*sin(9*x^3)
```

3.775.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int x^2 \cos(4x^3) \cos(5x^3)\,dx = \frac{\sin(x^3)}{6} + \frac{1}{54}\sin(9x^3)$$

input
```
Integrate[x^2*Cos[4*x^3]*Cos[5*x^3],x]
```

output
```
Sin[x^3]/6 + Sin[9*x^3]/54
```

3.775.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5083, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \cos\left(4x^3\right)\cos\left(5x^3\right)\,dx$$

$$\Big\downarrow \text{ 5083}$$

$$\int \left(\frac{1}{2}x^2\cos\left(x^3\right)+\frac{1}{2}x^2\cos\left(9x^3\right)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sin\left(x^3\right)}{6}+\frac{1}{54}\sin\left(9x^3\right)$$

input
```
Int[x^2*Cos[4*x^3]*Cos[5*x^3],x]
```

output
```
Sin[x^3]/6 + Sin[9*x^3]/54
```

3.775.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5083
```
Int[Cos[v_]^(p_.)*Cos[w_]^(q_.)*(x_)^(m_.), x_Symbol] :> Int[ExpandTrigRedu
ce[x^m, Cos[v]^p*Cos[w]^q, x], x] /; IGtQ[m, 0] && IGtQ[p, 0] && IGtQ[q, 0]
 && ((PolynomialQ[v, x] && PolynomialQ[w, x]) || (BinomialQ[{v, w}, x] && I
ndependentQ[Cancel[v/w], x]))
```

3.775.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{\sin(x^3)}{6}+\frac{\sin(9x^3)}{54}$	16
risch	$\frac{\sin(x^3)}{6}+\frac{\sin(9x^3)}{54}$	16
parallelrisch	$\frac{\sin(x^3)}{6}+\frac{\sin(9x^3)}{54}$	16
norman	$\frac{\frac{8\tan(2x^3)\tan\left(\frac{5x^3}{2}\right)^2}{27}-\frac{10\tan(2x^3)^2\tan\left(\frac{5x^3}{2}\right)}{27}-\frac{8\tan(2x^3)}{27}+\frac{10\tan\left(\frac{5x^3}{2}\right)}{27}}{\left(1+\tan(2x^3)^2\right)\left(1+\tan\left(\frac{5x^3}{2}\right)^2\right)}$	75

input
```
int(x^2*cos(4*x^3)*cos(5*x^3),x,method=_RETURNVERBOSE)
```

output
```
1/6*sin(x^3)+1/54*sin(9*x^3)
```

3.775.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 40 vs. 2(15) = 30.

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.11

$$\int x^2\cos(4x^3)\cos(5x^3)\,dx$$
$$=\frac{1}{27}\left(128\cos(x^3)^8-224\cos(x^3)^6+120\cos(x^3)^4-20\cos(x^3)^2+5\right)\sin(x^3)$$

input
```
integrate(x^2*cos(4*x^3)*cos(5*x^3),x, algorithm="fricas")
```

output
```
1/27*(128*cos(x^3)^8 - 224*cos(x^3)^6 + 120*cos(x^3)^4 - 20*cos(x^3)^2 + 5
)*sin(x^3)
```

3.775.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(14) = 28$.

Time = 0.60 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.68

$$\int x^2 \cos(4x^3) \cos(5x^3) \, dx = -\frac{4 \sin(4x^3) \cos(5x^3)}{27} + \frac{5 \sin(5x^3) \cos(4x^3)}{27}$$

input
```
integrate(x**2*cos(4*x**3)*cos(5*x**3),x)
```

output
```
-4*sin(4*x**3)*cos(5*x**3)/27 + 5*sin(5*x**3)*cos(4*x**3)/27
```

3.775.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int x^2 \cos(4x^3) \cos(5x^3) \, dx = \frac{1}{54} \sin(9x^3) + \frac{1}{6} \sin(x^3)$$

input
```
integrate(x^2*cos(4*x^3)*cos(5*x^3),x, algorithm="maxima")
```

output
```
1/54*sin(9*x^3) + 1/6*sin(x^3)
```

3.775.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(15) = 30$.

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.05

$$\int x^2 \cos(4x^3) \cos(5x^3) \, dx = \frac{128}{27} \sin(x^3)^9 - \frac{32}{3} \sin(x^3)^7 + 8 \sin(x^3)^5 - \frac{20}{9} \sin(x^3)^3 + \frac{1}{3} \sin(x^3)$$

input
```
integrate(x^2*cos(4*x^3)*cos(5*x^3),x, algorithm="giac")
```

output
```
128/27*sin(x^3)^9 - 32/3*sin(x^3)^7 + 8*sin(x^3)^5 - 20/9*sin(x^3)^3 + 1/3
*sin(x^3)
```

3.775.9 Mupad [B] (verification not implemented)

Time = 26.65 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int x^2 \cos\left(4x^3\right)\cos\left(5x^3\right)\,dx = \frac{\sin\left(x^3\right)}{6} + \frac{\sin\left(9\,x^3\right)}{54}$$

input
```
int(x^2*cos(4*x^3)*cos(5*x^3),x)
```

output
```
sin(x^3)/6 + sin(9*x^3)/54
```

3.776 $\int x^{14} \sin(x^3) \, dx$

3.776.1 Optimal result

Integrand size = 8, antiderivative size = 47

$$\int x^{14} \sin(x^3) \, dx = -8\cos(x^3) + 4x^6\cos(x^3) - \frac{1}{3}x^{12}\cos(x^3) - 8x^3\sin(x^3) + \frac{4}{3}x^9\sin(x^3)$$

output
```
-8*cos(x^3)+4*x^6*cos(x^3)-1/3*x^12*cos(x^3)-8*x^3*sin(x^3)+4/3*x^9*sin(x^
3)
```

3.776.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int x^{14} \sin(x^3) \, dx = -\frac{1}{3}(24 - 12x^6 + x^{12})\cos(x^3) + \frac{4}{3}x^3(-6 + x^6)\sin(x^3)$$

input
```
Integrate[x^14*Sin[x^3],x]
```

output
```
-1/3*((24 - 12*x^6 + x^12)*Cos[x^3]) + (4*x^3*(-6 + x^6)*Sin[x^3])/3
```

3.776.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.11, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.625$, Rules used = {3860, 3042, 3777, 3042, 3777, 25, 3042, 3777, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{14} \sin \left(x^3\right) \, dx$$

$$\Big\downarrow \text{3860}$$

$$\frac{1}{3} \int x^{12} \sin \left(x^3\right) dx^3$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3} \int x^{12} \sin \left(x^3\right) dx^3$$

$$\Big\downarrow \text{3777}$$

$$\frac{1}{3} \left(4 \int x^9 \cos \left(x^3\right) dx^3 - x^{12} \cos \left(x^3\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3} \left(4 \int x^9 \sin \left(x^3 + \frac{\pi}{2}\right) dx^3 - x^{12} \cos \left(x^3\right)\right)$$

$$\Big\downarrow \text{3777}$$

$$\frac{1}{3} \left(4 \left(3 \int -x^6 \sin \left(x^3\right) dx^3 + x^9 \sin \left(x^3\right)\right) - x^{12} \cos \left(x^3\right)\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3} \left(4 \left(x^9 \sin \left(x^3\right) - 3 \int x^6 \sin \left(x^3\right) dx^3\right) - x^{12} \cos \left(x^3\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3} \left(4 \left(x^9 \sin \left(x^3\right) - 3 \int x^6 \sin \left(x^3\right) dx^3\right) - x^{12} \cos \left(x^3\right)\right)$$

$$\Big\downarrow \text{3777}$$

$$\frac{1}{3} \left(4 \left(x^9 \sin \left(x^3\right) - 3 \left(2 \int x^3 \cos \left(x^3\right) dx^3 - x^6 \cos \left(x^3\right)\right)\right) - x^{12} \cos \left(x^3\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(4\left(x^9\sin\left(x^3\right)-3\left(2\int x^3\sin\left(x^3+\frac{\pi}{2}\right)dx^3-x^6\cos\left(x^3\right)\right)\right)-x^{12}\cos\left(x^3\right)\right)$$

$$\Big\downarrow \text{ 3777}$$

$$\frac{1}{3}\left(4\left(x^9\sin\left(x^3\right)-3\left(2\left(\int-\sin\left(x^3\right)dx^3+x^3\sin\left(x^3\right)\right)-x^6\cos\left(x^3\right)\right)\right)-x^{12}\cos\left(x^3\right)\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{3}\left(4\left(x^9\sin\left(x^3\right)-3\left(2\left(x^3\sin\left(x^3\right)-\int\sin\left(x^3\right)dx^3\right)-x^6\cos\left(x^3\right)\right)\right)-x^{12}\cos\left(x^3\right)\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{3}\left(4\left(x^9\sin\left(x^3\right)-3\left(2\left(x^3\sin\left(x^3\right)-\int\sin\left(x^3\right)dx^3\right)-x^6\cos\left(x^3\right)\right)\right)-x^{12}\cos\left(x^3\right)\right)$$

$$\Big\downarrow \text{ 3118}$$

$$\frac{1}{3}(4(x^9\sin\left(x^3\right)-3(2(x^3\sin\left(x^3\right)+\cos\left(x^3\right))-x^6\cos\left(x^3\right)))-x^{12}\cos\left(x^3\right))$$

input
```
Int[x^14*Sin[x^3],x]
```

output
```
(-(x^12*Cos[x^3]) + 4*(x^9*Sin[x^3] - 3*(-(x^6*Cos[x^3]) + 2*(Cos[x^3] + x
^3*Sin[x^3]))))/3
```

3.776.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

3.776.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.70

method	result	size
risch	$\left(-\frac{1}{3}x^{12}+4x^{6}-8\right)\cos\left(x^{3}\right)+\frac{4x^{3}\left(x^{6}-6\right)\sin\left(x^{3}\right)}{3}$	33
meijerg	$\frac{16\sqrt{\pi}\left(\frac{3}{2\sqrt{\pi}}-\frac{\left(\frac{3}{8}x^{12}-\frac{9}{2}x^{6}+9\right)\cos\left(x^{3}\right)}{6\sqrt{\pi}}-\frac{x^{3}\left(-\frac{3x^{6}}{2}+9\right)\sin\left(x^{3}\right)}{6\sqrt{\pi}}\right)}{3}$	52
parallelrisch	$\frac{\left(x^{12}-12x^{6}+48\right)\tan\left(\frac{x^{3}}{2}\right)^{2}+\left(8x^{9}-48x^{3}\right)\tan\left(\frac{x^{3}}{2}\right)-x^{12}+12x^{6}}{3\tan\left(\frac{x^{3}}{2}\right)^{2}+3}$	64

input
```
int(x^14*sin(x^3),x,method=_RETURNVERBOSE)
```

output
```
(-1/3*x^12+4*x^6-8)*cos(x^3)+4/3*x^3*(x^6-6)*sin(x^3)
```

3.776.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.68

$$\int x^{14}\sin\left(x^{3}\right)\,dx=-\frac{1}{3}\left(x^{12}-12\,x^{6}+24\right)\cos\left(x^{3}\right)+\frac{4}{3}\left(x^{9}-6\,x^{3}\right)\sin\left(x^{3}\right)$$

input
```
integrate(x^14*sin(x^3),x, algorithm="fricas")
```

output
```
-1/3*(x^12 - 12*x^6 + 24)*cos(x^3) + 4/3*(x^9 - 6*x^3)*sin(x^3)
```

3.776.6 Sympy [A] (verification not implemented)

Time = 4.44 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int x^{14}\sin\left(x^{3}\right)\,dx=-\frac{x^{12}\cos\left(x^{3}\right)}{3}+\frac{4x^{9}\sin\left(x^{3}\right)}{3}+4x^{6}\cos\left(x^{3}\right)-8x^{3}\sin\left(x^{3}\right)-8\cos\left(x^{3}\right)$$

input
```
integrate(x**14*sin(x**3),x)
```

output
```
-x**12*cos(x**3)/3 + 4*x**9*sin(x**3)/3 + 4*x**6*cos(x**3) - 8*x**3*sin(x*
*3) - 8*cos(x**3)
```

3.776.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.68

$$\int x^{14}\sin\left(x^{3}\right)\,dx=-\frac{1}{3}\left(x^{12}-12\,x^{6}+24\right)\cos\left(x^{3}\right)+\frac{4}{3}\left(x^{9}-6\,x^{3}\right)\sin\left(x^{3}\right)$$

input
```
integrate(x^14*sin(x^3),x, algorithm="maxima")
```

output
```
-1/3*(x^12 - 12*x^6 + 24)*cos(x^3) + 4/3*(x^9 - 6*x^3)*sin(x^3)
```

3.776.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.68

$$\int x^{14}\sin\left(x^{3}\right)\,dx=-\frac{1}{3}\left(x^{12}-12\,x^{6}+24\right)\cos\left(x^{3}\right)+\frac{4}{3}\left(x^{9}-6\,x^{3}\right)\sin\left(x^{3}\right)$$

input
```
integrate(x^14*sin(x^3),x, algorithm="giac")
```

output
```
-1/3*(x^12 - 12*x^6 + 24)*cos(x^3) + 4/3*(x^9 - 6*x^3)*sin(x^3)
```

3.776.9 Mupad [B] (verification not implemented)

Time = 26.93 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int x^{14}\sin\left(x^3\right)\,dx=4\,x^6\,\cos\left(x^3\right)-8\,\cos\left(x^3\right)-\frac{x^{12}\,\cos\left(x^3\right)}{3}-8\,x^3\,\sin\left(x^3\right)+\frac{4\,x^9\,\sin\left(x^3\right)}{3}$$

input
```
int(x^14*sin(x^3),x)
```

output
```
4*x^6*cos(x^3) - 8*cos(x^3) - (x^12*cos(x^3))/3 - 8*x^3*sin(x^3) + (4*x^9*
sin(x^3))/3
```

3.777 $\int e^{-3x^3} x^2 \sin(2x^3)\, dx$

3.777.1 Optimal result

Integrand size = 17, antiderivative size = 35

$$\int e^{-3x^3} x^2 \sin(2x^3)\, dx = -\frac{2}{39} e^{-3x^3} \cos(2x^3) - \frac{1}{13} e^{-3x^3} \sin(2x^3)$$

output
```
-2/39*cos(2*x^3)/exp(3*x^3)-1/13*sin(2*x^3)/exp(3*x^3)
```

3.777.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

$$\int e^{-3x^3} x^2 \sin(2x^3)\, dx = -\frac{1}{39} e^{-3x^3} (2\cos(2x^3) + 3\sin(2x^3))$$

input
```
Integrate[(x^2*Sin[2*x^3])/E^(3*x^3),x]
```

output
```
-1/39*(2*Cos[2*x^3] + 3*Sin[2*x^3])/E^(3*x^3)
```

3.777.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {7266, 4932}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-3x^3} x^2 \sin\left(2x^3\right) \, dx$$

$$\Big\downarrow \text{ 7266}$$

$$\frac{1}{3} \int e^{-3x^3} \sin\left(2x^3\right) dx^3$$

$$\Big\downarrow \text{ 4932}$$

$$\frac{1}{3} \left(-\frac{3}{13} e^{-3x^3} \sin\left(2x^3\right) - \frac{2}{13} e^{-3x^3} \cos\left(2x^3\right)\right)$$

input
```
Int[(x^2*Sin[2*x^3])/E^(3*x^3),x]
```

output
```
((-2*Cos[2*x^3])/(13*E^(3*x^3)) - (3*Sin[2*x^3])/(13*E^(3*x^3)))/3
```

3.777.3.1 Defintions of rubi rules used

rule 4932
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

3.777.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

method	result	size
parallelrisch	$-\frac{\mathrm{e}^{-3x^3}\left(2\cos\left(2x^3\right)+3\sin\left(2x^3\right)\right)}{39}$	26
norman	$\frac{\left(-\frac{2}{39}+\frac{2\tan\left(x^3\right)^2}{39}-\frac{2\tan\left(x^3\right)}{13}\right)\mathrm{e}^{-3x^3}}{1+\tan\left(x^3\right)^2}$	36
risch	$-\frac{\mathrm{e}^{(-3+2i)x^3}}{39}+\frac{i\mathrm{e}^{(-3+2i)x^3}}{26}-\frac{\mathrm{e}^{(-3-2i)x^3}}{39}-\frac{i\mathrm{e}^{(-3-2i)x^3}}{26}$	44

input
```
int(x^2*sin(2*x^3)/exp(3*x^3),x,method=_RETURNVERBOSE)
```

output
```
-1/39*exp(-3*x^3)*(2*cos(2*x^3)+3*sin(2*x^3))
```

3.777.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int e^{-3x^3}x^2\sin\left(2x^3\right)\,dx=-\frac{2}{39}\cos\left(2\,x^3\right)e^{\left(-3\,x^3\right)}-\frac{1}{13}\,e^{\left(-3\,x^3\right)}\sin\left(2\,x^3\right)$$

input
```
integrate(x^2*sin(2*x^3)/exp(3*x^3),x, algorithm="fricas")
```

output
```
-2/39*cos(2*x^3)*e^(-3*x^3) - 1/13*e^(-3*x^3)*sin(2*x^3)
```

3.777.6 Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int e^{-3x^3}x^2\sin\left(2x^3\right)\,dx=-\frac{e^{-3x^3}\sin\left(2x^3\right)}{13}-\frac{2e^{-3x^3}\cos\left(2x^3\right)}{39}$$

input
```
integrate(x**2*sin(2*x**3)/exp(3*x**3),x)
```

output
```
-exp(-3*x**3)*sin(2*x**3)/13 - 2*exp(-3*x**3)*cos(2*x**3)/39
```

3.777.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

$$\int e^{-3x^3}x^2\sin(2x^3)\,dx = -\frac{1}{39}\left(2\cos(2x^3)+3\sin(2x^3)\right)e^{(-3x^3)}$$

input
```
integrate(x^2*sin(2*x^3)/exp(3*x^3),x, algorithm="maxima")
```

output
```
-1/39*(2*cos(2*x^3) + 3*sin(2*x^3))*e^(-3*x^3)
```

3.777.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

$$\int e^{-3x^3}x^2\sin(2x^3)\,dx = -\frac{1}{39}\left(2\cos(2x^3)+3\sin(2x^3)\right)e^{(-3x^3)}$$

input
```
integrate(x^2*sin(2*x^3)/exp(3*x^3),x, algorithm="giac")
```

output
```
-1/39*(2*cos(2*x^3) + 3*sin(2*x^3))*e^(-3*x^3)
```

3.777.9 Mupad [B] (verification not implemented)

Time = 26.47 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

$$\int e^{-3x^3}x^2\sin(2x^3)\,dx = -\frac{e^{-3x^3}\left(2\cos(2x^3)+3\sin(2x^3)\right)}{39}$$

input
```
int(x^2*exp(-3*x^3)*sin(2*x^3),x)
```

output
```
-(exp(-3*x^3)*(2*cos(2*x^3) + 3*sin(2*x^3)))/39
```

3.778 $\int 2x\cos(x^2)\,dx$

3.778.1 Optimal result

Integrand size = 7, antiderivative size = 4

$$\int 2x\cos(x^2)\,dx = \sin(x^2)$$

output
```
sin(x^2)
```

3.778.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int 2x\cos(x^2)\,dx = \sin(x^2)$$

input
```
Integrate[2*x*Cos[x^2],x]
```

output
```
Sin[x^2]
```

3.778.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {27, 3861, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int 2x\cos\left(x^2\right)dx$$

$$\Big\downarrow \; 27$$

$$2\int x\cos\left(x^2\right)dx$$

$$\Big\downarrow \; 3861$$

$$\int \cos\left(x^2\right)dx^2$$

$$\Big\downarrow \; 3042$$

$$\int \sin\left(x^2+\frac{\pi}{2}\right)dx^2$$

$$\Big\downarrow \; 3117$$

$$\sin\left(x^2\right)$$

input
```
Int[2*x*Cos[x^2],x]
```

output
```
Sin[x^2]
```

3.778.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3861
```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

3.778.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

method	result	size
derivativedivides	$\sin\left(x^{2}\right)$	5
default	$\sin\left(x^{2}\right)$	5
meijerg	$\sin\left(x^{2}\right)$	5
risch	$\sin\left(x^{2}\right)$	5
parallelrisch	$\sin\left(x^{2}\right)$	5
norman	$\frac{2\tan\left(\frac{x^{2}}{2}\right)}{1+\tan\left(\frac{x^{2}}{2}\right)^{2}}$	21
parts	$\sqrt{2}\,\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{\sqrt{2}\,x}{\sqrt{\pi}}\right)x-\pi\left(\frac{\mathrm{FresnelC}\left(\frac{\sqrt{2}\,x}{\sqrt{\pi}}\right)\sqrt{2}\,x}{\sqrt{\pi}}-\frac{\sin\left(x^{2}\right)}{\pi}\right)$	49

input
```
int(2*x*cos(x^2),x,method=_RETURNVERBOSE)
```

output
```
sin(x^2)
```

3.778.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int 2x\cos\left(x^2\right)\,dx = \sin\left(x^2\right)$$

input
```
integrate(2*x*cos(x^2),x, algorithm="fricas")
```

output
```
sin(x^2)
```

3.778.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int 2x\cos\left(x^2\right)\,dx = \sin\left(x^2\right)$$

input
```
integrate(2*x*cos(x**2),x)
```

output
```
sin(x**2)
```

3.778.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int 2x\cos\left(x^2\right)\,dx = \sin\left(x^2\right)$$

input
```
integrate(2*x*cos(x^2),x, algorithm="maxima")
```

output
```
sin(x^2)
```

3.778.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int 2x \cos\left(x^2\right)\,dx = \sin\left(x^2\right)$$

input
```
integrate(2*x*cos(x^2),x, algorithm="giac")
```

output
```
sin(x^2)
```

3.778.9 Mupad [B] (verification not implemented)

Time = 26.52 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int 2x \cos\left(x^2\right)\,dx = \sin\left(x^2\right)$$

input
```
int(2*x*cos(x^2),x)
```

output
```
sin(x^2)
```

3.779 $\int 3x^2 \cos(7+x^3)\,dx$

3.779.1 Optimal result

Integrand size = 11, antiderivative size = 6

$$\int 3x^2 \cos(7+x^3)\,dx = \sin(7+x^3)$$

output
```
sin(x^3+7)
```

3.779.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int 3x^2 \cos(7+x^3)\,dx = \sin(7+x^3)$$

input
```
Integrate[3*x^2*Cos[7 + x^3],x]
```

output
```
Sin[7 + x^3]
```

3.779.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {27, 3861, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int 3x^2 \cos\left(x^3+7\right) dx$$

$$\Big\downarrow \text{ 27}$$

$$3\int x^2 \cos\left(x^3+7\right) dx$$

$$\Big\downarrow \text{ 3861}$$

$$\int \cos\left(x^3+7\right) dx^3$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin\left(x^3+\frac{\pi}{2}+7\right) dx^3$$

$$\Big\downarrow \text{ 3117}$$

$$\sin\left(x^3+7\right)$$

input
```
Int[3*x^2*Cos[7 + x^3],x]
```

output
```
Sin[7 + x^3]
```

3.779.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3861
```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

3.779.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

method	result	size
derivativedivides	$\sin\left(x^3+7\right)$	7
default	$\sin\left(x^3+7\right)$	7
risch	$\sin\left(x^3+7\right)$	7
parallelrisch	$\sin\left(x^3+7\right)$	7
norman	$\frac{2\tan\left(\frac{7}{2}+\frac{x^3}{2}\right)}{1+\tan\left(\frac{7}{2}+\frac{x^3}{2}\right)^2}$	25
meijerg	$\cos\left(7\right)\sin\left(x^3\right)-\sin\left(7\right)\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos\left(x^3\right)}{\sqrt{\pi}}\right)$	29

input
```
int(3*x^2*cos(x^3+7),x,method=_RETURNVERBOSE)
```

output
```
sin(x^3+7)
```

3.779.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int 3x^2\cos\left(7+x^3\right)\,dx=\sin\left(x^3+7\right)$$

input
```
integrate(3*x^2*cos(x^3+7),x, algorithm="fricas")
```

output
```
sin(x^3 + 7)
```

3.779.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int 3x^2 \cos\left(7+x^3\right)\,dx = \sin\left(x^3+7\right)$$

input
```
integrate(3*x**2*cos(x**3+7),x)
```

output
```
sin(x**3 + 7)
```

3.779.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int 3x^2 \cos\left(7+x^3\right)\,dx = \sin\left(x^3+7\right)$$

input
```
integrate(3*x^2*cos(x^3+7),x, algorithm="maxima")
```

output
```
sin(x^3 + 7)
```

3.779.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int 3x^2 \cos\left(7+x^3\right)\,dx = \sin\left(x^3+7\right)$$

input
```
integrate(3*x^2*cos(x^3+7),x, algorithm="giac")
```

output
```
sin(x^3 + 7)
```

3.779.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int 3x^2 \cos\left(7+x^3\right) \, dx = \sin\left(x^3+7\right)$$

input
```
int(3*x^2*cos(x^3 + 7),x)
```

output
```
sin(x^3 + 7)
```

3.780 $\int \left(\frac{1}{1+x^2}+\sin(x)\right)\,dx$

3.780.1 Optimal result

Integrand size = 10, antiderivative size = 7

$$\int \left(\frac{1}{1+x^2}+\sin(x)\right)\,dx=\arctan(x)-\cos(x)$$

output
```
arctan(x)-cos(x)
```

3.780.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \left(\frac{1}{1+x^2}+\sin(x)\right)\,dx=\arctan(x)-\cos(x)$$

input
```
Integrate[(1 + x^2)^(-1) + Sin[x],x]
```

output
```
ArcTan[x] - Cos[x]
```

3.780.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{1}{x^2+1}+\sin(x)\right) dx$$
$$\Big\downarrow \text{2009}$$
$$\arctan(x)-\cos(x)$$

input
```
Int[(1 + x^2)^(-1) + Sin[x],x]
```

output
```
ArcTan[x] - Cos[x]
```

3.780.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.780.4 Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

method	result	size
default	$\arctan\left(x\right)-\cos\left(x\right)$	8
risch	$\arctan\left(x\right)-\cos\left(x\right)$	8
parts	$\arctan\left(x\right)-\cos\left(x\right)$	8
meijerg	$\arctan\left(x\right)+\sqrt{\pi}\,\left(\frac{1}{\sqrt{\pi}}-\frac{\cos\left(x\right)}{\sqrt{\pi}}\right)$	19
parallelrisch	$\frac{i\ln\left(x+i\right)}{2}-\frac{i\ln\left(x-i\right)}{2}-\cos\left(x\right)-1$	23

input
```
int(1/(x^2+1)+sin(x),x,method=_RETURNVERBOSE)
```

output
```
arctan(x)-cos(x)
```

3.780.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \left(\frac{1}{1+x^2}+\sin(x)\right)\,dx=\arctan(x)-\cos(x)$$

input
```
integrate(1/(x^2+1)+sin(x),x, algorithm="fricas")
```

output
```
arctan(x) - cos(x)
```

3.780.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \left(\frac{1}{1+x^2}+\sin(x)\right)\,dx=-\cos(x)+\operatorname{atan}(x)$$

input
```
integrate(1/(x**2+1)+sin(x),x)
```

output
```
-cos(x) + atan(x)
```

3.780.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \left(\frac{1}{1+x^2}+\sin(x)\right)\,dx=\arctan(x)-\cos(x)$$

input
```
integrate(1/(x^2+1)+sin(x),x, algorithm="maxima")
```

output
```
arctan(x) - cos(x)
```

3.780.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \left(\frac{1}{1+x^2}+\sin(x)\right)\,dx=\arctan\left(x\right)-\cos\left(x\right)$$

input
```
integrate(1/(x^2+1)+sin(x),x, algorithm="giac")
```

output
```
arctan(x) - cos(x)
```

3.780.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \left(\frac{1}{1+x^2}+\sin(x)\right)\,dx=\mathrm{atan}(x)-\cos\left(x\right)$$

input
```
int(sin(x) + 1/(x^2 + 1),x)
```

output
```
atan(x) - cos(x)
```

3.781 $\int x \sin(1+x^2)\, dx$

3.781.1 Optimal result

Integrand size = 8, antiderivative size = 10

$$\int x \sin(1+x^2)\, dx = -\frac{1}{2}\cos(1+x^2)$$

output
```
-1/2*cos(x^2+1)
```

3.781.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 21 vs. 2(10) = 20.

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 2.10

$$\int x \sin(1+x^2)\, dx = -\frac{1}{2}\cos(1)\cos(x^2) + \frac{1}{2}\sin(1)\sin(x^2)$$

input
```
Integrate[x*Sin[1 + x^2],x]
```

output
```
-1/2*(Cos[1]*Cos[x^2]) + (Sin[1]*Sin[x^2])/2
```

3.781.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3860, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sin\left(x^2+1\right)\, dx$$

$$\Big\downarrow \text{3860}$$

$$\frac{1}{2}\int \sin\left(x^2+1\right)dx^2$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \sin\left(x^2+1\right)dx^2$$

$$\Big\downarrow \text{3118}$$

$$-\frac{1}{2}\cos\left(x^2+1\right)$$

input
```
Int[x*Sin[1 + x^2],x]
```

output
```
-1/2*Cos[1 + x^2]
```

3.781.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

3.781.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

method	result
derivativedivides	$-\frac{\cos(x^2+1)}{2}$
default	$-\frac{\cos(x^2+1)}{2}$
risch	$-\frac{\cos(x^2+1)}{2}$
parallelrisch	$-\frac{\cos(x^2+1)}{2}-\frac{1}{2}$
norman	$-\frac{1}{1+\tan\left(\frac{1}{2}+\frac{x^2}{2}\right)^2}$
meijerg	$\frac{\sin(1)\sin(x^2)}{2}+\frac{\cos(1)\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(x^2)}{\sqrt{\pi}}\right)}{2}$
parts	$\frac{\sqrt{2}\,\sin(1)\sqrt{\pi}\ \mathrm{FresnelC}\left(\frac{\sqrt{2}\,x}{\sqrt{\pi}}\right)x}{2}+\frac{\sqrt{2}\,\sqrt{\pi}\ \cos(1)\ \mathrm{FresnelS}\left(\frac{\sqrt{2}\,x}{\sqrt{\pi}}\right)x}{2}-\frac{\sqrt{2}\,\sqrt{\pi}\left(\frac{\cos(1)\sqrt{2}\,\sqrt{\pi}\left(\frac{\mathrm{FresnelS}\left(\frac{\sqrt{2}\,x}{\sqrt{\pi}}\right)\sqrt{2}\,x}{\sqrt{\pi}}+\right.}{2}\right.}{}$

input
```
int(x*sin(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2*cos(x^2+1)
```

3.781.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int x\sin\left(1+x^2\right)\,dx=-\frac{1}{2}\cos\left(x^2+1\right)$$

input
```
integrate(x*sin(x^2+1),x, algorithm="fricas")
```

output
```
-1/2*cos(x^2 + 1)
```

3.781.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int x\sin\left(1+x^2\right)\,dx=-\frac{\cos\left(x^2+1\right)}{2}$$

input
```
integrate(x*sin(x**2+1),x)
```

output
```
-cos(x**2 + 1)/2
```

3.781.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int x\sin\left(1+x^2\right)\,dx=-\frac{1}{2}\cos\left(x^2+1\right)$$

input
```
integrate(x*sin(x^2+1),x, algorithm="maxima")
```

output
```
-1/2*cos(x^2 + 1)
```

3.781.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int x \sin \left(1+x^2\right) dx = -\frac{1}{2} \cos \left(x^2+1\right)$$

input
```
integrate(x*sin(x^2+1),x, algorithm="giac")
```

output
```
-1/2*cos(x^2 + 1)
```

3.781.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int x \sin \left(1+x^2\right) dx = -\frac{\cos \left(x^2+1\right)}{2}$$

input
```
int(x*sin(x^2 + 1),x)
```

output
```
-cos(x^2 + 1)/2
```

3.782 $\int x \cos(1+x^2) \, dx$

3.782.1 Optimal result

Integrand size = 8, antiderivative size = 10

$$\int x \cos(1+x^2) \, dx = \frac{1}{2} \sin(1+x^2)$$

output
```
1/2*sin(x^2+1)
```

3.782.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int x \cos(1+x^2) \, dx = \frac{1}{2} \sin(1+x^2)$$

input
```
Integrate[x*Cos[1 + x^2],x]
```

output
```
Sin[1 + x^2]/2
```

3.782.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3861, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \cos \left(x^2+1\right) \, dx$$

$$\Big\downarrow \text{3861}$$

$$\frac{1}{2} \int \cos \left(x^2+1\right) dx^2$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2} \int \sin \left(x^2+\frac{\pi}{2}+1\right) dx^2$$

$$\Big\downarrow \text{3117}$$

$$\frac{1}{2} \sin \left(x^2+1\right)$$

input
```
Int[x*Cos[1 + x^2],x]
```

output
```
Sin[1 + x^2]/2
```

3.782.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3861
```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

3.782.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{\sin(x^2+1)}{2}$
default	$\frac{\sin(x^2+1)}{2}$
risch	$\frac{\sin(x^2+1)}{2}$
parallelrisch	$\frac{\sin(x^2+1)}{2}$
norman	$\frac{\tan\left(\frac{1}{2}+\frac{x^2}{2}\right)}{1+\tan\left(\frac{1}{2}+\frac{x^2}{2}\right)^2}$
meijerg	$\frac{\cos(1)\sin\left(x^2\right)}{2}-\frac{\sin(1)\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos\left(x^2\right)}{\sqrt{\pi}}\right)}{2}$
parts	$\frac{\sqrt{2}\,\cos(1)\sqrt{\pi}\,\text{FresnelC}\left(\frac{\sqrt{2}\,x}{\sqrt{\pi}}\right)x}{2}-\frac{\sqrt{2}\,\sqrt{\pi}\,\sin(1)\,\text{FresnelS}\left(\frac{\sqrt{2}\,x}{\sqrt{\pi}}\right)x}{2}-\frac{\sqrt{2}\,\sqrt{\pi}\left(\frac{\cos(1)\sqrt{2}\,\sqrt{\pi}\left(\frac{\text{FresnelC}\left(\frac{\sqrt{2}\,x}{\sqrt{\pi}}\right)\sqrt{2}\,x}{\sqrt{\pi}}-\cdots\right.}{2}\right.}{\cdots}$

input
```
int(x*cos(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*sin(x^2+1)
```

3.782.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int x\cos\left(1+x^2\right)\,dx=\frac{1}{2}\sin\left(x^2+1\right)$$

input
```
integrate(x*cos(x^2+1),x, algorithm="fricas")
```

output
```
1/2*sin(x^2 + 1)
```

3.782.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int x\cos\left(1+x^2\right)\,dx=\frac{\sin\left(x^2+1\right)}{2}$$

input
```
integrate(x*cos(x**2+1),x)
```

output
```
sin(x**2 + 1)/2
```

3.782.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int x\cos\left(1+x^2\right)\,dx=\frac{1}{2}\sin\left(x^2+1\right)$$

input
```
integrate(x*cos(x^2+1),x, algorithm="maxima")
```

output
```
1/2*sin(x^2 + 1)
```

3.782.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int x\cos(1+x^2)\,dx = \frac{1}{2}\sin(x^2+1)$$

input
```
integrate(x*cos(x^2+1),x, algorithm="giac")
```

output
```
1/2*sin(x^2 + 1)
```

3.782.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int x\cos(1+x^2)\,dx = \frac{\sin(x^2+1)}{2}$$

input
```
int(x*cos(x^2 + 1),x)
```

output
```
sin(x^2 + 1)/2
```

3.783 $\int (1+x^2\cos(x^3))\,dx$

3.783.1 Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \left(1+x^2\cos\left(x^3\right)\right)\,dx=x+\frac{\sin\left(x^3\right)}{3}$$

output
```
x+1/3*sin(x^3)
```

3.783.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \left(1+x^2\cos\left(x^3\right)\right)\,dx=x+\frac{\sin\left(x^3\right)}{3}$$

input
```
Integrate[1 + x^2*Cos[x^3],x]
```

output
```
x + Sin[x^3]/3
```

3.783.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x^2 \cos\left(x^3\right)+1\right) dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{\sin\left(x^3\right)}{3}+x$$

input
```
Int[1 + x^2*Cos[x^3],x]
```

output
```
x + Sin[x^3]/3
```

3.783.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.783.4 Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

method	result	size
default	$x+\frac{\sin\left(x^{3}\right)}{3}$	9
risch	$x+\frac{\sin\left(x^{3}\right)}{3}$	9
parallelrisch	$x+\frac{\sin\left(x^{3}\right)}{3}$	9
parts	$x+\frac{\sin\left(x^{3}\right)}{3}$	9
norman	$\frac{x+x\tan\left(\frac{x^{3}}{2}\right)^{2}+\frac{2\tan\left(\frac{x^{3}}{2}\right)}{3}}{1+\tan\left(\frac{x^{3}}{2}\right)^{2}}$	34

input
```
int(1+x^2*cos(x^3),x,method=_RETURNVERBOSE)
```

output
```
x+1/3*sin(x^3)
```

3.783.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int (1 + x^2 \cos(x^3)) \, dx = x + \frac{1}{3} \sin(x^3)$$

input
```
integrate(1+x^2*cos(x^3),x, algorithm="fricas")
```

output
```
x + 1/3*sin(x^3)
```

3.783.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int (1 + x^2 \cos(x^3)) \, dx = x + \frac{\sin(x^3)}{3}$$

input
```
integrate(1+x**2*cos(x**3),x)
```

output
```
x + sin(x**3)/3
```

3.783.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int (1 + x^2 \cos(x^3)) \, dx = x + \frac{1}{3} \sin(x^3)$$

input
```
integrate(1+x^2*cos(x^3),x, algorithm="maxima")
```

output
```
x + 1/3*sin(x^3)
```

3.783.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int (1 + x^2 \cos (x^3)) \, dx = x + \frac{1}{3} \sin (x^3)$$

input
```
integrate(1+x^2*cos(x^3),x, algorithm="giac")
```

output
```
x + 1/3*sin(x^3)
```

3.783.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int (1 + x^2 \cos (x^3)) \, dx = x + \frac{\sin (x^3)}{3}$$

input
```
int(x^2*cos(x^3) + 1,x)
```

output
```
x + sin(x^3)/3
```

3.784 $\int x^2 \sin(1+x^3)\, dx$

3.784.1 Optimal result

Integrand size = 10, antiderivative size = 10

$$\int x^2 \sin(1+x^3)\, dx = -\frac{1}{3}\cos(1+x^3)$$

output
```
-1/3*cos(x^3+1)
```

3.784.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 21 vs. 2(10) = 20.

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 2.10

$$\int x^2 \sin(1+x^3)\, dx = -\frac{1}{3}\cos(1)\cos(x^3) + \frac{1}{3}\sin(1)\sin(x^3)$$

input
```
Integrate[x^2*Sin[1 + x^3],x]
```

output
```
-1/3*(Cos[1]*Cos[x^3]) + (Sin[1]*Sin[x^3])/3
```

3.784.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3860, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sin \left(x^3+1\right) \, dx$$

$$\Big\downarrow \text{3860}$$

$$\frac{1}{3} \int \sin \left(x^3+1\right)dx^3$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3} \int \sin \left(x^3+1\right)dx^3$$

$$\Big\downarrow \text{3118}$$

$$-\frac{1}{3} \cos \left(x^3+1\right)$$

input
```
Int[x^2*Sin[1 + x^3],x]
```

output
```
-1/3*Cos[1 + x^3]
```

3.784.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

3.784.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$-\frac{\cos(x^3+1)}{3}$	9
default	$-\frac{\cos(x^3+1)}{3}$	9
parallelrisch	$-\frac{\cos(x^3+1)}{3}-\frac{1}{3}$	11
risch	$-\frac{\cos((x+1)(x^2-x+1))}{3}$	16
norman	$-\frac{2}{3\left(1+\tan\left(\frac{1}{2}+\frac{x^3}{2}\right)^2\right)}$	17
meijerg	$\frac{\sin(1)\sin(x^3)}{3}+\frac{\cos(1)\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(x^3)}{\sqrt{\pi}}\right)}{3}$	30

input
```
int(x^2*sin(x^3+1),x,method=_RETURNVERBOSE)
```

output
```
-1/3*cos(x^3+1)
```

3.784.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int x^2\sin\left(1+x^3\right)\,dx=-\frac{1}{3}\cos\left(x^3+1\right)$$

input
```
integrate(x^2*sin(x^3+1),x, algorithm="fricas")
```

output
```
-1/3*cos(x^3 + 1)
```

3.784.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int x^2 \sin(1+x^3)\,dx = -\frac{\cos(x^3+1)}{3}$$

input
```
integrate(x**2*sin(x**3+1),x)
```

output
```
-cos(x**3 + 1)/3
```

3.784.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int x^2 \sin(1+x^3)\,dx = -\frac{1}{3}\cos(x^3+1)$$

input
```
integrate(x^2*sin(x^3+1),x, algorithm="maxima")
```

output
```
-1/3*cos(x^3 + 1)
```

3.784.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int x^2 \sin(1+x^3)\,dx = -\frac{1}{3}\cos(x^3+1)$$

input
```
integrate(x^2*sin(x^3+1),x, algorithm="giac")
```

output
```
-1/3*cos(x^3 + 1)
```

3.784.9 Mupad [B] (verification not implemented)

Time = 27.29 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int x^2 \sin \left(1+x^3\right) \, dx = -\frac{\cos \left(x^3+1\right)}{3}$$

input
```
int(x^2*sin(x^3 + 1),x)
```

output
```
-cos(x^3 + 1)/3
```

3.785 $\int 12x^2 \cos(x^3)\, dx$

3.785.1 Optimal result

Integrand size = 9, antiderivative size = 6

$$\int 12x^2 \cos(x^3)\, dx = 4\sin(x^3)$$

output
```
4*sin(x^3)
```

3.785.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int 12x^2 \cos(x^3)\, dx = 4\sin(x^3)$$

input
```
Integrate[12*x^2*Cos[x^3],x]
```

output
```
4*Sin[x^3]
```

3.785.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {27, 3861, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int 12x^2 \cos\left(x^3\right) dx$$

$$\Big\downarrow \; 27$$

$$12 \int x^2 \cos\left(x^3\right) dx$$

$$\Big\downarrow \; 3861$$

$$4 \int \cos\left(x^3\right) dx^3$$

$$\Big\downarrow \; 3042$$

$$4 \int \sin\left(x^3 + \frac{\pi}{2}\right) dx^3$$

$$\Big\downarrow \; 3117$$

$$4 \sin\left(x^3\right)$$

input
```
Int[12*x^2*Cos[x^3],x]
```

output
```
4*Sin[x^3]
```

3.785.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3861
```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

3.785.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

method	result	size
derivativedivides	$4\sin\left(x^{3}\right)$	7
default	$4\sin\left(x^{3}\right)$	7
meijerg	$4\sin\left(x^{3}\right)$	7
risch	$4\sin\left(x^{3}\right)$	7
parallelrisch	$4\sin\left(x^{3}\right)$	7
norman	$\frac{8\tan\left(\frac{x^{3}}{2}\right)}{1+\tan\left(\frac{x^{3}}{2}\right)^{2}}$	21

input
```
int(12*x^2*cos(x^3),x,method=_RETURNVERBOSE)
```

output
```
4*sin(x^3)
```

3.785.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int 12x^2\cos\left(x^3\right)\,dx = 4\,\sin\left(x^3\right)$$

input
```
integrate(12*x^2*cos(x^3),x, algorithm="fricas")
```

output
```
4*sin(x^3)
```

3.785.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int 12x^2 \cos\left(x^3\right) \, dx = 4 \sin\left(x^3\right)$$

input
```
integrate(12*x**2*cos(x**3),x)
```

output
```
4*sin(x**3)
```

3.785.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int 12x^2 \cos\left(x^3\right) \, dx = 4 \sin\left(x^3\right)$$

input
```
integrate(12*x^2*cos(x^3),x, algorithm="maxima")
```

output
```
4*sin(x^3)
```

3.785.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int 12x^2 \cos\left(x^3\right) \, dx = 4 \sin\left(x^3\right)$$

input
```
integrate(12*x^2*cos(x^3),x, algorithm="giac")
```

output
```
4*sin(x^3)
```

3.785.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int 12x^2 \cos \left(x^3\right) \, dx = 4 \sin \left(x^3\right)$$

input
```
int(12*x^2*cos(x^3),x)
```

output
```
4*sin(x^3)
```

3.786 $\int (1+x)\sin(1+x)\,dx$

3.786.1 Optimal result

Integrand size = 8, antiderivative size = 14

$$\int (1+x)\sin(1+x)\,dx = -((1+x)\cos(1+x)) + \sin(1+x)$$

output
```
-(1+x)*cos(1+x)+sin(1+x)
```

3.786.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (1+x)\sin(1+x)\,dx = -((1+x)\cos(1+x)) + \sin(1+x)$$

input
```
Integrate[(1 + x)*Sin[1 + x],x]
```

output
```
-((1 + x)*Cos[1 + x]) + Sin[1 + x]
```

3.786.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x+1)\sin(x+1)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (x+1)\sin(x+1)dx$$

$$\Big\downarrow \text{ 3777}$$

$$\int \cos(x+1)dx-(x+1)\cos(x+1)$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin\left(x+\frac{\pi}{2}+1\right)dx-(x+1)\cos(x+1)$$

$$\Big\downarrow \text{ 3117}$$

$$\sin(x+1)-(x+1)\cos(x+1)$$

input
```
Int[(1 + x)*Sin[1 + x],x]
```

output
```
-((1 + x)*Cos[1 + x]) + Sin[1 + x]
```

3.786.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.786.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result
derivativedivides	$-(x+1)\cos(x+1)+\sin(x+1)$
default	$-(x+1)\cos(x+1)+\sin(x+1)$
risch	$(-x-1)\cos(x+1)+\sin(x+1)$
parts	$-\cos(x+1)\,x+\sin(x+1)-\cos(x+1)$
norman	$\frac{x\tan\left(\frac{x}{2}+\frac{1}{2}\right)^2-x+2\tan\left(\frac{x}{2}+\frac{1}{2}\right)-2}{1+\tan\left(\frac{x}{2}+\frac{1}{2}\right)^2}$
parallelrisch	$\frac{x\tan\left(\frac{x}{2}+\frac{1}{2}\right)^2-x+2\tan\left(\frac{x}{2}+\frac{1}{2}\right)-2}{1+\tan\left(\frac{x}{2}+\frac{1}{2}\right)^2}$
meijerg	$2\sin(1)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cos(x)}{2\sqrt{\pi}}+\frac{x\sin(x)}{2\sqrt{\pi}}\right)+2\cos(1)\sqrt{\pi}\left(-\frac{x\cos(x)}{2\sqrt{\pi}}+\frac{\sin(x)}{2\sqrt{\pi}}\right)+\sin(1)\sin(x$

input
```
int((x+1)*sin(x+1),x,method=_RETURNVERBOSE)
```

output
```
-(x+1)*cos(x+1)+sin(x+1)
```

3.786.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (1+x)\sin(1+x)\,dx = -(x+1)\cos(x+1)+\sin(x+1)$$

input
```
integrate((1+x)*sin(1+x),x, algorithm="fricas")
```

output
```
-(x + 1)*cos(x + 1) + sin(x + 1)
```

3.786.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int (1+x)\sin(1+x)\,dx = -x\cos(x+1) + \sin(x+1) - \cos(x+1)$$

input
```
integrate((1+x)*sin(1+x),x)
```

output
```
-x*cos(x + 1) + sin(x + 1) - cos(x + 1)
```

3.786.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (1+x)\sin(1+x)\,dx = -(x+1)\cos(x+1) + \sin(x+1)$$

input
```
integrate((1+x)*sin(1+x),x, algorithm="maxima")
```

output
```
-(x + 1)*cos(x + 1) + sin(x + 1)
```

3.786.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (1+x)\sin(1+x)\,dx = -(x+1)\cos(x+1) + \sin(x+1)$$

input
```
integrate((1+x)*sin(1+x),x, algorithm="giac")
```

output
```
-(x + 1)*cos(x + 1) + sin(x + 1)
```

3.786.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (1+x)\sin(1+x)\,dx = \sin\left(x+1\right)-\cos\left(x+1\right)\,\left(x+1\right)$$

input
```
int(sin(x + 1)*(x + 1),x)
```

output
```
sin(x + 1) - cos(x + 1)*(x + 1)
```

3.787 $\int x^5 \cos(x^3) \, dx$

3.787.1 Optimal result

Integrand size = 8, antiderivative size = 20

$$\int x^5 \cos(x^3) \, dx = \frac{\cos(x^3)}{3} + \frac{1}{3}x^3 \sin(x^3)$$

output
```
1/3*cos(x^3)+1/3*x^3*sin(x^3)
```

3.787.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x^5 \cos(x^3) \, dx = \frac{\cos(x^3)}{3} + \frac{1}{3}x^3 \sin(x^3)$$

input
```
Integrate[x^5*Cos[x^3],x]
```

output
```
Cos[x^3]/3 + (x^3*Sin[x^3])/3
```

3.787.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3861, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \cos \left(x^3\right) \, dx$$

$$\Big\downarrow \text{3861}$$

$$\frac{1}{3} \int x^3 \cos \left(x^3\right) dx^3$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3} \int x^3 \sin \left(x^3 + \frac{\pi}{2}\right) dx^3$$

$$\Big\downarrow \text{3777}$$

$$\frac{1}{3} \left(\int -\sin \left(x^3\right) dx^3 + x^3 \sin \left(x^3\right)\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3} \left(x^3 \sin \left(x^3\right) - \int \sin \left(x^3\right) dx^3\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3} \left(x^3 \sin \left(x^3\right) - \int \sin \left(x^3\right) dx^3\right)$$

$$\Big\downarrow \text{3118}$$

$$\frac{1}{3} \left(x^3 \sin \left(x^3\right) + \cos \left(x^3\right)\right)$$

input
```
Int[x^5*Cos[x^3],x]
```

output
```
(Cos[x^3] + x^3*Sin[x^3])/3
```

3.787.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3861
```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

3.787.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{\cos(x^3)}{3}+\frac{x^3\sin(x^3)}{3}$	17
default	$\frac{\cos(x^3)}{3}+\frac{x^3\sin(x^3)}{3}$	17
risch	$\frac{\cos(x^3)}{3}+\frac{x^3\sin(x^3)}{3}$	17
norman	$\frac{\frac{2\tan\left(\frac{x^3}{2}\right)x^3}{3}+\frac{2}{3}}{1+\tan\left(\frac{x^3}{2}\right)^2}$	27
parallelrisch	$\frac{2\tan\left(\frac{x^3}{2}\right)x^3+2}{3\tan\left(\frac{x^3}{2}\right)^2+3}$	29
meijerg	$\frac{2\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cos(x^3)}{2\sqrt{\pi}}+\frac{x^3\sin(x^3)}{2\sqrt{\pi}}\right)}{3}$	33

input
```
int(x^5*cos(x^3),x,method=_RETURNVERBOSE)
```

output
```
1/3*cos(x^3)+1/3*x^3*sin(x^3)
```

3.787.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int x^5 \cos\left(x^3\right)\,dx = \frac{1}{3}\,x^3 \sin\left(x^3\right) + \frac{1}{3}\,\cos\left(x^3\right)$$

input
```
integrate(x^5*cos(x^3),x, algorithm="fricas")
```

output
```
1/3*x^3*sin(x^3) + 1/3*cos(x^3)
```

3.787.6 Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int x^5 \cos\left(x^3\right)\,dx = \frac{x^3 \sin\left(x^3\right)}{3} + \frac{\cos\left(x^3\right)}{3}$$

input
```
integrate(x**5*cos(x**3),x)
```

output
```
x**3*sin(x**3)/3 + cos(x**3)/3
```

3.787.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int x^5 \cos\left(x^3\right)\,dx = \frac{1}{3}\,x^3 \sin\left(x^3\right) + \frac{1}{3}\,\cos\left(x^3\right)$$

input
```
integrate(x^5*cos(x^3),x, algorithm="maxima")
```

output
```
1/3*x^3*sin(x^3) + 1/3*cos(x^3)
```

3.787.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int x^5 \cos(x^3)\, dx = \frac{1}{3} x^3 \sin(x^3) + \frac{1}{3} \cos(x^3)$$

input
```
integrate(x^5*cos(x^3),x, algorithm="giac")
```

output
```
1/3*x^3*sin(x^3) + 1/3*cos(x^3)
```

3.787.9 Mupad [B] (verification not implemented)

Time = 27.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int x^5 \cos(x^3)\, dx = \frac{\cos(x^3)}{3} + \frac{x^3 \sin(x^3)}{3}$$

input
```
int(x^5*cos(x^3),x)
```

output
```
cos(x^3)/3 + (x^3*sin(x^3))/3
```

3.788 $\int e^{-3x}\cos(x)\,dx$

3.788.1 Optimal result

Integrand size = 8, antiderivative size = 23

$$\int e^{-3x}\cos(x)\,dx=-\frac{3}{10}e^{-3x}\cos(x)+\frac{1}{10}e^{-3x}\sin(x)$$

output
```
-3/10*cos(x)/exp(3*x)+1/10*sin(x)/exp(3*x)
```

3.788.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.70

$$\int e^{-3x}\cos(x)\,dx=\frac{1}{10}e^{-3x}(-3\cos(x)+\sin(x))$$

input
```
Integrate[Cos[x]/E^(3*x),x]
```

output
```
(-3*Cos[x] + Sin[x])/(10*E^(3*x))
```

3.788.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4933}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-3x}\cos(x)\,dx$$

$$\Big\downarrow \text{4933}$$

$$\frac{1}{10}e^{-3x}\sin(x)-\frac{3}{10}e^{-3x}\cos(x)$$

input
```
Int[Cos[x]/E^(3*x),x]
```

output
```
(-3*Cos[x])/(10*E^(3*x)) + Sin[x]/(10*E^(3*x))
```

3.788.3.1 Defintions of rubi rules used

rule 4933
```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

3.788.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.61

method	result	size
parallelrisch	$\frac{(-3\cos(x)+\sin(x))\,\mathrm{e}^{-3x}}{10}$	14
default	$-\frac{3\,\mathrm{e}^{-3x}\cos(x)}{10}+\frac{\mathrm{e}^{-3x}\sin(x)}{10}$	18
norman	$\frac{\left(-\frac{3}{10}+\frac{3\tan\left(\frac{x}{2}\right)^{2}}{10}+\frac{\tan\left(\frac{x}{2}\right)}{5}\right)\mathrm{e}^{-3x}}{1+\tan\left(\frac{x}{2}\right)^{2}}$	34
risch	$-\frac{3\,\mathrm{e}^{(-3+i)x}}{20}-\frac{i\mathrm{e}^{(-3+i)x}}{20}-\frac{3\,\mathrm{e}^{(-3-i)x}}{20}+\frac{i\mathrm{e}^{(-3-i)x}}{20}$	36

input
```
int(cos(x)/exp(3*x),x,method=_RETURNVERBOSE)
```

output
```
1/10*(-3*cos(x)+sin(x))*exp(-3*x)
```

3.788.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int e^{-3x}\cos(x)\,dx = -\frac{3}{10}\,\cos\left(x\right)e^{\left(-3\,x\right)} + \frac{1}{10}\,e^{\left(-3\,x\right)}\sin\left(x\right)$$

input
```
integrate(cos(x)/exp(3*x),x, algorithm="fricas")
```

output
```
-3/10*cos(x)*e^(-3*x) + 1/10*e^(-3*x)*sin(x)
```

3.788.6 Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int e^{-3x}\cos(x)\,dx = \frac{e^{-3x}\sin\left(x\right)}{10} - \frac{3e^{-3x}\cos\left(x\right)}{10}$$

input
```
integrate(cos(x)/exp(3*x),x)
```

output
```
exp(-3*x)*sin(x)/10 - 3*exp(-3*x)*cos(x)/10
```

3.788.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int e^{-3x}\cos(x)\,dx = -\frac{1}{10}\left(3\,\cos\left(x\right) - \sin\left(x\right)\right)e^{\left(-3\,x\right)}$$

input
```
integrate(cos(x)/exp(3*x),x, algorithm="maxima")
```

output
```
-1/10*(3*cos(x) - sin(x))*e^(-3*x)
```

3.788.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int e^{-3x}\cos(x)\,dx = -\frac{1}{10}\,(3\,\cos\left(x\right) - \sin\left(x\right))e^{\left(-3\,x\right)}$$

input
```
integrate(cos(x)/exp(3*x),x, algorithm="giac")
```

output
```
-1/10*(3*cos(x) - sin(x))*e^(-3*x)
```

3.788.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int e^{-3x}\cos(x)\,dx = -\frac{\mathrm{e}^{-3\,x}\,\left(3\,\cos\left(x\right) - \sin\left(x\right)\right)}{10}$$

input
```
int(exp(-3*x)*cos(x),x)
```

output
```
-(exp(-3*x)*(3*cos(x) - sin(x)))/10
```

3.789 $\int x^3 \sin(x^2)\, dx$

3.789.1 Optimal result

Integrand size = 8, antiderivative size = 20

$$\int x^3 \sin(x^2)\, dx = -\frac{1}{2}x^2\cos(x^2) + \frac{\sin(x^2)}{2}$$

output
```
-1/2*x^2*cos(x^2)+1/2*sin(x^2)
```

3.789.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x^3 \sin(x^2)\, dx = -\frac{1}{2}x^2\cos(x^2) + \frac{\sin(x^2)}{2}$$

input
```
Integrate[x^3*Sin[x^2],x]
```

output
```
-1/2*(x^2*Cos[x^2]) + Sin[x^2]/2
```

3.789.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {3860, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \sin \left(x^2\right) \, dx$$

$$\Big\downarrow \text{3860}$$

$$\frac{1}{2} \int x^2 \sin \left(x^2\right) dx^2$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2} \int x^2 \sin \left(x^2\right) dx^2$$

$$\Big\downarrow \text{3777}$$

$$\frac{1}{2} \left(\int \cos \left(x^2\right) dx^2 - x^2 \cos \left(x^2\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2} \left(\int \sin \left(x^2 + \frac{\pi}{2}\right) dx^2 - x^2 \cos \left(x^2\right)\right)$$

$$\Big\downarrow \text{3117}$$

$$\frac{1}{2} \left(\sin \left(x^2\right) - x^2 \cos \left(x^2\right)\right)$$

input
```
Int[x^3*Sin[x^2],x]
```

output
```
(-(x^2*Cos[x^2]) + Sin[x^2])/2
```

3.789.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

3.789.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$-\frac{x^{2} \cos\left(x^{2}\right)}{2}+\frac{\sin\left(x^{2}\right)}{2}$	17
default	$-\frac{x^{2} \cos\left(x^{2}\right)}{2}+\frac{\sin\left(x^{2}\right)}{2}$	17
risch	$-\frac{x^{2} \cos\left(x^{2}\right)}{2}+\frac{\sin\left(x^{2}\right)}{2}$	17
parallelrisch	$-\frac{x^{2} \cos\left(x^{2}\right)}{2}+\frac{\sin\left(x^{2}\right)}{2}$	17
meijerg	$\sqrt{\pi}\,\left(-\frac{x^{2} \cos\left(x^{2}\right)}{2 \sqrt{\pi}}+\frac{\sin\left(x^{2}\right)}{2 \sqrt{\pi}}\right)$	27
norman	$\frac{-\frac{x^{2}}{2}+\frac{x^{2} \tan\left(\frac{x^{2}}{2}\right)^{2}}{2}+\tan\left(\frac{x^{2}}{2}\right)}{1+\tan\left(\frac{x^{2}}{2}\right)^{2}}$	39
parts	$\frac{\sqrt{2}\, \sqrt{\pi}\, \operatorname{FresnelS}\left(\frac{\sqrt{2}\, x}{\sqrt{\pi}}\right) x^{3}}{2}-\frac{3 \pi^{2} \left(\frac{2 \operatorname{FresnelS}\left(\frac{\sqrt{2}\, x}{\sqrt{\pi}}\right) \sqrt{2}\, x^{3}}{3 \pi^{\frac{3}{2}}}+\frac{2 x^{2} \cos\left(x^{2}\right)}{3 \pi^{2}}-\frac{2 \sin\left(x^{2}\right)}{3 \pi^{2}}\right)}{4}$	69

input
```
int(x^3*sin(x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x^2*cos(x^2)+1/2*sin(x^2)
```

3.789.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int x^3 \sin\left(x^2\right)\,dx = -\frac{1}{2}\,x^2\cos\left(x^2\right) + \frac{1}{2}\,\sin\left(x^2\right)$$

input
```
integrate(x^3*sin(x^2),x, algorithm="fricas")
```

output
```
-1/2*x^2*cos(x^2) + 1/2*sin(x^2)
```

3.789.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int x^3 \sin\left(x^2\right)\,dx = -\frac{x^2\cos\left(x^2\right)}{2} + \frac{\sin\left(x^2\right)}{2}$$

input
```
integrate(x**3*sin(x**2),x)
```

output
```
-x**2*cos(x**2)/2 + sin(x**2)/2
```

3.789.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int x^3 \sin\left(x^2\right)\,dx = -\frac{1}{2}\,x^2\cos\left(x^2\right) + \frac{1}{2}\,\sin\left(x^2\right)$$

input
```
integrate(x^3*sin(x^2),x, algorithm="maxima")
```

output
```
-1/2*x^2*cos(x^2) + 1/2*sin(x^2)
```

3.789.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int x^3 \sin\left(x^2\right) \, dx = -\frac{1}{2} x^2 \cos\left(x^2\right) + \frac{1}{2} \sin\left(x^2\right)$$

input
```
integrate(x^3*sin(x^2),x, algorithm="giac")
```

output
```
-1/2*x^2*cos(x^2) + 1/2*sin(x^2)
```

3.789.9 Mupad [B] (verification not implemented)

Time = 26.82 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int x^3 \sin\left(x^2\right) \, dx = \frac{\sin\left(x^2\right)}{2} - \frac{x^2 \cos\left(x^2\right)}{2}$$

input
```
int(x^3*sin(x^2),x)
```

output
```
sin(x^2)/2 - (x^2*cos(x^2))/2
```

3.790 $\int x^3 \cos(x^2)\,dx$

3.790.1 Optimal result

Integrand size = 8, antiderivative size = 20

$$\int x^3 \cos(x^2)\,dx = \frac{\cos(x^2)}{2} + \frac{1}{2}x^2 \sin(x^2)$$

output
```
1/2*cos(x^2)+1/2*x^2*sin(x^2)
```

3.790.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x^3 \cos(x^2)\,dx = \frac{\cos(x^2)}{2} + \frac{1}{2}x^2 \sin(x^2)$$

input
```
Integrate[x^3*Cos[x^2],x]
```

output
```
Cos[x^2]/2 + (x^2*Sin[x^2])/2
```

3.790.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3861, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \cos \left(x^2\right) \, dx$$

$$\Big\downarrow \text{3861}$$

$$\frac{1}{2} \int x^2 \cos \left(x^2\right) dx^2$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2} \int x^2 \sin \left(x^2+\frac{\pi}{2}\right) dx^2$$

$$\Big\downarrow \text{3777}$$

$$\frac{1}{2} \left(\int -\sin \left(x^2\right) dx^2+x^2 \sin \left(x^2\right)\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2} \left(x^2 \sin \left(x^2\right)-\int \sin \left(x^2\right) dx^2\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2} \left(x^2 \sin \left(x^2\right)-\int \sin \left(x^2\right) dx^2\right)$$

$$\Big\downarrow \text{3118}$$

$$\frac{1}{2} \left(x^2 \sin \left(x^2\right)+\cos \left(x^2\right)\right)$$

input
```
Int[x^3*Cos[x^2],x]
```

output
```
(Cos[x^2] + x^2*Sin[x^2])/2
```

3.790.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3861
```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

3.790.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{\cos(x^2)}{2}+\frac{x^2\sin(x^2)}{2}$	17
default	$\frac{\cos(x^2)}{2}+\frac{x^2\sin(x^2)}{2}$	17
risch	$\frac{\cos(x^2)}{2}+\frac{x^2\sin(x^2)}{2}$	17
norman	$\frac{\tan\left(\frac{x^2}{2}\right)x^2+1}{1+\tan\left(\frac{x^2}{2}\right)^2}$	26
parallelrisch	$\frac{\tan\left(\frac{x^2}{2}\right)x^2+1}{1+\tan\left(\frac{x^2}{2}\right)^2}$	26
meijerg	$\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cos(x^2)}{2\sqrt{\pi}}+\frac{x^2\sin(x^2)}{2\sqrt{\pi}}\right)$	32
parts	$\frac{\sqrt{2}\,\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{\sqrt{2}\,x}{\sqrt{\pi}}\right)x^3}{2}-\frac{3\pi^2\left(\frac{2\,\mathrm{FresnelC}\left(\frac{\sqrt{2}\,x}{\sqrt{\pi}}\right)\sqrt{2}\,x^3}{3\pi^{\frac{3}{2}}}-\frac{2x^2\sin\left(x^2\right)}{3\pi^2}-\frac{2\cos\left(x^2\right)}{3\pi^2}\right)}{4}$	69

input
```
int(x^3*cos(x^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*cos(x^2)+1/2*x^2*sin(x^2)
```

3.790.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int x^3\cos\left(x^2\right)\,dx=\frac{1}{2}\,x^2\sin\left(x^2\right)+\frac{1}{2}\,\cos\left(x^2\right)$$

input
```
integrate(x^3*cos(x^2),x, algorithm="fricas")
```

output
```
1/2*x^2*sin(x^2) + 1/2*cos(x^2)
```

3.790.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int x^3 \cos\left(x^2\right)\,dx = \frac{x^2 \sin\left(x^2\right)}{2} + \frac{\cos\left(x^2\right)}{2}$$

input
```
integrate(x**3*cos(x**2),x)
```

output
```
x**2*sin(x**2)/2 + cos(x**2)/2
```

3.790.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int x^3 \cos\left(x^2\right)\,dx = \frac{1}{2}\,x^2 \sin\left(x^2\right) + \frac{1}{2}\,\cos\left(x^2\right)$$

input
```
integrate(x^3*cos(x^2),x, algorithm="maxima")
```

output
```
1/2*x^2*sin(x^2) + 1/2*cos(x^2)
```

3.790.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int x^3 \cos\left(x^2\right)\,dx = \frac{1}{2}\,x^2 \sin\left(x^2\right) + \frac{1}{2}\,\cos\left(x^2\right)$$

input
```
integrate(x^3*cos(x^2),x, algorithm="giac")
```

output
```
1/2*x^2*sin(x^2) + 1/2*cos(x^2)
```

3.790.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int x^3 \cos \left(x^2\right) \, dx = \frac{\cos \left(x^2\right)}{2} + \frac{x^2 \sin \left(x^2\right)}{2}$$

input
```
int(x^3*cos(x^2),x)
```

output
```
cos(x^2)/2 + (x^2*sin(x^2))/2
```

3.791 $\int \cos(x)\cos(2\sin(x))\,dx$

3.791.1 Optimal result

Integrand size = 8, antiderivative size = 9

$$\int \cos(x)\cos(2\sin(x))\,dx = \frac{1}{2}\sin(2\sin(x))$$

output
```
1/2*sin(2*sin(x))
```

3.791.2 Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \cos(x)\cos(2\sin(x))\,dx = \frac{1}{2}\sin(2\sin(x))$$

input
```
Integrate[Cos[x]*Cos[2*Sin[x]],x]
```

output
```
Sin[2*Sin[x]]/2
```

3.791.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4834, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\cos(2\sin(x))\,dx$$

$$\Big\downarrow \text{ 4834}$$

$$\int \cos(2\sin(x))d\sin(x)$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin\left(2\sin(x)+\frac{\pi}{2}\right)d\sin(x)$$

$$\Big\downarrow \text{ 3117}$$

$$\frac{1}{2}\sin(2\sin(x))$$

input
```
Int[Cos[x]*Cos[2*Sin[x]],x]
```

output
```
Sin[2*Sin[x]]/2
```

3.791.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 4834
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

3.791.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{\sin(2\sin(x))}{2}$	8
default	$\frac{\sin(2\sin(x))}{2}$	8
risch	$\frac{\sin(2\sin(x))}{2}$	8
parallelrisch	$\frac{\sin(2\sin(x))}{2}$	8
norman	$\frac{\tan\left(\frac{x}{2}\right)^{2}\tan\left(\frac{2\tan\left(\frac{x}{2}\right)}{1+\tan\left(\frac{x}{2}\right)^{2}}\right)+\tan\left(\frac{2\tan\left(\frac{x}{2}\right)}{1+\tan\left(\frac{x}{2}\right)^{2}}\right)}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)\left(1+\tan\left(\frac{2\tan\left(\frac{x}{2}\right)}{1+\tan\left(\frac{x}{2}\right)^{2}}\right)^{2}\right)}$	77

input
```
int(cos(x)*cos(2*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*sin(2*sin(x))
```

3.791.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(7) = 14.

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 2.11

$$\int \cos(x)\cos(2\sin(x))\,dx = \frac{1}{2}\sin\left(\frac{4\tan\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)^{2}+1}\right)$$

input
```
integrate(cos(x)*cos(2*sin(x)),x, algorithm="fricas")
```

output
```
1/2*sin(4*tan(1/2*x)/(tan(1/2*x)^2 + 1))
```

3.791.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \cos(x)\cos(2\sin(x))\,dx = \frac{\sin(2\sin(x))}{2}$$

input
```
integrate(cos(x)*cos(2*sin(x)),x)
```

output
```
sin(2*sin(x))/2
```

3.791.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \cos(x)\cos(2\sin(x))\,dx = \frac{1}{2}\sin(2\sin(x))$$

input
```
integrate(cos(x)*cos(2*sin(x)),x, algorithm="maxima")
```

output
```
1/2*sin(2*sin(x))
```

3.791.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \cos(x)\cos(2\sin(x))\,dx = \frac{1}{2}\sin(2\sin(x))$$

input
```
integrate(cos(x)*cos(2*sin(x)),x, algorithm="giac")
```

output
```
1/2*sin(2*sin(x))
```

3.791.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \cos(x)\cos(2\sin(x))\,dx = \frac{\sin\left(2\,\sin\left(x\right)\right)}{2}$$

input
```
int(cos(2*sin(x))*cos(x),x)
```

output
```
sin(2*sin(x))/2
```

3.792 $\int \frac{\cos(x)\sin(x)}{1+\cos^2(x)}\,dx$

3.792.1 Optimal result

Integrand size = 13, antiderivative size = 11

$$\int \frac{\cos(x)\sin(x)}{1+\cos^2(x)}\,dx = -\frac{1}{2}\log\left(1+\cos^2(x)\right)$$

output
```
-1/2*ln(1+cos(x)^2)
```

3.792.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)\sin(x)}{1+\cos^2(x)}\,dx = -\frac{1}{2}\log(3+\cos(2x))$$

input
```
Integrate[(Cos[x]*Sin[x])/(1 + Cos[x]^2),x]
```

output
```
-1/2*Log[3 + Cos[2*x]]
```

3.792.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3042, 4835, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)\cos(x)}{\cos^2(x)+1}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sin(x)\cos(x)}{\cos(x)^2+1}dx$$

$$\Big\downarrow \ 4835$$

$$-\int \frac{\cos(x)}{\cos^2(x)+1}d\cos(x)$$

$$\Big\downarrow \ 240$$

$$-\frac{1}{2}\log\left(\cos^2(x)+1\right)$$

input
```
Int[(Cos[x]*Sin[x])/(1 + Cos[x]^2),x]
```

output
```
-1/2*Log[1 + Cos[x]^2]
```

3.792.3.1 Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4835
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[1, Cos[c*(a +
b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

3.792.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$-\frac{\ln\left(\cos(x)^2+1\right)}{2}$	10
default	$-\frac{\ln\left(\cos(x)^2+1\right)}{2}$	10
norman	$-\frac{\ln\left(\tan\left(\frac{x}{2}\right)^4+1\right)}{2}+\ln\left(1+\tan\left(\frac{x}{2}\right)^2\right)$	22
risch	$ix-\frac{\ln\left(e^{4ix}+6\,e^{2ix}+1\right)}{2}$	23
parallelrisch	$\ln\left(\frac{\sqrt{2}}{\sqrt{\frac{3+\cos(2x)}{\cos(2x)+3+4\cos(x)}}}\right)+\ln\left(\frac{1}{\cos(x)+1}\right)$	35

input
```
int(cos(x)*sin(x)/(cos(x)^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(cos(x)^2+1)
```

3.792.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)\sin(x)}{1+\cos^2(x)}\,dx = -\frac{1}{2}\,\log\left(\frac{1}{2}\,\cos\left(x\right)^2+\frac{1}{2}\right)$$

input
```
integrate(cos(x)*sin(x)/(1+cos(x)^2),x, algorithm="fricas")
```

output
```
-1/2*log(1/2*cos(x)^2 + 1/2)
```

3.792.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{\cos(x)\sin(x)}{1+\cos^2(x)}\,dx = -\frac{\log\left(\cos^2(x)+1\right)}{2}$$

input
```
integrate(cos(x)*sin(x)/(1+cos(x)**2),x)
```

output
```
-log(cos(x)**2 + 1)/2
```

3.792.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{\cos(x)\sin(x)}{1+\cos^2(x)}\,dx = -\frac{1}{2}\log\left(\cos(x)^2+1\right)$$

input
```
integrate(cos(x)*sin(x)/(1+cos(x)^2),x, algorithm="maxima")
```

output
```
-1/2*log(cos(x)^2 + 1)
```

3.792.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{\cos(x)\sin(x)}{1+\cos^2(x)}\,dx = -\frac{1}{2}\log\left(\cos(x)^2+1\right)$$

input
```
integrate(cos(x)*sin(x)/(1+cos(x)^2),x, algorithm="giac")
```

output
```
-1/2*log(cos(x)^2 + 1)
```

3.792.9 Mupad [B] (verification not implemented)

Time = 26.78 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \frac{\cos(x)\sin(x)}{1+\cos^2(x)}\,dx = -\text{atanh}\left(\frac{16}{3\,\left(12\tan\left(x\right)^2+16\right)}-\frac{1}{3}\right)$$

input
```
int((cos(x)*sin(x))/(cos(x)^2 + 1),x)
```

output
```
-atanh(16/(3*(12*tan(x)^2 + 16)) - 1/3)
```

3.793 $\int (1+\cos(x))(x+\sin(x))^3\,dx$

3.793.1 Optimal result

Integrand size = 11, antiderivative size = 10

$$\int (1+\cos(x))(x+\sin(x))^3\,dx=\frac{1}{4}(x+\sin(x))^4$$

output
```
1/4*(x+sin(x))^4
```

3.793.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int (1+\cos(x))(x+\sin(x))^3\,dx=\frac{1}{4}(x+\sin(x))^4$$

input
```
Integrate[(1 + Cos[x])*(x + Sin[x])^3,x]
```

output
```
(x + Sin[x])^4/4
```

3.793.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {7237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x+\sin(x))^3(\cos(x)+1)\,dx$$

$$\Big\downarrow \text{ 7237}$$

$$\frac{1}{4}(x+\sin(x))^4$$

input
```
Int[(1 + Cos[x])*(x + Sin[x])^3,x]
```

output
```
(x + Sin[x])^4/4
```

3.793.3.1 Defintions of rubi rules used

rule 7237
```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[q*(y^(m + 1)/(m + 1)), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

3.793.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. 2(8) = 16.

Time = 3.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 5.10

method	result
risch	$\frac{x^4}{4}+\frac{3x^2}{4}+\frac{9}{16}+\frac{x(4x^2+3)\sin(x)}{4}+\frac{\cos(4x)}{32}-\frac{x\sin(3x)}{4}+2\left(-\frac{1}{16}-\frac{3x^2}{8}\right)\cos(2x)$
default	$\frac{\sin(x)^4}{4}+\sin(x)^3 x-\frac{3x^2\cos(x)^2}{2}+3x\left(\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)-\frac{3x^2}{2}+\sin(x)x^3+3x\left(-\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)+$
parts	$\frac{\sin(x)^4}{4}+\sin(x)^3 x-\frac{3x^2\cos(x)^2}{2}+3x\left(\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)-\frac{3x^2}{2}+\sin(x)x^3+3x\left(-\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)+$

input
```
int((cos(x)+1)*(x+sin(x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4+3/4*x^2+9/16+1/4*x*(4*x^2+3)*sin(x)+1/32*cos(4*x)-1/4*x*sin(3*x)+2
*(-1/16-3/8*x^2)*cos(2*x)
```

3.793.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. $2(8) = 16$.

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 4.50

$$\int (1+\cos(x))(x+\sin(x))^3\,dx = \frac{1}{4}\,x^4 + \frac{1}{4}\,\cos{(x)}^4 - \frac{1}{2}\,\left(3\,x^2+1\right)\cos{(x)}^2 + \frac{3}{2}\,x^2 + \left(x^3 - x\cos{(x)}^2 + x\right)\sin{(x)}$$

input
```
integrate((1+cos(x))*(x+sin(x))^3,x, algorithm="fricas")
```

output
```
1/4*x^4 + 1/4*cos(x)^4 - 1/2*(3*x^2 + 1)*cos(x)^2 + 3/2*x^2 + (x^3 - x*cos
(x)^2 + x)*sin(x)
```

3.793.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(7) = 14$.

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 3.60

$$\int (1+\cos(x))(x+\sin(x))^3\,dx = \frac{x^4}{4} + x^3\sin{(x)} + \frac{3x^2\sin^2{(x)}}{2} + x\sin^3{(x)} + \frac{\sin^4{(x)}}{4}$$

input
```
integrate((1+cos(x))*(x+sin(x))**3,x)
```

output
```
x**4/4 + x**3*sin(x) + 3*x**2*sin(x)**2/2 + x*sin(x)**3 + sin(x)**4/4
```

3.793.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int (1+\cos(x))(x+\sin(x))^3\,dx = \frac{1}{4}\,(x+\sin{(x)})^4$$

input
```
integrate((1+cos(x))*(x+sin(x))^3,x, algorithm="maxima")
```

output
```
1/4*(x + sin(x))^4
```

3.793.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(8) = 16.

Time = 0.26 (sec) , antiderivative size = 61, normalized size of antiderivative = 6.10

$$\int (1+\cos(x))(x+\sin(x))^3\,dx = \frac{1}{4}\,x^4 + \frac{3}{4}\,x^2 - \frac{1}{4}\,\left(3\,x^2-1\right)\cos{(2\,x)} - \frac{1}{4}\,x\sin{(3\,x)} + \frac{1}{4}\,\left(4\,x^3-21\,x\right)\sin{(x)} + 6\,x\sin{(x)} + \frac{1}{32}\,\cos{(4\,x)} - \frac{3}{8}\,\cos{(2\,x)}$$

input
```
integrate((1+cos(x))*(x+sin(x))^3,x, algorithm="giac")
```

output
```
1/4*x^4 + 3/4*x^2 - 1/4*(3*x^2 - 1)*cos(2*x) - 1/4*x*sin(3*x) + 1/4*(4*x^3
 - 21*x)*sin(x) + 6*x*sin(x) + 1/32*cos(4*x) - 3/8*cos(2*x)
```

3.793.9 Mupad [B] (verification not implemented)

Time = 26.61 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int (1+\cos(x))(x+\sin(x))^3\,dx = \frac{(x+\sin{(x)})^4}{4}$$

input
```
int((cos(x) + 1)*(x + sin(x))^3,x)
```

output
```
(x + sin(x))^4/4
```

3.794 $\int (1+\cos(x))\csc^2(x)\,dx$

3.794.1 Optimal result

Integrand size = 9, antiderivative size = 9

$$\int (1+\cos(x))\csc^2(x)\,dx = -\cot(x)-\csc(x)$$

output
```
-cot(x)-csc(x)
```

3.794.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int (1+\cos(x))\csc^2(x)\,dx = -\cot(x)-\csc(x)$$

input
```
Integrate[(1 + Cos[x])*Csc[x]^2,x]
```

output
```
-Cot[x] - Csc[x]
```

3.794.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {3042, 3148, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\cos(x)+1)\csc^2(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1-\sin\left(x-\frac{\pi}{2}\right)}{\cos\left(x-\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3148}$$

$$\int \csc^2(x)dx-\csc(x)$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(x)^2dx-\csc(x)$$

$$\Big\downarrow \text{4254}$$

$$-\int 1d\cot(x)-\csc(x)$$

$$\Big\downarrow \text{24}$$

$$-\cot(x)-\csc(x)$$

input
```
Int[(1 + Cos[x])*Csc[x]^2,x]
```

output
```
-Cot[x] - Csc[x]
```

3.794.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.794.4 Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$-\frac{1}{\tan\left(\frac{x}{2}\right)}$	9
parts	$-\cot\left(x\right)-\csc\left(x\right)$	10
default	$-\frac{1}{\sin\left(x\right)}-\cot\left(x\right)$	12
risch	$-\frac{2i}{{\mathrm e}^{i x}-1}$	13
norman	$\frac{-1-\tan\left(\frac{x}{2}\right)^{2}}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)\tan\left(\frac{x}{2}\right)}$	28

input
```
int((cos(x)+1)*csc(x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/tan(1/2*x)
```

3.794.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

$$\int (1+\cos(x))\csc^2(x)\,dx = -\frac{\cos(x)+1}{\sin(x)}$$

input
```
integrate((1+cos(x))*csc(x)^2,x, algorithm="fricas")
```

output
```
-(cos(x) + 1)/sin(x)
```

3.794.6 Sympy [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int (1+\cos(x))\csc^2(x)\,dx = -\cot(x) - \frac{1}{\sin(x)}$$

input
```
integrate((1+cos(x))*csc(x)**2,x)
```

output
```
-cot(x) - 1/sin(x)
```

3.794.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.44

$$\int (1+\cos(x))\csc^2(x)\,dx = -\frac{1}{\sin(x)} - \frac{1}{\tan(x)}$$

input
```
integrate((1+cos(x))*csc(x)^2,x, algorithm="maxima")
```

output
```
-1/sin(x) - 1/tan(x)
```

3.794.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int (1+\cos(x))\csc^2(x)\,dx = -\frac{1}{\tan\left(\frac{1}{2}x\right)}$$

input
```
integrate((1+cos(x))*csc(x)^2,x, algorithm="giac")
```

output
```
-1/tan(1/2*x)
```

3.794.9 Mupad [B] (verification not implemented)

Time = 26.36 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.67

$$\int (1+\cos(x))\csc^2(x)\,dx = -\cot\left(\frac{x}{2}\right)$$

input
```
int((cos(x) + 1)/sin(x)^2,x)
```

output
```
-cot(x/2)
```

3.795 $\int \sin(x)\tan^2(x)\,dx$

3.795.1 Optimal result

Integrand size = 7, antiderivative size = 5

$$\int \sin(x)\tan^2(x)\,dx = \cos(x) + \sec(x)$$

output
```
cos(x)+sec(x)
```

3.795.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \sin(x)\tan^2(x)\,dx = \cos(x) + \sec(x)$$

input
```
Integrate[Sin[x]*Tan[x]^2,x]
```

output
```
Cos[x] + Sec[x]
```

3.795.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3070, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\tan^2(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x)\tan(x)^2dx$$

$$\Big\downarrow \text{3070}$$

$$-\int \left(1-\cos^2(x)\right)\sec^2(x)d\cos(x)$$

$$\Big\downarrow \text{244}$$

$$-\int \left(\sec^2(x)-1\right)d\cos(x)$$

$$\Big\downarrow \text{2009}$$

$$\cos(x)+\sec(x)$$

input
```
Int[Sin[x]*Tan[x]^2,x]
```

output
```
Cos[x] + Sec[x]
```

3.795.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

3.795.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. $2(5) = 10$.

Time = 1.87 (sec) , antiderivative size = 20, normalized size of antiderivative = 4.00

method	result	size
default	$\frac{\sin(x)^4}{\cos(x)} + \left(2+\sin\left(x\right)^2\right)\cos\left(x\right)$	20
risch	$\frac{e^{3ix}+7\cos(x)+5i\sin(x)}{2\,e^{2ix}+2}$	27

input
```
int(sin(x)*tan(x)^2,x,method=_RETURNVERBOSE)
```

output
```
sin(x)^4/cos(x)+(2+sin(x)^2)*cos(x)
```

3.795.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11 vs. $2(5) = 10$.

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 2.20

$$\int \sin(x)\tan^2(x)\,dx = \frac{\cos\left(x\right)^2+1}{\cos\left(x\right)}$$

input
```
integrate(sin(x)*tan(x)^2,x, algorithm="fricas")
```

output
```
(cos(x)^2 + 1)/cos(x)
```

3.795.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.40

$$\int \sin(x)\tan^2(x)\,dx = \cos(x) + \frac{1}{\cos(x)}$$

input
```
integrate(sin(x)*tan(x)**2,x)
```

output
```
cos(x) + 1/cos(x)
```

3.795.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.40

$$\int \sin(x)\tan^2(x)\,dx = \frac{1}{\cos(x)} + \cos(x)$$

input
```
integrate(sin(x)*tan(x)^2,x, algorithm="maxima")
```

output
```
1/cos(x) + cos(x)
```

3.795.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.40

$$\int \sin(x)\tan^2(x)\,dx = \frac{1}{\cos(x)} + \cos(x)$$

input
```
integrate(sin(x)*tan(x)^2,x, algorithm="giac")
```

output
```
1/cos(x) + cos(x)
```

3.795.9 Mupad [B] (verification not implemented)

Time = 26.73 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.40

$$\int \sin(x)\tan^2(x)\,dx = \cos(x) + \frac{1}{\cos(x)}$$

input
```
int(sin(x)*tan(x)^2,x)
```

output
```
cos(x) + 1/cos(x)
```

3.796 $\int e^{\sin(x)} \sec^2(x) \left(x \cos^3(x) - \sin(x)\right) \, dx$

3.796.1 Optimal result

Integrand size = 20, antiderivative size = 13

$$\int e^{\sin(x)} \sec^2(x) \left(x \cos^3(x) - \sin(x)\right) \, dx = e^{\sin(x)}(-1 + x\cos(x)) \sec(x)$$

output
```
exp(sin(x))*(-1+x*cos(x))*sec(x)
```

3.796.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int e^{\sin(x)} \sec^2(x) \left(x \cos^3(x) - \sin(x)\right) \, dx = e^{\sin(x)}(-1 + x\cos(x)) \sec(x)$$

input
```
Integrate[E^Sin[x]*Sec[x]^2*(x*Cos[x]^3 - Sin[x]),x]
```

output
```
E^Sin[x]*(-1 + x*Cos[x])*Sec[x]
```

3.796.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\sin(x)} \sec^2(x) \left(x \cos^3(x) - \sin(x)\right) \, dx$$
$$\Big\downarrow \text{ 7293}$$
$$\int \left(x e^{\sin(x)} \cos(x) - e^{\sin(x)} \tan(x) \sec(x)\right) dx$$
$$\Big\downarrow \text{ 2009}$$
$$\int e^{\sin(x)} x \cos(x) dx - \int e^{\sin(x)} \sec(x) \tan(x) dx$$

input
```
Int[E^Sin[x]*Sec[x]^2*(x*Cos[x]^3 - Sin[x]),x]
```

output
```
$Aborted
```

3.796.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.796.4 Maple [A] (verified)

Time = 2.23 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

method	result	size
parallelrisch	$e^{\sin(x)}(-\sec(x) + x)$	11
risch	$\frac{\left(e^{2ix} x + x - 2\, e^{ix}\right) e^{\sin(x)}}{e^{2ix} + 1}$	30

input
```
int(exp(sin(x))*sec(x)^2*(x*cos(x)^3-sin(x)),x,method=_RETURNVERBOSE)
```

output
```
exp(sin(x))*(-sec(x)+x)
```

3.796.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int e^{\sin(x)}\sec^2(x)\left(x\cos^3(x)-\sin(x)\right)\,dx=\frac{(x\cos(x)-1)e^{\sin(x)}}{\cos(x)}$$

input
```
integrate(exp(sin(x))*sec(x)^2*(x*cos(x)^3-sin(x)),x, algorithm="fricas")
```

output
```
(x*cos(x) - 1)*e^sin(x)/cos(x)
```

3.796.6 Sympy [F(-1)]

Timed out.

$$\int e^{\sin(x)}\sec^2(x)\left(x\cos^3(x)-\sin(x)\right)\,dx=\text{Timed out}$$

input
```
integrate(exp(sin(x))*sec(x)**2*(x*cos(x)**3-sin(x)),x)
```

output
```
Timed out
```

3.796.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(12) = 24.

Time = 0.52 (sec) , antiderivative size = 88, normalized size of antiderivative = 6.77

$$\int e^{\sin(x)}\sec^2(x)\left(x\cos^3(x)-\sin(x)\right)\,dx$$
$$=\frac{x\cos(2x)^2e^{\sin(x)}+xe^{\sin(x)}\sin(2x)^2-2e^{\sin(x)}\sin(2x)\sin(x)+2\left(xe^{\sin(x)}-\cos(x)e^{\sin(x)}\right)\cos(2x)+xe}{\cos(2x)^2+\sin(2x)^2+2\cos(2x)+1}$$

input
```
integrate(exp(sin(x))*sec(x)^2*(x*cos(x)^3-sin(x)),x, algorithm="maxima")
```

output
```
(x*cos(2*x)^2*e^sin(x) + x*e^sin(x)*sin(2*x)^2 - 2*e^sin(x)*sin(2*x)*sin(x
) + 2*(x*e^sin(x) - cos(x)*e^sin(x))*cos(2*x) + x*e^sin(x) - 2*cos(x)*e^si
n(x))/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)
```

3.796.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 794 vs. $2(12) = 24$.

Time = 0.28 (sec) , antiderivative size = 794, normalized size of antiderivative = 61.08

$$\int e^{\sin(x)}\sec^2(x)\left(x\cos^3(x)-\sin(x)\right)\,dx = \text{Too large to display}$$

input
```
integrate(exp(sin(x))*sec(x)^2*(x*cos(x)^3-sin(x)),x, algorithm="giac")
```

output
```
(x*e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(3/2*x)^2*tan(1/2*x)^8 + e^(2*ta
n(1/2*x)/(tan(1/2*x)^2 + 1))*tan(3/2*x)^2*tan(1/2*x)^8 - 16*x*e^(2*tan(1/2
*x)/(tan(1/2*x)^2 + 1))*tan(3/2*x)^2*tan(1/2*x)^6 + 12*x*e^(2*tan(1/2*x)/(
tan(1/2*x)^2 + 1))*tan(3/2*x)*tan(1/2*x)^7 - x*e^(2*tan(1/2*x)/(tan(1/2*x)
^2 + 1))*tan(1/2*x)^8 - 14*e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(3/2*x)^
2*tan(1/2*x)^6 + 12*e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(3/2*x)*tan(1/2
*x)^7 - e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^8 + 30*x*e^(2*tan(1
/2*x)/(tan(1/2*x)^2 + 1))*tan(3/2*x)^2*tan(1/2*x)^4 - 52*x*e^(2*tan(1/2*x)
/(tan(1/2*x)^2 + 1))*tan(3/2*x)*tan(1/2*x)^5 + 16*x*e^(2*tan(1/2*x)/(tan(1
/2*x)^2 + 1))*tan(1/2*x)^6 - 28*e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(3/
2*x)*tan(1/2*x)^5 + 14*e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^6 -
16*x*e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(3/2*x)^2*tan(1/2*x)^2 + 52*x*
e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(3/2*x)*tan(1/2*x)^3 - 30*x*e^(2*ta
n(1/2*x)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^4 + 14*e^(2*tan(1/2*x)/(tan(1/2*x)
^2 + 1))*tan(3/2*x)^2*tan(1/2*x)^2 - 28*e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1)
)*tan(3/2*x)*tan(1/2*x)^3 + x*e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(3/2*
x)^2 - 12*x*e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(3/2*x)*tan(1/2*x) + 16
*x*e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^2 - e^(2*tan(1/2*x)/(tan
(1/2*x)^2 + 1))*tan(3/2*x)^2 + 12*e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(
3/2*x)*tan(1/2*x) - 14*e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^2...
```

3.796.9 Mupad [B] (verification not implemented)

Time = 26.89 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int e^{\sin(x)}\sec^2(x)\left(x\cos^3(x)-\sin(x)\right)\,dx=\frac{\mathrm{e}^{\sin(x)}\,(x\,\cos(x)-1)}{\cos(x)}$$

input
```
int(-(exp(sin(x))*(sin(x) - x*cos(x)^3))/cos(x)^2,x)
```

output
```
(exp(sin(x))*(x*cos(x) - 1))/cos(x)
```

3.797 $\int x \csc^2(x) \, dx$

3.797.1 Optimal result

Integrand size = 6, antiderivative size = 9

$$\int x \csc^2(x) \, dx = -x \cot(x) + \log(\sin(x))$$

output
```
-x*cot(x)+ln(sin(x))
```

3.797.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int x \csc^2(x) \, dx = -x \cot(x) + \log(\sin(x))$$

input
```
Integrate[x*Csc[x]^2,x]
```

output
```
-(x*Cot[x]) + Log[Sin[x]]
```

3.797.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {3042, 4672, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \csc^2(x) \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int x \csc(x)^2 dx$$

$$\Big\downarrow \text{ 4672}$$

$$\int \cot(x) dx - x \cot(x)$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\tan\left(x + \frac{\pi}{2}\right) dx - x \cot(x)$$

$$\Big\downarrow \text{ 25}$$

$$-\int \tan\left(x + \frac{\pi}{2}\right) dx - x \cot(x)$$

$$\Big\downarrow \text{ 3956}$$

$$\log(\sin(x)) - x \cot(x)$$

input
```
Int[x*Csc[x]^2,x]
```

output
```
-(x*Cot[x]) + Log[Sin[x]]
```

3.797.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.797.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

method	result	size
default	$-x \cot(x)+\ln(\sin(x))$	10
parallelrisch	$\ln\left(\frac{\csc(x)}{2}-\frac{\cot(x)}{2}\right)-\ln\left(\frac{1}{\cos(x)+1}\right)-x\cot(x)$	26
risch	$-2ix-\frac{2ix}{e^{2ix}-1}+\ln\left(e^{2ix}-1\right)$	27
norman	$\frac{-\frac{x}{2}+\frac{x\tan\left(\frac{x}{2}\right)^2}{2}}{\tan\left(\frac{x}{2}\right)}-\ln\left(1+\tan\left(\frac{x}{2}\right)^2\right)+\ln\left(\tan\left(\frac{x}{2}\right)\right)$	38

input
```
int(csc(x)^2*x,x,method=_RETURNVERBOSE)
```

output
```
-x*cot(x)+ln(sin(x))
```

3.797.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(9) = 18$.

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.22

$$\int x\csc^2(x)\,dx = -\frac{x\cos(x) - \log\left(\frac{1}{2}\sin(x)\right)\sin(x)}{\sin(x)}$$

input
```
integrate(x*csc(x)^2,x, algorithm="fricas")
```

output
```
-(x*cos(x) - log(1/2*sin(x))*sin(x))/sin(x)
```

3.797.6 Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.89

$$\int x\csc^2(x)\,dx = -x\cot(x) + \log(\sin(x))$$

input
```
integrate(x*csc(x)**2,x)
```

output
```
-x*cot(x) + log(sin(x))
```

3.797.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(9) = 18$.

Time = 0.18 (sec) , antiderivative size = 104, normalized size of antiderivative = 11.56

$$\int x\csc^2(x)\,dx$$
$$= \frac{\left(\cos(2x)^2 + \sin(2x)^2 - 2\cos(2x) + 1\right)\log\left(\cos(x)^2 + \sin(x)^2 + 2\cos(x) + 1\right) + \left(\cos(2x)^2 + \sin(2x)\right.}{2\left(\cos(2x)^2 + \sin(2x)^2 - 2\cos(2x)\right.}$$

input
```
integrate(x*csc(x)^2,x, algorithm="maxima")
```

output
```
1/2*((cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 +
2*cos(x) + 1) + (cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1)*log(cos(x)^2 +
sin(x)^2 - 2*cos(x) + 1) - 4*x*sin(2*x))/(cos(2*x)^2 + sin(2*x)^2 - 2*cos(
2*x) + 1)
```

3.797.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(9) = 18$.

Time = 0.31 (sec) , antiderivative size = 52, normalized size of antiderivative = 5.78

$$\int x\csc^2(x)\,dx = \frac{x\tan\left(\frac{1}{2}\,x\right)^2 + \log\left(\frac{16\,\tan\left(\frac{1}{2}\,x\right)^2}{\tan\left(\frac{1}{2}\,x\right)^4 + 2\,\tan\left(\frac{1}{2}\,x\right)^2 + 1}\right)\tan\left(\frac{1}{2}\,x\right) - x}{2\,\tan\left(\frac{1}{2}\,x\right)}$$

input
```
integrate(x*csc(x)^2,x, algorithm="giac")
```

output
```
1/2*(x*tan(1/2*x)^2 + log(16*tan(1/2*x)^2/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 +
 1))*tan(1/2*x) - x)/tan(1/2*x)
```

3.797.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int x\csc^2(x)\,dx = \ln\left(\sin\left(x\right)\right) - x\cot(x)$$

input
```
int(x/sin(x)^2,x)
```

output
```
log(sin(x)) - x*cot(x)
```

3.798 $\int \cos(x)\sin\left(\frac{\pi}{6}+x\right)\,dx$

3.798.1 Optimal result

Integrand size = 11, antiderivative size = 20

$$\int \cos(x)\sin\left(\frac{\pi}{6}+x\right)\,dx=\frac{x}{4}-\frac{1}{4}\cos\left(\frac{\pi}{6}+2x\right)$$

output
```
1/4*x-1/4*cos(1/6*Pi+2*x)
```

3.798.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \cos(x)\sin\left(\frac{\pi}{6}+x\right)\,dx=\frac{x}{4}-\frac{1}{4}\cos\left(\frac{\pi}{6}+2x\right)$$

input
```
Integrate[Cos[x]*Sin[Pi/6 + x],x]
```

output
```
x/4 - Cos[Pi/6 + 2*x]/4
```

3.798.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {5085, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin\left(x+\frac{\pi}{6}\right)\cos(x)\,dx$$

$$\Big\downarrow \text{5085}$$

$$\int \left(\frac{1}{2}\sin\left(2x+\frac{\pi}{6}\right)+\frac{1}{4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x}{4}-\frac{1}{4}\cos\left(2x+\frac{\pi}{6}\right)$$

input
```
Int[Cos[x]*Sin[Pi/6 + x],x]
```

output
```
x/4 - Cos[Pi/6 + 2*x]/4
```

3.798.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5085
```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

3.798.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{x}{4}-\frac{\cos\left(\frac{\pi}{6}+2x\right)}{4}$	15
risch	$\frac{x}{4}-\frac{\sqrt{3}\,\cos\left(2x\right)}{8}+\frac{\sin\left(2x\right)}{8}$	20
parallelrisch	$\frac{\sin\left(\frac{\pi}{3}+2x\right)}{8}-\frac{\cos\left(\frac{\pi}{6}+2x\right)}{8}-\frac{\sqrt{3}\,\cos\left(2x\right)}{8}+\frac{\sqrt{3}}{8}+\frac{x}{4}$	39
norman	$\frac{x\tan\left(\frac{\pi}{12}+\frac{x}{2}\right)+x\tan\left(\frac{x}{2}\right)\tan\left(\frac{\pi}{12}+\frac{x}{2}\right)^{2}+2\tan\left(\frac{x}{2}\right)\tan\left(\frac{\pi}{12}+\frac{x}{2}\right)-x\tan\left(\frac{x}{2}\right)-x\tan\left(\frac{x}{2}\right)^{2}\tan\left(\frac{\pi}{12}+\frac{x}{2}\right)}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)\left(1+\tan\left(\frac{\pi}{12}+\frac{x}{2}\right)^{2}\right)}$	91

input
```
int(cos(x)*sin(1/6*Pi+x),x,method=_RETURNVERBOSE)
```

output
```
1/4*x-1/4*cos(1/6*Pi+2*x)
```

3.798.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(14) = 28.

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int \cos(x)\sin\left(\frac{\pi}{6}+x\right)\,dx=-\frac{1}{4}\,\sqrt{3}\cos\left(\frac{1}{6}\,\pi+x\right)^{2}-\frac{1}{4}\,\cos\left(\frac{1}{6}\,\pi+x\right)\sin\left(\frac{1}{6}\,\pi+x\right)+\frac{1}{4}\,x$$

input
```
integrate(cos(x)*sin(1/6*pi+x),x, algorithm="fricas")
```

output
```
-1/4*sqrt(3)*cos(1/6*pi + x)^2 - 1/4*cos(1/6*pi + x)*sin(1/6*pi + x) + 1/4
*x
```

3.798.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. $2(12) = 24$.

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.85

$$\int \cos(x)\sin\left(\frac{\pi}{6}+x\right)\,dx = -\frac{x\sin(x)\cos\left(x+\frac{\pi}{6}\right)}{2}+\frac{x\sin\left(x+\frac{\pi}{6}\right)\cos(x)}{2}+\frac{\sin(x)\sin\left(x+\frac{\pi}{6}\right)}{2}$$

input
```
integrate(cos(x)*sin(1/6*pi+x),x)
```

output
```
-x*sin(x)*cos(x + pi/6)/2 + x*sin(x + pi/6)*cos(x)/2 + sin(x)*sin(x + pi/6
)/2
```

3.798.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \cos(x)\sin\left(\frac{\pi}{6}+x\right)\,dx = \frac{1}{4}\,x - \frac{1}{4}\,\cos\left(\frac{1}{6}\,\pi + 2\,x\right)$$

input
```
integrate(cos(x)*sin(1/6*pi+x),x, algorithm="maxima")
```

output
```
1/4*x - 1/4*cos(1/6*pi + 2*x)
```

3.798.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \cos(x)\sin\left(\frac{\pi}{6}+x\right)\,dx = \frac{1}{4}\,x - \frac{1}{4}\,\cos\left(\frac{1}{6}\,\pi + 2\,x\right)$$

input
```
integrate(cos(x)*sin(1/6*pi+x),x, algorithm="giac")
```

output
```
1/4*x - 1/4*cos(1/6*pi + 2*x)
```

3.798.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \cos(x)\sin\left(\frac{\pi}{6}+x\right)\,dx=\frac{x\,\sin\left(\frac{\Pi}{6}\right)}{2}-\frac{\cos\left(\frac{\Pi}{6}+2\,x\right)}{4}$$

input
```
int(cos(x)*sin(Pi/6 + x),x)
```

output
```
(x*sin(Pi/6))/2 - cos(Pi/6 + 2*x)/4
```

3.799 $\int x \sin^3(x^2)\, dx$

3.799.1 Optimal result

Integrand size = 8, antiderivative size = 19

$$\int x \sin^3(x^2)\, dx = -\frac{1}{2}\cos(x^2) + \frac{1}{6}\cos^3(x^2)$$

output
```
-1/2*cos(x^2)+1/6*cos(x^2)^3
```

3.799.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int x \sin^3(x^2)\, dx = -\frac{3}{8}\cos(x^2) + \frac{1}{24}\cos(3x^2)$$

input
```
Integrate[x*Sin[x^2]^3,x]
```

output
```
(-3*Cos[x^2])/8 + Cos[3*x^2]/24
```

3.799.3 Rubi [A] (warning: unable to verify)

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3860, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sin^3\left(x^2\right) \, dx$$

$$\Big\downarrow \text{3860}$$

$$\frac{1}{2} \int \sin^3\left(x^2\right) dx^2$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2} \int \sin\left(x^2\right)^3 dx^2$$

$$\Big\downarrow \text{3113}$$

$$-\frac{1}{2} \int \left(1-x^4\right) d\cos\left(x^2\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2} \left(\frac{x^6}{3} - \cos\left(x^2\right)\right)$$

input
```
Int[x*Sin[x^2]^3,x]
```

output
```
(x^6/3 - Cos[x^2])/2
```

3.799.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

3.799.4 Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$-\frac{\left(2+\sin\left(x^2\right)^2\right)\cos\left(x^2\right)}{6}$	15
default	$-\frac{\left(2+\sin\left(x^2\right)^2\right)\cos\left(x^2\right)}{6}$	15
risch	$-\frac{3\cos\left(x^2\right)}{8}+\frac{\cos\left(3x^2\right)}{24}$	16
parallelrisch	$-\frac{1}{3}-\frac{3\cos\left(x^2\right)}{8}+\frac{\cos\left(3x^2\right)}{24}$	17
norman	$\frac{-2\tan\left(\frac{x^2}{2}\right)^2-\frac{2}{3}}{\left(1+\tan\left(\frac{x^2}{2}\right)^2\right)^3}$	26

input
```
int(x*sin(x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(2+sin(x^2)^2)*cos(x^2)
```

3.799.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int x\sin^3\left(x^2\right)\,dx = \frac{1}{6}\,\cos\left(x^2\right)^3 - \frac{1}{2}\,\cos\left(x^2\right)$$

input
```
integrate(x*sin(x^2)^3,x, algorithm="fricas")
```

output
```
1/6*cos(x^2)^3 - 1/2*cos(x^2)
```

3.799.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int x\sin^3\left(x^2\right)\,dx = -\frac{\sin^2\left(x^2\right)\cos\left(x^2\right)}{2} - \frac{\cos^3\left(x^2\right)}{3}$$

input
```
integrate(x*sin(x**2)**3,x)
```

output
```
-sin(x**2)**2*cos(x**2)/2 - cos(x**2)**3/3
```

3.799.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int x\sin^3\left(x^2\right)\,dx = \frac{1}{24}\,\cos\left(3\,x^2\right) - \frac{3}{8}\,\cos\left(x^2\right)$$

input
```
integrate(x*sin(x^2)^3,x, algorithm="maxima")
```

output
```
1/24*cos(3*x^2) - 3/8*cos(x^2)
```

3.799.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int x\sin^3\left(x^2\right)\,dx = \frac{1}{6}\cos\left(x^2\right)^3 - \frac{1}{2}\cos\left(x^2\right)$$

input
```
integrate(x*sin(x^2)^3,x, algorithm="giac")
```

output
```
1/6*cos(x^2)^3 - 1/2*cos(x^2)
```

3.799.9 Mupad [B] (verification not implemented)

Time = 28.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int x\sin^3\left(x^2\right)\,dx = \frac{\cos\left(x^2\right)\left(\cos\left(x^2\right)^2-3\right)}{6}$$

input
```
int(x*sin(x^2)^3,x)
```

output
```
(cos(x^2)*(cos(x^2)^2 - 3))/6
```

3.800 $\int \sin^2(x) \tan(x) \, dx$

3.800.1 Optimal result

Integrand size = 7, antiderivative size = 14

$$\int \sin^2(x) \tan(x) \, dx = \frac{\cos^2(x)}{2} - \log(\cos(x))$$

output
```
1/2*cos(x)^2-ln(cos(x))
```

3.800.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \sin^2(x) \tan(x) \, dx = \frac{\cos^2(x)}{2} - \log(\cos(x))$$

input
```
Integrate[Sin[x]^2*Tan[x],x]
```

output
```
Cos[x]^2/2 - Log[Cos[x]]
```

3.800.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3042, 3070, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(x)\tan(x)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \sin(x)^2\tan(x)dx$$

$$\Big\downarrow \ 3070$$

$$-\int \left(1-\cos^2(x)\right)\sec(x)d\cos(x)$$

$$\Big\downarrow \ 244$$

$$-\int (\sec(x)-\cos(x))d\cos(x)$$

$$\Big\downarrow \ 2009$$

$$\frac{\cos^2(x)}{2}-\log(\cos(x))$$

input
```
Int[Sin[x]^2*Tan[x],x]
```

output
```
Cos[x]^2/2 - Log[Cos[x]]
```

3.800.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

3.800.4 Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{\sin(x)^2}{2}-\ln(\cos(x))$	13
risch	$ix+\frac{e^{2ix}}{8}+\frac{e^{-2ix}}{8}-\ln(e^{2ix}+1)$	30

input
```
int(sin(x)^2*tan(x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*sin(x)^2-ln(cos(x))
```

3.800.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \sin^2(x)\tan(x)\,dx = \frac{1}{2}\cos(x)^2 - \log(-\cos(x))$$

input
```
integrate(sin(x)^2*tan(x),x, algorithm="fricas")
```

output
```
1/2*cos(x)^2 - log(-cos(x))
```

3.800.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sin^2(x)\tan(x)\,dx = -\log\left(\cos\left(x\right)\right) + \frac{\cos^2\left(x\right)}{2}$$

input
```
integrate(sin(x)**2*tan(x),x)
```

output
```
-log(cos(x)) + cos(x)**2/2
```

3.800.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \sin^2(x)\tan(x)\,dx = -\frac{1}{2}\sin\left(x\right)^2 - \frac{1}{2}\log\left(\sin\left(x\right)^2 - 1\right)$$

input
```
integrate(sin(x)^2*tan(x),x, algorithm="maxima")
```

output
```
-1/2*sin(x)^2 - 1/2*log(sin(x)^2 - 1)
```

3.800.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \sin^2(x)\tan(x)\,dx = -\frac{1}{2}\sin\left(x\right)^2 - \frac{1}{2}\log\left(-\sin\left(x\right)^2 + 1\right)$$

input
```
integrate(sin(x)^2*tan(x),x, algorithm="giac")
```

output
```
-1/2*sin(x)^2 - 1/2*log(-sin(x)^2 + 1)
```

3.800.9 Mupad [B] (verification not implemented)

Time = 26.75 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \sin^2(x)\tan(x)\,dx = \frac{\cos(x)^2}{2} + \frac{\ln\left(\tan(x)^2+1\right)}{2}$$

input
```
int(sin(x)^2*tan(x),x)
```

output
```
log(tan(x)^2 + 1)/2 + cos(x)^2/2
```

3.801 $\int \cos^2(x)\cot^3(x)\,dx$

3.801.1 Optimal result

Integrand size = 9, antiderivative size = 22

$$\int \cos^2(x)\cot^3(x)\,dx = -\frac{1}{2}\csc^2(x) - 2\log(\sin(x)) + \frac{\sin^2(x)}{2}$$

output
```
-1/2*csc(x)^2-2*ln(sin(x))+1/2*sin(x)^2
```

3.801.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \cos^2(x)\cot^3(x)\,dx = \frac{1}{2}\left(-\csc^2(x) - 4\log(\sin(x)) + \sin^2(x)\right)$$

input
```
Integrate[Cos[x]^2*Cot[x]^3,x]
```

output
```
(-Csc[x]^2 - 4*Log[Sin[x]] + Sin[x]^2)/2
```

3.801.3 Rubi [A] (warning: unable to verify)

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 25, 3070, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(x)\cot^3(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\sin\left(x+\frac{\pi}{2}\right)^2\tan\left(x+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sin\left(x+\frac{\pi}{2}\right)^2\tan\left(x+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{3070}$$

$$\int \left(1-\sin^2(x)\right)^2\left(-\csc^3(x)\right)d(-\sin(x))$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\int \csc^2(x)(\sin(x)+1)^2d\sin^2(x)$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\int \left(\csc^2(x)+2\csc(x)+1\right)d\sin^2(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\sin^2(x)+\csc(x)-2\log\left(\sin^2(x)\right)\right)$$

input
```
Int[Cos[x]^2*Cot[x]^3,x]
```

output
```
(Csc[x] - 2*Log[Sin[x]^2] + Sin[x]^2)/2
```

3.801.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

3.801.4 Maple [A] (verified)

Time = 4.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.32

method	result	size
default	$-\frac{\cos(x)^6}{2\sin(x)^2}-\frac{\cos(x)^4}{2}-\cos(x)^2-2\ln(\sin(x))$	29
risch	$2ix-\frac{e^{2ix}}{8}-\frac{e^{-2ix}}{8}+\frac{2\,e^{2ix}}{(e^{2ix}-1)^2}-2\ln(e^{2ix}-1)$	46

input
```
int(cot(x)^3*cos(x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/sin(x)^2*cos(x)^6-1/2*cos(x)^4-cos(x)^2-2*ln(sin(x))
```

3.801.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 37 vs. 2(18) = 36.

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int \cos^2(x)\cot^3(x)\,dx = -\frac{2\cos(x)^4 - 3\cos(x)^2 + 8\left(\cos(x)^2 - 1\right)\log\left(\frac{1}{2}\sin(x)\right) - 1}{4\left(\cos(x)^2 - 1\right)}$$

input
```
integrate(cos(x)^2*cot(x)^3,x, algorithm="fricas")
```

output
```
-1/4*(2*cos(x)^4 - 3*cos(x)^2 + 8*(cos(x)^2 - 1)*log(1/2*sin(x)) - 1)/(cos
(x)^2 - 1)
```

3.801.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \cos^2(x)\cot^3(x)\,dx = -2\log(\sin(x)) + \frac{\sin^2(x)}{2} - \frac{1}{2\sin^2(x)}$$

input
```
integrate(cos(x)**2*cot(x)**3,x)
```

output
```
-2*log(sin(x)) + sin(x)**2/2 - 1/(2*sin(x)**2)
```

3.801.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \cos^2(x)\cot^3(x)\,dx = \frac{1}{2}\sin(x)^2 - \frac{1}{2\sin(x)^2} - \log\left(\sin(x)^2\right)$$

input
```
integrate(cos(x)^2*cot(x)^3,x, algorithm="maxima")
```

output
```
1/2*sin(x)^2 - 1/2/sin(x)^2 - log(sin(x)^2)
```

3.801.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27

$$\int \cos^2(x)\cot^3(x)\,dx = -\frac{1}{2}\cos(x)^2 + \frac{1}{2\left(\cos(x)^2 - 1\right)} - \log\left(-\cos(x)^2 + 1\right)$$

input
```
integrate(cos(x)^2*cot(x)^3,x, algorithm="giac")
```

output
```
-1/2*cos(x)^2 + 1/2/(cos(x)^2 - 1) - log(-cos(x)^2 + 1)
```

3.801.9 Mupad [B] (verification not implemented)

Time = 27.41 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.45

$$\int \cos^2(x)\cot^3(x)\,dx = \ln\left(\tan(x)^2 + 1\right) - 2\ln\left(\tan(x)\right) - \frac{\tan(x)^2 + \frac{1}{2}}{\tan(x)^4 + \tan(x)^2}$$

input
```
int(cos(x)^2*cot(x)^3,x)
```

output
```
log(tan(x)^2 + 1) - 2*log(tan(x)) - (tan(x)^2 + 1/2)/(tan(x)^2 + tan(x)^4)
```

3.802 $\int \sec(x)(1-\sin(x))\,dx$

3.802.1 Optimal result

Integrand size = 9, antiderivative size = 5

$$\int \sec(x)(1-\sin(x))\,dx = \log(1+\sin(x))$$

output
```
ln(1+sin(x))
```

3.802.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.40

$$\int \sec(x)(1-\sin(x))\,dx = \text{arctanh}(\sin(x)) + \log(\cos(x))$$

input
```
Integrate[Sec[x]*(1 - Sin[x]),x]
```

output
```
ArcTanh[Sin[x]] + Log[Cos[x]]
```

3.802.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (1-\sin(x))\sec(x)\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{1-\sin(x)}{\cos(x)}dx$$
$$\Big\downarrow \ 3146$$
$$-\int \frac{1}{\sin(x)+1}d(-\sin(x))$$
$$\Big\downarrow \ 16$$
$$\log(\sin(x)+1)$$

input
```
Int[Sec[x]*(1 - Sin[x]),x]
```

output
```
Log[1 + Sin[x]]
```

3.802.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

3.802.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

method	result	size
derivativedivides	$\ln(1+\sin(x))$	6
default	$\ln(1+\sin(x))$	6
risch	$-ix+2\ln(i+e^{ix})$	17
norman	$2\ln\left(\tan\left(\frac{x}{2}\right)+1\right)-\ln\left(1+\tan\left(\frac{x}{2}\right)^2\right)$	22
parallelrisch	$2\ln(\csc(x)-\cot(x)+1)-\ln\left(\frac{2}{\cos(x)+1}\right)$	24

input
```
int(sec(x)*(1-sin(x)),x,method=_RETURNVERBOSE)
```

output
```
ln(1+sin(x))
```

3.802.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \sec(x)(1-\sin(x))\,dx = \log(\sin(x)+1)$$

input
```
integrate(sec(x)*(1-sin(x)),x, algorithm="fricas")
```

output
```
log(sin(x) + 1)
```

3.802.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. $2(5) = 10$.

Time = 0.83 (sec) , antiderivative size = 12, normalized size of antiderivative = 2.40

$$\int \sec(x)(1-\sin(x))\,dx = \log\left(\tan\left(x\right)+\sec\left(x\right)\right)+\log\left(\cos\left(x\right)\right)$$

input
```
integrate(sec(x)*(1-sin(x)),x)
```

output
```
log(tan(x) + sec(x)) + log(cos(x))
```

3.802.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \sec(x)(1-\sin(x))\,dx = \log\left(\sin\left(x\right)+1\right)$$

input
```
integrate(sec(x)*(1-sin(x)),x, algorithm="maxima")
```

output
```
log(sin(x) + 1)
```

3.802.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \sec(x)(1-\sin(x))\,dx = \log\left(\sin\left(x\right)+1\right)$$

input
```
integrate(sec(x)*(1-sin(x)),x, algorithm="giac")
```

output
```
log(sin(x) + 1)
```

3.802.9 Mupad [B] (verification not implemented)

Time = 26.92 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \sec(x)(1-\sin(x))\,dx = \ln\left(\sin\left(x\right)+1\right)$$

input
```
int(-(sin(x) - 1)/cos(x),x)
```

output
```
log(sin(x) + 1)
```

3.803 $\int (1+\cos(x))\csc(x)\,dx$

3.803.1 Optimal result

Integrand size = 7, antiderivative size = 7

$$\int (1+\cos(x))\csc(x)\,dx=\log(1-\cos(x))$$

output
```
ln(1-cos(x))
```

3.803.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 23 vs. 2(7) = 14.

Time = 0.00 (sec) , antiderivative size = 23, normalized size of antiderivative = 3.29

$$\int (1+\cos(x))\csc(x)\,dx=-\log\left(\cos\left(\frac{x}{2}\right)\right)+\log(\cos(x))+\log\left(\sin\left(\frac{x}{2}\right)\right)+\log(\tan(x))$$

input
```
Integrate[(1 + Cos[x])*Csc[x],x]
```

output
```
-Log[Cos[x/2]] + Log[Cos[x]] + Log[Sin[x/2]] + Log[Tan[x]]
```

3.803.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\cos(x)+1)\csc(x)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1-\sin\left(x-\frac{\pi}{2}\right)}{\cos\left(x-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 3146$$

$$-\int \frac{1}{1-\cos(x)}d\cos(x)$$

$$\Big\downarrow \; 16$$

$$\log(1-\cos(x))$$

input
```
Int[(1 + Cos[x])*Csc[x],x]
```

output
```
Log[1 - Cos[x]]
```

3.803.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + 
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

3.803.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.86

method	result	size
default	$\ln(\sin(x)) + \ln(\csc(x) - \cot(x))$	13
parts	$-\ln(\csc(x)) - \ln(\cot(x) + \csc(x))$	15
risch	$-ix + 2\ln(e^{ix} - 1)$	16
norman	$2\ln\left(\tan\left(\frac{x}{2}\right)\right) - \ln\left(1 + \tan\left(\frac{x}{2}\right)^2\right)$	20
parallelrisch	$2\ln(\csc(x) - \cot(x)) - \ln\left(\frac{2}{\cos(x)+1}\right)$	23

input
```
int((cos(x)+1)*csc(x),x,method=_RETURNVERBOSE)
```

output
```
ln(sin(x))+ln(csc(x)-cot(x))
```

3.803.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int (1 + \cos(x)) \csc(x) \, dx = \log\left(-\frac{1}{2} \cos(x) + \frac{1}{2}\right)$$

input
```
integrate((1+cos(x))*csc(x),x, algorithm="fricas")
```

output
```
log(-1/2*cos(x) + 1/2)
```

3.803.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12 vs. $2(5) = 10$.

Time = 0.74 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.71

$$\int (1+\cos(x))\csc(x)\,dx = -\log(\cot(x)+\csc(x)) + \log(\sin(x))$$

input
```
integrate((1+cos(x))*csc(x),x)
```

output
```
-log(cot(x) + csc(x)) + log(sin(x))
```

3.803.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int (1+\cos(x))\csc(x)\,dx = \log(\cos(x)-1)$$

input
```
integrate((1+cos(x))*csc(x),x, algorithm="maxima")
```

output
```
log(cos(x) - 1)
```

3.803.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int (1+\cos(x))\csc(x)\,dx = \log(-\cos(x)+1)$$

input
```
integrate((1+cos(x))*csc(x),x, algorithm="giac")
```

output
```
log(-cos(x) + 1)
```

3.803.9 Mupad [B] (verification not implemented)

Time = 25.73 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int (1+\cos(x))\csc(x)\,dx = \ln(\cos(x)-1)$$

input
```
int((cos(x) + 1)/sin(x),x)
```

output
```
log(cos(x) - 1)
```

3.804 $\int \cos^2(x)\left(1-\tan^2(x)\right)\,dx$

3.804.1 Optimal result

Integrand size = 13, antiderivative size = 5

$$\int \cos^2(x)\left(1-\tan^2(x)\right)\,dx=\cos(x)\sin(x)$$

output
```
cos(x)*sin(x)
```

3.804.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.60

$$\int \cos^2(x)\left(1-\tan^2(x)\right)\,dx=\frac{1}{2}\sin(2x)$$

input
```
Integrate[Cos[x]^2*(1 - Tan[x]^2),x]
```

output
```
Sin[2*x]/2
```

3.804.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 11 vs. $2(5) = 10$.

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 2.20, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 4158, 297}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(x)\left(1-\tan^2(x)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1-\tan(x)^2}{\sec(x)^2}dx$$

$$\Big\downarrow \text{4158}$$

$$\int \frac{1-\tan^2(x)}{\left(\tan^2(x)+1\right)^2}d\tan(x)$$

$$\Big\downarrow \text{297}$$

$$\frac{\tan(x)}{\tan^2(x)+1}$$

input
```
Int[Cos[x]^2*(1 - Tan[x]^2),x]
```

output
```
Tan[x]/(1 + Tan[x]^2)
```

3.804.3.1 Defintions of rubi rules used

rule 297
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[c*
x*((a + b*x^2)^(p + 1)/a), x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d
, 0] && EqQ[a*d - b*c*(2*p + 3), 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

3.804.4 Maple [A] (verified)

Time = 3.15 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

method	result	size
default	$\cos(x)\sin(x)$	6
risch	$\frac{\sin(2x)}{2}$	7

input
```
int(cos(x)^2*(1-tan(x)^2),x,method=_RETURNVERBOSE)
```

output
```
cos(x)*sin(x)
```

3.804.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \cos^2(x)\left(1-\tan^2(x)\right)\,dx=\cos(x)\sin(x)$$

input
```
integrate(cos(x)^2*(1-tan(x)^2),x, algorithm="fricas")
```

output
```
cos(x)*sin(x)
```

3.804.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14 vs. $2(5) = 10$.

Time = 1.64 (sec) , antiderivative size = 14, normalized size of antiderivative = 2.80

$$\int \cos^2(x) \left(1 - \tan^2(x)\right) \, dx = \frac{\sin(x) \cos(x)}{2} + \frac{\sin(2x)}{4}$$

input
```
integrate(cos(x)**2*(1-tan(x)**2),x)
```

output
```
sin(x)*cos(x)/2 + sin(2*x)/4
```

3.804.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11 vs. $2(5) = 10$.

Time = 0.21 (sec) , antiderivative size = 11, normalized size of antiderivative = 2.20

$$\int \cos^2(x) \left(1 - \tan^2(x)\right) \, dx = \frac{\tan(x)}{\tan(x)^2 + 1}$$

input
```
integrate(cos(x)^2*(1-tan(x)^2),x, algorithm="maxima")
```

output
```
tan(x)/(tan(x)^2 + 1)
```

3.804.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.80

$$\int \cos^2(x) \left(1 - \tan^2(x)\right) \, dx = \frac{1}{\frac{1}{\tan(x)} + \tan(x)}$$

input
```
integrate(cos(x)^2*(1-tan(x)^2),x, algorithm="giac")
```

output
```
1/(1/tan(x) + tan(x))
```

3.804.9 Mupad [B] (verification not implemented)

Time = 25.92 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

$$\int \cos^2(x)\left(1-\tan^2(x)\right) dx = \frac{\sin(2x)}{2}$$

input
```
int(-cos(x)^2*(tan(x)^2 - 1),x)
```

output
```
sin(2*x)/2
```

3.805 $\int \csc(2x)(\cos(x)+\sin(x))\,dx$

3.805.1 Optimal result

Integrand size = 10, antiderivative size = 15

$$\int \csc(2x)(\cos(x)+\sin(x))\,dx = -\frac{1}{2}\operatorname{arctanh}(\cos(x))+\frac{1}{2}\operatorname{arctanh}(\sin(x))$$

output
```
-1/2*arctanh(cos(x))+1/2*arctanh(sin(x))
```

3.805.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.00

$$\int \csc(2x)(\cos(x)+\sin(x))\,dx = \frac{1}{2}\operatorname{arctanh}(\sin(x))-\frac{1}{2}\log\left(\cos\left(\frac{x}{2}\right)\right)+\frac{1}{2}\log\left(\sin\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[Csc[2*x]*(Cos[x] + Sin[x]),x]
```

output
```
ArcTanh[Sin[x]]/2 - Log[Cos[x/2]]/2 + Log[Sin[x/2]]/2
```

3.805.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 4901, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(2x)(\sin(x)+\cos(x))\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \csc(2x)(\sin(x)+\cos(x))dx$$
$$\Big\downarrow \text{4901}$$
$$\int (\cos(x)\csc(2x)+\sin(x)\csc(2x))dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{2}\text{arctanh}(\sin(x))-\frac{1}{2}\text{arctanh}(\cos(x))$$

input
```
Int[Csc[2*x]*(Cos[x] + Sin[x]),x]
```

output
```
-1/2*ArcTanh[Cos[x]] + ArcTanh[Sin[x]]/2
```

3.805.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4901
```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

3.805.4 Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

method	result	size
parts	$-\frac{\ln(\cot(x)+\csc(x))}{2}+\frac{\ln(\sec(x)+\tan(x))}{2}$	18
default	$\frac{\ln(\sec(x)+\tan(x))}{2}+\frac{\ln(\csc(x)-\cot(x))}{2}$	20
risch	$-\frac{\ln(e^{2ix}+(1-i)e^{ix}-i)}{2}+\frac{\ln(e^{2ix}+(-1+i)e^{ix}-i)}{2}$	42

input
```
int(csc(2*x)*(sin(x)+cos(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(cot(x)+csc(x))+1/2*ln(sec(x)+tan(x))
```

3.805.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(11) = 22.

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.33

$$\int \csc(2x)(\cos(x)+\sin(x))\,dx = -\frac{1}{4}\log\left(-\frac{1}{2}(\cos(x)+1)\sin(x)+\frac{1}{2}\cos(x)+\frac{1}{2}\right) + \frac{1}{4}\log\left(-\frac{1}{2}(\cos(x)-1)\sin(x)-\frac{1}{2}\cos(x)+\frac{1}{2}\right)$$

input
```
integrate(csc(2*x)*(cos(x)+sin(x)),x, algorithm="fricas")
```

output
```
-1/4*log(-1/2*(cos(x) + 1)*sin(x) + 1/2*cos(x) + 1/2) + 1/4*log(-1/2*(cos(
x) - 1)*sin(x) - 1/2*cos(x) + 1/2)
```

3.805.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(12) = 24.

Time = 0.73 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.13

$$\int \csc(2x)(\cos(x)+\sin(x))\,dx = -\frac{\log(\sin(x)-1)}{4}+\frac{\log(\sin(x)+1)}{4} + \frac{\log(\cos(x)-1)}{4}-\frac{\log(\cos(x)+1)}{4}$$

input
```
integrate(csc(2*x)*(cos(x)+sin(x)),x)
```

output
```
-log(sin(x) - 1)/4 + log(sin(x) + 1)/4 + log(cos(x) - 1)/4 - log(cos(x) +
1)/4
```

3.805.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(11) = 22$.

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 4.60

$$\begin{aligned}\int \csc(2x)(\cos(x)+\sin(x))\,dx = &-\frac{1}{4}\log\left(\cos(x)^2+\sin(x)^2+2\cos(x)+1\right)\\&+\frac{1}{4}\log\left(\cos(x)^2+\sin(x)^2-2\cos(x)+1\right)\\&+\frac{1}{4}\log\left(\cos(x)^2+\sin(x)^2+2\sin(x)+1\right)\\&-\frac{1}{4}\log\left(\cos(x)^2+\sin(x)^2-2\sin(x)+1\right)\end{aligned}$$

input
```
integrate(csc(2*x)*(cos(x)+sin(x)),x, algorithm="maxima")
```

output
```
-1/4*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/4*log(cos(x)^2 + sin(x)^2
 - 2*cos(x) + 1) + 1/4*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - 1/4*log(c
os(x)^2 + sin(x)^2 - 2*sin(x) + 1)
```

3.805.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(11) = 22$.

Time = 0.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.93

$$\begin{aligned}\int \csc(2x)(\cos(x)+\sin(x))\,dx = &\frac{1}{2}\log\left(\left|\tan\left(\frac{1}{2}x\right)+1\right|\right)\\&-\frac{1}{2}\log\left(\left|\tan\left(\frac{1}{2}x\right)-1\right|\right)+\frac{1}{2}\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)\end{aligned}$$

input
```
integrate(csc(2*x)*(cos(x)+sin(x)),x, algorithm="giac")
```

output
```
1/2*log(abs(tan(1/2*x) + 1)) - 1/2*log(abs(tan(1/2*x) - 1)) + 1/2*log(abs(
tan(1/2*x)))
```

3.805.9 Mupad [B] (verification not implemented)

Time = 27.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.60

$$\int \csc(2x)(\cos(x)+\sin(x))\,dx = \frac{\ln\left(\tan\left(\frac{x}{2}\right)^2+\tan\left(\frac{x}{2}\right)\right)}{2} - \frac{\ln\left(\tan\left(\frac{x}{2}\right)-1\right)}{2}$$

input
```
int((cos(x) + sin(x))/sin(2*x),x)
```

output
```
log(tan(x/2) + tan(x/2)^2)/2 - log(tan(x/2) - 1)/2
```

3.806 $\int \frac{\cos(x)(-3+2\sin(x))}{2-3\sin(x)+\sin^2(x)}\,dx$

3.806.1 Optimal result

Integrand size = 21, antiderivative size = 11

$$\int \frac{\cos(x)(-3+2\sin(x))}{2-3\sin(x)+\sin^2(x)}\,dx = \log\left(2-3\sin(x)+\sin^2(x)\right)$$

output
```
ln(2-3*sin(x)+sin(x)^2)
```

3.806.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \frac{\cos(x)(-3+2\sin(x))}{2-3\sin(x)+\sin^2(x)}\,dx = 2(\operatorname{arctanh}(3-2\sin(x))+\log(1-\sin(x)))$$

input
```
Integrate[(Cos[x]*(-3 + 2*Sin[x]))/(2 - 3*Sin[x] + Sin[x]^2),x]
```

output
```
2*(ArcTanh[3 - 2*Sin[x]] + Log[1 - Sin[x]])
```

3.806.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 4834, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2\sin(x)-3)\cos(x)}{\sin^2(x)-3\sin(x)+2}dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{(2\sin(x)-3)\cos(x)}{\sin(x)^2-3\sin(x)+2}dx$$
$$\Big\downarrow \; 4834$$
$$\int -\frac{3-2\sin(x)}{\sin^2(x)-3\sin(x)+2}d\sin(x)$$
$$\Big\downarrow \; 25$$
$$-\int \frac{3-2\sin(x)}{\sin^2(x)-3\sin(x)+2}d\sin(x)$$
$$\Big\downarrow \; 1103$$
$$\log\left(\sin^2(x)-3\sin(x)+2\right)$$

input
```
Int[(Cos[x]*(-3 + 2*Sin[x]))/(2 - 3*Sin[x] + Sin[x]^2),x]
```

output
```
Log[2 - 3*Sin[x] + Sin[x]^2]
```

3.806.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4834
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

3.806.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$\ln\left(2-3\sin\left(x\right)+\sin\left(x\right)^{2}\right)$	12
default	$\ln\left(2-3\sin\left(x\right)+\sin\left(x\right)^{2}\right)$	12
risch	$-2ix+2\ln\left(\mathrm{e}^{ix}-i\right)+\ln\left(-4i\mathrm{e}^{ix}+\mathrm{e}^{2ix}-1\right)$	33
norman	$2\ln\left(\tan\left(\frac{x}{2}\right)-1\right)-2\ln\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)+\ln\left(\tan\left(\frac{x}{2}\right)^{2}-\tan\left(\frac{x}{2}\right)+1\right)$	37
parallelrisch	$2\ln\left(-\cot\left(x\right)+\csc\left(x\right)-1\right)-2\ln\left(\frac{1}{\cos(x)+1}\right)+\ln\left(\frac{2-\sin(x)}{4\cos(x)+4}\right)$	38

input
```
int(cos(x)*(-3+2*sin(x))/(2-3*sin(x)+sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
ln(2-3*sin(x)+sin(x)^2)
```

3.806.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \frac{\cos(x)(-3+2\sin(x))}{2-3\sin(x)+\sin^2(x)}\,dx = \log\left(-\frac{1}{2}\sin\left(x\right)+1\right)+\log\left(-\sin\left(x\right)+1\right)$$

input
```
integrate(cos(x)*(-3+2*sin(x))/(2-3*sin(x)+sin(x)^2),x, algorithm="fricas"
)
```

output
```
log(-1/2*sin(x) + 1) + log(-sin(x) + 1)
```

3.806.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{\cos(x)(-3+2\sin(x))}{2-3\sin(x)+\sin^2(x)}\,dx = \log\left(\sin\left(x\right)-2\right)+\log\left(\sin\left(x\right)-1\right)$$

input
```
integrate(cos(x)*(-3+2*sin(x))/(2-3*sin(x)+sin(x)**2),x)
```

output
```
log(sin(x) - 2) + log(sin(x) - 1)
```

3.806.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)(-3+2\sin(x))}{2-3\sin(x)+\sin^2(x)}\,dx = \log\left(\sin\left(x\right)^2-3\,\sin\left(x\right)+2\right)$$

input
```
integrate(cos(x)*(-3+2*sin(x))/(2-3*sin(x)+sin(x)^2),x, algorithm="maxima"
)
```

output
```
log(sin(x)^2 - 3*sin(x) + 2)
```

3.806.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \frac{\cos(x)(-3+2\sin(x))}{2-3\sin(x)+\sin^2(x)}\,dx = \log\left(-\sin\left(x\right)+2\right)+\log\left(-\sin\left(x\right)+1\right)$$

input
```
integrate(cos(x)*(-3+2*sin(x))/(2-3*sin(x)+sin(x)^2),x, algorithm="giac")
```

output
```
log(-sin(x) + 2) + log(-sin(x) + 1)
```

3.806.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)(-3+2\sin(x))}{2-3\sin(x)+\sin^2(x)}\,dx = \ln\left(\sin(x)^2-3\sin(x)+2\right)$$

input
```
int((cos(x)*(2*sin(x) - 3))/(sin(x)^2 - 3*sin(x) + 2),x)
```

output
```
log(sin(x)^2 - 3*sin(x) + 2)
```

3.807 $\int \frac{\cos^2(x)\sin(x)}{5+\cos^2(x)}\,dx$

3.807.1 Optimal result

Integrand size = 15, antiderivative size = 20

$$\int \frac{\cos^2(x)\sin(x)}{5+\cos^2(x)}\,dx=\sqrt{5}\arctan\left(\frac{\cos(x)}{\sqrt{5}}\right)-\cos(x)$$

output
```
-cos(x)+arctan(1/5*cos(x)*5^(1/2))*5^(1/2)
```

3.807.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 82 vs. 2(20) = 40.

Time = 0.17 (sec) , antiderivative size = 82, normalized size of antiderivative = 4.10

$$\int \frac{\cos^2(x)\sin(x)}{5+\cos^2(x)}\,dx=\frac{1}{20}\left(-\sqrt{5}\arctan\left(\frac{\cos(x)}{\sqrt{5}}\right)+21\sqrt{5}\arctan\left(\frac{1}{\sqrt{5}}-\sqrt{\frac{6}{5}}\tan\left(\frac{x}{2}\right)\right)+21\sqrt{5}\arctan\left(\frac{1}{\sqrt{5}}+\sqrt{\frac{6}{5}}\tan\left(\frac{x}{2}\right)\right)-20\cos(x)\right)$$

input
```
Integrate[(Cos[x]^2*Sin[x])/(5 + Cos[x]^2),x]
```

output
```
(-(Sqrt[5]*ArcTan[Cos[x]/Sqrt[5]]) + 21*Sqrt[5]*ArcTan[1/Sqrt[5] - Sqrt[6/
5]*Tan[x/2]] + 21*Sqrt[5]*ArcTan[1/Sqrt[5] + Sqrt[6/5]*Tan[x/2]] - 20*Cos[
x])/20
```

3.807.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3042, 4835, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)\cos^2(x)}{\cos^2(x)+5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)\cos(x)^2}{\cos(x)^2+5}dx$$

$$\Big\downarrow \text{4835}$$

$$-\int \frac{\cos^2(x)}{\cos^2(x)+5}d\cos(x)$$

$$\Big\downarrow \text{262}$$

$$5\int \frac{1}{\cos^2(x)+5}d\cos(x)-\cos(x)$$

$$\Big\downarrow \text{216}$$

$$\sqrt{5}\arctan\left(\frac{\cos(x)}{\sqrt{5}}\right)-\cos(x)$$

input
```
Int[(Cos[x]^2*Sin[x])/(5 + Cos[x]^2),x]
```

output
```
Sqrt[5]*ArcTan[Cos[x]/Sqrt[5]] - Cos[x]
```

3.807.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4835
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[1, Cos[c*(a +
b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

3.807.4 Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$-\cos\left(x\right)+\arctan\left(\frac{\cos\left(x\right)\sqrt{5}}{5}\right)\sqrt{5}$	18
default	$-\cos\left(x\right)+\arctan\left(\frac{\cos\left(x\right)\sqrt{5}}{5}\right)\sqrt{5}$	18
risch	$-\frac{e^{ix}}{2}-\frac{e^{-ix}}{2}-\frac{i\sqrt{5}\,\ln\left(e^{2ix}-2i\sqrt{5}\,e^{ix}+1\right)}{2}+\frac{i\sqrt{5}\,\ln\left(e^{2ix}+2i\sqrt{5}\,e^{ix}+1\right)}{2}$	66

input
```
int(cos(x)^2*sin(x)/(5+cos(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-cos(x)+arctan(1/5*cos(x)*5^(1/2))*5^(1/2)
```

3.807.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{\cos^2(x)\sin(x)}{5+\cos^2(x)}\,dx = \sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}\cos(x)\right) - \cos(x)$$

input
```
integrate(cos(x)^2*sin(x)/(5+cos(x)^2),x, algorithm="fricas")
```

output
```
sqrt(5)*arctan(1/5*sqrt(5)*cos(x)) - cos(x)
```

3.807.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{\cos^2(x)\sin(x)}{5+\cos^2(x)}\,dx = -\cos(x) + \sqrt{5}\,\mathrm{atan}\left(\frac{\sqrt{5}\cos(x)}{5}\right)$$

input
```
integrate(cos(x)**2*sin(x)/(5+cos(x)**2),x)
```

output
```
-cos(x) + sqrt(5)*atan(sqrt(5)*cos(x)/5)
```

3.807.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{\cos^2(x)\sin(x)}{5+\cos^2(x)}\,dx = \sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}\cos(x)\right) - \cos(x)$$

input
```
integrate(cos(x)^2*sin(x)/(5+cos(x)^2),x, algorithm="maxima")
```

output
```
sqrt(5)*arctan(1/5*sqrt(5)*cos(x)) - cos(x)
```

3.807.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{\cos^2(x)\sin(x)}{5+\cos^2(x)}\,dx = \sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}\cos(x)\right) - \cos(x)$$

input
```
integrate(cos(x)^2*sin(x)/(5+cos(x)^2),x, algorithm="giac")
```

output
```
sqrt(5)*arctan(1/5*sqrt(5)*cos(x)) - cos(x)
```

3.807.9 Mupad [B] (verification not implemented)

Time = 26.86 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{\cos^2(x)\sin(x)}{5+\cos^2(x)}\,dx = \sqrt{5}\,\mathrm{atan}\left(\frac{\sqrt{5}\,\cos(x)}{5}\right) - \cos(x)$$

input
```
int((cos(x)^2*sin(x))/(cos(x)^2 + 5),x)
```

output
```
5^(1/2)*atan((5^(1/2)*cos(x))/5) - cos(x)
```

3.808 $\int \frac{\cos(x)}{\sin(x)+\sin^2(x)} dx$

3.808.1 Optimal result

Integrand size = 12, antiderivative size = 11

$$\int \frac{\cos(x)}{\sin(x)+\sin^2(x)} dx = \log(\sin(x)) - \log(1+\sin(x))$$

output
```
ln(sin(x))-ln(1+sin(x))
```

3.808.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{\sin(x)+\sin^2(x)} dx = \log(\sin(x)) - \log(1+\sin(x))$$

input
```
Integrate[Cos[x]/(Sin[x] + Sin[x]^2),x]
```

output
```
Log[Sin[x]] - Log[1 + Sin[x]]
```

3.808.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3739, 1080, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(x)}{\sin^2(x)+\sin(x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos(x)}{\sin(x)^2+\sin(x)}dx$$
$$\Big\downarrow \text{3739}$$
$$\int \frac{1}{\sin^2(x)+\sin(x)}d\sin(x)$$
$$\Big\downarrow \text{1080}$$
$$\int \left(\frac{1}{-\sin(x)-1}+\csc(x)\right)d\sin(x)$$
$$\Big\downarrow \text{2009}$$
$$\log(\sin(x))-\log(\sin(x)+1)$$

input
```
Int[Cos[x]/(Sin[x] + Sin[x]^2),x]
```

output
```
Log[Sin[x]] - Log[1 + Sin[x]]
```

3.808.3.1 Defintions of rubi rules used

rule 1080
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[x^p*
(b + c*x)^p, x], x] /; FreeQ[{b, c}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3739
```
Int[cos[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*sin[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*sin[(d_.) + (e_.)*(x_)])^(n2_.))^(p_.), x_Symbol
] :> Module[{g = FreeFactors[Sin[d + e*x], x]}, Simp[g/e   Subst[Int[(1 - g
^2*x^2)^((m - 1)/2)*(a + b*(f*g*x)^n + c*(f*g*x)^(2*n))^p, x], x, Sin[d + e
*x]/g], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[n2, 2*n] && Intege
rQ[(m - 1)/2]
```

3.808.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$\ln(\sin(x)) - \ln(1 + \sin(x))$	12
default	$\ln(\sin(x)) - \ln(1 + \sin(x))$	12
norman	$-2\ln\left(\tan\left(\frac{x}{2}\right) + 1\right) + \ln\left(\tan\left(\frac{x}{2}\right)\right)$	16
parallelrisch	$-2\ln\left(\tan\left(\frac{x}{2}\right) + 1\right) + \ln\left(\tan\left(\frac{x}{2}\right)\right)$	16
risch	$-2\ln(i + e^{ix}) + \ln(e^{2ix} - 1)$	21

input
```
int(cos(x)/(sin(x)+sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
ln(sin(x))-ln(1+sin(x))
```

3.808.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.18

$$\int \frac{\cos(x)}{\sin(x) + \sin^2(x)}\, dx = \log\left(\frac{1}{2}\sin(x)\right) - \log(\sin(x) + 1)$$

input
```
integrate(cos(x)/(sin(x)+sin(x)^2),x, algorithm="fricas")
```

output
```
log(1/2*sin(x)) - log(sin(x) + 1)
```

3.808.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

$$\int \frac{\cos(x)}{\sin(x)+\sin^2(x)}\,dx = -\log\left(\sin\left(x\right)+1\right)+\log\left(\sin\left(x\right)\right)$$

input
```
integrate(cos(x)/(sin(x)+sin(x)**2),x)
```

output
```
-log(sin(x) + 1) + log(sin(x))
```

3.808.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{\sin(x)+\sin^2(x)}\,dx = -\log\left(\sin\left(x\right)+1\right)+\log\left(\sin\left(x\right)\right)$$

input
```
integrate(cos(x)/(sin(x)+sin(x)^2),x, algorithm="maxima")
```

output
```
-log(sin(x) + 1) + log(sin(x))
```

3.808.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \frac{\cos(x)}{\sin(x)+\sin^2(x)}\,dx = -\log\left(\sin\left(x\right)+1\right)+\log\left(|\sin\left(x\right)|\right)$$

input
```
integrate(cos(x)/(sin(x)+sin(x)^2),x, algorithm="giac")
```

output
```
-log(sin(x) + 1) + log(abs(sin(x)))
```

3.808.9 Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{\cos(x)}{\sin(x) + \sin^2(x)}\, dx = -2\,\mathrm{atanh}(2\,\sin(x) + 1)$$

input
```
int(cos(x)/(sin(x) + sin(x)^2),x)
```

output
```
-2*atanh(2*sin(x) + 1)
```

3.809 $\int \frac{\cos(x)}{\sin(x)+\sin^{\sqrt{2}}(x)} dx$

3.809.1 Optimal result

Integrand size = 16, antiderivative size = 26

$$\int \frac{\cos(x)}{\sin(x)+\sin^{\sqrt{2}}(x)} dx = \log(\sin(x)) - \left(1+\sqrt{2}\right)\log\left(1+\sin^{-1+\sqrt{2}}(x)\right)$$

output
```
ln(sin(x))-ln(1+sin(x)^(2^(1/2)-1))*(1+2^(1/2))
```

3.809.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{\sin(x)+\sin^{\sqrt{2}}(x)} dx = \log(\sin(x)) - \left(1+\sqrt{2}\right)\log\left(1+\sin^{-1+\sqrt{2}}(x)\right)$$

input
```
Integrate[Cos[x]/(Sin[x] + Sin[x]^Sqrt[2]),x]
```

output
```
Log[Sin[x]] - (1 + Sqrt[2])*Log[1 + Sin[x]^(-1 + Sqrt[2])]
```

3.809.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.35, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3042, 4834, 798, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(x)}{\sin^{\sqrt{2}}(x)+\sin(x)}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\cos(x)}{\sin(x)^{\sqrt{2}}+\sin(x)}dx$$
$$\Big\downarrow \text{ 4834}$$
$$\int \frac{\csc(x)}{\sin^{\sqrt{2}-1}(x)+1}d\sin(x)$$
$$\Big\downarrow \text{ 798}$$
$$\left(1+\sqrt{2}\right)\int \frac{\csc(x)}{\sin^{-1+\sqrt{2}}(x)+1}d\sin^{-1+\sqrt{2}}(x)$$
$$\Big\downarrow \text{ 47}$$
$$\left(1+\sqrt{2}\right)\left(\int \csc(x)d\sin^{-1+\sqrt{2}}(x)-\int \frac{1}{\sin^{-1+\sqrt{2}}(x)+1}d\sin^{-1+\sqrt{2}}(x)\right)$$
$$\Big\downarrow \text{ 14}$$
$$\left(1+\sqrt{2}\right)\left(\log\left(\sin^{\sqrt{2}-1}(x)\right)-\int \frac{1}{\sin^{-1+\sqrt{2}}(x)+1}d\sin^{-1+\sqrt{2}}(x)\right)$$
$$\Big\downarrow \text{ 16}$$
$$\left(1+\sqrt{2}\right)\left(\log\left(\sin^{\sqrt{2}-1}(x)\right)-\log\left(\sin^{\sqrt{2}-1}(x)+1\right)\right)$$

input
```
Int[Cos[x]/(Sin[x] + Sin[x]^Sqrt[2]),x]
```

output
```
(1 + Sqrt[2])*(Log[Sin[x]^(-1 + Sqrt[2])] - Log[1 + Sin[x]^(-1 + Sqrt[2])]
)
```

3.809.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4834
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

3.809.4 Maple [A] (verified)

Time = 1.43 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19

method	result
derivativedivides	$\left(2+\sqrt{2}\right) \ln \left(\sin \left(x\right)\right)+\left(-1-\sqrt{2}\right) \ln \left(\sin \left(x\right)+{\mathrm e}^{\sqrt{2}\, \ln \left(\sin \left(x\right)\right)}\right)$
default	$\left(2+\sqrt{2}\right) \ln \left(\sin \left(x\right)\right)+\left(-1-\sqrt{2}\right) \ln \left(\sin \left(x\right)+{\mathrm e}^{\sqrt{2}\, \ln \left(\sin \left(x\right)\right)}\right)$
parallelrisch	$\sqrt{2}\, \left(-\ln \left(\frac{\sin \left(x\right)+\sin \left(x\right)^{\sqrt{2}}}{\cos \left(x\right)+1}\right)+\ln \left(\frac{\csc \left(x\right)}{2}-\frac{\cot \left(x\right)}{2}\right)\right)-\ln \left(\frac{4 \sin \left(x\right)+4 \sin \left(x\right)^{\sqrt{2}}}{\cos \left(x\right)+1}\right)+2 \ln \left(\csc \left(x\right)\right.$
risch	Expression too large to display

input
```
int(cos(x)/(sin(x)+sin(x)^(2^(1/2))),x,method=_RETURNVERBOSE)
```

output
```
(2+2^(1/2))*ln(sin(x))+(-1-2^(1/2))*ln(sin(x)+exp(2^(1/2)*ln(sin(x))))
```

3.809.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

$$\int \frac{\cos(x)}{\sin(x) + \sin^{\sqrt{2}}(x)}\,dx = -\left(\sqrt{2}+1\right)\log\left(\sin\left(x\right)^{\left(\sqrt{2}\right)} + \sin\left(x\right)\right) + \left(\sqrt{2}+2\right)\log\left(\sin\left(x\right)\right)$$

input
```
integrate(cos(x)/(sin(x)+sin(x)^(2^(1/2))),x, algorithm="fricas")
```

output
```
-(sqrt(2) + 1)*log(sin(x)^sqrt(2) + sin(x)) + (sqrt(2) + 2)*log(sin(x))
```

3.809.6 Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.38

$$\int \frac{\cos(x)}{\sin(x) + \sin^{\sqrt{2}}(x)}\,dx = -\frac{\log\left(\sin\left(x\right) + \sin^{\sqrt{2}}\left(x\right)\right)}{-1+\sqrt{2}} + \frac{\sqrt{2}\log\left(\sin\left(x\right)\right)}{-1+\sqrt{2}}$$

input
```
integrate(cos(x)/(sin(x)+sin(x)**(2**(1/2))),x)
```

output
```
-log(sin(x) + sin(x)**(sqrt(2)))/(-1 + sqrt(2)) + sqrt(2)*log(sin(x))/(-1
+ sqrt(2))
```

3.809.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.31

$$\int \frac{\cos(x)}{\sin(x) + \sin^{\sqrt{2}}(x)}\,dx = \frac{\sqrt{2}\log\left(\sin\left(x\right)\right)}{\sqrt{2}-1} - \frac{\log\left(\sin\left(x\right)^{\left(\sqrt{2}\right)} + \sin\left(x\right)\right)}{\sqrt{2}-1}$$

input
```
integrate(cos(x)/(sin(x)+sin(x)^(2^(1/2))),x, algorithm="maxima")
```

output
```
sqrt(2)*log(sin(x))/(sqrt(2) - 1) - log(sin(x)^sqrt(2) + sin(x))/(sqrt(2)
- 1)
```

3.809.8 Giac [F]

$$\int \frac{\cos(x)}{\sin(x) + \sin^{\sqrt{2}}(x)}\, dx = \int \frac{\cos(x)}{\sin(x)^{\left(\sqrt{2}\right)} + \sin(x)}\, dx$$

input
```
integrate(cos(x)/(sin(x)+sin(x)^(2^(1/2))),x, algorithm="giac")
```

output
```
integrate(cos(x)/(sin(x)^sqrt(2) + sin(x)), x)
```

3.809.9 Mupad [B] (verification not implemented)

Time = 27.36 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.12

$$\int \frac{\cos(x)}{\sin(x) + \sin^{\sqrt{2}}(x)}\, dx = \ln(\sin(x)) \left(\sqrt{2} + 2\right) - \frac{\ln\left(\sin(x) + \sin(x)^{\sqrt{2}}\right)}{\sqrt{2} - 1}$$

input
```
int(cos(x)/(sin(x) + sin(x)^(2^(1/2))),x)
```

output
```
log(sin(x))*(2^(1/2) + 2) - log(sin(x) + sin(x)^(2^(1/2)))/(2^(1/2) - 1)
```

3.810 $\int \frac{1}{2\sin(x)+\sin(2x)}\,dx$

3.810.1 Optimal result

Integrand size = 11, antiderivative size = 24

$$\int \frac{1}{2\sin(x)+\sin(2x)}\,dx = \frac{1}{4}\log\left(\tan\left(\frac{x}{2}\right)\right)+\frac{1}{8}\tan^2\left(\frac{x}{2}\right)$$

output
```
1/4*ln(tan(1/2*x))+1/8*tan(1/2*x)^2
```

3.810.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.62

$$\int \frac{1}{2\sin(x)+\sin(2x)}\,dx = \frac{1-2\cos^2\left(\frac{x}{2}\right)\left(\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)\right)\right)}{4(1+\cos(x))}$$

input
```
Integrate[(2*Sin[x] + Sin[2*x])^(-1),x]
```

output
```
(1 - 2*Cos[x/2]^2*(Log[Cos[x/2]] - Log[Sin[x/2]]))/(4*(1 + Cos[x]))
```

3.810.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {3042, 4826, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{2\sin(x)+\sin(2x)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{2\sin(x)+\sin(2x)}dx$$
$$\Big\downarrow \text{4826}$$
$$2\int \frac{1}{8}\cot\left(\frac{x}{2}\right)\left(\tan^2\left(\frac{x}{2}\right)+1\right)d\tan\left(\frac{x}{2}\right)$$
$$\Big\downarrow \text{27}$$
$$\frac{1}{4}\int \cot\left(\frac{x}{2}\right)\left(\tan^2\left(\frac{x}{2}\right)+1\right)d\tan\left(\frac{x}{2}\right)$$
$$\Big\downarrow \text{244}$$
$$\frac{1}{4}\int \left(\cot\left(\frac{x}{2}\right)+\tan\left(\frac{x}{2}\right)\right)d\tan\left(\frac{x}{2}\right)$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{4}\left(\frac{1}{2}\tan^2\left(\frac{x}{2}\right)+\log\left(\tan\left(\frac{x}{2}\right)\right)\right)$$

input
```
Int[(2*Sin[x] + Sin[2*x])^(-1),x]
```

output
```
(Log[Tan[x/2]] + Tan[x/2]^2/2)/4
```

3.810.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4826
```
Int[((a_.)*sin[(m_.)*((c_.) + (d_.)*(x_))] + (b_.)*sin[(n_.)*((c_.) + (d_.)
*(x_))])^(p_), x_Symbol] :> Simp[2/d   Subst[Int[Simplify[TrigExpand[a*Sin[
2*m*ArcTan[x]] + b*Sin[2*n*ArcTan[x]]]]^p/(1 + x^2), x], x, Tan[(1/2)*(c +
d*x)]], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m/2] && Inte
gerQ[(n - 1)/2]
```

3.810.4 Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{\ln(\cos(x)-1)}{8}+\frac{1}{4\cos(x)+4}-\frac{\ln(\cos(x)+1)}{8}$	24
risch	$\frac{e^{ix}}{2(e^{ix}+1)^2}-\frac{\ln(e^{ix}+1)}{4}+\frac{\ln(e^{ix}-1)}{4}$	38

input
```
int(1/(2*sin(x)+sin(2*x)),x,method=_RETURNVERBOSE)
```

output
```
1/8*ln(cos(x)-1)+1/4/(cos(x)+1)-1/8*ln(cos(x)+1)
```

3.810.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(16) = 32.

