Computer Algebra
Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

5-Inverse-trig-functions/5.3-Inverse-tangent/153-5.3.7-Inverse-tangent-functions

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [153]. This is test number [153].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (153)	0.00 (0)
Mathematica	98.69 (151)	1.31 (2)
Fricas	93.46 (143)	6.54 (10)
Maple	87.58 (134)	12.42 (19)
Maxima	56.86 (87)	43.14 (66)
Giac	38.56 (59)	61.44 (94)
Mupad	35.95 (55)	64.05 (98)
Sympy	33.33 (51)	66.67 (102)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

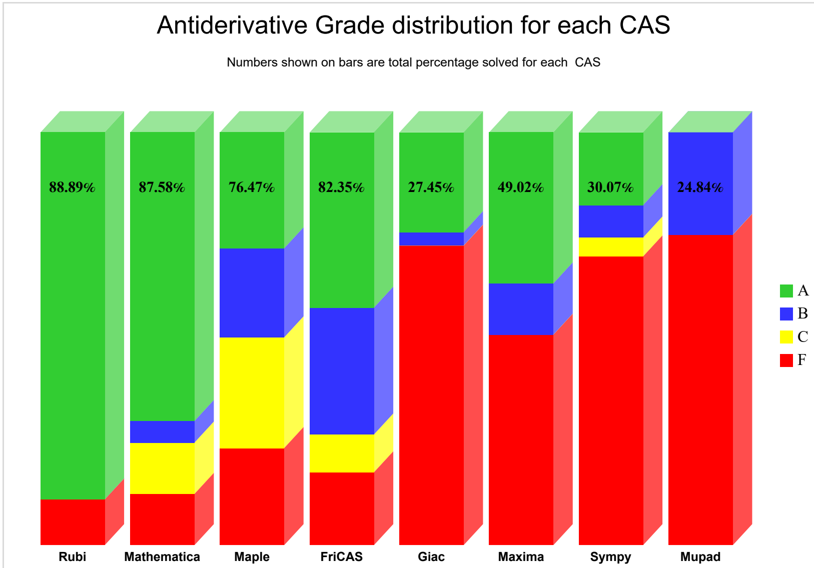
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

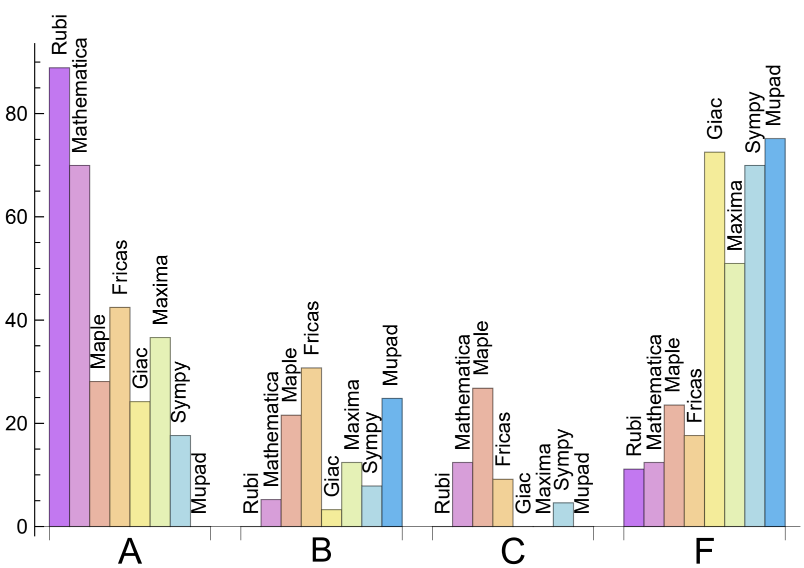
System	% A grade	% B grade	% C grade	% F grade
Rubi	88.235	0.000	0.654	11.111
Mathematica	69.935	5.229	12.418	12.418
Fricas	42.484	30.719	9.150	17.647
Maxima	36.601	12.418	0.000	50.980
Maple	28.105	21.569	26.797	23.529
Giac	24.183	3.268	0.000	72.549
Sympy	17.647	7.843	4.575	69.935
Mupad	0.000	24.837	0.000	75.163

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	2	100.00	0.00	0.00
Fricas	10	100.00	0.00	0.00
Maple	19	100.00	0.00	0.00
Maxima	66	80.30	1.52	18.18
Sympy	102	46.08	30.39	23.53
Giac	94	98.94	1.06	0.00
Mupad	98	0.00	100.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.27
Rubi	0.46
Mupad	0.78
Mathematica	1.15
Maxima	3.14
Maple	3.58
Giac	3.78
Sympy	12.80

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	44.58	1.10	25.00	1.00
Giac	56.32	1.12	34.00	0.95
Sympy	93.04	1.73	66.00	1.01
Maxima	110.82	1.49	68.00	1.00
Rubi	124.65	1.07	89.00	1.00
Mathematica	141.72	1.32	79.00	0.95
Fricas	241.43	1.71	83.00	1.29
Maple	927.45	5.37	163.00	1.86

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

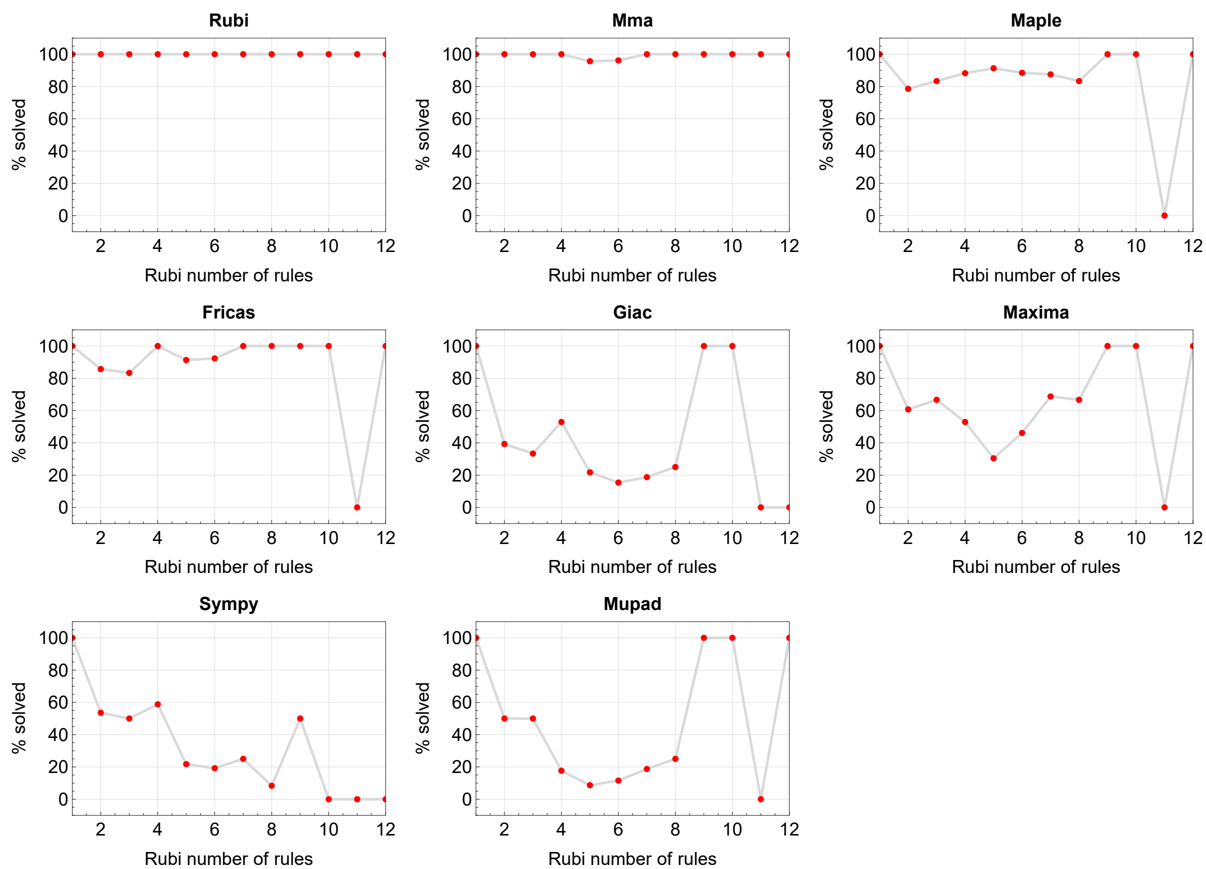


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

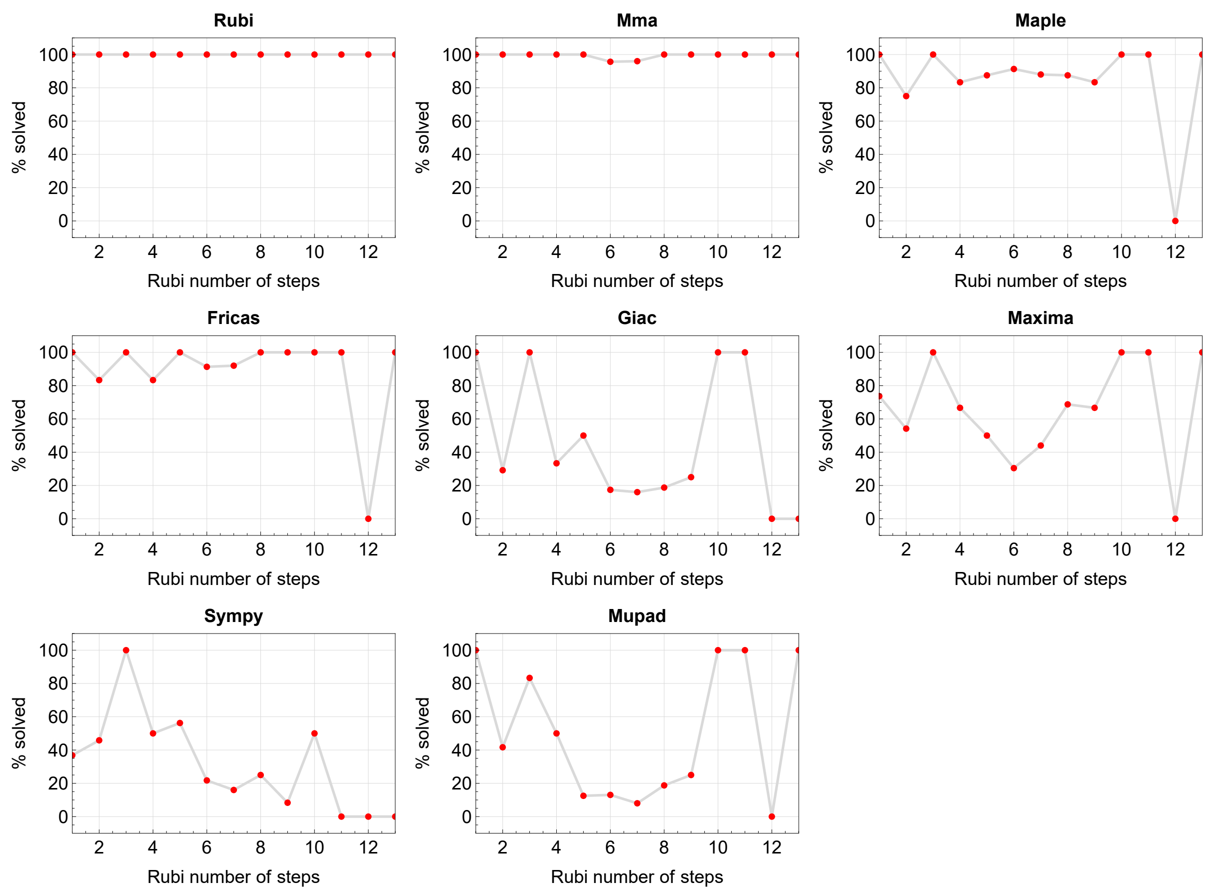


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

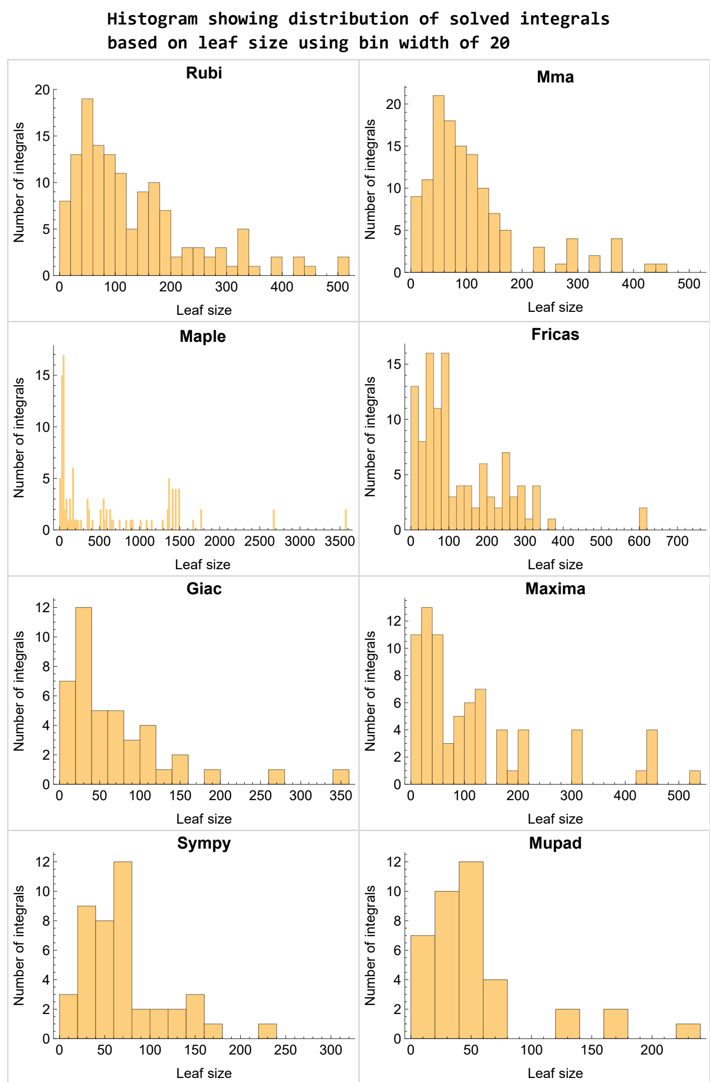


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

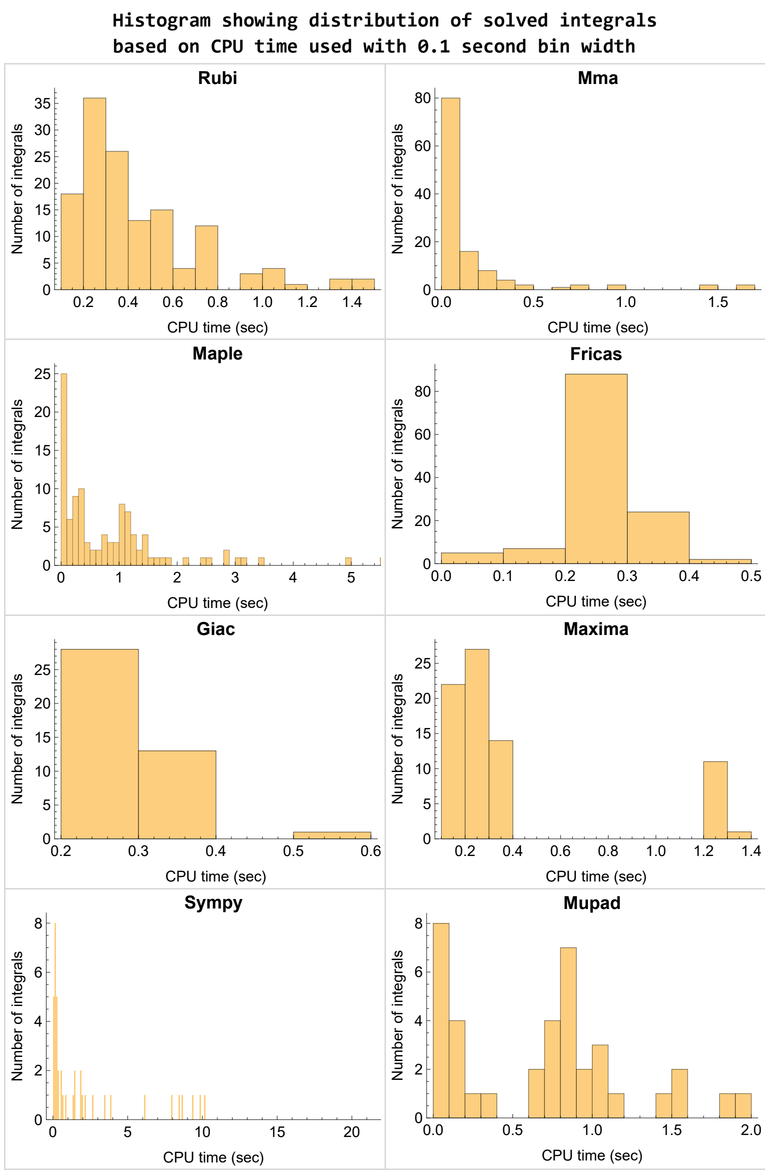


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

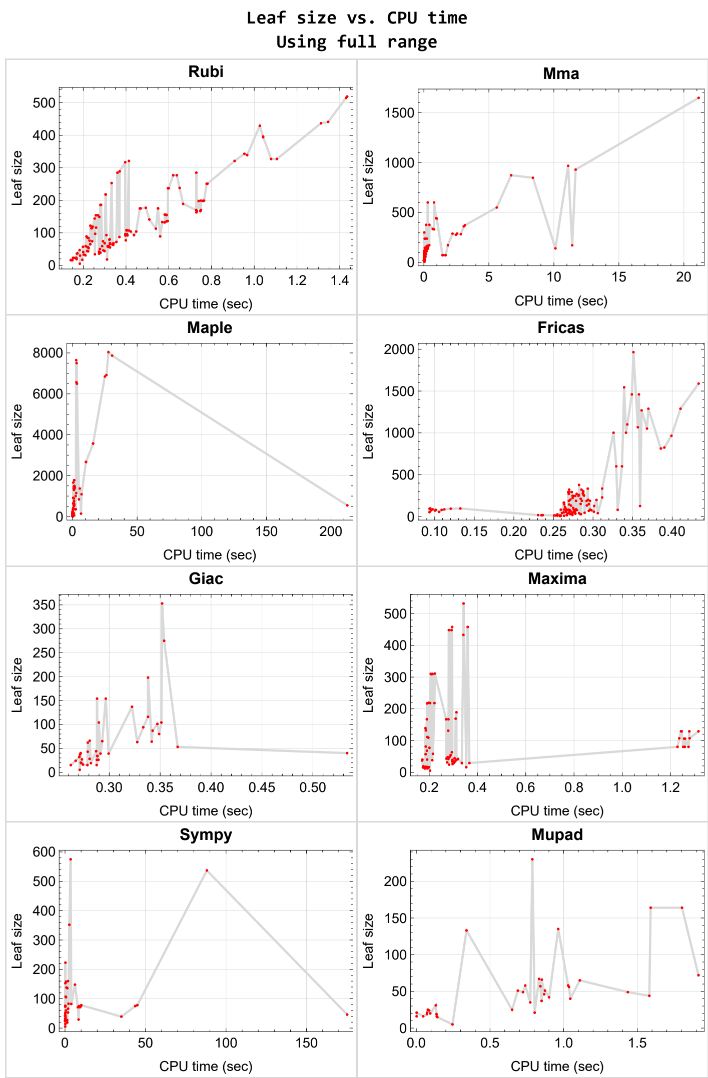


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{31, 35, 36, 51, 55, 59, 64, 68, 72, 80, 84, 88, 92, 97, 101, 105, 109\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi $\{\}$

Mathematica $\{\}$

Maple $\{\}$

Maxima $\{\}$

Fricas $\{\}$

Sympy $\{\}$

Giac $\{\}$

Mupad $\{\}$

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{1, 2\}$

Mathematica $\{50, 54, 58, 63, 67, 71\}$

Maple $\{48, 49, 52, 53, 56, 57, 61, 62, 65, 66, 69, 70, 74, 75, 76, 77, 78, 81, 82, 85, 86, 89, 90, 93, 94, 95, 98, 99, 102, 103, 106, 107, 146, 147, 148, 149, 150, 151, 152, 153\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
   r"""
   Return the tree size of this expression.
   """
   if expr not in SR:
      # deal with lists, tuples, vectors
      return 1 + sum(tree_size(a) for a in expr)
   expr = SR(expr)
   x, aa = expr.operator(), expr.operands()
   if x is None:
      return 1
   else:
      return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

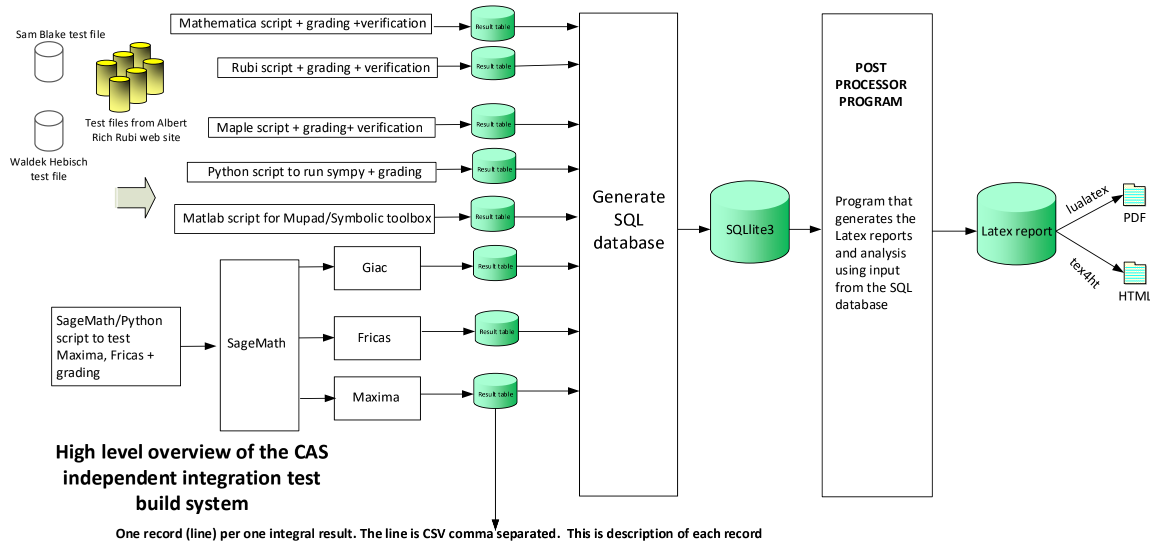
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,..}" which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

Nasser M. Abbasi
June 27, 2023
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 56, 57, 58, 60, 61, 62, 63, 65, 66, 67, 69, 70, 71, 73, 74, 75, 76, 77, 78, 79, 81, 82, 83, 85, 86, 87, 89, 90, 91, 93, 94, 95, 96, 98, 99, 100, 102, 103, 104, 106, 107, 108, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153 }

B grade { }

C grade { 6 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 30, 34, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 52, 53, 56, 57, 60, 61, 62, 65, 66, 69, 70, 73, 74, 75, 77, 78, 79, 81, 82, 83, 85, 86, 87, 89, 90, 91, 94, 95, 96, 98, 99, 100, 102, 103, 104, 106, 107, 108, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 152 }

B grade { 50, 54, 58, 63, 67, 71, 76, 93 }

C grade { 15, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 121, 148, 149, 150, 151, 153 }

F normal fail { 32, 33 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 8, 9, 10, 15, 16, 17, 30, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 60, 73, 111, 112, 116, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 132, 133, 134, 135, 136, 137, 138, 139 }

B grade { 3, 4, 5, 7, 11, 12, 13, 14, 32, 33, 34, 50, 54, 58, 63, 67, 71, 79, 83, 87, 91, 96, 100, 104, 108, 110, 113, 114, 115, 117, 118, 130, 131 }

C grade { 2, 48, 49, 52, 53, 56, 57, 61, 62, 65, 66, 69, 70, 74, 75, 76, 77, 78, 81, 82, 85, 86, 89, 90, 93, 94, 95, 98, 99, 102, 103, 106, 107, 146, 147, 148, 149, 150, 151, 152, 153 }

F normal fail { 6, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 140, 141, 142, 143, 144, 145 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 30, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 60, 65, 66, 67, 69, 70, 71, 74, 75, 111, 112, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 131, 132, 133, 137, 138, 139, 140, 143, 144, 145, 147 }

B grade { 48, 49, 50, 52, 53, 54, 56, 57, 58, 61, 62, 63, 73, 76, 77, 78, 79, 81, 82, 83, 85, 86, 87, 89, 90, 91, 93, 94, 95, 96, 98, 99, 100, 102, 103, 104, 106, 107, 108, 110, 113, 114, 134, 148, 151, 152, 153 }

C grade { 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 149, 150 }

F normal fail { 6, 32, 33, 34, 130, 135, 136, 141, 142, 146 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.5 Maxima

A grade { 1, 2, 7, 8, 9, 10, 11, 12, 13, 30, 37, 40, 41, 42, 43, 44, 45, 46, 47, 60, 85, 86, 87, 89, 90, 91, 102, 103, 104, 106, 107, 108, 116, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 138, 139, 147, 148, 149, 150, 151, 152 }

B grade { 14, 38, 39, 50, 52, 53, 54, 56, 57, 58, 63, 65, 66, 67, 69, 70, 71, 110, 113 }

C grade { }

F normal fail { 3, 4, 5, 6, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 32, 33, 34, 48, 49, 61, 62, 73, 74, 75, 76, 77, 78, 79, 81, 82, 83, 93, 94, 95, 96, 98, 99, 100, 111, 112, 114, 115, 117, 118, 135, 136, 137, 153 }

F(-1) timedout fail { 64 }

F(-2) exception fail { 55, 59, 68, 72, 134, 140, 141, 142, 143, 144, 145, 146 }

2.1.6 Giac

A grade { 1, 2, 3, 4, 5, 11, 12, 13, 14, 15, 16, 17, 30, 40, 42, 43, 44, 45, 46, 47, 60, 119, 120, 122, 123, 124, 126, 127, 128, 129, 131, 132, 133, 147, 148, 151, 152 }

B grade { 7, 8, 9, 10, 125 }

C grade { }

F normal fail { 6, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 32, 33, 34, 37, 38, 39, 41, 48, 49, 50, 52, 53, 54, 56, 57, 58, 61, 62, 63, 65, 66, 67, 69, 70, 71, 73, 74, 75, 77, 78, 79, 81, 82, 83, 85, 86, 87, 89, 90, 91, 93, 94, 95, 96, 98, 99, 100, 102, 103, 104, 106, 107, 108, 110, 111, 112, 113, 114, 115, 116, 117, 118, 121, 130, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 149, 150, 153 }

F(-1) timedout fail { 76 }

F(-2) exception fail { }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 14, 30, 38, 39, 40, 43, 44, 45, 47, 60, 110, 113, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 131, 132, 133, 134, 137, 138, 139, 147, 148, 149, 150, 151, 152 }

C grade { }

F normal fail { }

F(-1) timedout fail { 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 32, 33, 34, 37, 41, 42, 46, 48, 49, 50, 52, 53, 54, 56, 57, 58, 61, 62, 63, 65, 66, 67, 69, 70, 71, 73, 74, 75, 76, 77, 78, 79, 81, 82, 83, 85, 86, 87, 89, 90, 91, 93, 94, 95, 96, 98, 99, 100, 102, 103, 104, 106, 107, 108, 111, 112, 114, 115, 116, 117, 118, 130, 135, 136, 140, 141, 142, 143, 144, 145, 146, 153 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 1, 3, 4, 5, 7, 8, 11, 12, 13, 14, 15, 16, 17, 30, 38, 41, 43, 45, 46, 60, 119, 122, 123, 124, 127, 128, 129 }

B grade { 9, 10, 37, 39, 40, 42, 44, 47, 120, 121, 125, 131 }

C grade { 19, 20, 21, 22, 26, 27, 28 }

F normal fail { 6, 32, 33, 34, 50, 63, 73, 74, 75, 76, 77, 78, 79, 82, 83, 93, 94, 95, 96, 110, 111, 112, 113, 114, 115, 116, 117, 118, 130, 134, 135, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152 }

F(-1) timedout fail { 2, 18, 23, 24, 25, 29, 48, 49, 51, 55, 59, 61, 62, 64, 68, 72, 81, 84, 88, 92, 97, 98, 99, 100, 105, 109, 126, 132, 133, 136, 153 }

F(-2) exception fail { 52, 53, 54, 56, 57, 58, 65, 66, 67, 69, 70, 71, 85, 86, 87, 89, 90, 91, 102, 103, 104, 106, 107, 108 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	34	37	37	37	45	60	37	230
N.S.	1	0.81	0.88	0.88	0.88	1.07	1.43	0.88	5.48
time (sec)	N/A	0.274	0.015	0.286	0.171	0.289	0.562	0.271	0.787

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	36	40	140	40	58	0	40	58
N.S.	1	0.80	0.89	3.11	0.89	1.29	0.00	0.89	1.29
time (sec)	N/A	0.306	0.027	6.573	0.172	0.276	0.000	0.272	0.739

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	149	86	260	0	76	138	94	0
N.S.	1	1.03	0.60	1.81	0.00	0.53	0.96	0.65	0.00
time (sec)	N/A	0.290	0.055	0.031	0.000	0.280	0.895	0.333	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	119	74	212	0	65	107	80	0
N.S.	1	1.03	0.64	1.83	0.00	0.56	0.92	0.69	0.00
time (sec)	N/A	0.269	0.038	0.025	0.000	0.277	0.389	0.349	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	89	59	164	0	49	73	63	0
N.S.	1	1.01	0.67	1.86	0.00	0.56	0.83	0.72	0.00
time (sec)	N/A	0.242	0.028	0.023	0.000	0.275	0.126	0.327	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	C	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	288	189	171	0	0	0	0	0	0
N.S.	1	0.66	0.59	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.734	1.852	0.000	0.000	0.000	0.000	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	54	122	58	44	53	104	0
N.S.	1	1.00	0.95	2.14	1.02	0.77	0.93	1.82	0.00
time (sec)	N/A	0.215	0.028	0.023	0.216	0.274	1.872	0.351	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	84	67	69	68	58	83	198	0
N.S.	1	0.99	0.79	0.81	0.80	0.68	0.98	2.33	0.00
time (sec)	N/A	0.237	0.036	0.026	0.191	0.301	2.186	0.338	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	114	78	117	109	68	352	275	0
N.S.	1	1.01	0.69	1.04	0.96	0.60	3.12	2.43	0.00
time (sec)	N/A	0.257	0.043	0.024	0.197	0.295	2.676	0.354	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	144	89	165	132	80	575	353	0
N.S.	1	1.02	0.63	1.17	0.94	0.57	4.08	2.50	0.00
time (sec)	N/A	0.278	0.053	0.027	0.186	0.331	3.466	0.352	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	116	83	231	167	79	136	137	0
N.S.	1	0.94	0.67	1.86	1.35	0.64	1.10	1.10	0.00
time (sec)	N/A	0.286	0.076	0.038	0.190	0.270	1.434	0.322	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	97	72	183	139	68	105	101	0
N.S.	1	0.98	0.73	1.85	1.40	0.69	1.06	1.02	0.00
time (sec)	N/A	0.267	0.067	0.026	0.185	0.266	0.589	0.347	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	76	60	135	111	56	75	64	0
N.S.	1	1.03	0.81	1.82	1.50	0.76	1.01	0.86	0.00
time (sec)	N/A	0.255	0.070	0.023	0.192	0.266	0.276	0.342	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	47	43	86	77	41	41	40	35
N.S.	1	1.09	1.00	2.00	1.79	0.95	0.95	0.93	0.81
time (sec)	N/A	0.212	0.013	0.021	0.201	0.269	0.129	0.534	0.773

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	86	90	0	148	66	53	0
N.S.	1	1.00	1.46	1.53	0.00	2.51	1.12	0.90	0.00
time (sec)	N/A	0.242	0.042	0.023	0.000	0.295	1.930	0.367	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	85	101	130	0	198	82	87	0
N.S.	1	0.93	1.11	1.43	0.00	2.18	0.90	0.96	0.00
time (sec)	N/A	0.246	0.063	0.022	0.000	0.304	3.887	0.343	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	115	114	178	0	228	148	116	0
N.S.	1	0.97	0.96	1.50	0.00	1.92	1.24	0.97	0.00
time (sec)	N/A	0.261	0.083	0.027	0.000	0.312	6.188	0.338	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	211	218	170	0	0	96	0	0	0
N.S.	1	1.03	0.81	0.00	0.00	0.45	0.00	0.00	0.00
time (sec)	N/A	0.337	11.410	0.000	0.000	0.132	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	181	186	158	0	0	85	75	0	0
N.S.	1	1.03	0.87	0.00	0.00	0.47	0.41	0.00	0.00
time (sec)	N/A	0.298	0.312	0.000	0.000	0.112	43.609	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	153	154	147	0	0	71	75	0	0
N.S.	1	1.01	0.96	0.00	0.00	0.46	0.49	0.00	0.00
time (sec)	N/A	0.280	0.213	0.000	0.000	0.100	8.640	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	122	122	115	0	0	52	71	0	0
N.S.	1	1.00	0.94	0.00	0.00	0.43	0.58	0.00	0.00
time (sec)	N/A	0.250	0.093	0.000	0.000	0.093	7.972	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	156	154	150	0	0	73	78	0	0
N.S.	1	0.99	0.96	0.00	0.00	0.47	0.50	0.00	0.00
time (sec)	N/A	0.283	0.196	0.000	0.000	0.095	44.905	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	186	186	162	0	0	86	0	0	0
N.S.	1	1.00	0.87	0.00	0.00	0.46	0.00	0.00	0.00
time (sec)	N/A	0.308	0.223	0.000	0.000	0.097	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	216	218	171	0	0	97	0	0	0
N.S.	1	1.01	0.79	0.00	0.00	0.45	0.00	0.00	0.00
time (sec)	N/A	0.335	0.419	0.000	0.000	0.095	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	326	321	139	0	0	94	0	0	0
N.S.	1	0.98	0.43	0.00	0.00	0.29	0.00	0.00	0.00
time (sec)	N/A	0.432	10.120	0.000	0.000	0.120	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	296	289	119	0	0	79	75	0	0
N.S.	1	0.98	0.40	0.00	0.00	0.27	0.25	0.00	0.00
time (sec)	N/A	0.409	0.083	0.000	0.000	0.109	9.881	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	260	253	89	0	0	55	71	0	0
N.S.	1	0.97	0.34	0.00	0.00	0.21	0.27	0.00	0.00
time (sec)	N/A	0.368	0.081	0.000	0.000	0.106	9.308	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	298	285	121	0	0	80	78	0	0
N.S.	1	0.96	0.41	0.00	0.00	0.27	0.26	0.00	0.00
time (sec)	N/A	0.386	0.098	0.000	0.000	0.101	10.110	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	331	317	137	0	0	94	0	0	0
N.S.	1	0.96	0.41	0.00	0.00	0.28	0.00	0.00	0.00
time (sec)	N/A	0.419	0.086	0.000	0.000	0.094	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	43	42	47	41	43	42
N.S.	1	1.00	1.00	0.86	0.84	0.94	0.82	0.86	0.84
time (sec)	N/A	0.295	0.014	0.361	0.319	0.274	0.286	0.279	0.901

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	42	36	39	38	34	38	39
N.S.	1	1.00	1.05	0.90	0.98	0.95	0.85	0.95	0.98
time (sec)	N/A	0.250	0.135	0.415	0.963	0.292	3.748	0.387	1.017

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	B	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	431	429	0	1664	0	0	0	0	0
N.S.	1	1.00	0.00	3.86	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.142	0.000	0.580	0.000	0.000	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	B	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	283	285	0	916	0	0	0	0	0
N.S.	1	1.01	0.00	3.24	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.799	0.000	0.476	0.000	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	93	93	368	0	0	0	0	0
N.S.	1	0.95	0.95	3.76	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.329	0.029	0.324	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	42	36	39	48	61	38	39
N.S.	1	1.00	1.05	0.90	0.98	1.20	1.52	0.95	0.98
time (sec)	N/A	0.263	0.224	0.279	0.426	0.243	2.756	0.371	0.804

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	40	40	42	36	163	91	126	38	39
N.S.	1	1.00	1.05	0.90	4.08	2.28	3.15	0.95	0.98
time (sec)	N/A	0.250	1.019	0.296	0.536	0.245	6.120	0.517	1.553

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	34	41	38	33	158	0	0
N.S.	1	1.00	0.92	1.11	1.03	0.89	4.27	0.00	0.00
time (sec)	N/A	0.184	0.047	0.270	0.213	0.278	0.665	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	20	20	81	13	32	0	19
N.S.	1	1.00	0.87	0.87	3.52	0.57	1.39	0.00	0.83
time (sec)	N/A	0.162	0.014	0.114	0.186	0.253	0.102	0.000	0.137

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	20	20	57	13	76	0	19
N.S.	1	1.00	0.87	0.87	2.48	0.57	3.30	0.00	0.83
time (sec)	N/A	0.178	0.012	0.110	0.185	0.251	0.167	0.000	0.072

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	18	15	12	10	42	26	16
N.S.	1	1.00	1.12	0.94	0.75	0.62	2.62	1.62	1.00
time (sec)	N/A	0.156	0.006	0.101	0.182	0.260	0.066	0.289	0.047

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	19	23	42	8	34	0	0
N.S.	1	1.00	0.90	1.10	2.00	0.38	1.62	0.00	0.00
time (sec)	N/A	0.187	0.014	0.127	0.280	0.263	0.335	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	31	56	40	42	160	62	0
N.S.	1	1.00	0.86	1.56	1.11	1.17	4.44	1.72	0.00
time (sec)	N/A	0.187	0.035	0.317	0.191	0.261	1.857	0.279	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	20	26	17	17	26	19	25
N.S.	1	1.00	0.87	1.13	0.74	0.74	1.13	0.83	1.09
time (sec)	N/A	0.171	0.014	0.247	0.191	0.231	0.104	0.282	0.650

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	20	26	17	17	49	19	25
N.S.	1	1.00	0.87	1.13	0.74	0.74	2.13	0.83	1.09
time (sec)	N/A	0.180	0.014	0.221	0.184	0.236	0.162	0.273	0.078

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	18	22	15	15	24	15	21
N.S.	1	1.00	1.12	1.38	0.94	0.94	1.50	0.94	1.31
time (sec)	N/A	0.163	0.007	0.118	0.188	0.236	0.059	0.279	0.073

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	27	14	14	27	15	0
N.S.	1	1.00	1.00	1.42	0.74	0.74	1.42	0.79	0.00
time (sec)	N/A	0.185	0.014	0.168	0.191	0.258	1.351	0.262	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	18	15	12	10	42	26	16
N.S.	1	1.00	1.12	0.94	0.75	0.62	2.62	1.62	1.00
time (sec)	N/A	0.155	0.000	0.040	0.193	0.255	0.066	0.288	0.002

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	403	519	371	8039	0	1965	0	0	0
N.S.	1	1.29	0.92	19.95	0.00	4.88	0.00	0.00	0.00
time (sec)	N/A	1.598	3.162	27.782	0.000	0.351	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	305	396	281	7647	0	1545	0	0	0
N.S.	1	1.30	0.92	25.07	0.00	5.07	0.00	0.00	0.00
time (sec)	N/A	1.120	2.847	2.852	0.000	0.339	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	198	277	549	1001	433	1101	0	0	0
N.S.	1	1.40	2.77	5.06	2.19	5.56	0.00	0.00	0.00
time (sec)	N/A	0.675	5.607	2.408	0.342	0.343	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	17	0	17	17
N.S.	1	1.00	1.13	1.00	1.13	1.13	0.00	1.13	1.13
time (sec)	N/A	0.293	2.147	0.082	232.907	0.264	0.000	1.319	0.733

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	154	198	140	1487	310	322	0	0	0
N.S.	1	1.29	0.91	9.66	2.01	2.09	0.00	0.00	0.00
time (sec)	N/A	0.805	0.310	1.445	0.206	0.270	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	123	155	110	1452	218	271	0	0	0
N.S.	1	1.26	0.89	11.80	1.77	2.20	0.00	0.00	0.00
time (sec)	N/A	0.623	0.181	1.071	0.204	0.279	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	85	107	967	563	448	202	0	0	0
N.S.	1	1.26	11.38	6.62	5.27	2.38	0.00	0.00	0.00
time (sec)	N/A	0.454	11.097	1.112	0.281	0.271	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	0	37	0	21	22
N.S.	1	1.00	1.10	0.90	0.00	1.76	0.00	1.00	1.05
time (sec)	N/A	0.301	0.575	0.080	0.000	0.256	0.000	1.005	0.868

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	155	199	137	1488	310	322	0	0	0
N.S.	1	1.28	0.88	9.60	2.00	2.08	0.00	0.00	0.00
time (sec)	N/A	0.827	0.315	1.480	0.214	0.285	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	124	156	111	1453	218	271	0	0	0
N.S.	1	1.26	0.90	11.72	1.76	2.19	0.00	0.00	0.00
time (sec)	N/A	0.649	0.211	0.981	0.222	0.269	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	86	108	847	594	448	200	0	0	0
N.S.	1	1.26	9.85	6.91	5.21	2.33	0.00	0.00	0.00
time (sec)	N/A	0.440	8.388	1.169	0.293	0.294	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	19	0	37	0	21	22
N.S.	1	1.00	1.10	0.90	0.00	1.76	0.00	1.00	1.05
time (sec)	N/A	0.317	0.876	0.082	0.000	0.260	0.000	1.170	1.074

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	18	22	15	15	24	15	21
N.S.	1	1.00	1.12	1.38	0.94	0.94	1.50	0.94	1.31
time (sec)	N/A	0.160	0.001	0.054	0.174	0.235	0.059	0.288	0.002

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	399	515	360	7869	0	1589	0	0	0
N.S.	1	1.29	0.90	19.72	0.00	3.98	0.00	0.00	0.00
time (sec)	N/A	1.586	3.069	30.662	0.000	0.433	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	303	394	275	7501	0	1289	0	0	0
N.S.	1	1.30	0.91	24.76	0.00	4.25	0.00	0.00	0.00
time (sec)	N/A	1.110	2.469	3.151	0.000	0.411	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	198	277	1648	1145	532	965	0	0	0
N.S.	1	1.40	8.32	5.78	2.69	4.87	0.00	0.00	0.00
time (sec)	N/A	0.683	21.134	2.872	0.342	0.399	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-1)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	0	17	0	17	17
N.S.	1	1.00	1.13	1.00	0.00	1.13	0.00	1.13	1.13
time (sec)	N/A	0.286	1.983	0.138	0.000	0.257	0.000	3.784	0.785

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	154	198	136	1487	309	166	0	0	0
N.S.	1	1.29	0.88	9.66	2.01	1.08	0.00	0.00	0.00
time (sec)	N/A	0.799	0.282	1.420	0.210	0.263	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	123	155	110	1452	217	144	0	0	0
N.S.	1	1.26	0.89	11.80	1.76	1.17	0.00	0.00	0.00
time (sec)	N/A	0.638	0.195	1.101	0.191	0.264	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	A	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	85	107	929	584	458	112	0	0	0
N.S.	1	1.26	10.93	6.87	5.39	1.32	0.00	0.00	0.00
time (sec)	N/A	0.443	11.672	1.070	0.360	0.265	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	23	0	37	0	25	23
N.S.	1	1.00	1.10	1.10	0.00	1.76	0.00	1.19	1.10
time (sec)	N/A	0.292	0.539	0.125	0.000	0.260	0.000	1.798	1.062

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	155	199	140	1488	311	166	0	0	0
N.S.	1	1.28	0.90	9.60	2.01	1.07	0.00	0.00	0.00
time (sec)	N/A	0.824	0.291	1.497	0.223	0.263	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	124	156	110	1453	219	144	0	0	0
N.S.	1	1.26	0.89	11.72	1.77	1.16	0.00	0.00	0.00
time (sec)	N/A	0.633	0.191	1.098	0.197	0.270	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	B	A	F(-2)	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	86	108	872	625	458	112	0	0	0
N.S.	1	1.26	10.14	7.27	5.33	1.30	0.00	0.00	0.00
time (sec)	N/A	0.448	6.715	1.186	0.295	0.265	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	21	21	23	23	0	37	0	25	23
N.S.	1	1.00	1.10	1.10	0.00	1.76	0.00	1.19	1.10
time (sec)	N/A	0.305	0.540	0.135	0.000	0.259	0.000	2.043	1.121

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	56	52	0	58	0	0	0
N.S.	1	1.00	1.44	1.33	0.00	1.49	0.00	0.00	0.00
time (sec)	N/A	0.310	0.046	0.652	0.000	0.270	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	74	77	93	632	0	93	0	0	0
N.S.	1	1.04	1.26	8.54	0.00	1.26	0.00	0.00	0.00
time (sec)	N/A	0.445	0.042	0.561	0.000	0.265	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	108	113	121	658	0	125	0	0	0
N.S.	1	1.05	1.12	6.09	0.00	1.16	0.00	0.00	0.00
time (sec)	N/A	0.590	0.072	0.744	0.000	0.359	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	299	327	600	3570	0	1460	0	0	0
N.S.	1	1.09	2.01	11.94	0.00	4.88	0.00	0.00	0.00
time (sec)	N/A	1.150	0.785	15.921	0.000	0.349	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	229	251	375	2668	0	1002	0	0	0
N.S.	1	1.10	1.64	11.65	0.00	4.38	0.00	0.00	0.00
time (sec)	N/A	0.869	0.427	10.447	0.000	0.326	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	159	175	237	1776	0	600	0	0	0
N.S.	1	1.10	1.49	11.17	0.00	3.77	0.00	0.00	0.00
time (sec)	N/A	0.601	0.235	1.237	0.000	0.329	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	80	91	160	0	334	0	0	0
N.S.	1	1.08	1.23	2.16	0.00	4.51	0.00	0.00	0.00
time (sec)	N/A	0.356	0.024	0.825	0.000	0.312	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	17	14	3	17
N.S.	1	1.00	1.13	1.00	1.13	1.13	0.93	0.20	1.13
time (sec)	N/A	0.217	11.143	0.129	1.555	0.271	2.953	105.119	0.696

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	355	441	438	6917	0	1289	0	0	0
N.S.	1	1.24	1.23	19.48	0.00	3.63	0.00	0.00	0.00
time (sec)	N/A	1.441	0.990	26.221	0.000	0.370	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	267	343	330	6567	0	1067	0	0	0
N.S.	1	1.28	1.24	24.60	0.00	4.00	0.00	0.00	0.00
time (sec)	N/A	1.046	0.783	3.002	0.000	0.357	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	237	288	352	0	825	0	0	0
N.S.	1	1.36	1.66	2.02	0.00	4.74	0.00	0.00	0.00
time (sec)	N/A	0.637	2.551	2.177	0.000	0.390	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	17	0	17	17
N.S.	1	1.00	1.13	1.00	1.13	1.13	0.00	1.13	1.13
time (sec)	N/A	0.289	6.491	0.108	1.060	0.264	0.000	0.495	0.777

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	142	167	134	1406	129	293	0	0	0
N.S.	1	1.18	0.94	9.90	0.91	2.06	0.00	0.00	0.00
time (sec)	N/A	0.810	0.168	1.599	1.316	0.273	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	113	132	103	1370	106	247	0	0	0
N.S.	1	1.17	0.91	12.12	0.94	2.19	0.00	0.00	0.00
time (sec)	N/A	0.635	0.073	0.908	1.257	0.274	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	92	71	545	80	187	0	0	0
N.S.	1	1.16	0.90	6.90	1.01	2.37	0.00	0.00	0.00
time (sec)	N/A	0.453	1.656	1.027	1.229	0.296	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	19	21	17	72	37	0	19	20
N.S.	1	1.00	1.11	0.89	3.79	1.95	0.00	1.00	1.05
time (sec)	N/A	0.283	3.373	0.146	0.590	0.264	0.000	0.337	1.215

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	145	170	134	1409	129	293	0	0	0
N.S.	1	1.17	0.92	9.72	0.89	2.02	0.00	0.00	0.00
time (sec)	N/A	0.817	0.163	1.619	1.278	0.270	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	116	136	103	1373	107	247	0	0	0
N.S.	1	1.17	0.89	11.84	0.92	2.13	0.00	0.00	0.00
time (sec)	N/A	0.635	0.089	0.968	1.279	0.282	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	96	71	516	80	187	0	0	0
N.S.	1	1.17	0.87	6.29	0.98	2.28	0.00	0.00	0.00
time (sec)	N/A	0.451	1.679	1.015	1.274	0.269	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	77	37	0	19	20
N.S.	1	1.00	1.09	0.91	3.50	1.68	0.00	0.86	0.91
time (sec)	N/A	0.293	3.306	0.141	0.614	0.254	0.000	0.341	1.093

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	299	327	600	3570	0	1460	0	0	0
N.S.	1	1.09	2.01	11.94	0.00	4.88	0.00	0.00	0.00
time (sec)	N/A	1.222	0.299	15.857	0.000	0.358	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	229	251	375	2668	0	1002	0	0	0
N.S.	1	1.10	1.64	11.65	0.00	4.38	0.00	0.00	0.00
time (sec)	N/A	0.834	0.176	10.369	0.000	0.342	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	159	175	237	1777	0	600	0	0	0
N.S.	1	1.10	1.49	11.18	0.00	3.77	0.00	0.00	0.00
time (sec)	N/A	0.588	0.116	1.340	0.000	0.337	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	79	91	160	0	334	0	0	0
N.S.	1	1.08	1.25	2.19	0.00	4.58	0.00	0.00	0.00
time (sec)	N/A	0.352	0.021	0.837	0.000	0.293	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	17	0	3	17
N.S.	1	1.00	1.13	1.00	1.13	1.13	0.00	0.20	1.13
time (sec)	N/A	0.216	3.179	0.135	1.554	0.251	0.000	90.476	0.713

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	351	437	442	6845	0	1269	0	0	0
N.S.	1	1.25	1.26	19.50	0.00	3.62	0.00	0.00	0.00
time (sec)	N/A	1.436	0.940	24.977	0.000	0.361	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	265	339	334	6495	0	1051	0	0	0
N.S.	1	1.28	1.26	24.51	0.00	3.97	0.00	0.00	0.00
time (sec)	N/A	1.053	0.682	3.421	0.000	0.368	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	237	287	352	0	813	0	0	0
N.S.	1	1.36	1.65	2.02	0.00	4.67	0.00	0.00	0.00
time (sec)	N/A	0.645	2.203	2.573	0.000	0.386	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	15	15	17	15	17	17	14	17	17
N.S.	1	1.00	1.13	1.00	1.13	1.13	0.93	1.13	1.13
time (sec)	N/A	0.282	6.785	0.117	1.032	0.266	161.550	0.546	0.788

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	142	167	134	1405	129	293	0	0	0
N.S.	1	1.18	0.94	9.89	0.91	2.06	0.00	0.00	0.00
time (sec)	N/A	0.791	0.163	1.720	1.243	0.269	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	113	133	103	1369	106	247	0	0	0
N.S.	1	1.18	0.91	12.12	0.94	2.19	0.00	0.00	0.00
time (sec)	N/A	0.615	0.075	1.013	1.260	0.278	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	92	71	545	80	187	0	0	0
N.S.	1	1.16	0.90	6.90	1.01	2.37	0.00	0.00	0.00
time (sec)	N/A	0.422	1.476	1.153	1.252	0.286	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	19	19	21	17	77	37	0	19	20
N.S.	1	1.00	1.11	0.89	4.05	1.95	0.00	1.00	1.05
time (sec)	N/A	0.286	3.430	0.190	0.597	0.267	0.000	0.366	1.082

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	145	170	134	1410	129	293	0	0	0
N.S.	1	1.17	0.92	9.72	0.89	2.02	0.00	0.00	0.00
time (sec)	N/A	0.790	0.163	1.826	1.246	0.288	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	116	135	103	1374	107	247	0	0	0
N.S.	1	1.16	0.89	11.84	0.92	2.13	0.00	0.00	0.00
time (sec)	N/A	0.633	0.100	1.035	1.236	0.275	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	96	71	516	80	187	0	0	0
N.S.	1	1.17	0.87	6.29	0.98	2.28	0.00	0.00	0.00
time (sec)	N/A	0.437	1.430	1.144	1.259	0.292	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	20	72	37	0	19	20
N.S.	1	1.00	1.09	0.91	3.27	1.68	0.00	0.86	0.91
time (sec)	N/A	0.289	3.445	0.167	0.641	0.257	0.000	0.358	1.119

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	59	53	34	40	0	0	21
N.S.	1	1.00	1.90	1.71	1.10	1.29	0.00	0.00	0.68
time (sec)	N/A	0.230	0.018	0.307	0.303	0.277	0.000	0.000	0.803

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	57	50	44	0	65	0	0	0
N.S.	1	0.90	0.79	0.70	0.00	1.03	0.00	0.00	0.00
time (sec)	N/A	0.333	0.012	0.431	0.000	0.272	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	93	80	70	0	87	0	0	0
N.S.	1	1.02	0.88	0.77	0.00	0.96	0.00	0.00	0.00
time (sec)	N/A	0.467	0.013	0.644	0.000	0.279	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	43	83	95	63	103	0	0	37
N.S.	1	0.96	1.84	2.11	1.40	2.29	0.00	0.00	0.82
time (sec)	N/A	0.237	0.167	0.245	0.294	0.275	0.000	0.000	0.850

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	87	71	349	0	151	0	0	0
N.S.	1	0.96	0.78	3.84	0.00	1.66	0.00	0.00	0.00
time (sec)	N/A	0.384	0.013	0.378	0.000	0.284	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	141	115	407	0	187	0	0	0
N.S.	1	1.06	0.86	3.06	0.00	1.41	0.00	0.00	0.00
time (sec)	N/A	0.550	0.020	0.417	0.000	0.279	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	177	167	162	189	212	0	0	0
N.S.	1	0.90	0.85	0.83	0.96	1.08	0.00	0.00	0.00
time (sec)	N/A	0.536	0.310	0.740	0.313	0.284	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	238	236	672	0	304	0	0	0
N.S.	1	1.03	1.02	2.90	0.00	1.31	0.00	0.00	0.00
time (sec)	N/A	0.695	0.063	0.880	0.000	0.288	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	302	321	299	758	0	378	0	0	0
N.S.	1	1.06	0.99	2.51	0.00	1.25	0.00	0.00	0.00
time (sec)	N/A	0.969	0.033	1.247	0.000	0.282	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	33	25	23	19	28	19	20	20
N.S.	1	1.32	1.00	0.92	0.76	1.12	0.76	0.80	0.80
time (sec)	N/A	0.221	0.013	0.085	0.176	0.271	1.417	0.272	0.094

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	53	35	32	31	50	153	32	31
N.S.	1	1.18	0.78	0.71	0.69	1.11	3.40	0.71	0.69
time (sec)	N/A	0.236	0.025	0.267	0.274	0.263	0.220	0.271	0.132

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	74	81	54	54	77	223	0	46
N.S.	1	1.16	1.27	0.84	0.84	1.20	3.48	0.00	0.72
time (sec)	N/A	0.320	0.036	0.296	0.289	0.269	0.254	0.000	0.868

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	22	25	24	18	26	24	24
N.S.	1	1.00	0.73	0.83	0.80	0.60	0.87	0.80	0.80
time (sec)	N/A	0.194	0.019	0.023	0.282	0.274	0.089	0.267	0.087

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	5	5	5	5	5
N.S.	1	1.00	1.00	1.20	1.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.187	0.046	0.246	0.203	0.258	0.110	0.271	0.245

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	16	15	14	16	15
N.S.	1	1.00	1.00	0.82	0.94	0.88	0.82	0.94	0.88
time (sec)	N/A	0.210	0.073	0.247	0.199	0.267	0.194	0.274	0.143

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	16	26	53	104	16
N.S.	1	1.00	1.00	0.94	0.89	1.44	2.94	5.78	0.89
time (sec)	N/A	0.344	0.024	0.322	0.354	0.255	0.257	0.290	0.139

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	74	58	45	44	40	0	44	72
N.S.	1	1.09	0.85	0.66	0.65	0.59	0.00	0.65	1.06
time (sec)	N/A	0.263	0.042	0.032	0.305	0.306	0.000	0.287	1.915

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	65	53	40	39	35	46	39	65
N.S.	1	1.10	0.90	0.68	0.66	0.59	0.78	0.66	1.10
time (sec)	N/A	0.250	0.023	0.034	0.313	0.268	175.227	0.291	1.110

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	56	48	35	34	28	39	34	58
N.S.	1	1.12	0.96	0.70	0.68	0.56	0.78	0.68	1.16
time (sec)	N/A	0.237	0.022	0.034	0.308	0.285	34.992	0.288	1.030

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	41	31	28	26	22	29	27	40
N.S.	1	1.11	0.84	0.76	0.70	0.59	0.78	0.73	1.08
time (sec)	N/A	0.215	0.073	0.030	0.297	0.267	8.412	0.273	1.045

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	84	374	43	0	0	0	0
N.S.	1	1.00	2.00	8.90	1.02	0.00	0.00	0.00	0.00
time (sec)	N/A	0.295	0.140	0.765	0.271	0.000	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	44	40	57	29	28	537	28	44
N.S.	1	1.07	0.98	1.39	0.71	0.68	13.10	0.68	1.07
time (sec)	N/A	0.234	0.022	0.033	0.302	0.284	88.100	0.281	1.582

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	56	48	35	34	35	0	34	49
N.S.	1	1.12	0.96	0.70	0.68	0.70	0.00	0.68	0.98
time (sec)	N/A	0.236	0.023	0.045	0.295	0.273	0.000	0.288	1.435

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	65	51	40	39	40	0	39	56
N.S.	1	1.10	0.86	0.68	0.66	0.68	0.00	0.66	0.95
time (sec)	N/A	0.251	0.029	0.052	0.296	0.282	0.000	0.299	1.037

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	59	0	126	0	0	57
N.S.	1	1.00	1.00	0.94	0.00	2.00	0.00	0.00	0.90
time (sec)	N/A	0.372	0.044	0.758	0.000	0.302	0.000	0.000	0.842

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	59	57	0	0	0	0	0
N.S.	1	1.00	1.00	0.97	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.364	0.022	0.372	0.000	0.000	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	59	57	0	0	0	0	0
N.S.	1	1.00	1.00	0.97	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.322	0.016	0.382	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	55	0	83	0	0	49
N.S.	1	1.00	1.00	1.00	0.00	1.51	0.00	0.00	0.89
time (sec)	N/A	0.351	0.121	1.294	0.000	0.296	0.000	0.000	0.724

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	57	57	29	82	0	0	51
N.S.	1	1.00	1.00	1.00	0.51	1.44	0.00	0.00	0.89
time (sec)	N/A	0.342	0.019	0.356	0.335	0.259	0.000	0.000	0.689

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	59	57	29	82	0	0	51
N.S.	1	1.00	1.00	0.97	0.49	1.39	0.00	0.00	0.86
time (sec)	N/A	0.342	0.018	0.352	0.366	0.263	0.000	0.000	0.873

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	72	72	66	0	0	132	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	1.83	0.00	0.00	0.00
time (sec)	N/A	0.378	0.106	0.000	0.000	0.292	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	68	68	62	0	0	0	0	0	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.365	0.058	0.000	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	68	68	62	0	0	0	0	0	0
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.325	0.029	0.000	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	64	64	58	0	0	83	0	0	0
N.S.	1	1.00	0.91	0.00	0.00	1.30	0.00	0.00	0.00
time (sec)	N/A	0.364	0.051	0.000	0.000	0.276	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	66	66	60	0	0	82	0	0	0
N.S.	1	1.00	0.91	0.00	0.00	1.24	0.00	0.00	0.00
time (sec)	N/A	0.354	0.050	0.000	0.000	0.269	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	68	68	62	0	0	83	0	0	0
N.S.	1	1.00	0.91	0.00	0.00	1.22	0.00	0.00	0.00
time (sec)	N/A	0.353	0.044	0.000	0.000	0.266	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	101	89	79	1087	0	0	0	0	0
N.S.	1	0.88	0.78	10.76	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.608	0.105	6.954	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	48	46	61	1299	48	75	0	65	66
N.S.	1	0.96	1.27	27.06	1.00	1.56	0.00	1.35	1.38
time (sec)	N/A	0.314	0.057	1.111	0.284	0.298	0.000	0.293	0.852

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	B	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	103	104	146	1371	131	221	0	154	133
N.S.	1	1.01	1.42	13.31	1.27	2.15	0.00	1.50	1.29
time (sec)	N/A	0.489	0.083	5.592	0.279	0.274	0.000	0.297	0.341

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	C	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	180	175	89	1355	167	258	0	0	164
N.S.	1	0.97	0.49	7.53	0.93	1.43	0.00	0.00	0.91
time (sec)	N/A	0.507	0.066	1.384	0.270	0.282	0.000	0.000	1.591

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	C	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	180	175	89	1355	167	258	0	0	164
N.S.	1	0.97	0.49	7.53	0.93	1.43	0.00	0.00	0.91
time (sec)	N/A	0.505	0.064	1.226	0.280	0.288	0.000	0.000	1.803

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	A	B	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	103	104	145	838	169	276	0	154	135
N.S.	1	1.01	1.41	8.14	1.64	2.68	0.00	1.50	1.31
time (sec)	N/A	0.458	0.079	4.969	0.310	0.287	0.000	0.288	0.963

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	A	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	47	44	57	885	47	131	0	66	67
N.S.	1	0.94	1.21	18.83	1.00	2.79	0.00	1.40	1.43
time (sec)	N/A	0.330	0.051	1.017	0.276	0.272	0.000	0.281	0.833

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	163	163	116	547	0	250	0	0	0
N.S.	1	1.00	0.71	3.36	0.00	1.53	0.00	0.00	0.00
time (sec)	N/A	0.798	0.202	212.623	0.000	0.289	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [73] had the largest ratio of [1.66667000000000010]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	4	0.81	12	0.333
2	A	5	4	0.80	14	0.286
3	A	7	6	1.03	25	0.240
4	A	6	5	1.03	25	0.200
5	A	5	4	1.01	23	0.174
6	C	12	11	0.66	25	0.440
7	A	2	2	1.00	25	0.080
8	A	3	3	0.99	25	0.120
9	A	4	4	1.01	25	0.160
10	A	5	5	1.02	25	0.200
11	A	5	4	0.94	25	0.160
12	A	5	4	0.98	25	0.160
13	A	5	4	1.03	25	0.160
14	A	2	2	1.09	21	0.095
15	A	5	4	1.00	25	0.160
16	A	6	5	0.93	25	0.200
17	A	7	6	0.97	25	0.240
18	A	7	6	1.03	27	0.222
19	A	6	5	1.03	27	0.185
20	A	5	4	1.01	27	0.148
21	A	4	3	1.00	27	0.111
22	A	5	4	0.99	27	0.148

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	6	5	1.00	27	0.185
24	A	7	6	1.01	27	0.222
25	A	9	8	0.98	27	0.296
26	A	8	7	0.98	27	0.259
27	A	7	6	0.97	27	0.222
28	A	8	7	0.96	27	0.259
29	A	9	8	0.96	27	0.296
30	A	3	3	1.00	11	0.273
31	N/A	1	0	1.00	40	0.000
32	A	7	6	1.00	40	0.150
33	A	6	5	1.01	40	0.125
34	A	4	3	0.95	38	0.079
35	N/A	1	0	1.00	40	0.000
36	N/A	1	0	1.00	40	0.000
37	A	2	2	1.00	11	0.182
38	A	2	2	1.00	11	0.182
39	A	2	2	1.00	9	0.222
40	A	3	2	1.00	7	0.286
41	A	2	2	1.00	11	0.182
42	A	2	2	1.00	11	0.182
43	A	2	2	1.00	11	0.182
44	A	2	2	1.00	9	0.222
45	A	3	2	1.00	7	0.286
46	A	2	2	1.00	11	0.182
47	A	3	2	1.00	7	0.286
48	A	7	6	1.29	15	0.400
49	A	6	5	1.30	13	0.385
50	A	5	4	1.40	11	0.364
51	N/A	1	0	1.00	15	0.000
52	A	8	7	1.29	21	0.333
53	A	7	6	1.26	19	0.316
54	A	6	5	1.26	17	0.294
55	N/A	1	0	1.00	21	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
56	A	9	8	1.28	21	0.381
57	A	8	7	1.26	19	0.368
58	A	7	6	1.26	17	0.353
59	N/A	1	0	1.00	21	0.000
60	A	3	2	1.00	7	0.286
61	A	7	6	1.29	15	0.400
62	A	6	5	1.30	13	0.385
63	A	5	4	1.40	11	0.364
64	N/A	1	0	1.00	15	0.000
65	A	9	8	1.29	21	0.381
66	A	8	7	1.26	19	0.368
67	A	7	6	1.26	17	0.353
68	N/A	1	0	1.00	21	0.000
69	A	8	7	1.28	21	0.333
70	A	7	6	1.26	19	0.316
71	A	6	5	1.26	17	0.294
72	N/A	1	0	1.00	21	0.000
73	A	6	5	1.00	3	1.667
74	A	7	6	1.04	5	1.200
75	A	8	7	1.05	7	1.000
76	A	9	8	1.09	15	0.533
77	A	8	7	1.10	15	0.467
78	A	7	6	1.10	13	0.462
79	A	6	5	1.08	7	0.714
80	N/A	1	0	1.00	15	0.000
81	A	7	6	1.24	15	0.400
82	A	6	5	1.28	13	0.385
83	A	5	4	1.36	11	0.364
84	N/A	1	0	1.00	15	0.000
85	A	9	8	1.18	19	0.421
86	A	8	7	1.17	17	0.412
87	A	7	6	1.16	15	0.400
88	N/A	1	0	1.00	19	0.000

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
89	A	8	7	1.17	22	0.318
90	A	7	6	1.17	20	0.300
91	A	6	5	1.17	18	0.278
92	N/A	1	0	1.00	22	0.000
93	A	9	8	1.09	15	0.533
94	A	8	7	1.10	15	0.467
95	A	7	6	1.10	13	0.462
96	A	6	5	1.08	7	0.714
97	N/A	1	0	1.00	15	0.000
98	A	7	6	1.25	15	0.400
99	A	6	5	1.28	13	0.385
100	A	5	4	1.36	11	0.364
101	N/A	1	0	1.00	15	0.000
102	A	9	8	1.18	19	0.421
103	A	8	7	1.18	17	0.412
104	A	7	6	1.16	15	0.400
105	N/A	1	0	1.00	19	0.000
106	A	8	7	1.17	22	0.318
107	A	7	6	1.16	20	0.300
108	A	6	5	1.17	18	0.278
109	N/A	1	0	1.00	22	0.000
110	A	4	3	1.00	4	0.750
111	A	5	4	0.90	6	0.667
112	A	6	5	1.02	8	0.625
113	A	4	3	0.96	8	0.375
114	A	5	4	0.96	10	0.400
115	A	6	5	1.06	12	0.417
116	A	9	8	0.90	12	0.667
117	A	6	5	1.03	14	0.357
118	A	7	6	1.06	16	0.375
119	A	7	6	1.32	10	0.600
120	A	4	4	1.18	8	0.500
121	A	6	6	1.16	14	0.429

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
122	A	6	5	1.00	8	0.625
123	A	1	1	1.00	14	0.071
124	A	1	1	1.00	19	0.053
125	A	2	2	1.00	26	0.077
126	A	11	10	1.09	21	0.476
127	A	10	9	1.10	21	0.429
128	A	9	8	1.12	19	0.421
129	A	8	7	1.11	18	0.389
130	A	7	6	1.00	21	0.286
131	A	8	7	1.07	21	0.333
132	A	9	8	1.12	21	0.381
133	A	10	9	1.10	21	0.429
134	A	2	2	1.00	39	0.051
135	A	2	2	1.00	39	0.051
136	A	2	2	1.00	37	0.054
137	A	2	2	1.00	39	0.051
138	A	2	2	1.00	39	0.051
139	A	2	2	1.00	39	0.051
140	A	2	2	1.00	40	0.050
141	A	2	2	1.00	40	0.050
142	A	2	2	1.00	38	0.053
143	A	2	2	1.00	40	0.050
144	A	2	2	1.00	40	0.050
145	A	2	2	1.00	40	0.050
146	A	6	5	0.88	24	0.208
147	A	6	5	0.96	20	0.250
148	A	8	7	1.01	20	0.350
149	A	13	12	0.97	20	0.600
150	A	13	12	0.97	20	0.600
151	A	9	8	1.01	20	0.400
152	A	7	6	0.94	20	0.300
153	A	2	2	1.00	24	0.083

2.3. Detailed conclusion table specific for Rubi results

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int x^3 \arctan(a + bx^4) \, dx$

3.1.1 Optimal result

Integrand size = 12, antiderivative size = 42

$$\int x^3 \arctan\left(a + bx^4\right) \, dx = \frac{(a + bx^4) \arctan(a + bx^4)}{4b} - \frac{\log\left(1 + (a + bx^4)^2\right)}{8b}$$

output
```
1/4*(b*x^4+a)*arctan(b*x^4+a)/b-1/8*ln(1+(b*x^4+a)^2)/b
```

3.1.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int x^3 \arctan\left(a + bx^4\right) \, dx = -\frac{-2(a + bx^4) \arctan(a + bx^4) + \log\left(1 + (a + bx^4)^2\right)}{8b}$$

input
```
Integrate[x^3*ArcTan[a + b*x^4],x]
```

output
```
-1/8*(-2*(a + b*x^4)*ArcTan[a + b*x^4] + Log[1 + (a + b*x^4)^2])/b
```

3.1.3 Rubi [A] (warning: unable to verify)

Time = 0.27 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.81, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {7266, 5562, 5345, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \arctan\left(a + bx^4\right) \, dx$$

$$\Big\downarrow \ 7266$$

$$\frac{1}{4} \int \arctan\left(bx^4 + a\right) dx^4$$

$$\Big\downarrow \ 5562$$

$$\frac{\int \arctan\left(bx^4 + a\right) d\left(bx^4 + a\right)}{4b}$$

$$\Big\downarrow \ 5345$$

$$\frac{\left(a + bx^4\right) \arctan\left(a + bx^4\right) - \int \frac{bx^4+a}{x^8+1} d\left(bx^4 + a\right)}{4b}$$

$$\Big\downarrow \ 240$$

$$\frac{\left(a + bx^4\right) \arctan\left(a + bx^4\right) - \frac{1}{2} \log\left(x^8 + 1\right)}{4b}$$

input
```
Int[x^3*ArcTan[a + b*x^4],x]
```

output
```
((a + b*x^4)*ArcTan[a + b*x^4] - Log[1 + x^8]/2)/(4*b)
```

3.1.3.1 Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5562
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[1/d
  Subst[Int[(a + b*ArcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d},
 x] && IGtQ[p, 0]
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

3.1.4 Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{(b\,x^4+a)\arctan(b\,x^4+a)-\frac{\ln\left(1+\left(b\,x^4+a\right)^2\right)}{2}}{4b}$
default	$\frac{(b\,x^4+a)\arctan(b\,x^4+a)-\frac{\ln\left(1+\left(b\,x^4+a\right)^2\right)}{2}}{4b}$
parallelrisch	$-\frac{-2\arctan(b\,x^4+a)x^4b^2-2a\arctan(b\,x^4+a)b+\ln(b^2x^8+2ab\,x^4+a^2+1)b}{8b^2}$
parts	$\frac{x^4\arctan(b\,x^4+a)}{4}-b\left(\frac{\ln(b^2x^8+2ab\,x^4+a^2+1)}{8b^2}-\frac{a\arctan\left(\frac{2b^2x^4+2ab}{2b}\right)}{4b^2}\right)$
risch	$-\frac{ix^4\ln(1+i(b\,x^4+a))}{8}+\frac{ix^4\ln(1-i(b\,x^4+a))}{8}+\frac{a\arctan\left(\frac{x^4b}{a^2+1}+\frac{a^2b\,x^4}{a^2+1}+\frac{a^3}{a^2+1}+\frac{a}{a^2+1}\right)}{4b}-\frac{a\arctan(a)}{4b}-\frac{\ln(a^6$

input
```
int(x^3*arctan(b*x^4+a),x,method=_RETURNVERBOSE)
```

output
```
1/4/b*((b*x^4+a)*arctan(b*x^4+a)-1/2*ln(1+(b*x^4+a)^2))
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07

$$\int x^3\arctan\left(a+bx^4\right)\,dx=\frac{2\left(bx^4+a\right)\arctan\left(bx^4+a\right)-\log\left(b^2x^8+2\,abx^4+a^2+1\right)}{8\,b}$$

input
```
integrate(x^3*arctan(b*x^4+a),x, algorithm="fricas")
```

output
```
1/8*(2*(b*x^4 + a)*arctan(b*x^4 + a) - log(b^2*x^8 + 2*a*b*x^4 + a^2 + 1))
/b
```

3.1.6 Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.43

$$\int x^3 \arctan\left(a+bx^4\right)\,dx = \begin{cases} \frac{a\,\mathrm{atan}\,(a+bx^4)}{4b} + \frac{x^4\,\mathrm{atan}\,(a+bx^4)}{4} - \frac{\log\left(a^2+2abx^4+b^2x^8+1\right)}{8b} & \text{for } b \neq 0 \\ \frac{x^4\,\mathrm{atan}\,(a)}{4} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*atan(b*x**4+a),x)
```

output
```
Piecewise((a*atan(a + b*x**4)/(4*b) + x**4*atan(a + b*x**4)/4 - log(a**2 +
 2*a*b*x**4 + b**2*x**8 + 1)/(8*b), Ne(b, 0)), (x**4*atan(a)/4, True))
```

