Computer Algebra Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

5-Inverse-trig-functions/5.2-Inverse-cosine/147-5.2.5-Inverse-cosine-functions

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [118]. This is test number [147].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (118)	0.00 (0)
Mathematica	95.76 (113)	4.24 (5)
Maple	66.10 (78)	33.90 (40)
Giac	48.31 (57)	51.69 (61)
Fricas	43.22 (51)	56.78 (67)
Sympy	28.81 (34)	71.19 (84)
Maxima	26.27 (31)	73.73 (87)
Mupad	18.64 (22)	81.36 (96)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

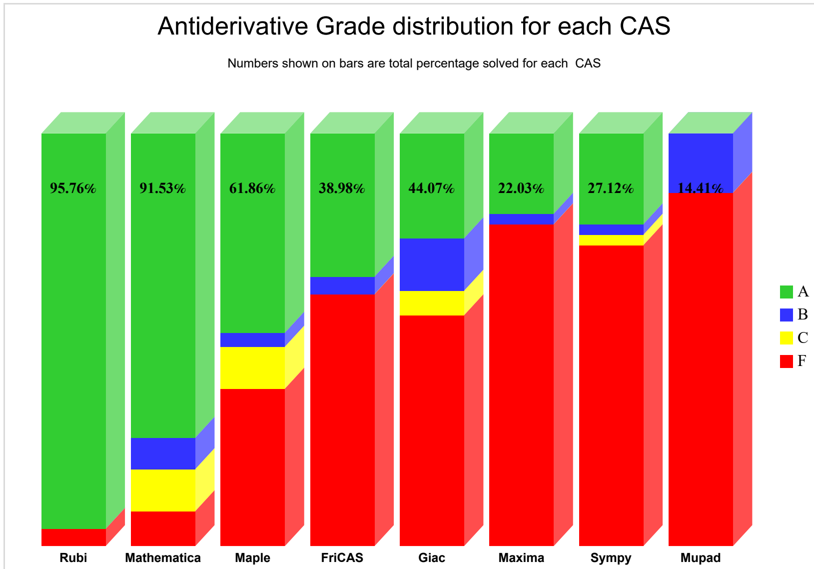
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

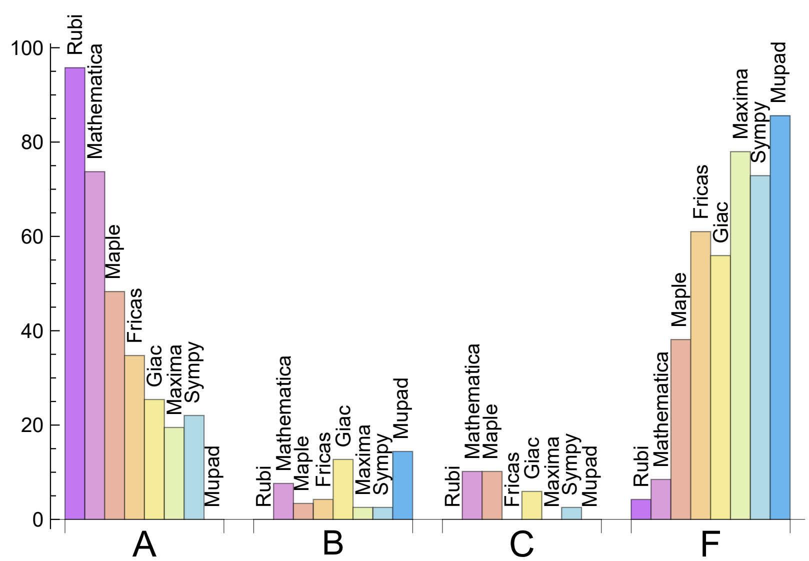
System	% A grade	% B grade	% C grade	% F grade
Rubi	95.763	0.000	0.000	4.237
Mathematica	73.729	7.627	10.169	8.475
Maple	48.305	3.390	10.169	38.136
Fricas	34.746	4.237	0.000	61.017
Giac	25.424	12.712	5.932	55.932
Sympy	22.034	2.542	2.542	72.881
Maxima	19.492	2.542	0.000	77.966
Mupad	0.000	14.407	0.000	85.593

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	5	100.00	0.00	0.00
Maple	40	100.00	0.00	0.00
Giac	61	75.41	0.00	24.59
Fricas	67	62.69	0.00	37.31
Sympy	84	83.33	13.10	3.57
Maxima	87	66.67	0.00	33.33
Mupad	96	0.00	100.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.25
Giac	0.36
Maxima	0.44
Rubi	0.46
Mupad	0.48
Mathematica	1.27
Maple	2.06
Sympy	6.20

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	44.82	1.09	39.00	0.93
Sympy	67.79	1.22	53.50	1.17
Maxima	70.77	1.27	41.00	0.89
Fricas	83.92	1.27	48.00	0.94
Giac	109.16	1.48	55.00	1.11
Rubi	148.86	0.96	87.00	1.00
Mathematica	270.33	1.26	85.00	0.97
Maple	385.05	1.34	72.50	1.14

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

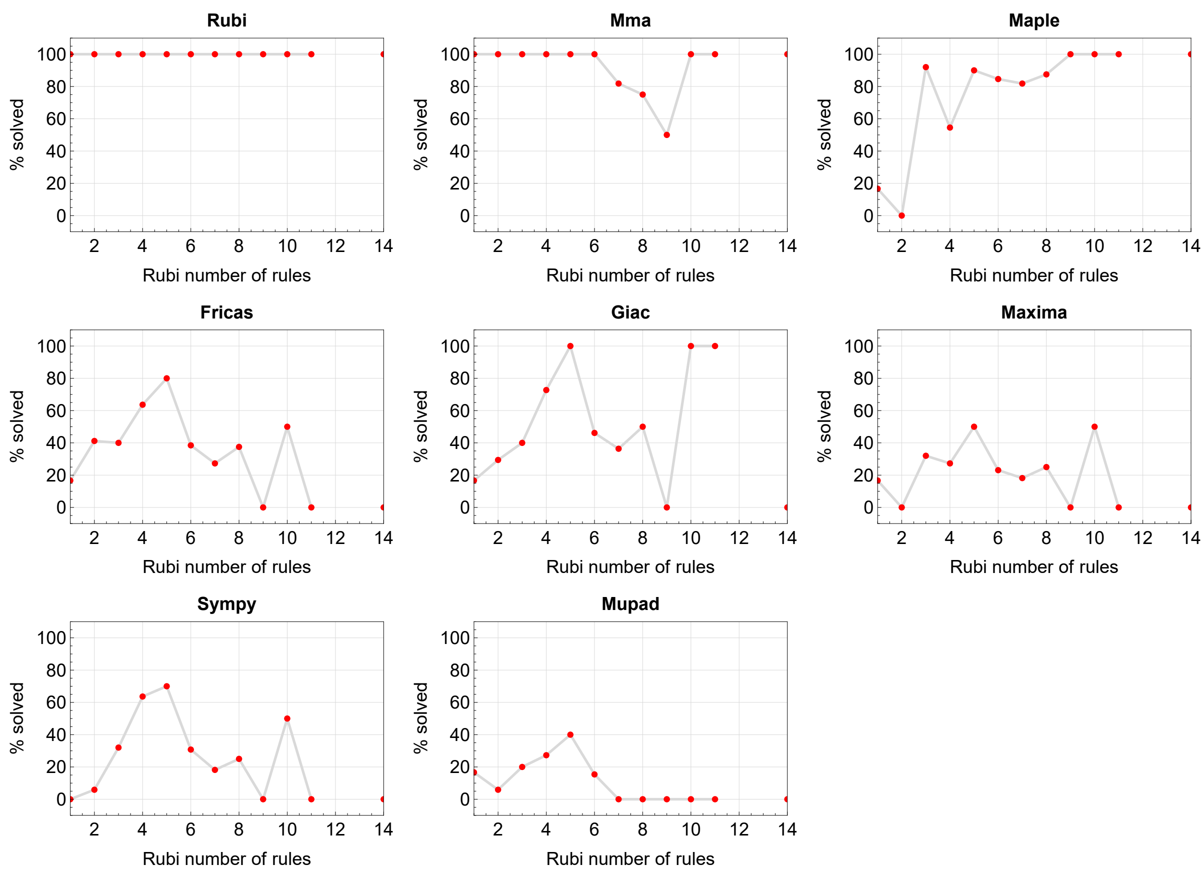


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

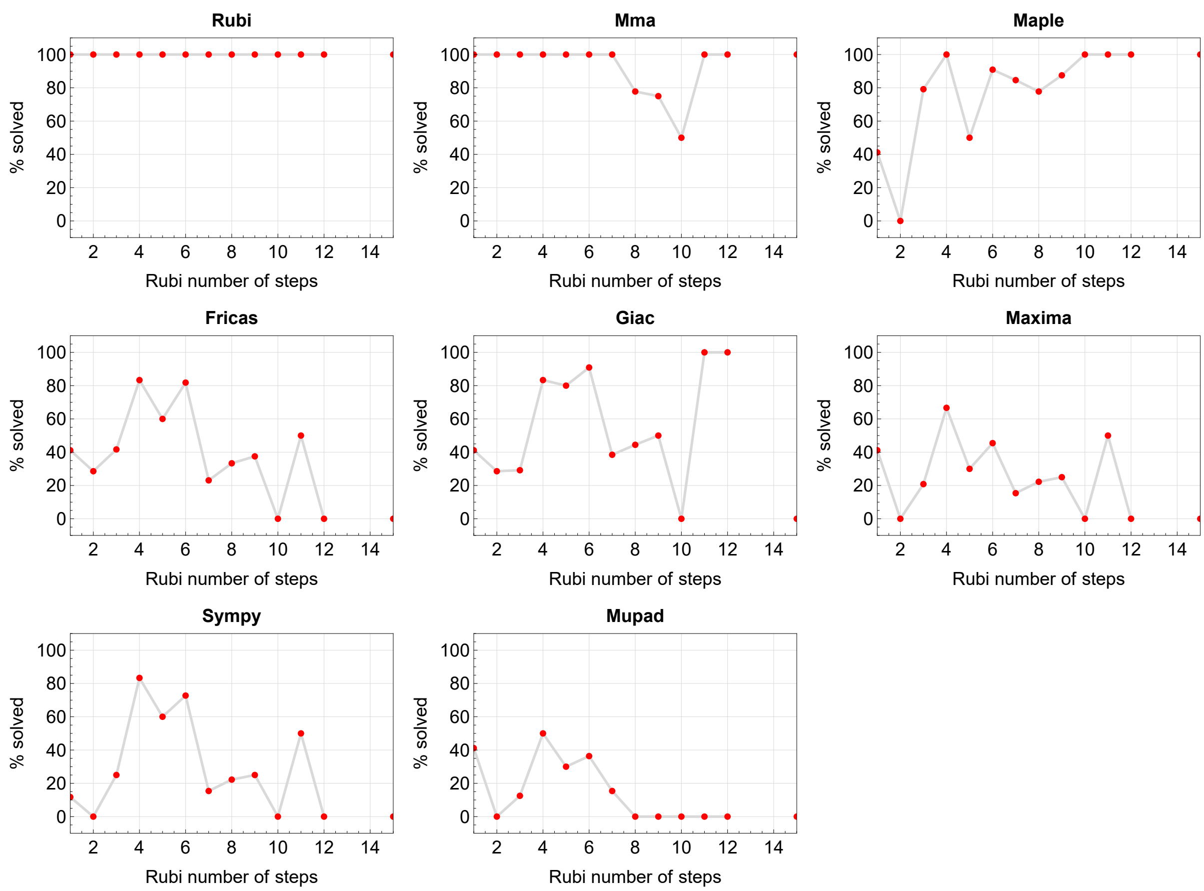


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

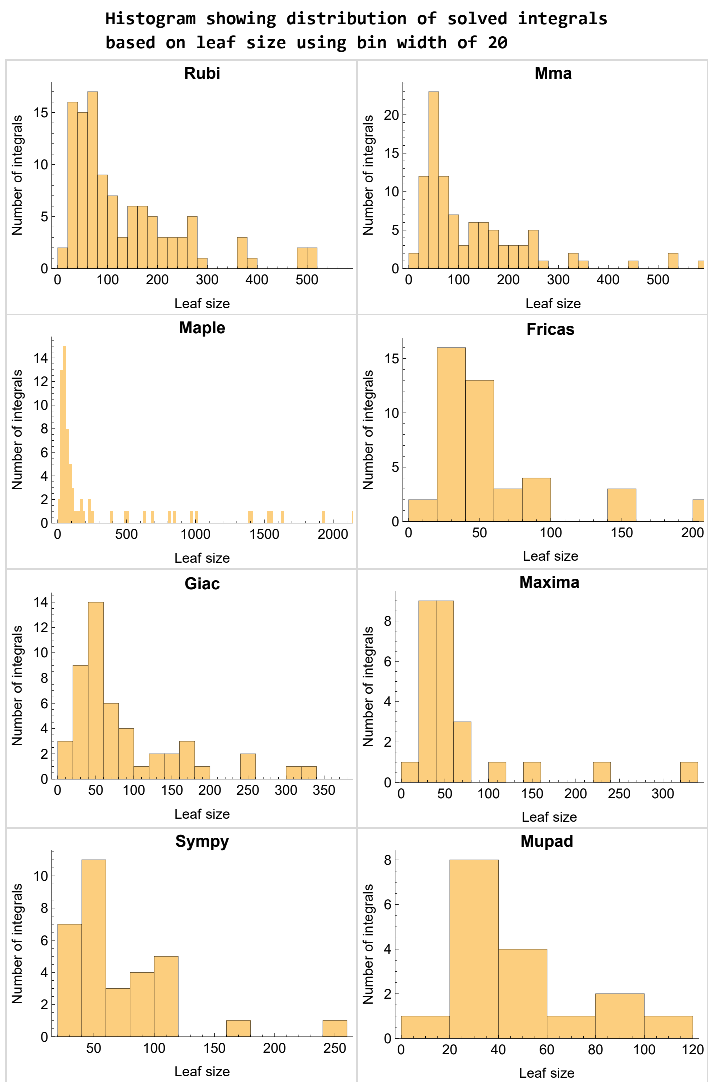


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

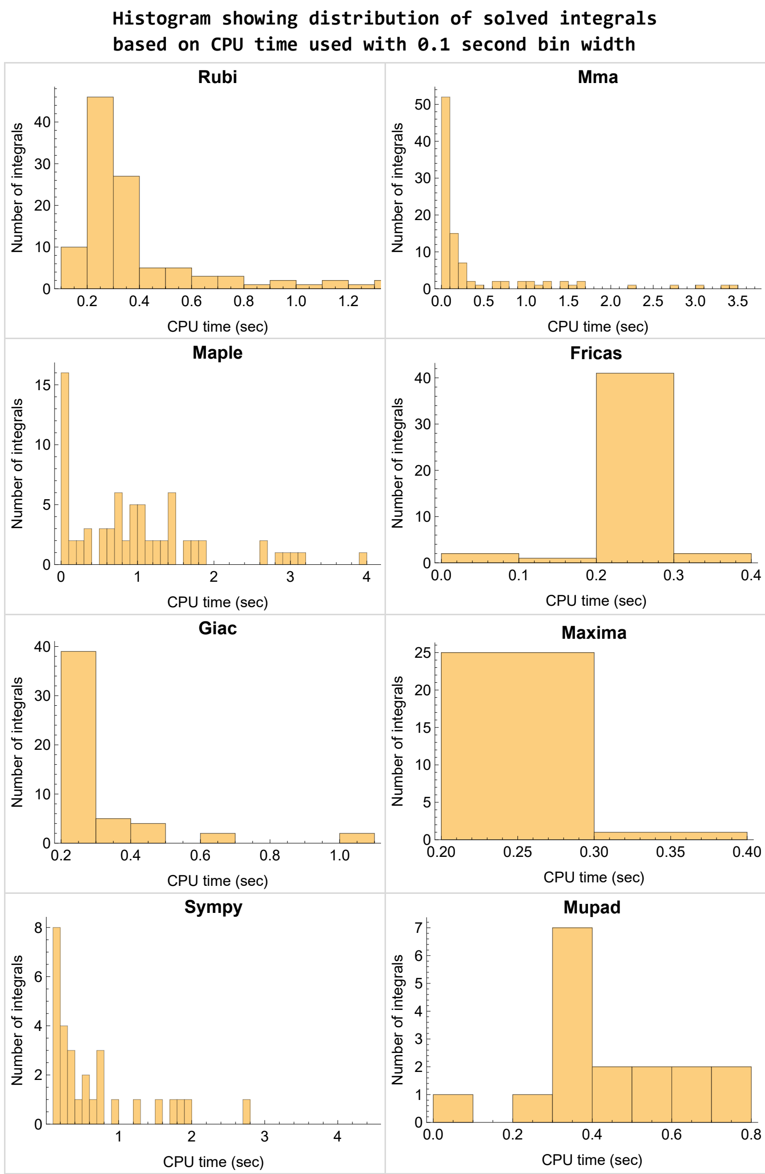


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

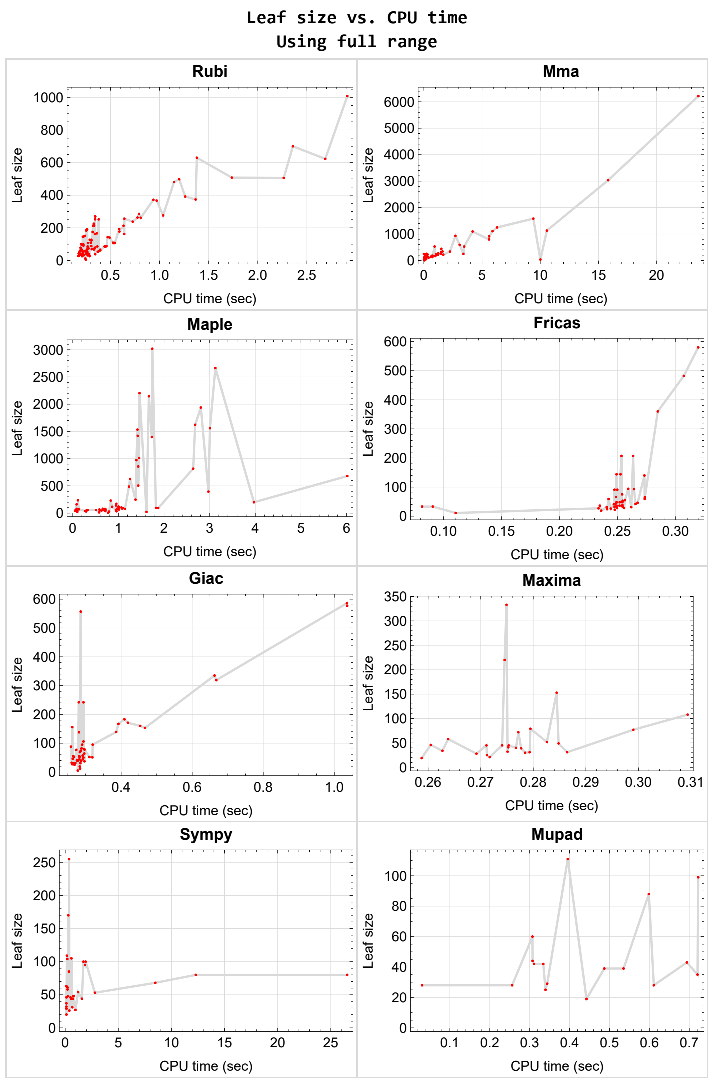


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{19, 23, 101, 105, 106\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{45, 71, 72, 107, 114\}$

Mathematica $\{4, 5, 9, 13, 17, 18, 21\}$

Maple $\{5\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

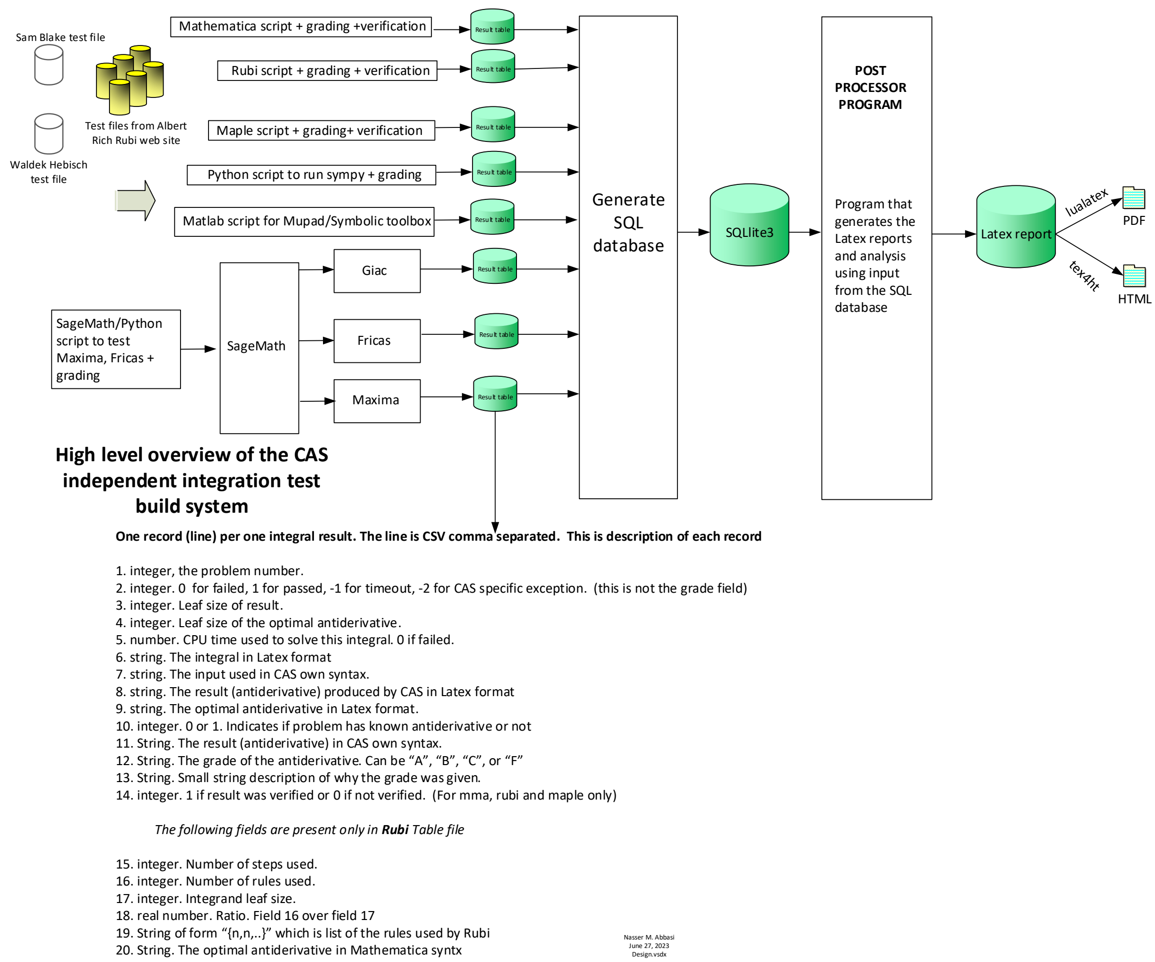
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 102, 103, 104, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 14, 15, 16, 22, 24, 25, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 45, 46, 47, 49, 51, 53, 54, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 108, 109, 110, 111, 112, 113, 116, 117, 118 }

B grade { 9, 13, 17, 18, 21, 27, 55, 69, 114 }

C grade { 37, 38, 39, 40, 41, 42, 43, 44, 48, 50, 52, 115 }

F normal fail { 20, 102, 103, 104, 107 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 4, 9, 13, 17, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 71, 76, 83, 103, 104, 107, 114, 115, 116 }

B grade { 5, 18, 52, 102 }

C grade { 1, 2, 3, 6, 7, 8, 10, 11, 12, 14, 15, 16 }

F normal fail { 20, 21, 22, 51, 70, 72, 73, 74, 75, 77, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 108, 109, 110, 111, 112, 113, 117, 118 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 24, 25, 26, 27, 32, 33, 46, 47, 48, 49, 50, 52, 53, 54, 57, 58, 59, 60, 61, 62, 64, 65, 66, 67, 68, 71, 72, 73, 74, 75, 76, 80, 81, 82, 83, 108, 109, 110, 111, 117, 118 }

B grade { 29, 30, 31, 55, 114 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 28, 34, 35, 36, 45, 51, 56, 63, 70, 77, 78, 79, 84, 85, 86, 102, 103, 104, 112, 113, 116 }

F(-1) timedout fail { }

F(-2) exception fail { 37, 38, 39, 40, 41, 42, 43, 44, 69, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 107, 115 }

2.1.5 Maxima

A grade { 16, 27, 46, 47, 49, 53, 54, 55, 57, 58, 59, 60, 61, 62, 64, 65, 66, 67, 68, 71, 72, 76, 83 }

B grade { 24, 25, 26 }

C grade { }

F normal fail { 1, 2, 3, 6, 7, 8, 10, 11, 12, 14, 15, 17, 18, 20, 21, 22, 28, 32, 33, 34, 35, 36, 43, 44, 45, 48, 50, 51, 52, 56, 63, 69, 70, 80, 81, 82, 84, 85, 86, 94, 95, 96, 97, 98, 99, 100, 102, 103, 104, 107, 108, 109, 110, 111, 112, 113, 114, 116 }

F(-1) timedout fail { }

F(-2) exception fail { 4, 5, 9, 13, 29, 30, 31, 37, 38, 39, 40, 41, 42, 73, 74, 75, 77, 78, 79, 87, 88, 89, 90, 91, 92, 93, 115, 117, 118 }

2.1.6 Giac

A grade { 25, 26, 27, 29, 32, 33, 34, 35, 36, 46, 47, 49, 53, 57, 58, 59, 60, 61, 62, 64, 68, 71, 72, 76, 83, 108, 109, 110, 111, 116 }

B grade { 24, 30, 31, 54, 55, 65, 66, 67, 73, 74, 75, 80, 81, 82, 114 }

C grade { 37, 38, 39, 40, 43, 44, 115 }

F normal fail { 14, 15, 16, 18, 20, 21, 22, 28, 41, 42, 45, 48, 50, 51, 52, 63, 69, 70, 77, 78, 79, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 102, 103, 104, 107, 112, 113, 117, 118 }

F(-1) timedout fail { }

F(-2) exception fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 17, 56 }

2.1.7 Mupad

A grade { }

B grade { 27, 32, 33, 47, 49, 54, 55, 57, 58, 62, 68, 71, 72, 76, 83, 114, 118 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 24, 25, 26, 28, 29, 30, 31, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48, 50, 51, 52, 53, 56, 59, 60, 61, 63, 64, 65, 66, 67, 69, 70, 73, 74, 75, 77, 78, 79, 80, 81, 82, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 102, 103, 104, 107, 108, 109, 110, 111, 112, 113, 115, 116, 117 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 26, 27, 32, 33, 46, 47, 48, 49, 50, 52, 54, 55, 57, 59, 60, 61, 62, 65, 66, 67, 68, 71, 108, 109, 110, 111 }

B grade { 24, 25, 72 }

C grade { 53, 58, 64 }

F normal fail { 1, 2, 3, 4, 5, 8, 9, 13, 17, 18, 20, 21, 22, 28, 29, 30, 31, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 51, 56, 63, 69, 70, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 107, 112, 113, 114, 115, 116, 117, 118 }

F(-1) timedout fail { 6, 7, 10, 11, 12, 19, 101, 102, 103, 104, 106 }

F(-2) exception fail { 14, 15, 16 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	670	372	442	1396	0	0	0	0	0
N.S.	1	0.56	0.66	2.08	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.950	1.505	1.733	0.000	0.000	0.000	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	450	263	320	973	0	0	0	0	0
N.S.	1	0.58	0.71	2.16	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.757	1.495	1.395	0.000	0.000	0.000	0.000	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	140	219	628	0	0	0	0	0
N.S.	1	0.59	0.92	2.64	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.483	1.672	1.256	0.000	0.000	0.000	0.000	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	725	506	1095	816	0	0	0	0	0
N.S.	1	0.70	1.51	1.13	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.210	4.191	2.639	0.000	0.000	0.000	0.000	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	No	No	TBD	TBD	TBD	TBD	TBD
size	851	623	1130	1939	0	0	0	0	0
N.S.	1	0.73	1.33	2.28	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.631	10.570	2.807	0.000	0.000	0.000	0.000	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	959	498	910	2146	0	0	0	0	0
N.S.	1	0.52	0.95	2.24	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.160	5.610	1.668	0.000	0.000	0.000	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	680	367	591	1533	0	0	0	0	0
N.S.	1	0.54	0.87	2.25	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.961	3.080	1.418	0.000	0.000	0.000	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	370	193	337	1012	0	0	0	0	0
N.S.	1	0.52	0.91	2.74	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.566	2.246	1.454	0.000	0.000	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	1064	700	3034	1559	0	0	0	0	0
N.S.	1	0.66	2.85	1.47	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.404	15.835	3.006	0.000	0.000	0.000	0.000	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1281	630	1582	3019	0	0	0	0	0
N.S.	1	0.49	1.23	2.36	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.376	9.407	1.742	0.000	0.000	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	940	481	794	2204	0	0	0	0	0
N.S.	1	0.51	0.84	2.34	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.105	5.603	1.463	0.000	0.000	0.000	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	517	256	526	1419	0	0	0	0	0
N.S.	1	0.50	1.02	2.74	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.608	3.467	1.424	0.000	0.000	0.000	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F(-2)	F	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	1637	1008	6216	2665	0	0	0	0	0
N.S.	1	0.62	3.80	1.63	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.856	23.588	3.125	0.000	0.000	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	450	262	342	856	0	0	0	0	0
N.S.	1	0.58	0.76	1.90	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.791	1.604	1.434	0.000	0.000	0.000	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	162	266	505	0	0	0	0	0
N.S.	1	0.60	0.99	1.87	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.618	1.237	1.435	0.000	0.000	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	88	172	247	108	0	0	0	0
N.S.	1	0.69	1.35	1.94	0.85	0.00	0.00	0.00	0.00
time (sec)	N/A	0.435	0.671	1.376	0.309	0.000	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	370	276	930	487	0	0	0	0	0
N.S.	1	0.75	2.51	1.32	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.016	2.714	1.232	0.000	0.000	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	496	374	1108	1622	0	0	0	0	0
N.S.	1	0.75	2.23	3.27	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.359	5.903	2.682	0.000	0.000	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	35	35	37	33	35	47	0	35	35
N.S.	1	1.00	1.06	0.94	1.00	1.34	0.00	1.00	1.00
time (sec)	N/A	0.325	0.181	22.416	1.993	0.267	0.000	0.434	0.304

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	496	508	0	0	0	0	0	0	0
N.S.	1	1.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.704	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	374	392	1248	0	0	0	0	0	0
N.S.	1	1.05	3.34	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.254	6.292	0.000	0.000	0.000	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	237	238	246	0	0	0	0	0	0
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.698	0.004	0.000	0.000	0.000	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	35	35	37	33	35	57	34	35	35
N.S.	1	1.00	1.06	0.94	1.00	1.63	0.97	1.00	1.00
time (sec)	N/A	0.354	0.367	56.020	1.012	0.259	10.067	0.415	0.285

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	166	104	235	333	94	255	242	0
N.S.	1	1.21	0.76	1.72	2.43	0.69	1.86	1.77	0.00
time (sec)	N/A	0.346	0.109	0.114	0.275	0.259	0.362	0.281	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	123	83	161	220	75	170	156	0
N.S.	1	1.31	0.88	1.71	2.34	0.80	1.81	1.66	0.00
time (sec)	N/A	0.296	0.093	0.085	0.275	0.254	0.293	0.262	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	86	69	78	153	59	104	88	0
N.S.	1	1.08	0.86	0.98	1.91	0.74	1.30	1.10	0.00
time (sec)	N/A	0.258	0.054	0.086	0.284	0.242	0.196	0.259	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	34	154	33	32	41	46	32	88
N.S.	1	0.94	4.28	0.92	0.89	1.14	1.28	0.89	2.44
time (sec)	N/A	0.200	0.095	0.083	0.275	0.248	0.113	0.261	0.599

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	228	199	0	0	0	0	0
N.S.	1	1.00	1.29	1.12	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.582	0.321	3.970	0.000	0.000	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	67	79	74	0	360	0	79	0
N.S.	1	1.06	1.25	1.17	0.00	5.71	0.00	1.25	0.00
time (sec)	N/A	0.257	0.069	0.700	0.000	0.285	0.000	0.296	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	109	126	116	0	482	0	242	0
N.S.	1	1.06	1.22	1.13	0.00	4.68	0.00	2.35	0.00
time (sec)	N/A	0.291	0.196	0.862	0.000	0.307	0.000	0.294	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	163	168	230	0	580	0	557	0
N.S.	1	1.13	1.17	1.60	0.00	4.03	0.00	3.87	0.00
time (sec)	N/A	0.330	0.203	0.832	0.000	0.319	0.000	0.286	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	79	74	71	0	66	109	78	60
N.S.	1	0.96	0.90	0.87	0.00	0.80	1.33	0.95	0.73
time (sec)	N/A	0.330	0.042	0.717	0.000	0.249	0.158	0.297	0.307

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	52	49	48	0	53	63	52	44
N.S.	1	1.11	1.04	1.02	0.00	1.13	1.34	1.11	0.94
time (sec)	N/A	0.259	0.029	0.696	0.000	0.254	0.108	0.311	0.307

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	0	0	0	12	0
N.S.	1	1.00	1.00	1.08	0.00	0.00	0.00	1.00	0.00
time (sec)	N/A	0.242	0.034	0.578	0.000	0.000	0.000	0.284	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	38	40	37	0	0	0	38	0
N.S.	1	0.95	1.00	0.92	0.00	0.00	0.00	0.95	0.00
time (sec)	N/A	0.339	0.064	0.714	0.000	0.000	0.000	0.287	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	58	65	53	0	0	0	57	0
N.S.	1	0.89	1.00	0.82	0.00	0.00	0.00	0.88	0.00
time (sec)	N/A	0.382	0.048	0.715	0.000	0.000	0.000	0.292	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	106	79	140	0	0	0	183	0
N.S.	1	0.95	0.71	1.26	0.00	0.00	0.00	1.65	0.00
time (sec)	N/A	0.529	0.036	0.947	0.000	0.000	0.000	0.409	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	84	76	105	0	0	0	139	0
N.S.	1	0.94	0.85	1.18	0.00	0.00	0.00	1.56	0.00
time (sec)	N/A	0.441	0.036	1.019	0.000	0.000	0.000	0.386	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	53	79	66	0	0	0	95	0
N.S.	1	0.96	1.44	1.20	0.00	0.00	0.00	1.73	0.00
time (sec)	N/A	0.361	0.036	1.013	0.000	0.000	0.000	0.320	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	78	28	0	0	0	51	0
N.S.	1	1.00	2.36	0.85	0.00	0.00	0.00	1.55	0.00
time (sec)	N/A	0.279	0.033	0.796	0.000	0.000	0.000	0.319	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	62	97	82	0	0	0	0	0
N.S.	1	0.97	1.52	1.28	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.379	0.055	0.976	0.000	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	86	139	120	0	0	0	0	0
N.S.	1	0.96	1.54	1.33	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.430	0.337	1.002	0.000	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-2)	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	108	128	93	0	0	0	167	0
N.S.	1	1.02	1.21	0.88	0.00	0.00	0.00	1.58	0.00
time (sec)	N/A	0.526	0.144	1.872	0.000	0.000	0.000	0.392	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-2)	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	109	133	95	0	0	0	171	0
N.S.	1	1.01	1.23	0.88	0.00	0.00	0.00	1.58	0.00
time (sec)	N/A	0.513	0.166	1.822	0.000	0.000	0.000	0.419	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	66	59	92	0	0	0	0	0
N.S.	1	0.97	0.87	1.35	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.382	0.074	1.100	0.000	0.000	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	30	33	30	26	31	27	0
N.S.	1	1.00	0.88	0.97	0.88	0.76	0.91	0.79	0.00
time (sec)	N/A	0.227	0.021	0.959	0.278	0.244	0.665	0.262	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	57	48	69	79	41	48	46	42
N.S.	1	1.12	0.94	1.35	1.55	0.80	0.94	0.90	0.82
time (sec)	N/A	0.212	0.033	0.134	0.279	0.247	0.263	0.264	0.334

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	61	63	79	0	33	48	0	0
N.S.	1	1.11	1.15	1.44	0.00	0.60	0.87	0.00	0.00
time (sec)	N/A	0.206	0.180	0.658	0.000	0.081	0.743	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	34	35	30	31	31	32	31	29
N.S.	1	0.97	1.00	0.86	0.89	0.89	0.91	0.89	0.83
time (sec)	N/A	0.213	0.017	0.094	0.286	0.262	0.109	0.271	0.344

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	46	34	65	0	33	44	0	0
N.S.	1	1.07	0.79	1.51	0.00	0.77	1.02	0.00	0.00
time (sec)	N/A	0.213	10.015	0.585	0.000	0.091	0.554	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	62	71	56	0	0	0	0	0	0
N.S.	1	1.15	0.90	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.316	0.034	0.000	0.000	0.000	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	40	57	0	11	44	0	0
N.S.	1	1.00	1.38	1.97	0.00	0.38	1.52	0.00	0.00
time (sec)	N/A	0.180	0.048	0.509	0.000	0.110	0.713	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	59	61	56	72	93	95	77	0
N.S.	1	1.02	1.05	0.97	1.24	1.60	1.64	1.33	0.00
time (sec)	N/A	0.241	0.059	0.325	0.277	0.264	1.859	0.274	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	33	39	28	32	48	64	28
N.S.	1	1.00	0.97	1.15	0.82	0.94	1.41	1.88	0.82
time (sec)	N/A	0.205	0.029	0.276	0.269	0.247	0.788	0.296	0.256

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	84	30	45	65	27	55	28
N.S.	1	1.00	3.11	1.11	1.67	2.41	1.00	2.04	1.04
time (sec)	N/A	0.218	0.121	0.292	0.274	0.273	0.961	0.266	0.611

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	67	60	77	0	0	0	0	0
N.S.	1	1.12	1.00	1.28	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.316	0.022	1.144	0.000	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	32	31	36	26	31	28
N.S.	1	1.00	1.00	1.07	1.03	1.20	0.87	1.03	0.93
time (sec)	N/A	0.212	0.025	0.091	0.279	0.235	0.388	0.285	0.030

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	57	50	47	77	47	100	44	42
N.S.	1	1.12	0.98	0.92	1.51	0.92	1.96	0.86	0.82
time (sec)	N/A	0.233	0.031	0.303	0.299	0.249	1.919	0.292	0.311

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	60	47	55	49	49	100	52	0
N.S.	1	1.07	0.84	0.98	0.88	0.88	1.79	0.93	0.00
time (sec)	N/A	0.247	0.036	0.306	0.285	0.254	1.713	0.283	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	91	46	53	52	36	54	52	0
N.S.	1	1.17	0.59	0.68	0.67	0.46	0.69	0.67	0.00
time (sec)	N/A	0.218	0.044	0.081	0.283	0.252	1.211	0.267	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	68	41	41	40	31	46	40	0
N.S.	1	1.13	0.68	0.68	0.67	0.52	0.77	0.67	0.00
time (sec)	N/A	0.196	0.035	0.095	0.277	0.241	0.493	0.277	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	40	43	26	25	24	29	25	35
N.S.	1	1.08	1.16	0.70	0.68	0.65	0.78	0.68	0.95
time (sec)	N/A	0.177	0.020	0.093	0.271	0.241	0.117	0.269	0.721

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	69	54	59	0	0	0	0	0
N.S.	1	1.23	0.96	1.05	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.305	0.035	0.946	0.000	0.000	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	24	22	21	22	44	40	0
N.S.	1	1.00	0.89	0.81	0.78	0.81	1.63	1.48	0.00
time (sec)	N/A	0.169	0.022	0.092	0.272	0.248	1.571	0.276	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	55	43	35	34	28	53	74	0
N.S.	1	1.10	0.86	0.70	0.68	0.56	1.06	1.48	0.00
time (sec)	N/A	0.182	0.028	0.082	0.263	0.250	2.795	0.286	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	78	37	47	46	33	68	106	0
N.S.	1	1.15	0.54	0.69	0.68	0.49	1.00	1.56	0.00
time (sec)	N/A	0.192	0.048	0.086	0.261	0.254	8.479	0.294	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	101	42	59	58	38	80	138	0
N.S.	1	1.17	0.49	0.69	0.67	0.44	0.93	1.60	0.00
time (sec)	N/A	0.199	0.056	0.095	0.264	0.254	26.546	0.281	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	26	25	20	19	19	20	19	19
N.S.	1	1.04	1.00	0.80	0.76	0.76	0.80	0.76	0.76
time (sec)	N/A	0.208	0.010	0.099	0.259	0.236	0.113	0.284	0.443

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	72	141	84	0	0	0	0	0
N.S.	1	1.06	2.07	1.24	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.333	0.159	1.080	0.000	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	62	71	56	0	0	0	0	0	0
N.S.	1	1.15	0.90	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.317	0.038	0.000	0.000	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	37	43	40	39	48	61	39	99
N.S.	1	0.79	0.91	0.85	0.83	1.02	1.30	0.83	2.11
time (sec)	N/A	0.258	0.034	0.099	0.278	0.249	0.213	0.278	0.722

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	A	B	A	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	48	39	43	0	41	59	80	41	111
N.S.	1	0.81	0.90	0.00	0.85	1.23	1.67	0.85	2.31
time (sec)	N/A	0.273	0.050	0.000	0.275	0.273	12.296	0.276	0.396

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	127	127	249	0	0	207	0	577	0
N.S.	1	1.00	1.96	0.00	0.00	1.63	0.00	4.54	0.00
time (sec)	N/A	0.295	0.232	0.000	0.000	0.253	0.000	1.036	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	110	107	162	0	0	144	0	335	0
N.S.	1	0.97	1.47	0.00	0.00	1.31	0.00	3.05	0.00
time (sec)	N/A	0.264	0.135	0.000	0.000	0.249	0.000	0.663	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	63	63	98	0	0	91	0	153	0
N.S.	1	1.00	1.56	0.00	0.00	1.44	0.00	2.43	0.00
time (sec)	N/A	0.192	0.070	0.000	0.000	0.250	0.000	0.467	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	41	45	45	48	0	55	39
N.S.	1	1.00	0.95	1.05	1.05	1.12	0.00	1.28	0.91
time (sec)	N/A	0.173	0.031	0.050	0.275	0.252	0.000	0.282	0.535

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	99	99	85	0	0	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.219	0.180	0.000	0.000	0.000	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	151	151	133	0	0	0	0	0	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.236	0.737	0.000	0.000	0.000	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	173	175	147	0	0	0	0	0	0
N.S.	1	1.01	0.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.313	0.208	0.000	0.000	0.000	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	127	127	249	0	0	207	0	586	0
N.S.	1	1.00	1.96	0.00	0.00	1.63	0.00	4.61	0.00
time (sec)	N/A	0.282	0.216	0.000	0.000	0.263	0.000	1.035	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	110	107	162	0	0	144	0	319	0
N.S.	1	0.97	1.47	0.00	0.00	1.31	0.00	2.90	0.00
time (sec)	N/A	0.259	0.121	0.000	0.000	0.252	0.000	0.668	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	63	63	98	0	0	91	0	160	0
N.S.	1	1.00	1.56	0.00	0.00	1.44	0.00	2.54	0.00
time (sec)	N/A	0.188	0.064	0.000	0.000	0.247	0.000	0.454	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	41	45	45	48	0	50	39
N.S.	1	1.00	0.95	1.05	1.05	1.12	0.00	1.16	0.91
time (sec)	N/A	0.170	0.027	0.060	0.271	0.254	0.000	0.294	0.487

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	98	98	85	0	0	0	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.199	0.147	0.000	0.000	0.000	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	149	149	131	0	0	0	0	0	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.224	0.640	0.000	0.000	0.000	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	171	174	149	0	0	0	0	0	0
N.S.	1	1.02	0.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.301	0.188	0.000	0.000	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	249	252	248	0	0	0	0	0	0
N.S.	1	1.01	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.372	3.400	0.000	0.000	0.000	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	207	213	192	0	0	0	0	0	0
N.S.	1	1.03	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.319	0.216	0.000	0.000	0.000	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	184	184	149	0	0	0	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.233	0.077	0.000	0.000	0.000	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	145	145	108	0	0	0	0	0	0
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.211	0.414	0.000	0.000	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	190	190	166	0	0	0	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.258	0.986	0.000	0.000	0.000	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	221	225	212	0	0	0	0	0	0
N.S.	1	1.02	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.322	1.206	0.000	0.000	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	269	270	234	0	0	0	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.336	1.050	0.000	0.000	0.000	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	249	252	248	0	0	0	0	0	0
N.S.	1	1.01	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.337	1.453	0.000	0.000	0.000	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	207	213	192	0	0	0	0	0	0
N.S.	1	1.03	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.320	0.185	0.000	0.000	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	184	184	149	0	0	0	0	0	0
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.233	0.085	0.000	0.000	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	145	145	109	0	0	0	0	0	0
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.220	0.246	0.000	0.000	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	190	190	181	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.244	0.756	0.000	0.000	0.000	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	221	225	212	0	0	0	0	0	0
N.S.	1	1.02	0.96	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.317	1.136	0.000	0.000	0.000	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	269	270	232	0	0	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.332	1.055	0.000	0.000	0.000	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	42	36	39	38	0	38	39
N.S.	1	1.00	1.05	0.90	0.98	0.95	0.00	0.95	0.98
time (sec)	N/A	0.226	0.282	2.844	1.008	0.290	0.000	0.951	0.740

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	279	285	0	681	0	0	0	0	0
N.S.	1	1.02	0.00	2.44	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.763	0.000	6.016	0.000	0.000	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	207	213	0	394	0	0	0	0	0
N.S.	1	1.03	0.00	1.90	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.600	0.000	2.973	0.000	0.000	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	141	143	0	171	0	0	0	0	0
N.S.	1	1.01	0.00	1.21	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.450	0.000	0.955	0.000	0.000	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	42	36	39	48	61	38	39
N.S.	1	1.00	1.05	0.90	0.98	1.20	1.52	0.95	0.98
time (sec)	N/A	0.227	0.471	1.430	0.406	0.254	133.802	0.446	0.482

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	42	36	293	91	0	38	39
N.S.	1	1.00	1.05	0.90	7.32	2.28	0.00	0.95	0.98
time (sec)	N/A	0.222	4.677	1.422	1.886	0.245	0.000	0.541	1.305

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	A	F	F(-2)	F	F	F(-1)
verified	N/A	No	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	84	86	0	102	0	0	0	0	0
N.S.	1	1.02	0.00	1.21	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.359	0.000	1.068	0.000	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	81	74	50	0	0	55	105	82	0
N.S.	1	0.91	0.62	0.00	0.00	0.68	1.30	1.01	0.00
time (sec)	N/A	0.269	0.201	0.000	0.000	0.256	0.583	0.289	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	82	76	50	0	0	46	85	69	0
N.S.	1	0.93	0.61	0.00	0.00	0.56	1.04	0.84	0.00
time (sec)	N/A	0.261	0.158	0.000	0.000	0.267	0.351	0.284	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	41	42	30	0	0	36	58	44	0
N.S.	1	1.02	0.73	0.00	0.00	0.88	1.41	1.07	0.00
time (sec)	N/A	0.244	0.051	0.000	0.000	0.249	0.219	0.283	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	39	42	32	0	0	28	37	31	0
N.S.	1	1.08	0.82	0.00	0.00	0.72	0.95	0.79	0.00
time (sec)	N/A	0.193	0.048	0.000	0.000	0.256	0.110	0.264	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	45	45	79	0	0	0	0	0	0
N.S.	1	1.00	1.76	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.251	0.072	0.000	0.000	0.000	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	87	88	55	0	0	0	0	0	0
N.S.	1	1.01	0.63	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.299	0.081	0.000	0.000	0.000	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	B	F	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	49	528	45	0	140	0	95	43
N.S.	1	1.02	11.00	0.94	0.00	2.92	0.00	1.98	0.90
time (sec)	N/A	0.250	0.926	0.629	0.000	0.273	0.000	0.290	0.694

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	56	21	0	0	0	37	0
N.S.	1	1.00	2.15	0.81	0.00	0.00	0.00	1.42	0.00
time (sec)	N/A	0.286	0.112	1.616	0.000	0.000	0.000	0.297	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	0	0	0	5	0
N.S.	1	1.00	1.00	1.20	0.00	0.00	0.00	1.00	0.00
time (sec)	N/A	0.249	0.056	0.776	0.000	0.000	0.000	0.278	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	39	39	39	0	0	42	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	1.08	0.00	0.00	0.00
time (sec)	N/A	0.269	0.067	0.000	0.000	0.265	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	31	31	25	0	0	27	0	0	25
N.S.	1	1.00	0.81	0.00	0.00	0.87	0.00	0.00	0.81
time (sec)	N/A	0.262	0.040	0.000	0.000	0.234	0.000	0.000	0.340

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [24] had the largest ratio of [1]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	3	0.56	31	0.097
2	A	3	3	0.58	31	0.097
3	A	3	3	0.59	29	0.103
4	A	7	7	0.70	31	0.226
5	A	7	7	0.73	31	0.226
6	A	3	3	0.52	31	0.097
7	A	3	3	0.54	31	0.097
8	A	3	3	0.52	29	0.103
9	A	3	3	0.66	31	0.097
10	A	3	3	0.49	31	0.097
11	A	3	3	0.51	31	0.097
12	A	3	3	0.50	29	0.103
13	A	3	3	0.62	31	0.097
14	A	3	3	0.58	31	0.097
15	A	3	3	0.60	31	0.097
16	A	3	3	0.69	29	0.103
17	A	10	9	0.75	31	0.290
18	A	15	14	0.75	31	0.452
19	N/A	1	0	1.00	35	0.000
20	A	9	8	1.02	35	0.229
21	A	8	7	1.05	33	0.212
22	A	8	7	1.00	25	0.280

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	N/A	1	0	1.00	35	0.000
24	A	11	10	1.21	10	1.000
25	A	8	7	1.31	10	0.700
26	A	9	8	1.08	8	1.000
27	A	4	3	0.94	6	0.500
28	A	9	8	1.00	10	0.800
29	A	6	5	1.06	10	0.500
30	A	8	7	1.06	10	0.700
31	A	9	8	1.13	10	0.800
32	A	6	5	0.96	8	0.625
33	A	5	4	1.11	8	0.500
34	A	5	4	1.00	8	0.500
35	A	6	5	0.95	8	0.625
36	A	7	6	0.89	8	0.750
37	A	9	8	0.95	10	0.800
38	A	8	7	0.94	10	0.700
39	A	7	6	0.96	10	0.600
40	A	6	5	1.00	10	0.500
41	A	7	6	0.97	10	0.600
42	A	8	7	0.96	10	0.700
43	A	12	11	1.02	14	0.786
44	A	11	10	1.01	15	0.667
45	A	9	8	0.97	19	0.421
46	A	3	3	1.00	14	0.214
47	A	6	5	1.12	10	0.500
48	A	4	4	1.11	10	0.400
49	A	4	3	0.97	8	0.375
50	A	6	6	1.07	6	1.000
51	A	7	6	1.15	10	0.600
52	A	3	3	1.00	10	0.300
53	A	7	6	1.02	10	0.600
54	A	3	3	1.00	8	0.375
55	A	6	5	1.00	6	0.833

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
56	A	7	6	1.12	10	0.600
57	A	3	3	1.00	10	0.300
58	A	6	5	1.12	10	0.500
59	A	6	5	1.07	10	0.500
60	A	9	8	1.17	10	0.800
61	A	8	7	1.13	8	0.875
62	A	7	6	1.08	6	1.000
63	A	7	6	1.23	10	0.600
64	A	3	3	1.00	10	0.300
65	A	4	4	1.10	10	0.400
66	A	5	5	1.15	10	0.500
67	A	6	6	1.17	10	0.600
68	A	4	3	1.04	12	0.250
69	A	7	6	1.06	10	0.600
70	A	7	6	1.15	10	0.600
71	A	5	4	0.79	12	0.333
72	A	5	4	0.81	14	0.286
73	A	3	3	1.00	14	0.214
74	A	2	2	0.97	14	0.143
75	A	2	2	1.00	14	0.143
76	A	1	1	1.00	12	0.083
77	A	1	1	1.00	14	0.071
78	A	1	1	1.00	14	0.071
79	A	2	2	1.01	14	0.143
80	A	3	3	1.00	14	0.214
81	A	2	2	0.97	14	0.143
82	A	2	2	1.00	14	0.143
83	A	1	1	1.00	12	0.083
84	A	1	1	1.00	14	0.071
85	A	1	1	1.00	14	0.071
86	A	2	2	1.02	14	0.143
87	A	2	2	1.01	16	0.125
88	A	2	2	1.03	16	0.125
89	A	1	1	1.00	16	0.062

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
90	A	1	1	1.00	16	0.062
91	A	1	1	1.00	16	0.062
92	A	2	2	1.02	16	0.125
93	A	2	2	1.00	16	0.125
94	A	2	2	1.01	16	0.125
95	A	2	2	1.03	16	0.125
96	A	1	1	1.00	16	0.062
97	A	1	1	1.00	16	0.062
98	A	1	1	1.00	16	0.062
99	A	2	2	1.02	16	0.125
100	A	2	2	1.00	16	0.125
101	N/A	1	0	1.00	40	0.000
102	A	10	9	1.02	40	0.225
103	A	9	8	1.03	40	0.200
104	A	8	7	1.01	38	0.184
105	N/A	1	0	1.00	40	0.000
106	N/A	1	0	1.00	40	0.000
107	A	8	7	1.02	10	0.700
108	A	5	4	0.91	10	0.400
109	A	5	4	0.93	10	0.400
110	A	6	5	1.02	8	0.625
111	A	3	2	1.08	6	0.333
112	A	5	4	1.00	10	0.400
113	A	5	4	1.01	10	0.400
114	A	7	6	1.02	10	0.600
115	A	5	4	1.00	19	0.211
116	A	4	3	1.00	17	0.176
117	A	3	2	1.00	26	0.077
118	A	3	2	1.00	26	0.077

2.3. Detailed conclusion table specific for Rubi results

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int (f+gx)^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx$

3.1.1 Optimal result

Integrand size = 31, antiderivative size = 670

$$\begin{aligned}\int (f+gx)^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx &= -\frac{bf^2gx\sqrt{d-c^2dx^2}}{c\sqrt{1-c^2x^2}}-\frac{2bg^3x\sqrt{d-c^2dx^2}}{15c^3\sqrt{1-c^2x^2}}+\frac{bcf^3x^2\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}-\frac{3bfg^2x^2\sqrt{d-c^2dx^2}}{16c\sqrt{1-c^2x^2}}\\
&+\frac{bcf^2gx^3\sqrt{d-c^2dx^2}}{3\sqrt{1-c^2x^2}}-\frac{bg^3x^3\sqrt{d-c^2dx^2}}{45c\sqrt{1-c^2x^2}}+\frac{3bcfg^2x^4\sqrt{d-c^2dx^2}}{16\sqrt{1-c^2x^2}}+\frac{bcg^3x^5\sqrt{d-c^2dx^2}}{25\sqrt{1-c^2x^2}}\\
&+\frac{1}{2}f^3x\sqrt{d-c^2dx^2}(a+b\arccos(cx))-\frac{3fg^2x\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{8c^2}\\
&+\frac{3}{4}fg^2x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))-\frac{f^2g(1-c^2x^2)\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{c^2}\\
&-\frac{g^3(1-c^2x^2)\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{3c^4}\\
&+\frac{g^3(1-c^2x^2)^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{5c^4}\\
&-\frac{f^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{4bc\sqrt{1-c^2x^2}}-\frac{3fg^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{16bc^3\sqrt{1-c^2x^2}}\end{aligned}$$

output
```
1/2*f^3*x*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)-3/8*f*g^2*x*(a+b*arccos(c
*x))*(-c^2*d*x^2+d)^(1/2)/c^2+3/4*f*g^2*x^3*(a+b*arccos(c*x))*(-c^2*d*x^2+
d)^(1/2)-f^2*g*(-c^2*x^2+1)*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)/c^2-1/3
*g^3*(-c^2*x^2+1)*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)/c^4+1/5*g^3*(-c^2
*x^2+1)^2*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)/c^4-b*f^2*g*x*(-c^2*d*x^2
+d)^(1/2)/c/(-c^2*x^2+1)^(1/2)-2/15*b*g^3*x*(-c^2*d*x^2+d)^(1/2)/c^3/(-c^2
*x^2+1)^(1/2)+1/4*b*c*f^3*x^2*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-3/16
*b*f*g^2*x^2*(-c^2*d*x^2+d)^(1/2)/c/(-c^2*x^2+1)^(1/2)+1/3*b*c*f^2*g*x^3*(
-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-1/45*b*g^3*x^3*(-c^2*d*x^2+d)^(1/2)
/c/(-c^2*x^2+1)^(1/2)+3/16*b*c*f*g^2*x^4*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)
^(1/2)+1/25*b*c*g^3*x^5*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-1/4*f^3*(a
+b*arccos(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/c/(-c^2*x^2+1)^(1/2)-3/16*f*g^2*(
a+b*arccos(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/c^3/(-c^2*x^2+1)^(1/2)
```

3.1.2 Mathematica [A] (verified)

Time = 1.50 (sec) , antiderivative size = 442, normalized size of antiderivative = 0.66

$$\int (f+gx)^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx$$

$$= \frac{240a\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}(-16g^3-c^2g(120f^2+45fgx+8g^2x^2)+6c^4x(10f^3+20f^2gx+15fg^2x^2+4g}{}$$

input
```
Integrate[(f + g*x)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]),x]
```

output
```
(240*a*Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2]*(-16*g^3 - c^2*g*(120*f^2 + 4
5*f*g*x + 8*g^2*x^2) + 6*c^4*x*(10*f^3 + 20*f^2*g*x + 15*f*g^2*x^2 + 4*g^3
*x^3)) - 3600*a*c*Sqrt[d]*f*(4*c^2*f^2 + 3*g^2)*Sqrt[1 - c^2*x^2]*ArcTan[(
c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] - 2400*b*c^2*f^2*g*Sqrt
[d - c^2*d*x^2]*(9*c*x + 12*(1 - c^2*x^2)^(3/2)*ArcCos[c*x] - Cos[3*ArcCos
[c*x]]) + 3600*b*c^3*f^3*Sqrt[d - c^2*d*x^2]*(Cos[2*ArcCos[c*x]] + 2*ArcCo
s[c*x]*(-ArcCos[c*x] + Sin[2*ArcCos[c*x]])) + 675*b*c*f*g^2*Sqrt[d - c^2*d
*x^2]*(-8*ArcCos[c*x]^2 + Cos[4*ArcCos[c*x]] + 4*ArcCos[c*x]*Sin[4*ArcCos[
c*x]]) - 8*b*g^3*Sqrt[d - c^2*d*x^2]*(16*c*x*(30 + 5*c^2*x^2 - 9*c^4*x^4)
+ 15*ArcCos[c*x]*(30*Sqrt[1 - c^2*x^2] - 5*Sin[3*ArcCos[c*x]] - 3*Sin[5*Ar
cCos[c*x]])))/(28800*c^4*Sqrt[1 - c^2*x^2])
```

3.1.3 Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 372, normalized size of antiderivative = 0.56, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {5277, 5263, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d-c^2dx^2}(f+gx)^3(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{5277}$$

$$\frac{\sqrt{d-c^2dx^2}\int(f+gx)^3\sqrt{1-c^2x^2}(a+b\arccos(cx))dx}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5263}$$

$$\frac{\sqrt{d-c^2dx^2}\int\left(\sqrt{1-c^2x^2}(a+b\arccos(cx))f^3+3gx\sqrt{1-c^2x^2}(a+b\arccos(cx))f^2+3g^2x^2\sqrt{1-c^2x^2}(a+b\arccos\right.}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{d-c^2dx^2}\left(-\frac{3fg^2(a+b\arccos(cx))^2}{16bc^3}+\frac{1}{2}f^3x\sqrt{1-c^2x^2}(a+b\arccos(cx))-\frac{f^2g\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))}{c^2}-\frac{3fg^2x\sqrt{1-c^2x}}{}\right.}{}$$

input
```
Int[(f + g*x)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]),x]
```

output
```
(Sqrt[d - c^2*d*x^2]*(-((b*f^2*g*x)/c) - (2*b*g^3*x)/(15*c^3) + (b*c*f^3*x
^2)/4 - (3*b*f*g^2*x^2)/(16*c) + (b*c*f^2*g*x^3)/3 - (b*g^3*x^3)/(45*c) +
(3*b*c*f*g^2*x^4)/16 + (b*c*g^3*x^5)/25 + (f^3*x*Sqrt[1 - c^2*x^2]*(a + b*
ArcCos[c*x]))/2 - (3*f*g^2*x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(8*c^2
) + (3*f*g^2*x^3*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/4 - (f^2*g*(1 - c^
2*x^2)^(3/2)*(a + b*ArcCos[c*x]))/c^2 - (g^3*(1 - c^2*x^2)^(3/2)*(a + b*Ar
cCos[c*x]))/(3*c^4) + (g^3*(1 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x]))/(5*c^4
) - (f^3*(a + b*ArcCos[c*x])^2)/(4*b*c) - (3*f*g^2*(a + b*ArcCos[c*x])^2)/
(16*b*c^3)))/Sqrt[1 - c^2*x^2]
```

3.1.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5263
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

rule 5277
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[Simp[(d + e*x^2)^p/(1 - c^2*x^2)^
p]   Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && Intege
rQ[p - 1/2] &&  !GtQ[d, 0]
```

3.1.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.73 (sec) , antiderivative size = 1396, normalized size of antiderivative = 2.08

method	result	size
default	Expression too large to display	1396
parts	Expression too large to display	1396

input
```
int((g*x+f)^3*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
a*(f^3*(1/2*x*(-c^2*d*x^2+d)^(1/2)+1/2*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2
)*x/(-c^2*d*x^2+d)^(1/2)))+g^3*(-1/5*x^2*(-c^2*d*x^2+d)^(3/2)/c^2/d-2/15/d
/c^4*(-c^2*d*x^2+d)^(3/2))+3*f*g^2*(-1/4*x*(-c^2*d*x^2+d)^(3/2)/c^2/d+1/4/
c^2*(1/2*x*(-c^2*d*x^2+d)^(1/2)+1/2*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x
/(-c^2*d*x^2+d)^(1/2))))-f^2*g*(-c^2*d*x^2+d)^(3/2)/c^2/d)+b*(1/16*(-d*(c^
2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c^3/(c^2*x^2-1)*arccos(c*x)^2*f*(4*c^2*
f^2+3*g^2)+1/800*(-d*(c^2*x^2-1))^(1/2)*(16*I*c^5*x^5*(-c^2*x^2+1)^(1/2)+1
6*c^6*x^6-20*I*(-c^2*x^2+1)^(1/2)*x^3*c^3-28*c^4*x^4+5*I*(-c^2*x^2+1)^(1/2
)*x*c+13*c^2*x^2-1)*g^3*(I+5*arccos(c*x))/c^4/(c^2*x^2-1)+3/256*(-d*(c^2*x
^2-1))^(1/2)*(8*I*(-c^2*x^2+1)^(1/2)*c^4*x^4+8*c^5*x^5-8*I*(-c^2*x^2+1)^(1
/2)*x^2*c^2-12*c^3*x^3+I*(-c^2*x^2+1)^(1/2)+4*c*x)*f*g^2*(4*arccos(c*x)+I)
/c^3/(c^2*x^2-1)+1/288*(-d*(c^2*x^2-1))^(1/2)*(4*I*c^3*x^3*(-c^2*x^2+1)^(1
/2)+4*c^4*x^4-3*I*(-c^2*x^2+1)^(1/2)*x*c-5*c^2*x^2+1)*g*(12*I*f^2*c^2+36*a
rccos(c*x)*c^2*f^2+I*g^2+3*arccos(c*x)*g^2)/c^4/(c^2*x^2-1)+1/16*(-d*(c^2*
x^2-1))^(1/2)*(2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+2*c^3*x^3-I*(-c^2*x^2+1)^(1/
2)-2*c*x)*f^3*(I+2*arccos(c*x))/c/(c^2*x^2-1)-1/16*(-d*(c^2*x^2-1))^(1/2)*
(I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*g*(6*I*f^2*c^2+6*arccos(c*x)*c^2*f^2+
I*g^2+arccos(c*x)*g^2)/c^4/(c^2*x^2-1)-1/16*(-d*(c^2*x^2-1))^(1/2)*(c^2*x^
2-I*(-c^2*x^2+1)^(1/2)*x*c-1)*g*(-6*I*f^2*c^2+6*arccos(c*x)*c^2*f^2-I*g^2+
arccos(c*x)*g^2)/c^4/(c^2*x^2-1)+1/16*(-d*(c^2*x^2-1))^(1/2)*(-2*I*(-c^...
```

3.1.5 Fricas [F]

$$\int (f+gx)^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx = \int \sqrt{-c^2dx^2+d}(gx+f)^3(b\arccos{(cx)}+a)\,dx$$

input
```
integrate((g*x+f)^3*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="f
ricas")
```

output
```
integral((a*g^3*x^3 + 3*a*f*g^2*x^2 + 3*a*f^2*g*x + a*f^3 + (b*g^3*x^3 + 3
*b*f*g^2*x^2 + 3*b*f^2*g*x + b*f^3)*arccos(c*x))*sqrt(-c^2*d*x^2 + d), x)
```

3.1.6 Sympy [F]

$$\int (f+gx)^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx = \int \sqrt{-d\,(cx-1)\,(cx+1)}(a+b\,\mathrm{acos}\,(cx))\,(f+gx)^3\,dx$$

input
```
integrate((g*x+f)**3*(a+b*acos(c*x))*(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acos(c*x))*(f + g*x)**3, x)
```

3.1.7 Maxima [F]

$$\int (f+gx)^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx = \int \sqrt{-c^2dx^2+d}(gx+f)^3(b\arccos(cx)+a)\,dx$$

input
```
integrate((g*x+f)^3*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="m
axima")
```

output
```
1/2*(sqrt(-c^2*d*x^2 + d)*x + sqrt(d)*arcsin(c*x)/c)*a*f^3 - 1/15*a*g^3*(3
*(-c^2*d*x^2 + d)^(3/2)*x^2/(c^2*d) + 2*(-c^2*d*x^2 + d)^(3/2)/(c^4*d)) +
3/8*a*f*g^2*(sqrt(-c^2*d*x^2 + d)*x/c^2 - 2*(-c^2*d*x^2 + d)^(3/2)*x/(c^2*
d) + sqrt(d)*arcsin(c*x)/c^3) - (-c^2*d*x^2 + d)^(3/2)*a*f^2*g/(c^2*d) + s
qrt(d)*integrate((b*g^3*x^3 + 3*b*f*g^2*x^2 + 3*b*f^2*g*x + b*f^3)*sqrt(c*
x + 1)*sqrt(-c*x + 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x), x)
```

3.1.8 Giac [F(-2)]

Exception generated.

$$\int (f+gx)^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((g*x+f)^3*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="g
iac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.1.9 Mupad [F(-1)]

Timed out.

$$\int (f+gx)^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx=\int (f+g\,x)^3\,(a+b\,\mathrm{acos}(c\,x))\,\sqrt{d-c^2\,d\,x^2}\,dx$$

input
```
int((f + g*x)^3*(a + b*acos(c*x))*(d - c^2*d*x^2)^(1/2),x)
```

output
```
int((f + g*x)^3*(a + b*acos(c*x))*(d - c^2*d*x^2)^(1/2), x)
```

3.2 $\int (f+gx)^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx$

3.2.1 Optimal result

Integrand size = 31, antiderivative size = 450

$$\begin{aligned}
&\int (f+gx)^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx\\
&=-\frac{2bfgx\sqrt{d-c^2dx^2}}{3c\sqrt{1-c^2x^2}}+\frac{bcf^2x^2\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}-\frac{bg^2x^2\sqrt{d-c^2dx^2}}{16c\sqrt{1-c^2x^2}}\\
&\quad+\frac{2bcfgx^3\sqrt{d-c^2dx^2}}{9\sqrt{1-c^2x^2}}+\frac{bcg^2x^4\sqrt{d-c^2dx^2}}{16\sqrt{1-c^2x^2}}+\frac{1}{2}f^2x\sqrt{d-c^2dx^2}(a+b\arccos(cx))\\
&\quad-\frac{g^2x\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{8c^2}+\frac{1}{4}g^2x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))\\
&\quad-\frac{2fg(1-c^2x^2)\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{3c^2}\\
&\quad-\frac{f^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{4bc\sqrt{1-c^2x^2}}-\frac{g^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{16bc^3\sqrt{1-c^2x^2}}
\end{aligned}$$

output
```
1/2*f^2*x*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)-1/8*g^2*x*(a+b*arccos(c*x
))*(-c^2*d*x^2+d)^(1/2)/c^2+1/4*g^2*x^3*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(
1/2)-2/3*f*g*(-c^2*x^2+1)*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)/c^2-2/3*b
*f*g*x*(-c^2*d*x^2+d)^(1/2)/c/(-c^2*x^2+1)^(1/2)+1/4*b*c*f^2*x^2*(-c^2*d*x
^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-1/16*b*g^2*x^2*(-c^2*d*x^2+d)^(1/2)/c/(-c^2
*x^2+1)^(1/2)+2/9*b*c*f*g*x^3*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)+1/16
*b*c*g^2*x^4*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-1/4*f^2*(a+b*arccos(c
*x))^2*(-c^2*d*x^2+d)^(1/2)/b/c/(-c^2*x^2+1)^(1/2)-1/16*g^2*(a+b*arccos(c*
x))^2*(-c^2*d*x^2+d)^(1/2)/b/c^3/(-c^2*x^2+1)^(1/2)
```

3.2.2 Mathematica [A] (verified)

Time = 1.49 (sec) , antiderivative size = 320, normalized size of antiderivative = 0.71

$$\int (f+gx)^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx$$
$$= \frac{48ac\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}(12c^2f^2x+16fg(-1+c^2x^2)+3g^2x(-1+2c^2x^2))-144a\sqrt{d}(4c^2f^2+g^2)\sqrt{1}}{}$$

input
```
Integrate[(f + g*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]),x]
```

output
```
(48*a*c*Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2]*(12*c^2*f^2*x + 16*f*g*(-1 +
 c^2*x^2) + 3*g^2*x*(-1 + 2*c^2*x^2)) - 144*a*Sqrt[d]*(4*c^2*f^2 + g^2)*Sq
rt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))]
 - 64*b*c*f*g*Sqrt[d - c^2*d*x^2]*(9*c*x + 12*(1 - c^2*x^2)^(3/2)*ArcCos[c
*x] - Cos[3*ArcCos[c*x]]) + 144*b*c^2*f^2*Sqrt[d - c^2*d*x^2]*(Cos[2*ArcCo
s[c*x]] + 2*ArcCos[c*x]*(-ArcCos[c*x] + Sin[2*ArcCos[c*x]])) + 9*b*g^2*Sqr
t[d - c^2*d*x^2]*(-8*ArcCos[c*x]^2 + Cos[4*ArcCos[c*x]] + 4*ArcCos[c*x]*Si
n[4*ArcCos[c*x]]))/(1152*c^3*Sqrt[1 - c^2*x^2])
```

3.2.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.58, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {5277, 5263, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d-c^2dx^2}(f+gx)^2(a+b\arccos(cx))\,dx$$
$$\Big\downarrow \; 5277$$
$$\frac{\sqrt{d-c^2dx^2}\int (f+gx)^2\sqrt{1-c^2x^2}(a+b\arccos(cx))dx}{\sqrt{1-c^2x^2}}$$
$$\Big\downarrow \; 5263$$
$$\frac{\sqrt{d-c^2dx^2}\int \left(\sqrt{1-c^2x^2}(a+b\arccos(cx))f^2+2gx\sqrt{1-c^2x^2}(a+b\arccos(cx))f+g^2x^2\sqrt{1-c^2x^2}(a+b\arccos(\right.}{\sqrt{1-c^2x^2}}$$

$\downarrow$ 2009

$$\frac{\sqrt{d-c^2dx^2}\left(-\frac{g^2(a+b\arccos(cx))^2}{16bc^3}+\frac{1}{2}f^2x\sqrt{1-c^2x^2}(a+b\arccos(cx))-\frac{2fg(1-c^2x^2)^{3/2}(a+b\arccos(cx))}{3c^2}-\frac{g^2x\sqrt{1-c^2x^2}(a+\dots}{8c^2}\right.}{\sqrt{\dots}}$$

input
```
Int[(f + g*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]),x]
```

output
```
(Sqrt[d - c^2*d*x^2]*((-2*b*f*g*x)/(3*c) + (b*c*f^2*x^2)/4 - (b*g^2*x^2)/(
16*c) + (2*b*c*f*g*x^3)/9 + (b*c*g^2*x^4)/16 + (f^2*x*Sqrt[1 - c^2*x^2]*(a
 + b*ArcCos[c*x]))/2 - (g^2*x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(8*c^
2) + (g^2*x^3*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/4 - (2*f*g*(1 - c^2*x
^2)^(3/2)*(a + b*ArcCos[c*x]))/(3*c^2) - (f^2*(a + b*ArcCos[c*x])^2)/(4*b*
c) - (g^2*(a + b*ArcCos[c*x])^2)/(16*b*c^3)))/Sqrt[1 - c^2*x^2]
```

3.2.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5263
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

rule 5277
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[Simp[(d + e*x^2)^p/(1 - c^2*x^2)^
p]   Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && Intege
rQ[p - 1/2] && !GtQ[d, 0]
```

3.2.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.40 (sec) , antiderivative size = 973, normalized size of antiderivative = 2.16

method	result
default	$a\left(f^{2}\left(\frac{x\sqrt{-c^{2}d\,x^{2}+d}}{2}+\frac{d\arctan\left(\frac{\sqrt{c^{2}d}\,x}{\sqrt{-c^{2}d\,x^{2}+d}}\right)}{2\sqrt{c^{2}d}}\right)+g^{2}\left(-\frac{x\left(-c^{2}d\,x^{2}+d\right)^{\frac{3}{2}}}{4c^{2}d}+\frac{\frac{x\sqrt{-c^{2}d\,x^{2}+d}}{2}+\frac{d\arctan\left(\frac{\sqrt{c^{2}d}\,x}{\sqrt{-c^{2}d\,x^{2}+d}}\right)}{2\sqrt{c^{2}d}}}{4c^{2}}\right)\right.$
parts	$a\left(f^{2}\left(\frac{x\sqrt{-c^{2}d\,x^{2}+d}}{2}+\frac{d\arctan\left(\frac{\sqrt{c^{2}d}\,x}{\sqrt{-c^{2}d\,x^{2}+d}}\right)}{2\sqrt{c^{2}d}}\right)+g^{2}\left(-\frac{x\left(-c^{2}d\,x^{2}+d\right)^{\frac{3}{2}}}{4c^{2}d}+\frac{\frac{x\sqrt{-c^{2}d\,x^{2}+d}}{2}+\frac{d\arctan\left(\frac{\sqrt{c^{2}d}\,x}{\sqrt{-c^{2}d\,x^{2}+d}}\right)}{2\sqrt{c^{2}d}}}{4c^{2}}\right)\right.$

input
```
int((g*x+f)^2*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
a*(f^2*(1/2*x*(-c^2*d*x^2+d)^(1/2)+1/2*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2
)*x/(-c^2*d*x^2+d)^(1/2)))+g^2*(-1/4*x*(-c^2*d*x^2+d)^(3/2)/c^2/d+1/4/c^2*
(1/2*x*(-c^2*d*x^2+d)^(1/2)+1/2*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c
^2*d*x^2+d)^(1/2))))-2/3*f*g*(-c^2*d*x^2+d)^(3/2)/c^2/d)+b*(1/16*(-d*(c^2*
x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c^3/(c^2*x^2-1)*arccos(c*x)^2*(4*c^2*f^2+
g^2)+1/256*(-d*(c^2*x^2-1))^(1/2)*(8*I*(-c^2*x^2+1)^(1/2)*c^4*x^4+8*c^5*x^
5-8*I*(-c^2*x^2+1)^(1/2)*x^2*c^2-12*c^3*x^3+I*(-c^2*x^2+1)^(1/2)+4*c*x)*g^
2*(4*arccos(c*x)+I)/c^3/(c^2*x^2-1)+1/36*(-d*(c^2*x^2-1))^(1/2)*(4*I*c^3*x
^3*(-c^2*x^2+1)^(1/2)+4*c^4*x^4-3*I*(-c^2*x^2+1)^(1/2)*x*c-5*c^2*x^2+1)*f*
g*(I+3*arccos(c*x))/c^2/(c^2*x^2-1)+1/16*(-d*(c^2*x^2-1))^(1/2)*(2*I*(-c^2
*x^2+1)^(1/2)*x^2*c^2+2*c^3*x^3-I*(-c^2*x^2+1)^(1/2)-2*c*x)*f^2*(I+2*arcco
s(c*x))/c/(c^2*x^2-1)-1/4*(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*x*c
+c^2*x^2-1)*f*g*(arccos(c*x)+I)/c^2/(c^2*x^2-1)-1/4*(-d*(c^2*x^2-1))^(1/2)
*(c^2*x^2-I*(-c^2*x^2+1)^(1/2)*x*c-1)*f*g*(arccos(c*x)-I)/c^2/(c^2*x^2-1)+
1/16*(-d*(c^2*x^2-1))^(1/2)*(-2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+2*c^3*x^3+I*(
-c^2*x^2+1)^(1/2)-2*c*x)*f^2*(-I+2*arccos(c*x))/c/(c^2*x^2-1)+1/36*(-d*(c^
2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2-4*I*c^3*x^3*(-c^2*x^2+1)^(1/2)+3*I*(-
c^2*x^2+1)^(1/2)*x*c+1)*f*g*(-I+3*arccos(c*x))/c^2/(c^2*x^2-1)+1/256*(-d*(
c^2*x^2-1))^(1/2)*(-8*I*(-c^2*x^2+1)^(1/2)*x^4*c^4+8*c^5*x^5+8*I*(-c^2*x^2
+1)^(1/2)*x^2*c^2-12*c^3*x^3-I*(-c^2*x^2+1)^(1/2)+4*c*x)*g^2*(-I+4*arcc...
```

3.2.5 Fricas [F]

$$\int (f+gx)^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx=\int \sqrt{-c^2dx^2+d}(gx+f)^2(b\arccos{(cx)}+a)\,dx$$

input
```
integrate((g*x+f)^2*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="f
ricas")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(a*g^2*x^2 + 2*a*f*g*x + a*f^2 + (b*g^2*x^2
+ 2*b*f*g*x + b*f^2)*arccos(c*x)), x)
```

3.2.6 Sympy [F]

$$\int (f+gx)^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx=\int \sqrt{-d\,(cx-1)\,(cx+1)}(a+b\operatorname{acos}{(cx)})\,(f+gx)^2\,dx$$

input
```
integrate((g*x+f)**2*(a+b*acos(c*x))*(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acos(c*x))*(f + g*x)**2, x)
```

3.2.7 Maxima [F]

$$\int (f+gx)^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx=\int \sqrt{-c^2dx^2+d}(gx+f)^2(b\arccos{(cx)}+a)\,dx$$

input
```
integrate((g*x+f)^2*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="m
axima")
```

output
```
1/2*(sqrt(-c^2*d*x^2 + d)*x + sqrt(d)*arcsin(c*x)/c)*a*f^2 + 1/8*a*g^2*(sq
rt(-c^2*d*x^2 + d)*x/c^2 - 2*(-c^2*d*x^2 + d)^(3/2)*x/(c^2*d) + sqrt(d)*ar
csin(c*x)/c^3) - 2/3*(-c^2*d*x^2 + d)^(3/2)*a*f*g/(c^2*d) + sqrt(d)*integr
ate((b*g^2*x^2 + 2*b*f*g*x + b*f^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(s
qrt(c*x + 1)*sqrt(-c*x + 1), c*x), x)
```

3.2.8 Giac [F(-2)]

Exception generated.

$$\int (f+gx)^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((g*x+f)^2*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="g
iac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.2.9 Mupad [F(-1)]

Timed out.

$$\int (f+gx)^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx = \int (f+g\,x)^2\,(a+b\,\mathrm{acos}(c\,x))\,\sqrt{d-c^2\,d\,x^2}\,dx$$

input
```
int((f + g*x)^2*(a + b*acos(c*x))*(d - c^2*d*x^2)^(1/2),x)
```

output
```
int((f + g*x)^2*(a + b*acos(c*x))*(d - c^2*d*x^2)^(1/2), x)
```

3.3 $\int (f+gx)\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx$

3.3.1 Optimal result

Integrand size = 29, antiderivative size = 238

$$\int (f+gx)\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx = -\frac{bgx\sqrt{d-c^2dx^2}}{3c\sqrt{1-c^2x^2}}+\frac{bcfx^2\sqrt{d-c^2dx^2}}{4\sqrt{1-c^2x^2}}+\frac{bcgx^3\sqrt{d-c^2dx^2}}{9\sqrt{1-c^2x^2}}+\frac{1}{2}fx\sqrt{d-c^2dx^2}(a+b\arccos(cx))-\frac{g(1-c^2x^2)\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{3c^2}-\frac{f\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{4bc\sqrt{1-c^2x^2}}$$

output
```
1/2*f*x*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)-1/3*g*(-c^2*x^2+1)*(a+b*arc
cos(c*x))*(-c^2*d*x^2+d)^(1/2)/c^2-1/3*b*g*x*(-c^2*d*x^2+d)^(1/2)/c/(-c^2*
x^2+1)^(1/2)+1/4*b*c*f*x^2*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)+1/9*b*c
*g*x^3*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-1/4*f*(a+b*arccos(c*x))^2*(
-c^2*d*x^2+d)^(1/2)/b/c/(-c^2*x^2+1)^(1/2)
```

3.3.2 Mathematica [A] (verified)

Time = 1.67 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.92

$$\int (f+gx)\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx$$

$$= \frac{12a\sqrt{d-c^2dx^2}(3c^2fx+2g(-1+c^2x^2))-36ac\sqrt{d}f\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right)+\frac{2bg\sqrt{d-c^2dx^2}\left(-9cx-12(1-c^2x^2)^{3/2}\right.}{\sqrt{1-c^2x}}}{72c^2}$$

input
```
Integrate[(f + g*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]),x]
```

output
```
(12*a*Sqrt[d - c^2*d*x^2]*(3*c^2*f*x + 2*g*(-1 + c^2*x^2)) - 36*a*c*Sqrt[d
]*f*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + (2*b*g*Sq
rt[d - c^2*d*x^2]*(-9*c*x - 12*(1 - c^2*x^2)^(3/2)*ArcCos[c*x] + Cos[3*Arc
Cos[c*x]]))/Sqrt[1 - c^2*x^2] + (9*b*c*f*Sqrt[d - c^2*d*x^2]*(-2*ArcCos[c*
x]^2 + Cos[2*ArcCos[c*x]] + 2*ArcCos[c*x]*Sin[2*ArcCos[c*x]]))/Sqrt[1 - c^
2*x^2])/(72*c^2)
```

3.3.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.59, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {5277, 5263, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d-c^2dx^2}(f+gx)(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \; 5277$$

$$\frac{\sqrt{d-c^2dx^2}\int (f+gx)\sqrt{1-c^2x^2}(a+b\arccos(cx))dx}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \; 5263$$

$$\frac{\sqrt{d-c^2dx^2}\int \left(f\sqrt{1-c^2x^2}(a+b\arccos(cx))+gx\sqrt{1-c^2x^2}(a+b\arccos(cx))\right)dx}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \; 2009$$

$$\frac{\sqrt{d-c^2dx^2}\left(\frac{1}{2}fx\sqrt{1-c^2x^2}(a+b\arccos(cx))-\frac{g(1-c^2x^2)^{3/2}(a+b\arccos(cx))}{3c^2}-\frac{f(a+b\arccos(cx))^2}{4bc}+\frac{1}{4}bcfx^2+\frac{1}{9}bcgx^3-\right.}{\sqrt{1-c^2x^2}}$$

input
```
Int[(f + g*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]),x]
```

output
```
(Sqrt[d - c^2*d*x^2]*(-1/3*(b*g*x)/c + (b*c*f*x^2)/4 + (b*c*g*x^3)/9 + (f*
x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/2 - (g*(1 - c^2*x^2)^(3/2)*(a + b
*ArcCos[c*x]))/(3*c^2) - (f*(a + b*ArcCos[c*x])^2)/(4*b*c)))/Sqrt[1 - c^2*
x^2]
```

3.3.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5263
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

rule 5277
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[Simp[(d + e*x^2)^p/(1 - c^2*x^2)^
p]   Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && Intege
rQ[p - 1/2] &&  !GtQ[d, 0]
```

3.3.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.26 (sec) , antiderivative size = 628, normalized size of antiderivative = 2.64

method	result
default	$\frac{afx\sqrt{-c^2dx^2+d}}{2}+\frac{afd\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2dx^2+d}}\right)}{2\sqrt{c^2d}}-\frac{ag(-c^2dx^2+d)^{\frac{3}{2}}}{3c^2d}+b\left(\frac{\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,\arccos(cx)^2f}{4c(c^2x^2-1)}+\frac{\sqrt{-d(c}{}\right.$
parts	$\frac{afx\sqrt{-c^2dx^2+d}}{2}+\frac{afd\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2dx^2+d}}\right)}{2\sqrt{c^2d}}-\frac{ag(-c^2dx^2+d)^{\frac{3}{2}}}{3c^2d}+b\left(\frac{\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,\arccos(cx)^2f}{4c(c^2x^2-1)}+\frac{\sqrt{-d(c}{}\right.$

input
```
int((g*x+f)*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
1/2*a*f*x*(-c^2*d*x^2+d)^(1/2)+1/2*a*f*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2
)*x/(-c^2*d*x^2+d)^(1/2))-1/3*a*g*(-c^2*d*x^2+d)^(3/2)/c^2/d+b*(1/4*(-d*(c
^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c/(c^2*x^2-1)*arccos(c*x)^2*f+1/72*(-d
*(c^2*x^2-1))^(1/2)*(4*I*c^3*x^3*(-c^2*x^2+1)^(1/2)+4*c^4*x^4-3*I*(-c^2*x^
2+1)^(1/2)*x*c-5*c^2*x^2+1)*g*(I+3*arccos(c*x))/c^2/(c^2*x^2-1)+1/16*(-d*(
c^2*x^2-1))^(1/2)*(2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+2*c^3*x^3-I*(-c^2*x^2+1)
^(1/2)-2*c*x)*f*(I+2*arccos(c*x))/c/(c^2*x^2-1)-1/8*(-d*(c^2*x^2-1))^(1/2)
*(I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*g*(arccos(c*x)+I)/c^2/(c^2*x^2-1)-1/
8*(-d*(c^2*x^2-1))^(1/2)*(c^2*x^2-I*(-c^2*x^2+1)^(1/2)*x*c-1)*g*(arccos(c*
x)-I)/c^2/(c^2*x^2-1)+1/16*(-d*(c^2*x^2-1))^(1/2)*(-2*I*(-c^2*x^2+1)^(1/2)
*x^2*c^2+2*c^3*x^3+I*(-c^2*x^2+1)^(1/2)-2*c*x)*f*(-I+2*arccos(c*x))/c/(c^2
*x^2-1)+1/72*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2-4*I*c^3*x^3*(-c^2
*x^2+1)^(1/2)+3*I*(-c^2*x^2+1)^(1/2)*x*c+1)*g*(-I+3*arccos(c*x))/c^2/(c^2*
x^2-1))
```

3.3.5 Fricas [F]

$$\int (f+gx)\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx=\int \sqrt{-c^2dx^2+d}(gx+f)(b\arccos(cx)+a)\,dx$$

input
```
integrate((g*x+f)*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="fri
cas")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(a*g*x + a*f + (b*g*x + b*f)*arccos(c*x)), x
)
```

3.3.6 Sympy [F]

$$\int (f+gx)\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx = \int \sqrt{-d\,(cx-1)\,(cx+1)}(a+b\,\mathrm{acos}\,(cx))\,(f+gx)\;dx$$

input
```
integrate((g*x+f)*(a+b*acos(c*x))*(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acos(c*x))*(f + g*x), x)
```

3.3.7 Maxima [F]

$$\int (f+gx)\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx = \int \sqrt{-c^2dx^2+d}(gx+f)(b\arccos{(cx)}+a)\,dx$$

input
```
integrate((g*x+f)*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="max
ima")
```

output
```
1/2*(sqrt(-c^2*d*x^2 + d)*x + sqrt(d)*arcsin(c*x)/c)*a*f + sqrt(d)*integra
te((b*g*x + b*f)*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(sqrt(c*x + 1)*sqrt(-
c*x + 1), c*x), x) - 1/3*(-c^2*d*x^2 + d)^(3/2)*a*g/(c^2*d)
```

3.3.8 Giac [F(-2)]

Exception generated.

$$\int (f+gx)\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((g*x+f)*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="gia
c")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.3.9 Mupad [F(-1)]

Timed out.

$$\int (f+gx)\sqrt{d-c^2dx^2}(a+b\arccos(cx))\,dx = \int (f+g\,x)\,\left(a+b\,\mathrm{acos}(c\,x)\right)\,\sqrt{d-c^2\,d\,x^2}\,dx$$

input
```
int((f + g*x)*(a + b*acos(c*x))*(d - c^2*d*x^2)^(1/2),x)
```

output
```
int((f + g*x)*(a + b*acos(c*x))*(d - c^2*d*x^2)^(1/2), x)
```

3.4 $\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{f+gx}\,dx$

3.4.1 Optimal result

Integrand size = 31, antiderivative size = 725

$$
\begin{aligned}
&\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{f+gx}\,dx \\
&= \frac{a\sqrt{d-c^2dx^2}}{g}+\frac{bcx\sqrt{d-c^2dx^2}}{g\sqrt{1-c^2x^2}}+\frac{b\sqrt{d-c^2dx^2}\arccos(cx)}{g} \\
&-\frac{cx\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2bg\sqrt{1-c^2x^2}}+\frac{\left(1-\frac{c^2f^2}{g^2}\right)\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2bc(f+gx)\sqrt{1-c^2x^2}} \\
&-\frac{\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2bc(f+gx)} \\
&-\frac{a\sqrt{c^2f^2-g^2}\sqrt{d-c^2dx^2}\arctan\left(\frac{g+c^2fx}{\sqrt{c^2f^2-g^2}\sqrt{1-c^2x^2}}\right)}{g^2\sqrt{1-c^2x^2}} \\
&-\frac{ib\sqrt{c^2f^2-g^2}\sqrt{d-c^2dx^2}\arccos(cx)\log\left(1+\frac{e^{i\arccos(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{g^2\sqrt{1-c^2x^2}} \\
&+\frac{ib\sqrt{c^2f^2-g^2}\sqrt{d-c^2dx^2}\arccos(cx)\log\left(1+\frac{e^{i\arccos(cx)}g}{cf+\sqrt{c^2f^2-g^2}}\right)}{g^2\sqrt{1-c^2x^2}} \\
&-\frac{b\sqrt{c^2f^2-g^2}\sqrt{d-c^2dx^2}\operatorname{PolyLog}\left(2,-\frac{e^{i\arccos(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{g^2\sqrt{1-c^2x^2}} \\
&+\frac{b\sqrt{c^2f^2-g^2}\sqrt{d-c^2dx^2}\operatorname{PolyLog}\left(2,-\frac{e^{i\arccos(cx)}g}{cf+\sqrt{c^2f^2-g^2}}\right)}{g^2\sqrt{1-c^2x^2}}
\end{aligned}
$$

output
```
a*(-c^2*d*x^2+d)^(1/2)/g+b*arccos(c*x)*(-c^2*d*x^2+d)^(1/2)/g+b*c*x*(-c^2*
d*x^2+d)^(1/2)/g/(-c^2*x^2+1)^(1/2)-1/2*c*x*(a+b*arccos(c*x))^2*(-c^2*d*x^
2+d)^(1/2)/b/g/(-c^2*x^2+1)^(1/2)+1/2*(1-c^2*f^2/g^2)*(a+b*arccos(c*x))^2*
(-c^2*d*x^2+d)^(1/2)/b/c/(g*x+f)/(-c^2*x^2+1)^(1/2)-a*arctan((c^2*f*x+g)/(
c^2*f^2-g^2)^(1/2)/(-c^2*x^2+1)^(1/2))*(c^2*f^2-g^2)^(1/2)*(-c^2*d*x^2+d)^
(1/2)/g^2/(-c^2*x^2+1)^(1/2)-I*b*arccos(c*x)*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2
))*g/(c*f-(c^2*f^2-g^2)^(1/2)))*(c^2*f^2-g^2)^(1/2)*(-c^2*d*x^2+d)^(1/2)/g
^2/(-c^2*x^2+1)^(1/2)+I*b*arccos(c*x)*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))*g/(c
*f+(c^2*f^2-g^2)^(1/2)))*(c^2*f^2-g^2)^(1/2)*(-c^2*d*x^2+d)^(1/2)/g^2/(-c^
2*x^2+1)^(1/2)-b*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2-g^2
)^(1/2)))*(c^2*f^2-g^2)^(1/2)*(-c^2*d*x^2+d)^(1/2)/g^2/(-c^2*x^2+1)^(1/2)+
b*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2)))*(c^2*
f^2-g^2)^(1/2)*(-c^2*d*x^2+d)^(1/2)/g^2/(-c^2*x^2+1)^(1/2)-1/2*(a+b*arccos
(c*x))^2*(-c^2*x^2+1)^(1/2)*(-c^2*d*x^2+d)^(1/2)/b/c/(g*x+f)
```

3.4.2 Mathematica [A] (warning: unable to verify)

Time = 4.19 (sec) , antiderivative size = 1095, normalized size of antiderivative = 1.51

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{f+gx}\,dx =$$

$$-\frac{-2ag\sqrt{d-c^2dx^2}+2ac\sqrt{d}f\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right)-2a\sqrt{d}\sqrt{-c^2f^2+g^2}\log(f+gx)+2a\sqrt{d}\sqrt{-c^2f^2+}}{}$$

input
```
Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(f + g*x),x]
```

output
```
-1/2*(-2*a*g*Sqrt[d - c^2*d*x^2] + 2*a*c*Sqrt[d]*f*ArcTan[(c*x*Sqrt[d - c^
2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] - 2*a*Sqrt[d]*Sqrt[-(c^2*f^2) + g^2]*L
og[f + g*x] + 2*a*Sqrt[d]*Sqrt[-(c^2*f^2) + g^2]*Log[d*(g + c^2*f*x) + Sqr
t[d]*Sqrt[-(c^2*f^2) + g^2]*Sqrt[d - c^2*d*x^2]] + b*Sqrt[d - c^2*d*x^2]*(
(-2*c*g*x)/Sqrt[1 - c^2*x^2] - 2*g*ArcCos[c*x] + (c*f*ArcCos[c*x]^2)/Sqrt[
1 - c^2*x^2] + (2*(-(c*f) + g)*(c*f + g)*(2*ArcCos[c*x]*ArcTanh[((c*f + g)
*Cot[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] - 2*ArcCos[-((c*f)/g)]*ArcTan
h[((-(c*f) + g)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + (ArcCos[-((c
*f)/g)] - (2*I)*ArcTanh[((c*f + g)*Cot[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g
^2]] + (2*I)*ArcTanh[((-(c*f) + g)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g
^2]])*Log[Sqrt[-(c^2*f^2) + g^2]/(Sqrt[2]*E^((I/2)*ArcCos[c*x])*Sqrt[g]*Sq
rt[c*(f + g*x)])] + (ArcCos[-((c*f)/g)] + (2*I)*(ArcTanh[((c*f + g)*Cot[Ar
cCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] - ArcTanh[((-(c*f) + g)*Tan[ArcCos[c
*x]/2])/Sqrt[-(c^2*f^2) + g^2]]))*Log[(E^((I/2)*ArcCos[c*x])*Sqrt[-(c^2*f^
2) + g^2])/(Sqrt[2]*Sqrt[g]*Sqrt[c*(f + g*x)])] - (ArcCos[-((c*f)/g)] - (2
*I)*ArcTanh[((-(c*f) + g)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])*Log
[((c*f + g)*((-I)*c*f + I*g + Sqrt[-(c^2*f^2) + g^2])*(-I + Tan[ArcCos[c*x
]/2]))/(g*(c*f + g + Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))] - (ArcCo
s[-((c*f)/g)] + (2*I)*ArcTanh[((-(c*f) + g)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2
*f^2) + g^2]])*Log[((c*f + g)*(I*c*f - I*g + Sqrt[-(c^2*f^2) + g^2])*(I...
```

3.4.3 Rubi [A] (verified)

Time = 2.21 (sec) , antiderivative size = 506, normalized size of antiderivative = 0.70, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {5277, 5265, 25, 5257, 25, 5299, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{f+gx}dx$$

$$\Big\downarrow \text{ 5277}$$

$$\frac{\sqrt{d-c^2dx^2}\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{f+gx}dx}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{ 5265}$$

$$\frac{\sqrt{d-c^2dx^2}\left(\frac{\int -\frac{\left(gx^2c^2+2fxc^2+g\right)(a+b\arccos(cx))^2}{(f+gx)^2}dx}{2bc}-\frac{(1-c^2x^2)(a+b\arccos(cx))^2}{2bc(f+gx)}\right)}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{d-c^2dx^2}\left(-\frac{\int \frac{\left(gx^2c^2+2fxc^2+g\right)(a+b\arccos(cx))^2}{(f+gx)^2}dx}{2bc}-\frac{(1-c^2x^2)(a+b\arccos(cx))^2}{2bc(f+gx)}\right)}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \; 5257$$

$$\frac{\sqrt{d-c^2dx^2}\left(-\frac{2bc\int -\frac{\left(\frac{1}{f+gx}-\frac{c^2\left(\frac{f^2}{f+gx}+gx\right)}{g^2}\right)(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx-\frac{\left(1-\frac{c^2f^2}{g^2}\right)(a+b\arccos(cx))^2}{f+gx}+\frac{c^2x(a+b\arccos(cx))^2}{g}}{2bc}-\frac{(1-c^2x^2)(a+b\arccos}{2bc(f+gx)}\right.}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{d-c^2dx^2}\left(-\frac{-2bc\int \frac{\left(\frac{1}{f+gx}-\frac{c^2\left(\frac{f^2}{f+gx}+gx\right)}{g^2}\right)(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx-\frac{\left(1-\frac{c^2f^2}{g^2}\right)(a+b\arccos(cx))^2}{f+gx}+\frac{c^2x(a+b\arccos(cx))^2}{g}}{2bc}-\frac{(1-c^2x^2)(a+b\arccos}{2bc(f+gx)}\right.}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \; 5299$$

$$\frac{\sqrt{d-c^2dx^2}\left(-\frac{-2bc\int\left(-\frac{b\arccos(cx)\left(f^2c^2+g^2x^2c^2+fgxc^2-g^2\right)}{g^2(f+gx)\sqrt{1-c^2x^2}}-\frac{a\left(f^2c^2+g^2x^2c^2+fgxc^2-g^2\right)}{g^2(f+gx)\sqrt{1-c^2x^2}}\right)dx-\frac{\left(1-\frac{c^2f^2}{g^2}\right)(a+b\arccos(cx))^2}{f+gx}+\frac{c^2x(a+b\arccos}{g}}{2bc}\right.}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \; 2009$$

$$\sqrt{d-c^2dx^2}\left(-\frac{(1-c^2x^2)(a+b\arccos(cx))^2}{2bc(f+gx)}-\frac{-2bc\left(-\frac{a\sqrt{c^2f^2-g^2}\arctan\left(\frac{c^2fx+g}{\sqrt{1-c^2x^2}\sqrt{c^2f^2-g^2}}\right)}{g^2}+\frac{a\sqrt{1-c^2x^2}}{g}-\frac{b\sqrt{c^2f^2-g^2}\operatorname{PolyLog}\left(2,-\frac{e^{i\,\mathrm{arc}}}{cf-\sqrt{}}\right.}{g^2}\right.}{}\right.$$

3.4. $\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{f+gx}\,dx$

input
```
Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(f + g*x),x]
```

output
```
(Sqrt[d - c^2*d*x^2]*(-1/2*((1 - c^2*x^2)*(a + b*ArcCos[c*x])^2)/(b*c*(f +
 g*x)) - ((c^2*x*(a + b*ArcCos[c*x])^2)/g - ((1 - (c^2*f^2)/g^2)*(a + b*Ar
cCos[c*x])^2)/(f + g*x) - 2*b*c*((b*c*x)/g + (a*Sqrt[1 - c^2*x^2])/g + (b*
Sqrt[1 - c^2*x^2]*ArcCos[c*x])/g - (a*Sqrt[c^2*f^2 - g^2]*ArcTan[(g + c^2*
f*x)/(Sqrt[c^2*f^2 - g^2]*Sqrt[1 - c^2*x^2])])/g^2 - (I*b*Sqrt[c^2*f^2 - g
^2]*ArcCos[c*x]*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2])]
)/g^2 + (I*b*Sqrt[c^2*f^2 - g^2]*ArcCos[c*x]*Log[1 + (E^(I*ArcCos[c*x])*g)
/(c*f + Sqrt[c^2*f^2 - g^2])])/g^2 - (b*Sqrt[c^2*f^2 - g^2]*PolyLog[2, -((
E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2]))])/g^2 + (b*Sqrt[c^2*f^2
- g^2]*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2]))])/g
^2))/(2*b*c)))/Sqrt[1 - c^2*x^2]
```

3.4.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5257
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.) + (g_.)*(x_) + (h_.)*(x
_)^2)^(p_.))/((d_) + (e_.)*(x_))^2, x_Symbol] :> With[{u = IntHide[(f + g*x
 + h*x^2)^p/(d + e*x)^2, x]}, Simp[(a + b*ArcCos[c*x])^n   u, x] + Simp[b*c
*n   Int[SimplifyIntegrand[u*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && IGtQ[n, 0] && IGtQ[
p, 0] && EqQ[e*g - 2*d*h, 0]
```

rule 5265
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_)*Sqrt[
(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f + g*x)^m)*(d + e*x^2)*((a + b*
ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1))), x] + Simp[1/(b*c*Sqrt[d]*(n +
1))   Int[(d*g*m + 2*e*f*x + e*g*(m + 2)*x^2)*(f + g*x)^(m - 1)*(a + b*ArcC
os[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[c^2*d +
e, 0] && ILtQ[m, 0] && GtQ[d, 0] && IGtQ[n, 0]
```

rule 5277
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[Simp[(d + e*x^2)^p/(1 - c^2*x^2)^
p]   Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && Intege
rQ[p - 1/2] &&  !GtQ[d, 0]
```

rule 5299
```
Int[(ArcCos[(c_.)*(x_)]*(b_.) + (a_))^(n_.)*(RFx_)*((d_) + (e_.)*(x_)^2)^(p
_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p, RFx*(a + b*ArcCos[c*x])
^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && RationalFunctionQ[RFx, x] && IGt
Q[n, 0] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2]
```

3.4.4 Maple [A] (verified)

Time = 2.64 (sec) , antiderivative size = 816, normalized size of antiderivative = 1.13

method	result
default	$\frac{a\left(\sqrt{-\left(x+\frac{f}{g}\right)^2 c^2 d+\frac{2c^2 df\left(x+\frac{f}{g}\right)}{g}-\frac{d\left(c^2f^2-g^2\right)}{g^2}}+\frac{c^2 df \arctan\left(\frac{\sqrt{c^2 d}\, x}{\sqrt{-\left(x+\frac{f}{g}\right)^2 c^2 d+\frac{2c^2 df\left(x+\frac{f}{g}\right)}{g}-\frac{d\left(c^2f^2-g^2\right)}{g^2}}}\right)}{g\sqrt{c^2 d}}+\frac{d\left(c^2f^2-g^2\right)\ln\left(-\frac{2d\left(c^{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{g}\cdots$
parts	$\frac{a\left(\sqrt{-\left(x+\frac{f}{g}\right)^2 c^2 d+\frac{2c^2 df\left(x+\frac{f}{g}\right)}{g}-\frac{d\left(c^2f^2-g^2\right)}{g^2}}+\frac{c^2 df \arctan\left(\frac{\sqrt{c^2 d}\, x}{\sqrt{-\left(x+\frac{f}{g}\right)^2 c^2 d+\frac{2c^2 df\left(x+\frac{f}{g}\right)}{g}-\frac{d\left(c^2f^2-g^2\right)}{g^2}}}\right)}{g\sqrt{c^2 d}}+\frac{d\left(c^2f^2-g^2\right)\ln\left(-\frac{2d\left(c^{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{g}\cdots$

input
```
int((a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)/(g*x+f),x,method=_RETURNVERBOSE
)
```

output
```
a/g*((-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2)+c^2*
d*f/g/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(
x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))+d*(c^2*f^2-g^2)/g^2/(-d*(c^2*f^2-g^2)/g
^2)^(1/2)*ln((-2*d*(c^2*f^2-g^2)/g^2+2*c^2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^
2)/g^2)^(1/2)*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(
1/2))/(x+f/g)))+b*(1/2*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-
1)*arccos(c*x)^2*f*c/g^2+1/2*(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*
x*c+c^2*x^2-1)*(arccos(c*x)+I)/(c^2*x^2-1)/g+1/2*(-d*(c^2*x^2-1))^(1/2)*(c
^2*x^2-I*(-c^2*x^2+1)^(1/2)*x*c-1)*(arccos(c*x)-I)/(c^2*x^2-1)/g+(-d*(c^2*
x^2-1))^(1/2)*(c^2*f^2-g^2)^(1/2)*(-c^2*x^2+1)^(1/2)*(I*arccos(c*x)*ln((-(
c*x+I*(-c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/2))/(-c*f+(c^2*f^2-g^2)^(
1/2)))-I*arccos(c*x)*ln(((c*x+I*(-c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2-g^2)^(1
/2))/(c*f+(c^2*f^2-g^2)^(1/2)))+dilog((-(c*x+I*(-c^2*x^2+1)^(1/2))*g-c*f+(
c^2*f^2-g^2)^(1/2))/(-c*f+(c^2*f^2-g^2)^(1/2)))-dilog(((c*x+I*(-c^2*x^2+1)
^(1/2))*g+c*f+(c^2*f^2-g^2)^(1/2))/(c*f+(c^2*f^2-g^2)^(1/2)))))/(c^2*x^2-1)
/g^2)
```

3.4.5 Fricas [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{f+gx}\,dx = \int \frac{\sqrt{-c^2dx^2+d}(b\arccos(cx)+a)}{gx+f}\,dx$$

input
```
integrate((a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)/(g*x+f),x, algorithm="fri
cas")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a)/(g*x + f), x)
```

3.4.6 Sympy [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{f+gx}\,dx = \int \frac{\sqrt{-d(cx-1)(cx+1)}(a+b\,\mathrm{acos}(cx))}{f+gx}\,dx$$

input
```
integrate((a+b*acos(c*x))*(-c**2*d*x**2+d)**(1/2)/(g*x+f),x)
```

output
```
Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acos(c*x))/(f + g*x), x)
```

3.4.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{f+gx}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)/(g*x+f),x, algorithm="max
ima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(g-c*f>0)', see `assume?` for mor
e details)
```

3.4.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{f+gx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)/(g*x+f),x, algorithm="gia
c")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.4.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{f+gx}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))\,\sqrt{d-c^2\,d\,x^2}}{f+g\,x}\,dx$$

input
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(1/2))/(f + g*x),x)
```

output
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(1/2))/(f + g*x), x)
```

3.5 $\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{(f+gx)^2}\,dx$

3.5.1 Optimal result

Integrand size = 31, antiderivative size = 851

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{(f+gx)^2}\,dx = -\frac{a\sqrt{d-c^2dx^2}}{g(f+gx)} - \frac{b\sqrt{d-c^2dx^2}\arccos(cx)}{g(f+gx)} + \frac{bc^3f^2\sqrt{d-c^2dx^2}\arccos(cx)^2}{2g^2(c^2f^2-g^2)\sqrt{1-c^2x^2}} - \frac{(g+c^2fx)^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2bc(c^2f^2-g^2)(f+gx)^2\sqrt{1-c^2x^2}} - \frac{\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2bc(f+gx)^2} - \frac{ac^3f^2\sqrt{d-c^2dx^2}\arcsin(cx)}{g^2(c^2f^2-g^2)\sqrt{1-c^2x^2}} + \frac{ac^2f\sqrt{d-c^2dx^2}\arctan\left(\frac{g+c^2fx}{\sqrt{c^2f^2-g^2}\sqrt{1-c^2x^2}}\right)}{g^2\sqrt{c^2f^2-g^2}\sqrt{1-c^2x^2}} + \frac{ibc^2f\sqrt{d-c^2dx^2}\arccos(cx)\log\left(1+\frac{e^{i\arccos(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{g^2\sqrt{c^2f^2-g^2}\sqrt{1-c^2x^2}} - \frac{ibc^2f\sqrt{d-c^2dx^2}\arccos(cx)\log\left(1+\frac{e^{i\arccos(cx)}g}{cf+\sqrt{c^2f^2-g^2}}\right)}{g^2\sqrt{c^2f^2-g^2}\sqrt{1-c^2x^2}} - \frac{bc\sqrt{d-c^2dx^2}\log(f+gx)}{g^2\sqrt{1-c^2x^2}} + \frac{bc^2f\sqrt{d-c^2dx^2}\operatorname{PolyLog}\left(2,-\frac{e^{i\arccos(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{g^2\sqrt{c^2f^2-g^2}\sqrt{1-c^2x^2}} - \frac{bc^2f\sqrt{d-c^2dx^2}\operatorname{PolyLog}\left(2,-\frac{e^{i\arccos(cx)}g}{cf+\sqrt{c^2f^2-g^2}}\right)}{g^2\sqrt{c^2f^2-g^2}\sqrt{1-c^2x^2}}$$

output
```
-a*(-c^2*d*x^2+d)^(1/2)/g/(g*x+f)-b*arccos(c*x)*(-c^2*d*x^2+d)^(1/2)/g/(g*
x+f)+1/2*b*c^3*f^2*arccos(c*x)^2*(-c^2*d*x^2+d)^(1/2)/g^2/(c^2*f^2-g^2)/(-
c^2*x^2+1)^(1/2)-1/2*(c^2*f*x+g)^2*(a+b*arccos(c*x))^2*(-c^2*d*x^2+d)^(1/2
)/b/c/(c^2*f^2-g^2)/(g*x+f)^2/(-c^2*x^2+1)^(1/2)-a*c^3*f^2*arcsin(c*x)*(-c
^2*d*x^2+d)^(1/2)/g^2/(c^2*f^2-g^2)/(-c^2*x^2+1)^(1/2)-b*c*ln(g*x+f)*(-c^2
*d*x^2+d)^(1/2)/g^2/(-c^2*x^2+1)^(1/2)+a*c^2*f*arctan((c^2*f*x+g)/(c^2*f^2
-g^2)^(1/2)/(-c^2*x^2+1)^(1/2))*(-c^2*d*x^2+d)^(1/2)/g^2/(c^2*f^2-g^2)^(1/
2)/(-c^2*x^2+1)^(1/2)+I*b*c^2*f*arccos(c*x)*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2)
)*g/(c*f-(c^2*f^2-g^2)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/g^2/(c^2*f^2-g^2)^(1/2
)/(-c^2*x^2+1)^(1/2)-I*b*c^2*f*arccos(c*x)*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))
*g/(c*f+(c^2*f^2-g^2)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/g^2/(c^2*f^2-g^2)^(1/2)
/(-c^2*x^2+1)^(1/2)+b*c^2*f*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))*g/(c*f-(
c^2*f^2-g^2)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/g^2/(c^2*f^2-g^2)^(1/2)/(-c^2*x^
2+1)^(1/2)-b*c^2*f*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2-g
^2)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/g^2/(c^2*f^2-g^2)^(1/2)/(-c^2*x^2+1)^(1/2
)-1/2*(a+b*arccos(c*x))^2*(-c^2*x^2+1)^(1/2)*(-c^2*d*x^2+d)^(1/2)/b/c/(g*x
+f)^2
```

3.5.2 Mathematica [A] (warning: unable to verify)

Time = 10.57 (sec) , antiderivative size = 1130, normalized size of antiderivative = 1.33

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{(f+gx)^2}\,dx = -\frac{a\sqrt{-d\left(-1+c^2x^2\right)}}{g(f+gx)}+\frac{ac\sqrt{d}\arctan\left(\frac{cx\sqrt{-d(-1+c^2x^2)}}{\sqrt{d}(-1+c^2x^2)}\right)}{g^2}$$
$$+\frac{ac^2\sqrt{d}f\log(f+gx)}{g^2\sqrt{-c^2f^2+g^2}}-\frac{ac^2\sqrt{d}f\log\left(dg+c^2dfx+\sqrt{d}\sqrt{-c^2f^2+g^2}\sqrt{-d\left(-1+c^2x^2\right)}\right)}{g^2\sqrt{-c^2f^2+g^2}}$$
$$-\frac{bc\sqrt{d\left(1-c^2x^2\right)}\left(\frac{2g\arccos(cx)}{cf+cgx}-\frac{\arccos(cx)^2}{\sqrt{1-c^2x^2}}+\frac{2\log\left(1+\frac{gx}{f}\right)}{\sqrt{1-c^2x^2}}+\frac{2cf\left(2\arccos(cx)\operatorname{arctanh}\left(\frac{(cf+g)\cot\left(\frac{1}{2}\arccos(cx)\right)}{\sqrt{-c^2f^2+g^2}}\right)-2\operatorname{arcc}\right.}{}\right.}{}$$

input
```
Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(f + g*x)^2,x]
```

output
```
-((a*Sqrt[-(d*(-1 + c^2*x^2))])/(g*(f + g*x))) + (a*c*Sqrt[d]*ArcTan[(c*x*
Sqrt[-(d*(-1 + c^2*x^2))])/(Sqrt[d]*(-1 + c^2*x^2))])/g^2 + (a*c^2*Sqrt[d]
*f*Log[f + g*x])/(g^2*Sqrt[-(c^2*f^2) + g^2]) - (a*c^2*Sqrt[d]*f*Log[d*g +
 c^2*d*f*x + Sqrt[d]*Sqrt[-(c^2*f^2) + g^2]*Sqrt[-(d*(-1 + c^2*x^2))]])/(g
^2*Sqrt[-(c^2*f^2) + g^2]) - (b*c*Sqrt[d*(1 - c^2*x^2)]*((2*g*ArcCos[c*x])
/(c*f + c*g*x) - ArcCos[c*x]^2/Sqrt[1 - c^2*x^2] + (2*Log[1 + (g*x)/f])/Sq
rt[1 - c^2*x^2] + (2*c*f*(2*ArcCos[c*x]*ArcTanh[((c*f + g)*Cot[ArcCos[c*x]
/2])/Sqrt[-(c^2*f^2) + g^2]] - 2*ArcCos[-((c*f)/g)]*ArcTanh[((-(c*f) + g)*
Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + (ArcCos[-((c*f)/g)] - (2*I)*
ArcTanh[((c*f + g)*Cot[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + (2*I)*Arc
Tanh[((-(c*f) + g)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])*Log[Sqrt[-
(c^2*f^2) + g^2]/(Sqrt[2]*E^((I/2)*ArcCos[c*x])*Sqrt[g]*Sqrt[c*f + c*g*x])
] + (ArcCos[-((c*f)/g)] + (2*I)*(ArcTanh[((c*f + g)*Cot[ArcCos[c*x]/2])/Sq
rt[-(c^2*f^2) + g^2]] - ArcTanh[((-(c*f) + g)*Tan[ArcCos[c*x]/2])/Sqrt[-(c
^2*f^2) + g^2]]))*Log[(E^((I/2)*ArcCos[c*x])*Sqrt[-(c^2*f^2) + g^2])/(Sqrt
[2]*Sqrt[g]*Sqrt[c*f + c*g*x])] - (ArcCos[-((c*f)/g)] - (2*I)*ArcTanh[((-(
c*f) + g)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])*Log[((c*f + g)*((-I
)*c*f + I*g + Sqrt[-(c^2*f^2) + g^2])*(-I + Tan[ArcCos[c*x]/2]))/(g*(c*f +
 g + Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))] - (ArcCos[-((c*f)/g)] +
(2*I)*ArcTanh[((-(c*f) + g)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]]...
```

3.5.3 Rubi [A] (verified)

Time = 2.63 (sec) , antiderivative size = 623, normalized size of antiderivative = 0.73, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {5277, 5265, 27, 5255, 27, 5299, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{(f+gx)^2}dx$$

$$\Big\downarrow \text{5277}$$

$$\frac{\sqrt{d-c^2dx^2}\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))}{(f+gx)^2}dx}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5265}$$

$$\frac{\sqrt{d-c^2dx^2}\left(\frac{\int -\frac{2\left(fxc^2+g\right)(a+b\arccos(cx))^2}{(f+gx)^3}dx}{2bc}-\frac{(1-c^2x^2)(a+b\arccos(cx))^2}{2bc(f+gx)^2}\right)}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d-c^2dx^2}\left(-\frac{\int \frac{\left(fxc^2+g\right)(a+b\arccos(cx))^2}{(f+gx)^3}dx}{bc}-\frac{(1-c^2x^2)(a+b\arccos(cx))^2}{2bc(f+gx)^2}\right)}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5255}$$

$$\frac{\sqrt{d-c^2dx^2}\left(-\frac{2bc\int \frac{\left(fxc^2+g\right)^2(a+b\arccos(cx))}{2\left(c^2f^2-g^2\right)(f+gx)^2\sqrt{1-c^2x^2}}dx+\frac{\left(c^2fx+g\right)^2(a+b\arccos(cx))^2}{2\left(c^2f^2-g^2\right)(f+gx)^2}}{bc}-\frac{(1-c^2x^2)(a+b\arccos(cx))^2}{2bc(f+gx)^2}\right)}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d-c^2dx^2}\left(-\frac{\frac{bc\int \frac{\left(fxc^2+g\right)^2(a+b\arccos(cx))}{(f+gx)^2\sqrt{1-c^2x^2}}dx}{c^2f^2-g^2}+\frac{\left(c^2fx+g\right)^2(a+b\arccos(cx))^2}{2\left(c^2f^2-g^2\right)(f+gx)^2}}{bc}-\frac{(1-c^2x^2)(a+b\arccos(cx))^2}{2bc(f+gx)^2}\right)}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5299}$$

$$\frac{\sqrt{d-c^2dx^2}\left(-\frac{\frac{bc\int \left(\frac{b\arccos(cx)\left(fxc^2+g\right)^2}{(f+gx)^2\sqrt{1-c^2x^2}}+\frac{a\left(fxc^2+g\right)^2}{(f+gx)^2\sqrt{1-c^2x^2}}\right)dx}{c^2f^2-g^2}+\frac{\left(c^2fx+g\right)^2(a+b\arccos(cx))^2}{2\left(c^2f^2-g^2\right)(f+gx)^2}}{bc}-\frac{(1-c^2x^2)(a+b\arccos(cx))^2}{2bc(f+gx)^2}\right)}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{2009}$$

3.5. $\displaystyle\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{(f+gx)^2}\,dx$

$$\sqrt{d-c^2dx^2}\left(-\frac{(1-c^2x^2)(a+b\arccos(cx))^2}{2bc(f+gx)^2}-\frac{\frac{\left(c^2fx+g\right)^2(a+b\arccos(cx))^2}{2\left(c^2f^2-g^2\right)(f+gx)^2}+\frac{bc\left(\frac{ac^3f^2\arcsin(cx)}{g^2}-\frac{ac^2f\sqrt{c^2f^2-g^2}\arctan\left(\frac{c^2fx+g}{\sqrt{1-c^2x^2}\sqrt{c^2f^2-g^2}}\right)}{g^2}\right.}{}}{}\right.$$

input
```
Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCos[c*x]))/(f + g*x)^2,x]
```

output
```
(Sqrt[d - c^2*d*x^2]*(-1/2*((1 - c^2*x^2)*(a + b*ArcCos[c*x])^2)/(b*c*(f +
 g*x)^2) - (((g + c^2*f*x)^2*(a + b*ArcCos[c*x])^2)/(2*(c^2*f^2 - g^2)*(f
+ g*x)^2) + (b*c*((a*(c*f - g)*(c*f + g)*Sqrt[1 - c^2*x^2])/(g*(f + g*x))
+ (b*(c*f - g)*(c*f + g)*Sqrt[1 - c^2*x^2]*ArcCos[c*x])/(g*(f + g*x)) - (b
*c^3*f^2*ArcCos[c*x]^2)/(2*g^2) + (a*c^3*f^2*ArcSin[c*x])/g^2 - (a*c^2*f*S
qrt[c^2*f^2 - g^2]*ArcTan[(g + c^2*f*x)/(Sqrt[c^2*f^2 - g^2]*Sqrt[1 - c^2*
x^2])])/g^2 - (I*b*c^2*f*Sqrt[c^2*f^2 - g^2]*ArcCos[c*x]*Log[1 + (E^(I*Arc
Cos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2])])/g^2 + (I*b*c^2*f*Sqrt[c^2*f^2 -
 g^2]*ArcCos[c*x]*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2]
)])/g^2 - b*c*(1 - (c^2*f^2)/g^2)*Log[f + g*x] - (b*c^2*f*Sqrt[c^2*f^2 - g
^2]*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2]))])/g^2
+ (b*c^2*f*Sqrt[c^2*f^2 - g^2]*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f + S
qrt[c^2*f^2 - g^2]))])/g^2))/(c^2*f^2 - g^2))/(b*c)))/Sqrt[1 - c^2*x^2]
```

3.5.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5255
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_))^(m_)*((f_.)
 + (g_.)*(x_))^(p_.), x_Symbol] :> With[{u = IntHide[(f + g*x)^p*(d + e*x)^
m, x]}, Simp[(a + b*ArcCos[c*x])^n   u, x] + Simp[b*c*n   Int[SimplifyInteg
rand[u*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]), x], x], x]] /; Free
Q[{a, b, c, d, e, f, g}, x] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[m, 0] && Lt
Q[m + p + 1, 0]
```

rule 5265
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_)*Sqrt[
(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f + g*x)^m)*(d + e*x^2)*((a + b*
ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1))), x] + Simp[1/(b*c*Sqrt[d]*(n +
1))   Int[(d*g*m + 2*e*f*x + e*g*(m + 2)*x^2)*(f + g*x)^(m - 1)*(a + b*ArcC
os[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[c^2*d +
e, 0] && ILtQ[m, 0] && GtQ[d, 0] && IGtQ[n, 0]
```

rule 5277
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[Simp[(d + e*x^2)^p/(1 - c^2*x^2)^
p]   Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && Intege
rQ[p - 1/2] &&  !GtQ[d, 0]
```

rule 5299
```
Int[(ArcCos[(c_.)*(x_)]*(b_.) + (a_))^(n_.)*(RFx_)*((d_) + (e_.)*(x_)^2)^(p
_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p, RFx*(a + b*ArcCos[c*x])
^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && RationalFunctionQ[RFx, x] && IGt
Q[n, 0] && EqQ[c^2*d + e, 0] && IntegerQ[p - 1/2]
```

3.5.4 Maple [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1938 vs. $2(813) = 1626$.

Time = 2.81 (sec) , antiderivative size = 1939, normalized size of antiderivative = 2.28

method	result	size
default	Expression too large to display	1939
parts	Expression too large to display	1939

input
```
int((a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)/(g*x+f)^2,x,method=_RETURNVERBO
SE)
```

output
```
a/g^2*(1/d/(c^2*f^2-g^2)*g^2/(x+f/g)*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)
-d*(c^2*f^2-g^2)/g^2)^(3/2)-c^2*f*g/(c^2*f^2-g^2)*((-(x+f/g)^2*c^2*d+2*c^2
*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2)+c^2*d*f/g/(c^2*d)^(1/2)*arctan((
c^2*d)^(1/2)*x/(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^
(1/2))+d*(c^2*f^2-g^2)/g^2/(-d*(c^2*f^2-g^2)/g^2)^(1/2)*ln((-2*d*(c^2*f^2-
g^2)/g^2+2*c^2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1/2)*(-(x+f/g)^2*c^
2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/g)))+2*c^2/(c^2*f
^2-g^2)*g^2*(-1/4*(-2*(x+f/g)*c^2*d+2*c^2*d*f/g)/c^2/d*(-(x+f/g)^2*c^2*d+2
*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2)-1/8*(4*c^2*d^2*(c^2*f^2-g^2)
/g^2-4*c^4*d^2*f^2/g^2)/c^2/d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-(x+f/
g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))))-1/2*b*(-d*(c^
2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^2-1)*arccos(c*x)^2*c/g^2-b*(-d*(
c^2*x^2-1))^(1/2)*arccos(c*x)/(c^2*x^2-1)/g^2/(g*x+f)*(-c^2*x^2+1)*x*c^2*f
-b*(-d*(c^2*x^2-1))^(1/2)*arccos(c*x)/(c^2*x^2-1)/g^2/(g*x+f)*x^3*c^4*f+I*
b*(-d*(c^2*x^2-1))^(1/2)*arccos(c*x)/(c^2*x^2-1)/g^2/(g*x+f)*(-c^2*x^2+1)^
(1/2)*c*f-b*(-d*(c^2*x^2-1))^(1/2)*arccos(c*x)/(c^2*x^2-1)/g/(g*x+f)*x^2*c
^2+I*b*(-d*(c^2*x^2-1))^(1/2)*arccos(c*x)/(c^2*x^2-1)/g/(g*x+f)*(-c^2*x^2+
1)^(1/2)*x*c+b*(-d*(c^2*x^2-1))^(1/2)*arccos(c*x)/(c^2*x^2-1)/g^2/(g*x+f)*
x*c^2*f+b*(-d*(c^2*x^2-1))^(1/2)*arccos(c*x)/(c^2*x^2-1)/g/(g*x+f)+I*b*c^2
*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*f^2-g^2)^(1/2)/(c^2*x^2...
```

3.5.5 Fricas [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{(f+gx)^2}\,dx = \int \frac{\sqrt{-c^2dx^2+d}(b\arccos(cx)+a)}{(gx+f)^2}\,dx$$

input
```
integrate((a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)/(g*x+f)^2,x, algorithm="f
ricas")
```

output
```
integral(sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a)/(g^2*x^2 + 2*f*g*x + f^2
), x)
```

3.5.6 Sympy [F]

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{(f+gx)^2}\,dx=\int \frac{\sqrt{-d\,(cx-1)\,(cx+1)}(a+b\,\mathrm{acos}\,(cx))}{(f+gx)^2}\,dx$$

input
```
integrate((a+b*acos(c*x))*(-c**2*d*x**2+d)**(1/2)/(g*x+f)**2,x)
```

output
```
Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acos(c*x))/(f + g*x)**2, x)
```

3.5.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{(f+gx)^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)/(g*x+f)^2,x, algorithm="m
axima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(g-c*f>0)', see `assume?` for mor
e details)
```

3.5.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{(f+gx)^2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)/(g*x+f)^2,x, algorithm="g
iac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.5.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{(f+gx)^2}\,dx=\int \frac{(a+b\,\mathrm{acos}(c\,x))\,\sqrt{d-c^2\,d\,x^2}}{(f+g\,x)^2}\,dx$$

input
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(1/2))/(f + g*x)^2,x)
```

output
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(1/2))/(f + g*x)^2, x)
```

3.6 $\int (f+gx)^3 \left(d-c^2dx^2\right)^{3/2} (a+b\arccos(cx))\,dx$

3.6.1 Optimal result

Integrand size = 31, antiderivative size = 959

$$\int (f+gx)^3 \left(d-c^2dx^2\right)^{3/2} (a+b\arccos(cx))\,dx =$$
$$-\frac{3bdf^2gx\sqrt{d-c^2dx^2}}{5c\sqrt{1-c^2x^2}}-\frac{2bdg^3x\sqrt{d-c^2dx^2}}{35c^3\sqrt{1-c^2x^2}}+\frac{5bcdf^3x^2\sqrt{d-c^2dx^2}}{16\sqrt{1-c^2x^2}}$$
$$-\frac{3bdfg^2x^2\sqrt{d-c^2dx^2}}{32c\sqrt{1-c^2x^2}}+\frac{2bcdf^2gx^3\sqrt{d-c^2dx^2}}{5\sqrt{1-c^2x^2}}-\frac{bdg^3x^3\sqrt{d-c^2dx^2}}{105c\sqrt{1-c^2x^2}}$$
$$-\frac{bc^3df^3x^4\sqrt{d-c^2dx^2}}{16\sqrt{1-c^2x^2}}+\frac{7bcdfg^2x^4\sqrt{d-c^2dx^2}}{32\sqrt{1-c^2x^2}}-\frac{3bc^3df^2gx^5\sqrt{d-c^2dx^2}}{25\sqrt{1-c^2x^2}}$$
$$+\frac{8bcdg^3x^5\sqrt{d-c^2dx^2}}{175\sqrt{1-c^2x^2}}-\frac{bc^3dfg^2x^6\sqrt{d-c^2dx^2}}{12\sqrt{1-c^2x^2}}-\frac{bc^3dg^3x^7\sqrt{d-c^2dx^2}}{49\sqrt{1-c^2x^2}}$$
$$+\frac{3}{8}df^3x\sqrt{d-c^2dx^2}(a+b\arccos(cx))-\frac{3dfg^2x\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{16c^2}$$
$$+\frac{3}{8}dfg^2x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))$$
$$+\frac{1}{4}df^3x\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}(a+b\arccos(cx))$$
$$+\frac{1}{2}dfg^2x^3\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}(a+b\arccos(cx))$$
$$-\frac{3df^2g(1-c^2x^2)^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{5c^2}$$
$$-\frac{dg^3(1-c^2x^2)^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{5c^4}$$
$$+\frac{dg^3(1-c^2x^2)^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{7c^4}$$
$$-\frac{3df^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{16bc\sqrt{1-c^2x^2}}-\frac{3dfg^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{32bc^3\sqrt{1-c^2x^2}}$$

output
```
3/8*d*f^3*x*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)-3/16*d*f*g^2*x*(a+b*arc
cos(c*x))*(-c^2*d*x^2+d)^(1/2)/c^2+3/8*d*f*g^2*x^3*(a+b*arccos(c*x))*(-c^2
*d*x^2+d)^(1/2)+1/4*d*f^3*x*(-c^2*x^2+1)*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^
(1/2)+1/2*d*f*g^2*x^3*(-c^2*x^2+1)*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)-
3/5*d*f^2*g*(-c^2*x^2+1)^2*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)/c^2-1/5*
d*g^3*(-c^2*x^2+1)^2*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)/c^4+1/7*d*g^3*
(-c^2*x^2+1)^3*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)/c^4-3/5*b*d*f^2*g*x*
(-c^2*d*x^2+d)^(1/2)/c/(-c^2*x^2+1)^(1/2)-2/35*b*d*g^3*x*(-c^2*d*x^2+d)^(1
/2)/c^3/(-c^2*x^2+1)^(1/2)+5/16*b*c*d*f^3*x^2*(-c^2*d*x^2+d)^(1/2)/(-c^2*x
^2+1)^(1/2)-3/32*b*d*f*g^2*x^2*(-c^2*d*x^2+d)^(1/2)/c/(-c^2*x^2+1)^(1/2)+2
/5*b*c*d*f^2*g*x^3*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-1/105*b*d*g^3*x
^3*(-c^2*d*x^2+d)^(1/2)/c/(-c^2*x^2+1)^(1/2)-1/16*b*c^3*d*f^3*x^4*(-c^2*d*
x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)+7/32*b*c*d*f*g^2*x^4*(-c^2*d*x^2+d)^(1/2)/
(-c^2*x^2+1)^(1/2)-3/25*b*c^3*d*f^2*g*x^5*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1
)^(1/2)+8/175*b*c*d*g^3*x^5*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-1/12*b
*c^3*d*f*g^2*x^6*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-1/49*b*c^3*d*g^3*
x^7*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-3/16*d*f^3*(a+b*arccos(c*x))^2
*(-c^2*d*x^2+d)^(1/2)/b/c/(-c^2*x^2+1)^(1/2)-3/32*d*f*g^2*(a+b*arccos(c*x)
)^2*(-c^2*d*x^2+d)^(1/2)/b/c^3/(-c^2*x^2+1)^(1/2)
```

3.6.2 Mathematica [A] (verified)

Time = 5.61 (sec) , antiderivative size = 910, normalized size of antiderivative = 0.95

$$\int (f+gx)^3 \left(d-c^2dx^2\right)^{3/2} (a+b\arccos(cx))\,dx = \frac{-88200bcdf(2c^2f^2+g^2)\sqrt{d-c^2dx^2}\arccos(cx)^2 - 176400acd^{3/2}f(2c^2f^2+g^2)\sqrt{1-c^2}}{}$$

input
```
Integrate[(f + g*x)^3*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]),x]
```

output
```
(-88200*b*c*d*f*(2*c^2*f^2 + g^2)*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]^2 - 1764
00*a*c*d^(3/2)*f*(2*c^2*f^2 + g^2)*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d -
c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] - d*Sqrt[d - c^2*d*x^2]*(352800*b*c^
3*f^2*g*x + 44100*b*c*g^3*x + 564480*a*c^2*f^2*g*Sqrt[1 - c^2*x^2] + 53760
*a*g^3*Sqrt[1 - c^2*x^2] - 588000*a*c^4*f^3*x*Sqrt[1 - c^2*x^2] + 176400*a
*c^2*f*g^2*x*Sqrt[1 - c^2*x^2] - 1128960*a*c^4*f^2*g*x^2*Sqrt[1 - c^2*x^2]
 + 26880*a*c^2*g^3*x^2*Sqrt[1 - c^2*x^2] + 235200*a*c^6*f^3*x^3*Sqrt[1 - c
^2*x^2] - 823200*a*c^4*f*g^2*x^3*Sqrt[1 - c^2*x^2] + 564480*a*c^6*f^2*g*x^
4*Sqrt[1 - c^2*x^2] - 215040*a*c^4*g^3*x^4*Sqrt[1 - c^2*x^2] + 470400*a*c^
6*f*g^2*x^5*Sqrt[1 - c^2*x^2] + 134400*a*c^6*g^3*x^6*Sqrt[1 - c^2*x^2] - 7
350*b*c*f*(16*c^2*f^2 + 3*g^2)*Cos[2*ArcCos[c*x]] - 4900*b*g*(12*c^2*f^2 +
 g^2)*Cos[3*ArcCos[c*x]] + 7350*b*c^3*f^3*Cos[4*ArcCos[c*x]] - 11025*b*c*f
*g^2*Cos[4*ArcCos[c*x]] + 7056*b*c^2*f^2*g*Cos[5*ArcCos[c*x]] - 588*b*g^3*
Cos[5*ArcCos[c*x]] + 2450*b*c*f*g^2*Cos[6*ArcCos[c*x]] + 300*b*g^3*Cos[7*A
rcCos[c*x]]) + 140*b*d*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]*(-4200*c^2*f^2*g*Sq
rt[1 - c^2*x^2] + 416*g^3*Sqrt[1 - c^2*x^2] + 6720*c^4*f^2*g*x^2*Sqrt[1 -
c^2*x^2] - 1256*c^2*g^3*x^2*Sqrt[1 - c^2*x^2] + 864*g^3*(1 - c^2*x^2)^(3/2
)*Cos[2*ArcCos[c*x]] + 120*g^3*(1 - c^2*x^2)^(3/2)*Cos[4*ArcCos[c*x]] + 16
80*c^3*f^3*Sin[2*ArcCos[c*x]] + 315*c*f*g^2*Sin[2*ArcCos[c*x]] - 420*c^2*f
^2*g*Sin[3*ArcCos[c*x]] + 140*g^3*Sin[3*ArcCos[c*x]] - 210*c^3*f^3*Sin[...
```

3.6.3 Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 498, normalized size of antiderivative = 0.52, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {5277, 5263, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d-c^2dx^2\right)^{3/2}(f+gx)^3(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{ 5277}$$

$$\frac{d\sqrt{d-c^2dx^2}\int (f+gx)^3\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))dx}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{ 5263}$$

$$\frac{d\sqrt{d-c^2dx^2}\int \left(\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))f^3+3gx\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))f^2+3g^2x^2\left(1-c^2x^2\right)^{3/2}\right.}{\sqrt{1-c^2x^2}}$$

$$\downarrow 2009$$

$$\frac{d\sqrt{d-c^2dx^2}\left(-\frac{3fg^2(a+b\arccos(cx))^2}{32bc^3}+\frac{1}{4}f^3x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{3}{8}f^3x\sqrt{1-c^2x^2}(a+b\arccos(cx))-\right.}{}$$

input
```
Int[(f + g*x)^3*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]),x]
```

output
```
(d*Sqrt[d - c^2*d*x^2]*((-3*b*f^2*g*x)/(5*c) - (2*b*g^3*x)/(35*c^3) + (5*b
*c*f^3*x^2)/16 - (3*b*f*g^2*x^2)/(32*c) + (2*b*c*f^2*g*x^3)/5 - (b*g^3*x^3
)/(105*c) - (b*c^3*f^3*x^4)/16 + (7*b*c*f*g^2*x^4)/32 - (3*b*c^3*f^2*g*x^5
)/25 + (8*b*c*g^3*x^5)/175 - (b*c^3*f*g^2*x^6)/12 - (b*c^3*g^3*x^7)/49 + (
3*f^3*x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/8 - (3*f*g^2*x*Sqrt[1 - c^2
*x^2]*(a + b*ArcCos[c*x]))/(16*c^2) + (3*f*g^2*x^3*Sqrt[1 - c^2*x^2]*(a +
b*ArcCos[c*x]))/8 + (f^3*x*(1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x]))/4 + (f
*g^2*x^3*(1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x]))/2 - (3*f^2*g*(1 - c^2*x^
2)^(5/2)*(a + b*ArcCos[c*x]))/(5*c^2) - (g^3*(1 - c^2*x^2)^(5/2)*(a + b*Ar
cCos[c*x]))/(5*c^4) + (g^3*(1 - c^2*x^2)^(7/2)*(a + b*ArcCos[c*x]))/(7*c^4
) - (3*f^3*(a + b*ArcCos[c*x])^2)/(16*b*c) - (3*f*g^2*(a + b*ArcCos[c*x])^
2)/(32*b*c^3)))/Sqrt[1 - c^2*x^2]
```

3.6.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5263
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

rule 5277
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[Simp[(d + e*x^2)^p/(1 - c^2*x^2)^
p]   Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && Intege
rQ[p - 1/2] &&  !GtQ[d, 0]
```