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \frac{1}{2\sin(x)+\sin(2x)}\,dx = -\frac{(\cos(x)+1)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)-(\cos(x)+1)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)-2}{8(\cos(x)+1)}$$

input
```
integrate(1/(2*sin(x)+sin(2*x)),x, algorithm="fricas")
```

output
```
-1/8*((cos(x) + 1)*log(1/2*cos(x) + 1/2) - (cos(x) + 1)*log(-1/2*cos(x) +
1/2) - 2)/(cos(x) + 1)
```

3.810.6 Sympy [F]

$$\int \frac{1}{2\sin(x)+\sin(2x)}\,dx = \int \frac{1}{2\sin(x)+\sin(2x)}\,dx$$

input
```
integrate(1/(2*sin(x)+sin(2*x)),x)
```

output
```
Integral(1/(2*sin(x) + sin(2*x)), x)
```

3.810.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 220 vs. 2(16) = 32.

Time = 0.21 (sec) , antiderivative size = 220, normalized size of antiderivative = 9.17

$$\int \frac{1}{2\sin(x)+\sin(2x)}\,dx = \frac{4\cos(2x)\cos(x)+8\cos(x)^2-(2(2\cos(x)+1)\cos(2x)+\cos(2x)^2+4\cos(x)^2+\sin(2x)^2+4\sin(}{}$$

input
```
integrate(1/(2*sin(x)+sin(2*x)),x, algorithm="maxima")
```

output
```
1/8*(4*cos(2*x)*cos(x) + 8*cos(x)^2 - (2*(2*cos(x) + 1)*cos(2*x) + cos(2*x
)^2 + 4*cos(x)^2 + sin(2*x)^2 + 4*sin(2*x)*sin(x) + 4*sin(x)^2 + 4*cos(x)
+ 1)*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + (2*(2*cos(x) + 1)*cos(2*x)
+ cos(2*x)^2 + 4*cos(x)^2 + sin(2*x)^2 + 4*sin(2*x)*sin(x) + 4*sin(x)^2 +
4*cos(x) + 1)*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) + 4*sin(2*x)*sin(x)
+ 8*sin(x)^2 + 4*cos(x))/(2*(2*cos(x) + 1)*cos(2*x) + cos(2*x)^2 + 4*cos(x
)^2 + sin(2*x)^2 + 4*sin(2*x)*sin(x) + 4*sin(x)^2 + 4*cos(x) + 1)
```

3.810.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17

$$\int \frac{1}{2\sin(x)+\sin(2x)}\,dx = -\frac{\cos(x)-1}{8(\cos(x)+1)} + \frac{1}{8}\log\left(-\frac{\cos(x)-1}{\cos(x)+1}\right)$$

input
```
integrate(1/(2*sin(x)+sin(2*x)),x, algorithm="giac")
```

output
```
-1/8*(cos(x) - 1)/(cos(x) + 1) + 1/8*log(-(cos(x) - 1)/(cos(x) + 1))
```

3.810.9 Mupad [B] (verification not implemented)

Time = 27.49 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \frac{1}{2\sin(x)+\sin(2x)}\,dx = \frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{4} + \frac{\tan\left(\frac{x}{2}\right)^2}{8}$$

input
```
int(1/(sin(2*x) + 2*sin(x)),x)
```

output
```
log(tan(x/2))/4 + tan(x/2)^2/8
```

3.811 $\int (-3+4x+x^2)\sin(2x)\,dx$

3.811.1 Optimal result

Integrand size = 13, antiderivative size = 40

$$\int (-3+4x+x^2)\sin(2x)\,dx=\frac{7}{4}\cos(2x)-2x\cos(2x)-\frac{1}{2}x^2\cos(2x)+\sin(2x)+\frac{1}{2}x\sin(2x)$$

output
```
7/4*cos(2*x)-2*x*cos(2*x)-1/2*x^2*cos(2*x)+sin(2*x)+1/2*x*sin(2*x)
```

3.811.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.72

$$\int (-3+4x+x^2)\sin(2x)\,dx=\frac{1}{4}\left((7-8x-2x^2)\cos(2x)+2(2+x)\sin(2x)\right)$$

input
```
Integrate[(-3 + 4*x + x^2)*Sin[2*x],x]
```

output
```
((7 - 8*x - 2*x^2)*Cos[2*x] + 2*(2 + x)*Sin[2*x])/4
```

3.811.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x^2+4x-3)\sin(2x)\,dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(x^2\sin(2x)+4x\sin(2x)-3\sin(2x)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2}x^2\cos(2x)+\frac{1}{2}x\sin(2x)+\sin(2x)-2x\cos(2x)+\frac{7}{4}\cos(2x)$$

input
```
Int[(-3 + 4*x + x^2)*Sin[2*x],x]
```

output
```
(7*Cos[2*x])/4 - 2*x*Cos[2*x] - (x^2*Cos[2*x])/2 + Sin[2*x] + (x*Sin[2*x])
/2
```

3.811.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.811.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.65

method	result	size
risch	$\left(-\frac{1}{2}x^{2}-2x+\frac{7}{4}\right)\cos\left(2x\right)+\frac{(2+x)\sin(2x)}{2}$	26
derivativedivides	$\frac{7\cos(2x)}{4}-2x\cos\left(2x\right)-\frac{x^{2}\cos(2x)}{2}+\sin\left(2x\right)+\frac{x\sin(2x)}{2}$	35
default	$\frac{7\cos(2x)}{4}-2x\cos\left(2x\right)-\frac{x^{2}\cos(2x)}{2}+\sin\left(2x\right)+\frac{x\sin(2x)}{2}$	35
parts	$\frac{7\cos(2x)}{4}-2x\cos\left(2x\right)-\frac{x^{2}\cos(2x)}{2}+\sin\left(2x\right)+\frac{x\sin(2x)}{2}$	35
parallelrisch	$\frac{(x^{2}+4x)\tan(x)^{2}+(2x+4)\tan(x)-x^{2}-4x+7}{2+2\tan(x)^{2}}$	42
norman	$\frac{x\tan(x)-2x-\frac{x^{2}}{2}+2x\tan(x)^{2}+\frac{x^{2}\tan(x)^{2}}{2}+2\tan(x)+\frac{7}{2}}{1+\tan(x)^{2}}$	44
meijerg	$\frac{\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\left(-2x^{2}+1\right)\cos(2x)}{2\sqrt{\pi}}+\frac{x\sin(2x)}{\sqrt{\pi}}\right)}{2}+2\sqrt{\pi}\left(-\frac{x\cos(2x)}{\sqrt{\pi}}+\frac{\sin(2x)}{2\sqrt{\pi}}\right)-\frac{3\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(2x)}{\sqrt{\pi}}\right)}{2}$	81

input
```
int((x^2+4*x-3)*sin(2*x),x,method=_RETURNVERBOSE)
```

output
```
(-1/2*x^2-2*x+7/4)*cos(2*x)+1/2*(2+x)*sin(2*x)
```

3.811.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.65

$$\int\left(-3+4x+x^{2}\right)\sin(2x)\,dx=-\frac{1}{4}\left(2\,x^{2}+8\,x-7\right)\cos\left(2\,x\right)+\frac{1}{2}\left(x+2\right)\sin\left(2\,x\right)$$

input
```
integrate((x^2+4*x-3)*sin(2*x),x, algorithm="fricas")
```

output
```
-1/4*(2*x^2 + 8*x - 7)*cos(2*x) + 1/2*(x + 2)*sin(2*x)
```

3.811.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int (-3+4x+x^2)\sin(2x)\,dx = -\frac{x^2\cos(2x)}{2}+\frac{x\sin(2x)}{2}-2x\cos(2x)+\sin(2x)+\frac{7\cos(2x)}{4}$$

input
```
integrate((x**2+4*x-3)*sin(2*x),x)
```

output
```
-x**2*cos(2*x)/2 + x*sin(2*x)/2 - 2*x*cos(2*x) + sin(2*x) + 7*cos(2*x)/4
```

3.811.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int (-3+4x+x^2)\sin(2x)\,dx = -\frac{1}{4}\left(2x^2-1\right)\cos(2x)-2x\cos(2x)+\frac{1}{2}x\sin(2x)+\frac{3}{2}\cos(2x)+\sin(2x)$$

input
```
integrate((x^2+4*x-3)*sin(2*x),x, algorithm="maxima")
```

output
```
-1/4*(2*x^2 - 1)*cos(2*x) - 2*x*cos(2*x) + 1/2*x*sin(2*x) + 3/2*cos(2*x) +
 sin(2*x)
```

3.811.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.65

$$\int (-3+4x+x^2)\sin(2x)\,dx = -\frac{1}{4}\left(2x^2+8x-7\right)\cos(2x)+\frac{1}{2}(x+2)\sin(2x)$$

input
```
integrate((x^2+4*x-3)*sin(2*x),x, algorithm="giac")
```

output
```
-1/4*(2*x^2 + 8*x - 7)*cos(2*x) + 1/2*(x + 2)*sin(2*x)
```

3.811.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \left(-3+4x+x^2\right) \sin(2x)\, dx = \frac{7 \cos(2x)}{4} + \sin(2x) - 2x \cos(2x) + \frac{x \sin(2x)}{2} - \frac{x^2 \cos(2x)}{2}$$

input
```
int(sin(2*x)*(4*x + x^2 - 3),x)
```

output
```
(7*cos(2*x))/4 + sin(2*x) - 2*x*cos(2*x) + (x*sin(2*x))/2 - (x^2*cos(2*x))
/2
```

3.812 $\int e^{-3x}\cos(4x)\,dx$

3.812.1 Optimal result

Integrand size = 10, antiderivative size = 27

$$\int e^{-3x}\cos(4x)\,dx = -\frac{3}{25}e^{-3x}\cos(4x)+\frac{4}{25}e^{-3x}\sin(4x)$$

output
```
-3/25*cos(4*x)/exp(3*x)+4/25*sin(4*x)/exp(3*x)
```

3.812.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int e^{-3x}\cos(4x)\,dx = \frac{1}{25}e^{-3x}(-3\cos(4x)+4\sin(4x))$$

input
```
Integrate[Cos[4*x]/E^(3*x),x]
```

output
```
(-3*Cos[4*x] + 4*Sin[4*x])/(25*E^(3*x))
```

3.812.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4933}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-3x}\cos(4x)\,dx$$

$$\Big\downarrow \text{4933}$$

$$\frac{4}{25}e^{-3x}\sin(4x)-\frac{3}{25}e^{-3x}\cos(4x)$$

input
```
Int[Cos[4*x]/E^(3*x),x]
```

output
```
(-3*Cos[4*x])/(25*E^(3*x)) + (4*Sin[4*x])/(25*E^(3*x))
```

3.812.3.1 Defintions of rubi rules used

rule 4933
```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] + Simp[e*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

3.812.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

method	result	size
parallelrisch	$\frac{{\mathrm e}^{-3 x} \left(-3 \cos \left(4 x \right)+4 \sin \left(4 x \right)\right)}{25}$	20
default	$-\frac{3 \,{\mathrm e}^{-3 x} \cos \left(4 x \right)}{25}+\frac{4 \,{\mathrm e}^{-3 x} \sin \left(4 x \right)}{25}$	22
norman	$\frac{\left(-\frac{3}{25}+\frac{3 \tan \left(2 x \right)^{2}}{25}+\frac{8 \tan \left(2 x \right)}{25}\right) {\mathrm e}^{-3 x}}{1+\tan \left(2 x \right)^{2}}$	34
risch	$-\frac{3 \,{\mathrm e}^{\left(-3+4 i\right) x}}{50}-\frac{2 i {\mathrm e}^{\left(-3+4 i\right) x}}{25}-\frac{3 \,{\mathrm e}^{\left(-3-4 i\right) x}}{50}+\frac{2 i {\mathrm e}^{\left(-3-4 i\right) x}}{25}$	36

input
```
int(cos(4*x)/exp(3*x),x,method=_RETURNVERBOSE)
```

output
```
1/25*exp(-3*x)*(-3*cos(4*x)+4*sin(4*x))
```

3.812.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int e^{-3x}\cos(4x)\,dx = -\frac{3}{25}\,\cos\left(4\,x\right)e^{\left(-3\,x\right)} + \frac{4}{25}\,e^{\left(-3\,x\right)}\sin\left(4\,x\right)$$

input
```
integrate(cos(4*x)/exp(3*x),x, algorithm="fricas")
```

output
```
-3/25*cos(4*x)*e^(-3*x) + 4/25*e^(-3*x)*sin(4*x)
```

3.812.6 Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int e^{-3x}\cos(4x)\,dx = \frac{4e^{-3x}\sin\left(4x\right)}{25} - \frac{3e^{-3x}\cos\left(4x\right)}{25}$$

input
```
integrate(cos(4*x)/exp(3*x),x)
```

output
```
4*exp(-3*x)*sin(4*x)/25 - 3*exp(-3*x)*cos(4*x)/25
```

3.812.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int e^{-3x}\cos(4x)\,dx = -\frac{1}{25}\,\left(3\,\cos\left(4\,x\right) - 4\,\sin\left(4\,x\right)\right)e^{\left(-3\,x\right)}$$

input
```
integrate(cos(4*x)/exp(3*x),x, algorithm="maxima")
```

output
```
-1/25*(3*cos(4*x) - 4*sin(4*x))*e^(-3*x)
```

3.812.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int e^{-3x}\cos(4x)\,dx = -\frac{1}{25}\,(3\,\cos\left(4\,x\right) - 4\,\sin\left(4\,x\right))e^{(-3\,x)}$$

input
```
integrate(cos(4*x)/exp(3*x),x, algorithm="giac")
```

output
```
-1/25*(3*cos(4*x) - 4*sin(4*x))*e^(-3*x)
```

3.812.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int e^{-3x}\cos(4x)\,dx = -\frac{\mathrm{e}^{-3\,x}\,(3\,\cos\left(4\,x\right) - 4\,\sin\left(4\,x\right))}{25}$$

input
```
int(cos(4*x)*exp(-3*x),x)
```

output
```
-(exp(-3*x)*(3*cos(4*x) - 4*sin(4*x)))/25
```

3.813 $\int \frac{\cos(x)\sin(x)}{\sqrt{1+\sin(x)}}\,dx$

3.813.1 Optimal result

Integrand size = 13, antiderivative size = 23

$$\int \frac{\cos(x)\sin(x)}{\sqrt{1+\sin(x)}}\,dx = -2\sqrt{1+\sin(x)}+\frac{2}{3}(1+\sin(x))^{3/2}$$

output
```
2/3*(1+sin(x))^(3/2)-2*(1+sin(x))^(1/2)
```

3.813.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int \frac{\cos(x)\sin(x)}{\sqrt{1+\sin(x)}}\,dx = \frac{2\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^2(-2+\sin(x))}{3\sqrt{1+\sin(x)}}$$

input
```
Integrate[(Cos[x]*Sin[x])/Sqrt[1 + Sin[x]],x]
```

output
```
(2*(Cos[x/2] + Sin[x/2])^2*(-2 + Sin[x]))/(3*Sqrt[1 + Sin[x]])
```

3.813.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 3312, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)\cos(x)}{\sqrt{\sin(x)+1}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin(x)\cos(x)}{\sqrt{\sin(x)+1}}dx$$
$$\Big\downarrow \text{3312}$$
$$\int \frac{\sin(x)}{\sqrt{\sin(x)+1}}d\sin(x)$$
$$\Big\downarrow \text{53}$$
$$\int \left(\sqrt{\sin(x)+1}-\frac{1}{\sqrt{\sin(x)+1}}\right)d\sin(x)$$
$$\Big\downarrow \text{2009}$$
$$\frac{2}{3}(\sin(x)+1)^{3/2}-2\sqrt{\sin(x)+1}$$

input
```
Int[(Cos[x]*Sin[x])/Sqrt[1 + Sin[x]],x]
```

output
```
-2*Sqrt[1 + Sin[x]] + (2*(1 + Sin[x])^(3/2))/3
```

3.813.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3312
```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[1/(b*f)   Su
bst[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a,
b, c, d, e, f, m, n}, x]
```

3.813.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{2(1+\sin(x))^{\frac{3}{2}}}{3}-2\sqrt{1+\sin\left(x\right)}$	18
default	$\frac{2(1+\sin(x))^{\frac{3}{2}}}{3}-2\sqrt{1+\sin\left(x\right)}$	18

input
```
int(cos(x)*sin(x)/(1+sin(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(1+sin(x))^(3/2)-2*(1+sin(x))^(1/2)
```

3.813.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.52

$$\int \frac{\cos(x)\sin(x)}{\sqrt{1+\sin(x)}}\,dx = \frac{2}{3}\sqrt{\sin\left(x\right)+1}(\sin\left(x\right)-2)$$

input
```
integrate(cos(x)*sin(x)/(1+sin(x))^(1/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(sin(x) + 1)*(sin(x) - 2)
```

3.813.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int \frac{\cos(x)\sin(x)}{\sqrt{1+\sin(x)}}\,dx = \frac{2\sqrt{\sin{(x)}+1}\sin{(x)}}{3} - \frac{4\sqrt{\sin{(x)}+1}}{3}$$

input
```
integrate(cos(x)*sin(x)/(1+sin(x))**(1/2),x)
```

output
```
2*sqrt(sin(x) + 1)*sin(x)/3 - 4*sqrt(sin(x) + 1)/3
```

3.813.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{\cos(x)\sin(x)}{\sqrt{1+\sin(x)}}\,dx = \frac{2}{3}\left(\sin{(x)}+1\right)^{\frac{3}{2}} - 2\sqrt{\sin{(x)}+1}$$

input
```
integrate(cos(x)*sin(x)/(1+sin(x))^(1/2),x, algorithm="maxima")
```

output
```
2/3*(sin(x) + 1)^(3/2) - 2*sqrt(sin(x) + 1)
```

3.813.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(17) = 34$.

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \frac{\cos(x)\sin(x)}{\sqrt{1+\sin(x)}}\,dx = \frac{2\left(2\sqrt{2}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}x\right)^{3} - 3\sqrt{2}\cos\left(-\frac{1}{4}\pi+\frac{1}{2}x\right)\right)}{3\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}x\right)\right)}$$

input
```
integrate(cos(x)*sin(x)/(1+sin(x))^(1/2),x, algorithm="giac")
```

output
```
2/3*(2*sqrt(2)*cos(-1/4*pi + 1/2*x)^3 - 3*sqrt(2)*cos(-1/4*pi + 1/2*x))/sg
n(cos(-1/4*pi + 1/2*x))
```

3.813.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.52

$$\int \frac{\cos(x)\sin(x)}{\sqrt{1+\sin(x)}}\,dx = \frac{2\sqrt{\sin(x)+1}\,(\sin(x)-2)}{3}$$

input
```
int((cos(x)*sin(x))/(sin(x) + 1)^(1/2),x)
```

output
```
(2*(sin(x) + 1)^(1/2)*(sin(x) - 2))/3
```

3.814 $\int (x+60\cos^5(x)\sin^4(x))\,dx$

3.814.1 Optimal result

Integrand size = 12, antiderivative size = 30

$$\int (x+60\cos^5(x)\sin^4(x))\,dx=\frac{x^2}{2}+12\sin^5(x)-\frac{120\sin^7(x)}{7}+\frac{20\sin^9(x)}{3}$$

output
```
1/2*x^2+12*sin(x)^5-120/7*sin(x)^7+20/3*sin(x)^9
```

3.814.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.53

$$\int (x+60\cos^5(x)\sin^4(x))\,dx=\frac{x^2}{2}+\frac{45\sin(x)}{32}-\frac{5}{16}\sin(3x)-\frac{3}{16}\sin(5x)+\frac{15}{448}\sin(7x)+\frac{5}{192}\sin(9x)$$

input
```
Integrate[x + 60*Cos[x]^5*Sin[x]^4,x]
```

output
```
x^2/2 + (45*Sin[x])/32 - (5*Sin[3*x])/16 - (3*Sin[5*x])/16 + (15*Sin[7*x])
/448 + (5*Sin[9*x])/192
```

3.814.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x+60\sin^4(x)\cos^5(x)\right)\,dx$$
$$\Big\downarrow \; 2009$$
$$\frac{x^2}{2}+\frac{20\sin^9(x)}{3}-\frac{120\sin^7(x)}{7}+12\sin^5(x)$$

input
```
Int[x + 60*Cos[x]^5*Sin[x]^4,x]
```

output
```
x^2/2 + 12*Sin[x]^5 - (120*Sin[x]^7)/7 + (20*Sin[x]^9)/3
```

3.814.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.814.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{x^{2}}{2}+12\sin\left(x\right)^{5}-\frac{120\sin\left(x\right)^{7}}{7}+\frac{20\sin\left(x\right)^{9}}{3}$	25
parts	$\frac{x^{2}}{2}+12\sin\left(x\right)^{5}-\frac{120\sin\left(x\right)^{7}}{7}+\frac{20\sin\left(x\right)^{9}}{3}$	25
risch	$\frac{x^{2}}{2}+\frac{45\sin\left(x\right)}{32}+\frac{5\sin\left(9x\right)}{192}+\frac{15\sin\left(7x\right)}{448}-\frac{3\sin\left(5x\right)}{16}-\frac{5\sin\left(3x\right)}{16}$	35
parallelrisch	$\frac{x^{2}}{2}+\frac{45\sin\left(x\right)}{32}+\frac{5\sin\left(9x\right)}{192}+\frac{15\sin\left(7x\right)}{448}-\frac{3\sin\left(5x\right)}{16}-\frac{5\sin\left(3x\right)}{16}$	35

input
```
int(x+60*cos(x)^5*sin(x)^4,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2+12*sin(x)^5-120/7*sin(x)^7+20/3*sin(x)^9
```

3.814.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20

$$\int \left(x+60\cos^5(x)\sin^4(x)\right)\,dx = \frac{1}{2}\,x^2+\frac{4}{21}\,\left(35\,\cos\left(x\right)^8-50\,\cos\left(x\right)^6+3\,\cos\left(x\right)^4+4\,\cos\left(x\right)^2+8\right)\sin\left(x\right)$$

input
```
integrate(x+60*cos(x)^5*sin(x)^4,x, algorithm="fricas")
```

output
```
1/2*x^2 + 4/21*(35*cos(x)^8 - 50*cos(x)^6 + 3*cos(x)^4 + 4*cos(x)^2 + 8)*s
in(x)
```

3.814.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int \left(x+60\cos^5(x)\sin^4(x)\right)\,dx = \frac{x^2}{2}+\frac{20\sin^9\left(x\right)}{3}-\frac{120\sin^7\left(x\right)}{7}+12\sin^5\left(x\right)$$

input
```
integrate(x+60*cos(x)**5*sin(x)**4,x)
```

output
```
x**2/2 + 20*sin(x)**9/3 - 120*sin(x)**7/7 + 12*sin(x)**5
```

3.814.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(x+60\cos^5(x)\sin^4(x)\right)\,dx = \frac{20}{3}\,\sin\left(x\right)^9-\frac{120}{7}\,\sin\left(x\right)^7+12\,\sin\left(x\right)^5+\frac{1}{2}\,x^2$$

input
```
integrate(x+60*cos(x)^5*sin(x)^4,x, algorithm="maxima")
```

output
```
20/3*sin(x)^9 - 120/7*sin(x)^7 + 12*sin(x)^5 + 1/2*x^2
```

3.814.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(x+60\cos^5(x)\sin^4(x)\right)\,dx=\frac{20}{3}\,\sin\left(x\right)^9-\frac{120}{7}\,\sin\left(x\right)^7+12\,\sin\left(x\right)^5+\frac{1}{2}\,x^2$$

input
```
integrate(x+60*cos(x)^5*sin(x)^4,x, algorithm="giac")
```

output
```
20/3*sin(x)^9 - 120/7*sin(x)^7 + 12*sin(x)^5 + 1/2*x^2
```

3.814.9 Mupad [B] (verification not implemented)

Time = 26.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(x+60\cos^5(x)\sin^4(x)\right)\,dx=\frac{x^2}{2}+\frac{20\sin\left(x\right)^9}{3}-\frac{120\sin\left(x\right)^7}{7}+12\sin\left(x\right)^5$$

input
```
int(x + 60*cos(x)^5*sin(x)^4,x)
```

output
```
12*sin(x)^5 - (120*sin(x)^7)/7 + (20*sin(x)^9)/3 + x^2/2
```

3.815 $\int \cos(x)(\sec(x)+\tan(x))\,dx$

3.815.1 Optimal result

Integrand size = 8, antiderivative size = 6

$$\int \cos(x)(\sec(x)+\tan(x))\,dx = x-\cos(x)$$

output
```
x-cos(x)
```

3.815.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \cos(x)(\sec(x)+\tan(x))\,dx = x-\cos(x)$$

input
```
Integrate[Cos[x]*(Sec[x] + Tan[x]),x]
```

output
```
x - Cos[x]
```

3.815.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3042, 3640, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)(\tan(x)+\sec(x))\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \cos(x)(\tan(x)+\sec(x))dx$$

$$\Big\downarrow \; 3640$$

$$\int (\sin(x)+1)dx$$

$$\Big\downarrow \; 2009$$

$$x-\cos(x)$$

input
```
Int[Cos[x]*(Sec[x] + Tan[x]),x]
```

output
```
x - Cos[x]
```

3.815.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3640
```
Int[cos[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] +
(c_.)*tan[(d_.) + (e_.)*(x_)])^(n_.), x_Symbol] :> Int[(b + a*Cos[d + e*x]
+ c*Sin[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[n]
```

3.815.4 Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

method	result	size
default	$x-\cos(x)$	7
risch	$x-\cos(x)$	7

input
```
int(cos(x)*(sec(x)+tan(x)),x,method=_RETURNVERBOSE)
```

output
```
x-cos(x)
```

3.815.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \cos(x)(\sec(x)+\tan(x))\,dx = x-\cos(x)$$

input
```
integrate(cos(x)*(sec(x)+tan(x)),x, algorithm="fricas")
```

output
```
x - cos(x)
```

3.815.6 Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.50

$$\int \cos(x)(\sec(x)+\tan(x))\,dx = x-\cos(x)$$

input
```
integrate(cos(x)*(sec(x)+tan(x)),x)
```

output
```
x - cos(x)
```

3.815.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \cos(x)(\sec(x)+\tan(x))\,dx = x - \cos(x)$$

input
```
integrate(cos(x)*(sec(x)+tan(x)),x, algorithm="maxima")
```

output
```
x - cos(x)
```

3.815.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14 vs. 2(6) = 12.

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 2.33

$$\int \cos(x)(\sec(x)+\tan(x))\,dx = x - \frac{2}{\tan\left(\frac{1}{2}x\right)^2+1}$$

input
```
integrate(cos(x)*(sec(x)+tan(x)),x, algorithm="giac")
```

output
```
x - 2/(tan(1/2*x)^2 + 1)
```

3.815.9 Mupad [B] (verification not implemented)

Time = 26.60 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \cos(x)(\sec(x)+\tan(x))\,dx = x - \cos(x)$$

input
```
int(cos(x)*(tan(x) + 1/cos(x)),x)
```

output
```
x - cos(x)
```

3.816 $\int \cos(x) \left(\sec^3(x) + \tan(x)\right) \, dx$

3.816.1 Optimal result

Integrand size = 10, antiderivative size = 7

$$\int \cos(x) \left(\sec^3(x) + \tan(x)\right) \, dx = -\cos(x) + \tan(x)$$

output
```
-cos(x)+tan(x)
```

3.816.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \cos(x) \left(\sec^3(x) + \tan(x)\right) \, dx = -\cos(x) + \tan(x)$$

input
```
Integrate[Cos[x]*(Sec[x]^3 + Tan[x]),x]
```

output
```
-Cos[x] + Tan[x]
```

3.816.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3042, 4901, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x) \left(\tan(x)+\sec^3(x)\right) \, dx$$

$$\Big\downarrow \; 3042$$

$$\int \cos(x) \left(\tan(x)+\sec(x)^3\right) dx$$

$$\Big\downarrow \; 4901$$

$$\int \left(\sin(x)+\sec^2(x)\right) dx$$

$$\Big\downarrow \; 2009$$

$$\tan(x)-\cos(x)$$

input
```
Int[Cos[x]*(Sec[x]^3 + Tan[x]),x]
```

output
```
-Cos[x] + Tan[x]
```

3.816.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4901
```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

3.816.4 Maple [A] (verified)

Time = 5.48 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

method	result	size
default	$-\cos(x)+\tan(x)$	8
risch	$\frac{2i}{e^{2ix}+1}-\cos(x)$	18

input
```
int(cos(x)*(sec(x)^3+tan(x)),x,method=_RETURNVERBOSE)
```

output
```
-cos(x)+tan(x)
```

3.816.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(7) = 14.

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 2.14

$$\int \cos(x)\left(\sec^3(x)+\tan(x)\right)\,dx = -\frac{\cos(x)^2-\sin(x)}{\cos(x)}$$

input
```
integrate(cos(x)*(sec(x)^3+tan(x)),x, algorithm="fricas")
```

output
```
-(cos(x)^2 - sin(x))/cos(x)
```

3.816.6 Sympy [A] (verification not implemented)

Time = 2.90 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

$$\int \cos(x)\left(\sec^3(x)+\tan(x)\right)\,dx = \frac{\sin(x)}{\cos(x)}-\cos(x)$$

input
```
integrate(cos(x)*(sec(x)**3+tan(x)),x)
```

output
```
sin(x)/cos(x) - cos(x)
```

3.816.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \cos(x) \left(\sec^3(x) + \tan(x)\right) \, dx = -\cos(x) + \tan(x)$$

input
```
integrate(cos(x)*(sec(x)^3+tan(x)),x, algorithm="maxima")
```

output
```
-cos(x) + tan(x)
```

3.816.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. 2(7) = 14.

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 4.29

$$\int \cos(x) \left(\sec^3(x) + \tan(x)\right) \, dx = -\frac{2 \left(\tan\left(\frac{1}{2}x\right)^3 + \tan\left(\frac{1}{2}x\right)^2 + \tan\left(\frac{1}{2}x\right) - 1\right)}{\tan\left(\frac{1}{2}x\right)^4 - 1}$$

input
```
integrate(cos(x)*(sec(x)^3+tan(x)),x, algorithm="giac")
```

output
```
-2*(tan(1/2*x)^3 + tan(1/2*x)^2 + tan(1/2*x) - 1)/(tan(1/2*x)^4 - 1)
```

3.816.9 Mupad [B] (verification not implemented)

Time = 25.43 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.71

$$\int \cos(x) \left(\sec^3(x) + \tan(x)\right) \, dx = \frac{\sin(x)}{\cos(x)} - \cos(x)$$

input
```
int(cos(x)*(tan(x) + 1/cos(x)^3),x)
```

output
```
sin(x)/cos(x) - cos(x)
```

3.817 $\int \frac{1}{2}(-\cot(x)\csc(x)+\csc^2(x))\,dx$

3.817.1 Optimal result

Integrand size = 15, antiderivative size = 13

$$\int \frac{1}{2}(-\cot(x)\csc(x)+\csc^2(x))\,dx=-\frac{\cot(x)}{2}+\frac{\csc(x)}{2}$$

output
```
-1/2*cot(x)+1/2*csc(x)
```

3.817.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{1}{2}(-\cot(x)\csc(x)+\csc^2(x))\,dx=\frac{1}{2}\tan\left(\frac{x}{2}\right)$$

input
```
Integrate[(-(Cot[x]*Csc[x]) + Csc[x]^2)/2,x]
```

output
```
Tan[x/2]/2
```

3.817.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {27, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{2}\left(\csc^2(x)-\cot(x)\csc(x)\right)\,dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \left(\csc^2(x)-\cot(x)\csc(x)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}(\csc(x)-\cot(x))$$

input
```
Int[(-(Cot[x]*Csc[x]) + Csc[x]^2)/2,x]
```

output
```
(-Cot[x] + Csc[x])/2
```

3.817.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.817.4 Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{\csc(x)}{2}-\frac{\cot(x)}{2}$	10
parts	$\frac{\csc(x)}{2}-\frac{\cot(x)}{2}$	10
risch	$\frac{i}{e^{ix}+1}$	13

input
```
int(-1/2*csc(x)*cot(x)+1/2*csc(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*csc(x)-1/2*cot(x)
```

3.817.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

$$\int \frac{1}{2}\left(-\cot(x)\csc(x)+\csc^2(x)\right)\,dx=\frac{\sin(x)}{2(\cos(x)+1)}$$

input
```
integrate(-1/2*cot(x)*csc(x)+1/2*csc(x)^2,x, algorithm="fricas")
```

output
```
1/2*sin(x)/(cos(x) + 1)
```

3.817.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{1}{2}\left(-\cot(x)\csc(x)+\csc^2(x)\right)\,dx=-\frac{\cos(x)}{2\sin(x)}+\frac{1}{2\sin(x)}$$

input
```
integrate(-1/2*cot(x)*csc(x)+1/2*csc(x)**2,x)
```

output
```
-cos(x)/(2*sin(x)) + 1/(2*sin(x))
```

3.817.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{2}\left(-\cot(x)\csc(x)+\csc^2(x)\right)\,dx=\frac{1}{2\,\sin\left(x\right)}-\frac{1}{2\,\tan\left(x\right)}$$

input
```
integrate(-1/2*cot(x)*csc(x)+1/2*csc(x)^2,x, algorithm="maxima")
```

output
```
1/2/sin(x) - 1/2/tan(x)
```

3.817.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{2}\left(-\cot(x)\csc(x)+\csc^2(x)\right)\,dx=\frac{1}{2\,\sin\left(x\right)}-\frac{1}{2\,\tan\left(x\right)}$$

input
```
integrate(-1/2*cot(x)*csc(x)+1/2*csc(x)^2,x, algorithm="giac")
```

output
```
1/2/sin(x) - 1/2/tan(x)
```

3.817.9 Mupad [B] (verification not implemented)

Time = 25.76 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.46

$$\int \frac{1}{2}\left(-\cot(x)\csc(x)+\csc^2(x)\right)\,dx=\frac{\tan\left(\frac{x}{2}\right)}{2}$$

input
```
int(1/(2*sin(x)^2) - cot(x)/(2*sin(x)),x)
```

output
```
tan(x/2)/2
```

3.818 $\int (-\csc^2(x)+\sin(2x))\,dx$

3.818.1 Optimal result

Integrand size = 11, antiderivative size = 11

$$\int (-\csc^2(x)+\sin(2x))\,dx=-\frac{1}{2}\cos(2x)+\cot(x)$$

output
```
-1/2*cos(2*x)+cot(x)
```

3.818.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int (-\csc^2(x)+\sin(2x))\,dx=-\frac{1}{2}\cos(2x)+\cot(x)$$

input
```
Integrate[-Csc[x]^2 + Sin[2*x],x]
```

output
```
-1/2*Cos[2*x] + Cot[x]
```

3.818.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\sin(2x)-\csc^2(x)\right)\,dx$$

$$\Big\downarrow \text{ 2009}$$

$$\cot(x)-\frac{1}{2}\cos(2x)$$

input
```
Int[-Csc[x]^2 + Sin[2*x],x]
```

output
```
-1/2*Cos[2*x] + Cot[x]
```

3.818.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.818.4 Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{\cos(2x)}{2}+\cot\left(x\right)$	10
parts	$-\frac{\cos(2x)}{2}+\cot\left(x\right)$	10
parallelrisch	$\frac{1}{2}-\frac{\cos(2x)}{2}+\cot\left(x\right)$	11
risch	$\frac{-{\mathrm e}^{4 i x}-1+8 i+2\cos(2x)}{4\,{\mathrm e}^{2 i x}-4}$	29
norman	$\frac{\frac{1}{2}-\tan\left(\frac{x}{2}\right)+\frac{\tan(x)^{2}}{2}-\frac{\tan\left(\frac{x}{2}\right)^{2}}{2}-\frac{\tan(x)^{2}\tan\left(\frac{x}{2}\right)^{2}}{2}}{\tan\left(\frac{x}{2}\right)\left(1+\tan(x)^{2}\right)}$	50

input
```
int(-csc(x)^2+sin(2*x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*cos(2*x)+cot(x)
```

3.818.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(9) = 18$.

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.00

$$\int \left(-\csc^2(x)+\sin(2x)\right)\, dx = -\frac{\left(2\, \cos\left(x\right)^{2}-1\right)\sin\left(x\right)-2\, \cos\left(x\right)}{2\, \sin\left(x\right)}$$

input
```
integrate(-csc(x)^2+sin(2*x),x, algorithm="fricas")
```

output
```
-1/2*((2*cos(x)^2 - 1)*sin(x) - 2*cos(x))/sin(x)
```

3.818.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

$$\int \left(-\csc^2(x)+\sin(2x)\right)\, dx = -\frac{\cos\left(2x\right)}{2}+\frac{\cos\left(x\right)}{\sin\left(x\right)}$$

input
```
integrate(-csc(x)**2+sin(2*x),x)
```

output
```
-cos(2*x)/2 + cos(x)/sin(x)
```

3.818.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \left(-\csc^2(x)+\sin(2x)\right)\, dx = \frac{1}{\tan\left(x\right)}-\frac{1}{2}\, \cos\left(2\, x\right)$$

input
```
integrate(-csc(x)^2+sin(2*x),x, algorithm="maxima")
```

output
```
1/tan(x) - 1/2*cos(2*x)
```

3.818.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \left(-\csc^2(x)+\sin(2x)\right)\,dx=\frac{1}{\tan(x)}-\frac{1}{2}\cos(2x)$$

input
```
integrate(-csc(x)^2+sin(2*x),x, algorithm="giac")
```

output
```
1/tan(x) - 1/2*cos(2*x)
```

3.818.9 Mupad [B] (verification not implemented)

Time = 26.63 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int \left(-\csc^2(x)+\sin(2x)\right)\,dx=\frac{\cos(x)}{\sin(x)}-\cos(x)^2$$

input
```
int(sin(2*x) - 1/sin(x)^2,x)
```

output
```
cos(x)/sin(x) - cos(x)^2
```

3.819 $\int (2\cot(2x) - 3\sin(3x))\,dx$

3.819.1 Optimal result

Integrand size = 13, antiderivative size = 10

$$\int (2\cot(2x) - 3\sin(3x))\,dx = \cos(3x) + \log(\sin(2x))$$

output
```
cos(3*x)+ln(sin(2*x))
```

3.819.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.50

$$\int (2\cot(2x) - 3\sin(3x))\,dx = \cos(3x) + \log(\cos(2x)) + \log(\tan(2x))$$

input
```
Integrate[2*Cot[2*x] - 3*Sin[3*x],x]
```

output
```
Cos[3*x] + Log[Cos[2*x]] + Log[Tan[2*x]]
```

3.819.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2\cot(2x) - 3\sin(3x))\,dx$$

$$\Big\downarrow \ 2009$$

$$\cos(3x) + \log(\sin(2x))$$

input
```
Int[2*Cot[2*x] - 3*Sin[3*x],x]
```

output
```
Cos[3*x] + Log[Sin[2*x]]
```

3.819.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.819.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.70

method	result	size
default	$-\frac{\ln\left(1+\cot(2x)^{2}\right)}{2}+\cos\left(3x\right)$	17
parts	$-\frac{\ln\left(1+\cot(2x)^{2}\right)}{2}+\cos\left(3x\right)$	17
risch	$-2ix+\ln\left(e^{4ix}-1\right)+\cos\left(3x\right)$	18
parallelrisch	$\ln\left(\tan\left(2x\right)\right)+\ln\left(\frac{1}{\sqrt{\sec(2x)^{2}}}\right)+1+\cos\left(3x\right)$	21
norman	$\frac{2}{1+\tan\left(\frac{3x}{2}\right)^{2}}-\frac{\ln\left(1+\tan(2x)^{2}\right)}{2}+\ln\left(\tan\left(2x\right)\right)$	30

input
```
int(2*cot(2*x)-3*sin(3*x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(1+cot(2*x)^2)+cos(3*x)
```

3.819.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.80

$$\int (2\cot(2x) - 3\sin(3x))\, dx = 4 \cos{(x)}^3 - 3 \cos{(x)} + \log\left(-\frac{1}{2} \cos{(x)} \sin{(x)}\right)$$

input
```
integrate(2*cot(2*x)-3*sin(3*x),x, algorithm="fricas")
```

output
```
4*cos(x)^3 - 3*cos(x) + log(-1/2*cos(x)*sin(x))
```

3.819.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int (2\cot(2x) - 3\sin(3x))\, dx = \log{(\sin{(2x)})} + \cos{(3x)}$$

input
```
integrate(2*cot(2*x)-3*sin(3*x),x)
```

output
```
log(sin(2*x)) + cos(3*x)
```

3.819.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int (2\cot(2x) - 3\sin(3x))\, dx = \cos{(3\,x)} + \log{(\sin{(2\,x)})}$$

input
```
integrate(2*cot(2*x)-3*sin(3*x),x, algorithm="maxima")
```

output
```
cos(3*x) + log(sin(2*x))
```

3.819.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int (2\cot(2x) - 3\sin(3x))\,dx = \cos(3x) + \log(|\sin(2x)|)$$

input
```
integrate(2*cot(2*x)-3*sin(3*x),x, algorithm="giac")
```

output
```
cos(3*x) + log(abs(sin(2*x)))
```

3.819.9 Mupad [B] (verification not implemented)

Time = 27.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.40

$$\int (2\cot(2x) - 3\sin(3x))\,dx = \cos(3x) + \ln\left(\cos\left(\frac{x}{2}\right)\,\left(\sin\left(\frac{x}{2}\right) - 2\sin\left(\frac{x}{2}\right)^3\right)\right)$$

input
```
int(2*cot(2*x) - 3*sin(3*x),x)
```

output
```
cos(3*x) + log(cos(x/2)*(sin(x/2) - 2*sin(x/2)^3))
```

3.820 $\int x \sin(2x^2)\, dx$

3.820.1 Optimal result

Integrand size = 8, antiderivative size = 10

$$\int x \sin(2x^2)\, dx = -\frac{1}{4}\cos(2x^2)$$

output
```
-1/4*cos(2*x^2)
```

3.820.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int x \sin(2x^2)\, dx = -\frac{1}{4}\cos(2x^2)$$

input
```
Integrate[x*Sin[2*x^2],x]
```

output
```
-1/4*Cos[2*x^2]
```

3.820.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3860, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sin\left(2x^2\right)\,dx$$

$$\Big\downarrow \text{3860}$$

$$\frac{1}{2}\int \sin\left(2x^2\right)dx^2$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \sin\left(2x^2\right)dx^2$$

$$\Big\downarrow \text{3118}$$

$$-\frac{1}{4}\cos\left(2x^2\right)$$

input
```
Int[x*Sin[2*x^2],x]
```

output
```
-1/4*Cos[2*x^2]
```

3.820.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

3.820.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$-\frac{\cos(2x^2)}{4}$	9
default	$-\frac{\cos(2x^2)}{4}$	9
risch	$-\frac{\cos(2x^2)}{4}$	9
parallelrisch	$-\frac{\cos(2x^2)}{4}-\frac{1}{4}$	11
norman	$-\frac{1}{2\left(1+\tan(x^2)^2\right)}$	13
meijerg	$\frac{\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(2x^2)}{\sqrt{\pi}}\right)}{4}$	21
parts	$\frac{\sqrt{\pi}\,\mathrm{FresnelS}\left(\frac{2x}{\sqrt{\pi}}\right)x}{2}-\frac{\pi\left(\frac{2\,\mathrm{FresnelS}\left(\frac{2x}{\sqrt{\pi}}\right)x}{\sqrt{\pi}}+\frac{\cos(2x^2)}{\pi}\right)}{4}$	42

input
```
int(x*sin(2*x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*cos(2*x^2)
```

3.820.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int x\sin\left(2x^2\right)\,dx=-\frac{1}{4}\,\cos\left(2\,x^2\right)$$

input
```
integrate(x*sin(2*x^2),x, algorithm="fricas")
```

output
```
-1/4*cos(2*x^2)
```

3.820.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int x\sin\left(2x^2\right)\,dx=-\frac{\cos\left(2x^2\right)}{4}$$

input
```
integrate(x*sin(2*x**2),x)
```

output
```
-cos(2*x**2)/4
```

3.820.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int x\sin\left(2x^2\right)\,dx=-\frac{1}{4}\cos\left(2\,x^2\right)$$

input
```
integrate(x*sin(2*x^2),x, algorithm="maxima")
```

output
```
-1/4*cos(2*x^2)
```

3.820.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int x\sin\left(2x^2\right)\,dx=-\frac{1}{4}\cos\left(2\,x^2\right)$$

input
```
integrate(x*sin(2*x^2),x, algorithm="giac")
```

output
```
-1/4*cos(2*x^2)
```

3.820.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int x\sin\left(2x^2\right)\,dx=\frac{\sin\left(x^2\right)^2}{2}$$

input
```
int(x*sin(2*x^2),x)
```

output
```
sin(x^2)^2/2
```

3.821 $\int -\cos(1-x)\sin(1-x)\sqrt{1+\sin^2(1-x)}\,dx$

3.821.1 Optimal result

Integrand size = 28, antiderivative size = 18

$$\int -\cos(1-x)\sin(1-x)\sqrt{1+\sin^2(1-x)}\,dx = \frac{1}{3}\left(1+\sin^2(1-x)\right)^{3/2}$$

output
```
1/3*(1+sin(-1+x)^2)^(3/2)
```

3.821.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int -\cos(1-x)\sin(1-x)\sqrt{1+\sin^2(1-x)}\,dx = \frac{1}{3}\left(1+\sin^2(1-x)\right)^{3/2}$$

input
```
Integrate[-(Cos[1 - x]*Sin[1 - x]*Sqrt[1 + Sin[1 - x]^2]),x]
```

output
```
(1 + Sin[1 - x]^2)^(3/2)/3
```

3.821.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {25, 3042, 3677, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(1-x)\sqrt{\sin^2(1-x)+1}(-\cos(1-x))\,dx$$
$$\Big\downarrow \; 25$$
$$-\int \cos(1-x)\sin(1-x)\sqrt{\sin^2(1-x)+1}dx$$
$$\Big\downarrow \; 3042$$
$$-\int \cos(1-x)\sin(1-x)\sqrt{\sin(1-x)^2+1}dx$$
$$\Big\downarrow \; 3677$$
$$\int \sin(1-x)\sqrt{\sin^2(1-x)+1}d\sin(1-x)$$
$$\Big\downarrow \; 241$$
$$\frac{1}{3}\left(\sin^2(1-x)+1\right)^{3/2}$$

input
```
Int[-(Cos[1 - x]*Sin[1 - x]*Sqrt[1 + Sin[1 - x]^2]),x]
```

output
```
(1 + Sin[1 - x]^2)^(3/2)/3
```

3.821.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3677
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a
_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFa
ctors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[(d*ff*x)^n*(1 - ff^2*x^2)^((
m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
d, e, f, n, p}, x] && IntegerQ[(m - 1)/2]
```

3.821.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.72

method	result	size
derivativedivides	$\frac{\left(1+\sin(x-1)^2\right)^{\frac{3}{2}}}{3}$	13
default	$\frac{\left(1+\sin(x-1)^2\right)^{\frac{3}{2}}}{3}$	13

input
```
int(cos(x-1)*sin(x-1)*(1+sin(x-1)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(1+sin(x-1)^2)^(3/2)
```

3.821.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int -\cos(1-x)\sin(1-x)\sqrt{1+\sin^2(1-x)}\,dx = \frac{1}{3}\left(-\cos\left(x-1\right)^2+2\right)^{\frac{3}{2}}$$

input
```
integrate(cos(-1+x)*sin(-1+x)*(1+sin(-1+x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(-cos(x - 1)^2 + 2)^(3/2)
```

3.821.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(12) = 24$.

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.78

$$\int -\cos(1-x)\sin(1-x)\sqrt{1+\sin^2(1-x)}\,dx = \frac{\sqrt{\sin^2(x-1)+1}\sin^2(x-1)}{3} + \frac{\sqrt{\sin^2(x-1)+1}}{3}$$

input
```
integrate(cos(-1+x)*sin(-1+x)*(1+sin(-1+x)**2)**(1/2),x)
```

output
```
sqrt(sin(x - 1)**2 + 1)*sin(x - 1)**2/3 + sqrt(sin(x - 1)**2 + 1)/3
```

3.821.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int -\cos(1-x)\sin(1-x)\sqrt{1+\sin^2(1-x)}\,dx = \frac{1}{3}\left(\sin(x-1)^2+1\right)^{\frac{3}{2}}$$

input
```
integrate(cos(-1+x)*sin(-1+x)*(1+sin(-1+x)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*(sin(x - 1)^2 + 1)^(3/2)
```

3.821.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int -\cos(1-x)\sin(1-x)\sqrt{1+\sin^2(1-x)}\,dx = \frac{1}{3}\left(\sin(x-1)^2+1\right)^{\frac{3}{2}}$$

input
```
integrate(cos(-1+x)*sin(-1+x)*(1+sin(-1+x)^2)^(1/2),x, algorithm="giac")
```

output
```
1/3*(sin(x - 1)^2 + 1)^(3/2)
```

3.821.9 Mupad [B] (verification not implemented)

Time = 26.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int -\cos(1-x)\sin(1-x)\sqrt{1+\sin^2(1-x)}\,dx = \frac{\left(\sin\left(x-1\right)^2+1\right)^{3/2}}{3}$$

input
```
int(cos(x - 1)*sin(x - 1)*(sin(x - 1)^2 + 1)^(1/2),x)
```

output
```
(sin(x - 1)^2 + 1)^(3/2)/3
```

3.822 $\int \frac{\cos\left(\frac{1}{x}\right)\sin\left(\frac{1}{x}\right)}{x^2}\,dx$

3.822.1 Optimal result

Integrand size = 12, antiderivative size = 10

$$\int \frac{\cos\left(\frac{1}{x}\right)\sin\left(\frac{1}{x}\right)}{x^2}\,dx = -\frac{1}{2}\sin^2\left(\frac{1}{x}\right)$$

output
```
-1/2*sin(1/x)^2
```

3.822.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{x}\right)\sin\left(\frac{1}{x}\right)}{x^2}\,dx = \frac{1}{2}\cos^2\left(\frac{1}{x}\right)$$

input
```
Integrate[(Cos[x^(-1)]*Sin[x^(-1)])/x^2,x]
```

output
```
Cos[x^(-1)]^2/2
```

3.822.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3922}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin\left(\frac{1}{x}\right)\cos\left(\frac{1}{x}\right)}{x^2}\,dx$$

$$\Big\downarrow \text{3922}$$

$$-\frac{1}{2}\sin^2\left(\frac{1}{x}\right)$$

input
```
Int[(Cos[x^(-1)]*Sin[x^(-1)])/x^2,x]
```

output
```
-1/2*Sin[x^(-1)]^2
```

3.822.3.1 Defintions of rubi rules used

rule 3922
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[
{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```

3.822.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$\frac{\cos\left(\frac{1}{x}\right)^{2}}{2}$	9
default	$\frac{\cos\left(\frac{1}{x}\right)^{2}}{2}$	9
risch	$\frac{\cos\left(\frac{2}{x}\right)}{4}$	9
parallelrisch	$\frac{3}{4}+\frac{\cos\left(\frac{2}{x}\right)}{4}$	11
meijerg	$-\frac{\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos\left(\frac{2}{x}\right)}{\sqrt{\pi}}\right)}{4}$	21
norman	$-\frac{2\tan\left(\frac{1}{2x}\right)^{2}}{\left(1+\tan\left(\frac{1}{2x}\right)^{2}\right)^{2}}$	23

input
```
int(cos(1/x)*sin(1/x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*cos(1/x)^2
```

3.822.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\cos\left(\frac{1}{x}\right)\sin\left(\frac{1}{x}\right)}{x^2}\,dx=\frac{1}{2}\cos\left(\frac{1}{x}\right)^2$$

input
```
integrate(cos(1/x)*sin(1/x)/x^2,x, algorithm="fricas")
```

output
```
1/2*cos(1/x)^2
```

3.822.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(8) = 16.

Time = 0.50 (sec) , antiderivative size = 31, normalized size of antiderivative = 3.10

$$\int \frac{\cos\left(\frac{1}{x}\right)\sin\left(\frac{1}{x}\right)}{x^2}\,dx = -\frac{2\tan^2\left(\frac{1}{2x}\right)}{\tan^4\left(\frac{1}{2x}\right)+2\tan^2\left(\frac{1}{2x}\right)+1}$$

input
```
integrate(cos(1/x)*sin(1/x)/x**2,x)
```

output
```
-2*tan(1/(2*x))**2/(tan(1/(2*x))**4 + 2*tan(1/(2*x))**2 + 1)
```

3.822.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\cos\left(\frac{1}{x}\right)\sin\left(\frac{1}{x}\right)}{x^2}\,dx = \frac{1}{2}\cos\left(\frac{1}{x}\right)^2$$

input
```
integrate(cos(1/x)*sin(1/x)/x^2,x, algorithm="maxima")
```

output
```
1/2*cos(1/x)^2
```

3.822.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\cos\left(\frac{1}{x}\right)\sin\left(\frac{1}{x}\right)}{x^2}\,dx = \frac{1}{2}\cos\left(\frac{1}{x}\right)^2$$

input
```
integrate(cos(1/x)*sin(1/x)/x^2,x, algorithm="giac")
```

output
```
1/2*cos(1/x)^2
```

3.822.9 Mupad [B] (verification not implemented)

Time = 26.62 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\cos\left(\frac{1}{x}\right)\sin\left(\frac{1}{x}\right)}{x^2}\,dx = \frac{\cos\left(\frac{1}{x}\right)^2}{2}$$

input
```
int((cos(1/x)*sin(1/x))/x^2,x)
```

output
```
cos(1/x)^2/2
```

3.823 $\int \cos\left(\frac{1}{2}(1+3x)\right)\sin^3\left(\frac{1}{2}(1+3x)\right)\,dx$

3.823.1 Optimal result

Integrand size = 23, antiderivative size = 16

$$\int \cos\left(\frac{1}{2}(1+3x)\right)\sin^3\left(\frac{1}{2}(1+3x)\right)\,dx=\frac{1}{6}\sin^4\left(\frac{1}{2}+\frac{3x}{2}\right)$$

output
```
1/6*sin(1/2+3/2*x)^4
```

3.823.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.56

$$\int \cos\left(\frac{1}{2}(1+3x)\right)\sin^3\left(\frac{1}{2}(1+3x)\right)\,dx=\frac{1}{2}\left(-\frac{1}{6}\cos(1+3x)+\frac{1}{24}\cos(2+6x)\right)$$

input
```
Integrate[Cos[(1 + 3*x)/2]*Sin[(1 + 3*x)/2]^3,x]
```

output
```
(-1/6*Cos[1 + 3*x] + Cos[2 + 6*x]/24)/2
```

3.823.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3042, 3044, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3\left(\frac{1}{2}(3x+1)\right)\cos\left(\frac{1}{2}(3x+1)\right)dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin\left(\frac{3x}{2}+\frac{1}{2}\right)^3\cos\left(\frac{3x}{2}+\frac{1}{2}\right)dx$$
$$\Big\downarrow \text{3044}$$
$$\frac{2}{3}\int \sin^3\left(\frac{3x}{2}+\frac{1}{2}\right)d\sin\left(\frac{3x}{2}+\frac{1}{2}\right)$$
$$\Big\downarrow \text{15}$$
$$\frac{1}{6}\sin^4\left(\frac{3x}{2}+\frac{1}{2}\right)$$

input
```
Int[Cos[(1 + 3*x)/2]*Sin[(1 + 3*x)/2]^3,x]
```

output
```
Sin[1/2 + (3*x)/2]^4/6
```

3.823.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

3.823.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.69

method	result	size
derivativedivides	$\frac{\sin\left(\frac{1}{2}+\frac{3x}{2}\right)^{4}}{6}$	11
default	$\frac{\sin\left(\frac{1}{2}+\frac{3x}{2}\right)^{4}}{6}$	11
risch	$-\frac{\cos(1+3x)}{12}+\frac{\cos(2+6x)}{48}$	18
parallelrisch	$\frac{\cos(2+6x)}{48}+\frac{1}{16}-\frac{\cos(1+3x)}{12}$	19
norman	$\frac{8\tan\left(\frac{1}{4}+\frac{3x}{4}\right)^{4}}{3\left(1+\tan\left(\frac{1}{4}+\frac{3x}{4}\right)^{2}\right)^{4}}$	23

input
```
int(cos(1/2+3/2*x)*sin(1/2+3/2*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/6*sin(1/2+3/2*x)^4
```

3.823.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. 2(10) = 20.

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int \cos\left(\frac{1}{2}(1+3x)\right)\sin^3\left(\frac{1}{2}(1+3x)\right)\,dx=\frac{1}{6}\cos\left(\frac{3}{2}x+\frac{1}{2}\right)^4-\frac{1}{3}\cos\left(\frac{3}{2}x+\frac{1}{2}\right)^2$$

input
```
integrate(cos(1/2+3/2*x)*sin(1/2+3/2*x)^3,x, algorithm="fricas")
```

output
```
1/6*cos(3/2*x + 1/2)^4 - 1/3*cos(3/2*x + 1/2)^2
```

3.823.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \cos\left(\frac{1}{2}(1+3x)\right)\sin^3\left(\frac{1}{2}(1+3x)\right)\,dx = \frac{\sin^4\left(\frac{3x}{2}+\frac{1}{2}\right)}{6}$$

input
```
integrate(cos(1/2+3/2*x)*sin(1/2+3/2*x)**3,x)
```

output
```
sin(3*x/2 + 1/2)**4/6
```

3.823.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \cos\left(\frac{1}{2}(1+3x)\right)\sin^3\left(\frac{1}{2}(1+3x)\right)\,dx = \frac{1}{6}\sin\left(\frac{3}{2}x+\frac{1}{2}\right)^4$$

input
```
integrate(cos(1/2+3/2*x)*sin(1/2+3/2*x)^3,x, algorithm="maxima")
```

output
```
1/6*sin(3/2*x + 1/2)^4
```

3.823.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \cos\left(\frac{1}{2}(1+3x)\right)\sin^3\left(\frac{1}{2}(1+3x)\right)\,dx = \frac{1}{6}\sin\left(\frac{3}{2}x+\frac{1}{2}\right)^4$$

input
```
integrate(cos(1/2+3/2*x)*sin(1/2+3/2*x)^3,x, algorithm="giac")
```

output
```
1/6*sin(3/2*x + 1/2)^4
```

3.823.9 Mupad [B] (verification not implemented)

Time = 26.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \cos\left(\frac{1}{2}(1+3x)\right)\sin^3\left(\frac{1}{2}(1+3x)\right)\,dx = \frac{\left(\frac{\cos(3\,x+1)}{2}-\frac{1}{2}\right)^2}{6}$$

input
```
int(cos((3*x)/2 + 1/2)*sin((3*x)/2 + 1/2)^3,x)
```

output
```
(cos(3*x + 1)/2 - 1/2)^2/6
```

3.824 $\int 4x \tan(x^2) \, dx$

3.824.1 Optimal result

Integrand size = 7, antiderivative size = 7

$$\int 4x \tan(x^2) \, dx = -2 \log(\cos(x^2))$$

output
```
-2*ln(cos(x^2))
```

3.824.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int 4x \tan(x^2) \, dx = -2 \log(\cos(x^2))$$

input
```
Integrate[4*x*Tan[x^2],x]
```

output
```
-2*Log[Cos[x^2]]
```

3.824.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {27, 4234, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int 4x \tan \left(x^2\right) dx$$

$$\Big\downarrow \text{27}$$

$$4 \int x \tan \left(x^2\right) dx$$

$$\Big\downarrow \text{4234}$$

$$2 \int \tan \left(x^2\right) dx^2$$

$$\Big\downarrow \text{3042}$$

$$2 \int \tan \left(x^2\right) dx^2$$

$$\Big\downarrow \text{3956}$$

$$-2 \log \left(\cos \left(x^2\right)\right)$$

input
```
Int[4*x*Tan[x^2],x]
```

output
```
-2*Log[Cos[x^2]]
```

3.824.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4234
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Tan[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Tan[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
 1)/n], 0] && IntegerQ[p]
```

3.824.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

method	result	size
derivativedivides	$-2\ln\left(\cos\left(x^{2}\right)\right)$	8
default	$-2\ln\left(\cos\left(x^{2}\right)\right)$	8
norman	$\ln\left(1+\tan\left(x^{2}\right)^{2}\right)$	10
parallelrisch	$\ln\left(1+\tan\left(x^{2}\right)^{2}\right)$	10
risch	$2ix^{2}-2\ln\left({\mathrm e}^{2ix^{2}}+1\right)$	20

input
```
int(4*x*tan(x^2),x,method=_RETURNVERBOSE)
```

output
```
-2*ln(cos(x^2))
```

3.824.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.86

$$\int 4x\tan\left(x^{2}\right)\,dx=-\log\left(\frac{1}{\tan\left(x^{2}\right)^{2}+1}\right)$$

input
```
integrate(4*x*tan(x^2),x, algorithm="fricas")
```

output
```
-log(1/(tan(x^2)^2 + 1))
```

3.824.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

$$\int 4x\tan\left(x^2\right)\,dx = \log\left(\tan^2\left(x^2\right)+1\right)$$

input
```
integrate(4*x*tan(x**2),x)
```

output
```
log(tan(x**2)**2 + 1)
```

3.824.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int 4x\tan\left(x^2\right)\,dx = 2\,\log\left(\sec\left(x^2\right)\right)$$

input
```
integrate(4*x*tan(x^2),x, algorithm="maxima")
```

output
```
2*log(sec(x^2))
```

3.824.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int 4x\tan\left(x^2\right)\,dx = \log\left(\tan\left(x^2\right)^2+1\right)$$

input
```
integrate(4*x*tan(x^2),x, algorithm="giac")
```

output
```
log(tan(x^2)^2 + 1)
```

3.824.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int 4x\tan\left(x^2\right)\,dx = \ln\left(\tan\left(x^2\right)^2 + 1\right)$$

input
```
int(4*x*tan(x^2),x)
```

output
```
log(tan(x^2)^2 + 1)
```

3.825 $\int x \sec(5-x^2)\,dx$

3.825.1 Optimal result

Integrand size = 10, antiderivative size = 13

$$\int x \sec(5-x^2)\,dx = -\frac{1}{2}\text{arctanh}\big(\sin(5-x^2)\big)$$

output
```
1/2*arctanh(sin(x^2-5))
```

3.825.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int x \sec(5-x^2)\,dx = -\frac{1}{2}\text{arctanh}\big(\sin(5-x^2)\big)$$

input
```
Integrate[x*Sec[5 - x^2],x]
```

output
```
-1/2*ArcTanh[Sin[5 - x^2]]
```

3.825.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4692, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sec \left(5-x^2\right) \, dx$$
$$\Big\downarrow \text{ 4692}$$
$$\frac{1}{2} \int \sec \left(5-x^2\right) dx^2$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{1}{2} \int \csc \left(-x^2+\frac{\pi}{2}+5\right) dx^2$$
$$\Big\downarrow \text{ 4257}$$
$$-\frac{1}{2} \text{arctanh}\left(\sin \left(5-x^2\right)\right)$$

input
```
Int[x*Sec[5 - x^2],x]
```

output
```
-1/2*ArcTanh[Sin[5 - x^2]]
```

3.825.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4692
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
 1)/n], 0] && IntegerQ[p]
```

3.825.4 Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

method	result	size
derivativedivides	$\frac{\ln(\sec(x^2-5)+\tan(x^2-5))}{2}$	17
default	$\frac{\ln(\sec(x^2-5)+\tan(x^2-5))}{2}$	17
norman	$-\frac{\ln\left(\tan\left(-\frac{5}{2}+\frac{x^2}{2}\right)-1\right)}{2}+\frac{\ln\left(\tan\left(-\frac{5}{2}+\frac{x^2}{2}\right)+1\right)}{2}$	28
parallelrisch	$\ln\left(\frac{1}{\sqrt{\tan\left(-\frac{5}{2}+\frac{x^2}{2}\right)-1}}\right)+\ln\left(\sqrt{\tan\left(-\frac{5}{2}+\frac{x^2}{2}\right)+1}\right)$	28
risch	$-\frac{\ln\left(e^{i(x^2-5)}-i\right)}{2}+\frac{\ln\left(i+e^{i(x^2-5)}\right)}{2}$	32

input
```
int(x*sec(x^2-5),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(sec(x^2-5)+tan(x^2-5))
```

3.825.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(9) = 18.

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.92

$$\int x\sec\left(5-x^2\right)\,dx=\frac{1}{4}\log\left(\sin\left(x^2-5\right)+1\right)-\frac{1}{4}\log\left(-\sin\left(x^2-5\right)+1\right)$$

input
```
integrate(x*sec(x^2-5),x, algorithm="fricas")
```

output
```
1/4*log(sin(x^2 - 5) + 1) - 1/4*log(-sin(x^2 - 5) + 1)
```

3.825.6 Sympy [A] (verification not implemented)

Time = 7.64 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int x\sec\left(5-x^2\right)\,dx=\frac{\log\left(\tan\left(x^2-5\right)+\sec\left(x^2-5\right)\right)}{2}$$

input
```
integrate(x*sec(x**2-5),x)
```

output
```
log(tan(x**2 - 5) + sec(x**2 - 5))/2
```

3.825.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.23

$$\int x\sec\left(5-x^2\right)\,dx=\frac{1}{2}\log\left(\sec\left(x^2-5\right)+\tan\left(x^2-5\right)\right)$$

input
```
integrate(x*sec(x^2-5),x, algorithm="maxima")
```

output
```
1/2*log(sec(x^2 - 5) + tan(x^2 - 5))
```

3.825.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(9) = 18.

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 3.15

$$\int x\sec\left(5-x^2\right)\,dx=\frac{1}{8}\log\left(\left|\frac{1}{\sin\left(x^2-5\right)}+\sin\left(x^2-5\right)+2\right|\right)-\frac{1}{8}\log\left(\left|\frac{1}{\sin\left(x^2-5\right)}+\sin\left(x^2-5\right)-2\right|\right)$$

input
```
integrate(x*sec(x^2-5),x, algorithm="giac")
```

output
```
1/8*log(abs(1/sin(x^2 - 5) + sin(x^2 - 5) + 2)) - 1/8*log(abs(1/sin(x^2 -
5) + sin(x^2 - 5) - 2))
```

3.825.9 Mupad [B] (verification not implemented)

Time = 26.88 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int x\sec\left(5-x^2\right)\,dx=-\mathrm{atan}\left(\mathrm{e}^{-5\mathrm{i}}\,\mathrm{e}^{x^2\,1\mathrm{i}}\right)\,1\mathrm{i}$$

input
```
int(x/cos(x^2 - 5),x)
```

output
```
-atan(exp(-5i)*exp(x^2*1i))*1i
```

3.826 $\int \frac{\csc\left(\frac{1}{x}\right)}{x^2}\,dx$

3.826.1 Optimal result

Integrand size = 8, antiderivative size = 5

$$\int \frac{\csc\left(\frac{1}{x}\right)}{x^2}\,dx = \operatorname{arctanh}\left(\cos\left(\frac{1}{x}\right)\right)$$

output
```
arctanh(cos(1/x))
```

3.826.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 21 vs. 2(5) = 10.

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 4.20

$$\int \frac{\csc\left(\frac{1}{x}\right)}{x^2}\,dx = \log\left(\cos\left(\frac{1}{2x}\right)\right) - \log\left(\sin\left(\frac{1}{2x}\right)\right)$$

input
```
Integrate[Csc[x^(-1)]/x^2,x]
```

output
```
Log[Cos[1/(2*x)]] - Log[Sin[1/(2*x)]]
```

3.826.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4693, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc\left(\frac{1}{x}\right)}{x^2}dx$$
$$\Big\downarrow \text{4693}$$
$$-\int \csc\left(\frac{1}{x}\right)d\frac{1}{x}$$
$$\Big\downarrow \text{3042}$$
$$-\int \csc\left(\frac{1}{x}\right)d\frac{1}{x}$$
$$\Big\downarrow \text{4257}$$
$$\text{arctanh}\left(\cos\left(\frac{1}{x}\right)\right)$$

input
```
Int[Csc[x^(-1)]/x^2,x]
```

output
```
ArcTanh[Cos[x^(-1)]]
```

3.826.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4693
```
Int[((a_.) + Csc[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Csc[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
 1)/n], 0] && IntegerQ[p]
```

3.826.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 2.00

method	result	size
norman	$-\ln\left(\tan\left(\frac{1}{2x}\right)\right)$	10
parallelrisch	$-\ln\left(\tan\left(\frac{1}{2x}\right)\right)$	10
derivativedivides	$\ln\left(\csc\left(\frac{1}{x}\right)+\cot\left(\frac{1}{x}\right)\right)$	11
default	$\ln\left(\csc\left(\frac{1}{x}\right)+\cot\left(\frac{1}{x}\right)\right)$	11
risch	$-\ln\left({\mathrm e}^{\frac{i}{x}}-1\right)+\ln\left({\mathrm e}^{\frac{i}{x}}+1\right)$	24

input
```
int(csc(1/x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-ln(tan(1/2/x))
```

3.826.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. 2(5) = 10.

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 4.60

$$\int \frac{\csc\left(\frac{1}{x}\right)}{x^2}\,dx = \frac{1}{2}\log\left(\frac{1}{2}\cos\left(\frac{1}{x}\right)+\frac{1}{2}\right)-\frac{1}{2}\log\left(-\frac{1}{2}\cos\left(\frac{1}{x}\right)+\frac{1}{2}\right)$$

input
```
integrate(csc(1/x)/x^2,x, algorithm="fricas")
```

output
```
1/2*log(1/2*cos(1/x) + 1/2) - 1/2*log(-1/2*cos(1/x) + 1/2)
```

3.826.6 Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 10, normalized size of antiderivative = 2.00

$$\int \frac{\csc\left(\frac{1}{x}\right)}{x^2}\,dx = \log\left(\cot\left(\frac{1}{x}\right) + \csc\left(\frac{1}{x}\right)\right)$$

input
```
integrate(csc(1/x)/x**2,x)
```

output
```
log(cot(1/x) + csc(1/x))
```

3.826.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 10, normalized size of antiderivative = 2.00

$$\int \frac{\csc\left(\frac{1}{x}\right)}{x^2}\,dx = \log\left(\cot\left(\frac{1}{x}\right) + \csc\left(\frac{1}{x}\right)\right)$$

input
```
integrate(csc(1/x)/x^2,x, algorithm="maxima")
```

output
```
log(cot(1/x) + csc(1/x))
```

3.826.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. 2(5) = 10.

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 8.60

$$\int \frac{\csc\left(\frac{1}{x}\right)}{x^2}\,dx = -\frac{1}{2}\log\left(\frac{4\,\tan\left(\frac{1}{2\,x}\right)^2}{\tan\left(\frac{1}{2\,x}\right)^2+1}\right) + \frac{1}{2}\log\left(\frac{4}{\tan\left(\frac{1}{2\,x}\right)^2+1}\right)$$

input
```
integrate(csc(1/x)/x^2,x, algorithm="giac")
```

output
```
-1/2*log(4*tan(1/2/x)^2/(tan(1/2/x)^2 + 1)) + 1/2*log(4/(tan(1/2/x)^2 + 1)
)
```

3.826.9 Mupad [B] (verification not implemented)

Time = 27.84 (sec) , antiderivative size = 31, normalized size of antiderivative = 6.20

$$\int \frac{\csc\left(\frac{1}{x}\right)}{x^2}\,dx = \ln\left(-\mathrm{e}^{1\mathrm{i}/x}\,2\mathrm{i} - 2\mathrm{i}\right) - \ln\left(-\mathrm{e}^{1\mathrm{i}/x}\,2\mathrm{i} + 2\mathrm{i}\right)$$

input
```
int(1/(x^2*sin(1/x)),x)
```

output
```
log(- exp(1i/x)*2i - 2i) - log(2i - exp(1i/x)*2i)
```

3.827 $\int (\csc(x)-\sec(x))(\cos(x)+\sin(x))\,dx$

3.827.1 Optimal result

Integrand size = 13, antiderivative size = 7

$$\int (\csc(x)-\sec(x))(\cos(x)+\sin(x))\,dx=\log(\cos(x))+\log(\sin(x))$$

output
```
ln(cos(x))+ln(sin(x))
```

3.827.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int (\csc(x)-\sec(x))(\cos(x)+\sin(x))\,dx=2\log(\cos(x))+\log(\tan(x))$$

input
```
Integrate[(Csc[x] - Sec[x])*(Cos[x] + Sin[x]),x]
```

output
```
2*Log[Cos[x]] + Log[Tan[x]]
```

3.827.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 19 vs. $2(7) = 14$.

Time = 0.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 2.71, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3042, 4889, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\sin(x)+\cos(x))(\csc(x)-\sec(x))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (\sin(x)+\cos(x))(\csc(x)-\sec(x))dx$$

$$\Big\downarrow \text{ 4889}$$

$$\int \frac{(1-\tan^2(x))\cot(x)}{\tan^2(x)+1}d\tan(x)$$

$$\Big\downarrow \text{ 354}$$

$$\frac{1}{2}\int \frac{\cot(x)\left(1-\tan^2(x)\right)}{\tan^2(x)+1}d\tan^2(x)$$

$$\Big\downarrow \text{ 86}$$

$$\frac{1}{2}\int \left(\cot(x)-\frac{2}{\tan^2(x)+1}\right)d\tan^2(x)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\log\left(\tan^2(x)\right)-2\log\left(\tan^2(x)+1\right)\right)$$

input
```
Int[(Csc[x] - Sec[x])*(Cos[x] + Sin[x]),x]
```

output
```
(Log[Tan[x]^2] - 2*Log[1 + Tan[x]^2])/2
```

3.827.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.827.4 Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

method	result
default	$\ln(\cos(x)) + \ln(\sin(x))$
parts	$\ln(\sin(x)) - \ln(\sec(x))$
risch	$-2ix + \ln(e^{4ix} - 1)$
norman	$-2\ln\left(1 + \tan\left(\frac{x}{2}\right)^2\right) + \ln\left(\tan\left(\frac{x}{2}\right)\right) + \ln\left(\tan\left(\frac{x}{2}\right) - 1\right) + \ln\left(\tan\left(\frac{x}{2}\right) + 1\right)$
parallelrisch	$-2\ln\left(\frac{1}{\cos(x)+1}\right) + \ln\left(\frac{\csc(x)}{4} - \frac{\cot(x)}{4}\right) + \ln(-\cot(x) + \csc(x) - 1) + \ln(\csc(x) - \cot(x) +$

input
```
int((csc(x)-sec(x))*(sin(x)+cos(x)),x,method=_RETURNVERBOSE)
```

output
```
ln(cos(x))+ln(sin(x))
```

3.827.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int (\csc(x) - \sec(x))(\cos(x) + \sin(x))\,dx = \log\left(-\frac{1}{2}\cos(x)\sin(x)\right)$$

input
```
integrate((csc(x)-sec(x))*(cos(x)+sin(x)),x, algorithm="fricas")
```

output
```
log(-1/2*cos(x)*sin(x))
```

3.827.6 Sympy [A] (verification not implemented)

Time = 1.16 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

$$\int (\csc(x) - \sec(x))(\cos(x) + \sin(x))\,dx = \log(\sin(x)) + \log(\cos(x))$$

input
```
integrate((csc(x)-sec(x))*(cos(x)+sin(x)),x)
```

output
```
log(sin(x)) + log(cos(x))
```

3.827.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(7) = 14.

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 2.14

$$\int (\csc(x) - \sec(x))(\cos(x) + \sin(x))\,dx = \frac{1}{2}\log\left(-\sin(x)^2 + 1\right) + \log(\sin(x))$$

input
```
integrate((csc(x)-sec(x))*(cos(x)+sin(x)),x, algorithm="maxima")
```

output
```
1/2*log(-sin(x)^2 + 1) + log(sin(x))
```

3.827.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16 vs. $2(7) = 14$.

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 2.29

$$\int (\csc(x) - \sec(x))(\cos(x) + \sin(x))\, dx = \frac{1}{2}\log\left(-\cos(x)^2 + 1\right) + \log\left(|\cos(x)|\right)$$

input
```
integrate((csc(x)-sec(x))*(cos(x)+sin(x)),x, algorithm="giac")
```

output
```
1/2*log(-cos(x)^2 + 1) + log(abs(cos(x)))
```

3.827.9 Mupad [B] (verification not implemented)

Time = 26.44 (sec) , antiderivative size = 26, normalized size of antiderivative = 3.71

$$\int (\csc(x) - \sec(x))(\cos(x) + \sin(x))\, dx = \ln\left(\tan\left(\frac{x}{2}\right)^3 - \tan\left(\frac{x}{2}\right)\right) - 2\ln\left(\tan\left(\frac{x}{2}\right)^2 + 1\right)$$

input
```
int(-(cos(x) + sin(x))*(1/cos(x) - 1/sin(x)),x)
```

output
```
log(tan(x/2)^3 - tan(x/2)) - 2*log(tan(x/2)^2 + 1)
```

3.828 $\int (-\cos(3x)\sin(2x)+\cos(2x)\sin(3x))\,dx$

3.828.1 Optimal result

Integrand size = 20, antiderivative size = 4

$$\int (-\cos(3x)\sin(2x)+\cos(2x)\sin(3x))\,dx=-\cos(x)$$

output
```
-cos(x)
```

3.828.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int (-\cos(3x)\sin(2x)+\cos(2x)\sin(3x))\,dx=-\cos(x)$$

input
```
Integrate[-(Cos[3*x]*Sin[2*x]) + Cos[2*x]*Sin[3*x],x]
```

output
```
-Cos[x]
```

3.828.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\sin(3x)\cos(2x) - \sin(2x)\cos(3x))\,dx$$

$$\Big\downarrow \text{2009}$$

$$-\cos(x)$$

input
```
Int[-(Cos[3*x]*Sin[2*x]) + Cos[2*x]*Sin[3*x],x]
```

output
```
-Cos[x]
```

3.828.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.828.4 Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

method	result	size
default	$-\cos(x)$	5
risch	$-\cos(x)$	5
parts	$-\cos(x)$	5
parallelrisch	$1-\cos(x)$	7
norman	$\frac{2\tan(x)^2+2\tan(\frac{3x}{2})^2-4\tan(x)\tan(\frac{3x}{2})}{\left(1+\tan(\frac{3x}{2})^2\right)\left(1+\tan(x)^2\right)}$	43

input
```
int(-cos(3*x)*sin(2*x)+cos(2*x)*sin(3*x),x,method=_RETURNVERBOSE)
```

output
```
-cos(x)
```

3.828.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int (-\cos(3x)\sin(2x)+\cos(2x)\sin(3x))\,dx = -\cos{(x)}$$

input
```
integrate(-cos(3*x)*sin(2*x)+cos(2*x)*sin(3*x),x, algorithm="fricas")
```

output
```
-cos(x)
```

3.828.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. 2(3) = 6.

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 5.00

$$\int (-\cos(3x)\sin(2x)+\cos(2x)\sin(3x))\,dx = -\sin{(2x)}\sin{(3x)} - \cos{(2x)}\cos{(3x)}$$

input
```
integrate(-cos(3*x)*sin(2*x)+cos(2*x)*sin(3*x),x)
```

output
```
-sin(2*x)*sin(3*x) - cos(2*x)*cos(3*x)
```

3.828.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int (-\cos(3x)\sin(2x)+\cos(2x)\sin(3x))\,dx = -\cos{(x)}$$

input
```
integrate(-cos(3*x)*sin(2*x)+cos(2*x)*sin(3*x),x, algorithm="maxima")
```

output
```
-cos(x)
```

3.828.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int (-\cos(3x)\sin(2x)+\cos(2x)\sin(3x))\,dx=-\cos(x)$$

input
```
integrate(-cos(3*x)*sin(2*x)+cos(2*x)*sin(3*x),x, algorithm="giac")
```

output
```
-cos(x)
```

3.828.9 Mupad [B] (verification not implemented)

Time = 27.41 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int (-\cos(3x)\sin(2x)+\cos(2x)\sin(3x))\,dx=-\cos(x)$$

input
```
int(cos(2*x)*sin(3*x) - cos(3*x)*sin(2*x),x)
```

output
```
-cos(x)
```

3.829 $\int 4x \sec^2(2x)\,dx$

3.829.1 Optimal result

Integrand size = 9, antiderivative size = 13

$$\int 4x \sec^2(2x)\,dx = \log(\cos(2x)) + 2x\tan(2x)$$

output
```
ln(cos(2*x))+2*x*tan(2*x)
```

3.829.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62

$$\int 4x \sec^2(2x)\,dx = 4\left(\frac{1}{4}\log(\cos(2x)) + \frac{1}{2}x\tan(2x)\right)$$

input
```
Integrate[4*x*Sec[2*x]^2,x]
```

output
```
4*(Log[Cos[2*x]]/4 + (x*Tan[2*x])/2)
```

3.829.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {27, 3042, 4672, 25, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int 4x\sec^2(2x)\,dx$$

$$\Big\downarrow \text{27}$$

$$4\int x\sec^2(2x)dx$$

$$\Big\downarrow \text{3042}$$

$$4\int x\csc\left(2x+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{4672}$$

$$4\left(\frac{1}{2}\int -\tan(2x)dx+\frac{1}{2}x\tan(2x)\right)$$

$$\Big\downarrow \text{25}$$

$$4\left(\frac{1}{2}x\tan(2x)-\frac{1}{2}\int \tan(2x)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$4\left(\frac{1}{2}x\tan(2x)-\frac{1}{2}\int \tan(2x)dx\right)$$

$$\Big\downarrow \text{3956}$$

$$4\left(\frac{1}{2}x\tan(2x)+\frac{1}{4}\log(\cos(2x))\right)$$

input
```
Int[4*x*Sec[2*x]^2,x]
```

output
```
4*(Log[Cos[2*x]]/4 + (x*Tan[2*x])/2)
```

3.829.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.829.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$\ln\left(\cos\left(2x\right)\right)+2x\tan\left(2x\right)$	14
default	$\ln\left(\cos\left(2x\right)\right)+2x\tan\left(2x\right)$	14
risch	$-4ix+\frac{4ix}{\mathrm{e}^{4ix}+1}+\ln\left(\mathrm{e}^{4ix}+1\right)$	27
norman	$-\frac{4x\tan(x)}{\tan(x)^{2}-1}-\ln\left(1+\tan\left(x\right)^{2}\right)+\ln\left(\tan\left(x\right)-1\right)+\ln\left(1+\tan\left(x\right)\right)$	34
parallelrisch	$\frac{\ln(\tan(x)-1)\cos(2x)+\ln(1+\tan(x))\cos(2x)-\ln\left(\sec(x)^{2}\right)\cos(2x)+2x\sin(2x)}{\cos(2x)}$	47

input
```
int(4*x*sec(2*x)^2,x,method=_RETURNVERBOSE)
```

output
```
ln(cos(2*x))+2*x*tan(2*x)
```

3.829.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(13) = 26$.

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.08

$$\int 4x\sec^2(2x)\,dx = \frac{\cos(2x)\log(-\cos(2x)) + 2x\sin(2x)}{\cos(2x)}$$

input
```
integrate(4*x*sec(2*x)^2,x, algorithm="fricas")
```

output
```
(cos(2*x)*log(-cos(2*x)) + 2*x*sin(2*x))/cos(2*x)
```

3.829.6 Sympy [F]

$$\int 4x\sec^2(2x)\,dx = 4\int x\sec^2(2x)\,dx$$

input
```
integrate(4*x*sec(2*x)**2,x)
```

output
```
4*Integral(x*sec(2*x)**2, x)
```

3.829.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(13) = 26$.

Time = 0.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 5.69

$$\int 4x\sec^2(2x)\,dx$$
$$= \frac{(\cos(4x)^2 + \sin(4x)^2 + 2\cos(4x) + 1)\log(\cos(4x)^2 + \sin(4x)^2 + 2\cos(4x) + 1) + 8x\sin(4x)}{2(\cos(4x)^2 + \sin(4x)^2 + 2\cos(4x) + 1)}$$

input
```
integrate(4*x*sec(2*x)^2,x, algorithm="maxima")
```

output
```
1/2*((cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)*log(cos(4*x)^2 + sin(4*x)^
2 + 2*cos(4*x) + 1) + 8*x*sin(4*x))/(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x)
+ 1)
```

3.829.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(13) = 26$.

Time = 0.29 (sec) , antiderivative size = 81, normalized size of antiderivative = 6.23

$$\int 4x\sec^2(2x)\,dx = \frac{\log\left(\frac{4\left(\tan(x)^4-2\tan(x)^2+1\right)}{\tan(x)^4+2\tan(x)^2+1}\right)\tan(x)^2 - 8x\tan(x) - \log\left(\frac{4\left(\tan(x)^4-2\tan(x)^2+1\right)}{\tan(x)^4+2\tan(x)^2+1}\right)}{2\left(\tan(x)^2-1\right)}$$

input
```
integrate(4*x*sec(2*x)^2,x, algorithm="giac")
```

output
```
1/2*(log(4*(tan(x)^4 - 2*tan(x)^2 + 1)/(tan(x)^4 + 2*tan(x)^2 + 1))*tan(x)
^2 - 8*x*tan(x) - log(4*(tan(x)^4 - 2*tan(x)^2 + 1)/(tan(x)^4 + 2*tan(x)^2
 + 1)))/(tan(x)^2 - 1)
```

3.829.9 Mupad [B] (verification not implemented)

Time = 26.95 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int 4x\sec^2(2x)\,dx = \ln(\cos(2x)) + 2x\tan(2x)$$

input
```
int((4*x)/cos(2*x)^2,x)
```

output
```
log(cos(2*x)) + 2*x*tan(2*x)
```

3.830 $\int 4\sin^2(x)\tan^2(x)\,dx$

3.830.1 Optimal result

Integrand size = 10, antiderivative size = 16

$$\int 4\sin^2(x)\tan^2(x)\,dx = -6x + 6\tan(x) - 2\sin^2(x)\tan(x)$$

output
```
-6*x+6*tan(x)-2*sin(x)^2*tan(x)
```

3.830.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int 4\sin^2(x)\tan^2(x)\,dx = 4\left(-\frac{3x}{2} + \frac{1}{4}\sin(2x) + \tan(x)\right)$$

input
```
Integrate[4*Sin[x]^2*Tan[x]^2,x]
```

output
```
4*((-3*x)/2 + Sin[2*x]/4 + Tan[x])
```

3.830.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {27, 3042, 3071, 252, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int 4\sin^2(x)\tan^2(x)\,dx$$

$$\Big\downarrow \text{27}$$

$$4\int \sin^2(x)\tan^2(x)dx$$

$$\Big\downarrow \text{3042}$$

$$4\int \sin(x)^2\tan(x)^2dx$$

$$\Big\downarrow \text{3071}$$

$$4\int \frac{\tan^4(x)}{(\tan^2(x)+1)^2}d\tan(x)$$

$$\Big\downarrow \text{252}$$

$$4\left(\frac{3}{2}\int \frac{\tan^2(x)}{\tan^2(x)+1}d\tan(x)-\frac{\tan^3(x)}{2(\tan^2(x)+1)}\right)$$

$$\Big\downarrow \text{262}$$

$$4\left(\frac{3}{2}\left(\tan(x)-\int \frac{1}{\tan^2(x)+1}d\tan(x)\right)-\frac{\tan^3(x)}{2(\tan^2(x)+1)}\right)$$

$$\Big\downarrow \text{216}$$

$$4\left(\frac{3}{2}(\tan(x)-\arctan(\tan(x)))-\frac{\tan^3(x)}{2(\tan^2(x)+1)}\right)$$

input
```
Int[4*Sin[x]^2*Tan[x]^2,x]
```

output
```
4*((3*(-ArcTan[Tan[x]] + Tan[x]))/2 - Tan[x]^3/(2*(1 + Tan[x]^2)))
```

3.830.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

3.830. $\int 4\sin^2(x)\tan^2(x)\,dx$

3.830.4 Maple [A] (verified)

Time = 2.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.75

method	result	size
default	$\frac{4\sin(x)^5}{\cos(x)}+4\left(\sin(x)^3+\frac{3\sin(x)}{2}\right)\cos(x)-6x$	28
risch	$-6x-\frac{i e^{2ix}}{2}+\frac{i e^{-2ix}}{2}+\frac{8i}{e^{2ix}+1}$	33

input
```
int(4*sin(x)^2*tan(x)^2,x,method=_RETURNVERBOSE)
```

output
```
4*sin(x)^5/cos(x)+4*(sin(x)^3+3/2*sin(x))*cos(x)-6*x
```

3.830.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int 4\sin^2(x)\tan^2(x)\,dx = -\frac{2\left(3x\cos(x)-\left(\cos(x)^2+2\right)\sin(x)\right)}{\cos(x)}$$

input
```
integrate(4*sin(x)^2*tan(x)^2,x, algorithm="fricas")
```

output
```
-2*(3*x*cos(x) - (cos(x)^2 + 2)*sin(x))/cos(x)
```

3.830.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int 4\sin^2(x)\tan^2(x)\,dx = -6x+\frac{4\sin^3(x)}{\cos(x)}+6\sin(x)\cos(x)$$

input
```
integrate(4*sin(x)**2*tan(x)**2,x)
```

output
```
-6*x + 4*sin(x)**3/cos(x) + 6*sin(x)*cos(x)
```

3.830.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int 4\sin^2(x)\tan^2(x)\,dx = -6\,x + \frac{2\,\tan\left(x\right)}{\tan\left(x\right)^2 + 1} + 4\,\tan\left(x\right)$$

input
```
integrate(4*sin(x)^2*tan(x)^2,x, algorithm="maxima")
```

output
```
-6*x + 2*tan(x)/(tan(x)^2 + 1) + 4*tan(x)
```

3.830.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25

$$\int 4\sin^2(x)\tan^2(x)\,dx = -6\,x + \frac{2\,\tan\left(x\right)}{\tan\left(x\right)^2 + 1} + 4\,\tan\left(x\right)$$

input
```
integrate(4*sin(x)^2*tan(x)^2,x, algorithm="giac")
```

output
```
-6*x + 2*tan(x)/(tan(x)^2 + 1) + 4*tan(x)
```

3.830.9 Mupad [B] (verification not implemented)

Time = 26.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int 4\sin^2(x)\tan^2(x)\,dx = 2\,\cos\left(x\right)\,\sin\left(x\right) - 6\,x + \frac{4\,\sin\left(x\right)}{\cos\left(x\right)}$$

input
```
int(4*sin(x)^2*tan(x)^2,x)
```

output
```
2*cos(x)*sin(x) - 6*x + (4*sin(x))/cos(x)
```

3.831 $\int \cos^4(x) \cot^2(x) \, dx$

3.831.1 Optimal result

Integrand size = 9, antiderivative size = 32

$$\int \cos^4(x) \cot^2(x) \, dx = -\frac{15x}{8} - \frac{15 \cot(x)}{8} + \frac{5}{8} \cos^2(x) \cot(x) + \frac{1}{4} \cos^4(x) \cot(x)$$

output
```
-15/8*x-15/8*cot(x)+5/8*cos(x)^2*cot(x)+1/4*cos(x)^4*cot(x)
```

3.831.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \cos^4(x) \cot^2(x) \, dx = -\frac{15x}{8} - \cot(x) - \frac{1}{2} \sin(2x) - \frac{1}{32} \sin(4x)$$

input
```
Integrate[Cos[x]^4*Cot[x]^2,x]
```

output
```
(-15*x)/8 - Cot[x] - Sin[2*x]/2 - Sin[4*x]/32
```

3.831.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.56, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 3071, 252, 252, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(x)\cot^2(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(x+\frac{\pi}{2}\right)^4\tan\left(x+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{3071}$$

$$-\int \frac{\cot^6(x)}{\left(\cot^2(x)+1\right)^3}d\cot(x)$$

$$\Big\downarrow \text{252}$$

$$\frac{\cot^5(x)}{4\left(\cot^2(x)+1\right)^2}-\frac{5}{4}\int \frac{\cot^4(x)}{\left(\cot^2(x)+1\right)^2}d\cot(x)$$

$$\Big\downarrow \text{252}$$

$$\frac{\cot^5(x)}{4\left(\cot^2(x)+1\right)^2}-\frac{5}{4}\left(\frac{3}{2}\int \frac{\cot^2(x)}{\cot^2(x)+1}d\cot(x)-\frac{\cot^3(x)}{2\left(\cot^2(x)+1\right)}\right)$$

$$\Big\downarrow \text{262}$$

$$\frac{\cot^5(x)}{4\left(\cot^2(x)+1\right)^2}-\frac{5}{4}\left(\frac{3}{2}\left(\cot(x)-\int \frac{1}{\cot^2(x)+1}d\cot(x)\right)-\frac{\cot^3(x)}{2\left(\cot^2(x)+1\right)}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{\cot^5(x)}{4\left(\cot^2(x)+1\right)^2}-\frac{5}{4}\left(\frac{3}{2}(\cot(x)-\arctan(\cot(x)))-\frac{\cot^3(x)}{2\left(\cot^2(x)+1\right)}\right)$$

input
```
Int[Cos[x]^4*Cot[x]^2,x]
```

output
```
Cot[x]^5/(4*(1 + Cot[x]^2)^2) - (5*((3*(-ArcTan[Cot[x]] + Cot[x]))/2 - Cot
[x]^3/(2*(1 + Cot[x]^2))))/4
```

3.831.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3071
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[I
nt[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, b*(Tan[e + f*x]/ff)],
x]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

3.831.4 Maple [A] (verified)

Time = 34.70 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06

method	result	size
default	$-\frac{\cos(x)^7}{\sin(x)}-\left(\cos(x)^5+\frac{5\cos(x)^3}{4}+\frac{15\cos(x)}{8}\right)\sin(x)-\frac{15x}{8}$	34
risch	$-\frac{15x}{8}+\frac{i e^{2ix}}{4}-\frac{i e^{-2ix}}{4}-\frac{2i}{e^{2ix}-1}-\frac{\sin(4x)}{32}$	39

input
```
int(cos(x)^4*cot(x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/sin(x)*cos(x)^7-(cos(x)^5+5/4*cos(x)^3+15/8*cos(x))*sin(x)-15/8*x
```

3.831.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \cos^4(x)\cot^2(x)\,dx = \frac{2\,\cos\left(x\right)^5 + 5\,\cos\left(x\right)^3 - 15\,x\sin\left(x\right) - 15\,\cos\left(x\right)}{8\,\sin\left(x\right)}$$

input
```
integrate(cos(x)^4*cot(x)^2,x, algorithm="fricas")
```

output
```
1/8*(2*cos(x)^5 + 5*cos(x)^3 - 15*x*sin(x) - 15*cos(x))/sin(x)
```

3.831.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12

$$\int \cos^4(x)\cot^2(x)\,dx = -\frac{15x}{8} - \frac{5\sin\left(x\right)\cos^3\left(x\right)}{4} - \frac{15\sin\left(x\right)\cos\left(x\right)}{8} - \frac{\cos^5\left(x\right)}{\sin\left(x\right)}$$

input
```
integrate(cos(x)**4*cot(x)**2,x)
```

output
```
-15*x/8 - 5*sin(x)*cos(x)**3/4 - 15*sin(x)*cos(x)/8 - cos(x)**5/sin(x)
```

3.831.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.09

$$\int \cos^4(x)\cot^2(x)\,dx = -\frac{15}{8}\,x - \frac{15\,\tan\left(x\right)^4 + 25\,\tan\left(x\right)^2 + 8}{8\left(\tan\left(x\right)^5 + 2\,\tan\left(x\right)^3 + \tan\left(x\right)\right)}$$

input
```
integrate(cos(x)^4*cot(x)^2,x, algorithm="maxima")
```

output
```
-15/8*x - 1/8*(15*tan(x)^4 + 25*tan(x)^2 + 8)/(tan(x)^5 + 2*tan(x)^3 + tan
(x))
```

3.831.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \cos^4(x)\cot^2(x)\,dx = -\frac{15}{8}\,x - \frac{7\,\tan{(x)}^3 + 9\,\tan{(x)}}{8\,\left(\tan{(x)}^2 + 1\right)^2} - \frac{1}{\tan{(x)}}$$

input
```
integrate(cos(x)^4*cot(x)^2,x, algorithm="giac")
```

output
```
-15/8*x - 1/8*(7*tan(x)^3 + 9*tan(x))/(tan(x)^2 + 1)^2 - 1/tan(x)
```

3.831.9 Mupad [B] (verification not implemented)

Time = 27.70 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \cos^4(x)\cot^2(x)\,dx = \frac{\frac{\cos(x)^5}{4} + \frac{5\cos(x)^3}{8} - \frac{15\,\cos(x)}{8}}{\sin{(x)}} - \frac{15\,x}{8}$$

input
```
int(cos(x)^4*cot(x)^2,x)
```

output
```
((5*cos(x)^3)/8 - (15*cos(x))/8 + cos(x)^5/4)/sin(x) - (15*x)/8
```

3.832 $\int 16\cos^2(x)\sin^2(x)\,dx$

3.832.1 Optimal result

Integrand size = 10, antiderivative size = 18

$$\int 16\cos^2(x)\sin^2(x)\,dx = 2x + 2\cos(x)\sin(x) - 4\cos^3(x)\sin(x)$$

output
```
2*x+2*cos(x)*sin(x)-4*cos(x)^3*sin(x)
```

3.832.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int 16\cos^2(x)\sin^2(x)\,dx = 4\left(\frac{x}{2} - \frac{1}{8}\sin(4x)\right)$$

input
```
Integrate[16*Cos[x]^2*Sin[x]^2,x]
```

output
```
4*(x/2 - Sin[4*x]/8)
```

3.832.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.72, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {27, 3042, 3048, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int 16\sin^2(x)\cos^2(x)\,dx$$

$$\Big\downarrow \text{ 27}$$

$$16\int \cos^2(x)\sin^2(x)dx$$

$$\Big\downarrow \text{ 3042}$$

$$16\int \cos(x)^2\sin(x)^2dx$$

$$\Big\downarrow \text{ 3048}$$

$$16\left(\frac{1}{4}\int \cos^2(x)dx-\frac{1}{4}\sin(x)\cos^3(x)\right)$$

$$\Big\downarrow \text{ 3042}$$

$$16\left(\frac{1}{4}\int \sin\left(x+\frac{\pi}{2}\right)^2dx-\frac{1}{4}\sin(x)\cos^3(x)\right)$$

$$\Big\downarrow \text{ 3115}$$

$$16\left(\frac{1}{4}\left(\frac{\int 1dx}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)$$

$$\Big\downarrow \text{ 24}$$

$$16\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)$$

input
```
Int[16*Cos[x]^2*Sin[x]^2,x]
```

output
```
16*(-1/4*(Cos[x]^3*Sin[x]) + (x/2 + (Cos[x]*Sin[x])/2)/4)
```

3.832.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.832.4 Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.61

method	result	size
risch	$2x-\frac{\sin(4x)}{2}$	11
parallelrisch	$2x-\frac{\sin(4x)}{2}$	11
default	$2x+2\cos\left(x\right)\sin\left(x\right)-4\cos\left(x\right)^{3}\sin\left(x\right)$	19
norman	$\frac{2x+28\tan\left(\frac{x}{2}\right)^{3}-28\tan\left(\frac{x}{2}\right)^{5}+4\tan\left(\frac{x}{2}\right)^{7}+8x\tan\left(\frac{x}{2}\right)^{2}+12x\tan\left(\frac{x}{2}\right)^{4}+8x\tan\left(\frac{x}{2}\right)^{6}+2x\tan\left(\frac{x}{2}\right)^{8}-4\tan\left(\frac{x}{2}\right)}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)^{4}}$	82

input
```
int(16*sin(x)^2*cos(x)^2,x,method=_RETURNVERBOSE)
```

output
```
2*x-1/2*sin(4*x)
```

3.832.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int 16\cos^2(x)\sin^2(x)\,dx = -2\left(2\cos(x)^3-\cos(x)\right)\sin(x)+2x$$

input
```
integrate(16*cos(x)^2*sin(x)^2,x, algorithm="fricas")
```

output
```
-2*(2*cos(x)^3 - cos(x))*sin(x) + 2*x
```

3.832.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int 16\cos^2(x)\sin^2(x)\,dx = 2x-\sin(2x)\cos(2x)$$

input
```
integrate(16*cos(x)**2*sin(x)**2,x)
```

output
```
2*x - sin(2*x)*cos(2*x)
```

3.832.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.56

$$\int 16\cos^2(x)\sin^2(x)\,dx = 2x-\frac{1}{2}\sin(4x)$$

input
```
integrate(16*cos(x)^2*sin(x)^2,x, algorithm="maxima")
```

output
```
2*x - 1/2*sin(4*x)
```

3.832.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.56

$$\int 16\cos^2(x)\sin^2(x)\,dx = 2\,x - \frac{1}{2}\,\sin\left(4\,x\right)$$

input
```
integrate(16*cos(x)^2*sin(x)^2,x, algorithm="giac")
```

output
```
2*x - 1/2*sin(4*x)
```

3.832.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int 16\cos^2(x)\sin^2(x)\,dx = 4\,\cos\left(x\right)\,\sin\left(x\right)^3 - 2\,\cos\left(x\right)\,\sin\left(x\right) + 2\,x$$

input
```
int(16*cos(x)^2*sin(x)^2,x)
```

output
```
2*x - 2*cos(x)*sin(x) + 4*cos(x)*sin(x)^3
```

3.833 $\int 8\cos^2(x)\sin^4(x)\,dx$

3.833.1 Optimal result

Integrand size = 10, antiderivative size = 34

$$\int 8\cos^2(x)\sin^4(x)\,dx = \frac{x}{2}+\frac{1}{2}\cos(x)\sin(x)-\cos^3(x)\sin(x)-\frac{4}{3}\cos^3(x)\sin^3(x)$$

output
```
1/2*x+1/2*cos(x)*sin(x)-cos(x)^3*sin(x)-4/3*cos(x)^3*sin(x)^3
```

3.833.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int 8\cos^2(x)\sin^4(x)\,dx = 8\left(\frac{x}{16}-\frac{1}{64}\sin(2x)-\frac{1}{64}\sin(4x)+\frac{1}{192}\sin(6x)\right)$$

input
```
Integrate[8*Cos[x]^2*Sin[x]^4,x]
```

output
```
8*(x/16 - Sin[2*x]/64 - Sin[4*x]/64 + Sin[6*x]/192)
```

3.833.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.41, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {27, 3042, 3048, 3042, 3048, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int 8\sin^4(x)\cos^2(x)\,dx$$

$$\Big\downarrow \text{27}$$

$$8\int \cos^2(x)\sin^4(x)dx$$

$$\Big\downarrow \text{3042}$$

$$8\int \cos(x)^2\sin(x)^4dx$$

$$\Big\downarrow \text{3048}$$

$$8\left(\frac{1}{2}\int \cos^2(x)\sin^2(x)dx-\frac{1}{6}\sin^3(x)\cos^3(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$8\left(\frac{1}{2}\int \cos(x)^2\sin(x)^2dx-\frac{1}{6}\sin^3(x)\cos^3(x)\right)$$

$$\Big\downarrow \text{3048}$$

$$8\left(\frac{1}{2}\left(\frac{1}{4}\int \cos^2(x)dx-\frac{1}{4}\sin(x)\cos^3(x)\right)-\frac{1}{6}\sin^3(x)\cos^3(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$8\left(\frac{1}{2}\left(\frac{1}{4}\int \sin\left(x+\frac{\pi}{2}\right)^2dx-\frac{1}{4}\sin(x)\cos^3(x)\right)-\frac{1}{6}\sin^3(x)\cos^3(x)\right)$$

$$\Big\downarrow \text{3115}$$

$$8\left(\frac{1}{2}\left(\frac{1}{4}\left(\frac{\int 1dx}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)-\frac{1}{6}\sin^3(x)\cos^3(x)\right)$$

$$\Big\downarrow \text{24}$$

$$8\left(\frac{1}{2}\left(\frac{1}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)-\frac{1}{4}\sin(x)\cos^3(x)\right)-\frac{1}{6}\sin^3(x)\cos^3(x)\right)$$

input
```
Int[8*Cos[x]^2*Sin[x]^4,x]
```

output
```
8*(-1/6*(Cos[x]^3*Sin[x]^3) + (-1/4*(Cos[x]^3*Sin[x]) + (x/2 + (Cos[x]*Sin
[x])/2)/4)/2)
```

3.833.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.833.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

method	result
risch	$\frac{x}{2}+\frac{\sin(6x)}{24}-\frac{\sin(4x)}{8}-\frac{\sin(2x)}{8}$
parallelrisch	$\frac{x}{2}+\frac{\sin(6x)}{24}-\frac{\sin(4x)}{8}-\frac{\sin(2x)}{8}$
default	$\frac{x}{2}+\frac{\cos(x)\sin(x)}{2}-\cos\left(x\right)^{3}\sin\left(x\right)-\frac{4\cos(x)^{3}\sin(x)^{3}}{3}$
norman	$\frac{\tan\left(\frac{x}{2}\right)^{11}+\frac{x}{2}-\frac{17\tan\left(\frac{x}{2}\right)^{3}}{3}+38\tan\left(\frac{x}{2}\right)^{5}-38\tan\left(\frac{x}{2}\right)^{7}+\frac{17\tan\left(\frac{x}{2}\right)^{9}}{3}+3x\tan\left(\frac{x}{2}\right)^{2}+\frac{15x\tan\left(\frac{x}{2}\right)^{4}}{2}+10x\tan\left(\frac{x}{2}\right)^{6}+\frac{15x\tan\left(\frac{x}{2}\right)^{8}}{2}+3x}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)^{6}}$

input
```
int(8*sin(x)^4*cos(x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x+1/24*sin(6*x)-1/8*sin(4*x)-1/8*sin(2*x)
```

3.833.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.74

$$\int 8\cos^2(x)\sin^4(x)\,dx=\frac{1}{6}\left(8\cos\left(x\right)^{5}-14\cos\left(x\right)^{3}+3\cos\left(x\right)\right)\sin\left(x\right)+\frac{1}{2}x$$

input
```
integrate(8*cos(x)^2*sin(x)^4,x, algorithm="fricas")
```

output
```
1/6*(8*cos(x)^5 - 14*cos(x)^3 + 3*cos(x))*sin(x) + 1/2*x
```

3.833.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int 8\cos^2(x)\sin^4(x)\,dx=\frac{x}{2}+\frac{4\sin^{5}\left(x\right)\cos\left(x\right)}{3}-\frac{\sin^{3}\left(x\right)\cos\left(x\right)}{3}-\frac{\sin\left(x\right)\cos\left(x\right)}{2}$$

input
```
integrate(8*cos(x)**2*sin(x)**4,x)
```

output
```
x/2 + 4*sin(x)**5*cos(x)/3 - sin(x)**3*cos(x)/3 - sin(x)*cos(x)/2
```

3.833.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.53

$$\int 8\cos^2(x)\sin^4(x)\,dx = -\frac{1}{6}\sin(2x)^3 + \frac{1}{2}x - \frac{1}{8}\sin(4x)$$

input
```
integrate(8*cos(x)^2*sin(x)^4,x, algorithm="maxima")
```

output
```
-1/6*sin(2*x)^3 + 1/2*x - 1/8*sin(4*x)
```

3.833.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.65

$$\int 8\cos^2(x)\sin^4(x)\,dx = \frac{1}{2}x + \frac{1}{24}\sin(6x) - \frac{1}{8}\sin(4x) - \frac{1}{8}\sin(2x)$$

input
```
integrate(8*cos(x)^2*sin(x)^4,x, algorithm="giac")
```

output
```
1/2*x + 1/24*sin(6*x) - 1/8*sin(4*x) - 1/8*sin(2*x)
```

3.833.9 Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

$$\int 8\cos^2(x)\sin^4(x)\,dx = \frac{4\cos(x)\sin(x)^5}{3} + \frac{x}{2} - \frac{\sin(2x)}{3} + \frac{\sin(4x)}{24}$$

input
```
int(8*cos(x)^2*sin(x)^4,x)
```

output
```
x/2 - sin(2*x)/3 + sin(4*x)/24 + (4*cos(x)*sin(x)^5)/3
```

3.834 $\int 35 \cos^3(x) \sin^4(x) \, dx$

3.834.1 Optimal result

Integrand size = 10, antiderivative size = 13

$$\int 35 \cos^3(x) \sin^4(x) \, dx = 7 \sin^5(x) - 5 \sin^7(x)$$

output
```
7*sin(x)^5-5*sin(x)^7
```

3.834.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 33 vs. 2(13) = 26.

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.54

$$\int 35 \cos^3(x) \sin^4(x) \, dx = 35 \left(\frac{3 \sin(x)}{64} - \frac{1}{64} \sin(3x) - \frac{1}{320} \sin(5x) + \frac{1}{448} \sin(7x) \right)$$

input
```
Integrate[35*Cos[x]^3*Sin[x]^4,x]
```

output
```
35*((3*Sin[x])/64 - Sin[3*x]/64 - Sin[5*x]/320 + Sin[7*x]/448)
```

3.834.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.46, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {27, 3042, 3044, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int 35\sin^4(x)\cos^3(x)\,dx$$

$$\Big\downarrow \ 27$$

$$35\int \cos^3(x)\sin^4(x)dx$$

$$\Big\downarrow \ 3042$$

$$35\int \cos(x)^3\sin(x)^4dx$$

$$\Big\downarrow \ 3044$$

$$35\int \sin^4(x)\left(1-\sin^2(x)\right)d\sin(x)$$

$$\Big\downarrow \ 244$$

$$35\int \left(\sin^4(x)-\sin^6(x)\right)d\sin(x)$$

$$\Big\downarrow \ 2009$$

$$35\left(\frac{\sin^5(x)}{5}-\frac{\sin^7(x)}{7}\right)$$

input
```
Int[35*Cos[x]^3*Sin[x]^4,x]
```

output
```
35*(Sin[x]^5/5 - Sin[x]^7/7)
```

3.834.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

3.834.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$7\sin\left(x\right)^{5}-5\sin\left(x\right)^{7}$	14
default	$7\sin\left(x\right)^{5}-5\sin\left(x\right)^{7}$	14
risch	$\frac{105\sin\left(x\right)}{64}+\frac{5\sin\left(7x\right)}{64}-\frac{7\sin\left(5x\right)}{64}-\frac{35\sin\left(3x\right)}{64}$	24
parallelrisch	$\frac{105\sin\left(x\right)}{64}+\frac{5\sin\left(7x\right)}{64}-\frac{7\sin\left(5x\right)}{64}-\frac{35\sin\left(3x\right)}{64}$	24
norman	$\frac{224\tan\left(\frac{x}{2}\right)^{5}-192\tan\left(\frac{x}{2}\right)^{7}+224\tan\left(\frac{x}{2}\right)^{9}}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)^{7}}$	37

input
```
int(35*cos(x)^3*sin(x)^4,x,method=_RETURNVERBOSE)
```

output
```
7*sin(x)^5-5*sin(x)^7
```

3.834.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62

$$\int 35\cos^3(x)\sin^4(x)\,dx = \left(5\,\cos\left(x\right)^6 - 8\,\cos\left(x\right)^4 + \cos\left(x\right)^2 + 2\right)\sin\left(x\right)$$

input
```
integrate(35*cos(x)^3*sin(x)^4,x, algorithm="fricas")
```

output
```
(5*cos(x)^6 - 8*cos(x)^4 + cos(x)^2 + 2)*sin(x)
```

3.834.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int 35\cos^3(x)\sin^4(x)\,dx = -5\sin^7\left(x\right) + 7\sin^5\left(x\right)$$

input
```
integrate(35*cos(x)**3*sin(x)**4,x)
```

output
```
-5*sin(x)**7 + 7*sin(x)**5
```

3.834.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int 35\cos^3(x)\sin^4(x)\,dx = -5\,\sin\left(x\right)^7 + 7\,\sin\left(x\right)^5$$

input
```
integrate(35*cos(x)^3*sin(x)^4,x, algorithm="maxima")
```

output
```
-5*sin(x)^7 + 7*sin(x)^5
```

3.834.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int 35\cos^3(x)\sin^4(x)\,dx = -5\,\sin{(x)}^7 + 7\,\sin{(x)}^5$$

input
```
integrate(35*cos(x)^3*sin(x)^4,x, algorithm="giac")
```

output
```
-5*sin(x)^7 + 7*sin(x)^5
```

3.834.9 Mupad [B] (verification not implemented)

Time = 27.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int 35\cos^3(x)\sin^4(x)\,dx = 7\sin{(x)}^5 - 5\sin{(x)}^7$$

input
```
int(35*cos(x)^3*sin(x)^4,x)
```

output
```
7*sin(x)^5 - 5*sin(x)^7
```

3.835 $\int 4 \cos^4(x) \sin^4(x) \, dx$

3.835.1 Optimal result

Integrand size = 10, antiderivative size = 46

$$\int 4 \cos^4(x) \sin^4(x) \, dx = \frac{3x}{32} + \frac{3}{32} \cos(x) \sin(x) + \frac{1}{16} \cos^3(x) \sin(x) - \frac{1}{4} \cos^5(x) \sin(x) - \frac{1}{2} \cos^5(x) \sin^3(x)$$

output
```
3/32*x+3/32*cos(x)*sin(x)+1/16*cos(x)^3*sin(x)-1/4*cos(x)^5*sin(x)-1/2*cos
(x)^5*sin(x)^3
```

3.835.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.52

$$\int 4 \cos^4(x) \sin^4(x) \, dx = 4 \left(\frac{3x}{128} - \frac{1}{128} \sin(4x) + \frac{\sin(8x)}{1024} \right)$$

input
```
Integrate[4*Cos[x]^4*Sin[x]^4,x]
```

output
```
4*((3*x)/128 - Sin[4*x]/128 + Sin[8*x]/1024)
```

3.835.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.37, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {27, 3042, 3048, 3042, 3048, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int 4\sin^4(x)\cos^4(x)\,dx$$

$$\Big\downarrow \text{27}$$

$$4\int \cos^4(x)\sin^4(x)dx$$

$$\Big\downarrow \text{3042}$$

$$4\int \cos(x)^4\sin(x)^4dx$$

$$\Big\downarrow \text{3048}$$

$$4\left(\frac{3}{8}\int \cos^4(x)\sin^2(x)dx-\frac{1}{8}\sin^3(x)\cos^5(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$4\left(\frac{3}{8}\int \cos(x)^4\sin(x)^2dx-\frac{1}{8}\sin^3(x)\cos^5(x)\right)$$

$$\Big\downarrow \text{3048}$$

$$4\left(\frac{3}{8}\left(\frac{1}{6}\int \cos^4(x)dx-\frac{1}{6}\sin(x)\cos^5(x)\right)-\frac{1}{8}\sin^3(x)\cos^5(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$4\left(\frac{3}{8}\left(\frac{1}{6}\int \sin\left(x+\frac{\pi}{2}\right)^4dx-\frac{1}{6}\sin(x)\cos^5(x)\right)-\frac{1}{8}\sin^3(x)\cos^5(x)\right)$$

$$\Big\downarrow \text{3115}$$

$$4\left(\frac{3}{8}\left(\frac{1}{6}\left(\frac{3}{4}\int \cos^2(x)dx+\frac{1}{4}\sin(x)\cos^3(x)\right)-\frac{1}{6}\sin(x)\cos^5(x)\right)-\frac{1}{8}\sin^3(x)\cos^5(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$4\left(\frac{3}{8}\left(\frac{1}{6}\left(\frac{3}{4}\int \sin\left(x+\frac{\pi}{2}\right)^2dx+\frac{1}{4}\sin(x)\cos^3(x)\right)-\frac{1}{6}\sin(x)\cos^5(x)\right)-\frac{1}{8}\sin^3(x)\cos^5(x)\right)$$

$$\Big\downarrow \text{3115}$$

$$4\left(\frac{3}{8}\left(\frac{1}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{1}{2}\sin(x)\cos(x)\right)+\frac{1}{4}\sin(x)\cos^3(x)\right)-\frac{1}{6}\sin(x)\cos^5(x)\right)-\frac{1}{8}\sin^3(x)\cos^5(x)\right)$$

$$\downarrow 24$$

$$4\left(\frac{3}{8}\left(\frac{1}{6}\left(\frac{1}{4}\sin(x)\cos^3(x)+\frac{3}{4}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)\right)-\frac{1}{6}\sin(x)\cos^5(x)\right)-\frac{1}{8}\sin^3(x)\cos^5(x)\right)$$

input
```
Int[4*Cos[x]^4*Sin[x]^4,x]
```

output
```
4*(-1/8*(Cos[x]^5*Sin[x]^3) + (3*(-1/6*(Cos[x]^5*Sin[x]) + ((Cos[x]^3*Sin[
x])/4 + (3*(x/2 + (Cos[x]*Sin[x])/2))/4)/6))/8)
```

3.835.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.835.4 Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.37

method	result
risch	$\frac{3x}{32}+\frac{\sin(8x)}{256}-\frac{\sin(4x)}{32}$
parallelrisch	$\frac{3x}{32}+\frac{\sin(8x)}{256}-\frac{\sin(4x)}{32}$
default	$-\frac{\cos(x)^5\sin(x)^3}{2}-\frac{\cos(x)^5\sin(x)}{4}+\frac{\left(\cos(x)^3+\frac{3\cos(x)}{2}\right)\sin(x)}{16}+\frac{3x}{32}$
norman	$\frac{\frac{3x}{32}-\frac{23\tan\left(\frac{x}{2}\right)^3}{16}+\frac{333\tan\left(\frac{x}{2}\right)^5}{16}-\frac{671\tan\left(\frac{x}{2}\right)^7}{16}+\frac{671\tan\left(\frac{x}{2}\right)^9}{16}-\frac{333\tan\left(\frac{x}{2}\right)^{11}}{16}+\frac{23\tan\left(\frac{x}{2}\right)^{13}}{16}+\frac{3\tan\left(\frac{x}{2}\right)^{15}}{16}+\frac{3x\tan\left(\frac{x}{2}\right)^2}{4}+\frac{21x\tan\left(\frac{x}{2}\right.}{8}}{\left(1+\tan\left(\frac{x}{2}\right)^2\right)^8}$

input
```
int(4*sin(x)^4*cos(x)^4,x,method=_RETURNVERBOSE)
```

output
```
3/32*x+1/256*sin(8*x)-1/32*sin(4*x)
```

3.835.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.67

$$\int 4\cos^4(x)\sin^4(x)\,dx=\frac{1}{32}\left(16\cos(x)^7-24\cos(x)^5+2\cos(x)^3+3\cos(x)\right)\sin(x)+\frac{3}{32}x$$

input
```
integrate(4*cos(x)^4*sin(x)^4,x, algorithm="fricas")
```

output
```
1/32*(16*cos(x)^7 - 24*cos(x)^5 + 2*cos(x)^3 + 3*cos(x))*sin(x) + 3/32*x
```

3.835.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.67

$$\int 4\cos^4(x)\sin^4(x)\,dx=\frac{3x}{32}-\frac{\sin^3(2x)\cos(2x)}{32}-\frac{3\sin(2x)\cos(2x)}{64}$$

input
```
integrate(4*cos(x)**4*sin(x)**4,x)
```

output
```
3*x/32 - sin(2*x)**3*cos(2*x)/32 - 3*sin(2*x)*cos(2*x)/64
```

3.835.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.35

$$\int 4\cos^4(x)\sin^4(x)\,dx = \frac{3}{32}\,x + \frac{1}{256}\,\sin\left(8\,x\right) - \frac{1}{32}\,\sin\left(4\,x\right)$$

input
```
integrate(4*cos(x)^4*sin(x)^4,x, algorithm="maxima")
```

output
```
3/32*x + 1/256*sin(8*x) - 1/32*sin(4*x)
```

3.835.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.35

$$\int 4\cos^4(x)\sin^4(x)\,dx = \frac{3}{32}\,x + \frac{1}{256}\,\sin\left(8\,x\right) - \frac{1}{32}\,\sin\left(4\,x\right)$$

input
```
integrate(4*cos(x)^4*sin(x)^4,x, algorithm="giac")
```

output
```
3/32*x + 1/256*sin(8*x) - 1/32*sin(4*x)
```

3.835.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

$$\int 4\cos^4(x)\sin^4(x)\,dx = \frac{3\,x}{32} - \frac{\sin\left(2\,x\right)}{16} + \frac{\sin\left(4\,x\right)}{128} + 4\sin\left(x\right)^5\left(\frac{\cos\left(x\right)^3}{8} + \frac{\cos\left(x\right)}{16}\right)$$

input
```
int(4*cos(x)^4*sin(x)^4,x)
```

output
```
(3*x)/32 - sin(2*x)/16 + sin(4*x)/128 + 4*sin(x)^5*(cos(x)/16 + cos(x)^3/8
)
```

3.836 $\int \frac{\cos(x)}{-\sin(x)+\sin^3(x)}\,dx$

3.836.1 Optimal result

Integrand size = 14, antiderivative size = 9

$$\int \frac{\cos(x)}{-\sin(x)+\sin^3(x)}\,dx = \log(\cos(x)) - \log(\sin(x))$$

output
```
ln(cos(x))-ln(sin(x))
```

3.836.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)}{-\sin(x)+\sin^3(x)}\,dx = \log(\cos(x)) - \log(\sin(x))$$

input
```
Integrate[Cos[x]/(-Sin[x] + Sin[x]^3),x]
```

output
```
Log[Cos[x]] - Log[Sin[x]]
```

3.836.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 21 vs. 2(9) = 18.

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 2.33, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4834, 25, 243, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(x)}{\sin^3(x)-\sin(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(x)}{\sin(x)^3-\sin(x)}dx$$

$$\Big\downarrow \text{4834}$$

$$\int -\frac{\csc(x)}{1-\sin^2(x)}d\sin(x)$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\csc(x)}{1-\sin^2(x)}d\sin(x)$$

$$\Big\downarrow \text{243}$$

$$-\frac{1}{2}\int \frac{\csc(x)}{1-\sin^2(x)}d\sin^2(x)$$

$$\Big\downarrow \text{47}$$

$$\frac{1}{2}\left(-\int \frac{1}{1-\sin^2(x)}d\sin^2(x)-\int \csc(x)d\sin^2(x)\right)$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{2}\left(-\int \frac{1}{1-\sin^2(x)}d\sin^2(x)-\log\left(\sin^2(x)\right)\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(\log\left(1-\sin^2(x)\right)-\log\left(\sin^2(x)\right)\right)$$

input
```
Int[Cos[x]/(-Sin[x] + Sin[x]^3),x]
```

3.836. $\int \frac{\cos(x)}{-\sin(x)+\sin^3(x)}dx$

output
```
(-Log[Sin[x]^2] + Log[1 - Sin[x]^2])/2
```

3.836.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4834
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

3.836. $\int \frac{\cos(x)}{-\sin(x)+\sin^3(x)}\,dx$

3.836.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.22

method	result	size
risch	$\ln\left(e^{2ix}+1\right)-\ln\left(e^{2ix}-1\right)$	20
derivativedivides	$-\ln\left(\sin\left(x\right)\right)+\frac{\ln(\sin(x)-1)}{2}+\frac{\ln(1+\sin(x))}{2}$	21
default	$-\ln\left(\sin\left(x\right)\right)+\frac{\ln(\sin(x)-1)}{2}+\frac{\ln(1+\sin(x))}{2}$	21
norman	$-\ln\left(\tan\left(\frac{x}{2}\right)\right)+\ln\left(\tan\left(\frac{x}{2}\right)-1\right)+\ln\left(\tan\left(\frac{x}{2}\right)+1\right)$	23
parallelrisch	$-\ln\left(\tan\left(\frac{x}{2}\right)\right)+\ln\left(\tan\left(\frac{x}{2}\right)-1\right)+\ln\left(\tan\left(\frac{x}{2}\right)+1\right)$	23

input
```
int(cos(x)/(-sin(x)+sin(x)^3),x,method=_RETURNVERBOSE)
```

output
```
ln(exp(2*I*x)+1)-ln(exp(2*I*x)-1)
```

3.836.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(9) = 18.

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 2.11

$$\int \frac{\cos(x)}{-\sin(x)+\sin^3(x)}\,dx = \frac{1}{2}\log\left(\cos\left(x\right)^2\right)-\frac{1}{2}\log\left(-\frac{1}{4}\cos\left(x\right)^2+\frac{1}{4}\right)$$

input
```
integrate(cos(x)/(-sin(x)+sin(x)^3),x, algorithm="fricas")
```

output
```
1/2*log(cos(x)^2) - 1/2*log(-1/4*cos(x)^2 + 1/4)
```

3.836.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(8) = 16$.

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.22

$$\int \frac{\cos(x)}{-\sin(x)+\sin^3(x)}\,dx = \frac{\log(\sin(x)-1)}{2} + \frac{\log(\sin(x)+1)}{2} - \log(\sin(x))$$

input
```
integrate(cos(x)/(-sin(x)+sin(x)**3),x)
```

output
```
log(sin(x) - 1)/2 + log(sin(x) + 1)/2 - log(sin(x))
```

3.836.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(9) = 18$.

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.22

$$\int \frac{\cos(x)}{-\sin(x)+\sin^3(x)}\,dx = \frac{1}{2}\log(\sin(x)+1) + \frac{1}{2}\log(\sin(x)-1) - \log(\sin(x))$$

input
```
integrate(cos(x)/(-sin(x)+sin(x)^3),x, algorithm="maxima")
```

output
```
1/2*log(sin(x) + 1) + 1/2*log(sin(x) - 1) - log(sin(x))
```

3.836.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.00

$$\int \frac{\cos(x)}{-\sin(x)+\sin^3(x)}\,dx = \frac{1}{2}\log\left(-\sin(x)^2+1\right) - \log(|\sin(x)|)$$

input
```
integrate(cos(x)/(-sin(x)+sin(x)^3),x, algorithm="giac")
```

output
```
1/2*log(-sin(x)^2 + 1) - log(abs(sin(x)))
```

3.836.9 Mupad [B] (verification not implemented)

Time = 28.61 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.44

$$\int \frac{\cos(x)}{-\sin(x)+\sin^3(x)}\,dx = \frac{\ln\left(\cos\left(x\right)^2\right)}{2} - \ln\left(\sin\left(x\right)\right)$$

input
```
int(-cos(x)/(sin(x) - sin(x)^3),x)
```

output
```
log(cos(x)^2)/2 - log(sin(x))
```

3.837 $\int (-1+2\cos^2(x)+\cos(x)\sin(x))\,dx$

3.837.1 Optimal result

Integrand size = 13, antiderivative size = 14

$$\int (-1+2\cos^2(x)+\cos(x)\sin(x))\,dx=\cos(x)\sin(x)+\frac{\sin^2(x)}{2}$$

output
```
cos(x)*sin(x)+1/2*sin(x)^2
```

3.837.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.21

$$\int (-1+2\cos^2(x)+\cos(x)\sin(x))\,dx=-\frac{1}{2}\cos^2(x)+\frac{1}{2}\sin(2x)$$

input
```
Integrate[-1 + 2*Cos[x]^2 + Cos[x]*Sin[x],x]
```

output
```
-1/2*Cos[x]^2 + Sin[2*x]/2
```

3.837.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(2\cos^2(x)+\sin(x)\cos(x)-1\right) dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{\sin^2(x)}{2}+\sin(x)\cos(x)$$

input
```
Int[-1 + 2*Cos[x]^2 + Cos[x]*Sin[x],x]
```

output
```
Cos[x]*Sin[x] + Sin[x]^2/2
```

3.837.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.837.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
default	$\cos(x)\sin(x)+\frac{\sin(x)^2}{2}$	13
parts	$\cos(x)\sin(x)+\frac{\sin(x)^2}{2}$	13
risch	$-\frac{\cos(2x)}{4}+\frac{\sin(2x)}{2}$	14
parallelrisch	$\frac{\sin(2x)}{2}+\frac{1}{4}-\frac{\cos(2x)}{4}$	15
meijerg	$\frac{\sin(2x)}{2}+\frac{\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(2x)}{\sqrt{\pi}}\right)}{4}$	26
norman	$\frac{2\tan\left(\frac{x}{2}\right)^2-2\tan\left(\frac{x}{2}\right)^3+2\tan\left(\frac{x}{2}\right)}{\left(1+\tan\left(\frac{x}{2}\right)^2\right)^2}$	35

input
```
int(-1+2*cos(x)^2+cos(x)*sin(x),x,method=_RETURNVERBOSE)
```

output
```
cos(x)*sin(x)+1/2*sin(x)^2
```

3.837.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \left(-1+2\cos^2(x)+\cos(x)\sin(x)\right)\,dx=-\frac{1}{2}\cos\left(x\right)^2+\cos\left(x\right)\sin\left(x\right)$$

input
```
integrate(-1+2*cos(x)^2+cos(x)*sin(x),x, algorithm="fricas")
```

output
```
-1/2*cos(x)^2 + cos(x)*sin(x)
```

3.837.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \left(-1+2\cos^2(x)+\cos(x)\sin(x)\right)\,dx=\frac{\sin^2\left(x\right)}{2}+\sin\left(x\right)\cos\left(x\right)$$

input
```
integrate(-1+2*cos(x)**2+cos(x)*sin(x),x)
```

output
```
sin(x)**2/2 + sin(x)*cos(x)
```

3.837.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \left(-1+2\cos^2(x)+\cos(x)\sin(x)\right)\,dx=-\frac{1}{2}\cos\left(x\right)^2+\frac{1}{2}\sin\left(2\,x\right)$$

input
```
integrate(-1+2*cos(x)^2+cos(x)*sin(x),x, algorithm="maxima")
```

output
```
-1/2*cos(x)^2 + 1/2*sin(2*x)
```

3.837.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \left(-1+2\cos^2(x)+\cos(x)\sin(x)\right)\,dx = -\frac{1}{2}\cos\left(x\right)^2+\frac{1}{2}\sin\left(2\,x\right)$$

input
```
integrate(-1+2*cos(x)^2+cos(x)*sin(x),x, algorithm="giac")
```

output
```
-1/2*cos(x)^2 + 1/2*sin(2*x)
```

3.837.9 Mupad [B] (verification not implemented)

Time = 26.40 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int \left(-1+2\cos^2(x)+\cos(x)\sin(x)\right)\,dx = -\frac{\cos\left(x\right)\,\left(\cos\left(x\right)-2\,\sin\left(x\right)\right)}{2}$$

input
```
int(cos(x)*sin(x) + 2*cos(x)^2 - 1,x)
```

output
```
-(cos(x)*(cos(x) - 2*sin(x)))/2
```

3.838 $\int (\cos^2(x) + \sin^2(x))\,dx$

3.838.1 Optimal result

Integrand size = 9, antiderivative size = 1

$$\int (\cos^2(x) + \sin^2(x))\,dx = x$$

output
```
x
```

3.838.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int (\cos^2(x) + \sin^2(x))\,dx = x$$

input
```
Integrate[Cos[x]^2 + Sin[x]^2,x]
```

output
```
x
```

3.838.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\sin^2(x)+\cos^2(x)\right) \, dx$$
$$\Big\downarrow \; 2009$$
$$x$$

input
```
Int[Cos[x]^2 + Sin[x]^2,x]
```

output
```
x
```

3.838.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.838.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 2, normalized size of antiderivative = 2.00

method	result	size
default	x	2
risch	x	2
parts	x	2
norman	$\frac{x+x\tan\left(\frac{x}{2}\right)^{4}+2x\tan\left(\frac{x}{2}\right)^{2}}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)^{2}}$	31

input
```
int(sin(x)^2+cos(x)^2,x,method=_RETURNVERBOSE)
```

output
```
x
```

3.838.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \left(\cos^2(x)+\sin^2(x)\right)\,dx=x$$

input
```
integrate(cos(x)^2+sin(x)^2,x, algorithm="fricas")
```

output
```
x
```

3.838.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 0, normalized size of antiderivative = 0.00

$$\int \left(\cos^2(x)+\sin^2(x)\right)\,dx=x$$

input
```
integrate(cos(x)**2+sin(x)**2,x)
```

output
```
x
```

3.838.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \left(\cos^2(x)+\sin^2(x)\right)\,dx=x$$

input
```
integrate(cos(x)^2+sin(x)^2,x, algorithm="maxima")
```

output
```
x
```

3.838.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \left(\cos^2(x)+\sin^2(x)\right)\,dx = x$$

input
```
integrate(cos(x)^2+sin(x)^2,x, algorithm="giac")
```

output
```
x
```

3.838.9 Mupad [B] (verification not implemented)

Time = 26.42 (sec) , antiderivative size = 1, normalized size of antiderivative = 1.00

$$\int \left(\cos^2(x)+\sin^2(x)\right)\,dx = x$$

input
```
int(cos(x)^2 + sin(x)^2,x)
```

output
```
x
```

3.839 $\int (-\cos^2(x)+\sin^2(x))\,dx$

3.839.1 Optimal result

Integrand size = 11, antiderivative size = 6

$$\int (-\cos^2(x)+\sin^2(x))\,dx=-\cos(x)\sin(x)$$

output
```
-cos(x)*sin(x)
```

3.839.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.33

$$\int (-\cos^2(x)+\sin^2(x))\,dx=-\frac{1}{2}\sin(2x)$$

input
```
Integrate[-Cos[x]^2 + Sin[x]^2,x]
```

output
```
-1/2*Sin[2*x]
```

3.839.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\sin^2(x)-\cos^2(x)\right) dx$$

$$\Big\downarrow \; 2009$$

$$\sin(x)(-\cos(x))$$

input
```
Int[-Cos[x]^2 + Sin[x]^2,x]
```

output
```
-(Cos[x]*Sin[x])
```

3.839.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.839.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

method	result	size
default	$-\cos\left(x\right)\sin\left(x\right)$	7
risch	$-\frac{\sin\left(2x\right)}{2}$	7
parallelrisch	$-\frac{\sin\left(2x\right)}{2}$	7
parts	$-\cos\left(x\right)\sin\left(x\right)$	7

input
```
int(-cos(x)^2+sin(x)^2,x,method=_RETURNVERBOSE)
```

output
```
-cos(x)*sin(x)
```

3.839.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \left(-\cos^2(x)+\sin^2(x)\right)\,dx=-\cos(x)\sin(x)$$

input
```
integrate(-cos(x)^2+sin(x)^2,x, algorithm="fricas")
```

output
```
-cos(x)*sin(x)
```

3.839.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

$$\int \left(-\cos^2(x)+\sin^2(x)\right)\,dx=-\sin(x)\cos(x)$$

input
```
integrate(-cos(x)**2+sin(x)**2,x)
```

output
```
-sin(x)*cos(x)
```

3.839.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \left(-\cos^2(x)+\sin^2(x)\right)\,dx=-\frac{1}{2}\sin(2x)$$

input
```
integrate(-cos(x)^2+sin(x)^2,x, algorithm="maxima")
```

output
```
-1/2*sin(2*x)
```

3.839.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \left(-\cos^2(x)+\sin^2(x)\right)\,dx=-\frac{1}{2}\sin(2x)$$

input
```
integrate(-cos(x)^2+sin(x)^2,x, algorithm="giac")
```

output
```
-1/2*sin(2*x)
```

3.839.9 Mupad [B] (verification not implemented)

Time = 26.15 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \left(-\cos^2(x)+\sin^2(x)\right)\,dx=-\frac{\sin(2x)}{2}$$

input
```
int(sin(x)^2 - cos(x)^2,x)
```

output
```
-sin(2*x)/2
```

3.840 $\int 2^{\sin(x)}\cos(x)\,dx$

3.840.1 Optimal result

Integrand size = 7, antiderivative size = 9

$$\int 2^{\sin(x)}\cos(x)\,dx=\frac{2^{\sin(x)}}{\log(2)}$$

output
```
2^sin(x)/ln(2)
```

3.840.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int 2^{\sin(x)}\cos(x)\,dx=\frac{2^{\sin(x)}}{\log(2)}$$

input
```
Integrate[2^Sin[x]*Cos[x],x]
```

output
```
2^Sin[x]/Log[2]
```

3.840.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4834, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int 2^{\sin(x)}\cos(x)\,dx$$

$$\Big\downarrow \text{ 4834}$$

$$\int 2^{\sin(x)}d\sin(x)$$

$$\Big\downarrow \text{ 2624}$$

$$\frac{2^{\sin(x)}}{\log(2)}$$

input
```
Int[2^Sin[x]*Cos[x],x]
```

output
```
2^Sin[x]/Log[2]
```

3.840.3.1 Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 4834
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

3.840.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

method	result	size
derivativedivides	$\frac{2^{\sin(x)}}{\ln(2)}$	10
default	$\frac{2^{\sin(x)}}{\ln(2)}$	10
risch	$\frac{2^{\sin(x)}}{\ln(2)}$	10
parallelrisch	$\frac{2^{\sin(x)}}{\ln(2)}$	10
norman	$\frac{\frac{e^{\frac{2\tan\left(\frac{x}{2}\right)\ln(2)}{1+\tan\left(\frac{x}{2}\right)^2}}}{\ln(2)}+\frac{\tan\left(\frac{x}{2}\right)^2 e^{\frac{2\tan\left(\frac{x}{2}\right)\ln(2)}{1+\tan\left(\frac{x}{2}\right)^2}}}{\ln(2)}}{1+\tan\left(\frac{x}{2}\right)^2}$	67

input
```
int(2^sin(x)*cos(x),x,method=_RETURNVERBOSE)
```

output
```
2^sin(x)/ln(2)
```

3.840.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int 2^{\sin(x)}\cos(x)\,dx = \frac{2^{\sin(x)}}{\log(2)}$$

input
```
integrate(2^sin(x)*cos(x),x, algorithm="fricas")
```

output
```
2^sin(x)/log(2)
```

3.840.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int 2^{\sin(x)}\cos(x)\,dx = \frac{2^{\sin{(x)}}}{\log{(2)}}$$

input
```
integrate(2**sin(x)*cos(x),x)
```

output
```
2**sin(x)/log(2)
```

3.840.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int 2^{\sin(x)}\cos(x)\,dx = \frac{2^{\sin(x)}}{\log{(2)}}$$

input
```
integrate(2^sin(x)*cos(x),x, algorithm="maxima")
```

output
```
2^sin(x)/log(2)
```

3.840.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int 2^{\sin(x)}\cos(x)\,dx = \frac{2^{\sin(x)}}{\log{(2)}}$$

input
```
integrate(2^sin(x)*cos(x),x, algorithm="giac")
```

output
```
2^sin(x)/log(2)
```

3.840.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int 2^{\sin(x)}\cos(x)\,dx=\frac{2^{\sin(x)}}{\ln(2)}$$

input
```
int(2^sin(x)*cos(x),x)
```

output
```
2^sin(x)/log(2)
```

3.841 $\int (\tan^3(x)+\tan^5(x))\,dx$

3.841.1 Optimal result

Integrand size = 9, antiderivative size = 8

$$\int (\tan^3(x)+\tan^5(x))\,dx=\frac{\tan^4(x)}{4}$$

output
```
1/4*tan(x)^4
```

3.841.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int (\tan^3(x)+\tan^5(x))\,dx=\frac{\tan^4(x)}{4}$$

input
```
Integrate[Tan[x]^3 + Tan[x]^5,x]
```

output
```
Tan[x]^4/4
```

3.841.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\tan^5(x)+\tan^3(x)\right) dx$$

$$\Big\downarrow \; 2009$$

$$\frac{\tan^4(x)}{4}$$

input
```
Int[Tan[x]^3 + Tan[x]^5,x]
```

output
```
Tan[x]^4/4
```

3.841.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.841.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{\tan(x)^4}{4}$	7
default	$\frac{\tan(x)^4}{4}$	7
norman	$\frac{\tan(x)^4}{4}$	7
parallelrisch	$\frac{\tan(x)^4}{4}$	7
parts	$\frac{\tan(x)^4}{4}$	7
risch	$-\frac{2\left(e^{6ix}+e^{2ix}\right)}{\left(e^{2ix}+1\right)^4}$	23

input
```
int(tan(x)^3+tan(x)^5,x,method=_RETURNVERBOSE)
```

output
```
1/4*tan(x)^4
```

3.841.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \left(\tan^3(x) + \tan^5(x)\right) \, dx = \frac{1}{4} \tan{(x)}^4$$

input
```
integrate(tan(x)^3+tan(x)^5,x, algorithm="fricas")
```

output
```
1/4*tan(x)^4
```

3.841.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(5) = 10.

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.75

$$\int \left(\tan^3(x) + \tan^5(x)\right) \, dx = -\frac{4\cos^2{(x)} - 1}{4\cos^4{(x)}} + \frac{1}{2\cos^2{(x)}}$$

input
```
integrate(tan(x)**3+tan(x)**5,x)
```

output
```
-(4*cos(x)**2 - 1)/(4*cos(x)**4) + 1/(2*cos(x)**2)
```

3.841.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(6) = 12.

Time = 0.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 4.38

$$\int \left(\tan^3(x) + \tan^5(x)\right) \, dx = \frac{4\sin{(x)}^2 - 3}{4\left(\sin{(x)}^4 - 2\sin{(x)}^2 + 1\right)} - \frac{1}{2\left(\sin{(x)}^2 - 1\right)}$$

input
```
integrate(tan(x)^3+tan(x)^5,x, algorithm="maxima")
```

output
```
1/4*(4*sin(x)^2 - 3)/(sin(x)^4 - 2*sin(x)^2 + 1) - 1/2/(sin(x)^2 - 1)
```

3.841.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \left(\tan^3(x)+\tan^5(x)\right)\,dx=\frac{1}{4}\,\tan\left(x\right)^4$$

input
```
integrate(tan(x)^3+tan(x)^5,x, algorithm="giac")
```

output
```
1/4*tan(x)^4
```

3.841.9 Mupad [B] (verification not implemented)

Time = 26.38 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \left(\tan^3(x)+\tan^5(x)\right)\,dx=\frac{\tan(x)^4}{4}$$

input
```
int(tan(x)^3 + tan(x)^5,x)
```

output
```
tan(x)^4/4
```

3.842 $\int x \sec(x)(2 + x \tan(x)) \, dx$

3.842.1 Optimal result

Integrand size = 10, antiderivative size = 6

$$\int x \sec(x)(2 + x \tan(x)) \, dx = x^2 \sec(x)$$

output
```
x^2*sec(x)
```

3.842.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 8.50

$$\int x \sec(x)(2+x \tan(x)) \, dx = x\left(-4i \arctan\left(e^{ix}\right)-2 \log\left(1-ie^{ix}\right)+2 \log\left(1+ie^{ix}\right)+x \sec(x)\right)$$

input
```
Integrate[x*Sec[x]*(2 + x*Tan[x]),x]
```

output
```
x*((-4*I)*ArcTan[E^(I*x)] - 2*Log[1 - I*E^(I*x)] + 2*Log[1 + I*E^(I*x)] +
x*Sec[x])
```

3.842.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(x\tan(x)+2)\sec(x)\,dx$$
$$\Big\downarrow \text{ 7293}$$
$$\int \left(x^2\tan(x)\sec(x)+2x\sec(x)\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$x^2\sec(x)$$

input
```
Int[x*Sec[x]*(2 + x*Tan[x]),x]
```

output
```
x^2*Sec[x]
```

3.842.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.842.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.50

method	result	size
default	$\frac{x^2}{\cos(x)}$	9
risch	$\frac{2x^2 e^{ix}}{e^{2ix}+1}$	20

input
```
int(x*sec(x)*(2+x*tan(x)),x,method=_RETURNVERBOSE)
```

output
```
x^2/cos(x)
```

3.842.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.33

$$\int x\sec(x)(2+x\tan(x))\,dx=\frac{x^2}{\cos(x)}$$

input
```
integrate(x*sec(x)*(2+x*tan(x)),x, algorithm="fricas")
```

output
```
x^2/cos(x)
```

3.842.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int x\sec(x)(2+x\tan(x))\,dx=x^2\sec(x)$$

input
```
integrate(x*sec(x)*(2+x*tan(x)),x)
```

output
```
x**2*sec(x)
```

3.842.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(6) = 12.

Time = 0.40 (sec) , antiderivative size = 51, normalized size of antiderivative = 8.50

$$\int x\sec(x)(2+x\tan(x))\,dx=\frac{2\left(x^2\cos(2x)\cos(x)+x^2\sin(2x)\sin(x)+x^2\cos(x)\right)}{\cos(2x)^2+\sin(2x)^2+2\cos(2x)+1}$$

input
```
integrate(x*sec(x)*(2+x*tan(x)),x, algorithm="maxima")
```

output
```
2*(x^2*cos(2*x)*cos(x) + x^2*sin(2*x)*sin(x) + x^2*cos(x))/(cos(2*x)^2 + s
in(2*x)^2 + 2*cos(2*x) + 1)
```

3.842.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(6) = 12$.

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 4.33

$$\int x\sec(x)(2+x\tan(x))\,dx = -\frac{x^2\tan\left(\frac{1}{2}x\right)^2 + x^2}{\tan\left(\frac{1}{2}x\right)^2 - 1}$$

input
```
integrate(x*sec(x)*(2+x*tan(x)),x, algorithm="giac")
```

output
```
-(x^2*tan(1/2*x)^2 + x^2)/(tan(1/2*x)^2 - 1)
```

3.842.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.33

$$\int x\sec(x)(2+x\tan(x))\,dx = \frac{x^2}{\cos(x)}$$

input
```
int((x*(x*tan(x) + 2))/cos(x),x)
```

output
```
x^2/cos(x)
```

3.843 $\int \frac{\cot(\sqrt{x})\csc(\sqrt{x})}{\sqrt{x}}\,dx$

3.843.1 Optimal result

Integrand size = 18, antiderivative size = 8

$$\int \frac{\cot(\sqrt{x})\csc(\sqrt{x})}{\sqrt{x}}\,dx = -2\csc(\sqrt{x})$$

output
```
-2*csc(x^(1/2))
```

3.843.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\cot(\sqrt{x})\csc(\sqrt{x})}{\sqrt{x}}\,dx = -2\csc(\sqrt{x})$$

input
```
Integrate[(Cot[Sqrt[x]]*Csc[Sqrt[x]])/Sqrt[x],x]
```

output
```
-2*Csc[Sqrt[x]]
```

3.843.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {7266, 3042, 25, 3086, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot \left(\sqrt{x}\right) \csc \left(\sqrt{x}\right)}{\sqrt{x}} \, dx$$
$$\Big\downarrow \ 7266$$
$$2 \int \cot \left(\sqrt{x}\right) \csc \left(\sqrt{x}\right) d\sqrt{x}$$
$$\Big\downarrow \ 3042$$
$$2 \int -\sec \left(\sqrt{x}-\frac{\pi}{2}\right) \tan \left(\sqrt{x}-\frac{\pi}{2}\right) d\sqrt{x}$$
$$\Big\downarrow \ 25$$
$$-2 \int \sec \left(\sqrt{x}-\frac{\pi}{2}\right) \tan \left(\sqrt{x}-\frac{\pi}{2}\right) d\sqrt{x}$$
$$\Big\downarrow \ 3086$$
$$-2 \int 1 d \csc \left(\sqrt{x}\right)$$
$$\Big\downarrow \ 24$$
$$-2 \csc \left(\sqrt{x}\right)$$

input
```
Int[(Cot[Sqrt[x]]*Csc[Sqrt[x]])/Sqrt[x],x]
```

output
```
-2*Csc[Sqrt[x]]
```

3.843.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

3.843.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$-2\csc\left(\sqrt{x}\right)$	7
default	$-2\csc\left(\sqrt{x}\right)$	7

input
```
int(1/x^(1/2)*cot(x^(1/2))*csc(x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-2*csc(x^(1/2))
```

3.843.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\cot\left(\sqrt{x}\right)\csc\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -\frac{2}{\sin\left(\sqrt{x}\right)}$$

input
```
integrate(cot(x^(1/2))*csc(x^(1/2))/x^(1/2),x, algorithm="fricas")
```

output
```
-2/sin(sqrt(x))
```

3.843.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\cot\left(\sqrt{x}\right)\csc\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -2\csc\left(\sqrt{x}\right)$$

input
```
integrate(cot(x**(1/2))*csc(x**(1/2))/x**(1/2),x)
```

output
```
-2*csc(sqrt(x))
```

3.843.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\cot\left(\sqrt{x}\right)\csc\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -\frac{2}{\sin\left(\sqrt{x}\right)}$$

input
```
integrate(cot(x^(1/2))*csc(x^(1/2))/x^(1/2),x, algorithm="maxima")
```

output
```
-2/sin(sqrt(x))
```

3.843.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\cot\left(\sqrt{x}\right)\csc\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -\frac{2}{\sin\left(\sqrt{x}\right)}$$

input
```
integrate(cot(x^(1/2))*csc(x^(1/2))/x^(1/2),x, algorithm="giac")
```

output
```
-2/sin(sqrt(x))
```

3.843.9 Mupad [B] (verification not implemented)

Time = 26.83 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\cot\left(\sqrt{x}\right)\csc\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -\frac{2}{\sin\left(\sqrt{x}\right)}$$

input
```
int(cot(x^(1/2))/(x^(1/2)*sin(x^(1/2))),x)
```

output
```
-2/sin(x^(1/2))
```

3.844 $\int \frac{\cos(\sqrt{x})\sin(\sqrt{x})}{\sqrt{x}}\,dx$

3.844.1 Optimal result

Integrand size = 18, antiderivative size = 8

$$\int \frac{\cos(\sqrt{x})\sin(\sqrt{x})}{\sqrt{x}}\,dx = \sin^2(\sqrt{x})$$

output
```
sin(x^(1/2))^2
```

3.844.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.50

$$\int \frac{\cos(\sqrt{x})\sin(\sqrt{x})}{\sqrt{x}}\,dx = -\frac{1}{2}\cos(2\sqrt{x})$$

input
```
Integrate[(Cos[Sqrt[x]]*Sin[Sqrt[x]])/Sqrt[x],x]
```

output
```
-1/2*Cos[2*Sqrt[x]]
```

3.844.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {3922}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin \left(\sqrt{x}\right) \cos \left(\sqrt{x}\right)}{\sqrt{x}} \, dx$$

$$\Big\downarrow \text{ 3922}$$

$$\sin^2\left(\sqrt{x}\right)$$

input
```
Int[(Cos[Sqrt[x]]*Sin[Sqrt[x]])/Sqrt[x],x]
```

output
```
Sin[Sqrt[x]]^2
```

3.844.3.1 Defintions of rubi rules used

rule 3922
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[
{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```

3.844.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

method	result	size
derivativedivides	$-\cos\left(\sqrt{x}\right)^2$	9
default	$-\cos\left(\sqrt{x}\right)^2$	9
meijerg	$\frac{\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos\left(2\sqrt{x}\right)}{\sqrt{\pi}}\right)}{2}$	21

input
```
int(cos(x^(1/2))*sin(x^(1/2))/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-cos(x^(1/2))^2
```

3.844.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\cos \left(\sqrt{x}\right) \sin \left(\sqrt{x}\right)}{\sqrt{x}} \, dx = -\cos \left(\sqrt{x}\right)^2$$

input
```
integrate(cos(x^(1/2))*sin(x^(1/2))/x^(1/2),x, algorithm="fricas")
```

output
```
-cos(sqrt(x))^2
```

3.844.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{\cos \left(\sqrt{x}\right) \sin \left(\sqrt{x}\right)}{\sqrt{x}} \, dx = \sin^2 \left(\sqrt{x}\right)$$

input
```
integrate(cos(x**(1/2))*sin(x**(1/2))/x**(1/2),x)
```

output
```
sin(sqrt(x))**2
```

3.844.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\cos \left(\sqrt{x}\right) \sin \left(\sqrt{x}\right)}{\sqrt{x}} \, dx = -\cos \left(\sqrt{x}\right)^2$$

input
```
integrate(cos(x^(1/2))*sin(x^(1/2))/x^(1/2),x, algorithm="maxima")
```

output
```
-cos(sqrt(x))^2
```

3.844.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\cos\left(\sqrt{x}\right)\sin\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \sin\left(\sqrt{x}\right)^2$$

input
```
integrate(cos(x^(1/2))*sin(x^(1/2))/x^(1/2),x, algorithm="giac")
```

output
```
sin(sqrt(x))^2
```

3.844.9 Mupad [B] (verification not implemented)

Time = 26.62 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\sqrt{x}\right)\sin\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -\cos\left(\sqrt{x}\right)^2$$

input
```
int((cos(x^(1/2))*sin(x^(1/2)))/x^(1/2),x)
```

output
```
-cos(x^(1/2))^2
```

3.845 $\int \frac{\sec(\sqrt{x})\tan(\sqrt{x})}{\sqrt{x}}\,dx$

3.845.1 Optimal result

Integrand size = 18, antiderivative size = 8

$$\int \frac{\sec\left(\sqrt{x}\right)\tan\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\sec\left(\sqrt{x}\right)$$

output
```
2*sec(x^(1/2))
```

3.845.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\sec\left(\sqrt{x}\right)\tan\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\sec\left(\sqrt{x}\right)$$

input
```
Integrate[(Sec[Sqrt[x]]*Tan[Sqrt[x]])/Sqrt[x],x]
```

output
```
2*Sec[Sqrt[x]]
```

3.845.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {7266, 3042, 3086, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan\left(\sqrt{x}\right)\sec\left(\sqrt{x}\right)}{\sqrt{x}}dx$$

$$\Big\downarrow \text{ 7266}$$

$$2\int \sec\left(\sqrt{x}\right)\tan\left(\sqrt{x}\right)d\sqrt{x}$$

$$\Big\downarrow \text{ 3042}$$

$$2\int \sec\left(\sqrt{x}\right)\tan\left(\sqrt{x}\right)d\sqrt{x}$$

$$\Big\downarrow \text{ 3086}$$

$$2\int 1d\sec\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{ 24}$$

$$2\sec\left(\sqrt{x}\right)$$

input
```
Int[(Sec[Sqrt[x]]*Tan[Sqrt[x]])/Sqrt[x],x]
```

output
```
2*Sec[Sqrt[x]]
```

3.845.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

3.845.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$2\sec\left(\sqrt{x}\right)$	7
default	$2\sec\left(\sqrt{x}\right)$	7

input
```
int(sec(x^(1/2))*tan(x^(1/2))/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*sec(x^(1/2))
```

3.845.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\sec\left(\sqrt{x}\right)\tan\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \frac{2}{\cos\left(\sqrt{x}\right)}$$

input
```
integrate(sec(x^(1/2))*tan(x^(1/2))/x^(1/2),x, algorithm="fricas")
```

output
```
2/cos(sqrt(x))
```

3.845.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{\sec\left(\sqrt{x}\right)\tan\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\sec\left(\sqrt{x}\right)$$

input
```
integrate(sec(x**(1/2))*tan(x**(1/2))/x**(1/2),x)
```

output
```
2*sec(sqrt(x))
```

3.845.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\sec\left(\sqrt{x}\right)\tan\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \frac{2}{\cos\left(\sqrt{x}\right)}$$

input
```
integrate(sec(x^(1/2))*tan(x^(1/2))/x^(1/2),x, algorithm="maxima")
```

output
```
2/cos(sqrt(x))
```

3.845.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\sec\left(\sqrt{x}\right)\tan\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \frac{2}{\cos\left(\sqrt{x}\right)}$$

input
```
integrate(sec(x^(1/2))*tan(x^(1/2))/x^(1/2),x, algorithm="giac")
```

output
```
2/cos(sqrt(x))
```

3.845.9 Mupad [B] (verification not implemented)

Time = 26.99 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\sec\left(\sqrt{x}\right)\tan\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \frac{2}{\cos\left(\sqrt{x}\right)}$$

input
```
int(tan(x^(1/2))/(x^(1/2)*cos(x^(1/2))),x)
```

output
```
2/cos(x^(1/2))
```

3.846 $\int \frac{\sin^2(x)}{a+b\sin(2x)}\,dx$

3.846.1 Optimal result

Integrand size = 15, antiderivative size = 55

$$\int \frac{\sin^2(x)}{a+b\sin(2x)}\,dx=\frac{\arctan\left(\frac{b+a\tan(x)}{\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}}-\frac{\log(a+b\sin(2x))}{4b}$$

output
```
-1/4*ln(a+b*sin(2*x))/b+1/2*arctan((b+a*tan(x))/(a^2-b^2)^(1/2))/(a^2-b^2)
^(1/2)
```

3.846.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \frac{\sin^2(x)}{a+b\sin(2x)}\,dx=\frac{\arctan\left(\frac{b+a\tan(x)}{\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}}-\frac{\log(a+b\sin(2x))}{4b}$$

input
```
Integrate[Sin[x]^2/(a + b*Sin[2*x]),x]
```

output
```
ArcTan[(b + a*Tan[x])/Sqrt[a^2 - b^2]]/(2*Sqrt[a^2 - b^2]) - Log[a + b*Sin
[2*x]]/(4*b)
```

3.846.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.65, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 4889, 2142, 27, 240, 1142, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(x)}{a+b\sin(2x)}\,dx$$

$$\Big\downarrow \textbf{3042}$$

$$\int \frac{\sin(x)^2}{a+b\sin(2x)}dx$$

$$\Big\downarrow \textbf{4889}$$

$$\int \frac{\tan^2(x)}{(\tan^2(x)+1)\left(a\tan^2(x)+a+2b\tan(x)\right)}d\tan(x)$$

$$\Big\downarrow \textbf{2142}$$

$$\frac{\int -\frac{2ab\tan(x)}{a\tan^2(x)+2b\tan(x)+a}d\tan(x)}{4b^2}+\frac{\int \frac{2b\tan(x)}{\tan^2(x)+1}d\tan(x)}{4b^2}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\int \frac{\tan(x)}{\tan^2(x)+1}d\tan(x)}{2b}-\frac{a\int \frac{\tan(x)}{a\tan^2(x)+2b\tan(x)+a}d\tan(x)}{2b}$$

$$\Big\downarrow \textbf{240}$$

$$\frac{\log\left(\tan^2(x)+1\right)}{4b}-\frac{a\int \frac{\tan(x)}{a\tan^2(x)+2b\tan(x)+a}d\tan(x)}{2b}$$

$$\Big\downarrow \textbf{1142}$$

$$\frac{\log\left(\tan^2(x)+1\right)}{4b}-\frac{a\left(\frac{\int \frac{2(b+a\tan(x))}{a\tan^2(x)+2b\tan(x)+a}d\tan(x)}{2a}-\frac{b\int \frac{1}{a\tan^2(x)+2b\tan(x)+a}d\tan(x)}{a}\right)}{2b}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\log\left(\tan^2(x)+1\right)}{4b}-\frac{a\left(\frac{\int \frac{b+a\tan(x)}{a\tan^2(x)+2b\tan(x)+a}d\tan(x)}{a}-\frac{b\int \frac{1}{a\tan^2(x)+2b\tan(x)+a}d\tan(x)}{a}\right)}{2b}$$

$$\Big\downarrow \textbf{1083}$$

$$\frac{\log\left(\tan^2(x)+1\right)}{4b}-\frac{a\left(\frac{2b\int\frac{1}{-(2b+2a\tan(x))^2-4\left(a^2-b^2\right)}d(2b+2a\tan(x))}{a}+\frac{\int\frac{b+a\tan(x)}{a\tan^2(x)+2b\tan(x)+a}d\tan(x)}{a}\right)}{2b}$$

$\Big\downarrow$ 217

$$\frac{\log\left(\tan^2(x)+1\right)}{4b}-\frac{a\left(\frac{\int\frac{b+a\tan(x)}{a\tan^2(x)+2b\tan(x)+a}d\tan(x)}{a}-\frac{b\arctan\left(\frac{2a\tan(x)+2b}{2\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}\right)}{2b}$$

$\Big\downarrow$ 1103

$$\frac{\log\left(\tan^2(x)+1\right)}{4b}-\frac{a\left(\frac{\log\left(a\tan^2(x)+a+2b\tan(x)\right)}{2a}-\frac{b\arctan\left(\frac{2a\tan(x)+2b}{2\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}\right)}{2b}$$

input
```
Int[Sin[x]^2/(a + b*Sin[2*x]),x]
```

output
```
Log[1 + Tan[x]^2]/(4*b) - (a*(-((b*ArcTan[(2*b + 2*a*Tan[x])/(2*Sqrt[a^2 -
 b^2])])/(a*Sqrt[a^2 - b^2])) + Log[a + 2*b*Tan[x] + a*Tan[x]^2]/(2*a)))/(
2*b)
```

3.846.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

3.846. $\int\frac{\sin^2(x)}{a+b\sin(2x)}\,dx$

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2142
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (f_.)*(x_)^2)), x_Sym
bol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x, 2]
, q = c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2}, Simp[1/q   Int[(A*c^2*d - a
*c*C*d + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f + c*(B*c*d - b*C*d + A*b*f -
 a*B*f)*x)/(a + b*x + c*x^2), x], x] + Simp[1/q   Int[(c*C*d^2 + b*B*d*f -
A*c*d*f - a*C*d*f + a*A*f^2 - f*(B*c*d - b*C*d + A*b*f - a*B*f)*x)/(d + f*x
^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, f}, x] && PolyQ[Px, x, 2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.846.4 Maple [A] (verified)

Time = 2.22 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.45

method	result
default	$-\frac{a\left(\frac{\ln\left(a\tan(x)^{2}+2b\tan(x)+a\right)}{2a}-\frac{b\arctan\left(\frac{2a\tan(x)+2b}{2\sqrt{a^{2}-b^{2}}}\right)}{a\sqrt{a^{2}-b^{2}}}\right)}{2b}+\frac{\ln\left(1+\tan(x)^{2}\right)}{4b}$
risch	$-\frac{ix}{2b}+\frac{ix\,a^{2}b}{a^{2}b^{2}-b^{4}}-\frac{ix\,b^{3}}{a^{2}b^{2}-b^{4}}-\frac{\ln\left(\mathrm{e}^{2ix}-\frac{-iab+\sqrt{-a^{2}b^{2}+b^{4}}}{b^{2}}\right)a^{2}}{4(a^{2}-b^{2})b}+\frac{b\ln\left(\mathrm{e}^{2ix}-\frac{-iab+\sqrt{-a^{2}b^{2}+b^{4}}}{b^{2}}\right)}{4a^{2}-4b^{2}}-\frac{\ln\left(\mathrm{e}^{2ix}-\frac{-iab+\sqrt{-a^{2}b^{2}}}{b^{2}}\right.}{4(a^{2}-b^{2}}$

3.846. $\int\frac{\sin^{2}(x)}{a+b\sin(2x)}\,dx$

input
```
int(sin(x)^2/(a+b*sin(2*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/b*(1/2/a*ln(a*tan(x)^2+2*b*tan(x)+a)-b/a/(a^2-b^2)^(1/2)*arctan(1/2
*(2*a*tan(x)+2*b)/(a^2-b^2)^(1/2)))+1/4/b*ln(1+tan(x)^2)
```

3.846.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. $2(47) = 94$.

Time = 0.28 (sec) , antiderivative size = 320, normalized size of antiderivative = 5.82

$$\int \frac{\sin^2(x)}{a+b\sin(2x)}\,dx = \left[-\frac{\sqrt{-a^2+b^2}b\log\left(-\frac{4\,(2\,a^2-b^2)\cos(x)^4-4\,ab\cos(x)\sin(x)-4\,(2\,a^2-b^2)\cos(x)^2+a^2-2\,b^2+2\,\left(2\,b\cos(x)^2+2\,\left(2\,a\cos(x)^3-a\cos(x)\right)s\right.}{4\,b^2\cos(x)^4-4\,b^2\cos(x)^2-4\,ab\cos(x)\sin(x)-a^2}\right.}{8\,(a^2b-b^3)}\right.$$

$$-\frac{2\sqrt{a^2-b^2}b\arctan\left(-\frac{(2\,a\cos(x)\sin(x)+b)\sqrt{a^2-b^2}}{2\,(a^2-b^2)\cos(x)^2-a^2+b^2}\right)+(a^2-b^2)\log\left(-4\,b^2\cos(x)^4+4\,b^2\cos(x)^2+4\,ab\cos(\right.}{8\,(a^2b-b^3)}$$

input
```
integrate(sin(x)^2/(a+b*sin(2*x)),x, algorithm="fricas")
```

output
```
[-1/8*(sqrt(-a^2 + b^2)*b*log(-(4*(2*a^2 - b^2)*cos(x)^4 - 4*a*b*cos(x)*si
n(x) - 4*(2*a^2 - b^2)*cos(x)^2 + a^2 - 2*b^2 + 2*(2*b*cos(x)^2 + 2*(2*a*c
os(x)^3 - a*cos(x))*sin(x) - b)*sqrt(-a^2 + b^2))/(4*b^2*cos(x)^4 - 4*b^2*
cos(x)^2 - 4*a*b*cos(x)*sin(x) - a^2)) + (a^2 - b^2)*log(-4*b^2*cos(x)^4 +
 4*b^2*cos(x)^2 + 4*a*b*cos(x)*sin(x) + a^2))/(a^2*b - b^3), -1/8*(2*sqrt(
a^2 - b^2)*b*arctan(-(2*a*cos(x)*sin(x) + b)*sqrt(a^2 - b^2)/(2*(a^2 - b^2
)*cos(x)^2 - a^2 + b^2)) + (a^2 - b^2)*log(-4*b^2*cos(x)^4 + 4*b^2*cos(x)^
2 + 4*a*b*cos(x)*sin(x) + a^2))/(a^2*b - b^3)]
```

3.846.6 Sympy [A] (verification not implemented)

Time = 2.91 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.25

$$\int \frac{\sin^2(x)}{a+b\sin(2x)}\,dx = -\begin{cases} \frac{\log\left(\frac{a}{b}+\sin(2x)\right)}{4b} & \text{for } b\neq 0 \\ \frac{\sin(2x)}{4a} & \text{otherwise} \end{cases} + \begin{cases} \tilde{\infty}\log(\tan(x)) & \text{for } a=0\wedge b=0 \\ \frac{\log(\tan(x))}{4b} & \text{for } a=0 \\ \frac{1}{2b\tan(x)-2b} & \text{for } a=-b \\ -\frac{1}{2b\tan(x)+2b} & \text{for } a=b \\ \frac{\log\left(\tan(x)+\frac{b}{a}-\frac{\sqrt{-a^2+b^2}}{a}\right)}{4\sqrt{-a^2+b^2}} - \frac{\log\left(\tan(x)+\frac{b}{a}+\frac{\sqrt{-a^2+b^2}}{a}\right)}{4\sqrt{-a^2+b^2}} & \text{otherwise} \end{cases}$$

input
```
integrate(sin(x)**2/(a+b*sin(2*x)),x)
```

output
```
-Piecewise((log(a/b + sin(2*x))/(4*b), Ne(b, 0)), (sin(2*x)/(4*a), True))
+ Piecewise((zoo*log(tan(x)), Eq(a, 0) & Eq(b, 0)), (log(tan(x))/(4*b), Eq
(a, 0)), (1/(2*b*tan(x) - 2*b), Eq(a, -b)), (-1/(2*b*tan(x) + 2*b), Eq(a,
b)), (log(tan(x) + b/a - sqrt(-a**2 + b**2)/a)/(4*sqrt(-a**2 + b**2)) - lo
g(tan(x) + b/a + sqrt(-a**2 + b**2)/a)/(4*sqrt(-a**2 + b**2)), True))
```

3.846.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sin^2(x)}{a+b\sin(2x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sin(x)^2/(a+b*sin(2*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.846.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.40

$$\int \frac{\sin^2(x)}{a+b\sin(2x)}\,dx = \frac{\pi\left\lfloor \frac{x}{\pi}+\frac{1}{2}\right\rfloor \operatorname{sgn}(a)+\arctan\left(\frac{a\tan(x)+b}{\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}} - \frac{\log\left(a\tan\left(x\right)^2+2\,b\tan\left(x\right)+a\right)}{4\,b} + \frac{\log\left(\tan\left(x\right)^2+1\right)}{4\,b}$$

input
```
integrate(sin(x)^2/(a+b*sin(2*x)),x, algorithm="giac")
```

output
```
1/2*(pi*floor(x/pi + 1/2)*sgn(a) + arctan((a*tan(x) + b)/sqrt(a^2 - b^2)))
/sqrt(a^2 - b^2) - 1/4*log(a*tan(x)^2 + 2*b*tan(x) + a)/b + 1/4*log(tan(x)
^2 + 1)/b
```

3.846.9 Mupad [B] (verification not implemented)

Time = 28.41 (sec) , antiderivative size = 1108, normalized size of antiderivative = 20.15

$$\int \frac{\sin^2(x)}{a+b\sin(2x)}\,dx = \frac{\ln\left(\tan(x)^2+1\right)}{4\,b} + \frac{\operatorname{atan}\left(\frac{2\tan(x)\left(a^2-b^2\right)^{3/2}\left(\frac{\left(4\,a^2\,b-2\,b^3\right)\left(2\,a\,b-\frac{\frac{8\,a\,b^3+\frac{\left(8\,a^2\,b-8\,b^3\right)\left(96\,a\,b^4-64\,a^3\,b^2\right)}{2\left(16\,b^4-16\,a^2\,b^2\right)}}{4\sqrt{a^2-b^2}}+\frac{\left(8\,a^2\,b-8\,b^3\right)\left(96\,a\,b^4-64\,a^3\,b^2\right)}{8\sqrt{a^2-b^2}\left(16\,b^4-16\,a^2\,b^2\right)}}{4\sqrt{a^2-b^2}}+\frac{\left(8\,a^2\,b-8\,b^3\right)\left(4\,a^3\cdots\right)}{\cdots}\cdots\right)}{a^3\left(4\,a^2-3\,b^2\right)\cdots}\cdots\right)}{\cdots}\right)}{\cdots} + \frac{\ln\left(a\tan(x)^2+2\,b\tan(x)+a\right)\,\left(8\,a^2\,b-8\,b^3\right)}{2\,\left(16\,b^4-16\,a^2\,b^2\right)}$$

input
```
int(sin(x)^2/(a + b*sin(2*x)),x)
```

output
```
log(tan(x)^2 + 1)/(4*b) + atan((2*tan(x)*(a^2 - b^2)^(3/2)*(((4*a^2*b - 2*
b^3)*(2*a*b - ((8*a*b^3 + ((8*a^2*b - 8*b^3)*(96*a*b^4 - 64*a^3*b^2))/(2*(
16*b^4 - 16*a^2*b^2)))/(4*(a^2 - b^2)^(1/2)) + ((8*a^2*b - 8*b^3)*(96*a*b^
4 - 64*a^3*b^2))/(8*(a^2 - b^2)^(1/2)*(16*b^4 - 16*a^2*b^2)))/(4*(a^2 - b^
2)^(1/2)) + ((8*a^2*b - 8*b^3)*(4*a^3 - 16*a*b^2 + ((8*a^2*b - 8*b^3)*(8*a
*b^3 + ((8*a^2*b - 8*b^3)*(96*a*b^4 - 64*a^3*b^2))/(2*(16*b^4 - 16*a^2*b^2
))))/(2*(16*b^4 - 16*a^2*b^2))))/(2*(16*b^4 - 16*a^2*b^2)) - ((8*a^2*b - 8
*b^3)*(96*a*b^4 - 64*a^3*b^2))/(32*(a^2 - b^2)*(16*b^4 - 16*a^2*b^2))))/(a
^3*(4*a^2 - 3*b^2)^2) - ((4*a^4 + 2*b^4 - 5*a^2*b^2)*((4*a^3 - 16*a*b^2 +
((8*a^2*b - 8*b^3)*(8*a*b^3 + ((8*a^2*b - 8*b^3)*(96*a*b^4 - 64*a^3*b^2))/
(2*(16*b^4 - 16*a^2*b^2))))/(2*(16*b^4 - 16*a^2*b^2)))/(4*(a^2 - b^2)^(1/2
)) - (96*a*b^4 - 64*a^3*b^2)/(64*(a^2 - b^2)^(3/2)) + ((8*a^2*b - 8*b^3)*(
(8*a*b^3 + ((8*a^2*b - 8*b^3)*(96*a*b^4 - 64*a^3*b^2))/(2*(16*b^4 - 16*a^2
*b^2)))/(4*(a^2 - b^2)^(1/2)) + ((8*a^2*b - 8*b^3)*(96*a*b^4 - 64*a^3*b^2)
)/(8*(a^2 - b^2)^(1/2)*(16*b^4 - 16*a^2*b^2))))/(2*(16*b^4 - 16*a^2*b^2)))
)/(a^3*(a^2 - b^2)^(1/2)*(4*a^2 - 3*b^2)^2)))/a + (2*(a^2 - b^2)*((6*a^2*b
 - (8*a^2*b^3*(8*a^2*b - 8*b^3)^2)/(16*b^4 - 16*a^2*b^2)^2)/(4*(a^2 - b^2)
^(1/2)) + (a^2*b^3)/(2*(a^2 - b^2)^(3/2)) - (4*a^2*b^3*(8*a^2*b - 8*b^3)^2
)/((a^2 - b^2)^(1/2)*(16*b^4 - 16*a^2*b^2)^2))*(4*a^4 + 2*b^4 - 5*a^2*b^2)
)/(a^4*(4*a^2 - 3*b^2)^2) - (2*(4*a^2*b - 2*b^3)*(a^2 - b^2)^(3/2)*(((8...
```

3.846. $\int \frac{\sin^2(x)}{a+b\sin(2x)}\,dx$

3.847 $\int \frac{\cos^2(x)}{a+b\sin(2x)}\,dx$

3.847.1 Optimal result

Integrand size = 15, antiderivative size = 55

$$\int \frac{\cos^2(x)}{a+b\sin(2x)}\,dx=\frac{\arctan\left(\frac{b+a\tan(x)}{\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}}+\frac{\log(a+b\sin(2x))}{4b}$$

output
```
1/4*ln(a+b*sin(2*x))/b+1/2*arctan((b+a*tan(x))/(a^2-b^2)^(1/2))/(a^2-b^2)^
(1/2)
```

3.847.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98

$$\int \frac{\cos^2(x)}{a+b\sin(2x)}\,dx=\frac{1}{4}\left(\frac{2\arctan\left(\frac{b+a\tan(x)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{\log(a+b\sin(2x))}{b}\right)$$

input
```
Integrate[Cos[x]^2/(a + b*Sin[2*x]),x]
```

output
```
((2*ArcTan[(b + a*Tan[x])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + Log[a + b*Si
n[2*x]]/b)/4
```

3.847.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.51, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 4889, 1312, 27, 240, 1142, 27, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(x)}{a+b\sin(2x)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(x)^2}{a+b\sin(2x)}dx$$

$$\Big\downarrow \text{ 4889}$$

$$\int \frac{1}{\left(\tan^2(x)+1\right)\left(a\tan^2(x)+a+2b\tan(x)\right)}d\tan(x)$$

$$\Big\downarrow \text{ 1312}$$

$$\frac{\int \frac{2b(2b+a\tan(x))}{a\tan^2(x)+2b\tan(x)+a}d\tan(x)}{4b^2}-\frac{\int \frac{2b\tan(x)}{\tan^2(x)+1}d\tan(x)}{4b^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{2b+a\tan(x)}{a\tan^2(x)+2b\tan(x)+a}d\tan(x)}{2b}-\frac{\int \frac{\tan(x)}{\tan^2(x)+1}d\tan(x)}{2b}$$

$$\Big\downarrow \text{ 240}$$

$$\frac{\int \frac{2b+a\tan(x)}{a\tan^2(x)+2b\tan(x)+a}d\tan(x)}{2b}-\frac{\log\left(\tan^2(x)+1\right)}{4b}$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{b\int \frac{1}{a\tan^2(x)+2b\tan(x)+a}d\tan(x)+\frac{1}{2}\int \frac{2(b+a\tan(x))}{a\tan^2(x)+2b\tan(x)+a}d\tan(x)}{2b}-\frac{\log\left(\tan^2(x)+1\right)}{4b}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{b\int \frac{1}{a\tan^2(x)+2b\tan(x)+a}d\tan(x)+\int \frac{b+a\tan(x)}{a\tan^2(x)+2b\tan(x)+a}d\tan(x)}{2b}-\frac{\log\left(\tan^2(x)+1\right)}{4b}$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{\int \frac{b+a\tan(x)}{a\tan^2(x)+2b\tan(x)+a}d\tan(x)-2b\int \frac{1}{-(2b+2a\tan(x))^2-4(a^2-b^2)}d(2b+2a\tan(x))}{2b}-\frac{\log\left(\tan^2(x)+1\right)}{4b}$$

$$\downarrow \text{217}$$

$$\frac{\int \frac{b+a\tan(x)}{a\tan^2(x)+2b\tan(x)+a}d\tan(x)+\frac{b\arctan\left(\frac{2a\tan(x)+2b}{2\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}}{2b}-\frac{\log\left(\tan^2(x)+1\right)}{4b}$$

$$\downarrow \text{1103}$$

$$\frac{\frac{b\arctan\left(\frac{2a\tan(x)+2b}{2\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{1}{2}\log\left(a\tan^2(x)+a+2b\tan(x)\right)}{2b}-\frac{\log\left(\tan^2(x)+1\right)}{4b}$$

input
```
Int[Cos[x]^2/(a + b*Sin[2*x]),x]
```

output
```
-1/4*Log[1 + Tan[x]^2]/b + ((b*ArcTan[(2*b + 2*a*Tan[x])/(2*Sqrt[a^2 - b^2
])])/Sqrt[a^2 - b^2] + Log[a + 2*b*Tan[x] + a*Tan[x]^2]/2)/(2*b)
```

3.847.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

3.847. $\int \frac{\cos^2(x)}{a+b\sin(2x)}\,dx$

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1312
```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (f_.)*(x_)^2)), x_Symbol]
 :> With[{q = c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2}, Simp[1/q   Int[(c^2
*d + b^2*f - a*c*f + b*c*f*x)/(a + b*x + c*x^2), x], x] - Simp[1/q   Int[(c
*d*f - a*f^2 + b*f^2*x)/(d + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c
, d, f}, x] && NeQ[b^2 - 4*a*c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.847.4 Maple [A] (verified)