3.1.7 Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int x^3 \arctan\left(a+bx^4\right)\,dx = \frac{2\,(bx^4+a)\arctan\left(bx^4+a\right) - \log\left(\left(bx^4+a\right)^2+1\right)}{8\,b}$$

input
```
integrate(x^3*arctan(b*x^4+a),x, algorithm="maxima")
```

output
```
1/8*(2*(b*x^4 + a)*arctan(b*x^4 + a) - log((b*x^4 + a)^2 + 1))/b
```

3.1.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int x^3 \arctan\left(a+bx^4\right)\,dx = \frac{2\,(bx^4+a)\arctan\left(bx^4+a\right) - \log\left(\left(bx^4+a\right)^2+1\right)}{8\,b}$$

input
```
integrate(x^3*arctan(b*x^4+a),x, algorithm="giac")
```

output
```
1/8*(2*(b*x^4 + a)*arctan(b*x^4 + a) - log((b*x^4 + a)^2 + 1))/b
```

3.1.9 Mupad [B] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 230, normalized size of antiderivative = 5.48

$$\int x^3 \arctan\left(a+bx^4\right)\,dx = \frac{x^4\,\mathrm{atan}(b\,x^4+a)}{4} - \frac{\ln\left(a^2+2\,a\,b\,x^4+b^2\,x^8+1\right)}{8\,b}$$
$$+\frac{a\,\mathrm{atan}\left(\frac{a}{a^6+3\,a^4+3\,a^2+1}+\frac{3\,a^3}{a^6+3\,a^4+3\,a^2+1}+\frac{3\,a^5}{a^6+3\,a^4+3\,a^2+1}+\frac{a^7}{a^6+3\,a^4+3\,a^2+1}+\frac{b\,x^4}{a^6+3\,a^4+3\,a^2+1}+\frac{3\,a^2\,b\,x^4}{a^6+3\,a^4+3\,a^2+1}+\cdots\right.}{4\,b}$$

input
```
int(x^3*atan(a + b*x^4),x)
```

output
```
(x^4*atan(a + b*x^4))/4 - log(a^2 + b^2*x^8 + 2*a*b*x^4 + 1)/(8*b) + (a*at
an(a/(3*a^2 + 3*a^4 + a^6 + 1) + (3*a^3)/(3*a^2 + 3*a^4 + a^6 + 1) + (3*a^
5)/(3*a^2 + 3*a^4 + a^6 + 1) + a^7/(3*a^2 + 3*a^4 + a^6 + 1) + (b*x^4)/(3*
a^2 + 3*a^4 + a^6 + 1) + (3*a^2*b*x^4)/(3*a^2 + 3*a^4 + a^6 + 1) + (3*a^4*
b*x^4)/(3*a^2 + 3*a^4 + a^6 + 1) + (a^6*b*x^4)/(3*a^2 + 3*a^4 + a^6 + 1)))
/(4*b)
```

3.2 $\int x^{-1+n} \arctan(a+bx^n)\, dx$

3.2.1 Optimal result

Integrand size = 14, antiderivative size = 45

$$\int x^{-1+n} \arctan(a+bx^n)\, dx = \frac{(a+bx^n)\arctan(a+bx^n)}{bn} - \frac{\log\left(1+(a+bx^n)^2\right)}{2bn}$$

output
```
(a+b*x^n)*arctan(a+b*x^n)/b/n-1/2*ln(1+(a+b*x^n)^2)/b/n
```

3.2.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.89

$$\int x^{-1+n} \arctan(a+bx^n)\, dx = -\frac{-2(a+bx^n)\arctan(a+bx^n)+\log\left(1+(a+bx^n)^2\right)}{2bn}$$

input
```
Integrate[x^(-1 + n)*ArcTan[a + b*x^n],x]
```

output
```
-1/2*(-2*(a + b*x^n)*ArcTan[a + b*x^n] + Log[1 + (a + b*x^n)^2])/(b*n)
```

3.2.3 Rubi [A] (warning: unable to verify)

Time = 0.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.80, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {7266, 5562, 5345, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{n-1} \arctan(a+bx^n)\,dx$$
$$\Big\downarrow \text{ 7266}$$
$$\frac{\int \arctan(bx^n+a)\,dx^n}{n}$$
$$\Big\downarrow \text{ 5562}$$
$$\frac{\int \arctan(bx^n+a)\,d(bx^n+a)}{bn}$$
$$\Big\downarrow \text{ 5345}$$
$$\frac{(a+bx^n)\arctan(a+bx^n)-\int \frac{bx^n+a}{x^{2n}+1}d(bx^n+a)}{bn}$$
$$\Big\downarrow \text{ 240}$$
$$\frac{(a+bx^n)\arctan(a+bx^n)-\frac{1}{2}\log(x^{2n}+1)}{bn}$$

input
```
Int[x^(-1 + n)*ArcTan[a + b*x^n],x]
```

output
```
((a + b*x^n)*ArcTan[a + b*x^n] - Log[1 + x^(2*n)]/2)/(b*n)
```

3.2.3.1 Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5562
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[1/d
  Subst[Int[(a + b*ArcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d},
 x] && IGtQ[p, 0]
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

3.2.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.57 (sec) , antiderivative size = 140, normalized size of antiderivative = 3.11

method	result	si
risch	$-\frac{i x^{n} \ln\left(1+i\left(a+b\,x^{n}\right)\right)}{2 n}+\frac{i x^{n} \ln\left(1-i\left(a+b\,x^{n}\right)\right)}{2 n}-\frac{\ln\left(x^{n}-\frac{i-a}{b}\right)}{2 b n}-\frac{\ln\left(\frac{i+a}{b}+x^{n}\right)}{2 b n}-\frac{i \ln\left(x^{n}-\frac{i-a}{b}\right) a}{2 b n}+\frac{i \ln\left(\frac{i+a}{b}+x^{n}\right) a}{2 b n}$	14

input
```
int(x^(-1+n)*arctan(a+b*x^n),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I/n*x^n*ln(1+I*(a+b*x^n))+1/2*I/n*x^n*ln(1-I*(a+b*x^n))-1/2/b/n*ln(x^
n-(I-a)/b)-1/2/b/n*ln((I+a)/b+x^n)-1/2*I/b/n*ln(x^n-(I-a)/b)*a+1/2*I/b/n*l
n((I+a)/b+x^n)*a
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.29

$$\int x^{-1+n} \arctan\left(a+bx^{n}\right)\,dx = \frac{2\,bx^{n} \arctan\left(bx^{n}+a\right)+2\,a \arctan\left(bx^{n}+a\right)-\log\left(b^{2}x^{2\,n}+2\,abx^{n}+a^{2}+1\right)}{2\,bn}$$

input
```
integrate(x^(-1+n)*arctan(a+b*x^n),x, algorithm="fricas")
```

output
```
1/2*(2*b*x^n*arctan(b*x^n + a) + 2*a*arctan(b*x^n + a) - log(b^2*x^(2*n) +
 2*a*b*x^n + a^2 + 1))/(b*n)
```

3.2.6 Sympy [F(-1)]

Timed out.

$$\int x^{-1+n} \arctan(a + bx^n) \, dx = \text{Timed out}$$

input
```
integrate(x**(-1+n)*atan(a+b*x**n),x)
```

output
```
Timed out
```

3.2.7 Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.89

$$\int x^{-1+n} \arctan(a + bx^n) \, dx = \frac{2(bx^n + a)\arctan(bx^n + a) - \log\left((bx^n + a)^2 + 1\right)}{2\,bn}$$

input
```
integrate(x^(-1+n)*arctan(a+b*x^n),x, algorithm="maxima")
```

output
```
1/2*(2*(b*x^n + a)*arctan(b*x^n + a) - log((b*x^n + a)^2 + 1))/(b*n)
```

3.2.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.89

$$\int x^{-1+n} \arctan(a + bx^n) \, dx = \frac{2(bx^n + a)\arctan(bx^n + a) - \log\left((bx^n + a)^2 + 1\right)}{2\,bn}$$

input
```
integrate(x^(-1+n)*arctan(a+b*x^n),x, algorithm="giac")
```

output
```
1/2*(2*(b*x^n + a)*arctan(b*x^n + a) - log((b*x^n + a)^2 + 1))/(b*n)
```

3.2.9 Mupad [B] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.29

$$\int x^{-1+n}\arctan\left(a+bx^n\right)\,dx=\frac{x^n\,\mathrm{atan}(a+b\,x^n)}{n}-\frac{\ln\left(a^2+b^2\,x^{2\,n}+2\,a\,b\,x^n+1\right)-2\,a\,\mathrm{atan}(a+b\,x^n)}{2\,b\,n}$$

input
```
int(x^(n - 1)*atan(a + b*x^n),x)
```

output
```
(x^n*atan(a + b*x^n))/n - (log(a^2 + b^2*x^(2*n) + 2*a*b*x^n + 1) - 2*a*at
an(a + b*x^n))/(2*b*n)
```

3.3 $\int x^5 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx$

3.3.1 Optimal result

Integrand size = 25, antiderivative size = 144

$$\int x^5 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \frac{5d^2x\sqrt{d+ex^2}}{96(-e)^{5/2}} + \frac{5dx^3\sqrt{d+ex^2}}{144(-e)^{3/2}} + \frac{x^5\sqrt{d+ex^2}}{36\sqrt{-e}} + \frac{1}{6}x^6 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) + \frac{5d^3\sqrt{-e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{96e^{7/2}}$$

output
```
1/6*x^6*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+5/96*d^3*arctanh(x*e^(1/2)/(e
*x^2+d)^(1/2))*(-e)^(1/2)/e^(7/2)+5/96*d^2*x*(e*x^2+d)^(1/2)/(-e)^(5/2)+5/
144*d*x^3*(e*x^2+d)^(1/2)/(-e)^(3/2)+1/36*x^5*(e*x^2+d)^(1/2)/(-e)^(1/2)
```

3.3.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.60

$$\int x^5 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \frac{\sqrt{-e}x\sqrt{d+ex^2}(-15d^2+10dex^2-8e^2x^4)+3(5d^3+16e^3x^6)\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{288e^3}$$

input
```
Integrate[x^5*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

output
```
(Sqrt[-e]*x*Sqrt[d + e*x^2]*(-15*d^2 + 10*d*e*x^2 - 8*e^2*x^4) + 3*(5*d^3
+ 16*e^3*x^6)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/(288*e^3)
```

3.3.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {5674, 262, 262, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx$$

$$\Big\downarrow \text{5674}$$

$$\frac{1}{6}x^6 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{1}{6}\sqrt{-e}\int \frac{x^6}{\sqrt{ex^2+d}}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{6}x^6 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{1}{6}\sqrt{-e}\left(\frac{x^5\sqrt{d+ex^2}}{6e} - \frac{5d\int \frac{x^4}{\sqrt{ex^2+d}}dx}{6e}\right)$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{6}x^6 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{1}{6}\sqrt{-e}\left(\frac{x^5\sqrt{d+ex^2}}{6e} - \frac{5d\left(\frac{x^3\sqrt{d+ex^2}}{4e} - \frac{3d\int \frac{x^2}{\sqrt{ex^2+d}}dx}{4e}\right)}{6e}\right)$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{6}x^6 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{1}{6}\sqrt{-e}\left(\frac{x^5\sqrt{d+ex^2}}{6e} - \frac{5d\left(\frac{x^3\sqrt{d+ex^2}}{4e} - \frac{3d\left(\frac{x\sqrt{d+ex^2}}{2e} - \frac{d\int \frac{1}{\sqrt{ex^2+d}}dx}{2e}\right)}{4e}\right)}{6e}\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{6}x^6\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{6}\sqrt{-e}\left(\frac{x^5\sqrt{d+ex^2}}{6e}-\frac{5d\left(\frac{x^3\sqrt{d+ex^2}}{4e}-\frac{3d\left(\frac{x\sqrt{d+ex^2}}{2e}-\frac{d\int\frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{2e}\right)}{4e}\right)}{6e}\right)$$

$\downarrow$ 219

$$\frac{1}{6}x^6\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{6}\sqrt{-e}\left(\frac{x^5\sqrt{d+ex^2}}{6e}-\frac{5d\left(\frac{x^3\sqrt{d+ex^2}}{4e}-\frac{3d\left(\frac{x\sqrt{d+ex^2}}{2e}-\frac{d\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2e^{3/2}}\right)}{4e}\right)}{6e}\right)$$

input
```
Int[x^5*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

output
```
(x^6*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/6 - (Sqrt[-e]*((x^5*Sqrt[d + e*
x^2])/(6*e) - (5*d*((x^3*Sqrt[d + e*x^2])/(4*e) - (3*d*((x*Sqrt[d + e*x^2]
)/(2*e) - (d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*e^(3/2))))/(4*e)))/(
6*e)))/6
```

3.3. $\int x^5\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx$

3.3.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 5674
```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Simp[c/(d*(m + 1))   Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ
[{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

3.3.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(110) = 220$.

Time = 0.03 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.81

method	result
default	$\frac{x^6 \arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{6} + \frac{\sqrt{-e}\,e\left(\frac{x^7\sqrt{e\,x^2+d}}{8e} - \frac{7d\left(\frac{x^5\sqrt{e\,x^2+d}}{6e} - \frac{5d\left(\frac{x^3\sqrt{e\,x^2+d}}{4e} - \frac{3d\left(\frac{x\sqrt{e\,x^2+d}}{2e} - \frac{d\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{2e^{\frac{3}{2}}}\right)}{4e}\right)}{6e}\right)}{8e}\right)}{6d} -$
parts	$\frac{x^6 \arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{6} + \frac{\sqrt{-e}\,e\left(\frac{x^7\sqrt{e\,x^2+d}}{8e} - \frac{7d\left(\frac{x^5\sqrt{e\,x^2+d}}{6e} - \frac{5d\left(\frac{x^3\sqrt{e\,x^2+d}}{4e} - \frac{3d\left(\frac{x\sqrt{e\,x^2+d}}{2e} - \frac{d\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{2e^{\frac{3}{2}}}\right)}{4e}\right)}{6e}\right)}{8e}\right)}{6d} -$

input
```
int(x^5*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/6*x^6*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+1/6*(-e)^(1/2)*e/d*(1/8*x^7/e
*(e*x^2+d)^(1/2)-7/8*d/e*(1/6*x^5/e*(e*x^2+d)^(1/2)-5/6*d/e*(1/4*x^3/e*(e*
x^2+d)^(1/2)-3/4*d/e*(1/2*x/e*(e*x^2+d)^(1/2)-1/2*d/e^(3/2)*ln(x*e^(1/2)+(
e*x^2+d)^(1/2))))))-1/6*(-e)^(1/2)/d*(1/8*x^5*(e*x^2+d)^(3/2)/e-5/8*d/e*(1
/6*x^3*(e*x^2+d)^(3/2)/e-1/2*d/e*(1/4*x*(e*x^2+d)^(3/2)/e-1/4*d/e*(1/2*x*(
e*x^2+d)^(1/2)+1/2*d/e^(1/2)*ln(x*e^(1/2)+(e*x^2+d)^(1/2))))))
```

3.3. $\int x^5 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx$

3.3.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.53

$$\int x^5 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx = -\frac{(8\,e^2x^5 - 10\,dex^3 + 15\,d^2x)\sqrt{ex^2+d}\sqrt{-e} - 3\,(16\,e^3x^6 + 5\,d^3)\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{288\,e^3}$$

input
```
integrate(x^5*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

output
```
-1/288*((8*e^2*x^5 - 10*d*e*x^3 + 15*d^2*x)*sqrt(e*x^2 + d)*sqrt(-e) - 3*(
16*e^3*x^6 + 5*d^3)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)))/e^3
```

3.3.6 Sympy [A] (verification not implemented)

Time = 0.89 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.96

$$\int x^5 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx = \begin{cases} \frac{5d^3 \operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{96e^3} - \frac{5d^2x\sqrt{-e}\sqrt{d+ex^2}}{96e^3} + \frac{5dx^3\sqrt{-e}\sqrt{d+ex^2}}{144e^2} + \frac{x^6 \operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{6} - \frac{x^5\sqrt{-e}\sqrt{d+ex^2}}{36e} & \text{for}\ e \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**5*atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)
```

output
```
Piecewise((5*d**3*atan(x*sqrt(-e)/sqrt(d + e*x**2))/(96*e**3) - 5*d**2*x*s
qrt(-e)*sqrt(d + e*x**2)/(96*e**3) + 5*d*x**3*sqrt(-e)*sqrt(d + e*x**2)/(1
44*e**2) + x**6*atan(x*sqrt(-e)/sqrt(d + e*x**2))/6 - x**5*sqrt(-e)*sqrt(d
 + e*x**2)/(36*e), Ne(e, 0)), (0, True))
```

3.3.7 Maxima [F]

$$\int x^5 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx = \int x^5 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)\,dx$$

input
```
integrate(x^5*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

output
```
1/6*x^6*arctan2(sqrt(-e)*x, sqrt(e*x^2 + d)) - d*sqrt(-e)*integrate(-1/6*s
qrt(e*x^2 + d)*x^6/(e^2*x^4 + d*e*x^2 - (e*x^2 + d)^2), x)
```

3.3.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.65

$$\int x^5 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx = \frac{1}{6}\,x^6 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right) - \frac{1}{288}\,\sqrt{-e^2x^2-de}\left(2\,x^2\left(\frac{4\,x^2}{e}-\frac{5\,d}{e^2}\right)+\frac{15\,d^2}{e^3}\right)x - \frac{5\,d^3 \arcsin\left(\frac{ex}{\sqrt{-de}}\right)\operatorname{sgn}(e)}{96\,e^2|e|}$$

input
```
integrate(x^5*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

output
```
1/6*x^6*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - 1/288*sqrt(-e^2*x^2 - d*e)*(2
*x^2*(4*x^2/e - 5*d/e^2) + 15*d^2/e^3)*x - 5/96*d^3*arcsin(e*x/sqrt(-d*e))
*sgn(e)/(e^2*abs(e))
```

3.3.9 Mupad [F(-1)]

Timed out.

$$\int x^5 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx = \int x^5 \operatorname{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)\,dx$$

input
```
int(x^5*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

output
```
int(x^5*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

3.4 $\int x^3 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx$

3.4.1 Optimal result

Integrand size = 25, antiderivative size = 116

$$\int x^3 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \frac{3dx\sqrt{d+ex^2}}{32(-e)^{3/2}} + \frac{x^3\sqrt{d+ex^2}}{16\sqrt{-e}} + \frac{1}{4}x^4 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{3d^2\sqrt{-e}\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{32e^{5/2}}$$

output
```
1/4*x^4*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))-3/32*d^2*arctanh(x*e^(1/2)/(e
*x^2+d)^(1/2))*(-e)^(1/2)/e^(5/2)+3/32*d*x*(e*x^2+d)^(1/2)/(-e)^(3/2)+1/16
*x^3*(e*x^2+d)^(1/2)/(-e)^(1/2)
```

3.4.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.64

$$\int x^3 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \frac{\sqrt{-e}x(3d-2ex^2)\sqrt{d+ex^2} + (-3d^2+8e^2x^4)\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{32e^2}$$

input
```
Integrate[x^3*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

output
```
(Sqrt[-e]*x*(3*d - 2*e*x^2)*Sqrt[d + e*x^2] + (-3*d^2 + 8*e^2*x^4)*ArcTan[
(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/(32*e^2)
```

3.4.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5674, 262, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx$$

$$\Big\downarrow \ 5674$$

$$\frac{1}{4}x^4 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{1}{4}\sqrt{-e}\int \frac{x^4}{\sqrt{ex^2+d}}dx$$

$$\Big\downarrow \ 262$$

$$\frac{1}{4}x^4 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{1}{4}\sqrt{-e}\left(\frac{x^3\sqrt{d+ex^2}}{4e} - \frac{3d\int \frac{x^2}{\sqrt{ex^2+d}}dx}{4e}\right)$$

$$\Big\downarrow \ 262$$

$$\frac{1}{4}x^4 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{1}{4}\sqrt{-e}\left(\frac{x^3\sqrt{d+ex^2}}{4e} - \frac{3d\left(\frac{x\sqrt{d+ex^2}}{2e} - \frac{d\int \frac{1}{\sqrt{ex^2+d}}dx}{2e}\right)}{4e}\right)$$

$$\Big\downarrow \ 224$$

$$\frac{1}{4}x^4 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{1}{4}\sqrt{-e}\left(\frac{x^3\sqrt{d+ex^2}}{4e} - \frac{3d\left(\frac{x\sqrt{d+ex^2}}{2e} - \frac{d\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{2e}\right)}{4e}\right)$$

$$\Big\downarrow \ 219$$

$$\frac{1}{4}x^4\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{4}\sqrt{-e}\left(\frac{x^3\sqrt{d+ex^2}}{4e}-\frac{3d\left(\frac{x\sqrt{d+ex^2}}{2e}-\frac{d\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2e^{3/2}}\right)}{4e}\right)$$

input
```
Int[x^3*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

output
```
(x^4*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/4 - (Sqrt[-e]*((x^3*Sqrt[d + e*
x^2])/(4*e) - (3*d*((x*Sqrt[d + e*x^2])/(2*e) - (d*ArcTanh[(Sqrt[e]*x)/Sqr
t[d + e*x^2]])/(2*e^(3/2))))/(4*e)))/4
```

3.4.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 5674
```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Simp[c/(d*(m + 1))   Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ
[{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

3.4.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. 2(88) = 176.

Time = 0.02 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.83

method	result
default	$\frac{x^4\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e x^2+d}}\right)}{4}+\frac{\sqrt{-e}\,e\left(\frac{x^5\sqrt{e x^2+d}}{6e}-\frac{5d\left(\frac{x^3\sqrt{e x^2+d}}{4e}-\frac{3d\left(\frac{x\sqrt{e x^2+d}}{2e}-\frac{d\ln\left(x\sqrt{e}+\sqrt{e x^2+d}\right)}{2e^{\frac{3}{2}}}\right)}{4e}\right)}{6e}\right)}{4d}-\frac{\sqrt{-e}\left(\frac{x^3\left(e x^2+d\right)^{\frac{3}{2}}}{6e}-\cdots\right)}{\cdots}$
parts	$\frac{x^4\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e x^2+d}}\right)}{4}+\frac{\sqrt{-e}\,e\left(\frac{x^5\sqrt{e x^2+d}}{6e}-\frac{5d\left(\frac{x^3\sqrt{e x^2+d}}{4e}-\frac{3d\left(\frac{x\sqrt{e x^2+d}}{2e}-\frac{d\ln\left(x\sqrt{e}+\sqrt{e x^2+d}\right)}{2e^{\frac{3}{2}}}\right)}{4e}\right)}{6e}\right)}{4d}-\frac{\sqrt{-e}\left(\frac{x^3\left(e x^2+d\right)^{\frac{3}{2}}}{6e}-\cdots\right)}{\cdots}$

input
```
int(x^3*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+1/4*(-e)^(1/2)*e/d*(1/6*x^5/e
*(e*x^2+d)^(1/2)-5/6*d/e*(1/4*x^3/e*(e*x^2+d)^(1/2)-3/4*d/e*(1/2*x/e*(e*x^
2+d)^(1/2)-1/2*d/e^(3/2)*ln(x*e^(1/2)+(e*x^2+d)^(1/2)))))-1/4*(-e)^(1/2)/d
*(1/6*x^3*(e*x^2+d)^(3/2)/e-1/2*d/e*(1/4*x*(e*x^2+d)^(3/2)/e-1/4*d/e*(1/2*
x*(e*x^2+d)^(1/2)+1/2*d/e^(1/2)*ln(x*e^(1/2)+(e*x^2+d)^(1/2)))))
```

3.4.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.56

$$\int x^3\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx$$
$$=-\frac{(2\,ex^3-3\,dx)\sqrt{ex^2+d}\sqrt{-e}-(8\,e^2x^4-3\,d^2)\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{32\,e^2}$$

input
```
integrate(x^3*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

output
```
-1/32*((2*e*x^3 - 3*d*x)*sqrt(e*x^2 + d)*sqrt(-e) - (8*e^2*x^4 - 3*d^2)*ar
ctan(sqrt(-e)*x/sqrt(e*x^2 + d)))/e^2
```

3.4.6 Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.92

$$\int x^3 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \begin{cases} -\frac{3d^2 \operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{32e^2} + \frac{3dx\sqrt{-e}\sqrt{d+ex^2}}{32e^2} + \frac{x^4 \operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{4} - \frac{x^3\sqrt{-e}\sqrt{d+ex^2}}{16e} & \text{for } e \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)
```

output
```
Piecewise((-3*d**2*atan(x*sqrt(-e)/sqrt(d + e*x**2))/(32*e**2) + 3*d*x*sqr
t(-e)*sqrt(d + e*x**2)/(32*e**2) + x**4*atan(x*sqrt(-e)/sqrt(d + e*x**2))/
4 - x**3*sqrt(-e)*sqrt(d + e*x**2)/(16*e), Ne(e, 0)), (0, True))
```

3.4.7 Maxima [F]

$$\int x^3 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \int x^3 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right) dx$$

input
```
integrate(x^3*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

output
```
1/4*x^4*arctan2(sqrt(-e)*x, sqrt(e*x^2 + d)) - d*sqrt(-e)*integrate(-1/4*s
qrt(e*x^2 + d)*x^4/(e^2*x^4 + d*e*x^2 - (e*x^2 + d)^2), x)
```

3.4.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.69

$$\int x^3 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \frac{1}{4}x^4 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right) - \frac{1}{32}\sqrt{-e^2x^2-dex}\left(\frac{2x^2}{e} - \frac{3d}{e^2}\right) + \frac{3d^2 \arcsin\left(\frac{ex}{\sqrt{-de}}\right)\mathrm{sgn}(e)}{32e|e|}$$

input
```
integrate(x^3*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

output
```
1/4*x^4*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - 1/32*sqrt(-e^2*x^2 - d*e)*x*(
2*x^2/e - 3*d/e^2) + 3/32*d^2*arcsin(e*x/sqrt(-d*e))*sgn(e)/(e*abs(e))
```

3.4.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \int x^3 \operatorname{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right) dx$$

input
```
int(x^3*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

output
```
int(x^3*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

3.5 $\int x \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx$

3.5.1 Optimal result

Integrand size = 23, antiderivative size = 88

$$\int x \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \frac{x\sqrt{d+ex^2}}{4\sqrt{-e}} + \frac{1}{2}x^2 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) + \frac{d\sqrt{-e}\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{4e^{3/2}}$$

output
```
1/2*x^2*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+1/4*d*arctanh(x*e^(1/2)/(e*x^
2+d)^(1/2))*(-e)^(1/2)/e^(3/2)+1/4*x*(e*x^2+d)^(1/2)/(-e)^(1/2)
```

3.5.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.67

$$\int x \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \frac{-\sqrt{-e}x\sqrt{d+ex^2} + (d+2ex^2)\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{4e}$$

input
```
Integrate[x*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

output
```
(-(Sqrt[-e]*x*Sqrt[d + e*x^2]) + (d + 2*e*x^2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d
+ e*x^2]])/(4*e)
```

3.5.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {5674, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx$$

$$\Big\downarrow \text{ 5674}$$

$$\frac{1}{2}x^2 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{1}{2}\sqrt{-e}\int \frac{x^2}{\sqrt{ex^2+d}}dx$$

$$\Big\downarrow \text{ 262}$$

$$\frac{1}{2}x^2 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{1}{2}\sqrt{-e}\left(\frac{x\sqrt{d+ex^2}}{2e} - \frac{d\int \frac{1}{\sqrt{ex^2+d}}dx}{2e}\right)$$

$$\Big\downarrow \text{ 224}$$

$$\frac{1}{2}x^2 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{1}{2}\sqrt{-e}\left(\frac{x\sqrt{d+ex^2}}{2e} - \frac{d\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{2e}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2}x^2 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{1}{2}\sqrt{-e}\left(\frac{x\sqrt{d+ex^2}}{2e} - \frac{d\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2e^{3/2}}\right)$$

input
```
Int[x*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

output
```
(x^2*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/2 - (Sqrt[-e]*((x*Sqrt[d + e*x^
2])/(2*e) - (d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*e^(3/2))))/2
```

3.5.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 5674
```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Simp[c/(d*(m + 1))   Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ
[{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

3.5.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(66) = 132$.

Time = 0.02 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.86

method	result
default	$\frac{x^2 \arctan\left(\frac{x\sqrt{-e}}{\sqrt{e x^2+d}}\right)}{2} + \frac{\sqrt{-e}\, e \left(\frac{x^3\sqrt{e x^2+d}}{4e} - \frac{3d\left(\frac{x\sqrt{e x^2+d}}{2e} - \frac{d \ln\left(x\sqrt{e}+\sqrt{e x^2+d}\right)}{2 e^{\frac{3}{2}}}\right)}{4e}\right)}{2d} - \frac{\sqrt{-e}\left(\frac{x\left(e x^2+d\right)^{\frac{3}{2}}}{4e} - \frac{d\left(\frac{x\sqrt{e x^2+d}}{2} + \frac{d \ln(}{}\right.}{4e}\right.}{2d}$
parts	$\frac{x^2 \arctan\left(\frac{x\sqrt{-e}}{\sqrt{e x^2+d}}\right)}{2} + \frac{\sqrt{-e}\, e \left(\frac{x^3\sqrt{e x^2+d}}{4e} - \frac{3d\left(\frac{x\sqrt{e x^2+d}}{2e} - \frac{d \ln\left(x\sqrt{e}+\sqrt{e x^2+d}\right)}{2 e^{\frac{3}{2}}}\right)}{4e}\right)}{2d} - \frac{\sqrt{-e}\left(\frac{x\left(e x^2+d\right)^{\frac{3}{2}}}{4e} - \frac{d\left(\frac{x\sqrt{e x^2+d}}{2} + \frac{d \ln(}{}\right.}{4e}\right.}{2d}$

input
```
int(x*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+1/2*(-e)^(1/2)*e/d*(1/4*x^3/e
*(e*x^2+d)^(1/2)-3/4*d/e*(1/2*x/e*(e*x^2+d)^(1/2)-1/2*d/e^(3/2)*ln(x*e^(1/
2)+(e*x^2+d)^(1/2))))-1/2*(-e)^(1/2)/d*(1/4*x*(e*x^2+d)^(3/2)/e-1/4*d/e*(1
/2*x*(e*x^2+d)^(1/2)+1/2*d/e^(1/2)*ln(x*e^(1/2)+(e*x^2+d)^(1/2))))
```

3.5.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.56

$$\int x \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = -\frac{\sqrt{ex^2+d}\sqrt{-e}x - (2\,ex^2+d)\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{4\,e}$$

input
```
integrate(x*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(e*x^2 + d)*sqrt(-e)*x - (2*e*x^2 + d)*arctan(sqrt(-e)*x/sqrt(e*
x^2 + d)))/e
```

3.5.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.83

$$\int x \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \begin{cases} \frac{d \operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{4e} + \frac{x^2 \operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{2} - \frac{x\sqrt{-e}\sqrt{d+ex^2}}{4e} & \text{for } e \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x*atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)
```

output
```
Piecewise((d*atan(x*sqrt(-e)/sqrt(d + e*x**2))/(4*e) + x**2*atan(x*sqrt(-e
)/sqrt(d + e*x**2))/2 - x*sqrt(-e)*sqrt(d + e*x**2)/(4*e), Ne(e, 0)), (0,
True))
```

3.5.7 Maxima [F]

$$\int x\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)dx = \int x\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)dx$$

input
```
integrate(x*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

output
```
1/2*x^2*arctan2(sqrt(-e)*x, sqrt(e*x^2 + d)) - d*sqrt(-e)*integrate(-1/2*s
qrt(e*x^2 + d)*x^2/(e^2*x^4 + d*e*x^2 - (e*x^2 + d)^2), x)
```

3.5.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

$$\int x\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)dx = \frac{1}{2}x^2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right) - \frac{d\arcsin\left(\frac{ex}{\sqrt{-de}}\right)\operatorname{sgn}(e)}{4|e|} - \frac{\sqrt{-e^2x^2-dex}}{4e}$$

input
```
integrate(x*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

output
```
1/2*x^2*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - 1/4*d*arcsin(e*x/sqrt(-d*e))*
sgn(e)/abs(e) - 1/4*sqrt(-e^2*x^2 - d*e)*x/e
```

3.5.9 Mupad [F(-1)]

Timed out.

$$\int x\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)dx = \int x\,\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)dx$$

input
```
int(x*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

output
```
int(x*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

3.6 $\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x}\,dx$

3.6.1 Optimal result

Integrand size = 25, antiderivative size = 288

$$\begin{aligned}\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x}\,dx =& -\frac{\sqrt{d}\sqrt{-e}\sqrt{1+\frac{ex^2}{d}}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2}{2\sqrt{e}\sqrt{d+ex^2}} \\ &+ \frac{\sqrt{d}\sqrt{-e}\sqrt{1+\frac{ex^2}{d}}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{\sqrt{e}\sqrt{d+ex^2}} \\ &- \frac{\sqrt{d}\sqrt{-e}\sqrt{1+\frac{ex^2}{d}}\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log(x)}{\sqrt{e}\sqrt{d+ex^2}} \\ &+ \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\log(x) \\ &+ \frac{\sqrt{d}\sqrt{-e}\sqrt{1+\frac{ex^2}{d}}\operatorname{PolyLog}\left(2,e^{2\operatorname{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)}{2\sqrt{e}\sqrt{d+ex^2}}\end{aligned}$$

output
```
arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))*ln(x)-1/2*arcsinh(x*e^(1/2)/d^(1/2))^
2*d^(1/2)*(-e)^(1/2)*(1+e*x^2/d)^(1/2)/e^(1/2)/(e*x^2+d)^(1/2)+arcsinh(x*e
^(1/2)/d^(1/2))*ln(1-(x*e^(1/2)/d^(1/2)+(1+e*x^2/d)^(1/2))^2)*d^(1/2)*(-e)
^(1/2)*(1+e*x^2/d)^(1/2)/e^(1/2)/(e*x^2+d)^(1/2)-arcsinh(x*e^(1/2)/d^(1/2)
)*ln(x)*d^(1/2)*(-e)^(1/2)*(1+e*x^2/d)^(1/2)/e^(1/2)/(e*x^2+d)^(1/2)+1/2*p
olylog(2,(x*e^(1/2)/d^(1/2)+(1+e*x^2/d)^(1/2))^2)*d^(1/2)*(-e)^(1/2)*(1+e*
x^2/d)^(1/2)/e^(1/2)/(e*x^2+d)^(1/2)
```

3.6.2 Mathematica [A] (verified)

Time = 1.85 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.59

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x}\,dx = \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\log(x) + \frac{\sqrt{-e}\sqrt{1+\frac{ex^2}{d}}\left(\operatorname{arcsinh}\left(\sqrt{\frac{e}{d}}x\right)^2+2\operatorname{arcsinh}\left(\sqrt{\frac{e}{d}}x\right)\log\left(1-e^{-2\operatorname{arcsinh}\left(\sqrt{\frac{e}{d}}x\right)}\right)-2\log(x)\log\left(\sqrt{\frac{e}{d}}x+\sqrt{}\right.\right.}{2\sqrt{\frac{e}{d}}\sqrt{d+ex^2}}$$

input
```
Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x,x]
```

output
```
ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]*Log[x] + (Sqrt[-e]*Sqrt[1 + (e*x^2)/d
]*(ArcSinh[Sqrt[e/d]*x]^2 + 2*ArcSinh[Sqrt[e/d]*x]*Log[1 - E^(-2*ArcSinh[S
qrt[e/d]*x])] - 2*Log[x]*Log[Sqrt[e/d]*x + Sqrt[1 + (e*x^2)/d]] - PolyLog[
2, E^(-2*ArcSinh[Sqrt[e/d]*x])]))/(2*Sqrt[e/d]*Sqrt[d + e*x^2])
```

3.6.3 Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.73 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.66, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {5672, 2764, 2762, 6190, 3042, 26, 4199, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x}dx$$

$$\Big\downarrow \text{5672}$$

$$\log(x)\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\sqrt{-e}\int\frac{\log(x)}{\sqrt{ex^2+d}}dx$$

$$\Big\downarrow \text{2764}$$

$$\log(x)\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{\sqrt{-e}\sqrt{\frac{ex^2}{d}+1}\int\frac{\log(x)}{\sqrt{\frac{ex^2}{d}+1}}dx}{\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{2762}$$

$$\log(x)\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{\sqrt{-e}\sqrt{\frac{ex^2}{d}+1}\left(\frac{\sqrt{d}\log(x)\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{\sqrt{d}\int\frac{\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{x}dx}{\sqrt{e}}\right)}{\sqrt{d+ex^2}}$$

$\downarrow$ 6190

$$\log(x)\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{\sqrt{-e}\sqrt{\frac{ex^2}{d}+1}\left(\frac{\sqrt{d}\log(x)\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{\sqrt{d}\int\frac{\sqrt{d}\sqrt{\frac{ex^2}{d}+1}\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}x}d\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}\right)}{\sqrt{d+ex^2}}$$

$\downarrow$ 3042

$$\log(x)\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{\sqrt{-e}\sqrt{\frac{ex^2}{d}+1}\left(\frac{\sqrt{d}\log(x)\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}-\frac{\sqrt{d}\int-i\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\tan\left(i\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)+\frac{\pi}{2}\right)d\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}\right)}{\sqrt{d+ex^2}}$$

$\downarrow$ 26

$$\log(x)\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{\sqrt{-e}\sqrt{\frac{ex^2}{d}+1}\left(\frac{\sqrt{d}\log(x)\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{i\sqrt{d}\int\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\tan\left(i\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)+\frac{\pi}{2}\right)d\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}\right)}{\sqrt{d+ex^2}}$$

$\downarrow$ 4199

$$\log(x)\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{\sqrt{-e}\sqrt{\frac{ex^2}{d}+1}\left(\frac{\sqrt{d}\log(x)\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{i\sqrt{d}\left(2i\int-\frac{e^{2\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{1-e^{2\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}}d\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)-\frac{1}{2}i\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2\right)}{\sqrt{e}}\right)}{\sqrt{d+ex^2}}$$

$\downarrow$ 25

3.6. $\displaystyle\int\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x}dx$

$$\log(x)\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{\sqrt{-e}\sqrt{\frac{ex^2}{d}+1}\left(\frac{\sqrt{d}\log(x)\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{i\sqrt{d}\left(-2i\int\frac{e^{2\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{1-e^{2\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}}d\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)-\frac{1}{2}i\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)^2\right)}{\sqrt{e}}\right)}{\sqrt{d+ex^2}}$$

$\downarrow$ 2620

$$\log(x)\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{\sqrt{-e}\sqrt{\frac{ex^2}{d}+1}\left(\frac{\sqrt{d}\log(x)\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{i\sqrt{d}\left(-2i\left(\frac{1}{2}\int\log\left(1-e^{2\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)d\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)-\frac{1}{2}\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\text{arc}\cdots}\right.\right.\right.}{\sqrt{e}}\right.}{\sqrt{d+ex^2}}$$

$\downarrow$ 2715

$$\log(x)\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{\sqrt{-e}\sqrt{\frac{ex^2}{d}+1}\left(\frac{\sqrt{d}\log(x)\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{i\sqrt{d}\left(-2i\left(\frac{1}{4}\int e^{-2\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\log\left(1-e^{2\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)de^{2\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}-\frac{1}{2}\text{arcsinh}\left(\cdots\right.\right.\right.}{\sqrt{e}}\right.}{\sqrt{d+ex^2}}$$

$\downarrow$ 2838

$$\log(x)\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{\sqrt{-e}\sqrt{\frac{ex^2}{d}+1}\left(\frac{\sqrt{d}\log(x)\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}}+\frac{i\sqrt{d}\left(-2i\left(-\frac{1}{4}\text{PolyLog}\left(2,e^{2\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)-\frac{1}{2}\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\log\left(1-e^{2\text{arcsinh}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}\right)\cdots\right.\right.}{\sqrt{e}}\right.}{\sqrt{d+ex^2}}$$

input
```
Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x,x]
```

3.6. $\int\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x}dx$

output
```
ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]*Log[x] - (Sqrt[-e]*Sqrt[1 + (e*x^2)/d
]*((Sqrt[d]*ArcSinh[(Sqrt[e]*x)/Sqrt[d]]*Log[x])/Sqrt[e] + (I*Sqrt[d]*((-1
/2*I)*ArcSinh[(Sqrt[e]*x)/Sqrt[d]]^2 - (2*I)*(-1/2*(ArcSinh[(Sqrt[e]*x)/Sq
rt[d]]*Log[1 - E^(2*ArcSinh[(Sqrt[e]*x)/Sqrt[d]])]) - PolyLog[2, E^(2*ArcS
inh[(Sqrt[e]*x)/Sqrt[d]])]/4)))/Sqrt[e]))/Sqrt[d + e*x^2]
```

3.6.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2762
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[ArcSinh[Rt[e, 2]*(x/Sqrt[d])]*((a + b*Log[c*x^n])/Rt[e, 2]), x]
 - Simp[b*(n/Rt[e, 2])   Int[ArcSinh[Rt[e, 2]*(x/Sqrt[d])]/x, x], x] /; Fre
eQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && PosQ[e]
```

rule 2764
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[Sqrt[1 + (e/d)*x^2]/Sqrt[d + e*x^2]   Int[(a + b*Log[c*x^n])/Sq
rt[1 + (e/d)*x^2], x], x] /; FreeQ[{a, b, c, d, e, n}, x] &&  !GtQ[d, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

3.6. $\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x}\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4199
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp
[2*I   Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x
))/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && In
tegerQ[4*k] && IGtQ[m, 0]
```

rule 5672
```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]/(x_), x_Symbol] :> Simp
[ArcTan[c*(x/Sqrt[a + b*x^2])]*Log[x], x] - Simp[c   Int[Log[x]/Sqrt[a + b*
x^2], x], x] /; FreeQ[{a, b, c}, x] && EqQ[b + c^2, 0]
```

rule 6190
```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Simp[1/b
 Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a
, b, c}, x] && IGtQ[n, 0]
```

3.6.4 Maple [F]

$$\int \frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{x}dx$$

input
```
int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x,x)
```

output
```
int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x,x)
```

3.6.5 Fricas [F]

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x}\,dx = \int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x}\,dx$$

input
```
integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x,x, algorithm="fricas")
```

output
```
integral(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x, x)
```

3.6.6 Sympy [F]

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x}\,dx = \int \frac{\operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{x}\,dx$$

input
```
integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x,x)
```

output
```
Integral(atan(x*sqrt(-e)/sqrt(d + e*x**2))/x, x)
```

3.6.7 Maxima [F]

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x}\,dx = \int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x}\,dx$$

input
```
integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x,x, algorithm="maxima")
```

output
```
integrate(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x, x)
```

3.6.8 Giac [F]

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x}\,dx = \int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x}\,dx$$

input
```
integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x,x, algorithm="giac")
```

output
```
integrate(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x, x)
```

3.6.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x}\,dx = \int \frac{\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)}{x}\,dx$$

input
```
int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x,x)
```

output
```
int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x, x)
```

3.7 $\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^3}\,dx$

3.7.1 Optimal result

Integrand size = 25, antiderivative size = 57

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^3}\,dx = -\frac{\sqrt{-e}\sqrt{d+ex^2}}{2dx} - \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{2x^2}$$

output
```
-1/2*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^2-1/2*(-e)^(1/2)*(e*x^2+d)^(1/
2)/d/x
```

3.7.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.95

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^3}\,dx = -\frac{\sqrt{-e}x\sqrt{d+ex^2} + d\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{2dx^2}$$

input
```
Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^3,x]
```

output
```
-1/2*(Sqrt[-e]*x*Sqrt[d + e*x^2] + d*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])
/(d*x^2)
```

3.7.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {5674, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^3}dx$$

$$\Big\downarrow \text{5674}$$

$$\frac{1}{2}\sqrt{-e}\int \frac{1}{x^2\sqrt{ex^2+d}}dx-\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{2x^2}$$

$$\Big\downarrow \text{242}$$

$$-\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{2x^2}-\frac{\sqrt{-e}\sqrt{d+ex^2}}{2dx}$$

input
```
Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^3,x]
```

output
```
-1/2*(Sqrt[-e]*Sqrt[d + e*x^2])/(d*x) - ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2
]]/(2*x^2)
```

3.7.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 5674
```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Simp[c/(d*(m + 1))   Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ
[{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

3.7.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(45) = 90.

Time = 0.02 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.14

method	result	size
default	$-\frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{2x^2}-\frac{\sqrt{-e}\,\sqrt{e}\,\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{2d}+\frac{\sqrt{-e}\left(-\frac{\left(e\,x^2+d\right)^{\frac{3}{2}}}{dx}+\frac{2e\left(\frac{x\sqrt{e\,x^2+d}}{2}+\frac{d\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{2\sqrt{e}}\right)}{d}\right)}{2d}$	122
parts	$-\frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{2x^2}-\frac{\sqrt{-e}\,\sqrt{e}\,\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{2d}+\frac{\sqrt{-e}\left(-\frac{\left(e\,x^2+d\right)^{\frac{3}{2}}}{dx}+\frac{2e\left(\frac{x\sqrt{e\,x^2+d}}{2}+\frac{d\ln\left(x\sqrt{e}+\sqrt{e\,x^2+d}\right)}{2\sqrt{e}}\right)}{d}\right)}{2d}$	122

input
```
int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^2-1/2*(-e)^(1/2)*e^(1/2)/d*ln(
x*e^(1/2)+(e*x^2+d)^(1/2))+1/2*(-e)^(1/2)/d*(-1/d/x*(e*x^2+d)^(3/2)+2*e/d*
(1/2*x*(e*x^2+d)^(1/2)+1/2*d/e^(1/2)*ln(x*e^(1/2)+(e*x^2+d)^(1/2))))
```

3.7.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.77

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^3}\,dx = -\frac{\sqrt{ex^2+d}\sqrt{-e}x+d\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{2\,dx^2}$$

input
```
integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^3,x, algorithm="fricas")
```

output
```
-1/2*(sqrt(e*x^2 + d)*sqrt(-e)*x + d*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)))/(
d*x^2)
```

3.7.6 Sympy [A] (verification not implemented)

Time = 1.87 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^3}\,dx = -\frac{\operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{2x^2} - \frac{\sqrt{e}\sqrt{-e}\sqrt{\frac{d}{ex^2}+1}}{2d}$$

input
```
integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**3,x)
```

output
```
-atan(x*sqrt(-e)/sqrt(d + e*x**2))/(2*x**2) - sqrt(e)*sqrt(-e)*sqrt(d/(e*x
**2) + 1)/(2*d)
```

3.7.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^3}\,dx = -\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{2\,x^2} - \frac{\sqrt{-e}ex^2 + d\sqrt{-e}}{2\,\sqrt{ex^2+d}dx}$$

input
```
integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^3,x, algorithm="maxima")
```

output
```
-1/2*arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^2 - 1/2*(sqrt(-e)*e*x^2 + d*sqrt
(-e))/(sqrt(e*x^2 + d)*d*x)
```

3.7.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(45) = 90.

Time = 0.35 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.82

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^3}\,dx = \frac{e^4x}{4\left(\sqrt{-de}e + \sqrt{-e^2x^2-de}|e|\right)d|e|} - \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{2\,x^2} - \frac{\sqrt{-de}e + \sqrt{-e^2x^2-de}|e|}{4\,dx|e|}$$

input
```
integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^3,x, algorithm="giac")
```

output
```
1/4*e^4*x/((sqrt(-d*e)*e + sqrt(-e^2*x^2 - d*e)*abs(e))*d*abs(e)) - 1/2*ar
ctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^2 - 1/4*(sqrt(-d*e)*e + sqrt(-e^2*x^2 -
 d*e)*abs(e))/(d*x*abs(e))
```

3.7.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^3}\,dx = \int \frac{\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^3}\,dx$$

input
```
int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^3,x)
```

output
```
int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^3, x)
```

3.8 $\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^5}\,dx$

3.8.1 Optimal result

Integrand size = 25, antiderivative size = 85

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^5}\,dx = -\frac{\sqrt{-e}\sqrt{d+ex^2}}{12dx^3} - \frac{(-e)^{3/2}\sqrt{d+ex^2}}{6d^2x} - \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{4x^4}$$

output
```
-1/4*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^4-1/6*(-e)^(3/2)*(e*x^2+d)^(1/
2)/d^2/x-1/12*(-e)^(1/2)*(e*x^2+d)^(1/2)/d/x^3
```

3.8.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.79

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^5}\,dx = \frac{\sqrt{-e}x\sqrt{d+ex^2}(-d+2ex^2) - 3d^2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{12d^2x^4}$$

input
```
Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^5,x]
```

output
```
(Sqrt[-e]*x*Sqrt[d + e*x^2]*(-d + 2*e*x^2) - 3*d^2*ArcTan[(Sqrt[-e]*x)/Sqr
t[d + e*x^2]])/(12*d^2*x^4)
```

3.8.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5674, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^5}dx$$

$$\Big\downarrow \text{5674}$$

$$\frac{1}{4}\sqrt{-e}\int \frac{1}{x^4\sqrt{ex^2+d}}dx-\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{4x^4}$$

$$\Big\downarrow \text{245}$$

$$\frac{1}{4}\sqrt{-e}\left(-\frac{2e\int \frac{1}{x^2\sqrt{ex^2+d}}dx}{3d}-\frac{\sqrt{d+ex^2}}{3dx^3}\right)-\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{4x^4}$$

$$\Big\downarrow \text{242}$$

$$\frac{1}{4}\sqrt{-e}\left(\frac{2e\sqrt{d+ex^2}}{3d^2x}-\frac{\sqrt{d+ex^2}}{3dx^3}\right)-\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{4x^4}$$

input
```
Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^5,x]
```

output
```
(Sqrt[-e]*(-1/3*Sqrt[d + e*x^2]/(d*x^3) + (2*e*Sqrt[d + e*x^2])/(3*d^2*x))
)/4 - ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/(4*x^4)
```

3.8.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 5674
```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Simp[c/(d*(m + 1))   Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ
[{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

3.8.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{4x^4}+\frac{\sqrt{-e}\,e\sqrt{e\,x^2+d}}{4d^2x}-\frac{\sqrt{-e}\,\left(e\,x^2+d\right)^{\frac{3}{2}}}{12d^2x^3}$	69
parts	$-\frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{4x^4}+\frac{\sqrt{-e}\,e\sqrt{e\,x^2+d}}{4d^2x}-\frac{\sqrt{-e}\,\left(e\,x^2+d\right)^{\frac{3}{2}}}{12d^2x^3}$	69

input
```
int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^4+1/4*(-e)^(1/2)*e/d^2/x*(e*x^
2+d)^(1/2)-1/12*(-e)^(1/2)/d^2/x^3*(e*x^2+d)^(3/2)
```

3.8.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.68

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^5}\,dx = -\frac{3\,d^2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)-(2\,ex^3-dx)\sqrt{ex^2+d}\sqrt{-e}}{12\,d^2x^4}$$

input
```
integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^5,x, algorithm="fricas")
```

output
```
-1/12*(3*d^2*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - (2*e*x^3 - d*x)*sqrt(e*x
^2 + d)*sqrt(-e))/(d^2*x^4)
```

3.8.6 Sympy [A] (verification not implemented)

Time = 2.19 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.98

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^5}\,dx = -\frac{\operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{4x^4} - \frac{\sqrt{e}\sqrt{-e}\sqrt{\frac{d}{ex^2}+1}}{12dx^2} + \frac{e^{\frac{3}{2}}\sqrt{-e}\sqrt{\frac{d}{ex^2}+1}}{6d^2}$$

input
```
integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**5,x)
```

output
```
-atan(x*sqrt(-e)/sqrt(d + e*x**2))/(4*x**4) - sqrt(e)*sqrt(-e)*sqrt(d/(e*x
**2) + 1)/(12*d*x**2) + e**(3/2)*sqrt(-e)*sqrt(d/(e*x**2) + 1)/(6*d**2)
```

3.8.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.80

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^5}\,dx = \frac{\sqrt{ex^2+d}\sqrt{-e}e}{4\,d^2x} - \frac{(ex^2+d)^{\frac{3}{2}}\sqrt{-e}}{12\,d^2x^3} - \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{4\,x^4}$$

input
```
integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^5,x, algorithm="maxima")
```

output
```
1/4*sqrt(e*x^2 + d)*sqrt(-e)*e/(d^2*x) - 1/12*(e*x^2 + d)^(3/2)*sqrt(-e)/(
d^2*x^3) - 1/4*arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^4
```

3.8.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 198 vs. $2(67) = 134$.

Time = 0.34 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.33

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^5}\,dx = -\frac{\left(e^3 + \frac{9\left(\sqrt{-de}e+\sqrt{-e^2x^2-de}|e|\right)^2}{ex^2}\right)e^6x^3}{96\left(\sqrt{-de}e+\sqrt{-e^2x^2-de}|e|\right)^3 d^2|e|} - \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{4\,x^4} + \frac{\frac{9\left(\sqrt{-de}e+\sqrt{-e^2x^2-de}|e|\right)d^4e^6}{x} + \frac{\left(\sqrt{-de}e+\sqrt{-e^2x^2-de}|e|\right)^3 d^4e^2}{x^3}}{96\,d^6e^5|e|}$$

input
```
integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^5,x, algorithm="giac")
```

output
```
-1/96*(e^3 + 9*(sqrt(-d*e)*e + sqrt(-e^2*x^2 - d*e)*abs(e))^2/(e*x^2))*e^6
*x^3/((sqrt(-d*e)*e + sqrt(-e^2*x^2 - d*e)*abs(e))^3*d^2*abs(e)) - 1/4*arc
tan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^4 + 1/96*(9*(sqrt(-d*e)*e + sqrt(-e^2*x^
2 - d*e)*abs(e))*d^4*e^6/x + (sqrt(-d*e)*e + sqrt(-e^2*x^2 - d*e)*abs(e))^
3*d^4*e^2/x^3)/(d^6*e^5*abs(e))
```

3.8.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^5}\,dx = \int \frac{\operatorname{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^5}\,dx$$

input
```
int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^5,x)
```

output
```
int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^5, x)
```

3.9 $\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^7}\,dx$

3.9.1 Optimal result

Integrand size = 25, antiderivative size = 113

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^7}\,dx = -\frac{\sqrt{-e}\sqrt{d+ex^2}}{30dx^5} - \frac{2(-e)^{3/2}\sqrt{d+ex^2}}{45d^2x^3} - \frac{4(-e)^{5/2}\sqrt{d+ex^2}}{45d^3x} - \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{6x^6}$$

output
```
-1/6*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^6-2/45*(-e)^(3/2)*(e*x^2+d)^(1
/2)/d^2/x^3-4/45*(-e)^(5/2)*(e*x^2+d)^(1/2)/d^3/x-1/30*(-e)^(1/2)*(e*x^2+d
)^(1/2)/d/x^5
```

3.9.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.69

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^7}\,dx = \frac{\sqrt{-e}x\sqrt{d+ex^2}(-3d^2+4dex^2-8e^2x^4) - 15d^3\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{90d^3x^6}$$

input
```
Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^7,x]
```

output
```
(Sqrt[-e]*x*Sqrt[d + e*x^2]*(-3*d^2 + 4*d*e*x^2 - 8*e^2*x^4) - 15*d^3*ArcT
an[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/(90*d^3*x^6)
```

3.9.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {5674, 245, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^7}dx$$

$$\Big\downarrow \text{5674}$$

$$\frac{1}{6}\sqrt{-e}\int \frac{1}{x^6\sqrt{ex^2+d}}dx-\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{6x^6}$$

$$\Big\downarrow \text{245}$$

$$\frac{1}{6}\sqrt{-e}\left(-\frac{4e\int \frac{1}{x^4\sqrt{ex^2+d}}dx}{5d}-\frac{\sqrt{d+ex^2}}{5dx^5}\right)-\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{6x^6}$$

$$\Big\downarrow \text{245}$$

$$\frac{1}{6}\sqrt{-e}\left(-\frac{4e\left(-\frac{2e\int \frac{1}{x^2\sqrt{ex^2+d}}dx}{3d}-\frac{\sqrt{d+ex^2}}{3dx^3}\right)}{5d}-\frac{\sqrt{d+ex^2}}{5dx^5}\right)-\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{6x^6}$$

$$\Big\downarrow \text{242}$$

$$\frac{1}{6}\sqrt{-e}\left(-\frac{4e\left(\frac{2e\sqrt{d+ex^2}}{3d^2x}-\frac{\sqrt{d+ex^2}}{3dx^3}\right)}{5d}-\frac{\sqrt{d+ex^2}}{5dx^5}\right)-\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{6x^6}$$

input
```
Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^7,x]
```

output
```
(Sqrt[-e]*(-1/5*Sqrt[d + e*x^2]/(d*x^5) - (4*e*(-1/3*Sqrt[d + e*x^2]/(d*x^
3) + (2*e*Sqrt[d + e*x^2])/(3*d^2*x)))/(5*d)))/6 - ArcTan[(Sqrt[-e]*x)/Sqr
t[d + e*x^2]]/(6*x^6)
```

3.9.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 5674
```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Simp[c/(d*(m + 1))   Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ
[{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

3.9.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.04

method	result	size
default	$-\frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{6x^6}-\frac{\sqrt{-e}\,e\left(-\frac{\sqrt{e\,x^2+d}}{3d\,x^3}+\frac{2e\sqrt{e\,x^2+d}}{3d^2x}\right)}{6d}+\frac{\sqrt{-e}\left(-\frac{\left(e\,x^2+d\right)^{\frac{3}{2}}}{5d\,x^5}+\frac{2e\left(e\,x^2+d\right)^{\frac{3}{2}}}{15d^2x^3}\right)}{6d}$	117
parts	$-\frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{6x^6}-\frac{\sqrt{-e}\,e\left(-\frac{\sqrt{e\,x^2+d}}{3d\,x^3}+\frac{2e\sqrt{e\,x^2+d}}{3d^2x}\right)}{6d}+\frac{\sqrt{-e}\left(-\frac{\left(e\,x^2+d\right)^{\frac{3}{2}}}{5d\,x^5}+\frac{2e\left(e\,x^2+d\right)^{\frac{3}{2}}}{15d^2x^3}\right)}{6d}$	117

input
```
int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^6-1/6*(-e)^(1/2)*e/d*(-1/3/d/x
^3*(e*x^2+d)^(1/2)+2/3*e/d^2/x*(e*x^2+d)^(1/2))+1/6*(-e)^(1/2)/d*(-1/5/d/x
^5*(e*x^2+d)^(3/2)+2/15*e/d^2/x^3*(e*x^2+d)^(3/2))
```

3.9.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.60

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^7}\,dx = -\frac{15\,d^3\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right) + (8\,e^2x^5 - 4\,dex^3 + 3\,d^2x)\sqrt{ex^2+d}\sqrt{-e}}{90\,d^3x^6}$$

input
```
integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^7,x, algorithm="fricas")
```

output
```
-1/90*(15*d^3*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) + (8*e^2*x^5 - 4*d*e*x^3
+ 3*d^2*x)*sqrt(e*x^2 + d)*sqrt(-e))/(d^3*x^6)
```

3.9.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 352 vs. 2(102) = 204.

Time = 2.68 (sec) , antiderivative size = 352, normalized size of antiderivative = 3.12

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^7}\,dx = -\frac{d^4e^{\frac{9}{2}}\sqrt{-e}\sqrt{\frac{d}{ex^2}+1}}{30d^5e^4x^4+60d^4e^5x^6+30d^3e^6x^8} - \frac{d^3e^{\frac{11}{2}}x^2\sqrt{-e}\sqrt{\frac{d}{ex^2}+1}}{45d^5e^4x^4+90d^4e^5x^6+45d^3e^6x^8} - \frac{d^2e^{\frac{13}{2}}x^4\sqrt{-e}\sqrt{\frac{d}{ex^2}+1}}{30d^5e^4x^4+60d^4e^5x^6+30d^3e^6x^8} - \frac{2de^{\frac{15}{2}}x^6\sqrt{-e}\sqrt{\frac{d}{ex^2}+1}}{15d^5e^4x^4+30d^4e^5x^6+15d^3e^6x^8} - \frac{4e^{\frac{17}{2}}x^8\sqrt{-e}\sqrt{\frac{d}{ex^2}+1}}{45d^5e^4x^4+90d^4e^5x^6+45d^3e^6x^8} - \frac{\operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{6x^6}$$

input
```
integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**7,x)
```

output
```
-d**4*e**(9/2)*sqrt(-e)*sqrt(d/(e*x**2) + 1)/(30*d**5*e**4*x**4 + 60*d**4*
e**5*x**6 + 30*d**3*e**6*x**8) - d**3*e**(11/2)*x**2*sqrt(-e)*sqrt(d/(e*x*
*2) + 1)/(45*d**5*e**4*x**4 + 90*d**4*e**5*x**6 + 45*d**3*e**6*x**8) - d**
2*e**(13/2)*x**4*sqrt(-e)*sqrt(d/(e*x**2) + 1)/(30*d**5*e**4*x**4 + 60*d**
4*e**5*x**6 + 30*d**3*e**6*x**8) - 2*d*e**(15/2)*x**6*sqrt(-e)*sqrt(d/(e*x
**2) + 1)/(15*d**5*e**4*x**4 + 30*d**4*e**5*x**6 + 15*d**3*e**6*x**8) - 4*
e**(17/2)*x**8*sqrt(-e)*sqrt(d/(e*x**2) + 1)/(45*d**5*e**4*x**4 + 90*d**4*
e**5*x**6 + 45*d**3*e**6*x**8) - atan(x*sqrt(-e)/sqrt(d + e*x**2))/(6*x**6
)
```

3.9.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.96

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^7}\,dx = -\frac{(2\,e^2x^4+dex^2-d^2)\sqrt{-e}e}{18\,\sqrt{ex^2+d}d^3x^3} - \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{6\,x^6} + \frac{(2\,e^2x^4-dex^2-3\,d^2)\sqrt{ex^2+d}\sqrt{-e}}{90\,d^3x^5}$$

input
```
integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^7,x, algorithm="maxima")
```

output
```
-1/18*(2*e^2*x^4 + d*e*x^2 - d^2)*sqrt(-e)*e/(sqrt(e*x^2 + d)*d^3*x^3) - 1
/6*arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^6 + 1/90*(2*e^2*x^4 - d*e*x^2 - 3*
d^2)*sqrt(e*x^2 + d)*sqrt(-e)/(d^3*x^5)
```

3.9.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. 2(89) = 178.

Time = 0.35 (sec) , antiderivative size = 275, normalized size of antiderivative = 2.43

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^7}\,dx = \frac{\left(3\,e^4+\frac{25\left(\sqrt{-d}e e+\sqrt{-e^2x^2-de}|e|\right)^2}{x^2}+\frac{150\left(\sqrt{-d}e e+\sqrt{-e^2x^2-de}|e|\right)^4}{e^4x^4}\right)e^{10}x^5}{2880\left(\sqrt{-d}e e+\sqrt{-e^2x^2-de}|e|\right)^5d^3|e|} - \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{6\,x^6} - \frac{\frac{150\left(\sqrt{-d}e e+\sqrt{-e^2x^2-de}|e|\right)d^{12}e^{16}}{x}+\frac{25\left(\sqrt{-d}e e+\sqrt{-e^2x^2-de}|e|\right)^3d^{12}e^{12}}{x^3}+\frac{3\left(\sqrt{-d}e e+\sqrt{-e^2x^2-de}|e|\right)^5d^{12}e^8}{x^5}}{2880\,d^{15}e^{14}|e|}$$

input
```
integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^7,x, algorithm="giac")
```

output
```
1/2880*(3*e^4 + 25*(sqrt(-d*e)*e + sqrt(-e^2*x^2 - d*e)*abs(e))^2/x^2 + 15
0*(sqrt(-d*e)*e + sqrt(-e^2*x^2 - d*e)*abs(e))^4/(e^4*x^4))*e^10*x^5/((sqr
t(-d*e)*e + sqrt(-e^2*x^2 - d*e)*abs(e))^5*d^3*abs(e)) - 1/6*arctan(sqrt(-
e)*x/sqrt(e*x^2 + d))/x^6 - 1/2880*(150*(sqrt(-d*e)*e + sqrt(-e^2*x^2 - d*
e)*abs(e))*d^12*e^16/x + 25*(sqrt(-d*e)*e + sqrt(-e^2*x^2 - d*e)*abs(e))^3
*d^12*e^12/x^3 + 3*(sqrt(-d*e)*e + sqrt(-e^2*x^2 - d*e)*abs(e))^5*d^12*e^8
/x^5)/(d^15*e^14*abs(e))
```

3.9.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^7}\,dx = \int \frac{\operatorname{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^7}\,dx$$

input
```
int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^7,x)
```

output
```
int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^7, x)
```

3.10 $\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^9}\,dx$

3.10.1 Optimal result

Integrand size = 25, antiderivative size = 141

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^9}\,dx = -\frac{\sqrt{-e}\sqrt{d+ex^2}}{56dx^7} - \frac{3(-e)^{3/2}\sqrt{d+ex^2}}{140d^2x^5} - \frac{(-e)^{5/2}\sqrt{d+ex^2}}{35d^3x^3} - \frac{2(-e)^{7/2}\sqrt{d+ex^2}}{35d^4x} - \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{8x^8}$$

output
```
-1/8*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^8-3/140*(-e)^(3/2)*(e*x^2+d)^(
1/2)/d^2/x^5-1/35*(-e)^(5/2)*(e*x^2+d)^(1/2)/d^3/x^3-2/35*(-e)^(7/2)*(e*x^
2+d)^(1/2)/d^4/x-1/56*(-e)^(1/2)*(e*x^2+d)^(1/2)/d/x^7
```

3.10.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.63

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^9}\,dx = \frac{\sqrt{-e}x\sqrt{d+ex^2}(-5d^3+6d^2ex^2-8de^2x^4+16e^3x^6) - 35d^4\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{280d^4x^8}$$

input
```
Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^9,x]
```

output
```
(Sqrt[-e]*x*Sqrt[d + e*x^2]*(-5*d^3 + 6*d^2*e*x^2 - 8*d*e^2*x^4 + 16*e^3*x
^6) - 35*d^4*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/(280*d^4*x^8)
```

3.10.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5674, 245, 245, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^9}dx$$

$$\Big\downarrow \text{5674}$$

$$\frac{1}{8}\sqrt{-e}\int \frac{1}{x^8\sqrt{ex^2+d}}dx-\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{8x^8}$$

$$\Big\downarrow \text{245}$$

$$\frac{1}{8}\sqrt{-e}\left(-\frac{6e\int \frac{1}{x^6\sqrt{ex^2+d}}dx}{7d}-\frac{\sqrt{d+ex^2}}{7dx^7}\right)-\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{8x^8}$$

$$\Big\downarrow \text{245}$$

$$\frac{1}{8}\sqrt{-e}\left(-\frac{6e\left(-\frac{4e\int \frac{1}{x^4\sqrt{ex^2+d}}dx}{5d}-\frac{\sqrt{d+ex^2}}{5dx^5}\right)}{7d}-\frac{\sqrt{d+ex^2}}{7dx^7}\right)-\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{8x^8}$$

$$\Big\downarrow \text{245}$$

$$\frac{1}{8}\sqrt{-e}\left(-\frac{6e\left(-\frac{4e\left(-\frac{2e\int \frac{1}{x^2\sqrt{ex^2+d}}dx}{3d}-\frac{\sqrt{d+ex^2}}{3dx^3}\right)}{5d}-\frac{\sqrt{d+ex^2}}{5dx^5}\right)}{7d}-\frac{\sqrt{d+ex^2}}{7dx^7}\right)-\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{8x^8}$$

$$\Big\downarrow \text{242}$$

$$\frac{1}{8}\sqrt{-e}\left(-\frac{6e\left(-\frac{4e\left(\frac{2e\sqrt{d+ex^2}}{3d^2x}-\frac{\sqrt{d+ex^2}}{3dx^3}\right)}{5d}-\frac{\sqrt{d+ex^2}}{5dx^5}\right)}{7d}-\frac{\sqrt{d+ex^2}}{7dx^7}\right)-\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{8x^8}$$

input
```
Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^9,x]
```

output
```
(Sqrt[-e]*(-1/7*Sqrt[d + e*x^2]/(d*x^7) - (6*e*(-1/5*Sqrt[d + e*x^2]/(d*x^
5) - (4*e*(-1/3*Sqrt[d + e*x^2]/(d*x^3) + (2*e*Sqrt[d + e*x^2])/(3*d^2*x))
)/(5*d)))/(7*d)))/8 - ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/(8*x^8)
```

3.10.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 5674
```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Simp[c/(d*(m + 1))   Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ
[{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

3.10.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.17

method	result
default	$-\frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{8x^8}-\frac{\sqrt{-e}\,e\left(-\frac{\sqrt{e\,x^2+d}}{5d\,x^5}-\frac{4e\left(-\frac{\sqrt{e\,x^2+d}}{3d\,x^3}+\frac{2e\sqrt{e\,x^2+d}}{3d^2x}\right)}{5d}\right)}{8d}+\frac{\sqrt{-e}\left(-\frac{\left(e\,x^2+d\right)^{\frac{3}{2}}}{7d\,x^7}-\frac{4e\left(-\frac{\left(e\,x^2+d\right)^{\frac{3}{2}}}{5d\,x^5}+\frac{2e\left(e\,x^2+d\right.}{15d^2x^3}\right.}{7d}\right.}{8d}$
parts	$-\frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{8x^8}-\frac{\sqrt{-e}\,e\left(-\frac{\sqrt{e\,x^2+d}}{5d\,x^5}-\frac{4e\left(-\frac{\sqrt{e\,x^2+d}}{3d\,x^3}+\frac{2e\sqrt{e\,x^2+d}}{3d^2x}\right)}{5d}\right)}{8d}+\frac{\sqrt{-e}\left(-\frac{\left(e\,x^2+d\right)^{\frac{3}{2}}}{7d\,x^7}-\frac{4e\left(-\frac{\left(e\,x^2+d\right)^{\frac{3}{2}}}{5d\,x^5}+\frac{2e\left(e\,x^2+d\right.}{15d^2x^3}\right.}{7d}\right.}{8d}$

input
```
int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/8*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^8-1/8*(-e)^(1/2)*e/d*(-1/5/d/x
^5*(e*x^2+d)^(1/2)-4/5*e/d*(-1/3/d/x^3*(e*x^2+d)^(1/2)+2/3*e/d^2/x*(e*x^2+
d)^(1/2)))+1/8*(-e)^(1/2)/d*(-1/7/d/x^7*(e*x^2+d)^(3/2)-4/7*e/d*(-1/5/d/x^
5*(e*x^2+d)^(3/2)+2/15*e/d^2/x^3*(e*x^2+d)^(3/2)))
```

3.10.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.57

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^9}\,dx$$
$$= -\frac{35\,d^4\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)-(16\,e^3x^7-8\,de^2x^5+6\,d^2ex^3-5\,d^3x)\sqrt{ex^2+d}\sqrt{-e}}{280\,d^4x^8}$$

input
```
integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^9,x, algorithm="fricas")
```

output
```
-1/280*(35*d^4*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - (16*e^3*x^7 - 8*d*e^2*
x^5 + 6*d^2*e*x^3 - 5*d^3*x)*sqrt(e*x^2 + d)*sqrt(-e))/(d^4*x^8)
```

3.10.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 575 vs. 2(128) = 256.

Time = 3.47 (sec) , antiderivative size = 575, normalized size of antiderivative = 4.08

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^9}\,dx = -\frac{5d^6e^{\frac{19}{2}}\sqrt{-e}\sqrt{\frac{d}{ex^2}+1}}{280d^7e^9x^6+840d^6e^{10}x^8+840d^5e^{11}x^{10}+280d^4e^{12}x^{12}} - \frac{9d^5e^{\frac{21}{2}}x^2\sqrt{-e}\sqrt{\frac{d}{ex^2}+1}}{280d^7e^9x^6+840d^6e^{10}x^8+840d^5e^{11}x^{10}+280d^4e^{12}x^{12}} - \frac{5d^4e^{\frac{23}{2}}x^4\sqrt{-e}\sqrt{\frac{d}{ex^2}+1}}{280d^7e^9x^6+840d^6e^{10}x^8+840d^5e^{11}x^{10}+280d^4e^{12}x^{12}} + \frac{5d^3e^{\frac{25}{2}}x^6\sqrt{-e}\sqrt{\frac{d}{ex^2}+1}}{280d^7e^9x^6+840d^6e^{10}x^8+840d^5e^{11}x^{10}+280d^4e^{12}x^{12}} + \frac{15d^2e^{\frac{27}{2}}x^8\sqrt{-e}\sqrt{\frac{d}{ex^2}+1}}{140d^7e^9x^6+420d^6e^{10}x^8+420d^5e^{11}x^{10}+140d^4e^{12}x^{12}} + \frac{5de^{\frac{29}{2}}x^{10}\sqrt{-e}\sqrt{\frac{d}{ex^2}+1}}{35d^7e^9x^6+105d^6e^{10}x^8+105d^5e^{11}x^{10}+35d^4e^{12}x^{12}} + \frac{2e^{\frac{31}{2}}x^{12}\sqrt{-e}\sqrt{\frac{d}{ex^2}+1}}{35d^7e^9x^6+105d^6e^{10}x^8+105d^5e^{11}x^{10}+35d^4e^{12}x^{12}} - \frac{\operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{8x^8}$$

input
```
integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**9,x)
```

output
```
-5*d**6*e**(19/2)*sqrt(-e)*sqrt(d/(e*x**2) + 1)/(280*d**7*e**9*x**6 + 840*
d**6*e**10*x**8 + 840*d**5*e**11*x**10 + 280*d**4*e**12*x**12) - 9*d**5*e*
*(21/2)*x**2*sqrt(-e)*sqrt(d/(e*x**2) + 1)/(280*d**7*e**9*x**6 + 840*d**6*
e**10*x**8 + 840*d**5*e**11*x**10 + 280*d**4*e**12*x**12) - 5*d**4*e**(23/
2)*x**4*sqrt(-e)*sqrt(d/(e*x**2) + 1)/(280*d**7*e**9*x**6 + 840*d**6*e**10
*x**8 + 840*d**5*e**11*x**10 + 280*d**4*e**12*x**12) + 5*d**3*e**(25/2)*x*
*6*sqrt(-e)*sqrt(d/(e*x**2) + 1)/(280*d**7*e**9*x**6 + 840*d**6*e**10*x**8
 + 840*d**5*e**11*x**10 + 280*d**4*e**12*x**12) + 15*d**2*e**(27/2)*x**8*s
qrt(-e)*sqrt(d/(e*x**2) + 1)/(140*d**7*e**9*x**6 + 420*d**6*e**10*x**8 + 4
20*d**5*e**11*x**10 + 140*d**4*e**12*x**12) + 5*d*e**(29/2)*x**10*sqrt(-e)
*sqrt(d/(e*x**2) + 1)/(35*d**7*e**9*x**6 + 105*d**6*e**10*x**8 + 105*d**5*
e**11*x**10 + 35*d**4*e**12*x**12) + 2*e**(31/2)*x**12*sqrt(-e)*sqrt(d/(e*
x**2) + 1)/(35*d**7*e**9*x**6 + 105*d**6*e**10*x**8 + 105*d**5*e**11*x**10
 + 35*d**4*e**12*x**12) - atan(x*sqrt(-e)/sqrt(d + e*x**2))/(8*x**8)
```

3.10.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.94

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^9}\,dx = \frac{(8\,e^3x^6+4\,de^2x^4-d^2ex^2+3\,d^3)\sqrt{-e}e}{120\,\sqrt{ex^2+d}d^4x^5} - \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{8\,x^8} - \frac{(8\,e^3x^6-4\,de^2x^4+3\,d^2ex^2+15\,d^3)\sqrt{ex^2+d}\sqrt{-e}}{840\,d^4x^7}$$

input
```
integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^9,x, algorithm="maxima")
```

output
```
1/120*(8*e^3*x^6 + 4*d*e^2*x^4 - d^2*e*x^2 + 3*d^3)*sqrt(-e)*e/(sqrt(e*x^2
 + d)*d^4*x^5) - 1/8*arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^8 - 1/840*(8*e^3
*x^6 - 4*d*e^2*x^4 + 3*d^2*e*x^2 + 15*d^3)*sqrt(e*x^2 + d)*sqrt(-e)/(d^4*x
^7)
```

3.10.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 353 vs. $2(111) = 222$.

Time = 0.35 (sec) , antiderivative size = 353, normalized size of antiderivative = 2.50

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^9}\,dx =$$

$$-\frac{\left(5\,e^5+\frac{49\left(\sqrt{-de}e+\sqrt{-e^2x^2-de}|e|\right)^2e}{x^2}+\frac{245\left(\sqrt{-de}e+\sqrt{-e^2x^2-de}|e|\right)^4}{e^3x^4}+\frac{1225\left(\sqrt{-de}e+\sqrt{-e^2x^2-de}|e|\right)^6}{e^7x^6}\right)e^{14}x^7}{35840\left(\sqrt{-de}e+\sqrt{-e^2x^2-de}|e|\right)^7d^4|e|}$$

$$-\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{8\,x^8}$$

$$+\frac{\frac{1225\left(\sqrt{-de}e+\sqrt{-e^2x^2-de}|e|\right)d^{24}e^{30}}{x}+\frac{245\left(\sqrt{-de}e+\sqrt{-e^2x^2-de}|e|\right)^3d^{24}e^{26}}{x^3}+\frac{49\left(\sqrt{-de}e+\sqrt{-e^2x^2-de}|e|\right)^5d^{24}e^{22}}{x^5}+\frac{5\left(\sqrt{-de}e+\cdots\right.}{\cdots}}{35840\,d^{28}e^{27}|e|}$$

input
```
integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^9,x, algorithm="giac")
```

output
```
-1/35840*(5*e^5 + 49*(sqrt(-d*e)*e + sqrt(-e^2*x^2 - d*e)*abs(e))^2*e/x^2
+ 245*(sqrt(-d*e)*e + sqrt(-e^2*x^2 - d*e)*abs(e))^4/(e^3*x^4) + 1225*(sqr
t(-d*e)*e + sqrt(-e^2*x^2 - d*e)*abs(e))^6/(e^7*x^6))*e^14*x^7/((sqrt(-d*e
)*e + sqrt(-e^2*x^2 - d*e)*abs(e))^7*d^4*abs(e)) - 1/8*arctan(sqrt(-e)*x/s
qrt(e*x^2 + d))/x^8 + 1/35840*(1225*(sqrt(-d*e)*e + sqrt(-e^2*x^2 - d*e)*a
bs(e))*d^24*e^30/x + 245*(sqrt(-d*e)*e + sqrt(-e^2*x^2 - d*e)*abs(e))^3*d^
24*e^26/x^3 + 49*(sqrt(-d*e)*e + sqrt(-e^2*x^2 - d*e)*abs(e))^5*d^24*e^22/
x^5 + 5*(sqrt(-d*e)*e + sqrt(-e^2*x^2 - d*e)*abs(e))^7*d^24*e^18/x^7)/(d^2
8*e^27*abs(e))
```

3.10.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^9}\,dx = \int \frac{\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^9}\,dx$$

input
```
int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^9,x)
```

output
```
int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^9, x)
```

3.11 $\int x^6 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx$

3.11.1 Optimal result

Integrand size = 25, antiderivative size = 124

$$\int x^6 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \frac{d^3\sqrt{d+ex^2}}{7(-e)^{7/2}} - \frac{d^2(d+ex^2)^{3/2}}{7(-e)^{7/2}} + \frac{3d(d+ex^2)^{5/2}}{35(-e)^{7/2}} - \frac{(d+ex^2)^{7/2}}{49(-e)^{7/2}} + \frac{1}{7}x^7 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)$$

output
```
-1/7*d^2*(e*x^2+d)^(3/2)/(-e)^(7/2)+3/35*d*(e*x^2+d)^(5/2)/(-e)^(7/2)-1/49
*(e*x^2+d)^(7/2)/(-e)^(7/2)+1/7*x^7*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+1
/7*d^3*(e*x^2+d)^(1/2)/(-e)^(7/2)
```

3.11.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.67

$$\int x^6 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \frac{\sqrt{d+ex^2}(16d^3-8d^2ex^2+6de^2x^4-5e^3x^6)}{245(-e)^{7/2}} + \frac{1}{7}x^7 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)$$

input
```
Integrate[x^6*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

output
```
(Sqrt[d + e*x^2]*(16*d^3 - 8*d^2*e*x^2 + 6*d*e^2*x^4 - 5*e^3*x^6))/(245*(-
e)^(7/2)) + (x^7*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/7
```

3.11.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {5674, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^6 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx$$

$$\Big\downarrow \ 5674$$

$$\frac{1}{7}x^7 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{1}{7}\sqrt{-e}\int \frac{x^7}{\sqrt{ex^2+d}}dx$$

$$\Big\downarrow \ 243$$

$$\frac{1}{7}x^7 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{1}{14}\sqrt{-e}\int \frac{x^6}{\sqrt{ex^2+d}}dx^2$$

$$\Big\downarrow \ 53$$

$$\frac{1}{7}x^7 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{1}{14}\sqrt{-e}\int \left(-\frac{d^3}{e^3\sqrt{ex^2+d}} + \frac{3\sqrt{ex^2+d}d^2}{e^3} - \frac{3(ex^2+d)^{3/2}d}{e^3} + \frac{(ex^2+d)^{5/2}}{e^3}\right)dx^2$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{7}x^7 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{1}{14}\sqrt{-e}\left(-\frac{2d^3\sqrt{d+ex^2}}{e^4} + \frac{2d^2(d+ex^2)^{3/2}}{e^4} + \frac{2(d+ex^2)^{7/2}}{7e^4} - \frac{6d(d+ex^2)^{5/2}}{5e^4}\right)$$

input
```
Int[x^6*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

output
```
-1/14*(Sqrt[-e]*((-2*d^3*Sqrt[d + e*x^2])/e^4 + (2*d^2*(d + e*x^2)^(3/2))/
e^4 - (6*d*(d + e*x^2)^(5/2))/(5*e^4) + (2*(d + e*x^2)^(7/2))/(7*e^4))) +
(x^7*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/7
```

3.11.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5674
```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Simp[c/(d*(m + 1))   Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ
[{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

3.11.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(94) = 188$.