3.6.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.67 (sec) , antiderivative size = 2146, normalized size of antiderivative = 2.24

method	result	size
default	Expression too large to display	2146
parts	Expression too large to display	2146

input
```
int((g*x+f)^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x,method=_RETURNVERBO
SE)
```

output
```
a*(f^3*(1/4*x*(-c^2*d*x^2+d)^(3/2)+3/4*d*(1/2*x*(-c^2*d*x^2+d)^(1/2)+1/2*d
/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))))+g^3*(-1/7*x^
2*(-c^2*d*x^2+d)^(5/2)/c^2/d-2/35/d/c^4*(-c^2*d*x^2+d)^(5/2))+3*f*g^2*(-1/
6*x*(-c^2*d*x^2+d)^(5/2)/c^2/d+1/6/c^2*(1/4*x*(-c^2*d*x^2+d)^(3/2)+3/4*d*(
1/2*x*(-c^2*d*x^2+d)^(1/2)+1/2*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^
2*d*x^2+d)^(1/2)))))-3/5*f^2*g/c^2/d*(-c^2*d*x^2+d)^(5/2))+b*(3/32*(-d*(c^
2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c^3/(c^2*x^2-1)*arccos(c*x)^2*f*(2*c^2*
f^2+g^2)*d-1/6272*(-d*(c^2*x^2-1))^(1/2)*(64*I*c^7*x^7*(-c^2*x^2+1)^(1/2)+
64*c^8*x^8-112*I*(-c^2*x^2+1)^(1/2)*x^5*c^5-144*c^6*x^6+56*I*(-c^2*x^2+1)^
(1/2)*x^3*c^3+104*c^4*x^4-7*I*(-c^2*x^2+1)^(1/2)*x*c-25*c^2*x^2+1)*g^3*(I+
7*arccos(c*x))*d/c^4/(c^2*x^2-1)-1/768*(-d*(c^2*x^2-1))^(1/2)*(32*I*(-c^2*
x^2+1)^(1/2)*c^6*x^6+32*c^7*x^7-48*I*(-c^2*x^2+1)^(1/2)*x^4*c^4-64*c^5*x^5
+18*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+38*c^3*x^3-I*(-c^2*x^2+1)^(1/2)-6*c*x)*f*
g^2*(I+6*arccos(c*x))*d/c^3/(c^2*x^2-1)-1/3200*(-d*(c^2*x^2-1))^(1/2)*(16*
I*c^5*x^5*(-c^2*x^2+1)^(1/2)+16*c^6*x^6-20*I*(-c^2*x^2+1)^(1/2)*x^3*c^3-28
*c^4*x^4+5*I*(-c^2*x^2+1)^(1/2)*x*c+13*c^2*x^2-1)*g*(60*arccos(c*x)*c^2*f^
2+12*I*f^2*c^2-5*arccos(c*x)*g^2-I*g^2)*d/c^4/(c^2*x^2-1)-1/512*(-d*(c^2*x
^2-1))^(1/2)*(8*I*(-c^2*x^2+1)^(1/2)*c^4*x^4+8*c^5*x^5-8*I*(-c^2*x^2+1)^(1
/2)*x^2*c^2-12*c^3*x^3+I*(-c^2*x^2+1)^(1/2)+4*c*x)*f*(2*I*c^2*f^2+8*arccos
(c*x)*c^2*f^2-3*I*g^2-12*arccos(c*x)*g^2)*d/c^3/(c^2*x^2-1)+1/384*(-d*(...
```

3.6.5 Fricas [F]

$$\int (f+gx)^3 \left(d-c^2dx^2\right)^{3/2} (a+b\arccos(cx))\,dx = \int \left(-c^2dx^2+d\right)^{\frac{3}{2}} (gx+f)^3 (b\arccos(cx)+a)\,dx$$

input
```
integrate((g*x+f)^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x, algorithm="f
ricas")
```

output
```
integral(-(a*c^2*d*g^3*x^5 + 3*a*c^2*d*f*g^2*x^4 - 3*a*d*f^2*g*x - a*d*f^3
 + (3*a*c^2*d*f^2*g - a*d*g^3)*x^3 + (a*c^2*d*f^3 - 3*a*d*f*g^2)*x^2 + (b*
c^2*d*g^3*x^5 + 3*b*c^2*d*f*g^2*x^4 - 3*b*d*f^2*g*x - b*d*f^3 + (3*b*c^2*d
*f^2*g - b*d*g^3)*x^3 + (b*c^2*d*f^3 - 3*b*d*f*g^2)*x^2)*arccos(c*x))*sqrt
(-c^2*d*x^2 + d), x)
```

3.6.6 Sympy [F(-1)]

Timed out.

$$\int (f+gx)^3 \left(d-c^2dx^2\right)^{3/2} (a+b\arccos(cx))\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)**3*(-c**2*d*x**2+d)**(3/2)*(a+b*acos(c*x)),x)
```

output
```
Timed out
```

3.6.7 Maxima [F]

$$\int (f+gx)^3 \left(d-c^2dx^2\right)^{3/2} (a+b\arccos(cx))\,dx = \int \left(-c^2dx^2+d\right)^{\frac{3}{2}} (gx+f)^3 (b\arccos(cx)+a)\,dx$$

input
```
integrate((g*x+f)^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x, algorithm="m
axima")
```

output
```
1/8*(2*(-c^2*d*x^2 + d)^(3/2)*x + 3*sqrt(-c^2*d*x^2 + d)*d*x + 3*d^(3/2)*a
rcsin(c*x)/c)*a*f^3 - 1/35*(5*(-c^2*d*x^2 + d)^(5/2)*x^2/(c^2*d) + 2*(-c^2
*d*x^2 + d)^(5/2)/(c^4*d))*a*g^3 + 1/16*a*f*g^2*(2*(-c^2*d*x^2 + d)^(3/2)*
x/c^2 - 8*(-c^2*d*x^2 + d)^(5/2)*x/(c^2*d) + 3*sqrt(-c^2*d*x^2 + d)*d*x/c^
2 + 3*d^(3/2)*arcsin(c*x)/c^3) - 3/5*(-c^2*d*x^2 + d)^(5/2)*a*f^2*g/(c^2*d
) + sqrt(d)*integrate(-(b*c^2*d*g^3*x^5 + 3*b*c^2*d*f*g^2*x^4 - 3*b*d*f^2*
g*x - b*d*f^3 + (3*b*c^2*d*f^2*g - b*d*g^3)*x^3 + (b*c^2*d*f^3 - 3*b*d*f*g
^2)*x^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1)
, c*x), x)
```

3.6.8 Giac [F(-2)]

Exception generated.

$$\int (f+gx)^3 \left(d-c^2dx^2\right)^{3/2} (a+b\arccos(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((g*x+f)^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x, algorithm="g
iac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.6.9 Mupad [F(-1)]

Timed out.

$$\int (f+gx)^3 \left(d-c^2dx^2\right)^{3/2} (a+b\arccos(cx))\,dx = \int (f+g\,x)^3\,(a+b\,\mathrm{acos}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{3/2}\,dx$$

input
```
int((f + g*x)^3*(a + b*acos(c*x))*(d - c^2*d*x^2)^(3/2),x)
```

output
```
int((f + g*x)^3*(a + b*acos(c*x))*(d - c^2*d*x^2)^(3/2), x)
```

3.7 $\int (f+gx)^2 \left(d-c^2dx^2\right)^{3/2} (a+b\arccos(cx))\,dx$

3.7.1 Optimal result

Integrand size = 31, antiderivative size = 680

$$\begin{aligned}
\int (f+gx)^2 \left(d-c^2dx^2\right)^{3/2} (a+b\arccos(cx))\,dx = & -\frac{2bdfgx\sqrt{d-c^2dx^2}}{5c\sqrt{1-c^2x^2}} \\
& +\frac{5bcdf^2x^2\sqrt{d-c^2dx^2}}{16\sqrt{1-c^2x^2}}-\frac{bdg^2x^2\sqrt{d-c^2dx^2}}{32c\sqrt{1-c^2x^2}}+\frac{4bcdfgx^3\sqrt{d-c^2dx^2}}{15\sqrt{1-c^2x^2}} \\
& -\frac{bc^3df^2x^4\sqrt{d-c^2dx^2}}{16\sqrt{1-c^2x^2}}+\frac{7bcdg^2x^4\sqrt{d-c^2dx^2}}{96\sqrt{1-c^2x^2}}-\frac{2bc^3dfgx^5\sqrt{d-c^2dx^2}}{25\sqrt{1-c^2x^2}} \\
& -\frac{bc^3dg^2x^6\sqrt{d-c^2dx^2}}{36\sqrt{1-c^2x^2}}+\frac{3}{8}df^2x\sqrt{d-c^2dx^2}(a+b\arccos(cx)) \\
& -\frac{dg^2x\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{16c^2}+\frac{1}{8}dg^2x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx)) \\
& +\frac{1}{4}df^2x\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}(a+b\arccos(cx)) \\
& +\frac{1}{6}dg^2x^3\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}(a+b\arccos(cx)) \\
& -\frac{2dfg\left(1-c^2x^2\right)^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{5c^2} \\
& -\frac{3df^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{16bc\sqrt{1-c^2x^2}}-\frac{dg^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{32bc^3\sqrt{1-c^2x^2}}
\end{aligned}$$

output
```
3/8*d*f^2*x*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)-1/16*d*g^2*x*(a+b*arcco
s(c*x))*(-c^2*d*x^2+d)^(1/2)/c^2+1/8*d*g^2*x^3*(a+b*arccos(c*x))*(-c^2*d*x
^2+d)^(1/2)+1/4*d*f^2*x*(-c^2*x^2+1)*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2
)+1/6*d*g^2*x^3*(-c^2*x^2+1)*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)-2/5*d*
f*g*(-c^2*x^2+1)^2*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)/c^2-2/5*b*d*f*g*
x*(-c^2*d*x^2+d)^(1/2)/c/(-c^2*x^2+1)^(1/2)+5/16*b*c*d*f^2*x^2*(-c^2*d*x^2
+d)^(1/2)/(-c^2*x^2+1)^(1/2)-1/32*b*d*g^2*x^2*(-c^2*d*x^2+d)^(1/2)/c/(-c^2
*x^2+1)^(1/2)+4/15*b*c*d*f*g*x^3*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-1
/16*b*c^3*d*f^2*x^4*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)+7/96*b*c*d*g^2
*x^4*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-2/25*b*c^3*d*f*g*x^5*(-c^2*d*
x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-1/36*b*c^3*d*g^2*x^6*(-c^2*d*x^2+d)^(1/2)/
(-c^2*x^2+1)^(1/2)-3/16*d*f^2*(a+b*arccos(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/c
/(-c^2*x^2+1)^(1/2)-1/32*d*g^2*(a+b*arccos(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/
c^3/(-c^2*x^2+1)^(1/2)
```

3.7.2 Mathematica [A] (verified)

Time = 3.08 (sec) , antiderivative size = 591, normalized size of antiderivative = 0.87

$$\int (f+gx)^2 \left(d-c^2dx^2\right)^{3/2} (a+b\arccos(cx))\,dx = \frac{-1800bd(6c^2f^2+g^2)\sqrt{d-c^2dx^2}\arccos(cx)^2-3600ad^{3/2}(6c^2f^2+g^2)\sqrt{1-c^2x^2}\,\mathrm{arctan}}{}$$

input
```
Integrate[(f + g*x)^2*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]),x]
```

output
```
(-1800*b*d*(6*c^2*f^2 + g^2)*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]^2 - 3600*a*d^
(3/2)*(6*c^2*f^2 + g^2)*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])
/(Sqrt[d]*(-1 + c^2*x^2))] - d*Sqrt[d - c^2*d*x^2]*(14400*b*c^2*f*g*x + 23
040*a*c*f*g*Sqrt[1 - c^2*x^2] - 36000*a*c^3*f^2*x*Sqrt[1 - c^2*x^2] + 3600
*a*c*g^2*x*Sqrt[1 - c^2*x^2] - 46080*a*c^3*f*g*x^2*Sqrt[1 - c^2*x^2] + 144
00*a*c^5*f^2*x^3*Sqrt[1 - c^2*x^2] - 16800*a*c^3*g^2*x^3*Sqrt[1 - c^2*x^2]
 + 23040*a*c^5*f*g*x^4*Sqrt[1 - c^2*x^2] + 9600*a*c^5*g^2*x^5*Sqrt[1 - c^2
*x^2] - 450*b*(16*c^2*f^2 + g^2)*Cos[2*ArcCos[c*x]] - 2400*b*c*f*g*Cos[3*A
rcCos[c*x]] + 450*b*c^2*f^2*Cos[4*ArcCos[c*x]] - 225*b*g^2*Cos[4*ArcCos[c*
x]] + 288*b*c*f*g*Cos[5*ArcCos[c*x]] + 50*b*g^2*Cos[6*ArcCos[c*x]]) + 60*b
*d*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]*(-400*c*f*g*Sqrt[1 - c^2*x^2] + 640*c^3
*f*g*x^2*Sqrt[1 - c^2*x^2] + 15*(16*c^2*f^2 + g^2)*Sin[2*ArcCos[c*x]] - 40
*c*f*g*Sin[3*ArcCos[c*x]] - 30*c^2*f^2*Sin[4*ArcCos[c*x]] + 15*g^2*Sin[4*A
rcCos[c*x]] - 24*c*f*g*Sin[5*ArcCos[c*x]] - 5*g^2*Sin[6*ArcCos[c*x]]))/(57
600*c^3*Sqrt[1 - c^2*x^2])
```

3.7.3 Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 367, normalized size of antiderivative = 0.54, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {5277, 5263, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d-c^2dx^2\right)^{3/2}(f+gx)^2(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \; 5277$$

$$\frac{d\sqrt{d-c^2dx^2}\int (f+gx)^2\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))dx}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \; 5263$$

$$\frac{d\sqrt{d-c^2dx^2}\int \left(\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))f^2+2gx\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))f+g^2x^2\left(1-c^2x^2\right)^{3/2}(a$$

$$\sqrt{1-c^2x^2}$$

$$\Big\downarrow \; 2009$$

$$d\sqrt{d-c^2dx^2}\left(-\frac{g^2(a+b\arccos(cx))^2}{32bc^3}+\frac{1}{4}f^2x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{3}{8}f^2x\sqrt{1-c^2x^2}(a+b\arccos(cx))-\frac{2}{}\right.$$

input
```
Int[(f + g*x)^2*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]),x]
```

output
```
(d*Sqrt[d - c^2*d*x^2]*((-2*b*f*g*x)/(5*c) + (5*b*c*f^2*x^2)/16 - (b*g^2*x
^2)/(32*c) + (4*b*c*f*g*x^3)/15 - (b*c^3*f^2*x^4)/16 + (7*b*c*g^2*x^4)/96
- (2*b*c^3*f*g*x^5)/25 - (b*c^3*g^2*x^6)/36 + (3*f^2*x*Sqrt[1 - c^2*x^2]*(
a + b*ArcCos[c*x]))/8 - (g^2*x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(16*
c^2) + (g^2*x^3*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/8 + (f^2*x*(1 - c^2
*x^2)^(3/2)*(a + b*ArcCos[c*x]))/4 + (g^2*x^3*(1 - c^2*x^2)^(3/2)*(a + b*A
rcCos[c*x]))/6 - (2*f*g*(1 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x]))/(5*c^2) -
 (3*f^2*(a + b*ArcCos[c*x])^2)/(16*b*c) - (g^2*(a + b*ArcCos[c*x])^2)/(32*
b*c^3)))/Sqrt[1 - c^2*x^2]
```

3.7.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5263
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

rule 5277
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[Simp[(d + e*x^2)^p/(1 - c^2*x^2)^
p]   Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && Intege
rQ[p - 1/2] &&  !GtQ[d, 0]
```

3.7.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.42 (sec) , antiderivative size = 1533, normalized size of antiderivative = 2.25

method	result	size
default	Expression too large to display	1533
parts	Expression too large to display	1533

input
```
int((g*x+f)^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x,method=_RETURNVERBO
SE)
```

output
```
a*(f^2*(1/4*x*(-c^2*d*x^2+d)^(3/2)+3/4*d*(1/2*x*(-c^2*d*x^2+d)^(1/2)+1/2*d
/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))))+g^2*(-1/6*x*
(-c^2*d*x^2+d)^(5/2)/c^2/d+1/6/c^2*(1/4*x*(-c^2*d*x^2+d)^(3/2)+3/4*d*(1/2*
x*(-c^2*d*x^2+d)^(1/2)+1/2*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*
x^2+d)^(1/2)))))-2/5*f*g/c^2/d*(-c^2*d*x^2+d)^(5/2))+b*(1/32*(-d*(c^2*x^2-
1))^(1/2)*(-c^2*x^2+1)^(1/2)/c^3/(c^2*x^2-1)*arccos(c*x)^2*(6*c^2*f^2+g^2)
*d-1/2304*(-d*(c^2*x^2-1))^(1/2)*(32*I*(-c^2*x^2+1)^(1/2)*c^6*x^6+32*c^7*x
^7-48*I*(-c^2*x^2+1)^(1/2)*x^4*c^4-64*c^5*x^5+18*I*(-c^2*x^2+1)^(1/2)*x^2*
c^2+38*c^3*x^3-I*(-c^2*x^2+1)^(1/2)-6*c*x)*g^2*(I+6*arccos(c*x))*d/c^3/(c^
2*x^2-1)-1/400*(-d*(c^2*x^2-1))^(1/2)*(16*I*c^5*x^5*(-c^2*x^2+1)^(1/2)+16*
c^6*x^6-20*I*(-c^2*x^2+1)^(1/2)*x^3*c^3-28*c^4*x^4+5*I*(-c^2*x^2+1)^(1/2)*
x*c+13*c^2*x^2-1)*f*g*(I+5*arccos(c*x))*d/c^2/(c^2*x^2-1)-1/512*(-d*(c^2*x
^2-1))^(1/2)*(8*I*(-c^2*x^2+1)^(1/2)*c^4*x^4+8*c^5*x^5-8*I*(-c^2*x^2+1)^(1
/2)*x^2*c^2-12*c^3*x^3+I*(-c^2*x^2+1)^(1/2)+4*c*x)*(8*arccos(c*x)*c^2*f^2+
2*I*c^2*f^2-4*arccos(c*x)*g^2-I*g^2)*d/c^3/(c^2*x^2-1)-1/8*(-d*(c^2*x^2-1)
)^(1/2)*(I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*f*g*(arccos(c*x)+I)*d/c^2/(c^
2*x^2-1)-1/8*(-d*(c^2*x^2-1))^(1/2)*(c^2*x^2-I*(-c^2*x^2+1)^(1/2)*x*c-1)*f
*g*(arccos(c*x)-I)*d/c^2/(c^2*x^2-1)+1/256*(-d*(c^2*x^2-1))^(1/2)*(-2*I*(-
c^2*x^2+1)^(1/2)*x^2*c^2+2*c^3*x^3+I*(-c^2*x^2+1)^(1/2)-2*c*x)*(-16*I*c^2*
f^2+32*arccos(c*x)*c^2*f^2-I*g^2+2*arccos(c*x)*g^2)*d/c^3/(c^2*x^2-1)+1...
```

3.7.5 Fricas [F]

$$\int (f+gx)^2 \left(d - c^2dx^2\right)^{3/2} (a+b\arccos(cx))\, dx = \int \left(-c^2dx^2+d\right)^{\frac{3}{2}}(gx+f)^2(b\arccos(cx)+a)\, dx$$

input
```
integrate((g*x+f)^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x, algorithm="f
ricas")
```

output
```
integral(-(a*c^2*d*g^2*x^4 + 2*a*c^2*d*f*g*x^3 - 2*a*d*f*g*x - a*d*f^2 + (
a*c^2*d*f^2 - a*d*g^2)*x^2 + (b*c^2*d*g^2*x^4 + 2*b*c^2*d*f*g*x^3 - 2*b*d*
f*g*x - b*d*f^2 + (b*c^2*d*f^2 - b*d*g^2)*x^2)*arccos(c*x))*sqrt(-c^2*d*x^
2 + d), x)
```

3.7.6 Sympy [F(-1)]

Timed out.

$$\int (f+gx)^2 \left(d-c^2dx^2\right)^{3/2} (a+b\arccos(cx))\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)**2*(-c**2*d*x**2+d)**(3/2)*(a+b*acos(c*x)),x)
```

output
```
Timed out
```

3.7.7 Maxima [F]

$$\int (f+gx)^2 \left(d-c^2dx^2\right)^{3/2} (a+b\arccos(cx))\,dx = \int \left(-c^2dx^2+d\right)^{\frac{3}{2}} (gx+f)^2 (b\arccos(cx)+a)\,dx$$

input
```
integrate((g*x+f)^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x, algorithm="m
axima")
```

output
```
1/8*(2*(-c^2*d*x^2 + d)^(3/2)*x + 3*sqrt(-c^2*d*x^2 + d)*d*x + 3*d^(3/2)*a
rcsin(c*x)/c)*a*f^2 + 1/48*a*g^2*(2*(-c^2*d*x^2 + d)^(3/2)*x/c^2 - 8*(-c^2
*d*x^2 + d)^(5/2)*x/(c^2*d) + 3*sqrt(-c^2*d*x^2 + d)*d*x/c^2 + 3*d^(3/2)*a
rcsin(c*x)/c^3) - 2/5*(-c^2*d*x^2 + d)^(5/2)*a*f*g/(c^2*d) + sqrt(d)*integ
rate(-(b*c^2*d*g^2*x^4 + 2*b*c^2*d*f*g*x^3 - 2*b*d*f*g*x - b*d*f^2 + (b*c^
2*d*f^2 - b*d*g^2)*x^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(sqrt(c*x + 1)
*sqrt(-c*x + 1), c*x), x)
```

3.7.8 Giac [F(-2)]

Exception generated.

$$\int (f+gx)^2 \left(d-c^2dx^2\right)^{3/2} (a+b\arccos(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((g*x+f)^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x, algorithm="g
iac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.7.9 Mupad [F(-1)]

Timed out.

$$\int (f+gx)^2 \left(d - c^2dx^2\right)^{3/2} (a+b\arccos(cx))\,dx = \int (f+g\,x)^2\,(a+b\,\mathrm{acos}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{3/2}\,dx$$

input
```
int((f + g*x)^2*(a + b*acos(c*x))*(d - c^2*d*x^2)^(3/2),x)
```

output
```
int((f + g*x)^2*(a + b*acos(c*x))*(d - c^2*d*x^2)^(3/2), x)
```

3.8 $\int (f+gx) \left(d-c^2dx^2\right)^{3/2} (a+b\arccos(cx)) \, dx$

3.8.1 Optimal result

Integrand size = 29, antiderivative size = 370

$$\begin{aligned}\int (f+gx) \left(d-c^2dx^2\right)^{3/2} (a+b\arccos(cx)) \, dx &= -\frac{bdgx\sqrt{d-c^2dx^2}}{5c\sqrt{1-c^2x^2}} \\ &+ \frac{5bcdfx^2\sqrt{d-c^2dx^2}}{16\sqrt{1-c^2x^2}} + \frac{2bcdgx^3\sqrt{d-c^2dx^2}}{15\sqrt{1-c^2x^2}} - \frac{bc^3dfx^4\sqrt{d-c^2dx^2}}{16\sqrt{1-c^2x^2}} \\ &- \frac{bc^3dgx^5\sqrt{d-c^2dx^2}}{25\sqrt{1-c^2x^2}} + \frac{3}{8}dfx\sqrt{d-c^2dx^2}(a+b\arccos(cx)) \\ &+ \frac{1}{4}dfx\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}(a+b\arccos(cx)) \\ &- \frac{dg\left(1-c^2x^2\right)^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{5c^2} - \frac{3df\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{16bc\sqrt{1-c^2x^2}}\end{aligned}$$

output
```
3/8*d*f*x*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)+1/4*d*f*x*(-c^2*x^2+1)*(a
+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)-1/5*d*g*(-c^2*x^2+1)^2*(a+b*arccos(c*
x))*(-c^2*d*x^2+d)^(1/2)/c^2-1/5*b*d*g*x*(-c^2*d*x^2+d)^(1/2)/c/(-c^2*x^2+
1)^(1/2)+5/16*b*c*d*f*x^2*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)+2/15*b*c
*d*g*x^3*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-1/16*b*c^3*d*f*x^4*(-c^2*
d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-1/25*b*c^3*d*g*x^5*(-c^2*d*x^2+d)^(1/2)/
(-c^2*x^2+1)^(1/2)-3/16*d*f*(a+b*arccos(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/c/(
-c^2*x^2+1)^(1/2)
```

3.8.2 Mathematica [A] (verified)

Time = 2.25 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.91

$$\int (f+gx)\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))\,dx = \frac{-1800bcdf\sqrt{d-c^2dx^2}\arccos(cx)^2-3600acd^{3/2}f\sqrt{1-c^2x^2}\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right)-d\sqrt{}$$

input
```
Integrate[(f + g*x)*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]),x]
```

output
```
(-1800*b*c*d*f*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]^2 - 3600*a*c*d^(3/2)*f*Sqrt
[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] -
 d*Sqrt[d - c^2*d*x^2]*(-1200*b*c*f*Cos[2*ArcCos[c*x]] - 200*b*g*Cos[3*Arc
Cos[c*x]] + 3*(400*b*c*g*x + 80*a*Sqrt[1 - c^2*x^2]*(8*g*(-1 + c^2*x^2)^2
+ 5*c^2*f*x*(-5 + 2*c^2*x^2)) + 25*b*c*f*Cos[4*ArcCos[c*x]] + 8*b*g*Cos[5*
ArcCos[c*x]])) + 20*b*d*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]*(-100*g*Sqrt[1 - c
^2*x^2] + 160*c^2*g*x^2*Sqrt[1 - c^2*x^2] + 120*c*f*Sin[2*ArcCos[c*x]] - 1
0*g*Sin[3*ArcCos[c*x]] - 15*c*f*Sin[4*ArcCos[c*x]] - 6*g*Sin[5*ArcCos[c*x]
]))/(9600*c^2*Sqrt[1 - c^2*x^2])
```

3.8.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.52, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {5277, 5263, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d-c^2dx^2\right)^{3/2}(f+gx)(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \; 5277$$

$$\frac{d\sqrt{d-c^2dx^2}\int (f+gx)\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))dx}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \; 5263$$

$$\frac{d\sqrt{d-c^2dx^2}\int \left(f(a+b\arccos(cx))\left(1-c^2x^2\right)^{3/2}+gx(a+b\arccos(cx))\left(1-c^2x^2\right)^{3/2}\right)dx}{\sqrt{1-c^2x^2}}$$

$$\downarrow \text{2009}$$

$$\frac{d\sqrt{d-c^2dx^2}\left(\frac{1}{4}fx(1-c^2x^2)^{3/2}(a+b\arccos(cx))+\frac{3}{8}fx\sqrt{1-c^2x^2}(a+b\arccos(cx))-\frac{g(1-c^2x^2)^{5/2}(a+b\arccos(cx))}{5c^2}\right.}{\sqrt{1-c^2x^2}}$$

input
```
Int[(f + g*x)*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]),x]
```

output
```
(d*Sqrt[d - c^2*d*x^2]*(-1/5*(b*g*x)/c + (5*b*c*f*x^2)/16 + (2*b*c*g*x^3)/
15 - (b*c^3*f*x^4)/16 - (b*c^3*g*x^5)/25 + (3*f*x*Sqrt[1 - c^2*x^2]*(a + b
*ArcCos[c*x]))/8 + (f*x*(1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x]))/4 - (g*(1
 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x]))/(5*c^2) - (3*f*(a + b*ArcCos[c*x])^
2)/(16*b*c)))/Sqrt[1 - c^2*x^2]
```

3.8.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5263
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

rule 5277
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[Simp[(d + e*x^2)^p/(1 - c^2*x^2)^
p]   Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && Intege
rQ[p - 1/2] && !GtQ[d, 0]
```

3.8.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.45 (sec) , antiderivative size = 1012, normalized size of antiderivative = 2.74

method	result
default	$\frac{afx(-c^2dx^2+d)^{\frac{3}{2}}}{4}+\frac{3afdx\sqrt{-c^2dx^2+d}}{8}+\frac{3af\,d^2\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2dx^2+d}}\right)}{8\sqrt{c^2d}}-\frac{ag(-c^2dx^2+d)^{\frac{5}{2}}}{5c^2d}+b\Bigg(\frac{3\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x}}{16c(c^2x^2}$
parts	$\frac{afx(-c^2dx^2+d)^{\frac{3}{2}}}{4}+\frac{3afdx\sqrt{-c^2dx^2+d}}{8}+\frac{3af\,d^2\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2dx^2+d}}\right)}{8\sqrt{c^2d}}-\frac{ag(-c^2dx^2+d)^{\frac{5}{2}}}{5c^2d}+b\Bigg(\frac{3\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x}}{16c(c^2x^2}$

input
```
int((g*x+f)*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE
)
```

output
```
1/4*a*f*x*(-c^2*d*x^2+d)^(3/2)+3/8*a*f*d*x*(-c^2*d*x^2+d)^(1/2)+3/8*a*f*d^
2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-1/5*a*g/c^2/d
*(-c^2*d*x^2+d)^(5/2)+b*(3/16*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c/
(c^2*x^2-1)*arccos(c*x)^2*f*d-1/800*(-d*(c^2*x^2-1))^(1/2)*(16*I*c^5*x^5*(
-c^2*x^2+1)^(1/2)+16*c^6*x^6-20*I*(-c^2*x^2+1)^(1/2)*x^3*c^3-28*c^4*x^4+5*
I*(-c^2*x^2+1)^(1/2)*x*c+13*c^2*x^2-1)*g*(I+5*arccos(c*x))*d/c^2/(c^2*x^2-
1)-1/256*(-d*(c^2*x^2-1))^(1/2)*(8*I*(-c^2*x^2+1)^(1/2)*c^4*x^4+8*c^5*x^5-
8*I*(-c^2*x^2+1)^(1/2)*x^2*c^2-12*c^3*x^3+I*(-c^2*x^2+1)^(1/2)+4*c*x)*f*(4
*arccos(c*x)+I)*d/c/(c^2*x^2-1)-1/16*(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1
)^(1/2)*x*c+c^2*x^2-1)*g*(arccos(c*x)+I)*d/c^2/(c^2*x^2-1)-1/16*(-d*(c^2*x
^2-1))^(1/2)*(c^2*x^2-I*(-c^2*x^2+1)^(1/2)*x*c-1)*g*(arccos(c*x)-I)*d/c^2/
(c^2*x^2-1)+1/16*(-d*(c^2*x^2-1))^(1/2)*(-2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+2
*c^3*x^3+I*(-c^2*x^2+1)^(1/2)-2*c*x)*f*(-I+2*arccos(c*x))*d/c/(c^2*x^2-1)+
1/96*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2-4*I*c^3*x^3*(-c^2*x^2+1)^
(1/2)+3*I*(-c^2*x^2+1)^(1/2)*x*c+1)*g*(-I+3*arccos(c*x))*d/c^2/(c^2*x^2-1)
-1/1200*(-d*(c^2*x^2-1))^(1/2)*(c^2*x^2-I*(-c^2*x^2+1)^(1/2)*x*c-1)*g*(11*
I+45*arccos(c*x))*cos(4*arccos(c*x))*d/c^2/(c^2*x^2-1)-1/600*(-d*(c^2*x^2-
1))^(1/2)*(c*x*(-c^2*x^2+1)^(1/2)+I*c^2*x^2-I)*g*(7*I+15*arccos(c*x))*sin(
4*arccos(c*x))*d/c^2/(c^2*x^2-1)-3/256*(-d*(c^2*x^2-1))^(1/2)*(c^2*x^2-I*(
-c^2*x^2+1)^(1/2)*x*c-1)*f*(5*I+12*arccos(c*x))*cos(3*arccos(c*x))*d/c/...
```

3.8.5 Fricas [F]

$$\int (f+gx)\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))\,dx=\int \left(-c^2dx^2+d\right)^{\frac{3}{2}}(gx+f)(b\arccos(cx)+a)\,dx$$

input
```
integrate((g*x+f)*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x, algorithm="fri
cas")
```

output
```
integral(-(a*c^2*d*g*x^3 + a*c^2*d*f*x^2 - a*d*g*x - a*d*f + (b*c^2*d*g*x^
3 + b*c^2*d*f*x^2 - b*d*g*x - b*d*f)*arccos(c*x))*sqrt(-c^2*d*x^2 + d), x)
```

3.8.6 Sympy [F]

$$\int (f+gx)\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))\,dx=\int (-d(cx-1)(cx+1))^{\frac{3}{2}}(a+b\operatorname{acos}(cx))(f+gx)\,dx$$

input
```
integrate((g*x+f)*(-c**2*d*x**2+d)**(3/2)*(a+b*acos(c*x)),x)
```

output
```
Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acos(c*x))*(f + g*x), x)
```

3.8.7 Maxima [F]

$$\int (f+gx)\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))\,dx=\int \left(-c^2dx^2+d\right)^{\frac{3}{2}}(gx+f)(b\arccos(cx)+a)\,dx$$

input
```
integrate((g*x+f)*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x, algorithm="max
ima")
```

output
```
1/8*(2*(-c^2*d*x^2 + d)^(3/2)*x + 3*sqrt(-c^2*d*x^2 + d)*d*x + 3*d^(3/2)*a
rcsin(c*x)/c)*a*f - 1/5*(-c^2*d*x^2 + d)^(5/2)*a*g/(c^2*d) + sqrt(d)*integ
rate(-(b*c^2*d*g*x^3 + b*c^2*d*f*x^2 - b*d*g*x - b*d*f)*sqrt(c*x + 1)*sqrt
(-c*x + 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x), x)
```

3.8.8 Giac [F(-2)]

Exception generated.

$$\int (f+gx)\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((g*x+f)*(-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x)),x, algorithm="gia
c")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.8.9 Mupad [F(-1)]

Timed out.

$$\int (f+gx)\left(d-c^2dx^2\right)^{3/2}(a+b\arccos(cx))\,dx = \int (f+g\,x)\,\left(a+b\,\mathrm{acos}(c\,x)\right)\,\left(d-c^2\,d\,x^2\right)^{3/2}\,dx$$

input
```
int((f + g*x)*(a + b*acos(c*x))*(d - c^2*d*x^2)^(3/2),x)
```

output
```
int((f + g*x)*(a + b*acos(c*x))*(d - c^2*d*x^2)^(3/2), x)
```

3.9 $\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{f+gx}\,dx$

3.9.1 Optimal result

Integrand size = 31, antiderivative size = 1064

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{f+gx}\,dx =$$
$$-\frac{ad(cf-g)(cf+g)\sqrt{d-c^2dx^2}}{g^3}+\frac{bcdx\sqrt{d-c^2dx^2}}{3g\sqrt{1-c^2x^2}}$$
$$-\frac{bcd(cf-g)(cf+g)x\sqrt{d-c^2dx^2}}{g^3\sqrt{1-c^2x^2}}+\frac{bc^3dfx^2\sqrt{d-c^2dx^2}}{4g^2\sqrt{1-c^2x^2}}$$
$$-\frac{bc^3dx^3\sqrt{d-c^2dx^2}}{9g\sqrt{1-c^2x^2}}-\frac{bd(cf-g)(cf+g)\sqrt{d-c^2dx^2}\arccos(cx)}{g^3}$$
$$+\frac{c^2dfx\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{2g^2}$$
$$+\frac{d(1-c^2x^2)\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{3g}$$
$$-\frac{cdf\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{4bg^2\sqrt{1-c^2x^2}}$$
$$+\frac{cd(cf-g)(cf+g)x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2bg^3\sqrt{1-c^2x^2}}$$
$$+\frac{d(c^2f^2-g^2)^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2bcg^4(f+gx)\sqrt{1-c^2x^2}}$$
$$+\frac{d(cf-g)(cf+g)\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2bcg^2(f+gx)}$$
$$+\frac{ad(c^2f^2-g^2)^{3/2}\sqrt{d-c^2dx^2}\arctan\left(\frac{g+c^2fx}{\sqrt{c^2f^2-g^2}\sqrt{1-c^2x^2}}\right)}{g^4\sqrt{1-c^2x^2}}$$
$$+\frac{ibd(c^2f^2-g^2)^{3/2}\sqrt{d-c^2dx^2}\arccos(cx)\log\left(1+\frac{e^{i\arccos(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{g^4\sqrt{1-c^2x^2}}$$
$$-\frac{ibd(c^2f^2-g^2)^{3/2}\sqrt{d-c^2dx^2}\arccos(cx)\log\left(1+\frac{e^{i\arccos(cx)}g}{cf+\sqrt{c^2f^2-g^2}}\right)}{g^4\sqrt{1-c^2x^2}}$$
$$+\frac{bd(c^2f^2-g^2)^{3/2}\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,-\frac{e^{i\arccos(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{g^4\sqrt{1-c^2x^2}}$$
$$-\frac{bd(c^2f^2-g^2)^{3/2}\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,-\frac{e^{i\arccos(cx)}g}{cf+\sqrt{c^2f^2-g^2}}\right)}{g^4\sqrt{1-c^2x^2}}$$

output
```
-a*d*(c*f-g)*(c*f+g)*(-c^2*d*x^2+d)^(1/2)/g^3-b*d*(c*f-g)*(c*f+g)*arccos(c
*x)*(-c^2*d*x^2+d)^(1/2)/g^3+1/2*c^2*d*f*x*(a+b*arccos(c*x))*(-c^2*d*x^2+d
)^(1/2)/g^2+1/3*d*(-c^2*x^2+1)*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)/g+1/
3*b*c*d*x*(-c^2*d*x^2+d)^(1/2)/g/(-c^2*x^2+1)^(1/2)-b*c*d*(c*f-g)*(c*f+g)*
x*(-c^2*d*x^2+d)^(1/2)/g^3/(-c^2*x^2+1)^(1/2)+1/4*b*c^3*d*f*x^2*(-c^2*d*x^
2+d)^(1/2)/g^2/(-c^2*x^2+1)^(1/2)-1/9*b*c^3*d*x^3*(-c^2*d*x^2+d)^(1/2)/g/(
-c^2*x^2+1)^(1/2)-1/4*c*d*f*(a+b*arccos(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/g^2
/(-c^2*x^2+1)^(1/2)+1/2*c*d*(c*f-g)*(c*f+g)*x*(a+b*arccos(c*x))^2*(-c^2*d*
x^2+d)^(1/2)/b/g^3/(-c^2*x^2+1)^(1/2)+1/2*d*(c^2*f^2-g^2)^2*(a+b*arccos(c*
x))^2*(-c^2*d*x^2+d)^(1/2)/b/c/g^4/(g*x+f)/(-c^2*x^2+1)^(1/2)+a*d*(c^2*f^2
-g^2)^(3/2)*arctan((c^2*f*x+g)/(c^2*f^2-g^2)^(1/2)/(-c^2*x^2+1)^(1/2))*(-c
^2*d*x^2+d)^(1/2)/g^4/(-c^2*x^2+1)^(1/2)+I*b*d*(c^2*f^2-g^2)^(3/2)*arccos(
c*x)*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2-g^2)^(1/2)))*(-c^2*d*
x^2+d)^(1/2)/g^4/(-c^2*x^2+1)^(1/2)-I*b*d*(c^2*f^2-g^2)^(3/2)*arccos(c*x)*
ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2)))*(-c^2*d*x^2+d
)^(1/2)/g^4/(-c^2*x^2+1)^(1/2)+b*d*(c^2*f^2-g^2)^(3/2)*polylog(2,-(c*x+I*(
-c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2-g^2)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/g^4/(
-c^2*x^2+1)^(1/2)-b*d*(c^2*f^2-g^2)^(3/2)*polylog(2,-(c*x+I*(-c^2*x^2+1)^(
1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/g^4/(-c^2*x^2+1)^(
1/2)+1/2*d*(c*f-g)*(c*f+g)*(a+b*arccos(c*x))^2*(-c^2*x^2+1)^(1/2)*(-c^2...
```

3.9.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 3034 vs. 2(1064) = 2128.

Time = 15.83 (sec) , antiderivative size = 3034, normalized size of antiderivative = 2.85

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{f+gx}\,dx = \text{Result too large to show}$$

input
```
Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]))/(f + g*x),x]
```

output
```
Sqrt[-(d*(-1 + c^2*x^2))]*((a*d*(-3*c^2*f^2 + 4*g^2))/(3*g^3) + (a*c^2*d*f
*x)/(2*g^2) - (a*c^2*d*x^2)/(3*g)) + (a*c*d^(3/2)*f*(2*c^2*f^2 - 3*g^2)*Ar
cTan[(c*x*Sqrt[-(d*(-1 + c^2*x^2))])/(Sqrt[d]*(-1 + c^2*x^2))])/(2*g^4) +
(a*d^(3/2)*(-(c^2*f^2) + g^2)^(3/2)*Log[f + g*x])/g^4 - (a*d^(3/2)*(-(c^2*
f^2) + g^2)^(3/2)*Log[d*g + c^2*d*f*x + Sqrt[d]*Sqrt[-(c^2*f^2) + g^2]*Sqr
t[-(d*(-1 + c^2*x^2))]])/g^4 - (b*d*Sqrt[d*(1 - c^2*x^2)]*((-2*c*g*x)/Sqrt
[1 - c^2*x^2] - 2*g*ArcCos[c*x] + (c*f*ArcCos[c*x]^2)/Sqrt[1 - c^2*x^2] +
(2*(-(c*f) + g)*(c*f + g)*(2*ArcCos[c*x]*ArcTanh[((c*f + g)*Cot[ArcCos[c*x
]/2])/Sqrt[-(c^2*f^2) + g^2]] - 2*ArcCos[-((c*f)/g)]*ArcTanh[((-(c*f) + g)
*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + (ArcCos[-((c*f)/g)] - (2*I)
*ArcTanh[((c*f + g)*Cot[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + (2*I)*Ar
cTanh[((-(c*f) + g)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])*Log[Sqrt[
-(c^2*f^2) + g^2]/(Sqrt[2]*E^((I/2)*ArcCos[c*x])*Sqrt[g]*Sqrt[c*f + c*g*x]
)] + (ArcCos[-((c*f)/g)] + (2*I)*(ArcTanh[((c*f + g)*Cot[ArcCos[c*x]/2])/S
qrt[-(c^2*f^2) + g^2]] - ArcTanh[((-(c*f) + g)*Tan[ArcCos[c*x]/2])/Sqrt[-(
c^2*f^2) + g^2]]))*Log[(E^((I/2)*ArcCos[c*x])*Sqrt[-(c^2*f^2) + g^2])/(Sqr
t[2]*Sqrt[g]*Sqrt[c*f + c*g*x])] - (ArcCos[-((c*f)/g)] - (2*I)*ArcTanh[((-
(c*f) + g)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])*Log[((c*f + g)*((-
I)*c*f + I*g + Sqrt[-(c^2*f^2) + g^2])*(-I + Tan[ArcCos[c*x]/2]))/(g*(c*f
+ g + Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))] - (ArcCos[-((c*f)/g)...
```

3.9.3 Rubi [A] (verified)

Time = 2.40 (sec) , antiderivative size = 700, normalized size of antiderivative = 0.66, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {5277, 5267, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{f+gx}dx$$

$$\Big\downarrow \; 5277$$

$$\frac{d\sqrt{d-c^2dx^2}\int \frac{(1-c^2x^2)^{3/2}(a+b\arccos(cx))}{f+gx}dx}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \; 5267$$

$$\frac{d\sqrt{d-c^2dx^2}\int \left(-\frac{x\sqrt{1-c^2x^2}(a+b\arccos(cx))c^2}{g}+\frac{f\sqrt{1-c^2x^2}(a+b\arccos(cx))c^2}{g^2}+\frac{(g^2-c^2f^2)\sqrt{1-c^2x^2}(a+b\arccos(cx))}{g^2(f+gx)}\right)dx}{\sqrt{1-c^2x^2}}$$

$\big\downarrow$ 2009

$$d\sqrt{d-c^2dx^2}\left(\frac{(1-c^2x^2)\,(c^2f^2-g^2)\,(a+b\arccos(cx))^2}{2bcg^2(f+gx)}+\frac{(c^2f^2-g^2)^2(a+b\arccos(cx))^2}{2bcg^4(f+gx)}+\frac{cx\,(c^2f^2-g^2)\,(a+b\arccos(cx))^2}{2bg^3}+\frac{c^2fx\sqrt{1-c^2x}\right.$$

input
```
Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCos[c*x]))/(f + g*x),x]
```

output
```
(d*Sqrt[d - c^2*d*x^2]*((b*c*x)/(3*g) - (b*c*(c^2*f^2 - g^2)*x)/g^3 + (b*c
^3*f*x^2)/(4*g^2) - (b*c^3*x^3)/(9*g) - (a*(c*f - g)*(c*f + g)*Sqrt[1 - c^
2*x^2])/g^3 - (b*(c*f - g)*(c*f + g)*Sqrt[1 - c^2*x^2]*ArcCos[c*x])/g^3 +
(c^2*f*x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(2*g^2) + ((1 - c^2*x^2)^(
3/2)*(a + b*ArcCos[c*x]))/(3*g) - (c*f*(a + b*ArcCos[c*x])^2)/(4*b*g^2) +
(c*(c^2*f^2 - g^2)*x*(a + b*ArcCos[c*x])^2)/(2*b*g^3) + ((c^2*f^2 - g^2)^2
*(a + b*ArcCos[c*x])^2)/(2*b*c*g^4*(f + g*x)) + ((c^2*f^2 - g^2)*(1 - c^2*
x^2)*(a + b*ArcCos[c*x])^2)/(2*b*c*g^2*(f + g*x)) + (a*(c^2*f^2 - g^2)^(3/
2)*ArcTan[(g + c^2*f*x)/(Sqrt[c^2*f^2 - g^2]*Sqrt[1 - c^2*x^2])])/g^4 + (I
*b*(c^2*f^2 - g^2)^(3/2)*ArcCos[c*x]*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f -
Sqrt[c^2*f^2 - g^2])])/g^4 - (I*b*(c^2*f^2 - g^2)^(3/2)*ArcCos[c*x]*Log[1
+ (E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/g^4 + (b*(c^2*f^2 -
g^2)^(3/2)*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2]))
])/g^4 - (b*(c^2*f^2 - g^2)^(3/2)*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f
+ Sqrt[c^2*f^2 - g^2]))])/g^4))/Sqrt[1 - c^2*x^2]
```

3.9.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5267
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[Sqrt[d + e*x^2]*(a
 + b*ArcCos[c*x])^n, (f + g*x)^m*(d + e*x^2)^(p - 1/2), x], x] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IGtQ[p + 1/2,
 0] && GtQ[d, 0] && IGtQ[n, 0]
```

rule 5277
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[Simp[(d + e*x^2)^p/(1 - c^2*x^2)^
p]   Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && Intege
rQ[p - 1/2] &&  !GtQ[d, 0]
```

3.9.4 Maple [A] (verified)

Time = 3.01 (sec) , antiderivative size = 1559, normalized size of antiderivative = 1.47

method	result	size
default	Expression too large to display	1559
parts	Expression too large to display	1559

input
```
int((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/(g*x+f),x,method=_RETURNVERBOSE
)
```

output
```
a/g*(1/3*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(3/2)+
c^2*d*f/g*(-1/4*(-2*(x+f/g)*c^2*d+2*c^2*d*f/g)/c^2/d*(-(x+f/g)^2*c^2*d+2*c
^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2)-1/8*(4*c^2*d^2*(c^2*f^2-g^2)/g
^2-4*c^4*d^2*f^2/g^2)/c^2/d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-(x+f/g)
^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2)))-d*(c^2*f^2-g^2)/
g^2*((-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2)+c^2*
d*f/g/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(
x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))+d*(c^2*f^2-g^2)/g^2/(-d*(c^2*f^2-g^2)/g
^2)^(1/2)*ln((-2*d*(c^2*f^2-g^2)/g^2+2*c^2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^
2)/g^2)^(1/2)*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(
1/2))/(x+f/g))))+b*(-1/4*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/(c^2*x^
2-1)*arccos(c*x)^2*f*(2*c^2*f^2-3*g^2)*c*d/g^4-1/72*(-d*(c^2*x^2-1))^(1/2)
*(4*I*c^3*x^3*(-c^2*x^2+1)^(1/2)+4*c^4*x^4-3*I*(-c^2*x^2+1)^(1/2)*x*c-5*c^
2*x^2+1)*(I+3*arccos(c*x))*d/(c^2*x^2-1)/g+1/16*(-d*(c^2*x^2-1))^(1/2)*(2*
I*(-c^2*x^2+1)^(1/2)*x^2*c^2+2*c^3*x^3-I*(-c^2*x^2+1)^(1/2)-2*c*x)*f*(I+2*
arccos(c*x))*c*d/(c^2*x^2-1)/g^2-1/8*(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1
)^(1/2)*x*c+c^2*x^2-1)*(4*arccos(c*x)*c^2*f^2+4*I*c^2*f^2-5*arccos(c*x)*g^
2-5*I*g^2)*d/(c^2*x^2-1)/g^3-1/8*(-d*(c^2*x^2-1))^(1/2)*(c^2*x^2-I*(-c^2*x
^2+1)^(1/2)*x*c-1)*(4*arccos(c*x)*c^2*f^2-4*I*c^2*f^2-5*arccos(c*x)*g^2+5*
I*g^2)*d/(c^2*x^2-1)/g^3+1/16*(-d*(c^2*x^2-1))^(1/2)*(-2*I*(-c^2*x^2+1)...
```

3.9.5 Fricas [F]

$$\int \frac{(d - c^2dx^2)^{3/2} (a + b \arccos(cx))}{f + gx} \, dx = \int \frac{(-c^2dx^2 + d)^{\frac{3}{2}} (b \arccos(cx) + a)}{gx + f} \, dx$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/(g*x+f),x, algorithm="fri
cas")
```

output
```
integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arccos(c*x))*sqrt(-c^2*
d*x^2 + d)/(g*x + f), x)
```

3.9.6 Sympy [F]

$$\int \frac{(d - c^2dx^2)^{3/2} (a + b \arccos(cx))}{f + gx} \, dx = \int \frac{(-d(cx - 1)(cx + 1))^{\frac{3}{2}} (a + b \operatorname{acos}(cx))}{f + gx} \, dx$$

input
```
integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acos(c*x))/(g*x+f),x)
```

output
```
Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acos(c*x))/(f + g*x), x)
```

3.9.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d - c^2dx^2)^{3/2} (a + b \arccos(cx))}{f + gx} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/(g*x+f),x, algorithm="max
ima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(g-c*f>0)', see `assume?` for mor
e details)
```

3.9.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{f+gx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccos(c*x))/(g*x+f),x, algorithm="gia
c")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.9.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\arccos(cx))}{f+gx}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))\,(d-c^2\,d\,x^2)^{3/2}}{f+g\,x}\,dx$$

input
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(3/2))/(f + g*x),x)
```

output
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(3/2))/(f + g*x), x)
```

3.10 $\int (f+gx)^3 \left(d-c^2dx^2\right)^{5/2} (a+b\arccos(cx))\,dx$

3.10.1 Optimal result

Integrand size = 31, antiderivative size = 1281

$$\int (f+gx)^3\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))\,dx=-\frac{3bd^2f^2gx\sqrt{d-c^2dx^2}}{7c\sqrt{1-c^2x^2}}$$
$$-\frac{2bd^2g^3x\sqrt{d-c^2dx^2}}{63c^3\sqrt{1-c^2x^2}}+\frac{25bcd^2f^3x^2\sqrt{d-c^2dx^2}}{96\sqrt{1-c^2x^2}}-\frac{15bd^2fg^2x^2\sqrt{d-c^2dx^2}}{256c\sqrt{1-c^2x^2}}$$
$$+\frac{3bcd^2f^2gx^3\sqrt{d-c^2dx^2}}{7\sqrt{1-c^2x^2}}-\frac{bd^2g^3x^3\sqrt{d-c^2dx^2}}{189c\sqrt{1-c^2x^2}}-\frac{5bc^3d^2f^3x^4\sqrt{d-c^2dx^2}}{96\sqrt{1-c^2x^2}}$$
$$+\frac{59bcd^2fg^2x^4\sqrt{d-c^2dx^2}}{256\sqrt{1-c^2x^2}}-\frac{9bc^3d^2f^2gx^5\sqrt{d-c^2dx^2}}{35\sqrt{1-c^2x^2}}+\frac{bcd^2g^3x^5\sqrt{d-c^2dx^2}}{21\sqrt{1-c^2x^2}}$$
$$-\frac{17bc^3d^2fg^2x^6\sqrt{d-c^2dx^2}}{96\sqrt{1-c^2x^2}}+\frac{3bc^5d^2f^2gx^7\sqrt{d-c^2dx^2}}{49\sqrt{1-c^2x^2}}-\frac{19bc^3d^2g^3x^7\sqrt{d-c^2dx^2}}{441\sqrt{1-c^2x^2}}$$
$$+\frac{3bc^5d^2fg^2x^8\sqrt{d-c^2dx^2}}{64\sqrt{1-c^2x^2}}+\frac{bc^5d^2g^3x^9\sqrt{d-c^2dx^2}}{81\sqrt{1-c^2x^2}}-\frac{bd^2f^3(1-c^2x^2)^{5/2}\sqrt{d-c^2dx^2}}{36c}$$
$$+\frac{5}{16}d^2f^3x\sqrt{d-c^2dx^2}(a+b\arccos(cx))-\frac{15d^2fg^2x\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{128c^2}$$
$$+\frac{15}{64}d^2fg^2x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))+\frac{5}{24}d^2f^3x\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}(a+b\arccos(cx))$$
$$+\frac{5}{16}d^2fg^2x^3\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}(a+b\arccos(cx))$$
$$+\frac{1}{6}d^2f^3x\left(1-c^2x^2\right)^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))$$
$$+\frac{3}{8}d^2fg^2x^3\left(1-c^2x^2\right)^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))$$
$$-\frac{3d^2f^2g(1-c^2x^2)^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{7c^2}$$
$$-\frac{d^2g^3(1-c^2x^2)^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{7c^4}$$
$$+\frac{d^2g^3(1-c^2x^2)^4\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{9c^4}$$
$$-\frac{5d^2f^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{32bc\sqrt{1-c^2x^2}}-\frac{15d^2fg^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{256bc^3\sqrt{1-c^2x^2}}$$

output
```
-5/96*b*c^3*d^2*f^3*x^4*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)+1/21*b*c*d
^2*g^3*x^5*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-19/441*b*c^3*d^2*g^3*x^
7*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)+1/81*b*c^5*d^2*g^3*x^9*(-c^2*d*x
^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-2/63*b*d^2*g^3*x*(-c^2*d*x^2+d)^(1/2)/c^3/(
-c^2*x^2+1)^(1/2)+25/96*b*c*d^2*f^3*x^2*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^
(1/2)-1/189*b*d^2*g^3*x^3*(-c^2*d*x^2+d)^(1/2)/c/(-c^2*x^2+1)^(1/2)-15/256
*d^2*f*g^2*(a+b*arccos(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/c^3/(-c^2*x^2+1)^(1/
2)-3/7*b*d^2*f^2*g*x*(-c^2*d*x^2+d)^(1/2)/c/(-c^2*x^2+1)^(1/2)-15/256*b*d^
2*f*g^2*x^2*(-c^2*d*x^2+d)^(1/2)/c/(-c^2*x^2+1)^(1/2)-1/36*b*d^2*f^3*(-c^2
*x^2+1)^(5/2)*(-c^2*d*x^2+d)^(1/2)/c-5/32*d^2*f^3*(a+b*arccos(c*x))^2*(-c^
2*d*x^2+d)^(1/2)/b/c/(-c^2*x^2+1)^(1/2)+3/7*b*c*d^2*f^2*g*x^3*(-c^2*d*x^2+
d)^(1/2)/(-c^2*x^2+1)^(1/2)+59/256*b*c*d^2*f*g^2*x^4*(-c^2*d*x^2+d)^(1/2)/
(-c^2*x^2+1)^(1/2)-9/35*b*c^3*d^2*f^2*g*x^5*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2
+1)^(1/2)-17/96*b*c^3*d^2*f*g^2*x^6*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2
)+3/49*b*c^5*d^2*f^2*g*x^7*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)+3/64*b*
c^5*d^2*f*g^2*x^8*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-15/128*d^2*f*g^2
*x*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)/c^2+5/16*d^2*f*g^2*x^3*(-c^2*x^2
+1)*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)+3/8*d^2*f*g^2*x^3*(-c^2*x^2+1)^
2*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)-3/7*d^2*f^2*g*(-c^2*x^2+1)^3*(a+b
*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)/c^2+15/64*d^2*f*g^2*x^3*(a+b*arccos(...
```

3.10.2 Mathematica [A] (verified)

Time = 9.41 (sec) , antiderivative size = 1582, normalized size of antiderivative = 1.23

$$\int (f+gx)^3 \left(d-c^2dx^2\right)^{5/2} (a+b\arccos(cx))\,dx = \text{Too large to display}$$

input
```
Integrate[(f + g*x)^3*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]),x]
```

output
```
Sqrt[-(d*(-1 + c^2*x^2))]*(-1/63*(a*d^2*g*(27*c^2*f^2 + 2*g^2))/c^4 + (a*d
^2*f*(88*c^2*f^2 - 15*g^2)*x)/(128*c^2) - (a*d^2*g*(-81*c^2*f^2 + g^2)*x^2
)/(63*c^2) - (a*d^2*f*(104*c^2*f^2 - 177*g^2)*x^3)/192 + (a*d^2*g*(-27*c^2
*f^2 + 5*g^2)*x^4)/21 + (a*c^2*d^2*f*(8*c^2*f^2 - 51*g^2)*x^5)/48 - (a*c^2
*d^2*g*(-27*c^2*f^2 + 19*g^2)*x^6)/63 + (3*a*c^4*d^2*f*g^2*x^7)/8 + (a*c^4
*d^2*g^3*x^8)/9) - (5*a*d^(5/2)*f*(8*c^2*f^2 + 3*g^2)*ArcTan[(c*x*Sqrt[-(d
*(-1 + c^2*x^2))])/(Sqrt[d]*(-1 + c^2*x^2))])/(128*c^3) - (b*d^2*f^2*g*Sqr
t[d*(1 - c^2*x^2)]*(9*c*x + 12*(1 - c^2*x^2)^(3/2)*ArcCos[c*x] - Cos[3*Arc
Cos[c*x]]))/(12*c^2*Sqrt[1 - c^2*x^2]) - (b*d^2*f^2*g*Sqrt[d*(1 - c^2*x^2)
]*(55125*c*x - 1225*Cos[3*ArcCos[c*x]] + 840*(1 - c^2*x^2)^(3/2)*ArcCos[c*
x]*(157 + 108*Cos[2*ArcCos[c*x]] + 15*Cos[4*ArcCos[c*x]]) - 1323*Cos[5*Arc
Cos[c*x]] - 225*Cos[7*ArcCos[c*x]]))/(235200*c^2*Sqrt[1 - c^2*x^2]) + (b*d
^2*g^3*Sqrt[d*(1 - c^2*x^2)]*(55125*c*x - 1225*Cos[3*ArcCos[c*x]] + 840*(1
 - c^2*x^2)^(3/2)*ArcCos[c*x]*(157 + 108*Cos[2*ArcCos[c*x]] + 15*Cos[4*Arc
Cos[c*x]]) - 1323*Cos[5*ArcCos[c*x]] - 225*Cos[7*ArcCos[c*x]]))/(352800*c^
4*Sqrt[1 - c^2*x^2]) + (b*d^2*f^3*Sqrt[d*(1 - c^2*x^2)]*(Cos[2*ArcCos[c*x]
] + 2*ArcCos[c*x]*(-ArcCos[c*x] + Sin[2*ArcCos[c*x]])))/(8*c*Sqrt[1 - c^2*
x^2]) + (b*d^2*f^3*Sqrt[d*(1 - c^2*x^2)]*(8*ArcCos[c*x]^2 - Cos[4*ArcCos[c
*x]] - 4*ArcCos[c*x]*Sin[4*ArcCos[c*x]]))/(64*c*Sqrt[1 - c^2*x^2]) - (3*b*
d^2*f*g^2*Sqrt[d*(1 - c^2*x^2)]*(8*ArcCos[c*x]^2 - Cos[4*ArcCos[c*x]] -...
```

3.10.3 Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 630, normalized size of antiderivative = 0.49, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {5277, 5263, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d-c^2dx^2\right)^{5/2}(f+gx)^3(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{5277}$$

$$\frac{d^2\sqrt{d-c^2dx^2}\int (f+gx)^3\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))dx}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5263}$$

$$\frac{d^2\sqrt{d-c^2dx^2}\int \Big(\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))f^3+3gx\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))f^2+3g^2x^2\left(1-c^2x^2\right)^{5/}}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{2009}$$

$$d^2\sqrt{d-c^2dx^2}\left(-\frac{15fg^2(a+b\arccos(cx))^2}{256bc^3}+\frac{1}{6}f^3x\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))+\frac{5}{24}f^3x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(c\right.$$

input
```
Int[(f + g*x)^3*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]),x]
```

output
```
(d^2*Sqrt[d - c^2*d*x^2]*((-3*b*f^2*g*x)/(7*c) - (2*b*g^3*x)/(63*c^3) + (2
5*b*c*f^3*x^2)/96 - (15*b*f*g^2*x^2)/(256*c) + (3*b*c*f^2*g*x^3)/7 - (b*g^
3*x^3)/(189*c) - (5*b*c^3*f^3*x^4)/96 + (59*b*c*f*g^2*x^4)/256 - (9*b*c^3*
f^2*g*x^5)/35 + (b*c*g^3*x^5)/21 - (17*b*c^3*f*g^2*x^6)/96 + (3*b*c^5*f^2*
g*x^7)/49 - (19*b*c^3*g^3*x^7)/441 + (3*b*c^5*f*g^2*x^8)/64 + (b*c^5*g^3*x
^9)/81 - (b*f^3*(1 - c^2*x^2)^3)/(36*c) + (5*f^3*x*Sqrt[1 - c^2*x^2]*(a +
b*ArcCos[c*x]))/16 - (15*f*g^2*x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(1
28*c^2) + (15*f*g^2*x^3*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/64 + (5*f^3
*x*(1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x]))/24 + (5*f*g^2*x^3*(1 - c^2*x^2
)^(3/2)*(a + b*ArcCos[c*x]))/16 + (f^3*x*(1 - c^2*x^2)^(5/2)*(a + b*ArcCos
[c*x]))/6 + (3*f*g^2*x^3*(1 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x]))/8 - (3*f
^2*g*(1 - c^2*x^2)^(7/2)*(a + b*ArcCos[c*x]))/(7*c^2) - (g^3*(1 - c^2*x^2)
^(7/2)*(a + b*ArcCos[c*x]))/(7*c^4) + (g^3*(1 - c^2*x^2)^(9/2)*(a + b*ArcC
os[c*x]))/(9*c^4) - (5*f^3*(a + b*ArcCos[c*x])^2)/(32*b*c) - (15*f*g^2*(a
+ b*ArcCos[c*x])^2)/(256*b*c^3)))/Sqrt[1 - c^2*x^2]
```

3.10.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5263
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

rule 5277
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[Simp[(d + e*x^2)^p/(1 - c^2*x^2)^
p]   Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && Intege
rQ[p - 1/2] &&  !GtQ[d, 0]
```

3.10.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.74 (sec) , antiderivative size = 3019, normalized size of antiderivative = 2.36

method	result	size
default	Expression too large to display	3019
parts	Expression too large to display	3019

input
```
int((g*x+f)^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x,method=_RETURNVERBO
SE)
```

output
```
a*(f^3*(1/6*x*(-c^2*d*x^2+d)^(5/2)+5/6*d*(1/4*x*(-c^2*d*x^2+d)^(3/2)+3/4*d
*(1/2*x*(-c^2*d*x^2+d)^(1/2)+1/2*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-
c^2*d*x^2+d)^(1/2)))))+g^3*(-1/9*x^2*(-c^2*d*x^2+d)^(7/2)/c^2/d-2/63/d/c^4
*(-c^2*d*x^2+d)^(7/2))+3*f*g^2*(-1/8*x*(-c^2*d*x^2+d)^(7/2)/c^2/d+1/8/c^2*
(1/6*x*(-c^2*d*x^2+d)^(5/2)+5/6*d*(1/4*x*(-c^2*d*x^2+d)^(3/2)+3/4*d*(1/2*x
*(-c^2*d*x^2+d)^(1/2)+1/2*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x
^2+d)^(1/2))))))-3/7*f^2*g*(-c^2*d*x^2+d)^(7/2)/c^2/d)+b*(5/256*(-d*(c^2*x
^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c^3/(c^2*x^2-1)*arccos(c*x)^2*f*(8*c^2*f^2
+3*g^2)*d^2+1/41472*(-d*(c^2*x^2-1))^(1/2)*(256*I*(-c^2*x^2+1)^(1/2)*x^9*c
^9+256*c^10*x^10-576*I*(-c^2*x^2+1)^(1/2)*x^7*c^7-704*c^8*x^8+432*I*(-c^2*
x^2+1)^(1/2)*x^5*c^5+688*c^6*x^6-120*I*(-c^2*x^2+1)^(1/2)*x^3*c^3-280*c^4*
x^4+9*I*(-c^2*x^2+1)^(1/2)*x*c+41*c^2*x^2-1)*g^3*(I+9*arccos(c*x))*d^2/c^4
/(c^2*x^2-1)+3/16384*(-d*(c^2*x^2-1))^(1/2)*(128*I*(-c^2*x^2+1)^(1/2)*x^8*
c^8+128*c^9*x^9-256*I*(-c^2*x^2+1)^(1/2)*x^6*c^6-320*c^7*x^7+160*I*(-c^2*x
^2+1)^(1/2)*x^4*c^4+272*c^5*x^5-32*I*(-c^2*x^2+1)^(1/2)*x^2*c^2-88*c^3*x^3
+I*(-c^2*x^2+1)^(1/2)+8*c*x)*f*g^2*(8*arccos(c*x)+I)*d^2/c^3/(c^2*x^2-1)+3
/25088*(-d*(c^2*x^2-1))^(1/2)*(64*I*c^7*x^7*(-c^2*x^2+1)^(1/2)+64*c^8*x^8-
112*I*(-c^2*x^2+1)^(1/2)*x^5*c^5-144*c^6*x^6+56*I*(-c^2*x^2+1)^(1/2)*x^3*c
^3+104*c^4*x^4-7*I*(-c^2*x^2+1)^(1/2)*x*c-25*c^2*x^2+1)*g*(28*arccos(c*x)*
c^2*f^2+4*I*c^2*f^2-7*arccos(c*x)*g^2-I*g^2)*d^2/c^4/(c^2*x^2-1)+1/2304...
```

3.10.5 Fricas [F]

$$\int (f+gx)^3 \left(d-c^2dx^2\right)^{5/2} (a+b\arccos(cx))\,dx = \int \left(-c^2dx^2+d\right)^{\frac{5}{2}}(gx+f)^3(b\arccos(cx)+a)\,dx$$

input
```
integrate((g*x+f)^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x, algorithm="f
ricas")
```

output
```
integral((a*c^4*d^2*g^3*x^7 + 3*a*c^4*d^2*f*g^2*x^6 + 3*a*d^2*f^2*g*x + a*
d^2*f^3 + (3*a*c^4*d^2*f^2*g - 2*a*c^2*d^2*g^3)*x^5 + (a*c^4*d^2*f^3 - 6*a
*c^2*d^2*f*g^2)*x^4 - (6*a*c^2*d^2*f^2*g - a*d^2*g^3)*x^3 - (2*a*c^2*d^2*f
^3 - 3*a*d^2*f*g^2)*x^2 + (b*c^4*d^2*g^3*x^7 + 3*b*c^4*d^2*f*g^2*x^6 + 3*b
*d^2*f^2*g*x + b*d^2*f^3 + (3*b*c^4*d^2*f^2*g - 2*b*c^2*d^2*g^3)*x^5 + (b*
c^4*d^2*f^3 - 6*b*c^2*d^2*f*g^2)*x^4 - (6*b*c^2*d^2*f^2*g - b*d^2*g^3)*x^3
 - (2*b*c^2*d^2*f^3 - 3*b*d^2*f*g^2)*x^2)*arccos(c*x))*sqrt(-c^2*d*x^2 + d
), x)
```

3.10.6 Sympy [F(-1)]

Timed out.

$$\int (f+gx)^3 \left(d-c^2dx^2\right)^{5/2} (a+b\arccos(cx))\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)**3*(-c**2*d*x**2+d)**(5/2)*(a+b*acos(c*x)),x)
```

output
```
Timed out
```

3.10.7 Maxima [F]

$$\int (f+gx)^3 \left(d - c^2dx^2\right)^{5/2} (a+b\arccos(cx))\,dx = \int \left(-c^2dx^2+d\right)^{\frac{5}{2}}(gx+f)^3(b\arccos(cx)+a)\,dx$$

input
```
integrate((g*x+f)^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x, algorithm="m
axima")
```

output
```
1/48*(8*(-c^2*d*x^2 + d)^(5/2)*x + 10*(-c^2*d*x^2 + d)^(3/2)*d*x + 15*sqrt
(-c^2*d*x^2 + d)*d^2*x + 15*d^(5/2)*arcsin(c*x)/c)*a*f^3 + 1/128*(8*(-c^2*
d*x^2 + d)^(5/2)*x/c^2 - 48*(-c^2*d*x^2 + d)^(7/2)*x/(c^2*d) + 10*(-c^2*d*
x^2 + d)^(3/2)*d*x/c^2 + 15*sqrt(-c^2*d*x^2 + d)*d^2*x/c^2 + 15*d^(5/2)*ar
csin(c*x)/c^3)*a*f*g^2 - 1/63*(7*(-c^2*d*x^2 + d)^(7/2)*x^2/(c^2*d) + 2*(-
c^2*d*x^2 + d)^(7/2)/(c^4*d))*a*g^3 - 3/7*(-c^2*d*x^2 + d)^(7/2)*a*f^2*g/(
c^2*d) + sqrt(d)*integrate((b*c^4*d^2*g^3*x^7 + 3*b*c^4*d^2*f*g^2*x^6 + 3*
b*d^2*f^2*g*x + b*d^2*f^3 + (3*b*c^4*d^2*f^2*g - 2*b*c^2*d^2*g^3)*x^5 + (b
*c^4*d^2*f^3 - 6*b*c^2*d^2*f*g^2)*x^4 - (6*b*c^2*d^2*f^2*g - b*d^2*g^3)*x^
3 - (2*b*c^2*d^2*f^3 - 3*b*d^2*f*g^2)*x^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)*ar
ctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x), x)
```

3.10.8 Giac [F(-2)]

Exception generated.

$$\int (f+gx)^3 \left(d-c^2dx^2\right)^{5/2} (a+b\arccos(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((g*x+f)^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x, algorithm="g
iac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.10.9 Mupad [F(-1)]

Timed out.

$$\int (f+gx)^3 \left(d - c^2dx^2\right)^{5/2} (a+b\arccos(cx))\,dx = \int (f+g\,x)^3\,(a+b\,\mathrm{acos}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{5/2}\,dx$$

input
```
int((f + g*x)^3*(a + b*acos(c*x))*(d - c^2*d*x^2)^(5/2),x)
```

output
```
int((f + g*x)^3*(a + b*acos(c*x))*(d - c^2*d*x^2)^(5/2), x)
```

3.11 $\int (f+gx)^2 \left(d-c^2dx^2\right)^{5/2} (a+b\arccos(cx))\,dx$

3.11.1 Optimal result

Integrand size = 31, antiderivative size = 940

$$\int (f+gx)^2\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))\,dx=-\frac{2bd^2fgx\sqrt{d-c^2dx^2}}{7c\sqrt{1-c^2x^2}}$$
$$+\frac{25bcd^2f^2x^2\sqrt{d-c^2dx^2}}{96\sqrt{1-c^2x^2}}-\frac{5bd^2g^2x^2\sqrt{d-c^2dx^2}}{256c\sqrt{1-c^2x^2}}+\frac{2bcd^2fgx^3\sqrt{d-c^2dx^2}}{7\sqrt{1-c^2x^2}}$$
$$-\frac{5bc^3d^2f^2x^4\sqrt{d-c^2dx^2}}{96\sqrt{1-c^2x^2}}+\frac{59bcd^2g^2x^4\sqrt{d-c^2dx^2}}{768\sqrt{1-c^2x^2}}-\frac{6bc^3d^2fgx^5\sqrt{d-c^2dx^2}}{35\sqrt{1-c^2x^2}}$$
$$-\frac{17bc^3d^2g^2x^6\sqrt{d-c^2dx^2}}{288\sqrt{1-c^2x^2}}+\frac{2bc^5d^2fgx^7\sqrt{d-c^2dx^2}}{49\sqrt{1-c^2x^2}}+\frac{bc^5d^2g^2x^8\sqrt{d-c^2dx^2}}{64\sqrt{1-c^2x^2}}$$
$$-\frac{bd^2f^2(1-c^2x^2)^{5/2}\sqrt{d-c^2dx^2}}{36c}+\frac{5}{16}d^2f^2x\sqrt{d-c^2dx^2}(a+b\arccos(cx))$$
$$-\frac{5d^2g^2x\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{128c^2}$$
$$+\frac{5}{64}d^2g^2x^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))$$
$$+\frac{5}{24}d^2f^2x\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}(a+b\arccos(cx))$$
$$+\frac{5}{48}d^2g^2x^3\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}(a+b\arccos(cx))$$
$$+\frac{1}{6}d^2f^2x\left(1-c^2x^2\right)^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))$$
$$+\frac{1}{8}d^2g^2x^3\left(1-c^2x^2\right)^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))$$
$$-\frac{2d^2fg(1-c^2x^2)^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{7c^2}$$
$$-\frac{5d^2f^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{32bc\sqrt{1-c^2x^2}}$$
$$-\frac{5d^2g^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{256bc^3\sqrt{1-c^2x^2}}$$

output
```
-1/36*b*d^2*f^2*(-c^2*x^2+1)^(5/2)*(-c^2*d*x^2+d)^(1/2)/c+5/16*d^2*f^2*x*(
a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)-5/128*d^2*g^2*x*(a+b*arccos(c*x))*(-
c^2*d*x^2+d)^(1/2)/c^2+5/64*d^2*g^2*x^3*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(
1/2)+5/24*d^2*f^2*x*(-c^2*x^2+1)*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)+5/
48*d^2*g^2*x^3*(-c^2*x^2+1)*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)+1/6*d^2
*f^2*x*(-c^2*x^2+1)^2*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)+1/8*d^2*g^2*x
^3*(-c^2*x^2+1)^2*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)-2/7*d^2*f*g*(-c^2
*x^2+1)^3*(a+b*arccos(c*x))*(-c^2*d*x^2+d)^(1/2)/c^2-2/7*b*d^2*f*g*x*(-c^2
*d*x^2+d)^(1/2)/c/(-c^2*x^2+1)^(1/2)+25/96*b*c*d^2*f^2*x^2*(-c^2*d*x^2+d)^
(1/2)/(-c^2*x^2+1)^(1/2)-5/256*b*d^2*g^2*x^2*(-c^2*d*x^2+d)^(1/2)/c/(-c^2*
x^2+1)^(1/2)+2/7*b*c*d^2*f*g*x^3*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-5
/96*b*c^3*d^2*f^2*x^4*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)+59/768*b*c*d
^2*g^2*x^4*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-6/35*b*c^3*d^2*f*g*x^5*
(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-17/288*b*c^3*d^2*g^2*x^6*(-c^2*d*x
^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)+2/49*b*c^5*d^2*f*g*x^7*(-c^2*d*x^2+d)^(1/2)
/(-c^2*x^2+1)^(1/2)+1/64*b*c^5*d^2*g^2*x^8*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+
1)^(1/2)-5/32*d^2*f^2*(a+b*arccos(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/c/(-c^2*x
^2+1)^(1/2)-5/256*d^2*g^2*(a+b*arccos(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/c^3/(
-c^2*x^2+1)^(1/2)
```

3.11.2 Mathematica [A] (verified)

Time = 5.60 (sec) , antiderivative size = 794, normalized size of antiderivative = 0.84

$$\int (f+gx)^2\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))\,dx = \frac{d^2\Big(-352800b(8c^2f^2+g^2)\sqrt{d-c^2dx^2}\arccos(cx)^2-705600a\sqrt{d}(8c^2f^2+g^2)\sqrt{1-c^2x^2}}{}$$

input
```
Integrate[(f + g*x)^2*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]),x]
```

output
```
(d^2*(-352800*b*(8*c^2*f^2 + g^2)*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]^2 - 7056
00*a*Sqrt[d]*(8*c^2*f^2 + g^2)*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d - c^2*
d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + Sqrt[d - c^2*d*x^2]*(-2822400*b*c^2*f*
g*x - 5160960*a*c*f*g*Sqrt[1 - c^2*x^2] + 12418560*a*c^3*f^2*x*Sqrt[1 - c^
2*x^2] - 705600*a*c*g^2*x*Sqrt[1 - c^2*x^2] + 15482880*a*c^3*f*g*x^2*Sqrt[
1 - c^2*x^2] - 9784320*a*c^5*f^2*x^3*Sqrt[1 - c^2*x^2] + 5550720*a*c^3*g^2
*x^3*Sqrt[1 - c^2*x^2] - 15482880*a*c^5*f*g*x^4*Sqrt[1 - c^2*x^2] + 301056
0*a*c^7*f^2*x^5*Sqrt[1 - c^2*x^2] - 6397440*a*c^5*g^2*x^5*Sqrt[1 - c^2*x^2
] + 5160960*a*c^7*f*g*x^6*Sqrt[1 - c^2*x^2] + 2257920*a*c^7*g^2*x^7*Sqrt[1
 - c^2*x^2] + 141120*b*(15*c^2*f^2 + g^2)*Cos[2*ArcCos[c*x]] + 564480*b*c*
f*g*Cos[3*ArcCos[c*x]] - 211680*b*c^2*f^2*Cos[4*ArcCos[c*x]] + 35280*b*g^2
*Cos[4*ArcCos[c*x]] - 112896*b*c*f*g*Cos[5*ArcCos[c*x]] + 15680*b*c^2*f^2*
Cos[6*ArcCos[c*x]] - 15680*b*g^2*Cos[6*ArcCos[c*x]] + 11520*b*c*f*g*Cos[7*
ArcCos[c*x]] + 2205*b*g^2*Cos[8*ArcCos[c*x]]) + 168*b*Sqrt[d - c^2*d*x^2]*
ArcCos[c*x]*(-58112*c*f*g*Sqrt[1 - c^2*x^2] + 111872*c^3*f*g*x^2*Sqrt[1 -
c^2*x^2] - 27648*c*f*g*(1 - c^2*x^2)^(3/2)*Cos[2*ArcCos[c*x]] - 3840*c*f*g
*(1 - c^2*x^2)^(3/2)*Cos[4*ArcCos[c*x]] + 25200*c^2*f^2*Sin[2*ArcCos[c*x]]
 + 1680*g^2*Sin[2*ArcCos[c*x]] - 8960*c*f*g*Sin[3*ArcCos[c*x]] - 5040*c^2*
f^2*Sin[4*ArcCos[c*x]] + 840*g^2*Sin[4*ArcCos[c*x]] - 5376*c*f*g*Sin[5*Arc
Cos[c*x]] + 560*c^2*f^2*Sin[6*ArcCos[c*x]] - 560*g^2*Sin[6*ArcCos[c*x]]...
```

3.11.3 Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 481, normalized size of antiderivative = 0.51, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {5277, 5263, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d-c^2dx^2\right)^{5/2}(f+gx)^2(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{5277}$$

$$\frac{d^2\sqrt{d-c^2dx^2}\int (f+gx)^2\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))dx}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5263}$$

$$\frac{d^2\sqrt{d-c^2dx^2}\int \Big(f^2(a+b\arccos(cx))\left(1-c^2x^2\right)^{5/2}+g^2x^2(a+b\arccos(cx))\left(1-c^2x^2\right)^{5/2}+2fgx(a+b\arccos(c$$
$$\sqrt{1-c^2x^2}$$

$$\Big\downarrow \text{2009}$$

$$d^2\sqrt{d-c^2dx^2}\left(-\frac{5g^2(a+b\arccos(cx))^2}{256bc^3}+\frac{1}{6}f^2x\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))+\frac{5}{24}f^2x\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx\right.$$

input
```
Int[(f + g*x)^2*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]),x]
```

output
```
(d^2*Sqrt[d - c^2*d*x^2]*((-2*b*f*g*x)/(7*c) + (25*b*c*f^2*x^2)/96 - (5*b*
g^2*x^2)/(256*c) + (2*b*c*f*g*x^3)/7 - (5*b*c^3*f^2*x^4)/96 + (59*b*c*g^2*
x^4)/768 - (6*b*c^3*f*g*x^5)/35 - (17*b*c^3*g^2*x^6)/288 + (2*b*c^5*f*g*x^
7)/49 + (b*c^5*g^2*x^8)/64 - (b*f^2*(1 - c^2*x^2)^3)/(36*c) + (5*f^2*x*Sqr
t[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/16 - (5*g^2*x*Sqrt[1 - c^2*x^2]*(a + b
*ArcCos[c*x]))/(128*c^2) + (5*g^2*x^3*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]
))/64 + (5*f^2*x*(1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x]))/24 + (5*g^2*x^3*
(1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x]))/48 + (f^2*x*(1 - c^2*x^2)^(5/2)*(
a + b*ArcCos[c*x]))/6 + (g^2*x^3*(1 - c^2*x^2)^(5/2)*(a + b*ArcCos[c*x]))/
8 - (2*f*g*(1 - c^2*x^2)^(7/2)*(a + b*ArcCos[c*x]))/(7*c^2) - (5*f^2*(a +
b*ArcCos[c*x])^2)/(32*b*c) - (5*g^2*(a + b*ArcCos[c*x])^2)/(256*b*c^3)))/S
qrt[1 - c^2*x^2]
```

3.11.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5263
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

rule 5277
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[Simp[(d + e*x^2)^p/(1 - c^2*x^2)^
p]   Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && Intege
rQ[p - 1/2] &&  !GtQ[d, 0]
```

3.11.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.46 (sec) , antiderivative size = 2204, normalized size of antiderivative = 2.34

method	result	size
default	Expression too large to display	2204
parts	Expression too large to display	2204

input
```
int((g*x+f)^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x,method=_RETURNVERBO
SE)
```

output
```
a*(f^2*(1/6*x*(-c^2*d*x^2+d)^(5/2)+5/6*d*(1/4*x*(-c^2*d*x^2+d)^(3/2)+3/4*d
*(1/2*x*(-c^2*d*x^2+d)^(1/2)+1/2*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-
c^2*d*x^2+d)^(1/2)))))+g^2*(-1/8*x*(-c^2*d*x^2+d)^(7/2)/c^2/d+1/8/c^2*(1/6
*x*(-c^2*d*x^2+d)^(5/2)+5/6*d*(1/4*x*(-c^2*d*x^2+d)^(3/2)+3/4*d*(1/2*x*(-c
^2*d*x^2+d)^(1/2)+1/2*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d
)^(1/2))))))-2/7*f*g*(-c^2*d*x^2+d)^(7/2)/c^2/d)+b*(-3/1024*(-d*(c^2*x^2-1
))^(1/2)*(c*x*(-c^2*x^2+1)^(1/2)+I*c^2*x^2-I)*(22*I*c^2*f^2+32*arccos(c*x)
*c^2*f^2+I*g^2+4*arccos(c*x)*g^2)*sin(3*arccos(c*x))*d^2/c^3/(c^2*x^2-1)+1
/16384*(-d*(c^2*x^2-1))^(1/2)*(128*I*(-c^2*x^2+1)^(1/2)*x^8*c^8+128*c^9*x^
9-256*I*(-c^2*x^2+1)^(1/2)*x^6*c^6-320*c^7*x^7+160*I*(-c^2*x^2+1)^(1/2)*x^
4*c^4+272*c^5*x^5-32*I*(-c^2*x^2+1)^(1/2)*x^2*c^2-88*c^3*x^3+I*(-c^2*x^2+1
)^(1/2)+8*c*x)*g^2*(8*arccos(c*x)+I)*d^2/c^3/(c^2*x^2-1)+1/3136*(-d*(c^2*x
^2-1))^(1/2)*(64*I*c^7*x^7*(-c^2*x^2+1)^(1/2)+64*c^8*x^8-112*I*(-c^2*x^2+1
)^(1/2)*x^5*c^5-144*c^6*x^6+56*I*(-c^2*x^2+1)^(1/2)*x^3*c^3+104*c^4*x^4-7*
I*(-c^2*x^2+1)^(1/2)*x*c-25*c^2*x^2+1)*f*g*(I+7*arccos(c*x))*d^2/c^2/(c^2*
x^2-1)+1/2304*(-d*(c^2*x^2-1))^(1/2)*(32*I*(-c^2*x^2+1)^(1/2)*c^6*x^6+32*c
^7*x^7-48*I*(-c^2*x^2+1)^(1/2)*x^4*c^4-64*c^5*x^5+18*I*(-c^2*x^2+1)^(1/2)*
x^2*c^2+38*c^3*x^3-I*(-c^2*x^2+1)^(1/2)-6*c*x)*(6*arccos(c*x)*c^2*f^2+I*c^
2*f^2-6*arccos(c*x)*g^2-I*g^2)*d^2/c^3/(c^2*x^2-1)-1/1024*(-d*(c^2*x^2-1))
^(1/2)*(8*I*(-c^2*x^2+1)^(1/2)*c^4*x^4+8*c^5*x^5-8*I*(-c^2*x^2+1)^(1/2)...
```

3.11.5 Fricas [F]

$$\int (f+gx)^2 \left(d-c^2dx^2\right)^{5/2} (a+b\arccos(cx))\,dx = \int \left(-c^2dx^2+d\right)^{\frac{5}{2}}(gx+f)^2(b\arccos(cx)+a)\,dx$$

input
```
integrate((g*x+f)^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x, algorithm="f
ricas")
```

output
```
integral((a*c^4*d^2*g^2*x^6 + 2*a*c^4*d^2*f*g*x^5 - 4*a*c^2*d^2*f*g*x^3 +
2*a*d^2*f*g*x + a*d^2*f^2 + (a*c^4*d^2*f^2 - 2*a*c^2*d^2*g^2)*x^4 - (2*a*c
^2*d^2*f^2 - a*d^2*g^2)*x^2 + (b*c^4*d^2*g^2*x^6 + 2*b*c^4*d^2*f*g*x^5 - 4
*b*c^2*d^2*f*g*x^3 + 2*b*d^2*f*g*x + b*d^2*f^2 + (b*c^4*d^2*f^2 - 2*b*c^2*
d^2*g^2)*x^4 - (2*b*c^2*d^2*f^2 - b*d^2*g^2)*x^2)*arccos(c*x))*sqrt(-c^2*d
*x^2 + d), x)
```

3.11.6 Sympy [F(-1)]

Timed out.

$$\int (f+gx)^2 \left(d-c^2dx^2\right)^{5/2} (a+b\arccos(cx))\,dx = \text{Timed out}$$

input
```
integrate((g*x+f)**2*(-c**2*d*x**2+d)**(5/2)*(a+b*acos(c*x)),x)
```

output
```
Timed out
```

3.11.7 Maxima [F]

$$\int (f+gx)^2 \left(d-c^2dx^2\right)^{5/2} (a+b\arccos(cx))\,dx = \int \left(-c^2dx^2+d\right)^{\frac{5}{2}}(gx+f)^2(b\arccos(cx)+a)\,dx$$

input
```
integrate((g*x+f)^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x, algorithm="m
axima")
```

output
```
1/48*(8*(-c^2*d*x^2 + d)^(5/2)*x + 10*(-c^2*d*x^2 + d)^(3/2)*d*x + 15*sqrt
(-c^2*d*x^2 + d)*d^2*x + 15*d^(5/2)*arcsin(c*x)/c)*a*f^2 + 1/384*(8*(-c^2*
d*x^2 + d)^(5/2)*x/c^2 - 48*(-c^2*d*x^2 + d)^(7/2)*x/(c^2*d) + 10*(-c^2*d*
x^2 + d)^(3/2)*d*x/c^2 + 15*sqrt(-c^2*d*x^2 + d)*d^2*x/c^2 + 15*d^(5/2)*ar
csin(c*x)/c^3)*a*g^2 - 2/7*(-c^2*d*x^2 + d)^(7/2)*a*f*g/(c^2*d) + sqrt(d)*
integrate((b*c^4*d^2*g^2*x^6 + 2*b*c^4*d^2*f*g*x^5 - 4*b*c^2*d^2*f*g*x^3 +
 2*b*d^2*f*g*x + b*d^2*f^2 + (b*c^4*d^2*f^2 - 2*b*c^2*d^2*g^2)*x^4 - (2*b*
c^2*d^2*f^2 - b*d^2*g^2)*x^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(sqrt(c*
x + 1)*sqrt(-c*x + 1), c*x), x)
```

3.11.8 Giac [F(-2)]

Exception generated.

$$\int (f+gx)^2 \left(d-c^2dx^2\right)^{5/2} (a+b\arccos(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((g*x+f)^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x, algorithm="g
iac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.11.9 Mupad [F(-1)]

Timed out.

$$\int (f+gx)^2 \left(d-c^2dx^2\right)^{5/2} (a+b\arccos(cx))\,dx = \int (f+g\,x)^2\,(a+b\,\mathrm{acos}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{5/2}\,dx$$

input
```
int((f + g*x)^2*(a + b*acos(c*x))*(d - c^2*d*x^2)^(5/2),x)
```

output
```
int((f + g*x)^2*(a + b*acos(c*x))*(d - c^2*d*x^2)^(5/2), x)
```

3.12 $\int (f+gx) \left(d-c^2dx^2\right)^{5/2} (a+b\arccos(cx)) \, dx$

3.12.1 Optimal result

Integrand size = 29, antiderivative size = 517

$$\begin{aligned}
&\int (f+gx) \left(d-c^2dx^2\right)^{5/2} (a+b\arccos(cx)) \, dx = \\
&-\frac{bd^2gx\sqrt{d-c^2dx^2}}{7c\sqrt{1-c^2x^2}}+\frac{25bcd^2fx^2\sqrt{d-c^2dx^2}}{96\sqrt{1-c^2x^2}}+\frac{bcd^2gx^3\sqrt{d-c^2dx^2}}{7\sqrt{1-c^2x^2}} \\
&-\frac{5bc^3d^2fx^4\sqrt{d-c^2dx^2}}{96\sqrt{1-c^2x^2}}-\frac{3bc^3d^2gx^5\sqrt{d-c^2dx^2}}{35\sqrt{1-c^2x^2}}+\frac{bc^5d^2gx^7\sqrt{d-c^2dx^2}}{49\sqrt{1-c^2x^2}} \\
&-\frac{bd^2f(1-c^2x^2)^{5/2}\sqrt{d-c^2dx^2}}{36c}+\frac{5}{16}d^2fx\sqrt{d-c^2dx^2}(a+b\arccos(cx)) \\
&+\frac{5}{24}d^2fx\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}(a+b\arccos(cx)) \\
&+\frac{1}{6}d^2fx\left(1-c^2x^2\right)^2\sqrt{d-c^2dx^2}(a+b\arccos(cx)) \\
&-\frac{d^2g(1-c^2x^2)^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{7c^2} \\
&-\frac{5d^2f\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{32bc\sqrt{1-c^2x^2}}
\end{aligned}$$

output
```
-1/36*b*d^2*f*(-c^2*x^2+1)^(5/2)*(-c^2*d*x^2+d)^(1/2)/c+5/16*d^2*f*x*(a+b*
arccos(c*x))*(-c^2*d*x^2+d)^(1/2)+5/24*d^2*f*x*(-c^2*x^2+1)*(a+b*arccos(c*
x))*(-c^2*d*x^2+d)^(1/2)+1/6*d^2*f*x*(-c^2*x^2+1)^2*(a+b*arccos(c*x))*(-c^
2*d*x^2+d)^(1/2)-1/7*d^2*g*(-c^2*x^2+1)^3*(a+b*arccos(c*x))*(-c^2*d*x^2+d)
^(1/2)/c^2-1/7*b*d^2*g*x*(-c^2*d*x^2+d)^(1/2)/c/(-c^2*x^2+1)^(1/2)+25/96*b
*c*d^2*f*x^2*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)+1/7*b*c*d^2*g*x^3*(-c
^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-5/96*b*c^3*d^2*f*x^4*(-c^2*d*x^2+d)^(
1/2)/(-c^2*x^2+1)^(1/2)-3/35*b*c^3*d^2*g*x^5*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^
2+1)^(1/2)+1/49*b*c^5*d^2*g*x^7*(-c^2*d*x^2+d)^(1/2)/(-c^2*x^2+1)^(1/2)-5/
32*d^2*f*(a+b*arccos(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/c/(-c^2*x^2+1)^(1/2)
```

3.12.2 Mathematica [A] (verified)

Time = 3.47 (sec) , antiderivative size = 526, normalized size of antiderivative = 1.02

$$\int (f+gx)\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))\,dx = \frac{d^2\left(-88200bcf\sqrt{d-c^2dx^2}\arccos(cx)^2-176400ac\sqrt{d}f\sqrt{1-c^2x^2}\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right)\right.}{}$$

input
```
Integrate[(f + g*x)*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]),x]
```

output
```
(d^2*(-88200*b*c*f*Sqrt[d - c^2*d*x^2]*ArcCos[c*x]^2 - 176400*a*c*Sqrt[d]*
f*Sqrt[1 - c^2*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^
2))] + Sqrt[d - c^2*d*x^2]*(-44100*b*c*g*x - 80640*a*g*Sqrt[1 - c^2*x^2] +
 388080*a*c^2*f*x*Sqrt[1 - c^2*x^2] + 241920*a*c^2*g*x^2*Sqrt[1 - c^2*x^2]
 - 305760*a*c^4*f*x^3*Sqrt[1 - c^2*x^2] - 241920*a*c^4*g*x^4*Sqrt[1 - c^2*
x^2] + 94080*a*c^6*f*x^5*Sqrt[1 - c^2*x^2] + 80640*a*c^6*g*x^6*Sqrt[1 - c^
2*x^2] + 66150*b*c*f*Cos[2*ArcCos[c*x]] + 8820*b*g*Cos[3*ArcCos[c*x]] - 66
15*b*c*f*Cos[4*ArcCos[c*x]] - 1764*b*g*Cos[5*ArcCos[c*x]] + 490*b*c*f*Cos[
6*ArcCos[c*x]] + 180*b*g*Cos[7*ArcCos[c*x]]) + 84*b*Sqrt[d - c^2*d*x^2]*Ar
cCos[c*x]*(-1816*g*Sqrt[1 - c^2*x^2] + 3496*c^2*g*x^2*Sqrt[1 - c^2*x^2] -
864*g*(1 - c^2*x^2)^(3/2)*Cos[2*ArcCos[c*x]] - 120*g*(1 - c^2*x^2)^(3/2)*C
os[4*ArcCos[c*x]] + 1575*c*f*Sin[2*ArcCos[c*x]] - 280*g*Sin[3*ArcCos[c*x]]
 - 315*c*f*Sin[4*ArcCos[c*x]] - 168*g*Sin[5*ArcCos[c*x]] + 35*c*f*Sin[6*Ar
cCos[c*x]])))/(564480*c^2*Sqrt[1 - c^2*x^2])
```

3.12.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.50, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {5277, 5263, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d-c^2dx^2\right)^{5/2}(f+gx)(a+b\arccos(cx))\,dx$$

$$\Big\downarrow \text{5277}$$

$$\frac{d^2\sqrt{d-c^2dx^2}\int (f+gx)\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))dx}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{5263}$$

$$\frac{d^2\sqrt{d-c^2dx^2}\int \left(f(a+b\arccos(cx))\left(1-c^2x^2\right)^{5/2}+gx(a+b\arccos(cx))\left(1-c^2x^2\right)^{5/2}\right)dx}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^2\sqrt{d-c^2dx^2}\left(\frac{1}{6}fx\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))+\frac{5}{24}fx\left(1-c^2x^2\right)^{3/2}(a+b\arccos(cx))+\frac{5}{16}fx\sqrt{1-c^2x^2}(a+\right.}{}$$

input
```
Int[(f + g*x)*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]),x]
```

output
```
(d^2*Sqrt[d - c^2*d*x^2]*(-1/7*(b*g*x)/c + (25*b*c*f*x^2)/96 + (b*c*g*x^3)
/7 - (5*b*c^3*f*x^4)/96 - (3*b*c^3*g*x^5)/35 + (b*c^5*g*x^7)/49 - (b*f*(1
- c^2*x^2)^3)/(36*c) + (5*f*x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/16 +
(5*f*x*(1 - c^2*x^2)^(3/2)*(a + b*ArcCos[c*x]))/24 + (f*x*(1 - c^2*x^2)^(5
/2)*(a + b*ArcCos[c*x]))/6 - (g*(1 - c^2*x^2)^(7/2)*(a + b*ArcCos[c*x]))/(
7*c^2) - (5*f*(a + b*ArcCos[c*x])^2)/(32*b*c)))/Sqrt[1 - c^2*x^2]
```

3.12.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5263
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

rule 5277
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[Simp[(d + e*x^2)^p/(1 - c^2*x^2)^
p]   Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && Intege
rQ[p - 1/2] &&  !GtQ[d, 0]
```

3.12.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.42 (sec) , antiderivative size = 1419, normalized size of antiderivative = 2.74

method	result	size
default	Expression too large to display	1419
parts	Expression too large to display	1419

input
```
int((g*x+f)*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE
)
```

output
```
1/6*a*f*x*(-c^2*d*x^2+d)^(5/2)+5/24*a*f*d*x*(-c^2*d*x^2+d)^(3/2)+5/16*a*f*
d^2*x*(-c^2*d*x^2+d)^(1/2)+5/16*a*f*d^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)
*x/(-c^2*d*x^2+d)^(1/2))-1/7*a*g*(-c^2*d*x^2+d)^(7/2)/c^2/d+b*(5/32*(-d*(c
^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c/(c^2*x^2-1)*arccos(c*x)^2*f*d^2+1/62
72*(-d*(c^2*x^2-1))^(1/2)*(64*I*c^7*x^7*(-c^2*x^2+1)^(1/2)+64*c^8*x^8-112*
I*(-c^2*x^2+1)^(1/2)*x^5*c^5-144*c^6*x^6+56*I*(-c^2*x^2+1)^(1/2)*x^3*c^3+1
04*c^4*x^4-7*I*(-c^2*x^2+1)^(1/2)*x*c-25*c^2*x^2+1)*g*(I+7*arccos(c*x))*d^
2/c^2/(c^2*x^2-1)+1/2304*(-d*(c^2*x^2-1))^(1/2)*(32*I*(-c^2*x^2+1)^(1/2)*c
^6*x^6+32*c^7*x^7-48*I*(-c^2*x^2+1)^(1/2)*x^4*c^4-64*c^5*x^5+18*I*(-c^2*x^
2+1)^(1/2)*x^2*c^2+38*c^3*x^3-I*(-c^2*x^2+1)^(1/2)-6*c*x)*f*(I+6*arccos(c*
x))*d^2/c/(c^2*x^2-1)-5/128*(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*x
*c+c^2*x^2-1)*g*(arccos(c*x)+I)*d^2/c^2/(c^2*x^2-1)-5/128*(-d*(c^2*x^2-1))
^(1/2)*(c^2*x^2-I*(-c^2*x^2+1)^(1/2)*x*c-1)*g*(arccos(c*x)-I)*d^2/c^2/(c^2
*x^2-1)+15/256*(-d*(c^2*x^2-1))^(1/2)*(-2*I*(-c^2*x^2+1)^(1/2)*x^2*c^2+2*c
^3*x^3+I*(-c^2*x^2+1)^(1/2)-2*c*x)*f*(-I+2*arccos(c*x))*d^2/c/(c^2*x^2-1)+
1/128*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2-4*I*c^3*x^3*(-c^2*x^2+1)
^(1/2)+3*I*(-c^2*x^2+1)^(1/2)*x*c+1)*g*(-I+3*arccos(c*x))*d^2/c^2/(c^2*x^2
-1)+1/7840*(-d*(c^2*x^2-1))^(1/2)*(c^2*x^2-I*(-c^2*x^2+1)^(1/2)*x*c-1)*g*(
11*I+70*arccos(c*x))*cos(6*arccos(c*x))*d^2/c^2/(c^2*x^2-1)+3/15680*(-d*(c
^2*x^2-1))^(1/2)*(c*x*(-c^2*x^2+1)^(1/2)+I*c^2*x^2-I)*g*(9*I+35*arccos(...
```

3.12.5 Fricas [F]

$$\int (f+gx)\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))\,dx=\int \left(-c^2dx^2+d\right)^{\frac{5}{2}}(gx+f)(b\arccos{(cx)}+a)\,dx$$

input
```
integrate((g*x+f)*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x, algorithm="fri
cas")
```

output
```
integral((a*c^4*d^2*g*x^5 + a*c^4*d^2*f*x^4 - 2*a*c^2*d^2*g*x^3 - 2*a*c^2*
d^2*f*x^2 + a*d^2*g*x + a*d^2*f + (b*c^4*d^2*g*x^5 + b*c^4*d^2*f*x^4 - 2*b
*c^2*d^2*g*x^3 - 2*b*c^2*d^2*f*x^2 + b*d^2*g*x + b*d^2*f)*arccos(c*x))*sqr
t(-c^2*d*x^2 + d), x)
```

3.12.6 Sympy [F(-1)]

Timed out.

$$\int (f+gx)\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))\,dx=\text{Timed out}$$

input
```
integrate((g*x+f)*(-c**2*d*x**2+d)**(5/2)*(a+b*acos(c*x)),x)
```

output
```
Timed out
```

3.12.7 Maxima [F]

$$\int (f+gx)\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))\,dx=\int \left(-c^2dx^2+d\right)^{\frac{5}{2}}(gx+f)(b\arccos(cx)+a)\,dx$$

input
```
integrate((g*x+f)*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x, algorithm="max
ima")
```

output
```
1/48*(8*(-c^2*d*x^2 + d)^(5/2)*x + 10*(-c^2*d*x^2 + d)^(3/2)*d*x + 15*sqrt
(-c^2*d*x^2 + d)*d^2*x + 15*d^(5/2)*arcsin(c*x)/c)*a*f - 1/7*(-c^2*d*x^2 +
 d)^(7/2)*a*g/(c^2*d) + sqrt(d)*integrate((b*c^4*d^2*g*x^5 + b*c^4*d^2*f*x
^4 - 2*b*c^2*d^2*g*x^3 - 2*b*c^2*d^2*f*x^2 + b*d^2*g*x + b*d^2*f)*sqrt(c*x
 + 1)*sqrt(-c*x + 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x), x)
```

3.12.8 Giac [F(-2)]

Exception generated.

$$\int (f+gx)\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((g*x+f)*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x)),x, algorithm="gia
c")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const ve
cteur & l) Error: Bad Argument Value
```

3.12.9 Mupad [F(-1)]

Timed out.

$$\int (f+gx)\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))\,dx=\int (f+g\,x)\,\left(a+b\,\mathrm{acos}(c\,x)\right)\,\left(d-c^2\,d\,x^2\right)^{5/2}\,dx$$

input
```
int((f + g*x)*(a + b*acos(c*x))*(d - c^2*d*x^2)^(5/2),x)
```

output
```
int((f + g*x)*(a + b*acos(c*x))*(d - c^2*d*x^2)^(5/2), x)
```

3.13 $\int \frac{\left(d-c^2 d x^2\right)^{5/2} (a+b \arccos (c x))}{f+g x} \, dx$

3.13.1 Optimal result

Integrand size = 31, antiderivative size = 1637

$$
\begin{aligned}
\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{f+gx}\,dx &= \frac{ad^2(c^2f^2-g^2)^2\sqrt{d-c^2dx^2}}{g^5} \\
&- \frac{2bcd^2x\sqrt{d-c^2dx^2}}{15g\sqrt{1-c^2x^2}} - \frac{bcd^2(c^2f^2-2g^2)\,x\sqrt{d-c^2dx^2}}{3g^3\sqrt{1-c^2x^2}} \\
&+ \frac{bcd^2(c^2f^2-g^2)^2\,x\sqrt{d-c^2dx^2}}{g^5\sqrt{1-c^2x^2}} + \frac{bc^3d^2fx^2\sqrt{d-c^2dx^2}}{16g^2\sqrt{1-c^2x^2}} \\
&- \frac{bc^3d^2f(c^2f^2-2g^2)\,x^2\sqrt{d-c^2dx^2}}{4g^4\sqrt{1-c^2x^2}} - \frac{bc^3d^2x^3\sqrt{d-c^2dx^2}}{45g\sqrt{1-c^2x^2}} \\
&+ \frac{bc^3d^2(c^2f^2-2g^2)\,x^3\sqrt{d-c^2dx^2}}{9g^3\sqrt{1-c^2x^2}} - \frac{bc^5d^2fx^4\sqrt{d-c^2dx^2}}{16g^2\sqrt{1-c^2x^2}} \\
&+ \frac{bc^5d^2x^5\sqrt{d-c^2dx^2}}{25g\sqrt{1-c^2x^2}} + \frac{bd^2(c^2f^2-g^2)^2\sqrt{d-c^2dx^2}\arccos(cx)}{g^5} \\
&+ \frac{c^2d^2fx\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{8g^2} \\
&- \frac{c^2d^2f(c^2f^2-2g^2)\,x\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{2g^4} \\
&- \frac{c^4d^2fx^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{4g^2} \\
&- \frac{d^2(1-c^2x^2)\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{3g} \\
&- \frac{d^2(c^2f^2-2g^2)(1-c^2x^2)\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{3g^3} \\
&+ \frac{d^2(1-c^2x^2)^2\sqrt{d-c^2dx^2}(a+b\arccos(cx))}{5g} \\
&+ \frac{cd^2f\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{16bg^2\sqrt{1-c^2x^2}} \\
&+ \frac{cd^2f(c^2f^2-2g^2)\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{4bg^4\sqrt{1-c^2x^2}} \\
&- \frac{cd^2(c^2f^2-g^2)^2\,x\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2bg^5\sqrt{1-c^2x^2}} \\
&- \frac{d^2(c^2f^2-g^2)^3\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2bcg^6(f+gx)\sqrt{1-c^2x^2}} \\
&- \frac{d^2(c^2f^2-g^2)^2\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}(a+b\arccos(cx))^2}{2bcg^4(f+gx)} \\
&- \frac{ad^2(c^2f^2-g^2)^{5/2}\sqrt{d-c^2dx^2}\arctan\left(\frac{g+c^2fx}{\sqrt{c^2f^2-g^2}\sqrt{1-c^2x^2}}\right)}{g^6\sqrt{1-c^2x^2}} \\
&- \frac{ibd^2(c^2f^2-g^2)^{5/2}\sqrt{d-c^2dx^2}\arccos(cx)\log\left(1+\frac{e^{i\arccos(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{g^6\sqrt{1-c^2x^2}}
\end{aligned}
$$

output
```
1/25*b*c^5*d^2*x^5*(-c^2*d*x^2+d)^(1/2)/g/(-c^2*x^2+1)^(1/2)-2/15*b*c*d^2*
x*(-c^2*d*x^2+d)^(1/2)/g/(-c^2*x^2+1)^(1/2)-1/45*b*c^3*d^2*x^3*(-c^2*d*x^2
+d)^(1/2)/g/(-c^2*x^2+1)^(1/2)+a*d^2*(c^2*f^2-g^2)^2*(-c^2*d*x^2+d)^(1/2)/
g^5+b*c*d^2*(c^2*f^2-g^2)^2*x*(-c^2*d*x^2+d)^(1/2)/g^5/(-c^2*x^2+1)^(1/2)-
a*d^2*(c^2*f^2-g^2)^(5/2)*arctan((c^2*f*x+g)/(c^2*f^2-g^2)^(1/2)/(-c^2*x^2
+1)^(1/2))*(-c^2*d*x^2+d)^(1/2)/g^6/(-c^2*x^2+1)^(1/2)-b*d^2*(c^2*f^2-g^2)
^(5/2)*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2-g^2)^(1/2)))*
(-c^2*d*x^2+d)^(1/2)/g^6/(-c^2*x^2+1)^(1/2)+b*d^2*(c^2*f^2-g^2)^(5/2)*poly
log(2,-(c*x+I*(-c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2)))*(-c^2*d*x^2
+d)^(1/2)/g^6/(-c^2*x^2+1)^(1/2)+I*b*d^2*(c^2*f^2-g^2)^(5/2)*arccos(c*x)*l
n(1+(c*x+I*(-c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2)))*(-c^2*d*x^2+d)
^(1/2)/g^6/(-c^2*x^2+1)^(1/2)+1/4*c*d^2*f*(c^2*f^2-2*g^2)*(a+b*arccos(c*x)
)^2*(-c^2*d*x^2+d)^(1/2)/b/g^4/(-c^2*x^2+1)^(1/2)-1/2*c*d^2*(c^2*f^2-g^2)^
2*x*(a+b*arccos(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/g^5/(-c^2*x^2+1)^(1/2)-1/2*
d^2*(c^2*f^2-g^2)^3*(a+b*arccos(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/c/g^6/(g*x+
f)/(-c^2*x^2+1)^(1/2)-1/2*d^2*(c^2*f^2-g^2)^2*(a+b*arccos(c*x))^2*(-c^2*x^
2+1)^(1/2)*(-c^2*d*x^2+d)^(1/2)/b/c/g^4/(g*x+f)-I*b*d^2*(c^2*f^2-g^2)^(5/2
)*arccos(c*x)*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2-g^2)^(1/2)))
*(-c^2*d*x^2+d)^(1/2)/g^6/(-c^2*x^2+1)^(1/2)+1/8*c^2*d^2*f*x*(a+b*arccos(c
*x))*(-c^2*d*x^2+d)^(1/2)/g^2-1/4*c^4*d^2*f*x^3*(a+b*arccos(c*x))*(-c^2...
```

3.13.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 6216 vs. $2(1637) = 3274$.

Time = 23.59 (sec) , antiderivative size = 6216, normalized size of antiderivative = 3.80

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{f+gx}\,dx = \text{Result too large to show}$$

input
```
Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]))/(f + g*x),x]
```

output
```
Result too large to show
```

3.13.3 Rubi [A] (verified)

Time = 2.86 (sec) , antiderivative size = 1008, normalized size of antiderivative = 0.62, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {5277, 5267, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}(a+b\arccos(cx))}{f+gx}dx$$

$$\Big\downarrow \; 5277$$

$$\frac{d^2\sqrt{d-c^2dx^2}\int \frac{\left(1-c^2x^2\right)^{5/2}(a+b\arccos(cx))}{f+gx}dx}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \; 5267$$

$$\frac{d^2\sqrt{d-c^2dx^2}\int \left(\frac{x^3\sqrt{1-c^2x^2}(a+b\arccos(cx))c^4}{g}-\frac{fx^2\sqrt{1-c^2x^2}(a+b\arccos(cx))c^4}{g^2}-\frac{f\left(c^2f^2-2g^2\right)\sqrt{1-c^2x^2}(a+b\arccos(cx))c^2}{g^4}+\frac{(c^2 \ldots}{} \right.}{\sqrt{1-c^2x^2}}$$

$$\Big\downarrow \; 2009$$

$$\frac{d^2\sqrt{d-c^2dx^2}\left(\frac{bx^5c^5}{25g}-\frac{bfx^4c^5}{16g^2}-\frac{fx^3\sqrt{1-c^2x^2}(a+b\arccos(cx))c^4}{4g^2}+\frac{b\left(c^2f^2-2g^2\right)x^3c^3}{9g^3}-\frac{bx^3c^3}{45g}-\frac{bf\left(c^2f^2-2g^2\right)x^2c^3}{4g^4}+\frac{bfx^2c^3}{16g^2}- \ldots\right.}{}$$

input
```
Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCos[c*x]))/(f + g*x),x]
```

output
```
(d^2*Sqrt[d - c^2*d*x^2]*((-2*b*c*x)/(15*g) - (b*c*(c^2*f^2 - 2*g^2)*x)/(3
*g^3) + (b*c*(c^2*f^2 - g^2)^2*x)/g^5 + (b*c^3*f*x^2)/(16*g^2) - (b*c^3*f*
(c^2*f^2 - 2*g^2)*x^2)/(4*g^4) - (b*c^3*x^3)/(45*g) + (b*c^3*(c^2*f^2 - 2*
g^2)*x^3)/(9*g^3) - (b*c^5*f*x^4)/(16*g^2) + (b*c^5*x^5)/(25*g) + (a*(c^2*
f^2 - g^2)^2*Sqrt[1 - c^2*x^2])/g^5 + (b*(c^2*f^2 - g^2)^2*Sqrt[1 - c^2*x^
2]*ArcCos[c*x])/g^5 + (c^2*f*x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(8*g
^2) - (c^2*f*(c^2*f^2 - 2*g^2)*x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(2
*g^4) - (c^4*f*x^3*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(4*g^2) - ((1 -
c^2*x^2)^(3/2)*(a + b*ArcCos[c*x]))/(3*g) - ((c^2*f^2 - 2*g^2)*(1 - c^2*x^
2)^(3/2)*(a + b*ArcCos[c*x]))/(3*g^3) + ((1 - c^2*x^2)^(5/2)*(a + b*ArcCos
[c*x]))/(5*g) + (c*f*(a + b*ArcCos[c*x])^2)/(16*b*g^2) + (c*f*(c^2*f^2 - 2
*g^2)*(a + b*ArcCos[c*x])^2)/(4*b*g^4) - (c*(c^2*f^2 - g^2)^2*x*(a + b*Arc
Cos[c*x])^2)/(2*b*g^5) - ((c^2*f^2 - g^2)^3*(a + b*ArcCos[c*x])^2)/(2*b*c*
g^6*(f + g*x)) - ((c^2*f^2 - g^2)^2*(1 - c^2*x^2)*(a + b*ArcCos[c*x])^2)/(
2*b*c*g^4*(f + g*x)) - (a*(c^2*f^2 - g^2)^(5/2)*ArcTan[(g + c^2*f*x)/(Sqrt
[c^2*f^2 - g^2]*Sqrt[1 - c^2*x^2])])/g^6 - (I*b*(c^2*f^2 - g^2)^(5/2)*ArcC
os[c*x]*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2])])/g^6 +
(I*b*(c^2*f^2 - g^2)^(5/2)*ArcCos[c*x]*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f
+ Sqrt[c^2*f^2 - g^2])])/g^6 - (b*(c^2*f^2 - g^2)^(5/2)*PolyLog[2, -((E^(I
*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2]))])/g^6 + (b*(c^2*f^2 - g^2...
```

3.13.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5267
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[Sqrt[d + e*x^2]*(a
 + b*ArcCos[c*x])^n, (f + g*x)^m*(d + e*x^2)^(p - 1/2), x], x] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IGtQ[p + 1/2,
 0] && GtQ[d, 0] && IGtQ[n, 0]
```

rule 5277
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[Simp[(d + e*x^2)^p/(1 - c^2*x^2)^
p]   Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && Intege
rQ[p - 1/2] &&  !GtQ[d, 0]
```

3.13. $\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{f+gx}\,dx$

3.13.4 Maple [A] (verified)

Time = 3.12 (sec) , antiderivative size = 2665, normalized size of antiderivative = 1.63

method	result	size
default	Expression too large to display	2665
parts	Expression too large to display	2665

input
```
int((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))/(g*x+f),x,method=_RETURNVERBOSE
)
```

output
```
a/g*(1/5*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(5/2)+
c^2*d*f/g*(-1/8*(-2*(x+f/g)*c^2*d+2*c^2*d*f/g)/c^2/d*(-(x+f/g)^2*c^2*d+2*c
^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(3/2)-3/16*(4*c^2*d^2*(c^2*f^2-g^2)/
g^2-4*c^4*d^2*f^2/g^2)/c^2/d*(-1/4*(-2*(x+f/g)*c^2*d+2*c^2*d*f/g)/c^2/d*(-
(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2)-1/8*(4*c^2*
d^2*(c^2*f^2-g^2)/g^2-4*c^4*d^2*f^2/g^2)/c^2/d/(c^2*d)^(1/2)*arctan((c^2*d
)^(1/2)*x/(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2)
)))-d*(c^2*f^2-g^2)/g^2*(1/3*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*
f^2-g^2)/g^2)^(3/2)+c^2*d*f/g*(-1/4*(-2*(x+f/g)*c^2*d+2*c^2*d*f/g)/c^2/d*(
-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2)-1/8*(4*c^2
*d^2*(c^2*f^2-g^2)/g^2-4*c^4*d^2*f^2/g^2)/c^2/d/(c^2*d)^(1/2)*arctan((c^2*
d)^(1/2)*x/(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2
)))-d*(c^2*f^2-g^2)/g^2*((-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-
g^2)/g^2)^(1/2)+c^2*d*f/g/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-(x+f/g)^2
*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))+d*(c^2*f^2-g^2)/g^2
/(-d*(c^2*f^2-g^2)/g^2)^(1/2)*ln((-2*d*(c^2*f^2-g^2)/g^2+2*c^2*d*f/g*(x+f/
g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1/2)*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*
(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/g)))))+b*(1/16*(-d*(c^2*x^2-1))^(1/2)*(-c^2
*x^2+1)^(1/2)/(c^2*x^2-1)*arccos(c*x)^2*f*(8*c^4*f^4-20*c^2*f^2*g^2+15*g^4
)*c*d^2/g^6+1/800*(-d*(c^2*x^2-1))^(1/2)*(16*I*c^5*x^5*(-c^2*x^2+1)^(1/...
```

3.13.5 Fricas [F]

$$\int \frac{(d-c^2dx^2)^{5/2}\,(a+b\arccos(cx))}{f+gx}\,dx = \int \frac{(-c^2dx^2+d)^{\frac{5}{2}}(b\arccos{(cx)}+a)}{gx+f}\,dx$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))/(g*x+f),x, algorithm="fri
cas")
```

output
```
integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c
^2*d^2*x^2 + b*d^2)*arccos(c*x))*sqrt(-c^2*d*x^2 + d)/(g*x + f), x)
```

3.13.6 Sympy [F]

$$\int \frac{(d-c^2dx^2)^{5/2}\,(a+b\arccos(cx))}{f+gx}\,dx = \int \frac{(-d(cx-1)\,(cx+1))^{\frac{5}{2}}\,(a+b\operatorname{acos}(cx))}{f+gx}\,dx$$

input
```
integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acos(c*x))/(g*x+f),x)
```

output
```
Integral((-d*(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acos(c*x))/(f + g*x), x)
```

3.13.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{5/2}\,(a+b\arccos(cx))}{f+gx}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))/(g*x+f),x, algorithm="max
ima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(g-c*f>0)', see `assume?` for mor
e details)
```

3.13.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(d-c^2dx^2)^{5/2}\,(a+b\arccos(cx))}{f+gx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccos(c*x))/(g*x+f),x, algorithm="gia
c")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

3.13.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(d-c^2dx^2)^{5/2}(a+b\arccos(cx))}{f+gx}\,dx=\int \frac{(a+b\,\mathrm{acos}(c\,x))\,\left(d-c^2\,d\,x^2\right)^{5/2}}{f+g\,x}\,dx$$

input
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(5/2))/(f + g*x),x)
```

output
```
int(((a + b*acos(c*x))*(d - c^2*d*x^2)^(5/2))/(f + g*x), x)
```

3.14 $\int \frac{(f+gx)^3(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx$

3.14.1 Optimal result

Integrand size = 31, antiderivative size = 450

$$\begin{aligned}\int \frac{(f+gx)^3(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx = &-\frac{3bf^2gx\sqrt{1-c^2x^2}}{c\sqrt{d-c^2dx^2}}-\frac{2bg^3x\sqrt{1-c^2x^2}}{3c^3\sqrt{d-c^2dx^2}}\\
&-\frac{3bfg^2x^2\sqrt{1-c^2x^2}}{4c\sqrt{d-c^2dx^2}}-\frac{bg^3x^3\sqrt{1-c^2x^2}}{9c\sqrt{d-c^2dx^2}}\\
&-\frac{3f^2g(1-c^2x^2)\,(a+b\arccos(cx))}{c^2\sqrt{d-c^2dx^2}}\\
&-\frac{2g^3(1-c^2x^2)\,(a+b\arccos(cx))}{3c^4\sqrt{d-c^2dx^2}}\\
&-\frac{3fg^2x(1-c^2x^2)\,(a+b\arccos(cx))}{2c^2\sqrt{d-c^2dx^2}}\\
&-\frac{g^3x^2(1-c^2x^2)\,(a+b\arccos(cx))}{3c^2\sqrt{d-c^2dx^2}}\\
&-\frac{f^3\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{2bc\sqrt{d-c^2dx^2}}\\
&-\frac{3fg^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{4bc^3\sqrt{d-c^2dx^2}}\end{aligned}$$

output
```
-3*f^2*g*(-c^2*x^2+1)*(a+b*arccos(c*x))/c^2/(-c^2*d*x^2+d)^(1/2)-2/3*g^3*(
-c^2*x^2+1)*(a+b*arccos(c*x))/c^4/(-c^2*d*x^2+d)^(1/2)-3/2*f*g^2*x*(-c^2*x
^2+1)*(a+b*arccos(c*x))/c^2/(-c^2*d*x^2+d)^(1/2)-1/3*g^3*x^2*(-c^2*x^2+1)*
(a+b*arccos(c*x))/c^2/(-c^2*d*x^2+d)^(1/2)-3*b*f^2*g*x*(-c^2*x^2+1)^(1/2)/
c/(-c^2*d*x^2+d)^(1/2)-2/3*b*g^3*x*(-c^2*x^2+1)^(1/2)/c^3/(-c^2*d*x^2+d)^(
1/2)-3/4*b*f*g^2*x^2*(-c^2*x^2+1)^(1/2)/c/(-c^2*d*x^2+d)^(1/2)-1/9*b*g^3*x
^3*(-c^2*x^2+1)^(1/2)/c/(-c^2*d*x^2+d)^(1/2)-1/2*f^3*(a+b*arccos(c*x))^2*(
-c^2*x^2+1)^(1/2)/b/c/(-c^2*d*x^2+d)^(1/2)-3/4*f*g^2*(a+b*arccos(c*x))^2*(
-c^2*x^2+1)^(1/2)/b/c^3/(-c^2*d*x^2+d)^(1/2)
```

3.14.2 Mathematica [A] (verified)

Time = 1.60 (sec) , antiderivative size = 342, normalized size of antiderivative = 0.76

$$\int \frac{(f+gx)^3(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx$$
$$= \frac{18bc\sqrt{d}f(2c^2f^2+3g^2)(-1+c^2x^2)\arccos(cx)^2-36acf(2c^2f^2+3g^2)\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}\arctan\left(\frac{cx\sqrt{\cdots}}{\sqrt{d}(-\cdots}\right.}{}$$

input
```
Integrate[((f + g*x)^3*(a + b*ArcCos[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

output
```
(18*b*c*Sqrt[d]*f*(2*c^2*f^2 + 3*g^2)*(-1 + c^2*x^2)*ArcCos[c*x]^2 - 36*a*
c*f*(2*c^2*f^2 + 3*g^2)*Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2]*ArcTan[(c*x*
Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + Sqrt[d]*g*(-1 + c^2*x^2)*
(8*b*c*x*(6*g^2 + c^2*(27*f^2 + g^2*x^2)) + 12*a*Sqrt[1 - c^2*x^2]*(4*g^2
+ c^2*(18*f^2 + 9*f*g*x + 2*g^2*x^2)) + 27*b*c*f*g*Cos[2*ArcCos[c*x]]) + 6
*b*Sqrt[d]*g*(-1 + c^2*x^2)*ArcCos[c*x]*(4*Sqrt[1 - c^2*x^2]*(2*g^2 + c^2*
(9*f^2 + g^2*x^2)) + 9*c*f*g*Sin[2*ArcCos[c*x]]))/(72*c^4*Sqrt[d]*Sqrt[1 -
 c^2*x^2]*Sqrt[d - c^2*d*x^2])
```

3.14.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.58, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {5277, 5263, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^3(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx$$

$$\Big\downarrow \text{ 5277}$$

$$\frac{\sqrt{1-c^2x^2}\int \frac{(f+gx)^3(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 5263}$$

$$\frac{\sqrt{1-c^2x^2}\int \left(\frac{(a+b\arccos(cx))f^3}{\sqrt{1-c^2x^2}}+\frac{3gx(a+b\arccos(cx))f^2}{\sqrt{1-c^2x^2}}+\frac{3g^2x^2(a+b\arccos(cx))f}{\sqrt{1-c^2x^2}}+\frac{g^3x^3(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}\right)dx}{\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{1-c^2x^2}\left(-\frac{3fg^2(a+b\arccos(cx))^2}{4bc^3}-\frac{3f^2g\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}-\frac{3fg^2x\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2c^2}-\frac{g^3x^2\sqrt{1-c^2x^2}(a+b\arccos(c$$

$$\sqrt{d-c^2dx^2}$$

input
```
Int[((f + g*x)^3*(a + b*ArcCos[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

output
```
(Sqrt[1 - c^2*x^2]*((-3*b*f^2*g*x)/c - (2*b*g^3*x)/(3*c^3) - (3*b*f*g^2*x^
2)/(4*c) - (b*g^3*x^3)/(9*c) - (3*f^2*g*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*
x]))/c^2 - (2*g^3*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(3*c^4) - (3*f*g^
2*x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(2*c^2) - (g^3*x^2*Sqrt[1 - c^2
*x^2]*(a + b*ArcCos[c*x]))/(3*c^2) - (f^3*(a + b*ArcCos[c*x])^2)/(2*b*c) -
 (3*f*g^2*(a + b*ArcCos[c*x])^2)/(4*b*c^3)))/Sqrt[d - c^2*d*x^2]
```

3.14.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5263
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

rule 5277
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[Simp[(d + e*x^2)^p/(1 - c^2*x^2)^
p]   Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && Intege
rQ[p - 1/2] &&  !GtQ[d, 0]
```

3.14.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.43 (sec) , antiderivative size = 856, normalized size of antiderivative = 1.90

method	result
default	$a\left(\frac{f^3\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2dx^2+d}}\right)}{\sqrt{c^2d}}+g^3\left(-\frac{x^2\sqrt{-c^2dx^2+d}}{3c^2d}-\frac{2\sqrt{-c^2dx^2+d}}{3d\,c^4}\right)+3f\,g^2\left(-\frac{x\sqrt{-c^2dx^2+d}}{2c^2d}+\frac{\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2dx^2+d}}\right)}{2c^2\sqrt{c^2d}}\right.\right.$
parts	$a\left(\frac{f^3\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2dx^2+d}}\right)}{\sqrt{c^2d}}+g^3\left(-\frac{x^2\sqrt{-c^2dx^2+d}}{3c^2d}-\frac{2\sqrt{-c^2dx^2+d}}{3d\,c^4}\right)+3f\,g^2\left(-\frac{x\sqrt{-c^2dx^2+d}}{2c^2d}+\frac{\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2dx^2+d}}\right)}{2c^2\sqrt{c^2d}}\right.\right.$

input
```
int((g*x+f)^3*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
a*(f^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+g^3*(-1/
3*x^2/c^2/d*(-c^2*d*x^2+d)^(1/2)-2/3/d/c^4*(-c^2*d*x^2+d)^(1/2))+3*f*g^2*(
-1/2*x/c^2/d*(-c^2*d*x^2+d)^(1/2)+1/2/c^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/
2)*x/(-c^2*d*x^2+d)^(1/2)))-3*f^2*g/c^2/d*(-c^2*d*x^2+d)^(1/2))+b*(1/4*(-d
*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c^3/d/(c^2*x^2-1)*arccos(c*x)^2*f*(
2*c^2*f^2+3*g^2)+1/144*(-d*(c^2*x^2-1))^(1/2)*(2*I*c*x*(-c^2*x^2+1)^(1/2)+
2*c^2*x^2-1)*g^3*(I+3*arccos(c*x))/c^4/d/(c^2*x^2-1)-3/8*(-d*(c^2*x^2-1))^
(1/2)*(I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*g*(4*I*c^2*f^2+4*arccos(c*x)*c^
2*f^2+I*g^2+arccos(c*x)*g^2)/c^4/d/(c^2*x^2-1)-3/8*(-d*(c^2*x^2-1))^(1/2)*
(c^2*x^2-I*(-c^2*x^2+1)^(1/2)*x*c-1)*g*(-4*I*c^2*f^2+4*arccos(c*x)*c^2*f^2
-I*g^2+arccos(c*x)*g^2)/c^4/d/(c^2*x^2-1)+1/144*(-d*(c^2*x^2-1))^(1/2)*(2*
c^2*x^2-2*I*c*x*(-c^2*x^2+1)^(1/2)-1)*g^3*(-I+3*arccos(c*x))/c^4/d/(c^2*x^
2-1)+3/8*(-d*(c^2*x^2-1))^(1/2)/c^2/d/(c^2*x^2-1)*f*g^2*arccos(c*x)*x-3/16
*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c^3/d/(c^2*x^2-1)*f*g^2-1/24*(-
d*(c^2*x^2-1))^(1/2)/c^4/d/(c^2*x^2-1)*arccos(c*x)*g^3*cos(4*arccos(c*x))+
1/72*(-d*(c^2*x^2-1))^(1/2)/c^4/d/(c^2*x^2-1)*g^3*sin(4*arccos(c*x))-3/8*(
-d*(c^2*x^2-1))^(1/2)/c^3/d/(c^2*x^2-1)*f*g^2*arccos(c*x)*cos(3*arccos(c*x
))+3/16*(-d*(c^2*x^2-1))^(1/2)/c^3/d/(c^2*x^2-1)*f*g^2*sin(3*arccos(c*x)))
```

3.14.5 Fricas [F]

$$\int \frac{(f+gx)^3(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{(gx+f)^3(b\arccos(cx)+a)}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate((g*x+f)^3*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="f
ricas")
```

output
```
integral(-(a*g^3*x^3 + 3*a*f*g^2*x^2 + 3*a*f^2*g*x + a*f^3 + (b*g^3*x^3 +
3*b*f*g^2*x^2 + 3*b*f^2*g*x + b*f^3)*arccos(c*x))*sqrt(-c^2*d*x^2 + d)/(c^
2*d*x^2 - d), x)
```