Time = 2.25 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.31

method	result
default	$-\frac{\ln\left(1+\tan(x)^{2}\right)}{4b}+\frac{\frac{\ln\left(a\tan(x)^{2}+2b\tan(x)+a\right)}{2}+\frac{b\arctan\left(\frac{2a\tan(x)+2b}{2\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}}}{2b}$
risch	$\frac{ix}{2b}-\frac{ix\,a^{2}b}{a^{2}b^{2}-b^{4}}+\frac{ix\,b^{3}}{a^{2}b^{2}-b^{4}}+\frac{\ln\left(e^{2ix}+\frac{iab+\sqrt{-a^{2}b^{2}+b^{4}}}{b^{2}}\right)a^{2}}{4(a^{2}-b^{2})b}-\frac{b\ln\left(e^{2ix}+\frac{iab+\sqrt{-a^{2}b^{2}+b^{4}}}{b^{2}}\right)}{4(a^{2}-b^{2})}+\frac{\ln\left(e^{2ix}+\frac{iab+\sqrt{-a^{2}b^{2}+b^{4}}}{b^{2}}\right)\sqrt{\ }}{4(a^{2}-b^{2})b}$

input
```
int(cos(x)^2/(a+b*sin(2*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/4/b*ln(1+tan(x)^2)+1/2/b*(1/2*ln(a*tan(x)^2+2*b*tan(x)+a)+b/(a^2-b^2)^(
1/2)*arctan(1/2*(2*a*tan(x)+2*b)/(a^2-b^2)^(1/2)))
```

3.847.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(47) = 94.

Time = 0.30 (sec) , antiderivative size = 322, normalized size of antiderivative = 5.85

$$\int \frac{\cos^2(x)}{a+b\sin(2x)}\,dx$$
$$= \left[-\frac{\sqrt{-a^2+b^2}b\log\left(-\frac{4\,(2\,a^2-b^2)\cos(x)^4-4\,ab\cos(x)\sin(x)-4\,(2\,a^2-b^2)\cos(x)^2+a^2-2\,b^2+2\left(2\,b\cos(x)^2+2\left(2\,a\cos(x)^3-a\cos(x)\right)\text{s}\right.}{4\,b^2\cos(x)^4-4\,b^2\cos(x)^2-4\,ab\cos(x)\sin(x)-a^2}}{8\,(a^2b-b^3)}\right.$$
$$-\frac{2\sqrt{a^2-b^2}b\arctan\left(-\frac{(2\,a\cos(x)\sin(x)+b)\sqrt{a^2-b^2}}{2\,(a^2-b^2)\cos(x)^2-a^2+b^2}\right)-(a^2-b^2)\log\left(-4\,b^2\cos(x)^4+4\,b^2\cos(x)^2+4\,ab\cos(\right.}{8\,(a^2b-b^3)}$$

input
```
integrate(cos(x)^2/(a+b*sin(2*x)),x, algorithm="fricas")
```

output
```
[-1/8*(sqrt(-a^2 + b^2)*b*log(-(4*(2*a^2 - b^2)*cos(x)^4 - 4*a*b*cos(x)*si
n(x) - 4*(2*a^2 - b^2)*cos(x)^2 + a^2 - 2*b^2 + 2*(2*b*cos(x)^2 + 2*(2*a*c
os(x)^3 - a*cos(x))*sin(x) - b)*sqrt(-a^2 + b^2))/(4*b^2*cos(x)^4 - 4*b^2*
cos(x)^2 - 4*a*b*cos(x)*sin(x) - a^2)) - (a^2 - b^2)*log(-4*b^2*cos(x)^4 +
 4*b^2*cos(x)^2 + 4*a*b*cos(x)*sin(x) + a^2))/(a^2*b - b^3), -1/8*(2*sqrt(
a^2 - b^2)*b*arctan(-(2*a*cos(x)*sin(x) + b)*sqrt(a^2 - b^2)/(2*(a^2 - b^2
)*cos(x)^2 - a^2 + b^2)) - (a^2 - b^2)*log(-4*b^2*cos(x)^4 + 4*b^2*cos(x)^
2 + 4*a*b*cos(x)*sin(x) + a^2))/(a^2*b - b^3)]
```

3.847.6 Sympy [A] (verification not implemented)

Time = 2.98 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.25

$$\int \frac{\cos^2(x)}{a+b\sin(2x)}\,dx = \begin{cases} \frac{\log\left(\frac{a}{b}+\sin(2x)\right)}{4b} & \text{for } b\neq 0 \\ \frac{\sin(2x)}{4a} & \text{otherwise} \end{cases} + \begin{cases} \tilde{\infty}\log(\tan(x)) & \text{for } a=0\wedge b=0 \\ \frac{\log(\tan(x))}{4b} & \text{for } a=0 \\ \frac{1}{2b\tan(x)-2b} & \text{for } a=-b \\ -\frac{1}{2b\tan(x)+2b} & \text{for } a=b \\ \frac{\log\left(\tan(x)+\frac{b}{a}-\frac{\sqrt{-a^2+b^2}}{a}\right)}{4\sqrt{-a^2+b^2}} - \frac{\log\left(\tan(x)+\frac{b}{a}+\frac{\sqrt{-a^2+b^2}}{a}\right)}{4\sqrt{-a^2+b^2}} & \text{otherwise} \end{cases}$$

input
```
integrate(cos(x)**2/(a+b*sin(2*x)),x)
```

output
```
Piecewise((log(a/b + sin(2*x))/(4*b), Ne(b, 0)), (sin(2*x)/(4*a), True)) +
 Piecewise((zoo*log(tan(x)), Eq(a, 0) & Eq(b, 0)), (log(tan(x))/(4*b), Eq(
a, 0)), (1/(2*b*tan(x) - 2*b), Eq(a, -b)), (-1/(2*b*tan(x) + 2*b), Eq(a, b
)), (log(tan(x) + b/a - sqrt(-a**2 + b**2)/a)/(4*sqrt(-a**2 + b**2)) - log
(tan(x) + b/a + sqrt(-a**2 + b**2)/a)/(4*sqrt(-a**2 + b**2)), True))
```

3.847.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^2(x)}{a+b\sin(2x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(x)^2/(a+b*sin(2*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.847.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.40

$$\int \frac{\cos^2(x)}{a+b\sin(2x)}\,dx = \frac{\pi\left\lfloor \frac{x}{\pi}+\frac{1}{2}\right\rfloor \operatorname{sgn}(a)+\arctan\left(\frac{a\tan(x)+b}{\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}} + \frac{\log\left(a\tan(x)^2+2\,b\tan(x)+a\right)}{4\,b} - \frac{\log\left(\tan(x)^2+1\right)}{4\,b}$$

input
```
integrate(cos(x)^2/(a+b*sin(2*x)),x, algorithm="giac")
```

output
```
1/2*(pi*floor(x/pi + 1/2)*sgn(a) + arctan((a*tan(x) + b)/sqrt(a^2 - b^2)))
/sqrt(a^2 - b^2) + 1/4*log(a*tan(x)^2 + 2*b*tan(x) + a)/b - 1/4*log(tan(x)
^2 + 1)/b
```

3.847.9 Mupad [B] (verification not implemented)

Time = 26.73 (sec) , antiderivative size = 1374, normalized size of antiderivative = 24.98

$$\int \frac{\cos^2(x)}{a+b\sin(2x)}\,dx = \text{Too large to display}$$

input
```
int(cos(x)^2/(a + b*sin(2*x)),x)
```

output
```
- log(tan(x)^2 + 1)/(4*b) - atan((2*tan(x)*((6*b*(2*a^2 - 3*b^2)*(((24*a*b
^3 - ((8*a^2*b - 8*b^3)*(96*a*b^4 - 64*a^3*b^2))/(2*(16*b^4 - 16*a^2*b^2))
)/(4*(a^2 - b^2)^(1/2)) - ((8*a^2*b - 8*b^3)*(96*a*b^4 - 64*a^3*b^2))/(8*(
a^2 - b^2)^(1/2)*(16*b^4 - 16*a^2*b^2)))/(4*(a^2 - b^2)^(1/2)) + ((8*a^2*b
 - 8*b^3)*(4*a^3 - ((8*a^2*b - 8*b^3)*(24*a*b^3 - ((8*a^2*b - 8*b^3)*(96*a
*b^4 - 64*a^3*b^2))/(2*(16*b^4 - 16*a^2*b^2))))/(2*(16*b^4 - 16*a^2*b^2))
)/(2*(16*b^4 - 16*a^2*b^2)) - ((8*a^2*b - 8*b^3)*(96*a*b^4 - 64*a^3*b^2))/
(32*(a^2 - b^2)*(16*b^4 - 16*a^2*b^2))))/(a^3*(4*a^2 - 3*b^2)^2) - (((96*a
*b^4 - 64*a^3*b^2)/(64*(a^2 - b^2)^(3/2)) - (4*a^3 - ((8*a^2*b - 8*b^3)*(2
4*a*b^3 - ((8*a^2*b - 8*b^3)*(96*a*b^4 - 64*a^3*b^2))/(2*(16*b^4 - 16*a^2*
b^2))))/(2*(16*b^4 - 16*a^2*b^2)))/(4*(a^2 - b^2)^(1/2)) + ((8*a^2*b - 8*b
^3)*((24*a*b^3 - ((8*a^2*b - 8*b^3)*(96*a*b^4 - 64*a^3*b^2))/(2*(16*b^4 -
16*a^2*b^2)))/(4*(a^2 - b^2)^(1/2)) - ((8*a^2*b - 8*b^3)*(96*a*b^4 - 64*a^
3*b^2))/(8*(a^2 - b^2)^(1/2)*(16*b^4 - 16*a^2*b^2))))/(2*(16*b^4 - 16*a^2*
b^2)))*(4*a^4 + 18*b^4 - 21*a^2*b^2))/(a^3*(a^2 - b^2)^(1/2)*(4*a^2 - 3*b^
2)^2))*(a^2 - b^2)^(3/2))/a - (2*(a^2 - b^2)*((a^2*b^3)/(2*(a^2 - b^2)^(3/
2)) - (2*a^2*b - ((8*a^2*b - 8*b^3)*(16*a^2*b^2 - (16*a^2*b^3*(8*a^2*b - 8
*b^3))/(16*b^4 - 16*a^2*b^2)))/(2*(16*b^4 - 16*a^2*b^2)))/(4*(a^2 - b^2)^(
1/2)) + ((8*a^2*b - 8*b^3)*((16*a^2*b^2 - (16*a^2*b^3*(8*a^2*b - 8*b^3))/(
16*b^4 - 16*a^2*b^2))/(4*(a^2 - b^2)^(1/2)) - (4*a^2*b^3*(8*a^2*b - 8*b...
```

3.847. $\int \frac{\cos^2(x)}{a+b\sin(2x)}\,dx$

3.848 $\int \frac{\sin^2(x)}{a+b\cos(2x)}\,dx$

3.848.1 Optimal result

Integrand size = 15, antiderivative size = 52

$$\int \frac{\sin^2(x)}{a+b\cos(2x)}\,dx = -\frac{x}{2b}+\frac{\sqrt{a+b}\arctan\left(\frac{\sqrt{a-b}\tan(x)}{\sqrt{a+b}}\right)}{2\sqrt{a-b}b}$$

output
```
-1/2*x/b+1/2*arctan((a-b)^(1/2)*tan(x)/(a+b)^(1/2))*(a+b)^(1/2)/b/(a-b)^(1
/2)
```

3.848.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.92

$$\int \frac{\sin^2(x)}{a+b\cos(2x)}\,dx = -\frac{x+\frac{(a+b)\operatorname{arctanh}\left(\frac{(a-b)\tan(x)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}}{2b}$$

input
```
Integrate[Sin[x]^2/(a + b*Cos[2*x]),x]
```

output
```
-1/2*(x + ((a + b)*ArcTanh[((a - b)*Tan[x])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 +
 b^2])/b
```

3.848.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.27, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 4889, 1450, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(x)}{a+b\cos(2x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)^2}{a+b\cos(2x)}dx$$

$$\Big\downarrow \text{4889}$$

$$\int \frac{\tan^2(x)}{(a-b)\tan^4(x)+2a\tan^2(x)+a+b}d\tan(x)$$

$$\Big\downarrow \text{1450}$$

$$\frac{1}{2}\left(1-\frac{a}{b}\right)\int \frac{1}{(a-b)\tan^2(x)+a-b}d\tan(x)+\frac{(a+b)\int \frac{1}{(a-b)\tan^2(x)+a+b}d\tan(x)}{2b}$$

$$\Big\downarrow \text{218}$$

$$\frac{\left(1-\frac{a}{b}\right)\arctan(\tan(x))}{2(a-b)}+\frac{\sqrt{a+b}\arctan\left(\frac{\sqrt{a-b}\tan(x)}{\sqrt{a+b}}\right)}{2b\sqrt{a-b}}$$

input
```
Int[Sin[x]^2/(a + b*Cos[2*x]),x]
```

output
```
((1 - a/b)*ArcTan[Tan[x]])/(2*(a - b)) + (Sqrt[a + b]*ArcTan[(Sqrt[a - b]*
Tan[x])/Sqrt[a + b]])/(2*Sqrt[a - b]*b)
```

3.848.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1450
```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^2/2)*(b/q + 1)   Int[(d*x)^(m - 2)/(b/
2 + q/2 + c*x^2), x], x] - Simp[(d^2/2)*(b/q - 1)   Int[(d*x)^(m - 2)/(b/2
- q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] &&
 GeQ[m, 2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.848.4 Maple [A] (verified)

Time = 2.23 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{\arctan(\tan(x))}{2b}+\frac{(a+b)\arctan\left(\frac{(a-b)\tan(x)}{\sqrt{(a+b)(a-b)}}\right)}{2b\sqrt{(a+b)(a-b)}}$	49
risch	$-\frac{x}{2b}-\frac{\sqrt{-(a+b)(a-b)}\,\ln\left(\mathrm{e}^{2ix}+\frac{i\sqrt{-(a+b)(a-b)}+a}{b}\right)}{4(a-b)b}+\frac{\sqrt{-(a+b)(a-b)}\,\ln\left(\mathrm{e}^{2ix}-\frac{i\sqrt{-(a+b)(a-b)}-a}{b}\right)}{4(a-b)b}$	115

input
```
int(sin(x)^2/(a+b*cos(2*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b*arctan(tan(x))+1/2*(a+b)/b/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(x)/
((a+b)*(a-b))^(1/2))
```

3.848.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 225, normalized size of antiderivative = 4.33

$$\int \frac{\sin^2(x)}{a+b\cos(2x)}\,dx = \left[\frac{\sqrt{-\frac{a+b}{a-b}}\log\left(\frac{4\,(2\,a^2-b^2)\cos(x)^4-4\,(2\,a^2-ab-b^2)\cos(x)^2-4\left(2\,(a^2-ab)\cos(x)^3-(a^2-2\,ab+b^2)\cos(x)\right)\sqrt{-\frac{a+b}{a-b}}\sin(x)+a^2-2\,ab+b^2}{4\,b^2\cos(x)^4+4\,(ab-b^2)\cos(x)^2+a^2-2\,ab+b^2}\right)}{8\,b}\right.$$
$$\left.-\frac{\sqrt{\frac{a+b}{a-b}}\arctan\left(\frac{\left(2\,a\cos(x)^2-a+b\right)\sqrt{\frac{a+b}{a-b}}}{2\,(a+b)\cos(x)\sin(x)}\right)+2\,x}{4\,b}\right]$$

input
```
integrate(sin(x)^2/(a+b*cos(2*x)),x, algorithm="fricas")
```

output
```
[1/8*(sqrt(-(a + b)/(a - b))*log((4*(2*a^2 - b^2)*cos(x)^4 - 4*(2*a^2 - a*
b - b^2)*cos(x)^2 - 4*(2*(a^2 - a*b)*cos(x)^3 - (a^2 - 2*a*b + b^2)*cos(x)
)*sqrt(-(a + b)/(a - b))*sin(x) + a^2 - 2*a*b + b^2)/(4*b^2*cos(x)^4 + 4*(
a*b - b^2)*cos(x)^2 + a^2 - 2*a*b + b^2)) - 4*x)/b, -1/4*(sqrt((a + b)/(a
- b))*arctan(1/2*(2*a*cos(x)^2 - a + b)*sqrt((a + b)/(a - b))/((a + b)*cos
(x)*sin(x))) + 2*x)/b]
```

3.848.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(39) = 78$.

Time = 16.32 (sec) , antiderivative size = 432, normalized size of antiderivative = 8.31

$$\int \frac{\sin^2(x)}{a+b\cos(2x)}\,dx$$
$$= \begin{cases} \tilde{\infty}\left(-\frac{\log(\tan(x)-1)}{2}+\frac{\log(\tan(x)+1)}{2}\right) & \text{for } a=0 \wedge b=0 \\ \frac{\tan(x)}{4b} & \text{for } a=b \\ \frac{1}{4b\tan(x)} & \text{for } a=-b \\ \frac{\log\left(-\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan(x)\right)}{4a\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-4b\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}-\frac{\log\left(\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan(x)\right)}{4a\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-4b\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}} & \text{otherwise} \end{cases}$$
$$-\begin{cases} \tilde{\infty}x \\ \frac{x}{2b}-\frac{\tan(x)}{4b} \\ \frac{x}{2b}+\frac{1}{4b\tan(x)} \\ \frac{\sin(2x)}{4a} \\ \frac{2ax\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}{4ab\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-4b^2\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}-\frac{a\log\left(-\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan(x)\right)}{4ab\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-4b^2\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}+\frac{a\log\left(\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan(x)\right)}{4ab\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-4b^2\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}-\frac{2\cdots}{4ab\sqrt{-\frac{a}{a-\cdots}}} \end{cases}$$

input
```
integrate(sin(x)**2/(a+b*cos(2*x)),x)
```

output
```
Piecewise((zoo*(-log(tan(x) - 1)/2 + log(tan(x) + 1)/2), Eq(a, 0) & Eq(b,
0)), (tan(x)/(4*b), Eq(a, b)), (1/(4*b*tan(x)), Eq(a, -b)), (log(-sqrt(-a/
(a - b) - b/(a - b)) + tan(x))/(4*a*sqrt(-a/(a - b) - b/(a - b)) - 4*b*sqr
t(-a/(a - b) - b/(a - b))) - log(sqrt(-a/(a - b) - b/(a - b)) + tan(x))/(4
*a*sqrt(-a/(a - b) - b/(a - b)) - 4*b*sqrt(-a/(a - b) - b/(a - b))), True)
) - Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (x/(2*b) - tan(x)/(4*b), Eq(a,
 b)), (x/(2*b) + 1/(4*b*tan(x)), Eq(a, -b)), (sin(2*x)/(4*a), Eq(b, 0)), (
2*a*x*sqrt(-a/(a - b) - b/(a - b))/(4*a*b*sqrt(-a/(a - b) - b/(a - b)) - 4
*b**2*sqrt(-a/(a - b) - b/(a - b))) - a*log(-sqrt(-a/(a - b) - b/(a - b))
+ tan(x))/(4*a*b*sqrt(-a/(a - b) - b/(a - b)) - 4*b**2*sqrt(-a/(a - b) - b
/(a - b))) + a*log(sqrt(-a/(a - b) - b/(a - b)) + tan(x))/(4*a*b*sqrt(-a/(
a - b) - b/(a - b)) - 4*b**2*sqrt(-a/(a - b) - b/(a - b))) - 2*b*x*sqrt(-a
/(a - b) - b/(a - b))/(4*a*b*sqrt(-a/(a - b) - b/(a - b)) - 4*b**2*sqrt(-a
/(a - b) - b/(a - b))), True))
```

3.848. $\int \frac{\sin^2(x)}{a+b\cos(2x)}\,dx$

3.848.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sin^2(x)}{a + b\cos(2x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sin(x)^2/(a+b*cos(2*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.848.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(40) = 80.

Time = 0.29 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.71

$$\int \frac{\sin^2(x)}{a + b\cos(2x)}\,dx = \frac{\sqrt{a^2 - b^2}\left(\pi\left\lfloor\frac{x}{\pi} + \frac{1}{2}\right\rfloor + \arctan\left(\frac{2\tan(x)}{\sqrt{\frac{4a + \sqrt{-16(a+b)(a-b)+16a^2}}{a-b}}}\right)\right)|a - b|}{2(a^2 - 2ab + b^2)|b|} - \frac{\pi\left\lfloor\frac{x}{\pi} + \frac{1}{2}\right\rfloor + \arctan\left(\frac{2\tan(x)}{\sqrt{\frac{4a - \sqrt{-16(a+b)(a-b)+16a^2}}{a-b}}}\right)}{2|b|}$$

input
```
integrate(sin(x)^2/(a+b*cos(2*x)),x, algorithm="giac")
```

output
```
1/2*sqrt(a^2 - b^2)*(pi*floor(x/pi + 1/2) + arctan(2*tan(x)/sqrt((4*a + sq
rt(-16*(a + b)*(a - b) + 16*a^2))/(a - b))))*abs(a - b)/((a^2 - 2*a*b + b^
2)*abs(b)) - 1/2*(pi*floor(x/pi + 1/2) + arctan(2*tan(x)/sqrt((4*a - sqrt(
-16*(a + b)*(a - b) + 16*a^2))/(a - b))))/abs(b)
```

3.848.9 Mupad [B] (verification not implemented)

Time = 26.85 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.08

$$\int \frac{\sin^2(x)}{a+b\cos(2x)}\,dx = \frac{\operatorname{atan}\left(\frac{2\,b^3\tan(x)}{2\,a^2\,b-2\,b^3}-\frac{2\,a^2\,b\tan(x)}{2\,a^2\,b-2\,b^3}\right)}{2\,b}+\frac{\operatorname{atanh}\left(\frac{a\tan(x)}{\sqrt{b^2-a^2}}-\frac{b\tan(x)}{\sqrt{b^2-a^2}}\right)\sqrt{b^2-a^2}}{2\,(a\,b-b^2)}$$

input
```
int(sin(x)^2/(a + b*cos(2*x)),x)
```

output
```
atan((2*b^3*tan(x))/(2*a^2*b - 2*b^3) - (2*a^2*b*tan(x))/(2*a^2*b - 2*b^3)
)/(2*b) + (atanh((a*tan(x))/(b^2 - a^2)^(1/2) - (b*tan(x))/(b^2 - a^2)^(1/
2))*(b^2 - a^2)^(1/2))/(2*(a*b - b^2))
```

3.849 $\int \frac{\cos^2(x)}{a+b\cos(2x)}\,dx$

3.849.1 Optimal result

Integrand size = 15, antiderivative size = 52

$$\int \frac{\cos^2(x)}{a+b\cos(2x)}\,dx=\frac{x}{2b}-\frac{\sqrt{a-b}\arctan\left(\frac{\sqrt{a-b}\tan(x)}{\sqrt{a+b}}\right)}{2b\sqrt{a+b}}$$

output
```
1/2*x/b-1/2*arctan((a-b)^(1/2)*tan(x)/(a+b)^(1/2))*(a-b)^(1/2)/b/(a+b)^(1/
2)
```

3.849.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96

$$\int \frac{\cos^2(x)}{a+b\cos(2x)}\,dx=\frac{x+\frac{(a-b)\text{arctanh}\left(\frac{(a-b)\tan(x)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}}{2b}$$

input
```
Integrate[Cos[x]^2/(a + b*Cos[2*x]),x]
```

output
```
(x + ((a - b)*ArcTanh[((a - b)*Tan[x])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2]
)/(2*b)
```

3.849.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3042, 4889, 1406, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(x)}{a+b\cos(2x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(x)^2}{a+b\cos(2x)}dx$$

$$\Big\downarrow \text{4889}$$

$$\int \frac{1}{(a-b)\tan^4(x)+2a\tan^2(x)+a+b}d\tan(x)$$

$$\Big\downarrow \text{1406}$$

$$\frac{(a-b)\int \frac{1}{(a-b)\tan^2(x)+a-b}d\tan(x)}{2b}-\frac{(a-b)\int \frac{1}{(a-b)\tan^2(x)+a+b}d\tan(x)}{2b}$$

$$\Big\downarrow \text{218}$$

$$\frac{\arctan(\tan(x))}{2b}-\frac{\sqrt{a-b}\arctan\left(\frac{\sqrt{a-b}\tan(x)}{\sqrt{a+b}}\right)}{2b\sqrt{a+b}}$$

input
```
Int[Cos[x]^2/(a + b*Cos[2*x]),x]
```

output
```
ArcTan[Tan[x]]/(2*b) - (Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Tan[x])/Sqrt[a + b
]])/(2*b*Sqrt[a + b])
```

3.849.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.849.4 Maple [A] (verified)

Time = 2.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{(-a+b)\arctan\left(\frac{(a-b)\tan(x)}{\sqrt{(a+b)(a-b)}}\right)}{2b\sqrt{(a+b)(a-b)}}+\frac{\arctan(\tan(x))}{2b}$	51
risch	$\frac{x}{2b}+\frac{\sqrt{-(a+b)(a-b)}\,\ln\left(e^{2ix}+\frac{i\sqrt{-(a+b)(a-b)}+a}{b}\right)}{4(a+b)b}-\frac{\sqrt{-(a+b)(a-b)}\,\ln\left(e^{2ix}-\frac{i\sqrt{-(a+b)(a-b)}-a}{b}\right)}{4(a+b)b}$	111

input
```
int(cos(x)^2/(a+b*cos(2*x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-a+b)/b/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(x)/((a+b)*(a-b))^(1/2))+
1/2/b*arctan(tan(x))
```

3.849.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 224, normalized size of antiderivative = 4.31

$$\int \frac{\cos^2(x)}{a+b\cos(2x)}\,dx$$
$$= \left[\frac{\sqrt{-\frac{a-b}{a+b}}\log\left(\frac{4\,(2\,a^2-b^2)\cos(x)^4-4\,(2\,a^2-ab-b^2)\cos(x)^2+4\left(2\,(a^2+ab)\cos(x)^3-(a^2-b^2)\cos(x)\right)\sqrt{-\frac{a-b}{a+b}}\sin(x)+a^2-2\,ab+b^2}{4\,b^2\cos(x)^4+4\,(ab-b^2)\cos(x)^2+a^2-2\,ab+b^2}\right)+4}{8\,b}\right.$$
$$\left.-\frac{\sqrt{\frac{a-b}{a+b}}\arctan\left(-\frac{\left(2\,a\cos(x)^2-a+b\right)\sqrt{\frac{a-b}{a+b}}}{2\,(a-b)\cos(x)\sin(x)}\right)-2\,x}{4\,b}\right]$$

input
```
integrate(cos(x)^2/(a+b*cos(2*x)),x, algorithm="fricas")
```

output
```
[1/8*(sqrt(-(a - b)/(a + b))*log((4*(2*a^2 - b^2)*cos(x)^4 - 4*(2*a^2 - a*
b - b^2)*cos(x)^2 + 4*(2*(a^2 + a*b)*cos(x)^3 - (a^2 - b^2)*cos(x))*sqrt(-
(a - b)/(a + b))*sin(x) + a^2 - 2*a*b + b^2)/(4*b^2*cos(x)^4 + 4*(a*b - b^
2)*cos(x)^2 + a^2 - 2*a*b + b^2)) + 4*x)/b, -1/4*(sqrt((a - b)/(a + b))*ar
ctan(-1/2*(2*a*cos(x)^2 - a + b)*sqrt((a - b)/(a + b))/((a - b)*cos(x)*sin
(x))) - 2*x)/b]
```

3.849.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. 2(39) = 78.

Time = 16.28 (sec) , antiderivative size = 432, normalized size of antiderivative = 8.31

$$\int \frac{\cos^2(x)}{a+b\cos(2x)}\,dx$$
$$= \begin{cases} \tilde{\infty}\left(-\frac{\log(\tan(x)-1)}{2}+\frac{\log(\tan(x)+1)}{2}\right) & \text{for } a=0\wedge b=0 \\ \frac{\tan(x)}{4b} & \text{for } a=b \\ \frac{1}{4b\tan(x)} & \text{for } a=-b \\ \frac{\log\left(-\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan(x)\right)}{4a\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-4b\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}-\frac{\log\left(\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan(x)\right)}{4a\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-4b\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}} & \text{otherwise} \end{cases}$$
$$+ \begin{cases} \tilde{\infty}x \\ \frac{x}{2b}-\frac{\tan(x)}{4b} \\ \frac{x}{2b}+\frac{1}{4b\tan(x)} \\ \frac{\sin(2x)}{4a} \\ \frac{2ax\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}{4ab\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-4b^2\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}-\frac{a\log\left(-\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan(x)\right)}{4ab\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-4b^2\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}+\frac{a\log\left(\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}+\tan(x)\right)}{4ab\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}-4b^2\sqrt{-\frac{a}{a-b}-\frac{b}{a-b}}}-\frac{2\cdots}{4ab\sqrt{-\frac{a}{a-\cdots}}} \end{cases}$$

input
```
integrate(cos(x)**2/(a+b*cos(2*x)),x)
```

output
```
Piecewise((zoo*(-log(tan(x) - 1)/2 + log(tan(x) + 1)/2), Eq(a, 0) & Eq(b,
0)), (tan(x)/(4*b), Eq(a, b)), (1/(4*b*tan(x)), Eq(a, -b)), (log(-sqrt(-a/
(a - b) - b/(a - b)) + tan(x))/(4*a*sqrt(-a/(a - b) - b/(a - b)) - 4*b*sqr
t(-a/(a - b) - b/(a - b))) - log(sqrt(-a/(a - b) - b/(a - b)) + tan(x))/(4
*a*sqrt(-a/(a - b) - b/(a - b)) - 4*b*sqrt(-a/(a - b) - b/(a - b))), True)
) + Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (x/(2*b) - tan(x)/(4*b), Eq(a,
 b)), (x/(2*b) + 1/(4*b*tan(x)), Eq(a, -b)), (sin(2*x)/(4*a), Eq(b, 0)), (
2*a*x*sqrt(-a/(a - b) - b/(a - b))/(4*a*b*sqrt(-a/(a - b) - b/(a - b)) - 4
*b**2*sqrt(-a/(a - b) - b/(a - b))) - a*log(-sqrt(-a/(a - b) - b/(a - b))
+ tan(x))/(4*a*b*sqrt(-a/(a - b) - b/(a - b)) - 4*b**2*sqrt(-a/(a - b) - b
/(a - b))) + a*log(sqrt(-a/(a - b) - b/(a - b)) + tan(x))/(4*a*b*sqrt(-a/(
a - b) - b/(a - b)) - 4*b**2*sqrt(-a/(a - b) - b/(a - b))) - 2*b*x*sqrt(-a
/(a - b) - b/(a - b))/(4*a*b*sqrt(-a/(a - b) - b/(a - b)) - 4*b**2*sqrt(-a
/(a - b) - b/(a - b))), True))
```

3.849. $\int \frac{\cos^2(x)}{a+b\cos(2x)}\,dx$

3.849.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cos^2(x)}{a+b\cos(2x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cos(x)^2/(a+b*cos(2*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

3.849.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(40) = 80$.

Time = 0.30 (sec) , antiderivative size = 159, normalized size of antiderivative = 3.06

$$\int \frac{\cos^2(x)}{a+b\cos(2x)}\,dx = -\frac{\sqrt{a^2-b^2}\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\tan(x)}{\sqrt{\frac{4a+\sqrt{-16(a+b)(a-b)+16a^2}}{a-b}}}\right)\right)|a-b|}{2\left((a-b)b^2+(a^2-ab)|b|\right)} - \frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\tan(x)}{\sqrt{\frac{4a-\sqrt{-16(a+b)(a-b)+16a^2}}{a-b}}}\right)\right)(a-b)}{2\left(b^2-a|b|\right)}$$

input
```
integrate(cos(x)^2/(a+b*cos(2*x)),x, algorithm="giac")
```

output
```
-1/2*sqrt(a^2 - b^2)*(pi*floor(x/pi + 1/2) + arctan(2*tan(x)/sqrt((4*a + s
qrt(-16*(a + b)*(a - b) + 16*a^2))/(a - b))))*abs(a - b)/((a - b)*b^2 + (a
^2 - a*b)*abs(b)) - 1/2*(pi*floor(x/pi + 1/2) + arctan(2*tan(x)/sqrt((4*a
- sqrt(-16*(a + b)*(a - b) + 16*a^2))/(a - b))))*(a - b)/(b^2 - a*abs(b))
```

3.849.9 Mupad [B] (verification not implemented)

Time = 26.61 (sec) , antiderivative size = 684, normalized size of antiderivative = 13.15

$$\int \frac{\cos^2(x)}{a+b\cos(2x)}\,dx = \frac{\operatorname{atan}\left(\frac{2\,a^2\tan(x)}{2\,a^2-4\,a\,b+2\,b^2}+\frac{2\,b^2\tan(x)}{2\,a^2-4\,a\,b+2\,b^2}-\frac{4\,a\,b\tan(x)}{2\,a^2-4\,a\,b+2\,b^2}\right)}{2\,b} + \frac{\operatorname{atan}\left(\frac{\frac{\left(\frac{\tan(x)\left(4\,a^3-12\,a^2\,b+12\,a\,b^2-4\,b^3\right)}{4}+\frac{\sqrt{b^2-a^2}\left(4\,b^4-8\,a\,b^3+4\,a^2\,b^2+\frac{\tan(x)\sqrt{b^2-a^2}\left(64\,a^3\,b^2-128\,a^2\,b^3+64\,a\,b^4\right)}{16\left(b^2+a\,b\right)}\right)}{4\left(b^2+a\,b\right)}\right)\sqrt{b^2-a^2}\,1\mathrm{i}}{b^2+a\,b}+\cdots}{\frac{\left(\frac{\tan(x)\left(4\,a^3-12\,a^2\,b+12\,a\,b^2-4\,b^3\right)}{4}+\frac{\sqrt{b^2-a^2}\left(4\,b^4-8\,a\,b^3+4\,a^2\,b^2+\frac{\tan(x)\sqrt{b^2-a^2}\left(64\,a^3\,b^2-128\,a^2\,b^3+64\,a\,b^4\right)}{16\left(b^2+a\,b\right)}\right)}{4\left(b^2+a\,b\right)}\right)\sqrt{b^2-a^2}}{b^2+a\,b}-\cdots}\right)\cdots}{2\left(b^2+a\cdots\right.}$$

input
```
int(cos(x)^2/(a + b*cos(2*x)),x)
```

output
```
atan((2*a^2*tan(x))/(2*a^2 - 4*a*b + 2*b^2) + (2*b^2*tan(x))/(2*a^2 - 4*a*
b + 2*b^2) - (4*a*b*tan(x))/(2*a^2 - 4*a*b + 2*b^2))/(2*b) + (atan((((tan
(x)*(12*a*b^2 - 12*a^2*b + 4*a^3 - 4*b^3))/4 + ((b^2 - a^2)^(1/2)*(4*b^4 -
 8*a*b^3 + 4*a^2*b^2 + (tan(x)*(b^2 - a^2)^(1/2)*(64*a*b^4 - 128*a^2*b^3 +
 64*a^3*b^2))/(16*(a*b + b^2))))/(4*(a*b + b^2)))*(b^2 - a^2)^(1/2)*1i)/(a
*b + b^2) + (((tan(x)*(12*a*b^2 - 12*a^2*b + 4*a^3 - 4*b^3))/4 + ((b^2 - a
^2)^(1/2)*(8*a*b^3 - 4*b^4 - 4*a^2*b^2 + (tan(x)*(b^2 - a^2)^(1/2)*(64*a*b
^4 - 128*a^2*b^3 + 64*a^3*b^2))/(16*(a*b + b^2))))/(4*(a*b + b^2)))*(b^2 -
 a^2)^(1/2)*1i)/(a*b + b^2))/((((tan(x)*(12*a*b^2 - 12*a^2*b + 4*a^3 - 4*b
^3))/4 + ((b^2 - a^2)^(1/2)*(4*b^4 - 8*a*b^3 + 4*a^2*b^2 + (tan(x)*(b^2 -
a^2)^(1/2)*(64*a*b^4 - 128*a^2*b^3 + 64*a^3*b^2))/(16*(a*b + b^2))))/(4*(a
*b + b^2)))*(b^2 - a^2)^(1/2))/(a*b + b^2) - (((tan(x)*(12*a*b^2 - 12*a^2*
b + 4*a^3 - 4*b^3))/4 + ((b^2 - a^2)^(1/2)*(8*a*b^3 - 4*b^4 - 4*a^2*b^2 +
(tan(x)*(b^2 - a^2)^(1/2)*(64*a*b^4 - 128*a^2*b^3 + 64*a^3*b^2))/(16*(a*b
+ b^2))))/(4*(a*b + b^2)))*(b^2 - a^2)^(1/2))/(a*b + b^2)))*(b^2 - a^2)^(1
/2)*1i)/(2*(a*b + b^2))
```

3.850 $\int \frac{\tan(c+dx)}{\sqrt{a\sin^2(c+dx)}}\,dx$

3.850.1 Optimal result

Integrand size = 21, antiderivative size = 30

$$\int \frac{\tan(c+dx)}{\sqrt{a\sin^2(c+dx)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a\sin^2(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

output
```
arctanh((a*sin(d*x+c)^2)^(1/2)/a^(1/2))/d/a^(1/2)
```

3.850.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \frac{\tan(c+dx)}{\sqrt{a\sin^2(c+dx)}}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))\sin(c+dx)}{d\sqrt{a\sin^2(c+dx)}}$$

input
```
Integrate[Tan[c + d*x]/Sqrt[a*Sin[c + d*x]^2],x]
```

output
```
(ArcTanh[Sin[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a*Sin[c + d*x]^2])
```

3.850.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3684, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(c+dx)}{\sqrt{a\sin^2(c+dx)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\tan(c+dx)}{\sqrt{a\sin(c+dx)^2}}dx$$

$$\Big\downarrow \text{ 3684}$$

$$\frac{\int \frac{1}{\sqrt{a\sin^2(c+dx)}(1-\sin^2(c+dx))}d\sin^2(c+dx)}{2d}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{\int \frac{1}{1-\frac{\sin^4(c+dx)}{a}}d\sqrt{a\sin^2(c+dx)}}{ad}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{a\sin^2(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

input
```
Int[Tan[c + d*x]/Sqrt[a*Sin[c + d*x]^2],x]
```

output
```
ArcTanh[Sqrt[a*Sin[c + d*x]^2]/Sqrt[a]]/(Sqrt[a]*d)
```

3.850.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3684
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m + 1
)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
 + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Inte
gerQ[(m - 1)/2] && IntegerQ[n/2]
```

3.850.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{\sin(dx+c)\,\operatorname{arctanh}(\sin(dx+c))}{\sqrt{a\sin(dx+c)^{2}}\,d}$	30
risch	$-\frac{2\ln\left(e^{idx}-ie^{-ic}\right)\sin(dx+c)}{d\sqrt{-\left(e^{2i(dx+c)}-1\right)^{2}a\,e^{-2i(dx+c)}}}+\frac{2\ln\left(e^{idx}+ie^{-ic}\right)\sin(dx+c)}{d\sqrt{-\left(e^{2i(dx+c)}-1\right)^{2}a\,e^{-2i(dx+c)}}}$	110

input
```
int(tan(d*x+c)/(a*sin(d*x+c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(a*sin(d*x+c)^2)^(1/2)*sin(d*x+c)*arctanh(sin(d*x+c))/d
```

3.850.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 91, normalized size of antiderivative = 3.03

$$\int \frac{\tan(c+dx)}{\sqrt{a\sin^2(c+dx)}}\,dx = \left[\frac{\sqrt{-a\cos(dx+c)^2+a}\log\left(-\frac{\sin(dx+c)+1}{\sin(dx+c)-1}\right)}{2\,ad\sin(dx+c)}, -\frac{\sqrt{-a}\arctan\left(\frac{\sqrt{-a\cos(dx+c)^2+a}\sqrt{-a}}{a}\right)}{ad}\right]$$

input
```
integrate(tan(d*x+c)/(a*sin(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(-a*cos(d*x + c)^2 + a)*log(-(sin(d*x + c) + 1)/(sin(d*x + c) - 1
))/(a*d*sin(d*x + c)), -sqrt(-a)*arctan(sqrt(-a*cos(d*x + c)^2 + a)*sqrt(-
a)/a)/(a*d)]
```

3.850.6 Sympy [F]

$$\int \frac{\tan(c+dx)}{\sqrt{a\sin^2(c+dx)}}\,dx = \int \frac{\tan(c+dx)}{\sqrt{a\sin^2(c+dx)}}\,dx$$

input
```
integrate(tan(d*x+c)/(a*sin(d*x+c)**2)**(1/2),x)
```

output
```
Integral(tan(c + d*x)/sqrt(a*sin(c + d*x)**2), x)
```

3.850.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(24) = 48.

Time = 0.34 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.53

$$\int \frac{\tan(c+dx)}{\sqrt{a\sin^2(c+dx)}}\,dx = \frac{\frac{(-1)^{2\,a\sin(dx+c)}\log\left(-\frac{a\sin(dx+c)}{\sin(dx+c)+1}\right)}{\sqrt{a}} + \frac{(-1)^{2\,a\sin(dx+c)}\log\left(-\frac{a\sin(dx+c)}{\sin(dx+c)-1}\right)}{\sqrt{a}}}{2\,d}$$

input
```
integrate(tan(d*x+c)/(a*sin(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*((-1)^(2*a*sin(d*x + c))*log(-a*sin(d*x + c)/(sin(d*x + c) + 1))/sqrt(
a) + (-1)^(2*a*sin(d*x + c))*log(-a*sin(d*x + c)/(sin(d*x + c) - 1))/sqrt(
a))/d
```

3.850.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(24) = 48$.

Time = 0.41 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.87

$$\int \frac{\tan(c+dx)}{\sqrt{a\sin^2(c+dx)}}\,dx = \frac{\log\left(\left|\frac{1}{\sin(dx+c)}+\sin(dx+c)+2\right|\right)-\log\left(\left|\frac{1}{\sin(dx+c)}+\sin(dx+c)-2\right|\right)}{4\sqrt{a}d\operatorname{sgn}(\sin(dx+c))}$$

input
```
integrate(tan(d*x+c)/(a*sin(d*x+c)^2)^(1/2),x, algorithm="giac")
```

output
```
1/4*(log(abs(1/sin(d*x + c) + sin(d*x + c) + 2)) - log(abs(1/sin(d*x + c)
+ sin(d*x + c) - 2)))/(sqrt(a)*d*sgn(sin(d*x + c)))
```

3.850.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan(c+dx)}{\sqrt{a\sin^2(c+dx)}}\,dx = \int \frac{\tan(c+d\,x)}{\sqrt{a\sin(c+d\,x)^2}}\,dx$$

input
```
int(tan(c + d*x)/(a*sin(c + d*x)^2)^(1/2),x)
```

output
```
int(tan(c + d*x)/(a*sin(c + d*x)^2)^(1/2), x)
```

3.851 $\int \frac{\cot(c+dx)}{\sqrt{a\cos^2(c+dx)}}\,dx$

3.851.1 Optimal result

Integrand size = 21, antiderivative size = 31

$$\int \frac{\cot(c+dx)}{\sqrt{a\cos^2(c+dx)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a\cos^2(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

output
```
-arctanh((a*cos(d*x+c)^2)^(1/2)/a^(1/2))/d/a^(1/2)
```

3.851.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{\cot(c+dx)}{\sqrt{a\cos^2(c+dx)}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a\cos^2(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

input
```
Integrate[Cot[c + d*x]/Sqrt[a*Cos[c + d*x]^2],x]
```

output
```
-(ArcTanh[Sqrt[a*Cos[c + d*x]^2]/Sqrt[a]]/(Sqrt[a]*d))
```

3.851.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 25, 3684, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(c+dx)}{\sqrt{a\cos^2(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\tan\left(c+dx+\frac{\pi}{2}\right)}{\sqrt{a\sin\left(c+dx+\frac{\pi}{2}\right)^2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\tan\left(\frac{1}{2}(2c+\pi)+dx\right)}{\sqrt{a\sin\left(\frac{1}{2}(2c+\pi)+dx\right)^2}}dx$$

$$\Big\downarrow \text{3684}$$

$$-\frac{\int \frac{1}{\sqrt{a\cos^2(c+dx)}(1-\cos^2(c+dx))}d\cos^2(c+dx)}{2d}$$

$$\Big\downarrow \text{73}$$

$$-\frac{\int \frac{1}{1-\frac{\cos^4(c+dx)}{a}}d\sqrt{a\cos^2(c+dx)}}{ad}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\text{arctanh}\left(\frac{\sqrt{a\cos^2(c+dx)}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

input
```
Int[Cot[c + d*x]/Sqrt[a*Cos[c + d*x]^2],x]
```

output
```
-(ArcTanh[Sqrt[a*Cos[c + d*x]^2]/Sqrt[a]]/(Sqrt[a]*d))
```

3.851.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3684
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Simp[ff^((m + 1
)/2)/(2*f)   Subst[Int[x^((m - 1)/2)*((b*ff^(n/2)*x^(n/2))^p/(1 - ff*x)^((m
 + 1)/2)), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Inte
gerQ[(m - 1)/2] && IntegerQ[n/2]
```

3.851.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

method	result	size
default	$-\frac{\cos(dx+c)\operatorname{arctanh}(\cos(dx+c))}{\sqrt{a\cos(dx+c)^2}\, d}$	31
risch	$\frac{2\ln\left(\mathrm{e}^{idx}-\mathrm{e}^{-ic}\right)\cos(dx+c)}{d\sqrt{a\left(\mathrm{e}^{2i(dx+c)}+1\right)^2\mathrm{e}^{-2i(dx+c)}}}-\frac{2\ln\left(\mathrm{e}^{idx}+\mathrm{e}^{-ic}\right)\cos(dx+c)}{d\sqrt{a\left(\mathrm{e}^{2i(dx+c)}+1\right)^2\mathrm{e}^{-2i(dx+c)}}}$	104

input
```
int(cot(d*x+c)/(a*cos(d*x+c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(a*cos(d*x+c)^2)^(1/2)*cos(d*x+c)*arctanh(cos(d*x+c))/d
```

3.851.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.71

$$\int \frac{\cot(c+dx)}{\sqrt{a\cos^2(c+dx)}}\,dx = \left[-\frac{\sqrt{a\cos\left(dx+c\right)^2}\log\left(-\frac{\cos(dx+c)+1}{\cos(dx+c)-1}\right)}{2\,ad\cos\left(dx+c\right)}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{a\cos(dx+c)^2}\sqrt{-a}}{a}\right)}{ad}\right]$$

input
```
integrate(cot(d*x+c)/(a*cos(d*x+c)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(a*cos(d*x + c)^2)*log(-(cos(d*x + c) + 1)/(cos(d*x + c) - 1))/(
a*d*cos(d*x + c)), sqrt(-a)*arctan(sqrt(a*cos(d*x + c)^2)*sqrt(-a)/a)/(a*d
)]
```

3.851.6 Sympy [F]

$$\int \frac{\cot(c+dx)}{\sqrt{a\cos^2(c+dx)}}\,dx = \int \frac{\cot\left(c+dx\right)}{\sqrt{a\cos^2\left(c+dx\right)}}\,dx$$

input
```
integrate(cot(d*x+c)/(a*cos(d*x+c)**2)**(1/2),x)
```

output
```
Integral(cot(c + d*x)/sqrt(a*cos(c + d*x)**2), x)
```

3.851.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(25) = 50$.

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.65

$$\int \frac{\cot(c+dx)}{\sqrt{a\cos^2(c+dx)}}\,dx = -\frac{\log\left(\frac{2\sqrt{-a\sin(dx+c)^2+a}\sqrt{a}}{|\sin(dx+c)|}+\frac{2\,a}{|\sin(dx+c)|}\right)}{\sqrt{a}d}$$

input
```
integrate(cot(d*x+c)/(a*cos(d*x+c)^2)^(1/2),x, algorithm="maxima")
```

output
```
-log(2*sqrt(-a*sin(d*x + c)^2 + a)*sqrt(a)/abs(sin(d*x + c)) + 2*a/abs(sin
(d*x + c)))/(sqrt(a)*d)
```

3.851.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{\cot(c+dx)}{\sqrt{a\cos^2(c+dx)}}\,dx = \frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{\sqrt{a}d\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}$$

input
```
integrate(cot(d*x+c)/(a*cos(d*x+c)^2)^(1/2),x, algorithm="giac")
```

output
```
log(abs(tan(1/2*d*x + 1/2*c)))/(sqrt(a)*d*sgn(cos(d*x + c)))
```

3.851.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot(c+dx)}{\sqrt{a\cos^2(c+dx)}}\,dx = \int \frac{\cot(c+d\,x)}{\sqrt{a\cos\left(c+d\,x\right)^2}}\,dx$$

input
```
int(cot(c + d*x)/(a*cos(c + d*x)^2)^(1/2),x)
```

output
```
int(cot(c + d*x)/(a*cos(c + d*x)^2)^(1/2), x)
```

3.852 $\int \frac{x\cos(x^2)}{\sqrt{\sin(x^2)}}\,dx$

3.852.1 Optimal result

Integrand size = 14, antiderivative size = 8

$$\int \frac{x\cos(x^2)}{\sqrt{\sin(x^2)}}\,dx = \sqrt{\sin(x^2)}$$

output
```
sin(x^2)^(1/2)
```

3.852.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{x\cos(x^2)}{\sqrt{\sin(x^2)}}\,dx = \sqrt{\sin(x^2)}$$

input
```
Integrate[(x*Cos[x^2])/Sqrt[Sin[x^2]],x]
```

output
```
Sqrt[Sin[x^2]]
```

3.852.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {3922}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\cos(x^2)}{\sqrt{\sin(x^2)}}dx$$
$$\Big\downarrow \text{3922}$$
$$\sqrt{\sin(x^2)}$$

input
```
Int[(x*Cos[x^2])/Sqrt[Sin[x^2]],x]
```

output
```
Sqrt[Sin[x^2]]
```

3.852.3.1 Defintions of rubi rules used

rule 3922
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[
{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```

3.852.4 Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\sqrt{\sin(x^2)}$	7
default	$\sqrt{\sin(x^2)}$	7
risch	$\sqrt{\sin(x^2)}$	7

input
```
int(x*cos(x^2)/sin(x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
sin(x^2)^(1/2)
```

3.852.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{x\cos(x^2)}{\sqrt{\sin(x^2)}}\,dx = \sqrt{\sin(x^2)}$$

input
```
integrate(x*cos(x^2)/sin(x^2)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(sin(x^2))
```

3.852.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{x\cos(x^2)}{\sqrt{\sin(x^2)}}\,dx = \sqrt{\sin(x^2)}$$

input
```
integrate(x*cos(x**2)/sin(x**2)**(1/2),x)
```

output
```
sqrt(sin(x**2))
```

3.852.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{x\cos(x^2)}{\sqrt{\sin(x^2)}}\,dx = \sqrt{\sin(x^2)}$$

input
```
integrate(x*cos(x^2)/sin(x^2)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(sin(x^2))
```

3.852.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{x \cos(x^2)}{\sqrt{\sin(x^2)}} \, dx = \sqrt{\sin(x^2)}$$

input
```
integrate(x*cos(x^2)/sin(x^2)^(1/2),x, algorithm="giac")
```

output
```
sqrt(sin(x^2))
```

3.852.9 Mupad [B] (verification not implemented)

Time = 27.19 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{x \cos(x^2)}{\sqrt{\sin(x^2)}} \, dx = \sqrt{\sin(x^2)}$$

input
```
int((x*cos(x^2))/sin(x^2)^(1/2),x)
```

output
```
sin(x^2)^(1/2)
```

3.853 $\int \frac{\cos(x)}{\sqrt{1-\cos(2x)}}\,dx$

3.853.1 Optimal result

Integrand size = 15, antiderivative size = 19

$$\int \frac{\cos(x)}{\sqrt{1-\cos(2x)}}\,dx = \frac{\log(\sin(x))\sin(x)}{\sqrt{2}\sqrt{\sin^2(x)}}$$

output
```
1/2*ln(sin(x))*sin(x)*2^(1/2)/(sin(x)^2)^(1/2)
```

3.853.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \frac{\cos(x)}{\sqrt{1-\cos(2x)}}\,dx = \frac{(\log(\cos(x))+\log(\tan(x)))\sin(x)}{\sqrt{1-\cos(2x)}}$$

input
```
Integrate[Cos[x]/Sqrt[1 - Cos[2*x]],x]
```

output
```
((Log[Cos[x]] + Log[Tan[x]])*Sin[x])/Sqrt[1 - Cos[2*x]]
```

3.853.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4856, 27, 20, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(x)}{\sqrt{1-\cos(2x)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos(x)}{\sqrt{1-\cos(2x)}}dx$$
$$\Big\downarrow \text{4856}$$
$$\int \frac{1}{\sqrt{2}\sqrt{\sin^2(x)}}d\sin(x)$$
$$\Big\downarrow \text{27}$$
$$\frac{\int \frac{1}{\sqrt{\sin^2(x)}}d\sin(x)}{\sqrt{2}}$$
$$\Big\downarrow \text{20}$$
$$\frac{\sin(x)\int \csc(x)d\sin(x)}{\sqrt{2}\sqrt{\sin^2(x)}}$$
$$\Big\downarrow \text{14}$$
$$\frac{\sin(x)\log(\sin(x))}{\sqrt{2}\sqrt{\sin^2(x)}}$$

input
```
Int[Cos[x]/Sqrt[1 - Cos[2*x]],x]
```

output
```
(Log[Sin[x]]*Sin[x])/(Sqrt[2]*Sqrt[Sin[x]^2])
```

3.853.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 20
```
Int[((a_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a*x^n)^p/x^(n*p)   Int[x^(n*
p), x], x] /; FreeQ[{a, n, p}, x] &&  !IntegerQ[p]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4856
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[d/(b*c)   Subst[Int[SubstFor[1, Sin[c*(a + b
*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

3.853.4 Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.32

method	result	size
default	$\frac{\sin(x)(\ln(\cos(x)-1)+\ln(\cos(x)+1))\sqrt{2}}{2\sqrt{2-2\cos(2x)}}$	25
risch	$-\frac{i\sqrt{2}\,x\sin(x)}{\sqrt{-(\mathrm{e}^{2ix}-1)^2\mathrm{e}^{-2ix}}}+\frac{\sqrt{2}\,\ln(\mathrm{e}^{2ix}-1)\sin(x)}{\sqrt{-(\mathrm{e}^{2ix}-1)^2\mathrm{e}^{-2ix}}}$	61

input
```
int(cos(x)/(1-cos(2*x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*sin(x)*(ln(cos(x)-1)+ln(cos(x)+1))*2^(1/2)/(sin(x)^2)^(1/2)
```

3.853.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{\cos(x)}{\sqrt{1-\cos(2x)}}\,dx = \frac{\sqrt{-2\cos(x)^2+2}\log\left(\frac{1}{2}\sin(x)\right)}{2\sin(x)}$$

input
```
integrate(cos(x)/(1-cos(2*x))^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(-2*cos(x)^2 + 2)*log(1/2*sin(x))/sin(x)
```

3.853.6 Sympy [F]

$$\int \frac{\cos(x)}{\sqrt{1-\cos(2x)}}\,dx = \int \frac{\cos(x)}{\sqrt{1-\cos(2x)}}\,dx$$

input
```
integrate(cos(x)/(1-cos(2*x))**(1/2),x)
```

output
```
Integral(cos(x)/sqrt(1 - cos(2*x)), x)
```

3.853.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(16) = 32.

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.16

$$\int \frac{\cos(x)}{\sqrt{1-\cos(2x)}}\,dx = \frac{1}{4}\sqrt{2}\log\left(\cos(x)^2+\sin(x)^2+2\cos(x)+1\right) + \frac{1}{4}\sqrt{2}\log\left(\cos(x)^2+\sin(x)^2-2\cos(x)+1\right)$$

input
```
integrate(cos(x)/(1-cos(2*x))^(1/2),x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/4*sqrt(2)*log(cos(
x)^2 + sin(x)^2 - 2*cos(x) + 1)
```

3.853.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{\cos(x)}{\sqrt{1-\cos(2x)}}\,dx = \frac{\sqrt{2}\log\left(|\sin\left(x\right)|\right)}{2\,\mathrm{sgn}\left(\sin\left(x\right)\right)}$$

input
```
integrate(cos(x)/(1-cos(2*x))^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*log(abs(sin(x)))/sgn(sin(x))
```

3.853.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(x)}{\sqrt{1-\cos(2x)}}\,dx = \int \frac{\cos\left(x\right)}{\sqrt{1-\cos\left(2\,x\right)}}\,dx$$

input
```
int(cos(x)/(1 - cos(2*x))^(1/2),x)
```

output
```
int(cos(x)/(1 - cos(2*x))^(1/2), x)
```

3.854 $\int \frac{\cos^2(\log(x))\sin^2(\log(x))}{x}\,dx$

3.854.1 Optimal result

Integrand size = 14, antiderivative size = 29

$$\int \frac{\cos^2(\log(x))\sin^2(\log(x))}{x}\,dx = \frac{\log(x)}{8} + \frac{1}{8}\cos(\log(x))\sin(\log(x)) - \frac{1}{4}\cos^3(\log(x))\sin(\log(x))$$

output
```
1/8*ln(x)+1/8*cos(ln(x))*sin(ln(x))-1/4*cos(ln(x))^3*sin(ln(x))
```

3.854.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.55

$$\int \frac{\cos^2(\log(x))\sin^2(\log(x))}{x}\,dx = \frac{\log(x)}{8} - \frac{1}{32}\sin(4\log(x))$$

input
```
Integrate[(Cos[Log[x]]^2*Sin[Log[x]]^2)/x,x]
```

output
```
Log[x]/8 - Sin[4*Log[x]]/32
```

3.854.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3039, 3042, 3048, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(\log(x))\cos^2(\log(x))}{x}dx$$
$$\Big\downarrow \text{3039}$$
$$\int \sin^2(\log(x))\cos^2(\log(x))d\log(x)$$
$$\Big\downarrow \text{3042}$$
$$\int \sin(\log(x))^2\cos(\log(x))^2d\log(x)$$
$$\Big\downarrow \text{3048}$$
$$\frac{1}{4}\int \cos^2(\log(x))d\log(x)-\frac{1}{4}\sin(\log(x))\cos^3(\log(x))$$
$$\Big\downarrow \text{3042}$$
$$\frac{1}{4}\int \sin\left(\log(x)+\frac{\pi}{2}\right)^2d\log(x)-\frac{1}{4}\sin(\log(x))\cos^3(\log(x))$$
$$\Big\downarrow \text{3115}$$
$$\frac{1}{4}\left(\frac{\int 1d\log(x)}{2}+\frac{1}{2}\sin(\log(x))\cos(\log(x))\right)-\frac{1}{4}\sin(\log(x))\cos^3(\log(x))$$
$$\Big\downarrow \text{24}$$
$$\frac{1}{4}\left(\frac{\log(x)}{2}+\frac{1}{2}\sin(\log(x))\cos(\log(x))\right)-\frac{1}{4}\sin(\log(x))\cos^3(\log(x))$$

input
```
Int[(Cos[Log[x]]^2*Sin[Log[x]]^2)/x,x]
```

output
```
-1/4*(Cos[Log[x]]^3*Sin[Log[x]]) + (Log[x]/2 + (Cos[Log[x]]*Sin[Log[x]])/2
)/4
```

3.854.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3048
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Simp[a^2*((m - 1)/(m + n))   Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
 && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

3.854.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.45

method	result	size
parallelrisch	$\ln\left(x^{\frac{1}{8}}\right)-\frac{\sin(4\ln(x))}{32}$	13
derivativedivides	$\frac{\ln(x)}{8}+\frac{\cos(\ln(x))\sin(\ln(x))}{8}-\frac{\cos(\ln(x))^{3}\sin(\ln(x))}{4}$	24
default	$\frac{\ln(x)}{8}+\frac{\cos(\ln(x))\sin(\ln(x))}{8}-\frac{\cos(\ln(x))^{3}\sin(\ln(x))}{4}$	24
risch	$\frac{\ln(x)}{8}+\frac{i x^{4i}}{64}-\frac{i x^{-4i}}{64}$	24

input
```
int(cos(ln(x))^2*sin(ln(x))^2/x,x,method=_RETURNVERBOSE)
```

output
```
ln(x^(1/8))-1/32*sin(4*ln(x))
```

3.854.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{\cos^2(\log(x))\sin^2(\log(x))}{x}\,dx = -\frac{1}{8}\left(2\cos\left(\log\left(x\right)\right)^3 - \cos\left(\log\left(x\right)\right)\right)\sin\left(\log\left(x\right)\right) + \frac{1}{8}\log\left(x\right)$$

input
```
integrate(cos(log(x))^2*sin(log(x))^2/x,x, algorithm="fricas")
```

output
```
-1/8*(2*cos(log(x))^3 - cos(log(x)))*sin(log(x)) + 1/8*log(x)
```

3.854.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 476 vs. 2(29) = 58.

Time = 8.13 (sec) , antiderivative size = 476, normalized size of antiderivative = 16.41

$$\int \frac{\cos^2(\log(x))\sin^2(\log(x))}{x}\,dx$$
$$= \frac{\log(x)\tan^8\left(\frac{\log(x)}{2}\right)}{8\tan^8\left(\frac{\log(x)}{2}\right)+32\tan^6\left(\frac{\log(x)}{2}\right)+48\tan^4\left(\frac{\log(x)}{2}\right)+32\tan^2\left(\frac{\log(x)}{2}\right)+8}$$
$$+\frac{4\log(x)\tan^6\left(\frac{\log(x)}{2}\right)}{8\tan^8\left(\frac{\log(x)}{2}\right)+32\tan^6\left(\frac{\log(x)}{2}\right)+48\tan^4\left(\frac{\log(x)}{2}\right)+32\tan^2\left(\frac{\log(x)}{2}\right)+8}$$
$$+\frac{6\log(x)\tan^4\left(\frac{\log(x)}{2}\right)}{8\tan^8\left(\frac{\log(x)}{2}\right)+32\tan^6\left(\frac{\log(x)}{2}\right)+48\tan^4\left(\frac{\log(x)}{2}\right)+32\tan^2\left(\frac{\log(x)}{2}\right)+8}$$
$$+\frac{4\log(x)\tan^2\left(\frac{\log(x)}{2}\right)}{8\tan^8\left(\frac{\log(x)}{2}\right)+32\tan^6\left(\frac{\log(x)}{2}\right)+48\tan^4\left(\frac{\log(x)}{2}\right)+32\tan^2\left(\frac{\log(x)}{2}\right)+8}$$
$$+\frac{\log(x)}{8\tan^8\left(\frac{\log(x)}{2}\right)+32\tan^6\left(\frac{\log(x)}{2}\right)+48\tan^4\left(\frac{\log(x)}{2}\right)+32\tan^2\left(\frac{\log(x)}{2}\right)+8}$$
$$+\frac{2\tan^7\left(\frac{\log(x)}{2}\right)}{8\tan^8\left(\frac{\log(x)}{2}\right)+32\tan^6\left(\frac{\log(x)}{2}\right)+48\tan^4\left(\frac{\log(x)}{2}\right)+32\tan^2\left(\frac{\log(x)}{2}\right)+8}$$
$$-\frac{14\tan^5\left(\frac{\log(x)}{2}\right)}{8\tan^8\left(\frac{\log(x)}{2}\right)+32\tan^6\left(\frac{\log(x)}{2}\right)+48\tan^4\left(\frac{\log(x)}{2}\right)+32\tan^2\left(\frac{\log(x)}{2}\right)+8}$$
$$+\frac{14\tan^3\left(\frac{\log(x)}{2}\right)}{8\tan^8\left(\frac{\log(x)}{2}\right)+32\tan^6\left(\frac{\log(x)}{2}\right)+48\tan^4\left(\frac{\log(x)}{2}\right)+32\tan^2\left(\frac{\log(x)}{2}\right)+8}$$
$$-\frac{2\tan\left(\frac{\log(x)}{2}\right)}{8\tan^8\left(\frac{\log(x)}{2}\right)+32\tan^6\left(\frac{\log(x)}{2}\right)+48\tan^4\left(\frac{\log(x)}{2}\right)+32\tan^2\left(\frac{\log(x)}{2}\right)+8}$$

input
```
integrate(cos(ln(x))**2*sin(ln(x))**2/x,x)
```

3.854. $\int \frac{\cos^2(\log(x))\sin^2(\log(x))}{x}\,dx$

output
```
log(x)*tan(log(x)/2)**8/(8*tan(log(x)/2)**8 + 32*tan(log(x)/2)**6 + 48*tan
(log(x)/2)**4 + 32*tan(log(x)/2)**2 + 8) + 4*log(x)*tan(log(x)/2)**6/(8*ta
n(log(x)/2)**8 + 32*tan(log(x)/2)**6 + 48*tan(log(x)/2)**4 + 32*tan(log(x)
/2)**2 + 8) + 6*log(x)*tan(log(x)/2)**4/(8*tan(log(x)/2)**8 + 32*tan(log(x
)/2)**6 + 48*tan(log(x)/2)**4 + 32*tan(log(x)/2)**2 + 8) + 4*log(x)*tan(lo
g(x)/2)**2/(8*tan(log(x)/2)**8 + 32*tan(log(x)/2)**6 + 48*tan(log(x)/2)**4
 + 32*tan(log(x)/2)**2 + 8) + log(x)/(8*tan(log(x)/2)**8 + 32*tan(log(x)/2
)**6 + 48*tan(log(x)/2)**4 + 32*tan(log(x)/2)**2 + 8) + 2*tan(log(x)/2)**7
/(8*tan(log(x)/2)**8 + 32*tan(log(x)/2)**6 + 48*tan(log(x)/2)**4 + 32*tan(
log(x)/2)**2 + 8) - 14*tan(log(x)/2)**5/(8*tan(log(x)/2)**8 + 32*tan(log(x
)/2)**6 + 48*tan(log(x)/2)**4 + 32*tan(log(x)/2)**2 + 8) + 14*tan(log(x)/2
)**3/(8*tan(log(x)/2)**8 + 32*tan(log(x)/2)**6 + 48*tan(log(x)/2)**4 + 32*
tan(log(x)/2)**2 + 8) - 2*tan(log(x)/2)/(8*tan(log(x)/2)**8 + 32*tan(log(x
)/2)**6 + 48*tan(log(x)/2)**4 + 32*tan(log(x)/2)**2 + 8)
```

3.854.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.41

$$\int \frac{\cos^2(\log(x))\sin^2(\log(x))}{x}\,dx = \frac{1}{8}\log(x) - \frac{1}{32}\sin(4\log(x))$$

input
```
integrate(cos(log(x))^2*sin(log(x))^2/x,x, algorithm="maxima")
```

output
```
1/8*log(x) - 1/32*sin(4*log(x))
```

3.854.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.41

$$\int \frac{\cos^2(\log(x))\sin^2(\log(x))}{x}\,dx = \frac{1}{8}\log(x) - \frac{1}{32}\sin(4\log(x))$$

input
```
integrate(cos(log(x))^2*sin(log(x))^2/x,x, algorithm="giac")
```

output
```
1/8*log(x) - 1/32*sin(4*log(x))
```

3.854.9 Mupad [B] (verification not implemented)

Time = 26.87 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.41

$$\int \frac{\cos^2(\log(x))\sin^2(\log(x))}{x}\,dx = \frac{\ln(x)}{8} - \frac{\sin(4\ln(x))}{32}$$

input
```
int((cos(log(x))^2*sin(log(x))^2)/x,x)
```

output
```
log(x)/8 - sin(4*log(x))/32
```

3.855 $\int \frac{\sin^3(x)}{\cos^3(x)+\sin^3(x)} dx$

3.855.1 Optimal result

Integrand size = 16, antiderivative size = 29

$$\int \frac{\sin^3(x)}{\cos^3(x)+\sin^3(x)}\,dx=\frac{x}{2}-\frac{1}{6}\log(\cos(x)+\sin(x))+\frac{1}{3}\log(2-\sin(2x))$$

output
```
1/2*x-1/6*ln(cos(x)+sin(x))+1/3*ln(2-sin(2*x))
```

3.855.2 Mathematica [A] (verified)

Time = 5.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\sin^3(x)}{\cos^3(x)+\sin^3(x)}\,dx=\frac{x}{2}-\frac{1}{6}\log(\cos(x)+\sin(x))+\frac{1}{3}\log(2-\sin(2x))$$

input
```
Integrate[Sin[x]^3/(Cos[x]^3 + Sin[x]^3),x]
```

output
```
x/2 - Log[Cos[x] + Sin[x]]/6 + Log[2 - Sin[2*x]]/3
```

3.855.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.48, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 4889, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(x)}{\sin^3(x)+\cos^3(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)^3}{\sin(x)^3+\cos(x)^3}dx$$

$$\Big\downarrow \text{4889}$$

$$\int \frac{\tan^3(x)}{\tan^5(x)+\tan^3(x)+\tan^2(x)+1}d\tan(x)$$

$$\Big\downarrow \text{2462}$$

$$\int \left(\frac{1-\tan(x)}{2\left(\tan^2(x)+1\right)}+\frac{2\tan(x)-1}{3\left(\tan^2(x)-\tan(x)+1\right)}-\frac{1}{6(\tan(x)+1)}\right)d\tan(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\arctan(\tan(x))-\frac{1}{4}\log\left(\tan^2(x)+1\right)+\frac{1}{3}\log\left(\tan^2(x)-\tan(x)+1\right)-\frac{1}{6}\log(\tan(x)+1)$$

input
```
Int[Sin[x]^3/(Cos[x]^3 + Sin[x]^3),x]
```

output
```
ArcTan[Tan[x]]/2 - Log[1 + Tan[x]]/6 - Log[1 + Tan[x]^2]/4 + Log[1 - Tan[x
] + Tan[x]^2]/3
```

3.855.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.855.4 Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.24

method	result
default	$-\frac{\ln\left(1+\tan(x)^2\right)}{4}+\frac{\arctan(\tan(x))}{2}-\frac{\ln(1+\tan(x))}{6}+\frac{\ln\left(\tan(x)^2-\tan(x)+1\right)}{3}$
risch	$\frac{x}{2}-\frac{ix}{2}-\frac{\ln\left(i+\mathrm{e}^{2ix}\right)}{6}+\frac{\ln\left(\mathrm{e}^{4ix}-4i\mathrm{e}^{2ix}-1\right)}{3}$
parallelrisch	$\ln\left(\left(-\frac{-2+\sin(2x)}{\cos(2x)+3+4\cos(x)}\right)^{\frac{1}{3}}\right)+\ln\left(\frac{1}{\left(-\frac{\sin(x)+\cos(x)}{\cos(x)+1}\right)^{\frac{1}{6}}}\right)+\ln\left(\frac{1}{\sqrt{\frac{1}{\cos(x)+1}}}\right)+\frac{x}{2}$
norman	$\frac{\frac{x}{2}+\frac{3x\tan\left(\frac{x}{2}\right)^2}{2}+\frac{3x\tan\left(\frac{x}{2}\right)^4}{2}+\frac{x\tan\left(\frac{x}{2}\right)^6}{2}}{\left(1+\tan\left(\frac{x}{2}\right)^2\right)^3}-\frac{\ln\left(1+\tan\left(\frac{x}{2}\right)^2\right)}{2}-\frac{\ln\left(\tan\left(\frac{x}{2}\right)^2-2\tan\left(\frac{x}{2}\right)-1\right)}{6}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)^4+2\tan\left(\frac{x}{2}\right)^3+2\right.}{3}$

input
```
int(sin(x)^3/(cos(x)^3+sin(x)^3),x,method=_RETURNVERBOSE)
```

output
```
-1/4*ln(1+tan(x)^2)+1/2*arctan(tan(x))-1/6*ln(1+tan(x))+1/3*ln(tan(x)^2-ta
n(x)+1)
```

3.855.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{\sin^3(x)}{\cos^3(x)+\sin^3(x)}\,dx = \frac{1}{2}\,x - \frac{1}{12}\,\log\left(2\,\cos\left(x\right)\sin\left(x\right)+1\right) + \frac{1}{3}\,\log\left(-\cos\left(x\right)\sin\left(x\right)+1\right)$$

input
```
integrate(sin(x)^3/(cos(x)^3+sin(x)^3),x, algorithm="fricas")
```

output
```
1/2*x - 1/12*log(2*cos(x)*sin(x) + 1) + 1/3*log(-cos(x)*sin(x) + 1)
```

3.855.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10

$$\int \frac{\sin^3(x)}{\cos^3(x)+\sin^3(x)}\,dx = \frac{x}{2} - \frac{\log\left(\sin\left(x\right)+\cos\left(x\right)\right)}{6} + \frac{\log\left(\sin^2\left(x\right)-\sin\left(x\right)\cos\left(x\right)+\cos^2\left(x\right)\right)}{3}$$

input
```
integrate(sin(x)**3/(cos(x)**3+sin(x)**3),x)
```

output
```
x/2 - log(sin(x) + cos(x))/6 + log(sin(x)**2 - sin(x)*cos(x) + cos(x)**2)/
3
```

3.855.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(23) = 46.

Time = 0.30 (sec) , antiderivative size = 103, normalized size of antiderivative = 3.55

$$\int \frac{\sin^3(x)}{\cos^3(x)+\sin^3(x)}\,dx = \arctan\left(\frac{\sin(x)}{\cos(x)+1}\right) + \frac{1}{3}\log\left(-\frac{2\sin(x)}{\cos(x)+1} + \frac{2\sin(x)^2}{(\cos(x)+1)^2} + \frac{2\sin(x)^3}{(\cos(x)+1)^3} + \frac{\sin(x)^4}{(\cos(x)+1)^4} + 1\right) - \frac{1}{6}\log\left(-\frac{2\sin(x)}{\cos(x)+1} + \frac{\sin(x)^2}{(\cos(x)+1)^2} - 1\right) - \frac{1}{2}\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2} + 1\right)$$

input
```
integrate(sin(x)^3/(cos(x)^3+sin(x)^3),x, algorithm="maxima")
```

output
```
arctan(sin(x)/(cos(x) + 1)) + 1/3*log(-2*sin(x)/(cos(x) + 1) + 2*sin(x)^2/
(cos(x) + 1)^2 + 2*sin(x)^3/(cos(x) + 1)^3 + sin(x)^4/(cos(x) + 1)^4 + 1)
- 1/6*log(-2*sin(x)/(cos(x) + 1) + sin(x)^2/(cos(x) + 1)^2 - 1) - 1/2*log(
sin(x)^2/(cos(x) + 1)^2 + 1)
```

3.855.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17

$$\int \frac{\sin^3(x)}{\cos^3(x)+\sin^3(x)}\,dx = \frac{1}{2}x + \frac{1}{3}\log\left(\tan(x)^2 - \tan(x) + 1\right) - \frac{1}{4}\log\left(\tan(x)^2 + 1\right) - \frac{1}{6}\log\left(|\tan(x) + 1|\right)$$

input
```
integrate(sin(x)^3/(cos(x)^3+sin(x)^3),x, algorithm="giac")
```

output
```
1/2*x + 1/3*log(tan(x)^2 - tan(x) + 1) - 1/4*log(tan(x)^2 + 1) - 1/6*log(a
bs(tan(x) + 1))
```

3.855.9 Mupad [B] (verification not implemented)

Time = 26.57 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.55

$$\int \frac{\sin^3(x)}{\cos^3(x)+\sin^3(x)}\,dx = \frac{x}{2}-\frac{\ln\left(\frac{1}{\cos\left(\frac{x}{2}\right)^2}\right)}{2}+\frac{\ln\left(\frac{\sin(2\,x)-2}{\cos\left(\frac{x}{2}\right)^4}\right)}{3}-\frac{\ln\left(\frac{\sin\left(x+\frac{\pi}{4}\right)}{\cos\left(\frac{x}{2}\right)^2}\right)}{6}$$

input
```
int(sin(x)^3/(cos(x)^3 + sin(x)^3),x)
```

output
```
x/2 - log(1/cos(x/2)^2)/2 + log((sin(2*x) - 2)/cos(x/2)^4)/3 - log(sin(x +
 pi/4)/cos(x/2)^2)/6
```

3.856 $\int \frac{\cos^3(x)}{\cos^3(x)+\sin^3(x)}\,dx$

3.856.1 Optimal result

Integrand size = 16, antiderivative size = 29

$$\int \frac{\cos^3(x)}{\cos^3(x)+\sin^3(x)}\,dx = \frac{x}{2}+\frac{1}{6}\log(\cos(x)+\sin(x))-\frac{1}{3}\log(2-\sin(2x))$$

output
```
1/2*x+1/6*ln(cos(x)+sin(x))-1/3*ln(2-sin(2*x))
```

3.856.2 Mathematica [A] (verified)

Time = 5.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\cos^3(x)}{\cos^3(x)+\sin^3(x)}\,dx = \frac{x}{2}+\frac{1}{6}\log(\cos(x)+\sin(x))-\frac{1}{3}\log(2-\sin(2x))$$

input
```
Integrate[Cos[x]^3/(Cos[x]^3 + Sin[x]^3),x]
```

output
```
x/2 + Log[Cos[x] + Sin[x]]/6 - Log[2 - Sin[2*x]]/3
```

3.856.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.48, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 4889, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(x)}{\sin^3(x)+\cos^3(x)}dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{\cos(x)^3}{\sin(x)^3+\cos(x)^3}dx$$
$$\Big\downarrow \ 4889$$
$$\int \frac{1}{\tan^5(x)+\tan^3(x)+\tan^2(x)+1}d\tan(x)$$
$$\Big\downarrow \ 2462$$
$$\int \left(\frac{1-2\tan(x)}{3\left(\tan^2(x)-\tan(x)+1\right)}+\frac{\tan(x)+1}{2\left(\tan^2(x)+1\right)}+\frac{1}{6(\tan(x)+1)}\right)d\tan(x)$$
$$\Big\downarrow \ 2009$$
$$\frac{1}{2}\arctan(\tan(x))+\frac{1}{4}\log\left(\tan^2(x)+1\right)-\frac{1}{3}\log\left(\tan^2(x)-\tan(x)+1\right)+\frac{1}{6}\log(\tan(x)+1)$$

input
```
Int[Cos[x]^3/(Cos[x]^3 + Sin[x]^3),x]
```

output
```
ArcTan[Tan[x]]/2 + Log[1 + Tan[x]]/6 + Log[1 + Tan[x]^2]/4 - Log[1 - Tan[x
] + Tan[x]^2]/3
```

3.856.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.856.4 Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.24

method	result
default	$\frac{\ln(1+\tan(x))}{6}-\frac{\ln\left(\tan(x)^2-\tan(x)+1\right)}{3}+\frac{\ln\left(1+\tan(x)^2\right)}{4}+\frac{\arctan(\tan(x))}{2}$
risch	$\frac{x}{2}+\frac{ix}{2}+\frac{\ln(i+e^{2ix})}{6}-\frac{\ln(e^{4ix}-4ie^{2ix}-1)}{3}$
parallelrisch	$\ln\left(\frac{1}{\left(-\frac{-2+\sin(2x)}{\cos(2x)+3+4\cos(x)}\right)^{\frac{1}{3}}}\right)+\ln\left(\left(-\frac{\sin(x)+\cos(x)}{\cos(x)+1}\right)^{\frac{1}{6}}\right)+\ln\left(\sqrt{\frac{1}{\cos(x)+1}}\right)+\frac{x}{2}$
norman	$\frac{\frac{x}{2}+\frac{3x\tan(\frac{x}{2})^2}{2}+\frac{3x\tan(\frac{x}{2})^4}{2}+\frac{x\tan(\frac{x}{2})^6}{2}}{\left(1+\tan(\frac{x}{2})^2\right)^3}+\frac{\ln\left(1+\tan(\frac{x}{2})^2\right)}{2}+\frac{\ln\left(\tan(\frac{x}{2})^2-2\tan(\frac{x}{2})-1\right)}{6}-\frac{\ln\left(\tan(\frac{x}{2})^4+2\tan(\frac{x}{2})^3+2\right.}{3}$

input
```
int(cos(x)^3/(cos(x)^3+sin(x)^3),x,method=_RETURNVERBOSE)
```

output
```
1/6*ln(1+tan(x))-1/3*ln(tan(x)^2-tan(x)+1)+1/4*ln(1+tan(x)^2)+1/2*arctan(t
an(x))
```

3.856.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{\cos^3(x)}{\cos^3(x)+\sin^3(x)}\,dx = \frac{1}{2}\,x + \frac{1}{12}\,\log\left(2\,\cos\left(x\right)\sin\left(x\right)+1\right) - \frac{1}{3}\,\log\left(-\cos\left(x\right)\sin\left(x\right)+1\right)$$

input
```
integrate(cos(x)^3/(cos(x)^3+sin(x)^3),x, algorithm="fricas")
```

output
```
1/2*x + 1/12*log(2*cos(x)*sin(x) + 1) - 1/3*log(-cos(x)*sin(x) + 1)
```

3.856.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10

$$\int \frac{\cos^3(x)}{\cos^3(x)+\sin^3(x)}\,dx = \frac{x}{2} + \frac{\log\left(\sin\left(x\right)+\cos\left(x\right)\right)}{6} - \frac{\log\left(\sin^2\left(x\right)-\sin\left(x\right)\cos\left(x\right)+\cos^2\left(x\right)\right)}{3}$$

input
```
integrate(cos(x)**3/(cos(x)**3+sin(x)**3),x)
```

output
```
x/2 + log(sin(x) + cos(x))/6 - log(sin(x)**2 - sin(x)*cos(x) + cos(x)**2)/
3
```

3.856.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(23) = 46$.

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 3.55

$$\int \frac{\cos^3(x)}{\cos^3(x)+\sin^3(x)}\,dx = \arctan\left(\frac{\sin(x)}{\cos(x)+1}\right) - \frac{1}{3}\log\left(-\frac{2\sin(x)}{\cos(x)+1} + \frac{2\sin(x)^2}{(\cos(x)+1)^2} + \frac{2\sin(x)^3}{(\cos(x)+1)^3} + \frac{\sin(x)^4}{(\cos(x)+1)^4} + 1\right) + \frac{1}{6}\log\left(-\frac{2\sin(x)}{\cos(x)+1} + \frac{\sin(x)^2}{(\cos(x)+1)^2} - 1\right) + \frac{1}{2}\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2} + 1\right)$$

input
```
integrate(cos(x)^3/(cos(x)^3+sin(x)^3),x, algorithm="maxima")
```

output
```
arctan(sin(x)/(cos(x) + 1)) - 1/3*log(-2*sin(x)/(cos(x) + 1) + 2*sin(x)^2/
(cos(x) + 1)^2 + 2*sin(x)^3/(cos(x) + 1)^3 + sin(x)^4/(cos(x) + 1)^4 + 1)
+ 1/6*log(-2*sin(x)/(cos(x) + 1) + sin(x)^2/(cos(x) + 1)^2 - 1) + 1/2*log(
sin(x)^2/(cos(x) + 1)^2 + 1)
```

3.856.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17

$$\int \frac{\cos^3(x)}{\cos^3(x)+\sin^3(x)}\,dx = \frac{1}{2}x - \frac{1}{3}\log\left(\tan(x)^2 - \tan(x) + 1\right) + \frac{1}{4}\log\left(\tan(x)^2 + 1\right) + \frac{1}{6}\log\left(|\tan(x) + 1|\right)$$

input
```
integrate(cos(x)^3/(cos(x)^3+sin(x)^3),x, algorithm="giac")
```

output
```
1/2*x - 1/3*log(tan(x)^2 - tan(x) + 1) + 1/4*log(tan(x)^2 + 1) + 1/6*log(a
bs(tan(x) + 1))
```

3.856.9 Mupad [B] (verification not implemented)

Time = 26.40 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.55

$$\int \frac{\cos^3(x)}{\cos^3(x)+\sin^3(x)}\,dx = \frac{x}{2}+\frac{\ln\left(\frac{1}{\cos\left(\frac{x}{2}\right)^2}\right)}{2}-\frac{\ln\left(\frac{\sin(2\,x)-2}{\cos\left(\frac{x}{2}\right)^4}\right)}{3}+\frac{\ln\left(\frac{\sin\left(x+\frac{\pi}{4}\right)}{\cos\left(\frac{x}{2}\right)^2}\right)}{6}$$

input
```
int(cos(x)^3/(cos(x)^3 + sin(x)^3),x)
```

output
```
x/2 + log(1/cos(x/2)^2)/2 - log((sin(2*x) - 2)/cos(x/2)^4)/3 + log(sin(x +
 pi/4)/cos(x/2)^2)/6
```

3.857 $\int \frac{\sec(x)}{-5+\cos^2(x)+4\sin(x)}\,dx$

3.857.1 Optimal result

Integrand size = 15, antiderivative size = 44

$$\int \frac{\sec(x)}{-5+\cos^2(x)+4\sin(x)}\,dx = \frac{1}{2}\log(1-\sin(x))-\frac{4}{9}\log(2-\sin(x))-\frac{1}{18}\log(1+\sin(x))+\frac{1}{3(2-\sin(x))}$$

output
```
1/2*ln(1-sin(x))-4/9*ln(2-sin(x))-1/18*ln(1+sin(x))+1/3/(2-sin(x))
```

3.857.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \frac{\sec(x)}{-5+\cos^2(x)+4\sin(x)}\,dx = \frac{1}{18}\left(9\log(1-\sin(x))-8\log(2-\sin(x))-\log(1+\sin(x))-\frac{6}{-2+\sin(x)}\right)$$

input
```
Integrate[Sec[x]/(-5 + Cos[x]^2 + 4*Sin[x]),x]
```

output
```
(9*Log[1 - Sin[x]] - 8*Log[2 - Sin[x]] - Log[1 + Sin[x]] - 6/(-2 + Sin[x])
)/18
```

3.857.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4878, 25, 477, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(x)}{4\sin(x)+\cos^2(x)-5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(x)}{4\sin(x)+\cos(x)^2-5}dx$$

$$\Big\downarrow \text{4878}$$

$$\int -\frac{1}{(2-\sin(x))^2\left(1-\sin^2(x)\right)}d\sin(x)$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{(2-\sin(x))^2\left(1-\sin^2(x)\right)}d\sin(x)$$

$$\Big\downarrow \text{477}$$

$$-\int \left(-\frac{4}{9(2-\sin(x))}+\frac{1}{18(\sin(x)+1)}-\frac{1}{3(2-\sin(x))^2}+\frac{1}{2(1-\sin(x))}\right)d\sin(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3(2-\sin(x))}+\frac{1}{2}\log(1-\sin(x))-\frac{4}{9}\log(2-\sin(x))-\frac{1}{18}\log(\sin(x)+1)$$

input
```
Int[Sec[x]/(-5 + Cos[x]^2 + 4*Sin[x]),x]
```

output
```
Log[1 - Sin[x]]/2 - (4*Log[2 - Sin[x]])/9 - Log[1 + Sin[x]]/18 + 1/(3*(2 -
 Sin[x]))
```

3.857.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 477
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ExpandIntegrand[(c + d*x)^n*(1 - Rt[-b/a, 2]*x)^p*(1 + Rt[-b/a, 2
]*x)^p, x], x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[n] &
& NiceSqrtQ[-b/a] &&  !FractionalPowerFactorQ[Rt[-b/a, 2]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

3.857.4 Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.70

method	result	size
default	$-\frac{1}{3(\sin(x)-2)}-\frac{4\ln(\sin(x)-2)}{9}+\frac{\ln(\sin(x)-1)}{2}-\frac{\ln(1+\sin(x))}{18}$	31
norman	$\frac{\tan(\frac{x}{2})}{6\tan(\frac{x}{2})^2-6\tan(\frac{x}{2})+6}-\frac{\ln(\tan(\frac{x}{2})+1)}{9}-\frac{4\ln\left(\tan(\frac{x}{2})^2-\tan(\frac{x}{2})+1\right)}{9}+\ln\left(\tan\left(\frac{x}{2}\right)-1\right)$	57
risch	$-\frac{2ie^{ix}}{3(-4ie^{ix}+e^{2ix}-1)}+\ln\left(e^{ix}-i\right)-\frac{\ln(i+e^{ix})}{9}-\frac{4\ln(-4ie^{ix}+e^{2ix}-1)}{9}$	65
parallelrisch	$\frac{(-8\sin(x)+16)\ln\left(\frac{2-\sin(x)}{\cos(x)+1}\right)+(18\sin(x)-36)\ln(-\cot(x)+\csc(x)-1)+(-2\sin(x)+4)\ln(\csc(x)-\cot(x)+1)-3\sin(x)}{18\sin(x)-36}$	68

input
```
int(sec(x)/(-5+cos(x)^2+4*sin(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/3/(sin(x)-2)-4/9*ln(sin(x)-2)+1/2*ln(sin(x)-1)-1/18*ln(1+sin(x))
```

3.857.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int \frac{\sec(x)}{-5+\cos^2(x)+4\sin(x)}\,dx = -\frac{(\sin(x)-2)\log(\sin(x)+1)+8(\sin(x)-2)\log\left(-\frac{1}{2}\sin(x)+1\right)-9(\sin(x)-2)\log(-\sin(x)+1)}{18(\sin(x)-2)}$$

input
```
integrate(sec(x)/(-5+cos(x)^2+4*sin(x)),x, algorithm="fricas")
```

output
```
-1/18*((sin(x) - 2)*log(sin(x) + 1) + 8*(sin(x) - 2)*log(-1/2*sin(x) + 1)
- 9*(sin(x) - 2)*log(-sin(x) + 1) + 6)/(sin(x) - 2)
```

3.857.6 Sympy [F]

$$\int \frac{\sec(x)}{-5+\cos^2(x)+4\sin(x)}\,dx = \int \frac{\sec(x)}{4\sin(x)+\cos^2(x)-5}\,dx$$

input
```
integrate(sec(x)/(-5+cos(x)**2+4*sin(x)),x)
```

output
```
Integral(sec(x)/(4*sin(x) + cos(x)**2 - 5), x)
```

3.857.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.68

$$\int \frac{\sec(x)}{-5+\cos^2(x)+4\sin(x)}\,dx = -\frac{1}{3(\sin(x)-2)}-\frac{1}{18}\log(\sin(x)+1)+\frac{1}{2}\log(\sin(x)-1)-\frac{4}{9}\log(\sin(x)-2)$$

input
```
integrate(sec(x)/(-5+cos(x)^2+4*sin(x)),x, algorithm="maxima")
```

output
```
-1/3/(sin(x) - 2) - 1/18*log(sin(x) + 1) + 1/2*log(sin(x) - 1) - 4/9*log(s
in(x) - 2)
```

3.857.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.77

$$\int \frac{\sec(x)}{-5+\cos^2(x)+4\sin(x)}\,dx = -\frac{1}{3(\sin(x)-2)} - \frac{1}{18}\log(\sin(x)+1) - \frac{4}{9}\log(-\sin(x)+2) + \frac{1}{2}\log(-\sin(x)+1)$$

input
```
integrate(sec(x)/(-5+cos(x)^2+4*sin(x)),x, algorithm="giac")
```

output
```
-1/3/(sin(x) - 2) - 1/18*log(sin(x) + 1) - 4/9*log(-sin(x) + 2) + 1/2*log(
-sin(x) + 1)
```

3.857.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.73

$$\int \frac{\sec(x)}{-5+\cos^2(x)+4\sin(x)}\,dx = \frac{\ln(\sin(x)-1)}{2} - \frac{\ln(\sin(x)+1)}{18} - \frac{4\ln(\sin(x)-2)}{9} - \frac{1}{3(\sin(x)-2)}$$

input
```
int(1/(cos(x)*(4*sin(x) + cos(x)^2 - 5)),x)
```

output
```
log(sin(x) - 1)/2 - log(sin(x) + 1)/18 - (4*log(sin(x) - 2))/9 - 1/(3*(sin
(x) - 2))
```

3.858 $\int \frac{1}{\cos^{\frac{3}{2}}(x)\sqrt{3\cos(x)+\sin(x)}}\,dx$

3.858.1 Optimal result

Integrand size = 18, antiderivative size = 19

$$\int \frac{1}{\cos^{\frac{3}{2}}(x)\sqrt{3\cos(x)+\sin(x)}}\,dx = \frac{2\sqrt{3\cos(x)+\sin(x)}}{\sqrt{\cos(x)}}$$

output
```
2*(3*cos(x)+sin(x))^(1/2)/cos(x)^(1/2)
```

3.858.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{\cos^{\frac{3}{2}}(x)\sqrt{3\cos(x)+\sin(x)}}\,dx = \frac{2\sqrt{3\cos(x)+\sin(x)}}{\sqrt{\cos(x)}}$$

input
```
Integrate[1/(Cos[x]^(3/2)*Sqrt[3*Cos[x] + Sin[x]]),x]
```

output
```
(2*Sqrt[3*Cos[x] + Sin[x]])/Sqrt[Cos[x]]
```

3.858.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 100 vs. 2(19) = 38.

Time = 0.87 (sec) , antiderivative size = 100, normalized size of antiderivative = 5.26, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4902, 2058, 7270, 2136, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cos^{\frac{3}{2}}(x)\sqrt{\sin(x)+3\cos(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos(x)^{3/2}\sqrt{\sin(x)+3\cos(x)}}dx$$

$$\Big\downarrow \text{4902}$$

$$2\int \frac{1}{\left(1-\tan^2\left(\frac{x}{2}\right)\right)\sqrt{\frac{-3\tan^2\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)+3}{\tan^2\left(\frac{x}{2}\right)+1}}\sqrt{\frac{1-\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}}}d\tan\left(\frac{x}{2}\right)$$

$$\Big\downarrow \text{2058}$$

$$\frac{2\sqrt{1-\tan^2\left(\frac{x}{2}\right)}\int \frac{\sqrt{\tan^2\left(\frac{x}{2}\right)+1}}{\left(1-\tan^2\left(\frac{x}{2}\right)\right)^{3/2}\sqrt{\frac{-3\tan^2\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)+3}{\tan^2\left(\frac{x}{2}\right)+1}}}d\tan\left(\frac{x}{2}\right)}{\sqrt{\frac{1-\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}}\sqrt{\tan^2\left(\frac{x}{2}\right)+1}}$$

$$\Big\downarrow \text{7270}$$

$$\frac{2\sqrt{-3\tan^2\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)+3}\sqrt{1-\tan^2\left(\frac{x}{2}\right)}\int \frac{\tan^2\left(\frac{x}{2}\right)+1}{\sqrt{-3\tan^2\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)+3}\left(1-\tan^2\left(\frac{x}{2}\right)\right)^{3/2}}d\tan\left(\frac{x}{2}\right)}{\sqrt{\frac{-3\tan^2\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)+3}{\tan^2\left(\frac{x}{2}\right)+1}}\sqrt{\frac{1-\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}}\left(\tan^2\left(\frac{x}{2}\right)+1\right)}$$

$$\Big\downarrow \text{2136}$$

$$\frac{2\sqrt{-3\tan^2\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)+3}\sqrt{1-\tan^2\left(\frac{x}{2}\right)}\left(\frac{1}{8}\int 0d\tan\left(\frac{x}{2}\right)+\frac{\sqrt{-3\tan^2\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)+3}}{\sqrt{1-\tan^2\left(\frac{x}{2}\right)}}\right)}{\sqrt{\frac{-3\tan^2\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)+3}{\tan^2\left(\frac{x}{2}\right)+1}}\sqrt{\frac{1-\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}}\left(\tan^2\left(\frac{x}{2}\right)+1\right)}$$

$$\Big\downarrow \text{24}$$

$$\frac{2\left(-3\tan^2\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)+3\right)}{\sqrt{\frac{-3\tan^2\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)+3}{\tan^2\left(\frac{x}{2}\right)+1}}\sqrt{\frac{1-\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}}\left(\tan^2\left(\frac{x}{2}\right)+1\right)}$$

input
```
Int[1/(Cos[x]^(3/2)*Sqrt[3*Cos[x] + Sin[x]]),x]
```

output
```
(2*(3 + 2*Tan[x/2] - 3*Tan[x/2]^2))/(Sqrt[(3 + 2*Tan[x/2] - 3*Tan[x/2]^2)/
(1 + Tan[x/2]^2)]*Sqrt[(1 - Tan[x/2]^2)/(1 + Tan[x/2]^2)]*(1 + Tan[x/2]^2)
)
```

3.858.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 2136
```
Int[(Px_)*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_
), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[P
x, x, 2]}, Simp[(a + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(q + 1)/((-4*a*c)*(a
*c*e^2 + (c*d - a*f)^2)*(p + 1)))*((A*c - a*C)*(2*a*c*e) + ((-a)*B)*(2*c^2*
d - c*(2*a*f)) + c*(A*(2*c^2*d - c*(2*a*f)) - B*(-2*a*c*e) + C*(-2*a*(c*d -
 a*f)))*x), x] + Simp[1/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1))   Int[
(a + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(-2*A*c - 2*a*C)*((c*d - a*f)^
2 - ((-a)*e)*(c*e))*(p + 1) + (2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*
f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e) + ((-a)*B)*(2*c
^2*d - c*((Plus[2])*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e) + ((-a
)*B)*(2*c^2*d + (-c)*((Plus[2])*a*f)))*(p + q + 2) - (2*(A*c*(c*d - a*f) -
a*(c*C*d - B*c*e - a*C*f)))*((-c)*e*(2*p + q + 4)))*x - c*f*(2*(A*c*(c*d -
a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; Free
Q[{a, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[a*c*e^2 +
(c*d - a*f)^2, 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.858. $\int \frac{1}{\cos^{\frac{3}{2}}(x)\sqrt{3\cos(x)+\sin(x)}}\,dx$

rule 4902
```
Int[u_, x_Symbol] :> With[{w = Block[{$ShowSteps = False, $StepCounter = Nu
ll}, Int[SubstFor[1/(1 + FreeFactors[Tan[FunctionOfTrig[u, x]/2], x]^2*x^2)
, Tan[FunctionOfTrig[u, x]/2]/FreeFactors[Tan[FunctionOfTrig[u, x]/2], x],
u, x], x]]}, Module[{v = FunctionOfTrig[u, x], d}, Simp[d = FreeFactors[Tan
[v/2], x]; 2*(d/Coefficient[v, x, 1])   Subst[Int[SubstFor[1/(1 + d^2*x^2),
 Tan[v/2]/d, u, x], x], x, Tan[v/2]/d], x]] /; CalculusFreeQ[w, x]] /; Inve
rseFunctionFreeQ[u, x] &&  !FalseQ[FunctionOfTrig[u, x]]
```

rule 7270
```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Simp[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p])))   Int[u*v
^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !Free
Q[v, x] &&  !FreeQ[w, x]
```

3.858.4 Maple [A] (verified)

Time = 5.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{2\sqrt{3\cos(x)+\sin(x)}}{\sqrt{\cos(x)}}$	16

input
```
int(1/cos(x)^(3/2)/(3*cos(x)+sin(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(3*cos(x)+sin(x))^(1/2)/cos(x)^(1/2)
```

3.858.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{\cos^{\frac{3}{2}}(x)\sqrt{3\cos(x)+\sin(x)}}\,dx = \frac{2\sqrt{3\cos(x)+\sin(x)}}{\sqrt{\cos(x)}}$$

input
```
integrate(1/cos(x)^(3/2)/(3*cos(x)+sin(x))^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(3*cos(x) + sin(x))/sqrt(cos(x))
```

3.858.6 Sympy [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(x)\sqrt{3\cos(x)+\sin(x)}}\,dx = \int \frac{1}{\sqrt{\sin\left(x\right)+3\cos\left(x\right)}\cos^{\frac{3}{2}}\left(x\right)}\,dx$$

input
```
integrate(1/cos(x)**(3/2)/(3*cos(x)+sin(x))**(1/2),x)
```

output
```
Integral(1/(sqrt(sin(x) + 3*cos(x))*cos(x)**(3/2)), x)
```

3.858.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 145 vs. $2(15) = 30$.

Time = 0.34 (sec) , antiderivative size = 145, normalized size of antiderivative = 7.63

$$\int \frac{1}{\cos^{\frac{3}{2}}(x)\sqrt{3\cos(x)+\sin(x)}}\,dx = \frac{2\left(\frac{2\sin(x)}{\cos(x)+1} - \frac{6\sin(x)^2}{(\cos(x)+1)^2} - \frac{2\sin(x)^3}{(\cos(x)+1)^3} + \frac{3\sin(x)^4}{(\cos(x)+1)^4} + 3\right)\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)^2}{\sqrt{\frac{2\sin(x)}{\cos(x)+1} - \frac{3\sin(x)^2}{(\cos(x)+1)^2} + 3}\left(\frac{\sin(x)}{\cos(x)+1}+1\right)^{\frac{3}{2}}\left(-\frac{\sin(x)}{\cos(x)+1}+1\right)^{\frac{3}{2}}\left(\frac{2\sin(x)^2}{(\cos(x)+1)^2} + \frac{\sin(x)^4}{(\cos(x)+1)^4}+1\right)}$$

input
```
integrate(1/cos(x)^(3/2)/(3*cos(x)+sin(x))^(1/2),x, algorithm="maxima")
```

output
```
2*(2*sin(x)/(cos(x) + 1) - 6*sin(x)^2/(cos(x) + 1)^2 - 2*sin(x)^3/(cos(x)
+ 1)^3 + 3*sin(x)^4/(cos(x) + 1)^4 + 3)*(sin(x)^2/(cos(x) + 1)^2 + 1)^2/(s
qrt(2*sin(x)/(cos(x) + 1) - 3*sin(x)^2/(cos(x) + 1)^2 + 3)*(sin(x)/(cos(x)
 + 1) + 1)^(3/2)*(-sin(x)/(cos(x) + 1) + 1)^(3/2)*(2*sin(x)^2/(cos(x) + 1)
^2 + sin(x)^4/(cos(x) + 1)^4 + 1))
```

3.858.8 Giac [F]

$$\int \frac{1}{\cos^{\frac{3}{2}}(x)\sqrt{3\cos(x)+\sin(x)}}\,dx = \int \frac{1}{\sqrt{3\cos\left(x\right)+\sin\left(x\right)}\cos\left(x\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cos(x)^(3/2)/(3*cos(x)+sin(x))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(3*cos(x) + sin(x))*cos(x)^(3/2)), x)
```

3.858.9 Mupad [B] (verification not implemented)

Time = 26.86 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{1}{\cos^{\frac{3}{2}}(x)\sqrt{3\cos(x)+\sin(x)}}\,dx = \frac{2\sqrt{3\,\cos(x)+\sin(x)}}{\sqrt{\cos(x)}}$$

input
```
int(1/(cos(x)^(3/2)*(3*cos(x) + sin(x))^(1/2)),x)
```

output
```
(2*(3*cos(x) + sin(x))^(1/2))/cos(x)^(1/2)
```

3.859 $\int \frac{\csc(x)\sqrt{\cos(x)+\sin(x)}}{\cos^{\frac{3}{2}}(x)} dx$

3.859.1 Optimal result

Integrand size = 18, antiderivative size = 44

$$\int \frac{\csc(x)\sqrt{\cos(x)+\sin(x)}}{\cos^{\frac{3}{2}}(x)} dx = -\log(\sin(x)) + 2\log\left(-\sqrt{\cos(x)}+\sqrt{\cos(x)+\sin(x)}\right) + \frac{2\sqrt{\cos(x)+\sin(x)}}{\sqrt{\cos(x)}}$$

output
```
-ln(sin(x))+2*ln(-cos(x)^(1/2)+(cos(x)+sin(x))^(1/2))+2*(cos(x)+sin(x))^(1
/2)/cos(x)^(1/2)
```

3.859.2 Mathematica [A] (warning: unable to verify)

Time = 0.94 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.55

$$\int \frac{\csc(x)\sqrt{\cos(x)+\sin(x)}}{\cos^{\frac{3}{2}}(x)} dx = \frac{2\left(\cos(x)+\sin(x)-\operatorname{arctanh}\left(\frac{\sqrt{\cos(x)}}{\sqrt{\cos(x)+\sqrt{\sin^2(x)}}}\right)\sqrt{\cos(x)}\sqrt{\cos(x)+\sqrt{\sin^2(x)}}\right)}{\sqrt{\cos(x)}\sqrt{\cos(x)+\sin(x)}}$$

input
```
Integrate[(Csc[x]*Sqrt[Cos[x] + Sin[x]])/Cos[x]^(3/2),x]
```

output
```
(2*(Cos[x] + Sin[x] - ArcTanh[Sqrt[Cos[x]]/Sqrt[Cos[x] + Sqrt[Sin[x]^2]]]*
Sqrt[Cos[x]]*Sqrt[Cos[x] + Sqrt[Sin[x]^2]]))/(Sqrt[Cos[x]]*Sqrt[Cos[x] + S
in[x]])
```

3.859.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(x)\sqrt{\sin(x)+\cos(x)}}{\cos^{\frac{3}{2}}(x)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{\sin(x)+\cos(x)}}{\sin(x)\cos(x)^{3/2}}dx$$

$$\Big\downarrow \; 4902$$

$$2\int \frac{\cot\left(\frac{x}{2}\right)\sqrt{\frac{-\tan^2\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)+1}{\tan^2\left(\frac{x}{2}\right)+1}}}{2\left(\frac{1-\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}\right)^{3/2}}d\tan\left(\frac{x}{2}\right)$$

$$\Big\downarrow \; 27$$

$$\int \frac{\sqrt{\frac{-\tan^2\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)+1}{\tan^2\left(\frac{x}{2}\right)+1}}\cot\left(\frac{x}{2}\right)}{\left(\frac{1-\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}\right)^{3/2}}d\tan\left(\frac{x}{2}\right)$$

$$\Big\downarrow \; 2058$$

$$\frac{\sqrt{1-\tan^2\left(\frac{x}{2}\right)}\int \frac{\cot\left(\frac{x}{2}\right)\sqrt{\frac{-\tan^2\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)+1}{\tan^2\left(\frac{x}{2}\right)+1}}\left(\tan^2\left(\frac{x}{2}\right)+1\right)^{3/2}}{\left(1-\tan^2\left(\frac{x}{2}\right)\right)^{3/2}}d\tan\left(\frac{x}{2}\right)}{\sqrt{\frac{1-\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}}\sqrt{\tan^2\left(\frac{x}{2}\right)+1}}$$

$$\Big\downarrow \; 7270$$

$$\frac{\sqrt{1-\tan^2\left(\frac{x}{2}\right)}\sqrt{\frac{-\tan^2\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)+1}{\tan^2\left(\frac{x}{2}\right)+1}}\int \frac{\cot\left(\frac{x}{2}\right)\sqrt{-\tan^2\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)+1}\left(\tan^2\left(\frac{x}{2}\right)+1\right)}{\left(1-\tan^2\left(\frac{x}{2}\right)\right)^{3/2}}d\tan\left(\frac{x}{2}\right)}{\sqrt{-\tan^2\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)+1}\sqrt{\frac{1-\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}}}$$

$$\Big\downarrow \; 7293$$

$$\frac{\sqrt{1-\tan^2\left(\frac{x}{2}\right)}\sqrt{\frac{-\tan^2\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)+1}{\tan^2\left(\frac{x}{2}\right)+1}}\int\left(\frac{\sqrt{-\tan^2\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)+1}\cot\left(\frac{x}{2}\right)}{\left(1-\tan^2\left(\frac{x}{2}\right)\right)^{3/2}}+\frac{\tan\left(\frac{x}{2}\right)\sqrt{-\tan^2\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)+1}}{\left(1-\tan^2\left(\frac{x}{2}\right)\right)^{3/2}}\right)d\tan\left(\frac{x}{2}\right)}{\sqrt{-\tan^2\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)+1}\sqrt{\frac{1-\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+1}}}$$

$$\Big\downarrow \ 2009$$

$$\frac{\sqrt{1-\tan^2\left(\frac{x}{2}\right)}\sqrt{\frac{-\tan^2\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)+1}{\tan^2\left(\frac{x}{2}\right)+1}}\left(\int\frac{\cot\left(\frac{x}{2}\right)\sqrt{-\tan^2\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)+1}}{\left(1-\tan^2\left(\frac{x}{2}\right)\right)^{3/2}}d\tan\left(\frac{x}{2}\right)-\frac{2^{3/4}\sqrt{\sqrt{2}-1}\sqrt{1-\tan\left(\frac{x}{2}\right)}\sqrt{\left(1+\sqrt{2}\right)\tan\left(\frac{x}{2}\right.}}{\sqrt{\frac{-\tan\left(\right.}{\tan}}}\right.}{}$$

input
```
Int[(Csc[x]*Sqrt[Cos[x] + Sin[x]])/Cos[x]^(3/2),x]
```

output
```
$Aborted
```

3.859.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.859. $\int \frac{\csc(x)\sqrt{\cos(x)+\sin(x)}}{\cos^{\frac{3}{2}}(x)}\,dx$

rule 4902
```
Int[u_, x_Symbol] :> With[{w = Block[{$ShowSteps = False, $StepCounter = Nu
ll}, Int[SubstFor[1/(1 + FreeFactors[Tan[FunctionOfTrig[u, x]/2], x]^2*x^2)
, Tan[FunctionOfTrig[u, x]/2]/FreeFactors[Tan[FunctionOfTrig[u, x]/2], x],
u, x], x]]}, Module[{v = FunctionOfTrig[u, x], d}, Simp[d = FreeFactors[Tan
[v/2], x]; 2*(d/Coefficient[v, x, 1])   Subst[Int[SubstFor[1/(1 + d^2*x^2),
 Tan[v/2]/d, u, x], x], x, Tan[v/2]/d], x]] /; CalculusFreeQ[w, x]] /; Inve
rseFunctionFreeQ[u, x] &&  !FalseQ[FunctionOfTrig[u, x]]
```

rule 7270
```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Simp[a^IntPart[p
]*((a*v^m*w^n)^FracPart[p]/(v^(m*FracPart[p])*w^(n*FracPart[p])))   Int[u*v
^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !Free
Q[v, x] &&  !FreeQ[w, x]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.859.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. $2(36) = 72$.

Time = 10.20 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.82

method	result
default	$\frac{\left(2\cos(x)\sqrt{\frac{\cos(x)(\sin(x)+\cos(x))}{(\cos(x)+1)^2}}+\cos(x)\ln\left(2\cot(x)\sqrt{\frac{\cos(x)(\sin(x)+\cos(x))}{(\cos(x)+1)^2}}-2\cot(x)-1+2\csc(x)\sqrt{\frac{\cos(x)(\sin(x)+\cos(x))}{(\cos(x)+1)^2}}\right)+2\sqrt{\frac{\cos(x)}{}}\right.}{(\cos(x)+1)\sqrt{\frac{\cos(x)(\sin(x)+\cos(x))}{(\cos(x)+1)^2}}\sqrt{\cos(x)}}$

input
```
int(csc(x)*(sin(x)+cos(x))^(1/2)/cos(x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(2*cos(x)*(cos(x)*(sin(x)+cos(x))/(cos(x)+1)^2)^(1/2)+cos(x)*ln(2*cot(x)*(
cos(x)*(sin(x)+cos(x))/(cos(x)+1)^2)^(1/2)-2*cot(x)-1+2*csc(x)*(cos(x)*(si
n(x)+cos(x))/(cos(x)+1)^2)^(1/2))+2*(cos(x)*(sin(x)+cos(x))/(cos(x)+1)^2)^
(1/2))*(sin(x)+cos(x))^(1/2)/(cos(x)+1)/(cos(x)*(sin(x)+cos(x))/(cos(x)+1)
^2)^(1/2)/cos(x)^(1/2)
```

3.859.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. 2(36) = 72.

Time = 0.26 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.18

$$\int \frac{\csc(x)\sqrt{\cos(x)+\sin(x)}}{\cos^{\frac{3}{2}}(x)}\,dx =$$
$$-\frac{\cos(x)\log\left((2\cos(x)+\sin(x))\sqrt{\cos(x)+\sin(x)}\sqrt{\cos(x)}+\frac{7}{4}\cos(x)^2+2\cos(x)\sin(x)+\frac{1}{4}\right)-\cos}{}$$

input
```
integrate(csc(x)*(cos(x)+sin(x))^(1/2)/cos(x)^(3/2),x, algorithm="fricas")
```

output
```
-1/4*(cos(x)*log((2*cos(x) + sin(x))*sqrt(cos(x) + sin(x))*sqrt(cos(x)) +
7/4*cos(x)^2 + 2*cos(x)*sin(x) + 1/4) - cos(x)*log(-(2*cos(x) + sin(x))*sq
rt(cos(x) + sin(x))*sqrt(cos(x)) + 7/4*cos(x)^2 + 2*cos(x)*sin(x) + 1/4) -
 8*sqrt(cos(x) + sin(x))*sqrt(cos(x)))/cos(x)
```

3.859.6 Sympy [F]

$$\int \frac{\csc(x)\sqrt{\cos(x)+\sin(x)}}{\cos^{\frac{3}{2}}(x)}\,dx = \int \frac{\sqrt{\sin(x)+\cos(x)}\csc(x)}{\cos^{\frac{3}{2}}(x)}\,dx$$

input
```
integrate(csc(x)*(cos(x)+sin(x))**(1/2)/cos(x)**(3/2),x)
```

output
```
Integral(sqrt(sin(x) + cos(x))*csc(x)/cos(x)**(3/2), x)
```

3.859.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 518 vs. 2(36) = 72.

Time = 0.49 (sec) , antiderivative size = 518, normalized size of antiderivative = 11.77

$$\int \frac{\csc(x)\sqrt{\cos(x)+\sin(x)}}{\cos^{\frac{3}{2}}(x)}\,dx$$
$$= \frac{4\left((2\cos(2x)+\sin(2x))\cos\left(\frac{1}{2}\arctan(-\cos(4x)+\sin(4x)+2\sin(2x)+1,\cos(4x)+2\cos(2x)+\right.\right.}{}$$

input
```
integrate(csc(x)*(cos(x)+sin(x))^(1/2)/cos(x)^(3/2),x, algorithm="maxima")
```

output
```
4*((2*cos(2*x) + sin(2*x))*cos(1/2*arctan2(-cos(4*x) + sin(4*x) + 2*sin(2*
x) + 1, cos(4*x) + 2*cos(2*x) + sin(4*x) + 1))^3 + (2*cos(2*x) + sin(2*x))
*cos(1/2*arctan2(-cos(4*x) + sin(4*x) + 2*sin(2*x) + 1, cos(4*x) + 2*cos(2
*x) + sin(4*x) + 1))*sin(1/2*arctan2(-cos(4*x) + sin(4*x) + 2*sin(2*x) + 1
, cos(4*x) + 2*cos(2*x) + sin(4*x) + 1))^2 - (cos(2*x) - 2*sin(2*x) + 1)*s
in(1/2*arctan2(-cos(4*x) + sin(4*x) + 2*sin(2*x) + 1, cos(4*x) + 2*cos(2*x
) + sin(4*x) + 1))^3 - (cos(2*x) - sin(2*x) - 1)*cos(1/2*arctan2(-cos(4*x)
 + sin(4*x) + 2*sin(2*x) + 1, cos(4*x) + 2*cos(2*x) + sin(4*x) + 1)) - ((c
os(2*x) - 2*sin(2*x) + 1)*cos(1/2*arctan2(-cos(4*x) + sin(4*x) + 2*sin(2*x
) + 1, cos(4*x) + 2*cos(2*x) + sin(4*x) + 1))^2 + cos(2*x) + sin(2*x) - 1)
*sin(1/2*arctan2(-cos(4*x) + sin(4*x) + 2*sin(2*x) + 1, cos(4*x) + 2*cos(2
*x) + sin(4*x) + 1)))/((4*(cos(2*x) - sin(2*x))*cos(4*x) + 2*cos(4*x)^2 +
4*cos(2*x)^2 + 4*(cos(2*x) + sin(2*x) + 1)*sin(4*x) + 2*sin(4*x)^2 + 4*sin
(2*x)^2 + 4*cos(2*x) + 4*sin(2*x) + 2)^(1/4)*(cos(1/2*arctan2(-cos(4*x) +
sin(4*x) + 2*sin(2*x) + 1, cos(4*x) + 2*cos(2*x) + sin(4*x) + 1))^2 + sin(
1/2*arctan2(-cos(4*x) + sin(4*x) + 2*sin(2*x) + 1, cos(4*x) + 2*cos(2*x) +
 sin(4*x) + 1))^2))
```

3.859.8 Giac [F]

$$\int \frac{\csc(x)\sqrt{\cos(x)+\sin(x)}}{\cos^{\frac{3}{2}}(x)}\,dx = \int \frac{\sqrt{\cos\left(x\right)+\sin\left(x\right)}\csc\left(x\right)}{\cos\left(x\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(csc(x)*(cos(x)+sin(x))^(1/2)/cos(x)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(cos(x) + sin(x))*csc(x)/cos(x)^(3/2), x)
```

3.859.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(x)\sqrt{\cos(x)+\sin(x)}}{\cos^{\frac{3}{2}}(x)}\,dx = \int \frac{\sqrt{\cos(x)+\sin(x)}}{\cos(x)^{3/2}\,\sin(x)}\,dx$$

input
```
int((cos(x) + sin(x))^(1/2)/(cos(x)^(3/2)*sin(x)),x)
```

output
```
int((cos(x) + sin(x))^(1/2)/(cos(x)^(3/2)*sin(x)), x)
```

3.860 $\int \frac{\cos(x)+\sin(x)}{\sqrt{1+\sin(2x)}}\,dx$

3.860.1 Optimal result

Integrand size = 16, antiderivative size = 19

$$\int \frac{\cos(x)+\sin(x)}{\sqrt{1+\sin(2x)}}\,dx = \frac{x\sqrt{1+\sin(2x)}}{\cos(x)+\sin(x)}$$

output
```
x*(1+sin(2*x))^(1/2)/(cos(x)+sin(x))
```

3.860.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{\cos(x)+\sin(x)}{\sqrt{1+\sin(2x)}}\,dx = \frac{x(\cos(x)+\sin(x))}{\sqrt{1+\sin(2x)}}$$

input
```
Integrate[(Cos[x] + Sin[x])/Sqrt[1 + Sin[2*x]],x]
```

output
```
(x*(Cos[x] + Sin[x]))/Sqrt[1 + Sin[2*x]]
```

3.860.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 72 vs. $2(19) = 38$.

Time = 1.47 (sec) , antiderivative size = 72, normalized size of antiderivative = 3.79, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3042, 4901, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)+\cos(x)}{\sqrt{\sin(2x)+1}}dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{\sin(x)+\cos(x)}{\sqrt{\sin(2x)+1}}dx$$
$$\Big\downarrow \ 4901$$
$$\int \left(\frac{\sin(x)}{\sqrt{\sin(2x)+1}}+\frac{\cos(x)}{\sqrt{\sin(2x)+1}}\right)dx$$
$$\Big\downarrow \ 2009$$
$$\frac{2\cos^2\left(\frac{x}{2}\right)\left(-\tan^2\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)+1\right)\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{\sqrt{\cos^4\left(\frac{x}{2}\right)\left(-\tan^2\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)+1\right)^2}}$$

input
```
Int[(Cos[x] + Sin[x])/Sqrt[1 + Sin[2*x]],x]
```

output
```
(2*ArcTan[Tan[x/2]]*Cos[x/2]^2*(1 + 2*Tan[x/2] - Tan[x/2]^2))/Sqrt[Cos[x/2
]^4*(1 + 2*Tan[x/2] - Tan[x/2]^2)^2]
```

3.860.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4901
```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

3.860.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 8.31 (sec) , antiderivative size = 12132, normalized size of antiderivative = 638.53

method	result	size
default	Expression too large to display	12132
parts	Expression too large to display	13332

input
```
int((sin(x)+cos(x))/(1+sin(2*x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.860.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.16

$$\int \frac{\cos(x) + \sin(x)}{\sqrt{1 + \sin(2x)}}\, dx = -x$$

input
```
integrate((cos(x)+sin(x))/(1+sin(2*x))^(1/2),x, algorithm="fricas")
```

output
```
-x
```

3.860.6 Sympy [F]

$$\int \frac{\cos(x) + \sin(x)}{\sqrt{1 + \sin(2x)}}\, dx = \int \frac{\sin(x) + \cos(x)}{\sqrt{\sin(2x) + 1}}\, dx$$

input
```
integrate((cos(x)+sin(x))/(1+sin(2*x))**(1/2),x)
```

output
```
Integral((sin(x) + cos(x))/sqrt(sin(2*x) + 1), x)
```

3.860.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 329 vs. 2(17) = 34.

Time = 0.35 (sec) , antiderivative size = 329, normalized size of antiderivative = 17.32

$$\int \frac{\cos(x)+\sin(x)}{\sqrt{1+\sin(2x)}}\,dx$$
$$= \frac{1}{16}\sqrt{2}\Big(2\sqrt{2}\arctan\left(\sin\left(2x\right)+1,\cos\left(2x\right)\right)+\sqrt{2}\log\left(\cos\left(2x\right)^2+\sin\left(2x\right)^2+2\sin\left(2x\right)+1\right)+4\left(\cos(4\right.$$
$$+\frac{1}{16}\sqrt{2}\Big(2\sqrt{2}\arctan\left(\sin\left(2x\right)+1,\cos\left(2x\right)\right)-\sqrt{2}\log\left(\cos\left(2x\right)^2+\sin\left(2x\right)^2+2\sin\left(2x\right)+1\right)-4\left(\cos\right.$$

input
```
integrate((cos(x)+sin(x))/(1+sin(2*x))^(1/2),x, algorithm="maxima")
```

output
```
1/16*sqrt(2)*(2*sqrt(2)*arctan2(sin(2*x) + 1, cos(2*x)) + sqrt(2)*log(cos(
2*x)^2 + sin(2*x)^2 + 2*sin(2*x) + 1) + 4*(cos(4*x)^2 + 4*cos(2*x)^2 + 4*c
os(2*x)*sin(4*x) + sin(4*x)^2 - 4*cos(4*x)*sin(2*x) + 4*sin(2*x)^2)^(1/4)*
(cos(1/2*arctan2(cos(4*x) - 2*sin(2*x), 2*cos(2*x) + sin(4*x)))*sin(2*x) +
 cos(2*x)*sin(1/2*arctan2(cos(4*x) - 2*sin(2*x), 2*cos(2*x) + sin(4*x)))))
 + 1/16*sqrt(2)*(2*sqrt(2)*arctan2(sin(2*x) + 1, cos(2*x)) - sqrt(2)*log(c
os(2*x)^2 + sin(2*x)^2 + 2*sin(2*x) + 1) - 4*(cos(4*x)^2 + 4*cos(2*x)^2 +
4*cos(2*x)*sin(4*x) + sin(4*x)^2 - 4*cos(4*x)*sin(2*x) + 4*sin(2*x)^2)^(1/
4)*(cos(2*x)*cos(1/2*arctan2(cos(4*x) - 2*sin(2*x), 2*cos(2*x) + sin(4*x))
) - sin(2*x)*sin(1/2*arctan2(cos(4*x) - 2*sin(2*x), 2*cos(2*x) + sin(4*x))
)))
```

3.860.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.58

$$\int \frac{\cos(x)+\sin(x)}{\sqrt{1+\sin(2x)}}\,dx = \frac{x}{\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+x\right)\right)}$$

input
```
integrate((cos(x)+sin(x))/(1+sin(2*x))^(1/2),x, algorithm="giac")
```

output
```
x/sgn(cos(-1/4*pi + x))
```

3.860.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(x)+\sin(x)}{\sqrt{1+\sin(2x)}}\,dx=\int \frac{\cos\left(x\right)+\sin\left(x\right)}{\sqrt{\sin\left(2\,x\right)+1}}\,dx$$

input
```
int((cos(x) + sin(x))/(sin(2*x) + 1)^(1/2),x)
```

output
```
int((cos(x) + sin(x))/(sin(2*x) + 1)^(1/2), x)
```

3.861 $\int \sec(x)\sqrt{\sec(x)+\tan(x)}\,dx$

3.861.1 Optimal result

Integrand size = 12, antiderivative size = 13

$$\int \sec(x)\sqrt{\sec(x)+\tan(x)}\,dx = 2\sqrt{\sec(x)(1+\sin(x))}$$

output
```
2*(sec(x)*(1+sin(x)))^(1/2)
```

3.861.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 37 vs. 2(13) = 26.