Time = 0.04 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.86

method	result
default	$\frac{x^{7} \arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^{2}+d}}\right)}{7}-\frac{\sqrt{-e}\left(\frac{x^{6}\left(e\,x^{2}+d\right)^{\frac{3}{2}}}{9e}-\frac{2d\left(\frac{x^{4}\left(e\,x^{2}+d\right)^{\frac{3}{2}}}{7e}-\frac{4d\left(\frac{x^{2}\left(e\,x^{2}+d\right)^{\frac{3}{2}}}{5e}-\frac{2d\left(e\,x^{2}+d\right)^{\frac{3}{2}}}{15e^{2}}\right)}{7e}\right)}{3e}\right)}{7d}+\frac{\sqrt{-e}\,e\left(\frac{x^{8}\sqrt{e\,x^{2}+d}}{9e}-\cdots\right.}{\cdots}$
parts	$\frac{x^{7} \arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^{2}+d}}\right)}{7}-\frac{\sqrt{-e}\left(\frac{x^{6}\left(e\,x^{2}+d\right)^{\frac{3}{2}}}{9e}-\frac{2d\left(\frac{x^{4}\left(e\,x^{2}+d\right)^{\frac{3}{2}}}{7e}-\frac{4d\left(\frac{x^{2}\left(e\,x^{2}+d\right)^{\frac{3}{2}}}{5e}-\frac{2d\left(e\,x^{2}+d\right)^{\frac{3}{2}}}{15e^{2}}\right)}{7e}\right)}{3e}\right)}{7d}+\frac{\sqrt{-e}\,e\left(\frac{x^{8}\sqrt{e\,x^{2}+d}}{9e}-\cdots\right.}{\cdots}$

input
```
int(x^6*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/7*x^7*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))-1/7*(-e)^(1/2)/d*(1/9*x^6*(e*
x^2+d)^(3/2)/e-2/3*d/e*(1/7*x^4*(e*x^2+d)^(3/2)/e-4/7*d/e*(1/5*x^2*(e*x^2+
d)^(3/2)/e-2/15*d/e^2*(e*x^2+d)^(3/2))))+1/7*(-e)^(1/2)*e/d*(1/9*x^8/e*(e*
x^2+d)^(1/2)-8/9*d/e*(1/7*x^6/e*(e*x^2+d)^(1/2)-6/7*d/e*(1/5*x^4/e*(e*x^2+
d)^(1/2)-4/5*d/e*(1/3*x^2/e*(e*x^2+d)^(1/2)-2/3*d/e^2*(e*x^2+d)^(1/2)))))
```

3.11.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.64

$$\int x^6 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx$$
$$= \frac{35\,e^4x^7\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)-(5\,e^3x^6-6\,de^2x^4+8\,d^2ex^2-16\,d^3)\sqrt{ex^2+d}\sqrt{-e}}{245\,e^4}$$

input
```
integrate(x^6*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

output
```
1/245*(35*e^4*x^7*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - (5*e^3*x^6 - 6*d*e^
2*x^4 + 8*d^2*e*x^2 - 16*d^3)*sqrt(e*x^2 + d)*sqrt(-e))/e^4
```

3.11.6 Sympy [A] (verification not implemented)

Time = 1.43 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.10

$$\int x^6 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx$$
$$= \begin{cases} \frac{16d^3\sqrt{-e}\sqrt{d+ex^2}}{245e^4} - \frac{8d^2x^2\sqrt{-e}\sqrt{d+ex^2}}{245e^3} + \frac{6dx^4\sqrt{-e}\sqrt{d+ex^2}}{245e^2} + \frac{x^7 \operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{7} - \frac{x^6\sqrt{-e}\sqrt{d+ex^2}}{49e} & \text{for } e \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**6*atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)
```

output
```
Piecewise((16*d**3*sqrt(-e)*sqrt(d + e*x**2)/(245*e**4) - 8*d**2*x**2*sqrt
(-e)*sqrt(d + e*x**2)/(245*e**3) + 6*d*x**4*sqrt(-e)*sqrt(d + e*x**2)/(245
*e**2) + x**7*atan(x*sqrt(-e)/sqrt(d + e*x**2))/7 - x**6*sqrt(-e)*sqrt(d +
 e*x**2)/(49*e), Ne(e, 0)), (0, True))
```

3.11.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.35

$$\int x^6 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \frac{1}{7}x^7 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)$$
$$- \frac{\left(35\,(ex^2+d)^{\frac{9}{2}} - 135\,(ex^2+d)^{\frac{7}{2}}d + 189\,(ex^2+d)^{\frac{5}{2}}d^2 - 105\,(ex^2+d)^{\frac{3}{2}}d^3\right)\sqrt{-e}}{2205\,de^4}$$
$$+ \frac{\left(35\,(ex^2+d)^{\frac{9}{2}} - 180\,(ex^2+d)^{\frac{7}{2}}d + 378\,(ex^2+d)^{\frac{5}{2}}d^2 - 420\,(ex^2+d)^{\frac{3}{2}}d^3 + 315\,\sqrt{ex^2+d}d^4\right)\sqrt{-e}}{2205\,de^4}$$

input
```
integrate(x^6*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

output
```
1/7*x^7*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - 1/2205*(35*(e*x^2 + d)^(9/2)
- 135*(e*x^2 + d)^(7/2)*d + 189*(e*x^2 + d)^(5/2)*d^2 - 105*(e*x^2 + d)^(3
/2)*d^3)*sqrt(-e)/(d*e^4) + 1/2205*(35*(e*x^2 + d)^(9/2) - 180*(e*x^2 + d)
^(7/2)*d + 378*(e*x^2 + d)^(5/2)*d^2 - 420*(e*x^2 + d)^(3/2)*d^3 + 315*sqr
t(e*x^2 + d)*d^4)*sqrt(-e)/(d*e^4)
```

3.11.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.10

$$\int x^6 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx$$
$$= \frac{1}{7}x^7 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right) + \frac{\sqrt{-e^2x^2-de}d^3}{7e^4}$$
$$+ \frac{35(-e^2x^2-de)^{\frac{3}{2}}d^2e^2 + 21(e^2x^2+de)^2\sqrt{-e^2x^2-de}de - 5(e^2x^2+de)^3\sqrt{-e^2x^2-de}}{245e^7}$$

input
```
integrate(x^6*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

output
```
1/7*x^7*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) + 1/7*sqrt(-e^2*x^2 - d*e)*d^3/
e^4 + 1/245*(35*(-e^2*x^2 - d*e)^(3/2)*d^2*e^2 + 21*(e^2*x^2 + d*e)^2*sqrt
(-e^2*x^2 - d*e)*d*e - 5*(e^2*x^2 + d*e)^3*sqrt(-e^2*x^2 - d*e))/e^7
```

3.11.9 Mupad [F(-1)]

Timed out.

$$\int x^6 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \int x^6 \operatorname{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right) dx$$

input
```
int(x^6*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

output
```
int(x^6*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

3.12 $\int x^4 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx$

3.12.1 Optimal result

Integrand size = 25, antiderivative size = 99

$$\int x^4 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \frac{d^2\sqrt{d+ex^2}}{5(-e)^{5/2}} - \frac{2d(d+ex^2)^{3/2}}{15(-e)^{5/2}} + \frac{(d+ex^2)^{5/2}}{25(-e)^{5/2}} + \frac{1}{5}x^5 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)$$

output
```
-2/15*d*(e*x^2+d)^(3/2)/(-e)^(5/2)+1/25*(e*x^2+d)^(5/2)/(-e)^(5/2)+1/5*x^5
*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+1/5*d^2*(e*x^2+d)^(1/2)/(-e)^(5/2)
```

3.12.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.73

$$\int x^4 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \frac{\sqrt{d+ex^2}(8d^2-4dex^2+3e^2x^4)}{75(-e)^{5/2}} + \frac{1}{5}x^5 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)$$

input
```
Integrate[x^4*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

output
```
(Sqrt[d + e*x^2]*(8*d^2 - 4*d*e*x^2 + 3*e^2*x^4))/(75*(-e)^(5/2)) + (x^5*A
rcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/5
```

3.12.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {5674, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx$$

$$\Big\downarrow \text{5674}$$

$$\frac{1}{5}x^5 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{5}\sqrt{-e}\int \frac{x^5}{\sqrt{ex^2+d}}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{5}x^5 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{10}\sqrt{-e}\int \frac{x^4}{\sqrt{ex^2+d}}dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{5}x^5 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{10}\sqrt{-e}\int \left(\frac{d^2}{e^2\sqrt{ex^2+d}}-\frac{2\sqrt{ex^2+d}d}{e^2}+\frac{(ex^2+d)^{3/2}}{e^2}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}x^5 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{1}{10}\sqrt{-e}\left(\frac{2d^2\sqrt{d+ex^2}}{e^3}+\frac{2(d+ex^2)^{5/2}}{5e^3}-\frac{4d(d+ex^2)^{3/2}}{3e^3}\right)$$

input
```
Int[x^4*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

output
```
-1/10*(Sqrt[-e]*((2*d^2*Sqrt[d + e*x^2])/e^3 - (4*d*(d + e*x^2)^(3/2))/(3*
e^3) + (2*(d + e*x^2)^(5/2))/(5*e^3))) + (x^5*ArcTan[(Sqrt[-e]*x)/Sqrt[d +
 e*x^2]])/5
```

3.12.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5674
```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Simp[c/(d*(m + 1))   Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ
[{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

3.12.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(75) = 150$.

Time = 0.03 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.85

method	result
default	$\frac{x^5 \arctan\left(\frac{x\sqrt{-e}}{\sqrt{e x^2+d}}\right)}{5} + \frac{\sqrt{-e}\, e \left(\frac{x^6\sqrt{e x^2+d}}{7e} - \frac{6d\left(\frac{x^4\sqrt{e x^2+d}}{5e} - \frac{4d\left(\frac{x^2\sqrt{e x^2+d}}{3e} - \frac{2d\sqrt{e x^2+d}}{3e^2}\right)}{5e}\right)}{7e}\right)}{5d} - \frac{\sqrt{-e}\left(\frac{x^4\left(e x^2+d\right)^{\frac{3}{2}}}{7e} - \frac{4d\left(x^2 \cdots\right.}{\cdots}\right.}{5 \cdots}$
parts	$\frac{x^5 \arctan\left(\frac{x\sqrt{-e}}{\sqrt{e x^2+d}}\right)}{5} + \frac{\sqrt{-e}\, e \left(\frac{x^6\sqrt{e x^2+d}}{7e} - \frac{6d\left(\frac{x^4\sqrt{e x^2+d}}{5e} - \frac{4d\left(\frac{x^2\sqrt{e x^2+d}}{3e} - \frac{2d\sqrt{e x^2+d}}{3e^2}\right)}{5e}\right)}{7e}\right)}{5d} - \frac{\sqrt{-e}\left(\frac{x^4\left(e x^2+d\right)^{\frac{3}{2}}}{7e} - \frac{4d\left(x^2 \cdots\right.}{\cdots}\right.}{5 \cdots}$

input
```
int(x^4*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/5*x^5*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+1/5*(-e)^(1/2)*e/d*(1/7*x^6/e
*(e*x^2+d)^(1/2)-6/7*d/e*(1/5*x^4/e*(e*x^2+d)^(1/2)-4/5*d/e*(1/3*x^2/e*(e*
x^2+d)^(1/2)-2/3*d/e^2*(e*x^2+d)^(1/2))))-1/5*(-e)^(1/2)/d*(1/7*x^4*(e*x^2
+d)^(3/2)/e-4/7*d/e*(1/5*x^2*(e*x^2+d)^(3/2)/e-2/15*d/e^2*(e*x^2+d)^(3/2))
)
```

3.12.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.69

$$\int x^4 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \frac{15\,e^3x^5\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right) - (3\,e^2x^4 - 4\,dex^2 + 8\,d^2)\sqrt{ex^2+d}\sqrt{-e}}{75\,e^3}$$

input
```
integrate(x^4*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

output
```
1/75*(15*e^3*x^5*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - (3*e^2*x^4 - 4*d*e*x
^2 + 8*d^2)*sqrt(e*x^2 + d)*sqrt(-e))/e^3
```

3.12.6 Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.06

$$\int x^4 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \begin{cases} -\frac{8d^2\sqrt{-e}\sqrt{d+ex^2}}{75e^3} + \frac{4dx^2\sqrt{-e}\sqrt{d+ex^2}}{75e^2} + \frac{x^5\,\mathrm{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{5} - \frac{x^4\sqrt{-e}\sqrt{d+ex^2}}{25e} & \text{for } e \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**4*atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)
```

output
```
Piecewise((-8*d**2*sqrt(-e)*sqrt(d + e*x**2)/(75*e**3) + 4*d*x**2*sqrt(-e)
*sqrt(d + e*x**2)/(75*e**2) + x**5*atan(x*sqrt(-e)/sqrt(d + e*x**2))/5 - x
**4*sqrt(-e)*sqrt(d + e*x**2)/(25*e), Ne(e, 0)), (0, True))
```

3.12.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.40

$$\int x^4 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx$$
$$= \frac{1}{5}x^5\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right) - \frac{\left(15\,(ex^2+d)^{\frac{7}{2}} - 42\,(ex^2+d)^{\frac{5}{2}}d + 35\,(ex^2+d)^{\frac{3}{2}}d^2\right)\sqrt{-e}}{525\,de^3}$$
$$+ \frac{\left(5\,(ex^2+d)^{\frac{7}{2}} - 21\,(ex^2+d)^{\frac{5}{2}}d + 35\,(ex^2+d)^{\frac{3}{2}}d^2 - 35\,\sqrt{ex^2+d}d^3\right)\sqrt{-e}}{175\,de^3}$$

input
```
integrate(x^4*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

output
```
1/5*x^5*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - 1/525*(15*(e*x^2 + d)^(7/2) -
 42*(e*x^2 + d)^(5/2)*d + 35*(e*x^2 + d)^(3/2)*d^2)*sqrt(-e)/(d*e^3) + 1/1
75*(5*(e*x^2 + d)^(7/2) - 21*(e*x^2 + d)^(5/2)*d + 35*(e*x^2 + d)^(3/2)*d^
2 - 35*sqrt(e*x^2 + d)*d^3)*sqrt(-e)/(d*e^3)
```

3.12.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.02

$$\int x^4 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx = \frac{1}{5}x^5\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right) - \frac{\sqrt{-e^2x^2-de}d^2}{5\,e^3}$$
$$- \frac{10\,(-e^2x^2-de)^{\frac{3}{2}}de + 3\,(e^2x^2+de)^2\sqrt{-e^2x^2-de}}{75\,e^5}$$

input
```
integrate(x^4*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

output
```
1/5*x^5*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - 1/5*sqrt(-e^2*x^2 - d*e)*d^2/
e^3 - 1/75*(10*(-e^2*x^2 - d*e)^(3/2)*d*e + 3*(e^2*x^2 + d*e)^2*sqrt(-e^2*
x^2 - d*e))/e^5
```

3.12.9 Mupad [F(-1)]

Timed out.

$$\int x^4 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \int x^4 \operatorname{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right) dx$$

input
```
int(x^4*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

output
```
int(x^4*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

3.13 $\int x^2 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx$

3.13.1 Optimal result

Integrand size = 25, antiderivative size = 74

$$\int x^2 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \frac{d\sqrt{d+ex^2}}{3(-e)^{3/2}} - \frac{(d+ex^2)^{3/2}}{9(-e)^{3/2}} + \frac{1}{3}x^3 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)$$

output
```
-1/9*(e*x^2+d)^(3/2)/(-e)^(3/2)+1/3*x^3*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2
))+1/3*d*(e*x^2+d)^(1/2)/(-e)^(3/2)
```

3.13.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.81

$$\int x^2 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \frac{1}{9}\left(\frac{(2d-ex^2)\sqrt{d+ex^2}}{(-e)^{3/2}} + 3x^3 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\right)$$

input
```
Integrate[x^2*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

output
```
(((2*d - e*x^2)*Sqrt[d + e*x^2])/(-e)^(3/2) + 3*x^3*ArcTan[(Sqrt[-e]*x)/Sq
rt[d + e*x^2]])/9
```

3.13.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {5674, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx$$

$$\Big\downarrow \text{ 5674}$$

$$\frac{1}{3}x^3 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{1}{3}\sqrt{-e}\int \frac{x^3}{\sqrt{ex^2+d}}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{3}x^3 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{1}{6}\sqrt{-e}\int \frac{x^2}{\sqrt{ex^2+d}}dx^2$$

$$\Big\downarrow \text{ 53}$$

$$\frac{1}{3}x^3 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{1}{6}\sqrt{-e}\int \left(\frac{\sqrt{ex^2+d}}{e} - \frac{d}{e\sqrt{ex^2+d}}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}x^3 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{1}{6}\sqrt{-e}\left(\frac{2\left(d+ex^2\right)^{3/2}}{3e^2} - \frac{2d\sqrt{d+ex^2}}{e^2}\right)$$

input
```
Int[x^2*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

output
```
-1/6*(Sqrt[-e]*((-2*d*Sqrt[d + e*x^2])/e^2 + (2*(d + e*x^2)^(3/2))/(3*e^2)
)) + (x^3*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/3
```

3.13.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5674
```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Simp[c/(d*(m + 1))   Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ
[{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

3.13.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(56) = 112$.

Time = 0.02 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.82

method	result	size
default	$\frac{x^{3} \arctan\left(\frac{x \sqrt{-e}}{\sqrt{e\,x^{2}+d}}\right)}{3}+\frac{\sqrt{-e}\,e \left(\frac{x^{4} \sqrt{e\,x^{2}+d}}{5e}-\frac{4d \left(\frac{x^{2} \sqrt{e\,x^{2}+d}}{3e}-\frac{2d \sqrt{e\,x^{2}+d}}{3e^{2}}\right)}{5e}\right)}{3d}-\frac{\sqrt{-e}\left(\frac{x^{2} \left(e\,x^{2}+d\right)^{\frac{3}{2}}}{5e}-\frac{2d \left(e\,x^{2}+d\right)^{\frac{3}{2}}}{15e^{2}}\right)}{3d}$	135
parts	$\frac{x^{3} \arctan\left(\frac{x \sqrt{-e}}{\sqrt{e\,x^{2}+d}}\right)}{3}+\frac{\sqrt{-e}\,e \left(\frac{x^{4} \sqrt{e\,x^{2}+d}}{5e}-\frac{4d \left(\frac{x^{2} \sqrt{e\,x^{2}+d}}{3e}-\frac{2d \sqrt{e\,x^{2}+d}}{3e^{2}}\right)}{5e}\right)}{3d}-\frac{\sqrt{-e}\left(\frac{x^{2} \left(e\,x^{2}+d\right)^{\frac{3}{2}}}{5e}-\frac{2d \left(e\,x^{2}+d\right)^{\frac{3}{2}}}{15e^{2}}\right)}{3d}$	135

input
```
int(x^2*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+1/3*(-e)^(1/2)*e/d*(1/5*x^4/e
*(e*x^2+d)^(1/2)-4/5*d/e*(1/3*x^2/e*(e*x^2+d)^(1/2)-2/3*d/e^2*(e*x^2+d)^(1
/2)))-1/3*(-e)^(1/2)/d*(1/5*x^2*(e*x^2+d)^(3/2)/e-2/15*d/e^2*(e*x^2+d)^(3/
2))
```

3.13.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.76

$$\int x^2 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \frac{3\,e^2x^3\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right) - \sqrt{ex^2+d}(ex^2-2\,d)\sqrt{-e}}{9\,e^2}$$

input
```
integrate(x^2*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

output
```
1/9*(3*e^2*x^3*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - sqrt(e*x^2 + d)*(e*x^2
 - 2*d)*sqrt(-e))/e^2
```

3.13.6 Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.01

$$\int x^2 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \begin{cases} \frac{2d\sqrt{-e}\sqrt{d+ex^2}}{9e^2} + \frac{x^3\,\mathrm{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{3} - \frac{x^2\sqrt{-e}\sqrt{d+ex^2}}{9e} & \text{for } e \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)
```

output
```
Piecewise((2*d*sqrt(-e)*sqrt(d + e*x**2)/(9*e**2) + x**3*atan(x*sqrt(-e)/s
qrt(d + e*x**2))/3 - x**2*sqrt(-e)*sqrt(d + e*x**2)/(9*e), Ne(e, 0)), (0,
True))
```

3.13.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.50

$$\int x^2 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \frac{1}{3}\,x^3\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right) - \frac{\left(3\,(ex^2+d)^{\frac{5}{2}} - 5\,(ex^2+d)^{\frac{3}{2}}d\right)\sqrt{-e}}{45\,de^2} + \frac{\left(3\,(ex^2+d)^{\frac{5}{2}} - 10\,(ex^2+d)^{\frac{3}{2}}d + 15\,\sqrt{ex^2+d}d^2\right)\sqrt{-e}}{45\,de^2}$$

input
```
integrate(x^2*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

output
```
1/3*x^3*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - 1/45*(3*(e*x^2 + d)^(5/2) - 5
*(e*x^2 + d)^(3/2)*d)*sqrt(-e)/(d*e^2) + 1/45*(3*(e*x^2 + d)^(5/2) - 10*(e
*x^2 + d)^(3/2)*d + 15*sqrt(e*x^2 + d)*d^2)*sqrt(-e)/(d*e^2)
```

3.13.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int x^2 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \frac{1}{3}x^3 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right) + \frac{\sqrt{-e^2x^2-ded}}{3e^2} + \frac{(-e^2x^2-de)^{\frac{3}{2}}}{9e^3}$$

input
```
integrate(x^2*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

output
```
1/3*x^3*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) + 1/3*sqrt(-e^2*x^2 - d*e)*d/e^
2 + 1/9*(-e^2*x^2 - d*e)^(3/2)/e^3
```

3.13.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \int x^2 \operatorname{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right) dx$$

input
```
int(x^2*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

output
```
int(x^2*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

3.14 $\int \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx$

3.14.1 Optimal result

Integrand size = 21, antiderivative size = 43

$$\int \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \frac{\sqrt{d+ex^2}}{\sqrt{-e}} + x\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)$$

output
```
x*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+(e*x^2+d)^(1/2)/(-e)^(1/2)
```

3.14.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \frac{\sqrt{d+ex^2}}{\sqrt{-e}} + x\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)$$

input
```
Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

output
```
Sqrt[d + e*x^2]/Sqrt[-e] + x*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]
```

3.14.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5670, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)dx$$

$$\Big\downarrow \text{5670}$$

$$x\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\sqrt{-e}\int \frac{x}{\sqrt{ex^2+d}}dx$$

$$\Big\downarrow \text{241}$$

$$x\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{\sqrt{-e}\sqrt{d+ex^2}}{e}$$

input
```
Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

output
```
-((Sqrt[-e]*Sqrt[d + e*x^2])/e) + x*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]
```

3.14.3.1 Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5670
```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]], x_Symbol] :> Simp[x*Ar
cTan[(c*x)/Sqrt[a + b*x^2]], x] - Simp[c   Int[x/Sqrt[a + b*x^2], x], x] /;
 FreeQ[{a, b, c}, x] && EqQ[b + c^2, 0]
```

3.14.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(35) = 70$.

Time = 0.02 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.00

method	result	size
default	$x\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)+\frac{\sqrt{-e}\,e\left(\frac{x^2\sqrt{e\,x^2+d}}{3e}-\frac{2d\sqrt{e\,x^2+d}}{3e^2}\right)}{d}-\frac{\sqrt{-e}\,(e\,x^2+d)^{\frac{3}{2}}}{3de}$	86
parts	$x\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)+\frac{\sqrt{-e}\,e\left(\frac{x^2\sqrt{e\,x^2+d}}{3e}-\frac{2d\sqrt{e\,x^2+d}}{3e^2}\right)}{d}-\frac{\sqrt{-e}\,(e\,x^2+d)^{\frac{3}{2}}}{3de}$	86

input
```
int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
x*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+(-e)^(1/2)*e/d*(1/3*x^2/e*(e*x^2+d)
^(1/2)-2/3*d/e^2*(e*x^2+d)^(1/2))-1/3*(-e)^(1/2)/d*(e*x^2+d)^(3/2)/e
```

3.14.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx = \frac{ex\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)-\sqrt{ex^2+d}\sqrt{-e}}{e}$$

input
```
integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="fricas")
```

output
```
(e*x*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - sqrt(e*x^2 + d)*sqrt(-e))/e
```

3.14.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx = \begin{cases} x\,\mathrm{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)-\frac{\sqrt{-e}\sqrt{d+ex^2}}{e} & \text{for } e\neq 0\\ 0 & \text{otherwise}\end{cases}$$

input
```
integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)
```

output
```
Piecewise((x*atan(x*sqrt(-e)/sqrt(d + e*x**2)) - sqrt(-e)*sqrt(d + e*x**2)
/e, Ne(e, 0)), (0, True))
```

3.14.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 77 vs. $2(35) = 70$.

Time = 0.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.79

$$\int \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx = x\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right) - \frac{(ex^2+d)^{\frac{3}{2}}\sqrt{-e}}{3\,de} + \frac{\left((ex^2+d)^{\frac{3}{2}} - 3\sqrt{ex^2+d}d\right)\sqrt{-e}}{3\,de}$$

input
```
integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxima")
```

output
```
x*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - 1/3*(e*x^2 + d)^(3/2)*sqrt(-e)/(d*e
) + 1/3*((e*x^2 + d)^(3/2) - 3*sqrt(e*x^2 + d)*d)*sqrt(-e)/(d*e)
```

3.14.8 Giac [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx = x\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right) - \frac{\sqrt{-e^2x^2-de}}{e}$$

input
```
integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac")
```

output
```
x*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - sqrt(-e^2*x^2 - d*e)/e
```

3.14.9 Mupad [B] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx = \frac{\sqrt{e\,x^2+d}}{\sqrt{-e}} + x\,\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)$$

input
```
int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

output
```
(d + e*x^2)^(1/2)/(-e)^(1/2) + x*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))
```

3.15 $\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^2}\,dx$

3.15.1 Optimal result

Integrand size = 25, antiderivative size = 59

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^2}\,dx = -\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x} - \frac{\sqrt{-e}\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}}$$

output
```
-arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x-arctanh((e*x^2+d)^(1/2)/d^(1/2))*(
-e)^(1/2)/d^(1/2)
```

3.15.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.46

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^2}\,dx = -\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x} + \frac{i\sqrt{e}\log\left(\frac{2i\sqrt{d}}{\sqrt{e}x} - \frac{2\sqrt{-e}\sqrt{d+ex^2}}{ex}\right)}{\sqrt{d}}$$

input
```
Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^2,x]
```

output
```
-(ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x) + (I*Sqrt[e]*Log[((2*I)*Sqrt[d])
/(Sqrt[e]*x) - (2*Sqrt[-e]*Sqrt[d + e*x^2])/(e*x)])/Sqrt[d]
```

3.15.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {5674, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^2}dx$$

$$\Big\downarrow \; 5674$$

$$\sqrt{-e}\int \frac{1}{x\sqrt{ex^2+d}}dx-\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x}$$

$$\Big\downarrow \; 243$$

$$\frac{1}{2}\sqrt{-e}\int \frac{1}{x^2\sqrt{ex^2+d}}dx^2-\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x}$$

$$\Big\downarrow \; 73$$

$$\frac{\sqrt{-e}\int \frac{1}{\frac{x^4}{e}-\frac{d}{e}}d\sqrt{ex^2+d}}{e}-\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x}$$

$$\Big\downarrow \; 221$$

$$-\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x}-\frac{\sqrt{-e}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}}$$

input
```
Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^2,x]
```

output
```
-(ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x) - (Sqrt[-e]*ArcTanh[Sqrt[d + e*x
^2]/Sqrt[d]])/Sqrt[d]
```

3.15.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5674
```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Simp[c/(d*(m + 1))   Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ
[{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

3.15.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.53

method	result	size
default	$-\frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{x}-\frac{\sqrt{-e}\,\sqrt{e\,x^2+d}}{d}+\frac{\sqrt{-e}\,\left(\sqrt{e\,x^2+d}-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{e\,x^2+d}}{x}\right)\right)}{d}$	90
parts	$-\frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{x}-\frac{\sqrt{-e}\,\sqrt{e\,x^2+d}}{d}+\frac{\sqrt{-e}\,\left(\sqrt{e\,x^2+d}-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{e\,x^2+d}}{x}\right)\right)}{d}$	90

input
```
int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x-(-e)^(1/2)/d*(e*x^2+d)^(1/2)+(-e)^
(1/2)/d*((e*x^2+d)^(1/2)-d^(1/2)*ln((2*d+2*d^(1/2)*(e*x^2+d)^(1/2))/x))
```

3.15.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.51

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^2}\,dx = \left[\frac{x\sqrt{-\frac{e}{d}}\log\left(-\frac{e^2x^2+2\sqrt{ex^2+d}d\sqrt{-e}\sqrt{-\frac{e}{d}}+2\,de}{x^2}\right)-2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{2\,x}, -\frac{x\sqrt{\frac{e}{d}}\arctan\left(\frac{\sqrt{ex^2+d}d\sqrt{-e}\sqrt{\frac{e}{d}}}{e^2x^2+de}\right)+\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x}\right]$$

input
```
integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^2,x, algorithm="fricas")
```

output
```
[1/2*(x*sqrt(-e/d)*log(-(e^2*x^2 + 2*sqrt(e*x^2 + d)*d*sqrt(-e)*sqrt(-e/d)
 + 2*d*e)/x^2) - 2*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)))/x, -(x*sqrt(e/d)*ar
ctan(sqrt(e*x^2 + d)*d*sqrt(-e)*sqrt(e/d)/(e^2*x^2 + d*e)) + arctan(sqrt(-
e)*x/sqrt(e*x^2 + d)))/x]
```

3.15.6 Sympy [A] (verification not implemented)

Time = 1.93 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.12

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^2}\,dx = \frac{\sqrt{-e}\left(\begin{cases}\frac{2\,\mathrm{atan}\left(\frac{\sqrt{d+ex^2}}{\sqrt{-d}}\right)}{\sqrt{-d}} & \text{for } e\neq 0\\ \frac{\log\left(x^2\right)}{\sqrt{d}} & \text{otherwise}\end{cases}\right)}{2}-\frac{\mathrm{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{x}$$

input
```
integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**2,x)
```

output
```
sqrt(-e)*Piecewise((2*atan(sqrt(d + e*x**2)/sqrt(-d))/sqrt(-d), Ne(e, 0)),
 (log(x**2)/sqrt(d), True))/2 - atan(x*sqrt(-e)/sqrt(d + e*x**2))/x
```

3.15.7 Maxima [F]

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^2}\,dx = \int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x^2}\,dx$$

input
```
integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^2,x, algorithm="maxima")
```

output
```
(d*sqrt(-e)*x*integrate(-sqrt(e*x^2 + d)/(e^2*x^5 + d*e*x^3 - (e*x^3 + d*x
)*(e*x^2 + d)), x) - arctan2(sqrt(-e)*x, sqrt(e*x^2 + d)))/x
```

3.15.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^2}\,dx = -\frac{e\arctan\left(\frac{\sqrt{-e^2x^2-de}}{\sqrt{de}}\right)}{\sqrt{de}} - \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x}$$

input
```
integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^2,x, algorithm="giac")
```

output
```
-e*arctan(sqrt(-e^2*x^2 - d*e)/sqrt(d*e))/sqrt(d*e) - arctan(sqrt(-e)*x/sq
rt(e*x^2 + d))/x
```

3.15.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^2}\,dx = \int \frac{\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^2}\,dx$$

input
```
int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^2,x)
```

output
```
int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^2, x)
```

3.16 $\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^4}\,dx$

3.16.1 Optimal result

Integrand size = 25, antiderivative size = 91

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^4}\,dx = -\frac{\sqrt{-e}\sqrt{d+ex^2}}{6dx^2} - \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{3x^3} - \frac{(-e)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{6d^{3/2}}$$

output
```
-1/3*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^3-1/6*(-e)^(3/2)*arctanh((e*x^
2+d)^(1/2)/d^(1/2))/d^(3/2)-1/6*(-e)^(1/2)*(e*x^2+d)^(1/2)/d/x^2
```

3.16.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.11

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^4}\,dx = -\frac{\sqrt{-e}\sqrt{d+ex^2}}{6dx^2} + \frac{e^{3/2}\arctan\left(\frac{\sqrt{d}\sqrt{-e}}{\sqrt{e}\sqrt{d+ex^2}}\right)}{6d^{3/2}} - \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{3x^3}$$

input
```
Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^4,x]
```

output
```
-1/6*(Sqrt[-e]*Sqrt[d + e*x^2])/(d*x^2) + (e^(3/2)*ArcTan[(Sqrt[d]*Sqrt[-e
])/(Sqrt[e]*Sqrt[d + e*x^2])])/(6*d^(3/2)) - ArcTan[(Sqrt[-e]*x)/Sqrt[d +
e*x^2]]/(3*x^3)
```

3.16.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5674, 243, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^4}dx$$

$$\Big\downarrow \text{5674}$$

$$\frac{1}{3}\sqrt{-e}\int \frac{1}{x^3\sqrt{ex^2+d}}dx-\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{3x^3}$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{6}\sqrt{-e}\int \frac{1}{x^4\sqrt{ex^2+d}}dx^2-\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{3x^3}$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{6}\sqrt{-e}\left(-\frac{e\int \frac{1}{x^2\sqrt{ex^2+d}}dx^2}{2d}-\frac{\sqrt{d+ex^2}}{dx^2}\right)-\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{3x^3}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{6}\sqrt{-e}\left(-\frac{\int \frac{1}{\frac{x^4}{e}-\frac{d}{e}}d\sqrt{ex^2+d}}{d}-\frac{\sqrt{d+ex^2}}{dx^2}\right)-\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{3x^3}$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{6}\sqrt{-e}\left(\frac{e\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{\sqrt{d+ex^2}}{dx^2}\right)-\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{3x^3}$$

input
```
Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^4,x]
```

output
```
-1/3*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^3 + (Sqrt[-e]*(-(Sqrt[d + e*x^
2]/(d*x^2)) + (e*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/d^(3/2)))/6
```

3.16.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5674
```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Simp[c/(d*(m + 1))   Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ
[{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

3.16.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.43

method	result	size
default	$-\frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^{2}+d}}\right)}{3x^{3}}+\frac{\sqrt{-e}\,e\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{e\,x^{2}+d}}{x}\right)}{3d^{\frac{3}{2}}}+\frac{\sqrt{-e}\left(-\frac{\left(e\,x^{2}+d\right)^{\frac{3}{2}}}{2d\,x^{2}}+\frac{e\left(\sqrt{e\,x^{2}+d}-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{e\,x^{2}+d}}{x}\right)\right)}{2d}\right)}{3d}$	130
parts	$-\frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^{2}+d}}\right)}{3x^{3}}+\frac{\sqrt{-e}\,e\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{e\,x^{2}+d}}{x}\right)}{3d^{\frac{3}{2}}}+\frac{\sqrt{-e}\left(-\frac{\left(e\,x^{2}+d\right)^{\frac{3}{2}}}{2d\,x^{2}}+\frac{e\left(\sqrt{e\,x^{2}+d}-\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{e\,x^{2}+d}}{x}\right)\right)}{2d}\right)}{3d}$	130

---

input
```
int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^3+1/3*(-e)^(1/2)*e/d^(3/2)*ln(
(2*d+2*d^(1/2)*(e*x^2+d)^(1/2))/x)+1/3*(-e)^(1/2)/d*(-1/2/d/x^2*(e*x^2+d)^
(3/2)+1/2*e/d*((e*x^2+d)^(1/2)-d^(1/2)*ln((2*d+2*d^(1/2)*(e*x^2+d)^(1/2))/
x)))
```

3.16.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.18

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^4}\,dx = \left[\frac{ex^3\sqrt{-\frac{e}{d}}\log\left(-\frac{e^2x^2-2\sqrt{ex^2+d}d\sqrt{-e}\sqrt{-\frac{e}{d}}+2\,de}{x^2}\right)-2\sqrt{ex^2+d}\sqrt{-e}x-4\,d\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{12\,dx^3}, \frac{ex^3\sqrt{\frac{e}{d}}\arctan\right.$$

input
```
integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^4,x, algorithm="fricas")
```

output
```
[1/12*(e*x^3*sqrt(-e/d)*log(-(e^2*x^2 - 2*sqrt(e*x^2 + d)*d*sqrt(-e)*sqrt(
-e/d) + 2*d*e)/x^2) - 2*sqrt(e*x^2 + d)*sqrt(-e)*x - 4*d*arctan(sqrt(-e)*x
/sqrt(e*x^2 + d)))/(d*x^3), 1/6*(e*x^3*sqrt(e/d)*arctan(sqrt(e*x^2 + d)*d*
sqrt(-e)*sqrt(e/d)/(e^2*x^2 + d*e)) - sqrt(e*x^2 + d)*sqrt(-e)*x - 2*d*arc
tan(sqrt(-e)*x/sqrt(e*x^2 + d)))/(d*x^3)]
```

3.16.6 Sympy [A] (verification not implemented)

Time = 3.89 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.90

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^4}\,dx = -\frac{\operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{3x^3} - \frac{\sqrt{e}\sqrt{-e}\sqrt{\frac{d}{ex^2}+1}}{6dx} + \frac{e\sqrt{-e}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{e}x}\right)}{6d^{\frac{3}{2}}}$$

input
```
integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**4,x)
```

output
```
-atan(x*sqrt(-e)/sqrt(d + e*x**2))/(3*x**3) - sqrt(e)*sqrt(-e)*sqrt(d/(e*x
**2) + 1)/(6*d*x) + e*sqrt(-e)*asinh(sqrt(d)/(sqrt(e)*x))/(6*d**(3/2))
```

3.16.7 Maxima [F]

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^4}\,dx = \int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x^4}\,dx$$

input
```
integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^4,x, algorithm="maxima")
```

output
```
1/3*(3*d*sqrt(-e)*x^3*integrate(-1/3*sqrt(e*x^2 + d)/(e^2*x^7 + d*e*x^5 -
(e*x^5 + d*x^3)*(e*x^2 + d)), x) - arctan2(sqrt(-e)*x, sqrt(e*x^2 + d)))/x
^3
```

3.16.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.96

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^4}\,dx = \frac{\frac{e^3\arctan\left(\frac{\sqrt{-e^2x^2-de}}{\sqrt{de}}\right)}{\sqrt{ded}} - \frac{\sqrt{-e^2x^2-dee}}{dx^2}}{6\,e} - \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{3\,x^3}$$

input
```
integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^4,x, algorithm="giac")
```

output
```
1/6*(e^3*arctan(sqrt(-e^2*x^2 - d*e)/sqrt(d*e))/(sqrt(d*e)*d) - sqrt(-e^2*
x^2 - d*e)*e/(d*x^2))/e - 1/3*arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^3
```

3.16.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^4}\,dx = \int \frac{\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^4}\,dx$$

input
```
int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^4,x)
```

output
```
int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^4, x)
```

3.17 $\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^6}\,dx$

3.17.1 Optimal result

Integrand size = 25, antiderivative size = 119

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^6}\,dx = -\frac{\sqrt{-e}\sqrt{d+ex^2}}{20dx^4} - \frac{3(-e)^{3/2}\sqrt{d+ex^2}}{40d^2x^2} - \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{5x^5} - \frac{3(-e)^{5/2}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{40d^{5/2}}$$

output
```
-1/5*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^5-3/40*(-e)^(5/2)*arctanh((e*x
^2+d)^(1/2)/d^(1/2))/d^(5/2)-3/40*(-e)^(3/2)*(e*x^2+d)^(1/2)/d^2/x^2-1/20*
(-e)^(1/2)*(e*x^2+d)^(1/2)/d/x^4
```

3.17.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.96

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^6}\,dx = \sqrt{-e}\left(-\frac{1}{20dx^4} + \frac{3e}{40d^2x^2}\right)\sqrt{d+ex^2} - \frac{3e^{5/2}\arctan\left(\frac{\sqrt{d}\sqrt{-e}}{\sqrt{e}\sqrt{d+ex^2}}\right)}{40d^{5/2}} - \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{5x^5}$$

input
```
Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^6,x]
```

output
```
Sqrt[-e]*(-1/20*1/(d*x^4) + (3*e)/(40*d^2*x^2))*Sqrt[d + e*x^2] - (3*e^(5/
2)*ArcTan[(Sqrt[d]*Sqrt[-e])/(Sqrt[e]*Sqrt[d + e*x^2])])/(40*d^(5/2)) - Ar
cTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/(5*x^5)
```

3.17.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {5674, 243, 52, 52, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^6}dx$$

$$\Big\downarrow \text{5674}$$

$$\frac{1}{5}\sqrt{-e}\int \frac{1}{x^5\sqrt{ex^2+d}}dx-\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{5x^5}$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{10}\sqrt{-e}\int \frac{1}{x^6\sqrt{ex^2+d}}dx^2-\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{5x^5}$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{10}\sqrt{-e}\left(-\frac{3e\int \frac{1}{x^4\sqrt{ex^2+d}}dx^2}{4d}-\frac{\sqrt{d+ex^2}}{2dx^4}\right)-\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{5x^5}$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{10}\sqrt{-e}\left(-\frac{3e\left(-\frac{e\int \frac{1}{x^2\sqrt{ex^2+d}}dx^2}{2d}-\frac{\sqrt{d+ex^2}}{dx^2}\right)}{4d}-\frac{\sqrt{d+ex^2}}{2dx^4}\right)-\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{5x^5}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{10}\sqrt{-e}\left(-\frac{3e\left(-\frac{\int \frac{1}{\frac{x^4}{e}-\frac{d}{e}}d\sqrt{ex^2+d}}{d}-\frac{\sqrt{d+ex^2}}{dx^2}\right)}{4d}-\frac{\sqrt{d+ex^2}}{2dx^4}\right)-\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{5x^5}$$

$$\Big\downarrow \; 221$$

$$\frac{1}{10}\sqrt{-e}\left(-\frac{3e\left(\frac{e\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{\sqrt{d+ex^2}}{dx^2}\right)}{4d}-\frac{\sqrt{d+ex^2}}{2dx^4}\right)-\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{5x^5}$$

input
```
Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^6,x]
```

output
```
-1/5*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^5 + (Sqrt[-e]*(-1/2*Sqrt[d + e
*x^2]/(d*x^4) - (3*e*(-(Sqrt[d + e*x^2]/(d*x^2)) + (e*ArcTanh[Sqrt[d + e*x
^2]/Sqrt[d]])/d^(3/2)))/(4*d)))/10
```

3.17.3.1 Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5674
```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Simp[c/(d*(m + 1))   Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ
[{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

3.17.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.50

method	result
default	$-\frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e x^2+d}}\right)}{5x^5}-\frac{\sqrt{-e}\, e\left(-\frac{\sqrt{e x^2+d}}{2 d x^2}+\frac{e \ln\left(\frac{2d+2\sqrt{d}\sqrt{e x^2+d}}{x}\right)}{2 d^{\frac{3}{2}}}\right)}{5d}+\frac{\sqrt{-e}\left(-\frac{\left(e x^2+d\right)^{\frac{3}{2}}}{4 d x^4}-\frac{e\left(-\frac{\left(e x^2+d\right)^{\frac{3}{2}}}{2 d x^2}+\frac{e\left(\sqrt{e x^2+d}-\sqrt{d}\cdots\right.}{\cdots}\right.}{4d}\cdots\right.}{5d}\cdots$
parts	$-\frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e x^2+d}}\right)}{5x^5}-\frac{\sqrt{-e}\, e\left(-\frac{\sqrt{e x^2+d}}{2 d x^2}+\frac{e \ln\left(\frac{2d+2\sqrt{d}\sqrt{e x^2+d}}{x}\right)}{2 d^{\frac{3}{2}}}\right)}{5d}+\frac{\sqrt{-e}\left(-\frac{\left(e x^2+d\right)^{\frac{3}{2}}}{4 d x^4}-\frac{e\left(-\frac{\left(e x^2+d\right)^{\frac{3}{2}}}{2 d x^2}+\frac{e\left(\sqrt{e x^2+d}-\sqrt{d}\cdots\right.}{\cdots}\right.}{4d}\cdots\right.}{5d}\cdots$

input
```
int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^5-1/5*(-e)^(1/2)*e/d*(-1/2/d/x
^2*(e*x^2+d)^(1/2)+1/2*e/d^(3/2)*ln((2*d+2*d^(1/2)*(e*x^2+d)^(1/2))/x))+1/
5*(-e)^(1/2)/d*(-1/4/d/x^4*(e*x^2+d)^(3/2)-1/4*e/d*(-1/2/d/x^2*(e*x^2+d)^(
3/2)+1/2*e/d*((e*x^2+d)^(1/2)-d^(1/2)*ln((2*d+2*d^(1/2)*(e*x^2+d)^(1/2))/x
))))
```

3.17.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.92

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^6}\,dx$$
$$= \left[\frac{3\,e^2x^5\sqrt{-\frac{e}{d}}\log\left(-\frac{e^2x^2+2\,\sqrt{ex^2+d}d\sqrt{-e}\sqrt{-\frac{e}{d}}+2\,de}{x^2}\right)-16\,d^2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)+2\,(3\,ex^3-2\,dx)\sqrt{ex^2+d}\sqrt{-e}}{80\,d^2x^5},\right.$$
$$\left.-\frac{3\,e^2x^5\sqrt{\frac{e}{d}}\arctan\left(\frac{\sqrt{ex^2+d}d\sqrt{-e}\sqrt{\frac{e}{d}}}{e^2x^2+de}\right)+8\,d^2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)-(3\,ex^3-2\,dx)\sqrt{ex^2+d}\sqrt{-e}}{40\,d^2x^5}\right]$$

input
```
integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^6,x, algorithm="fricas")
```

output
```
[1/80*(3*e^2*x^5*sqrt(-e/d)*log(-(e^2*x^2 + 2*sqrt(e*x^2 + d)*d*sqrt(-e)*s
qrt(-e/d) + 2*d*e)/x^2) - 16*d^2*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) + 2*(3
*e*x^3 - 2*d*x)*sqrt(e*x^2 + d)*sqrt(-e))/(d^2*x^5), -1/40*(3*e^2*x^5*sqrt
(e/d)*arctan(sqrt(e*x^2 + d)*d*sqrt(-e)*sqrt(e/d)/(e^2*x^2 + d*e)) + 8*d^2
*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - (3*e*x^3 - 2*d*x)*sqrt(e*x^2 + d)*sq
rt(-e))/(d^2*x^5)]
```

3.17.6 Sympy [A] (verification not implemented)

Time = 6.19 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.24

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^6}\,dx = -\frac{\operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{5x^5} - \frac{\sqrt{-e}}{20\sqrt{e}x^5\sqrt{\frac{d}{ex^2}+1}} + \frac{\sqrt{e}\sqrt{-e}}{40dx^3\sqrt{\frac{d}{ex^2}+1}} + \frac{3e^{\frac{3}{2}}\sqrt{-e}}{40d^2x\sqrt{\frac{d}{ex^2}+1}} - \frac{3e^2\sqrt{-e}\operatorname{asinh}\left(\frac{\sqrt{d}}{\sqrt{e}x}\right)}{40d^{\frac{5}{2}}}$$

input
```
integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**6,x)
```

output
```
-atan(x*sqrt(-e)/sqrt(d + e*x**2))/(5*x**5) - sqrt(-e)/(20*sqrt(e)*x**5*sq
rt(d/(e*x**2) + 1)) + sqrt(e)*sqrt(-e)/(40*d*x**3*sqrt(d/(e*x**2) + 1)) +
3*e**(3/2)*sqrt(-e)/(40*d**2*x*sqrt(d/(e*x**2) + 1)) - 3*e**2*sqrt(-e)*asi
nh(sqrt(d)/(sqrt(e)*x))/(40*d**(5/2))
```

3.17.7 Maxima [F]

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^6}\,dx = \int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x^6}\,dx$$

input
```
integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^6,x, algorithm="maxima")
```

output
```
1/5*(5*d*sqrt(-e)*x^5*integrate(-1/5*sqrt(e*x^2 + d)/(e^2*x^9 + d*e*x^7 -
(e*x^7 + d*x^5)*(e*x^2 + d)), x) - arctan2(sqrt(-e)*x, sqrt(e*x^2 + d)))/x
^5
```

3.17.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.97

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^6}\,dx = -\frac{\frac{3\,e^4\arctan\left(\frac{\sqrt{-e^2x^2-de}}{\sqrt{de}}\right)}{\sqrt{de}d^2} + \frac{5\,\sqrt{-e^2x^2-de}de^5+3\,(-e^2x^2-de)^{\frac{3}{2}}e^4}{d^2e^4x^4}}{40\,e} - \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{5\,x^5}$$

input
```
integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^6,x, algorithm="giac")
```

output
```
-1/40*(3*e^4*arctan(sqrt(-e^2*x^2 - d*e)/sqrt(d*e))/(sqrt(d*e)*d^2) + (5*s
qrt(-e^2*x^2 - d*e)*d*e^5 + 3*(-e^2*x^2 - d*e)^(3/2)*e^4)/(d^2*e^4*x^4))/e
 - 1/5*arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^5
```

3.17.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^6}\,dx = \int \frac{\operatorname{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^6}\,dx$$

input
```
int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^6,x)
```

output
```
int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^6, x)
```

3.18 $\int x^{9/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx$

3.18.1 Optimal result

Integrand size = 27, antiderivative size = 211

$$\int x^{9/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)dx=\frac{60d^2\sqrt{x}\sqrt{d+ex^2}}{847(-e)^{5/2}}+\frac{36dx^{5/2}\sqrt{d+ex^2}}{847(-e)^{3/2}}+\frac{4x^{9/2}\sqrt{d+ex^2}}{121\sqrt{-e}}+\frac{2}{11}x^{11/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)+\frac{30d^{11/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{847e^{13/4}\sqrt{d+ex^2}}$$

output
```
2/11*x^(11/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+36/847*d*x^(5/2)*(e*x^2
+d)^(1/2)/(-e)^(3/2)+4/121*x^(9/2)*(e*x^2+d)^(1/2)/(-e)^(1/2)+60/847*d^2*x
^(1/2)*(e*x^2+d)^(1/2)/(-e)^(5/2)+30/847*d^(11/4)*(cos(2*arctan(e^(1/4)*x^
(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticF(
sin(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(-e)^(1/2)*(d^(1/2)+x*
e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/e^(13/4)/(e*x^2+d)^(1/2)
```

3.18.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.41 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.81

$$\int x^{9/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx=\frac{4\sqrt{x}\sqrt{d+ex^2}(15d^2-9dex^2+7e^2x^4)}{847(-e)^{5/2}}+\frac{2}{11}x^{11/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{60id^3\sqrt{1+\frac{d}{ex^2}}\,x\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right),-1\right)}{847\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}(-e)^{5/2}\sqrt{d+ex^2}}$$

input
```
Integrate[x^(9/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

output
```
(4*Sqrt[x]*Sqrt[d + e*x^2]*(15*d^2 - 9*d*e*x^2 + 7*e^2*x^4))/(847*(-e)^(5/
2)) + (2*x^(11/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/11 - (((60*I)/847)
*d^3*Sqrt[1 + d/(e*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[d])/Sqrt[e]]/S
qrt[x]], -1])/(Sqrt[(I*Sqrt[d])/Sqrt[e]]*(-e)^(5/2)*Sqrt[d + e*x^2])
```

3.18.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5674, 262, 262, 262, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{9/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx$$

$$\Big\downarrow \text{5674}$$

$$\frac{2}{11}x^{11/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{2}{11}\sqrt{-e}\int\frac{x^{11/2}}{\sqrt{ex^2+d}}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{2}{11}x^{11/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{2}{11}\sqrt{-e}\left(\frac{2x^{9/2}\sqrt{d+ex^2}}{11e}-\frac{9d\int\frac{x^{7/2}}{\sqrt{ex^2+d}}dx}{11e}\right)$$

$$\Big\downarrow \text{262}$$

$$\frac{2}{11}x^{11/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{2}{11}\sqrt{-e}\left(\frac{2x^{9/2}\sqrt{d+ex^2}}{11e}-\frac{9d\left(\frac{2x^{5/2}\sqrt{d+ex^2}}{7e}-\frac{5d\int\frac{x^{3/2}}{\sqrt{ex^2+d}}dx}{7e}\right)}{11e}\right)$$

$$\Big\downarrow \textbf{262}$$

$$\frac{2}{11}x^{11/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{2}{11}\sqrt{-e}\left(\frac{2x^{9/2}\sqrt{d+ex^2}}{11e}-\frac{9d\left(\frac{2x^{5/2}\sqrt{d+ex^2}}{7e}-\frac{5d\left(\frac{2\sqrt{x}\sqrt{d+ex^2}}{3e}-\frac{d\int\frac{1}{\sqrt{x}\sqrt{ex^2+d}}dx}{3e}\right)}{7e}\right)}{11e}\right)$$

$$\Big\downarrow \textbf{266}$$

$$\frac{2}{11}x^{11/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{2}{11}\sqrt{-e}\left(\frac{2x^{9/2}\sqrt{d+ex^2}}{11e}-\frac{9d\left(\frac{2x^{5/2}\sqrt{d+ex^2}}{7e}-\frac{5d\left(\frac{2\sqrt{x}\sqrt{d+ex^2}}{3e}-\frac{2d\int\frac{1}{\sqrt{ex^2+d}}d\sqrt{x}}{3e}\right)}{7e}\right)}{11e}\right)$$

$$\Big\downarrow \textbf{761}$$

3.18. $\int x^{9/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx$

$$\frac{2}{11}x^{11/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{2}{11}\sqrt{-e}\left(\frac{2x^{9/2}\sqrt{d+ex^2}}{11e}-\frac{9d\left(\frac{2x^{5/2}\sqrt{d+ex^2}}{7e}-\frac{5d\left(\frac{2\sqrt{x}\sqrt{d+ex^2}}{3e}-\frac{d^{3/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{3e^{5/4}\sqrt{d+ex^2}}\right)}{7e}\right)}{11e}\right)$$

input
```
Int[x^(9/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

output
```
(2*x^(11/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/11 - (2*Sqrt[-e]*((2*x^(
9/2)*Sqrt[d + e*x^2])/(11*e) - (9*d*((2*x^(5/2)*Sqrt[d + e*x^2])/(7*e) - (
5*d*((2*Sqrt[x]*Sqrt[d + e*x^2])/(3*e) - (d^(3/4)*(Sqrt[d] + Sqrt[e]*x)*Sq
rt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x
])/d^(1/4)], 1/2])/(3*e^(5/4)*Sqrt[d + e*x^2])))/(7*e)))/(11*e)))/11
```

3.18.3.1 Defintions of rubi rules used

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 5674
```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Simp[c/(d*(m + 1))   Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ
[{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

3.18.4 Maple [F]

$$\int x^{\frac{9}{2}} \arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^{2}+d}}\right) dx$$

input
```
int(x^(9/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
```

output
```
int(x^(9/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
```

3.18.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.13 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.45

$$\int x^{9/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)dx = \frac{2\left(77\,e^4x^{\frac{11}{2}}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)+30\,d^3\sqrt{-e}\sqrt{e}\text{weierstrassPInverse}\left(-\frac{4\,d}{e},0,x\right.}{847\,e^4}$$

input
```
integrate(x^(9/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="frica
s")
```

output
```
2/847*(77*e^4*x^(11/2)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) + 30*d^3*sqrt(-e
)*sqrt(e)*weierstrassPInverse(-4*d/e, 0, x) - 2*(7*e^3*x^4 - 9*d*e^2*x^2 +
 15*d^2*e)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt(x))/e^4
```

3.18.6 Sympy [F(-1)]

Timed out.

$$\int x^{9/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx=\text{Timed out}$$

input
```
integrate(x**(9/2)*atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)
```

output
```
Timed out
```

3.18.7 Maxima [F]

$$\int x^{9/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx=\int x^{\frac{9}{2}}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)\,dx$$

input
```
integrate(x^(9/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxim
a")
```

output
```
2/11*x^(11/2)*arctan2(sqrt(-e)*x, sqrt(e*x^2 + d)) - 2*d*sqrt(-e)*integrat
e(-1/11*x*e^(1/2*log(e*x^2 + d) + 9/2*log(x))/(e^2*x^4 + d*e*x^2 - (e*x^2
+ d)^2), x)
```

3.18.8 Giac [F]

$$\int x^{9/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx=\int x^{\frac{9}{2}}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)\,dx$$

input
```
integrate(x^(9/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac"
)
```

output
```
integrate(x^(9/2)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)), x)
```

3.18.9 Mupad [F(-1)]

Timed out.

$$\int x^{9/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx=\int x^{9/2}\,\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)\,dx$$

input
```
int(x^(9/2)*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

output
```
int(x^(9/2)*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

3.19 $\int x^{5/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx$

3.19.1 Optimal result

Integrand size = 27, antiderivative size = 181

$$\int x^{5/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx=\frac{20d\sqrt{x}\sqrt{d+ex^2}}{147(-e)^{3/2}}+\frac{4x^{5/2}\sqrt{d+ex^2}}{49\sqrt{-e}}+\frac{2}{7}x^{7/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{10d^{7/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{147e^{9/4}\sqrt{d+ex^2}}$$

output
```
2/7*x^(7/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+4/49*x^(5/2)*(e*x^2+d)^(1
/2)/(-e)^(1/2)+20/147*d*x^(1/2)*(e*x^2+d)^(1/2)/(-e)^(3/2)-10/147*d^(7/4)*
(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1
/2)/d^(1/4)))*EllipticF(sin(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2)
)*(-e)^(1/2)*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/e
^(9/4)/(e*x^2+d)^(1/2)
```

3.19.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.87

$$\int x^{5/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx=\frac{2}{147}\sqrt{x}\left(\frac{2(5d-3ex^2)\sqrt{d+ex^2}}{(-e)^{3/2}}+21x^3\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\right)-\frac{20id^2\sqrt{1+\frac{d}{ex^2}}x\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right),-1\right)}{147\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}(-e)^{3/2}\sqrt{d+ex^2}}$$

input
```
Integrate[x^(5/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

output
```
(2*Sqrt[x]*((2*(5*d - 3*e*x^2)*Sqrt[d + e*x^2])/(-e)^(3/2) + 21*x^3*ArcTan
[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]))/147 - (((20*I)/147)*d^2*Sqrt[1 + d/(e*x^2
)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[d])/Sqrt[e]]/Sqrt[x]], -1])/(Sqrt[(I
*Sqrt[d])/Sqrt[e]]*(-e)^(3/2)*Sqrt[d + e*x^2])
```

3.19.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5674, 262, 262, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx$$

$$\Big\downarrow \text{ 5674}$$

$$\frac{2}{7}x^{7/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{2}{7}\sqrt{-e}\int\frac{x^{7/2}}{\sqrt{ex^2+d}}dx$$

$$\Big\downarrow \text{ 262}$$

$$\frac{2}{7}x^{7/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{2}{7}\sqrt{-e}\left(\frac{2x^{5/2}\sqrt{d+ex^2}}{7e}-\frac{5d\int\frac{x^{3/2}}{\sqrt{ex^2+d}}dx}{7e}\right)$$

$$\Big\downarrow \text{ 262}$$

$$\frac{2}{7}x^{7/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{2}{7}\sqrt{-e}\left(\frac{2x^{5/2}\sqrt{d+ex^2}}{7e}-\frac{5d\left(\frac{2\sqrt{x}\sqrt{d+ex^2}}{3e}-\frac{d\int\frac{1}{\sqrt{x}\sqrt{ex^2+d}}dx}{3e}\right)}{7e}\right)$$

$$\Big\downarrow \text{ 266}$$

$$\frac{2}{7}x^{7/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{2}{7}\sqrt{-e}\left(\frac{2x^{5/2}\sqrt{d+ex^2}}{7e}-\frac{5d\left(\frac{2\sqrt{x}\sqrt{d+ex^2}}{3e}-\frac{2d\int\frac{1}{\sqrt{ex^2+d}}d\sqrt{x}}{3e}\right)}{7e}\right)$$

$$\Big\downarrow \text{ 761}$$

$$\frac{2}{7}x^{7/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{2}{7}\sqrt{-e}\left(\frac{2x^{5/2}\sqrt{d+ex^2}}{7e}-\frac{5d\left(\frac{2\sqrt{x}\sqrt{d+ex^2}}{3e}-\frac{d^{3/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{3e^{5/4}\sqrt{d+ex^2}}\right)}{7e}\right)$$

input
```
Int[x^(5/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

output
```
(2*x^(7/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/7 - (2*Sqrt[-e]*((2*x^(5/
2)*Sqrt[d + e*x^2])/(7*e) - (5*d*((2*Sqrt[x]*Sqrt[d + e*x^2])/(3*e) - (d^(
3/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*Ellip
ticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(3*e^(5/4)*Sqrt[d + e*x^2]
)))/(7*e)))/7
```

3.19.3.1 Defintions of rubi rules used

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 5674
```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Simp[c/(d*(m + 1))   Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ
[{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

3.19.4 Maple [F]

$$\int x^{\frac{5}{2}} \arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right) dx$$

input
```
int(x^(5/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
```

output
```
int(x^(5/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
```

3.19.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.47

$$\int x^{5/2} \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \frac{2\left(21\,e^3x^{\frac{7}{2}}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right) - 10\,d^2\sqrt{-e}\sqrt{e}\text{weierstrassPInverse}\left(-\frac{4\,d}{e}, 0, x\right.}{147\,e^3}$$

input
```
integrate(x^(5/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="frica
s")
```

output
```
2/147*(21*e^3*x^(7/2)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - 10*d^2*sqrt(-e)
*sqrt(e)*weierstrassPInverse(-4*d/e, 0, x) - 2*(3*e^2*x^2 - 5*d*e)*sqrt(e*
x^2 + d)*sqrt(-e)*sqrt(x))/e^3
```

3.19.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 43.61 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.41

$$\int x^{5/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx=\frac{2x^{\frac{7}{2}}\operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{7}-\frac{x^{\frac{9}{2}}\sqrt{-e}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{9}{4}\\\frac{13}{4}\end{matrix}\middle|\frac{ex^2e^{i\pi}}{d}\right)}{7\sqrt{d}\Gamma\left(\frac{13}{4}\right)}$$

input
```
integrate(x**(5/2)*atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)
```

output
```
2*x**(7/2)*atan(x*sqrt(-e)/sqrt(d + e*x**2))/7 - x**(9/2)*sqrt(-e)*gamma(9
/4)*hyper((1/2, 9/4), (13/4,), e*x**2*exp_polar(I*pi)/d)/(7*sqrt(d)*gamma(
13/4))
```

3.19.7 Maxima [F]

$$\int x^{5/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx=\int x^{\frac{5}{2}}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)\,dx$$

input
```
integrate(x^(5/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxim
a")
```

output
```
2/7*x^(7/2)*arctan2(sqrt(-e)*x, sqrt(e*x^2 + d)) - 2*d*sqrt(-e)*integrate(
-1/7*x*e^(1/2*log(e*x^2 + d) + 5/2*log(x))/(e^2*x^4 + d*e*x^2 - (e*x^2 + d
)^2), x)
```

3.19.8 Giac [F]

$$\int x^{5/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx=\int x^{\frac{5}{2}}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)\,dx$$

input
```
integrate(x^(5/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac"
)
```

output
```
integrate(x^(5/2)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)), x)
```

3.19.9 Mupad [F(-1)]

Timed out.

$$\int x^{5/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx=\int x^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)\,dx$$

input
```
int(x^(5/2)*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

output
```
int(x^(5/2)*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

3.20 $\int \sqrt{x} \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx$

3.20.1 Optimal result

Integrand size = 27, antiderivative size = 153

$$\int \sqrt{x} \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \frac{4\sqrt{x}\sqrt{d+ex^2}}{9\sqrt{-e}} + \frac{2}{3}x^{3/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) + \frac{2d^{3/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{9e^{5/4}\sqrt{d+ex^2}}$$

output
```
2/3*x^(3/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+4/9*x^(1/2)*(e*x^2+d)^(1/
2)/(-e)^(1/2)+2/9*d^(3/4)*(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)
/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticF(sin(2*arctan(e^(1/4)*x^(
1/2)/d^(1/4))),1/2*2^(1/2))*(-e)^(1/2)*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(
1/2)+x*e^(1/2))^2)^(1/2)/e^(5/4)/(e*x^2+d)^(1/2)
```

3.20.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.96

$$\int \sqrt{x}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)dx = \frac{4\sqrt{x}\sqrt{d+ex^2}}{9\sqrt{-e}} + \frac{2}{3}x^{3/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{4id\sqrt{1+\frac{d}{ex^2}}\,x\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right),-1\right)}{9\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\sqrt{-e}\sqrt{d+ex^2}}$$

input
```
Integrate[Sqrt[x]*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

output
```
(4*Sqrt[x]*Sqrt[d + e*x^2])/(9*Sqrt[-e]) + (2*x^(3/2)*ArcTan[(Sqrt[-e]*x)/
Sqrt[d + e*x^2]])/3 - (((4*I)/9)*d*Sqrt[1 + d/(e*x^2)]*x*EllipticF[I*ArcSi
nh[Sqrt[(I*Sqrt[d])/Sqrt[e]]/Sqrt[x]], -1])/(Sqrt[(I*Sqrt[d])/Sqrt[e]]*Sqr
t[-e]*Sqrt[d + e*x^2])
```

3.20.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {5674, 262, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)dx$$

$$\Big\downarrow \text{ 5674}$$

$$\frac{2}{3}x^{3/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{2}{3}\sqrt{-e}\int \frac{x^{3/2}}{\sqrt{ex^2+d}}dx$$

$$\Big\downarrow \text{ 262}$$

$$\frac{2}{3}x^{3/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{2}{3}\sqrt{-e}\left(\frac{2\sqrt{x}\sqrt{d+ex^2}}{3e} - \frac{d\int \frac{1}{\sqrt{x}\sqrt{ex^2+d}}dx}{3e}\right)$$

$$\Big\downarrow 266$$

$$\frac{2}{3}x^{3/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{2}{3}\sqrt{-e}\left(\frac{2\sqrt{x}\sqrt{d+ex^2}}{3e}-\frac{2d\int\frac{1}{\sqrt{ex^2+d}}d\sqrt{x}}{3e}\right)$$

$$\Big\downarrow 761$$

$$\frac{2}{3}x^{3/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{2}{3}\sqrt{-e}\left(\frac{2\sqrt{x}\sqrt{d+ex^2}}{3e}-\frac{d^{3/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{3e^{5/4}\sqrt{d+ex^2}}\right)$$

input
```
Int[Sqrt[x]*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

output
```
(2*x^(3/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/3 - (2*Sqrt[-e]*((2*Sqrt[
x]*Sqrt[d + e*x^2])/(3*e) - (d^(3/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2
)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)],
1/2])/(3*e^(5/4)*Sqrt[d + e*x^2])))/3
```

3.20.3.1 Defintions of rubi rules used

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 5674
```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Simp[c/(d*(m + 1))   Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ
[{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

3.20.4 Maple [F]

$$\int \sqrt{x}\, \arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^{2}+d}}\right)dx$$

input
```
int(x^(1/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
```

output
```
int(x^(1/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
```

3.20.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.46

$$\int \sqrt{x} \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \frac{2\left(3\,e^{2}x^{\frac{3}{2}}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^{2}+d}}\right)+2\,d\sqrt{-e}\sqrt{e}\mathrm{weierstrassPInverse}\left(-\frac{4\,d}{e},0,x\right)-2\,\sqrt{ex^{2}+d}\sqrt{-e}e\sqrt{x}\right)}{9\,e^{2}}$$

input
```
integrate(x^(1/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="frica
s")
```

output
```
2/9*(3*e^2*x^(3/2)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) + 2*d*sqrt(-e)*sqrt(
e)*weierstrassPInverse(-4*d/e, 0, x) - 2*sqrt(e*x^2 + d)*sqrt(-e)*e*sqrt(x
))/e^2
```

3.20.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 8.64 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.49

$$\int \sqrt{x}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)dx = \frac{2x^{\frac{3}{2}}\operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{3} - \frac{x^{\frac{5}{2}}\sqrt{-e}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{ex^2e^{i\pi}}{d}\right)}{3\sqrt{d}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate(x**(1/2)*atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)
```

output
```
2*x**(3/2)*atan(x*sqrt(-e)/sqrt(d + e*x**2))/3 - x**(5/2)*sqrt(-e)*gamma(5
/4)*hyper((1/2, 5/4), (9/4,), e*x**2*exp_polar(I*pi)/d)/(3*sqrt(d)*gamma(9
/4))
```

3.20.7 Maxima [F]

$$\int \sqrt{x}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)dx = \int \sqrt{x}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)dx$$

input
```
integrate(x^(1/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxim
a")
```

output
```
2/3*x^(3/2)*arctan2(sqrt(-e)*x, sqrt(e*x^2 + d)) - 2*d*sqrt(-e)*integrate(
-1/3*x*e^(1/2*log(e*x^2 + d) + 1/2*log(x))/(e^2*x^4 + d*e*x^2 - (e*x^2 + d
)^2), x)
```

3.20.8 Giac [F]

$$\int \sqrt{x}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)dx = \int \sqrt{x}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)dx$$

input
```
integrate(x^(1/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac"
)
```

output
```
integrate(sqrt(x)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)), x)
```