3.14.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^3(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((g*x+f)**3*(a+b*acos(c*x))/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real zoo
```

3.14.7 Maxima [F]

$$\int \frac{(f+gx)^3(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{(gx+f)^3(b\arccos(cx)+a)}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate((g*x+f)^3*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="m
axima")
```

output
```
-1/3*a*g^3*(sqrt(-c^2*d*x^2 + d)*x^2/(c^2*d) + 2*sqrt(-c^2*d*x^2 + d)/(c^4
*d)) - 3/2*a*f*g^2*(sqrt(-c^2*d*x^2 + d)*x/(c^2*d) - arcsin(c*x)/(c^3*sqrt
(d))) + b*f^3*arccos(c*x)*arcsin(c*x)/(c*sqrt(d)) + 1/2*b*f^3*arcsin(c*x)^
2/(c*sqrt(d)) - 3*b*f^2*g*x/(c*sqrt(d)) + a*f^3*arcsin(c*x)/(c*sqrt(d)) -
3*sqrt(-c^2*d*x^2 + d)*b*f^2*g*arccos(c*x)/(c^2*d) - 3*sqrt(-c^2*d*x^2 + d
)*a*f^2*g/(c^2*d) - sqrt(d)*integrate((b*g^3*x^3 + 3*b*f*g^2*x^2)*sqrt(c*x
 + 1)*sqrt(-c*x + 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/(c^2*d*x^2
 - d), x)
```

3.14.8 Giac [F]

$$\int \frac{(f+gx)^3(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{(gx+f)^3(b\arccos(cx)+a)}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate((g*x+f)^3*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((g*x + f)^3*(b*arccos(c*x) + a)/sqrt(-c^2*d*x^2 + d), x)
```

3.14.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^3(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{(f+g\,x)^3\,(a+b\,\mathrm{acos}(c\,x))}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

input
```
int(((f + g*x)^3*(a + b*acos(c*x)))/(d - c^2*d*x^2)^(1/2),x)
```

output
```
int(((f + g*x)^3*(a + b*acos(c*x)))/(d - c^2*d*x^2)^(1/2), x)
```

3.15 $\int \frac{(f+gx)^2(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx$

3.15.1 Optimal result

Integrand size = 31, antiderivative size = 270

$$\int \frac{(f+gx)^2(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx = -\frac{2bfgx\sqrt{1-c^2x^2}}{c\sqrt{d-c^2dx^2}} - \frac{bg^2x^2\sqrt{1-c^2x^2}}{4c\sqrt{d-c^2dx^2}} - \frac{2fg(1-c^2x^2)\,(a+b\arccos(cx))}{c^2\sqrt{d-c^2dx^2}} - \frac{g^2x(1-c^2x^2)\,(a+b\arccos(cx))}{2c^2\sqrt{d-c^2dx^2}} - \frac{f^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{2bc\sqrt{d-c^2dx^2}} - \frac{g^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{4bc^3\sqrt{d-c^2dx^2}}$$

output
```
-2*f*g*(-c^2*x^2+1)*(a+b*arccos(c*x))/c^2/(-c^2*d*x^2+d)^(1/2)-1/2*g^2*x*(
-c^2*x^2+1)*(a+b*arccos(c*x))/c^2/(-c^2*d*x^2+d)^(1/2)-2*b*f*g*x*(-c^2*x^2
+1)^(1/2)/c/(-c^2*d*x^2+d)^(1/2)-1/4*b*g^2*x^2*(-c^2*x^2+1)^(1/2)/c/(-c^2*
d*x^2+d)^(1/2)-1/2*f^2*(a+b*arccos(c*x))^2*(-c^2*x^2+1)^(1/2)/b/c/(-c^2*d*
x^2+d)^(1/2)-1/4*g^2*(a+b*arccos(c*x))^2*(-c^2*x^2+1)^(1/2)/b/c^3/(-c^2*d*
x^2+d)^(1/2)
```

3.15.2 Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.99

$$\int \frac{(f+gx)^2(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx$$
$$= \frac{2b\sqrt{d}(2c^2f^2+g^2)\left(-1+c^2x^2\right)\arccos(cx)^2-4a(2c^2f^2+g^2)\sqrt{1-c^2x^2}\sqrt{d-c^2dx^2}\arctan\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(-1+c^2x^2)}\right)-$$

input
```
Integrate[((f + g*x)^2*(a + b*ArcCos[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

output
```
(2*b*Sqrt[d]*(2*c^2*f^2 + g^2)*(-1 + c^2*x^2)*ArcCos[c*x]^2 - 4*a*(2*c^2*f
^2 + g^2)*Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d
*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + Sqrt[d]*g*(-1 + c^2*x^2)*(4*c*(4*b*c*f*
x + a*(4*f + g*x)*Sqrt[1 - c^2*x^2]) + b*g*Cos[2*ArcCos[c*x]]) + 2*b*Sqrt[
d]*g*(-1 + c^2*x^2)*ArcCos[c*x]*(8*c*f*Sqrt[1 - c^2*x^2] + g*Sin[2*ArcCos[
c*x]]))/(8*c^3*Sqrt[d]*Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2])
```

3.15.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.60, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {5277, 5263, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)^2(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx$$
$$\Big\downarrow \; 5277$$
$$\frac{\sqrt{1-c^2x^2}\int \frac{(f+gx)^2(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{\sqrt{d-c^2dx^2}}$$
$$\Big\downarrow \; 5263$$
$$\frac{\sqrt{1-c^2x^2}\int \left(\frac{(a+b\arccos(cx))f^2}{\sqrt{1-c^2x^2}}+\frac{2gx(a+b\arccos(cx))f}{\sqrt{1-c^2x^2}}+\frac{g^2x^2(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}\right)dx}{\sqrt{d-c^2dx^2}}$$
$$\Big\downarrow \; 2009$$

$$\frac{\sqrt{1-c^2x^2}\left(-\frac{g^2(a+b\arccos(cx))^2}{4bc^3}-\frac{2fg\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}-\frac{g^2x\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2c^2}-\frac{f^2(a+b\arccos(cx))^2}{2bc}-\frac{2bfgx}{c}-\right.}{\sqrt{d-c^2dx^2}}$$

input
```
Int[((f + g*x)^2*(a + b*ArcCos[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

output
```
(Sqrt[1 - c^2*x^2]*((-2*b*f*g*x)/c - (b*g^2*x^2)/(4*c) - (2*f*g*Sqrt[1 - c
^2*x^2]*(a + b*ArcCos[c*x]))/c^2 - (g^2*x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[
c*x]))/(2*c^2) - (f^2*(a + b*ArcCos[c*x])^2)/(2*b*c) - (g^2*(a + b*ArcCos[
c*x])^2)/(4*b*c^3)))/Sqrt[d - c^2*d*x^2]
```

3.15.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5263
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

rule 5277
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[Simp[(d + e*x^2)^p/(1 - c^2*x^2)^
p]   Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && Intege
rQ[p - 1/2] &&  !GtQ[d, 0]
```

3.15.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.44 (sec) , antiderivative size = 505, normalized size of antiderivative = 1.87

method	result
default	$a\left(\frac{f^2\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{\sqrt{c^2d}}+g^2\left(-\frac{x\sqrt{-c^2d\,x^2+d}}{2c^2d}+\frac{\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{2c^2\sqrt{c^2d}}\right)-\frac{2fg\sqrt{-c^2d\,x^2+d}}{c^2d}\right)+b\left(\frac{\sqrt{-d(c^2x^2-}}{}\right.$
parts	$a\left(\frac{f^2\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{\sqrt{c^2d}}+g^2\left(-\frac{x\sqrt{-c^2d\,x^2+d}}{2c^2d}+\frac{\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{2c^2\sqrt{c^2d}}\right)-\frac{2fg\sqrt{-c^2d\,x^2+d}}{c^2d}\right)+b\left(\frac{\sqrt{-d(c^2x^2-}}{}\right.$

input
```
int((g*x+f)^2*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
a*(f^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+g^2*(-1/
2*x/c^2/d*(-c^2*d*x^2+d)^(1/2)+1/2/c^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*
x/(-c^2*d*x^2+d)^(1/2)))-2*f*g/c^2/d*(-c^2*d*x^2+d)^(1/2))+b*(1/4*(-d*(c^2
*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c^3/d/(c^2*x^2-1)*arccos(c*x)^2*(2*c^2*f
^2+g^2)-(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2)*x*c+c^2*x^2-1)*f*g*(a
rccos(c*x)+I)/c^2/d/(c^2*x^2-1)-(-d*(c^2*x^2-1))^(1/2)*(c^2*x^2-I*(-c^2*x^
2+1)^(1/2)*x*c-1)*f*g*(arccos(c*x)-I)/c^2/d/(c^2*x^2-1)+1/8*(-d*(c^2*x^2-1
))^(1/2)/c^2/d/(c^2*x^2-1)*arccos(c*x)*g^2*x-1/16*(-d*(c^2*x^2-1))^(1/2)*(
-c^2*x^2+1)^(1/2)/c^3/d/(c^2*x^2-1)*g^2-1/8*(-d*(c^2*x^2-1))^(1/2)/c^3/d/(
c^2*x^2-1)*arccos(c*x)*g^2*cos(3*arccos(c*x))+1/16*(-d*(c^2*x^2-1))^(1/2)/
c^3/d/(c^2*x^2-1)*g^2*sin(3*arccos(c*x)))
```

3.15.5 Fricas [F]

$$\int \frac{(f+gx)^2(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx=\int \frac{(gx+f)^2(b\arccos(cx)+a)}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate((g*x+f)^2*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="f
ricas")
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(a*g^2*x^2 + 2*a*f*g*x + a*f^2 + (b*g^2*x^2
 + 2*b*f*g*x + b*f^2)*arccos(c*x))/(c^2*d*x^2 - d), x)
```

3.15.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{(f+gx)^2(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((g*x+f)**2*(a+b*acos(c*x))/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real zoo
```

3.15.7 Maxima [F]

$$\int \frac{(f+gx)^2(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{(gx+f)^2(b\arccos(cx)+a)}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate((g*x+f)^2*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="m
axima")
```

output
```
-1/2*a*g^2*(sqrt(-c^2*d*x^2 + d)*x/(c^2*d) - arcsin(c*x)/(c^3*sqrt(d))) +
b*f^2*arccos(c*x)*arcsin(c*x)/(c*sqrt(d)) + 1/2*b*f^2*arcsin(c*x)^2/(c*sqr
t(d)) + b*g^2*integrate(x^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/(sq
rt(c*x + 1)*sqrt(-c*x + 1)), x)/sqrt(d) - 2*b*f*g*x/(c*sqrt(d)) + a*f^2*ar
csin(c*x)/(c*sqrt(d)) - 2*sqrt(-c^2*d*x^2 + d)*b*f*g*arccos(c*x)/(c^2*d) -
 2*sqrt(-c^2*d*x^2 + d)*a*f*g/(c^2*d)
```

3.15.8 Giac [F]

$$\int \frac{(f+gx)^2(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{(gx+f)^2(b\arccos(cx)+a)}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate((g*x+f)^2*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((g*x + f)^2*(b*arccos(c*x) + a)/sqrt(-c^2*d*x^2 + d), x)
```

3.15.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)^2(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{(f+g\,x)^2\,(a+b\,\mathrm{acos}(c\,x))}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

input
```
int(((f + g*x)^2*(a + b*acos(c*x)))/(d - c^2*d*x^2)^(1/2),x)
```

output
```
int(((f + g*x)^2*(a + b*acos(c*x)))/(d - c^2*d*x^2)^(1/2), x)
```

3.16 $\int \frac{(f+gx)(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx$

3.16.1 Optimal result

Integrand size = 29, antiderivative size = 127

$$\int \frac{(f+gx)(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx = -\frac{bgx\sqrt{1-c^2x^2}}{c\sqrt{d-c^2dx^2}} - \frac{g(1-c^2x^2)\,(a+b\arccos(cx))}{c^2\sqrt{d-c^2dx^2}} - \frac{f\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{2bc\sqrt{d-c^2dx^2}}$$

output
```
-g*(-c^2*x^2+1)*(a+b*arccos(c*x))/c^2/(-c^2*d*x^2+d)^(1/2)-b*g*x*(-c^2*x^2
+1)^(1/2)/c/(-c^2*d*x^2+d)^(1/2)-1/2*f*(a+b*arccos(c*x))^2*(-c^2*x^2+1)^(1
/2)/b/c/(-c^2*d*x^2+d)^(1/2)
```

3.16.2 Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.35

$$\int \frac{(f+gx)(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx = \frac{-2\sqrt{d}g\left(a-ac^2x^2+bcx\sqrt{1-c^2x^2}\right)+2b\sqrt{d}g(-1+c^2x^2)\arccos(cx)-bc\sqrt{d}f\sqrt{1-c^2x^2}\arccos(cx)^2-2c}{2c^2\sqrt{d}\sqrt{d-c^2dx^2}}$$

input
```
Integrate[((f + g*x)*(a + b*ArcCos[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

output
```
(-2*Sqrt[d]*g*(a - a*c^2*x^2 + b*c*x*Sqrt[1 - c^2*x^2]) + 2*b*Sqrt[d]*g*(-
1 + c^2*x^2)*ArcCos[c*x] - b*c*Sqrt[d]*f*Sqrt[1 - c^2*x^2]*ArcCos[c*x]^2 -
 2*a*c*f*Sqrt[d - c^2*d*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1
 + c^2*x^2))])/(2*c^2*Sqrt[d]*Sqrt[d - c^2*d*x^2])
```

3.16.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.69, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {5277, 5263, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(f+gx)(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx$$

$$\Big\downarrow \; 5277$$

$$\frac{\sqrt{1-c^2x^2}\int \frac{(f+gx)(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}dx}{\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \; 5263$$

$$\frac{\sqrt{1-c^2x^2}\int \left(\frac{f(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}+\frac{gx(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}\right)dx}{\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \; 2009$$

$$\frac{\sqrt{1-c^2x^2}\left(-\frac{g\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c^2}-\frac{f(a+b\arccos(cx))^2}{2bc}-\frac{bgx}{c}\right)}{\sqrt{d-c^2dx^2}}$$

input
```
Int[((f + g*x)*(a + b*ArcCos[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

output
```
(Sqrt[1 - c^2*x^2]*(-((b*g*x)/c) - (g*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]
))/c^2 - (f*(a + b*ArcCos[c*x])^2)/(2*b*c)))/Sqrt[d - c^2*d*x^2]
```

3.16.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5263
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^p*(a +
 b*ArcCos[c*x])^n, (f + g*x)^m, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& EqQ[c^2*d + e, 0] && IGtQ[m, 0] && IntegerQ[p + 1/2] && GtQ[d, 0] && IGtQ
[n, 0] && (m == 1 || p > 0 || (n == 1 && p > -1) || (m == 2 && p < -2))
```

rule 5277
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[Simp[(d + e*x^2)^p/(1 - c^2*x^2)^
p]   Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && Intege
rQ[p - 1/2] &&  !GtQ[d, 0]
```

3.16.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.38 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.94

method	result
default	$\frac{af\arctan\left(\frac{\sqrt{c^2 d}\,x}{\sqrt{-c^2 d\,x^2+d}}\right)}{\sqrt{c^2 d}}-\frac{ag\sqrt{-c^2 d\,x^2+d}}{c^2 d}+b\left(\frac{\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,\arccos(cx)^2 f}{2cd(c^2x^2-1)}-\frac{\sqrt{-d(c^2x^2-1)}\,\left(icx\sqrt{-c^2x^2+1}+\right.}{2c^2d(c^2x^2}\right.$
parts	$\frac{af\arctan\left(\frac{\sqrt{c^2 d}\,x}{\sqrt{-c^2 d\,x^2+d}}\right)}{\sqrt{c^2 d}}-\frac{ag\sqrt{-c^2 d\,x^2+d}}{c^2 d}+b\left(\frac{\sqrt{-d(c^2x^2-1)}\,\sqrt{-c^2x^2+1}\,\arccos(cx)^2 f}{2cd(c^2x^2-1)}-\frac{\sqrt{-d(c^2x^2-1)}\,\left(icx\sqrt{-c^2x^2+1}+\right.}{2c^2d(c^2x^2}\right.$

input
```
int((g*x+f)*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
a*f/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-a*g/c^2/d*(
-c^2*d*x^2+d)^(1/2)+b*(1/2*(-d*(c^2*x^2-1))^(1/2)*(-c^2*x^2+1)^(1/2)/c/d/(
c^2*x^2-1)*arccos(c*x)^2*f-1/2*(-d*(c^2*x^2-1))^(1/2)*(I*(-c^2*x^2+1)^(1/2
)*x*c+c^2*x^2-1)*g*(arccos(c*x)+I)/c^2/d/(c^2*x^2-1)-1/2*(-d*(c^2*x^2-1))^
(1/2)*(c^2*x^2-I*(-c^2*x^2+1)^(1/2)*x*c-1)*g*(arccos(c*x)-I)/c^2/d/(c^2*x^
2-1))
```

3.16.5 Fricas [F]

$$\int \frac{(f+gx)(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx = \int \frac{(gx+f)(b\arccos(cx)+a)}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate((g*x+f)*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="fri
cas")
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(a*g*x + a*f + (b*g*x + b*f)*arccos(c*x))/(
c^2*d*x^2 - d), x)
```

3.16.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{(f+gx)(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((g*x+f)*(a+b*acos(c*x))/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real zoo
```

3.16.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.85

$$\int \frac{(f+gx)(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx = \frac{bf\arccos(cx)\arcsin(cx)}{c\sqrt{d}} + \frac{bf\arcsin(cx)^2}{2\,c\sqrt{d}} - \frac{bgx}{c\sqrt{d}} + \frac{af\arcsin(cx)}{c\sqrt{d}} - \frac{\sqrt{-c^2dx^2+d}bg\arccos(cx)}{c^2d} - \frac{\sqrt{-c^2dx^2+d}ag}{c^2d}$$

input
```
integrate((g*x+f)*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="max
ima")
```

output
```
b*f*arccos(c*x)*arcsin(c*x)/(c*sqrt(d)) + 1/2*b*f*arcsin(c*x)^2/(c*sqrt(d)
) - b*g*x/(c*sqrt(d)) + a*f*arcsin(c*x)/(c*sqrt(d)) - sqrt(-c^2*d*x^2 + d)
*b*g*arccos(c*x)/(c^2*d) - sqrt(-c^2*d*x^2 + d)*a*g/(c^2*d)
```

3.16.8 Giac [F]

$$\int \frac{(f+gx)(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx=\int \frac{(gx+f)(b\arccos(cx)+a)}{\sqrt{-c^2dx^2+d}}\,dx$$

input
```
integrate((g*x+f)*(a+b*arccos(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="gia
c")
```

output
```
integrate((g*x + f)*(b*arccos(c*x) + a)/sqrt(-c^2*d*x^2 + d), x)
```

3.16.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(f+gx)(a+b\arccos(cx))}{\sqrt{d-c^2dx^2}}\,dx=\int \frac{(f+g\,x)\,(a+b\,\mathrm{acos}(c\,x))}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

input
```
int(((f + g*x)*(a + b*acos(c*x)))/(d - c^2*d*x^2)^(1/2),x)
```

output
```
int(((f + g*x)*(a + b*acos(c*x)))/(d - c^2*d*x^2)^(1/2), x)
```

3.17 $\int \frac{a+b\arccos(cx)}{(f+gx)\sqrt{d-c^2dx^2}}\,dx$

3.17.1 Optimal result

Integrand size = 31, antiderivative size = 370

$$\int \frac{a+b\arccos(cx)}{(f+gx)\sqrt{d-c^2dx^2}}\,dx = \frac{i\sqrt{1-c^2x^2}(a+b\arccos(cx))\log\left(1+\frac{e^{i\arccos(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{\sqrt{c^2f^2-g^2}\sqrt{d-c^2dx^2}} - \frac{i\sqrt{1-c^2x^2}(a+b\arccos(cx))\log\left(1+\frac{e^{i\arccos(cx)}g}{cf+\sqrt{c^2f^2-g^2}}\right)}{\sqrt{c^2f^2-g^2}\sqrt{d-c^2dx^2}} + \frac{b\sqrt{1-c^2x^2}\,\mathrm{PolyLog}\left(2,-\frac{e^{i\arccos(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{\sqrt{c^2f^2-g^2}\sqrt{d-c^2dx^2}} - \frac{b\sqrt{1-c^2x^2}\,\mathrm{PolyLog}\left(2,-\frac{e^{i\arccos(cx)}g}{cf+\sqrt{c^2f^2-g^2}}\right)}{\sqrt{c^2f^2-g^2}\sqrt{d-c^2dx^2}}$$

output
```
I*(a+b*arccos(c*x))*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2-g^2)^(
1/2)))*(-c^2*x^2+1)^(1/2)/(c^2*f^2-g^2)^(1/2)/(-c^2*d*x^2+d)^(1/2)-I*(a+b*
arccos(c*x))*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2)))*
(-c^2*x^2+1)^(1/2)/(c^2*f^2-g^2)^(1/2)/(-c^2*d*x^2+d)^(1/2)+b*polylog(2,-(
c*x+I*(-c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2-g^2)^(1/2)))*(-c^2*x^2+1)^(1/2)/
(c^2*f^2-g^2)^(1/2)/(-c^2*d*x^2+d)^(1/2)-b*polylog(2,-(c*x+I*(-c^2*x^2+1)^
(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2)))*(-c^2*x^2+1)^(1/2)/(c^2*f^2-g^2)^(1/2)
/(-c^2*d*x^2+d)^(1/2)
```

3.17.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 930 vs. 2(370) = 740.

Time = 2.71 (sec) , antiderivative size = 930, normalized size of antiderivative = 2.51

$$\int \frac{a+b\arccos(cx)}{(f+gx)\sqrt{d-c^2dx^2}}\,dx$$

$$= \frac{\frac{a\log(f+gx)}{\sqrt{d}} - \frac{a\log\left(d(g+c^2fx)+\sqrt{d}\sqrt{-c^2f^2+g^2}\sqrt{d-c^2dx^2}\right)}{\sqrt{d}} - \frac{b\sqrt{1-c^2x^2}\left(2\arccos(cx)\operatorname{arctanh}\left(\frac{(cf+g)\cot\left(\frac{1}{2}\arccos(cx)\right)}{\sqrt{-c^2f^2+g^2}}\right)-2\arccos\left(-\right.\right.}{}}{}$$

input
```
Integrate[(a + b*ArcCos[c*x])/((f + g*x)*Sqrt[d - c^2*d*x^2]),x]
```

output
```
((a*Log[f + g*x])/Sqrt[d] - (a*Log[d*(g + c^2*f*x) + Sqrt[d]*Sqrt[-(c^2*f^
2) + g^2]*Sqrt[d - c^2*d*x^2]])/Sqrt[d] - (b*Sqrt[1 - c^2*x^2]*(2*ArcCos[c
*x]*ArcTanh[((c*f + g)*Cot[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] - 2*Arc
Cos[-((c*f)/g)]*ArcTanh[((-(c*f) + g)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2)
+ g^2]] + (ArcCos[-((c*f)/g)] - (2*I)*ArcTanh[((c*f + g)*Cot[ArcCos[c*x]/2
])/Sqrt[-(c^2*f^2) + g^2]] + (2*I)*ArcTanh[((-(c*f) + g)*Tan[ArcCos[c*x]/2
])/Sqrt[-(c^2*f^2) + g^2]])*Log[Sqrt[-(c^2*f^2) + g^2]/(Sqrt[2]*E^((I/2)*A
rcCos[c*x])*Sqrt[g]*Sqrt[c*(f + g*x)])] + (ArcCos[-((c*f)/g)] + (2*I)*(Arc
Tanh[((c*f + g)*Cot[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] - ArcTanh[((-(
c*f) + g)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]]))*Log[(E^((I/2)*ArcC
os[c*x])*Sqrt[-(c^2*f^2) + g^2])/(Sqrt[2]*Sqrt[g]*Sqrt[c*(f + g*x)])] - (A
rcCos[-((c*f)/g)] - (2*I)*ArcTanh[((-(c*f) + g)*Tan[ArcCos[c*x]/2])/Sqrt[-
(c^2*f^2) + g^2]])*Log[((c*f + g)*((-I)*c*f + I*g + Sqrt[-(c^2*f^2) + g^2]
)*(-I + Tan[ArcCos[c*x]/2]))/(g*(c*f + g + Sqrt[-(c^2*f^2) + g^2]*Tan[ArcC
os[c*x]/2]))] - (ArcCos[-((c*f)/g)] + (2*I)*ArcTanh[((-(c*f) + g)*Tan[ArcC
os[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])*Log[((c*f + g)*(I*c*f - I*g + Sqrt[-(
c^2*f^2) + g^2])*(I + Tan[ArcCos[c*x]/2]))/(g*(c*f + g + Sqrt[-(c^2*f^2) +
 g^2]*Tan[ArcCos[c*x]/2]))] + I*(PolyLog[2, ((c*f - I*Sqrt[-(c^2*f^2) + g^
2])*(c*f + g - Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))/(g*(c*f + g + S
qrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))] - PolyLog[2, ((c*f + I*Sqrt...
```

3.17.3 Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.75, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.290, Rules used = {5277, 5273, 3042, 3802, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{\sqrt{d-c^2dx^2}(f+gx)}dx$$

$$\Big\downarrow \text{5277}$$

$$\frac{\sqrt{1-c^2x^2}\int \frac{a+b\arccos(cx)}{(f+gx)\sqrt{1-c^2x^2}}dx}{\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{5273}$$

$$-\frac{\sqrt{1-c^2x^2}\int \frac{a+b\arccos(cx)}{cf+cgx}d\arccos(cx)}{\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sqrt{1-c^2x^2}\int \frac{a+b\arccos(cx)}{cf+g\sin\left(\arccos(cx)+\frac{\pi}{2}\right)}d\arccos(cx)}{\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{3802}$$

$$-\frac{2\sqrt{1-c^2x^2}\int \frac{e^{i\arccos(cx)}(a+b\arccos(cx))}{2ce^{i\arccos(cx)}f+e^{2i\arccos(cx)}g+g}d\arccos(cx)}{\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{2694}$$

$$-\frac{2\sqrt{1-c^2x^2}\left(\frac{g\int \frac{e^{i\arccos(cx)}(a+b\arccos(cx))}{2\left(cf+e^{i\arccos(cx)}g-\sqrt{c^2f^2-g^2}\right)}d\arccos(cx)}{\sqrt{c^2f^2-g^2}}-\frac{g\int \frac{e^{i\arccos(cx)}(a+b\arccos(cx))}{2\left(cf+e^{i\arccos(cx)}g+\sqrt{c^2f^2-g^2}\right)}d\arccos(cx)}{\sqrt{c^2f^2-g^2}}\right)}{\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2\sqrt{1-c^2x^2}\left(\frac{g\int \frac{e^{i\arccos(cx)}(a+b\arccos(cx))}{cf+e^{i\arccos(cx)}g-\sqrt{c^2f^2-g^2}}d\arccos(cx)}{2\sqrt{c^2f^2-g^2}}-\frac{g\int \frac{e^{i\arccos(cx)}(a+b\arccos(cx))}{cf+e^{i\arccos(cx)}g+\sqrt{c^2f^2-g^2}}d\arccos(cx)}{2\sqrt{c^2f^2-g^2}}\right)}{\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{2\sqrt{1-c^2x^2}\left(\frac{g\left(\frac{ib\int \log\left(\frac{e^{i\arccos(cx)}g}{cf-\sqrt{c^2f^2-g^2}}+1\right)d\arccos(cx)}{g}-\frac{i(a+b\arccos(cx))\log\left(1+\frac{ge^{i\arccos(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{g}\right)}{2\sqrt{c^2f^2-g^2}}-\frac{g\left(\frac{ib\int\log\left(\frac{e^{i\arccos(cx)}g}{cf+\sqrt{c^2f^2-g^2}}+1\right)d\arccos\cdots}{g}\cdots\right)}{2\cdots}\right.}{\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \; 2715$$

$$-\frac{2\sqrt{1-c^2x^2}\left(\frac{g\left(\frac{b\int e^{-i\arccos(cx)}\log\left(\frac{e^{i\arccos(cx)}g}{cf-\sqrt{c^2f^2-g^2}}+1\right)de^{i\arccos(cx)}}{g}-\frac{i(a+b\arccos(cx))\log\left(1+\frac{ge^{i\arccos(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{g}\right)}{2\sqrt{c^2f^2-g^2}}-\frac{g\left(\frac{b\int e^{-i\arccos(cx)}\log\cdots}{\cdots}\right.}{\cdots}\right.}{\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \; 2838$$

$$-\frac{2\sqrt{1-c^2x^2}\left(\frac{g\left(-\frac{i(a+b\arccos(cx))\log\left(1+\frac{ge^{i\arccos(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{g}-\frac{b\,\mathrm{PolyLog}\left(2,-\frac{e^{i\arccos(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{g}\right)}{2\sqrt{c^2f^2-g^2}}-\frac{g\left(-\frac{i(a+b\arccos(cx))\log\left(1+\frac{ge^{i\arccos(cx)}}{\sqrt{c^2f^2-g^2}+cf}\cdots\right)}{g}\cdots\right.}{2\sqrt{c^2f^2-\cdots}}\right.}{\sqrt{d-c^2dx^2}}$$

input
```
Int[(a + b*ArcCos[c*x])/((f + g*x)*Sqrt[d - c^2*d*x^2]),x]
```

output
```
(-2*Sqrt[1 - c^2*x^2]*((g*(((-I)*(a + b*ArcCos[c*x])*Log[1 + (E^(I*ArcCos[
c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2])])/g - (b*PolyLog[2, -((E^(I*ArcCos[c*
x])*g)/(c*f - Sqrt[c^2*f^2 - g^2]))])/g))/(2*Sqrt[c^2*f^2 - g^2]) - (g*(((
-I)*(a + b*ArcCos[c*x])*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2
- g^2])])/g - (b*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 -
g^2]))])/g))/(2*Sqrt[c^2*f^2 - g^2])))/Sqrt[d - c^2*d*x^2]
```

3.17. $\int \frac{a+b\arccos(cx)}{(f+gx)\sqrt{d-c^2dx^2}}\,dx$

3.17.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3802
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*E^(I*Pi*(k - 1/2))*(E^(I*(e +
f*x))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(
e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ
[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 5273
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/Sq
rt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[-(c^(m + 1)*Sqrt[d])^(-1)   Subs
t[Int[(a + b*x)^n*(c*f + g*Cos[x])^m, x], x, ArcCos[c*x]], x] /; FreeQ[{a,
b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && GtQ[d, 0] &
& (GtQ[m, 0] || IGtQ[n, 0])
```

rule 5277
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[Simp[(d + e*x^2)^p/(1 - c^2*x^2)^
p]   Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && Intege
rQ[p - 1/2] &&  !GtQ[d, 0]
```

3.17.4 Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 487, normalized size of antiderivative = 1.32

method	result
default	$-\frac{a\ln\left(\frac{-\frac{2d\left(c^2f^2-g^2\right)}{g^2}+\frac{2c^2df\left(x+\frac{f}{g}\right)}{g}+2\sqrt{-\frac{d\left(c^2f^2-g^2\right)}{g^2}}\sqrt{-\left(x+\frac{f}{g}\right)^2c^2d+\frac{2c^2df\left(x+\frac{f}{g}\right)}{g}-\frac{d\left(c^2f^2-g^2\right)}{g^2}}}{x+\frac{f}{g}}\right)}{g\sqrt{-\frac{d\left(c^2f^2-g^2\right)}{g^2}}}-\frac{b\sqrt{-d(c^2x^2-1)}\sqrt{-c^2x^2-}}{}$
parts	$-\frac{a\ln\left(\frac{-\frac{2d\left(c^2f^2-g^2\right)}{g^2}+\frac{2c^2df\left(x+\frac{f}{g}\right)}{g}+2\sqrt{-\frac{d\left(c^2f^2-g^2\right)}{g^2}}\sqrt{-\left(x+\frac{f}{g}\right)^2c^2d+\frac{2c^2df\left(x+\frac{f}{g}\right)}{g}-\frac{d\left(c^2f^2-g^2\right)}{g^2}}}{x+\frac{f}{g}}\right)}{g\sqrt{-\frac{d\left(c^2f^2-g^2\right)}{g^2}}}-\frac{b\sqrt{-d(c^2x^2-1)}\sqrt{-c^2x^2-}}{}$

input
```
int((a+b*arccos(c*x))/(g*x+f)/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBOSE
)
```

output
```
-a/g/(-d*(c^2*f^2-g^2)/g^2)^(1/2)*ln((-2*d*(c^2*f^2-g^2)/g^2+2*c^2*d*f/g*(
x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1/2)*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g
)-d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/g))-b*(-d*(c^2*x^2-1))^(1/2)/(c^2*f^2-g
^2)^(1/2)*(-c^2*x^2+1)^(1/2)*(I*arccos(c*x)*ln((-(c*x+I*(-c^2*x^2+1)^(1/2)
)*g-c*f+(c^2*f^2-g^2)^(1/2))/(-c*f+(c^2*f^2-g^2)^(1/2)))-I*arccos(c*x)*ln(
((c*x+I*(-c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2-g^2)^(1/2))/(c*f+(c^2*f^2-g^2)^
(1/2)))+dilog((-(c*x+I*(-c^2*x^2+1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/2))/(-c*
f+(c^2*f^2-g^2)^(1/2)))-dilog(((c*x+I*(-c^2*x^2+1)^(1/2))*g+c*f+(c^2*f^2-g
^2)^(1/2))/(c*f+(c^2*f^2-g^2)^(1/2))))/d/(c^2*x^2-1)
```

3.17.5 Fricas [F]

$$\int \frac{a+b\arccos(cx)}{(f+gx)\sqrt{d-c^2dx^2}}\,dx=\int \frac{b\arccos{(cx)}+a}{\sqrt{-c^2dx^2+d}(gx+f)}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(g*x+f)/(-c^2*d*x^2+d)^(1/2),x, algorithm="fri
cas")
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a)/(c^2*d*g*x^3 + c^2*d*f*
x^2 - d*g*x - d*f), x)
```

3.17.6 Sympy [F]

$$\int \frac{a+b\arccos(cx)}{(f+gx)\sqrt{d-c^2dx^2}}\,dx=\int \frac{a+b\,\mathrm{acos}\,(cx)}{\sqrt{-d\,(cx-1)\,(cx+1)}\,(f+gx)}\,dx$$

input
```
integrate((a+b*acos(c*x))/(g*x+f)/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*acos(c*x))/(sqrt(-d*(c*x - 1)*(c*x + 1))*(f + g*x)), x)
```

3.17.7 Maxima [F]

$$\int \frac{a + b\arccos(cx)}{(f + gx)\sqrt{d - c^2dx^2}}\,dx = \int \frac{b\arccos{(cx)} + a}{\sqrt{-c^2dx^2 + d}(gx + f)}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(g*x+f)/(-c^2*d*x^2+d)^(1/2),x, algorithm="max
ima")
```

output
```
integrate((b*arccos(c*x) + a)/(sqrt(-c^2*d*x^2 + d)*(g*x + f)), x)
```

3.17.8 Giac [F(-2)]

Exception generated.

$$\int \frac{a + b\arccos(cx)}{(f + gx)\sqrt{d - c^2dx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccos(c*x))/(g*x+f)/(-c^2*d*x^2+d)^(1/2),x, algorithm="gia
c")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.17.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\arccos(cx)}{(f + gx)\sqrt{d - c^2dx^2}}\,dx = \int \frac{a + b\,\mathrm{acos}(c\,x)}{(f + g\,x)\,\sqrt{d - c^2\,d\,x^2}}\,dx$$

input
```
int((a + b*acos(c*x))/((f + g*x)*(d - c^2*d*x^2)^(1/2)),x)
```

output
```
int((a + b*acos(c*x))/((f + g*x)*(d - c^2*d*x^2)^(1/2)), x)
```

3.18 $\int \frac{a+b\arccos(cx)}{(f+gx)^2\sqrt{d-c^2dx^2}}\,dx$

3.18.1 Optimal result

Integrand size = 31, antiderivative size = 496

$$\begin{aligned}\int \frac{a+b\arccos(cx)}{(f+gx)^2\sqrt{d-c^2dx^2}}\,dx &= \frac{g(1-c^2x^2)\,(a+b\arccos(cx))}{(c^2f^2-g^2)\,(f+gx)\sqrt{d-c^2dx^2}}\\ &+ \frac{ic^2f\sqrt{1-c^2x^2}(a+b\arccos(cx))\log\left(1+\frac{e^{i\arccos(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{(c^2f^2-g^2)^{3/2}\sqrt{d-c^2dx^2}}\\ &- \frac{ic^2f\sqrt{1-c^2x^2}(a+b\arccos(cx))\log\left(1+\frac{e^{i\arccos(cx)}g}{cf+\sqrt{c^2f^2-g^2}}\right)}{(c^2f^2-g^2)^{3/2}\sqrt{d-c^2dx^2}}\\ &+ \frac{bc\sqrt{1-c^2x^2}\log(f+gx)}{(c^2f^2-g^2)\sqrt{d-c^2dx^2}}\\ &+ \frac{bc^2f\sqrt{1-c^2x^2}\,\mathrm{PolyLog}\left(2,-\frac{e^{i\arccos(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{(c^2f^2-g^2)^{3/2}\sqrt{d-c^2dx^2}}\\ &- \frac{bc^2f\sqrt{1-c^2x^2}\,\mathrm{PolyLog}\left(2,-\frac{e^{i\arccos(cx)}g}{cf+\sqrt{c^2f^2-g^2}}\right)}{(c^2f^2-g^2)^{3/2}\sqrt{d-c^2dx^2}}\end{aligned}$$

output
```
g*(-c^2*x^2+1)*(a+b*arccos(c*x))/(c^2*f^2-g^2)/(g*x+f)/(-c^2*d*x^2+d)^(1/2
)+b*c*ln(g*x+f)*(-c^2*x^2+1)^(1/2)/(c^2*f^2-g^2)/(-c^2*d*x^2+d)^(1/2)+I*c^
2*f*(a+b*arccos(c*x))*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2-g^2)
^(1/2)))*(-c^2*x^2+1)^(1/2)/(c^2*f^2-g^2)^(3/2)/(-c^2*d*x^2+d)^(1/2)-I*c^2
*f*(a+b*arccos(c*x))*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^
(1/2)))*(-c^2*x^2+1)^(1/2)/(c^2*f^2-g^2)^(3/2)/(-c^2*d*x^2+d)^(1/2)+b*c^2*
f*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2-g^2)^(1/2)))*(-c^2
*x^2+1)^(1/2)/(c^2*f^2-g^2)^(3/2)/(-c^2*d*x^2+d)^(1/2)-b*c^2*f*polylog(2,-
(c*x+I*(-c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2)))*(-c^2*x^2+1)^(1/2)
/(c^2*f^2-g^2)^(3/2)/(-c^2*d*x^2+d)^(1/2)
```

3.18.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1108 vs. $2(496) = 992$.

Time = 5.90 (sec) , antiderivative size = 1108, normalized size of antiderivative = 2.23

$$\int \frac{a+b\arccos(cx)}{(f+gx)^2\sqrt{d-c^2dx^2}}\,dx = -\frac{ag\sqrt{d-c^2dx^2}}{d\left(-c^2f^2+g^2\right)(f+gx)} - \frac{ac^2f\log(f+gx)}{\sqrt{d}\left(-c^2f^2+g^2\right)^{3/2}}$$
$$-\frac{ac^2f\log\left(d(g+c^2fx)+\sqrt{d}\sqrt{-c^2f^2+g^2}\sqrt{d-c^2dx^2}\right)}{\sqrt{d}(cf-g)(cf+g)\sqrt{-c^2f^2+g^2}}$$
$$-\frac{bc\sqrt{1-c^2x^2}\left(-\frac{g\sqrt{1-c^2x^2}\arccos(cx)}{(cf-g)(cf+g)(cf+cgx)}-\frac{\log\left(1+\frac{gx}{f}\right)}{c^2f^2-g^2}-\frac{cf\left(2\arccos(cx)\text{arctanh}\left(\frac{(cf+g)\cot\left(\frac{1}{2}\arccos(cx)\right)}{\sqrt{-c^2f^2+g^2}}\right)-2\arccos\left(-\frac{cf}{g}\right)a\right.}{}\right.}{}$$

input
```
Integrate[(a + b*ArcCos[c*x])/((f + g*x)^2*Sqrt[d - c^2*d*x^2]),x]
```

output
```
-((a*g*Sqrt[d - c^2*d*x^2])/(d*(-(c^2*f^2) + g^2)*(f + g*x))) - (a*c^2*f*L
og[f + g*x])/(Sqrt[d]*(-(c^2*f^2) + g^2)^(3/2)) - (a*c^2*f*Log[d*(g + c^2*
f*x) + Sqrt[d]*Sqrt[-(c^2*f^2) + g^2]*Sqrt[d - c^2*d*x^2]])/(Sqrt[d]*(c*f
- g)*(c*f + g)*Sqrt[-(c^2*f^2) + g^2]) - (b*c*Sqrt[1 - c^2*x^2]*(-((g*Sqrt
[1 - c^2*x^2]*ArcCos[c*x])/((c*f - g)*(c*f + g)*(c*f + c*g*x))) - Log[1 +
(g*x)/f]/(c^2*f^2 - g^2) - (c*f*(2*ArcCos[c*x]*ArcTanh[((c*f + g)*Cot[ArcC
os[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] - 2*ArcCos[-((c*f)/g)]*ArcTanh[((-(c*f
) + g)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + (ArcCos[-((c*f)/g)] -
 (2*I)*ArcTanh[((c*f + g)*Cot[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]] + (2
*I)*ArcTanh[((-(c*f) + g)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])*Log
[Sqrt[-(c^2*f^2) + g^2]/(Sqrt[2]*E^((I/2)*ArcCos[c*x])*Sqrt[g]*Sqrt[c*(f +
 g*x)])] + (ArcCos[-((c*f)/g)] + (2*I)*(ArcTanh[((c*f + g)*Cot[ArcCos[c*x]
/2])/Sqrt[-(c^2*f^2) + g^2]] - ArcTanh[((-(c*f) + g)*Tan[ArcCos[c*x]/2])/S
qrt[-(c^2*f^2) + g^2]]))*Log[(E^((I/2)*ArcCos[c*x])*Sqrt[-(c^2*f^2) + g^2]
)/(Sqrt[2]*Sqrt[g]*Sqrt[c*(f + g*x)])] - (ArcCos[-((c*f)/g)] - (2*I)*ArcTa
nh[((-(c*f) + g)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g^2]])*Log[((c*f +
g)*((-I)*c*f + I*g + Sqrt[-(c^2*f^2) + g^2])*(-I + Tan[ArcCos[c*x]/2]))/(g
*(c*f + g + Sqrt[-(c^2*f^2) + g^2]*Tan[ArcCos[c*x]/2]))] - (ArcCos[-((c*f)
/g)] + (2*I)*ArcTanh[((-(c*f) + g)*Tan[ArcCos[c*x]/2])/Sqrt[-(c^2*f^2) + g
^2]])*Log[((c*f + g)*(I*c*f - I*g + Sqrt[-(c^2*f^2) + g^2])*(I + Tan[Ar...
```

3.18.3 Rubi [A] (verified)

Time = 1.36 (sec) , antiderivative size = 374, normalized size of antiderivative = 0.75, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.452$, Rules used = {5277, 5273, 3042, 3805, 25, 3042, 3147, 16, 3802, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos(cx)}{\sqrt{d-c^2dx^2}(f+gx)^2}\,dx$$

$$\Big\downarrow \text{5277}$$

$$\frac{\sqrt{1-c^2x^2}\int \frac{a+b\arccos(cx)}{(f+gx)^2\sqrt{1-c^2x^2}}dx}{\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{5273}$$

$$-\frac{c\sqrt{1-c^2x^2}\int \frac{a+b\arccos(cx)}{(cf+cgx)^2}d\arccos(cx)}{\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{c\sqrt{1-c^2x^2}\int \frac{a+b\arccos(cx)}{\left(cf+g\sin\left(\arccos(cx)+\frac{\pi}{2}\right)\right)^2}d\arccos(cx)}{\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \; 3805$$

$$-\frac{c\sqrt{1-c^2x^2}\left(\frac{cf\int \frac{a+b\arccos(cx)}{cf+cgx}d\arccos(cx)}{c^2f^2-g^2}-\frac{bg\int -\frac{\sqrt{1-c^2x^2}}{cf+cgx}d\arccos(cx)}{c^2f^2-g^2}-\frac{g\sqrt{1-c^2x^2}(a+b\arccos(cx))}{(c^2f^2-g^2)(cf+cgx)}\right)}{\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \; 25$$

$$-\frac{c\sqrt{1-c^2x^2}\left(\frac{cf\int \frac{a+b\arccos(cx)}{cf+cgx}d\arccos(cx)}{c^2f^2-g^2}+\frac{bg\int \frac{\sqrt{1-c^2x^2}}{cf+cgx}d\arccos(cx)}{c^2f^2-g^2}-\frac{g\sqrt{1-c^2x^2}(a+b\arccos(cx))}{(c^2f^2-g^2)(cf+cgx)}\right)}{\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{c\sqrt{1-c^2x^2}\left(\frac{cf\int \frac{a+b\arccos(cx)}{cf+g\sin\left(\arccos(cx)+\frac{\pi}{2}\right)}d\arccos(cx)}{c^2f^2-g^2}+\frac{bg\int \frac{\cos\left(\arccos(cx)-\frac{\pi}{2}\right)}{cf-g\sin\left(\arccos(cx)-\frac{\pi}{2}\right)}d\arccos(cx)}{c^2f^2-g^2}-\frac{g\sqrt{1-c^2x^2}(a+b\arccos(cx))}{(c^2f^2-g^2)(cf+cgx)}\right)}{\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \; 3147$$

$$-\frac{c\sqrt{1-c^2x^2}\left(\frac{cf\int \frac{a+b\arccos(cx)}{cf+g\sin\left(\arccos(cx)+\frac{\pi}{2}\right)}d\arccos(cx)}{c^2f^2-g^2}-\frac{b\int \frac{1}{cf+cgx}d(cgx)}{c^2f^2-g^2}-\frac{g\sqrt{1-c^2x^2}(a+b\arccos(cx))}{(c^2f^2-g^2)(cf+cgx)}\right)}{\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \; 16$$

$$-\frac{c\sqrt{1-c^2x^2}\left(\frac{cf\int \frac{a+b\arccos(cx)}{cf+g\sin\left(\arccos(cx)+\frac{\pi}{2}\right)}d\arccos(cx)}{c^2f^2-g^2}-\frac{g\sqrt{1-c^2x^2}(a+b\arccos(cx))}{(c^2f^2-g^2)(cf+cgx)}-\frac{b\log(cf+cgx)}{c^2f^2-g^2}\right)}{\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \; 3802$$

$$-\frac{c\sqrt{1-c^2x^2}\left(\frac{2cf\int \frac{e^{i\arccos(cx)}(a+b\arccos(cx))}{2ce^{i\arccos(cx)}f+e^{2i\arccos(cx)}g+g}d\arccos(cx)}{c^2f^2-g^2}-\frac{g\sqrt{1-c^2x^2}(a+b\arccos(cx))}{(c^2f^2-g^2)(cf+cgx)}-\frac{b\log(cf+cgx)}{c^2f^2-g^2}\right)}{\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \; 2694$$

3.18. $\displaystyle\int \frac{a+b\arccos(cx)}{(f+gx)^2\sqrt{d-c^2dx^2}}\,dx$

$$-\frac{c\sqrt{1-c^2x^2}\left(\frac{2cf\left(\frac{g\int \frac{e^{i\arccos(cx)}(a+b\arccos(cx))}{2\left(cf+e^{i\arccos(cx)}g-\sqrt{c^2f^2-g^2}\right)}d\arccos(cx)}{\sqrt{c^2f^2-g^2}}-\frac{g\int \frac{e^{i\arccos(cx)}(a+b\arccos(cx))}{2\left(cf+e^{i\arccos(cx)}g+\sqrt{c^2f^2-g^2}\right)}d\arccos(cx)}{\sqrt{c^2f^2-g^2}}\right)}{c^2f^2-g^2}-\frac{g\sqrt{1-c^2x^2}(a+b\,\mathrm{arc}}{(c^2f^2-g^2)(cf+}\right.}{\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{c\sqrt{1-c^2x^2}\left(\frac{2cf\left(\frac{g\int \frac{e^{i\arccos(cx)}(a+b\arccos(cx))}{cf+e^{i\arccos(cx)}g-\sqrt{c^2f^2-g^2}}d\arccos(cx)}{2\sqrt{c^2f^2-g^2}}-\frac{g\int \frac{e^{i\arccos(cx)}(a+b\arccos(cx))}{cf+e^{i\arccos(cx)}g+\sqrt{c^2f^2-g^2}}d\arccos(cx)}{2\sqrt{c^2f^2-g^2}}\right)}{c^2f^2-g^2}-\frac{g\sqrt{1-c^2x^2}(a+b\arccos(cx)}{(c^2f^2-g^2)(cf+cgx)}\right.}{\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{c\sqrt{1-c^2x^2}\left(\frac{2cf\left(\frac{g\left(\frac{ib\int \log\left(\frac{e^{i\arccos(cx)}g}{cf-\sqrt{c^2f^2-g^2}}+1\right)d\arccos(cx)}{g}-\frac{i(a+b\arccos(cx))\log\left(1+\frac{ge^{i\arccos(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{g}\right)}{2\sqrt{c^2f^2-g^2}}-\frac{g\left(\frac{ib\int \log\left(\frac{e^{i\arccos(cx)}g}{cf+\sqrt{c^2f^2-g^2}}+1\right)d\,\mathrm{arcc}}{g}\right.}{}\right.}{c^2f^2-g^2}\right.}{\sqrt{d-c^2dx^2}}$$

$$\Big\downarrow \text{2715}$$

3.18. $\int \frac{a+b\arccos(cx)}{(f+gx)^2\sqrt{d-c^2dx^2}}\,dx$

$$-\frac{c\sqrt{1-c^2x^2}\left(\frac{2cf\left(\frac{g\left(\frac{b\int e^{-i\arccos(cx)}\log\left(\frac{e^{i\arccos(cx)}g}{cf-\sqrt{c^2f^2-g^2}}+1\right)de^{i\arccos(cx)}}{g}-\frac{i(a+b\arccos(cx))\log\left(1+\frac{ge^{i\arccos(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{g}\right)}{2\sqrt{c^2f^2-g^2}}-\frac{g\left(\frac{b\int e^{-i\arccos(cx)}\cdots}{\cdots}\right)}{\cdots}\right)}{c^2f^2-g^2}\cdots\right)}{\sqrt{d-c^2d\cdots}}$$

$$\Big\downarrow \text{ 2838}$$

$$-\frac{c\sqrt{1-c^2x^2}\left(\frac{2cf\left(\frac{g\left(-\frac{i(a+b\arccos(cx))\log\left(1+\frac{ge^{i\arccos(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{g}-\frac{b\,\mathrm{PolyLog}\left(2,-\frac{e^{i\arccos(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{g}\right)}{2\sqrt{c^2f^2-g^2}}-\frac{g\left(-\frac{i(a+b\arccos(cx))\log\left(1+\frac{ge^{i\arccos(cx)}}{\sqrt{c^2f^2-g^2}+c\cdots}\right)}{g}\cdots\right)}{2\sqrt{c^2f^2\cdots}}\right)}{c^2f^2-g^2}\cdots\right)}{\sqrt{d-c^2dx^2}}$$

input
```
Int[(a + b*ArcCos[c*x])/((f + g*x)^2*Sqrt[d - c^2*d*x^2]),x]
```

output
```
-((c*Sqrt[1 - c^2*x^2]*(-((g*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/((c^2*
f^2 - g^2)*(c*f + c*g*x))) - (b*Log[c*f + c*g*x])/(c^2*f^2 - g^2) + (2*c*f
*((g*(((-I)*(a + b*ArcCos[c*x])*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[
c^2*f^2 - g^2])])/g - (b*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^
2*f^2 - g^2]))])/g))/(2*Sqrt[c^2*f^2 - g^2]) - (g*(((-I)*(a + b*ArcCos[c*x
])*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/g - (b*Poly
Log[2, -((E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2]))])/g))/(2*Sqrt[
c^2*f^2 - g^2])))/(c^2*f^2 - g^2)))/Sqrt[d - c^2*d*x^2])
```

3.18. $\int \frac{a+b\arccos(cx)}{(f+gx)^2\sqrt{d-c^2dx^2}}\,dx$

3.18.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 3802
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[2   Int[(c + d*x)^m*E^(I*Pi*(k - 1/2))*(E^(I*(e +
f*x))/(b + 2*a*E^(I*Pi*(k - 1/2))*E^(I*(e + f*x)) - b*E^(2*I*k*Pi)*E^(2*I*(
e + f*x)))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[2*k] && NeQ
[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3805
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Simp[a/(a^2 - b^2)   Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x]
, x] - Simp[b*d*(m/(f*(a^2 - b^2)))   Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(
a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 -
b^2, 0] && IGtQ[m, 0]
```

rule 5273
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.))/Sq
rt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[-(c^(m + 1)*Sqrt[d])^(-1)   Subs
t[Int[(a + b*x)^n*(c*f + g*Cos[x])^m, x], x, ArcCos[c*x]], x] /; FreeQ[{a,
b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && GtQ[d, 0] &
& (GtQ[m, 0] || IGtQ[n, 0])
```

rule 5277
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_) + (g_.)*(x_))^(m_.)*((d_
) + (e_.)*(x_)^2)^(p_), x_Symbol] :> Simp[Simp[(d + e*x^2)^p/(1 - c^2*x^2)^
p]   Int[(f + g*x)^m*(1 - c^2*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] /; FreeQ
[{a, b, c, d, e, f, g, n}, x] && EqQ[c^2*d + e, 0] && IntegerQ[m] && Intege
rQ[p - 1/2] &&  !GtQ[d, 0]
```

3.18. $\int \frac{a+b\arccos(cx)}{(f+gx)^2\sqrt{d-c^2dx^2}}\,dx$

3.18.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1621 vs. 2(492) = 984.

Time = 2.68 (sec) , antiderivative size = 1622, normalized size of antiderivative = 3.27

method	result	size
default	Expression too large to display	1622
parts	Expression too large to display	1622

input
```
int((a+b*arccos(c*x))/(g*x+f)^2/(-c^2*d*x^2+d)^(1/2),x,method=_RETURNVERBO
SE)
```

output
```
a/d/(c^2*f^2-g^2)/(x+f/g)*(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2
-g^2)/g^2)^(1/2)-a/g*c^2*f/(c^2*f^2-g^2)/(-d*(c^2*f^2-g^2)/g^2)^(1/2)*ln((
-2*d*(c^2*f^2-g^2)/g^2+2*c^2*d*f/g*(x+f/g)+2*(-d*(c^2*f^2-g^2)/g^2)^(1/2)*
(-(x+f/g)^2*c^2*d+2*c^2*d*f/g*(x+f/g)-d*(c^2*f^2-g^2)/g^2)^(1/2))/(x+f/g))
+b*(-d*(c^2*x^2-1))^(1/2)*arccos(c*x)/d/(c^2*x^2-1)/(c^2*f^2-g^2)/(g*x+f)*
(-c^2*x^2+1)*x*c^2*f+b*(-d*(c^2*x^2-1))^(1/2)*arccos(c*x)/d/(c^2*x^2-1)/(c
^2*f^2-g^2)/(g*x+f)*x^3*c^4*f+I*b*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2
)/d/(c^2*x^2-1)/(c^2*f^2-g^2)^(3/2)*c^2*arccos(c*x)*ln(((c*x+I*(-c^2*x^2+1
)^(1/2))*g+c*f+(c^2*f^2-g^2)^(1/2))/(c*f+(c^2*f^2-g^2)^(1/2)))*f+b*(-d*(c^
2*x^2-1))^(1/2)*arccos(c*x)/d/(c^2*x^2-1)/(c^2*f^2-g^2)/(g*x+f)*x^2*c^2*g-
I*b*(-d*(c^2*x^2-1))^(1/2)*arccos(c*x)/d/(c^2*x^2-1)/(c^2*f^2-g^2)/(g*x+f)
*(-c^2*x^2+1)^(1/2)*c*f-b*(-d*(c^2*x^2-1))^(1/2)*arccos(c*x)/d/(c^2*x^2-1)
/(c^2*f^2-g^2)/(g*x+f)*x*c^2*f-b*(-d*(c^2*x^2-1))^(1/2)*arccos(c*x)/d/(c^2
*x^2-1)/(c^2*f^2-g^2)/(g*x+f)*g-I*b*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1
/2)/d/(c^2*x^2-1)/(c^2*f^2-g^2)^(3/2)*c^2*arccos(c*x)*ln((-(c*x+I*(-c^2*x^
2+1)^(1/2))*g-c*f+(c^2*f^2-g^2)^(1/2))/(-c*f+(c^2*f^2-g^2)^(1/2)))*f-I*b*(
-d*(c^2*x^2-1))^(1/2)*arccos(c*x)/d/(c^2*x^2-1)/(c^2*f^2-g^2)/(g*x+f)*(-c^
2*x^2+1)^(1/2)*x*c*g+2*b*(-c^2*x^2+1)^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d/(c^2*
x^2-1)/(c^2*f^2-g^2)^2*c^3*ln(c*x+I*(-c^2*x^2+1)^(1/2))*f^2-b*(-c^2*x^2+1)
^(1/2)*(-d*(c^2*x^2-1))^(1/2)/d/(c^2*x^2-1)/(c^2*f^2-g^2)^2*c^3*ln((c*x...
```

3.18.5 Fricas [F]

$$\int \frac{a+b\arccos(cx)}{(f+gx)^2\sqrt{d-c^2dx^2}}\,dx=\int \frac{b\arccos{(cx)}+a}{\sqrt{-c^2dx^2+d}(gx+f)^2}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(g*x+f)^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="f
ricas")
```

output
```
integral(-sqrt(-c^2*d*x^2 + d)*(b*arccos(c*x) + a)/(c^2*d*g^2*x^4 + 2*c^2*
d*f*g*x^3 - 2*d*f*g*x - d*f^2 + (c^2*d*f^2 - d*g^2)*x^2), x)
```

3.18.6 Sympy [F]

$$\int \frac{a+b\arccos(cx)}{(f+gx)^2\sqrt{d-c^2dx^2}}\,dx=\int \frac{a+b\operatorname{acos}{(cx)}}{\sqrt{-d\,(cx-1)\,(cx+1)}\,(f+gx)^2}\,dx$$

input
```
integrate((a+b*acos(c*x))/(g*x+f)**2/(-c**2*d*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*acos(c*x))/(sqrt(-d*(c*x - 1)*(c*x + 1))*(f + g*x)**2), x)
```

3.18.7 Maxima [F]

$$\int \frac{a+b\arccos(cx)}{(f+gx)^2\sqrt{d-c^2dx^2}}\,dx=\int \frac{b\arccos{(cx)}+a}{\sqrt{-c^2dx^2+d}(gx+f)^2}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(g*x+f)^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="m
axima")
```

output
```
integrate((b*arccos(c*x) + a)/(sqrt(-c^2*d*x^2 + d)*(g*x + f)^2), x)
```

3.18.8 Giac [F]

$$\int \frac{a+b\arccos(cx)}{(f+gx)^2\sqrt{d-c^2dx^2}}\,dx = \int \frac{b\arccos{(cx)}+a}{\sqrt{-c^2dx^2+d}(gx+f)^2}\,dx$$

input
```
integrate((a+b*arccos(c*x))/(g*x+f)^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="g
iac")
```

output
```
integrate((b*arccos(c*x) + a)/(sqrt(-c^2*d*x^2 + d)*(g*x + f)^2), x)
```

3.18.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{(f+gx)^2\sqrt{d-c^2dx^2}}\,dx = \int \frac{a+b\,\mathrm{acos}(c\,x)}{\left(f+g\,x\right)^2\,\sqrt{d-c^2\,d\,x^2}}\,dx$$

input
```
int((a + b*acos(c*x))/((f + g*x)^2*(d - c^2*d*x^2)^(1/2)),x)
```

output
```
int((a + b*acos(c*x))/((f + g*x)^2*(d - c^2*d*x^2)^(1/2)), x)
```

3.19 $\int \frac{(a+b\arccos(cx))^n\log(h(f+gx)^m)}{\sqrt{1-c^2x^2}}\,dx$

3.19.1 Optimal result

Integrand size = 35, antiderivative size = 35

$$\int \frac{(a+b\arccos(cx))^n\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx=\text{Int}\left(\frac{(a+b\arccos(cx))^n\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}},x\right)$$

output
```
Unintegrable((a+b*arccos(c*x))^n*ln(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x)
```

3.19.2 Mathematica [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{(a+b\arccos(cx))^n\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx=\int \frac{(a+b\arccos(cx))^n\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx$$

input
```
Integrate[((a + b*ArcCos[c*x])^n*Log[h*(f + g*x)^m])/Sqrt[1 - c^2*x^2],x]
```

output
```
Integrate[((a + b*ArcCos[c*x])^n*Log[h*(f + g*x)^m])/Sqrt[1 - c^2*x^2], x]
```

3.19.3 Rubi [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {5301}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^n \log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx$$
$$\Big\downarrow \ 5301$$
$$\int \frac{(a+b\arccos(cx))^n \log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}dx$$

input
```
Int[((a + b*ArcCos[c*x])^n*Log[h*(f + g*x)^m])/Sqrt[1 - c^2*x^2],x]
```

output
```
$Aborted
```

3.19.3.1 Defintions of rubi rules used

rule 5301
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcCos[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.19.4 Maple [N/A] (verified)

Not integrable

Time = 22.42 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{(a+b\arccos(cx))^n \ln\left(h(gx+f)^m\right)}{\sqrt{-c^2x^2+1}}dx$$

input
```
int((a+b*arccos(c*x))^n*ln(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x)
```

output
```
int((a+b*arccos(c*x))^n*ln(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x)
```

3.19.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.34

$$\int \frac{(a+b\arccos(cx))^n \log(h(f+gx)^m)}{\sqrt{1-c^2x^2}}\,dx = \int \frac{(b\arccos(cx)+a)^n \log((gx+f)^m h)}{\sqrt{-c^2x^2+1}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^n*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algor
ithm="fricas")
```

output
```
integral(-sqrt(-c^2*x^2 + 1)*(b*arccos(c*x) + a)^n*log((g*x + f)^m*h)/(c^2
*x^2 - 1), x)
```

3.19.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^n \log(h(f+gx)^m)}{\sqrt{1-c^2x^2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*acos(c*x))**n*ln(h*(g*x+f)**m)/(-c**2*x**2+1)**(1/2),x)
```

output
```
Timed out
```

3.19.7 Maxima [N/A]

Not integrable

Time = 1.99 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arccos(cx))^n \log(h(f+gx)^m)}{\sqrt{1-c^2x^2}}\,dx = \int \frac{(b\arccos(cx)+a)^n \log((gx+f)^m h)}{\sqrt{-c^2x^2+1}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^n*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algor
ithm="maxima")
```

output
```
integrate((b*arccos(c*x) + a)^n*log((g*x + f)^m*h)/sqrt(-c^2*x^2 + 1), x)
```

3.19.8 Giac [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arccos(cx))^n\log(h(f+gx)^m)}{\sqrt{1-c^2x^2}}\,dx=\int \frac{(b\arccos(cx)+a)^n\log((gx+f)^mh)}{\sqrt{-c^2x^2+1}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^n*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algor
ithm="giac")
```

output
```
integrate((b*arccos(c*x) + a)^n*log((g*x + f)^m*h)/sqrt(-c^2*x^2 + 1), x)
```

3.19.9 Mupad [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arccos(cx))^n\log(h(f+gx)^m)}{\sqrt{1-c^2x^2}}\,dx=\int \frac{\ln\left(h\,{\left(f+g\,x\right)}^m\right)\,{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}^n}{\sqrt{1-c^2\,x^2}}\,dx$$

input
```
int((log(h*(f + g*x)^m)*(a + b*acos(c*x))^n)/(1 - c^2*x^2)^(1/2),x)
```

output
```
int((log(h*(f + g*x)^m)*(a + b*acos(c*x))^n)/(1 - c^2*x^2)^(1/2), x)
```

3.20 $\int \frac{(a+b\arccos(cx))^2\log(h(f+gx)^m)}{\sqrt{1-c^2x^2}}\,dx$

3.20.1 Optimal result

Integrand size = 35, antiderivative size = 496

$$
\begin{aligned}
&\int \frac{(a+b\arccos(cx))^2\log(h(f+gx)^m)}{\sqrt{1-c^2x^2}}\,dx \\
&= -\frac{im(a+b\arccos(cx))^4}{12b^2c}+\frac{m(a+b\arccos(cx))^3\log\left(1+\frac{e^{i\arccos(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{3bc} \\
&\quad+\frac{m(a+b\arccos(cx))^3\log\left(1+\frac{e^{i\arccos(cx)}g}{cf+\sqrt{c^2f^2-g^2}}\right)}{3bc}-\frac{(a+b\arccos(cx))^3\log(h(f+gx)^m)}{3bc} \\
&\quad-\frac{im(a+b\arccos(cx))^2\operatorname{PolyLog}\left(2,-\frac{e^{i\arccos(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{c} \\
&\quad-\frac{im(a+b\arccos(cx))^2\operatorname{PolyLog}\left(2,-\frac{e^{i\arccos(cx)}g}{cf+\sqrt{c^2f^2-g^2}}\right)}{c} \\
&\quad+\frac{2bm(a+b\arccos(cx))\operatorname{PolyLog}\left(3,-\frac{e^{i\arccos(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{c} \\
&\quad+\frac{2bm(a+b\arccos(cx))\operatorname{PolyLog}\left(3,-\frac{e^{i\arccos(cx)}g}{cf+\sqrt{c^2f^2-g^2}}\right)}{c} \\
&\quad+\frac{2ib^2m\operatorname{PolyLog}\left(4,-\frac{e^{i\arccos(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{c}+\frac{2ib^2m\operatorname{PolyLog}\left(4,-\frac{e^{i\arccos(cx)}g}{cf+\sqrt{c^2f^2-g^2}}\right)}{c}
\end{aligned}
$$

output
```
-1/12*I*m*(a+b*arccos(c*x))^4/b^2/c-1/3*(a+b*arccos(c*x))^3*ln(h*(g*x+f)^m
)/b/c+1/3*m*(a+b*arccos(c*x))^3*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))*g/(c*f-(c^
2*f^2-g^2)^(1/2)))/b/c+1/3*m*(a+b*arccos(c*x))^3*ln(1+(c*x+I*(-c^2*x^2+1)^
(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2)))/b/c-I*m*(a+b*arccos(c*x))^2*polylog(2,
-(c*x+I*(-c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2-g^2)^(1/2)))/c-I*m*(a+b*arccos
(c*x))^2*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2))
)/c+2*b*m*(a+b*arccos(c*x))*polylog(3,-(c*x+I*(-c^2*x^2+1)^(1/2))*g/(c*f-(
c^2*f^2-g^2)^(1/2)))/c+2*b*m*(a+b*arccos(c*x))*polylog(3,-(c*x+I*(-c^2*x^2
+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2)))/c+2*I*b^2*m*polylog(4,-(c*x+I*(-c^
2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2-g^2)^(1/2)))/c+2*I*b^2*m*polylog(4,-(c*x+I
*(-c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2)))/c
```

3.20.2 Mathematica [F]

$$\int \frac{(a+b\arccos(cx))^2\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx=\int \frac{(a+b\arccos(cx))^2\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx$$

input
```
Integrate[((a + b*ArcCos[c*x])^2*Log[h*(f + g*x)^m])/Sqrt[1 - c^2*x^2],x]
```

output
```
Integrate[((a + b*ArcCos[c*x])^2*Log[h*(f + g*x)^m])/Sqrt[1 - c^2*x^2], x]
```

3.20.3 Rubi [A] (verified)

Time = 1.70 (sec) , antiderivative size = 508, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {5279, 5241, 5031, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))^2\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx$$

$$\Big\downarrow \text{5279}$$

$$\frac{gm\int \frac{(a+b\arccos(cx))^3}{f+gx}dx}{3bc}-\frac{(a+b\arccos(cx))^3\log\left(h(f+gx)^m\right)}{3bc}$$

$$\Big\downarrow \text{5241}$$

$$-\frac{gm\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^3}{cf+cgx}d\arccos(cx)}{3bc}-\frac{(a+b\arccos(cx))^3\log\left(h(f+gx)^m\right)}{3bc}$$

$\Big\downarrow$ 5031

$$-\frac{(a+b\arccos(cx))^3\log\left(h(f+gx)^m\right)}{3bc}-\frac{gm\left(-i\int \frac{e^{i\arccos(cx)}(a+b\arccos(cx))^3}{cf+e^{i\arccos(cx)}g-\sqrt{c^2f^2-g^2}}d\arccos(cx)-i\int \frac{e^{i\arccos(cx)}(a+b\arccos(cx))^3}{cf+e^{i\arccos(cx)}g+\sqrt{c^2f^2-g^2}}d\arccos(cx)+\frac{i(a+b\arccos(cx))^4}{4bg}\right)}{3bc}$$

$\Big\downarrow$ 2620

$$-\frac{(a+b\arccos(cx))^3\log\left(h(f+gx)^m\right)}{3bc}-\frac{gm\left(-i\left(\frac{3ib\int(a+b\arccos(cx))^2\log\left(\frac{e^{i\arccos(cx)}g}{cf-\sqrt{c^2f^2-g^2}}+1\right)d\arccos(cx)}{g}-\frac{i(a+b\arccos(cx))^3\log\left(1+\frac{ge^{i\arccos(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{g}\right)-i\left(\frac{3ib\int(a+b\arcc}{}\right.\right.}{3bc}$$

$\Big\downarrow$ 3011

$$-\frac{(a+b\arccos(cx))^3\log\left(h(f+gx)^m\right)}{3bc}-\frac{gm\left(-i\left(\frac{3ib\left(i(a+b\arccos(cx))^2\text{PolyLog}\left(2,-\frac{e^{i\arccos(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)-2ib\int(a+b\arccos(cx))\text{PolyLog}\left(2,-\frac{e^{i\arccos(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)d\arccos(cx)\right)}{g}-\frac{i(a+b}{}\right.\right.}{}$$

$\Big\downarrow$ 7163

$$-\frac{(a+b\arccos(cx))^3\log\left(h(f+gx)^m\right)}{3bc}-\frac{gm\left(-i\left(\frac{3ib\left(i(a+b\arccos(cx))^2\text{PolyLog}\left(2,-\frac{e^{i\arccos(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)-2ib\left(ib\int\text{PolyLog}\left(3,-\frac{e^{i\arccos(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)d\arccos(cx)-i(a+b\arccos(cx))\text{PolyL}\right.\right.}{g}\right.\right.}{}$$

$\Big\downarrow$ 2720

$$-\frac{(a+b\arccos(cx))^3\log\left(h(f+gx)^m\right)}{3bc}-\frac{gm\left(-i\left(\frac{3ib\left(i(a+b\arccos(cx))^2\text{PolyLog}\left(2,-\frac{e^{i\arccos(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)-2ib\left(b\int e^{-i\arccos(cx)}\text{PolyLog}\left(3,-\frac{e^{i\arccos(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)de^{i\arccos(cx)}-i(a+b\arcc\right.\right.}{g}\right.\right.}{}$$

$\Big\downarrow$ 7143

3.20. $\int \frac{(a+b\arccos(cx))^2\log(h(f+gx)^m)}{\sqrt{1-c^2x^2}}\,dx$

$$-\frac{(a+b\arccos(cx))^3\log\left(h(f+gx)^m\right)}{3bc}-$$

$$gm\Bigg(-i\Bigg(\frac{3ib\Big(i(a+b\arccos(cx))^2\,\mathrm{PolyLog}\Big(2,-\frac{e^{i\arccos(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\Big)-2ib\Big(b\,\mathrm{PolyLog}\Big(4,-\frac{e^{i\arccos(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\Big)-i(a+b\arccos(cx))\,\mathrm{PolyLog}\Big(3,-\frac{e^{i\arccos(cx)}}{cf-\sqrt{c}}}{g}$$

input
```
Int[((a + b*ArcCos[c*x])^2*Log[h*(f + g*x)^m])/Sqrt[1 - c^2*x^2],x]
```

output
```
-1/3*((a + b*ArcCos[c*x])^3*Log[h*(f + g*x)^m])/(b*c) - (g*m*(((I/4)*(a +
b*ArcCos[c*x])^4)/(b*g) - I*(((-I)*(a + b*ArcCos[c*x])^3*Log[1 + (E^(I*Arc
Cos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2])])/g + ((3*I)*b*(I*(a + b*ArcCos[c
*x])^2*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2]))] -
(2*I)*b*((-I)*(a + b*ArcCos[c*x])*PolyLog[3, -((E^(I*ArcCos[c*x])*g)/(c*f
- Sqrt[c^2*f^2 - g^2]))] + b*PolyLog[4, -((E^(I*ArcCos[c*x])*g)/(c*f - Sqr
t[c^2*f^2 - g^2]))])))/g) - I*(((-I)*(a + b*ArcCos[c*x])^3*Log[1 + (E^(I*A
rcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/g + ((3*I)*b*(I*(a + b*ArcCos
[c*x])^2*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2]))]
- (2*I)*b*((-I)*(a + b*ArcCos[c*x])*PolyLog[3, -((E^(I*ArcCos[c*x])*g)/(c*
f + Sqrt[c^2*f^2 - g^2]))] + b*PolyLog[4, -((E^(I*ArcCos[c*x])*g)/(c*f + S
qrt[c^2*f^2 - g^2]))])))/g)))/(3*b*c)
```

3.20.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5031
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)])/(Cos[(c_.) + (d_.)
*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[I*((e + f*x)^(m + 1)/(b*f*(m + 1)))
, x] + (-Simp[I   Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] +
b*E^(I*(c + d*x)))), x], x] - Simp[I   Int[(e + f*x)^m*(E^(I*(c + d*x))/(a
+ Rt[a^2 - b^2, 2] + b*E^(I*(c + d*x)))), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[m, 0] && PosQ[a^2 - b^2]
```

rule 5241
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Subst[Int[(a + b*x)^n*(Sin[x]/(c*d + e*Cos[x])), x], x, ArcCos[c*x]] /
; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

rule 5279
```
Int[(Log[(h_.)*((f_.) + (g_.)*(x_))^(m_.)]*((a_.) + ArcCos[(c_.)*(x_)]*(b_.
))^(n_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[(-Log[h*(f + g*x)^m]
)*((a + b*ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1))), x] + Simp[g*(m/(b*c*
Sqrt[d]*(n + 1)))   Int[(a + b*ArcCos[c*x])^(n + 1)/(f + g*x), x], x] /; Fr
eeQ[{a, b, c, d, e, f, g, h, m}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && IG
tQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.20. $\int \frac{(a+b\arccos(cx))^2\log(h(f+gx)^m)}{\sqrt{1-c^2x^2}}\,dx$

3.20.4 Maple [F]

$$\int \frac{(a+b\arccos(cx))^2 \ln(h(gx+f)^m)}{\sqrt{-c^2x^2+1}}dx$$

input
```
int((a+b*arccos(c*x))^2*ln(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x)
```

output
```
int((a+b*arccos(c*x))^2*ln(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x)
```

3.20.5 Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2 \log(h(f+gx)^m)}{\sqrt{1-c^2x^2}}\,dx = \int \frac{(b\arccos(cx)+a)^2 \log((gx+f)^m h)}{\sqrt{-c^2x^2+1}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algor
ithm="fricas")
```

output
```
integral(-sqrt(-c^2*x^2 + 1)*(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)
*log((g*x + f)^m*h)/(c^2*x^2 - 1), x)
```

3.20.6 Sympy [F]

$$\int \frac{(a+b\arccos(cx))^2 \log(h(f+gx)^m)}{\sqrt{1-c^2x^2}}\,dx = \int \frac{(a+b\operatorname{acos}(cx))^2 \log(h(f+gx)^m)}{\sqrt{-(cx-1)(cx+1)}}\,dx$$

input
```
integrate((a+b*acos(c*x))**2*ln(h*(g*x+f)**m)/(-c**2*x**2+1)**(1/2),x)
```

output
```
Integral((a + b*acos(c*x))**2*log(h*(f + g*x)**m)/sqrt(-(c*x - 1)*(c*x + 1
)), x)
```

3.20.7 Maxima [F]

$$\int \frac{(a+b\arccos(cx))^2\log(h(f+gx)^m)}{\sqrt{1-c^2x^2}}\,dx = \int \frac{(b\arccos(cx)+a)^2\log((gx+f)^m h)}{\sqrt{-c^2x^2+1}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algor
ithm="maxima")
```

output
```
(b^2*c*integrate(arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2/(sqrt(c*x +
1)*sqrt(-c*x + 1)), x)*log(h) + 2*a*b*c*integrate(arctan2(sqrt(c*x + 1)*sq
rt(-c*x + 1), c*x)/(sqrt(c*x + 1)*sqrt(-c*x + 1)), x)*log(h) + b^2*c*integ
rate(arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2*log((g*x + f)^m)/(sqrt(c
*x + 1)*sqrt(-c*x + 1)), x) + 2*a*b*c*integrate(arctan2(sqrt(c*x + 1)*sqrt
(-c*x + 1), c*x)*log((g*x + f)^m)/(sqrt(c*x + 1)*sqrt(-c*x + 1)), x) + a^2
*c*integrate(log((g*x + f)^m)/(sqrt(c*x + 1)*sqrt(-c*x + 1)), x) + a^2*arc
tan2(c*x, sqrt(-c^2*x^2 + 1))*log(h))/c
```

3.20.8 Giac [F]

$$\int \frac{(a+b\arccos(cx))^2\log(h(f+gx)^m)}{\sqrt{1-c^2x^2}}\,dx = \int \frac{(b\arccos(cx)+a)^2\log((gx+f)^m h)}{\sqrt{-c^2x^2+1}}\,dx$$

input
```
integrate((a+b*arccos(c*x))^2*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algor
ithm="giac")
```

output
```
integrate((b*arccos(c*x) + a)^2*log((g*x + f)^m*h)/sqrt(-c^2*x^2 + 1), x)
```

3.20.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2\log(h(f+gx)^m)}{\sqrt{1-c^2x^2}}\,dx = \int \frac{\ln\left(h\,(f+g\,x)^m\right)\,\left(a+b\,\mathrm{acos}(c\,x)\right)^2}{\sqrt{1-c^2\,x^2}}\,dx$$

input
```
int((log(h*(f + g*x)^m)*(a + b*acos(c*x))^2)/(1 - c^2*x^2)^(1/2),x)
```

output
```
int((log(h*(f + g*x)^m)*(a + b*acos(c*x))^2)/(1 - c^2*x^2)^(1/2), x)
```

3.21 $\int \frac{(a+b\arccos(cx))\log(h(f+gx)^m)}{\sqrt{1-c^2x^2}}\,dx$

3.21.1 Optimal result

Integrand size = 33, antiderivative size = 374

$$\begin{aligned}
\int \frac{(a+b\arccos(cx))\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx
&= -\frac{im(a+b\arccos(cx))^3}{6b^2c}+\frac{m(a+b\arccos(cx))^2\log\left(1+\frac{e^{i\arccos(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{2bc}\\
&\quad+\frac{m(a+b\arccos(cx))^2\log\left(1+\frac{e^{i\arccos(cx)}g}{cf+\sqrt{c^2f^2-g^2}}\right)}{2bc}-\frac{(a+b\arccos(cx))^2\log\left(h(f+gx)^m\right)}{2bc}\\
&\quad-\frac{im(a+b\arccos(cx))\operatorname{PolyLog}\left(2,-\frac{e^{i\arccos(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{c}\\
&\quad-\frac{im(a+b\arccos(cx))\operatorname{PolyLog}\left(2,-\frac{e^{i\arccos(cx)}g}{cf+\sqrt{c^2f^2-g^2}}\right)}{c}\\
&\quad+\frac{bm\operatorname{PolyLog}\left(3,-\frac{e^{i\arccos(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{c}+\frac{bm\operatorname{PolyLog}\left(3,-\frac{e^{i\arccos(cx)}g}{cf+\sqrt{c^2f^2-g^2}}\right)}{c}
\end{aligned}$$

output
```
-1/6*I*m*(a+b*arccos(c*x))^3/b^2/c-1/2*(a+b*arccos(c*x))^2*ln(h*(g*x+f)^m)
/b/c+1/2*m*(a+b*arccos(c*x))^2*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))*g/(c*f-(c^2
*f^2-g^2)^(1/2)))/b/c+1/2*m*(a+b*arccos(c*x))^2*ln(1+(c*x+I*(-c^2*x^2+1)^(
1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2)))/b/c-I*m*(a+b*arccos(c*x))*polylog(2,-(c
*x+I*(-c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2-g^2)^(1/2)))/c-I*m*(a+b*arccos(c*
x))*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2)))/c+b
*m*polylog(3,-(c*x+I*(-c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2-g^2)^(1/2)))/c+b*
m*polylog(3,-(c*x+I*(-c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2)))/c
```

3.21.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1248 vs. 2(374) = 748.

Time = 6.29 (sec) , antiderivative size = 1248, normalized size of antiderivative = 3.34

$$\int \frac{(a+b\arccos(cx))\log(h(f+gx)^m)}{\sqrt{1-c^2x^2}}\,dx$$
$$= \frac{-3iam\arccos(cx)^2 - ibm\arccos(cx)^3 + 24iam\arcsin\left(\frac{\sqrt{1+\frac{cf}{g}}}{\sqrt{2}}\right)\arctan\left(\frac{(cf-g)\tan\left(\frac{1}{2}\arccos(cx)\right)}{\sqrt{c^2f^2-g^2}}\right) + 3bm\operatorname{arcc}}{}$$

input
```
Integrate[((a + b*ArcCos[c*x])*Log[h*(f + g*x)^m])/Sqrt[1 - c^2*x^2],x]
```

output
```
((-3*I)*a*m*ArcCos[c*x]^2 - I*b*m*ArcCos[c*x]^3 + (24*I)*a*m*ArcSin[Sqrt[1
 + (c*f)/g]/Sqrt[2]]*ArcTan[((c*f - g)*Tan[ArcCos[c*x]/2])/Sqrt[c^2*f^2 -
g^2]] + 3*b*m*ArcCos[c*x]^2*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*
f^2 - g^2])] + 6*a*m*ArcCos[c*x]*Log[1 + (E^(I*ArcCos[c*x])*(c*f - Sqrt[c^
2*f^2 - g^2]))/g] + 3*b*m*ArcCos[c*x]^2*Log[1 + (E^(I*ArcCos[c*x])*(c*f -
Sqrt[c^2*f^2 - g^2]))/g] + 12*a*m*ArcSin[Sqrt[1 + (c*f)/g]/Sqrt[2]]*Log[1
+ (E^(I*ArcCos[c*x])*(c*f - Sqrt[c^2*f^2 - g^2]))/g] + 12*b*m*ArcCos[c*x]*
ArcSin[Sqrt[1 + (c*f)/g]/Sqrt[2]]*Log[1 + (E^(I*ArcCos[c*x])*(c*f - Sqrt[c
^2*f^2 - g^2]))/g] + 3*b*m*ArcCos[c*x]^2*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*
f + Sqrt[c^2*f^2 - g^2])] + 6*a*m*ArcCos[c*x]*Log[1 + (E^(I*ArcCos[c*x])*(
c*f + Sqrt[c^2*f^2 - g^2]))/g] + 3*b*m*ArcCos[c*x]^2*Log[1 + (E^(I*ArcCos[
c*x])*(c*f + Sqrt[c^2*f^2 - g^2]))/g] - 12*a*m*ArcSin[Sqrt[1 + (c*f)/g]/Sq
rt[2]]*Log[1 + (E^(I*ArcCos[c*x])*(c*f + Sqrt[c^2*f^2 - g^2]))/g] - 12*b*m
*ArcCos[c*x]*ArcSin[Sqrt[1 + (c*f)/g]/Sqrt[2]]*Log[1 + (E^(I*ArcCos[c*x])*
(c*f + Sqrt[c^2*f^2 - g^2]))/g] - 6*a*m*ArcCos[c*x]*Log[f + g*x] - 6*a*m*A
rcSin[c*x]*Log[f + g*x] - 3*b*ArcCos[c*x]^2*Log[h*(f + g*x)^m] + 6*a*ArcSi
n[c*x]*Log[h*(f + g*x)^m] - 3*b*m*ArcCos[c*x]^2*Log[1 + ((c*f - Sqrt[c^2*f
^2 - g^2])*(c*x + I*Sqrt[1 - c^2*x^2]))/g] - 12*b*m*ArcCos[c*x]*ArcSin[Sqr
t[1 + (c*f)/g]/Sqrt[2]]*Log[1 + ((c*f - Sqrt[c^2*f^2 - g^2])*(c*x + I*Sqrt
[1 - c^2*x^2]))/g] - 3*b*m*ArcCos[c*x]^2*Log[1 + ((c*f + Sqrt[c^2*f^2 -...
```

3.21.3 Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {5279, 5241, 5031, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arccos(cx))\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx$$

$$\Big\downarrow \text{5279}$$

$$\frac{gm\int \frac{(a+b\arccos(cx))^2}{f+gx}dx}{2bc}-\frac{(a+b\arccos(cx))^2\log\left(h(f+gx)^m\right)}{2bc}$$

$$\Big\downarrow \text{5241}$$

$$-\frac{gm\int \frac{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{cf+cgx}d\arccos(cx)}{2bc}-\frac{(a+b\arccos(cx))^2\log\left(h(f+gx)^m\right)}{2bc}$$

$$\Big\downarrow \text{5031}$$

$$-\frac{(a+b\arccos(cx))^2\log\left(h(f+gx)^m\right)}{2bc}-\frac{gm\left(-i\int \frac{e^{i\arccos(cx)}(a+b\arccos(cx))^2}{cf+e^{i\arccos(cx)}g-\sqrt{c^2f^2-g^2}}d\arccos(cx)-i\int \frac{e^{i\arccos(cx)}(a+b\arccos(cx))^2}{cf+e^{i\arccos(cx)}g+\sqrt{c^2f^2-g^2}}d\arccos(cx)+\frac{i(a+b\arccos(cx))^3}{3bg}\right)}{2bc}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{(a+b\arccos(cx))^2\log\left(h(f+gx)^m\right)}{2bc}-\frac{gm\left(-i\left(\frac{2ib\int(a+b\arccos(cx))\log\left(\frac{e^{i\arccos(cx)}g}{cf-\sqrt{c^2f^2-g^2}}+1\right)d\arccos(cx)}{g}-\frac{i(a+b\arccos(cx))^2\log\left(1+\frac{ge^{i\arccos(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{g}\right)-i\left(\frac{2ib\int(a+b\arccos}{}\right.\right.}{2bc}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{(a+b\arccos(cx))^2\log\left(h(f+gx)^m\right)}{2bc}-\frac{gm\left(-i\left(\frac{2ib\left(i(a+b\arccos(cx))\text{PolyLog}\left(2,-\frac{e^{i\arccos(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)-ib\int\text{PolyLog}\left(2,-\frac{e^{i\arccos(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)d\arccos(cx)\right)}{g}-\frac{i(a+b\arccos(cx))^2\log\left(1\right.}{g}\right.\right.}{}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{(a+b\arccos(cx))^2\log\left(h(f+gx)^m\right)}{2bc}-$$

$$\frac{gm\left(-i\left(\frac{2ib\left(i(a+b\arccos(cx))\operatorname{PolyLog}\left(2,-\frac{e^{i\arccos(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)-b\int e^{-i\arccos(cx)}\operatorname{PolyLog}\left(2,-\frac{e^{i\arccos(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)de^{i\arccos(cx)}\right)}{g}-\frac{i(a+b\arccos\ldots}{\ldots}\right.\right.}{\ldots}$$

$\downarrow$ 7143

$$-\frac{(a+b\arccos(cx))^2\log\left(h(f+gx)^m\right)}{2bc}-$$

$$\frac{gm\left(-i\left(\frac{2ib\left(i(a+b\arccos(cx))\operatorname{PolyLog}\left(2,-\frac{e^{i\arccos(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)-b\operatorname{PolyLog}\left(3,-\frac{e^{i\arccos(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)\right)}{g}-\frac{i(a+b\arccos(cx))^2\log\left(1+\frac{ge^{i\arccos(cx)}}{cf-\sqrt{c^2f^2-g\ldots}}\right.}{g}\right.\right.}{\ldots}$$

input
```
Int[((a + b*ArcCos[c*x])*Log[h*(f + g*x)^m])/Sqrt[1 - c^2*x^2],x]
```

output
```
-1/2*((a + b*ArcCos[c*x])^2*Log[h*(f + g*x)^m])/(b*c) - (g*m*(((I/3)*(a +
b*ArcCos[c*x])^3)/(b*g) - I*(((-I)*(a + b*ArcCos[c*x])^2*Log[1 + (E^(I*Arc
Cos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2])])/g + ((2*I)*b*(I*(a + b*ArcCos[c
*x])*PolyLog[2, -((E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2]))] - b*
PolyLog[3, -((E^(I*ArcCos[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2]))]))/g) - I*
(((-I)*(a + b*ArcCos[c*x])^2*Log[1 + (E^(I*ArcCos[c*x])*g)/(c*f + Sqrt[c^2
*f^2 - g^2])])/g + ((2*I)*b*(I*(a + b*ArcCos[c*x])*PolyLog[2, -((E^(I*ArcC
os[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2]))] - b*PolyLog[3, -((E^(I*ArcCos[c*
x])*g)/(c*f + Sqrt[c^2*f^2 - g^2]))]))/g)))/(2*b*c)
```

3.21.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5031
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)])/(Cos[(c_.) + (d_.)
*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[I*((e + f*x)^(m + 1)/(b*f*(m + 1)))
, x] + (-Simp[I   Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] +
b*E^(I*(c + d*x)))), x], x] - Simp[I   Int[(e + f*x)^m*(E^(I*(c + d*x))/(a
+ Rt[a^2 - b^2, 2] + b*E^(I*(c + d*x)))), x], x]) /; FreeQ[{a, b, c, d, e,
f}, x] && IGtQ[m, 0] && PosQ[a^2 - b^2]
```

rule 5241
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Subst[Int[(a + b*x)^n*(Sin[x]/(c*d + e*Cos[x])), x], x, ArcCos[c*x]] /
; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

rule 5279
```
Int[(Log[(h_.)*((f_.) + (g_.)*(x_))^(m_.)]*((a_.) + ArcCos[(c_.)*(x_)]*(b_.
))^(n_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] :> Simp[(-Log[h*(f + g*x)^m]
)*((a + b*ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1))), x] + Simp[g*(m/(b*c*
Sqrt[d]*(n + 1)))   Int[(a + b*ArcCos[c*x])^(n + 1)/(f + g*x), x], x] /; Fr
eeQ[{a, b, c, d, e, f, g, h, m}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && IG
tQ[n, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.21. $\int \frac{(a+b\arccos(cx))\log(h(f+gx)^m)}{\sqrt{1-c^2x^2}}\,dx$

3.21.4 Maple [F]

$$\int \frac{(a+b\arccos(cx))\ln\left(h(gx+f)^m\right)}{\sqrt{-c^2x^2+1}}dx$$

input
```
int((a+b*arccos(c*x))*ln(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x)
```

output
```
int((a+b*arccos(c*x))*ln(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x)
```

3.21.5 Fricas [F]

$$\int \frac{(a+b\arccos(cx))\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx = \int \frac{(b\arccos(cx)+a)\log\left((gx+f)^m h\right)}{\sqrt{-c^2x^2+1}}\,dx$$

input
```
integrate((a+b*arccos(c*x))*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algorit
hm="fricas")
```

output
```
integral(-sqrt(-c^2*x^2 + 1)*(b*arccos(c*x) + a)*log((g*x + f)^m*h)/(c^2*x
^2 - 1), x)
```

3.21.6 Sympy [F]

$$\int \frac{(a+b\arccos(cx))\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx = \int \frac{(a+b\,\mathrm{acos}\,(cx))\log\left(h(f+gx)^m\right)}{\sqrt{-(cx-1)(cx+1)}}\,dx$$

input
```
integrate((a+b*acos(c*x))*ln(h*(g*x+f)**m)/(-c**2*x**2+1)**(1/2),x)
```

output
```
Integral((a + b*acos(c*x))*log(h*(f + g*x)**m)/sqrt(-(c*x - 1)*(c*x + 1)),
 x)
```

3.21.7 Maxima [F]

$$\int \frac{(a+b\arccos(cx))\log(h(f+gx)^m)}{\sqrt{1-c^2x^2}}\,dx = \int \frac{(b\arccos(cx)+a)\log((gx+f)^m h)}{\sqrt{-c^2x^2+1}}\,dx$$

input
```
integrate((a+b*arccos(c*x))*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algorit
hm="maxima")
```

output
```
(b*c*integrate(arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/(sqrt(c*x + 1)*s
qrt(-c*x + 1)), x)*log(h) + b*c*integrate(arctan2(sqrt(c*x + 1)*sqrt(-c*x
+ 1), c*x)*log((g*x + f)^m)/(sqrt(c*x + 1)*sqrt(-c*x + 1)), x) + a*c*integ
rate(log((g*x + f)^m)/(sqrt(c*x + 1)*sqrt(-c*x + 1)), x) + a*arctan2(c*x,
sqrt(-c^2*x^2 + 1))*log(h))/c
```

3.21.8 Giac [F]

$$\int \frac{(a+b\arccos(cx))\log(h(f+gx)^m)}{\sqrt{1-c^2x^2}}\,dx = \int \frac{(b\arccos(cx)+a)\log((gx+f)^m h)}{\sqrt{-c^2x^2+1}}\,dx$$

input
```
integrate((a+b*arccos(c*x))*log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algorit
hm="giac")
```

output
```
integrate((b*arccos(c*x) + a)*log((g*x + f)^m*h)/sqrt(-c^2*x^2 + 1), x)
```

3.21.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))\log(h(f+gx)^m)}{\sqrt{1-c^2x^2}}\,dx = \int \frac{\ln\left(h\,(f+g\,x)^m\right)\,(a+b\,\mathrm{acos}(c\,x))}{\sqrt{1-c^2\,x^2}}\,dx$$

input
```
int((log(h*(f + g*x)^m)*(a + b*acos(c*x)))/(1 - c^2*x^2)^(1/2),x)
```

output
```
int((log(h*(f + g*x)^m)*(a + b*acos(c*x)))/(1 - c^2*x^2)^(1/2), x)
```

3.22 $\int \frac{\log(h(f+gx)^m)}{\sqrt{1-c^2x^2}}\,dx$

3.22.1 Optimal result

Integrand size = 25, antiderivative size = 237

$$\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx = \frac{im\arcsin(cx)^2}{2c} - \frac{m\arcsin(cx)\log\left(1-\frac{ie^{i\arcsin(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{c} - \frac{m\arcsin(cx)\log\left(1-\frac{ie^{i\arcsin(cx)}g}{cf+\sqrt{c^2f^2-g^2}}\right)}{c} + \frac{\arcsin(cx)\log\left(h(f+gx)^m\right)}{c} + \frac{im\,\mathrm{PolyLog}\left(2,\frac{ie^{i\arcsin(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{c} + \frac{im\,\mathrm{PolyLog}\left(2,\frac{ie^{i\arcsin(cx)}g}{cf+\sqrt{c^2f^2-g^2}}\right)}{c}$$

output

```
1/2*I*m*arcsin(c*x)^2/c+arcsin(c*x)*ln(h*(g*x+f)^m)/c-m*arcsin(c*x)*ln(1-I
*(I*c*x+(-c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2-g^2)^(1/2)))/c-m*arcsin(c*x)*l
n(1-I*(I*c*x+(-c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2)))/c+I*m*polylo
g(2,I*(I*c*x+(-c^2*x^2+1)^(1/2))*g/(c*f-(c^2*f^2-g^2)^(1/2)))/c+I*m*polylo
g(2,I*(I*c*x+(-c^2*x^2+1)^(1/2))*g/(c*f+(c^2*f^2-g^2)^(1/2)))/c
```

3.22.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.04

$$\int \frac{\log (h(f+gx)^m)}{\sqrt{1-c^2x^2}}\,dx = \frac{im\arcsin(cx)^2}{2c} - \frac{m\arcsin(cx)\log\left(1-\frac{ice^{i\arcsin(cx)}g}{c^2f-c\sqrt{c^2f^2-g^2}}\right)}{c} - \frac{m\arcsin(cx)\log\left(1-\frac{ice^{i\arcsin(cx)}g}{c^2f+c\sqrt{c^2f^2-g^2}}\right)}{c} + \frac{\arcsin(cx)\log(h(f+gx)^m)}{c} + \frac{im\,\text{PolyLog}\left(2,\frac{ie^{i\arcsin(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{c} + \frac{im\,\text{PolyLog}\left(2,\frac{ie^{i\arcsin(cx)}g}{cf+\sqrt{c^2f^2-g^2}}\right)}{c}$$

input
```
Integrate[Log[h*(f + g*x)^m]/Sqrt[1 - c^2*x^2],x]
```

output
```
((I/2)*m*ArcSin[c*x]^2)/c - (m*ArcSin[c*x]*Log[1 - (I*c*E^(I*ArcSin[c*x])*
g)/(c^2*f - c*Sqrt[c^2*f^2 - g^2])])/c - (m*ArcSin[c*x]*Log[1 - (I*c*E^(I*
ArcSin[c*x])*g)/(c^2*f + c*Sqrt[c^2*f^2 - g^2])])/c + (ArcSin[c*x]*Log[h*(
f + g*x)^m])/c + (I*m*PolyLog[2, (I*E^(I*ArcSin[c*x])*g)/(c*f - Sqrt[c^2*f
^2 - g^2])])/c + (I*m*PolyLog[2, (I*E^(I*ArcSin[c*x])*g)/(c*f + Sqrt[c^2*f
^2 - g^2])])/c
```

3.22.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2851, 27, 5240, 5030, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log (h(f+gx)^m)}{\sqrt{1-c^2x^2}}\,dx$$

$$\Big\downarrow \ 2851$$

$$\frac{\arcsin(cx)\log(h(f+gx)^m)}{c} - gm\int \frac{\arcsin(cx)}{c(f+gx)}dx$$

$$\Big\downarrow \ 27$$

$$\frac{\arcsin(cx)\log\left(h(f+gx)^m\right)}{c}-\frac{gm\int\frac{\arcsin(cx)}{f+gx}dx}{c}$$

$\downarrow$ 5240

$$\frac{\arcsin(cx)\log\left(h(f+gx)^m\right)}{c}-\frac{gm\int\frac{\sqrt{1-c^2x^2}\arcsin(cx)}{cf+cgx}d\arcsin(cx)}{c}$$

$\downarrow$ 5030

$$\frac{\arcsin(cx)\log\left(h(f+gx)^m\right)}{c}-\frac{gm\left(\int\frac{e^{i\arcsin(cx)}\arcsin(cx)}{cf-ie^{i\arcsin(cx)}g-\sqrt{c^2f^2-g^2}}d\arcsin(cx)+\int\frac{e^{i\arcsin(cx)}\arcsin(cx)}{cf-ie^{i\arcsin(cx)}g+\sqrt{c^2f^2-g^2}}d\arcsin(cx)-\frac{i\arcsin(cx)^2}{2g}\right)}{c}$$

$\downarrow$ 2620

$$\frac{\arcsin(cx)\log\left(h(f+gx)^m\right)}{c}-\frac{gm\left(-\frac{\int\log\left(1-\frac{ie^{i\arcsin(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)d\arcsin(cx)}{g}-\frac{\int\log\left(1-\frac{ie^{i\arcsin(cx)}g}{cf+\sqrt{c^2f^2-g^2}}\right)d\arcsin(cx)}{g}+\frac{\arcsin(cx)\log\left(1-\frac{ige^{i\arcsin(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{g}+\frac{\arcsin(cx)\log}{}\right.}{c}$$

$\downarrow$ 2715

$$\frac{\arcsin(cx)\log\left(h(f+gx)^m\right)}{c}-\frac{gm\left(\frac{i\int e^{-i\arcsin(cx)}\log\left(1-\frac{ie^{i\arcsin(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)de^{i\arcsin(cx)}}{g}+\frac{i\int e^{-i\arcsin(cx)}\log\left(1-\frac{ie^{i\arcsin(cx)}g}{cf+\sqrt{c^2f^2-g^2}}\right)de^{i\arcsin(cx)}}{g}+\frac{\arcsin(cx)\log\left(1-\frac{ige^{i a}}{cf-\sqrt{}}\right.}{g}\right.}{c}$$

$\downarrow$ 2838

$$\frac{\arcsin(cx)\log\left(h(f+gx)^m\right)}{c}-\frac{gm\left(-\frac{i\,\text{PolyLog}\left(2,\frac{ie^{i\arcsin(cx)}g}{cf-\sqrt{c^2f^2-g^2}}\right)}{g}-\frac{i\,\text{PolyLog}\left(2,\frac{ie^{i\arcsin(cx)}g}{cf+\sqrt{c^2f^2-g^2}}\right)}{g}+\frac{\arcsin(cx)\log\left(1-\frac{ige^{i\arcsin(cx)}}{cf-\sqrt{c^2f^2-g^2}}\right)}{g}+\frac{\arcsin(cx)\log\left(1-\frac{ige^{i\arcsin(}}{\sqrt{c^2f^2-g^2}}\right.}{g}\right.}{c}$$

input
```
Int[Log[h*(f + g*x)^m]/Sqrt[1 - c^2*x^2],x]
```

3.22. $\int\frac{\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}dx$

output
```
(ArcSin[c*x]*Log[h*(f + g*x)^m])/c - (g*m*(((-1/2*I)*ArcSin[c*x]^2)/g + (A
rcSin[c*x]*Log[1 - (I*E^(I*ArcSin[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2])])/g
 + (ArcSin[c*x]*Log[1 - (I*E^(I*ArcSin[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2]
)])/g - (I*PolyLog[2, (I*E^(I*ArcSin[c*x])*g)/(c*f - Sqrt[c^2*f^2 - g^2])]
)/g - (I*PolyLog[2, (I*E^(I*ArcSin[c*x])*g)/(c*f + Sqrt[c^2*f^2 - g^2])])/
g))/c
```

3.22.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2851
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/Sqrt[(f_) + (g_.)*
(x_)^2], x_Symbol] :> With[{u = IntHide[1/Sqrt[f + g*x^2], x]}, Simp[u*(a +
 b*Log[c*(d + e*x)^n]), x] - Simp[b*e*n   Int[SimplifyIntegrand[u/(d + e*x)
, x], x], x]] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && GtQ[f, 0]
```

rule 5030
```
Int[(Cos[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin[
(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-I)*((e + f*x)^(m + 1)/(b*f*(m + 1
))), x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(a - Rt[a^2 - b^2, 2] - I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(a + Rt[a^2 - b^2, 2]
 - I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
 && PosQ[a^2 - b^2]
```

rule 5240
```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Subst[Int[(a + b*x)^n*(Cos[x]/(c*d + e*Sin[x])), x], x, ArcSin[c*x]] /;
 FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

3.22.4 Maple [F]

$$\int \frac{\ln\left(h(gx+f)^m\right)}{\sqrt{-c^2x^2+1}}dx$$

input
```
int(ln(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x)
```

output
```
int(ln(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x)
```

3.22.5 Fricas [F]

$$\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx = \int \frac{\log\left((gx+f)^m h\right)}{\sqrt{-c^2x^2+1}}\,dx$$

input
```
integrate(log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-c^2*x^2 + 1)*log((g*x + f)^m*h)/(c^2*x^2 - 1), x)
```

3.22.6 Sympy [F]

$$\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx = \int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{-\left(cx-1\right)\left(cx+1\right)}}\,dx$$

input
```
integrate(ln(h*(g*x+f)**m)/(-c**2*x**2+1)**(1/2),x)
```

output
```
Integral(log(h*(f + g*x)**m)/sqrt(-(c*x - 1)*(c*x + 1)), x)
```

3.22.7 Maxima [F]

$$\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx = \int \frac{\log\left((gx+f)^m h\right)}{\sqrt{-c^2x^2+1}}\,dx$$

input
```
integrate(log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(log((g*x + f)^m*h)/sqrt(-c^2*x^2 + 1), x)
```

3.22.8 Giac [F]

$$\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx = \int \frac{\log\left((gx+f)^m h\right)}{\sqrt{-c^2x^2+1}}\,dx$$

input
```
integrate(log(h*(g*x+f)^m)/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(log((g*x + f)^m*h)/sqrt(-c^2*x^2 + 1), x)
```

3.22.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}}\,dx = \int \frac{\ln\left(h\,(f+g\,x)^m\right)}{\sqrt{1-c^2\,x^2}}\,dx$$

input
```
int(log(h*(f + g*x)^m)/(1 - c^2*x^2)^(1/2),x)
```

output
```
int(log(h*(f + g*x)^m)/(1 - c^2*x^2)^(1/2), x)
```

3.23 $\int \frac{\log(h(f+gx)^m)}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx$

3.23.1 Optimal result

Integrand size = 35, antiderivative size = 35

$$\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = \text{Int}\left(\frac{\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}(a+b\arccos(cx))},x\right)$$

output
```
Unintegrable(ln(h*(g*x+f)^m)/(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2),x)
```

3.23.2 Mathematica [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = \int \frac{\log\left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx$$

input
```
Integrate[Log[h*(f + g*x)^m]/(Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])),x]
```

output
```
Integrate[Log[h*(f + g*x)^m]/(Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])), x]
```

3.23.3 Rubi [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {5301}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}dx$$
$$\Big\downarrow \text{ 5301}$$
$$\int \frac{\log \left(h(f+gx)^m\right)}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}dx$$

input
```
Int[Log[h*(f + g*x)^m]/(Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])),x]
```

output
```
$Aborted
```

3.23.3.1 Defintions of rubi rules used

rule 5301
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(u_.), x_Symbol] :> Unintegrab
le[u*(a + b*ArcCos[c*x])^n, x] /; FreeQ[{a, b, c, n}, x]
```

3.23.4 Maple [N/A] (verified)

Not integrable

Time = 56.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{\ln \left(h(gx+f)^m\right)}{(a+b\arccos(cx))\sqrt{-c^2x^2+1}}dx$$

input
```
int(ln(h*(g*x+f)^m)/(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2),x)
```

output
```
int(ln(h*(g*x+f)^m)/(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2),x)
```

3.23.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.63

$$\int \frac{\log(h(f+gx)^m)}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = \int \frac{\log\left((gx+f)^m h\right)}{\sqrt{-c^2x^2+1}(b\arccos(cx)+a)}\,dx$$

input
```
integrate(log(h*(g*x+f)^m)/(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2),x, algorit
hm="fricas")
```

output
```
integral(-sqrt(-c^2*x^2 + 1)*log((g*x + f)^m*h)/(a*c^2*x^2 + (b*c^2*x^2 -
b)*arccos(c*x) - a), x)
```

3.23.6 Sympy [N/A]

Not integrable

Time = 10.07 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97

$$\int \frac{\log(h(f+gx)^m)}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = \int \frac{\log(h(f+gx)^m)}{\sqrt{-(cx-1)(cx+1)}\,(a+b\operatorname{acos}(cx))}\,dx$$

input
```
integrate(ln(h*(g*x+f)**m)/(a+b*acos(c*x))/(-c**2*x**2+1)**(1/2),x)
```

output
```
Integral(log(h*(f + g*x)**m)/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acos(c*x))
), x)
```

3.23.7 Maxima [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{\log(h(f+gx)^m)}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = \int \frac{\log\left((gx+f)^m h\right)}{\sqrt{-c^2x^2+1}(b\arccos(cx)+a)}\,dx$$

input
```
integrate(log(h*(g*x+f)^m)/(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2),x, algorit
hm="maxima")
```

output
```
integrate(log((g*x + f)^m*h)/(sqrt(-c^2*x^2 + 1)*(b*arccos(c*x) + a)), x)
```

3.23.8 Giac [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{\log (h(f+gx)^m)}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = \int \frac{\log ((gx+f)^m h)}{\sqrt{-c^2x^2+1}(b\arccos (cx)+a)}\,dx$$

input
```
integrate(log(h*(g*x+f)^m)/(a+b*arccos(c*x))/(-c^2*x^2+1)^(1/2),x, algorit
hm="giac")
```

output
```
integrate(log((g*x + f)^m*h)/(sqrt(-c^2*x^2 + 1)*(b*arccos(c*x) + a)), x)
```

3.23.9 Mupad [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{\log (h(f+gx)^m)}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx = \int \frac{\ln \left(h\,{\left(f+g\,x\right)}^m\right)}{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)\,\sqrt{1-c^2\,x^2}}\,dx$$

input
```
int(log(h*(f + g*x)^m)/((a + b*acos(c*x))*(1 - c^2*x^2)^(1/2)),x)
```

output
```
int(log(h*(f + g*x)^m)/((a + b*acos(c*x))*(1 - c^2*x^2)^(1/2)), x)
```

3.24 $\int x^3 \arccos(a+bx)\,dx$

3.24.1 Optimal result

Integrand size = 10, antiderivative size = 137

$$\int x^3 \arccos(a+bx)\,dx = \frac{7ax^2\sqrt{1-(a+bx)^2}}{48b^2} - \frac{x^3\sqrt{1-(a+bx)^2}}{16b} + \frac{(4a(16+19a^2)-(9+26a^2)(a+bx))\sqrt{1-(a+bx)^2}}{96b^4} + \frac{1}{4}x^4\arccos(a+bx) + \frac{(3+24a^2+8a^4)\arcsin(a+bx)}{32b^4}$$

output
```
1/4*x^4*arccos(b*x+a)+1/32*(8*a^4+24*a^2+3)*arcsin(b*x+a)/b^4+7/48*a*x^2*(
1-(b*x+a)^2)^(1/2)/b^2-1/16*x^3*(1-(b*x+a)^2)^(1/2)/b+1/96*(4*a*(19*a^2+16
)-(26*a^2+9)*(b*x+a))*(1-(b*x+a)^2)^(1/2)/b^4
```

3.24.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.76

$$\int x^3 \arccos(a+bx)\,dx = \frac{\sqrt{1-a^2-2abx-b^2x^2}(55a+50a^3-9bx-26a^2bx+14ab^2x^2-6b^3x^3)+24b^4x^4\arccos(a+bx)+3(3+}{96b^4}$$

input
```
Integrate[x^3*ArcCos[a + b*x],x]
```

output
```
(Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2]*(55*a + 50*a^3 - 9*b*x - 26*a^2*b*x + 1
4*a*b^2*x^2 - 6*b^3*x^3) + 24*b^4*x^4*ArcCos[a + b*x] + 3*(3 + 24*a^2 + 8*
a^4)*ArcSin[a + b*x])/(96*b^4)
```

3.24.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.21, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5305, 25, 27, 5243, 497, 25, 687, 25, 676, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \arccos(a+bx)\,dx$$

$$\Big\downarrow \text{5305}$$

$$\frac{\int x^3 \arccos(a+bx)d(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -x^3 \arccos(a+bx)d(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int -b^3x^3 \arccos(a+bx)d(a+bx)}{b^4}$$

$$\Big\downarrow \text{5243}$$

$$-\frac{-\frac{1}{4}\int \frac{b^4x^4}{\sqrt{1-(a+bx)^2}}d(a+bx)-\frac{1}{4}b^4x^4\arccos(a+bx)}{b^4}$$

$$\Big\downarrow \text{497}$$

$$-\frac{\frac{1}{4}\left(\frac{1}{4}\int -\frac{b^2x^2\left(4a^2-7(a+bx)a+3\right)}{\sqrt{1-(a+bx)^2}}d(a+bx)+\frac{1}{4}b^3x^3\sqrt{1-(a+bx)^2}\right)-\frac{1}{4}b^4x^4\arccos(a+bx)}{b^4}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\frac{1}{4}\left(\frac{1}{4}b^3x^3\sqrt{1-(a+bx)^2}-\frac{1}{4}\int \frac{b^2x^2\left(4a^2-7(a+bx)a+3\right)}{\sqrt{1-(a+bx)^2}}d(a+bx)\right)-\frac{1}{4}b^4x^4\arccos(a+bx)}{b^4}$$

$$\Big\downarrow \text{687}$$

$$-\frac{\frac{1}{4}\left(\frac{1}{4}\left(\frac{1}{3}\int\frac{bx\left(a\left(12a^2+23\right)-\left(26a^2+9\right)(a+bx)\right)}{\sqrt{1-(a+bx)^2}}d(a+bx)-\frac{7}{3}ab^2x^2\sqrt{1-(a+bx)^2}\right)+\frac{1}{4}b^3x^3\sqrt{1-(a+bx)^2}\right)-\frac{1}{4}b^4x^4\operatorname{ar}}{b^4}$$

$\Big\downarrow$ 25

$$-\frac{\frac{1}{4}\left(\frac{1}{4}\left(-\frac{1}{3}\int-\frac{bx\left(a\left(12a^2+23\right)-\left(26a^2+9\right)(a+bx)\right)}{\sqrt{1-(a+bx)^2}}d(a+bx)-\frac{7}{3}ab^2x^2\sqrt{1-(a+bx)^2}\right)+\frac{1}{4}b^3x^3\sqrt{1-(a+bx)^2}\right)-\frac{1}{4}b^4x}{b^4}$$

$\Big\downarrow$ 676

$$-\frac{\frac{1}{4}\left(\frac{1}{4}\left(\frac{1}{3}\left(-\frac{3}{2}\left(8a^4+24a^2+3\right)\int\frac{1}{\sqrt{1-(a+bx)^2}}d(a+bx)-2a\left(19a^2+16\right)\sqrt{1-(a+bx)^2}+\frac{1}{2}\left(26a^2+9\right)(a+bx)\sqrt{}\right.\right.\right.}{b^4}$$

$\Big\downarrow$ 223

$$-\frac{\frac{1}{4}\left(\frac{1}{4}\left(\frac{1}{3}\left(-2a\left(19a^2+16\right)\sqrt{1-(a+bx)^2}+\frac{1}{2}\left(26a^2+9\right)(a+bx)\sqrt{1-(a+bx)^2}-\frac{3}{2}\left(8a^4+24a^2+3\right)\arcsin(a\right.\right.\right.}{b^4}$$

input
```
Int[x^3*ArcCos[a + b*x],x]
```

output
```
-((-1/4*(b^4*x^4*ArcCos[a + b*x]) + ((b^3*x^3*Sqrt[1 - (a + b*x)^2])/4 + (
(-7*a*b^2*x^2*Sqrt[1 - (a + b*x)^2])/3 + (-2*a*(16 + 19*a^2)*Sqrt[1 - (a +
 b*x)^2] + ((9 + 26*a^2)*(a + b*x)*Sqrt[1 - (a + b*x)^2])/2 - (3*(3 + 24*a
^2 + 8*a^4)*ArcSin[a + b*x])/2)/3)/4)/4)/b^4)
```

3.24.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 687
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp
[c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x
] /; FreeQ[{a, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] &&
 (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && Eq
Q[f, 0])
```

rule 5243
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(e*(m + 1))), x] +
Simp[b*c*(n/(e*(m + 1)))   Int[(d + e*x)^(m + 1)*((a + b*ArcCos[c*x])^(n -
1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
 && NeQ[m, -1]
```

rule 5305
```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.24. $\int x^3 \arccos(a+bx)\,dx$

3.24.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.72

method	result
derivativedivides	$\frac{\frac{\arccos(bx+a)a^4}{4} - \arccos(bx+a)a^3(bx+a) + \frac{3\arccos(bx+a)a^2(bx+a)^2}{2} - \arccos(bx+a)a(bx+a)^3 + \frac{\arccos(bx+a)(bx+a)^4}{4} + \frac{a^4 \operatorname{arc}\cdots}{\cdots}}{\cdots}$
default	$\frac{\frac{\arccos(bx+a)a^4}{4} - \arccos(bx+a)a^3(bx+a) + \frac{3\arccos(bx+a)a^2(bx+a)^2}{2} - \arccos(bx+a)a(bx+a)^3 + \frac{\arccos(bx+a)(bx+a)^4}{4} + \frac{a^4 \operatorname{arc}\cdots}{\cdots}}{\cdots}$
parts	$\frac{x^4\arccos(bx+a)}{4} + \frac{b\left(-\frac{x^3\sqrt{-b^2x^2-2abx-a^2+1}}{4b^2} - \frac{7a\left(-\frac{x^2\sqrt{-b^2x^2-2abx-a^2+1}}{3b^2} - \frac{5a\left(-\frac{x\sqrt{-b^2x^2-2abx-a^2+1}}{2b^2} - \frac{3a\left(-\frac{\sqrt{\cdots}}{\cdots}\cdots\right)}{\cdots}\right)}{\cdots}\right)}{\cdots}\right)}{\cdots}$

input
```
int(x^3*arccos(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^4*(1/4*arccos(b*x+a)*a^4-arccos(b*x+a)*a^3*(b*x+a)+3/2*arccos(b*x+a)*a
^2*(b*x+a)^2-arccos(b*x+a)*a*(b*x+a)^3+1/4*arccos(b*x+a)*(b*x+a)^4+1/4*a^4
*arcsin(b*x+a)-1/16*(b*x+a)^3*(1-(b*x+a)^2)^(1/2)-3/32*(b*x+a)*(1-(b*x+a)^
2)^(1/2)+3/32*arcsin(b*x+a)+a^3*(1-(b*x+a)^2)^(1/2)+3/2*a^2*(-1/2*(b*x+a)*
(1-(b*x+a)^2)^(1/2)+1/2*arcsin(b*x+a))-a*(-1/3*(b*x+a)^2*(1-(b*x+a)^2)^(1/
2)-2/3*(1-(b*x+a)^2)^(1/2)))
```

3.24.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.69

$$\int x^3 \arccos(a+bx)\,dx$$
$$= \frac{3\,(8\,b^4x^4 - 8\,a^4 - 24\,a^2 - 3)\arccos\left(bx+a\right) - (6\,b^3x^3 - 14\,ab^2x^2 - 50\,a^3 + (26\,a^2+9)bx - 55\,a)\sqrt{-b^2x^2}}{96\,b^4}$$

input
```
integrate(x^3*arccos(b*x+a),x, algorithm="fricas")
```

output
```
1/96*(3*(8*b^4*x^4 - 8*a^4 - 24*a^2 - 3)*arccos(b*x + a) - (6*b^3*x^3 - 14
*a*b^2*x^2 - 50*a^3 + (26*a^2 + 9)*b*x - 55*a)*sqrt(-b^2*x^2 - 2*a*b*x - a
^2 + 1))/b^4
```

3.24.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(117) = 234$.

Time = 0.36 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.86

$$\int x^3 \arccos(a+bx)\,dx$$
$$= \begin{cases} -\frac{a^4 \operatorname{acos}(a+bx)}{4b^4} + \frac{25a^3\sqrt{-a^2-2abx-b^2x^2+1}}{48b^4} - \frac{13a^2x\sqrt{-a^2-2abx-b^2x^2+1}}{48b^3} - \frac{3a^2 \operatorname{acos}(a+bx)}{4b^4} + \frac{7ax^2\sqrt{-a^2-2abx-b^2x^2+1}}{48b^2} + 55 \\ \frac{x^4 \operatorname{acos}(a)}{4} \end{cases}$$

input
```
integrate(x**3*acos(b*x+a),x)
```

output
```
Piecewise((-a**4*acos(a + b*x)/(4*b**4) + 25*a**3*sqrt(-a**2 - 2*a*b*x - b
**2*x**2 + 1)/(48*b**4) - 13*a**2*x*sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1)/
(48*b**3) - 3*a**2*acos(a + b*x)/(4*b**4) + 7*a*x**2*sqrt(-a**2 - 2*a*b*x
- b**2*x**2 + 1)/(48*b**2) + 55*a*sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1)/(9
6*b**4) + x**4*acos(a + b*x)/4 - x**3*sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1
)/(16*b) - 3*x*sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1)/(32*b**3) - 3*acos(a
+ b*x)/(32*b**4), Ne(b, 0)), (x**4*acos(a)/4, True))
```

3.24.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(120) = 240$.

Time = 0.27 (sec) , antiderivative size = 333, normalized size of antiderivative = 2.43

$$\int x^3 \arccos(a+bx)\,dx = \frac{1}{4}x^4 \arccos(bx+a) - \frac{1}{96}\left(\frac{6\sqrt{-b^2x^2-2\,abx-a^2+1}x^3}{b^2} - \frac{14\sqrt{-b^2x^2-2\,abx-a^2+1}ax^2}{b^3} + \frac{105\,a^4\arcsin\left(-\frac{b^2x+ab}{\sqrt{a^2b^2-(a^2-1)b^2}}\right.}{b^5}\right.$$

input
```
integrate(x^3*arccos(b*x+a),x, algorithm="maxima")
```

output
```
1/4*x^4*arccos(b*x + a) - 1/96*(6*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*x^3/b
^2 - 14*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*a*x^2/b^3 + 105*a^4*arcsin(-(b^
2*x + a*b)/sqrt(a^2*b^2 - (a^2 - 1)*b^2))/b^5 + 35*sqrt(-b^2*x^2 - 2*a*b*x
 - a^2 + 1)*a^2*x/b^4 - 90*(a^2 - 1)*a^2*arcsin(-(b^2*x + a*b)/sqrt(a^2*b^
2 - (a^2 - 1)*b^2))/b^5 - 105*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*a^3/b^5 -
 9*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a^2 - 1)*x/b^4 + 9*(a^2 - 1)^2*arcs
in(-(b^2*x + a*b)/sqrt(a^2*b^2 - (a^2 - 1)*b^2))/b^5 + 55*sqrt(-b^2*x^2 -
2*a*b*x - a^2 + 1)*(a^2 - 1)*a/b^5)*b
```

3.24.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. $2(120) = 240$.

Time = 0.28 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.77

$$\int x^3 \arccos(a+bx)\,dx = \frac{(bx+a)^4 \arccos(bx+a)}{4\,b^4} - \frac{(bx+a)^3 a \arccos(bx+a)}{b^4} + \frac{3\,(bx+a)^2 a^2 \arccos(bx+a)}{2\,b^4} - \frac{(bx+a)a^3 \arccos(bx+a)}{b^4} - \frac{\sqrt{-(bx+a)^2+1}(bx+a)^3}{16\,b^4} + \frac{\sqrt{-(bx+a)^2+1}(bx+a)^2 a}{3\,b^4} - \frac{3\sqrt{-(bx+a)^2+1}(bx+a)a^2}{4\,b^4} + \frac{\sqrt{-(bx+a)^2+1}a^3}{b^4} - \frac{3\,a^2 \arccos(bx+a)}{4\,b^4} - \frac{3\sqrt{-(bx+a)^2+1}(bx+a)}{32\,b^4} + \frac{2\sqrt{-(bx+a)^2+1}a}{3\,b^4} - \frac{3\arccos(bx+a)}{32\,b^4}$$

input
```
integrate(x^3*arccos(b*x+a),x, algorithm="giac")
```

output
```
1/4*(b*x + a)^4*arccos(b*x + a)/b^4 - (b*x + a)^3*a*arccos(b*x + a)/b^4 +
3/2*(b*x + a)^2*a^2*arccos(b*x + a)/b^4 - (b*x + a)*a^3*arccos(b*x + a)/b^
4 - 1/16*sqrt(-(b*x + a)^2 + 1)*(b*x + a)^3/b^4 + 1/3*sqrt(-(b*x + a)^2 +
1)*(b*x + a)^2*a/b^4 - 3/4*sqrt(-(b*x + a)^2 + 1)*(b*x + a)*a^2/b^4 + sqrt
(-(b*x + a)^2 + 1)*a^3/b^4 - 3/4*a^2*arccos(b*x + a)/b^4 - 3/32*sqrt(-(b*x
 + a)^2 + 1)*(b*x + a)/b^4 + 2/3*sqrt(-(b*x + a)^2 + 1)*a/b^4 - 3/32*arcco
s(b*x + a)/b^4
```

3.24.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \arccos(a+bx)\,dx = \int x^3 \operatorname{acos}(a+b\,x)\,dx$$

input
```
int(x^3*acos(a + b*x),x)
```

output
```
int(x^3*acos(a + b*x), x)
```

3.25 $\int x^2 \arccos(a+bx)\,dx$

3.25.1 Optimal result

Integrand size = 10, antiderivative size = 94

$$\int x^2 \arccos(a+bx)\,dx = -\frac{x^2\sqrt{1-(a+bx)^2}}{9b} - \frac{(4+11a^2-5abx)\sqrt{1-(a+bx)^2}}{18b^3} + \frac{1}{3}x^3\arccos(a+bx) - \frac{a(3+2a^2)\arcsin(a+bx)}{6b^3}$$

output
```
1/3*x^3*arccos(b*x+a)-1/6*a*(2*a^2+3)*arcsin(b*x+a)/b^3-1/9*x^2*(1-(b*x+a)
^2)^(1/2)/b-1/18*(-5*a*b*x+11*a^2+4)*(1-(b*x+a)^2)^(1/2)/b^3
```

3.25.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.88

$$\int x^2 \arccos(a+bx)\,dx = -\frac{\sqrt{1-a^2-2abx-b^2x^2}(4+11a^2-5abx+2b^2x^2) - 6b^3x^3\arccos(a+bx) + 3a(3+2a^2)\arcsin(a+bx)}{18b^3}$$

input
```
Integrate[x^2*ArcCos[a + b*x],x]
```

output
```
-1/18*(Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2]*(4 + 11*a^2 - 5*a*b*x + 2*b^2*x^2
) - 6*b^3*x^3*ArcCos[a + b*x] + 3*a*(3 + 2*a^2)*ArcSin[a + b*x])/b^3
```

3.25.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.31, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {5305, 27, 5243, 497, 25, 676, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arccos(a+bx)\,dx$$

$$\Big\downarrow \text{ 5305}$$

$$\frac{\int x^2 \arccos(a+bx)d(a+bx)}{b}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int b^2x^2 \arccos(a+bx)d(a+bx)}{b^3}$$

$$\Big\downarrow \text{ 5243}$$

$$\frac{\frac{1}{3}b^3x^3\arccos(a+bx)-\frac{1}{3}\int -\frac{b^3x^3}{\sqrt{1-(a+bx)^2}}d(a+bx)}{b^3}$$

$$\Big\downarrow \text{ 497}$$

$$\frac{\frac{1}{3}\left(\frac{1}{3}\int \frac{bx(3a^2-5(a+bx)a+2)}{\sqrt{1-(a+bx)^2}}d(a+bx)-\frac{1}{3}b^2x^2\sqrt{1-(a+bx)^2}\right)+\frac{1}{3}b^3x^3\arccos(a+bx)}{b^3}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{1}{3}\left(-\frac{1}{3}\int -\frac{bx(3a^2-5(a+bx)a+2)}{\sqrt{1-(a+bx)^2}}d(a+bx)-\frac{1}{3}b^2x^2\sqrt{1-(a+bx)^2}\right)+\frac{1}{3}b^3x^3\arccos(a+bx)}{b^3}$$

$$\Big\downarrow \text{ 676}$$

$$\frac{\frac{1}{3}\left(\frac{1}{3}\left(-\frac{3}{2}a(2a^2+3)\int \frac{1}{\sqrt{1-(a+bx)^2}}d(a+bx)-2(4a^2+1)\sqrt{1-(a+bx)^2}+\frac{5}{2}a(a+bx)\sqrt{1-(a+bx)^2}\right)-\frac{1}{3}b^2x^2\sqrt{\ldots}\right.}{b^3}$$

$$\Big\downarrow \text{ 223}$$

$$\frac{\frac{1}{3}\left(\frac{1}{3}\left(-\frac{3}{2}a(2a^2+3)\arcsin(a+bx)-2(4a^2+1)\sqrt{1-(a+bx)^2}+\frac{5}{2}a(a+bx)\sqrt{1-(a+bx)^2}\right)-\frac{1}{3}b^2x^2\sqrt{1-(a\ldots}\right.}{b^3}$$

input
```
Int[x^2*ArcCos[a + b*x],x]
```

output
```
((b^3*x^3*ArcCos[a + b*x])/3 + (-1/3*(b^2*x^2*Sqrt[1 - (a + b*x)^2]) + (-2
*(1 + 4*a^2)*Sqrt[1 - (a + b*x)^2] + (5*a*(a + b*x)*Sqrt[1 - (a + b*x)^2])
/2 - (3*a*(3 + 2*a^2)*ArcSin[a + b*x])/2)/3)/3)/b^3
```

3.25.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 676
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(e*f + d*g)*((a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + (Sim
p[e*g*x*((a + c*x^2)^(p + 1)/(c*(2*p + 3))), x] - Simp[(a*e*g - c*d*f*(2*p
+ 3))/(c*(2*p + 3))   Int[(a + c*x^2)^p, x], x]) /; FreeQ[{a, c, d, e, f, g
, p}, x] &&  !LeQ[p, -1]
```

rule 5243
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(e*(m + 1))), x] +
Simp[b*c*(n/(e*(m + 1)))   Int[(d + e*x)^(m + 1)*((a + b*ArcCos[c*x])^(n -
1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
 && NeQ[m, -1]
```

rule 5305
```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.25.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.71

method	result
derivativedivides	$\frac{-\frac{\arccos(bx+a)a^3}{3}+\arccos(bx+a)a^2(bx+a)-\arccos(bx+a)a(bx+a)^2+\frac{\arccos(bx+a)(bx+a)^3}{3}-\frac{a^3\arcsin(bx+a)}{3}-\frac{(bx+a)^2\sqrt{1-}}{9}}{b^3}$
default	$\frac{-\frac{\arccos(bx+a)a^3}{3}+\arccos(bx+a)a^2(bx+a)-\arccos(bx+a)a(bx+a)^2+\frac{\arccos(bx+a)(bx+a)^3}{3}-\frac{a^3\arcsin(bx+a)}{3}-\frac{(bx+a)^2\sqrt{1-}}{9}}{b^3}$
parts	$\frac{x^3\arccos(bx+a)}{3}+\frac{b\left(-\frac{x^2\sqrt{-b^2x^2-2abx-a^2+1}}{3b^2}-\frac{5a\left(-\frac{x\sqrt{-b^2x^2-2abx-a^2+1}}{2b^2}-\frac{3a\left(-\frac{\sqrt{-b^2x^2-2abx-a^2+1}}{b^2}-\frac{a\arctan\left(\frac{-}{\sqrt{}}\right)}{}\right)}{2b}\right)}{3b}\right)}{}$

input
```
int(x^2*arccos(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(-1/3*arccos(b*x+a)*a^3+arccos(b*x+a)*a^2*(b*x+a)-arccos(b*x+a)*a*(b
*x+a)^2+1/3*arccos(b*x+a)*(b*x+a)^3-1/3*a^3*arcsin(b*x+a)-1/9*(b*x+a)^2*(1
-(b*x+a)^2)^(1/2)-2/9*(1-(b*x+a)^2)^(1/2)-a^2*(1-(b*x+a)^2)^(1/2)-a*(-1/2*
(b*x+a)*(1-(b*x+a)^2)^(1/2)+1/2*arcsin(b*x+a)))
```

3.25.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.80

$$\int x^2 \arccos(a+bx)\,dx$$
$$= \frac{3\,(2\,b^3x^3 + 2\,a^3 + 3\,a)\arccos\left(bx+a\right) - (2\,b^2x^2 - 5\,abx + 11\,a^2 + 4)\sqrt{-b^2x^2 - 2\,abx - a^2 + 1}}{18\,b^3}$$

input
```
integrate(x^2*arccos(b*x+a),x, algorithm="fricas")
```

output
```
1/18*(3*(2*b^3*x^3 + 2*a^3 + 3*a)*arccos(b*x + a) - (2*b^2*x^2 - 5*a*b*x +
 11*a^2 + 4)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1))/b^3
```

3.25.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(83) = 166.

Time = 0.29 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.81

$$\int x^2 \arccos(a+bx)\,dx$$
$$= \begin{cases} \frac{a^3 \operatorname{acos}\left(a+bx\right)}{3b^3} - \frac{11a^2\sqrt{-a^2-2abx-b^2x^2+1}}{18b^3} + \frac{5ax\sqrt{-a^2-2abx-b^2x^2+1}}{18b^2} + \frac{a \operatorname{acos}\left(a+bx\right)}{2b^3} + \frac{x^3 \operatorname{acos}\left(a+bx\right)}{3} - \frac{x^2\sqrt{-a^2-2abx-b^2x^2}}{9b} \\ \frac{x^3 \operatorname{acos}\left(a\right)}{3} \end{cases}$$

input
```
integrate(x**2*acos(b*x+a),x)
```

output
```
Piecewise((a**3*acos(a + b*x)/(3*b**3) - 11*a**2*sqrt(-a**2 - 2*a*b*x - b*
*2*x**2 + 1)/(18*b**3) + 5*a*x*sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1)/(18*b
**2) + a*acos(a + b*x)/(2*b**3) + x**3*acos(a + b*x)/3 - x**2*sqrt(-a**2 -
 2*a*b*x - b**2*x**2 + 1)/(9*b) - 2*sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1)/
(9*b**3), Ne(b, 0)), (x**3*acos(a)/3, True))
```

3.25.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 220 vs. 2(82) = 164.