Time = 0.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.85

$$\int \sec(x)\sqrt{\sec(x)+\tan(x)}\,dx = 2\sqrt{\frac{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)}}$$

input
```
Integrate[Sec[x]*Sqrt[Sec[x] + Tan[x]],x]
```

output
```
2*Sqrt[(Cos[x/2] + Sin[x/2])/(Cos[x/2] - Sin[x/2])]
```

3.861.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 4897, 3042, 4900, 3042, 3184, 3042, 3150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sec(x)\sqrt{\tan(x)+\sec(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(x)\sqrt{\tan(x)+\sec(x)}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \sec(x)\sqrt{(\sin(x)+1)\sec(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(x)\sqrt{(\sin(x)+1)\sec(x)}dx$$

$$\Big\downarrow \text{4900}$$

$$\frac{\sqrt{(\sin(x)+1)\sec(x)}\int \sec^{\frac{3}{2}}(x)\sqrt{\sin(x)+1}dx}{\sqrt{\sin(x)+1}\sqrt{\sec(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{(\sin(x)+1)\sec(x)}\int \sec(x)^{3/2}\sqrt{\sin(x)+1}dx}{\sqrt{\sin(x)+1}\sqrt{\sec(x)}}$$

$$\Big\downarrow \text{3184}$$

$$\frac{\sqrt{\cos(x)}\sqrt{(\sin(x)+1)\sec(x)}\int \frac{\sqrt{\sin(x)+1}}{\cos^{\frac{3}{2}}(x)}dx}{\sqrt{\sin(x)+1}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cos(x)}\sqrt{(\sin(x)+1)\sec(x)}\int \frac{\sqrt{\sin(x)+1}}{\cos(x)^{3/2}}dx}{\sqrt{\sin(x)+1}}$$

$$\Big\downarrow \text{3150}$$

$$2\sqrt{(\sin(x)+1)\sec(x)}$$

3.861. $\int \sec(x)\sqrt{\sec(x)+\tan(x)}\,dx$

input
```
Int[Sec[x]*Sqrt[Sec[x] + Tan[x]],x]
```

output
```
2*Sqrt[Sec[x]*(1 + Sin[x])]
```

3.861.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3150
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] &&  !ILtQ[p, 0]
```

rule 3184
```
Int[((g_.)*sec[(e_.) + (f_.)*(x_)])^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Simp[g^(2*IntPart[p])*(g*Cos[e + f*x])^FracPart[p]
*(g*Sec[e + f*x])^FracPart[p]   Int[(a + b*Sin[e + f*x])^m/(g*Cos[e + f*x])
^p, x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

rule 4900
```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Simp[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p]))   Int[uu*vv^(m*p)*ww^(n*p), x]
, x]] /; FreeQ[{m, n, p}, x] &&  !IntegerQ[p] && ( !InertTrigFreeQ[v] ||  !
InertTrigFreeQ[w])
```

3.861.4 Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$2\sqrt{\sec\left(x\right)+\tan\left(x\right)}$	10
default	$2\sqrt{\sec\left(x\right)+\tan\left(x\right)}$	10

input
```
int(sec(x)*(sec(x)+tan(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(sec(x)+tan(x))^(1/2)
```

3.861.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62

$$\int \sec(x)\sqrt{\sec(x)+\tan(x)}\,dx = 2\sqrt{\frac{\cos\left(x\right)+\sin\left(x\right)+1}{\cos\left(x\right)-\sin\left(x\right)+1}}$$

input
```
integrate(sec(x)*(sec(x)+tan(x))^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt((cos(x) + sin(x) + 1)/(cos(x) - sin(x) + 1))
```

3.861.6 Sympy [F]

$$\int \sec(x)\sqrt{\sec(x)+\tan(x)}\,dx = \int \sqrt{\tan\left(x\right)+\sec\left(x\right)}\sec\left(x\right)\,dx$$

input
```
integrate(sec(x)*(sec(x)+tan(x))**(1/2),x)
```

output
```
Integral(sqrt(tan(x) + sec(x))*sec(x), x)
```

3.861.7 Maxima [F]

$$\int \sec(x)\sqrt{\sec(x)+\tan(x)}\,dx = \int \sqrt{\sec(x)+\tan(x)}\sec(x)\,dx$$

input
```
integrate(sec(x)*(sec(x)+tan(x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(sec(x) + tan(x))*sec(x), x)
```

3.861.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(11) = 22$.

Time = 0.37 (sec) , antiderivative size = 55, normalized size of antiderivative = 4.23

$$\int \sec(x)\sqrt{\sec(x)+\tan(x)}\,dx = -\frac{4\,\mathrm{sgn}\left(-\tan\left(\frac{1}{2}\,x\right)^3-\tan\left(\frac{1}{2}\,x\right)^2-\tan\left(\frac{1}{2}\,x\right)-1\right)\mathrm{sgn}(\cos(x))}{\frac{\sqrt{-\tan\left(\frac{1}{2}\,x\right)^2+1}-1}{\tan\left(\frac{1}{2}\,x\right)}+1}$$

input
```
integrate(sec(x)*(sec(x)+tan(x))^(1/2),x, algorithm="giac")
```

output
```
-4*sgn(-tan(1/2*x)^3 - tan(1/2*x)^2 - tan(1/2*x) - 1)*sgn(cos(x))/((sqrt(-
tan(1/2*x)^2 + 1) - 1)/tan(1/2*x) + 1)
```

3.861.9 Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \sec(x)\sqrt{\sec(x)+\tan(x)}\,dx = 2\sqrt{\frac{1}{\cos(x)}}\sqrt{\sin(x)+1}$$

input
```
int((tan(x) + 1/cos(x))^(1/2)/cos(x),x)
```

output
```
2*(1/cos(x))^(1/2)*(sin(x) + 1)^(1/2)
```

3.862 $\int \sec(x)\sqrt{4+3\sec(x)}\tan(x)\,dx$

3.862.1 Optimal result

Integrand size = 15, antiderivative size = 14

$$\int \sec(x)\sqrt{4+3\sec(x)}\tan(x)\,dx = \frac{2}{9}(4+3\sec(x))^{3/2}$$

output
```
2/9*(4+3*sec(x))^(3/2)
```

3.862.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \sec(x)\sqrt{4+3\sec(x)}\tan(x)\,dx = \frac{2}{9}(4+3\sec(x))^{3/2}$$

input
```
Integrate[Sec[x]*Sqrt[4 + 3*Sec[x]]*Tan[x],x]
```

output
```
(2*(4 + 3*Sec[x])^(3/2))/9
```

3.862.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 4839, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(x)\sec(x)\sqrt{3\sec(x)+4}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(x)\sec(x)\sqrt{3\sec(x)+4}dx$$

$$\Big\downarrow \text{4839}$$

$$-\int \sec^2(x)\sqrt{3\sec(x)+4}d\cos(x)$$

$$\Big\downarrow \text{793}$$

$$\frac{2}{9}(3\sec(x)+4)^{3/2}$$

input
```
Int[Sec[x]*Sqrt[4 + 3*Sec[x]]*Tan[x],x]
```

output
```
(2*(4 + 3*Sec[x])^(3/2))/9
```

3.862.3.1 Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4839
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-(b*c)^(-1)   Subst[Int[SubstFor[1/x, Cos[c*
(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
 + b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, t
an])
```

3.862.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\frac{2(4+3\sec(x))^{\frac{3}{2}}}{9}$	11
default	$\frac{2(4+3\sec(x))^{\frac{3}{2}}}{9}$	11

input
```
int(sec(x)*(4+3*sec(x))^(1/2)*tan(x),x,method=_RETURNVERBOSE)
```

output
```
2/9*(4+3*sec(x))^(3/2)
```

3.862.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(10) = 20.

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.79

$$\int \sec(x)\sqrt{4+3\sec(x)}\tan(x)\,dx = \frac{2\sqrt{\frac{4\cos(x)+3}{\cos(x)}}(4\cos(x)+3)}{9\cos(x)}$$

input
```
integrate(sec(x)*(4+3*sec(x))^(1/2)*tan(x),x, algorithm="fricas")
```

output
```
2/9*sqrt((4*cos(x) + 3)/cos(x))*(4*cos(x) + 3)/cos(x)
```

3.862.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(12) = 24.

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.07

$$\int \sec(x)\sqrt{4+3\sec(x)}\tan(x)\,dx = \frac{2\sqrt{3\sec(x)+4}\sec(x)}{3} + \frac{8\sqrt{3\sec(x)+4}}{9}$$

input
```
integrate(sec(x)*(4+3*sec(x))**(1/2)*tan(x),x)
```

output
```
2*sqrt(3*sec(x) + 4)*sec(x)/3 + 8*sqrt(3*sec(x) + 4)/9
```

3.862.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \sec(x)\sqrt{4+3\sec(x)}\tan(x)\,dx = \frac{2}{9}\,(3\,\sec(x)+4)^{\frac{3}{2}}$$

input
```
integrate(sec(x)*(4+3*sec(x))^(1/2)*tan(x),x, algorithm="maxima")
```

output
```
2/9*(3*sec(x) + 4)^(3/2)
```

3.862.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(10) = 20.

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 4.86

$$\int \sec(x)\sqrt{4+3\sec(x)}\tan(x)\,dx$$
$$= \frac{2\left(4\left(\sqrt{4\cos(x)^2+3\cos(x)}-2\cos(x)\right)^2-6\sqrt{4\cos(x)^2+3\cos(x)}+12\cos(x)+3\right)\operatorname{sgn}(\cos(x))}{\left(\sqrt{4\cos(x)^2+3\cos(x)}-2\cos(x)\right)^3}$$

input
```
integrate(sec(x)*(4+3*sec(x))^(1/2)*tan(x),x, algorithm="giac")
```

output
```
2*(4*(sqrt(4*cos(x)^2 + 3*cos(x)) - 2*cos(x))^2 - 6*sqrt(4*cos(x)^2 + 3*co
s(x)) + 12*cos(x) + 3)*sgn(cos(x))/(sqrt(4*cos(x)^2 + 3*cos(x)) - 2*cos(x)
)^3
```

3.862.9 Mupad [B] (verification not implemented)

Time = 26.60 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.07

$$\int \sec(x)\sqrt{4+3\sec(x)}\tan(x)\,dx = \frac{8\sqrt{\frac{3}{\cos(x)}+4}}{9} + \frac{2\sqrt{\frac{3}{\cos(x)}+4}}{3\,\cos(x)}$$

input
```
int((tan(x)*(3/cos(x) + 4)^(1/2))/cos(x),x)
```

output
```
(8*(3/cos(x) + 4)^(1/2))/9 + (2*(3/cos(x) + 4)^(1/2))/(3*cos(x))
```

3.863 $\int \sec(x)\sqrt{1+\sec(x)}\tan^3(x)\,dx$

3.863.1 Optimal result

Integrand size = 15, antiderivative size = 25

$$\int \sec(x)\sqrt{1+\sec(x)}\tan^3(x)\,dx = -\frac{4}{5}(1+\sec(x))^{5/2}+\frac{2}{7}(1+\sec(x))^{7/2}$$

output
```
-4/5*(1+sec(x))^(5/2)+2/7*(1+sec(x))^(7/2)
```

3.863.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20

$$\int \sec(x)\sqrt{1+\sec(x)}\tan^3(x)\,dx = -\frac{8}{35}\cos^4\left(\frac{x}{2}\right)(-5+9\cos(x))\sec^3(x)\sqrt{1+\sec(x)}$$

input
```
Integrate[Sec[x]*Sqrt[1 + Sec[x]]*Tan[x]^3,x]
```

output
```
(-8*Cos[x/2]^4*(-5 + 9*Cos[x])*Sec[x]^3*Sqrt[1 + Sec[x]])/35
```

3.863.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {3042, 4873, 1894, 1388, 946, 25, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(x)\sec(x)\sqrt{\sec(x)+1}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(x)^3\sec(x)\sqrt{\sec(x)+1}dx$$

$$\Big\downarrow \text{4873}$$

$$-\int \left(1-\cos^2(x)\right)\sec^4(x)\sqrt{\sec(x)+1}d\cos(x)$$

$$\Big\downarrow \text{1894}$$

$$-\int \sec^2(x)\sqrt{\sec(x)+1}\left(\sec^2(x)-1\right)d\cos(x)$$

$$\Big\downarrow \text{1388}$$

$$-\int (\sec(x)-1)\sec^2(x)(\sec(x)+1)^{3/2}d\cos(x)$$

$$\Big\downarrow \text{946}$$

$$\int -\left((1-\sec(x))(\sec(x)+1)^{3/2}\right)d\sec(x)$$

$$\Big\downarrow \text{25}$$

$$-\int (1-\sec(x))(\sec(x)+1)^{3/2}d\sec(x)$$

$$\Big\downarrow \text{53}$$

$$-\int \left(2(\sec(x)+1)^{3/2}-(\sec(x)+1)^{5/2}\right)d\sec(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{7}(\sec(x)+1)^{7/2}-\frac{4}{5}(\sec(x)+1)^{5/2}$$

input
```
Int[Sec[x]*Sqrt[1 + Sec[x]]*Tan[x]^3,x]
```

output
```
(-4*(1 + Sec[x])^(5/2))/5 + (2*(1 + Sec[x])^(7/2))/7
```

3.863.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

rule 1894
```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_.))
^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n))^p, x]
/; FreeQ[{a, c, d, e, m, n, q}, x] && EqQ[mn2, -2*n] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4873
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Cos[c*(a + b*x)], x]}, Simp[-(b*c*d^(n - 1))^(-1)   Subst[Int[Subst
For[(1 - d^2*x^2)^((n - 1)/2)/x^n, Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*
(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b,
 c}, x] && IntegerQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Tan] || EqQ[F, tan]
)
```

3.863.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

method	result	size
derivativedivides	$-\frac{4(1+\sec(x))^{\frac{5}{2}}}{5}+\frac{2(1+\sec(x))^{\frac{7}{2}}}{7}$	18
default	$-\frac{4(1+\sec(x))^{\frac{5}{2}}}{5}+\frac{2(1+\sec(x))^{\frac{7}{2}}}{7}$	18

input
```
int(sec(x)*(1+sec(x))^(1/2)*tan(x)^3,x,method=_RETURNVERBOSE)
```

output
```
-4/5*(1+sec(x))^(5/2)+2/7*(1+sec(x))^(7/2)
```

3.863.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. 2(17) = 34.

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.40

$$\int \sec(x)\sqrt{1+\sec(x)}\tan^3(x)\,dx = -\frac{2\left(9\cos(x)^3+13\cos(x)^2-\cos(x)-5\right)\sqrt{\frac{\cos(x)+1}{\cos(x)}}}{35\cos(x)^3}$$

input
```
integrate(sec(x)*(1+sec(x))^(1/2)*tan(x)^3,x, algorithm="fricas")
```

output
```
-2/35*(9*cos(x)^3 + 13*cos(x)^2 - cos(x) - 5)*sqrt((cos(x) + 1)/cos(x))/co
s(x)^3
```

3.863.6 Sympy [F]

$$\int \sec(x)\sqrt{1+\sec(x)}\tan^3(x)\,dx = \int \sqrt{\sec(x)+1}\tan^3(x)\sec(x)\,dx$$

input
```
integrate(sec(x)*(1+sec(x))**(1/2)*tan(x)**3,x)
```

output
```
Integral(sqrt(sec(x) + 1)*tan(x)**3*sec(x), x)
```

3.863.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \sec(x)\sqrt{1+\sec(x)}\tan^3(x)\,dx = \frac{2}{7}\left(\frac{1}{\cos(x)}+1\right)^{\frac{7}{2}} - \frac{4}{5}\left(\frac{1}{\cos(x)}+1\right)^{\frac{5}{2}}$$

input
```
integrate(sec(x)*(1+sec(x))^(1/2)*tan(x)^3,x, algorithm="maxima")
```

output
```
2/7*(1/cos(x) + 1)^(7/2) - 4/5*(1/cos(x) + 1)^(5/2)
```

3.863.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(17) = 34.

Time = 0.27 (sec) , antiderivative size = 128, normalized size of antiderivative = 5.12

$$\int \sec(x)\sqrt{1+\sec(x)}\tan^3(x)\,dx =$$
$$-\frac{2\left(35\left(\sqrt{\cos(x)^2+\cos(x)}-\cos(x)\right)^6 - 35\left(\sqrt{\cos(x)^2+\cos(x)}-\cos(x)\right)^5 - 35\left(\sqrt{\cos(x)^2+co\right.\right.}{}$$

input
```
integrate(sec(x)*(1+sec(x))^(1/2)*tan(x)^3,x, algorithm="giac")
```

output
```
-2/35*(35*(sqrt(cos(x)^2 + cos(x)) - cos(x))^6 - 35*(sqrt(cos(x)^2 + cos(x
)) - cos(x))^5 - 35*(sqrt(cos(x)^2 + cos(x)) - cos(x))^4 + 105*(sqrt(cos(x
)^2 + cos(x)) - cos(x))^3 - 91*(sqrt(cos(x)^2 + cos(x)) - cos(x))^2 + 35*s
qrt(cos(x)^2 + cos(x)) - 35*cos(x) - 5)*sgn(cos(x))/(sqrt(cos(x)^2 + cos(x
)) - cos(x))^7
```

3.863.9 Mupad [B] (verification not implemented)

Time = 27.38 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \sec(x)\sqrt{1+\sec(x)}\tan^3(x)\,dx = -\frac{2\,{\left(\cos\left(x\right)+1\right)}^{5/2}\,\sqrt{\frac{1}{\cos\left(x\right)}}\,\left(9\,\cos\left(x\right)-5\right)}{35\,{\cos\left(x\right)}^3}$$

input
```
int((tan(x)^3*(1/cos(x) + 1)^(1/2))/cos(x),x)
```

output
```
-(2*(cos(x) + 1)^(5/2)*(1/cos(x))^(1/2)*(9*cos(x) - 5))/(35*cos(x)^3)
```

3.864 $\int \cot^3(x)\csc(x)\sqrt{1+\csc(x)}\,dx$

3.864.1 Optimal result

Integrand size = 15, antiderivative size = 25

$$\int \cot^3(x)\csc(x)\sqrt{1+\csc(x)}\,dx = \frac{4}{5}(1+\csc(x))^{5/2} - \frac{2}{7}(1+\csc(x))^{7/2}$$

output
```
4/5*(1+csc(x))^(5/2)-2/7*(1+csc(x))^(7/2)
```

3.864.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

$$\int \cot^3(x)\csc(x)\sqrt{1+\csc(x)}\,dx = -\frac{2}{35}(1+\csc(x))^{5/2}(-9+5\csc(x))$$

input
```
Integrate[Cot[x]^3*Csc[x]*Sqrt[1 + Csc[x]],x]
```

output
```
(-2*(1 + Csc[x])^(5/2)*(-9 + 5*Csc[x]))/35
```

3.864.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {3042, 4872, 1894, 1388, 946, 25, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(x)\csc(x)\sqrt{\csc(x)+1}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cot(x)^3\csc(x)\sqrt{\csc(x)+1}dx$$

$$\Big\downarrow \text{4872}$$

$$\int \left(1-\sin^2(x)\right)\csc^4(x)\sqrt{\csc(x)+1}d\sin(x)$$

$$\Big\downarrow \text{1894}$$

$$\int \csc^2(x)\sqrt{\csc(x)+1}\left(\csc^2(x)-1\right)d\sin(x)$$

$$\Big\downarrow \text{1388}$$

$$\int (\csc(x)-1)\csc^2(x)(\csc(x)+1)^{3/2}d\sin(x)$$

$$\Big\downarrow \text{946}$$

$$-\int -\left((1-\csc(x))(\csc(x)+1)^{3/2}\right)d\csc(x)$$

$$\Big\downarrow \text{25}$$

$$\int (1-\csc(x))(\csc(x)+1)^{3/2}d\csc(x)$$

$$\Big\downarrow \text{53}$$

$$\int \left(2(\csc(x)+1)^{3/2}-(\csc(x)+1)^{5/2}\right)d\csc(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{4}{5}(\csc(x)+1)^{5/2}-\frac{2}{7}(\csc(x)+1)^{7/2}$$

input
```
Int[Cot[x]^3*Csc[x]*Sqrt[1 + Csc[x]],x]
```

output
```
(4*(1 + Csc[x])^(5/2))/5 - (2*(1 + Csc[x])^(7/2))/7
```

3.864.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 946
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Simp[1/n   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n],
x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n
+ 1, 0]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

rule 1894
```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_.))
^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n))^p, x]
/; FreeQ[{a, c, d, e, m, n, q}, x] && EqQ[mn2, -2*n] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4872
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Sin[c*(a + b*x)], x]}, Simp[1/(b*c*d^(n - 1))   Subst[Int[SubstFor[
(1 - d^2*x^2)^((n - 1)/2)/x^n, Sin[c*(a + b*x)]/d, u, x], x], x, Sin[c*(a +
 b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c},
 x] && IntegerQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cot] || EqQ[F, cot])
```

3.864.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

method	result	size
derivativedivides	$\frac{4(\csc(x)+1)^{\frac{5}{2}}}{5}-\frac{2(\csc(x)+1)^{\frac{7}{2}}}{7}$	18
default	$\frac{4(\csc(x)+1)^{\frac{5}{2}}}{5}-\frac{2(\csc(x)+1)^{\frac{7}{2}}}{7}$	18

input
```
int(cot(x)^3*csc(x)*(csc(x)+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
4/5*(csc(x)+1)^(5/2)-2/7*(csc(x)+1)^(7/2)
```

3.864.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. 2(17) = 34.

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.76

$$\int \cot^3(x)\csc(x)\sqrt{1+\csc(x)}\,dx=\frac{2\left(13\cos\left(x\right)^2+\left(9\cos\left(x\right)^2-8\right)\sin\left(x\right)-8\right)\sqrt{\frac{\sin(x)+1}{\sin(x)}}}{35\left(\cos\left(x\right)^2-1\right)\sin\left(x\right)}$$

input
```
integrate(cot(x)^3*csc(x)*(1+csc(x))^(1/2),x, algorithm="fricas")
```

output
```
2/35*(13*cos(x)^2 + (9*cos(x)^2 - 8)*sin(x) - 8)*sqrt((sin(x) + 1)/sin(x))
/((cos(x)^2 - 1)*sin(x))
```

3.864.6 Sympy [F]

$$\int \cot^3(x)\csc(x)\sqrt{1+\csc(x)}\,dx = \int \sqrt{\csc(x)+1}\cot^3(x)\csc(x)\,dx$$

input
```
integrate(cot(x)**3*csc(x)*(1+csc(x))**(1/2),x)
```

output
```
Integral(sqrt(csc(x) + 1)*cot(x)**3*csc(x), x)
```

3.864.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

$$\int \cot^3(x)\csc(x)\sqrt{1+\csc(x)}\,dx = -\frac{2}{7}\left(\frac{1}{\sin(x)}+1\right)^{\frac{7}{2}} + \frac{4}{5}\left(\frac{1}{\sin(x)}+1\right)^{\frac{5}{2}}$$

input
```
integrate(cot(x)^3*csc(x)*(1+csc(x))^(1/2),x, algorithm="maxima")
```

output
```
-2/7*(1/sin(x) + 1)^(7/2) + 4/5*(1/sin(x) + 1)^(5/2)
```

3.864.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(17) = 34.

Time = 0.27 (sec) , antiderivative size = 128, normalized size of antiderivative = 5.12

$$\int \cot^3(x)\csc(x)\sqrt{1+\csc(x)}\,dx$$
$$= \frac{2\left(35\left(\sqrt{\sin(x)^2+\sin(x)}-\sin(x)\right)^6 - 35\left(\sqrt{\sin(x)^2+\sin(x)}-\sin(x)\right)^5 - 35\left(\sqrt{\sin(x)^2+\sin(x)}\right.\right.}{}$$

input
```
integrate(cot(x)^3*csc(x)*(1+csc(x))^(1/2),x, algorithm="giac")
```

output
```
2/35*(35*(sqrt(sin(x)^2 + sin(x)) - sin(x))^6 - 35*(sqrt(sin(x)^2 + sin(x)
) - sin(x))^5 - 35*(sqrt(sin(x)^2 + sin(x)) - sin(x))^4 + 105*(sqrt(sin(x)
^2 + sin(x)) - sin(x))^3 - 91*(sqrt(sin(x)^2 + sin(x)) - sin(x))^2 + 35*sq
rt(sin(x)^2 + sin(x)) - 35*sin(x) - 5)*sgn(sin(x))/(sqrt(sin(x)^2 + sin(x)
) - sin(x))^7
```

3.864.9 Mupad [B] (verification not implemented)

Time = 27.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \cot^3(x)\csc(x)\sqrt{1+\csc(x)}\,dx = \frac{2\,(\sin(x)+1)^{5/2}\,\sqrt{\frac{1}{\sin(x)}}\,(9\,\sin(x)-5)}{35\sin(x)^3}$$

input
```
int((cot(x)^3*(1/sin(x) + 1)^(1/2))/sin(x),x)
```

output
```
(2*(sin(x) + 1)^(5/2)*(1/sin(x))^(1/2)*(9*sin(x) - 5))/(35*sin(x)^3)
```

3.865 $\int \sqrt{\csc(x)}(x\cos(x) - 4\sec(x)\tan(x))\,dx$

3.865.1 Optimal result

Integrand size = 18, antiderivative size = 20

$$\int \sqrt{\csc(x)}(x\cos(x) - 4\sec(x)\tan(x))\,dx = \frac{2x}{\sqrt{\csc(x)}} - \frac{4\sec(x)}{\csc^{\frac{3}{2}}(x)}$$

output
```
-4*sec(x)/csc(x)^(3/2)+2*x/csc(x)^(1/2)
```

3.865.2 Mathematica [A] (verified)

Time = 1.52 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \sqrt{\csc(x)}(x\cos(x) - 4\sec(x)\tan(x))\,dx = \frac{2(x\csc(x) - 2\sec(x))}{\csc^{\frac{3}{2}}(x)}$$

input
```
Integrate[Sqrt[Csc[x]]*(x*Cos[x] - 4*Sec[x]*Tan[x]),x]
```

output
```
(2*(x*Csc[x] - 2*Sec[x]))/Csc[x]^(3/2)
```

3.865.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\csc(x)}(x\cos(x) - 4\tan(x)\sec(x))\,dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(x\cos(x)\sqrt{\csc(x)} - \frac{4\sec^2(x)}{\sqrt{\csc(x)}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2x}{\sqrt{\csc(x)}} - \frac{4\sec(x)}{\csc^{\frac{3}{2}}(x)}$$

input
```
Int[Sqrt[Csc[x]]*(x*Cos[x] - 4*Sec[x]*Tan[x]),x]
```

output
```
(2*x)/Sqrt[Csc[x]] - (4*Sec[x])/Csc[x]^(3/2)
```

3.865.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.865.4 Maple [F]

$$\int \sqrt{\csc\left(x\right)}\, \left(x\cos\left(x\right)-4\sec\left(x\right)\tan\left(x\right)\right)dx$$

input
```
int(csc(x)^(1/2)*(x*cos(x)-4*sec(x)*tan(x)),x)
```

output
```
int(csc(x)^(1/2)*(x*cos(x)-4*sec(x)*tan(x)),x)
```

3.865.5 Fricas [F(-2)]

Exception generated.

$$\int \sqrt{\csc(x)}(x\cos(x)-4\sec(x)\tan(x))\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(csc(x)^(1/2)*(x*cos(x)-4*sec(x)*tan(x)),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.865.6 Sympy [F]

$$\int \sqrt{\csc(x)}(x\cos(x)-4\sec(x)\tan(x))\, dx = \int \left(x\cos\left(x\right)-4\tan\left(x\right)\sec\left(x\right)\right)\sqrt{\csc\left(x\right)}\, dx$$

input
```
integrate(csc(x)**(1/2)*(x*cos(x)-4*sec(x)*tan(x)),x)
```

output
```
Integral((x*cos(x) - 4*tan(x)*sec(x))*sqrt(csc(x)), x)
```

3.865.7 Maxima [F]

$$\int \sqrt{\csc(x)}(x\cos(x) - 4\sec(x)\tan(x))\,dx = \int (x\cos(x) - 4\sec(x)\tan(x))\sqrt{\csc(x)}\,dx$$

input
```
integrate(csc(x)^(1/2)*(x*cos(x)-4*sec(x)*tan(x)),x, algorithm="maxima")
```

output
```
integrate((x*cos(x) - 4*sec(x)*tan(x))*sqrt(csc(x)), x)
```

3.865.8 Giac [F]

$$\int \sqrt{\csc(x)}(x\cos(x) - 4\sec(x)\tan(x))\,dx = \int (x\cos(x) - 4\sec(x)\tan(x))\sqrt{\csc(x)}\,dx$$

input
```
integrate(csc(x)^(1/2)*(x*cos(x)-4*sec(x)*tan(x)),x, algorithm="giac")
```

output
```
integrate((x*cos(x) - 4*sec(x)*tan(x))*sqrt(csc(x)), x)
```

3.865.9 Mupad [B] (verification not implemented)

Time = 26.74 (sec) , antiderivative size = 77, normalized size of antiderivative = 3.85

$$\int \sqrt{\csc(x)}(x\cos(x) - 4\sec(x)\tan(x))\,dx$$
$$= \frac{(4\cos(x)^3 - 4\cos(x) + 2x\cos(x)^2\sin(x) - \sin(x)\,4i - x\cos(x)^3\,2i + \cos(x)^2\sin(x)\,4i + x\cos(x)\,2i}{\cos(x)\sin(x)\sqrt{\frac{1}{\sin(x)}}\,(-\sin(x) + \cos(x)\,1i)}$$

input
```
int(-(1/sin(x))^(1/2)*((4*tan(x))/cos(x) - x*cos(x)),x)
```

output
```
((4*cos(x)^3 - sin(x)*4i - x*cos(x)^3*2i - 4*cos(x) + cos(x)^2*sin(x)*4i +
 x*cos(x)*2i + 2*x*cos(x)^2*sin(x))*1i)/(cos(x)*sin(x)*(1/sin(x))^(1/2)*(c
os(x)*1i - sin(x)))
```

3.866 $\int \cot(x)\sqrt{-1+\csc^2(x)}\left(1-\sin^2(x)\right)^3\,dx$

3.866.1 Optimal result

Integrand size = 23, antiderivative size = 76

$$\int \cot(x)\sqrt{-1+\csc^2(x)}\left(1-\sin^2(x)\right)^3\,dx = -\frac{35}{16}\sqrt{\cot^2(x)}+\frac{35}{48}\cos^2(x)\sqrt{\cot^2(x)}+\frac{7}{24}\cos^4(x)\sqrt{\cot^2(x)}+\frac{1}{6}\cos^6(x)\sqrt{\cot^2(x)}-\frac{35}{16}x\sqrt{\cot^2(x)}\tan(x)$$

output
```
-35/16*(cot(x)^2)^(1/2)+35/48*cos(x)^2*(cot(x)^2)^(1/2)+7/24*cos(x)^4*(cot
(x)^2)^(1/2)+1/6*cos(x)^6*(cot(x)^2)^(1/2)-35/16*x*(cot(x)^2)^(1/2)*tan(x)
```

3.866.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.34 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.03

$$\int \cot(x)\sqrt{-1+\csc^2(x)}\left(1-\sin^2(x)\right)^3\,dx = \frac{1}{288}\cot^8(x)\sqrt{\cot^2(x)}\left(-27+36\cos(2x)-9\cos(4x)+40\,\text{Hypergeometric2F1}\left(2,\frac{9}{2},\frac{11}{2},-\cot^2(x)\right)+32\,\text{Hypergeometric2F1}\left(3,\frac{9}{2},\frac{11}{2},-\cot^2(x)\right)-32\,\text{Hypergeometric2F1}\left(4,\frac{9}{2},\frac{11}{2},-\cot^2(x)\right)\right)$$

input
```
Integrate[Cot[x]*Sqrt[-1 + Csc[x]^2]*(1 - Sin[x]^2)^3,x]
```

output
```
(Cot[x]^8*Sqrt[Cot[x]^2]*(-27 + 36*Cos[2*x] - 9*Cos[4*x] + 40*Hypergeometr
ic2F1[2, 9/2, 11/2, -Cot[x]^2] + 32*Hypergeometric2F1[3, 9/2, 11/2, -Cot[x
]^2] - 32*Hypergeometric2F1[4, 9/2, 11/2, -Cot[x]^2]))/288
```

3.866.3 Rubi [A] (warning: unable to verify)

Time = 0.40 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.25, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {3042, 3654, 3042, 4860, 1016, 798, 51, 51, 51, 60, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(1-\sin^2(x)\right)^3 \cot(x)\sqrt{\csc^2(x)-1}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \left(1-\sin(x)^2\right)^3 \cot(x)\sqrt{\csc(x)^2-1}dx$$

$$\Big\downarrow \ 3654$$

$$\int \cos^6(x)\cot(x)\sqrt{\csc^2(x)-1}dx$$

$$\Big\downarrow \ 3042$$

$$\int \cos(x)^6\cot(x)\sqrt{\csc(x)^2-1}dx$$

$$\Big\downarrow \ 4860$$

$$\int \left(1-\sin^2(x)\right)^3 \csc(x)\sqrt{\csc^2(x)-1}d\sin(x)$$

$$\Big\downarrow \ 1016$$

$$\int \sin^5(x)\left(\csc^2(x)-1\right)^{7/2}d\sin(x)$$

$$\Big\downarrow \ 798$$

$$-\frac{1}{2}\int \csc^4(x)\left(\csc^2(x)-1\right)^{7/2}d\csc^2(x)$$

$$\Big\downarrow \ 51$$

$$\frac{1}{2}\left(\frac{1}{3}\csc^3(x)\left(\csc^2(x)-1\right)^{7/2}-\frac{7}{6}\int\csc^3(x)\left(\csc^2(x)-1\right)^{5/2}d\csc^2(x)\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{1}{3}\csc^3(x)\left(\csc^2(x)-1\right)^{7/2}-\frac{7}{6}\left(\frac{5}{4}\int\csc^2(x)\left(\csc^2(x)-1\right)^{3/2}d\csc^2(x)-\frac{1}{2}\csc^2(x)\left(\csc^2(x)-1\right)^{5/2}\right)\right)$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}\left(\frac{1}{3}\csc^3(x)\left(\csc^2(x)-1\right)^{7/2}-\frac{7}{6}\left(\frac{5}{4}\left(\frac{3}{2}\int\csc(x)\sqrt{\csc^2(x)-1}d\csc^2(x)-\csc(x)\left(\csc^2(x)-1\right)^{3/2}\right)-\frac{1}{2}\csc^2(x)\left(\right.\right.\right.$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{1}{3}\csc^3(x)\left(\csc^2(x)-1\right)^{7/2}-\frac{7}{6}\left(\frac{5}{4}\left(\frac{3}{2}\left(2\sqrt{\csc^2(x)-1}-\int\frac{\csc(x)}{\sqrt{\csc^2(x)-1}}d\csc^2(x)\right)-\csc(x)\left(\csc^2(x)-1\right)^{3/2}\right.\right.\right.$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{1}{3}\csc^3(x)\left(\csc^2(x)-1\right)^{7/2}-\frac{7}{6}\left(\frac{5}{4}\left(\frac{3}{2}\left(2\sqrt{\csc^2(x)-1}-2\int\frac{1}{\csc^4(x)+1}d\sqrt{\csc^2(x)-1}\right)-\csc(x)\left(\csc^2(x)-\right.\right.\right.\right.$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(\frac{1}{3}\csc^3(x)\left(\csc^2(x)-1\right)^{7/2}-\frac{7}{6}\left(\frac{5}{4}\left(\frac{3}{2}\left(2\sqrt{\csc^2(x)-1}-2\arctan\left(\sqrt{\csc^2(x)-1}\right)\right)-\csc(x)\left(\csc^2(x)-1\right)^{3/2}\right)\right.\right.$$

input
```
Int[Cot[x]*Sqrt[-1 + Csc[x]^2]*(1 - Sin[x]^2)^3,x]
```

output
```
((Csc[x]^3*(-1 + Csc[x]^2)^(7/2))/3 - (7*(-1/2*(Csc[x]^2*(-1 + Csc[x]^2)^(
5/2)) + (5*(-(Csc[x]*(-1 + Csc[x]^2)^(3/2)) + (3*(-2*ArcTan[Sqrt[-1 + Csc[
x]^2]] + 2*Sqrt[-1 + Csc[x]^2]))/2))/4))/6)/2
```

3.866. $\int \cot(x)\sqrt{-1+\csc^2(x)}\left(1-\sin^2(x)\right)^3\,dx$

3.866.3.1 Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 1016
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.866. $\int \cot(x)\sqrt{-1+\csc^2(x)}\left(1-\sin^2(x)\right)^3\,dx$

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 4860
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[1/(b*c)   Subst[Int[SubstFor[1/x, Sin[c*(a +
 b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b
*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

3.866.4 Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.49

method	result
default	$\frac{\sqrt{\cot(x)^{2}}\left(8\cos(x)^{6}+14\cos(x)^{4}+35\cos(x)^{2}-105x\tan(x)-105\right)\sqrt{4}}{96}$
risch	$\frac{35i\sqrt{-\frac{\left(e^{2ix}+1\right)^{2}}{\left(e^{2ix}-1\right)^{2}}}\left(e^{2ix}-1\right)x}{16\left(e^{2ix}+1\right)}+\frac{\sqrt{-\frac{\left(e^{2ix}+1\right)^{2}}{\left(e^{2ix}-1\right)^{2}}}\,e^{8ix}}{384\,e^{2ix}+384}-\frac{47\sqrt{-\frac{\left(e^{2ix}+1\right)^{2}}{\left(e^{2ix}-1\right)^{2}}}\,e^{2ix}}{128\left(e^{2ix}+1\right)}-\frac{47\sqrt{-\frac{\left(e^{2ix}+1\right)^{2}}{\left(e^{2ix}-1\right)^{2}}}\left(1-e^{-2ix}\right)}{128\left(e^{2ix}+1\right)}-\frac{5\sqrt{-\frac{\left(e^{2ix}+1\right.}{\left(e^{2ix}-1\right.}}}{128(e^{2ix}}$

input
```
int(cot(x)*(1-sin(x)^2)^3*(csc(x)^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/96*(cot(x)^2)^(1/2)*(8*cos(x)^6+14*cos(x)^4+35*cos(x)^2-105*x*tan(x)-105
)*4^(1/2)
```

3.866.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.45

$$\int \cot(x)\sqrt{-1+\csc^2(x)}\left(1-\sin^2(x)\right)^3\,dx$$
$$=-\frac{8\,\cos\left(x\right)^{7}+14\,\cos\left(x\right)^{5}+35\,\cos\left(x\right)^{3}-105\,x\sin\left(x\right)-105\,\cos\left(x\right)}{48\,\sin\left(x\right)}$$

input
```
integrate(cot(x)*(1-sin(x)^2)^3*(-1+csc(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/48*(8*cos(x)^7 + 14*cos(x)^5 + 35*cos(x)^3 - 105*x*sin(x) - 105*cos(x))
/sin(x)
```

3.866.6 Sympy [F(-1)]

Timed out.

$$\int \cot(x)\sqrt{-1+\csc^2(x)}\left(1-\sin^2(x)\right)^3\,dx = \text{Timed out}$$

input
```
integrate(cot(x)*(1-sin(x)**2)**3*(-1+csc(x)**2)**(1/2),x)
```

output
```
Timed out
```

3.866.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(56) = 112$.

Time = 0.30 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.79

$$\int \cot(x)\sqrt{-1+\csc^2(x)}\left(1-\sin^2(x)\right)^3\,dx$$
$$= -\frac{3}{2}\sqrt{\frac{1}{\sin(x)^2}-1}\sin(x)^2 - \sqrt{\frac{1}{\sin(x)^2}-1} + \frac{3\left(\frac{1}{\sin(x)^2}-1\right)^{\frac{5}{2}} + 8\left(\frac{1}{\sin(x)^2}-1\right)^{\frac{3}{2}} - 3\sqrt{\frac{1}{\sin(x)^2}-1}}{48\left(\left(\frac{1}{\sin(x)^2}-1\right)^3 + 3\left(\frac{1}{\sin(x)^2}-1\right)^2 + \frac{3}{\sin(x)^2}-2\right)} - \frac{3\left(\left(\frac{1}{\sin(x)^2}-1\right)^{\frac{3}{2}} - \sqrt{\frac{1}{\sin(x)^2}-1}\right)}{8\left(\left(\frac{1}{\sin(x)^2}-1\right)^2 + \frac{2}{\sin(x)^2}-1\right)} + \frac{35}{16}\arctan\left(\sqrt{\frac{1}{\sin(x)^2}-1}\right)$$

input
```
integrate(cot(x)*(1-sin(x)^2)^3*(-1+csc(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-3/2*sqrt(1/sin(x)^2 - 1)*sin(x)^2 - sqrt(1/sin(x)^2 - 1) + 1/48*(3*(1/sin
(x)^2 - 1)^(5/2) + 8*(1/sin(x)^2 - 1)^(3/2) - 3*sqrt(1/sin(x)^2 - 1))/((1/
sin(x)^2 - 1)^3 + 3*(1/sin(x)^2 - 1)^2 + 3/sin(x)^2 - 2) - 3/8*((1/sin(x)^
2 - 1)^(3/2) - sqrt(1/sin(x)^2 - 1))/((1/sin(x)^2 - 1)^2 + 2/sin(x)^2 - 1)
 + 35/16*arctan(sqrt(1/sin(x)^2 - 1))
```

3.866.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.28

$$\int \cot(x)\sqrt{-1+\csc^2(x)}\left(1-\sin^2(x)\right)^3 \, dx =$$
$$-\frac{1}{48}\left(\left(2\left(4\sin(x)^2-19\right)\sin(x)^2+87\right)\sqrt{-\sin(x)^2+1}\sin(x)-105\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor-x\right)(-1)^{\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor}+\frac{2}{}\right.$$

input
```
integrate(cot(x)*(1-sin(x)^2)^3*(-1+csc(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/48*((2*(4*sin(x)^2 - 19)*sin(x)^2 + 87)*sqrt(-sin(x)^2 + 1)*sin(x) - 10
5*(pi*floor(x/pi + 1/2) - x)*(-1)^floor(x/pi + 1/2) + 24*(sqrt(-sin(x)^2 +
 1) - 1)/sin(x) - 24*sin(x)/(sqrt(-sin(x)^2 + 1) - 1))*sgn(sin(x))
```

3.866.9 Mupad [F(-1)]

Timed out.

$$\int \cot(x)\sqrt{-1+\csc^2(x)}\left(1-\sin^2(x)\right)^3 \, dx = \int -\cot(x)\sqrt{\frac{1}{\sin(x)^2}-1}\left(\sin(x)^2-1\right)^3 \, dx$$

input
```
int(-cot(x)*(1/sin(x)^2 - 1)^(1/2)*(sin(x)^2 - 1)^3,x)
```

output
```
int(-cot(x)*(1/sin(x)^2 - 1)^(1/2)*(sin(x)^2 - 1)^3, x)
```

3.867 $\int \cos(x)\sqrt{-1+\csc^2(x)}\left(1-\sin^2(x)\right)^3\,dx$

3.867.1 Optimal result

Integrand size = 23, antiderivative size = 81

$$\int \cos(x)\sqrt{-1+\csc^2(x)}\left(1-\sin^2(x)\right)^3\,dx = \sqrt{\cot^2(x)}\sin(x)+\frac{1}{3}\cos^2(x)\sqrt{\cot^2(x)}\sin(x)+\frac{1}{5}\cos^4(x)\sqrt{\cot^2(x)}\sin(x)+\frac{1}{7}\cos^6(x)\sqrt{\cot^2(x)}\sin(x)-\operatorname{arctanh}(\cos(x))\sqrt{\cot^2(x)}\tan(x)$$

output
```
sin(x)*(cot(x)^2)^(1/2)+1/3*cos(x)^2*sin(x)*(cot(x)^2)^(1/2)+1/5*cos(x)^4*
sin(x)*(cot(x)^2)^(1/2)+1/7*cos(x)^6*sin(x)*(cot(x)^2)^(1/2)-arctanh(cos(x
))*(cot(x)^2)^(1/2)*tan(x)
```

3.867.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.49

$$\int \cos(x)\sqrt{-1+\csc^2(x)}\left(1-\sin^2(x)\right)^3\,dx = -\frac{\sqrt{\cot^2(x)}\,\text{Hypergeometric2F1}\left(-\frac{7}{2},-\frac{7}{2},-\frac{5}{2},\csc^2(x)\right)\sin^7(x)}{7\sqrt{-\cot^2(x)}}$$

input
```
Integrate[Cos[x]*Sqrt[-1 + Csc[x]^2]*(1 - Sin[x]^2)^3,x]
```

output
```
-1/7*(Sqrt[Cot[x]^2]*Hypergeometric2F1[-7/2, -7/2, -5/2, Csc[x]^2]*Sin[x]^
7)/Sqrt[-Cot[x]^2]
```

3.867.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.54, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 3654, 3042, 4609, 3042, 4141, 3042, 25, 3072, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(1-\sin^2(x)\right)^3 \cos(x) \sqrt{\csc^2(x)-1} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(1-\sin(x)^2\right)^3 \cos(x) \sqrt{\csc(x)^2-1} dx$$

$$\Big\downarrow \text{3654}$$

$$\int \cos^7(x) \sqrt{\csc^2(x)-1} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(x+\frac{\pi}{2}\right)^7 \sqrt{\sec\left(x+\frac{\pi}{2}\right)^2-1} dx$$

$$\Big\downarrow \text{4609}$$

$$\int \cos^7(x) \sqrt{\cot^2(x)} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(x+\frac{\pi}{2}\right)^7 \sqrt{\tan\left(x+\frac{\pi}{2}\right)^2} dx$$

$$\Big\downarrow \text{4141}$$

$$\tan(x) \sqrt{\cot^2(x)} \int \cos^7(x) \cot(x) dx$$

$$\Big\downarrow \text{3042}$$

$$\tan(x)\sqrt{\cot^2(x)}\int -\sin\left(x+\frac{\pi}{2}\right)^7\tan\left(x+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{25}$$

$$\tan(x)\left(-\sqrt{\cot^2(x)}\right)\int \sin\left(x+\frac{\pi}{2}\right)^7\tan\left(x+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3072}$$

$$\tan(x)\left(-\sqrt{\cot^2(x)}\right)\int \frac{\cos^8(x)}{1-\cos^2(x)}d\cos(x)$$

$$\Big\downarrow \text{254}$$

$$\tan(x)\left(-\sqrt{\cot^2(x)}\right)\int \left(-\cos^6(x)-\cos^4(x)-\cos^2(x)+\frac{1}{1-\cos^2(x)}-1\right)d\cos(x)$$

$$\Big\downarrow \text{2009}$$

$$-\left(\tan(x)\sqrt{\cot^2(x)}\left(\operatorname{arctanh}(\cos(x))-\frac{1}{7}\cos^7(x)-\frac{\cos^5(x)}{5}-\frac{\cos^3(x)}{3}-\cos(x)\right)\right)$$

input
```
Int[Cos[x]*Sqrt[-1 + Csc[x]^2]*(1 - Sin[x]^2)^3,x]
```

output
```
-((ArcTanh[Cos[x]] - Cos[x] - Cos[x]^3/3 - Cos[x]^5/5 - Cos[x]^7/7)*Sqrt[C
ot[x]^2]*Tan[x])
```

3.867.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.867. $\int \cos(x)\sqrt{-1+\csc^2(x)}(1-\sin^2(x))^3\,dx$

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 4141
```
Int[(u_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Tan[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Tan[e + f*x]^
n)^FracPart[p]/(Tan[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Ta
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4609
```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(b*tan[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

3.867.4 Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.59

method	result
default	$\frac{\tan(x)\sqrt{\cot(x)^2}\left(15\cos(x)^7+21\cos(x)^5+35\cos(x)^3+105\cos(x)+105\ln(\csc(x)-\cot(x))+176\right)\sqrt{4}}{210}$
risch	$\frac{i\sqrt{-\frac{(e^{2ix}+1)^2}{(e^{2ix}-1)^2}}\ln(e^{ix}+1)e^{2ix}}{e^{2ix}+1}-\frac{i\sqrt{-\frac{(e^{2ix}+1)^2}{(e^{2ix}-1)^2}}\ln(e^{ix}-1)e^{2ix}}{e^{2ix}+1}-\frac{i\sqrt{-\frac{(e^{2ix}+1)^2}{(e^{2ix}-1)^2}}e^{9ix}}{896(e^{2ix}+1)}+\frac{121i\sqrt{-\frac{(e^{2ix}+1)^2}{(e^{2ix}-1)^2}}e^{-ix}}{192(e^{2ix}+1)}-$

input
```
int(cos(x)*(1-sin(x)^2)^3*(csc(x)^2-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/210*tan(x)*(cot(x)^2)^(1/2)*(15*cos(x)^7+21*cos(x)^5+35*cos(x)^3+105*cos
(x)+105*ln(csc(x)-cot(x))+176)*4^(1/2)
```

3.867.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.51

$$\int \cos(x)\sqrt{-1+\csc^2(x)}\left(1-\sin^2(x)\right)^3\,dx = -\frac{1}{7}\cos(x)^7 - \frac{1}{5}\cos(x)^5 - \frac{1}{3}\cos(x)^3 - \cos(x) + \frac{1}{2}\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right) - \frac{1}{2}\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)$$

input
```
integrate(cos(x)*(1-sin(x)^2)^3*(-1+csc(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/7*cos(x)^7 - 1/5*cos(x)^5 - 1/3*cos(x)^3 - cos(x) + 1/2*log(1/2*cos(x)
+ 1/2) - 1/2*log(-1/2*cos(x) + 1/2)
```

3.867.6 Sympy [F(-1)]

Timed out.

$$\int \cos(x)\sqrt{-1+\csc^2(x)}\left(1-\sin^2(x)\right)^3\,dx = \text{Timed out}$$

input
```
integrate(cos(x)*(1-sin(x)**2)**3*(-1+csc(x)**2)**(1/2),x)
```

output
```
Timed out
```

3.867.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.06

$$\int \cos(x)\sqrt{-1+\csc^2(x)}\left(1-\sin^2(x)\right)^3\,dx = \frac{1}{7}\left(\frac{1}{\sin(x)^2}-1\right)^{\frac{7}{2}}\sin(x)^7 + \frac{1}{5}\left(\frac{1}{\sin(x)^2}-1\right)^{\frac{5}{2}}\sin(x)^5 + \frac{1}{3}\left(\frac{1}{\sin(x)^2}-1\right)^{\frac{3}{2}}\sin(x)^3 + \sqrt{\frac{1}{\sin(x)^2}-1}\sin(x) - \frac{1}{2}\log\left(\sqrt{\frac{1}{\sin(x)^2}-1}\sin(x)+1\right) + \frac{1}{2}\log\left(\sqrt{\frac{1}{\sin(x)^2}-1}\sin(x)-1\right)$$

input
```
integrate(cos(x)*(1-sin(x)^2)^3*(-1+csc(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/7*(1/sin(x)^2 - 1)^(7/2)*sin(x)^7 + 1/5*(1/sin(x)^2 - 1)^(5/2)*sin(x)^5
+ 1/3*(1/sin(x)^2 - 1)^(3/2)*sin(x)^3 + sqrt(1/sin(x)^2 - 1)*sin(x) - 1/2*
log(sqrt(1/sin(x)^2 - 1)*sin(x) + 1) + 1/2*log(sqrt(1/sin(x)^2 - 1)*sin(x)
 - 1)
```

3.867.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.54

$$\int \cos(x)\sqrt{-1+\csc^2(x)}\left(1-\sin^2(x)\right)^3\,dx$$
$$= \frac{1}{210}\left(30\cos(x)^7 + 42\cos(x)^5 + 70\cos(x)^3 + 210\cos(x) - 105\log(\cos(x)+1) + 105\log(-\cos(x)+1\right.$$

input
```
integrate(cos(x)*(1-sin(x)^2)^3*(-1+csc(x)^2)^(1/2),x, algorithm="giac")
```

output
```
1/210*(30*cos(x)^7 + 42*cos(x)^5 + 70*cos(x)^3 + 210*cos(x) - 105*log(cos(
x) + 1) + 105*log(-cos(x) + 1))*sgn(sin(x))
```

3.867.9 Mupad [F(-1)]

Timed out.

$$\int \cos(x)\sqrt{-1+\csc^2(x)}\left(1-\sin^2(x)\right)^3\,dx = -\int \cos(x)\,\sqrt{\frac{1}{\sin(x)^2}-1}\,\left(\sin(x)^2-1\right)^3\,dx$$

input
```
int(-cos(x)*(1/sin(x)^2 - 1)^(1/2)*(sin(x)^2 - 1)^3,x)
```

output
```
-int(cos(x)*(1/sin(x)^2 - 1)^(1/2)*(sin(x)^2 - 1)^3, x)
```

3.868 $\int \frac{x\csc(x)\sec(x)}{\sqrt{a\sec^2(x)}}\,dx$

3.868.1 Optimal result

Integrand size = 16, antiderivative size = 76

$$\int \frac{x\csc(x)\sec(x)}{\sqrt{a\sec^2(x)}}\,dx = -\frac{2x\operatorname{arctanh}(e^{ix})\sec(x)}{\sqrt{a\sec^2(x)}} + \frac{i\operatorname{PolyLog}(2,-e^{ix})\sec(x)}{\sqrt{a\sec^2(x)}} - \frac{i\operatorname{PolyLog}(2,e^{ix})\sec(x)}{\sqrt{a\sec^2(x)}}$$

output
```
-2*x*arctanh(exp(I*x))*sec(x)/(a*sec(x)^2)^(1/2)+I*polylog(2,-exp(I*x))*se
c(x)/(a*sec(x)^2)^(1/2)-I*polylog(2,exp(I*x))*sec(x)/(a*sec(x)^2)^(1/2)
```

3.868.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.91

$$\int \frac{x\csc(x)\sec(x)}{\sqrt{a\sec^2(x)}}\,dx = \frac{(x(\log(1-e^{ix})-\log(1+e^{ix}))+i\operatorname{PolyLog}(2,-e^{ix})-i\operatorname{PolyLog}(2,e^{ix}))\sec(x)}{\sqrt{a\sec^2(x)}}$$

input
```
Integrate[(x*Csc[x]*Sec[x])/Sqrt[a*Sec[x]^2],x]
```

output
```
((x*(Log[1 - E^(I*x)] - Log[1 + E^(I*x)]) + I*PolyLog[2, -E^(I*x)] - I*Pol
yLog[2, E^(I*x)])*Sec[x])/Sqrt[a*Sec[x]^2]
```

3.868.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.70, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {7271, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\csc(x)\sec(x)}{\sqrt{a\sec^2(x)}}dx$$

$$\Big\downarrow \text{7271}$$

$$\frac{\sec(x)\int x\csc(x)dx}{\sqrt{a\sec^2(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sec(x)\int x\csc(x)dx}{\sqrt{a\sec^2(x)}}$$

$$\Big\downarrow \text{4671}$$

$$\frac{\sec(x)\left(-\int \log\left(1-e^{ix}\right)dx+\int \log\left(1+e^{ix}\right)dx-2x\text{arctanh}\left(e^{ix}\right)\right)}{\sqrt{a\sec^2(x)}}$$

$$\Big\downarrow \text{2715}$$

$$\frac{\sec(x)\left(i\int e^{-ix}\log\left(1-e^{ix}\right)de^{ix}-i\int e^{-ix}\log\left(1+e^{ix}\right)de^{ix}-2x\text{arctanh}\left(e^{ix}\right)\right)}{\sqrt{a\sec^2(x)}}$$

$$\Big\downarrow \text{2838}$$

$$\frac{\sec(x)\left(-2x\text{arctanh}\left(e^{ix}\right)+i\,\text{PolyLog}\left(2,-e^{ix}\right)-i\,\text{PolyLog}\left(2,e^{ix}\right)\right)}{\sqrt{a\sec^2(x)}}$$

input
```
Int[(x*Csc[x]*Sec[x])/Sqrt[a*Sec[x]^2],x]
```

output
```
((-2*x*ArcTanh[E^(I*x)] + I*PolyLog[2, -E^(I*x)] - I*PolyLog[2, E^(I*x)])*
Sec[x])/Sqrt[a*Sec[x]^2]
```

3.868.3.1 Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

3.868.4 Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.09

method	result	size
risch	$-\frac{2 i \,{\mathrm e}^{i x} \left(-\frac{i x \ln \left({\mathrm e}^{i x}+1\right)}{2}-\frac{\operatorname{polylog}\left(2,-{\mathrm e}^{i x}\right)}{2}+\frac{i x \ln \left(1-{\mathrm e}^{i x}\right)}{2}+\frac{\operatorname{polylog}\left(2,{\mathrm e}^{i x}\right)}{2}\right)}{\sqrt{\frac{a \,{\mathrm e}^{2 i x}}{\left({\mathrm e}^{2 i x}+1\right)^{2}}}\, \left({\mathrm e}^{2 i x}+1\right)}$	83

input
```
int(x*csc(x)*sec(x)/(a*sec(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*I/(a*exp(2*I*x)/(exp(2*I*x)+1)^2)^(1/2)/(exp(2*I*x)+1)*exp(I*x)*(-1/2*I
*x*ln(exp(I*x)+1)-1/2*polylog(2,-exp(I*x))+1/2*I*x*ln(1-exp(I*x))+1/2*poly
log(2,exp(I*x)))
```

3.868.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(55) = 110.

Time = 0.26 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.63

$$\int \frac{x\csc(x)\sec(x)}{\sqrt{a\sec^2(x)}}\,dx = -\frac{(x\cos(x)\log(\cos(x)+i\sin(x)+1)+x\cos(x)\log(\cos(x)-i\sin(x)+1)-x\cos(x)\log(-\cos(x)+\cdots}{\cdots}$$

input
```
integrate(x*csc(x)*sec(x)/(a*sec(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(x*cos(x)*log(cos(x) + I*sin(x) + 1) + x*cos(x)*log(cos(x) - I*sin(x)
 + 1) - x*cos(x)*log(-cos(x) + I*sin(x) + 1) - x*cos(x)*log(-cos(x) - I*si
n(x) + 1) + I*cos(x)*dilog(cos(x) + I*sin(x)) - I*cos(x)*dilog(cos(x) - I*
sin(x)) + I*cos(x)*dilog(-cos(x) + I*sin(x)) - I*cos(x)*dilog(-cos(x) - I*
sin(x)))*sqrt(a/cos(x)^2)/a
```

3.868.6 Sympy [F]

$$\int \frac{x\csc(x)\sec(x)}{\sqrt{a\sec^2(x)}}\,dx = \int \frac{x\csc(x)\sec(x)}{\sqrt{a\sec^2(x)}}\,dx$$

input
```
integrate(x*csc(x)*sec(x)/(a*sec(x)**2)**(1/2),x)
```

output
```
Integral(x*csc(x)*sec(x)/sqrt(a*sec(x)**2), x)
```

3.868.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.04

$$\int \frac{x\csc(x)\sec(x)}{\sqrt{a\sec^2(x)}}\,dx = -\frac{2i\,x\arctan\left(\sin\left(x\right),\cos\left(x\right)+1\right)+2i\,x\arctan\left(\sin\left(x\right),-\cos\left(x\right)+1\right)+x\log\left(\cos\left(x\right)^2+\sin\left(x\right)^2+2\,\text{co}\right.}{2\sqrt{a}}$$

input
```
integrate(x*csc(x)*sec(x)/(a*sec(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*(2*I*x*arctan2(sin(x), cos(x) + 1) + 2*I*x*arctan2(sin(x), -cos(x) +
1) + x*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) - x*log(cos(x)^2 + sin(x)^2
 - 2*cos(x) + 1) - 2*I*dilog(-e^(I*x)) + 2*I*dilog(e^(I*x)))/sqrt(a)
```

3.868.8 Giac [F]

$$\int \frac{x\csc(x)\sec(x)}{\sqrt{a\sec^2(x)}}\,dx = \int \frac{x\csc\left(x\right)\sec\left(x\right)}{\sqrt{a\sec\left(x\right)^2}}\,dx$$

input
```
integrate(x*csc(x)*sec(x)/(a*sec(x)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(x*csc(x)*sec(x)/sqrt(a*sec(x)^2), x)
```

3.868.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\csc(x)\sec(x)}{\sqrt{a\sec^2(x)}}\,dx = \int \frac{x}{\cos\left(x\right)\,\sin\left(x\right)\,\sqrt{\frac{a}{\cos(x)^2}}}\,dx$$

input
```
int(x/(cos(x)*sin(x)*(a/cos(x)^2)^(1/2)),x)
```

output
```
int(x/(cos(x)*sin(x)*(a/cos(x)^2)^(1/2)), x)
```

3.869 $\int \frac{x^2 \csc(x) \sec(x)}{\sqrt{a \sec^2(x)}} \, dx$

3.869.1 Optimal result

Integrand size = 18, antiderivative size = 128

$$\begin{aligned}\int \frac{x^2 \csc(x) \sec(x)}{\sqrt{a \sec^2(x)}} \, dx = & -\frac{2x^2 \operatorname{arctanh}(e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}} + \frac{2ix \operatorname{PolyLog}(2, -e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}} \\ & - \frac{2ix \operatorname{PolyLog}(2, e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}} \\ & - \frac{2 \operatorname{PolyLog}(3, -e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}} + \frac{2 \operatorname{PolyLog}(3, e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}}\end{aligned}$$

output
```
-2*x^2*arctanh(exp(I*x))*sec(x)/(a*sec(x)^2)^(1/2)+2*I*x*polylog(2,-exp(I*
x))*sec(x)/(a*sec(x)^2)^(1/2)-2*I*x*polylog(2,exp(I*x))*sec(x)/(a*sec(x)^2
)^(1/2)-2*polylog(3,-exp(I*x))*sec(x)/(a*sec(x)^2)^(1/2)+2*polylog(3,exp(I
*x))*sec(x)/(a*sec(x)^2)^(1/2)
```

3.869.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.77

$$\int \frac{x^2 \csc(x) \sec(x)}{\sqrt{a \sec^2(x)}} \, dx = \frac{(x^2 \log(1 - e^{ix}) - x^2 \log(1 + e^{ix}) + 2ix \operatorname{PolyLog}(2, -e^{ix}) - 2ix \operatorname{PolyLog}(2, e^{ix}) - 2 \operatorname{PolyLog}(3, -e^{ix}) +}{\sqrt{a \sec^2(x)}}$$

input
```
Integrate[(x^2*Csc[x]*Sec[x])/Sqrt[a*Sec[x]^2],x]
```

output
```
((x^2*Log[1 - E^(I*x)] - x^2*Log[1 + E^(I*x)] + (2*I)*x*PolyLog[2, -E^(I*x
)] - (2*I)*x*PolyLog[2, E^(I*x)] - 2*PolyLog[3, -E^(I*x)] + 2*PolyLog[3, E
^(I*x)])*Sec[x])/Sqrt[a*Sec[x]^2]
```

3.869.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.68, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {7271, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \csc(x) \sec(x)}{\sqrt{a \sec^2(x)}} \, dx$$

$$\Big\downarrow \text{ 7271}$$

$$\frac{\sec(x) \int x^2 \csc(x) dx}{\sqrt{a \sec^2(x)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sec(x) \int x^2 \csc(x) dx}{\sqrt{a \sec^2(x)}}$$

$$\Big\downarrow \text{ 4671}$$

$$\frac{\sec(x) \left(-2 \int x \log\left(1-e^{ix}\right) dx + 2 \int x \log\left(1+e^{ix}\right) dx - 2x^2 \text{arctanh}\left(e^{ix}\right)\right)}{\sqrt{a \sec^2(x)}}$$

$$\Big\downarrow \text{ 3011}$$

$$\frac{\sec(x) \left(2\left(ix \text{PolyLog}\left(2,-e^{ix}\right) - i \int \text{PolyLog}\left(2,-e^{ix}\right) dx\right) - 2\left(ix \text{PolyLog}\left(2,e^{ix}\right) - i \int \text{PolyLog}\left(2,e^{ix}\right) dx\right) - 2x\right.}{\sqrt{a \sec^2(x)}}$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{\sec(x) \left(2\left(ix \text{PolyLog}\left(2,-e^{ix}\right) - \int e^{-ix} \text{PolyLog}\left(2,-e^{ix}\right) de^{ix}\right) - 2\left(ix \text{PolyLog}\left(2,e^{ix}\right) - \int e^{-ix} \text{PolyLog}\left(2,e^{ix}\right)\right.\right.}{\sqrt{a \sec^2(x)}}$$

$$\Big\downarrow \text{ 7143}$$

$$\frac{\sec(x)\left(-2x^2\text{arctanh}(e^{ix})+2(ix\,\text{PolyLog}\left(2,-e^{ix}\right)-\text{PolyLog}\left(3,-e^{ix}\right))-2(ix\,\text{PolyLog}\left(2,e^{ix}\right)-\text{PolyLog}\left(3,e\right.\right.}{\sqrt{a\sec^2(x)}}$$

input
```
Int[(x^2*Csc[x]*Sec[x])/Sqrt[a*Sec[x]^2],x]
```

output
```
((-2*x^2*ArcTanh[E^(I*x)] + 2*(I*x*PolyLog[2, -E^(I*x)] - PolyLog[3, -E^(I
*x)]) - 2*(I*x*PolyLog[2, E^(I*x)] - PolyLog[3, E^(I*x)]))*Sec[x])/Sqrt[a*
Sec[x]^2]
```

3.869.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(Eq
Q[v, x] && EqQ[m, 1])
```

3.869.4 Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.83

method	result	size
risch	$-\frac{2\,\mathrm{e}^{ix}\left(\frac{x^{2}\ln\left(\mathrm{e}^{ix}+1\right)}{2}-ix\,\mathrm{polylog}\left(2,-\mathrm{e}^{ix}\right)+\mathrm{polylog}\left(3,-\mathrm{e}^{ix}\right)-\frac{x^{2}\ln\left(1-\mathrm{e}^{ix}\right)}{2}+ix\,\mathrm{polylog}\left(2,\mathrm{e}^{ix}\right)-\mathrm{polylog}\left(3,\mathrm{e}^{ix}\right)\right)}{\sqrt{\frac{a\,\mathrm{e}^{2ix}}{\left(\mathrm{e}^{2ix}+1\right)^{2}}}\,\left(\mathrm{e}^{2ix}+1\right)}$	106

input
```
int(x^2*csc(x)*sec(x)/(a*sec(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/(a*exp(2*I*x)/(exp(2*I*x)+1)^2)^(1/2)/(exp(2*I*x)+1)*exp(I*x)*(1/2*x^2*
ln(exp(I*x)+1)-I*x*polylog(2,-exp(I*x))+polylog(3,-exp(I*x))-1/2*x^2*ln(1-
exp(I*x))+I*x*polylog(2,exp(I*x))-polylog(3,exp(I*x)))
```

3.869.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 227 vs. 2(97) = 194.

Time = 0.28 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.77

$$\int \frac{x^2\csc(x)\sec(x)}{\sqrt{a\sec^2(x)}}\,dx$$
$$=\frac{2\sqrt{\frac{a}{\cos(x)^2}}\cos(x)\,\mathrm{polylog}(3,\cos(x)+i\sin(x))+2\sqrt{\frac{a}{\cos(x)^2}}\cos(x)\,\mathrm{polylog}(3,\cos(x)-i\sin(x))-2\sqrt{\frac{}{\mathrm{co}}}}{}$$

input
```
integrate(x^2*csc(x)*sec(x)/(a*sec(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(2*sqrt(a/cos(x)^2)*cos(x)*polylog(3, cos(x) + I*sin(x)) + 2*sqrt(a/co
s(x)^2)*cos(x)*polylog(3, cos(x) - I*sin(x)) - 2*sqrt(a/cos(x)^2)*cos(x)*p
olylog(3, -cos(x) + I*sin(x)) - 2*sqrt(a/cos(x)^2)*cos(x)*polylog(3, -cos(
x) - I*sin(x)) - (x^2*cos(x)*log(cos(x) + I*sin(x) + 1) + x^2*cos(x)*log(c
os(x) - I*sin(x) + 1) - x^2*cos(x)*log(-cos(x) + I*sin(x) + 1) - x^2*cos(x
)*log(-cos(x) - I*sin(x) + 1) + 2*I*x*cos(x)*dilog(cos(x) + I*sin(x)) - 2*
I*x*cos(x)*dilog(cos(x) - I*sin(x)) + 2*I*x*cos(x)*dilog(-cos(x) + I*sin(x
)) - 2*I*x*cos(x)*dilog(-cos(x) - I*sin(x)))*sqrt(a/cos(x)^2))/a
```

3.869.6 Sympy [F]

$$\int \frac{x^2 \csc(x) \sec(x)}{\sqrt{a \sec^2(x)}}\, dx = \int \frac{x^2 \csc{(x)} \sec{(x)}}{\sqrt{a \sec^2{(x)}}}\, dx$$

input
```
integrate(x**2*csc(x)*sec(x)/(a*sec(x)**2)**(1/2),x)
```

output
```
Integral(x**2*csc(x)*sec(x)/sqrt(a*sec(x)**2), x)
```

3.869.7 Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.84

$$\int \frac{x^2 \csc(x) \sec(x)}{\sqrt{a \sec^2(x)}}\, dx = -\frac{2i\, x^2 \arctan\left(\sin\left(x\right), \cos\left(x\right) + 1\right) + 2i\, x^2 \arctan\left(\sin\left(x\right), -\cos\left(x\right) + 1\right) + x^2 \log\left(\cos\left(x\right)^2 + \sin\left(x\right)^2 + 2\right.}{}$$

input
```
integrate(x^2*csc(x)*sec(x)/(a*sec(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*(2*I*x^2*arctan2(sin(x), cos(x) + 1) + 2*I*x^2*arctan2(sin(x), -cos(x
) + 1) + x^2*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) - x^2*log(cos(x)^2 +
sin(x)^2 - 2*cos(x) + 1) - 4*I*x*dilog(-e^(I*x)) + 4*I*x*dilog(e^(I*x)) +
4*polylog(3, -e^(I*x)) - 4*polylog(3, e^(I*x)))/sqrt(a)
```

3.869.8 Giac [F]

$$\int \frac{x^2 \csc(x) \sec(x)}{\sqrt{a \sec^2(x)}} \, dx = \int \frac{x^2 \csc(x) \sec(x)}{\sqrt{a \sec(x)^2}} \, dx$$

input
```
integrate(x^2*csc(x)*sec(x)/(a*sec(x)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2*csc(x)*sec(x)/sqrt(a*sec(x)^2), x)
```

3.869.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \csc(x) \sec(x)}{\sqrt{a \sec^2(x)}} \, dx = \int \frac{x^2}{\cos(x) \sin(x) \sqrt{\frac{a}{\cos(x)^2}}} \, dx$$

input
```
int(x^2/(cos(x)*sin(x)*(a/cos(x)^2)^(1/2)),x)
```

output
```
int(x^2/(cos(x)*sin(x)*(a/cos(x)^2)^(1/2)), x)
```

3.870 $\int \frac{x^3 \csc(x) \sec(x)}{\sqrt{a \sec^2(x)}} \, dx$

3.870.1 Optimal result

Integrand size = 18, antiderivative size = 186

$$\begin{aligned}\int \frac{x^3 \csc(x) \sec(x)}{\sqrt{a \sec^2(x)}} \, dx = &-\frac{2x^3 \operatorname{arctanh}(e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}} + \frac{3ix^2 \operatorname{PolyLog}(2, -e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}} \\ &- \frac{3ix^2 \operatorname{PolyLog}(2, e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}} \\ &- \frac{6x \operatorname{PolyLog}(3, -e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}} + \frac{6x \operatorname{PolyLog}(3, e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}} \\ &- \frac{6i \operatorname{PolyLog}(4, -e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}} + \frac{6i \operatorname{PolyLog}(4, e^{ix}) \sec(x)}{\sqrt{a \sec^2(x)}}\end{aligned}$$

output
```
-2*x^3*arctanh(exp(I*x))*sec(x)/(a*sec(x)^2)^(1/2)+3*I*x^2*polylog(2,-exp(
I*x))*sec(x)/(a*sec(x)^2)^(1/2)-3*I*x^2*polylog(2,exp(I*x))*sec(x)/(a*sec(
x)^2)^(1/2)-6*x*polylog(3,-exp(I*x))*sec(x)/(a*sec(x)^2)^(1/2)+6*x*polylog
(3,exp(I*x))*sec(x)/(a*sec(x)^2)^(1/2)-6*I*polylog(4,-exp(I*x))*sec(x)/(a*
sec(x)^2)^(1/2)+6*I*polylog(4,exp(I*x))*sec(x)/(a*sec(x)^2)^(1/2)
```

3.870.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.79

$$\int \frac{x^3 \csc(x) \sec(x)}{\sqrt{a \sec^2(x)}} \, dx = -\frac{i(\pi^4 - 2x^4 + 8ix^3 \log(1 - e^{-ix}) - 8ix^3 \log(1 + e^{ix}) - 24x^2 \operatorname{PolyLog}(2, e^{-ix}) - 24x^2 \operatorname{PolyLog}(2, -e^{ix})}{8\sqrt{a s}}$$

input
```
Integrate[(x^3*Csc[x]*Sec[x])/Sqrt[a*Sec[x]^2],x]
```

output
```
((-1/8*I)*(Pi^4 - 2*x^4 + (8*I)*x^3*Log[1 - E^((-I)*x)] - (8*I)*x^3*Log[1
+ E^(I*x)] - 24*x^2*PolyLog[2, E^((-I)*x)] - 24*x^2*PolyLog[2, -E^(I*x)] +
 (48*I)*x*PolyLog[3, E^((-I)*x)] - (48*I)*x*PolyLog[3, -E^(I*x)] + 48*Poly
Log[4, E^((-I)*x)] + 48*PolyLog[4, -E^(I*x)])*Sec[x])/Sqrt[a*Sec[x]^2]
```

3.870.3 Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.68, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {7271, 3042, 4671, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \csc(x) \sec(x)}{\sqrt{a \sec^2(x)}} \, dx$$

$$\Big\downarrow \text{ 7271}$$

$$\frac{\sec(x) \int x^3 \csc(x) dx}{\sqrt{a \sec^2(x)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sec(x) \int x^3 \csc(x) dx}{\sqrt{a \sec^2(x)}}$$

$$\Big\downarrow \text{ 4671}$$

$$\frac{\sec(x) \left(-3 \int x^2 \log(1 - e^{ix}) \, dx + 3 \int x^2 \log(1 + e^{ix}) \, dx - 2x^3 \operatorname{arctanh}(e^{ix})\right)}{\sqrt{a \sec^2(x)}}$$

$$\downarrow \text{3011}$$

$$\frac{\sec(x)\left(3\left(ix^2\operatorname{PolyLog}\left(2,-e^{ix}\right)-2i\int x\operatorname{PolyLog}\left(2,-e^{ix}\right)dx\right)-3\left(ix^2\operatorname{PolyLog}\left(2,e^{ix}\right)-2i\int x\operatorname{PolyLog}\left(2,e^{ix}\right)d\right.}{\sqrt{a\sec^2(x)}}$$

$$\downarrow \text{7163}$$

$$\frac{\sec(x)\left(3\left(ix^2\operatorname{PolyLog}\left(2,-e^{ix}\right)-2i\left(i\int\operatorname{PolyLog}\left(3,-e^{ix}\right)dx-ix\operatorname{PolyLog}\left(3,-e^{ix}\right)\right)\right)-3\left(ix^2\operatorname{PolyLog}\left(2,e^{ix}\right)-\right.\right.}{\sqrt{a\sec^2(x)}}$$

$$\downarrow \text{2720}$$

$$\frac{\sec(x)\left(3\left(ix^2\operatorname{PolyLog}\left(2,-e^{ix}\right)-2i\left(\int e^{-ix}\operatorname{PolyLog}\left(3,-e^{ix}\right)de^{ix}-ix\operatorname{PolyLog}\left(3,-e^{ix}\right)\right)\right)-3\left(ix^2\operatorname{PolyLog}\left(2,e\right.\right.\right.}{\sqrt{a\sec^2(x)}}$$

$$\downarrow \text{7143}$$

$$\frac{\sec(x)\left(-2x^3\operatorname{arctanh}\left(e^{ix}\right)+3\left(ix^2\operatorname{PolyLog}\left(2,-e^{ix}\right)-2i\left(\operatorname{PolyLog}\left(4,-e^{ix}\right)-ix\operatorname{PolyLog}\left(3,-e^{ix}\right)\right)\right)-3\left(ix^2\operatorname{Po}\right.\right.}{\sqrt{a\sec^2(x)}}$$

input
```
Int[(x^3*Csc[x]*Sec[x])/Sqrt[a*Sec[x]^2],x]
```

output
```
((-2*x^3*ArcTanh[E^(I*x)] + 3*(I*x^2*PolyLog[2, -E^(I*x)] - (2*I)*((-I)*x*
PolyLog[3, -E^(I*x)] + PolyLog[4, -E^(I*x)])) - 3*(I*x^2*PolyLog[2, E^(I*x
)] - (2*I)*((-I)*x*PolyLog[3, E^(I*x)] + PolyLog[4, E^(I*x)])))*Sec[x])/Sq
rt[a*Sec[x]^2]
```

3.870.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.870. $\int \frac{x^3\csc(x)\sec(x)}{\sqrt{a\sec^2(x)}}\,dx$

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(Eq
Q[v, x] && EqQ[m, 1])
```

3.870. $\int \frac{x^3 \csc(x) \sec(x)}{\sqrt{a \sec^2(x)}}\,dx$

3.870.4 Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.74

method	result
risch	$\frac{2i e^{ix}\left(\frac{i x^3 \ln\left(e^{ix}+1\right)}{2}+\frac{3x^2 \operatorname{polylog}\left(2,-e^{ix}\right)}{2}+3ix \operatorname{polylog}\left(3,-e^{ix}\right)-3\operatorname{polylog}\left(4,-e^{ix}\right)-\frac{i x^3 \ln\left(1-e^{ix}\right)}{2}-\frac{3x^2 \operatorname{polylog}\left(2,e^{ix}\right)}{2}-3ix \operatorname{polylog}\right.}{\sqrt{\frac{a\, e^{2ix}}{\left(e^{2ix}+1\right)^2}}\left(e^{2ix}+1\right)}$

input
```
int(x^3*csc(x)*sec(x)/(a*sec(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*I/(a*exp(2*I*x)/(exp(2*I*x)+1)^2)^(1/2)/(exp(2*I*x)+1)*exp(I*x)*(1/2*I*x
^3*ln(exp(I*x)+1)+3/2*x^2*polylog(2,-exp(I*x))+3*I*x*polylog(3,-exp(I*x))-
3*polylog(4,-exp(I*x))-1/2*I*x^3*ln(1-exp(I*x))-3/2*x^2*polylog(2,exp(I*x)
)-3*I*x*polylog(3,exp(I*x))+3*polylog(4,exp(I*x)))
```

3.870.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 327 vs. 2(141) = 282.

Time = 0.30 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.76

$$\int \frac{x^3 \csc(x)\sec(x)}{\sqrt{a\sec^2(x)}}\,dx$$
$$= \frac{6\,x\sqrt{\frac{a}{\cos(x)^2}}\cos(x)\operatorname{polylog}(3,\cos(x)+i\,\sin(x))+6\,x\sqrt{\frac{a}{\cos(x)^2}}\cos(x)\operatorname{polylog}(3,\cos(x)-i\,\sin(x))-6\,x}{}$$

input
```
integrate(x^3*csc(x)*sec(x)/(a*sec(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(6*x*sqrt(a/cos(x)^2)*cos(x)*polylog(3, cos(x) + I*sin(x)) + 6*x*sqrt(
a/cos(x)^2)*cos(x)*polylog(3, cos(x) - I*sin(x)) - 6*x*sqrt(a/cos(x)^2)*co
s(x)*polylog(3, -cos(x) + I*sin(x)) - 6*x*sqrt(a/cos(x)^2)*cos(x)*polylog(
3, -cos(x) - I*sin(x)) + 6*I*sqrt(a/cos(x)^2)*cos(x)*polylog(4, cos(x) + I
*sin(x)) - 6*I*sqrt(a/cos(x)^2)*cos(x)*polylog(4, cos(x) - I*sin(x)) + 6*I
*sqrt(a/cos(x)^2)*cos(x)*polylog(4, -cos(x) + I*sin(x)) - 6*I*sqrt(a/cos(x
)^2)*cos(x)*polylog(4, -cos(x) - I*sin(x)) - (x^3*cos(x)*log(cos(x) + I*si
n(x) + 1) + x^3*cos(x)*log(cos(x) - I*sin(x) + 1) - x^3*cos(x)*log(-cos(x)
 + I*sin(x) + 1) - x^3*cos(x)*log(-cos(x) - I*sin(x) + 1) + 3*I*x^2*cos(x)
*dilog(cos(x) + I*sin(x)) - 3*I*x^2*cos(x)*dilog(cos(x) - I*sin(x)) + 3*I*
x^2*cos(x)*dilog(-cos(x) + I*sin(x)) - 3*I*x^2*cos(x)*dilog(-cos(x) - I*si
n(x)))*sqrt(a/cos(x)^2))/a
```

3.870.6 Sympy [F]

$$\int \frac{x^3 \csc(x) \sec(x)}{\sqrt{a \sec^2(x)}}\, dx = \int \frac{x^3 \csc(x) \sec(x)}{\sqrt{a \sec^2(x)}}\, dx$$

input
```
integrate(x**3*csc(x)*sec(x)/(a*sec(x)**2)**(1/2),x)
```

output
```
Integral(x**3*csc(x)*sec(x)/sqrt(a*sec(x)**2), x)
```

3.870.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.70

$$\int \frac{x^3 \csc(x) \sec(x)}{\sqrt{a \sec^2(x)}}\, dx = -\frac{2i\, x^3 \arctan(\sin(x), \cos(x) + 1) + 2i\, x^3 \arctan(\sin(x), -\cos(x) + 1) + x^3 \log\left(\cos(x)^2 + \sin(x)^2 + 2\right.}{}$$

input
```
integrate(x^3*csc(x)*sec(x)/(a*sec(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*(2*I*x^3*arctan2(sin(x), cos(x) + 1) + 2*I*x^3*arctan2(sin(x), -cos(x
) + 1) + x^3*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) - x^3*log(cos(x)^2 +
sin(x)^2 - 2*cos(x) + 1) - 6*I*x^2*dilog(-e^(I*x)) + 6*I*x^2*dilog(e^(I*x)
) + 12*x*polylog(3, -e^(I*x)) - 12*x*polylog(3, e^(I*x)) + 12*I*polylog(4,
 -e^(I*x)) - 12*I*polylog(4, e^(I*x)))/sqrt(a)
```

3.870.8 Giac [F]

$$\int \frac{x^3 \csc(x) \sec(x)}{\sqrt{a \sec^2(x)}} \, dx = \int \frac{x^3 \csc(x) \sec(x)}{\sqrt{a \sec(x)^2}} \, dx$$

input
```
integrate(x^3*csc(x)*sec(x)/(a*sec(x)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^3*csc(x)*sec(x)/sqrt(a*sec(x)^2), x)
```

3.870.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \csc(x) \sec(x)}{\sqrt{a \sec^2(x)}} \, dx = \int \frac{x^3}{\cos(x) \sin(x) \sqrt{\frac{a}{\cos(x)^2}}} \, dx$$

input
```
int(x^3/(cos(x)*sin(x)*(a/cos(x)^2)^(1/2)),x)
```

output
```
int(x^3/(cos(x)*sin(x)*(a/cos(x)^2)^(1/2)), x)
```

3.871 $\int \frac{x \csc(x) \sec(x)}{\sqrt{a \sec^4(x)}} \, dx$

3.871.1 Optimal result

Integrand size = 16, antiderivative size = 81

$$\int \frac{x \csc(x) \sec(x)}{\sqrt{a \sec^4(x)}} \, dx = -\frac{ix^2 \sec^2(x)}{2\sqrt{a \sec^4(x)}} + \frac{x \log\left(1 - e^{2ix}\right) \sec^2(x)}{\sqrt{a \sec^4(x)}} - \frac{i \operatorname{PolyLog}\left(2, e^{2ix}\right) \sec^2(x)}{2\sqrt{a \sec^4(x)}}$$

output
```
-1/2*I*x^2*sec(x)^2/(a*sec(x)^4)^(1/2)+x*ln(1-exp(2*I*x))*sec(x)^2/(a*sec(
x)^4)^(1/2)-1/2*I*polylog(2,exp(2*I*x))*sec(x)^2/(a*sec(x)^4)^(1/2)
```

3.871.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.62

$$\int \frac{x \csc(x) \sec(x)}{\sqrt{a \sec^4(x)}} \, dx = -\frac{i\left(x\left(x + 2i \log\left(1 - e^{2ix}\right)\right) + \operatorname{PolyLog}\left(2, e^{2ix}\right)\right) \sec^2(x)}{2\sqrt{a \sec^4(x)}}$$

input
```
Integrate[(x*Csc[x]*Sec[x])/Sqrt[a*Sec[x]^4],x]
```

output
```
((-1/2*I)*(x*(x + (2*I)*Log[1 - E^((2*I)*x)]) + PolyLog[2, E^((2*I)*x)])*S
ec[x]^2)/Sqrt[a*Sec[x]^4]
```

3.871.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.78, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {7271, 3042, 25, 4200, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \csc(x) \sec(x)}{\sqrt{a \sec^4(x)}} \, dx$$

$$\Big\downarrow \text{ 7271}$$

$$\frac{\sec^2(x) \int x \cot(x) dx}{\sqrt{a \sec^4(x)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sec^2(x) \int -x \tan\left(x+\frac{\pi}{2}\right) dx}{\sqrt{a \sec^4(x)}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\sec^2(x) \int x \tan\left(x+\frac{\pi}{2}\right) dx}{\sqrt{a \sec^4(x)}}$$

$$\Big\downarrow \text{ 4200}$$

$$-\frac{\sec^2(x) \left(\frac{ix^2}{2}-2i \int -\frac{e^{2ix} x}{1-e^{2ix}} dx\right)}{\sqrt{a \sec^4(x)}}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\sec^2(x) \left(2i \int \frac{e^{2ix} x}{1-e^{2ix}} dx+\frac{ix^2}{2}\right)}{\sqrt{a \sec^4(x)}}$$

$$\Big\downarrow \text{ 2620}$$

$$-\frac{\sec^2(x) \left(2i\left(\frac{1}{2} i x \log\left(1-e^{2ix}\right)-\frac{1}{2} i \int \log\left(1-e^{2ix}\right) dx\right)+\frac{ix^2}{2}\right)}{\sqrt{a \sec^4(x)}}$$

$$\Big\downarrow \text{ 2715}$$

$$-\frac{\sec^2(x) \left(2i\left(\frac{1}{2} i x \log\left(1-e^{2ix}\right)-\frac{1}{4} \int e^{-2ix} \log\left(1-e^{2ix}\right) de^{2ix}\right)+\frac{ix^2}{2}\right)}{\sqrt{a \sec^4(x)}}$$

$$\Big\downarrow \text{ 2838}$$

$$-\frac{\sec^2(x)\left(2i\left(\frac{1}{4}\operatorname{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{2}ix\log\left(1-e^{2ix}\right)\right)+\frac{ix^2}{2}\right)}{\sqrt{a\sec^4(x)}}$$

input
```
Int[(x*Csc[x]*Sec[x])/Sqrt[a*Sec[x]^4],x]
```

output
```
-((((I/2)*x^2 + (2*I)*((I/2)*x*Log[1 - E^((2*I)*x)] + PolyLog[2, E^((2*I)*
x)]/4))*Sec[x]^2)/Sqrt[a*Sec[x]^4])
```

3.871.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4200
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

3.871. $\int \frac{x\csc(x)\sec(x)}{\sqrt{a\sec^4(x)}}\,dx$

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

3.871.4 Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.57

method	result	size
risch	$\frac{i \mathrm{e}^{2 i x} x^{2}}{2 \sqrt{\frac{a \,\mathrm{e}^{4 i x}}{\left(\mathrm{e}^{2 i x}+1\right)^{4}}}\, \left(\mathrm{e}^{2 i x}+1\right)^{2}}-\frac{2 i \mathrm{e}^{2 i x} \left(\frac{x^{2}}{2}+\frac{i x \ln \left(\mathrm{e}^{i x}+1\right)}{2}+\frac{\operatorname{polylog}\left(2,-\mathrm{e}^{i x}\right)}{2}+\frac{i x \ln \left(1-\mathrm{e}^{i x}\right)}{2}+\frac{\operatorname{polylog}\left(2,\mathrm{e}^{i x}\right)}{2}\right)}{\sqrt{\frac{a \,\mathrm{e}^{4 i x}}{\left(\mathrm{e}^{2 i x}+1\right)^{4}}}\, \left(\mathrm{e}^{2 i x}+1\right)^{2}}$	127

input
```
int(x*csc(x)*sec(x)/(a*sec(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*I/(a*exp(4*I*x)/(exp(2*I*x)+1)^4)^(1/2)/(exp(2*I*x)+1)^2*exp(2*I*x)*x^
2-2*I/(a*exp(4*I*x)/(exp(2*I*x)+1)^4)^(1/2)/(exp(2*I*x)+1)^2*exp(2*I*x)*(1
/2*x^2+1/2*I*x*ln(exp(I*x)+1)+1/2*polylog(2,-exp(I*x))+1/2*I*x*ln(1-exp(I*
x))+1/2*polylog(2,exp(I*x)))
```

3.871.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(60) = 120.

Time = 0.28 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.70

$$\int \frac{x \csc(x) \sec(x)}{\sqrt{a \sec^4(x)}}\, dx = \frac{\left(x \cos\left(x\right)^{2} \log\left(\cos\left(x\right) + i\, \sin\left(x\right) + 1\right) + x \cos\left(x\right)^{2} \log\left(\cos\left(x\right) - i\, \sin\left(x\right) + 1\right) + x \cos\left(x\right)^{2} \log\left(-\cos\left(x\right) \cdots\right.}{\cdots}$$

input
```
integrate(x*csc(x)*sec(x)/(a*sec(x)^4)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(x*cos(x)^2*log(cos(x) + I*sin(x) + 1) + x*cos(x)^2*log(cos(x) - I*sin
(x) + 1) + x*cos(x)^2*log(-cos(x) + I*sin(x) + 1) + x*cos(x)^2*log(-cos(x)
 - I*sin(x) + 1) - I*cos(x)^2*dilog(cos(x) + I*sin(x)) + I*cos(x)^2*dilog(
cos(x) - I*sin(x)) + I*cos(x)^2*dilog(-cos(x) + I*sin(x)) - I*cos(x)^2*dil
og(-cos(x) - I*sin(x)))*sqrt(a/cos(x)^4)/a
```

3.871.6 Sympy [F]

$$\int \frac{x\csc(x)\sec(x)}{\sqrt{a\sec^4(x)}}\,dx = \int \frac{x\csc(x)\sec(x)}{\sqrt{a\sec^4(x)}}\,dx$$

input
```
integrate(x*csc(x)*sec(x)/(a*sec(x)**4)**(1/2),x)
```

output
```
Integral(x*csc(x)*sec(x)/sqrt(a*sec(x)**4), x)
```

3.871.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.02

$$\int \frac{x\csc(x)\sec(x)}{\sqrt{a\sec^4(x)}}\,dx = \frac{-i\,x^2 + 2i\,x\arctan(\sin(x),\cos(x)+1) - 2i\,x\arctan(\sin(x),-\cos(x)+1) + x\log\left(\cos(x)^2+\sin(x)^2 + \dots\right.}{2\sqrt{a}}$$

input
```
integrate(x*csc(x)*sec(x)/(a*sec(x)^4)^(1/2),x, algorithm="maxima")
```

output
```
1/2*(-I*x^2 + 2*I*x*arctan2(sin(x), cos(x) + 1) - 2*I*x*arctan2(sin(x), -c
os(x) + 1) + x*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + x*log(cos(x)^2 +
sin(x)^2 - 2*cos(x) + 1) - 2*I*dilog(-e^(I*x)) - 2*I*dilog(e^(I*x)))/sqrt(
a)
```

3.871.8 Giac [F]

$$\int \frac{x\csc(x)\sec(x)}{\sqrt{a\sec^4(x)}}\,dx = \int \frac{x\csc(x)\sec(x)}{\sqrt{a\sec(x)^4}}\,dx$$

input
```
integrate(x*csc(x)*sec(x)/(a*sec(x)^4)^(1/2),x, algorithm="giac")
```

output
```
integrate(x*csc(x)*sec(x)/sqrt(a*sec(x)^4), x)
```

3.871.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\csc(x)\sec(x)}{\sqrt{a\sec^4(x)}}\,dx = \int \frac{x}{\cos(x)\,\sin(x)\,\sqrt{\frac{a}{\cos(x)^4}}}\,dx$$

input
```
int(x/(cos(x)*sin(x)*(a/cos(x)^4)^(1/2)),x)
```

output
```
int(x/(cos(x)*sin(x)*(a/cos(x)^4)^(1/2)), x)
```

3.872 $\int \frac{x^2 \csc(x)\sec(x)}{\sqrt{a\sec^4(x)}}\,dx$

3.872.1 Optimal result

Integrand size = 18, antiderivative size = 109

$$\int \frac{x^2\csc(x)\sec(x)}{\sqrt{a\sec^4(x)}}\,dx = -\frac{ix^3\sec^2(x)}{3\sqrt{a\sec^4(x)}}+\frac{x^2\log\left(1-e^{2ix}\right)\sec^2(x)}{\sqrt{a\sec^4(x)}} - \frac{ix\,\text{PolyLog}\left(2,e^{2ix}\right)\sec^2(x)}{\sqrt{a\sec^4(x)}}+\frac{\text{PolyLog}\left(3,e^{2ix}\right)\sec^2(x)}{2\sqrt{a\sec^4(x)}}$$

output
```
-1/3*I*x^3*sec(x)^2/(a*sec(x)^4)^(1/2)+x^2*ln(1-exp(2*I*x))*sec(x)^2/(a*se
c(x)^4)^(1/2)-I*x*polylog(2,exp(2*I*x))*sec(x)^2/(a*sec(x)^4)^(1/2)+1/2*po
lylog(3,exp(2*I*x))*sec(x)^2/(a*sec(x)^4)^(1/2)
```

3.872.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.69

$$\int \frac{x^2\csc(x)\sec(x)}{\sqrt{a\sec^4(x)}}\,dx = \frac{\left(-i\pi^3+8ix^3+24x^2\log\left(1-e^{-2ix}\right)+24ix\,\text{PolyLog}\left(2,e^{-2ix}\right)+12\,\text{PolyLog}\left(3,e^{-2ix}\right)\right)\sec^2(x)}{24\sqrt{a\sec^4(x)}}$$

input
```
Integrate[(x^2*Csc[x]*Sec[x])/Sqrt[a*Sec[x]^4],x]
```

output
```
(((-I)*Pi^3 + (8*I)*x^3 + 24*x^2*Log[1 - E^((-2*I)*x)] + (24*I)*x*PolyLog[
2, E^((-2*I)*x)] + 12*PolyLog[3, E^((-2*I)*x)])*Sec[x]^2)/(24*Sqrt[a*Sec[x
]^4])
```

3.872.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.79, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {7271, 3042, 25, 4200, 25, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \csc(x)\sec(x)}{\sqrt{a\sec^4(x)}}dx$$

$$\Big\downarrow \text{7271}$$

$$\frac{\sec^2(x)\int x^2\cot(x)dx}{\sqrt{a\sec^4(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sec^2(x)\int -x^2\tan\left(x+\frac{\pi}{2}\right)dx}{\sqrt{a\sec^4(x)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sec^2(x)\int x^2\tan\left(x+\frac{\pi}{2}\right)dx}{\sqrt{a\sec^4(x)}}$$

$$\Big\downarrow \text{4200}$$

$$-\frac{\sec^2(x)\left(\frac{ix^3}{3}-2i\int -\frac{e^{2ix}x^2}{1-e^{2ix}}dx\right)}{\sqrt{a\sec^4(x)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sec^2(x)\left(2i\int \frac{e^{2ix}x^2}{1-e^{2ix}}dx+\frac{ix^3}{3}\right)}{\sqrt{a\sec^4(x)}}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{\sec^2(x)\left(2i\left(\frac{1}{2}ix^2\log\left(1-e^{2ix}\right)-i\int x\log\left(1-e^{2ix}\right)dx\right)+\frac{ix^3}{3}\right)}{\sqrt{a\sec^4(x)}}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{\sec^2(x)\left(2i\left(\frac{1}{2}ix^2\log\left(1-e^{2ix}\right)-i\left(\frac{1}{2}ix\operatorname{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{2}i\int\operatorname{PolyLog}\left(2,e^{2ix}\right)dx\right)\right)+\frac{ix^3}{3}\right)}{\sqrt{a\sec^4(x)}}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{\sec^2(x)\left(2i\left(\frac{1}{2}ix^2\log\left(1-e^{2ix}\right)-i\left(\frac{1}{2}ix\operatorname{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\operatorname{PolyLog}\left(2,e^{2ix}\right)de^{2ix}\right)\right)+\frac{ix^3}{3}\right)}{\sqrt{a\sec^4(x)}}$$

$$\Big\downarrow \text{7143}$$

$$-\frac{\sec^2(x)\left(2i\left(\frac{1}{2}ix^2\log\left(1-e^{2ix}\right)-i\left(\frac{1}{2}ix\operatorname{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{4}\operatorname{PolyLog}\left(3,e^{2ix}\right)\right)\right)+\frac{ix^3}{3}\right)}{\sqrt{a\sec^4(x)}}$$

input
```
Int[(x^2*Csc[x]*Sec[x])/Sqrt[a*Sec[x]^4],x]
```

output
```
-((((I/3)*x^3 + (2*I)*((I/2)*x^2*Log[1 - E^((2*I)*x)] - I*((I/2)*x*PolyLog
[2, E^((2*I)*x)] - PolyLog[3, E^((2*I)*x)]/4)))*Sec[x]^2)/Sqrt[a*Sec[x]^4]
)
```

3.872.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

3.872. $\int \frac{x^2\csc(x)\sec(x)}{\sqrt{a\sec^4(x)}}\,dx$

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4200
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(Eq
Q[v, x] && EqQ[m, 1])
```

3.872.4 Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.40

method	result
risch	$\frac{i e^{2ix} x^3}{3\sqrt{\frac{a\,e^{4ix}}{\left(e^{2ix}+1\right)^4}}\left(e^{2ix}+1\right)^2} - \frac{2\,e^{2ix}\left(\frac{ix^3}{3}-\frac{x^2\ln\left(e^{ix}+1\right)}{2}+ix\,\mathrm{polylog}\left(2,-e^{ix}\right)-\mathrm{polylog}\left(3,-e^{ix}\right)-\frac{x^2\ln\left(1-e^{ix}\right)}{2}+ix\,\mathrm{polylog}\left(2,e^{ix}\right)-\mathrm{po}\right.}{\sqrt{\frac{a\,e^{4ix}}{\left(e^{2ix}+1\right)^4}}\left(e^{2ix}+1\right)^2}$

input
```
int(x^2*csc(x)*sec(x)/(a*sec(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*I/(a*exp(4*I*x)/(exp(2*I*x)+1)^4)^(1/2)/(exp(2*I*x)+1)^2*exp(2*I*x)*x^
3-2/(a*exp(4*I*x)/(exp(2*I*x)+1)^4)^(1/2)/(exp(2*I*x)+1)^2*exp(2*I*x)*(1/3
*I*x^3-1/2*x^2*ln(exp(I*x)+1)+I*x*polylog(2,-exp(I*x))-polylog(3,-exp(I*x)
)-1/2*x^2*ln(1-exp(I*x))+I*x*polylog(2,exp(I*x))-polylog(3,exp(I*x)))
```

3.872.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 248 vs. 2(83) = 166.

Time = 0.27 (sec) , antiderivative size = 248, normalized size of antiderivative = 2.28

$$\int \frac{x^2 \csc(x) \sec(x)}{\sqrt{a \sec^4(x)}} \, dx$$
$$= \frac{2 \sqrt{\frac{a}{\cos(x)^4}} \cos(x)^2 \operatorname{polylog}(3, \cos(x) + i \sin(x)) + 2 \sqrt{\frac{a}{\cos(x)^4}} \cos(x)^2 \operatorname{polylog}(3, \cos(x) - i \sin(x)) + 2 \sqrt{}}{}$$

input
```
integrate(x^2*csc(x)*sec(x)/(a*sec(x)^4)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(2*sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, cos(x) + I*sin(x)) + 2*sqrt(a/
cos(x)^4)*cos(x)^2*polylog(3, cos(x) - I*sin(x)) + 2*sqrt(a/cos(x)^4)*cos(
x)^2*polylog(3, -cos(x) + I*sin(x)) + 2*sqrt(a/cos(x)^4)*cos(x)^2*polylog(
3, -cos(x) - I*sin(x)) + (x^2*cos(x)^2*log(cos(x) + I*sin(x) + 1) + x^2*co
s(x)^2*log(cos(x) - I*sin(x) + 1) + x^2*cos(x)^2*log(-cos(x) + I*sin(x) +
1) + x^2*cos(x)^2*log(-cos(x) - I*sin(x) + 1) - 2*I*x*cos(x)^2*dilog(cos(x
) + I*sin(x)) + 2*I*x*cos(x)^2*dilog(cos(x) - I*sin(x)) + 2*I*x*cos(x)^2*d
ilog(-cos(x) + I*sin(x)) - 2*I*x*cos(x)^2*dilog(-cos(x) - I*sin(x)))*sqrt(
a/cos(x)^4))/a
```

3.872.6 Sympy [F]

$$\int \frac{x^2 \csc(x) \sec(x)}{\sqrt{a \sec^4(x)}} \, dx = \int \frac{x^2 \csc(x) \sec(x)}{\sqrt{a \sec^4(x)}} \, dx$$

input
```
integrate(x**2*csc(x)*sec(x)/(a*sec(x)**4)**(1/2),x)
```

output
```
Integral(x**2*csc(x)*sec(x)/sqrt(a*sec(x)**4), x)
```

3.872.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.04

$$\int \frac{x^2 \csc(x) \sec(x)}{\sqrt{a \sec^4(x)}} \, dx = \frac{-2i x^3 + 6i x^2 \arctan(\sin(x), \cos(x) + 1) - 6i x^2 \arctan(\sin(x), -\cos(x) + 1) + 3 x^2 \log(\cos(x)^2 + \sin$$

input
```
integrate(x^2*csc(x)*sec(x)/(a*sec(x)^4)^(1/2),x, algorithm="maxima")
```

output
```
1/6*(-2*I*x^3 + 6*I*x^2*arctan2(sin(x), cos(x) + 1) - 6*I*x^2*arctan2(sin(
x), -cos(x) + 1) + 3*x^2*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 3*x^2*l
og(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) - 12*I*x*dilog(-e^(I*x)) - 12*I*x*d
ilog(e^(I*x)) + 12*polylog(3, -e^(I*x)) + 12*polylog(3, e^(I*x)))/sqrt(a)
```

3.872.8 Giac [F]

$$\int \frac{x^2 \csc(x) \sec(x)}{\sqrt{a \sec^4(x)}} \, dx = \int \frac{x^2 \csc(x) \sec(x)}{\sqrt{a \sec(x)^4}} \, dx$$

input
```
integrate(x^2*csc(x)*sec(x)/(a*sec(x)^4)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2*csc(x)*sec(x)/sqrt(a*sec(x)^4), x)
```

3.872.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \csc(x) \sec(x)}{\sqrt{a \sec^4(x)}} \, dx = \int \frac{x^2}{\cos(x) \sin(x) \sqrt{\frac{a}{\cos(x)^4}}} \, dx$$

input
```
int(x^2/(cos(x)*sin(x)*(a/cos(x)^4)^(1/2)),x)
```

output
```
int(x^2/(cos(x)*sin(x)*(a/cos(x)^4)^(1/2)), x)
```

3.873 $\int \frac{x^3 \csc(x)\sec(x)}{\sqrt{a \sec^4(x)}}\,dx$

3.873.1 Optimal result

Integrand size = 18, antiderivative size = 143

$$\int \frac{x^3 \csc(x)\sec(x)}{\sqrt{a \sec^4(x)}}\,dx = -\frac{i x^4 \sec^2(x)}{4\sqrt{a \sec^4(x)}} + \frac{x^3 \log\left(1-e^{2ix}\right)\sec^2(x)}{\sqrt{a \sec^4(x)}} - \frac{3 i x^2 \operatorname{PolyLog}\left(2,e^{2ix}\right)\sec^2(x)}{2\sqrt{a \sec^4(x)}} + \frac{3x \operatorname{PolyLog}\left(3,e^{2ix}\right)\sec^2(x)}{2\sqrt{a \sec^4(x)}} + \frac{3i \operatorname{PolyLog}\left(4,e^{2ix}\right)\sec^2(x)}{4\sqrt{a \sec^4(x)}}$$

output
```
-1/4*I*x^4*sec(x)^2/(a*sec(x)^4)^(1/2)+x^3*ln(1-exp(2*I*x))*sec(x)^2/(a*se
c(x)^4)^(1/2)-3/2*I*x^2*polylog(2,exp(2*I*x))*sec(x)^2/(a*sec(x)^4)^(1/2)+
3/2*x*polylog(3,exp(2*I*x))*sec(x)^2/(a*sec(x)^4)^(1/2)+3/4*I*polylog(4,ex
p(2*I*x))*sec(x)^2/(a*sec(x)^4)^(1/2)
```

3.873.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.61

$$\int \frac{x^3 \csc(x)\sec(x)}{\sqrt{a \sec^4(x)}}\,dx = -\frac{i\left(\pi^4 - 16x^4 + 64 i x^3 \log\left(1-e^{-2ix}\right) - 96x^2 \operatorname{PolyLog}\left(2,e^{-2ix}\right) + 96 i x \operatorname{PolyLog}\left(3,e^{-2ix}\right) + 48 \operatorname{PolyLog}\left(4\right.\right.}{64\sqrt{a \sec^4(x)}}$$

input
```
Integrate[(x^3*Csc[x]*Sec[x])/Sqrt[a*Sec[x]^4],x]
```

output
```
((-1/64*I)*(Pi^4 - 16*x^4 + (64*I)*x^3*Log[1 - E^((-2*I)*x)] - 96*x^2*Poly
Log[2, E^((-2*I)*x)] + (96*I)*x*PolyLog[3, E^((-2*I)*x)] + 48*PolyLog[4, E
^((-2*I)*x)])*Sec[x]^2)/Sqrt[a*Sec[x]^4]
```

3.873.3 Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.78, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {7271, 3042, 25, 4200, 25, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \csc(x)\sec(x)}{\sqrt{a\sec^4(x)}}dx$$

$$\Big\downarrow \text{7271}$$

$$\frac{\sec^2(x)\int x^3\cot(x)dx}{\sqrt{a\sec^4(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sec^2(x)\int -x^3\tan\left(x+\frac{\pi}{2}\right)dx}{\sqrt{a\sec^4(x)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sec^2(x)\int x^3\tan\left(x+\frac{\pi}{2}\right)dx}{\sqrt{a\sec^4(x)}}$$

$$\Big\downarrow \text{4200}$$

$$-\frac{\sec^2(x)\left(\frac{ix^4}{4}-2i\int -\frac{e^{2ix}x^3}{1-e^{2ix}}dx\right)}{\sqrt{a\sec^4(x)}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\sec^2(x)\left(2i\int \frac{e^{2ix}x^3}{1-e^{2ix}}dx+\frac{ix^4}{4}\right)}{\sqrt{a\sec^4(x)}}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{\sec^2(x)\left(2i\left(\frac{1}{2}ix^3\log\left(1-e^{2ix}\right)-\frac{3}{2}i\int x^2\log\left(1-e^{2ix}\right)dx\right)+\frac{ix^4}{4}\right)}{\sqrt{a\sec^4(x)}}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{\sec^2(x)\left(2i\left(\frac{1}{2}ix^3\log\left(1-e^{2ix}\right)-\frac{3}{2}i\left(\frac{1}{2}ix^2\,\text{PolyLog}\left(2,e^{2ix}\right)-i\int x\,\text{PolyLog}\left(2,e^{2ix}\right)dx\right)\right)+\frac{ix^4}{4}\right)}{\sqrt{a\sec^4(x)}}$$

$$\Big\downarrow \text{7163}$$

$$-\frac{\sec^2(x)\left(2i\left(\frac{1}{2}ix^3\log\left(1-e^{2ix}\right)-\frac{3}{2}i\left(\frac{1}{2}ix^2\,\text{PolyLog}\left(2,e^{2ix}\right)-i\left(\frac{1}{2}i\int \text{PolyLog}\left(3,e^{2ix}\right)dx-\frac{1}{2}ix\,\text{PolyLog}\left(3,e^{2ix}\right)\right.\right.\right.\right.}{\sqrt{a\sec^4(x)}}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{\sec^2(x)\left(2i\left(\frac{1}{2}ix^3\log\left(1-e^{2ix}\right)-\frac{3}{2}i\left(\frac{1}{2}ix^2\,\text{PolyLog}\left(2,e^{2ix}\right)-i\left(\frac{1}{4}\int e^{-2ix}\,\text{PolyLog}\left(3,e^{2ix}\right)de^{2ix}-\frac{1}{2}ix\,\text{PolyLog}\left(\right.\right.\right.\right.\right.}{\sqrt{a\sec^4(x)}}$$

$$\Big\downarrow \text{7143}$$

$$-\frac{\sec^2(x)\left(2i\left(\frac{1}{2}ix^3\log\left(1-e^{2ix}\right)-\frac{3}{2}i\left(\frac{1}{2}ix^2\,\text{PolyLog}\left(2,e^{2ix}\right)-i\left(\frac{1}{4}\text{PolyLog}\left(4,e^{2ix}\right)-\frac{1}{2}ix\,\text{PolyLog}\left(3,e^{2ix}\right)\right)\right)\right)+\right.}{\sqrt{a\sec^4(x)}}$$

input
```
Int[(x^3*Csc[x]*Sec[x])/Sqrt[a*Sec[x]^4],x]
```

output
```
-((((I/4)*x^4 + (2*I)*((I/2)*x^3*Log[1 - E^((2*I)*x)] - ((3*I)/2)*((I/2)*x
^2*PolyLog[2, E^((2*I)*x)] - I*((-1/2*I)*x*PolyLog[3, E^((2*I)*x)] + PolyL
og[4, E^((2*I)*x)]/4))))*Sec[x]^2)/Sqrt[a*Sec[x]^4])
```

3.873. $\int \frac{x^3\csc(x)\sec(x)}{\sqrt{a\sec^4(x)}}\,dx$

3.873.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4200
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.873. $\int \frac{x^3 \csc(x) \sec(x)}{\sqrt{a \sec^4(x)}} \, dx$

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

3.873.4 Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.27

method	result
risch	$\frac{i e^{2ix} x^4}{4\sqrt{\frac{a\,e^{4ix}}{\left(e^{2ix}+1\right)^4}}\left(e^{2ix}+1\right)^2}+\frac{2i e^{2ix}\left(-\frac{x^4}{4}-\frac{i x^3 \ln\left(e^{ix}+1\right)}{2}-\frac{3x^2\,\mathrm{polylog}\left(2,-e^{ix}\right)}{2}-3ix\,\mathrm{polylog}\left(3,-e^{ix}\right)+3\,\mathrm{polylog}\left(4,-e^{ix}\right)-\frac{i x^3 \ln\left(1-e\right.}{2}\right.}{\sqrt{\frac{a\,e^{4ix}}{\left(e^{2ix}+1\right)^4}}\left(e^{2ix}+1\right)^2}$

input
```
int(x^3*csc(x)*sec(x)/(a*sec(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*I/(a*exp(4*I*x)/(exp(2*I*x)+1)^4)^(1/2)/(exp(2*I*x)+1)^2*exp(2*I*x)*x^
4+2*I/(a*exp(4*I*x)/(exp(2*I*x)+1)^4)^(1/2)/(exp(2*I*x)+1)^2*exp(2*I*x)*(-
1/4*x^4-1/2*I*x^3*ln(exp(I*x)+1)-3/2*x^2*polylog(2,-exp(I*x))-3*I*x*polylo
g(3,-exp(I*x))+3*polylog(4,-exp(I*x))-1/2*I*x^3*ln(1-exp(I*x))-3/2*x^2*pol
ylog(2,exp(I*x))-3*I*x*polylog(3,exp(I*x))+3*polylog(4,exp(I*x)))
```

3.873.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 356 vs. 2(106) = 212.

Time = 0.28 (sec) , antiderivative size = 356, normalized size of antiderivative = 2.49

$$\int \frac{x^3 \csc(x)\sec(x)}{\sqrt{a\sec^4(x)}}\,dx$$
$$= \frac{6\,x\sqrt{\frac{a}{\cos(x)^4}}\cos(x)^2\,\mathrm{polylog}(3,\cos(x)+i\,\sin(x))+6\,x\sqrt{\frac{a}{\cos(x)^4}}\cos(x)^2\,\mathrm{polylog}(3,\cos(x)-i\,\sin(x))+6}{}$$

input
```
integrate(x^3*csc(x)*sec(x)/(a*sec(x)^4)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(6*x*sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, cos(x) + I*sin(x)) + 6*x*sqr
t(a/cos(x)^4)*cos(x)^2*polylog(3, cos(x) - I*sin(x)) + 6*x*sqrt(a/cos(x)^4
)*cos(x)^2*polylog(3, -cos(x) + I*sin(x)) + 6*x*sqrt(a/cos(x)^4)*cos(x)^2*
polylog(3, -cos(x) - I*sin(x)) + 6*I*sqrt(a/cos(x)^4)*cos(x)^2*polylog(4,
cos(x) + I*sin(x)) - 6*I*sqrt(a/cos(x)^4)*cos(x)^2*polylog(4, cos(x) - I*s
in(x)) - 6*I*sqrt(a/cos(x)^4)*cos(x)^2*polylog(4, -cos(x) + I*sin(x)) + 6*
I*sqrt(a/cos(x)^4)*cos(x)^2*polylog(4, -cos(x) - I*sin(x)) + (x^3*cos(x)^2
*log(cos(x) + I*sin(x) + 1) + x^3*cos(x)^2*log(cos(x) - I*sin(x) + 1) + x^
3*cos(x)^2*log(-cos(x) + I*sin(x) + 1) + x^3*cos(x)^2*log(-cos(x) - I*sin(
x) + 1) - 3*I*x^2*cos(x)^2*dilog(cos(x) + I*sin(x)) + 3*I*x^2*cos(x)^2*dil
og(cos(x) - I*sin(x)) + 3*I*x^2*cos(x)^2*dilog(-cos(x) + I*sin(x)) - 3*I*x
^2*cos(x)^2*dilog(-cos(x) - I*sin(x)))*sqrt(a/cos(x)^4))/a
```

3.873.6 Sympy [F]

$$\int \frac{x^3 \csc(x)\sec(x)}{\sqrt{a\sec^4(x)}}\,dx = \int \frac{x^3 \csc(x)\sec(x)}{\sqrt{a\sec^4(x)}}\,dx$$

input
```
integrate(x**3*csc(x)*sec(x)/(a*sec(x)**4)**(1/2),x)
```

output
```
Integral(x**3*csc(x)*sec(x)/sqrt(a*sec(x)**4), x)
```

3.873.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.96

$$\int \frac{x^3 \csc(x) \sec(x)}{\sqrt{a \sec^4(x)}} \, dx$$
$$= \frac{-i\, x^4 + 4i\, x^3 \arctan(\sin(x), \cos(x) + 1) - 4i\, x^3 \arctan(\sin(x), -\cos(x) + 1) + 2\, x^3 \log(\cos(x)^2 + \sin(x)^2 + \cdots}{\cdots}$$

input
```
integrate(x^3*csc(x)*sec(x)/(a*sec(x)^4)^(1/2),x, algorithm="maxima")
```

output
```
1/4*(-I*x^4 + 4*I*x^3*arctan2(sin(x), cos(x) + 1) - 4*I*x^3*arctan2(sin(x)
, -cos(x) + 1) + 2*x^3*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 2*x^3*log
(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) - 12*I*x^2*dilog(-e^(I*x)) - 12*I*x^2
*dilog(e^(I*x)) + 24*x*polylog(3, -e^(I*x)) + 24*x*polylog(3, e^(I*x)) + 2
4*I*polylog(4, -e^(I*x)) + 24*I*polylog(4, e^(I*x)))/sqrt(a)
```

3.873.8 Giac [F]

$$\int \frac{x^3 \csc(x) \sec(x)}{\sqrt{a \sec^4(x)}} \, dx = \int \frac{x^3 \csc(x) \sec(x)}{\sqrt{a \sec(x)^4}} \, dx$$

input
```
integrate(x^3*csc(x)*sec(x)/(a*sec(x)^4)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^3*csc(x)*sec(x)/sqrt(a*sec(x)^4), x)
```

3.873.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \csc(x) \sec(x)}{\sqrt{a \sec^4(x)}} \, dx = \int \frac{x^3}{\cos(x) \sin(x) \sqrt{\frac{a}{\cos(x)^4}}} \, dx$$

input
```
int(x^3/(cos(x)*sin(x)*(a/cos(x)^4)^(1/2)),x)
```

output
```
int(x^3/(cos(x)*sin(x)*(a/cos(x)^4)^(1/2)), x)
```

3.874 $\int x\csc(x)\sec(x)\sqrt{a\sec^2(x)}\,dx$

3.874.1 Optimal result

Integrand size = 16, antiderivative size = 105

$$\begin{aligned}\int x\csc(x)\sec(x)\sqrt{a\sec^2(x)}\,dx &= x\sqrt{a\sec^2(x)}-2x\text{arctanh}\left(e^{ix}\right)\cos(x)\sqrt{a\sec^2(x)}\\&\quad-\text{arctanh}(\sin(x))\cos(x)\sqrt{a\sec^2(x)}\\&\quad+i\cos(x)\,\text{PolyLog}\left(2,-e^{ix}\right)\sqrt{a\sec^2(x)}\\&\quad-i\cos(x)\,\text{PolyLog}\left(2,e^{ix}\right)\sqrt{a\sec^2(x)}\end{aligned}$$

output
```
x*(a*sec(x)^2)^(1/2)-2*x*arctanh(exp(I*x))*cos(x)*(a*sec(x)^2)^(1/2)-arcta
nh(sin(x))*cos(x)*(a*sec(x)^2)^(1/2)+I*cos(x)*polylog(2,-exp(I*x))*(a*sec(
x)^2)^(1/2)-I*cos(x)*polylog(2,exp(I*x))*(a*sec(x)^2)^(1/2)
```

3.874.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.03

$$\begin{aligned}\int x\csc(x)\sec(x)\sqrt{a\sec^2(x)}\,dx &= \Big(x+x\cos(x)\left(\log\left(1-e^{ix}\right)-\log\left(1+e^{ix}\right)\right)\\&\quad+\cos(x)\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)\\&\quad-\cos(x)\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\\&\quad+i\cos(x)\left(\text{PolyLog}\left(2,-e^{ix}\right)\right.\\&\quad\left.-\text{PolyLog}\left(2,e^{ix}\right)\right)\Big)\sqrt{a\sec^2(x)}\end{aligned}$$

input
```
Integrate[x*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^2],x]
```

output
```
(x + x*Cos[x]*(Log[1 - E^(I*x)] - Log[1 + E^(I*x)]) + Cos[x]*Log[Cos[x/2]
- Sin[x/2]] - Cos[x]*Log[Cos[x/2] + Sin[x/2]] + I*Cos[x]*(PolyLog[2, -E^(I
*x)] - PolyLog[2, E^(I*x)]))*Sqrt[a*Sec[x]^2]
```

3.874.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.59, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {7271, 4920, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\csc(x)\sec(x)\sqrt{a\sec^2(x)}\,dx$$

$$\Big\downarrow \text{7271}$$

$$\cos(x)\sqrt{a\sec^2(x)}\int x\csc(x)\sec^2(x)dx$$

$$\Big\downarrow \text{4920}$$

$$\cos(x)\sqrt{a\sec^2(x)}(-\int(\sec(x)-\text{arctanh}(\cos(x)))dx-x\text{arctanh}(\cos(x))+x\sec(x))$$

$$\Big\downarrow \text{2009}$$

$$\cos(x)\sqrt{a\sec^2(x)}\left(-2x\text{arctanh}\left(e^{ix}\right)-\text{arctanh}(\sin(x))+i\,\text{PolyLog}\left(2,-e^{ix}\right)-i\,\text{PolyLog}\left(2,e^{ix}\right)+x\sec(x)\right)$$

input
```
Int[x*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^2],x]
```

output
```
Cos[x]*Sqrt[a*Sec[x]^2]*(-2*x*ArcTanh[E^(I*x)] - ArcTanh[Sin[x]] + I*PolyL
og[2, -E^(I*x)] - I*PolyLog[2, E^(I*x)] + x*Sec[x])
```

3.874.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4920
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(Eq
Q[v, x] && EqQ[m, 1])
```

3.874.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 289 vs. 2(87) = 174.

Time = 1.12 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.76

method	result
risch	$2\sqrt{\frac{a\,\mathrm{e}^{2ix}}{\left(\mathrm{e}^{2ix}+1\right)^{2}}}\,x+4\sqrt{\frac{a\,\mathrm{e}^{2ix}}{\left(\mathrm{e}^{2ix}+1\right)^{2}}}\left(i\left(\frac{\ln\left(\mathrm{e}^{ix}-1\right)}{4}+\frac{\arctan\left(\mathrm{e}^{ix}\right)}{2}-\frac{\ln\left(\mathrm{e}^{ix}+1\right)}{4}\right)-i\left(-\frac{\mathrm{dilog}\left(\mathrm{e}^{ix}\right)}{4}-\frac{x\ln\left(1+i\mathrm{e}^{ix}\right)}{4}+\right.\right.$

input
```
int(x*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(a*exp(2*I*x)/(exp(2*I*x)+1)^2)^(1/2)*x+4*(a*exp(2*I*x)/(exp(2*I*x)+1)^2
)^(1/2)*(I*(1/4*ln(exp(I*x)-1)+1/2*arctan(exp(I*x))-1/4*ln(exp(I*x)+1))-I*
(-1/4*dilog(exp(I*x))-1/4*x*ln(1+I*exp(I*x))+1/4*x*ln(1-I*exp(I*x))+1/4*I*
dilog(1+I*exp(I*x))-1/4*I*dilog(1-I*exp(I*x))-1/4*dilog(exp(I*x)+1)-1/4*I*
x*ln(exp(I*x)+1))-I*(-1/4*dilog(exp(I*x))+1/4*x*ln(1+I*exp(I*x))-1/4*x*ln(
1-I*exp(I*x))-1/4*I*dilog(1+I*exp(I*x))+1/4*I*dilog(1-I*exp(I*x))-1/4*dilo
g(exp(I*x)+1)-1/4*I*x*ln(exp(I*x)+1))-I*(1/4*ln(exp(I*x)-1)-1/2*arctan(exp
(I*x))-1/4*ln(exp(I*x)+1)))*cos(x)
```

3.874.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.33

$$\int x\csc(x)\sec(x)\sqrt{a\sec^2(x)}\,dx =$$
$$-\frac{1}{2}\Big(x\cos(x)\log(\cos(x)+i\sin(x)+1)+x\cos(x)\log(\cos(x)-i\sin(x)+1)-x\cos(x)\log(-\cos(x$$

input
```
integrate(x*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*(x*cos(x)*log(cos(x) + I*sin(x) + 1) + x*cos(x)*log(cos(x) - I*sin(x)
 + 1) - x*cos(x)*log(-cos(x) + I*sin(x) + 1) - x*cos(x)*log(-cos(x) - I*si
n(x) + 1) + I*cos(x)*dilog(cos(x) + I*sin(x)) - I*cos(x)*dilog(cos(x) - I*
sin(x)) + I*cos(x)*dilog(-cos(x) + I*sin(x)) - I*cos(x)*dilog(-cos(x) - I*
sin(x)) + cos(x)*log(-(sin(x) + 1)/(sin(x) - 1)) - 2*x)*sqrt(a/cos(x)^2)
```

3.874.6 Sympy [F]

$$\int x\csc(x)\sec(x)\sqrt{a\sec^2(x)}\,dx = \int x\sqrt{a\sec^2(x)}\csc(x)\sec(x)\,dx$$

input
```
integrate(x*csc(x)*sec(x)*(a*sec(x)**2)**(1/2),x)
```

output
```
Integral(x*sqrt(a*sec(x)**2)*csc(x)*sec(x), x)
```

3.874.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(80) = 160.

Time = 0.31 (sec) , antiderivative size = 294, normalized size of antiderivative = 2.80

$$\int x\csc(x)\sec(x)\sqrt{a\sec^2(x)}\,dx$$
$$= \frac{(2(\cos(2x)+i\sin(2x)+1)\arctan(\cos(x),\sin(x)+1)+2(\cos(2x)+i\sin(2x)+1)\arctan(\cos(x),}{}$$

input
```
integrate(x*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
(2*(cos(2*x) + I*sin(2*x) + 1)*arctan2(cos(x), sin(x) + 1) + 2*(cos(2*x) +
 I*sin(2*x) + 1)*arctan2(cos(x), -sin(x) + 1) - 2*(x*cos(2*x) + I*x*sin(2*
x) + x)*arctan2(sin(x), cos(x) + 1) - 2*(x*cos(2*x) + I*x*sin(2*x) + x)*ar
ctan2(sin(x), -cos(x) + 1) - 4*I*x*cos(x) + 2*(cos(2*x) + I*sin(2*x) + 1)*
dilog(-e^(I*x)) - 2*(cos(2*x) + I*sin(2*x) + 1)*dilog(e^(I*x)) - (-I*x*cos
(2*x) + x*sin(2*x) - I*x)*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) - (I*x*c
os(2*x) - x*sin(2*x) + I*x)*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) - (-I*
cos(2*x) + sin(2*x) - I)*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - (I*cos(
2*x) - sin(2*x) + I)*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1) + 4*x*sin(x))
*sqrt(a)/(-2*I*cos(2*x) + 2*sin(2*x) - 2*I)
```

3.874.8 Giac [F]

$$\int x\csc(x)\sec(x)\sqrt{a\sec^2(x)}\,dx = \int \sqrt{a\sec(x)^2}x\csc(x)\sec(x)\,dx$$

input
```
integrate(x*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*sec(x)^2)*x*csc(x)*sec(x), x)
```

3.874.9 Mupad [F(-1)]

Timed out.

$$\int x\csc(x)\sec(x)\sqrt{a\sec^2(x)}\,dx = \int \frac{x\sqrt{\frac{a}{\cos(x)^2}}}{\cos(x)\,\sin(x)}\,dx$$

input
```
int((x*(a/cos(x)^2)^(1/2))/(cos(x)*sin(x)),x)
```

output
```
int((x*(a/cos(x)^2)^(1/2))/(cos(x)*sin(x)), x)
```

3.875 $\int x^2 \csc(x) \sec(x) \sqrt{a \sec^2(x)}\, dx$

3.875.1 Optimal result

Integrand size = 18, antiderivative size = 225

$$\begin{aligned}\int x^2 \csc(x) \sec(x) \sqrt{a \sec^2(x)}\, dx &= x^2 \sqrt{a \sec^2(x)} + 4ix \arctan\left(e^{ix}\right) \cos(x) \sqrt{a \sec^2(x)} \\ &\quad - 2x^2 \text{arctanh}\left(e^{ix}\right) \cos(x) \sqrt{a \sec^2(x)} \\ &\quad + 2ix \cos(x) \text{PolyLog}\left(2, -e^{ix}\right) \sqrt{a \sec^2(x)} \\ &\quad - 2i \cos(x) \text{PolyLog}\left(2, -ie^{ix}\right) \sqrt{a \sec^2(x)} \\ &\quad + 2i \cos(x) \text{PolyLog}\left(2, ie^{ix}\right) \sqrt{a \sec^2(x)} \\ &\quad - 2ix \cos(x) \text{PolyLog}\left(2, e^{ix}\right) \sqrt{a \sec^2(x)} \\ &\quad - 2 \cos(x) \text{PolyLog}\left(3, -e^{ix}\right) \sqrt{a \sec^2(x)} \\ &\quad + 2 \cos(x) \text{PolyLog}\left(3, e^{ix}\right) \sqrt{a \sec^2(x)}\end{aligned}$$

output
```
x^2*(a*sec(x)^2)^(1/2)+4*I*x*arctan(exp(I*x))*cos(x)*(a*sec(x)^2)^(1/2)-2*
x^2*arctanh(exp(I*x))*cos(x)*(a*sec(x)^2)^(1/2)+2*I*x*cos(x)*polylog(2,-ex
p(I*x))*(a*sec(x)^2)^(1/2)-2*I*cos(x)*polylog(2,-I*exp(I*x))*(a*sec(x)^2)^
(1/2)+2*I*cos(x)*polylog(2,I*exp(I*x))*(a*sec(x)^2)^(1/2)-2*I*x*cos(x)*pol
ylog(2,exp(I*x))*(a*sec(x)^2)^(1/2)-2*cos(x)*polylog(3,-exp(I*x))*(a*sec(x
)^2)^(1/2)+2*cos(x)*polylog(3,exp(I*x))*(a*sec(x)^2)^(1/2)
```

3.875.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.77

$$\int x^2\csc(x)\sec(x)\sqrt{a\sec^2(x)}\,dx = \left(x^2+x^2\cos(x)\left(\log\left(1-e^{ix}\right)-\log\left(1+e^{ix}\right)\right)\right. \\ -2\cos(x)\left(x\left(\log\left(1-ie^{ix}\right)-\log\left(1+ie^{ix}\right)\right)\right. \\ \left.+i\left(\text{PolyLog}\left(2,-ie^{ix}\right)-\text{PolyLog}\left(2,ie^{ix}\right)\right)\right) \\ +2ix\cos(x)\left(\text{PolyLog}\left(2,-e^{ix}\right)-\text{PolyLog}\left(2,e^{ix}\right)\right) \\ +2\cos(x)\left(-\text{PolyLog}\left(3,-e^{ix}\right)\right. \\ \left.\left.+\text{PolyLog}\left(3,e^{ix}\right)\right)\right)\sqrt{a\sec^2(x)}$$

input
```
Integrate[x^2*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^2],x]
```

output
```
(x^2 + x^2*Cos[x]*(Log[1 - E^(I*x)] - Log[1 + E^(I*x)]) - 2*Cos[x]*(x*(Log
[1 - I*E^(I*x)] - Log[1 + I*E^(I*x)]) + I*(PolyLog[2, (-I)*E^(I*x)] - Poly
Log[2, I*E^(I*x)])) + (2*I)*x*Cos[x]*(PolyLog[2, -E^(I*x)] - PolyLog[2, E^
(I*x)]) + 2*Cos[x]*(-PolyLog[3, -E^(I*x)] + PolyLog[3, E^(I*x)]))*Sqrt[a*S
ec[x]^2]
```

3.875.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.68, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {7271, 4920, 25, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\csc(x)\sec(x)\sqrt{a\sec^2(x)}\,dx$$

$$\Big\downarrow \text{7271}$$

$$\cos(x)\sqrt{a\sec^2(x)}\int x^2\csc(x)\sec^2(x)dx$$

$$\Big\downarrow \text{4920}$$

$$\cos(x)\sqrt{a\sec^2(x)}\left(-2\int -x(\text{arctanh}(\cos(x))-\sec(x))dx+x^2(-\text{arctanh}(\cos(x)))+x^2\sec(x)\right)$$

$$\Big\downarrow \text{25}$$

$$\cos(x)\sqrt{a\sec^2(x)}\left(2\int x(\operatorname{arctanh}(\cos(x))-\sec(x))dx+x^2(-\operatorname{arctanh}(\cos(x)))+x^2\sec(x)\right)$$

$$\Big\downarrow \text{ 2010}$$

$$\cos(x)\sqrt{a\sec^2(x)}\left(2\int (x\operatorname{arctanh}(\cos(x))-x\sec(x))dx+x^2(-\operatorname{arctanh}(\cos(x)))+x^2\sec(x)\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\cos(x)\sqrt{a\sec^2(x)}\left(2\left(2ix\arctan\left(e^{ix}\right)+x^2\left(-\operatorname{arctanh}\left(e^{ix}\right)\right)+\frac{1}{2}x^2\operatorname{arctanh}(\cos(x))+ix\operatorname{PolyLog}\left(2,-e^{ix}\right)-ix\operatorname{Po}\right.\right.$$

input
```
Int[x^2*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^2],x]
```

output
```
Cos[x]*Sqrt[a*Sec[x]^2]*(-(x^2*ArcTanh[Cos[x]]) + 2*((2*I)*x*ArcTan[E^(I*x
)] - x^2*ArcTanh[E^(I*x)] + (x^2*ArcTanh[Cos[x]])/2 + I*x*PolyLog[2, -E^(I
*x)] - I*PolyLog[2, (-I)*E^(I*x)] + I*PolyLog[2, I*E^(I*x)] - I*x*PolyLog[
2, E^(I*x)] - PolyLog[3, -E^(I*x)] + PolyLog[3, E^(I*x)]) + x^2*Sec[x])
```

3.875.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 4920
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

3.875.4 Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.89

method	result
risch	$2\sqrt{\frac{a\,\mathrm{e}^{2ix}}{(\mathrm{e}^{2ix}+1)^2}}\,x^2-4i\sqrt{\frac{a\,\mathrm{e}^{2ix}}{(\mathrm{e}^{2ix}+1)^2}}\left(2i\left(\frac{x\ln(1+i\mathrm{e}^{ix})}{2}-\frac{x\ln(1-i\mathrm{e}^{ix})}{2}-\frac{i\,\mathrm{dilog}(1+i\mathrm{e}^{ix})}{2}+\frac{i\,\mathrm{dilog}(1-i\mathrm{e}^{ix})}{2}\right)-\frac{i\left(-\frac{ix^3}{3}\right.}{}\right.$

input
```
int(x^2*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(a*exp(2*I*x)/(exp(2*I*x)+1)^2)^(1/2)*x^2-4*I*(a*exp(2*I*x)/(exp(2*I*x)+
1)^2)^(1/2)*(2*I*(1/2*x*ln(1+I*exp(I*x))-1/2*x*ln(1-I*exp(I*x))-1/2*I*dilo
g(1+I*exp(I*x))+1/2*I*dilog(1-I*exp(I*x)))-1/2*I*(-1/3*I*x^3+x^2*ln(exp(I*
x)+1)-2*I*x*polylog(2,-exp(I*x))+2*polylog(3,-exp(I*x)))-1/2*I*(1/3*I*x^3-
x^2*ln(1-exp(I*x))+2*I*x*polylog(2,exp(I*x))-2*polylog(3,exp(I*x))))*cos(x
)
```

3.875.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 337 vs. 2(165) = 330.

Time = 0.30 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.50

$$\int x^2\csc(x)\sec(x)\sqrt{a\sec^2(x)}\,dx=\sqrt{\frac{a}{\cos(x)^2}}\cos(x)\,\mathrm{polylog}(3,\cos(x)+i\sin(x))$$
$$+\sqrt{\frac{a}{\cos(x)^2}}\cos(x)\,\mathrm{polylog}(3,\cos(x)-i\sin(x))$$
$$-\sqrt{\frac{a}{\cos(x)^2}}\cos(x)\,\mathrm{polylog}(3,-\cos(x)+i\sin(x))$$
$$-\sqrt{\frac{a}{\cos(x)^2}}\cos(x)\,\mathrm{polylog}(3,-\cos(x)-i\sin(x))$$
$$-\frac{1}{2}\left(x^2\cos(x)\log(\cos(x)+i\sin(x)+1)+x^2\cos(x)\log(\cos(x)-i\sin(x)+1)-x^2\cos(x)\log(-\cos\right.$$

input
```
integrate(x^2*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(a/cos(x)^2)*cos(x)*polylog(3, cos(x) + I*sin(x)) + sqrt(a/cos(x)^2)*c
os(x)*polylog(3, cos(x) - I*sin(x)) - sqrt(a/cos(x)^2)*cos(x)*polylog(3, -
cos(x) + I*sin(x)) - sqrt(a/cos(x)^2)*cos(x)*polylog(3, -cos(x) - I*sin(x)
) - 1/2*(x^2*cos(x)*log(cos(x) + I*sin(x) + 1) + x^2*cos(x)*log(cos(x) - I
*sin(x) + 1) - x^2*cos(x)*log(-cos(x) + I*sin(x) + 1) - x^2*cos(x)*log(-co
s(x) - I*sin(x) + 1) + 2*I*x*cos(x)*dilog(cos(x) + I*sin(x)) - 2*I*x*cos(x
)*dilog(cos(x) - I*sin(x)) + 2*I*x*cos(x)*dilog(-cos(x) + I*sin(x)) - 2*I*
x*cos(x)*dilog(-cos(x) - I*sin(x)) + 2*x*cos(x)*log(I*cos(x) + sin(x) + 1)
 - 2*x*cos(x)*log(I*cos(x) - sin(x) + 1) + 2*x*cos(x)*log(-I*cos(x) + sin(
x) + 1) - 2*x*cos(x)*log(-I*cos(x) - sin(x) + 1) - 2*x^2 - 2*I*cos(x)*dilo
g(I*cos(x) + sin(x)) - 2*I*cos(x)*dilog(I*cos(x) - sin(x)) + 2*I*cos(x)*di
log(-I*cos(x) + sin(x)) + 2*I*cos(x)*dilog(-I*cos(x) - sin(x)))*sqrt(a/cos
(x)^2)
```

3.875.6 Sympy [F]

$$\int x^2 \csc(x)\sec(x)\sqrt{a\sec^2(x)}\,dx = \int x^2 \sqrt{a\sec^2{(x)}}\csc{(x)}\sec{(x)}\,dx$$

input
```
integrate(x**2*csc(x)*sec(x)*(a*sec(x)**2)**(1/2),x)
```

output
```
Integral(x**2*sqrt(a*sec(x)**2)*csc(x)*sec(x), x)
```

3.875.7 Maxima [F]

$$\int x^2 \csc(x)\sec(x)\sqrt{a\sec^2(x)}\,dx = \int \sqrt{a\sec{(x)}^2}x^2\csc{(x)}\sec{(x)}\,dx$$

input
```
integrate(x^2*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-(4*I*x^2*cos(x) - 4*x^2*sin(x) + 2*(x^2*cos(2*x) + I*x^2*sin(2*x) + x^2)*
arctan2(sin(x), cos(x) + 1) + 2*(x^2*cos(2*x) + I*x^2*sin(2*x) + x^2)*arct
an2(sin(x), -cos(x) + 1) - 4*(x*cos(2*x) + I*x*sin(2*x) + x)*dilog(-e^(I*x
)) + 4*(x*cos(2*x) + I*x*sin(2*x) + x)*dilog(e^(I*x)) - 8*(I*cos(2*x) - si
n(2*x) + I)*integrate((x*cos(2*x)*cos(x) + x*sin(2*x)*sin(x) + x*cos(x))/(
cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1), x) - 8*(cos(2*x) + I*sin(2*x) +
 1)*integrate((x*cos(x)*sin(2*x) - x*cos(2*x)*sin(x) - x*sin(x))/(cos(2*x)
^2 + sin(2*x)^2 + 2*cos(2*x) + 1), x) + (-I*x^2*cos(2*x) + x^2*sin(2*x) -
I*x^2)*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + (I*x^2*cos(2*x) - x^2*sin
(2*x) + I*x^2)*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) - 4*(I*cos(2*x) - s
in(2*x) + I)*polylog(3, -e^(I*x)) - 4*(-I*cos(2*x) + sin(2*x) - I)*polylog
(3, e^(I*x)))*sqrt(a)/(-2*I*cos(2*x) + 2*sin(2*x) - 2*I)
```

3.875.8 Giac [F]

$$\int x^2 \csc(x)\sec(x)\sqrt{a\sec^2(x)}\,dx = \int \sqrt{a\sec(x)^2}x^2\csc(x)\sec(x)\,dx$$

input
```
integrate(x^2*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*sec(x)^2)*x^2*csc(x)*sec(x), x)
```

3.875.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \csc(x)\sec(x)\sqrt{a\sec^2(x)}\,dx = \int \frac{x^2\sqrt{\frac{a}{\cos(x)^2}}}{\cos(x)\sin(x)}\,dx$$

input
```
int((x^2*(a/cos(x)^2)^(1/2))/(cos(x)*sin(x)),x)
```

output
```
int((x^2*(a/cos(x)^2)^(1/2))/(cos(x)*sin(x)), x)
```

3.876 $\int x^3 \csc(x) \sec(x) \sqrt{a \sec^2(x)}\, dx$

3.876.1 Optimal result

Integrand size = 18, antiderivative size = 341

$$
\begin{aligned}
\int x^3 \csc(x) \sec(x) \sqrt{a \sec^2(x)}\, dx &= x^3 \sqrt{a \sec^2(x)} + 6ix^2 \arctan\left(e^{ix}\right) \cos(x) \sqrt{a \sec^2(x)} \\
&\quad - 2x^3 \operatorname{arctanh}\left(e^{ix}\right) \cos(x) \sqrt{a \sec^2(x)} \\
&\quad + 3ix^2 \cos(x) \operatorname{PolyLog}\left(2, -e^{ix}\right) \sqrt{a \sec^2(x)} \\
&\quad - 6ix \cos(x) \operatorname{PolyLog}\left(2, -ie^{ix}\right) \sqrt{a \sec^2(x)} \\
&\quad + 6ix \cos(x) \operatorname{PolyLog}\left(2, ie^{ix}\right) \sqrt{a \sec^2(x)} \\
&\quad - 3ix^2 \cos(x) \operatorname{PolyLog}\left(2, e^{ix}\right) \sqrt{a \sec^2(x)} \\
&\quad - 6x \cos(x) \operatorname{PolyLog}\left(3, -e^{ix}\right) \sqrt{a \sec^2(x)} \\
&\quad + 6 \cos(x) \operatorname{PolyLog}\left(3, -ie^{ix}\right) \sqrt{a \sec^2(x)} \\
&\quad - 6 \cos(x) \operatorname{PolyLog}\left(3, ie^{ix}\right) \sqrt{a \sec^2(x)} \\
&\quad + 6x \cos(x) \operatorname{PolyLog}\left(3, e^{ix}\right) \sqrt{a \sec^2(x)} \\
&\quad - 6i \cos(x) \operatorname{PolyLog}\left(4, -e^{ix}\right) \sqrt{a \sec^2(x)} \\
&\quad + 6i \cos(x) \operatorname{PolyLog}\left(4, e^{ix}\right) \sqrt{a \sec^2(x)}
\end{aligned}
$$

output
```
x^3*(a*sec(x)^2)^(1/2)+6*I*x^2*arctan(exp(I*x))*cos(x)*(a*sec(x)^2)^(1/2)-
2*x^3*arctanh(exp(I*x))*cos(x)*(a*sec(x)^2)^(1/2)+3*I*x^2*cos(x)*polylog(2
,-exp(I*x))*(a*sec(x)^2)^(1/2)-6*I*x*cos(x)*polylog(2,-I*exp(I*x))*(a*sec(
x)^2)^(1/2)+6*I*x*cos(x)*polylog(2,I*exp(I*x))*(a*sec(x)^2)^(1/2)-3*I*x^2*
cos(x)*polylog(2,exp(I*x))*(a*sec(x)^2)^(1/2)-6*x*cos(x)*polylog(3,-exp(I*
x))*(a*sec(x)^2)^(1/2)+6*cos(x)*polylog(3,-I*exp(I*x))*(a*sec(x)^2)^(1/2)-
6*cos(x)*polylog(3,I*exp(I*x))*(a*sec(x)^2)^(1/2)+6*x*cos(x)*polylog(3,exp
(I*x))*(a*sec(x)^2)^(1/2)-6*I*cos(x)*polylog(4,-exp(I*x))*(a*sec(x)^2)^(1/
2)+6*I*cos(x)*polylog(4,exp(I*x))*(a*sec(x)^2)^(1/2)
```

3.876.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.85

$$\begin{aligned}\int x^3\csc(x)\sec(x)\sqrt{a\sec^2(x)}\,dx &= \frac{1}{8}\big(8x^3-i\pi^4\cos(x)+2ix^4\cos(x)+8x^3\cos(x)\log\left(1-e^{-ix}\right)\\ &\quad-24x^2\cos(x)\log\left(1-ie^{ix}\right)+24x^2\cos(x)\log\left(1+ie^{ix}\right)\\ &\quad-8x^3\cos(x)\log\left(1+e^{ix}\right)+24ix^2\cos(x)\operatorname{PolyLog}\left(2,e^{-ix}\right)\\ &\quad+24ix^2\cos(x)\operatorname{PolyLog}\left(2,-e^{ix}\right)\\ &\quad-48ix\cos(x)\operatorname{PolyLog}\left(2,-ie^{ix}\right)\\ &\quad+48ix\cos(x)\operatorname{PolyLog}\left(2,ie^{ix}\right)\\ &\quad+48x\cos(x)\operatorname{PolyLog}\left(3,e^{-ix}\right)\\ &\quad-48x\cos(x)\operatorname{PolyLog}\left(3,-e^{ix}\right)\\ &\quad+48\cos(x)\operatorname{PolyLog}\left(3,-ie^{ix}\right)\\ &\quad-48\cos(x)\operatorname{PolyLog}\left(3,ie^{ix}\right)\\ &\quad-48i\cos(x)\operatorname{PolyLog}\left(4,e^{-ix}\right)\\ &\quad-48i\cos(x)\operatorname{PolyLog}\left(4,-e^{ix}\right)\big)\sqrt{a\sec^2(x)}\end{aligned}$$

input
```
Integrate[x^3*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^2],x]
```

output
```
((8*x^3 - I*Pi^4*Cos[x] + (2*I)*x^4*Cos[x] + 8*x^3*Cos[x]*Log[1 - E^((-I)*
x)] - 24*x^2*Cos[x]*Log[1 - I*E^(I*x)] + 24*x^2*Cos[x]*Log[1 + I*E^(I*x)]
- 8*x^3*Cos[x]*Log[1 + E^(I*x)] + (24*I)*x^2*Cos[x]*PolyLog[2, E^((-I)*x)]
 + (24*I)*x^2*Cos[x]*PolyLog[2, -E^(I*x)] - (48*I)*x*Cos[x]*PolyLog[2, (-I
)*E^(I*x)] + (48*I)*x*Cos[x]*PolyLog[2, I*E^(I*x)] + 48*x*Cos[x]*PolyLog[3
, E^((-I)*x)] - 48*x*Cos[x]*PolyLog[3, -E^(I*x)] + 48*Cos[x]*PolyLog[3, (-
I)*E^(I*x)] - 48*Cos[x]*PolyLog[3, I*E^(I*x)] - (48*I)*Cos[x]*PolyLog[4, E
^((-I)*x)] - (48*I)*Cos[x]*PolyLog[4, -E^(I*x)])*Sqrt[a*Sec[x]^2])/8
```

3.876.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.66, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {7271, 4920, 25, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \csc(x)\sec(x)\sqrt{a\sec^2(x)}\,dx$$

$$\Big\downarrow \text{ 7271}$$

$$\cos(x)\sqrt{a\sec^2(x)}\int x^3\csc(x)\sec^2(x)dx$$

$$\Big\downarrow \text{ 4920}$$

$$\cos(x)\sqrt{a\sec^2(x)}\left(-3\int -x^2(\text{arctanh}(\cos(x))-\sec(x))dx+x^3(-\text{arctanh}(\cos(x)))+x^3\sec(x)\right)$$

$$\Big\downarrow \text{ 25}$$

$$\cos(x)\sqrt{a\sec^2(x)}\left(3\int x^2(\text{arctanh}(\cos(x))-\sec(x))dx+x^3(-\text{arctanh}(\cos(x)))+x^3\sec(x)\right)$$

$$\Big\downarrow \text{ 2010}$$

$$\cos(x)\sqrt{a\sec^2(x)}\left(3\int \left(x^2\text{arctanh}(\cos(x))-x^2\sec(x)\right)dx+x^3(-\text{arctanh}(\cos(x)))+x^3\sec(x)\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\cos(x)\sqrt{a\sec^2(x)}\left(3\left(2ix^2\arctan\left(e^{ix}\right)-\frac{2}{3}x^3\text{arctanh}\left(e^{ix}\right)+\frac{1}{3}x^3\text{arctanh}(\cos(x))+ix^2\,\text{PolyLog}\left(2,-e^{ix}\right)-ix^2\,\text{P}\right.\right.$$

input
```
Int[x^3*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^2],x]
```

output
```
Cos[x]*Sqrt[a*Sec[x]^2]*(-(x^3*ArcTanh[Cos[x]]) + 3*((2*I)*x^2*ArcTan[E^(I
*x)] - (2*x^3*ArcTanh[E^(I*x)])/3 + (x^3*ArcTanh[Cos[x]])/3 + I*x^2*PolyLo
g[2, -E^(I*x)] - (2*I)*x*PolyLog[2, (-I)*E^(I*x)] + (2*I)*x*PolyLog[2, I*E
^(I*x)] - I*x^2*PolyLog[2, E^(I*x)] - 2*x*PolyLog[3, -E^(I*x)] + 2*PolyLog
[3, (-I)*E^(I*x)] - 2*PolyLog[3, I*E^(I*x)] + 2*x*PolyLog[3, E^(I*x)] - (2
*I)*PolyLog[4, -E^(I*x)] + (2*I)*PolyLog[4, E^(I*x)]) + x^3*Sec[x])
```

3.876.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 4920
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(Eq
Q[v, x] && EqQ[m, 1])
```

3.876.4 Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.73

method	result
risch	$2\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^{2}}}\,x^{3}+4\sqrt{\frac{a\,e^{2ix}}{\left(e^{2ix}+1\right)^{2}}}\left(\frac{3x^{2}\ln\left(1+ie^{ix}\right)}{2}-3ix\,\text{polylog}\left(2,-ie^{ix}\right)+3\,\text{polylog}\left(3,-ie^{ix}\right)-\frac{3x^{2}\ln(}{}\right.$

input
```
int(x^3*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(a*exp(2*I*x)/(exp(2*I*x)+1)^2)^(1/2)*x^3+4*(a*exp(2*I*x)/(exp(2*I*x)+1)
^2)^(1/2)*(3/2*x^2*ln(1+I*exp(I*x))-3*I*x*polylog(2,-I*exp(I*x))+3*polylog
(3,-I*exp(I*x))-3/2*x^2*ln(1-I*exp(I*x))+3*I*x*polylog(2,I*exp(I*x))-3*pol
ylog(3,I*exp(I*x))+1/2*I*(1/4*x^4+I*x^3*ln(exp(I*x)+1)+3*x^2*polylog(2,-ex
p(I*x))+6*I*x*polylog(3,-exp(I*x))-6*polylog(4,-exp(I*x)))+1/2*I*(-1/4*x^4
-I*x^3*ln(1-exp(I*x))-3*x^2*polylog(2,exp(I*x))-6*I*x*polylog(3,exp(I*x))+
6*polylog(4,exp(I*x))))*cos(x)
```

3.876.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 539 vs. 2(253) = 506.