3.20.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{x} \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \int \sqrt{x}\, \mathrm{atan}\left(\frac{\sqrt{-e}\, x}{\sqrt{e\, x^2+d}}\right) dx$$

input
```
int(x^(1/2)*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

output
```
int(x^(1/2)*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

3.21 $\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{3/2}}\,dx$

3.21.1 Optimal result

Integrand size = 27, antiderivative size = 122

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{3/2}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}} + \frac{2\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{\sqrt[4]{d}\sqrt[4]{e}\sqrt{d+ex^2}}$$

output
```
-2*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(1/2)+2*(cos(2*arctan(e^(1/4)*x^
(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticF(
sin(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(-e)^(1/2)*(d^(1/2)+x*
e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/d^(1/4)/e^(1/4)/(e*x^2+d)
^(1/2)
```

3.21.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.94

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{3/2}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}} + \frac{4i\sqrt{-e}\sqrt{1+\frac{d}{ex^2}}x\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right),-1\right)}{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\sqrt{d+ex^2}}$$

input
```
Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(3/2),x]
```

output
```
(-2*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/Sqrt[x] + ((4*I)*Sqrt[-e]*Sqrt[1
 + d/(e*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[d])/Sqrt[e]]/Sqrt[x]], -1
])/(Sqrt[(I*Sqrt[d])/Sqrt[e]]*Sqrt[d + e*x^2])
```

3.21.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5674, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{3/2}}dx$$

$$\Big\downarrow \text{ 5674}$$

$$2\sqrt{-e}\int \frac{1}{\sqrt{x}\sqrt{ex^2+d}}dx - \frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}$$

$$\Big\downarrow \text{ 266}$$

$$4\sqrt{-e}\int \frac{1}{\sqrt{ex^2+d}}d\sqrt{x} - \frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}$$

$$\Big\downarrow \text{761}$$

$$\frac{2\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{\sqrt[4]{d}\sqrt[4]{e}\sqrt{d+ex^2}}-\frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}$$

input
```
Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(3/2),x]
```

output
```
(-2*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/Sqrt[x] + (2*Sqrt[-e]*(Sqrt[d] +
 Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(
e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(d^(1/4)*e^(1/4)*Sqrt[d + e*x^2])
```

3.21.3.1 Defintions of rubi rules used

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 5674
```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Simp[c/(d*(m + 1))   Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ
[{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

3.21.4 Maple [F]

$$\int \frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{x^{\frac{3}{2}}}dx$$

input
```
int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(3/2),x)
```

output
```
int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(3/2),x)
```

3.21.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.43

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{3/2}}\,dx = \frac{2\left(2\sqrt{-e}\sqrt{e}x\text{weierstrassPInverse}\left(-\frac{4d}{e},0,x\right)-e\sqrt{x}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)\right)}{ex}$$

input
```
integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(3/2),x, algorithm="frica
s")
```

output
```
2*(2*sqrt(-e)*sqrt(e)*x*weierstrassPInverse(-4*d/e, 0, x) - e*sqrt(x)*arct
an(sqrt(-e)*x/sqrt(e*x^2 + d)))/(e*x)
```

3.21.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 7.97 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.58

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{3/2}}\,dx = -\frac{2\operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{\sqrt{x}} + \frac{\sqrt{x}\sqrt{-e}\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{ex^2e^{i\pi}}{d}\right)}{\sqrt{d}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**(3/2),x)
```

output
```
-2*atan(x*sqrt(-e)/sqrt(d + e*x**2))/sqrt(x) + sqrt(x)*sqrt(-e)*gamma(1/4)
*hyper((1/4, 1/2), (5/4,), e*x**2*exp_polar(I*pi)/d)/(sqrt(d)*gamma(5/4))
```

3.21.7 Maxima [F]

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{3/2}}\,dx = \int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(3/2),x, algorithm="maxim
a")
```

output
```
2*(d*sqrt(-e)*sqrt(x)*integrate(-sqrt(e*x^2 + d)*x/((e^2*x^4 + d*e*x^2)*x^
(3/2) - (e*x^2 + d)*e^(log(e*x^2 + d) + 3/2*log(x))), x) - arctan2(sqrt(-e
)*x, sqrt(e*x^2 + d)))/sqrt(x)
```

3.21.8 Giac [F]

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{3/2}}\,dx = \int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(3/2),x, algorithm="giac"
)
```

output
```
integrate(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^(3/2), x)
```

3.21.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{3/2}}\,dx = \int \frac{\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^{3/2}}\,dx$$

input
```
int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^(3/2),x)
```

output
```
int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^(3/2), x)
```

3.22 $\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{7/2}}\,dx$

3.22.1 Optimal result

Integrand size = 27, antiderivative size = 156

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{7/2}}\,dx = -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{15dx^{3/2}} - \frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{5x^{5/2}} - \frac{2\sqrt{-e}e^{3/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{15d^{5/4}\sqrt{d+ex^2}}$$

output
```
-2/5*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(5/2)-4/15*(-e)^(1/2)*(e*x^2+d
)^(1/2)/d/x^(3/2)-2/15*e^(3/4)*(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^
(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticF(sin(2*arctan(e^(1/4
)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(-e)^(1/2)*(d^(1/2)+x*e^(1/2))*((e*x^2+d)
/(d^(1/2)+x*e^(1/2))^2)^(1/2)/d^(5/4)/(e*x^2+d)^(1/2)
```

3.22.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.96

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{7/2}}\,dx = -\frac{2\left(2\sqrt{-e}x\sqrt{d+ex^2}+3d\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\right)}{15dx^{5/2}} + \frac{4i(-e)^{3/2}\sqrt{1+\frac{d}{ex^2}}x\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right),-1\right)}{15d\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\sqrt{d+ex^2}}$$

input
```
Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(7/2),x]
```

output
```
(-2*(2*Sqrt[-e]*x*Sqrt[d + e*x^2] + 3*d*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2
]]))/(15*d*x^(5/2)) + (((4*I)/15)*(-e)^(3/2)*Sqrt[1 + d/(e*x^2)]*x*Ellipti
cF[I*ArcSinh[Sqrt[(I*Sqrt[d])/Sqrt[e]]/Sqrt[x]], -1])/(d*Sqrt[(I*Sqrt[d])/
Sqrt[e]]*Sqrt[d + e*x^2])
```

3.22.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {5674, 264, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{7/2}}\,dx$$

$$\Big\downarrow \text{5674}$$

$$\frac{2}{5}\sqrt{-e}\int \frac{1}{x^{5/2}\sqrt{ex^2+d}}dx - \frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{5x^{5/2}}$$

$$\Big\downarrow \text{264}$$

$$\frac{2}{5}\sqrt{-e}\left(-\frac{e\int \frac{1}{\sqrt{x}\sqrt{ex^2+d}}dx}{3d} - \frac{2\sqrt{d+ex^2}}{3dx^{3/2}}\right) - \frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{5x^{5/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{2}{5}\sqrt{-e}\left(-\frac{2e\int \frac{1}{\sqrt{ex^2+d}}d\sqrt{x}}{3d}-\frac{2\sqrt{d+ex^2}}{3dx^{3/2}}\right)-\frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{5x^{5/2}}$$

$$\Big\downarrow \text{761}$$

$$\frac{2}{5}\sqrt{-e}\left(-\frac{e^{3/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{3d^{5/4}\sqrt{d+ex^2}}-\frac{2\sqrt{d+ex^2}}{3dx^{3/2}}\right)-\frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{5x^{5/2}}$$

input
```
Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(7/2),x]
```

output
```
(-2*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/(5*x^(5/2)) + (2*Sqrt[-e]*((-2*S
qrt[d + e*x^2])/(3*d*x^(3/2)) - (e^(3/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e
*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4
)], 1/2])/(3*d^(5/4)*Sqrt[d + e*x^2])))/5
```

3.22.3.1 Defintions of rubi rules used

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 5674
```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Simp[c/(d*(m + 1))   Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ
[{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

3.22.4 Maple [F]

$$\int \frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{x^{\frac{7}{2}}}dx$$

input
```
int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(7/2),x)
```

output
```
int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(7/2),x)
```

3.22.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.47

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{7/2}}\,dx = -\frac{2\left(2\sqrt{-e}\sqrt{e}x^3\text{weierstrassPInverse}\left(-\frac{4\,d}{e},0,x\right)+2\sqrt{ex^2+d}\sqrt{-e}x^{\frac{3}{2}}+3\,d\sqrt{x}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)\right)}{15\,dx^3}$$

input
```
integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(7/2),x, algorithm="frica
s")
```

output
```
-2/15*(2*sqrt(-e)*sqrt(e)*x^3*weierstrassPInverse(-4*d/e, 0, x) + 2*sqrt(e
*x^2 + d)*sqrt(-e)*x^(3/2) + 3*d*sqrt(x)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)
))/(d*x^3)
```

3.22.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 44.90 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.50

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{7/2}}\,dx = -\frac{2\,\mathrm{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{5x^{\frac{5}{2}}} + \frac{\sqrt{-e}\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{ex^2e^{i\pi}}{d}\right)}{5\sqrt{d}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**(7/2),x)
```

output
```
-2*atan(x*sqrt(-e)/sqrt(d + e*x**2))/(5*x**(5/2)) + sqrt(-e)*gamma(-3/4)*h
yper((-3/4, 1/2), (1/4,), e*x**2*exp_polar(I*pi)/d)/(5*sqrt(d)*x**(3/2)*ga
mma(1/4))
```

3.22.7 Maxima [F]

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{7/2}}\,dx = \int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{7}{2}}}\,dx$$

input
```
integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(7/2),x, algorithm="maxim
a")
```

output
```
2/5*(5*d*sqrt(-e)*x^(5/2)*integrate(-1/5*sqrt(e*x^2 + d)*x/((e^2*x^4 + d*e
*x^2)*x^(7/2) - (e*x^2 + d)*e^(log(e*x^2 + d) + 7/2*log(x))), x) - arctan2
(sqrt(-e)*x, sqrt(e*x^2 + d)))/x^(5/2)
```

3.22.8 Giac [F]

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{7/2}}\,dx = \int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{7}{2}}}\,dx$$

input
```
integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(7/2),x, algorithm="giac"
)
```

output
```
integrate(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^(7/2), x)
```

3.22.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{7/2}}\,dx = \int \frac{\operatorname{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^{7/2}}\,dx$$

input
```
int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^(7/2),x)
```

output
```
int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^(7/2), x)
```

3.23 $\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{11/2}}\,dx$

3.23.1 Optimal result

Integrand size = 27, antiderivative size = 186

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{11/2}}\,dx = -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{63dx^{7/2}} - \frac{20(-e)^{3/2}\sqrt{d+ex^2}}{189d^2x^{3/2}} - \frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{9x^{9/2}} + \frac{10\sqrt{-e}e^{7/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{189d^{9/4}\sqrt{d+ex^2}}$$

output
```
-2/9*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(9/2)-20/189*(-e)^(3/2)*(e*x^2
+d)^(1/2)/d^2/x^(3/2)-4/63*(-e)^(1/2)*(e*x^2+d)^(1/2)/d/x^(7/2)+10/189*e^(
7/4)*(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)
*x^(1/2)/d^(1/4)))*EllipticF(sin(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^
(1/2))*(-e)^(1/2)*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1
/2)/d^(9/4)/(e*x^2+d)^(1/2)
```

3.23.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.87

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{11/2}}\,dx = \frac{4\sqrt{-e}x\sqrt{d+ex^2}(-3d+5ex^2)-42d^2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{189d^2x^{9/2}} + \frac{20i(-e)^{5/2}\sqrt{1+\frac{d}{ex^2}}x\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right),-1\right)}{189d^2\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\sqrt{d+ex^2}}$$

input
```
Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(11/2),x]
```

output
```
(4*Sqrt[-e]*x*Sqrt[d + e*x^2]*(-3*d + 5*e*x^2) - 42*d^2*ArcTan[(Sqrt[-e]*x
)/Sqrt[d + e*x^2]])/(189*d^2*x^(9/2)) + (((20*I)/189)*(-e)^(5/2)*Sqrt[1 +
d/(e*x^2)]*x*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[d])/Sqrt[e]]/Sqrt[x]], -1])/
(d^2*Sqrt[(I*Sqrt[d])/Sqrt[e]]*Sqrt[d + e*x^2])
```

3.23.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {5674, 264, 264, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{11/2}}dx$$

$$\Big\downarrow \text{5674}$$

$$\frac{2}{9}\sqrt{-e}\int \frac{1}{x^{9/2}\sqrt{ex^2+d}}dx - \frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{9x^{9/2}}$$

$$\Big\downarrow \text{264}$$

$$\frac{2}{9}\sqrt{-e}\left(-\frac{5e\int \frac{1}{x^{5/2}\sqrt{ex^2+d}}dx}{7d} - \frac{2\sqrt{d+ex^2}}{7dx^{7/2}}\right) - \frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{9x^{9/2}}$$

$$\Big\downarrow \; 264$$

$$\frac{2}{9}\sqrt{-e}\left(-\frac{5e\left(-\frac{e\int \frac{1}{\sqrt{x}\sqrt{ex^2+d}}dx}{3d}-\frac{2\sqrt{d+ex^2}}{3dx^{3/2}}\right)}{7d}-\frac{2\sqrt{d+ex^2}}{7dx^{7/2}}\right)-\frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{9x^{9/2}}$$

$$\Big\downarrow \; 266$$

$$\frac{2}{9}\sqrt{-e}\left(-\frac{5e\left(-\frac{2e\int \frac{1}{\sqrt{ex^2+d}}d\sqrt{x}}{3d}-\frac{2\sqrt{d+ex^2}}{3dx^{3/2}}\right)}{7d}-\frac{2\sqrt{d+ex^2}}{7dx^{7/2}}\right)-\frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{9x^{9/2}}$$

$$\Big\downarrow \; 761$$

$$\frac{2}{9}\sqrt{-e}\left(-\frac{5e\left(-\frac{e^{3/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{3d^{5/4}\sqrt{d+ex^2}}-\frac{2\sqrt{d+ex^2}}{3dx^{3/2}}\right)}{7d}-\frac{2\sqrt{d+ex^2}}{7dx^{7/2}}\right)-\frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{9x^{9/2}}$$

input
```
Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(11/2),x]
```

output
```
(-2*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/(9*x^(9/2)) + (2*Sqrt[-e]*((-2*S
qrt[d + e*x^2])/(7*d*x^(7/2)) - (5*e*((-2*Sqrt[d + e*x^2])/(3*d*x^(3/2)) -
 (e^(3/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*
EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(3*d^(5/4)*Sqrt[d + e
*x^2])))/(7*d)))/9
```

3.23. $\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{11/2}}dx$

3.23.3.1 Defintions of rubi rules used

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 5674
```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Simp[c/(d*(m + 1))   Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ
[{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

3.23.4 Maple [F]

$$\int \frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^{2}+d}}\right)}{x^{\frac{11}{2}}}dx$$

input
```
int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(11/2),x)
```

output
```
int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(11/2),x)
```

3.23.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.46

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{11/2}}\,dx = \frac{2\left(10\sqrt{-e}e^{\frac{3}{2}}x^5\text{weierstrassPInverse}\left(-\frac{4d}{e},0,x\right)-21\,d^2\sqrt{x}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)+2\,(5\right.}{189\,d^2x^5}$$

input
```
integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(11/2),x, algorithm="fric
as")
```

output
```
2/189*(10*sqrt(-e)*e^(3/2)*x^5*weierstrassPInverse(-4*d/e, 0, x) - 21*d^2*
sqrt(x)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) + 2*(5*e*x^3 - 3*d*x)*sqrt(e*x^
2 + d)*sqrt(-e)*sqrt(x))/(d^2*x^5)
```

3.23.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**(11/2),x)
```

output
```
Timed out
```

3.23.7 Maxima [F]

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{11/2}}\,dx = \int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{11}{2}}}\,dx$$

input
```
integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(11/2),x, algorithm="maxi
ma")
```

output
```
2/9*(9*d*sqrt(-e)*x^(9/2)*integrate(-1/9*sqrt(e*x^2 + d)*x/((e^2*x^4 + d*e
*x^2)*x^(11/2) - (e*x^2 + d)*e^(log(e*x^2 + d) + 11/2*log(x))), x) - arcta
n2(sqrt(-e)*x, sqrt(e*x^2 + d)))/x^(9/2)
```

3.23.8 Giac [F]

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{11/2}}\,dx = \int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{11}{2}}}\,dx$$

input
```
integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(11/2),x, algorithm="giac
")
```

output
```
integrate(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^(11/2), x)
```

3.23.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{11/2}}\,dx = \int \frac{\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^{11/2}}\,dx$$

input
```
int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^(11/2),x)
```

output
```
int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^(11/2), x)
```

3.24 $\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{15/2}}\,dx$

3.24.1 Optimal result

Integrand size = 27, antiderivative size = 216

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{15/2}}\,dx = -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{143dx^{11/2}} - \frac{36(-e)^{3/2}\sqrt{d+ex^2}}{1001d^2x^{7/2}} - \frac{60(-e)^{5/2}\sqrt{d+ex^2}}{1001d^3x^{3/2}} - \frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{13x^{13/2}} - \frac{30\sqrt{-e}e^{11/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{1001d^{13/4}\sqrt{d+ex^2}}$$

output
```
-2/13*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(13/2)-36/1001*(-e)^(3/2)*(e*
x^2+d)^(1/2)/d^2/x^(7/2)-60/1001*(-e)^(5/2)*(e*x^2+d)^(1/2)/d^3/x^(3/2)-4/
143*(-e)^(1/2)*(e*x^2+d)^(1/2)/d/x^(11/2)-30/1001*e^(11/4)*(cos(2*arctan(e
^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*E
llipticF(sin(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(-e)^(1/2)*(d
^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/d^(13/4)/(e*x^2+
d)^(1/2)
```

3.24.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.79

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{15/2}}\,dx = \frac{2\left(-\frac{2\sqrt{-e}\sqrt{d+ex^2}(7d^2x-9dex^3+15e^2x^5)}{d^3} - 77\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) + \frac{30i(-e)^{7/2}\sqrt{1+\frac{d}{ex^2}}x^{15/2}\,\mathrm{Ellip}}{d^3\sqrt{\frac{i\sqrt{}}{\sqrt{e}}}}\right.}{1001x^{13/2}}$$

input
```
Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(15/2),x]
```

output
```
(2*((-2*Sqrt[-e]*Sqrt[d + e*x^2]*(7*d^2*x - 9*d*e*x^3 + 15*e^2*x^5))/d^3 -
 77*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]] + ((30*I)*(-e)^(7/2)*Sqrt[1 + d/(
e*x^2)]*x^(15/2)*EllipticF[I*ArcSinh[Sqrt[(I*Sqrt[d])/Sqrt[e]]/Sqrt[x]], -
1])/(d^3*Sqrt[(I*Sqrt[d])/Sqrt[e]]*Sqrt[d + e*x^2])))/(1001*x^(13/2))
```

3.24.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5674, 264, 264, 264, 266, 761}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{15/2}}dx$$

$$\Big\downarrow \text{5674}$$

$$\frac{2}{13}\sqrt{-e}\int \frac{1}{x^{13/2}\sqrt{ex^2+d}}dx - \frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{13x^{13/2}}$$

$$\Big\downarrow \text{264}$$

$$\frac{2}{13}\sqrt{-e}\left(-\frac{9e\int \frac{1}{x^{9/2}\sqrt{ex^2+d}}dx}{11d} - \frac{2\sqrt{d+ex^2}}{11dx^{11/2}}\right) - \frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{13x^{13/2}}$$

$$\Big\downarrow \text{264}$$

$$\frac{2}{13}\sqrt{-e}\left(-\frac{9e\left(-\frac{5e\int\frac{1}{x^{5/2}\sqrt{ex^2+d}}dx}{7d}-\frac{2\sqrt{d+ex^2}}{7dx^{7/2}}\right)}{11d}-\frac{2\sqrt{d+ex^2}}{11dx^{11/2}}\right)-\frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{13x^{13/2}}$$

$$\downarrow 264$$

$$\frac{2}{13}\sqrt{-e}\left(-\frac{9e\left(-\frac{5e\left(-\frac{e\int\frac{1}{\sqrt{x}\sqrt{ex^2+d}}dx}{3d}-\frac{2\sqrt{d+ex^2}}{3dx^{3/2}}\right)}{7d}-\frac{2\sqrt{d+ex^2}}{7dx^{7/2}}\right)}{11d}-\frac{2\sqrt{d+ex^2}}{11dx^{11/2}}\right)-\frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{13x^{13/2}}$$

$$\downarrow 266$$

$$\frac{2}{13}\sqrt{-e}\left(-\frac{9e\left(-\frac{5e\left(-\frac{2e\int\frac{1}{\sqrt{ex^2+d}}d\sqrt{x}}{3d}-\frac{2\sqrt{d+ex^2}}{3dx^{3/2}}\right)}{7d}-\frac{2\sqrt{d+ex^2}}{7dx^{7/2}}\right)}{11d}-\frac{2\sqrt{d+ex^2}}{11dx^{11/2}}\right)-\frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{13x^{13/2}}$$

$$\downarrow 761$$

$$\frac{2}{13}\sqrt{-e}\left(-\frac{9e\left(-\frac{5e\left(-\frac{e^{3/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{3d^{5/4}\sqrt{d+ex^2}}-\frac{2\sqrt{d+ex^2}}{3dx^{3/2}}\right)}{7d}-\frac{2\sqrt{d+ex^2}}{7dx^{7/2}}\right)}{11d}-\frac{2\sqrt{d+ex^2}}{11dx^{11/2}}\right)-\frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{13x^{13/2}}$$

3.24. $\int\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{15/2}}dx$

input
```
Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(15/2),x]
```

output
```
(-2*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/(13*x^(13/2)) + (2*Sqrt[-e]*((-2
*Sqrt[d + e*x^2])/(11*d*x^(11/2)) - (9*e*((-2*Sqrt[d + e*x^2])/(7*d*x^(7/2
)) - (5*e*((-2*Sqrt[d + e*x^2])/(3*d*x^(3/2)) - (e^(3/4)*(Sqrt[d] + Sqrt[e
]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)
*Sqrt[x])/d^(1/4)], 1/2])/(3*d^(5/4)*Sqrt[d + e*x^2])))/(7*d)))/(11*d)))/1
3
```

3.24.3.1 Defintions of rubi rules used

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 5674
```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Simp[c/(d*(m + 1))   Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ
[{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

3.24.4 Maple [F]

$$\int \frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{x^{\frac{15}{2}}}dx$$

input
```
int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(15/2),x)
```

output
```
int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(15/2),x)
```

3.24.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.45

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{15/2}}\,dx = -\frac{2\left(30\sqrt{-e}e^{\frac{5}{2}}x^7\text{weierstrassPInverse}\left(-\frac{4d}{e},0,x\right)+77\,d^3\sqrt{x}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)+2\left(15\,e^2x^5-9\,dex^3+7\,d^2x\right)\right.}{1001\,d^3x^7}$$

input
```
integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(15/2),x, algorithm="fric
as")
```

output
```
-2/1001*(30*sqrt(-e)*e^(5/2)*x^7*weierstrassPInverse(-4*d/e, 0, x) + 77*d^
3*sqrt(x)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) + 2*(15*e^2*x^5 - 9*d*e*x^3 +
 7*d^2*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt(x))/(d^3*x^7)
```

3.24.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{15/2}}\,dx = \text{Timed out}$$

input
```
integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**(15/2),x)
```

output
```
Timed out
```

3.24.7 Maxima [F]

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{15/2}}\,dx = \int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{15}{2}}}\,dx$$

input
```
integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(15/2),x, algorithm="maxi
ma")
```

output
```
2/13*(13*d*sqrt(-e)*x^(13/2)*integrate(-1/13*sqrt(e*x^2 + d)*x/((e^2*x^4 +
 d*e*x^2)*x^(15/2) - (e*x^2 + d)*e^(log(e*x^2 + d) + 15/2*log(x))), x) - a
rctan2(sqrt(-e)*x, sqrt(e*x^2 + d)))/x^(13/2)
```

3.24.8 Giac [F]

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{15/2}}\,dx = \int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{15}{2}}}\,dx$$

input
```
integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(15/2),x, algorithm="giac
")
```

output
```
integrate(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^(15/2), x)
```

3.24.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{15/2}}\,dx = \int \frac{\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^{15/2}}\,dx$$

input
```
int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^(15/2),x)
```

output
```
int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^(15/2), x)
```

3.25 $\int x^{7/2} \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx$

3.25.1 Optimal result

Integrand size = 27, antiderivative size = 326

$$\int x^{7/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)dx=\frac{28dx^{3/2}\sqrt{d+ex^2}}{405(-e)^{3/2}}+\frac{4x^{7/2}\sqrt{d+ex^2}}{81\sqrt{-e}}-\frac{28d^2\sqrt{-e}\sqrt{x}\sqrt{d+ex^2}}{135e^{5/2}\left(\sqrt{d}+\sqrt{e}x\right)}+\frac{2}{9}x^{9/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)+\frac{28d^{9/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{135e^{11/4}\sqrt{d+ex^2}}-\frac{14d^{9/4}\sqrt{-e}}{}$$

output
```
2/9*x^(9/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+28/405*d*x^(3/2)*(e*x^2+d
)^(1/2)/(-e)^(3/2)+4/81*x^(7/2)*(e*x^2+d)^(1/2)/(-e)^(1/2)-28/135*d^2*(-e)
^(1/2)*x^(1/2)*(e*x^2+d)^(1/2)/e^(5/2)/(d^(1/2)+x*e^(1/2))+28/135*d^(9/4)*
(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1
/2)/d^(1/4)))*EllipticE(sin(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2)
)*(-e)^(1/2)*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/e
^(11/4)/(e*x^2+d)^(1/2)-14/135*d^(9/4)*(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/
4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticF(sin(2*arcta
n(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(-e)^(1/2)*(d^(1/2)+x*e^(1/2))*((
e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/e^(11/4)/(e*x^2+d)^(1/2)
```

3.25.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.12 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.43

$$\int x^{7/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx=\frac{2x^{3/2}\left(2\sqrt{-e}(7d^2+2dex^2-5e^2x^4)+45e^2x^3\sqrt{d+ex^2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\right.}{405e^2\sqrt{d+ex^2}}$$

input
```
Integrate[x^(7/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

output
```
(2*x^(3/2)*(2*Sqrt[-e]*(7*d^2 + 2*d*e*x^2 - 5*e^2*x^4) + 45*e^2*x^3*Sqrt[d
 + e*x^2]*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]] - 14*d^2*Sqrt[-e]*Sqrt[1 +
(e*x^2)/d]*Hypergeometric2F1[1/2, 3/4, 7/4, -((e*x^2)/d)]))/(405*e^2*Sqrt[
d + e*x^2])
```

3.25.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 321, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {5674, 262, 262, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{7/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx$$

$$\Big\downarrow \text{5674}$$

$$\frac{2}{9}x^{9/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{2}{9}\sqrt{-e}\int\frac{x^{9/2}}{\sqrt{ex^2+d}}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{2}{9}x^{9/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{2}{9}\sqrt{-e}\left(\frac{2x^{7/2}\sqrt{d+ex^2}}{9e}-\frac{7d\int\frac{x^{5/2}}{\sqrt{ex^2+d}}dx}{9e}\right)$$

$$\Big\downarrow \text{262}$$

$$\frac{2}{9}x^{9/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{2}{9}\sqrt{-e}\left(\frac{2x^{7/2}\sqrt{d+ex^2}}{9e}-\frac{7d\left(\frac{2x^{3/2}\sqrt{d+ex^2}}{5e}-\frac{3d\int\frac{\sqrt{x}}{\sqrt{ex^2+d}}dx}{5e}\right)}{9e}\right)$$

$\downarrow$ 266

$$\frac{2}{9}x^{9/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{2}{9}\sqrt{-e}\left(\frac{2x^{7/2}\sqrt{d+ex^2}}{9e}-\frac{7d\left(\frac{2x^{3/2}\sqrt{d+ex^2}}{5e}-\frac{6d\int\frac{x}{\sqrt{ex^2+d}}d\sqrt{x}}{5e}\right)}{9e}\right)$$

$\downarrow$ 834

$$\frac{2}{9}x^{9/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{2}{9}\sqrt{-e}\left(\frac{2x^{7/2}\sqrt{d+ex^2}}{9e}-\frac{7d\left(\frac{2x^{3/2}\sqrt{d+ex^2}}{5e}-\frac{6d\left(\frac{\sqrt{d}\int\frac{1}{\sqrt{ex^2+d}}d\sqrt{x}}{\sqrt{e}}-\frac{\sqrt{d}\int\frac{\sqrt{d}-\sqrt{e}x}{\sqrt{d}\sqrt{ex^2+d}}d\sqrt{x}}{\sqrt{e}}\right)}{5e}\right)}{9e}\right)$$

$\downarrow$ 27

$$\frac{2}{9}x^{9/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{2}{9}\sqrt{-e}\left(\frac{2x^{7/2}\sqrt{d+ex^2}}{9e}-\frac{7d\left(\frac{2x^{3/2}\sqrt{d+ex^2}}{5e}-\frac{6d\left(\frac{\sqrt{d}\int\frac{1}{\sqrt{ex^2+d}}d\sqrt{x}}{\sqrt{e}}-\frac{\int\frac{\sqrt{d}-\sqrt{e}x}{\sqrt{ex^2+d}}d\sqrt{x}}{\sqrt{e}}\right)}{5e}\right)}{9e}\right)$$

$\downarrow$ 761

3.25. $\int x^{7/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx$

$$\frac{2}{9}x^{9/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{2}{9}\sqrt{-e}\left(\frac{2x^{7/2}\sqrt{d+ex^2}}{9e}-\frac{7d\left(\frac{2x^{3/2}\sqrt{d+ex^2}}{5e}-\frac{6d\left(\frac{\sqrt[4]{d}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{2e^{3/4}\sqrt{d+ex^2}}-\frac{\int\frac{\sqrt{d}-\sqrt{e}x}{\sqrt{ex^2+d}}d\sqrt{x}}{\sqrt{e}}\right)}{5e}\right)}{9e}\right)$$

$\downarrow$ 1510

$$\frac{2}{9}x^{9/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{2}{9}\sqrt{-e}\left(\frac{2x^{7/2}\sqrt{d+ex^2}}{9e}-\frac{7d\left(\frac{2x^{3/2}\sqrt{d+ex^2}}{5e}-\frac{6d\left(\frac{\sqrt[4]{d}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{2e^{3/4}\sqrt{d+ex^2}}-\frac{\frac{\sqrt[4]{d}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\cdots}}{\cdots}}{\cdots}\right)}{5e}\right)}{9e}\right)$$

3.25. $\int x^{7/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx$

input
```
Int[x^(7/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

output
```
(2*x^(9/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/9 - (2*Sqrt[-e]*((2*x^(7/
2)*Sqrt[d + e*x^2])/(9*e) - (7*d*((2*x^(3/2)*Sqrt[d + e*x^2])/(5*e) - (6*d
*(-((-((Sqrt[x]*Sqrt[d + e*x^2])/(Sqrt[d] + Sqrt[e]*x)) + (d^(1/4)*(Sqrt[d
] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticE[2*ArcTa
n[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(e^(1/4)*Sqrt[d + e*x^2]))/Sqrt[e]) +
(d^(1/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*E
llipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(2*e^(3/4)*Sqrt[d + e*
x^2])))/(5*e)))/(9*e)))/9
```

3.25.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 5674
```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Simp[c/(d*(m + 1))   Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ
[{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

3.25.4 Maple [F]

$$\int x^{\frac{7}{2}} \arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^{2}+d}}\right)dx$$

input
```
int(x^(7/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
```

output
```
int(x^(7/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
```

3.25.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.12 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.29

$$\int x^{7/2} \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx = \frac{2\left(45\,e^{3}x^{\frac{9}{2}}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^{2}+d}}\right)+42\,d^{2}\sqrt{-e}\sqrt{e}\text{weierstrassZeta}\left(-\frac{4\,d}{e},0,\text{weiers}\right.\right.}{405\,e^{3}}$$

input
```
integrate(x^(7/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="frica
s")
```

output
```
2/405*(45*e^3*x^(9/2)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) + 42*d^2*sqrt(-e)
*sqrt(e)*weierstrassZeta(-4*d/e, 0, weierstrassPInverse(-4*d/e, 0, x)) - 2
*(5*e^2*x^3 - 7*d*e*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt(x))/e^3
```

3.25.6 Sympy [F(-1)]

Timed out.

$$\int x^{7/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)dx = \text{Timed out}$$

input
```
integrate(x**(7/2)*atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)
```

output
```
Timed out
```

3.25.7 Maxima [F]

$$\int x^{7/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)dx = \int x^{\frac{7}{2}}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)dx$$

input
```
integrate(x^(7/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxim
a")
```

output
```
2/9*x^(9/2)*arctan2(sqrt(-e)*x, sqrt(e*x^2 + d)) - 2*d*sqrt(-e)*integrate(
-1/9*x*e^(1/2*log(e*x^2 + d) + 7/2*log(x))/(e^2*x^4 + d*e*x^2 - (e*x^2 + d
)^2), x)
```

3.25.8 Giac [F]

$$\int x^{7/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)dx = \int x^{\frac{7}{2}}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)dx$$

input
```
integrate(x^(7/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac"
)
```

output
```
integrate(x^(7/2)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)), x)
```

3.25.9 Mupad [F(-1)]

Timed out.

$$\int x^{7/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx=\int x^{7/2}\,\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)\,dx$$

input
```
int(x^(7/2)*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

output
```
int(x^(7/2)*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

3.26 $\int x^{3/2} \arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx$

3.26.1 Optimal result

Integrand size = 27, antiderivative size = 296

$$\int x^{3/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) dx = \frac{4x^{3/2}\sqrt{d+ex^2}}{25\sqrt{-e}} + \frac{12d\sqrt{-e}\sqrt{x}\sqrt{d+ex^2}}{25e^{3/2}\left(\sqrt{d}+\sqrt{e}x\right)} + \frac{2}{5}x^{5/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{12d^{5/4}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{25e^{7/4}\sqrt{d+ex^2}} + \frac{6d^{5/4}\sqrt{-e}(}{}$$

output
```
2/5*x^(5/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))+4/25*x^(3/2)*(e*x^2+d)^(1
/2)/(-e)^(1/2)+12/25*d*(-e)^(1/2)*x^(1/2)*(e*x^2+d)^(1/2)/e^(3/2)/(d^(1/2)
+x*e^(1/2))-12/25*d^(5/4)*(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)
/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticE(sin(2*arctan(e^(1/4)*x^(
1/2)/d^(1/4))),1/2*2^(1/2))*(-e)^(1/2)*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(
1/2)+x*e^(1/2))^2)^(1/2)/e^(7/4)/(e*x^2+d)^(1/2)+6/25*d^(5/4)*(cos(2*arcta
n(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))
)*EllipticF(sin(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(-e)^(1/2)
*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/e^(7/4)/(e*x^
2+d)^(1/2)
```

3.26.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.40

$$\int x^{3/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)dx=\frac{2x^{3/2}\left(-2\sqrt{-e}(d+ex^2)+5ex\sqrt{d+ex^2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)+2d\sqrt{-e}\sqrt{1+\right.}{25e\sqrt{d+ex^2}}$$

input
```
Integrate[x^(3/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

output
```
(2*x^(3/2)*(-2*Sqrt[-e]*(d + e*x^2) + 5*e*x*Sqrt[d + e*x^2]*ArcTan[(Sqrt[-
e]*x)/Sqrt[d + e*x^2]] + 2*d*Sqrt[-e]*Sqrt[1 + (e*x^2)/d]*Hypergeometric2F
1[1/2, 3/4, 7/4, -((e*x^2)/d)]))/(25*e*Sqrt[d + e*x^2])
```

3.26.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {5674, 262, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)dx$$

$$\Big\downarrow \text{ 5674}$$

$$\frac{2}{5}x^{5/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{2}{5}\sqrt{-e}\int\frac{x^{5/2}}{\sqrt{ex^2+d}}dx$$

$$\Big\downarrow \text{ 262}$$

$$\frac{2}{5}x^{5/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{2}{5}\sqrt{-e}\left(\frac{2x^{3/2}\sqrt{d+ex^2}}{5e}-\frac{3d\int\frac{\sqrt{x}}{\sqrt{ex^2+d}}dx}{5e}\right)$$

$$\Big\downarrow \text{ 266}$$

$$\frac{2}{5}x^{5/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{2}{5}\sqrt{-e}\left(\frac{2x^{3/2}\sqrt{d+ex^2}}{5e}-\frac{6d\int\frac{x}{\sqrt{ex^2+d}}d\sqrt{x}}{5e}\right)$$

$$\Big\downarrow \text{ 834}$$

$$\frac{2}{5}x^{5/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{2}{5}\sqrt{-e}\left(\frac{2x^{3/2}\sqrt{d+ex^2}}{5e}-\frac{6d\left(\frac{\sqrt{d}\int\frac{1}{\sqrt{ex^2+d}}d\sqrt{x}}{\sqrt{e}}-\frac{\sqrt{d}\int\frac{\sqrt{d}-\sqrt{e}x}{\sqrt{d}\sqrt{ex^2+d}}d\sqrt{x}}{\sqrt{e}}\right)}{5e}\right)$$

$\downarrow$ 27

$$\frac{2}{5}x^{5/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{2}{5}\sqrt{-e}\left(\frac{2x^{3/2}\sqrt{d+ex^2}}{5e}-\frac{6d\left(\frac{\sqrt{d}\int\frac{1}{\sqrt{ex^2+d}}d\sqrt{x}}{\sqrt{e}}-\frac{\int\frac{\sqrt{d}-\sqrt{e}x}{\sqrt{ex^2+d}}d\sqrt{x}}{\sqrt{e}}\right)}{5e}\right)$$

$\downarrow$ 761

$$\frac{2}{5}x^{5/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{2}{5}\sqrt{-e}\left(\frac{2x^{3/2}\sqrt{d+ex^2}}{5e}-\frac{6d\left(\frac{\sqrt[4]{d}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{2e^{3/4}\sqrt{d+ex^2}}-\frac{\int\frac{\sqrt{d}-\sqrt{e}x}{\sqrt{ex^2+d}}d\sqrt{x}}{\sqrt{e}}\right)}{5e}\right)$$

$\downarrow$ 1510

$$\frac{2}{5}x^{5/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-\frac{2}{5}\sqrt{-e}\left(\frac{2x^{3/2}\sqrt{d+ex^2}}{5e}-\frac{6d\left(\frac{\sqrt[4]{d}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{2e^{3/4}\sqrt{d+ex^2}}-\frac{\frac{\sqrt[4]{d}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}E\left(2\,a\right.}{\sqrt[4]{e}\sqrt{d+ex^2}}}{\sqrt{}}\right.}{5e}\right.$$

input
```
Int[x^(3/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]],x]
```

3.26. $\int x^{3/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx$

output
```
(2*x^(5/2)*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/5 - (2*Sqrt[-e]*((2*x^(3/
2)*Sqrt[d + e*x^2])/(5*e) - (6*d*(-((-((Sqrt[x]*Sqrt[d + e*x^2])/(Sqrt[d]
+ Sqrt[e]*x)) + (d^(1/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] +
 Sqrt[e]*x)^2]*EllipticE[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(e^(1/
4)*Sqrt[d + e*x^2]))/Sqrt[e]) + (d^(1/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e
*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4
)], 1/2])/(2*e^(3/4)*Sqrt[d + e*x^2])))/(5*e)))/5
```

3.26.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 5674
```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Simp[c/(d*(m + 1))   Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ
[{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

3.26.4 Maple [F]

$$\int x^{\frac{3}{2}} \arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^{2}+d}}\right)dx$$

input
```
int(x^(3/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
```

output
```
int(x^(3/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x)
```

3.26.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.27

$$\int x^{3/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)dx = \frac{2\left(5\,e^{2}x^{\frac{5}{2}}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^{2}+d}}\right)-2\,\sqrt{ex^{2}+d}\sqrt{-e}ex^{\frac{3}{2}}-6\,d\sqrt{-e}\sqrt{e}\text{weierstrass}\right.}{25\,e^{2}}$$

input
```
integrate(x^(3/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="frica
s")
```

output
```
2/25*(5*e^2*x^(5/2)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - 2*sqrt(e*x^2 + d)
*sqrt(-e)*e*x^(3/2) - 6*d*sqrt(-e)*sqrt(e)*weierstrassZeta(-4*d/e, 0, weie
rstrassPInverse(-4*d/e, 0, x)))/e^2
```

3.26.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.88 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.25

$$\int x^{3/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx=\frac{2x^{\frac{5}{2}}\operatorname{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{5}-\frac{x^{\frac{7}{2}}\sqrt{-e}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{ex^2e^{i\pi}}{d}\right)}{5\sqrt{d}\Gamma\left(\frac{11}{4}\right)}$$

input
```
integrate(x**(3/2)*atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2)),x)
```

output
```
2*x**(5/2)*atan(x*sqrt(-e)/sqrt(d + e*x**2))/5 - x**(7/2)*sqrt(-e)*gamma(7
/4)*hyper((1/2, 7/4), (11/4,), e*x**2*exp_polar(I*pi)/d)/(5*sqrt(d)*gamma(
11/4))
```

3.26.7 Maxima [F]

$$\int x^{3/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx=\int x^{\frac{3}{2}}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)\,dx$$

input
```
integrate(x^(3/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="maxim
a")
```

output
```
2/5*x^(5/2)*arctan2(sqrt(-e)*x, sqrt(e*x^2 + d)) - 2*d*sqrt(-e)*integrate(
-1/5*x*e^(1/2*log(e*x^2 + d) + 3/2*log(x))/(e^2*x^4 + d*e*x^2 - (e*x^2 + d
)^2), x)
```

3.26.8 Giac [F]

$$\int x^{3/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx=\int x^{\frac{3}{2}}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)\,dx$$

input
```
integrate(x^(3/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2)),x, algorithm="giac"
)
```

output
```
integrate(x^(3/2)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)), x)
```

3.26.9 Mupad [F(-1)]

Timed out.

$$\int x^{3/2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)\,dx=\int x^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)\,dx$$

input
```
int(x^(3/2)*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)),x)
```

output
```
int(x^(3/2)*atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2)), x)
```

3.27 $\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}\,dx$

3.27.1 Optimal result

Integrand size = 27, antiderivative size = 260

$$\begin{aligned}
&\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}\,dx \\
&= -\frac{4\sqrt{-e}\sqrt{x}\sqrt{d+ex^2}}{\sqrt{e}\left(\sqrt{d}+\sqrt{e}x\right)}+2\sqrt{x}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) \\
&\quad+\frac{4\sqrt[4]{d}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{e^{3/4}\sqrt{d+ex^2}} \\
&\quad-\frac{2\sqrt[4]{d}\sqrt{-e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{e^{3/4}\sqrt{d+ex^2}}
\end{aligned}$$

output
```
2*x^(1/2)*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))-4*(-e)^(1/2)*x^(1/2)*(e*x^2
+d)^(1/2)/e^(1/2)/(d^(1/2)+x*e^(1/2))+4*d^(1/4)*(cos(2*arctan(e^(1/4)*x^(1
/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticE(si
n(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(-e)^(1/2)*(d^(1/2)+x*e^
(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/e^(3/4)/(e*x^2+d)^(1/2)-2*d
^(1/4)*(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/
4)*x^(1/2)/d^(1/4)))*EllipticF(sin(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*
2^(1/2))*(-e)^(1/2)*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^
(1/2)/e^(3/4)/(e*x^2+d)^(1/2)
```

3.27.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.34

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}\,dx = 2\sqrt{x}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - \frac{4\sqrt{-e}x^{3/2}\sqrt{1+\frac{ex^2}{d}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\frac{ex^2}{d}\right)}{3\sqrt{d+ex^2}}$$

input
```
Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/Sqrt[x],x]
```

output
```
2*Sqrt[x]*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]] - (4*Sqrt[-e]*x^(3/2)*Sqrt[
1 + (e*x^2)/d]*Hypergeometric2F1[1/2, 3/4, 7/4, -((e*x^2)/d)])/(3*Sqrt[d +
 e*x^2])
```

3.27.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5674, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}\,dx$$

$$\Big\downarrow \text{ 5674}$$

$$2\sqrt{x}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - 2\sqrt{-e}\int \frac{\sqrt{x}}{\sqrt{ex^2+d}}dx$$

$$\Big\downarrow \text{ 266}$$

$$2\sqrt{x}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right) - 4\sqrt{-e}\int \frac{x}{\sqrt{ex^2+d}}d\sqrt{x}$$

$$\Big\downarrow \text{ 834}$$

$$2\sqrt{x}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-4\sqrt{-e}\left(\frac{\sqrt{d}\int\frac{1}{\sqrt{ex^2+d}}d\sqrt{x}}{\sqrt{e}}-\frac{\sqrt{d}\int\frac{\sqrt{d}-\sqrt{e}x}{\sqrt{d}\sqrt{ex^2+d}}d\sqrt{x}}{\sqrt{e}}\right)$$

$$\Big\downarrow \; 27$$

$$2\sqrt{x}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-4\sqrt{-e}\left(\frac{\sqrt{d}\int\frac{1}{\sqrt{ex^2+d}}d\sqrt{x}}{\sqrt{e}}-\frac{\int\frac{\sqrt{d}-\sqrt{e}x}{\sqrt{ex^2+d}}d\sqrt{x}}{\sqrt{e}}\right)$$

$$\Big\downarrow \; 761$$

$$2\sqrt{x}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-4\sqrt{-e}\left(\frac{\sqrt[4]{d}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{2e^{3/4}\sqrt{d+ex^2}}-\frac{\int\frac{\sqrt{d}-\sqrt{e}x}{\sqrt{ex^2+d}}d\sqrt{x}}{\sqrt{e}}\right)$$

$$\Big\downarrow \; 1510$$

$$2\sqrt{x}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-4\sqrt{-e}\left(\frac{\sqrt[4]{d}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{2e^{3/4}\sqrt{d+ex^2}}-\frac{\frac{\sqrt[4]{d}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{\sqrt[4]{e}\sqrt{d+ex^2}}-\frac{\sqrt{x}\sqrt{d+ex^2}}{\sqrt{d}+\sqrt{e}x}}{\sqrt{e}}\right)$$

input
```
Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/Sqrt[x],x]
```

output
```
2*Sqrt[x]*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]] - 4*Sqrt[-e]*(-((-((Sqrt[x]
*Sqrt[d + e*x^2])/(Sqrt[d] + Sqrt[e]*x)) + (d^(1/4)*(Sqrt[d] + Sqrt[e]*x)*
Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticE[2*ArcTan[(e^(1/4)*Sqrt
[x])/d^(1/4)], 1/2])/(e^(1/4)*Sqrt[d + e*x^2]))/Sqrt[e]) + (d^(1/4)*(Sqrt[
d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcT
an[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(2*e^(3/4)*Sqrt[d + e*x^2]))
```

3.27. $\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}\,dx$

3.27.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 5674
```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Simp[c/(d*(m + 1))   Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ
[{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

3.27.4 Maple [F]

$$\int \frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{\sqrt{x}}dx$$

input
```
int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(1/2),x)
```

output
```
int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(1/2),x)
```

3.27.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.21

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}\,dx = \frac{2\left(e\sqrt{x}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)+2\sqrt{-e}\sqrt{e}\text{weierstrassZeta}\left(-\frac{4\,d}{e},0,\text{weierstrassPInverse}\left(-\frac{4\,d}{e},0,x\right)\right)\right)}{e}$$

input
```
integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(1/2),x, algorithm="frica
s")
```

output
```
2*(e*sqrt(x)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) + 2*sqrt(-e)*sqrt(e)*weier
strassZeta(-4*d/e, 0, weierstrassPInverse(-4*d/e, 0, x)))/e
```

3.27.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 9.31 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.27

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}\,dx = 2\sqrt{x}\,\text{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right) - \frac{x^{\frac{3}{2}}\sqrt{-e}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{ex^2e^{i\pi}}{d}\right)}{\sqrt{d}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**(1/2),x)
```

output
```
2*sqrt(x)*atan(x*sqrt(-e)/sqrt(d + e*x**2)) - x**(3/2)*sqrt(-e)*gamma(3/4)
*hyper((1/2, 3/4), (7/4,), e*x**2*exp_polar(I*pi)/d)/(sqrt(d)*gamma(7/4))
```

3.27.7 Maxima [F]

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}\,dx = \int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{\sqrt{x}}\,dx$$

input
```
integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(1/2),x, algorithm="maxim
a")
```

output
```
-2*d*sqrt(-e)*integrate(sqrt(e*x^2 + d)*x/((e*x^2 + d)*e^(log(e*x^2 + d) +
 1/2*log(x)) - (e^2*x^4 + d*e*x^2)*sqrt(x)), x) + 2*sqrt(x)*arctan2(sqrt(-
e)*x, sqrt(e*x^2 + d))
```

3.27.8 Giac [F]

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}\,dx = \int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{\sqrt{x}}\,dx$$

input
```
integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(1/2),x, algorithm="giac"
)
```

output
```
integrate(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/sqrt(x), x)
```

3.27.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{x}}\,dx = \int \frac{\operatorname{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)}{\sqrt{x}}\,dx$$

input
```
int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^(1/2),x)
```

output
```
int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^(1/2), x)
```

3.28 $\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{5/2}}\,dx$

3.28.1 Optimal result

Integrand size = 27, antiderivative size = 298

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{5/2}}\,dx = -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{3d\sqrt{x}}+\frac{4\sqrt{-e^2}\sqrt{x}\sqrt{d+ex^2}}{3d\left(\sqrt{d}+\sqrt{e}x\right)} - \frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}} - \frac{4\sqrt{-e}\sqrt[4]{e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{3d^{3/4}\sqrt{d+ex^2}} + \frac{2\sqrt{-e}\sqrt[4]{e}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{3d^{3/4}\sqrt{d+ex^2}}$$

output
```
-2/3*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(3/2)-4/3*(-e)^(1/2)*(e*x^2+d)
^(1/2)/d/x^(1/2)+4/3*(-e^2)^(1/2)*x^(1/2)*(e*x^2+d)^(1/2)/d/(d^(1/2)+x*e^(
1/2))-4/3*e^(1/4)*(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*a
rctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticE(sin(2*arctan(e^(1/4)*x^(1/2)/d^(
1/4))),1/2*2^(1/2))*(-e)^(1/2)*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e
^(1/2))^2)^(1/2)/d^(3/4)/(e*x^2+d)^(1/2)+2/3*e^(1/4)*(cos(2*arctan(e^(1/4)
*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*Ellipti
cF(sin(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(-e)^(1/2)*(d^(1/2)
+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/d^(3/4)/(e*x^2+d)^(1/2
)
```

3.28.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.10 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.41

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{5/2}}\,dx =$$
$$-\frac{2\left(6\sqrt{-e}x(d+ex^2)+3d\sqrt{d+ex^2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)+2(-e)^{3/2}x^3\sqrt{1+\frac{ex^2}{d}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\right.\right.}{9dx^{3/2}\sqrt{d+ex^2}}$$

input
```
Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(5/2),x]
```

output
```
(-2*(6*Sqrt[-e]*x*(d + e*x^2) + 3*d*Sqrt[d + e*x^2]*ArcTan[(Sqrt[-e]*x)/Sq
rt[d + e*x^2]] + 2*(-e)^(3/2)*x^3*Sqrt[1 + (e*x^2)/d]*Hypergeometric2F1[1/
2, 3/4, 7/4, -((e*x^2)/d)]))/(9*d*x^(3/2)*Sqrt[d + e*x^2])
```

3.28.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {5674, 264, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{5/2}}dx$$

$$\Big\downarrow \text{ 5674}$$

$$\frac{2}{3}\sqrt{-e}\int \frac{1}{x^{3/2}\sqrt{ex^2+d}}dx-\frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}}$$

$$\Big\downarrow \text{ 264}$$

$$\frac{2}{3}\sqrt{-e}\left(\frac{e\int \frac{\sqrt{x}}{\sqrt{ex^2+d}}dx}{d}-\frac{2\sqrt{d+ex^2}}{d\sqrt{x}}\right)-\frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{2}{3}\sqrt{-e}\left(\frac{2e\int\frac{x}{\sqrt{ex^2+d}}d\sqrt{x}}{d}-\frac{2\sqrt{d+ex^2}}{d\sqrt{x}}\right)-\frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}}$$

$\big\downarrow$ 834

$$\frac{2}{3}\sqrt{-e}\left(\frac{2e\left(\frac{\sqrt{d}\int\frac{1}{\sqrt{ex^2+d}}d\sqrt{x}}{\sqrt{e}}-\frac{\sqrt{d}\int\frac{\sqrt{d}-\sqrt{e}x}{\sqrt{d}\sqrt{ex^2+d}}d\sqrt{x}}{\sqrt{e}}\right)}{d}-\frac{2\sqrt{d+ex^2}}{d\sqrt{x}}\right)-\frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}}$$

$\big\downarrow$ 27

$$\frac{2}{3}\sqrt{-e}\left(\frac{2e\left(\frac{\sqrt{d}\int\frac{1}{\sqrt{ex^2+d}}d\sqrt{x}}{\sqrt{e}}-\frac{\int\frac{\sqrt{d}-\sqrt{e}x}{\sqrt{ex^2+d}}d\sqrt{x}}{\sqrt{e}}\right)}{d}-\frac{2\sqrt{d+ex^2}}{d\sqrt{x}}\right)-\frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}}$$

$\big\downarrow$ 761

$$\frac{2}{3}\sqrt{-e}\left(\frac{2e\left(\frac{\sqrt[4]{d}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{2e^{3/4}\sqrt{d+ex^2}}-\frac{\int\frac{\sqrt{d}-\sqrt{e}x}{\sqrt{ex^2+d}}d\sqrt{x}}{\sqrt{e}}\right)}{d}-\frac{2\sqrt{d+ex^2}}{d\sqrt{x}}\right)-\frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}}$$

$\big\downarrow$ 1510

3.28. $\int\frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{5/2}}\,dx$

$$\frac{2}{3}\sqrt{-e}\left(\frac{2e\left(\frac{\sqrt[4]{d}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{2e^{3/4}\sqrt{d+ex^2}}-\frac{\frac{\sqrt[4]{d}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{\sqrt[4]{e}\sqrt{d+ex^2}}-\cdots}{\sqrt{e}}\right.}{d}\right.$$

$$\frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{3x^{3/2}}$$

input
```
Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(5/2),x]
```

output
```
(-2*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/(3*x^(3/2)) + (2*Sqrt[-e]*((-2*S
qrt[d + e*x^2])/(d*Sqrt[x]) + (2*e*(-((-((Sqrt[x]*Sqrt[d + e*x^2])/(Sqrt[d
] + Sqrt[e]*x)) + (d^(1/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d]
 + Sqrt[e]*x)^2]*EllipticE[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(e^(
1/4)*Sqrt[d + e*x^2]))/Sqrt[e]) + (d^(1/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d +
 e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1
/4)], 1/2])/(2*e^(3/4)*Sqrt[d + e*x^2])))/d))/3
```

3.28.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 5674
```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Simp[c/(d*(m + 1))   Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ
[{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

3.28.4 Maple [F]

$$\int \frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{x^{\frac{5}{2}}}dx$$

input
```
int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(5/2),x)
```

output
```
int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(5/2),x)
```

3.28.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.27

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{5/2}}\,dx =$$
$$-\frac{2\left(2\sqrt{-e}\sqrt{e}x^2\text{weierstrassZeta}\left(-\frac{4d}{e},0,\text{weierstrassPInverse}\left(-\frac{4d}{e},0,x\right)\right)+2\sqrt{ex^2+d}\sqrt{-e}x^{\frac{3}{2}}+d\sqrt{x}\,\text{arct}\right.}{3\,dx^2}$$

input
```
integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(5/2),x, algorithm="frica
s")
```

output
```
-2/3*(2*sqrt(-e)*sqrt(e)*x^2*weierstrassZeta(-4*d/e, 0, weierstrassPInvers
e(-4*d/e, 0, x)) + 2*sqrt(e*x^2 + d)*sqrt(-e)*x^(3/2) + d*sqrt(x)*arctan(s
qrt(-e)*x/sqrt(e*x^2 + d)))/(d*x^2)
```

3.28.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.26

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{5/2}}\,dx = -\frac{2\,\text{atan}\left(\frac{x\sqrt{-e}}{\sqrt{d+ex^2}}\right)}{3x^{\frac{3}{2}}}+\frac{\sqrt{-e}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{ex^2e^{i\pi}}{d}\right)}{3\sqrt{d}\sqrt{x}\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**(5/2),x)
```

output
```
-2*atan(x*sqrt(-e)/sqrt(d + e*x**2))/(3*x**(3/2)) + sqrt(-e)*gamma(-1/4)*h
yper((-1/4, 1/2), (3/4,), e*x**2*exp_polar(I*pi)/d)/(3*sqrt(d)*sqrt(x)*gam
ma(3/4))
```

3.28.7 Maxima [F]

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{5/2}}\,dx = \int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(5/2),x, algorithm="maxim
a")
```

output
```
2/3*(3*d*sqrt(-e)*x^(3/2)*integrate(-1/3*sqrt(e*x^2 + d)*x/((e^2*x^4 + d*e
*x^2)*x^(5/2) - (e*x^2 + d)*e^(log(e*x^2 + d) + 5/2*log(x))), x) - arctan2
(sqrt(-e)*x, sqrt(e*x^2 + d)))/x^(3/2)
```

3.28.8 Giac [F]

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{5/2}}\,dx = \int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(5/2),x, algorithm="giac"
)
```

output
```
integrate(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^(5/2), x)
```

3.28.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{5/2}}\,dx = \int \frac{\operatorname{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^{5/2}}\,dx$$

input
```
int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^(5/2),x)
```

output
```
int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^(5/2), x)
```

3.29 $\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{9/2}}\,dx$

3.29.1 Optimal result

Integrand size = 27, antiderivative size = 331

$$\begin{aligned}
\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{9/2}}\,dx &= -\frac{4\sqrt{-e}\sqrt{d+ex^2}}{35dx^{5/2}}-\frac{12(-e)^{3/2}\sqrt{d+ex^2}}{35d^2\sqrt{x}} \\
&\quad -\frac{12\sqrt{-e}e^{3/2}\sqrt{x}\sqrt{d+ex^2}}{35d^2\left(\sqrt{d}+\sqrt{e}x\right)}-\frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}} \\
&\quad +\frac{12\sqrt{-e}e^{5/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{35d^{7/4}\sqrt{d+ex^2}} \\
&\quad -\frac{6\sqrt{-e}e^{5/4}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\operatorname{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{35d^{7/4}\sqrt{d+ex^2}}
\end{aligned}$$

output
```
-2/7*arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(7/2)-4/35*(-e)^(1/2)*(e*x^2+d
)^(1/2)/d/x^(5/2)-12/35*(-e)^(3/2)*(e*x^2+d)^(1/2)/d^2/x^(1/2)-12/35*e^(3/
2)*(-e)^(1/2)*x^(1/2)*(e*x^2+d)^(1/2)/d^2/(d^(1/2)+x*e^(1/2))+12/35*e^(5/4
)*(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^
(1/2)/d^(1/4)))*EllipticE(sin(2*arctan(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/
2))*(-e)^(1/2)*(d^(1/2)+x*e^(1/2))*((e*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)
/d^(7/4)/(e*x^2+d)^(1/2)-6/35*e^(5/4)*(cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4
)))^2)^(1/2)/cos(2*arctan(e^(1/4)*x^(1/2)/d^(1/4)))*EllipticF(sin(2*arctan
(e^(1/4)*x^(1/2)/d^(1/4))),1/2*2^(1/2))*(-e)^(1/2)*(d^(1/2)+x*e^(1/2))*((e
*x^2+d)/(d^(1/2)+x*e^(1/2))^2)^(1/2)/d^(7/4)/(e*x^2+d)^(1/2)
```

3.29.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.09 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.41

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{9/2}}\,dx = \frac{4\sqrt{-e}x(-d^2+2dex^2+3e^2x^4)-10d^2\sqrt{d+ex^2}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)-4(-e)^{5/2}x^5\sqrt{1}}{35d^2x^{7/2}\sqrt{d+ex^2}}$$

input
```
Integrate[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(9/2),x]
```

output
```
(4*Sqrt[-e]*x*(-d^2 + 2*d*e*x^2 + 3*e^2*x^4) - 10*d^2*Sqrt[d + e*x^2]*ArcT
an[(Sqrt[-e]*x)/Sqrt[d + e*x^2]] - 4*(-e)^(5/2)*x^5*Sqrt[1 + (e*x^2)/d]*Hy
pergeometric2F1[1/2, 3/4, 7/4, -((e*x^2)/d)])/(35*d^2*x^(7/2)*Sqrt[d + e*x
^2])
```

3.29.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 317, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {5674, 264, 264, 266, 834, 27, 761, 1510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{9/2}}dx$$

$$\Big\downarrow \text{5674}$$

$$\frac{2}{7}\sqrt{-e}\int \frac{1}{x^{7/2}\sqrt{ex^2+d}}dx-\frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}}$$

$$\Big\downarrow \text{264}$$

$$\frac{2}{7}\sqrt{-e}\left(-\frac{3e\int \frac{1}{x^{3/2}\sqrt{ex^2+d}}dx}{5d}-\frac{2\sqrt{d+ex^2}}{5dx^{5/2}}\right)-\frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}}$$

$$\Big\downarrow \text{264}$$

$$\frac{2}{7}\sqrt{-e}\left(-\frac{3e\left(\frac{e\int \frac{\sqrt{x}}{\sqrt{ex^2+d}}dx}{d}-\frac{2\sqrt{d+ex^2}}{d\sqrt{x}}\right)}{5d}-\frac{2\sqrt{d+ex^2}}{5dx^{5/2}}\right)-\frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{2}{7}\sqrt{-e}\left(-\frac{3e\left(\frac{2e\int \frac{x}{\sqrt{ex^2+d}}d\sqrt{x}}{d}-\frac{2\sqrt{d+ex^2}}{d\sqrt{x}}\right)}{5d}-\frac{2\sqrt{d+ex^2}}{5dx^{5/2}}\right)-\frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}}$$

$$\Big\downarrow \text{834}$$

$$\frac{2}{7}\sqrt{-e}\left(-\frac{3e\left(\frac{2e\left(\frac{\sqrt{d}\int \frac{1}{\sqrt{ex^2+d}}d\sqrt{x}}{\sqrt{e}}-\frac{\sqrt{d}\int \frac{\sqrt{d}-\sqrt{e}x}{\sqrt{d}\sqrt{ex^2+d}}d\sqrt{x}}{\sqrt{e}}\right)}{d}-\frac{2\sqrt{d+ex^2}}{d\sqrt{x}}\right)}{5d}-\frac{2\sqrt{d+ex^2}}{5dx^{5/2}}\right)-\frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}}$$

$$\Big\downarrow \text{27}$$

3.29. $\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{9/2}}dx$

$$\frac{2}{7}\sqrt{-e}\left(-\frac{3e\left(\frac{2e\left(\frac{\sqrt{d}\int \frac{1}{\sqrt{ex^2+d}}d\sqrt{x}}{\sqrt{e}}-\frac{\int \frac{\sqrt{d}-\sqrt{e}x}{\sqrt{ex^2+d}}d\sqrt{x}}{\sqrt{e}}\right)}{d}-\frac{2\sqrt{d+ex^2}}{d\sqrt{x}}\right)}{5d}-\frac{2\sqrt{d+ex^2}}{5dx^{5/2}}\right)-\frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}}$$

$\downarrow$ 761

$$\frac{2}{7}\sqrt{-e}\left(-\frac{3e\left(\frac{2e\left(\frac{\sqrt[4]{d}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{2e^{3/4}\sqrt{d+ex^2}}-\frac{\int \frac{\sqrt{d}-\sqrt{e}x}{\sqrt{ex^2+d}}d\sqrt{x}}{\sqrt{e}}\right)}{d}-\frac{2\sqrt{d+ex^2}}{d\sqrt{x}}\right)}{5d}-\frac{2\sqrt{d+ex^2}}{5dx^{5/2}}\right)-\frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}}$$

$\downarrow$ 1510

3.29. $\displaystyle\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{9/2}}\,dx$

$$\frac{2}{7}\sqrt{-e}\left(-\frac{3e\left(\frac{2e\left(\frac{\sqrt[4]{d}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}\,\text{EllipticF}\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right),\frac{1}{2}\right)}{2e^{3/4}\sqrt{d+ex^2}}-\frac{\frac{\sqrt[4]{d}\left(\sqrt{d}+\sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d}+\sqrt{e}x\right)^2}}E\left(2\arctan\left(\frac{\sqrt[4]{e}\sqrt{x}}{\sqrt[4]{d}}\right)|\frac{1}{2}\right)}{\sqrt[4]{e}\sqrt{d+ex^2}}-\sqrt{}}{\sqrt{e}}\right)}{d}}{5d}\right.$$

$$\frac{2\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{7x^{7/2}}$$

input
```
Int[ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]]/x^(9/2),x]
```

output
```
(-2*ArcTan[(Sqrt[-e]*x)/Sqrt[d + e*x^2]])/(7*x^(7/2)) + (2*Sqrt[-e]*((-2*S
qrt[d + e*x^2])/(5*d*x^(5/2)) - (3*e*((-2*Sqrt[d + e*x^2])/(d*Sqrt[x]) + (
2*e*(-((-((Sqrt[x]*Sqrt[d + e*x^2])/(Sqrt[d] + Sqrt[e]*x)) + (d^(1/4)*(Sqr
t[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2]*EllipticE[2*Ar
cTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(e^(1/4)*Sqrt[d + e*x^2]))/Sqrt[e])
 + (d^(1/4)*(Sqrt[d] + Sqrt[e]*x)*Sqrt[(d + e*x^2)/(Sqrt[d] + Sqrt[e]*x)^2
]*EllipticF[2*ArcTan[(e^(1/4)*Sqrt[x])/d^(1/4)], 1/2])/(2*e^(3/4)*Sqrt[d +
 e*x^2])))/d))/(5*d)))/7
```

3.29. $\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{9/2}}\,dx$

3.29.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 761
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 834
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 2]}, S
imp[1/q   Int[1/Sqrt[a + b*x^4], x], x] - Simp[1/q   Int[(1 - q*x^2)/Sqrt[a
 + b*x^4], x], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

rule 1510
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

rule 5674
```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*((d_.)*(x_))^(m_.), x_S
ymbol] :> Simp[(d*x)^(m + 1)*(ArcTan[(c*x)/Sqrt[a + b*x^2]]/(d*(m + 1))), x
] - Simp[c/(d*(m + 1))   Int[(d*x)^(m + 1)/Sqrt[a + b*x^2], x], x] /; FreeQ
[{a, b, c, d, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

3.29.4 Maple [F]

$$\int \frac{\arctan\left(\frac{x\sqrt{-e}}{\sqrt{e\,x^2+d}}\right)}{x^{\frac{9}{2}}}dx$$

input
```
int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(9/2),x)
```

output
```
int(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(9/2),x)
```

3.29.5 Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.28

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{9/2}}\,dx = \frac{2\left(6\sqrt{-e}e^{\frac{3}{2}}x^4\text{weierstrassZeta}\left(-\frac{4\,d}{e},0,\text{weierstrassPInverse}\left(-\frac{4\,d}{e},0,x\right)\right)-5\,d^2\sqrt{x}\right.}{35\,d^2x^4}$$

input
```
integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(9/2),x, algorithm="frica
s")
```

output
```
2/35*(6*sqrt(-e)*e^(3/2)*x^4*weierstrassZeta(-4*d/e, 0, weierstrassPInvers
e(-4*d/e, 0, x)) - 5*d^2*sqrt(x)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) + 2*(3
*e*x^3 - d*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt(x))/(d^2*x^4)
```

3.29.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate(atan(x*(-e)**(1/2)/(e*x**2+d)**(1/2))/x**(9/2),x)
```

output
```
Timed out
```

3.29.7 Maxima [F]

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{9/2}}\,dx = \int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{9}{2}}}\,dx$$

input
```
integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(9/2),x, algorithm="maxim
a")
```

output
```
2/7*(7*d*sqrt(-e)*x^(7/2)*integrate(-1/7*sqrt(e*x^2 + d)*x/((e^2*x^4 + d*e
*x^2)*x^(9/2) - (e*x^2 + d)*e^(log(e*x^2 + d) + 9/2*log(x))), x) - arctan2
(sqrt(-e)*x, sqrt(e*x^2 + d)))/x^(7/2)
```

3.29.8 Giac [F]

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{9/2}}\,dx = \int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)}{x^{\frac{9}{2}}}\,dx$$

input
```
integrate(arctan(x*(-e)^(1/2)/(e*x^2+d)^(1/2))/x^(9/2),x, algorithm="giac"
)
```

output
```
integrate(arctan(sqrt(-e)*x/sqrt(e*x^2 + d))/x^(9/2), x)
```

3.29.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan\left(\frac{\sqrt{-e}x}{\sqrt{d+ex^2}}\right)}{x^{9/2}}\,dx = \int \frac{\mathrm{atan}\left(\frac{\sqrt{-e}\,x}{\sqrt{e\,x^2+d}}\right)}{x^{9/2}}\,dx$$

input
```
int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^(9/2),x)
```

output
```
int(atan(((-e)^(1/2)*x)/(d + e*x^2)^(1/2))/x^(9/2), x)
```

3.30 $\int \frac{\arctan(1+x+x^2)}{x^2} \, dx$

3.30.1 Optimal result

Integrand size = 11, antiderivative size = 50

$$\int \frac{\arctan(1+x+x^2)}{x^2} \, dx = \frac{1}{2}\arctan(1+x) - \frac{\arctan(1+x+x^2)}{x} + \frac{\log(x)}{2} - \frac{1}{2}\log(1+x^2) + \frac{1}{4}\log(2+2x+x^2)$$

output
```
1/2*arctan(1+x)-arctan(x^2+x+1)/x+1/2*ln(x)-1/2*ln(x^2+1)+1/4*ln(x^2+2*x+2
)
```

3.30.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(1+x+x^2)}{x^2} \, dx = \frac{1}{2}\arctan(1+x) - \frac{\arctan(1+x+x^2)}{x} + \frac{\log(x)}{2} - \frac{1}{2}\log(1+x^2) + \frac{1}{4}\log(2+2x+x^2)$$

input
```
Integrate[ArcTan[1 + x + x^2]/x^2,x]
```

output
```
ArcTan[1 + x]/2 - ArcTan[1 + x + x^2]/x + Log[x]/2 - Log[1 + x^2]/2 + Log[
2 + 2*x + x^2]/4
```

3.30.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {5728, 2462, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(x^2+x+1)}{x^2}\,dx$$

$$\Big\downarrow \ 5728$$

$$\int \frac{2x+1}{x(x^4+2x^3+3x^2+2x+2)}dx - \frac{\arctan(x^2+x+1)}{x}$$

$$\Big\downarrow \ 2462$$

$$\int \left(-\frac{x}{x^2+1}+\frac{x+2}{2(x^2+2x+2)}+\frac{1}{2x}\right)dx - \frac{\arctan(x^2+x+1)}{x}$$

$$\Big\downarrow \ 2009$$

$$-\frac{\arctan(x^2+x+1)}{x}+\frac{1}{2}\arctan(x+1)-\frac{1}{2}\log(x^2+1)+\frac{1}{4}\log(x^2+2x+2)+\frac{\log(x)}{2}$$

input
```
Int[ArcTan[1 + x + x^2]/x^2,x]
```

output
```
ArcTan[1 + x]/2 - ArcTan[1 + x + x^2]/x + Log[x]/2 - Log[1 + x^2]/2 + Log[
2 + 2*x + x^2]/4
```

3.30.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2462
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{Qx = Factor[Px]}, Int[ExpandIntegr
and[u*Qx^p, x], x] /;  !SumQ[NonfreeFactors[Qx, x]]] /; PolyQ[Px, x] && GtQ
[Expon[Px, x], 2] &&  !BinomialQ[Px, x] &&  !TrinomialQ[Px, x] && ILtQ[p, 0
] && RationalFunctionQ[u, x]
```

rule 5728
```
Int[((a_.) + ArcTan[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(c + d*x)^(m + 1)*((a + b*ArcTan[u])/(d*(m + 1))), x] - Simp[b/(d*(m + 1)
)   Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(1 + u^2)), x], x], x]
 /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &
&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m +
 1, x]]
```

3.30.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

method	result
default	$\frac{\arctan(1+x)}{2}-\frac{\arctan(x^2+x+1)}{x}+\frac{\ln(x)}{2}-\frac{\ln(x^2+1)}{2}+\frac{\ln(x^2+2x+2)}{4}$
parts	$\frac{\arctan(1+x)}{2}-\frac{\arctan(x^2+x+1)}{x}+\frac{\ln(x)}{2}-\frac{\ln(x^2+1)}{2}+\frac{\ln(x^2+2x+2)}{4}$
risch	$\frac{i\ln(1+i(x^2+x+1))}{2x}-\frac{i(i\ln(x+1-i)x+i\ln(x+1+i)x+2i\ln(x)x-2i\ln(x^2+1)x+\ln(x+1-i)x-\ln(x+1+i)x+2\ln(1-i(x^2+x}{4x}$
parallelrisch	$\frac{-4i\ln(i+x)x+7i\ln(x+1+i)x-7i\ln(x+1-i)x+4i\ln(x-i)x+6\ln(x)x-6\ln(x-i)x-6\ln(i+x)x+3\ln(x+1-i)x+3\ln(x+1+i)x-}{12x}$

input
```
int(arctan(x^2+x+1)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*arctan(1+x)-arctan(x^2+x+1)/x+1/2*ln(x)-1/2*ln(x^2+1)+1/4*ln(x^2+2*x+2
)
```

3.30.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int \frac{\arctan(1+x+x^2)}{x^2}\,dx = \frac{2\,x\arctan(x+1)+x\log(x^2+2\,x+2)-2\,x\log(x^2+1)+2\,x\log(x)-4\,\arctan(x^2+x+1)}{4\,x}$$

input
```
integrate(arctan(x^2+x+1)/x^2,x, algorithm="fricas")
```

output
```
1/4*(2*x*arctan(x + 1) + x*log(x^2 + 2*x + 2) - 2*x*log(x^2 + 1) + 2*x*log
(x) - 4*arctan(x^2 + x + 1))/x
```

3.30.6 Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82

$$\int \frac{\arctan(1+x+x^2)}{x^2}\,dx = \frac{\log(x)}{2} - \frac{\log(x^2+1)}{2} + \frac{\log(x^2+2x+2)}{4} + \frac{\operatorname{atan}(x+1)}{2} - \frac{\operatorname{atan}(x^2+x+1)}{x}$$

input
```
integrate(atan(x**2+x+1)/x**2,x)
```

output
```
log(x)/2 - log(x**2 + 1)/2 + log(x**2 + 2*x + 2)/4 + atan(x + 1)/2 - atan(
x**2 + x + 1)/x
```

3.30.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int \frac{\arctan(1+x+x^2)}{x^2}\,dx = -\frac{\arctan(x^2+x+1)}{x} + \frac{1}{2}\arctan(x+1) + \frac{1}{4}\log\left(x^2+2x+2\right) - \frac{1}{2}\log\left(x^2+1\right) + \frac{1}{2}\log(x)$$

input
```
integrate(arctan(x^2+x+1)/x^2,x, algorithm="maxima")
```

output
```
-arctan(x^2 + x + 1)/x + 1/2*arctan(x + 1) + 1/4*log(x^2 + 2*x + 2) - 1/2*
log(x^2 + 1) + 1/2*log(x)
```

3.30.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

$$\int \frac{\arctan(1+x+x^2)}{x^2}\,dx = -\frac{\arctan(x^2+x+1)}{x} + \frac{1}{2}\arctan(x+1) + \frac{1}{4}\log\left(x^2+2x+2\right) - \frac{1}{2}\log\left(x^2+1\right) + \frac{1}{2}\log(|x|)$$

input
```
integrate(arctan(x^2+x+1)/x^2,x, algorithm="giac")
```

output
```
-arctan(x^2 + x + 1)/x + 1/2*arctan(x + 1) + 1/4*log(x^2 + 2*x + 2) - 1/2*
log(x^2 + 1) + 1/2*log(abs(x))
```

3.30.9 Mupad [B] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int \frac{\arctan (1+x+x^2)}{x^2} \, dx = \frac{\mathrm{atan}(x+1)}{2} + \frac{\ln (x^2+2\,x+2)}{4} - \frac{\ln (x^2+1)}{2} + \frac{\ln (x)}{2} - \frac{\mathrm{atan}(x^2+x+1)}{x}$$

input
```
int(atan(x + x^2 + 1)/x^2,x)
```

output
```
atan(x + 1)/2 + log(2*x + x^2 + 2)/4 - log(x^2 + 1)/2 + log(x)/2 - atan(x
+ x^2 + 1)/x
```

3.31 $\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$

3.31.1 Optimal result

Integrand size = 40, antiderivative size = 40

$$\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=\operatorname{Int}\left(\frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2},x\right)$$

output
```
Unintegrable((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

3.31.2 Mathematica [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$$

input
```
Integrate[(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
```

output
```
Integrate[(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2), x]
```

3.31.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7234}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^n}{1-c^2x^2}dx$$

$$\Big\downarrow \text{ 7234}$$

$$\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^n}{1-c^2x^2}dx$$

input
```
Int[(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^n/(1 - c^2*x^2),x]
```

output
```
$Aborted
```

3.31.3.1 Defintions of rubi rules used

rule 7234
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_)/((A_) + (C_.)*(x_)^2), x_Symbol] :> Unintegrable[(a + b*F[(c*
Sqrt[d + e*x])/Sqrt[f + g*x]])^n/(A + C*x^2), x] /; FreeQ[{a, b, c, d, e, f
, g, A, C, F, n}, x] && EqQ[C*d*f - A*e*g, 0] && EqQ[e*f + d*g, 0] &&  !IGt
Q[n, 0]
```

3.31.4 Maple [N/A] (verified)

Not integrable

Time = 0.42 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{\left(a+b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^n}{-c^2x^2+1}dx$$

input
```
int((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

output
```
int((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x)
```

3.31.5 Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=\int -\frac{\left(b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, alg
orithm="fricas")
```

output
```
integral(-(b*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x)
```

3.31.6 Sympy [N/A]

Not integrable

Time = 3.75 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.85

$$\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=-\int \frac{\left(a+b\,\mathrm{atan}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^n}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*atan((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**n/(-c**2*x**2+1),x)
```

output
```
-Integral((a + b*atan(sqrt(-c*x + 1)/sqrt(c*x + 1)))**n/(c**2*x**2 - 1), x
)
```

3.31.7 Maxima [N/A]

Not integrable

Time = 0.96 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=\int -\frac{\left(b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, alg
orithm="maxima")
```

output
```
-integrate((b*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x
)
```

3.31.8 Giac [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=\int -\frac{\left(b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^n}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^n/(-c^2*x^2+1),x, alg
orithm="giac")
```

output
```
integrate(-(b*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^n/(c^2*x^2 - 1), x
)
```

3.31.9 Mupad [N/A]

Not integrable

Time = 1.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx=-\int \frac{\left(a+b\,\mathrm{atan}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^n}{c^2\,x^2-1}\,dx$$

input
```
int(-(a + b*atan((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^n/(c^2*x^2 - 1),x)
```

output
```
-int((a + b*atan((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^n/(c^2*x^2 - 1), x)
```

3.31. $\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^n}{1-c^2x^2}\,dx$

3.32 $\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$

3.32.1 Optimal result

Integrand size = 40, antiderivative size = 431

$$\begin{aligned}
&\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx \\
&= -\frac{2\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3\operatorname{arctanh}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} \\
&\quad+\frac{3ib\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{2c} \\
&\quad-\frac{3ib\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{2c} \\
&\quad+\frac{3b^2\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\operatorname{PolyLog}\left(3,1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{2c} \\
&\quad-\frac{3b^2\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\operatorname{PolyLog}\left(3,-1+\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{2c} \\
&\quad-\frac{3ib^3\operatorname{PolyLog}\left(4,1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{4c}+\frac{3ib^3\operatorname{PolyLog}\left(4,-1+\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{4c}
\end{aligned}$$

output
```
2*(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3*arctanh(-1+2/(1+I*(-c*x+1)^
(1/2)/(c*x+1)^(1/2)))/c+3/2*I*b*(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))
^2*polylog(2,1-2/(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c-3/2*I*b*(a+b*arctan
((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2*polylog(2,-1+2/(1+I*(-c*x+1)^(1/2)/(c*x+
1)^(1/2)))/c+3/2*b^2*(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))*polylog(3,
1-2/(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c-3/2*b^2*(a+b*arctan((-c*x+1)^(1/
2)/(c*x+1)^(1/2)))*polylog(3,-1+2/(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c-3/
4*I*b^3*polylog(4,1-2/(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c+3/4*I*b^3*poly
log(4,-1+2/(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c
```

3.32.2 Mathematica [F]

$$\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx=\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$$

input
```
Integrate[(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
```

output
```
Integrate[(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2), x]
```

3.32.3 Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {7232, 5357, 5523, 5529, 5533, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{1-c^2x^2}\,dx$$

$$\Big\downarrow \text{7232}$$

$$-\frac{\int \frac{\sqrt{cx+1}\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3}{\sqrt{1-cx}}d\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}{c}$$

$$\Big\downarrow \text{5357}$$

$$-\frac{2\text{arctanh}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3-6b\int\frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2\text{arctanh}\left(1-\frac{2}{\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}+1}\right)}{\frac{1-cx}{cx+1}+1}d\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}{c}$$

$\big\downarrow$ 5523

$$-\frac{2\text{arctanh}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3-6b\left(\frac{1}{2}\int\frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2\log\left(2-\frac{2}{\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}+1}\right)}{\frac{1-cx}{cx+1}+1}d\frac{\sqrt{1-cx}}{\sqrt{cx+1}}-\frac{1}{2}\int\right.}{c}$$

$\big\downarrow$ 5529

$$-\frac{2\text{arctanh}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3-6b\left(\frac{1}{2}\left(\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}+1}\right)\left(a+b\arctan\left(\frac{\sqrt{1-}}{\sqrt{cx-}}\right.\right.\right.\right.}{}$$

$\big\downarrow$ 5533

$$-\frac{2\text{arctanh}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3-6b\left(\frac{1}{2}\left(\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}+1}\right)\left(a+b\arctan\left(\frac{\sqrt{1-}}{\sqrt{cx-}}\right.\right.\right.\right.}{}$$

$\big\downarrow$ 7164

$$-\frac{2\text{arctanh}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^3-6b\left(\frac{1}{2}\left(\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}+1}\right)\left(a+b\arctan\left(\frac{\sqrt{1-c}}{\sqrt{cx+}}\right.\right.\right.\right.}{}$$

input
```
Int[(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3/(1 - c^2*x^2),x]
```

3.32. $\int\frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx$

output
```
-((2*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^3*ArcTanh[1 - 2/(1 + (I*S
qrt[1 - c*x])/Sqrt[1 + c*x])] - 6*b*(((I/2)*(a + b*ArcTan[Sqrt[1 - c*x]/Sq
rt[1 + c*x]])^2*PolyLog[2, 1 - 2/(1 + (I*Sqrt[1 - c*x])/Sqrt[1 + c*x])] -
I*b*((I/2)*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[3, 1 - 2/(1
 + (I*Sqrt[1 - c*x])/Sqrt[1 + c*x])] + (b*PolyLog[4, 1 - 2/(1 + (I*Sqrt[1
- c*x])/Sqrt[1 + c*x])])/4))/2 + ((-1/2*I)*(a + b*ArcTan[Sqrt[1 - c*x]/Sqr
t[1 + c*x]])^2*PolyLog[2, -1 + 2/(1 + (I*Sqrt[1 - c*x])/Sqrt[1 + c*x])] +
I*b*((I/2)*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[3, -1 + 2/(
1 + (I*Sqrt[1 - c*x])/Sqrt[1 + c*x])] + (b*PolyLog[4, -1 + 2/(1 + (I*Sqrt[
1 - c*x])/Sqrt[1 + c*x])])/4))/2))/c)
```

3.32.3.1 Defintions of rubi rules used

rule 5357
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Simp[2*b*c*p   Int[(a + b
*ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
 FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

rule 5523
```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[1/2   Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e
*x^2)), x], x] - Simp[1/2   Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] &&
EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5533
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1
, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &&
EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

rule 7232
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Simp[2*e*(g/(C*(e*f - d
*g)))   Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]],
x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] &&
EqQ[e*f + d*g, 0] && IGtQ[n, 0]
```

3.32.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1663 vs. $2(360) = 720$.