Time = 0.27 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.34

$$\int x^2 \arccos(a+bx)\,dx = \frac{1}{3}x^3\arccos(bx+a) - \frac{1}{18}b\left(\frac{2\sqrt{-b^2x^2-2abx-a^2+1}x^2}{b^2} - \frac{15a^3\arcsin\left(-\frac{b^2x+ab}{\sqrt{a^2b^2-(a^2-1)b^2}}\right)}{b^4} - \frac{5\sqrt{-b^2x^2-2abx-a^2+1}ax}{b^3}\right.$$

input
```
integrate(x^2*arccos(b*x+a),x, algorithm="maxima")
```

output
```
1/3*x^3*arccos(b*x + a) - 1/18*b*(2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*x^2
/b^2 - 15*a^3*arcsin(-(b^2*x + a*b)/sqrt(a^2*b^2 - (a^2 - 1)*b^2))/b^4 - 5
*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*a*x/b^3 + 9*(a^2 - 1)*a*arcsin(-(b^2*x
 + a*b)/sqrt(a^2*b^2 - (a^2 - 1)*b^2))/b^4 + 15*sqrt(-b^2*x^2 - 2*a*b*x -
a^2 + 1)*a^2/b^4 - 4*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a^2 - 1)/b^4)
```

3.25.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.66

$$\int x^2 \arccos(a+bx)\,dx = \frac{(bx+a)^3\arccos(bx+a)}{3b^3} - \frac{(bx+a)^2a\arccos(bx+a)}{b^3} + \frac{(bx+a)a^2\arccos(bx+a)}{b^3} - \frac{\sqrt{-(bx+a)^2+1}(bx+a)^2}{9b^3} + \frac{\sqrt{-(bx+a)^2+1}(bx+a)a}{2b^3} - \frac{\sqrt{-(bx+a)^2+1}a^2}{b^3} + \frac{a\arccos(bx+a)}{2b^3} - \frac{2\sqrt{-(bx+a)^2+1}}{9b^3}$$

input
```
integrate(x^2*arccos(b*x+a),x, algorithm="giac")
```

output
```
1/3*(b*x + a)^3*arccos(b*x + a)/b^3 - (b*x + a)^2*a*arccos(b*x + a)/b^3 +
(b*x + a)*a^2*arccos(b*x + a)/b^3 - 1/9*sqrt(-(b*x + a)^2 + 1)*(b*x + a)^2
/b^3 + 1/2*sqrt(-(b*x + a)^2 + 1)*(b*x + a)*a/b^3 - sqrt(-(b*x + a)^2 + 1)
*a^2/b^3 + 1/2*a*arccos(b*x + a)/b^3 - 2/9*sqrt(-(b*x + a)^2 + 1)/b^3
```

3.25.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \arccos(a+bx)\,dx = \int x^2\,\mathrm{acos}(a+b\,x)\,dx$$

input
```
int(x^2*acos(a + b*x),x)
```

output
```
int(x^2*acos(a + b*x), x)
```

3.26 $\int x \arccos(a + bx) \, dx$

3.26.1 Optimal result

Integrand size = 8, antiderivative size = 80

$$\int x \arccos(a+bx) \, dx = \frac{3a\sqrt{1-(a+bx)^2}}{4b^2} - \frac{x\sqrt{1-(a+bx)^2}}{4b} + \frac{1}{2}x^2 \arccos(a+bx) + \frac{(1+2a^2)\arcsin(a+bx)}{4b^2}$$

output
```
1/2*x^2*arccos(b*x+a)+1/4*(2*a^2+1)*arcsin(b*x+a)/b^2+3/4*a*(1-(b*x+a)^2)^
(1/2)/b^2-1/4*x*(1-(b*x+a)^2)^(1/2)/b
```

3.26.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.86

$$\int x \arccos(a+bx) \, dx = \frac{(3a-bx)\sqrt{1-a^2-2abx-b^2x^2} + 2b^2x^2 \arccos(a+bx) + (1+2a^2)\arcsin(a+bx)}{4b^2}$$

input
```
Integrate[x*ArcCos[a + b*x],x]
```

output
```
((3*a - b*x)*Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2] + 2*b^2*x^2*ArcCos[a + b*x]
 + (1 + 2*a^2)*ArcSin[a + b*x])/(4*b^2)
```

3.26.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5305, 25, 27, 5243, 497, 25, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arccos(a+bx)\,dx$$

$$\Big\downarrow \; 5305$$

$$\frac{\int x\arccos(a+bx)d(a+bx)}{b}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int -x\arccos(a+bx)d(a+bx)}{b}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int -bx\arccos(a+bx)d(a+bx)}{b^2}$$

$$\Big\downarrow \; 5243$$

$$-\frac{-\frac{1}{2}\int \frac{b^2x^2}{\sqrt{1-(a+bx)^2}}d(a+bx)-\frac{1}{2}b^2x^2\arccos(a+bx)}{b^2}$$

$$\Big\downarrow \; 497$$

$$-\frac{\frac{1}{2}\left(\frac{1}{2}\int -\frac{2a^2-3(a+bx)a+1}{\sqrt{1-(a+bx)^2}}d(a+bx)+\frac{1}{2}bx\sqrt{1-(a+bx)^2}\right)-\frac{1}{2}b^2x^2\arccos(a+bx)}{b^2}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{1}{2}\left(\frac{1}{2}bx\sqrt{1-(a+bx)^2}-\frac{1}{2}\int \frac{2a^2-3(a+bx)a+1}{\sqrt{1-(a+bx)^2}}d(a+bx)\right)-\frac{1}{2}b^2x^2\arccos(a+bx)}{b^2}$$

$$\Big\downarrow \; 455$$

$$-\frac{\frac{1}{2}\left(\frac{1}{2}\left(-(2a^2+1)\int \frac{1}{\sqrt{1-(a+bx)^2}}d(a+bx)-3a\sqrt{1-(a+bx)^2}\right)+\frac{1}{2}bx\sqrt{1-(a+bx)^2}\right)-\frac{1}{2}b^2x^2\arccos(a+bx)}{b^2}$$

$$\Big\downarrow \; 223$$

$$-\frac{\frac{1}{2}\left(\frac{1}{2}\left(-\left(2a^2+1\right)\arcsin(a+bx)-3a\sqrt{1-(a+bx)^2}\right)+\frac{1}{2}bx\sqrt{1-(a+bx)^2}\right)-\frac{1}{2}b^2x^2\arccos(a+bx)}{b^2}$$

input
```
Int[x*ArcCos[a + b*x],x]
```

output
```
-((-1/2*(b^2*x^2*ArcCos[a + b*x]) + ((b*x*Sqrt[1 - (a + b*x)^2])/2 + (-3*a
*Sqrt[1 - (a + b*x)^2] - (1 + 2*a^2)*ArcSin[a + b*x])/2)/2)/b^2)
```

3.26.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 497
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n - 1)*((a + b*x^2)^(p + 1)/(b*(n + 2*p + 1))), x] + Simp[1/(b
*(n + 2*p + 1))   Int[(c + d*x)^(n - 2)*(a + b*x^2)^p*Simp[b*c^2*(n + 2*p +
 1) - a*d^2*(n - 1) + 2*b*c*d*(n + p)*x, x], x], x] /; FreeQ[{a, b, c, d, n
, p}, x] && If[RationalQ[n], GtQ[n, 1], SumSimplerQ[n, -2]] && NeQ[n + 2*p
+ 1, 0] && IntQuadraticQ[a, 0, b, c, d, n, p, x]
```

rule 5243
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(e*(m + 1))), x] +
Simp[b*c*(n/(e*(m + 1)))   Int[(d + e*x)^(m + 1)*((a + b*ArcCos[c*x])^(n -
1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
 && NeQ[m, -1]
```

rule 5305
```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.26.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{\frac{\arccos(bx+a)(bx+a)^2}{2}-\arccos(bx+a)a(bx+a)-\frac{(bx+a)\sqrt{1-(bx+a)^2}}{4}+\frac{\arcsin(bx+a)}{4}+a\sqrt{1-(bx+a)^2}}{b^2}$
default	$\frac{\frac{\arccos(bx+a)(bx+a)^2}{2}-\arccos(bx+a)a(bx+a)-\frac{(bx+a)\sqrt{1-(bx+a)^2}}{4}+\frac{\arcsin(bx+a)}{4}+a\sqrt{1-(bx+a)^2}}{b^2}$
parts	$\frac{x^2\arccos(bx+a)}{2}+\frac{b\left(-\frac{x\sqrt{-b^2x^2-2abx-a^2+1}}{2b^2}-\frac{3a\left(-\frac{\sqrt{-b^2x^2-2abx-a^2+1}}{b^2}-\frac{a\arctan\left(\frac{\sqrt{b^2}\left(x+\frac{a}{b}\right)}{\sqrt{-b^2x^2-2abx-a^2+1}}\right)}{b\sqrt{b^2}}\right)}{2b}+\frac{(-a^2}{}\right.}{2}$

input
```
int(x*arccos(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^2*(1/2*arccos(b*x+a)*(b*x+a)^2-arccos(b*x+a)*a*(b*x+a)-1/4*(b*x+a)*(1-
(b*x+a)^2)^(1/2)+1/4*arcsin(b*x+a)+a*(1-(b*x+a)^2)^(1/2))
```

3.26.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.74

$$\int x\arccos(a+bx)\,dx$$
$$=\frac{(2\,b^2x^2-2\,a^2-1)\arccos{(bx+a)}-\sqrt{-b^2x^2-2\,abx-a^2+1}(bx-3\,a)}{4\,b^2}$$

input
```
integrate(x*arccos(b*x+a),x, algorithm="fricas")
```

output
```
1/4*((2*b^2*x^2 - 2*a^2 - 1)*arccos(b*x + a) - sqrt(-b^2*x^2 - 2*a*b*x - a
^2 + 1)*(b*x - 3*a))/b^2
```

3.26.6 Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.30

$$\int x\arccos(a+bx)\,dx$$
$$=\begin{cases} -\frac{a^2\operatorname{acos}(a+bx)}{2b^2}+\frac{3a\sqrt{-a^2-2abx-b^2x^2+1}}{4b^2}+\frac{x^2\operatorname{acos}(a+bx)}{2}-\frac{x\sqrt{-a^2-2abx-b^2x^2+1}}{4b}-\frac{\operatorname{acos}(a+bx)}{4b^2} & \text{for } b\neq 0\\ \frac{x^2\operatorname{acos}(a)}{2} & \text{otherwise}\end{cases}$$

input
```
integrate(x*acos(b*x+a),x)
```

output
```
Piecewise((-a**2*acos(a + b*x)/(2*b**2) + 3*a*sqrt(-a**2 - 2*a*b*x - b**2*
x**2 + 1)/(4*b**2) + x**2*acos(a + b*x)/2 - x*sqrt(-a**2 - 2*a*b*x - b**2*
x**2 + 1)/(4*b) - acos(a + b*x)/(4*b**2), Ne(b, 0)), (x**2*acos(a)/2, True
))
```

3.26.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(68) = 136.

Time = 0.28 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.91

$$\int x\arccos(a+bx)\,dx=\frac{1}{2}x^2\arccos(bx+a)$$
$$-\frac{1}{4}b\left(\frac{3a^2\arcsin\left(-\frac{b^2x+ab}{\sqrt{a^2b^2-(a^2-1)b^2}}\right)}{b^3}+\frac{\sqrt{-b^2x^2-2abx-a^2+1}x}{b^2}-\frac{(a^2-1)\arcsin\left(-\frac{b^2x+ab}{\sqrt{a^2b^2-(a^2-1)b^2}}\right)}{b^3}\right.$$

input
```
integrate(x*arccos(b*x+a),x, algorithm="maxima")
```

output
```
1/2*x^2*arccos(b*x + a) - 1/4*b*(3*a^2*arcsin(-(b^2*x + a*b)/sqrt(a^2*b^2
- (a^2 - 1)*b^2))/b^3 + sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*x/b^2 - (a^2 -
1)*arcsin(-(b^2*x + a*b)/sqrt(a^2*b^2 - (a^2 - 1)*b^2))/b^3 - 3*sqrt(-b^2*
x^2 - 2*a*b*x - a^2 + 1)*a/b^3)
```

3.26.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.10

$$\int x \arccos(a+bx)\,dx = \frac{(bx+a)^2 \arccos(bx+a)}{2\,b^2} - \frac{(bx+a)a \arccos(bx+a)}{b^2} - \frac{\sqrt{-(bx+a)^2+1}(bx+a)}{4\,b^2} + \frac{\sqrt{-(bx+a)^2+1}a}{b^2} - \frac{\arccos(bx+a)}{4\,b^2}$$

input
```
integrate(x*arccos(b*x+a),x, algorithm="giac")
```

output
```
1/2*(b*x + a)^2*arccos(b*x + a)/b^2 - (b*x + a)*a*arccos(b*x + a)/b^2 - 1/
4*sqrt(-(b*x + a)^2 + 1)*(b*x + a)/b^2 + sqrt(-(b*x + a)^2 + 1)*a/b^2 - 1/
4*arccos(b*x + a)/b^2
```

3.26.9 Mupad [F(-1)]

Timed out.

$$\int x \arccos(a+bx)\,dx = \int x \,\mathrm{acos}(a+b\,x)\,dx$$

input
```
int(x*acos(a + b*x),x)
```

output
```
int(x*acos(a + b*x), x)
```

3.27 $\int \arccos(a + bx)\,dx$

3.27.1 Optimal result

Integrand size = 6, antiderivative size = 36

$$\int \arccos(a+bx)\,dx = -\frac{\sqrt{1-(a+bx)^2}}{b} + \frac{(a+bx)\arccos(a+bx)}{b}$$

output
```
(b*x+a)*arccos(b*x+a)/b-(1-(b*x+a)^2)^(1/2)/b
```

3.27.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 154 vs. 2(36) = 72.

Time = 0.09 (sec) , antiderivative size = 154, normalized size of antiderivative = 4.28

$$\int \arccos(a+bx)\,dx = x\arccos(a+bx) - \frac{2b\sqrt{1-a^2-2abx-b^2x^2} + 2ab\arctan\left(\frac{\sqrt{-b^2}x-\sqrt{1-a^2-2abx-b^2x^2}}{a}\right) + a\sqrt{-b^2}\log\left(-1+2abx+2b^2x^2+2\right.}{2b^2}$$

input
```
Integrate[ArcCos[a + b*x],x]
```

output
```
x*ArcCos[a + b*x] - (2*b*Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2] + 2*a*b*ArcTan[
(Sqrt[-b^2]*x - Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2])/a] + a*Sqrt[-b^2]*Log[-
1 + 2*a*b*x + 2*b^2*x^2 + 2*Sqrt[-b^2]*x*Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2]
])/(2*b^2)
```

3.27.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5303, 5131, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arccos(a+bx)\,dx$$

$$\Big\downarrow \; 5303$$

$$\frac{\int \arccos(a+bx)d(a+bx)}{b}$$

$$\Big\downarrow \; 5131$$

$$\frac{\int \frac{a+bx}{\sqrt{1-(a+bx)^2}}d(a+bx)+(a+bx)\arccos(a+bx)}{b}$$

$$\Big\downarrow \; 241$$

$$\frac{(a+bx)\arccos(a+bx)-\sqrt{1-(a+bx)^2}}{b}$$

input
```
Int[ArcCos[a + b*x],x]
```

output
```
(-Sqrt[1 - (a + b*x)^2] + (a + b*x)*ArcCos[a + b*x])/b
```

3.27.3.1 Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5131
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Simp[b*c*n   Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5303
```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
  Subst[Int[(a + b*ArcCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
n}, x]
```

3.27.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{(bx+a)\arccos(bx+a)-\sqrt{1-(bx+a)^{2}}}{b}$	33
default	$\frac{(bx+a)\arccos(bx+a)-\sqrt{1-(bx+a)^{2}}}{b}$	33
parts	$x\arccos\left(bx+a\right)+b\left(-\frac{\sqrt{-b^{2}x^{2}-2abx-a^{2}+1}}{b^{2}}-\frac{a\arctan\left(\frac{\sqrt{b^{2}}\left(x+\frac{a}{b}\right)}{\sqrt{-b^{2}x^{2}-2abx-a^{2}+1}}\right)}{b\sqrt{b^{2}}}\right)$	87

input
```
int(arccos(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*((b*x+a)*arccos(b*x+a)-(1-(b*x+a)^2)^(1/2))
```

3.27.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.14

$$\int\arccos(a+bx)\,dx=\frac{(bx+a)\arccos\left(bx+a\right)-\sqrt{-b^{2}x^{2}-2\,abx-a^{2}+1}}{b}$$

input
```
integrate(arccos(b*x+a),x, algorithm="fricas")
```

output
```
((b*x + a)*arccos(b*x + a) - sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1))/b
```

3.27.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.28

$$\int \arccos(a+bx)\,dx = \begin{cases} \frac{a \operatorname{acos}(a+bx)}{b} + x \operatorname{acos}(a+bx) - \frac{\sqrt{-a^2-2abx-b^2x^2+1}}{b} & \text{for } b \neq 0 \\ x \operatorname{acos}(a) & \text{otherwise} \end{cases}$$

input
```
integrate(acos(b*x+a),x)
```

output
```
Piecewise((a*acos(a + b*x)/b + x*acos(a + b*x) - sqrt(-a**2 - 2*a*b*x - b*
*2*x**2 + 1)/b, Ne(b, 0)), (x*acos(a), True))
```

3.27.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \arccos(a+bx)\,dx = \frac{(bx+a)\arccos(bx+a) - \sqrt{-(bx+a)^2+1}}{b}$$

input
```
integrate(arccos(b*x+a),x, algorithm="maxima")
```

output
```
((b*x + a)*arccos(b*x + a) - sqrt(-(b*x + a)^2 + 1))/b
```

3.27.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \arccos(a+bx)\,dx = \frac{(bx+a)\arccos(bx+a) - \sqrt{-(bx+a)^2+1}}{b}$$

input
```
integrate(arccos(b*x+a),x, algorithm="giac")
```

output
```
((b*x + a)*arccos(b*x + a) - sqrt(-(b*x + a)^2 + 1))/b
```

3.27.9 Mupad [B] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.44

$$\int \arccos(a+bx)\,dx = x\,\mathrm{acos}(a+b\,x) - \frac{\sqrt{-a^2-2\,a\,b\,x-b^2\,x^2+1}}{b} - \frac{a\,\ln\left(\sqrt{-a^2-2\,a\,b\,x-b^2\,x^2+1}-\frac{x\,b^2+a\,b}{\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

input
```
int(acos(a + b*x),x)
```

output
```
x*acos(a + b*x) - (1 - b^2*x^2 - 2*a*b*x - a^2)^(1/2)/b - (a*log((1 - b^2*
x^2 - 2*a*b*x - a^2)^(1/2) - (a*b + b^2*x)/(-b^2)^(1/2)))/(-b^2)^(1/2)
```

3.28 $\int \frac{\arccos(a+bx)}{x} dx$

3.28.1 Optimal result

Integrand size = 10, antiderivative size = 177

$$\int \frac{\arccos(a+bx)}{x} dx = -\frac{1}{2}i\arccos(a+bx)^2 + \arccos(a+bx)\log\left(1-\frac{e^{i\arccos(a+bx)}}{a-i\sqrt{1-a^2}}\right) + \arccos(a+bx)\log\left(1-\frac{e^{i\arccos(a+bx)}}{a+i\sqrt{1-a^2}}\right) - i\,\mathrm{PolyLog}\left(2,\frac{e^{i\arccos(a+bx)}}{a-i\sqrt{1-a^2}}\right) - i\,\mathrm{PolyLog}\left(2,\frac{e^{i\arccos(a+bx)}}{a+i\sqrt{1-a^2}}\right)$$

output
```
-1/2*I*arccos(b*x+a)^2+arccos(b*x+a)*ln(1-(b*x+a+I*(1-(b*x+a)^2)^(1/2))/(a
-I*(-a^2+1)^(1/2)))+arccos(b*x+a)*ln(1-(b*x+a+I*(1-(b*x+a)^2)^(1/2))/(a+I*
(-a^2+1)^(1/2)))-I*polylog(2,(b*x+a+I*(1-(b*x+a)^2)^(1/2))/(a-I*(-a^2+1)^(
1/2)))-I*polylog(2,(b*x+a+I*(1-(b*x+a)^2)^(1/2))/(a+I*(-a^2+1)^(1/2)))
```

3.28.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.29

$$\int \frac{\arccos(a+bx)}{x} dx = -\frac{1}{2}i\arccos(a+bx)^2 - 4i\arcsin\left(\frac{\sqrt{1-a}}{\sqrt{2}}\right)\arctan\left(\frac{(1+a)\tan\left(\frac{1}{2}\arccos(a+bx)\right)}{\sqrt{-1+a^2}}\right) + \left(\arccos(a+bx) - 2\arcsin\left(\frac{\sqrt{1-a}}{\sqrt{2}}\right)\right)\log\left(1 + \left(-a+\sqrt{-1+a^2}\right)e^{i\arccos(a+bx)}\right) + \left(\arccos(a+bx) + 2\arcsin\left(\frac{\sqrt{1-a}}{\sqrt{2}}\right)\right)\log\left(1-\left(a+\sqrt{-1+a^2}\right)e^{i\arccos(a+bx)}\right) - i\left(\text{PolyLog}\left(2,\left(a-\sqrt{-1+a^2}\right)e^{i\arccos(a+bx)}\right) + \text{PolyLog}\left(2,\left(a+\sqrt{-1+a^2}\right)e^{i\arccos(a+bx)}\right)\right)$$

input
```
Integrate[ArcCos[a + b*x]/x,x]
```

output
```
(-1/2*I)*ArcCos[a + b*x]^2 - (4*I)*ArcSin[Sqrt[1 - a]/Sqrt[2]]*ArcTan[((1
+ a)*Tan[ArcCos[a + b*x]/2])/Sqrt[-1 + a^2]] + (ArcCos[a + b*x] - 2*ArcSin
[Sqrt[1 - a]/Sqrt[2]])*Log[1 + (-a + Sqrt[-1 + a^2])*E^(I*ArcCos[a + b*x])
] + (ArcCos[a + b*x] + 2*ArcSin[Sqrt[1 - a]/Sqrt[2]])*Log[1 - (a + Sqrt[-1
 + a^2])*E^(I*ArcCos[a + b*x])] - I*(PolyLog[2, (a - Sqrt[-1 + a^2])*E^(I*
ArcCos[a + b*x])] + PolyLog[2, (a + Sqrt[-1 + a^2])*E^(I*ArcCos[a + b*x])]
)
```

3.28.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {5305, 25, 27, 5241, 5033, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(a+bx)}{x} dx$$

$$\Big\downarrow \text{5305}$$

$$\frac{\int \frac{\arccos(a+bx)}{x}d(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -\frac{\arccos(a+bx)}{x}d(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\int -\frac{\arccos(a+bx)}{bx}d(a+bx)$$

$$\Big\downarrow \text{5241}$$

$$\int -\frac{\sqrt{1-(a+bx)^2}\arccos(a+bx)}{bx}d\arccos(a+bx)$$

$$\Big\downarrow \text{5033}$$

$$\int \frac{e^{i\arccos(a+bx)}\arccos(a+bx)}{ia-ie^{i\arccos(a+bx)}-\sqrt{1-a^2}}d\arccos(a+bx)+\int \frac{e^{i\arccos(a+bx)}\arccos(a+bx)}{ia-ie^{i\arccos(a+bx)}+\sqrt{1-a^2}}d\arccos(a+bx)-\frac{1}{2}i\arccos(a+bx)^2$$

$$\Big\downarrow \text{2620}$$

$$-\int \log\left(1-\frac{e^{i\arccos(a+bx)}}{a-i\sqrt{1-a^2}}\right)d\arccos(a+bx)-\int \log\left(1-\frac{e^{i\arccos(a+bx)}}{a+i\sqrt{1-a^2}}\right)d\arccos(a+bx)+\arccos(a+bx)\log\left(1-\frac{e^{i\arccos(a+bx)}}{a-i\sqrt{1-a^2}}\right)+\arccos(a+bx)\log\left(1-\frac{e^{i\arccos(a+bx)}}{a+i\sqrt{1-a^2}}\right)-\frac{1}{2}i\arccos(a+bx)^2$$

$$\Big\downarrow \text{2715}$$

$$i\int e^{-i\arccos(a+bx)}\log\left(1-\frac{e^{i\arccos(a+bx)}}{a-i\sqrt{1-a^2}}\right)de^{i\arccos(a+bx)}+i\int e^{-i\arccos(a+bx)}\log\left(1-\frac{e^{i\arccos(a+bx)}}{a+i\sqrt{1-a^2}}\right)de^{i\arccos(a+bx)}+\arccos(a+bx)\log\left(1-\frac{e^{i\arccos(a+bx)}}{a-i\sqrt{1-a^2}}\right)+\arccos(a+bx)\log\left(1-\frac{e^{i\arccos(a+bx)}}{a+i\sqrt{1-a^2}}\right)-\frac{1}{2}i\arccos(a+bx)^2$$

$$\Big\downarrow \text{2838}$$

$$-i\,\text{PolyLog}\left(2,\frac{e^{i\arccos(a+bx)}}{a-i\sqrt{1-a^2}}\right)-i\,\text{PolyLog}\left(2,\frac{e^{i\arccos(a+bx)}}{a+i\sqrt{1-a^2}}\right)+\arccos(a+bx)\log\left(1-\frac{e^{i\arccos(a+bx)}}{a-i\sqrt{1-a^2}}\right)+\arccos(a+bx)\log\left(1-\frac{e^{i\arccos(a+bx)}}{a+i\sqrt{1-a^2}}\right)-\frac{1}{2}i\arccos(a+bx)^2$$

3.28. $\int \frac{\arccos(a+bx)}{x}\,dx$

input
```
Int[ArcCos[a + b*x]/x,x]
```

output
```
(-1/2*I)*ArcCos[a + b*x]^2 + ArcCos[a + b*x]*Log[1 - E^(I*ArcCos[a + b*x])
/(a - I*Sqrt[1 - a^2])] + ArcCos[a + b*x]*Log[1 - E^(I*ArcCos[a + b*x])/(a
 + I*Sqrt[1 - a^2])] - I*PolyLog[2, E^(I*ArcCos[a + b*x])/(a - I*Sqrt[1 -
a^2])] - I*PolyLog[2, E^(I*ArcCos[a + b*x])/(a + I*Sqrt[1 - a^2])]
```

3.28.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5033
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)])/(Cos[(c_.) + (d_.)
*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[I*((e + f*x)^(m + 1)/(b*f*(m + 1)))
, x] + (Int[(e + f*x)^m*(E^(I*(c + d*x))/(I*a - Rt[-a^2 + b^2, 2] + I*b*E^(
I*(c + d*x)))), x] + Int[(e + f*x)^m*(E^(I*(c + d*x))/(I*a + Rt[-a^2 + b^2,
 2] + I*b*E^(I*(c + d*x)))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m,
 0] && NegQ[a^2 - b^2]
```

rule 5241
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Subst[Int[(a + b*x)^n*(Sin[x]/(c*d + e*Cos[x])), x], x, ArcCos[c*x]] /
; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

rule 5305
```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.28.4 Maple [A] (verified)

Time = 3.97 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.12

method	result
derivativedivides	$-\frac{i \arccos(bx+a)^2}{2} + \arccos(bx+a) \ln\left(\frac{\sqrt{a^2-1}-bx-i\sqrt{1-(bx+a)^2}}{a+\sqrt{a^2-1}}\right) + \arccos(bx+a) \ln\left(\frac{\sqrt{a^2-1}}{}\right.$
default	$-\frac{i \arccos(bx+a)^2}{2} + \arccos(bx+a) \ln\left(\frac{\sqrt{a^2-1}-bx-i\sqrt{1-(bx+a)^2}}{a+\sqrt{a^2-1}}\right) + \arccos(bx+a) \ln\left(\frac{\sqrt{a^2-1}}{}\right.$

input
```
int(arccos(b*x+a)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*arccos(b*x+a)^2+arccos(b*x+a)*ln(((a^2-1)^(1/2)-b*x-I*(1-(b*x+a)^2)
^(1/2))/(a+(a^2-1)^(1/2)))+arccos(b*x+a)*ln(((a^2-1)^(1/2)+b*x+I*(1-(b*x+a
)^2)^(1/2))/(-a+(a^2-1)^(1/2)))-I*dilog(((a^2-1)^(1/2)-b*x-I*(1-(b*x+a)^2)
^(1/2))/(a+(a^2-1)^(1/2)))-I*dilog(((a^2-1)^(1/2)+b*x+I*(1-(b*x+a)^2)^(1/2
))/(-a+(a^2-1)^(1/2)))
```

3.28.5 Fricas [F]

$$\int \frac{\arccos(a+bx)}{x} dx = \int \frac{\arccos(bx+a)}{x} dx$$

input
```
integrate(arccos(b*x+a)/x,x, algorithm="fricas")
```

output
```
integral(arccos(b*x + a)/x, x)
```

3.28.6 Sympy [F]

$$\int \frac{\arccos(a+bx)}{x}\,dx = \int \frac{\operatorname{acos}(a+bx)}{x}\,dx$$

input
```
integrate(acos(b*x+a)/x,x)
```

output
```
Integral(acos(a + b*x)/x, x)
```

3.28.7 Maxima [F]

$$\int \frac{\arccos(a+bx)}{x}\,dx = \int \frac{\arccos(bx+a)}{x}\,dx$$

input
```
integrate(arccos(b*x+a)/x,x, algorithm="maxima")
```

output
```
integrate(arccos(b*x + a)/x, x)
```

3.28.8 Giac [F]

$$\int \frac{\arccos(a+bx)}{x}\,dx = \int \frac{\arccos(bx+a)}{x}\,dx$$

input
```
integrate(arccos(b*x+a)/x,x, algorithm="giac")
```

output
```
integrate(arccos(b*x + a)/x, x)
```

3.28.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(a+bx)}{x}\,dx = \int \frac{\operatorname{acos}(a+b\,x)}{x}\,dx$$

input
```
int(acos(a + b*x)/x,x)
```

output
```
int(acos(a + b*x)/x, x)
```

3.29 $\int \frac{\arccos(a+bx)}{x^2} dx$

3.29.1 Optimal result

Integrand size = 10, antiderivative size = 63

$$\int \frac{\arccos(a+bx)}{x^2} dx = -\frac{\arccos(a+bx)}{x} + \frac{b\text{arctanh}\left(\frac{1-a(a+bx)}{\sqrt{1-a^2}\sqrt{1-(a+bx)^2}}\right)}{\sqrt{1-a^2}}$$

output
```
-arccos(b*x+a)/x+b*arctanh((1-a*(b*x+a))/(-a^2+1)^(1/2)/(1-(b*x+a)^2)^(1/2
))/(-a^2+1)^(1/2)
```

3.29.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.25

$$\int \frac{\arccos(a+bx)}{x^2} dx = -\frac{\arccos(a+bx)}{x} + \frac{b\left(-\log(x)+\log\left(1-a^2-abx+\sqrt{1-a^2}\sqrt{1-a^2-2abx-b^2x^2}\right)\right)}{\sqrt{1-a^2}}$$

input
```
Integrate[ArcCos[a + b*x]/x^2,x]
```

output
```
-(ArcCos[a + b*x]/x) + (b*(-Log[x] + Log[1 - a^2 - a*b*x + Sqrt[1 - a^2]*S
qrt[1 - a^2 - 2*a*b*x - b^2*x^2]]))/Sqrt[1 - a^2]
```

3.29.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5305, 27, 5243, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(a+bx)}{x^2}dx$$

$$\Big\downarrow \text{5305}$$

$$\frac{\int \frac{\arccos(a+bx)}{x^2}d(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$b\int \frac{\arccos(a+bx)}{b^2x^2}d(a+bx)$$

$$\Big\downarrow \text{5243}$$

$$b\left(\int -\frac{1}{bx\sqrt{1-(a+bx)^2}}d(a+bx)-\frac{\arccos(a+bx)}{bx}\right)$$

$$\Big\downarrow \text{488}$$

$$b\left(-\int \frac{1}{-a^2-\frac{(a(a+bx)-1)^2}{1-(a+bx)^2}+1}d\frac{a(a+bx)-1}{\sqrt{1-(a+bx)^2}}-\frac{\arccos(a+bx)}{bx}\right)$$

$$\Big\downarrow \text{219}$$

$$b\left(-\frac{\text{arctanh}\left(\frac{a(a+bx)-1}{\sqrt{1-a^2}\sqrt{1-(a+bx)^2}}\right)}{\sqrt{1-a^2}}-\frac{\arccos(a+bx)}{bx}\right)$$

input
```
Int[ArcCos[a + b*x]/x^2,x]
```

output
```
b*(-(ArcCos[a + b*x]/(b*x)) - ArcTanh[(-1 + a*(a + b*x))/(Sqrt[1 - a^2]*Sq
rt[1 - (a + b*x)^2])]/Sqrt[1 - a^2])
```

3.29.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 5243
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(e*(m + 1))), x] +
Simp[b*c*(n/(e*(m + 1)))   Int[(d + e*x)^(m + 1)*((a + b*ArcCos[c*x])^(n -
1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
 && NeQ[m, -1]
```

rule 5305
```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.29.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.17

method	result	size
parts	$-\frac{\arccos(bx+a)}{x}+\frac{b\ln\left(\frac{-2a^2+2-2abx+2\sqrt{-a^2+1}\,\sqrt{-b^2x^2-2abx-a^2+1}}{x}\right)}{\sqrt{-a^2+1}}$	74
derivativedivides	$b\left(-\frac{\arccos(bx+a)}{bx}+\frac{\ln\left(\frac{-2a^2+2-2abx+2\sqrt{-a^2+1}\,\sqrt{-b^2x^2-2abx-a^2+1}}{bx}\right)}{\sqrt{-a^2+1}}\right)$	81
default	$b\left(-\frac{\arccos(bx+a)}{bx}+\frac{\ln\left(\frac{-2a^2+2-2abx+2\sqrt{-a^2+1}\,\sqrt{-b^2x^2-2abx-a^2+1}}{bx}\right)}{\sqrt{-a^2+1}}\right)$	81

input
```
int(arccos(b*x+a)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-arccos(b*x+a)/x+b/(-a^2+1)^(1/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-
b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x)
```

3.29.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(57) = 114$.

Time = 0.28 (sec) , antiderivative size = 360, normalized size of antiderivative = 5.71

$$\int \frac{\arccos(a+bx)}{x^2}\,dx = \left[-\frac{\sqrt{-a^2+1}bx\log\left(\frac{(2a^2-1)b^2x^2+2a^4+4(a^3-a)bx-2\sqrt{-b^2x^2-2abx-a^2+1}(abx+a^2-1)\sqrt{-a^2+1}-4a^2+2}{x^2}\right)+2(a^2-1)x\,\mathrm{ar}}{2(a^2-1)x}\right.$$
$$-\frac{\sqrt{a^2-1}bx\arctan\left(\frac{\sqrt{-b^2x^2-2abx-a^2+1}(abx+a^2-1)\sqrt{a^2-1}}{(a^2-1)b^2x^2+a^4+2(a^3-a)bx-2a^2+1}\right)+(a^2-1)x\arctan\left(\frac{\sqrt{-b^2x^2-2abx-a^2+1}(bx+a)}{b^2x^2+2abx+a^2-1}\right)+(a}{(a^2-1)x}$$

input
```
integrate(arccos(b*x+a)/x^2,x, algorithm="fricas")
```

output
```
[-1/2*(sqrt(-a^2 + 1)*b*x*log(((2*a^2 - 1)*b^2*x^2 + 2*a^4 + 4*(a^3 - a)*b
*x - 2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x + a^2 - 1)*sqrt(-a^2 + 1)
 - 4*a^2 + 2)/x^2) + 2*(a^2 - 1)*x*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 +
1)*(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 - 1)) + 2*(a^2 - (a^2 - 1)*x - 1)*ar
ccos(b*x + a))/((a^2 - 1)*x), -(sqrt(a^2 - 1)*b*x*arctan(sqrt(-b^2*x^2 - 2
*a*b*x - a^2 + 1)*(a*b*x + a^2 - 1)*sqrt(a^2 - 1)/((a^2 - 1)*b^2*x^2 + a^4
 + 2*(a^3 - a)*b*x - 2*a^2 + 1)) + (a^2 - 1)*x*arctan(sqrt(-b^2*x^2 - 2*a*
b*x - a^2 + 1)*(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 - 1)) + (a^2 - (a^2 - 1)
*x - 1)*arccos(b*x + a))/((a^2 - 1)*x)]
```

3.29.6 Sympy [F]

$$\int \frac{\arccos(a+bx)}{x^2}\,dx = \int \frac{\operatorname{acos}(a+bx)}{x^2}\,dx$$

input
```
integrate(acos(b*x+a)/x**2,x)
```

output
```
Integral(acos(a + b*x)/x**2, x)
```

3.29.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\arccos(a+bx)}{x^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(arccos(b*x+a)/x^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a-1>0)', see `assume?` for more
details)Is
```

3.29.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.25

$$\int \frac{\arccos(a+bx)}{x^2}\,dx = -\frac{2\,b^2\arctan\left(\frac{\frac{\left(\sqrt{-b^2x^2-2\,abx-a^2+1}|b|+b\right)a}{b^2x+ab}-1}{\sqrt{a^2-1}}\right)}{\sqrt{a^2-1}|b|}-\frac{\arccos\left(bx+a\right)}{x}$$

input
```
integrate(arccos(b*x+a)/x^2,x, algorithm="giac")
```

output
```
-2*b^2*arctan(((sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)*a/(b^2*x +
a*b) - 1)/sqrt(a^2 - 1))/(sqrt(a^2 - 1)*abs(b)) - arccos(b*x + a)/x
```

3.29.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(a+bx)}{x^2}\,dx = \int \frac{\operatorname{acos}(a+b\,x)}{x^2}\,dx$$

input
```
int(acos(a + b*x)/x^2,x)
```

output
```
int(acos(a + b*x)/x^2, x)
```

3.30 $\int \frac{\arccos(a+bx)}{x^3}\,dx$

3.30.1 Optimal result

Integrand size = 10, antiderivative size = 103

$$\int \frac{\arccos(a+bx)}{x^3}\,dx = \frac{b\sqrt{1-(a+bx)^2}}{2(1-a^2)\,x} - \frac{\arccos(a+bx)}{2x^2} + \frac{ab^2\operatorname{arctanh}\left(\frac{1-a(a+bx)}{\sqrt{1-a^2}\sqrt{1-(a+bx)^2}}\right)}{2(1-a^2)^{3/2}}$$

output
```
-1/2*arccos(b*x+a)/x^2+1/2*a*b^2*arctanh((1-a*(b*x+a))/(-a^2+1)^(1/2)/(1-(
b*x+a)^2)^(1/2))/(-a^2+1)^(3/2)+1/2*b*(1-(b*x+a)^2)^(1/2)/(-a^2+1)/x
```

3.30.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.22

$$\int \frac{\arccos(a+bx)}{x^3}\,dx = -\frac{\arccos(a+bx) - \frac{bx\left(\sqrt{1-a^2}\sqrt{1-a^2-2abx-b^2x^2} - abx\log(x) + abx\log\left(1-a^2-abx+\sqrt{1-a^2}\sqrt{1-a^2-2abx-b^2x^2}\right)\right)}{(1-a^2)^{3/2}}}{2x^2}$$

input
```
Integrate[ArcCos[a + b*x]/x^3,x]
```

output
```
-1/2*(ArcCos[a + b*x] - (b*x*(Sqrt[1 - a^2]*Sqrt[1 - a^2 - 2*a*b*x - b^2*x
^2] - a*b*x*Log[x] + a*b*x*Log[1 - a^2 - a*b*x + Sqrt[1 - a^2]*Sqrt[1 - a^
2 - 2*a*b*x - b^2*x^2]]))/(1 - a^2)^(3/2))/x^2
```

3.30.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {5305, 25, 27, 5243, 491, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(a+bx)}{x^3}\,dx$$

$$\Big\downarrow \text{ 5305}$$

$$\frac{\int \frac{\arccos(a+bx)}{x^3}d(a+bx)}{b}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int -\frac{\arccos(a+bx)}{x^3}d(a+bx)}{b}$$

$$\Big\downarrow \text{ 27}$$

$$-b^2\int -\frac{\arccos(a+bx)}{b^3x^3}d(a+bx)$$

$$\Big\downarrow \text{ 5243}$$

$$-b^2\left(\frac{1}{2}\int \frac{1}{b^2x^2\sqrt{1-(a+bx)^2}}d(a+bx)+\frac{\arccos(a+bx)}{2b^2x^2}\right)$$

$$\Big\downarrow \text{ 491}$$

$$-b^2\left(\frac{1}{2}\left(-\frac{a\int -\frac{1}{bx\sqrt{1-(a+bx)^2}}d(a+bx)}{1-a^2}-\frac{\sqrt{1-(a+bx)^2}}{(1-a^2)\,bx}\right)+\frac{\arccos(a+bx)}{2b^2x^2}\right)$$

$$\Big\downarrow \text{ 488}$$

$$-b^2\left(\frac{1}{2}\left(\frac{a\int \frac{1}{-a^2-\frac{(a(a+bx)-1)^2}{1-(a+bx)^2}+1}d\frac{a(a+bx)-1}{\sqrt{1-(a+bx)^2}}}{1-a^2}-\frac{\sqrt{1-(a+bx)^2}}{(1-a^2)\,bx}\right)+\frac{\arccos(a+bx)}{2b^2x^2}\right)$$

$$\Big\downarrow \text{ 219}$$

$$-b^2\left(\frac{1}{2}\left(\frac{a\,\text{arctanh}\left(\frac{a(a+bx)-1}{\sqrt{1-a^2}\sqrt{1-(a+bx)^2}}\right)}{(1-a^2)^{3/2}}-\frac{\sqrt{1-(a+bx)^2}}{(1-a^2)\,bx}\right)+\frac{\arccos(a+bx)}{2b^2x^2}\right)$$

input
```
Int[ArcCos[a + b*x]/x^3,x]
```

output
```
-(b^2*(ArcCos[a + b*x]/(2*b^2*x^2) + (-(Sqrt[1 - (a + b*x)^2]/((1 - a^2)*b
*x)) + (a*ArcTanh[(-1 + a*(a + b*x))/(Sqrt[1 - a^2]*Sqrt[1 - (a + b*x)^2])
])/(1 - a^2)^(3/2))/2))
```

3.30.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 491
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b*(c/(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, n, p}, x] && EqQ[n + 2*p + 3, 0]
```

rule 5243
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(e*(m + 1))), x] +
Simp[b*c*(n/(e*(m + 1)))   Int[(d + e*x)^(m + 1)*((a + b*ArcCos[c*x])^(n -
1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
 && NeQ[m, -1]
```

rule 5305
```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.30.4 Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.13

method	result	size
parts	$-\frac{\arccos(bx+a)}{2x^2}-\frac{b\left(-\frac{\sqrt{-b^2x^2-2abx-a^2+1}}{(-a^2+1)x}-\frac{ab\ln\left(\frac{-2a^2+2-2abx+2\sqrt{-a^2+1}\,\sqrt{-b^2x^2-2abx-a^2+1}}{x}\right)}{(-a^2+1)^{\frac{3}{2}}}\right)}{2}$	116
derivativedivides	$b^2\left(-\frac{\arccos(bx+a)}{2b^2x^2}+\frac{\sqrt{-b^2x^2-2abx-a^2+1}}{2(-a^2+1)bx}+\frac{a\ln\left(\frac{-2a^2+2-2abx+2\sqrt{-a^2+1}\,\sqrt{-b^2x^2-2abx-a^2+1}}{bx}\right)}{2(-a^2+1)^{\frac{3}{2}}}\right)$	124
default	$b^2\left(-\frac{\arccos(bx+a)}{2b^2x^2}+\frac{\sqrt{-b^2x^2-2abx-a^2+1}}{2(-a^2+1)bx}+\frac{a\ln\left(\frac{-2a^2+2-2abx+2\sqrt{-a^2+1}\,\sqrt{-b^2x^2-2abx-a^2+1}}{bx}\right)}{2(-a^2+1)^{\frac{3}{2}}}\right)$	124

input
```
int(arccos(b*x+a)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*arccos(b*x+a)/x^2-1/2*b*(-1/(-a^2+1)/x*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)
-a*b/(-a^2+1)^(3/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*
x-a^2+1)^(1/2))/x))
```

3.30.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. 2(87) = 174.

Time = 0.31 (sec) , antiderivative size = 482, normalized size of antiderivative = 4.68

$$\int \frac{\arccos(a+bx)}{x^3}\,dx$$
$$=\left[-\frac{\sqrt{-a^2+1}ab^2x^2\log\left(\frac{(2\,a^2-1)b^2x^2+2\,a^4+4\,(a^3-a)bx+2\,\sqrt{-b^2x^2-2\,abx-a^2+1}(abx+a^2-1)\sqrt{-a^2+1}-4\,a^2+2}{x^2}\right)+2\,(a^4-2\,a\right.$$

input
```
integrate(arccos(b*x+a)/x^3,x, algorithm="fricas")
```

output
```
[-1/4*(sqrt(-a^2 + 1)*a*b^2*x^2*log(((2*a^2 - 1)*b^2*x^2 + 2*a^4 + 4*(a^3
- a)*b*x + 2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x + a^2 - 1)*sqrt(-a^
2 + 1) - 4*a^2 + 2)/x^2) + 2*(a^4 - 2*a^2 + 1)*x^2*arctan(sqrt(-b^2*x^2 -
2*a*b*x - a^2 + 1)*(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 - 1)) + 2*sqrt(-b^2*
x^2 - 2*a*b*x - a^2 + 1)*(a^2 - 1)*b*x + 2*(a^4 - (a^4 - 2*a^2 + 1)*x^2 -
2*a^2 + 1)*arccos(b*x + a))/((a^4 - 2*a^2 + 1)*x^2), 1/2*(sqrt(a^2 - 1)*a*
b^2*x^2*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x + a^2 - 1)*sqrt(a
^2 - 1)/((a^2 - 1)*b^2*x^2 + a^4 + 2*(a^3 - a)*b*x - 2*a^2 + 1)) - (a^4 -
2*a^2 + 1)*x^2*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x + a)/(b^2*x^
2 + 2*a*b*x + a^2 - 1)) - sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a^2 - 1)*b*x
 - (a^4 - (a^4 - 2*a^2 + 1)*x^2 - 2*a^2 + 1)*arccos(b*x + a))/((a^4 - 2*a^
2 + 1)*x^2)]
```

3.30.6 Sympy [F]

$$\int \frac{\arccos(a + bx)}{x^3} \, dx = \int \frac{\operatorname{acos}(a + bx)}{x^3} \, dx$$

input
```
integrate(acos(b*x+a)/x**3,x)
```

output
```
Integral(acos(a + b*x)/x**3, x)
```

3.30.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\arccos(a + bx)}{x^3} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(arccos(b*x+a)/x^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a-1>0)', see `assume?` for more
details)Is
```

3.30.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. $2(87) = 174$.

Time = 0.29 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.35

$$\int \frac{\arccos(a+bx)}{x^3}\,dx = \left(\frac{ab^2\arctan\left(\frac{\frac{\left(\sqrt{-b^2x^2-2\,abx-a^2+1}|b|+b\right)a}{b^2x+ab}-1}{\sqrt{a^2-1}}\right)}{(a^2|b|-|b|)\sqrt{a^2-1}} - \frac{ab^2-\frac{\left(\sqrt{-b^2x^2-2\,abx-a^2+1}|b|+b\right)b^2}{b^2x+ab}}{(a^3|b|-a|b|)\left(\frac{\left(\sqrt{-b^2x^2-2\,abx-a^2+1}|b|+b\right)^2a}{(b^2x+ab)^2}+a-\frac{2\left(\sqrt{-b^2x^2-2\,abx-\right.}{b^2x+ab}\right.}\right. - \frac{\arccos(bx+a)}{2\,x^2}$$

input
```
integrate(arccos(b*x+a)/x^3,x, algorithm="giac")
```

output
```
(a*b^2*arctan(((sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)*a/(b^2*x +
a*b) - 1)/sqrt(a^2 - 1))/((a^2*abs(b) - abs(b))*sqrt(a^2 - 1)) - (a*b^2 -
(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)*b^2/(b^2*x + a*b))/((a^3*a
bs(b) - a*abs(b))*((sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^2*a/(b^
2*x + a*b)^2 + a - 2*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)/(b^2*
x + a*b))))*b - 1/2*arccos(b*x + a)/x^2
```

3.30.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(a+bx)}{x^3}\,dx = \int \frac{\mathrm{acos}(a+b\,x)}{x^3}\,dx$$

input
```
int(acos(a + b*x)/x^3,x)
```

output
```
int(acos(a + b*x)/x^3, x)
```

3.31 $\int \frac{\arccos(a+bx)}{x^4}\,dx$

3.31.1 Optimal result

Integrand size = 10, antiderivative size = 144

$$\int \frac{\arccos(a+bx)}{x^4}\,dx = \frac{b\sqrt{1-(a+bx)^2}}{6(1-a^2)x^2} + \frac{ab^2\sqrt{1-(a+bx)^2}}{2(1-a^2)^2x} - \frac{\arccos(a+bx)}{3x^3} + \frac{(1+2a^2)\,b^3\operatorname{arctanh}\left(\frac{1-a(a+bx)}{\sqrt{1-a^2}\sqrt{1-(a+bx)^2}}\right)}{6(1-a^2)^{5/2}}$$

output
```
-1/3*arccos(b*x+a)/x^3+1/6*(2*a^2+1)*b^3*arctanh((1-a*(b*x+a))/(-a^2+1)^(1
/2)/(1-(b*x+a)^2)^(1/2))/(-a^2+1)^(5/2)+1/6*b*(1-(b*x+a)^2)^(1/2)/(-a^2+1)
/x^2+1/2*a*b^2*(1-(b*x+a)^2)^(1/2)/(-a^2+1)^2/x
```

3.31.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.17

$$\int \frac{\arccos(a+bx)}{x^4}\,dx$$
$$= \frac{\sqrt{1-a^2}bx(1-a^2+3abx)\sqrt{1-a^2-2abx-b^2x^2} - 2(1-a^2)^{5/2}\arccos(a+bx) - (1+2a^2)\,b^3x^3\log(x) -}{6(1-a^2)^{5/2}x^3}$$

input
```
Integrate[ArcCos[a + b*x]/x^4,x]
```

output
```
(Sqrt[1 - a^2]*b*x*(1 - a^2 + 3*a*b*x)*Sqrt[1 - a^2 - 2*a*b*x - b^2*x^2] -
 2*(1 - a^2)^(5/2)*ArcCos[a + b*x] - (1 + 2*a^2)*b^3*x^3*Log[x] + (1 + 2*a
^2)*b^3*x^3*Log[1 - a^2 - a*b*x + Sqrt[1 - a^2]*Sqrt[1 - a^2 - 2*a*b*x - b
^2*x^2]])/(6*(1 - a^2)^(5/2)*x^3)
```

3.31.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {5305, 27, 5243, 498, 25, 679, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(a+bx)}{x^4}\,dx$$

$$\Big\downarrow \text{5305}$$

$$\frac{\int \frac{\arccos(a+bx)}{x^4}d(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$b^3\int \frac{\arccos(a+bx)}{b^4x^4}d(a+bx)$$

$$\Big\downarrow \text{5243}$$

$$b^3\left(\frac{1}{3}\int -\frac{1}{b^3x^3\sqrt{1-(a+bx)^2}}d(a+bx)-\frac{\arccos(a+bx)}{3b^3x^3}\right)$$

$$\Big\downarrow \text{498}$$

$$b^3\left(\frac{1}{3}\left(\frac{\int -\frac{3a+bx}{b^2x^2\sqrt{1-(a+bx)^2}}d(a+bx)}{2\left(1-a^2\right)}+\frac{\sqrt{1-(a+bx)^2}}{2\left(1-a^2\right)b^2x^2}\right)-\frac{\arccos(a+bx)}{3b^3x^3}\right)$$

$$\Big\downarrow \text{25}$$

$$b^3\left(\frac{1}{3}\left(\frac{\sqrt{1-(a+bx)^2}}{2\left(1-a^2\right)b^2x^2}-\frac{\int \frac{3a+bx}{b^2x^2\sqrt{1-(a+bx)^2}}d(a+bx)}{2\left(1-a^2\right)}\right)-\frac{\arccos(a+bx)}{3b^3x^3}\right)$$

$$\Big\downarrow \text{679}$$

$$b^3\left(\frac{1}{3}\left(\frac{\sqrt{1-(a+bx)^2}}{2(1-a^2)b^2x^2}-\frac{-\frac{(2a^2+1)\int -\frac{1}{bx\sqrt{1-(a+bx)^2}}d(a+bx)}{1-a^2}-\frac{3a\sqrt{1-(a+bx)^2}}{(1-a^2)bx}}{2(1-a^2)}\right)-\frac{\arccos(a+bx)}{3b^3x^3}\right)$$

$$\Big\downarrow \; 488$$

$$b^3\left(\frac{1}{3}\left(\frac{\sqrt{1-(a+bx)^2}}{2(1-a^2)b^2x^2}-\frac{\frac{(2a^2+1)\int \frac{1}{-a^2-\frac{(a(a+bx)-1)^2}{1-(a+bx)^2}+1}d\frac{a(a+bx)-1}{\sqrt{1-(a+bx)^2}}}{1-a^2}-\frac{3a\sqrt{1-(a+bx)^2}}{(1-a^2)bx}}{2(1-a^2)}\right)-\frac{\arccos(a+bx)}{3b^3x^3}\right)$$

$$\Big\downarrow \; 219$$

$$b^3\left(\frac{1}{3}\left(\frac{\sqrt{1-(a+bx)^2}}{2(1-a^2)b^2x^2}-\frac{\frac{(2a^2+1)\text{arctanh}\left(\frac{a(a+bx)-1}{\sqrt{1-a^2}\sqrt{1-(a+bx)^2}}\right)}{(1-a^2)^{3/2}}-\frac{3a\sqrt{1-(a+bx)^2}}{(1-a^2)bx}}{2(1-a^2)}\right)-\frac{\arccos(a+bx)}{3b^3x^3}\right)$$

input
```
Int[ArcCos[a + b*x]/x^4,x]
```

output
```
b^3*(-1/3*ArcCos[a + b*x]/(b^3*x^3) + (Sqrt[1 - (a + b*x)^2]/(2*(1 - a^2)*
b^2*x^2) - ((-3*a*Sqrt[1 - (a + b*x)^2])/((1 - a^2)*b*x) + ((1 + 2*a^2)*Ar
cTanh[(-1 + a*(a + b*x))/(Sqrt[1 - a^2]*Sqrt[1 - (a + b*x)^2])])/(1 - a^2)
^(3/2))/(2*(1 - a^2)))/3)
```

3.31.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 498
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
d*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + S
imp[b/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)^p*(c*(n
 + 1) - d*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[n
, -1] && ((LtQ[n, -1] && IntQuadraticQ[a, 0, b, c, d, n, p, x]) || (SumSimp
lerQ[n, 1] && IntegerQ[p]) || ILtQ[Simplify[n + 2*p + 3], 0])
```

rule 679
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[(c*d*f + a*e*g)/(c*d^2 + a*e^2)
 Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
 p}, x] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 5243
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(e*(m + 1))), x] +
Simp[b*c*(n/(e*(m + 1)))   Int[(d + e*x)^(m + 1)*((a + b*ArcCos[c*x])^(n -
1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
 && NeQ[m, -1]
```

rule 5305
```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.31. $\int \frac{\arccos(a+bx)}{x^4} \, dx$

3.31.4 Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.60

method	result
parts	$-\frac{\arccos(bx+a)}{3x^3}-\frac{b\left(-\frac{\sqrt{-b^2x^2-2abx-a^2+1}}{2\left(-a^2+1\right)x^2}+\frac{3ab\left(-\frac{\sqrt{-b^2x^2-2abx-a^2+1}}{\left(-a^2+1\right)x}-\frac{ab\ln\left(\frac{-2a^2+2-2abx+2\sqrt{-a^2+1}\,\sqrt{-b^2x^2-2a\cdots}}{x}\right)}{\left(-a^2+1\right)^{\frac{3}{2}}}\cdots\right)}{2\left(-a^2+1\right)}\cdots\right)}{3}\cdots$
derivativedivides	$b^3\left(-\frac{\arccos(bx+a)}{3b^3x^3}+\frac{\sqrt{-b^2x^2-2abx-a^2+1}}{6(-a^2+1)b^2x^2}-\frac{a\left(-\frac{\sqrt{-b^2x^2-2abx-a^2+1}}{\left(-a^2+1\right)bx}-\frac{a\ln\left(\frac{-2a^2+2-2abx+2\sqrt{-a^2+1}\,\sqrt{-b^2x^2\cdots}}{bx}\right)}{\left(-a^2+1\right)^{\frac{3}{2}}}\cdots\right)}{2(-a^2+1)}\cdots\right)$
default	$b^3\left(-\frac{\arccos(bx+a)}{3b^3x^3}+\frac{\sqrt{-b^2x^2-2abx-a^2+1}}{6(-a^2+1)b^2x^2}-\frac{a\left(-\frac{\sqrt{-b^2x^2-2abx-a^2+1}}{\left(-a^2+1\right)bx}-\frac{a\ln\left(\frac{-2a^2+2-2abx+2\sqrt{-a^2+1}\,\sqrt{-b^2x^2\cdots}}{bx}\right)}{\left(-a^2+1\right)^{\frac{3}{2}}}\cdots\right)}{2(-a^2+1)}\cdots\right)$

input
```
int(arccos(b*x+a)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*arccos(b*x+a)/x^3-1/3*b*(-1/2/(-a^2+1)/x^2*(-b^2*x^2-2*a*b*x-a^2+1)^(
1/2)+3/2*a*b/(-a^2+1)*(-1/(-a^2+1)/x*(-b^2*x^2-2*a*b*x-a^2+1)^(1/2)-a*b/(-
a^2+1)^(3/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1
)^(1/2))/x))-1/2*b^2/(-a^2+1)^(3/2)*ln((-2*a^2+2-2*a*b*x+2*(-a^2+1)^(1/2)*
(-b^2*x^2-2*a*b*x-a^2+1)^(1/2))/x))
```

3.31.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(122) = 244$.

Time = 0.32 (sec) , antiderivative size = 580, normalized size of antiderivative = 4.03

$$\int \frac{\arccos(a+bx)}{x^4}\,dx$$
$$= \left[-\frac{(2\,a^2+1)\sqrt{-a^2+1}b^3x^3\log\left(\frac{(2\,a^2-1)b^2x^2+2\,a^4+4\,(a^3-a)bx-2\,\sqrt{-b^2x^2-2\,abx-a^2+1}(abx+a^2-1)\sqrt{-a^2+1}-4\,a^2+2}{x^2}\right)+4\,\cdots}{\cdots}\right.$$
$$\left.-\frac{(2\,a^2+1)\sqrt{a^2-1}b^3x^3\arctan\left(\frac{\sqrt{-b^2x^2-2\,abx-a^2+1}(abx+a^2-1)\sqrt{a^2-1}}{(a^2-1)b^2x^2+a^4+2\,(a^3-a)bx-2\,a^2+1}\right)+2\,(a^6-3\,a^4+3\,a^2-1)x^3\arctan\left(\cdots\right)}{\cdots}\right.$$

input
```
integrate(arccos(b*x+a)/x^4,x, algorithm="fricas")
```

output
```
[-1/12*((2*a^2 + 1)*sqrt(-a^2 + 1)*b^3*x^3*log(((2*a^2 - 1)*b^2*x^2 + 2*a^
4 + 4*(a^3 - a)*b*x - 2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x + a^2 -
1)*sqrt(-a^2 + 1) - 4*a^2 + 2)/x^2) + 4*(a^6 - 3*a^4 + 3*a^2 - 1)*x^3*arct
an(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 -
 1)) + 4*(a^6 - 3*a^4 - (a^6 - 3*a^4 + 3*a^2 - 1)*x^3 + 3*a^2 - 1)*arccos(
b*x + a) - 2*(3*(a^3 - a)*b^2*x^2 - (a^4 - 2*a^2 + 1)*b*x)*sqrt(-b^2*x^2 -
 2*a*b*x - a^2 + 1))/((a^6 - 3*a^4 + 3*a^2 - 1)*x^3), -1/6*((2*a^2 + 1)*sq
rt(a^2 - 1)*b^3*x^3*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*(a*b*x + a^2
 - 1)*sqrt(a^2 - 1)/((a^2 - 1)*b^2*x^2 + a^4 + 2*(a^3 - a)*b*x - 2*a^2 + 1
)) + 2*(a^6 - 3*a^4 + 3*a^2 - 1)*x^3*arctan(sqrt(-b^2*x^2 - 2*a*b*x - a^2
+ 1)*(b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 - 1)) + 2*(a^6 - 3*a^4 - (a^6 - 3*
a^4 + 3*a^2 - 1)*x^3 + 3*a^2 - 1)*arccos(b*x + a) - (3*(a^3 - a)*b^2*x^2 -
 (a^4 - 2*a^2 + 1)*b*x)*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1))/((a^6 - 3*a^4
+ 3*a^2 - 1)*x^3)]
```

3.31.6 Sympy [F]

$$\int \frac{\arccos(a+bx)}{x^4}\,dx = \int \frac{\operatorname{acos}(a+bx)}{x^4}\,dx$$

input
```
integrate(acos(b*x+a)/x**4,x)
```

output
```
Integral(acos(a + b*x)/x**4, x)
```

3.31.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\arccos(a+bx)}{x^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(arccos(b*x+a)/x^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a-1>0)', see `assume?` for more
details)Is
```

3.31.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 557 vs. $2(122) = 244$.

Time = 0.29 (sec) , antiderivative size = 557, normalized size of antiderivative = 3.87

$$\int \frac{\arccos(a+bx)}{x^4}\,dx = -\frac{1}{3}\,b\left(\frac{(2\,a^2b^3+b^3)\arctan\left(\frac{\frac{\left(\sqrt{-b^2x^2-2\,abx-a^2+1}|b|+b\right)a}{b^2x+ab}-1}{\sqrt{a^2-1}}\right)}{(a^4|b|-2\,a^2|b|+|b|)\sqrt{a^2-1}} - \frac{\frac{4\left(\sqrt{-b^2x^2-2\,abx-a^2+1}|b|+b\right)^2a^4b^3}{(b^2x+ab)^2}+4\,a^4b^3-\frac{11\left(\sqrt{-b\right.}}{}}{}\right. - \frac{\arccos(bx+a)}{3\,x^3}$$

input
```
integrate(arccos(b*x+a)/x^4,x, algorithm="giac")
```

output
```
-1/3*b*((2*a^2*b^3 + b^3)*arctan(((sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(
b) + b)*a/(b^2*x + a*b) - 1)/sqrt(a^2 - 1))/((a^4*abs(b) - 2*a^2*abs(b) +
abs(b))*sqrt(a^2 - 1)) - (4*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b
)^2*a^4*b^3/(b^2*x + a*b)^2 + 4*a^4*b^3 - 11*(sqrt(-b^2*x^2 - 2*a*b*x - a^
2 + 1)*abs(b) + b)*a^3*b^3/(b^2*x + a*b) - 5*(sqrt(-b^2*x^2 - 2*a*b*x - a^
2 + 1)*abs(b) + b)^3*a^3*b^3/(b^2*x + a*b)^3 + 7*(sqrt(-b^2*x^2 - 2*a*b*x
- a^2 + 1)*abs(b) + b)^2*a^2*b^3/(b^2*x + a*b)^2 - a^2*b^3 + 2*(sqrt(-b^2*
x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)*a*b^3/(b^2*x + a*b) + 2*(sqrt(-b^2*x^
2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^3*a*b^3/(b^2*x + a*b)^3 - 2*(sqrt(-b^2*
x^2 - 2*a*b*x - a^2 + 1)*abs(b) + b)^2*b^3/(b^2*x + a*b)^2)/((a^6*abs(b) -
 2*a^4*abs(b) + a^2*abs(b))*((sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b) +
b)^2*a/(b^2*x + a*b)^2 + a - 2*(sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1)*abs(b)
+ b)/(b^2*x + a*b))^2)) - 1/3*arccos(b*x + a)/x^3
```

3.31.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(a+bx)}{x^4} \, dx = \int \frac{\mathrm{acos}(a+b\,x)}{x^4} \, dx$$

input
```
int(acos(a + b*x)/x^4,x)
```

output
```
int(acos(a + b*x)/x^4, x)
```

3.32 $\int \arccos(a+bx)^3\,dx$

3.32.1 Optimal result

Integrand size = 8, antiderivative size = 82

$$\int \arccos(a+bx)^3\,dx = \frac{6\sqrt{1-(a+bx)^2}}{b} - \frac{6(a+bx)\arccos(a+bx)}{b} - \frac{3\sqrt{1-(a+bx)^2}\arccos(a+bx)^2}{b} + \frac{(a+bx)\arccos(a+bx)^3}{b}$$

output
```
-6*(b*x+a)*arccos(b*x+a)/b+(b*x+a)*arccos(b*x+a)^3/b+6*(1-(b*x+a)^2)^(1/2)
/b-3*arccos(b*x+a)^2*(1-(b*x+a)^2)^(1/2)/b
```

3.32.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.90

$$\int \arccos(a+bx)^3\,dx = \frac{6\sqrt{1-(a+bx)^2} - 6(a+bx)\arccos(a+bx) - 3\sqrt{1-(a+bx)^2}\arccos(a+bx)^2 + (a+bx)\arccos(a+b}{b}$$

input
```
Integrate[ArcCos[a + b*x]^3,x]
```

output
```
(6*Sqrt[1 - (a + b*x)^2] - 6*(a + b*x)*ArcCos[a + b*x] - 3*Sqrt[1 - (a + b
*x)^2]*ArcCos[a + b*x]^2 + (a + b*x)*ArcCos[a + b*x]^3)/b
```

3.32.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5303, 5131, 5183, 5131, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arccos(a+bx)^3\,dx$$

$$\Big\downarrow \text{ 5303}$$

$$\frac{\int \arccos(a+bx)^3 d(a+bx)}{b}$$

$$\Big\downarrow \text{ 5131}$$

$$\frac{3\int \frac{(a+bx)\arccos(a+bx)^2}{\sqrt{1-(a+bx)^2}}d(a+bx)+(a+bx)\arccos(a+bx)^3}{b}$$

$$\Big\downarrow \text{ 5183}$$

$$\frac{3\left(-2\int \arccos(a+bx)d(a+bx)-\sqrt{1-(a+bx)^2}\arccos(a+bx)^2\right)+(a+bx)\arccos(a+bx)^3}{b}$$

$$\Big\downarrow \text{ 5131}$$

$$\frac{3\left(-2\left(\int \frac{a+bx}{\sqrt{1-(a+bx)^2}}d(a+bx)+(a+bx)\arccos(a+bx)\right)-\sqrt{1-(a+bx)^2}\arccos(a+bx)^2\right)+(a+bx)\arccos(a}{b}$$

$$\Big\downarrow \text{ 241}$$

$$\frac{(a+bx)\arccos(a+bx)^3+3\left(-\sqrt{1-(a+bx)^2}\arccos(a+bx)^2-2\left((a+bx)\arccos(a+bx)-\sqrt{1-(a+bx)^2}\right)\right)}{b}$$

input
```
Int[ArcCos[a + b*x]^3,x]
```

output
```
((a + b*x)*ArcCos[a + b*x]^3 + 3*(-(Sqrt[1 - (a + b*x)^2]*ArcCos[a + b*x]^
2) - 2*(-Sqrt[1 - (a + b*x)^2] + (a + b*x)*ArcCos[a + b*x])))/b
```

3.32.3.1 Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5131
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Simp[b*c*n   Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5303
```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
  Subst[Int[(a + b*ArcCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
n}, x]
```

3.32.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.87

method	result	size
derivativedivides	$\frac{\arccos(bx+a)^3(bx+a)-3\arccos(bx+a)^2\sqrt{1-(bx+a)^2}+6\sqrt{1-(bx+a)^2}-6(bx+a)\arccos(bx+a)}{b}$	71
default	$\frac{\arccos(bx+a)^3(bx+a)-3\arccos(bx+a)^2\sqrt{1-(bx+a)^2}+6\sqrt{1-(bx+a)^2}-6(bx+a)\arccos(bx+a)}{b}$	71

input
```
int(arccos(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(arccos(b*x+a)^3*(b*x+a)-3*arccos(b*x+a)^2*(1-(b*x+a)^2)^(1/2)+6*(1-(b
*x+a)^2)^(1/2)-6*(b*x+a)*arccos(b*x+a))
```

3.32.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.80

$$\int \arccos(a+bx)^3\,dx$$
$$= \frac{(bx+a)\arccos{(bx+a)}^3 - 6\,(bx+a)\arccos{(bx+a)} - 3\,\sqrt{-b^2x^2 - 2\,abx - a^2 + 1}\left(\arccos{(bx+a)}^2 - 2\right)}{b}$$

input
```
integrate(arccos(b*x+a)^3,x, algorithm="fricas")
```

output
```
((b*x + a)*arccos(b*x + a)^3 - 6*(b*x + a)*arccos(b*x + a) - 3*sqrt(-b^2*x
^2 - 2*a*b*x - a^2 + 1)*(arccos(b*x + a)^2 - 2))/b
```

3.32.6 Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.33

$$\int \arccos(a+bx)^3\,dx$$
$$= \begin{cases} \frac{a\operatorname{acos}^3{(a+bx)}}{b} - \frac{6a\operatorname{acos}{(a+bx)}}{b} + x\operatorname{acos}^3{(a+bx)} - 6x\operatorname{acos}{(a+bx)} - \frac{3\sqrt{-a^2-2abx-b^2x^2+1}\operatorname{acos}^2{(a+bx)}}{b} + \frac{6\sqrt{-a^2-} \\ x\operatorname{acos}^3{(a)} \end{cases}$$

input
```
integrate(acos(b*x+a)**3,x)
```

output
```
Piecewise((a*acos(a + b*x)**3/b - 6*a*acos(a + b*x)/b + x*acos(a + b*x)**3
 - 6*x*acos(a + b*x) - 3*sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1)*acos(a + b*
x)**2/b + 6*sqrt(-a**2 - 2*a*b*x - b**2*x**2 + 1)/b, Ne(b, 0)), (x*acos(a)
**3, True))
```

3.32.7 Maxima [F]

$$\int \arccos(a+bx)^3\,dx = \int \arccos\left(bx+a\right)^3\,dx$$

input
```
integrate(arccos(b*x+a)^3,x, algorithm="maxima")
```

output
```
x*arctan2(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1), b*x + a)^3 - 3*b*integrate
(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*x*arctan2(sqrt(b*x + a + 1)*sqrt(-b*
x - a + 1), b*x + a)^2/(b^2*x^2 + 2*a*b*x + a^2 - 1), x)
```

3.32.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.95

$$\int \arccos(a+bx)^3\,dx = \frac{(bx+a)\arccos\left(bx+a\right)^3}{b} - \frac{3\sqrt{-(bx+a)^2+1}\arccos\left(bx+a\right)^2}{b} - \frac{6\,(bx+a)\arccos\left(bx+a\right)}{b} + \frac{6\sqrt{-(bx+a)^2+1}}{b}$$

input
```
integrate(arccos(b*x+a)^3,x, algorithm="giac")
```

output
```
(b*x + a)*arccos(b*x + a)^3/b - 3*sqrt(-(b*x + a)^2 + 1)*arccos(b*x + a)^2
/b - 6*(b*x + a)*arccos(b*x + a)/b + 6*sqrt(-(b*x + a)^2 + 1)/b
```

3.32.9 Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.73

$$\int \arccos(a+bx)^3\,dx = -\frac{\left(3\,\mathrm{acos}(a+b\,x)^2-6\right)\,\sqrt{1-(a+b\,x)^2}}{b} - \frac{\left(6\,\mathrm{acos}(a+b\,x)-\mathrm{acos}(a+b\,x)^3\right)\,(a+b\,x)}{b}$$

input
```
int(acos(a + b*x)^3,x)
```

output
```
- ((3*acos(a + b*x)^2 - 6)*(1 - (a + b*x)^2)^(1/2))/b - ((6*acos(a + b*x)
- acos(a + b*x)^3)*(a + b*x))/b
```

3.32. $\int \arccos(a+bx)^3\,dx$

3.33 $\int \arccos(a+bx)^2\,dx$

3.33.1 Optimal result

Integrand size = 8, antiderivative size = 47

$$\int \arccos(a+bx)^2\,dx = -2x - \frac{2\sqrt{1-(a+bx)^2}\arccos(a+bx)}{b} + \frac{(a+bx)\arccos(a+bx)^2}{b}$$

output
```
-2*x+(b*x+a)*arccos(b*x+a)^2/b-2*arccos(b*x+a)*(1-(b*x+a)^2)^(1/2)/b
```

3.33.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04

$$\int \arccos(a+bx)^2\,dx = \frac{-2(a+bx)-2\sqrt{1-(a+bx)^2}\arccos(a+bx)+(a+bx)\arccos(a+bx)^2}{b}$$

input
```
Integrate[ArcCos[a + b*x]^2,x]
```

output
```
(-2*(a + b*x) - 2*Sqrt[1 - (a + b*x)^2]*ArcCos[a + b*x] + (a + b*x)*ArcCos
[a + b*x]^2)/b
```

3.33.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5303, 5131, 5183, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arccos(a+bx)^2\,dx$$

$$\Big\downarrow \text{5303}$$

$$\frac{\int \arccos(a+bx)^2 d(a+bx)}{b}$$

$$\Big\downarrow \text{5131}$$

$$\frac{2\int \frac{(a+bx)\arccos(a+bx)}{\sqrt{1-(a+bx)^2}}d(a+bx)+(a+bx)\arccos(a+bx)^2}{b}$$

$$\Big\downarrow \text{5183}$$

$$\frac{2\left(-\int 1d(a+bx)-\sqrt{1-(a+bx)^2}\arccos(a+bx)\right)+(a+bx)\arccos(a+bx)^2}{b}$$

$$\Big\downarrow \text{24}$$

$$\frac{(a+bx)\arccos(a+bx)^2+2\left(-\sqrt{1-(a+bx)^2}\arccos(a+bx)-a-bx\right)}{b}$$

input
```
Int[ArcCos[a + b*x]^2,x]
```

output
```
((a + b*x)*ArcCos[a + b*x]^2 + 2*(-a - b*x - Sqrt[1 - (a + b*x)^2]*ArcCos[
a + b*x]))/b
```

3.33.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 5131
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Simp[b*c*n   Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5303
```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
  Subst[Int[(a + b*ArcCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
n}, x]
```

3.33.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

method	result	size
derivativedivides	$\frac{\arccos(bx+a)^2(bx+a)-2bx-2a-2\arccos(bx+a)\sqrt{1-(bx+a)^2}}{b}$	48
default	$\frac{\arccos(bx+a)^2(bx+a)-2bx-2a-2\arccos(bx+a)\sqrt{1-(bx+a)^2}}{b}$	48

input
```
int(arccos(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(arccos(b*x+a)^2*(b*x+a)-2*b*x-2*a-2*arccos(b*x+a)*(1-(b*x+a)^2)^(1/2)
)
```

3.33.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.13

$$\int \arccos(a+bx)^2\,dx = \frac{(bx+a)\arccos(bx+a)^2 - 2\,bx - 2\sqrt{-b^2x^2-2\,abx-a^2+1}\arccos(bx+a)}{b}$$

input
```
integrate(arccos(b*x+a)^2,x, algorithm="fricas")
```

output
```
((b*x + a)*arccos(b*x + a)^2 - 2*b*x - 2*sqrt(-b^2*x^2 - 2*a*b*x - a^2 + 1
)*arccos(b*x + a))/b
```

3.33.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.34

$$\int \arccos(a+bx)^2\,dx = \begin{cases} \frac{a\operatorname{acos}^2(a+bx)}{b} + x\operatorname{acos}^2(a+bx) - 2x - \frac{2\sqrt{-a^2-2abx-b^2x^2+1}\operatorname{acos}(a+bx)}{b} & \text{for } b\neq 0 \\ x\operatorname{acos}^2(a) & \text{otherwise} \end{cases}$$

input
```
integrate(acos(b*x+a)**2,x)
```

output
```
Piecewise((a*acos(a + b*x)**2/b + x*acos(a + b*x)**2 - 2*x - 2*sqrt(-a**2
- 2*a*b*x - b**2*x**2 + 1)*acos(a + b*x)/b, Ne(b, 0)), (x*acos(a)**2, True
))
```

3.33.7 Maxima [F]

$$\int \arccos(a+bx)^2\,dx = \int \arccos(bx+a)^2\,dx$$

input
```
integrate(arccos(b*x+a)^2,x, algorithm="maxima")
```

output
```
x*arctan2(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1), b*x + a)^2 - 2*b*integrate
(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1)*x*arctan2(sqrt(b*x + a + 1)*sqrt(-b*
x - a + 1), b*x + a)/(b^2*x^2 + 2*a*b*x + a^2 - 1), x)
```

3.33.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.11

$$\int \arccos(a+bx)^2\,dx = \frac{(bx+a)\arccos{(bx+a)}^2}{b} - \frac{2\sqrt{-(bx+a)^2+1}\arccos{(bx+a)}}{b} - \frac{2\,(bx+a)}{b}$$

input
```
integrate(arccos(b*x+a)^2,x, algorithm="giac")
```

output
```
(b*x + a)*arccos(b*x + a)^2/b - 2*sqrt(-(b*x + a)^2 + 1)*arccos(b*x + a)/b
 - 2*(b*x + a)/b
```

3.33.9 Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int \arccos(a+bx)^2\,dx = \frac{\left(\mathrm{acos}(a+b\,x)^2-2\right)\,\left(a+b\,x\right)}{b} - \frac{2\,\mathrm{acos}(a+b\,x)\,\sqrt{1-(a+b\,x)^2}}{b}$$

input
```
int(acos(a + b*x)^2,x)
```

output
```
((acos(a + b*x)^2 - 2)*(a + b*x))/b - (2*acos(a + b*x)*(1 - (a + b*x)^2)^(
1/2))/b
```

3.34 $\int \frac{1}{\arccos(a+bx)}\,dx$

3.34.1 Optimal result

Integrand size = 8, antiderivative size = 12

$$\int \frac{1}{\arccos(a+bx)}\,dx = -\frac{\text{Si}(\arccos(a+bx))}{b}$$

output
```
-Si(arccos(b*x+a))/b
```

3.34.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{\arccos(a+bx)}\,dx = -\frac{\text{Si}(\arccos(a+bx))}{b}$$

input
```
Integrate[ArcCos[a + b*x]^(-1),x]
```

output
```
-(SinIntegral[ArcCos[a + b*x]]/b)
```

3.34.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5303, 5135, 3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arccos(a+bx)}dx$$
$$\Big\downarrow \text{5303}$$
$$\frac{\int \frac{1}{\arccos(a+bx)}d(a+bx)}{b}$$
$$\Big\downarrow \text{5135}$$
$$-\frac{\int \frac{\sqrt{1-(a+bx)^2}}{\arccos(a+bx)}d\arccos(a+bx)}{b}$$
$$\Big\downarrow \text{3042}$$
$$-\frac{\int \frac{\sin(\arccos(a+bx))}{\arccos(a+bx)}d\arccos(a+bx)}{b}$$
$$\Big\downarrow \text{3780}$$
$$-\frac{\text{Si}(\arccos(a+bx))}{b}$$

input
```
Int[ArcCos[a + b*x]^(-1),x]
```

output
```
-(SinIntegral[ArcCos[a + b*x]]/b)
```

3.34.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 5135
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[-(b*c)^(-1)
  Subst[Int[x^n*Sin[-a/b + x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a,
 b, c, n}, x]
```

rule 5303
```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
  Subst[Int[(a + b*ArcCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
n}, x]
```

3.34.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$-\frac{\mathrm{Si}(\arccos(bx+a))}{b}$	13
default	$-\frac{\mathrm{Si}(\arccos(bx+a))}{b}$	13

input
```
int(1/arccos(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-Si(arccos(b*x+a))/b
```

3.34.5 Fricas [F]

$$\int \frac{1}{\arccos(a+bx)}\,dx = \int \frac{1}{\arccos(bx+a)}\,dx$$

input
```
integrate(1/arccos(b*x+a),x, algorithm="fricas")
```

output
```
integral(1/arccos(b*x + a), x)
```

3.34.6 Sympy [F]

$$\int \frac{1}{\arccos(a+bx)}\,dx = \int \frac{1}{\operatorname{acos}(a+bx)}\,dx$$

input
```
integrate(1/acos(b*x+a),x)
```

output
```
Integral(1/acos(a + b*x), x)
```

3.34.7 Maxima [F]

$$\int \frac{1}{\arccos(a+bx)}\,dx = \int \frac{1}{\arccos(bx+a)}\,dx$$

input
```
integrate(1/arccos(b*x+a),x, algorithm="maxima")
```

output
```
integrate(1/arccos(b*x + a), x)
```

3.34.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{\arccos(a+bx)}\,dx = -\frac{\operatorname{Si}(\arccos(bx+a))}{b}$$

input
```
integrate(1/arccos(b*x+a),x, algorithm="giac")
```

output
```
-sin_integral(arccos(b*x + a))/b
```

3.34.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\arccos(a+bx)}\,dx = \int \frac{1}{\mathrm{acos}\,(a+b\,x)}\,dx$$

input
```
int(1/acos(a + b*x),x)
```

output
```
int(1/acos(a + b*x), x)
```

3.35 $\int \frac{1}{\arccos(a+bx)^2} dx$

3.35.1 Optimal result

Integrand size = 8, antiderivative size = 40

$$\int \frac{1}{\arccos(a+bx)^2} dx = \frac{\sqrt{1-(a+bx)^2}}{b\arccos(a+bx)} - \frac{\text{CosIntegral}(\arccos(a+bx))}{b}$$

output
```
-Ci(arccos(b*x+a))/b+(1-(b*x+a)^2)^(1/2)/b/arccos(b*x+a)
```

3.35.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{1}{\arccos(a+bx)^2} dx = \frac{\sqrt{1-(a+bx)^2}}{b\arccos(a+bx)} - \frac{\text{CosIntegral}(\arccos(a+bx))}{b}$$

input
```
Integrate[ArcCos[a + b*x]^(-2),x]
```

output
```
Sqrt[1 - (a + b*x)^2]/(b*ArcCos[a + b*x]) - CosIntegral[ArcCos[a + b*x]]/b
```

3.35.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5303, 5133, 5225, 3042, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arccos(a+bx)^2}dx$$

$$\Big\downarrow \text{ 5303}$$

$$\frac{\int \frac{1}{\arccos(a+bx)^2}d(a+bx)}{b}$$

$$\Big\downarrow \text{ 5133}$$

$$\frac{\int \frac{a+bx}{\sqrt{1-(a+bx)^2}\arccos(a+bx)}d(a+bx)+\frac{\sqrt{1-(a+bx)^2}}{\arccos(a+bx)}}{b}$$

$$\Big\downarrow \text{ 5225}$$

$$\frac{\frac{\sqrt{1-(a+bx)^2}}{\arccos(a+bx)}-\int \frac{a+bx}{\arccos(a+bx)}d\arccos(a+bx)}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{\sqrt{1-(a+bx)^2}}{\arccos(a+bx)}-\int \frac{\sin\left(\arccos(a+bx)+\frac{\pi}{2}\right)}{\arccos(a+bx)}d\arccos(a+bx)}{b}$$

$$\Big\downarrow \text{ 3783}$$

$$\frac{\frac{\sqrt{1-(a+bx)^2}}{\arccos(a+bx)}-\text{CosIntegral}(\arccos(a+bx))}{b}$$

input
```
Int[ArcCos[a + b*x]^(-2),x]
```

output
```
(Sqrt[1 - (a + b*x)^2]/ArcCos[a + b*x] - CosIntegral[ArcCos[a + b*x]])/b
```

3.35.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 5133
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-Sqrt[1 - c
^2*x^2])*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c/(b*(n + 1
))   Int[x*((a + b*ArcCos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && LtQ[n, -1]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

rule 5303
```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
  Subst[Int[(a + b*ArcCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
n}, x]
```

3.35.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{\frac{\sqrt{1-(bx+a)^2}}{\arccos(bx+a)}-\mathrm{Ci}(\arccos(bx+a))}{b}$	37
default	$\frac{\frac{\sqrt{1-(bx+a)^2}}{\arccos(bx+a)}-\mathrm{Ci}(\arccos(bx+a))}{b}$	37

input
```
int(1/arccos(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*((1-(b*x+a)^2)^(1/2)/arccos(b*x+a)-Ci(arccos(b*x+a)))
```

3.35.5 Fricas [F]

$$\int \frac{1}{\arccos(a+bx)^2}\,dx = \int \frac{1}{\arccos\left(bx+a\right)^2}\,dx$$

input
```
integrate(1/arccos(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(arccos(b*x + a)^(-2), x)
```

3.35.6 Sympy [F]

$$\int \frac{1}{\arccos(a+bx)^2}\,dx = \int \frac{1}{\operatorname{acos}^2\left(a+bx\right)}\,dx$$

input
```
integrate(1/acos(b*x+a)**2,x)
```

output
```
Integral(acos(a + b*x)**(-2), x)
```

3.35.7 Maxima [F]

$$\int \frac{1}{\arccos(a+bx)^2}\,dx = \int \frac{1}{\arccos\left(bx+a\right)^2}\,dx$$

input
```
integrate(1/arccos(b*x+a)^2,x, algorithm="maxima")
```

output
```
-(b*arctan2(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1), b*x + a)*integrate(sqrt(
b*x + a + 1)*(b*x + a)*sqrt(-b*x - a + 1)/((b^2*x^2 + 2*a*b*x + a^2 - 1)*a
rctan2(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1), b*x + a)), x) - sqrt(b*x + a
+ 1)*sqrt(-b*x - a + 1))/(b*arctan2(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1),
b*x + a))
```

3.35.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{1}{\arccos(a+bx)^2} dx = -\frac{\mathrm{Ci}\left(\arccos\left(bx+a\right)\right)}{b} + \frac{\sqrt{-\left(bx+a\right)^2+1}}{b\arccos\left(bx+a\right)}$$

input
```
integrate(1/arccos(b*x+a)^2,x, algorithm="giac")
```

output
```
-cos_integral(arccos(b*x + a))/b + sqrt(-(b*x + a)^2 + 1)/(b*arccos(b*x +
a))
```

3.35.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\arccos(a+bx)^2} dx = \int \frac{1}{\mathrm{acos}\left(a+b\,x\right)^2} \,dx$$

input
```
int(1/acos(a + b*x)^2,x)
```

output
```
int(1/acos(a + b*x)^2, x)
```

3.36 $\int \frac{1}{\arccos(a+bx)^3}\,dx$

3.36.1 Optimal result

Integrand size = 8, antiderivative size = 65

$$\int \frac{1}{\arccos(a+bx)^3}\,dx=\frac{\sqrt{1-(a+bx)^2}}{2b\arccos(a+bx)^2}+\frac{a+bx}{2b\arccos(a+bx)}+\frac{\mathrm{Si}(\arccos(a+bx))}{2b}$$

output
```
1/2*(b*x+a)/b/arccos(b*x+a)+1/2*Si(arccos(b*x+a))/b+1/2*(1-(b*x+a)^2)^(1/2
)/b/arccos(b*x+a)^2
```

3.36.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int \frac{1}{\arccos(a+bx)^3}\,dx=\frac{\sqrt{1-(a+bx)^2}}{2b\arccos(a+bx)^2}+\frac{a+bx}{2b\arccos(a+bx)}+\frac{\mathrm{Si}(\arccos(a+bx))}{2b}$$

input
```
Integrate[ArcCos[a + b*x]^(-3),x]
```

output
```
Sqrt[1 - (a + b*x)^2]/(2*b*ArcCos[a + b*x]^2) + (a + b*x)/(2*b*ArcCos[a +
b*x]) + SinIntegral[ArcCos[a + b*x]]/(2*b)
```

3.36.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {5303, 5133, 5223, 5135, 3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arccos(a+bx)^3}dx$$

$$\Big\downarrow \; 5303$$

$$\frac{\int \frac{1}{\arccos(a+bx)^3}d(a+bx)}{b}$$

$$\Big\downarrow \; 5133$$

$$\frac{\frac{1}{2}\int \frac{a+bx}{\sqrt{1-(a+bx)^2}\arccos(a+bx)^2}d(a+bx)+\frac{\sqrt{1-(a+bx)^2}}{2\arccos(a+bx)^2}}{b}$$

$$\Big\downarrow \; 5223$$

$$\frac{\frac{1}{2}\left(\frac{a+bx}{\arccos(a+bx)}-\int \frac{1}{\arccos(a+bx)}d(a+bx)\right)+\frac{\sqrt{1-(a+bx)^2}}{2\arccos(a+bx)^2}}{b}$$

$$\Big\downarrow \; 5135$$

$$\frac{\frac{1}{2}\left(\int \frac{\sqrt{1-(a+bx)^2}}{\arccos(a+bx)}d\arccos(a+bx)+\frac{a+bx}{\arccos(a+bx)}\right)+\frac{\sqrt{1-(a+bx)^2}}{2\arccos(a+bx)^2}}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{1}{2}\left(\int \frac{\sin(\arccos(a+bx))}{\arccos(a+bx)}d\arccos(a+bx)+\frac{a+bx}{\arccos(a+bx)}\right)+\frac{\sqrt{1-(a+bx)^2}}{2\arccos(a+bx)^2}}{b}$$

$$\Big\downarrow \; 3780$$

$$\frac{\frac{1}{2}\left(\text{Si}(\arccos(a+bx))+\frac{a+bx}{\arccos(a+bx)}\right)+\frac{\sqrt{1-(a+bx)^2}}{2\arccos(a+bx)^2}}{b}$$

input
```
Int[ArcCos[a + b*x]^(-3),x]
```

output
```
(Sqrt[1 - (a + b*x)^2]/(2*ArcCos[a + b*x]^2) + ((a + b*x)/ArcCos[a + b*x]
+ SinIntegral[ArcCos[a + b*x]])/2)/b
```

3.36.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 5133
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-Sqrt[1 - c
^2*x^2])*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c/(b*(n + 1
))   Int[x*((a + b*ArcCos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && LtQ[n, -1]
```

rule 5135
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[-(b*c)^(-1)
  Subst[Int[x^n*Sin[-a/b + x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a,
 b, c, n}, x]
```

rule 5223
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Simp[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a + b
*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2
*d + e, 0] && LtQ[n, -1]
```

rule 5303
```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
  Subst[Int[(a + b*ArcCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
n}, x]
```

3.36.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{\frac{\sqrt{1-(bx+a)^2}}{2\arccos(bx+a)^2}+\frac{bx+a}{2\arccos(bx+a)}+\frac{\mathrm{Si}(\arccos(bx+a))}{2}}{b}$	53
default	$\frac{\frac{\sqrt{1-(bx+a)^2}}{2\arccos(bx+a)^2}+\frac{bx+a}{2\arccos(bx+a)}+\frac{\mathrm{Si}(\arccos(bx+a))}{2}}{b}$	53

input
```
int(1/arccos(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/2*(1-(b*x+a)^2)^(1/2)/arccos(b*x+a)^2+1/2/arccos(b*x+a)*(b*x+a)+1/2
*Si(arccos(b*x+a)))
```

3.36.5 Fricas [F]

$$\int \frac{1}{\arccos(a+bx)^3}\,dx = \int \frac{1}{\arccos{(bx+a)}^3}\,dx$$

input
```
integrate(1/arccos(b*x+a)^3,x, algorithm="fricas")
```

output
```
integral(arccos(b*x + a)^(-3), x)
```

3.36.6 Sympy [F]

$$\int \frac{1}{\arccos(a+bx)^3}\,dx = \int \frac{1}{\operatorname{acos}^3{(a+bx)}}\,dx$$

input
```
integrate(1/acos(b*x+a)**3,x)
```

output
```
Integral(acos(a + b*x)**(-3), x)
```

3.36.7 Maxima [F]

$$\int \frac{1}{\arccos(a+bx)^3} \, dx = \int \frac{1}{\arccos\left(bx+a\right)^3} \, dx$$

input
```
integrate(1/arccos(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/2*(b*arctan2(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1), b*x + a)^2*integrate
(1/arctan2(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1), b*x + a), x) - (b*x + a)*
arctan2(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1), b*x + a) - sqrt(b*x + a + 1)
*sqrt(-b*x - a + 1))/(b*arctan2(sqrt(b*x + a + 1)*sqrt(-b*x - a + 1), b*x
+ a)^2)
```

3.36.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int \frac{1}{\arccos(a+bx)^3} \, dx = \frac{\mathrm{Si}\left(\arccos\left(bx+a\right)\right)}{2\,b} + \frac{bx+a}{2\,b\arccos\left(bx+a\right)} + \frac{\sqrt{-\left(bx+a\right)^2+1}}{2\,b\arccos\left(bx+a\right)^2}$$

input
```
integrate(1/arccos(b*x+a)^3,x, algorithm="giac")
```

output
```
1/2*sin_integral(arccos(b*x + a))/b + 1/2*(b*x + a)/(b*arccos(b*x + a)) +
1/2*sqrt(-(b*x + a)^2 + 1)/(b*arccos(b*x + a)^2)
```

3.36.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\arccos(a+bx)^3} \, dx = \int \frac{1}{\mathrm{acos}\left(a+b\,x\right)^3} \, dx$$

input
```
int(1/acos(a + b*x)^3,x)
```

output
```
int(1/acos(a + b*x)^3, x)
```

3.37 $\int \arccos(a+bx)^{5/2}\,dx$

3.37.1 Optimal result

Integrand size = 10, antiderivative size = 111

$$\int \arccos(a+bx)^{5/2}\,dx = -\frac{15(a+bx)\sqrt{\arccos(a+bx)}}{4b} - \frac{5\sqrt{1-(a+bx)^2}\arccos(a+bx)^{3/2}}{2b} + \frac{(a+bx)\arccos(a+bx)^{5/2}}{b} + \frac{15\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(a+bx)}\right)}{4b}$$

output
```
(b*x+a)*arccos(b*x+a)^(5/2)/b+15/8*FresnelC(2^(1/2)/Pi^(1/2)*arccos(b*x+a)
^(1/2))*2^(1/2)*Pi^(1/2)/b-5/2*arccos(b*x+a)^(3/2)*(1-(b*x+a)^2)^(1/2)/b-1
5/4*(b*x+a)*arccos(b*x+a)^(1/2)/b
```

3.37.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.71

$$\int \arccos(a+bx)^{5/2}\,dx = -\frac{i\left(\sqrt{-i\arccos(a+bx)}\,\Gamma\left(\frac{7}{2},-i\arccos(a+bx)\right)-\sqrt{i\arccos(a+bx)}\,\Gamma\left(\frac{7}{2},i\arccos(a+bx)\right)\right)}{2b\sqrt{\arccos(a+bx)}}$$

input
```
Integrate[ArcCos[a + b*x]^(5/2),x]
```

output
```
((-1/2*I)*(Sqrt[(-I)*ArcCos[a + b*x]]*Gamma[7/2, (-I)*ArcCos[a + b*x]] - S
qrt[I*ArcCos[a + b*x]]*Gamma[7/2, I*ArcCos[a + b*x]]))/(b*Sqrt[ArcCos[a +
b*x]])
```

3.37.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {5303, 5131, 5183, 5131, 5225, 3042, 3785, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arccos(a+bx)^{5/2}\,dx$$

$$\Big\downarrow \; 5303$$

$$\frac{\int \arccos(a+bx)^{5/2}d(a+bx)}{b}$$

$$\Big\downarrow \; 5131$$

$$\frac{\frac{5}{2}\int \frac{(a+bx)\arccos(a+bx)^{3/2}}{\sqrt{1-(a+bx)^2}}d(a+bx)+(a+bx)\arccos(a+bx)^{5/2}}{b}$$

$$\Big\downarrow \; 5183$$

$$\frac{\frac{5}{2}\left(-\frac{3}{2}\int \sqrt{\arccos(a+bx)}d(a+bx)-\sqrt{1-(a+bx)^2}\arccos(a+bx)^{3/2}\right)+(a+bx)\arccos(a+bx)^{5/2}}{b}$$

$$\Big\downarrow \; 5131$$

$$\frac{\frac{5}{2}\left(-\frac{3}{2}\left(\frac{1}{2}\int \frac{a+bx}{\sqrt{1-(a+bx)^2}\sqrt{\arccos(a+bx)}}d(a+bx)+(a+bx)\sqrt{\arccos(a+bx)}\right)-\sqrt{1-(a+bx)^2}\arccos(a+bx)^{3/2}\right)+}{b}$$

$$\Big\downarrow \; 5225$$

$$\frac{\frac{5}{2}\left(-\frac{3}{2}\left((a+bx)\sqrt{\arccos(a+bx)}-\frac{1}{2}\int \frac{a+bx}{\sqrt{\arccos(a+bx)}}d\arccos(a+bx)\right)-\sqrt{1-(a+bx)^2}\arccos(a+bx)^{3/2}\right)+(a}{b}$$

$\big\downarrow$ 3042

$$\frac{\frac{5}{2}\left(-\frac{3}{2}\left((a+bx)\sqrt{\arccos(a+bx)}-\frac{1}{2}\int\frac{\sin\left(\arccos(a+bx)+\frac{\pi}{2}\right)}{\sqrt{\arccos(a+bx)}}d\arccos(a+bx)\right)-\sqrt{1-(a+bx)^2}\arccos(a+bx)^{3/2}\right)}{b}$$

$\big\downarrow$ 3785

$$\frac{\frac{5}{2}\left(-\frac{3}{2}\left((a+bx)\sqrt{\arccos(a+bx)}-\int(a+bx)d\sqrt{\arccos(a+bx)}\right)-\sqrt{1-(a+bx)^2}\arccos(a+bx)^{3/2}\right)+(a+bx}{b}$$

$\big\downarrow$ 3833

$$\frac{\frac{5}{2}\left(-\frac{3}{2}\left((a+bx)\sqrt{\arccos(a+bx)}-\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(a+bx)}\right)\right)-\sqrt{1-(a+bx)^2}\arccos(a+bx)^{3/2}\right)}{b}$$

input
```
Int[ArcCos[a + b*x]^(5/2),x]
```

output
```
((a + b*x)*ArcCos[a + b*x]^(5/2) + (5*(-(Sqrt[1 - (a + b*x)^2]*ArcCos[a +
b*x]^(3/2)) - (3*((a + b*x)*Sqrt[ArcCos[a + b*x]] - Sqrt[Pi/2]*FresnelC[Sq
rt[2/Pi]*Sqrt[ArcCos[a + b*x]]]))/2))/2)/b
```

3.37.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5131
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Simp[b*c*n   Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

rule 5303
```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
  Subst[Int[(a + b*ArcCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
n}, x]
```

3.37.4 Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.26

method	result
default	$-\frac{\sqrt{2}\left(-4\arccos(bx+a)^{\frac{5}{2}}\sqrt{2}\sqrt{\pi}\,bx-4\arccos(bx+a)^{\frac{5}{2}}\sqrt{2}\sqrt{\pi}\,a+10\arccos(bx+a)^{\frac{3}{2}}\sqrt{2}\sqrt{\pi}\sqrt{-b^2x^2-2abx-a^2+1}+15\sqrt{2}\sqrt{\arccos(bx}\right.}{8b\sqrt{\pi}}$

input
```
int(arccos(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8/b*2^(1/2)*(-4*arccos(b*x+a)^(5/2)*2^(1/2)*Pi^(1/2)*b*x-4*arccos(b*x+a
)^(5/2)*2^(1/2)*Pi^(1/2)*a+10*arccos(b*x+a)^(3/2)*2^(1/2)*Pi^(1/2)*(-b^2*x
^2-2*a*b*x-a^2+1)^(1/2)+15*2^(1/2)*arccos(b*x+a)^(1/2)*Pi^(1/2)*b*x+15*2^(
1/2)*arccos(b*x+a)^(1/2)*Pi^(1/2)*a-15*Pi*FresnelC(2^(1/2)/Pi^(1/2)*arccos
(b*x+a)^(1/2)))/Pi^(1/2)
```

3.37.5 Fricas [F(-2)]

Exception generated.

$$\int \arccos(a+bx)^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arccos(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.37.6 Sympy [F]

$$\int \arccos(a+bx)^{5/2}\,dx = \int \operatorname{acos}^{\frac{5}{2}}(a+bx)\,dx$$

input
```
integrate(acos(b*x+a)**(5/2),x)
```

output
```
Integral(acos(a + b*x)**(5/2), x)
```

3.37.7 Maxima [F(-2)]

Exception generated.

$$\int \arccos(a+bx)^{5/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arccos(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.37.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.65

$$\int \arccos(a+bx)^{5/2}\,dx = \frac{\arccos(bx+a)^{\frac{5}{2}} e^{(i \arccos(bx+a))}}{2b} + \frac{\arccos(bx+a)^{\frac{5}{2}} e^{(-i \arccos(bx+a))}}{2b} + \frac{5i \arccos(bx+a)^{\frac{3}{2}} e^{(i \arccos(bx+a))}}{4b} - \frac{5i \arccos(bx+a)^{\frac{3}{2}} e^{(-i \arccos(bx+a))}}{4b} - \frac{(15i+15)\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\arccos(bx+a)}\right)}{32b} + \frac{(15i-15)\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{\arccos(bx+a)}\right)}{32b} - \frac{15\sqrt{\arccos(bx+a)}e^{(i \arccos(bx+a))}}{8b} - \frac{15\sqrt{\arccos(bx+a)}e^{(-i \arccos(bx+a))}}{8b}$$

input
```
integrate(arccos(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
1/2*arccos(b*x + a)^(5/2)*e^(I*arccos(b*x + a))/b + 1/2*arccos(b*x + a)^(5
/2)*e^(-I*arccos(b*x + a))/b + 5/4*I*arccos(b*x + a)^(3/2)*e^(I*arccos(b*x
 + a))/b - 5/4*I*arccos(b*x + a)^(3/2)*e^(-I*arccos(b*x + a))/b - (15/32*I
 + 15/32)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(arccos(b*x + a))
)/b + (15/32*I - 15/32)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(a
rccos(b*x + a)))/b - 15/8*sqrt(arccos(b*x + a))*e^(I*arccos(b*x + a))/b -
15/8*sqrt(arccos(b*x + a))*e^(-I*arccos(b*x + a))/b
```

3.37.9 Mupad [F(-1)]

Timed out.

$$\int \arccos(a+bx)^{5/2}\,dx = \int \operatorname{acos}(a+b\,x)^{5/2}\,dx$$

input
```
int(acos(a + b*x)^(5/2),x)
```

output
```
int(acos(a + b*x)^(5/2), x)
```

3.38 $\int \arccos(a+bx)^{3/2}\,dx$

3.38.1 Optimal result

Integrand size = 10, antiderivative size = 89

$$\int \arccos(a+bx)^{3/2}\,dx = -\frac{3\sqrt{1-(a+bx)^2}\sqrt{\arccos(a+bx)}}{2b} + \frac{(a+bx)\arccos(a+bx)^{3/2}}{b} + \frac{3\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(a+bx)}\right)}{2b}$$

output
```
(b*x+a)*arccos(b*x+a)^(3/2)/b+3/4*FresnelS(2^(1/2)/Pi^(1/2)*arccos(b*x+a)^
(1/2))*2^(1/2)*Pi^(1/2)/b-3/2*(1-(b*x+a)^2)^(1/2)*arccos(b*x+a)^(1/2)/b
```

3.38.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.85

$$\int \arccos(a+bx)^{3/2}\,dx = -\frac{\sqrt{-i\arccos(a+bx)}\,\Gamma\left(\frac{5}{2},-i\arccos(a+bx)\right)+\sqrt{i\arccos(a+bx)}\,\Gamma\left(\frac{5}{2},i\arccos(a+bx)\right)}{2b\sqrt{\arccos(a+bx)}}$$

input
```
Integrate[ArcCos[a + b*x]^(3/2),x]
```

output
```
-1/2*(Sqrt[(-I)*ArcCos[a + b*x]]*Gamma[5/2, (-I)*ArcCos[a + b*x]] + Sqrt[I
*ArcCos[a + b*x]]*Gamma[5/2, I*ArcCos[a + b*x]])/(b*Sqrt[ArcCos[a + b*x]])
```

3.38.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {5303, 5131, 5183, 5135, 3042, 3786, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arccos(a+bx)^{3/2}\,dx$$

$$\Big\downarrow \text{5303}$$

$$\frac{\int \arccos(a+bx)^{3/2}d(a+bx)}{b}$$

$$\Big\downarrow \text{5131}$$

$$\frac{\frac{3}{2}\int \frac{(a+bx)\sqrt{\arccos(a+bx)}}{\sqrt{1-(a+bx)^2}}d(a+bx)+(a+bx)\arccos(a+bx)^{3/2}}{b}$$

$$\Big\downarrow \text{5183}$$

$$\frac{\frac{3}{2}\left(-\frac{1}{2}\int \frac{1}{\sqrt{\arccos(a+bx)}}d(a+bx)-\sqrt{1-(a+bx)^2}\sqrt{\arccos(a+bx)}\right)+(a+bx)\arccos(a+bx)^{3/2}}{b}$$

$$\Big\downarrow \text{5135}$$

$$\frac{\frac{3}{2}\left(\frac{1}{2}\int \frac{\sqrt{1-(a+bx)^2}}{\sqrt{\arccos(a+bx)}}d\arccos(a+bx)-\sqrt{1-(a+bx)^2}\sqrt{\arccos(a+bx)}\right)+(a+bx)\arccos(a+bx)^{3/2}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3}{2}\left(\frac{1}{2}\int \frac{\sin(\arccos(a+bx))}{\sqrt{\arccos(a+bx)}}d\arccos(a+bx)-\sqrt{1-(a+bx)^2}\sqrt{\arccos(a+bx)}\right)+(a+bx)\arccos(a+bx)^{3/2}}{b}$$

$$\Big\downarrow \text{3786}$$

$$\frac{\frac{3}{2}\left(\int \sqrt{1-(a+bx)^2}d\sqrt{\arccos(a+bx)}-\sqrt{1-(a+bx)^2}\sqrt{\arccos(a+bx)}\right)+(a+bx)\arccos(a+bx)^{3/2}}{b}$$

$$\Big\downarrow \text{3832}$$

$$\frac{\frac{3}{2}\left(\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(a+bx)}\right)-\sqrt{1-(a+bx)^2}\sqrt{\arccos(a+bx)}\right)+(a+bx)\arccos(a+bx)^{3/2}}{b}$$

input
```
Int[ArcCos[a + b*x]^(3/2),x]
```

output
```
((a + b*x)*ArcCos[a + b*x]^(3/2) + (3*(-(Sqrt[1 - (a + b*x)^2]*Sqrt[ArcCos
[a + b*x]]) + Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcCos[a + b*x]]]))/2)/b
```

3.38.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5131
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Simp[b*c*n   Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5135
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[-(b*c)^(-1)
  Subst[Int[x^n*Sin[-a/b + x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a,
 b, c, n}, x]
```

rule 5183
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Simp[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p]   I
nt[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

rule 5303
```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
  Subst[Int[(a + b*ArcCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
n}, x]
```

3.38.4 Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.18

method	result
default	$-\frac{\sqrt{2}\left(-2\arccos(bx+a)^{\frac{3}{2}}\sqrt{2}\sqrt{\pi}\,bx-2\arccos(bx+a)^{\frac{3}{2}}\sqrt{2}\sqrt{\pi}\,a+3\sqrt{2}\,\sqrt{\arccos(bx+a)}\,\sqrt{\pi}\,\sqrt{-b^2x^2-2abx-a^2+1}-3\pi\ \text{FresnelS}\left(\frac{\sqrt{2}}{\cdots}\right.\right.}{4b\sqrt{\pi}}$

input
```
int(arccos(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/b*2^(1/2)*(-2*arccos(b*x+a)^(3/2)*2^(1/2)*Pi^(1/2)*b*x-2*arccos(b*x+a
)^(3/2)*2^(1/2)*Pi^(1/2)*a+3*2^(1/2)*arccos(b*x+a)^(1/2)*Pi^(1/2)*(-b^2*x^
2-2*a*b*x-a^2+1)^(1/2)-3*Pi*FresnelS(2^(1/2)/Pi^(1/2)*arccos(b*x+a)^(1/2))
)/Pi^(1/2)
```

3.38.5 Fricas [F(-2)]

Exception generated.

$$\int \arccos(a+bx)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arccos(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.38.6 Sympy [F]

$$\int \arccos(a+bx)^{3/2}\,dx = \int \operatorname{acos}^{\frac{3}{2}}(a+bx)\,dx$$

input
```
integrate(acos(b*x+a)**(3/2),x)
```

output
```
Integral(acos(a + b*x)**(3/2), x)
```

3.38.7 Maxima [F(-2)]

Exception generated.

$$\int \arccos(a+bx)^{3/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arccos(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.38.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.56

$$\int \arccos(a+bx)^{3/2}\,dx = \frac{\arccos(bx+a)^{\frac{3}{2}}e^{(i\arccos(bx+a))}}{2b} + \frac{\arccos(bx+a)^{\frac{3}{2}}e^{(-i\arccos(bx+a))}}{2b} + \frac{(3i-3)\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\arccos(bx+a)}\right)}{16b} - \frac{(3i+3)\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{\arccos(bx+a)}\right)}{16b} + \frac{3i\sqrt{\arccos(bx+a)}e^{(i\arccos(bx+a))}}{4b} - \frac{3i\sqrt{\arccos(bx+a)}e^{(-i\arccos(bx+a))}}{4b}$$

input
```
integrate(arccos(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
1/2*arccos(b*x + a)^(3/2)*e^(I*arccos(b*x + a))/b + 1/2*arccos(b*x + a)^(3
/2)*e^(-I*arccos(b*x + a))/b + (3/16*I - 3/16)*sqrt(2)*sqrt(pi)*erf((1/2*I
 - 1/2)*sqrt(2)*sqrt(arccos(b*x + a)))/b - (3/16*I + 3/16)*sqrt(2)*sqrt(pi
)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(arccos(b*x + a)))/b + 3/4*I*sqrt(arccos(
b*x + a))*e^(I*arccos(b*x + a))/b - 3/4*I*sqrt(arccos(b*x + a))*e^(-I*arcc
os(b*x + a))/b
```

3.38.9 Mupad [F(-1)]

Timed out.

$$\int \arccos(a+bx)^{3/2}\,dx = \int \mathrm{acos}(a+b\,x)^{3/2}\,dx$$

input
```
int(acos(a + b*x)^(3/2),x)
```

output
```
int(acos(a + b*x)^(3/2), x)
```

3.39 $\int \sqrt{\arccos(a+bx)}\,dx$

3.39.1 Optimal result

Integrand size = 10, antiderivative size = 55

$$\int \sqrt{\arccos(a+bx)}\,dx = \frac{(a+bx)\sqrt{\arccos(a+bx)}}{b} - \frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(a+bx)}\right)}{b}$$

output
```
-1/2*FresnelC(2^(1/2)/Pi^(1/2)*arccos(b*x+a)^(1/2))*2^(1/2)*Pi^(1/2)/b+(b*
x+a)*arccos(b*x+a)^(1/2)/b
```

3.39.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.44

$$\int \sqrt{\arccos(a+bx)}\,dx = \frac{i\left(\sqrt{-i\arccos(a+bx)}\,\Gamma\left(\frac{3}{2},-i\arccos(a+bx)\right)-\sqrt{i\arccos(a+bx)}\,\Gamma\left(\frac{3}{2},i\arccos(a+bx)\right)\right)}{2b\sqrt{\arccos(a+bx)}}$$

input
```
Integrate[Sqrt[ArcCos[a + b*x]],x]
```

output
```
((I/2)*(Sqrt[(-I)*ArcCos[a + b*x]]*Gamma[3/2, (-I)*ArcCos[a + b*x]] - Sqrt
[I*ArcCos[a + b*x]]*Gamma[3/2, I*ArcCos[a + b*x]]))/(b*Sqrt[ArcCos[a + b*x
]])
```

3.39.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5303, 5131, 5225, 3042, 3785, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\arccos(a+bx)}\,dx$$

$$\Big\downarrow \text{ 5303}$$

$$\frac{\int \sqrt{\arccos(a+bx)}d(a+bx)}{b}$$

$$\Big\downarrow \text{ 5131}$$

$$\frac{\frac{1}{2}\int \frac{a+bx}{\sqrt{1-(a+bx)^2}\sqrt{\arccos(a+bx)}}d(a+bx)+(a+bx)\sqrt{\arccos(a+bx)}}{b}$$

$$\Big\downarrow \text{ 5225}$$

$$\frac{(a+bx)\sqrt{\arccos(a+bx)}-\frac{1}{2}\int \frac{a+bx}{\sqrt{\arccos(a+bx)}}d\arccos(a+bx)}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(a+bx)\sqrt{\arccos(a+bx)}-\frac{1}{2}\int \frac{\sin\left(\arccos(a+bx)+\frac{\pi}{2}\right)}{\sqrt{\arccos(a+bx)}}d\arccos(a+bx)}{b}$$

$$\Big\downarrow \text{ 3785}$$

$$\frac{(a+bx)\sqrt{\arccos(a+bx)}-\int (a+bx)d\sqrt{\arccos(a+bx)}}{b}$$

$$\Big\downarrow \text{ 3833}$$

$$\frac{(a+bx)\sqrt{\arccos(a+bx)}-\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(a+bx)}\right)}{b}$$

input
```
Int[Sqrt[ArcCos[a + b*x]],x]
```

output
```
((a + b*x)*Sqrt[ArcCos[a + b*x]] - Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[Arc
Cos[a + b*x]]])/b
```

3.39.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5131
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Simp[b*c*n   Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

rule 5303
```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
  Subst[Int[(a + b*ArcCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
n}, x]
```

3.39.4 Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.20

method	result	size
default	$\frac{-\sqrt{2}\,\sqrt{\arccos\left(bx+a\right)}\,\sqrt{\pi}\,\operatorname{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{\arccos\left(bx+a\right)}}{\sqrt{\pi}}\right)+2\arccos\left(bx+a\right)bx+2\arccos\left(bx+a\right)a}{2b\sqrt{\arccos\left(bx+a\right)}}$	66

input
```
int(arccos(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/b/arccos(b*x+a)^(1/2)*(-2^(1/2)*arccos(b*x+a)^(1/2)*Pi^(1/2)*FresnelC(
2^(1/2)/Pi^(1/2)*arccos(b*x+a)^(1/2))+2*arccos(b*x+a)*b*x+2*arccos(b*x+a)*
a)
```

3.39.5 Fricas [F(-2)]

Exception generated.

$$\int \sqrt{\arccos(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arccos(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.39.6 Sympy [F]

$$\int \sqrt{\arccos(a+bx)}\,dx = \int \sqrt{\operatorname{acos}(a+bx)}\,dx$$

input
```
integrate(acos(b*x+a)**(1/2),x)
```

output
```
Integral(sqrt(acos(a + b*x)), x)
```

3.39.7 Maxima [F(-2)]

Exception generated.

$$\int \sqrt{\arccos(a+bx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arccos(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.39.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.73

$$\int \sqrt{\arccos(a+bx)}\,dx = \frac{(i+1)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\arccos(bx+a)}\right)}{8\,b} - \frac{(i-1)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{\arccos(bx+a)}\right)}{8\,b} + \frac{\sqrt{\arccos(bx+a)}e^{(i\arccos(bx+a))}}{2\,b} + \frac{\sqrt{\arccos(bx+a)}e^{(-i\arccos(bx+a))}}{2\,b}$$

input
```
integrate(arccos(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
(1/8*I + 1/8)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(arccos(b*x +
 a)))/b - (1/8*I - 1/8)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(a
rccos(b*x + a)))/b + 1/2*sqrt(arccos(b*x + a))*e^(I*arccos(b*x + a))/b + 1
/2*sqrt(arccos(b*x + a))*e^(-I*arccos(b*x + a))/b
```

3.39.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{\arccos(a+bx)}\,dx = \int \sqrt{\mathrm{acos}\left(a+b\,x\right)}\,dx$$

input
```
int(acos(a + b*x)^(1/2),x)
```

output
```
int(acos(a + b*x)^(1/2), x)
```

3.40 $\int \frac{1}{\sqrt{\arccos(a+bx)}}\,dx$

3.40.1 Optimal result

Integrand size = 10, antiderivative size = 33

$$\int \frac{1}{\sqrt{\arccos(a+bx)}}\,dx = -\frac{\sqrt{2\pi}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(a+bx)}\right)}{b}$$

output
```
-FresnelS(2^(1/2)/Pi^(1/2)*arccos(b*x+a)^(1/2))*2^(1/2)*Pi^(1/2)/b
```

3.40.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.36

$$\int \frac{1}{\sqrt{\arccos(a+bx)}}\,dx = -\frac{-\sqrt{-i\arccos(a+bx)}\,\Gamma\left(\frac{1}{2},-i\arccos(a+bx)\right)-\sqrt{i\arccos(a+bx)}\,\Gamma\left(\frac{1}{2},i\arccos(a+bx)\right)}{2b\sqrt{\arccos(a+bx)}}$$

input
```
Integrate[1/Sqrt[ArcCos[a + b*x]],x]
```

output
```
-1/2*(-(Sqrt[(-I)*ArcCos[a + b*x]]*Gamma[1/2, (-I)*ArcCos[a + b*x]]) - Sqr
t[I*ArcCos[a + b*x]]*Gamma[1/2, I*ArcCos[a + b*x]])/(b*Sqrt[ArcCos[a + b*x
]])
```

3.40.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5303, 5135, 3042, 3786, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\arccos (a+b x)}}dx$$

$$\Big\downarrow \text{ 5303}$$

$$\frac{\int \frac{1}{\sqrt{\arccos (a+b x)}}d(a+b x)}{b}$$

$$\Big\downarrow \text{ 5135}$$

$$-\frac{\int \frac{\sqrt{1-(a+b x)^2}}{\sqrt{\arccos (a+b x)}}d\arccos (a+b x)}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\int \frac{\sin (\arccos (a+b x))}{\sqrt{\arccos (a+b x)}}d\arccos (a+b x)}{b}$$

$$\Big\downarrow \text{ 3786}$$

$$-\frac{2 \int \sqrt{1-(a+b x)^2}d\sqrt{\arccos (a+b x)}}{b}$$

$$\Big\downarrow \text{ 3832}$$

$$-\frac{\sqrt{2 \pi } \operatorname{FresnelS}\left(\sqrt{\frac{2}{\pi }} \sqrt{\arccos (a+b x)}\right)}{b}$$

input
```
Int[1/Sqrt[ArcCos[a + b*x]],x]
```

output
```
-((Sqrt[2*Pi]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcCos[a + b*x]]])/b)
```

3.40.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5135
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[-(b*c)^(-1)
  Subst[Int[x^n*Sin[-a/b + x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a,
 b, c, n}, x]
```

rule 5303
```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
  Subst[Int[(a + b*ArcCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
n}, x]
```

3.40.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

method	result	size
default	$-\frac{\operatorname{FresnelS}\left(\frac{\sqrt{2}\,\sqrt{\arccos(bx+a)}}{\sqrt{\pi}}\right)\sqrt{2}\,\sqrt{\pi}}{b}$	28

input
```
int(1/arccos(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-FresnelS(2^(1/2)/Pi^(1/2)*arccos(b*x+a)^(1/2))*2^(1/2)*Pi^(1/2)/b
```

3.40.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{\arccos(a+bx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/arccos(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.40.6 Sympy [F]

$$\int \frac{1}{\sqrt{\arccos(a+bx)}}\,dx = \int \frac{1}{\sqrt{\operatorname{acos}(a+bx)}}\,dx$$

input
```
integrate(1/acos(b*x+a)**(1/2),x)
```

output
```
Integral(1/sqrt(acos(a + b*x)), x)
```

3.40.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{\arccos(a+bx)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/arccos(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.40.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.55

$$\int \frac{1}{\sqrt{\arccos(a+bx)}}\,dx = -\frac{(i-1)\,\sqrt{2}\sqrt{\pi}\,\text{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\,\sqrt{2}\sqrt{\arccos\left(bx+a\right)}\right)}{4\,b} + \frac{(i+1)\,\sqrt{2}\sqrt{\pi}\,\text{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\,\sqrt{2}\sqrt{\arccos\left(bx+a\right)}\right)}{4\,b}$$

input
```
integrate(1/arccos(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
-(1/4*I - 1/4)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(arccos(b*x
+ a)))/b + (1/4*I + 1/4)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(
arccos(b*x + a)))/b
```

3.40.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\arccos(a+bx)}}\,dx = \int \frac{1}{\sqrt{\mathrm{acos}\left(a+b\,x\right)}}\,dx$$

input
```
int(1/acos(a + b*x)^(1/2),x)
```

output
```
int(1/acos(a + b*x)^(1/2), x)
```

3.41 $\int \frac{1}{\arccos(a+bx)^{3/2}}\,dx$

3.41.1 Optimal result

Integrand size = 10, antiderivative size = 64

$$\int \frac{1}{\arccos(a+bx)^{3/2}}\,dx=\frac{2\sqrt{1-(a+bx)^2}}{b\sqrt{\arccos(a+bx)}}-\frac{2\sqrt{2\pi}\,\mathrm{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(a+bx)}\right)}{b}$$

output
```
-2*FresnelC(2^(1/2)/Pi^(1/2)*arccos(b*x+a)^(1/2))*2^(1/2)*Pi^(1/2)/b+2*(1-
(b*x+a)^2)^(1/2)/b/arccos(b*x+a)^(1/2)
```

3.41.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.52

$$\int \frac{1}{\arccos(a+bx)^{3/2}}\,dx = -\frac{-2\sqrt{1-(a+bx)^2}-i\sqrt{-i\arccos(a+bx)}\Gamma\left(\frac{1}{2},-i\arccos(a+bx)\right)+i\sqrt{i\arccos(a+bx)}\Gamma\left(\frac{1}{2},i\arccos(a+\right.}{b\sqrt{\arccos(a+bx)}}$$

input
```
Integrate[ArcCos[a + b*x]^(-3/2),x]
```

output
```
-((-2*Sqrt[1 - (a + b*x)^2] - I*Sqrt[(-I)*ArcCos[a + b*x]]*Gamma[1/2, (-I)
*ArcCos[a + b*x]] + I*Sqrt[I*ArcCos[a + b*x]]*Gamma[1/2, I*ArcCos[a + b*x]
])/(b*Sqrt[ArcCos[a + b*x]]))
```

3.41.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5303, 5133, 5225, 3042, 3785, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arccos(a+bx)^{3/2}}\,dx$$

$$\Big\downarrow \text{5303}$$

$$\frac{\int \frac{1}{\arccos(a+bx)^{3/2}}d(a+bx)}{b}$$

$$\Big\downarrow \text{5133}$$

$$\frac{2\int \frac{a+bx}{\sqrt{1-(a+bx)^2}\sqrt{\arccos(a+bx)}}d(a+bx)+\frac{2\sqrt{1-(a+bx)^2}}{\sqrt{\arccos(a+bx)}}}{b}$$

$$\Big\downarrow \text{5225}$$

$$\frac{\frac{2\sqrt{1-(a+bx)^2}}{\sqrt{\arccos(a+bx)}}-2\int \frac{a+bx}{\sqrt{\arccos(a+bx)}}d\arccos(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2\sqrt{1-(a+bx)^2}}{\sqrt{\arccos(a+bx)}}-2\int \frac{\sin\left(\arccos(a+bx)+\frac{\pi}{2}\right)}{\sqrt{\arccos(a+bx)}}d\arccos(a+bx)}{b}$$

$$\Big\downarrow \text{3785}$$

$$\frac{\frac{2\sqrt{1-(a+bx)^2}}{\sqrt{\arccos(a+bx)}}-4\int (a+bx)d\sqrt{\arccos(a+bx)}}{b}$$

$$\Big\downarrow \text{3833}$$

$$\frac{\frac{2\sqrt{1-(a+bx)^2}}{\sqrt{\arccos(a+bx)}}-2\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(a+bx)}\right)}{b}$$

input
```
Int[ArcCos[a + b*x]^(-3/2),x]
```

output
```
((2*Sqrt[1 - (a + b*x)^2])/Sqrt[ArcCos[a + b*x]] - 2*Sqrt[2*Pi]*FresnelC[S
qrt[2/Pi]*Sqrt[ArcCos[a + b*x]]])/b
```

3.41.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5133
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-Sqrt[1 - c
^2*x^2])*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c/(b*(n + 1
))   Int[x*((a + b*ArcCos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && LtQ[n, -1]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

rule 5303
```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
  Subst[Int[(a + b*ArcCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
n}, x]
```

3.41.4 Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.28

method	result	size
default	$\frac{\sqrt{2}\left(\sqrt{2}\,\sqrt{\arccos(bx+a)}\,\sqrt{\pi}\,\sqrt{-b^2x^2-2abx-a^2+1}-2\arccos(bx+a)\pi\ \mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{\arccos(bx+a)}}{\sqrt{\pi}}\right)\right)}{b\sqrt{\pi}\,\arccos(bx+a)}$	82

3.41. $\int \frac{1}{\arccos(a+bx)^{3/2}}\,dx$

input
```
int(1/arccos(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/b*2^(1/2)*(2^(1/2)*arccos(b*x+a)^(1/2)*Pi^(1/2)*(-b^2*x^2-2*a*b*x-a^2+1)
^(1/2)-2*arccos(b*x+a)*Pi*FresnelC(2^(1/2)/Pi^(1/2)*arccos(b*x+a)^(1/2)))/
Pi^(1/2)/arccos(b*x+a)
```

3.41.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\arccos(a+bx)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/arccos(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.41.6 Sympy [F]

$$\int \frac{1}{\arccos(a+bx)^{3/2}}\,dx = \int \frac{1}{\operatorname{acos}^{\frac{3}{2}}(a+bx)}\,dx$$

input
```
integrate(1/acos(b*x+a)**(3/2),x)
```

output
```
Integral(acos(a + b*x)**(-3/2), x)
```

3.41.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\arccos(a+bx)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/arccos(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.41.8 Giac [F]

$$\int \frac{1}{\arccos(a+bx)^{3/2}}\,dx = \int \frac{1}{\arccos\left(bx+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/arccos(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(arccos(b*x + a)^(-3/2), x)
```

3.41.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\arccos(a+bx)^{3/2}}\,dx = \int \frac{1}{\mathrm{acos}\left(a+b\,x\right)^{3/2}}\,dx$$

input
```
int(1/acos(a + b*x)^(3/2),x)
```

output
```
int(1/acos(a + b*x)^(3/2), x)
```

3.42 $\int \frac{1}{\arccos(a+bx)^{5/2}}\,dx$

3.42.1 Optimal result

Integrand size = 10, antiderivative size = 90

$$\int \frac{1}{\arccos(a+bx)^{5/2}}\,dx = \frac{2\sqrt{1-(a+bx)^2}}{3b\arccos(a+bx)^{3/2}} + \frac{4(a+bx)}{3b\sqrt{\arccos(a+bx)}} + \frac{4\sqrt{2\pi}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(a+bx)}\right)}{3b}$$

output
```
4/3*FresnelS(2^(1/2)/Pi^(1/2)*arccos(b*x+a)^(1/2))*2^(1/2)*Pi^(1/2)/b+2/3*
(1-(b*x+a)^2)^(1/2)/b/arccos(b*x+a)^(3/2)+4/3*(b*x+a)/b/arccos(b*x+a)^(1/2
)
```

3.42.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.54

$$\int \frac{1}{\arccos(a+bx)^{5/2}}\,dx = -\frac{2\left(-\sqrt{1-(a+bx)^2} - e^{-i\arccos(a+bx)}\arccos(a+bx) - e^{i\arccos(a+bx)}\arccos(a+bx) + i(-i\arccos(a+bx))^3}{3b\arccos(a+bx)^{3/2}}$$

input
```
Integrate[ArcCos[a + b*x]^(-5/2),x]
```

output
```
(-2*(-Sqrt[1 - (a + b*x)^2] - ArcCos[a + b*x]/E^(I*ArcCos[a + b*x]) - E^(I
*ArcCos[a + b*x])*ArcCos[a + b*x] + I*((-I)*ArcCos[a + b*x])^(3/2)*Gamma[1
/2, (-I)*ArcCos[a + b*x]] - I*(I*ArcCos[a + b*x])^(3/2)*Gamma[1/2, I*ArcCo
s[a + b*x]]))/(3*b*ArcCos[a + b*x]^(3/2))
```

3.42.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {5303, 5133, 5223, 5135, 3042, 3786, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arccos(a+bx)^{5/2}}\,dx$$

$$\Big\downarrow \text{5303}$$

$$\frac{\int \frac{1}{\arccos(a+bx)^{5/2}}d(a+bx)}{b}$$

$$\Big\downarrow \text{5133}$$

$$\frac{\frac{2}{3}\int \frac{a+bx}{\sqrt{1-(a+bx)^2}\arccos(a+bx)^{3/2}}d(a+bx)+\frac{2\sqrt{1-(a+bx)^2}}{3\arccos(a+bx)^{3/2}}}{b}$$

$$\Big\downarrow \text{5223}$$

$$\frac{\frac{2}{3}\left(\frac{2(a+bx)}{\sqrt{\arccos(a+bx)}}-2\int \frac{1}{\sqrt{\arccos(a+bx)}}d(a+bx)\right)+\frac{2\sqrt{1-(a+bx)^2}}{3\arccos(a+bx)^{3/2}}}{b}$$

$$\Big\downarrow \text{5135}$$

$$\frac{\frac{2}{3}\left(2\int \frac{\sqrt{1-(a+bx)^2}}{\sqrt{\arccos(a+bx)}}d\arccos(a+bx)+\frac{2(a+bx)}{\sqrt{\arccos(a+bx)}}\right)+\frac{2\sqrt{1-(a+bx)^2}}{3\arccos(a+bx)^{3/2}}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2}{3}\left(2\int \frac{\sin(\arccos(a+bx))}{\sqrt{\arccos(a+bx)}}d\arccos(a+bx)+\frac{2(a+bx)}{\sqrt{\arccos(a+bx)}}\right)+\frac{2\sqrt{1-(a+bx)^2}}{3\arccos(a+bx)^{3/2}}}{b}$$

$$\Big\downarrow \text{3786}$$

$$\frac{\frac{2}{3}\left(4\int\sqrt{1-(a+bx)^2}d\sqrt{\arccos(a+bx)}+\frac{2(a+bx)}{\sqrt{\arccos(a+bx)}}\right)+\frac{2\sqrt{1-(a+bx)^2}}{3\arccos(a+bx)^{3/2}}}{b}$$

$$\Big\downarrow \text{3832}$$

$$\frac{\frac{2}{3}\left(2\sqrt{2\pi}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(a+bx)}\right)+\frac{2(a+bx)}{\sqrt{\arccos(a+bx)}}\right)+\frac{2\sqrt{1-(a+bx)^2}}{3\arccos(a+bx)^{3/2}}}{b}$$

input
```
Int[ArcCos[a + b*x]^(-5/2),x]
```

output
```
((2*Sqrt[1 - (a + b*x)^2])/(3*ArcCos[a + b*x]^(3/2)) + (2*((2*(a + b*x))/S
qrt[ArcCos[a + b*x]] + 2*Sqrt[2*Pi]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcCos[a + b*
x]]]))/3)/b
```

3.42.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5133
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-Sqrt[1 - c
^2*x^2])*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - Simp[c/(b*(n + 1
))   Int[x*((a + b*ArcCos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ
[{a, b, c}, x] && LtQ[n, -1]
```

rule 5135
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[-(b*c)^(-1)
  Subst[Int[x^n*Sin[-a/b + x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a,
 b, c, n}, x]
```

rule 5223
```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Simp[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]   Int[(f*x)^(m - 1)*(a + b
*ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2
*d + e, 0] && LtQ[n, -1]
```

rule 5303
```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
  Subst[Int[(a + b*ArcCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
n}, x]
```

3.42.4 Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.33

method	result
default	$\frac{\sqrt{2}\left(4\arccos(bx+a)^2\pi\ \mathrm{FresnelS}\left(\frac{\sqrt{2}\sqrt{\arccos(bx+a)}}{\sqrt{\pi}}\right)+2\arccos(bx+a)^{\frac{3}{2}}\sqrt{2}\sqrt{\pi}\,bx+2\arccos(bx+a)^{\frac{3}{2}}\sqrt{2}\sqrt{\pi}\,a+\sqrt{2}\sqrt{\arccos(bx+a)}\ \sqrt{}\right.}{3b\sqrt{\pi}\ \arccos(bx+a)^2}$

input
```
int(1/arccos(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/b*2^(1/2)/Pi^(1/2)*(4*arccos(b*x+a)^2*Pi*FresnelS(2^(1/2)/Pi^(1/2)*arc
cos(b*x+a)^(1/2))+2*arccos(b*x+a)^(3/2)*2^(1/2)*Pi^(1/2)*b*x+2*arccos(b*x+
a)^(3/2)*2^(1/2)*Pi^(1/2)*a+2^(1/2)*arccos(b*x+a)^(1/2)*Pi^(1/2)*(-b^2*x^2
-2*a*b*x-a^2+1)^(1/2))/arccos(b*x+a)^2
```

3.42.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\arccos(a+bx)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/arccos(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.42.6 Sympy [F]

$$\int \frac{1}{\arccos(a+bx)^{5/2}}\,dx = \int \frac{1}{\operatorname{acos}^{\frac{5}{2}}(a+bx)}\,dx$$

input
```
integrate(1/acos(b*x+a)**(5/2),x)
```

output
```
Integral(acos(a + b*x)**(-5/2), x)
```

3.42.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\arccos(a+bx)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/arccos(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.42.8 Giac [F]

$$\int \frac{1}{\arccos(a+bx)^{5/2}}\,dx = \int \frac{1}{\arccos(bx+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/arccos(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
integrate(arccos(b*x + a)^(-5/2), x)
```

3.42.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\arccos(a+bx)^{5/2}}\,dx = \int \frac{1}{\mathrm{acos}\left(a+b\,x\right)^{5/2}}\,dx$$

input
```
int(1/acos(a + b*x)^(5/2),x)
```

output
```
int(1/acos(a + b*x)^(5/2), x)
```

3.43 $\int \frac{1}{\sqrt{a+b\arccos(c+dx)}}\,dx$

3.43.1 Optimal result

Integrand size = 14, antiderivative size = 106

$$\int \frac{1}{\sqrt{a+b\arccos(c+dx)}}\,dx=-\frac{\sqrt{2\pi}\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}d}+\frac{\sqrt{2\pi}\,\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(c+dx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{\sqrt{b}d}$$

output
```
-cos(a/b)*FresnelS(2^(1/2)/Pi^(1/2)*(a+b*arccos(d*x+c))^(1/2)/b^(1/2))*2^(
1/2)*Pi^(1/2)/d/b^(1/2)+FresnelC(2^(1/2)/Pi^(1/2)*(a+b*arccos(d*x+c))^(1/2
)/b^(1/2))*sin(a/b)*2^(1/2)*Pi^(1/2)/d/b^(1/2)
```

3.43.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.21

$$\int \frac{1}{\sqrt{a+b\arccos(c+dx)}}\,dx=\frac{e^{-\frac{ia}{b}}\left(\sqrt{-\frac{i(a+b\arccos(c+dx))}{b}}\Gamma\left(\frac{1}{2},-\frac{i(a+b\arccos(c+dx))}{b}\right)+e^{\frac{2ia}{b}}\sqrt{\frac{i(a+b\arccos(c+dx))}{b}}\Gamma\left(\frac{1}{2},\frac{i(a+b\arccos(c+dx))}{b}\right)\right)}{2d\sqrt{a+b\arccos(c+dx)}}$$

input
```
Integrate[1/Sqrt[a + b*ArcCos[c + d*x]],x]
```

output
```
(Sqrt[((-I)*(a + b*ArcCos[c + d*x]))/b]*Gamma[1/2, ((-I)*(a + b*ArcCos[c +
 d*x]))/b] + E^(((2*I)*a)/b)*Sqrt[(I*(a + b*ArcCos[c + d*x]))/b]*Gamma[1/2
, (I*(a + b*ArcCos[c + d*x]))/b])/(2*d*E^((I*a)/b)*Sqrt[a + b*ArcCos[c + d
*x]])
```