Time = 0.33 (sec) , antiderivative size = 539, normalized size of antiderivative = 1.58

$$\int x^3 \csc(x)\sec(x)\sqrt{a\sec^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
3*x*sqrt(a/cos(x)^2)*cos(x)*polylog(3, cos(x) + I*sin(x)) + 3*x*sqrt(a/cos
(x)^2)*cos(x)*polylog(3, cos(x) - I*sin(x)) - 3*x*sqrt(a/cos(x)^2)*cos(x)*
polylog(3, -cos(x) + I*sin(x)) - 3*x*sqrt(a/cos(x)^2)*cos(x)*polylog(3, -c
os(x) - I*sin(x)) + 3*I*sqrt(a/cos(x)^2)*cos(x)*polylog(4, cos(x) + I*sin(
x)) - 3*I*sqrt(a/cos(x)^2)*cos(x)*polylog(4, cos(x) - I*sin(x)) + 3*I*sqrt
(a/cos(x)^2)*cos(x)*polylog(4, -cos(x) + I*sin(x)) - 3*I*sqrt(a/cos(x)^2)*
cos(x)*polylog(4, -cos(x) - I*sin(x)) + 3*sqrt(a/cos(x)^2)*cos(x)*polylog(
3, I*cos(x) + sin(x)) - 3*sqrt(a/cos(x)^2)*cos(x)*polylog(3, I*cos(x) - si
n(x)) + 3*sqrt(a/cos(x)^2)*cos(x)*polylog(3, -I*cos(x) + sin(x)) - 3*sqrt(
a/cos(x)^2)*cos(x)*polylog(3, -I*cos(x) - sin(x)) - 1/2*(x^3*cos(x)*log(co
s(x) + I*sin(x) + 1) + x^3*cos(x)*log(cos(x) - I*sin(x) + 1) - x^3*cos(x)*
log(-cos(x) + I*sin(x) + 1) - x^3*cos(x)*log(-cos(x) - I*sin(x) + 1) + 3*I
*x^2*cos(x)*dilog(cos(x) + I*sin(x)) - 3*I*x^2*cos(x)*dilog(cos(x) - I*sin
(x)) + 3*I*x^2*cos(x)*dilog(-cos(x) + I*sin(x)) - 3*I*x^2*cos(x)*dilog(-co
s(x) - I*sin(x)) + 3*x^2*cos(x)*log(I*cos(x) + sin(x) + 1) - 3*x^2*cos(x)*
log(I*cos(x) - sin(x) + 1) + 3*x^2*cos(x)*log(-I*cos(x) + sin(x) + 1) - 3*
x^2*cos(x)*log(-I*cos(x) - sin(x) + 1) - 2*x^3 - 6*I*x*cos(x)*dilog(I*cos(
x) + sin(x)) - 6*I*x*cos(x)*dilog(I*cos(x) - sin(x)) + 6*I*x*cos(x)*dilog(
-I*cos(x) + sin(x)) + 6*I*x*cos(x)*dilog(-I*cos(x) - sin(x)))*sqrt(a/cos(x
)^2)
```

3.876.6 Sympy [F]

$$\int x^3 \csc(x)\sec(x)\sqrt{a\sec^2(x)}\,dx = \int x^3\sqrt{a\sec^2{(x)}}\csc{(x)}\sec{(x)}\,dx$$

input
```
integrate(x**3*csc(x)*sec(x)*(a*sec(x)**2)**(1/2),x)
```

output
```
Integral(x**3*sqrt(a*sec(x)**2)*csc(x)*sec(x), x)
```

3.876.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 567 vs. 2(253) = 506.

Time = 0.33 (sec) , antiderivative size = 567, normalized size of antiderivative = 1.66

$$\int x^3 \csc(x)\sec(x)\sqrt{a\sec^2(x)}\,dx = -\frac{\left(4i\,x^3\cos{(x)} - 4\,x^3\sin{(x)} - 6\left(x^2\cos{(2\,x)} + i\,x^2\sin{(2\,x)} + x^2\right)\arctan{(\cos{(x)}\,,\sin{(x)}+1)} - 6\left(x^2\,\mathrm{co}\right.}{}$$

input
```
integrate(x^3*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-(4*I*x^3*cos(x) - 4*x^3*sin(x) - 6*(x^2*cos(2*x) + I*x^2*sin(2*x) + x^2)*
arctan2(cos(x), sin(x) + 1) - 6*(x^2*cos(2*x) + I*x^2*sin(2*x) + x^2)*arct
an2(cos(x), -sin(x) + 1) + 2*(x^3*cos(2*x) + I*x^3*sin(2*x) + x^3)*arctan2
(sin(x), cos(x) + 1) + 2*(x^3*cos(2*x) + I*x^3*sin(2*x) + x^3)*arctan2(sin
(x), -cos(x) + 1) - 12*(x*cos(2*x) + I*x*sin(2*x) + x)*dilog(I*e^(I*x)) +
12*(x*cos(2*x) + I*x*sin(2*x) + x)*dilog(-I*e^(I*x)) - 6*(x^2*cos(2*x) + I
*x^2*sin(2*x) + x^2)*dilog(-e^(I*x)) + 6*(x^2*cos(2*x) + I*x^2*sin(2*x) +
x^2)*dilog(e^(I*x)) + (-I*x^3*cos(2*x) + x^3*sin(2*x) - I*x^3)*log(cos(x)^
2 + sin(x)^2 + 2*cos(x) + 1) + (I*x^3*cos(2*x) - x^3*sin(2*x) + I*x^3)*log
(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) - 3*(I*x^2*cos(2*x) - x^2*sin(2*x) +
I*x^2)*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - 3*(-I*x^2*cos(2*x) + x^2*
sin(2*x) - I*x^2)*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1) + 12*(cos(2*x) +
 I*sin(2*x) + 1)*polylog(4, -e^(I*x)) - 12*(cos(2*x) + I*sin(2*x) + 1)*pol
ylog(4, e^(I*x)) - 12*(I*cos(2*x) - sin(2*x) + I)*polylog(3, I*e^(I*x)) -
12*(-I*cos(2*x) + sin(2*x) - I)*polylog(3, -I*e^(I*x)) - 12*(I*x*cos(2*x)
- x*sin(2*x) + I*x)*polylog(3, -e^(I*x)) - 12*(-I*x*cos(2*x) + x*sin(2*x)
- I*x)*polylog(3, e^(I*x)))*sqrt(a)/(-2*I*cos(2*x) + 2*sin(2*x) - 2*I)
```

3.876.8 Giac [F]

$$\int x^3 \csc(x) \sec(x) \sqrt{a \sec^2(x)}\, dx = \int \sqrt{a \sec(x)^2} x^3 \csc(x) \sec(x)\, dx$$

input
```
integrate(x^3*csc(x)*sec(x)*(a*sec(x)^2)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*sec(x)^2)*x^3*csc(x)*sec(x), x)
```

3.876.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \csc(x) \sec(x) \sqrt{a \sec^2(x)}\, dx = \int \frac{x^3 \sqrt{\frac{a}{\cos(x)^2}}}{\cos(x) \sin(x)}\, dx$$

input
```
int((x^3*(a/cos(x)^2)^(1/2))/(cos(x)*sin(x)),x)
```

output
```
int((x^3*(a/cos(x)^2)^(1/2))/(cos(x)*sin(x)), x)
```

3.877 $\int x \csc(x) \sec(x) \sqrt{a \sec^4(x)} \, dx$

3.877.1 Optimal result

Integrand size = 16, antiderivative size = 142

$$\begin{aligned}\int x \csc(x) \sec(x) \sqrt{a \sec^4(x)} \, dx = & \frac{1}{2} x \cos^2(x) \sqrt{a \sec^4(x)} \\ & - 2x\operatorname{arctanh}\left(e^{2ix}\right) \cos^2(x) \sqrt{a \sec^4(x)} \\ & + \frac{1}{2} i \cos^2(x) \operatorname{PolyLog}\left(2, -e^{2ix}\right) \sqrt{a \sec^4(x)} \\ & - \frac{1}{2} i \cos^2(x) \operatorname{PolyLog}\left(2, e^{2ix}\right) \sqrt{a \sec^4(x)} \\ & - \frac{1}{2} \cos(x) \sqrt{a \sec^4(x)} \sin(x) + \frac{1}{2} x \sqrt{a \sec^4(x)} \sin^2(x)\end{aligned}$$

output
```
1/2*x*cos(x)^2*(a*sec(x)^4)^(1/2)-2*x*arctanh(exp(2*I*x))*cos(x)^2*(a*sec(
x)^4)^(1/2)+1/2*I*cos(x)^2*polylog(2,-exp(2*I*x))*(a*sec(x)^4)^(1/2)-1/2*I
*cos(x)^2*polylog(2,exp(2*I*x))*(a*sec(x)^4)^(1/2)-1/2*cos(x)*sin(x)*(a*se
c(x)^4)^(1/2)+1/2*x*sin(x)^2*(a*sec(x)^4)^(1/2)
```

3.877.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.60

$$\int x\csc(x)\sec(x)\sqrt{a\sec^4(x)}\,dx = \frac{1}{2}\cos^2(x)\sqrt{a\sec^4(x)}\left(2x\log\left(1-e^{2ix}\right)-2x\log\left(1+e^{2ix}\right)+i\,\text{PolyLog}\left(2,-e^{2ix}\right)-i\,\text{PolyLog}\left(2,e^{2ix}\right)+x\sec^2(x)-\tan(x)\right)$$

input
```
Integrate[x*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^4],x]
```

output
```
(Cos[x]^2*Sqrt[a*Sec[x]^4]*(2*x*Log[1 - E^((2*I)*x)] - 2*x*Log[1 + E^((2*I
)*x)] + I*PolyLog[2, -E^((2*I)*x)] - I*PolyLog[2, E^((2*I)*x)] + x*Sec[x]^
2 - Tan[x]))/2
```

3.877.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.56, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {7271, 4920, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\csc(x)\sec(x)\sqrt{a\sec^4(x)}\,dx$$

$$\Big\downarrow \text{7271}$$

$$\cos^2(x)\sqrt{a\sec^4(x)}\int x\csc(x)\sec^3(x)dx$$

$$\Big\downarrow \text{4920}$$

$$\cos^2(x)\sqrt{a\sec^4(x)}\left(-\int\left(\frac{\tan^2(x)}{2}+\log(\tan(x))\right)dx+\frac{1}{2}x\tan^2(x)+x\log(\tan(x))\right)$$

$$\Big\downarrow \text{2009}$$

$$\cos^2(x)\sqrt{a\sec^4(x)}\left(-2x\text{arctanh}\left(e^{2ix}\right)+\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2ix}\right)-\frac{1}{2}i\,\text{PolyLog}\left(2,e^{2ix}\right)+\frac{x}{2}+\frac{1}{2}x\tan^2(x)-\frac{\tan(x)}{2}\right.$$

input
```
Int[x*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^4],x]
```

output
```
Cos[x]^2*Sqrt[a*Sec[x]^4]*(x/2 - 2*x*ArcTanh[E^((2*I)*x)] + (I/2)*PolyLog[
2, -E^((2*I)*x)] - (I/2)*PolyLog[2, E^((2*I)*x)] - Tan[x]/2 + (x*Tan[x]^2)
/2)
```

3.877.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4920
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(Eq
Q[v, x] && EqQ[m, 1])
```

3.877.4 Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.99

method	result
risch	$\sqrt{\frac{a\,e^{4ix}}{(e^{2ix}+1)^4}}\,(-i+2x-ie^{-2ix})-4i\sqrt{\frac{a\,e^{4ix}}{(e^{2ix}+1)^4}}\,e^{-2ix}(e^{2ix}+1)^2\,(\frac{ix\ln(e^{ix}+1)}{4}+\frac{\operatorname{polylog}(2,-e^{ix})}{4}+\frac{ix\ln(1-}{4}$

input
```
int(x*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(a*exp(4*I*x)/(exp(2*I*x)+1)^4)^(1/2)*(-I+2*x-I*exp(-2*I*x))-4*I*(a*exp(4*
I*x)/(exp(2*I*x)+1)^4)^(1/2)*exp(-2*I*x)*(exp(2*I*x)+1)^2*(1/4*I*x*ln(exp(
I*x)+1)+1/4*polylog(2,-exp(I*x))+1/4*I*x*ln(1-exp(I*x))+1/4*polylog(2,exp(
I*x))-1/4*I*x*ln(exp(2*I*x)+1)-1/8*polylog(2,-exp(2*I*x)))
```

3.877.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(105) = 210$.

Time = 0.29 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.90

$$\int x\csc(x)\sec(x)\sqrt{a\sec^4(x)}\,dx$$
$$= \frac{1}{2}\left(x\cos(x)^2\log(\cos(x) + i\sin(x) + 1) + x\cos(x)^2\log(\cos(x) - i\sin(x) + 1) - x\cos(x)^2\log(i\cos(x)\right.$$

input
```
integrate(x*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(x*cos(x)^2*log(cos(x) + I*sin(x) + 1) + x*cos(x)^2*log(cos(x) - I*sin
(x) + 1) - x*cos(x)^2*log(I*cos(x) + sin(x) + 1) - x*cos(x)^2*log(I*cos(x)
 - sin(x) + 1) - x*cos(x)^2*log(-I*cos(x) + sin(x) + 1) - x*cos(x)^2*log(-
I*cos(x) - sin(x) + 1) + x*cos(x)^2*log(-cos(x) + I*sin(x) + 1) + x*cos(x)
^2*log(-cos(x) - I*sin(x) + 1) - I*cos(x)^2*dilog(cos(x) + I*sin(x)) + I*c
os(x)^2*dilog(cos(x) - I*sin(x)) - I*cos(x)^2*dilog(I*cos(x) + sin(x)) + I
*cos(x)^2*dilog(I*cos(x) - sin(x)) + I*cos(x)^2*dilog(-I*cos(x) + sin(x))
- I*cos(x)^2*dilog(-I*cos(x) - sin(x)) + I*cos(x)^2*dilog(-cos(x) + I*sin(
x)) - I*cos(x)^2*dilog(-cos(x) - I*sin(x)) - cos(x)*sin(x) + x)*sqrt(a/cos
(x)^4)
```

3.877.6 Sympy [F]

$$\int x\csc(x)\sec(x)\sqrt{a\sec^4(x)}\,dx = \int x\sqrt{a\sec^4(x)}\csc(x)\sec(x)\,dx$$

input
```
integrate(x*csc(x)*sec(x)*(a*sec(x)**4)**(1/2),x)
```

output
```
Integral(x*sqrt(a*sec(x)**4)*csc(x)*sec(x), x)
```

3.877.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 423 vs. 2(105) = 210.

Time = 0.32 (sec) , antiderivative size = 423, normalized size of antiderivative = 2.98

$$\int x\csc(x)\sec(x)\sqrt{a\sec^4(x)}\,dx = -\frac{(2\,(x\cos\,(4\,x)+2\,x\cos\,(2\,x)+i\,x\sin\,(4\,x)+2i\,x\sin\,(2\,x)+x)\arctan\,(\sin\,(2\,x)\,,\cos\,(2\,x)+1)-2\,(x\cos}{}$$

input
```
integrate(x*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x, algorithm="maxima")
```

output
```
-(2*(x*cos(4*x) + 2*x*cos(2*x) + I*x*sin(4*x) + 2*I*x*sin(2*x) + x)*arctan
2(sin(2*x), cos(2*x) + 1) - 2*(x*cos(4*x) + 2*x*cos(2*x) + I*x*sin(4*x) +
2*I*x*sin(2*x) + x)*arctan2(sin(x), cos(x) + 1) + 2*(x*cos(4*x) + 2*x*cos(
2*x) + I*x*sin(4*x) + 2*I*x*sin(2*x) + x)*arctan2(sin(x), -cos(x) + 1) - 2
*(-2*I*x - 1)*cos(2*x) - (cos(4*x) + 2*cos(2*x) + I*sin(4*x) + 2*I*sin(2*x
) + 1)*dilog(-e^(2*I*x)) + 2*(cos(4*x) + 2*cos(2*x) + I*sin(4*x) + 2*I*sin
(2*x) + 1)*dilog(-e^(I*x)) + 2*(cos(4*x) + 2*cos(2*x) + I*sin(4*x) + 2*I*s
in(2*x) + 1)*dilog(e^(I*x)) + (-I*x*cos(4*x) - 2*I*x*cos(2*x) + x*sin(4*x)
 + 2*x*sin(2*x) - I*x)*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1) + (I*
x*cos(4*x) + 2*I*x*cos(2*x) - x*sin(4*x) - 2*x*sin(2*x) + I*x)*log(cos(x)^
2 + sin(x)^2 + 2*cos(x) + 1) + (I*x*cos(4*x) + 2*I*x*cos(2*x) - x*sin(4*x)
 - 2*x*sin(2*x) + I*x)*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) - 2*(2*x -
I)*sin(2*x) + 2)*sqrt(a)/(-2*I*cos(4*x) - 4*I*cos(2*x) + 2*sin(4*x) + 4*si
n(2*x) - 2*I)
```

3.877.8 Giac [F]

$$\int x\csc(x)\sec(x)\sqrt{a\sec^4(x)}\,dx = \int\sqrt{a\sec\,(x)^4}x\csc\,(x)\sec\,(x)\;dx$$

input
```
integrate(x*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*sec(x)^4)*x*csc(x)*sec(x), x)
```

3.877.9 Mupad [F(-1)]

Timed out.

$$\int x\csc(x)\sec(x)\sqrt{a\sec^4(x)}\,dx=\int\frac{x\sqrt{\frac{a}{\cos(x)^4}}}{\cos(x)\,\sin(x)}\,dx$$

input
```
int((x*(a/cos(x)^4)^(1/2))/(cos(x)*sin(x)),x)
```

output
```
int((x*(a/cos(x)^4)^(1/2))/(cos(x)*sin(x)), x)
```

3.878 $\int x^2 \csc(x) \sec(x) \sqrt{a \sec^4(x)} \, dx$

3.878.1 Optimal result

Integrand size = 18, antiderivative size = 220

$$\begin{aligned}\int x^2 \csc(x) \sec(x) \sqrt{a \sec^4(x)} \, dx = {} & \frac{1}{2} x^2 \cos^2(x) \sqrt{a \sec^4(x)} \\ & - 2x^2 \operatorname{arctanh}\left(e^{2ix}\right) \cos^2(x) \sqrt{a \sec^4(x)} \\ & - \cos^2(x) \log(\cos(x)) \sqrt{a \sec^4(x)} \\ & + ix \cos^2(x) \operatorname{PolyLog}\left(2, -e^{2ix}\right) \sqrt{a \sec^4(x)} \\ & - ix \cos^2(x) \operatorname{PolyLog}\left(2, e^{2ix}\right) \sqrt{a \sec^4(x)} \\ & - \frac{1}{2} \cos^2(x) \operatorname{PolyLog}\left(3, -e^{2ix}\right) \sqrt{a \sec^4(x)} \\ & + \frac{1}{2} \cos^2(x) \operatorname{PolyLog}\left(3, e^{2ix}\right) \sqrt{a \sec^4(x)} \\ & - x \cos(x) \sqrt{a \sec^4(x)} \sin(x) + \frac{1}{2} x^2 \sqrt{a \sec^4(x)} \sin^2(x)\end{aligned}$$

output
```
1/2*x^2*cos(x)^2*(a*sec(x)^4)^(1/2)-2*x^2*arctanh(exp(2*I*x))*cos(x)^2*(a*
sec(x)^4)^(1/2)-cos(x)^2*ln(cos(x))*(a*sec(x)^4)^(1/2)+I*x*cos(x)^2*polylo
g(2,-exp(2*I*x))*(a*sec(x)^4)^(1/2)-I*x*cos(x)^2*polylog(2,exp(2*I*x))*(a*
sec(x)^4)^(1/2)-1/2*cos(x)^2*polylog(3,-exp(2*I*x))*(a*sec(x)^4)^(1/2)+1/2
*cos(x)^2*polylog(3,exp(2*I*x))*(a*sec(x)^4)^(1/2)-x*cos(x)*sin(x)*(a*sec(
x)^4)^(1/2)+1/2*x^2*sin(x)^2*(a*sec(x)^4)^(1/2)
```

3.878.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.63

$$\int x^2\csc(x)\sec(x)\sqrt{a\sec^4(x)}\,dx = \frac{1}{24}\cos^2(x)\sqrt{a\sec^4(x)}\left(-i\pi^3+16ix^3+24x^2\log\left(1-e^{-2ix}\right)-24x^2\log\left(1+e^{2ix}\right)-24\log(\cos(x))+24ix\operatorname{PolyLog}\left(2,e^{-2ix}\right)+24ix\operatorname{PolyLog}\left(2,-e^{2ix}\right)+12\operatorname{PolyLog}\left(3,e^{-2ix}\right)-12\operatorname{PolyLog}\left(3,-e^{2ix}\right)+12x^2\sec^2(x)-24x\tan(x)\right)$$

input
```
Integrate[x^2*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^4],x]
```

output
```
(Cos[x]^2*Sqrt[a*Sec[x]^4]*((-I)*Pi^3 + (16*I)*x^3 + 24*x^2*Log[1 - E^((-2
*I)*x)] - 24*x^2*Log[1 + E^((2*I)*x)] - 24*Log[Cos[x]] + (24*I)*x*PolyLog[
2, E^((-2*I)*x)] + (24*I)*x*PolyLog[2, -E^((2*I)*x)] + 12*PolyLog[3, E^((-
2*I)*x)] - 12*PolyLog[3, -E^((2*I)*x)] + 12*x^2*Sec[x]^2 - 24*x*Tan[x]))/2
4
```

3.878.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.52, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {7271, 4920, 27, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\csc(x)\sec(x)\sqrt{a\sec^4(x)}\,dx$$

$$\Big\downarrow \text{7271}$$

$$\cos^2(x)\sqrt{a\sec^4(x)}\int x^2\csc(x)\sec^3(x)dx$$

$$\Big\downarrow \text{4920}$$

$$\cos^2(x)\sqrt{a\sec^4(x)}\left(-2\int \frac{1}{2}x\left(\tan^2(x)+2\log(\tan(x))\right)dx+\frac{1}{2}x^2\tan^2(x)+x^2\log(\tan(x))\right)$$

$$\Big\downarrow \text{27}$$

$$\cos^2(x)\sqrt{a\sec^4(x)}\left(-\int x\left(\tan^2(x)+2\log(\tan(x))\right)dx+\frac{1}{2}x^2\tan^2(x)+x^2\log(\tan(x))\right)$$

$\downarrow$ 2010

$$\cos^2(x)\sqrt{a\sec^4(x)}\left(-\int\left(x\tan^2(x)+2x\log(\tan(x))\right)dx+\frac{1}{2}x^2\tan^2(x)+x^2\log(\tan(x))\right)$$

$\downarrow$ 2009

$$\cos^2(x)\sqrt{a\sec^4(x)}\left(-2x^2\text{arctanh}\left(e^{2ix}\right)+ix\,\text{PolyLog}\left(2,-e^{2ix}\right)-ix\,\text{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{2}\text{PolyLog}\left(3,-e^{2ix}\right)+\frac{1}{2}\right.$$

input
```
Int[x^2*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^4],x]
```

output
```
Cos[x]^2*Sqrt[a*Sec[x]^4]*(x^2/2 - 2*x^2*ArcTanh[E^((2*I)*x)] - Log[Cos[x]
] + I*x*PolyLog[2, -E^((2*I)*x)] - I*x*PolyLog[2, E^((2*I)*x)] - PolyLog[3
, -E^((2*I)*x)]/2 + PolyLog[3, E^((2*I)*x)]/2 - x*Tan[x] + (x^2*Tan[x]^2)/
2)
```

3.878.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 4920
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

3.878.4 Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.87

method	result
risch	$2\sqrt{\frac{a\,\mathrm{e}^{4ix}}{(\mathrm{e}^{2ix}+1)^4}}\,x(x-i-i\mathrm{e}^{-2ix})+2\sqrt{\frac{a\,\mathrm{e}^{4ix}}{(\mathrm{e}^{2ix}+1)^4}}\,\mathrm{e}^{-2ix}(\mathrm{e}^{2ix}+1)^2\left(-\frac{\ln(\mathrm{e}^{2ix}+1)}{2}+\ln(\mathrm{e}^{ix})+\frac{x^2\ln(\mathrm{e}^{ix}+1)}{2}-\right.$

input
```
int(x^2*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(a*exp(4*I*x)/(exp(2*I*x)+1)^4)^(1/2)*x*(x-I-I*exp(-2*I*x))+2*(a*exp(4*I
*x)/(exp(2*I*x)+1)^4)^(1/2)*exp(-2*I*x)*(exp(2*I*x)+1)^2*(-1/2*ln(exp(2*I*
x)+1)+ln(exp(I*x))+1/2*x^2*ln(exp(I*x)+1)-I*x*polylog(2,-exp(I*x))+polylog
(3,-exp(I*x))+1/2*x^2*ln(1-exp(I*x))-I*x*polylog(2,exp(I*x))+polylog(3,exp
(I*x))-1/2*x^2*ln(exp(2*I*x)+1)+1/2*I*x*polylog(2,-exp(2*I*x))-1/4*polylog
(3,-exp(2*I*x)))
```

3.878.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 550 vs. 2(173) = 346.

Time = 0.32 (sec) , antiderivative size = 550, normalized size of antiderivative = 2.50

$$\int x^2\csc(x)\sec(x)\sqrt{a\sec^4(x)}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, cos(x) + I*sin(x)) + sqrt(a/cos(x)^4)
*cos(x)^2*polylog(3, cos(x) - I*sin(x)) - sqrt(a/cos(x)^4)*cos(x)^2*polylo
g(3, I*cos(x) + sin(x)) - sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, I*cos(x) -
sin(x)) - sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, -I*cos(x) + sin(x)) - sqrt(
a/cos(x)^4)*cos(x)^2*polylog(3, -I*cos(x) - sin(x)) + sqrt(a/cos(x)^4)*cos
(x)^2*polylog(3, -cos(x) + I*sin(x)) + sqrt(a/cos(x)^4)*cos(x)^2*polylog(3
, -cos(x) - I*sin(x)) + 1/2*(x^2*cos(x)^2*log(cos(x) + I*sin(x) + 1) + x^2
*cos(x)^2*log(cos(x) - I*sin(x) + 1) - x^2*cos(x)^2*log(I*cos(x) + sin(x)
+ 1) - x^2*cos(x)^2*log(I*cos(x) - sin(x) + 1) - x^2*cos(x)^2*log(-I*cos(x
) + sin(x) + 1) - x^2*cos(x)^2*log(-I*cos(x) - sin(x) + 1) + x^2*cos(x)^2*
log(-cos(x) + I*sin(x) + 1) + x^2*cos(x)^2*log(-cos(x) - I*sin(x) + 1) - 2
*I*x*cos(x)^2*dilog(cos(x) + I*sin(x)) + 2*I*x*cos(x)^2*dilog(cos(x) - I*s
in(x)) - 2*I*x*cos(x)^2*dilog(I*cos(x) + sin(x)) + 2*I*x*cos(x)^2*dilog(I*
cos(x) - sin(x)) + 2*I*x*cos(x)^2*dilog(-I*cos(x) + sin(x)) - 2*I*x*cos(x)
^2*dilog(-I*cos(x) - sin(x)) + 2*I*x*cos(x)^2*dilog(-cos(x) + I*sin(x)) -
2*I*x*cos(x)^2*dilog(-cos(x) - I*sin(x)) - cos(x)^2*log(cos(x) + I*sin(x)
+ I) - cos(x)^2*log(cos(x) - I*sin(x) + I) - cos(x)^2*log(-cos(x) + I*sin(
x) + I) - cos(x)^2*log(-cos(x) - I*sin(x) + I) - 2*x*cos(x)*sin(x) + x^2)*
sqrt(a/cos(x)^4)
```

3.878.6 Sympy [F]

$$\int x^2 \csc(x) \sec(x) \sqrt{a \sec^4(x)}\, dx = \int x^2 \sqrt{a \sec^4{(x)}} \csc{(x)} \sec{(x)}\, dx$$

input
```
integrate(x**2*csc(x)*sec(x)*(a*sec(x)**4)**(1/2),x)
```

output
```
Integral(x**2*sqrt(a*sec(x)**4)*csc(x)*sec(x), x)
```

3.878.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 639 vs. $2(173) = 346$.

Time = 0.36 (sec) , antiderivative size = 639, normalized size of antiderivative = 2.90

$$\int x^2 \csc(x) \sec(x) \sqrt{a \sec^4(x)}\, dx = \text{Too large to display}$$

input
```
integrate(x^2*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x, algorithm="maxima")
```

output
```
-(2*(x^2 + (x^2 + 1)*cos(4*x) + 2*(x^2 + 1)*cos(2*x) - (-I*x^2 - I)*sin(4*
x) - 2*(-I*x^2 - I)*sin(2*x) + 1)*arctan2(sin(2*x), cos(2*x) + 1) - 2*(x^2
*cos(4*x) + 2*x^2*cos(2*x) + I*x^2*sin(4*x) + 2*I*x^2*sin(2*x) + x^2)*arct
an2(sin(x), cos(x) + 1) + 2*(x^2*cos(4*x) + 2*x^2*cos(2*x) + I*x^2*sin(4*x
) + 2*I*x^2*sin(2*x) + x^2)*arctan2(sin(x), -cos(x) + 1) - 4*x*cos(4*x) -
4*(-I*x^2 + x)*cos(2*x) - 2*(x*cos(4*x) + 2*x*cos(2*x) + I*x*sin(4*x) + 2*
I*x*sin(2*x) + x)*dilog(-e^(2*I*x)) + 4*(x*cos(4*x) + 2*x*cos(2*x) + I*x*s
in(4*x) + 2*I*x*sin(2*x) + x)*dilog(-e^(I*x)) + 4*(x*cos(4*x) + 2*x*cos(2*
x) + I*x*sin(4*x) + 2*I*x*sin(2*x) + x)*dilog(e^(I*x)) + (-I*x^2 + (-I*x^2
 - I)*cos(4*x) - 2*(I*x^2 + I)*cos(2*x) + (x^2 + 1)*sin(4*x) + 2*(x^2 + 1)
*sin(2*x) - I)*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1) + (I*x^2*cos(
4*x) + 2*I*x^2*cos(2*x) - x^2*sin(4*x) - 2*x^2*sin(2*x) + I*x^2)*log(cos(x
)^2 + sin(x)^2 + 2*cos(x) + 1) + (I*x^2*cos(4*x) + 2*I*x^2*cos(2*x) - x^2*
sin(4*x) - 2*x^2*sin(2*x) + I*x^2)*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)
 + (-I*cos(4*x) - 2*I*cos(2*x) + sin(4*x) + 2*sin(2*x) - I)*polylog(3, -e^
(2*I*x)) - 4*(-I*cos(4*x) - 2*I*cos(2*x) + sin(4*x) + 2*sin(2*x) - I)*poly
log(3, -e^(I*x)) - 4*(-I*cos(4*x) - 2*I*cos(2*x) + sin(4*x) + 2*sin(2*x) -
 I)*polylog(3, e^(I*x)) - 4*I*x*sin(4*x) - 4*(x^2 + I*x)*sin(2*x))*sqrt(a)
/(-2*I*cos(4*x) - 4*I*cos(2*x) + 2*sin(4*x) + 4*sin(2*x) - 2*I)
```

3.878.8 Giac [F]

$$\int x^2 \csc(x) \sec(x) \sqrt{a \sec^4(x)} \, dx = \int \sqrt{a \sec(x)^4} x^2 \csc(x) \sec(x) \, dx$$

input
```
integrate(x^2*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*sec(x)^4)*x^2*csc(x)*sec(x), x)
```

3.878.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \csc(x) \sec(x) \sqrt{a \sec^4(x)}\,dx = \int \frac{x^2 \sqrt{\frac{a}{\cos(x)^4}}}{\cos(x) \sin(x)}\,dx$$

input
```
int((x^2*(a/cos(x)^4)^(1/2))/(cos(x)*sin(x)),x)
```

output
```
int((x^2*(a/cos(x)^4)^(1/2))/(cos(x)*sin(x)), x)
```

3.879 $\int x^3 \csc(x) \sec(x) \sqrt{a \sec^4(x)} \, dx$

3.879.1 Optimal result

Integrand size = 18, antiderivative size = 356

$$\begin{aligned}\int x^3 \csc(x) \sec(x) \sqrt{a \sec^4(x)} \, dx = {} & \frac{3}{2} i x^2 \cos^2(x) \sqrt{a \sec^4(x)} + \frac{1}{2} x^3 \cos^2(x) \sqrt{a \sec^4(x)} \\ & - 2x^3 \operatorname{arctanh}\left(e^{2ix}\right) \cos^2(x) \sqrt{a \sec^4(x)} \\ & - 3x \cos^2(x) \log\left(1 + e^{2ix}\right) \sqrt{a \sec^4(x)} \\ & + \frac{3}{2} i \cos^2(x) \operatorname{PolyLog}\left(2, -e^{2ix}\right) \sqrt{a \sec^4(x)} \\ & + \frac{3}{2} i x^2 \cos^2(x) \operatorname{PolyLog}\left(2, -e^{2ix}\right) \sqrt{a \sec^4(x)} \\ & - \frac{3}{2} i x^2 \cos^2(x) \operatorname{PolyLog}\left(2, e^{2ix}\right) \sqrt{a \sec^4(x)} \\ & - \frac{3}{2} x \cos^2(x) \operatorname{PolyLog}\left(3, -e^{2ix}\right) \sqrt{a \sec^4(x)} \\ & + \frac{3}{2} x \cos^2(x) \operatorname{PolyLog}\left(3, e^{2ix}\right) \sqrt{a \sec^4(x)} \\ & - \frac{3}{4} i \cos^2(x) \operatorname{PolyLog}\left(4, -e^{2ix}\right) \sqrt{a \sec^4(x)} \\ & + \frac{3}{4} i \cos^2(x) \operatorname{PolyLog}\left(4, e^{2ix}\right) \sqrt{a \sec^4(x)} \\ & - \frac{3}{2} x^2 \cos(x) \sqrt{a \sec^4(x)} \sin(x) + \frac{1}{2} x^3 \sqrt{a \sec^4(x)} \sin^2(x)\end{aligned}$$

output
```
3/2*I*x^2*cos(x)^2*(a*sec(x)^4)^(1/2)+1/2*x^3*cos(x)^2*(a*sec(x)^4)^(1/2)-
2*x^3*arctanh(exp(2*I*x))*cos(x)^2*(a*sec(x)^4)^(1/2)-3*x*cos(x)^2*ln(1+ex
p(2*I*x))*(a*sec(x)^4)^(1/2)+3/2*I*cos(x)^2*polylog(2,-exp(2*I*x))*(a*sec(
x)^4)^(1/2)+3/2*I*x^2*cos(x)^2*polylog(2,-exp(2*I*x))*(a*sec(x)^4)^(1/2)-3
/2*I*x^2*cos(x)^2*polylog(2,exp(2*I*x))*(a*sec(x)^4)^(1/2)-3/2*x*cos(x)^2*
polylog(3,-exp(2*I*x))*(a*sec(x)^4)^(1/2)+3/2*x*cos(x)^2*polylog(3,exp(2*I
*x))*(a*sec(x)^4)^(1/2)-3/4*I*cos(x)^2*polylog(4,-exp(2*I*x))*(a*sec(x)^4)
^(1/2)+3/4*I*cos(x)^2*polylog(4,exp(2*I*x))*(a*sec(x)^4)^(1/2)-3/2*x^2*cos
(x)*sin(x)*(a*sec(x)^4)^(1/2)+1/2*x^3*sin(x)^2*(a*sec(x)^4)^(1/2)
```

3.879.2 Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.54

$$\int x^3 \csc(x) \sec(x) \sqrt{a \sec^4(x)} \, dx = \frac{1}{64} \cos^2(x) \sqrt{a \sec^4(x)} \big(-i\pi^4 + 96ix^2 + 32ix^4 + 64x^3 \log\left(1 - e^{-2ix}\right) - 192x \log\left(1 + e^{2ix}\right) - 64x^3 \log\left(1 + e^{2ix}\right) + 96ix^2 \operatorname{PolyLog}\left(2, e^{-2ix}\right) + 96i(1+x^2) \operatorname{PolyLog}\left(2, -e^{2ix}\right) + 96x \operatorname{PolyLog}\left(3, e^{-2ix}\right) - 96x \operatorname{PolyLog}\left(3, -e^{2ix}\right) - 48i \operatorname{PolyLog}\left(4, e^{-2ix}\right) - 48i \operatorname{PolyLog}\left(4, -e^{2ix}\right) + 32x^3 \sec^2(x) - 96x^2 \tan(x)\big)$$

input
```
Integrate[x^3*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^4],x]
```

output
```
(Cos[x]^2*Sqrt[a*Sec[x]^4]*((-I)*Pi^4 + (96*I)*x^2 + (32*I)*x^4 + 64*x^3*L
og[1 - E^((-2*I)*x)] - 192*x*Log[1 + E^((2*I)*x)] - 64*x^3*Log[1 + E^((2*I
)*x)] + (96*I)*x^2*PolyLog[2, E^((-2*I)*x)] + (96*I)*(1 + x^2)*PolyLog[2,
-E^((2*I)*x)] + 96*x*PolyLog[3, E^((-2*I)*x)] - 96*x*PolyLog[3, -E^((2*I)*
x)] - (48*I)*PolyLog[4, E^((-2*I)*x)] - (48*I)*PolyLog[4, -E^((2*I)*x)] +
32*x^3*Sec[x]^2 - 96*x^2*Tan[x]))/64
```

3.879.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.57, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {7271, 4920, 27, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \csc(x) \sec(x) \sqrt{a \sec^4(x)} \, dx$$

$$\Big\downarrow \text{7271}$$

$$\cos^2(x) \sqrt{a \sec^4(x)} \int x^3 \csc(x) \sec^3(x) dx$$

$$\Big\downarrow \text{4920}$$

$$\cos^2(x) \sqrt{a \sec^4(x)} \left(-3 \int \frac{1}{2} x^2 \left(\tan^2(x) + 2 \log(\tan(x))\right) dx + \frac{1}{2} x^3 \tan^2(x) + x^3 \log(\tan(x))\right)$$

$$\Big\downarrow \text{27}$$

$$\cos^2(x) \sqrt{a \sec^4(x)} \left(-\frac{3}{2} \int x^2 \left(\tan^2(x) + 2 \log(\tan(x))\right) dx + \frac{1}{2} x^3 \tan^2(x) + x^3 \log(\tan(x))\right)$$

$$\Big\downarrow \text{2010}$$

$$\cos^2(x) \sqrt{a \sec^4(x)} \left(-\frac{3}{2} \int \left(\tan^2(x) x^2 + 2 \log(\tan(x)) x^2\right) dx + \frac{1}{2} x^3 \tan^2(x) + x^3 \log(\tan(x))\right)$$

$$\Big\downarrow \text{2009}$$

$$\cos^2(x) \sqrt{a \sec^4(x)} \left(-\frac{3}{2} \left(\frac{4}{3} x^3 \text{arctanh}\left(e^{2ix}\right) - i x^2 \text{PolyLog}\left(2, -e^{2ix}\right) + i x^2 \text{PolyLog}\left(2, e^{2ix}\right) + x \text{PolyLog}\left(3, -e^{2i}\right.\right.\right.$$

input
```
Int[x^3*Csc[x]*Sec[x]*Sqrt[a*Sec[x]^4],x]
```

output
```
Cos[x]^2*Sqrt[a*Sec[x]^4]*(x^3*Log[Tan[x]] + (x^3*Tan[x]^2)/2 - (3*((-I)*x
^2 - x^3/3 + (4*x^3*ArcTanh[E^((2*I)*x)])/3 + 2*x*Log[1 + E^((2*I)*x)] + (
2*x^3*Log[Tan[x]])/3 - I*PolyLog[2, -E^((2*I)*x)] - I*x^2*PolyLog[2, -E^((
2*I)*x)] + I*x^2*PolyLog[2, E^((2*I)*x)] + x*PolyLog[3, -E^((2*I)*x)] - x*
PolyLog[3, E^((2*I)*x)] + (I/2)*PolyLog[4, -E^((2*I)*x)] - (I/2)*PolyLog[4
, E^((2*I)*x)] + x^2*Tan[x]))/2)
```

3.879.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 4920
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

3.879.4 Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.71

method	result
risch	$\sqrt{\frac{a\,e^{4ix}}{\left(e^{2ix}+1\right)^{4}}}\,x^{2}\left(2x-3i-3i\,e^{-2ix}\right)-2i\sqrt{\frac{a\,e^{4ix}}{\left(e^{2ix}+1\right)^{4}}}\,e^{-2ix}\left(e^{2ix}+1\right)^{2}\left(-\frac{3x^{2}}{2}-\frac{3ix\ln\left(e^{2ix}+1\right)}{2}-\frac{3\,\mathrm{polylog}(2,}{4}\right.$

input
```
int(x^3*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(a*exp(4*I*x)/(exp(2*I*x)+1)^4)^(1/2)*x^2*(2*x-3*I-3*I*exp(-2*I*x))-2*I*(a
*exp(4*I*x)/(exp(2*I*x)+1)^4)^(1/2)*exp(-2*I*x)*(exp(2*I*x)+1)^2*(-3/2*x^2
-3/2*I*x*ln(exp(2*I*x)+1)-3/4*polylog(2,-exp(2*I*x))+1/2*I*x^3*ln(exp(I*x)
+1)+3/2*x^2*polylog(2,-exp(I*x))+3*I*x*polylog(3,-exp(I*x))-3*polylog(4,-e
xp(I*x))+1/2*I*x^3*ln(1-exp(I*x))+3/2*x^2*polylog(2,exp(I*x))+3*I*x*polylo
g(3,exp(I*x))-3*polylog(4,exp(I*x))-1/2*I*x^3*ln(exp(2*I*x)+1)-3/4*x^2*pol
ylog(2,-exp(2*I*x))-3/4*I*x*polylog(3,-exp(2*I*x))+3/8*polylog(4,-exp(2*I*
x)))
```

3.879.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 740 vs. 2(266) = 532.

Time = 0.35 (sec) , antiderivative size = 740, normalized size of antiderivative = 2.08

$$\int x^3 \csc(x) \sec(x) \sqrt{a \sec^4(x)}\, dx = \text{Too large to display}$$

input
```
integrate(x^3*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x, algorithm="fricas")
```

output
```
3*x*sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, cos(x) + I*sin(x)) + 3*x*sqrt(a/c
os(x)^4)*cos(x)^2*polylog(3, cos(x) - I*sin(x)) - 3*x*sqrt(a/cos(x)^4)*cos
(x)^2*polylog(3, I*cos(x) + sin(x)) - 3*x*sqrt(a/cos(x)^4)*cos(x)^2*polylo
g(3, I*cos(x) - sin(x)) - 3*x*sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, -I*cos(
x) + sin(x)) - 3*x*sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, -I*cos(x) - sin(x)
) + 3*x*sqrt(a/cos(x)^4)*cos(x)^2*polylog(3, -cos(x) + I*sin(x)) + 3*x*sqr
t(a/cos(x)^4)*cos(x)^2*polylog(3, -cos(x) - I*sin(x)) + 3*I*sqrt(a/cos(x)^
4)*cos(x)^2*polylog(4, cos(x) + I*sin(x)) - 3*I*sqrt(a/cos(x)^4)*cos(x)^2*
polylog(4, cos(x) - I*sin(x)) + 3*I*sqrt(a/cos(x)^4)*cos(x)^2*polylog(4, I
*cos(x) + sin(x)) - 3*I*sqrt(a/cos(x)^4)*cos(x)^2*polylog(4, I*cos(x) - si
n(x)) - 3*I*sqrt(a/cos(x)^4)*cos(x)^2*polylog(4, -I*cos(x) + sin(x)) + 3*I
*sqrt(a/cos(x)^4)*cos(x)^2*polylog(4, -I*cos(x) - sin(x)) - 3*I*sqrt(a/cos
(x)^4)*cos(x)^2*polylog(4, -cos(x) + I*sin(x)) + 3*I*sqrt(a/cos(x)^4)*cos(
x)^2*polylog(4, -cos(x) - I*sin(x)) + 1/2*(x^3*cos(x)^2*log(cos(x) + I*sin
(x) + 1) + x^3*cos(x)^2*log(cos(x) - I*sin(x) + 1) + x^3*cos(x)^2*log(-cos
(x) + I*sin(x) + 1) + x^3*cos(x)^2*log(-cos(x) - I*sin(x) + 1) - 3*I*x^2*c
os(x)^2*dilog(cos(x) + I*sin(x)) + 3*I*x^2*cos(x)^2*dilog(cos(x) - I*sin(x
)) + 3*I*x^2*cos(x)^2*dilog(-cos(x) + I*sin(x)) - 3*I*x^2*cos(x)^2*dilog(-
cos(x) - I*sin(x)) - 3*(I*x^2 + I)*cos(x)^2*dilog(I*cos(x) + sin(x)) - 3*(
-I*x^2 - I)*cos(x)^2*dilog(I*cos(x) - sin(x)) - 3*(-I*x^2 - I)*cos(x)^2...
```

3.879.6 Sympy [F]

$$\int x^3 \csc(x)\sec(x)\sqrt{a\sec^4(x)}\,dx = \int x^3\sqrt{a\sec^4(x)}\csc(x)\sec(x)\,dx$$

input
```
integrate(x**3*csc(x)*sec(x)*(a*sec(x)**4)**(1/2),x)
```

output
```
Integral(x**3*sqrt(a*sec(x)**4)*csc(x)*sec(x), x)
```

3.879.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 852 vs. 2(266) = 532.

Time = 0.42 (sec) , antiderivative size = 852, normalized size of antiderivative = 2.39

$$\int x^3 \csc(x)\sec(x)\sqrt{a\sec^4(x)}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x, algorithm="maxima")
```

output
```
(18*x^2*cos(4*x) + 18*I*x^2*sin(4*x) - 2*(4*x^3 + (4*x^3 + 9*x)*cos(4*x) +
 2*(4*x^3 + 9*x)*cos(2*x) - (-4*I*x^3 - 9*I*x)*sin(4*x) - 2*(-4*I*x^3 - 9*
I*x)*sin(2*x) + 9*x)*arctan2(sin(2*x), cos(2*x) + 1) + 6*(x^3*cos(4*x) + 2
*x^3*cos(2*x) + I*x^3*sin(4*x) + 2*I*x^3*sin(2*x) + x^3)*arctan2(sin(x), c
os(x) + 1) - 6*(x^3*cos(4*x) + 2*x^3*cos(2*x) + I*x^3*sin(4*x) + 2*I*x^3*s
in(2*x) + x^3)*arctan2(sin(x), -cos(x) + 1) + 6*(-2*I*x^3 + 3*x^2)*cos(2*x
) + 3*(4*x^2 + (4*x^2 + 3)*cos(4*x) + 2*(4*x^2 + 3)*cos(2*x) + (4*I*x^2 +
3*I)*sin(4*x) + 2*(4*I*x^2 + 3*I)*sin(2*x) + 3)*dilog(-e^(2*I*x)) - 18*(x^
2*cos(4*x) + 2*x^2*cos(2*x) + I*x^2*sin(4*x) + 2*I*x^2*sin(2*x) + x^2)*dil
og(-e^(I*x)) - 18*(x^2*cos(4*x) + 2*x^2*cos(2*x) + I*x^2*sin(4*x) + 2*I*x^
2*sin(2*x) + x^2)*dilog(e^(I*x)) - (-4*I*x^3 + (-4*I*x^3 - 9*I*x)*cos(4*x)
 - 2*(4*I*x^3 + 9*I*x)*cos(2*x) + (4*x^3 + 9*x)*sin(4*x) + 2*(4*x^3 + 9*x)
*sin(2*x) - 9*I*x)*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1) + 3*(-I*x
^3*cos(4*x) - 2*I*x^3*cos(2*x) + x^3*sin(4*x) + 2*x^3*sin(2*x) - I*x^3)*lo
g(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 3*(-I*x^3*cos(4*x) - 2*I*x^3*cos(2
*x) + x^3*sin(4*x) + 2*x^3*sin(2*x) - I*x^3)*log(cos(x)^2 + sin(x)^2 - 2*c
os(x) + 1) - 6*(cos(4*x) + 2*cos(2*x) + I*sin(4*x) + 2*I*sin(2*x) + 1)*pol
ylog(4, -e^(2*I*x)) + 36*(cos(4*x) + 2*cos(2*x) + I*sin(4*x) + 2*I*sin(2*x
) + 1)*polylog(4, -e^(I*x)) + 36*(cos(4*x) + 2*cos(2*x) + I*sin(4*x) + 2*I
*sin(2*x) + 1)*polylog(4, e^(I*x)) + 12*(I*x*cos(4*x) + 2*I*x*cos(2*x) ...
```

3.879.8 Giac [F]

$$\int x^3 \csc(x) \sec(x) \sqrt{a \sec^4(x)}\, dx = \int \sqrt{a \sec(x)^4} x^3 \csc(x) \sec(x)\, dx$$

input
```
integrate(x^3*csc(x)*sec(x)*(a*sec(x)^4)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*sec(x)^4)*x^3*csc(x)*sec(x), x)
```

3.879.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \csc(x) \sec(x) \sqrt{a \sec^4(x)}\, dx = \int \frac{x^3 \sqrt{\frac{a}{\cos(x)^4}}}{\cos(x) \sin(x)}\, dx$$

input
```
int((x^3*(a/cos(x)^4)^(1/2))/(cos(x)*sin(x)),x)
```

output
```
int((x^3*(a/cos(x)^4)^(1/2))/(cos(x)*sin(x)), x)
```

3.880 $\int \sin(x)\sin(2x)\sin(3x)\,dx$

3.880.1 Optimal result

Integrand size = 11, antiderivative size = 25

$$\int \sin(x)\sin(2x)\sin(3x)\,dx = -\frac{1}{8}\cos(2x) - \frac{1}{16}\cos(4x) + \frac{1}{24}\cos(6x)$$

output
```
-1/8*cos(2*x)-1/16*cos(4*x)+1/24*cos(6*x)
```

3.880.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \sin(x)\sin(2x)\sin(3x)\,dx = -\frac{1}{8}\cos(2x) - \frac{1}{16}\cos(4x) + \frac{1}{24}\cos(6x)$$

input
```
Integrate[Sin[x]*Sin[2*x]*Sin[3*x],x]
```

output
```
-1/8*Cos[2*x] - Cos[4*x]/16 + Cos[6*x]/24
```

3.880.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 4855, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\sin(2x)\sin(3x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x)\sin(2x)\sin(3x)dx$$

$$\Big\downarrow \text{4855}$$

$$\int \left(\frac{1}{4}\sin(2x)+\frac{1}{4}\sin(4x)-\frac{1}{4}\sin(6x)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{8}\cos(2x)-\frac{1}{16}\cos(4x)+\frac{1}{24}\cos(6x)$$

input
```
Int[Sin[x]*Sin[2*x]*Sin[3*x],x]
```

output
```
-1/8*Cos[2*x] - Cos[4*x]/16 + Cos[6*x]/24
```

3.880.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4855
```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.)*(H_)[(e_.
) + (f_.)*(x_)]^(r_.), x_Symbol] :> Int[ExpandTrigReduce[ActivateTrig[F[a +
 b*x]^p*G[c + d*x]^q*H[e + f*x]^r], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && (EqQ[H,
sin] || EqQ[H, cos]) && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

3.880.4 Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{\cos(2x)}{8}-\frac{\cos(4x)}{16}+\frac{\cos(6x)}{24}$	20
risch	$-\frac{\cos(2x)}{8}-\frac{\cos(4x)}{16}+\frac{\cos(6x)}{24}$	20
parallelrisch	$-\frac{29}{48}+\frac{\cos(6x)}{24}-\frac{\cos(4x)}{16}-\frac{\cos(2x)}{8}$	21

input
```
int(sin(x)*sin(2*x)*sin(3*x),x,method=_RETURNVERBOSE)
```

output
```
-1/8*cos(2*x)-1/16*cos(4*x)+1/24*cos(6*x)
```

3.880.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.68

$$\int \sin(x)\sin(2x)\sin(3x)\,dx=\frac{4}{3}\cos(x)^6-\frac{5}{2}\cos(x)^4+\cos(x)^2$$

input
```
integrate(sin(x)*sin(2*x)*sin(3*x),x, algorithm="fricas")
```

output
```
4/3*cos(x)^6 - 5/2*cos(x)^4 + cos(x)^2
```

3.880.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(19) = 38.

Time = 0.87 (sec) , antiderivative size = 112, normalized size of antiderivative = 4.48

$$\begin{aligned}\int \sin(x)\sin(2x)\sin(3x)\,dx=&\ \frac{x\sin(x)\sin(2x)\sin(3x)}{4}+\frac{x\sin(x)\cos(2x)\cos(3x)}{4}\\&+\frac{x\sin(2x)\cos(x)\cos(3x)}{4}\\&-\frac{x\sin(3x)\cos(x)\cos(2x)}{4}-\frac{\sin(x)\sin(2x)\cos(3x)}{3}\\&+\frac{\sin(x)\sin(3x)\cos(2x)}{8}-\frac{\cos(x)\cos(2x)\cos(3x)}{24}\end{aligned}$$

input
```
integrate(sin(x)*sin(2*x)*sin(3*x),x)
```

output
```
x*sin(x)*sin(2*x)*sin(3*x)/4 + x*sin(x)*cos(2*x)*cos(3*x)/4 + x*sin(2*x)*c
os(x)*cos(3*x)/4 - x*sin(3*x)*cos(x)*cos(2*x)/4 - sin(x)*sin(2*x)*cos(3*x)
/3 + sin(x)*sin(3*x)*cos(2*x)/8 - cos(x)*cos(2*x)*cos(3*x)/24
```

3.880.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \sin(x)\sin(2x)\sin(3x)\,dx = \frac{1}{24}\cos(6x) - \frac{1}{16}\cos(4x) - \frac{1}{8}\cos(2x)$$

input
```
integrate(sin(x)*sin(2*x)*sin(3*x),x, algorithm="maxima")
```

output
```
1/24*cos(6*x) - 1/16*cos(4*x) - 1/8*cos(2*x)
```

3.880.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.52

$$\int \sin(x)\sin(2x)\sin(3x)\,dx = -\frac{4}{3}\sin(x)^6 + \frac{3}{2}\sin(x)^4$$

input
```
integrate(sin(x)*sin(2*x)*sin(3*x),x, algorithm="giac")
```

output
```
-4/3*sin(x)^6 + 3/2*sin(x)^4
```

3.880.9 Mupad [B] (verification not implemented)

Time = 26.89 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.56

$$\int \sin(x)\sin(2x)\sin(3x)\,dx = -\frac{\sin(x)^4\,(8\sin(x)^2 - 9)}{6}$$

input
```
int(sin(2*x)*sin(3*x)*sin(x),x)
```

output
```
-(sin(x)^4*(8*sin(x)^2 - 9))/6
```

3.881 $\int \cos(x)\cos(2x)\cos(3x)\,dx$

3.881.1 Optimal result

Integrand size = 11, antiderivative size = 30

$$\int \cos(x)\cos(2x)\cos(3x)\,dx=\frac{x}{4}+\frac{1}{8}\sin(2x)+\frac{1}{16}\sin(4x)+\frac{1}{24}\sin(6x)$$

output
```
1/4*x+1/8*sin(2*x)+1/16*sin(4*x)+1/24*sin(6*x)
```

3.881.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \cos(x)\cos(2x)\cos(3x)\,dx=\frac{x}{4}+\frac{1}{8}\sin(2x)+\frac{1}{16}\sin(4x)+\frac{1}{24}\sin(6x)$$

input
```
Integrate[Cos[x]*Cos[2*x]*Cos[3*x],x]
```

output
```
x/4 + Sin[2*x]/8 + Sin[4*x]/16 + Sin[6*x]/24
```

3.881.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 4855, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(x)\cos(2x)\cos(3x)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \cos(x)\cos(2x)\cos(3x)dx$$

$$\Big\downarrow \ 4855$$

$$\int \left(\frac{1}{4}\cos(2x)+\frac{1}{4}\cos(4x)+\frac{1}{4}\cos(6x)+\frac{1}{4}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{x}{4}+\frac{1}{8}\sin(2x)+\frac{1}{16}\sin(4x)+\frac{1}{24}\sin(6x)$$

input
```
Int[Cos[x]*Cos[2*x]*Cos[3*x],x]
```

output
```
x/4 + Sin[2*x]/8 + Sin[4*x]/16 + Sin[6*x]/24
```

3.881.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4855
```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.)*(H_)[(e_.
) + (f_.)*(x_)]^(r_.), x_Symbol] :> Int[ExpandTrigReduce[ActivateTrig[F[a +
 b*x]^p*G[c + d*x]^q*H[e + f*x]^r], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && (EqQ[H,
sin] || EqQ[H, cos]) && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

3.881.4 Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{x}{4}+\frac{\sin(2x)}{8}+\frac{\sin(4x)}{16}+\frac{\sin(6x)}{24}$	23
risch	$\frac{x}{4}+\frac{\sin(2x)}{8}+\frac{\sin(4x)}{16}+\frac{\sin(6x)}{24}$	23
parallelrisch	$\frac{x}{4}+\frac{\sin(2x)}{8}+\frac{\sin(4x)}{16}+\frac{\sin(6x)}{24}$	23

input
```
int(cos(x)*cos(2*x)*cos(3*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*x+1/8*sin(2*x)+1/16*sin(4*x)+1/24*sin(6*x)
```

3.881.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \cos(x)\cos(2x)\cos(3x)\,dx = \frac{1}{12}\left(16\,\cos\left(x\right)^{5}-10\,\cos\left(x\right)^{3}+3\,\cos\left(x\right)\right)\sin\left(x\right)+\frac{1}{4}\,x$$

input
```
integrate(cos(x)*cos(2*x)*cos(3*x),x, algorithm="fricas")
```

output
```
1/12*(16*cos(x)^5 - 10*cos(x)^3 + 3*cos(x))*sin(x) + 1/4*x
```

3.881.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(22) = 44.

Time = 0.86 (sec) , antiderivative size = 114, normalized size of antiderivative = 3.80

$$\begin{aligned}\int \cos(x)\cos(2x)\cos(3x)\,dx = &-\frac{x\sin\left(x\right)\sin\left(2x\right)\cos\left(3x\right)}{4}+\frac{x\sin\left(x\right)\sin\left(3x\right)\cos\left(2x\right)}{4}\\ &+\frac{x\sin\left(2x\right)\sin\left(3x\right)\cos\left(x\right)}{4}+\frac{x\cos\left(x\right)\cos\left(2x\right)\cos\left(3x\right)}{4}\\ &+\frac{\sin\left(x\right)\sin\left(2x\right)\sin\left(3x\right)}{6}+\frac{\sin\left(x\right)\cos\left(2x\right)\cos\left(3x\right)}{8}\\ &+\frac{5\sin\left(3x\right)\cos\left(x\right)\cos\left(2x\right)}{24}\end{aligned}$$

input
```
integrate(cos(x)*cos(2*x)*cos(3*x),x)
```

output
```
-x*sin(x)*sin(2*x)*cos(3*x)/4 + x*sin(x)*sin(3*x)*cos(2*x)/4 + x*sin(2*x)*
sin(3*x)*cos(x)/4 + x*cos(x)*cos(2*x)*cos(3*x)/4 + sin(x)*sin(2*x)*sin(3*x
)/6 + sin(x)*cos(2*x)*cos(3*x)/8 + 5*sin(3*x)*cos(x)*cos(2*x)/24
```

3.881.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \cos(x)\cos(2x)\cos(3x)\,dx = \frac{1}{4}\,x + \frac{1}{24}\,\sin\left(6\,x\right) + \frac{1}{16}\,\sin\left(4\,x\right) + \frac{1}{8}\,\sin\left(2\,x\right)$$

input
```
integrate(cos(x)*cos(2*x)*cos(3*x),x, algorithm="maxima")
```

output
```
1/4*x + 1/24*sin(6*x) + 1/16*sin(4*x) + 1/8*sin(2*x)
```

3.881.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \cos(x)\cos(2x)\cos(3x)\,dx = \frac{1}{4}\,x + \frac{1}{24}\,\sin\left(6\,x\right) + \frac{1}{16}\,\sin\left(4\,x\right) + \frac{1}{8}\,\sin\left(2\,x\right)$$

input
```
integrate(cos(x)*cos(2*x)*cos(3*x),x, algorithm="giac")
```

output
```
1/4*x + 1/24*sin(6*x) + 1/16*sin(4*x) + 1/8*sin(2*x)
```

3.881.9 Mupad [B] (verification not implemented)

Time = 26.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \cos(x)\cos(2x)\cos(3x)\,dx = \frac{x}{4} + \frac{\sin\left(2\,x\right)}{8} + \frac{\sin\left(4\,x\right)}{16} + \frac{\sin\left(6\,x\right)}{24}$$

input
```
int(cos(2*x)*cos(3*x)*cos(x),x)
```

output
```
x/4 + sin(2*x)/8 + sin(4*x)/16 + sin(6*x)/24
```

3.882 $\int \cos(x)\sin(2x)\sin(3x)\,dx$

3.882.1 Optimal result

Integrand size = 11, antiderivative size = 30

$$\int \cos(x)\sin(2x)\sin(3x)\,dx = \frac{x}{4} + \frac{1}{8}\sin(2x) - \frac{1}{16}\sin(4x) - \frac{1}{24}\sin(6x)$$

output
```
1/4*x+1/8*sin(2*x)-1/16*sin(4*x)-1/24*sin(6*x)
```

3.882.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \cos(x)\sin(2x)\sin(3x)\,dx = \frac{x}{4} + \frac{1}{8}\sin(2x) - \frac{1}{16}\sin(4x) - \frac{1}{24}\sin(6x)$$

input
```
Integrate[Cos[x]*Sin[2*x]*Sin[3*x],x]
```

output
```
x/4 + Sin[2*x]/8 - Sin[4*x]/16 - Sin[6*x]/24
```

3.882.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 4855, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(2x)\sin(3x)\cos(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(2x)\sin(3x)\cos(x)dx$$

$$\Big\downarrow \text{4855}$$

$$\int \left(\frac{1}{4}\cos(2x)-\frac{1}{4}\cos(4x)-\frac{1}{4}\cos(6x)+\frac{1}{4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x}{4}+\frac{1}{8}\sin(2x)-\frac{1}{16}\sin(4x)-\frac{1}{24}\sin(6x)$$

input
```
Int[Cos[x]*Sin[2*x]*Sin[3*x],x]
```

output
```
x/4 + Sin[2*x]/8 - Sin[4*x]/16 - Sin[6*x]/24
```

3.882.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4855
```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.)*(H_)[(e_.
) + (f_.)*(x_)]^(r_.), x_Symbol] :> Int[ExpandTrigReduce[ActivateTrig[F[a +
 b*x]^p*G[c + d*x]^q*H[e + f*x]^r], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && (EqQ[H,
sin] || EqQ[H, cos]) && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

3.882.4 Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{x}{4}+\frac{\sin(2x)}{8}-\frac{\sin(4x)}{16}-\frac{\sin(6x)}{24}$	23
risch	$\frac{x}{4}+\frac{\sin(2x)}{8}-\frac{\sin(4x)}{16}-\frac{\sin(6x)}{24}$	23
parallelrisch	$\frac{x}{4}+\frac{\sin(2x)}{8}-\frac{\sin(4x)}{16}-\frac{\sin(6x)}{24}$	23

input
```
int(cos(x)*sin(2*x)*sin(3*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*x+1/8*sin(2*x)-1/16*sin(4*x)-1/24*sin(6*x)
```

3.882.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \cos(x)\sin(2x)\sin(3x)\,dx = -\frac{1}{12}\left(16\cos(x)^5-10\cos(x)^3-3\cos(x)\right)\sin(x)+\frac{1}{4}x$$

input
```
integrate(cos(x)*sin(2*x)*sin(3*x),x, algorithm="fricas")
```

output
```
-1/12*(16*cos(x)^5 - 10*cos(x)^3 - 3*cos(x))*sin(x) + 1/4*x
```

3.882.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(22) = 44.

Time = 0.89 (sec) , antiderivative size = 116, normalized size of antiderivative = 3.87

$$\begin{aligned}\int \cos(x)\sin(2x)\sin(3x)\,dx = &-\frac{x\sin(x)\sin(2x)\cos(3x)}{4}+\frac{x\sin(x)\sin(3x)\cos(2x)}{4}\\&+\frac{x\sin(2x)\sin(3x)\cos(x)}{4}+\frac{x\cos(x)\cos(2x)\cos(3x)}{4}\\&+\frac{\sin(x)\sin(2x)\sin(3x)}{3}+\frac{3\sin(x)\cos(2x)\cos(3x)}{8}\\&-\frac{5\sin(3x)\cos(x)\cos(2x)}{24}\end{aligned}$$

input
```
integrate(cos(x)*sin(2*x)*sin(3*x),x)
```

output
```
-x*sin(x)*sin(2*x)*cos(3*x)/4 + x*sin(x)*sin(3*x)*cos(2*x)/4 + x*sin(2*x)*
sin(3*x)*cos(x)/4 + x*cos(x)*cos(2*x)*cos(3*x)/4 + sin(x)*sin(2*x)*sin(3*x
)/3 + 3*sin(x)*cos(2*x)*cos(3*x)/8 - 5*sin(3*x)*cos(x)*cos(2*x)/24
```

3.882.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \cos(x)\sin(2x)\sin(3x)\,dx = \frac{1}{4}\,x - \frac{1}{24}\,\sin\left(6\,x\right) - \frac{1}{16}\,\sin\left(4\,x\right) + \frac{1}{8}\,\sin\left(2\,x\right)$$

input
```
integrate(cos(x)*sin(2*x)*sin(3*x),x, algorithm="maxima")
```

output
```
1/4*x - 1/24*sin(6*x) - 1/16*sin(4*x) + 1/8*sin(2*x)
```

3.882.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \cos(x)\sin(2x)\sin(3x)\,dx = \frac{1}{4}\,x - \frac{1}{24}\,\sin\left(6\,x\right) - \frac{1}{16}\,\sin\left(4\,x\right) + \frac{1}{8}\,\sin\left(2\,x\right)$$

input
```
integrate(cos(x)*sin(2*x)*sin(3*x),x, algorithm="giac")
```

output
```
1/4*x - 1/24*sin(6*x) - 1/16*sin(4*x) + 1/8*sin(2*x)
```

3.882.9 Mupad [B] (verification not implemented)

Time = 26.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \cos(x)\sin(2x)\sin(3x)\,dx = \frac{x}{4} + \frac{\sin\left(2\,x\right)}{8} - \frac{\sin\left(4\,x\right)}{16} - \frac{\sin\left(6\,x\right)}{24}$$

input
```
int(sin(2*x)*sin(3*x)*cos(x),x)
```

output
```
x/4 + sin(2*x)/8 - sin(4*x)/16 - sin(6*x)/24
```

3.883 $\int \cos(2x)\cos(3x)\sin(x)\,dx$

3.883.1 Optimal result

Integrand size = 11, antiderivative size = 25

$$\int \cos(2x)\cos(3x)\sin(x)\,dx = -\frac{1}{8}\cos(2x) + \frac{1}{16}\cos(4x) - \frac{1}{24}\cos(6x)$$

output
```
-1/8*cos(2*x)+1/16*cos(4*x)-1/24*cos(6*x)
```

3.883.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \cos(2x)\cos(3x)\sin(x)\,dx = -\frac{1}{8}\cos(2x) + \frac{1}{16}\cos(4x) - \frac{1}{24}\cos(6x)$$

input
```
Integrate[Cos[2*x]*Cos[3*x]*Sin[x],x]
```

output
```
-1/8*Cos[2*x] + Cos[4*x]/16 - Cos[6*x]/24
```

3.883.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 4855, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\cos(2x)\cos(3x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x)\cos(2x)\cos(3x)dx$$

$$\Big\downarrow \text{4855}$$

$$\int \left(\frac{1}{4}\sin(2x)-\frac{1}{4}\sin(4x)+\frac{1}{4}\sin(6x)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{8}\cos(2x)+\frac{1}{16}\cos(4x)-\frac{1}{24}\cos(6x)$$

input
```
Int[Cos[2*x]*Cos[3*x]*Sin[x],x]
```

output
```
-1/8*Cos[2*x] + Cos[4*x]/16 - Cos[6*x]/24
```

3.883.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4855
```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.)*(H_)[(e_.
) + (f_.)*(x_)]^(r_.), x_Symbol] :> Int[ExpandTrigReduce[ActivateTrig[F[a +
 b*x]^p*G[c + d*x]^q*H[e + f*x]^r], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && (EqQ[H,
sin] || EqQ[H, cos]) && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

3.883.4 Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
default	$-\frac{\cos(2x)}{8}+\frac{\cos(4x)}{16}-\frac{\cos(6x)}{24}$	20
risch	$-\frac{\cos(2x)}{8}+\frac{\cos(4x)}{16}-\frac{\cos(6x)}{24}$	20
parallelrisch	$-\frac{7}{48}-\frac{\cos(6x)}{24}+\frac{\cos(4x)}{16}-\frac{\cos(2x)}{8}$	21

input
```
int(cos(2*x)*cos(3*x)*sin(x),x,method=_RETURNVERBOSE)
```

output
```
-1/8*cos(2*x)+1/16*cos(4*x)-1/24*cos(6*x)
```

3.883.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \cos(2x)\cos(3x)\sin(x)\,dx = -\frac{4}{3}\cos(x)^6+\frac{5}{2}\cos(x)^4-\frac{3}{2}\cos(x)^2$$

input
```
integrate(cos(2*x)*cos(3*x)*sin(x),x, algorithm="fricas")
```

output
```
-4/3*cos(x)^6 + 5/2*cos(x)^4 - 3/2*cos(x)^2
```

3.883.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(19) = 38.

Time = 0.86 (sec) , antiderivative size = 114, normalized size of antiderivative = 4.56

$$\begin{aligned}\int \cos(2x)\cos(3x)\sin(x)\,dx &= \frac{x\sin(x)\sin(2x)\sin(3x)}{4}+\frac{x\sin(x)\cos(2x)\cos(3x)}{4}\\ &+\frac{x\sin(2x)\cos(x)\cos(3x)}{4}\\ &-\frac{x\sin(3x)\cos(x)\cos(2x)}{4}-\frac{\sin(x)\sin(2x)\cos(3x)}{6}\\ &+\frac{3\sin(x)\sin(3x)\cos(2x)}{8}+\frac{\cos(x)\cos(2x)\cos(3x)}{24}\end{aligned}$$

input
```
integrate(cos(2*x)*cos(3*x)*sin(x),x)
```

output
```
x*sin(x)*sin(2*x)*sin(3*x)/4 + x*sin(x)*cos(2*x)*cos(3*x)/4 + x*sin(2*x)*c
os(x)*cos(3*x)/4 - x*sin(3*x)*cos(x)*cos(2*x)/4 - sin(x)*sin(2*x)*cos(3*x)
/6 + 3*sin(x)*sin(3*x)*cos(2*x)/8 + cos(x)*cos(2*x)*cos(3*x)/24
```

3.883.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \cos(2x)\cos(3x)\sin(x)\,dx = -\frac{1}{24}\cos(6x) + \frac{1}{16}\cos(4x) - \frac{1}{8}\cos(2x)$$

input
```
integrate(cos(2*x)*cos(3*x)*sin(x),x, algorithm="maxima")
```

output
```
-1/24*cos(6*x) + 1/16*cos(4*x) - 1/8*cos(2*x)
```

3.883.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \cos(2x)\cos(3x)\sin(x)\,dx = \frac{4}{3}\sin(x)^6 - \frac{3}{2}\sin(x)^4 + \frac{1}{2}\sin(x)^2$$

input
```
integrate(cos(2*x)*cos(3*x)*sin(x),x, algorithm="giac")
```

output
```
4/3*sin(x)^6 - 3/2*sin(x)^4 + 1/2*sin(x)^2
```

3.883.9 Mupad [B] (verification not implemented)

Time = 26.57 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \cos(2x)\cos(3x)\sin(x)\,dx = \frac{4\sin(x)^6}{3} - \frac{3\sin(x)^4}{2} + \frac{\sin(x)^2}{2}$$

input
```
int(cos(2*x)*cos(3*x)*sin(x),x)
```

output
```
sin(x)^2/2 - (3*sin(x)^4)/2 + (4*sin(x)^6)/3
```

3.884 $\int x \sin(x^2)\,dx$

3.884.1 Optimal result

Integrand size = 6, antiderivative size = 8

$$\int x \sin(x^2)\,dx = -\frac{1}{2}\cos(x^2)$$

output
```
-1/2*cos(x^2)
```

3.884.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int x \sin(x^2)\,dx = -\frac{1}{2}\cos(x^2)$$

input
```
Integrate[x*Sin[x^2],x]
```

output
```
-1/2*Cos[x^2]
```

3.884.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3860, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sin\left(x^2\right) dx$$

$$\Big\downarrow \text{ 3860}$$

$$\frac{1}{2}\int \sin\left(x^2\right) dx^2$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}\int \sin\left(x^2\right) dx^2$$

$$\Big\downarrow \text{ 3118}$$

$$-\frac{1}{2}\cos\left(x^2\right)$$

input
```
Int[x*Sin[x^2],x]
```

output
```
-1/2*Cos[x^2]
```

3.884.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3860
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

3.884.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$-\frac{\cos\left(x^{2}\right)}{2}$	7
default	$-\frac{\cos\left(x^{2}\right)}{2}$	7
risch	$-\frac{\cos\left(x^{2}\right)}{2}$	7
parallelrisch	$-\frac{\cos\left(x^{2}\right)}{2}-\frac{1}{2}$	9
norman	$-\frac{1}{1+\tan\left(\frac{x^{2}}{2}\right)^{2}}$	15
meijerg	$\frac{\sqrt{\pi}\,\left(\frac{1}{\sqrt{\pi}}-\frac{\cos\left(x^{2}\right)}{\sqrt{\pi}}\right)}{2}$	19
parts	$\frac{\sqrt{2}\,\sqrt{\pi}\,\operatorname{FresnelS}\left(\frac{\sqrt{2}\,x}{\sqrt{\pi}}\right)x}{2}-\frac{\pi\left(\frac{\operatorname{FresnelS}\left(\frac{\sqrt{2}\,x}{\sqrt{\pi}}\right)\sqrt{2}\,x}{\sqrt{\pi}}+\frac{\cos\left(x^{2}\right)}{\pi}\right)}{2}$	49

input
```
int(x*sin(x^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*cos(x^2)
```

3.884.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int x\sin\left(x^2\right)\,dx=-\frac{1}{2}\,\cos\left(x^2\right)$$

input
```
integrate(x*sin(x^2),x, algorithm="fricas")
```

output
```
-1/2*cos(x^2)
```

3.884.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int x\sin\left(x^2\right)\,dx = -\frac{\cos\left(x^2\right)}{2}$$

input
```
integrate(x*sin(x**2),x)
```

output
```
-cos(x**2)/2
```

3.884.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int x\sin\left(x^2\right)\,dx = -\frac{1}{2}\,\cos\left(x^2\right)$$

input
```
integrate(x*sin(x^2),x, algorithm="maxima")
```

output
```
-1/2*cos(x^2)
```

3.884.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int x\sin\left(x^2\right)\,dx = -\frac{1}{2}\,\cos\left(x^2\right)$$

input
```
integrate(x*sin(x^2),x, algorithm="giac")
```

output
```
-1/2*cos(x^2)
```

3.884.9 Mupad [B] (verification not implemented)

Time = 26.71 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int x\sin\left(x^2\right)\,dx=-\frac{\cos\left(x^2\right)}{2}$$

input
```
int(x*sin(x^2),x)
```

output
```
-cos(x^2)/2
```

3.885 $\int (-\cos(x)+\sin(x))(\cos(x)+\sin(x))^5\,dx$

3.885.1 Optimal result

Integrand size = 15, antiderivative size = 11

$$\int (-\cos(x)+\sin(x))(\cos(x)+\sin(x))^5\,dx = -\frac{1}{6}(\cos(x)+\sin(x))^6$$

output
```
-1/6*(cos(x)+sin(x))^6
```

3.885.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int (-\cos(x)+\sin(x))(\cos(x)+\sin(x))^5\,dx = -\frac{1}{6}(\cos(x)+\sin(x))^6$$

input
```
Integrate[(-Cos[x] + Sin[x])*(Cos[x] + Sin[x])^5,x]
```

output
```
-1/6*(Cos[x] + Sin[x])^6
```

3.885.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3042, 3624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\sin(x)-\cos(x))(\sin(x)+\cos(x))^5\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (\sin(x)-\cos(x))(\sin(x)+\cos(x))^5dx$$

$$\Big\downarrow \text{ 3624}$$

$$-\frac{1}{6}(\sin(x)+\cos(x))^6$$

input
```
Int[(-Cos[x] + Sin[x])*(Cos[x] + Sin[x])^5,x]
```

output
```
-1/6*(Cos[x] + Sin[x])^6
```

3.885.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3624
```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(n_.)*(
cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :
> Simp[(c*B - b*C)*((b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)/(e*(n + 1)*(b
^2 + c^2))), x] /; FreeQ[{b, c, d, e, B, C}, x] && NeQ[n, -1] && NeQ[b^2 +
c^2, 0] && EqQ[b*B + c*C, 0]
```

3.885.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(9) = 18.

Time = 2.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.82

method	result
risch	$\frac{\sin(6x)}{24}+\frac{\cos(4x)}{4}-\frac{5\sin(2x)}{8}$
parallelrisch	$-\frac{21}{20}-\frac{5\sin(2x)}{8}+\frac{\sin(6x)}{24}+\frac{\cos(4x)}{4}$
norman	$\frac{-8\tan\left(\frac{x}{2}\right)^{2}-\frac{50\tan\left(\frac{x}{2}\right)^{3}}{3}+28\tan\left(\frac{x}{2}\right)^{5}+16\tan\left(\frac{x}{2}\right)^{6}-28\tan\left(\frac{x}{2}\right)^{7}+\frac{50\tan\left(\frac{x}{2}\right)^{9}}{3}-8\tan\left(\frac{x}{2}\right)^{10}+2\tan\left(\frac{x}{2}\right)^{11}-2\tan\left(\frac{x}{2}\right)}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)^{6}}$
default	$\frac{2\sin(x)^{6}}{3}-\frac{5\cos(x)^{3}\sin(x)^{3}}{6}-\frac{5\cos(x)^{3}\sin(x)}{8}+\frac{5\cos(x)\sin(x)}{16}+\frac{5\cos(x)^{5}\sin(x)}{6}-\frac{5\left(\cos(x)^{3}+\frac{3\cos(x)}{2}\right)\sin(x)}{24}+$
parts	$\frac{2\sin(x)^{6}}{3}-\frac{5\cos(x)^{3}\sin(x)^{3}}{6}-\frac{5\cos(x)^{3}\sin(x)}{8}+\frac{5\cos(x)\sin(x)}{16}+\frac{5\cos(x)^{5}\sin(x)}{6}-\frac{5\left(\cos(x)^{3}+\frac{3\cos(x)}{2}\right)\sin(x)}{24}+$

input
```
int((-cos(x)+sin(x))*(sin(x)+cos(x))^5,x,method=_RETURNVERBOSE)
```

output
```
1/24*sin(6*x)+1/4*cos(4*x)-5/8*sin(2*x)
```

3.885.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. 2(9) = 18.

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 3.09

$$\int(-\cos(x)+\sin(x))(\cos(x)+\sin(x))^5\,dx=2\cos(x)^4-2\cos(x)^2+\frac{1}{3}\left(4\cos(x)^5-4\cos(x)^3-3\cos(x)\right)\sin(x)$$

input
```
integrate((-cos(x)+sin(x))*(cos(x)+sin(x))^5,x, algorithm="fricas")
```

output
```
2*cos(x)^4 - 2*cos(x)^2 + 1/3*(4*cos(x)^5 - 4*cos(x)^3 - 3*cos(x))*sin(x)
```

3.885.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(10) = 20.

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 4.18

$$\int (-\cos(x)+\sin(x))(\cos(x)+\sin(x))^5\,dx = \frac{2\sin^6(x)}{3} - \sin^5(x)\cos(x) - \frac{10\sin^3(x)\cos^3(x)}{3} - \sin(x)\cos^5(x) + \frac{2\cos^6(x)}{3}$$

input
```
integrate((-cos(x)+sin(x))*(cos(x)+sin(x))**5,x)
```

output
```
2*sin(x)**6/3 - sin(x)**5*cos(x) - 10*sin(x)**3*cos(x)**3/3 - sin(x)*cos(x
)**5 + 2*cos(x)**6/3
```

3.885.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int (-\cos(x)+\sin(x))(\cos(x)+\sin(x))^5\,dx = -\frac{1}{6}(\cos(x)+\sin(x))^6$$

input
```
integrate((-cos(x)+sin(x))*(cos(x)+sin(x))^5,x, algorithm="maxima")
```

output
```
-1/6*(cos(x) + sin(x))^6
```

3.885.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 19 vs. 2(9) = 18.

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int (-\cos(x)+\sin(x))(\cos(x)+\sin(x))^5\,dx = \frac{1}{4}\cos(4x) + \frac{1}{24}\sin(6x) - \frac{5}{8}\sin(2x)$$

input
```
integrate((-cos(x)+sin(x))*(cos(x)+sin(x))^5,x, algorithm="giac")
```

output
```
1/4*cos(4*x) + 1/24*sin(6*x) - 5/8*sin(2*x)
```

3.885.9 Mupad [B] (verification not implemented)

Time = 26.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.82

$$\int (-\cos(x)+\sin(x))(\cos(x)+\sin(x))^5 \, dx = -\frac{\sin(2x)\, \left(\sin(2x)^2+3\sin(2x)+3\right)}{6}$$

input
```
int(-(cos(x) + sin(x))^5*(cos(x) - sin(x)),x)
```

output
```
-(sin(2*x)*(3*sin(2*x) + sin(2*x)^2 + 3))/6
```

3.886 $\int 2x \sec^2(x) \tan(x) \, dx$

3.886.1 Optimal result

Integrand size = 9, antiderivative size = 11

$$\int 2x \sec^2(x) \tan(x) \, dx = x \sec^2(x) - \tan(x)$$

output
```
x*sec(x)^2-tan(x)
```

3.886.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.64

$$\int 2x \sec^2(x) \tan(x) \, dx = 2\left(\frac{1}{2}x \sec^2(x) - \frac{\tan(x)}{2}\right)$$

input
```
Integrate[2*x*Sec[x]^2*Tan[x],x]
```

output
```
2*((x*Sec[x]^2)/2 - Tan[x]/2)
```

3.886.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.64, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {27, 4244, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int 2x\tan(x)\sec^2(x)\,dx$$
$$\Big\downarrow \text{27}$$
$$2\int x\sec^2(x)\tan(x)dx$$
$$\Big\downarrow \text{4244}$$
$$2\left(\frac{1}{2}x\sec^2(x)-\frac{1}{2}\int \sec^2(x)dx\right)$$
$$\Big\downarrow \text{3042}$$
$$2\left(\frac{1}{2}x\sec^2(x)-\frac{1}{2}\int \csc\left(x+\frac{\pi}{2}\right)^2dx\right)$$
$$\Big\downarrow \text{4254}$$
$$2\left(\frac{1}{2}\int 1d(-\tan(x))+\frac{1}{2}x\sec^2(x)\right)$$
$$\Big\downarrow \text{24}$$
$$2\left(\frac{1}{2}x\sec^2(x)-\frac{\tan(x)}{2}\right)$$

input
```
Int[2*x*Sec[x]^2*Tan[x],x]
```

output
```
2*((x*Sec[x]^2)/2 - Tan[x]/2)
```

3.886.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4244
```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tan[(a_.) + (b_.)*(x_)^(
n_.)]^(q_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sec[a + b*x^n]^p/(b*n*p)), x]
 - Simp[(m - n + 1)/(b*n*p)   Int[x^(m - n)*Sec[a + b*x^n]^p, x], x] /; Fre
eQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m, n] && EqQ[q, 1]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.886.4 Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
default	$\frac{x}{\cos\left(x\right)^{2}}-\tan\left(x\right)$	12
risch	$\frac{-2 i \,{\mathrm e}^{2 i x}+4 \,{\mathrm e}^{2 i x} x -2 i}{\left({\mathrm e}^{2 i x}+1\right)^{2}}$	31

input
```
int(2*x*sec(x)^2*tan(x),x,method=_RETURNVERBOSE)
```

output
```
x/cos(x)^2-tan(x)
```

3.886.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int 2x\sec^2(x)\tan(x)\,dx = -\frac{\cos(x)\sin(x) - x}{\cos(x)^2}$$

input
```
integrate(2*x*sec(x)^2*tan(x),x, algorithm="fricas")
```

output
```
-(cos(x)*sin(x) - x)/cos(x)^2
```

3.886.6 Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.73

$$\int 2x\sec^2(x)\tan(x)\,dx = x\sec^2(x) - \tan(x)$$

input
```
integrate(2*x*sec(x)**2*tan(x),x)
```

output
```
x*sec(x)**2 - tan(x)
```

3.886.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. 2(11) = 22.

Time = 0.21 (sec) , antiderivative size = 133, normalized size of antiderivative = 12.09

$$\int 2x\sec^2(x)\tan(x)\,dx = \frac{2\left(4x\cos(2x)^2 + 4x\sin(2x)^2 + (2x\cos(2x) + \sin(2x))\cos(4x) + 2x\cos(2x) + (2x\sin(2x) - \cos(2\right.}{2(2\cos(2x) + 1)\cos(4x) + \cos(4x)^2 + 4\cos(2x)^2 + \sin(4x)^2 + 4\sin(4x)\sin(2x) + 4\sin(2}$$

input
```
integrate(2*x*sec(x)^2*tan(x),x, algorithm="maxima")
```

output
```
2*(4*x*cos(2*x)^2 + 4*x*sin(2*x)^2 + (2*x*cos(2*x) + sin(2*x))*cos(4*x) +
2*x*cos(2*x) + (2*x*sin(2*x) - cos(2*x) - 1)*sin(4*x) - sin(2*x))/(2*(2*co
s(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 4*cos(2*x)^2 + sin(4*x)^2 + 4*sin(4*x)
*sin(2*x) + 4*sin(2*x)^2 + 4*cos(2*x) + 1)
```

3.886.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(11) = 22$.

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 4.73

$$\int 2x\sec^2(x)\tan(x)\,dx = \frac{x\tan\left(\frac{1}{2}x\right)^4 + 2x\tan\left(\frac{1}{2}x\right)^2 + 2\tan\left(\frac{1}{2}x\right)^3 + x - 2\tan\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)^4 - 2\tan\left(\frac{1}{2}x\right)^2 + 1}$$

input
```
integrate(2*x*sec(x)^2*tan(x),x, algorithm="giac")
```

output
```
(x*tan(1/2*x)^4 + 2*x*tan(1/2*x)^2 + 2*tan(1/2*x)^3 + x - 2*tan(1/2*x))/(t
an(1/2*x)^4 - 2*tan(1/2*x)^2 + 1)
```

3.886.9 Mupad [B] (verification not implemented)

Time = 26.48 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.45

$$\int 2x\sec^2(x)\tan(x)\,dx = \frac{2x - \sin(2x)}{2\cos(x)^2}$$

input
```
int((2*x*tan(x))/cos(x)^2,x)
```

output
```
(2*x - sin(2*x))/(2*cos(x)^2)
```

3.887 $\int \frac{1+\cos^2(x)}{1+\cos(2x)}\,dx$

3.887.1 Optimal result

Integrand size = 15, antiderivative size = 12

$$\int \frac{1+\cos^2(x)}{1+\cos(2x)}\,dx=\frac{x}{2}+\frac{\tan(x)}{2}$$

output
```
1/2*x+1/2*tan(x)
```

3.887.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1+\cos^2(x)}{1+\cos(2x)}\,dx=\frac{x}{2}+\frac{\tan(x)}{2}$$

input
```
Integrate[(1 + Cos[x]^2)/(1 + Cos[2*x]),x]
```

output
```
x/2 + Tan[x]/2
```

3.887.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4889, 27, 299, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(x)+1}{\cos(2x)+1}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos(x)^2+1}{\cos(2x)+1}dx$$
$$\Big\downarrow \text{4889}$$
$$\int \frac{\tan^2(x)+2}{2\left(\tan^2(x)+1\right)}d\tan(x)$$
$$\Big\downarrow \text{27}$$
$$\frac{1}{2}\int \frac{\tan^2(x)+2}{\tan^2(x)+1}d\tan(x)$$
$$\Big\downarrow \text{299}$$
$$\frac{1}{2}\left(\int \frac{1}{\tan^2(x)+1}d\tan(x)+\tan(x)\right)$$
$$\Big\downarrow \text{216}$$
$$\frac{1}{2}(\arctan(\tan(x))+\tan(x))$$

input
```
Int[(1 + Cos[x]^2)/(1 + Cos[2*x]),x]
```

output
```
(ArcTan[Tan[x]] + Tan[x])/2
```

3.887.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.887.4 Maple [A] (verified)

Time = 4.72 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{x}{2}+\frac{\tan(x)}{2}$	9
parts	$\frac{x}{2}+\frac{\tan(x)}{2}$	9
risch	$\frac{x}{2}+\frac{i}{{\mathrm e}^{2 i x}+1}$	17

input
```
int((cos(x)^2+1)/(1+cos(2*x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x+1/2*tan(x)
```

3.887.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{1+\cos^2(x)}{1+\cos(2x)}\,dx = \frac{x\cos(x)+\sin(x)}{2\cos(x)}$$

input
```
integrate((1+cos(x)^2)/(1+cos(2*x)),x, algorithm="fricas")
```

output
```
1/2*(x*cos(x) + sin(x))/cos(x)
```

3.887.6 Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.58

$$\int \frac{1+\cos^2(x)}{1+\cos(2x)}\,dx = \frac{x}{2}+\frac{\tan(x)}{2}$$

input
```
integrate((1+cos(x)**2)/(1+cos(2*x)),x)
```

output
```
x/2 + tan(x)/2
```

3.887.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. 2(8) = 16.

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \frac{1+\cos^2(x)}{1+\cos(2x)}\,dx = \frac{1}{2}x+\frac{\sin(2x)}{2(\cos(2x)+1)}$$

input
```
integrate((1+cos(x)^2)/(1+cos(2*x)),x, algorithm="maxima")
```

output
```
1/2*x + 1/2*sin(2*x)/(cos(2*x) + 1)
```

3.887.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1+\cos^2(x)}{1+\cos(2x)}\,dx = \frac{1}{2}\,x + \frac{1}{2}\,\tan\left(x\right)$$

input
```
integrate((1+cos(x)^2)/(1+cos(2*x)),x, algorithm="giac")
```

output
```
1/2*x + 1/2*tan(x)
```

3.887.9 Mupad [B] (verification not implemented)

Time = 26.59 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1+\cos^2(x)}{1+\cos(2x)}\,dx = \frac{x}{2} + \frac{\tan(x)}{2}$$

input
```
int((cos(x)^2 + 1)/(cos(2*x) + 1),x)
```

output
```
x/2 + tan(x)/2
```

3.888 $\int \frac{\sin(x)}{\cos^3(x)-\cos^5(x)}\,dx$

3.888.1 Optimal result

Integrand size = 16, antiderivative size = 12

$$\int \frac{\sin(x)}{\cos^3(x)-\cos^5(x)}\,dx = \log(\tan(x)) + \frac{\tan^2(x)}{2}$$

output
```
ln(tan(x))+1/2*tan(x)^2
```

3.888.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

$$\int \frac{\sin(x)}{\cos^3(x)-\cos^5(x)}\,dx = -\log(\cos(x)) + \log(\sin(x)) + \frac{\sec^2(x)}{2}$$

input
```
Integrate[Sin[x]/(Cos[x]^3 - Cos[x]^5),x]
```

output
```
-Log[Cos[x]] + Log[Sin[x]] + Sec[x]^2/2
```

3.888.3 Rubi [A] (warning: unable to verify)

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3042, 4835, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)}{\cos^3(x)-\cos^5(x)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin(x)}{\cos(x)^3-\cos(x)^5}dx$$

$$\Big\downarrow \; 4835$$

$$-\int \frac{\sec^3(x)}{1-\cos^2(x)}d\cos(x)$$

$$\Big\downarrow \; 243$$

$$-\frac{1}{2}\int \frac{\sec^2(x)}{1-\cos^2(x)}d\cos^2(x)$$

$$\Big\downarrow \; 54$$

$$-\frac{1}{2}\int \left(\sec^2(x)+\sec(x)+\frac{1}{1-\cos^2(x)}\right)d\cos^2(x)$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(\sec(x)-\log\left(\cos^2(x)\right)+\log\left(1-\cos^2(x)\right)\right)$$

input
```
Int[Sin[x]/(Cos[x]^3 - Cos[x]^5),x]
```

output
```
(-Log[Cos[x]^2] + Log[1 - Cos[x]^2] + Sec[x])/2
```

3.888.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4835
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-d/(b*c)   Subst[Int[SubstFor[1, Cos[c*(a +
b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

3.888.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(10) = 20$.

Time = 0.49 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.25

method	result
derivativedivides	$\frac{\ln(\cos(x)-1)}{2}+\frac{1}{2\cos(x)^{2}}-\ln\left(\cos\left(x\right)\right)+\frac{\ln(\cos(x)+1)}{2}$
default	$\frac{\ln(\cos(x)-1)}{2}+\frac{1}{2\cos(x)^{2}}-\ln\left(\cos\left(x\right)\right)+\frac{\ln(\cos(x)+1)}{2}$
risch	$\frac{2\,e^{2ix}}{(e^{2ix}+1)^{2}}-\ln\left(e^{2ix}+1\right)+\ln\left(e^{2ix}-1\right)$
parallelrisch	$-\ln\left(\csc\left(x\right)-\cot\left(x\right)+1\right)-\ln\left(-\cot\left(x\right)+\csc\left(x\right)-1\right)+\ln\left(\csc\left(x\right)-\cot\left(x\right)\right)+\frac{\tan(}{2}$
norman	$\frac{2\tan\left(\frac{x}{2}\right)^{3}+2\tan\left(\frac{x}{2}\right)^{5}}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)\left(\tan\left(\frac{x}{2}\right)^{2}-1\right)^{2}\tan\left(\frac{x}{2}\right)}-\ln\left(\tan\left(\frac{x}{2}\right)-1\right)-\ln\left(\tan\left(\frac{x}{2}\right)+1\right)+\ln\left(\tan\left(\frac{x}{2}\right)\right)$

input
```
int(sin(x)/(cos(x)^3-cos(x)^5),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(cos(x)-1)+1/2/cos(x)^2-ln(cos(x))+1/2*ln(cos(x)+1)
```

3.888.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(10) = 20$.

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.75

$$\int \frac{\sin(x)}{\cos^3(x) - \cos^5(x)}\,dx = -\frac{\cos(x)^2 \log\left(\cos(x)^2\right) - \cos(x)^2 \log\left(-\frac{1}{4}\cos(x)^2 + \frac{1}{4}\right) - 1}{2\cos(x)^2}$$

input
```
integrate(sin(x)/(cos(x)^3-cos(x)^5),x, algorithm="fricas")
```

output
```
-1/2*(cos(x)^2*log(cos(x)^2) - cos(x)^2*log(-1/4*cos(x)^2 + 1/4) - 1)/cos(
x)^2
```

3.888.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(10) = 20$.

Time = 0.33 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.42

$$\int \frac{\sin(x)}{\cos^3(x) - \cos^5(x)}\,dx = \frac{\log(\cos(x) - 1)}{2} + \frac{\log(\cos(x) + 1)}{2} - \log(\cos(x)) + \frac{1}{2\cos^2(x)}$$

input
```
integrate(sin(x)/(cos(x)**3-cos(x)**5),x)
```

output
```
log(cos(x) - 1)/2 + log(cos(x) + 1)/2 - log(cos(x)) + 1/(2*cos(x)**2)
```

3.888.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(10) = 20$.

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.17

$$\int \frac{\sin(x)}{\cos^3(x) - \cos^5(x)}\, dx = \frac{1}{2\cos(x)^2} + \frac{1}{2}\log(\cos(x)+1) + \frac{1}{2}\log(\cos(x)-1) - \log(\cos(x))$$

input
```
integrate(sin(x)/(cos(x)^3-cos(x)^5),x, algorithm="maxima")
```

output
```
1/2/cos(x)^2 + 1/2*log(cos(x) + 1) + 1/2*log(cos(x) - 1) - log(cos(x))
```

3.888.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(10) = 20$.

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.00

$$\int \frac{\sin(x)}{\cos^3(x) - \cos^5(x)}\, dx = \frac{1}{2\cos(x)^2} + \frac{1}{2}\log\left(-\cos(x)^2 + 1\right) - \log(|\cos(x)|)$$

input
```
integrate(sin(x)/(cos(x)^3-cos(x)^5),x, algorithm="giac")
```

output
```
1/2/cos(x)^2 + 1/2*log(-cos(x)^2 + 1) - log(abs(cos(x)))
```

3.888.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.58

$$\int \frac{\sin(x)}{\cos^3(x) - \cos^5(x)}\, dx = \frac{\ln\left(\sin(x)^2\right)}{2} - \ln(\cos(x)) + \frac{1}{2\cos(x)^2}$$

input
```
int(sin(x)/(cos(x)^3 - cos(x)^5),x)
```

output
```
log(sin(x)^2)/2 - log(cos(x)) + 1/(2*cos(x)^2)
```

3.889 $\int \sec(x)\left(5-11\sec^5(x)\right)^2\tan(x)\,dx$

3.889.1 Optimal result

Integrand size = 15, antiderivative size = 19

$$\int \sec(x)\left(5-11\sec^5(x)\right)^2\tan(x)\,dx=25\sec(x)-\frac{55\sec^6(x)}{3}+11\sec^{11}(x)$$

output
```
25*sec(x)-55/3*sec(x)^6+11*sec(x)^11
```

3.889.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \sec(x)\left(5-11\sec^5(x)\right)^2\tan(x)\,dx=25\sec(x)-\frac{55\sec^6(x)}{3}+11\sec^{11}(x)$$

input
```
Integrate[Sec[x]*(5 - 11*Sec[x]^5)^2*Tan[x],x]
```

output
```
25*Sec[x] - (55*Sec[x]^6)/3 + 11*Sec[x]^11
```

3.889.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3042, 4839, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(x)\sec(x)\left(5-11\sec^5(x)\right)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(x)\sec(x)\left(5-11\sec(x)^5\right)^2\,dx$$

$$\Big\downarrow \text{4839}$$

$$-\int \left(11-5\cos^5(x)\right)^2\sec^{12}(x)d\cos(x)$$

$$\Big\downarrow \text{802}$$

$$-\int \left(121\sec^{12}(x)-110\sec^7(x)+25\sec^2(x)\right)d\cos(x)$$

$$\Big\downarrow \text{2009}$$

$$11\sec^{11}(x)-\frac{55\sec^6(x)}{3}+25\sec(x)$$

input
```
Int[Sec[x]*(5 - 11*Sec[x]^5)^2*Tan[x],x]
```

output
```
25*Sec[x] - (55*Sec[x]^6)/3 + 11*Sec[x]^11
```

3.889.3.1 Defintions of rubi rules used

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4839
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, Simp[-(b*c)^(-1)   Subst[Int[SubstFor[1/x, Cos[c*
(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
 + b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, t
an])
```

3.889.4 Maple [A] (verified)

Time = 249.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result
derivativedivides	$25\sec(x)-\frac{55\sec(x)^{6}}{3}+11\sec(x)^{11}$
default	$25\sec(x)-\frac{55\sec(x)^{6}}{3}+11\sec(x)^{11}$
parts	$25\sec(x)-\frac{55\sec(x)^{6}}{3}+11\sec(x)^{11}$
risch	$\frac{50\,\mathrm{e}^{21ix}+500\,\mathrm{e}^{19ix}+2250\,\mathrm{e}^{17ix}-\frac{3520\,\mathrm{e}^{16ix}}{3}+6000\,\mathrm{e}^{15ix}-\frac{17600\,\mathrm{e}^{14ix}}{3}+10500\,\mathrm{e}^{13ix}-\frac{35200\,\mathrm{e}^{12ix}}{3}+35128\,\mathrm{e}^{11ix}-\frac{35200\,\mathrm{e}^{10ix}}{3}}{\left(\mathrm{e}^{2ix}+1\right)^{11}}$

input
```
int(sec(x)*(5-11*sec(x)^5)^2*tan(x),x,method=_RETURNVERBOSE)
```

output
```
25*sec(x)-55/3*sec(x)^6+11*sec(x)^11
```

3.889.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \sec(x)\left(5-11\sec^5(x)\right)^2\tan(x)\,dx=\frac{75\cos(x)^{10}-55\cos(x)^{5}+33}{3\cos(x)^{11}}$$

input
```
integrate(sec(x)*(5-11*sec(x)^5)^2*tan(x),x, algorithm="fricas")
```

output
```
1/3*(75*cos(x)^10 - 55*cos(x)^5 + 33)/cos(x)^11
```

3.889.6 Sympy [A] (verification not implemented)

Time = 2.52 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \sec(x)\left(5-11\sec^5(x)\right)^2\tan(x)\,dx = 11\sec^{11}(x) - \frac{55\sec^6(x)}{3} + 25\sec(x)$$

input
```
integrate(sec(x)*(5-11*sec(x)**5)**2*tan(x),x)
```

output
```
11*sec(x)**11 - 55*sec(x)**6/3 + 25*sec(x)
```

3.889.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \sec(x)\left(5-11\sec^5(x)\right)^2\tan(x)\,dx = \frac{75\cos(x)^{10} - 55\cos(x)^5 + 33}{3\cos(x)^{11}}$$

input
```
integrate(sec(x)*(5-11*sec(x)^5)^2*tan(x),x, algorithm="maxima")
```

output
```
1/3*(75*cos(x)^10 - 55*cos(x)^5 + 33)/cos(x)^11
```

3.889.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \sec(x)\left(5-11\sec^5(x)\right)^2\tan(x)\,dx = \frac{75\cos(x)^{10} - 55\cos(x)^5 + 33}{3\cos(x)^{11}}$$

input
```
integrate(sec(x)*(5-11*sec(x)^5)^2*tan(x),x, algorithm="giac")
```

output
```
1/3*(75*cos(x)^10 - 55*cos(x)^5 + 33)/cos(x)^11
```

3.889.9 Mupad [B] (verification not implemented)

Time = 27.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \sec(x)\left(5-11\sec^5(x)\right)^2\tan(x)\,dx=\frac{25\cos(x)^{10}-\frac{55\cos(x)^5}{3}+11}{\cos(x)^{11}}$$

input
```
int((tan(x)*(11/cos(x)^5 - 5)^2)/cos(x),x)
```

output
```
(25*cos(x)^10 - (55*cos(x)^5)/3 + 11)/cos(x)^11
```

3.890 $\int \sin^3(5x)\tan^3(5x)\,dx$

3.890.1 Optimal result

Integrand size = 13, antiderivative size = 44

$$\int \sin^3(5x)\tan^3(5x)\,dx = -\frac{1}{2}\text{arctanh}(\sin(5x))+\frac{1}{2}\sin(5x)+\frac{1}{6}\sin^3(5x)+\frac{1}{10}\sin^3(5x)\tan^2(5x)$$

output
```
-1/2*arctanh(sin(5*x))+1/2*sin(5*x)+1/6*sin(5*x)^3+1/10*sin(5*x)^3*tan(5*x
)^2
```

3.890.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.18

$$\int \sin^3(5x)\tan^3(5x)\,dx = -\frac{1}{2}\text{arctanh}(\sin(5x)) + \frac{1}{2}\sec(5x)\tan(5x) - \frac{1}{3}\sin(5x)\tan^2(5x) - \frac{1}{15}\sin^3(5x)\tan^2(5x)$$

input
```
Integrate[Sin[5*x]^3*Tan[5*x]^3,x]
```

output
```
-1/2*ArcTanh[Sin[5*x]] + (Sec[5*x]*Tan[5*x])/2 - (Sin[5*x]*Tan[5*x]^2)/3 -
 (Sin[5*x]^3*Tan[5*x]^2)/15
```

3.890.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.20, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3042, 3072, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(5x)\tan^3(5x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(5x)^3\tan(5x)^3dx$$

$$\Big\downarrow \text{3072}$$

$$\frac{1}{5}\int \frac{\sin^6(5x)}{(1-\sin^2(5x))^2}d\sin(5x)$$

$$\Big\downarrow \text{252}$$

$$\frac{1}{5}\left(\frac{\sin^5(5x)}{2(1-\sin^2(5x))}-\frac{5}{2}\int \frac{\sin^4(5x)}{1-\sin^2(5x)}d\sin(5x)\right)$$

$$\Big\downarrow \text{254}$$

$$\frac{1}{5}\left(\frac{\sin^5(5x)}{2(1-\sin^2(5x))}-\frac{5}{2}\int \left(-\sin^2(5x)+\frac{1}{1-\sin^2(5x)}-1\right)d\sin(5x)\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}\left(\frac{\sin^5(5x)}{2(1-\sin^2(5x))}-\frac{5}{2}\left(\text{arctanh}(\sin(5x))-\frac{1}{3}\sin^3(5x)-\sin(5x)\right)\right)$$

input
```
Int[Sin[5*x]^3*Tan[5*x]^3,x]
```

output
```
(Sin[5*x]^5/(2*(1 - Sin[5*x]^2)) - (5*(ArcTanh[Sin[5*x]] - Sin[5*x] - Sin[
5*x]^3/3))/2)/5
```

3.890.3.1 Defintions of rubi rules used

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

3.890.4 Maple [A] (verified)

Time = 3.52 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.14

method	result	size
derivativedivides	$\frac{\sin(5x)^7}{10\cos(5x)^2}+\frac{\sin(5x)^5}{10}+\frac{\sin(5x)^3}{6}+\frac{\sin(5x)}{2}-\frac{\ln(\sec(5x)+\tan(5x))}{2}$	50
default	$\frac{\sin(5x)^7}{10\cos(5x)^2}+\frac{\sin(5x)^5}{10}+\frac{\sin(5x)^3}{6}+\frac{\sin(5x)}{2}-\frac{\ln(\sec(5x)+\tan(5x))}{2}$	50
risch	$\frac{i e^{15ix}}{120}-\frac{9i e^{5ix}}{40}+\frac{9i e^{-5ix}}{40}-\frac{i e^{-15ix}}{120}-\frac{i(e^{15ix}-e^{5ix})}{5(e^{10ix}+1)^2}-\frac{\ln(i+e^{5ix})}{2}+\frac{\ln(e^{5ix}-i)}{2}$	81

input
```
int(sin(5*x)^3*tan(5*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/10*sin(5*x)^7/cos(5*x)^2+1/10*sin(5*x)^5+1/6*sin(5*x)^3+1/2*sin(5*x)-1/2
*ln(sec(5*x)+tan(5*x))
```

3.890.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.48

$$\int \sin^3(5x)\tan^3(5x)\,dx = -\frac{15\,\cos\left(5\,x\right)^{2}\log\left(\sin\left(5\,x\right)+1\right)-15\,\cos\left(5\,x\right)^{2}\log\left(-\sin\left(5\,x\right)+1\right)+2\left(2\,\cos\left(5\,x\right)^{4}-14\,\cos\left(5\,x\right)^{2}-3\right.}{60\,\cos\left(5\,x\right)^{2}}$$

input
```
integrate(sin(5*x)^3*tan(5*x)^3,x, algorithm="fricas")
```

output
```
-1/60*(15*cos(5*x)^2*log(sin(5*x) + 1) - 15*cos(5*x)^2*log(-sin(5*x) + 1)
+ 2*(2*cos(5*x)^4 - 14*cos(5*x)^2 - 3)*sin(5*x))/cos(5*x)^2
```

3.890.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.16

$$\int \sin^3(5x)\tan^3(5x)\,dx = \frac{\log(\sin(5x)-1)}{4} - \frac{\log(\sin(5x)+1)}{4} + \frac{\sin^3(5x)}{15} + \frac{2\sin(5x)}{5} - \frac{\sin(5x)}{5\cdot(2\sin^2(5x)-2)}$$

input
```
integrate(sin(5*x)**3*tan(5*x)**3,x)
```

output
```
log(sin(5*x) - 1)/4 - log(sin(5*x) + 1)/4 + sin(5*x)**3/15 + 2*sin(5*x)/5
- sin(5*x)/(5*(2*sin(5*x)**2 - 2))
```

3.890.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.11

$$\int \sin^3(5x)\tan^3(5x)\,dx = \frac{1}{15}\,\sin\left(5\,x\right)^{3} - \frac{\sin\left(5\,x\right)}{10\left(\sin\left(5\,x\right)^{2}-1\right)} - \frac{1}{4}\,\log\left(\sin\left(5\,x\right)+1\right) + \frac{1}{4}\,\log\left(\sin\left(5\,x\right)-1\right) + \frac{2}{5}\,\sin\left(5\,x\right)$$

input
```
integrate(sin(5*x)^3*tan(5*x)^3,x, algorithm="maxima")
```

output
```
1/15*sin(5*x)^3 - 1/10*sin(5*x)/(sin(5*x)^2 - 1) - 1/4*log(sin(5*x) + 1) +
 1/4*log(sin(5*x) - 1) + 2/5*sin(5*x)
```

3.890.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.16

$$\int \sin^3(5x)\tan^3(5x)\,dx = \frac{1}{15}\sin(5x)^3 - \frac{\sin(5x)}{10\left(\sin(5x)^2 - 1\right)} - \frac{1}{4}\log(\sin(5x) + 1) + \frac{1}{4}\log(-\sin(5x) + 1) + \frac{2}{5}\sin(5x)$$

input
```
integrate(sin(5*x)^3*tan(5*x)^3,x, algorithm="giac")
```

output
```
1/15*sin(5*x)^3 - 1/10*sin(5*x)/(sin(5*x)^2 - 1) - 1/4*log(sin(5*x) + 1) +
 1/4*log(-sin(5*x) + 1) + 2/5*sin(5*x)
```

3.890.9 Mupad [B] (verification not implemented)

Time = 26.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.57

$$\int \sin^3(5x)\tan^3(5x)\,dx = \frac{5\tan\left(\frac{5x}{2}\right)^9 + \frac{20\tan\left(\frac{5x}{2}\right)^7}{3} - \frac{22\tan\left(\frac{5x}{2}\right)^5}{3} + \frac{20\tan\left(\frac{5x}{2}\right)^3}{3} + 5\tan\left(\frac{5x}{2}\right)}{5\left(\tan\left(\frac{5x}{2}\right)^2 - 1\right)^2\left(\tan\left(\frac{5x}{2}\right)^2 + 1\right)^3} - \operatorname{atanh}\left(\tan\left(\frac{5x}{2}\right)\right)$$

input
```
int(sin(5*x)^3*tan(5*x)^3,x)
```

output
```
(5*tan((5*x)/2) + (20*tan((5*x)/2)^3)/3 - (22*tan((5*x)/2)^5)/3 + (20*tan(
(5*x)/2)^7)/3 + 5*tan((5*x)/2)^9)/(5*(tan((5*x)/2)^2 - 1)^2*(tan((5*x)/2)^
2 + 1)^3) - atanh(tan((5*x)/2))
```

3.891 $\int \sin^3(5x)\tan^4(5x)\,dx$

3.891.1 Optimal result

Integrand size = 13, antiderivative size = 37

$$\int \sin^3(5x)\tan^4(5x)\,dx = -\frac{3}{5}\cos(5x)+\frac{1}{15}\cos^3(5x)-\frac{3}{5}\sec(5x)+\frac{1}{15}\sec^3(5x)$$

output
```
-3/5*cos(5*x)+1/15*cos(5*x)^3-3/5*sec(5*x)+1/15*sec(5*x)^3
```

3.891.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \sin^3(5x)\tan^4(5x)\,dx = -\frac{11}{20}\cos(5x)+\frac{1}{60}\cos(15x)-\frac{3}{5}\sec(5x)+\frac{1}{15}\sec^3(5x)$$

input
```
Integrate[Sin[5*x]^3*Tan[5*x]^4,x]
```

output
```
(-11*Cos[5*x])/20 + Cos[15*x]/60 - (3*Sec[5*x])/5 + Sec[5*x]^3/15
```

3.891.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 3070, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(5x)\tan^4(5x)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin(5x)^3\tan(5x)^4dx$$

$$\Big\downarrow \; 3070$$

$$-\frac{1}{5}\int \left(1-\cos^2(5x)\right)^3\sec^4(5x)d\cos(5x)$$

$$\Big\downarrow \; 244$$

$$-\frac{1}{5}\int \left(\sec^4(5x)-3\sec^2(5x)-\cos^2(5x)+3\right)d\cos(5x)$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{5}\left(\frac{1}{3}\cos^3(5x)-3\cos(5x)+\frac{1}{3}\sec^3(5x)-3\sec(5x)\right)$$

input
```
Int[Sin[5*x]^3*Tan[5*x]^4,x]
```

output
```
(-3*Cos[5*x] + Cos[5*x]^3/3 - 3*Sec[5*x] + Sec[5*x]^3/3)/5
```

3.891.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

3.891.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.54

method	result	size
risch	$\frac{e^{45ix}-30\,e^{35ix}-273\,e^{25ix}-420\,e^{15ix}+e^{-15ix}-303\cos(5x)-243i\sin(5x)}{120(e^{10ix}+1)^3}$	57
derivativedivides	$\frac{\sin(5x)^8}{15\cos(5x)^3}-\frac{\sin(5x)^8}{3\cos(5x)}-\frac{\left(\frac{16}{5}+\sin(5x)^6+\frac{6\sin(5x)^4}{5}+\frac{8\sin(5x)^2}{5}\right)\cos(5x)}{3}$	60
default	$\frac{\sin(5x)^8}{15\cos(5x)^3}-\frac{\sin(5x)^8}{3\cos(5x)}-\frac{\left(\frac{16}{5}+\sin(5x)^6+\frac{6\sin(5x)^4}{5}+\frac{8\sin(5x)^2}{5}\right)\cos(5x)}{3}$	60

input
```
int(sin(5*x)^3*tan(5*x)^4,x,method=_RETURNVERBOSE)
```

output
```
1/120*(exp(45*I*x)-30*exp(35*I*x)-273*exp(25*I*x)-420*exp(15*I*x)+exp(-15*
I*x)-303*cos(5*x)-243*I*sin(5*x))/(exp(10*I*x)+1)^3
```

3.891.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \sin^3(5x)\tan^4(5x)\,dx = \frac{\cos(5x)^6-9\cos(5x)^4-9\cos(5x)^2+1}{15\cos(5x)^3}$$

input
```
integrate(sin(5*x)^3*tan(5*x)^4,x, algorithm="fricas")
```

output
```
1/15*(cos(5*x)^6 - 9*cos(5*x)^4 - 9*cos(5*x)^2 + 1)/cos(5*x)^3
```

3.891.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int \sin^3(5x)\tan^4(5x)\,dx = \frac{1-9\cos^2(5x)}{15\cos^3(5x)} + \frac{\cos^3(5x)}{15} - \frac{3\cos(5x)}{5}$$

input
```
integrate(sin(5*x)**3*tan(5*x)**4,x)
```

output
```
(1 - 9*cos(5*x)**2)/(15*cos(5*x)**3) + cos(5*x)**3/15 - 3*cos(5*x)/5
```

3.891.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \sin^3(5x)\tan^4(5x)\,dx = \frac{1}{15}\cos(5x)^3 - \frac{9\cos(5x)^2-1}{15\cos(5x)^3} - \frac{3}{5}\cos(5x)$$

input
```
integrate(sin(5*x)^3*tan(5*x)^4,x, algorithm="maxima")
```

output
```
1/15*cos(5*x)^3 - 1/15*(9*cos(5*x)^2 - 1)/cos(5*x)^3 - 3/5*cos(5*x)
```

3.891.8 Giac [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \sin^3(5x)\tan^4(5x)\,dx = \frac{1}{15}\cos(5x)^3 - \frac{9\cos(5x)^2-1}{15\cos(5x)^3} - \frac{3}{5}\cos(5x)$$

input
```
integrate(sin(5*x)^3*tan(5*x)^4,x, algorithm="giac")
```

output
```
1/15*cos(5*x)^3 - 1/15*(9*cos(5*x)^2 - 1)/cos(5*x)^3 - 3/5*cos(5*x)
```

3.891.9 Mupad [B] (verification not implemented)

Time = 26.45 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

$$\int \sin^3(5x)\tan^4(5x)\,dx = \frac{(\cos(5x)+1)^4\left(\cos(5x)^2-4\cos(5x)+1\right)}{15\cos(5x)^3}$$

input
```
int(sin(5*x)^3*tan(5*x)^4,x)
```

output
```
((cos(5*x) + 1)^4*(cos(5*x)^2 - 4*cos(5*x) + 1))/(15*cos(5*x)^3)
```

3.892 $\int \sin^5(6x)\tan^3(6x)\,dx$

3.892.1 Optimal result

Integrand size = 13, antiderivative size = 54

$$\int \sin^5(6x)\tan^3(6x)\,dx = -\frac{7}{12}\operatorname{arctanh}(\sin(6x)) + \frac{7}{12}\sin(6x) + \frac{7}{36}\sin^3(6x) + \frac{7}{60}\sin^5(6x) + \frac{1}{12}\sin^5(6x)\tan^2(6x)$$

output
```
-7/12*arctanh(sin(6*x))+7/12*sin(6*x)+7/36*sin(6*x)^3+7/60*sin(6*x)^5+1/12
*sin(6*x)^5*tan(6*x)^2
```

3.892.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.26

$$\int \sin^5(6x)\tan^3(6x)\,dx = -\frac{7}{12}\operatorname{arctanh}(\sin(6x)) + \frac{7}{12}\sec(6x)\tan(6x) - \frac{7}{18}\sin(6x)\tan^2(6x) - \frac{7}{90}\sin^3(6x)\tan^2(6x) - \frac{1}{30}\sin^5(6x)\tan^2(6x)$$

input
```
Integrate[Sin[6*x]^5*Tan[6*x]^3,x]
```

output
```
(-7*ArcTanh[Sin[6*x]])/12 + (7*Sec[6*x]*Tan[6*x])/12 - (7*Sin[6*x]*Tan[6*x
]^2)/18 - (7*Sin[6*x]^3*Tan[6*x]^2)/90 - (Sin[6*x]^5*Tan[6*x]^2)/30
```

3.892.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3042, 3072, 252, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^5(6x)\tan^3(6x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(6x)^5\tan(6x)^3dx$$

$$\Big\downarrow \text{3072}$$

$$\frac{1}{6}\int \frac{\sin^8(6x)}{(1-\sin^2(6x))^2}d\sin(6x)$$

$$\Big\downarrow \text{252}$$

$$\frac{1}{6}\left(\frac{\sin^7(6x)}{2(1-\sin^2(6x))}-\frac{7}{2}\int \frac{\sin^6(6x)}{1-\sin^2(6x)}d\sin(6x)\right)$$

$$\Big\downarrow \text{254}$$

$$\frac{1}{6}\left(\frac{\sin^7(6x)}{2(1-\sin^2(6x))}-\frac{7}{2}\int \left(-\sin^4(6x)-\sin^2(6x)+\frac{1}{1-\sin^2(6x)}-1\right)d\sin(6x)\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{6}\left(\frac{\sin^7(6x)}{2(1-\sin^2(6x))}-\frac{7}{2}\left(\text{arctanh}(\sin(6x))-\frac{1}{5}\sin^5(6x)-\frac{1}{3}\sin^3(6x)-\sin(6x)\right)\right)$$

input
```
Int[Sin[6*x]^5*Tan[6*x]^3,x]
```

output
```
(Sin[6*x]^7/(2*(1 - Sin[6*x]^2)) - (7*(ArcTanh[Sin[6*x]] - Sin[6*x] - Sin[
6*x]^3/3 - Sin[6*x]^5/5))/2)/6
```

3.892.3.1 Defintions of rubi rules used

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

3.892.4 Maple [A] (verified)

Time = 35.37 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07

method	result	s
derivativedivides	$\frac{\sin(6x)^9}{12\cos(6x)^2}+\frac{\sin(6x)^7}{12}+\frac{7\sin(6x)^5}{60}+\frac{7\sin(6x)^3}{36}+\frac{7\sin(6x)}{12}-\frac{7\ln(\sec(6x)+\tan(6x))}{12}$	5
default	$\frac{\sin(6x)^9}{12\cos(6x)^2}+\frac{\sin(6x)^7}{12}+\frac{7\sin(6x)^5}{60}+\frac{7\sin(6x)^3}{36}+\frac{7\sin(6x)}{12}-\frac{7\ln(\sec(6x)+\tan(6x))}{12}$	5
risch	$\frac{11i\,e^{18ix}}{576}-\frac{29i\,e^{6ix}}{96}+\frac{29i\,e^{-6ix}}{96}-\frac{11i\,e^{-18ix}}{576}-\frac{i(e^{18ix}-e^{6ix})}{6(e^{12ix}+1)^2}+\frac{7\ln(e^{6ix}-i)}{12}-\frac{7\ln(i+e^{6ix})}{12}+\frac{\sin(30x)}{480}$	8

input
```
int(sin(6*x)^5*tan(6*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/12*sin(6*x)^9/cos(6*x)^2+1/12*sin(6*x)^7+7/60*sin(6*x)^5+7/36*sin(6*x)^3
+7/12*sin(6*x)-7/12*ln(sec(6*x)+tan(6*x))
```

3.892.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.35

$$\int \sin^5(6x)\tan^3(6x)\,dx = -\frac{105\cos(6x)^2\log(\sin(6x)+1) - 105\cos(6x)^2\log(-\sin(6x)+1) - 2\left(6\cos(6x)^6 - 32\cos(6x)^4 + \right.}{360\cos(6x)^2}$$

input

```
integrate(sin(6*x)^5*tan(6*x)^3,x, algorithm="fricas")
```

output

```
-1/360*(105*cos(6*x)^2*log(sin(6*x) + 1) - 105*cos(6*x)^2*log(-sin(6*x) +
1) - 2*(6*cos(6*x)^6 - 32*cos(6*x)^4 + 116*cos(6*x)^2 + 15)*sin(6*x))/cos(
6*x)^2
```

3.892.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.13

$$\int \sin^5(6x)\tan^3(6x)\,dx = \frac{7\log(\sin(6x)-1)}{24} - \frac{7\log(\sin(6x)+1)}{24} + \frac{\sin^5(6x)}{30} + \frac{\sin^3(6x)}{9} + \frac{\sin(6x)}{2} - \frac{\sin(6x)}{6\cdot(2\sin^2(6x)-2)}$$

input

```
integrate(sin(6*x)**5*tan(6*x)**3,x)
```

output

```
7*log(sin(6*x) - 1)/24 - 7*log(sin(6*x) + 1)/24 + sin(6*x)**5/30 + sin(6*x
)**3/9 + sin(6*x)/2 - sin(6*x)/(6*(2*sin(6*x)**2 - 2))
```

3.892.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.06

$$\int \sin^5(6x)\tan^3(6x)\,dx = \frac{1}{30}\sin(6x)^5 + \frac{1}{9}\sin(6x)^3 - \frac{\sin(6x)}{12\left(\sin(6x)^2-1\right)} - \frac{7}{24}\log(\sin(6x)+1) + \frac{7}{24}\log(\sin(6x)-1) + \frac{1}{2}\sin(6x)$$

input
```
integrate(sin(6*x)^5*tan(6*x)^3,x, algorithm="maxima")
```

output
```
1/30*sin(6*x)^5 + 1/9*sin(6*x)^3 - 1/12*sin(6*x)/(sin(6*x)^2 - 1) - 7/24*l
og(sin(6*x) + 1) + 7/24*log(sin(6*x) - 1) + 1/2*sin(6*x)
```

3.892.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09

$$\int \sin^5(6x)\tan^3(6x)\,dx = \frac{1}{30}\sin(6x)^5 + \frac{1}{9}\sin(6x)^3 - \frac{\sin(6x)}{12\left(\sin(6x)^2 - 1\right)} - \frac{7}{24}\log(\sin(6x) + 1) + \frac{7}{24}\log(-\sin(6x) + 1) + \frac{1}{2}\sin(6x)$$

input
```
integrate(sin(6*x)^5*tan(6*x)^3,x, algorithm="giac")
```

output
```
1/30*sin(6*x)^5 + 1/9*sin(6*x)^3 - 1/12*sin(6*x)/(sin(6*x)^2 - 1) - 7/24*l
og(sin(6*x) + 1) + 7/24*log(-sin(6*x) + 1) + 1/2*sin(6*x)
```

3.892.9 Mupad [B] (verification not implemented)

Time = 31.82 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.57

$$\int \sin^5(6x)\tan^3(6x)\,dx = \frac{7\tan(3x)^{13} + \frac{70\tan(3x)^{11}}{3} + \frac{77\tan(3x)^9}{5} - \frac{412\tan(3x)^7}{15} + \frac{77\tan(3x)^5}{5} + \frac{70\tan(3x)^3}{3} + 7\tan(3x)}{6\left(\tan(3x)^2 - 1\right)^2\left(\tan(3x)^2 + 1\right)^5} - \frac{7\,\mathrm{atanh}(\tan(3x))}{6}$$

input
```
int(sin(6*x)^5*tan(6*x)^3,x)
```

output
```
(7*tan(3*x) + (70*tan(3*x)^3)/3 + (77*tan(3*x)^5)/5 - (412*tan(3*x)^7)/15
+ (77*tan(3*x)^9)/5 + (70*tan(3*x)^11)/3 + 7*tan(3*x)^13)/(6*(tan(3*x)^2 -
 1)^2*(tan(3*x)^2 + 1)^5) - (7*atanh(tan(3*x)))/6
```

3.893 $\int \left(-1+\sec^2(2x)\right)^3 \sin(2x)\,dx$

3.893.1 Optimal result

Integrand size = 15, antiderivative size = 37

$$\int \left(-1+\sec^2(2x)\right)^3 \sin(2x)\,dx=\frac{1}{2}\cos(2x)+\frac{3}{2}\sec(2x)-\frac{1}{2}\sec^3(2x)+\frac{1}{10}\sec^5(2x)$$

output
```
1/2*cos(2*x)+3/2*sec(2*x)-1/2*sec(2*x)^3+1/10*sec(2*x)^5
```

3.893.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \left(-1+\sec^2(2x)\right)^3 \sin(2x)\,dx=\frac{1}{2}\cos(2x)+\frac{3}{2}\sec(2x)-\frac{1}{2}\sec^3(2x)+\frac{1}{10}\sec^5(2x)$$

input
```
Integrate[(-1 + Sec[2*x]^2)^3*Sin[2*x],x]
```

output
```
Cos[2*x]/2 + (3*Sec[2*x])/2 - Sec[2*x]^3/2 + Sec[2*x]^5/10
```

3.893.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 4608, 3042, 3070, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(2x)\left(\sec^2(2x)-1\right)^3 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(2x)\left(\sec(2x)^2-1\right)^3 dx$$

$$\Big\downarrow \text{4608}$$

$$\int \sin(2x)\tan^6(2x)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(2x)\tan(2x)^6dx$$

$$\Big\downarrow \text{3070}$$

$$-\frac{1}{2}\int \left(1-\cos^2(2x)\right)^3\sec^6(2x)d\cos(2x)$$

$$\Big\downarrow \text{244}$$

$$-\frac{1}{2}\int \left(\sec^6(2x)-3\sec^4(2x)+3\sec^2(2x)-1\right)d\cos(2x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\cos(2x)+\frac{1}{5}\sec^5(2x)-\sec^3(2x)+3\sec(2x)\right)$$

input
```
Int[(-1 + Sec[2*x]^2)^3*Sin[2*x],x]
```

output
```
(Cos[2*x] + 3*Sec[2*x] - Sec[2*x]^3 + Sec[2*x]^5/5)/2
```

3.893.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

rule 4608
```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
b^p   Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.893.4 Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

method	result	size
parts	$\frac{\cos(2x)}{2}+\frac{3\sec(2x)}{2}-\frac{\sec(2x)^{3}}{2}+\frac{\sec(2x)^{5}}{10}$	30
derivativedivides	$\frac{\cos(2x)}{2}+\frac{3}{2\cos(2x)}-\frac{1}{2\cos(2x)^{3}}+\frac{1}{10\cos(2x)^{5}}$	32
default	$\frac{\cos(2x)}{2}+\frac{3}{2\cos(2x)}-\frac{1}{2\cos(2x)^{3}}+\frac{1}{10\cos(2x)^{5}}$	32
norman	$\frac{-16\tan(x)^{4}+\frac{64\tan(x)^{2}}{5}-\frac{16}{5}}{\left(\tan(x)^{2}-1\right)^{5}\left(1+\tan(x)^{2}\right)}$	32
risch	$\frac{5\,e^{22ix}+90\,e^{18ix}+235\,e^{14ix}+364\,e^{10ix}+235\,e^{6ix}+95\cos(2x)+85i\sin(2x)}{20(e^{4ix}+1)^{5}}$	61
parallelrisch	$\frac{182+320\cos(2x)+32\cos(10x)+235\cos(4x)+90\cos(8x)+5\cos(12x)+160\cos(6x)}{20\cos(10x)+100\cos(6x)+200\cos(2x)}$	61

input
```
int((-1+sec(2*x)^2)^3*sin(2*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*cos(2*x)+3/2*sec(2*x)-1/2*sec(2*x)^3+1/10*sec(2*x)^5
```

3.893.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int \left(-1+\sec^2(2x)\right)^3 \sin(2x)\,dx = \frac{5\,\cos\left(2\,x\right)^6+15\,\cos\left(2\,x\right)^4-5\,\cos\left(2\,x\right)^2+1}{10\,\cos\left(2\,x\right)^5}$$

input
```
integrate((-1+sec(2*x)^2)^3*sin(2*x),x, algorithm="fricas")
```

output
```
1/10*(5*cos(2*x)^6 + 15*cos(2*x)^4 - 5*cos(2*x)^2 + 1)/cos(2*x)^5
```

3.893.6 Sympy [A] (verification not implemented)

Time = 123.55 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int \left(-1+\sec^2(2x)\right)^3 \sin(2x)\,dx = \frac{\cos(2x)}{2} - \frac{1}{2\left(2\cos^2(x)-1\right)^3} + \frac{1}{10\left(2\cos^2(x)-1\right)^5} + \frac{3}{4\left(\cos^2(x)-\frac{1}{2}\right)}$$

input
```
integrate((-1+sec(2*x)**2)**3*sin(2*x),x)
```

output
```
cos(2*x)/2 - 1/(2*(2*cos(x)**2 - 1)**3) + 1/(10*(2*cos(x)**2 - 1)**5) + 3/
(4*(cos(x)**2 - 1/2))
```

3.893.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \left(-1+\sec^2(2x)\right)^3 \sin(2x)\,dx = \frac{3}{2\,\cos\left(2\,x\right)} - \frac{1}{2\,\cos\left(2\,x\right)^3} + \frac{1}{10\,\cos\left(2\,x\right)^5} + \frac{1}{2}\,\cos\left(2\,x\right)$$

input
```
integrate((-1+sec(2*x)^2)^3*sin(2*x),x, algorithm="maxima")
```

output
```
3/2/cos(2*x) - 1/2/cos(2*x)^3 + 1/10/cos(2*x)^5 + 1/2*cos(2*x)
```

3.893.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \left(-1+\sec^2(2x)\right)^3 \sin(2x)\,dx = \frac{15\,\cos\left(2\,x\right)^{4}-5\,\cos\left(2\,x\right)^{2}+1}{10\,\cos\left(2\,x\right)^{5}}+\frac{1}{2}\,\cos\left(2\,x\right)$$

input
```
integrate((-1+sec(2*x)^2)^3*sin(2*x),x, algorithm="giac")
```

output
```
1/10*(15*cos(2*x)^4 - 5*cos(2*x)^2 + 1)/cos(2*x)^5 + 1/2*cos(2*x)
```

3.893.9 Mupad [B] (verification not implemented)

Time = 26.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \left(-1+\sec^2(2x)\right)^3 \sin(2x)\,dx = \frac{\cos\left(2\,x\right)}{2}+\frac{3\cos\left(2\,x\right)^4-\cos\left(2\,x\right)^2+\frac{1}{5}}{2\cos\left(2\,x\right)^5}$$

input
```
int(sin(2*x)*(1/cos(2*x)^2 - 1)^3,x)
```

output
```
cos(2*x)/2 + (3*cos(2*x)^4 - cos(2*x)^2 + 1/5)/(2*cos(2*x)^5)
```

3.894 $\int \sin(x)\tan^5(x)\,dx$

3.894.1 Optimal result

Integrand size = 7, antiderivative size = 34

$$\int \sin(x)\tan^5(x)\,dx = \frac{15}{8}\operatorname{arctanh}(\sin(x)) - \frac{15\sin(x)}{8} - \frac{5}{8}\sin(x)\tan^2(x) + \frac{1}{4}\sin(x)\tan^4(x)$$

output
```
15/8*arctanh(sin(x))-15/8*sin(x)-5/8*sin(x)*tan(x)^2+1/4*sin(x)*tan(x)^4
```

3.894.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

$$\int \sin(x)\tan^5(x)\,dx = \frac{15}{8}\operatorname{arctanh}(\sin(x)) + \frac{15}{8}\sec(x)\tan(x) - \frac{15}{4}\sec^3(x)\tan(x) + 5\sec(x)\tan^3(x) - \sin(x)\tan^4(x)$$

input
```
Integrate[Sin[x]*Tan[x]^5,x]
```

output
```
(15*ArcTanh[Sin[x]])/8 + (15*Sec[x]*Tan[x])/8 - (15*Sec[x]^3*Tan[x])/4 + 5
*Sec[x]*Tan[x]^3 - Sin[x]*Tan[x]^4
```

3.894.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.59, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {3042, 3072, 252, 252, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\tan^5(x)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(x)\tan(x)^5dx$$

$$\Big\downarrow \text{ 3072}$$

$$\int \frac{\sin^6(x)}{\left(1-\sin^2(x)\right)^3}d\sin(x)$$

$$\Big\downarrow \text{ 252}$$

$$\frac{\sin^5(x)}{4\left(1-\sin^2(x)\right)^2}-\frac{5}{4}\int \frac{\sin^4(x)}{\left(1-\sin^2(x)\right)^2}d\sin(x)$$

$$\Big\downarrow \text{ 252}$$

$$\frac{\sin^5(x)}{4\left(1-\sin^2(x)\right)^2}-\frac{5}{4}\left(\frac{\sin^3(x)}{2\left(1-\sin^2(x)\right)}-\frac{3}{2}\int \frac{\sin^2(x)}{1-\sin^2(x)}d\sin(x)\right)$$

$$\Big\downarrow \text{ 262}$$

$$\frac{\sin^5(x)}{4\left(1-\sin^2(x)\right)^2}-\frac{5}{4}\left(\frac{\sin^3(x)}{2\left(1-\sin^2(x)\right)}-\frac{3}{2}\left(\int \frac{1}{1-\sin^2(x)}d\sin(x)-\sin(x)\right)\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\sin^5(x)}{4\left(1-\sin^2(x)\right)^2}-\frac{5}{4}\left(\frac{\sin^3(x)}{2\left(1-\sin^2(x)\right)}-\frac{3}{2}(\text{arctanh}(\sin(x))-\sin(x))\right)$$

input
```
Int[Sin[x]*Tan[x]^5,x]
```

output
```
Sin[x]^5/(4*(1 - Sin[x]^2)^2) - (5*((-3*(ArcTanh[Sin[x]] - Sin[x]))/2 + Si
n[x]^3/(2*(1 - Sin[x]^2))))/4
```

3.894.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3072
```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   Subst[Int[
(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x
]] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

3.894.4 Maple [A] (verified)

Time = 3.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

method	result	size
default	$\frac{\sin(x)^7}{4\cos(x)^4}-\frac{3\sin(x)^7}{8\cos(x)^2}-\frac{3\sin(x)^5}{8}-\frac{5\sin(x)^3}{8}-\frac{15\sin(x)}{8}+\frac{15\ln(\sec(x)+\tan(x))}{8}$	46
risch	$\frac{i e^{ix}}{2}-\frac{i e^{-ix}}{2}+\frac{i(9\,e^{7ix}+e^{5ix}-e^{3ix}-9\,e^{ix})}{4(e^{2ix}+1)^4}+\frac{15\ln(i+e^{ix})}{8}-\frac{15\ln(e^{ix}-i)}{8}$	79

input
```
int(sin(x)*tan(x)^5,x,method=_RETURNVERBOSE)
```

output
```
1/4*sin(x)^7/cos(x)^4-3/8*sin(x)^7/cos(x)^2-3/8*sin(x)^5-5/8*sin(x)^3-15/8
*sin(x)+15/8*ln(sec(x)+tan(x))
```

3.894.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.44

$$\int \sin(x)\tan^5(x)\,dx = \frac{15\cos(x)^4\log(\sin(x)+1) - 15\cos(x)^4\log(-\sin(x)+1) - 2\left(8\cos(x)^4 + 9\cos(x)^2 - 2\right)\sin(x)}{16\cos(x)^4}$$

input
```
integrate(sin(x)*tan(x)^5,x, algorithm="fricas")
```

output
```
1/16*(15*cos(x)^4*log(sin(x) + 1) - 15*cos(x)^4*log(-sin(x) + 1) - 2*(8*co
s(x)^4 + 9*cos(x)^2 - 2)*sin(x))/cos(x)^4
```

3.894.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.44

$$\int \sin(x)\tan^5(x)\,dx = -\frac{-9\sin^3(x) + 7\sin(x)}{8\sin^4(x) - 16\sin^2(x) + 8} - \frac{15\log(\sin(x)-1)}{16} + \frac{15\log(\sin(x)+1)}{16} - \sin(x)$$

input
```
integrate(sin(x)*tan(x)**5,x)
```

output
```
-(-9*sin(x)**3 + 7*sin(x))/(8*sin(x)**4 - 16*sin(x)**2 + 8) - 15*log(sin(x
) - 1)/16 + 15*log(sin(x) + 1)/16 - sin(x)
```

3.894.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.35

$$\int \sin(x)\tan^5(x)\,dx = \frac{9\sin(x)^3-7\sin(x)}{8\left(\sin(x)^4-2\sin(x)^2+1\right)}+\frac{15}{16}\log(\sin(x)+1) -\frac{15}{16}\log(\sin(x)-1)-\sin(x)$$

input
```
integrate(sin(x)*tan(x)^5,x, algorithm="maxima")
```

output
```
1/8*(9*sin(x)^3 - 7*sin(x))/(sin(x)^4 - 2*sin(x)^2 + 1) + 15/16*log(sin(x)
 + 1) - 15/16*log(sin(x) - 1) - sin(x)
```

3.894.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

$$\int \sin(x)\tan^5(x)\,dx = \frac{9\sin(x)^3-7\sin(x)}{8\left(\sin(x)^2-1\right)^2}+\frac{15}{16}\log(\sin(x)+1) -\frac{15}{16}\log(-\sin(x)+1)-\sin(x)$$

input
```
integrate(sin(x)*tan(x)^5,x, algorithm="giac")
```

output
```
1/8*(9*sin(x)^3 - 7*sin(x))/(sin(x)^2 - 1)^2 + 15/16*log(sin(x) + 1) - 15/
16*log(-sin(x) + 1) - sin(x)
```

3.894.9 Mupad [B] (verification not implemented)

Time = 26.81 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.03

$$\int \sin(x)\tan^5(x)\,dx = \frac{15\,\mathrm{atanh}\left(\tan\left(\frac{x}{2}\right)\right)}{4} -\frac{\frac{15\tan\left(\frac{x}{2}\right)^9}{4}-10\tan\left(\frac{x}{2}\right)^7+\frac{9\tan\left(\frac{x}{2}\right)^5}{2}-10\tan\left(\frac{x}{2}\right)^3+\frac{15\tan\left(\frac{x}{2}\right)}{4}}{\left(\tan\left(\frac{x}{2}\right)^2-1\right)^4\left(\tan\left(\frac{x}{2}\right)^2+1\right)}$$

input
```
int(sin(x)*tan(x)^5,x)
```

output
```
(15*atanh(tan(x/2)))/4 - ((15*tan(x/2))/4 - 10*tan(x/2)^3 + (9*tan(x/2)^5)
/2 - 10*tan(x/2)^7 + (15*tan(x/2)^9)/4)/((tan(x/2)^2 - 1)^4*(tan(x/2)^2 +
1))
```

3.894. $\int \sin(x)\tan^5(x)\,dx$

3.895 $\int \cos^5(2x)\cot^4(2x)\,dx$

3.895.1 Optimal result

Integrand size = 13, antiderivative size = 43

$$\int \cos^5(2x)\cot^4(2x)\,dx = 2\csc(2x)-\frac{1}{6}\csc^3(2x)+3\sin(2x)-\frac{2}{3}\sin^3(2x)+\frac{1}{10}\sin^5(2x)$$

output
```
2*csc(2*x)-1/6*csc(2*x)^3+3*sin(2*x)-2/3*sin(2*x)^3+1/10*sin(2*x)^5
```

3.895.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \cos^5(2x)\cot^4(2x)\,dx = 2\csc(2x)-\frac{1}{6}\csc^3(2x)+3\sin(2x)-\frac{2}{3}\sin^3(2x)+\frac{1}{10}\sin^5(2x)$$

input
```
Integrate[Cos[2*x]^5*Cot[2*x]^4,x]
```

output
```
2*Csc[2*x] - Csc[2*x]^3/6 + 3*Sin[2*x] - (2*Sin[2*x]^3)/3 + Sin[2*x]^5/10
```

3.895.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 3070, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^5(2x)\cot^4(2x)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin\left(2x+\frac{\pi}{2}\right)^5\tan\left(2x+\frac{\pi}{2}\right)^4dx$$

$$\Big\downarrow \; 3070$$

$$-\frac{1}{2}\int \csc^4(2x)\left(1-\sin^2(2x)\right)^4d(-\sin(2x))$$

$$\Big\downarrow \; 244$$

$$-\frac{1}{2}\int \left(\csc^4(2x)-4\csc^2(2x)+\sin^4(2x)-4\sin^2(2x)+6\right)d(-\sin(2x))$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(\frac{1}{5}\sin^5(2x)-\frac{4}{3}\sin^3(2x)+6\sin(2x)-\frac{1}{3}\csc^3(2x)+4\csc(2x)\right)$$

input
```
Int[Cos[2*x]^5*Cot[2*x]^4,x]
```

output
```
(4*Csc[2*x] - Csc[2*x]^3/3 + 6*Sin[2*x] - (4*Sin[2*x]^3)/3 + Sin[2*x]^5/5)
/2
```

3.895.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

3.895.4 Maple [A] (verified)

Time = 4.93 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.58

$$-\frac{\cos\left(2x\right)^{10}}{6\sin\left(2x\right)^{3}}+\frac{7\cos\left(2x\right)^{10}}{6\sin\left(2x\right)}+\frac{7\left(\frac{128}{35}+\cos\left(2x\right)^{8}+\frac{8\cos(2x)^{6}}{7}+\frac{48\cos(2x)^{4}}{35}+\frac{64\cos(2x)^{2}}{35}\right)\sin\left(2x\right)}{6}$$

input
```
int(cos(2*x)^5*cot(2*x)^4,x)
```

output
```
-1/6/sin(2*x)^3*cos(2*x)^10+7/6/sin(2*x)*cos(2*x)^10+7/6*(128/35+cos(2*x)^
8+8/7*cos(2*x)^6+48/35*cos(2*x)^4+64/35*cos(2*x)^2)*sin(2*x)
```

3.895.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.21

$$\int \cos^5(2x)\cot^4(2x)\,dx=-\frac{3\cos\left(2\,x\right)^{8}+8\cos\left(2\,x\right)^{6}+48\cos\left(2\,x\right)^{4}-192\cos\left(2\,x\right)^{2}+128}{30\left(\cos\left(2\,x\right)^{2}-1\right)\sin\left(2\,x\right)}$$

input
```
integrate(cos(2*x)^5*cot(2*x)^4,x, algorithm="fricas")
```

output
```
-1/30*(3*cos(2*x)^8 + 8*cos(2*x)^6 + 48*cos(2*x)^4 - 192*cos(2*x)^2 + 128)
/((cos(2*x)^2 - 1)*sin(2*x))
```

3.895.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \cos^5(2x)\cot^4(2x)\,dx = \frac{12\sin^2{(2x)}-1}{6\sin^3{(2x)}} + \frac{\sin^5{(2x)}}{10} - \frac{2\sin^3{(2x)}}{3} + 3\sin{(2x)}$$

input
```
integrate(cos(2*x)**5*cot(2*x)**4,x)
```

output
```
(12*sin(2*x)**2 - 1)/(6*sin(2*x)**3) + sin(2*x)**5/10 - 2*sin(2*x)**3/3 +
3*sin(2*x)
```

3.895.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int \cos^5(2x)\cot^4(2x)\,dx = \frac{1}{10}\sin{(2x)}^5 - \frac{2}{3}\sin{(2x)}^3 + \frac{12\sin{(2x)}^2-1}{6\sin{(2x)}^3} + 3\sin{(2x)}$$

input
```
integrate(cos(2*x)^5*cot(2*x)^4,x, algorithm="maxima")
```

output
```
1/10*sin(2*x)^5 - 2/3*sin(2*x)^3 + 1/6*(12*sin(2*x)^2 - 1)/sin(2*x)^3 + 3*
sin(2*x)
```

3.895.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int \cos^5(2x)\cot^4(2x)\,dx = \frac{1}{10}\sin{(2x)}^5 - \frac{2}{3}\sin{(2x)}^3 + \frac{12\sin{(2x)}^2-1}{6\sin{(2x)}^3} + 3\sin{(2x)}$$

input
```
integrate(cos(2*x)^5*cot(2*x)^4,x, algorithm="giac")
```

output
```
1/10*sin(2*x)^5 - 2/3*sin(2*x)^3 + 1/6*(12*sin(2*x)^2 - 1)/sin(2*x)^3 + 3*
sin(2*x)
```

3.895.9 Mupad [B] (verification not implemented)

Time = 26.54 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \cos^5(2x)\cot^4(2x)\,dx = \frac{3\sin(2x)^8 - 20\sin(2x)^6 + 90\sin(2x)^4 + 60\sin(2x)^2 - 5}{30\sin(2x)^3}$$

input
```
int(cos(2*x)^5*cot(2*x)^4,x)
```

output
```
(60*sin(2*x)^2 + 90*sin(2*x)^4 - 20*sin(2*x)^6 + 3*sin(2*x)^8 - 5)/(30*sin
(2*x)^3)
```

3.896 $\int \cos(3x)\left(-1+\csc^2(3x)\right)^3\left(1-\sin^2(3x)\right)^5\,dx$

3.896.1 Optimal result

Integrand size = 27, antiderivative size = 87

$$\int \cos(3x)\left(-1+\csc^2(3x)\right)^3\left(1-\sin^2(3x)\right)^5\,dx = -\frac{28}{3}\csc(3x)+\frac{8}{9}\csc^3(3x)-\frac{1}{15}\csc^5(3x)-\frac{56}{3}\sin(3x)+\frac{70}{9}\sin^3(3x)-\frac{56}{15}\sin^5(3x)+\frac{4}{3}\sin^7(3x)-\frac{8}{27}\sin^9(3x)+\frac{1}{33}\sin^{11}(3x)$$

output
```
-28/3*csc(3*x)+8/9*csc(3*x)^3-1/15*csc(3*x)^5-56/3*sin(3*x)+70/9*sin(3*x)^
3-56/15*sin(3*x)^5+4/3*sin(3*x)^7-8/27*sin(3*x)^9+1/33*sin(3*x)^11
```

3.896.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int \cos(3x)\left(-1+\csc^2(3x)\right)^3\left(1-\sin^2(3x)\right)^5\,dx = -\frac{28}{3}\csc(3x)+\frac{8}{9}\csc^3(3x)-\frac{1}{15}\csc^5(3x)-\frac{56}{3}\sin(3x)+\frac{70}{9}\sin^3(3x)-\frac{56}{15}\sin^5(3x)+\frac{4}{3}\sin^7(3x)-\frac{8}{27}\sin^9(3x)+\frac{1}{33}\sin^{11}(3x)$$

input
```
Integrate[Cos[3*x]*(-1 + Csc[3*x]^2)^3*(1 - Sin[3*x]^2)^5,x]
```

output
```
(-28*Csc[3*x])/3 + (8*Csc[3*x]^3)/9 - Csc[3*x]^5/15 - (56*Sin[3*x])/3 + (7
0*Sin[3*x]^3)/9 - (56*Sin[3*x]^5)/15 + (4*Sin[3*x]^7)/3 - (8*Sin[3*x]^9)/2
7 + Sin[3*x]^11/33
```

3.896.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3654, 25, 3042, 4608, 3042, 3070, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(1-\sin^2(3x)\right)^5 \cos(3x) \left(\csc^2(3x)-1\right)^3 \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(1-\sin(3x)^2\right)^5 \cos(3x) \left(\csc(3x)^2-1\right)^3 dx$$

$$\Big\downarrow \text{3654}$$

$$\int -\cos^{11}(3x) \left(1-\csc^2(3x)\right)^3 dx$$

$$\Big\downarrow \text{25}$$

$$-\int \cos^{11}(3x) \left(1-\csc^2(3x)\right)^3 dx$$

$$\Big\downarrow \text{3042}$$

$$-\int \left(1-\sec\left(3x+\frac{\pi}{2}\right)^2\right)^3 \sin\left(3x+\frac{\pi}{2}\right)^{11} dx$$

$$\Big\downarrow \text{4608}$$

$$\int \cos^{11}(3x) \cot^6(3x) dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(3x+\frac{\pi}{2}\right)^{11} \tan\left(3x+\frac{\pi}{2}\right)^6 dx$$

$$\Big\downarrow \text{3070}$$

$$-\frac{1}{3}\int \csc^6(3x) \left(1-\sin^2(3x)\right)^8 d(-\sin(3x))$$

$$\Big\downarrow \text{244}$$

$$-\frac{1}{3}\int \left(\sin^{10}(3x)-8\sin^{8}(3x)+28\sin^{6}(3x)-56\sin^{4}(3x)+70\sin^{2}(3x)+\csc^{6}(3x)-8\csc^{4}(3x)+28\csc^{2}(3x)-5\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{1}{11}\sin^{11}(3x)-\frac{8}{9}\sin^{9}(3x)+4\sin^{7}(3x)-\frac{56}{5}\sin^{5}(3x)+\frac{70}{3}\sin^{3}(3x)-56\sin(3x)-\frac{1}{5}\csc^{5}(3x)+\frac{8}{3}\csc^{3}(3x)-2\right.$$

input
```
Int[Cos[3*x]*(-1 + Csc[3*x]^2)^3*(1 - Sin[3*x]^2)^5,x]
```

output
```
(-28*Csc[3*x] + (8*Csc[3*x]^3)/3 - Csc[3*x]^5/5 - 56*Sin[3*x] + (70*Sin[3*
x]^3)/3 - (56*Sin[3*x]^5)/5 + 4*Sin[3*x]^7 - (8*Sin[3*x]^9)/9 + Sin[3*x]^1
1/11)/3
```

3.896.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

3.896. $\int \cos(3x)\left(-1+\csc^2(3x)\right)^3\left(1-\sin^2(3x)\right)^5\,dx$