Time = 0.58 (sec) , antiderivative size = 1664, normalized size of antiderivative = 3.86

method	result	size
default	Expression too large to display	1664
parts	Expression too large to display	1664

input
```
int((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x,method=_RE
TURNVERBOSE)
```

output
```
-1/2*a^3/c*ln(c*x-1)+1/2*a^3/c*ln(c*x+1)-b^3*(1/c*arctan((-c*x+1)^(1/2)/(c
*x+1)^(1/2))^3*ln(1-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1
)^(1/2))-3*I/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))^2*polylog(2,(1+I*(-c*x
+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))+6/c*arctan((-c*x+1)^(
1/2)/(c*x+1)^(1/2))*polylog(3,(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)
/(c*x+1)+1)^(1/2))+6*I/c*polylog(4,(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c
*x+1)/(c*x+1)+1)^(1/2))-1/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))^3*ln(1+(1
+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))+3/2*I/c*arctan((-
c*x+1)^(1/2)/(c*x+1)^(1/2))^2*polylog(2,-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2)
)^2/((-c*x+1)/(c*x+1)+1))-3/2/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polyl
og(3,-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))-3/4*I/c*p
olylog(4,-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))+1/c*a
rctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))^3*ln(1+(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/
2))/((-c*x+1)/(c*x+1)+1)^(1/2))-3*I/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))
^2*polylog(2,-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2
))+6/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(3,-(1+I*(-c*x+1)^(1/2)
/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))+6*I/c*polylog(4,-(1+I*(-c*x+1)
^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2)))-3*a*b^2*(1/c*arctan((-c
*x+1)^(1/2)/(c*x+1)^(1/2))^2*ln(1-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*
x+1)/(c*x+1)+1)^(1/2))-2*I/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polyl...
```

3.32.5 Fricas [F]

$$\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx=\int -\frac{\left(b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^3}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, alg
orithm="fricas")
```

output
```
integral(-(b^3*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1))^3 + 3*a*b^2*arctan(sqr
t(-c*x + 1)/sqrt(c*x + 1))^2 + 3*a^2*b*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1)
) + a^3)/(c^2*x^2 - 1), x)
```

3.32.6 Sympy [F]

$$\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = -\int \frac{a^3}{c^2x^2-1}\,dx - \int \frac{b^3\operatorname{atan}^3\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx - \int \frac{3ab^2\operatorname{atan}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx - \int \frac{3a^2b\operatorname{atan}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*atan((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**3/(-c**2*x**2+1),x)
```

output
```
-Integral(a**3/(c**2*x**2 - 1), x) - Integral(b**3*atan(sqrt(-c*x + 1)/sqr
t(c*x + 1))**3/(c**2*x**2 - 1), x) - Integral(3*a*b**2*atan(sqrt(-c*x + 1)
/sqrt(c*x + 1))**2/(c**2*x**2 - 1), x) - Integral(3*a**2*b*atan(sqrt(-c*x
+ 1)/sqrt(c*x + 1))/(c**2*x**2 - 1), x)
```

3.32.7 Maxima [F]

$$\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx = \int -\frac{\left(b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^3}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, alg
orithm="maxima")
```

output
```
1/2*a^3*(log(c*x + 1)/c - log(c*x - 1)/c) + 1/64*(4*(b^3*log(c*x + 1) - b^
3*log(-c*x + 1))*arctan2(sqrt(-c*x + 1), sqrt(c*x + 1))^3 - 3*(b^3*log(2)^
2*log(c*x + 1) - b^3*log(2)^2*log(-c*x + 1))*arctan2(sqrt(-c*x + 1), sqrt(
c*x + 1)) - 64*c*integrate(1/128*(112*b^3*arctan2(sqrt(-c*x + 1), sqrt(c*x
 + 1))^3 + 384*a*b^2*arctan2(sqrt(-c*x + 1), sqrt(c*x + 1))^2 - 3*(b^3*log
(2)^2*log(c*x + 1) - b^3*log(2)^2*log(-c*x + 1) - 4*(b^3*log(c*x + 1) - b^
3*log(-c*x + 1))*arctan2(sqrt(-c*x + 1), sqrt(c*x + 1))^2)*sqrt(c*x + 1)*s
qrt(-c*x + 1) + 12*(b^3*log(2)^2 + 32*a^2*b)*arctan2(sqrt(-c*x + 1), sqrt(
c*x + 1)))/(c^2*x^2 - 1), x))/c
```

3.32.8 Giac [F]

$$\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx=\int -\frac{\left(b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^3}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^3/(-c^2*x^2+1),x, alg
orithm="giac")
```

output
```
integrate(-(b*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^3/(c^2*x^2 - 1), x
)
```

3.32.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^3}{1-c^2x^2}\,dx=\int -\frac{\left(a+b\,\mathrm{atan}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^3}{c^2\,x^2-1}\,dx$$

input
```
int(-(a + b*atan((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3/(c^2*x^2 - 1),x)
```

output
```
int(-(a + b*atan((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^3/(c^2*x^2 - 1), x)
```

3.33 $\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx$

3.33.1 Optimal result

Integrand size = 40, antiderivative size = 283

$$\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx = -\frac{2\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2\operatorname{arctanh}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{ib\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\operatorname{PolyLog}\left(2,1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} - \frac{ib\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)\operatorname{PolyLog}\left(2,-1+\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{c} + \frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{2c} - \frac{b^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}}\right)}{2c}$$

output
```
2*(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2*arctanh(-1+2/(1+I*(-c*x+1)^
(1/2)/(c*x+1)^(1/2)))/c+I*b*(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))*pol
ylog(2,1-2/(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c-I*b*(a+b*arctan((-c*x+1)^
(1/2)/(c*x+1)^(1/2)))*polylog(2,-1+2/(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c
+1/2*b^2*polylog(3,1-2/(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c-1/2*b^2*polyl
og(3,-1+2/(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2)))/c
```

3.33.2 Mathematica [F]

$$\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx=\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx$$

input
```
Integrate[(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```

output
```
Integrate[(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2), x]
```

3.33.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {7232, 5357, 5523, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{1-c^2x^2}dx$$

$$\Big\downarrow \; 7232$$

$$-\frac{\int \frac{\sqrt{cx+1}\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}{\sqrt{1-cx}}d\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}{c}$$

$$\Big\downarrow \; 5357$$

$$-\frac{2\text{arctanh}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2-4b\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)\text{arctanh}\left(1-\frac{2}{\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}+1}\right)}{\frac{1-cx}{cx+1}+1}d\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}{c}$$

$$\Big\downarrow \; 5523$$

$$-\frac{2\text{arctanh}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2-4b\left(\frac{1}{2}\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)\log\left(2-\frac{2}{\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}+1}\right)}{\frac{1-cx}{cx+1}+1}d\frac{\sqrt{1-cx}}{\sqrt{cx+1}}-\frac{1}{2}\int\right.}{c}$$

$\Big\downarrow$ 5529

$$-\frac{2\text{arctanh}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2-4b\left(\frac{1}{2}\left(\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}+1}\right)\left(a+b\arctan\left(\frac{\sqrt{1-}}{\sqrt{cx}}\right.\right.\right.\right.}{}$$

$\Big\downarrow$ 7164

$$-\frac{2\text{arctanh}\left(1-\frac{2}{1+\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}}\right)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2-4b\left(\frac{1}{2}\left(\frac{1}{2}i\,\text{PolyLog}\left(2,1-\frac{2}{\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}+1}\right)\left(a+b\arctan\left(\frac{\sqrt{1-c}}{\sqrt{cx+}}\right.\right.\right.\right.}{}$$

input
```
Int[(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2/(1 - c^2*x^2),x]
```

output
```
-((2*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2*ArcTanh[1 - 2/(1 + (I*S
qrt[1 - c*x])/Sqrt[1 + c*x])] - 4*b*(((I/2)*(a + b*ArcTan[Sqrt[1 - c*x]/Sq
rt[1 + c*x]])*PolyLog[2, 1 - 2/(1 + (I*Sqrt[1 - c*x])/Sqrt[1 + c*x])] + (b
*PolyLog[3, 1 - 2/(1 + (I*Sqrt[1 - c*x])/Sqrt[1 + c*x])])/4)/2 + ((-1/2*I)
*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])*PolyLog[2, -1 + 2/(1 + (I*Sqr
t[1 - c*x])/Sqrt[1 + c*x])] - (b*PolyLog[3, -1 + 2/(1 + (I*Sqrt[1 - c*x])/
Sqrt[1 + c*x])])/4)/2))/c)
```

3.33.3.1 Defintions of rubi rules used

rule 5357
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Simp[2*b*c*p   Int[(a + b
*ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
 FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

rule 5523
```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[1/2   Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e
*x^2)), x], x] - Simp[1/2   Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] &&
EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

rule 7232
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Simp[2*e*(g/(C*(e*f - d
*g)))   Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]],
x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] &&
EqQ[e*f + d*g, 0] && IGtQ[n, 0]
```

3.33.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 915 vs. $2(240) = 480$.

Time $= 0.48$ (sec) , antiderivative size $= 916$, normalized size of antiderivative $= 3.24$

method	result
default	$-\frac{a^2\ln(cx-1)}{2c}+\frac{a^2\ln(cx+1)}{2c}-b^2\left(\frac{\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2\ln\left(1-\frac{1+\frac{i\sqrt{-cx+1}}{\sqrt{cx+1}}}{\sqrt{\frac{-cx+1}{cx+1}+1}}\right)}{c}-\frac{2i\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\operatorname{polylog}\left(2,\frac{1+\frac{i\sqrt{-cx+1}}{\sqrt{cx+1}}}{\sqrt{\frac{-cx+1}{cx+1}+1}}\right.}{c}\right.$
parts	$-\frac{a^2\ln(cx-1)}{2c}+\frac{a^2\ln(cx+1)}{2c}-b^2\left(\frac{\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)^2\ln\left(1-\frac{1+\frac{i\sqrt{-cx+1}}{\sqrt{cx+1}}}{\sqrt{\frac{-cx+1}{cx+1}+1}}\right)}{c}-\frac{2i\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\operatorname{polylog}\left(2,\frac{1+\frac{i\sqrt{-cx+1}}{\sqrt{cx+1}}}{\sqrt{\frac{-cx+1}{cx+1}+1}}\right.}{c}\right.$

input
```
int((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x,method=_RE
TURNVERBOSE)
```

output
```
-1/2*a^2/c*ln(c*x-1)+1/2*a^2/c*ln(c*x+1)-b^2*(1/c*arctan((-c*x+1)^(1/2)/(c
*x+1)^(1/2))^2*ln(1-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1
)^(1/2))-2*I/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,(1+I*(-c*x+1
)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))+2/c*polylog(3,(1+I*(-c*
x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))-1/c*arctan((-c*x+1)^
(1/2)/(c*x+1)^(1/2))^2*ln(1+(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)
/(c*x+1)+1))+I/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,-(1+I*(-c*
x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))-1/2/c*polylog(3,-(1+I*(-
c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))+1/c*arctan((-c*x+1)^(1
/2)/(c*x+1)^(1/2))^2*ln(1+(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*
x+1)+1)^(1/2))-2*I/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))*polylog(2,-(1+I*
(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))+2/c*polylog(3,-(
1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2)))-2*a*b*(1/c*
arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln(1-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2
))/((-c*x+1)/(c*x+1)+1)^(1/2))-I/c*polylog(2,(1+I*(-c*x+1)^(1/2)/(c*x+1)^(
1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))-1/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))
*ln(1+(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))+1/2*I/c*p
olylog(2,-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))+1/c*a
rctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln(1+(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2)
)/((-c*x+1)/(c*x+1)+1)^(1/2))-I/c*polylog(2,-(1+I*(-c*x+1)^(1/2)/(c*x+1...
```

3.33.5 Fricas [F]

$$\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx=\int -\frac{\left(b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, alg
orithm="fricas")
```

output
```
integral(-(b^2*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1))^2 + 2*a*b*arctan(sqrt(
-c*x + 1)/sqrt(c*x + 1)) + a^2)/(c^2*x^2 - 1), x)
```

3.33.6 Sympy [F]

$$\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx = -\int \frac{a^2}{c^2x^2-1}\,dx - \int \frac{b^2\operatorname{atan}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx - \int \frac{2ab\operatorname{atan}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*atan((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2/(-c**2*x**2+1),x)
```

output
```
-Integral(a**2/(c**2*x**2 - 1), x) - Integral(b**2*atan(sqrt(-c*x + 1)/sqr
t(c*x + 1))**2/(c**2*x**2 - 1), x) - Integral(2*a*b*atan(sqrt(-c*x + 1)/sq
rt(c*x + 1))/(c**2*x**2 - 1), x)
```

3.33.7 Maxima [F]

$$\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx = \int -\frac{\left(b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, alg
orithm="maxima")
```

output
```
1/2*a^2*(log(c*x + 1)/c - log(c*x - 1)/c) - 1/32*(b^2*log(2)^2*log(c*x + 1
) - b^2*log(2)^2*log(-c*x + 1) - 4*(b^2*log(c*x + 1) - b^2*log(-c*x + 1))*
arctan2(sqrt(-c*x + 1), sqrt(c*x + 1))^2 - (b^2*(log(c*x + 1)/c - log(c*x
- 1)/c)*log(2)^2 - 64*b^2*integrate(1/16*sqrt(c*x + 1)*sqrt(-c*x + 1)*arct
an(sqrt(-c*x + 1)/sqrt(c*x + 1))*log(c*x + 1)/(c^2*x^2 - 1), x) + 64*b^2*i
ntegrate(1/16*sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan(sqrt(-c*x + 1)/sqrt(c*x
+ 1))*log(-c*x + 1)/(c^2*x^2 - 1), x) - 384*b^2*integrate(1/16*arctan(sqrt
(-c*x + 1)/sqrt(c*x + 1))^2/(c^2*x^2 - 1), x) - 1024*a*b*integrate(1/16*ar
ctan(sqrt(-c*x + 1)/sqrt(c*x + 1))/(c^2*x^2 - 1), x))*c)/c
```

3.33.8 Giac [F]

$$\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx=\int -\frac{\left(b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2/(-c^2*x^2+1),x, alg
orithm="giac")
```

output
```
integrate(-(b*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^2/(c^2*x^2 - 1), x
)
```

3.33.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}{1-c^2x^2}\,dx=\int -\frac{\left(a+b\,\mathrm{atan}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^2}{c^2\,x^2-1}\,dx$$

input
```
int(-(a + b*atan((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2/(c^2*x^2 - 1),x)
```

output
```
int(-(a + b*atan((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2/(c^2*x^2 - 1), x)
```

3.34 $\int \frac{a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx$

3.34.1 Optimal result

Integrand size = 38, antiderivative size = 98

$$\int \frac{a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = -\frac{a\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}-\frac{ib\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{2c}+\frac{ib\,\text{PolyLog}\left(2,\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{2c}$$

output
```
-a*ln((-c*x+1)^(1/2)/(c*x+1)^(1/2))/c-1/2*I*b*polylog(2,-I*(-c*x+1)^(1/2)/
(c*x+1)^(1/2))/c+1/2*I*b*polylog(2,I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/c
```

3.34.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.95

$$\int \frac{a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx = -\frac{a\log\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)+\frac{1}{2}ib\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}\right)-\frac{1}{2}ib\,\text{PolyLog}\left(2,\frac{i\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{c}$$

input
```
Integrate[(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

output
```
-((a*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]] + (I/2)*b*PolyLog[2, ((-I)*Sqrt[1 -
c*x])/Sqrt[1 + c*x]] - (I/2)*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]]
)/c)
```

3.34.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {7232, 5355, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{1-c^2x^2}dx$$

$$\Big\downarrow \text{7232}$$

$$-\frac{\int \frac{\sqrt{cx+1}\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}{\sqrt{1-cx}}d\frac{\sqrt{1-cx}}{\sqrt{cx+1}}}{c}$$

$$\Big\downarrow \text{5355}$$

$$-\frac{\frac{1}{2}ib\int \frac{\sqrt{cx+1}\log\left(1-\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{\sqrt{1-cx}}d\frac{\sqrt{1-cx}}{\sqrt{cx+1}}-\frac{1}{2}ib\int \frac{\sqrt{cx+1}\log\left(\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}+1\right)}{\sqrt{1-cx}}d\frac{\sqrt{1-cx}}{\sqrt{cx+1}}+a\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{c}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{a\log\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)+\frac{1}{2}ib\,\text{PolyLog}\left(2,-\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}\right)-\frac{1}{2}ib\,\text{PolyLog}\left(2,\frac{i\sqrt{1-cx}}{\sqrt{cx+1}}\right)}{c}$$

input
```
Int[(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])/(1 - c^2*x^2),x]
```

output
```
-((a*Log[Sqrt[1 - c*x]/Sqrt[1 + c*x]] + (I/2)*b*PolyLog[2, ((-I)*Sqrt[1 -
c*x])/Sqrt[1 + c*x]] - (I/2)*b*PolyLog[2, (I*Sqrt[1 - c*x])/Sqrt[1 + c*x]]
)/c)
```

3.34.3.1 Defintions of rubi rules used

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5355
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Simp[I*(b/2)   Int[Log[1 - I*c*x]/x, x], x] - Simp[I*(b/2)   Int[Log[1
+ I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

rule 7232
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Simp[2*e*(g/(C*(e*f - d
*g)))   Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]],
x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] &&
EqQ[e*f + d*g, 0] && IGtQ[n, 0]
```

3.34.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 367 vs. $2(78) = 156$.

Time = 0.32 (sec) , antiderivative size = 368, normalized size of antiderivative = 3.76

method	result
default	$-\frac{a\ln(cx-1)}{2c}+\frac{a\ln(cx+1)}{2c}-b\left(\frac{\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\ln\left(1-\frac{1+\frac{i\sqrt{-cx+1}}{\sqrt{cx+1}}}{\sqrt{\frac{-cx+1}{cx+1}+1}}\right)}{c}-\frac{i\,\mathrm{polylog}\left(2,\frac{1+\frac{i\sqrt{-cx+1}}{\sqrt{cx+1}}}{\sqrt{\frac{-cx+1}{cx+1}+1}}\right)}{c}-\frac{\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{}\right.$
parts	$-\frac{a\ln(cx-1)}{2c}+\frac{a\ln(cx+1)}{2c}-b\left(\frac{\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\ln\left(1-\frac{1+\frac{i\sqrt{-cx+1}}{\sqrt{cx+1}}}{\sqrt{\frac{-cx+1}{cx+1}+1}}\right)}{c}-\frac{i\,\mathrm{polylog}\left(2,\frac{1+\frac{i\sqrt{-cx+1}}{\sqrt{cx+1}}}{\sqrt{\frac{-cx+1}{cx+1}+1}}\right)}{c}-\frac{\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{}\right.$

input
```
int((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x,method=_RETU
RNVERBOSE)
```

output
```
-1/2*a/c*ln(c*x-1)+1/2*a/c*ln(c*x+1)-b*(1/c*arctan((-c*x+1)^(1/2)/(c*x+1)^
(1/2))*ln(1-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))
-I/c*polylog(2,(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/
2))-1/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))*ln(1+(1+I*(-c*x+1)^(1/2)/(c*x
+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))+1/2*I/c*polylog(2,-(1+I*(-c*x+1)^(1/2)/
(c*x+1)^(1/2))^2/((-c*x+1)/(c*x+1)+1))+1/c*arctan((-c*x+1)^(1/2)/(c*x+1)^(
1/2))*ln(1+(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/2))-
I/c*polylog(2,-(1+I*(-c*x+1)^(1/2)/(c*x+1)^(1/2))/((-c*x+1)/(c*x+1)+1)^(1/
2)))
```

3.34.5 Fricas [F]

$$\int \frac{a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx=\int -\frac{b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algor
ithm="fricas")
```

output
```
integral(-(b*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)/(c^2*x^2 - 1), x)
```

3.34.6 Sympy [F]

$$\int \frac{a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx=-\int \frac{a}{c^2x^2-1}\,dx-\int \frac{b\,\mathrm{atan}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*atan((-c*x+1)**(1/2)/(c*x+1)**(1/2)))/(-c**2*x**2+1),x)
```

output
```
-Integral(a/(c**2*x**2 - 1), x) - Integral(b*atan(sqrt(-c*x + 1)/sqrt(c*x
+ 1))/(c**2*x**2 - 1), x)
```

3.34.7 Maxima [F]

$$\int \frac{a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx=\int -\frac{b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algor
ithm="maxima")
```

output
```
1/2*a*(log(c*x + 1)/c - log(c*x - 1)/c) + 1/2*((log(c*x + 1) - log(-c*x +
1))*arctan2(sqrt(-c*x + 1), sqrt(c*x + 1)) - 2*c*integrate(1/2*(e^(1/2*log
(c*x + 1) + 1/2*log(-c*x + 1))*log(c*x + 1) - e^(1/2*log(c*x + 1) + 1/2*lo
g(-c*x + 1))*log(-c*x + 1))/((c^2*x^2 - 1)*(c*x + 1) - (c^2*x^2 - 1)*(c*x
- 1)), x))*b/c
```

3.34.8 Giac [F]

$$\int \frac{a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx=\int -\frac{b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a}{c^2x^2-1}\,dx$$

input
```
integrate((a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))/(-c^2*x^2+1),x, algor
ithm="giac")
```

output
```
integrate(-(b*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)/(c^2*x^2 - 1), x)
```

3.34.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)}{1-c^2x^2}\,dx=\int -\frac{a+b\,\mathrm{atan}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)}{c^2\,x^2-1}\,dx$$

input
```
int(-(a + b*atan((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))/(c^2*x^2 - 1),x)
```

output
```
int(-(a + b*atan((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))/(c^2*x^2 - 1), x)
```

3.35 $\int \frac{1}{(1-c^2x^2)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$

3.35.1 Optimal result

Integrand size = 40, antiderivative size = 40

$$\int \frac{1}{(1-c^2x^2)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx=\text{Int}\left(\frac{1}{(1-c^2x^2)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)},x\right)$$

output
```
Unintegrable(1/(-c^2*x^2+1)/(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

3.35.2 Mathematica [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{1}{(1-c^2x^2)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx=\int \frac{1}{(1-c^2x^2)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$

input
```
Integrate[1/((1 - c^2*x^2)*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]
```

output
```
Integrate[1/((1 - c^2*x^2)*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])), x
]
```

3.35.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7234}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-c^2x^2)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}dx$$
$$\Big\downarrow \text{7234}$$
$$\int \frac{1}{(1-c^2x^2)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)}dx$$

input
```
Int[1/((1 - c^2*x^2)*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])),x]
```

output
```
$Aborted
```

3.35.3.1 Defintions of rubi rules used

rule 7234
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_)/((A_) + (C_.)*(x_)^2), x_Symbol] :> Unintegrable[(a + b*F[(c*
Sqrt[d + e*x])/Sqrt[f + g*x]])^n/(A + C*x^2), x] /; FreeQ[{a, b, c, d, e, f
, g, A, C, F, n}, x] && EqQ[C*d*f - A*e*g, 0] && EqQ[e*f + d*g, 0] &&  !IGt
Q[n, 0]
```

3.35.4 Maple [N/A] (verified)

Not integrable

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{1}{(-c^2x^2+1)\left(a+b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)}dx$$

input
```
int(1/(-c^2*x^2+1)/(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

output
```
int(1/(-c^2*x^2+1)/(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x)
```

3.35.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

$$\int \frac{1}{(1-c^2x^2)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx=\int -\frac{1}{(c^2x^2-1)\left(b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)/(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, alg
orithm="fricas")
```

output
```
integral(-1/(a*c^2*x^2 + (b*c^2*x^2 - b)*arctan(sqrt(-c*x + 1)/sqrt(c*x +
1)) - a), x)
```

3.35.6 Sympy [N/A]

Not integrable

Time = 2.76 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.52

$$\int \frac{1}{(1-c^2x^2)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$$
$$=-\int \frac{1}{ac^2x^2-a+bc^2x^2\,\mathrm{atan}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)-b\,\mathrm{atan}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}\,dx$$

input
```
integrate(1/(-c**2*x**2+1)/(a+b*atan((-c*x+1)**(1/2)/(c*x+1)**(1/2))),x)
```

output
```
-Integral(1/(a*c**2*x**2 - a + b*c**2*x**2*atan(sqrt(-c*x + 1)/sqrt(c*x +
1)) - b*atan(sqrt(-c*x + 1)/sqrt(c*x + 1))), x)
```

3.35.7 Maxima [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{1}{(1-c^2x^2)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = \int -\frac{1}{(c^2x^2-1)\left(b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)/(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, alg
orithm="maxima")
```

output
```
-integrate(1/((c^2*x^2 - 1)*(b*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)),
 x)
```

3.35.8 Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{1}{(1-c^2x^2)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = \int -\frac{1}{(c^2x^2-1)\left(b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)/(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2))),x, alg
orithm="giac")
```

output
```
integrate(-1/((c^2*x^2 - 1)*(b*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)),
 x)
```

3.35.9 Mupad [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{1}{(1-c^2x^2)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx = -\int \frac{1}{\left(a+b\,\mathrm{atan}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)\,(c^2\,x^2-1)}\,dx$$

input
```
int(-1/((a + b*atan((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))*(c^2*x^2 - 1)),x)
```

output
```
-int(1/((a + b*atan((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))*(c^2*x^2 - 1)), x)
```

3.35. $\int \frac{1}{(1-c^2x^2)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)}\,dx$

3.36 $\int \frac{1}{(1-c^2x^2)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$

3.36.1 Optimal result

Integrand size = 40, antiderivative size = 40

$$\int \frac{1}{(1-c^2x^2)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = \text{Int}\left(\frac{1}{(1-c^2x^2)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2},x\right)$$

output
```
Unintegrable(1/(-c^2*x^2+1)/(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x
)
```

3.36.2 Mathematica [N/A]

Not integrable

Time = 1.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{1}{(1-c^2x^2)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = \int \frac{1}{(1-c^2x^2)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx$$

input
```
Integrate[1/((1 - c^2*x^2)*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),
x]
```

output
```
Integrate[1/((1 - c^2*x^2)*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),
 x]
```

3.36.3 Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7234}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-c^2x^2)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}dx$$

$$\Big\downarrow \text{ 7234}$$

$$\int \frac{1}{(1-c^2x^2)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{cx+1}}\right)\right)^2}dx$$

input
```
Int[1/((1 - c^2*x^2)*(a + b*ArcTan[Sqrt[1 - c*x]/Sqrt[1 + c*x]])^2),x]
```

output
```
$Aborted
```

3.36.3.1 Defintions of rubi rules used

rule 7234
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_)/((A_) + (C_.)*(x_)^2), x_Symbol] :> Unintegrable[(a + b*F[(c*
Sqrt[d + e*x])/Sqrt[f + g*x]])^n/(A + C*x^2), x] /; FreeQ[{a, b, c, d, e, f
, g, A, C, F, n}, x] && EqQ[C*d*f - A*e*g, 0] && EqQ[e*f + d*g, 0] &&  !IGt
Q[n, 0]
```

3.36.4 Maple [N/A] (verified)

Not integrable

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.90

$$\int \frac{1}{(-c^2x^2+1)\left(a+b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)\right)^2}dx$$

input
```
int(1/(-c^2*x^2+1)/(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

output
```
int(1/(-c^2*x^2+1)/(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x)
```

3.36.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 91, normalized size of antiderivative = 2.28

$$\int \frac{1}{(1-c^2x^2)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = \int -\frac{1}{(c^2x^2-1)\left(b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)/(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, a
lgorithm="fricas")
```

output
```
integral(-1/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arctan(sqrt(-c*x + 1)/sqrt(
c*x + 1))^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arctan(sqrt(-c*x + 1)/sqrt(c*x +
 1))), x)
```

3.36.6 Sympy [N/A]

Not integrable

Time = 6.12 (sec) , antiderivative size = 126, normalized size of antiderivative = 3.15

$$\int \frac{1}{(1-c^2x^2)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = -\int \frac{1}{a^2c^2x^2-a^2+2abc^2x^2\operatorname{atan}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)-2ab\operatorname{atan}\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+b^2c^2x^2\operatorname{atan}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)-b^2\operatorname{atan}^2\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)}\,dx$$

input
```
integrate(1/(-c**2*x**2+1)/(a+b*atan((-c*x+1)**(1/2)/(c*x+1)**(1/2)))**2,x
)
```

output
```
-Integral(1/(a**2*c**2*x**2 - a**2 + 2*a*b*c**2*x**2*atan(sqrt(-c*x + 1)/s
qrt(c*x + 1)) - 2*a*b*atan(sqrt(-c*x + 1)/sqrt(c*x + 1)) + b**2*c**2*x**2*
atan(sqrt(-c*x + 1)/sqrt(c*x + 1))**2 - b**2*atan(sqrt(-c*x + 1)/sqrt(c*x
+ 1))**2), x)
```

3.36.7 Maxima [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 163, normalized size of antiderivative = 4.08

$$\int \frac{1}{(1-c^2x^2)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = \int -\frac{1}{(c^2x^2-1)\left(b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)/(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, a
lgorithm="maxima")
```

output
```
2*(2*(b^2*c^2*arctan2(sqrt(-c*x + 1), sqrt(c*x + 1)) + a*b*c^2)*sqrt(c*x +
 1)*sqrt(-c*x + 1)*integrate(1/2*x/((a*b*c^2*x^2 - a*b + (b^2*c^2*x^2 - b^
2)*arctan2(sqrt(-c*x + 1), sqrt(c*x + 1)))*sqrt(c*x + 1)*sqrt(-c*x + 1)),
x) + 1)/((b^2*c*arctan2(sqrt(-c*x + 1), sqrt(c*x + 1)) + a*b*c)*sqrt(c*x +
 1)*sqrt(-c*x + 1))
```

3.36.8 Giac [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int \frac{1}{(1-c^2x^2)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = \int -\frac{1}{(c^2x^2-1)\left(b\arctan\left(\frac{\sqrt{-cx+1}}{\sqrt{cx+1}}\right)+a\right)^2}\,dx$$

input
```
integrate(1/(-c^2*x^2+1)/(a+b*arctan((-c*x+1)^(1/2)/(c*x+1)^(1/2)))^2,x, a
lgorithm="giac")
```

output
```
integrate(-1/((c^2*x^2 - 1)*(b*arctan(sqrt(-c*x + 1)/sqrt(c*x + 1)) + a)^2
), x)
```

3.36.9 Mupad [N/A]

Not integrable

Time = 1.55 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{1}{(1-c^2x^2)\left(a+b\arctan\left(\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right)\right)^2}\,dx = -\int \frac{1}{\left(a+b\,\mathrm{atan}\left(\frac{\sqrt{1-c\,x}}{\sqrt{c\,x+1}}\right)\right)^2\,(c^2\,x^2-1)}\,dx$$

input
```
int(-1/((a + b*atan((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2*(c^2*x^2 - 1)),x)
```

output
```
-int(1/((a + b*atan((1 - c*x)^(1/2)/(c*x + 1)^(1/2)))^2*(c^2*x^2 - 1)), x)
```

3.37 $\int x^m \arctan(\tan(a+bx))\,dx$

3.37.1 Optimal result

Integrand size = 11, antiderivative size = 37

$$\int x^m \arctan(\tan(a+bx))\,dx = -\frac{bx^{2+m}}{2+3m+m^2} + \frac{x^{1+m}\arctan(\tan(a+bx))}{1+m}$$

output
```
-b*x^(2+m)/(m^2+3*m+2)+x^(1+m)*arctan(tan(b*x+a))/(1+m)
```

3.37.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

$$\int x^m \arctan(\tan(a+bx))\,dx = x^m\left(\frac{bx^2}{2+m} + \frac{x(-bx+\arctan(\tan(a+bx)))}{1+m}\right)$$

input
```
Integrate[x^m*ArcTan[Tan[a + b*x]],x]
```

output
```
x^m*((b*x^2)/(2 + m) + (x*(-(b*x) + ArcTan[Tan[a + b*x]]))/(1 + m))
```

3.37.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \arctan(\tan(a+bx))\,dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{x^{m+1}\arctan(\tan(a+bx))}{m+1}-\frac{b\int x^{m+1}dx}{m+1}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{x^{m+1}\arctan(\tan(a+bx))}{m+1}-\frac{bx^{m+2}}{(m+1)(m+2)}$$

input
```
Int[x^m*ArcTan[Tan[a + b*x]],x]
```

output
```
-((b*x^(2 + m))/((1 + m)*(2 + m))) + (x^(1 + m)*ArcTan[Tan[a + b*x]])/(1 +
 m)
```

3.37.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.37.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.11

method	result
default	$\frac{b\,x^{2} \mathrm{e}^{m \ln(x)}}{2+m}+\frac{(\arctan(\tan(bx+a))-bx)x\,\mathrm{e}^{m \ln(x)}}{1+m}$
parallelrisch	$-\frac{-2\arctan(\tan(bx+a))x^{m}x+b\,x^{m}x^{2}-x\,x^{m}\arctan(\tan(bx+a))m}{(1+m)(2+m)}$
risch	$-\frac{ix\,x^{m}\ln\left(\mathrm{e}^{i(bx+a)}\right)}{1+m}-\frac{x\left(2\pi\,\mathrm{csgn}\left(\frac{i\mathrm{e}^{2i(bx+a)}}{\mathrm{e}^{2i(bx+a)}+1}\right)^{3}+2\pi\,\mathrm{csgn}\left(i\mathrm{e}^{2i(bx+a)}\right)^{3}+4bx+\pi m\,\mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2i(bx+a)}+1}\right)\mathrm{csgn}\left(i\mathrm{e}^{2i(bx+a)}\right)}{}$

input
```
int(x^m*arctan(tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
b/(2+m)*x^2*exp(m*ln(x))+(arctan(tan(b*x+a))-b*x)/(1+m)*x*exp(m*ln(x))
```

3.37.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int x^m \arctan(\tan(a+bx))\,dx = \frac{((bm+b)x^2+(am+2\,a)x)x^m}{m^2+3\,m+2}$$

input
```
integrate(x^m*arctan(tan(b*x+a)),x, algorithm="fricas")
```

output
```
((b*m + b)*x^2 + (a*m + 2*a)*x)*x^m/(m^2 + 3*m + 2)
```

3.37.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(31) = 62$.

Time = 0.67 (sec) , antiderivative size = 158, normalized size of antiderivative = 4.27

$$\int x^m \arctan(\tan(a+bx))\,dx$$
$$= \begin{cases} b\log(x) - \frac{\operatorname{atan}(\tan(a+bx)) + \pi\left\lfloor\frac{a+bx-\frac{\pi}{2}}{\pi}\right\rfloor}{x} & \text{for}\: m = -2 \\ -bx\log(x) + bx + \left(\operatorname{atan}(\tan(a+bx)) + 2\pi\left\lfloor\frac{a+bx-\frac{\pi}{2}}{\pi}\right\rfloor\right)\log(x) & \text{for}\: m = -1 \\ -\frac{bx^2x^m}{m^2+3m+2} + \frac{mxx^m\left(\operatorname{atan}(\tan(a+bx))+\pi\left\lfloor\frac{a+bx-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{m^2+3m+2} + \frac{2xx^m\left(\operatorname{atan}(\tan(a+bx))+\pi\left\lfloor\frac{a+bx-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{m^2+3m+2} & \text{otherwise} \end{cases}$$

input
```
integrate(x**m*atan(tan(b*x+a)),x)
```

output
```
Piecewise((b*log(x) - (atan(tan(a + b*x)) + pi*floor((a + b*x - pi/2)/pi))
/x, Eq(m, -2)), (-b*x*log(x) + b*x + (atan(tan(a + b*x)) + 2*pi*floor((a +
 b*x - pi/2)/pi))*log(x), Eq(m, -1)), (-b*x**2*x**m/(m**2 + 3*m + 2) + m*x
*x**m*(atan(tan(a + b*x)) + pi*floor((a + b*x - pi/2)/pi))/(m**2 + 3*m + 2
) + 2*x*x**m*(atan(tan(a + b*x)) + pi*floor((a + b*x - pi/2)/pi))/(m**2 +
3*m + 2), True))
```

3.37.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

$$\int x^m \arctan(\tan(a+bx))\,dx = -\frac{bx^2x^m}{(m+2)(m+1)} + \frac{x^{m+1}\arctan\left(\tan\left(bx+a\right)\right)}{m+1}$$

input
```
integrate(x^m*arctan(tan(b*x+a)),x, algorithm="maxima")
```

output
```
-b*x^2*x^m/((m + 2)*(m + 1)) + x^(m + 1)*arctan(tan(b*x + a))/(m + 1)
```

3.37.8 Giac [F]

$$\int x^m \arctan(\tan(a+bx))\,dx = \int x^m \arctan\left(\tan\left(bx+a\right)\right)\,dx$$

input
```
integrate(x^m*arctan(tan(b*x+a)),x, algorithm="giac")
```

output
```
sage0*x
```

3.37.9 Mupad [F(-1)]

Timed out.

$$\int x^m \arctan(\tan(a+bx))\,dx = \int x^m \operatorname{atan}(\tan(a+b\,x))\,dx$$

input
```
int(x^m*atan(tan(a + b*x)),x)
```

output
```
int(x^m*atan(tan(a + b*x)), x)
```

3.38 $\int x^2 \arctan(\tan(a+bx))\,dx$

3.38.1 Optimal result

Integrand size = 11, antiderivative size = 23

$$\int x^2 \arctan(\tan(a+bx))\,dx = -\frac{bx^4}{12} + \frac{1}{3}x^3 \arctan(\tan(a+bx))$$

output
```
-1/12*b*x^4+1/3*x^3*arctan(tan(b*x+a))
```

3.38.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int x^2 \arctan(\tan(a+bx))\,dx = -\frac{1}{12}x^3(bx - 4\arctan(\tan(a+bx)))$$

input
```
Integrate[x^2*ArcTan[Tan[a + b*x]],x]
```

output
```
-1/12*(x^3*(b*x - 4*ArcTan[Tan[a + b*x]]))
```

3.38.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(\tan(a+bx))\,dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{1}{3}x^3\arctan(\tan(a+bx)) - \frac{b\int x^3dx}{3}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{1}{3}x^3\arctan(\tan(a+bx)) - \frac{bx^4}{12}$$

input
```
Int[x^2*ArcTan[Tan[a + b*x]],x]
```

output
```
-1/12*(b*x^4) + (x^3*ArcTan[Tan[a + b*x]])/3
```

3.38.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.38.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result
default	$-\frac{b\,x^{4}}{12}+\frac{x^{3}\arctan\left(\tan\left(bx+a\right)\right)}{3}$
parallelrisch	$-\frac{b\,x^{4}}{12}+\frac{x^{3}\arctan\left(\tan\left(bx+a\right)\right)}{3}$
parts	$-\frac{b\,x^{4}}{12}+\frac{x^{3}\arctan\left(\tan\left(bx+a\right)\right)}{3}$
risch	$-\frac{i x^{3}\ln\left(\mathrm{e}^{i\left(bx+a\right)}\right)}{3}-\frac{\pi\,x^{3}\operatorname{csgn}\left(\frac{i}{\mathrm{e}^{2i\left(bx+a\right)}+1}\right)\operatorname{csgn}\left(i\mathrm{e}^{2i\left(bx+a\right)}\right)\operatorname{csgn}\left(\frac{i\mathrm{e}^{2i\left(bx+a\right)}}{\mathrm{e}^{2i\left(bx+a\right)}+1}\right)}{12}+\frac{\pi\,x^{3}\operatorname{csgn}\left(\frac{i}{\mathrm{e}^{2i\left(bx+a\right)}+1}\right)\operatorname{csgn}\left(\frac{i\mathrm{e}}{\mathrm{e}^{2i}}\right.}{12}$

input
```
int(x^2*arctan(tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/12*b*x^4+1/3*x^3*arctan(tan(b*x+a))
```

3.38.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.57

$$\int x^2\arctan(\tan(a+bx))\,dx=\frac{1}{4}\,bx^4+\frac{1}{3}\,ax^3$$

input
```
integrate(x^2*arctan(tan(b*x+a)),x, algorithm="fricas")
```

output
```
1/4*b*x^4 + 1/3*a*x^3
```

3.38.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.39

$$\int x^2\arctan(\tan(a+bx))\,dx=-\frac{bx^4}{12}+\frac{x^3\left(\operatorname{atan}\left(\tan\left(a+bx\right)\right)+\pi\left\lfloor\frac{a+bx-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{3}$$

input
```
integrate(x**2*atan(tan(b*x+a)),x)
```

output
```
-b*x**4/12 + x**3*(atan(tan(a + b*x)) + pi*floor((a + b*x - pi/2)/pi))/3
```

3.38.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(19) = 38.

Time = 0.19 (sec) , antiderivative size = 81, normalized size of antiderivative = 3.52

$$\int x^2 \arctan(\tan(a+bx))\,dx = \frac{\frac{4\left((bx+a)^3-3\,(bx+a)^2a+3\,(bx+a)a^2\right)\arctan(\tan(bx+a))}{b^2}-\frac{(bx+a)^4-4\,(bx+a)^3a+6\,(bx+a)^2a^2}{b^2}}{12\,b}$$

input
```
integrate(x^2*arctan(tan(b*x+a)),x, algorithm="maxima")
```

output
```
1/12*(4*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arctan(tan(b*x +
 a))/b^2 - ((b*x + a)^4 - 4*(b*x + a)^3*a + 6*(b*x + a)^2*a^2)/b^2)/b
```

3.38.8 Giac [F]

$$\int x^2 \arctan(\tan(a+bx))\,dx = \int x^2 \arctan\left(\tan\left(bx+a\right)\right)\,dx$$

input
```
integrate(x^2*arctan(tan(b*x+a)),x, algorithm="giac")
```

output
```
sage0*x
```

3.38.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int x^2 \arctan(\tan(a+bx))\,dx = \frac{x^3\,\mathrm{atan}(\tan(a+b\,x))}{3}-\frac{b\,x^4}{12}$$

input
```
int(x^2*atan(tan(a + b*x)),x)
```

output
```
(x^3*atan(tan(a + b*x)))/3 - (b*x^4)/12
```

3.39 $\int x \arctan(\tan(a+bx))\,dx$

3.39.1 Optimal result

Integrand size = 9, antiderivative size = 23

$$\int x \arctan(\tan(a+bx))\,dx = -\frac{bx^3}{6}+\frac{1}{2}x^2\arctan(\tan(a+bx))$$

output
```
-1/6*b*x^3+1/2*x^2*arctan(tan(b*x+a))
```

3.39.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int x \arctan(\tan(a+bx))\,dx = -\frac{1}{6}x^2(bx-3\arctan(\tan(a+bx)))$$

input
```
Integrate[x*ArcTan[Tan[a + b*x]],x]
```

output
```
-1/6*(x^2*(b*x - 3*ArcTan[Tan[a + b*x]]))
```

3.39.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5694, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan(\tan(a+bx))\,dx$$

$$\Big\downarrow \text{5694}$$

$$\frac{1}{2}x^2\arctan(\tan(a+bx)) - \frac{1}{2}ib\int -ix^2dx$$

$$\Big\downarrow \text{15}$$

$$\frac{1}{2}x^2\arctan(\tan(a+bx)) - \frac{bx^3}{6}$$

input
```
Int[x*ArcTan[Tan[a + b*x]],x]
```

output
```
-1/6*(b*x^3) + (x^2*ArcTan[Tan[a + b*x]])/2
```

3.39.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 5694
```
Int[ArcTan[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Tan[a + b*x]]/(f*(m +
1))), x] - Simp[I*(b/(f*(m + 1)))   Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(2
*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && E
qQ[(c + I*d)^2, -1]
```

3.39.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result
default	$-\frac{b\,x^{3}}{6}+\frac{x^{2} \arctan \left(\tan \left(b x +a \right)\right)}{2}$
parallelrisch	$-\frac{b\,x^{3}}{6}+\frac{x^{2} \arctan \left(\tan \left(b x +a \right)\right)}{2}$
parts	$-\frac{b\,x^{3}}{6}+\frac{x^{2} \arctan \left(\tan \left(b x +a \right)\right)}{2}$
risch	$-\frac{i x^{2} \ln \left({\mathrm e}^{i \left(b x +a \right)}\right)}{2}-\frac{\pi \,x^{2} \operatorname{csgn}\left(\frac{i}{{\mathrm e}^{2 i \left(b x +a \right)}+1}\right) \operatorname{csgn}\left(i {\mathrm e}^{2 i \left(b x +a \right)}\right) \operatorname{csgn}\left(\frac{i {\mathrm e}^{2 i \left(b x +a \right)}}{{\mathrm e}^{2 i \left(b x +a \right)}+1}\right)}{8}+\frac{\pi \,x^{2} \operatorname{csgn}\left(\frac{i}{{\mathrm e}^{2 i \left(b x +a \right)}+1}\right) \operatorname{csgn}\left(\frac{i e}{e^{2i}}\right.}{8}$

input
```
int(x*arctan(tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/6*b*x^3+1/2*x^2*arctan(tan(b*x+a))
```

3.39.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.57

$$\int x \arctan(\tan(a+bx))\,dx = \frac{1}{3}\,bx^{3} + \frac{1}{2}\,ax^{2}$$

input
```
integrate(x*arctan(tan(b*x+a)),x, algorithm="fricas")
```

output
```
1/3*b*x^3 + 1/2*a*x^2
```

3.39.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. 2(19) = 38.

Time = 0.17 (sec) , antiderivative size = 76, normalized size of antiderivative = 3.30

$$\int x \arctan(\tan(a+bx))\,dx$$
$$= \begin{cases} \frac{x\left(\operatorname{atan}\left(\tan\left(a+bx\right)\right)+\pi\left\lfloor\frac{a+bx-\frac{\pi}{2}}{\pi}\right\rfloor\right)^{2}}{2b} - \frac{\left(\operatorname{atan}\left(\tan\left(a+bx\right)\right)+\pi\left\lfloor\frac{a+bx-\frac{\pi}{2}}{\pi}\right\rfloor\right)^{3}}{6b^{2}} & \text{for}\ b \neq 0 \\ \frac{x^{2}\left(\operatorname{atan}\left(\tan\left(a\right)\right)+\pi\left\lfloor\frac{a-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x*atan(tan(b*x+a)),x)
```

output
```
Piecewise((x*(atan(tan(a + b*x)) + pi*floor((a + b*x - pi/2)/pi))**2/(2*b)
 - (atan(tan(a + b*x)) + pi*floor((a + b*x - pi/2)/pi))**3/(6*b**2), Ne(b,
 0)), (x**2*(atan(tan(a)) + pi*floor((a - pi/2)/pi))/2, True))
```

3.39.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(19) = 38$.

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.48

$$\int x \arctan(\tan(a+bx))\,dx = \frac{\frac{3\left(\left(bx+a\right)^2-2\,\left(bx+a\right)a\right)\arctan\left(\tan\left(bx+a\right)\right)}{b}-\frac{\left(bx+a\right)^3-3\,\left(bx+a\right)^2a}{b}}{6\,b}$$

input
```
integrate(x*arctan(tan(b*x+a)),x, algorithm="maxima")
```

output
```
1/6*(3*((b*x + a)^2 - 2*(b*x + a)*a)*arctan(tan(b*x + a))/b - ((b*x + a)^3
 - 3*(b*x + a)^2*a)/b)/b
```

3.39.8 Giac [F]

$$\int x \arctan(\tan(a+bx))\,dx = \int x \arctan\left(\tan\left(bx+a\right)\right)\,dx$$

input
```
integrate(x*arctan(tan(b*x+a)),x, algorithm="giac")
```

output
```
sage0*x
```

3.39.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int x \arctan(\tan(a+bx))\,dx = \frac{x^2\,\mathrm{atan}(\tan(a+b\,x))}{2} - \frac{b\,x^3}{6}$$

input
```
int(x*atan(tan(a + b*x)),x)
```

output
```
(x^2*atan(tan(a + b*x)))/2 - (b*x^3)/6
```

3.40 $\int \arctan(\tan(a+bx))\,dx$

3.40.1 Optimal result

Integrand size = 7, antiderivative size = 16

$$\int \arctan(\tan(a+bx))\,dx = \frac{\arctan(\tan(a+bx))^2}{2b}$$

output
```
1/2*arctan(tan(b*x+a))^2/b
```

3.40.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \arctan(\tan(a+bx))\,dx = -\frac{bx^2}{2} + x\arctan(\tan(a+bx))$$

input
```
Integrate[ArcTan[Tan[a + b*x]],x]
```

output
```
-1/2*(b*x^2) + x*ArcTan[Tan[a + b*x]]
```

3.40.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(\tan(a+bx))\,dx$$

$$\Big\downarrow \text{2588}$$

$$\frac{\int \arctan(\tan(a+bx))d\arctan(\tan(a+bx))}{b}$$

$$\Big\downarrow \text{15}$$

$$\frac{\arctan(\tan(a+bx))^2}{2b}$$

input
```
Int[ArcTan[Tan[a + b*x]],x]
```

output
```
ArcTan[Tan[a + b*x]]^2/(2*b)
```

3.40.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

3.40.4 Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{\arctan(\tan(bx+a))^2}{2b}$
default	$\frac{\arctan(\tan(bx+a))^2}{2b}$
parallelrisch	$-\frac{x^2 b}{2}+x\arctan\left(\tan\left(bx+a\right)\right)$
parts	$-\frac{x^2 b}{2}+x\arctan\left(\tan\left(bx+a\right)\right)$
risch	$-ix\ln\left(\mathrm{e}^{i(bx+a)}\right)-\frac{\pi x\ \mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2i(bx+a)}+1}\right)\mathrm{csgn}\left(i\mathrm{e}^{2i(bx+a)}\right)\mathrm{csgn}\left(\frac{i\mathrm{e}^{2i(bx+a)}}{\mathrm{e}^{2i(bx+a)}+1}\right)}{4}+\frac{\pi x\ \mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2i(bx+a)}+1}\right)\mathrm{cs}}{4}$

input
```
int(arctan(tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctan(tan(b*x+a))^2/b
```

3.40.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \arctan(\tan(a+bx))\,dx=\frac{1}{2}\,bx^2+ax$$

input
```
integrate(arctan(tan(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*b*x^2 + a*x
```

3.40.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(12) = 24.

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.62

$$\int \arctan(\tan(a+bx))\,dx=\begin{cases}\frac{\left(\operatorname{atan}\left(\tan\left(a+bx\right)\right)+\pi\left\lfloor\frac{a+bx-\frac{\pi}{2}}{\pi}\right\rfloor\right)^2}{2b} & \text{for } b\neq 0\\ x\left(\operatorname{atan}\left(\tan\left(a\right)\right)+\pi\left\lfloor\frac{a-\frac{\pi}{2}}{\pi}\right\rfloor\right) & \text{otherwise}\end{cases}$$

input
```
integrate(atan(tan(b*x+a)),x)
```

output
```
Piecewise(((atan(tan(a + b*x)) + pi*floor((a + b*x - pi/2)/pi))**2/(2*b),
Ne(b, 0)), (x*(atan(tan(a)) + pi*floor((a - pi/2)/pi)), True))
```

3.40.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \arctan(\tan(a+bx))\,dx = \frac{(bx+a)^2}{2\,b}$$

input
```
integrate(arctan(tan(b*x+a)),x, algorithm="maxima")
```

output
```
1/2*(b*x + a)^2/b
```

3.40.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int \arctan(\tan(a+bx))\,dx = \frac{1}{2}\,bx^2 - \pi x\left\lfloor\frac{bx+a}{\pi}+\frac{1}{2}\right\rfloor + ax$$

input
```
integrate(arctan(tan(b*x+a)),x, algorithm="giac")
```

output
```
1/2*b*x^2 - pi*x*floor((b*x + a)/pi + 1/2) + a*x
```

3.40.9 Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \arctan(\tan(a+bx))\,dx = x\,\mathrm{atan}(\tan(a+b\,x)) - \frac{b\,x^2}{2}$$

input
```
int(atan(tan(a + b*x)),x)
```

output
```
x*atan(tan(a + b*x)) - (b*x^2)/2
```

3.41 $\int \frac{\arctan(\tan(a+bx))}{x} \, dx$

3.41.1 Optimal result

Integrand size = 11, antiderivative size = 21

$$\int \frac{\arctan(\tan(a+bx))}{x} \, dx = bx - (bx - \arctan(\tan(a+bx)))\log(x)$$

output
```
b*x-(b*x-arctan(tan(b*x+a)))*ln(x)
```

3.41.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{\arctan(\tan(a+bx))}{x} \, dx = bx + (-bx + \arctan(\tan(a+bx)))\log(x)$$

input
```
Integrate[ArcTan[Tan[a + b*x]]/x,x]
```

output
```
b*x + (-(b*x) + ArcTan[Tan[a + b*x]])*Log[x]
```

3.41.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2589, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(\tan(a+bx))}{x} \, dx$$

$$\Big\downarrow \; 2589$$

$$bx-(bx-\arctan(\tan(a+bx)))\int \frac{1}{x}dx$$

$$\Big\downarrow \; 14$$

$$bx-\log(x)(bx-\arctan(\tan(a+bx)))$$

input
```
Int[ArcTan[Tan[a + b*x]]/x,x]
```

output
```
b*x - (b*x - ArcTan[Tan[a + b*x]])*Log[x]
```

3.41.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2589
```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Simp[(b*u - a*v)/a   Int[1/u, x], x] /; NeQ[b*u -
a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

3.41.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

method	result
default	$\ln(x)\arctan(\tan(bx+a)) - b(\ln(x)x - x)$
parts	$\ln(x)\arctan(\tan(bx+a)) - b(\ln(x)x - x)$
risch	$-i\ln(x)\ln\left(e^{i(bx+a)}\right) - \ln(x)bx + bx - \frac{\pi\left(\operatorname{csgn}\left(\frac{i}{e^{2i(bx+a)}+1}\right)\operatorname{csgn}\left(ie^{2i(bx+a)}\right)\operatorname{csgn}\left(\frac{ie^{2i(bx+a)}}{e^{2i(bx+a)}+1}\right) - \operatorname{csgn}\left(\frac{i}{e^{2i(bx+}\right.\right.}{}$

input
```
int(arctan(tan(b*x+a))/x,x,method=_RETURNVERBOSE)
```

output
```
ln(x)*arctan(tan(b*x+a))-b*(ln(x)*x-x)
```

3.41.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.38

$$\int \frac{\arctan(\tan(a+bx))}{x}\,dx = bx + a\log(x)$$

input
```
integrate(arctan(tan(b*x+a))/x,x, algorithm="fricas")
```

output
```
b*x + a*log(x)
```

3.41.6 Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.62

$$\int \frac{\arctan(\tan(a+bx))}{x}\,dx = -bx\log(x) + bx + \left(\operatorname{atan}(\tan(a+bx)) + \pi\left\lfloor\frac{a+bx-\frac{\pi}{2}}{\pi}\right\rfloor\right)\log(x)$$

input
```
integrate(atan(tan(b*x+a))/x,x)
```

output
```
-b*x*log(x) + b*x + (atan(tan(a + b*x)) + pi*floor((a + b*x - pi/2)/pi))*l
og(x)
```

3.41.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.00

$$\int \frac{\arctan(\tan(a+bx))}{x}\,dx = \frac{b\arctan\left(\tan\left(bx+a\right)\right)\log\left(bx\right) + \left(bx - \left(bx+a\right)\log\left(bx\right) + a\log\left(bx\right) + a\right)b}{b}$$

input
```
integrate(arctan(tan(b*x+a))/x,x, algorithm="maxima")
```

output
```
(b*arctan(tan(b*x + a))*log(b*x) + (b*x - (b*x + a)*log(b*x) + a*log(b*x)
+ a)*b)/b
```

3.41.8 Giac [F]

$$\int \frac{\arctan(\tan(a+bx))}{x}\,dx = \int \frac{\arctan\left(\tan\left(bx+a\right)\right)}{x}\,dx$$

input
```
integrate(arctan(tan(b*x+a))/x,x, algorithm="giac")
```

output
```
sage0*x
```

3.41.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(\tan(a+bx))}{x}\,dx = \int \frac{\mathrm{atan}(\tan(a+b\,x))}{x}\,dx$$

input
```
int(atan(tan(a + b*x))/x,x)
```

output
```
int(atan(tan(a + b*x))/x, x)
```

3.42 $\int x^m \arctan(\cot(a+bx))\,dx$

3.42.1 Optimal result

Integrand size = 11, antiderivative size = 36

$$\int x^m \arctan(\cot(a+bx))\,dx = \frac{bx^{2+m}}{2+3m+m^2} + \frac{x^{1+m}\arctan(\cot(a+bx))}{1+m}$$

output
```
b*x^(2+m)/(m^2+3*m+2)+x^(1+m)*(1/2*Pi-arccot(cot(b*x+a)))/(1+m)
```

3.42.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int x^m \arctan(\cot(a+bx))\,dx = \frac{x^{1+m}(bx+(2+m)\arctan(\cot(a+bx)))}{(1+m)(2+m)}$$

input
```
Integrate[x^m*ArcTan[Cot[a + b*x]],x]
```

output
```
(x^(1 + m)*(b*x + (2 + m)*ArcTan[Cot[a + b*x]]))/((1 + m)*(2 + m))
```

3.42.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \arctan(\cot(a+bx))\,dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{b\int x^{m+1}dx}{m+1}+\frac{x^{m+1}\arctan(\cot(a+bx))}{m+1}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{x^{m+1}\arctan(\cot(a+bx))}{m+1}+\frac{bx^{m+2}}{(m+1)(m+2)}$$

input
```
Int[x^m*ArcTan[Cot[a + b*x]],x]
```

output
```
(b*x^(2 + m))/((1 + m)*(2 + m)) + (x^(1 + m)*ArcTan[Cot[a + b*x]])/(1 + m)
```

3.42.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.42.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.56

method	result
default	$\frac{\pi x^{1+m}}{2+2m} - \frac{b x^2 e^{m\ln(x)}}{2+m} - \frac{(\operatorname{arccot}(\cot(bx+a))-bx)x\,e^{m\ln(x)}}{1+m}$
parts	$\frac{\pi x^{1+m}}{2+2m} - \frac{b x^2 e^{m\ln(x)}}{2+m} - \frac{(\operatorname{arccot}(\cot(bx+a))-bx)x\,e^{m\ln(x)}}{1+m}$
parallelrisch	$\frac{2\pi x\,x^m - 4\operatorname{arccot}(\cot(bx+a))x^m x + 2b\,x^m x^2 + \pi x\,x^m m - 2\operatorname{arccot}(\cot(bx+a))x^m x m}{2(1+m)(2+m)}$
risch	$\frac{i x\,x^m \ln(e^{i(bx+a)})}{1+m} + \frac{x\left(4\pi+2\pi m\,\operatorname{csgn}\left(\frac{i}{e^{2i(bx+a)}-1}\right)^3+\pi m\,\operatorname{csgn}\left(\frac{ie^{2i(bx+a)}}{e^{2i(bx+a)}-1}\right)^3+2m\pi-2\pi\operatorname{csgn}\left(ie^{2i(bx+a)}\right)\operatorname{csgn}\left(\frac{ie^{2i(}}{e^{2i(bx}}\right.\right.}{}$

input
```
int(x^m*(1/2*Pi-arccot(cot(b*x+a))),x,method=_RETURNVERBOSE)
```

output
```
1/2*Pi*x^(1+m)/(1+m)-b/(2+m)*x^2*exp(m*ln(x))-(arccot(cot(b*x+a))-b*x)/(1+
m)*x*exp(m*ln(x))
```

3.42.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.17

$$\int x^m \arctan(\cot(a+bx))\,dx = -\frac{(2\,(bm+b)x^2 - (\pi(m+2) - 2\,am - 4\,a)x)x^m}{2\,(m^2+3\,m+2)}$$

input
```
integrate(x^m*(1/2*pi-arccot(cot(b*x+a))),x, algorithm="fricas")
```

output
```
-1/2*(2*(b*m + b)*x^2 - (pi*(m + 2) - 2*a*m - 4*a)*x)*x^m/(m^2 + 3*m + 2)
```

3.42.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(34) = 68.