3.43.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {5303, 5135, 25, 3042, 3787, 25, 3042, 3785, 3786, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\arccos(c+dx)}}dx$$

$$\Big\downarrow \text{5303}$$

$$\frac{\int \frac{1}{\sqrt{a+b\arccos(c+dx)}}d(c+dx)}{d}$$

$$\Big\downarrow \text{5135}$$

$$-\frac{\int -\frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(c+dx)}{b}\right)}{\sqrt{a+b\arccos(c+dx)}}d(a+b\arccos(c+dx))}{bd}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(c+dx)}{b}\right)}{\sqrt{a+b\arccos(c+dx)}}d(a+b\arccos(c+dx))}{bd}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(\frac{a}{b}-\frac{a+b\arccos(c+dx)}{b}\right)}{\sqrt{a+b\arccos(c+dx)}}d(a+b\arccos(c+dx))}{bd}$$

$$\Big\downarrow \text{3787}$$

$$-\frac{-\sin\left(\frac{a}{b}\right)\int \frac{\cos\left(\frac{a+b\arccos(c+dx)}{b}\right)}{\sqrt{a+b\arccos(c+dx)}}d(a+b\arccos(c+dx))-\cos\left(\frac{a}{b}\right)\int -\frac{\sin\left(\frac{a+b\arccos(c+dx)}{b}\right)}{\sqrt{a+b\arccos(c+dx)}}d(a+b\arccos(c+dx))}{bd}$$

$$\Big\downarrow \ 25$$

$$-\frac{\cos\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a+b\arccos(c+dx)}{b}\right)}{\sqrt{a+b\arccos(c+dx)}}d(a+b\arccos(c+dx))-\sin\left(\frac{a}{b}\right)\int \frac{\cos\left(\frac{a+b\arccos(c+dx)}{b}\right)}{\sqrt{a+b\arccos(c+dx)}}d(a+b\arccos(c+dx))}{bd}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\cos\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a+b\arccos(c+dx)}{b}\right)}{\sqrt{a+b\arccos(c+dx)}}d(a+b\arccos(c+dx))-\sin\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a+b\arccos(c+dx)}{b}+\frac{\pi}{2}\right)}{\sqrt{a+b\arccos(c+dx)}}d(a+b\arccos(c+dx))}{bd}$$

$$\Big\downarrow \ 3785$$

$$-\frac{\cos\left(\frac{a}{b}\right)\int \frac{\sin\left(\frac{a+b\arccos(c+dx)}{b}\right)}{\sqrt{a+b\arccos(c+dx)}}d(a+b\arccos(c+dx))-2\sin\left(\frac{a}{b}\right)\int \cos\left(\frac{a+b\arccos(c+dx)}{b}\right)d\sqrt{a+b\arccos(c+dx)}}{bd}$$

$$\Big\downarrow \ 3786$$

$$-\frac{2\cos\left(\frac{a}{b}\right)\int \sin\left(\frac{a+b\arccos(c+dx)}{b}\right)d\sqrt{a+b\arccos(c+dx)}-2\sin\left(\frac{a}{b}\right)\int \cos\left(\frac{a+b\arccos(c+dx)}{b}\right)d\sqrt{a+b\arccos(c+dx)}}{bd}$$

$$\Big\downarrow \ 3832$$

$$-\frac{\sqrt{2\pi}\sqrt{b}\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(c+dx)}}{\sqrt{b}}\right)-2\sin\left(\frac{a}{b}\right)\int \cos\left(\frac{a+b\arccos(c+dx)}{b}\right)d\sqrt{a+b\arccos(c+dx)}}{bd}$$

$$\Big\downarrow \ 3833$$

$$-\frac{\sqrt{2\pi}\sqrt{b}\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(c+dx)}}{\sqrt{b}}\right)-\sqrt{2\pi}\sqrt{b}\sin\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(c+dx)}}{\sqrt{b}}\right)}{bd}$$

input
```
Int[1/Sqrt[a + b*ArcCos[c + d*x]],x]
```

output
```
-((Sqrt[b]*Sqrt[2*Pi]*Cos[a/b]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c +
d*x]])/Sqrt[b]] - Sqrt[b]*Sqrt[2*Pi]*FresnelC[(Sqrt[2/Pi]*Sqrt[a + b*ArcCo
s[c + d*x]])/Sqrt[b]]*Sin[a/b])/(b*d))
```

3.43. $\int \frac{1}{\sqrt{a+b\arccos(c+dx)}}\,dx$

3.43.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3787
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Cos
[(d*e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Simp[Sin[(
d*e - c*f)/d]   Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5135
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[-(b*c)^(-1)
  Subst[Int[x^n*Sin[-a/b + x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a,
 b, c, n}, x]
```

rule 5303
```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
  Subst[Int[(a + b*ArcCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
n}, x]
```

3.43. $\int \frac{1}{\sqrt{a+b\arccos(c+dx)}}\,dx$

3.43.4 Maple [A] (verified)

Time = 1.87 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{\sqrt{2}\,\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\left(\cos\left(\frac{a}{b}\right)\operatorname{FresnelS}\left(\frac{\sqrt{2}\,\sqrt{a+b\arccos(dx+c)}}{\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,b}\right)+\sin\left(\frac{a}{b}\right)\operatorname{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{a+b\arccos(dx+c)}}{\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,b}\right)\right)}{d}$	93

input
```
int(1/(a+b*arccos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2^(1/2)*Pi^(1/2)*(-1/b)^(1/2)*(cos(a/b)*FresnelS(2^(1/2)/Pi^(1/2)/(-1/b)^(
1/2)/b*(a+b*arccos(d*x+c))^(1/2))+sin(a/b)*FresnelC(2^(1/2)/Pi^(1/2)/(-1/b
)^(1/2)/b*(a+b*arccos(d*x+c))^(1/2)))/d
```

3.43.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+b\arccos(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arccos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.43.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+b\arccos(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+b\operatorname{acos}(c+dx)}}\,dx$$

input
```
integrate(1/(a+b*acos(d*x+c))**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*acos(c + d*x)), x)
```

3.43.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+b\arccos(c+dx)}}\,dx=\int \frac{1}{\sqrt{b\arccos\left(dx+c\right)+a}}\,dx$$

input
```
integrate(1/(a+b*arccos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*arccos(d*x + c) + a), x)
```

3.43.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.58

$$\int \frac{1}{\sqrt{a+b\arccos(c+dx)}}\,dx = \frac{i\sqrt{\pi}\,\mathrm{erf}\left(-\frac{i\sqrt{2}\sqrt{b\arccos(dx+c)+a}}{2\sqrt{|b|}}-\frac{\sqrt{2}\sqrt{b\arccos(dx+c)+a}\sqrt{|b|}}{2b}\right)e^{\left(\frac{ia}{b}\right)}}{d\left(\frac{i\sqrt{2}b}{\sqrt{|b|}}+\sqrt{2}\sqrt{|b|}\right)} - \frac{i\sqrt{\pi}\,\mathrm{erf}\left(\frac{i\sqrt{2}\sqrt{b\arccos(dx+c)+a}}{2\sqrt{|b|}}-\frac{\sqrt{2}\sqrt{b\arccos(dx+c)+a}\sqrt{|b|}}{2b}\right)e^{\left(-\frac{ia}{b}\right)}}{d\left(-\frac{i\sqrt{2}b}{\sqrt{|b|}}+\sqrt{2}\sqrt{|b|}\right)}$$

input
```
integrate(1/(a+b*arccos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
I*sqrt(pi)*erf(-1/2*I*sqrt(2)*sqrt(b*arccos(d*x + c) + a)/sqrt(abs(b)) - 1
/2*sqrt(2)*sqrt(b*arccos(d*x + c) + a)*sqrt(abs(b))/b)*e^(I*a/b)/(d*(I*sqr
t(2)*b/sqrt(abs(b)) + sqrt(2)*sqrt(abs(b)))) - I*sqrt(pi)*erf(1/2*I*sqrt(2
)*sqrt(b*arccos(d*x + c) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(d*x
 + c) + a)*sqrt(abs(b))/b)*e^(-I*a/b)/(d*(-I*sqrt(2)*b/sqrt(abs(b)) + sqrt
(2)*sqrt(abs(b))))
```

3.43.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\arccos(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+b\,\mathrm{acos}\left(c+d\,x\right)}}\,dx$$

input
```
int(1/(a + b*acos(c + d*x))^(1/2),x)
```

output
```
int(1/(a + b*acos(c + d*x))^(1/2), x)
```

3.44 $\int \frac{1}{\sqrt{a-b\arccos(c+dx)}}\,dx$

3.44.1 Optimal result

Integrand size = 15, antiderivative size = 108

$$\int \frac{1}{\sqrt{a-b\arccos(c+dx)}}\,dx = -\frac{\sqrt{2\pi}\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a-b\arccos(c+dx)}}{\sqrt{b}}\right)}{\sqrt{b}d} + \frac{\sqrt{2\pi}\,\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a-b\arccos(c+dx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{\sqrt{b}d}$$

output
```
-cos(a/b)*FresnelS(2^(1/2)/Pi^(1/2)*(a-b*arccos(d*x+c))^(1/2)/b^(1/2))*2^(
1/2)*Pi^(1/2)/d/b^(1/2)+FresnelC(2^(1/2)/Pi^(1/2)*(a-b*arccos(d*x+c))^(1/2
)/b^(1/2))*sin(a/b)*2^(1/2)*Pi^(1/2)/d/b^(1/2)
```

3.44.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.23

$$\int \frac{1}{\sqrt{a-b\arccos(c+dx)}}\,dx = \frac{e^{-\frac{ia}{b}}\left(\sqrt{-\frac{i(a-b\arccos(c+dx))}{b}}\,\Gamma\left(\frac{1}{2},-\frac{i(a-b\arccos(c+dx))}{b}\right)+e^{\frac{2ia}{b}}\sqrt{\frac{i(a-b\arccos(c+dx))}{b}}\,\Gamma\left(\frac{1}{2},\frac{i(a-b\arccos(c+dx))}{b}\right)\right)}{2d\sqrt{a-b\arccos(c+dx)}}$$

input
```
Integrate[1/Sqrt[a - b*ArcCos[c + d*x]],x]
```

output
```
(Sqrt[((-I)*(a - b*ArcCos[c + d*x]))/b]*Gamma[1/2, ((-I)*(a - b*ArcCos[c +
 d*x]))/b] + E^(((2*I)*a)/b)*Sqrt[(I*(a - b*ArcCos[c + d*x]))/b]*Gamma[1/2
, (I*(a - b*ArcCos[c + d*x]))/b])/(2*d*E^((I*a)/b)*Sqrt[a - b*ArcCos[c + d
*x]])
```

3.44.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5303, 5135, 3042, 3787, 25, 3042, 3785, 3786, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a-b\arccos(c+dx)}}dx$$

$$\Big\downarrow \text{ 5303}$$

$$\frac{\int \frac{1}{\sqrt{a-b\arccos(c+dx)}}d(c+dx)}{d}$$

$$\Big\downarrow \text{ 5135}$$

$$\frac{\int \frac{\sin\left(\frac{a}{b}-\frac{a-b\arccos(c+dx)}{b}\right)}{\sqrt{a-b\arccos(c+dx)}}d(a-b\arccos(c+dx))}{bd}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{\sin\left(\frac{a}{b}-\frac{a-b\arccos(c+dx)}{b}\right)}{\sqrt{a-b\arccos(c+dx)}}d(a-b\arccos(c+dx))}{bd}$$

$$\Big\downarrow \text{ 3787}$$

$$\frac{\sin\left(\frac{a}{b}\right)\int \frac{\cos\left(\frac{a-b\arccos(c+dx)}{b}\right)}{\sqrt{a-b\arccos(c+dx)}}d(a-b\arccos(c+dx))+\cos\left(\frac{a}{b}\right)\int -\frac{\sin\left(\frac{a-b\arccos(c+dx)}{b}\right)}{\sqrt{a-b\arccos(c+dx)}}d(a-b\arccos(c+dx))}{bd}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\sin\left(\frac{a}{b}\right)\int\frac{\cos\left(\frac{a-b\arccos(c+dx)}{b}\right)}{\sqrt{a-b\arccos(c+dx)}}d(a-b\arccos(c+dx))-\cos\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{a-b\arccos(c+dx)}{b}\right)}{\sqrt{a-b\arccos(c+dx)}}d(a-b\arccos(c+dx))}{bd}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sin\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{a-b\arccos(c+dx)}{b}+\frac{\pi}{2}\right)}{\sqrt{a-b\arccos(c+dx)}}d(a-b\arccos(c+dx))-\cos\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{a-b\arccos(c+dx)}{b}\right)}{\sqrt{a-b\arccos(c+dx)}}d(a-b\arccos(c+dx))}{bd}$$

$$\Big\downarrow \text{ 3785}$$

$$\frac{2\sin\left(\frac{a}{b}\right)\int\cos\left(\frac{a-b\arccos(c+dx)}{b}\right)d\sqrt{a-b\arccos(c+dx)}-\cos\left(\frac{a}{b}\right)\int\frac{\sin\left(\frac{a-b\arccos(c+dx)}{b}\right)}{\sqrt{a-b\arccos(c+dx)}}d(a-b\arccos(c+dx))}{bd}$$

$$\Big\downarrow \text{ 3786}$$

$$\frac{2\sin\left(\frac{a}{b}\right)\int\cos\left(\frac{a-b\arccos(c+dx)}{b}\right)d\sqrt{a-b\arccos(c+dx)}-2\cos\left(\frac{a}{b}\right)\int\sin\left(\frac{a-b\arccos(c+dx)}{b}\right)d\sqrt{a-b\arccos(c+dx)}}{bd}$$

$$\Big\downarrow \text{ 3832}$$

$$\frac{2\sin\left(\frac{a}{b}\right)\int\cos\left(\frac{a-b\arccos(c+dx)}{b}\right)d\sqrt{a-b\arccos(c+dx)}-\sqrt{2\pi}\sqrt{b}\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a-b\arccos(c+dx)}}{\sqrt{b}}\right)}{bd}$$

$$\Big\downarrow \text{ 3833}$$

$$\frac{\sqrt{2\pi}\sqrt{b}\sin\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a-b\arccos(c+dx)}}{\sqrt{b}}\right)-\sqrt{2\pi}\sqrt{b}\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a-b\arccos(c+dx)}}{\sqrt{b}}\right)}{bd}$$

input
```
Int[1/Sqrt[a - b*ArcCos[c + d*x]],x]
```

output
```
(-(Sqrt[b]*Sqrt[2*Pi]*Cos[a/b]*FresnelS[(Sqrt[2/Pi]*Sqrt[a - b*ArcCos[c +
d*x]])/Sqrt[b]]) + Sqrt[b]*Sqrt[2*Pi]*FresnelC[(Sqrt[2/Pi]*Sqrt[a - b*ArcC
os[c + d*x]])/Sqrt[b]]*Sin[a/b])/(b*d)
```

3.44. $\int\frac{1}{\sqrt{a-b\arccos(c+dx)}}\,dx$

3.44.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3787
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Cos
[(d*e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Simp[Sin[(
d*e - c*f)/d]   Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5135
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[-(b*c)^(-1)
  Subst[Int[x^n*Sin[-a/b + x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a,
 b, c, n}, x]
```

rule 5303
```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
  Subst[Int[(a + b*ArcCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
n}, x]
```

3.44. $\int \frac{1}{\sqrt{a-b\arccos(c+dx)}}\,dx$

3.44.4 Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.88

method	result	size
default	$\frac{\sqrt{2}\,\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\left(\cos\left(\frac{a}{b}\right)\mathrm{FresnelS}\left(\frac{\sqrt{2}\,\sqrt{a-b\arccos(dx+c)}}{\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,b}\right)+\sin\left(\frac{a}{b}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{a-b\arccos(dx+c)}}{\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,b}\right)\right)}{d}$	95

input
```
int(1/(a-b*arccos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2^(1/2)*Pi^(1/2)*(-1/b)^(1/2)*(cos(a/b)*FresnelS(2^(1/2)/Pi^(1/2)/(-1/b)^(
1/2)/b*(a-b*arccos(d*x+c))^(1/2))+sin(a/b)*FresnelC(2^(1/2)/Pi^(1/2)/(-1/b
)^(1/2)/b*(a-b*arccos(d*x+c))^(1/2)))/d
```

3.44.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a-b\arccos(c+dx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a-b*arccos(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.44.6 Sympy [F]

$$\int \frac{1}{\sqrt{a-b\arccos(c+dx)}}\,dx = \int \frac{1}{\sqrt{a-b\operatorname{acos}(c+dx)}}\,dx$$

input
```
integrate(1/(a-b*acos(d*x+c))**(1/2),x)
```

output
```
Integral(1/sqrt(a - b*acos(c + d*x)), x)
```

3.44.7 Maxima [F]

$$\int \frac{1}{\sqrt{a-b\arccos(c+dx)}}\,dx=\int \frac{1}{\sqrt{-b\arccos\left(dx+c\right)+a}}\,dx$$

input
```
integrate(1/(a-b*arccos(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-b*arccos(d*x + c) + a), x)
```

3.44.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.58

$$\int \frac{1}{\sqrt{a-b\arccos(c+dx)}}\,dx = \frac{i\sqrt{\pi}\,\mathrm{erf}\left(-\frac{i\sqrt{2}\sqrt{-b\arccos(dx+c)+a}}{2\sqrt{|b|}}-\frac{\sqrt{2}\sqrt{-b\arccos(dx+c)+a}\sqrt{|b|}}{2\,b}\right)e^{\left(\frac{i\,a}{b}\right)}}{d\left(\frac{i\sqrt{2}b}{\sqrt{|b|}}+\sqrt{2}\sqrt{|b|}\right)} - \frac{i\sqrt{\pi}\,\mathrm{erf}\left(\frac{i\sqrt{2}\sqrt{-b\arccos(dx+c)+a}}{2\sqrt{|b|}}-\frac{\sqrt{2}\sqrt{-b\arccos(dx+c)+a}\sqrt{|b|}}{2\,b}\right)e^{\left(-\frac{i\,a}{b}\right)}}{d\left(-\frac{i\sqrt{2}b}{\sqrt{|b|}}+\sqrt{2}\sqrt{|b|}\right)}$$

input
```
integrate(1/(a-b*arccos(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
I*sqrt(pi)*erf(-1/2*I*sqrt(2)*sqrt(-b*arccos(d*x + c) + a)/sqrt(abs(b)) -
1/2*sqrt(2)*sqrt(-b*arccos(d*x + c) + a)*sqrt(abs(b))/b)*e^(I*a/b)/(d*(I*s
qrt(2)*b/sqrt(abs(b)) + sqrt(2)*sqrt(abs(b)))) - I*sqrt(pi)*erf(1/2*I*sqrt
(2)*sqrt(-b*arccos(d*x + c) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(-b*arccos
(d*x + c) + a)*sqrt(abs(b))/b)*e^(-I*a/b)/(d*(-I*sqrt(2)*b/sqrt(abs(b)) +
sqrt(2)*sqrt(abs(b))))
```

3.44.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a-b\arccos(c+dx)}}\,dx=\int \frac{1}{\sqrt{a-b\,\mathrm{acos}\,(c+d\,x)}}\,dx$$

input
```
int(1/(a - b*acos(c + d*x))^(1/2),x)
```

output
```
int(1/(a - b*acos(c + d*x))^(1/2), x)
```

3.45 $\int \frac{\arccos(a+bx)}{\frac{ad}{b}+dx} dx$

3.45.1 Optimal result

Integrand size = 19, antiderivative size = 68

$$\int \frac{\arccos(a+bx)}{\frac{ad}{b}+dx} dx = -\frac{i\arccos(a+bx)^2}{2d} + \frac{\arccos(a+bx)\log\left(1+e^{2i\arccos(a+bx)}\right)}{d} - \frac{i\,\text{PolyLog}\left(2,-e^{2i\arccos(a+bx)}\right)}{2d}$$

output
```
-1/2*I*arccos(b*x+a)^2/d+arccos(b*x+a)*ln(1+(b*x+a+I*(1-(b*x+a)^2)^(1/2))^
2)/d-1/2*I*polylog(2,-(b*x+a+I*(1-(b*x+a)^2)^(1/2))^2)/d
```

3.45.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.87

$$\int \frac{\arccos(a+bx)}{\frac{ad}{b}+dx} dx = -\frac{i\left(\arccos(a+bx)\left(\arccos(a+bx)+2i\log\left(1+e^{2i\arccos(a+bx)}\right)\right)+\text{PolyLog}\left(2,-e^{2i\arccos(a+bx)}\right)\right)}{2d}$$

input
```
Integrate[ArcCos[a + b*x]/((a*d)/b + d*x),x]
```

output
```
((-1/2*I)*(ArcCos[a + b*x]*(ArcCos[a + b*x] + (2*I)*Log[1 + E^((2*I)*ArcCo
s[a + b*x])]) + PolyLog[2, -E^((2*I)*ArcCos[a + b*x])]))/d
```

3.45.3 Rubi [A] (warning: unable to verify)

Time = 0.38 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {5305, 27, 5137, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(a+bx)}{\frac{ad}{b}+dx}\,dx$$

$$\Big\downarrow \; 5305$$

$$\frac{\int \frac{b\arccos(a+bx)}{d(a+bx)}d(a+bx)}{b}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\arccos(a+bx)}{a+bx}d(a+bx)}{d}$$

$$\Big\downarrow \; 5137$$

$$-\frac{\int \frac{\sqrt{1-(a+bx)^2}\arccos(a+bx)}{a+bx}d\arccos(a+bx)}{d}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\int \arccos(a+bx)\tan(\arccos(a+bx))d\arccos(a+bx)}{d}$$

$$\Big\downarrow \; 4202$$

$$-\frac{\frac{1}{2}i\arccos(a+bx)^2-2i\int \frac{e^{2i\arccos(a+bx)}\arccos(a+bx)}{1+e^{2i\arccos(a+bx)}}d\arccos(a+bx)}{d}$$

$$\Big\downarrow \; 2620$$

$$-\frac{\frac{1}{2}i\arccos(a+bx)^2-2i\left(\frac{1}{2}i\int \log\left(1+e^{2i\arccos(a+bx)}\right)d\arccos(a+bx)-\frac{1}{2}i\arccos(a+bx)\log\left(1+e^{2i\arccos(a+bx)}\right.\right.}{d}$$

$$\Big\downarrow \; 2715$$

$$-\frac{\frac{1}{2}i\arccos(a+bx)^2-2i\left(\frac{1}{4}\int e^{-2i\arccos(a+bx)}\log\left(1+e^{2i\arccos(a+bx)}\right)de^{2i\arccos(a+bx)}-\frac{1}{2}i\arccos(a+bx)\log\left(1+e\right.\right.}{d}$$

$$\Big\downarrow \; 2838$$

$$-\frac{\frac{1}{2}i\arccos(a+bx)^2-2i\left(-\frac{1}{4}\operatorname{PolyLog}(2,-a-bx)-\frac{1}{2}i\arccos(a+bx)\log\left(1+e^{2i\arccos(a+bx)}\right)\right)}{d}$$

input
```
Int[ArcCos[a + b*x]/((a*d)/b + d*x),x]
```

output
```
-(((I/2)*ArcCos[a + b*x]^2 - (2*I)*((-1/2*I)*ArcCos[a + b*x]*Log[1 + E^((2
*I)*ArcCos[a + b*x])] - PolyLog[2, -a - b*x]/4))/d)
```

3.45.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5137
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

rule 5305
```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]
```

3.45.4 Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.35

method	result	size
derivativedivides	$\frac{-\frac{ib \arccos(bx+a)^2}{2d}+\frac{b \arccos(bx+a) \ln\left(1+\left(bx+a+i\sqrt{1-(bx+a)^2}\right)^2\right)}{d}-\frac{ib\,\mathrm{polylog}\left(2,-\left(bx+a+i\sqrt{1-(bx+a)^2}\right)^2\right)}{2d}}{b}$	92
default	$\frac{-\frac{ib \arccos(bx+a)^2}{2d}+\frac{b \arccos(bx+a) \ln\left(1+\left(bx+a+i\sqrt{1-(bx+a)^2}\right)^2\right)}{d}-\frac{ib\,\mathrm{polylog}\left(2,-\left(bx+a+i\sqrt{1-(bx+a)^2}\right)^2\right)}{2d}}{b}$	92

input
```
int(arccos(b*x+a)/(a*d/b+d*x),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/2*I*b/d*arccos(b*x+a)^2+b/d*arccos(b*x+a)*ln(1+(b*x+a+I*(1-(b*x+a)
^2)^(1/2))^2)-1/2*I*b/d*polylog(2,-(b*x+a+I*(1-(b*x+a)^2)^(1/2))^2))
```

3.45.5 Fricas [F]

$$\int \frac{\arccos(a+bx)}{\frac{ad}{b}+dx}\,dx=\int \frac{\arccos(bx+a)}{dx+\frac{ad}{b}}\,dx$$

input
```
integrate(arccos(b*x+a)/(a*d/b+d*x),x, algorithm="fricas")
```

output
```
integral(b*arccos(b*x + a)/(b*d*x + a*d), x)
```

3.45.6 Sympy [F]

$$\int \frac{\arccos(a+bx)}{\frac{ad}{b}+dx}\,dx = \frac{b\int \frac{\operatorname{acos}(a+bx)}{a+bx}\,dx}{d}$$

input
```
integrate(acos(b*x+a)/(a*d/b+d*x),x)
```

output
```
b*Integral(acos(a + b*x)/(a + b*x), x)/d
```

3.45.7 Maxima [F]

$$\int \frac{\arccos(a+bx)}{\frac{ad}{b}+dx}\,dx = \int \frac{\arccos(bx+a)}{dx+\frac{ad}{b}}\,dx$$

input
```
integrate(arccos(b*x+a)/(a*d/b+d*x),x, algorithm="maxima")
```

output
```
integrate(arccos(b*x + a)/(d*x + a*d/b), x)
```

3.45.8 Giac [F]

$$\int \frac{\arccos(a+bx)}{\frac{ad}{b}+dx}\,dx = \int \frac{\arccos(bx+a)}{dx+\frac{ad}{b}}\,dx$$

input
```
integrate(arccos(b*x+a)/(a*d/b+d*x),x, algorithm="giac")
```

output
```
integrate(arccos(b*x + a)/(d*x + a*d/b), x)
```

3.45.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(a+bx)}{\frac{ad}{b}+dx}\,dx=\int \frac{\mathrm{acos}(a+b\,x)}{d\,x+\frac{a\,d}{b}}\,dx$$

input
```
int(acos(a + b*x)/(d*x + (a*d)/b),x)
```

output
```
int(acos(a + b*x)/(d*x + (a*d)/b), x)
```

3.46 $\int \sqrt{1-x^2} \arccos(x) \, dx$

3.46.1 Optimal result

Integrand size = 14, antiderivative size = 34

$$\int \sqrt{1-x^2} \arccos(x) \, dx = \frac{x^2}{4} + \frac{1}{2} x \sqrt{1-x^2} \arccos(x) - \frac{\arccos(x)^2}{4}$$

output
```
1/4*x^2-1/4*arccos(x)^2+1/2*x*arccos(x)*(-x^2+1)^(1/2)
```

3.46.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \sqrt{1-x^2} \arccos(x) \, dx = \frac{1}{4} \left(x^2 + 2x\sqrt{1-x^2} \arccos(x) - \arccos(x)^2\right)$$

input
```
Integrate[Sqrt[1 - x^2]*ArcCos[x],x]
```

output
```
(x^2 + 2*x*Sqrt[1 - x^2]*ArcCos[x] - ArcCos[x]^2)/4
```

3.46.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5157, 15, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{1-x^2} \arccos(x) \, dx$$

$$\Big\downarrow \; 5157$$

$$\frac{1}{2} \int \frac{\arccos(x)}{\sqrt{1-x^2}} dx + \frac{\int x dx}{2} + \frac{1}{2} x \sqrt{1-x^2} \arccos(x)$$

$$\Big\downarrow \; 15$$

$$\frac{1}{2} \int \frac{\arccos(x)}{\sqrt{1-x^2}} dx + \frac{1}{2} \sqrt{1-x^2} x \arccos(x) + \frac{x^2}{4}$$

$$\Big\downarrow \; 5153$$

$$\frac{1}{2} \sqrt{1-x^2} x \arccos(x) - \frac{\arccos(x)^2}{4} + \frac{x^2}{4}$$

input
```
Int[Sqrt[1 - x^2]*ArcCos[x],x]
```

output
```
x^2/4 + (x*Sqrt[1 - x^2]*ArcCos[x])/2 - ArcCos[x]^2/4
```

3.46.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5157
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[x*Sqrt[d + e*x^2]*((a + b*ArcCos[c*x])^n/2), x] + (Simp[(1/2
)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2]]   Int[(a + b*ArcCos[c*x])^n/Sqrt[
1 - c^2*x^2], x], x] + Simp[b*c*(n/2)*Simp[Sqrt[d + e*x^2]/Sqrt[1 - c^2*x^2
]]   Int[x*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x
] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
```

3.46.4 Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

method	result	size
default	$-\frac{\arccos(x)\left(-\sqrt{-x^2+1}\,x+\arccos(x)\right)}{2}+\frac{\arccos(x)^2}{4}+\frac{x^2}{4}-\frac{1}{4}$	33

input
```
int(arccos(x)*(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*arccos(x)*(-(-x^2+1)^(1/2)*x+arccos(x))+1/4*arccos(x)^2+1/4*x^2-1/4
```

3.46.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \sqrt{1-x^2}\arccos(x)\,dx=\frac{1}{2}\sqrt{-x^2+1}x\arccos{(x)}+\frac{1}{4}x^2-\frac{1}{4}\arccos{(x)}^2$$

input
```
integrate(arccos(x)*(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(-x^2 + 1)*x*arccos(x) + 1/4*x^2 - 1/4*arccos(x)^2
```

3.46.6 Sympy [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \sqrt{1-x^2}\arccos(x)\,dx = \frac{x^2}{4} + \left(\frac{x\sqrt{1-x^2}}{2} + \frac{\operatorname{asin}(x)}{2}\right)\operatorname{acos}(x) + \frac{\operatorname{asin}^2(x)}{4}$$

input
```
integrate(acos(x)*(-x**2+1)**(1/2),x)
```

output
```
x**2/4 + (x*sqrt(1 - x**2)/2 + asin(x)/2)*acos(x) + asin(x)**2/4
```

3.46.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \sqrt{1-x^2}\arccos(x)\,dx = \frac{1}{4}x^2 + \frac{1}{2}\left(\sqrt{-x^2+1}x + \arcsin(x)\right)\arccos(x) + \frac{1}{4}\arcsin(x)^2$$

input
```
integrate(arccos(x)*(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
1/4*x^2 + 1/2*(sqrt(-x^2 + 1)*x + arcsin(x))*arccos(x) + 1/4*arcsin(x)^2
```

3.46.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int \sqrt{1-x^2}\arccos(x)\,dx = \frac{1}{2}\sqrt{-x^2+1}x\arccos(x) + \frac{1}{4}x^2 - \frac{1}{4}\arccos(x)^2 - \frac{1}{8}$$

input
```
integrate(arccos(x)*(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(-x^2 + 1)*x*arccos(x) + 1/4*x^2 - 1/4*arccos(x)^2 - 1/8
```

3.46.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{1-x^2} \arccos(x)\,dx = \int \operatorname{acos}(x)\,\sqrt{1-x^2}\,dx$$

input
```
int(acos(x)*(1 - x^2)^(1/2),x)
```

output
```
int(acos(x)*(1 - x^2)^(1/2), x)
```

3.47 $\int x^3 \arccos(ax^2)\,dx$

3.47.1 Optimal result

Integrand size = 10, antiderivative size = 51

$$\int x^3 \arccos(ax^2)\,dx = -\frac{x^2\sqrt{1-a^2x^4}}{8a} + \frac{1}{4}x^4 \arccos(ax^2) + \frac{\arcsin(ax^2)}{8a^2}$$

output
```
1/4*x^4*arccos(a*x^2)+1/8*arcsin(a*x^2)/a^2-1/8*x^2*(-a^2*x^4+1)^(1/2)/a
```

3.47.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

$$\int x^3 \arccos(ax^2)\,dx = \frac{-ax^2\sqrt{1-a^2x^4} + 2a^2x^4 \arccos(ax^2) + \arcsin(ax^2)}{8a^2}$$

input
```
Integrate[x^3*ArcCos[a*x^2],x]
```

output
```
(-(a*x^2*Sqrt[1 - a^2*x^4]) + 2*a^2*x^4*ArcCos[a*x^2] + ArcSin[a*x^2])/(8*
a^2)
```

3.47.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5342, 27, 807, 262, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \arccos\left(ax^2\right) \, dx$$

$$\Big\downarrow \text{5342}$$

$$\frac{1}{4} \int \frac{2ax^5}{\sqrt{1-a^2x^4}}dx + \frac{1}{4}x^4 \arccos\left(ax^2\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}a \int \frac{x^5}{\sqrt{1-a^2x^4}}dx + \frac{1}{4}x^4 \arccos\left(ax^2\right)$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{4}a \int \frac{x^4}{\sqrt{1-a^2x^4}}dx^2 + \frac{1}{4}x^4 \arccos\left(ax^2\right)$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{4}a\left(\frac{\int \frac{1}{\sqrt{1-a^2x^4}}dx^2}{2a^2} - \frac{x^2\sqrt{1-a^2x^4}}{2a^2}\right) + \frac{1}{4}x^4 \arccos\left(ax^2\right)$$

$$\Big\downarrow \text{223}$$

$$\frac{1}{4}a\left(\frac{\arcsin\left(ax^2\right)}{2a^3} - \frac{x^2\sqrt{1-a^2x^4}}{2a^2}\right) + \frac{1}{4}x^4 \arccos\left(ax^2\right)$$

input
```
Int[x^3*ArcCos[a*x^2],x]
```

output
```
(x^4*ArcCos[a*x^2])/4 + (a*(-1/2*(x^2*Sqrt[1 - a^2*x^4])/a^2 + ArcSin[a*x^
2]/(2*a^3)))/4
```

3.47.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 5342
```
Int[((a_.) + ArcCos[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(c + d*x)^(m + 1)*((a + b*ArcCos[u])/(d*(m + 1))), x] + Simp[b/(d*(m + 1)
)   Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 - u^2]), x], x]
, x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u,
 x]
```

3.47.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.35

method	result	size
default	$\frac{x^{4} \arccos \left(a \,x^{2}\right)}{4}+\frac{a \left(-\frac{x^{2} \sqrt{-a^{2} x^{4}+1}}{4 a^{2}}+\frac{\arctan \left(\frac{\sqrt{a^{2}}\, x^{2}}{\sqrt{-a^{2} x^{4}+1}}\right)}{4 a^{2} \sqrt{a^{2}}}\right)}{2}$	69
parts	$\frac{x^{4} \arccos \left(a \,x^{2}\right)}{4}+\frac{a \left(-\frac{x^{2} \sqrt{-a^{2} x^{4}+1}}{4 a^{2}}+\frac{\arctan \left(\frac{\sqrt{a^{2}}\, x^{2}}{\sqrt{-a^{2} x^{4}+1}}\right)}{4 a^{2} \sqrt{a^{2}}}\right)}{2}$	69

input
```
int(x^3*arccos(a*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4*arccos(a*x^2)+1/2*a*(-1/4*x^2/a^2*(-a^2*x^4+1)^(1/2)+1/4/a^2/(a^2)
^(1/2)*arctan((a^2)^(1/2)*x^2/(-a^2*x^4+1)^(1/2)))
```

3.47.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.80

$$\int x^3 \arccos\left(ax^2\right)\, dx = -\frac{\sqrt{-a^2x^4+1}ax^2 - \left(2\, a^2x^4 - 1\right)\arccos\left(ax^2\right)}{8\, a^2}$$

input
```
integrate(x^3*arccos(a*x^2),x, algorithm="fricas")
```

output
```
-1/8*(sqrt(-a^2*x^4 + 1)*a*x^2 - (2*a^2*x^4 - 1)*arccos(a*x^2))/a^2
```

3.47.6 Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

$$\int x^3 \arccos\left(ax^2\right)\, dx = \begin{cases} \frac{x^4 \operatorname{acos}\left(ax^2\right)}{4} - \frac{x^2\sqrt{-a^2x^4+1}}{8a} - \frac{\operatorname{acos}\left(ax^2\right)}{8a^2} & \text{for}\ a \neq 0 \\ \frac{\pi x^4}{8} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*acos(a*x**2),x)
```

output
```
Piecewise((x**4*acos(a*x**2)/4 - x**2*sqrt(-a**2*x**4 + 1)/(8*a) - acos(a*
x**2)/(8*a**2), Ne(a, 0)), (pi*x**4/8, True))
```

3.47.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.55

$$\int x^3 \arccos\left(ax^2\right)\,dx = \frac{1}{4}\,x^4 \arccos\left(ax^2\right) - \frac{1}{8}\,a\left(\frac{\arctan\left(\frac{\sqrt{-a^2x^4+1}}{ax^2}\right)}{a^3} + \frac{\sqrt{-a^2x^4+1}}{\left(a^4 - \frac{(a^2x^4-1)a^2}{x^4}\right)x^2}\right)$$

input
```
integrate(x^3*arccos(a*x^2),x, algorithm="maxima")
```

output
```
1/4*x^4*arccos(a*x^2) - 1/8*a*(arctan(sqrt(-a^2*x^4 + 1)/(a*x^2))/a^3 + sq
rt(-a^2*x^4 + 1)/((a^4 - (a^2*x^4 - 1)*a^2/x^4)*x^2))
```

3.47.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

$$\int x^3 \arccos\left(ax^2\right)\,dx = \frac{2\,a^2x^4 \arccos\left(ax^2\right) - \sqrt{-a^2x^4+1}ax^2 - \arccos\left(ax^2\right)}{8\,a^2}$$

input
```
integrate(x^3*arccos(a*x^2),x, algorithm="giac")
```

output
```
1/8*(2*a^2*x^4*arccos(a*x^2) - sqrt(-a^2*x^4 + 1)*a*x^2 - arccos(a*x^2))/a
^2
```

3.47.9 Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int x^3 \arccos\left(ax^2\right)\,dx = \frac{\mathrm{acos}(a\,x^2)\,\left(2\,a^2\,x^4-1\right)}{8\,a^2} - \frac{x^2\,\sqrt{1-a^2\,x^4}}{8\,a}$$

input
```
int(x^3*acos(a*x^2),x)
```

output
```
(acos(a*x^2)*(2*a^2*x^4 - 1))/(8*a^2) - (x^2*(1 - a^2*x^4)^(1/2))/(8*a)
```

3.48 $\int x^2 \arccos(ax^2)\,dx$

3.48.1 Optimal result

Integrand size = 10, antiderivative size = 55

$$\int x^2 \arccos(ax^2)\,dx = -\frac{2x\sqrt{1-a^2x^4}}{9a} + \frac{1}{3}x^3\arccos(ax^2) + \frac{2\,\text{EllipticF}\left(\arcsin\left(\sqrt{a}x\right),-1\right)}{9a^{3/2}}$$

output
```
1/3*x^3*arccos(a*x^2)+2/9*EllipticF(x*a^(1/2),I)/a^(3/2)-2/9*x*(-a^2*x^4+1
)^(1/2)/a
```

3.48.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.15

$$\int x^2 \arccos(ax^2)\,dx = \frac{1}{9}\left(-\frac{2x\sqrt{1-a^2x^4}}{a} + 3x^3\arccos(ax^2) + \frac{2i\,\text{EllipticF}\left(i\,\text{arcsinh}\left(\sqrt{-a}x\right),-1\right)}{(-a)^{3/2}}\right)$$

input
```
Integrate[x^2*ArcCos[a*x^2],x]
```

output
```
((-2*x*Sqrt[1 - a^2*x^4])/a + 3*x^3*ArcCos[a*x^2] + ((2*I)*EllipticF[I*Arc
Sinh[Sqrt[-a]*x], -1])/(-a)^(3/2))/9
```

3.48.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5342, 27, 843, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arccos \left(ax^2\right) \, dx$$

$$\Big\downarrow \text{ 5342}$$

$$\frac{1}{3}\int \frac{2ax^4}{\sqrt{1-a^2x^4}}dx+\frac{1}{3}x^3 \arccos \left(ax^2\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2}{3}a\int \frac{x^4}{\sqrt{1-a^2x^4}}dx+\frac{1}{3}x^3 \arccos \left(ax^2\right)$$

$$\Big\downarrow \text{ 843}$$

$$\frac{2}{3}a\left(\frac{\int \frac{1}{\sqrt{1-a^2x^4}}dx}{3a^2}-\frac{x\sqrt{1-a^2x^4}}{3a^2}\right)+\frac{1}{3}x^3 \arccos \left(ax^2\right)$$

$$\Big\downarrow \text{ 762}$$

$$\frac{2}{3}a\left(\frac{\text{EllipticF}\left(\arcsin \left(\sqrt{a}x\right),-1\right)}{3a^{5/2}}-\frac{x\sqrt{1-a^2x^4}}{3a^2}\right)+\frac{1}{3}x^3 \arccos \left(ax^2\right)$$

input
```
Int[x^2*ArcCos[a*x^2],x]
```

output
```
(x^3*ArcCos[a*x^2])/3 + (2*a*(-1/3*(x*Sqrt[1 - a^2*x^4])/a^2 + EllipticF[A
rcSin[Sqrt[a]*x], -1]/(3*a^(5/2))))/3
```

3.48.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 5342
```
Int[((a_.) + ArcCos[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(c + d*x)^(m + 1)*((a + b*ArcCos[u])/(d*(m + 1))), x] + Simp[b/(d*(m + 1)
)   Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 - u^2]), x], x]
, x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u,
 x]
```

3.48.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.44

method	result	size
default	$\frac{x^{3} \arccos\left(a\,x^{2}\right)}{3}+\frac{2 a \left(-\frac{x \sqrt{-a^{2} x^{4}+1}}{3 a^{2}}+\frac{\sqrt{-a\,x^{2}+1}\,\sqrt{a\,x^{2}+1}\,\operatorname{EllipticF}\left(x \sqrt{a}, i\right)}{3 a^{\frac{5}{2}} \sqrt{-a^{2} x^{4}+1}}\right)}{3}$	79
parts	$\frac{x^{3} \arccos\left(a\,x^{2}\right)}{3}+\frac{2 a \left(-\frac{x \sqrt{-a^{2} x^{4}+1}}{3 a^{2}}+\frac{\sqrt{-a\,x^{2}+1}\,\sqrt{a\,x^{2}+1}\,\operatorname{EllipticF}\left(x \sqrt{a}, i\right)}{3 a^{\frac{5}{2}} \sqrt{-a^{2} x^{4}+1}}\right)}{3}$	79

input
```
int(x^2*arccos(a*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*arccos(a*x^2)+2/3*a*(-1/3*x/a^2*(-a^2*x^4+1)^(1/2)+1/3/a^(5/2)*(-a
*x^2+1)^(1/2)*(a*x^2+1)^(1/2)/(-a^2*x^4+1)^(1/2)*EllipticF(x*a^(1/2),I))
```

3.48.5 Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.60

$$\int x^2 \arccos\left(ax^2\right)\,dx = \frac{3\,ax^3 \arccos\left(ax^2\right) - 2\sqrt{-a^2x^4+1}x}{9\,a}$$

input
```
integrate(x^2*arccos(a*x^2),x, algorithm="fricas")
```

output
```
1/9*(3*a*x^3*arccos(a*x^2) - 2*sqrt(-a^2*x^4 + 1)*x)/a
```

3.48.6 Sympy [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87

$$\int x^2 \arccos\left(ax^2\right)\,dx = \frac{ax^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle| a^2x^4e^{2i\pi}\right)}{6\Gamma\left(\frac{9}{4}\right)} + \frac{x^3\operatorname{acos}\left(ax^2\right)}{3}$$

input
```
integrate(x**2*acos(a*x**2),x)
```

output
```
a*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), a**2*x**4*exp_polar(2*I*pi))/(
6*gamma(9/4)) + x**3*acos(a*x**2)/3
```

3.48.7 Maxima [F]

$$\int x^2 \arccos\left(ax^2\right)\,dx = \int x^2 \arccos\left(ax^2\right)\,dx$$

input
```
integrate(x^2*arccos(a*x^2),x, algorithm="maxima")
```

output
```
1/3*x^3*arctan2(sqrt(a*x^2 + 1)*sqrt(-a*x^2 + 1), a*x^2) - 2*a*integrate(1
/3*x^4*e^(1/2*log(a*x^2 + 1) + 1/2*log(-a*x^2 + 1))/(a^4*x^8 - a^2*x^4 + (
a^2*x^4 - 1)*e^(log(a*x^2 + 1) + log(-a*x^2 + 1))), x)
```

3.48.8 Giac [F]

$$\int x^2 \arccos \left(ax^2\right) \, dx = \int x^2 \arccos \left(ax^2\right) \, dx$$

input
```
integrate(x^2*arccos(a*x^2),x, algorithm="giac")
```

output
```
integrate(x^2*arccos(a*x^2), x)
```

3.48.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \arccos \left(ax^2\right) \, dx = \int x^2 \operatorname{acos}\left(a\,x^2\right) \, dx$$

input
```
int(x^2*acos(a*x^2),x)
```

output
```
int(x^2*acos(a*x^2), x)
```

3.49 $\int x \arccos(ax^2) \, dx$

3.49.1 Optimal result

Integrand size = 8, antiderivative size = 35

$$\int x \arccos(ax^2) \, dx = -\frac{\sqrt{1-a^2x^4}}{2a} + \frac{1}{2}x^2 \arccos(ax^2)$$

output
```
1/2*x^2*arccos(a*x^2)-1/2*(-a^2*x^4+1)^(1/2)/a
```

3.49.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int x \arccos(ax^2) \, dx = -\frac{\sqrt{1-a^2x^4}}{2a} + \frac{1}{2}x^2 \arccos(ax^2)$$

input
```
Integrate[x*ArcCos[a*x^2],x]
```

output
```
-1/2*Sqrt[1 - a^2*x^4]/a + (x^2*ArcCos[a*x^2])/2
```

3.49.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {7266, 5131, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arccos\left(ax^2\right) dx$$

$$\Big\downarrow \text{7266}$$

$$\frac{1}{2}\int \arccos\left(ax^2\right) dx^2$$

$$\Big\downarrow \text{5131}$$

$$\frac{1}{2}\left(a\int \frac{x^2}{\sqrt{1-a^2x^4}}dx^2 + x^2\arccos\left(ax^2\right)\right)$$

$$\Big\downarrow \text{241}$$

$$\frac{1}{2}\left(x^2\arccos\left(ax^2\right) - \frac{\sqrt{1-a^2x^4}}{a}\right)$$

input
```
Int[x*ArcCos[a*x^2],x]
```

output
```
(-(Sqrt[1 - a^2*x^4]/a) + x^2*ArcCos[a*x^2])/2
```

3.49.3.1 Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5131
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Simp[b*c*n   Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

3.49.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

method	result	size
parts	$\frac{x^2 \arccos(a\,x^2)}{2}-\frac{\sqrt{-a^2x^4+1}}{2a}$	30
derivativedivides	$\frac{a\,x^2 \arccos(a\,x^2)-\sqrt{-a^2x^4+1}}{2a}$	32
default	$\frac{a\,x^2 \arccos(a\,x^2)-\sqrt{-a^2x^4+1}}{2a}$	32

input
```
int(x*arccos(a*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*arccos(a*x^2)-1/2*(-a^2*x^4+1)^(1/2)/a
```

3.49.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int x \arccos\left(ax^2\right)\,dx = \frac{ax^2\arccos\left(ax^2\right)-\sqrt{-a^2x^4+1}}{2\,a}$$

input
```
integrate(x*arccos(a*x^2),x, algorithm="fricas")
```

output
```
1/2*(a*x^2*arccos(a*x^2) - sqrt(-a^2*x^4 + 1))/a
```

3.49.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int x \arccos(ax^2)\ dx = \begin{cases} \frac{x^2 \operatorname{acos}(ax^2)}{2} - \frac{\sqrt{-a^2x^4+1}}{2a} & \text{for } a \neq 0 \\ \frac{\pi x^2}{4} & \text{otherwise} \end{cases}$$

input
```
integrate(x*acos(a*x**2),x)
```

output
```
Piecewise((x**2*acos(a*x**2)/2 - sqrt(-a**2*x**4 + 1)/(2*a), Ne(a, 0)), (p
i*x**2/4, True))
```

3.49.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int x \arccos(ax^2)\ dx = \frac{ax^2 \arccos(ax^2) - \sqrt{-a^2x^4+1}}{2\,a}$$

input
```
integrate(x*arccos(a*x^2),x, algorithm="maxima")
```

output
```
1/2*(a*x^2*arccos(a*x^2) - sqrt(-a^2*x^4 + 1))/a
```

3.49.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int x \arccos(ax^2)\ dx = \frac{ax^2 \arccos(ax^2) - \sqrt{-a^2x^4+1}}{2\,a}$$

input
```
integrate(x*arccos(a*x^2),x, algorithm="giac")
```

output
```
1/2*(a*x^2*arccos(a*x^2) - sqrt(-a^2*x^4 + 1))/a
```

3.49.9 Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int x \arccos\left(ax^2\right)\,dx = \frac{x^2\,\mathrm{acos}(a\,x^2)}{2} - \frac{\sqrt{1-a^2\,x^4}}{2\,a}$$

input
```
int(x*acos(a*x^2),x)
```

output
```
(x^2*acos(a*x^2))/2 - (1 - a^2*x^4)^(1/2)/(2*a)
```

3.50 $\int \arccos(ax^2)\,dx$

3.50.1 Optimal result

Integrand size = 6, antiderivative size = 43

$$\int \arccos(ax^2)\,dx = x\arccos(ax^2) + \frac{2E\left(\arcsin\left(\sqrt{a}x\right)\middle|-1\right)}{\sqrt{a}} - \frac{2\,\text{EllipticF}\left(\arcsin\left(\sqrt{a}x\right),-1\right)}{\sqrt{a}}$$

output
```
x*arccos(a*x^2)+2*EllipticE(x*a^(1/2),I)/a^(1/2)-2*EllipticF(x*a^(1/2),I)/
a^(1/2)
```

3.50.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.79

$$\int \arccos(ax^2)\,dx = x\arccos(ax^2) + \frac{2}{3}ax^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},a^2x^4\right)$$

input
```
Integrate[ArcCos[a*x^2],x]
```

output
```
x*ArcCos[a*x^2] + (2*a*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, a^2*x^4])/3
```

3.50.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {5340, 27, 836, 762, 1388, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arccos\left(ax^2\right) dx$$

$$\Big\downarrow \text{5340}$$

$$\int \frac{2ax^2}{\sqrt{1-a^2x^4}}dx + x\arccos\left(ax^2\right)$$

$$\Big\downarrow \text{27}$$

$$2a\int \frac{x^2}{\sqrt{1-a^2x^4}}dx + x\arccos\left(ax^2\right)$$

$$\Big\downarrow \text{836}$$

$$2a\left(\frac{\int \frac{ax^2+1}{\sqrt{1-a^2x^4}}dx}{a} - \frac{\int \frac{1}{\sqrt{1-a^2x^4}}dx}{a}\right) + x\arccos\left(ax^2\right)$$

$$\Big\downarrow \text{762}$$

$$2a\left(\frac{\int \frac{ax^2+1}{\sqrt{1-a^2x^4}}dx}{a} - \frac{\text{EllipticF}\left(\arcsin\left(\sqrt{a}x\right),-1\right)}{a^{3/2}}\right) + x\arccos\left(ax^2\right)$$

$$\Big\downarrow \text{1388}$$

$$2a\left(\frac{\int \frac{\sqrt{ax^2+1}}{\sqrt{1-ax^2}}dx}{a} - \frac{\text{EllipticF}\left(\arcsin\left(\sqrt{a}x\right),-1\right)}{a^{3/2}}\right) + x\arccos\left(ax^2\right)$$

$$\Big\downarrow \text{327}$$

$$2a\left(\frac{E\left(\arcsin\left(\sqrt{a}x\right)|-1\right)}{a^{3/2}} - \frac{\text{EllipticF}\left(\arcsin\left(\sqrt{a}x\right),-1\right)}{a^{3/2}}\right) + x\arccos\left(ax^2\right)$$

input
```
Int[ArcCos[a*x^2],x]
```

output
```
x*ArcCos[a*x^2] + 2*a*(EllipticE[ArcSin[Sqrt[a]*x], -1]/a^(3/2) - Elliptic
F[ArcSin[Sqrt[a]*x], -1]/a^(3/2))
```

3.50.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

rule 5340
```
Int[ArcCos[u_], x_Symbol] :> Simp[x*ArcCos[u], x] + Int[SimplifyIntegrand[x
*(D[u, x]/Sqrt[1 - u^2]), x], x] /; InverseFunctionFreeQ[u, x] &&  !Functio
nOfExponentialQ[u, x]
```

3.50.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.51

method	result	size
default	$x \arccos\left(a\,x^{2}\right)-\frac{2 \sqrt{-a\,x^{2}+1}\,\sqrt{a\,x^{2}+1}\,\left(\operatorname{EllipticF}\left(x \sqrt{a}, i\right)-\operatorname{EllipticE}\left(x \sqrt{a}, i\right)\right)}{\sqrt{a}\,\sqrt{-a^{2} x^{4}+1}}$	65
parts	$x \arccos\left(a\,x^{2}\right)-\frac{2 \sqrt{-a\,x^{2}+1}\,\sqrt{a\,x^{2}+1}\,\left(\operatorname{EllipticF}\left(x \sqrt{a}, i\right)-\operatorname{EllipticE}\left(x \sqrt{a}, i\right)\right)}{\sqrt{a}\,\sqrt{-a^{2} x^{4}+1}}$	65

input
```
int(arccos(a*x^2),x,method=_RETURNVERBOSE)
```

output
```
x*arccos(a*x^2)-2/a^(1/2)*(-a*x^2+1)^(1/2)*(a*x^2+1)^(1/2)/(-a^2*x^4+1)^(1
/2)*(EllipticF(x*a^(1/2),I)-EllipticE(x*a^(1/2),I))
```

3.50.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

$$\int \arccos \left(ax^2\right) \, dx = \frac{ax^2 \arccos \left(ax^2\right) - 2\sqrt{-a^2x^4+1}}{ax}$$

input
```
integrate(arccos(a*x^2),x, algorithm="fricas")
```

output
```
(a*x^2*arccos(a*x^2) - 2*sqrt(-a^2*x^4 + 1))/(a*x)
```

3.50.6 Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02

$$\int \arccos \left(ax^2\right) \, dx = \frac{ax^3\Gamma\left(\frac{3}{4}\right) {}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{3}{4} \\ \frac{7}{4} \end{matrix}\middle| a^2x^4e^{2i\pi}\right)}{2\Gamma\left(\frac{7}{4}\right)} + x \operatorname{acos}\left(ax^2\right)$$

input
```
integrate(acos(a*x**2),x)
```

output
```
a*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), a**2*x**4*exp_polar(2*I*pi))/(
2*gamma(7/4)) + x*acos(a*x**2)
```

3.50.7 Maxima [F]

$$\int \arccos\left(ax^2\right)\,dx = \int \arccos\left(ax^2\right)\,dx$$

input
```
integrate(arccos(a*x^2),x, algorithm="maxima")
```

output
```
x*arctan2(sqrt(a*x^2 + 1)*sqrt(-a*x^2 + 1), a*x^2) - 2*a*integrate(x^2*e^(
1/2*log(a*x^2 + 1) + 1/2*log(-a*x^2 + 1))/(a^4*x^8 - a^2*x^4 + (a^2*x^4 -
1)*e^(log(a*x^2 + 1) + log(-a*x^2 + 1))), x)
```

3.50.8 Giac [F]

$$\int \arccos\left(ax^2\right)\,dx = \int \arccos\left(ax^2\right)\,dx$$

input
```
integrate(arccos(a*x^2),x, algorithm="giac")
```

output
```
integrate(arccos(a*x^2), x)
```

3.50.9 Mupad [F(-1)]

Timed out.

$$\int \arccos\left(ax^2\right)\,dx = \int \mathrm{acos}\left(a\,x^2\right)\,dx$$

input
```
int(acos(a*x^2),x)
```

output
```
int(acos(a*x^2), x)
```

3.51 $\int \frac{\arccos(ax^2)}{x} \, dx$

3.51.1 Optimal result

Integrand size = 10, antiderivative size = 62

$$\int \frac{\arccos(ax^2)}{x} \, dx = -\frac{1}{4} i \arccos(ax^2)^2 + \frac{1}{2} \arccos(ax^2) \log\left(1 + e^{2i \arccos(ax^2)}\right) - \frac{1}{4} i \operatorname{PolyLog}\left(2, -e^{2i \arccos(ax^2)}\right)$$

output
```
-1/4*I*arccos(a*x^2)^2+1/2*arccos(a*x^2)*ln(1+(a*x^2+I*(-a^2*x^4+1)^(1/2))
^2)-1/4*I*polylog(2,-(a*x^2+I*(-a^2*x^4+1)^(1/2))^2)
```

3.51.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.90

$$\int \frac{\arccos(ax^2)}{x} \, dx = -\frac{1}{4} i \left(\arccos(ax^2) \left(\arccos(ax^2) + 2i \log\left(1 + e^{2i \arccos(ax^2)}\right)\right) + \operatorname{PolyLog}\left(2, -e^{2i \arccos(ax^2)}\right)\right)$$

input
```
Integrate[ArcCos[a*x^2]/x,x]
```

output
```
(-1/4*I)*(ArcCos[a*x^2]*(ArcCos[a*x^2] + (2*I)*Log[1 + E^((2*I)*ArcCos[a*x
^2])]) + PolyLog[2, -E^((2*I)*ArcCos[a*x^2])])
```

3.51.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5330, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax^2)}{x}dx$$

$$\Big\downarrow \text{5330}$$

$$-\frac{1}{2}\int \frac{\sqrt{1-a^2x^4}\arccos(ax^2)}{ax^2}d\arccos(ax^2)$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{2}\int \arccos(ax^2)\tan(\arccos(ax^2))\,d\arccos(ax^2)$$

$$\Big\downarrow \text{4202}$$

$$\frac{1}{2}\left(2i\int \frac{e^{2i\arccos(ax^2)}\arccos(ax^2)}{1+e^{2i\arccos(ax^2)}}d\arccos(ax^2)-\frac{1}{2}i\arccos(ax^2)^2\right)$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{2}\left(2i\left(\frac{1}{2}i\int \log\left(1+e^{2i\arccos(ax^2)}\right)d\arccos(ax^2)-\frac{1}{2}i\arccos(ax^2)\log\left(1+e^{2i\arccos(ax^2)}\right)\right)-\frac{1}{2}i\arccos(ax^2)^2\right)$$

$$\Big\downarrow \text{2715}$$

$$\frac{1}{2}\left(2i\left(\frac{1}{4}\int e^{-2i\arccos(ax^2)}\log\left(1+e^{2i\arccos(ax^2)}\right)de^{2i\arccos(ax^2)}-\frac{1}{2}i\arccos(ax^2)\log\left(1+e^{2i\arccos(ax^2)}\right)\right)-\frac{1}{2}i\arccos(ax^2)^2\right)$$

$$\Big\downarrow \text{2838}$$

$$\frac{1}{2}\left(2i\left(-\frac{1}{4}\text{PolyLog}\left(2,-e^{2i\arccos(ax^2)}\right)-\frac{1}{2}i\arccos(ax^2)\log\left(1+e^{2i\arccos(ax^2)}\right)\right)-\frac{1}{2}i\arccos(ax^2)^2\right)$$

input
```
Int[ArcCos[a*x^2]/x,x]
```

output
```
((-1/2*I)*ArcCos[a*x^2]^2 + (2*I)*((-1/2*I)*ArcCos[a*x^2]*Log[1 + E^((2*I)
*ArcCos[a*x^2])] - PolyLog[2, -E^((2*I)*ArcCos[a*x^2])]/4))/2
```

3.51.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5330
```
Int[ArcCos[(a_.)*(x_)^(p_)]^(n_.)/(x_), x_Symbol] :> Simp[-p^(-1)   Subst[I
nt[x^n*Tan[x], x], x, ArcCos[a*x^p]], x] /; FreeQ[{a, p}, x] && IGtQ[n, 0]
```

3.51.4 Maple [F]

$$\int \frac{\arccos\left(a\,x^{2}\right)}{x}dx$$

input
```
int(arccos(a*x^2)/x,x)
```

output
```
int(arccos(a*x^2)/x,x)
```

3.51.5 Fricas [F]

$$\int \frac{\arccos(ax^2)}{x} dx = \int \frac{\arccos(ax^2)}{x} dx$$

input
```
integrate(arccos(a*x^2)/x,x, algorithm="fricas")
```

output
```
integral(arccos(a*x^2)/x, x)
```

3.51.6 Sympy [F]

$$\int \frac{\arccos(ax^2)}{x} dx = \int \frac{\operatorname{acos}(ax^2)}{x} dx$$

input
```
integrate(acos(a*x**2)/x,x)
```

output
```
Integral(acos(a*x**2)/x, x)
```

3.51.7 Maxima [F]

$$\int \frac{\arccos(ax^2)}{x} dx = \int \frac{\arccos(ax^2)}{x} dx$$

input
```
integrate(arccos(a*x^2)/x,x, algorithm="maxima")
```

output
```
integrate(arccos(a*x^2)/x, x)
```

3.51.8 Giac [F]

$$\int \frac{\arccos (ax^2)}{x} \, dx = \int \frac{\arccos (ax^2)}{x} \, dx$$

input
```
integrate(arccos(a*x^2)/x,x, algorithm="giac")
```

output
```
integrate(arccos(a*x^2)/x, x)
```

3.51.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos (ax^2)}{x} \, dx = \int \frac{\mathrm{acos}(a\,x^2)}{x} \, dx$$

input
```
int(acos(a*x^2)/x,x)
```

output
```
int(acos(a*x^2)/x, x)
```

3.52 $\int \frac{\arccos(ax^2)}{x^2} dx$

3.52.1 Optimal result

Integrand size = 10, antiderivative size = 29

$$\int \frac{\arccos(ax^2)}{x^2} dx = -\frac{\arccos(ax^2)}{x} - 2\sqrt{a}\,\text{EllipticF}\left(\arcsin\left(\sqrt{a}x\right), -1\right)$$

output
```
-arccos(a*x^2)/x-2*EllipticF(x*a^(1/2),I)*a^(1/2)
```

3.52.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.38

$$\int \frac{\arccos(ax^2)}{x^2} dx = -\frac{\arccos(ax^2) + 2i\sqrt{-a}x\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{-a}x\right), -1\right)}{x}$$

input
```
Integrate[ArcCos[a*x^2]/x^2,x]
```

output
```
-((ArcCos[a*x^2] + (2*I)*Sqrt[-a]*x*EllipticF[I*ArcSinh[Sqrt[-a]*x], -1])/
x)
```

3.52.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5342, 27, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax^2)}{x^2}\,dx$$

$$\Big\downarrow \text{ 5342}$$

$$-\int \frac{2a}{\sqrt{1-a^2x^4}}dx-\frac{\arccos(ax^2)}{x}$$

$$\Big\downarrow \text{ 27}$$

$$-2a\int \frac{1}{\sqrt{1-a^2x^4}}dx-\frac{\arccos(ax^2)}{x}$$

$$\Big\downarrow \text{ 762}$$

$$-\frac{\arccos(ax^2)}{x}-2\sqrt{a}\,\text{EllipticF}\left(\arcsin\left(\sqrt{a}x\right),-1\right)$$

input
```
Int[ArcCos[a*x^2]/x^2,x]
```

output
```
-(ArcCos[a*x^2]/x) - 2*Sqrt[a]*EllipticF[ArcSin[Sqrt[a]*x], -1]
```

3.52.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 5342
```
Int[((a_.) + ArcCos[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(c + d*x)^(m + 1)*((a + b*ArcCos[u])/(d*(m + 1))), x] + Simp[b/(d*(m + 1)
)   Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 - u^2]), x], x]
, x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u,
 x]
```

3.52.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(25) = 50.

Time = 0.51 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.97

method	result	size
default	$-\frac{\arccos(a\,x^2)}{x}-\frac{2\sqrt{a}\,\sqrt{-a\,x^2+1}\,\sqrt{a\,x^2+1}\,\operatorname{EllipticF}(x\sqrt{a},i)}{\sqrt{-a^2x^4+1}}$	57
parts	$-\frac{\arccos(a\,x^2)}{x}-\frac{2\sqrt{a}\,\sqrt{-a\,x^2+1}\,\sqrt{a\,x^2+1}\,\operatorname{EllipticF}(x\sqrt{a},i)}{\sqrt{-a^2x^4+1}}$	57

input
```
int(arccos(a*x^2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-arccos(a*x^2)/x-2*a^(1/2)*(-a*x^2+1)^(1/2)*(a*x^2+1)^(1/2)/(-a^2*x^4+1)^(
1/2)*EllipticF(x*a^(1/2),I)
```

3.52.5 Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.38

$$\int \frac{\arccos(ax^2)}{x^2}\,dx = -\frac{\arccos(ax^2)}{x}$$

input
```
integrate(arccos(a*x^2)/x^2,x, algorithm="fricas")
```

output
```
-arccos(a*x^2)/x
```

3.52.6 Sympy [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.52

$$\int \frac{\arccos(ax^2)}{x^2}\,dx = -\frac{ax\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|a^2x^4e^{2i\pi}\right)}{2\Gamma\left(\frac{5}{4}\right)} - \frac{\operatorname{acos}(ax^2)}{x}$$

input
```
integrate(acos(a*x**2)/x**2,x)
```

output
```
-a*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), a**2*x**4*exp_polar(2*I*pi))/(2*
gamma(5/4)) - acos(a*x**2)/x
```

3.52.7 Maxima [F]

$$\int \frac{\arccos(ax^2)}{x^2}\,dx = \int \frac{\arccos(ax^2)}{x^2}\,dx$$

input
```
integrate(arccos(a*x^2)/x^2,x, algorithm="maxima")
```

output
```
(2*a*x*integrate(e^(1/2*log(a*x^2 + 1) + 1/2*log(-a*x^2 + 1))/(a^4*x^8 - a
^2*x^4 + (a^2*x^4 - 1)*e^(log(a*x^2 + 1) + log(-a*x^2 + 1))), x) - arctan2
(sqrt(a*x^2 + 1)*sqrt(-a*x^2 + 1), a*x^2))/x
```

3.52.8 Giac [F]

$$\int \frac{\arccos(ax^2)}{x^2}\,dx = \int \frac{\arccos(ax^2)}{x^2}\,dx$$

input
```
integrate(arccos(a*x^2)/x^2,x, algorithm="giac")
```

output
```
integrate(arccos(a*x^2)/x^2, x)
```

3.52.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos (ax^2)}{x^2} \, dx = \int \frac{\mathrm{acos}(a\,x^2)}{x^2} \, dx$$

input
```
int(acos(a*x^2)/x^2,x)
```

output
```
int(acos(a*x^2)/x^2, x)
```

3.53 $\int x^2 \arccos\left(\frac{a}{x}\right) dx$

3.53.1 Optimal result

Integrand size = 10, antiderivative size = 58

$$\int x^2 \arccos\left(\frac{a}{x}\right) dx = -\frac{1}{6}a\sqrt{1-\frac{a^2}{x^2}}x^2 + \frac{1}{3}x^3\sec^{-1}\left(\frac{x}{a}\right) - \frac{1}{6}a^3\text{arctanh}\left(\sqrt{1-\frac{a^2}{x^2}}\right)$$

output
```
1/3*x^3*arcsec(x/a)-1/6*a^3*arctanh((1-a^2/x^2)^(1/2))-1/6*a*x^2*(1-a^2/x^
2)^(1/2)
```

3.53.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.05

$$\int x^2 \arccos\left(\frac{a}{x}\right) dx = \frac{1}{3}x^3\arccos\left(\frac{a}{x}\right) - \frac{1}{6}a\left(\sqrt{1-\frac{a^2}{x^2}}x^2 + a^2\log\left(\left(1+\sqrt{1-\frac{a^2}{x^2}}\right)x\right)\right)$$

input
```
Integrate[x^2*ArcCos[a/x],x]
```

output
```
(x^3*ArcCos[a/x])/3 - (a*(Sqrt[1 - a^2/x^2]*x^2 + a^2*Log[(1 + Sqrt[1 - a^
2/x^2])*x]))/6
```

3.53.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5332, 5743, 798, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arccos\left(\frac{a}{x}\right) dx$$

$$\Big\downarrow \text{5332}$$

$$\int x^2 \sec^{-1}\left(\frac{x}{a}\right) dx$$

$$\Big\downarrow \text{5743}$$

$$\frac{1}{3}x^3 \sec^{-1}\left(\frac{x}{a}\right) - \frac{1}{3}a \int \frac{x}{\sqrt{1-\frac{a^2}{x^2}}}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{6}a \int \frac{x^4}{\sqrt{1-\frac{a^2}{x^2}}}d\frac{1}{x^2} + \frac{1}{3}x^3 \sec^{-1}\left(\frac{x}{a}\right)$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{6}a\left(\frac{1}{2}a^2 \int \frac{x^2}{\sqrt{1-\frac{a^2}{x^2}}}d\frac{1}{x^2} - x^2\sqrt{1-\frac{a^2}{x^2}}\right) + \frac{1}{3}x^3 \sec^{-1}\left(\frac{x}{a}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{6}a\left(x^2\left(-\sqrt{1-\frac{a^2}{x^2}}\right) - \int \frac{1}{\frac{1}{a^2}-\frac{1}{a^2x^4}}d\sqrt{1-\frac{a^2}{x^2}}\right) + \frac{1}{3}x^3 \sec^{-1}\left(\frac{x}{a}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{6}a\left(a^2\left(-\text{arctanh}\left(\sqrt{1-\frac{a^2}{x^2}}\right)\right) - x^2\sqrt{1-\frac{a^2}{x^2}}\right) + \frac{1}{3}x^3 \sec^{-1}\left(\frac{x}{a}\right)$$

input
```
Int[x^2*ArcCos[a/x],x]
```

output
```
(x^3*ArcSec[x/a])/3 + (a*(-(Sqrt[1 - a^2/x^2]*x^2) - a^2*ArcTanh[Sqrt[1 -
a^2/x^2]]))/6
```

3.53.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 5332
```
Int[ArcCos[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int[
u*ArcSec[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

rule 5743
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcSec[c*x])/(d*(m + 1))), x] - Simp[b*(d/(c*(m + 1
)))   Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

3.53.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97

method	result	size
derivativedivides	$-a^3\left(-\frac{x^3 \arccos\left(\frac{a}{x}\right)}{3a^3}+\frac{x^2\sqrt{1-\frac{a^2}{x^2}}}{6a^2}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{1-\frac{a^2}{x^2}}}\right)}{6}\right)$	56
default	$-a^3\left(-\frac{x^3 \arccos\left(\frac{a}{x}\right)}{3a^3}+\frac{x^2\sqrt{1-\frac{a^2}{x^2}}}{6a^2}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{1-\frac{a^2}{x^2}}}\right)}{6}\right)$	56
parts	$\frac{x^3 \arccos\left(\frac{a}{x}\right)}{3}-\frac{a\sqrt{-a^2+x^2}\left(a^2\ln\left(x+\sqrt{-a^2+x^2}\right)+x\sqrt{-a^2+x^2}\right)}{6\sqrt{-\frac{a^2-x^2}{x^2}}\,x}$	78

input

```
int(x^2*arccos(a/x),x,method=_RETURNVERBOSE)
```

output

```
-a^3*(-1/3/a^3*x^3*arccos(a/x)+1/6/a^2*x^2*(1-a^2/x^2)^(1/2)+1/6*arctanh(1
/(1-a^2/x^2)^(1/2)))
```

3.53.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.60

$$\int x^2 \arccos\left(\frac{a}{x}\right)\,dx = \frac{1}{6}\,a^3\log\left(x\sqrt{-\frac{a^2-x^2}{x^2}}-x\right)-\frac{1}{6}\,ax^2\sqrt{-\frac{a^2-x^2}{x^2}} + \frac{1}{3}\left(x^3-1\right)\arccos\left(\frac{a}{x}\right)+\frac{2}{3}\arctan\left(\frac{x\sqrt{-\frac{a^2-x^2}{x^2}}-x}{a}\right)$$

input

```
integrate(x^2*arccos(a/x),x, algorithm="fricas")
```

output

```
1/6*a^3*log(x*sqrt(-(a^2 - x^2)/x^2) - x) - 1/6*a*x^2*sqrt(-(a^2 - x^2)/x^
2) + 1/3*(x^3 - 1)*arccos(a/x) + 2/3*arctan((x*sqrt(-(a^2 - x^2)/x^2) - x)
/a)
```

3.53.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.86 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.64

$$\int x^2 \arccos\left(\frac{a}{x}\right) dx = -\frac{a\left(\begin{cases} \frac{a^2 \operatorname{acosh}\left(\frac{x}{a}\right)}{2} - \frac{ax}{2\sqrt{-1+\frac{x^2}{a^2}}} + \frac{x^3}{2a\sqrt{-1+\frac{x^2}{a^2}}} & \text{for } \left|\frac{x^2}{a^2}\right| > 1 \\ -\frac{ia^2 \operatorname{asin}\left(\frac{x}{a}\right)}{2} + \frac{iax\sqrt{1-\frac{x^2}{a^2}}}{2} & \text{otherwise} \end{cases}\right)}{3} + \frac{x^3 \operatorname{acos}\left(\frac{a}{x}\right)}{3}$$

input
```
integrate(x**2*acos(a/x),x)
```

output
```
-a*Piecewise((a**2*acosh(x/a)/2 - a*x/(2*sqrt(-1 + x**2/a**2)) + x**3/(2*a
*sqrt(-1 + x**2/a**2)), Abs(x**2/a**2) > 1), (-I*a**2*asin(x/a)/2 + I*a*x*
sqrt(1 - x**2/a**2)/2, True))/3 + x**3*acos(a/x)/3
```

3.53.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.24

$$\int x^2 \arccos\left(\frac{a}{x}\right) dx = \frac{1}{3} x^3 \arccos\left(\frac{a}{x}\right) - \frac{1}{12}\left(a^2 \log\left(\sqrt{-\frac{a^2}{x^2}+1}+1\right) - a^2 \log\left(\sqrt{-\frac{a^2}{x^2}+1}-1\right) + 2x^2\sqrt{-\frac{a^2}{x^2}+1}\right)a$$

input
```
integrate(x^2*arccos(a/x),x, algorithm="maxima")
```

output
```
1/3*x^3*arccos(a/x) - 1/12*(a^2*log(sqrt(-a^2/x^2 + 1) + 1) - a^2*log(sqrt
(-a^2/x^2 + 1) - 1) + 2*x^2*sqrt(-a^2/x^2 + 1))*a
```

3.53.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.33

$$\int x^2 \arccos\left(\frac{a}{x}\right) dx = -\frac{a^4\left(\frac{2x^2\sqrt{-\frac{a^2}{x^2}+1}}{a^2} + \log\left(\sqrt{-\frac{a^2}{x^2}+1}+1\right) - \log\left(-\sqrt{-\frac{a^2}{x^2}+1}+1\right)\right) - 4ax^3 \arccos\left(\frac{a}{x}\right)}{12a}$$

input
```
integrate(x^2*arccos(a/x),x, algorithm="giac")
```

output
```
-1/12*(a^4*(2*x^2*sqrt(-a^2/x^2 + 1)/a^2 + log(sqrt(-a^2/x^2 + 1) + 1) - l
og(-sqrt(-a^2/x^2 + 1) + 1)) - 4*a*x^3*arccos(a/x))/a
```

3.53.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \arccos\left(\frac{a}{x}\right) dx = \int x^2 \operatorname{acos}\left(\frac{a}{x}\right) dx$$

input
```
int(x^2*acos(a/x),x)
```

output
```
int(x^2*acos(a/x), x)
```

3.54 $\int x \arccos\left(\frac{a}{x}\right) dx$

3.54.1 Optimal result

Integrand size = 8, antiderivative size = 34

$$\int x \arccos\left(\frac{a}{x}\right) dx = -\frac{1}{2}a\sqrt{1-\frac{a^2}{x^2}}x + \frac{1}{2}x^2 \sec^{-1}\left(\frac{x}{a}\right)$$

output
```
1/2*x^2*arcsec(x/a)-1/2*a*x*(1-a^2/x^2)^(1/2)
```

3.54.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int x \arccos\left(\frac{a}{x}\right) dx = \frac{1}{2}\left(-a\sqrt{1-\frac{a^2}{x^2}}x + x^2 \arccos\left(\frac{a}{x}\right)\right)$$

input
```
Integrate[x*ArcCos[a/x],x]
```

output
```
(-(a*Sqrt[1 - a^2/x^2]*x) + x^2*ArcCos[a/x])/2
```

3.54.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5332, 5743, 746}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arccos\left(\frac{a}{x}\right) dx$$

$$\Big\downarrow \text{5332}$$

$$\int x \sec^{-1}\left(\frac{x}{a}\right) dx$$

$$\Big\downarrow \text{5743}$$

$$\frac{1}{2}x^2 \sec^{-1}\left(\frac{x}{a}\right) - \frac{1}{2}a \int \frac{1}{\sqrt{1-\frac{a^2}{x^2}}}dx$$

$$\Big\downarrow \text{746}$$

$$\frac{1}{2}x^2 \sec^{-1}\left(\frac{x}{a}\right) - \frac{1}{2}ax\sqrt{1-\frac{a^2}{x^2}}$$

input
```
Int[x*ArcCos[a/x],x]
```

output
```
-1/2*(a*Sqrt[1 - a^2/x^2]*x) + (x^2*ArcSec[x/a])/2
```

3.54.3.1 Defintions of rubi rules used

rule 746
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

rule 5332
```
Int[ArcCos[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int[
u*ArcSec[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

rule 5743
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcSec[c*x])/(d*(m + 1))), x] - Simp[b*(d/(c*(m + 1
)))   Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

3.54.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.15

method	result	size
derivativedivides	$-a^{2}\left(-\frac{x^{2} \arccos\left(\frac{a}{x}\right)}{2 a^{2}}+\frac{x \sqrt{1-\frac{a^{2}}{x^{2}}}}{2 a}\right)$	39
default	$-a^{2}\left(-\frac{x^{2} \arccos\left(\frac{a}{x}\right)}{2 a^{2}}+\frac{x \sqrt{1-\frac{a^{2}}{x^{2}}}}{2 a}\right)$	39
parts	$\frac{x^{2} \arccos\left(\frac{a}{x}\right)}{2}+\frac{a\left(a^{2}-x^{2}\right)}{2 x \sqrt{-\frac{a^{2}-x^{2}}{x^{2}}}}$	44

input
```
int(x*arccos(a/x),x,method=_RETURNVERBOSE)
```

output
```
-a^2*(-1/2/a^2*x^2*arccos(a/x)+1/2/a*x*(1-a^2/x^2)^(1/2))
```

3.54.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int x \arccos\left(\frac{a}{x}\right) dx = \frac{1}{2} x^{2} \arccos\left(\frac{a}{x}\right) - \frac{1}{2} a x \sqrt{-\frac{a^{2}-x^{2}}{x^{2}}}$$

input
```
integrate(x*arccos(a/x),x, algorithm="fricas")
```

output
```
1/2*x^2*arccos(a/x) - 1/2*a*x*sqrt(-(a^2 - x^2)/x^2)
```

3.54.6 Sympy [A] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.41

$$\int x\arccos\left(\frac{a}{x}\right)\,dx=-\frac{a\left(\begin{cases}a\sqrt{-1+\frac{x^2}{a^2}} & \text{for } \left|\frac{x^2}{a^2}\right|>1\\ ia\sqrt{1-\frac{x^2}{a^2}} & \text{otherwise}\end{cases}\right)}{2}+\frac{x^2\operatorname{acos}\left(\frac{a}{x}\right)}{2}$$

input
```
integrate(x*acos(a/x),x)
```

output
```
-a*Piecewise((a*sqrt(-1 + x**2/a**2), Abs(x**2/a**2) > 1), (I*a*sqrt(1 - x
**2/a**2), True))/2 + x**2*acos(a/x)/2
```

3.54.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int x\arccos\left(\frac{a}{x}\right)\,dx=\frac{1}{2}\,x^2\arccos\left(\frac{a}{x}\right)-\frac{1}{2}\,ax\sqrt{-\frac{a^2}{x^2}+1}$$

input
```
integrate(x*arccos(a/x),x, algorithm="maxima")
```

output
```
1/2*x^2*arccos(a/x) - 1/2*a*x*sqrt(-a^2/x^2 + 1)
```

3.54.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 64 vs. 2(28) = 56.

Time = 0.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.88

$$\int x\arccos\left(\frac{a}{x}\right)\,dx=-\frac{a^3\left(\frac{x\left(\sqrt{-\frac{a^2}{x^2}+1}-1\right)}{a}-\frac{a}{x\left(\sqrt{-\frac{a^2}{x^2}+1}-1\right)}\right)-2\,ax^2\arccos\left(\frac{a}{x}\right)}{4\,a}$$

input
```
integrate(x*arccos(a/x),x, algorithm="giac")
```

output
```
-1/4*(a^3*(x*(sqrt(-a^2/x^2 + 1) - 1)/a - a/(x*(sqrt(-a^2/x^2 + 1) - 1)))
- 2*a*x^2*arccos(a/x))/a
```

3.54.9 Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int x \arccos\left(\frac{a}{x}\right) dx = \frac{x^2 \operatorname{acos}\left(\frac{a}{x}\right)}{2} - \frac{a\, x\, \sqrt{1 - \frac{a^2}{x^2}}}{2}$$

input
```
int(x*acos(a/x),x)
```

output
```
(x^2*acos(a/x))/2 - (a*x*(1 - a^2/x^2)^(1/2))/2
```

3.55 $\int \arccos\left(\frac{a}{x}\right) dx$

3.55.1 Optimal result

Integrand size = 6, antiderivative size = 27

$$\int \arccos\left(\frac{a}{x}\right) dx = x\sec^{-1}\left(\frac{x}{a}\right) - a\text{arctanh}\left(\sqrt{1-\frac{a^2}{x^2}}\right)$$

output
```
x*arcsec(x/a)-a*arctanh((1-a^2/x^2)^(1/2))
```

3.55.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 84 vs. 2(27) = 54.

Time = 0.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 3.11

$$\int \arccos\left(\frac{a}{x}\right) dx = x\arccos\left(\frac{a}{x}\right) - \frac{a\sqrt{-a^2+x^2}\left(-\log\left(1-\frac{x}{\sqrt{-a^2+x^2}}\right)+\log\left(1+\frac{x}{\sqrt{-a^2+x^2}}\right)\right)}{2\sqrt{1-\frac{a^2}{x^2}}x}$$

input
```
Integrate[ArcCos[a/x],x]
```

output
```
x*ArcCos[a/x] - (a*Sqrt[-a^2 + x^2]*(-Log[1 - x/Sqrt[-a^2 + x^2]] + Log[1
+ x/Sqrt[-a^2 + x^2]]))/(2*Sqrt[1 - a^2/x^2]*x)
```

3.55.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5332, 5737, 798, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arccos\left(\frac{a}{x}\right)dx$$

$$\Big\downarrow \text{5332}$$

$$\int \sec^{-1}\left(\frac{x}{a}\right)dx$$

$$\Big\downarrow \text{5737}$$

$$x\sec^{-1}\left(\frac{x}{a}\right)-a\int \frac{1}{\sqrt{1-\frac{a^2}{x^2}}x}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{2}a\int \frac{x^2}{\sqrt{1-\frac{a^2}{x^2}}}d\frac{1}{x^2}+x\sec^{-1}\left(\frac{x}{a}\right)$$

$$\Big\downarrow \text{73}$$

$$x\sec^{-1}\left(\frac{x}{a}\right)-\frac{\int \frac{1}{\frac{1}{a^2}-\frac{1}{a^2x^4}}d\sqrt{1-\frac{a^2}{x^2}}}{a}$$

$$\Big\downarrow \text{221}$$

$$x\sec^{-1}\left(\frac{x}{a}\right)-a\text{arctanh}\left(\sqrt{1-\frac{a^2}{x^2}}\right)$$

input
```
Int[ArcCos[a/x],x]
```

output
```
x*ArcSec[x/a] - a*ArcTanh[Sqrt[1 - a^2/x^2]]
```

3.55.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 5332
```
Int[ArcCos[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int[
u*ArcSec[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

rule 5737
```
Int[ArcSec[(c_.)*(x_)], x_Symbol] :> Simp[x*ArcSec[c*x], x] - Simp[1/c   In
t[1/(x*Sqrt[1 - 1/(c^2*x^2)]), x], x] /; FreeQ[c, x]
```

3.55.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

method	result	size
derivativedivides	$-a\left(-\frac{x\arccos\left(\frac{a}{x}\right)}{a}+\operatorname{arctanh}\left(\frac{1}{\sqrt{1-\frac{a^2}{x^2}}}\right)\right)$	30
default	$-a\left(-\frac{x\arccos\left(\frac{a}{x}\right)}{a}+\operatorname{arctanh}\left(\frac{1}{\sqrt{1-\frac{a^2}{x^2}}}\right)\right)$	30
parts	$x\arccos\left(\frac{a}{x}\right)-\frac{a\sqrt{-a^2+x^2}\,\ln\left(x+\sqrt{-a^2+x^2}\right)}{\sqrt{-\frac{a^2-x^2}{x^2}}\,x}$	57

input
```
int(arccos(a/x),x,method=_RETURNVERBOSE)
```

output
```
-a*(-1/a*x*arccos(a/x)+arctanh(1/(1-a^2/x^2)^(1/2)))
```

3.55.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(25) = 50.

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.41

$$\int \arccos\left(\frac{a}{x}\right)\,dx = (x-1)\arccos\left(\frac{a}{x}\right) + a\log\left(x\sqrt{-\frac{a^2-x^2}{x^2}}-x\right) + 2\arctan\left(\frac{x\sqrt{-\frac{a^2-x^2}{x^2}}-x}{a}\right)$$

input
```
integrate(arccos(a/x),x, algorithm="fricas")
```

output
```
(x - 1)*arccos(a/x) + a*log(x*sqrt(-(a^2 - x^2)/x^2) - x) + 2*arctan((x*sq
rt(-(a^2 - x^2)/x^2) - x)/a)
```

3.55.6 Sympy [A] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \arccos\left(\frac{a}{x}\right)\,dx = -a\left(\begin{cases}\operatorname{acosh}\left(\frac{x}{a}\right) & \text{for } \left|\frac{x^2}{a^2}\right|>1\\ -i\operatorname{asin}\left(\frac{x}{a}\right) & \text{otherwise}\end{cases}\right) + x\operatorname{acos}\left(\frac{a}{x}\right)$$

input
```
integrate(acos(a/x),x)
```

output
```
-a*Piecewise((acosh(x/a), Abs(x**2/a**2) > 1), (-I*asin(x/a), True)) + x*a
cos(a/x)
```

3.55.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.67

$$\int \arccos\left(\frac{a}{x}\right)\,dx = -\frac{1}{2}\,a\left(\log\left(\sqrt{-\frac{a^2}{x^2}+1}+1\right)-\log\left(\sqrt{-\frac{a^2}{x^2}+1}-1\right)\right)+x\arccos\left(\frac{a}{x}\right)$$

input
```
integrate(arccos(a/x),x, algorithm="maxima")
```

output
```
-1/2*a*(log(sqrt(-a^2/x^2 + 1) + 1) - log(sqrt(-a^2/x^2 + 1) - 1)) + x*arc
cos(a/x)
```

3.55.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(25) = 50.

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.04

$$\int \arccos\left(\frac{a}{x}\right)\,dx = -\frac{a^2\left(\log\left(\sqrt{-\frac{a^2}{x^2}+1}+1\right)-\log\left(-\sqrt{-\frac{a^2}{x^2}+1}+1\right)\right)-2\,ax\arccos\left(\frac{a}{x}\right)}{2\,a}$$

input
```
integrate(arccos(a/x),x, algorithm="giac")
```

output
```
-1/2*(a^2*(log(sqrt(-a^2/x^2 + 1) + 1) - log(-sqrt(-a^2/x^2 + 1) + 1)) - 2
*a*x*arccos(a/x))/a
```

3.55.9 Mupad [B] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

$$\int \arccos\left(\frac{a}{x}\right)\,dx = x\,\mathrm{acos}\left(\frac{a}{x}\right)-a\,\mathrm{sign}(x)\,\ln\left(x+\sqrt{x^2-a^2}\right)$$

input
```
int(acos(a/x),x)
```

output
```
x*acos(a/x) - a*sign(x)*log(x + (x^2 - a^2)^(1/2))
```

3.56 $\int \frac{\arccos\left(\frac{a}{x}\right)}{x}\,dx$

3.56.1 Optimal result

Integrand size = 10, antiderivative size = 60

$$\int \frac{\arccos\left(\frac{a}{x}\right)}{x}\,dx = \frac{1}{2}i\arccos\left(\frac{a}{x}\right)^2 - \arccos\left(\frac{a}{x}\right)\log\left(1+e^{2i\arccos\left(\frac{a}{x}\right)}\right) + \frac{1}{2}i\,\mathrm{PolyLog}\left(2,-e^{2i\arccos\left(\frac{a}{x}\right)}\right)$$

output
```
1/2*I*arccos(a/x)^2-arccos(a/x)*ln(1+(a/x+I*(1-a^2/x^2)^(1/2))^2)+1/2*I*po
lylog(2,-(a/x+I*(1-a^2/x^2)^(1/2))^2)
```

3.56.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \frac{\arccos\left(\frac{a}{x}\right)}{x}\,dx = \frac{1}{2}i\arccos\left(\frac{a}{x}\right)^2 - \arccos\left(\frac{a}{x}\right)\log\left(1+e^{2i\arccos\left(\frac{a}{x}\right)}\right) + \frac{1}{2}i\,\mathrm{PolyLog}\left(2,-e^{2i\arccos\left(\frac{a}{x}\right)}\right)$$

input
```
Integrate[ArcCos[a/x]/x,x]
```

output
```
(I/2)*ArcCos[a/x]^2 - ArcCos[a/x]*Log[1 + E^((2*I)*ArcCos[a/x])] + (I/2)*P
olyLog[2, -E^((2*I)*ArcCos[a/x])]
```

3.56.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5330, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos\left(\frac{a}{x}\right)}{x}dx$$

$$\Big\downarrow \text{5330}$$

$$\int \frac{x\sqrt{1-\frac{a^2}{x^2}}\arccos\left(\frac{a}{x}\right)}{a}d\arccos\left(\frac{a}{x}\right)$$

$$\Big\downarrow \text{3042}$$

$$\int \arccos\left(\frac{a}{x}\right)\tan\left(\arccos\left(\frac{a}{x}\right)\right)d\arccos\left(\frac{a}{x}\right)$$

$$\Big\downarrow \text{4202}$$

$$\frac{1}{2}i\arccos\left(\frac{a}{x}\right)^2-2i\int \frac{e^{2i\arccos\left(\frac{a}{x}\right)}\arccos\left(\frac{a}{x}\right)}{1+e^{2i\arccos\left(\frac{a}{x}\right)}}d\arccos\left(\frac{a}{x}\right)$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{2}i\arccos\left(\frac{a}{x}\right)^2-2i\left(\frac{1}{2}i\int \log\left(1+e^{2i\arccos\left(\frac{a}{x}\right)}\right)d\arccos\left(\frac{a}{x}\right)-\frac{1}{2}i\arccos\left(\frac{a}{x}\right)\log\left(1+e^{2i\arccos\left(\frac{a}{x}\right)}\right)\right)$$

$$\Big\downarrow \text{2715}$$

$$\frac{1}{2}i\arccos\left(\frac{a}{x}\right)^2-2i\left(\frac{1}{4}\int e^{-2i\arccos\left(\frac{a}{x}\right)}\log\left(1+e^{2i\arccos\left(\frac{a}{x}\right)}\right)de^{2i\arccos\left(\frac{a}{x}\right)}-\frac{1}{2}i\arccos\left(\frac{a}{x}\right)\log\left(1+e^{2i\arccos\left(\frac{a}{x}\right)}\right)\right)$$

$$\Big\downarrow \text{2838}$$

$$\frac{1}{2}i\arccos\left(\frac{a}{x}\right)^2-2i\left(-\frac{1}{4}\text{PolyLog}\left(2,-e^{2i\arccos\left(\frac{a}{x}\right)}\right)-\frac{1}{2}i\arccos\left(\frac{a}{x}\right)\log\left(1+e^{2i\arccos\left(\frac{a}{x}\right)}\right)\right)$$

input
```
Int[ArcCos[a/x]/x,x]
```

output
```
(I/2)*ArcCos[a/x]^2 - (2*I)*((-1/2*I)*ArcCos[a/x]*Log[1 + E^((2*I)*ArcCos[
a/x])] - PolyLog[2, -E^((2*I)*ArcCos[a/x])]/4)
```

3.56.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5330
```
Int[ArcCos[(a_.)*(x_)^(p_)]^(n_.)/(x_), x_Symbol] :> Simp[-p^(-1)   Subst[I
nt[x^n*Tan[x], x], x, ArcCos[a*x^p]], x] /; FreeQ[{a, p}, x] && IGtQ[n, 0]
```

3.56.4 Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.28

method	result	size
derivativedivides	$\frac{i \arccos\left(\frac{a}{x}\right)^{2}}{2}-\arccos\left(\frac{a}{x}\right) \ln\left(1+\left(\frac{a}{x}+i \sqrt{1-\frac{a^{2}}{x^{2}}}\right)^{2}\right)+\frac{i \operatorname{polylog}\left(2,-\left(\frac{a}{x}+i \sqrt{1-\frac{a^{2}}{x^{2}}}\right)^{2}\right)}{2}$	77
default	$\frac{i \arccos\left(\frac{a}{x}\right)^{2}}{2}-\arccos\left(\frac{a}{x}\right) \ln\left(1+\left(\frac{a}{x}+i \sqrt{1-\frac{a^{2}}{x^{2}}}\right)^{2}\right)+\frac{i \operatorname{polylog}\left(2,-\left(\frac{a}{x}+i \sqrt{1-\frac{a^{2}}{x^{2}}}\right)^{2}\right)}{2}$	77

input
```
int(arccos(a/x)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*I*arccos(a/x)^2-arccos(a/x)*ln(1+(a/x+I*(1-a^2/x^2)^(1/2))^2)+1/2*I*po
lylog(2,-(a/x+I*(1-a^2/x^2)^(1/2))^2)
```

3.56.5 Fricas [F]

$$\int \frac{\arccos\left(\frac{a}{x}\right)}{x}\,dx=\int \frac{\arccos\left(\frac{a}{x}\right)}{x}\,dx$$

input
```
integrate(arccos(a/x)/x,x, algorithm="fricas")
```

output
```
integral(arccos(a/x)/x, x)
```

3.56.6 Sympy [F]

$$\int \frac{\arccos\left(\frac{a}{x}\right)}{x}\,dx=\int \frac{\operatorname{acos}\left(\frac{a}{x}\right)}{x}\,dx$$

input
```
integrate(acos(a/x)/x,x)
```

output
```
Integral(acos(a/x)/x, x)
```

3.56.7 Maxima [F]

$$\int \frac{\arccos\left(\frac{a}{x}\right)}{x}\,dx = \int \frac{\arccos\left(\frac{a}{x}\right)}{x}\,dx$$

input
```
integrate(arccos(a/x)/x,x, algorithm="maxima")
```

output
```
-I*a^2*integrate(-log(x)/(a^2*x - x^3), x) - a*integrate(-sqrt(a + x)*sqrt
(-a + x)*log(x)/(a^2*x - x^3), x) + arctan(sqrt(a + x)*sqrt(-a + x)/a)*log
(x) - 1/2*I*log(x^2)*log(x) + 1/2*I*log(x)^2 + 1/2*I*log(x)*log((a + x)/a)
 + 1/2*I*log(x)*log((a - x)/a) + 1/2*I*dilog(x/a) + 1/2*I*dilog(-x/a)
```

3.56.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\arccos\left(\frac{a}{x}\right)}{x}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arccos(a/x)/x,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Limit: Max order reached or unable to make series expan
sion Error: Bad Argument Value
```

3.56.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos\left(\frac{a}{x}\right)}{x}\,dx = \int \frac{\mathrm{acos}\left(\frac{a}{x}\right)}{x}\,dx$$

input
```
int(acos(a/x)/x,x)
```

output
```
int(acos(a/x)/x, x)
```

3.57 $\int \frac{\arccos\left(\frac{a}{x}\right)}{x^2}\,dx$

3.57.1 Optimal result

Integrand size = 10, antiderivative size = 30

$$\int \frac{\arccos\left(\frac{a}{x}\right)}{x^2}\,dx = \frac{\sqrt{1-\frac{a^2}{x^2}}}{a} - \frac{\sec^{-1}\left(\frac{x}{a}\right)}{x}$$

output
```
-arcsec(x/a)/x+(1-a^2/x^2)^(1/2)/a
```

3.57.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\arccos\left(\frac{a}{x}\right)}{x^2}\,dx = \frac{\sqrt{1-\frac{a^2}{x^2}}}{a} - \frac{\arccos\left(\frac{a}{x}\right)}{x}$$

input
```
Integrate[ArcCos[a/x]/x^2,x]
```

output
```
Sqrt[1 - a^2/x^2]/a - ArcCos[a/x]/x
```

3.57.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5332, 5743, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos\left(\frac{a}{x}\right)}{x^2}dx$$
$$\Big\downarrow \text{5332}$$
$$\int \frac{\sec^{-1}\left(\frac{x}{a}\right)}{x^2}dx$$
$$\Big\downarrow \text{5743}$$
$$a\int \frac{1}{\sqrt{1-\frac{a^2}{x^2}}x^3}dx-\frac{\sec^{-1}\left(\frac{x}{a}\right)}{x}$$
$$\Big\downarrow \text{793}$$
$$\frac{\sqrt{1-\frac{a^2}{x^2}}}{a}-\frac{\sec^{-1}\left(\frac{x}{a}\right)}{x}$$

input
```
Int[ArcCos[a/x]/x^2,x]
```

output
```
Sqrt[1 - a^2/x^2]/a - ArcSec[x/a]/x
```

3.57.3.1 Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 5332
```
Int[ArcCos[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int[
u*ArcSec[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

rule 5743
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcSec[c*x])/(d*(m + 1))), x] - Simp[b*(d/(c*(m + 1
)))   Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

3.57.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

method	result	size
derivativedivides	$-\frac{\frac{a \arccos\left(\frac{a}{x}\right)}{x}-\sqrt{1-\frac{a^{2}}{x^{2}}}}{a}$	32
default	$-\frac{\frac{a \arccos\left(\frac{a}{x}\right)}{x}-\sqrt{1-\frac{a^{2}}{x^{2}}}}{a}$	32
parts	$-\frac{\arccos\left(\frac{a}{x}\right)}{x}-\frac{a^{2}-x^{2}}{a\sqrt{-\frac{a^{2}-x^{2}}{x^{2}}}\, x^{2}}$	46

input
```
int(arccos(a/x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/a*(a/x*arccos(a/x)-(1-a^2/x^2)^(1/2))
```

3.57.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20

$$\int \frac{\arccos\left(\frac{a}{x}\right)}{x^2}\, dx = -\frac{a \arccos\left(\frac{a}{x}\right) - x\sqrt{-\frac{a^2-x^2}{x^2}}}{ax}$$

input
```
integrate(arccos(a/x)/x^2,x, algorithm="fricas")
```

output
```
-(a*arccos(a/x) - x*sqrt(-(a^2 - x^2)/x^2))/(a*x)
```

3.57.6 Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \frac{\arccos\left(\frac{a}{x}\right)}{x^2}\,dx = \begin{cases} -\frac{\operatorname{acos}\left(\frac{a}{x}\right)}{x} + \frac{\sqrt{-\frac{a^2}{x^2}+1}}{a} & \text{for } a \neq 0 \\ -\frac{\pi}{2x} & \text{otherwise} \end{cases}$$

input
```
integrate(acos(a/x)/x**2,x)
```

output
```
Piecewise((-acos(a/x)/x + sqrt(-a**2/x**2 + 1)/a, Ne(a, 0)), (-pi/(2*x), T
rue))
```

3.57.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \frac{\arccos\left(\frac{a}{x}\right)}{x^2}\,dx = -\frac{\frac{a\arccos\left(\frac{a}{x}\right)}{x} - \sqrt{-\frac{a^2}{x^2}+1}}{a}$$

input
```
integrate(arccos(a/x)/x^2,x, algorithm="maxima")
```

output
```
-(a*arccos(a/x)/x - sqrt(-a^2/x^2 + 1))/a
```

3.57.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.03

$$\int \frac{\arccos\left(\frac{a}{x}\right)}{x^2}\,dx = -\frac{\frac{a\arccos\left(\frac{a}{x}\right)}{x} - \sqrt{-\frac{a^2}{x^2}+1}}{a}$$

input
```
integrate(arccos(a/x)/x^2,x, algorithm="giac")
```

output
```
-(a*arccos(a/x)/x - sqrt(-a^2/x^2 + 1))/a
```

3.57.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{\arccos\left(\frac{a}{x}\right)}{x^2}\,dx = \frac{\sqrt{1-\frac{a^2}{x^2}}}{a} - \frac{\mathrm{acos}\left(\frac{a}{x}\right)}{x}$$

input
```
int(acos(a/x)/x^2,x)
```

output
```
(1 - a^2/x^2)^(1/2)/a - acos(a/x)/x
```

3.58 $\int \frac{\arccos\left(\frac{a}{x}\right)}{x^3}\,dx$

3.58.1 Optimal result

Integrand size = 10, antiderivative size = 51

$$\int \frac{\arccos\left(\frac{a}{x}\right)}{x^3}\,dx=\frac{\sqrt{1-\frac{a^2}{x^2}}}{4ax}-\frac{\csc^{-1}\left(\frac{x}{a}\right)}{4a^2}-\frac{\sec^{-1}\left(\frac{x}{a}\right)}{2x^2}$$

output
```
-1/4*arccsc(x/a)/a^2-1/2*arcsec(x/a)/x^2+1/4*(1-a^2/x^2)^(1/2)/a/x
```

3.58.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.98

$$\int \frac{\arccos\left(\frac{a}{x}\right)}{x^3}\,dx=\frac{a\sqrt{1-\frac{a^2}{x^2}}x-2a^2\arccos\left(\frac{a}{x}\right)-x^2\arcsin\left(\frac{a}{x}\right)}{4a^2x^2}$$

input
```
Integrate[ArcCos[a/x]/x^3,x]
```

output
```
(a*Sqrt[1 - a^2/x^2]*x - 2*a^2*ArcCos[a/x] - x^2*ArcSin[a/x])/(4*a^2*x^2)
```

3.58.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5332, 5743, 858, 262, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos\left(\frac{a}{x}\right)}{x^3}dx$$

$$\Big\downarrow \text{5332}$$

$$\int \frac{\sec^{-1}\left(\frac{x}{a}\right)}{x^3}dx$$

$$\Big\downarrow \text{5743}$$

$$\frac{1}{2}a\int \frac{1}{\sqrt{1-\frac{a^2}{x^2}}x^4}dx-\frac{\sec^{-1}\left(\frac{x}{a}\right)}{2x^2}$$

$$\Big\downarrow \text{858}$$

$$-\frac{1}{2}a\int \frac{1}{\sqrt{1-\frac{a^2}{x^2}}x^2}d\frac{1}{x}-\frac{\sec^{-1}\left(\frac{x}{a}\right)}{2x^2}$$

$$\Big\downarrow \text{262}$$

$$-\frac{1}{2}a\left(\frac{\int \frac{1}{\sqrt{1-\frac{a^2}{x^2}}}d\frac{1}{x}}{2a^2}-\frac{\sqrt{1-\frac{a^2}{x^2}}}{2a^2x}\right)-\frac{\sec^{-1}\left(\frac{x}{a}\right)}{2x^2}$$

$$\Big\downarrow \text{223}$$

$$-\frac{1}{2}a\left(\frac{\arcsin\left(\frac{a}{x}\right)}{2a^3}-\frac{\sqrt{1-\frac{a^2}{x^2}}}{2a^2x}\right)-\frac{\sec^{-1}\left(\frac{x}{a}\right)}{2x^2}$$

input
```
Int[ArcCos[a/x]/x^3,x]
```

output
```
-1/2*ArcSec[x/a]/x^2 - (a*(-1/2*Sqrt[1 - a^2/x^2]/(a^2*x) + ArcSin[a/x]/(2
*a^3)))/2
```

3.58.3.1 Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 858
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

rule 5332
```
Int[ArcCos[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int[
u*ArcSec[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

rule 5743
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcSec[c*x])/(d*(m + 1))), x] - Simp[b*(d/(c*(m + 1
)))   Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

3.58.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$-\frac{\frac{a^{2} \arccos\left(\frac{a}{x}\right)}{2 x^{2}}-\frac{a \sqrt{1-\frac{a^{2}}{x^{2}}}}{4 x}+\frac{\arcsin\left(\frac{a}{x}\right)}{4}}{a^{2}}$	47
default	$-\frac{\frac{a^{2} \arccos\left(\frac{a}{x}\right)}{2 x^{2}}-\frac{a \sqrt{1-\frac{a^{2}}{x^{2}}}}{4 x}+\frac{\arcsin\left(\frac{a}{x}\right)}{4}}{a^{2}}$	47
parts	$-\frac{\arccos\left(\frac{a}{x}\right)}{2 x^{2}}+\frac{\sqrt{-a^{2}+x^{2}}\, \left(-\ln\left(\frac{-2 a^{2}+2 \sqrt{-a^{2}}\, \sqrt{-a^{2}+x^{2}}}{x}\right) x^{2}+\sqrt{-a^{2}}\, \sqrt{-a^{2}+x^{2}}\right)}{4 a \sqrt{-\frac{a^{2}-x^{2}}{x^{2}}}\, x^{3} \sqrt{-a^{2}}}$	111

input
```
int(arccos(a/x)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/a^2*(1/2*a^2/x^2*arccos(a/x)-1/4*a/x*(1-a^2/x^2)^(1/2)+1/4*arcsin(a/x))
```

3.58.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

$$\int \frac{\arccos\left(\frac{a}{x}\right)}{x^3}\,dx = \frac{ax\sqrt{-\frac{a^2-x^2}{x^2}} - (2\,a^2 - x^2)\arccos\left(\frac{a}{x}\right)}{4\,a^2x^2}$$

input
```
integrate(arccos(a/x)/x^3,x, algorithm="fricas")
```

output
```
1/4*(a*x*sqrt(-(a^2 - x^2)/x^2) - (2*a^2 - x^2)*arccos(a/x))/(a^2*x^2)
```

3.58.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.92 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.96

$$\int \frac{\arccos\left(\frac{a}{x}\right)}{x^3}\,dx = \frac{a\left(\begin{cases} \frac{i\sqrt{\frac{a^2}{x^2}-1}}{2a^2x} + \frac{i\operatorname{acosh}\left(\frac{a}{x}\right)}{2a^3} & \text{for}\ \left|\frac{a^2}{x^2}\right| > 1 \\ -\frac{1}{2x^3\sqrt{-\frac{a^2}{x^2}+1}} + \frac{1}{2a^2x\sqrt{-\frac{a^2}{x^2}+1}} - \frac{\operatorname{asin}\left(\frac{a}{x}\right)}{2a^3} & \text{otherwise} \end{cases}\right)}{2} - \frac{\operatorname{acos}\left(\frac{a}{x}\right)}{2x^2}$$

input
```
integrate(acos(a/x)/x**3,x)
```

output
```
a*Piecewise((I*sqrt(a**2/x**2 - 1)/(2*a**2*x) + I*acosh(a/x)/(2*a**3), Abs
(a**2/x**2) > 1), (-1/(2*x**3*sqrt(-a**2/x**2 + 1)) + 1/(2*a**2*x*sqrt(-a*
*2/x**2 + 1)) - asin(a/x)/(2*a**3), True))/2 - acos(a/x)/(2*x**2)
```

3.58.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.51

$$\int \frac{\arccos\left(\frac{a}{x}\right)}{x^3}\,dx = -\frac{1}{4}\,a\left(\frac{x\sqrt{-\frac{a^2}{x^2}+1}}{a^2x^2\left(\frac{a^2}{x^2}-1\right)-a^4} - \frac{\arctan\left(\frac{x\sqrt{-\frac{a^2}{x^2}+1}}{a}\right)}{a^3}\right) - \frac{\arccos\left(\frac{a}{x}\right)}{2\,x^2}$$

input
```
integrate(arccos(a/x)/x^3,x, algorithm="maxima")
```

output
```
-1/4*a*(x*sqrt(-a^2/x^2 + 1)/(a^2*x^2*(a^2/x^2 - 1) - a^4) - arctan(x*sqrt
(-a^2/x^2 + 1)/a)/a^3) - 1/2*arccos(a/x)/x^2
```

3.58.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.86

$$\int \frac{\arccos\left(\frac{a}{x}\right)}{x^3}\,dx = \frac{\frac{\arccos\left(\frac{a}{x}\right)}{a} - \frac{2\,a\arccos\left(\frac{a}{x}\right)}{x^2} + \frac{\sqrt{-\frac{a^2}{x^2}+1}}{x}}{4\,a}$$

input
```
integrate(arccos(a/x)/x^3,x, algorithm="giac")
```

output
```
1/4*(arccos(a/x)/a - 2*a*arccos(a/x)/x^2 + sqrt(-a^2/x^2 + 1)/x)/a
```

3.58.9 Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int \frac{\arccos\left(\frac{a}{x}\right)}{x^3}\,dx = \frac{\sqrt{1-\frac{a^2}{x^2}}}{4\,a\,x} - \frac{\mathrm{acos}\left(\frac{a}{x}\right)\,\left(\frac{2\,a^2}{x^2}-1\right)}{4\,a^2}$$

input
```
int(acos(a/x)/x^3,x)
```

output
```
(1 - a^2/x^2)^(1/2)/(4*a*x) - (acos(a/x)*((2*a^2)/x^2 - 1))/(4*a^2)
```

3.59 $\int \frac{\arccos\left(\frac{a}{x}\right)}{x^4} dx$

3.59.1 Optimal result

Integrand size = 10, antiderivative size = 56

$$\int \frac{\arccos\left(\frac{a}{x}\right)}{x^4} dx = \frac{\sqrt{1-\frac{a^2}{x^2}}}{3a^3} - \frac{\left(1-\frac{a^2}{x^2}\right)^{3/2}}{9a^3} - \frac{\sec^{-1}\left(\frac{x}{a}\right)}{3x^3}$$

output
```
-1/9*(1-a^2/x^2)^(3/2)/a^3-1/3*arcsec(x/a)/x^3+1/3*(1-a^2/x^2)^(1/2)/a^3
```

3.59.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.84

$$\int \frac{\arccos\left(\frac{a}{x}\right)}{x^4} dx = \frac{\sqrt{1-\frac{a^2}{x^2}}x(a^2+2x^2) - 3a^3\arccos\left(\frac{a}{x}\right)}{9a^3x^3}$$

input
```
Integrate[ArcCos[a/x]/x^4,x]
```

output
```
(Sqrt[1 - a^2/x^2]*x*(a^2 + 2*x^2) - 3*a^3*ArcCos[a/x])/(9*a^3*x^3)
```

3.59.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5332, 5743, 798, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos\left(\frac{a}{x}\right)}{x^4}dx$$

$$\Big\downarrow \text{5332}$$

$$\int \frac{\sec^{-1}\left(\frac{x}{a}\right)}{x^4}dx$$

$$\Big\downarrow \text{5743}$$

$$\frac{1}{3}a\int \frac{1}{\sqrt{1-\frac{a^2}{x^2}}x^5}dx-\frac{\sec^{-1}\left(\frac{x}{a}\right)}{3x^3}$$

$$\Big\downarrow \text{798}$$

$$-\frac{1}{6}a\int \frac{1}{\sqrt{1-\frac{a^2}{x^2}}x^2}d\frac{1}{x^2}-\frac{\sec^{-1}\left(\frac{x}{a}\right)}{3x^3}$$

$$\Big\downarrow \text{53}$$

$$-\frac{1}{6}a\int \left(\frac{1}{a^2\sqrt{1-\frac{a^2}{x^2}}}-\frac{\sqrt{1-\frac{a^2}{x^2}}}{a^2}\right)d\frac{1}{x^2}-\frac{\sec^{-1}\left(\frac{x}{a}\right)}{3x^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{6}a\left(\frac{2\left(1-\frac{a^2}{x^2}\right)^{3/2}}{3a^4}-\frac{2\sqrt{1-\frac{a^2}{x^2}}}{a^4}\right)-\frac{\sec^{-1}\left(\frac{x}{a}\right)}{3x^3}$$

input
```
Int[ArcCos[a/x]/x^4,x]
```

output
```
-1/6*(a*((-2*Sqrt[1 - a^2/x^2])/a^4 + (2*(1 - a^2/x^2)^(3/2))/(3*a^4))) -
ArcSec[x/a]/(3*x^3)
```