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 4608
```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
b^p   Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.896.4 Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{\sin(3x)^{11}}{33}-\frac{8\sin(3x)^{9}}{27}+\frac{4\sin(3x)^{7}}{3}-\frac{56\sin(3x)^{5}}{15}+\frac{70\sin(3x)^{3}}{9}-\frac{56\sin(3x)}{3}-\frac{28}{3\sin(3x)}+\frac{8}{9\sin(3x)^{3}}-$
default	$\frac{\sin(3x)^{11}}{33}-\frac{8\sin(3x)^{9}}{27}+\frac{4\sin(3x)^{7}}{3}-\frac{56\sin(3x)^{5}}{15}+\frac{70\sin(3x)^{3}}{9}-\frac{56\sin(3x)}{3}-\frac{28}{3\sin(3x)}+\frac{8}{9\sin(3x)^{3}}-$
risch	$\frac{23ie^{27ix}}{55296}+\frac{37ie^{21ix}}{6144}+\frac{1909ie^{15ix}}{30720}+\frac{5197ie^{9ix}}{9216}+\frac{22379ie^{3ix}}{3072}-\frac{22379ie^{-3ix}}{3072}-\frac{5197ie^{-9ix}}{9216}-\frac{1909ie^{-15ix}}{30720}-$
parallelrisch	$-99\tan\left(\frac{3x}{2}\right)^{27}+3696\tan\left(\frac{3x}{2}\right)^{25}-159720\tan\left(\frac{3x}{2}\right)^{23}-4010160\tan\left(\frac{3x}{2}\right)^{21}-27678420\tan\left(\frac{3x}{2}\right)^{19}-105790608\tan\left(\frac{3x}{2}\right)^{17}$

input
```
int(cos(3*x)*(-1+csc(3*x)^2)^3*(1-sin(3*x)^2)^5,x,method=_RETURNVERBOSE)
```

output
```
1/33*sin(3*x)^11-8/27*sin(3*x)^9+4/3*sin(3*x)^7-56/15*sin(3*x)^5+70/9*sin(
3*x)^3-56/3*sin(3*x)-28/3/sin(3*x)+8/9/sin(3*x)^3-1/15/sin(3*x)^5
```

3.896.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.06

$$\int\cos(3x)\left(-1+\csc^2(3x)\right)^3\left(1-\sin^2(3x)\right)^5\,dx$$
$$=\frac{45\cos(3x)^{16}+80\cos(3x)^{14}+160\cos(3x)^{12}+384\cos(3x)^{10}+1280\cos(3x)^{8}+10240\cos(3x)^{6}-6}{1485\left(\cos(3x)^{4}-2\cos(3x)^{2}+1\right)\sin(3x)}$$

input
```
integrate(cos(3*x)*(-1+csc(3*x)^2)^3*(1-sin(3*x)^2)^5,x, algorithm="fricas
")
```

output
```
1/1485*(45*cos(3*x)^16 + 80*cos(3*x)^14 + 160*cos(3*x)^12 + 384*cos(3*x)^1
0 + 1280*cos(3*x)^8 + 10240*cos(3*x)^6 - 61440*cos(3*x)^4 + 81920*cos(3*x)
^2 - 32768)/((cos(3*x)^4 - 2*cos(3*x)^2 + 1)*sin(3*x))
```

3.896.6 Sympy [F(-1)]

Timed out.

$$\int \cos(3x) \left(-1+\csc^2(3x)\right)^3 \left(1-\sin^2(3x)\right)^5 \, dx = \text{Timed out}$$

input
```
integrate(cos(3*x)*(-1+csc(3*x)**2)**3*(1-sin(3*x)**2)**5,x)
```

output
```
Timed out
```

3.896.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.84

$$\int \cos(3x) \left(-1+\csc^2(3x)\right)^3 \left(1-\sin^2(3x)\right)^5 \, dx = \frac{1}{33} \sin\left(3x\right)^{11} - \frac{8}{27} \sin\left(3x\right)^{9} + \frac{4}{3} \sin\left(3x\right)^{7} - \frac{56}{15} \sin\left(3x\right)^{5} + \frac{70}{9} \sin\left(3x\right)^{3} - \frac{420 \sin\left(3x\right)^{4} - 40 \sin\left(3x\right)^{2} + 3}{45 \sin\left(3x\right)^{5}} - \frac{56}{3} \sin\left(3x\right)$$

input
```
integrate(cos(3*x)*(-1+csc(3*x)^2)^3*(1-sin(3*x)^2)^5,x, algorithm="maxima
")
```

output
```
1/33*sin(3*x)^11 - 8/27*sin(3*x)^9 + 4/3*sin(3*x)^7 - 56/15*sin(3*x)^5 + 7
0/9*sin(3*x)^3 - 1/45*(420*sin(3*x)^4 - 40*sin(3*x)^2 + 3)/sin(3*x)^5 - 56
/3*sin(3*x)
```

3.896.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.84

$$\int \cos(3x) \left(-1+\csc^2(3x)\right)^3 \left(1-\sin^2(3x)\right)^5 \, dx$$
$$= \frac{1}{33} \sin(3x)^{11} - \frac{8}{27} \sin(3x)^9 + \frac{4}{3} \sin(3x)^7 - \frac{56}{15} \sin(3x)^5 + \frac{70}{9} \sin(3x)^3 - \frac{420 \sin(3x)^4 - 40 \sin(3x)^2 + 3}{45 \sin(3x)^5} - \frac{56}{3} \sin(3x)$$

input
```
integrate(cos(3*x)*(-1+csc(3*x)^2)^3*(1-sin(3*x)^2)^5,x, algorithm="giac")
```

output
```
1/33*sin(3*x)^11 - 8/27*sin(3*x)^9 + 4/3*sin(3*x)^7 - 56/15*sin(3*x)^5 + 7
0/9*sin(3*x)^3 - 1/45*(420*sin(3*x)^4 - 40*sin(3*x)^2 + 3)/sin(3*x)^5 - 56
/3*sin(3*x)
```

3.896.9 Mupad [B] (verification not implemented)

Time = 25.90 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.85

$$\int \cos(3x) \left(-1+\csc^2(3x)\right)^3 \left(1-\sin^2(3x)\right)^5 \, dx =$$
$$-\frac{-45 \sin(3x)^{16} + 440 \sin(3x)^{14} - 1980 \sin(3x)^{12} + 5544 \sin(3x)^{10} - 11550 \sin(3x)^8 + 27720 \sin(3x)}{1485 \sin(3x)^5}$$

input
```
int(-cos(3*x)*(1/sin(3*x)^2 - 1)^3*(sin(3*x)^2 - 1)^5,x)
```

output
```
-(13860*sin(3*x)^4 - 1320*sin(3*x)^2 + 27720*sin(3*x)^6 - 11550*sin(3*x)^8
 + 5544*sin(3*x)^10 - 1980*sin(3*x)^12 + 440*sin(3*x)^14 - 45*sin(3*x)^16
+ 99)/(1485*sin(3*x)^5)
```

3.897 $\int \cot(2x)\left(-1+\csc^2(2x)\right)^2\left(1-\sin^2(2x)\right)^2 dx$

3.897.1 Optimal result

Integrand size = 27, antiderivative size = 42

$$\int \cot(2x)\left(-1+\csc^2(2x)\right)^2\left(1-\sin^2(2x)\right)^2 dx = \csc^2(2x)-\frac{1}{8}\csc^4(2x)+3\log(\sin(2x))-\sin^2(2x)+\frac{1}{8}\sin^4(2x)$$

output
```
csc(2*x)^2-1/8*csc(2*x)^4+3*ln(sin(2*x))-sin(2*x)^2+1/8*sin(2*x)^4
```

3.897.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \cot(2x)\left(-1+\csc^2(2x)\right)^2\left(1-\sin^2(2x)\right)^2 dx = \csc^2(2x)-\frac{1}{8}\csc^4(2x)+3\log(\sin(2x))-\sin^2(2x)+\frac{1}{8}\sin^4(2x)$$

input
```
Integrate[Cot[2*x]*(-1 + Csc[2*x]^2)^2*(1 - Sin[2*x]^2)^2,x]
```

output
```
Csc[2*x]^2 - Csc[2*x]^4/8 + 3*Log[Sin[2*x]] - Sin[2*x]^2 + Sin[2*x]^4/8
```

3.897.3 Rubi [A] (warning: unable to verify)

Time = 0.35 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {3042, 3654, 3042, 4860, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(1-\sin^2(2x)\right)^2 \cot(2x) \left(\csc^2(2x)-1\right)^2 \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(1-\sin(2x)^2\right)^2 \cot(2x) \left(\csc(2x)^2-1\right)^2 \, dx$$

$$\Big\downarrow \text{3654}$$

$$\int \cos^4(2x) \cot(2x) \left(1-\csc^2(2x)\right)^2 \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \cos(2x)^4 \cot(2x) \left(1-\csc(2x)^2\right)^2 \, dx$$

$$\Big\downarrow \text{4860}$$

$$\frac{1}{2} \int \csc^5(2x) \left(1-\sin^2(2x)\right)^4 d\sin(2x)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{4} \int \csc^3(2x) \left(1-\sin^2(2x)\right)^4 d\sin^2(2x)$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{4} \int \left(\csc^3(2x)-4\csc^2(2x)+6\csc(2x)+\sin^2(2x)-4\right) d\sin^2(2x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4} \left(\frac{1}{2}\sin^4(2x)-4\sin^2(2x)-\frac{1}{2}\csc^2(2x)+4\csc(2x)+6\log\left(\sin^2(2x)\right)\right)$$

input
```
Int[Cot[2*x]*(-1 + Csc[2*x]^2)^2*(1 - Sin[2*x]^2)^2,x]
```

output
```
(4*Csc[2*x] - Csc[2*x]^2/2 + 6*Log[Sin[2*x]^2] - 4*Sin[2*x]^2 + Sin[2*x]^4
/2)/4
```

3.897.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 4860
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[1/(b*c)   Subst[Int[SubstFor[1/x, Sin[c*(a +
 b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b
*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

3.897.4 Maple [A] (verified)

Time = 14.66 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{\sin(2x)^4}{8}+\cos\left(2x\right)^2+3\ln\left(\sin\left(2x\right)\right)+\frac{1}{\sin(2x)^2}-\frac{1}{8\sin(2x)^4}$	37
default	$\frac{\sin(2x)^4}{8}+\cos\left(2x\right)^2+3\ln\left(\sin\left(2x\right)\right)+\frac{1}{\sin(2x)^2}-\frac{1}{8\sin(2x)^4}$	37
risch	$-6ix+\frac{\mathrm{e}^{8ix}}{128}+\frac{7\,\mathrm{e}^{4ix}}{32}+\frac{7\,\mathrm{e}^{-4ix}}{32}+\frac{\mathrm{e}^{-8ix}}{128}-\frac{2\left(2\,\mathrm{e}^{12ix}-3\,\mathrm{e}^{8ix}+2\,\mathrm{e}^{4ix}\right)}{\left(\mathrm{e}^{4ix}-1\right)^4}+3\ln\left(\mathrm{e}^{4ix}-1\right)$	77

input
```
int(cot(2*x)*(-1+csc(2*x)^2)^2*(1-sin(2*x)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*sin(2*x)^4+cos(2*x)^2+3*ln(sin(2*x))+1/sin(2*x)^2-1/8/sin(2*x)^4
```

3.897.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(38) = 76$.

Time = 0.26 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.88

$$\int \cot(2x)\left(-1+\csc^2(2x)\right)^2\left(1-\sin^2(2x)\right)^2\,dx$$
$$= \frac{8\cos\left(2x\right)^8 + 32\cos\left(2x\right)^6 - 115\cos\left(2x\right)^4 + 38\cos\left(2x\right)^2 + 192\left(\cos\left(2x\right)^4 - 2\cos\left(2x\right)^2 + 1\right)\log\left(\frac{1}{2}\sin\right.}{64\left(\cos\left(2x\right)^4 - 2\cos\left(2x\right)^2 + 1\right)}$$

input
```
integrate(cot(2*x)*(-1+csc(2*x)^2)^2*(1-sin(2*x)^2)^2,x, algorithm="fricas
")
```

output
```
1/64*(8*cos(2*x)^8 + 32*cos(2*x)^6 - 115*cos(2*x)^4 + 38*cos(2*x)^2 + 192*
(cos(2*x)^4 - 2*cos(2*x)^2 + 1)*log(1/2*sin(2*x)) + 29)/(cos(2*x)^4 - 2*co
s(2*x)^2 + 1)
```

3.897.6 Sympy [F(-1)]

Timed out.

$$\int \cot(2x)\left(-1+\csc^2(2x)\right)^2\left(1-\sin^2(2x)\right)^2\,dx = \text{Timed out}$$

input
```
integrate(cot(2*x)*(-1+csc(2*x)**2)**2*(1-sin(2*x)**2)**2,x)
```

output
```
Timed out
```

3.897.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.05

$$\int \cot(2x) \left(-1+\csc^2(2x)\right)^2 \left(1-\sin^2(2x)\right)^2 \, dx = \frac{1}{8} \sin\left(2x\right)^4 - \sin\left(2x\right)^2 + \frac{8 \sin\left(2x\right)^2 - 1}{8 \sin\left(2x\right)^4} + \frac{3}{2} \log\left(\sin\left(2x\right)^2\right)$$

input
```
integrate(cot(2*x)*(-1+csc(2*x)^2)^2*(1-sin(2*x)^2)^2,x, algorithm="maxima
")
```

output
```
1/8*sin(2*x)^4 - sin(2*x)^2 + 1/8*(8*sin(2*x)^2 - 1)/sin(2*x)^4 + 3/2*log(
sin(2*x)^2)
```

3.897.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.24

$$\int \cot(2x) \left(-1+\csc^2(2x)\right)^2 \left(1-\sin^2(2x)\right)^2 \, dx = \frac{1}{8} \cos\left(2x\right)^4 + \frac{3}{4} \cos\left(2x\right)^2 - \frac{8 \cos\left(2x\right)^2 - 7}{8 \left(\cos\left(2x\right)^2 - 1\right)^2} + \frac{3}{2} \log\left(-\cos\left(2x\right)^2 + 1\right)$$

input
```
integrate(cot(2*x)*(-1+csc(2*x)^2)^2*(1-sin(2*x)^2)^2,x, algorithm="giac")
```

output
```
1/8*cos(2*x)^4 + 3/4*cos(2*x)^2 - 1/8*(8*cos(2*x)^2 - 7)/(cos(2*x)^2 - 1)^
2 + 3/2*log(-cos(2*x)^2 + 1)
```

3.897.9 Mupad [B] (verification not implemented)

Time = 26.99 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.69

$$\int \cot(2x) \left(-1+\csc^2(2x)\right)^2 \left(1-\sin^2(2x)\right)^2 \, dx = 3 \ln\left(\tan(2x)\right) - \frac{3 \ln\left(\tan(2x)^2 + 1\right)}{2} + \frac{3 \tan(2x)^6 + \frac{9 \tan(2x)^4}{2} + \tan(2x)^2 - \frac{1}{4}}{2 \left(\tan\left(2x\right)^8 + 2 \tan\left(2x\right)^6 + \tan\left(2x\right)^4\right)}$$

input
```
int(cot(2*x)*(1/sin(2*x)^2 - 1)^2*(sin(2*x)^2 - 1)^2,x)
```

output
```
3*log(tan(2*x)) - (3*log(tan(2*x)^2 + 1))/2 + (tan(2*x)^2 + (9*tan(2*x)^4)
/2 + 3*tan(2*x)^6 - 1/4)/(2*(tan(2*x)^4 + 2*tan(2*x)^6 + tan(2*x)^8))
```

3.897. $\int \cot(2x)\left(-1+\csc^2(2x)\right)^2\left(1-\sin^2(2x)\right)^2\,dx$

3.898 $\int \cos(2x)\left(-1+\csc^2(2x)\right)^4\left(1-\sin^2(2x)\right)^2\,dx$

3.898.1 Optimal result

Integrand size = 27, antiderivative size = 63

$$\int \cos(2x)\left(-1+\csc^2(2x)\right)^4\left(1-\sin^2(2x)\right)^2\,dx = 10\csc(2x)-\frac{5}{2}\csc^3(2x)+\frac{3}{5}\csc^5(2x)-\frac{1}{14}\csc^7(2x)+\frac{15}{2}\sin(2x)-\sin^3(2x)+\frac{1}{10}\sin^5(2x)$$

output
```
10*csc(2*x)-5/2*csc(2*x)^3+3/5*csc(2*x)^5-1/14*csc(2*x)^7+15/2*sin(2*x)-si
n(2*x)^3+1/10*sin(2*x)^5
```

3.898.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \cos(2x)\left(-1+\csc^2(2x)\right)^4\left(1-\sin^2(2x)\right)^2\,dx = 10\csc(2x)-\frac{5}{2}\csc^3(2x)+\frac{3}{5}\csc^5(2x)-\frac{1}{14}\csc^7(2x)+\frac{15}{2}\sin(2x)-\sin^3(2x)+\frac{1}{10}\sin^5(2x)$$

input
```
Integrate[Cos[2*x]*(-1 + Csc[2*x]^2)^4*(1 - Sin[2*x]^2)^2,x]
```

output
```
10*Csc[2*x] - (5*Csc[2*x]^3)/2 + (3*Csc[2*x]^5)/5 - Csc[2*x]^7/14 + (15*Si
n[2*x])/2 - Sin[2*x]^3 + Sin[2*x]^5/10
```

3.898.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {3042, 3654, 3042, 4608, 3042, 3070, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(1-\sin^2(2x)\right)^2 \cos(2x) \left(\csc^2(2x)-1\right)^4 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(1-\sin(2x)^2\right)^2 \cos(2x) \left(\csc(2x)^2-1\right)^4 dx$$

$$\Big\downarrow \text{3654}$$

$$\int \cos^5(2x) \left(1-\csc^2(2x)\right)^4 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(2x+\frac{\pi}{2}\right)^5 \left(1-\sec\left(2x+\frac{\pi}{2}\right)^2\right)^4 dx$$

$$\Big\downarrow \text{4608}$$

$$\int \cos^5(2x) \cot^8(2x) dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(2x+\frac{\pi}{2}\right)^5 \tan\left(2x+\frac{\pi}{2}\right)^8 dx$$

$$\Big\downarrow \text{3070}$$

$$-\frac{1}{2}\int \csc^8(2x) \left(1-\sin^2(2x)\right)^6 d(-\sin(2x))$$

$$\Big\downarrow \text{244}$$

$$-\frac{1}{2}\int \left(\csc^8(2x)-6\csc^6(2x)+15\csc^4(2x)-20\csc^2(2x)+\sin^4(2x)-6\sin^2(2x)+15\right) d(-\sin(2x))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{5}\sin^5(2x)-2\sin^3(2x)+15\sin(2x)-\frac{1}{7}\csc^7(2x)+\frac{6}{5}\csc^5(2x)-5\csc^3(2x)+20\csc(2x)\right)$$

input
```
Int[Cos[2*x]*(-1 + Csc[2*x]^2)^4*(1 - Sin[2*x]^2)^2,x]
```

output
```
(20*Csc[2*x] - 5*Csc[2*x]^3 + (6*Csc[2*x]^5)/5 - Csc[2*x]^7/7 + 15*Sin[2*x
] - 2*Sin[2*x]^3 + Sin[2*x]^5/5)/2
```

3.898.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 4608
```
Int[(u_.)*((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
b^p   Int[ActivateTrig[u*tan[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

3.898. $\int \cos(2x)\left(-1+\csc^2(2x)\right)^4\left(1-\sin^2(2x)\right)^2\,dx$

3.898.4 Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.78

method	result
parallelrisch	$\frac{\sec(x)^7 \csc(x)^7 (6062 \cos(16x)+429065 \cos(8x)-100940 \cos(12x)+7 \cos(24x)+196 \cos(20x)-952952 \cos(4x)+608322)}{18350080}$
derivativedivides	$\frac{\sin(2x)^5}{10}-\sin(2x)^3+\frac{15\sin(2x)}{2}+\frac{10}{\sin(2x)}-\frac{5}{2\sin(2x)^3}+\frac{3}{5\sin(2x)^5}-\frac{1}{14\sin(2x)^7}$
default	$\frac{\sin(2x)^5}{10}-\sin(2x)^3+\frac{15\sin(2x)}{2}+\frac{10}{\sin(2x)}-\frac{5}{2\sin(2x)^3}+\frac{3}{5\sin(2x)^5}-\frac{1}{14\sin(2x)^7}$
risch	$-\frac{i e^{10ix}}{320}-\frac{7i e^{6ix}}{64}-\frac{109i e^{2ix}}{32}+\frac{109i e^{-2ix}}{32}+\frac{7i e^{-6ix}}{64}+\frac{i e^{-10ix}}{320}+\frac{4i(175\, e^{26ix}-875\, e^{22ix}+2093\, e^{18ix}-2706\, e^{\cdots}}{35(e^{4ix}-\cdots}$

input
```
int(cos(2*x)*(-1+csc(2*x)^2)^4*(1-sin(2*x)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/18350080*sec(x)^7*csc(x)^7*(6062*cos(16*x)+429065*cos(8*x)-100940*cos(12
*x)+7*cos(24*x)+196*cos(20*x)-952952*cos(4*x)+608322)
```

3.898.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.33

$$\int \cos(2x)\left(-1+\csc^2(2x)\right)^4\left(1-\sin^2(2x)\right)^2\,dx = -\frac{7\cos(2x)^{12}+28\cos(2x)^{10}+280\cos(2x)^8-2240\cos(2x)^6+4480\cos(2x)^4-3584\cos(2x)^2+10\cdots}{70\left(\cos(2x)^6-3\cos(2x)^4+3\cos(2x)^2-1\right)\sin(2x)}$$

input
```
integrate(cos(2*x)*(-1+csc(2*x)^2)^4*(1-sin(2*x)^2)^2,x, algorithm="fricas
")
```

output
```
-1/70*(7*cos(2*x)^12 + 28*cos(2*x)^10 + 280*cos(2*x)^8 - 2240*cos(2*x)^6 +
 4480*cos(2*x)^4 - 3584*cos(2*x)^2 + 1024)/((cos(2*x)^6 - 3*cos(2*x)^4 + 3
*cos(2*x)^2 - 1)*sin(2*x))
```

3.898.6 Sympy [F(-1)]

Timed out.

$$\int \cos(2x)\left(-1+\csc^2(2x)\right)^4\left(1-\sin^2(2x)\right)^2\,dx = \text{Timed out}$$

input
```
integrate(cos(2*x)*(-1+csc(2*x)**2)**4*(1-sin(2*x)**2)**2,x)
```

output
```
Timed out
```

3.898.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \cos(2x)\left(-1+\csc^2(2x)\right)^4\left(1-\sin^2(2x)\right)^2\,dx = \frac{1}{10}\sin(2x)^5 - \sin(2x)^3 + \frac{700\sin(2x)^6 - 175\sin(2x)^4 + 42\sin(2x)^2 - 5}{70\sin(2x)^7} + \frac{15}{2}\sin(2x)$$

input
```
integrate(cos(2*x)*(-1+csc(2*x)^2)^4*(1-sin(2*x)^2)^2,x, algorithm="maxima
")
```

output
```
1/10*sin(2*x)^5 - sin(2*x)^3 + 1/70*(700*sin(2*x)^6 - 175*sin(2*x)^4 + 42*
sin(2*x)^2 - 5)/sin(2*x)^7 + 15/2*sin(2*x)
```

3.898.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \cos(2x)\left(-1+\csc^2(2x)\right)^4\left(1-\sin^2(2x)\right)^2\,dx = \frac{1}{10}\sin(2x)^5 - \sin(2x)^3 + \frac{700\sin(2x)^6 - 175\sin(2x)^4 + 42\sin(2x)^2 - 5}{70\sin(2x)^7} + \frac{15}{2}\sin(2x)$$

input
```
integrate(cos(2*x)*(-1+csc(2*x)^2)^4*(1-sin(2*x)^2)^2,x, algorithm="giac")
```

output
```
1/10*sin(2*x)^5 - sin(2*x)^3 + 1/70*(700*sin(2*x)^6 - 175*sin(2*x)^4 + 42*
sin(2*x)^2 - 5)/sin(2*x)^7 + 15/2*sin(2*x)
```

3.898.9 Mupad [B] (verification not implemented)

Time = 26.37 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \cos(2x) \left(-1+\csc^2(2x)\right)^4 \left(1-\sin^2(2x)\right)^2 \, dx = \frac{\frac{\sin(2\,x)^{12}}{10}-\sin\left(2\,x\right)^{10}+\frac{15\,\sin(2\,x)^8}{2}+10\,\sin\left(2\,x\right)^6-\frac{5\,\sin(2\,x)^4}{2}+\frac{3\,\sin(2\,x)^2}{5}-\frac{1}{14}}{\sin\left(2\,x\right)^7}$$

input
```
int(cos(2*x)*(1/sin(2*x)^2 - 1)^4*(sin(2*x)^2 - 1)^2,x)
```

output
```
((3*sin(2*x)^2)/5 - (5*sin(2*x)^4)/2 + 10*sin(2*x)^6 + (15*sin(2*x)^8)/2 -
 sin(2*x)^10 + sin(2*x)^12/10 - 1/14)/sin(2*x)^7
```

3.899 $\int \cot(3x)\left(-1+\csc^2(3x)\right)^3\left(1-\sin^2(3x)\right)^2\,dx$

3.899.1 Optimal result

Integrand size = 27, antiderivative size = 60

$$\int \cot(3x)\left(-1+\csc^2(3x)\right)^3\left(1-\sin^2(3x)\right)^2\,dx = -\frac{5}{3}\csc^2(3x)+\frac{5}{12}\csc^4(3x)-\frac{1}{18}\csc^6(3x)-\frac{10}{3}\log(\sin(3x))+\frac{5}{6}\sin^2(3x)-\frac{1}{12}\sin^4(3x)$$

output
```
-5/3*csc(3*x)^2+5/12*csc(3*x)^4-1/18*csc(3*x)^6-10/3*ln(sin(3*x))+5/6*sin(
3*x)^2-1/12*sin(3*x)^4
```

3.899.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \cot(3x)\left(-1+\csc^2(3x)\right)^3\left(1-\sin^2(3x)\right)^2\,dx = -\frac{5}{3}\csc^2(3x)+\frac{5}{12}\csc^4(3x)-\frac{1}{18}\csc^6(3x)-\frac{10}{3}\log(\sin(3x))+\frac{5}{6}\sin^2(3x)-\frac{1}{12}\sin^4(3x)$$

input
```
Integrate[Cot[3*x]*(-1 + Csc[3*x]^2)^3*(1 - Sin[3*x]^2)^2,x]
```

output
```
(-5*Csc[3*x]^2)/3 + (5*Csc[3*x]^4)/12 - Csc[3*x]^6/18 - (10*Log[Sin[3*x]])
/3 + (5*Sin[3*x]^2)/6 - Sin[3*x]^4/12
```

3.899.3 Rubi [A] (warning: unable to verify)

Time = 0.36 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3654, 25, 3042, 4860, 25, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(1-\sin^2(3x)\right)^2 \cot(3x) \left(\csc^2(3x)-1\right)^3 \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(1-\sin(3x)^2\right)^2 \cot(3x) \left(\csc(3x)^2-1\right)^3 dx$$

$$\Big\downarrow \text{3654}$$

$$\int -\cos^4(3x)\cot(3x)\left(1-\csc^2(3x)\right)^3 dx$$

$$\Big\downarrow \text{25}$$

$$-\int \cos^4(3x)\cot(3x)\left(1-\csc^2(3x)\right)^3 dx$$

$$\Big\downarrow \text{3042}$$

$$-\int \cos(3x)^4\cot(3x)\left(1-\csc(3x)^2\right)^3 dx$$

$$\Big\downarrow \text{4860}$$

$$-\frac{1}{3}\int -\csc^7(3x)\left(1-\sin^2(3x)\right)^5 d\sin(3x)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\int \csc^7(3x)\left(1-\sin^2(3x)\right)^5 d\sin(3x)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{6}\int \csc^4(3x)\left(1-\sin^2(3x)\right)^5 d\sin^2(3x)$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{6}\int \left(\csc^4(3x)-5\csc^3(3x)+10\csc^2(3x)-10\csc(3x)-\sin^2(3x)+5\right) d\sin^2(3x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{6}\left(-\frac{1}{2}\sin^4(3x)+5\sin^2(3x)-\frac{1}{3}\csc^3(3x)+\frac{5}{2}\csc^2(3x)-10\csc(3x)-10\log\left(\sin^2(3x)\right)\right)$$

input
```
Int[Cot[3*x]*(-1 + Csc[3*x]^2)^3*(1 - Sin[3*x]^2)^2,x]
```

output
```
(-10*Csc[3*x] + (5*Csc[3*x]^2)/2 - Csc[3*x]^3/3 - 10*Log[Sin[3*x]^2] + 5*S
in[3*x]^2 - Sin[3*x]^4/2)/6
```

3.899.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 4860
```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Simp[1/(b*c)   Subst[Int[SubstFor[1/x, Sin[c*(a +
 b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b
*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

3.899. $\int \cot(3x)\left(-1+\csc^2(3x)\right)^3\left(1-\sin^2(3x)\right)^2\,dx$

3.899.4 Maple [A] (verified)

Time = 45.95 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.82

method	result
derivativedivides	$-\frac{\sin(3x)^4}{12}-\frac{5\cos(3x)^2}{6}-\frac{10\ln(\sin(3x))}{3}-\frac{5}{3\sin(3x)^2}+\frac{5}{12\sin(3x)^4}-\frac{1}{18\sin(3x)^6}$
default	$-\frac{\sin(3x)^4}{12}-\frac{5\cos(3x)^2}{6}-\frac{10\ln(\sin(3x))}{3}-\frac{5}{3\sin(3x)^2}+\frac{5}{12\sin(3x)^4}-\frac{1}{18\sin(3x)^6}$
risch	$10ix-\frac{e^{12ix}}{192}-\frac{3e^{6ix}}{16}-\frac{3e^{-6ix}}{16}-\frac{e^{-12ix}}{192}+\frac{\frac{20e^{30ix}}{3}-20e^{24ix}+\frac{272e^{18ix}}{9}-20e^{12ix}+\frac{20e^{6ix}}{3}}{(e^{6ix}-1)^6}-\frac{10\ln(e^{6ix}-1)}{3}$

input
```
int(cot(3*x)*(-1+csc(3*x)^2)^3*(1-sin(3*x)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/12*sin(3*x)^4-5/6*cos(3*x)^2-10/3*ln(sin(3*x))-5/3/sin(3*x)^2+5/12/sin(
3*x)^4-1/18/sin(3*x)^6
```

3.899.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(48) = 96.

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.72

$$\int \cot(3x)\left(-1+\csc^2(3x)\right)^3\left(1-\sin^2(3x)\right)^2\,dx =$$
$$-\frac{24\cos(3x)^{10}+120\cos(3x)^8-609\cos(3x)^6+387\cos(3x)^4+333\cos(3x)^2+960\left(\cos(3x)^6-3\cos\right.}{288\left(\cos(3x)^6-3\cos(3x)^4+3\cos(3x)^2-1\right)}$$

input
```
integrate(cot(3*x)*(-1+csc(3*x)^2)^3*(1-sin(3*x)^2)^2,x, algorithm="fricas
")
```

output
```
-1/288*(24*cos(3*x)^10 + 120*cos(3*x)^8 - 609*cos(3*x)^6 + 387*cos(3*x)^4
+ 333*cos(3*x)^2 + 960*(cos(3*x)^6 - 3*cos(3*x)^4 + 3*cos(3*x)^2 - 1)*log(
1/2*sin(3*x)) - 271)/(cos(3*x)^6 - 3*cos(3*x)^4 + 3*cos(3*x)^2 - 1)
```

3.899.6 Sympy [F(-1)]

Timed out.

$$\int \cot(3x) \left(-1+\csc^2(3x)\right)^3 \left(1-\sin^2(3x)\right)^2 \, dx = \text{Timed out}$$

input
```
integrate(cot(3*x)*(-1+csc(3*x)**2)**3*(1-sin(3*x)**2)**2,x)
```

output
```
Timed out
```

3.899.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$\int \cot(3x) \left(-1+\csc^2(3x)\right)^3 \left(1-\sin^2(3x)\right)^2 \, dx = -\frac{1}{12} \sin\left(3x\right)^4 + \frac{5}{6} \sin\left(3x\right)^2 - \frac{60 \sin\left(3x\right)^4 - 15 \sin\left(3x\right)^2 + 2}{36 \sin\left(3x\right)^6} - \frac{5}{3} \log\left(\sin\left(3x\right)^2\right)$$

input
```
integrate(cot(3*x)*(-1+csc(3*x)^2)^3*(1-sin(3*x)^2)^2,x, algorithm="maxima
")
```

output
```
-1/12*sin(3*x)^4 + 5/6*sin(3*x)^2 - 1/36*(60*sin(3*x)^4 - 15*sin(3*x)^2 +
2)/sin(3*x)^6 - 5/3*log(sin(3*x)^2)
```

3.899.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \cot(3x) \left(-1+\csc^2(3x)\right)^3 \left(1-\sin^2(3x)\right)^2 \, dx = -\frac{1}{12} \sin\left(3x\right)^4 + \frac{5}{6} \sin\left(3x\right)^2 + \frac{110 \sin\left(3x\right)^6 - 60 \sin\left(3x\right)^4 + 15 \sin\left(3x\right)^2 - 2}{36 \sin\left(3x\right)^6} - \frac{5}{3} \log\left(\sin\left(3x\right)^2\right)$$

input
```
integrate(cot(3*x)*(-1+csc(3*x)^2)^3*(1-sin(3*x)^2)^2,x, algorithm="giac")
```

output
```
-1/12*sin(3*x)^4 + 5/6*sin(3*x)^2 + 1/36*(110*sin(3*x)^6 - 60*sin(3*x)^4 +
 15*sin(3*x)^2 - 2)/sin(3*x)^6 - 5/3*log(sin(3*x)^2)
```

3.899.9 Mupad [B] (verification not implemented)

Time = 28.17 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.40

$$\int \cot(3x) \left(-1+\csc^2(3x)\right)^3 \left(1-\sin^2(3x)\right)^2 \, dx$$
$$= \frac{\ln\left(\left(\tan(3\,x)^2+1\right)^5\right)}{3} - \frac{10\,\ln\left(\tan(3\,x)\right)}{3} - \frac{5\tan(3\,x)^8 + \frac{15\tan(3\,x)^6}{2} + \frac{5\tan(3\,x)^4}{3} - \frac{5\tan(3\,x)^2}{12} + \frac{1}{6}}{3\,\left(\tan\left(3\,x\right)^{10} + 2\tan\left(3\,x\right)^8 + \tan\left(3\,x\right)^6\right)}$$

input
```
int(cot(3*x)*(1/sin(3*x)^2 - 1)^3*(sin(3*x)^2 - 1)^2,x)
```

output
```
log((tan(3*x)^2 + 1)^5)/3 - (10*log(tan(3*x)))/3 - ((5*tan(3*x)^4)/3 - (5*
tan(3*x)^2)/12 + (15*tan(3*x)^6)/2 + 5*tan(3*x)^8 + 1/6)/(3*(tan(3*x)^6 +
2*tan(3*x)^8 + tan(3*x)^10))
```

3.900 $\int (1+\cot^2(9x))^2 (1+\tan^2(9x))^3 \, dx$

3.900.1 Optimal result

Integrand size = 21, antiderivative size = 47

$$\int (1+\cot^2(9x))^2 (1+\tan^2(9x))^3 \, dx = -\frac{4}{9}\cot(9x) - \frac{1}{27}\cot^3(9x) + \frac{2}{3}\tan(9x) + \frac{4}{27}\tan^3(9x) + \frac{1}{45}\tan^5(9x)$$

output
```
-4/9*cot(9*x)-1/27*cot(9*x)^3+2/3*tan(9*x)+4/27*tan(9*x)^3+1/45*tan(9*x)^5
```

3.900.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.26

$$\int (1+\cot^2(9x))^2 (1+\tan^2(9x))^3 \, dx = -\frac{11}{27}\cot(9x) - \frac{1}{27}\cot(9x)\csc^2(9x) + \frac{73}{135}\tan(9x) + \frac{14}{135}\sec^2(9x)\tan(9x) + \frac{1}{45}\sec^4(9x)\tan(9x)$$

input
```
Integrate[(1 + Cot[9*x]^2)^2*(1 + Tan[9*x]^2)^3,x]
```

output
```
(-11*Cot[9*x])/27 - (Cot[9*x]*Csc[9*x]^2)/27 + (73*Tan[9*x])/135 + (14*Sec
[9*x]^2*Tan[9*x])/135 + (Sec[9*x]^4*Tan[9*x])/45
```

3.900.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3042, 4140, 3042, 4140, 3042, 3100, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\tan^2(9x)+1\right)^3 \left(\cot^2(9x)+1\right)^2 \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(\tan(9x)^2+1\right)^3 \left(\cot(9x)^2+1\right)^2 dx$$

$$\Big\downarrow \text{4140}$$

$$\int \left(\cot^2(9x)+1\right)^2 \sec^6(9x)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(\tan\left(9x+\frac{\pi}{2}\right)^2+1\right)^2}{\sin\left(9x+\frac{\pi}{2}\right)^6}dx$$

$$\Big\downarrow \text{4140}$$

$$\int \csc^4(9x)\sec^6(9x)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc(9x)^4\sec(9x)^6dx$$

$$\Big\downarrow \text{3100}$$

$$\frac{1}{9}\int \cot^4(9x)\left(\tan^2(9x)+1\right)^4 d\tan(9x)$$

$$\Big\downarrow \text{244}$$

$$\frac{1}{9}\int \left(\cot^4(9x)+4\cot^2(9x)+\tan^4(9x)+4\tan^2(9x)+6\right) d\tan(9x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{9}\left(\frac{1}{5}\tan^5(9x)+\frac{4}{3}\tan^3(9x)+6\tan(9x)-\frac{1}{3}\cot^3(9x)-4\cot(9x)\right)$$

input
```
Int[(1 + Cot[9*x]^2)^2*(1 + Tan[9*x]^2)^3,x]
```

output
```
(-4*Cot[9*x] - Cot[9*x]^3/3 + 6*Tan[9*x] + (4*Tan[9*x]^3)/3 + Tan[9*x]^5/5
)/9
```

3.900.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3100
```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[1/f   Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]]
, x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

3.900.4 Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$-\frac{4\cot(9x)}{9}-\frac{\cot(9x)^3}{27}+\frac{2\tan(9x)}{3}+\frac{4\tan(9x)^3}{27}+\frac{\tan(9x)^5}{45}$	38
default	$-\frac{4\cot(9x)}{9}-\frac{\cot(9x)^3}{27}+\frac{2\tan(9x)}{3}+\frac{4\tan(9x)^3}{27}+\frac{\tan(9x)^5}{45}$	38
parallelrisch	$-\frac{4\cot(9x)}{9}-\frac{\cot(9x)^3}{27}+\frac{2\tan(9x)}{3}+\frac{4\tan(9x)^3}{27}+\frac{\tan(9x)^5}{45}$	38
norman	$\frac{-\frac{1}{27}-\frac{4\tan(9x)^2}{9}+\frac{2\tan(9x)^4}{3}+\frac{4\tan(9x)^6}{27}+\frac{\tan(9x)^8}{45}}{\tan(9x)^3}$	42
risch	$\frac{256i(6\,e^{54ix}+2\,e^{36ix}-2\,e^{18ix}-1)}{135(e^{18ix}+1)^5(e^{18ix}-1)^3}$	45
parts	$x+\frac{\tan(9x)^5}{45}+\frac{4\tan(9x)^3}{27}+\frac{2\tan(9x)}{3}-\frac{\arctan(\tan(9x))}{9}-\frac{4\cot(9x)}{9}-\frac{\cot(9x)^3}{27}$	46

input
```
int((1+cot(9*x)^2)^2*(1+tan(9*x)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-4/9*cot(9*x)-1/27*cot(9*x)^3+2/3*tan(9*x)+4/27*tan(9*x)^3+1/45*tan(9*x)^5
```

3.900.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \left(1+\cot^2(9x)\right)^2\left(1+\tan^2(9x)\right)^3\,dx = \frac{3\,\tan\left(9\,x\right)^8+20\,\tan\left(9\,x\right)^6+90\,\tan\left(9\,x\right)^4-60\,\tan\left(9\,x\right)^2-5}{135\,\tan\left(9\,x\right)^3}$$

input
```
integrate((1+cot(9*x)^2)^2*(1+tan(9*x)^2)^3,x, algorithm="fricas")
```

output
```
1/135*(3*tan(9*x)^8 + 20*tan(9*x)^6 + 90*tan(9*x)^4 - 60*tan(9*x)^2 - 5)/t
an(9*x)^3
```

3.900.6 Sympy [A] (verification not implemented)

Time = 3.63 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int \left(1+\cot^2(9x)\right)^2 \left(1+\tan^2(9x)\right)^3 \, dx = \frac{\tan^5(9x)}{45} + \frac{4\tan^3(9x)}{27} + \frac{2\tan(9x)}{3} - \frac{4}{9\tan(9x)} - \frac{1}{27\tan^3(9x)}$$

input
```
integrate((1+cot(9*x)**2)**2*(1+tan(9*x)**2)**3,x)
```

output
```
tan(9*x)**5/45 + 4*tan(9*x)**3/27 + 2*tan(9*x)/3 - 4/(9*tan(9*x)) - 1/(27*
tan(9*x)**3)
```

3.900.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int \left(1+\cot^2(9x)\right)^2 \left(1+\tan^2(9x)\right)^3 \, dx = \frac{1}{45}\tan(9x)^5 + \frac{4}{27}\tan(9x)^3 - \frac{12\tan(9x)^2+1}{27\tan(9x)^3} + \frac{2}{3}\tan(9x)$$

input
```
integrate((1+cot(9*x)^2)^2*(1+tan(9*x)^2)^3,x, algorithm="maxima")
```

output
```
1/45*tan(9*x)^5 + 4/27*tan(9*x)^3 - 1/27*(12*tan(9*x)^2 + 1)/tan(9*x)^3 +
2/3*tan(9*x)
```

3.900.8 Giac [A] (verification not implemented)

Time = 69.70 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int \left(1+\cot^2(9x)\right)^2 \left(1+\tan^2(9x)\right)^3 \, dx = \frac{1}{45}\tan(9x)^5 + \frac{4}{27}\tan(9x)^3 - \frac{12\tan(9x)^2+1}{27\tan(9x)^3} + \frac{2}{3}\tan(9x)$$

input
```
integrate((1+cot(9*x)^2)^2*(1+tan(9*x)^2)^3,x, algorithm="giac")
```

output
```
1/45*tan(9*x)^5 + 4/27*tan(9*x)^3 - 1/27*(12*tan(9*x)^2 + 1)/tan(9*x)^3 +
2/3*tan(9*x)
```

3.900.9 Mupad [B] (verification not implemented)

Time = 29.50 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \left(1+\cot^2(9x)\right)^2 \left(1+\tan^2(9x)\right)^3 \, dx = \frac{3\tan(9\,x)^8+20\tan(9\,x)^6+90\tan(9\,x)^4-60\tan(9\,x)^2-5}{135\tan\left(9\,x\right)^3}$$

input
```
int((tan(9*x)^2 + 1)^3*(cot(9*x)^2 + 1)^2,x)
```

output
```
(90*tan(9*x)^4 - 60*tan(9*x)^2 + 20*tan(9*x)^6 + 3*tan(9*x)^8 - 5)/(135*ta
n(9*x)^3)
```

3.901 $\int \frac{\cos(x)(9-7\sin^3(x))^2}{1-\sin^2(x)}\,dx$

3.901.1 Optimal result

Integrand size = 23, antiderivative size = 43

$$\int \frac{\cos(x)\,(9-7\sin^3(x))^2}{1-\sin^2(x)}\,dx = -2\log(1-\sin(x))+128\log(1+\sin(x)) - 49\sin(x)+63\sin^2(x)-\frac{49\sin^3(x)}{3}-\frac{49\sin^5(x)}{5}$$

output
```
-2*ln(1-sin(x))+128*ln(1+sin(x))-49*sin(x)+63*sin(x)^2-49/3*sin(x)^3-49/5*
sin(x)^5
```

3.901.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{\cos(x)\,(9-7\sin^3(x))^2}{1-\sin^2(x)}\,dx = 130\text{arctanh}(\sin(x))-63\cos^2(x)+126\log(\cos(x)) - 49\sin(x)-\frac{49\sin^3(x)}{3}-\frac{49\sin^5(x)}{5}$$

input
```
Integrate[(Cos[x]*(9 - 7*Sin[x]^3)^2)/(1 - Sin[x]^2),x]
```

output
```
130*ArcTanh[Sin[x]] - 63*Cos[x]^2 + 126*Log[Cos[x]] - 49*Sin[x] - (49*Sin[
x]^3)/3 - (49*Sin[x]^5)/5
```

3.901.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 3654, 3042, 3702, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(9-7\sin^3(x))^2\cos(x)}{1-\sin^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(9-7\sin(x)^3)^2\cos(x)}{1-\sin(x)^2}dx$$

$$\Big\downarrow \text{3654}$$

$$\int (9-7\sin^3(x))^2\sec(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(9-7\sin(x)^3)^2}{\cos(x)}dx$$

$$\Big\downarrow \text{3702}$$

$$\int \frac{(9-7\sin^3(x))^2}{1-\sin^2(x)}d\sin(x)$$

$$\Big\downarrow \text{2341}$$

$$\int \left(-49\sin^4(x)-49\sin^2(x)+\frac{2(65-63\sin(x))}{1-\sin^2(x)}+126\sin(x)-49\right)d\sin(x)$$

$$\Big\downarrow \text{2009}$$

$$130\text{arctanh}(\sin(x))-\frac{49}{5}\sin^5(x)-\frac{49\sin^3(x)}{3}+63\sin^2(x)-49\sin(x)+63\log\left(1-\sin^2(x)\right)$$

input
```
Int[(Cos[x]*(9 - 7*Sin[x]^3)^2)/(1 - Sin[x]^2),x]
```

output
```
130*ArcTanh[Sin[x]] + 63*Log[1 - Sin[x]^2] - 49*Sin[x] + 63*Sin[x]^2 - (49
*Sin[x]^3)/3 - (49*Sin[x]^5)/5
```

3.901. $\int \frac{\cos(x)(9-7\sin^3(x))^2}{1-\sin^2(x)}dx$

3.901.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3654
```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f,
p}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

rule 3702
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Si
mp[ff/f   Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
 Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m -
1)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

3.901.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.88

method	result
derivativedivides	$-\frac{49\sin(x)^5}{5}-\frac{49\sin(x)^3}{3}+63\sin(x)^2-49\sin(x)-2\ln(\sin(x)-1)+128\ln(1+\sin(x))$
default	$-\frac{49\sin(x)^5}{5}-\frac{49\sin(x)^3}{3}+63\sin(x)^2-49\sin(x)-2\ln(\sin(x)-1)+128\ln(1+\sin(x))$
parallelrisch	$-\frac{441}{10}-126\ln(2)-126\ln\left(\frac{1}{\cos(x)+1}\right)+256\ln(\csc(x)-\cot(x)+1)-4\ln(-\cot(x)+$
risch	$-126ix+\frac{539i e^{ix}}{16}-\frac{539i e^{-ix}}{16}-4\ln(e^{ix}-i)+256\ln(i+e^{ix})-\frac{49\sin(5x)}{80}+\frac{343\sin(3x)}{48}-\frac{63}{}$
norman	$\frac{-1260\tan(\frac{x}{2})^6-1008\tan(\frac{x}{2})^4-252\tan(\frac{x}{2})^2+252\tan(\frac{x}{2})^{14}+1008\tan(\frac{x}{2})^{12}+1260\tan(\frac{x}{2})^{10}+\frac{1862\tan(\frac{x}{2})^3}{3}+\frac{7938\tan(}{5}}{(1+\tan(\frac{x}{2})^2)^7(\tan(\frac{x}{2})^2-}$

input
```
int(cos(x)*(9-7*sin(x)^3)^2/(1-sin(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-49/5*sin(x)^5-49/3*sin(x)^3+63*sin(x)^2-49*sin(x)-2*ln(sin(x)-1)+128*ln(1
+sin(x))
```

3.901.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int \frac{\cos(x)\left(9-7\sin^3(x)\right)^2}{1-\sin^2(x)}\,dx = -63\,\cos\left(x\right)^2 - \frac{49}{15}\left(3\,\cos\left(x\right)^4 - 11\,\cos\left(x\right)^2 + 23\right)\sin\left(x\right) + 128\,\log\left(\sin\left(x\right)+1\right) - 2\,\log\left(-\sin\left(x\right)+1\right)$$

input
```
integrate(cos(x)*(9-7*sin(x)^3)^2/(1-sin(x)^2),x, algorithm="fricas")
```

output
```
-63*cos(x)^2 - 49/15*(3*cos(x)^4 - 11*cos(x)^2 + 23)*sin(x) + 128*log(sin(
x) + 1) - 2*log(-sin(x) + 1)
```

3.901.6 Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02

$$\int \frac{\cos(x)\left(9-7\sin^3(x)\right)^2}{1-\sin^2(x)}\,dx = -2\log\left(\sin\left(x\right)-1\right) + 128\log\left(\sin\left(x\right)+1\right) - \frac{49\sin^5\left(x\right)}{5} - \frac{49\sin^3\left(x\right)}{3} + 63\sin^2\left(x\right) - 49\sin\left(x\right)$$

input
```
integrate(cos(x)*(9-7*sin(x)**3)**2/(1-sin(x)**2),x)
```

output
```
-2*log(sin(x) - 1) + 128*log(sin(x) + 1) - 49*sin(x)**5/5 - 49*sin(x)**3/3
 + 63*sin(x)**2 - 49*sin(x)
```

3.901.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{\cos(x)\left(9-7\sin^3(x)\right)^2}{1-\sin^2(x)}\,dx = -\frac{49}{5}\sin(x)^5-\frac{49}{3}\sin(x)^3+63\sin(x)^2+128\log(\sin(x)+1)-2\log(\sin(x)-1)-49\sin(x)$$

input
```
integrate(cos(x)*(9-7*sin(x)^3)^2/(1-sin(x)^2),x, algorithm="maxima")
```

output
```
-49/5*sin(x)^5 - 49/3*sin(x)^3 + 63*sin(x)^2 + 128*log(sin(x) + 1) - 2*log
(sin(x) - 1) - 49*sin(x)
```

3.901.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{\cos(x)\left(9-7\sin^3(x)\right)^2}{1-\sin^2(x)}\,dx = -\frac{49}{5}\sin(x)^5-\frac{49}{3}\sin(x)^3+63\sin(x)^2+128\log(\sin(x)+1)-2\log(-\sin(x)+1)-49\sin(x)$$

input
```
integrate(cos(x)*(9-7*sin(x)^3)^2/(1-sin(x)^2),x, algorithm="giac")
```

output
```
-49/5*sin(x)^5 - 49/3*sin(x)^3 + 63*sin(x)^2 + 128*log(sin(x) + 1) - 2*log
(-sin(x) + 1) - 49*sin(x)
```

3.901.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{\cos(x)\left(9-7\sin^3(x)\right)^2}{1-\sin^2(x)}\,dx = 128\ln(\sin(x)+1)-2\ln(\sin(x)-1)-49\sin(x)+63\sin(x)^2-\frac{49\sin(x)^3}{3}-\frac{49\sin(x)^5}{5}$$

input
```
int(-(cos(x)*(7*sin(x)^3 - 9)^2)/(sin(x)^2 - 1),x)
```

output
```
128*log(sin(x) + 1) - 2*log(sin(x) - 1) - 49*sin(x) + 63*sin(x)^2 - (49*si
n(x)^3)/3 - (49*sin(x)^5)/5
```

3.902 $\int \cos^4(2x)\cot^5(2x)\,dx$

3.902.1 Optimal result

Integrand size = 13, antiderivative size = 42

$$\int \cos^4(2x)\cot^5(2x)\,dx = \csc^2(2x) - \frac{1}{8}\csc^4(2x) + 3\log(\sin(2x)) - \sin^2(2x) + \frac{1}{8}\sin^4(2x)$$

output
```
csc(2*x)^2-1/8*csc(2*x)^4+3*ln(sin(2*x))-sin(2*x)^2+1/8*sin(2*x)^4
```

3.902.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \cos^4(2x)\cot^5(2x)\,dx = \csc^2(2x) - \frac{1}{8}\csc^4(2x) + 3\log(\sin(2x)) - \sin^2(2x) + \frac{1}{8}\sin^4(2x)$$

input
```
Integrate[Cos[2*x]^4*Cot[2*x]^5,x]
```

output
```
Csc[2*x]^2 - Csc[2*x]^4/8 + 3*Log[Sin[2*x]] - Sin[2*x]^2 + Sin[2*x]^4/8
```

3.902.3 Rubi [A] (warning: unable to verify)

Time = 0.22 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {3042, 25, 3070, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^4(2x)\cot^5(2x)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int -\sin\left(2x+\frac{\pi}{2}\right)^4\tan\left(2x+\frac{\pi}{2}\right)^5dx$$

$$\Big\downarrow \ 25$$

$$-\int \sin\left(2x+\frac{\pi}{2}\right)^4\tan\left(2x+\frac{\pi}{2}\right)^5dx$$

$$\Big\downarrow \ 3070$$

$$\frac{1}{2}\int -\csc^5(2x)\left(1-\sin^2(2x)\right)^4d(-\sin(2x))$$

$$\Big\downarrow \ 243$$

$$\frac{1}{4}\int -\csc^3(2x)(\sin(2x)+1)^4d\sin^2(2x)$$

$$\Big\downarrow \ 49$$

$$\frac{1}{4}\int \left(-\csc^3(2x)-4\csc^2(2x)-6\csc(2x)+\sin^2(2x)-4\right)d\sin^2(2x)$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{4}\left(\frac{1}{2}\sin^2(2x)+4\sin(2x)-\frac{1}{2}\csc^2(2x)-4\csc(2x)+6\log\left(\sin^2(2x)\right)\right)$$

input
```
Int[Cos[2*x]^4*Cot[2*x]^5,x]
```

output
```
(-4*Csc[2*x] - Csc[2*x]^2/2 + 6*Log[Sin[2*x]^2] + 4*Sin[2*x] + Sin[2*x]^2/
2)/4
```

3.902.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3070
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f
*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

3.902.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.64

$$-\frac{\cos \left(2x\right)^{10}}{8 \sin \left(2x\right)^{4}}+\frac{3 \cos \left(2x\right)^{10}}{8 \sin \left(2x\right)^{2}}+\frac{3 \cos \left(2x\right)^{8}}{8}+\frac{\cos \left(2x\right)^{6}}{2}+\frac{3 \cos \left(2x\right)^{4}}{4}+\frac{3 \cos \left(2x\right)^{2}}{2}+3 \ln \left(\sin \left(2x\right)\right)$$

input
```
int(cos(2*x)^4*cot(2*x)^5,x)
```

output
```
-1/8/sin(2*x)^4*cos(2*x)^10+3/8/sin(2*x)^2*cos(2*x)^10+3/8*cos(2*x)^8+1/2*
cos(2*x)^6+3/4*cos(2*x)^4+3/2*cos(2*x)^2+3*ln(sin(2*x))
```

3.902.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(38) = 76.

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.88

$$\int \cos^4(2x)\cot^5(2x)\,dx$$
$$= \frac{8\cos(2x)^8 + 32\cos(2x)^6 - 115\cos(2x)^4 + 38\cos(2x)^2 + 192\left(\cos(2x)^4 - 2\cos(2x)^2 + 1\right)\log\left(\frac{1}{2}\sin\right.}{64\left(\cos(2x)^4 - 2\cos(2x)^2 + 1\right)}$$

input
```
integrate(cos(2*x)^4*cot(2*x)^5,x, algorithm="fricas")
```

output
```
1/64*(8*cos(2*x)^8 + 32*cos(2*x)^6 - 115*cos(2*x)^4 + 38*cos(2*x)^2 + 192*
(cos(2*x)^4 - 2*cos(2*x)^2 + 1)*log(1/2*sin(2*x)) + 29)/(cos(2*x)^4 - 2*co
s(2*x)^2 + 1)
```

3.902.6 Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

$$\int \cos^4(2x)\cot^5(2x)\,dx = \frac{8\sin^2(2x) - 1}{8\sin^4(2x)} + 3\log(\sin(2x)) + \frac{\sin^4(2x)}{8} - \sin^2(2x)$$

input
```
integrate(cos(2*x)**4*cot(2*x)**5,x)
```

output
```
(8*sin(2*x)**2 - 1)/(8*sin(2*x)**4) + 3*log(sin(2*x)) + sin(2*x)**4/8 - si
n(2*x)**2
```

3.902.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.05

$$\int \cos^4(2x)\cot^5(2x)\,dx = \frac{1}{8}\sin(2x)^4 - \sin(2x)^2 + \frac{8\sin(2x)^2 - 1}{8\sin(2x)^4} + \frac{3}{2}\log\left(\sin(2x)^2\right)$$

input
```
integrate(cos(2*x)^4*cot(2*x)^5,x, algorithm="maxima")
```

output
```
1/8*sin(2*x)^4 - sin(2*x)^2 + 1/8*(8*sin(2*x)^2 - 1)/sin(2*x)^4 + 3/2*log(
sin(2*x)^2)
```

3.902.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.24

$$\int \cos^4(2x)\cot^5(2x)\,dx = \frac{1}{8}\cos(2x)^4 + \frac{3}{4}\cos(2x)^2 - \frac{8\cos(2x)^2 - 7}{8\left(\cos(2x)^2 - 1\right)^2} + \frac{3}{2}\log\left(-\cos(2x)^2 + 1\right)$$

input
```
integrate(cos(2*x)^4*cot(2*x)^5,x, algorithm="giac")
```

output
```
1/8*cos(2*x)^4 + 3/4*cos(2*x)^2 - 1/8*(8*cos(2*x)^2 - 7)/(cos(2*x)^2 - 1)^
2 + 3/2*log(-cos(2*x)^2 + 1)
```

3.902.9 Mupad [B] (verification not implemented)

Time = 27.18 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.69

$$\int \cos^4(2x)\cot^5(2x)\,dx = 3\ln(\tan(2x)) - \frac{3\ln\left(\tan(2x)^2 + 1\right)}{2} + \frac{3\tan(2x)^6 + \frac{9\tan(2x)^4}{2} + \tan(2x)^2 - \frac{1}{4}}{2\left(\tan(2x)^8 + 2\tan(2x)^6 + \tan(2x)^4\right)}$$

input
```
int(cos(2*x)^4*cot(2*x)^5,x)
```

output
```
3*log(tan(2*x)) - (3*log(tan(2*x)^2 + 1))/2 + (tan(2*x)^2 + (9*tan(2*x)^4)
/2 + 3*tan(2*x)^6 - 1/4)/(2*(tan(2*x)^4 + 2*tan(2*x)^6 + tan(2*x)^8))
```

3.903 $\int \frac{\sec(x)\tan^2(x)}{4+3\sec(x)}\,dx$

3.903.1 Optimal result

Integrand size = 15, antiderivative size = 74

$$\int \frac{\sec(x)\tan^2(x)}{4+3\sec(x)}\,dx = -\frac{4}{9}\text{arctanh}(\sin(x)) - \frac{1}{9}\sqrt{7}\log\left(\sqrt{7}\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right) + \frac{1}{9}\sqrt{7}\log\left(\sqrt{7}\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right) + \frac{\tan(x)}{3}$$

output
```
-4/9*arctanh(sin(x))-1/9*ln(-sin(1/2*x)+cos(1/2*x)*7^(1/2))*7^(1/2)+1/9*ln
(sin(1/2*x)+cos(1/2*x)*7^(1/2))*7^(1/2)+1/3*tan(x)
```

3.903.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.85

$$\int \frac{\sec(x)\tan^2(x)}{4+3\sec(x)}\,dx = \frac{1}{9}\left(2\sqrt{7}\text{arctanh}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{7}}\right) + 4\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right) - 4\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right) + 3\tan(x)\right)$$

input
```
Integrate[(Sec[x]*Tan[x]^2)/(4 + 3*Sec[x]),x]
```

output
```
(2*Sqrt[7]*ArcTanh[Tan[x/2]/Sqrt[7]] + 4*Log[Cos[x/2] - Sin[x/2]] - 4*Log[
Cos[x/2] + Sin[x/2]] + 3*Tan[x])/9
```

3.903.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.55, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.867, Rules used = {3042, 4897, 3042, 3202, 3042, 3535, 25, 3042, 3480, 3042, 3138, 219, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(x)\sec(x)}{3\sec(x)+4}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(x)^2\sec(x)}{3\sec(x)+4}dx$$

$$\Big\downarrow \text{4897}$$

$$\int \frac{\tan^2(x)}{4\cos(x)+3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(3-4\sin\left(x-\frac{\pi}{2}\right)\right)\tan\left(x-\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3202}$$

$$\int \frac{\left(1-\cos^2(x)\right)\sec^2(x)}{4\cos(x)+3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1-\sin\left(x+\frac{\pi}{2}\right)^2}{\sin\left(x+\frac{\pi}{2}\right)^2\left(4\sin\left(x+\frac{\pi}{2}\right)+3\right)}dx$$

$$\Big\downarrow \text{3535}$$

$$\frac{1}{3}\int -\frac{(3\cos(x)+4)\sec(x)}{4\cos(x)+3}dx+\frac{\tan(x)}{3}$$

$$\Big\downarrow \text{25}$$

$$\frac{\tan(x)}{3}-\frac{1}{3}\int \frac{(3\cos(x)+4)\sec(x)}{4\cos(x)+3}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan(x)}{3}-\frac{1}{3}\int \frac{3\sin\left(x+\frac{\pi}{2}\right)+4}{\sin\left(x+\frac{\pi}{2}\right)\left(4\sin\left(x+\frac{\pi}{2}\right)+3\right)}dx$$

$$\Big\downarrow \text{3480}$$

$$\frac{1}{3}\left(\frac{7}{3}\int \frac{1}{4\cos(x)+3}dx-\frac{4\int \sec(x)dx}{3}\right)+\frac{\tan(x)}{3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\left(\frac{7}{3}\int \frac{1}{4\sin\left(x+\frac{\pi}{2}\right)+3}dx-\frac{4}{3}\int \csc\left(x+\frac{\pi}{2}\right)dx\right)+\frac{\tan(x)}{3}$$

$$\Big\downarrow \text{3138}$$

$$\frac{1}{3}\left(\frac{14}{3}\int \frac{1}{7-\tan^2\left(\frac{x}{2}\right)}d\tan\left(\frac{x}{2}\right)-\frac{4}{3}\int \csc\left(x+\frac{\pi}{2}\right)dx\right)+\frac{\tan(x)}{3}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}\left(\frac{2}{3}\sqrt{7}\text{arctanh}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{7}}\right)-\frac{4}{3}\int \csc\left(x+\frac{\pi}{2}\right)dx\right)+\frac{\tan(x)}{3}$$

$$\Big\downarrow \text{4257}$$

$$\frac{1}{3}\left(\frac{2}{3}\sqrt{7}\text{arctanh}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{7}}\right)-\frac{4}{3}\text{arctanh}(\sin(x))\right)+\frac{\tan(x)}{3}$$

input
```
Int[(Sec[x]*Tan[x]^2)/(4 + 3*Sec[x]),x]
```

output
```
((-4*ArcTanh[Sin[x]])/3 + (2*Sqrt[7]*ArcTanh[Tan[x/2]/Sqrt[7]])/3)/3 + Tan
[x]/3
```

3.903.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3202
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((1 - Sin[e + f*x]^2)/Sin[e + f*x]^
2), x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0]
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3535
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(A*b^2 + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*S
in[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m
+ 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n
+ 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d
*(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||
 !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a,
 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.903. $\int \frac{\sec(x)\tan^2(x)}{4+3\sec(x)}\,dx$

3.903.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.74

method	result	size
default	$-\frac{1}{3\left(\tan\left(\frac{x}{2}\right)+1\right)}-\frac{4\ln\left(\tan\left(\frac{x}{2}\right)+1\right)}{9}-\frac{1}{3\left(\tan\left(\frac{x}{2}\right)-1\right)}+\frac{4\ln\left(\tan\left(\frac{x}{2}\right)-1\right)}{9}+\frac{2\sqrt{7}\,\operatorname{arctanh}\left(\frac{\tan\left(\frac{x}{2}\right)\sqrt{7}}{7}\right)}{9}$	55
risch	$\frac{2i}{3\left(\mathrm{e}^{2ix}+1\right)}-\frac{\sqrt{7}\,\ln\left(\mathrm{e}^{ix}-\frac{i\sqrt{7}}{4}+\frac{3}{4}\right)}{9}+\frac{\sqrt{7}\,\ln\left(\mathrm{e}^{ix}+\frac{i\sqrt{7}}{4}+\frac{3}{4}\right)}{9}+\frac{4\ln\left(\mathrm{e}^{ix}-i\right)}{9}-\frac{4\ln\left(i+\mathrm{e}^{ix}\right)}{9}$	74

input
```
int(sec(x)*tan(x)^2/(4+3*sec(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/3/(tan(1/2*x)+1)-4/9*ln(tan(1/2*x)+1)-1/3/(tan(1/2*x)-1)+4/9*ln(tan(1/2
*x)-1)+2/9*7^(1/2)*arctanh(1/7*tan(1/2*x)*7^(1/2))
```

3.903.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.11

$$\int \frac{\sec(x)\tan^2(x)}{4+3\sec(x)}\,dx$$
$$= \frac{\sqrt{7}\cos\left(x\right)\log\left(\frac{2\cos(x)^2+2\left(3\sqrt{7}\cos(x)+4\sqrt{7}\right)\sin(x)+24\cos(x)+23}{16\cos(x)^2+24\cos(x)+9}\right)-4\cos\left(x\right)\log\left(\sin\left(x\right)+1\right)+4\cos\left(x\right)\log\left(-\right.}{18\cos\left(x\right)}$$

input
```
integrate(sec(x)*tan(x)^2/(4+3*sec(x)),x, algorithm="fricas")
```

output
```
1/18*(sqrt(7)*cos(x)*log((2*cos(x)^2 + 2*(3*sqrt(7)*cos(x) + 4*sqrt(7))*si
n(x) + 24*cos(x) + 23)/(16*cos(x)^2 + 24*cos(x) + 9)) - 4*cos(x)*log(sin(x
) + 1) + 4*cos(x)*log(-sin(x) + 1) + 6*sin(x))/cos(x)
```

3.903.6 Sympy [F]

$$\int \frac{\sec(x)\tan^2(x)}{4+3\sec(x)}\,dx = \int \frac{\tan^2(x)\sec(x)}{3\sec(x)+4}\,dx$$

input
```
integrate(sec(x)*tan(x)**2/(4+3*sec(x)),x)
```

output
```
Integral(tan(x)**2*sec(x)/(3*sec(x) + 4), x)
```

3.903.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.23

$$\int \frac{\sec(x)\tan^2(x)}{4+3\sec(x)}\,dx = -\frac{1}{9}\sqrt{7}\log\left(-\frac{\sqrt{7}-\frac{\sin(x)}{\cos(x)+1}}{\sqrt{7}+\frac{\sin(x)}{\cos(x)+1}}\right) - \frac{2\sin(x)}{3\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}-1\right)(\cos(x)+1)} - \frac{4}{9}\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right) + \frac{4}{9}\log\left(\frac{\sin(x)}{\cos(x)+1}-1\right)$$

input
```
integrate(sec(x)*tan(x)^2/(4+3*sec(x)),x, algorithm="maxima")
```

output
```
-1/9*sqrt(7)*log(-(sqrt(7) - sin(x)/(cos(x) + 1))/(sqrt(7) + sin(x)/(cos(x
) + 1))) - 2/3*sin(x)/((sin(x)^2/(cos(x) + 1)^2 - 1)*(cos(x) + 1)) - 4/9*l
og(sin(x)/(cos(x) + 1) + 1) + 4/9*log(sin(x)/(cos(x) + 1) - 1)
```

3.903.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.97

$$\int \frac{\sec(x)\tan^2(x)}{4+3\sec(x)}\,dx = -\frac{1}{9}\sqrt{7}\log\left(\frac{\left|-2\sqrt{7}+2\tan\left(\frac{1}{2}x\right)\right|}{\left|2\sqrt{7}+2\tan\left(\frac{1}{2}x\right)\right|}\right) - \frac{2\tan\left(\frac{1}{2}x\right)}{3\left(\tan\left(\frac{1}{2}x\right)^2-1\right)} - \frac{4}{9}\log\left(\left|\tan\left(\frac{1}{2}x\right)+1\right|\right) + \frac{4}{9}\log\left(\left|\tan\left(\frac{1}{2}x\right)-1\right|\right)$$

input
```
integrate(sec(x)*tan(x)^2/(4+3*sec(x)),x, algorithm="giac")
```

output
```
-1/9*sqrt(7)*log(abs(-2*sqrt(7) + 2*tan(1/2*x))/abs(2*sqrt(7) + 2*tan(1/2*
x))) - 2/3*tan(1/2*x)/(tan(1/2*x)^2 - 1) - 4/9*log(abs(tan(1/2*x) + 1)) +
4/9*log(abs(tan(1/2*x) - 1))
```

3.903.9 Mupad [B] (verification not implemented)

Time = 27.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.55

$$\int \frac{\sec(x)\tan^2(x)}{4+3\sec(x)}\,dx = \frac{2\sqrt{7}\,\mathrm{atanh}\left(\frac{\sqrt{7}\tan\left(\frac{x}{2}\right)}{7}\right)}{9} - \frac{2\tan\left(\frac{x}{2}\right)}{3\left(\tan\left(\frac{x}{2}\right)^2-1\right)} - \frac{8\,\mathrm{atanh}\left(\tan\left(\frac{x}{2}\right)\right)}{9}$$

input
```
int(tan(x)^2/(cos(x)*(3/cos(x) + 4)),x)
```

output
```
(2*7^(1/2)*atanh((7^(1/2)*tan(x/2))/7))/9 - (2*tan(x/2))/(3*(tan(x/2)^2 -
1)) - (8*atanh(tan(x/2)))/9
```

3.904 $\int x \sec(1+x) \tan(1+x) \, dx$

3.904.1 Optimal result

Integrand size = 10, antiderivative size = 14

$$\int x \sec(1+x) \tan(1+x) \, dx = -\operatorname{arctanh}(\sin(1+x)) + x \sec(1+x)$$

output
```
-arctanh(sin(1+x))+x*sec(1+x)
```

3.904.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 47 vs. 2(14) = 28.

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 3.36

$$\int x \sec(1+x) \tan(1+x) \, dx = \log\left(\cos\left(\frac{1+x}{2}\right) - \sin\left(\frac{1+x}{2}\right)\right) - \log\left(\cos\left(\frac{1+x}{2}\right) + \sin\left(\frac{1+x}{2}\right)\right) + x \sec(1+x)$$

input
```
Integrate[x*Sec[1 + x]*Tan[1 + x],x]
```

output
```
Log[Cos[(1 + x)/2] - Sin[(1 + x)/2]] - Log[Cos[(1 + x)/2] + Sin[(1 + x)/2]
] + x*Sec[1 + x]
```

3.904.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4244, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \tan(x+1)\sec(x+1)\,dx$$
$$\Big\downarrow \text{ 4244}$$
$$x\sec(x+1) - \int \sec(x+1)dx$$
$$\Big\downarrow \text{ 3042}$$
$$x\sec(x+1) - \int \csc\left(x+\frac{\pi}{2}+1\right)dx$$
$$\Big\downarrow \text{ 4257}$$
$$x\sec(x+1) - \text{arctanh}(\sin(x+1))$$

input
```
Int[x*Sec[1 + x]*Tan[1 + x],x]
```

output
```
-ArcTanh[Sin[1 + x]] + x*Sec[1 + x]
```

3.904.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4244
```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tan[(a_.) + (b_.)*(x_)^(
n_.)]^(q_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sec[a + b*x^n]^p/(b*n*p)), x]
 - Simp[(m - n + 1)/(b*n*p)   Int[x^(m - n)*Sec[a + b*x^n]^p, x], x] /; Fre
eQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m, n] && EqQ[q, 1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.904.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(14) = 28$.

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.29

method	result	size
derivativedivides	$\frac{x+1}{\cos(x+1)} - \ln(\sec(x+1) + \tan(x+1)) - \frac{1}{\cos(x+1)}$	32
default	$\frac{x+1}{\cos(x+1)} - \ln(\sec(x+1) + \tan(x+1)) - \frac{1}{\cos(x+1)}$	32
risch	$\frac{2x\,e^{i(x+1)}}{e^{2i(x+1)}+1} + \ln\left(e^{i(x+1)} - i\right) - \ln\left(e^{i(x+1)} + i\right)$	47

input
```
int(x*sec(x+1)*tan(x+1),x,method=_RETURNVERBOSE)
```

output
```
(x+1)/cos(x+1)-ln(sec(x+1)+tan(x+1))-1/cos(x+1)
```

3.904.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(14) = 28$.

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.79

$$\int x\sec(1+x)\tan(1+x)\,dx = -\frac{\cos(x+1)\log(\sin(x+1)+1) - \cos(x+1)\log(-\sin(x+1)+1) - 2x}{2\cos(x+1)}$$

input
```
integrate(x*sec(1+x)*tan(1+x),x, algorithm="fricas")
```

output
```
-1/2*(cos(x + 1)*log(sin(x + 1) + 1) - cos(x + 1)*log(-sin(x + 1) + 1) - 2
*x)/cos(x + 1)
```

3.904.6 Sympy [A] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.21

$$\int x\sec(1+x)\tan(1+x)\,dx = x\sec(x+1) - \log(\tan(x+1)+\sec(x+1))$$

input
```
integrate(x*sec(1+x)*tan(1+x),x)
```

output
```
x*sec(x + 1) - log(tan(x + 1) + sec(x + 1))
```

3.904.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 176 vs. 2(14) = 28.

Time = 0.28 (sec) , antiderivative size = 176, normalized size of antiderivative = 12.57

$$\int x\sec(1+x)\tan(1+x)\,dx$$
$$= \frac{4\,(x+1)\cos(2\,x+2)\cos(x+1) + 4\,(x+1)\sin(2\,x+2)\sin(x+1) + 4\,(x+1)\cos(x+1) - (\cos(2\,x+}{}$$
$$- \frac{1}{\cos(x+1)}$$

input
```
integrate(x*sec(1+x)*tan(1+x),x, algorithm="maxima")
```

output
```
1/2*(4*(x + 1)*cos(2*x + 2)*cos(x + 1) + 4*(x + 1)*sin(2*x + 2)*sin(x + 1)
 + 4*(x + 1)*cos(x + 1) - (cos(2*x + 2)^2 + sin(2*x + 2)^2 + 2*cos(2*x + 2
) + 1)*log(cos(x + 1)^2 + sin(x + 1)^2 + 2*sin(x + 1) + 1) + (cos(2*x + 2)
^2 + sin(2*x + 2)^2 + 2*cos(2*x + 2) + 1)*log(cos(x + 1)^2 + sin(x + 1)^2
- 2*sin(x + 1) + 1))/(cos(2*x + 2)^2 + sin(2*x + 2)^2 + 2*cos(2*x + 2) + 1
) - 1/cos(x + 1)
```

3.904.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 949 vs. $2(14) = 28$.

Time = 0.45 (sec) , antiderivative size = 949, normalized size of antiderivative = 67.79

$$\int x\sec(1+x)\tan(1+x)\,dx = \text{Too large to display}$$

input
```
integrate(x*sec(1+x)*tan(1+x),x, algorithm="giac")
```

output
```
1/2*(2*x*tan(1/2)^2*tan(1/2*x)^2 + log(2*(tan(1/2)^2*tan(1/2*x)^2 + 2*tan(
1/2)^2*tan(1/2*x) + 2*tan(1/2)*tan(1/2*x)^2 + tan(1/2)^2 + tan(1/2*x)^2 -
2*tan(1/2) - 2*tan(1/2*x) + 1)/(tan(1/2)^2*tan(1/2*x)^2 + tan(1/2)^2 + tan
(1/2*x)^2 + 1))*tan(1/2)^2*tan(1/2*x)^2 - log(2*(tan(1/2)^2*tan(1/2*x)^2 -
 2*tan(1/2)^2*tan(1/2*x) - 2*tan(1/2)*tan(1/2*x)^2 + tan(1/2)^2 + tan(1/2*
x)^2 + 2*tan(1/2) + 2*tan(1/2*x) + 1)/(tan(1/2)^2*tan(1/2*x)^2 + tan(1/2)^
2 + tan(1/2*x)^2 + 1))*tan(1/2)^2*tan(1/2*x)^2 + 2*x*tan(1/2)^2 - log(2*(t
an(1/2)^2*tan(1/2*x)^2 + 2*tan(1/2)^2*tan(1/2*x) + 2*tan(1/2)*tan(1/2*x)^2
 + tan(1/2)^2 + tan(1/2*x)^2 - 2*tan(1/2) - 2*tan(1/2*x) + 1)/(tan(1/2)^2*
tan(1/2*x)^2 + tan(1/2)^2 + tan(1/2*x)^2 + 1))*tan(1/2)^2 + log(2*(tan(1/2
)^2*tan(1/2*x)^2 - 2*tan(1/2)^2*tan(1/2*x) - 2*tan(1/2)*tan(1/2*x)^2 + tan
(1/2)^2 + tan(1/2*x)^2 + 2*tan(1/2) + 2*tan(1/2*x) + 1)/(tan(1/2)^2*tan(1/
2*x)^2 + tan(1/2)^2 + tan(1/2*x)^2 + 1))*tan(1/2)^2 - 4*log(2*(tan(1/2)^2*
tan(1/2*x)^2 + 2*tan(1/2)^2*tan(1/2*x) + 2*tan(1/2)*tan(1/2*x)^2 + tan(1/2
)^2 + tan(1/2*x)^2 - 2*tan(1/2) - 2*tan(1/2*x) + 1)/(tan(1/2)^2*tan(1/2*x)
^2 + tan(1/2)^2 + tan(1/2*x)^2 + 1))*tan(1/2)*tan(1/2*x) + 4*log(2*(tan(1/
2)^2*tan(1/2*x)^2 - 2*tan(1/2)^2*tan(1/2*x) - 2*tan(1/2)*tan(1/2*x)^2 + ta
n(1/2)^2 + tan(1/2*x)^2 + 2*tan(1/2) + 2*tan(1/2*x) + 1)/(tan(1/2)^2*tan(1
/2*x)^2 + tan(1/2)^2 + tan(1/2*x)^2 + 1))*tan(1/2)*tan(1/2*x) + 2*x*tan(1/
2*x)^2 - log(2*(tan(1/2)^2*tan(1/2*x)^2 + 2*tan(1/2)^2*tan(1/2*x) + 2*t...
```

3.904.9 Mupad [B] (verification not implemented)

Time = 26.92 (sec) , antiderivative size = 34, normalized size of antiderivative = 2.43

$$\int x\sec(1+x)\tan(1+x)\,dx = \frac{2\,x\,\cos\left(x+1\right)}{\cos\left(2\,x+2\right)+1} + \mathrm{atan}(\cos\left(x+1\right)+\sin\left(x+1\right)\,1\mathrm{i})\,2\mathrm{i}$$

input
```
int((x*tan(x + 1))/cos(x + 1),x)
```

output
```
atan(cos(x + 1) + sin(x + 1)*1i)*2i + (2*x*cos(x + 1))/(cos(2*x + 2) + 1)
```

3.905 $\int \frac{\sin(2x)}{\sqrt{9-\sin^2(x)}}\,dx$

3.905.1 Optimal result

Integrand size = 17, antiderivative size = 14

$$\int \frac{\sin(2x)}{\sqrt{9-\sin^2(x)}}\,dx = -2\sqrt{9-\sin^2(x)}$$

output
```
-2*(9-sin(x)^2)^(1/2)
```

3.905.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\sin(2x)}{\sqrt{9-\sin^2(x)}}\,dx = -2\sqrt{9-\sin^2(x)}$$

input
```
Integrate[Sin[2*x]/Sqrt[9 - Sin[x]^2],x]
```

output
```
-2*Sqrt[9 - Sin[x]^2]
```

3.905.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3042, 4878, 27, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(2x)}{\sqrt{9-\sin^2(x)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(2x)}{\sqrt{9-\sin(x)^2}}dx$$

$$\Big\downarrow \text{ 4878}$$

$$\int \frac{2\sin(x)}{\sqrt{9-\sin^2(x)}}d\sin(x)$$

$$\Big\downarrow \text{ 27}$$

$$2\int \frac{\sin(x)}{\sqrt{9-\sin^2(x)}}d\sin(x)$$

$$\Big\downarrow \text{ 241}$$

$$-2\sqrt{9-\sin^2(x)}$$

input
```
Int[Sin[2*x]/Sqrt[9 - Sin[x]^2],x]
```

output
```
-2*Sqrt[9 - Sin[x]^2]
```

3.905.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

3.905.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$-2\sqrt{9-\sin\left(x\right)^{2}}$	13
default	$-2\sqrt{9-\sin\left(x\right)^{2}}$	13

input
```
int(sin(2*x)/(9-sin(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(9-sin(x)^2)^(1/2)
```

3.905.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{\sin(2x)}{\sqrt{9-\sin^2(x)}}\,dx = -2\,\sqrt{\cos\left(x\right)^{2}+8}$$

input
```
integrate(sin(2*x)/(9-sin(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(cos(x)^2 + 8)
```

3.905.6 Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\sin(2x)}{\sqrt{9-\sin^2(x)}}\,dx = -2\sqrt{9-\sin^2(x)}$$

input
```
integrate(sin(2*x)/(9-sin(x)**2)**(1/2),x)
```

output
```
-2*sqrt(9 - sin(x)**2)
```

3.905.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\sin(2x)}{\sqrt{9-\sin^2(x)}}\,dx = -2\sqrt{-\sin(x)^2+9}$$

input
```
integrate(sin(2*x)/(9-sin(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-2*sqrt(-sin(x)^2 + 9)
```

3.905.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\sin(2x)}{\sqrt{9-\sin^2(x)}}\,dx = -2\sqrt{-\sin(x)^2+9}$$

input
```
integrate(sin(2*x)/(9-sin(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-2*sqrt(-sin(x)^2 + 9)
```

3.905.9 Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{\sin(2x)}{\sqrt{9-\sin^2(x)}}\,dx = -2\sqrt{\cos(x)^2+8}$$

input
```
int(sin(2*x)/(9 - sin(x)^2)^(1/2),x)
```

output
```
-2*(cos(x)^2 + 8)^(1/2)
```

3.906 $\int \frac{\sin(2x)}{\sqrt{9-\cos^4(x)}}\,dx$

3.906.1 Optimal result

Integrand size = 17, antiderivative size = 11

$$\int \frac{\sin(2x)}{\sqrt{9-\cos^4(x)}}\,dx = -\arcsin\left(\frac{\cos^2(x)}{3}\right)$$

output
```
-arcsin(1/3*cos(x)^2)
```

3.906.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\sin(2x)}{\sqrt{9-\cos^4(x)}}\,dx = -\arcsin\left(\frac{\cos^2(x)}{3}\right)$$

input
```
Integrate[Sin[2*x]/Sqrt[9 - Cos[x]^4],x]
```

output
```
-ArcSin[Cos[x]^2/3]
```

3.906.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3042, 4878, 27, 1432, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(2x)}{\sqrt{9-\cos^4(x)}} \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(2x)}{\sqrt{9-\cos(x)^4}}dx$$

$$\Big\downarrow \text{ 4878}$$

$$\int \frac{2\sin(x)}{\sqrt{-\sin^4(x)+2\sin^2(x)+8}}d\sin(x)$$

$$\Big\downarrow \text{ 27}$$

$$2\int \frac{\sin(x)}{\sqrt{-\sin^4(x)+2\sin^2(x)+8}}d\sin(x)$$

$$\Big\downarrow \text{ 1432}$$

$$\int \frac{1}{\sqrt{-\sin^4(x)+2\sin^2(x)+8}}d\sin^2(x)$$

$$\Big\downarrow \text{ 1090}$$

$$-\frac{1}{6}\int \frac{1}{\sqrt{1-\frac{\sin^4(x)}{36}}}d\left(2-2\sin^2(x)\right)$$

$$\Big\downarrow \text{ 223}$$

$$-\arcsin\left(\frac{1}{6}\left(2-2\sin^2(x)\right)\right)$$

input
```
Int[Sin[2*x]/Sqrt[9 - Cos[x]^4],x]
```

output
```
-ArcSin[(2 - 2*Sin[x]^2)/6]
```

3.906.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1432
```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[1/2
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4878
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Sin[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1, Sin[v]/d,
 u/Cos[v], x], x], x, Sin[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[NonfreeF
actors[Sin[v], x], u/Cos[v], x]]
```

3.906.4 Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$-\arcsin\left(\frac{\cos(x)^2}{3}\right)$	10
default	$-\arcsin\left(\frac{\cos(x)^2}{3}\right)$	10

input
```
int(sin(2*x)/(9-cos(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-arcsin(1/3*cos(x)^2)
```

3.906.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(9) = 18$.

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.18

$$\int \frac{\sin(2x)}{\sqrt{9-\cos^4(x)}}\,dx = \arctan\left(\frac{\sqrt{-\cos(x)^4+9}\cos(x)^2}{\cos(x)^4-9}\right)$$

input
```
integrate(sin(2*x)/(9-cos(x)^4)^(1/2),x, algorithm="fricas")
```

output
```
arctan(sqrt(-cos(x)^4 + 9)*cos(x)^2/(cos(x)^4 - 9))
```

3.906.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sin(2x)}{\sqrt{9-\cos^4(x)}}\,dx = \text{Timed out}$$

input
```
integrate(sin(2*x)/(9-cos(x)**4)**(1/2),x)
```

output
```
Timed out
```

3.906.7 Maxima [F]

$$\int \frac{\sin(2x)}{\sqrt{9-\cos^4(x)}}\,dx = \int \frac{\sin(2x)}{\sqrt{-\cos(x)^4+9}}\,dx$$

input
```
integrate(sin(2*x)/(9-cos(x)^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sin(2*x)/sqrt(-cos(x)^4 + 9), x)
```

3.906.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.82

$$\int \frac{\sin(2x)}{\sqrt{9-\cos^4(x)}}\,dx = -\arcsin\left(\frac{1}{3}\cos(x)^2\right)$$

input
```
integrate(sin(2*x)/(9-cos(x)^4)^(1/2),x, algorithm="giac")
```

output
```
-arcsin(1/3*cos(x)^2)
```

3.906.9 Mupad [B] (verification not implemented)

Time = 27.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.64

$$\int \frac{\sin(2x)}{\sqrt{9-\cos^4(x)}}\,dx = -\mathrm{atan}\left(\frac{\cos(x)^2}{\sqrt{9-\cos(x)^4}}\right)$$

input
```
int(sin(2*x)/(9 - cos(x)^4)^(1/2),x)
```

output
```
-atan(cos(x)^2/(9 - cos(x)^4)^(1/2))
```

3.907 $\int \frac{\cos\left(\frac{1}{x}\right)}{x^5}\,dx$

3.907.1 Optimal result

Integrand size = 8, antiderivative size = 34

$$\int \frac{\cos\left(\frac{1}{x}\right)}{x^5}\,dx = 6\cos\left(\frac{1}{x}\right) - \frac{3\cos\left(\frac{1}{x}\right)}{x^2} - \frac{\sin\left(\frac{1}{x}\right)}{x^3} + \frac{6\sin\left(\frac{1}{x}\right)}{x}$$

output
```
6*cos(1/x)-3*cos(1/x)/x^2-sin(1/x)/x^3+6*sin(1/x)/x
```

3.907.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{\cos\left(\frac{1}{x}\right)}{x^5}\,dx = \frac{3(-1+2x^2)\cos\left(\frac{1}{x}\right)}{x^2} + \frac{(-1+6x^2)\sin\left(\frac{1}{x}\right)}{x^3}$$

input
```
Integrate[Cos[x^(-1)]/x^5,x]
```

output
```
(3*(-1 + 2*x^2)*Cos[x^(-1)])/x^2 + ((-1 + 6*x^2)*Sin[x^(-1)])/x^3
```

3.907.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.09, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.375, Rules used = {3861, 3042, 3777, 25, 3042, 3777, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos\left(\frac{1}{x}\right)}{x^5}dx$$

$$\Big\downarrow \text{3861}$$

$$-\int \frac{\cos\left(\frac{1}{x}\right)}{x^3}d\frac{1}{x}$$

$$\Big\downarrow \text{3042}$$

$$-\int \frac{\sin\left(\frac{\pi}{2}+\frac{1}{x}\right)}{x^3}d\frac{1}{x}$$

$$\Big\downarrow \text{3777}$$

$$-3\int -\frac{\sin\left(\frac{1}{x}\right)}{x^2}d\frac{1}{x}-\frac{\sin\left(\frac{1}{x}\right)}{x^3}$$

$$\Big\downarrow \text{25}$$

$$3\int \frac{\sin\left(\frac{1}{x}\right)}{x^2}d\frac{1}{x}-\frac{\sin\left(\frac{1}{x}\right)}{x^3}$$

$$\Big\downarrow \text{3042}$$

$$3\int \frac{\sin\left(\frac{1}{x}\right)}{x^2}d\frac{1}{x}-\frac{\sin\left(\frac{1}{x}\right)}{x^3}$$

$$\Big\downarrow \text{3777}$$

$$3\left(2\int \frac{\cos\left(\frac{1}{x}\right)}{x}d\frac{1}{x}-\frac{\cos\left(\frac{1}{x}\right)}{x^2}\right)-\frac{\sin\left(\frac{1}{x}\right)}{x^3}$$

$$\Big\downarrow \text{3042}$$

$$3\left(2\int \frac{\sin\left(\frac{\pi}{2}+\frac{1}{x}\right)}{x}d\frac{1}{x}-\frac{\cos\left(\frac{1}{x}\right)}{x^2}\right)-\frac{\sin\left(\frac{1}{x}\right)}{x^3}$$

$$\Big\downarrow \text{3777}$$

$$3\left(2\left(\int -\sin\left(\frac{1}{x}\right)d\frac{1}{x}+\frac{\sin\left(\frac{1}{x}\right)}{x}\right)-\frac{\cos\left(\frac{1}{x}\right)}{x^2}\right)-\frac{\sin\left(\frac{1}{x}\right)}{x^3}$$

$$\Big\downarrow \text{25}$$

$$3\left(2\left(\frac{\sin\left(\frac{1}{x}\right)}{x}-\int\sin\left(\frac{1}{x}\right)d\frac{1}{x}\right)-\frac{\cos\left(\frac{1}{x}\right)}{x^2}\right)-\frac{\sin\left(\frac{1}{x}\right)}{x^3}$$

$$\Big\downarrow \text{3042}$$

$$3\left(2\left(\frac{\sin\left(\frac{1}{x}\right)}{x}-\int\sin\left(\frac{1}{x}\right)d\frac{1}{x}\right)-\frac{\cos\left(\frac{1}{x}\right)}{x^2}\right)-\frac{\sin\left(\frac{1}{x}\right)}{x^3}$$

$$\Big\downarrow \text{3118}$$

$$3\left(2\left(\frac{\sin\left(\frac{1}{x}\right)}{x}+\cos\left(\frac{1}{x}\right)\right)-\frac{\cos\left(\frac{1}{x}\right)}{x^2}\right)-\frac{\sin\left(\frac{1}{x}\right)}{x^3}$$

input
```
Int[Cos[x^(-1)]/x^5,x]
```

output
```
-(Sin[x^(-1)]/x^3) + 3*(-(Cos[x^(-1)]/x^2) + 2*(Cos[x^(-1)] + Sin[x^(-1)]/
x))
```

3.907.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.907. $\int \frac{\cos\left(\frac{1}{x}\right)}{x^5}\,dx$

rule 3861
```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

3.907.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

method	result	size
risch	$\frac{3(2x^2-1)\cos\left(\frac{1}{x}\right)}{x^2}+\frac{(6x^2-1)\sin\left(\frac{1}{x}\right)}{x^3}$	33
derivativedivides	$6\cos\left(\frac{1}{x}\right)-\frac{3\cos\left(\frac{1}{x}\right)}{x^2}-\frac{\sin\left(\frac{1}{x}\right)}{x^3}+\frac{6\sin\left(\frac{1}{x}\right)}{x}$	35
default	$6\cos\left(\frac{1}{x}\right)-\frac{3\cos\left(\frac{1}{x}\right)}{x^2}-\frac{\sin\left(\frac{1}{x}\right)}{x^3}+\frac{6\sin\left(\frac{1}{x}\right)}{x}$	35
meijerg	$-8\sqrt{\pi}\left(\frac{3}{4\sqrt{\pi}}-\frac{\left(-\frac{3}{2x^2}+3\right)\cos\left(\frac{1}{x}\right)}{4\sqrt{\pi}}-\frac{\left(-\frac{1}{2x^2}+3\right)\sin\left(\frac{1}{x}\right)}{4\sqrt{\pi}\,x}\right)$	47
parallelrisch	$\frac{12x^3+12\tan\left(\frac{1}{2x}\right)x^2+3\tan\left(\frac{1}{2x}\right)^2x-3x-2\tan\left(\frac{1}{2x}\right)}{x^3\left(1+\tan\left(\frac{1}{2x}\right)^2\right)}$	56
norman	$\frac{12x^4-3x^2-2x\tan\left(\frac{1}{2x}\right)+3x^2\tan\left(\frac{1}{2x}\right)^2+12x^3\tan\left(\frac{1}{2x}\right)}{\left(1+\tan\left(\frac{1}{2x}\right)^2\right)x^4}$	61

input
```
int(cos(1/x)/x^5,x,method=_RETURNVERBOSE)
```

output
```
3/x^2*(2*x^2-1)*cos(1/x)+(6*x^2-1)/x^3*sin(1/x)
```

3.907.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{\cos\left(\frac{1}{x}\right)}{x^5}\,dx=\frac{3\,(2\,x^3-x)\cos\left(\frac{1}{x}\right)+(6\,x^2-1)\sin\left(\frac{1}{x}\right)}{x^3}$$

input
```
integrate(cos(1/x)/x^5,x, algorithm="fricas")
```

output
```
(3*(2*x^3 - x)*cos(1/x) + (6*x^2 - 1)*sin(1/x))/x^3
```

3.907.6 Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{\cos\left(\frac{1}{x}\right)}{x^5}\,dx = 6\cos\left(\frac{1}{x}\right) + \frac{6\sin\left(\frac{1}{x}\right)}{x} - \frac{3\cos\left(\frac{1}{x}\right)}{x^2} - \frac{\sin\left(\frac{1}{x}\right)}{x^3}$$

input
```
integrate(cos(1/x)/x**5,x)
```

output
```
6*cos(1/x) + 6*sin(1/x)/x - 3*cos(1/x)/x**2 - sin(1/x)/x**3
```

3.907.7 Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.56

$$\int \frac{\cos\left(\frac{1}{x}\right)}{x^5}\,dx = \frac{1}{2}\,\Gamma\left(4, \frac{i}{x}\right) + \frac{1}{2}\,\Gamma\left(4, -\frac{i}{x}\right)$$

input
```
integrate(cos(1/x)/x^5,x, algorithm="maxima")
```

output
```
1/2*gamma(4, I/x) + 1/2*gamma(4, -I/x)
```

3.907.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(\frac{1}{x}\right)}{x^5}\,dx = \frac{6\sin\left(\frac{1}{x}\right)}{x} - \frac{3\cos\left(\frac{1}{x}\right)}{x^2} - \frac{\sin\left(\frac{1}{x}\right)}{x^3} + 6\cos\left(\frac{1}{x}\right)$$

input
```
integrate(cos(1/x)/x^5,x, algorithm="giac")
```

output
```
6*sin(1/x)/x - 3*cos(1/x)/x^2 - sin(1/x)/x^3 + 6*cos(1/x)
```

3.907.9 Mupad [B] (verification not implemented)

Time = 26.51 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int \frac{\cos\left(\frac{1}{x}\right)}{x^5}\,dx = 6\,\cos\left(\frac{1}{x}\right) - \frac{\sin\left(\frac{1}{x}\right) + 3\,x\,\cos\left(\frac{1}{x}\right) - 6\,x^2\,\sin\left(\frac{1}{x}\right)}{x^3}$$

input
```
int(cos(1/x)/x^5,x)
```

output
```
6*cos(1/x) - (sin(1/x) + 3*x*cos(1/x) - 6*x^2*sin(1/x))/x^3
```

3.908 $\int \cos^3(1+x)\sin^3(1+x)\,dx$

3.908.1 Optimal result

Integrand size = 13, antiderivative size = 21

$$\int \cos^3(1+x)\sin^3(1+x)\,dx=\frac{1}{4}\sin^4(1+x)-\frac{1}{6}\sin^6(1+x)$$

output
```
1/4*sin(1+x)^4-1/6*sin(1+x)^6
```

3.908.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \cos^3(1+x)\sin^3(1+x)\,dx=\frac{1}{8}\left(-\frac{3}{8}\cos(2(1+x))+\frac{1}{24}\cos(6(1+x))\right)$$

input
```
Integrate[Cos[1 + x]^3*Sin[1 + x]^3,x]
```

output
```
((-3*Cos[2*(1 + x)])/8 + Cos[6*(1 + x)]/24)/8
```

3.908.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 3044, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(x+1)\cos^3(x+1)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(x+1)^3\cos(x+1)^3dx$$

$$\Big\downarrow \text{3044}$$

$$\int \sin^3(x+1)\left(1-\sin^2(x+1)\right)d\sin(x+1)$$

$$\Big\downarrow \text{244}$$

$$\int \left(\sin^3(x+1)-\sin^5(x+1)\right)d\sin(x+1)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\sin^4(x+1)-\frac{1}{6}\sin^6(x+1)$$

input
```
Int[Cos[1 + x]^3*Sin[1 + x]^3,x]
```

output
```
Sin[1 + x]^4/4 - Sin[1 + x]^6/6
```

3.908.3.1 Defintions of rubi rules used

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3044
```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Simp[1/(a*f)   Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a
*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(I
ntegerQ[(m - 1)/2] && LtQ[0, m, n])
```

3.908.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{\sin(x+1)^4}{4}-\frac{\sin(x+1)^6}{6}$	18
default	$\frac{\sin(x+1)^4}{4}-\frac{\sin(x+1)^6}{6}$	18
risch	$\frac{\cos(6x+6)}{192}-\frac{3\cos(2x+2)}{64}$	18
parallelrisch	$\frac{7}{40}+\frac{\cos(6x+6)}{192}-\frac{3\cos(2x+2)}{64}$	19
norman	$\frac{4\tan\left(\frac{x}{2}+\frac{1}{2}\right)^4+4\tan\left(\frac{x}{2}+\frac{1}{2}\right)^8-\frac{8\tan\left(\frac{x}{2}+\frac{1}{2}\right)^6}{3}}{\left(1+\tan\left(\frac{x}{2}+\frac{1}{2}\right)^2\right)^6}$	45

input
```
int(cos(x+1)^3*sin(x+1)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*sin(x+1)^4-1/6*sin(x+1)^6
```

3.908.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \cos^3(1+x)\sin^3(1+x)\,dx = \frac{1}{6}\cos(x+1)^6 - \frac{1}{4}\cos(x+1)^4$$

input
```
integrate(cos(1+x)^3*sin(1+x)^3,x, algorithm="fricas")
```

output
```
1/6*cos(x + 1)^6 - 1/4*cos(x + 1)^4
```

3.908.6 Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int \cos^3(1+x)\sin^3(1+x)\,dx = -\frac{\sin^2(x+1)\cos^4(x+1)}{4} - \frac{\cos^6(x+1)}{12}$$

input
```
integrate(cos(1+x)**3*sin(1+x)**3,x)
```

output
```
-sin(x + 1)**2*cos(x + 1)**4/4 - cos(x + 1)**6/12
```

3.908.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \cos^3(1+x)\sin^3(1+x)\,dx = -\frac{1}{6}\sin(x+1)^6 + \frac{1}{4}\sin(x+1)^4$$

input
```
integrate(cos(1+x)^3*sin(1+x)^3,x, algorithm="maxima")
```

output
```
-1/6*sin(x + 1)^6 + 1/4*sin(x + 1)^4
```

3.908.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \cos^3(1+x)\sin^3(1+x)\,dx = -\frac{1}{6}\sin(x+1)^6 + \frac{1}{4}\sin(x+1)^4$$

input
```
integrate(cos(1+x)^3*sin(1+x)^3,x, algorithm="giac")
```

output
```
-1/6*sin(x + 1)^6 + 1/4*sin(x + 1)^4
```

3.908.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.86

$$\int \cos^3(1+x)\sin^3(1+x)\,dx = -\frac{\sin(x+1)^4\left(2\sin(x+1)^2-3\right)}{12}$$

input
```
int(cos(x + 1)^3*sin(x + 1)^3,x)
```

output
```
-(sin(x + 1)^4*(2*sin(x + 1)^2 - 3))/12
```

3.909 $\int (1+2x)^3 \sin^2(1+2x)\,dx$

3.909.1 Optimal result

Integrand size = 16, antiderivative size = 99

$$\int (1+2x)^3 \sin^2(1+2x)\,dx = -\frac{3x}{4}-\frac{3x^2}{4}+\frac{1}{16}(1+2x)^4+\frac{3}{8}(1+2x)\cos(1+2x)\sin(1+2x) -\frac{1}{4}(1+2x)^3\cos(1+2x)\sin(1+2x) -\frac{3}{16}\sin^2(1+2x)+\frac{3}{8}(1+2x)^2\sin^2(1+2x)$$

output
```
-3/4*x-3/4*x^2+1/16*(1+2*x)^4+3/8*(1+2*x)*cos(1+2*x)*sin(1+2*x)-1/4*(1+2*x
)^3*cos(1+2*x)*sin(1+2*x)-3/16*sin(1+2*x)^2+3/8*(1+2*x)^2*sin(1+2*x)^2
```

3.909.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.56

$$\int (1+2x)^3 \sin^2(1+2x)\,dx = \frac{1}{32}\left(-3\left(1+8x+8x^2\right)\cos(2+4x) +2(1+2x)\left((1+2x)^3+\left(1-8x-8x^2\right)\sin(2+4x)\right)\right)$$

input
```
Integrate[(1 + 2*x)^3*Sin[1 + 2*x]^2,x]
```

output
```
(-3*(1 + 8*x + 8*x^2)*Cos[2 + 4*x] + 2*(1 + 2*x)*((1 + 2*x)^3 + (1 - 8*x -
 8*x^2)*Sin[2 + 4*x]))/32
```

3.909.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3042, 3792, 17, 3042, 3791, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2x+1)^3\sin^2(2x+1)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (2x+1)^3\sin(2x+1)^2dx$$

$$\Big\downarrow \text{3792}$$

$$\frac{1}{2}\int (2x+1)^3dx-\frac{3}{2}\int (2x+1)\sin^2(2x+1)dx+\frac{3}{8}(2x+1)^2\sin^2(2x+1)-\frac{1}{4}(2x+1)^3\sin(2x+1)\cos(2x+1)$$

$$\Big\downarrow \text{17}$$

$$-\frac{3}{2}\int (2x+1)\sin^2(2x+1)dx+\frac{1}{16}(2x+1)^4+\frac{3}{8}(2x+1)^2\sin^2(2x+1)-\frac{1}{4}(2x+1)^3\sin(2x+1)\cos(2x+1)$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3}{2}\int (2x+1)\sin(2x+1)^2dx+\frac{1}{16}(2x+1)^4+\frac{3}{8}(2x+1)^2\sin^2(2x+1)-\frac{1}{4}(2x+1)^3\sin(2x+1)\cos(2x+1)$$

$$\Big\downarrow \text{3791}$$

$$-\frac{3}{2}\left(\frac{1}{2}\int (2x+1)dx+\frac{1}{8}\sin^2(2x+1)-\frac{1}{4}(2x+1)\sin(2x+1)\cos(2x+1)\right)+\frac{1}{16}(2x+1)^4+\frac{3}{8}(2x+1)^2\sin^2(2x+1)-\frac{1}{4}(2x+1)^3\sin(2x+1)\cos(2x+1)$$

$$\Big\downarrow \text{17}$$

$$\frac{1}{16}(2x+1)^4+\frac{3}{8}(2x+1)^2\sin^2(2x+1)-\frac{3}{2}\left(\frac{1}{8}(2x+1)^2+\frac{1}{8}\sin^2(2x+1)-\frac{1}{4}(2x+1)\sin(2x+1)\cos(2x+1)\right)-\frac{1}{4}(2x+1)^3\sin(2x+1)\cos(2x+1)$$

input
```
Int[(1 + 2*x)^3*Sin[1 + 2*x]^2,x]
```

output
```
(1 + 2*x)^4/16 - ((1 + 2*x)^3*Cos[1 + 2*x]*Sin[1 + 2*x])/4 + (3*(1 + 2*x)^
2*Sin[1 + 2*x]^2)/8 - (3*((1 + 2*x)^2/8 - ((1 + 2*x)*Cos[1 + 2*x]*Sin[1 +
2*x])/4 + Sin[1 + 2*x]^2/8))/2
```

3.909.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

3.909.4 Maple [A] (verified)

Time = 1.62 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.61

method	result
risch	$x^4+2x^3+\frac{3x^2}{2}+\frac{x}{2}+\frac{1}{16}-\frac{3(8x^2+8x+1)\cos(2+4x)}{32}-\frac{(16x^3+24x^2+6x-1)\sin(2+4x)}{16}$
derivativedivides	$\frac{(1+2x)^3\left(-\frac{\cos(1+2x)\sin(1+2x)}{2}+\frac{1}{2}+x\right)}{2}-\frac{3(1+2x)^2\cos(1+2x)^2}{8}+\frac{3(1+2x)\left(\frac{\cos(1+2x)\sin(1+2x)}{2}+\frac{1}{2}+x\right)}{4}-\frac{3(1+2x}{16}$
default	$\frac{(1+2x)^3\left(-\frac{\cos(1+2x)\sin(1+2x)}{2}+\frac{1}{2}+x\right)}{2}-\frac{3(1+2x)^2\cos(1+2x)^2}{8}+\frac{3(1+2x)\left(\frac{\cos(1+2x)\sin(1+2x)}{2}+\frac{1}{2}+x\right)}{4}-\frac{3(1+2x}{16}$
norman	$x^4+x^4\tan(x+\frac{1}{2})^4+\frac{3\tan(x+\frac{1}{2})^2}{4}-\frac{x}{4}+\frac{3x^2}{4}+2x^3-\frac{\tan(x+\frac{1}{2})^3}{4}-\frac{3x\tan(x+\frac{1}{2})}{2}+\frac{11x\tan(x+\frac{1}{2})^2}{2}+\frac{3x\tan(x+\frac{1}{2})^3}{2}-\frac{x\tan(x+}{4}$

input
```
int((1+2*x)^3*sin(1+2*x)^2,x,method=_RETURNVERBOSE)
```

output
```
x^4+2*x^3+3/2*x^2+1/2*x+1/16-3/32*(8*x^2+8*x+1)*cos(2+4*x)-1/16*(16*x^3+24
*x^2+6*x-1)*sin(2+4*x)
```

3.909.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.67

$$\int (1+2x)^3\sin^2(1+2x)\,dx = x^4+2\,x^3-\frac{3}{16}\left(8\,x^2+8\,x+1\right)\cos\left(2\,x+1\right)^2 -\frac{1}{8}\left(16\,x^3+24\,x^2+6\,x-1\right)\cos\left(2\,x+1\right)\sin\left(2\,x+1\right) +\frac{9}{4}\,x^2+\frac{5}{4}\,x$$

input
```
integrate((1+2*x)^3*sin(1+2*x)^2,x, algorithm="fricas")
```

output
```
x^4 + 2*x^3 - 3/16*(8*x^2 + 8*x + 1)*cos(2*x + 1)^2 - 1/8*(16*x^3 + 24*x^2
 + 6*x - 1)*cos(2*x + 1)*sin(2*x + 1) + 9/4*x^2 + 5/4*x
```

3.909.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(94) = 188$.

Time = 0.19 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.91

$$\int (1+2x)^3\sin^2(1+2x)\,dx = x^4\sin^2(2x+1) + x^4\cos^2(2x+1) + 2x^3\sin^2(2x+1) - 2x^3\sin(2x+1)\cos(2x+1) + 2x^3\cos^2(2x+1) + \frac{9x^2\sin^2(2x+1)}{4} - 3x^2\sin(2x+1)\cos(2x+1) + \frac{3x^2\cos^2(2x+1)}{4} + \frac{5x\sin^2(2x+1)}{4} - \frac{3x\sin(2x+1)\cos(2x+1)}{4} - \frac{x\cos^2(2x+1)}{4} + \frac{3\sin^2(2x+1)}{16} + \frac{\sin(2x+1)\cos(2x+1)}{8}$$

input
```
integrate((1+2*x)**3*sin(1+2*x)**2,x)
```

output
```
x**4*sin(2*x + 1)**2 + x**4*cos(2*x + 1)**2 + 2*x**3*sin(2*x + 1)**2 - 2*x
**3*sin(2*x + 1)*cos(2*x + 1) + 2*x**3*cos(2*x + 1)**2 + 9*x**2*sin(2*x +
1)**2/4 - 3*x**2*sin(2*x + 1)*cos(2*x + 1) + 3*x**2*cos(2*x + 1)**2/4 + 5*
x*sin(2*x + 1)**2/4 - 3*x*sin(2*x + 1)*cos(2*x + 1)/4 - x*cos(2*x + 1)**2/
4 + 3*sin(2*x + 1)**2/16 + sin(2*x + 1)*cos(2*x + 1)/8
```

3.909.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.52

$$\int (1+2x)^3\sin^2(1+2x)\,dx = \frac{1}{16}(2x+1)^4 - \frac{3}{32}\left(2(2x+1)^2-1\right)\cos(4x+2) - \frac{1}{16}\left(2(2x+1)^3-6x-3\right)\sin(4x+2)$$

input
```
integrate((1+2*x)^3*sin(1+2*x)^2,x, algorithm="maxima")
```

output
```
1/16*(2*x + 1)^4 - 3/32*(2*(2*x + 1)^2 - 1)*cos(4*x + 2) - 1/16*(2*(2*x +
1)^3 - 6*x - 3)*sin(4*x + 2)
```

3.909.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.59

$$\int (1+2x)^3\sin^2(1+2x)\,dx = x^4+2\,x^3+\frac{3}{2}\,x^2-\frac{3}{32}\left(8\,x^2+8\,x+1\right)\cos\left(4\,x+2\right) - \frac{1}{16}\left(16\,x^3+24\,x^2+6\,x-1\right)\sin\left(4\,x+2\right)+\frac{1}{2}\,x$$

input
```
integrate((1+2*x)^3*sin(1+2*x)^2,x, algorithm="giac")
```

output
```
x^4 + 2*x^3 + 3/2*x^2 - 3/32*(8*x^2 + 8*x + 1)*cos(4*x + 2) - 1/16*(16*x^3
 + 24*x^2 + 6*x - 1)*sin(4*x + 2) + 1/2*x
```

3.909.9 Mupad [B] (verification not implemented)

Time = 26.54 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.70

$$\int (1+2x)^3\sin^2(1+2x)\,dx = \frac{3\,\sin\left(4\,x+2\right)\,\left(2\,x+1\right)}{16}-\frac{3\sin\left(2\,x+1\right)^2}{16}+\frac{\left(2\,x+1\right)^4}{16} - \frac{\sin\left(4\,x+2\right)\,\left(2\,x+1\right)^3}{8}+\frac{3\left(2\,x+1\right)^2\left(2\sin\left(2\,x+1\right)^2-1\right)}{16}$$

input
```
int(sin(2*x + 1)^2*(2*x + 1)^3,x)
```

output
```
(3*sin(4*x + 2)*(2*x + 1))/16 - (3*sin(2*x + 1)^2)/16 + (2*x + 1)^4/16 - (
sin(4*x + 2)*(2*x + 1)^3)/8 + (3*(2*x + 1)^2*(2*sin(2*x + 1)^2 - 1))/16
```

3.910 $\int \frac{-1+\sec(x)}{1-\tan(x)}\,dx$

3.910.1 Optimal result

Integrand size = 13, antiderivative size = 37

$$\int \frac{-1+\sec(x)}{1-\tan(x)}\,dx=-\frac{x}{2}+\frac{\operatorname{arctanh}\left(\frac{\cos(x)(1+\tan(x))}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{1}{2}\log(\cos(x)-\sin(x))$$

output
```
-1/2*x+1/2*ln(cos(x)-sin(x))+1/2*arctanh(1/2*cos(x)*(1+tan(x))*2^(1/2))*2^
(1/2)
```

3.910.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.08

$$\int \frac{-1+\sec(x)}{1-\tan(x)}\,dx=\frac{1}{2}\left(-x+(2-2i)\sqrt[4]{-1}\operatorname{arctanh}\left(\frac{1+\tan\left(\frac{x}{2}\right)}{\sqrt{2}}\right)+\log(\cos(x)-\sin(x))\right)$$

input
```
Integrate[(-1 + Sec[x])/(1 - Tan[x]),x]
```

output
```
(-x + (2 - 2*I)*(-1)^(1/4)*ArcTanh[(1 + Tan[x/2])/Sqrt[2]] + Log[Cos[x] -
Sin[x]])/2
```

3.910.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3042, 4901, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(x)-1}{1-\tan(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(x)-1}{1-\tan(x)}dx$$

$$\Big\downarrow \text{4901}$$

$$\int \left(\frac{1}{\tan(x)-1}-\frac{\sec(x)}{\tan(x)-1}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\operatorname{arctanh}\left(\frac{\cos(x)(\tan(x)+1)}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{x}{2}+\frac{1}{2}\log(\cos(x)-\sin(x))$$

input
```
Int[(-1 + Sec[x])/(1 - Tan[x]),x]
```

output
```
-1/2*x + ArcTanh[(Cos[x]*(1 + Tan[x]))/Sqrt[2]]/Sqrt[2] + Log[Cos[x] - Sin
[x]]/2
```

3.910.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4901
```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

3.910.4 Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.11

method	result	size
parts	$\sqrt{2}\, \operatorname{arctanh}\left(\frac{\left(2 \tan \left(\frac{x}{2}\right)+2\right) \sqrt{2}}{4}\right)+\frac{\ln \left(\tan \left(x \right)-1\right)}{2}-\frac{\ln \left(1+\tan \left(x \right)^{2}\right)}{4}-\frac{\arctan \left(\tan \left(x \right)\right)}{2}$	41
default	$\frac{\ln \left(\tan \left(\frac{x}{2}\right)^{2}+2 \tan \left(\frac{x}{2}\right)-1\right)}{2}+\sqrt{2}\, \operatorname{arctanh}\left(\frac{\left(2 \tan \left(\frac{x}{2}\right)+2\right) \sqrt{2}}{4}\right)-\frac{\ln \left(1+\tan \left(\frac{x}{2}\right)^{2}\right)}{2}-\arctan \left(\tan \left(\frac{x}{2}\right)\right)$	55
risch	$-\frac{x}{2}-\frac{i x}{2}+\frac{\ln \left({\mathrm e}^{i x}+\frac{i \sqrt{2}}{2}+\frac{\sqrt{2}}{2}\right)}{2}+\frac{\ln \left({\mathrm e}^{i x}+\frac{i \sqrt{2}}{2}+\frac{\sqrt{2}}{2}\right) \sqrt{2}}{2}+\frac{\ln \left({\mathrm e}^{i x}-\frac{i \sqrt{2}}{2}-\frac{\sqrt{2}}{2}\right)}{2}-\frac{\ln \left({\mathrm e}^{i x}-\frac{i \sqrt{2}}{2}-\frac{\sqrt{2}}{2}\right) \sqrt{2}}{2}$	95

input
```
int((sec(x)-1)/(1-tan(x)),x,method=_RETURNVERBOSE)
```

output
```
2^(1/2)*arctanh(1/4*(2*tan(1/2*x)+2)*2^(1/2))+1/2*ln(tan(x)-1)-1/4*ln(1+ta
n(x)^2)-1/2*arctan(tan(x))
```

3.910.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.38

$$\int \frac{-1+\sec(x)}{1-\tan(x)}\,dx=\frac{1}{4}\,\sqrt{2}\log\left(\frac{2\,\left(\sqrt{2}+\cos\left(x\right)\right)\sin\left(x\right)+2\,\sqrt{2}\cos\left(x\right)+3}{2\,\cos\left(x\right)\sin\left(x\right)-1}\right)-\frac{1}{2}\,x+\frac{1}{4}\,\log\left(-2\,\cos\left(x\right)\sin\left(x\right)+1\right)$$

input
```
integrate((-1+sec(x))/(1-tan(x)),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*log((2*(sqrt(2) + cos(x))*sin(x) + 2*sqrt(2)*cos(x) + 3)/(2*co
s(x)*sin(x) - 1)) - 1/2*x + 1/4*log(-2*cos(x)*sin(x) + 1)
```

3.910.6 Sympy [F]

$$\int \frac{-1+\sec(x)}{1-\tan(x)}\,dx = -\int \frac{\sec(x)}{\tan(x)-1}\,dx - \int \left(-\frac{1}{\tan(x)-1}\right)dx$$

input
```
integrate((-1+sec(x))/(1-tan(x)),x)
```

output
```
-Integral(sec(x)/(tan(x) - 1), x) - Integral(-1/(tan(x) - 1), x)
```

3.910.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.59

$$\int \frac{-1+\sec(x)}{1-\tan(x)}\,dx = -\frac{1}{2}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{\sin(x)}{\cos(x)+1}-1}{\sqrt{2}+\frac{\sin(x)}{\cos(x)+1}+1}\right) - \frac{1}{2}x - \frac{1}{4}\log\left(\tan(x)^2+1\right) + \frac{1}{2}\log\left(\tan(x)-1\right)$$

input
```
integrate((-1+sec(x))/(1-tan(x)),x, algorithm="maxima")
```

output
```
-1/2*sqrt(2)*log(-(sqrt(2) - sin(x)/(cos(x) + 1) - 1)/(sqrt(2) + sin(x)/(c
os(x) + 1) + 1)) - 1/2*x - 1/4*log(tan(x)^2 + 1) + 1/2*log(tan(x) - 1)
```

3.910.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(31) = 62.

Time = 0.31 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.89

$$\int \frac{-1+\sec(x)}{1-\tan(x)}\,dx = -\frac{1}{2}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\tan\left(\frac{1}{2}x\right)+2\right|}{\left|2\sqrt{2}+2\tan\left(\frac{1}{2}x\right)+2\right|}\right) - \frac{1}{2}x - \frac{1}{2}\log\left(\tan\left(\frac{1}{2}x\right)^2+1\right) + \frac{1}{2}\log\left(\left|\tan\left(\frac{1}{2}x\right)^2+2\tan\left(\frac{1}{2}x\right)-1\right|\right)$$

input
```
integrate((-1+sec(x))/(1-tan(x)),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*log(abs(-2*sqrt(2) + 2*tan(1/2*x) + 2)/abs(2*sqrt(2) + 2*tan(
1/2*x) + 2)) - 1/2*x - 1/2*log(tan(1/2*x)^2 + 1) + 1/2*log(abs(tan(1/2*x)^
2 + 2*tan(1/2*x) - 1))
```

3.910.9 Mupad [B] (verification not implemented)

Time = 26.13 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.73

$$\int \frac{-1+\sec(x)}{1-\tan(x)}\,dx = \ln\left(\tan\left(\frac{x}{2}\right)+\sqrt{2}+1\right)\left(\frac{\sqrt{2}}{2}+\frac{1}{2}\right)-\ln\left(\tan\left(\frac{x}{2}\right)-\sqrt{2}+1\right)\left(\frac{\sqrt{2}}{2}-\frac{1}{2}\right) + \ln\left(\tan\left(\frac{x}{2}\right)-\mathrm{i}\right)\left(-\frac{1}{2}+\frac{1}{2}\mathrm{i}\right)+\ln\left(\tan\left(\frac{x}{2}\right)+1\mathrm{i}\right)\left(-\frac{1}{2}-\frac{1}{2}\mathrm{i}\right)$$

input
```
int(-(1/cos(x) - 1)/(tan(x) - 1),x)
```

output
```
log(tan(x/2) + 2^(1/2) + 1)*(2^(1/2)/2 + 1/2) - log(tan(x/2) + 1i)*(1/2 +
1i/2) - log(tan(x/2) - 2^(1/2) + 1)*(2^(1/2)/2 - 1/2) - log(tan(x/2) - 1i)
*(1/2 - 1i/2)
```

3.911 $\int x^2 \cos(3x) \cos(5x) \, dx$

3.911.1 Optimal result

Integrand size = 12, antiderivative size = 57

$$\int x^2 \cos(3x) \cos(5x) \, dx = \frac{1}{4} x \cos(2x) + \frac{1}{64} x \cos(8x) - \frac{1}{8} \sin(2x) + \frac{1}{4} x^2 \sin(2x) - \frac{1}{512} \sin(8x) + \frac{1}{16} x^2 \sin(8x)$$

output
```
1/4*x*cos(2*x)+1/64*x*cos(8*x)-1/8*sin(2*x)+1/4*x^2*sin(2*x)-1/512*sin(8*x
)+1/16*x^2*sin(8*x)
```

3.911.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int x^2 \cos(3x) \cos(5x) \, dx = \frac{1}{512} \left(128 x \cos(2x) + 8 x \cos(8x) - 64 \sin(2x) + 128 x^2 \sin(2x) - \sin(8x) + 32 x^2 \sin(8x)\right)$$

input
```
Integrate[x^2*Cos[3*x]*Cos[5*x],x]
```

output
```
(128*x*Cos[2*x] + 8*x*Cos[8*x] - 64*Sin[2*x] + 128*x^2*Sin[2*x] - Sin[8*x]
 + 32*x^2*Sin[8*x])/512
```

3.911.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4929, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \cos(3x)\cos(5x)\,dx$$

$$\Big\downarrow \text{ 4929}$$

$$\int \left(\frac{1}{2}x^2\cos(2x)+\frac{1}{2}x^2\cos(8x)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}x^2\sin(2x)+\frac{1}{16}x^2\sin(8x)-\frac{1}{8}\sin(2x)-\frac{1}{512}\sin(8x)+\frac{1}{4}x\cos(2x)+\frac{1}{64}x\cos(8x)$$

input
```
Int[x^2*Cos[3*x]*Cos[5*x],x]
```

output
```
(x*Cos[2*x])/4 + (x*Cos[8*x])/64 - Sin[2*x]/8 + (x^2*Sin[2*x])/4 - Sin[8*x
]/512 + (x^2*Sin[8*x])/16
```

3.911.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4929
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*Cos[(c_.) + (d_.)*(x_)]^(q_.)*((e_.) + (f
_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandTrigReduce[(e + f*x)^m, Cos[a + b*x
]^p*Cos[c + d*x]^q, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] &&
 IGtQ[q, 0] && IntegerQ[m]
```

3.911.4 Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.74

method	result
risch	$\frac{x\cos(8x)}{64}+\frac{(32x^2-1)\sin(8x)}{512}+\frac{x\cos(2x)}{4}+\frac{(2x^2-1)\sin(2x)}{8}$
default	$\frac{x\cos(2x)}{4}+\frac{x\cos(8x)}{64}-\frac{\sin(2x)}{8}+\frac{x^2\sin(2x)}{4}-\frac{\sin(8x)}{512}+\frac{x^2\sin(8x)}{16}$
parallelrisch	$\frac{x\cos(2x)}{4}+\frac{x\cos(8x)}{64}-\frac{\sin(2x)}{8}+\frac{x^2\sin(2x)}{4}-\frac{\sin(8x)}{512}+\frac{x^2\sin(8x)}{16}$
norman	$\frac{\frac{17x}{64}-\frac{17x\tan\left(\frac{5x}{2}\right)^2}{64}-\frac{3x^2\tan\left(\frac{3x}{2}\right)}{8}+\frac{5x^2\tan\left(\frac{5x}{2}\right)}{8}-\frac{63\tan\left(\frac{3x}{2}\right)\tan\left(\frac{5x}{2}\right)^2}{256}-\frac{17\tan\left(\frac{3x}{2}\right)^2x}{64}+\frac{65\tan\left(\frac{3x}{2}\right)^2\tan\left(\frac{5x}{2}\right)}{256}+\frac{3x^2\tan\left(\frac{3x}{2}\right)\tan}{8}}{\left(1+\tan\left(\frac{3x}{2}\right)^2\right)\left(1+\tan\left(\frac{5x}{2}\right.\right.}$

input
```
int(x^2*cos(3*x)*cos(5*x),x,method=_RETURNVERBOSE)
```

output
```
1/64*x*cos(8*x)+1/512*(32*x^2-1)*sin(8*x)+1/4*x*cos(2*x)+1/8*(2*x^2-1)*sin
(2*x)
```

3.911.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.28

$$\int x^2\cos(3x)\cos(5x)\,dx = 2\,x\cos\left(x\right)^8-4\,x\cos\left(x\right)^6+\frac{5}{2}\,x\cos\left(x\right)^4$$
$$+\frac{1}{64}\left(16\left(32\,x^2-1\right)\cos\left(x\right)^7-24\left(32\,x^2-1\right)\cos\left(x\right)^5+10\left(32\,x^2-1\right)\cos\left(x\right)^3-15\,\cos\left(x\right)\right)\sin\left(x\right)$$
$$-\frac{15}{64}\,x$$

input
```
integrate(x^2*cos(3*x)*cos(5*x),x, algorithm="fricas")
```

output
```
2*x*cos(x)^8 - 4*x*cos(x)^6 + 5/2*x*cos(x)^4 + 1/64*(16*(32*x^2 - 1)*cos(x
)^7 - 24*(32*x^2 - 1)*cos(x)^5 + 10*(32*x^2 - 1)*cos(x)^3 - 15*cos(x))*sin
(x) - 15/64*x
```

3.911.6 Sympy [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.58

$$\int x^2\cos(3x)\cos(5x)\,dx = -\frac{3x^2\sin(3x)\cos(5x)}{16}+\frac{5x^2\sin(5x)\cos(3x)}{16}+\frac{15x\sin(3x)\sin(5x)}{64}+\frac{17x\cos(3x)\cos(5x)}{64}+\frac{63\sin(3x)\cos(5x)}{512}-\frac{65\sin(5x)\cos(3x)}{512}$$

input
```
integrate(x**2*cos(3*x)*cos(5*x),x)
```

output
```
-3*x**2*sin(3*x)*cos(5*x)/16 + 5*x**2*sin(5*x)*cos(3*x)/16 + 15*x*sin(3*x)
*sin(5*x)/64 + 17*x*cos(3*x)*cos(5*x)/64 + 63*sin(3*x)*cos(5*x)/512 - 65*s
in(5*x)*cos(3*x)/512
```

3.911.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.72

$$\int x^2\cos(3x)\cos(5x)\,dx = \frac{1}{64}\,x\cos(8\,x)+\frac{1}{4}\,x\cos(2\,x)+\frac{1}{512}\,\left(32\,x^2-1\right)\sin(8\,x)+\frac{1}{8}\,\left(2\,x^2-1\right)\sin(2\,x)$$

input
```
integrate(x^2*cos(3*x)*cos(5*x),x, algorithm="maxima")
```

output
```
1/64*x*cos(8*x) + 1/4*x*cos(2*x) + 1/512*(32*x^2 - 1)*sin(8*x) + 1/8*(2*x^
2 - 1)*sin(2*x)
```

3.911.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.72

$$\int x^2\cos(3x)\cos(5x)\,dx = \frac{1}{64}\,x\cos(8\,x)+\frac{1}{4}\,x\cos(2\,x)+\frac{1}{512}\,\left(32\,x^2-1\right)\sin(8\,x)+\frac{1}{8}\,\left(2\,x^2-1\right)\sin(2\,x)$$

input
```
integrate(x^2*cos(3*x)*cos(5*x),x, algorithm="giac")
```

output
```
1/64*x*cos(8*x) + 1/4*x*cos(2*x) + 1/512*(32*x^2 - 1)*sin(8*x) + 1/8*(2*x^
2 - 1)*sin(2*x)
```

3.911.9 Mupad [B] (verification not implemented)

Time = 26.36 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.79

$$\int x^2\cos(3x)\cos(5x)\,dx = \frac{x\,\cos\left(2\,x\right)}{4}-\frac{\sin\left(8\,x\right)}{512}-\frac{\sin\left(2\,x\right)}{8}+\frac{x\,\cos\left(8\,x\right)}{64}+\frac{x^2\,\sin\left(2\,x\right)}{4}+\frac{x^2\,\sin\left(8\,x\right)}{16}$$

input
```
int(x^2*cos(3*x)*cos(5*x),x)
```

output
```
(x*cos(2*x))/4 - sin(8*x)/512 - sin(2*x)/8 + (x*cos(8*x))/64 + (x^2*sin(2*
x))/4 + (x^2*sin(8*x))/16
```

3.912 $\int \frac{\cos(x)+\sin(x)}{\sqrt{\cos(x)}\sqrt{\sin(x)}}\,dx$

3.912.1 Optimal result

Integrand size = 18, antiderivative size = 57

$$\int \frac{\cos(x)+\sin(x)}{\sqrt{\cos(x)}\sqrt{\sin(x)}}\,dx=-\sqrt{2}\arctan\left(1-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)+\sqrt{2}\arctan\left(1+\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)$$

output
```
-arctan(1-2^(1/2)*sin(x)^(1/2)/cos(x)^(1/2))*2^(1/2)+arctan(1+2^(1/2)*sin(
x)^(1/2)/cos(x)^(1/2))*2^(1/2)
```

3.912.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.19

$$\int \frac{\cos(x)+\sin(x)}{\sqrt{\cos(x)}\sqrt{\sin(x)}}\,dx$$
$$=\frac{2\sqrt[4]{\cos^2(x)}\sqrt{\sin(x)}\left(3\cos(x)\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{4},\frac{5}{4},\sin^2(x)\right)+\sqrt{\cos^2(x)}\,\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{4}\right.\right.}{3\cos^{\frac{3}{2}}(x)}$$

input
```
Integrate[(Cos[x] + Sin[x])/(Sqrt[Cos[x]]*Sqrt[Sin[x]]),x]
```

output
```
(2*(Cos[x]^2)^(1/4)*Sqrt[Sin[x]]*(3*Cos[x]*Hypergeometric2F1[1/4, 1/4, 5/4
, Sin[x]^2] + Sqrt[Cos[x]^2]*Hypergeometric2F1[3/4, 3/4, 7/4, Sin[x]^2]*Si
n[x]))/(3*Cos[x]^(3/2))
```

3.912.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 243 vs. 2(57) = 114.

Time = 0.46 (sec) , antiderivative size = 243, normalized size of antiderivative = 4.26, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3586, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)+\cos(x)}{\sqrt{\sin(x)}\sqrt{\cos(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)+\cos(x)}{\sqrt{\sin(x)}\sqrt{\cos(x)}}dx$$

$$\Big\downarrow \text{3586}$$

$$\int \left(\frac{\sqrt{\cos(x)}}{\sqrt{\sin(x)}}+\frac{\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{\cos(x)}}{\sqrt{\sin(x)}}\right)}{\sqrt{2}}-\frac{\arctan\left(\frac{\sqrt{2}\sqrt{\cos(x)}}{\sqrt{\sin(x)}}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}\right)}{\sqrt{2}}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1\right)}{\sqrt{2}}+\frac{\log\left(\tan(x)-\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(x)+\frac{\sqrt{2}\sqrt{\sin(x)}}{\sqrt{\cos(x)}}+1\right)}{2\sqrt{2}}-\frac{\log\left(\cot(x)-\frac{\sqrt{2}\sqrt{\cos(x)}}{\sqrt{\sin(x)}}+1\right)}{2\sqrt{2}}+\frac{\log\left(\cot(x)+\frac{\sqrt{2}\sqrt{\cos(x)}}{\sqrt{\sin(x)}}+1\right)}{2\sqrt{2}}$$

input
```
Int[(Cos[x] + Sin[x])/(Sqrt[Cos[x]]*Sqrt[Sin[x]]),x]
```

output
```
ArcTan[1 - (Sqrt[2]*Sqrt[Cos[x]])/Sqrt[Sin[x]]]/Sqrt[2] - ArcTan[1 + (Sqrt
[2]*Sqrt[Cos[x]])/Sqrt[Sin[x]]]/Sqrt[2] - ArcTan[1 - (Sqrt[2]*Sqrt[Sin[x]]
)/Sqrt[Cos[x]]]/Sqrt[2] + ArcTan[1 + (Sqrt[2]*Sqrt[Sin[x]])/Sqrt[Cos[x]]]/
Sqrt[2] - Log[1 + Cot[x] - (Sqrt[2]*Sqrt[Cos[x]])/Sqrt[Sin[x]]]/(2*Sqrt[2]
) + Log[1 + Cot[x] + (Sqrt[2]*Sqrt[Cos[x]])/Sqrt[Sin[x]]]/(2*Sqrt[2]) + Lo
g[1 - (Sqrt[2]*Sqrt[Sin[x]])/Sqrt[Cos[x]] + Tan[x]]/(2*Sqrt[2]) - Log[1 +
(Sqrt[2]*Sqrt[Sin[x]])/Sqrt[Cos[x]] + Tan[x]]/(2*Sqrt[2])
```

3.912.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3586
```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_.), x_Symbol] :> In
t[ExpandTrig[cos[c + d*x]^m*sin[c + d*x]^n*(a*cos[c + d*x] + b*sin[c + d*x]
)^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0]
```

3.912.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(41) = 82$.

Time = 24.51 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.68

method	result
default	$-\frac{\sqrt{2}\left(\arctan\left(\frac{\sqrt{2}\,\sin(x)\sqrt{\frac{\cos(x)\sin(x)}{(\cos(x)+1)^{2}}}+\cos(x)-1}{\cos(x)-1}\right)+\arctan\left(\frac{\sqrt{2}\,\sin(x)\sqrt{\frac{\cos(x)\sin(x)}{(\cos(x)+1)^{2}}}-\cos(x)+1}{\cos(x)-1}\right)\right)(\cos(x)-1)\sqrt{\cos(x)}}{\sqrt{\frac{\cos(x)\sin(x)}{(\cos(x)+1)^{2}}}\,\sin(x)^{\frac{3}{2}}}$
parts	$-\frac{\sqrt{\cos(x)}\,(\cos(x)-1)\left(\ln\left(-2\cot(x)\sqrt{2}\,\sqrt{\frac{\cos(x)\sin(x)}{(\cos(x)+1)^{2}}}-2\csc(x)\sqrt{2}\,\sqrt{\frac{\cos(x)\sin(x)}{(\cos(x)+1)^{2}}}+2\cot(x)+2\right)+2\arctan\left(\frac{\sqrt{2}\,\sin(x)\sqrt{\frac{\cos(x)\sin(x)}{(\cos(x)+1)^{2}}}}{\cos(x)-1}\right.\right.}{4\sin(x)}$

input
```
int((sin(x)+cos(x))/cos(x)^(1/2)/sin(x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2^(1/2)*(arctan((2^(1/2)*sin(x)*(cos(x)*sin(x)/(cos(x)+1)^2)^(1/2)+cos(x)
-1)/(cos(x)-1))+arctan((2^(1/2)*sin(x)*(cos(x)*sin(x)/(cos(x)+1)^2)^(1/2)-
cos(x)+1)/(cos(x)-1)))*(cos(x)-1)*cos(x)^(1/2)/(cos(x)*sin(x)/(cos(x)+1)^2
)^(1/2)/sin(x)^(3/2)
```

3.912.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(41) = 82$.

Time = 0.26 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.49

$$\int \frac{\cos(x)+\sin(x)}{\sqrt{\cos(x)}\sqrt{\sin(x)}}\,dx = -\frac{1}{4}\sqrt{2}\arctan\left(-\frac{\left(32\sqrt{2}\cos(x)^4-32\sqrt{2}\cos(x)^2+16\sqrt{2}\cos(x)\sin(x)-\sqrt{2}\right)\sqrt{\cos(x)}\sqrt{\sin(x)}}{8\left(4\cos(x)^5-3\cos(x)^3-\left(4\cos(x)^4-5\cos(x)^2\right)\sin(x)-\cos(x)\right)}\right)$$

input
```
integrate((cos(x)+sin(x))/cos(x)^(1/2)/sin(x)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*sqrt(2)*arctan(-1/8*(32*sqrt(2)*cos(x)^4 - 32*sqrt(2)*cos(x)^2 + 16*s
qrt(2)*cos(x)*sin(x) - sqrt(2))*sqrt(cos(x))*sqrt(sin(x))/(4*cos(x)^5 - 3*
cos(x)^3 - (4*cos(x)^4 - 5*cos(x)^2)*sin(x) - cos(x)))
```

3.912.6 Sympy [F]

$$\int \frac{\cos(x)+\sin(x)}{\sqrt{\cos(x)}\sqrt{\sin(x)}}\,dx = \int \frac{\sin(x)+\cos(x)}{\sqrt{\sin(x)}\sqrt{\cos(x)}}\,dx$$

input
```
integrate((cos(x)+sin(x))/cos(x)**(1/2)/sin(x)**(1/2),x)
```

output
```
Integral((sin(x) + cos(x))/(sqrt(sin(x))*sqrt(cos(x))), x)
```

3.912.7 Maxima [F]

$$\int \frac{\cos(x)+\sin(x)}{\sqrt{\cos(x)}\sqrt{\sin(x)}}\,dx = \int \frac{\cos(x)+\sin(x)}{\sqrt{\cos(x)}\sqrt{\sin(x)}}\,dx$$

input
```
integrate((cos(x)+sin(x))/cos(x)^(1/2)/sin(x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((cos(x) + sin(x))/(sqrt(cos(x))*sqrt(sin(x))), x)
```

3.912.8 Giac [F]

$$\int \frac{\cos(x) + \sin(x)}{\sqrt{\cos(x)}\sqrt{\sin(x)}}\, dx = \int \frac{\cos(x) + \sin(x)}{\sqrt{\cos(x)}\sqrt{\sin(x)}}\, dx$$

input
```
integrate((cos(x)+sin(x))/cos(x)^(1/2)/sin(x)^(1/2),x, algorithm="giac")
```

output
```
integrate((cos(x) + sin(x))/(sqrt(cos(x))*sqrt(sin(x))), x)
```

3.912.9 Mupad [B] (verification not implemented)

Time = 28.44 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \frac{\cos(x) + \sin(x)}{\sqrt{\cos(x)}\sqrt{\sin(x)}}\, dx = -\frac{2\sqrt{\cos(x)}\sin(x)^{3/2}\, {}_2F_1\left(\frac{1}{4}, \frac{1}{4};\ \frac{5}{4};\ \cos(x)^2\right)}{\left(\sin(x)^2\right)^{3/4}} - \frac{2\cos(x)^{3/2}\sqrt{\sin(x)}\, {}_2F_1\left(\frac{3}{4}, \frac{3}{4};\ \frac{7}{4};\ \cos(x)^2\right)}{3\left(\sin(x)^2\right)^{1/4}}$$

input
```
int((cos(x) + sin(x))/(cos(x)^(1/2)*sin(x)^(1/2)),x)
```

output
```
- (2*cos(x)^(1/2)*sin(x)^(3/2)*hypergeom([1/4, 1/4], 5/4, cos(x)^2))/(sin(
x)^2)^(3/4) - (2*cos(x)^(3/2)*sin(x)^(1/2)*hypergeom([3/4, 3/4], 7/4, cos(
x)^2))/(3*(sin(x)^2)^(1/4))
```

3.913 $\int \sec^2(x)(1+\sin(x))\,dx$

3.913.1 Optimal result

Integrand size = 9, antiderivative size = 5

$$\int \sec^2(x)(1+\sin(x))\,dx = \sec(x)+\tan(x)$$

output
```
sec(x)+tan(x)
```

3.913.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \sec^2(x)(1+\sin(x))\,dx = \sec(x)+\tan(x)$$

input
```
Integrate[Sec[x]^2*(1 + Sin[x]),x]
```

output
```
Sec[x] + Tan[x]
```

3.913.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {3042, 3148, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (\sin(x)+1)\sec^2(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(x)+1}{\cos(x)^2}dx$$

$$\Big\downarrow \text{3148}$$

$$\int \sec^2(x)dx+\sec(x)$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(x+\frac{\pi}{2}\right)^2dx+\sec(x)$$

$$\Big\downarrow \text{4254}$$

$$\sec(x)-\int 1d(-\tan(x))$$

$$\Big\downarrow \text{24}$$

$$\tan(x)+\sec(x)$$

input
```
Int[Sec[x]^2*(1 + Sin[x]),x]
```

output
```
Sec[x] + Tan[x]
```

3.913.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3148
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Simp[a   Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
 (IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.913.4 Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.60

method	result	size
default	$\tan(x)+\frac{1}{\cos(x)}$	8
parallelrisch	$-\frac{2}{\tan(\frac{x}{2})-1}$	11
risch	$\frac{2}{e^{ix}-i}$	13
norman	$\frac{-2\tan(\frac{x}{2})^2-2\tan(\frac{x}{2})^3-2\tan(\frac{x}{2})-2}{\left(1+\tan(\frac{x}{2})^2\right)\left(\tan(\frac{x}{2})^2-1\right)}$	46

input
```
int(sec(x)^2*(1+sin(x)),x,method=_RETURNVERBOSE)
```

output
```
tan(x)+1/cos(x)
```

3.913.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17 vs. 2(5) = 10.

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 3.40

$$\int \sec^2(x)(1+\sin(x))\,dx = \frac{\cos(x)+\sin(x)+1}{\cos(x)-\sin(x)+1}$$

input
```
integrate(sec(x)^2*(1+sin(x)),x, algorithm="fricas")
```

output
```
(cos(x) + sin(x) + 1)/(cos(x) - sin(x) + 1)
```

3.913.6 Sympy [A] (verification not implemented)

Time = 1.14 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.40

$$\int \sec^2(x)(1+\sin(x))\,dx = \tan(x) + \frac{1}{\cos(x)}$$

input
```
integrate(sec(x)**2*(1+sin(x)),x)
```

output
```
tan(x) + 1/cos(x)
```

3.913.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.40

$$\int \sec^2(x)(1+\sin(x))\,dx = \frac{1}{\cos(x)} + \tan(x)$$

input
```
integrate(sec(x)^2*(1+sin(x)),x, algorithm="maxima")
```

output
```
1/cos(x) + tan(x)
```

3.913.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 10, normalized size of antiderivative = 2.00

$$\int \sec^2(x)(1+\sin(x))\,dx = -\frac{2}{\tan\left(\frac{1}{2}\,x\right)-1}$$

input
```
integrate(sec(x)^2*(1+sin(x)),x, algorithm="giac")
```

output
```
-2/(tan(1/2*x) - 1)
```

3.913.9 Mupad [B] (verification not implemented)

Time = 26.48 (sec) , antiderivative size = 10, normalized size of antiderivative = 2.00

$$\int \sec^2(x)(1+\sin(x))\,dx = -\frac{2}{\tan\left(\frac{x}{2}\right)-1}$$

input
```
int((sin(x) + 1)/cos(x)^2,x)
```

output
```
-2/(tan(x/2) - 1)
```

3.914 $\int (10x^9 \cos(x^5 \log(x)) - x^{10}(x^4 + 5x^4 \log(x)) \sin(x^5$

3.914.1 Optimal result

Integrand size = 36, antiderivative size = 11

$$\int \left(10x^9 \cos\left(x^5 \log(x)\right) - x^{10}\left(x^4 + 5x^4 \log(x)\right) \sin\left(x^5 \log(x)\right)\right) \, dx = x^{10} \cos\left(x^5 \log(x)\right)$$

output
```
x^10*cos(x^5*ln(x))
```

3.914.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \left(10x^9 \cos\left(x^5 \log(x)\right) - x^{10}\left(x^4 + 5x^4 \log(x)\right) \sin\left(x^5 \log(x)\right)\right) \, dx = x^{10} \cos\left(x^5 \log(x)\right)$$

input
```
Integrate[10*x^9*Cos[x^5*Log[x]] - x^10*(x^4 + 5*x^4*Log[x])*Sin[x^5*Log[x
]],x]
```

output
```
x^10*Cos[x^5*Log[x]]
```

3.914.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(10 x^9 \cos \left(x^5 \log (x)\right)-x^{10} \left(x^4+5 x^4 \log (x)\right) \sin \left(x^5 \log (x)\right)\right) \, dx$$

$$\Big\downarrow \ 2009$$

$$-\int x^{14} \sin \left(x^5 \log (x)\right)dx-5 \int x^{14} \log (x) \sin \left(x^5 \log (x)\right)dx+10 \int x^9 \cos \left(x^5 \log (x)\right)dx$$

input
```
Int[10*x^9*Cos[x^5*Log[x]] - x^10*(x^4 + 5*x^4*Log[x])*Sin[x^5*Log[x]],x]
```

output
```
$Aborted
```

3.914.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.914.4 Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

method	result	size
parallelrisch	$x^{10} \cos \left(2 x^5 \ln \left(\sqrt{x}\right)\right)$	15
risch	$\frac{x^{10} x^{i x^5}}{2}+\frac{x^{10} x^{-i x^5}}{2}$	30

input
```
int(10*x^9*cos(x^5*ln(x))-x^10*(x^4+5*x^4*ln(x))*sin(x^5*ln(x)),x,method=_
RETURNVERBOSE)
```

output
```
x^10*cos(2*x^5*ln(x^(1/2)))
```

3.914.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \left(10x^9 \cos \left(x^5 \log(x)\right) - x^{10}\left(x^4 + 5x^4 \log(x)\right) \sin \left(x^5 \log(x)\right)\right) \, dx = x^{10} \cos \left(x^5 \log \left(x\right)\right)$$

input
```
integrate(10*x^9*cos(x^5*log(x))-x^10*(x^4+5*x^4*log(x))*sin(x^5*log(x)),x
, algorithm="fricas")
```

output
```
x^10*cos(x^5*log(x))
```

3.914.6 Sympy [F]

$$\int \left(10x^9 \cos \left(x^5 \log(x)\right) - x^{10}\left(x^4 + 5x^4 \log(x)\right) \sin \left(x^5 \log(x)\right)\right) \, dx$$
$$= -\int \left(-10x^9 \cos \left(x^5 \log \left(x\right)\right)\right) \, dx - \int x^{14} \sin \left(x^5 \log \left(x\right)\right) \, dx - \int 5x^{14} \log \left(x\right) \sin \left(x^5 \log \left(x\right)\right) \, dx$$

input
```
integrate(10*x**9*cos(x**5*ln(x))-x**10*(x**4+5*x**4*ln(x))*sin(x**5*ln(x)
),x)
```

output
```
-Integral(-10*x**9*cos(x**5*log(x)), x) - Integral(x**14*sin(x**5*log(x)),
 x) - Integral(5*x**14*log(x)*sin(x**5*log(x)), x)
```

3.914.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \left(10x^9 \cos \left(x^5 \log(x)\right) - x^{10}\left(x^4 + 5x^4 \log(x)\right) \sin \left(x^5 \log(x)\right)\right) \, dx = x^{10} \cos \left(x^5 \log \left(x\right)\right)$$

input
```
integrate(10*x^9*cos(x^5*log(x))-x^10*(x^4+5*x^4*log(x))*sin(x^5*log(x)),x
, algorithm="maxima")
```

output
```
x^10*cos(x^5*log(x))
```

3.914.8 Giac [F(-1)]

Timed out.

$$\int \left(10x^9\cos\left(x^5\log(x)\right)-x^{10}\left(x^4+5x^4\log(x)\right)\sin\left(x^5\log(x)\right)\right)\,dx=\text{Timed out}$$

input
```
integrate(10*x^9*cos(x^5*log(x))-x^10*(x^4+5*x^4*log(x))*sin(x^5*log(x)),x
, algorithm="giac")
```

output
```
Timed out
```

3.914.9 Mupad [B] (verification not implemented)

Time = 27.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \left(10x^9\cos\left(x^5\log(x)\right)-x^{10}\left(x^4+5x^4\log(x)\right)\sin\left(x^5\log(x)\right)\right)\,dx=x^{10}\,\cos\left(x^5\,\ln\left(x\right)\right)$$

input
```
int(10*x^9*cos(x^5*log(x)) - x^10*sin(x^5*log(x))*(5*x^4*log(x) + x^4),x)
```

output
```
x^10*cos(x^5*log(x))
```

3.915 $\int \cos^2\left(\frac{x}{2}\right)\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx$

3.915.1 Optimal result

Integrand size = 21, antiderivative size = 27

$$\int \cos^2\left(\frac{x}{2}\right)\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx=\frac{x}{2}-\frac{\cos(x)}{2}-\log\left(\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)\right)$$

output
```
1/2*x-1/2*cos(x)-ln(cos(1/4*Pi+1/2*x))
```

3.915.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \cos^2\left(\frac{x}{2}\right)\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx=\frac{1}{2}\left(x+2\text{arctanh}\left(\cot\left(\frac{x}{2}\right)\right)-\cos(x)-\log(\cos(x))\right)$$

input
```
Integrate[Cos[x/2]^2*Tan[Pi/4 + x/2],x]
```

output
```
(x + 2*ArcTanh[Cot[x/2]] - Cos[x] - Log[Cos[x]])/2
```

3.915.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2\left(\frac{x}{2}\right)\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)dx$$
$$\Big\downarrow \ 7299$$
$$\int \cos^2\left(\frac{x}{2}\right)\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)dx$$

input
```
Int[Cos[x/2]^2*Tan[Pi/4 + x/2],x]
```

output
```
$Aborted
```

3.915.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.915.4 Maple [A] (verified)

Time = 3.52 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{x}{2}+\frac{\ln(\sec(x)+\tan(x))}{2}-\frac{\cos(x)}{2}-\frac{\ln(\cos(x))}{2}$	22
risch	$\frac{x}{2}+\frac{ix}{2}-\frac{e^{ix}}{4}-\frac{e^{-ix}}{4}-\ln\left(e^{ix}-i\right)$	34

input
```
int(cos(1/2*x)^2*tan(1/4*Pi+1/2*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*x+1/2*ln(sec(x)+tan(x))-1/2*cos(x)-1/2*ln(cos(x))
```

3.915.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \cos^2\left(\frac{x}{2}\right)\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx=-\cos\left(\frac{1}{2}\,x\right)^2+\frac{1}{2}\,x-\frac{1}{2}\,\log\left(-2\,\cos\left(\frac{1}{2}\,x\right)\sin\left(\frac{1}{2}\,x\right)+1\right)$$

input
```
integrate(cos(1/2*x)^2*tan(1/4*pi+1/2*x),x, algorithm="fricas")
```

output
```
-cos(1/2*x)^2 + 1/2*x - 1/2*log(-2*cos(1/2*x)*sin(1/2*x) + 1)
```

3.915.6 Sympy [F]

$$\int \cos^2\left(\frac{x}{2}\right)\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx=\int \cos^2\left(\frac{x}{2}\right)\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\,dx$$

input
```
integrate(cos(1/2*x)**2*tan(1/4*pi+1/2*x),x)
```

output
```
Integral(cos(x/2)**2*tan(x/2 + pi/4), x)
```

3.915.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(19) = 38.

Time = 0.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.74

$$\int \cos^2\left(\frac{x}{2}\right)\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx$$
$$=\frac{2\,x\cos\left(x\right)^2+2\,x\sin\left(x\right)^2-\cos\left(2\,x\right)\cos\left(x\right)-2\,\left(\cos\left(x\right)^2+\sin\left(x\right)^2\right)\log\left(\cos\left(x\right)^2+\sin\left(x\right)^2-2\,\sin\left(x\right)+\right.}{4\,\left(\cos\left(x\right)^2+\sin\left(x\right)^2\right)}$$

input
```
integrate(cos(1/2*x)^2*tan(1/4*pi+1/2*x),x, algorithm="maxima")
```

output
```
1/4*(2*x*cos(x)^2 + 2*x*sin(x)^2 - cos(2*x)*cos(x) - 2*(cos(x)^2 + sin(x)^
2)*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1) - sin(2*x)*sin(x) - cos(x))/(co
s(x)^2 + sin(x)^2)
```

3.915.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(19) = 38$.

Time = 0.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.44

$$\int \cos^2\left(\frac{x}{2}\right)\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx$$
$$= \frac{x\tan\left(\frac{1}{2}\,x\right)^2-\log\left(\frac{2\left(\tan\left(\frac{1}{2}\,x\right)^2-2\,\tan\left(\frac{1}{2}\,x\right)+1\right)}{\tan\left(\frac{1}{2}\,x\right)^2+1}\right)\tan\left(\frac{1}{2}\,x\right)^2+\tan\left(\frac{1}{2}\,x\right)^2+x-\log\left(\frac{2\left(\tan\left(\frac{1}{2}\,x\right)^2-2\,\tan\left(\frac{1}{2}\,x\right)+1\right)}{\tan\left(\frac{1}{2}\,x\right)^2+1}\right)}{2\left(\tan\left(\frac{1}{2}\,x\right)^2+1\right)}$$

input
```
integrate(cos(1/2*x)^2*tan(1/4*pi+1/2*x),x, algorithm="giac")
```

output
```
1/2*(x*tan(1/2*x)^2 - log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*x)^
2 + 1))*tan(1/2*x)^2 + tan(1/2*x)^2 + x - log(2*(tan(1/2*x)^2 - 2*tan(1/2*
x) + 1)/(tan(1/2*x)^2 + 1)) - 1)/(tan(1/2*x)^2 + 1)
```

3.915.9 Mupad [B] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.41

$$\int \cos^2\left(\frac{x}{2}\right)\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx = -2\,\ln\left(\mathrm{e}^{\frac{\Pi\,1\mathrm{i}}{2}}\,\mathrm{e}^{x\,1\mathrm{i}}+1\right)\,\sin\left(\frac{\Pi}{4}\right)^2+x\,\mathrm{e}^{\frac{\Pi\,1\mathrm{i}}{4}}\,\sin\left(\frac{\Pi}{4}\right)-\frac{\cos\left(x\right)}{2}$$

input
```
int(cos(x/2)^2*tan(Pi/4 + x/2),x)
```

output
```
x*sin(Pi/4)*exp((Pi*1i)/4) - 2*sin(Pi/4)^2*log(exp((Pi*1i)/2)*exp(x*1i) +
1) - cos(x)/2
```

3.916 $\int (2+3x)^2 \sin^3(x) \, dx$

3.916.1 Optimal result

Integrand size = 12, antiderivative size = 65

$$\int (2+3x)^2 \sin^3(x) \, dx = 14\cos(x) - \frac{2}{3}(2+3x)^2\cos(x) - \frac{2\cos^3(x)}{3} + 4(2+3x)\sin(x) - \frac{1}{3}(2+3x)^2\cos(x)\sin^2(x) + \frac{2}{3}(2+3x)\sin^3(x)$$

output
```
14*cos(x)-2/3*(2+3*x)^2*cos(x)-2/3*cos(x)^3+4*(2+3*x)*sin(x)-1/3*(2+3*x)^2
*cos(x)*sin(x)^2+2/3*(2+3*x)*sin(x)^3
```

3.916.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77

$$\int (2+3x)^2 \sin^3(x) \, dx = \frac{1}{12}\left(-9\left(-14+12x+9x^2\right)\cos(x) + \left(2+12x+9x^2\right)\cos(3x) - 2(2+3x)(-27\sin(x)+\sin(3x))\right)$$

input
```
Integrate[(2 + 3*x)^2*Sin[x]^3,x]
```

output
```
(-9*(-14 + 12*x + 9*x^2)*Cos[x] + (2 + 12*x + 9*x^2)*Cos[3*x] - 2*(2 + 3*x
)*(-27*Sin[x] + Sin[3*x]))/12
```

3.916.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.15, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {3042, 3792, 3042, 3113, 2009, 3777, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (3x+2)^2 \sin^3(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (3x+2)^2 \sin(x)^3dx$$

$$\Big\downarrow \text{3792}$$

$$-2\int \sin^3(x)dx+\frac{2}{3}\int (3x+2)^2 \sin(x)dx+\frac{2}{3}(3x+2)\sin^3(x)-\frac{1}{3}(3x+2)^2\sin^2(x)\cos(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\int (3x+2)^2 \sin(x)dx-2\int \sin(x)^3dx+\frac{2}{3}(3x+2)\sin^3(x)-\frac{1}{3}(3x+2)^2\sin^2(x)\cos(x)$$

$$\Big\downarrow \text{3113}$$

$$\frac{2}{3}\int (3x+2)^2 \sin(x)dx+2\int \left(1-\cos^2(x)\right)d\cos(x)+\frac{2}{3}(3x+2)\sin^3(x)-\frac{1}{3}(3x+2)^2\sin^2(x)\cos(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{3}\int (3x+2)^2 \sin(x)dx+\frac{2}{3}(3x+2)\sin^3(x)+2\left(\cos(x)-\frac{\cos^3(x)}{3}\right)-\frac{1}{3}(3x+2)^2\sin^2(x)\cos(x)$$

$$\Big\downarrow \text{3777}$$

$$\frac{2}{3}\left(6\int (3x+2)\cos(x)dx-(3x+2)^2\cos(x)\right)+\frac{2}{3}(3x+2)\sin^3(x)+2\left(\cos(x)-\frac{\cos^3(x)}{3}\right)-\frac{1}{3}(3x+2)^2\sin^2(x)\cos(x)$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\left(6\int (3x+2)\sin\left(x+\frac{\pi}{2}\right)dx-(3x+2)^2\cos(x)\right)+\frac{2}{3}(3x+2)\sin^3(x)+2\left(\cos(x)-\frac{\cos^3(x)}{3}\right)-\frac{1}{3}(3x+2)^2\sin^2(x)\cos(x)$$

$$\Big\downarrow \text{3777}$$

$$\frac{2}{3}\left(6(3\int -\sin(x)dx+(3x+2)\sin(x))-(3x+2)^2\cos(x)\right)+\frac{2}{3}(3x+2)\sin^3(x)+2\left(\cos(x)-\frac{\cos^3(x)}{3}\right)-\frac{1}{3}(3x+2)^2\sin^2(x)\cos(x)$$

$$\Big\downarrow \; 25$$

$$\frac{2}{3}\left(6((3x+2)\sin(x)-3\int \sin(x)dx)-(3x+2)^2\cos(x)\right)+\frac{2}{3}(3x+2)\sin^3(x)+2\left(\cos(x)-\frac{\cos^3(x)}{3}\right)-\frac{1}{3}(3x+2)^2\sin^2(x)\cos(x)$$

$$\Big\downarrow \; 3042$$

$$\frac{2}{3}\left(6((3x+2)\sin(x)-3\int \sin(x)dx)-(3x+2)^2\cos(x)\right)+\frac{2}{3}(3x+2)\sin^3(x)+2\left(\cos(x)-\frac{\cos^3(x)}{3}\right)-\frac{1}{3}(3x+2)^2\sin^2(x)\cos(x)$$

$$\Big\downarrow \; 3118$$

$$\frac{2}{3}(3x+2)\sin^3(x)+2\left(\cos(x)-\frac{\cos^3(x)}{3}\right)-\frac{1}{3}(3x+2)^2\sin^2(x)\cos(x)+\frac{2}{3}(6((3x+2)\sin(x)+3\cos(x))-(3x+2)^2\cos(x))$$

input
```
Int[(2 + 3*x)^2*Sin[x]^3,x]
```

output
```
2*(Cos[x] - Cos[x]^3/3) - ((2 + 3*x)^2*Cos[x]*Sin[x]^2)/3 + (2*(2 + 3*x)*S
in[x]^3)/3 + (2*(-((2 + 3*x)^2*Cos[x]) + 6*(3*Cos[x] + (2 + 3*x)*Sin[x])))
/3
```

3.916.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

3.916.4 Maple [A] (verified)

Time = 2.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

method	result
risch	$\left(-\frac{27}{4}x^{2}-9x+\frac{21}{2}\right)\cos\left(x\right)+\frac{9(2+3x)\sin(x)}{2}+\left(\frac{3}{4}x^{2}+x+\frac{1}{6}\right)\cos\left(3x\right)-\frac{(2+3x)\sin(3x)}{6}$
default	$-3x^{2}\left(2+\sin\left(x\right)^{2}\right)\cos\left(x\right)+12\cos\left(x\right)+12x\sin\left(x\right)+2\sin\left(x\right)^{3}x-\frac{2\left(2+\sin(x)^{2}\right)\cos(x)}{3}-4x\left(2+\mathrm{s}\right.$
norman	$\frac{24\tan\left(\frac{x}{2}\right)^{4}+40\tan\left(\frac{x}{2}\right)^{2}-8x-6x^{2}+\frac{128\tan\left(\frac{x}{2}\right)^{3}}{3}+16\tan\left(\frac{x}{2}\right)^{5}+24x\tan\left(\frac{x}{2}\right)-24x\tan\left(\frac{x}{2}\right)^{2}+64x\tan\left(\frac{x}{2}\right)^{3}+24x\tan\left(\frac{x}{2}\right)^{4}+24x\tan\left(\frac{x}{2}\right)^{5}}{\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)^{3}}$

input
```
int((2+3*x)^2*sin(x)^3,x,method=_RETURNVERBOSE)
```

output
```
(-27/4*x^2-9*x+21/2)*cos(x)+9/2*(2+3*x)*sin(x)+(3/4*x^2+x+1/6)*cos(3*x)-1/
6*(2+3*x)*sin(3*x)
```

3.916.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.77

$$\int (2+3x)^2 \sin^3(x)\,dx = \frac{1}{3}\left(9\,x^2+12\,x+2\right)\cos(x)^3 - \left(9\,x^2+12\,x-10\right)\cos(x) - \frac{2}{3}\left((3\,x+2)\cos(x)^2 - 21\,x - 14\right)\sin(x)$$

input
```
integrate((2+3*x)^2*sin(x)^3,x, algorithm="fricas")
```

output
```
1/3*(9*x^2 + 12*x + 2)*cos(x)^3 - (9*x^2 + 12*x - 10)*cos(x) - 2/3*((3*x +
 2)*cos(x)^2 - 21*x - 14)*sin(x)
```

3.916.6 Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.54

$$\int (2+3x)^2 \sin^3(x)\,dx = -9x^2\sin^2(x)\cos(x) - 6x^2\cos^3(x) + 14x\sin^3(x) - 12x\sin^2(x)\cos(x) + 12x\sin(x)\cos^2(x) - 8x\cos^3(x) + \frac{28\sin^3(x)}{3} + 10\sin^2(x)\cos(x) + 8\sin(x)\cos^2(x) + \frac{32\cos^3(x)}{3}$$

input
```
integrate((2+3*x)**2*sin(x)**3,x)
```

output
```
-9*x**2*sin(x)**2*cos(x) - 6*x**2*cos(x)**3 + 14*x*sin(x)**3 - 12*x*sin(x)
**2*cos(x) + 12*x*sin(x)*cos(x)**2 - 8*x*cos(x)**3 + 28*sin(x)**3/3 + 10*s
in(x)**2*cos(x) + 8*sin(x)*cos(x)**2 + 32*cos(x)**3/3
```

3.916.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.02

$$\int (2+3x)^2 \sin^3(x)\,dx = \frac{4}{3}\cos(x)^3 + \frac{1}{12}\left(9\,x^2-2\right)\cos(3\,x) + x\cos(3\,x) - \frac{27}{4}\left(x^2-2\right)\cos(x) - 9\,x\cos(x) - \frac{1}{2}\,x\sin(3\,x) + \frac{27}{2}\,x\sin(x) - 4\cos(x) - \frac{1}{3}\sin(3\,x) + 9\sin(x)$$

input
```
integrate((2+3*x)^2*sin(x)^3,x, algorithm="maxima")
```

output
```
4/3*cos(x)^3 + 1/12*(9*x^2 - 2)*cos(3*x) + x*cos(3*x) - 27/4*(x^2 - 2)*cos
(x) - 9*x*cos(x) - 1/2*x*sin(3*x) + 27/2*x*sin(x) - 4*cos(x) - 1/3*sin(3*x
) + 9*sin(x)
```

3.916.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.78

$$\int (2+3x)^2 \sin^3(x)\,dx = \frac{1}{12}\left(9\,x^2+12\,x+2\right)\cos\left(3\,x\right)-\frac{3}{4}\left(9\,x^2+12\,x-14\right)\cos\left(x\right) - \frac{1}{6}\left(3\,x+2\right)\sin\left(3\,x\right)+\frac{9}{2}\left(3\,x+2\right)\sin\left(x\right)$$

input
```
integrate((2+3*x)^2*sin(x)^3,x, algorithm="giac")
```

output
```
1/12*(9*x^2 + 12*x + 2)*cos(3*x) - 3/4*(9*x^2 + 12*x - 14)*cos(x) - 1/6*(3
*x + 2)*sin(3*x) + 9/2*(3*x + 2)*sin(x)
```

3.916.9 Mupad [B] (verification not implemented)

Time = 26.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int (2+3x)^2 \sin^3(x)\,dx = 10\,\cos\left(x\right)+\frac{28\,\sin\left(x\right)}{3}-9\,x^2\,\cos\left(x\right)+4\,x\cos\left(x\right)^3 + \frac{2\cos\left(x\right)^3}{3}+3\,x^2\cos\left(x\right)^3-\frac{4\cos\left(x\right)^2\,\sin\left(x\right)}{3} - 12\,x\,\cos\left(x\right)+14\,x\,\sin\left(x\right)-2\,x\cos\left(x\right)^2\,\sin\left(x\right)$$

input
```
int(sin(x)^3*(3*x + 2)^2,x)
```

output
```
10*cos(x) + (28*sin(x))/3 - 9*x^2*cos(x) + 4*x*cos(x)^3 + (2*cos(x)^3)/3 +
 3*x^2*cos(x)^3 - (4*cos(x)^2*sin(x))/3 - 12*x*cos(x) + 14*x*sin(x) - 2*x*
cos(x)^2*sin(x)
```

3.917 $\int \sec^{1+m}(x)\sin(x)\,dx$

3.917.1 Optimal result

Integrand size = 9, antiderivative size = 8

$$\int \sec^{1+m}(x)\sin(x)\,dx = \frac{\sec^m(x)}{m}$$

output
```
sec(x)^m/m
```

3.917.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \sec^{1+m}(x)\sin(x)\,dx = \frac{\sec^m(x)}{m}$$

input
```
Integrate[Sec[x]^(1 + m)*Sin[x],x]
```

output
```
Sec[x]^m/m
```

3.917.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3102, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\sec^{m+1}(x)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(x)^{m+1}}{\csc(x)}dx$$

$$\Big\downarrow \text{3102}$$

$$\int \sec^{m-1}(x)d\sec(x)$$

$$\Big\downarrow \text{15}$$

$$\frac{\sec^m(x)}{m}$$

input
```
Int[Sec[x]^(1 + m)*Sin[x],x]
```

output
```
Sec[x]^m/m
```

3.917.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3102
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Simp[1/(f*a^n)   Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/
2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1
)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

3.917.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.12

$$\frac{e^{(1+m)\ln(\sec(x))}}{m\sec(x)}$$

input
```
int(sec(x)^(1+m)*sin(x),x)
```

output
```
1/m*exp((1+m)*ln(sec(x)))/sec(x)
```

3.917.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.75

$$\int \sec^{1+m}(x)\sin(x)\,dx = \frac{\frac{1}{\cos(x)}^{m+1}\cos(x)}{m}$$

input
```
integrate(sec(x)^(1+m)*sin(x),x, algorithm="fricas")
```

output
```
(1/cos(x))^(m + 1)*cos(x)/m
```

3.917.6 Sympy [F]

$$\int \sec^{1+m}(x)\sin(x)\,dx = \int \sin(x)\sec^{m+1}(x)\,dx$$

input
```
integrate(sec(x)**(1+m)*sin(x),x)
```

output
```
Integral(sin(x)*sec(x)**(m + 1), x)
```

3.917.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \sec^{1+m}(x)\sin(x)\,dx = \frac{\cos(x)^{-m}}{m}$$

input
```
integrate(sec(x)^(1+m)*sin(x),x, algorithm="maxima")
```

output
```
cos(x)^(-m)/m
```

3.917.8 Giac [F]

$$\int \sec^{1+m}(x)\sin(x)\,dx = \int \sec(x)^{m+1}\sin(x)\,dx$$

input
```
integrate(sec(x)^(1+m)*sin(x),x, algorithm="giac")
```

output
```
integrate(sec(x)^(m + 1)*sin(x), x)
```

3.917.9 Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \sec^{1+m}(x)\sin(x)\,dx = \frac{\left(\frac{1}{\cos(x)}\right)^m}{m}$$

input
```
int(sin(x)*(1/cos(x))^(m + 1),x)
```

output
```
(1/cos(x))^m/m
```

3.918 $\int \cos^n(a+bx)\sin^{-2-n}(a+bx)\,dx$

3.918.1 Optimal result

Integrand size = 21, antiderivative size = 32

$$\int \cos^n(a+bx)\sin^{-2-n}(a+bx)\,dx = -\frac{\cos^{1+n}(a+bx)\sin^{-1-n}(a+bx)}{b(1+n)}$$

output
```
-cos(b*x+a)^(1+n)*sin(b*x+a)^(-1-n)/b/(1+n)
```

3.918.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \cos^n(a+bx)\sin^{-2-n}(a+bx)\,dx = -\frac{\cos^{1+n}(a+bx)\sin^{-1-n}(a+bx)}{b(1+n)}$$

input
```
Integrate[Cos[a + b*x]^n*Sin[a + b*x]^(-2 - n),x]
```

output
```
-((Cos[a + b*x]^(1 + n)*Sin[a + b*x]^(-1 - n))/(b*(1 + n)))
```

3.918.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3042, 3043}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^{-n-2}(a+bx)\cos^n(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(a+bx)^{-n-2}\cos(a+bx)^n dx$$

$$\Big\downarrow \text{3043}$$

$$-\frac{\sin^{-n-1}(a+bx)\cos^{n+1}(a+bx)}{b(n+1)}$$

input
```
Int[Cos[a + b*x]^n*Sin[a + b*x]^(-2 - n),x]
```

output
```
-((Cos[a + b*x]^(1 + n)*Sin[a + b*x]^(-1 - n))/(b*(1 + n)))
```

3.918.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3043
```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(
m_.), x_Symbol] :> Simp[(a*Sin[e + f*x])^(m + 1)*((b*Cos[e + f*x])^(n + 1)/
(a*b*f*(m + 1))), x] /; FreeQ[{a, b, e, f, m, n}, x] && EqQ[m + n + 2, 0] &
& NeQ[m, -1]
```

3.918.4 Maple [F]

$$\int \cos(xb+a)^n \sin(xb+a)^{-2-n}\,dx$$

input
```
int(cos(b*x+a)^n*sin(b*x+a)^(-2-n),x)
```

output
```
int(cos(b*x+a)^n*sin(b*x+a)^(-2-n),x)
```

3.918.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.28

$$\int \cos^n(a+bx)\sin^{-2-n}(a+bx)\,dx = -\frac{\cos(bx+a)^n \sin(bx+a)^{-n-2}\cos(bx+a)\sin(bx+a)}{bn+b}$$

input
```
integrate(cos(b*x+a)^n*sin(b*x+a)^(-2-n),x, algorithm="fricas")
```

output
```
-cos(b*x + a)^n*sin(b*x + a)^(-n - 2)*cos(b*x + a)*sin(b*x + a)/(b*n + b)
```

3.918.6 Sympy [F]

$$\int \cos^n(a+bx)\sin^{-2-n}(a+bx)\,dx = \int \sin^{-n-2}(a+bx)\cos^n(a+bx)\,dx$$

input
```
integrate(cos(b*x+a)**n*sin(b*x+a)**(-2-n),x)
```

output
```
Integral(sin(a + b*x)**(-n - 2)*cos(a + b*x)**n, x)
```

3.918.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. 2(32) = 64.

Time = 0.30 (sec) , antiderivative size = 125, normalized size of antiderivative = 3.91

$$\int \cos^n(a+bx)\sin^{-2-n}(a+bx)\,dx = \frac{2\left(\frac{\sin(bx+a)^2}{(\cos(bx+a)+1)^2}-1\right)(\cos(bx+a)+1)e^{\left(n\log\left(\frac{\sin(bx+a)}{\cos(bx+a)+1}+1\right)-n\log\left(\frac{\sin(bx+a)}{\cos(bx+a)+1}\right)+n\log\left(-\frac{\sin(bx+a)}{\cos(bx+a)+1}+1\right)\right)}}{(2^{n+2}n+2^{n+2})b\sin(bx+a)}$$

input
```
integrate(cos(b*x+a)^n*sin(b*x+a)^(-2-n),x, algorithm="maxima")
```

output
```
2*(sin(b*x + a)^2/(cos(b*x + a) + 1)^2 - 1)*(cos(b*x + a) + 1)*e^(n*log(si
n(b*x + a)/(cos(b*x + a) + 1) + 1) - n*log(sin(b*x + a)/(cos(b*x + a) + 1)
) + n*log(-sin(b*x + a)/(cos(b*x + a) + 1) + 1))/((2^(n + 2)*n + 2^(n + 2)
)*b*sin(b*x + a))
```

3.918.8 Giac [F]

$$\int \cos^n(a+bx)\sin^{-2-n}(a+bx)\,dx = \int \cos(bx+a)^n\sin(bx+a)^{-n-2}\,dx$$

input
```
integrate(cos(b*x+a)^n*sin(b*x+a)^(-2-n),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^n*sin(b*x + a)^(-n - 2), x)
```

3.918.9 Mupad [B] (verification not implemented)

Time = 26.67 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.41

$$\int \cos^n(a+bx)\sin^{-2-n}(a+bx)\,dx = -\frac{\cos(a+b\,x)^n\,\sin(2\,a+2\,b\,x)}{2\,b\sin(a+b\,x)^n\,\sin(a+b\,x)^2\,(n+1)}$$

input
```
int(cos(a + b*x)^n/sin(a + b*x)^(n + 2),x)
```

output
```
-(cos(a + b*x)^n*sin(2*a + 2*b*x))/(2*b*sin(a + b*x)^n*sin(a + b*x)^2*(n +
 1))
```

3.919 $\int \frac{1}{\sec(x)+\sin(x)\tan(x)}\,dx$

3.919.1 Optimal result

Integrand size = 10, antiderivative size = 3

$$\int \frac{1}{\sec(x)+\sin(x)\tan(x)}\,dx = \arctan(\sin(x))$$

output
```
arctan(sin(x))
```

3.919.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sec(x)+\sin(x)\tan(x)}\,dx = \arctan(\sin(x))$$

input
```
Integrate[(Sec[x] + Sin[x]*Tan[x])^(-1),x]
```

output
```
ArcTan[Sin[x]]
```

3.919.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4897, 3042, 3669, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sec(x)+\sin(x)\tan(x)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sec(x)+\sin(x)\tan(x)}dx$$
$$\Big\downarrow \text{4897}$$
$$\int \frac{\cos(x)}{\sin^2(x)+1}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos(x)}{\sin(x)^2+1}dx$$
$$\Big\downarrow \text{3669}$$
$$\int \frac{1}{\sin^2(x)+1}d\sin(x)$$
$$\Big\downarrow \text{216}$$
$$\arctan(\sin(x))$$

input
```
Int[(Sec[x] + Sin[x]*Tan[x])^(-1),x]
```

output
```
ArcTan[Sin[x]]
```

3.919.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3669
```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f   S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]
/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