Time = 1.86 (sec) , antiderivative size = 160, normalized size of antiderivative = 4.44

$$\int x^m \arctan(\cot(a+bx))\,dx$$
$$= \begin{cases} -b\log(x) + \frac{\operatorname{acot}(\cot(a+bx))}{x} - \frac{\pi}{2x} & \text{for } m=-2 \\ bx\log(x) - bx - \log(x)\operatorname{acot}(\cot(a+bx)) + \frac{\pi\log(x)}{2} & \text{for } m=-1 \\ \frac{2bx^2x^m}{2m^2+6m+4} - \frac{2mxx^m\operatorname{acot}(\cot(a+bx))}{2m^2+6m+4} + \frac{\pi mxx^m}{2m^2+6m+4} - \frac{4xx^m\operatorname{acot}(\cot(a+bx))}{2m^2+6m+4} + \frac{2\pi xx^m}{2m^2+6m+4} & \text{otherwise} \end{cases}$$

input
```
integrate(x**m*(1/2*pi-acot(cot(b*x+a))),x)
```

output
```
Piecewise((-b*log(x) + acot(cot(a + b*x))/x - pi/(2*x), Eq(m, -2)), (b*x*l
og(x) - b*x - log(x)*acot(cot(a + b*x)) + pi*log(x)/2, Eq(m, -1)), (2*b*x*
*2*x**m/(2*m**2 + 6*m + 4) - 2*m*x*x**m*acot(cot(a + b*x))/(2*m**2 + 6*m +
 4) + pi*m*x*x**m/(2*m**2 + 6*m + 4) - 4*x*x**m*acot(cot(a + b*x))/(2*m**2
 + 6*m + 4) + 2*pi*x*x**m/(2*m**2 + 6*m + 4), True))
```

3.42.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11

$$\int x^m \arctan(\cot(a+bx))\,dx = -\frac{bx^{m+2}}{m+2} + \frac{\pi x^{m+1}}{2\,(m+1)} - \frac{ax^{m+1}}{m+1}$$

input
```
integrate(x^m*(1/2*pi-arccot(cot(b*x+a))),x, algorithm="maxima")
```

output
```
-b*x^(m + 2)/(m + 2) + 1/2*pi*x^(m + 1)/(m + 1) - a*x^(m + 1)/(m + 1)
```

3.42.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.72

$$\int x^m \arctan(\cot(a+bx))\,dx$$
$$= -\frac{2\,bmx^2x^m - \pi mxx^m + 2\,amxx^m + 2\,bx^2x^m - 2\,\pi xx^m + 4\,axx^m}{2\,(m^2+3\,m+2)}$$

input
```
integrate(x^m*(1/2*pi-arccot(cot(b*x+a))),x, algorithm="giac")
```

output
```
-1/2*(2*b*m*x^2*x^m - pi*m*x*x^m + 2*a*m*x*x^m + 2*b*x^2*x^m - 2*pi*x*x^m
+ 4*a*x*x^m)/(m^2 + 3*m + 2)
```

3.42.9 Mupad [F(-1)]

Timed out.

$$\int x^m \arctan(\cot(a+bx))\,dx = \int x^m \left(\frac{\Pi}{2} - \operatorname{acot}(\cot(a+b\,x))\right)\,dx$$

input
```
int(x^m*(Pi/2 - acot(cot(a + b*x))),x)
```

output
```
int(x^m*(Pi/2 - acot(cot(a + b*x))), x)
```

3.43 $\int x^2 \arctan(\cot(a+bx))\,dx$

3.43.1 Optimal result

Integrand size = 11, antiderivative size = 23

$$\int x^2 \arctan(\cot(a+bx))\,dx = \frac{bx^4}{12} + \frac{1}{3}x^3 \arctan(\cot(a+bx))$$

output
```
1/12*b*x^4+1/3*x^3*(1/2*Pi-arccot(cot(b*x+a)))
```

3.43.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int x^2 \arctan(\cot(a+bx))\,dx = \frac{1}{12}x^3(bx + 4\arctan(\cot(a+bx)))$$

input
```
Integrate[x^2*ArcTan[Cot[a + b*x]],x]
```

output
```
(x^3*(b*x + 4*ArcTan[Cot[a + b*x]]))/12
```

3.43.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2599, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(\cot(a+bx))\,dx$$

$$\Big\downarrow \text{ 2599}$$

$$\frac{b\int x^3dx}{3}+\frac{1}{3}x^3\arctan(\cot(a+bx))$$

$$\Big\downarrow \text{ 15}$$

$$\frac{1}{3}x^3\arctan(\cot(a+bx))+\frac{bx^4}{12}$$

input
```
Int[x^2*ArcTan[Cot[a + b*x]],x]
```

output
```
(b*x^4)/12 + (x^3*ArcTan[Cot[a + b*x]])/3
```

3.43.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2599
```
Int[(u_)^(m_)*(v_)^(n_.), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Sim
plify[D[v, x]]}, Simp[u^(m + 1)*(v^n/(a*(m + 1))), x] - Simp[b*(n/(a*(m + 1
)))   Int[u^(m + 1)*v^(n - 1), x], x] /; NeQ[b*u - a*v, 0]] /; FreeQ[{m, n}
, x] && PiecewiseLinearQ[u, v, x] && NeQ[m, -1] && ((LtQ[m, -1] && GtQ[n, 0
] &&  !(ILtQ[m + n, -2] && (FractionQ[m] || GeQ[2*n + m + 1, 0]))) || (IGtQ
[n, 0] && IGtQ[m, 0] && LeQ[n, m]) || (IGtQ[n, 0] &&  !IntegerQ[m]) || (ILt
Q[m, 0] &&  !IntegerQ[n]))
```

3.43.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

method	result
parallelrisch	$-\frac{x^3 \operatorname{arccot}(\cot(bx+a))}{3}+\frac{\pi x^3}{6}+\frac{b x^4}{12}$
default	$\frac{\pi x^3}{6}-\frac{x^3 \operatorname{arccot}(\cot(bx+a))}{3}-\frac{-\frac{(bx+a)^4}{4}+a(bx+a)^3-\frac{3a^2(bx+a)^2}{2}+a^3(bx+a)}{3b^3}$
parts	$\frac{\pi x^3}{6}-\frac{x^3 \operatorname{arccot}(\cot(bx+a))}{3}-\frac{-\frac{(bx+a)^4}{4}+a(bx+a)^3-\frac{3a^2(bx+a)^2}{2}+a^3(bx+a)}{3b^3}$
risch	$\frac{i x^3 \ln(e^{i(bx+a)})}{3}+\frac{\pi x^3 \operatorname{csgn}(i e^{i(bx+a)})^2 \operatorname{csgn}(i e^{2i(bx+a)})}{12}-\frac{\pi x^3 \operatorname{csgn}(i e^{i(bx+a)}) \operatorname{csgn}(i e^{2i(bx+a)})^2}{6}+\frac{\pi x^3 \operatorname{csgn}(i e^{2i(bx+}}{12}$

input
```
int(x^2*(1/2*Pi-arccot(cot(b*x+a))),x,method=_RETURNVERBOSE)
```

output
```
-1/3*x^3*arccot(cot(b*x+a))+1/6*Pi*x^3+1/12*b*x^4
```

3.43.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int x^2 \arctan(\cot(a+bx))\,dx = -\frac{1}{4}\,bx^4+\frac{1}{6}\,(\pi-2\,a)x^3$$

input
```
integrate(x^2*(1/2*pi-arccot(cot(b*x+a))),x, algorithm="fricas")
```

output
```
-1/4*b*x^4 + 1/6*(pi - 2*a)*x^3
```

3.43.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

$$\int x^2 \arctan(\cot(a+bx))\,dx = \frac{bx^4}{12}-\frac{x^3 \operatorname{acot}(\cot(a+bx))}{3}+\frac{\pi x^3}{6}$$

input
```
integrate(x**2*(1/2*pi-acot(cot(b*x+a))),x)
```

output
```
b*x**4/12 - x**3*acot(cot(a + b*x))/3 + pi*x**3/6
```

3.43.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int x^2 \arctan(\cot(a+bx))\,dx = -\frac{1}{4}\,bx^4 + \frac{1}{6}\,(\pi - 2\,a)x^3$$

input
```
integrate(x^2*(1/2*pi-arccot(cot(b*x+a))),x, algorithm="maxima")
```

output
```
-1/4*b*x^4 + 1/6*(pi - 2*a)*x^3
```

3.43.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int x^2 \arctan(\cot(a+bx))\,dx = -\frac{1}{4}\,bx^4 + \frac{1}{6}\,\pi x^3 - \frac{1}{3}\,ax^3$$

input
```
integrate(x^2*(1/2*pi-arccot(cot(b*x+a))),x, algorithm="giac")
```

output
```
-1/4*b*x^4 + 1/6*pi*x^3 - 1/3*a*x^3
```

3.43.9 Mupad [B] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int x^2 \arctan(\cot(a+bx))\,dx = \frac{\Pi\,x^3}{6} + \frac{b\,x^4}{12} - \frac{x^3\,\mathrm{acot}(\cot(a+b\,x))}{3}$$

input
```
int(x^2*(Pi/2 - acot(cot(a + b*x))),x)
```

output
```
(Pi*x^3)/6 + (b*x^4)/12 - (x^3*acot(cot(a + b*x)))/3
```

3.44 $\int x \arctan(\cot(a+bx))\,dx$

3.44.1 Optimal result

Integrand size = 9, antiderivative size = 23

$$\int x \arctan(\cot(a+bx))\,dx = \frac{bx^3}{6} + \frac{1}{2}x^2 \arctan(\cot(a+bx))$$

output
```
1/6*b*x^3+1/2*x^2*(1/2*Pi-arccot(cot(b*x+a)))
```

3.44.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int x \arctan(\cot(a+bx))\,dx = \frac{1}{6}x^2(bx + 3\arctan(\cot(a+bx)))$$

input
```
Integrate[x*ArcTan[Cot[a + b*x]],x]
```

output
```
(x^2*(b*x + 3*ArcTan[Cot[a + b*x]]))/6
```

3.44.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5696, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan(\cot(a+bx))\,dx$$

$$\Big\downarrow \; 5696$$

$$\frac{1}{2}x^2\arctan(\cot(a+bx))-\frac{1}{2}ib\int ix^2dx$$

$$\Big\downarrow \; 15$$

$$\frac{1}{2}x^2\arctan(\cot(a+bx))+\frac{bx^3}{6}$$

input
```
Int[x*ArcTan[Cot[a + b*x]],x]
```

output
```
(b*x^3)/6 + (x^2*ArcTan[Cot[a + b*x]])/2
```

3.44.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 5696
```
Int[ArcTan[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Cot[a + b*x]]/(f*(m +
1))), x] - Simp[I*(b/(f*(m + 1)))   Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(2
*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && E
qQ[(c - I*d)^2, -1]
```

3.44.4 Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.13

method	result
parallelrisch	$\frac{b\,x^3}{6}-\frac{x^2\,\mathrm{arccot}(\cot(bx+a))}{2}+\frac{\pi\,x^2}{4}$
default	$\frac{\pi\,x^2}{4}-\frac{x^2\,\mathrm{arccot}(\cot(bx+a))}{2}-\frac{-\frac{(bx+a)^3}{3}+(bx+a)^2 a-a^2(bx+a)}{2b^2}$
parts	$\frac{\pi\,x^2}{4}-\frac{x^2\,\mathrm{arccot}(\cot(bx+a))}{2}-\frac{-\frac{(bx+a)^3}{3}+(bx+a)^2 a-a^2(bx+a)}{2b^2}$
risch	$\frac{ix^2\ln\left(\mathrm{e}^{i(bx+a)}\right)}{2}+\frac{\pi\,x^2\,\mathrm{csgn}\left(i\mathrm{e}^{i(bx+a)}\right)^2\mathrm{csgn}\left(i\mathrm{e}^{2i(bx+a)}\right)}{8}-\frac{\pi\,x^2\,\mathrm{csgn}\left(i\mathrm{e}^{i(bx+a)}\right)\mathrm{csgn}\left(i\mathrm{e}^{2i(bx+a)}\right)^2}{4}+\frac{\pi\,x^2\,\mathrm{csgn}\left(i\mathrm{e}^{2i(bx+}\right.}{8}$

input
```
int(x*(1/2*Pi-arccot(cot(b*x+a))),x,method=_RETURNVERBOSE)
```

output
```
1/6*b*x^3-1/2*x^2*arccot(cot(b*x+a))+1/4*Pi*x^2
```

3.44.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int x\arctan(\cot(a+bx))\,dx=-\frac{1}{3}\,bx^3+\frac{1}{4}\,(\pi-2\,a)x^2$$

input
```
integrate(x*(1/2*pi-arccot(cot(b*x+a))),x, algorithm="fricas")
```

output
```
-1/3*b*x^3 + 1/4*(pi - 2*a)*x^2
```

3.44.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(22) = 44.

Time = 0.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.13

$$\int x\arctan(\cot(a+bx))\,dx=\begin{cases}\frac{\pi x^2}{4}-\frac{x\,\mathrm{acot}^2\left(\cot\left(a+bx\right)\right)}{2b}+\frac{\mathrm{acot}^3\left(\cot\left(a+bx\right)\right)}{6b^2} & \text{for } b\neq 0\\ \frac{x^2\left(-\,\mathrm{acot}\left(\cot\left(a\right)\right)+\frac{\pi}{2}\right)}{2} & \text{otherwise}\end{cases}$$

input
```
integrate(x*(1/2*pi-acot(cot(b*x+a))),x)
```

output
```
Piecewise((pi*x**2/4 - x*acot(cot(a + b*x))**2/(2*b) + acot(cot(a + b*x))*
*3/(6*b**2), Ne(b, 0)), (x**2*(-acot(cot(a)) + pi/2)/2, True))
```

3.44.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int x\arctan(\cot(a+bx))\,dx=-\frac{1}{3}\,bx^3+\frac{1}{4}\,(\pi-2\,a)x^2$$

input
```
integrate(x*(1/2*pi-arccot(cot(b*x+a))),x, algorithm="maxima")
```

output
```
-1/3*b*x^3 + 1/4*(pi - 2*a)*x^2
```

3.44.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int x\arctan(\cot(a+bx))\,dx=-\frac{1}{3}\,bx^3+\frac{1}{4}\,\pi x^2-\frac{1}{2}\,ax^2$$

input
```
integrate(x*(1/2*pi-arccot(cot(b*x+a))),x, algorithm="giac")
```

output
```
-1/3*b*x^3 + 1/4*pi*x^2 - 1/2*a*x^2
```

3.44.9 Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int x\arctan(\cot(a+bx))\,dx=\frac{\Pi\,x^2}{4}+\frac{b\,x^3}{6}-\frac{x^2\,\mathrm{acot}(\cot(a+b\,x))}{2}$$

input
```
int(x*(Pi/2 - acot(cot(a + b*x))),x)
```

output
```
(Pi*x^2)/4 + (b*x^3)/6 - (x^2*acot(cot(a + b*x)))/2
```

3.45 $\int \arctan(\cot(a+bx))\,dx$

3.45.1 Optimal result

Integrand size = 7, antiderivative size = 16

$$\int \arctan(\cot(a+bx))\,dx = -\frac{\arctan(\cot(a+bx))^2}{2b}$$

output
```
-1/2*(1/2*Pi-arccot(cot(b*x+a)))^2/b
```

3.45.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \arctan(\cot(a+bx))\,dx = \frac{bx^2}{2} + x\arctan(\cot(a+bx))$$

input
```
Integrate[ArcTan[Cot[a + b*x]],x]
```

output
```
(b*x^2)/2 + x*ArcTan[Cot[a + b*x]]
```

3.45.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(\cot(a+bx))\,dx$$

$$\Big\downarrow \text{ 2588}$$

$$-\frac{\int \arctan(\cot(a+bx))d\arctan(\cot(a+bx))}{b}$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{\arctan(\cot(a+bx))^2}{2b}$$

input
```
Int[ArcTan[Cot[a + b*x]],x]
```

output
```
-1/2*ArcTan[Cot[a + b*x]]^2/b
```

3.45.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

3.45.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

method	result
parallelrisch	$\frac{x^{2} b}{2}-x \operatorname{arccot}\left(\cot \left(b x+a\right)\right)+\frac{\pi x}{2}$
derivativedivides	$\frac{-\pi\left(\frac{\pi}{2}-\operatorname{arccot}\left(\cot \left(b x+a\right)\right)\right)-\operatorname{arccot}\left(\cot \left(b x+a\right)\right)^{2}}{2 b}$
default	$\frac{\pi x}{2}-\frac{-\left(\frac{\pi}{2}-\operatorname{arccot}\left(\cot \left(b x+a\right)\right)\right) \operatorname{arccot}\left(\cot \left(b x+a\right)\right)-\frac{\left(\frac{\pi}{2}-\operatorname{arccot}\left(\cot \left(b x+a\right)\right)\right)^{2}}{2}}{b}$
parts	$\frac{\pi x}{2}-\frac{-\left(\frac{\pi}{2}-\operatorname{arccot}\left(\cot \left(b x+a\right)\right)\right) \operatorname{arccot}\left(\cot \left(b x+a\right)\right)-\frac{\left(\frac{\pi}{2}-\operatorname{arccot}\left(\cot \left(b x+a\right)\right)\right)^{2}}{2}}{b}$
risch	$\frac{\pi x}{2}+i x \ln\left({\mathrm e}^{i \left(b x+a\right)}\right)+\frac{\pi x \,\operatorname{csgn}\left(i {\mathrm e}^{i \left(b x+a\right)}\right)^{2} \operatorname{csgn}\left(i {\mathrm e}^{2 i \left(b x+a\right)}\right)}{4}-\frac{\pi x \,\operatorname{csgn}\left(i {\mathrm e}^{i \left(b x+a\right)}\right) \operatorname{csgn}\left(i {\mathrm e}^{2 i \left(b x+a\right)}\right)^{2}}{2}+\pi x$ c

input
```
int(1/2*Pi-arccot(cot(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*b-x*arccot(cot(b*x+a))+1/2*Pi*x
```

3.45.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \arctan(\cot(a+bx))\,dx = -\frac{1}{2}\,bx^2+\frac{1}{2}\,(\pi-2\,a)x$$

input
```
integrate(1/2*pi-arccot(cot(b*x+a)),x, algorithm="fricas")
```

output
```
-1/2*b*x^2 + 1/2*(pi - 2*a)*x
```

3.45.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int \arctan(\cot(a+bx))\,dx = \frac{\pi x}{2}-\begin{cases}\frac{\operatorname{acot}^{2}\left(\cot\left(a+bx\right)\right)}{2b} & \text{for}\ b\neq 0\\ x\operatorname{acot}\left(\cot\left(a\right)\right) & \text{otherwise}\end{cases}$$

input
```
integrate(1/2*pi-acot(cot(b*x+a)),x)
```

output
```
pi*x/2 - Piecewise((acot(cot(a + b*x))**2/(2*b), Ne(b, 0)), (x*acot(cot(a)
), True))
```

3.45.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \arctan(\cot(a+bx))\,dx = -\frac{1}{2}\,bx^2 + \frac{1}{2}\,\pi x - ax$$

input
```
integrate(1/2*pi-arccot(cot(b*x+a)),x, algorithm="maxima")
```

output
```
-1/2*b*x^2 + 1/2*pi*x - a*x
```

3.45.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \arctan(\cot(a+bx))\,dx = -\frac{1}{2}\,bx^2 + \frac{1}{2}\,\pi x - ax$$

input
```
integrate(1/2*pi-arccot(cot(b*x+a)),x, algorithm="giac")
```

output
```
-1/2*b*x^2 + 1/2*pi*x - a*x
```

3.45.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int \arctan(\cot(a+bx))\,dx = \frac{\Pi\,x}{2} - x\,\mathrm{acot}(\cot(a+b\,x)) + \frac{b\,x^2}{2}$$

input
```
int(Pi/2 - acot(cot(a + b*x)),x)
```

output
```
(Pi*x)/2 - x*acot(cot(a + b*x)) + (b*x^2)/2
```

3.46 $\int \frac{\arctan(\cot(a+bx))}{x} dx$

3.46.1 Optimal result

Integrand size = 11, antiderivative size = 19

$$\int \frac{\arctan(\cot(a+bx))}{x} dx = -bx + (bx + \arctan(\cot(a+bx))) \log(x)$$

output
```
-b*x+(b*x+1/2*Pi-arccot(cot(b*x+a)))*ln(x)
```

3.46.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(\cot(a+bx))}{x} dx = -bx + (bx + \arctan(\cot(a+bx))) \log(x)$$

input
```
Integrate[ArcTan[Cot[a + b*x]]/x,x]
```

output
```
-(b*x) + (b*x + ArcTan[Cot[a + b*x]])*Log[x]
```

3.46.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2589, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(\cot(a+bx))}{x} \, dx$$

$$\Big\downarrow \text{ 2589}$$

$$(\arctan(\cot(a+bx))+bx)\int \frac{1}{x}dx-bx$$

$$\Big\downarrow \text{ 14}$$

$$\log(x)(\arctan(\cot(a+bx))+bx)-bx$$

input
```
Int[ArcTan[Cot[a + b*x]]/x,x]
```

output
```
-(b*x) + (b*x + ArcTan[Cot[a + b*x]])*Log[x]
```

3.46.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2589
```
Int[(v_)/(u_), x_Symbol] :> With[{a = Simplify[D[u, x]], b = Simplify[D[v,
x]]}, Simp[b*(x/a), x] - Simp[(b*u - a*v)/a   Int[1/u, x], x] /; NeQ[b*u -
a*v, 0]] /; PiecewiseLinearQ[u, v, x]
```

3.46.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42

method	result
default	$\frac{\pi \ln(x)}{2} - bx - (\operatorname{arccot}(\cot(bx+a)) - bx)\ln(x)$
parts	$\frac{\pi \ln(x)}{2} - bx - (\operatorname{arccot}(\cot(bx+a)) - bx)\ln(x)$
risch	$i\ln(x)\ln\left(\mathrm{e}^{i(bx+a)}\right) + \ln(x)bx - bx + \frac{\pi\left(\operatorname{csgn}\left(i\mathrm{e}^{i(bx+a)}\right)^2 \operatorname{csgn}\left(i\mathrm{e}^{2i(bx+a)}\right) - 2\operatorname{csgn}\left(i\mathrm{e}^{i(bx+a)}\right)\operatorname{csgn}\left(i\mathrm{e}^{2i(bx+a)}\right)^2 + \operatorname{csg}\right.}{}$

input
```
int((1/2*Pi-arccot(cot(b*x+a)))/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*Pi*ln(x)-b*x-(arccot(cot(b*x+a))-b*x)*ln(x)
```

3.46.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{\arctan(\cot(a+bx))}{x}\,dx = -bx + \frac{1}{2}(\pi - 2a)\log(x)$$

input
```
integrate((1/2*pi-arccot(cot(b*x+a)))/x,x, algorithm="fricas")
```

output
```
-b*x + 1/2*(pi - 2*a)*log(x)
```

3.46.6 Sympy [A] (verification not implemented)

Time = 1.35 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42

$$\int \frac{\arctan(\cot(a+bx))}{x}\,dx = bx\log(x) - bx - \log(x)\operatorname{acot}(\cot(a+bx)) + \frac{\pi\log(x)}{2}$$

input
```
integrate((1/2*pi-acot(cot(b*x+a)))/x,x)
```

output
```
b*x*log(x) - b*x - log(x)*acot(cot(a + b*x)) + pi*log(x)/2
```

3.46.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int \frac{\arctan(\cot(a+bx))}{x}\,dx = -bx + \frac{1}{2}\,(\pi - 2\,a)\log{(x)}$$

input
```
integrate((1/2*pi-arccot(cot(b*x+a)))/x,x, algorithm="maxima")
```

output
```
-b*x + 1/2*(pi - 2*a)*log(x)
```

3.46.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \frac{\arctan(\cot(a+bx))}{x}\,dx = -bx + \frac{1}{2}\,(\pi - 2\,a)\log{(|x|)}$$

input
```
integrate((1/2*pi-arccot(cot(b*x+a)))/x,x, algorithm="giac")
```

output
```
-b*x + 1/2*(pi - 2*a)*log(abs(x))
```

3.46.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(\cot(a+bx))}{x}\,dx = \int \frac{\frac{\Pi}{2} - \mathrm{acot}(\cot(a+b\,x))}{x}\,dx$$

input
```
int((Pi/2 - acot(cot(a + b*x)))/x,x)
```

output
```
int((Pi/2 - acot(cot(a + b*x)))/x, x)
```

3.47 $\int \arctan(\tan(a+bx))\,dx$

3.47.1 Optimal result

Integrand size = 7, antiderivative size = 16

$$\int \arctan(\tan(a+bx))\,dx = \frac{\arctan(\tan(a+bx))^2}{2b}$$

output
```
1/2*arctan(tan(b*x+a))^2/b
```

3.47.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \arctan(\tan(a+bx))\,dx = -\frac{bx^2}{2} + x\arctan(\tan(a+bx))$$

input
```
Integrate[ArcTan[Tan[a + b*x]],x]
```

output
```
-1/2*(b*x^2) + x*ArcTan[Tan[a + b*x]]
```

3.47.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(\tan(a+bx))\,dx$$

$$\Big\downarrow \text{2588}$$

$$\frac{\int \arctan(\tan(a+bx))d\arctan(\tan(a+bx))}{b}$$

$$\Big\downarrow \text{15}$$

$$\frac{\arctan(\tan(a+bx))^2}{2b}$$

input
```
Int[ArcTan[Tan[a + b*x]],x]
```

output
```
ArcTan[Tan[a + b*x]]^2/(2*b)
```

3.47.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

3.47.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{\arctan(\tan(bx+a))^2}{2b}$
default	$\frac{\arctan(\tan(bx+a))^2}{2b}$
parallelrisch	$-\frac{x^2 b}{2}+x\arctan\left(\tan\left(bx+a\right)\right)$
parts	$-\frac{x^2 b}{2}+x\arctan\left(\tan\left(bx+a\right)\right)$
risch	$-ix\ln\left(\mathrm{e}^{i(bx+a)}\right)-\frac{\pi x\ \mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2i(bx+a)}+1}\right)\mathrm{csgn}\left(i\mathrm{e}^{2i(bx+a)}\right)\mathrm{csgn}\left(\frac{i\mathrm{e}^{2i(bx+a)}}{\mathrm{e}^{2i(bx+a)}+1}\right)}{4}+\frac{\pi x\ \mathrm{csgn}\left(\frac{i}{\mathrm{e}^{2i(bx+a)}+1}\right)\mathrm{cs}}{4}$

input
```
int(arctan(tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctan(tan(b*x+a))^2/b
```

3.47.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int \arctan(\tan(a+bx))\,dx=\frac{1}{2}\,bx^2+ax$$

input
```
integrate(arctan(tan(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*b*x^2 + a*x
```

3.47.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. 2(12) = 24.

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.62

$$\int \arctan(\tan(a+bx))\,dx=\begin{cases}\frac{\left(\operatorname{atan}\left(\tan\left(a+bx\right)\right)+\pi\left\lfloor\frac{a+bx-\frac{\pi}{2}}{\pi}\right\rfloor\right)^2}{2b} & \text{for } b\neq 0\\ x\left(\operatorname{atan}\left(\tan\left(a\right)\right)+\pi\left\lfloor\frac{a-\frac{\pi}{2}}{\pi}\right\rfloor\right) & \text{otherwise}\end{cases}$$

input
```
integrate(atan(tan(b*x+a)),x)
```

output
```
Piecewise(((atan(tan(a + b*x)) + pi*floor((a + b*x - pi/2)/pi))**2/(2*b),
Ne(b, 0)), (x*(atan(tan(a)) + pi*floor((a - pi/2)/pi)), True))
```

3.47.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \arctan(\tan(a+bx))\,dx = \frac{(bx+a)^2}{2\,b}$$

input
```
integrate(arctan(tan(b*x+a)),x, algorithm="maxima")
```

output
```
1/2*(b*x + a)^2/b
```

3.47.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

$$\int \arctan(\tan(a+bx))\,dx = \frac{1}{2}\,bx^2 - \pi x\left\lfloor\frac{bx+a}{\pi} + \frac{1}{2}\right\rfloor + ax$$

input
```
integrate(arctan(tan(b*x+a)),x, algorithm="giac")
```

output
```
1/2*b*x^2 - pi*x*floor((b*x + a)/pi + 1/2) + a*x
```

3.47.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \arctan(\tan(a+bx))\,dx = x\,\mathrm{atan}(\tan(a+b\,x)) - \frac{b\,x^2}{2}$$

input
```
int(atan(tan(a + b*x)),x)
```

output
```
x*atan(tan(a + b*x)) - (b*x^2)/2
```

3.48 $\int x^2 \arctan(c + d\tan(a + bx))\, dx$

3.48.1 Optimal result

Integrand size = 15, antiderivative size = 403

$$\begin{aligned}\int x^2 \arctan(c + d\tan(a + bx))\, dx = {} & \frac{1}{3}x^3 \arctan(c + d\tan(a + bx)) \\ & + \frac{1}{6}ix^3 \log\left(1 + \frac{(1 + ic + d)e^{2ia+2ibx}}{1 + ic - d}\right) \\ & - \frac{1}{6}ix^3 \log\left(1 + \frac{(c + i(1 - d))e^{2ia+2ibx}}{c + i(1 + d)}\right) \\ & + \frac{x^2 \operatorname{PolyLog}\left(2, -\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)}{4b} \\ & - \frac{x^2 \operatorname{PolyLog}\left(2, -\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)}{4b} \\ & + \frac{ix \operatorname{PolyLog}\left(3, -\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)}{4b^2} \\ & - \frac{ix \operatorname{PolyLog}\left(3, -\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)}{4b^2} \\ & - \frac{\operatorname{PolyLog}\left(4, -\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)}{8b^3} \\ & + \frac{\operatorname{PolyLog}\left(4, -\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)}{8b^3}\end{aligned}$$

output
```
1/3*x^3*arctan(c+d*tan(b*x+a))+1/6*I*x^3*ln(1+(1+I*c+d)*exp(2*I*a+2*I*b*x)
/(1+I*c-d))-1/6*I*x^3*ln(1+(c+I*(1-d))*exp(2*I*a+2*I*b*x)/(c+I*(1+d)))+1/4
*x^2*polylog(2,-(1+I*c+d)*exp(2*I*a+2*I*b*x)/(1+I*c-d))/b-1/4*x^2*polylog(
2,-(c+I*(1-d))*exp(2*I*a+2*I*b*x)/(c+I*(1+d)))/b+1/4*I*x*polylog(3,-(1+I*c
+d)*exp(2*I*a+2*I*b*x)/(1+I*c-d))/b^2-1/4*I*x*polylog(3,-(c+I*(1-d))*exp(2
*I*a+2*I*b*x)/(c+I*(1+d)))/b^2-1/8*polylog(4,-(1+I*c+d)*exp(2*I*a+2*I*b*x)
/(1+I*c-d))/b^3+1/8*polylog(4,-(c+I*(1-d))*exp(2*I*a+2*I*b*x)/(c+I*(1+d)))
/b^3
```

3.48.2 Mathematica [A] (verified)

Time = 3.16 (sec) , antiderivative size = 371, normalized size of antiderivative = 0.92

$$\int x^2 \arctan(c + d \tan(a + bx)) \, dx$$
$$= \frac{8b^3x^3 \arctan(c + d\tan(a + bx)) + 4ib^3x^3 \log\left(1 + \frac{(c+i(-1+d))e^{-2i(a+bx)}}{c-i(1+d)}\right) - 4ib^3x^3 \log\left(1 + \frac{(c+i(1+d))e^{-2i(a+bx)}}{i+c-id}\right.}{}$$

input
```
Integrate[x^2*ArcTan[c + d*Tan[a + b*x]],x]
```

output
```
(8*b^3*x^3*ArcTan[c + d*Tan[a + b*x]] + (4*I)*b^3*x^3*Log[1 + (c + I*(-1 +
 d))/((c - I*(1 + d))*E^((2*I)*(a + b*x)))] - (4*I)*b^3*x^3*Log[1 + (c + I
*(1 + d))/((I + c - I*d)*E^((2*I)*(a + b*x)))] + 6*b^2*x^2*PolyLog[2, (-c
- I*(1 + d))/((c - I*(-1 + d))*E^((2*I)*(a + b*x)))] - 6*b^2*x^2*PolyLog[2
, (I - c - I*d)/((c - I*(1 + d))*E^((2*I)*(a + b*x)))] - (6*I)*b*x*PolyLog
[3, (-c - I*(1 + d))/((I + c - I*d)*E^((2*I)*(a + b*x)))] + (6*I)*b*x*Poly
Log[3, (I - c - I*d)/((c - I*(1 + d))*E^((2*I)*(a + b*x)))] - 3*PolyLog[4,
 (-c - I*(1 + d))/((I + c - I*d)*E^((2*I)*(a + b*x)))] + 3*PolyLog[4, (I -
 c - I*d)/((c - I*(1 + d))*E^((2*I)*(a + b*x)))])/(24*b^3)
```

3.48.3 Rubi [A] (verified)

Time = 1.60 (sec) , antiderivative size = 519, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5698, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(d\tan(a+bx)+c)\,dx$$

$$\Big\downarrow \text{5698}$$

$$\frac{1}{3}b(-ic-d+1)\int \frac{e^{2ia+2ibx}x^3}{-ic+(-ic-d+1)e^{2ia+2ibx}+d+1}dx-\frac{1}{3}b(ic+d+1)\int \frac{e^{2ia+2ibx}x^3}{ic+(ic+d+1)e^{2ia+2ibx}-d+1}dx+\frac{1}{3}x^3\arctan(d\tan(a+bx)+c)$$

$$\Big\downarrow \text{2620}$$

$$-\frac{1}{3}b(ic+d+1)\left(\frac{3\int x^2\log\left(\frac{e^{2ia+2ibx}(ic+d+1)}{ic-d+1}+1\right)dx}{2b(c-i(d+1))}-\frac{x^3\log\left(1+\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{2b(c-i(d+1))}\right)+\frac{1}{3}b(-ic-d+1)\left(\frac{x^3\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{2b(c+i(1-d))}-\frac{3\int x^2\log\left(\frac{e^{2ia+2ibx}(c+i(1-d))}{c+i(d+1)}+1\right)dx}{2b(c+i(1-d))}\right)+\frac{1}{3}x^3\arctan(d\tan(a+bx)+c)$$

$$\Big\downarrow \text{3011}$$

$$-\frac{1}{3}b(ic+d+1)\left(\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{2b}-\frac{i\int x\operatorname{PolyLog}\left(2,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)dx}{b}\right)}{2b(c-i(d+1))}-\frac{x^3\log\left(1+\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{2b(c-i(d+1))}\right)+\frac{1}{3}b(-ic-d+1)\left(\frac{x^3\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{2b(c+i(1-d))}-\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{2b}-\frac{i\int x\operatorname{PolyLog}\left(2,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)dx}{b}\right)}{2b(c+i(1-d))}\right)+\frac{1}{3}x^3\arctan(d\tan(a+bx)+c)$$

$$\Big\downarrow \text{7163}$$

3.48. $\int x^2 \arctan(c+d\tan(a+bx))\,dx$

$$-\frac{1}{3}b(ic+d+$$

$$1)\left(\frac{3\left(\frac{ix^2\,\text{PolyLog}\left(2,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{2b}-\frac{i\left(\frac{i\int\text{PolyLog}\left(3,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)dx}{2b}-\frac{ix\,\text{PolyLog}\left(3,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{2b}\right)}{b}\right)}{2b(c-i(d+1))}-\frac{x^3\log\right.$$

$$\frac{1}{3}b(-ic-d+$$

$$1)\left(\frac{x^3\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{2b(c+i(1-d))}-\frac{3\left(\frac{ix^2\,\text{PolyLog}\left(2,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{2b}-\frac{i\left(\frac{i\int\text{PolyLog}\left(3,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)dx}{2b}-\frac{ix\,\text{Pol}\right.}{b}\right.}{2b(c+i(1-d))}\right.$$

$$\frac{1}{3}x^3\arctan(d\tan(a+bx)+c)$$

$$\big\downarrow \text{2720}$$

3.48. $\int x^2\arctan(c+d\tan(a+bx))\,dx$

$$-\frac{1}{3}b(ic+d+1)\left(\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{2b}-\frac{i\left(\frac{\int e^{-2ia-2ibx}\operatorname{PolyLog}\left(3,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)de^{2ia+2ibx}}{4b^2}-\frac{ix\operatorname{PolyLog}\left(3,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right.}{2b}\right.}{b}\right.}{2b(c-i(d+1))}\right.$$

$$\frac{1}{3}b(-ic-d+1)\left(\frac{x^3\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{2b(c+i(1-d))}-\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{2b}-\frac{i\left(\frac{\int e^{-2ia-2ibx}\operatorname{PolyLog}\left(3,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{4b^2}\right.}{}\right.}{2b(c+i(1-d))}\right.$$

$$\frac{1}{3}x^3\arctan(d\tan(a+bx)+c)$$

$$\Big\downarrow \text{7143}$$

3.48. $\int x^2\arctan(c+d\tan(a+bx))\,dx$

$$\frac{1}{3}x^3\arctan(d\tan(a+bx)+c)-\frac{1}{3}b(ic+d+1)\left(\frac{3\left(\frac{ix^2\,\text{PolyLog}\left(2,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{2b}-\frac{i\left(\frac{\text{PolyLog}\left(4,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{4b^2}-\frac{ix\,\text{PolyLog}\left(3,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{2b}\right)}{b}\right)}{2b(c-i(d+1))}-\frac{x^3\log\left(1\right.}{2b(}\right.$$

$$\frac{1}{3}b(-ic-d+1)\left(\frac{x^3\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{2b(c+i(1-d))}-\frac{3\left(\frac{ix^2\,\text{PolyLog}\left(2,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{2b}-\frac{i\left(\frac{\text{PolyLog}\left(4,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{4b^2}-\frac{ix\,\text{PolyLog}}{}\right.}{b}\right.}{2b(c+i(1-d))}\right.$$

input
```
Int[x^2*ArcTan[c + d*Tan[a + b*x]],x]
```

output
```
(x^3*ArcTan[c + d*Tan[a + b*x]])/3 - (b*(1 + I*c + d)*(-1/2*(x^3*Log[1 + (
(1 + I*c + d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c - d)])/(b*(c - I*(1 + d)))
 + (3*(((I/2)*x^2*PolyLog[2, -(((1 + I*c + d)*E^((2*I)*a + (2*I)*b*x))/(1
+ I*c - d))])/b - (I*(((-1/2*I)*x*PolyLog[3, -(((1 + I*c + d)*E^((2*I)*a +
 (2*I)*b*x))/(1 + I*c - d))])/b + PolyLog[4, -(((1 + I*c + d)*E^((2*I)*a +
 (2*I)*b*x))/(1 + I*c - d))]/(4*b^2)))/b))/(2*b*(c - I*(1 + d)))))/3 + (b*
(1 - I*c - d)*((x^3*Log[1 + ((c + I*(1 - d))*E^((2*I)*a + (2*I)*b*x))/(c +
 I*(1 + d))])/(2*b*(c + I*(1 - d))) - (3*(((I/2)*x^2*PolyLog[2, -(((c + I*
(1 - d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 + d)))])/b - (I*(((-1/2*I)*x*P
olyLog[3, -(((c + I*(1 - d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 + d)))])/b
 + PolyLog[4, -(((c + I*(1 - d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 + d)))
]/(4*b^2)))/b))/(2*b*(c + I*(1 - d)))))/3
```

3.48. $\int x^2\arctan(c+d\tan(a+bx))\,dx$

3.48.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5698
```
Int[ArcTan[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Tan[a + b*x]]/(f*(m +
1))), x] + (Simp[b*((1 - I*c - d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(E^(
2*I*a + 2*I*b*x)/(1 - I*c + d + (1 - I*c - d)*E^(2*I*a + 2*I*b*x))), x], x]
 - Simp[b*((1 + I*c + d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(E^(2*I*a + 2
*I*b*x)/(1 + I*c - d + (1 + I*c + d)*E^(2*I*a + 2*I*b*x))), x], x]) /; Free
Q[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c + I*d)^2, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.48.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 27.78 (sec) , antiderivative size = 8039, normalized size of antiderivative = 19.95

method	result	size
risch	Expression too large to display	8039

input
```
int(x^2*arctan(c+d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.48.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1965 vs. 2(290) = 580.

Time = 0.35 (sec) , antiderivative size = 1965, normalized size of antiderivative = 4.88

$$\int x^2 \arctan(c + d\tan(a + bx))\, dx = \text{Too large to display}$$

input
```
integrate(x^2*arctan(c+d*tan(b*x+a)),x, algorithm="fricas")
```

output
```
1/48*(16*b^3*x^3*arctan(d*tan(b*x + a) + c) + 6*b^2*x^2*dilog(2*((I*c*d -
d^2 + d)*tan(b*x + a)^2 - c^2 - I*c*d + (I*c^2 - 2*c*d - I*d^2 + I)*tan(b*
x + a) + d - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d +
1) + 1) - 6*b^2*x^2*dilog(2*((I*c*d - d^2 - d)*tan(b*x + a)^2 - c^2 - I*c*
d + (I*c^2 - 2*c*d - I*d^2 + I)*tan(b*x + a) - d - 1)/((c^2 + d^2 + 2*d +
1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d + 1) + 1) + 6*b^2*x^2*dilog(2*((-I*c*d
 - d^2 + d)*tan(b*x + a)^2 - c^2 + I*c*d + (-I*c^2 - 2*c*d + I*d^2 - I)*ta
n(b*x + a) + d - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*
d + 1) + 1) - 6*b^2*x^2*dilog(2*((-I*c*d - d^2 - d)*tan(b*x + a)^2 - c^2 +
 I*c*d + (-I*c^2 - 2*c*d + I*d^2 - I)*tan(b*x + a) - d - 1)/((c^2 + d^2 +
2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d + 1) + 1) + 4*I*a^3*log(((I*c*d
+ d^2 + d)*tan(b*x + a)^2 - c^2 + I*c*d + (I*c^2 + I*d^2 + 2*I*d + I)*tan(
b*x + a) - d - 1)/(tan(b*x + a)^2 + 1)) - 4*I*a^3*log(((I*c*d + d^2 - d)*t
an(b*x + a)^2 - c^2 + I*c*d + (I*c^2 + I*d^2 - 2*I*d + I)*tan(b*x + a) + d
 - 1)/(tan(b*x + a)^2 + 1)) + 4*I*a^3*log(((I*c*d - d^2 + d)*tan(b*x + a)^
2 + c^2 + I*c*d + (I*c^2 + I*d^2 - 2*I*d + I)*tan(b*x + a) - d + 1)/(tan(b
*x + a)^2 + 1)) - 4*I*a^3*log(((I*c*d - d^2 - d)*tan(b*x + a)^2 + c^2 + I*
c*d + (I*c^2 + I*d^2 + 2*I*d + I)*tan(b*x + a) + d + 1)/(tan(b*x + a)^2 +
1)) + 6*I*b*x*polylog(3, ((c^2 + 2*I*c*d - d^2 + 1)*tan(b*x + a)^2 - c^2 -
 2*I*c*d + d^2 - 2*(-I*c^2 + 2*c*d + I*d^2 - I)*tan(b*x + a) - 1)/((c^2...
```

3.48.6 Sympy [F(-1)]

Timed out.

$$\int x^2 \arctan(c + d\tan(a + bx))\,dx = \text{Timed out}$$

input
```
integrate(x**2*atan(c+d*tan(b*x+a)),x)
```

output
```
Timed out
```

3.48.7 Maxima [F]

$$\int x^2 \arctan(c + d\tan(a + bx))\,dx = \int x^2 \arctan\left(d\tan\left(bx + a\right) + c\right)\,dx$$

input
```
integrate(x^2*arctan(c+d*tan(b*x+a)),x, algorithm="maxima")
```

output
```
1/6*x^3*arctan2(c*cos(2*b*x + 2*a) + (d + 1)*sin(2*b*x + 2*a) + c, (d + 1)
*cos(2*b*x + 2*a) - c*sin(2*b*x + 2*a) - d + 1) + 1/6*x^3*arctan2(c*cos(2*
b*x + 2*a) + (d - 1)*sin(2*b*x + 2*a) + c, -(d - 1)*cos(2*b*x + 2*a) + c*s
in(2*b*x + 2*a) + d + 1) + 4*b*d*integrate(-1/3*(2*(c^2 + d^2 + 1)*x^3*cos
(2*b*x + 2*a)^2 + 2*c*d*x^3*sin(2*b*x + 2*a) + 2*(c^2 + d^2 + 1)*x^3*sin(2
*b*x + 2*a)^2 + (c^2 - d^2 + 1)*x^3*cos(2*b*x + 2*a) - (2*c*d*x^3*sin(2*b*
x + 2*a) - (c^2 - d^2 + 1)*x^3*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d
*x^3*cos(2*b*x + 2*a) + (c^2 - d^2 + 1)*x^3*sin(2*b*x + 2*a))*sin(4*b*x +
4*a))/(c^4 + d^4 + 2*(c^2 - 1)*d^2 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2
+ 1)*cos(4*b*x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*cos(
2*b*x + 2*a)^2 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 + 1)*sin(4*b*x + 4*a
)^2 + 4*(c^4 + d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*sin(2*b*x + 2*a)^2 + 2*c
^2 + 2*(c^4 + d^4 - 2*(3*c^2 + 1)*d^2 + 2*c^2 + 2*(c^4 - d^4 + 2*c^2 + 1)*
cos(2*b*x + 2*a) - 4*(c*d^3 + (c^3 + c)*d)*sin(2*b*x + 2*a) + 1)*cos(4*b*x
 + 4*a) + 4*(c^4 - d^4 + 2*c^2 + 1)*cos(2*b*x + 2*a) - 4*(2*c*d^3 - 2*(c^3
 + c)*d - 2*(c*d^3 + (c^3 + c)*d)*cos(2*b*x + 2*a) - (c^4 - d^4 + 2*c^2 +
1)*sin(2*b*x + 2*a))*sin(4*b*x + 4*a) + 8*(c*d^3 + (c^3 + c)*d)*sin(2*b*x
+ 2*a) + 1), x)
```

3.48.8 Giac [F]

$$\int x^2 \arctan(c + d\tan(a + bx))\,dx = \int x^2 \arctan\left(d\tan\left(bx + a\right) + c\right)\,dx$$

input
```
integrate(x^2*arctan(c+d*tan(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*arctan(d*tan(b*x + a) + c), x)
```

3.48.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \arctan(c + d\tan(a + bx))\,dx = \int x^2 \,\mathrm{atan}(c + d\tan(a + b\,x))\;dx$$

input
```
int(x^2*atan(c + d*tan(a + b*x)),x)
```

output
```
int(x^2*atan(c + d*tan(a + b*x)), x)
```

3.49 $\int x \arctan(c + d\tan(a + bx))\,dx$

3.49.1 Optimal result

Integrand size = 13, antiderivative size = 305

$$\begin{aligned}\int x \arctan(c+d\tan(a+bx))\,dx &= \frac{1}{2}x^2\arctan(c+d\tan(a+bx)) \\ &+ \frac{1}{4}ix^2\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) \\ &- \frac{1}{4}ix^2\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right) \\ &+ \frac{x\,\mathrm{PolyLog}\left(2,-\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)}{4b} \\ &- \frac{x\,\mathrm{PolyLog}\left(2,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)}{4b} \\ &+ \frac{i\,\mathrm{PolyLog}\left(3,-\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)}{8b^2} \\ &- \frac{i\,\mathrm{PolyLog}\left(3,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)}{8b^2}\end{aligned}$$

output
```
1/2*x^2*arctan(c+d*tan(b*x+a))+1/4*I*x^2*ln(1+(1+I*c+d)*exp(2*I*a+2*I*b*x)
/(1+I*c-d))-1/4*I*x^2*ln(1+(c+I*(1-d))*exp(2*I*a+2*I*b*x)/(c+I*(1+d)))+1/4
*x*polylog(2,-(1+I*c+d)*exp(2*I*a+2*I*b*x)/(1+I*c-d))/b-1/4*x*polylog(2,-(
c+I*(1-d))*exp(2*I*a+2*I*b*x)/(c+I*(1+d)))/b+1/8*I*polylog(3,-(1+I*c+d)*ex
p(2*I*a+2*I*b*x)/(1+I*c-d))/b^2-1/8*I*polylog(3,-(c+I*(1-d))*exp(2*I*a+2*I
*b*x)/(c+I*(1+d)))/b^2
```

3.49.2 Mathematica [A] (verified)

Time = 2.85 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.92

$$\int x\arctan(c+d\tan(a+bx))\,dx$$
$$=\frac{4b^2x^2\arctan(c+d\tan(a+bx))+2ib^2x^2\log\left(1+\frac{(c+i(-1+d))e^{-2i(a+bx)}}{c-i(1+d)}\right)-2ib^2x^2\log\left(1+\frac{(c+i(1+d))e^{-2i(a+bx)}}{i+c-id}\right.}{}$$

input
```
Integrate[x*ArcTan[c + d*Tan[a + b*x]],x]
```

output
```
(4*b^2*x^2*ArcTan[c + d*Tan[a + b*x]] + (2*I)*b^2*x^2*Log[1 + (c + I*(-1 +
 d))/((c - I*(1 + d))*E^((2*I)*(a + b*x)))] - (2*I)*b^2*x^2*Log[1 + (c + I
*(1 + d))/((I + c - I*d)*E^((2*I)*(a + b*x)))] + 2*b*x*PolyLog[2, (-c - I*
(1 + d))/((c - I*(-1 + d))*E^((2*I)*(a + b*x)))] - 2*b*x*PolyLog[2, (I - c
 - I*d)/((c - I*(1 + d))*E^((2*I)*(a + b*x)))] - I*PolyLog[3, (-c - I*(1 +
 d))/((I + c - I*d)*E^((2*I)*(a + b*x)))] + I*PolyLog[3, (I - c - I*d)/((c
 - I*(1 + d))*E^((2*I)*(a + b*x)))])/(8*b^2)
```

3.49.3 Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.30, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5698, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\arctan(d\tan(a+bx)+c)\,dx$$

$$\Big\downarrow \text{5698}$$

$$\frac{1}{2}b(-ic-d+1)\int\frac{e^{2ia+2ibx}x^2}{-ic+(-ic-d+1)e^{2ia+2ibx}+d+1}dx-\frac{1}{2}b(ic+d+1)\int\frac{e^{2ia+2ibx}x^2}{ic+(ic+d+1)e^{2ia+2ibx}-d+1}dx+\frac{1}{2}x^2\arctan(d\tan(a+bx)+c)$$

$$\Big\downarrow \text{2620}$$

$$-\frac{1}{2}b(ic+d+1)\left(\frac{\int x\log\left(\frac{e^{2ia+2ibx}(ic+d+1)}{ic-d+1}+1\right)dx}{b(c-i(d+1))}-\frac{x^2\log\left(1+\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{2b(c-i(d+1))}\right)+\frac{1}{2}b(-ic-d+1)\left(\frac{x^2\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{2b(c+i(1-d))}-\frac{\int x\log\left(\frac{e^{2ia+2ibx}(c+i(1-d))}{c+i(d+1)}+1\right)dx}{b(c+i(1-d))}\right)+\frac{1}{2}x^2\arctan(d\tan(a+bx)+c)$$

$\downarrow$ 3011

$$-\frac{1}{2}b(ic+d+1)\left(\frac{\frac{ix\,\text{PolyLog}\left(2,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{2b}-\frac{i\int \text{PolyLog}\left(2,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)dx}{2b}}{b(c-i(d+1))}-\frac{x^2\log\left(1+\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{2b(c-i(d+1))}\right)+\frac{1}{2}b(-ic-d+1)\left(\frac{x^2\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{2b(c+i(1-d))}-\frac{\frac{ix\,\text{PolyLog}\left(2,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{2b}-\frac{i\int \text{PolyLog}\left(2,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)dx}{2b}}{b(c+i(1-d))}\right)+\frac{1}{2}x^2\arctan(d\tan(a+bx)+c)$$

$\downarrow$ 2720

$$-\frac{1}{2}b(ic+d+1)\left(\frac{\frac{ix\,\text{PolyLog}\left(2,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{2b}-\frac{\int e^{-2ia-2ibx}\,\text{PolyLog}\left(2,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)de^{2ia+2ibx}}{4b^2}}{b(c-i(d+1))}-\frac{x^2\log\left(1+\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{2b(c-i(d+1))}\right)+\frac{1}{2}b(-ic-d+1)\left(\frac{x^2\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{2b(c+i(1-d))}-\frac{\frac{ix\,\text{PolyLog}\left(2,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{2b}-\frac{\int e^{-2ia-2ibx}\,\text{PolyLog}\left(2,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)de^{2ia+2ibx}}{4b^2}}{b(c+i(1-d))}\right)+\frac{1}{2}x^2\arctan(d\tan(a+bx)+c)$$

$\downarrow$ 7143

3.49. $\int x\arctan(c+d\tan(a+bx))\,dx$

$$\frac{1}{2}x^2\arctan(d\tan(a+bx)+c)-\frac{1}{2}b(ic+d+1)\left(\frac{\frac{ix\operatorname{PolyLog}\left(2,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{2b}-\frac{\operatorname{PolyLog}\left(3,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{4b^2}}{b(c-i(d+1))}-\frac{x^2\log\left(1+\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{2b(c-i(d+1))}\right)+\frac{1}{2}b(-ic-d+1)\left(\frac{x^2\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{2b(c+i(1-d))}-\frac{\frac{ix\operatorname{PolyLog}\left(2,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{2b}-\frac{\operatorname{PolyLog}\left(3,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{4b^2}}{b(c+i(1-d))}\right)$$

input
```
Int[x*ArcTan[c + d*Tan[a + b*x]],x]
```

output
```
(x^2*ArcTan[c + d*Tan[a + b*x]])/2 - (b*(1 + I*c + d)*(-1/2*(x^2*Log[1 + (
(1 + I*c + d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c - d)])/(b*(c - I*(1 + d)))
 + (((I/2)*x*PolyLog[2, -(((1 + I*c + d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c
 - d))])/b - PolyLog[3, -(((1 + I*c + d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c
 - d))]/(4*b^2))/(b*(c - I*(1 + d)))))/2 + (b*(1 - I*c - d)*((x^2*Log[1 +
((c + I*(1 - d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 + d))])/(2*b*(c + I*(1
 - d))) - (((I/2)*x*PolyLog[2, -(((c + I*(1 - d))*E^((2*I)*a + (2*I)*b*x))
/(c + I*(1 + d)))])/b - PolyLog[3, -(((c + I*(1 - d))*E^((2*I)*a + (2*I)*b
*x))/(c + I*(1 + d)))]/(4*b^2))/(b*(c + I*(1 - d)))))/2
```

3.49.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5698
```
Int[ArcTan[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Tan[a + b*x]]/(f*(m +
1))), x] + (Simp[b*((1 - I*c - d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(E^(
2*I*a + 2*I*b*x)/(1 - I*c + d + (1 - I*c - d)*E^(2*I*a + 2*I*b*x))), x], x]
 - Simp[b*((1 + I*c + d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(E^(2*I*a + 2
*I*b*x)/(1 + I*c - d + (1 + I*c + d)*E^(2*I*a + 2*I*b*x))), x], x]) /; Free
Q[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c + I*d)^2, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.49.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.85 (sec) , antiderivative size = 7647, normalized size of antiderivative = 25.07

method	result	size
risch	Expression too large to display	7647

input
```
int(x*arctan(c+d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.49.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1545 vs. 2(217) = 434.

Time = 0.34 (sec) , antiderivative size = 1545, normalized size of antiderivative = 5.07

$$\int x\arctan(c+d\tan(a+bx))\,dx = \text{Too large to display}$$

input
```
integrate(x*arctan(c+d*tan(b*x+a)),x, algorithm="fricas")
```

output
```
1/16*(8*b^2*x^2*arctan(d*tan(b*x + a) + c) + 2*b*x*dilog(2*((I*c*d - d^2 +
 d)*tan(b*x + a)^2 - c^2 - I*c*d + (I*c^2 - 2*c*d - I*d^2 + I)*tan(b*x + a
) + d - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1) +
1) - 2*b*x*dilog(2*((I*c*d - d^2 - d)*tan(b*x + a)^2 - c^2 - I*c*d + (I*c^
2 - 2*c*d - I*d^2 + I)*tan(b*x + a) - d - 1)/((c^2 + d^2 + 2*d + 1)*tan(b*
x + a)^2 + c^2 + d^2 + 2*d + 1) + 1) + 2*b*x*dilog(2*((-I*c*d - d^2 + d)*t
an(b*x + a)^2 - c^2 + I*c*d + (-I*c^2 - 2*c*d + I*d^2 - I)*tan(b*x + a) +
d - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1) + 1) -
 2*b*x*dilog(2*((-I*c*d - d^2 - d)*tan(b*x + a)^2 - c^2 + I*c*d + (-I*c^2
- 2*c*d + I*d^2 - I)*tan(b*x + a) - d - 1)/((c^2 + d^2 + 2*d + 1)*tan(b*x
+ a)^2 + c^2 + d^2 + 2*d + 1) + 1) - 2*I*a^2*log(((I*c*d + d^2 + d)*tan(b*
x + a)^2 - c^2 + I*c*d + (I*c^2 + I*d^2 + 2*I*d + I)*tan(b*x + a) - d - 1)
/(tan(b*x + a)^2 + 1)) + 2*I*a^2*log(((I*c*d + d^2 - d)*tan(b*x + a)^2 - c
^2 + I*c*d + (I*c^2 + I*d^2 - 2*I*d + I)*tan(b*x + a) + d - 1)/(tan(b*x +
a)^2 + 1)) - 2*I*a^2*log(((I*c*d - d^2 + d)*tan(b*x + a)^2 + c^2 + I*c*d +
 (I*c^2 + I*d^2 - 2*I*d + I)*tan(b*x + a) - d + 1)/(tan(b*x + a)^2 + 1)) +
 2*I*a^2*log(((I*c*d - d^2 - d)*tan(b*x + a)^2 + c^2 + I*c*d + (I*c^2 + I*
d^2 + 2*I*d + I)*tan(b*x + a) + d + 1)/(tan(b*x + a)^2 + 1)) - 2*(I*b^2*x^
2 - I*a^2)*log(-2*((I*c*d - d^2 + d)*tan(b*x + a)^2 - c^2 - I*c*d + (I*c^2
 - 2*c*d - I*d^2 + I)*tan(b*x + a) + d - 1)/((c^2 + d^2 - 2*d + 1)*tan(...
```

3.49.6 Sympy [F(-1)]

Timed out.

$$\int x\arctan(c+d\tan(a+bx))\,dx = \text{Timed out}$$

input
```
integrate(x*atan(c+d*tan(b*x+a)),x)
```

output
```
Timed out
```

3.49.7 Maxima [F]

$$\int x\arctan(c+d\tan(a+bx))\,dx=\int x\arctan\left(d\tan\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(x*arctan(c+d*tan(b*x+a)),x, algorithm="maxima")
```

output
```
1/4*x^2*arctan2(c*cos(2*b*x + 2*a) + (d + 1)*sin(2*b*x + 2*a) + c, (d + 1)
*cos(2*b*x + 2*a) - c*sin(2*b*x + 2*a) - d + 1) + 1/4*x^2*arctan2(c*cos(2*
b*x + 2*a) + (d - 1)*sin(2*b*x + 2*a) + c, -(d - 1)*cos(2*b*x + 2*a) + c*s
in(2*b*x + 2*a) + d + 1) + 2*b*d*integrate(-(2*(c^2 + d^2 + 1)*x^2*cos(2*b
*x + 2*a)^2 + 2*c*d*x^2*sin(2*b*x + 2*a) + 2*(c^2 + d^2 + 1)*x^2*sin(2*b*x
 + 2*a)^2 + (c^2 - d^2 + 1)*x^2*cos(2*b*x + 2*a) - (2*c*d*x^2*sin(2*b*x +
2*a) - (c^2 - d^2 + 1)*x^2*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x^2
*cos(2*b*x + 2*a) + (c^2 - d^2 + 1)*x^2*sin(2*b*x + 2*a))*sin(4*b*x + 4*a)
)/(c^4 + d^4 + 2*(c^2 - 1)*d^2 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 + 1)
*cos(4*b*x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*cos(2*b*
x + 2*a)^2 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 + 1)*sin(4*b*x + 4*a)^2
+ 4*(c^4 + d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*sin(2*b*x + 2*a)^2 + 2*c^2 +
 2*(c^4 + d^4 - 2*(3*c^2 + 1)*d^2 + 2*c^2 + 2*(c^4 - d^4 + 2*c^2 + 1)*cos(
2*b*x + 2*a) - 4*(c*d^3 + (c^3 + c)*d)*sin(2*b*x + 2*a) + 1)*cos(4*b*x + 4
*a) + 4*(c^4 - d^4 + 2*c^2 + 1)*cos(2*b*x + 2*a) - 4*(2*c*d^3 - 2*(c^3 + c
)*d - 2*(c*d^3 + (c^3 + c)*d)*cos(2*b*x + 2*a) - (c^4 - d^4 + 2*c^2 + 1)*s
in(2*b*x + 2*a))*sin(4*b*x + 4*a) + 8*(c*d^3 + (c^3 + c)*d)*sin(2*b*x + 2*
a) + 1), x)
```

3.49.8 Giac [F]

$$\int x\arctan(c+d\tan(a+bx))\,dx=\int x\arctan\left(d\tan\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(x*arctan(c+d*tan(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*arctan(d*tan(b*x + a) + c), x)
```

3.49.9 Mupad [F(-1)]

Timed out.

$$\int x \arctan(c + d \tan(a + bx)) \, dx = \int x \operatorname{atan}(c + d \tan(a + b\,x)) \, dx$$

input
```
int(x*atan(c + d*tan(a + b*x)),x)
```

output
```
int(x*atan(c + d*tan(a + b*x)), x)
```

3.50 $\int \arctan(c+d\tan(a+bx))\,dx$

3.50.1 Optimal result

Integrand size = 11, antiderivative size = 198

$$\begin{aligned}\int \arctan(c+d\tan(a+bx))\,dx = {} & x\arctan(c+d\tan(a+bx)) \\ & + \frac{1}{2}ix\log\left(1+\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right) \\ & - \frac{1}{2}ix\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right) \\ & + \frac{\operatorname{PolyLog}\left(2,-\frac{(1+ic+d)e^{2ia+2ibx}}{1+ic-d}\right)}{4b} \\ & - \frac{\operatorname{PolyLog}\left(2,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(1+d)}\right)}{4b}\end{aligned}$$

output
```
x*arctan(c+d*tan(b*x+a))+1/2*I*x*ln(1+(1+I*c+d)*exp(2*I*a+2*I*b*x)/(1+I*c-
d))-1/2*I*x*ln(1+(c+I*(1-d))*exp(2*I*a+2*I*b*x)/(c+I*(1+d)))+1/4*polylog(2
,-(1+I*c+d)*exp(2*I*a+2*I*b*x)/(1+I*c-d))/b-1/4*polylog(2,-(c+I*(1-d))*exp
(2*I*a+2*I*b*x)/(c+I*(1+d)))/b
```

3.50.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 549 vs. $2(198) = 396$.

Time = 5.61 (sec) , antiderivative size = 549, normalized size of antiderivative = 2.77

$$\int \arctan(c+d\tan(a+bx))\,dx = x\arctan(c+d\tan(a+bx)) - \frac{x\left(4a\sqrt{-d^2}\arctan(c+d\tan(a+bx)) - id\log(1-i\tan(a+bx))\log\left(\frac{-cd+\sqrt{-d^2}-d^2\tan(a+bx)}{-cd+id^2+\sqrt{-d^2}}\right)+id\log(1\right.}{}$$

input
```
Integrate[ArcTan[c + d*Tan[a + b*x]],x]
```

output
```
x*ArcTan[c + d*Tan[a + b*x]] - (x*(4*a*Sqrt[-d^2]*ArcTan[c + d*Tan[a + b*x
]] - I*d*Log[1 - I*Tan[a + b*x]]*Log[(-(c*d) + Sqrt[-d^2] - d^2*Tan[a + b*
x])/(-(c*d) + I*d^2 + Sqrt[-d^2])] + I*d*Log[1 + I*Tan[a + b*x]]*Log[(c*d
- Sqrt[-d^2] + d^2*Tan[a + b*x])/(c*d + I*d^2 - Sqrt[-d^2])] + I*d*Log[1 -
 I*Tan[a + b*x]]*Log[(c*d + Sqrt[-d^2] + d^2*Tan[a + b*x])/(c*d - I*d^2 +
Sqrt[-d^2])] - I*d*Log[1 + I*Tan[a + b*x]]*Log[(c*d + Sqrt[-d^2] + d^2*Tan
[a + b*x])/(c*d + I*d^2 + Sqrt[-d^2])] - I*d*PolyLog[2, (d^2*(1 - I*Tan[a
+ b*x]))/(I*c*d + d^2 - I*Sqrt[-d^2])] + I*d*PolyLog[2, (d^2*(1 - I*Tan[a
+ b*x]))/(I*c*d + d^2 + I*Sqrt[-d^2])] + I*d*PolyLog[2, (d^2*(1 + I*Tan[a
+ b*x]))/((-I)*c*d + d^2 + I*Sqrt[-d^2])] - I*d*PolyLog[2, (d^2*(1 + I*Tan
[a + b*x]))/(d^2 - I*(c*d + Sqrt[-d^2]))]))/(2*Sqrt[-d^2]*(2*a - I*Log[1 -
 I*Tan[a + b*x]] + I*Log[1 + I*Tan[a + b*x]]))
```

3.50.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.40, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {5690, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(d\tan(a+bx)+c)\,dx$$

$$\Big\downarrow \; 5690$$

$$b(-ic-d+1)\int \frac{e^{2ia+2ibx}x}{-ic+(-ic-d+1)e^{2ia+2ibx}+d+1}dx-b(ic+d+1)\int \frac{e^{2ia+2ibx}x}{ic+(ic+d+1)e^{2ia+2ibx}-d+1}dx+x\arctan(d\tan(a+bx)+c)$$

$\Big\downarrow$ 2620

$$-b(ic+d+1)\left(\frac{\int \log\left(\frac{e^{2ia+2ibx}(ic+d+1)}{ic-d+1}+1\right)dx}{2b(c-i(d+1))}-\frac{x\log\left(1+\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{2b(c-i(d+1))}\right)+b(-ic-d+1)\left(\frac{x\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{2b(c+i(1-d))}-\frac{\int \log\left(\frac{e^{2ia+2ibx}(c+i(1-d))}{c+i(d+1)}+1\right)dx}{2b(c+i(1-d))}\right)+x\arctan(d\tan(a+bx)+c)$$

$\Big\downarrow$ 2715

$$-b(ic+d+1)\left(-\frac{i\int e^{-2ia-2ibx}\log\left(\frac{e^{2ia+2ibx}(ic+d+1)}{ic-d+1}+1\right)de^{2ia+2ibx}}{4b^2(c-i(d+1))}-\frac{x\log\left(1+\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{2b(c-i(d+1))}\right)+b(-ic-d+1)\left(\frac{i\int e^{-2ia-2ibx}\log\left(\frac{e^{2ia+2ibx}(c+i(1-d))}{c+i(d+1)}+1\right)de^{2ia+2ibx}}{4b^2(c+i(1-d))}+\frac{x\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{2b(c+i(1-d))}\right)+x\arctan(d\tan(a+bx)+c)$$

$\Big\downarrow$ 2838

$$x\arctan(d\tan(a+bx)+c)-b(ic+d+1)\left(\frac{i\,\text{PolyLog}\left(2,-\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{4b^2(c-i(d+1))}-\frac{x\log\left(1+\frac{(ic+d+1)e^{2ia+2ibx}}{ic-d+1}\right)}{2b(c-i(d+1))}\right)+b(-ic-d+1)\left(\frac{x\log\left(1+\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{2b(c+i(1-d))}-\frac{i\,\text{PolyLog}\left(2,-\frac{(c+i(1-d))e^{2ia+2ibx}}{c+i(d+1)}\right)}{4b^2(c+i(1-d))}\right)$$

input
```
Int[ArcTan[c + d*Tan[a + b*x]],x]
```

output
```
x*ArcTan[c + d*Tan[a + b*x]] - b*(1 + I*c + d)*(-1/2*(x*Log[1 + ((1 + I*c
+ d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c - d)])/(b*(c - I*(1 + d))) + ((I/4)
*PolyLog[2, -(((1 + I*c + d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c - d))])/(b^
2*(c - I*(1 + d)))) + b*(1 - I*c - d)*((x*Log[1 + ((c + I*(1 - d))*E^((2*I
)*a + (2*I)*b*x))/(c + I*(1 + d))])/(2*b*(c + I*(1 - d))) - ((I/4)*PolyLog
[2, -(((c + I*(1 - d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 + d)))])/(b^2*(c
 + I*(1 - d))))
```

3.50. $\int \arctan(c+d\tan(a+bx))\,dx$

3.50.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5690
```
Int[ArcTan[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcT
an[c + d*Tan[a + b*x]], x] + (Simp[b*(1 - I*c - d)   Int[x*(E^(2*I*a + 2*I*
b*x)/(1 - I*c + d + (1 - I*c - d)*E^(2*I*a + 2*I*b*x))), x], x] - Simp[b*(1
 + I*c + d)   Int[x*(E^(2*I*a + 2*I*b*x)/(1 + I*c - d + (1 + I*c + d)*E^(2*
I*a + 2*I*b*x))), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[(c + I*d)^2, -1]
```

3.50.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1000 vs. $2(168) = 336$.

Time = 2.41 (sec) , antiderivative size = 1001, normalized size of antiderivative = 5.06

method	result	size
derivativedivides	Expression too large to display	1001
default	Expression too large to display	1001
risch	Expression too large to display	4973

input
```
int(arctan(c+d*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b/d*(d*arctan(tan(b*x+a))*arctan(c+d*tan(b*x+a))-d^2*(1/2*I/d*arctan(-(c
+d*tan(b*x+a))/d+c/d)*ln(1-(c-I*d-I)*(1+I*((c+d*tan(b*x+a))/d-c/d))^2/(((c
+d*tan(b*x+a))/d-c/d)^2+1)/(-I*d+I-c))-1/2/d*arctan(-(c+d*tan(b*x+a))/d+c/
d)^2-1/4/d*polylog(2,(c-I*d-I)*(1+I*((c+d*tan(b*x+a))/d-c/d))^2/(((c+d*tan
(b*x+a))/d-c/d)^2+1)/(-I*d+I-c))+1/2/(I+c+I*d)*ln(1-(c-I*d+I)*(1+I*((c+d*t
an(b*x+a))/d-c/d))^2/(((c+d*tan(b*x+a))/d-c/d)^2+1)/(-I*d-I-c))*arctan(-(c
+d*tan(b*x+a))/d+c/d)+1/2/d/(I+c+I*d)*ln(1-(c-I*d+I)*(1+I*((c+d*tan(b*x+a)
)/d-c/d))^2/(((c+d*tan(b*x+a))/d-c/d)^2+1)/(-I*d-I-c))*arctan(-(c+d*tan(b*
x+a))/d+c/d)-1/2*I/d/(I+c+I*d)*ln(1-(c-I*d+I)*(1+I*((c+d*tan(b*x+a))/d-c/d
))^2/(((c+d*tan(b*x+a))/d-c/d)^2+1)/(-I*d-I-c))*c*arctan(-(c+d*tan(b*x+a))
/d+c/d)+1/2*I/(I+c+I*d)*arctan(-(c+d*tan(b*x+a))/d+c/d)^2+1/4*I/(I+c+I*d)*
polylog(2,(c-I*d+I)*(1+I*((c+d*tan(b*x+a))/d-c/d))^2/(((c+d*tan(b*x+a))/d-
c/d)^2+1)/(-I*d-I-c))+1/2*I/d/(I+c+I*d)*arctan(-(c+d*tan(b*x+a))/d+c/d)^2+
1/2/d/(I+c+I*d)*c*arctan(-(c+d*tan(b*x+a))/d+c/d)^2+1/4*I/d/(I+c+I*d)*poly
log(2,(c-I*d+I)*(1+I*((c+d*tan(b*x+a))/d-c/d))^2/(((c+d*tan(b*x+a))/d-c/d)
^2+1)/(-I*d-I-c))+1/4/d/(I+c+I*d)*polylog(2,(c-I*d+I)*(1+I*((c+d*tan(b*x+a
))/d-c/d))^2/(((c+d*tan(b*x+a))/d-c/d)^2+1)/(-I*d-I-c))*c))
```

3.50.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1101 vs. $2(141) = 282$.

Time = 0.34 (sec) , antiderivative size = 1101, normalized size of antiderivative = 5.56

$$\int \arctan(c + d\tan(a + bx))\,dx = \text{Too large to display}$$

input
```
integrate(arctan(c+d*tan(b*x+a)),x, algorithm="fricas")
```

output
```
1/8*(8*b*x*arctan(d*tan(b*x + a) + c) - 2*(I*b*x + I*a)*log(-2*((I*c*d - d
^2 + d)*tan(b*x + a)^2 - c^2 - I*c*d + (I*c^2 - 2*c*d - I*d^2 + I)*tan(b*x
 + a) + d - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 - 2*d + 1
)) - 2*(-I*b*x - I*a)*log(-2*((I*c*d - d^2 - d)*tan(b*x + a)^2 - c^2 - I*c
*d + (I*c^2 - 2*c*d - I*d^2 + I)*tan(b*x + a) - d - 1)/((c^2 + d^2 + 2*d +
 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d + 1)) - 2*(-I*b*x - I*a)*log(-2*((-I*
c*d - d^2 + d)*tan(b*x + a)^2 - c^2 + I*c*d + (-I*c^2 - 2*c*d + I*d^2 - I)
*tan(b*x + a) + d - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 -
 2*d + 1)) - 2*(I*b*x + I*a)*log(-2*((-I*c*d - d^2 - d)*tan(b*x + a)^2 - c
^2 + I*c*d + (-I*c^2 - 2*c*d + I*d^2 - I)*tan(b*x + a) - d - 1)/((c^2 + d^
2 + 2*d + 1)*tan(b*x + a)^2 + c^2 + d^2 + 2*d + 1)) + 2*I*a*log(((I*c*d +
d^2 + d)*tan(b*x + a)^2 - c^2 + I*c*d + (I*c^2 + I*d^2 + 2*I*d + I)*tan(b*
x + a) - d - 1)/(tan(b*x + a)^2 + 1)) - 2*I*a*log(((I*c*d + d^2 - d)*tan(b
*x + a)^2 - c^2 + I*c*d + (I*c^2 + I*d^2 - 2*I*d + I)*tan(b*x + a) + d - 1
)/(tan(b*x + a)^2 + 1)) + 2*I*a*log(((I*c*d - d^2 + d)*tan(b*x + a)^2 + c^
2 + I*c*d + (I*c^2 + I*d^2 - 2*I*d + I)*tan(b*x + a) - d + 1)/(tan(b*x + a
)^2 + 1)) - 2*I*a*log(((I*c*d - d^2 - d)*tan(b*x + a)^2 + c^2 + I*c*d + (I
*c^2 + I*d^2 + 2*I*d + I)*tan(b*x + a) + d + 1)/(tan(b*x + a)^2 + 1)) + di
log(2*((I*c*d - d^2 + d)*tan(b*x + a)^2 - c^2 - I*c*d + (I*c^2 - 2*c*d - I
*d^2 + I)*tan(b*x + a) + d - 1)/((c^2 + d^2 - 2*d + 1)*tan(b*x + a)^2 +...
```

3.50.6 Sympy [F]

$$\int \arctan(c + d\tan(a + bx))\,dx = \int \operatorname{atan}\left(c + d\tan\left(a + bx\right)\right)dx$$

input
```
integrate(atan(c+d*tan(b*x+a)),x)
```

output
```
Integral(atan(c + d*tan(a + b*x)), x)
```

3.50.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 433 vs. $2(141) = 282$.