3.59.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5332
```
Int[ArcCos[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int[
u*ArcSec[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

rule 5743
```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(d*x)^(m + 1)*((a + b*ArcSec[c*x])/(d*(m + 1))), x] - Simp[b*(d/(c*(m + 1
)))   Int[(d*x)^(m - 1)/Sqrt[1 - 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d,
 m}, x] && NeQ[m, -1]
```

3.59.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.98

method	result	size
derivativedivides	$-\frac{\frac{a^3 \arccos\left(\frac{a}{x}\right)}{3x^3}-\frac{a^2\sqrt{1-\frac{a^2}{x^2}}}{9x^2}-\frac{2\sqrt{1-\frac{a^2}{x^2}}}{9}}{a^3}$	55
default	$-\frac{\frac{a^3 \arccos\left(\frac{a}{x}\right)}{3x^3}-\frac{a^2\sqrt{1-\frac{a^2}{x^2}}}{9x^2}-\frac{2\sqrt{1-\frac{a^2}{x^2}}}{9}}{a^3}$	55
parts	$-\frac{\arccos\left(\frac{a}{x}\right)}{3x^3}-\frac{(a^2-x^2)(a^2+2x^2)}{9a^3\sqrt{-\frac{a^2-x^2}{x^2}}\, x^4}$	55

input
```
int(arccos(a/x)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/a^3*(1/3*a^3/x^3*arccos(a/x)-1/9*a^2/x^2*(1-a^2/x^2)^(1/2)-2/9*(1-a^2/x
^2)^(1/2))
```

3.59.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.88

$$\int \frac{\arccos\left(\frac{a}{x}\right)}{x^4}\,dx = -\frac{3\,a^3\arccos\left(\frac{a}{x}\right) - (a^2x + 2\,x^3)\sqrt{-\frac{a^2-x^2}{x^2}}}{9\,a^3x^3}$$

input
```
integrate(arccos(a/x)/x^4,x, algorithm="fricas")
```

output
```
-1/9*(3*a^3*arccos(a/x) - (a^2*x + 2*x^3)*sqrt(-(a^2 - x^2)/x^2))/(a^3*x^3
)
```

3.59.6 Sympy [A] (verification not implemented)

Time = 1.71 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.79

$$\int \frac{\arccos\left(\frac{a}{x}\right)}{x^4}\,dx = \frac{a\left(\begin{cases} \frac{\sqrt{-1+\frac{x^2}{a^2}}}{3ax^3} + \frac{2\sqrt{-1+\frac{x^2}{a^2}}}{3a^3x} & \text{for}\ \left|\frac{x^2}{a^2}\right| > 1 \\ \frac{i\sqrt{1-\frac{x^2}{a^2}}}{3ax^3} + \frac{2i\sqrt{1-\frac{x^2}{a^2}}}{3a^3x} & \text{otherwise} \end{cases}\right)}{3} - \frac{\operatorname{acos}\left(\frac{a}{x}\right)}{3x^3}$$

input
```
integrate(acos(a/x)/x**4,x)
```

output
```
a*Piecewise((sqrt(-1 + x**2/a**2)/(3*a*x**3) + 2*sqrt(-1 + x**2/a**2)/(3*a
**3*x), Abs(x**2/a**2) > 1), (I*sqrt(1 - x**2/a**2)/(3*a*x**3) + 2*I*sqrt(
1 - x**2/a**2)/(3*a**3*x), True))/3 - acos(a/x)/(3*x**3)
```

3.59.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.88

$$\int \frac{\arccos\left(\frac{a}{x}\right)}{x^4}\,dx = -\frac{1}{9}\,a\left(\frac{\left(-\frac{a^2}{x^2}+1\right)^{\frac{3}{2}}}{a^4} - \frac{3\sqrt{-\frac{a^2}{x^2}+1}}{a^4}\right) - \frac{\arccos\left(\frac{a}{x}\right)}{3\,x^3}$$

input
```
integrate(arccos(a/x)/x^4,x, algorithm="maxima")
```

output
```
-1/9*a*((-a^2/x^2 + 1)^(3/2)/a^4 - 3*sqrt(-a^2/x^2 + 1)/a^4) - 1/3*arccos(
a/x)/x^3
```

3.59.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.93

$$\int \frac{\arccos\left(\frac{a}{x}\right)}{x^4}\,dx = -\frac{\frac{3\,a\arccos\left(\frac{a}{x}\right)}{x^3} - \frac{2\sqrt{-\frac{a^2}{x^2}+1}}{a^2} - \frac{\sqrt{-\frac{a^2}{x^2}+1}}{x^2}}{9\,a}$$

input
```
integrate(arccos(a/x)/x^4,x, algorithm="giac")
```

output
```
-1/9*(3*a*arccos(a/x)/x^3 - 2*sqrt(-a^2/x^2 + 1)/a^2 - sqrt(-a^2/x^2 + 1)/
x^2)/a
```

3.59.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos\left(\frac{a}{x}\right)}{x^4}\,dx = \int \frac{\mathrm{acos}\left(\frac{a}{x}\right)}{x^4}\,dx$$

input
```
int(acos(a/x)/x^4,x)
```

output
```
int(acos(a/x)/x^4, x)
```

3.60 $\int x^2 \arccos\left(\sqrt{x}\right)\,dx$

3.60.1 Optimal result

Integrand size = 10, antiderivative size = 78

$$\int x^2 \arccos\left(\sqrt{x}\right)\,dx = -\frac{5}{48}\sqrt{1-x}\sqrt{x} - \frac{5}{72}\sqrt{1-x}x^{3/2} - \frac{1}{18}\sqrt{1-x}x^{5/2} + \frac{1}{3}x^3\arccos\left(\sqrt{x}\right) - \frac{5}{96}\arcsin(1-2x)$$

output
```
1/3*x^3*arccos(x^(1/2))+5/96*arcsin(-1+2*x)-5/72*x^(3/2)*(1-x)^(1/2)-1/18*
x^(5/2)*(1-x)^(1/2)-5/48*(1-x)^(1/2)*x^(1/2)
```

3.60.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.59

$$\int x^2 \arccos\left(\sqrt{x}\right)\,dx = \frac{1}{144}\left(-\sqrt{-((-1+x)x)}(15+10x+8x^2) + 48x^3\arccos\left(\sqrt{x}\right) + 15\arcsin\left(\sqrt{x}\right)\right)$$

input
```
Integrate[x^2*ArcCos[Sqrt[x]],x]
```

output
```
(-(Sqrt[-((-1 + x)*x)]*(15 + 10*x + 8*x^2)) + 48*x^3*ArcCos[Sqrt[x]] + 15*
ArcSin[Sqrt[x]])/144
```

3.60.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.17, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {5342, 27, 60, 60, 60, 62, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arccos\left(\sqrt{x}\right) dx$$

$$\Big\downarrow \text{ 5342}$$

$$\frac{1}{3}\int \frac{x^{5/2}}{2\sqrt{1-x}}dx+\frac{1}{3}x^3 \arccos\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{6}\int \frac{x^{5/2}}{\sqrt{1-x}}dx+\frac{1}{3}x^3 \arccos\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{6}\left(\frac{5}{6}\int \frac{x^{3/2}}{\sqrt{1-x}}dx-\frac{1}{3}\sqrt{1-x}x^{5/2}\right)+\frac{1}{3}x^3 \arccos\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{6}\left(\frac{5}{6}\left(\frac{3}{4}\int \frac{\sqrt{x}}{\sqrt{1-x}}dx-\frac{1}{2}\sqrt{1-x}x^{3/2}\right)-\frac{1}{3}\sqrt{1-x}x^{5/2}\right)+\frac{1}{3}x^3 \arccos\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{6}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-x}\sqrt{x}}dx-\sqrt{1-x}\sqrt{x}\right)-\frac{1}{2}\sqrt{1-x}x^{3/2}\right)-\frac{1}{3}\sqrt{1-x}x^{5/2}\right)+\frac{1}{3}x^3 \arccos\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{ 62}$$

$$\frac{1}{6}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{x-x^2}}dx-\sqrt{1-x}\sqrt{x}\right)-\frac{1}{2}\sqrt{1-x}x^{3/2}\right)-\frac{1}{3}\sqrt{1-x}x^{5/2}\right)+\frac{1}{3}x^3 \arccos\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{ 1090}$$

$$\frac{1}{6}\left(\frac{5}{6}\left(\frac{3}{4}\left(-\frac{1}{2}\int \frac{1}{\sqrt{1-(1-2x)^2}}d(1-2x)-\sqrt{1-x}\sqrt{x}\right)-\frac{1}{2}\sqrt{1-x}x^{3/2}\right)-\frac{1}{3}\sqrt{1-x}x^{5/2}\right)+ \frac{1}{3}x^3 \arccos\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{ 223}$$

$$\frac{1}{3}x^3\arccos\left(\sqrt{x}\right)+\frac{1}{6}\left(\frac{5}{6}\left(\frac{3}{4}\left(-\frac{1}{2}\arcsin(1-2x)-\sqrt{1-x}\sqrt{x}\right)-\frac{1}{2}\sqrt{1-x}x^{3/2}\right)-\frac{1}{3}\sqrt{1-x}x^{5/2}\right)$$

input
```
Int[x^2*ArcCos[Sqrt[x]],x]
```

output
```
(x^3*ArcCos[Sqrt[x]])/3 + (-1/3*(Sqrt[1 - x]*x^(5/2)) + (5*(-1/2*(Sqrt[1 -
 x]*x^(3/2)) + (3*(-(Sqrt[1 - x]*Sqrt[x]) - ArcSin[1 - 2*x]/2))/4))/6)/6
```

3.60.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 62
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 5342
```
Int[((a_.) + ArcCos[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(c + d*x)^(m + 1)*((a + b*ArcCos[u])/(d*(m + 1))), x] + Simp[b/(d*(m + 1)
)   Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 - u^2]), x], x]
, x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u,
 x]
```

3.60.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.68

method	result	size
derivativedivides	$\frac{x^3 \arccos(\sqrt{x})}{3} - \frac{x^{\frac{5}{2}}\sqrt{1-x}}{18} - \frac{5x^{\frac{3}{2}}\sqrt{1-x}}{72} - \frac{5\sqrt{1-x}\,\sqrt{x}}{48} + \frac{5\arcsin(\sqrt{x})}{48}$	53
default	$\frac{x^3 \arccos(\sqrt{x})}{3} - \frac{x^{\frac{5}{2}}\sqrt{1-x}}{18} - \frac{5x^{\frac{3}{2}}\sqrt{1-x}}{72} - \frac{5\sqrt{1-x}\,\sqrt{x}}{48} + \frac{5\arcsin(\sqrt{x})}{48}$	53
parts	$\frac{x^3 \arccos(\sqrt{x})}{3} - \frac{x^{\frac{5}{2}}\sqrt{1-x}}{18} - \frac{5x^{\frac{3}{2}}\sqrt{1-x}}{72} - \frac{5\sqrt{1-x}\,\sqrt{x}}{48} + \frac{5\sqrt{x(1-x)}\,\arcsin(-1+2x)}{96\sqrt{x}\,\sqrt{1-x}}$	74

input
```
int(x^2*arccos(x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*arccos(x^(1/2))-1/18*x^(5/2)*(1-x)^(1/2)-5/72*x^(3/2)*(1-x)^(1/2)-
5/48*(1-x)^(1/2)*x^(1/2)+5/48*arcsin(x^(1/2))
```

3.60.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.46

$$\int x^2 \arccos\left(\sqrt{x}\right)\,dx = -\frac{1}{144}\left(8\,x^2 + 10\,x + 15\right)\sqrt{x}\sqrt{-x+1} + \frac{1}{48}\left(16\,x^3 - 5\right)\arccos\left(\sqrt{x}\right)$$

input
```
integrate(x^2*arccos(x^(1/2)),x, algorithm="fricas")
```

output
```
-1/144*(8*x^2 + 10*x + 15)*sqrt(x)*sqrt(-x + 1) + 1/48*(16*x^3 - 5)*arccos
(sqrt(x))
```

3.60.6 Sympy [A] (verification not implemented)

Time = 1.21 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.69

$$\int x^2 \arccos\left(\sqrt{x}\right)\, dx = \frac{x^3 \operatorname{acos}\left(\sqrt{x}\right)}{3} + \frac{\sqrt{1-x}\left(-\frac{x^{\frac{5}{2}}}{6} - \frac{5x^{\frac{3}{2}}}{24} - \frac{5\sqrt{x}}{16}\right)}{3} + \frac{5\operatorname{asin}\left(\sqrt{x}\right)}{48}$$

input
```
integrate(x**2*acos(x**(1/2)),x)
```

output
```
x**3*acos(sqrt(x))/3 + sqrt(1 - x)*(-x**(5/2)/6 - 5*x**(3/2)/24 - 5*sqrt(x
)/16)/3 + 5*asin(sqrt(x))/48
```

3.60.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.67

$$\int x^2 \arccos\left(\sqrt{x}\right)\, dx = \frac{1}{3}\, x^3 \arccos\left(\sqrt{x}\right) - \frac{1}{18}\, x^{\frac{5}{2}} \sqrt{-x+1} - \frac{5}{72}\, x^{\frac{3}{2}} \sqrt{-x+1} - \frac{5}{48}\, \sqrt{x}\sqrt{-x+1} + \frac{5}{48}\, \arcsin\left(\sqrt{x}\right)$$

input
```
integrate(x^2*arccos(x^(1/2)),x, algorithm="maxima")
```

output
```
1/3*x^3*arccos(sqrt(x)) - 1/18*x^(5/2)*sqrt(-x + 1) - 5/72*x^(3/2)*sqrt(-x
 + 1) - 5/48*sqrt(x)*sqrt(-x + 1) + 5/48*arcsin(sqrt(x))
```

3.60.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.67

$$\int x^2 \arccos\left(\sqrt{x}\right)\, dx = \frac{1}{3}\, x^3 \arccos\left(\sqrt{x}\right) - \frac{1}{18}\, x^{\frac{5}{2}} \sqrt{-x+1} - \frac{5}{72}\, x^{\frac{3}{2}} \sqrt{-x+1} - \frac{5}{48}\, \sqrt{x}\sqrt{-x+1} - \frac{5}{48}\, \arccos\left(\sqrt{x}\right)$$

input
```
integrate(x^2*arccos(x^(1/2)),x, algorithm="giac")
```

output
```
1/3*x^3*arccos(sqrt(x)) - 1/18*x^(5/2)*sqrt(-x + 1) - 5/72*x^(3/2)*sqrt(-x
 + 1) - 5/48*sqrt(x)*sqrt(-x + 1) - 5/48*arccos(sqrt(x))
```

3.60.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \arccos\left(\sqrt{x}\right)\,dx = \int x^2\,\mathrm{acos}\left(\sqrt{x}\right)\,dx$$

input
```
int(x^2*acos(x^(1/2)),x)
```

output
```
int(x^2*acos(x^(1/2)), x)
```

3.61 $\int x \arccos \left(\sqrt{x}\right) \, dx$

3.61.1 Optimal result

Integrand size = 8, antiderivative size = 60

$$\int x \arccos \left(\sqrt{x}\right) \, dx = -\frac{3}{16}\sqrt{1-x}\sqrt{x}-\frac{1}{8}\sqrt{1-x}x^{3/2}+\frac{1}{2}x^2 \arccos \left(\sqrt{x}\right)-\frac{3}{32}\arcsin(1-2x)$$

output
```
1/2*x^2*arccos(x^(1/2))+3/32*arcsin(-1+2*x)-1/8*x^(3/2)*(1-x)^(1/2)-3/16*(
1-x)^(1/2)*x^(1/2)
```

3.61.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.68

$$\int x \arccos \left(\sqrt{x}\right) \, dx = \frac{1}{16}\left(-\sqrt{-((-1+x)x)}(3+2x)+8x^2 \arccos \left(\sqrt{x}\right)+3 \arcsin \left(\sqrt{x}\right)\right)$$

input
```
Integrate[x*ArcCos[Sqrt[x]],x]
```

output
```
(-(Sqrt[-((-1 + x)*x)]*(3 + 2*x)) + 8*x^2*ArcCos[Sqrt[x]] + 3*ArcSin[Sqrt[
x]])/16
```

3.61.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {5342, 27, 60, 60, 62, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arccos\left(\sqrt{x}\right) dx$$

$$\Big\downarrow \text{5342}$$

$$\frac{1}{2}\int \frac{x^{3/2}}{2\sqrt{1-x}}dx + \frac{1}{2}x^2\arccos\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\int \frac{x^{3/2}}{\sqrt{1-x}}dx + \frac{1}{2}x^2\arccos\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4}\left(\frac{3}{4}\int \frac{\sqrt{x}}{\sqrt{1-x}}dx - \frac{1}{2}\sqrt{1-x}x^{3/2}\right) + \frac{1}{2}x^2\arccos\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-x}\sqrt{x}}dx - \sqrt{1-x}\sqrt{x}\right) - \frac{1}{2}\sqrt{1-x}x^{3/2}\right) + \frac{1}{2}x^2\arccos\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{62}$$

$$\frac{1}{4}\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{\sqrt{x-x^2}}dx - \sqrt{1-x}\sqrt{x}\right) - \frac{1}{2}\sqrt{1-x}x^{3/2}\right) + \frac{1}{2}x^2\arccos\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{4}\left(\frac{3}{4}\left(-\frac{1}{2}\int \frac{1}{\sqrt{1-(1-2x)^2}}d(1-2x) - \sqrt{1-x}\sqrt{x}\right) - \frac{1}{2}\sqrt{1-x}x^{3/2}\right) + \frac{1}{2}x^2\arccos\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{223}$$

$$\frac{1}{2}x^2\arccos\left(\sqrt{x}\right) + \frac{1}{4}\left(\frac{3}{4}\left(-\frac{1}{2}\arcsin(1-2x) - \sqrt{1-x}\sqrt{x}\right) - \frac{1}{2}\sqrt{1-x}x^{3/2}\right)$$

input
```
Int[x*ArcCos[Sqrt[x]],x]
```

output
```
(x^2*ArcCos[Sqrt[x]])/2 + (-1/2*(Sqrt[1 - x]*x^(3/2)) + (3*(-(Sqrt[1 - x]*
Sqrt[x]) - ArcSin[1 - 2*x]/2))/4)/4
```

3.61.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 62
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 5342
```
Int[((a_.) + ArcCos[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(c + d*x)^(m + 1)*((a + b*ArcCos[u])/(d*(m + 1))), x] + Simp[b/(d*(m + 1)
)   Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 - u^2]), x], x]
, x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u,
 x]
```

3.61.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.68

method	result	size
derivativedivides	$\frac{x^2 \arccos(\sqrt{x})}{2} - \frac{x^{\frac{3}{2}}\sqrt{1-x}}{8} - \frac{3\sqrt{1-x}\,\sqrt{x}}{16} + \frac{3\arcsin(\sqrt{x})}{16}$	41
default	$\frac{x^2 \arccos(\sqrt{x})}{2} - \frac{x^{\frac{3}{2}}\sqrt{1-x}}{8} - \frac{3\sqrt{1-x}\,\sqrt{x}}{16} + \frac{3\arcsin(\sqrt{x})}{16}$	41
parts	$\frac{x^2 \arccos(\sqrt{x})}{2} - \frac{x^{\frac{3}{2}}\sqrt{1-x}}{8} - \frac{3\sqrt{1-x}\,\sqrt{x}}{16} + \frac{3\sqrt{x(1-x)}\,\arcsin(-1+2x)}{32\sqrt{x}\,\sqrt{1-x}}$	62

input
```
int(x*arccos(x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*arccos(x^(1/2))-1/8*x^(3/2)*(1-x)^(1/2)-3/16*(1-x)^(1/2)*x^(1/2)+3
/16*arcsin(x^(1/2))
```

3.61.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.52

$$\int x \arccos\left(\sqrt{x}\right) dx = -\frac{1}{16}(2x+3)\sqrt{x}\sqrt{-x+1} + \frac{1}{16}\left(8x^2-3\right)\arccos\left(\sqrt{x}\right)$$

input
```
integrate(x*arccos(x^(1/2)),x, algorithm="fricas")
```

output
```
-1/16*(2*x + 3)*sqrt(x)*sqrt(-x + 1) + 1/16*(8*x^2 - 3)*arccos(sqrt(x))
```

3.61.6 Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.77

$$\int x \arccos\left(\sqrt{x}\right) dx = \frac{x^2 \operatorname{acos}\left(\sqrt{x}\right)}{2} + \frac{\sqrt{1-x}\left(-\frac{x^{\frac{3}{2}}}{4} - \frac{3\sqrt{x}}{8}\right)}{2} + \frac{3\operatorname{asin}\left(\sqrt{x}\right)}{16}$$

input
```
integrate(x*acos(x**(1/2)),x)
```

output
```
x**2*acos(sqrt(x))/2 + sqrt(1 - x)*(-x**(3/2)/4 - 3*sqrt(x)/8)/2 + 3*asin(
sqrt(x))/16
```

3.61.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.67

$$\int x\arccos\left(\sqrt{x}\right)\,dx = \frac{1}{2}\,x^2\arccos\left(\sqrt{x}\right) - \frac{1}{8}\,x^{\frac{3}{2}}\sqrt{-x+1} - \frac{3}{16}\,\sqrt{x}\sqrt{-x+1} + \frac{3}{16}\,\arcsin\left(\sqrt{x}\right)$$

input
```
integrate(x*arccos(x^(1/2)),x, algorithm="maxima")
```

output
```
1/2*x^2*arccos(sqrt(x)) - 1/8*x^(3/2)*sqrt(-x + 1) - 3/16*sqrt(x)*sqrt(-x
+ 1) + 3/16*arcsin(sqrt(x))
```

3.61.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.67

$$\int x\arccos\left(\sqrt{x}\right)\,dx = \frac{1}{2}\,x^2\arccos\left(\sqrt{x}\right) - \frac{1}{8}\,x^{\frac{3}{2}}\sqrt{-x+1} - \frac{3}{16}\,\sqrt{x}\sqrt{-x+1} - \frac{3}{16}\,\arccos\left(\sqrt{x}\right)$$

input
```
integrate(x*arccos(x^(1/2)),x, algorithm="giac")
```

output
```
1/2*x^2*arccos(sqrt(x)) - 1/8*x^(3/2)*sqrt(-x + 1) - 3/16*sqrt(x)*sqrt(-x
+ 1) - 3/16*arccos(sqrt(x))
```

3.61.9 Mupad [F(-1)]

Timed out.

$$\int x\arccos\left(\sqrt{x}\right)\,dx = \int x\,\mathrm{acos}\left(\sqrt{x}\right)\,dx$$

input
```
int(x*acos(x^(1/2)),x)
```

output
```
int(x*acos(x^(1/2)), x)
```

3.62 $\int \arccos\left(\sqrt{x}\right)\, dx$

3.62.1 Optimal result

Integrand size = 6, antiderivative size = 37

$$\int \arccos\left(\sqrt{x}\right)\, dx = -\frac{1}{2}\sqrt{1-x}\sqrt{x} + x\arccos\left(\sqrt{x}\right) - \frac{1}{4}\arcsin(1-2x)$$

output
```
x*arccos(x^(1/2))+1/4*arcsin(-1+2*x)-1/2*(1-x)^(1/2)*x^(1/2)
```

3.62.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.16

$$\int \arccos\left(\sqrt{x}\right)\, dx = -\frac{1}{2}\sqrt{-((-1+x)x)} + x\arccos\left(\sqrt{x}\right) + \arctan\left(\frac{\sqrt{x}}{-1+\sqrt{1-x}}\right)$$

input
```
Integrate[ArcCos[Sqrt[x]],x]
```

output
```
-1/2*Sqrt[-((-1 + x)*x)] + x*ArcCos[Sqrt[x]] + ArcTan[Sqrt[x]/(-1 + Sqrt[1
 - x])]
```

3.62.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5340, 27, 60, 62, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arccos\left(\sqrt{x}\right)\,dx$$

$$\Big\downarrow \text{5340}$$

$$\int \frac{\sqrt{x}}{2\sqrt{1-x}}dx+x\arccos\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{1-x}}dx+x\arccos\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\sqrt{1-x}\sqrt{x}}dx-\sqrt{1-x}\sqrt{x}\right)+x\arccos\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{62}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{\sqrt{x-x^2}}dx-\sqrt{1-x}\sqrt{x}\right)+x\arccos\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{2}\left(-\frac{1}{2}\int \frac{1}{\sqrt{1-(1-2x)^2}}d(1-2x)-\sqrt{1-x}\sqrt{x}\right)+x\arccos\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{223}$$

$$x\arccos\left(\sqrt{x}\right)+\frac{1}{2}\left(-\frac{1}{2}\arcsin(1-2x)-\sqrt{1-x}\sqrt{x}\right)$$

input
```
Int[ArcCos[Sqrt[x]],x]
```

output
```
x*ArcCos[Sqrt[x]] + (-(Sqrt[1 - x]*Sqrt[x]) - ArcSin[1 - 2*x]/2)/2
```

3.62.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 62
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 5340
```
Int[ArcCos[u_], x_Symbol] :> Simp[x*ArcCos[u], x] + Int[SimplifyIntegrand[x
*(D[u, x]/Sqrt[1 - u^2]), x], x] /; InverseFunctionFreeQ[u, x] &&  !Functio
nOfExponentialQ[u, x]
```

3.62.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$x \arccos\left(\sqrt{x}\right)-\frac{\sqrt{1-x}\,\sqrt{x}}{2}+\frac{\arcsin\left(\sqrt{x}\right)}{2}$	26
default	$x \arccos\left(\sqrt{x}\right)-\frac{\sqrt{1-x}\,\sqrt{x}}{2}+\frac{\arcsin\left(\sqrt{x}\right)}{2}$	26
parts	$x \arccos\left(\sqrt{x}\right)-\frac{\sqrt{1-x}\,\sqrt{x}}{2}+\frac{\sqrt{x(1-x)}\,\arcsin(-1+2x)}{4\sqrt{x}\,\sqrt{1-x}}$	47

input
```
int(arccos(x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
x*arccos(x^(1/2))-1/2*(1-x)^(1/2)*x^(1/2)+1/2*arcsin(x^(1/2))
```

3.62.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.65

$$\int \arccos \left(\sqrt{x}\right) \, dx = \frac{1}{2} \, (2 \, x - 1) \arccos \left(\sqrt{x}\right) - \frac{1}{2} \, \sqrt{x}\sqrt{-x + 1}$$

input
```
integrate(arccos(x^(1/2)),x, algorithm="fricas")
```

output
```
1/2*(2*x - 1)*arccos(sqrt(x)) - 1/2*sqrt(x)*sqrt(-x + 1)
```

3.62.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \arccos \left(\sqrt{x}\right) \, dx = -\frac{\sqrt{x}\sqrt{1-x}}{2} + x \operatorname{acos}\left(\sqrt{x}\right) - \frac{\operatorname{acos}\left(\sqrt{x}\right)}{2}$$

input
```
integrate(acos(x**(1/2)),x)
```

output
```
-sqrt(x)*sqrt(1 - x)/2 + x*acos(sqrt(x)) - acos(sqrt(x))/2
```

3.62.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.68

$$\int \arccos \left(\sqrt{x}\right) \, dx = x \arccos \left(\sqrt{x}\right) - \frac{1}{2} \, \sqrt{x}\sqrt{-x + 1} + \frac{1}{2} \, \arcsin \left(\sqrt{x}\right)$$

input
```
integrate(arccos(x^(1/2)),x, algorithm="maxima")
```

output
```
x*arccos(sqrt(x)) - 1/2*sqrt(x)*sqrt(-x + 1) + 1/2*arcsin(sqrt(x))
```

3.62.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.68

$$\int \arccos\left(\sqrt{x}\right)\,dx = x\arccos\left(\sqrt{x}\right) - \frac{1}{2}\sqrt{x}\sqrt{-x+1} - \frac{1}{2}\arccos\left(\sqrt{x}\right)$$

input
```
integrate(arccos(x^(1/2)),x, algorithm="giac")
```

output
```
x*arccos(sqrt(x)) - 1/2*sqrt(x)*sqrt(-x + 1) - 1/2*arccos(sqrt(x))
```

3.62.9 Mupad [B] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \arccos\left(\sqrt{x}\right)\,dx = \operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt{1-x}-1}\right) + x\,\operatorname{acos}\left(\sqrt{x}\right) - \frac{\sqrt{x}\,\sqrt{1-x}}{2}$$

input
```
int(acos(x^(1/2)),x)
```

output
```
atan(x^(1/2)/((1 - x)^(1/2) - 1)) + x*acos(x^(1/2)) - (x^(1/2)*(1 - x)^(1/
2))/2
```

3.63 $\int \frac{\arccos(\sqrt{x})}{x}\,dx$

3.63.1 Optimal result

Integrand size = 10, antiderivative size = 56

$$\int \frac{\arccos\left(\sqrt{x}\right)}{x}\,dx = -i\arccos\left(\sqrt{x}\right)^2 + 2\arccos\left(\sqrt{x}\right)\log\left(1+e^{2i\arccos(\sqrt{x})}\right) - i\,\text{PolyLog}\left(2,-e^{2i\arccos(\sqrt{x})}\right)$$

output
```
-I*arccos(x^(1/2))^2+2*arccos(x^(1/2))*ln(1+(x^(1/2)+I*(1-x)^(1/2))^2)-I*p
olylog(2,-(x^(1/2)+I*(1-x)^(1/2))^2)
```

3.63.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int \frac{\arccos\left(\sqrt{x}\right)}{x}\,dx = -i\left(\arccos\left(\sqrt{x}\right)\left(\arccos\left(\sqrt{x}\right)+2i\log\left(1+e^{2i\arccos(\sqrt{x})}\right)\right) + \text{PolyLog}\left(2,-e^{2i\arccos(\sqrt{x})}\right)\right)$$

input
```
Integrate[ArcCos[Sqrt[x]]/x,x]
```

output
```
(-I)*(ArcCos[Sqrt[x]]*(ArcCos[Sqrt[x]] + (2*I)*Log[1 + E^((2*I)*ArcCos[Sqr
t[x]])]) + PolyLog[2, -E^((2*I)*ArcCos[Sqrt[x]])])
```

3.63.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5330, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos\left(\sqrt{x}\right)}{x}dx$$

$$\Big\downarrow \text{5330}$$

$$-2\int \frac{\sqrt{1-x}\arccos\left(\sqrt{x}\right)}{\sqrt{x}}d\arccos\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{3042}$$

$$-2\int \arccos\left(\sqrt{x}\right)\tan\left(\arccos\left(\sqrt{x}\right)\right)d\arccos\left(\sqrt{x}\right)$$

$$\Big\downarrow \text{4202}$$

$$-2\left(\frac{1}{2}i\arccos\left(\sqrt{x}\right)^2-2i\int \frac{e^{2i\arccos\left(\sqrt{x}\right)}\arccos\left(\sqrt{x}\right)}{1+e^{2i\arccos\left(\sqrt{x}\right)}}d\arccos\left(\sqrt{x}\right)\right)$$

$$\Big\downarrow \text{2620}$$

$$-2\left(\frac{1}{2}i\arccos\left(\sqrt{x}\right)^2-2i\left(\frac{1}{2}i\int \log\left(1+e^{2i\arccos\left(\sqrt{x}\right)}\right)d\arccos\left(\sqrt{x}\right)-\frac{1}{2}i\arccos\left(\sqrt{x}\right)\log\left(1+e^{2i\arccos\left(\sqrt{x}\right)}\right)\right)\right)$$

$$\Big\downarrow \text{2715}$$

$$-2\left(\frac{1}{2}i\arccos\left(\sqrt{x}\right)^2-2i\left(\frac{1}{4}\int e^{-2i\arccos\left(\sqrt{x}\right)}\log\left(1+e^{2i\arccos\left(\sqrt{x}\right)}\right)de^{2i\arccos\left(\sqrt{x}\right)}-\frac{1}{2}i\arccos\left(\sqrt{x}\right)\log\left(1+e^{2i\arccos\left(\sqrt{x}\right)}\right)\right)\right)$$

$$\Big\downarrow \text{2838}$$

$$-2\left(\frac{1}{2}i\arccos\left(\sqrt{x}\right)^2-2i\left(-\frac{1}{4}\text{PolyLog}\left(2,-e^{2i\arccos\left(\sqrt{x}\right)}\right)-\frac{1}{2}i\arccos\left(\sqrt{x}\right)\log\left(1+e^{2i\arccos\left(\sqrt{x}\right)}\right)\right)\right)$$

input
```
Int[ArcCos[Sqrt[x]]/x,x]
```

output
```
-2*((I/2)*ArcCos[Sqrt[x]]^2 - (2*I)*((-1/2*I)*ArcCos[Sqrt[x]]*Log[1 + E^((
2*I)*ArcCos[Sqrt[x]])] - PolyLog[2, -E^((2*I)*ArcCos[Sqrt[x]])]/4))
```

3.63.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5330
```
Int[ArcCos[(a_.)*(x_)^(p_)]^(n_.)/(x_), x_Symbol] :> Simp[-p^(-1)   Subst[I
nt[x^n*Tan[x], x], x, ArcCos[a*x^p]], x] /; FreeQ[{a, p}, x] && IGtQ[n, 0]
```

3.63.4 Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.05

method	result
derivativedivides	$-i \arccos\left(\sqrt{x}\right)^2 + 2\arccos\left(\sqrt{x}\right)\ln\left(1+\left(\sqrt{x}+i\sqrt{1-x}\right)^2\right) - i\,\mathrm{polylog}\left(2, -\left(\sqrt{x}+i\right.\right.$
default	$-i \arccos\left(\sqrt{x}\right)^2 + 2\arccos\left(\sqrt{x}\right)\ln\left(1+\left(\sqrt{x}+i\sqrt{1-x}\right)^2\right) - i\,\mathrm{polylog}\left(2, -\left(\sqrt{x}+i\right.\right.$

input
```
int(arccos(x^(1/2))/x,x,method=_RETURNVERBOSE)
```

output
```
-I*arccos(x^(1/2))^2+2*arccos(x^(1/2))*ln(1+(x^(1/2)+I*(1-x)^(1/2))^2)-I*p
olylog(2,-(x^(1/2)+I*(1-x)^(1/2))^2)
```

3.63.5 Fricas [F]

$$\int \frac{\arccos\left(\sqrt{x}\right)}{x}\,dx = \int \frac{\arccos\left(\sqrt{x}\right)}{x}\,dx$$

input
```
integrate(arccos(x^(1/2))/x,x, algorithm="fricas")
```

output
```
integral(arccos(sqrt(x))/x, x)
```

3.63.6 Sympy [F]

$$\int \frac{\arccos\left(\sqrt{x}\right)}{x}\,dx = \int \frac{\operatorname{acos}\left(\sqrt{x}\right)}{x}\,dx$$

input
```
integrate(acos(x**(1/2))/x,x)
```

output
```
Integral(acos(sqrt(x))/x, x)
```

3.63.7 Maxima [F]

$$\int \frac{\arccos\left(\sqrt{x}\right)}{x}\,dx = \int \frac{\arccos\left(\sqrt{x}\right)}{x}\,dx$$

input
```
integrate(arccos(x^(1/2))/x,x, algorithm="maxima")
```

output
```
integrate(arccos(sqrt(x))/x, x)
```

3.63.8 Giac [F]

$$\int \frac{\arccos\left(\sqrt{x}\right)}{x}\,dx = \int \frac{\arccos\left(\sqrt{x}\right)}{x}\,dx$$

input
```
integrate(arccos(x^(1/2))/x,x, algorithm="giac")
```

output
```
integrate(arccos(sqrt(x))/x, x)
```

3.63.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos\left(\sqrt{x}\right)}{x}\,dx = \int \frac{\mathrm{acos}\left(\sqrt{x}\right)}{x}\,dx$$

input
```
int(acos(x^(1/2))/x,x)
```

output
```
int(acos(x^(1/2))/x, x)
```

3.64 $\int \frac{\arccos(\sqrt{x})}{x^2}\,dx$

3.64.1 Optimal result

Integrand size = 10, antiderivative size = 27

$$\int \frac{\arccos(\sqrt{x})}{x^2}\,dx = \frac{\sqrt{1-x}}{\sqrt{x}} - \frac{\arccos(\sqrt{x})}{x}$$

output
```
-arccos(x^(1/2))/x+(1-x)^(1/2)/x^(1/2)
```

3.64.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{\arccos(\sqrt{x})}{x^2}\,dx = \frac{\sqrt{x-x^2} - \arccos(\sqrt{x})}{x}$$

input
```
Integrate[ArcCos[Sqrt[x]]/x^2,x]
```

output
```
(Sqrt[x - x^2] - ArcCos[Sqrt[x]])/x
```

3.64.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5342, 27, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos\left(\sqrt{x}\right)}{x^2} \, dx$$

$$\Big\downarrow \text{5342}$$

$$-\int \frac{1}{2\sqrt{1-x}x^{3/2}}dx-\frac{\arccos\left(\sqrt{x}\right)}{x}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{2}\int \frac{1}{\sqrt{1-x}x^{3/2}}dx-\frac{\arccos\left(\sqrt{x}\right)}{x}$$

$$\Big\downarrow \text{48}$$

$$\frac{\sqrt{1-x}}{\sqrt{x}}-\frac{\arccos\left(\sqrt{x}\right)}{x}$$

input
```
Int[ArcCos[Sqrt[x]]/x^2,x]
```

output
```
Sqrt[1 - x]/Sqrt[x] - ArcCos[Sqrt[x]]/x
```

3.64.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 5342
```
Int[((a_.) + ArcCos[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(c + d*x)^(m + 1)*((a + b*ArcCos[u])/(d*(m + 1))), x] + Simp[b/(d*(m + 1)
)   Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 - u^2]), x], x]
, x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u,
 x]
```

3.64.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$-\frac{\arccos(\sqrt{x})}{x}+\frac{\sqrt{1-x}}{\sqrt{x}}$	22
default	$-\frac{\arccos(\sqrt{x})}{x}+\frac{\sqrt{1-x}}{\sqrt{x}}$	22
parts	$-\frac{\arccos(\sqrt{x})}{x}+\frac{\sqrt{1-x}}{\sqrt{x}}$	22

input
```
int(arccos(x^(1/2))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-arccos(x^(1/2))/x+(1-x)^(1/2)/x^(1/2)
```

3.64.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{\arccos\left(\sqrt{x}\right)}{x^2}\,dx=\frac{\sqrt{x}\sqrt{-x+1}-\arccos\left(\sqrt{x}\right)}{x}$$

input
```
integrate(arccos(x^(1/2))/x^2,x, algorithm="fricas")
```

output
```
(sqrt(x)*sqrt(-x + 1) - arccos(sqrt(x)))/x
```

3.64.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.57 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.63

$$\int \frac{\arccos\left(\sqrt{x}\right)}{x^2}\,dx = -\frac{\begin{cases} -\frac{2i\sqrt{x-1}}{\sqrt{x}} & \text{for } |x|>1 \\ -\frac{2\sqrt{1-x}}{\sqrt{x}} & \text{otherwise} \end{cases}}{2} - \frac{\operatorname{acos}\left(\sqrt{x}\right)}{x}$$

input
```
integrate(acos(x**(1/2))/x**2,x)
```

output
```
-Piecewise((-2*I*sqrt(x - 1)/sqrt(x), Abs(x) > 1), (-2*sqrt(1 - x)/sqrt(x)
, True))/2 - acos(sqrt(x))/x
```

3.64.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.78

$$\int \frac{\arccos\left(\sqrt{x}\right)}{x^2}\,dx = \frac{\sqrt{-x+1}}{\sqrt{x}} - \frac{\arccos\left(\sqrt{x}\right)}{x}$$

input
```
integrate(arccos(x^(1/2))/x^2,x, algorithm="maxima")
```

output
```
sqrt(-x + 1)/sqrt(x) - arccos(sqrt(x))/x
```

3.64.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.48

$$\int \frac{\arccos\left(\sqrt{x}\right)}{x^2}\,dx = \frac{\sqrt{-x+1}-1}{2\sqrt{x}} - \frac{\arccos\left(\sqrt{x}\right)}{x} - \frac{\sqrt{x}}{2\left(\sqrt{-x+1}-1\right)}$$

input
```
integrate(arccos(x^(1/2))/x^2,x, algorithm="giac")
```

output
```
1/2*(sqrt(-x + 1) - 1)/sqrt(x) - arccos(sqrt(x))/x - 1/2*sqrt(x)/(sqrt(-x
+ 1) - 1)
```

3.64.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos\left(\sqrt{x}\right)}{x^2} dx = \int \frac{\mathrm{acos}\left(\sqrt{x}\right)}{x^2} dx$$

input
```
int(acos(x^(1/2))/x^2,x)
```

output
```
int(acos(x^(1/2))/x^2, x)
```

3.65 $\int \frac{\arccos(\sqrt{x})}{x^3} \, dx$

3.65.1 Optimal result

Integrand size = 10, antiderivative size = 50

$$\int \frac{\arccos\left(\sqrt{x}\right)}{x^3} \, dx = \frac{\sqrt{1-x}}{6x^{3/2}} + \frac{\sqrt{1-x}}{3\sqrt{x}} - \frac{\arccos\left(\sqrt{x}\right)}{2x^2}$$

output
```
-1/2*arccos(x^(1/2))/x^2+1/6*(1-x)^(1/2)/x^(3/2)+1/3*(1-x)^(1/2)/x^(1/2)
```

3.65.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

$$\int \frac{\arccos\left(\sqrt{x}\right)}{x^3} \, dx = \left(\frac{1}{6x^{3/2}} + \frac{1}{3\sqrt{x}}\right)\sqrt{1-x} - \frac{\arccos\left(\sqrt{x}\right)}{2x^2}$$

input
```
Integrate[ArcCos[Sqrt[x]]/x^3,x]
```

output
```
(1/(6*x^(3/2)) + 1/(3*Sqrt[x]))*Sqrt[1 - x] - ArcCos[Sqrt[x]]/(2*x^2)
```

3.65.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5342, 27, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos\left(\sqrt{x}\right)}{x^3}\,dx$$

$$\Big\downarrow \text{ 5342}$$

$$-\frac{1}{2}\int \frac{1}{2\sqrt{1-x}x^{5/2}}dx-\frac{\arccos\left(\sqrt{x}\right)}{2x^2}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{1}{4}\int \frac{1}{\sqrt{1-x}x^{5/2}}dx-\frac{\arccos\left(\sqrt{x}\right)}{2x^2}$$

$$\Big\downarrow \text{ 55}$$

$$\frac{1}{4}\left(\frac{2\sqrt{1-x}}{3x^{3/2}}-\frac{2}{3}\int \frac{1}{\sqrt{1-x}x^{3/2}}dx\right)-\frac{\arccos\left(\sqrt{x}\right)}{2x^2}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{1}{4}\left(\frac{2\sqrt{1-x}}{3x^{3/2}}+\frac{4\sqrt{1-x}}{3\sqrt{x}}\right)-\frac{\arccos\left(\sqrt{x}\right)}{2x^2}$$

input
```
Int[ArcCos[Sqrt[x]]/x^3,x]
```

output
```
((2*Sqrt[1 - x])/(3*x^(3/2)) + (4*Sqrt[1 - x])/(3*Sqrt[x]))/4 - ArcCos[Sqr
t[x]]/(2*x^2)
```

3.65.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 5342
```
Int[((a_.) + ArcCos[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(c + d*x)^(m + 1)*((a + b*ArcCos[u])/(d*(m + 1))), x] + Simp[b/(d*(m + 1)
)   Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 - u^2]), x], x]
, x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u,
 x]
```

3.65.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$-\frac{\arccos(\sqrt{x})}{2x^2}+\frac{\sqrt{1-x}}{6x^{\frac{3}{2}}}+\frac{\sqrt{1-x}}{3\sqrt{x}}$	35
default	$-\frac{\arccos(\sqrt{x})}{2x^2}+\frac{\sqrt{1-x}}{6x^{\frac{3}{2}}}+\frac{\sqrt{1-x}}{3\sqrt{x}}$	35
parts	$-\frac{\arccos(\sqrt{x})}{2x^2}+\frac{\sqrt{1-x}}{6x^{\frac{3}{2}}}+\frac{\sqrt{1-x}}{3\sqrt{x}}$	35

input
```
int(arccos(x^(1/2))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*arccos(x^(1/2))/x^2+1/6*(1-x)^(1/2)/x^(3/2)+1/3*(1-x)^(1/2)/x^(1/2)
```

3.65.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.56

$$\int \frac{\arccos\left(\sqrt{x}\right)}{x^3}\,dx = \frac{(2\,x+1)\sqrt{x}\sqrt{-x+1} - 3\,\arccos\left(\sqrt{x}\right)}{6\,x^2}$$

input
```
integrate(arccos(x^(1/2))/x^3,x, algorithm="fricas")
```

output
```
1/6*((2*x + 1)*sqrt(x)*sqrt(-x + 1) - 3*arccos(sqrt(x)))/x^2
```

3.65.6 Sympy [A] (verification not implemented)

Time = 2.79 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06

$$\int \frac{\arccos\left(\sqrt{x}\right)}{x^3}\,dx = -\frac{\begin{cases} -\frac{\sqrt{1-x}}{\sqrt{x}} - \frac{(1-x)^{\frac{3}{2}}}{3x^{\frac{3}{2}}} & \text{for}\ \sqrt{x} > -1 \wedge \sqrt{x} < 1 \end{cases}}{2} - \frac{\operatorname{acos}\left(\sqrt{x}\right)}{2x^2}$$

input
```
integrate(acos(x**(1/2))/x**3,x)
```

output
```
-Piecewise((-sqrt(1 - x)/sqrt(x) - (1 - x)**(3/2)/(3*x**(3/2)), (sqrt(x) >
 -1) & (sqrt(x) < 1)))/2 - acos(sqrt(x))/(2*x**2)
```

3.65.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.68

$$\int \frac{\arccos\left(\sqrt{x}\right)}{x^3}\,dx = \frac{\sqrt{-x+1}}{3\,\sqrt{x}} + \frac{\sqrt{-x+1}}{6\,x^{\frac{3}{2}}} - \frac{\arccos\left(\sqrt{x}\right)}{2\,x^2}$$

input
```
integrate(arccos(x^(1/2))/x^3,x, algorithm="maxima")
```

output
```
1/3*sqrt(-x + 1)/sqrt(x) + 1/6*sqrt(-x + 1)/x^(3/2) - 1/2*arccos(sqrt(x))/
x^2
```

3.65.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(34) = 68$.

Time = 0.29 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.48

$$\int \frac{\arccos\left(\sqrt{x}\right)}{x^3}\,dx = \frac{\left(\sqrt{-x+1}-1\right)^3}{48\,x^{\frac{3}{2}}} + \frac{3\left(\sqrt{-x+1}-1\right)}{16\,\sqrt{x}} - \frac{x^{\frac{3}{2}}\left(\frac{9\left(\sqrt{-x+1}-1\right)^2}{x}+1\right)}{48\left(\sqrt{-x+1}-1\right)^3} - \frac{\arccos\left(\sqrt{x}\right)}{2\,x^2}$$

input
```
integrate(arccos(x^(1/2))/x^3,x, algorithm="giac")
```

output
```
1/48*(sqrt(-x + 1) - 1)^3/x^(3/2) + 3/16*(sqrt(-x + 1) - 1)/sqrt(x) - 1/48
*x^(3/2)*(9*(sqrt(-x + 1) - 1)^2/x + 1)/(sqrt(-x + 1) - 1)^3 - 1/2*arccos(
sqrt(x))/x^2
```

3.65.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos\left(\sqrt{x}\right)}{x^3}\,dx = \int \frac{\mathrm{acos}\left(\sqrt{x}\right)}{x^3}\,dx$$

input
```
int(acos(x^(1/2))/x^3,x)
```

output
```
int(acos(x^(1/2))/x^3, x)
```

3.66 $\int \frac{\arccos(\sqrt{x})}{x^4}\,dx$

3.66.1 Optimal result

Integrand size = 10, antiderivative size = 68

$$\int \frac{\arccos(\sqrt{x})}{x^4}\,dx = \frac{\sqrt{1-x}}{15x^{5/2}} + \frac{4\sqrt{1-x}}{45x^{3/2}} + \frac{8\sqrt{1-x}}{45\sqrt{x}} - \frac{\arccos(\sqrt{x})}{3x^3}$$

output
```
-1/3*arccos(x^(1/2))/x^3+1/15*(1-x)^(1/2)/x^(5/2)+4/45*(1-x)^(1/2)/x^(3/2)
+8/45*(1-x)^(1/2)/x^(1/2)
```

3.66.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.54

$$\int \frac{\arccos(\sqrt{x})}{x^4}\,dx = \frac{\sqrt{-((-1+x)x)}(3+4x+8x^2) - 15\arccos(\sqrt{x})}{45x^3}$$

input
```
Integrate[ArcCos[Sqrt[x]]/x^4,x]
```

output
```
(Sqrt[-((-1 + x)*x)]*(3 + 4*x + 8*x^2) - 15*ArcCos[Sqrt[x]])/(45*x^3)
```

3.66.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5342, 27, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos\left(\sqrt{x}\right)}{x^4} \, dx$$

$$\Big\downarrow \text{ 5342}$$

$$-\frac{1}{3}\int \frac{1}{2\sqrt{1-x}x^{7/2}}dx-\frac{\arccos\left(\sqrt{x}\right)}{3x^3}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{1}{6}\int \frac{1}{\sqrt{1-x}x^{7/2}}dx-\frac{\arccos\left(\sqrt{x}\right)}{3x^3}$$

$$\Big\downarrow \text{ 55}$$

$$\frac{1}{6}\left(\frac{2\sqrt{1-x}}{5x^{5/2}}-\frac{4}{5}\int \frac{1}{\sqrt{1-x}x^{5/2}}dx\right)-\frac{\arccos\left(\sqrt{x}\right)}{3x^3}$$

$$\Big\downarrow \text{ 55}$$

$$\frac{1}{6}\left(\frac{2\sqrt{1-x}}{5x^{5/2}}-\frac{4}{5}\left(\frac{2}{3}\int \frac{1}{\sqrt{1-x}x^{3/2}}dx-\frac{2\sqrt{1-x}}{3x^{3/2}}\right)\right)-\frac{\arccos\left(\sqrt{x}\right)}{3x^3}$$

$$\Big\downarrow \text{ 48}$$

$$\frac{1}{6}\left(\frac{2\sqrt{1-x}}{5x^{5/2}}-\frac{4}{5}\left(-\frac{2\sqrt{1-x}}{3x^{3/2}}-\frac{4\sqrt{1-x}}{3\sqrt{x}}\right)\right)-\frac{\arccos\left(\sqrt{x}\right)}{3x^3}$$

input
```
Int[ArcCos[Sqrt[x]]/x^4,x]
```

output
```
((-4*((-2*Sqrt[1 - x])/(3*x^(3/2)) - (4*Sqrt[1 - x])/(3*Sqrt[x])))/5 + (2*
Sqrt[1 - x])/(5*x^(5/2)))/6 - ArcCos[Sqrt[x]]/(3*x^3)
```

3.66.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 5342
```
Int[((a_.) + ArcCos[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(c + d*x)^(m + 1)*((a + b*ArcCos[u])/(d*(m + 1))), x] + Simp[b/(d*(m + 1)
)   Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 - u^2]), x], x]
, x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u,
 x]
```

3.66.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.69

method	result	size
derivativedivides	$-\frac{\arccos\left(\sqrt{x}\right)}{3x^{3}}+\frac{\sqrt{1-x}}{15x^{\frac{5}{2}}}+\frac{4\sqrt{1-x}}{45x^{\frac{3}{2}}}+\frac{8\sqrt{1-x}}{45\sqrt{x}}$	47
default	$-\frac{\arccos\left(\sqrt{x}\right)}{3x^{3}}+\frac{\sqrt{1-x}}{15x^{\frac{5}{2}}}+\frac{4\sqrt{1-x}}{45x^{\frac{3}{2}}}+\frac{8\sqrt{1-x}}{45\sqrt{x}}$	47
parts	$-\frac{\arccos\left(\sqrt{x}\right)}{3x^{3}}+\frac{\sqrt{1-x}}{15x^{\frac{5}{2}}}+\frac{4\sqrt{1-x}}{45x^{\frac{3}{2}}}+\frac{8\sqrt{1-x}}{45\sqrt{x}}$	47

input
```
int(arccos(x^(1/2))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*arccos(x^(1/2))/x^3+1/15*(1-x)^(1/2)/x^(5/2)+4/45*(1-x)^(1/2)/x^(3/2)
+8/45*(1-x)^(1/2)/x^(1/2)
```

3.66.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.49

$$\int \frac{\arccos\left(\sqrt{x}\right)}{x^4}\,dx = \frac{(8\,x^2+4\,x+3)\sqrt{x}\sqrt{-x+1}-15\,\arccos\left(\sqrt{x}\right)}{45\,x^3}$$

input
```
integrate(arccos(x^(1/2))/x^4,x, algorithm="fricas")
```

output
```
1/45*((8*x^2 + 4*x + 3)*sqrt(x)*sqrt(-x + 1) - 15*arccos(sqrt(x)))/x^3
```

3.66.6 Sympy [A] (verification not implemented)

Time = 8.48 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00

$$\int \frac{\arccos\left(\sqrt{x}\right)}{x^4}\,dx = -\frac{\begin{cases} -\frac{\sqrt{1-x}}{\sqrt{x}}-\frac{2(1-x)^{\frac{3}{2}}}{3x^{\frac{3}{2}}}-\frac{(1-x)^{\frac{5}{2}}}{5x^{\frac{5}{2}}} & \text{for } \sqrt{x}>-1\wedge\sqrt{x}<1 \end{cases}}{3}-\frac{\operatorname{acos}\left(\sqrt{x}\right)}{3x^3}$$

input
```
integrate(acos(x**(1/2))/x**4,x)
```

output
```
-Piecewise((-sqrt(1 - x)/sqrt(x) - 2*(1 - x)**(3/2)/(3*x**(3/2)) - (1 - x)
**(5/2)/(5*x**(5/2)), (sqrt(x) > -1) & (sqrt(x) < 1)))/3 - acos(sqrt(x))/(
3*x**3)
```

3.66.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.68

$$\int \frac{\arccos\left(\sqrt{x}\right)}{x^4}\,dx = \frac{8\,\sqrt{-x+1}}{45\,\sqrt{x}}+\frac{4\,\sqrt{-x+1}}{45\,x^{\frac{3}{2}}}+\frac{\sqrt{-x+1}}{15\,x^{\frac{5}{2}}}-\frac{\arccos\left(\sqrt{x}\right)}{3\,x^3}$$

input
```
integrate(arccos(x^(1/2))/x^4,x, algorithm="maxima")
```

output
```
8/45*sqrt(-x + 1)/sqrt(x) + 4/45*sqrt(-x + 1)/x^(3/2) + 1/15*sqrt(-x + 1)/
x^(5/2) - 1/3*arccos(sqrt(x))/x^3
```

3.66.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(46) = 92$.

Time = 0.29 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.56

$$\int \frac{\arccos\left(\sqrt{x}\right)}{x^4}\,dx = \frac{\left(\sqrt{-x+1}-1\right)^5}{480\,x^{\frac{5}{2}}} + \frac{5\left(\sqrt{-x+1}-1\right)^3}{288\,x^{\frac{3}{2}}} + \frac{5\left(\sqrt{-x+1}-1\right)}{48\,\sqrt{x}} - \frac{\left(\frac{150\left(\sqrt{-x+1}-1\right)^4}{x^2} + \frac{25\left(\sqrt{-x+1}-1\right)^2}{x} + 3\right)x^{\frac{5}{2}}}{1440\left(\sqrt{-x+1}-1\right)^5} - \frac{\arccos\left(\sqrt{x}\right)}{3\,x^3}$$

input
```
integrate(arccos(x^(1/2))/x^4,x, algorithm="giac")
```

output
```
1/480*(sqrt(-x + 1) - 1)^5/x^(5/2) + 5/288*(sqrt(-x + 1) - 1)^3/x^(3/2) +
5/48*(sqrt(-x + 1) - 1)/sqrt(x) - 1/1440*(150*(sqrt(-x + 1) - 1)^4/x^2 + 2
5*(sqrt(-x + 1) - 1)^2/x + 3)*x^(5/2)/(sqrt(-x + 1) - 1)^5 - 1/3*arccos(sq
rt(x))/x^3
```

3.66.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos\left(\sqrt{x}\right)}{x^4}\,dx = \int \frac{\mathrm{acos}\left(\sqrt{x}\right)}{x^4}\,dx$$

input
```
int(acos(x^(1/2))/x^4,x)
```

output
```
int(acos(x^(1/2))/x^4, x)
```

3.67 $\int \frac{\arccos(\sqrt{x})}{x^5} dx$

3.67.1 Optimal result

Integrand size = 10, antiderivative size = 86

$$\int \frac{\arccos(\sqrt{x})}{x^5} dx = \frac{\sqrt{1-x}}{28x^{7/2}} + \frac{3\sqrt{1-x}}{70x^{5/2}} + \frac{2\sqrt{1-x}}{35x^{3/2}} + \frac{4\sqrt{1-x}}{35\sqrt{x}} - \frac{\arccos(\sqrt{x})}{4x^4}$$

output
```
-1/4*arccos(x^(1/2))/x^4+1/28*(1-x)^(1/2)/x^(7/2)+3/70*(1-x)^(1/2)/x^(5/2)
+2/35*(1-x)^(1/2)/x^(3/2)+4/35*(1-x)^(1/2)/x^(1/2)
```

3.67.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.49

$$\int \frac{\arccos(\sqrt{x})}{x^5} dx = \frac{\sqrt{-((-1+x)x)}(5+6x+8x^2+16x^3) - 35\arccos(\sqrt{x})}{140x^4}$$

input
```
Integrate[ArcCos[Sqrt[x]]/x^5,x]
```

output
```
(Sqrt[-((-1 + x)*x)]*(5 + 6*x + 8*x^2 + 16*x^3) - 35*ArcCos[Sqrt[x]])/(140
*x^4)
```

3.67.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5342, 27, 55, 55, 55, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos\left(\sqrt{x}\right)}{x^5}\,dx$$

$$\Big\downarrow \text{5342}$$

$$-\frac{1}{4}\int \frac{1}{2\sqrt{1-x}x^{9/2}}dx-\frac{\arccos\left(\sqrt{x}\right)}{4x^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{8}\int \frac{1}{\sqrt{1-x}x^{9/2}}dx-\frac{\arccos\left(\sqrt{x}\right)}{4x^4}$$

$$\Big\downarrow \text{55}$$

$$\frac{1}{8}\left(\frac{2\sqrt{1-x}}{7x^{7/2}}-\frac{6}{7}\int \frac{1}{\sqrt{1-x}x^{7/2}}dx\right)-\frac{\arccos\left(\sqrt{x}\right)}{4x^4}$$

$$\Big\downarrow \text{55}$$

$$\frac{1}{8}\left(\frac{2\sqrt{1-x}}{7x^{7/2}}-\frac{6}{7}\left(\frac{4}{5}\int \frac{1}{\sqrt{1-x}x^{5/2}}dx-\frac{2\sqrt{1-x}}{5x^{5/2}}\right)\right)-\frac{\arccos\left(\sqrt{x}\right)}{4x^4}$$

$$\Big\downarrow \text{55}$$

$$\frac{1}{8}\left(\frac{2\sqrt{1-x}}{7x^{7/2}}-\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int \frac{1}{\sqrt{1-x}x^{3/2}}dx-\frac{2\sqrt{1-x}}{3x^{3/2}}\right)-\frac{2\sqrt{1-x}}{5x^{5/2}}\right)\right)-\frac{\arccos\left(\sqrt{x}\right)}{4x^4}$$

$$\Big\downarrow \text{48}$$

$$\frac{1}{8}\left(\frac{2\sqrt{1-x}}{7x^{7/2}}-\frac{6}{7}\left(\frac{4}{5}\left(-\frac{2\sqrt{1-x}}{3x^{3/2}}-\frac{4\sqrt{1-x}}{3\sqrt{x}}\right)-\frac{2\sqrt{1-x}}{5x^{5/2}}\right)\right)-\frac{\arccos\left(\sqrt{x}\right)}{4x^4}$$

input
```
Int[ArcCos[Sqrt[x]]/x^5,x]
```

output
```
((-6*((4*((-2*Sqrt[1 - x])/(3*x^(3/2)) - (4*Sqrt[1 - x])/(3*Sqrt[x])))/5 -
 (2*Sqrt[1 - x])/(5*x^(5/2))))/7 + (2*Sqrt[1 - x])/(7*x^(7/2)))/8 - ArcCos
[Sqrt[x]]/(4*x^4)
```

3.67.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 55
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*(S
implify[m + n + 2]/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^Simplify[m + 1]*(
c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && ILtQ[Simplify[m + n +
 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] && (EqQ[a, 0] || (NeQ[
c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimplerQ[m, 1] ||  !SumSimp
lerQ[n, 1])
```

rule 5342
```
Int[((a_.) + ArcCos[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(c + d*x)^(m + 1)*((a + b*ArcCos[u])/(d*(m + 1))), x] + Simp[b/(d*(m + 1)
)   Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/Sqrt[1 - u^2]), x], x]
, x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u,
x] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u,
 x]
```

3.67.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.69

method	result	size
derivativedivides	$-\frac{\arccos\left(\sqrt{x}\right)}{4x^{4}}+\frac{\sqrt{1-x}}{28x^{\frac{7}{2}}}+\frac{3\sqrt{1-x}}{70x^{\frac{5}{2}}}+\frac{2\sqrt{1-x}}{35x^{\frac{3}{2}}}+\frac{4\sqrt{1-x}}{35\sqrt{x}}$	59
default	$-\frac{\arccos\left(\sqrt{x}\right)}{4x^{4}}+\frac{\sqrt{1-x}}{28x^{\frac{7}{2}}}+\frac{3\sqrt{1-x}}{70x^{\frac{5}{2}}}+\frac{2\sqrt{1-x}}{35x^{\frac{3}{2}}}+\frac{4\sqrt{1-x}}{35\sqrt{x}}$	59
parts	$-\frac{\arccos\left(\sqrt{x}\right)}{4x^{4}}+\frac{\sqrt{1-x}}{28x^{\frac{7}{2}}}+\frac{3\sqrt{1-x}}{70x^{\frac{5}{2}}}+\frac{2\sqrt{1-x}}{35x^{\frac{3}{2}}}+\frac{4\sqrt{1-x}}{35\sqrt{x}}$	59

input
```
int(arccos(x^(1/2))/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*arccos(x^(1/2))/x^4+1/28*(1-x)^(1/2)/x^(7/2)+3/70*(1-x)^(1/2)/x^(5/2)
+2/35*(1-x)^(1/2)/x^(3/2)+4/35*(1-x)^(1/2)/x^(1/2)
```

3.67.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.44

$$\int \frac{\arccos(\sqrt{x})}{x^5}\,dx = \frac{(16\,x^3+8\,x^2+6\,x+5)\sqrt{x}\sqrt{-x+1}-35\,\arccos(\sqrt{x})}{140\,x^4}$$

input
```
integrate(arccos(x^(1/2))/x^5,x, algorithm="fricas")
```

output
```
1/140*((16*x^3 + 8*x^2 + 6*x + 5)*sqrt(x)*sqrt(-x + 1) - 35*arccos(sqrt(x)
))/x^4
```

3.67.6 Sympy [A] (verification not implemented)

Time = 26.55 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.93

$$\int \frac{\arccos(\sqrt{x})}{x^5}\,dx = -\frac{\begin{cases} -\frac{\sqrt{1-x}}{\sqrt{x}}-\frac{(1-x)^{\frac{3}{2}}}{x^{\frac{3}{2}}}-\frac{3(1-x)^{\frac{5}{2}}}{5x^{\frac{5}{2}}}-\frac{(1-x)^{\frac{7}{2}}}{7x^{\frac{7}{2}}} & \text{for } \sqrt{x}>-1\wedge\sqrt{x}<1 \end{cases}}{4} - \frac{\operatorname{acos}(\sqrt{x})}{4x^4}$$

input
```
integrate(acos(x**(1/2))/x**5,x)
```

output
```
-Piecewise((-sqrt(1 - x)/sqrt(x) - (1 - x)**(3/2)/x**(3/2) - 3*(1 - x)**(5
/2)/(5*x**(5/2)) - (1 - x)**(7/2)/(7*x**(7/2)), (sqrt(x) > -1) & (sqrt(x)
< 1)))/4 - acos(sqrt(x))/(4*x**4)
```

3.67.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.67

$$\int \frac{\arccos\left(\sqrt{x}\right)}{x^5}\,dx = \frac{4\,\sqrt{-x+1}}{35\,\sqrt{x}} + \frac{2\,\sqrt{-x+1}}{35\,x^{\frac{3}{2}}} + \frac{3\,\sqrt{-x+1}}{70\,x^{\frac{5}{2}}} + \frac{\sqrt{-x+1}}{28\,x^{\frac{7}{2}}} - \frac{\arccos\left(\sqrt{x}\right)}{4\,x^4}$$

input
```
integrate(arccos(x^(1/2))/x^5,x, algorithm="maxima")
```

output
```
4/35*sqrt(-x + 1)/sqrt(x) + 2/35*sqrt(-x + 1)/x^(3/2) + 3/70*sqrt(-x + 1)/
x^(5/2) + 1/28*sqrt(-x + 1)/x^(7/2) - 1/4*arccos(sqrt(x))/x^4
```

3.67.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(58) = 116.

Time = 0.28 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.60

$$\begin{aligned}\int \frac{\arccos\left(\sqrt{x}\right)}{x^5}\,dx &= \frac{\left(\sqrt{-x+1}-1\right)^7}{3584\,x^{\frac{7}{2}}} + \frac{7\left(\sqrt{-x+1}-1\right)^5}{2560\,x^{\frac{5}{2}}} \\ &\quad + \frac{7\left(\sqrt{-x+1}-1\right)^3}{512\,x^{\frac{3}{2}}} + \frac{35\left(\sqrt{-x+1}-1\right)}{512\,\sqrt{x}} \\ &\quad - \frac{\left(\frac{1225\left(\sqrt{-x+1}-1\right)^6}{x^3} + \frac{245\left(\sqrt{-x+1}-1\right)^4}{x^2} + \frac{49\left(\sqrt{-x+1}-1\right)^2}{x} + 5\right)x^{\frac{7}{2}}}{17920\left(\sqrt{-x+1}-1\right)^7} \\ &\quad - \frac{\arccos\left(\sqrt{x}\right)}{4\,x^4}\end{aligned}$$

input
```
integrate(arccos(x^(1/2))/x^5,x, algorithm="giac")
```

output
```
1/3584*(sqrt(-x + 1) - 1)^7/x^(7/2) + 7/2560*(sqrt(-x + 1) - 1)^5/x^(5/2)
+ 7/512*(sqrt(-x + 1) - 1)^3/x^(3/2) + 35/512*(sqrt(-x + 1) - 1)/sqrt(x) -
 1/17920*(1225*(sqrt(-x + 1) - 1)^6/x^3 + 245*(sqrt(-x + 1) - 1)^4/x^2 + 4
9*(sqrt(-x + 1) - 1)^2/x + 5)*x^(7/2)/(sqrt(-x + 1) - 1)^7 - 1/4*arccos(sq
rt(x))/x^4
```

3.67.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos\left(\sqrt{x}\right)}{x^5}\,dx = \int \frac{\mathrm{acos}\left(\sqrt{x}\right)}{x^5}\,dx$$

input
```
int(acos(x^(1/2))/x^5,x)
```

output
```
int(acos(x^(1/2))/x^5, x)
```

3.68 $\int \frac{\arccos(\sqrt{x})}{\sqrt{x}}\,dx$

3.68.1 Optimal result

Integrand size = 12, antiderivative size = 25

$$\int \frac{\arccos\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -2\sqrt{1-x}+2\sqrt{x}\arccos\left(\sqrt{x}\right)$$

output
```
-2*(1-x)^(1/2)+2*arccos(x^(1/2))*x^(1/2)
```

3.68.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\arccos\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = -2\sqrt{1-x}+2\sqrt{x}\arccos\left(\sqrt{x}\right)$$

input
```
Integrate[ArcCos[Sqrt[x]]/Sqrt[x],x]
```

output
```
-2*Sqrt[1 - x] + 2*Sqrt[x]*ArcCos[Sqrt[x]]
```

3.68.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {7266, 5131, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos\left(\sqrt{x}\right)}{\sqrt{x}}dx$$
$$\Big\downarrow \text{7266}$$
$$2\int \arccos\left(\sqrt{x}\right)d\sqrt{x}$$
$$\Big\downarrow \text{5131}$$
$$2\left(\int \frac{\sqrt{x}}{\sqrt{1-x}}d\sqrt{x}+\sqrt{x}\arccos\left(\sqrt{x}\right)\right)$$
$$\Big\downarrow \text{241}$$
$$2\left(\sqrt{x}\arccos\left(\sqrt{x}\right)-\sqrt{1-x}\right)$$

input
```
Int[ArcCos[Sqrt[x]]/Sqrt[x],x]
```

output
```
2*(-Sqrt[1 - x] + Sqrt[x]*ArcCos[Sqrt[x]])
```

3.68.3.1 Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5131
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Simp[b*c*n   Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

3.68.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$-2\sqrt{1-x}+2\arccos\left(\sqrt{x}\right)\sqrt{x}$	20
default	$-2\sqrt{1-x}+2\arccos\left(\sqrt{x}\right)\sqrt{x}$	20

input
```
int(arccos(x^(1/2))/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(1-x)^(1/2)+2*arccos(x^(1/2))*x^(1/2)
```

3.68.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{\arccos\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\,\sqrt{x}\arccos\left(\sqrt{x}\right) - 2\,\sqrt{-x+1}$$

input
```
integrate(arccos(x^(1/2))/x^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(x)*arccos(sqrt(x)) - 2*sqrt(-x + 1)
```

3.68.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{\arccos\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\sqrt{x}\,\mathrm{acos}\left(\sqrt{x}\right) - 2\sqrt{1-x}$$

input
```
integrate(acos(x**(1/2))/x**(1/2),x)
```

output
```
2*sqrt(x)*acos(sqrt(x)) - 2*sqrt(1 - x)
```

3.68.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{\arccos\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\sqrt{x}\arccos\left(\sqrt{x}\right) - 2\sqrt{-x+1}$$

input
```
integrate(arccos(x^(1/2))/x^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(x)*arccos(sqrt(x)) - 2*sqrt(-x + 1)
```

3.68.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{\arccos\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\sqrt{x}\arccos\left(\sqrt{x}\right) - 2\sqrt{-x+1}$$

input
```
integrate(arccos(x^(1/2))/x^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(x)*arccos(sqrt(x)) - 2*sqrt(-x + 1)
```

3.68.9 Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int \frac{\arccos\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\sqrt{x}\,\mathrm{acos}\left(\sqrt{x}\right) - 2\sqrt{1-x}$$

input
```
int(acos(x^(1/2))/x^(1/2),x)
```

output
```
2*x^(1/2)*acos(x^(1/2)) - 2*(1 - x)^(1/2)
```

3.69 $\int \frac{\arccos(ax^n)}{x} dx$

3.69.1 Optimal result

Integrand size = 10, antiderivative size = 68

$$\int \frac{\arccos(ax^n)}{x} dx = -\frac{i\arccos(ax^n)^2}{2n} + \frac{\arccos(ax^n)\log\left(1+e^{2i\arccos(ax^n)}\right)}{n} - \frac{i\,\text{PolyLog}\left(2,-e^{2i\arccos(ax^n)}\right)}{2n}$$

output
```
-1/2*I*arccos(a*x^n)^2/n+arccos(a*x^n)*ln(1+(a*x^n+I*(1-a^2*(x^n)^2)^(1/2)
)^2)/n-1/2*I*polylog(2,-(a*x^n+I*(1-a^2*(x^n)^2)^(1/2))^2)/n
```

3.69.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 141 vs. 2(68) = 136.

Time = 0.16 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.07

$$\int \frac{\arccos(ax^n)}{x} dx = \arccos(ax^n)\log(x) + \frac{a\left(-\text{arcsinh}\left(\sqrt{-a^2}x^n\right)^2 - 2\text{arcsinh}\left(\sqrt{-a^2}x^n\right)\log\left(1-e^{-2\text{arcsinh}\left(\sqrt{-a^2}x^n\right)}\right) + 2n\log(x)\log\left(\sqrt{-a^2}x^n +\right.\right.}{2\sqrt{-a^2}n}$$

input
```
Integrate[ArcCos[a*x^n]/x,x]
```

output
```
ArcCos[a*x^n]*Log[x] + (a*(-ArcSinh[Sqrt[-a^2]*x^n]^2 - 2*ArcSinh[Sqrt[-a^
2]*x^n]*Log[1 - E^(-2*ArcSinh[Sqrt[-a^2]*x^n])] + 2*n*Log[x]*Log[Sqrt[-a^2
]*x^n + Sqrt[1 - a^2*x^(2*n)]] + PolyLog[2, E^(-2*ArcSinh[Sqrt[-a^2]*x^n])
]))/(2*Sqrt[-a^2]*n)
```

3.69.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5330, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax^n)}{x}\,dx$$

$$\Big\downarrow \text{5330}$$

$$-\frac{\int \frac{x^{-n}\sqrt{1-a^2x^{2n}}\arccos(ax^n)}{a}d\arccos(ax^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \arccos(ax^n)\tan(\arccos(ax^n))\,d\arccos(ax^n)}{n}$$

$$\Big\downarrow \text{4202}$$

$$-\frac{\frac{1}{2}i\arccos(ax^n)^2-2i\int \frac{e^{2i\arccos(ax^n)}\arccos(ax^n)}{1+e^{2i\arccos(ax^n)}}d\arccos(ax^n)}{n}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{\frac{1}{2}i\arccos(ax^n)^2-2i\left(\frac{1}{2}i\int \log\left(1+e^{2i\arccos(ax^n)}\right)d\arccos(ax^n)-\frac{1}{2}i\arccos(ax^n)\log\left(1+e^{2i\arccos(ax^n)}\right)\right)}{n}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{\frac{1}{2}i\arccos(ax^n)^2-2i\left(\frac{1}{4}\int e^{-2i\arccos(ax^n)}\log\left(1+e^{2i\arccos(ax^n)}\right)de^{2i\arccos(ax^n)}-\frac{1}{2}i\arccos(ax^n)\log\left(1+e^{2i\arccos(\right.\right.}{n}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{\frac{1}{2}i\arccos(ax^n)^2-2i\left(-\frac{1}{4}\operatorname{PolyLog}\left(2,-e^{2i\arccos(ax^n)}\right)-\frac{1}{2}i\arccos(ax^n)\log\left(1+e^{2i\arccos(ax^n)}\right)\right)}{n}$$

input
```
Int[ArcCos[a*x^n]/x,x]
```

output
```
-(((I/2)*ArcCos[a*x^n]^2 - (2*I)*((-1/2*I)*ArcCos[a*x^n]*Log[1 + E^((2*I)*
ArcCos[a*x^n])] - PolyLog[2, -E^((2*I)*ArcCos[a*x^n])]/4))/n)
```

3.69.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5330
```
Int[ArcCos[(a_.)*(x_)^(p_)]^(n_.)/(x_), x_Symbol] :> Simp[-p^(-1)   Subst[I
nt[x^n*Tan[x], x], x, ArcCos[a*x^p]], x] /; FreeQ[{a, p}, x] && IGtQ[n, 0]
```

3.69.4 Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.24

method	result	size
derivativedivides	$\frac{-\frac{i\arccos(a\,x^n)^2}{2}+\arccos(a\,x^n)\ln\left(1+\left(a\,x^n+i\sqrt{1-a^2x^{2n}}\right)^2\right)-\frac{i\,\mathrm{polylog}\left(2,-\left(a\,x^n+i\sqrt{1-a^2x^{2n}}\right)^2\right)}{2}}{n}$	84
default	$\frac{-\frac{i\arccos(a\,x^n)^2}{2}+\arccos(a\,x^n)\ln\left(1+\left(a\,x^n+i\sqrt{1-a^2x^{2n}}\right)^2\right)-\frac{i\,\mathrm{polylog}\left(2,-\left(a\,x^n+i\sqrt{1-a^2x^{2n}}\right)^2\right)}{2}}{n}$	84

input
```
int(arccos(a*x^n)/x,x,method=_RETURNVERBOSE)
```

output
```
1/n*(-1/2*I*arccos(a*x^n)^2+arccos(a*x^n)*ln(1+(a*x^n+I*(1-a^2*(x^n)^2)^(1
/2))^2)-1/2*I*polylog(2,-(a*x^n+I*(1-a^2*(x^n)^2)^(1/2))^2))
```

3.69.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{\arccos(ax^n)}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arccos(a*x^n)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.69.6 Sympy [F]

$$\int \frac{\arccos(ax^n)}{x}\,dx = \int \frac{\operatorname{acos}(ax^n)}{x}\,dx$$

input
```
integrate(acos(a*x**n)/x,x)
```

output
```
Integral(acos(a*x**n)/x, x)
```

3.69.7 Maxima [F]

$$\int \frac{\arccos(ax^n)}{x}\,dx = \int \frac{\arccos(ax^n)}{x}\,dx$$

input
```
integrate(arccos(a*x^n)/x,x, algorithm="maxima")
```

output
```
-a*n*integrate(sqrt(a*x^n + 1)*sqrt(-a*x^n + 1)*x^n*log(x)/(a^2*x*x^(2*n)
- x), x) + arctan(sqrt(a*x^n + 1)*sqrt(-a*x^n + 1)/(a*x^n))*log(x)
```

3.69.8 Giac [F]

$$\int \frac{\arccos(ax^n)}{x}\,dx = \int \frac{\arccos(ax^n)}{x}\,dx$$

input
```
integrate(arccos(a*x^n)/x,x, algorithm="giac")
```

output
```
integrate(arccos(a*x^n)/x, x)
```

3.69.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax^n)}{x}\,dx = \int \frac{\mathrm{acos}(a\,x^n)}{x}\,dx$$

input
```
int(acos(a*x^n)/x,x)
```

output
```
int(acos(a*x^n)/x, x)
```

3.70 $\int \frac{\arccos(ax^5)}{x} dx$

3.70.1 Optimal result

Integrand size = 10, antiderivative size = 62

$$\int \frac{\arccos(ax^5)}{x} dx = -\frac{1}{10} i \arccos(ax^5)^2 + \frac{1}{5} \arccos(ax^5) \log\left(1 + e^{2i\arccos(ax^5)}\right) - \frac{1}{10} i \operatorname{PolyLog}\left(2, -e^{2i\arccos(ax^5)}\right)$$

output
```
-1/10*I*arccos(a*x^5)^2+1/5*arccos(a*x^5)*ln(1+(a*x^5+I*(-a^2*x^10+1)^(1/2
))^2)-1/10*I*polylog(2,-(a*x^5+I*(-a^2*x^10+1)^(1/2))^2)
```

3.70.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.90

$$\int \frac{\arccos(ax^5)}{x} dx = -\frac{1}{10} i \left(\arccos(ax^5) \left(\arccos(ax^5) + 2i \log\left(1 + e^{2i\arccos(ax^5)}\right)\right) + \operatorname{PolyLog}\left(2, -e^{2i\arccos(ax^5)}\right)\right)$$

input
```
Integrate[ArcCos[a*x^5]/x,x]
```

output
```
(-1/10*I)*(ArcCos[a*x^5]*(ArcCos[a*x^5] + (2*I)*Log[1 + E^((2*I)*ArcCos[a*
x^5])]) + PolyLog[2, -E^((2*I)*ArcCos[a*x^5])])
```

3.70.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5330, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos(ax^5)}{x}\,dx$$

$$\Big\downarrow \text{5330}$$

$$-\frac{1}{5}\int \frac{\sqrt{1-a^2x^{10}}\arccos(ax^5)}{ax^5}d\arccos(ax^5)$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{5}\int \arccos(ax^5)\tan(\arccos(ax^5))\,d\arccos(ax^5)$$

$$\Big\downarrow \text{4202}$$

$$\frac{1}{5}\left(2i\int \frac{e^{2i\arccos(ax^5)}\arccos(ax^5)}{1+e^{2i\arccos(ax^5)}}d\arccos(ax^5)-\frac{1}{2}i\arccos(ax^5)^2\right)$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{5}\left(2i\left(\frac{1}{2}i\int \log\left(1+e^{2i\arccos(ax^5)}\right)d\arccos(ax^5)-\frac{1}{2}i\arccos(ax^5)\log\left(1+e^{2i\arccos(ax^5)}\right)\right)-\frac{1}{2}i\arccos(ax^5)^2\right)$$

$$\Big\downarrow \text{2715}$$

$$\frac{1}{5}\left(2i\left(\frac{1}{4}\int e^{-2i\arccos(ax^5)}\log\left(1+e^{2i\arccos(ax^5)}\right)de^{2i\arccos(ax^5)}-\frac{1}{2}i\arccos(ax^5)\log\left(1+e^{2i\arccos(ax^5)}\right)\right)-\frac{1}{2}i\,\text{ar}$$

$$\Big\downarrow \text{2838}$$

$$\frac{1}{5}\left(2i\left(-\frac{1}{4}\text{PolyLog}\left(2,-e^{2i\arccos(ax^5)}\right)-\frac{1}{2}i\arccos(ax^5)\log\left(1+e^{2i\arccos(ax^5)}\right)\right)-\frac{1}{2}i\arccos(ax^5)^2\right)$$

input
```
Int[ArcCos[a*x^5]/x,x]
```

output
```
((-1/2*I)*ArcCos[a*x^5]^2 + (2*I)*((-1/2*I)*ArcCos[a*x^5]*Log[1 + E^((2*I)
*ArcCos[a*x^5])] - PolyLog[2, -E^((2*I)*ArcCos[a*x^5])]/4))/5
```

3.70.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5330
```
Int[ArcCos[(a_.)*(x_)^(p_)]^(n_.)/(x_), x_Symbol] :> Simp[-p^(-1)   Subst[I
nt[x^n*Tan[x], x], x, ArcCos[a*x^p]], x] /; FreeQ[{a, p}, x] && IGtQ[n, 0]
```

3.70.4 Maple [F]

$$\int \frac{\arccos\left(a\,x^5\right)}{x}dx$$

input
```
int(arccos(a*x^5)/x,x)
```

output
```
int(arccos(a*x^5)/x,x)
```

3.70.5 Fricas [F]

$$\int \frac{\arccos(ax^5)}{x}\,dx = \int \frac{\arccos(ax^5)}{x}\,dx$$

input
```
integrate(arccos(a*x^5)/x,x, algorithm="fricas")
```

output
```
integral(arccos(a*x^5)/x, x)
```

3.70.6 Sympy [F]

$$\int \frac{\arccos(ax^5)}{x}\,dx = \int \frac{\operatorname{acos}(ax^5)}{x}\,dx$$

input
```
integrate(acos(a*x**5)/x,x)
```

output
```
Integral(acos(a*x**5)/x, x)
```

3.70.7 Maxima [F]

$$\int \frac{\arccos(ax^5)}{x}\,dx = \int \frac{\arccos(ax^5)}{x}\,dx$$

input
```
integrate(arccos(a*x^5)/x,x, algorithm="maxima")
```

output
```
integrate(arccos(a*x^5)/x, x)
```

3.70.8 Giac [F]

$$\int \frac{\arccos(ax^5)}{x}\,dx = \int \frac{\arccos(ax^5)}{x}\,dx$$

input
```
integrate(arccos(a*x^5)/x,x, algorithm="giac")
```

output
```
integrate(arccos(a*x^5)/x, x)
```

3.70.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax^5)}{x}\,dx = \int \frac{\mathrm{acos}(a\,x^5)}{x}\,dx$$

input
```
int(acos(a*x^5)/x,x)
```

output
```
int(acos(a*x^5)/x, x)
```

3.71 $\int x^3 \arccos(a+bx^4)\,dx$

3.71.1 Optimal result

Integrand size = 12, antiderivative size = 47

$$\int x^3 \arccos(a+bx^4)\,dx = -\frac{\sqrt{1-(a+bx^4)^2}}{4b}+\frac{(a+bx^4)\arccos(a+bx^4)}{4b}$$

output
```
1/4*(b*x^4+a)*arccos(b*x^4+a)/b-1/4*(1-(b*x^4+a)^2)^(1/2)/b
```

3.71.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int x^3 \arccos(a+bx^4)\,dx = \frac{-\sqrt{1-(a+bx^4)^2}+(a+bx^4)\arccos(a+bx^4)}{4b}$$

input
```
Integrate[x^3*ArcCos[a + b*x^4],x]
```

output
```
(-Sqrt[1 - (a + b*x^4)^2] + (a + b*x^4)*ArcCos[a + b*x^4])/(4*b)
```

3.71.3 Rubi [A] (warning: unable to verify)

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {7266, 5303, 5131, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \arccos\left(a+bx^4\right) dx$$

$$\Big\downarrow \ 7266$$

$$\frac{1}{4}\int \arccos\left(bx^4+a\right) dx^4$$

$$\Big\downarrow \ 5303$$

$$\frac{\int \arccos\left(bx^4+a\right) d\left(bx^4+a\right)}{4b}$$

$$\Big\downarrow \ 5131$$

$$\frac{\int \frac{bx^4+a}{\sqrt{1-x^8}} d\left(bx^4+a\right)+\left(a+bx^4\right)\arccos\left(a+bx^4\right)}{4b}$$

$$\Big\downarrow \ 241$$

$$\frac{\left(a+bx^4\right)\arccos\left(a+bx^4\right)-\sqrt{1-x^8}}{4b}$$

input
```
Int[x^3*ArcCos[a + b*x^4],x]
```

output
```
(-Sqrt[1 - x^8] + (a + b*x^4)*ArcCos[a + b*x^4])/(4*b)
```

3.71.3.1 Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5131
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Simp[b*c*n   Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5303
```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
  Subst[Int[(a + b*ArcCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
n}, x]
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

3.71.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{(b\,x^4+a)\arccos(b\,x^4+a)-\sqrt{1-(b\,x^4+a)^2}}{4b}$	40
default	$\frac{(b\,x^4+a)\arccos(b\,x^4+a)-\sqrt{1-(b\,x^4+a)^2}}{4b}$	40

input
```
int(x^3*arccos(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4/b*((b*x^4+a)*arccos(b*x^4+a)-(1-(b*x^4+a)^2)^(1/2))
```

3.71.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int x^3 \arccos\left(a+bx^4\right)\,dx = \frac{(bx^4+a)\arccos\left(bx^4+a\right)-\sqrt{-b^2x^8-2\,abx^4-a^2+1}}{4\,b}$$

input
```
integrate(x^3*arccos(b*x^4+a),x, algorithm="fricas")
```

output
```
1/4*((b*x^4 + a)*arccos(b*x^4 + a) - sqrt(-b^2*x^8 - 2*a*b*x^4 - a^2 + 1))
/b
```

3.71.6 Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.30

$$\int x^3 \arccos\left(a+bx^4\right)\,dx = \begin{cases} \frac{a\operatorname{acos}(a+bx^4)}{4b} + \frac{x^4\operatorname{acos}(a+bx^4)}{4} - \frac{\sqrt{-a^2-2abx^4-b^2x^8+1}}{4b} & \text{for } b\neq 0 \\ \frac{x^4\operatorname{acos}(a)}{4} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*acos(b*x**4+a),x)
```

output
```
Piecewise((a*acos(a + b*x**4)/(4*b) + x**4*acos(a + b*x**4)/4 - sqrt(-a**2
 - 2*a*b*x**4 - b**2*x**8 + 1)/(4*b), Ne(b, 0)), (x**4*acos(a)/4, True))
```

3.71.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int x^3 \arccos\left(a+bx^4\right)\,dx = \frac{(bx^4+a)\arccos\left(bx^4+a\right) - \sqrt{-\left(bx^4+a\right)^2+1}}{4\,b}$$

input
```
integrate(x^3*arccos(b*x^4+a),x, algorithm="maxima")
```

output
```
1/4*((b*x^4 + a)*arccos(b*x^4 + a) - sqrt(-(b*x^4 + a)^2 + 1))/b
```

3.71.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int x^3 \arccos\left(a+bx^4\right)\,dx = \frac{(bx^4+a)\arccos\left(bx^4+a\right) - \sqrt{-\left(bx^4+a\right)^2+1}}{4\,b}$$

input
```
integrate(x^3*arccos(b*x^4+a),x, algorithm="giac")
```

output
```
1/4*((b*x^4 + a)*arccos(b*x^4 + a) - sqrt(-(b*x^4 + a)^2 + 1))/b
```

3.71.9 Mupad [B] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.11

$$\int x^3 \arccos\left(a+bx^4\right)\,dx = \frac{x^4\,\mathrm{acos}(b\,x^4+a)}{4} - \frac{\sqrt{-a^2-2\,a\,b\,x^4-b^2\,x^8+1}}{4\,b} - \frac{a\,\ln\left(\sqrt{-a^2-2\,a\,b\,x^4-b^2\,x^8+1}-\frac{b^2\,x^4+a\,b}{\sqrt{-b^2}}\right)}{4\,\sqrt{-b^2}}$$

input
```
int(x^3*acos(a + b*x^4),x)
```

output
```
(x^4*acos(a + b*x^4))/4 - (1 - b^2*x^8 - 2*a*b*x^4 - a^2)^(1/2)/(4*b) - (a
*log((1 - b^2*x^8 - 2*a*b*x^4 - a^2)^(1/2) - (a*b + b^2*x^4)/(-b^2)^(1/2))
)/(4*(-b^2)^(1/2))
```

3.72 $\int x^{-1+n} \arccos(a + bx^n)\, dx$

3.72.1 Optimal result

Integrand size = 14, antiderivative size = 48

$$\int x^{-1+n} \arccos(a + bx^n)\, dx = -\frac{\sqrt{1-(a+bx^n)^2}}{bn} + \frac{(a+bx^n)\arccos(a+bx^n)}{bn}$$

output
```
(a+b*x^n)*arccos(a+b*x^n)/b/n-(1-(a+b*x^n)^2)^(1/2)/b/n
```

3.72.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

$$\int x^{-1+n} \arccos(a + bx^n)\, dx = \frac{-\sqrt{1-(a+bx^n)^2} + (a+bx^n)\arccos(a+bx^n)}{bn}$$

input
```
Integrate[x^(-1 + n)*ArcCos[a + b*x^n],x]
```

output
```
(-Sqrt[1 - (a + b*x^n)^2] + (a + b*x^n)*ArcCos[a + b*x^n])/(b*n)
```

3.72.3 Rubi [A] (warning: unable to verify)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.81, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {7266, 5303, 5131, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{n-1} \arccos(a+bx^n)\,dx$$

$$\Big\downarrow \text{ 7266}$$

$$\frac{\int \arccos(bx^n+a)\,dx^n}{n}$$

$$\Big\downarrow \text{ 5303}$$

$$\frac{\int \arccos(bx^n+a)\,d(bx^n+a)}{bn}$$

$$\Big\downarrow \text{ 5131}$$

$$\frac{\int \frac{bx^n+a}{\sqrt{1-x^{2n}}}d(bx^n+a)+(a+bx^n)\arccos(a+bx^n)}{bn}$$

$$\Big\downarrow \text{ 241}$$

$$\frac{(a+bx^n)\arccos(a+bx^n)-\sqrt{1-x^{2n}}}{bn}$$

input
```
Int[x^(-1 + n)*ArcCos[a + b*x^n],x]
```

output
```
(-Sqrt[1 - x^(2*n)] + (a + b*x^n)*ArcCos[a + b*x^n])/(b*n)
```

3.72.3.1 Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5131
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Simp[b*c*n   Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
 c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

rule 5303
```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[1/d
  Subst[Int[(a + b*ArcCos[x])^n, x], x, c + d*x], x] /; FreeQ[{a, b, c, d,
n}, x]
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

3.72.4 Maple [F]

$$\int x^{n-1} \arccos\left(a+b\,x^{n}\right)dx$$

input
```
int(x^(n-1)*arccos(a+b*x^n),x)
```

output
```
int(x^(n-1)*arccos(a+b*x^n),x)
```

3.72.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.23

$$\int x^{-1+n}\arccos\left(a+bx^{n}\right)\,dx$$
$$=\frac{bx^{n}\arccos\left(bx^{n}+a\right)+a\arccos\left(bx^{n}+a\right)-\sqrt{-b^{2}x^{2\,n}-2\,abx^{n}-a^{2}+1}}{bn}$$

input
```
integrate(x^(-1+n)*arccos(a+b*x^n),x, algorithm="fricas")
```

output
```
(b*x^n*arccos(b*x^n + a) + a*arccos(b*x^n + a) - sqrt(-b^2*x^(2*n) - 2*a*b
*x^n - a^2 + 1))/(b*n)
```

3.72.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(34) = 68$.

Time = 12.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.67

$$\int x^{-1+n}\arccos\left(a+bx^n\right)\,dx = \begin{cases} \log\left(x\right)\operatorname{acos}\left(a\right) & \text{for}\ b=0 \wedge n=0 \\ \frac{xx^{n-1}\operatorname{acos}\left(a\right)}{n} & \text{for}\ b=0 \\ \log\left(x\right)\operatorname{acos}\left(a+b\right) & \text{for}\ n=0 \\ \frac{a\operatorname{acos}\left(a+bx^n\right)}{bn}+\frac{x^n\operatorname{acos}\left(a+bx^n\right)}{n}-\frac{\sqrt{-a^2-2abx^n-b^2x^{2n}+1}}{bn} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(-1+n)*acos(a+b*x**n),x)
```

output
```
Piecewise((log(x)*acos(a), Eq(b, 0) & Eq(n, 0)), (x*x**(n - 1)*acos(a)/n,
Eq(b, 0)), (log(x)*acos(a + b), Eq(n, 0)), (a*acos(a + b*x**n)/(b*n) + x**
n*acos(a + b*x**n)/n - sqrt(-a**2 - 2*a*b*x**n - b**2*x**(2*n) + 1)/(b*n),
 True))
```

3.72.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int x^{-1+n}\arccos\left(a+bx^n\right)\,dx = \frac{(bx^n+a)\arccos\left(bx^n+a\right)-\sqrt{-(bx^n+a)^2+1}}{bn}$$

input
```
integrate(x^(-1+n)*arccos(a+b*x^n),x, algorithm="maxima")
```

output
```
((b*x^n + a)*arccos(b*x^n + a) - sqrt(-(b*x^n + a)^2 + 1))/(b*n)
```

3.72.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int x^{-1+n}\arccos(a+bx^n)\,dx=\frac{(bx^n+a)\arccos(bx^n+a)-\sqrt{-(bx^n+a)^2+1}}{bn}$$

input
```
integrate(x^(-1+n)*arccos(a+b*x^n),x, algorithm="giac")
```

output
```
((b*x^n + a)*arccos(b*x^n + a) - sqrt(-(b*x^n + a)^2 + 1))/(b*n)
```

3.72.9 Mupad [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.31

$$\int x^{-1+n}\arccos(a+bx^n)\,dx=\frac{x^n\,\mathrm{acos}(a+b\,x^n)}{n}-\frac{\sqrt{1-b^2\,x^{2\,n}-2\,a\,b\,x^n-a^2}}{b\,n}-\frac{a\,\ln\left(\sqrt{1-b^2\,x^{2\,n}-2\,a\,b\,x^n-a^2}-\frac{a\,b+b^2\,x^n}{\sqrt{-b^2}}\right)}{n\,\sqrt{-b^2}}$$

input
```
int(x^(n - 1)*acos(a + b*x^n),x)
```

output
```
(x^n*acos(a + b*x^n))/n - (1 - b^2*x^(2*n) - 2*a*b*x^n - a^2)^(1/2)/(b*n)
- (a*log((1 - b^2*x^(2*n) - 2*a*b*x^n - a^2)^(1/2) - (a*b + b^2*x^n)/(-b^2
)^(1/2)))/(n*(-b^2)^(1/2))
```

3.73 $\int (a+b\arccos(1+dx^2))^4 \, dx$

3.73.1 Optimal result

Integrand size = 14, antiderivative size = 127

$$\int (a+b\arccos(1+dx^2))^4 \, dx = 384b^4x + \frac{192b^3\sqrt{-2dx^2-d^2x^4}(a+b\arccos(1+dx^2))}{dx} - 48b^2x(a+b\arccos(1+dx^2))^2 - \frac{8b\sqrt{-2dx^2-d^2x^4}(a+b\arccos(1+dx^2))^3}{dx} + x(a+b\arccos(1+dx^2))^4$$

output
```
384*b^4*x-48*b^2*x*(a+b*arccos(d*x^2+1))^2+x*(a+b*arccos(d*x^2+1))^4+192*b
^3*(a+b*arccos(d*x^2+1))*(-d^2*x^4-2*d*x^2)^(1/2)/d/x-8*b*(a+b*arccos(d*x^
2+1))^3*(-d^2*x^4-2*d*x^2)^(1/2)/d/x
```

3.73.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.96

$$\int (a+b\arccos(1+dx^2))^4 \, dx$$

$$= \frac{(a^4-48a^2b^2+384b^4)\,dx^2 - 8ab(a^2-24b^2)\sqrt{-dx^2(2+dx^2)} + 4b\Big(a^3dx^2 - 24ab^2dx^2 - 6a^2b\sqrt{-dx^2(2+}}{}$$

input
```
Integrate[(a + b*ArcCos[1 + d*x^2])^4,x]
```

output
```
((a^4 - 48*a^2*b^2 + 384*b^4)*d*x^2 - 8*a*b*(a^2 - 24*b^2)*Sqrt[-(d*x^2*(2
 + d*x^2))] + 4*b*(a^3*d*x^2 - 24*a*b^2*d*x^2 - 6*a^2*b*Sqrt[-(d*x^2*(2 +
d*x^2))] + 48*b^3*Sqrt[-(d*x^2*(2 + d*x^2))])*ArcCos[1 + d*x^2] + 6*b^2*(a
^2*d*x^2 - 8*b^2*d*x^2 - 4*a*b*Sqrt[-(d*x^2*(2 + d*x^2))])*ArcCos[1 + d*x^
2]^2 + 4*b^3*(a*d*x^2 - 2*b*Sqrt[-(d*x^2*(2 + d*x^2))])*ArcCos[1 + d*x^2]^
3 + b^4*d*x^2*ArcCos[1 + d*x^2]^4)/(d*x)
```

3.73.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5314, 5314, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b \arccos \left(d x^2+1\right)\right)^4 dx$$

$$\Big\downarrow \text{ 5314}$$

$$-48 b^2 \int \left(a+b \arccos \left(d x^2+1\right)\right)^2 dx-\frac{8 b \sqrt{-d^2 x^4-2 d x^2} \left(a+b \arccos \left(d x^2+1\right)\right)^3}{d x}+x \left(a+b \arccos \left(d x^2+1\right)\right)^4$$

$$\Big\downarrow \text{ 5314}$$

$$-48 b^2 \left(-8 b^2 \int 1 dx-\frac{4 b \sqrt{-d^2 x^4-2 d x^2} \left(a+b \arccos \left(d x^2+1\right)\right)}{d x}+x \left(a+b \arccos \left(d x^2+1\right)\right)^2\right)-\frac{8 b \sqrt{-d^2 x^4-2 d x^2} \left(a+b \arccos \left(d x^2+1\right)\right)^3}{d x}+x \left(a+b \arccos \left(d x^2+1\right)\right)^4$$

$$\Big\downarrow \text{ 24}$$

$$-48 b^2 \left(-\frac{4 b \sqrt{-d^2 x^4-2 d x^2} \left(a+b \arccos \left(d x^2+1\right)\right)}{d x}+x \left(a+b \arccos \left(d x^2+1\right)\right)^2-8 b^2 x\right)-\frac{8 b \sqrt{-d^2 x^4-2 d x^2} \left(a+b \arccos \left(d x^2+1\right)\right)^3}{d x}+x \left(a+b \arccos \left(d x^2+1\right)\right)^4$$

input
```
Int[(a + b*ArcCos[1 + d*x^2])^4,x]
```

output
```
(-8*b*Sqrt[-2*d*x^2 - d^2*x^4]*(a + b*ArcCos[1 + d*x^2])^3)/(d*x) + x*(a +
 b*ArcCos[1 + d*x^2])^4 - 48*b^2*(-8*b^2*x - (4*b*Sqrt[-2*d*x^2 - d^2*x^4]
*(a + b*ArcCos[1 + d*x^2]))/(d*x) + x*(a + b*ArcCos[1 + d*x^2])^2)
```

3.73.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 5314
```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcCos[c + d*x^2])^n, x] + (-Simp[2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*((
a + b*ArcCos[c + d*x^2])^(n - 1)/(d*x)), x] - Simp[4*b^2*n*(n - 1)   Int[(a
 + b*ArcCos[c + d*x^2])^(n - 2), x], x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c
^2, 1] && GtQ[n, 1]
```

3.73.4 Maple [F]

$$\int \left(a + b \arccos\left(d\,x^2 + 1\right)\right)^4 dx$$

input
```
int((a+b*arccos(d*x^2+1))^4,x)
```

output
```
int((a+b*arccos(d*x^2+1))^4,x)
```

3.73.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.63

$$\int \left(a + b \arccos\left(1 + dx^2\right)\right)^4 \, dx$$
$$= \frac{b^4 dx^2 \arccos\left(dx^2+1\right)^4 + 4\,ab^3 dx^2 \arccos\left(dx^2+1\right)^3 + 6\left(a^2b^2 - 8\,b^4\right)dx^2 \arccos\left(dx^2+1\right)^2 + 4\left(a^3b - 24\,a\right.}{}$$

input
```
integrate((a+b*arccos(d*x^2+1))^4,x, algorithm="fricas")
```

output
```
(b^4*d*x^2*arccos(d*x^2 + 1)^4 + 4*a*b^3*d*x^2*arccos(d*x^2 + 1)^3 + 6*(a^
2*b^2 - 8*b^4)*d*x^2*arccos(d*x^2 + 1)^2 + 4*(a^3*b - 24*a*b^3)*d*x^2*arcc
os(d*x^2 + 1) + (a^4 - 48*a^2*b^2 + 384*b^4)*d*x^2 - 8*(b^4*arccos(d*x^2 +
 1)^3 + 3*a*b^3*arccos(d*x^2 + 1)^2 + a^3*b - 24*a*b^3 + 3*(a^2*b^2 - 8*b^
4)*arccos(d*x^2 + 1))*sqrt(-d^2*x^4 - 2*d*x^2))/(d*x)
```

3.73.6 Sympy [F]

$$\int \left(a+b\arccos\left(1+dx^2\right)\right)^4\,dx=\int \left(a+b\operatorname{acos}\left(dx^2+1\right)\right)^4\,dx$$

input
```
integrate((a+b*acos(d*x**2+1))**4,x)
```

output
```
Integral((a + b*acos(d*x**2 + 1))**4, x)
```

3.73.7 Maxima [F(-2)]

Exception generated.

$$\int \left(a+b\arccos\left(1+dx^2\right)\right)^4\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(d*x^2+1))^4,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: sign: argument cannot be imagi
nary; found sqrt((-_SAGE_VAR_d*_SAGE_VAR_x^2)-2)
```

3.73.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 577 vs. $2(123) = 246$.

Time = 1.04 (sec) , antiderivative size = 577, normalized size of antiderivative = 4.54

$$\int \left(a+b\arccos\left(1+dx^2\right)\right)^4\,dx$$

$$= 4\left(x\arccos\left(dx^2+1\right)+\frac{2\sqrt{2}\sqrt{-d}\mathrm{sgn}(x)}{d}-\frac{2\sqrt{-d^2x^2-2\,d}}{d\mathrm{sgn}\left(x\right)}\right)a^3b$$

$$+6\left(x\arccos\left(dx^2+1\right)^2-\frac{8\sqrt{2}\sqrt{-d}\mathrm{sgn}(x)}{|d|}-\frac{4\left(\sqrt{-d^2x^2-2\,d}\arccos\left(dx^2+1\right)-\frac{2\left(\sqrt{2}\sqrt{-d}-\sqrt{d^2x^2}\right)d}{|d|}\right)}{d\mathrm{sgn}\left(x\right)}\right.$$

$$+4\left(x\arccos\left(dx^2+1\right)^3-\frac{24\left(\sqrt{2}\pi\sqrt{-d}|d|+2\sqrt{2}\sqrt{-d}d\right)\mathrm{sgn}(x)}{d^2}-\frac{6\left(\sqrt{-d^2x^2-2\,d}\arccos\left(dx^2+1\right)^2\right.}{}\right.$$

$$+\left(x\arccos\left(dx^2+1\right)^4-\frac{48\left(\sqrt{2}\pi^2\sqrt{-d}-8\sqrt{2}\sqrt{-d}\right)\mathrm{sgn}(x)}{|d|}-\frac{8\left(\sqrt{-d^2x^2-2\,d}\arccos\left(dx^2+1\right)^3-\frac{6}{}\right.}{}\right.$$

$$+a^4x$$

input
```
integrate((a+b*arccos(d*x^2+1))^4,x, algorithm="giac")
```

3.73. $\int \left(a+b\arccos\left(1+dx^2\right)\right)^4\,dx$

output
```
4*(x*arccos(d*x^2 + 1) + 2*sqrt(2)*sqrt(-d)*sgn(x)/d - 2*sqrt(-d^2*x^2 - 2
*d)/(d*sgn(x)))*a^3*b + 6*(x*arccos(d*x^2 + 1)^2 - 8*sqrt(2)*sqrt(-d)*sgn(
x)/abs(d) - 4*(sqrt(-d^2*x^2 - 2*d)*arccos(d*x^2 + 1) - 2*(sqrt(2)*sqrt(-d
) - sqrt(d^2*x^2))*d/abs(d))/(d*sgn(x)))*a^2*b^2 + 4*(x*arccos(d*x^2 + 1)^
3 - 24*(sqrt(2)*pi*sqrt(-d)*abs(d) + 2*sqrt(2)*sqrt(-d)*d)*sgn(x)/d^2 - 6*
(sqrt(-d^2*x^2 - 2*d)*arccos(d*x^2 + 1)^2 + 4*(sqrt(d^2*x^2)*arccos((d^2*x
^2 + d)/d) + 2*(sqrt(2)*sqrt(-d) - sqrt(-d^2*x^2 - 2*d))*d/abs(d) - (sqrt(
2)*pi*sqrt(-d)*abs(d) + 2*sqrt(2)*sqrt(-d)*d)/abs(d))*d/abs(d))/(d*sgn(x))
)*a*b^3 + (x*arccos(d*x^2 + 1)^4 - 48*(sqrt(2)*pi^2*sqrt(-d) - 8*sqrt(2)*s
qrt(-d))*sgn(x)/abs(d) - 8*(sqrt(-d^2*x^2 - 2*d)*arccos(d*x^2 + 1)^3 - 6*(
sqrt(2)*pi^2*sqrt(-d) - sqrt(d^2*x^2)*arccos((d^2*x^2 + d)/d)^2 - 8*sqrt(2
)*sqrt(-d) + 2*(pi*sqrt(-d^2*x^2 - 2*d) + 2*sqrt(-d^2*x^2 - 2*d)*arcsin(-(
d^2*x^2 + d)/d) - 4*(sqrt(2)*sqrt(-d) - sqrt(d^2*x^2))*d/abs(d) + 4*sqrt(2
)*sqrt(-d)*d/abs(d))*d/abs(d))*d/abs(d))/(d*sgn(x)))*b^4 + a^4*x
```

3.73.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+b\arccos\left(1+dx^2\right)\right)^4\,dx=\int \left(a+b\,\mathrm{acos}\left(d\,x^2+1\right)\right)^4\,dx$$

input
```
int((a + b*acos(d*x^2 + 1))^4,x)
```

output
```
int((a + b*acos(d*x^2 + 1))^4, x)
```

3.74 $\int \left(a+b\arccos\left(1+dx^2\right)\right)^3\,dx$

3.74.1 Optimal result

Integrand size = 14, antiderivative size = 110

$$\int \left(a+b\arccos\left(1+dx^2\right)\right)^3\,dx=-24ab^2x+\frac{48b^3\sqrt{-2dx^2-d^2x^4}}{dx}-24b^3x\arccos\left(1+dx^2\right)-\frac{6b\sqrt{-2dx^2-d^2x^4}\left(a+b\arccos\left(1+dx^2\right)\right)^2}{dx}+x\left(a+b\arccos\left(1+dx^2\right)\right)^3$$

output
```
-24*a*b^2*x-24*b^3*x*arccos(d*x^2+1)+x*(a+b*arccos(d*x^2+1))^3+48*b^3*(-d^
2*x^4-2*d*x^2)^(1/2)/d/x-6*b*(a+b*arccos(d*x^2+1))^2*(-d^2*x^4-2*d*x^2)^(1
/2)/d/x
```

3.74.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.47

$$\int \left(a+b\arccos\left(1+dx^2\right)\right)^3\,dx$$
$$=\frac{a(a^2-24b^2)\,dx^2-6b(a^2-8b^2)\,\sqrt{-dx^2\left(2+dx^2\right)}+3b\left(a^2dx^2-8b^2dx^2-4ab\sqrt{-dx^2\left(2+dx^2\right)}\right)\arccos\left(\right.}{dx}$$

input
```
Integrate[(a + b*ArcCos[1 + d*x^2])^3,x]
```

output
```
(a*(a^2 - 24*b^2)*d*x^2 - 6*b*(a^2 - 8*b^2)*Sqrt[-(d*x^2*(2 + d*x^2))] + 3
*b*(a^2*d*x^2 - 8*b^2*d*x^2 - 4*a*b*Sqrt[-(d*x^2*(2 + d*x^2))])*ArcCos[1 +
 d*x^2] + 3*b^2*(a*d*x^2 - 2*b*Sqrt[-(d*x^2*(2 + d*x^2))])*ArcCos[1 + d*x^
2]^2 + b^3*d*x^2*ArcCos[1 + d*x^2]^3)/(d*x)
```

3.74.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5314, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\arccos\left(dx^2+1\right)\right)^3 dx$$

$$\Big\downarrow \text{5314}$$

$$-24b^2\int \left(a+b\arccos\left(dx^2+1\right)\right)dx-\frac{6b\sqrt{-d^2x^4-2dx^2}\left(a+b\arccos\left(dx^2+1\right)\right)^2}{dx}+x\left(a+b\arccos\left(dx^2+1\right)\right)^3$$

$$\Big\downarrow \text{2009}$$

$$-24b^2\left(ax+bx\arccos\left(dx^2+1\right)-\frac{2b\sqrt{-d^2x^4-2dx^2}}{dx}\right)-\frac{6b\sqrt{-d^2x^4-2dx^2}\left(a+b\arccos\left(dx^2+1\right)\right)^2}{dx}+x\left(a+b\arccos\left(dx^2+1\right)\right)^3$$

input
```
Int[(a + b*ArcCos[1 + d*x^2])^3,x]
```

output
```
(-6*b*Sqrt[-2*d*x^2 - d^2*x^4]*(a + b*ArcCos[1 + d*x^2])^2)/(d*x) + x*(a +
 b*ArcCos[1 + d*x^2])^3 - 24*b^2*(a*x - (2*b*Sqrt[-2*d*x^2 - d^2*x^4])/(d*
x) + b*x*ArcCos[1 + d*x^2])
```

3.74.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5314
```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcCos[c + d*x^2])^n, x] + (-Simp[2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*((
a + b*ArcCos[c + d*x^2])^(n - 1)/(d*x)), x] - Simp[4*b^2*n*(n - 1)   Int[(a
 + b*ArcCos[c + d*x^2])^(n - 2), x], x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c
^2, 1] && GtQ[n, 1]
```

3.74.4 Maple [F]

$$\int \left(a+b\arccos\left(d\,x^{2}+1\right)\right)^{3}dx$$

input
```
int((a+b*arccos(d*x^2+1))^3,x)
```

output
```
int((a+b*arccos(d*x^2+1))^3,x)
```

3.74.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.31

$$\int \left(a+b\arccos\left(1+dx^2\right)\right)^3\,dx$$
$$=\frac{b^3dx^2\arccos\left(dx^2+1\right)^3+3\,ab^2dx^2\arccos\left(dx^2+1\right)^2+3\left(a^2b-8\,b^3\right)dx^2\arccos\left(dx^2+1\right)+\left(a^3-24\,ab^2\right)d}{dx}$$

input
```
integrate((a+b*arccos(d*x^2+1))^3,x, algorithm="fricas")
```

output
```
(b^3*d*x^2*arccos(d*x^2 + 1)^3 + 3*a*b^2*d*x^2*arccos(d*x^2 + 1)^2 + 3*(a^
2*b - 8*b^3)*d*x^2*arccos(d*x^2 + 1) + (a^3 - 24*a*b^2)*d*x^2 - 6*sqrt(-d^
2*x^4 - 2*d*x^2)*(b^3*arccos(d*x^2 + 1)^2 + 2*a*b^2*arccos(d*x^2 + 1) + a^
2*b - 8*b^3))/(d*x)
```

3.74.6 Sympy [F]

$$\int \left(a+b\arccos\left(1+dx^2\right)\right)^3\,dx=\int \left(a+b\,\mathrm{acos}\left(dx^2+1\right)\right)^3\,dx$$

input
```
integrate((a+b*acos(d*x**2+1))**3,x)
```

output
```
Integral((a + b*acos(d*x**2 + 1))**3, x)
```

3.74.7 Maxima [F(-2)]

Exception generated.

$$\int \left(a+b\arccos\left(1+dx^2\right)\right)^3\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(d*x^2+1))^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: sign: argument cannot be imagi
nary; found sqrt((-_SAGE_VAR_d*_SAGE_VAR_x^2)-2)
```

3.74.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(106) = 212$.

Time = 0.66 (sec) , antiderivative size = 335, normalized size of antiderivative = 3.05

$$\int \left(a+b\arccos\left(1+dx^2\right)\right)^3 \, dx$$
$$= 3\left(x\arccos\left(dx^2+1\right)+\frac{2\sqrt{2}\sqrt{-d}\mathrm{sgn}(x)}{d}-\frac{2\sqrt{-d^2x^2-2d}}{d\mathrm{sgn}(x)}\right)a^2b$$
$$+3\left(x\arccos\left(dx^2+1\right)^2-\frac{8\sqrt{2}\sqrt{-d}\mathrm{sgn}(x)}{|d|}-\frac{4\left(\sqrt{-d^2x^2-2d}\arccos\left(dx^2+1\right)-\frac{2\left(\sqrt{2}\sqrt{-d}-\sqrt{d^2x^2}\right)d}{|d|}\right)}{d\mathrm{sgn}(x)}\right)$$
$$+\left(x\arccos\left(dx^2+1\right)^3-\frac{24\left(\sqrt{2}\pi\sqrt{-d}|d|+2\sqrt{2}\sqrt{-d}d\right)\mathrm{sgn}(x)}{d^2}-\frac{6\left(\sqrt{-d^2x^2-2d}\arccos\left(dx^2+1\right)^2+\right.}{}\right.$$
$$+a^3x$$

input
```
integrate((a+b*arccos(d*x^2+1))^3,x, algorithm="giac")
```

output
```
3*(x*arccos(d*x^2 + 1) + 2*sqrt(2)*sqrt(-d)*sgn(x)/d - 2*sqrt(-d^2*x^2 - 2
*d)/(d*sgn(x)))*a^2*b + 3*(x*arccos(d*x^2 + 1)^2 - 8*sqrt(2)*sqrt(-d)*sgn(
x)/abs(d) - 4*(sqrt(-d^2*x^2 - 2*d)*arccos(d*x^2 + 1) - 2*(sqrt(2)*sqrt(-d
) - sqrt(d^2*x^2))*d/abs(d))/(d*sgn(x)))*a*b^2 + (x*arccos(d*x^2 + 1)^3 -
24*(sqrt(2)*pi*sqrt(-d)*abs(d) + 2*sqrt(2)*sqrt(-d)*d)*sgn(x)/d^2 - 6*(sqr
t(-d^2*x^2 - 2*d)*arccos(d*x^2 + 1)^2 + 4*(sqrt(d^2*x^2)*arccos((d^2*x^2 +
 d)/d) + 2*(sqrt(2)*sqrt(-d) - sqrt(-d^2*x^2 - 2*d))*d/abs(d) - (sqrt(2)*p
i*sqrt(-d)*abs(d) + 2*sqrt(2)*sqrt(-d)*d)/abs(d))*d/abs(d))/(d*sgn(x)))*b^
3 + a^3*x
```

3.74.9 Mupad [F(-1)]

Timed out.

$$\int \left(a + b \arccos\left(1 + dx^2\right)\right)^3 \, dx = \int \left(a + b \operatorname{acos}\left(d\,x^2 + 1\right)\right)^3 \, dx$$

input
```
int((a + b*acos(d*x^2 + 1))^3,x)
```

output
```
int((a + b*acos(d*x^2 + 1))^3, x)
```

3.75 $\int (a + b \arccos(1 + dx^2))^2 \, dx$

3.75.1 Optimal result

Integrand size = 14, antiderivative size = 63

$$\int (a + b \arccos(1 + dx^2))^2 \, dx = -8b^2 x - \frac{4b\sqrt{-2dx^2 - d^2x^4}(a + b \arccos(1 + dx^2))}{dx} + x(a + b \arccos(1 + dx^2))^2$$

output
```
-8*b^2*x+x*(a+b*arccos(d*x^2+1))^2-4*b*(a+b*arccos(d*x^2+1))*(-d^2*x^4-2*d
*x^2)^(1/2)/d/x
```

3.75.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.56

$$\int (a + b \arccos(1 + dx^2))^2 \, dx = (a^2 - 8b^2) x - \frac{4ab\sqrt{-dx^2(2 + dx^2)}}{dx} + \frac{2b\left(adx^2 - 2b\sqrt{-dx^2(2 + dx^2)}\right) \arccos(1 + dx^2)}{dx} + b^2 x \arccos(1 + dx^2)^2$$

input
```
Integrate[(a + b*ArcCos[1 + d*x^2])^2,x]
```

output
```
(a^2 - 8*b^2)*x - (4*a*b*Sqrt[-(d*x^2*(2 + d*x^2))])/(d*x) + (2*b*(a*d*x^2
 - 2*b*Sqrt[-(d*x^2*(2 + d*x^2))])*ArcCos[1 + d*x^2])/(d*x) + b^2*x*ArcCos
[1 + d*x^2]^2
```

3.75.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5314, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\arccos(dx^2+1))^2 \, dx$$

$$\Big\downarrow \; 5314$$

$$-8b^2\int 1dx-\frac{4b\sqrt{-d^2x^4-2dx^2}(a+b\arccos(dx^2+1))}{dx}+x(a+b\arccos(dx^2+1))^2$$

$$\Big\downarrow \; 24$$

$$-\frac{4b\sqrt{-d^2x^4-2dx^2}(a+b\arccos(dx^2+1))}{dx}+x(a+b\arccos(dx^2+1))^2-8b^2x$$

input
```
Int[(a + b*ArcCos[1 + d*x^2])^2,x]
```

output
```
-8*b^2*x - (4*b*Sqrt[-2*d*x^2 - d^2*x^4]*(a + b*ArcCos[1 + d*x^2]))/(d*x)
+ x*(a + b*ArcCos[1 + d*x^2])^2
```

3.75.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 5314
```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcCos[c + d*x^2])^n, x] + (-Simp[2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*((
a + b*ArcCos[c + d*x^2])^(n - 1)/(d*x)), x] - Simp[4*b^2*n*(n - 1)   Int[(a
 + b*ArcCos[c + d*x^2])^(n - 2), x], x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c
^2, 1] && GtQ[n, 1]
```

3.75.4 Maple [F]

$$\int \left(a+b\arccos\left(d\,x^{2}+1\right)\right)^{2}dx$$

input
```
int((a+b*arccos(d*x^2+1))^2,x)
```

output
```
int((a+b*arccos(d*x^2+1))^2,x)
```

3.75.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.44

$$\int \left(a+b\arccos\left(1+dx^2\right)\right)^2\,dx$$
$$=\frac{b^2dx^2\arccos\left(dx^2+1\right)^2+2\,abdx^2\arccos\left(dx^2+1\right)+\left(a^2-8\,b^2\right)dx^2-4\,\sqrt{-d^2x^4-2\,dx^2}\left(b^2\arccos\left(dx^2+\right.\right.}{dx}$$

input
```
integrate((a+b*arccos(d*x^2+1))^2,x, algorithm="fricas")
```

output
```
(b^2*d*x^2*arccos(d*x^2 + 1)^2 + 2*a*b*d*x^2*arccos(d*x^2 + 1) + (a^2 - 8*
b^2)*d*x^2 - 4*sqrt(-d^2*x^4 - 2*d*x^2)*(b^2*arccos(d*x^2 + 1) + a*b))/(d*
x)
```

3.75.6 Sympy [F]

$$\int \left(a+b\arccos\left(1+dx^2\right)\right)^2\,dx=\int \left(a+b\,\mathrm{acos}\left(dx^2+1\right)\right)^2\,dx$$

input
```
integrate((a+b*acos(d*x**2+1))**2,x)
```

output
```
Integral((a + b*acos(d*x**2 + 1))**2, x)
```

3.75.7 Maxima [F(-2)]

Exception generated.

$$\int \left(a+b\arccos\left(1+dx^2\right)\right)^2\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(d*x^2+1))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: sign: argument cannot be imagi
nary; found sqrt((-_SAGE_VAR_d*_SAGE_VAR_x^2)-2)
```

3.75.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. 2(61) = 122.

Time = 0.47 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.43

$$\int \left(a+b\arccos\left(1+dx^2\right)\right)^2\,dx = 2\left(x\arccos\left(dx^2+1\right)+\frac{2\sqrt{2}\sqrt{-d}\mathrm{sgn}(x)}{d}-\frac{2\sqrt{-d^2x^2-2\,d}}{d\mathrm{sgn}\left(x\right)}\right)ab + \left(x\arccos\left(dx^2+1\right)^2-\frac{8\sqrt{2}\sqrt{-d}\mathrm{sgn}(x)}{|d|}-\frac{4\left(\sqrt{-d^2x^2-2\,d}\arccos\left(dx^2+1\right)-\frac{2\left(\sqrt{2}\sqrt{-d}-\sqrt{d^2x^2}\right)d}{|d|}\right)}{d\mathrm{sgn}\left(x\right)}\right)b + a^2x$$

input
```
integrate((a+b*arccos(d*x^2+1))^2,x, algorithm="giac")
```

output
```
2*(x*arccos(d*x^2 + 1) + 2*sqrt(2)*sqrt(-d)*sgn(x)/d - 2*sqrt(-d^2*x^2 - 2
*d)/(d*sgn(x)))*a*b + (x*arccos(d*x^2 + 1)^2 - 8*sqrt(2)*sqrt(-d)*sgn(x)/a
bs(d) - 4*(sqrt(-d^2*x^2 - 2*d)*arccos(d*x^2 + 1) - 2*(sqrt(2)*sqrt(-d) -
sqrt(d^2*x^2))*d/abs(d))/(d*sgn(x)))*b^2 + a^2*x
```

3.75.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+b\arccos\left(1+dx^2\right)\right)^2\,dx=\int \left(a+b\,\mathrm{acos}\left(d\,x^2+1\right)\right)^2\,dx$$

input
```
int((a + b*acos(d*x^2 + 1))^2,x)
```

output
```
int((a + b*acos(d*x^2 + 1))^2, x)
```

3.76 $\int (a+b\arccos(1+dx^2))\,dx$

3.76.1 Optimal result

Integrand size = 12, antiderivative size = 43

$$\int (a+b\arccos(1+dx^2))\,dx = ax - \frac{2b\sqrt{-2dx^2-d^2x^4}}{dx} + bx\arccos(1+dx^2)$$

output
```
a*x+b*x*arccos(d*x^2+1)-2*b*(-d^2*x^4-2*d*x^2)^(1/2)/d/x
```

3.76.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int (a+b\arccos(1+dx^2))\,dx = ax - \frac{2b\sqrt{-dx^2(2+dx^2)}}{dx} + bx\arccos(1+dx^2)$$

input
```
Integrate[a + b*ArcCos[1 + d*x^2],x]
```

output
```
a*x - (2*b*Sqrt[-(d*x^2*(2 + d*x^2))])/(d*x) + b*x*ArcCos[1 + d*x^2]
```

3.76.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\arccos\left(dx^2+1\right)\right)\,dx$$

$$\Big\downarrow \ 2009$$

$$ax+bx\arccos\left(dx^2+1\right)-\frac{2b\sqrt{-d^2x^4-2dx^2}}{dx}$$

input
```
Int[a + b*ArcCos[1 + d*x^2],x]
```

output
```
a*x - (2*b*Sqrt[-2*d*x^2 - d^2*x^4])/(d*x) + b*x*ArcCos[1 + d*x^2]
```

3.76.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.76.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

method	result	size
default	$ax+b\left(x\arccos\left(d\,x^2+1\right)+\frac{2x\left(d\,x^2+2\right)}{\sqrt{-d^2x^4-2d\,x^2}}\right)$	45
parts	$ax+b\left(x\arccos\left(d\,x^2+1\right)+\frac{2x\left(d\,x^2+2\right)}{\sqrt{-d^2x^4-2d\,x^2}}\right)$	45

input
```
int(a+b*arccos(d*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
a*x+b*(x*arccos(d*x^2+1)+2/(-d^2*x^4-2*d*x^2)^(1/2)*x*(d*x^2+2))
```

3.76.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12

$$\int \left(a+b\arccos\left(1+dx^2\right)\right)\,dx=\frac{bdx^2\arccos\left(dx^2+1\right)+adx^2-2\,\sqrt{-d^2x^4-2\,dx^2}b}{dx}$$

input
```
integrate(a+b*arccos(d*x^2+1),x, algorithm="fricas")
```

output
```
(b*d*x^2*arccos(d*x^2 + 1) + a*d*x^2 - 2*sqrt(-d^2*x^4 - 2*d*x^2)*b)/(d*x)
```

3.76.6 Sympy [F]

$$\int \left(a+b\arccos\left(1+dx^2\right)\right)\,dx=\int \left(a+b\,\mathrm{acos}\left(dx^2+1\right)\right)\,dx$$

input
```
integrate(a+b*acos(d*x**2+1),x)
```

output
```
Integral(a + b*acos(d*x**2 + 1), x)
```

3.76.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

$$\int \left(a+b\arccos\left(1+dx^2\right)\right)\,dx=\left(x\arccos\left(dx^2+1\right)+\frac{2\left(d^{\frac{3}{2}}x^2+2\sqrt{d}\right)}{\sqrt{-dx^2-2}d}\right)b+ax$$

input
```
integrate(a+b*arccos(d*x^2+1),x, algorithm="maxima")
```

output
```
(x*arccos(d*x^2 + 1) + 2*(d^(3/2)*x^2 + 2*sqrt(d))/(sqrt(-d*x^2 - 2)*d))*b
 + a*x
```

3.76.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.28

$$\int \left(a+b\arccos\left(1+dx^2\right)\right)\,dx = \left(x\arccos\left(dx^2+1\right)+\frac{2\sqrt{2}\sqrt{-d}\mathrm{sgn}(x)}{d}-\frac{2\sqrt{-d^2x^2-2d}}{d\mathrm{sgn}\left(x\right)}\right)b + ax$$

input
```
integrate(a+b*arccos(d*x^2+1),x, algorithm="giac")
```

output
```
(x*arccos(d*x^2 + 1) + 2*sqrt(2)*sqrt(-d)*sgn(x)/d - 2*sqrt(-d^2*x^2 - 2*d
)/(d*sgn(x)))*b + a*x
```

3.76.9 Mupad [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \left(a+b\arccos\left(1+dx^2\right)\right)\,dx = a\,x+b\,x\,\mathrm{acos}\left(d\,x^2+1\right)-\frac{2\,b\,\sqrt{1-{\left(d\,x^2+1\right)}^2}}{d\,x}$$

input
```
int(a + b*acos(d*x^2 + 1),x)
```

output
```
a*x + b*x*acos(d*x^2 + 1) - (2*b*(1 - (d*x^2 + 1)^2)^(1/2))/(d*x)
```

3.77 $\int \frac{1}{a+b\arccos(1+dx^2)}\,dx$

3.77.1 Optimal result

Integrand size = 14, antiderivative size = 99

$$\int \frac{1}{a+b\arccos(1+dx^2)}\,dx = \frac{x\cos\left(\frac{a}{2b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(1+dx^2)}{2b}\right)}{\sqrt{2}b\sqrt{-dx^2}} + \frac{x\sin\left(\frac{a}{2b}\right)\text{Si}\left(\frac{a+b\arccos(1+dx^2)}{2b}\right)}{\sqrt{2}b\sqrt{-dx^2}}$$

output
```
1/2*x*Ci(1/2*(a+b*arccos(d*x^2+1))/b)*cos(1/2*a/b)/b*2^(1/2)/(-d*x^2)^(1/2
)+1/2*x*Si(1/2*(a+b*arccos(d*x^2+1))/b)*sin(1/2*a/b)/b*2^(1/2)/(-d*x^2)^(1
/2)
```

3.77.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.86

$$\int \frac{1}{a+b\arccos(1+dx^2)}\,dx = -\frac{\sin\left(\frac{1}{2}\arccos(1+dx^2)\right)\left(\cos\left(\frac{a}{2b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(1+dx^2)}{2b}\right)+\sin\left(\frac{a}{2b}\right)\text{Si}\left(\frac{a+b\arccos(1+dx^2)}{2b}\right)\right)}{bdx}$$

input
```
Integrate[(a + b*ArcCos[1 + d*x^2])^(-1),x]
```

output
```
-((Sin[ArcCos[1 + d*x^2]/2]*(Cos[a/(2*b)]*CosIntegral[(a + b*ArcCos[1 + d*
x^2])/(2*b)] + Sin[a/(2*b)]*SinIntegral[(a + b*ArcCos[1 + d*x^2])/(2*b)]))
/(b*d*x))
```

3.77.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {5316}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\arccos\left(dx^2+1\right)}dx$$

$$\Big\downarrow \; 5316$$

$$\frac{x\cos\left(\frac{a}{2b}\right)\text{CosIntegral}\left(\frac{a+b\arccos\left(dx^2+1\right)}{2b}\right)}{\sqrt{2}b\sqrt{-dx^2}}+\frac{x\sin\left(\frac{a}{2b}\right)\text{Si}\left(\frac{a+b\arccos\left(dx^2+1\right)}{2b}\right)}{\sqrt{2}b\sqrt{-dx^2}}$$

input
```
Int[(a + b*ArcCos[1 + d*x^2])^(-1),x]
```

output
```
(x*Cos[a/(2*b)]*CosIntegral[(a + b*ArcCos[1 + d*x^2])/(2*b)])/(Sqrt[2]*b*S
qrt[-(d*x^2)]) + (x*Sin[a/(2*b)]*SinIntegral[(a + b*ArcCos[1 + d*x^2])/(2*
b)])/(Sqrt[2]*b*Sqrt[-(d*x^2)])
```

3.77.3.1 Defintions of rubi rules used

rule 5316
```
Int[((a_.) + ArcCos[1 + (d_.)*(x_)^2]*(b_.))^(-1), x_Symbol] :> Simp[x*Cos[
a/(2*b)]*(CosIntegral[(a + b*ArcCos[1 + d*x^2])/(2*b)]/(Sqrt[2]*b*Sqrt[(-d)
*x^2])), x] + Simp[x*Sin[a/(2*b)]*(SinIntegral[(a + b*ArcCos[1 + d*x^2])/(2
*b)]/(Sqrt[2]*b*Sqrt[(-d)*x^2])), x] /; FreeQ[{a, b, d}, x]
```

3.77.4 Maple [F]

$$\int \frac{1}{a + b \arccos \left(d\,x^{2}+1\right)}dx$$

input
```
int(1/(a+b*arccos(d*x^2+1)),x)
```

output
```
int(1/(a+b*arccos(d*x^2+1)),x)
```

3.77.5 Fricas [F]

$$\int \frac{1}{a + b \arccos (1 + dx^2)} \, dx = \int \frac{1}{b \arccos (dx^2 + 1) + a} \, dx$$

input
```
integrate(1/(a+b*arccos(d*x^2+1)),x, algorithm="fricas")
```

output
```
integral(1/(b*arccos(d*x^2 + 1) + a), x)
```

3.77.6 Sympy [F]

$$\int \frac{1}{a + b \arccos (1 + dx^2)} \, dx = \int \frac{1}{a + b \operatorname{acos} (dx^2 + 1)} \, dx$$

input
```
integrate(1/(a+b*acos(d*x**2+1)),x)
```

output
```
Integral(1/(a + b*acos(d*x**2 + 1)), x)
```

3.77.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a+b\arccos(1+dx^2)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a+b*arccos(d*x^2+1)),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: sign: argument cannot be imagi
nary; found sqrt((-_SAGE_VAR_d*_SAGE_VAR_x^2)-2)
```

3.77.8 Giac [F]

$$\int \frac{1}{a+b\arccos(1+dx^2)}\,dx = \int \frac{1}{b\arccos(dx^2+1)+a}\,dx$$

input
```
integrate(1/(a+b*arccos(d*x^2+1)),x, algorithm="giac")
```

output
```
integrate(1/(b*arccos(d*x^2 + 1) + a), x)
```

3.77.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{a+b\arccos(1+dx^2)}\,dx = \int \frac{1}{a+b\,\mathrm{acos}(d\,x^2+1)}\,dx$$

input
```
int(1/(a + b*acos(d*x^2 + 1)),x)
```

output
```
int(1/(a + b*acos(d*x^2 + 1)), x)
```

3.78 $\int \frac{1}{(a+b\arccos(1+dx^2))^2}\,dx$

3.78.1 Optimal result

Integrand size = 14, antiderivative size = 151

$$\int \frac{1}{(a+b\arccos(1+dx^2))^2}\,dx = \frac{\sqrt{-2dx^2-d^2x^4}}{2bdx\,(a+b\arccos(1+dx^2))} + \frac{x\,\operatorname{CosIntegral}\left(\frac{a+b\arccos(1+dx^2)}{2b}\right)\sin\left(\frac{a}{2b}\right)}{2\sqrt{2}b^2\sqrt{-dx^2}} - \frac{x\cos\left(\frac{a}{2b}\right)\operatorname{Si}\left(\frac{a+b\arccos(1+dx^2)}{2b}\right)}{2\sqrt{2}b^2\sqrt{-dx^2}}$$

output
```
-1/4*x*cos(1/2*a/b)*Si(1/2*(a+b*arccos(d*x^2+1))/b)/b^2*2^(1/2)/(-d*x^2)^(
1/2)+1/4*x*Ci(1/2*(a+b*arccos(d*x^2+1))/b)*sin(1/2*a/b)/b^2*2^(1/2)/(-d*x^
2)^(1/2)+1/2*(-d^2*x^4-2*d*x^2)^(1/2)/b/d/x/(a+b*arccos(d*x^2+1))
```

3.78.2 Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a+b\arccos(1+dx^2))^2}\,dx = \frac{\sqrt{-dx^2(2+dx^2)}\left(\frac{b}{a+b\arccos(1+dx^2)} - \frac{\cos\left(\frac{1}{2}\arccos(1+dx^2)\right)\left(\operatorname{CosIntegral}\left(\frac{a+b\arccos(1+dx^2)}{2b}\right)\sin\left(\frac{a}{2b}\right)-\cos\left(\frac{a}{2b}\right)\operatorname{Si}\left(\frac{a+b\arccos(1\cdots}{2b}\right.\cdots}{2+dx^2}\cdots\right.}{2b^2dx}$$

input
```
Integrate[(a + b*ArcCos[1 + d*x^2])^(-2),x]
```

output
```
(Sqrt[-(d*x^2*(2 + d*x^2))]*(b/(a + b*ArcCos[1 + d*x^2]) - (Cos[ArcCos[1 +
 d*x^2]/2]*(CosIntegral[(a + b*ArcCos[1 + d*x^2])/(2*b)]*Sin[a/(2*b)] - Co
s[a/(2*b)]*SinIntegral[(a + b*ArcCos[1 + d*x^2])/(2*b)]))/(2 + d*x^2)))/(2
*b^2*d*x)
```

3.78.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {5325}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\arccos(dx^2+1))^2}dx$$

$$\Big\downarrow \text{5325}$$

$$\frac{x\sin\left(\frac{a}{2b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(dx^2+1)}{2b}\right)}{2\sqrt{2}b^2\sqrt{-dx^2}}-\frac{x\cos\left(\frac{a}{2b}\right)\text{Si}\left(\frac{a+b\arccos(dx^2+1)}{2b}\right)}{2\sqrt{2}b^2\sqrt{-dx^2}}+\frac{\sqrt{-d^2x^4-2dx^2}}{2bdx\,(a+b\arccos(dx^2+1))}$$

input
```
Int[(a + b*ArcCos[1 + d*x^2])^(-2),x]
```

output
```
Sqrt[-2*d*x^2 - d^2*x^4]/(2*b*d*x*(a + b*ArcCos[1 + d*x^2])) + (x*CosInteg
ral[(a + b*ArcCos[1 + d*x^2])/(2*b)]*Sin[a/(2*b)])/(2*Sqrt[2]*b^2*Sqrt[-(d
*x^2)]) - (x*Cos[a/(2*b)]*SinIntegral[(a + b*ArcCos[1 + d*x^2])/(2*b)])/(2
*Sqrt[2]*b^2*Sqrt[-(d*x^2)])
```

3.78.3.1 Defintions of rubi rules used

rule 5325
```
Int[((a_.) + ArcCos[1 + (d_.)*(x_)^2]*(b_.))^(-2), x_Symbol] :> Simp[Sqrt[-
2*d*x^2 - d^2*x^4]/(2*b*d*x*(a + b*ArcCos[1 + d*x^2])), x] + (Simp[x*Sin[a/
(2*b)]*(CosIntegral[(a + b*ArcCos[1 + d*x^2])/(2*b)]/(2*Sqrt[2]*b^2*Sqrt[(-
d)*x^2])), x] - Simp[x*Cos[a/(2*b)]*(SinIntegral[(a + b*ArcCos[1 + d*x^2])/
(2*b)]/(2*Sqrt[2]*b^2*Sqrt[(-d)*x^2])), x]) /; FreeQ[{a, b, d}, x]
```

3.78.4 Maple [F]

$$\int \frac{1}{\left(a+b\arccos\left(d\,x^{2}+1\right)\right)^{2}}dx$$

input
```
int(1/(a+b*arccos(d*x^2+1))^2,x)
```

output
```
int(1/(a+b*arccos(d*x^2+1))^2,x)
```

3.78.5 Fricas [F]

$$\int \frac{1}{(a+b\arccos(1+dx^2))^2}\,dx = \int \frac{1}{(b\arccos(dx^2+1)+a)^2}\,dx$$

input
```
integrate(1/(a+b*arccos(d*x^2+1))^2,x, algorithm="fricas")
```

output
```
integral(1/(b^2*arccos(d*x^2 + 1)^2 + 2*a*b*arccos(d*x^2 + 1) + a^2), x)
```

3.78.6 Sympy [F]

$$\int \frac{1}{(a+b\arccos(1+dx^2))^2}\,dx = \int \frac{1}{(a+b\operatorname{acos}(dx^2+1))^2}\,dx$$

input
```
integrate(1/(a+b*acos(d*x**2+1))**2,x)
```

output
```
Integral((a + b*acos(d*x**2 + 1))**(-2), x)
```

3.78.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+b\arccos\left(1+dx^2\right)\right)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a+b*arccos(d*x^2+1))^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: sign: argument cannot be imagi
nary; found sqrt((-_SAGE_VAR_d*_SAGE_VAR_x^2)-2)
```

3.78.8 Giac [F]

$$\int \frac{1}{\left(a+b\arccos\left(1+dx^2\right)\right)^2}\,dx = \int \frac{1}{\left(b\arccos\left(dx^2+1\right)+a\right)^2}\,dx$$

input
```
integrate(1/(a+b*arccos(d*x^2+1))^2,x, algorithm="giac")
```

output
```
integrate((b*arccos(d*x^2 + 1) + a)^(-2), x)
```

3.78.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\arccos\left(1+dx^2\right)\right)^2}\,dx = \int \frac{1}{\left(a+b\,\mathrm{acos}\left(d\,x^2+1\right)\right)^2}\,dx$$

input
```
int(1/(a + b*acos(d*x^2 + 1))^2,x)
```

output
```
int(1/(a + b*acos(d*x^2 + 1))^2, x)
```

3.79 $\int \frac{1}{(a+b\arccos(1+dx^2))^3}\,dx$

3.79.1 Optimal result

Integrand size = 14, antiderivative size = 173

$$\int \frac{1}{(a+b\arccos(1+dx^2))^3}\,dx = \frac{\sqrt{-2dx^2-d^2x^4}}{4bdx\,(a+b\arccos(1+dx^2))^2} + \frac{x}{8b^2\,(a+b\arccos(1+dx^2))} - \frac{x\cos\left(\frac{a}{2b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(1+dx^2)}{2b}\right)}{8\sqrt{2}b^3\sqrt{-dx^2}} - \frac{x\sin\left(\frac{a}{2b}\right)\text{Si}\left(\frac{a+b\arccos(1+dx^2)}{2b}\right)}{8\sqrt{2}b^3\sqrt{-dx^2}}$$

output
```
1/8*x/b^2/(a+b*arccos(d*x^2+1))-1/16*x*Ci(1/2*(a+b*arccos(d*x^2+1))/b)*cos
(1/2*a/b)/b^3*2^(1/2)/(-d*x^2)^(1/2)-1/16*x*Si(1/2*(a+b*arccos(d*x^2+1))/b
)*sin(1/2*a/b)/b^3*2^(1/2)/(-d*x^2)^(1/2)+1/4*(-d^2*x^4-2*d*x^2)^(1/2)/b/d
/x/(a+b*arccos(d*x^2+1))^2
```

3.79.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.85

$$\int \frac{1}{(a+b\arccos(1+dx^2))^3}\,dx$$
$$=\frac{\frac{2b^2\sqrt{-dx^2(2+dx^2)}}{d(a+b\arccos(1+dx^2))^2}+\frac{bx^2}{a+b\arccos(1+dx^2)}+\frac{\sin\left(\frac{1}{2}\arccos(1+dx^2)\right)\left(\cos\left(\frac{a}{2b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(1+dx^2)}{2b}\right)+\sin\left(\frac{a}{2b}\right)\text{Si}\left(\frac{a+b\arccos(1+dx^2)}{2b}\right)\right)}{d}}{8b^3x}$$

input
```
Integrate[(a + b*ArcCos[1 + d*x^2])^(-3),x]
```

output
```
((2*b^2*Sqrt[-(d*x^2*(2 + d*x^2))])/(d*(a + b*ArcCos[1 + d*x^2])^2) + (b*x
^2)/(a + b*ArcCos[1 + d*x^2]) + (Sin[ArcCos[1 + d*x^2]/2]*(Cos[a/(2*b)]*Co
sIntegral[(a + b*ArcCos[1 + d*x^2])/(2*b)] + Sin[a/(2*b)]*SinIntegral[(a +
 b*ArcCos[1 + d*x^2])/(2*b)]))/d)/(8*b^3*x)
```

3.79.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5328, 5316}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\arccos(dx^2+1))^3}\,dx$$
$$\Big\downarrow \; 5328$$
$$-\frac{\int \frac{1}{a+b\arccos(dx^2+1)}dx}{8b^2}+\frac{x}{8b^2(a+b\arccos(dx^2+1))}+\frac{\sqrt{-d^2x^4-2dx^2}}{4bdx(a+b\arccos(dx^2+1))^2}$$
$$\Big\downarrow \; 5316$$
$$-\frac{\frac{x\cos\left(\frac{a}{2b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(dx^2+1)}{2b}\right)}{\sqrt{2}b\sqrt{-dx^2}}+\frac{x\sin\left(\frac{a}{2b}\right)\text{Si}\left(\frac{a+b\arccos(dx^2+1)}{2b}\right)}{\sqrt{2}b\sqrt{-dx^2}}}{8b^2}+\frac{x}{8b^2(a+b\arccos(dx^2+1))}+\frac{\sqrt{-d^2x^4-2dx^2}}{4bdx(a+b\arccos(dx^2+1))^2}$$

input
```
Int[(a + b*ArcCos[1 + d*x^2])^(-3),x]
```

output
```
Sqrt[-2*d*x^2 - d^2*x^4]/(4*b*d*x*(a + b*ArcCos[1 + d*x^2])^2) + x/(8*b^2*
(a + b*ArcCos[1 + d*x^2])) - ((x*Cos[a/(2*b)]*CosIntegral[(a + b*ArcCos[1
+ d*x^2])/(2*b)])/(Sqrt[2]*b*Sqrt[-(d*x^2)]) + (x*Sin[a/(2*b)]*SinIntegral
[(a + b*ArcCos[1 + d*x^2])/(2*b)])/(Sqrt[2]*b*Sqrt[-(d*x^2)]))/(8*b^2)
```

3.79.3.1 Defintions of rubi rules used

rule 5316
```
Int[((a_.) + ArcCos[1 + (d_.)*(x_)^2]*(b_.))^(-1), x_Symbol] :> Simp[x*Cos[
a/(2*b)]*(CosIntegral[(a + b*ArcCos[1 + d*x^2])/(2*b)]/(Sqrt[2]*b*Sqrt[(-d)
*x^2])), x] + Simp[x*Sin[a/(2*b)]*(SinIntegral[(a + b*ArcCos[1 + d*x^2])/(2
*b)]/(Sqrt[2]*b*Sqrt[(-d)*x^2])), x] /; FreeQ[{a, b, d}, x]
```

rule 5328
```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
(a + b*ArcCos[c + d*x^2])^(n + 2)/(4*b^2*(n + 1)*(n + 2))), x] + (-Simp[Sqr
t[-2*c*d*x^2 - d^2*x^4]*((a + b*ArcCos[c + d*x^2])^(n + 1)/(2*b*d*(n + 1)*x
)), x] - Simp[1/(4*b^2*(n + 1)*(n + 2))   Int[(a + b*ArcCos[c + d*x^2])^(n
+ 2), x], x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && LtQ[n, -1] && NeQ
[n, -2]
```

3.79.4 Maple [F]

$$\int \frac{1}{\left(a+b\arccos\left(d\,x^{2}+1\right)\right)^{3}}dx$$

input
```
int(1/(a+b*arccos(d*x^2+1))^3,x)
```

output
```
int(1/(a+b*arccos(d*x^2+1))^3,x)
```

3.79.5 Fricas [F]

$$\int \frac{1}{\left(a+b\arccos\left(1+dx^2\right)\right)^3}\,dx=\int \frac{1}{\left(b\arccos\left(dx^2+1\right)+a\right)^3}\,dx$$

input
```
integrate(1/(a+b*arccos(d*x^2+1))^3,x, algorithm="fricas")
```

output
```
integral(1/(b^3*arccos(d*x^2 + 1)^3 + 3*a*b^2*arccos(d*x^2 + 1)^2 + 3*a^2*
b*arccos(d*x^2 + 1) + a^3), x)
```

3.79.6 Sympy [F]

$$\int \frac{1}{\left(a+b\arccos\left(1+dx^2\right)\right)^3}\,dx=\int \frac{1}{\left(a+b\,\mathrm{acos}\left(dx^2+1\right)\right)^3}\,dx$$

input
```
integrate(1/(a+b*acos(d*x**2+1))**3,x)
```

output
```
Integral((a + b*acos(d*x**2 + 1))**(-3), x)
```

3.79.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+b\arccos\left(1+dx^2\right)\right)^3}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a+b*arccos(d*x^2+1))^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: sign: argument cannot be imagi
nary; found sqrt((-_SAGE_VAR_d*_SAGE_VAR_x^2)-2)
```

3.79.8 Giac [F]

$$\int \frac{1}{(a+b\arccos(1+dx^2))^3}\,dx = \int \frac{1}{(b\arccos(dx^2+1)+a)^3}\,dx$$

input
```
integrate(1/(a+b*arccos(d*x^2+1))^3,x, algorithm="giac")
```

output
```
integrate((b*arccos(d*x^2 + 1) + a)^(-3), x)
```

3.79.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\arccos(1+dx^2))^3}\,dx = \int \frac{1}{(a+b\,\mathrm{acos}(d\,x^2+1))^3}\,dx$$

input
```
int(1/(a + b*acos(d*x^2 + 1))^3,x)
```

output
```
int(1/(a + b*acos(d*x^2 + 1))^3, x)
```

3.80 $\int (a+b\arccos(-1+dx^2))^4\,dx$

3.80.1 Optimal result

Integrand size = 14, antiderivative size = 127

$$\int (a+b\arccos(-1+dx^2))^4\,dx = 384b^4x+\frac{192b^3\sqrt{2dx^2-d^2x^4}(a+b\arccos(-1+dx^2))}{dx} - 48b^2x(a+b\arccos(-1+dx^2))^2 - \frac{8b\sqrt{2dx^2-d^2x^4}(a+b\arccos(-1+dx^2))^3}{dx} + x(a+b\arccos(-1+dx^2))^4$$

output
```
384*b^4*x-48*b^2*x*(a+b*arccos(d*x^2-1))^2+x*(a+b*arccos(d*x^2-1))^4+192*b
^3*(a+b*arccos(d*x^2-1))*(-d^2*x^4+2*d*x^2)^(1/2)/d/x-8*b*(a+b*arccos(d*x^
2-1))^3*(-d^2*x^4+2*d*x^2)^(1/2)/d/x
```

3.80.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.96

$$\int (a+b\arccos(-1+dx^2))^4\,dx = \frac{(a^4-48a^2b^2+384b^4)\,dx^2-8ab(a^2-24b^2)\sqrt{dx^2(2-dx^2)}+4b\Big(a^3dx^2-24ab^2dx^2-6a^2b\sqrt{-dx^2(-2+}}{}$$

input
```
Integrate[(a + b*ArcCos[-1 + d*x^2])^4,x]
```

output
```
((a^4 - 48*a^2*b^2 + 384*b^4)*d*x^2 - 8*a*b*(a^2 - 24*b^2)*Sqrt[d*x^2*(2 -
 d*x^2)] + 4*b*(a^3*d*x^2 - 24*a*b^2*d*x^2 - 6*a^2*b*Sqrt[-(d*x^2*(-2 + d*
x^2))] + 48*b^3*Sqrt[-(d*x^2*(-2 + d*x^2))])*ArcCos[-1 + d*x^2] + 6*b^2*(a
^2*d*x^2 - 8*b^2*d*x^2 - 4*a*b*Sqrt[-(d*x^2*(-2 + d*x^2))])*ArcCos[-1 + d*
x^2]^2 + 4*b^3*(a*d*x^2 - 2*b*Sqrt[-(d*x^2*(-2 + d*x^2))])*ArcCos[-1 + d*x
^2]^3 + b^4*d*x^2*ArcCos[-1 + d*x^2]^4)/(d*x)
```

3.80.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5314, 5314, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b \arccos \left(dx^2-1\right)\right)^4 dx$$

$$\Big\downarrow \text{5314}$$

$$-48b^2\int \left(a+b \arccos \left(dx^2-1\right)\right)^2 dx-\frac{8b\sqrt{2dx^2-d^2x^4}\left(a+b \arccos \left(dx^2-1\right)\right)^3}{dx}+x\left(a+b \arccos \left(dx^2-1\right)\right)^4$$

$$\Big\downarrow \text{5314}$$

$$-48b^2\left(-8b^2\int 1dx-\frac{4b\sqrt{2dx^2-d^2x^4}\left(a+b \arccos \left(dx^2-1\right)\right)}{dx}+x\left(a+b \arccos \left(dx^2-1\right)\right)^2\right)-\frac{8b\sqrt{2dx^2-d^2x^4}\left(a+b \arccos \left(dx^2-1\right)\right)^3}{dx}+x\left(a+b \arccos \left(dx^2-1\right)\right)^4$$

$$\Big\downarrow \text{24}$$

$$-48b^2\left(-\frac{4b\sqrt{2dx^2-d^2x^4}\left(a+b \arccos \left(dx^2-1\right)\right)}{dx}+x\left(a+b \arccos \left(dx^2-1\right)\right)^2-8b^2x\right)-\frac{8b\sqrt{2dx^2-d^2x^4}\left(a+b \arccos \left(dx^2-1\right)\right)^3}{dx}+x\left(a+b \arccos \left(dx^2-1\right)\right)^4$$

input
```
Int[(a + b*ArcCos[-1 + d*x^2])^4,x]
```

output
```
(-8*b*Sqrt[2*d*x^2 - d^2*x^4]*(a + b*ArcCos[-1 + d*x^2])^3)/(d*x) + x*(a +
 b*ArcCos[-1 + d*x^2])^4 - 48*b^2*(-8*b^2*x - (4*b*Sqrt[2*d*x^2 - d^2*x^4]
*(a + b*ArcCos[-1 + d*x^2]))/(d*x) + x*(a + b*ArcCos[-1 + d*x^2])^2)
```

3.80.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 5314
```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcCos[c + d*x^2])^n, x] + (-Simp[2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*((
a + b*ArcCos[c + d*x^2])^(n - 1)/(d*x)), x] - Simp[4*b^2*n*(n - 1)   Int[(a
 + b*ArcCos[c + d*x^2])^(n - 2), x], x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c
^2, 1] && GtQ[n, 1]
```

3.80.4 Maple [F]

$$\int \left(a +b \arccos \left(d \,x^{2}-1\right)\right)^{4}d x$$

input
```
int((a+b*arccos(d*x^2-1))^4,x)
```

output
```
int((a+b*arccos(d*x^2-1))^4,x)
```

3.80.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.63

$$\int \left(a+b\arccos\left(-1+dx^2\right)\right)^4\,dx$$
$$= \frac{b^4dx^2\arccos\left(dx^2-1\right)^4+4\,ab^3dx^2\arccos\left(dx^2-1\right)^3+6\left(a^2b^2-8\,b^4\right)dx^2\arccos\left(dx^2-1\right)^2+4\left(a^3b-24\,a\right.}{}$$

input
```
integrate((a+b*arccos(d*x^2-1))^4,x, algorithm="fricas")
```

output
```
(b^4*d*x^2*arccos(d*x^2 - 1)^4 + 4*a*b^3*d*x^2*arccos(d*x^2 - 1)^3 + 6*(a^
2*b^2 - 8*b^4)*d*x^2*arccos(d*x^2 - 1)^2 + 4*(a^3*b - 24*a*b^3)*d*x^2*arcc
os(d*x^2 - 1) + (a^4 - 48*a^2*b^2 + 384*b^4)*d*x^2 - 8*(b^4*arccos(d*x^2 -
 1)^3 + 3*a*b^3*arccos(d*x^2 - 1)^2 + a^3*b - 24*a*b^3 + 3*(a^2*b^2 - 8*b^
4)*arccos(d*x^2 - 1))*sqrt(-d^2*x^4 + 2*d*x^2))/(d*x)
```

3.80.6 Sympy [F]

$$\int \left(a+b\arccos\left(-1+dx^2\right)\right)^4\,dx=\int \left(a+b\operatorname{acos}\left(dx^2-1\right)\right)^4\,dx$$

input
```
integrate((a+b*acos(d*x**2-1))**4,x)
```

output
```
Integral((a + b*acos(d*x**2 - 1))**4, x)
```

3.80.7 Maxima [F]

$$\int \left(a+b\arccos\left(-1+dx^2\right)\right)^4\,dx=\int \left(b\arccos\left(dx^2-1\right)+a\right)^4\,dx$$

input
```
integrate((a+b*arccos(d*x^2-1))^4,x, algorithm="maxima")
```

output
```
b^4*x*arctan2(sqrt(-d*x^2 + 2)*sqrt(d)*x, d*x^2 - 1)^4 + 4*(x*arccos(d*x^2
 - 1) + 2*(d^(3/2)*x^2 - 2*sqrt(d))/(sqrt(-d*x^2 + 2)*d))*a^3*b + a^4*x -
integrate(2*(4*sqrt(-d*x^2 + 2)*b^4*sqrt(d)*x*arctan2(sqrt(-d*x^2 + 2)*sqr
t(d)*x, d*x^2 - 1)^3 - 2*(a*b^3*d*x^2 - 2*a*b^3)*arctan2(sqrt(-d*x^2 + 2)*
sqrt(d)*x, d*x^2 - 1)^3 - 3*(a^2*b^2*d*x^2 - 2*a^2*b^2)*arctan2(sqrt(-d*x^
2 + 2)*sqrt(d)*x, d*x^2 - 1)^2)/(d*x^2 - 2), x)
```

3.80.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 586 vs. $2(123) = 246$.