rule 4897
```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

3.919.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
derivativedivides	$\arctan\left(\sin\left(x\right)\right)$	4
default	$\arctan\left(\sin\left(x\right)\right)$	4
risch	$\frac{i\ln\left(\mathrm{e}^{2 i x}-2\,\mathrm{e}^{i x}-1\right)}{2}-\frac{i\ln\left(\mathrm{e}^{2 i x}+2\,\mathrm{e}^{i x}-1\right)}{2}$	38

input
```
int(1/(sec(x)+sin(x)*tan(x)),x,method=_RETURNVERBOSE)
```

output
```
arctan(sin(x))
```

3.919.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sec(x) + \sin(x)\tan(x)}\,dx = \arctan\left(\sin\left(x\right)\right)$$

input
```
integrate(1/(sec(x)+sin(x)*tan(x)),x, algorithm="fricas")
```

output
```
arctan(sin(x))
```

3.919.6 Sympy [F]

$$\int \frac{1}{\sec(x) + \sin(x)\tan(x)}\,dx = \int \frac{1}{\sin\left(x\right)\tan\left(x\right) + \sec\left(x\right)}\,dx$$

input
```
integrate(1/(sec(x)+sin(x)*tan(x)),x)
```

output
```
Integral(1/(sin(x)*tan(x) + sec(x)), x)
```

3.919.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(3) = 6.

Time = 0.22 (sec) , antiderivative size = 45, normalized size of antiderivative = 15.00

$$\int \frac{1}{\sec(x) + \sin(x)\tan(x)}\,dx = \frac{1}{2}\arctan\left(\sin\left(2x\right) + 2\sin\left(x\right), \cos\left(2x\right) + 2\cos\left(x\right) - 1\right) - \frac{1}{2}\arctan\left(\sin\left(2x\right) - 2\sin\left(x\right), \cos\left(2x\right) - 2\cos\left(x\right) - 1\right)$$

input
```
integrate(1/(sec(x)+sin(x)*tan(x)),x, algorithm="maxima")
```

output
```
1/2*arctan2(sin(2*x) + 2*sin(x), cos(2*x) + 2*cos(x) - 1) - 1/2*arctan2(si
n(2*x) - 2*sin(x), cos(2*x) - 2*cos(x) - 1)
```

3.919.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sec(x)+\sin(x)\tan(x)}\,dx=\arctan\left(\sin\left(x\right)\right)$$

input
```
integrate(1/(sec(x)+sin(x)*tan(x)),x, algorithm="giac")
```

output
```
arctan(sin(x))
```

3.919.9 Mupad [B] (verification not implemented)

Time = 27.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 8.67

$$\int \frac{1}{\sec(x)+\sin(x)\tan(x)}\,dx=\mathrm{atan}\left(\frac{\tan\left(\frac{x}{2}\right)^3}{2}+\frac{5\tan\left(\frac{x}{2}\right)}{2}\right)-\mathrm{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{2}\right)$$

input
```
int(1/(sin(x)*tan(x) + 1/cos(x)),x)
```

output
```
atan((5*tan(x/2))/2 + tan(x/2)^3/2) - atan(tan(x/2)/2)
```

3.920 $\int (a + bx + cx^2) \sin(x) \, dx$

3.920.1 Optimal result

Integrand size = 13, antiderivative size = 35

$$\int (a + bx + cx^2) \sin(x) \, dx = -a\cos(x) + 2c\cos(x) - bx\cos(x) - cx^2\cos(x) + b\sin(x) + 2cx\sin(x)$$

output
```
-a*cos(x)+2*c*cos(x)-b*x*cos(x)-c*x^2*cos(x)+b*sin(x)+2*c*x*sin(x)
```

3.920.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int (a+bx+cx^2) \sin(x) \, dx = -a\cos(x) - bx\cos(x) - c(-2+x^2)\cos(x) + b\sin(x) + 2cx\sin(x)$$

input
```
Integrate[(a + b*x + c*x^2)*Sin[x],x]
```

output
```
-(a*Cos[x]) - b*x*Cos[x] - c*(-2 + x^2)*Cos[x] + b*Sin[x] + 2*c*x*Sin[x]
```

3.920.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(x)\left(a+bx+cx^2\right)\,dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(a\sin(x)+bx\sin(x)+cx^2\sin(x)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-a\cos(x)+b\sin(x)-bx\cos(x)-cx^2\cos(x)+2cx\sin(x)+2c\cos(x)$$

input
```
Int[(a + b*x + c*x^2)*Sin[x],x]
```

output
```
-(a*Cos[x]) + 2*c*Cos[x] - b*x*Cos[x] - c*x^2*Cos[x] + b*Sin[x] + 2*c*x*Si
n[x]
```

3.920.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.920.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

method	result
risch	$\left(-c\,x^{2}-x b-a+2 c\right) \cos \left(x\right)+\left(2 c x+b\right) \sin \left(x\right)$
parts	$-c\,x^{2} \cos \left(x\right)-b x \cos \left(x\right)-a \cos \left(x\right)+2 c \left(\cos \left(x\right)+x \sin \left(x\right)\right)+b \sin \left(x\right)$
default	$c \left(-x^{2} \cos \left(x\right)+2 \cos \left(x\right)+2 x \sin \left(x\right)\right)+b \left(\sin \left(x\right)-x \cos \left(x\right)\right)-a \cos \left(x\right)$
parallelrisch	$\left(-c\,x^{2}-x b-a+2 c\right) \cos \left(x\right)+\left(2 c x+b\right) \sin \left(x\right)-a+2 c$
norman	$\frac{c\,x^{2} \tan \left(\frac{x}{2}\right)^{2}+x b \tan \left(\frac{x}{2}\right)^{2}+2 b \tan \left(\frac{x}{2}\right)-c\,x^{2}-x b+4 c x \tan \left(\frac{x}{2}\right)-2 a+4 c}{1+\tan \left(\frac{x}{2}\right)^{2}}$
meijerg	$4 c \sqrt{\pi}\, \left(-\frac{1}{2 \sqrt{\pi}}+\frac{\left(-\frac{x^{2}}{2}+1\right) \cos \left(x\right)}{2 \sqrt{\pi}}+\frac{x \sin \left(x\right)}{2 \sqrt{\pi}}\right)+2 b \sqrt{\pi}\, \left(-\frac{x \cos \left(x\right)}{2 \sqrt{\pi}}+\frac{\sin \left(x\right)}{2 \sqrt{\pi}}\right)+a \sqrt{\pi}\, \left(\frac{1}{\sqrt{\pi}}-\frac{\cos \left(x\right)}{\sqrt{\pi}}\right)$

input
```
int((c*x^2+b*x+a)*sin(x),x,method=_RETURNVERBOSE)
```

output
```
(-c*x^2-b*x-a+2*c)*cos(x)+(2*c*x+b)*sin(x)
```

3.920.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \left(a+bx+cx^2\right)\sin(x)\,dx=-\left(cx^{2}+bx+a-2\,c\right)\cos\left(x\right)+\left(2\,cx+b\right)\sin\left(x\right)$$

input
```
integrate((c*x^2+b*x+a)*sin(x),x, algorithm="fricas")
```

output
```
-(c*x^2 + b*x + a - 2*c)*cos(x) + (2*c*x + b)*sin(x)
```

3.920.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\begin{aligned}\int \left(a+bx+cx^2\right)\sin(x)\,dx=&-a\cos\left(x\right)-bx\cos\left(x\right)+b\sin\left(x\right)\\&-cx^{2}\cos\left(x\right)+2cx\sin\left(x\right)+2c\cos\left(x\right)\end{aligned}$$

input
```
integrate((c*x**2+b*x+a)*sin(x),x)
```

output
```
-a*cos(x) - b*x*cos(x) + b*sin(x) - c*x**2*cos(x) + 2*c*x*sin(x) + 2*c*cos
(x)
```

3.920.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int (a+bx+cx^2)\sin(x)\,dx = -(x\cos(x)-\sin(x))b - \left(\left(x^2-2\right)\cos(x)-2\,x\sin(x)\right)c - a\cos(x)$$

input
```
integrate((c*x^2+b*x+a)*sin(x),x, algorithm="maxima")
```

output
```
-(x*cos(x) - sin(x))*b - ((x^2 - 2)*cos(x) - 2*x*sin(x))*c - a*cos(x)
```

3.920.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int (a+bx+cx^2)\sin(x)\,dx = -\left(cx^2+bx+a-2\,c\right)\cos(x)+(2\,cx+b)\sin(x)$$

input
```
integrate((c*x^2+b*x+a)*sin(x),x, algorithm="giac")
```

output
```
-(c*x^2 + b*x + a - 2*c)*cos(x) + (2*c*x + b)*sin(x)
```

3.920.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int (a+bx+cx^2)\sin(x)\,dx = b\,\sin(x)-\cos(x)\,(a-2\,c)-b\,x\,\cos(x)+2\,c\,x\,\sin(x)-c\,x^2\,\cos(x)$$

input
```
int(sin(x)*(a + b*x + c*x^2),x)
```

output
```
b*sin(x) - cos(x)*(a - 2*c) - b*x*cos(x) + 2*c*x*sin(x) - c*x^2*cos(x)
```

3.921 $\int \frac{\sin(x^5)}{x} dx$

3.921.1 Optimal result

Integrand size = 8, antiderivative size = 8

$$\int \frac{\sin(x^5)}{x} dx = \frac{\text{Si}(x^5)}{5}$$

output
```
1/5*Si(x^5)
```

3.921.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\sin(x^5)}{x} dx = \frac{\text{Si}(x^5)}{5}$$

input
```
Integrate[Sin[x^5]/x,x]
```

output
```
SinIntegral[x^5]/5
```

3.921.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3856}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x^5)}{x} \, dx$$

$$\Big\downarrow \, 3856$$

$$\frac{\text{Si}(x^5)}{5}$$

input
```
Int[Sin[x^5]/x,x]
```

output
```
SinIntegral[x^5]/5
```

3.921.3.1 Defintions of rubi rules used

rule 3856
```
Int[Sin[(d_.)*(x_)^(n_)]/(x_), x_Symbol] :> Simp[SinIntegral[d*x^n]/n, x] /
; FreeQ[{d, n}, x]
```

3.921.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{\text{Si}(x^5)}{5}$	7
default	$\frac{\text{Si}(x^5)}{5}$	7
meijerg	$\frac{\text{Si}(x^5)}{5}$	7

input
```
int(sin(x^5)/x,x,method=_RETURNVERBOSE)
```

output
```
1/5*Si(x^5)
```

3.921.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\sin(x^5)}{x}\,dx = \frac{1}{5}\operatorname{Si}(x^5)$$

input
```
integrate(sin(x^5)/x,x, algorithm="fricas")
```

output
```
1/5*sin_integral(x^5)
```

3.921.6 Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \frac{\sin(x^5)}{x}\,dx = \frac{\operatorname{Si}(x^5)}{5}$$

input
```
integrate(sin(x**5)/x,x)
```

output
```
Si(x**5)/5
```

3.921.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 2.12

$$\int \frac{\sin(x^5)}{x}\,dx = -\frac{1}{10}i\operatorname{Ei}(i\,x^5) + \frac{1}{10}i\operatorname{Ei}(-i\,x^5)$$

input
```
integrate(sin(x^5)/x,x, algorithm="maxima")
```

output
```
-1/10*I*Ei(I*x^5) + 1/10*I*Ei(-I*x^5)
```

3.921.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\sin(x^5)}{x} \, dx = \frac{1}{5} \operatorname{Si}(x^5)$$

input
```
integrate(sin(x^5)/x,x, algorithm="giac")
```

output
```
1/5*sin_integral(x^5)
```

3.921.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(x^5)}{x} \, dx = \frac{\operatorname{sinint}(x^5)}{5}$$

input
```
int(sin(x^5)/x,x)
```

output
```
sinint(x^5)/5
```

3.922 $\int \frac{\sin(2^x)}{1+2^x}\,dx$

3.922.1 Optimal result

Integrand size = 12, antiderivative size = 37

$$\int \frac{\sin(2^x)}{1+2^x}\,dx=\frac{\text{CosIntegral}(1+2^x)\sin(1)}{\log(2)}+\frac{\text{Si}(2^x)}{\log(2)}-\frac{\cos(1)\text{Si}(1+2^x)}{\log(2)}$$

output
```
Si(2^x)/ln(2)-cos(1)*Si(1+2^x)/ln(2)+Ci(1+2^x)*sin(1)/ln(2)
```

3.922.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{\sin(2^x)}{1+2^x}\,dx=\frac{\text{CosIntegral}(1+2^x)\sin(1)+\text{Si}(2^x)-\cos(1)\text{Si}(1+2^x)}{\log(2)}$$

input
```
Integrate[Sin[2^x]/(1 + 2^x),x]
```

output
```
(CosIntegral[1 + 2^x]*Sin[1] + SinIntegral[2^x] - Cos[1]*SinIntegral[1 + 2
^x])/Log[2]
```

3.922.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2720, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(2^x)}{2^x+1}dx$$
$$\Big\downarrow \ 2720$$
$$\frac{\int \frac{2^{-x}\sin(2^x)}{1+2^x}d2^x}{\log(2)}$$
$$\Big\downarrow \ 7293$$
$$\frac{\int \left(2^{-x}\sin(2^x)-\frac{\sin(2^x)}{1+2^x}\right)d2^x}{\log(2)}$$
$$\Big\downarrow \ 2009$$
$$\frac{\sin(1)\text{CosIntegral}(1+2^x)+\text{Si}(2^x)-\cos(1)\text{Si}(1+2^x)}{\log(2)}$$

input
```
Int[Sin[2^x]/(1 + 2^x),x]
```

output
```
(CosIntegral[1 + 2^x]*Sin[1] + SinIntegral[2^x] - Cos[1]*SinIntegral[1 + 2
^x])/Log[2]
```

3.922.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.922.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{-\operatorname{Si}(1+2^x)\cos(1)+\operatorname{Ci}(1+2^x)\sin(1)+\operatorname{Si}(2^x)}{\ln(2)}$	30
default	$\frac{-\operatorname{Si}(1+2^x)\cos(1)+\operatorname{Ci}(1+2^x)\sin(1)+\operatorname{Si}(2^x)}{\ln(2)}$	30
risch	$-\frac{i\operatorname{Ei}_1(-i2^x-i)\mathrm{e}^{-i}}{2\ln(2)}+\frac{i\operatorname{Ei}_1(i2^x+i)\mathrm{e}^{i}}{2\ln(2)}+\frac{i\operatorname{Ei}_1(-i2^x)}{2\ln(2)}-\frac{i\operatorname{Ei}_1(i2^x)}{2\ln(2)}$	74

input
```
int(sin(2^x)/(1+2^x),x,method=_RETURNVERBOSE)
```

output
```
1/ln(2)*(-Si(1+2^x)*cos(1)+Ci(1+2^x)*sin(1)+Si(2^x))
```

3.922.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{\sin(2^x)}{1+2^x}\,dx = \frac{\operatorname{Ci}(2^x+1)\sin(1)-\cos(1)\operatorname{Si}(2^x+1)+\operatorname{Si}(2^x)}{\log(2)}$$

input
```
integrate(sin(2^x)/(1+2^x),x, algorithm="fricas")
```

output
```
(cos_integral(2^x + 1)*sin(1) - cos(1)*sin_integral(2^x + 1) + sin_integra
l(2^x))/log(2)
```

3.922.6 Sympy [F]

$$\int \frac{\sin(2^x)}{1+2^x}\,dx = \int \frac{\sin(2^x)}{2^x+1}\,dx$$

input
```
integrate(sin(2**x)/(1+2**x),x)
```

output
```
Integral(sin(2**x)/(2**x + 1), x)
```

3.922.7 Maxima [F]

$$\int \frac{\sin(2^x)}{1+2^x}\,dx = \int \frac{\sin(2^x)}{2^x+1}\,dx$$

input
```
integrate(sin(2^x)/(1+2^x),x, algorithm="maxima")
```

output
```
integrate(sin(2^x)/(2^x + 1), x)
```

3.922.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{\sin(2^x)}{1+2^x}\,dx = \frac{\mathrm{Ci}(2^x+1)\sin(1) - \cos(1)\,\mathrm{Si}(2^x+1) + \mathrm{Si}(2^x)}{\log(2)}$$

input
```
integrate(sin(2^x)/(1+2^x),x, algorithm="giac")
```

output
```
(cos_integral(2^x + 1)*sin(1) - cos(1)*sin_integral(2^x + 1) + sin_integra
l(2^x))/log(2)
```

3.922.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sin(2^x)}{1+2^x}\,dx = \int \frac{\sin(2^x)}{2^x+1}\,dx$$

input
```
int(sin(2^x)/(2^x + 1),x)
```

output
```
int(sin(2^x)/(2^x + 1), x)
```

3.923 $\int x \cos(2x^2) \sin^{\frac{3}{4}}(2x^2)\, dx$

3.923.1 Optimal result

Integrand size = 18, antiderivative size = 14

$$\int x \cos(2x^2) \sin^{\frac{3}{4}}(2x^2)\, dx = \frac{1}{7} \sin^{\frac{7}{4}}(2x^2)$$

output
```
1/7*sin(2*x^2)^(7/4)
```

3.923.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int x \cos(2x^2) \sin^{\frac{3}{4}}(2x^2)\, dx = \frac{1}{7} \sin^{\frac{7}{4}}(2x^2)$$

input
```
Integrate[x*Cos[2*x^2]*Sin[2*x^2]^(3/4),x]
```

output
```
Sin[2*x^2]^(7/4)/7
```

3.923.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {3922}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sin^{\frac{3}{4}}\left(2x^2\right)\cos\left(2x^2\right)\,dx$$

$$\Big\downarrow \text{3922}$$

$$\frac{1}{7}\sin^{\frac{7}{4}}\left(2x^2\right)$$

input
```
Int[x*Cos[2*x^2]*Sin[2*x^2]^(3/4),x]
```

output
```
Sin[2*x^2]^(7/4)/7
```

3.923.3.1 Defintions of rubi rules used

rule 3922
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[
{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```

3.923.4 Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\frac{\sin\left(2x^2\right)^{\frac{7}{4}}}{7}$	11
default	$\frac{\sin\left(2x^2\right)^{\frac{7}{4}}}{7}$	11

input
```
int(x*cos(2*x^2)*sin(2*x^2)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
1/7*sin(2*x^2)^(7/4)
```

3.923.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int x\cos\left(2x^2\right)\sin^{\frac{3}{4}}\left(2x^2\right)\,dx=\frac{1}{7}\,\sin\left(2\,x^2\right)^{\frac{7}{4}}$$

input
```
integrate(x*cos(2*x^2)*sin(2*x^2)^(3/4),x, algorithm="fricas")
```

output
```
1/7*sin(2*x^2)^(7/4)
```

3.923.6 Sympy [A] (verification not implemented)

Time = 5.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int x\cos\left(2x^2\right)\sin^{\frac{3}{4}}\left(2x^2\right)\,dx=\frac{\sin^{\frac{7}{4}}\left(2x^2\right)}{7}$$

input
```
integrate(x*cos(2*x**2)*sin(2*x**2)**(3/4),x)
```

output
```
sin(2*x**2)**(7/4)/7
```

3.923.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int x\cos\left(2x^2\right)\sin^{\frac{3}{4}}\left(2x^2\right)\,dx=\frac{1}{7}\,\sin\left(2\,x^2\right)^{\frac{7}{4}}$$

input
```
integrate(x*cos(2*x^2)*sin(2*x^2)^(3/4),x, algorithm="maxima")
```

output
```
1/7*sin(2*x^2)^(7/4)
```

3.923.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int x\cos\left(2x^2\right)\sin^{\frac{3}{4}}\left(2x^2\right)\,dx = \frac{1}{7}\sin\left(2\,x^2\right)^{\frac{7}{4}}$$

input
```
integrate(x*cos(2*x^2)*sin(2*x^2)^(3/4),x, algorithm="giac")
```

output
```
1/7*sin(2*x^2)^(7/4)
```

3.923.9 Mupad [B] (verification not implemented)

Time = 27.75 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.93

$$\int x\cos\left(2x^2\right)\sin^{\frac{3}{4}}\left(2x^2\right)\,dx = -\frac{\cos\left(2\,x^2\right)^2\sin\left(2\,x^2\right)^{7/4}\,{}_2F_1\left(\frac{1}{8},1;\ 2;\ \cos\left(2\,x^2\right)^2\right)}{8\left(\sin\left(2\,x^2\right)^2\right)^{7/8}}$$

input
```
int(x*cos(2*x^2)*sin(2*x^2)^(3/4),x)
```

output
```
-(cos(2*x^2)^2*sin(2*x^2)^(7/4)*hypergeom([1/8, 1], 2, cos(2*x^2)^2))/(8*(
sin(2*x^2)^2)^(7/8))
```

3.924 $\int x \sec^2(x^2) \tan^2(x^2) \, dx$

3.924.1 Optimal result

Integrand size = 14, antiderivative size = 10

$$\int x \sec^2(x^2) \tan^2(x^2) \, dx = \frac{1}{6} \tan^3(x^2)$$

output
```
1/6*tan(x^2)^3
```

3.924.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int x \sec^2(x^2) \tan^2(x^2) \, dx = \frac{1}{6} \tan^3(x^2)$$

input
```
Integrate[x*Sec[x^2]^2*Tan[x^2]^2,x]
```

output
```
Tan[x^2]^3/6
```

3.924.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {7237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \tan^2\left(x^2\right) \sec^2\left(x^2\right) \, dx$$

$$\Big\downarrow \text{7237}$$

$$\frac{1}{6} \tan^3\left(x^2\right)$$

input
```
Int[x*Sec[x^2]^2*Tan[x^2]^2,x]
```

output
```
Tan[x^2]^3/6
```

3.924.3.1 Defintions of rubi rules used

rule 7237
```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[q*(y^(m + 1)/(m + 1)), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

3.924.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$\frac{\tan\left(x^2\right)^3}{6}$	9
default	$\frac{\tan\left(x^2\right)^3}{6}$	9
risch	$-\frac{i\left(3\,{\rm e}^{4 i x^2}+1\right)}{3\left({\rm e}^{2 i x^2}+1\right)^3}$	26

input
```
int(x*sec(x^2)^2*tan(x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*tan(x^2)^3
```

3.924.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. 2(8) = 16.

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 2.00

$$\int x\sec^2\left(x^2\right)\tan^2\left(x^2\right)\,dx = -\frac{\left(\cos\left(x^2\right)^2-1\right)\sin\left(x^2\right)}{6\,\cos\left(x^2\right)^3}$$

input
```
integrate(x*sec(x^2)^2*tan(x^2)^2,x, algorithm="fricas")
```

output
```
-1/6*(cos(x^2)^2 - 1)*sin(x^2)/cos(x^2)^3
```

3.924.6 Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.70

$$\int x\sec^2\left(x^2\right)\tan^2\left(x^2\right)\,dx = \frac{\tan^3\left(x^2\right)}{6}$$

input
```
integrate(x*sec(x**2)**2*tan(x**2)**2,x)
```

output
```
tan(x**2)**3/6
```

3.924.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int x\sec^2\left(x^2\right)\tan^2\left(x^2\right)\,dx = \frac{1}{6}\,\tan\left(x^2\right)^3$$

input
```
integrate(x*sec(x^2)^2*tan(x^2)^2,x, algorithm="maxima")
```

output
```
1/6*tan(x^2)^3
```

3.924.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int x\sec^2\left(x^2\right)\tan^2\left(x^2\right)\,dx=\frac{1}{6}\tan\left(x^2\right)^3$$

input
```
integrate(x*sec(x^2)^2*tan(x^2)^2,x, algorithm="giac")
```

output
```
1/6*tan(x^2)^3
```

3.924.9 Mupad [B] (verification not implemented)

Time = 26.97 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.90

$$\int x\sec^2\left(x^2\right)\tan^2\left(x^2\right)\,dx=\frac{\tan(x^2)}{6\cos\left(x^2\right)^2}-\frac{\tan(x^2)}{6}$$

input
```
int((x*tan(x^2)^2)/cos(x^2)^2,x)
```

output
```
tan(x^2)/(6*cos(x^2)^2) - tan(x^2)/6
```

3.925 $\int x^2 \cos^7\left(a+bx^3\right)\sin\left(a+bx^3\right)\,dx$

3.925.1 Optimal result

Integrand size = 22, antiderivative size = 17

$$\int x^2 \cos^7\left(a+bx^3\right)\sin\left(a+bx^3\right)\,dx=-\frac{\cos^8\left(a+bx^3\right)}{24b}$$

output
```
-1/24*cos(b*x^3+a)^8/b
```

3.925.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int x^2 \cos^7\left(a+bx^3\right)\sin\left(a+bx^3\right)\,dx=-\frac{\cos^8\left(a+bx^3\right)}{24b}$$

input
```
Integrate[x^2*Cos[a + b*x^3]^7*Sin[a + b*x^3],x]
```

output
```
-1/24*Cos[a + b*x^3]^8/b
```

3.925.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {3923}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sin\left(a+bx^3\right) \cos^7\left(a+bx^3\right) \, dx$$

$$\Big\downarrow \text{3923}$$

$$-\frac{\cos^8\left(a+bx^3\right)}{24b}$$

input
```
Int[x^2*Cos[a + b*x^3]^7*Sin[a + b*x^3],x]
```

output
```
-1/24*Cos[a + b*x^3]^8/b
```

3.925.3.1 Defintions of rubi rules used

rule 3923
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> Simp[-Cos[a + b*x^n]^(p + 1)/(b*n*(p + 1)), x] /; FreeQ
[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```

3.925.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$-\frac{\cos\left(b\,x^{3}+a\right)^{8}}{24b}$	16
default	$-\frac{\cos\left(b\,x^{3}+a\right)^{8}}{24b}$	16
risch	$-\frac{\cos\left(8b\,x^{3}+8a\right)}{3072b}-\frac{\cos\left(6b\,x^{3}+6a\right)}{384b}-\frac{7\cos\left(4b\,x^{3}+4a\right)}{768b}-\frac{7\cos\left(2b\,x^{3}+2a\right)}{384b}$	66
parallelrisch	$\frac{\frac{2\tan\left(\frac{a}{2}+\frac{b\,x^{3}}{2}\right)^{14}}{3}+\frac{14\tan\left(\frac{a}{2}+\frac{b\,x^{3}}{2}\right)^{10}}{3}+\frac{14\tan\left(\frac{a}{2}+\frac{b\,x^{3}}{2}\right)^{6}}{3}+\frac{2\tan\left(\frac{a}{2}+\frac{b\,x^{3}}{2}\right)^{2}}{3}}{b\left(1+\tan\left(\frac{a}{2}+\frac{b\,x^{3}}{2}\right)^{2}\right)^{8}}$	80

input
```
int(x^2*cos(b*x^3+a)^7*sin(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/24*cos(b*x^3+a)^8/b
```

3.925.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int x^2 \cos^7\left(a+bx^3\right)\sin\left(a+bx^3\right)\,dx = -\frac{\cos\left(bx^3+a\right)^8}{24\,b}$$

input
```
integrate(x^2*cos(b*x^3+a)^7*sin(b*x^3+a),x, algorithm="fricas")
```

output
```
-1/24*cos(b*x^3 + a)^8/b
```

3.925.6 Sympy [A] (verification not implemented)

Time = 1.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int x^2 \cos^7\left(a+bx^3\right)\sin\left(a+bx^3\right)\,dx = \begin{cases} -\frac{\cos^8\left(a+bx^3\right)}{24b} & \text{for } b \neq 0 \\ \frac{x^3\sin\left(a\right)\cos^7\left(a\right)}{3} & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*cos(b*x**3+a)**7*sin(b*x**3+a),x)
```

output
```
Piecewise((-cos(a + b*x**3)**8/(24*b), Ne(b, 0)), (x**3*sin(a)*cos(a)**7/3
, True))
```

3.925.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int x^2 \cos^7(a + bx^3) \sin(a + bx^3) \, dx = -\frac{\cos(bx^3 + a)^8}{24\,b}$$

input
```
integrate(x^2*cos(b*x^3+a)^7*sin(b*x^3+a),x, algorithm="maxima")
```

output
```
-1/24*cos(b*x^3 + a)^8/b
```

3.925.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int x^2 \cos^7(a + bx^3) \sin(a + bx^3) \, dx = -\frac{\cos(bx^3 + a)^8}{24\,b}$$

input
```
integrate(x^2*cos(b*x^3+a)^7*sin(b*x^3+a),x, algorithm="giac")
```

output
```
-1/24*cos(b*x^3 + a)^8/b
```

3.925.9 Mupad [B] (verification not implemented)

Time = 26.99 (sec) , antiderivative size = 56, normalized size of antiderivative = 3.29

$$\int x^2 \cos^7(a + bx^3) \sin(a + bx^3) \, dx = -\frac{56\,\cos(2\,b\,x^3 + 2\,a) + 28\,\cos(4\,b\,x^3 + 4\,a) + 8\,\cos(6\,b\,x^3 + 6\,a) + \cos(8\,b\,x^3 + 8\,a)}{3072\,b}$$

input
```
int(x^2*cos(a + b*x^3)^7*sin(a + b*x^3),x)
```

output
```
-(56*cos(2*a + 2*b*x^3) + 28*cos(4*a + 4*b*x^3) + 8*cos(6*a + 6*b*x^3) + c
os(8*a + 8*b*x^3))/(3072*b)
```

3.926 $\int x^5 \cos^7(a+bx^3)\sin(a+bx^3)\,dx$

3.926.1 Optimal result

Integrand size = 22, antiderivative size = 129

$$\int x^5 \cos^7(a+bx^3)\sin(a+bx^3)\,dx = \frac{35x^3}{3072b} - \frac{x^3\cos^8(a+bx^3)}{24b} + \frac{35\cos(a+bx^3)\sin(a+bx^3)}{3072b^2} + \frac{35\cos^3(a+bx^3)\sin(a+bx^3)}{4608b^2} + \frac{7\cos^5(a+bx^3)\sin(a+bx^3)}{1152b^2} + \frac{\cos^7(a+bx^3)\sin(a+bx^3)}{192b^2}$$

output
```
35/3072*x^3/b-1/24*x^3*cos(b*x^3+a)^8/b+35/3072*cos(b*x^3+a)*sin(b*x^3+a)/
b^2+35/4608*cos(b*x^3+a)^3*sin(b*x^3+a)/b^2+7/1152*cos(b*x^3+a)^5*sin(b*x^
3+a)/b^2+1/192*cos(b*x^3+a)^7*sin(b*x^3+a)/b^2
```

3.926.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.93

$$\int x^5 \cos^7\left(a+bx^3\right)\sin\left(a+bx^3\right)\,dx$$
$$=\frac{-1344bx^3\cos\left(2(a+bx^3)\right)-672bx^3\cos\left(4(a+bx^3)\right)-192bx^3\cos\left(6(a+bx^3)\right)-24bx^3\cos\left(8(a+bx^3)\right)+}{73728b^2}$$

input
```
Integrate[x^5*Cos[a + b*x^3]^7*Sin[a + b*x^3],x]
```

output
```
(-1344*b*x^3*Cos[2*(a + b*x^3)] - 672*b*x^3*Cos[4*(a + b*x^3)] - 192*b*x^3
*Cos[6*(a + b*x^3)] - 24*b*x^3*Cos[8*(a + b*x^3)] + 672*Sin[2*(a + b*x^3)]
 + 168*Sin[4*(a + b*x^3)] + 32*Sin[6*(a + b*x^3)] + 3*Sin[8*(a + b*x^3)])/
(73728*b^2)
```

3.926.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.16, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3925, 3861, 3042, 3115, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\sin\left(a+bx^3\right)\cos^7\left(a+bx^3\right)\,dx$$

$$\Big\downarrow \text{3925}$$

$$\frac{\int x^2\cos^8\left(bx^3+a\right)dx}{8b}-\frac{x^3\cos^8\left(a+bx^3\right)}{24b}$$

$$\Big\downarrow \text{3861}$$

$$\frac{\int \cos^8\left(bx^3+a\right)dx^3}{24b}-\frac{x^3\cos^8\left(a+bx^3\right)}{24b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sin\left(bx^3+a+\frac{\pi}{2}\right)^8dx^3}{24b}-\frac{x^3\cos^8\left(a+bx^3\right)}{24b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{7}{8}\int \cos^6\left(bx^3+a\right)dx^3+\frac{\sin\left(a+bx^3\right)\cos^7\left(a+bx^3\right)}{8b}}{24b}-\frac{x^3\cos^8\left(a+bx^3\right)}{24b}$$

$\big\downarrow$ 3042

$$\frac{\frac{7}{8}\int \sin\left(bx^3+a+\frac{\pi}{2}\right)^6dx^3+\frac{\sin\left(a+bx^3\right)\cos^7\left(a+bx^3\right)}{8b}}{24b}-\frac{x^3\cos^8\left(a+bx^3\right)}{24b}$$

$\big\downarrow$ 3115

$$\frac{\frac{7}{8}\left(\frac{5}{6}\int \cos^4\left(bx^3+a\right)dx^3+\frac{\sin\left(a+bx^3\right)\cos^5\left(a+bx^3\right)}{6b}\right)+\frac{\sin\left(a+bx^3\right)\cos^7\left(a+bx^3\right)}{8b}}{24b}-\frac{x^3\cos^8\left(a+bx^3\right)}{24b}$$

$\big\downarrow$ 3042

$$\frac{\frac{7}{8}\left(\frac{5}{6}\int \sin\left(bx^3+a+\frac{\pi}{2}\right)^4dx^3+\frac{\sin\left(a+bx^3\right)\cos^5\left(a+bx^3\right)}{6b}\right)+\frac{\sin\left(a+bx^3\right)\cos^7\left(a+bx^3\right)}{8b}}{24b}-\frac{x^3\cos^8\left(a+bx^3\right)}{24b}$$

$\big\downarrow$ 3115

$$\frac{\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \cos^2\left(bx^3+a\right)dx^3+\frac{\sin\left(a+bx^3\right)\cos^3\left(a+bx^3\right)}{4b}\right)+\frac{\sin\left(a+bx^3\right)\cos^5\left(a+bx^3\right)}{6b}\right)+\frac{\sin\left(a+bx^3\right)\cos^7\left(a+bx^3\right)}{8b}}{24b}-\frac{x^3\cos^8\left(a+bx^3\right)}{24b}$$

$\big\downarrow$ 3042

$$\frac{\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \sin\left(bx^3+a+\frac{\pi}{2}\right)^2dx^3+\frac{\sin\left(a+bx^3\right)\cos^3\left(a+bx^3\right)}{4b}\right)+\frac{\sin\left(a+bx^3\right)\cos^5\left(a+bx^3\right)}{6b}\right)+\frac{\sin\left(a+bx^3\right)\cos^7\left(a+bx^3\right)}{8b}}{24b}-\frac{x^3\cos^8\left(a+bx^3\right)}{24b}$$

$\big\downarrow$ 3115

$$\frac{\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx^3}{2}+\frac{\sin\left(a+bx^3\right)\cos\left(a+bx^3\right)}{2b}\right)+\frac{\sin\left(a+bx^3\right)\cos^3\left(a+bx^3\right)}{4b}\right)+\frac{\sin\left(a+bx^3\right)\cos^5\left(a+bx^3\right)}{6b}\right)+\frac{\sin\left(a+bx^3\right)\cos^7\left(a+bx^3\right)}{8b}}{24b}-\frac{x^3\cos^8\left(a+bx^3\right)}{24b}$$

$\big\downarrow$ 24

$$\frac{\frac{\sin\left(a+bx^3\right)\cos^7\left(a+bx^3\right)}{8b}+\frac{7}{8}\left(\frac{\sin\left(a+bx^3\right)\cos^5\left(a+bx^3\right)}{6b}+\frac{5}{6}\left(\frac{\sin\left(a+bx^3\right)\cos^3\left(a+bx^3\right)}{4b}+\frac{3}{4}\left(\frac{\sin\left(a+bx^3\right)\cos\left(a+bx^3\right)}{2b}+\frac{x^3}{2}\right)\right)\right)}{24b}-\frac{x^3\cos^8\left(a+bx^3\right)}{24b}$$

3.926. $\int x^5\cos^7\left(a+bx^3\right)\sin\left(a+bx^3\right)\,dx$

input
```
Int[x^5*Cos[a + b*x^3]^7*Sin[a + b*x^3],x]
```

output
```
-1/24*(x^3*Cos[a + b*x^3]^8)/b + ((Cos[a + b*x^3]^7*Sin[a + b*x^3])/(8*b)
+ (7*((Cos[a + b*x^3]^5*Sin[a + b*x^3])/(6*b) + (5*((Cos[a + b*x^3]^3*Sin[
a + b*x^3])/(4*b) + (3*(x^3/2 + (Cos[a + b*x^3]*Sin[a + b*x^3])/(2*b)))/4)
)/6))/8)/(24*b)
```

3.926.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3861
```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

rule 3925
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> Simp[(-x^(m - n + 1))*(Cos[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] + Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Cos[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

3.926.4 Maple [A] (verified)

Time = 1.99 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.98

method	result
parallelrisch	$\frac{-24x^{3}\cos(8b\,x^{3}+8a)b-192x^{3}\cos(6b\,x^{3}+6a)b-672x^{3}\cos(4b\,x^{3}+4a)b-1344\cos(2b\,x^{3}+2a)x^{3}b+3\sin(8b\,x^{3}+8a)+32\sin(6b\,x}{73728b^{2}}$
risch	$-\frac{x^{3}\cos(8b\,x^{3}+8a)}{3072b}+\frac{\sin(8b\,x^{3}+8a)}{24576b^{2}}-\frac{x^{3}\cos(6b\,x^{3}+6a)}{384b}+\frac{\sin(6b\,x^{3}+6a)}{2304b^{2}}-\frac{7x^{3}\cos(4b\,x^{3}+4a)}{768b}+\frac{7\sin(4b\,x^{3}+4a)}{3072b^{2}}$
default	$\frac{-\frac{x^{3}}{24b}+\frac{\tan(4b\,x^{3}+4a)}{96b^{2}}+\frac{x^{3}\tan(4b\,x^{3}+4a)^{2}}{24b}}{128+128\tan(4b\,x^{3}+4a)^{2}}+\frac{-\frac{7x^{3}}{3b}+\frac{7\tan(b\,x^{3}+a)}{3b^{2}}+\frac{7x^{3}\tan(b\,x^{3}+a)^{2}}{3b}}{128+128\tan(b\,x^{3}+a)^{2}}+\frac{-\frac{7x^{3}}{6b}+\frac{7\tan(2b\,x^{3}+2a)}{12b^{2}}+\frac{7x^{3}\tan(2b}{6}}{128+128\tan(2b\,x^{3}+2a)^{2}}$

input
```
int(x^5*cos(b*x^3+a)^7*sin(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/73728*(-24*x^3*cos(8*b*x^3+8*a)*b-192*x^3*cos(6*b*x^3+6*a)*b-672*x^3*cos
(4*b*x^3+4*a)*b-1344*cos(2*b*x^3+2*a)*x^3*b+3*sin(8*b*x^3+8*a)+32*sin(6*b*
x^3+6*a)+168*sin(4*b*x^3+4*a)+672*sin(2*b*x^3+2*a))/b^2
```

3.926.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.66

$$\int x^{5}\cos^{7}\left(a+bx^{3}\right)\sin\left(a+bx^{3}\right)\,dx=$$
$$-\frac{384\,bx^{3}\cos\left(bx^{3}+a\right)^{8}-105\,bx^{3}-\left(48\,\cos\left(bx^{3}+a\right)^{7}+56\,\cos\left(bx^{3}+a\right)^{5}+70\,\cos\left(bx^{3}+a\right)^{3}+105\,\cos\right.}{9216\,b^{2}}$$

input
```
integrate(x^5*cos(b*x^3+a)^7*sin(b*x^3+a),x, algorithm="fricas")
```

output
```
-1/9216*(384*b*x^3*cos(b*x^3 + a)^8 - 105*b*x^3 - (48*cos(b*x^3 + a)^7 + 5
6*cos(b*x^3 + a)^5 + 70*cos(b*x^3 + a)^3 + 105*cos(b*x^3 + a))*sin(b*x^3 +
 a))/b^2
```

3.926.6 Sympy [A] (verification not implemented)

Time = 3.41 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.87

$$\int x^5\cos^7\left(a+bx^3\right)\sin\left(a+bx^3\right)\,dx$$
$$=\begin{cases}\frac{35x^3\sin^8(a+bx^3)}{3072b}+\frac{35x^3\sin^6(a+bx^3)\cos^2(a+bx^3)}{768b}+\frac{35x^3\sin^4(a+bx^3)\cos^4(a+bx^3)}{512b}+\frac{35x^3\sin^2(a+bx^3)\cos^6(a+bx^3)}{768b}-\frac{31x^3\cos}{10}\\ \frac{x^6\sin(a)\cos^7(a)}{6}\end{cases}$$

input
```
integrate(x**5*cos(b*x**3+a)**7*sin(b*x**3+a),x)
```

output
```
Piecewise((35*x**3*sin(a + b*x**3)**8/(3072*b) + 35*x**3*sin(a + b*x**3)**
6*cos(a + b*x**3)**2/(768*b) + 35*x**3*sin(a + b*x**3)**4*cos(a + b*x**3)*
*4/(512*b) + 35*x**3*sin(a + b*x**3)**2*cos(a + b*x**3)**6/(768*b) - 31*x*
*3*cos(a + b*x**3)**8/(1024*b) + 35*sin(a + b*x**3)**7*cos(a + b*x**3)/(30
72*b**2) + 385*sin(a + b*x**3)**5*cos(a + b*x**3)**3/(9216*b**2) + 511*sin
(a + b*x**3)**3*cos(a + b*x**3)**5/(9216*b**2) + 31*sin(a + b*x**3)*cos(a
+ b*x**3)**7/(1024*b**2), Ne(b, 0)), (x**6*sin(a)*cos(a)**7/6, True))
```

3.926.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.98

$$\int x^5\cos^7\left(a+bx^3\right)\sin\left(a+bx^3\right)\,dx =$$
$$-\frac{24\,bx^3\cos\left(8\,bx^3+8\,a\right)+192\,bx^3\cos\left(6\,bx^3+6\,a\right)+672\,bx^3\cos\left(4\,bx^3+4\,a\right)+1344\,bx^3\cos\left(2\,bx^3+2\,a\right.}{73728\,b^2}$$

input
```
integrate(x^5*cos(b*x^3+a)^7*sin(b*x^3+a),x, algorithm="maxima")
```

output
```
-1/73728*(24*b*x^3*cos(8*b*x^3 + 8*a) + 192*b*x^3*cos(6*b*x^3 + 6*a) + 672
*b*x^3*cos(4*b*x^3 + 4*a) + 1344*b*x^3*cos(2*b*x^3 + 2*a) - 3*sin(8*b*x^3
+ 8*a) - 32*sin(6*b*x^3 + 6*a) - 168*sin(4*b*x^3 + 4*a) - 672*sin(2*b*x^3
+ 2*a))/b^2
```

3.926.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.20

$$\int x^5 \cos^7(a + bx^3) \sin(a + bx^3)\, dx = \frac{a\cos(bx^3 + a)^8}{24\, b^2} - \frac{24\,(bx^3 + a)\cos(8\,bx^3 + 8\,a) + 192\,(bx^3 + a)\cos(6\,bx^3 + 6\,a) + 672\,(bx^3 + a)\cos(4\,bx^3 + 4\,a) + 1344\,(}{}$$

input
```
integrate(x^5*cos(b*x^3+a)^7*sin(b*x^3+a),x, algorithm="giac")
```

output
```
1/24*a*cos(b*x^3 + a)^8/b^2 - 1/73728*(24*(b*x^3 + a)*cos(8*b*x^3 + 8*a) +
 192*(b*x^3 + a)*cos(6*b*x^3 + 6*a) + 672*(b*x^3 + a)*cos(4*b*x^3 + 4*a) +
 1344*(b*x^3 + a)*cos(2*b*x^3 + 2*a) - 3*sin(8*b*x^3 + 8*a) - 32*sin(6*b*x
^3 + 6*a) - 168*sin(4*b*x^3 + 4*a) - 672*sin(2*b*x^3 + 2*a))/b^2
```

3.926.9 Mupad [B] (verification not implemented)

Time = 27.28 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.14

$$\int x^5 \cos^7(a + bx^3) \sin(a + bx^3)\, dx$$

$$= \frac{168\,\sin(2\,b\,x^3 + 2\,a) + 42\,\sin(4\,b\,x^3 + 4\,a) + 8\,\sin(6\,b\,x^3 + 6\,a) + \frac{3\,\sin(8\,b\,x^3 + 8\,a)}{4} + 336\,b\,x^3\,\Big(2\sin(b\,x^3 + a}{}$$

input
```
int(x^5*cos(a + b*x^3)^7*sin(a + b*x^3),x)
```

output
```
(168*sin(2*a + 2*b*x^3) + 42*sin(4*a + 4*b*x^3) + 8*sin(6*a + 6*b*x^3) + (
3*sin(8*a + 8*b*x^3))/4 + 336*b*x^3*(2*sin(a + b*x^3)^2 - 1) + 168*b*x^3*(
2*sin(2*a + 2*b*x^3)^2 - 1) + 48*b*x^3*(2*sin(3*a + 3*b*x^3)^2 - 1) + 6*b*
x^3*(2*sin(4*a + 4*b*x^3)^2 - 1))/(18432*b^2)
```

3.927 $\int x^5 \sec^7(a+bx^3)\tan(a+bx^3)\,dx$

3.927.1 Optimal result

Integrand size = 22, antiderivative size = 110

$$\int x^5 \sec^7(a+bx^3)\tan(a+bx^3)\,dx = -\frac{5\text{arctanh}(\sin(a+bx^3))}{336b^2}+\frac{x^3\sec^7(a+bx^3)}{21b}-\frac{5\sec(a+bx^3)\tan(a+bx^3)}{336b^2}-\frac{5\sec^3(a+bx^3)\tan(a+bx^3)}{504b^2}-\frac{\sec^5(a+bx^3)\tan(a+bx^3)}{126b^2}$$

output
```
-5/336*arctanh(sin(b*x^3+a))/b^2+1/21*x^3*sec(b*x^3+a)^7/b-5/336*sec(b*x^3
+a)*tan(b*x^3+a)/b^2-5/504*sec(b*x^3+a)^3*tan(b*x^3+a)/b^2-1/126*sec(b*x^3
+a)^5*tan(b*x^3+a)/b^2
```

3.927.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 352 vs. 2(110) = 220.

Time = 0.79 (sec) , antiderivative size = 352, normalized size of antiderivative = 3.20

$$\int x^5 \sec^7(a+bx^3)\tan(a+bx^3)\,dx$$
$$= \frac{\sec^7(a+bx^3)\left(3072bx^3+105\cos(5(a+bx^3))\log\left(\cos\left(\frac{1}{2}(a+bx^3)\right)-\sin\left(\frac{1}{2}(a+bx^3)\right)\right)+15\cos(7(a+b\ldots\right.}{}$$

input
```
Integrate[x^5*Sec[a + b*x^3]^7*Tan[a + b*x^3],x]
```

output
```
(Sec[a + b*x^3]^7*(3072*b*x^3 + 105*Cos[5*(a + b*x^3)]*Log[Cos[(a + b*x^3)
/2] - Sin[(a + b*x^3)/2]] + 15*Cos[7*(a + b*x^3)]*Log[Cos[(a + b*x^3)/2] -
 Sin[(a + b*x^3)/2]] + 525*Cos[a + b*x^3]*(Log[Cos[(a + b*x^3)/2] - Sin[(a
 + b*x^3)/2]] - Log[Cos[(a + b*x^3)/2] + Sin[(a + b*x^3)/2]]) + 315*Cos[3*
(a + b*x^3)]*(Log[Cos[(a + b*x^3)/2] - Sin[(a + b*x^3)/2]] - Log[Cos[(a +
b*x^3)/2] + Sin[(a + b*x^3)/2]]) - 105*Cos[5*(a + b*x^3)]*Log[Cos[(a + b*x
^3)/2] + Sin[(a + b*x^3)/2]] - 15*Cos[7*(a + b*x^3)]*Log[Cos[(a + b*x^3)/2
] + Sin[(a + b*x^3)/2]] - 566*Sin[2*(a + b*x^3)] - 200*Sin[4*(a + b*x^3)]
- 30*Sin[6*(a + b*x^3)]))/(64512*b^2)
```

3.927.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.16, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {4244, 4692, 3042, 4255, 3042, 4255, 3042, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \tan \left(a+bx^3\right) \sec ^7\left(a+bx^3\right) \, dx$$

$$\Big\downarrow \text{4244}$$

$$\frac{x^3 \sec ^7\left(a+bx^3\right)}{21 b}-\frac{\int x^2 \sec ^7\left(bx^3+a\right)dx}{7 b}$$

$$\Big\downarrow \text{4692}$$

$$\frac{x^3 \sec ^7\left(a+bx^3\right)}{21 b}-\frac{\int \sec ^7\left(bx^3+a\right)dx^3}{21 b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x^3 \sec ^7\left(a+bx^3\right)}{21 b}-\frac{\int \csc \left(bx^3+a+\frac{\pi }{2}\right)^7dx^3}{21 b}$$

$$\Big\downarrow \text{4255}$$

$$\frac{x^3 \sec ^7\left(a+bx^3\right)}{21 b}-\frac{\frac{5}{6} \int \sec ^5\left(bx^3+a\right)dx^3+\frac{\tan \left(a+bx^3\right) \sec ^5\left(a+bx^3\right)}{6 b}}{21 b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x^3 \sec^7\left(a+bx^3\right)}{21b} - \frac{\frac{5}{6}\int \csc\left(bx^3+a+\frac{\pi}{2}\right)^5 dx^3 + \frac{\tan(a+bx^3)\sec^5(a+bx^3)}{6b}}{21b}$$

$$\Big\downarrow \text{4255}$$

$$\frac{x^3 \sec^7\left(a+bx^3\right)}{21b} - \frac{\frac{5}{6}\left(\frac{3}{4}\int \sec^3\left(bx^3+a\right) dx^3 + \frac{\tan(a+bx^3)\sec^3(a+bx^3)}{4b}\right) + \frac{\tan(a+bx^3)\sec^5(a+bx^3)}{6b}}{21b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x^3 \sec^7\left(a+bx^3\right)}{21b} - \frac{\frac{5}{6}\left(\frac{3}{4}\int \csc\left(bx^3+a+\frac{\pi}{2}\right)^3 dx^3 + \frac{\tan(a+bx^3)\sec^3(a+bx^3)}{4b}\right) + \frac{\tan(a+bx^3)\sec^5(a+bx^3)}{6b}}{21b}$$

$$\Big\downarrow \text{4255}$$

$$\frac{x^3 \sec^7\left(a+bx^3\right)}{21b} - \frac{\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \sec\left(bx^3+a\right) dx^3 + \frac{\tan(a+bx^3)\sec(a+bx^3)}{2b}\right) + \frac{\tan(a+bx^3)\sec^3(a+bx^3)}{4b}\right) + \frac{\tan(a+bx^3)\sec^5(a+bx^3)}{6b}}{21b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x^3 \sec^7\left(a+bx^3\right)}{21b} - \frac{\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \csc\left(bx^3+a+\frac{\pi}{2}\right) dx^3 + \frac{\tan(a+bx^3)\sec(a+bx^3)}{2b}\right) + \frac{\tan(a+bx^3)\sec^3(a+bx^3)}{4b}\right) + \frac{\tan(a+bx^3)\sec^5(a+bx^3)}{6b}}{21b}$$

$$\Big\downarrow \text{4257}$$

$$\frac{x^3 \sec^7\left(a+bx^3\right)}{21b} - \frac{\frac{5}{6}\left(\frac{3}{4}\left(\frac{\text{arctanh}(\sin(a+bx^3))}{2b} + \frac{\tan(a+bx^3)\sec(a+bx^3)}{2b}\right) + \frac{\tan(a+bx^3)\sec^3(a+bx^3)}{4b}\right) + \frac{\tan(a+bx^3)\sec^5(a+bx^3)}{6b}}{21b}$$

input
```
Int[x^5*Sec[a + b*x^3]^7*Tan[a + b*x^3],x]
```

output
```
(x^3*Sec[a + b*x^3]^7)/(21*b) - ((Sec[a + b*x^3]^5*Tan[a + b*x^3])/(6*b) +
 (5*((Sec[a + b*x^3]^3*Tan[a + b*x^3])/(4*b) + (3*(ArcTanh[Sin[a + b*x^3]]
/(2*b) + (Sec[a + b*x^3]*Tan[a + b*x^3])/(2*b)))/4))/6)/(21*b)
```

3.927. $\int x^5 \sec^7\left(a+bx^3\right) \tan\left(a+bx^3\right) dx$

3.927.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4244
```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tan[(a_.) + (b_.)*(x_)^(
n_.)]^(q_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sec[a + b*x^n]^p/(b*n*p)), x]
 - Simp[(m - n + 1)/(b*n*p)   Int[x^(m - n)*Sec[a + b*x^n]^p, x], x] /; Fre
eQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m, n] && EqQ[q, 1]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4692
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
 1)/n], 0] && IntegerQ[p]
```

3.927.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 52.38 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.45

method	result
risch	$\frac{i\left(15\,\mathrm{e}^{13i\left(b\,x^{3}+a\right)}-3072ib\,x^{3}\mathrm{e}^{7i\left(b\,x^{3}+a\right)}+100\,\mathrm{e}^{11i\left(b\,x^{3}+a\right)}+283\,\mathrm{e}^{9i\left(b\,x^{3}+a\right)}-283\,\mathrm{e}^{5i\left(b\,x^{3}+a\right)}-100\,\mathrm{e}^{3i\left(b\,x^{3}+a\right)}-15\,\mathrm{e}^{i\left(b\,x^{3}+a\right)}\right)}{504b^{2}\left(\mathrm{e}^{2i\left(b\,x^{3}+a\right)}+1\right)^{7}}+$

input
```
int(x^5*sec(b*x^3+a)^7*tan(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/504*I/b^2/(exp(2*I*(b*x^3+a))+1)^7*(15*exp(13*I*(b*x^3+a))-3072*I*b*x^3*
exp(7*I*(b*x^3+a))+100*exp(11*I*(b*x^3+a))+283*exp(9*I*(b*x^3+a))-283*exp(
5*I*(b*x^3+a))-100*exp(3*I*(b*x^3+a))-15*exp(I*(b*x^3+a)))+5/336/b^2*ln(ex
p(I*(b*x^3+a))-I)-5/336/b^2*ln(exp(I*(b*x^3+a))+I)
```

3.927.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.05

$$\int x^5 \sec^7\left(a+bx^3\right) \tan\left(a+bx^3\right) \, dx = -\frac{15 \cos\left(bx^3+a\right)^7 \log\left(\sin\left(bx^3+a\right)+1\right) - 15 \cos\left(bx^3+a\right)^7 \log\left(-\sin\left(bx^3+a\right)+1\right) - 96\, bx^3 + 2\left(15 \cos\right.}{2016\, b^2 \cos\left(bx^3+a\right)^7}$$

input
```
integrate(x^5*sec(b*x^3+a)^7*tan(b*x^3+a),x, algorithm="fricas")
```

output
```
-1/2016*(15*cos(b*x^3 + a)^7*log(sin(b*x^3 + a) + 1) - 15*cos(b*x^3 + a)^7
*log(-sin(b*x^3 + a) + 1) - 96*b*x^3 + 2*(15*cos(b*x^3 + a)^5 + 10*cos(b*x
^3 + a)^3 + 8*cos(b*x^3 + a))*sin(b*x^3 + a))/(b^2*cos(b*x^3 + a)^7)
```

3.927.6 Sympy [F]

$$\int x^5 \sec^7\left(a+bx^3\right) \tan\left(a+bx^3\right) \, dx = \int x^5 \tan\left(a+bx^3\right) \sec^7\left(a+bx^3\right) \, dx$$

input
```
integrate(x**5*sec(b*x**3+a)**7*tan(b*x**3+a),x)
```

output
```
Integral(x**5*tan(a + b*x**3)*sec(a + b*x**3)**7, x)
```

3.927.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3830 vs. $2(100) = 200$.

Time = 0.46 (sec) , antiderivative size = 3830, normalized size of antiderivative = 34.82

$$\int x^5 \sec^7\left(a+bx^3\right)\tan\left(a+bx^3\right)\,dx = \text{Too large to display}$$

input
```
integrate(x^5*sec(b*x^3+a)^7*tan(b*x^3+a),x, algorithm="maxima")
```

output
```
1/2016*(4*(3072*b*x^3*cos(7*b*x^3 + 7*a) - 15*sin(13*b*x^3 + 13*a) - 100*s
in(11*b*x^3 + 11*a) - 283*sin(9*b*x^3 + 9*a) + 283*sin(5*b*x^3 + 5*a) + 10
0*sin(3*b*x^3 + 3*a) + 15*sin(b*x^3 + a))*cos(14*b*x^3 + 14*a) + 420*(sin(
12*b*x^3 + 12*a) + 3*sin(10*b*x^3 + 10*a) + 5*sin(8*b*x^3 + 8*a) + 5*sin(6
*b*x^3 + 6*a) + 3*sin(4*b*x^3 + 4*a) + sin(2*b*x^3 + 2*a))*cos(13*b*x^3 +
13*a) + 28*(3072*b*x^3*cos(7*b*x^3 + 7*a) - 100*sin(11*b*x^3 + 11*a) - 283
*sin(9*b*x^3 + 9*a) + 283*sin(5*b*x^3 + 5*a) + 100*sin(3*b*x^3 + 3*a) + 15
*sin(b*x^3 + a))*cos(12*b*x^3 + 12*a) + 2800*(3*sin(10*b*x^3 + 10*a) + 5*s
in(8*b*x^3 + 8*a) + 5*sin(6*b*x^3 + 6*a) + 3*sin(4*b*x^3 + 4*a) + sin(2*b*
x^3 + 2*a))*cos(11*b*x^3 + 11*a) + 84*(3072*b*x^3*cos(7*b*x^3 + 7*a) - 283
*sin(9*b*x^3 + 9*a) + 283*sin(5*b*x^3 + 5*a) + 100*sin(3*b*x^3 + 3*a) + 15
*sin(b*x^3 + a))*cos(10*b*x^3 + 10*a) + 7924*(5*sin(8*b*x^3 + 8*a) + 5*sin
(6*b*x^3 + 6*a) + 3*sin(4*b*x^3 + 4*a) + sin(2*b*x^3 + 2*a))*cos(9*b*x^3 +
 9*a) + 140*(3072*b*x^3*cos(7*b*x^3 + 7*a) + 283*sin(5*b*x^3 + 5*a) + 100*
sin(3*b*x^3 + 3*a) + 15*sin(b*x^3 + a))*cos(8*b*x^3 + 8*a) + 12288*(35*b*x
^3*cos(6*b*x^3 + 6*a) + 21*b*x^3*cos(4*b*x^3 + 4*a) + 7*b*x^3*cos(2*b*x^3
+ 2*a) + b*x^3)*cos(7*b*x^3 + 7*a) + 140*(283*sin(5*b*x^3 + 5*a) + 100*sin
(3*b*x^3 + 3*a) + 15*sin(b*x^3 + a))*cos(6*b*x^3 + 6*a) - 7924*(3*sin(4*b*
x^3 + 4*a) + sin(2*b*x^3 + 2*a))*cos(5*b*x^3 + 5*a) + 420*(20*sin(3*b*x^3
+ 3*a) + 3*sin(b*x^3 + a))*cos(4*b*x^3 + 4*a) + 15*(2*(7*cos(12*b*x^3 +...
```

3.927.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1363 vs. $2(100) = 200$.

Time = 0.85 (sec) , antiderivative size = 1363, normalized size of antiderivative = 12.39

$$\int x^5 \sec^7\left(a+bx^3\right)\tan\left(a+bx^3\right)\,dx = \text{Too large to display}$$

input
```
integrate(x^5*sec(b*x^3+a)^7*tan(b*x^3+a),x, algorithm="giac")
```

output
```
-1/2016*(96*(b*x^3 + a)*tan(1/2*b*x^3 + 1/2*a)^14 + 15*log(2*(tan(1/2*b*x^
3 + 1/2*a)^2 + 2*tan(1/2*b*x^3 + 1/2*a) + 1)/(tan(1/2*b*x^3 + 1/2*a)^2 + 1
))*tan(1/2*b*x^3 + 1/2*a)^14 - 15*log(2*(tan(1/2*b*x^3 + 1/2*a)^2 - 2*tan(
1/2*b*x^3 + 1/2*a) + 1)/(tan(1/2*b*x^3 + 1/2*a)^2 + 1))*tan(1/2*b*x^3 + 1/
2*a)^14 + 672*(b*x^3 + a)*tan(1/2*b*x^3 + 1/2*a)^12 - 105*log(2*(tan(1/2*b
*x^3 + 1/2*a)^2 + 2*tan(1/2*b*x^3 + 1/2*a) + 1)/(tan(1/2*b*x^3 + 1/2*a)^2
+ 1))*tan(1/2*b*x^3 + 1/2*a)^12 + 105*log(2*(tan(1/2*b*x^3 + 1/2*a)^2 - 2*
tan(1/2*b*x^3 + 1/2*a) + 1)/(tan(1/2*b*x^3 + 1/2*a)^2 + 1))*tan(1/2*b*x^3
+ 1/2*a)^12 + 132*tan(1/2*b*x^3 + 1/2*a)^13 + 2016*(b*x^3 + a)*tan(1/2*b*x
^3 + 1/2*a)^10 + 315*log(2*(tan(1/2*b*x^3 + 1/2*a)^2 + 2*tan(1/2*b*x^3 + 1
/2*a) + 1)/(tan(1/2*b*x^3 + 1/2*a)^2 + 1))*tan(1/2*b*x^3 + 1/2*a)^10 - 315
*log(2*(tan(1/2*b*x^3 + 1/2*a)^2 - 2*tan(1/2*b*x^3 + 1/2*a) + 1)/(tan(1/2*
b*x^3 + 1/2*a)^2 + 1))*tan(1/2*b*x^3 + 1/2*a)^10 - 112*tan(1/2*b*x^3 + 1/2
*a)^11 + 3360*(b*x^3 + a)*tan(1/2*b*x^3 + 1/2*a)^8 - 525*log(2*(tan(1/2*b*
x^3 + 1/2*a)^2 + 2*tan(1/2*b*x^3 + 1/2*a) + 1)/(tan(1/2*b*x^3 + 1/2*a)^2 +
 1))*tan(1/2*b*x^3 + 1/2*a)^8 + 525*log(2*(tan(1/2*b*x^3 + 1/2*a)^2 - 2*ta
n(1/2*b*x^3 + 1/2*a) + 1)/(tan(1/2*b*x^3 + 1/2*a)^2 + 1))*tan(1/2*b*x^3 +
1/2*a)^8 + 340*tan(1/2*b*x^3 + 1/2*a)^9 + 3360*(b*x^3 + a)*tan(1/2*b*x^3 +
 1/2*a)^6 + 525*log(2*(tan(1/2*b*x^3 + 1/2*a)^2 + 2*tan(1/2*b*x^3 + 1/2*a)
 + 1)/(tan(1/2*b*x^3 + 1/2*a)^2 + 1))*tan(1/2*b*x^3 + 1/2*a)^6 - 525*lo...
```

3.927. $\int x^5 \sec^7(a+bx^3)\tan(a+bx^3)\,dx$

3.927.9 Mupad [B] (verification not implemented)

Time = 43.08 (sec) , antiderivative size = 730, normalized size of antiderivative = 6.64

$$\int x^5 \sec^7\left(a+bx^3\right)\tan\left(a+bx^3\right)\,dx$$

$$= -\frac{\frac{8\,\mathrm{e}^{1\mathrm{i}\,b\,x^3+a\,1\mathrm{i}}\,\left(15\,b\,x^3-8\mathrm{i}\right)}{315\,b^2}-\frac{8\,\mathrm{e}^{3\mathrm{i}\,b\,x^3+a\,3\mathrm{i}}\,\left(35\,b\,x^3-12\mathrm{i}\right)}{315\,b^2}}{5\,\mathrm{e}^{2\mathrm{i}\,b\,x^3+a\,2\mathrm{i}}+10\,\mathrm{e}^{4\mathrm{i}\,b\,x^3+a\,4\mathrm{i}}+10\,\mathrm{e}^{6\mathrm{i}\,b\,x^3+a\,6\mathrm{i}}+5\,\mathrm{e}^{8\mathrm{i}\,b\,x^3+a\,8\mathrm{i}}+\mathrm{e}^{10\mathrm{i}\,b\,x^3+a\,10\mathrm{i}}+1}$$
$$+\frac{5\,\ln\left(x^2\,\left(\mathrm{e}^{1\mathrm{i}\,b\,x^3+a\,1\mathrm{i}}-\mathrm{i}\right)\right)}{336\,b^2}-\frac{5\,\ln\left(x^2\,\left(\mathrm{e}^{1\mathrm{i}\,b\,x^3+a\,1\mathrm{i}}+1\mathrm{i}\right)\right)}{336\,b^2}$$
$$-\frac{\frac{16\,\mathrm{e}^{3\mathrm{i}\,b\,x^3+a\,3\mathrm{i}}\,\left(5\,b\,x^3-\mathrm{i}\right)}{63\,b^2}-\frac{16\,\mathrm{e}^{5\mathrm{i}\,b\,x^3+a\,5\mathrm{i}}\,\left(7\,b\,x^3-\mathrm{i}\right)}{63\,b^2}}{6\,\mathrm{e}^{2\mathrm{i}\,b\,x^3+a\,2\mathrm{i}}+15\,\mathrm{e}^{4\mathrm{i}\,b\,x^3+a\,4\mathrm{i}}+20\,\mathrm{e}^{6\mathrm{i}\,b\,x^3+a\,6\mathrm{i}}+15\,\mathrm{e}^{8\mathrm{i}\,b\,x^3+a\,8\mathrm{i}}+6\,\mathrm{e}^{10\mathrm{i}\,b\,x^3+a\,10\mathrm{i}}+\mathrm{e}^{12\mathrm{i}\,b\,x^3+a\,12\mathrm{i}}+1}$$
$$-\frac{\frac{64\,x^3\,\mathrm{e}^{5\mathrm{i}\,b\,x^3+a\,5\mathrm{i}}}{21\,b}-\frac{64\,x^3\,\mathrm{e}^{7\mathrm{i}\,b\,x^3+a\,7\mathrm{i}}}{21\,b}}{7\,\mathrm{e}^{2\mathrm{i}\,b\,x^3+a\,2\mathrm{i}}+21\,\mathrm{e}^{4\mathrm{i}\,b\,x^3+a\,4\mathrm{i}}+35\,\mathrm{e}^{6\mathrm{i}\,b\,x^3+a\,6\mathrm{i}}+35\,\mathrm{e}^{8\mathrm{i}\,b\,x^3+a\,8\mathrm{i}}+21\,\mathrm{e}^{10\mathrm{i}\,b\,x^3+a\,10\mathrm{i}}+7\,\mathrm{e}^{12\mathrm{i}\,b\,x^3+a\,12\mathrm{i}}+\mathrm{e}^{14\mathrm{i}\,b\,x^3+a\,14\mathrm{i}}}$$
$$+\frac{\mathrm{e}^{1\mathrm{i}\,b\,x^3+a\,1\mathrm{i}}\,1\mathrm{i}}{63\,b^2\,\left(3\,\mathrm{e}^{2\mathrm{i}\,b\,x^3+a\,2\mathrm{i}}+3\,\mathrm{e}^{4\mathrm{i}\,b\,x^3+a\,4\mathrm{i}}+\mathrm{e}^{6\mathrm{i}\,b\,x^3+a\,6\mathrm{i}}+1\right)}$$
$$+\frac{\mathrm{e}^{1\mathrm{i}\,b\,x^3+a\,1\mathrm{i}}\,5\mathrm{i}}{168\,b^2\,\left(\mathrm{e}^{2\mathrm{i}\,b\,x^3+a\,2\mathrm{i}}+1\right)}+\frac{\mathrm{e}^{1\mathrm{i}\,b\,x^3+a\,1\mathrm{i}}\,5\mathrm{i}}{252\,b^2\,\left(2\,\mathrm{e}^{2\mathrm{i}\,b\,x^3+a\,2\mathrm{i}}+\mathrm{e}^{4\mathrm{i}\,b\,x^3+a\,4\mathrm{i}}+1\right)}$$
$$+\frac{2\,\mathrm{e}^{1\mathrm{i}\,b\,x^3+a\,1\mathrm{i}}\,\left(60\,b\,x^3-47\mathrm{i}\right)}{315\,b^2\,\left(4\,\mathrm{e}^{2\mathrm{i}\,b\,x^3+a\,2\mathrm{i}}+6\,\mathrm{e}^{4\mathrm{i}\,b\,x^3+a\,4\mathrm{i}}+4\,\mathrm{e}^{6\mathrm{i}\,b\,x^3+a\,6\mathrm{i}}+\mathrm{e}^{8\mathrm{i}\,b\,x^3+a\,8\mathrm{i}}+1\right)}$$

input
```
int((x^5*tan(a + b*x^3))/cos(a + b*x^3)^7,x)
```

output
```
(5*log(x^2*(exp(a*1i + b*x^3*1i) - 1i)))/(336*b^2) - ((8*exp(a*1i + b*x^3*
1i)*(15*b*x^3 - 8i))/(315*b^2) - (8*exp(a*3i + b*x^3*3i)*(35*b*x^3 - 12i))
/(315*b^2))/(5*exp(a*2i + b*x^3*2i) + 10*exp(a*4i + b*x^3*4i) + 10*exp(a*6
i + b*x^3*6i) + 5*exp(a*8i + b*x^3*8i) + exp(a*10i + b*x^3*10i) + 1) - (5*
log(x^2*(exp(a*1i + b*x^3*1i) + 1i)))/(336*b^2) - ((16*exp(a*3i + b*x^3*3i
)*(5*b*x^3 - 1i))/(63*b^2) - (16*exp(a*5i + b*x^3*5i)*(7*b*x^3 - 1i))/(63*
b^2))/(6*exp(a*2i + b*x^3*2i) + 15*exp(a*4i + b*x^3*4i) + 20*exp(a*6i + b*
x^3*6i) + 15*exp(a*8i + b*x^3*8i) + 6*exp(a*10i + b*x^3*10i) + exp(a*12i +
 b*x^3*12i) + 1) - ((64*x^3*exp(a*5i + b*x^3*5i))/(21*b) - (64*x^3*exp(a*7
i + b*x^3*7i))/(21*b))/(7*exp(a*2i + b*x^3*2i) + 21*exp(a*4i + b*x^3*4i) +
 35*exp(a*6i + b*x^3*6i) + 35*exp(a*8i + b*x^3*8i) + 21*exp(a*10i + b*x^3*
10i) + 7*exp(a*12i + b*x^3*12i) + exp(a*14i + b*x^3*14i) + 1) + (exp(a*1i
+ b*x^3*1i)*1i)/(63*b^2*(3*exp(a*2i + b*x^3*2i) + 3*exp(a*4i + b*x^3*4i) +
 exp(a*6i + b*x^3*6i) + 1)) + (exp(a*1i + b*x^3*1i)*5i)/(168*b^2*(exp(a*2i
 + b*x^3*2i) + 1)) + (exp(a*1i + b*x^3*1i)*5i)/(252*b^2*(2*exp(a*2i + b*x^
3*2i) + exp(a*4i + b*x^3*4i) + 1)) + (2*exp(a*1i + b*x^3*1i)*(60*b*x^3 - 4
7i))/(315*b^2*(4*exp(a*2i + b*x^3*2i) + 6*exp(a*4i + b*x^3*4i) + 4*exp(a*6
i + b*x^3*6i) + exp(a*8i + b*x^3*8i) + 1))
```

3.927. $\int x^5 \sec^7(a+bx^3) \tan(a+bx^3)\,dx$

3.928 $\int \frac{\sec^2\left(\frac{1}{x}\right)}{x^2}\,dx$

3.928.1 Optimal result

Integrand size = 10, antiderivative size = 6

$$\int \frac{\sec^2\left(\frac{1}{x}\right)}{x^2}\,dx = -\tan\left(\frac{1}{x}\right)$$

output
```
-tan(1/x)
```

3.928.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2\left(\frac{1}{x}\right)}{x^2}\,dx = -\tan\left(\frac{1}{x}\right)$$

input
```
Integrate[Sec[x^(-1)]^2/x^2,x]
```

output
```
-Tan[x^(-1)]
```

3.928.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4692, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2\left(\frac{1}{x}\right)}{x^2} \, dx$$

$$\Big\downarrow \; 4692$$

$$-\int \sec^2\left(\frac{1}{x}\right)d\frac{1}{x}$$

$$\Big\downarrow \; 3042$$

$$-\int \csc\left(\frac{\pi}{2}+\frac{1}{x}\right)^2d\frac{1}{x}$$

$$\Big\downarrow \; 4254$$

$$\int 1d\left(-\tan\left(\frac{1}{x}\right)\right)$$

$$\Big\downarrow \; 24$$

$$-\tan\left(\frac{1}{x}\right)$$

input
```
Int[Sec[x^(-1)]^2/x^2,x]
```

output
```
-Tan[x^(-1)]
```

3.928.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4692
```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sec[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sec[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IGtQ[Simplify[(m +
 1)/n], 0] && IntegerQ[p]
```

3.928.4 Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

method	result	size
derivativedivides	$-\tan\left(\frac{1}{x}\right)$	7
default	$-\tan\left(\frac{1}{x}\right)$	7
risch	$-\frac{2i}{e^{\frac{2i}{x}}+1}$	15
norman	$\frac{2\tan\left(\frac{1}{2x}\right)}{\tan\left(\frac{1}{2x}\right)^{2}-1}$	21
parallelrisch	$\frac{2\tan\left(\frac{1}{2x}\right)}{\tan\left(\frac{1}{2x}\right)^{2}-1}$	21

input
```
int(sec(1/x)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-tan(1/x)
```

3.928.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 2.00

$$\int \frac{\sec^2\left(\frac{1}{x}\right)}{x^2}\,dx = -\frac{\sin\left(\frac{1}{x}\right)}{\cos\left(\frac{1}{x}\right)}$$

input
```
integrate(sec(1/x)^2/x^2,x, algorithm="fricas")
```

output
```
-sin(1/x)/cos(1/x)
```

3.928.6 Sympy [F]

$$\int \frac{\sec^2\left(\frac{1}{x}\right)}{x^2}\,dx = \int \frac{\sec^2\left(\frac{1}{x}\right)}{x^2}\,dx$$

input
```
integrate(sec(1/x)**2/x**2,x)
```

output
```
Integral(sec(1/x)**2/x**2, x)
```

3.928.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(6) = 12$.

Time $= 0.20$ (sec) , antiderivative size $= 36$, normalized size of antiderivative $= 6.00$

$$\int \frac{\sec^2\left(\frac{1}{x}\right)}{x^2}\,dx = -\frac{2\sin\left(\frac{2}{x}\right)}{\cos\left(\frac{2}{x}\right)^2 + \sin\left(\frac{2}{x}\right)^2 + 2\cos\left(\frac{2}{x}\right) + 1}$$

input
```
integrate(sec(1/x)^2/x^2,x, algorithm="maxima")
```

output
```
-2*sin(2/x)/(cos(2/x)^2 + sin(2/x)^2 + 2*cos(2/x) + 1)
```

3.928.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(6) = 12$.

Time $= 0.26$ (sec) , antiderivative size $= 20$, normalized size of antiderivative $= 3.33$

$$\int \frac{\sec^2\left(\frac{1}{x}\right)}{x^2}\,dx = \frac{2\tan\left(\frac{1}{2x}\right)}{\tan\left(\frac{1}{2x}\right)^2 - 1}$$

input
```
integrate(sec(1/x)^2/x^2,x, algorithm="giac")
```

output
```
2*tan(1/2/x)/(tan(1/2/x)^2 - 1)
```

3.928.9 Mupad [B] (verification not implemented)

Time = 26.92 (sec) , antiderivative size = 14, normalized size of antiderivative = 2.33

$$\int \frac{\sec^2\left(\frac{1}{x}\right)}{x^2}\,dx = -\frac{2i}{e^{\frac{2i}{x}}+1}$$

input
```
int(1/(x^2*cos(1/x)^2),x)
```

output
```
-2i/(exp(2i/x) + 1)
```

3.929 $\int 3x^2 \cos(x^3) \, dx$

3.929.1 Optimal result

Integrand size = 9, antiderivative size = 4

$$\int 3x^2 \cos(x^3) \, dx = \sin(x^3)$$

output
```
sin(x^3)
```

3.929.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int 3x^2 \cos(x^3) \, dx = \sin(x^3)$$

input
```
Integrate[3*x^2*Cos[x^3],x]
```

output
```
Sin[x^3]
```

3.929.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {27, 3861, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int 3x^2 \cos\left(x^3\right) dx$$
$$\Big\downarrow \ 27$$
$$3 \int x^2 \cos\left(x^3\right) dx$$
$$\Big\downarrow \ 3861$$
$$\int \cos\left(x^3\right) dx^3$$
$$\Big\downarrow \ 3042$$
$$\int \sin\left(x^3 + \frac{\pi}{2}\right) dx^3$$
$$\Big\downarrow \ 3117$$
$$\sin\left(x^3\right)$$

input
```
Int[3*x^2*Cos[x^3],x]
```

output
```
Sin[x^3]
```

3.929.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3861
```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^
p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[
(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[
(m + 1)/n], 0]))
```

3.929.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.25

method	result	size
derivativedivides	$\sin\left(x^3\right)$	5
default	$\sin\left(x^3\right)$	5
meijerg	$\sin\left(x^3\right)$	5
risch	$\sin\left(x^3\right)$	5
parallelrisch	$\sin\left(x^3\right)$	5
norman	$\frac{2\tan\left(\frac{x^3}{2}\right)}{1+\tan\left(\frac{x^3}{2}\right)^2}$	21

input
```
int(3*x^2*cos(x^3),x,method=_RETURNVERBOSE)
```

output
```
sin(x^3)
```

3.929.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int 3x^2 \cos\left(x^3\right) \, dx = \sin\left(x^3\right)$$

input
```
integrate(3*x^2*cos(x^3),x, algorithm="fricas")
```

output
```
sin(x^3)
```

3.929.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.75

$$\int 3x^2\cos\left(x^3\right)\,dx = \sin\left(x^3\right)$$

input
```
integrate(3*x**2*cos(x**3),x)
```

output
```
sin(x**3)
```

3.929.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int 3x^2\cos\left(x^3\right)\,dx = \sin\left(x^3\right)$$

input
```
integrate(3*x^2*cos(x^3),x, algorithm="maxima")
```

output
```
sin(x^3)
```

3.929.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int 3x^2\cos\left(x^3\right)\,dx = \sin\left(x^3\right)$$

input
```
integrate(3*x^2*cos(x^3),x, algorithm="giac")
```

output
```
sin(x^3)
```

3.929.9 Mupad [B] (verification not implemented)

Time = 27.68 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.00

$$\int 3x^2 \cos\left(x^3\right)\,dx = \sin\left(x^3\right)$$

input
```
int(3*x^2*cos(x^3),x)
```

output
```
sin(x^3)
```

3.930 $\int (1+2x)\sec^2(1+2x)\,dx$

3.930.1 Optimal result

Integrand size = 14, antiderivative size = 27

$$\int (1+2x)\sec^2(1+2x)\,dx=\frac{1}{2}\log(\cos(1+2x))+\frac{1}{2}(1+2x)\tan(1+2x)$$

output
```
1/2*ln(cos(1+2*x))+1/2*(1+2*x)*tan(1+2*x)
```

3.930.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

$$\int (1+2x)\sec^2(1+2x)\,dx=\frac{1}{2}\log(\cos(1+2x))+\frac{1}{2}\tan(1+2x)+x\tan(1+2x)$$

input
```
Integrate[(1 + 2*x)*Sec[1 + 2*x]^2,x]
```

output
```
Log[Cos[1 + 2*x]]/2 + Tan[1 + 2*x]/2 + x*Tan[1 + 2*x]
```

3.930.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3042, 4672, 25, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2x+1)\sec^2(2x+1)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (2x+1)\csc\left(2x+\frac{\pi}{2}+1\right)^2dx$$

$$\Big\downarrow \text{4672}$$

$$\int -\tan(2x+1)dx+\frac{1}{2}(2x+1)\tan(2x+1)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}(2x+1)\tan(2x+1)-\int \tan(2x+1)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}(2x+1)\tan(2x+1)-\int \tan(2x+1)dx$$

$$\Big\downarrow \text{3956}$$

$$\frac{1}{2}(2x+1)\tan(2x+1)+\frac{1}{2}\log(\cos(2x+1))$$

input
```
Int[(1 + 2*x)*Sec[1 + 2*x]^2,x]
```

output
```
Log[Cos[1 + 2*x]]/2 + ((1 + 2*x)*Tan[1 + 2*x])/2
```

3.930.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

3.930.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

method	result	s
derivativedivides	$\frac{\ln(\cos(1+2x))}{2}+\frac{(1+2x)\tan(1+2x)}{2}$	2
default	$\frac{\ln(\cos(1+2x))}{2}+\frac{(1+2x)\tan(1+2x)}{2}$	2
risch	$-2ix-i+\frac{i(1+2x)}{e^{2i(1+2x)}+1}+\frac{\ln(e^{2i(1+2x)}+1)}{2}$	4
norman	$\frac{-2x\tan(x+\frac{1}{2})-\tan(x+\frac{1}{2})}{\tan(x+\frac{1}{2})^2-1}+\frac{\ln(\tan(x+\frac{1}{2})-1)}{2}+\frac{\ln(\tan(x+\frac{1}{2})+1)}{2}-\frac{\ln\left(1+\tan(x+\frac{1}{2})^2\right)}{2}$	5
parallelrisch	$\frac{\ln(\tan(x+\frac{1}{2})-1)\cos(1+2x)+\ln(\tan(x+\frac{1}{2})+1)\cos(1+2x)-\ln\left(\sec(x+\frac{1}{2})^2\right)\cos(1+2x)+2x\sin(1+2x)+\sin(1+2x)}{2\cos(1+2x)}$	7

input
```
int((1+2*x)*sec(1+2*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(cos(1+2*x))+1/2*(1+2*x)*tan(1+2*x)
```

3.930.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.44

$$\int (1+2x)\sec^2(1+2x)\,dx = \frac{\cos(2x+1)\log(-\cos(2x+1)) + (2x+1)\sin(2x+1)}{2\cos(2x+1)}$$

input
```
integrate((1+2*x)*sec(1+2*x)^2,x, algorithm="fricas")
```

output
```
1/2*(cos(2*x + 1)*log(-cos(2*x + 1)) + (2*x + 1)*sin(2*x + 1))/cos(2*x + 1
)
```

3.930.6 Sympy [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int (1+2x)\sec^2(1+2x)\,dx = \frac{(2x+1)\tan(2x+1)}{2} + \frac{\log(\cos(2x+1))}{2}$$

input
```
integrate((1+2*x)*sec(1+2*x)**2,x)
```

output
```
(2*x + 1)*tan(2*x + 1)/2 + log(cos(2*x + 1))/2
```

3.930.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. 2(23) = 46.

Time = 0.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 3.63

$$\int (1+2x)\sec^2(1+2x)\,dx$$
$$= \frac{(\cos(4x+2)^2 + \sin(4x+2)^2 + 2\cos(4x+2) + 1)\log(\cos(4x+2)^2 + \sin(4x+2)^2 + 2\cos(4x+2)}{4(\cos(4x+2)^2 + \sin(4x+2)^2 + 2\cos(4x+2) + 1)}$$

input
```
integrate((1+2*x)*sec(1+2*x)^2,x, algorithm="maxima")
```

output
```
1/4*((cos(4*x + 2)^2 + sin(4*x + 2)^2 + 2*cos(4*x + 2) + 1)*log(cos(4*x +
2)^2 + sin(4*x + 2)^2 + 2*cos(4*x + 2) + 1) + 4*(2*x + 1)*sin(4*x + 2))/(c
os(4*x + 2)^2 + sin(4*x + 2)^2 + 2*cos(4*x + 2) + 1)
```

3.930.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 908 vs. $2(23) = 46$.

Time = 0.40 (sec) , antiderivative size = 908, normalized size of antiderivative = 33.63

$$\int (1 + 2x)\sec^2(1 + 2x)\,dx = \text{Too large to display}$$

input
```
integrate((1+2*x)*sec(1+2*x)^2,x, algorithm="giac")
```

output
```
1/4*(log(4*(tan(1/2)^4*tan(x)^4 - 2*tan(1/2)^4*tan(x)^2 - 8*tan(1/2)^3*tan
(x)^3 - 2*tan(1/2)^2*tan(x)^4 + tan(1/2)^4 + 8*tan(1/2)^3*tan(x) + 20*tan(
1/2)^2*tan(x)^2 + 8*tan(1/2)*tan(x)^3 + tan(x)^4 - 2*tan(1/2)^2 - 8*tan(1/
2)*tan(x) - 2*tan(x)^2 + 1)/(tan(1/2)^4*tan(x)^4 + 2*tan(1/2)^4*tan(x)^2 +
 2*tan(1/2)^2*tan(x)^4 + tan(1/2)^4 + 4*tan(1/2)^2*tan(x)^2 + tan(x)^4 + 2
*tan(1/2)^2 + 2*tan(x)^2 + 1))*tan(1/2)^2*tan(x)^2 - 8*x*tan(1/2)^2*tan(x)
 - 8*x*tan(1/2)*tan(x)^2 - log(4*(tan(1/2)^4*tan(x)^4 - 2*tan(1/2)^4*tan(x
)^2 - 8*tan(1/2)^3*tan(x)^3 - 2*tan(1/2)^2*tan(x)^4 + tan(1/2)^4 + 8*tan(1
/2)^3*tan(x) + 20*tan(1/2)^2*tan(x)^2 + 8*tan(1/2)*tan(x)^3 + tan(x)^4 - 2
*tan(1/2)^2 - 8*tan(1/2)*tan(x) - 2*tan(x)^2 + 1)/(tan(1/2)^4*tan(x)^4 + 2
*tan(1/2)^4*tan(x)^2 + 2*tan(1/2)^2*tan(x)^4 + tan(1/2)^4 + 4*tan(1/2)^2*t
an(x)^2 + tan(x)^4 + 2*tan(1/2)^2 + 2*tan(x)^2 + 1))*tan(1/2)^2 - 4*log(4*
(tan(1/2)^4*tan(x)^4 - 2*tan(1/2)^4*tan(x)^2 - 8*tan(1/2)^3*tan(x)^3 - 2*t
an(1/2)^2*tan(x)^4 + tan(1/2)^4 + 8*tan(1/2)^3*tan(x) + 20*tan(1/2)^2*tan(
x)^2 + 8*tan(1/2)*tan(x)^3 + tan(x)^4 - 2*tan(1/2)^2 - 8*tan(1/2)*tan(x) -
 2*tan(x)^2 + 1)/(tan(1/2)^4*tan(x)^4 + 2*tan(1/2)^4*tan(x)^2 + 2*tan(1/2)
^2*tan(x)^4 + tan(1/2)^4 + 4*tan(1/2)^2*tan(x)^2 + tan(x)^4 + 2*tan(1/2)^2
 + 2*tan(x)^2 + 1))*tan(1/2)*tan(x) - 4*tan(1/2)^2*tan(x) - log(4*(tan(1/2
)^4*tan(x)^4 - 2*tan(1/2)^4*tan(x)^2 - 8*tan(1/2)^3*tan(x)^3 - 2*tan(1/2)^
2*tan(x)^4 + tan(1/2)^4 + 8*tan(1/2)^3*tan(x) + 20*tan(1/2)^2*tan(x)^2 ...
```

3.930.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int (1 + 2x)\sec^2(1 + 2x)\,dx = \frac{\ln\left(\cos\left(2\,x+1\right)\right)}{2} + \frac{\tan(2\,x+1)\,\left(2\,x+1\right)}{2}$$

input
```
int((2*x + 1)/cos(2*x + 1)^2,x)
```

output
```
log(cos(2*x + 1))/2 + (tan(2*x + 1)*(2*x + 1))/2
```

3.931 $\int \left(\frac{x^4}{b\sqrt{x^3+3\sin(a+bx)}}+\frac{x^2\cos(a+bx)}{\sqrt{x^3+3\sin(a+bx)}}+\frac{4x\sqrt{x^3+3\sin(a+bx)}}{3b}\right.$

3.931.1 Optimal result

Integrand size = 74, antiderivative size = 26

$$\int \left(\frac{x^4}{b\sqrt{x^3+3\sin(a+bx)}}+\frac{x^2\cos(a+bx)}{\sqrt{x^3+3\sin(a+bx)}}+\frac{4x\sqrt{x^3+3\sin(a+bx)}}{3b}\right)\,dx = \frac{2x^2\sqrt{x^3+3\sin(a+bx)}}{3b}$$

output
```
2/3*x^2*(x^3+3*sin(b*x+a))^(1/2)/b
```

3.931.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \left(\frac{x^4}{b\sqrt{x^3+3\sin(a+bx)}}+\frac{x^2\cos(a+bx)}{\sqrt{x^3+3\sin(a+bx)}}+\frac{4x\sqrt{x^3+3\sin(a+bx)}}{3b}\right)\,dx = \frac{2x^2\sqrt{x^3+3\sin(a+bx)}}{3b}$$

input
```
Integrate[x^4/(b*Sqrt[x^3 + 3*Sin[a + b*x]]) + (x^2*Cos[a + b*x])/Sqrt[x^3
 + 3*Sin[a + b*x]] + (4*x*Sqrt[x^3 + 3*Sin[a + b*x]])/(3*b),x]
```

output
```
(2*x^2*Sqrt[x^3 + 3*Sin[a + b*x]])/(3*b)
```

3.931.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{4x\sqrt{3\sin(a+bx)+x^3}}{3b}+\frac{x^4}{b\sqrt{3\sin(a+bx)+x^3}}+\frac{x^2\cos(a+bx)}{\sqrt{3\sin(a+bx)+x^3}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{4\int x\sqrt{x^3+3\sin(a+bx)}dx}{3b}+\frac{\int \frac{x^4}{\sqrt{x^3+3\sin(a+bx)}}dx}{b}+\int \frac{x^2\cos(a+bx)}{\sqrt{x^3+3\sin(a+bx)}}dx$$

input
```
Int[x^4/(b*Sqrt[x^3 + 3*Sin[a + b*x]]) + (x^2*Cos[a + b*x])/Sqrt[x^3 + 3*S
in[a + b*x]] + (4*x*Sqrt[x^3 + 3*Sin[a + b*x]])/(3*b),x]
```

output
```
$Aborted
```

3.931.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.931.4 Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

method	result	size
risch	$\frac{\sqrt{2x^3+6\sin(xb+a)}\,\sqrt{2}\,x^2}{3b}$	28

input
```
int(x^4/b/(x^3+3*sin(b*x+a))^(1/2)+x^2*cos(b*x+a)/(x^3+3*sin(b*x+a))^(1/2)
+4/3*x*(x^3+3*sin(b*x+a))^(1/2)/b,x,method=_RETURNVERBOSE)
```

output
```
1/3*(2*x^3+6*sin(b*x+a))^(1/2)/b*2^(1/2)*x^2
```

3.931.5 Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{x^4}{b\sqrt{x^3+3\sin(a+bx)}}+\frac{x^2\cos(a+bx)}{\sqrt{x^3+3\sin(a+bx)}}+\frac{4x\sqrt{x^3+3\sin(a+bx)}}{3b}\right)dx$$
$$= \text{Exception raised: TypeError}$$

input
```
integrate(x^4/b/(x^3+3*sin(b*x+a))^(1/2)+x^2*cos(b*x+a)/(x^3+3*sin(b*x+a))
^(1/2)+4/3*x*(x^3+3*sin(b*x+a))^(1/2)/b,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.931.6 Sympy [F]

$$\int \left(\frac{x^4}{b\sqrt{x^3+3\sin(a+bx)}}+\frac{x^2\cos(a+bx)}{\sqrt{x^3+3\sin(a+bx)}}+\frac{4x\sqrt{x^3+3\sin(a+bx)}}{3b}\right)dx$$
$$= \frac{\int \frac{7x^4}{\sqrt{x^3+3\sin(a+bx)}}\,dx+\int \frac{12x\sin(a+bx)}{\sqrt{x^3+3\sin(a+bx)}}\,dx+\int \frac{3bx^2\cos(a+bx)}{\sqrt{x^3+3\sin(a+bx)}}\,dx}{3b}$$

input
```
integrate(x**4/b/(x**3+3*sin(b*x+a))**(1/2)+x**2*cos(b*x+a)/(x**3+3*sin(b*
x+a))**(1/2)+4/3*x*(x**3+3*sin(b*x+a))**(1/2)/b,x)
```

output
```
(Integral(7*x**4/sqrt(x**3 + 3*sin(a + b*x)), x) + Integral(12*x*sin(a + b
*x)/sqrt(x**3 + 3*sin(a + b*x)), x) + Integral(3*b*x**2*cos(a + b*x)/sqrt(
x**3 + 3*sin(a + b*x)), x))/(3*b)
```

3.931.7 Maxima [F]

$$\int \left(\frac{x^4}{b\sqrt{x^3+3\sin(a+bx)}}+\frac{x^2\cos(a+bx)}{\sqrt{x^3+3\sin(a+bx)}}+\frac{4x\sqrt{x^3+3\sin(a+bx)}}{3b}\right)dx$$
$$= \int \frac{x^4}{\sqrt{x^3+3\sin(bx+a)}\,b}+\frac{x^2\cos(bx+a)}{\sqrt{x^3+3\sin(bx+a)}}+\frac{4\sqrt{x^3+3\sin(bx+a)}\,x}{3\,b}\,dx$$

input
```
integrate(x^4/b/(x^3+3*sin(b*x+a))^(1/2)+x^2*cos(b*x+a)/(x^3+3*sin(b*x+a))
^(1/2)+4/3*x*(x^3+3*sin(b*x+a))^(1/2)/b,x, algorithm="maxima")
```

output
```
integrate(x^4/(sqrt(x^3 + 3*sin(b*x + a))*b) + x^2*cos(b*x + a)/sqrt(x^3 +
 3*sin(b*x + a)) + 4/3*sqrt(x^3 + 3*sin(b*x + a))*x/b, x)
```

3.931.8 Giac [F]

$$\int\left(\frac{x^4}{b\sqrt{x^3+3\sin(a+bx)}}+\frac{x^2\cos(a+bx)}{\sqrt{x^3+3\sin(a+bx)}}+\frac{4x\sqrt{x^3+3\sin(a+bx)}}{3b}\right)dx$$
$$=\int\frac{x^4}{\sqrt{x^3+3\sin(bx+a)}b}+\frac{x^2\cos(bx+a)}{\sqrt{x^3+3\sin(bx+a)}}+\frac{4\sqrt{x^3+3\sin(bx+a)}x}{3b}\,dx$$

input
```
integrate(x^4/b/(x^3+3*sin(b*x+a))^(1/2)+x^2*cos(b*x+a)/(x^3+3*sin(b*x+a))
^(1/2)+4/3*x*(x^3+3*sin(b*x+a))^(1/2)/b,x, algorithm="giac")
```

output
```
integrate(x^4/(sqrt(x^3 + 3*sin(b*x + a))*b) + x^2*cos(b*x + a)/sqrt(x^3 +
 3*sin(b*x + a)) + 4/3*sqrt(x^3 + 3*sin(b*x + a))*x/b, x)
```

3.931.9 Mupad [B] (verification not implemented)

Time = 28.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int\left(\frac{x^4}{b\sqrt{x^3+3\sin(a+bx)}}+\frac{x^2\cos(a+bx)}{\sqrt{x^3+3\sin(a+bx)}}+\frac{4x\sqrt{x^3+3\sin(a+bx)}}{3b}\right)dx$$
$$=\frac{2x^2\sqrt{3\sin(a+bx)+x^3}}{3b}$$

input
```
int(x^4/(b*(3*sin(a + b*x) + x^3)^(1/2)) + (x^2*cos(a + b*x))/(3*sin(a + b
*x) + x^3)^(1/2) + (4*x*(3*sin(a + b*x) + x^3)^(1/2))/(3*b),x)
```

output
```
(2*x^2*(3*sin(a + b*x) + x^3)^(1/2))/(3*b)
```

3.932 $\int \frac{x^2 \cos(a+bx)}{\sqrt{x^3+3 \sin(a+bx)}} \, dx$

3.932.1 Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^2 \cos(a+bx)}{\sqrt{x^3+3 \sin(a+bx)}} \, dx = \text{Int}\left(\frac{x^2 \cos(a+bx)}{\sqrt{x^3+3 \sin(a+bx)}}, x\right)$$

output
```
CannotIntegrate(x^2*cos(b*x+a)/(x^3+3*sin(b*x+a))^(1/2),x)
```

3.932.2 Mathematica [N/A]

Not integrable

Time = 5.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^2 \cos(a+bx)}{\sqrt{x^3+3 \sin(a+bx)}} \, dx = \int \frac{x^2 \cos(a+bx)}{\sqrt{x^3+3 \sin(a+bx)}} \, dx$$

input
```
Integrate[(x^2*Cos[a + b*x])/Sqrt[x^3 + 3*Sin[a + b*x]],x]
```

output
```
Integrate[(x^2*Cos[a + b*x])/Sqrt[x^3 + 3*Sin[a + b*x]], x]
```

3.932.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \cos(a+bx)}{\sqrt{3\sin(a+bx)+x^3}}\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \frac{x^2 \cos(a+bx)}{\sqrt{3\sin(a+bx)+x^3}}dx$$

input
```
Int[(x^2*Cos[a + b*x])/Sqrt[x^3 + 3*Sin[a + b*x]],x]
```

output
```
$Aborted
```

3.932.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.932.4 Maple [N/A] (verified)

Not integrable

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^2 \cos(xb+a)}{\sqrt{x^3+3\sin(xb+a)}}dx$$

input
```
int(x^2*cos(b*x+a)/(x^3+3*sin(b*x+a))^(1/2),x)
```

output
```
int(x^2*cos(b*x+a)/(x^3+3*sin(b*x+a))^(1/2),x)
```

3.932.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2 \cos(a+bx)}{\sqrt{x^3+3\sin(a+bx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*cos(b*x+a)/(x^3+3*sin(b*x+a))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

3.932.6 Sympy [N/A]

Not integrable

Time = 1.88 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x^2 \cos(a+bx)}{\sqrt{x^3+3\sin(a+bx)}}\,dx = \int \frac{x^2 \cos{(a+bx)}}{\sqrt{x^3+3\sin{(a+bx)}}}\,dx$$

input
```
integrate(x**2*cos(b*x+a)/(x**3+3*sin(b*x+a))**(1/2),x)
```

output
```
Integral(x**2*cos(a + b*x)/sqrt(x**3 + 3*sin(a + b*x)), x)
```

3.932.7 Maxima [N/A]

Not integrable

Time = 1.41 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x^2 \cos(a+bx)}{\sqrt{x^3+3\sin(a+bx)}}\,dx = \int \frac{x^2 \cos{(bx+a)}}{\sqrt{x^3+3\,\sin{(bx+a)}}}\,dx$$

input
```
integrate(x^2*cos(b*x+a)/(x^3+3*sin(b*x+a))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2*cos(b*x + a)/sqrt(x^3 + 3*sin(b*x + a)), x)
```

3.932.8 Giac [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x^2\cos(a+bx)}{\sqrt{x^3+3\sin(a+bx)}}\,dx=\int \frac{x^2\cos\left(bx+a\right)}{\sqrt{x^3+3\,\sin\left(bx+a\right)}}\,dx$$

input
```
integrate(x^2*cos(b*x+a)/(x^3+3*sin(b*x+a))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2*cos(b*x + a)/sqrt(x^3 + 3*sin(b*x + a)), x)
```

3.932.9 Mupad [N/A]

Not integrable

Time = 27.61 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x^2\cos(a+bx)}{\sqrt{x^3+3\sin(a+bx)}}\,dx=\int \frac{x^2\,\cos\left(a+b\,x\right)}{\sqrt{3\,\sin\left(a+b\,x\right)+x^3}}\,dx$$

input
```
int((x^2*cos(a + b*x))/(3*sin(a + b*x) + x^3)^(1/2),x)
```

output
```
int((x^2*cos(a + b*x))/(3*sin(a + b*x) + x^3)^(1/2), x)
```

3.933 $\int \frac{\cos(x)+\sin(x)}{e^{-x}+\sin(x)} \, dx$

3.933.1 Optimal result

Integrand size = 16, antiderivative size = 9

$$\int \frac{\cos(x)+\sin(x)}{e^{-x}+\sin(x)} \, dx = \log\left(1+e^x\sin(x)\right)$$

output
```
ln(1+exp(x)*sin(x))
```

3.933.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)+\sin(x)}{e^{-x}+\sin(x)} \, dx = \log\left(1+e^x\sin(x)\right)$$

input
```
Integrate[(Cos[x] + Sin[x])/(E^(-x) + Sin[x]),x]
```

output
```
Log[1 + E^x*Sin[x]]
```

3.933.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)+\cos(x)}{e^{-x}+\sin(x)}\,dx$$
$$\Big\downarrow \text{ 7293}$$
$$\int \left(\cot(x)-\frac{(\cot(x)+1)\csc(x)}{e^x+\csc(x)}+1\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$-\int \frac{1}{e^x\sin(x)+1}dx-\int \frac{\cot(x)}{e^x\sin(x)+1}dx+x+\log(\sin(x))$$

input
```
Int[(Cos[x] + Sin[x])/(E^(-x) + Sin[x]),x]
```

output
```
$Aborted
```

3.933.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.933.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 2.67

method	result	size
risch	$-ix+\ln\left(\mathrm{e}^{2ix}+2i\mathrm{e}^{(-1+i)x}-1\right)+x$	24
parallelrisch	$x-\ln\left(\frac{1}{\cos(x)+1}\right)+\ln\left(\frac{\mathrm{e}^{-x}+\sin(x)}{\cos(x)+1}\right)$	27
norman	$\frac{x+x\tan\left(\frac{x}{2}\right)^{2}}{1+\tan\left(\frac{x}{2}\right)^{2}}-\ln\left(1+\tan\left(\frac{x}{2}\right)^{2}\right)+\ln\left(\mathrm{e}^{-x}\tan\left(\frac{x}{2}\right)^{2}+\mathrm{e}^{-x}+2\tan\left(\frac{x}{2}\right)\right)$	57

input
```
int((sin(x)+cos(x))/(exp(-x)+sin(x)),x,method=_RETURNVERBOSE)
```

output
```
-I*x+ln(exp(2*I*x)+2*I*exp((-1+I)*x)-1)+x
```

3.933.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

$$\int \frac{\cos(x)+\sin(x)}{e^{-x}+\sin(x)}\,dx = x+\log\left(e^{(-x)}+\sin(x)\right)$$

input
```
integrate((cos(x)+sin(x))/(exp(-x)+sin(x)),x, algorithm="fricas")
```

output
```
x + log(e^(-x) + sin(x))
```

3.933.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

$$\int \frac{\cos(x)+\sin(x)}{e^{-x}+\sin(x)}\,dx = x+\log\left(\sin(x)+e^{-x}\right)$$

input
```
integrate((cos(x)+sin(x))/(exp(-x)+sin(x)),x)
```

output
```
x + log(sin(x) + exp(-x))
```

3.933.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(8) = 16.

Time = 0.33 (sec) , antiderivative size = 82, normalized size of antiderivative = 9.11

$$\int \frac{\cos(x)+\sin(x)}{e^{-x}+\sin(x)}\,dx = x + \frac{1}{2}\log\left(\left(\cos(2x)^2e^{(2x)}+4\cos(x)e^{x}\sin(2x)+e^{(2x)}\sin(2x)^2-2\left(2e^{x}\sin(x)+e^{(2x)}\right)\cos(2x)+4\cos\right.\right.$$

input
```
integrate((cos(x)+sin(x))/(exp(-x)+sin(x)),x, algorithm="maxima")
```

output
```
x + 1/2*log((cos(2*x)^2*e^(2*x) + 4*cos(x)*e^x*sin(2*x) + e^(2*x)*sin(2*x)
^2 - 2*(2*e^x*sin(x) + e^(2*x))*cos(2*x) + 4*cos(x)^2 + 4*e^x*sin(x) + 4*s
in(x)^2 + e^(2*x))*e^(-2*x))
```

3.933.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(8) = 16$.

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 9.22

$$\int \frac{\cos(x)+\sin(x)}{e^{-x}+\sin(x)}\,dx = x + \frac{1}{2}\log\left(\frac{4\left(e^{(-2x)}\tan\left(\frac{1}{2}x\right)^4 + 4e^{(-x)}\tan\left(\frac{1}{2}x\right)^3 + 2e^{(-2x)}\tan\left(\frac{1}{2}x\right)^2 + 4e^{(-x)}\tan\left(\frac{1}{2}x\right) + 4\tan\left(\frac{1}{2}x\right)^2 + \right.}{\tan\left(\frac{1}{2}x\right)^4 + 2\tan\left(\frac{1}{2}x\right)^2 + 1}$$

input
```
integrate((cos(x)+sin(x))/(exp(-x)+sin(x)),x, algorithm="giac")
```

output
```
x + 1/2*log(4*(e^(-2*x)*tan(1/2*x)^4 + 4*e^(-x)*tan(1/2*x)^3 + 2*e^(-2*x)*
tan(1/2*x)^2 + 4*e^(-x)*tan(1/2*x) + 4*tan(1/2*x)^2 + e^(-2*x))/(tan(1/2*x
)^4 + 2*tan(1/2*x)^2 + 1))
```

3.933.9 Mupad [B] (verification not implemented)

Time = 26.76 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.11

$$\int \frac{\cos(x)+\sin(x)}{e^{-x}+\sin(x)}\,dx = x + \ln\left(\mathrm{e}^{-x} + \sin(x)\right)$$

input
```
int((cos(x) + sin(x))/(exp(-x) + sin(x)),x)
```

output
```
x + log(exp(-x) + sin(x))
```

3.934 $\int \sin(c+dx) \left(a \sin^2(c+dx)+b \sin^3(c+dx)\right) \, dx$

3.934.1 Optimal result

Integrand size = 28, antiderivative size = 77

$$\int \sin(c+dx) \left(a \sin^2(c+dx)+b \sin^3(c+dx)\right) \, dx = \frac{3bx}{8}-\frac{a \cos(c+dx)}{d}+\frac{a \cos^3(c+dx)}{3d}-\frac{3b \cos(c+dx) \sin(c+dx)}{8d}-\frac{b \cos(c+dx) \sin^3(c+dx)}{4d}$$

output
```
3/8*b*x-a*cos(d*x+c)/d+1/3*a*cos(d*x+c)^3/d-3/8*b*cos(d*x+c)*sin(d*x+c)/d-
1/4*b*cos(d*x+c)*sin(d*x+c)^3/d
```

3.934.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int \sin(c+dx) \left(a \sin^2(c+dx)+b \sin^3(c+dx)\right) \, dx = \frac{3b(c+dx)}{8d}-\frac{3a \cos(c+dx)}{4d}+\frac{a \cos(3(c+dx))}{12d}-\frac{b \sin(2(c+dx))}{4d}+\frac{b \sin(4(c+dx))}{32d}$$

input
```
Integrate[Sin[c + d*x]*(a*Sin[c + d*x]^2 + b*Sin[c + d*x]^3),x]
```

output
```
(3*b*(c + d*x))/(8*d) - (3*a*Cos[c + d*x])/(4*d) + (a*Cos[3*(c + d*x)])/(1
2*d) - (b*Sin[2*(c + d*x)])/(4*d) + (b*Sin[4*(c + d*x)])/(32*d)
```

3.934.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {3042, 4893, 3042, 3227, 3042, 3113, 2009, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(c+dx)\left(a\sin^2(c+dx)+b\sin^3(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(c+dx)\left(a\sin(c+dx)^2+b\sin(c+dx)^3\right)dx$$

$$\Big\downarrow \text{4893}$$

$$\int \sin^3(c+dx)(a+b\sin(c+dx))dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(c+dx)^3(a+b\sin(c+dx))dx$$

$$\Big\downarrow \text{3227}$$

$$a\int \sin^3(c+dx)dx+b\int \sin^4(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$a\int \sin(c+dx)^3dx+b\int \sin(c+dx)^4dx$$

$$\Big\downarrow \text{3113}$$

$$b\int \sin(c+dx)^4dx-\frac{a\int\left(1-\cos^2(c+dx)\right)d\cos(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$b\int \sin(c+dx)^4dx-\frac{a\left(\cos(c+dx)-\frac{1}{3}\cos^3(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3115}$$

$$b\left(\frac{3}{4}\int \sin^2(c+dx)dx-\frac{\sin^3(c+dx)\cos(c+dx)}{4d}\right)-\frac{a\left(\cos(c+dx)-\frac{1}{3}\cos^3(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$b\left(\frac{3}{4}\int \sin(c+dx)^2dx-\frac{\sin^3(c+dx)\cos(c+dx)}{4d}\right)-\frac{a\left(\cos(c+dx)-\frac{1}{3}\cos^3(c+dx)\right)}{d}$$

$$\Big\downarrow \text{3115}$$

$$b\left(\frac{3}{4}\left(\frac{\int 1dx}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{\sin^3(c+dx)\cos(c+dx)}{4d}\right)-\frac{a\left(\cos(c+dx)-\frac{1}{3}\cos^3(c+dx)\right)}{d}$$

$$\Big\downarrow \text{24}$$

$$b\left(\frac{3}{4}\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{\sin^3(c+dx)\cos(c+dx)}{4d}\right)-\frac{a\left(\cos(c+dx)-\frac{1}{3}\cos^3(c+dx)\right)}{d}$$

input
```
Int[Sin[c + d*x]*(a*Sin[c + d*x]^2 + b*Sin[c + d*x]^3),x]
```

output
```
-((a*(Cos[c + d*x] - Cos[c + d*x]^3/3))/d) + b*(-1/4*(Cos[c + d*x]*Sin[c +
 d*x]^3)/d + (3*(x/2 - (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/4)
```

3.934.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 4893
```
Int[(u_)*((a_)*(F_)[(c_.) + (d_.)*(x_)]^(p_.) + (b_.)*(F_)[(c_.) + (d_.)*(x
_)]^(q_.))^(n_.), x_Symbol] :> Int[ActivateTrig[u*F[c + d*x]^(n*p)*(a + b*F
[c + d*x]^(q - p))^n], x] /; FreeQ[{a, b, c, d, p, q}, x] && InertTrigQ[F]
&& IntegerQ[n] && PosQ[q - p]
```

3.934.4 Maple [A] (verified)

Time = 1.99 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{b\left(-\frac{\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)-\frac{a\left(2+\sin(dx+c)^2\right)\cos(dx+c)}{3}}{d}$
default	$\frac{b\left(-\frac{\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)-\frac{a\left(2+\sin(dx+c)^2\right)\cos(dx+c)}{3}}{d}$
parallelrisch	$\frac{36dxb-72\cos(dx+c)a+3b\sin(4dx+4c)+8a\cos(3dx+3c)-24b\sin(2dx+2c)-64a}{96d}$
parts	$-\frac{a\left(2+\sin(dx+c)^2\right)\cos(dx+c)}{3d}+\frac{b\left(-\frac{\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$
risch	$\frac{3xb}{8}-\frac{3a\cos(dx+c)}{4d}+\frac{b\sin(4dx+4c)}{32d}+\frac{a\cos(3dx+3c)}{12d}-\frac{b\sin(2dx+2c)}{4d}$
norman	$\frac{\frac{3xb}{8}-\frac{4a}{3d}-\frac{3b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}-\frac{11b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{4d}+\frac{11b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{4d}+\frac{3b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{4d}+\frac{3xb\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2}+\frac{9xb\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{4}+}{\left(1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2\right)^4}$

input
```
int(sin(d*x+c)*(a*sin(d*x+c)^2+b*sin(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(b*(-1/4*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*c)-1/3*a
*(2+sin(d*x+c)^2)*cos(d*x+c))
```

3.934.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.78

$$\int \sin(c+dx)\left(a\sin^2(c+dx)+b\sin^3(c+dx)\right)\,dx$$
$$=\frac{8\,a\cos{(dx+c)}^3+9\,bdx-24\,a\cos{(dx+c)}+3\,\left(2\,b\cos{(dx+c)}^3-5\,b\cos{(dx+c)}\right)\sin{(dx+c)}}{24\,d}$$

input
```
integrate(sin(d*x+c)*(a*sin(d*x+c)^2+b*sin(d*x+c)^3),x, algorithm="fricas"
)
```

output
```
1/24*(8*a*cos(d*x + c)^3 + 9*b*d*x - 24*a*cos(d*x + c) + 3*(2*b*cos(d*x +
c)^3 - 5*b*cos(d*x + c))*sin(d*x + c))/d
```

3.934.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. 2(70) = 140.

Time = 0.17 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.95

$$\int \sin(c+dx)\left(a\sin^2(c+dx)+b\sin^3(c+dx)\right)\,dx$$
$$=\begin{cases}-\frac{a\sin^2{(c+dx)}\cos{(c+dx)}}{d}-\frac{2a\cos^3{(c+dx)}}{3d}+\frac{3bx\sin^4{(c+dx)}}{8}+\frac{3bx\sin^2{(c+dx)}\cos^2{(c+dx)}}{4}+\frac{3bx\cos^4{(c+dx)}}{8}-\frac{5b\sin^3{(c+dx)}\cos}{8d} \\ x(a\sin^2{(c)}+b\sin^3{(c)})\sin{(c)}\end{cases}$$

input
```
integrate(sin(d*x+c)*(a*sin(d*x+c)**2+b*sin(d*x+c)**3),x)
```

output
```
Piecewise((-a*sin(c + d*x)**2*cos(c + d*x)/d - 2*a*cos(c + d*x)**3/(3*d) +
 3*b*x*sin(c + d*x)**4/8 + 3*b*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*b*x
*cos(c + d*x)**4/8 - 5*b*sin(c + d*x)**3*cos(c + d*x)/(8*d) - 3*b*sin(c +
d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a*sin(c)**2 + b*sin(c)**3)*sin(
c), True))
```

3.934.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.74

$$\int \sin(c+dx) \left(a\sin^2(c+dx)+b\sin^3(c+dx)\right) \, dx = \frac{32\left(\cos\left(dx+c\right)^3-3\,\cos\left(dx+c\right)\right)a+3\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)-8\,\sin\left(2\,dx+2\,c\right)\right)b}{96\,d}$$

input
```
integrate(sin(d*x+c)*(a*sin(d*x+c)^2+b*sin(d*x+c)^3),x, algorithm="maxima"
)
```

output
```
1/96*(32*(cos(d*x + c)^3 - 3*cos(d*x + c))*a + 3*(12*d*x + 12*c + sin(4*d*
x + 4*c) - 8*sin(2*d*x + 2*c))*b)/d
```

3.934.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.81

$$\int \sin(c+dx) \left(a\sin^2(c+dx)+b\sin^3(c+dx)\right) \, dx = \frac{3}{8}\,bx+\frac{a\cos\left(3\,dx+3\,c\right)}{12\,d}-\frac{3\,a\cos\left(dx+c\right)}{4\,d}+\frac{b\sin\left(4\,dx+4\,c\right)}{32\,d}-\frac{b\sin\left(2\,dx+2\,c\right)}{4\,d}$$

input
```
integrate(sin(d*x+c)*(a*sin(d*x+c)^2+b*sin(d*x+c)^3),x, algorithm="giac")
```

output
```
3/8*b*x + 1/12*a*cos(3*d*x + 3*c)/d - 3/4*a*cos(d*x + c)/d + 1/32*b*sin(4*
d*x + 4*c)/d - 1/4*b*sin(2*d*x + 2*c)/d
```

3.934.9 Mupad [B] (verification not implemented)

Time = 29.71 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.44

$$\int \sin(c+dx) \left(a\sin^2(c+dx)+b\sin^3(c+dx)\right) \, dx = \frac{3\,b\,x}{8} - \frac{-\frac{3\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}-\frac{11\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}+4\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{11\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4}+\frac{16\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+\frac{3\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}+ \cdots}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)^4}$$

input
```
int(sin(c + d*x)*(a*sin(c + d*x)^2 + b*sin(c + d*x)^3),x)
```

output
```
(3*b*x)/8 - ((4*a)/3 + (3*b*tan(c/2 + (d*x)/2))/4 + (16*a*tan(c/2 + (d*x)/
2)^2)/3 + 4*a*tan(c/2 + (d*x)/2)^4 + (11*b*tan(c/2 + (d*x)/2)^3)/4 - (11*b
*tan(c/2 + (d*x)/2)^5)/4 - (3*b*tan(c/2 + (d*x)/2)^7)/4)/(d*(tan(c/2 + (d*
x)/2)^2 + 1)^4)
```

3.934. $\int \sin(c+dx)\left(a\sin^2(c+dx)+b\sin^3(c+dx)\right)\,dx$

3.935 $\int \sin(c+dx)\left(a\sin^2(c+dx)+b\sin^3(c+dx)\right)^2\,dx$

3.935.1 Optimal result

Integrand size = 30, antiderivative size = 161

$$\begin{aligned}&\int \sin(c+dx)\left(a\sin^2(c+dx)+b\sin^3(c+dx)\right)^2\,dx\\&=\frac{5abx}{8}-\frac{(a^2+b^2)\cos(c+dx)}{d}+\frac{(2a^2+3b^2)\cos^3(c+dx)}{3d}\\&\quad-\frac{(a^2+3b^2)\cos^5(c+dx)}{5d}+\frac{b^2\cos^7(c+dx)}{7d}-\frac{5ab\cos(c+dx)\sin(c+dx)}{8d}\\&\quad-\frac{5ab\cos(c+dx)\sin^3(c+dx)}{12d}-\frac{ab\cos(c+dx)\sin^5(c+dx)}{3d}\end{aligned}$$

output
```
5/8*a*b*x-(a^2+b^2)*cos(d*x+c)/d+1/3*(2*a^2+3*b^2)*cos(d*x+c)^3/d-1/5*(a^2
+3*b^2)*cos(d*x+c)^5/d+1/7*b^2*cos(d*x+c)^7/d-5/8*a*b*cos(d*x+c)*sin(d*x+c
)/d-5/12*a*b*cos(d*x+c)*sin(d*x+c)^3/d-1/3*a*b*cos(d*x+c)*sin(d*x+c)^5/d
```

3.935.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.83

$$\int \sin(c+dx)\left(a\sin^2(c+dx)+b\sin^3(c+dx)\right)^2\,dx$$
$$=\frac{4200abc+4200abdx-525(8a^2+7b^2)\cos(c+dx)+35(20a^2+21b^2)\cos(3(c+dx))-84a^2\cos(5(c+dx)}{}$$

input
```
Integrate[Sin[c + d*x]*(a*Sin[c + d*x]^2 + b*Sin[c + d*x]^3)^2,x]
```

output
```
(4200*a*b*c + 4200*a*b*d*x - 525*(8*a^2 + 7*b^2)*Cos[c + d*x] + 35*(20*a^2
 + 21*b^2)*Cos[3*(c + d*x)] - 84*a^2*Cos[5*(c + d*x)] - 147*b^2*Cos[5*(c +
 d*x)] + 15*b^2*Cos[7*(c + d*x)] - 3150*a*b*Sin[2*(c + d*x)] + 630*a*b*Sin
[4*(c + d*x)] - 70*a*b*Sin[6*(c + d*x)])/(6720*d)
```

3.935.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {3042, 4893, 3042, 3268, 3042, 3115, 3042, 3115, 3042, 3115, 24, 3492, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(c+dx)\left(a\sin^2(c+dx)+b\sin^3(c+dx)\right)^2 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(c+dx)\left(a\sin(c+dx)^2+b\sin(c+dx)^3\right)^2 dx$$

$$\Big\downarrow \text{4893}$$

$$\int \sin^5(c+dx)(a+b\sin(c+dx))^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(c+dx)^5(a+b\sin(c+dx))^2dx$$

$$\Big\downarrow \text{3268}$$

$$\int \sin^5(c+dx)\left(a^2+b^2\sin^2(c+dx)\right)dx+2ab\int \sin^6(c+dx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(c+dx)^5\left(a^2+b^2\sin(c+dx)^2\right)dx+2ab\int \sin(c+dx)^6dx$$

$$\Big\downarrow \text{3115}$$

$$\int \sin(c+dx)^5\left(a^2+b^2\sin(c+dx)^2\right)dx+2ab\left(\frac{5}{6}\int \sin^4(c+dx)dx-\frac{\sin^5(c+dx)\cos(c+dx)}{6d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(c+dx)^5 \left(a^2+b^2\sin(c+dx)^2\right) dx + 2ab\left(\frac{5}{6}\int \sin(c+dx)^4 dx - \frac{\sin^5(c+dx)\cos(c+dx)}{6d}\right)$$

$$\Big\downarrow \text{3115}$$

$$\int \sin(c+dx)^5 \left(a^2+b^2\sin(c+dx)^2\right) dx + 2ab\left(\frac{5}{6}\left(\frac{3}{4}\int \sin^2(c+dx)dx - \frac{\sin^3(c+dx)\cos(c+dx)}{4d}\right) - \frac{\sin^5(c+dx)\cos(c+dx)}{6d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(c+dx)^5 \left(a^2+b^2\sin(c+dx)^2\right) dx + 2ab\left(\frac{5}{6}\left(\frac{3}{4}\int \sin(c+dx)^2dx - \frac{\sin^3(c+dx)\cos(c+dx)}{4d}\right) - \frac{\sin^5(c+dx)\cos(c+dx)}{6d}\right)$$

$$\Big\downarrow \text{3115}$$

$$\int \sin(c+dx)^5 \left(a^2+b^2\sin(c+dx)^2\right) dx + 2ab\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2} - \frac{\sin(c+dx)\cos(c+dx)}{2d}\right) - \frac{\sin^3(c+dx)\cos(c+dx)}{4d}\right) - \frac{\sin^5(c+dx)\cos(c+dx)}{6d}\right)$$

$$\Big\downarrow \text{24}$$

$$\int \sin(c+dx)^5 \left(a^2+b^2\sin(c+dx)^2\right) dx + 2ab\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{x}{2} - \frac{\sin(c+dx)\cos(c+dx)}{2d}\right) - \frac{\sin^3(c+dx)\cos(c+dx)}{4d}\right) - \frac{\sin^5(c+dx)\cos(c+dx)}{6d}\right)$$

$$\Big\downarrow \text{3492}$$

$$2ab\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{x}{2} - \frac{\sin(c+dx)\cos(c+dx)}{2d}\right) - \frac{\sin^3(c+dx)\cos(c+dx)}{4d}\right) - \frac{\sin^5(c+dx)\cos(c+dx)}{6d}\right) - \frac{\int \left(1-\cos^2(c+dx)\right)^2 \left(a^2+b^2-b^2\cos^2(c+dx)\right) d\cos(c+dx)}{d}$$

$$\Big\downarrow \text{290}$$

$$2ab\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{x}{2} - \frac{\sin(c+dx)\cos(c+dx)}{2d}\right) - \frac{\sin^3(c+dx)\cos(c+dx)}{4d}\right) - \frac{\sin^5(c+dx)\cos(c+dx)}{6d}\right) - \frac{\int \left(-b^2\cos^6(c+dx) + \left(a^2+3b^2\right)\cos^4(c+dx) - \left(2a^2+3b^2\right)\cos^2(c+dx) + a^2\left(\frac{b^2}{a^2}+1\right)\right) d\cos(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

3.935. $\int \sin(c+dx)\left(a\sin^2(c+dx)+b\sin^3(c+dx)\right)^2 \, dx$

$$2ab\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{\sin^3(c+dx)\cos(c+dx)}{4d}\right)-\frac{\sin^5(c+dx)\cos(c+dx)}{6d}\right)-\frac{\frac{1}{5}(a^2+3b^2)\cos^5(c+dx)-\frac{1}{3}(2a^2+3b^2)\cos^3(c+dx)+(a^2+b^2)\cos(c+dx)-\frac{1}{7}b^2\cos^7(c+dx)}{d}$$

input
```
Int[Sin[c + d*x]*(a*Sin[c + d*x]^2 + b*Sin[c + d*x]^3)^2,x]
```

output
```
-(((a^2 + b^2)*Cos[c + d*x] - ((2*a^2 + 3*b^2)*Cos[c + d*x]^3)/3 + ((a^2 +
 3*b^2)*Cos[c + d*x]^5)/5 - (b^2*Cos[c + d*x]^7)/7)/d) + 2*a*b*(-1/6*(Cos[
c + d*x]*Sin[c + d*x]^5)/d + (5*(-1/4*(Cos[c + d*x]*Sin[c + d*x]^3)/d + (3
*(x/2 - (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/4))/6)
```

3.935.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3268
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)])^2, x_Symbol] :> Simp[2*c*(d/b)   Int[(b*Sin[e + f*x])^(m + 1), x], x]
+ Int[(b*Sin[e + f*x])^m*(c^2 + d^2*Sin[e + f*x]^2), x] /; FreeQ[{b, c, d,
e, f, m}, x]
```

rule 3492
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Simp[-f^(-1)   Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2
), x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

rule 4893
```
Int[(u_)*((a_)*(F_)[(c_.) + (d_.)*(x_)]^(p_.) + (b_.)*(F_)[(c_.) + (d_.)*(x
_)]^(q_.))^(n_.), x_Symbol] :> Int[ActivateTrig[u*F[c + d*x]^(n*p)*(a + b*F
[c + d*x]^(q - p))^n], x] /; FreeQ[{a, b, c, d, p, q}, x] && InertTrigQ[F]
&& IntegerQ[n] && PosQ[q - p]
```

3.935.4 Maple [A] (verified)

Time = 2.76 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{-\frac{b^2\left(\frac{16}{5}+\sin(dx+c)^6+\frac{6\sin(dx+c)^4}{5}+\frac{8\sin(dx+c)^2}{5}\right)\cos(dx+c)}{7}+2ab\left(-\frac{\left(\sin(dx+c)^5+\frac{5\sin(dx+c)^3}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{6}+\right.}{d}$
default	$\frac{-\frac{b^2\left(\frac{16}{5}+\sin(dx+c)^6+\frac{6\sin(dx+c)^4}{5}+\frac{8\sin(dx+c)^2}{5}\right)\cos(dx+c)}{7}+2ab\left(-\frac{\left(\sin(dx+c)^5+\frac{5\sin(dx+c)^3}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{6}+\right.}{d}$
parts	$-\frac{a^2\left(\frac{8}{3}+\sin(dx+c)^4+\frac{4\sin(dx+c)^2}{3}\right)\cos(dx+c)}{5d}-\frac{b^2\left(\frac{16}{5}+\sin(dx+c)^6+\frac{6\sin(dx+c)^4}{5}+\frac{8\sin(dx+c)^2}{5}\right)\cos(dx+c)}{7d}+\frac{2a}{}$
parallelrisch	$\frac{(700a^2+735b^2)\cos(3dx+3c)+(-84a^2-147b^2)\cos(5dx+5c)+15b^2\cos(7dx+7c)-3150ab\sin(2dx+2c)+630ab\sin(4dx+}{6720d}$
risch	$\frac{5xab}{8}-\frac{5a^2\cos(dx+c)}{8d}-\frac{35\cos(dx+c)b^2}{64d}+\frac{b^2\cos(7dx+7c)}{448d}-\frac{ab\sin(6dx+6c)}{96d}-\frac{\cos(5dx+5c)a^2}{80d}-\frac{7\cos(5dx+}{320d}$
norman	$\frac{-\frac{112a^2+96b^2}{105d}+\frac{5xab}{8}-\frac{32a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{3d}-\frac{(80a^2+96b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{3d}-\frac{(112a^2+96b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{15d}-\frac{(112a^2+96b^2)\tan\left(\frac{dx}{2}\right.}{5d}}{}$

input
```
int(sin(d*x+c)*(a*sin(d*x+c)^2+b*sin(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/7*b^2*(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*
x+c)+2*a*b*(-1/6*(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c
)+5/16*d*x+5/16*c)-1/5*a^2*(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))
```

3.935.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.76

$$\int \sin(c+dx)\left(a\sin^2(c+dx)+b\sin^3(c+dx)\right)^2\,dx$$
$$= \frac{120\,b^2\cos\left(dx+c\right)^7 - 168\,(a^2+3\,b^2)\cos\left(dx+c\right)^5 + 525\,abdx + 280\,(2\,a^2+3\,b^2)\cos\left(dx+c\right)^3 - 840\,(a^2+}{840\,d}$$

input
```
integrate(sin(d*x+c)*(a*sin(d*x+c)^2+b*sin(d*x+c)^3)^2,x, algorithm="frica
s")
```

output
```
1/840*(120*b^2*cos(d*x + c)^7 - 168*(a^2 + 3*b^2)*cos(d*x + c)^5 + 525*a*b
*d*x + 280*(2*a^2 + 3*b^2)*cos(d*x + c)^3 - 840*(a^2 + b^2)*cos(d*x + c) -
 35*(8*a*b*cos(d*x + c)^5 - 26*a*b*cos(d*x + c)^3 + 33*a*b*cos(d*x + c))*s
in(d*x + c))/d
```

3.935.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(148) = 296$.