Time = 0.34 (sec) , antiderivative size = 433, normalized size of antiderivative = 2.19

$$\int \arctan(c+d\tan(a+bx))\,dx = \frac{d\left(\frac{8(bx+a)\arctan\left(\frac{d^2\tan(bx+a)+cd}{d}\right)}{d} - \frac{4(bx+a)\arctan\left(\frac{cd+\left(d^2+d\right)\tan(bx+a)}{c^2+d^2+2d+1}, \frac{cd\tan(bx+a)+c^2+d+1}{c^2+d^2+2d+1}\right) - 4(bx+a)\arctan\left(\frac{cd+\left(d^2-d\right)\tan(bx+}{c^2+d^2-2d+1}\right.}{}\right.}{}$$

input
```
integrate(arctan(c+d*tan(b*x+a)),x, algorithm="maxima")
```

output
```
1/8*(d*(8*(b*x + a)*arctan((d^2*tan(b*x + a) + c*d)/d)/d - (4*(b*x + a)*ar
ctan2((c*d + (d^2 + d)*tan(b*x + a))/(c^2 + d^2 + 2*d + 1), (c*d*tan(b*x +
 a) + c^2 + d + 1)/(c^2 + d^2 + 2*d + 1)) - 4*(b*x + a)*arctan2((c*d + (d^
2 - d)*tan(b*x + a))/(c^2 + d^2 - 2*d + 1), (c*d*tan(b*x + a) + c^2 - d +
1)/(c^2 + d^2 - 2*d + 1)) + log(tan(b*x + a)^2 + 1)*log((d^2*tan(b*x + a)^
2 + 2*c*d*tan(b*x + a) + c^2 + 1)/(c^2 + d^2 + 2*d + 1)) - log(tan(b*x + a
)^2 + 1)*log((d^2*tan(b*x + a)^2 + 2*c*d*tan(b*x + a) + c^2 + 1)/(c^2 + d^
2 - 2*d + 1)) + 2*dilog(-(I*d*tan(b*x + a) - d)/(I*c + d + 1)) - 2*dilog(-
(I*d*tan(b*x + a) - d)/(I*c + d - 1)) + 2*dilog((I*d*tan(b*x + a) + d)/(-I
*c + d + 1)) - 2*dilog((I*d*tan(b*x + a) + d)/(-I*c + d - 1)))/d) + 8*(b*x
 + a)*arctan(d*tan(b*x + a) + c) - 8*(b*x + a)*arctan((d^2*tan(b*x + a) +
c*d)/d))/b
```

3.50.8 Giac [F]

$$\int \arctan(c+d\tan(a+bx))\,dx = \int \arctan\left(d\tan\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(arctan(c+d*tan(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arctan(d*tan(b*x + a) + c), x)
```

3.50.9 Mupad [F(-1)]

Timed out.

$$\int \arctan(c + d\tan(a + bx))\,dx = \int \operatorname{atan}(c + d\tan(a + b\,x))\ dx$$

input
```
int(atan(c + d*tan(a + b*x)),x)
```

output
```
int(atan(c + d*tan(a + b*x)), x)
```

3.51 $\int \frac{\arctan(c+d\tan(a+bx))}{x}\,dx$

3.51.1 Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{\arctan(c+d\tan(a+bx))}{x}\,dx=\text{Int}\left(\frac{\arctan(c+d\tan(a+bx))}{x},x\right)$$

output
```
CannotIntegrate(arctan(c+d*tan(b*x+a))/x,x)
```

3.51.2 Mathematica [N/A]

Not integrable

Time = 2.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\arctan(c+d\tan(a+bx))}{x}\,dx=\int \frac{\arctan(c+d\tan(a+bx))}{x}\,dx$$

input
```
Integrate[ArcTan[c + d*Tan[a + b*x]]/x,x]
```

output
```
Integrate[ArcTan[c + d*Tan[a + b*x]]/x, x]
```

3.51.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(d\tan(a+bx)+c)}{x} \, dx$$
$$\Big\downarrow \ 7299$$
$$\int \frac{\arctan(d\tan(a+bx)+c)}{x}dx$$

input
```
Int[ArcTan[c + d*Tan[a + b*x]]/x,x]
```

output
```
$Aborted
```

3.51.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.51.4 Maple [N/A] (verified)

Not integrable

Time = 0.08 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\arctan \left(c+d\tan \left(bx+a\right)\right)}{x}dx$$

input
```
int(arctan(c+d*tan(b*x+a))/x,x)
```

output
```
int(arctan(c+d*tan(b*x+a))/x,x)
```

3.51.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\arctan(c + d\tan(a + bx))}{x} \, dx = \int \frac{\arctan\left(d\tan\left(bx + a\right) + c\right)}{x} \, dx$$

input
```
integrate(arctan(c+d*tan(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(arctan(d*tan(b*x + a) + c)/x, x)
```

3.51.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan(c + d\tan(a + bx))}{x} \, dx = \text{Timed out}$$

input
```
integrate(atan(c+d*tan(b*x+a))/x,x)
```

output
```
Timed out
```

3.51.7 Maxima [N/A]

Not integrable

Time = 232.91 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\arctan(c + d\tan(a + bx))}{x} \, dx = \int \frac{\arctan\left(d\tan\left(bx + a\right) + c\right)}{x} \, dx$$

input
```
integrate(arctan(c+d*tan(b*x+a))/x,x, algorithm="maxima")
```

output
```
integrate(arctan(d*tan(b*x + a) + c)/x, x)
```

3.51.8 Giac [N/A]

Not integrable

Time = 1.32 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\arctan(c + d\tan(a + bx))}{x}\,dx = \int \frac{\arctan\left(d\tan\left(bx + a\right) + c\right)}{x}\,dx$$

input
```
integrate(arctan(c+d*tan(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(arctan(d*tan(b*x + a) + c)/x, x)
```

3.51.9 Mupad [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\arctan(c + d\tan(a + bx))}{x}\,dx = \int \frac{\mathrm{atan}(c + d\tan(a + b\,x))}{x}\,dx$$

input
```
int(atan(c + d*tan(a + b*x))/x,x)
```

output
```
int(atan(c + d*tan(a + b*x))/x, x)
```

3.52 $\int x^2 \arctan(c+(1+ic)\tan(a+bx))\,dx$

3.52.1 Optimal result

Integrand size = 21, antiderivative size = 154

$$\int x^2 \arctan(c+(1+ic)\tan(a+bx))\,dx = -\frac{bx^4}{12}+\frac{1}{3}x^3\arctan(c+(1+ic)\tan(a+bx))-\frac{1}{6}ix^3\log\left(1-ice^{2ia+2ibx}\right) -\frac{x^2\,\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}-\frac{ix\,\text{PolyLog}\left(3,ice^{2ia+2ibx}\right)}{4b^2}+\frac{\text{PolyLog}\left(4,ice^{2ia+2ibx}\right)}{8b^3}$$

output
```
-1/12*b*x^4+1/3*x^3*arctan(c+(1+I*c)*tan(b*x+a))-1/6*I*x^3*ln(1-I*c*exp(2*
I*a+2*I*b*x))-1/4*x^2*polylog(2,I*c*exp(2*I*a+2*I*b*x))/b-1/4*I*x*polylog(
3,I*c*exp(2*I*a+2*I*b*x))/b^2+1/8*polylog(4,I*c*exp(2*I*a+2*I*b*x))/b^3
```

3.52.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.91

$$\int x^2 \arctan(c+(1+ic)\tan(a+bx))\,dx = \frac{1}{3}x^3\arctan(c+(1+ic)\tan(a+bx)) - \frac{4ib^3x^3\log\left(1+\frac{ie^{-2i(a+bx)}}{c}\right)-6b^2x^2\,\text{PolyLog}\left(2,-\frac{ie^{-2i(a+bx)}}{c}\right)+6ibx\,\text{PolyLog}\left(3,-\frac{ie^{-2i(a+bx)}}{c}\right)+3\,\text{PolyL}}{24b^3}$$

input
```
Integrate[x^2*ArcTan[c + (1 + I*c)*Tan[a + b*x]],x]
```

output
```
(x^3*ArcTan[c + (1 + I*c)*Tan[a + b*x]])/3 - ((4*I)*b^3*x^3*Log[1 + I/(c*E
^((2*I)*(a + b*x)))] - 6*b^2*x^2*PolyLog[2, (-I)/(c*E^((2*I)*(a + b*x)))]
+ (6*I)*b*x*PolyLog[3, (-I)/(c*E^((2*I)*(a + b*x)))] + 3*PolyLog[4, (-I)/(
c*E^((2*I)*(a + b*x)))])/(24*b^3)
```

3.52.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.29, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5694, 2615, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(c+(1+ic)\tan(a+bx))\,dx$$

$$\Big\downarrow \text{5694}$$

$$\frac{1}{3}x^3\arctan(c+(1+ic)\tan(a+bx))-\frac{1}{3}ib\int \frac{x^3}{e^{2ia+2ibx}c+i}dx$$

$$\Big\downarrow \text{2615}$$

$$\frac{1}{3}x^3\arctan(c+(1+ic)\tan(a+bx))-\frac{1}{3}ib\left(ic\int \frac{e^{2ia+2ibx}x^3}{e^{2ia+2ibx}c+i}dx-\frac{ix^4}{4}\right)$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{3}x^3\arctan(c+(1+ic)\tan(a+bx))-\frac{1}{3}ib\left(ic\left(\frac{3i\int x^2\log\left(1-ice^{2ia+2ibx}\right)dx}{2bc}-\frac{ix^3\log\left(1-ice^{2ia+2ibx}\right)}{2bc}\right)-\frac{ix^4}{4}\right)$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{3}x^3\arctan(c+(1+ic)\tan(a+bx))-\frac{1}{3}ib\left(ic\left(\frac{3i\left(\frac{ix^2\operatorname{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{2b}-\frac{i\int x\operatorname{PolyLog}\left(2,ice^{2ia+2ibx}\right)dx}{b}\right)}{2bc}-\frac{ix^3\log\left(1-ice^{2ia+2ibx}\right)}{2bc}\right)-\frac{ix^4}{4}\right)$$

$$\Big\downarrow \text{7163}$$

$$\frac{1}{3}x^3\arctan(c+(1+ic)\tan(a+bx))-$$
$$\frac{1}{3}ib\left(ic\left(\frac{3i\left(\frac{ix^2\operatorname{PolyLog}(2,ice^{2ia+2ibx})}{2b}-\frac{i\left(\frac{i\int\operatorname{PolyLog}(3,ice^{2ia+2ibx})dx}{2b}-\frac{ix\operatorname{PolyLog}(3,ice^{2ia+2ibx})}{2b}\right)}{b}\right)}{2bc}-\frac{ix^3\log(1-ice^{2ia+2ibx})}{2bc}\right.\right.$$

$\downarrow$ 2720

$$\frac{1}{3}x^3\arctan(c+(1+ic)\tan(a+bx))-$$
$$\frac{1}{3}ib\left(ic\left(\frac{3i\left(\frac{ix^2\operatorname{PolyLog}(2,ice^{2ia+2ibx})}{2b}-\frac{i\left(\frac{\int e^{-2ia-2ibx}\operatorname{PolyLog}(3,ice^{2ia+2ibx})de^{2ia+2ibx}}{4b^2}-\frac{ix\operatorname{PolyLog}(3,ice^{2ia+2ibx})}{2b}\right)}{b}\right)}{2bc}-\frac{ix^3\log(1\right.\right.$$

$\downarrow$ 7143

$$\frac{1}{3}x^3\arctan(c+(1+ic)\tan(a+bx))-$$
$$\frac{1}{3}ib\left(ic\left(\frac{3i\left(\frac{ix^2\operatorname{PolyLog}(2,ice^{2ia+2ibx})}{2b}-\frac{i\left(\frac{\operatorname{PolyLog}(4,ice^{2ia+2ibx})}{4b^2}-\frac{ix\operatorname{PolyLog}(3,ice^{2ia+2ibx})}{2b}\right)}{b}\right)}{2bc}-\frac{ix^3\log(1-ice^{2ia+2ibx})}{2bc}\right)\right.$$

input
```
Int[x^2*ArcTan[c + (1 + I*c)*Tan[a + b*x]],x]
```

output
```
(x^3*ArcTan[c + (1 + I*c)*Tan[a + b*x]])/3 - (I/3)*b*((-1/4*I)*x^4 + I*c*(
((-1/2*I)*x^3*Log[1 - I*c*E^((2*I)*a + (2*I)*b*x)])/(b*c) + (((3*I)/2)*(((
I/2)*x^2*PolyLog[2, I*c*E^((2*I)*a + (2*I)*b*x)])/b - (I*(((-1/2*I)*x*Poly
Log[3, I*c*E^((2*I)*a + (2*I)*b*x)])/b + PolyLog[4, I*c*E^((2*I)*a + (2*I)
*b*x)]/(4*b^2)))/b))/(b*c)))
```

3.52. $\int x^2\arctan(c+(1+ic)\tan(a+bx))\,dx$

3.52.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5694
```
Int[ArcTan[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Tan[a + b*x]]/(f*(m +
1))), x] - Simp[I*(b/(f*(m + 1)))   Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(2
*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && E
qQ[(c + I*d)^2, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.52.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.44 (sec) , antiderivative size = 1487, normalized size of antiderivative = 9.66

method	result	size
risch	Expression too large to display	1487

input
```
int(x^2*arctan(c+(I*c+1)*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/3*I*x^3*ln(exp(I*(b*x+a)))-1/2*I/b^2*a^2*ln(1+I*exp(I*(b*x+a))*(-I*c)^(
1/2))*x-1/2*I/b^3*a^3*ln(1+I*exp(I*(b*x+a))*(-I*c)^(1/2))-1/2*I/b^2*a^2*ln
(1-I*exp(I*(b*x+a))*(-I*c)^(1/2))*x-1/6*I*x^3*ln(1-I*exp(2*I*(b*x+a))*c)-1
/4*I*x*polylog(3,I*exp(2*I*(b*x+a))*c)/b^2-1/4*x^2*polylog(2,I*exp(2*I*(b*
x+a))*c)/b+1/4/b^3*polylog(2,I*exp(2*I*(b*x+a))*c)*a^2+1/2*I/b^2*ln(1-I*ex
p(2*I*(b*x+a))*c)*x*a^2+1/8*polylog(4,I*exp(2*I*(b*x+a))*c)/b^3+1/6*I*x^3*
ln(exp(2*I*(b*x+a))*c+I)+1/12*I*(-2*I*Pi-I*Pi*csgn(I/(exp(2*I*(b*x+a))+1))
*csgn(I*(exp(2*I*(b*x+a))*c+I))*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*
x+a))+1))-2*ln(c-I)-I*Pi*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1
))*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))^2-I*Pi*csgn(I*(exp(2*
I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I
*(b*x+a))+1))+I*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(c-I))*csgn(I*(c-I)
/(exp(2*I*(b*x+a))+1))+I*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*(c-I)/(exp(2*I
*(b*x+a))+1))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))+I*Pi*csg
n(exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))^2+I*Pi*csgn(I*(c-I)/(exp(2*
I*(b*x+a))+1))^3+I*Pi*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))^
3+I*Pi*csgn(I*exp(2*I*(b*x+a)))^3-I*Pi*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(
2*I*(b*x+a))+1))^3-I*Pi*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))^
3+I*Pi*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))^2-I*Pi*csgn(exp(2
*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))^3+I*Pi*csgn(I*(exp(2*I*(b*x+a))...
```

3.52.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 322 vs. 2(107) = 214.

Time = 0.27 (sec) , antiderivative size = 322, normalized size of antiderivative = 2.09

$$\int x^2 \arctan(c+(1+ic)\tan(a+bx))\,dx = -\frac{b^4x^4 - 2i\,b^3x^3\log\left(-\frac{\left(ce^{(2i\,bx+2i\,a)}+i\right)e^{(-2i\,bx-2i\,a)}}{c-i}\right) + 6\,b^2x^2\text{Li}_2\left(\frac{1}{2}\sqrt{4i\,c}e^{(i\,bx+i\,a)}\right) + 6\,b^2x^2\text{Li}_2\left(-\frac{1}{2}\sqrt{4i\,c}e^{(i\,bx-}\right.}{}$$

input
```
integrate(x^2*arctan(c+(1+I*c)*tan(b*x+a)),x, algorithm="fricas")
```

output
```
-1/12*(b^4*x^4 - 2*I*b^3*x^3*log(-(c*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x
- 2*I*a)/(c - I)) + 6*b^2*x^2*dilog(1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) + 6*b
^2*x^2*dilog(-1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) - a^4 - 2*I*a^3*log(1/2*(2*
c*e^(I*b*x + I*a) + I*sqrt(4*I*c))/c) - 2*I*a^3*log(1/2*(2*c*e^(I*b*x + I*
a) - I*sqrt(4*I*c))/c) + 12*I*b*x*polylog(3, 1/2*sqrt(4*I*c)*e^(I*b*x + I*
a)) + 12*I*b*x*polylog(3, -1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) + 2*(I*b^3*x^3
 + I*a^3)*log(1/2*sqrt(4*I*c)*e^(I*b*x + I*a) + 1) + 2*(I*b^3*x^3 + I*a^3)
*log(-1/2*sqrt(4*I*c)*e^(I*b*x + I*a) + 1) - 12*polylog(4, 1/2*sqrt(4*I*c)
*e^(I*b*x + I*a)) - 12*polylog(4, -1/2*sqrt(4*I*c)*e^(I*b*x + I*a)))/b^3
```

3.52.6 Sympy [F(-2)]

Exception generated.

$$\int x^2 \arctan(c+(1+ic)\tan(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

input
```
integrate(x**2*atan(c+(1+I*c)*tan(b*x+a)),x)
```

output
```
Exception raised: CoercionFailed >> Cannot convert _t0**2 + exp(2*I*a) of
type <class 'sympy.core.add.Add'> to QQ_I[x,b,_t0,exp(I*a)]
```

3.52.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 310 vs. 2(107) = 214.

Time = 0.21 (sec) , antiderivative size = 310, normalized size of antiderivative = 2.01

$$\int x^2 \arctan(c + (1 + ic)\tan(a + bx))\,dx$$
$$= \frac{\frac{4\left((bx+a)^3-3\,(bx+a)^2a+3\,(bx+a)a^2\right)\arctan((i\,c+1)\tan(bx+a)+c)}{b^2} - \frac{\left(-3i\,(bx+a)^4+12i\,(bx+a)^3a-18i\,(bx+a)^2a^2-2\left(4i\,(bx+a)^3-9i\,(bx+a\right.\right.}{}}{}$$

input
```
integrate(x^2*arctan(c+(1+I*c)*tan(b*x+a)),x, algorithm="maxima")
```

output
```
1/12*(4*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arctan((I*c + 1)
*tan(b*x + a) + c)/b^2 - (-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b*
x + a)^2*a^2 - 2*(4*I*(b*x + a)^3 - 9*I*(b*x + a)^2*a + 9*I*(b*x + a)*a^2)
*arctan2(c*cos(2*b*x + 2*a), c*sin(2*b*x + 2*a) + 1) - 3*(4*I*(b*x + a)^2
- 6*I*(b*x + a)*a + 3*I*a^2)*dilog(I*c*e^(2*I*b*x + 2*I*a)) + (4*(b*x + a)
^3 - 9*(b*x + a)^2*a + 9*(b*x + a)*a^2)*log(c^2*cos(2*b*x + 2*a)^2 + c^2*s
in(2*b*x + 2*a)^2 + 2*c*sin(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polylog(3, I
*c*e^(2*I*b*x + 2*I*a)) + 6*I*polylog(4, I*c*e^(2*I*b*x + 2*I*a)))*(I*c +
1)/(b^2*(c - I)))/b
```

3.52.8 Giac [F]

$$\int x^2 \arctan(c + (1 + ic)\tan(a + bx))\,dx = \int x^2 \arctan\left((i\,c + 1)\tan\left(bx + a\right) + c\right)\,dx$$

input
```
integrate(x^2*arctan(c+(1+I*c)*tan(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*arctan((I*c + 1)*tan(b*x + a) + c), x)
```

3.52.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \arctan(c + (1 + ic)\tan(a + bx))\,dx = \int x^2\,\mathrm{atan}(c + \tan(a + b\,x)\,\,(1 + c\,1\mathrm{i}))\,dx$$

input
```
int(x^2*atan(c + tan(a + b*x)*(c*1i + 1)),x)
```

output
```
int(x^2*atan(c + tan(a + b*x)*(c*1i + 1)), x)
```

3.53 $\int x \arctan(c+(1+ic)\tan(a+bx))\,dx$

3.53.1 Optimal result

Integrand size = 19, antiderivative size = 123

$$\int x \arctan(c+(1+ic)\tan(a+bx))\,dx = -\frac{bx^3}{6}+\frac{1}{2}x^2\arctan(c+(1+ic)\tan(a+bx)) - \frac{1}{4}ix^2\log\left(1-ice^{2ia+2ibx}\right) - \frac{x\,\mathrm{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b} - \frac{i\,\mathrm{PolyLog}\left(3,ice^{2ia+2ibx}\right)}{8b^2}$$

output
```
-1/6*b*x^3+1/2*x^2*arctan(c+(1+I*c)*tan(b*x+a))-1/4*I*x^2*ln(1-I*c*exp(2*I
*a+2*I*b*x))-1/4*x*polylog(2,I*c*exp(2*I*a+2*I*b*x))/b-1/8*I*polylog(3,I*c
*exp(2*I*a+2*I*b*x))/b^2
```

3.53.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.89

$$\int x \arctan(c+(1+ic)\tan(a+bx))\,dx = \frac{1}{2}x^2\arctan(c+(1+ic)\tan(a+bx)) - \frac{i\left(2b^2x^2\log\left(1+\frac{ie^{-2i(a+bx)}}{c}\right)+2ibx\,\mathrm{PolyLog}\left(2,-\frac{ie^{-2i(a+bx)}}{c}\right)+\mathrm{PolyLog}\left(3,-\frac{ie^{-2i(a+bx)}}{c}\right)\right)}{8b^2}$$

input
```
Integrate[x*ArcTan[c + (1 + I*c)*Tan[a + b*x]],x]
```

output
```
(x^2*ArcTan[c + (1 + I*c)*Tan[a + b*x]])/2 - ((I/8)*(2*b^2*x^2*Log[1 + I/(
c*E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, (-I)/(c*E^((2*I)*(a + b*x))
)] + PolyLog[3, (-I)/(c*E^((2*I)*(a + b*x)))]))/b^2
```

3.53.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.26, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5694, 2615, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\arctan(c+(1+ic)\tan(a+bx))\,dx$$

$$\Big\downarrow \; 5694$$

$$\frac{1}{2}x^2\arctan(c+(1+ic)\tan(a+bx))-\frac{1}{2}ib\int \frac{x^2}{e^{2ia+2ibx}c+i}dx$$

$$\Big\downarrow \; 2615$$

$$\frac{1}{2}x^2\arctan(c+(1+ic)\tan(a+bx))-\frac{1}{2}ib\left(ic\int \frac{e^{2ia+2ibx}x^2}{e^{2ia+2ibx}c+i}dx-\frac{ix^3}{3}\right)$$

$$\Big\downarrow \; 2620$$

$$\frac{1}{2}x^2\arctan(c+(1+ic)\tan(a+bx))-\frac{1}{2}ib\left(ic\left(\frac{i\int x\log\left(1-ice^{2ia+2ibx}\right)dx}{bc}-\frac{ix^2\log\left(1-ice^{2ia+2ibx}\right)}{2bc}\right)-\frac{ix^3}{3}\right)$$

$$\Big\downarrow \; 3011$$

$$\frac{1}{2}x^2\arctan(c+(1+ic)\tan(a+bx))-\frac{1}{2}ib\left(ic\left(\frac{i\left(\frac{ix\,\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{2b}-\frac{i\int \text{PolyLog}\left(2,ice^{2ia+2ibx}\right)dx}{2b}\right)}{bc}-\frac{ix^2\log\left(1-ice^{2ia+2ibx}\right)}{2bc}\right)-\frac{ix^3}{3}\right)$$

$$\Big\downarrow \; 2720$$

$$\frac{1}{2}x^2\arctan(c+(1+ic)\tan(a+bx)) - \frac{1}{2}ib\left(ic\left(\frac{i\left(\frac{ix\operatorname{PolyLog}(2,ice^{2ia+2ibx})}{2b}-\frac{\int e^{-2ia-2ibx}\operatorname{PolyLog}(2,ice^{2ia+2ibx})de^{2ia+2ibx}}{4b^2}\right)}{bc}-\frac{ix^2\log\left(1-ice^{2ia+2ibx}\right)}{2bc}\right)-\frac{ix^3}{3}\right)$$

$$\Big\downarrow \text{ 7143}$$

$$\frac{1}{2}x^2\arctan(c+(1+ic)\tan(a+bx)) - \frac{1}{2}ib\left(ic\left(\frac{i\left(\frac{ix\operatorname{PolyLog}(2,ice^{2ia+2ibx})}{2b}-\frac{\operatorname{PolyLog}(3,ice^{2ia+2ibx})}{4b^2}\right)}{bc}-\frac{ix^2\log\left(1-ice^{2ia+2ibx}\right)}{2bc}\right)-\frac{ix^3}{3}\right)$$

input
```
Int[x*ArcTan[c + (1 + I*c)*Tan[a + b*x]],x]
```

output
```
(x^2*ArcTan[c + (1 + I*c)*Tan[a + b*x]])/2 - (I/2)*b*((-1/3*I)*x^3 + I*c*(
((-1/2*I)*x^2*Log[1 - I*c*E^((2*I)*a + (2*I)*b*x)])/(b*c) + (I*(((I/2)*x*P
olyLog[2, I*c*E^((2*I)*a + (2*I)*b*x)])/b - PolyLog[3, I*c*E^((2*I)*a + (2
*I)*b*x)]/(4*b^2)))/(b*c)))
```

3.53.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5694
```
Int[ArcTan[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Tan[a + b*x]]/(f*(m +
1))), x] - Simp[I*(b/(f*(m + 1)))   Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(2
*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && E
qQ[(c + I*d)^2, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.53.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.07 (sec) , antiderivative size = 1452, normalized size of antiderivative = 11.80

method	result	size
risch	Expression too large to display	1452

input
```
int(x*arctan(c+(I*c+1)*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/4*I*x^2*ln(exp(2*I*(b*x+a))*c+I)-1/6*b*x^3-1/4*I*ln(1-I*exp(2*I*(b*x+a))
*c)*x^2+1/8*I*(-2*I*Pi-I*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(
b*x+a))*c+I))*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))-2*ln(c-I
)-I*Pi*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))*csgn(exp(2*I*(b
*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))^2-I*Pi*csgn(I*(exp(2*I*(b*x+a))*c+I)/(e
xp(2*I*(b*x+a))+1))*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))+I*Pi
*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(c-I))*csgn(I*(c-I)/(exp(2*I*(b*x+a))
+1))+I*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))*csgn
(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))+I*Pi*csgn(exp(2*I*(b*x+a))
*(c-I)/(exp(2*I*(b*x+a))+1))^2+I*Pi*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))^3+I
*Pi*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))^3+I*Pi*csgn(I*exp(
2*I*(b*x+a)))^3-I*Pi*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))^3
-I*Pi*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))^3+I*Pi*csgn((exp(2
*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))^2-I*Pi*csgn(exp(2*I*(b*x+a))*(c-I)/
(exp(2*I*(b*x+a))+1))^3+I*Pi*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a
))+1))*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))+1))^2-I*Pi*csgn(I*(c-
I)/(exp(2*I*(b*x+a))+1))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1
))^2-I*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(c-I)/(exp(2*I*(b*x+a))+1))^
2+I*Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+a)))-I*Pi*csgn(I*exp(2
*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))+1))^2-I*Pi...
```

3.53.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 271 vs. 2(85) = 170.

Time = 0.28 (sec) , antiderivative size = 271, normalized size of antiderivative = 2.20

$$\int x \arctan(c + (1 + ic) \tan(a + bx))\, dx = -\frac{2\, b^3 x^3 - 3i\, b^2 x^2 \log\left(-\frac{\left(ce^{(2i\, bx+2i\, a)}+i\right)e^{(-2i\, bx-2i\, a)}}{c-i}\right) + 2\, a^3 + 6\, bx\mathrm{Li}_2\left(\frac{1}{2}\sqrt{4i\, c}e^{(i\, bx+i\, a)}\right) + 6\, bx\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{4i\, c}\right.}{}$$

input
```
integrate(x*arctan(c+(1+I*c)*tan(b*x+a)),x, algorithm="fricas")
```

output
```
-1/12*(2*b^3*x^3 - 3*I*b^2*x^2*log(-(c*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*
x - 2*I*a)/(c - I)) + 2*a^3 + 6*b*x*dilog(1/2*sqrt(4*I*c)*e^(I*b*x + I*a))
 + 6*b*x*dilog(-1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) + 3*I*a^2*log(1/2*(2*c*e^
(I*b*x + I*a) + I*sqrt(4*I*c))/c) + 3*I*a^2*log(1/2*(2*c*e^(I*b*x + I*a) -
 I*sqrt(4*I*c))/c) + 3*(I*b^2*x^2 - I*a^2)*log(1/2*sqrt(4*I*c)*e^(I*b*x +
I*a) + 1) + 3*(I*b^2*x^2 - I*a^2)*log(-1/2*sqrt(4*I*c)*e^(I*b*x + I*a) + 1
) + 6*I*polylog(3, 1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) + 6*I*polylog(3, -1/2*
sqrt(4*I*c)*e^(I*b*x + I*a)))/b^2
```

3.53.6 Sympy [F(-2)]

Exception generated.

$$\int x \arctan(c + (1 + ic)\tan(a + bx))\,dx = \text{Exception raised: CoercionFailed}$$

input
```
integrate(x*atan(c+(1+I*c)*tan(b*x+a)),x)
```

output
```
Exception raised: CoercionFailed >> Cannot convert _t0**2 + exp(2*I*a) of
type <class 'sympy.core.add.Add'> to QQ_I[x,b,_t0,exp(I*a)]
```

3.53.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(85) = 170$.

Time = 0.20 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.77

$$\int x \arctan(c + (1 + ic)\tan(a + bx))\,dx$$
$$= \frac{\frac{6\left((bx+a)^2-2\,(bx+a)a\right)\arctan((i\,c+1)\tan(bx+a)+c)}{b} - \frac{\left(-4i\,(bx+a)^3+12i\,(bx+a)^2a-6i\,bx\mathrm{Li}_2\left(i\,ce^{(2i\,bx+2i\,a)}\right)-6\left(i\,(bx+a)^2-2i\,(bx+a)a\right)\,\mathrm{a}}{}}{}$$

input
```
integrate(x*arctan(c+(1+I*c)*tan(b*x+a)),x, algorithm="maxima")
```

output
```
1/12*(6*((b*x + a)^2 - 2*(b*x + a)*a)*arctan((I*c + 1)*tan(b*x + a) + c)/b
 - (-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*dilog(I*c*e^(2*I*b*x +
 2*I*a)) - 6*(I*(b*x + a)^2 - 2*I*(b*x + a)*a)*arctan2(c*cos(2*b*x + 2*a),
 c*sin(2*b*x + 2*a) + 1) + 3*((b*x + a)^2 - 2*(b*x + a)*a)*log(c^2*cos(2*b
*x + 2*a)^2 + c^2*sin(2*b*x + 2*a)^2 + 2*c*sin(2*b*x + 2*a) + 1) + 3*polyl
og(3, I*c*e^(2*I*b*x + 2*I*a)))*(I*c + 1)/(b*(c - I)))/b
```

3.53.8 Giac [F]

$$\int x \arctan(c+(1+ic)\tan(a+bx))\,dx=\int x \arctan\left((i\,c+1)\tan\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(x*arctan(c+(1+I*c)*tan(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*arctan((I*c + 1)*tan(b*x + a) + c), x)
```

3.53.9 Mupad [F(-1)]

Timed out.

$$\int x \arctan(c+(1+ic)\tan(a+bx))\,dx=\int x\,\mathrm{atan}(c+\tan(a+b\,x)\,\left(1+c\,1\mathrm{i}\right))\,dx$$

input
```
int(x*atan(c + tan(a + b*x)*(c*1i + 1)),x)
```

output
```
int(x*atan(c + tan(a + b*x)*(c*1i + 1)), x)
```

3.54 $\int \arctan(c+(1+ic)\tan(a+bx))\,dx$

3.54.1 Optimal result

Integrand size = 17, antiderivative size = 85

$$\int \arctan(c+(1+ic)\tan(a+bx))\,dx = -\frac{bx^2}{2}+x\arctan(c+(1+ic)\tan(a+bx)) -\frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right)-\frac{\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}$$

output
```
-1/2*b*x^2+x*arctan(c+(1+I*c)*tan(b*x+a))-1/2*I*x*ln(1-I*c*exp(2*I*a+2*I*b
*x))-1/4*polylog(2,I*c*exp(2*I*a+2*I*b*x))/b
```

3.54.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 967 vs. 2(85) = 170.

Time = 11.10 (sec) , antiderivative size = 967, normalized size of antiderivative = 11.38

$$\int \arctan(c+(1+ic)\tan(a+bx))\,dx = x\arctan(c+(1+ic)\tan(a+bx)) + \frac{}{((i+c)\cos(a+bx)+(1+ic)\sin(a+bx))\left(2bx-i\log\left(1-\frac{\sec(bx)((-i+c)\cos(a)+i(i+c)\sin(a))(\cos(a+bx)-i\sin(a}{2c}\right.\right.}$$

input
```
Integrate[ArcTan[c + (1 + I*c)*Tan[a + b*x]],x]
```

output
```
x*ArcTan[c + (1 + I*c)*Tan[a + b*x]] + (I*x*((2*I)*b*x*Log[2*Cos[b*x]*(Cos
[b*x] - I*Sin[b*x])] - Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((I + c)*Cos[a +
b*x] + (1 + I*c)*Sin[a + b*x]))/(2*c)]*Log[1 - I*Tan[b*x]] + Log[(Sec[b*x]
*((1 - I*c)*Cos[a + b*x] + (-I + c)*Sin[a + b*x]))/(2*Cos[a] - (2*I)*Sin[a
])]*Log[1 + I*Tan[b*x]] - PolyLog[2, -Cos[2*b*x] + I*Sin[2*b*x]] - PolyLog
[2, (Sec[b*x]*((-I + c)*Cos[a] + I*(I + c)*Sin[a])*(Cos[a + b*x] - I*Sin[a
 + b*x]))/(2*c)] + PolyLog[2, (Sec[b*x]*((1 + I*c)*Cos[a] - (I + c)*Sin[a]
)*(Cos[a + b*x] + I*Sin[a + b*x]))/2])*Sec[a + b*x]^2*(Cos[b*x] + I*Sin[b*
x])*(I*Cos[b*x] + Sin[b*x])*((1 - I*c)*Cos[a + b*x] + (-I + c)*Sin[a + b*x
]))/(((I + c)*Cos[a + b*x] + (1 + I*c)*Sin[a + b*x])*(2*b*x - I*Log[1 - (S
ec[b*x]*((-I + c)*Cos[a] + I*(I + c)*Sin[a])*(Cos[a + b*x] - I*Sin[a + b*x
]))/(2*c)] - I*Log[1 + (Sec[b*x]*((-1 - I*c)*Cos[a] + (I + c)*Sin[a])*(Cos
[a + b*x] + I*Sin[a + b*x]))/2] - (I*(-I + c)*Cos[a + b*x]*(Log[1 - I*Tan[
b*x]] - Log[1 + I*Tan[b*x]]))/((I + c)*Cos[a + b*x] + (1 + I*c)*Sin[a + b*
x]) + ((I + c)*(Log[1 - I*Tan[b*x]] - Log[1 + I*Tan[b*x]])*Sin[a + b*x])/(
(I + c)*Cos[a + b*x] + (1 + I*c)*Sin[a + b*x]) - (2*I)*b*x*Tan[b*x] + Log[
1 - (Sec[b*x]*((-I + c)*Cos[a] + I*(I + c)*Sin[a])*(Cos[a + b*x] - I*Sin[a
 + b*x]))/(2*c)]*Tan[b*x] - Log[1 + (Sec[b*x]*((-1 - I*c)*Cos[a] + (I + c)
*Sin[a])*(Cos[a + b*x] + I*Sin[a + b*x]))/2]*Tan[b*x] - Log[1 - I*Tan[b*x]
]*Tan[b*x] + Cos[a]^2*Log[1 + I*Tan[b*x]]*Tan[b*x] + Log[1 + I*Tan[b*x]...
```

3.54.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.26, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {5686, 2615, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(c+(1+ic)\tan(a+bx))\,dx$$

$$\Big\downarrow \text{5686}$$

$$x\arctan(c+(1+ic)\tan(a+bx))-ib\int \frac{x}{e^{2ia+2ibx}c+i}dx$$

$$\Big\downarrow \text{2615}$$

$$x\arctan(c+(1+ic)\tan(a+bx))-ib\left(ic\int \frac{e^{2ia+2ibx}x}{e^{2ia+2ibx}c+i}dx-\frac{ix^2}{2}\right)$$

$$\Big\downarrow \text{2620}$$

$$x\arctan(c+(1+ic)\tan(a+bx))-ib\left(ic\left(\frac{i\int\log\left(1-ice^{2ia+2ibx}\right)dx}{2bc}-\frac{ix\log\left(1-ice^{2ia+2ibx}\right)}{2bc}\right)-\frac{ix^2}{2}\right)$$

$$\Big\downarrow \text{2715}$$

$$x\arctan(c+(1+ic)\tan(a+bx))-ib\left(ic\left(\frac{\int e^{-2ia-2ibx}\log\left(1-ice^{2ia+2ibx}\right)de^{2ia+2ibx}}{4b^2c}-\frac{ix\log\left(1-ice^{2ia+2ibx}\right)}{2bc}\right)-\frac{ix^2}{2}\right)$$

$$\Big\downarrow \text{2838}$$

$$x\arctan(c+(1+ic)\tan(a+bx))-ib\left(ic\left(-\frac{\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b^2c}-\frac{ix\log\left(1-ice^{2ia+2ibx}\right)}{2bc}\right)-\frac{ix^2}{2}\right)$$

input
```
Int[ArcTan[c + (1 + I*c)*Tan[a + b*x]],x]
```

output
```
x*ArcTan[c + (1 + I*c)*Tan[a + b*x]] - I*b*((-1/2*I)*x^2 + I*c*(((-1/2*I)*
x*Log[1 - I*c*E^((2*I)*a + (2*I)*b*x)])/(b*c) - PolyLog[2, I*c*E^((2*I)*a
+ (2*I)*b*x)]/(4*b^2*c)))
```

3.54.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5686
```
Int[ArcTan[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcT
an[c + d*Tan[a + b*x]], x] - Simp[I*b   Int[x/(c + I*d + c*E^(2*I*a + 2*I*b
*x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c + I*d)^2, -1]
```

3.54.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 562 vs. $2(69) = 138$.

Time = 1.11 (sec) , antiderivative size = 563, normalized size of antiderivative = 6.62

method	result
derivativedivides	$\frac{\arctan(c+(ic+1)\tan(bx+a))\ln(-c+(ic+1)\tan(bx+a)+i)c^2}{2i-2c} - \frac{2i\arctan(c+(ic+1)\tan(bx+a))\ln(-c+(ic+1)\tan(bx+a)+i)c}{2i-2c} - \arctan(c \ldots$
default	$\frac{\arctan(c+(ic+1)\tan(bx+a))\ln(-c+(ic+1)\tan(bx+a)+i)c^2}{2i-2c} - \frac{2i\arctan(c+(ic+1)\tan(bx+a))\ln(-c+(ic+1)\tan(bx+a)+i)c}{2i-2c} - \arctan(c \ldots$
risch	Expression too large to display

input
```
int(arctan(c+(I*c+1)*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b/(I*c+1)*(arctan(c+(I*c+1)*tan(b*x+a))/(2*I-2*c)*ln(-c+(I*c+1)*tan(b*x+
a)+I)*c^2-2*I*arctan(c+(I*c+1)*tan(b*x+a))/(2*I-2*c)*ln(-c+(I*c+1)*tan(b*x
+a)+I)*c-arctan(c+(I*c+1)*tan(b*x+a))/(2*I-2*c)*ln(-c+(I*c+1)*tan(b*x+a)+I
)-arctan(c+(I*c+1)*tan(b*x+a))/(2*I-2*c)*ln(-I+c+(I*c+1)*tan(b*x+a))*c^2+2
*I*arctan(c+(I*c+1)*tan(b*x+a))/(2*I-2*c)*ln(-I+c+(I*c+1)*tan(b*x+a))*c+ar
ctan(c+(I*c+1)*tan(b*x+a))/(2*I-2*c)*ln(-I+c+(I*c+1)*tan(b*x+a))-(I*c+1)^2
*(1/2/(I-c)*(-1/4*I*ln(-I+c+(I*c+1)*tan(b*x+a))^2+1/2*I*(dilog(-1/2*I*(c+(
I*c+1)*tan(b*x+a)+I))+ln(-I+c+(I*c+1)*tan(b*x+a))*ln(-1/2*I*(c+(I*c+1)*tan
(b*x+a)+I))))-1/2/(I-c)*(1/2*I*(dilog(1/2*(c+(I*c+1)*tan(b*x+a)+I)/c)+ln(-
c+(I*c+1)*tan(b*x+a)+I)*ln(1/2*(c+(I*c+1)*tan(b*x+a)+I)/c))-1/2*I*(dilog((
-I+c+(I*c+1)*tan(b*x+a))/(-2*I+2*c))+ln(-c+(I*c+1)*tan(b*x+a)+I)*ln((-I+c+
(I*c+1)*tan(b*x+a))/(-2*I+2*c))))))
```

3.54.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(60) = 120$.

Time = 0.27 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.38

$$\int \arctan(c + (1 + ic)\tan(a + bx))\,dx =$$

$$-\frac{b^2x^2 - i\,bx\log\left(-\frac{(ce^{(2i\,bx+2i\,a)}+i)e^{(-2i\,bx-2i\,a)}}{c-i}\right) - a^2 - (-i\,bx - i\,a)\log\left(\frac{1}{2}\sqrt{4i\,c}e^{(i\,bx+i\,a)} + 1\right) - (-i\,bx - i}{}$$

input
```
integrate(arctan(c+(1+I*c)*tan(b*x+a)),x, algorithm="fricas")
```

output
```
-1/2*(b^2*x^2 - I*b*x*log(-(c*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x - 2*I*a
)/(c - I)) - a^2 - (-I*b*x - I*a)*log(1/2*sqrt(4*I*c)*e^(I*b*x + I*a) + 1)
 - (-I*b*x - I*a)*log(-1/2*sqrt(4*I*c)*e^(I*b*x + I*a) + 1) - I*a*log(1/2*
(2*c*e^(I*b*x + I*a) + I*sqrt(4*I*c))/c) - I*a*log(1/2*(2*c*e^(I*b*x + I*a
) - I*sqrt(4*I*c))/c) + dilog(1/2*sqrt(4*I*c)*e^(I*b*x + I*a)) + dilog(-1/
2*sqrt(4*I*c)*e^(I*b*x + I*a)))/b
```

3.54.6 Sympy [F(-2)]

Exception generated.

$$\int \arctan(c + (1 + ic)\tan(a + bx))\,dx = \text{Exception raised: CoercionFailed}$$

input
```
integrate(atan(c+(1+I*c)*tan(b*x+a)),x)
```

output
```
Exception raised: CoercionFailed >> Cannot convert _t0**2 + exp(2*I*a) of
type <class 'sympy.core.add.Add'> to QQ_I[b,_t0,exp(I*a)]
```

3.54.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 448 vs. $2(60) = 120$.

Time = 0.28 (sec) , antiderivative size = 448, normalized size of antiderivative = 5.27

$$\int \arctan(c + (1 + ic)\tan(a + bx))\, dx =$$

$$-\frac{(i\,c+1)\left(\frac{4i\,(bx+a)\log\left(-\frac{2\left(-i\,c^2+\left(c^2-2i\,c-1\right)\tan(bx+a)-2\,c+i\right)}{2i\,c^2-2\left(c^2-2i\,c-1\right)\tan(bx+a)+2i}\right)}{i\,c+1} - \frac{i\left(4\,(bx+a)(\log(-i\,c^2+(c^2-2i\,c-1)\tan(bx+a)-2\,c+i)-\log(-i\,c\right.}{}\right.}{}$$

input
```
integrate(arctan(c+(1+I*c)*tan(b*x+a)),x, algorithm="maxima")
```

output
```
-1/8*((I*c + 1)*(4*I*(b*x + a)*log(-2*(-I*c^2 + (c^2 - 2*I*c - 1)*tan(b*x
+ a) - 2*c + I)/(2*I*c^2 - 2*(c^2 - 2*I*c - 1)*tan(b*x + a) + 2*I))/(I*c +
 1) - I*(4*(b*x + a)*(log(-I*c^2 + (c^2 - 2*I*c - 1)*tan(b*x + a) - 2*c +
I) - log(-I*c^2 + (c^2 - 2*I*c - 1)*tan(b*x + a) - I)) + I*log(-I*c^2 + (c
^2 - 2*I*c - 1)*tan(b*x + a) - 2*c + I)^2 - 2*I*log(-I*c^2 + (c^2 - 2*I*c
- 1)*tan(b*x + a) - I)*log(-1/2*(c - I)*tan(b*x + a) + 1/2*I*c + 1/2) + 2*
I*log(-I*c^2 + (c^2 - 2*I*c - 1)*tan(b*x + a) - I)*log(-1/2*((I*c + 1)*tan
(b*x + a) + c + I)/c + 1) - 2*I*log(-I*c^2 + (c^2 - 2*I*c - 1)*tan(b*x + a
) - 2*c + I)*log(-1/2*I*tan(b*x + a) + 1/2) - 2*I*dilog(1/2*(c - I)*tan(b*
x + a) - 1/2*I*c + 1/2) + 2*I*dilog(1/2*((I*c + 1)*tan(b*x + a) + c + I)/c
) - 2*I*dilog(1/2*I*tan(b*x + a) + 1/2))/(I*c + 1)) - 8*(b*x + a)*arctan((
I*c + 1)*tan(b*x + a) + c) + 4*(-I*b*x - I*a)*log(-2*(-I*c^2 + (c^2 - 2*I*
c - 1)*tan(b*x + a) - 2*c + I)/(2*I*c^2 - 2*(c^2 - 2*I*c - 1)*tan(b*x + a)
 + 2*I)))/b
```

3.54.8 Giac [F]

$$\int \arctan(c + (1 + ic)\tan(a + bx))\, dx = \int \arctan\left((i\,c+1)\tan\left(bx+a\right)+c\right)\, dx$$

input
```
integrate(arctan(c+(1+I*c)*tan(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arctan((I*c + 1)*tan(b*x + a) + c), x)
```

3.54.9 Mupad [F(-1)]

Timed out.

$$\int \arctan(c+(1+ic)\tan(a+bx))\,dx=\int \operatorname{atan}(c+\tan(a+b\,x)\,\left(1+c\,1\mathrm{i}\right))\,dx$$

input
```
int(atan(c + tan(a + b*x)*(c*1i + 1)),x)
```

output
```
int(atan(c + tan(a + b*x)*(c*1i + 1)), x)
```

3.55 $\int \frac{\arctan(c+(1+ic)\tan(a+bx))}{x}\,dx$

3.55.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{\arctan(c+(1+ic)\tan(a+bx))}{x}\,dx = \text{Int}\left(\frac{\arctan(c+(1+ic)\tan(a+bx))}{x},x\right)$$

output
```
CannotIntegrate(arctan(c+(1+I*c)*tan(b*x+a))/x,x)
```

3.55.2 Mathematica [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{\arctan(c+(1+ic)\tan(a+bx))}{x}\,dx = \int \frac{\arctan(c+(1+ic)\tan(a+bx))}{x}\,dx$$

input
```
Integrate[ArcTan[c + (1 + I*c)*Tan[a + b*x]]/x,x]
```

output
```
Integrate[ArcTan[c + (1 + I*c)*Tan[a + b*x]]/x, x]
```

3.55.3 Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(c+(1+ic)\tan(a+bx))}{x} \, dx$$
$$\Big\downarrow \; 7299$$
$$\int \frac{\arctan(c+(1+ic)\tan(a+bx))}{x}dx$$

input
```
Int[ArcTan[c + (1 + I*c)*Tan[a + b*x]]/x,x]
```

output
```
$Aborted
```

3.55.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.55.4 Maple [N/A] (verified)

Not integrable

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{\arctan\left(c+\left(ic+1\right)\tan\left(bx+a\right)\right)}{x}dx$$

input
```
int(arctan(c+(I*c+1)*tan(b*x+a))/x,x)
```

output
```
int(arctan(c+(I*c+1)*tan(b*x+a))/x,x)
```

3.55.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.76

$$\int \frac{\arctan(c+(1+ic)\tan(a+bx))}{x}\,dx = \int \frac{\arctan\left((i\,c+1)\tan\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arctan(c+(1+I*c)*tan(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(1/2*I*log(-(c*e^(2*I*b*x + 2*I*a) + I)*e^(-2*I*b*x - 2*I*a)/(c -
I))/x, x)
```

3.55.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan(c+(1+ic)\tan(a+bx))}{x}\,dx = \text{Timed out}$$

input
```
integrate(atan(c+(1+I*c)*tan(b*x+a))/x,x)
```

output
```
Timed out
```

3.55.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(c+(1+ic)\tan(a+bx))}{x}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(arctan(c+(1+I*c)*tan(b*x+a))/x,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c-1>0)', see `assume?` for more
details)Is
```

3.55.8 Giac [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(c+(1+ic)\tan(a+bx))}{x}\,dx = \int \frac{\arctan\left((i\,c+1)\tan\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arctan(c+(1+I*c)*tan(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(arctan((I*c + 1)*tan(b*x + a) + c)/x, x)
```

3.55.9 Mupad [N/A]

Not integrable

Time = 0.87 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{\arctan(c+(1+ic)\tan(a+bx))}{x}\,dx = \int \frac{\mathrm{atan}\left(c+\tan\left(a+b\,x\right)\,\left(1+c\,1{}\mathrm{i}\right)\right)}{x}\,dx$$

input
```
int(atan(c + tan(a + b*x)*(c*1i + 1))/x,x)
```

output
```
int(atan(c + tan(a + b*x)*(c*1i + 1))/x, x)
```

3.56 $\int x^2 \arctan(c+(-1+ic)\tan(a+bx))\,dx$

3.56.1 Optimal result

Integrand size = 21, antiderivative size = 155

$$\begin{aligned}\int x^2 \arctan(c+(-1+ic)\tan(a+bx))\,dx &= \frac{bx^4}{12}+\frac{1}{3}x^3\arctan(c-(1-ic)\tan(a+bx))\\ &+\frac{1}{6}ix^3\log\left(1+ice^{2ia+2ibx}\right)\\ &+\frac{x^2\,\mathrm{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}\\ &+\frac{ix\,\mathrm{PolyLog}\left(3,-ice^{2ia+2ibx}\right)}{4b^2}\\ &-\frac{\mathrm{PolyLog}\left(4,-ice^{2ia+2ibx}\right)}{8b^3}\end{aligned}$$

output
```
1/12*b*x^4+1/3*x^3*arctan(c-(1-I*c)*tan(b*x+a))+1/6*I*x^3*ln(1+I*c*exp(2*I
*a+2*I*b*x))+1/4*x^2*polylog(2,-I*c*exp(2*I*a+2*I*b*x))/b+1/4*I*x*polylog(
3,-I*c*exp(2*I*a+2*I*b*x))/b^2-1/8*polylog(4,-I*c*exp(2*I*a+2*I*b*x))/b^3
```

3.56.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.88

$$\int x^2 \arctan(c+(-1+ic)\tan(a+bx))\,dx = \frac{1}{24}\left(8x^3\arctan(c+i(i+c)\tan(a+bx)) + 4ix^3\log\left(1-\frac{ie^{-2i(a+bx)}}{c}\right) - \frac{6x^2\operatorname{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{c}\right)}{b} + \frac{6ix\operatorname{PolyLog}\left(3,\frac{ie^{-2i(a+bx)}}{c}\right)}{b^2} + \frac{3\operatorname{PolyLog}\left(4,\frac{ie^{-2i(a+bx)}}{c}\right)}{b^3}\right)$$

input
```
Integrate[x^2*ArcTan[c + (-1 + I*c)*Tan[a + b*x]],x]
```

output
```
(8*x^3*ArcTan[c + I*(I + c)*Tan[a + b*x]] + (4*I)*x^3*Log[1 - I/(c*E^((2*I
)*(a + b*x)))] - (6*x^2*PolyLog[2, I/(c*E^((2*I)*(a + b*x)))])/b + ((6*I)*
x*PolyLog[3, I/(c*E^((2*I)*(a + b*x)))])/b^2 + (3*PolyLog[4, I/(c*E^((2*I)
*(a + b*x)))])/b^3)/24
```

3.56.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.28, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {5694, 25, 2615, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(c+(-1+ic)\tan(a+bx))\,dx$$

$$\Big\downarrow \text{ 5694}$$

$$\frac{1}{3}x^3\arctan(c-(1-ic)\tan(a+bx)) - \frac{1}{3}ib\int -\frac{x^3}{i-ce^{2ia+2ibx}}dx$$

$\downarrow$ 25

$$\frac{1}{3}ib\int \frac{x^3}{i-ce^{2ia+2ibx}}dx+\frac{1}{3}x^3\arctan(c-(1-ic)\tan(a+bx))$$

$\downarrow$ 2615

$$\frac{1}{3}ib\left(-ic\int \frac{e^{2ia+2ibx}x^3}{i-ce^{2ia+2ibx}}dx-\frac{ix^4}{4}\right)+\frac{1}{3}x^3\arctan(c-(1-ic)\tan(a+bx))$$

$\downarrow$ 2620

$$\frac{1}{3}ib\left(-ic\left(\frac{ix^3\log\left(1+ice^{2ia+2ibx}\right)}{2bc}-\frac{3i\int x^2\log\left(ie^{2ia+2ibx}c+1\right)dx}{2bc}\right)-\frac{ix^4}{4}\right)+\frac{1}{3}x^3\arctan(c-(1-ic)\tan(a+bx))$$

$\downarrow$ 3011

$$\frac{1}{3}ib\left(-ic\left(\frac{ix^3\log\left(1+ice^{2ia+2ibx}\right)}{2bc}-\frac{3i\left(\frac{ix^2\operatorname{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{2b}-\frac{i\int x\operatorname{PolyLog}\left(2,-ice^{2ia+2ibx}\right)dx}{b}\right)}{2bc}\right)-\frac{ix^4}{4}\right)+\frac{1}{3}x^3\arctan(c-(1-ic)\tan(a+bx))$$

$\downarrow$ 7163

$$\frac{1}{3}ib\left(-ic\left(\frac{ix^3\log\left(1+ice^{2ia+2ibx}\right)}{2bc}-\frac{3i\left(\frac{ix^2\operatorname{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{2b}-\frac{i\left(\frac{i\int \operatorname{PolyLog}\left(3,-ice^{2ia+2ibx}\right)dx}{2b}-\frac{ix\operatorname{PolyLog}\left(3,-ice^{2ia+2ib\right.}}{2b}\right.}{b}\right.}{2bc}\right.\right.$$

$$\frac{1}{3}x^3\arctan(c-(1-ic)\tan(a+bx))$$

$\downarrow$ 2720

$$\frac{1}{3}ib\left(-ic\left(\frac{ix^3\log\left(1+ice^{2ia+2ibx}\right)}{2bc}-\frac{3i\left(\frac{ix^2\operatorname{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{2b}-\frac{i\left(\frac{\int e^{-2ia-2ibx}\operatorname{PolyLog}\left(3,-ice^{2ia+2ibx}\right)de^{2ia+2ibx}}{4b^2}-\frac{ix\operatorname{Pol}}{}\right.}{b}\right.}{2bc}\right.\right.$$

$$\frac{1}{3}x^3\arctan(c-(1-ic)\tan(a+bx))$$

3.56. $\int x^2\arctan(c+(-1+ic)\tan(a+bx))\,dx$

$$\downarrow \text{7143}$$

$$\frac{1}{3}x^3\arctan(c-(1-ic)\tan(a+bx))+\frac{1}{3}ib\left(-ic\left(\frac{ix^3\log\left(1+ice^{2ia+2ibx}\right)}{2bc}-\frac{3i\left(\frac{ix^2\operatorname{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{2b}-\frac{i\left(\frac{\operatorname{PolyLog}\left(4,-ice^{2ia+2ibx}\right)}{4b^2}-\frac{ix\operatorname{PolyLog}\left(3,-ice^{2ia+2ibx}\right)}{2b}\right)}{b}\right)}{2bc}\right.\right.$$

input
```
Int[x^2*ArcTan[c + (-1 + I*c)*Tan[a + b*x]],x]
```

output
```
(x^3*ArcTan[c - (1 - I*c)*Tan[a + b*x]])/3 + (I/3)*b*((-1/4*I)*x^4 - I*c*(
((I/2)*x^3*Log[1 + I*c*E^((2*I)*a + (2*I)*b*x)])/(b*c) - (((3*I)/2)*(((I/2
)*x^2*PolyLog[2, (-I)*c*E^((2*I)*a + (2*I)*b*x)])/b - (I*(((-1/2*I)*x*Poly
Log[3, (-I)*c*E^((2*I)*a + (2*I)*b*x)])/b + PolyLog[4, (-I)*c*E^((2*I)*a +
 (2*I)*b*x)]/(4*b^2)))/b))/(b*c)))
```

3.56.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5694
```
Int[ArcTan[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Tan[a + b*x]]/(f*(m +
1))), x] - Simp[I*(b/(f*(m + 1)))   Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(2
*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && E
qQ[(c + I*d)^2, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.56.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.48 (sec) , antiderivative size = 1488, normalized size of antiderivative = 9.60

method	result	size
risch	Expression too large to display	1488

input
```
int(x^2*arctan(c+(-1+I*c)*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/12*I*(-I*Pi*csgn(I*exp(2*I*(b*x+a)))^3-2*I*Pi+I*Pi*csgn(I/(exp(2*I*(b*x+
a))+1)*(I+c))*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))^2+2*ln(I
+c)-I*Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+a)))+I*Pi*csgn(I*(ex
p(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))^3-I*Pi*csgn(I*exp(2*I*(b*x+a))*(
I+c)/(exp(2*I*(b*x+a))+1))^3+I*Pi*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(
b*x+a))+1))*csgn((exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))-I*Pi*csgn(ex
p(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))^3-I*Pi*csgn(I/(exp(2*I*(b*x+a))
+1)*(I+c))^3-I*Pi*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))*csgn
(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))+I*Pi*csgn((exp(2*I*(b*x+a))*
c-I)/(exp(2*I*(b*x+a))+1))^2-I*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(I+c
))*csgn(I/(exp(2*I*(b*x+a))+1)*(I+c))-I*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I
/(exp(2*I*(b*x+a))+1)*(I+c))*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a
))+1))+I*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))*c-I))*cs
gn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))-I*Pi*csgn(I/(exp(2*I*(b*
x+a))+1))*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))^2-I*Pi*csgn(
I*(exp(2*I*(b*x+a))*c-I))*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+
1))^2+I*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I/(exp(2*I*(b*x+a))+1)*(I+c))
^2+I*Pi*csgn(I*(I+c))*csgn(I/(exp(2*I*(b*x+a))+1)*(I+c))^2+I*Pi*csgn(I*exp
(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))^2+I*Pi*
csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))^2-I*Pi*csgn((exp(2*I*...
```

3.56.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(108) = 216$.

Time = 0.29 (sec) , antiderivative size = 322, normalized size of antiderivative = 2.08

$$\int x^2 \arctan(c + (-1 + ic)\tan(a + bx))\,dx$$
$$= \frac{b^4x^4 + 2i\,b^3x^3\log\left(-\frac{(c+i)e^{(2i\,bx+2i\,a)}}{ce^{(2i\,bx+2i\,a)}-i}\right) + 6\,b^2x^2\mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4i\,c}e^{(i\,bx+i\,a)}\right) + 6\,b^2x^2\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{-4i\,c}e^{(i\,bx+i\,a)}\right) - a^4}{}$$

input
```
integrate(x^2*arctan(c+(-1+I*c)*tan(b*x+a)),x, algorithm="fricas")
```

output
```
1/12*(b^4*x^4 + 2*I*b^3*x^3*log(-(c + I)*e^(2*I*b*x + 2*I*a)/(c*e^(2*I*b*x
 + 2*I*a) - I)) + 6*b^2*x^2*dilog(1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)) + 6*b^
2*x^2*dilog(-1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)) - a^4 - 2*I*a^3*log(1/2*(2*
c*e^(I*b*x + I*a) + I*sqrt(-4*I*c))/c) - 2*I*a^3*log(1/2*(2*c*e^(I*b*x + I
*a) - I*sqrt(-4*I*c))/c) + 12*I*b*x*polylog(3, 1/2*sqrt(-4*I*c)*e^(I*b*x +
 I*a)) + 12*I*b*x*polylog(3, -1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)) - 2*(-I*b^
3*x^3 - I*a^3)*log(1/2*sqrt(-4*I*c)*e^(I*b*x + I*a) + 1) - 2*(-I*b^3*x^3 -
 I*a^3)*log(-1/2*sqrt(-4*I*c)*e^(I*b*x + I*a) + 1) - 12*polylog(4, 1/2*sqr
t(-4*I*c)*e^(I*b*x + I*a)) - 12*polylog(4, -1/2*sqrt(-4*I*c)*e^(I*b*x + I*
a)))/b^3
```

3.56.6 Sympy [F(-2)]

Exception generated.

$$\int x^2 \arctan(c + (-1 + ic)\tan(a + bx))\,dx = \text{Exception raised: CoercionFailed}$$

input
```
integrate(x**2*atan(c+(-1+I*c)*tan(b*x+a)),x)
```

output
```
Exception raised: CoercionFailed >> Cannot convert _t0**2 + exp(2*I*a) of
type <class 'sympy.core.add.Add'> to QQ_I[x,b,_t0,exp(I*a)]
```

3.56.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 310 vs. 2(108) = 216.

Time = 0.21 (sec) , antiderivative size = 310, normalized size of antiderivative = 2.00

$$\int x^2 \arctan(c + (-1 + ic)\tan(a + bx))\,dx = \frac{\frac{4\left((bx+a)^3 - 3\,(bx+a)^2 a + 3\,(bx+a)a^2\right)\arctan((i\,c-1)\tan(bx+a)+c)}{b^2} + \frac{\left(-3i\,(bx+a)^4 + 12i\,(bx+a)^3 a - 18i\,(bx+a)^2 a^2 - 2\left(-4i\,(bx+a)^3 + 9i\,(bx+\right.\right.}{}$$

input
```
integrate(x^2*arctan(c+(-1+I*c)*tan(b*x+a)),x, algorithm="maxima")
```

output
```
1/12*(4*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arctan((I*c - 1)
*tan(b*x + a) + c)/b^2 + (-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b*
x + a)^2*a^2 - 2*(-4*I*(b*x + a)^3 + 9*I*(b*x + a)^2*a - 9*I*(b*x + a)*a^2
)*arctan2(c*cos(2*b*x + 2*a), -c*sin(2*b*x + 2*a) + 1) - 3*(4*I*(b*x + a)^
2 - 6*I*(b*x + a)*a + 3*I*a^2)*dilog(-I*c*e^(2*I*b*x + 2*I*a)) + (4*(b*x +
 a)^3 - 9*(b*x + a)^2*a + 9*(b*x + a)*a^2)*log(c^2*cos(2*b*x + 2*a)^2 + c^
2*sin(2*b*x + 2*a)^2 - 2*c*sin(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polylog(3
, -I*c*e^(2*I*b*x + 2*I*a)) + 6*I*polylog(4, -I*c*e^(2*I*b*x + 2*I*a)))*(I
*c - 1)/(b^2*(c + I)))/b
```

3.56.8 Giac [F]

$$\int x^2 \arctan(c+(-1+ic)\tan(a+bx))\,dx = \int x^2 \arctan\left((i\,c-1)\tan\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(x^2*arctan(c+(-1+I*c)*tan(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*arctan((I*c - 1)*tan(b*x + a) + c), x)
```

3.56.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \arctan(c+(-1+ic)\tan(a+bx))\,dx = \int x^2\,\mathrm{atan}(c+\tan(a+b\,x)\,\left(-1+c\,1\mathrm{i}\right))\,dx$$

input
```
int(x^2*atan(c + tan(a + b*x)*(c*1i - 1)),x)
```

output
```
int(x^2*atan(c + tan(a + b*x)*(c*1i - 1)), x)
```

3.57 $\int x \arctan(c+(-1+ic)\tan(a+bx))\,dx$

3.57.1 Optimal result

Integrand size = 19, antiderivative size = 124

$$\int x\arctan(c+(-1+ic)\tan(a+bx))\,dx=\frac{bx^3}{6}+\frac{1}{2}x^2\arctan(c-(1-ic)\tan(a+bx))+\frac{1}{4}ix^2\log\left(1+ice^{2ia+2ibx}\right)+\frac{x\,\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}+\frac{i\,\text{PolyLog}\left(3,-ice^{2ia+2ibx}\right)}{8b^2}$$

output
```
1/6*b*x^3+1/2*x^2*arctan(c-(1-I*c)*tan(b*x+a))+1/4*I*x^2*ln(1+I*c*exp(2*I*
a+2*I*b*x))+1/4*x*polylog(2,-I*c*exp(2*I*a+2*I*b*x))/b+1/8*I*polylog(3,-I*
c*exp(2*I*a+2*I*b*x))/b^2
```

3.57.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.90

$$\int x\arctan(c+(-1+ic)\tan(a+bx))\,dx=\frac{1}{2}x^2\arctan(c+i(i+c)\tan(a+bx))+\frac{i\left(2b^2x^2\log\left(1-\frac{ie^{-2i(a+bx)}}{c}\right)+2ibx\,\text{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{c}\right)+\text{PolyLog}\left(3,\frac{ie^{-2i(a+bx)}}{c}\right)\right)}{8b^2}$$

input
```
Integrate[x*ArcTan[c + (-1 + I*c)*Tan[a + b*x]],x]
```

output
```
(x^2*ArcTan[c + I*(I + c)*Tan[a + b*x]])/2 + ((I/8)*(2*b^2*x^2*Log[1 - I/(
c*E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, I/(c*E^((2*I)*(a + b*x)))]
+ PolyLog[3, I/(c*E^((2*I)*(a + b*x)))]))/b^2
```

3.57.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.26, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {5694, 25, 2615, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\arctan(c+(-1+ic)\tan(a+bx))\,dx$$

$$\Big\downarrow \text{5694}$$

$$\frac{1}{2}x^2\arctan(c-(1-ic)\tan(a+bx))-\frac{1}{2}ib\int -\frac{x^2}{i-ce^{2ia+2ibx}}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}ib\int \frac{x^2}{i-ce^{2ia+2ibx}}dx+\frac{1}{2}x^2\arctan(c-(1-ic)\tan(a+bx))$$

$$\Big\downarrow \text{2615}$$

$$\frac{1}{2}ib\left(-ic\int \frac{e^{2ia+2ibx}x^2}{i-ce^{2ia+2ibx}}dx-\frac{ix^3}{3}\right)+\frac{1}{2}x^2\arctan(c-(1-ic)\tan(a+bx))$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{2}ib\left(-ic\left(\frac{ix^2\log\left(1+ice^{2ia+2ibx}\right)}{2bc}-\frac{i\int x\log\left(ie^{2ia+2ibx}c+1\right)dx}{bc}\right)-\frac{ix^3}{3}\right)+\frac{1}{2}x^2\arctan(c-(1-ic)\tan(a+bx))$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{2}ib\left(-ic\left(\frac{ix^2\log\left(1+ice^{2ia+2ibx}\right)}{2bc}-\frac{i\left(\frac{ix\,\text{PolyLog}(2,-ice^{2ia+2ibx})}{2b}-\frac{i\int \text{PolyLog}(2,-ice^{2ia+2ibx})dx}{2b}\right)}{bc}\right)-\frac{ix^3}{3}\right)+\frac{1}{2}x^2\arctan(c-(1-ic)\tan(a+bx))$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{2}ib\left(-ic\left(\frac{ix^2\log\left(1+ice^{2ia+2ibx}\right)}{2bc}-\frac{i\left(\frac{ix\,\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{2b}-\frac{\int e^{-2ia-2ibx}\,\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)de^{2ia+2ibx}}{4b^2}\right)}{bc}\right)-\frac{ix^3}{3}\right)+\frac{1}{2}x^2\arctan(c-(1-ic)\tan(a+bx))$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{2}x^2\arctan(c-(1-ic)\tan(a+bx))+\frac{1}{2}ib\left(-ic\left(\frac{ix^2\log\left(1+ice^{2ia+2ibx}\right)}{2bc}-\frac{i\left(\frac{ix\,\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{2b}-\frac{\text{PolyLog}\left(3,-ice^{2ia+2ibx}\right)}{4b^2}\right)}{bc}\right)-\frac{ix^3}{3}\right)$$

input
```
Int[x*ArcTan[c + (-1 + I*c)*Tan[a + b*x]],x]
```

output
```
(x^2*ArcTan[c - (1 - I*c)*Tan[a + b*x]])/2 + (I/2)*b*((-1/3*I)*x^3 - I*c*(
((I/2)*x^2*Log[1 + I*c*E^((2*I)*a + (2*I)*b*x)])/(b*c) - (I*(((I/2)*x*Poly
Log[2, (-I)*c*E^((2*I)*a + (2*I)*b*x)])/b - PolyLog[3, (-I)*c*E^((2*I)*a +
 (2*I)*b*x)]/(4*b^2)))/(b*c)))
```

3.57.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5694
```
Int[ArcTan[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Tan[a + b*x]]/(f*(m +
1))), x] - Simp[I*(b/(f*(m + 1)))   Int[(e + f*x)^(m + 1)/(c + I*d + c*E^(2
*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && E
qQ[(c + I*d)^2, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.57.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.98 (sec) , antiderivative size = 1453, normalized size of antiderivative = 11.72

method	result	size
risch	Expression too large to display	1453

input
```
int(x*arctan(c+(-1+I*c)*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/2*I/b*ln(I*exp(2*I*(b*x+a))*c+1)*a*x+1/8*I/b^2*polylog(3,-I*exp(2*I*(b*x
+a))*c)-1/2*I/b*a*ln(1+I*exp(I*(b*x+a))*(I*c)^(1/2))*x+1/8*I*(-I*Pi*csgn(I
*exp(2*I*(b*x+a)))^3-2*I*Pi+I*Pi*csgn(I/(exp(2*I*(b*x+a))+1)*(I+c))*csgn(I
*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))^2+2*ln(I+c)-I*Pi*csgn(I*exp(
I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+a)))+I*Pi*csgn(I*(exp(2*I*(b*x+a))*c-I)/
(exp(2*I*(b*x+a))+1))^3-I*Pi*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a
))+1))^3+I*Pi*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))*csgn((ex
p(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))-I*Pi*csgn(exp(2*I*(b*x+a))*(I+c)
/(exp(2*I*(b*x+a))+1))^3-I*Pi*csgn(I/(exp(2*I*(b*x+a))+1)*(I+c))^3-I*Pi*cs
gn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))*csgn(exp(2*I*(b*x+a))*(I
+c)/(exp(2*I*(b*x+a))+1))+I*Pi*csgn((exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a
))+1))^2-I*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(I+c))*csgn(I/(exp(2*I*(
b*x+a))+1)*(I+c))-I*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I/(exp(2*I*(b*x+a))+1
)*(I+c))*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))+1))+I*Pi*csgn(I/(
exp(2*I*(b*x+a))+1))*csgn(I*(exp(2*I*(b*x+a))*c-I))*csgn(I*(exp(2*I*(b*x+a
))*c-I)/(exp(2*I*(b*x+a))+1))-I*Pi*csgn(I/(exp(2*I*(b*x+a))+1))*csgn(I*(ex
p(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))^2-I*Pi*csgn(I*(exp(2*I*(b*x+a))*
c-I))*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))+1))^2+I*Pi*csgn(I/(e
xp(2*I*(b*x+a))+1))*csgn(I/(exp(2*I*(b*x+a))+1)*(I+c))^2+I*Pi*csgn(I*(I+c)
)*csgn(I/(exp(2*I*(b*x+a))+1)*(I+c))^2+I*Pi*csgn(I*exp(2*I*(b*x+a)))*cs...
```

3.57.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(86) = 172$.

Time = 0.27 (sec) , antiderivative size = 271, normalized size of antiderivative = 2.19

$$\int x \arctan(c + (-1 + ic)\tan(a + bx))\,dx$$
$$= \frac{2\,b^3x^3 + 3i\,b^2x^2\log\left(-\frac{(c+i)e^{(2i\,bx+2i\,a)}}{ce^{(2i\,bx+2i\,a)}-i}\right) + 2\,a^3 + 6\,bx\mathrm{Li}_2\left(\frac{1}{2}\,\sqrt{-4i\,c}e^{(i\,bx+i\,a)}\right) + 6\,bx\mathrm{Li}_2\left(-\frac{1}{2}\,\sqrt{-4i\,c}e^{(i\,bx+i\,a)}\right)}{}$$

input
```
integrate(x*arctan(c+(-1+I*c)*tan(b*x+a)),x, algorithm="fricas")
```

output
```
1/12*(2*b^3*x^3 + 3*I*b^2*x^2*log(-(c + I)*e^(2*I*b*x + 2*I*a)/(c*e^(2*I*b
*x + 2*I*a) - I)) + 2*a^3 + 6*b*x*dilog(1/2*sqrt(-4*I*c)*e^(I*b*x + I*a))
+ 6*b*x*dilog(-1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)) + 3*I*a^2*log(1/2*(2*c*e^
(I*b*x + I*a) + I*sqrt(-4*I*c))/c) + 3*I*a^2*log(1/2*(2*c*e^(I*b*x + I*a)
- I*sqrt(-4*I*c))/c) - 3*(-I*b^2*x^2 + I*a^2)*log(1/2*sqrt(-4*I*c)*e^(I*b*
x + I*a) + 1) - 3*(-I*b^2*x^2 + I*a^2)*log(-1/2*sqrt(-4*I*c)*e^(I*b*x + I*
a) + 1) + 6*I*polylog(3, 1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)) + 6*I*polylog(3
, -1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)))/b^2
```

3.57.6 Sympy [F(-2)]

Exception generated.

$$\int x\arctan(c+(-1+ic)\tan(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

input
```
integrate(x*atan(c+(-1+I*c)*tan(b*x+a)),x)
```

output
```
Exception raised: CoercionFailed >> Cannot convert _t0**2 + exp(2*I*a) of
type <class 'sympy.core.add.Add'> to QQ_I[x,b,_t0,exp(I*a)]
```

3.57.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(86) = 172.