Time = 1.04 (sec) , antiderivative size = 586, normalized size of antiderivative = 4.61

$$\int \left(a + b\arccos\left(-1 + dx^2\right)\right)^4 \, dx$$

$$= 4\left(x\arccos\left(dx^2 - 1\right) + \frac{2\sqrt{2}\operatorname{sgn}(x)}{\sqrt{d}} - \frac{2\sqrt{-d^2x^2 + 2d}}{d\operatorname{sgn}(x)}\right)a^3b$$

$$+6\left(x\arccos\left(dx^2 - 1\right)^2 + \frac{4\left(\sqrt{2}\pi\sqrt{d}|d| - 2\sqrt{2}d^{\frac{3}{2}}\right)\operatorname{sgn}(x)}{d|d|} - \frac{4\left(\sqrt{-d^2x^2 + 2d}\arccos\left(dx^2 - 1\right) - \frac{2\left(\sqrt{2}\cdots\right.}{\cdots}\cdots\right.}{d\operatorname{sgn}(x)}\cdots\right.$$

$$+4\left(x\arccos\left(dx^2 - 1\right)^3 + \frac{6\left(\sqrt{2}\pi^2\sqrt{d} - 8\sqrt{2}\sqrt{d}\right)\operatorname{sgn}(x)}{d} - \frac{6\left(\sqrt{-d^2x^2 + 2d}\arccos\left(dx^2 - 1\right)^2 + \frac{4\left(\sqrt{\cdots}\right.}{\cdots}\cdots\right.}{ds\cdots}\cdots\right.$$

$$+\left(x\arccos\left(dx^2 - 1\right)^4 + \frac{8\left(\sqrt{2}\pi^3\sqrt{d}|d| - 24\sqrt{2}\pi\sqrt{d}|d| + 48\sqrt{2}d^{\frac{3}{2}}\right)\operatorname{sgn}(x)}{d|d|} - \frac{8\left(\sqrt{-d^2x^2 + 2d}\arccos\cdots\right.}{\cdots}\cdots\right.$$

$$+ a^4x$$

input
```
integrate((a+b*arccos(d*x^2-1))^4,x, algorithm="giac")
```

output
```
4*(x*arccos(d*x^2 - 1) + 2*sqrt(2)*sgn(x)/sqrt(d) - 2*sqrt(-d^2*x^2 + 2*d)
/(d*sgn(x)))*a^3*b + 6*(x*arccos(d*x^2 - 1)^2 + 4*(sqrt(2)*pi*sqrt(d)*abs(
d) - 2*sqrt(2)*d^(3/2))*sgn(x)/(d*abs(d)) - 4*(sqrt(-d^2*x^2 + 2*d)*arccos
(d*x^2 - 1) - 2*(sqrt(2)*sqrt(d) - sqrt(d^2*x^2))*d/abs(d))/(d*sgn(x)))*a^
2*b^2 + 4*(x*arccos(d*x^2 - 1)^3 + 6*(sqrt(2)*pi^2*sqrt(d) - 8*sqrt(2)*sqr
t(d))*sgn(x)/d - 6*(sqrt(-d^2*x^2 + 2*d)*arccos(d*x^2 - 1)^2 + 4*(sqrt(d^2
*x^2)*arccos((d^2*x^2 - d)/d) + 2*(sqrt(2)*sqrt(d) - sqrt(-d^2*x^2 + 2*d))
*d/abs(d) - 2*sqrt(2)*d^(3/2)/abs(d))*d/abs(d))/(d*sgn(x)))*a*b^3 + (x*arc
cos(d*x^2 - 1)^4 + 8*(sqrt(2)*pi^3*sqrt(d)*abs(d) - 24*sqrt(2)*pi*sqrt(d)*
abs(d) + 48*sqrt(2)*d^(3/2))*sgn(x)/(d*abs(d)) - 8*(sqrt(-d^2*x^2 + 2*d)*a
rccos(d*x^2 - 1)^3 + 6*(sqrt(d^2*x^2)*arccos((d^2*x^2 - d)/d)^2 - 2*(pi*sq
rt(-d^2*x^2 + 2*d) + 2*sqrt(-d^2*x^2 + 2*d)*arcsin(-(d^2*x^2 - d)/d) - 4*(
sqrt(2)*sqrt(d) - sqrt(d^2*x^2))*d/abs(d) - 2*(sqrt(2)*pi*sqrt(d)*abs(d) -
 2*sqrt(2)*d^(3/2))/abs(d))*d/abs(d) - 4*(sqrt(2)*pi*sqrt(d)*abs(d) - 2*sq
rt(2)*d^(3/2))/d)*d/abs(d))/(d*sgn(x)))*b^4 + a^4*x
```

3.80.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+b\arccos\left(-1+dx^2\right)\right)^4\,dx=\int \left(a+b\,\mathrm{acos}\left(d\,x^2-1\right)\right)^4\,dx$$

input
```
int((a + b*acos(d*x^2 - 1))^4,x)
```

output
```
int((a + b*acos(d*x^2 - 1))^4, x)
```

3.81 $\int \left(a+b \arccos \left(-1+d x^2\right)\right)^3 \, dx$

3.81.1 Optimal result

Integrand size = 14, antiderivative size = 110

$$\int \left(a+b \arccos \left(-1+d x^2\right)\right)^3 \, dx = -24 a b^2 x+\frac{48 b^3 \sqrt{2 d x^2-d^2 x^4}}{d x}-24 b^3 x \arccos \left(-1+d x^2\right) -\frac{6 b \sqrt{2 d x^2-d^2 x^4} \left(a+b \arccos \left(-1+d x^2\right)\right)^2}{d x} +x \left(a+b \arccos \left(-1+d x^2\right)\right)^3$$

output
```
-24*a*b^2*x-24*b^3*x*arccos(d*x^2-1)+x*(a+b*arccos(d*x^2-1))^3+48*b^3*(-d^
2*x^4+2*d*x^2)^(1/2)/d/x-6*b*(a+b*arccos(d*x^2-1))^2*(-d^2*x^4+2*d*x^2)^(1
/2)/d/x
```

3.81.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.47

$$\int \left(a+b \arccos \left(-1+d x^2\right)\right)^3 \, dx = \frac{a (a^2-24 b^2) d x^2-6 b (a^2-8 b^2) \sqrt{d x^2 \left(2-d x^2\right)}+3 b \left(a^2 d x^2-8 b^2 d x^2-4 a b \sqrt{-d x^2 \left(-2+d x^2\right)}\right) \arccos (\ldots}{d x}$$

input
```
Integrate[(a + b*ArcCos[-1 + d*x^2])^3,x]
```

output
```
(a*(a^2 - 24*b^2)*d*x^2 - 6*b*(a^2 - 8*b^2)*Sqrt[d*x^2*(2 - d*x^2)] + 3*b*
(a^2*d*x^2 - 8*b^2*d*x^2 - 4*a*b*Sqrt[-(d*x^2*(-2 + d*x^2))])*ArcCos[-1 +
d*x^2] + 3*b^2*(a*d*x^2 - 2*b*Sqrt[-(d*x^2*(-2 + d*x^2))])*ArcCos[-1 + d*x
^2]^2 + b^3*d*x^2*ArcCos[-1 + d*x^2]^3)/(d*x)
```

3.81.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5314, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\arccos\left(dx^2-1\right)\right)^3 dx$$

$$\Big\downarrow \text{5314}$$

$$-24b^2\int \left(a+b\arccos\left(dx^2-1\right)\right)dx-\frac{6b\sqrt{2dx^2-d^2x^4}\left(a+b\arccos\left(dx^2-1\right)\right)^2}{dx}+x\left(a+b\arccos\left(dx^2-1\right)\right)^3$$

$$\Big\downarrow \text{2009}$$

$$-24b^2\left(ax+bx\arccos\left(dx^2-1\right)-\frac{2b\sqrt{2dx^2-d^2x^4}}{dx}\right)-\frac{6b\sqrt{2dx^2-d^2x^4}\left(a+b\arccos\left(dx^2-1\right)\right)^2}{dx}+x\left(a+b\arccos\left(dx^2-1\right)\right)^3$$

input
```
Int[(a + b*ArcCos[-1 + d*x^2])^3,x]
```

output
```
(-6*b*Sqrt[2*d*x^2 - d^2*x^4]*(a + b*ArcCos[-1 + d*x^2])^2)/(d*x) + x*(a +
 b*ArcCos[-1 + d*x^2])^3 - 24*b^2*(a*x - (2*b*Sqrt[2*d*x^2 - d^2*x^4])/(d*
x) + b*x*ArcCos[-1 + d*x^2])
```

3.81.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5314
```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcCos[c + d*x^2])^n, x] + (-Simp[2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*((
a + b*ArcCos[c + d*x^2])^(n - 1)/(d*x)), x] - Simp[4*b^2*n*(n - 1)   Int[(a
 + b*ArcCos[c + d*x^2])^(n - 2), x], x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c
^2, 1] && GtQ[n, 1]
```

3.81.4 Maple [F]

$$\int \left(a+b\arccos\left(d\,x^{2}-1\right)\right)^{3}dx$$

input
```
int((a+b*arccos(d*x^2-1))^3,x)
```

output
```
int((a+b*arccos(d*x^2-1))^3,x)
```

3.81.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.31

$$\int \left(a+b\arccos\left(-1+dx^2\right)\right)^3\,dx$$
$$=\frac{b^3dx^2\arccos\left(dx^2-1\right)^3+3\,ab^2dx^2\arccos\left(dx^2-1\right)^2+3\left(a^2b-8\,b^3\right)dx^2\arccos\left(dx^2-1\right)+\left(a^3-24\,ab^2\right)d}{dx}$$

input
```
integrate((a+b*arccos(d*x^2-1))^3,x, algorithm="fricas")
```

output
```
(b^3*d*x^2*arccos(d*x^2 - 1)^3 + 3*a*b^2*d*x^2*arccos(d*x^2 - 1)^2 + 3*(a^
2*b - 8*b^3)*d*x^2*arccos(d*x^2 - 1) + (a^3 - 24*a*b^2)*d*x^2 - 6*sqrt(-d^
2*x^4 + 2*d*x^2)*(b^3*arccos(d*x^2 - 1)^2 + 2*a*b^2*arccos(d*x^2 - 1) + a^
2*b - 8*b^3))/(d*x)
```

3.81.6 Sympy [F]

$$\int \left(a+b\arccos\left(-1+dx^2\right)\right)^3\,dx=\int \left(a+b\operatorname{acos}\left(dx^2-1\right)\right)^3\,dx$$

input
```
integrate((a+b*acos(d*x**2-1))**3,x)
```

output
```
Integral((a + b*acos(d*x**2 - 1))**3, x)
```

3.81.7 Maxima [F]

$$\int \left(a+b\arccos\left(-1+dx^2\right)\right)^3\,dx=\int \left(b\arccos\left(dx^2-1\right)+a\right)^3\,dx$$

input
```
integrate((a+b*arccos(d*x^2-1))^3,x, algorithm="maxima")
```

output
```
b^3*x*arctan2(sqrt(-d*x^2 + 2)*sqrt(d)*x, d*x^2 - 1)^3 + 3*(x*arccos(d*x^2
 - 1) + 2*(d^(3/2)*x^2 - 2*sqrt(d))/(sqrt(-d*x^2 + 2)*d))*a^2*b + a^3*x -
integrate(3*(2*sqrt(-d*x^2 + 2)*b^3*sqrt(d)*x*arctan2(sqrt(-d*x^2 + 2)*sqr
t(d)*x, d*x^2 - 1)^2 - (a*b^2*d*x^2 - 2*a*b^2)*arctan2(sqrt(-d*x^2 + 2)*sq
rt(d)*x, d*x^2 - 1)^2)/(d*x^2 - 2), x)
```

3.81.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 319 vs. $2(106) = 212$.

Time = 0.67 (sec) , antiderivative size = 319, normalized size of antiderivative = 2.90

$$\int \left(a+b\arccos\left(-1+dx^2\right)\right)^3\,dx$$
$$=3\left(x\arccos\left(dx^2-1\right)+\frac{2\sqrt{2}\mathrm{sgn}(x)}{\sqrt{d}}-\frac{2\sqrt{-d^2x^2+2d}}{d\mathrm{sgn}\left(x\right)}\right)a^2b$$
$$+3\left(x\arccos\left(dx^2-1\right)^2+\frac{4\left(\sqrt{2}\pi\sqrt{d}|d|-2\sqrt{2}d^{\frac{3}{2}}\right)\mathrm{sgn}(x)}{d|d|}-\frac{4\left(\sqrt{-d^2x^2+2d}\arccos\left(dx^2-1\right)-\frac{2\left(\sqrt{2}\right.}{}\right.}{d\mathrm{sgn}\left(x\right)}\right.$$
$$+\left(x\arccos\left(dx^2-1\right)^3+\frac{6\left(\sqrt{2}\pi^2\sqrt{d}-8\sqrt{2}\sqrt{d}\right)\mathrm{sgn}(x)}{d}-\frac{6\left(\sqrt{-d^2x^2+2d}\arccos\left(dx^2-1\right)^2+\frac{4\left(\sqrt{d^2\right.}{}\right.}{d\mathrm{sgn}}\right.$$
$$+a^3x$$

input
```
integrate((a+b*arccos(d*x^2-1))^3,x, algorithm="giac")
```

output
```
3*(x*arccos(d*x^2 - 1) + 2*sqrt(2)*sgn(x)/sqrt(d) - 2*sqrt(-d^2*x^2 + 2*d)
/(d*sgn(x)))*a^2*b + 3*(x*arccos(d*x^2 - 1)^2 + 4*(sqrt(2)*pi*sqrt(d)*abs(
d) - 2*sqrt(2)*d^(3/2))*sgn(x)/(d*abs(d)) - 4*(sqrt(-d^2*x^2 + 2*d)*arccos
(d*x^2 - 1) - 2*(sqrt(2)*sqrt(d) - sqrt(d^2*x^2))*d/abs(d))/(d*sgn(x)))*a*
b^2 + (x*arccos(d*x^2 - 1)^3 + 6*(sqrt(2)*pi^2*sqrt(d) - 8*sqrt(2)*sqrt(d)
)*sgn(x)/d - 6*(sqrt(-d^2*x^2 + 2*d)*arccos(d*x^2 - 1)^2 + 4*(sqrt(d^2*x^2
)*arccos((d^2*x^2 - d)/d) + 2*(sqrt(2)*sqrt(d) - sqrt(-d^2*x^2 + 2*d))*d/a
bs(d) - 2*sqrt(2)*d^(3/2)/abs(d))*d/abs(d))/(d*sgn(x)))*b^3 + a^3*x
```

3.81.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+b\arccos\left(-1+dx^2\right)\right)^3\,dx=\int \left(a+b\,\mathrm{acos}\left(d\,x^2-1\right)\right)^3\,dx$$

input
```
int((a + b*acos(d*x^2 - 1))^3,x)
```

output
```
int((a + b*acos(d*x^2 - 1))^3, x)
```

3.82 $\int \left(a+b \arccos \left(-1+d x^2\right)\right)^2 \, dx$

3.82.1 Optimal result

Integrand size = 14, antiderivative size = 63

$$\int \left(a+b \arccos \left(-1+d x^2\right)\right)^2 \, dx=-8 b^2 x-\frac{4 b \sqrt{2 d x^2-d^2 x^4} \left(a+b \arccos \left(-1+d x^2\right)\right)}{d x}+x \left(a+b \arccos \left(-1+d x^2\right)\right)^2$$

output
```
-8*b^2*x+x*(a+b*arccos(d*x^2-1))^2-4*b*(a+b*arccos(d*x^2-1))*(-d^2*x^4+2*d
*x^2)^(1/2)/d/x
```

3.82.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.56

$$\int \left(a+b \arccos \left(-1+d x^2\right)\right)^2 \, dx=\left(a^2-8 b^2\right) x-\frac{4 a b \sqrt{-d x^2 \left(-2+d x^2\right)}}{d x}+\frac{2 b \left(a d x^2-2 b \sqrt{-d x^2 \left(-2+d x^2\right)}\right) \arccos \left(-1+d x^2\right)}{d x}+b^2 x \arccos \left(-1+d x^2\right)^2$$

input
```
Integrate[(a + b*ArcCos[-1 + d*x^2])^2,x]
```

output
```
(a^2 - 8*b^2)*x - (4*a*b*Sqrt[-(d*x^2*(-2 + d*x^2))])/(d*x) + (2*b*(a*d*x^
2 - 2*b*Sqrt[-(d*x^2*(-2 + d*x^2))])*ArcCos[-1 + d*x^2])/(d*x) + b^2*x*Arc
Cos[-1 + d*x^2]^2
```

3.82.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5314, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\arccos(dx^2-1))^2\,dx$$

$$\Big\downarrow \text{ 5314}$$

$$-8b^2\int 1dx-\frac{4b\sqrt{2dx^2-d^2x^4}(a+b\arccos(dx^2-1))}{dx}+x(a+b\arccos(dx^2-1))^2$$

$$\Big\downarrow \text{ 24}$$

$$-\frac{4b\sqrt{2dx^2-d^2x^4}(a+b\arccos(dx^2-1))}{dx}+x(a+b\arccos(dx^2-1))^2-8b^2x$$

input
```
Int[(a + b*ArcCos[-1 + d*x^2])^2,x]
```

output
```
-8*b^2*x - (4*b*Sqrt[2*d*x^2 - d^2*x^4]*(a + b*ArcCos[-1 + d*x^2]))/(d*x)
+ x*(a + b*ArcCos[-1 + d*x^2])^2
```

3.82.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 5314
```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcCos[c + d*x^2])^n, x] + (-Simp[2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*((
a + b*ArcCos[c + d*x^2])^(n - 1)/(d*x)), x] - Simp[4*b^2*n*(n - 1)   Int[(a
 + b*ArcCos[c + d*x^2])^(n - 2), x], x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c
^2, 1] && GtQ[n, 1]
```

3.82.4 Maple [F]

$$\int \left(a+b\arccos\left(d\,x^{2}-1\right)\right)^{2}dx$$

input
```
int((a+b*arccos(d*x^2-1))^2,x)
```

output
```
int((a+b*arccos(d*x^2-1))^2,x)
```

3.82.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.44

$$\int \left(a+b\arccos\left(-1+dx^2\right)\right)^2\,dx = \frac{b^2dx^2\arccos\left(dx^2-1\right)^2+2\,abdx^2\arccos\left(dx^2-1\right)+(a^2-8\,b^2)dx^2-4\,\sqrt{-d^2x^4+2\,dx^2}(b^2\arccos\left(dx^2-\right.}{dx}$$

input
```
integrate((a+b*arccos(d*x^2-1))^2,x, algorithm="fricas")
```

output
```
(b^2*d*x^2*arccos(d*x^2 - 1)^2 + 2*a*b*d*x^2*arccos(d*x^2 - 1) + (a^2 - 8*
b^2)*d*x^2 - 4*sqrt(-d^2*x^4 + 2*d*x^2)*(b^2*arccos(d*x^2 - 1) + a*b))/(d*
x)
```

3.82.6 Sympy [F]

$$\int \left(a+b\arccos\left(-1+dx^2\right)\right)^2\,dx = \int \left(a+b\,\mathrm{acos}\left(dx^2-1\right)\right)^2\,dx$$

input
```
integrate((a+b*acos(d*x**2-1))**2,x)
```

output
```
Integral((a + b*acos(d*x**2 - 1))**2, x)
```

3.82.7 Maxima [F]

$$\int \left(a+b\arccos\left(-1+dx^2\right)\right)^2\,dx=\int \left(b\arccos\left(dx^2-1\right)+a\right)^2\,dx$$

input
```
integrate((a+b*arccos(d*x^2-1))^2,x, algorithm="maxima")
```

output
```
2*(x*arccos(d*x^2 - 1) + 2*(d^(3/2)*x^2 - 2*sqrt(d))/(sqrt(-d*x^2 + 2)*d))
*a*b + (x*arctan2(sqrt(-d*x^2 + 2)*sqrt(d)*x, d*x^2 - 1)^2 - 4*sqrt(d)*int
egrate(sqrt(-d*x^2 + 2)*x*arctan2(sqrt(-d*x^2 + 2)*sqrt(d)*x, d*x^2 - 1)/(
d*x^2 - 2), x))*b^2 + a^2*x
```

3.82.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(61)=122$.

Time = 0.45 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.54

$$\int \left(a+b\arccos\left(-1+dx^2\right)\right)^2\,dx$$
$$=2\left(x\arccos\left(dx^2-1\right)+\frac{2\sqrt{2}\mathrm{sgn}(x)}{\sqrt{d}}-\frac{2\sqrt{-d^2x^2+2d}}{d\mathrm{sgn}\left(x\right)}\right)ab$$
$$+\left(x\arccos\left(dx^2-1\right)^2+\frac{4\left(\sqrt{2}\pi\sqrt{d}|d|-2\sqrt{2}d^{\frac{3}{2}}\right)\mathrm{sgn}(x)}{d|d|}-\frac{4\left(\sqrt{-d^2x^2+2d}\arccos\left(dx^2-1\right)-\frac{2\left(\sqrt{2}\sqrt{\ldots}\right.}{\ldots}\right.}{d\mathrm{sgn}\left(x\right)}\right.$$
$$+a^2x$$

input
```
integrate((a+b*arccos(d*x^2-1))^2,x, algorithm="giac")
```

output
```
2*(x*arccos(d*x^2 - 1) + 2*sqrt(2)*sgn(x)/sqrt(d) - 2*sqrt(-d^2*x^2 + 2*d)
/(d*sgn(x)))*a*b + (x*arccos(d*x^2 - 1)^2 + 4*(sqrt(2)*pi*sqrt(d)*abs(d) -
 2*sqrt(2)*d^(3/2))*sgn(x)/(d*abs(d)) - 4*(sqrt(-d^2*x^2 + 2*d)*arccos(d*x
^2 - 1) - 2*(sqrt(2)*sqrt(d) - sqrt(d^2*x^2))*d/abs(d))/(d*sgn(x)))*b^2 +
a^2*x
```

3.82.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+b\arccos\left(-1+dx^2\right)\right)^2\,dx=\int \left(a+b\,\mathrm{acos}\left(d\,x^2-1\right)\right)^2\,dx$$

input
```
int((a + b*acos(d*x^2 - 1))^2,x)
```

output
```
int((a + b*acos(d*x^2 - 1))^2, x)
```

3.83 $\int \left(a+b \arccos \left(-1+d x^2\right)\right) \, dx$

3.83.1 Optimal result

Integrand size = 12, antiderivative size = 43

$$\int \left(a+b \arccos \left(-1+d x^2\right)\right) \, dx=a x-\frac{2 b \sqrt{2 d x^2-d^2 x^4}}{d x}+b x \arccos \left(-1+d x^2\right)$$

output
```
a*x+b*x*arccos(d*x^2-1)-2*b*(-d^2*x^4+2*d*x^2)^(1/2)/d/x
```

3.83.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int \left(a+b \arccos \left(-1+d x^2\right)\right) \, dx=a x-\frac{2 b \sqrt{-d x^2 \left(-2+d x^2\right)}}{d x}+b x \arccos \left(-1+d x^2\right)$$

input
```
Integrate[a + b*ArcCos[-1 + d*x^2],x]
```

output
```
a*x - (2*b*Sqrt[-(d*x^2*(-2 + d*x^2))])/(d*x) + b*x*ArcCos[-1 + d*x^2]
```

3.83.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\arccos\left(dx^2-1\right)\right)\,dx$$

$$\Big\downarrow \; 2009$$

$$ax+bx\arccos\left(dx^2-1\right)-\frac{2b\sqrt{2dx^2-d^2x^4}}{dx}$$

input
```
Int[a + b*ArcCos[-1 + d*x^2],x]
```

output
```
a*x - (2*b*Sqrt[2*d*x^2 - d^2*x^4])/(d*x) + b*x*ArcCos[-1 + d*x^2]
```

3.83.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.83.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

method	result	size
default	$ax+b\left(x\arccos\left(d\,x^2-1\right)+\frac{2x\left(d\,x^2-2\right)}{\sqrt{-d^2x^4+2d\,x^2}}\right)$	45
parts	$ax+b\left(x\arccos\left(d\,x^2-1\right)+\frac{2x\left(d\,x^2-2\right)}{\sqrt{-d^2x^4+2d\,x^2}}\right)$	45

input
```
int(a+b*arccos(d*x^2-1),x,method=_RETURNVERBOSE)
```

output
```
a*x+b*(x*arccos(d*x^2-1)+2/(-d^2*x^4+2*d*x^2)^(1/2)*x*(d*x^2-2))
```

3.83.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12

$$\int \left(a+b\arccos\left(-1+dx^2\right)\right)\,dx=\frac{bdx^2\arccos\left(dx^2-1\right)+adx^2-2\sqrt{-d^2x^4+2\,dx^2}b}{dx}$$

input
```
integrate(a+b*arccos(d*x^2-1),x, algorithm="fricas")
```

output
```
(b*d*x^2*arccos(d*x^2 - 1) + a*d*x^2 - 2*sqrt(-d^2*x^4 + 2*d*x^2)*b)/(d*x)
```

3.83.6 Sympy [F]

$$\int \left(a+b\arccos\left(-1+dx^2\right)\right)\,dx=\int \left(a+b\operatorname{acos}\left(dx^2-1\right)\right)\,dx$$

input
```
integrate(a+b*acos(d*x**2-1),x)
```

output
```
Integral(a + b*acos(d*x**2 - 1), x)
```

3.83.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

$$\int \left(a+b\arccos\left(-1+dx^2\right)\right)\,dx=\left(x\arccos\left(dx^2-1\right)+\frac{2\left(d^{\frac{3}{2}}x^2-2\sqrt{d}\right)}{\sqrt{-dx^2+2}d}\right)b+ax$$

input
```
integrate(a+b*arccos(d*x^2-1),x, algorithm="maxima")
```

output
```
(x*arccos(d*x^2 - 1) + 2*(d^(3/2)*x^2 - 2*sqrt(d))/(sqrt(-d*x^2 + 2)*d))*b
 + a*x
```

3.83.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.16

$$\int \left(a+b\arccos\left(-1+dx^2\right)\right)\,dx=\left(x\arccos\left(dx^2-1\right)+\frac{2\sqrt{2}\mathrm{sgn}(x)}{\sqrt{d}}-\frac{2\sqrt{-d^2x^2+2d}}{d\mathrm{sgn}\left(x\right)}\right)b+ax$$

input
```
integrate(a+b*arccos(d*x^2-1),x, algorithm="giac")
```

output
```
(x*arccos(d*x^2 - 1) + 2*sqrt(2)*sgn(x)/sqrt(d) - 2*sqrt(-d^2*x^2 + 2*d)/(
d*sgn(x)))*b + a*x
```

3.83.9 Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \left(a+b\arccos\left(-1+dx^2\right)\right)\,dx=a\,x+b\,x\,\mathrm{acos}\left(d\,x^2-1\right)-\frac{2\,b\,\sqrt{1-{\left(d\,x^2-1\right)}^2}}{d\,x}$$

input
```
int(a + b*acos(d*x^2 - 1),x)
```

output
```
a*x + b*x*acos(d*x^2 - 1) - (2*b*(1 - (d*x^2 - 1)^2)^(1/2))/(d*x)
```

3.84 $\int \frac{1}{a+b\arccos(-1+dx^2)}\,dx$

3.84.1 Optimal result

Integrand size = 14, antiderivative size = 98

$$\int \frac{1}{a+b\arccos(-1+dx^2)}\,dx=\frac{x\,\text{CosIntegral}\left(\frac{a+b\arccos(-1+dx^2)}{2b}\right)\sin\left(\frac{a}{2b}\right)}{\sqrt{2}b\sqrt{dx^2}}-\frac{x\cos\left(\frac{a}{2b}\right)\text{Si}\left(\frac{a+b\arccos(-1+dx^2)}{2b}\right)}{\sqrt{2}b\sqrt{dx^2}}$$

output
```
-1/2*x*cos(1/2*a/b)*Si(1/2*(a+b*arccos(d*x^2-1))/b)/b*2^(1/2)/(d*x^2)^(1/2
)+1/2*x*Ci(1/2*(a+b*arccos(d*x^2-1))/b)*sin(1/2*a/b)/b*2^(1/2)/(d*x^2)^(1/
2)
```

3.84.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.87

$$\int \frac{1}{a+b\arccos(-1+dx^2)}\,dx=\frac{\cos\left(\frac{1}{2}\arccos(-1+dx^2)\right)\left(\text{CosIntegral}\left(\frac{a+b\arccos(-1+dx^2)}{2b}\right)\sin\left(\frac{a}{2b}\right)-\cos\left(\frac{a}{2b}\right)\text{Si}\left(\frac{a+b\arccos(-1+dx^2)}{2b}\right)\right)}{bdx}$$

input
```
Integrate[(a + b*ArcCos[-1 + d*x^2])^(-1),x]
```

output
```
(Cos[ArcCos[-1 + d*x^2]/2]*(CosIntegral[(a + b*ArcCos[-1 + d*x^2])/(2*b)]*
Sin[a/(2*b)] - Cos[a/(2*b)]*SinIntegral[(a + b*ArcCos[-1 + d*x^2])/(2*b)])
)/(b*d*x)
```

3.84.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {5317}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\arccos\left(dx^2-1\right)}dx$$

$$\Big\downarrow \text{5317}$$

$$\frac{x\sin\left(\frac{a}{2b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(dx^2-1)}{2b}\right)}{\sqrt{2}b\sqrt{dx^2}}-\frac{x\cos\left(\frac{a}{2b}\right)\text{Si}\left(\frac{a+b\arccos(dx^2-1)}{2b}\right)}{\sqrt{2}b\sqrt{dx^2}}$$

input
```
Int[(a + b*ArcCos[-1 + d*x^2])^(-1),x]
```

output
```
(x*CosIntegral[(a + b*ArcCos[-1 + d*x^2])/(2*b)]*Sin[a/(2*b)])/(Sqrt[2]*b*
Sqrt[d*x^2]) - (x*Cos[a/(2*b)]*SinIntegral[(a + b*ArcCos[-1 + d*x^2])/(2*b
)])/(Sqrt[2]*b*Sqrt[d*x^2])
```

3.84.3.1 Defintions of rubi rules used

rule 5317
```
Int[((a_.) + ArcCos[-1 + (d_.)*(x_)^2]*(b_.))^(-1), x_Symbol] :> Simp[x*Sin
[a/(2*b)]*(CosIntegral[(a + b*ArcCos[-1 + d*x^2])/(2*b)]/(Sqrt[2]*b*Sqrt[d*
x^2])), x] - Simp[x*Cos[a/(2*b)]*(SinIntegral[(a + b*ArcCos[-1 + d*x^2])/(2
*b)]/(Sqrt[2]*b*Sqrt[d*x^2])), x] /; FreeQ[{a, b, d}, x]
```

3.84.4 Maple [F]

$$\int \frac{1}{a+b\arccos\left(d\,x^{2}-1\right)}dx$$

input
```
int(1/(a+b*arccos(d*x^2-1)),x)
```

output
```
int(1/(a+b*arccos(d*x^2-1)),x)
```

3.84.5 Fricas [F]

$$\int \frac{1}{a+b\arccos\left(-1+dx^2\right)}\,dx=\int \frac{1}{b\arccos\left(dx^2-1\right)+a}\,dx$$

input
```
integrate(1/(a+b*arccos(d*x^2-1)),x, algorithm="fricas")
```

output
```
integral(1/(b*arccos(d*x^2 - 1) + a), x)
```

3.84.6 Sympy [F]

$$\int \frac{1}{a+b\arccos\left(-1+dx^2\right)}\,dx=\int \frac{1}{a+b\,\mathrm{acos}\left(dx^2-1\right)}\,dx$$

input
```
integrate(1/(a+b*acos(d*x**2-1)),x)
```

output
```
Integral(1/(a + b*acos(d*x**2 - 1)), x)
```

3.84.7 Maxima [F]

$$\int \frac{1}{a + b\arccos(-1 + dx^2)}\,dx = \int \frac{1}{b\arccos(dx^2 - 1) + a}\,dx$$

input
```
integrate(1/(a+b*arccos(d*x^2-1)),x, algorithm="maxima")
```

output
```
integrate(1/(b*arccos(d*x^2 - 1) + a), x)
```

3.84.8 Giac [F]

$$\int \frac{1}{a + b\arccos(-1 + dx^2)}\,dx = \int \frac{1}{b\arccos(dx^2 - 1) + a}\,dx$$

input
```
integrate(1/(a+b*arccos(d*x^2-1)),x, algorithm="giac")
```

output
```
integrate(1/(b*arccos(d*x^2 - 1) + a), x)
```

3.84.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{a + b\arccos(-1 + dx^2)}\,dx = \int \frac{1}{a + b\,\mathrm{acos}(d\,x^2 - 1)}\,dx$$

input
```
int(1/(a + b*acos(d*x^2 - 1)),x)
```

output
```
int(1/(a + b*acos(d*x^2 - 1)), x)
```

3.85 $\int \frac{1}{\left(a+b\arccos\left(-1+dx^2\right)\right)^2}\,dx$

3.85.1 Optimal result

Integrand size = 14, antiderivative size = 149

$$\int \frac{1}{\left(a+b\arccos\left(-1+dx^2\right)\right)^2}\,dx=\frac{\sqrt{2dx^2-d^2x^4}}{2bdx\left(a+b\arccos\left(-1+dx^2\right)\right)}-\frac{x\cos\left(\frac{a}{2b}\right)\text{CosIntegral}\left(\frac{a+b\arccos\left(-1+dx^2\right)}{2b}\right)}{2\sqrt{2}b^2\sqrt{dx^2}}-\frac{x\sin\left(\frac{a}{2b}\right)\text{Si}\left(\frac{a+b\arccos\left(-1+dx^2\right)}{2b}\right)}{2\sqrt{2}b^2\sqrt{dx^2}}$$

output
```
-1/4*x*Ci(1/2*(a+b*arccos(d*x^2-1))/b)*cos(1/2*a/b)/b^2*2^(1/2)/(d*x^2)^(1
/2)-1/4*x*Si(1/2*(a+b*arccos(d*x^2-1))/b)*sin(1/2*a/b)/b^2*2^(1/2)/(d*x^2)
^(1/2)+1/2*(-d^2*x^4+2*d*x^2)^(1/2)/b/d/x/(a+b*arccos(d*x^2-1))
```

3.85.2 Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.88

$$\int \frac{1}{\left(a+b\arccos\left(-1+dx^2\right)\right)^2}\,dx = \frac{\sqrt{-dx^2\left(-2+dx^2\right)}\left(\frac{b}{a+b\arccos(-1+dx^2)}+\frac{\sin\left(\frac{1}{2}\arccos(-1+dx^2)\right)\left(\cos\left(\frac{a}{2b}\right)\text{CosIntegral}\left(\frac{a+b\arccos\left(-1+dx^2\right)}{2b}\right)+\sin\left(\frac{a}{2b}\right)\text{Si}\left(\frac{a+b\,\mathrm{a}\ldots}{}\right.\right.}{-2+dx^2}\right.}{2b^2dx}$$

input
```
Integrate[(a + b*ArcCos[-1 + d*x^2])^(-2),x]
```

output
```
(Sqrt[-(d*x^2*(-2 + d*x^2))]*(b/(a + b*ArcCos[-1 + d*x^2]) + (Sin[ArcCos[-
1 + d*x^2]/2]*(Cos[a/(2*b)]*CosIntegral[(a + b*ArcCos[-1 + d*x^2])/(2*b)]
+ Sin[a/(2*b)]*SinIntegral[(a + b*ArcCos[-1 + d*x^2])/(2*b)]))/(-2 + d*x^2
)))/(2*b^2*d*x)
```

3.85.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {5326}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\arccos(dx^2-1))^2}dx$$

$$\Big\downarrow \text{5326}$$

$$-\frac{x\cos\left(\frac{a}{2b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(dx^2-1)}{2b}\right)}{2\sqrt{2}b^2\sqrt{dx^2}}-\frac{x\sin\left(\frac{a}{2b}\right)\text{Si}\left(\frac{a+b\arccos(dx^2-1)}{2b}\right)}{2\sqrt{2}b^2\sqrt{dx^2}}+\frac{\sqrt{2dx^2-d^2x^4}}{2bdx(a+b\arccos(dx^2-1))}$$

input
```
Int[(a + b*ArcCos[-1 + d*x^2])^(-2),x]
```

output
```
Sqrt[2*d*x^2 - d^2*x^4]/(2*b*d*x*(a + b*ArcCos[-1 + d*x^2])) - (x*Cos[a/(2
*b)]*CosIntegral[(a + b*ArcCos[-1 + d*x^2])/(2*b)])/(2*Sqrt[2]*b^2*Sqrt[d*
x^2]) - (x*Sin[a/(2*b)]*SinIntegral[(a + b*ArcCos[-1 + d*x^2])/(2*b)])/(2*
Sqrt[2]*b^2*Sqrt[d*x^2])
```

3.85.3.1 Defintions of rubi rules used

rule 5326
```
Int[((a_.) + ArcCos[-1 + (d_.)*(x_)^2]*(b_.))^(-2), x_Symbol] :> Simp[Sqrt[
2*d*x^2 - d^2*x^4]/(2*b*d*x*(a + b*ArcCos[-1 + d*x^2])), x] + (-Simp[x*Cos[
a/(2*b)]*(CosIntegral[(a + b*ArcCos[-1 + d*x^2])/(2*b)]/(2*Sqrt[2]*b^2*Sqrt
[d*x^2])), x] - Simp[x*Sin[a/(2*b)]*(SinIntegral[(a + b*ArcCos[-1 + d*x^2])
/(2*b)]/(2*Sqrt[2]*b^2*Sqrt[d*x^2])), x]) /; FreeQ[{a, b, d}, x]
```

3.85.4 Maple [F]

$$\int \frac{1}{\left(a+b\arccos\left(d\,x^{2}-1\right)\right)^{2}}dx$$

input
```
int(1/(a+b*arccos(d*x^2-1))^2,x)
```

output
```
int(1/(a+b*arccos(d*x^2-1))^2,x)
```

3.85.5 Fricas [F]

$$\int \frac{1}{(a+b\arccos(-1+dx^2))^2}\,dx = \int \frac{1}{(b\arccos(dx^2-1)+a)^2}\,dx$$

input
```
integrate(1/(a+b*arccos(d*x^2-1))^2,x, algorithm="fricas")
```

output
```
integral(1/(b^2*arccos(d*x^2 - 1)^2 + 2*a*b*arccos(d*x^2 - 1) + a^2), x)
```

3.85.6 Sympy [F]

$$\int \frac{1}{(a+b\arccos(-1+dx^2))^2}\,dx = \int \frac{1}{(a+b\,\mathrm{acos}\,(dx^2-1))^2}\,dx$$

input
```
integrate(1/(a+b*acos(d*x**2-1))**2,x)
```

output
```
Integral((a + b*acos(d*x**2 - 1))**(-2), x)
```

3.85.7 Maxima [F]

$$\int \frac{1}{(a+b\arccos(-1+dx^2))^2}\,dx = \int \frac{1}{(b\arccos(dx^2-1)+a)^2}\,dx$$

input
```
integrate(1/(a+b*arccos(d*x^2-1))^2,x, algorithm="maxima")
```

output
```
-1/2*(2*(b^2*d*arctan2(sqrt(-d*x^2 + 2)*sqrt(d)*x, d*x^2 - 1) + a*b*d)*sqr
t(d)*integrate(1/2*sqrt(-d*x^2 + 2)*x/(a*b*d*x^2 - 2*a*b + (b^2*d*x^2 - 2*
b^2)*arctan2(sqrt(-d*x^2 + 2)*sqrt(d)*x, d*x^2 - 1)), x) - sqrt(-d*x^2 + 2
)*sqrt(d))/(b^2*d*arctan2(sqrt(-d*x^2 + 2)*sqrt(d)*x, d*x^2 - 1) + a*b*d)
```

3.85.8 Giac [F]

$$\int \frac{1}{(a+b\arccos(-1+dx^2))^2}\,dx = \int \frac{1}{(b\arccos(dx^2-1)+a)^2}\,dx$$

input
```
integrate(1/(a+b*arccos(d*x^2-1))^2,x, algorithm="giac")
```

output
```
integrate((b*arccos(d*x^2 - 1) + a)^(-2), x)
```

3.85.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\arccos(-1+dx^2))^2}\,dx = \int \frac{1}{(a+b\,\mathrm{acos}(d\,x^2-1))^2}\,dx$$

input
```
int(1/(a + b*acos(d*x^2 - 1))^2,x)
```

output
```
int(1/(a + b*acos(d*x^2 - 1))^2, x)
```

3.86 $\int \frac{1}{(a+b\arccos(-1+dx^2))^3}\,dx$

3.86.1 Optimal result

Integrand size = 14, antiderivative size = 171

$$\int \frac{1}{(a+b\arccos(-1+dx^2))^3}\,dx = \frac{\sqrt{2dx^2-d^2x^4}}{4bdx\,(a+b\arccos(-1+dx^2))^2} + \frac{x}{8b^2\,(a+b\arccos(-1+dx^2))} - \frac{x\,\text{CosIntegral}\left(\frac{a+b\arccos(-1+dx^2)}{2b}\right)\sin\left(\frac{a}{2b}\right)}{8\sqrt{2}b^3\sqrt{dx^2}} + \frac{x\cos\left(\frac{a}{2b}\right)\text{Si}\left(\frac{a+b\arccos(-1+dx^2)}{2b}\right)}{8\sqrt{2}b^3\sqrt{dx^2}}$$

output
```
1/8*x/b^2/(a+b*arccos(d*x^2-1))+1/16*x*cos(1/2*a/b)*Si(1/2*(a+b*arccos(d*x
^2-1))/b)/b^3*2^(1/2)/(d*x^2)^(1/2)-1/16*x*Ci(1/2*(a+b*arccos(d*x^2-1))/b)
*sin(1/2*a/b)/b^3*2^(1/2)/(d*x^2)^(1/2)+1/4*(-d^2*x^4+2*d*x^2)^(1/2)/b/d/x
/(a+b*arccos(d*x^2-1))^2
```

3.86.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.87

$$\int \frac{1}{(a+b\arccos(-1+dx^2))^3}\,dx$$
$$= \frac{\frac{2b^2\sqrt{-dx^2(-2+dx^2)}}{d(a+b\arccos(-1+dx^2))^2}+\frac{bx^2}{a+b\arccos(-1+dx^2)}-\frac{\cos\left(\frac{1}{2}\arccos(-1+dx^2)\right)\left(\operatorname{CosIntegral}\left(\frac{a+b\arccos(-1+dx^2)}{2b}\right)\sin\left(\frac{a}{2b}\right)-\cos\left(\frac{a}{2b}\right)\operatorname{Si}\left(\frac{a+b\arccos(-1+dx^2)}{2b}\right)\right)}{d}}{8b^3x}$$

input
```
Integrate[(a + b*ArcCos[-1 + d*x^2])^(-3),x]
```

output
```
((2*b^2*Sqrt[-(d*x^2*(-2 + d*x^2))])/(d*(a + b*ArcCos[-1 + d*x^2])^2) + (b
*x^2)/(a + b*ArcCos[-1 + d*x^2]) - (Cos[ArcCos[-1 + d*x^2]/2]*(CosIntegral
[(a + b*ArcCos[-1 + d*x^2])/(2*b)]*Sin[a/(2*b)] - Cos[a/(2*b)]*SinIntegral
[(a + b*ArcCos[-1 + d*x^2])/(2*b)]))/d)/(8*b^3*x)
```

3.86.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5328, 5317}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\arccos(dx^2-1))^3}\,dx$$
$$\Big\downarrow \text{5328}$$
$$-\frac{\int \frac{1}{a+b\arccos(dx^2-1)}dx}{8b^2}+\frac{x}{8b^2(a+b\arccos(dx^2-1))}+\frac{\sqrt{2dx^2-d^2x^4}}{4bdx(a+b\arccos(dx^2-1))^2}$$
$$\Big\downarrow \text{5317}$$
$$-\frac{\frac{x\sin\left(\frac{a}{2b}\right)\operatorname{CosIntegral}\left(\frac{a+b\arccos(dx^2-1)}{2b}\right)}{\sqrt{2}b\sqrt{dx^2}}-\frac{x\cos\left(\frac{a}{2b}\right)\operatorname{Si}\left(\frac{a+b\arccos(dx^2-1)}{2b}\right)}{\sqrt{2}b\sqrt{dx^2}}}{8b^2}+\frac{x}{8b^2(a+b\arccos(dx^2-1))}+\frac{\sqrt{2dx^2-d^2x^4}}{4bdx(a+b\arccos(dx^2-1))^2}$$

input
```
Int[(a + b*ArcCos[-1 + d*x^2])^(-3),x]
```

output
```
Sqrt[2*d*x^2 - d^2*x^4]/(4*b*d*x*(a + b*ArcCos[-1 + d*x^2])^2) + x/(8*b^2*
(a + b*ArcCos[-1 + d*x^2])) - ((x*CosIntegral[(a + b*ArcCos[-1 + d*x^2])/(
2*b)]*Sin[a/(2*b)])/(Sqrt[2]*b*Sqrt[d*x^2]) - (x*Cos[a/(2*b)]*SinIntegral[
(a + b*ArcCos[-1 + d*x^2])/(2*b)])/(Sqrt[2]*b*Sqrt[d*x^2]))/(8*b^2)
```

3.86.3.1 Defintions of rubi rules used

rule 5317
```
Int[((a_.) + ArcCos[-1 + (d_.)*(x_)^2]*(b_.))^(-1), x_Symbol] :> Simp[x*Sin
[a/(2*b)]*(CosIntegral[(a + b*ArcCos[-1 + d*x^2])/(2*b)]/(Sqrt[2]*b*Sqrt[d*
x^2])), x] - Simp[x*Cos[a/(2*b)]*(SinIntegral[(a + b*ArcCos[-1 + d*x^2])/(2
*b)]/(Sqrt[2]*b*Sqrt[d*x^2])), x] /; FreeQ[{a, b, d}, x]
```

rule 5328
```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
(a + b*ArcCos[c + d*x^2])^(n + 2)/(4*b^2*(n + 1)*(n + 2))), x] + (-Simp[Sqr
t[-2*c*d*x^2 - d^2*x^4]*((a + b*ArcCos[c + d*x^2])^(n + 1)/(2*b*d*(n + 1)*x
)), x] - Simp[1/(4*b^2*(n + 1)*(n + 2))   Int[(a + b*ArcCos[c + d*x^2])^(n
+ 2), x], x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && LtQ[n, -1] && NeQ
[n, -2]
```

3.86.4 Maple [F]

$$\int \frac{1}{\left(a+b\arccos\left(d\,x^{2}-1\right)\right)^{3}}dx$$

input
```
int(1/(a+b*arccos(d*x^2-1))^3,x)
```

output
```
int(1/(a+b*arccos(d*x^2-1))^3,x)
```

3.86.5 Fricas [F]

$$\int \frac{1}{(a+b\arccos(-1+dx^2))^3}\,dx = \int \frac{1}{(b\arccos(dx^2-1)+a)^3}\,dx$$

input
```
integrate(1/(a+b*arccos(d*x^2-1))^3,x, algorithm="fricas")
```

output
```
integral(1/(b^3*arccos(d*x^2 - 1)^3 + 3*a*b^2*arccos(d*x^2 - 1)^2 + 3*a^2*
b*arccos(d*x^2 - 1) + a^3), x)
```

3.86.6 Sympy [F]

$$\int \frac{1}{(a+b\arccos(-1+dx^2))^3}\,dx = \int \frac{1}{(a+b\operatorname{acos}(dx^2-1))^3}\,dx$$

input
```
integrate(1/(a+b*acos(d*x**2-1))**3,x)
```

output
```
Integral((a + b*acos(d*x**2 - 1))**(-3), x)
```

3.86.7 Maxima [F]

$$\int \frac{1}{(a+b\arccos(-1+dx^2))^3}\,dx = \int \frac{1}{(b\arccos(dx^2-1)+a)^3}\,dx$$

input
```
integrate(1/(a+b*arccos(d*x^2-1))^3,x, algorithm="maxima")
```

output
```
1/8*(b*d*x*arctan2(sqrt(-d*x^2 + 2)*sqrt(d)*x, d*x^2 - 1) + a*d*x + 2*sqrt
(-d*x^2 + 2)*b*sqrt(d) - 8*(b^4*d*arctan2(sqrt(-d*x^2 + 2)*sqrt(d)*x, d*x^
2 - 1)^2 + 2*a*b^3*d*arctan2(sqrt(-d*x^2 + 2)*sqrt(d)*x, d*x^2 - 1) + a^2*
b^2*d)*integrate(1/8/(b^3*arctan2(sqrt(-d*x^2 + 2)*sqrt(d)*x, d*x^2 - 1) +
 a*b^2), x))/(b^4*d*arctan2(sqrt(-d*x^2 + 2)*sqrt(d)*x, d*x^2 - 1)^2 + 2*a
*b^3*d*arctan2(sqrt(-d*x^2 + 2)*sqrt(d)*x, d*x^2 - 1) + a^2*b^2*d)
```

3.86.8 Giac [F]

$$\int \frac{1}{\left(a+b\arccos\left(-1+dx^2\right)\right)^3}\,dx=\int \frac{1}{\left(b\arccos\left(dx^2-1\right)+a\right)^3}\,dx$$

input
```
integrate(1/(a+b*arccos(d*x^2-1))^3,x, algorithm="giac")
```

output
```
integrate((b*arccos(d*x^2 - 1) + a)^(-3), x)
```

3.86.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\arccos\left(-1+dx^2\right)\right)^3}\,dx=\int \frac{1}{\left(a+b\,\mathrm{acos}\left(d\,x^2-1\right)\right)^3}\,dx$$

input
```
int(1/(a + b*acos(d*x^2 - 1))^3,x)
```

output
```
int(1/(a + b*acos(d*x^2 - 1))^3, x)
```

3.87 $\int (a + b \arccos(1 + dx^2))^{5/2} \, dx$

3.87.1 Optimal result

Integrand size = 16, antiderivative size = 249

$$\begin{aligned}\int (a + b \arccos(1 + dx^2))^{5/2} \, dx =& -\frac{5b\sqrt{-2dx^2 - d^2x^4}(a + b \arccos(1 + dx^2))^{3/2}}{dx} + x(a + b \arccos(1 + dx^2))^{5/2} \\ & - \frac{30\sqrt{\pi} \cos\left(\frac{a}{2b}\right) \text{FresnelS}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\arccos(1+dx^2)}}{\sqrt{\pi}}\right) \sin\left(\frac{1}{2}\arccos(1 + dx^2)\right)}{\left(\frac{1}{b}\right)^{5/2} dx} \\ & + \frac{30\sqrt{\pi} \, \text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\arccos(1+dx^2)}}{\sqrt{\pi}}\right) \sin\left(\frac{a}{2b}\right) \sin\left(\frac{1}{2}\arccos(1 + dx^2)\right)}{\left(\frac{1}{b}\right)^{5/2} dx} \\ & + \frac{30b^2\sqrt{a + b \arccos(1 + dx^2)} \sin^2\left(\frac{1}{2}\arccos(1 + dx^2)\right)}{dx}\end{aligned}$$

output
```
x*(a+b*arccos(d*x^2+1))^(5/2)-30*cos(1/2*a/b)*FresnelS((1/b)^(1/2)*(a+b*ar
ccos(d*x^2+1))^(1/2)/Pi^(1/2))*sin(1/2*arccos(d*x^2+1))*Pi^(1/2)/(1/b)^(5/
2)/d/x+30*FresnelC((1/b)^(1/2)*(a+b*arccos(d*x^2+1))^(1/2)/Pi^(1/2))*sin(1
/2*a/b)*sin(1/2*arccos(d*x^2+1))*Pi^(1/2)/(1/b)^(5/2)/d/x-5*b*(a+b*arccos(
d*x^2+1))^(3/2)*(-d^2*x^4-2*d*x^2)^(1/2)/d/x+30*b^2*sin(1/2*arccos(d*x^2+1
))^2*(a+b*arccos(d*x^2+1))^(1/2)/d/x
```

3.87.2 Mathematica [A] (verified)

Time = 3.40 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.00

$$\int \left(a+b\arccos\left(1+dx^2\right)\right)^{5/2}\,dx =$$
$$-\frac{2\sin\left(\frac{1}{2}\arccos\left(1+dx^2\right)\right)\left(15b^{5/2}\sqrt{\pi}\cos\left(\frac{a}{2b}\right)\text{FresnelS}\left(\frac{\sqrt{a+b\arccos(1+dx^2)}}{\sqrt{b}\sqrt{\pi}}\right)-15b^{5/2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{\sqrt{a+b\arccos}}{\sqrt{b}\sqrt{}}\right.}{}$$

input
```
Integrate[(a + b*ArcCos[1 + d*x^2])^(5/2),x]
```

output
```
(-2*Sin[ArcCos[1 + d*x^2]/2]*(15*b^(5/2)*Sqrt[Pi]*Cos[a/(2*b)]*FresnelS[Sq
rt[a + b*ArcCos[1 + d*x^2]]/(Sqrt[b]*Sqrt[Pi])] - 15*b^(5/2)*Sqrt[Pi]*Fres
nelC[Sqrt[a + b*ArcCos[1 + d*x^2]]/(Sqrt[b]*Sqrt[Pi])]*Sin[a/(2*b)] + Sqrt
[a + b*ArcCos[1 + d*x^2]]*(5*a*b*Cos[ArcCos[1 + d*x^2]/2] + (a^2 - 15*b^2)
*Sin[ArcCos[1 + d*x^2]/2] + b^2*ArcCos[1 + d*x^2]^2*Sin[ArcCos[1 + d*x^2]/
2] + b*ArcCos[1 + d*x^2]*(5*b*Cos[ArcCos[1 + d*x^2]/2] + 2*a*Sin[ArcCos[1
+ d*x^2]/2]))))/(d*x)
```

3.87.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5314, 5311}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\arccos\left(dx^2+1\right)\right)^{5/2}\,dx$$

$$\Big\downarrow \ 5314$$

$$-15b^2\int \sqrt{a+b\arccos\left(dx^2+1\right)}dx-\frac{5b\sqrt{-d^2x^4-2dx^2}\left(a+b\arccos\left(dx^2+1\right)\right)^{3/2}}{dx}+x\left(a+b\arccos\left(dx^2+1\right)\right)^{5/2}$$

$$\Big\downarrow \ 5311$$

$$-15b^2\left(-\frac{2\sqrt{\pi}\sin\left(\frac{a}{2b}\right)\sin\left(\frac{1}{2}\arccos\left(dx^2+1\right)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\arccos(dx^2+1)}}{\sqrt{\pi}}\right)}{\sqrt{\frac{1}{b}}dx}+\frac{2\sqrt{\pi}\cos\left(\frac{a}{2b}\right)\sin\left(\frac{1}{2}\arccos\left(dx\right.\right.}{}\right.$$
$$\frac{5b\sqrt{-d^2x^4-2dx^2}\left(a+b\arccos\left(dx^2+1\right)\right)^{3/2}}{dx}+x\left(a+b\arccos\left(dx^2+1\right)\right)^{5/2}$$

input
```
Int[(a + b*ArcCos[1 + d*x^2])^(5/2),x]
```

output
```
(-5*b*Sqrt[-2*d*x^2 - d^2*x^4]*(a + b*ArcCos[1 + d*x^2])^(3/2))/(d*x) + x*
(a + b*ArcCos[1 + d*x^2])^(5/2) - 15*b^2*((2*Sqrt[Pi]*Cos[a/(2*b)]*Fresnel
S[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]*Sin[ArcCos[1 + d*
x^2]/2])/(Sqrt[b^(-1)]*d*x) - (2*Sqrt[Pi]*FresnelC[(Sqrt[b^(-1)]*Sqrt[a +
b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b)]*Sin[ArcCos[1 + d*x^2]/2])/(Sq
rt[b^(-1)]*d*x) - (2*Sqrt[a + b*ArcCos[1 + d*x^2]]*Sin[ArcCos[1 + d*x^2]/2
]^2)/(d*x))
```

3.87.3.1 Defintions of rubi rules used

rule 5311
```
Int[Sqrt[(a_.) + ArcCos[1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[-2*Sqrt
[a + b*ArcCos[1 + d*x^2]]*(Sin[ArcCos[1 + d*x^2]/2]^2/(d*x)), x] + (-Simp[2
*Sqrt[Pi]*Sin[a/(2*b)]*Sin[ArcCos[1 + d*x^2]/2]*(FresnelC[Sqrt[1/(Pi*b)]*Sq
rt[a + b*ArcCos[1 + d*x^2]]]/(Sqrt[1/b]*d*x)), x] + Simp[2*Sqrt[Pi]*Cos[a/(
2*b)]*Sin[ArcCos[1 + d*x^2]/2]*(FresnelS[Sqrt[1/(Pi*b)]*Sqrt[a + b*ArcCos[1
 + d*x^2]]]/(Sqrt[1/b]*d*x)), x]) /; FreeQ[{a, b, d}, x]
```

rule 5314
```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcCos[c + d*x^2])^n, x] + (-Simp[2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*((
a + b*ArcCos[c + d*x^2])^(n - 1)/(d*x)), x] - Simp[4*b^2*n*(n - 1)   Int[(a
 + b*ArcCos[c + d*x^2])^(n - 2), x], x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c
^2, 1] && GtQ[n, 1]
```

3.87.4 Maple [F]

$$\int \left(a+b \arccos \left(d \,x^{2}+1\right)\right)^{\frac{5}{2}}d x$$

input
```
int((a+b*arccos(d*x^2+1))^(5/2),x)
```

output
```
int((a+b*arccos(d*x^2+1))^(5/2),x)
```

3.87.5 Fricas [F(-2)]

Exception generated.

$$\int \left(a+b\arccos \left(1+dx^2\right)\right)^{5/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccos(d*x^2+1))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.87.6 Sympy [F]

$$\int \left(a+b\arccos \left(1+dx^2\right)\right)^{5/2}\,dx=\int \left(a+b\operatorname{acos}\left(dx^{2}+1\right)\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*acos(d*x**2+1))**(5/2),x)
```

output
```
Integral((a + b*acos(d*x**2 + 1))**(5/2), x)
```

3.87.7 Maxima [F(-2)]

Exception generated.

$$\int \left(a+b\arccos\left(1+dx^2\right)\right)^{5/2} dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(d*x^2+1))^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: sign: argument cannot be imagi
nary; found sqrt((-_SAGE_VAR_d*_SAGE_VAR_x^2)-2)
```

3.87.8 Giac [F]

$$\int \left(a+b\arccos\left(1+dx^2\right)\right)^{5/2} dx = \int \left(b\arccos\left(dx^2+1\right)+a\right)^{\frac{5}{2}} dx$$

input
```
integrate((a+b*arccos(d*x^2+1))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arccos(d*x^2 + 1) + a)^(5/2), x)
```

3.87.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+b\arccos\left(1+dx^2\right)\right)^{5/2} dx = \int \left(a+b\,\mathrm{acos}\left(d\,x^2+1\right)\right)^{5/2} dx$$

input
```
int((a + b*acos(d*x^2 + 1))^(5/2),x)
```

output
```
int((a + b*acos(d*x^2 + 1))^(5/2), x)
```

3.88 $\int (a+b\arccos(1+dx^2))^{3/2}\,dx$

3.88.1 Optimal result

Integrand size = 16, antiderivative size = 207

$$\int (a+b\arccos(1+dx^2))^{3/2}\,dx = -\frac{3b\sqrt{-2dx^2-d^2x^4}\sqrt{a+b\arccos(1+dx^2)}}{dx}+x\left(a+b\arccos(1+dx^2)\right)^{3/2}$$
$$+\frac{6\sqrt{\pi}\cos\left(\frac{a}{2b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\arccos(1+dx^2)}}{\sqrt{\pi}}\right)\sin\left(\frac{1}{2}\arccos(1+dx^2)\right)}{\left(\frac{1}{b}\right)^{3/2}dx}$$
$$+\frac{6\sqrt{\pi}\,\text{FresnelS}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\arccos(1+dx^2)}}{\sqrt{\pi}}\right)\sin\left(\frac{a}{2b}\right)\sin\left(\frac{1}{2}\arccos(1+dx^2)\right)}{\left(\frac{1}{b}\right)^{3/2}dx}$$

output
```
x*(a+b*arccos(d*x^2+1))^(3/2)+6*cos(1/2*a/b)*FresnelC((1/b)^(1/2)*(a+b*arc
cos(d*x^2+1))^(1/2)/Pi^(1/2))*sin(1/2*arccos(d*x^2+1))*Pi^(1/2)/(1/b)^(3/2
)/d/x+6*FresnelS((1/b)^(1/2)*(a+b*arccos(d*x^2+1))^(1/2)/Pi^(1/2))*sin(1/2
*a/b)*sin(1/2*arccos(d*x^2+1))*Pi^(1/2)/(1/b)^(3/2)/d/x-3*b*(-d^2*x^4-2*d*
x^2)^(1/2)*(a+b*arccos(d*x^2+1))^(1/2)/d/x
```

3.88.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.93

$$\int (a+b\arccos(1+dx^2))^{3/2}\,dx =$$

$$-\frac{2\sin\left(\frac{1}{2}\arccos(1+dx^2)\right)\left(-3b^{3/2}\sqrt{\pi}\cos\left(\frac{a}{2b}\right)\text{FresnelC}\left(\frac{\sqrt{a+b\arccos(1+dx^2)}}{\sqrt{b}\sqrt{\pi}}\right)-3b^{3/2}\sqrt{\pi}\,\text{FresnelS}\left(\frac{\sqrt{a+b\arccos}}{\sqrt{b}\sqrt{}}\right.}{}$$

input
```
Integrate[(a + b*ArcCos[1 + d*x^2])^(3/2),x]
```

output
```
(-2*Sin[ArcCos[1 + d*x^2]/2]*(-3*b^(3/2)*Sqrt[Pi]*Cos[a/(2*b)]*FresnelC[Sq
rt[a + b*ArcCos[1 + d*x^2]]/(Sqrt[b]*Sqrt[Pi])] - 3*b^(3/2)*Sqrt[Pi]*Fresn
elS[Sqrt[a + b*ArcCos[1 + d*x^2]]/(Sqrt[b]*Sqrt[Pi])]*Sin[a/(2*b)] + Sqrt[
a + b*ArcCos[1 + d*x^2]]*(3*b*Cos[ArcCos[1 + d*x^2]/2] + a*Sin[ArcCos[1 +
d*x^2]/2] + b*ArcCos[1 + d*x^2]*Sin[ArcCos[1 + d*x^2]/2])))/(d*x)
```

3.88.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.03, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5314, 5319}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\arccos(dx^2+1))^{3/2}\,dx$$

$$\Big\downarrow \text{5314}$$

$$-3b^2\int \frac{1}{\sqrt{a+b\arccos(dx^2+1)}}dx-\frac{3b\sqrt{-d^2x^4-2dx^2}\sqrt{a+b\arccos(dx^2+1)}}{dx}+x(a+b\arccos(dx^2+1))^{3/2}$$

$$\Big\downarrow \text{5319}$$

$$-3b^2\left(-\frac{2\sqrt{\pi}\sqrt{\frac{1}{b}}\cos\left(\frac{a}{2b}\right)\sin\left(\frac{1}{2}\arccos\left(dx^2+1\right)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\arccos(dx^2+1)}}{\sqrt{\pi}}\right)}{dx}-\frac{2\sqrt{\pi}\sqrt{\frac{1}{b}}\sin\left(\frac{a}{2b}\right)\sin\left(\frac{1}{2}\text{arcc}\right.}{}\right.$$

$$\frac{3b\sqrt{-d^2x^4-2dx^2}\sqrt{a+b\arccos\left(dx^2+1\right)}}{dx}+x\left(a+b\arccos\left(dx^2+1\right)\right)^{3/2}$$

input
```
Int[(a + b*ArcCos[1 + d*x^2])^(3/2),x]
```

output
```
(-3*b*Sqrt[-2*d*x^2 - d^2*x^4]*Sqrt[a + b*ArcCos[1 + d*x^2]])/(d*x) + x*(a
 + b*ArcCos[1 + d*x^2])^(3/2) - 3*b^2*((-2*Sqrt[b^(-1)]*Sqrt[Pi]*Cos[a/(2*
b)]*FresnelC[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]*Sin[Ar
cCos[1 + d*x^2]/2])/(d*x) - (2*Sqrt[b^(-1)]*Sqrt[Pi]*FresnelS[(Sqrt[b^(-1)
]*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b)]*Sin[ArcCos[1 + d*x
^2]/2])/(d*x))
```

3.88.3.1 Defintions of rubi rules used

rule 5314
```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcCos[c + d*x^2])^n, x] + (-Simp[2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*((
a + b*ArcCos[c + d*x^2])^(n - 1)/(d*x)), x] - Simp[4*b^2*n*(n - 1)   Int[(a
 + b*ArcCos[c + d*x^2])^(n - 2), x], x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c
^2, 1] && GtQ[n, 1]
```

rule 5319
```
Int[1/Sqrt[(a_.) + ArcCos[1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[-2*Sq
rt[Pi/b]*Cos[a/(2*b)]*Sin[ArcCos[1 + d*x^2]/2]*(FresnelC[Sqrt[1/(Pi*b)]*Sqr
t[a + b*ArcCos[1 + d*x^2]]]/(d*x)), x] - Simp[2*Sqrt[Pi/b]*Sin[a/(2*b)]*Sin
[ArcCos[1 + d*x^2]/2]*(FresnelS[Sqrt[1/(Pi*b)]*Sqrt[a + b*ArcCos[1 + d*x^2]
]]/(d*x)), x] /; FreeQ[{a, b, d}, x]
```

3.88.4 Maple [F]

$$\int \left(a+b \arccos \left(d \,x^{2}+1\right)\right)^{\frac{3}{2}}d x$$

input
```
int((a+b*arccos(d*x^2+1))^(3/2),x)
```

output
```
int((a+b*arccos(d*x^2+1))^(3/2),x)
```

3.88.5 Fricas [F(-2)]

Exception generated.

$$\int \left(a+b\arccos \left(1+dx^2\right)\right)^{3/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccos(d*x^2+1))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.88.6 Sympy [F]

$$\int \left(a+b\arccos \left(1+dx^2\right)\right)^{3/2} dx = \int \left(a+b\operatorname{acos}\left(dx^{2}+1\right)\right)^{\frac{3}{2}} dx$$

input
```
integrate((a+b*acos(d*x**2+1))**(3/2),x)
```

output
```
Integral((a + b*acos(d*x**2 + 1))**(3/2), x)
```

3.88.7 Maxima [F(-2)]

Exception generated.

$$\int \left(a + b \arccos\left(1 + dx^2\right)\right)^{3/2} dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(d*x^2+1))^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: sign: argument cannot be imagi
nary; found sqrt((-_SAGE_VAR_d*_SAGE_VAR_x^2)-2)
```

3.88.8 Giac [F]

$$\int \left(a + b \arccos\left(1 + dx^2\right)\right)^{3/2} dx = \int \left(b \arccos\left(dx^2 + 1\right) + a\right)^{\frac{3}{2}} dx$$

input
```
integrate((a+b*arccos(d*x^2+1))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccos(d*x^2 + 1) + a)^(3/2), x)
```

3.88.9 Mupad [F(-1)]

Timed out.

$$\int \left(a + b \arccos\left(1 + dx^2\right)\right)^{3/2} dx = \int \left(a + b \operatorname{acos}\left(d\,x^2 + 1\right)\right)^{3/2} dx$$

input
```
int((a + b*acos(d*x^2 + 1))^(3/2),x)
```

output
```
int((a + b*acos(d*x^2 + 1))^(3/2), x)
```

3.89 $\int \sqrt{a+b\arccos(1+dx^2)}\,dx$

3.89.1 Optimal result

Integrand size = 16, antiderivative size = 184

$$\int \sqrt{a+b\arccos(1+dx^2)}\,dx = \frac{2\sqrt{\pi}\cos\left(\frac{a}{2b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\arccos(1+dx^2)}}{\sqrt{\pi}}\right)\sin\left(\frac{1}{2}\arccos(1+dx^2)\right)}{\sqrt{\frac{1}{b}}dx} - \frac{2\sqrt{\pi}\,\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\arccos(1+dx^2)}}{\sqrt{\pi}}\right)\sin\left(\frac{a}{2b}\right)\sin\left(\frac{1}{2}\arccos(1+dx^2)\right)}{\sqrt{\frac{1}{b}}dx} - \frac{2\sqrt{a+b\arccos(1+dx^2)}\sin^2\left(\frac{1}{2}\arccos(1+dx^2)\right)}{dx}$$

output
```
2*cos(1/2*a/b)*FresnelS((1/b)^(1/2)*(a+b*arccos(d*x^2+1))^(1/2)/Pi^(1/2))*
sin(1/2*arccos(d*x^2+1))*Pi^(1/2)/d/x/(1/b)^(1/2)-2*FresnelC((1/b)^(1/2)*(
a+b*arccos(d*x^2+1))^(1/2)/Pi^(1/2))*sin(1/2*a/b)*sin(1/2*arccos(d*x^2+1))
*Pi^(1/2)/d/x/(1/b)^(1/2)-2*sin(1/2*arccos(d*x^2+1))^2*(a+b*arccos(d*x^2+1
))^(1/2)/d/x
```

3.89.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.81

$$\int \sqrt{a+b\arccos\left(1+dx^2\right)}\,dx =$$

$$-\frac{2\sin\left(\frac{1}{2}\arccos\left(1+dx^2\right)\right)\left(-\sqrt{b}\sqrt{\pi}\cos\left(\frac{a}{2b}\right)\text{FresnelS}\left(\frac{\sqrt{a+b\arccos(1+dx^2)}}{\sqrt{b}\sqrt{\pi}}\right)+\sqrt{b}\sqrt{\pi}\,\text{FresnelC}\left(\frac{\sqrt{a+b\arccos(}}{\sqrt{b}\sqrt{\pi}}\right.\right.}{dx}$$

input
```
Integrate[Sqrt[a + b*ArcCos[1 + d*x^2]],x]
```

output
```
(-2*Sin[ArcCos[1 + d*x^2]/2]*(-(Sqrt[b]*Sqrt[Pi]*Cos[a/(2*b)]*FresnelS[Sqr
t[a + b*ArcCos[1 + d*x^2]]/(Sqrt[b]*Sqrt[Pi])]) + Sqrt[b]*Sqrt[Pi]*Fresnel
C[Sqrt[a + b*ArcCos[1 + d*x^2]]/(Sqrt[b]*Sqrt[Pi])]*Sin[a/(2*b)] + Sqrt[a
+ b*ArcCos[1 + d*x^2]]*Sin[ArcCos[1 + d*x^2]/2]))/(d*x)
```

3.89.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {5311}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\arccos\left(dx^2+1\right)}\,dx$$

$$\Big\downarrow \text{ 5311}$$

$$-\frac{2\sqrt{\pi}\sin\left(\frac{a}{2b}\right)\sin\left(\frac{1}{2}\arccos\left(dx^2+1\right)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\arccos(dx^2+1)}}{\sqrt{\pi}}\right)}{\sqrt{\frac{1}{b}}dx}+\frac{2\sqrt{\pi}\cos\left(\frac{a}{2b}\right)\sin\left(\frac{1}{2}\arccos\left(dx^2+1\right)\right)\text{FresnelS}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\arccos(dx^2+1)}}{\sqrt{\pi}}\right)}{\sqrt{\frac{1}{b}}dx}-\frac{2\sin^2\left(\frac{1}{2}\arccos\left(dx^2+1\right)\right)\sqrt{a+b\arccos\left(dx^2+1\right)}}{dx}$$

input
```
Int[Sqrt[a + b*ArcCos[1 + d*x^2]],x]
```

output
```
(2*Sqrt[Pi]*Cos[a/(2*b)]*FresnelS[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[1 + d*x^
2]])/Sqrt[Pi]]*Sin[ArcCos[1 + d*x^2]/2])/(Sqrt[b^(-1)]*d*x) - (2*Sqrt[Pi]*
FresnelC[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b
)]*Sin[ArcCos[1 + d*x^2]/2])/(Sqrt[b^(-1)]*d*x) - (2*Sqrt[a + b*ArcCos[1 +
 d*x^2]]*Sin[ArcCos[1 + d*x^2]/2]^2)/(d*x)
```

3.89.3.1 Defintions of rubi rules used

rule 5311
```
Int[Sqrt[(a_.) + ArcCos[1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[-2*Sqrt
[a + b*ArcCos[1 + d*x^2]]*(Sin[ArcCos[1 + d*x^2]/2]^2/(d*x)), x] + (-Simp[2
*Sqrt[Pi]*Sin[a/(2*b)]*Sin[ArcCos[1 + d*x^2]/2]*(FresnelC[Sqrt[1/(Pi*b)]*Sq
rt[a + b*ArcCos[1 + d*x^2]]]/(Sqrt[1/b]*d*x)), x] + Simp[2*Sqrt[Pi]*Cos[a/(
2*b)]*Sin[ArcCos[1 + d*x^2]/2]*(FresnelS[Sqrt[1/(Pi*b)]*Sqrt[a + b*ArcCos[1
 + d*x^2]]]/(Sqrt[1/b]*d*x)), x]) /; FreeQ[{a, b, d}, x]
```

3.89.4 Maple [F]

$$\int \sqrt{a+b \arccos \left(d\, x^{2}+1\right)}dx$$

input
```
int((a+b*arccos(d*x^2+1))^(1/2),x)
```

output
```
int((a+b*arccos(d*x^2+1))^(1/2),x)
```

3.89.5 Fricas [F(-2)]

Exception generated.

$$\int \sqrt{a+b \arccos \left(1+dx^2\right)}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccos(d*x^2+1))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.89.6 Sympy [F]

$$\int \sqrt{a+b\arccos(1+dx^2)}\,dx = \int \sqrt{a+b\,\mathrm{acos}\,(dx^2+1)}\,dx$$

input
```
integrate((a+b*acos(d*x**2+1))**(1/2),x)
```

output
```
Integral(sqrt(a + b*acos(d*x**2 + 1)), x)
```

3.89.7 Maxima [F(-2)]

Exception generated.

$$\int \sqrt{a+b\arccos(1+dx^2)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arccos(d*x^2+1))^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: sign: argument cannot be imagi
nary; found sqrt((-_SAGE_VAR_d*_SAGE_VAR_x^2)-2)
```

3.89.8 Giac [F]

$$\int \sqrt{a+b\arccos(1+dx^2)}\,dx = \int \sqrt{b\arccos(dx^2+1)+a}\,dx$$

input
```
integrate((a+b*arccos(d*x^2+1))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*arccos(d*x^2 + 1) + a), x)
```

3.89.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\arccos\left(1+dx^2\right)}\,dx=\int \sqrt{a+b\,\mathrm{acos}\left(d\,x^2+1\right)}\,dx$$

input
```
int((a + b*acos(d*x^2 + 1))^(1/2),x)
```

output
```
int((a + b*acos(d*x^2 + 1))^(1/2), x)
```

3.90 $\int \frac{1}{\sqrt{a+b\arccos(1+dx^2)}}\,dx$

3.90.1 Optimal result

Integrand size = 16, antiderivative size = 145

$$\int \frac{1}{\sqrt{a+b\arccos(1+dx^2)}}\,dx = -\frac{2\sqrt{\frac{1}{b}}\sqrt{\pi}\cos\left(\frac{a}{2b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\arccos(1+dx^2)}}{\sqrt{\pi}}\right)\sin\left(\frac{1}{2}\arccos(1+dx^2)\right)}{dx} - \frac{2\sqrt{\frac{1}{b}}\sqrt{\pi}\,\text{FresnelS}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\arccos(1+dx^2)}}{\sqrt{\pi}}\right)\sin\left(\frac{a}{2b}\right)\sin\left(\frac{1}{2}\arccos(1+dx^2)\right)}{dx}$$

output
```
-2*cos(1/2*a/b)*FresnelC((1/b)^(1/2)*(a+b*arccos(d*x^2+1))^(1/2)/Pi^(1/2))
*sin(1/2*arccos(d*x^2+1))*(1/b)^(1/2)*Pi^(1/2)/d/x-2*FresnelS((1/b)^(1/2)*
(a+b*arccos(d*x^2+1))^(1/2)/Pi^(1/2))*sin(1/2*a/b)*sin(1/2*arccos(d*x^2+1)
)*(1/b)^(1/2)*Pi^(1/2)/d/x
```

3.90.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.74

$$\int \frac{1}{\sqrt{a+b\arccos(1+dx^2)}}\,dx = -\frac{2\sqrt{\pi}\left(\cos\left(\frac{a}{2b}\right)\text{FresnelC}\left(\frac{\sqrt{a+b\arccos(1+dx^2)}}{\sqrt{b}\sqrt{\pi}}\right)+\text{FresnelS}\left(\frac{\sqrt{a+b\arccos(1+dx^2)}}{\sqrt{b}\sqrt{\pi}}\right)\sin\left(\frac{a}{2b}\right)\right)\sin\left(\frac{1}{2}\arccos(1+d\right.}{\sqrt{b}dx}$$

input
```
Integrate[1/Sqrt[a + b*ArcCos[1 + d*x^2]],x]
```

output
```
(-2*Sqrt[Pi]*(Cos[a/(2*b)]*FresnelC[Sqrt[a + b*ArcCos[1 + d*x^2]]/(Sqrt[b]
*Sqrt[Pi])] + FresnelS[Sqrt[a + b*ArcCos[1 + d*x^2]]/(Sqrt[b]*Sqrt[Pi])]*S
in[a/(2*b)])*Sin[ArcCos[1 + d*x^2]/2])/(Sqrt[b]*d*x)
```

3.90.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {5319}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\arccos(dx^2+1)}}dx$$

$$\Big\downarrow \ 5319$$

$$-\frac{2\sqrt{\pi}\sqrt{\frac{1}{b}}\cos\left(\frac{a}{2b}\right)\sin\left(\frac{1}{2}\arccos\left(dx^2+1\right)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\arccos(dx^2+1)}}{\sqrt{\pi}}\right)}{dx}-\frac{2\sqrt{\pi}\sqrt{\frac{1}{b}}\sin\left(\frac{a}{2b}\right)\sin\left(\frac{1}{2}\arccos\left(dx^2+1\right)\right)\text{FresnelS}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\arccos(dx^2+1)}}{\sqrt{\pi}}\right)}{dx}$$

input
```
Int[1/Sqrt[a + b*ArcCos[1 + d*x^2]],x]
```

output
```
(-2*Sqrt[b^(-1)]*Sqrt[Pi]*Cos[a/(2*b)]*FresnelC[(Sqrt[b^(-1)]*Sqrt[a + b*A
rcCos[1 + d*x^2]])/Sqrt[Pi]]*Sin[ArcCos[1 + d*x^2]/2])/(d*x) - (2*Sqrt[b^(
-1)]*Sqrt[Pi]*FresnelS[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[P
i]]*Sin[a/(2*b)]*Sin[ArcCos[1 + d*x^2]/2])/(d*x)
```

3.90.3.1 Defintions of rubi rules used

rule 5319
```
Int[1/Sqrt[(a_.) + ArcCos[1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[-2*Sq
rt[Pi/b]*Cos[a/(2*b)]*Sin[ArcCos[1 + d*x^2]/2]*(FresnelC[Sqrt[1/(Pi*b)]*Sqr
t[a + b*ArcCos[1 + d*x^2]]]/(d*x)), x] - Simp[2*Sqrt[Pi/b]*Sin[a/(2*b)]*Sin
[ArcCos[1 + d*x^2]/2]*(FresnelS[Sqrt[1/(Pi*b)]*Sqrt[a + b*ArcCos[1 + d*x^2]
]]/(d*x)), x] /; FreeQ[{a, b, d}, x]
```

3.90.4 Maple [F]

$$\int \frac{1}{\sqrt{a+b\arccos\left(d\,x^{2}+1\right)}}dx$$

input
```
int(1/(a+b*arccos(d*x^2+1))^(1/2),x)
```

output
```
int(1/(a+b*arccos(d*x^2+1))^(1/2),x)
```

3.90.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+b\arccos\left(1+dx^{2}\right)}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arccos(d*x^2+1))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.90.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+b\arccos\left(1+dx^2\right)}}\,dx=\int \frac{1}{\sqrt{a+b\,\mathrm{acos}\left(dx^2+1\right)}}\,dx$$

input
```
integrate(1/(a+b*acos(d*x**2+1))**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*acos(d*x**2 + 1)), x)
```

3.90.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+b\arccos\left(1+dx^2\right)}}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a+b*arccos(d*x^2+1))^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: sign: argument cannot be imagi
nary; found sqrt((-_SAGE_VAR_d*_SAGE_VAR_x^2)-2)
```

3.90.8 Giac [F]

$$\int \frac{1}{\sqrt{a+b\arccos\left(1+dx^2\right)}}\,dx=\int \frac{1}{\sqrt{b\arccos\left(dx^2+1\right)+a}}\,dx$$

input
```
integrate(1/(a+b*arccos(d*x^2+1))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*arccos(d*x^2 + 1) + a), x)
```

3.90.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\arccos\left(1+dx^2\right)}}\,dx=\int \frac{1}{\sqrt{a+b\,\mathrm{acos}\left(d\,x^2+1\right)}}\,dx$$

input
```
int(1/(a + b*acos(d*x^2 + 1))^(1/2),x)
```

output
```
int(1/(a + b*acos(d*x^2 + 1))^(1/2), x)
```

3.91 $\int \frac{1}{(a+b\arccos(1+dx^2))^{3/2}}\,dx$

3.91.1 Optimal result

Integrand size = 16, antiderivative size = 190

$$\int \frac{1}{(a+b\arccos(1+dx^2))^{3/2}}\,dx = \frac{\sqrt{-2dx^2-d^2x^4}}{bdx\sqrt{a+b\arccos(1+dx^2)}} + \frac{2\left(\frac{1}{b}\right)^{3/2}\sqrt{\pi}\cos\left(\frac{a}{2b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\arccos(1+dx^2)}}{\sqrt{\pi}}\right)\sin\left(\frac{1}{2}\arccos(1+dx^2)\right)}{dx} - \frac{2\left(\frac{1}{b}\right)^{3/2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\arccos(1+dx^2)}}{\sqrt{\pi}}\right)\sin\left(\frac{a}{2b}\right)\sin\left(\frac{1}{2}\arccos(1+dx^2)\right)}{dx}$$

output
```
2*(1/b)^(3/2)*cos(1/2*a/b)*FresnelS((1/b)^(1/2)*(a+b*arccos(d*x^2+1))^(1/2
)/Pi^(1/2))*sin(1/2*arccos(d*x^2+1))*Pi^(1/2)/d/x-2*(1/b)^(3/2)*FresnelC((
1/b)^(1/2)*(a+b*arccos(d*x^2+1))^(1/2)/Pi^(1/2))*sin(1/2*a/b)*sin(1/2*arcc
os(d*x^2+1))*Pi^(1/2)/d/x+(-d^2*x^4-2*d*x^2)^(1/2)/b/d/x/(a+b*arccos(d*x^2
+1))^(1/2)
```

3.91.2 Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.87

$$\int \frac{1}{(a+b\arccos(1+dx^2))^{3/2}}\,dx = \frac{\frac{\sqrt{b}\sqrt{-dx^2(2+dx^2)}}{\sqrt{a+b\arccos(1+dx^2)}} + 2\sqrt{\pi}\cos\left(\frac{a}{2b}\right)\text{FresnelS}\left(\frac{\sqrt{a+b\arccos(1+dx^2)}}{\sqrt{b}\sqrt{\pi}}\right)\sin\left(\frac{1}{2}\text{arcco}\right.}{}$$

input
```
Integrate[(a + b*ArcCos[1 + d*x^2])^(-3/2),x]
```

output
```
((Sqrt[b]*Sqrt[-(d*x^2*(2 + d*x^2))])/Sqrt[a + b*ArcCos[1 + d*x^2]] + 2*Sq
rt[Pi]*Cos[a/(2*b)]*FresnelS[Sqrt[a + b*ArcCos[1 + d*x^2]]/(Sqrt[b]*Sqrt[P
i])]*Sin[ArcCos[1 + d*x^2]/2] - 2*Sqrt[Pi]*FresnelC[Sqrt[a + b*ArcCos[1 +
d*x^2]]/(Sqrt[b]*Sqrt[Pi])]*Sin[a/(2*b)]*Sin[ArcCos[1 + d*x^2]/2])/(b^(3/2
)*d*x)
```

3.91.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {5322}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\arccos(dx^2+1))^{3/2}}\,dx$$

$$\Big\downarrow \; 5322$$

$$\frac{\sqrt{-d^2x^4-2dx^2}}{bdx\sqrt{a+b\arccos(dx^2+1)}} - \frac{2\sqrt{\pi}\left(\frac{1}{b}\right)^{3/2}\sin\left(\frac{a}{2b}\right)\sin\left(\frac{1}{2}\arccos(dx^2+1)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\arccos(dx^2+1)}}{\sqrt{\pi}}\right)}{dx} + \frac{2\sqrt{\pi}\left(\frac{1}{b}\right)^{3/2}\cos\left(\frac{a}{2b}\right)\sin\left(\frac{1}{2}\arccos(dx^2+1)\right)\text{FresnelS}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\arccos(dx^2+1)}}{\sqrt{\pi}}\right)}{dx}$$

input
```
Int[(a + b*ArcCos[1 + d*x^2])^(-3/2),x]
```

output
```
Sqrt[-2*d*x^2 - d^2*x^4]/(b*d*x*Sqrt[a + b*ArcCos[1 + d*x^2]]) + (2*(b^(-1
))^(3/2)*Sqrt[Pi]*Cos[a/(2*b)]*FresnelS[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[1
+ d*x^2]])/Sqrt[Pi]]*Sin[ArcCos[1 + d*x^2]/2])/(d*x) - (2*(b^(-1))^(3/2)*S
qrt[Pi]*FresnelC[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]*Si
n[a/(2*b)]*Sin[ArcCos[1 + d*x^2]/2])/(d*x)
```

3.91.3.1 Defintions of rubi rules used

rule 5322
```
Int[((a_.) + ArcCos[1 + (d_.)*(x_)^2]*(b_.))^(-3/2), x_Symbol] :> Simp[Sqrt
[-2*d*x^2 - d^2*x^4]/(b*d*x*Sqrt[a + b*ArcCos[1 + d*x^2]]), x] + (-Simp[2*(
1/b)^(3/2)*Sqrt[Pi]*Sin[a/(2*b)]*Sin[ArcCos[1 + d*x^2]/2]*(FresnelC[Sqrt[1/
(Pi*b)]*Sqrt[a + b*ArcCos[1 + d*x^2]]]/(d*x)), x] + Simp[2*(1/b)^(3/2)*Sqrt
[Pi]*Cos[a/(2*b)]*Sin[ArcCos[1 + d*x^2]/2]*(FresnelS[Sqrt[1/(Pi*b)]*Sqrt[a
+ b*ArcCos[1 + d*x^2]]]/(d*x)), x]) /; FreeQ[{a, b, d}, x]
```

3.91.4 Maple [F]

$$\int \frac{1}{\left(a+b \arccos \left(d\,x^{2}+1\right)\right)^{\frac{3}{2}}}d x$$

input
```
int(1/(a+b*arccos(d*x^2+1))^(3/2),x)
```

output
```
int(1/(a+b*arccos(d*x^2+1))^(3/2),x)
```

3.91.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\arccos(1+dx^2))^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arccos(d*x^2+1))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.91.6 Sympy [F]

$$\int \frac{1}{(a+b\arccos(1+dx^2))^{3/2}}\,dx = \int \frac{1}{(a+b\operatorname{acos}(dx^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*acos(d*x**2+1))**(3/2),x)
```

output
```
Integral((a + b*acos(d*x**2 + 1))**(-3/2), x)
```

3.91.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\arccos(1+dx^2))^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a+b*arccos(d*x^2+1))^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: sign: argument cannot be imagi
nary; found sqrt((-_SAGE_VAR_d*_SAGE_VAR_x^2)-2)
```

3.91.8 Giac [F]

$$\int \frac{1}{(a+b\arccos(1+dx^2))^{3/2}}\,dx = \int \frac{1}{(b\arccos(dx^2+1)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*arccos(d*x^2+1))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccos(d*x^2 + 1) + a)^(-3/2), x)
```

3.91.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\arccos(1+dx^2))^{3/2}}\,dx = \int \frac{1}{(a+b\operatorname{acos}(d\,x^2+1))^{3/2}}\,dx$$

input
```
int(1/(a + b*acos(d*x^2 + 1))^(3/2),x)
```

output
```
int(1/(a + b*acos(d*x^2 + 1))^(3/2), x)
```

3.92 $\int \frac{1}{\left(a+b\arccos\left(1+dx^2\right)\right)^{5/2}}\,dx$

3.92.1 Optimal result

Integrand size = 16, antiderivative size = 221

$$\int \frac{1}{\left(a+b\arccos\left(1+dx^2\right)\right)^{5/2}}\,dx = \frac{\sqrt{-2dx^2-d^2x^4}}{3bdx\left(a+b\arccos\left(1+dx^2\right)\right)^{3/2}} + \frac{x}{3b^2\sqrt{a+b\arccos\left(1+dx^2\right)}} + \frac{2\left(\frac{1}{b}\right)^{5/2}\sqrt{\pi}\cos\left(\frac{a}{2b}\right)\operatorname{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\arccos(1+dx^2)}}{\sqrt{\pi}}\right)\sin\left(\frac{1}{2}\arccos\left(1+dx^2\right)\right)}{3dx} + \frac{2\left(\frac{1}{b}\right)^{5/2}\sqrt{\pi}\operatorname{FresnelS}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\arccos(1+dx^2)}}{\sqrt{\pi}}\right)\sin\left(\frac{a}{2b}\right)\sin\left(\frac{1}{2}\arccos\left(1+dx^2\right)\right)}{3dx}$$

output
```
2/3*(1/b)^(5/2)*cos(1/2*a/b)*FresnelC((1/b)^(1/2)*(a+b*arccos(d*x^2+1))^(1
/2)/Pi^(1/2))*sin(1/2*arccos(d*x^2+1))*Pi^(1/2)/d/x+2/3*(1/b)^(5/2)*Fresne
lS((1/b)^(1/2)*(a+b*arccos(d*x^2+1))^(1/2)/Pi^(1/2))*sin(1/2*a/b)*sin(1/2*
arccos(d*x^2+1))*Pi^(1/2)/d/x+1/3*(-d^2*x^4-2*d*x^2)^(1/2)/b/d/x/(a+b*arcc
os(d*x^2+1))^(3/2)+1/3*x/b^2/(a+b*arccos(d*x^2+1))^(1/2)
```

3.92.2 Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.96

$$\int \frac{1}{(a+b\arccos(1+dx^2))^{5/2}}\,dx = \frac{2\sin\left(\frac{1}{2}\arccos(1+dx^2)\right)\left(\sqrt{\pi}(a+b\arccos(1+dx^2))^{3/2}\cos\left(\frac{a}{2b}\right)\text{Fresnel}\right.}{}$$

input
```
Integrate[(a + b*ArcCos[1 + d*x^2])^(-5/2),x]
```

output
```
(2*Sin[ArcCos[1 + d*x^2]/2]*(Sqrt[Pi]*(a + b*ArcCos[1 + d*x^2])^(3/2)*Cos[
a/(2*b)]*FresnelC[Sqrt[a + b*ArcCos[1 + d*x^2]]/(Sqrt[b]*Sqrt[Pi])] + Sqrt
[Pi]*(a + b*ArcCos[1 + d*x^2])^(3/2)*FresnelS[Sqrt[a + b*ArcCos[1 + d*x^2]
]/(Sqrt[b]*Sqrt[Pi])]*Sin[a/(2*b)] + Sqrt[b]*(b*Cos[ArcCos[1 + d*x^2]/2] -
 (a + b*ArcCos[1 + d*x^2])*Sin[ArcCos[1 + d*x^2]/2])))/(3*b^(5/2)*d*x*(a +
 b*ArcCos[1 + d*x^2])^(3/2))
```

3.92.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5328, 5319}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\arccos(dx^2+1))^{5/2}}\,dx$$

$$\Big\downarrow \text{5328}$$

$$-\frac{\int \frac{1}{\sqrt{a+b\arccos(dx^2+1)}}dx}{3b^2}+\frac{x}{3b^2\sqrt{a+b\arccos(dx^2+1)}}+\frac{\sqrt{-d^2x^4-2dx^2}}{3bdx(a+b\arccos(dx^2+1))^{3/2}}$$

$$\Big\downarrow \text{5319}$$

$$-\frac{-\frac{2\sqrt{\pi}\sqrt{\frac{1}{b}}\cos\left(\frac{a}{2b}\right)\sin\left(\frac{1}{2}\arccos(dx^2+1)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\arccos(dx^2+1)}}{\sqrt{\pi}}\right)}{dx}-\frac{2\sqrt{\pi}\sqrt{\frac{1}{b}}\sin\left(\frac{a}{2b}\right)\sin\left(\frac{1}{2}\arccos(dx^2+1)\right)\text{FresnelS}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b}}{}\right.}{dx}}{3b^2}$$

$$\frac{x}{3b^2\sqrt{a+b\arccos(dx^2+1)}}+\frac{\sqrt{-d^2x^4-2dx^2}}{3bdx(a+b\arccos(dx^2+1))^{3/2}}$$

input
```
Int[(a + b*ArcCos[1 + d*x^2])^(-5/2),x]
```

output
```
Sqrt[-2*d*x^2 - d^2*x^4]/(3*b*d*x*(a + b*ArcCos[1 + d*x^2])^(3/2)) + x/(3*
b^2*Sqrt[a + b*ArcCos[1 + d*x^2]]) - ((-2*Sqrt[b^(-1)]*Sqrt[Pi]*Cos[a/(2*b
)]*FresnelC[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]*Sin[Arc
Cos[1 + d*x^2]/2])/(d*x) - (2*Sqrt[b^(-1)]*Sqrt[Pi]*FresnelS[(Sqrt[b^(-1)]
*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b)]*Sin[ArcCos[1 + d*x^
2]/2])/(d*x))/(3*b^2)
```

3.92.3.1 Defintions of rubi rules used

rule 5319
```
Int[1/Sqrt[(a_.) + ArcCos[1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[-2*Sq
rt[Pi/b]*Cos[a/(2*b)]*Sin[ArcCos[1 + d*x^2]/2]*(FresnelC[Sqrt[1/(Pi*b)]*Sqr
t[a + b*ArcCos[1 + d*x^2]]]/(d*x)), x] - Simp[2*Sqrt[Pi/b]*Sin[a/(2*b)]*Sin
[ArcCos[1 + d*x^2]/2]*(FresnelS[Sqrt[1/(Pi*b)]*Sqrt[a + b*ArcCos[1 + d*x^2]
]]/(d*x)), x] /; FreeQ[{a, b, d}, x]
```

rule 5328
```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
(a + b*ArcCos[c + d*x^2])^(n + 2)/(4*b^2*(n + 1)*(n + 2))), x] + (-Simp[Sqr
t[-2*c*d*x^2 - d^2*x^4]*((a + b*ArcCos[c + d*x^2])^(n + 1)/(2*b*d*(n + 1)*x
)), x] - Simp[1/(4*b^2*(n + 1)*(n + 2))   Int[(a + b*ArcCos[c + d*x^2])^(n
+ 2), x], x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && LtQ[n, -1] && NeQ
[n, -2]
```

3.92.4 Maple [F]

$$\int \frac{1}{\left(a+b \arccos \left(d\,x^{2}+1\right)\right)^{\frac{5}{2}}}dx$$

input
```
int(1/(a+b*arccos(d*x^2+1))^(5/2),x)
```

output
```
int(1/(a+b*arccos(d*x^2+1))^(5/2),x)
```

3.92.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a + b \arccos(1 + dx^2))^{5/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arccos(d*x^2+1))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.92.6 Sympy [F]

$$\int \frac{1}{(a + b \arccos(1 + dx^2))^{5/2}} \, dx = \int \frac{1}{(a + b \operatorname{acos}(dx^2 + 1))^{\frac{5}{2}}} \, dx$$

input
```
integrate(1/(a+b*acos(d*x**2+1))**(5/2),x)
```

output
```
Integral((a + b*acos(d*x**2 + 1))**(-5/2), x)
```

3.92.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a + b \arccos(1 + dx^2))^{5/2}} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a+b*arccos(d*x^2+1))^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: sign: argument cannot be imagi
nary; found sqrt((-_SAGE_VAR_d*_SAGE_VAR_x^2)-2)
```

3.92.8 Giac [F]

$$\int \frac{1}{(a+b\arccos(1+dx^2))^{5/2}}\,dx = \int \frac{1}{(b\arccos(dx^2+1)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*arccos(d*x^2+1))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arccos(d*x^2 + 1) + a)^(-5/2), x)
```

3.92.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\arccos(1+dx^2))^{5/2}}\,dx = \int \frac{1}{(a+b\,\mathrm{acos}(d\,x^2+1))^{5/2}}\,dx$$

input
```
int(1/(a + b*acos(d*x^2 + 1))^(5/2),x)
```

output
```
int(1/(a + b*acos(d*x^2 + 1))^(5/2), x)
```

3.93 $\int \frac{1}{(a+b \arccos(1+dx^2))^{7/2}} \, dx$

3.93.1 Optimal result

Integrand size = 16, antiderivative size = 269

$$\begin{aligned}\int \frac{1}{(a+b\arccos(1+dx^2))^{7/2}}\,dx &= \frac{\sqrt{-2dx^2-d^2x^4}}{5bdx\,(a+b\arccos(1+dx^2))^{5/2}} \\ &+ \frac{x}{15b^2\,(a+b\arccos(1+dx^2))^{3/2}} - \frac{\sqrt{-2dx^2-d^2x^4}}{15b^3dx\sqrt{a+b\arccos(1+dx^2)}} \\ &- \frac{2\left(\frac{1}{b}\right)^{7/2}\sqrt{\pi}\cos\left(\frac{a}{2b}\right)\operatorname{FresnelS}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\arccos(1+dx^2)}}{\sqrt{\pi}}\right)\sin\left(\frac{1}{2}\arccos(1+dx^2)\right)}{15dx} \\ &+ \frac{2\left(\frac{1}{b}\right)^{7/2}\sqrt{\pi}\operatorname{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\arccos(1+dx^2)}}{\sqrt{\pi}}\right)\sin\left(\frac{a}{2b}\right)\sin\left(\frac{1}{2}\arccos(1+dx^2)\right)}{15dx}\end{aligned}$$

output
```
1/15*x/b^2/(a+b*arccos(d*x^2+1))^(3/2)-2/15*(1/b)^(7/2)*cos(1/2*a/b)*Fresn
elS((1/b)^(1/2)*(a+b*arccos(d*x^2+1))^(1/2)/Pi^(1/2))*sin(1/2*arccos(d*x^2
+1))*Pi^(1/2)/d/x+2/15*(1/b)^(7/2)*FresnelC((1/b)^(1/2)*(a+b*arccos(d*x^2+
1))^(1/2)/Pi^(1/2))*sin(1/2*a/b)*sin(1/2*arccos(d*x^2+1))*Pi^(1/2)/d/x+1/5
*(-d^2*x^4-2*d*x^2)^(1/2)/b/d/x/(a+b*arccos(d*x^2+1))^(5/2)-1/15*(-d^2*x^4
-2*d*x^2)^(1/2)/b^3/d/x/(a+b*arccos(d*x^2+1))^(1/2)
```

3.93.2 Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.87

$$\int \frac{1}{(a+b\arccos(1+dx^2))^{7/2}}\,dx = \frac{2\sin\left(\frac{1}{2}\arccos(1+dx^2)\right)\left(-\sqrt{\pi}(a+b\arccos(1+dx^2))^{5/2}\cos\left(\frac{a}{2b}\right)\text{Fresn}\right.}{}$$

input
```
Integrate[(a + b*ArcCos[1 + d*x^2])^(-7/2),x]
```

output
```
(2*Sin[ArcCos[1 + d*x^2]/2]*(-(Sqrt[Pi]*(a + b*ArcCos[1 + d*x^2])^(5/2)*Co
s[a/(2*b)]*FresnelS[Sqrt[a + b*ArcCos[1 + d*x^2]]/(Sqrt[b]*Sqrt[Pi])]) + S
qrt[Pi]*(a + b*ArcCos[1 + d*x^2])^(5/2)*FresnelC[Sqrt[a + b*ArcCos[1 + d*x
^2]]/(Sqrt[b]*Sqrt[Pi])]*Sin[a/(2*b)] + Sqrt[b]*(-((-3*b^2 + (a + b*ArcCos
[1 + d*x^2])^2)*Cos[ArcCos[1 + d*x^2]/2]) - b*(a + b*ArcCos[1 + d*x^2])*Si
n[ArcCos[1 + d*x^2]/2])))/(15*b^(7/2)*d*x*(a + b*ArcCos[1 + d*x^2])^(5/2))
```

3.93.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5328, 5322}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\arccos(dx^2+1))^{7/2}}\,dx$$

$$\Big\downarrow \text{5328}$$

$$-\frac{\int \frac{1}{(a+b\arccos(dx^2+1))^{3/2}}dx}{15b^2}+\frac{x}{15b^2(a+b\arccos(dx^2+1))^{3/2}}+\frac{\sqrt{-d^2x^4-2dx^2}}{5bdx(a+b\arccos(dx^2+1))^{5/2}}$$

$$\Big\downarrow \text{5322}$$

$$-\frac{\frac{\sqrt{-d^2x^4-2dx^2}}{bdx\sqrt{a+b\arccos(dx^2+1)}}-\frac{2\sqrt{\pi}\left(\frac{1}{b}\right)^{3/2}\sin\left(\frac{a}{2b}\right)\sin\left(\frac{1}{2}\arccos(dx^2+1)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\arccos(dx^2+1)}}{\sqrt{\pi}}\right)}{dx}+\frac{2\sqrt{\pi}\left(\frac{1}{b}\right)^{3/2}\cos\left(\frac{a}{2b}\right)\sin\left(\frac{1}{2}\text{arc}\right.}{}}{15b^2}$$

$$\frac{x}{15b^2(a+b\arccos(dx^2+1))^{3/2}}+\frac{\sqrt{-d^2x^4-2dx^2}}{5bdx(a+b\arccos(dx^2+1))^{5/2}}$$

input
```
Int[(a + b*ArcCos[1 + d*x^2])^(-7/2),x]
```

output
```
Sqrt[-2*d*x^2 - d^2*x^4]/(5*b*d*x*(a + b*ArcCos[1 + d*x^2])^(5/2)) + x/(15
*b^2*(a + b*ArcCos[1 + d*x^2])^(3/2)) - (Sqrt[-2*d*x^2 - d^2*x^4]/(b*d*x*S
qrt[a + b*ArcCos[1 + d*x^2]]) + (2*(b^(-1))^(3/2)*Sqrt[Pi]*Cos[a/(2*b)]*Fr
esnelS[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]*Sin[ArcCos[1
 + d*x^2]/2])/(d*x) - (2*(b^(-1))^(3/2)*Sqrt[Pi]*FresnelC[(Sqrt[b^(-1)]*Sq
rt[a + b*ArcCos[1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b)]*Sin[ArcCos[1 + d*x^2]/
2])/(d*x))/(15*b^2)
```

3.93.3.1 Defintions of rubi rules used

rule 5322
```
Int[((a_.) + ArcCos[1 + (d_.)*(x_)^2]*(b_.))^(-3/2), x_Symbol] :> Simp[Sqrt
[-2*d*x^2 - d^2*x^4]/(b*d*x*Sqrt[a + b*ArcCos[1 + d*x^2]]), x] + (-Simp[2*(
1/b)^(3/2)*Sqrt[Pi]*Sin[a/(2*b)]*Sin[ArcCos[1 + d*x^2]/2]*(FresnelC[Sqrt[1/
(Pi*b)]*Sqrt[a + b*ArcCos[1 + d*x^2]]]/(d*x)), x] + Simp[2*(1/b)^(3/2)*Sqrt
[Pi]*Cos[a/(2*b)]*Sin[ArcCos[1 + d*x^2]/2]*(FresnelS[Sqrt[1/(Pi*b)]*Sqrt[a
+ b*ArcCos[1 + d*x^2]]]/(d*x)), x]) /; FreeQ[{a, b, d}, x]
```

rule 5328
```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
(a + b*ArcCos[c + d*x^2])^(n + 2)/(4*b^2*(n + 1)*(n + 2))), x] + (-Simp[Sqr
t[-2*c*d*x^2 - d^2*x^4]*((a + b*ArcCos[c + d*x^2])^(n + 1)/(2*b*d*(n + 1)*x
)), x] - Simp[1/(4*b^2*(n + 1)*(n + 2))   Int[(a + b*ArcCos[c + d*x^2])^(n
+ 2), x], x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && LtQ[n, -1] && NeQ
[n, -2]
```

3.93.4 Maple [F]

$$\int \frac{1}{\left(a+b\arccos\left(d\,x^{2}+1\right)\right)^{\frac{7}{2}}}dx$$

input
```
int(1/(a+b*arccos(d*x^2+1))^(7/2),x)
```

output
```
int(1/(a+b*arccos(d*x^2+1))^(7/2),x)
```

3.93.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\arccos(1+dx^2))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arccos(d*x^2+1))^(7/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.93.6 Sympy [F]

$$\int \frac{1}{(a+b\arccos(1+dx^2))^{7/2}}\,dx = \int \frac{1}{(a+b\,\mathrm{acos}\,(dx^2+1))^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+b*acos(d*x**2+1))**(7/2),x)
```

output
```
Integral((a + b*acos(d*x**2 + 1))**(-7/2), x)
```

3.93.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\arccos(1+dx^2))^{7/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a+b*arccos(d*x^2+1))^(7/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: sign: argument cannot be imagi
nary; found sqrt((-_SAGE_VAR_d*_SAGE_VAR_x^2)-2)
```

3.93.8 Giac [F]

$$\int \frac{1}{\left(a+b\arccos\left(1+dx^2\right)\right)^{7/2}}\,dx=\int \frac{1}{\left(b\arccos\left(dx^2+1\right)+a\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+b*arccos(d*x^2+1))^(7/2),x, algorithm="giac")
```

output
```
integrate((b*arccos(d*x^2 + 1) + a)^(-7/2), x)
```

3.93.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\arccos\left(1+dx^2\right)\right)^{7/2}}\,dx=\int \frac{1}{\left(a+b\,\mathrm{acos}\left(d\,x^2+1\right)\right)^{7/2}}\,dx$$

input
```
int(1/(a + b*acos(d*x^2 + 1))^(7/2),x)
```

output
```
int(1/(a + b*acos(d*x^2 + 1))^(7/2), x)
```

3.94 $\int \left(a+b \arccos \left(-1+d x^2\right)\right)^{5/2} dx$

3.94.1 Optimal result

Integrand size = 16, antiderivative size = 249

$$\begin{aligned}\int \left(a+b \arccos \left(-1+d x^2\right)\right)^{5/2} dx = & -\frac{5b\sqrt{2dx^2-d^2x^4}\left(a+b\arccos\left(-1+dx^2\right)\right)^{3/2}}{dx}+x\left(a+b\arccos\left(-1+dx^2\right)\right)^{5/2} \\ & -\frac{30b^2\sqrt{a+b\arccos\left(-1+dx^2\right)}\cos^2\left(\frac{1}{2}\arccos\left(-1+dx^2\right)\right)}{dx} \\ & +\frac{30\sqrt{\pi}\cos\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\arccos\left(-1+dx^2\right)\right)\operatorname{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\arccos(-1+dx^2)}}{\sqrt{\pi}}\right)}{\left(\frac{1}{b}\right)^{5/2}dx} \\ & +\frac{30\sqrt{\pi}\cos\left(\frac{1}{2}\arccos\left(-1+dx^2\right)\right)\operatorname{FresnelS}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\arccos(-1+dx^2)}}{\sqrt{\pi}}\right)\sin\left(\frac{a}{2b}\right)}{\left(\frac{1}{b}\right)^{5/2}dx}\end{aligned}$$

output
```
x*(a+b*arccos(d*x^2-1))^(5/2)+30*cos(1/2*a/b)*cos(1/2*arccos(d*x^2-1))*Fre
snelC((1/b)^(1/2)*(a+b*arccos(d*x^2-1))^(1/2)/Pi^(1/2))*Pi^(1/2)/(1/b)^(5/
2)/d/x+30*cos(1/2*arccos(d*x^2-1))*FresnelS((1/b)^(1/2)*(a+b*arccos(d*x^2-
1))^(1/2)/Pi^(1/2))*sin(1/2*a/b)*Pi^(1/2)/(1/b)^(5/2)/d/x-5*b*(a+b*arccos(
d*x^2-1))^(3/2)*(-d^2*x^4+2*d*x^2)^(1/2)/d/x-30*b^2*cos(1/2*arccos(d*x^2-1
))^2*(a+b*arccos(d*x^2-1))^(1/2)/d/x
```

3.94.2 Mathematica [A] (verified)

Time = 1.45 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.00

$$\int (a+b\arccos(-1+dx^2))^{5/2}\,dx=\frac{2\cos\left(\frac{1}{2}\arccos(-1+dx^2)\right)\left(15b^{5/2}\sqrt{\pi}\cos\left(\frac{a}{2b}\right)\text{FresnelC}\left(\frac{\sqrt{a+b\arccos(-1+dx^2)}}{\sqrt{b}\sqrt{\pi}}\right)+15b^{5/2}\sqrt{\pi}\,\text{Fr}$$

input
```
Integrate[(a + b*ArcCos[-1 + d*x^2])^(5/2),x]
```

output
```
(2*Cos[ArcCos[-1 + d*x^2]/2]*(15*b^(5/2)*Sqrt[Pi]*Cos[a/(2*b)]*FresnelC[Sq
rt[a + b*ArcCos[-1 + d*x^2]]/(Sqrt[b]*Sqrt[Pi])] + 15*b^(5/2)*Sqrt[Pi]*Fre
snelS[Sqrt[a + b*ArcCos[-1 + d*x^2]]/(Sqrt[b]*Sqrt[Pi])]*Sin[a/(2*b)] + Sq
rt[a + b*ArcCos[-1 + d*x^2]]*((a^2 - 15*b^2)*Cos[ArcCos[-1 + d*x^2]/2] + b
^2*ArcCos[-1 + d*x^2]^2*Cos[ArcCos[-1 + d*x^2]/2] - 5*a*b*Sin[ArcCos[-1 +
d*x^2]/2] + b*ArcCos[-1 + d*x^2]*(2*a*Cos[ArcCos[-1 + d*x^2]/2] - 5*b*Sin[
ArcCos[-1 + d*x^2]/2]))))/(d*x)
```

3.94.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5314, 5312}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\arccos(dx^2-1))^{5/2}\,dx$$

$$\Big\downarrow \; 5314$$

$$-15b^2\int\sqrt{a+b\arccos(dx^2-1)}dx-\frac{5b\sqrt{2dx^2-d^2x^4}(a+b\arccos(dx^2-1))^{3/2}}{dx}+x(a+b\arccos(dx^2-1))^{5/2}$$

$$\Big\downarrow \; 5312$$

$$-15b^2\left(-\frac{2\sqrt{\pi}\cos\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\arccos\left(dx^2-1\right)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\arccos(dx^2-1)}}{\sqrt{\pi}}\right)}{\sqrt{\frac{1}{b}}dx}-\frac{2\sqrt{\pi}\sin\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\arccos\left(dx^2\right.\right.}{}\right.$$

$$\frac{5b\sqrt{2dx^2-d^2x^4}\left(a+b\arccos\left(dx^2-1\right)\right)^{3/2}}{dx}+x\left(a+b\arccos\left(dx^2-1\right)\right)^{5/2}$$

input
```
Int[(a + b*ArcCos[-1 + d*x^2])^(5/2),x]
```

output
```
(-5*b*Sqrt[2*d*x^2 - d^2*x^4]*(a + b*ArcCos[-1 + d*x^2])^(3/2))/(d*x) + x*
(a + b*ArcCos[-1 + d*x^2])^(5/2) - 15*b^2*((2*Sqrt[a + b*ArcCos[-1 + d*x^2
]]*Cos[ArcCos[-1 + d*x^2]/2]^2)/(d*x) - (2*Sqrt[Pi]*Cos[a/(2*b)]*Cos[ArcCo
s[-1 + d*x^2]/2]*FresnelC[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sq
rt[Pi]])/(Sqrt[b^(-1)]*d*x) - (2*Sqrt[Pi]*Cos[ArcCos[-1 + d*x^2]/2]*Fresne
lS[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b)])/(
Sqrt[b^(-1)]*d*x))
```

3.94.3.1 Defintions of rubi rules used

rule 5312
```
Int[Sqrt[(a_.) + ArcCos[-1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[2*Sqrt
[a + b*ArcCos[-1 + d*x^2]]*(Cos[(1/2)*ArcCos[-1 + d*x^2]]^2/(d*x)), x] + (-
Simp[2*Sqrt[Pi]*Cos[a/(2*b)]*Cos[ArcCos[-1 + d*x^2]/2]*(FresnelC[Sqrt[1/(Pi
*b)]*Sqrt[a + b*ArcCos[-1 + d*x^2]]]/(Sqrt[1/b]*d*x)), x] - Simp[2*Sqrt[Pi]
*Sin[a/(2*b)]*Cos[ArcCos[-1 + d*x^2]/2]*(FresnelS[Sqrt[1/(Pi*b)]*Sqrt[a + b
*ArcCos[-1 + d*x^2]]]/(Sqrt[1/b]*d*x)), x]) /; FreeQ[{a, b, d}, x]
```

rule 5314
```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcCos[c + d*x^2])^n, x] + (-Simp[2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*((
a + b*ArcCos[c + d*x^2])^(n - 1)/(d*x)), x] - Simp[4*b^2*n*(n - 1)   Int[(a
 + b*ArcCos[c + d*x^2])^(n - 2), x], x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c
^2, 1] && GtQ[n, 1]
```

3.94.4 Maple [F]

$$\int \left(a+b \arccos \left(d \,x^{2}-1\right)\right)^{\frac{5}{2}}d x$$

input
```
int((a+b*arccos(d*x^2-1))^(5/2),x)
```

output
```
int((a+b*arccos(d*x^2-1))^(5/2),x)
```

3.94.5 Fricas [F(-2)]

Exception generated.

$$\int \left(a+b\arccos\left(-1+dx^2\right)\right)^{5/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccos(d*x^2-1))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.94.6 Sympy [F]

$$\int \left(a+b\arccos\left(-1+dx^2\right)\right)^{5/2}\,dx=\int \left(a+b\,\text{acos}\left(dx^2-1\right)\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*acos(d*x**2-1))**(5/2),x)
```

output
```
Integral((a + b*acos(d*x**2 - 1))**(5/2), x)
```

3.94.7 Maxima [F]

$$\int \left(a+b\arccos\left(-1+dx^2\right)\right)^{5/2}\,dx=\int \left(b\arccos\left(dx^2-1\right)+a\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*arccos(d*x^2-1))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*arccos(d*x^2 - 1) + a)^(5/2), x)
```

3.94.8 Giac [F]

$$\int \left(a+b\arccos\left(-1+dx^2\right)\right)^{5/2}\,dx=\int \left(b\arccos\left(dx^2-1\right)+a\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*arccos(d*x^2-1))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arccos(d*x^2 - 1) + a)^(5/2), x)
```

3.94.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+b\arccos\left(-1+dx^2\right)\right)^{5/2}\,dx=\int \left(a+b\,\mathrm{acos}\left(d\,x^2-1\right)\right)^{5/2}\,dx$$

input
```
int((a + b*acos(d*x^2 - 1))^(5/2),x)
```

output
```
int((a + b*acos(d*x^2 - 1))^(5/2), x)
```

3.95 $\int \left(a+b\arccos\left(-1+dx^2\right)\right)^{3/2}\,dx$

3.95.1 Optimal result

Integrand size = 16, antiderivative size = 207

$$\int \left(a+b\arccos\left(-1+dx^2\right)\right)^{3/2}\,dx = -\frac{3b\sqrt{2dx^2-d^2x^4}\sqrt{a+b\arccos\left(-1+dx^2\right)}}{dx}+x\left(a+b\arccos\left(-1+dx^2\right)\right)^{3/2} + \frac{6\sqrt{\pi}\cos\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\arccos\left(-1+dx^2\right)\right)\operatorname{FresnelS}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\arccos(-1+dx^2)}}{\sqrt{\pi}}\right)}{\left(\frac{1}{b}\right)^{3/2}dx} - \frac{6\sqrt{\pi}\cos\left(\frac{1}{2}\arccos\left(-1+dx^2\right)\right)\operatorname{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\arccos(-1+dx^2)}}{\sqrt{\pi}}\right)\sin\left(\frac{a}{2b}\right)}{\left(\frac{1}{b}\right)^{3/2}dx}$$

output
```
x*(a+b*arccos(d*x^2-1))^(3/2)+6*cos(1/2*a/b)*cos(1/2*arccos(d*x^2-1))*Fres
nelS((1/b)^(1/2)*(a+b*arccos(d*x^2-1))^(1/2)/Pi^(1/2))*Pi^(1/2)/(1/b)^(3/2
)/d/x-6*cos(1/2*arccos(d*x^2-1))*FresnelC((1/b)^(1/2)*(a+b*arccos(d*x^2-1)
)^(1/2)/Pi^(1/2))*sin(1/2*a/b)*Pi^(1/2)/(1/b)^(3/2)/d/x-3*b*(-d^2*x^4+2*d*
x^2)^(1/2)*(a+b*arccos(d*x^2-1))^(1/2)/d/x
```

3.95.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.93

$$\int (a+b\arccos(-1+dx^2))^{3/2}\,dx = \frac{2\cos\left(\frac{1}{2}\arccos(-1+dx^2)\right)\left(3b^{3/2}\sqrt{\pi}\cos\left(\frac{a}{2b}\right)\text{FresnelS}\left(\frac{\sqrt{a+b\arccos(-1+dx^2)}}{\sqrt{b}\sqrt{\pi}}\right)-3b^{3/2}\sqrt{\pi}\,\text{Fres}\right.}{}$$

input
```
Integrate[(a + b*ArcCos[-1 + d*x^2])^(3/2),x]
```

output
```
(2*Cos[ArcCos[-1 + d*x^2]/2]*(3*b^(3/2)*Sqrt[Pi]*Cos[a/(2*b)]*FresnelS[Sqr
t[a + b*ArcCos[-1 + d*x^2]]/(Sqrt[b]*Sqrt[Pi])] - 3*b^(3/2)*Sqrt[Pi]*Fresn
elC[Sqrt[a + b*ArcCos[-1 + d*x^2]]/(Sqrt[b]*Sqrt[Pi])]*Sin[a/(2*b)] + Sqrt
[a + b*ArcCos[-1 + d*x^2]]*(a*Cos[ArcCos[-1 + d*x^2]/2] + b*ArcCos[-1 + d*
x^2]*Cos[ArcCos[-1 + d*x^2]/2] - 3*b*Sin[ArcCos[-1 + d*x^2]/2])))/(d*x)
```

3.95.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.03, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5314, 5320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\arccos(dx^2-1))^{3/2}\,dx$$

$$\Big\downarrow \text{5314}$$

$$-3b^2\int \frac{1}{\sqrt{a+b\arccos(dx^2-1)}}dx - \frac{3b\sqrt{2dx^2-d^2x^4}\sqrt{a+b\arccos(dx^2-1)}}{dx} + x(a+b\arccos(dx^2-1))^{3/2}$$

$$\Big\downarrow \text{5320}$$

$$-3b^2\left(\frac{2\sqrt{\pi}\sqrt{\frac{1}{b}}\sin\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\arccos\left(dx^2-1\right)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\arccos(dx^2-1)}}{\sqrt{\pi}}\right)}{dx}-\frac{2\sqrt{\pi}\sqrt{\frac{1}{b}}\cos\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\text{arcc}\right.}{}\right.$$

$$\frac{3b\sqrt{2dx^2-d^2x^4}\sqrt{a+b\arccos\left(dx^2-1\right)}}{dx}+x\left(a+b\arccos\left(dx^2-1\right)\right)^{3/2}$$

input
```
Int[(a + b*ArcCos[-1 + d*x^2])^(3/2),x]
```

output
```
(-3*b*Sqrt[2*d*x^2 - d^2*x^4]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/(d*x) + x*(a
 + b*ArcCos[-1 + d*x^2])^(3/2) - 3*b^2*((-2*Sqrt[b^(-1)]*Sqrt[Pi]*Cos[a/(2
*b)]*Cos[ArcCos[-1 + d*x^2]/2]*FresnelS[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1
 + d*x^2]])/Sqrt[Pi]])/(d*x) + (2*Sqrt[b^(-1)]*Sqrt[Pi]*Cos[ArcCos[-1 + d*
x^2]/2]*FresnelC[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]]*S
in[a/(2*b)])/(d*x))
```

3.95.3.1 Defintions of rubi rules used

rule 5314
```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
a + b*ArcCos[c + d*x^2])^n, x] + (-Simp[2*b*n*Sqrt[-2*c*d*x^2 - d^2*x^4]*((
a + b*ArcCos[c + d*x^2])^(n - 1)/(d*x)), x] - Simp[4*b^2*n*(n - 1)   Int[(a
 + b*ArcCos[c + d*x^2])^(n - 2), x], x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c
^2, 1] && GtQ[n, 1]
```

rule 5320
```
Int[1/Sqrt[(a_.) + ArcCos[-1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[2*Sq
rt[Pi/b]*Sin[a/(2*b)]*Cos[ArcCos[-1 + d*x^2]/2]*(FresnelC[Sqrt[1/(Pi*b)]*Sq
rt[a + b*ArcCos[-1 + d*x^2]]]/(d*x)), x] - Simp[2*Sqrt[Pi/b]*Cos[a/(2*b)]*C
os[ArcCos[-1 + d*x^2]/2]*(FresnelS[Sqrt[1/(Pi*b)]*Sqrt[a + b*ArcCos[-1 + d*
x^2]]]/(d*x)), x] /; FreeQ[{a, b, d}, x]
```

3.95.4 Maple [F]

$$\int \left(a+b\arccos\left(d\,x^{2}-1\right)\right)^{\frac{3}{2}}dx$$

input
```
int((a+b*arccos(d*x^2-1))^(3/2),x)
```

output
```
int((a+b*arccos(d*x^2-1))^(3/2),x)
```

3.95.5 Fricas [F(-2)]

Exception generated.

$$\int \left(a+b\arccos\left(-1+dx^2\right)\right)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccos(d*x^2-1))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.95.6 Sympy [F]

$$\int \left(a+b\arccos\left(-1+dx^2\right)\right)^{3/2}\,dx = \int \left(a+b\,\mathrm{acos}\left(dx^2-1\right)\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*acos(d*x**2-1))**(3/2),x)
```

output
```
Integral((a + b*acos(d*x**2 - 1))**(3/2), x)
```

3.95.7 Maxima [F]

$$\int \left(a+b\arccos\left(-1+dx^2\right)\right)^{3/2} dx = \int \left(b\arccos\left(dx^2-1\right)+a\right)^{\frac{3}{2}} dx$$

input
```
integrate((a+b*arccos(d*x^2-1))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*arccos(d*x^2 - 1) + a)^(3/2), x)
```

3.95.8 Giac [F]

$$\int \left(a+b\arccos\left(-1+dx^2\right)\right)^{3/2} dx = \int \left(b\arccos\left(dx^2-1\right)+a\right)^{\frac{3}{2}} dx$$

input
```
integrate((a+b*arccos(d*x^2-1))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccos(d*x^2 - 1) + a)^(3/2), x)
```

3.95.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+b\arccos\left(-1+dx^2\right)\right)^{3/2} dx = \int \left(a+b\,\mathrm{acos}\left(d\,x^2-1\right)\right)^{3/2} dx$$

input
```
int((a + b*acos(d*x^2 - 1))^(3/2),x)
```

output
```
int((a + b*acos(d*x^2 - 1))^(3/2), x)
```

3.96 $\int \sqrt{a+b\arccos(-1+dx^2)}\,dx$

3.96.1 Optimal result

Integrand size = 16, antiderivative size = 184

$$\begin{aligned}\int \sqrt{a+b\arccos(-1+dx^2)}\,dx &= \frac{2\sqrt{a+b\arccos(-1+dx^2)}\cos^2\left(\frac{1}{2}\arccos(-1+dx^2)\right)}{dx} \\ &\quad - \frac{2\sqrt{\pi}\cos\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\arccos(-1+dx^2)\right)\operatorname{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\arccos(-1+dx^2)}}{\sqrt{\pi}}\right)}{\sqrt{\frac{1}{b}}dx} \\ &\quad - \frac{2\sqrt{\pi}\cos\left(\frac{1}{2}\arccos(-1+dx^2)\right)\operatorname{FresnelS}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\arccos(-1+dx^2)}}{\sqrt{\pi}}\right)\sin\left(\frac{a}{2b}\right)}{\sqrt{\frac{1}{b}}dx}\end{aligned}$$

output
```
-2*cos(1/2*a/b)*cos(1/2*arccos(d*x^2-1))*FresnelC((1/b)^(1/2)*(a+b*arccos(
d*x^2-1))^(1/2)/Pi^(1/2))*Pi^(1/2)/d/x/(1/b)^(1/2)-2*cos(1/2*arccos(d*x^2-
1))*FresnelS((1/b)^(1/2)*(a+b*arccos(d*x^2-1))^(1/2)/Pi^(1/2))*sin(1/2*a/b
)*Pi^(1/2)/d/x/(1/b)^(1/2)+2*cos(1/2*arccos(d*x^2-1))^2*(a+b*arccos(d*x^2-
1))^(1/2)/d/x
```

3.96.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.81

$$\int \sqrt{a+b\arccos\left(-1+dx^2\right)}\,dx =$$

$$-\frac{2\cos\left(\frac{1}{2}\arccos\left(-1+dx^2\right)\right)\left(-\sqrt{a+b\arccos\left(-1+dx^2\right)}\cos\left(\frac{1}{2}\arccos\left(-1+dx^2\right)\right)+\sqrt{b}\sqrt{\pi}\cos\left(\frac{a}{2b}\right)\text{Fr}\right.}{dx}$$

input
```
Integrate[Sqrt[a + b*ArcCos[-1 + d*x^2]],x]
```

output
```
(-2*Cos[ArcCos[-1 + d*x^2]/2]*(-(Sqrt[a + b*ArcCos[-1 + d*x^2]]*Cos[ArcCos
[-1 + d*x^2]/2]) + Sqrt[b]*Sqrt[Pi]*Cos[a/(2*b)]*FresnelC[Sqrt[a + b*ArcCo
s[-1 + d*x^2]]/(Sqrt[b]*Sqrt[Pi])] + Sqrt[b]*Sqrt[Pi]*FresnelS[Sqrt[a + b*
ArcCos[-1 + d*x^2]]/(Sqrt[b]*Sqrt[Pi])]*Sin[a/(2*b)]))/(d*x)
```

3.96.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {5312}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\arccos\left(dx^2-1\right)}\,dx$$

$$\Big\downarrow \text{5312}$$

$$-\frac{2\sqrt{\pi}\cos\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\arccos\left(dx^2-1\right)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\arccos(dx^2-1)}}{\sqrt{\pi}}\right)}{\sqrt{\frac{1}{b}}dx}-\frac{2\sqrt{\pi}\sin\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\arccos\left(dx^2-1\right)\right)\text{FresnelS}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\arccos(dx^2-1)}}{\sqrt{\pi}}\right)}{\sqrt{\frac{1}{b}}dx}+\frac{2\cos^2\left(\frac{1}{2}\arccos\left(dx^2-1\right)\right)\sqrt{a+b\arccos\left(dx^2-1\right)}}{dx}$$

input
```
Int[Sqrt[a + b*ArcCos[-1 + d*x^2]],x]
```

output
```
(2*Sqrt[a + b*ArcCos[-1 + d*x^2]]*Cos[ArcCos[-1 + d*x^2]/2]^2)/(d*x) - (2*
Sqrt[Pi]*Cos[a/(2*b)]*Cos[ArcCos[-1 + d*x^2]/2]*FresnelC[(Sqrt[b^(-1)]*Sqr
t[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]])/(Sqrt[b^(-1)]*d*x) - (2*Sqrt[Pi]*C
os[ArcCos[-1 + d*x^2]/2]*FresnelS[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1 + d*x
^2]])/Sqrt[Pi]]*Sin[a/(2*b)])/(Sqrt[b^(-1)]*d*x)
```

3.96.3.1 Defintions of rubi rules used

rule 5312
```
Int[Sqrt[(a_.) + ArcCos[-1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[2*Sqrt
[a + b*ArcCos[-1 + d*x^2]]*(Cos[(1/2)*ArcCos[-1 + d*x^2]]^2/(d*x)), x] + (-
Simp[2*Sqrt[Pi]*Cos[a/(2*b)]*Cos[ArcCos[-1 + d*x^2]/2]*(FresnelC[Sqrt[1/(Pi
*b)]*Sqrt[a + b*ArcCos[-1 + d*x^2]]]/(Sqrt[1/b]*d*x)), x] - Simp[2*Sqrt[Pi]
*Sin[a/(2*b)]*Cos[ArcCos[-1 + d*x^2]/2]*(FresnelS[Sqrt[1/(Pi*b)]*Sqrt[a + b
*ArcCos[-1 + d*x^2]]]/(Sqrt[1/b]*d*x)), x]) /; FreeQ[{a, b, d}, x]
```

3.96.4 Maple [F]

$$\int \sqrt{a+b\arccos\left(d\,x^{2}-1\right)}dx$$

input
```
int((a+b*arccos(d*x^2-1))^(1/2),x)
```

output
```
int((a+b*arccos(d*x^2-1))^(1/2),x)
```

3.96.5 Fricas [F(-2)]

Exception generated.

$$\int \sqrt{a+b\arccos\left(-1+dx^2\right)}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+b*arccos(d*x^2-1))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.96.6 Sympy [F]

$$\int \sqrt{a+b\arccos\left(-1+dx^2\right)}\,dx=\int \sqrt{a+b\,\mathrm{acos}\left(dx^2-1\right)}\,dx$$

input
```
integrate((a+b*acos(d*x**2-1))**(1/2),x)
```

output
```
Integral(sqrt(a + b*acos(d*x**2 - 1)), x)
```

3.96.7 Maxima [F]

$$\int \sqrt{a+b\arccos\left(-1+dx^2\right)}\,dx=\int \sqrt{b\arccos\left(dx^2-1\right)+a}\,dx$$

input
```
integrate((a+b*arccos(d*x^2-1))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*arccos(d*x^2 - 1) + a), x)
```

3.96.8 Giac [F]

$$\int \sqrt{a+b\arccos\left(-1+dx^2\right)}\,dx=\int \sqrt{b\arccos\left(dx^2-1\right)+a}\,dx$$

input
```
integrate((a+b*arccos(d*x^2-1))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*arccos(d*x^2 - 1) + a), x)
```

3.96.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\arccos\left(-1+dx^2\right)}\,dx = \int \sqrt{a+b\,\mathrm{acos}\left(d\,x^2-1\right)}\,dx$$

input
```
int((a + b*acos(d*x^2 - 1))^(1/2),x)
```

output
```
int((a + b*acos(d*x^2 - 1))^(1/2), x)
```

3.97 $\int \frac{1}{\sqrt{a+b\arccos(-1+dx^2)}}\,dx$

3.97.1 Optimal result

Integrand size = 16, antiderivative size = 145

$$\int \frac{1}{\sqrt{a+b\arccos(-1+dx^2)}}\,dx = -\frac{2\sqrt{\frac{1}{b}}\sqrt{\pi}\cos\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\arccos(-1+dx^2)\right)\operatorname{FresnelS}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\arccos(-1+dx^2)}}{\sqrt{\pi}}\right)}{dx} + \frac{2\sqrt{\frac{1}{b}}\sqrt{\pi}\cos\left(\frac{1}{2}\arccos(-1+dx^2)\right)\operatorname{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\arccos(-1+dx^2)}}{\sqrt{\pi}}\right)\sin\left(\frac{a}{2b}\right)}{dx}$$

output
```
-2*cos(1/2*a/b)*cos(1/2*arccos(d*x^2-1))*FresnelS((1/b)^(1/2)*(a+b*arccos(
d*x^2-1))^(1/2)/Pi^(1/2))*(1/b)^(1/2)*Pi^(1/2)/d/x+2*cos(1/2*arccos(d*x^2-
1))*FresnelC((1/b)^(1/2)*(a+b*arccos(d*x^2-1))^(1/2)/Pi^(1/2))*sin(1/2*a/b
)*(1/b)^(1/2)*Pi^(1/2)/d/x
```

3.97.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{a+b\arccos(-1+dx^2)}}\,dx = -\frac{2\sqrt{\pi}\cos\left(\frac{1}{2}\arccos(-1+dx^2)\right)\left(\cos\left(\frac{a}{2b}\right)\text{FresnelS}\left(\frac{\sqrt{a+b\arccos(-1+dx^2)}}{\sqrt{b}\sqrt{\pi}}\right)-\text{FresnelC}\left(\frac{\sqrt{a+b\arccos(-1+dx^2)}}{\sqrt{b}\sqrt{\pi}}\right)\right.}{\sqrt{b}dx}$$

input
```
Integrate[1/Sqrt[a + b*ArcCos[-1 + d*x^2]],x]
```

output
```
(-2*Sqrt[Pi]*Cos[ArcCos[-1 + d*x^2]/2]*(Cos[a/(2*b)]*FresnelS[Sqrt[a + b*A
rcCos[-1 + d*x^2]]/(Sqrt[b]*Sqrt[Pi])] - FresnelC[Sqrt[a + b*ArcCos[-1 + d
*x^2]]/(Sqrt[b]*Sqrt[Pi])]*Sin[a/(2*b)]))/(Sqrt[b]*d*x)
```

3.97.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {5320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\arccos(dx^2-1)}}dx$$

$$\Big\downarrow \text{5320}$$

$$\frac{2\sqrt{\pi}\sqrt{\frac{1}{b}}\sin\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\arccos(dx^2-1)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\arccos(dx^2-1)}}{\sqrt{\pi}}\right)}{dx}-\frac{2\sqrt{\pi}\sqrt{\frac{1}{b}}\cos\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\arccos(dx^2-1)\right)\text{FresnelS}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\arccos(dx^2-1)}}{\sqrt{\pi}}\right)}{dx}$$

input
```
Int[1/Sqrt[a + b*ArcCos[-1 + d*x^2]],x]
```

output
```
(-2*Sqrt[b^(-1)]*Sqrt[Pi]*Cos[a/(2*b)]*Cos[ArcCos[-1 + d*x^2]/2]*FresnelS[
(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]])/(d*x) + (2*Sqrt[b
^(-1)]*Sqrt[Pi]*Cos[ArcCos[-1 + d*x^2]/2]*FresnelC[(Sqrt[b^(-1)]*Sqrt[a +
b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b)])/(d*x)
```

3.97.3.1 Defintions of rubi rules used

rule 5320
```
Int[1/Sqrt[(a_.) + ArcCos[-1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[2*Sq
rt[Pi/b]*Sin[a/(2*b)]*Cos[ArcCos[-1 + d*x^2]/2]*(FresnelC[Sqrt[1/(Pi*b)]*Sq
rt[a + b*ArcCos[-1 + d*x^2]]]/(d*x)), x] - Simp[2*Sqrt[Pi/b]*Cos[a/(2*b)]*C
os[ArcCos[-1 + d*x^2]/2]*(FresnelS[Sqrt[1/(Pi*b)]*Sqrt[a + b*ArcCos[-1 + d*
x^2]]]/(d*x)), x] /; FreeQ[{a, b, d}, x]
```

3.97.4 Maple [F]

$$\int \frac{1}{\sqrt{a+b \arccos\left(d\,x^{2}-1\right)}}dx$$

input
```
int(1/(a+b*arccos(d*x^2-1))^(1/2),x)
```

output
```
int(1/(a+b*arccos(d*x^2-1))^(1/2),x)
```

3.97.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+b\arccos\left(-1+dx^2\right)}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arccos(d*x^2-1))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.97.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+b\arccos(-1+dx^2)}}\,dx = \int \frac{1}{\sqrt{a+b\,\mathrm{acos}\,(dx^2-1)}}\,dx$$

input
```
integrate(1/(a+b*acos(d*x**2-1))**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*acos(d*x**2 - 1)), x)
```

3.97.7 Maxima [F]

$$\int \frac{1}{\sqrt{a+b\arccos(-1+dx^2)}}\,dx = \int \frac{1}{\sqrt{b\arccos(dx^2-1)+a}}\,dx$$

input
```
integrate(1/(a+b*arccos(d*x^2-1))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*arccos(d*x^2 - 1) + a), x)
```

3.97.8 Giac [F]

$$\int \frac{1}{\sqrt{a+b\arccos(-1+dx^2)}}\,dx = \int \frac{1}{\sqrt{b\arccos(dx^2-1)+a}}\,dx$$

input
```
integrate(1/(a+b*arccos(d*x^2-1))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*arccos(d*x^2 - 1) + a), x)
```

3.97.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\arccos\left(-1+dx^2\right)}}\,dx=\int \frac{1}{\sqrt{a+b\,\mathrm{acos}\left(d\,x^2-1\right)}}\,dx$$

input
```
int(1/(a + b*acos(d*x^2 - 1))^(1/2),x)
```

output
```
int(1/(a + b*acos(d*x^2 - 1))^(1/2), x)
```

3.98 $\int \frac{1}{\left(a+b\arccos\left(-1+dx^2\right)\right)^{3/2}}\,dx$

3.98.1 Optimal result

Integrand size = 16, antiderivative size = 190

$$\int \frac{1}{\left(a+b\arccos\left(-1+dx^2\right)\right)^{3/2}}\,dx = \frac{\sqrt{2dx^2-d^2x^4}}{bdx\sqrt{a+b\arccos\left(-1+dx^2\right)}} - \frac{2\left(\frac{1}{b}\right)^{3/2}\sqrt{\pi}\cos\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\arccos\left(-1+dx^2\right)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\arccos(-1+dx^2)}}{\sqrt{\pi}}\right)}{dx} - \frac{2\left(\frac{1}{b}\right)^{3/2}\sqrt{\pi}\cos\left(\frac{1}{2}\arccos\left(-1+dx^2\right)\right)\text{FresnelS}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\arccos(-1+dx^2)}}{\sqrt{\pi}}\right)\sin\left(\frac{a}{2b}\right)}{dx}$$

output
```
-2*(1/b)^(3/2)*cos(1/2*a/b)*cos(1/2*arccos(d*x^2-1))*FresnelC((1/b)^(1/2)*
(a+b*arccos(d*x^2-1))^(1/2)/Pi^(1/2))*Pi^(1/2)/d/x-2*(1/b)^(3/2)*cos(1/2*a
rccos(d*x^2-1))*FresnelS((1/b)^(1/2)*(a+b*arccos(d*x^2-1))^(1/2)/Pi^(1/2))
*sin(1/2*a/b)*Pi^(1/2)/d/x+(-d^2*x^4+2*d*x^2)^(1/2)/b/d/x/(a+b*arccos(d*x^
2-1))^(1/2)
```

3.98.2 Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.95

$$\int \frac{1}{(a+b\arccos(-1+dx^2))^{3/2}}\,dx = -\frac{2\cos\left(\frac{1}{2}\arccos(-1+dx^2)\right)\left(\sqrt{\pi}\sqrt{a+b\arccos(-1+dx^2)}\cos\left(\frac{a}{2b}\right)\text{FresnelC}\left(\frac{\sqrt{a+b\arccos(-1+dx^2)}}{\sqrt{b}\sqrt{\pi}}\right)+\sqrt{\pi}\sqrt{\cdots}\right.}{b^{3/2}dx\sqrt{a+b\arccos(\cdots}}$$

input
```
Integrate[(a + b*ArcCos[-1 + d*x^2])^(-3/2),x]
```

output
```
(-2*Cos[ArcCos[-1 + d*x^2]/2]*(Sqrt[Pi]*Sqrt[a + b*ArcCos[-1 + d*x^2]]*Cos
[a/(2*b)]*FresnelC[Sqrt[a + b*ArcCos[-1 + d*x^2]]/(Sqrt[b]*Sqrt[Pi])] + Sq
rt[Pi]*Sqrt[a + b*ArcCos[-1 + d*x^2]]*FresnelS[Sqrt[a + b*ArcCos[-1 + d*x^
2]]/(Sqrt[b]*Sqrt[Pi])]*Sin[a/(2*b)] - Sqrt[b]*Sin[ArcCos[-1 + d*x^2]/2]))
/(b^(3/2)*d*x*Sqrt[a + b*ArcCos[-1 + d*x^2]])
```

3.98.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {5323}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\arccos(dx^2-1))^{3/2}}\,dx$$

$$\Big\downarrow \; 5323$$

$$\frac{\sqrt{2dx^2-d^2x^4}}{bdx\sqrt{a+b\arccos(dx^2-1)}}-\frac{2\sqrt{\pi}\left(\frac{1}{b}\right)^{3/2}\cos\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\arccos(dx^2-1)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\arccos(dx^2-1)}}{\sqrt{\pi}}\right)}{dx}-\frac{2\sqrt{\pi}\left(\frac{1}{b}\right)^{3/2}\sin\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\arccos(dx^2-1)\right)\text{FresnelS}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\arccos(dx^2-1)}}{\sqrt{\pi}}\right)}{dx}$$

input
```
Int[(a + b*ArcCos[-1 + d*x^2])^(-3/2),x]
```

output
```
Sqrt[2*d*x^2 - d^2*x^4]/(b*d*x*Sqrt[a + b*ArcCos[-1 + d*x^2]]) - (2*(b^(-1
))^(3/2)*Sqrt[Pi]*Cos[a/(2*b)]*Cos[ArcCos[-1 + d*x^2]/2]*FresnelC[(Sqrt[b^
(-1)]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]])/(d*x) - (2*(b^(-1))^(3/2)
*Sqrt[Pi]*Cos[ArcCos[-1 + d*x^2]/2]*FresnelS[(Sqrt[b^(-1)]*Sqrt[a + b*ArcC
os[-1 + d*x^2]])/Sqrt[Pi]]*Sin[a/(2*b)])/(d*x)
```

3.98.3.1 Defintions of rubi rules used

rule 5323
```
Int[((a_.) + ArcCos[-1 + (d_.)*(x_)^2]*(b_.))^(-3/2), x_Symbol] :> Simp[Sqr
t[2*d*x^2 - d^2*x^4]/(b*d*x*Sqrt[a + b*ArcCos[-1 + d*x^2]]), x] + (-Simp[2*
(1/b)^(3/2)*Sqrt[Pi]*Cos[a/(2*b)]*Cos[ArcCos[-1 + d*x^2]/2]*(FresnelC[Sqrt[
1/(Pi*b)]*Sqrt[a + b*ArcCos[-1 + d*x^2]]]/(d*x)), x] - Simp[2*(1/b)^(3/2)*S
qrt[Pi]*Sin[a/(2*b)]*Cos[ArcCos[-1 + d*x^2]/2]*(FresnelS[Sqrt[1/(Pi*b)]*Sqr
t[a + b*ArcCos[-1 + d*x^2]]]/(d*x)), x]) /; FreeQ[{a, b, d}, x]
```

3.98.4 Maple [F]

$$\int \frac{1}{\left(a+b\arccos\left(d\,x^{2}-1\right)\right)^{\frac{3}{2}}}dx$$

input
```
int(1/(a+b*arccos(d*x^2-1))^(3/2),x)
```

output
```
int(1/(a+b*arccos(d*x^2-1))^(3/2),x)
```

3.98.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\left(a+b\arccos\left(-1+dx^2\right)\right)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arccos(d*x^2-1))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.98.6 Sympy [F]

$$\int \frac{1}{(a+b\arccos(-1+dx^2))^{3/2}}\,dx = \int \frac{1}{(a+b\,\mathrm{acos}\,(dx^2-1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*acos(d*x**2-1))**(3/2),x)
```

output
```
Integral((a + b*acos(d*x**2 - 1))**(-3/2), x)
```

3.98.7 Maxima [F]

$$\int \frac{1}{(a+b\arccos(-1+dx^2))^{3/2}}\,dx = \int \frac{1}{(b\arccos(dx^2-1)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*arccos(d*x^2-1))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*arccos(d*x^2 - 1) + a)^(-3/2), x)
```

3.98.8 Giac [F]

$$\int \frac{1}{(a+b\arccos(-1+dx^2))^{3/2}}\,dx = \int \frac{1}{(b\arccos(dx^2-1)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*arccos(d*x^2-1))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arccos(d*x^2 - 1) + a)^(-3/2), x)
```