Time = 0.50 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.02

$$\int \sin(c+dx)\left(a\sin^2(c+dx)+b\sin^3(c+dx)\right)^2\,dx$$
$$= \begin{cases} -\frac{a^2\sin^4(c+dx)\cos(c+dx)}{d} - \frac{4a^2\sin^2(c+dx)\cos^3(c+dx)}{3d} - \frac{8a^2\cos^5(c+dx)}{15d} + \frac{5abx\sin^6(c+dx)}{8} + \frac{15abx\sin^4(c+dx)\cos^2(c+dx)}{8} + \\ x(a\sin^2(c)+b\sin^3(c))^2\sin(c) \end{cases}$$

input
```
integrate(sin(d*x+c)*(a*sin(d*x+c)**2+b*sin(d*x+c)**3)**2,x)
```

output
```
Piecewise((-a**2*sin(c + d*x)**4*cos(c + d*x)/d - 4*a**2*sin(c + d*x)**2*c
os(c + d*x)**3/(3*d) - 8*a**2*cos(c + d*x)**5/(15*d) + 5*a*b*x*sin(c + d*x
)**6/8 + 15*a*b*x*sin(c + d*x)**4*cos(c + d*x)**2/8 + 15*a*b*x*sin(c + d*x
)**2*cos(c + d*x)**4/8 + 5*a*b*x*cos(c + d*x)**6/8 - 11*a*b*sin(c + d*x)**
5*cos(c + d*x)/(8*d) - 5*a*b*sin(c + d*x)**3*cos(c + d*x)**3/(3*d) - 5*a*b
*sin(c + d*x)*cos(c + d*x)**5/(8*d) - b**2*sin(c + d*x)**6*cos(c + d*x)/d
- 2*b**2*sin(c + d*x)**4*cos(c + d*x)**3/d - 8*b**2*sin(c + d*x)**2*cos(c
+ d*x)**5/(5*d) - 16*b**2*cos(c + d*x)**7/(35*d), Ne(d, 0)), (x*(a*sin(c)*
*2 + b*sin(c)**3)**2*sin(c), True))
```

3.935.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.81

$$\int \sin(c+dx)\left(a\sin^2(c+dx)+b\sin^3(c+dx)\right)^2\,dx =$$
$$-\frac{224\left(3\cos(dx+c)^5-10\cos(dx+c)^3+15\cos(dx+c)\right)a^2-35\left(4\sin(2dx+2c)^3+60dx+60c+\right.}{}$$

input
```
integrate(sin(d*x+c)*(a*sin(d*x+c)^2+b*sin(d*x+c)^3)^2,x, algorithm="maxim
a")
```

output
```
-1/3360*(224*(3*cos(d*x + c)^5 - 10*cos(d*x + c)^3 + 15*cos(d*x + c))*a^2
- 35*(4*sin(2*d*x + 2*c)^3 + 60*d*x + 60*c + 9*sin(4*d*x + 4*c) - 48*sin(2
*d*x + 2*c))*a*b - 96*(5*cos(d*x + c)^7 - 21*cos(d*x + c)^5 + 35*cos(d*x +
 c)^3 - 35*cos(d*x + c))*b^2)/d
```

3.935.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.89

$$\begin{aligned}\int \sin(c+dx)\left(a\sin^2(c+dx)+b\sin^3(c+dx)\right)^2\,dx &= \frac{5}{8}abx+\frac{b^2\cos(7dx+7c)}{448d}-\frac{ab\sin(6dx+6c)}{96d}+\frac{3ab\sin(4dx+4c)}{32d}\\&\quad-\frac{15ab\sin(2dx+2c)}{32d}-\frac{(4a^2+7b^2)\cos(5dx+5c)}{320d}\\&\quad+\frac{(20a^2+21b^2)\cos(3dx+3c)}{192d}-\frac{5(8a^2+7b^2)\cos(dx+c)}{64d}\end{aligned}$$

input
```
integrate(sin(d*x+c)*(a*sin(d*x+c)^2+b*sin(d*x+c)^3)^2,x, algorithm="giac"
)
```

output
```
5/8*a*b*x + 1/448*b^2*cos(7*d*x + 7*c)/d - 1/96*a*b*sin(6*d*x + 6*c)/d + 3
/32*a*b*sin(4*d*x + 4*c)/d - 15/32*a*b*sin(2*d*x + 2*c)/d - 1/320*(4*a^2 +
 7*b^2)*cos(5*d*x + 5*c)/d + 1/192*(20*a^2 + 21*b^2)*cos(3*d*x + 3*c)/d -
5/64*(8*a^2 + 7*b^2)*cos(d*x + c)/d
```

3.935.9 Mupad [B] (verification not implemented)

Time = 30.88 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.30

$$\int \sin(c+dx)\left(a\sin^2(c+dx)+b\sin^3(c+dx)\right)^2\,dx=\frac{5\,a\,b\,x}{8}$$
$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\left(\frac{80\,a^2}{3}+32\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(\frac{112\,a^2}{15}+\frac{32\,b^2}{5}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(\frac{112\,a^2}{5}+\frac{96\,b^2}{5}\right)+\frac{32\,a^2\tan(}{3}$$

input
```
int(sin(c + d*x)*(a*sin(c + d*x)^2 + b*sin(c + d*x)^3)^2,x)
```

output
```
(5*a*b*x)/8 - (tan(c/2 + (d*x)/2)^6*((80*a^2)/3 + 32*b^2) + tan(c/2 + (d*x
)/2)^2*((112*a^2)/15 + (32*b^2)/5) + tan(c/2 + (d*x)/2)^4*((112*a^2)/5 + (
96*b^2)/5) + (32*a^2*tan(c/2 + (d*x)/2)^8)/3 + (16*a^2)/15 + (32*b^2)/35 +
 (25*a*b*tan(c/2 + (d*x)/2)^3)/3 + (283*a*b*tan(c/2 + (d*x)/2)^5)/12 - (28
3*a*b*tan(c/2 + (d*x)/2)^9)/12 - (25*a*b*tan(c/2 + (d*x)/2)^11)/3 - (5*a*b
*tan(c/2 + (d*x)/2)^13)/4 + (5*a*b*tan(c/2 + (d*x)/2))/4)/(d*(tan(c/2 + (d
*x)/2)^2 + 1)^7)
```

3.936 $\int \sin(c+dx) \left(a\sin(c+dx)+b\sin^2(c+dx)+c\sin^3(c+dx)\right)$

3.936.1 Optimal result

Integrand size = 36, antiderivative size = 89

$$\int \sin(c+dx) \left(a\sin(c+dx)+b\sin^2(c+dx)+c\sin^3(c+dx)\right)\,dx = \frac{1}{8}(4a+3c)x-\frac{b\cos(c+dx)}{d}+\frac{b\cos^3(c+dx)}{3d} - \frac{(4a+3c)\cos(c+dx)\sin(c+dx)}{8d}-\frac{c\cos(c+dx)\sin^3(c+dx)}{4d}$$

output
```
1/8*(4*a+3*c)*x-b*cos(d*x+c)/d+1/3*b*cos(d*x+c)^3/d-1/8*(4*a+3*c)*cos(d*x+
c)*sin(d*x+c)/d-1/4*c*cos(d*x+c)*sin(d*x+c)^3/d
```

3.936.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.18

$$\int \sin(c+dx) \left(a\sin(c+dx)+b\sin^2(c+dx)+c\sin^3(c+dx)\right)\,dx = \frac{a(c+dx)}{2d}+\frac{3c(c+dx)}{8d}-\frac{3b\cos(c+dx)}{4d}+\frac{b\cos(3(c+dx))}{12d} - \frac{a\sin(2(c+dx))}{4d}-\frac{c\sin(2(c+dx))}{4d}+\frac{c\sin(4(c+dx))}{32d}$$

input
```
Integrate[Sin[c + d*x]*(a*Sin[c + d*x] + b*Sin[c + d*x]^2 + c*Sin[c + d*x]
^3),x]
```

output
```
(a*(c + d*x))/(2*d) + (3*c*(c + d*x))/(8*d) - (3*b*Cos[c + d*x])/(4*d) + (
b*Cos[3*(c + d*x)])/(12*d) - (a*Sin[2*(c + d*x)])/(4*d) - (c*Sin[2*(c + d*
x)])/(4*d) + (c*Sin[4*(c + d*x)])/(32*d)
```

3.936.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.97, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.306, Rules used = {3042, 4725, 3042, 3502, 3042, 3227, 3042, 3113, 2009, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(c+dx)\left(a\sin(c+dx)+b\sin^2(c+dx)+c\sin^3(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(c+dx)\left(a\sin(c+dx)+b\sin(c+dx)^2+c\sin(c+dx)^3\right)dx$$

$$\Big\downarrow \text{4725}$$

$$\int \sin^2(c+dx)\left(a+b\sin(c+dx)+c\sin^2(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(c+dx)^2\left(a+b\sin(c+dx)+c\sin(c+dx)^2\right)dx$$

$$\Big\downarrow \text{3502}$$

$$\frac{1}{4}\int \sin^2(c+dx)(4a+3c+4b\sin(c+dx))dx-\frac{c\sin^3(c+dx)\cos(c+dx)}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\int \sin(c+dx)^2(4a+3c+4b\sin(c+dx))dx-\frac{c\sin^3(c+dx)\cos(c+dx)}{4d}$$

$$\Big\downarrow \text{3227}$$

$$\frac{1}{4}\left((4a+3c)\int \sin^2(c+dx)dx+4b\int \sin^3(c+dx)dx\right)-\frac{c\sin^3(c+dx)\cos(c+dx)}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{4}\left((4a+3c)\int \sin(c+dx)^2dx+4b\int \sin(c+dx)^3dx\right)-\frac{c\sin^3(c+dx)\cos(c+dx)}{4d}$$

$$\Big\downarrow \text{3113}$$

$$\frac{1}{4}\left((4a+3c)\int \sin(c+dx)^2dx-\frac{4b\int\left(1-\cos^2(c+dx)\right)d\cos(c+dx)}{d}\right)-\frac{c\sin^3(c+dx)\cos(c+dx)}{4d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\left((4a+3c)\int \sin(c+dx)^2dx-\frac{4b\left(\cos(c+dx)-\frac{1}{3}\cos^3(c+dx)\right)}{d}\right)-\frac{c\sin^3(c+dx)\cos(c+dx)}{4d}$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{4}\left((4a+3c)\left(\frac{\int 1dx}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{4b\left(\cos(c+dx)-\frac{1}{3}\cos^3(c+dx)\right)}{d}\right)-\frac{c\sin^3(c+dx)\cos(c+dx)}{4d}$$

$$\Big\downarrow \text{24}$$

$$\frac{1}{4}\left((4a+3c)\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{4b\left(\cos(c+dx)-\frac{1}{3}\cos^3(c+dx)\right)}{d}\right)-\frac{c\sin^3(c+dx)\cos(c+dx)}{4d}$$

input
```
Int[Sin[c + d*x]*(a*Sin[c + d*x] + b*Sin[c + d*x]^2 + c*Sin[c + d*x]^3),x]
```

output
```
-1/4*(c*Cos[c + d*x]*Sin[c + d*x]^3)/d + ((-4*b*(Cos[c + d*x] - Cos[c + d*
x]^3/3))/d + (4*a + 3*c)*(x/2 - (Cos[c + d*x]*Sin[c + d*x])/(2*d)))/4
```

3.936.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.936. $\int \sin(c+dx)\left(a\sin(c+dx)+b\sin^2(c+dx)+c\sin^3(c+dx)\right)\,dx$

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 4725
```
Int[(u_)*((A_.)*sin[(a_.) + (b_.)*(x_)]^(n_.) + (B_.)*sin[(a_.) + (b_.)*(x_
)]^(n1_) + (C_.)*sin[(a_.) + (b_.)*(x_)]^(n2_)), x_Symbol] :> Int[ActivateT
rig[u]*Sin[a + b*x]^n*(A + B*Sin[a + b*x] + C*Sin[a + b*x]^2), x] /; FreeQ[
{a, b, A, B, C, n}, x] && EqQ[n1, n + 1] && EqQ[n2, n + 2]
```

3.936.4 Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.79

method	result
parallelrisch	$\frac{(-24a-24c)\sin(2dx+2c)+48dxa+36cdx-72\cos(dx+c)b+8b\cos(3dx+3c)+3c\sin(4dx+4c)-64b}{96d}$
risch	$\frac{ax}{2}+\frac{3cx}{8}-\frac{3b\cos(dx+c)}{4d}+\frac{c\sin(4dx+4c)}{32d}+\frac{b\cos(3dx+3c)}{12d}-\frac{\sin(2dx+2c)a}{4d}-\frac{\sin(2dx+2c)c}{4d}$
derivativedivides	$\frac{c\left(-\frac{\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)-\frac{b\left(2+\sin(dx+c)^2\right)\cos(dx+c)}{3}+a\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
default	$\frac{c\left(-\frac{\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)-\frac{b\left(2+\sin(dx+c)^2\right)\cos(dx+c)}{3}+a\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
parts	$\frac{a\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{b\left(2+\sin(dx+c)^2\right)\cos(dx+c)}{3d}+\frac{c\left(-\frac{\left(\sin(dx+c)^3+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\right.}{d}$
norman	$\frac{(\frac{a}{2}+\frac{3c}{8})x+(2a+\frac{3c}{2})x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2+(2a+\frac{3c}{2})x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^6+(3a+\frac{9c}{4})x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4+(\frac{a}{2}+\frac{3c}{8})x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8-\frac{4b}{3d}}{\left(1+\tan\right.}$

input
```
int(sin(d*x+c)*(sin(d*x+c)*a+b*sin(d*x+c)^2+c*sin(d*x+c)^3),x,method=_RETU
RNVERBOSE)
```

output
```
1/96*((-24*a-24*c)*sin(2*d*x+2*c)+48*d*x*a+36*c*d*x-72*cos(d*x+c)*b+8*b*co
s(3*d*x+3*c)+3*c*sin(4*d*x+4*c)-64*b)/d
```

3.936.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.81

$$\int \sin(c+dx)\left(a\sin(c+dx)+b\sin^2(c+dx)+c\sin^3(c+dx)\right)\,dx$$
$$=\frac{8\,b\cos\left(dx+c\right)^3+3\,(4\,a+3\,c)dx-24\,b\cos\left(dx+c\right)+3\,\left(2\,c\cos\left(dx+c\right)^3-(4\,a+5\,c)\cos\left(dx+c\right)\right)\sin\left(\right.}{24\,d}$$

input
```
integrate(sin(d*x+c)*(a*sin(d*x+c)+b*sin(d*x+c)^2+c*sin(d*x+c)^3),x, algor
ithm="fricas")
```

output
```
1/24*(8*b*cos(d*x + c)^3 + 3*(4*a + 3*c)*d*x - 24*b*cos(d*x + c) + 3*(2*c*
cos(d*x + c)^3 - (4*a + 5*c)*cos(d*x + c))*sin(d*x + c))/d
```

3.936.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(76) = 152$.

Time = 0.18 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.26

$$\int \sin(c+dx)\left(a\sin(c+dx)+b\sin^2(c+dx)+c\sin^3(c+dx)\right)\,dx$$
$$=\begin{cases} \frac{ax\sin^2(c+dx)}{2}+\frac{ax\cos^2(c+dx)}{2}-\frac{a\sin(c+dx)\cos(c+dx)}{2d}-\frac{b\sin^2(c+dx)\cos(c+dx)}{d}-\frac{2b\cos^3(c+dx)}{3d}+\frac{3cx\sin^4(c+dx)}{8}+\frac{3cx\sin \\ x(a\sin(c)+b\sin^2(c)+c\sin^3(c))\sin(c) \end{cases}$$

input
```
integrate(sin(d*x+c)*(a*sin(d*x+c)+b*sin(d*x+c)**2+c*sin(d*x+c)**3),x)
```

output
```
Piecewise((a*x*sin(c + d*x)**2/2 + a*x*cos(c + d*x)**2/2 - a*sin(c + d*x)*
cos(c + d*x)/(2*d) - b*sin(c + d*x)**2*cos(c + d*x)/d - 2*b*cos(c + d*x)**
3/(3*d) + 3*c*x*sin(c + d*x)**4/8 + 3*c*x*sin(c + d*x)**2*cos(c + d*x)**2/
4 + 3*c*x*cos(c + d*x)**4/8 - 5*c*sin(c + d*x)**3*cos(c + d*x)/(8*d) - 3*c
*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a*sin(c) + b*sin(c)**2
 + c*sin(c)**3)*sin(c), True))
```

3.936.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.89

$$\int \sin(c+dx)\left(a\sin(c+dx)+b\sin^2(c+dx)+c\sin^3(c+dx)\right)\,dx$$
$$=\frac{24\,(2\,dx+2\,c-\sin\left(2\,dx+2\,c\right))a+32\,\left(\cos\left(dx+c\right)^3-3\,\cos\left(dx+c\right)\right)b+3\,(12\,dx+12\,c+\sin\left(4\,dx+4}{96\,d}$$

input
```
integrate(sin(d*x+c)*(a*sin(d*x+c)+b*sin(d*x+c)^2+c*sin(d*x+c)^3),x, algor
ithm="maxima")
```

output
```
1/96*(24*(2*d*x + 2*c - sin(2*d*x + 2*c))*a + 32*(cos(d*x + c)^3 - 3*cos(d
*x + c))*b + 3*(12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*d*x + 2*c))*c)/
d
```

3.936.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.79

$$\int \sin(c+dx)\left(a\sin(c+dx)+b\sin^2(c+dx)+c\sin^3(c+dx)\right)\,dx$$
$$=\frac{1}{8}\,(4\,a+3\,c)x+\frac{b\cos\left(3\,dx+3\,c\right)}{12\,d}-\frac{3\,b\cos\left(dx+c\right)}{4\,d}+\frac{c\sin\left(4\,dx+4\,c\right)}{32\,d}-\frac{(a+c)\sin\left(2\,dx+2\,c\right)}{4\,d}$$

input
```
integrate(sin(d*x+c)*(a*sin(d*x+c)+b*sin(d*x+c)^2+c*sin(d*x+c)^3),x, algor
ithm="giac")
```

output
```
1/8*(4*a + 3*c)*x + 1/12*b*cos(3*d*x + 3*c)/d - 3/4*b*cos(d*x + c)/d + 1/3
2*c*sin(4*d*x + 4*c)/d - 1/4*(a + c)*sin(2*d*x + 2*c)/d
```

3.936.9 Mupad [B] (verification not implemented)

Time = 27.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.82

$$\int \sin(c+dx)\left(a\sin(c+dx)+b\sin^2(c+dx)+c\sin^3(c+dx)\right)\,dx$$
$$=\frac{2\,b\,\cos\left(3\,c+3\,d\,x\right)-18\,b\,\cos\left(c+d\,x\right)-6\,a\,\sin\left(2\,c+2\,d\,x\right)-6\,c\,\sin\left(2\,c+2\,d\,x\right)+\frac{3\,c\,\sin\left(4\,c+4\,d\,x\right)}{4}+12\,a}{24\,d}$$

input
```
int(sin(c + d*x)*(a*sin(c + d*x) + b*sin(c + d*x)^2 + c*sin(c + d*x)^3),x)
```

output
```
(2*b*cos(3*c + 3*d*x) - 18*b*cos(c + d*x) - 6*a*sin(2*c + 2*d*x) - 6*c*sin
(2*c + 2*d*x) + (3*c*sin(4*c + 4*d*x))/4 + 12*a*d*x + 9*c*d*x)/(24*d)
```

3.937 $\int \sin(c+dx) \left(a \sin(c+dx) + b \sin^2(c+dx) + c \sin^3(c$

3.937.1 Optimal result

Integrand size = 38, antiderivative size = 288

$$\begin{aligned}
&\int \sin(c+dx) \left(a \sin(c+dx)+b \sin^2(c+dx)+c \sin^3(c+dx)\right)^2 \, dx \\
&= \frac{3abx}{4}+\frac{5bcx}{8}-\frac{a^2 \cos(c+dx)}{d}-\frac{c^2 \cos(c+dx)}{d}-\frac{(b^2+2ac) \cos(c+dx)}{d}+\frac{a^2 \cos^3(c+dx)}{3d} \\
&\quad+\frac{c^2 \cos^3(c+dx)}{d}+\frac{2(b^2+2ac) \cos^3(c+dx)}{3d}-\frac{3c^2 \cos^5(c+dx)}{5d}-\frac{(b^2+2ac) \cos^5(c+dx)}{5d} \\
&\quad+\frac{c^2 \cos^7(c+dx)}{7d}-\frac{3ab \cos(c+dx) \sin(c+dx)}{4d}-\frac{5bc \cos(c+dx) \sin(c+dx)}{8d} \\
&\quad-\frac{ab \cos(c+dx) \sin^3(c+dx)}{2d}-\frac{5bc \cos(c+dx) \sin^3(c+dx)}{12d}-\frac{bc \cos(c+dx) \sin^5(c+dx)}{3d}
\end{aligned}$$

output
```
3/4*a*b*x+5/8*b*c*x-a^2*cos(d*x+c)/d-c^2*cos(d*x+c)/d-(2*a*c+b^2)*cos(d*x+
c)/d+1/3*a^2*cos(d*x+c)^3/d+c^2*cos(d*x+c)^3/d+2/3*(2*a*c+b^2)*cos(d*x+c)^
3/d-3/5*c^2*cos(d*x+c)^5/d-1/5*(2*a*c+b^2)*cos(d*x+c)^5/d+1/7*c^2*cos(d*x+
c)^7/d-3/4*a*b*cos(d*x+c)*sin(d*x+c)/d-5/8*b*c*cos(d*x+c)*sin(d*x+c)/d-1/2
*a*b*cos(d*x+c)*sin(d*x+c)^3/d-5/12*b*c*cos(d*x+c)*sin(d*x+c)^3/d-1/3*b*c*
cos(d*x+c)*sin(d*x+c)^5/d
```

3.937.2 Mathematica [A] (verified)

Time = 1.66 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.58

$$\int \sin(c+dx)\left(a\sin(c+dx)+b\sin^2(c+dx)+c\sin^3(c+dx)\right)^2\,dx$$
$$=\frac{840b(6a+5c)(c+dx)-105(48a^2+40b^2+80ac+35c^2)\cos(c+dx)+35(16a^2+20b^2+40ac+21c^2)\text{co}}{}$$

input
```
Integrate[Sin[c + d*x]*(a*Sin[c + d*x] + b*Sin[c + d*x]^2 + c*Sin[c + d*x]
^3)^2,x]
```

output
```
(840*b*(6*a + 5*c)*(c + d*x) - 105*(48*a^2 + 40*b^2 + 80*a*c + 35*c^2)*Cos
[c + d*x] + 35*(16*a^2 + 20*b^2 + 40*a*c + 21*c^2)*Cos[3*(c + d*x)] - 21*(
4*b^2 + c*(8*a + 7*c))*Cos[5*(c + d*x)] + 15*c^2*Cos[7*(c + d*x)] - 210*b*
(16*a + 15*c)*Sin[2*(c + d*x)] + 210*b*(2*a + 3*c)*Sin[4*(c + d*x)] - 70*b
*c*Sin[6*(c + d*x)])/(6720*d)
```

3.937.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {3042, 4894, 3042, 3737, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(c+dx)\left(a\sin(c+dx)+b\sin^2(c+dx)+c\sin^3(c+dx)\right)^2\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin(c+dx)\left(a\sin(c+dx)+b\sin(c+dx)^2+c\sin(c+dx)^3\right)^2dx$$
$$\Big\downarrow \text{4894}$$
$$\int \sin^3(c+dx)\left(a+b\sin(c+dx)+c\sin^2(c+dx)\right)^2dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin(c+dx)^3\left(a+b\sin(c+dx)+c\sin(c+dx)^2\right)^2dx$$

$$\Big\downarrow \ 3737$$

$$\int \left(a^2\sin^3(c+dx)+\left(2ac+b^2\right)\sin^5(c+dx)+2ab\sin^4(c+dx)+2bc\sin^6(c+dx)+c^2\sin^7(c+dx)\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{a^2\cos^3(c+dx)}{3d}-\frac{a^2\cos(c+dx)}{d}-\frac{\left(2ac+b^2\right)\cos^5(c+dx)}{5d}+\frac{2\left(2ac+b^2\right)\cos^3(c+dx)}{3d}-\frac{\left(2ac+b^2\right)\cos(c+dx)}{d}-\frac{ab\sin^3(c+dx)\cos(c+dx)}{2d}-\frac{3ab\sin(c+dx)\cos(c+dx)}{4d}+\frac{3abx}{4}-\frac{bc\sin^5(c+dx)\cos(c+dx)}{3d}-\frac{5bc\sin^3(c+dx)\cos(c+dx)}{12d}-\frac{5bc\sin(c+dx)\cos(c+dx)}{8d}+\frac{5bcx}{8}+\frac{c^2\cos^7(c+dx)}{7d}-\frac{3c^2\cos^5(c+dx)}{5d}+\frac{c^2\cos^3(c+dx)}{d}-\frac{c^2\cos(c+dx)}{d}$$

input
```
Int[Sin[c + d*x]*(a*Sin[c + d*x] + b*Sin[c + d*x]^2 + c*Sin[c + d*x]^3)^2,
x]
```

output
```
(3*a*b*x)/4 + (5*b*c*x)/8 - (a^2*Cos[c + d*x])/d - (c^2*Cos[c + d*x])/d -
((b^2 + 2*a*c)*Cos[c + d*x])/d + (a^2*Cos[c + d*x]^3)/(3*d) + (c^2*Cos[c +
 d*x]^3)/d + (2*(b^2 + 2*a*c)*Cos[c + d*x]^3)/(3*d) - (3*c^2*Cos[c + d*x]^
5)/(5*d) - ((b^2 + 2*a*c)*Cos[c + d*x]^5)/(5*d) + (c^2*Cos[c + d*x]^7)/(7*
d) - (3*a*b*Cos[c + d*x]*Sin[c + d*x])/(4*d) - (5*b*c*Cos[c + d*x]*Sin[c +
 d*x])/(8*d) - (a*b*Cos[c + d*x]*Sin[c + d*x]^3)/(2*d) - (5*b*c*Cos[c + d*
x]*Sin[c + d*x]^3)/(12*d) - (b*c*Cos[c + d*x]*Sin[c + d*x]^5)/(3*d)
```

3.937.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3737
```
Int[sin[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*sin[(d_.) + (e_.)*(x_)]^(n
_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]^(n2_.))^(p_), x_Symbol] :> Int[ExpandTr
ig[sin[d + e*x]^m*(a + b*sin[d + e*x]^n + c*sin[d + e*x]^(2*n))^p, x], x] /
; FreeQ[{a, b, c, d, e}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && Integ
ersQ[m, n, p]
```

rule 4894
```
Int[(u_)*((a_)*(F_)[(d_.) + (e_.)*(x_)]^(p_.) + (b_.)*(F_)[(d_.) + (e_.)*(x
_)]^(q_.) + (c_.)*(F_)[(d_.) + (e_.)*(x_)]^(r_.))^(n_.), x_Symbol] :> Int[A
ctivateTrig[u*F[d + e*x]^(n*p)*(a + b*F[d + e*x]^(q - p) + c*F[d + e*x]^(r
- p))^n], x] /; FreeQ[{a, b, c, d, e, p, q, r}, x] && InertTrigQ[F] && Inte
gerQ[n] && PosQ[q - p] && PosQ[r - p]
```

3.937.4 Maple [A] (verified)

Time = 3.72 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.63

method	result
parallelrisch	$\frac{(560a^2+1400ac+700b^2+735c^2)\cos(3dx+3c)+(-168ac-84b^2-147c^2)\cos(5dx+5c)-3360b(a+\frac{15c}{16})\sin(2dx+2c)+420b\ldots}{\ldots}$
parts	$-\frac{a^2\left(2+\sin(dx+c)^2\right)\cos(dx+c)}{3d}-\frac{c^2\left(\frac{16}{5}+\sin(dx+c)^6+\frac{6\sin(dx+c)^4}{5}+\frac{8\sin(dx+c)^2}{5}\right)\cos(dx+c)}{7d}-\frac{(2ac+b^2)\left(\frac{8}{3}+\sin\ldots\right.}{\ldots}$
derivativedivides	$\frac{-\frac{c^2\left(\frac{16}{5}+\sin(dx+c)^6+\frac{6\sin(dx+c)^4}{5}+\frac{8\sin(dx+c)^2}{5}\right)\cos(dx+c)}{7}+2bc\left(-\frac{\left(\sin(dx+c)^5+\frac{5\sin(dx+c)^3}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{6}+\ldots\right.}{\ldots}$
default	$\frac{-\frac{c^2\left(\frac{16}{5}+\sin(dx+c)^6+\frac{6\sin(dx+c)^4}{5}+\frac{8\sin(dx+c)^2}{5}\right)\cos(dx+c)}{7}+2bc\left(-\frac{\left(\sin(dx+c)^5+\frac{5\sin(dx+c)^3}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{6}+\ldots\right.}{\ldots}$
risch	$\frac{3xab}{4}+\frac{5bcx}{8}-\frac{3a^2\cos(dx+c)}{4d}-\frac{5\cos(dx+c)ac}{4d}-\frac{5\cos(dx+c)b^2}{8d}-\frac{35c^2\cos(dx+c)}{64d}+\frac{c^2\cos(7dx+7c)}{448d}-\frac{bc\,s\ldots}{\ldots}$
norman	$\frac{-\frac{140a^2+224ac+112b^2+96c^2}{105d}+\frac{b(6a+5c)x}{8}-\frac{4a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{10}}{d}-\frac{(52a^2+64ac+32b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^8}{3d}-\frac{(88a^2+160ac+80b^2+96c^2)\ldots}{3d}\ldots}{\ldots}$

input
```
int(sin(d*x+c)*(sin(d*x+c)*a+b*sin(d*x+c)^2+c*sin(d*x+c)^3)^2,x,method=_RE
TURNVERBOSE)
```

output
```
1/6720*((560*a^2+1400*a*c+700*b^2+735*c^2)*cos(3*d*x+3*c)+(-168*a*c-84*b^2
-147*c^2)*cos(5*d*x+5*c)-3360*b*(a+15/16*c)*sin(2*d*x+2*c)+420*b*(a+3/2*c)
*sin(4*d*x+4*c)+15*c^2*cos(7*d*x+7*c)-70*b*c*sin(6*d*x+6*c)+(-5040*a^2-840
0*a*c-4200*b^2-3675*c^2)*cos(d*x+c)-3072*c^2+(4200*b*d*x-7168*a)*c+5040*x*
a*b*d-4480*a^2-3584*b^2)/d
```

3.937.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.56

$$\int \sin(c+dx)\left(a\sin(c+dx)+b\sin^2(c+dx)+c\sin^3(c+dx)\right)^2\,dx$$
$$= \frac{120\,c^2\cos\left(dx+c\right)^7-168\left(b^2+2\,ac+3\,c^2\right)\cos\left(dx+c\right)^5+280\left(a^2+2\,b^2+4\,ac+3\,c^2\right)\cos\left(dx+c\right)^3+10}{}$$

input
```
integrate(sin(d*x+c)*(a*sin(d*x+c)+b*sin(d*x+c)^2+c*sin(d*x+c)^3)^2,x, alg
orithm="fricas")
```

output
```
1/840*(120*c^2*cos(d*x + c)^7 - 168*(b^2 + 2*a*c + 3*c^2)*cos(d*x + c)^5 +
 280*(a^2 + 2*b^2 + 4*a*c + 3*c^2)*cos(d*x + c)^3 + 105*(6*a*b + 5*b*c)*d*
x - 840*(a^2 + b^2 + 2*a*c + c^2)*cos(d*x + c) - 35*(8*b*c*cos(d*x + c)^5
- 2*(6*a*b + 13*b*c)*cos(d*x + c)^3 + 3*(10*a*b + 11*b*c)*cos(d*x + c))*si
n(d*x + c))/d
```

3.937.6 Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.88

$$\int \sin(c+dx)\left(a\sin(c+dx)+b\sin^2(c+dx)+c\sin^3(c+dx)\right)^2\,dx$$
$$= \begin{cases} -\frac{a^2\sin^2(c+dx)\cos(c+dx)}{d}-\frac{2a^2\cos^3(c+dx)}{3d}+\frac{3abx\sin^4(c+dx)}{4}+\frac{3abx\sin^2(c+dx)\cos^2(c+dx)}{2}+\frac{3abx\cos^4(c+dx)}{4}-\frac{5ab\sin^3(c+}{} \\ x\left(a\sin(c)+b\sin^2(c)+c\sin^3(c)\right)^2\sin(c) \end{cases}$$

input
```
integrate(sin(d*x+c)*(a*sin(d*x+c)+b*sin(d*x+c)**2+c*sin(d*x+c)**3)**2,x)
```

output
```
Piecewise((-a**2*sin(c + d*x)**2*cos(c + d*x)/d - 2*a**2*cos(c + d*x)**3/(
3*d) + 3*a*b*x*sin(c + d*x)**4/4 + 3*a*b*x*sin(c + d*x)**2*cos(c + d*x)**2
/2 + 3*a*b*x*cos(c + d*x)**4/4 - 5*a*b*sin(c + d*x)**3*cos(c + d*x)/(4*d)
- 3*a*b*sin(c + d*x)*cos(c + d*x)**3/(4*d) - 2*a*c*sin(c + d*x)**4*cos(c +
 d*x)/d - 8*a*c*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - 16*a*c*cos(c + d*x
)**5/(15*d) - b**2*sin(c + d*x)**4*cos(c + d*x)/d - 4*b**2*sin(c + d*x)**2
*cos(c + d*x)**3/(3*d) - 8*b**2*cos(c + d*x)**5/(15*d) + 5*b*c*x*sin(c + d
*x)**6/8 + 15*b*c*x*sin(c + d*x)**4*cos(c + d*x)**2/8 + 15*b*c*x*sin(c + d
*x)**2*cos(c + d*x)**4/8 + 5*b*c*x*cos(c + d*x)**6/8 - 11*b*c*sin(c + d*x)
**5*cos(c + d*x)/(8*d) - 5*b*c*sin(c + d*x)**3*cos(c + d*x)**3/(3*d) - 5*b
*c*sin(c + d*x)*cos(c + d*x)**5/(8*d) - c**2*sin(c + d*x)**6*cos(c + d*x)/
d - 2*c**2*sin(c + d*x)**4*cos(c + d*x)**3/d - 8*c**2*sin(c + d*x)**2*cos(
c + d*x)**5/(5*d) - 16*c**2*cos(c + d*x)**7/(35*d), Ne(d, 0)), (x*(a*sin(c
) + b*sin(c)**2 + c*sin(c)**3)**2*sin(c), True))
```

3.937.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.76

$$\int \sin(c+dx)\left(a\sin(c+dx)+b\sin^2(c+dx)+c\sin^3(c+dx)\right)^2\,dx$$
$$=\frac{1120\left(\cos\left(dx+c\right)^3-3\,\cos\left(dx+c\right)\right)a^2+210\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)-8\,\sin\left(2\,dx+2\,c\right)\right)ab-22\cdots}{}$$

input
```
integrate(sin(d*x+c)*(a*sin(d*x+c)+b*sin(d*x+c)^2+c*sin(d*x+c)^3)^2,x, alg
orithm="maxima")
```

output
```
1/3360*(1120*(cos(d*x + c)^3 - 3*cos(d*x + c))*a^2 + 210*(12*d*x + 12*c +
sin(4*d*x + 4*c) - 8*sin(2*d*x + 2*c))*a*b - 224*(3*cos(d*x + c)^5 - 10*co
s(d*x + c)^3 + 15*cos(d*x + c))*b^2 - 448*(3*cos(d*x + c)^5 - 10*cos(d*x +
 c)^3 + 15*cos(d*x + c))*a*c + 35*(4*sin(2*d*x + 2*c)^3 + 60*d*x + 60*c +
9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*b*c + 96*(5*cos(d*x + c)^7 - 21*
cos(d*x + c)^5 + 35*cos(d*x + c)^3 - 35*cos(d*x + c))*c^2)/d
```

3.937.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.65

$$\int \sin(c+dx)\left(a\sin(c+dx)+b\sin^2(c+dx)+c\sin^3(c+dx)\right)^2\,dx$$
$$=\frac{1}{8}(6ab+5bc)x+\frac{c^2\cos(7dx+7c)}{448d}-\frac{bc\sin(6dx+6c)}{96d}-\frac{(4b^2+8ac+7c^2)\cos(5dx+5c)}{320d}+\frac{(16a^2+20b^2+40ac+21c^2)\cos(3dx+3c)}{192d}-\frac{(48a^2+40b^2+80ac+35c^2)\cos(dx+c)}{64d}+\frac{(2ab+3bc)\sin(4dx+4c)}{32d}-\frac{(16ab+15bc)\sin(2dx+2c)}{32d}$$

input
```
integrate(sin(d*x+c)*(a*sin(d*x+c)+b*sin(d*x+c)^2+c*sin(d*x+c)^3)^2,x, alg
orithm="giac")
```

output
```
1/8*(6*a*b + 5*b*c)*x + 1/448*c^2*cos(7*d*x + 7*c)/d - 1/96*b*c*sin(6*d*x
+ 6*c)/d - 1/320*(4*b^2 + 8*a*c + 7*c^2)*cos(5*d*x + 5*c)/d + 1/192*(16*a^
2 + 20*b^2 + 40*a*c + 21*c^2)*cos(3*d*x + 3*c)/d - 1/64*(48*a^2 + 40*b^2 +
 80*a*c + 35*c^2)*cos(d*x + c)/d + 1/32*(2*a*b + 3*b*c)*sin(4*d*x + 4*c)/d
 - 1/32*(16*a*b + 15*b*c)*sin(2*d*x + 2*c)/d
```

3.937.9 Mupad [B] (verification not implemented)

Time = 28.97 (sec) , antiderivative size = 456, normalized size of antiderivative = 1.58

$$\int \sin(c+dx)\left(a\sin(c+dx)+b\sin^2(c+dx)+c\sin^3(c+dx)\right)^2\,dx$$
$$=\frac{b\,\mathrm{atan}\left(\frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(6a+5c)}{4\left(\frac{3ab}{2}+\frac{5bc}{4}\right)}\right)(6a+5c)}{4d}-\frac{b\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)-\frac{dx}{2}\right)(6a+5c)}{4d}-\frac{\frac{32ac}{15}-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}\left(\frac{3ab}{2}+\frac{5bc}{4}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}\left(10ab+\frac{25bc}{3}\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\left(10ab+\frac{25bc}{3}\right)+\mathrm{ta}}{}$$

input
```
int(sin(c + d*x)*(a*sin(c + d*x) + b*sin(c + d*x)^2 + c*sin(c + d*x)^3)^2,
x)
```

output
```
(b*atan((b*tan(c/2 + (d*x)/2)*(6*a + 5*c))/(4*((3*a*b)/2 + (5*b*c)/4)))*(6
*a + 5*c))/(4*d) - (b*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2)*(6*a + 5*c))/(4
*d) - ((32*a*c)/15 - tan(c/2 + (d*x)/2)^13*((3*a*b)/2 + (5*b*c)/4) + tan(c
/2 + (d*x)/2)^3*(10*a*b + (25*b*c)/3) - tan(c/2 + (d*x)/2)^11*(10*a*b + (2
5*b*c)/3) + tan(c/2 + (d*x)/2)^5*((31*a*b)/2 + (283*b*c)/12) - tan(c/2 + (
d*x)/2)^9*((31*a*b)/2 + (283*b*c)/12) + 4*a^2*tan(c/2 + (d*x)/2)^10 + tan(
c/2 + (d*x)/2)^8*((64*a*c)/3 + (52*a^2)/3 + (32*b^2)/3) + tan(c/2 + (d*x)/
2)^6*((160*a*c)/3 + (88*a^2)/3 + (80*b^2)/3 + 32*c^2) + tan(c/2 + (d*x)/2)
^2*((224*a*c)/15 + (28*a^2)/3 + (112*b^2)/15 + (32*c^2)/5) + tan(c/2 + (d*
x)/2)^4*((224*a*c)/5 + 24*a^2 + (112*b^2)/5 + (96*c^2)/5) + (4*a^2)/3 + (1
6*b^2)/15 + (32*c^2)/35 + tan(c/2 + (d*x)/2)*((3*a*b)/2 + (5*b*c)/4))/(d*(
7*tan(c/2 + (d*x)/2)^2 + 21*tan(c/2 + (d*x)/2)^4 + 35*tan(c/2 + (d*x)/2)^6
 + 35*tan(c/2 + (d*x)/2)^8 + 21*tan(c/2 + (d*x)/2)^10 + 7*tan(c/2 + (d*x)/
2)^12 + tan(c/2 + (d*x)/2)^14 + 1))
```

3.937. $\int \sin(c+dx)\left(a\sin(c+dx)+b\sin^2(c+dx)+c\sin^3(c+dx)\right)^2\,dx$

3.938 $\int \sin(c+dx)\left(a+\frac{b}{\sqrt{\sin(c+dx)}}+c\sin(c+dx)\right)dx$

3.938.1 Optimal result

Integrand size = 29, antiderivative size = 61

$$\int \sin(c+dx)\left(a+\frac{b}{\sqrt{\sin(c+dx)}}+c\sin(c+dx)\right)dx = \frac{cx}{2}-\frac{a\cos(c+dx)}{d}+\frac{2bE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{d}-\frac{c\cos(c+dx)\sin(c+dx)}{2d}$$

output
```
1/2*c*x-a*cos(d*x+c)/d-2*b*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1
/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))/d-1/2*c*cos(d*
x+c)*sin(d*x+c)/d
```

3.938.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.90

$$\int \sin(c+dx)\left(a+\frac{b}{\sqrt{\sin(c+dx)}}+c\sin(c+dx)\right)dx = \frac{-4a\cos(c+dx)-8bE\left(\frac{1}{4}(-2c+\pi-2dx)\middle|2\right)+c(2c+2dx-\sin(2(c+dx)))}{4d}$$

input
```
Integrate[Sin[c + d*x]*(a + b/Sqrt[Sin[c + d*x]] + c*Sin[c + d*x]),x]
```

output
```
(-4*a*Cos[c + d*x] - 8*b*EllipticE[(-2*c + Pi - 2*d*x)/4, 2] + c*(2*c + 2*
d*x - Sin[2*(c + d*x)]))/(4*d)
```

3.938.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {3042, 4895, 3042, 4901, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(c+dx)\left(a+\frac{b}{\sqrt{\sin(c+dx)}}+c\sin(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(c+dx)\left(a+\frac{b}{\sqrt{\sin(c+dx)}}+c\sin(c+dx)\right)dx$$

$$\Big\downarrow \text{4895}$$

$$\int \sqrt{\sin(c+dx)}\left(a\sqrt{\sin(c+dx)}+b+c\sin^{\frac{3}{2}}(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin(c+dx)}\left(a\sqrt{\sin(c+dx)}+b+c\sin(c+dx)^{3/2}\right)dx$$

$$\Big\downarrow \text{4901}$$

$$\int \left(a\sin(c+dx)+b\sqrt{\sin(c+dx)}+c\sin^2(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\cos(c+dx)}{d}+\frac{2bE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)|2\right)}{d}-\frac{c\sin(c+dx)\cos(c+dx)}{2d}+\frac{cx}{2}$$

input
```
Int[Sin[c + d*x]*(a + b/Sqrt[Sin[c + d*x]] + c*Sin[c + d*x]),x]
```

output
```
(c*x)/2 - (a*Cos[c + d*x])/d + (2*b*EllipticE[(c - Pi/2 + d*x)/2, 2])/d -
(c*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

3.938.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4895
```
Int[(u_)*((a_) + (b_.)*(F_)[(d_.) + (e_.)*(x_)]^(p_.) + (c_.)*(F_)[(d_.) +
(e_.)*(x_)]^(q_.))^(n_.), x_Symbol] :> Int[ActivateTrig[u*F[d + e*x]^(n*p)*
(b + a/F[d + e*x]^p + c*F[d + e*x]^(q - p))^n], x] /; FreeQ[{a, b, c, d, e,
 p, q}, x] && InertTrigQ[F] && IntegerQ[n] && NegQ[p]
```

rule 4901
```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

3.938.4 Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.16

method	result
default	$-\frac{a\cos(dx+c)}{d}+\frac{c\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{b\sqrt{\sin(dx+c)+1}\,\sqrt{-2\sin(dx+c)+2}\,\sqrt{-\sin(dx+c)}\,\left(2\,\mathrm{EllipticE}\left(\sqrt{\sin(dx+\right.\right.}{\cos(dx+c)\sqrt{\sin(dx+c)}\,d}$
parts	$-\frac{a\cos(dx+c)}{d}+\frac{c\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{b\sqrt{\sin(dx+c)+1}\,\sqrt{-2\sin(dx+c)+2}\,\sqrt{-\sin(dx+c)}\,\left(2\,\mathrm{EllipticE}\left(\sqrt{\sin(dx+\right.\right.}{\cos(dx+c)\sqrt{\sin(dx+c)}\,d}$

input
```
int(sin(d*x+c)*(a+c*sin(d*x+c)+b/sin(d*x+c)^(1/2)),x,method=_RETURNVERBOSE
)
```

output
```
-a*cos(d*x+c)/d+c/d*(-1/2*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)-b*(sin(d*x+
c)+1)^(1/2)*(-2*sin(d*x+c)+2)^(1/2)*(-sin(d*x+c))^(1/2)*(2*EllipticE((sin(
d*x+c)+1)^(1/2),1/2*2^(1/2))-EllipticF((sin(d*x+c)+1)^(1/2),1/2*2^(1/2)))/
cos(d*x+c)/sin(d*x+c)^(1/2)/d
```

3.938.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.77

$$\int \sin(c+dx)\left(a+\frac{b}{\sqrt{\sin(c+dx)}}+c\sin(c+dx)\right)\,dx =$$

$$-\frac{c\cos\left(dx+c\right)\sin\left(dx+c\right)-2i\sqrt{2}\sqrt{-i}b\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,\cos\left(dx+c\right)+i}{}$$

input
```
integrate(sin(d*x+c)*(a+c*sin(d*x+c)+b/sin(d*x+c)^(1/2)),x, algorithm="fri
cas")
```

output
```
-1/2*(c*cos(d*x + c)*sin(d*x + c) - 2*I*sqrt(2)*sqrt(-I)*b*weierstrassZeta
(4, 0, weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c))) + 2*I*sqr
t(2)*sqrt(I)*b*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(d*x + c
) - I*sin(d*x + c))) - c*arctan(sin(d*x + c)/cos(d*x + c)) + 2*a*cos(d*x +
 c))/d
```

3.938.6 Sympy [F]

$$\int \sin(c+dx)\left(a+\frac{b}{\sqrt{\sin(c+dx)}}+c\sin(c+dx)\right)\,dx = \int \left(a\sqrt{\sin\left(c+dx\right)}+b+c\sin^{\frac{3}{2}}\left(c+dx\right)\right)\sqrt{\sin\left(c+dx\right)}\,dx$$

input
```
integrate(sin(d*x+c)*(a+c*sin(d*x+c)+b/sin(d*x+c)**(1/2)),x)
```

output
```
Integral((a*sqrt(sin(c + d*x)) + b + c*sin(c + d*x)**(3/2))*sqrt(sin(c + d
*x)), x)
```

3.938.7 Maxima [F]

$$\int \sin(c+dx)\left(a+\frac{b}{\sqrt{\sin(c+dx)}}+c\sin(c+dx)\right)\,dx$$
$$=\int \left(c\sin\left(dx+c\right)+a+\frac{b}{\sqrt{\sin\left(dx+c\right)}}\right)\sin\left(dx+c\right)\,dx$$

input
```
integrate(sin(d*x+c)*(a+c*sin(d*x+c)+b/sin(d*x+c)^(1/2)),x, algorithm="max
ima")
```

output
```
1/4*(2*c*d*x - 4*a*cos(d*x + c) + 2*d*integrate(-(((b*cos(3/2*d*x + 3/2*c)
 - b*cos(1/2*d*x + 1/2*c) - b*sin(3/2*d*x + 3/2*c) - b*sin(1/2*d*x + 1/2*c
))*cos(1/2*arctan2(sin(d*x + c), -cos(d*x + c) + 1)) - (b*cos(3/2*d*x + 3/
2*c) - b*cos(1/2*d*x + 1/2*c) + b*sin(3/2*d*x + 3/2*c) + b*sin(1/2*d*x + 1
/2*c))*sin(1/2*arctan2(sin(d*x + c), -cos(d*x + c) + 1)))*cos(1/2*arctan2(
sin(d*x + c), cos(d*x + c) + 1)) + ((b*cos(3/2*d*x + 3/2*c) - b*cos(1/2*d*
x + 1/2*c) + b*sin(3/2*d*x + 3/2*c) + b*sin(1/2*d*x + 1/2*c))*cos(1/2*arct
an2(sin(d*x + c), -cos(d*x + c) + 1)) + (b*cos(3/2*d*x + 3/2*c) - b*cos(1/
2*d*x + 1/2*c) - b*sin(3/2*d*x + 3/2*c) - b*sin(1/2*d*x + 1/2*c))*sin(1/2*
arctan2(sin(d*x + c), -cos(d*x + c) + 1)))*sin(1/2*arctan2(sin(d*x + c), c
os(d*x + c) + 1)))/((cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(d*x + c) + 1)
^(1/4)*(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*cos(d*x + c) + 1)^(1/4)), x) -
 c*sin(2*d*x + 2*c))/d
```

3.938.8 Giac [F]

$$\int \sin(c+dx)\left(a+\frac{b}{\sqrt{\sin(c+dx)}}+c\sin(c+dx)\right)\,dx$$
$$=\int \left(c\sin\left(dx+c\right)+a+\frac{b}{\sqrt{\sin\left(dx+c\right)}}\right)\sin\left(dx+c\right)\,dx$$

input
```
integrate(sin(d*x+c)*(a+c*sin(d*x+c)+b/sin(d*x+c)^(1/2)),x, algorithm="gia
c")
```

output
```
integrate((c*sin(d*x + c) + a + b/sqrt(sin(d*x + c)))*sin(d*x + c), x)
```

3.938.9 Mupad [B] (verification not implemented)

Time = 27.98 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \sin(c+dx)\left(a+\frac{b}{\sqrt{\sin(c+dx)}}+c\sin(c+dx)\right)\,dx$$
$$=\frac{c\,x}{2}-\frac{c\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}-\frac{a\,\cos\left(c+d\,x\right)}{d}+\frac{2\,b\,\mathrm{E}\left(\frac{c}{2}-\frac{\pi}{4}+\frac{d\,x}{2}\middle|2\right)}{d}$$

input
```
int(sin(c + d*x)*(a + c*sin(c + d*x) + b/sin(c + d*x)^(1/2)),x)
```

output
```
(c*x)/2 - (c*sin(2*c + 2*d*x))/(4*d) - (a*cos(c + d*x))/d + (2*b*ellipticE
(c/2 - pi/4 + (d*x)/2, 2))/d
```

3.939 $\int \sin(c+dx) \left(a+\frac{b}{\sqrt{\sin(c+dx)}}+c\sin(c+dx)\right)^2 dx$

3.939.1 Optimal result

Integrand size = 31, antiderivative size = 148

$$\begin{aligned}\int \sin(c+dx) \left(a+\frac{b}{\sqrt{\sin(c+dx)}}+c\sin(c+dx)\right)^2 dx &= b^2x+acx-\frac{a^2\cos(c+dx)}{d}-\frac{c^2\cos(c+dx)}{d}+\frac{c^2\cos^3(c+dx)}{3d}\\ &+\frac{4abE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{d}+\frac{4bc\,\text{EllipticF}\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right),2\right)}{3d}\\ &-\frac{4bc\cos(c+dx)\sqrt{\sin(c+dx)}}{3d}-\frac{ac\cos(c+dx)\sin(c+dx)}{d}\end{aligned}$$

output
```
b^2*x+a*c*x-a^2*cos(d*x+c)/d-c^2*cos(d*x+c)/d+1/3*c^2*cos(d*x+c)^3/d-4*a*b
*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(c
os(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))/d-4/3*b*c*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^
(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2
))/d-a*c*cos(d*x+c)*sin(d*x+c)/d-4/3*b*c*cos(d*x+c)*sin(d*x+c)^(1/2)/d
```

3.939.2 Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.93

$$\int \sin(c+dx)\left(a+\frac{b}{\sqrt{\sin(c+dx)}}+c\sin(c+dx)\right)^2 dx$$
$$= \frac{12b^2c+12ac^2+12b^2dx+12acdx-12a^2\cos(c+dx)-9c^2\cos(c+dx)+c^2\cos(3(c+dx))-48abE\left(\frac{1}{4}(-\right.}{}$$

input
```
Integrate[Sin[c + d*x]*(a + b/Sqrt[Sin[c + d*x]] + c*Sin[c + d*x])^2,x]
```

output
```
(12*b^2*c + 12*a*c^2 + 12*b^2*d*x + 12*a*c*d*x - 12*a^2*Cos[c + d*x] - 9*c
^2*Cos[c + d*x] + c^2*Cos[3*(c + d*x)] - 48*a*b*EllipticE[(-2*c + Pi - 2*d
*x)/4, 2] - 16*b*c*EllipticF[(-2*c + Pi - 2*d*x)/4, 2] - 16*b*c*Cos[c + d*
x]*Sqrt[Sin[c + d*x]] - 6*a*c*Sin[2*(c + d*x)])/(12*d)
```

3.939.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3042, 4895, 3042, 4901, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(c+dx)\left(a+\frac{b}{\sqrt{\sin(c+dx)}}+c\sin(c+dx)\right)^2 dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sin(c+dx)\left(a+\frac{b}{\sqrt{\sin(c+dx)}}+c\sin(c+dx)\right)^2 dx$$
$$\Big\downarrow \text{4895}$$
$$\int \left(a\sqrt{\sin(c+dx)}+b+c\sin^{\frac{3}{2}}(c+dx)\right)^2 dx$$
$$\Big\downarrow \text{3042}$$
$$\int \left(a\sqrt{\sin(c+dx)}+b+c\sin(c+dx)^{3/2}\right)^2 dx$$

$$\downarrow \text{4901}$$

$$\int \left(a^2\sin(c+dx)+2ab\sqrt{\sin(c+dx)}+2ac\sin^2(c+dx)+b^2+2bc\sin^{\frac{3}{2}}(c+dx)+c^2\sin^3(c+dx)\right)dx$$

$$\downarrow \text{2009}$$

$$-\frac{a^2\cos(c+dx)}{d}+\frac{4abE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{d}-\frac{ac\sin(c+dx)\cos(c+dx)}{d}+acx+b^2x+\frac{4bc\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{3d}-\frac{4bc\sqrt{\sin(c+dx)}\cos(c+dx)}{3d}+\frac{c^2\cos^3(c+dx)}{3d}-\frac{c^2\cos(c+dx)}{d}$$

input
```
Int[Sin[c + d*x]*(a + b/Sqrt[Sin[c + d*x]] + c*Sin[c + d*x])^2,x]
```

output
```
b^2*x + a*c*x - (a^2*Cos[c + d*x])/d - (c^2*Cos[c + d*x])/d + (c^2*Cos[c +
 d*x]^3)/(3*d) + (4*a*b*EllipticE[(c - Pi/2 + d*x)/2, 2])/d + (4*b*c*Ellip
ticF[(c - Pi/2 + d*x)/2, 2])/(3*d) - (4*b*c*Cos[c + d*x]*Sqrt[Sin[c + d*x]
])/(3*d) - (a*c*Cos[c + d*x]*Sin[c + d*x])/d
```

3.939.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4895
```
Int[(u_)*((a_) + (b_.)*(F_)[(d_.) + (e_.)*(x_)]^(p_.) + (c_.)*(F_)[(d_.) +
(e_.)*(x_)]^(q_.))^(n_.), x_Symbol] :> Int[ActivateTrig[u*F[d + e*x]^(n*p)*
(b + a/F[d + e*x]^p + c*F[d + e*x]^(q - p))^n], x] /; FreeQ[{a, b, c, d, e,
 p, q}, x] && InertTrigQ[F] && IntegerQ[n] && NegQ[p]
```

rule 4901
```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

3.939.4 Maple [A] (verified)

Time = 2.60 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.73

method	result
parts	$x\,b^{2}-\frac{c^{2}\left(2+\sin(dx+c)^{2}\right)\cos(dx+c)}{3d}-\frac{a^{2}\cos(dx+c)}{d}+\frac{2ac\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2ab\sqrt{\sin(dx+c)+1}\,\sqrt{-2\sin(d}}{}$
default	$x\,b^{2}-\frac{c^{2}\left(2+\sin(dx+c)^{2}\right)\cos(dx+c)}{3d}-\frac{a^{2}\cos(dx+c)}{d}+\frac{2ac\left(-\frac{\sin(dx+c)\cos(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2b\left(6a\sqrt{\sin(dx+c)+1}\,\sqrt{-2\sin}\right.}{}$

input
```
int(sin(d*x+c)*(a+c*sin(d*x+c)+b/sin(d*x+c)^(1/2))^2,x,method=_RETURNVERBO
SE)
```

output
```
x*b^2-1/3*c^2/d*(2+sin(d*x+c)^2)*cos(d*x+c)-a^2*cos(d*x+c)/d+2*a*c/d*(-1/2
*sin(d*x+c)*cos(d*x+c)+1/2*d*x+1/2*c)-2*a*b*(sin(d*x+c)+1)^(1/2)*(-2*sin(d
*x+c)+2)^(1/2)*(-sin(d*x+c))^(1/2)*(2*EllipticE((sin(d*x+c)+1)^(1/2),1/2*2
^(1/2))-EllipticF((sin(d*x+c)+1)^(1/2),1/2*2^(1/2)))/cos(d*x+c)/sin(d*x+c)
^(1/2)/d+2*c*b*(1/3*(sin(d*x+c)+1)^(1/2)*(-2*sin(d*x+c)+2)^(1/2)*(-sin(d*x
+c))^(1/2)*EllipticF((sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-2/3*cos(d*x+c)^2*si
n(d*x+c))/cos(d*x+c)/sin(d*x+c)^(1/2)/d
```

3.939.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.42

$$\int \sin(c+dx)\left(a+\frac{b}{\sqrt{\sin(c+dx)}}+c\sin(c+dx)\right)^2\,dx$$
$$=\frac{c^2\cos{(dx+c)}^3-3\,ac\cos{(dx+c)}\sin{(dx+c)}+2\,\sqrt{2}\sqrt{-i}bc\text{weierstrassPInverse}(4,0,\cos{(dx+c)}+i\,\sin}{}$$

input
```
integrate(sin(d*x+c)*(a+c*sin(d*x+c)+b/sin(d*x+c)^(1/2))^2,x, algorithm="f
ricas")
```

output
```
1/3*(c^2*cos(d*x + c)^3 - 3*a*c*cos(d*x + c)*sin(d*x + c) + 2*sqrt(2)*sqrt
(-I)*b*c*weierstrassPInverse(4, 0, cos(d*x + c) + I*sin(d*x + c)) + 2*sqrt
(2)*sqrt(I)*b*c*weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c)) +
 6*I*sqrt(2)*sqrt(-I)*a*b*weierstrassZeta(4, 0, weierstrassPInverse(4, 0,
cos(d*x + c) + I*sin(d*x + c))) - 6*I*sqrt(2)*sqrt(I)*a*b*weierstrassZeta(
4, 0, weierstrassPInverse(4, 0, cos(d*x + c) - I*sin(d*x + c))) - 4*b*c*co
s(d*x + c)*sqrt(sin(d*x + c)) + 3*(b^2 + a*c)*arctan(sin(d*x + c)/cos(d*x
+ c)) - 3*(a^2 + c^2)*cos(d*x + c))/d
```

3.939.6 Sympy [F]

$$\int \sin(c+dx)\left(a+\frac{b}{\sqrt{\sin(c+dx)}}+c\sin(c+dx)\right)^2\,dx$$
$$=\int \left(a+\frac{b}{\sqrt{\sin\left(c+dx\right)}}+c\sin\left(c+dx\right)\right)^2\sin\left(c+dx\right)dx$$

input
```
integrate(sin(d*x+c)*(a+c*sin(d*x+c)+b/sin(d*x+c)**(1/2))**2,x)
```

output
```
Integral((a + b/sqrt(sin(c + d*x)) + c*sin(c + d*x))**2*sin(c + d*x), x)
```

3.939.7 Maxima [F]

$$\int \sin(c+dx)\left(a+\frac{b}{\sqrt{\sin(c+dx)}}+c\sin(c+dx)\right)^2\,dx$$
$$=\int \left(c\sin\left(dx+c\right)+a+\frac{b}{\sqrt{\sin\left(dx+c\right)}}\right)^2\sin\left(dx+c\right)\,dx$$

input
```
integrate(sin(d*x+c)*(a+c*sin(d*x+c)+b/sin(d*x+c)^(1/2))^2,x, algorithm="m
axima")
```

output
```
1/12*(12*b^2*c + 12*(b^2 + a*c)*d*x + c^2*cos(3*d*x + 3*c) - 6*a*c*sin(2*d
*x + 2*c) - 3*sqrt(2)*d*integrate((((sqrt(2)*b*c*cos(5/2*d*x + 5/2*c) + sq
rt(2)*b*c*cos(3/2*d*x + 3/2*c) - 2*sqrt(2)*b*c*cos(1/2*d*x + 1/2*c) - 2*sq
rt(2)*a*b*sin(3/2*d*x + 3/2*c) - 2*sqrt(2)*a*b*sin(1/2*d*x + 1/2*c))*cos(1
/2*arctan2(sin(d*x + c), -cos(d*x + c) + 1)) - (2*sqrt(2)*a*b*cos(3/2*d*x
+ 3/2*c) - 2*sqrt(2)*a*b*cos(1/2*d*x + 1/2*c) + sqrt(2)*b*c*sin(5/2*d*x +
5/2*c) - sqrt(2)*b*c*sin(3/2*d*x + 3/2*c) - 2*sqrt(2)*b*c*sin(1/2*d*x + 1/
2*c))*sin(1/2*arctan2(sin(d*x + c), -cos(d*x + c) + 1)))*cos(1/2*arctan2(s
in(d*x + c), cos(d*x + c) + 1)) + ((2*sqrt(2)*a*b*cos(3/2*d*x + 3/2*c) - 2
*sqrt(2)*a*b*cos(1/2*d*x + 1/2*c) + sqrt(2)*b*c*sin(5/2*d*x + 5/2*c) - sqr
t(2)*b*c*sin(3/2*d*x + 3/2*c) - 2*sqrt(2)*b*c*sin(1/2*d*x + 1/2*c))*cos(1/
2*arctan2(sin(d*x + c), -cos(d*x + c) + 1)) + (sqrt(2)*b*c*cos(5/2*d*x + 5
/2*c) + sqrt(2)*b*c*cos(3/2*d*x + 3/2*c) - 2*sqrt(2)*b*c*cos(1/2*d*x + 1/2
*c) - 2*sqrt(2)*a*b*sin(3/2*d*x + 3/2*c) - 2*sqrt(2)*a*b*sin(1/2*d*x + 1/2
*c))*sin(1/2*arctan2(sin(d*x + c), -cos(d*x + c) + 1)))*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c) + 1)))/((cos(d*x + c)^2 + sin(d*x + c)^2 + 2*cos(
d*x + c) + 1)^(1/4)*(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*cos(d*x + c) + 1)
^(1/4)), x) - 3*sqrt(2)*d*integrate((((2*sqrt(2)*a*b*cos(3/2*d*x + 3/2*c)
- 2*sqrt(2)*a*b*cos(1/2*d*x + 1/2*c) + sqrt(2)*b*c*sin(5/2*d*x + 5/2*c) -
sqrt(2)*b*c*sin(3/2*d*x + 3/2*c) - 2*sqrt(2)*b*c*sin(1/2*d*x + 1/2*c))*...
```

3.939.8 Giac [F]

$$\int \sin(c+dx)\left(a+\frac{b}{\sqrt{\sin(c+dx)}}+c\sin(c+dx)\right)^2 dx$$
$$=\int \left(c\sin(dx+c)+a+\frac{b}{\sqrt{\sin(dx+c)}}\right)^2 \sin(dx+c)\,dx$$

input
```
integrate(sin(d*x+c)*(a+c*sin(d*x+c)+b/sin(d*x+c)^(1/2))^2,x, algorithm="g
iac")
```

output
```
integrate((c*sin(d*x + c) + a + b/sqrt(sin(d*x + c)))^2*sin(d*x + c), x)
```

3.939.9 Mupad [B] (verification not implemented)

Time = 31.51 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.87

$$\int \sin(c+dx)\left(a+\frac{b}{\sqrt{\sin(c+dx)}}+c\sin(c+dx)\right)^2\,dx$$
$$= b^2\,x-\frac{a^2\,\cos\left(c+d\,x\right)}{d}-\frac{a\,c\,\left(\sin\left(2\,c+2\,d\,x\right)-2\,d\,x\right)}{2\,d}$$
$$+\frac{4\,a\,b\,\mathrm{E}\left(\frac{c}{2}-\frac{\pi}{4}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{c^2\,\cos\left(c+d\,x\right)\,\left({\cos\left(c+d\,x\right)}^2-3\right)}{3\,d}$$
$$-\frac{2\,b\,c\,\cos\left(c+d\,x\right)\,{\sin\left(c+d\,x\right)}^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{2};\ {\cos\left(c+d\,x\right)}^2\right)}{d\,{\left({\sin\left(c+d\,x\right)}^2\right)}^{5/4}}$$

input
```
int(sin(c + d*x)*(a + c*sin(c + d*x) + b/sin(c + d*x)^(1/2))^2,x)
```

output
```
b^2*x - (a^2*cos(c + d*x))/d - (a*c*(sin(2*c + 2*d*x) - 2*d*x))/(2*d) + (4
*a*b*ellipticE(c/2 - pi/4 + (d*x)/2, 2))/d + (c^2*cos(c + d*x)*(cos(c + d*
x)^2 - 3))/(3*d) - (2*b*c*cos(c + d*x)*sin(c + d*x)^(5/2)*hypergeom([-1/4,
 1/2], 3/2, cos(c + d*x)^2))/(d*(sin(c + d*x)^2)^(5/4))
```

3.940 $\int f^{a+bx}(\cos(c+dx)+i\sin(c+dx))^n\,dx$

3.940.1 Optimal result

Integrand size = 27, antiderivative size = 34

$$\int f^{a+bx}(\cos(c+dx)+i\sin(c+dx))^n\,dx=\frac{\left(e^{i(c+dx)}\right)^nf^{a+bx}}{idn+b\log(f)}$$

output
```
exp(I*(d*x+c))^n*f^(b*x+a)/(I*d*n+b*ln(f))
```

3.940.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.26

$$\int f^{a+bx}(\cos(c+dx)+i\sin(c+dx))^n\,dx=-\frac{if^{a+bx}(\cos(c+dx)+i\sin(c+dx))^n}{dn-ib\log(f)}$$

input
```
Integrate[f^(a + b*x)*(Cos[c + d*x] + I*Sin[c + d*x])^n,x]
```

output
```
((-I)*f^(a + b*x)*(Cos[c + d*x] + I*Sin[c + d*x])^n)/(d*n - I*b*Log[f])
```

3.940.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.82, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {5125, 2717, 2725, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+bx}(\cos(c+dx)+i\sin(c+dx))^n\,dx$$

$$\Big\downarrow \text{ 5125}$$

$$\int f^{a+bx}\left(e^{i(c+dx)}\right)^n dx$$

$$\Big\downarrow \text{ 2717}$$

$$e^{-in(c+dx)}\left(e^{i(c+dx)}\right)^n\int e^{in(c+dx)}f^{a+bx}dx$$

$$\Big\downarrow \text{ 2725}$$

$$e^{-in(c+dx)}\left(e^{i(c+dx)}\right)^n\int e^{icn+a\log(f)+x(idn+b\log(f))}dx$$

$$\Big\downarrow \text{ 2624}$$

$$\frac{f^a\left(e^{i(c+dx)}\right)^n\exp(x(b\log(f)+idn)-in(c+dx)+icn)}{b\log(f)+idn}$$

input
```
Int[f^(a + b*x)*(Cos[c + d*x] + I*Sin[c + d*x])^n,x]
```

output
```
(E^(I*c*n - I*n*(c + d*x) + x*(I*d*n + b*Log[f]))*(E^(I*(c + d*x)))^n*f^a)
/(I*d*n + b*Log[f])
```

3.940.3.1 Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 2717
```
Int[(u_.)*((a_.)*(F_)^(v_))^(n_), x_Symbol] :> Simp[(a*F^v)^n/F^(n*v)   Int
[u*F^(n*v), x], x] /; FreeQ[{F, a, n}, x] && !IntegerQ[n]
```

rule 2725
```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

rule 5125
```
Int[(u_.)*(Cos[v_]*(a_.) + (b_.)*Sin[v_])^(n_.), x_Symbol] :> Int[u*(a/E^((
a/b)*v))^n, x] /; FreeQ[{a, b, n}, x] && EqQ[a^2 + b^2, 0]
```

3.940.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

method	result	size
risch	$\frac{\left(e^{i(dx+c)}\right)^{n} f^{xb+a}}{idn+b\ln(f)}$	32
norman	$\frac{e^{(xb+a)\ln(f)} e^{n\ln\left(\frac{2i\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}+\frac{1-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\right)}}{idn+b\ln(f)}$	86

input
```
int(f^(b*x+a)*(I*sin(d*x+c)+cos(d*x+c))^n,x,method=_RETURNVERBOSE)
```

output
```
exp(I*(d*x+c))^n*f^(b*x+a)/(I*d*n+b*ln(f))
```

3.940.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int f^{a+bx}(\cos(c+dx)+i\sin(c+dx))^n\,dx=\frac{f^{bx+a}e^{(i\,dnx+i\,cn)}}{i\,dn+b\log{(f)}}$$

input
```
integrate(f^(b*x+a)*(cos(d*x+c)+I*sin(d*x+c))^n,x, algorithm="fricas")
```

output
```
f^(b*x + a)*e^(I*d*n*x + I*c*n)/(I*d*n + b*log(f))
```

3.940.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 73 vs. 2(26) = 52.

Time = 0.48 (sec) , antiderivative size = 73, normalized size of antiderivative = 2.15

$$\int f^{a+bx}(\cos(c+dx)+i\sin(c+dx))^n\,dx = \begin{cases} \frac{f^{a+bx}(i\sin(c+dx)+\cos(c+dx))^n}{b\log(f)+idn} & \text{for } b\neq -\frac{idn}{\log(f)} \\ f^{a-\frac{idnx}{\log(f)}}x(i\sin(c+dx)+\cos(c+dx))^n & \text{otherwise} \end{cases}$$

input
```
integrate(f**(b*x+a)*(cos(d*x+c)+I*sin(d*x+c))**n,x)
```

output
```
Piecewise((f**(a + b*x)*(I*sin(c + d*x) + cos(c + d*x))**n/(b*log(f) + I*d
*n), Ne(b, -I*d*n/log(f))), (f**(a - I*d*n*x/log(f))*x*(I*sin(c + d*x) + c
os(c + d*x))**n, True))
```

3.940.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.47

$$\int f^{a+bx}(\cos(c+dx)+i\sin(c+dx))^n\,dx = \frac{-i\,f^{bx}f^a\cos(dnx+cn)+f^{bx}f^a\sin(dnx+cn)}{dn-i\,b\log(f)}$$

input
```
integrate(f^(b*x+a)*(cos(d*x+c)+I*sin(d*x+c))^n,x, algorithm="maxima")
```

output
```
(-I*f^(b*x)*f^a*cos(d*n*x + c*n) + f^(b*x)*f^a*sin(d*n*x + c*n))/(d*n - I*
b*log(f))
```

3.940.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int f^{a+bx}(\cos(c+dx)+i\sin(c+dx))^n\,dx = \frac{f^a e^{(i\,dnx+bx\log(f)+i\,cn)}}{i\,dn+b\log(f)}$$

input
```
integrate(f^(b*x+a)*(cos(d*x+c)+I*sin(d*x+c))^n,x, algorithm="giac")
```

output
```
f^a*e^(I*d*n*x + b*x*log(f) + I*c*n)/(I*d*n + b*log(f))
```

3.940.9 Mupad [B] (verification not implemented)

Time = 27.67 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int f^{a+bx}(\cos(c+dx)+i\sin(c+dx))^n\,dx = -\frac{f^{a+b\,x}\,\left(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\right)^n\,1\mathrm{i}}{d\,n-b\,\ln(f)\,1\mathrm{i}}$$

input
```
int(f^(a + b*x)*(cos(c + d*x) + sin(c + d*x)*1i)^n,x)
```

output
```
-(f^(a + b*x)*exp(c*1i + d*x*1i)^n*1i)/(d*n - b*log(f)*1i)
```

3.941 $\int f^{a+bx}(\cos(c+dx)-i\sin(c+dx))^n\,dx$

3.941.1 Optimal result

Integrand size = 27, antiderivative size = 36

$$\int f^{a+bx}(\cos(c+dx)-i\sin(c+dx))^n\,dx=-\frac{\left(e^{-i(c+dx)}\right)^n f^{a+bx}}{idn-b\log(f)}$$

output
```
-exp(-I*(d*x+c))^n*f^(b*x+a)/(I*d*n-b*ln(f))
```

3.941.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.19

$$\int f^{a+bx}(\cos(c+dx)-i\sin(c+dx))^n\,dx=\frac{if^{a+bx}(\cos(c+dx)-i\sin(c+dx))^n}{dn+ib\log(f)}$$

input
```
Integrate[f^(a + b*x)*(Cos[c + d*x] - I*Sin[c + d*x])^n,x]
```

output
```
(I*f^(a + b*x)*(Cos[c + d*x] - I*Sin[c + d*x])^n)/(d*n + I*b*Log[f])
```

3.941.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.83, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {5125, 2717, 2725, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+bx}(\cos(c+dx)-i\sin(c+dx))^n\,dx$$

$$\Big\downarrow \text{5125}$$

$$\int f^{a+bx}\left(e^{-i(c+dx)}\right)^n dx$$

$$\Big\downarrow \text{2717}$$

$$e^{in(c+dx)}\left(e^{-i(c+dx)}\right)^n\int e^{-in(c+dx)}f^{a+bx}dx$$

$$\Big\downarrow \text{2725}$$

$$e^{in(c+dx)}\left(e^{-i(c+dx)}\right)^n\int \exp(-icn+a\log(f)-x(idn-b\log(f)))dx$$

$$\Big\downarrow \text{2624}$$

$$-\frac{f^a\left(e^{-i(c+dx)}\right)^n\exp(-x(-b\log(f)+idn)+in(c+dx)-icn)}{-b\log(f)+idn}$$

input
```
Int[f^(a + b*x)*(Cos[c + d*x] - I*Sin[c + d*x])^n,x]
```

output
```
-((E^((-I)*c*n + I*n*(c + d*x) - x*(I*d*n - b*Log[f]))*(E^((-I)*(c + d*x))
)^n*f^a)/(I*d*n - b*Log[f]))
```

3.941.3.1 Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 2717
```
Int[(u_.)*((a_.)*(F_)^(v_))^(n_), x_Symbol] :> Simp[(a*F^v)^n/F^(n*v)   Int
[u*F^(n*v), x], x] /; FreeQ[{F, a, n}, x] && !IntegerQ[n]
```

rule 2725
```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

rule 5125
```
Int[(u_.)*(Cos[v_]*(a_.) + (b_.)*Sin[v_])^(n_.), x_Symbol] :> Int[u*(a/E^((
a/b)*v))^n, x] /; FreeQ[{a, b, n}, x] && EqQ[a^2 + b^2, 0]
```

3.941.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

method	result	size
risch	$\frac{f^{xb+a}\left(\mathrm{e}^{i(dx+c)}\right)^{-n}}{-idn+b\ln(f)}$	34
norman	$-\frac{\mathrm{e}^{(xb+a)\ln(f)}\mathrm{e}^{n\ln\left(-\frac{2i\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}+\frac{1-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{1+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\right)}}{idn-b\ln(f)}$	88

input
```
int(f^(b*x+a)*(-I*sin(d*x+c)+cos(d*x+c))^n,x,method=_RETURNVERBOSE)
```

output
```
1/(-I*d*n+b*ln(f))*f^(b*x+a)*exp(I*(d*x+c))^(-n)
```

3.941.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int f^{a+bx}(\cos(c+dx)-i\sin(c+dx))^n\,dx=\frac{f^{bx+a}e^{(-i\,dnx-i\,cn)}}{-i\,dn+b\log(f)}$$

input
```
integrate(f^(b*x+a)*(cos(d*x+c)-I*sin(d*x+c))^n,x, algorithm="fricas")
```

output
```
f^(b*x + a)*e^(-I*d*n*x - I*c*n)/(-I*d*n + b*log(f))
```

3.941.6 Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(29) = 58.

Time = 0.51 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.97

$$\int f^{a+bx}(\cos(c+dx)-i\sin(c+dx))^n\,dx$$
$$=\begin{cases}\frac{f^{a+bx}(-i\sin{(c+dx)}+\cos{(c+dx)})^n}{b\log{(f)}-idn} & \text{for } b\neq\frac{idn}{\log{(f)}}\\ f^{a+\frac{idnx}{\log{(f)}}}x(-i\sin{(c+dx)}+\cos{(c+dx)})^n & \text{otherwise}\end{cases}$$

input
```
integrate(f**(b*x+a)*(cos(d*x+c)-I*sin(d*x+c))**n,x)
```

output
```
Piecewise((f**(a + b*x)*(-I*sin(c + d*x) + cos(c + d*x))**n/(b*log(f) - I*
d*n), Ne(b, I*d*n/log(f))), (f**(a + I*d*n*x/log(f))*x*(-I*sin(c + d*x) +
cos(c + d*x))**n, True))
```

3.941.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.72

$$\int f^{a+bx}(\cos(c+dx)-i\sin(c+dx))^n\,dx$$
$$=\frac{f^{bx}f^a\cos{(dnx)}-i\,f^{bx}f^a\sin{(dnx)}}{(-i\,dn+b\log{(f)})\cos{(cn)}+(dn+i\,b\log{(f)})\sin{(cn)}}$$

input
```
integrate(f^(b*x+a)*(cos(d*x+c)-I*sin(d*x+c))^n,x, algorithm="maxima")
```

output
```
(f^(b*x)*f^a*cos(d*n*x) - I*f^(b*x)*f^a*sin(d*n*x))/((-I*d*n + b*log(f))*c
os(c*n) + (d*n + I*b*log(f))*sin(c*n))
```

3.941.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int f^{a+bx}(\cos(c+dx)-i\sin(c+dx))^n\,dx=\frac{f^a e^{(-i\,dnx+bx\log(f)-i\,cn)}}{-i\,dn+b\log(f)}$$

input
```
integrate(f^(b*x+a)*(cos(d*x+c)-I*sin(d*x+c))^n,x, algorithm="giac")
```

output
```
f^a*e^(-I*d*n*x + b*x*log(f) - I*c*n)/(-I*d*n + b*log(f))
```

3.941.9 Mupad [B] (verification not implemented)

Time = 27.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int f^{a+bx}(\cos(c+dx)-i\sin(c+dx))^n\,dx=-\frac{f^{a+b\,x}\,\left(\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}\right)^n}{-b\,\ln(f)+d\,n\,1\mathrm{i}}$$

input
```
int(f^(a + b*x)*(cos(c + d*x) - sin(c + d*x)*1i)^n,x)
```

output
```
-(f^(a + b*x)*exp(- c*1i - d*x*1i)^n)/(d*n*1i - b*log(f))
```

3.942 $\int \frac{\cos^5(a+bx)-\sin^5(a+bx)}{\cos^5(a+bx)+\sin^5(a+bx)} \, dx$

3.942.1 Optimal result

Integrand size = 39, antiderivative size = 120

$$\begin{aligned}\int \frac{\cos^5(a+bx)-\sin^5(a+bx)}{\cos^5(a+bx)+\sin^5(a+bx)} \, dx = {} & \frac{\log(\cos(a+bx))}{b} + \frac{\log(1+\tan(a+bx))}{5b} \\ & - \frac{4\log\left(2-\left(1-\sqrt{5}\right)\tan(a+bx)+2\tan^2(a+bx)\right)}{5\left(1-\sqrt{5}\right)b} \\ & - \frac{4\log\left(2-\left(1+\sqrt{5}\right)\tan(a+bx)+2\tan^2(a+bx)\right)}{5\left(1+\sqrt{5}\right)b}\end{aligned}$$

output
```
ln(cos(b*x+a))/b+1/5*ln(1+tan(b*x+a))/b-4/5*ln(2-(-5^(1/2)+1)*tan(b*x+a)+2
*tan(b*x+a)^2)/b/(-5^(1/2)+1)-4/5*ln(2-(5^(1/2)+1)*tan(b*x+a)+2*tan(b*x+a)
^2)/b/(5^(1/2)+1)
```

3.942.2 Mathematica [A] (verified)

Time = 8.95 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.61

$$\int \frac{\cos^5(a+bx)-\sin^5(a+bx)}{\cos^5(a+bx)+\sin^5(a+bx)} \, dx$$
$$= \frac{\log(\cos(a+bx)+\sin(a+bx)) - \left(-1+\sqrt{5}\right)\log\left(1-\sqrt{5}+\sin(2(a+bx))\right) + \left(1+\sqrt{5}\right)\log\left(1+\sqrt{5}+\sin\right.}{5b}$$

input
```
Integrate[(Cos[a + b*x]^5 - Sin[a + b*x]^5)/(Cos[a + b*x]^5 + Sin[a + b*x]
^5),x]
```

output
```
(Log[Cos[a + b*x] + Sin[a + b*x]] - (-1 + Sqrt[5])*Log[1 - Sqrt[5] + Sin[2
*(a + b*x)]] + (1 + Sqrt[5])*Log[1 + Sqrt[5] + Sin[2*(a + b*x)]])/(5*b)
```

3.942.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {3042, 4889, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^5(a+bx)-\sin^5(a+bx)}{\sin^5(a+bx)+\cos^5(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(a+bx)^5-\sin(a+bx)^5}{\sin(a+bx)^5+\cos(a+bx)^5}dx$$

$$\Big\downarrow \text{4889}$$

$$\frac{\int \frac{1-\tan^5(a+bx)}{\tan^7(a+bx)+\tan^5(a+bx)+\tan^2(a+bx)+1}d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{2462}$$

$$\frac{\int \left(-\frac{\tan(a+bx)}{\tan^2(a+bx)+1}+\frac{1}{5(\tan(a+bx)+1)}+\frac{2(2\tan^3(a+bx)-4\tan^2(a+bx)+\tan(a+bx)+2)}{5(\tan^4(a+bx)-\tan^3(a+bx)+\tan^2(a+bx)-\tan(a+bx)+1)}\right)d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{2}\log\left(\tan^2(a+bx)+1\right)-\frac{4\log\left(2\tan^2(a+bx)-\left(1-\sqrt{5}\right)\tan(a+bx)+2\right)}{5\left(1-\sqrt{5}\right)}-\frac{4\log\left(2\tan^2(a+bx)-\left(1+\sqrt{5}\right)\tan(a+bx)+2\right)}{5\left(1+\sqrt{5}\right)}+\frac{1}{5}\log(\text{t}}{b}$$

input
```
Int[(Cos[a + b*x]^5 - Sin[a + b*x]^5)/(Cos[a + b*x]^5 + Sin[a + b*x]^5),x]
```

output
```
(Log[1 + Tan[a + b*x]]/5 - Log[1 + Tan[a + b*x]^2]/2 - (4*Log[2 - (1 - Sqr
t[5])*Tan[a + b*x] + 2*Tan[a + b*x]^2])/(5*(1 - Sqrt[5])) - (4*Log[2 - (1
+ Sqrt[5])*Tan[a + b*x] + 2*Tan[a + b*x]^2])/(5*(1 + Sqrt[5])))/b
```

3.942.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.942.4 Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{-\frac{4\left(\frac{\sqrt{5}}{4}-\frac{1}{4}\right)\ln\left(-\sqrt{5}\,\tan(xb+a)+2\tan(xb+a)^2-\tan(xb+a)+2\right)}{5}+\frac{4\left(\frac{\sqrt{5}}{4}+\frac{1}{4}\right)\ln\left(\sqrt{5}\,\tan(xb+a)+2\tan(xb+a)^2-\tan(xb+a)+2\right)}{5}+\ln}{b}$
default	$\frac{-\frac{4\left(\frac{\sqrt{5}}{4}-\frac{1}{4}\right)\ln\left(-\sqrt{5}\,\tan(xb+a)+2\tan(xb+a)^2-\tan(xb+a)+2\right)}{5}+\frac{4\left(\frac{\sqrt{5}}{4}+\frac{1}{4}\right)\ln\left(\sqrt{5}\,\tan(xb+a)+2\tan(xb+a)^2-\tan(xb+a)+2\right)}{5}+\ln}{b}$
risch	$-ix-\frac{2ia}{b}+\frac{\ln\left(e^{2i(xb+a)}+i\right)}{5b}+\frac{\ln\left(e^{4i(xb+a)}+2i\left(\sqrt{5}+1\right)e^{2i(xb+a)}-1\right)}{5b}+\frac{\ln\left(e^{4i(xb+a)}+2i\left(\sqrt{5}+1\right)e^{2i(xb+a)}-\right.}{5b}$

3.942. $\int \frac{\cos^5(a+bx)-\sin^5(a+bx)}{\cos^5(a+bx)+\sin^5(a+bx)}\,dx$

input
```
int((cos(b*x+a)^5-sin(b*x+a)^5)/(cos(b*x+a)^5+sin(b*x+a)^5),x,method=_RETU
RNVERBOSE)
```

output
```
1/b*(-4/5*(1/4*5^(1/2)-1/4)*ln(-5^(1/2)*tan(b*x+a)+2*tan(b*x+a)^2-tan(b*x+
a)+2)+4/5*(1/4*5^(1/2)+1/4)*ln(5^(1/2)*tan(b*x+a)+2*tan(b*x+a)^2-tan(b*x+a
)+2)+1/5*ln(1+tan(b*x+a))-1/2*ln(1+tan(b*x+a)^2))
```

3.942.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.25

$$\int \frac{\cos^5(a+bx)-\sin^5(a+bx)}{\cos^5(a+bx)+\sin^5(a+bx)}\,dx$$
$$= \frac{2\sqrt{5}\log\left(-\frac{2\cos(bx+a)^4-2\left(\sqrt{5}+1\right)\cos(bx+a)\sin(bx+a)-2\cos(bx+a)^2-\sqrt{5}-3}{\cos(bx+a)^4-\cos(bx+a)^2-\cos(bx+a)\sin(bx+a)+1}\right)+2\log\left(\cos(bx+a)^4-\cos(bx+a)^2-\right.}{10\,b}$$

input
```
integrate((cos(b*x+a)^5-sin(b*x+a)^5)/(cos(b*x+a)^5+sin(b*x+a)^5),x, algor
ithm="fricas")
```

output
```
1/10*(2*sqrt(5)*log(-(2*cos(b*x + a)^4 - 2*(sqrt(5) + 1)*cos(b*x + a)*sin(
b*x + a) - 2*cos(b*x + a)^2 - sqrt(5) - 3)/(cos(b*x + a)^4 - cos(b*x + a)^
2 - cos(b*x + a)*sin(b*x + a) + 1)) + 2*log(cos(b*x + a)^4 - cos(b*x + a)^
2 - cos(b*x + a)*sin(b*x + a) + 1) + log(2*cos(b*x + a)*sin(b*x + a) + 1))
/b
```

3.942.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. 2(100) = 200.

Time = 84.52 (sec) , antiderivative size = 379, normalized size of antiderivative = 3.16

$$\int \frac{\cos^5(a+bx)-\sin^5(a+bx)}{\cos^5(a+bx)+\sin^5(a+bx)}\,dx$$
$$= \begin{cases} -\frac{47\log(\sin(a+bx)+\cos(a+bx))}{-235b+105\sqrt{5}b}+\frac{21\sqrt{5}\log(\sin(a+bx)+\cos(a+bx))}{-235b+105\sqrt{5}b}-\frac{26\sqrt{5}\log\left(16\sin^2(a+bx)-8\sin(a+bx)\cos(a+bx)+8\sqrt{5}\sin(a+\right.}{-235b+105\sqrt{5}b} \\ \frac{x(-\sin^5(a)+\cos^5(a))}{\sin^5(a)+\cos^5(a)} \end{cases}$$

input
```
integrate((cos(b*x+a)**5-sin(b*x+a)**5)/(cos(b*x+a)**5+sin(b*x+a)**5),x)
```

output
```
Piecewise((-47*log(sin(a + b*x) + cos(a + b*x))/(-235*b + 105*sqrt(5)*b) +
 21*sqrt(5)*log(sin(a + b*x) + cos(a + b*x))/(-235*b + 105*sqrt(5)*b) - 26
*sqrt(5)*log(16*sin(a + b*x)**2 - 8*sin(a + b*x)*cos(a + b*x) + 8*sqrt(5)*
sin(a + b*x)*cos(a + b*x) + 16*cos(a + b*x)**2)/(-235*b + 105*sqrt(5)*b) +
 58*log(16*sin(a + b*x)**2 - 8*sin(a + b*x)*cos(a + b*x) + 8*sqrt(5)*sin(a
 + b*x)*cos(a + b*x) + 16*cos(a + b*x)**2)/(-235*b + 105*sqrt(5)*b) - 152*
log(16*sin(a + b*x)**2 - 8*sqrt(5)*sin(a + b*x)*cos(a + b*x) - 8*sin(a + b
*x)*cos(a + b*x) + 16*cos(a + b*x)**2)/(-235*b + 105*sqrt(5)*b) + 68*sqrt(
5)*log(16*sin(a + b*x)**2 - 8*sqrt(5)*sin(a + b*x)*cos(a + b*x) - 8*sin(a
+ b*x)*cos(a + b*x) + 16*cos(a + b*x)**2)/(-235*b + 105*sqrt(5)*b), Ne(b,
0)), (x*(-sin(a)**5 + cos(a)**5)/(sin(a)**5 + cos(a)**5), True))
```

3.942.7 Maxima [F]

$$\int \frac{\cos^5(a+bx)-\sin^5(a+bx)}{\cos^5(a+bx)+\sin^5(a+bx)}\,dx=\int \frac{\cos(bx+a)^5-\sin(bx+a)^5}{\cos(bx+a)^5+\sin(bx+a)^5}\,dx$$

input
```
integrate((cos(b*x+a)^5-sin(b*x+a)^5)/(cos(b*x+a)^5+sin(b*x+a)^5),x, algor
ithm="maxima")
```

output
```
integrate((cos(b*x + a)^5 - sin(b*x + a)^5)/(cos(b*x + a)^5 + sin(b*x + a)
^5), x)
```

3.942.8 Giac [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.07

$$\int \frac{\cos^5(a+bx)-\sin^5(a+bx)}{\cos^5(a+bx)+\sin^5(a+bx)}\,dx=$$
$$-\frac{2\sqrt{5}\log\left(-\frac{1}{2}\left(\sqrt{5}+1\right)\tan(bx+a)+\tan(bx+a)^2+1\right)-2\sqrt{5}\log\left(\frac{1}{2}\left(\sqrt{5}-1\right)\tan(bx+a)+\tan(bx+a)^2+\dots\right.}{}$$

input
```
integrate((cos(b*x+a)^5-sin(b*x+a)^5)/(cos(b*x+a)^5+sin(b*x+a)^5),x, algor
ithm="giac")
```

output
```
-1/10*(2*sqrt(5)*log(-1/2*(sqrt(5) + 1)*tan(b*x + a) + tan(b*x + a)^2 + 1)
 - 2*sqrt(5)*log(1/2*(sqrt(5) - 1)*tan(b*x + a) + tan(b*x + a)^2 + 1) - 2*
log(tan(b*x + a)^4 - tan(b*x + a)^3 + tan(b*x + a)^2 - tan(b*x + a) + 1) +
 5*log(tan(b*x + a)^2 + 1) - 2*log(abs(tan(b*x + a) + 1)))/b
```

3.942.9 Mupad [B] (verification not implemented)

Time = 28.34 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.88

$$\int \frac{\cos^5(a+bx)-\sin^5(a+bx)}{\cos^5(a+bx)+\sin^5(a+bx)}\,dx$$
$$= \frac{\ln\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2-2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)-1\right)}{5\,b}-\frac{\ln\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2+1\right)}{b}$$
$$+\frac{\ln\left(2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2-\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^3+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^4+\sqrt{5}\tan\left(\frac{a}{2}+\frac{bx}{2}\right)-\sqrt{5}\tan\left(\frac{a}{2}+\frac{bx}{2}\right.}{5\,b}$$
$$-\frac{\ln\left(2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2-\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^3+\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^4-\sqrt{5}\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+\sqrt{5}\tan\left(\frac{a}{2}+\frac{bx}{2}\right.}{5\,b}$$

input
```
int((cos(a + b*x)^5 - sin(a + b*x)^5)/(cos(a + b*x)^5 + sin(a + b*x)^5),x)
```

output
```
log(tan(a/2 + (b*x)/2)^2 - 2*tan(a/2 + (b*x)/2) - 1)/(5*b) - log(tan(a/2 +
 (b*x)/2)^2 + 1)/b + (log(2*tan(a/2 + (b*x)/2)^2 - tan(a/2 + (b*x)/2) + ta
n(a/2 + (b*x)/2)^3 + tan(a/2 + (b*x)/2)^4 + 5^(1/2)*tan(a/2 + (b*x)/2) - 5
^(1/2)*tan(a/2 + (b*x)/2)^3 + 1)*(5^(1/2) + 1))/(5*b) - (log(2*tan(a/2 + (
b*x)/2)^2 - tan(a/2 + (b*x)/2) + tan(a/2 + (b*x)/2)^3 + tan(a/2 + (b*x)/2)
^4 - 5^(1/2)*tan(a/2 + (b*x)/2) + 5^(1/2)*tan(a/2 + (b*x)/2)^3 + 1)*(5^(1/
2) - 1))/(5*b)
```

3.943 $\int \frac{\cos^4(a+bx)-\sin^4(a+bx)}{\cos^4(a+bx)+\sin^4(a+bx)}\,dx$

3.943.1 Optimal result

Integrand size = 39, antiderivative size = 72

$$\int \frac{\cos^4(a+bx)-\sin^4(a+bx)}{\cos^4(a+bx)+\sin^4(a+bx)}\,dx = -\frac{\log\left(1-\sqrt{2}\tan(a+bx)+\tan^2(a+bx)\right)}{2\sqrt{2}b} + \frac{\log\left(1+\sqrt{2}\tan(a+bx)+\tan^2(a+bx)\right)}{2\sqrt{2}b}$$

output
```
-1/4*ln(1-2^(1/2)*tan(b*x+a)+tan(b*x+a)^2)/b*2^(1/2)+1/4*ln(1+2^(1/2)*tan(
b*x+a)+tan(b*x+a)^2)/b*2^(1/2)
```

3.943.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.35

$$\int \frac{\cos^4(a+bx)-\sin^4(a+bx)}{\cos^4(a+bx)+\sin^4(a+bx)}\,dx = \frac{\text{arctanh}\left(\frac{\sin(2a+2bx)}{\sqrt{2}}\right)}{\sqrt{2}b}$$

input
```
Integrate[(Cos[a + b*x]^4 - Sin[a + b*x]^4)/(Cos[a + b*x]^4 + Sin[a + b*x]
^4),x]
```

output
```
ArcTanh[Sin[2*a + 2*b*x]/Sqrt[2]]/(Sqrt[2]*b)
```

3.943.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3042, 4889, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^4(a+bx)-\sin^4(a+bx)}{\sin^4(a+bx)+\cos^4(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos(a+bx)^4-\sin(a+bx)^4}{\sin(a+bx)^4+\cos(a+bx)^4}dx$$

$$\Big\downarrow \text{4889}$$

$$\frac{\int \frac{1-\tan^2(a+bx)}{\tan^4(a+bx)+1}d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{1479}$$

$$\frac{-\frac{\int -\frac{\sqrt{2}-2\tan(a+bx)}{\tan^2(a+bx)-\sqrt{2}\tan(a+bx)+1}d\tan(a+bx)}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\tan(a+bx)+1\right)}{\tan^2(a+bx)+\sqrt{2}\tan(a+bx)+1}d\tan(a+bx)}{2\sqrt{2}}}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{\sqrt{2}-2\tan(a+bx)}{\tan^2(a+bx)-\sqrt{2}\tan(a+bx)+1}d\tan(a+bx)}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\tan(a+bx)+1\right)}{\tan^2(a+bx)+\sqrt{2}\tan(a+bx)+1}d\tan(a+bx)}{2\sqrt{2}}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\sqrt{2}-2\tan(a+bx)}{\tan^2(a+bx)-\sqrt{2}\tan(a+bx)+1}d\tan(a+bx)}{2\sqrt{2}}+\frac{1}{2}\int \frac{\sqrt{2}\tan(a+bx)+1}{\tan^2(a+bx)+\sqrt{2}\tan(a+bx)+1}d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\frac{\log\left(\tan^2(a+bx)+\sqrt{2}\tan(a+bx)+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan^2(a+bx)-\sqrt{2}\tan(a+bx)+1\right)}{2\sqrt{2}}}{b}$$

input
```
Int[(Cos[a + b*x]^4 - Sin[a + b*x]^4)/(Cos[a + b*x]^4 + Sin[a + b*x]^4),x]
```

output
```
(-1/2*Log[1 - Sqrt[2]*Tan[a + b*x] + Tan[a + b*x]^2]/Sqrt[2] + Log[1 + Sqr
t[2]*Tan[a + b*x] + Tan[a + b*x]^2]/(2*Sqrt[2]))/b
```

3.943.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.943. $\int \frac{\cos^4(a+bx)-\sin^4(a+bx)}{\cos^4(a+bx)+\sin^4(a+bx)}\,dx$

3.943.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00

method	result
risch	$\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{4i(xb+a)}+2i\sqrt{2}\,\mathrm{e}^{2i(xb+a)}-1\right)}{4b}-\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{4i(xb+a)}-2i\sqrt{2}\,\mathrm{e}^{2i(xb+a)}-1\right)}{4b}$
derivativedivides	$\frac{\frac{\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\,\tan(xb+a)+\tan(xb+a)^2}{1-\sqrt{2}\,\tan(xb+a)+\tan(xb+a)^2}\right)+2\arctan\left(\sqrt{2}\,\tan(xb+a)+1\right)+2\arctan\left(\sqrt{2}\,\tan(xb+a)-1\right)\right)}{8}-\frac{\sqrt{2}\left(\ln\left(\frac{1-\sqrt{2}\,\tan(xb+a)+}{1+\sqrt{2}\,\tan(xb+a)+}\right.\right.}{}}{b}$
default	$\frac{\frac{\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\,\tan(xb+a)+\tan(xb+a)^2}{1-\sqrt{2}\,\tan(xb+a)+\tan(xb+a)^2}\right)+2\arctan\left(\sqrt{2}\,\tan(xb+a)+1\right)+2\arctan\left(\sqrt{2}\,\tan(xb+a)-1\right)\right)}{8}-\frac{\sqrt{2}\left(\ln\left(\frac{1-\sqrt{2}\,\tan(xb+a)+}{1+\sqrt{2}\,\tan(xb+a)+}\right.\right.}{}}{b}$

input
```
int((cos(b*x+a)^4-sin(b*x+a)^4)/(cos(b*x+a)^4+sin(b*x+a)^4),x,method=_RETU
RNVERBOSE)
```

output
```
1/4*2^(1/2)/b*ln(exp(4*I*(b*x+a))+2*I*2^(1/2)*exp(2*I*(b*x+a))-1)-1/4*2^(1
/2)/b*ln(exp(4*I*(b*x+a))-2*I*2^(1/2)*exp(2*I*(b*x+a))-1)
```

3.943.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.03

$$\int \frac{\cos^4(a+bx)-\sin^4(a+bx)}{\cos^4(a+bx)+\sin^4(a+bx)}\,dx=\frac{\sqrt{2}\log\left(-\frac{2\cos(bx+a)^4-2\sqrt{2}\cos(bx+a)\sin(bx+a)-2\cos(bx+a)^2-1}{2\cos(bx+a)^4-2\cos(bx+a)^2+1}\right)}{4b}$$

input
```
integrate((cos(b*x+a)^4-sin(b*x+a)^4)/(cos(b*x+a)^4+sin(b*x+a)^4),x, algor
ithm="fricas")
```

output
```
1/4*sqrt(2)*log(-(2*cos(b*x + a)^4 - 2*sqrt(2)*cos(b*x + a)*sin(b*x + a) -
 2*cos(b*x + a)^2 - 1)/(2*cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1))/b
```

3.943.6 Sympy [A] (verification not implemented)

Time = 1.72 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.69

$$\int \frac{\cos^4(a+bx)-\sin^4(a+bx)}{\cos^4(a+bx)+\sin^4(a+bx)}\,dx$$
$$= \begin{cases} -\frac{\sqrt{2}\log\left(4\sin^2(a+bx)-4\sqrt{2}\sin(a+bx)\cos(a+bx)+4\cos^2(a+bx)\right)}{4b} + \frac{\sqrt{2}\log\left(4\sin^2(a+bx)+4\sqrt{2}\sin(a+bx)\cos(a+bx)+4\cos^2(a+bx)\right)}{4b} \\ \frac{x(-\sin^4(a)+\cos^4(a))}{\sin^4(a)+\cos^4(a)} \end{cases}$$

input
```
integrate((cos(b*x+a)**4-sin(b*x+a)**4)/(cos(b*x+a)**4+sin(b*x+a)**4),x)
```

output
```
Piecewise((-sqrt(2)*log(4*sin(a + b*x)**2 - 4*sqrt(2)*sin(a + b*x)*cos(a +
 b*x) + 4*cos(a + b*x)**2)/(4*b) + sqrt(2)*log(4*sin(a + b*x)**2 + 4*sqrt(
2)*sin(a + b*x)*cos(a + b*x) + 4*cos(a + b*x)**2)/(4*b), Ne(b, 0)), (x*(-s
in(a)**4 + cos(a)**4)/(sin(a)**4 + cos(a)**4), True))
```

3.943.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.81

$$\int \frac{\cos^4(a+bx)-\sin^4(a+bx)}{\cos^4(a+bx)+\sin^4(a+bx)}\,dx$$
$$= \frac{\sqrt{2}\log\left(\tan(bx+a)^2+\sqrt{2}\tan(bx+a)+1\right)-\sqrt{2}\log\left(\tan(bx+a)^2-\sqrt{2}\tan(bx+a)+1\right)}{4\,b}$$

input
```
integrate((cos(b*x+a)^4-sin(b*x+a)^4)/(cos(b*x+a)^4+sin(b*x+a)^4),x, algor
ithm="maxima")
```

output
```
1/4*(sqrt(2)*log(tan(b*x + a)^2 + sqrt(2)*tan(b*x + a) + 1) - sqrt(2)*log(
tan(b*x + a)^2 - sqrt(2)*tan(b*x + a) + 1))/b
```

3.943.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.67

$$\int \frac{\cos^4(a+bx)-\sin^4(a+bx)}{\cos^4(a+bx)+\sin^4(a+bx)}\,dx = -\frac{\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\sin(2bx+2a)\right|}{\left|2\sqrt{2}+2\sin(2bx+2a)\right|}\right)}{4b}$$

input
```
integrate((cos(b*x+a)^4-sin(b*x+a)^4)/(cos(b*x+a)^4+sin(b*x+a)^4),x, algor
ithm="giac")
```

output
```
-1/4*sqrt(2)*log(abs(-2*sqrt(2) + 2*sin(2*b*x + 2*a))/abs(2*sqrt(2) + 2*si
n(2*b*x + 2*a)))/b
```

3.943.9 Mupad [B] (verification not implemented)

Time = 25.95 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.32

$$\int \frac{\cos^4(a+bx)-\sin^4(a+bx)}{\cos^4(a+bx)+\sin^4(a+bx)}\,dx = \frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\sin(2a+2bx)}{2}\right)}{2b}$$

input
```
int((cos(a + b*x)^4 - sin(a + b*x)^4)/(cos(a + b*x)^4 + sin(a + b*x)^4),x)
```

output
```
(2^(1/2)*atanh((2^(1/2)*sin(2*a + 2*b*x))/2))/(2*b)
```

3.944 $\int \frac{\cos^3(a+bx)-\sin^3(a+bx)}{\cos^3(a+bx)+\sin^3(a+bx)}\,dx$

3.944.1 Optimal result

Integrand size = 39, antiderivative size = 55

$$\int \frac{\cos^3(a+bx)-\sin^3(a+bx)}{\cos^3(a+bx)+\sin^3(a+bx)}\,dx = -\frac{\log(\cos(a+bx))}{b}+\frac{\log(1+\tan(a+bx))}{3b} - \frac{2\log\left(1-\tan(a+bx)+\tan^2(a+bx)\right)}{3b}$$

output
```
-ln(cos(b*x+a))/b+1/3*ln(1+tan(b*x+a))/b-2/3*ln(1-tan(b*x+a)+tan(b*x+a)^2)
/b
```

3.944.2 Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.76

$$\int \frac{\cos^3(a+bx)-\sin^3(a+bx)}{\cos^3(a+bx)+\sin^3(a+bx)}\,dx = \frac{\log(\cos(a+bx)+\sin(a+bx))}{3b}-\frac{2\log(2-\sin(2(a+bx)))}{3b}$$

input
```
Integrate[(Cos[a + b*x]^3 - Sin[a + b*x]^3)/(Cos[a + b*x]^3 + Sin[a + b*x]
^3),x]
```

output
```
Log[Cos[a + b*x] + Sin[a + b*x]]/(3*b) - (2*Log[2 - Sin[2*(a + b*x)]])/(3*
b)
```

3.944.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {3042, 4889, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^3(a+bx)-\sin^3(a+bx)}{\sin^3(a+bx)+\cos^3(a+bx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(a+bx)^3-\sin(a+bx)^3}{\sin(a+bx)^3+\cos(a+bx)^3}dx$$

$$\Big\downarrow \text{ 4889}$$

$$\frac{\int \frac{1-\tan^3(a+bx)}{\tan^5(a+bx)+\tan^3(a+bx)+\tan^2(a+bx)+1}d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{ 2462}$$

$$\frac{\int \left(\frac{\tan(a+bx)}{\tan^2(a+bx)+1}+\frac{1}{3(\tan(a+bx)+1)}-\frac{2(2\tan(a+bx)-1)}{3(\tan^2(a+bx)-\tan(a+bx)+1)}\right)d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{1}{2}\log\left(\tan^2(a+bx)+1\right)-\frac{2}{3}\log\left(\tan^2(a+bx)-\tan(a+bx)+1\right)+\frac{1}{3}\log(\tan(a+bx)+1)}{b}$$

input
```
Int[(Cos[a + b*x]^3 - Sin[a + b*x]^3)/(Cos[a + b*x]^3 + Sin[a + b*x]^3),x]
```

output
```
(Log[1 + Tan[a + b*x]]/3 + Log[1 + Tan[a + b*x]^2]/2 - (2*Log[1 - Tan[a + 
b*x] + Tan[a + b*x]^2])/3)/b
```

3.944.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.944.4 Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{-\frac{2\ln\left(1-\tan(xb+a)+\tan(xb+a)^2\right)}{3}+\frac{\ln(1+\tan(xb+a))}{3}+\frac{\ln\left(1+\tan(xb+a)^2\right)}{2}}{b}$
default	$\frac{-\frac{2\ln\left(1-\tan(xb+a)+\tan(xb+a)^2\right)}{3}+\frac{\ln(1+\tan(xb+a))}{3}+\frac{\ln\left(1+\tan(xb+a)^2\right)}{2}}{b}$
risch	$ix+\frac{2ia}{b}+\frac{\ln\left(e^{2i(xb+a)}+i\right)}{3b}-\frac{2\ln\left(e^{4i(xb+a)}-4ie^{2i(xb+a)}-1\right)}{3b}$
parallelrisch	$\frac{3\ln\left(\sec\left(\frac{a}{2}+\frac{xb}{2}\right)^2\right)+\ln\left(\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2-2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)-1\right)-2\ln\left(\sec\left(\frac{a}{2}+\frac{xb}{2}\right)^4-4\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\sec\left(\frac{a}{2}\right.\right.}{3b}$
norman	$\frac{\ln\left(1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2\right)}{b}+\frac{\ln\left(\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2-2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)-1\right)}{3b}-\frac{2\ln\left(\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^4+2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^3+2\tan\left(\frac{a}{2}+\right.\right.}{3b}$

input
```
int((cos(b*x+a)^3-sin(b*x+a)^3)/(cos(b*x+a)^3+sin(b*x+a)^3),x,method=_RETU
RNVERBOSE)
```

output
```
1/b*(-2/3*ln(1-tan(b*x+a)+tan(b*x+a)^2)+1/3*ln(1+tan(b*x+a))+1/2*ln(1+tan(
b*x+a)^2))
```

3.944.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.76

$$\int \frac{\cos^3(a+bx)-\sin^3(a+bx)}{\cos^3(a+bx)+\sin^3(a+bx)}\,dx = \frac{\log\left(2\,\cos\left(bx+a\right)\sin\left(bx+a\right)+1\right)-4\,\log\left(-\cos\left(bx+a\right)\sin\left(bx+a\right)+1\right)}{6\,b}$$

input
```
integrate((cos(b*x+a)^3-sin(b*x+a)^3)/(cos(b*x+a)^3+sin(b*x+a)^3),x, algor
ithm="fricas")
```

output
```
1/6*(log(2*cos(b*x + a)*sin(b*x + a) + 1) - 4*log(-cos(b*x + a)*sin(b*x +
a) + 1))/b
```

3.944.6 Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.38

$$\int \frac{\cos^3(a+bx)-\sin^3(a+bx)}{\cos^3(a+bx)+\sin^3(a+bx)}\,dx = \begin{cases} \frac{\log\left(\sin\left(a+bx\right)+\cos\left(a+bx\right)\right)}{3b}-\frac{2\log\left(\sin^2\left(a+bx\right)-\sin\left(a+bx\right)\cos\left(a+bx\right)+\cos^2\left(a+bx\right)\right)}{3b} & \text{for } b\neq 0 \\ \frac{x\left(-\sin^3\left(a\right)+\cos^3\left(a\right)\right)}{\sin^3\left(a\right)+\cos^3\left(a\right)} & \text{otherwise} \end{cases}$$

input
```
integrate((cos(b*x+a)**3-sin(b*x+a)**3)/(cos(b*x+a)**3+sin(b*x+a)**3),x)
```

output
```
Piecewise((log(sin(a + b*x) + cos(a + b*x))/(3*b) - 2*log(sin(a + b*x)**2
- sin(a + b*x)*cos(a + b*x) + cos(a + b*x)**2)/(3*b), Ne(b, 0)), (x*(-sin(
a)**3 + cos(a)**3)/(sin(a)**3 + cos(a)**3), True))
```

3.944.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(51) = 102$.

Time = 0.31 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.80

$$\int \frac{\cos^3(a+bx)-\sin^3(a+bx)}{\cos^3(a+bx)+\sin^3(a+bx)}\,dx = -\frac{2\log\left(-\frac{2\sin(bx+a)}{\cos(bx+a)+1}+\frac{2\sin(bx+a)^2}{(\cos(bx+a)+1)^2}+\frac{2\sin(bx+a)^3}{(\cos(bx+a)+1)^3}+\frac{\sin(bx+a)^4}{(\cos(bx+a)+1)^4}+1\right)-\log\left(-\frac{2\sin(bx+a)}{\cos(bx+a)+1}+\frac{\sin(bx+a)}{(\cos(bx+a)+}\right.}{3b}$$

input
```
integrate((cos(b*x+a)^3-sin(b*x+a)^3)/(cos(b*x+a)^3+sin(b*x+a)^3),x, algor
ithm="maxima")
```

output
```
-1/3*(2*log(-2*sin(b*x + a)/(cos(b*x + a) + 1) + 2*sin(b*x + a)^2/(cos(b*x
 + a) + 1)^2 + 2*sin(b*x + a)^3/(cos(b*x + a) + 1)^3 + sin(b*x + a)^4/(cos
(b*x + a) + 1)^4 + 1) - log(-2*sin(b*x + a)/(cos(b*x + a) + 1) + sin(b*x +
 a)^2/(cos(b*x + a) + 1)^2 - 1) - 3*log(sin(b*x + a)^2/(cos(b*x + a) + 1)^
2 + 1))/b
```

3.944.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

$$\int \frac{\cos^3(a+bx)-\sin^3(a+bx)}{\cos^3(a+bx)+\sin^3(a+bx)}\,dx = -\frac{4\log\left(\tan(bx+a)^2-\tan(bx+a)+1\right)-3\log\left(\tan(bx+a)^2+1\right)-2\log\left(|\tan(bx+a)+1|\right)}{6b}$$

input
```
integrate((cos(b*x+a)^3-sin(b*x+a)^3)/(cos(b*x+a)^3+sin(b*x+a)^3),x, algor
ithm="giac")
```

output
```
-1/6*(4*log(tan(b*x + a)^2 - tan(b*x + a) + 1) - 3*log(tan(b*x + a)^2 + 1)
 - 2*log(abs(tan(b*x + a) + 1)))/b
```

3.944.9 Mupad [B] (verification not implemented)

Time = 27.04 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.91

$$\int \frac{\cos^3(a+bx)-\sin^3(a+bx)}{\cos^3(a+bx)+\sin^3(a+bx)}\,dx$$
$$= \frac{\ln\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2+1\right)}{b}+\frac{\ln\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2-2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)-1\right)}{3b}$$
$$-\frac{2\ln\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^4+2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^3+2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)^2-2\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+1\right)}{3b}$$

input
```
int((cos(a + b*x)^3 - sin(a + b*x)^3)/(cos(a + b*x)^3 + sin(a + b*x)^3),x)
```

output
```
log(tan(a/2 + (b*x)/2)^2 + 1)/b + log(tan(a/2 + (b*x)/2)^2 - 2*tan(a/2 + (
b*x)/2) - 1)/(3*b) - (2*log(2*tan(a/2 + (b*x)/2)^2 - 2*tan(a/2 + (b*x)/2)
+ 2*tan(a/2 + (b*x)/2)^3 + tan(a/2 + (b*x)/2)^4 + 1))/(3*b)
```

3.945 $\int \frac{\cos^2(a+bx)-\sin^2(a+bx)}{\cos^2(a+bx)+\sin^2(a+bx)} \, dx$

3.945.1 Optimal result

Integrand size = 39, antiderivative size = 16

$$\int \frac{\cos^2(a+bx)-\sin^2(a+bx)}{\cos^2(a+bx)+\sin^2(a+bx)} \, dx = \frac{\cos(a+bx)\sin(a+bx)}{b}$$

output
```
cos(b*x+a)*sin(b*x+a)/b
```

3.945.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 33 vs. 2(16) = 32.

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.06

$$\int \frac{\cos^2(a+bx)-\sin^2(a+bx)}{\cos^2(a+bx)+\sin^2(a+bx)} \, dx = \frac{\cos(2bx)\sin(2a)}{2b} + \frac{\cos(2a)\sin(2bx)}{2b}$$

input
```
Integrate[(Cos[a + b*x]^2 - Sin[a + b*x]^2)/(Cos[a + b*x]^2 + Sin[a + b*x]
^2),x]
```

output
```
(Cos[2*b*x]*Sin[2*a])/(2*b) + (Cos[2*a]*Sin[2*b*x])/(2*b)
```

3.945.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3042, 4880, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(a+bx)-\sin^2(a+bx)}{\sin^2(a+bx)+\cos^2(a+bx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\cos(a+bx)^2-\sin(a+bx)^2}{\sin(a+bx)^2+\cos(a+bx)^2}dx$$

$$\Big\downarrow \text{ 4880}$$

$$\int \left(\cos^2(a+bx)-\sin^2(a+bx)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sin(a+bx)\cos(a+bx)}{b}$$

input
```
Int[(Cos[a + b*x]^2 - Sin[a + b*x]^2)/(Cos[a + b*x]^2 + Sin[a + b*x]^2),x]
```

output
```
(Cos[a + b*x]*Sin[a + b*x])/b
```

3.945.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4880
```
Int[(u_.)*((a_.) + cos[(d_.) + (e_.)*(x_)]^2*(b_.) + (c_.)*sin[(d_.) + (e_.
)*(x_)]^2)^(p_.), x_Symbol] :> Simp[(a + c)^p   Int[ActivateTrig[u], x], x]
 /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b - c, 0]
```

3.945.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
risch	$\frac{\sin(2xb+2a)}{2b}$	15
parallelrisch	$\frac{\sin(2xb+2a)}{2b}$	15
derivativedivides	$\frac{\cos(xb+a)\sin(xb+a)}{b}$	17
default	$\frac{\cos(xb+a)\sin(xb+a)}{b}$	17
norman	$\frac{\frac{2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{b}+\frac{2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^3}{b}-\frac{2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^5}{b}-\frac{2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^7}{b}}{\left(1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2\right)^4}$	80

input
```
int((cos(b*x+a)^2-sin(b*x+a)^2)/(cos(b*x+a)^2+sin(b*x+a)^2),x,method=_RETU
RNVERBOSE)
```

output
```
1/2*sin(2*b*x+2*a)/b
```

3.945.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\cos^2(a+bx)-\sin^2(a+bx)}{\cos^2(a+bx)+\sin^2(a+bx)}\,dx = \frac{\cos(bx+a)\sin(bx+a)}{b}$$

input
```
integrate((cos(b*x+a)^2-sin(b*x+a)^2)/(cos(b*x+a)^2+sin(b*x+a)^2),x, algor
ithm="fricas")
```

output
```
cos(b*x + a)*sin(b*x + a)/b
```

3.945.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(14) = 28$.

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.00

$$\int \frac{\cos^2(a+bx)-\sin^2(a+bx)}{\cos^2(a+bx)+\sin^2(a+bx)}\,dx = \frac{\sin(a+bx)\cos(a+bx)}{b\sin^2(a+bx)+b\cos^2(a+bx)}$$

input
```
integrate((cos(b*x+a)**2-sin(b*x+a)**2)/(cos(b*x+a)**2+sin(b*x+a)**2),x)
```

output
```
sin(a + b*x)*cos(a + b*x)/(b*sin(a + b*x)**2 + b*cos(a + b*x)**2)
```

3.945.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{\cos^2(a+bx)-\sin^2(a+bx)}{\cos^2(a+bx)+\sin^2(a+bx)}\,dx = \frac{\tan(bx+a)}{\left(\tan(bx+a)^2+1\right)b}$$

input
```
integrate((cos(b*x+a)^2-sin(b*x+a)^2)/(cos(b*x+a)^2+sin(b*x+a)^2),x, algor
ithm="maxima")
```

output
```
tan(b*x + a)/((tan(b*x + a)^2 + 1)*b)
```

3.945.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\cos^2(a+bx)-\sin^2(a+bx)}{\cos^2(a+bx)+\sin^2(a+bx)}\,dx = \frac{\sin(2\,bx+2\,a)}{2\,b}$$

input
```
integrate((cos(b*x+a)^2-sin(b*x+a)^2)/(cos(b*x+a)^2+sin(b*x+a)^2),x, algor
ithm="giac")
```

output
```
1/2*sin(2*b*x + 2*a)/b
```

3.945.9 Mupad [B] (verification not implemented)

Time = 26.99 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\cos^2(a+bx)-\sin^2(a+bx)}{\cos^2(a+bx)+\sin^2(a+bx)}\,dx=\frac{\sin\left(2\,a+2\,b\,x\right)}{2\,b}$$

input
```
int((cos(a + b*x)^2 - sin(a + b*x)^2)/(cos(a + b*x)^2 + sin(a + b*x)^2),x)
```

output
```
sin(2*a + 2*b*x)/(2*b)
```

3.946 $\int \frac{\cos(a+bx)-\sin(a+bx)}{\cos(a+bx)+\sin(a+bx)} \, dx$

3.946.1 Optimal result

Integrand size = 31, antiderivative size = 18

$$\int \frac{\cos(a+bx)-\sin(a+bx)}{\cos(a+bx)+\sin(a+bx)} \, dx = \frac{\log(\cos(a+bx)+\sin(a+bx))}{b}$$

output
```
ln(cos(b*x+a)+sin(b*x+a))/b
```

3.946.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\cos(a+bx)-\sin(a+bx)}{\cos(a+bx)+\sin(a+bx)} \, dx = \frac{\log(\cos(a+bx)+\sin(a+bx))}{b}$$

input
```
Integrate[(Cos[a + b*x] - Sin[a + b*x])/(Cos[a + b*x] + Sin[a + b*x]),x]
```

output
```
Log[Cos[a + b*x] + Sin[a + b*x]]/b
```

3.946.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {3042, 3612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(a+bx)-\sin(a+bx)}{\sin(a+bx)+\cos(a+bx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\cos(a+bx)-\sin(a+bx)}{\sin(a+bx)+\cos(a+bx)}dx$$

$$\Big\downarrow \; 3612$$

$$\frac{\log(\sin(a+bx)+\cos(a+bx))}{b}$$

input
```
Int[(Cos[a + b*x] - Sin[a + b*x])/(Cos[a + b*x] + Sin[a + b*x]),x]
```

output
```
Log[Cos[a + b*x] + Sin[a + b*x]]/b
```

3.946.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3612
```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

3.946.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$\frac{\ln(\cos(xb+a)+\sin(xb+a))}{b}$	19
default	$\frac{\ln(\cos(xb+a)+\sin(xb+a))}{b}$	19
risch	$-ix-\frac{2ia}{b}+\frac{\ln(\mathrm{e}^{2i(xb+a)}+i)}{b}$	30
parallelrisch	$\frac{-\ln\left(\sec\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}\right)+\ln\left(\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}-2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)-1\right)}{b}$	45
norman	$\frac{\ln\left(\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}-2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)-1\right)}{b}-\frac{\ln\left(1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}\right)}{b}$	50

input
```
int((cos(b*x+a)-sin(b*x+a))/(cos(b*x+a)+sin(b*x+a)),x,method=_RETURNVERBOS
E)
```

output
```
ln(cos(b*x+a)+sin(b*x+a))/b
```

3.946.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\cos(a+bx)-\sin(a+bx)}{\cos(a+bx)+\sin(a+bx)}\,dx=\frac{\log\left(2\cos\left(bx+a\right)\sin\left(bx+a\right)+1\right)}{2\,b}$$

input
```
integrate((cos(b*x+a)-sin(b*x+a))/(cos(b*x+a)+sin(b*x+a)),x, algorithm="fr
icas")
```

output
```
1/2*log(2*cos(b*x + a)*sin(b*x + a) + 1)/b
```

3.946.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(15) = 30$.

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.72

$$\int \frac{\cos(a+bx)-\sin(a+bx)}{\cos(a+bx)+\sin(a+bx)}\,dx = \begin{cases} \frac{\log\left(\sin\left(a+bx\right)+\cos\left(a+bx\right)\right)}{b} & \text{for}\ b\neq 0 \\ \frac{x(-\sin\left(a\right)+\cos\left(a\right))}{\sin\left(a\right)+\cos\left(a\right)} & \text{otherwise} \end{cases}$$

input
```
integrate((cos(b*x+a)-sin(b*x+a))/(cos(b*x+a)+sin(b*x+a)),x)
```

output
```
Piecewise((log(sin(a + b*x) + cos(a + b*x))/b, Ne(b, 0)), (x*(-sin(a) + co
s(a))/(sin(a) + cos(a)), True))
```

3.946.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\cos(a+bx)-\sin(a+bx)}{\cos(a+bx)+\sin(a+bx)}\,dx = \frac{\log\left(\cos\left(bx+a\right)+\sin\left(bx+a\right)\right)}{b}$$

input
```
integrate((cos(b*x+a)-sin(b*x+a))/(cos(b*x+a)+sin(b*x+a)),x, algorithm="ma
xima")
```

output
```
log(cos(b*x + a) + sin(b*x + a))/b
```

3.946.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.61

$$\int \frac{\cos(a+bx)-\sin(a+bx)}{\cos(a+bx)+\sin(a+bx)}\,dx = -\frac{\log\left(\tan\left(bx+a\right)^{2}+1\right)-2\,\log\left(\left|\tan\left(bx+a\right)+1\right|\right)}{2\,b}$$

input
```
integrate((cos(b*x+a)-sin(b*x+a))/(cos(b*x+a)+sin(b*x+a)),x, algorithm="gi
ac")
```

output
```
-1/2*(log(tan(b*x + a)^2 + 1) - 2*log(abs(tan(b*x + a) + 1)))/b
```

3.946.9 Mupad [B] (verification not implemented)

Time = 27.44 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.78

$$\int \frac{\cos(a+bx)-\sin(a+bx)}{\cos(a+bx)+\sin(a+bx)}\,dx = \frac{2\,\mathrm{atanh}\left(\frac{128\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)+128}{16\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2+32\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)+48}-3\right)}{b}$$

input
```
int((cos(a + b*x) - sin(a + b*x))/(cos(a + b*x) + sin(a + b*x)),x)
```

output
```
(2*atanh((128*tan(a/2 + (b*x)/2) + 128)/(32*tan(a/2 + (b*x)/2) + 16*tan(a/
2 + (b*x)/2)^2 + 48) - 3))/b
```

3.947 $\int \frac{-\csc(a+bx)+\sec(a+bx)}{\csc(a+bx)+\sec(a+bx)}\,dx$

3.947.1 Optimal result

Integrand size = 31, antiderivative size = 19

$$\int \frac{-\csc(a+bx)+\sec(a+bx)}{\csc(a+bx)+\sec(a+bx)}\,dx = -\frac{\log(\cos(a+bx)+\sin(a+bx))}{b}$$

output
```
-ln(cos(b*x+a)+sin(b*x+a))/b
```

3.947.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{-\csc(a+bx)+\sec(a+bx)}{\csc(a+bx)+\sec(a+bx)}\,dx = -\frac{\log(\cos(a+bx)+\sin(a+bx))}{b}$$

input
```
Integrate[(-Csc[a + b*x] + Sec[a + b*x])/(Csc[a + b*x] + Sec[a + b*x]),x]
```

output
```
-(Log[Cos[a + b*x] + Sin[a + b*x]]/b)
```

3.947.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.63, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3042, 4889, 25, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec(a+bx)-\csc(a+bx)}{\csc(a+bx)+\sec(a+bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(a+bx)-\csc(a+bx)}{\csc(a+bx)+\sec(a+bx)}dx$$

$$\Big\downarrow \text{4889}$$

$$\frac{\int -\frac{1-\tan(a+bx)}{(\tan(a+bx)+1)(\tan^2(a+bx)+1)}d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{1-\tan(a+bx)}{(\tan(a+bx)+1)(\tan^2(a+bx)+1)}d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{657}$$

$$-\frac{\int \left(\frac{1}{\tan(a+bx)+1}-\frac{\tan(a+bx)}{\tan^2(a+bx)+1}\right)d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}\log\left(\tan^2(a+bx)+1\right)-\log(\tan(a+bx)+1)}{b}$$

input
```
Int[(-Csc[a + b*x] + Sec[a + b*x])/(Csc[a + b*x] + Sec[a + b*x]),x]
```

output
```
(-Log[1 + Tan[a + b*x]] + Log[1 + Tan[a + b*x]^2]/2)/b
```

3.947.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.947.4 Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.58

method	result	size
derivativedivides	$\frac{-\ln(1+\tan(xb+a))+\frac{\ln\left(1+\tan(xb+a)^2\right)}{2}}{b}$	30
default	$\frac{-\ln(1+\tan(xb+a))+\frac{\ln\left(1+\tan(xb+a)^2\right)}{2}}{b}$	30
risch	$ix+\frac{2ia}{b}-\frac{\ln\left(e^{2i(xb+a)}+i\right)}{b}$	31
parallelrisch	$\frac{\ln\left(\sec\left(\frac{a}{2}+\frac{xb}{2}\right)^2\right)-\ln\left(\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2-2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)-1\right)}{b}$	45
norman	$\frac{\ln\left(1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2\right)}{b}-\frac{\ln\left(\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2-2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)-1\right)}{b}$	50

3.947. $\int \frac{-\csc(a+bx)+\sec(a+bx)}{\csc(a+bx)+\sec(a+bx)}\,dx$

input
```
int((-csc(b*x+a)+sec(b*x+a))/(csc(b*x+a)+sec(b*x+a)),x,method=_RETURNVERBO
SE)
```

output
```
1/b*(-ln(1+tan(b*x+a))+1/2*ln(1+tan(b*x+a)^2))
```

3.947.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \frac{-\csc(a+bx)+\sec(a+bx)}{\csc(a+bx)+\sec(a+bx)}\,dx = -\frac{\log\left(2\,\cos\left(bx+a\right)\sin\left(bx+a\right)+1\right)}{2\,b}$$

input
```
integrate((-csc(b*x+a)+sec(b*x+a))/(csc(b*x+a)+sec(b*x+a)),x, algorithm="f
ricas")
```

output
```
-1/2*log(2*cos(b*x + a)*sin(b*x + a) + 1)/b
```

3.947.6 Sympy [F]

$$\int \frac{-\csc(a+bx)+\sec(a+bx)}{\csc(a+bx)+\sec(a+bx)}\,dx = -\int \frac{\csc\left(a+bx\right)}{\csc\left(a+bx\right)+\sec\left(a+bx\right)}\,dx - \int \left(-\frac{\sec\left(a+bx\right)}{\csc\left(a+bx\right)+\sec\left(a+bx\right)}\right)dx$$

input
```
integrate((-csc(b*x+a)+sec(b*x+a))/(csc(b*x+a)+sec(b*x+a)),x)
```

output
```
-Integral(csc(a + b*x)/(csc(a + b*x) + sec(a + b*x)), x) - Integral(-sec(a
 + b*x)/(csc(a + b*x) + sec(a + b*x)), x)
```

3.947.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(19) = 38.

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 3.68

$$\int \frac{-\csc(a+bx)+\sec(a+bx)}{\csc(a+bx)+\sec(a+bx)}\,dx = -\frac{\log\left(-\frac{2\sin(bx+a)}{\cos(bx+a)+1}+\frac{\sin(bx+a)^2}{(\cos(bx+a)+1)^2}-1\right)-\log\left(\frac{\sin(bx+a)^2}{(\cos(bx+a)+1)^2}+1\right)}{b}$$

input
```
integrate((-csc(b*x+a)+sec(b*x+a))/(csc(b*x+a)+sec(b*x+a)),x, algorithm="m
axima")
```

output
```
-(log(-2*sin(b*x + a)/(cos(b*x + a) + 1) + sin(b*x + a)^2/(cos(b*x + a) +
1)^2 - 1) - log(sin(b*x + a)^2/(cos(b*x + a) + 1)^2 + 1))/b
```

3.947.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.53

$$\int \frac{-\csc(a+bx)+\sec(a+bx)}{\csc(a+bx)+\sec(a+bx)}\,dx = \frac{\log\left(\tan(bx+a)^2+1\right)-2\log\left(|\tan(bx+a)+1|\right)}{2b}$$

input
```
integrate((-csc(b*x+a)+sec(b*x+a))/(csc(b*x+a)+sec(b*x+a)),x, algorithm="g
iac")
```

output
```
1/2*(log(tan(b*x + a)^2 + 1) - 2*log(abs(tan(b*x + a) + 1)))/b
```

3.947.9 Mupad [B] (verification not implemented)

Time = 27.57 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.63

$$\int \frac{-\csc(a+bx)+\sec(a+bx)}{\csc(a+bx)+\sec(a+bx)}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{128\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)+128}{16\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2+32\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)+48}-3\right)}{b}$$

input
```
int((1/cos(a + b*x) - 1/sin(a + b*x))/(1/cos(a + b*x) + 1/sin(a + b*x)),x)
```

output
```
-(2*atanh((128*tan(a/2 + (b*x)/2) + 128)/(32*tan(a/2 + (b*x)/2) + 16*tan(a
/2 + (b*x)/2)^2 + 48) - 3))/b
```

3.947. $\int \frac{-\csc(a+bx)+\sec(a+bx)}{\csc(a+bx)+\sec(a+bx)}\,dx$

3.948 $\int \frac{-\csc^2(a+bx)+\sec^2(a+bx)}{\csc^2(a+bx)+\sec^2(a+bx)}\,dx$

3.948.1 Optimal result

Integrand size = 39, antiderivative size = 17

$$\int \frac{-\csc^2(a+bx)+\sec^2(a+bx)}{\csc^2(a+bx)+\sec^2(a+bx)}\,dx = -\frac{\cos(a+bx)\sin(a+bx)}{b}$$

output
```
-cos(b*x+a)*sin(b*x+a)/b
```

3.948.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.94

$$\int \frac{-\csc^2(a+bx)+\sec^2(a+bx)}{\csc^2(a+bx)+\sec^2(a+bx)}\,dx = -\frac{\cos(2bx)\sin(2a)}{2b} - \frac{\cos(2a)\sin(2bx)}{2b}$$

input
```
Integrate[(-Csc[a + b*x]^2 + Sec[a + b*x]^2)/(Csc[a + b*x]^2 + Sec[a + b*x
]^2),x]
```

output
```
-1/2*(Cos[2*b*x]*Sin[2*a])/b - (Cos[2*a]*Sin[2*b*x])/(2*b)
```

3.948.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.35, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {3042, 4889, 25, 297}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^2(a+bx)-\csc^2(a+bx)}{\csc^2(a+bx)+\sec^2(a+bx)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sec(a+bx)^2-\csc(a+bx)^2}{\csc(a+bx)^2+\sec(a+bx)^2}dx$$

$$\Big\downarrow \; 4889$$

$$\frac{\int -\frac{1-\tan^2(a+bx)}{\left(\tan^2(a+bx)+1\right)^2}d\tan(a+bx)}{b}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{1-\tan^2(a+bx)}{\left(\tan^2(a+bx)+1\right)^2}d\tan(a+bx)}{b}$$

$$\Big\downarrow \; 297$$

$$-\frac{\tan(a+bx)}{b\left(\tan^2(a+bx)+1\right)}$$

input
```
Int[(-Csc[a + b*x]^2 + Sec[a + b*x]^2)/(Csc[a + b*x]^2 + Sec[a + b*x]^2),x
]
```

output
```
-(Tan[a + b*x]/(b*(1 + Tan[a + b*x]^2)))
```

3.948.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 297
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[c*
x*((a + b*x^2)^(p + 1)/a), x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d
, 0] && EqQ[a*d - b*c*(2*p + 3), 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.948.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

method	result	size
risch	$-\frac{\sin(2xb+2a)}{2b}$	15
parallelrisch	$-\frac{\sin(2xb+2a)}{2b}$	15
derivativedivides	$-\frac{\cos(xb+a)\sin(xb+a)}{b}$	18
default	$-\frac{\cos(xb+a)\sin(xb+a)}{b}$	18
norman	$\frac{\frac{2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}}{b}-\frac{4\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{6}}{b}+\frac{2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{10}}{b}}{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\left(\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}-1\right)\left(1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}\right)^{4}}$	92

input
```
int((-csc(b*x+a)^2+sec(b*x+a)^2)/(csc(b*x+a)^2+sec(b*x+a)^2),x,method=_RET
URNVERBOSE)
```

output
```
-1/2*sin(2*b*x+2*a)/b
```

3.948.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{-\csc^2(a+bx)+\sec^2(a+bx)}{\csc^2(a+bx)+\sec^2(a+bx)}\,dx=-\frac{\cos(bx+a)\sin(bx+a)}{b}$$

input
```
integrate((-csc(b*x+a)^2+sec(b*x+a)^2)/(csc(b*x+a)^2+sec(b*x+a)^2),x, algo
rithm="fricas")
```

output
```
-cos(b*x + a)*sin(b*x + a)/b
```

3.948.6 Sympy [F]

$$\int \frac{-\csc^2(a+bx)+\sec^2(a+bx)}{\csc^2(a+bx)+\sec^2(a+bx)}\,dx=-\int \frac{\csc^2(a+bx)}{\csc^2(a+bx)+\sec^2(a+bx)}\,dx-\int \left(-\frac{\sec^2(a+bx)}{\csc^2(a+bx)+\sec^2(a+bx)}\right)dx$$

input
```
integrate((-csc(b*x+a)**2+sec(b*x+a)**2)/(csc(b*x+a)**2+sec(b*x+a)**2),x)
```

output
```
-Integral(csc(a + b*x)**2/(csc(a + b*x)**2 + sec(a + b*x)**2), x) - Integr
al(-sec(a + b*x)**2/(csc(a + b*x)**2 + sec(a + b*x)**2), x)
```

3.948.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.35

$$\int \frac{-\csc^2(a+bx)+\sec^2(a+bx)}{\csc^2(a+bx)+\sec^2(a+bx)}\,dx=-\frac{\tan(bx+a)}{\left(\tan(bx+a)^2+1\right)b}$$

input
```
integrate((-csc(b*x+a)^2+sec(b*x+a)^2)/(csc(b*x+a)^2+sec(b*x+a)^2),x, algo
rithm="maxima")
```

output
```
-tan(b*x + a)/((tan(b*x + a)^2 + 1)*b)
```

3.948.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{-\csc^2(a+bx)+\sec^2(a+bx)}{\csc^2(a+bx)+\sec^2(a+bx)}\,dx = -\frac{\sin(2\,bx+2\,a)}{2\,b}$$

input
```
integrate((-csc(b*x+a)^2+sec(b*x+a)^2)/(csc(b*x+a)^2+sec(b*x+a)^2),x, algo
rithm="giac")
```

output
```
-1/2*sin(2*b*x + 2*a)/b
```

3.948.9 Mupad [B] (verification not implemented)

Time = 26.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{-\csc^2(a+bx)+\sec^2(a+bx)}{\csc^2(a+bx)+\sec^2(a+bx)}\,dx = -\frac{\sin(2\,a+2\,b\,x)}{2\,b}$$

input
```
int((1/cos(a + b*x)^2 - 1/sin(a + b*x)^2)/(1/cos(a + b*x)^2 + 1/sin(a + b*
x)^2),x)
```

output
```
-sin(2*a + 2*b*x)/(2*b)
```

3.949 $\int \frac{-\csc^3(a+bx)+\sec^3(a+bx)}{\csc^3(a+bx)+\sec^3(a+bx)}\,dx$

3.949.1 Optimal result

Integrand size = 39, antiderivative size = 54

$$\int \frac{-\csc^3(a+bx)+\sec^3(a+bx)}{\csc^3(a+bx)+\sec^3(a+bx)}\,dx=\frac{\log(\cos(a+bx))}{b}-\frac{\log(1+\tan(a+bx))}{3b}+\frac{2\log\left(1-\tan(a+bx)+\tan^2(a+bx)\right)}{3b}$$

output
```
ln(cos(b*x+a))/b-1/3*ln(1+tan(b*x+a))/b+2/3*ln(1-tan(b*x+a)+tan(b*x+a)^2)/
b
```

3.949.2 Mathematica [A] (verified)

Time = 11.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int \frac{-\csc^3(a+bx)+\sec^3(a+bx)}{\csc^3(a+bx)+\sec^3(a+bx)}\,dx=-\frac{\log(\cos(a+bx)+\sin(a+bx))}{3b}+\frac{2\log(2-\sin(2(a+bx)))}{3b}$$

input
```
Integrate[(-Csc[a + b*x]^3 + Sec[a + b*x]^3)/(Csc[a + b*x]^3 + Sec[a + b*x
]^3),x]
```

output
```
-1/3*Log[Cos[a + b*x] + Sin[a + b*x]]/b + (2*Log[2 - Sin[2*(a + b*x)]])/(3
*b)
```

3.949.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {3042, 4889, 25, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^3(a+bx)-\csc^3(a+bx)}{\csc^3(a+bx)+\sec^3(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(a+bx)^3-\csc(a+bx)^3}{\csc(a+bx)^3+\sec(a+bx)^3}dx$$

$$\Big\downarrow \text{4889}$$

$$\frac{\int -\frac{1-\tan^3(a+bx)}{(\tan^2(a+bx)+1)(\tan^3(a+bx)+1)}d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{1-\tan^3(a+bx)}{(\tan^2(a+bx)+1)(\tan^3(a+bx)+1)}d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{7276}$$

$$-\frac{\int \left(\frac{\tan(a+bx)}{\tan^2(a+bx)+1}+\frac{1}{3(\tan(a+bx)+1)}-\frac{2(2\tan(a+bx)-1)}{3(\tan^2(a+bx)-\tan(a+bx)+1)}\right)d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{2}\log\left(\tan^2(a+bx)+1\right)+\frac{2}{3}\log\left(\tan^2(a+bx)-\tan(a+bx)+1\right)-\frac{1}{3}\log(\tan(a+bx)+1)}{b}$$

input
```
Int[(-Csc[a + b*x]^3 + Sec[a + b*x]^3)/(Csc[a + b*x]^3 + Sec[a + b*x]^3),x
]
```

output
```
(-1/3*Log[1 + Tan[a + b*x]] - Log[1 + Tan[a + b*x]^2]/2 + (2*Log[1 - Tan[a
 + b*x] + Tan[a + b*x]^2])/3)/b
```

3.949. $\int \frac{-\csc^3(a+bx)+\sec^3(a+bx)}{\csc^3(a+bx)+\sec^3(a+bx)}\,dx$

3.949.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.949.4 Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{-\frac{\ln\left(1+\tan(xb+a)^2\right)}{2}+\frac{2\ln\left(1-\tan(xb+a)+\tan(xb+a)^2\right)}{3}-\frac{\ln(1+\tan(xb+a))}{3}}{b}$
default	$\frac{-\frac{\ln\left(1+\tan(xb+a)^2\right)}{2}+\frac{2\ln\left(1-\tan(xb+a)+\tan(xb+a)^2\right)}{3}-\frac{\ln(1+\tan(xb+a))}{3}}{b}$
risch	$-ix-\frac{2ia}{b}-\frac{\ln(e^{2i(xb+a)}+i)}{3b}+\frac{2\ln(e^{4i(xb+a)}-4ie^{2i(xb+a)}-1)}{3b}$
parallelrisch	$\frac{-3\ln\left(\sec\left(\frac{a}{2}+\frac{xb}{2}\right)^2\right)-\ln\left(\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2-2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)-1\right)+2\ln\left(\sec\left(\frac{a}{2}+\frac{xb}{2}\right)^4-4\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\sec\left(\right.\right.}{3b}$
norman	$-\frac{\ln\left(1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2\right)}{b}-\frac{\ln\left(\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2-2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)-1\right)}{3b}+\frac{2\ln\left(\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^4+2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^3+2\tan\left(\frac{a}{2}\right.\right.}{3b}$

3.949. $\int \frac{-\csc^3(a+bx)+\sec^3(a+bx)}{\csc^3(a+bx)+\sec^3(a+bx)}\,dx$

input
```
int((-csc(b*x+a)^3+sec(b*x+a)^3)/(csc(b*x+a)^3+sec(b*x+a)^3),x,method=_RET
URNVERBOSE)
```

output
```
1/b*(-1/2*ln(1+tan(b*x+a)^2)+2/3*ln(1-tan(b*x+a)+tan(b*x+a)^2)-1/3*ln(1+ta
n(b*x+a)))
```

3.949.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int \frac{-\csc^3(a+bx)+\sec^3(a+bx)}{\csc^3(a+bx)+\sec^3(a+bx)}\,dx = -\frac{\log\left(2\cos\left(bx+a\right)\sin\left(bx+a\right)+1\right)-4\log\left(-\cos\left(bx+a\right)\sin\left(bx+a\right)+1\right)}{6\,b}$$

input
```
integrate((-csc(b*x+a)^3+sec(b*x+a)^3)/(csc(b*x+a)^3+sec(b*x+a)^3),x, algo
rithm="fricas")
```

output
```
-1/6*(log(2*cos(b*x + a)*sin(b*x + a) + 1) - 4*log(-cos(b*x + a)*sin(b*x +
 a) + 1))/b
```

3.949.6 Sympy [F]

$$\int \frac{-\csc^3(a+bx)+\sec^3(a+bx)}{\csc^3(a+bx)+\sec^3(a+bx)}\,dx = -\int \frac{\csc^3\left(a+bx\right)}{\csc^3\left(a+bx\right)+\sec^3\left(a+bx\right)}\,dx - \int \left(-\frac{\sec^3\left(a+bx\right)}{\csc^3\left(a+bx\right)+\sec^3\left(a+bx\right)}\right)dx$$

input
```
integrate((-csc(b*x+a)**3+sec(b*x+a)**3)/(csc(b*x+a)**3+sec(b*x+a)**3),x)
```

output
```
-Integral(csc(a + b*x)**3/(csc(a + b*x)**3 + sec(a + b*x)**3), x) - Integr
al(-sec(a + b*x)**3/(csc(a + b*x)**3 + sec(a + b*x)**3), x)
```

3.949.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(50) = 100$.

Time = 0.29 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.85

$$\int \frac{-\csc^3(a+bx)+\sec^3(a+bx)}{\csc^3(a+bx)+\sec^3(a+bx)}\,dx = \frac{2\log\left(-\frac{2\sin(bx+a)}{\cos(bx+a)+1}+\frac{2\sin(bx+a)^2}{(\cos(bx+a)+1)^2}+\frac{2\sin(bx+a)^3}{(\cos(bx+a)+1)^3}+\frac{\sin(bx+a)^4}{(\cos(bx+a)+1)^4}+1\right)-\log\left(-\frac{2\sin(bx+a)}{\cos(bx+a)+1}+\frac{\sin(bx+a)^2}{(\cos(bx+a)+1)^2}\right.}{3\,b}$$

input
```
integrate((-csc(b*x+a)^3+sec(b*x+a)^3)/(csc(b*x+a)^3+sec(b*x+a)^3),x, algo
rithm="maxima")
```

output
```
1/3*(2*log(-2*sin(b*x + a)/(cos(b*x + a) + 1) + 2*sin(b*x + a)^2/(cos(b*x
+ a) + 1)^2 + 2*sin(b*x + a)^3/(cos(b*x + a) + 1)^3 + sin(b*x + a)^4/(cos(
b*x + a) + 1)^4 + 1) - log(-2*sin(b*x + a)/(cos(b*x + a) + 1) + sin(b*x +
a)^2/(cos(b*x + a) + 1)^2 - 1) - 3*log(sin(b*x + a)^2/(cos(b*x + a) + 1)^2
 + 1))/b
```

3.949.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96

$$\int \frac{-\csc^3(a+bx)+\sec^3(a+bx)}{\csc^3(a+bx)+\sec^3(a+bx)}\,dx = \frac{4\log\left(\tan(bx+a)^2-\tan(bx+a)+1\right)-3\log\left(\tan(bx+a)^2+1\right)-2\log\left(|\tan(bx+a)+1|\right)}{6\,b}$$

input
```
integrate((-csc(b*x+a)^3+sec(b*x+a)^3)/(csc(b*x+a)^3+sec(b*x+a)^3),x, algo
rithm="giac")
```

output
```
1/6*(4*log(tan(b*x + a)^2 - tan(b*x + a) + 1) - 3*log(tan(b*x + a)^2 + 1)
- 2*log(abs(tan(b*x + a) + 1)))/b
```

3.949.9 Mupad [B] (verification not implemented)

Time = 26.95 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.96

$$\int \frac{-\csc^3(a+bx)+\sec^3(a+bx)}{\csc^3(a+bx)+\sec^3(a+bx)}\,dx = \frac{2\ln\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^4+2\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^3+2\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2-2\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)+1\right)}{3\,b} - \frac{\ln\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2-2\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)-1\right)}{3\,b} - \frac{\ln\left(\tan\left(\frac{a}{2}+\frac{b\,x}{2}\right)^2+1\right)}{b}$$

input
```
int((1/cos(a + b*x)^3 - 1/sin(a + b*x)^3)/(1/cos(a + b*x)^3 + 1/sin(a + b*
x)^3),x)
```

output
```
(2*log(2*tan(a/2 + (b*x)/2)^2 - 2*tan(a/2 + (b*x)/2) + 2*tan(a/2 + (b*x)/2
)^3 + tan(a/2 + (b*x)/2)^4 + 1))/(3*b) - log(tan(a/2 + (b*x)/2)^2 - 2*tan(
a/2 + (b*x)/2) - 1)/(3*b) - log(tan(a/2 + (b*x)/2)^2 + 1)/b
```

3.950 $\int \frac{-\csc^4(a+bx)+\sec^4(a+bx)}{\csc^4(a+bx)+\sec^4(a+bx)}\,dx$

3.950.1 Optimal result

Integrand size = 39, antiderivative size = 72

$$\int \frac{-\csc^4(a+bx)+\sec^4(a+bx)}{\csc^4(a+bx)+\sec^4(a+bx)}\,dx = \frac{\log\left(1-\sqrt{2}\tan(a+bx)+\tan^2(a+bx)\right)}{2\sqrt{2}b} - \frac{\log\left(1+\sqrt{2}\tan(a+bx)+\tan^2(a+bx)\right)}{2\sqrt{2}b}$$

output
```
1/4*ln(1-2^(1/2)*tan(b*x+a)+tan(b*x+a)^2)/b*2^(1/2)-1/4*ln(1+2^(1/2)*tan(b
*x+a)+tan(b*x+a)^2)/b*2^(1/2)
```

3.950.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.36

$$\int \frac{-\csc^4(a+bx)+\sec^4(a+bx)}{\csc^4(a+bx)+\sec^4(a+bx)}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sin(2a+2bx)}{\sqrt{2}}\right)}{\sqrt{2}b}$$

input
```
Integrate[(-Csc[a + b*x]^4 + Sec[a + b*x]^4)/(Csc[a + b*x]^4 + Sec[a + b*x
]^4),x]
```

output
```
-(ArcTanh[Sin[2*a + 2*b*x]/Sqrt[2]]/(Sqrt[2]*b))
```

3.950.3 Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3042, 4889, 25, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sec^4(a+bx)-\csc^4(a+bx)}{\csc^4(a+bx)+\sec^4(a+bx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(a+bx)^4-\csc(a+bx)^4}{\csc(a+bx)^4+\sec(a+bx)^4}dx$$

$$\Big\downarrow \text{4889}$$

$$\frac{\int -\frac{1-\tan^2(a+bx)}{\tan^4(a+bx)+1}d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{1-\tan^2(a+bx)}{\tan^4(a+bx)+1}d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{1479}$$

$$\frac{\frac{\int -\frac{\sqrt{2}-2\tan(a+bx)}{\tan^2(a+bx)-\sqrt{2}\tan(a+bx)+1}d\tan(a+bx)}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(\sqrt{2}\tan(a+bx)+1\right)}{\tan^2(a+bx)+\sqrt{2}\tan(a+bx)+1}d\tan(a+bx)}{2\sqrt{2}}}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{\int \frac{\sqrt{2}-2\tan(a+bx)}{\tan^2(a+bx)-\sqrt{2}\tan(a+bx)+1}d\tan(a+bx)}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(\sqrt{2}\tan(a+bx)+1\right)}{\tan^2(a+bx)+\sqrt{2}\tan(a+bx)+1}d\tan(a+bx)}{2\sqrt{2}}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{\int \frac{\sqrt{2}-2\tan(a+bx)}{\tan^2(a+bx)-\sqrt{2}\tan(a+bx)+1}d\tan(a+bx)}{2\sqrt{2}}-\frac{1}{2}\int \frac{\sqrt{2}\tan(a+bx)+1}{\tan^2(a+bx)+\sqrt{2}\tan(a+bx)+1}d\tan(a+bx)}{b}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\frac{\log\left(\tan^2(a+bx)-\sqrt{2}\tan(a+bx)+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan^2(a+bx)+\sqrt{2}\tan(a+bx)+1\right)}{2\sqrt{2}}}{b}$$

3.950. $\int \frac{-\csc^4(a+bx)+\sec^4(a+bx)}{\csc^4(a+bx)+\sec^4(a+bx)}dx$

input
```
Int[(-Csc[a + b*x]^4 + Sec[a + b*x]^4)/(Csc[a + b*x]^4 + Sec[a + b*x]^4),x
]
```

output
```
(Log[1 - Sqrt[2]*Tan[a + b*x] + Tan[a + b*x]^2]/(2*Sqrt[2]) - Log[1 + Sqrt
[2]*Tan[a + b*x] + Tan[a + b*x]^2]/(2*Sqrt[2]))/b
```

3.950.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4889
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, With[{d = FreeFactors
[Tan[v], x]}, Simp[d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d], x]] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x]] /; InverseFunctionFreeQ[u, x] &&  !MatchQ[
u, (v_.)*((c_.)*tan[w_]^(n_.)*tan[z_]^(n_.))^(p_.) /; FreeQ[{c, p}, x] && I
ntegerQ[n] && LinearQ[w, x] && EqQ[z, 2*w]]
```

3.950. $\int \frac{-\csc^4(a+bx)+\sec^4(a+bx)}{\csc^4(a+bx)+\sec^4(a+bx)}\,dx$

3.950.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.02 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00

method	result
risch	$\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{4i(xb+a)}-2i\sqrt{2}\,\mathrm{e}^{2i(xb+a)}-1\right)}{4b}-\frac{\sqrt{2}\,\ln\left(\mathrm{e}^{4i(xb+a)}+2i\sqrt{2}\,\mathrm{e}^{2i(xb+a)}-1\right)}{4b}$
derivativedivides	$\frac{-\frac{\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\tan(xb+a)+\tan(xb+a)^2}{1-\sqrt{2}\tan(xb+a)+\tan(xb+a)^2}\right)+2\arctan\left(\sqrt{2}\tan(xb+a)+1\right)+2\arctan\left(\sqrt{2}\tan(xb+a)-1\right)\right)}{8}+\frac{\sqrt{2}\left(\ln\left(\frac{1-\sqrt{2}\tan(xb+a)}{1+\sqrt{2}\tan(xb+a)}\right.\right.}{}}{b}$
default	$\frac{-\frac{\sqrt{2}\left(\ln\left(\frac{1+\sqrt{2}\tan(xb+a)+\tan(xb+a)^2}{1-\sqrt{2}\tan(xb+a)+\tan(xb+a)^2}\right)+2\arctan\left(\sqrt{2}\tan(xb+a)+1\right)+2\arctan\left(\sqrt{2}\tan(xb+a)-1\right)\right)}{8}+\frac{\sqrt{2}\left(\ln\left(\frac{1-\sqrt{2}\tan(xb+a)}{1+\sqrt{2}\tan(xb+a)}\right.\right.}{}}{b}$

input
```
int((-csc(b*x+a)^4+sec(b*x+a)^4)/(csc(b*x+a)^4+sec(b*x+a)^4),x,method=_RET
URNVERBOSE)
```

output
```
1/4*2^(1/2)/b*ln(exp(4*I*(b*x+a))-2*I*2^(1/2)*exp(2*I*(b*x+a))-1)-1/4*2^(1
/2)/b*ln(exp(4*I*(b*x+a))+2*I*2^(1/2)*exp(2*I*(b*x+a))-1)
```

3.950.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.03

$$\int \frac{-\csc^4(a+bx)+\sec^4(a+bx)}{\csc^4(a+bx)+\sec^4(a+bx)}\,dx = \frac{\sqrt{2}\log\left(-\frac{2\cos(bx+a)^4+2\sqrt{2}\cos(bx+a)\sin(bx+a)-2\cos(bx+a)^2-1}{2\cos(bx+a)^4-2\cos(bx+a)^2+1}\right)}{4b}$$

input
```
integrate((-csc(b*x+a)^4+sec(b*x+a)^4)/(csc(b*x+a)^4+sec(b*x+a)^4),x, algo
rithm="fricas")
```

output
```
1/4*sqrt(2)*log(-(2*cos(b*x + a)^4 + 2*sqrt(2)*cos(b*x + a)*sin(b*x + a) -
 2*cos(b*x + a)^2 - 1)/(2*cos(b*x + a)^4 - 2*cos(b*x + a)^2 + 1))/b
```

3.950.6 Sympy [F]

$$\int \frac{-\csc^4(a+bx)+\sec^4(a+bx)}{\csc^4(a+bx)+\sec^4(a+bx)}\,dx = -\int \frac{\csc^4(a+bx)}{\csc^4(a+bx)+\sec^4(a+bx)}\,dx - \int \left(-\frac{\sec^4(a+bx)}{\csc^4(a+bx)+\sec^4(a+bx)}\right)dx$$

input
```
integrate((-csc(b*x+a)**4+sec(b*x+a)**4)/(csc(b*x+a)**4+sec(b*x+a)**4),x)
```

output
```
-Integral(csc(a + b*x)**4/(csc(a + b*x)**4 + sec(a + b*x)**4), x) - Integr
al(-sec(a + b*x)**4/(csc(a + b*x)**4 + sec(a + b*x)**4), x)
```

3.950.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.81

$$\int \frac{-\csc^4(a+bx)+\sec^4(a+bx)}{\csc^4(a+bx)+\sec^4(a+bx)}\,dx = -\frac{\sqrt{2}\log\left(\tan(bx+a)^2+\sqrt{2}\tan(bx+a)+1\right)-\sqrt{2}\log\left(\tan(bx+a)^2-\sqrt{2}\tan(bx+a)+1\right)}{4\,b}$$

input
```
integrate((-csc(b*x+a)^4+sec(b*x+a)^4)/(csc(b*x+a)^4+sec(b*x+a)^4),x, algo
rithm="maxima")
```

output
```
-1/4*(sqrt(2)*log(tan(b*x + a)^2 + sqrt(2)*tan(b*x + a) + 1) - sqrt(2)*log
(tan(b*x + a)^2 - sqrt(2)*tan(b*x + a) + 1))/b
```

3.950.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.67

$$\int \frac{-\csc^4(a+bx)+\sec^4(a+bx)}{\csc^4(a+bx)+\sec^4(a+bx)}\,dx = \frac{\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\sin(2bx+2a)\right|}{\left|2\sqrt{2}+2\sin(2bx+2a)\right|}\right)}{4\,b}$$

input
```
integrate((-csc(b*x+a)^4+sec(b*x+a)^4)/(csc(b*x+a)^4+sec(b*x+a)^4),x, algo
rithm="giac")
```

output
```
1/4*sqrt(2)*log(abs(-2*sqrt(2) + 2*sin(2*b*x + 2*a))/abs(2*sqrt(2) + 2*sin
(2*b*x + 2*a)))/b
```

3.950.9 Mupad [B] (verification not implemented)

Time = 26.87 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.32

$$\int \frac{-\csc^4(a+bx)+\sec^4(a+bx)}{\csc^4(a+bx)+\sec^4(a+bx)}\,dx = -\frac{\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sin(2\,a+2\,b\,x)}{2}\right)}{2\,b}$$

input
```
int((1/cos(a + b*x)^4 - 1/sin(a + b*x)^4)/(1/cos(a + b*x)^4 + 1/sin(a + b*
x)^4),x)
```

output
```
-(2^(1/2)*atanh((2^(1/2)*sin(2*a + 2*b*x))/2))/(2*b)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
      return 1
  elif isinstance(expn,list):
    return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
  elif  is_sqrt(expn):
    if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
      return 1
    else:
      return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
  elif isinstance(expn,Pow):   #type(expn,'`^`')
    if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
      return expnType(expn.args[0])   #ExpnType(op(1,expn))
    elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
        return 1
      else:
        return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    else:
      return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
  elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
    m1 = expnType(expn.args[0])
    m2 = expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
    return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
  elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
  elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  elif is_appell_function(expn.func):
    m1 = max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  elif isinstance(expn,RootSum):
    m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
    return max(7,m1)
  elif str(expn).find("Integral") != -1:
    m1 = max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
  else:
    return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#             Albert Rich to use with Sagemath. This is used to
#             grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#             'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#             issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
        return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```