3.98.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\arccos(-1+dx^2))^{3/2}}\,dx = \int \frac{1}{(a+b\,\mathrm{acos}\,(d\,x^2-1))^{3/2}}\,dx$$

input
```
int(1/(a + b*acos(d*x^2 - 1))^(3/2),x)
```

output
```
int(1/(a + b*acos(d*x^2 - 1))^(3/2), x)
```

3.99 $\int \frac{1}{\left(a+b\arccos\left(-1+dx^2\right)\right)^{5/2}}\,dx$

3.99.1 Optimal result

Integrand size = 16, antiderivative size = 221

$$\begin{aligned}\int \frac{1}{(a+b\arccos(-1+dx^2))^{5/2}}\,dx &= \frac{\sqrt{2dx^2-d^2x^4}}{3bdx\,(a+b\arccos(-1+dx^2))^{3/2}} \\ &+ \frac{x}{3b^2\sqrt{a+b\arccos(-1+dx^2)}} \\ &+ \frac{2\left(\frac{1}{b}\right)^{5/2}\sqrt{\pi}\cos\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\arccos(-1+dx^2)\right)\operatorname{FresnelS}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\arccos(-1+dx^2)}}{\sqrt{\pi}}\right)}{3dx} \\ &- \frac{2\left(\frac{1}{b}\right)^{5/2}\sqrt{\pi}\cos\left(\frac{1}{2}\arccos(-1+dx^2)\right)\operatorname{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\arccos(-1+dx^2)}}{\sqrt{\pi}}\right)\sin\left(\frac{a}{2b}\right)}{3dx}\end{aligned}$$

output
```
2/3*(1/b)^(5/2)*cos(1/2*a/b)*cos(1/2*arccos(d*x^2-1))*FresnelS((1/b)^(1/2)
*(a+b*arccos(d*x^2-1))^(1/2)/Pi^(1/2))*Pi^(1/2)/d/x-2/3*(1/b)^(5/2)*cos(1/
2*arccos(d*x^2-1))*FresnelC((1/b)^(1/2)*(a+b*arccos(d*x^2-1))^(1/2)/Pi^(1/
2))*sin(1/2*a/b)*Pi^(1/2)/d/x+1/3*(-d^2*x^4+2*d*x^2)^(1/2)/b/d/x/(a+b*arcc
os(d*x^2-1))^(3/2)+1/3*x/b^2/(a+b*arccos(d*x^2-1))^(1/2)
```

3.99.2 Mathematica [A] (verified)

Time = 1.14 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.96

$$\int \frac{1}{(a+b\arccos(-1+dx^2))^{5/2}}\,dx = \frac{2\cos\left(\frac{1}{2}\arccos(-1+dx^2)\right)\left(\sqrt{\pi}(a+b\arccos(-1+dx^2))^{3/2}\cos\left(\frac{a}{2b}\right)\text{F}\right.}{}$$

input
```
Integrate[(a + b*ArcCos[-1 + d*x^2])^(-5/2),x]
```

output
```
(2*Cos[ArcCos[-1 + d*x^2]/2]*(Sqrt[Pi]*(a + b*ArcCos[-1 + d*x^2])^(3/2)*Co
s[a/(2*b)]*FresnelS[Sqrt[a + b*ArcCos[-1 + d*x^2]]/(Sqrt[b]*Sqrt[Pi])] - S
qrt[Pi]*(a + b*ArcCos[-1 + d*x^2])^(3/2)*FresnelC[Sqrt[a + b*ArcCos[-1 + d
*x^2]]/(Sqrt[b]*Sqrt[Pi])]*Sin[a/(2*b)] + Sqrt[b]*((a + b*ArcCos[-1 + d*x^
2])*Cos[ArcCos[-1 + d*x^2]/2] + b*Sin[ArcCos[-1 + d*x^2]/2])))/(3*b^(5/2)*
d*x*(a + b*ArcCos[-1 + d*x^2])^(3/2))
```

3.99.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5328, 5320}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\arccos(dx^2-1))^{5/2}}dx$$

$$\Big\downarrow \text{5328}$$

$$-\frac{\int \frac{1}{\sqrt{a+b\arccos(dx^2-1)}}dx}{3b^2} + \frac{x}{3b^2\sqrt{a+b\arccos(dx^2-1)}} + \frac{\sqrt{2dx^2-d^2x^4}}{3bdx(a+b\arccos(dx^2-1))^{3/2}}$$

$$\Big\downarrow \text{5320}$$

$$-\frac{\frac{2\sqrt{\pi}\sqrt{\frac{1}{b}}\sin\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\arccos(dx^2-1)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\arccos(dx^2-1)}}{\sqrt{\pi}}\right)}{dx} - \frac{2\sqrt{\pi}\sqrt{\frac{1}{b}}\cos\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\arccos(dx^2-1)\right)\text{FresnelS}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\text{ ar}}}{\sqrt{}}\right.}{dx}}{3b^2}$$

$$\frac{x}{3b^2\sqrt{a+b\arccos(dx^2-1)}} + \frac{\sqrt{2dx^2-d^2x^4}}{3bdx(a+b\arccos(dx^2-1))^{3/2}}$$

input
```
Int[(a + b*ArcCos[-1 + d*x^2])^(-5/2),x]
```

output
```
Sqrt[2*d*x^2 - d^2*x^4]/(3*b*d*x*(a + b*ArcCos[-1 + d*x^2])^(3/2)) + x/(3*
b^2*Sqrt[a + b*ArcCos[-1 + d*x^2]]) - ((-2*Sqrt[b^(-1)]*Sqrt[Pi]*Cos[a/(2*
b)]*Cos[ArcCos[-1 + d*x^2]/2]*FresnelS[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1
+ d*x^2]])/Sqrt[Pi]])/(d*x) + (2*Sqrt[b^(-1)]*Sqrt[Pi]*Cos[ArcCos[-1 + d*x
^2]/2]*FresnelC[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]]*Si
n[a/(2*b)])/(d*x))/(3*b^2)
```

3.99.3.1 Defintions of rubi rules used

rule 5320
```
Int[1/Sqrt[(a_.) + ArcCos[-1 + (d_.)*(x_)^2]*(b_.)], x_Symbol] :> Simp[2*Sq
rt[Pi/b]*Sin[a/(2*b)]*Cos[ArcCos[-1 + d*x^2]/2]*(FresnelC[Sqrt[1/(Pi*b)]*Sq
rt[a + b*ArcCos[-1 + d*x^2]]]/(d*x)), x] - Simp[2*Sqrt[Pi/b]*Cos[a/(2*b)]*C
os[ArcCos[-1 + d*x^2]/2]*(FresnelS[Sqrt[1/(Pi*b)]*Sqrt[a + b*ArcCos[-1 + d*
x^2]]]/(d*x)), x] /; FreeQ[{a, b, d}, x]
```

rule 5328
```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
(a + b*ArcCos[c + d*x^2])^(n + 2)/(4*b^2*(n + 1)*(n + 2))), x] + (-Simp[Sqr
t[-2*c*d*x^2 - d^2*x^4]*((a + b*ArcCos[c + d*x^2])^(n + 1)/(2*b*d*(n + 1)*x
)), x] - Simp[1/(4*b^2*(n + 1)*(n + 2))   Int[(a + b*ArcCos[c + d*x^2])^(n
+ 2), x], x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && LtQ[n, -1] && NeQ
[n, -2]
```

3.99.4 Maple [F]

$$\int \frac{1}{\left(a+b\arccos\left(d\,x^{2}-1\right)\right)^{\frac{5}{2}}}dx$$

input
```
int(1/(a+b*arccos(d*x^2-1))^(5/2),x)
```

output
```
int(1/(a+b*arccos(d*x^2-1))^(5/2),x)
```

3.99.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\arccos(-1+dx^2))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arccos(d*x^2-1))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.99.6 Sympy [F]

$$\int \frac{1}{(a+b\arccos(-1+dx^2))^{5/2}}\,dx = \int \frac{1}{(a+b\operatorname{acos}(dx^2-1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*acos(d*x**2-1))**(5/2),x)
```

output
```
Integral((a + b*acos(d*x**2 - 1))**(-5/2), x)
```

3.99.7 Maxima [F]

$$\int \frac{1}{(a+b\arccos(-1+dx^2))^{5/2}}\,dx = \int \frac{1}{(b\arccos(dx^2-1)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*arccos(d*x^2-1))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*arccos(d*x^2 - 1) + a)^(-5/2), x)
```

3.99.8 Giac [F]

$$\int \frac{1}{(a+b\arccos(-1+dx^2))^{5/2}}\,dx=\int \frac{1}{(b\arccos(dx^2-1)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*arccos(d*x^2-1))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arccos(d*x^2 - 1) + a)^(-5/2), x)
```

3.99.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\arccos(-1+dx^2))^{5/2}}\,dx=\int \frac{1}{(a+b\,\mathrm{acos}(d\,x^2-1))^{5/2}}\,dx$$

input
```
int(1/(a + b*acos(d*x^2 - 1))^(5/2),x)
```

output
```
int(1/(a + b*acos(d*x^2 - 1))^(5/2), x)
```

3.100 $\int \frac{1}{\left(a+b\arccos\left(-1+dx^2\right)\right)^{7/2}}\,dx$

3.100.1 Optimal result

Integrand size = 16, antiderivative size = 269

$$\begin{aligned}\int \frac{1}{\left(a+b\arccos\left(-1+dx^2\right)\right)^{7/2}}\,dx &= \frac{\sqrt{2dx^2-d^2x^4}}{5bdx\left(a+b\arccos\left(-1+dx^2\right)\right)^{5/2}} \\ &+ \frac{x}{15b^2\left(a+b\arccos\left(-1+dx^2\right)\right)^{3/2}} - \frac{\sqrt{2dx^2-d^2x^4}}{15b^3dx\sqrt{a+b\arccos\left(-1+dx^2\right)}} \\ &+ \frac{2\left(\frac{1}{b}\right)^{7/2}\sqrt{\pi}\cos\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\arccos\left(-1+dx^2\right)\right)\operatorname{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\arccos(-1+dx^2)}}{\sqrt{\pi}}\right)}{15dx} \\ &+ \frac{2\left(\frac{1}{b}\right)^{7/2}\sqrt{\pi}\cos\left(\frac{1}{2}\arccos\left(-1+dx^2\right)\right)\operatorname{FresnelS}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\arccos(-1+dx^2)}}{\sqrt{\pi}}\right)\sin\left(\frac{a}{2b}\right)}{15dx}\end{aligned}$$

output
```
1/15*x/b^2/(a+b*arccos(d*x^2-1))^(3/2)+2/15*(1/b)^(7/2)*cos(1/2*a/b)*cos(1
/2*arccos(d*x^2-1))*FresnelC((1/b)^(1/2)*(a+b*arccos(d*x^2-1))^(1/2)/Pi^(1
/2))*Pi^(1/2)/d/x+2/15*(1/b)^(7/2)*cos(1/2*arccos(d*x^2-1))*FresnelS((1/b)
^(1/2)*(a+b*arccos(d*x^2-1))^(1/2)/Pi^(1/2))*sin(1/2*a/b)*Pi^(1/2)/d/x+1/5
*(-d^2*x^4+2*d*x^2)^(1/2)/b/d/x/(a+b*arccos(d*x^2-1))^(5/2)-1/15*(-d^2*x^4
+2*d*x^2)^(1/2)/b^3/d/x/(a+b*arccos(d*x^2-1))^(1/2)
```

3.100.2 Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.86

$$\int \frac{1}{(a+b\arccos(-1+dx^2))^{7/2}}\,dx = \frac{2\cos\left(\frac{1}{2}\arccos(-1+dx^2)\right)\left(\sqrt{\pi}(a+b\arccos(-1+dx^2))^{5/2}\cos\left(\frac{a}{2b}\right)\mathrm{F}\right.}{}$$

input
```
Integrate[(a + b*ArcCos[-1 + d*x^2])^(-7/2),x]
```

output
```
(2*Cos[ArcCos[-1 + d*x^2]/2]*(Sqrt[Pi]*(a + b*ArcCos[-1 + d*x^2])^(5/2)*Co
s[a/(2*b)]*FresnelC[Sqrt[a + b*ArcCos[-1 + d*x^2]]/(Sqrt[b]*Sqrt[Pi])] + S
qrt[Pi]*(a + b*ArcCos[-1 + d*x^2])^(5/2)*FresnelS[Sqrt[a + b*ArcCos[-1 + d
*x^2]]/(Sqrt[b]*Sqrt[Pi])]*Sin[a/(2*b)] + Sqrt[b]*(b*(a + b*ArcCos[-1 + d*
x^2])*Cos[ArcCos[-1 + d*x^2]/2] - (-3*b^2 + (a + b*ArcCos[-1 + d*x^2])^2)*
Sin[ArcCos[-1 + d*x^2]/2])))/(15*b^(7/2)*d*x*(a + b*ArcCos[-1 + d*x^2])^(5
/2))
```

3.100.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5328, 5323}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\arccos(dx^2-1))^{7/2}}dx$$

$$\Big\downarrow \text{5328}$$

$$-\frac{\int \frac{1}{(a+b\arccos(dx^2-1))^{3/2}}dx}{15b^2}+\frac{x}{15b^2(a+b\arccos(dx^2-1))^{3/2}}+\frac{\sqrt{2dx^2-d^2x^4}}{5bdx(a+b\arccos(dx^2-1))^{5/2}}$$

$$\Big\downarrow \text{5323}$$

$$-\frac{\frac{\sqrt{2dx^2-d^2x^4}}{bdx\sqrt{a+b\arccos(dx^2-1)}}-\frac{2\sqrt{\pi}\left(\frac{1}{b}\right)^{3/2}\cos\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\arccos(dx^2-1)\right)\text{FresnelC}\left(\frac{\sqrt{\frac{1}{b}}\sqrt{a+b\arccos(dx^2-1)}}{\sqrt{\pi}}\right)}{dx}-\frac{2\sqrt{\pi}\left(\frac{1}{b}\right)^{3/2}\sin\left(\frac{a}{2b}\right)\cos\left(\frac{1}{2}\text{arc}\ldots}{}}{15b^2}$$

$$\frac{x}{15b^2\left(a+b\arccos\left(dx^2-1\right)\right)^{3/2}}+\frac{\sqrt{2dx^2-d^2x^4}}{5bdx\left(a+b\arccos\left(dx^2-1\right)\right)^{5/2}}$$

input
```
Int[(a + b*ArcCos[-1 + d*x^2])^(-7/2),x]
```

output
```
Sqrt[2*d*x^2 - d^2*x^4]/(5*b*d*x*(a + b*ArcCos[-1 + d*x^2])^(5/2)) + x/(15
*b^2*(a + b*ArcCos[-1 + d*x^2])^(3/2)) - (Sqrt[2*d*x^2 - d^2*x^4]/(b*d*x*S
qrt[a + b*ArcCos[-1 + d*x^2]]) - (2*(b^(-1))^(3/2)*Sqrt[Pi]*Cos[a/(2*b)]*C
os[ArcCos[-1 + d*x^2]/2]*FresnelC[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1 + d*x
^2]])/Sqrt[Pi]])/(d*x) - (2*(b^(-1))^(3/2)*Sqrt[Pi]*Cos[ArcCos[-1 + d*x^2]
/2]*FresnelS[(Sqrt[b^(-1)]*Sqrt[a + b*ArcCos[-1 + d*x^2]])/Sqrt[Pi]]*Sin[a
/(2*b)])/(d*x))/(15*b^2)
```

3.100.3.1 Defintions of rubi rules used

rule 5323
```
Int[((a_.) + ArcCos[-1 + (d_.)*(x_)^2]*(b_.))^(-3/2), x_Symbol] :> Simp[Sqr
t[2*d*x^2 - d^2*x^4]/(b*d*x*Sqrt[a + b*ArcCos[-1 + d*x^2]]), x] + (-Simp[2*
(1/b)^(3/2)*Sqrt[Pi]*Cos[a/(2*b)]*Cos[ArcCos[-1 + d*x^2]/2]*(FresnelC[Sqrt[
1/(Pi*b)]*Sqrt[a + b*ArcCos[-1 + d*x^2]]]/(d*x)), x] - Simp[2*(1/b)^(3/2)*S
qrt[Pi]*Sin[a/(2*b)]*Cos[ArcCos[-1 + d*x^2]/2]*(FresnelS[Sqrt[1/(Pi*b)]*Sqr
t[a + b*ArcCos[-1 + d*x^2]]]/(d*x)), x]) /; FreeQ[{a, b, d}, x]
```

rule 5328
```
Int[((a_.) + ArcCos[(c_) + (d_.)*(x_)^2]*(b_.))^(n_), x_Symbol] :> Simp[x*(
(a + b*ArcCos[c + d*x^2])^(n + 2)/(4*b^2*(n + 1)*(n + 2))), x] + (-Simp[Sqr
t[-2*c*d*x^2 - d^2*x^4]*((a + b*ArcCos[c + d*x^2])^(n + 1)/(2*b*d*(n + 1)*x
)), x] - Simp[1/(4*b^2*(n + 1)*(n + 2))   Int[(a + b*ArcCos[c + d*x^2])^(n
+ 2), x], x]) /; FreeQ[{a, b, c, d}, x] && EqQ[c^2, 1] && LtQ[n, -1] && NeQ
[n, -2]
```

3.100.4 Maple [F]

$$\int \frac{1}{\left(a+b \arccos \left(d\, x^{2}-1\right)\right)^{\frac{7}{2}}}d x$$

input
```
int(1/(a+b*arccos(d*x^2-1))^(7/2),x)
```

output
```
int(1/(a+b*arccos(d*x^2-1))^(7/2),x)
```

3.100.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\arccos(-1+dx^2))^{7/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+b*arccos(d*x^2-1))^(7/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.100.6 Sympy [F]

$$\int \frac{1}{(a+b\arccos(-1+dx^2))^{7/2}}\,dx = \int \frac{1}{\left(a+b\,\mathrm{acos}\left(dx^{2}-1\right)\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+b*acos(d*x**2-1))**(7/2),x)
```

output
```
Integral((a + b*acos(d*x**2 - 1))**(-7/2), x)
```

3.100.7 Maxima [F]

$$\int \frac{1}{(a + b \arccos(-1 + dx^2))^{7/2}} \, dx = \int \frac{1}{(b \arccos(dx^2 - 1) + a)^{\frac{7}{2}}} \, dx$$

input
```
integrate(1/(a+b*arccos(d*x^2-1))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*arccos(d*x^2 - 1) + a)^(-7/2), x)
```

3.100.8 Giac [F]

$$\int \frac{1}{(a + b \arccos(-1 + dx^2))^{7/2}} \, dx = \int \frac{1}{(b \arccos(dx^2 - 1) + a)^{\frac{7}{2}}} \, dx$$

input
```
integrate(1/(a+b*arccos(d*x^2-1))^(7/2),x, algorithm="giac")
```

output
```
integrate((b*arccos(d*x^2 - 1) + a)^(-7/2), x)
```

3.100.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a + b \arccos(-1 + dx^2))^{7/2}} \, dx = \int \frac{1}{(a + b \operatorname{acos}(d x^2 - 1))^{7/2}} \, dx$$

input
```
int(1/(a + b*acos(d*x^2 - 1))^(7/2),x)
```

output
```
int(1/(a + b*acos(d*x^2 - 1))^(7/2), x)
```

3.101 $\int \frac{\left(a+b \arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2} dx$

3.101.1 Optimal result

Integrand size = 40, antiderivative size = 40

$$\int \frac{\left(a+b \arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2} dx = \text{Int}\left(\frac{\left(a+b \arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}, x\right)$$

output
```
Unintegrable((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

3.101.2 Mathematica [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{\left(a+b \arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2} dx = \int \frac{\left(a+b \arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2} dx$$

input
```
Integrate[(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
```

output
```
Integrate[(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2), x]
```

3.101.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7234}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^n}{1-c^2x^2}dx$$
$$\Big\downarrow \text{ 7234}$$
$$\int \frac{\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^n}{1-c^2x^2}dx$$

input
```
Int[(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
```

output
```
$Aborted
```

3.101.3.1 Defintions of rubi rules used

rule 7234
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_)/((A_) + (C_.)*(x_)^2), x_Symbol] :> Unintegrable[(a + b*F[(c*
Sqrt[d + e*x])/Sqrt[f + g*x]])^n/(A + C*x^2), x] /; FreeQ[{a, b, c, d, e, f
, g, A, C, F, n}, x] && EqQ[C*d*f - A*e*g, 0] && EqQ[e*f + d*g, 0] &&  !IGt
Q[n, 0]
```

3.101.4 Maple [N/A] (verified)

Not integrable

Time = 2.84 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{\left(a+b\arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^n}{-c^2x^2+1}dx$$

input
```
int((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

output
```
int((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

3.101.5 Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=\int -\frac{\left(b\arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, alg
orithm="fricas")
```

output
```
integral(-(b*arccos(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x)
```

3.101.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=\text{Timed out}$$

input
```
integrate((a+b*acos((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**n/(-c**2*x**2+1),x)
```

output
```
Timed out
```

3.101.7 Maxima [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=\int -\frac{\left(b\arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, alg
orithm="maxima")
```

output
```
-integrate((b*arccos(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x
)
```

3.101.8 Giac [N/A]

Not integrable

Time = 0.95 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=\int -\frac{\left(b\arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, alg
orithm="giac")
```

output
```
integrate(-(b*arccos(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x
)
```

3.101.9 Mupad [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=-\int \frac{\left(a+b\,\mathrm{acos}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^n}{c^2\,x^2-1}\,dx$$

input
```
int(-(a + b*acos((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^n/(c^2*x^2 - 1),x)
```

output
```
-int((a + b*acos((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^n/(c^2*x^2 - 1), x)
```

3.102 $\int \frac{\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$

3.102.1 Optimal result

Integrand size = 40, antiderivative size = 279

$$\int \frac{\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = \frac{i\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^4}{4bc} - \frac{\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\log\left(1+e^{2i\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} + \frac{3ib\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\operatorname{PolyLog}\left(2,-e^{2i\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{2c} - \frac{3b^2\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\operatorname{PolyLog}\left(3,-e^{2i\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{2c} - \frac{3ib^3\operatorname{PolyLog}\left(4,-e^{2i\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{4c}$$

output
```
1/4*I*(a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^4/b/c-(a+b*arccos((-c*x+1
)^(1/2)/(c*x+1)^(1/2)))^3*ln(1+((-c*x+1)^(1/2)/(c*x+1)^(1/2)+I*(1-(-c*x+1)
/(c*x+1))^(1/2))^2)/c+3/2*I*b*(a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2
*polylog(2,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+I*(1-(-c*x+1)/(c*x+1))^(1/2))^2)
/c-3/2*b^2*(a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))*polylog(3,-((-c*x+1)
^(1/2)/(c*x+1)^(1/2)+I*(1-(-c*x+1)/(c*x+1))^(1/2))^2)/c-3/4*I*b^3*polylog(
4,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+I*(1-(-c*x+1)/(c*x+1))^(1/2))^2)/c
```

3.102.2 Mathematica [F]

$$\int \frac{\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx=\int \frac{\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$$

input
```
Integrate[(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
```

output
```
Integrate[(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2), x]
```

3.102.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.225, Rules used = {7232, 5137, 3042, 4202, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{1-c^2x^2}\,dx$$

$$\Big\downarrow \ 7232$$

$$-\frac{\int \frac{\sqrt{cx+1}\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{\sqrt{1-cx}}d\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}{c}$$

$$\Big\downarrow \ 5137$$

$$\frac{\int \frac{\sqrt{cx+1}\sqrt{1-\frac{1-cx}{cx+1}}\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{\sqrt{1-cx}}d\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{c}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3\tan\left(\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)d\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{c}$$

$$\Big\downarrow \ 4202$$

$$\frac{\frac{i\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^4}{4b}-2i\int \frac{e^{2i\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{1+e^{2i\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}}d\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{c}$$

$$\Big\downarrow \text{2620}$$

$$\frac{\frac{i\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^4}{4b}-2i\left(\frac{3}{2}ib\int\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2\log\left(1+e^{2i\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)d\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)-\frac{1}{2}i\log\left(1\right.\right.}{c}$$

$$\Big\downarrow \text{3011}$$

$$\frac{i\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^4}{4b}-2i\left(\frac{3}{2}ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2i\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2-ib\int\left(a+b\arccos\left(\right.\right.\right.\right.$$

$$\Big\downarrow \text{7163}$$

$$\frac{i\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^4}{4b}-2i\left(\frac{3}{2}ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2i\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2-ib\left(\frac{1}{2}ib\int\text{PolyLog}\left(\right.\right.\right.\right.\right.$$

$$\Big\downarrow \text{2720}$$

$$\frac{i\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^4}{4b}-2i\left(\frac{3}{2}ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2i\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2-ib\left(\frac{1}{4}b\int\frac{\sqrt{cx+1}\,\text{Poly}}{}\right.\right.\right.$$

$$\Big\downarrow \text{7143}$$

$$\frac{i\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^4}{4b}-2i\left(\frac{3}{2}ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2i\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2-ib\left(\frac{1}{4}b\,\text{PolyLog}\left(4,\right.\right.\right.\right.$$

input
```
Int[(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
```

output
```
(((I/4)*(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^4)/b - (2*I)*((-1/2*I)
*(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3*Log[1 + E^((2*I)*ArcCos[Sqr
t[1 - c*x]/Sqrt[1 + c*x]])] + ((3*I)/2)*b*((I/2)*(a + b*ArcCos[Sqrt[1 - c*
x]/Sqrt[1 + c*x]])^2*PolyLog[2, -E^((2*I)*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*
x]])] - I*b*((-1/2*I)*(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[
3, -E^((2*I)*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])] + (b*PolyLog[4, -E^((2*
I)*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/4))))/c
```

3.102. $\int \frac{\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$

3.102.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5137
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.102. $\int \frac{\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

rule 7232
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Simp[2*e*(g/(C*(e*f - d
*g)))   Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]],
x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] &&
EqQ[e*f + d*g, 0] && IGtQ[n, 0]
```

3.102.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 680 vs. $2(308) = 616$.

Time = 6.02 (sec) , antiderivative size = 681, normalized size of antiderivative = 2.44

method	result
default	$-\frac{a^3\ln(cx-1)}{2c}+\frac{a^3\ln(cx+1)}{2c}-b^3\left(-\frac{i\arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^4}{4c}+\frac{\arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^3\ln\left(1+\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}+i\sqrt{1-\frac{-cx+1}{cx+1}}\right)^2\right)}{c}-\frac{3i\,a}{}\right.$
parts	$-\frac{a^3\ln(cx-1)}{2c}+\frac{a^3\ln(cx+1)}{2c}-b^3\left(-\frac{i\arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^4}{4c}+\frac{\arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^3\ln\left(1+\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}+i\sqrt{1-\frac{-cx+1}{cx+1}}\right)^2\right)}{c}-\frac{3i\,a}{}\right.$

input
```
int((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x,method=_RE
TURNVERBOSE)
```

output
```
-1/2*a^3/c*ln(c*x-1)+1/2*a^3/c*ln(c*x+1)-b^3*(-1/4*I/c*arccos((-c*x+1)^(1/
2)/(c*x+1)^(1/2))^4+1/c*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2))^3*ln(1+((-c*x
+1)^(1/2)/(c*x+1)^(1/2)+I*(1-(-c*x+1)/(c*x+1))^(1/2))^2)-3/2*I/c*arccos((-
c*x+1)^(1/2)/(c*x+1)^(1/2))^2*polylog(2,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+I*(
1-(-c*x+1)/(c*x+1))^(1/2))^2)+3/2/c*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2))*p
olylog(3,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+I*(1-(-c*x+1)/(c*x+1))^(1/2))^2)+3
/4*I/c*polylog(4,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+I*(1-(-c*x+1)/(c*x+1))^(1/
2))^2))-3*a*b^2*(-1/3*I/c*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2))^3+1/c*arcco
s((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(1+((-c*x+1)^(1/2)/(c*x+1)^(1/2)+I*(1-
(-c*x+1)/(c*x+1))^(1/2))^2)-I/c*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polyl
og(2,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+I*(1-(-c*x+1)/(c*x+1))^(1/2))^2)+1/2/c
*polylog(3,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+I*(1-(-c*x+1)/(c*x+1))^(1/2))^2)
)-3*a^2*b*(-1/2*I/c*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2+1/c*arccos((-c*
x+1)^(1/2)/(c*x+1)^(1/2))*ln(1+((-c*x+1)^(1/2)/(c*x+1)^(1/2)+I*(1-(-c*x+1)
/(c*x+1))^(1/2))^2)-1/2*I/c*polylog(2,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+I*(1-
(-c*x+1)/(c*x+1))^(1/2))^2))
```

3.102.5 Fricas [F]

$$\int \frac{\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx=\int -\frac{\left(b\arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^3}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, alg
orithm="fricas")
```

output
```
integral(-(b^3*arccos(sqrt(-c*x + 1)/sqrt(c*x + 1))^3 + 3*a*b^2*arccos(sqr
t(-c*x + 1)/sqrt(c*x + 1))^2 + 3*a^2*b*arccos(sqrt(-c*x + 1)/sqrt(c*x + 1)
) + a^3)/(c^2*x^2 - 1), x)
```

3.102.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx=\text{Timed out}$$

input
```
integrate((a+b*acos((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**3/(-c**2*x**2+1),x)
```

output
```
Timed out
```

3.102.7 Maxima [F]

$$\int \frac{\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx=\int -\frac{\left(b\arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^3}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, alg
orithm="maxima")
```

output
```
1/2*a^3*(log(c*x + 1)/c - log(c*x - 1)/c) - integrate((b^3*arctan2(sqrt(2)
*sqrt(c)*sqrt(x), sqrt(-c*x + 1))^3 + 3*a*b^2*arctan2(sqrt(2)*sqrt(c)*sqrt
(x), sqrt(-c*x + 1))^2 + 3*a^2*b*arctan2(sqrt(2)*sqrt(c)*sqrt(x), sqrt(-c*
x + 1)))/(c^2*x^2 - 1), x)
```

3.102.8 Giac [F]

$$\int \frac{\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx=\int -\frac{\left(b\arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^3}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, alg
orithm="giac")
```

output
```
integrate(-(b*arccos(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^3/(c^2*x^2 - 1), x
)
```

3.102.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = \int -\frac{\left(a+b\,\mathrm{acos}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^3}{c^2\,x^2-1}\,dx$$

input
```
int(-(a + b*acos((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3/(c^2*x^2 - 1),x)
```

output
```
int(-(a + b*acos((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3/(c^2*x^2 - 1), x)
```

3.103 $\int \frac{\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx$

3.103.1 Optimal result

Integrand size = 40, antiderivative size = 207

$$\begin{aligned}\int \frac{\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx &= \frac{i\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{3bc} \\ &\quad - \frac{\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\log\left(1+e^{2i\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} \\ &\quad + \frac{ib\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\operatorname{PolyLog}\left(2,-e^{2i\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} \\ &\quad - \frac{b^2\operatorname{PolyLog}\left(3,-e^{2i\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{2c}\end{aligned}$$

output
```
1/3*I*(a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/b/c-(a+b*arccos((-c*x+1
)^(1/2)/(c*x+1)^(1/2)))^2*ln(1+((-c*x+1)^(1/2)/(c*x+1)^(1/2)+I*(1-(-c*x+1)
/(c*x+1))^(1/2))^2)/c+I*b*(a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))*polyl
og(2,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+I*(1-(-c*x+1)/(c*x+1))^(1/2))^2)/c-1/2
*b^2*polylog(3,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+I*(1-(-c*x+1)/(c*x+1))^(1/2)
)^2)/c
```

3.103.2 Mathematica [F]

$$\int \frac{\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx=\int \frac{\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx$$

input
```
Integrate[(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```

output
```
Integrate[(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2), x]
```

3.103.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {7232, 5137, 3042, 4202, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{1-c^2x^2}\,dx$$

$$\Big\downarrow \ 7232$$

$$-\frac{\int \frac{\sqrt{cx+1}\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{\sqrt{1-cx}}d\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}{c}$$

$$\Big\downarrow \ 5137$$

$$\frac{\int \frac{\sqrt{cx+1}\sqrt{1-\frac{1-cx}{cx+1}}\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{\sqrt{1-cx}}d\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{c}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2\tan\left(\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)d\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{c}$$

$$\Big\downarrow \ 4202$$

$$\frac{\frac{i\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{3b}-2i\int \frac{e^{2i\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{1+e^{2i\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}}d\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{c}$$

$$\Big\downarrow \text{2620}$$

$$\frac{\frac{i\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{3b}-2i\left(ib\int\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)\log\left(1+e^{2i\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)d\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)-\frac{1}{2}i\log\left(1+\right.\right.}{c}$$

$$\Big\downarrow \text{3011}$$

$$\frac{\frac{i\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{3b}-2i\left(ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2i\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)-\frac{1}{2}ib\int\text{PolyLog}\left(2,-e\right.\right.\right.}{c}$$

$$\Big\downarrow \text{2720}$$

$$\frac{\frac{i\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{3b}-2i\left(ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2i\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)-\frac{1}{4}b\int\frac{\sqrt{cx+1}\,\text{PolyLog}\left(2,\right.}{\sqrt{}}\right.\right.}{c}$$

$$\Big\downarrow \text{7143}$$

$$\frac{\frac{i\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{3b}-2i\left(ib\left(\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2i\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)-\frac{1}{4}b\,\text{PolyLog}\left(3,-e^{2i\,a}\right.\right.\right.}{c}$$

input
```
Int[(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```

output
```
(((I/3)*(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3)/b - (2*I)*((-1/2*I)
*(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*Log[1 + E^((2*I)*ArcCos[Sqr
t[1 - c*x]/Sqrt[1 + c*x]])] + I*b*((I/2)*(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[
1 + c*x]])*PolyLog[2, -E^((2*I)*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])] - (b
*PolyLog[3, -E^((2*I)*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])])/4)))/c
```

3.103. $\int \frac{\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx$

3.103.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5137
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.103. $\int \frac{\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx$

rule 7232
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Simp[2*e*(g/(C*(e*f - d
*g)))   Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]],
x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] &&
EqQ[e*f + d*g, 0] && IGtQ[n, 0]
```

3.103.4 Maple [A] (verified)

Time = 2.97 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.90

method	result
default	$-\frac{a^{2} \ln \left(c x-1\right)}{2 c}+\frac{a^{2} \ln \left(c x+1\right)}{2 c}-b^{2} \left(-\frac{i \arccos \left(\frac{\sqrt{-c x+1}}{\sqrt{c x+1}}\right)^{3}}{3 c}+\frac{\arccos \left(\frac{\sqrt{-c x+1}}{\sqrt{c x+1}}\right)^{2} \ln \left(1+\left(\frac{\sqrt{-c x+1}}{\sqrt{c x+1}}+i \sqrt{1-\frac{-c x+1}{c x+1}}\right)^{2}\right)}{c}-\frac{i \operatorname{ar}}{}\right.$
parts	$-\frac{a^{2} \ln \left(c x-1\right)}{2 c}+\frac{a^{2} \ln \left(c x+1\right)}{2 c}-b^{2} \left(-\frac{i \arccos \left(\frac{\sqrt{-c x+1}}{\sqrt{c x+1}}\right)^{3}}{3 c}+\frac{\arccos \left(\frac{\sqrt{-c x+1}}{\sqrt{c x+1}}\right)^{2} \ln \left(1+\left(\frac{\sqrt{-c x+1}}{\sqrt{c x+1}}+i \sqrt{1-\frac{-c x+1}{c x+1}}\right)^{2}\right)}{c}-\frac{i \operatorname{ar}}{}\right.$

input
```
int((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x,method=_RE
TURNVERBOSE)
```

output
```
-1/2*a^2/c*ln(c*x-1)+1/2*a^2/c*ln(c*x+1)-b^2*(-1/3*I/c*arccos((-c*x+1)^(1/
2)/(c*x+1)^(1/2))^3+1/c*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(1+((-c*x
+1)^(1/2)/(c*x+1)^(1/2)+I*(1-(-c*x+1)/(c*x+1))^(1/2))^2)-I/c*arccos((-c*x+
1)^(1/2)/(c*x+1)^(1/2))*polylog(2,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+I*(1-(-c*
x+1)/(c*x+1))^(1/2))^2)+1/2/c*polylog(3,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+I*(
1-(-c*x+1)/(c*x+1))^(1/2))^2))+I*a*b/c*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)
)^2-2*a*b/c*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln(1+((-c*x+1)^(1/2)/(c*x
+1)^(1/2)+I*(1-(-c*x+1)/(c*x+1))^(1/2))^2)+I*a*b/c*polylog(2,-((-c*x+1)^(1
/2)/(c*x+1)^(1/2)+I*(1-(-c*x+1)/(c*x+1))^(1/2))^2)
```

3.103.5 Fricas [F]

$$\int \frac{\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx=\int -\frac{\left(b\arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, alg
orithm="fricas")
```

output
```
integral(-(b^2*arccos(sqrt(-c*x + 1)/sqrt(c*x + 1))^2 + 2*a*b*arccos(sqrt(
-c*x + 1)/sqrt(c*x + 1)) + a^2)/(c^2*x^2 - 1), x)
```

3.103.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx=\text{Timed out}$$

input
```
integrate((a+b*acos((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2/(-c**2*x**2+1),x)
```

output
```
Timed out
```

3.103.7 Maxima [F]

$$\int \frac{\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx=\int -\frac{\left(b\arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, alg
orithm="maxima")
```

output
```
1/2*a^2*(log(c*x + 1)/c - log(c*x - 1)/c) - integrate((b^2*arctan2(sqrt(2)
*sqrt(c)*sqrt(x), sqrt(-c*x + 1))^2 + 2*a*b*arctan2(sqrt(2)*sqrt(c)*sqrt(x
), sqrt(-c*x + 1)))/(c^2*x^2 - 1), x)
```

3.103.8 Giac [F]

$$\int \frac{\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx=\int -\frac{\left(b\arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, alg
orithm="giac")
```

output
```
integrate(-(b*arccos(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^2/(c^2*x^2 - 1), x
)
```

3.103.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx=\int -\frac{\left(a+b\,\mathrm{acos}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^2}{c^2\,x^2-1}\,dx$$

input
```
int(-(a + b*acos((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2/(c^2*x^2 - 1),x)
```

output
```
int(-(a + b*acos((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2/(c^2*x^2 - 1), x)
```

3.104 $\int \frac{a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx$

3.104.1 Optimal result

Integrand size = 38, antiderivative size = 141

$$\int \frac{a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = \frac{i\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{2bc} - \frac{\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\log\left(1+e^{2i\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{c} + \frac{ib\,\text{PolyLog}\left(2,-e^{2i\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}\right)}{2c}$$

output
```
1/2*I*(a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/b/c-(a+b*arccos((-c*x+1
)^(1/2)/(c*x+1)^(1/2)))*ln(1+((-c*x+1)^(1/2)/(c*x+1)^(1/2)+I*(1-(-c*x+1)/(
c*x+1))^(1/2))^2)/c+1/2*I*b*polylog(2,-((-c*x+1)^(1/2)/(c*x+1)^(1/2)+I*(1-
(-c*x+1)/(c*x+1))^(1/2))^2)/c
```

3.104.2 Mathematica [F]

$$\int \frac{a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx=\int \frac{a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx$$

input
```
Integrate[(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

output
```
Integrate[(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2), x]
```

3.104.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.184, Rules used = {7232, 5137, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{1-c^2x^2}\,dx$$

$$\Big\downarrow \text{7232}$$

$$-\frac{\int \frac{\sqrt{cx+1}\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{\sqrt{1-cx}}d\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}{c}$$

$$\Big\downarrow \text{5137}$$

$$\frac{\int \frac{\sqrt{cx+1}\sqrt{1-\frac{1-cx}{cx+1}}\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{\sqrt{1-cx}}d\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{c}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)\tan\left(\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)d\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{c}$$

$$\Big\downarrow \text{4202}$$

$$\frac{\frac{i\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{2b}-2i\int \frac{e^{2i\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{1+e^{2i\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}}d\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{c}$$

$$\Big\downarrow \text{2620}$$

$$\frac{\frac{i\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{2b}-2i\left(\frac{1}{2}ib\int\log\left(1+e^{2i\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)d\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)-\frac{1}{2}i\log\left(1+e^{2i\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+\right.\right.}{c}$$

$$\Big\downarrow \text{2715}$$

$$\frac{\frac{i\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{2b}-2i\left(\frac{1}{4}b\int\frac{\sqrt{cx+1}\log\left(1+e^{2i\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)}{\sqrt{1-cx}}de^{2i\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}-\frac{1}{2}i\log\left(1+e^{2i\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a-\right.\right.}{c}$$

$$\Big\downarrow \text{2838}$$

$$\frac{\frac{i\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{2b}-2i\left(-\frac{1}{2}i\log\left(1+e^{2i\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}\right)\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)-\frac{1}{4}b\,\text{PolyLog}\left(2,-e^{2i\arccos\left(\frac{\sqrt{1}}{\sqrt{c}}\right.}\right.\right.}{c}$$

input
```
Int[(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

output
```
(((I/2)*(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2)/b - (2*I)*((-1/2*I)
*(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*Log[1 + E^((2*I)*ArcCos[Sqrt[
1 - c*x]/Sqrt[1 + c*x]])] - (b*PolyLog[2, -E^((2*I)*ArcCos[Sqrt[1 - c*x]/S
qrt[1 + c*x]])])/4))/c
```

3.104.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

3.104. $\int \frac{a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx$

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5137
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

rule 7232
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Simp[2*e*(g/(C*(e*f - d
*g)))   Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]],
x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] &&
EqQ[e*f + d*g, 0] && IGtQ[n, 0]
```

3.104.4 Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.21

method	result
default	$-\frac{a\ln(cx-1)}{2c}+\frac{a\ln(cx+1)}{2c}+\frac{ib\arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^{2}}{2c}-\frac{b\arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\ln\left(1+\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}+i\sqrt{1-\frac{-cx+1}{cx+1}}\right)^{2}\right)}{c}+\frac{ib\,\mathrm{polylog}\left(2\right.}{}$
parts	$-\frac{a\ln(cx-1)}{2c}+\frac{a\ln(cx+1)}{2c}+\frac{ib\arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^{2}}{2c}-\frac{b\arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\ln\left(1+\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}+i\sqrt{1-\frac{-cx+1}{cx+1}}\right)^{2}\right)}{c}+\frac{ib\,\mathrm{polylog}\left(2\right.}{}$

input
```
int((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x,method=_RETU
RNVERBOSE)
```

output
```
-1/2*a/c*ln(c*x-1)+1/2*a/c*ln(c*x+1)+1/2*I*b/c*arccos((-c*x+1)^(1/2)/(c*x+
1)^(1/2))^2-b/c*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln(1+((-c*x+1)^(1/2)/
(c*x+1)^(1/2)+I*(1-(-c*x+1)/(c*x+1))^(1/2))^2)+1/2*I*b*polylog(2,-((-c*x+1
)^(1/2)/(c*x+1)^(1/2)+I*(1-(-c*x+1)/(c*x+1))^(1/2))^2)/c
```

3.104.5 Fricas [F]

$$\int \frac{a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx=\int -\frac{b\arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algor
ithm="fricas")
```

output
```
integral(-(b*arccos(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)/(c^2*x^2 - 1), x)
```

3.104.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx=\text{Timed out}$$

input
```
integrate((a+b*acos((-c*x+1)**(1/2)/(c*x+1)**(1/2)))/(-c**2*x**2+1),x)
```

output
```
Timed out
```

3.104.7 Maxima [F]

$$\int \frac{a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx=\int -\frac{b\arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algor
ithm="maxima")
```

output
```
1/2*a*(log(c*x + 1)/c - log(c*x - 1)/c) - b*integrate(arctan2(sqrt(2)*sqrt
(c)*sqrt(x), sqrt(-c*x + 1))/(c^2*x^2 - 1), x)
```

3.104.8 Giac [F]

$$\int \frac{a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx=\int -\frac{b\arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algor
ithm="giac")
```

output
```
integrate(-(b*arccos(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)/(c^2*x^2 - 1), x)
```

3.104.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx=\int -\frac{a+b\,\mathrm{acos}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)}{c^2\,x^2-1}\,dx$$

input
```
int(-(a + b*acos((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))/(c^2*x^2 - 1),x)
```

output
```
int(-(a + b*acos((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))/(c^2*x^2 - 1), x)
```

3.105 $\int \frac{1}{(1-c^2x^2)\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$

3.105.1 Optimal result

Integrand size = 40, antiderivative size = 40

$$\int \frac{1}{(1-c^2x^2)\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = \text{Int}\left(\frac{1}{(1-c^2x^2)\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)},x\right)$$

output
```
Unintegrable(1/(-c^2*x^2+1)/(a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

3.105.2 Mathematica [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{1}{(1-c^2x^2)\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = \int \frac{1}{(1-c^2x^2)\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

input
```
Integrate[1/((1 - c^2*x^2)*(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]
```

output
```
Integrate[1/((1 - c^2*x^2)*(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])), x
]
```

3.105.3 Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7234}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-c^2x^2)\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}dx$$

$$\Big\downarrow \; 7234$$

$$\int \frac{1}{(1-c^2x^2)\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}dx$$

input
```
Int[1/((1 - c^2*x^2)*(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]
```

output
```
$Aborted
```

3.105.3.1 Defintions of rubi rules used

rule 7234
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_)/((A_) + (C_.)*(x_)^2), x_Symbol] :> Unintegrable[(a + b*F[(c*
Sqrt[d + e*x])/Sqrt[f + g*x]])^n/(A + C*x^2), x] /; FreeQ[{a, b, c, d, e, f
, g, A, C, F, n}, x] && EqQ[C*d*f - A*e*g, 0] && EqQ[e*f + d*g, 0] && !IGt
Q[n, 0]
```

3.105.4 Maple [N/A] (verified)

Not integrable

Time = 1.43 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{1}{(-c^2x^2+1)\left(a+b\arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)}dx$$

input
```
int(1/(-c^2*x^2+1)/(a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

output
```
int(1/(-c^2*x^2+1)/(a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

3.105.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

$$\int \frac{1}{(1-c^2x^2)\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = \int -\frac{1}{(c^2x^2-1)\left(b\arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)/(a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, alg
orithm="fricas")
```

output
```
integral(-1/(a*c^2*x^2 + (b*c^2*x^2 - b)*arccos(sqrt(-c*x + 1)/sqrt(c*x +
1)) - a), x)
```

3.105.6 Sympy [N/A]

Not integrable

Time = 133.80 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.52

$$\int \frac{1}{(1-c^2x^2)\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$
$$= -\int \frac{1}{ac^2x^2-a+bc^2x^2\operatorname{acos}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)-b\operatorname{acos}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}\,dx$$

input
```
integrate(1/(-c**2*x**2+1)/(a+b*acos((-c*x+1)**(1/2)/(c*x+1)**(1/2))),x)
```

output
```
-Integral(1/(a*c**2*x**2 - a + b*c**2*x**2*acos(sqrt(-c*x + 1)/sqrt(c*x +
1)) - b*acos(sqrt(-c*x + 1)/sqrt(c*x + 1))), x)
```

3.105.7 Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{1}{(1-c^2x^2)\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx=\int -\frac{1}{(c^2x^2-1)\left(b\arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)/(a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, alg
orithm="maxima")
```

output
```
-integrate(1/((c^2*x^2 - 1)*(b*arccos(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)),
 x)
```

3.105.8 Giac [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{1}{(1-c^2x^2)\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx=\int -\frac{1}{(c^2x^2-1)\left(b\arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)/(a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, alg
orithm="giac")
```

output
```
integrate(-1/((c^2*x^2 - 1)*(b*arccos(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)),
 x)
```

3.105.9 Mupad [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{1}{(1-c^2x^2)\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx=-\int \frac{1}{\left(a+b\,\mathrm{acos}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)\,(c^2\,x^2-1)}\,dx$$

input
```
int(-1/((a + b*acos((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))*(c^2*x^2 - 1)),x)
```

output
```
-int(1/((a + b*acos((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))*(c^2*x^2 - 1)), x)
```

3.105. $\int \frac{1}{(1-c^2x^2)\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$

3.106 $\int \frac{1}{(1-c^2x^2)\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$

3.106.1 Optimal result

Integrand size = 40, antiderivative size = 40

$$\int \frac{1}{(1-c^2x^2)\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx=\text{Int}\left(\frac{1}{(1-c^2x^2)\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2},x\right)$$

output
```
Unintegrable(1/(-c^2*x^2+1)/(a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x
)
```

3.106.2 Mathematica [N/A]

Not integrable

Time = 4.68 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{1}{(1-c^2x^2)\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx=\int \frac{1}{(1-c^2x^2)\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

input
```
Integrate[1/((1 - c^2*x^2)*(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),
x]
```

output
```
Integrate[1/((1 - c^2*x^2)*(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),
 x]
```

3.106.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7234}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-c^2x^2)\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}dx$$

$$\Big\downarrow \text{7234}$$

$$\int \frac{1}{(1-c^2x^2)\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}dx$$

input
```
Int[1/((1 - c^2*x^2)*(a + b*ArcCos[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),x]
```

output
```
$Aborted
```

3.106.3.1 Defintions of rubi rules used

rule 7234
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]]])^(n_)/((A_) + (C_.)*(x_)^2), x_Symbol] :> Unintegrable[(a + b*F[(c*
Sqrt[d + e*x])/Sqrt[f + g*x]])^n/(A + C*x^2), x] /; FreeQ[{a, b, c, d, e, f
, g, A, C, F, n}, x] && EqQ[C*d*f - A*e*g, 0] && EqQ[e*f + d*g, 0] &&  !IGt
Q[n, 0]
```

3.106.4 Maple [N/A] (verified)

Not integrable

Time = 1.42 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{1}{(-c^2x^2+1)\left(a+b\arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^2}dx$$

input
```
int(1/(-c^2*x^2+1)/(a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

output
```
int(1/(-c^2*x^2+1)/(a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

3.106.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.28

$$\int \frac{1}{(1-c^2x^2)\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = \int -\frac{1}{(c^2x^2-1)\left(b\arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)/(a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, a
lgorithm="fricas")
```

output
```
integral(-1/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arccos(sqrt(-c*x + 1)/sqrt(
c*x + 1))^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arccos(sqrt(-c*x + 1)/sqrt(c*x +
 1))), x)
```

3.106.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(1-c^2x^2)\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(-c**2*x**2+1)/(a+b*acos((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2,x
)
```

output
```
Timed out
```

3.106.7 Maxima [N/A]

Not integrable

Time = 1.89 (sec) , antiderivative size = 293, normalized size of antiderivative = 7.32

$$\int \frac{1}{(1-c^2x^2)\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = \int -\frac{1}{(c^2x^2-1)\left(b\arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)/(a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, a
lgorithm="maxima")
```

output
```
((sqrt(2)*b^2*c*arctan2(sqrt(2)*sqrt(c)*sqrt(x), sqrt(-c*x + 1)) + sqrt(2)
*a*b*c - (sqrt(2)*b^2*c^2*arctan2(sqrt(2)*sqrt(c)*sqrt(x), sqrt(-c*x + 1))
 + sqrt(2)*a*b*c^2)*x)*sqrt(c)*integrate(1/2*sqrt(-c*x + 1)*sqrt(x)/((b^2*
c^3*arctan2(sqrt(2)*sqrt(c)*sqrt(x), sqrt(-c*x + 1)) + a*b*c^3)*x^3 - 2*(b
^2*c^2*arctan2(sqrt(2)*sqrt(c)*sqrt(x), sqrt(-c*x + 1)) + a*b*c^2)*x^2 + (
b^2*c*arctan2(sqrt(2)*sqrt(c)*sqrt(x), sqrt(-c*x + 1)) + a*b*c)*x), x) - s
qrt(2)*sqrt(-c*x + 1)*sqrt(c)*sqrt(x))/(b^2*c*arctan2(sqrt(2)*sqrt(c)*sqrt
(x), sqrt(-c*x + 1)) + a*b*c - (b^2*c^2*arctan2(sqrt(2)*sqrt(c)*sqrt(x), s
qrt(-c*x + 1)) + a*b*c^2)*x)
```

3.106.8 Giac [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{1}{(1-c^2x^2)\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = \int -\frac{1}{(c^2x^2-1)\left(b\arccos\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)/(a+b*arccos((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, a
lgorithm="giac")
```

output
```
integrate(-1/((c^2*x^2 - 1)*(b*arccos(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^2
), x)
```

3.106.9 Mupad [N/A]

Not integrable

Time = 1.31 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{1}{(1-c^2x^2)\left(a+b\arccos\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = -\int \frac{1}{\left(a+b\,\mathrm{acos}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^2\,(c^2\,x^2-1)}\,dx$$

input

```
int(-1/((a + b*acos((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2*(c^2*x^2 - 1)),x)
```

output

```
-int(1/((a + b*acos((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2*(c^2*x^2 - 1)), x)
```

3.107 $\int \arccos\left(ce^{a+bx}\right)\,dx$

3.107.1 Optimal result

Integrand size = 10, antiderivative size = 84

$$\int \arccos\left(ce^{a+bx}\right)\,dx = -\frac{i\arccos\left(ce^{a+bx}\right)^2}{2b} + \frac{\arccos\left(ce^{a+bx}\right)\log\left(1+e^{2i\arccos\left(ce^{a+bx}\right)}\right)}{b} - \frac{i\,\text{PolyLog}\left(2,-e^{2i\arccos\left(ce^{a+bx}\right)}\right)}{2b}$$

output
```
-1/2*I*arccos(c*exp(b*x+a))^2/b+arccos(c*exp(b*x+a))*ln(1+(c*exp(b*x+a)+I*
(1-c^2*exp(b*x+a)^2)^(1/2))^2)/b-1/2*I*polylog(2,-(c*exp(b*x+a)+I*(1-c^2*e
xp(b*x+a)^2)^(1/2))^2)/b
```

3.107.2 Mathematica [F]

$$\int \arccos\left(ce^{a+bx}\right)\,dx = \int \arccos\left(ce^{a+bx}\right)\,dx$$

input
```
Integrate[ArcCos[c*E^(a + b*x)],x]
```

output
```
Integrate[ArcCos[c*E^(a + b*x)], x]
```

3.107.3 Rubi [A] (warning: unable to verify)

Time = 0.36 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {2720, 5137, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arccos\left(ce^{a+bx}\right)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int e^{-a-bx}\arccos\left(ce^{a+bx}\right)de^{a+bx}}{b}$$

$$\Big\downarrow \text{5137}$$

$$-\frac{\int \frac{e^{-a-bx}\sqrt{1-c^2e^{2a+2bx}}\arccos\left(ce^{a+bx}\right)}{c}d\arccos\left(ce^{a+bx}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \arccos\left(ce^{a+bx}\right)\tan\left(\arccos\left(ce^{a+bx}\right)\right)d\arccos\left(ce^{a+bx}\right)}{b}$$

$$\Big\downarrow \text{4202}$$

$$-\frac{\frac{1}{2}ie^{2a+2bx}-2i\int \frac{e^{a+bx+2i\arccos\left(ce^{a+bx}\right)}}{1+e^{2i\arccos\left(ce^{a+bx}\right)}}d\arccos\left(ce^{a+bx}\right)}{b}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{\frac{1}{2}ie^{2a+2bx}-2i\left(\frac{1}{2}i\int \log\left(1+e^{2i\arccos\left(ce^{a+bx}\right)}\right)d\arccos\left(ce^{a+bx}\right)-\frac{1}{2}i\arccos\left(ce^{a+bx}\right)\log\left(1+e^{2i\arccos\left(ce^{a+bx}\right)}\right)\right)}{b}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{\frac{1}{2}ie^{2a+2bx}-2i\left(\frac{1}{4}\int e^{-a-bx}\log\left(1+e^{2i\arccos\left(ce^{a+bx}\right)}\right)de^{2i\arccos\left(ce^{a+bx}\right)}-\frac{1}{2}i\arccos\left(ce^{a+bx}\right)\log\left(1+e^{2i\arccos\left(ce^{a+bx}\right)}\right)\right)}{b}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{\frac{1}{2}ie^{2a+2bx}-2i\left(-\frac{1}{4}\operatorname{PolyLog}\left(2,-e^{2i\arccos\left(ce^{a+bx}\right)}\right)-\frac{1}{2}i\arccos\left(ce^{a+bx}\right)\log\left(1+e^{2i\arccos\left(ce^{a+bx}\right)}\right)\right)}{b}$$

input
```
Int[ArcCos[c*E^(a + b*x)],x]
```

output
```
-(((I/2)*E^(2*a + 2*b*x) - (2*I)*((-1/2*I)*ArcCos[c*E^(a + b*x)]*Log[1 + E
^((2*I)*ArcCos[c*E^(a + b*x)])] - PolyLog[2, -E^((2*I)*ArcCos[c*E^(a + b*x
)])]/4))/b)
```

3.107.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 5137
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

3.107.4 Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.21

method	result
derivativedivides	$\frac{-\frac{i \arccos\left(c\,e^{bx+a}\right)^2}{2}+\arccos\left(c\,e^{bx+a}\right)\ln\left(1+\left(c\,e^{bx+a}+i\sqrt{1-c^2 e^{2bx+2a}}\right)^2\right)-\frac{i\,\mathrm{polylog}\left(2,-\left(c\,e^{bx+a}+i\sqrt{1-c^2 e^{2bx+2a}}\right)^2\right)}{2}}{b}$
default	$\frac{-\frac{i \arccos\left(c\,e^{bx+a}\right)^2}{2}+\arccos\left(c\,e^{bx+a}\right)\ln\left(1+\left(c\,e^{bx+a}+i\sqrt{1-c^2 e^{2bx+2a}}\right)^2\right)-\frac{i\,\mathrm{polylog}\left(2,-\left(c\,e^{bx+a}+i\sqrt{1-c^2 e^{2bx+2a}}\right)^2\right)}{2}}{b}$

input
```
int(arccos(c*exp(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/2*I*arccos(c*exp(b*x+a))^2+arccos(c*exp(b*x+a))*ln(1+(c*exp(b*x+a)
+I*(1-c^2*exp(b*x+a)^2)^(1/2))^2)-1/2*I*polylog(2,-(c*exp(b*x+a)+I*(1-c^2*
exp(b*x+a)^2)^(1/2))^2))
```

3.107.5 Fricas [F(-2)]

Exception generated.

$$\int \arccos\left(ce^{a+bx}\right)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arccos(c*exp(b*x+a)),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.107.6 Sympy [F]

$$\int \arccos\left(ce^{a+bx}\right)\,dx = \int \operatorname{acos}\left(ce^{a+bx}\right)dx$$

input
```
integrate(acos(c*exp(b*x+a)),x)
```

output
```
Integral(acos(c*exp(a + b*x)), x)
```

3.107.7 Maxima [F]

$$\int \arccos\left(ce^{a+bx}\right)\,dx = \int \arccos\left(ce^{(bx+a)}\right)\,dx$$

input
```
integrate(arccos(c*exp(b*x+a)),x, algorithm="maxima")
```

output
```
-1/2*(2*I*b^2*c^2*integrate(x*e^(2*b*x + 2*a)/(c^4*e^(4*b*x + 4*a) - c^2*e
^(2*b*x + 2*a) + (c^2*e^(2*b*x + 2*a) - 1)*e^(log(c*e^(b*x + a) + 1) + log
(-c*e^(b*x + a) + 1))), x) + 2*b^2*c*integrate(x*e^(b*x + a + 1/2*log(c*e^
(b*x + a) + 1) + 1/2*log(-c*e^(b*x + a) + 1))/(c^4*e^(4*b*x + 4*a) - c^2*e
^(2*b*x + 2*a) + (c^2*e^(2*b*x + 2*a) - 1)*e^(log(c*e^(b*x + a) + 1) + log
(-c*e^(b*x + a) + 1))), x) - 2*b*x*arctan(sqrt(c*e^(b*x + a) + 1)*sqrt(-c*
e^(b*x + a) + 1)*e^(-b*x - a)/c) - I*b*x*log(c*e^(b*x + a) + 1) - I*b*x*lo
g(-c*e^(b*x + a) + 1) - I*dilog(c*e^(b*x + a)) - I*dilog(-c*e^(b*x + a)))/
b
```

3.107.8 Giac [F]

$$\int \arccos\left(ce^{a+bx}\right)\,dx = \int \arccos\left(ce^{(bx+a)}\right)\,dx$$

input
```
integrate(arccos(c*exp(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arccos(c*e^(b*x + a)), x)
```

3.107.9 Mupad [F(-1)]

Timed out.

$$\int \arccos\left(ce^{a+bx}\right)\,dx = \int \mathrm{acos}\left(c\,\mathrm{e}^{a+b\,x}\right)\,dx$$

input
```
int(acos(c*exp(a + b*x)),x)
```

output
```
int(acos(c*exp(a + b*x)), x)
```

3.108 $\int e^{\arccos(ax)}x^3\,dx$

3.108.1 Optimal result

Integrand size = 10, antiderivative size = 81

$$\int e^{\arccos(ax)}x^3\,dx=\frac{e^{\arccos(ax)}\cos(2\arccos(ax))}{10a^4}+\frac{e^{\arccos(ax)}\cos(4\arccos(ax))}{34a^4}-\frac{e^{\arccos(ax)}\sin(2\arccos(ax))}{20a^4}-\frac{e^{\arccos(ax)}\sin(4\arccos(ax))}{136a^4}$$

output
```
1/10*exp(arccos(a*x))*cos(2*arccos(a*x))/a^4+1/34*exp(arccos(a*x))*cos(4*a
rccos(a*x))/a^4-1/20*exp(arccos(a*x))*sin(2*arccos(a*x))/a^4-1/136*exp(arc
cos(a*x))*sin(4*arccos(a*x))/a^4
```

3.108.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.62

$$\int e^{\arccos(ax)}x^3\,dx=-\frac{e^{\arccos(ax)}(-68\cos(2\arccos(ax))-20\cos(4\arccos(ax))+34\sin(2\arccos(ax))+5\sin(4\arccos(ax)))}{680a^4}$$

input
```
Integrate[E^ArcCos[a*x]*x^3,x]
```

output
```
-1/680*(E^ArcCos[a*x]*(-68*Cos[2*ArcCos[a*x]] - 20*Cos[4*ArcCos[a*x]] + 34
*Sin[2*ArcCos[a*x]] + 5*Sin[4*ArcCos[a*x]]))/a^4
```

3.108.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5336, 27, 4972, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 e^{\arccos(ax)}\,dx$$

$$\Big\downarrow \text{ 5336}$$

$$-\frac{\int e^{\arccos(ax)} x^3 \sqrt{1-a^2x^2}d\arccos(ax)}{a}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int a^3 e^{\arccos(ax)} x^3 \sqrt{1-a^2x^2}d\arccos(ax)}{a^4}$$

$$\Big\downarrow \text{ 4972}$$

$$-\frac{\int \left(\frac{1}{4}e^{\arccos(ax)}\sin(2\arccos(ax))+\frac{1}{8}e^{\arccos(ax)}\sin(4\arccos(ax))\right)d\arccos(ax)}{a^4}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\frac{1}{20}e^{\arccos(ax)}\sin(2\arccos(ax))+\frac{1}{136}e^{\arccos(ax)}\sin(4\arccos(ax))-\frac{1}{10}e^{\arccos(ax)}\cos(2\arccos(ax))-\frac{1}{34}e^{\arccos(ax)}\text{co}}{a^4}$$

input
```
Int[E^ArcCos[a*x]*x^3,x]
```

output
```
-((-1/10*(E^ArcCos[a*x]*Cos[2*ArcCos[a*x]]) - (E^ArcCos[a*x]*Cos[4*ArcCos[
a*x]])/34 + (E^ArcCos[a*x]*Sin[2*ArcCos[a*x]])/20 + (E^ArcCos[a*x]*Sin[4*A
rcCos[a*x]])/136)/a^4)
```

3.108.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4972
```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

rule 5336
```
Int[(u_.)*(f_)^(ArcCos[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> Simp[
-b^(-1)   Subst[Int[(u /. x -> -a/b + Cos[x]/b)*f^(c*x^n)*Sin[x], x], x, Ar
cCos[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[n, 0]
```

3.108.4 Maple [F]

$$\int \mathrm{e}^{\arccos(ax)} x^3 dx$$

input
```
int(exp(arccos(a*x))*x^3,x)
```

output
```
int(exp(arccos(a*x))*x^3,x)
```

3.108.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.68

$$\int e^{\arccos(ax)} x^3 \, dx = \frac{\left(20\, a^4x^4 - 3\, a^2x^2 - (5\, a^3x^3 + 6\, ax)\sqrt{-a^2x^2+1} - 6\right)e^{(\arccos(ax))}}{85\, a^4}$$

input
```
integrate(exp(arccos(a*x))*x^3,x, algorithm="fricas")
```

output
```
1/85*(20*a^4*x^4 - 3*a^2*x^2 - (5*a^3*x^3 + 6*a*x)*sqrt(-a^2*x^2 + 1) - 6)
*e^(arccos(a*x))/a^4
```

3.108.6 Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.30

$$\int e^{\arccos(ax)}x^3\,dx = \begin{cases} \frac{4x^4e^{\operatorname{acos}(ax)}}{17} - \frac{x^3\sqrt{-a^2x^2+1}e^{\operatorname{acos}(ax)}}{17a} - \frac{3x^2e^{\operatorname{acos}(ax)}}{85a^2} - \frac{6x\sqrt{-a^2x^2+1}e^{\operatorname{acos}(ax)}}{85a^3} - \frac{6e^{\operatorname{acos}(ax)}}{85a^4} & \text{for } a \neq 0 \\ \frac{x^4e^{\frac{\pi}{2}}}{4} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(acos(a*x))*x**3,x)
```

output
```
Piecewise((4*x**4*exp(acos(a*x))/17 - x**3*sqrt(-a**2*x**2 + 1)*exp(acos(a
*x))/(17*a) - 3*x**2*exp(acos(a*x))/(85*a**2) - 6*x*sqrt(-a**2*x**2 + 1)*e
xp(acos(a*x))/(85*a**3) - 6*exp(acos(a*x))/(85*a**4), Ne(a, 0)), (x**4*exp
(pi/2)/4, True))
```

3.108.7 Maxima [F]

$$\int e^{\arccos(ax)}x^3\,dx = \int x^3e^{(\arccos(ax))}\,dx$$

input
```
integrate(exp(arccos(a*x))*x^3,x, algorithm="maxima")
```

output
```
integrate(x^3*e^(arccos(a*x)), x)
```

3.108.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.01

$$\int e^{\arccos(ax)}x^3\,dx = \frac{4}{17}x^4e^{(\arccos(ax))} - \frac{\sqrt{-a^2x^2+1}x^3e^{(\arccos(ax))}}{17\,a} - \frac{3\,x^2e^{(\arccos(ax))}}{85\,a^2} - \frac{6\,\sqrt{-a^2x^2+1}xe^{(\arccos(ax))}}{85\,a^3} - \frac{6\,e^{(\arccos(ax))}}{85\,a^4}$$

input
```
integrate(exp(arccos(a*x))*x^3,x, algorithm="giac")
```

output
```
4/17*x^4*e^(arccos(a*x)) - 1/17*sqrt(-a^2*x^2 + 1)*x^3*e^(arccos(a*x))/a -
 3/85*x^2*e^(arccos(a*x))/a^2 - 6/85*sqrt(-a^2*x^2 + 1)*x*e^(arccos(a*x))/
a^3 - 6/85*e^(arccos(a*x))/a^4
```

3.108.9 Mupad [F(-1)]

Timed out.

$$\int e^{\arccos(ax)}x^3\,dx = \int x^3\,\mathrm{e}^{\mathrm{acos}(a\,x)}\,dx$$

input
```
int(x^3*exp(acos(a*x)),x)
```

output
```
int(x^3*exp(acos(a*x)), x)
```

3.109 $\int e^{\arccos(ax)}x^2\,dx$

3.109.1 Optimal result

Integrand size = 10, antiderivative size = 82

$$\int e^{\arccos(ax)}x^2\,dx = \frac{e^{\arccos(ax)}x}{8a^2} - \frac{e^{\arccos(ax)}\sqrt{1-a^2x^2}}{8a^3} + \frac{3e^{\arccos(ax)}\cos(3\arccos(ax))}{40a^3} - \frac{e^{\arccos(ax)}\sin(3\arccos(ax))}{40a^3}$$

output
```
1/8*exp(arccos(a*x))*x/a^2+3/40*exp(arccos(a*x))*cos(3*arccos(a*x))/a^3-1/
40*exp(arccos(a*x))*sin(3*arccos(a*x))/a^3-1/8*exp(arccos(a*x))*(-a^2*x^2+
1)^(1/2)/a^3
```

3.109.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.61

$$\int e^{\arccos(ax)}x^2\,dx = -\frac{e^{\arccos(ax)}\left(-5ax+5\sqrt{1-a^2x^2}-3\cos(3\arccos(ax))+\sin(3\arccos(ax))\right)}{40a^3}$$

input
```
Integrate[E^ArcCos[a*x]*x^2,x]
```

output
```
-1/40*(E^ArcCos[a*x]*(-5*a*x + 5*Sqrt[1 - a^2*x^2] - 3*Cos[3*ArcCos[a*x]]
+ Sin[3*ArcCos[a*x]]))/a^3
```

3.109.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5336, 27, 4972, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 e^{\arccos(ax)} \, dx$$

$$\Big\downarrow \; 5336$$

$$-\frac{\int e^{\arccos(ax)} x^2 \sqrt{1-a^2x^2} d\arccos(ax)}{a}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int a^2 e^{\arccos(ax)} x^2 \sqrt{1-a^2x^2} d\arccos(ax)}{a^3}$$

$$\Big\downarrow \; 4972$$

$$-\frac{\int \left(\frac{1}{4} e^{\arccos(ax)} \sin(3\arccos(ax)) + \frac{1}{4} e^{\arccos(ax)} \sqrt{1-a^2x^2}\right) d\arccos(ax)}{a^3}$$

$$\Big\downarrow \; 2009$$

$$-\frac{\frac{1}{8}\sqrt{1-a^2x^2} e^{\arccos(ax)} - \frac{1}{8} a x e^{\arccos(ax)} + \frac{1}{40} e^{\arccos(ax)} \sin(3\arccos(ax)) - \frac{3}{40} e^{\arccos(ax)} \cos(3\arccos(ax))}{a^3}$$

input
```
Int[E^ArcCos[a*x]*x^2,x]
```

output
```
-((-1/8*(a*E^ArcCos[a*x]*x) + (E^ArcCos[a*x]*Sqrt[1 - a^2*x^2])/8 - (3*E^A
rcCos[a*x]*Cos[3*ArcCos[a*x]])/40 + (E^ArcCos[a*x]*Sin[3*ArcCos[a*x]])/40)
/a^3)
```

3.109.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4972
```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

rule 5336
```
Int[(u_.)*(f_)^(ArcCos[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> Simp[
-b^(-1)   Subst[Int[(u /. x -> -a/b + Cos[x]/b)*f^(c*x^n)*Sin[x], x], x, Ar
cCos[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[n, 0]
```

3.109.4 Maple [F]

$$\int \mathrm{e}^{\arccos(ax)} x^2 dx$$

input
```
int(exp(arccos(a*x))*x^2,x)
```

output
```
int(exp(arccos(a*x))*x^2,x)
```

3.109.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.56

$$\int e^{\arccos(ax)} x^2 \, dx = \frac{\left(3\, a^3 x^3 - ax - (a^2x^2+1)\sqrt{-a^2x^2+1}\right) e^{(\arccos(ax))}}{10\, a^3}$$

input
```
integrate(exp(arccos(a*x))*x^2,x, algorithm="fricas")
```

output
```
1/10*(3*a^3*x^3 - a*x - (a^2*x^2 + 1)*sqrt(-a^2*x^2 + 1))*e^(arccos(a*x))/
a^3
```

3.109.6 Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.04

$$\int e^{\arccos(ax)}x^2\,dx = \begin{cases} \frac{3x^3e^{\operatorname{acos}(ax)}}{10} - \frac{x^2\sqrt{-a^2x^2+1}e^{\operatorname{acos}(ax)}}{10a} - \frac{xe^{\operatorname{acos}(ax)}}{10a^2} - \frac{\sqrt{-a^2x^2+1}e^{\operatorname{acos}(ax)}}{10a^3} & \text{for } a \neq 0 \\ \frac{x^3e^{\frac{\pi}{2}}}{3} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(acos(a*x))*x**2,x)
```

output
```
Piecewise((3*x**3*exp(acos(a*x))/10 - x**2*sqrt(-a**2*x**2 + 1)*exp(acos(a
*x))/(10*a) - x*exp(acos(a*x))/(10*a**2) - sqrt(-a**2*x**2 + 1)*exp(acos(a
*x))/(10*a**3), Ne(a, 0)), (x**3*exp(pi/2)/3, True))
```

3.109.7 Maxima [F]

$$\int e^{\arccos(ax)}x^2\,dx = \int x^2e^{(\arccos(ax))}\,dx$$

input
```
integrate(exp(arccos(a*x))*x^2,x, algorithm="maxima")
```

output
```
integrate(x^2*e^(arccos(a*x)), x)
```

3.109.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.84

$$\int e^{\arccos(ax)}x^2\,dx = \frac{3}{10}\,x^3e^{(\arccos(ax))} - \frac{\sqrt{-a^2x^2+1}x^2e^{(\arccos(ax))}}{10\,a} - \frac{xe^{(\arccos(ax))}}{10\,a^2} - \frac{\sqrt{-a^2x^2+1}e^{(\arccos(ax))}}{10\,a^3}$$

input
```
integrate(exp(arccos(a*x))*x^2,x, algorithm="giac")
```

output
```
3/10*x^3*e^(arccos(a*x)) - 1/10*sqrt(-a^2*x^2 + 1)*x^2*e^(arccos(a*x))/a -
 1/10*x*e^(arccos(a*x))/a^2 - 1/10*sqrt(-a^2*x^2 + 1)*e^(arccos(a*x))/a^3
```

3.109.9 Mupad [F(-1)]

Timed out.

$$\int e^{\arccos(ax)}x^2\,dx = \int x^2\,\mathrm{e}^{\mathrm{acos}(a\,x)}\,dx$$

input
```
int(x^2*exp(acos(a*x)),x)
```

output
```
int(x^2*exp(acos(a*x)), x)
```

3.110 $\int e^{\arccos(ax)} x \, dx$

3.110.1 Optimal result

Integrand size = 8, antiderivative size = 41

$$\int e^{\arccos(ax)} x \, dx = \frac{e^{\arccos(ax)} \cos(2 \arccos(ax))}{5a^2} - \frac{e^{\arccos(ax)} \sin(2 \arccos(ax))}{10a^2}$$

output
```
1/5*exp(arccos(a*x))*cos(2*arccos(a*x))/a^2-1/10*exp(arccos(a*x))*sin(2*ar
ccos(a*x))/a^2
```

3.110.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

$$\int e^{\arccos(ax)} x \, dx = -\frac{e^{\arccos(ax)}(-2 \cos(2 \arccos(ax)) + \sin(2 \arccos(ax)))}{10a^2}$$

input
```
Integrate[E^ArcCos[a*x]*x,x]
```

output
```
-1/10*(E^ArcCos[a*x]*(-2*Cos[2*ArcCos[a*x]] + Sin[2*ArcCos[a*x]]))/a^2
```

3.110.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5336, 27, 4972, 27, 4932}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int xe^{\arccos(ax)}\,dx$$

$$\Big\downarrow \text{5336}$$

$$-\frac{\int e^{\arccos(ax)}x\sqrt{1-a^2x^2}d\arccos(ax)}{a}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int ae^{\arccos(ax)}x\sqrt{1-a^2x^2}d\arccos(ax)}{a^2}$$

$$\Big\downarrow \text{4972}$$

$$-\frac{\int \frac{1}{2}e^{\arccos(ax)}\sin(2\arccos(ax))d\arccos(ax)}{a^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int e^{\arccos(ax)}\sin(2\arccos(ax))d\arccos(ax)}{2a^2}$$

$$\Big\downarrow \text{4932}$$

$$-\frac{\frac{1}{5}e^{\arccos(ax)}\sin(2\arccos(ax))-\frac{2}{5}e^{\arccos(ax)}\cos(2\arccos(ax))}{2a^2}$$

input
```
Int[E^ArcCos[a*x]*x,x]
```

output
```
-1/2*((-2*E^ArcCos[a*x]*Cos[2*ArcCos[a*x]])/5 + (E^ArcCos[a*x]*Sin[2*ArcCo
s[a*x]])/5)/a^2
```

3.110.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 4932
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

rule 4972
```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

rule 5336
```
Int[(u_.)*(f_)^(ArcCos[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> Simp[
-b^(-1)   Subst[Int[(u /. x -> -a/b + Cos[x]/b)*f^(c*x^n)*Sin[x], x], x, Ar
cCos[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[n, 0]
```

3.110.4 Maple [F]

$$\int \mathrm{e}^{\arccos(ax)} x dx$$

input
```
int(exp(arccos(a*x))*x,x)
```

output
```
int(exp(arccos(a*x))*x,x)
```

3.110.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int e^{\arccos(ax)} x \, dx = \frac{\left(2 \, a^2 x^2 - \sqrt{-a^2 x^2 + 1} a x - 1\right) e^{(\arccos(ax))}}{5 \, a^2}$$

input
```
integrate(exp(arccos(a*x))*x,x, algorithm="fricas")
```

output
```
1/5*(2*a^2*x^2 - sqrt(-a^2*x^2 + 1)*a*x - 1)*e^(arccos(a*x))/a^2
```

3.110.6 Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.41

$$\int e^{\arccos(ax)}x\,dx = \begin{cases} \frac{2x^2e^{\operatorname{acos}(ax)}}{5} - \frac{x\sqrt{-a^2x^2+1}e^{\operatorname{acos}(ax)}}{5a} - \frac{e^{\operatorname{acos}(ax)}}{5a^2} & \text{for } a \neq 0 \\ \frac{x^2e^{\frac{\pi}{2}}}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(acos(a*x))*x,x)
```

output
```
Piecewise((2*x**2*exp(acos(a*x))/5 - x*sqrt(-a**2*x**2 + 1)*exp(acos(a*x))
/(5*a) - exp(acos(a*x))/(5*a**2), Ne(a, 0)), (x**2*exp(pi/2)/2, True))
```

3.110.7 Maxima [F]

$$\int e^{\arccos(ax)}x\,dx = \int xe^{(\arccos(ax))}\,dx$$

input
```
integrate(exp(arccos(a*x))*x,x, algorithm="maxima")
```

output
```
integrate(x*e^(arccos(a*x)), x)
```

3.110.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07

$$\int e^{\arccos(ax)}x\,dx = \frac{2}{5}x^2e^{(\arccos(ax))} - \frac{\sqrt{-a^2x^2+1}xe^{(\arccos(ax))}}{5\,a} - \frac{e^{(\arccos(ax))}}{5\,a^2}$$

input
```
integrate(exp(arccos(a*x))*x,x, algorithm="giac")
```

output
```
2/5*x^2*e^(arccos(a*x)) - 1/5*sqrt(-a^2*x^2 + 1)*x*e^(arccos(a*x))/a - 1/5
*e^(arccos(a*x))/a^2
```

3.110.9 Mupad [F(-1)]

Timed out.

$$\int e^{\arccos(ax)}x\,dx = \int x\,\mathrm{e}^{\mathrm{acos}(a\,x)}\,dx$$

input
```
int(x*exp(acos(a*x)),x)
```

output
```
int(x*exp(acos(a*x)), x)
```

3.111 $\int e^{\arccos(ax)}\,dx$

3.111.1 Optimal result

Integrand size = 6, antiderivative size = 39

$$\int e^{\arccos(ax)}\,dx=\frac{1}{2}e^{\arccos(ax)}x-\frac{e^{\arccos(ax)}\sqrt{1-a^2x^2}}{2a}$$

output
```
1/2*exp(arccos(a*x))*x-1/2*exp(arccos(a*x))*(-a^2*x^2+1)^(1/2)/a
```

3.111.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int e^{\arccos(ax)}\,dx=-\frac{e^{\arccos(ax)}\left(-ax+\sqrt{1-a^2x^2}\right)}{2a}$$

input
```
Integrate[E^ArcCos[a*x],x]
```

output
```
-1/2*(E^ArcCos[a*x]*(-(a*x) + Sqrt[1 - a^2*x^2]))/a
```

3.111.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5336, 4932}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{\arccos(ax)}\,dx$$

$$\Big\downarrow \text{5336}$$

$$-\frac{\int e^{\arccos(ax)}\sqrt{1-a^2x^2}d\arccos(ax)}{a}$$

$$\Big\downarrow \text{4932}$$

$$-\frac{\frac{1}{2}\sqrt{1-a^2x^2}e^{\arccos(ax)}-\frac{1}{2}axe^{\arccos(ax)}}{a}$$

input
```
Int[E^ArcCos[a*x],x]
```

output
```
-((-1/2*(a*E^ArcCos[a*x]*x) + (E^ArcCos[a*x]*Sqrt[1 - a^2*x^2])/2)/a)
```

3.111.3.1 Defintions of rubi rules used

rule 4932
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sin[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x
] - Simp[e*F^(c*(a + b*x))*(Cos[d + e*x]/(e^2 + b^2*c^2*Log[F]^2)), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

rule 5336
```
Int[(u_.)*(f_)^(ArcCos[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> Simp[
-b^(-1)   Subst[Int[(u /. x -> -a/b + Cos[x]/b)*f^(c*x^n)*Sin[x], x], x, Ar
cCos[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[n, 0]
```

3.111.4 Maple [F]

$$\int e^{\arccos(ax)}dx$$

input
```
int(exp(arccos(a*x)),x)
```

output
```
int(exp(arccos(a*x)),x)
```

3.111.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

$$\int e^{\arccos(ax)}\,dx = \frac{\left(ax - \sqrt{-a^2x^2+1}\right)e^{(\arccos(ax))}}{2\,a}$$

input
```
integrate(exp(arccos(a*x)),x, algorithm="fricas")
```

output
```
1/2*(a*x - sqrt(-a^2*x^2 + 1))*e^(arccos(a*x))/a
```

3.111.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int e^{\arccos(ax)}\,dx = \begin{cases} \frac{xe^{\operatorname{acos}(ax)}}{2} - \frac{\sqrt{-a^2x^2+1}e^{\operatorname{acos}(ax)}}{2a} & \text{for } a \neq 0 \\ xe^{\frac{\pi}{2}} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(acos(a*x)),x)
```

output
```
Piecewise((x*exp(acos(a*x))/2 - sqrt(-a**2*x**2 + 1)*exp(acos(a*x))/(2*a),
 Ne(a, 0)), (x*exp(pi/2), True))
```

3.111.7 Maxima [F]

$$\int e^{\arccos(ax)}\,dx = \int e^{(\arccos(ax))}\,dx$$

input
```
integrate(exp(arccos(a*x)),x, algorithm="maxima")
```

output
```
integrate(e^(arccos(a*x)), x)
```

3.111.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int e^{\arccos(ax)}\,dx = \frac{1}{2}\,xe^{(\arccos(ax))} - \frac{\sqrt{-a^2x^2+1}e^{(\arccos(ax))}}{2\,a}$$

input
```
integrate(exp(arccos(a*x)),x, algorithm="giac")
```

output
```
1/2*x*e^(arccos(a*x)) - 1/2*sqrt(-a^2*x^2 + 1)*e^(arccos(a*x))/a
```

3.111.9 Mupad [F(-1)]

Timed out.

$$\int e^{\arccos(ax)}\,dx = \int e^{\mathrm{acos}(a\,x)}\,dx$$

input
```
int(exp(acos(a*x)),x)
```

output
```
int(exp(acos(a*x)), x)
```

3.112 $\int \frac{e^{\arccos(ax)}}{x} \, dx$

3.112.1 Optimal result

Integrand size = 10, antiderivative size = 45

$$\int \frac{e^{\arccos(ax)}}{x} \, dx = ie^{\arccos(ax)} - 2ie^{\arccos(ax)} \text{Hypergeometric2F1}\left(-\frac{i}{2}, 1, 1 - \frac{i}{2}, -e^{2i \arccos(ax)}\right)$$

output
```
I*exp(arccos(a*x))-2*I*exp(arccos(a*x))*hypergeom([1, -1/2*I],[1-1/2*I],-(
a*x+I*(-a^2*x^2+1)^(1/2))^2)
```

3.112.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.76

$$\int \frac{e^{\arccos(ax)}}{x} \, dx = i\left(-e^{\arccos(ax)} \text{Hypergeometric2F1}\left(-\frac{i}{2}, 1, 1 - \frac{i}{2}, -e^{2i \arccos(ax)}\right) + \left(\frac{1}{5} - \frac{2i}{5}\right) e^{(1+2i) \arccos(ax)} \text{Hypergeometric2F1}\left(1, 1 - \frac{i}{2}, 2 - \frac{i}{2}, -e^{2i \arccos(ax)}\right)\right)$$

input
```
Integrate[E^ArcCos[a*x]/x,x]
```

output
```
I*(-(E^ArcCos[a*x]*Hypergeometric2F1[-1/2*I, 1, 1 - I/2, -E^((2*I)*ArcCos[
a*x])]) + (1/5 - (2*I)/5)*E^((1 + 2*I)*ArcCos[a*x])*Hypergeometric2F1[1, 1
 - I/2, 2 - I/2, -E^((2*I)*ArcCos[a*x])])
```

3.112.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5336, 27, 4942, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\arccos(ax)}}{x} dx$$

$$\Big\downarrow \ 5336$$

$$-\frac{\int \frac{e^{\arccos(ax)}\sqrt{1-a^2x^2}}{x}d\arccos(ax)}{a}$$

$$\Big\downarrow \ 27$$

$$-\int \frac{e^{\arccos(ax)}\sqrt{1-a^2x^2}}{ax}d\arccos(ax)$$

$$\Big\downarrow \ 4942$$

$$-i\int \left(\frac{2e^{\arccos(ax)}}{1+e^{2i\arccos(ax)}}-e^{\arccos(ax)}\right)d\arccos(ax)$$

$$\Big\downarrow \ 2009$$

$$-i\left(-e^{\arccos(ax)}+2e^{\arccos(ax)}\text{Hypergeometric2F1}\left(-\frac{i}{2},1,1-\frac{i}{2},-e^{2i\arccos(ax)}\right)\right)$$

input
```
Int[E^ArcCos[a*x]/x,x]
```

output
```
(-I)*(-E^ArcCos[a*x] + 2*E^ArcCos[a*x]*Hypergeometric2F1[-1/2*I, 1, 1 - I/
2, -E^((2*I)*ArcCos[a*x])])
```

3.112.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4942
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[I^n   Int[ExpandIntegrand[F^(c*(a + b*x))*((1 - E^(2*I*(d + e*x
)))^n/(1 + E^(2*I*(d + e*x)))^n), x], x], x] /; FreeQ[{F, a, b, c, d, e}, x
] && IntegerQ[n]
```

rule 5336
```
Int[(u_.)*(f_)^(ArcCos[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> Simp[
-b^(-1)   Subst[Int[(u /. x -> -a/b + Cos[x]/b)*f^(c*x^n)*Sin[x], x], x, Ar
cCos[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[n, 0]
```

3.112.4 Maple [F]

$$\int \frac{\mathrm{e}^{\arccos(ax)}}{x}dx$$

input
```
int(exp(arccos(a*x))/x,x)
```

output
```
int(exp(arccos(a*x))/x,x)
```

3.112.5 Fricas [F]

$$\int \frac{e^{\arccos(ax)}}{x}\,dx = \int \frac{e^{(\arccos(ax))}}{x}\,dx$$

input
```
integrate(exp(arccos(a*x))/x,x, algorithm="fricas")
```

output
```
integral(e^(arccos(a*x))/x, x)
```

3.112.6 Sympy [F]

$$\int \frac{e^{\arccos(ax)}}{x} \, dx = \int \frac{e^{\operatorname{acos}(ax)}}{x} \, dx$$

input
```
integrate(exp(acos(a*x))/x,x)
```

output
```
Integral(exp(acos(a*x))/x, x)
```

3.112.7 Maxima [F]

$$\int \frac{e^{\arccos(ax)}}{x} \, dx = \int \frac{e^{(\arccos(ax))}}{x} \, dx$$

input
```
integrate(exp(arccos(a*x))/x,x, algorithm="maxima")
```

output
```
integrate(e^(arccos(a*x))/x, x)
```

3.112.8 Giac [F]

$$\int \frac{e^{\arccos(ax)}}{x} \, dx = \int \frac{e^{(\arccos(ax))}}{x} \, dx$$

input
```
integrate(exp(arccos(a*x))/x,x, algorithm="giac")
```

output
```
integrate(e^(arccos(a*x))/x, x)
```

3.112.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\arccos(ax)}}{x}\,dx = \int \frac{\mathrm{e}^{\mathrm{acos}(a\,x)}}{x}\,dx$$

input
```
int(exp(acos(a*x))/x,x)
```

output
```
int(exp(acos(a*x))/x, x)
```

3.113 $\int \frac{e^{\arccos(ax)}}{x^2} dx$

3.113.1 Optimal result

Integrand size = 10, antiderivative size = 87

$$\int \frac{e^{\arccos(ax)}}{x^2} dx = (1+i)ae^{(1+i)\arccos(ax)} \text{Hypergeometric2F1}\left(\frac{1}{2}-\frac{i}{2},1,\frac{3}{2}-\frac{i}{2},-e^{2i\arccos(ax)}\right) - (2+2i)ae^{(1+i)\arccos(ax)} \text{Hypergeometric2F1}\left(\frac{1}{2}-\frac{i}{2},2,\frac{3}{2}-\frac{i}{2},-e^{2i\arccos(ax)}\right)$$

output
```
(1+I)*a*exp((1+I)*arccos(a*x))*hypergeom([1, 1/2-1/2*I],[3/2-1/2*I],-(a*x+
I*(-a^2*x^2+1)^(1/2))^2)-(2+2*I)*a*exp((1+I)*arccos(a*x))*hypergeom([2, 1/
2-1/2*I],[3/2-1/2*I],-(a*x+I*(-a^2*x^2+1)^(1/2))^2)
```

3.113.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.63

$$\int \frac{e^{\arccos(ax)}}{x^2} dx = -\frac{e^{\arccos(ax)}}{x} + (1-i)ae^{(1+i)\arccos(ax)} \text{Hypergeometric2F1}\left(\frac{1}{2}-\frac{i}{2},1,\frac{3}{2}-\frac{i}{2},-e^{2i\arccos(ax)}\right)$$

input
```
Integrate[E^ArcCos[a*x]/x^2,x]
```

output
```
-(E^ArcCos[a*x]/x) + (1 - I)*a*E^((1 + I)*ArcCos[a*x])*Hypergeometric2F1[1
/2 - I/2, 1, 3/2 - I/2, -E^((2*I)*ArcCos[a*x])]
```

3.113.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5336, 27, 4974, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{\arccos(ax)}}{x^2}\,dx$$

$$\Big\downarrow \ 5336$$

$$-\frac{\int \frac{e^{\arccos(ax)}\sqrt{1-a^2x^2}}{x^2}d\arccos(ax)}{a}$$

$$\Big\downarrow \ 27$$

$$-a\int \frac{e^{\arccos(ax)}\sqrt{1-a^2x^2}}{a^2x^2}d\arccos(ax)$$

$$\Big\downarrow \ 4974$$

$$-a\int \left(\frac{4ie^{(1+i)\arccos(ax)}}{\left(1+e^{2i\arccos(ax)}\right)^2}-\frac{2ie^{(1+i)\arccos(ax)}}{1+e^{2i\arccos(ax)}}\right)d\arccos(ax)$$

$$\Big\downarrow \ 2009$$

$$-a\left((2+2i)e^{(1+i)\arccos(ax)}\text{Hypergeometric2F1}\left(\frac{1}{2}-\frac{i}{2},2,\frac{3}{2}-\frac{i}{2},-e^{2i\arccos(ax)}\right)-(1+i)e^{(1+i)\arccos(ax)}\text{Hyperge}\right.$$

input
```
Int[E^ArcCos[a*x]/x^2,x]
```

output
```
-(a*((-1 - I)*E^((1 + I)*ArcCos[a*x])*Hypergeometric2F1[1/2 - I/2, 1, 3/2
- I/2, -E^((2*I)*ArcCos[a*x])] + (2 + 2*I)*E^((1 + I)*ArcCos[a*x])*Hyperge
ometric2F1[1/2 - I/2, 2, 3/2 - I/2, -E^((2*I)*ArcCos[a*x])]))
```

3.113.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4974
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(G_)[(d_.) + (e_.)*(x_)]^(m_.)*(H_)[(
d_.) + (e_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigToExp[F^(c*(a + b*x)),
 G[d + e*x]^m*H[d + e*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e}, x] && IGtQ[
m, 0] && IGtQ[n, 0] && TrigQ[G] && TrigQ[H]
```

rule 5336
```
Int[(u_.)*(f_)^(ArcCos[(a_.) + (b_.)*(x_)]^(n_.)*(c_.)), x_Symbol] :> Simp[
-b^(-1)   Subst[Int[(u /. x -> -a/b + Cos[x]/b)*f^(c*x^n)*Sin[x], x], x, Ar
cCos[a + b*x]], x] /; FreeQ[{a, b, c, f}, x] && IGtQ[n, 0]
```

3.113.4 Maple [F]

$$\int \frac{\mathrm{e}^{\arccos(ax)}}{x^2}dx$$

input
```
int(exp(arccos(a*x))/x^2,x)
```

output
```
int(exp(arccos(a*x))/x^2,x)
```

3.113.5 Fricas [F]

$$\int \frac{e^{\arccos(ax)}}{x^2}\,dx = \int \frac{e^{(\arccos(ax))}}{x^2}\,dx$$

input
```
integrate(exp(arccos(a*x))/x^2,x, algorithm="fricas")
```

output
```
integral(e^(arccos(a*x))/x^2, x)
```

3.113.6 Sympy [F]

$$\int \frac{e^{\arccos(ax)}}{x^2}\,dx = \int \frac{e^{\operatorname{acos}(ax)}}{x^2}\,dx$$

input
```
integrate(exp(acos(a*x))/x**2,x)
```

output
```
Integral(exp(acos(a*x))/x**2, x)
```

3.113.7 Maxima [F]

$$\int \frac{e^{\arccos(ax)}}{x^2}\,dx = \int \frac{e^{(\arccos(ax))}}{x^2}\,dx$$

input
```
integrate(exp(arccos(a*x))/x^2,x, algorithm="maxima")
```

output
```
integrate(e^(arccos(a*x))/x^2, x)
```

3.113.8 Giac [F]

$$\int \frac{e^{\arccos(ax)}}{x^2}\,dx = \int \frac{e^{(\arccos(ax))}}{x^2}\,dx$$

input
```
integrate(exp(arccos(a*x))/x^2,x, algorithm="giac")
```

output
```
integrate(e^(arccos(a*x))/x^2, x)
```

3.113.9 Mupad [F(-1)]

Timed out.

$$\int \frac{e^{\arccos(ax)}}{x^2}\,dx = \int \frac{\mathrm{e}^{\mathrm{acos}(a\,x)}}{x^2}\,dx$$

input
```
int(exp(acos(a*x))/x^2,x)
```

output
```
int(exp(acos(a*x))/x^2, x)
```

3.114 $\int \arccos\left(\frac{c}{a+bx}\right)\,dx$

3.114.1 Optimal result

Integrand size = 10, antiderivative size = 48

$$\int \arccos\left(\frac{c}{a+bx}\right)\,dx = \frac{(a+bx)\sec^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b} - \frac{c\,\text{arctanh}\left(\sqrt{1-\frac{c^2}{(a+bx)^2}}\right)}{b}$$

output
```
(b*x+a)*arcsec(a/c+b*x/c)/b-c*arctanh((1-c^2/(b*x+a)^2)^(1/2))/b
```

3.114.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 528 vs. 2(48) = 96.

Time = 0.93 (sec) , antiderivative size = 528, normalized size of antiderivative = 11.00

$$\int \arccos\left(\frac{c}{a+bx}\right)\,dx = x\arccos\left(\frac{c}{a+bx}\right) + \frac{(a+bx)\sqrt{\frac{a^2-c^2+2abx+b^2x^2}{(a+bx)^2}}\left(\left(-c+\sqrt{-a^2+c^2}\right)\sqrt{-a^2+2c\left(c+\sqrt{-a^2+c^2}\right)}\arctan\left(\frac{b\sqrt{-a^2+2c\left(c+\sqrt{-c\right.}\right.}}{a\left(\sqrt{a^2-c^2}-\sqrt{a^2-c^2+2\right.}}\right.\right.}{\ldots}$$

input
```
Integrate[ArcCos[c/(a + b*x)],x]
```

output
```
x*ArcCos[c/(a + b*x)] + ((a + b*x)*Sqrt[(a^2 - c^2 + 2*a*b*x + b^2*x^2)/(a
 + b*x)^2]*((-c + Sqrt[-a^2 + c^2])*Sqrt[-a^2 + 2*c*(c + Sqrt[-a^2 + c^2])
]*ArcTan[(b*Sqrt[-a^2 + 2*c*(c + Sqrt[-a^2 + c^2])]*x)/(a*(Sqrt[a^2 - c^2]
 - Sqrt[a^2 - c^2 + 2*a*b*x + b^2*x^2]))] + (c + Sqrt[-a^2 + c^2])*Sqrt[a^
2 + 2*c*(-c + Sqrt[-a^2 + c^2])]*ArcTanh[(b*Sqrt[a^2 - 2*c^2 + 2*c*Sqrt[-a
^2 + c^2]]*x)/(a*Sqrt[a^2 - c^2] - a*Sqrt[a^2 - c^2 + 2*a*b*x + b^2*x^2])]
 + a*(a*ArcTan[(b^2*c*Sqrt[a^2 - c^2]*x^2)/(a^4 + a^3*b*x + b^2*c^2*x^2 -
a^2*(c^2 + Sqrt[a^2 - c^2]*Sqrt[a^2 - c^2 + 2*a*b*x + b^2*x^2]))] + c*(-Lo
g[Sqrt[a^2 - c^2] - b*x - Sqrt[a^2 - c^2 + 2*a*b*x + b^2*x^2]] + Log[b^2*(
Sqrt[a^2 - c^2] + b*x - Sqrt[a^2 - c^2 + 2*a*b*x + b^2*x^2])]))))/(a*b*Sqr
t[a^2 - c^2 + 2*a*b*x + b^2*x^2])
```

3.114.3 Rubi [A] (warning: unable to verify)

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5332, 5773, 895, 798, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arccos\left(\frac{c}{a+bx}\right)dx$$

$$\Big\downarrow \text{5332}$$

$$\int \sec^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)dx$$

$$\Big\downarrow \text{5773}$$

$$\frac{(a+bx)\sec^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b}-\int \frac{1}{\left(\frac{a}{c}+\frac{bx}{c}\right)\sqrt{1-\frac{1}{\left(\frac{a}{c}+\frac{bx}{c}\right)^2}}}dx$$

$$\Big\downarrow \text{895}$$

$$\frac{(a+bx)\sec^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b}-\frac{c\int \frac{1}{\left(\frac{a}{c}+\frac{bx}{c}\right)\sqrt{1-\frac{1}{\left(\frac{a}{c}+\frac{bx}{c}\right)^2}}}d\left(\frac{a}{c}+\frac{bx}{c}\right)}{b}$$

$$\Big\downarrow \text{798}$$

$$\frac{c\int \frac{\left(\frac{a}{c}+\frac{bx}{c}\right)^2}{\sqrt{-\frac{a}{c}-\frac{bx}{c}+1}}d\frac{1}{\left(\frac{a}{c}+\frac{bx}{c}\right)^2}}{2b}+\frac{(a+bx)\sec^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b}$$

$$\Big\downarrow \; 73$$

$$\frac{(a+bx)\sec^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b}-\frac{c\int \frac{1}{1-\frac{1}{\left(\frac{a}{c}+\frac{bx}{c}\right)^4}}d\sqrt{-\frac{a}{c}-\frac{bx}{c}+1}}{b}$$

$$\Big\downarrow \; 219$$

$$\frac{(a+bx)\sec^{-1}\left(\frac{a}{c}+\frac{bx}{c}\right)}{b}-\frac{c\operatorname{arctanh}\left(\sqrt{-\frac{a}{c}-\frac{bx}{c}+1}\right)}{b}$$

input
```
Int[ArcCos[c/(a + b*x)],x]
```

output
```
((a + b*x)*ArcSec[a/c + (b*x)/c])/b - (c*ArcTanh[Sqrt[1 - a/c - (b*x)/c]])
/b
```

3.114.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 895
```
Int[(u_)^(m_.)*((a_) + (b_.)*(v_)^(n_))^(p_.), x_Symbol] :> Simp[u^m/(Coeff
icient[v, x, 1]*v^m)   Subst[Int[x^m*(a + b*x^n)^p, x], x, v], x] /; FreeQ[
{a, b, m, n, p}, x] && LinearPairQ[u, v, x]
```

rule 5332
```
Int[ArcCos[(c_.)/((a_.) + (b_.)*(x_)^(n_.))]^(m_.)*(u_.), x_Symbol] :> Int[
u*ArcSec[a/c + b*(x^n/c)]^m, x] /; FreeQ[{a, b, c, n, m}, x]
```

rule 5773
```
Int[ArcSec[(c_) + (d_.)*(x_)], x_Symbol] :> Simp[(c + d*x)*(ArcSec[c + d*x]
/d), x] - Int[1/((c + d*x)*Sqrt[1 - 1/(c + d*x)^2]), x] /; FreeQ[{c, d}, x]
```

3.114.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94

method	result
derivativedivides	$-\frac{c\left(-\frac{(bx+a)\arccos\left(\frac{c}{bx+a}\right)}{c}+\operatorname{arctanh}\left(\frac{1}{\sqrt{1-\frac{c^2}{(bx+a)^2}}}\right)\right)}{b}$
default	$-\frac{c\left(-\frac{(bx+a)\arccos\left(\frac{c}{bx+a}\right)}{c}+\operatorname{arctanh}\left(\frac{1}{\sqrt{1-\frac{c^2}{(bx+a)^2}}}\right)\right)}{b}$
parts	$x\arccos\left(\frac{c}{bx+a}\right)-\frac{c\sqrt{b^2x^2+2abx+a^2-c^2}\left(\ln\left(\frac{b^2x+\sqrt{b^2x^2+2abx+a^2-c^2}\,\sqrt{b^2}+ab}{\sqrt{b^2}}\right)b\sqrt{-c^2}+a\ln\left(\frac{2\left(\sqrt{-c^2}\,\sqrt{b^2x^2+2 \cdots}\right.}{bx \cdots}\right.\right.}{b\sqrt{\frac{b^2x^2+2abx+a^2-c^2}{(bx+a)^2}}\,(bx+a)\sqrt{b^2}\,\sqrt{-c^2}}$

input
```
int(arccos(c/(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/b*c*(-1/c*(b*x+a)*arccos(c/(b*x+a))+arctanh(1/(1-c^2/(b*x+a)^2)^(1/2)))
```

3.114.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(46) = 92.

Time = 0.27 (sec) , antiderivative size = 140, normalized size of antiderivative = 2.92

$$\int \arccos\left(\frac{c}{a+bx}\right)\,dx$$
$$=\frac{bx\arccos\left(\frac{c}{bx+a}\right)+2\,a\arctan\left(-\frac{bx-(bx+a)\sqrt{\frac{b^2x^2+2\,abx+a^2-c^2}{b^2x^2+2\,abx+a^2}}+a}{c}\right)+c\log\left(-bx+(bx+a)\sqrt{\frac{b^2x^2+2\,abx+a^2-c^2}{b^2x^2+2\,abx+a^2}}\right. - \cdots}{b}$$

input
```
integrate(arccos(c/(b*x+a)),x, algorithm="fricas")
```

output
```
(b*x*arccos(c/(b*x + a)) + 2*a*arctan(-(b*x - (b*x + a)*sqrt((b^2*x^2 + 2*
a*b*x + a^2 - c^2)/(b^2*x^2 + 2*a*b*x + a^2)) + a)/c) + c*log(-b*x + (b*x
+ a)*sqrt((b^2*x^2 + 2*a*b*x + a^2 - c^2)/(b^2*x^2 + 2*a*b*x + a^2)) - a))
/b
```

3.114.6 Sympy [F]

$$\int \arccos\left(\frac{c}{a+bx}\right) dx = \int \operatorname{acos}\left(\frac{c}{a+bx}\right) dx$$

input
```
integrate(acos(c/(b*x+a)),x)
```

output
```
Integral(acos(c/(a + b*x)), x)
```

3.114.7 Maxima [F]

$$\int \arccos\left(\frac{c}{a+bx}\right) dx = \int \arccos\left(\frac{c}{bx+a}\right) dx$$

input
```
integrate(arccos(c/(b*x+a)),x, algorithm="maxima")
```

output
```
x*arctan(sqrt(b*x + a + c)*sqrt(b*x + a - c)/c) - integrate((b^2*c*x^2 + a
*b*c*x)*e^(1/2*log(b*x + a + c) + 1/2*log(b*x + a - c))/(b^2*c^2*x^2 + 2*a
*b*c^2*x + a^2*c^2 - c^4 + (b^2*x^2 + 2*a*b*x + a^2 - c^2)*e^(log(b*x + a
+ c) + log(b*x + a - c))), x)
```

3.114.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(46) = 92$.

Time = 0.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.98

$$\int \arccos\left(\frac{c}{a+bx}\right)\,dx = -\frac{b\left(\frac{c^2\left(\log\left(\sqrt{-\frac{c^2}{(bx+a)^2}+1}+1\right)-\log\left(-\sqrt{-\frac{c^2}{(bx+a)^2}+1}+1\right)\right)}{b^2}-\frac{2\,(bx+a)c\arccos\left(-\frac{c}{(bx+a)\left(\frac{a}{bx+a}-1\right)-a}\right)}{b^2}\right)}{2\,c}$$

input
```
integrate(arccos(c/(b*x+a)),x, algorithm="giac")
```

output
```
-1/2*b*(c^2*(log(sqrt(-c^2/(b*x + a)^2 + 1) + 1) - log(-sqrt(-c^2/(b*x + a
)^2 + 1) + 1))/b^2 - 2*(b*x + a)*c*arccos(-c/((b*x + a)*(a/(b*x + a) - 1)
- a))/b^2)/c
```

3.114.9 Mupad [B] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

$$\int \arccos\left(\frac{c}{a+bx}\right)\,dx = \frac{\operatorname{acos}\left(\frac{c}{a+b\,x}\right)\,(a+b\,x)}{b} - \frac{c\,\operatorname{atanh}\left(\frac{1}{\sqrt{1-\frac{c^2}{(a+b\,x)^2}}}\right)}{b}$$

input
```
int(acos(c/(a + b*x)),x)
```

output
```
(acos(c/(a + b*x))*(a + b*x))/b - (c*atanh(1/(1 - c^2/(a + b*x)^2)^(1/2)))
/b
```

3.115 $\int \frac{x}{\sqrt{1-x^2}\sqrt{\arccos(x)}}\,dx$

3.115.1 Optimal result

Integrand size = 19, antiderivative size = 26

$$\int \frac{x}{\sqrt{1-x^2}\sqrt{\arccos(x)}}\,dx = -\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(x)}\right)$$

output
```
-FresnelC(2^(1/2)/Pi^(1/2)*arccos(x)^(1/2))*2^(1/2)*Pi^(1/2)
```

3.115.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.15

$$\int \frac{x}{\sqrt{1-x^2}\sqrt{\arccos(x)}}\,dx = \frac{i\left(\sqrt{-i\arccos(x)}\Gamma\left(\frac{1}{2},-i\arccos(x)\right)-\sqrt{i\arccos(x)}\Gamma\left(\frac{1}{2},i\arccos(x)\right)\right)}{2\sqrt{\arccos(x)}}$$

input
```
Integrate[x/(Sqrt[1 - x^2]*Sqrt[ArcCos[x]]),x]
```

output
```
((I/2)*(Sqrt[(-I)*ArcCos[x]]*Gamma[1/2, (-I)*ArcCos[x]] - Sqrt[I*ArcCos[x]
]*Gamma[1/2, I*ArcCos[x]]))/Sqrt[ArcCos[x]]
```

3.115.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {5225, 3042, 3785, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{1-x^2}\sqrt{\arccos(x)}}dx$$

$$\Big\downarrow \; 5225$$

$$-\int \frac{x}{\sqrt{\arccos(x)}}d\arccos(x)$$

$$\Big\downarrow \; 3042$$

$$-\int \frac{\sin\left(\arccos(x)+\frac{\pi}{2}\right)}{\sqrt{\arccos(x)}}d\arccos(x)$$

$$\Big\downarrow \; 3785$$

$$-2\int xd\sqrt{\arccos(x)}$$

$$\Big\downarrow \; 3833$$

$$-\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(x)}\right)$$

input
```
Int[x/(Sqrt[1 - x^2]*Sqrt[ArcCos[x]]),x]
```

output
```
-(Sqrt[2*Pi]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcCos[x]]])
```

3.115.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

3.115.4 Maple [A] (verified)

Time = 1.62 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.81

method	result	size
default	$-\operatorname{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{\arccos(x)}}{\sqrt{\pi}}\right)\sqrt{2}\,\sqrt{\pi}$	21

input
```
int(x/(-x^2+1)^(1/2)/arccos(x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-FresnelC(2^(1/2)/Pi^(1/2)*arccos(x)^(1/2))*2^(1/2)*Pi^(1/2)
```

3.115.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{1-x^2}\sqrt{\arccos(x)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(-x^2+1)^(1/2)/arccos(x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

3.115.6 Sympy [F]

$$\int \frac{x}{\sqrt{1-x^2}\sqrt{\arccos(x)}}\,dx = \int \frac{x}{\sqrt{-(x-1)(x+1)}\sqrt{\operatorname{acos}(x)}}\,dx$$

input
```
integrate(x/(-x**2+1)**(1/2)/acos(x)**(1/2),x)
```

output
```
Integral(x/(sqrt(-(x - 1)*(x + 1))*sqrt(acos(x))), x)
```

3.115.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{1-x^2}\sqrt{\arccos(x)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x/(-x^2+1)^(1/2)/arccos(x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.115.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.42

$$\int \frac{x}{\sqrt{1-x^2}\sqrt{\arccos(x)}}\,dx = \left(\frac{1}{4}i+\frac{1}{4}\right)\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\arccos(x)}\right) - \left(\frac{1}{4}i-\frac{1}{4}\right)\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{\arccos(x)}\right)$$

input
```
integrate(x/(-x^2+1)^(1/2)/arccos(x)^(1/2),x, algorithm="giac")
```

output
```
(1/4*I + 1/4)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(arccos(x)))
- (1/4*I - 1/4)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(arccos(x)
))
```

3.115.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{1-x^2}\sqrt{\arccos(x)}}\,dx = \int \frac{x}{\sqrt{\mathrm{acos}\left(x\right)}\,\sqrt{1-x^2}}\,dx$$

input
```
int(x/(acos(x)^(1/2)*(1 - x^2)^(1/2)),x)
```

output
```
int(x/(acos(x)^(1/2)*(1 - x^2)^(1/2)), x)
```

3.116 $\int \frac{x}{\sqrt{1-x^2}\arccos(x)}\,dx$

3.116.1 Optimal result

Integrand size = 17, antiderivative size = 5

$$\int \frac{x}{\sqrt{1-x^2}\arccos(x)}\,dx = -\operatorname{CosIntegral}(\arccos(x))$$

output
```
-Ci(arccos(x))
```

3.116.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{1-x^2}\arccos(x)}\,dx = -\operatorname{CosIntegral}(\arccos(x))$$

input
```
Integrate[x/(Sqrt[1 - x^2]*ArcCos[x]),x]
```

output
```
-CosIntegral[ArcCos[x]]
```

3.116.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {5225, 3042, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{1-x^2}\arccos(x)}dx$$

$$\Big\downarrow \; 5225$$

$$-\int \frac{x}{\arccos(x)}d\arccos(x)$$

$$\Big\downarrow \; 3042$$

$$-\int \frac{\sin\left(\arccos(x)+\frac{\pi}{2}\right)}{\arccos(x)}d\arccos(x)$$

$$\Big\downarrow \; 3783$$

$$-\operatorname{CosIntegral}(\arccos(x))$$

input
```
Int[x/(Sqrt[1 - x^2]*ArcCos[x]),x]
```

output
```
-CosIntegral[ArcCos[x]]
```

3.116.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 5225
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Simp[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p]   Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
 x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

3.116.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

method	result	size
default	$-\operatorname{Ci}(\arccos(x))$	6

input
```
int(x/arccos(x)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-Ci(arccos(x))
```

3.116.5 Fricas [F]

$$\int \frac{x}{\sqrt{1-x^2}\arccos(x)}\,dx = \int \frac{x}{\sqrt{-x^2+1}\arccos(x)}\,dx$$

input
```
integrate(x/arccos(x)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
integral(-sqrt(-x^2 + 1)*x/((x^2 - 1)*arccos(x)), x)
```

3.116.6 Sympy [F]

$$\int \frac{x}{\sqrt{1-x^2}\arccos(x)}\,dx = \int \frac{x}{\sqrt{-(x-1)(x+1)}\operatorname{acos}(x)}\,dx$$

input
```
integrate(x/acos(x)/(-x**2+1)**(1/2),x)
```

output
```
Integral(x/(sqrt(-(x - 1)*(x + 1))*acos(x)), x)
```

3.116.7 Maxima [F]

$$\int \frac{x}{\sqrt{1-x^2}\arccos(x)}\,dx = \int \frac{x}{\sqrt{-x^2+1}\arccos(x)}\,dx$$

input
```
integrate(x/arccos(x)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(-x^2 + 1)*arccos(x)), x)
```

3.116.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{1-x^2}\arccos(x)}\,dx = -\operatorname{Ci}(\arccos(x))$$

input
```
integrate(x/arccos(x)/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
-cos_integral(arccos(x))
```

3.116.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{1-x^2}\arccos(x)}\,dx = \int \frac{x}{\operatorname{acos}(x)\sqrt{1-x^2}}\,dx$$

input
```
int(x/(acos(x)*(1 - x^2)^(1/2)),x)
```

output
```
int(x/(acos(x)*(1 - x^2)^(1/2)), x)
```

3.117 $\int \frac{\arccos\left(\sqrt{1+bx^2}\right)^n}{\sqrt{1+bx^2}}\,dx$

3.117.1 Optimal result

Integrand size = 26, antiderivative size = 39

$$\int \frac{\arccos\left(\sqrt{1+bx^2}\right)^n}{\sqrt{1+bx^2}}\,dx = -\frac{\sqrt{-bx^2}\arccos\left(\sqrt{1+bx^2}\right)^{1+n}}{b(1+n)x}$$

output
```
-arccos((b*x^2+1)^(1/2))^(1+n)*(-b*x^2)^(1/2)/b/(1+n)/x
```

3.117.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{\arccos\left(\sqrt{1+bx^2}\right)^n}{\sqrt{1+bx^2}}\,dx = -\frac{\sqrt{-bx^2}\arccos\left(\sqrt{1+bx^2}\right)^{1+n}}{b(1+n)x}$$

input
```
Integrate[ArcCos[Sqrt[1 + b*x^2]]^n/Sqrt[1 + b*x^2],x]
```

output
```
-((Sqrt[-(b*x^2)]*ArcCos[Sqrt[1 + b*x^2]]^(1 + n))/(b*(1 + n)*x))
```

3.117.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {5334, 5153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arccos\left(\sqrt{bx^2+1}\right)^n}{\sqrt{bx^2+1}}dx$$

$$\Big\downarrow \text{5334}$$

$$\frac{\sqrt{-bx^2}\int \frac{\arccos\left(\sqrt{bx^2+1}\right)^n}{\sqrt{-bx^2}}d\sqrt{bx^2+1}}{bx}$$

$$\Big\downarrow \text{5153}$$

$$-\frac{\sqrt{-bx^2}\arccos\left(\sqrt{bx^2+1}\right)^{n+1}}{b(n+1)x}$$

input
```
Int[ArcCos[Sqrt[1 + b*x^2]]^n/Sqrt[1 + b*x^2],x]
```

output
```
-((Sqrt[-(b*x^2)]*ArcCos[Sqrt[1 + b*x^2]]^(1 + n))/(b*(1 + n)*x))
```

3.117.3.1 Defintions of rubi rules used

rule 5153
```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

rule 5334
```
Int[ArcCos[Sqrt[1 + (b_.)*(x_)^2]]^(n_.)/Sqrt[1 + (b_.)*(x_)^2], x_Symbol]
:> Simp[Sqrt[(-b)*x^2]/(b*x)   Subst[Int[ArcCos[x]^n/Sqrt[1 - x^2], x], x,
Sqrt[1 + b*x^2]], x] /; FreeQ[{b, n}, x]
```

3.117.4 Maple [F]

$$\int \frac{\arccos\left(\sqrt{b\,x^{2}+1}\right)^{n}}{\sqrt{b\,x^{2}+1}}dx$$

input
```
int(arccos((b*x^2+1)^(1/2))^n/(b*x^2+1)^(1/2),x)
```

output
```
int(arccos((b*x^2+1)^(1/2))^n/(b*x^2+1)^(1/2),x)
```

3.117.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

$$\int \frac{\arccos\left(\sqrt{1+bx^2}\right)^n}{\sqrt{1+bx^2}}\,dx = -\frac{\sqrt{-bx^2}\arccos\left(\sqrt{bx^2+1}\right)^n\arccos\left(\sqrt{bx^2+1}\right)}{(bn+b)x}$$

input
```
integrate(arccos((b*x^2+1)^(1/2))^n/(b*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(-b*x^2)*arccos(sqrt(b*x^2 + 1))^n*arccos(sqrt(b*x^2 + 1))/((b*n + b)
*x)
```

3.117.6 Sympy [F]

$$\int \frac{\arccos\left(\sqrt{1+bx^2}\right)^n}{\sqrt{1+bx^2}}\,dx = \begin{cases} \tilde{\infty}x & \text{for } b=0\wedge n=-1 \\ 0^n x & \text{for } b=0 \\ \int \frac{1}{\sqrt{bx^2+1}\operatorname{acos}\left(\sqrt{bx^2+1}\right)}\,dx & \text{for } n=-1 \\ -\frac{\sqrt{-bx^2}\operatorname{acos}\left(\sqrt{bx^2+1}\right)\operatorname{acos}^n\left(\sqrt{bx^2+1}\right)}{bnx+bx} & \text{otherwise} \end{cases}$$

input
```
integrate(acos((b*x**2+1)**(1/2))**n/(b*x**2+1)**(1/2),x)
```

output
```
Piecewise((zoo*x, Eq(b, 0) & Eq(n, -1)), (0**n*x, Eq(b, 0)), (Integral(1/(
sqrt(b*x**2 + 1)*acos(sqrt(b*x**2 + 1))), x), Eq(n, -1)), (-sqrt(-b*x**2)*
acos(sqrt(b*x**2 + 1))*acos(sqrt(b*x**2 + 1))**n/(b*n*x + b*x), True))
```

3.117.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\arccos\left(\sqrt{1+bx^2}\right)^n}{\sqrt{1+bx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arccos((b*x^2+1)^(1/2))^n/(b*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: sign: argument cannot be imagi
nary; found sqrt(-_SAGE_VAR_b)
```

3.117.8 Giac [F]

$$\int \frac{\arccos\left(\sqrt{1+bx^2}\right)^n}{\sqrt{1+bx^2}}\,dx = \int \frac{\arccos\left(\sqrt{bx^2+1}\right)^n}{\sqrt{bx^2+1}}\,dx$$

input
```
integrate(arccos((b*x^2+1)^(1/2))^n/(b*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

3.117.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos\left(\sqrt{1+bx^2}\right)^n}{\sqrt{1+bx^2}}\,dx = \int \frac{\mathrm{acos}\left(\sqrt{b\,x^2+1}\right)^n}{\sqrt{b\,x^2+1}}\,dx$$

input
```
int(acos((b*x^2 + 1)^(1/2))^n/(b*x^2 + 1)^(1/2),x)
```

output
```
int(acos((b*x^2 + 1)^(1/2))^n/(b*x^2 + 1)^(1/2), x)
```

3.118 $\int \frac{1}{\sqrt{1+bx^2}\arccos\left(\sqrt{1+bx^2}\right)}\,dx$

3.118.1 Optimal result

Integrand size = 26, antiderivative size = 31

$$\int \frac{1}{\sqrt{1+bx^2}\arccos\left(\sqrt{1+bx^2}\right)}\,dx = -\frac{\sqrt{-bx^2}\log\left(\arccos\left(\sqrt{1+bx^2}\right)\right)}{bx}$$

output
```
-ln(arccos((b*x^2+1)^(1/2)))*(-b*x^2)^(1/2)/b/x
```

3.118.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{1+bx^2}\arccos\left(\sqrt{1+bx^2}\right)}\,dx = \frac{x\log\left(\arccos\left(\sqrt{1+bx^2}\right)\right)}{\sqrt{-bx^2}}$$

input
```
Integrate[1/(Sqrt[1 + b*x^2]*ArcCos[Sqrt[1 + b*x^2]]),x]
```

output
```
(x*Log[ArcCos[Sqrt[1 + b*x^2]]])/Sqrt[-(b*x^2)]
```

3.118.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {5334, 5151}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{bx^2+1}\arccos\left(\sqrt{bx^2+1}\right)}dx$$

$$\Big\downarrow \text{5334}$$

$$\frac{\sqrt{-bx^2}\int \frac{1}{\sqrt{-bx^2}\arccos\left(\sqrt{bx^2+1}\right)}d\sqrt{bx^2+1}}{bx}$$

$$\Big\downarrow \text{5151}$$

$$-\frac{\sqrt{-bx^2}\log\left(\arccos\left(\sqrt{bx^2+1}\right)\right)}{bx}$$

input
```
Int[1/(Sqrt[1 + b*x^2]*ArcCos[Sqrt[1 + b*x^2]]),x]
```

output
```
-((Sqrt[-(b*x^2)]*Log[ArcCos[Sqrt[1 + b*x^2]]])/(b*x))
```

3.118.3.1 Defintions of rubi rules used

rule 5151
```
Int[1/(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*Sqrt[(d_) + (e_.)*(x_)^2]), x_Sym
bol] :> Simp[(-(b*c)^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*(Log[a +
 b*ArcCos[c*x]]/(b*c*Sqrt[d])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*
d + e, 0]
```

rule 5334
```
Int[ArcCos[Sqrt[1 + (b_.)*(x_)^2]]^(n_.)/Sqrt[1 + (b_.)*(x_)^2], x_Symbol]
:> Simp[Sqrt[(-b)*x^2]/(b*x)   Subst[Int[ArcCos[x]^n/Sqrt[1 - x^2], x], x,
Sqrt[1 + b*x^2]], x] /; FreeQ[{b, n}, x]
```

3.118.4 Maple [F]

$$\int \frac{1}{\arccos\left(\sqrt{b\,x^{2}+1}\right)\sqrt{b\,x^{2}+1}}dx$$

input
```
int(1/arccos((b*x^2+1)^(1/2))/(b*x^2+1)^(1/2),x)
```

output
```
int(1/arccos((b*x^2+1)^(1/2))/(b*x^2+1)^(1/2),x)
```

3.118.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{1}{\sqrt{1+bx^2}\arccos\left(\sqrt{1+bx^2}\right)}\,dx = -\frac{\sqrt{-bx^2}\log\left(\arccos\left(\sqrt{bx^2+1}\right)\right)}{bx}$$

input
```
integrate(1/arccos((b*x^2+1)^(1/2))/(b*x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(-b*x^2)*log(arccos(sqrt(b*x^2 + 1)))/(b*x)
```

3.118.6 Sympy [F]

$$\int \frac{1}{\sqrt{1+bx^2}\arccos\left(\sqrt{1+bx^2}\right)}\,dx = \int \frac{1}{\sqrt{bx^2+1}\,\mathrm{acos}\left(\sqrt{bx^2+1}\right)}\,dx$$

input
```
integrate(1/acos((b*x**2+1)**(1/2))/(b*x**2+1)**(1/2),x)
```

output
```
Integral(1/(sqrt(b*x**2 + 1)*acos(sqrt(b*x**2 + 1))), x)
```

3.118.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{1+bx^2}\arccos\left(\sqrt{1+bx^2}\right)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/arccos((b*x^2+1)^(1/2))/(b*x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: sign: argument cannot be imagi
nary; found sqrt(-_SAGE_VAR_b)
```

3.118.8 Giac [F]

$$\int \frac{1}{\sqrt{1+bx^2}\arccos\left(\sqrt{1+bx^2}\right)}\,dx = \int \frac{1}{\sqrt{bx^2+1}\arccos\left(\sqrt{bx^2+1}\right)}\,dx$$

input
```
integrate(1/arccos((b*x^2+1)^(1/2))/(b*x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(b*x^2 + 1)*arccos(sqrt(b*x^2 + 1))), x)
```

3.118.9 Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{1+bx^2}\arccos\left(\sqrt{1+bx^2}\right)}\,dx = \frac{\ln\left(\operatorname{acos}\left(\sqrt{b\,x^2+1}\right)\right)\sqrt{x^2}}{\sqrt{-b}\,x}$$

input
```
int(1/(acos((b*x^2 + 1)^(1/2))*(b*x^2 + 1)^(1/2)),x)
```

output
```
(log(acos((b*x^2 + 1)^(1/2)))*(x^2)^(1/2))/((-b)^(1/2)*x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)



(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                convert(leaf_count_result,string),"$ vs. $2(",
                                convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
              #is checked before calling the grading function that is passed.
              #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
return grade, grade_annotation
```