Time = 0.22 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.76

$$\int x\arctan(c+(-1+ic)\tan(a+bx))\,dx$$
$$=\frac{\frac{6\left((bx+a)^2-2\,(bx+a)a\right)\arctan((i\,c-1)\tan(bx+a)+c)}{b}+\frac{\left(-4i\,(bx+a)^3+12i\,(bx+a)^2a-6i\,bx\mathrm{Li}_2\left(-i\,ce^{(2i\,bx+2i\,a)}\right)-6\left(-i\,(bx+a)^2+2i\,(bx+a)a\right.}{}}{}$$

input
```
integrate(x*arctan(c+(-1+I*c)*tan(b*x+a)),x, algorithm="maxima")
```

output
```
1/12*(6*((b*x + a)^2 - 2*(b*x + a)*a)*arctan((I*c - 1)*tan(b*x + a) + c)/b
 + (-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*dilog(-I*c*e^(2*I*b*x
+ 2*I*a)) - 6*(-I*(b*x + a)^2 + 2*I*(b*x + a)*a)*arctan2(c*cos(2*b*x + 2*a
), -c*sin(2*b*x + 2*a) + 1) + 3*((b*x + a)^2 - 2*(b*x + a)*a)*log(c^2*cos(
2*b*x + 2*a)^2 + c^2*sin(2*b*x + 2*a)^2 - 2*c*sin(2*b*x + 2*a) + 1) + 3*po
lylog(3, -I*c*e^(2*I*b*x + 2*I*a)))*(I*c - 1)/(b*(c + I)))/b
```

3.57.8 Giac [F]

$$\int x\arctan(c+(-1+ic)\tan(a+bx))\,dx=\int x\arctan\left((i\,c-1)\tan\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(x*arctan(c+(-1+I*c)*tan(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*arctan((I*c - 1)*tan(b*x + a) + c), x)
```

3.57.9 Mupad [F(-1)]

Timed out.

$$\int x\arctan(c+(-1+ic)\tan(a+bx))\,dx=\int x\,\mathrm{atan}(c+\tan(a+b\,x)\,\left(-1+c\,1\mathrm{i}\right))\,dx$$

input
```
int(x*atan(c + tan(a + b*x)*(c*1i - 1)),x)
```

output
```
int(x*atan(c + tan(a + b*x)*(c*1i - 1)), x)
```

3.58 $\int \arctan(c+(-1+ic)\tan(a+bx))\,dx$

3.58.1 Optimal result

Integrand size = 17, antiderivative size = 86

$$\int \arctan(c+(-1+ic)\tan(a+bx))\,dx=\frac{bx^2}{2}+x\arctan(c-(1-ic)\tan(a+bx))+\frac{1}{2}ix\log\left(1+ice^{2ia+2ibx}\right)+\frac{\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}$$

output
```
1/2*b*x^2+x*arctan(c-(1-I*c)*tan(b*x+a))+1/2*I*x*ln(1+I*c*exp(2*I*a+2*I*b*
x))+1/4*polylog(2,-I*c*exp(2*I*a+2*I*b*x))/b
```

3.58.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 847 vs. 2(86) = 172.

Time = 8.39 (sec) , antiderivative size = 847, normalized size of antiderivative = 9.85

$$\int \arctan(c+(-1+ic)\tan(a+bx))\,dx=x\arctan(c+i(i+c)\tan(a+bx))+\frac{ix\left(-2ibx\log(2\cos(bx)(\cos(bx)-i\sin(bx)))+\log\left(\frac{\sec(bx)(\cos(a)\cdots}{\cdots}\right.\right.}{((-i+c)\cos(a+bx)+i(i+c)\sin(a+bx))\left(-2bx+i\log\left(1-\frac{\sec(bx)((i+c)\cos(a)+(1+ic)\sin(a))(\cos(a+bx)-i\sin\cdots}{2c}\right.\right.}$$

input
```
Integrate[ArcTan[c + (-1 + I*c)*Tan[a + b*x]],x]
```

output
```
x*ArcTan[c + I*(I + c)*Tan[a + b*x]] + (I*x*((-2*I)*b*x*Log[2*Cos[b*x]*(Co
s[b*x] - I*Sin[b*x])] + Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((-I + c)*Cos[a
+ b*x] + I*(I + c)*Sin[a + b*x]))/(2*c)]*Log[1 - I*Tan[b*x]] - Log[(Sec[b*
x]*(Cos[a] + I*Sin[a])*((1 + I*c)*Cos[a + b*x] - (I + c)*Sin[a + b*x]))/2]
*Log[1 + I*Tan[b*x]] + PolyLog[2, -Cos[2*b*x] + I*Sin[2*b*x]] + PolyLog[2,
 (Sec[b*x]*((I + c)*Cos[a] + (1 + I*c)*Sin[a])*(Cos[a + b*x] - I*Sin[a + b
*x]))/(2*c)] - PolyLog[2, ((Cos[a] + I*Sin[a])*((I + c)*Cos[a] + (1 + I*c)
*Sin[a])*(-I + Tan[b*x]))/2])*Sec[a + b*x]*(Cos[b*x] + I*Sin[b*x])*(I*Cos[
b*x] + Sin[b*x]))/(((-I + c)*Cos[a + b*x] + I*(I + c)*Sin[a + b*x])*(-2*b*
x + I*Log[1 - (Sec[b*x]*((I + c)*Cos[a] + (1 + I*c)*Sin[a])*(Cos[a + b*x]
- I*Sin[a + b*x]))/(2*c)] + (I*(I + c)*Cos[a + b*x]*(Log[1 - I*Tan[b*x]] -
 Log[1 + I*Tan[b*x]]))/((-I + c)*Cos[a + b*x] + I*(I + c)*Sin[a + b*x]) +
((1 + I*c)*(Log[1 - I*Tan[b*x]] - Log[1 + I*Tan[b*x]])*Sin[a + b*x])/((-1
- I*c)*Cos[a + b*x] + (I + c)*Sin[a + b*x]) + (2*I)*b*x*Tan[b*x] - Log[1 -
 (Sec[b*x]*((I + c)*Cos[a] + (1 + I*c)*Sin[a])*(Cos[a + b*x] - I*Sin[a + b
*x]))/(2*c)]*Tan[b*x] + Log[1 - I*Tan[b*x]]*Tan[b*x] - Log[1 + I*Tan[b*x]]
*Tan[b*x] - (Log[(Sec[b*x]*(Cos[a] + I*Sin[a])*((1 + I*c)*Cos[a + b*x] - (
I + c)*Sin[a + b*x]))/2]*Sec[b*x]^2)/(-I + Tan[b*x]) + (Log[1 - ((Cos[a] +
 I*Sin[a])*((I + c)*Cos[a] + (1 + I*c)*Sin[a])*(-I + Tan[b*x]))/2]*Sec[b*x
]^2)/(-I + Tan[b*x]) + (Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((-I + c)*Cos...
```

3.58.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.26, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {5686, 25, 2615, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(c+(-1+ic)\tan(a+bx))\,dx$$

$$\Big\downarrow \text{5686}$$

$$x\arctan(c-(1-ic)\tan(a+bx))-ib\int -\frac{x}{i-ce^{2ia+2ibx}}dx$$

$$\Big\downarrow \text{25}$$

$$ib\int \frac{x}{i-ce^{2ia+2ibx}}dx+x\arctan(c-(1-ic)\tan(a+bx))$$

$$\Big\downarrow \text{2615}$$

$$ib\left(-ic\int \frac{e^{2ia+2ibx}x}{i-ce^{2ia+2ibx}}dx-\frac{ix^2}{2}\right)+x\arctan(c-(1-ic)\tan(a+bx))$$

$$\Big\downarrow \text{2620}$$

$$ib\left(-ic\left(\frac{ix\log\left(1+ice^{2ia+2ibx}\right)}{2bc}-\frac{i\int\log\left(ie^{2ia+2ibx}c+1\right)dx}{2bc}\right)-\frac{ix^2}{2}\right)+x\arctan(c-(1-ic)\tan(a+bx))$$

$$\Big\downarrow \text{2715}$$

$$ib\left(-ic\left(\frac{ix\log\left(1+ice^{2ia+2ibx}\right)}{2bc}-\frac{\int e^{-2ia-2ibx}\log\left(ie^{2ia+2ibx}c+1\right)de^{2ia+2ibx}}{4b^2c}\right)-\frac{ix^2}{2}\right)+x\arctan(c-(1-ic)\tan(a+bx))$$

$$\Big\downarrow \text{2838}$$

$$x\arctan(c-(1-ic)\tan(a+bx))+ib\left(-ic\left(\frac{\operatorname{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b^2c}+\frac{ix\log\left(1+ice^{2ia+2ibx}\right)}{2bc}\right)-\frac{ix^2}{2}\right)$$

input
```
Int[ArcTan[c + (-1 + I*c)*Tan[a + b*x]],x]
```

output
```
x*ArcTan[c - (1 - I*c)*Tan[a + b*x]] + I*b*((-1/2*I)*x^2 - I*c*(((I/2)*x*L
og[1 + I*c*E^((2*I)*a + (2*I)*b*x)])/(b*c) + PolyLog[2, (-I)*c*E^((2*I)*a
+ (2*I)*b*x)]/(4*b^2*c)))
```

3.58.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5686
```
Int[ArcTan[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcT
an[c + d*Tan[a + b*x]], x] - Simp[I*b   Int[x/(c + I*d + c*E^(2*I*a + 2*I*b
*x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c + I*d)^2, -1]
```

3.58.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 593 vs. $2(70) = 140$.

Time = 1.17 (sec) , antiderivative size = 594, normalized size of antiderivative = 6.91

method	result
derivativedivides	$\frac{\frac{\arctan(c+(ic-1)\tan(bx+a))\ln(i+c+(ic-1)\tan(bx+a))c^2}{2i+2c}+\frac{2i\arctan(c+(ic-1)\tan(bx+a))\ln(i+c+(ic-1)\tan(bx+a))c}{2i+2c}-\frac{\arctan(c+(\cdots}{}}{}$
default	$\frac{\frac{\arctan(c+(ic-1)\tan(bx+a))\ln(i+c+(ic-1)\tan(bx+a))c^2}{2i+2c}+\frac{2i\arctan(c+(ic-1)\tan(bx+a))\ln(i+c+(ic-1)\tan(bx+a))c}{2i+2c}-\frac{\arctan(c+(\cdots}{}}{}$
risch	Expression too large to display

input
```
int(arctan(c+(-1+I*c)*tan(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b/(-1+I*c)*(arctan(c+(-1+I*c)*tan(b*x+a))/(2*I+2*c)*ln(I+c+(-1+I*c)*tan(
b*x+a))*c^2+2*I*arctan(c+(-1+I*c)*tan(b*x+a))/(2*I+2*c)*ln(I+c+(-1+I*c)*ta
n(b*x+a))*c-arctan(c+(-1+I*c)*tan(b*x+a))/(2*I+2*c)*ln(I+c+(-1+I*c)*tan(b*
x+a))-arctan(c+(-1+I*c)*tan(b*x+a))/(2*I+2*c)*ln(c-(-1+I*c)*tan(b*x+a)+I)*
c^2-2*I*arctan(c+(-1+I*c)*tan(b*x+a))/(2*I+2*c)*ln(c-(-1+I*c)*tan(b*x+a)+I
)*c+arctan(c+(-1+I*c)*tan(b*x+a))/(2*I+2*c)*ln(c-(-1+I*c)*tan(b*x+a)+I)+(-
1+I*c)^2*(1/2/(I+c)*(1/4*I*ln(I+c+(-1+I*c)*tan(b*x+a))^2-1/2*I*((ln(I+c+(-
1+I*c)*tan(b*x+a))-ln(-1/2*I*(I+c+(-1+I*c)*tan(b*x+a))))*ln(-1/2*I*(I-c-(-
1+I*c)*tan(b*x+a)))-dilog(-1/2*I*(I+c+(-1+I*c)*tan(b*x+a)))))-1/2/(I+c)*(1
/2*I*(dilog((-I-c-(-1+I*c)*tan(b*x+a))/(-2*I-2*c))+ln(c-(-1+I*c)*tan(b*x+a
)+I)*ln((-I-c-(-1+I*c)*tan(b*x+a))/(-2*I-2*c)))-1/2*I*(dilog(-1/2*(I-c-(-1
+I*c)*tan(b*x+a))/c)+ln(c-(-1+I*c)*tan(b*x+a)+I)*ln(-1/2*(I-c-(-1+I*c)*tan
(b*x+a))/c)))))
```

3.58.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(61) = 122$.

Time = 0.29 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.33

$$\int \arctan(c + (-1 + ic)\tan(a + bx))\,dx$$
$$= \frac{b^2x^2 + i\,bx\log\left(-\frac{(c+i)e^{(2i\,bx+2i\,a)}}{ce^{(2i\,bx+2i\,a)}-i}\right) - a^2 + (i\,bx + i\,a)\log\left(\frac{1}{2}\sqrt{-4i\,c}e^{(i\,bx+i\,a)} + 1\right) + (i\,bx + i\,a)\log\left(-\frac{1}{2}\sqrt{-4}\right.}{}$$

input
```
integrate(arctan(c+(-1+I*c)*tan(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*(b^2*x^2 + I*b*x*log(-(c + I)*e^(2*I*b*x + 2*I*a)/(c*e^(2*I*b*x + 2*I*
a) - I)) - a^2 + (I*b*x + I*a)*log(1/2*sqrt(-4*I*c)*e^(I*b*x + I*a) + 1) +
 (I*b*x + I*a)*log(-1/2*sqrt(-4*I*c)*e^(I*b*x + I*a) + 1) - I*a*log(1/2*(2
*c*e^(I*b*x + I*a) + I*sqrt(-4*I*c))/c) - I*a*log(1/2*(2*c*e^(I*b*x + I*a)
 - I*sqrt(-4*I*c))/c) + dilog(1/2*sqrt(-4*I*c)*e^(I*b*x + I*a)) + dilog(-1
/2*sqrt(-4*I*c)*e^(I*b*x + I*a)))/b
```

3.58.6 Sympy [F(-2)]

Exception generated.

$$\int \arctan(c + (-1 + ic)\tan(a + bx))\,dx = \text{Exception raised: CoercionFailed}$$

input
```
integrate(atan(c+(-1+I*c)*tan(b*x+a)),x)
```

output
```
Exception raised: CoercionFailed >> Cannot convert _t0**2 + exp(2*I*a) of
type <class 'sympy.core.add.Add'> to QQ_I[b,_t0,exp(I*a)]
```

3.58.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 448 vs. 2(61) = 122.

Time = 0.29 (sec) , antiderivative size = 448, normalized size of antiderivative = 5.21

$$\int \arctan(c + (-1 + ic)\tan(a + bx))\,dx =$$

$$-\frac{(i\,c-1)\left(\frac{4i\,(bx+a)\log\left(-\frac{2\left(-i\,c^2+\left(c^2+2i\,c-1\right)\tan(bx+a)-i\right)}{2i\,c^2-2\left(c^2+2i\,c-1\right)\tan(bx+a)-4\,c-2i}\right)}{i\,c-1}+\frac{i\left(4\,(bx+a)(\log(-i\,c^2+(c^2+2i\,c-1)\tan(bx+a)+2\,c+i)-\log(-i\,c^2\cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
integrate(arctan(c+(-1+I*c)*tan(b*x+a)),x, algorithm="maxima")
```

output
```
-1/8*((I*c - 1)*(4*I*(b*x + a)*log(-2*(-I*c^2 + (c^2 + 2*I*c - 1)*tan(b*x
+ a) - I)/(2*I*c^2 - 2*(c^2 + 2*I*c - 1)*tan(b*x + a) - 4*c - 2*I))/(I*c -
 1) + I*(4*(b*x + a)*(log(-I*c^2 + (c^2 + 2*I*c - 1)*tan(b*x + a) + 2*c +
I) - log(-I*c^2 + (c^2 + 2*I*c - 1)*tan(b*x + a) - I)) + I*log(-I*c^2 + (c
^2 + 2*I*c - 1)*tan(b*x + a) + 2*c + I)^2 - 2*I*log(-I*c^2 + (c^2 + 2*I*c
- 1)*tan(b*x + a) - I)*log(1/2*(c + I)*tan(b*x + a) - 1/2*I*c + 1/2) + 2*I
*log(-I*c^2 + (c^2 + 2*I*c - 1)*tan(b*x + a) - I)*log(-1/2*((I*c - 1)*tan(
b*x + a) + c - I)/c + 1) - 2*I*log(-I*c^2 + (c^2 + 2*I*c - 1)*tan(b*x + a)
 + 2*c + I)*log(-1/2*I*tan(b*x + a) + 1/2) - 2*I*dilog(-1/2*(c + I)*tan(b*
x + a) + 1/2*I*c + 1/2) + 2*I*dilog(1/2*((I*c - 1)*tan(b*x + a) + c - I)/c
) - 2*I*dilog(1/2*I*tan(b*x + a) + 1/2))/(I*c - 1)) - 8*(b*x + a)*arctan((
I*c - 1)*tan(b*x + a) + c) + 4*(-I*b*x - I*a)*log(-2*(-I*c^2 + (c^2 + 2*I*
c - 1)*tan(b*x + a) - I)/(2*I*c^2 - 2*(c^2 + 2*I*c - 1)*tan(b*x + a) - 4*c
 - 2*I)))/b
```

3.58.8 Giac [F]

$$\int \arctan(c+(-1+ic)\tan(a+bx))\,dx=\int \arctan\left((i\,c-1)\tan\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(arctan(c+(-1+I*c)*tan(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arctan((I*c - 1)*tan(b*x + a) + c), x)
```

3.58.9 Mupad [F(-1)]

Timed out.

$$\int \arctan(c+(-1+ic)\tan(a+bx))\,dx=\int \mathrm{atan}\left(c+\tan\left(a+b\,x\right)\,\left(-1+c\,1\mathrm{i}\right)\right)\,dx$$

input
```
int(atan(c + tan(a + b*x)*(c*1i - 1)),x)
```

output
```
int(atan(c + tan(a + b*x)*(c*1i - 1)), x)
```

3.59 $\int \frac{\arctan(c+(-1+ic)\tan(a+bx))}{x} dx$

3.59.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{\arctan(c+(-1+ic)\tan(a+bx))}{x} dx = \text{Int}\left(\frac{\arctan(c+(-1+ic)\tan(a+bx))}{x}, x\right)$$

output
```
CannotIntegrate(arctan(c+(-1+I*c)*tan(b*x+a))/x,x)
```

3.59.2 Mathematica [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{\arctan(c+(-1+ic)\tan(a+bx))}{x} dx = \int \frac{\arctan(c+(-1+ic)\tan(a+bx))}{x} dx$$

input
```
Integrate[ArcTan[c + (-1 + I*c)*Tan[a + b*x]]/x,x]
```

output
```
Integrate[ArcTan[c + (-1 + I*c)*Tan[a + b*x]]/x, x]
```

3.59.3 Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(c+(-1+ic)\tan(a+bx))}{x} \, dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \frac{\arctan(c+(-1+ic)\tan(a+bx))}{x}dx$$

input
```
Int[ArcTan[c + (-1 + I*c)*Tan[a + b*x]]/x,x]
```

output
```
$Aborted
```

3.59.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.59.4 Maple [N/A] (verified)

Not integrable

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{\arctan \left(c+\left(i c-1\right) \tan \left(b x+a\right)\right)}{x}dx$$

input
```
int(arctan(c+(-1+I*c)*tan(b*x+a))/x,x)
```

output
```
int(arctan(c+(-1+I*c)*tan(b*x+a))/x,x)
```

3.59.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.76

$$\int \frac{\arctan(c+(-1+ic)\tan(a+bx))}{x}\,dx = \int \frac{\arctan\left((i\,c-1)\tan\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arctan(c+(-1+I*c)*tan(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(1/2*I*log(-(c + I)*e^(2*I*b*x + 2*I*a)/(c*e^(2*I*b*x + 2*I*a) - I
))/x, x)
```

3.59.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan(c+(-1+ic)\tan(a+bx))}{x}\,dx = \text{Timed out}$$

input
```
integrate(atan(c+(-1+I*c)*tan(b*x+a))/x,x)
```

output
```
Timed out
```

3.59.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(c+(-1+ic)\tan(a+bx))}{x}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(arctan(c+(-1+I*c)*tan(b*x+a))/x,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c-1>0)', see `assume?` for more
details)Is
```

3.59.8 Giac [N/A]

Not integrable

Time = 1.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(c+(-1+ic)\tan(a+bx))}{x}\,dx = \int \frac{\arctan\left((i\,c-1)\tan\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arctan(c+(-1+I*c)*tan(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(arctan((I*c - 1)*tan(b*x + a) + c)/x, x)
```

3.59.9 Mupad [N/A]

Not integrable

Time = 1.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{\arctan(c+(-1+ic)\tan(a+bx))}{x}\,dx = \int \frac{\mathrm{atan}(c+\tan(a+b\,x)\,\left(-1+c\,1\mathrm{i}\right))}{x}\,dx$$

input
```
int(atan(c + tan(a + b*x)*(c*1i - 1))/x,x)
```

output
```
int(atan(c + tan(a + b*x)*(c*1i - 1))/x, x)
```

3.60 $\int \arctan(\cot(a+bx))\,dx$

3.60.1 Optimal result

Integrand size = 7, antiderivative size = 16

$$\int \arctan(\cot(a+bx))\,dx = -\frac{\arctan(\cot(a+bx))^2}{2b}$$

output
```
-1/2*(1/2*Pi-arccot(cot(b*x+a)))^2/b
```

3.60.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \arctan(\cot(a+bx))\,dx = \frac{bx^2}{2} + x\arctan(\cot(a+bx))$$

input
```
Integrate[ArcTan[Cot[a + b*x]],x]
```

output
```
(b*x^2)/2 + x*ArcTan[Cot[a + b*x]]
```

3.60.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2588, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(\cot(a+bx))\,dx$$

$$\Big\downarrow \text{2588}$$

$$-\frac{\int \arctan(\cot(a+bx))d\arctan(\cot(a+bx))}{b}$$

$$\Big\downarrow \text{15}$$

$$-\frac{\arctan(\cot(a+bx))^2}{2b}$$

input
```
Int[ArcTan[Cot[a + b*x]],x]
```

output
```
-1/2*ArcTan[Cot[a + b*x]]^2/b
```

3.60.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2588
```
Int[(u_)^(m_.), x_Symbol] :> With[{c = Simplify[D[u, x]]}, Simp[1/c   Subst
[Int[x^m, x], x, u], x]] /; FreeQ[m, x] && PiecewiseLinearQ[u, x]
```

3.60.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

method	result
parallelrisch	$\frac{x^2 b}{2}-x\ \mathrm{arccot}\left(\cot\left(bx+a\right)\right)+\frac{\pi x}{2}$
derivativedivides	$\frac{-\pi\left(\frac{\pi}{2}-\mathrm{arccot}(\cot(bx+a))\right)-\mathrm{arccot}(\cot(bx+a))^2}{2b}$
default	$\frac{\pi x}{2}-\frac{-\left(\frac{\pi}{2}-\mathrm{arccot}(\cot(bx+a))\right)\mathrm{arccot}(\cot(bx+a))-\frac{\left(\frac{\pi}{2}-\mathrm{arccot}(\cot(bx+a))\right)^2}{2}}{b}$
parts	$\frac{\pi x}{2}-\frac{-\left(\frac{\pi}{2}-\mathrm{arccot}(\cot(bx+a))\right)\mathrm{arccot}(\cot(bx+a))-\frac{\left(\frac{\pi}{2}-\mathrm{arccot}(\cot(bx+a))\right)^2}{2}}{b}$
risch	$\frac{\pi x}{2}+ix\ln\left(\mathrm{e}^{i(bx+a)}\right)+\frac{\pi x\,\mathrm{csgn}\left(i\mathrm{e}^{i(bx+a)}\right)^2\mathrm{csgn}\left(i\mathrm{e}^{2i(bx+a)}\right)}{4}-\frac{\pi x\ \mathrm{csgn}\left(i\mathrm{e}^{i(bx+a)}\right)\mathrm{csgn}\left(i\mathrm{e}^{2i(bx+a)}\right)^2}{2}+\frac{\pi x\ \mathrm{c}}{}$

input
```
int(1/2*Pi-arccot(cot(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*b-x*arccot(cot(b*x+a))+1/2*Pi*x
```

3.60.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \arctan(\cot(a+bx))\,dx = -\frac{1}{2}\,bx^2+\frac{1}{2}\,(\pi-2\,a)x$$

input
```
integrate(1/2*pi-arccot(cot(b*x+a)),x, algorithm="fricas")
```

output
```
-1/2*b*x^2 + 1/2*(pi - 2*a)*x
```

3.60.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.50

$$\int \arctan(\cot(a+bx))\,dx = \frac{\pi x}{2}-\begin{cases}\frac{\mathrm{acot}^2\left(\cot\left(a+bx\right)\right)}{2b} & \text{for } b\neq 0\\ x\,\mathrm{acot}\left(\cot\left(a\right)\right) & \text{otherwise}\end{cases}$$

input
```
integrate(1/2*pi-acot(cot(b*x+a)),x)
```

output
```
pi*x/2 - Piecewise((acot(cot(a + b*x))**2/(2*b), Ne(b, 0)), (x*acot(cot(a)
), True))
```

3.60.7 Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \arctan(\cot(a+bx))\,dx = -\frac{1}{2}\,bx^2 + \frac{1}{2}\,\pi x - ax$$

input
```
integrate(1/2*pi-arccot(cot(b*x+a)),x, algorithm="maxima")
```

output
```
-1/2*b*x^2 + 1/2*pi*x - a*x
```

3.60.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \arctan(\cot(a+bx))\,dx = -\frac{1}{2}\,bx^2 + \frac{1}{2}\,\pi x - ax$$

input
```
integrate(1/2*pi-arccot(cot(b*x+a)),x, algorithm="giac")
```

output
```
-1/2*b*x^2 + 1/2*pi*x - a*x
```

3.60.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.31

$$\int \arctan(\cot(a+bx))\,dx = \frac{\Pi\,x}{2} - x\,\mathrm{acot}(\cot(a+b\,x)) + \frac{b\,x^2}{2}$$

input
```
int(Pi/2 - acot(cot(a + b*x)),x)
```

output
```
(Pi*x)/2 - x*acot(cot(a + b*x)) + (b*x^2)/2
```

3.61 $\int x^2 \arctan(c + d\cot(a + bx))\,dx$

3.61.1 Optimal result

Integrand size = 15, antiderivative size = 399

$$\begin{aligned}\int x^2 \arctan(c + d\cot(a + bx))\,dx = {} & \frac{1}{3}x^3 \arctan(c + d\cot(a + bx)) \\ & + \frac{1}{6}ix^3 \log\left(1 - \frac{(1 + ic - d)e^{2ia+2ibx}}{1 + ic + d}\right) \\ & - \frac{1}{6}ix^3 \log\left(1 - \frac{(c + i(1 + d))e^{2ia+2ibx}}{c + i(1 - d)}\right) \\ & + \frac{x^2 \operatorname{PolyLog}\left(2, \frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)}{4b} \\ & - \frac{x^2 \operatorname{PolyLog}\left(2, \frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b} \\ & + \frac{ix \operatorname{PolyLog}\left(3, \frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)}{4b^2} \\ & - \frac{ix \operatorname{PolyLog}\left(3, \frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b^2} \\ & - \frac{\operatorname{PolyLog}\left(4, \frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)}{8b^3} \\ & + \frac{\operatorname{PolyLog}\left(4, \frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)}{8b^3}\end{aligned}$$

output
```
1/3*x^3*arctan(c+d*cot(b*x+a))+1/6*I*x^3*ln(1-(1+I*c-d)*exp(2*I*a+2*I*b*x)
/(1+I*c+d))-1/6*I*x^3*ln(1-(c+I*(1+d))*exp(2*I*a+2*I*b*x)/(c+I*(1-d)))+1/4
*x^2*polylog(2,(1+I*c-d)*exp(2*I*a+2*I*b*x)/(1+I*c+d))/b-1/4*x^2*polylog(2
,(c+I*(1+d))*exp(2*I*a+2*I*b*x)/(c+I*(1-d)))/b+1/4*I*x*polylog(3,(1+I*c-d)
*exp(2*I*a+2*I*b*x)/(1+I*c+d))/b^2-1/4*I*x*polylog(3,(c+I*(1+d))*exp(2*I*a
+2*I*b*x)/(c+I*(1-d)))/b^2-1/8*polylog(4,(1+I*c-d)*exp(2*I*a+2*I*b*x)/(1+I
*c+d))/b^3+1/8*polylog(4,(c+I*(1+d))*exp(2*I*a+2*I*b*x)/(c+I*(1-d)))/b^3
```

3.61.2 Mathematica [A] (verified)

Time = 3.07 (sec) , antiderivative size = 360, normalized size of antiderivative = 0.90

$$\int x^2 \arctan(c + d\cot(a + bx))\,dx$$
$$= \frac{8b^3x^3\arctan(c + d\cot(a + bx)) + 4ib^3x^3\log\left(1 + \frac{(-c+i(1+d))e^{-2i(a+bx)}}{c+i(-1+d)}\right) - 4ib^3x^3\log\left(1 + \frac{(-c+i(-1+d))e^{-2i(a+}}{c+i(1+d)}\right.}{}$$

input
```
Integrate[x^2*ArcTan[c + d*Cot[a + b*x]],x]
```

output
```
(8*b^3*x^3*ArcTan[c + d*Cot[a + b*x]] + (4*I)*b^3*x^3*Log[1 + (-c + I*(1 +
 d))/((c + I*(-1 + d))*E^((2*I)*(a + b*x)))] - (4*I)*b^3*x^3*Log[1 + (-c +
 I*(-1 + d))/((c + I*(1 + d))*E^((2*I)*(a + b*x)))] - 6*b^2*x^2*PolyLog[2,
 (c - I*(1 + d))/((c + I*(-1 + d))*E^((2*I)*(a + b*x)))] + 6*b^2*x^2*PolyL
og[2, (I + c - I*d)/((c + I*(1 + d))*E^((2*I)*(a + b*x)))] + (6*I)*b*x*Pol
yLog[3, (c - I*(1 + d))/((c + I*(-1 + d))*E^((2*I)*(a + b*x)))] - (6*I)*b*
x*PolyLog[3, (I + c - I*d)/((c + I*(1 + d))*E^((2*I)*(a + b*x)))] + 3*Poly
Log[4, (c - I*(1 + d))/((c + I*(-1 + d))*E^((2*I)*(a + b*x)))] - 3*PolyLog
[4, (I + c - I*d)/((c + I*(1 + d))*E^((2*I)*(a + b*x)))])/(24*b^3)
```

3.61.3 Rubi [A] (verified)

Time = 1.59 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5700, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(d\cot(a+bx)+c)\,dx$$

$$\Big\downarrow \text{5700}$$

$$\frac{1}{3}b(ic-d+1)\int \frac{e^{2ia+2ibx}x^3}{ic-(ic-d+1)e^{2ia+2ibx}+d+1}dx-\frac{1}{3}b(-ic+d+1)\int \frac{e^{2ia+2ibx}x^3}{-ic-(-ic+d+1)e^{2ia+2ibx}-d+1}dx+\frac{1}{3}x^3\arctan(d\cot(a+bx)+c)$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{3}b(ic-d+1)\left(\frac{x^3\log\left(1-\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{2b(c-i(1-d))}-\frac{3\int x^2\log\left(1-\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)dx}{2b(c-i(1-d))}\right)-\frac{1}{3}b(-ic+d+1)\left(\frac{3\int x^2\log\left(1-\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)dx}{2b(c+i(d+1))}-\frac{x^3\log\left(1-\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{2b(c+i(d+1))}\right)+\frac{1}{3}x^3\arctan(d\cot(a+bx)+c)$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{3}b(ic-d+1)\left(\frac{x^3\log\left(1-\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{2b(c-i(1-d))}-\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{2b}-\frac{i\int x\operatorname{PolyLog}\left(2,\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)dx}{b}\right)}{2b(c-i(1-d))}\right)-\frac{1}{3}b(-ic+d+1)\left(\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{2b}-\frac{i\int x\operatorname{PolyLog}\left(2,\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)dx}{b}\right)}{2b(c+i(d+1))}-\frac{x^3\log\left(1-\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{2b(c+i(d+1))}\right)+\frac{1}{3}x^3\arctan(d\cot(a+bx)+c)$$

$$\Big\downarrow \text{7163}$$

3.61. $\int x^2 \arctan(c+d\cot(a+bx))\,dx$

$$\frac{1}{3}b(ic-d+$$

$$1)\left(\frac{x^3\log\left(1-\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{2b(c-i(1-d))}-\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{2b}-\frac{i\left(\frac{i\int\operatorname{PolyLog}\left(3,\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)dx}{2b}-\frac{ix\operatorname{PolyLog}\left(3,\right.}{}\right.}{b}\right.}{2b(c-i(1-d))}\right.$$

$$\frac{1}{3}b(-ic+d+$$

$$1)\left(\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{2b}-\frac{i\left(\frac{i\int\operatorname{PolyLog}\left(3,\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)dx}{2b}-\frac{ix\operatorname{PolyLog}\left(3,\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{2b}\right)}{b}\right)}{2b(c+i(d+1))}-\frac{x^3\log}{}\right.$$

$$\frac{1}{3}x^3\arctan(d\cot(a+bx)+c)$$

$$\Big\downarrow \text{2720}$$

3.61. $\int x^2\arctan(c+d\cot(a+bx))\,dx$

$$\frac{1}{3}b(ic-d+1)\left(\frac{x^3\log\left(1-\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{2b(c-i(1-d))}-\frac{3\left(\frac{ix^2\text{PolyLog}\left(2,\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{2b}-\frac{i\left(\frac{\int e^{-2ia-2ibx}\text{PolyLog}\left(3,\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)de^{2ia+2i}}{4b^2}\right.}{b}\right.}{2b(c-i(1-d))}\right)$$

$$\frac{1}{3}b(-ic+d+1)\left(\frac{3\left(\frac{ix^2\text{PolyLog}\left(2,\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{2b}-\frac{i\left(\frac{\int e^{-2ia-2ibx}\text{PolyLog}\left(3,\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)de^{2ia+2ibx}}{4b^2}-\frac{ix\text{PolyLog}\left(3,\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{2b}\right.}{b}\right.}{2b(c+i(d+1))}\right)$$

$$\frac{1}{3}x^3\arctan(d\cot(a+bx)+c)$$

$$\Big\downarrow \text{7143}$$

3.61. $\int x^2\arctan(c+d\cot(a+bx))\,dx$

$$\frac{1}{3}x^3\arctan(d\cot(a+bx)+c)+\frac{1}{3}b(ic-d+1)\left(\frac{x^3\log\left(1-\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{2b(c-i(1-d))}-\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{2b}-\frac{i\left(\frac{\operatorname{PolyLog}\left(4,\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{4b^2}-\frac{ix\operatorname{PolyLog}\left(3,\frac{(ic-d}{}\right.}{2b}\right.}{b}\right.}{2b(c-i(1-d))}\right.$$

$$\frac{1}{3}b(-ic+d+1)\left(\frac{3\left(\frac{ix^2\operatorname{PolyLog}\left(2,\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{2b}-\frac{i\left(\frac{\operatorname{PolyLog}\left(4,\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b^2}-\frac{ix\operatorname{PolyLog}\left(3,\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{2b}\right)}{b}\right)}{2b(c+i(d+1))}-\frac{x^3\log\left(1\right.}{2b(}\right.$$

input
```
Int[x^2*ArcTan[c + d*Cot[a + b*x]],x]
```

output
```
(x^3*ArcTan[c + d*Cot[a + b*x]])/3 + (b*(1 + I*c - d)*((x^3*Log[1 - ((1 +
I*c - d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c + d)])/(2*b*(c - I*(1 - d))) -
(3*(((I/2)*x^2*PolyLog[2, ((1 + I*c - d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c
 + d)])/b - (I*(((-1/2*I)*x*PolyLog[3, ((1 + I*c - d)*E^((2*I)*a + (2*I)*b
*x))/(1 + I*c + d)])/b + PolyLog[4, ((1 + I*c - d)*E^((2*I)*a + (2*I)*b*x)
)/(1 + I*c + d)]/(4*b^2)))/b))/(2*b*(c - I*(1 - d)))))/3 - (b*(1 - I*c + d
)*(-1/2*(x^3*Log[1 - ((c + I*(1 + d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 -
 d))])/(b*(c + I*(1 + d))) + (3*(((I/2)*x^2*PolyLog[2, ((c + I*(1 + d))*E^
((2*I)*a + (2*I)*b*x))/(c + I*(1 - d))])/b - (I*(((-1/2*I)*x*PolyLog[3, ((
c + I*(1 + d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 - d))])/b + PolyLog[4, (
(c + I*(1 + d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 - d))]/(4*b^2)))/b))/(2
*b*(c + I*(1 + d)))))/3
```

3.61. $\int x^2\arctan(c+d\cot(a+bx))\,dx$

3.61.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5700
```
Int[ArcTan[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Cot[a + b*x]]/(f*(m +
1))), x] + (Simp[b*((1 + I*c - d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(E^(
2*I*a + 2*I*b*x)/(1 + I*c + d - (1 + I*c - d)*E^(2*I*a + 2*I*b*x))), x], x]
 - Simp[b*((1 - I*c + d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(E^(2*I*a + 2
*I*b*x)/(1 - I*c - d - (1 - I*c + d)*E^(2*I*a + 2*I*b*x))), x], x]) /; Free
Q[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c - I*d)^2, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.61.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 30.66 (sec) , antiderivative size = 7869, normalized size of antiderivative = 19.72

method	result	size
risch	Expression too large to display	7869

input
```
int(x^2*arctan(c+d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.61.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1589 vs. 2(283) = 566.

Time = 0.43 (sec) , antiderivative size = 1589, normalized size of antiderivative = 3.98

$$\int x^2 \arctan(c + d\cot(a + bx))\, dx = \text{Too large to display}$$

input
```
integrate(x^2*arctan(c+d*cot(b*x+a)),x, algorithm="fricas")
```

output
```
1/48*(16*b^3*x^3*arctan(d*cot(b*x + a) + c) + 6*b^2*x^2*dilog(-(c^2 + d^2
- (c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d^2 - I
)*sin(2*b*x + 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1) + 1) + 6*b^2*x^2*dilog
(-(c^2 + d^2 - (c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d
 - I*d^2 + I)*sin(2*b*x + 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1) + 1) - 6*b
^2*x^2*dilog(-(c^2 + d^2 - (c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-
I*c^2 + 2*c*d + I*d^2 - I)*sin(2*b*x + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d +
1) + 1) - 6*b^2*x^2*dilog(-(c^2 + d^2 - (c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*
x + 2*a) + (I*c^2 + 2*c*d - I*d^2 + I)*sin(2*b*x + 2*a) - 2*d + 1)/(c^2 +
d^2 - 2*d + 1) + 1) - 4*I*a^3*log(1/2*c^2 + I*c*d - 1/2*d^2 - 1/2*(c^2 + d
^2 + 2*d + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 + 2*I*d + I)*sin(2*b*x
 + 2*a) + 1/2) + 4*I*a^3*log(1/2*c^2 + I*c*d - 1/2*d^2 - 1/2*(c^2 + d^2 -
2*d + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 - 2*I*d + I)*sin(2*b*x + 2*
a) + 1/2) + 4*I*a^3*log(-1/2*c^2 + I*c*d + 1/2*d^2 + 1/2*(c^2 + d^2 + 2*d
+ 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 + 2*I*d + I)*sin(2*b*x + 2*a) -
 1/2) - 4*I*a^3*log(-1/2*c^2 + I*c*d + 1/2*d^2 + 1/2*(c^2 + d^2 - 2*d + 1)
*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 - 2*I*d + I)*sin(2*b*x + 2*a) - 1/2
) + 6*I*b*x*polylog(3, ((c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^
2 - 2*c*d - I*d^2 + I)*sin(2*b*x + 2*a))/(c^2 + d^2 + 2*d + 1)) - 6*I*b*x*
polylog(3, ((c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 - 2*c*d...
```

3.61.6 Sympy [F(-1)]

Timed out.

$$\int x^2 \arctan(c + d\cot(a + bx))\, dx = \text{Timed out}$$

input
```
integrate(x**2*atan(c+d*cot(b*x+a)),x)
```

output
```
Timed out
```

3.61.7 Maxima [F]

$$\int x^2 \arctan(c + d\cot(a + bx))\,dx = \int x^2 \arctan\left(d\cot\left(bx + a\right) + c\right)\,dx$$

input
```
integrate(x^2*arctan(c+d*cot(b*x+a)),x, algorithm="maxima")
```

output
```
-1/6*x^3*arctan2(-c*cos(2*b*x + 2*a) + (d + 1)*sin(2*b*x + 2*a) + c, (d +
1)*cos(2*b*x + 2*a) + c*sin(2*b*x + 2*a) + d - 1) - 1/6*x^3*arctan2(-c*cos
(2*b*x + 2*a) + (d - 1)*sin(2*b*x + 2*a) + c, -(d - 1)*cos(2*b*x + 2*a) -
c*sin(2*b*x + 2*a) - d - 1) + 4*b*d*integrate(1/3*(2*(c^2 + d^2 + 1)*x^3*c
os(2*b*x + 2*a)^2 + 2*c*d*x^3*sin(2*b*x + 2*a) + 2*(c^2 + d^2 + 1)*x^3*sin
(2*b*x + 2*a)^2 - (c^2 - d^2 + 1)*x^3*cos(2*b*x + 2*a) - (2*c*d*x^3*sin(2*
b*x + 2*a) + (c^2 - d^2 + 1)*x^3*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c
*d*x^3*cos(2*b*x + 2*a) - (c^2 - d^2 + 1)*x^3*sin(2*b*x + 2*a))*sin(4*b*x
+ 4*a))/(c^4 + d^4 + 2*(c^2 - 1)*d^2 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^
2 + 1)*cos(4*b*x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*co
s(2*b*x + 2*a)^2 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 + 1)*sin(4*b*x + 4
*a)^2 + 4*(c^4 + d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*sin(2*b*x + 2*a)^2 + 2
*c^2 + 2*(c^4 + d^4 - 2*(3*c^2 + 1)*d^2 + 2*c^2 - 2*(c^4 - d^4 + 2*c^2 + 1
)*cos(2*b*x + 2*a) - 4*(c*d^3 + (c^3 + c)*d)*sin(2*b*x + 2*a) + 1)*cos(4*b
*x + 4*a) - 4*(c^4 - d^4 + 2*c^2 + 1)*cos(2*b*x + 2*a) + 4*(2*c*d^3 - 2*(c
^3 + c)*d + 2*(c*d^3 + (c^3 + c)*d)*cos(2*b*x + 2*a) - (c^4 - d^4 + 2*c^2
+ 1)*sin(2*b*x + 2*a))*sin(4*b*x + 4*a) + 8*(c*d^3 + (c^3 + c)*d)*sin(2*b*
x + 2*a) + 1), x)
```

3.61.8 Giac [F]

$$\int x^2 \arctan(c + d\cot(a + bx))\,dx = \int x^2 \arctan\left(d\cot\left(bx + a\right) + c\right)\,dx$$

input
```
integrate(x^2*arctan(c+d*cot(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*arctan(d*cot(b*x + a) + c), x)
```

3.61.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \arctan(c + d\cot(a + bx))\,dx = \int x^2\,\mathrm{atan}(c + d\cot(a + b\,x))\;dx$$

input
```
int(x^2*atan(c + d*cot(a + b*x)),x)
```

output
```
int(x^2*atan(c + d*cot(a + b*x)), x)
```

3.62 $\int x \arctan(c + d\cot(a + bx))\,dx$

3.62.1 Optimal result

Integrand size = 13, antiderivative size = 303

$$\begin{aligned}\int x\arctan(c+d\cot(a+bx))\,dx &= \frac{1}{2}x^2\arctan(c+d\cot(a+bx)) \\ &\quad+\frac{1}{4}ix^2\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right) \\ &\quad-\frac{1}{4}ix^2\log\left(1-\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right) \\ &\quad+\frac{x\,\mathrm{PolyLog}\left(2,\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)}{4b} \\ &\quad-\frac{x\,\mathrm{PolyLog}\left(2,\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b} \\ &\quad+\frac{i\,\mathrm{PolyLog}\left(3,\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)}{8b^2} \\ &\quad-\frac{i\,\mathrm{PolyLog}\left(3,\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)}{8b^2}\end{aligned}$$

output
```
1/2*x^2*arctan(c+d*cot(b*x+a))+1/4*I*x^2*ln(1-(1+I*c-d)*exp(2*I*a+2*I*b*x)
/(1+I*c+d))-1/4*I*x^2*ln(1-(c+I*(1+d))*exp(2*I*a+2*I*b*x)/(c+I*(1-d)))+1/4
*x*polylog(2,(1+I*c-d)*exp(2*I*a+2*I*b*x)/(1+I*c+d))/b-1/4*x*polylog(2,(c+
I*(1+d))*exp(2*I*a+2*I*b*x)/(c+I*(1-d)))/b+1/8*I*polylog(3,(1+I*c-d)*exp(2
*I*a+2*I*b*x)/(1+I*c+d))/b^2-1/8*I*polylog(3,(c+I*(1+d))*exp(2*I*a+2*I*b*x
)/(c+I*(1-d)))/b^2
```

3.62.2 Mathematica [A] (verified)

Time = 2.47 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.91

$$\int x\arctan(c+d\cot(a+bx))\,dx$$
$$=\frac{4b^2x^2\arctan(c+d\cot(a+bx))+2ib^2x^2\log\left(1+\frac{(-c+i(1+d))e^{-2i(a+bx)}}{c+i(-1+d)}\right)-2ib^2x^2\log\left(1+\frac{(-c+i(-1+d))e^{-2i(a+}}{c+i(1+d)}\right.}{}$$

input
```
Integrate[x*ArcTan[c + d*Cot[a + b*x]],x]
```

output
```
(4*b^2*x^2*ArcTan[c + d*Cot[a + b*x]] + (2*I)*b^2*x^2*Log[1 + (-c + I*(1 +
 d))/((c + I*(-1 + d))*E^((2*I)*(a + b*x)))] - (2*I)*b^2*x^2*Log[1 + (-c +
 I*(-1 + d))/((c + I*(1 + d))*E^((2*I)*(a + b*x)))] - 2*b*x*PolyLog[2, (c
- I*(1 + d))/((c + I*(-1 + d))*E^((2*I)*(a + b*x)))] + 2*b*x*PolyLog[2, (I
 + c - I*d)/((c + I*(1 + d))*E^((2*I)*(a + b*x)))] + I*PolyLog[3, (c - I*(
1 + d))/((c + I*(-1 + d))*E^((2*I)*(a + b*x)))] - I*PolyLog[3, (I + c - I*
d)/((c + I*(1 + d))*E^((2*I)*(a + b*x)))])/(8*b^2)
```

3.62.3 Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.30, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {5700, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\arctan(d\cot(a+bx)+c)\,dx$$
$$\Big\downarrow \text{5700}$$
$$\frac{1}{2}b(ic-d+1)\int\frac{e^{2ia+2ibx}x^2}{ic-(ic-d+1)e^{2ia+2ibx}+d+1}dx-\frac{1}{2}b(-ic+d+1)\int\frac{e^{2ia+2ibx}x^2}{-ic-(-ic+d+1)e^{2ia+2ibx}-d+1}dx+\frac{1}{2}x^2\arctan(d\cot(a+bx)+c)$$
$$\Big\downarrow \text{2620}$$

$$\frac{1}{2}b(ic-d+1)\left(\frac{x^2\log\left(1-\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{2b(c-i(1-d))}-\frac{\int x\log\left(1-\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)dx}{b(c-i(1-d))}\right)-\frac{1}{2}b(-ic+d+1)\left(\frac{\int x\log\left(1-\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)dx}{b(c+i(d+1))}-\frac{x^2\log\left(1-\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{2b(c+i(d+1))}\right)+\frac{1}{2}x^2\arctan(d\cot(a+bx)+c)$$

$$\Big\downarrow \text{ 3011}$$

$$\frac{1}{2}b(ic-d+1)\left(\frac{x^2\log\left(1-\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{2b(c-i(1-d))}-\frac{\frac{ix\operatorname{PolyLog}\left(2,\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{2b}-\frac{i\int \operatorname{PolyLog}\left(2,\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)dx}{2b}}{b(c-i(1-d))}\right)-\frac{1}{2}b(-ic+d+1)\left(\frac{\frac{ix\operatorname{PolyLog}\left(2,\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{2b}-\frac{i\int \operatorname{PolyLog}\left(2,\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)dx}{2b}}{b(c+i(d+1))}-\frac{x^2\log\left(1-\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{2b(c+i(d+1))}\right)+\frac{1}{2}x^2\arctan(d\cot(a+bx)+c)$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{1}{2}b(ic-d+1)\left(\frac{x^2\log\left(1-\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{2b(c-i(1-d))}-\frac{\frac{ix\operatorname{PolyLog}\left(2,\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{2b}-\frac{\int e^{-2ia-2ibx}\operatorname{PolyLog}\left(2,\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)de^{2ia+2ibx}}{4b^2}}{b(c-i(1-d))}\right)$$
$$\frac{1}{2}b(-ic+d+1)\left(\frac{\frac{ix\operatorname{PolyLog}\left(2,\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{2b}-\frac{\int e^{-2ia-2ibx}\operatorname{PolyLog}\left(2,\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)de^{2ia+2ibx}}{4b^2}}{b(c+i(d+1))}-\frac{x^2\log\left(1-\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right.}{2b(c+i(d+1))}\right.$$
$$\frac{1}{2}x^2\arctan(d\cot(a+bx)+c)$$

$$\Big\downarrow \text{ 7143}$$

3.62. $\int x\arctan(c+d\cot(a+bx))\,dx$

$$\frac{1}{2}x^2\arctan(d\cot(a+bx)+c)+\frac{1}{2}b(ic-d+1)\left(\frac{x^2\log\left(1-\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{2b(c-i(1-d))}-\frac{\frac{ix\operatorname{PolyLog}\left(2,\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{2b}-\frac{\operatorname{PolyLog}\left(3,\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{4b^2}}{b(c-i(1-d))}\right)-\frac{1}{2}b(-ic+d+1)\left(\frac{\frac{ix\operatorname{PolyLog}\left(2,\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{2b}-\frac{\operatorname{PolyLog}\left(3,\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b^2}}{b(c+i(d+1))}-\frac{x^2\log\left(1-\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{2b(c+i(d+1))}\right)$$

input
```
Int[x*ArcTan[c + d*Cot[a + b*x]],x]
```

output
```
(x^2*ArcTan[c + d*Cot[a + b*x]])/2 + (b*(1 + I*c - d)*((x^2*Log[1 - ((1 +
I*c - d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c + d)])/(2*b*(c - I*(1 - d))) -
(((I/2)*x*PolyLog[2, ((1 + I*c - d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c + d)
])/b - PolyLog[3, ((1 + I*c - d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c + d)]/(
4*b^2))/(b*(c - I*(1 - d)))))/2 - (b*(1 - I*c + d)*(-1/2*(x^2*Log[1 - ((c
+ I*(1 + d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 - d))])/(b*(c + I*(1 + d))
) + (((I/2)*x*PolyLog[2, ((c + I*(1 + d))*E^((2*I)*a + (2*I)*b*x))/(c + I*
(1 - d))])/b - PolyLog[3, ((c + I*(1 + d))*E^((2*I)*a + (2*I)*b*x))/(c + I
*(1 - d))]/(4*b^2))/(b*(c + I*(1 + d)))))/2
```

3.62.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5700
```
Int[ArcTan[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Cot[a + b*x]]/(f*(m +
1))), x] + (Simp[b*((1 + I*c - d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(E^(
2*I*a + 2*I*b*x)/(1 + I*c + d - (1 + I*c - d)*E^(2*I*a + 2*I*b*x))), x], x]
 - Simp[b*((1 - I*c + d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(E^(2*I*a + 2
*I*b*x)/(1 - I*c - d - (1 - I*c + d)*E^(2*I*a + 2*I*b*x))), x], x]) /; Free
Q[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[(c - I*d)^2, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.62.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.15 (sec) , antiderivative size = 7501, normalized size of antiderivative = 24.76

method	result	size
risch	Expression too large to display	7501

input
```
int(x*arctan(c+d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.62.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1289 vs. $2(213) = 426$.

Time = 0.41 (sec) , antiderivative size = 1289, normalized size of antiderivative = 4.25

$$\int x \arctan(c + d \cot(a + bx)) \, dx = \text{Too large to display}$$

input
```
integrate(x*arctan(c+d*cot(b*x+a)),x, algorithm="fricas")
```

output
```
1/16*(8*b^2*x^2*arctan(d*cot(b*x + a) + c) + 2*b*x*dilog(-(c^2 + d^2 - (c^
2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d^2 - I)*sin
(2*b*x + 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1) + 1) + 2*b*x*dilog(-(c^2 +
d^2 - (c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d - I*d^2
+ I)*sin(2*b*x + 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1) + 1) - 2*b*x*dilog(
-(c^2 + d^2 - (c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d
 + I*d^2 - I)*sin(2*b*x + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1) + 1) - 2*b
*x*dilog(-(c^2 + d^2 - (c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2
 + 2*c*d - I*d^2 + I)*sin(2*b*x + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1) +
1) + 2*I*a^2*log(1/2*c^2 + I*c*d - 1/2*d^2 - 1/2*(c^2 + d^2 + 2*d + 1)*cos
(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 + 2*I*d + I)*sin(2*b*x + 2*a) + 1/2) -
2*I*a^2*log(1/2*c^2 + I*c*d - 1/2*d^2 - 1/2*(c^2 + d^2 - 2*d + 1)*cos(2*b*
x + 2*a) + 1/2*(I*c^2 + I*d^2 - 2*I*d + I)*sin(2*b*x + 2*a) + 1/2) - 2*I*a
^2*log(-1/2*c^2 + I*c*d + 1/2*d^2 + 1/2*(c^2 + d^2 + 2*d + 1)*cos(2*b*x +
2*a) + 1/2*(I*c^2 + I*d^2 + 2*I*d + I)*sin(2*b*x + 2*a) - 1/2) + 2*I*a^2*l
og(-1/2*c^2 + I*c*d + 1/2*d^2 + 1/2*(c^2 + d^2 - 2*d + 1)*cos(2*b*x + 2*a)
 + 1/2*(I*c^2 + I*d^2 - 2*I*d + I)*sin(2*b*x + 2*a) - 1/2) - 2*(-I*b^2*x^2
 + I*a^2)*log((c^2 + d^2 - (c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-
I*c^2 + 2*c*d + I*d^2 - I)*sin(2*b*x + 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d +
1)) - 2*(I*b^2*x^2 - I*a^2)*log((c^2 + d^2 - (c^2 - 2*I*c*d - d^2 + 1)*...
```

3.62.6 Sympy [F(-1)]

Timed out.

$$\int x \arctan(c + d \cot(a + bx)) \, dx = \text{Timed out}$$

input
```
integrate(x*atan(c+d*cot(b*x+a)),x)
```

output
```
Timed out
```

3.62.7 Maxima [F]

$$\int x \arctan(c + d\cot(a + bx))\,dx = \int x \arctan\left(d\cot\left(bx + a\right) + c\right)\,dx$$

input
```
integrate(x*arctan(c+d*cot(b*x+a)),x, algorithm="maxima")
```

output
```
-1/4*x^2*arctan2(-c*cos(2*b*x + 2*a) + (d + 1)*sin(2*b*x + 2*a) + c, (d +
1)*cos(2*b*x + 2*a) + c*sin(2*b*x + 2*a) + d - 1) - 1/4*x^2*arctan2(-c*cos
(2*b*x + 2*a) + (d - 1)*sin(2*b*x + 2*a) + c, -(d - 1)*cos(2*b*x + 2*a) -
c*sin(2*b*x + 2*a) - d - 1) + 2*b*d*integrate((2*(c^2 + d^2 + 1)*x^2*cos(2
*b*x + 2*a)^2 + 2*c*d*x^2*sin(2*b*x + 2*a) + 2*(c^2 + d^2 + 1)*x^2*sin(2*b
*x + 2*a)^2 - (c^2 - d^2 + 1)*x^2*cos(2*b*x + 2*a) - (2*c*d*x^2*sin(2*b*x
+ 2*a) + (c^2 - d^2 + 1)*x^2*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*c*d*x
^2*cos(2*b*x + 2*a) - (c^2 - d^2 + 1)*x^2*sin(2*b*x + 2*a))*sin(4*b*x + 4*
a))/(c^4 + d^4 + 2*(c^2 - 1)*d^2 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 +
1)*cos(4*b*x + 4*a)^2 + 4*(c^4 + d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*cos(2*
b*x + 2*a)^2 + (c^4 + d^4 + 2*(c^2 - 1)*d^2 + 2*c^2 + 1)*sin(4*b*x + 4*a)^
2 + 4*(c^4 + d^4 + 2*(c^2 + 1)*d^2 + 2*c^2 + 1)*sin(2*b*x + 2*a)^2 + 2*c^2
 + 2*(c^4 + d^4 - 2*(3*c^2 + 1)*d^2 + 2*c^2 - 2*(c^4 - d^4 + 2*c^2 + 1)*co
s(2*b*x + 2*a) - 4*(c*d^3 + (c^3 + c)*d)*sin(2*b*x + 2*a) + 1)*cos(4*b*x +
 4*a) - 4*(c^4 - d^4 + 2*c^2 + 1)*cos(2*b*x + 2*a) + 4*(2*c*d^3 - 2*(c^3 +
 c)*d + 2*(c*d^3 + (c^3 + c)*d)*cos(2*b*x + 2*a) - (c^4 - d^4 + 2*c^2 + 1)
*sin(2*b*x + 2*a))*sin(4*b*x + 4*a) + 8*(c*d^3 + (c^3 + c)*d)*sin(2*b*x +
2*a) + 1), x)
```

3.62.8 Giac [F]

$$\int x \arctan(c + d\cot(a + bx))\,dx = \int x \arctan\left(d\cot\left(bx + a\right) + c\right)\,dx$$

input
```
integrate(x*arctan(c+d*cot(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*arctan(d*cot(b*x + a) + c), x)
```

3.62.9 Mupad [F(-1)]

Timed out.

$$\int x \arctan(c + d\cot(a + bx))\, dx = \int x\,\mathrm{atan}(c + d\cot(a + b\,x))\ dx$$

input
```
int(x*atan(c + d*cot(a + b*x)),x)
```

output
```
int(x*atan(c + d*cot(a + b*x)), x)
```

3.63 $\int \arctan(c+d\cot(a+bx))\,dx$

3.63.1 Optimal result

Integrand size = 11, antiderivative size = 198

$$\int \arctan(c+d\cot(a+bx))\,dx = x\arctan(c+d\cot(a+bx)) + \frac{1}{2}ix\log\left(1-\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right) - \frac{1}{2}ix\log\left(1-\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right) + \frac{\operatorname{PolyLog}\left(2,\frac{(1+ic-d)e^{2ia+2ibx}}{1+ic+d}\right)}{4b} - \frac{\operatorname{PolyLog}\left(2,\frac{(c+i(1+d))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b}$$

output
```
x*arctan(c+d*cot(b*x+a))+1/2*I*x*ln(1-(1+I*c-d)*exp(2*I*a+2*I*b*x)/(1+I*c+
d))-1/2*I*x*ln(1-(c+I*(1+d))*exp(2*I*a+2*I*b*x)/(c+I*(1-d)))+1/4*polylog(2
,(1+I*c-d)*exp(2*I*a+2*I*b*x)/(1+I*c+d))/b-1/4*polylog(2,(c+I*(1+d))*exp(2
*I*a+2*I*b*x)/(c+I*(1-d)))/b
```

3.63.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1648 vs. $2(198) = 396$.

Time = 21.13 (sec) , antiderivative size = 1648, normalized size of antiderivative = 8.32

$$\int \arctan(c + d\cot(a + bx))\,dx = \text{Too large to display}$$

input
```
Integrate[ArcTan[c + d*Cot[a + b*x]],x]
```

output
```
x*ArcTan[c + d*Cot[a + b*x]] + (d*(4*a*Sqrt[-d^2]*ArcTan[(c*d + Tan[a + b*
x] + c^2*Tan[a + b*x])/d] + I*d*Log[1 + I*Tan[a + b*x]]*Log[(c*d - Sqrt[-d
^2] + Tan[a + b*x] + c^2*Tan[a + b*x])/(I + I*c^2 + c*d - Sqrt[-d^2])] + I
*d*Log[1 - I*Tan[a + b*x]]*Log[(c*d + Sqrt[-d^2] + Tan[a + b*x] + c^2*Tan[
a + b*x])/(-I - I*c^2 + c*d + Sqrt[-d^2])] - I*d*Log[1 + I*Tan[a + b*x]]*L
og[(c*d + Sqrt[-d^2] + Tan[a + b*x] + c^2*Tan[a + b*x])/(I + I*c^2 + c*d +
 Sqrt[-d^2])] - I*d*Log[1 - I*Tan[a + b*x]]*Log[(-(c*d) + Sqrt[-d^2] - (1
+ c^2)*Tan[a + b*x])/(I + I*c^2 - c*d + Sqrt[-d^2])] - I*d*PolyLog[2, ((1
+ c^2)*(1 - I*Tan[a + b*x]))/(1 + c^2 + I*c*d - I*Sqrt[-d^2])] + I*d*PolyL
og[2, ((1 + c^2)*(1 - I*Tan[a + b*x]))/(1 + c^2 + I*c*d + I*Sqrt[-d^2])] -
 I*d*PolyLog[2, ((1 + c^2)*(1 + I*Tan[a + b*x]))/(1 + c^2 - I*c*d - I*Sqrt
[-d^2])] + I*d*PolyLog[2, ((1 + c^2)*(1 + I*Tan[a + b*x]))/(1 + c^2 - I*c*
d + I*Sqrt[-d^2])])*((2*a)/(b*(-1 - c^2 - d^2 + Cos[2*(a + b*x)] + c^2*Cos
[2*(a + b*x)] - d^2*Cos[2*(a + b*x)] - 2*c*d*Sin[2*(a + b*x)])) - (2*(a +
b*x))/(b*(-1 - c^2 - d^2 + Cos[2*(a + b*x)] + c^2*Cos[2*(a + b*x)] - d^2*C
os[2*(a + b*x)] - 2*c*d*Sin[2*(a + b*x)]))))/((d*Log[1 - ((1 + c^2)*(1 - I
*Tan[a + b*x]))/(1 + c^2 + I*c*d - I*Sqrt[-d^2])]*Sec[a + b*x]^2)/(1 - I*T
an[a + b*x]) - (d*Log[1 - ((1 + c^2)*(1 - I*Tan[a + b*x]))/(1 + c^2 + I*c*
d + I*Sqrt[-d^2])]*Sec[a + b*x]^2)/(1 - I*Tan[a + b*x]) + (d*Log[(c*d + Sq
rt[-d^2] + Tan[a + b*x] + c^2*Tan[a + b*x])/(-I - I*c^2 + c*d + Sqrt[-d...
```

3.63.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.40, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {5692, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(d\cot(a+bx)+c)\,dx$$

$$\Big\downarrow \; 5692$$

$$b(ic-d+1)\int \frac{e^{2ia+2ibx}x}{ic-(ic-d+1)e^{2ia+2ibx}+d+1}dx-b(-ic+d+1)\int \frac{e^{2ia+2ibx}x}{-ic-(-ic+d+1)e^{2ia+2ibx}-d+1}dx+x\arctan(d\cot(a+bx)+c)$$

$$\Big\downarrow \; 2620$$

$$b(ic-d+1)\left(\frac{x\log\left(1-\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{2b(c-i(1-d))}-\frac{\int \log\left(1-\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)dx}{2b(c-i(1-d))}\right)-b(-ic+d+1)\left(\frac{\int \log\left(1-\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)dx}{2b(c+i(d+1))}-\frac{x\log\left(1-\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{2b(c+i(d+1))}\right)+x\arctan(d\cot(a+bx)+c)$$

$$\Big\downarrow \; 2715$$

$$b(ic-d+1)\left(\frac{i\int e^{-2ia-2ibx}\log\left(1-\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)de^{2ia+2ibx}}{4b^2(c-i(1-d))}+\frac{x\log\left(1-\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{2b(c-i(1-d))}\right)-b(-ic+d+1)\left(-\frac{i\int e^{-2ia-2ibx}\log\left(1-\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)de^{2ia+2ibx}}{4b^2(c+i(d+1))}-\frac{x\log\left(1-\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{2b(c+i(d+1))}\right)+x\arctan(d\cot(a+bx)+c)$$

$$\Big\downarrow \; 2838$$

$$x\arctan(d\cot(a+bx)+c)+b(ic-d+1)\left(\frac{x\log\left(1-\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{2b(c-i(1-d))}-\frac{i\,\text{PolyLog}\left(2,\frac{(ic-d+1)e^{2ia+2ibx}}{ic+d+1}\right)}{4b^2(c-i(1-d))}\right)-b(-ic+d+1)\left(\frac{i\,\text{PolyLog}\left(2,\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{4b^2(c+i(d+1))}-\frac{x\log\left(1-\frac{(c+i(d+1))e^{2ia+2ibx}}{c+i(1-d)}\right)}{2b(c+i(d+1))}\right)$$

input
```
Int[ArcTan[c + d*Cot[a + b*x]],x]
```

3.63. $\int \arctan(c+d\cot(a+bx))\,dx$

output
```
x*ArcTan[c + d*Cot[a + b*x]] + b*(1 + I*c - d)*((x*Log[1 - ((1 + I*c - d)*
E^((2*I)*a + (2*I)*b*x))/(1 + I*c + d)])/(2*b*(c - I*(1 - d))) - ((I/4)*Po
lyLog[2, ((1 + I*c - d)*E^((2*I)*a + (2*I)*b*x))/(1 + I*c + d)])/(b^2*(c -
 I*(1 - d)))) - b*(1 - I*c + d)*(-1/2*(x*Log[1 - ((c + I*(1 + d))*E^((2*I)
*a + (2*I)*b*x))/(c + I*(1 - d))])/(b*(c + I*(1 + d))) + ((I/4)*PolyLog[2,
 ((c + I*(1 + d))*E^((2*I)*a + (2*I)*b*x))/(c + I*(1 - d))])/(b^2*(c + I*(
1 + d))))
```

3.63.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5692
```
Int[ArcTan[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*ArcT
an[c + d*Cot[a + b*x]], x] + (Simp[b*(1 + I*c - d)   Int[x*(E^(2*I*a + 2*I*
b*x)/(1 + I*c + d - (1 + I*c - d)*E^(2*I*a + 2*I*b*x))), x], x] - Simp[b*(1
 - I*c + d)   Int[x*(E^(2*I*a + 2*I*b*x)/(1 - I*c - d - (1 - I*c + d)*E^(2*
I*a + 2*I*b*x))), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[(c + I*d)^2, -1]
```

3.63.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1144 vs. 2(168) = 336.

Time = 2.87 (sec) , antiderivative size = 1145, normalized size of antiderivative = 5.78

method	result	size
derivativedivides	Expression too large to display	1145
default	Expression too large to display	1145
risch	Expression too large to display	4986

input
```
int(arctan(c+d*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b/d*(-d*(1/2*Pi-arccot(cot(b*x+a)))*arctan(c+d*cot(b*x+a))+d^2*(-1/d*arc
tan(d*((c+d*cot(b*x+a))/d-c/d)+c)*arctan(-(c+d*cot(b*x+a))/d+c/d)-1/d^2*(-
1/2*I*d*arctan(d*((c+d*cot(b*x+a))/d-c/d)+c)*ln(1-(I+c+I*d)*(1+I*(d*((c+d*
cot(b*x+a))/d-c/d)+c))^2/((d*((c+d*cot(b*x+a))/d-c/d)+c)^2+1)/(-I*d+I-c))-
1/2*d*arctan(d*((c+d*cot(b*x+a))/d-c/d)+c)^2-1/4*d*polylog(2,(I+c+I*d)*(1+
I*(d*((c+d*cot(b*x+a))/d-c/d)+c))^2/((d*((c+d*cot(b*x+a))/d-c/d)+c)^2+1)/(
-I*d+I-c))+1/2*I*d^2*ln(1-(c-I*d+I)*(1+I*(d*((c+d*cot(b*x+a))/d-c/d)+c))^2
/((d*((c+d*cot(b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))*arctan(d*((c+d*cot(b*x+a)
)/d-c/d)+c)/(1+I*c+d)+1/2*I*d*ln(1-(c-I*d+I)*(1+I*(d*((c+d*cot(b*x+a))/d-c
/d)+c))^2/((d*((c+d*cot(b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))*arctan(d*((c+d*c
ot(b*x+a))/d-c/d)+c)/(1+I*c+d)+1/2*I*d/(c-I*d-I)*ln(1-(c-I*d+I)*(1+I*(d*((
c+d*cot(b*x+a))/d-c/d)+c))^2/((d*((c+d*cot(b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c
))*c*arctan(d*((c+d*cot(b*x+a))/d-c/d)+c)+1/2*d^2*arctan(d*((c+d*cot(b*x+a
))/d-c/d)+c)^2/(1+I*c+d)+1/4*d^2*polylog(2,(c-I*d+I)*(1+I*(d*((c+d*cot(b*x
+a))/d-c/d)+c))^2/((d*((c+d*cot(b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))/(1+I*c+d
)+1/2*d*arctan(d*((c+d*cot(b*x+a))/d-c/d)+c)^2/(1+I*c+d)+1/2*d/(c-I*d-I)*c
*arctan(d*((c+d*cot(b*x+a))/d-c/d)+c)^2+1/4*d*polylog(2,(c-I*d+I)*(1+I*(d*
((c+d*cot(b*x+a))/d-c/d)+c))^2/((d*((c+d*cot(b*x+a))/d-c/d)+c)^2+1)/(I*d+I
-c))/(1+I*c+d)+1/4*d/(c-I*d-I)*polylog(2,(c-I*d+I)*(1+I*(d*((c+d*cot(b*x+a
))/d-c/d)+c))^2/((d*((c+d*cot(b*x+a))/d-c/d)+c)^2+1)/(I*d+I-c))*c)))
```

3.63.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 965 vs. $2(140) = 280$.

Time = 0.40 (sec) , antiderivative size = 965, normalized size of antiderivative = 4.87

$$\int \arctan(c + d\cot(a + bx))\,dx = \text{Too large to display}$$

input
```
integrate(arctan(c+d*cot(b*x+a)),x, algorithm="fricas")
```

output
```
1/8*(8*b*x*arctan(d*cot(b*x + a) + c) - 2*I*a*log(1/2*c^2 + I*c*d - 1/2*d^
2 - 1/2*(c^2 + d^2 + 2*d + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 + 2*I*
d + I)*sin(2*b*x + 2*a) + 1/2) + 2*I*a*log(1/2*c^2 + I*c*d - 1/2*d^2 - 1/2
*(c^2 + d^2 - 2*d + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 - 2*I*d + I)*
sin(2*b*x + 2*a) + 1/2) + 2*I*a*log(-1/2*c^2 + I*c*d + 1/2*d^2 + 1/2*(c^2
+ d^2 + 2*d + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 + 2*I*d + I)*sin(2*
b*x + 2*a) - 1/2) - 2*I*a*log(-1/2*c^2 + I*c*d + 1/2*d^2 + 1/2*(c^2 + d^2
- 2*d + 1)*cos(2*b*x + 2*a) + 1/2*(I*c^2 + I*d^2 - 2*I*d + I)*sin(2*b*x +
2*a) - 1/2) - 2*(-I*b*x - I*a)*log((c^2 + d^2 - (c^2 + 2*I*c*d - d^2 + 1)*
cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d^2 - I)*sin(2*b*x + 2*a) + 2*d + 1
)/(c^2 + d^2 + 2*d + 1)) - 2*(I*b*x + I*a)*log((c^2 + d^2 - (c^2 - 2*I*c*d
 - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d - I*d^2 + I)*sin(2*b*x + 2*a
) + 2*d + 1)/(c^2 + d^2 + 2*d + 1)) - 2*(I*b*x + I*a)*log((c^2 + d^2 - (c^
2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d^2 - I)*sin
(2*b*x + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1)) - 2*(-I*b*x - I*a)*log((c^
2 + d^2 - (c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d - I*
d^2 + I)*sin(2*b*x + 2*a) - 2*d + 1)/(c^2 + d^2 - 2*d + 1)) + dilog(-(c^2
+ d^2 - (c^2 + 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (-I*c^2 + 2*c*d + I*d
^2 - I)*sin(2*b*x + 2*a) + 2*d + 1)/(c^2 + d^2 + 2*d + 1) + 1) + dilog(-(c
^2 + d^2 - (c^2 - 2*I*c*d - d^2 + 1)*cos(2*b*x + 2*a) + (I*c^2 + 2*c*d ...
```

3.63.6 Sympy [F]

$$\int \arctan(c + d\cot(a + bx))\,dx = \int \operatorname{atan}\left(c + d\cot\left(a + bx\right)\right)dx$$

input
```
integrate(atan(c+d*cot(b*x+a)),x)
```

output
```
Integral(atan(c + d*cot(a + b*x)), x)
```

3.63.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 532 vs. 2(140) = 280.

Time = 0.34 (sec) , antiderivative size = 532, normalized size of antiderivative = 2.69

$$\int \arctan(c + d\cot(a + bx))\, dx =$$

$$-\frac{d\left(\frac{8\,(bx+a)\arctan\left(\frac{cd+\left(c^2+1\right)\tan(bx+a)}{d}\right)}{d} - \frac{8\,(bx+a)\arctan\left(\frac{cd+\left(c^2+1\right)\tan(bx+a)}{d}\right)-4\arctan\left(\frac{cd+\left(c^2+1\right)\tan(bx+a)}{d}\right)\arctan\left(\frac{cd+\left(c^2\cdots\right.}{c^2\cdots}\right.}{\cdots}\right.}{\cdots}$$

input
```
integrate(arctan(c+d*cot(b*x+a)),x, algorithm="maxima")
```

output
```
-1/8*(d*(8*(b*x + a)*arctan((c*d + (c^2 + 1)*tan(b*x + a))/d)/d - (8*(b*x
+ a)*arctan((c*d + (c^2 + 1)*tan(b*x + a))/d) - 4*arctan((c*d + (c^2 + 1)*
tan(b*x + a))/d)*arctan2((c*d + (c^2 + d + 1)*tan(b*x + a))/(c^2 + d^2 + 2
*d + 1), -(c*d*tan(b*x + a) - c^2 - d - 1)/(c^2 + d^2 + 2*d + 1)) + 4*arct
an((c*d + (c^2 + 1)*tan(b*x + a))/d)*arctan2(-(c*d + (c^2 - d + 1)*tan(b*x
 + a))/(c^2 + d^2 - 2*d + 1), -(c*d*tan(b*x + a) - c^2 + d - 1)/(c^2 + d^2
 - 2*d + 1)) - (log(((c^2 + 1)*tan(b*x + a)^2 + c^2 + 1)/(c^2 + d^2 + 2*d
+ 1)) - log(((c^2 + 1)*tan(b*x + a)^2 + c^2 + 1)/(c^2 + d^2 - 2*d + 1)))*l
og((c^2 + 1)*d^2 + 2*(c^3 + c)*d*tan(b*x + a) + (c^4 + 2*c^2 + 1)*tan(b*x
+ a)^2) - 2*dilog(((I*c - 1)*tan(b*x + a) + I*d)/(c + I*d + I)) + 2*dilog(
((I*c + 1)*tan(b*x + a) + I*d)/(c + I*d - I)) + 2*dilog(-((I*c - 1)*tan(b*
x + a) + I*d)/(c - I*d + I)) - 2*dilog(-((I*c + 1)*tan(b*x + a) + I*d)/(c
- I*d - I)))/d) - 8*(b*x + a)*arctan(c + d/tan(b*x + a)) - 8*(b*x + a)*arc
tan((c*d + (c^2 + 1)*tan(b*x + a))/d))/b
```

3.63.8 Giac [F]

$$\int \arctan(c + d\cot(a + bx))\, dx = \int \arctan\left(d\cot\left(bx + a\right) + c\right)\, dx$$

input
```
integrate(arctan(c+d*cot(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arctan(d*cot(b*x + a) + c), x)
```

3.63.9 Mupad [F(-1)]

Timed out.

$$\int \arctan(c + d\cot(a + bx))\,dx = \int \operatorname{atan}(c + d\cot(a + b\,x))\,dx$$

input
```
int(atan(c + d*cot(a + b*x)),x)
```

output
```
int(atan(c + d*cot(a + b*x)), x)
```

3.64 $\int \frac{\arctan(c+d\cot(a+bx))}{x}\,dx$

3.64.1 Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{\arctan(c+d\cot(a+bx))}{x}\,dx=\operatorname{Int}\left(\frac{\arctan(c+d\cot(a+bx))}{x},x\right)$$

output
```
CannotIntegrate(arctan(c+d*cot(b*x+a))/x,x)
```

3.64.2 Mathematica [N/A]

Not integrable

Time = 1.98 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\arctan(c+d\cot(a+bx))}{x}\,dx=\int \frac{\arctan(c+d\cot(a+bx))}{x}\,dx$$

input
```
Integrate[ArcTan[c + d*Cot[a + b*x]]/x,x]
```

output
```
Integrate[ArcTan[c + d*Cot[a + b*x]]/x, x]
```

3.64.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(d\cot(a+bx)+c)}{x} \, dx$$
$$\Big\downarrow \; 7299$$
$$\int \frac{\arctan(d\cot(a+bx)+c)}{x} dx$$

input
```
Int[ArcTan[c + d*Cot[a + b*x]]/x,x]
```

output
```
$Aborted
```

3.64.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.64.4 Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\arctan\left(c+d\cot\left(bx+a\right)\right)}{x}dx$$

input
```
int(arctan(c+d*cot(b*x+a))/x,x)
```

output
```
int(arctan(c+d*cot(b*x+a))/x,x)
```

3.64.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\arctan(c+d\cot(a+bx))}{x}\,dx = \int \frac{\arctan\left(d\cot\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arctan(c+d*cot(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(arctan(d*cot(b*x + a) + c)/x, x)
```

3.64.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan(c+d\cot(a+bx))}{x}\,dx = \text{Timed out}$$

input
```
integrate(atan(c+d*cot(b*x+a))/x,x)
```

output
```
Timed out
```

3.64.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\arctan(c+d\cot(a+bx))}{x}\,dx = \text{Timed out}$$

input
```
integrate(arctan(c+d*cot(b*x+a))/x,x, algorithm="maxima")
```

output
```
Timed out
```

3.64.8 Giac [N/A]

Not integrable

Time = 3.78 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\arctan(c + d\cot(a + bx))}{x}\,dx = \int \frac{\arctan\left(d\cot\left(bx + a\right) + c\right)}{x}\,dx$$

input
```
integrate(arctan(c+d*cot(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(arctan(d*cot(b*x + a) + c)/x, x)
```

3.64.9 Mupad [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\arctan(c + d\cot(a + bx))}{x}\,dx = \int \frac{\mathrm{atan}(c + d\cot(a + b\,x))}{x}\,dx$$

input
```
int(atan(c + d*cot(a + b*x))/x,x)
```

output
```
int(atan(c + d*cot(a + b*x))/x, x)
```

3.65 $\int x^2 \arctan(c+(1-ic)\cot(a+bx))\,dx$

3.65.1 Optimal result

Integrand size = 21, antiderivative size = 154

$$\int x^2 \arctan(c+(1-ic)\cot(a+bx))\,dx = \frac{bx^4}{12}+\frac{1}{3}x^3\arctan(c+(1-ic)\cot(a+bx)) + \frac{1}{6}ix^3\log\left(1-ice^{2ia+2ibx}\right) + \frac{x^2\operatorname{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b} + \frac{ix\operatorname{PolyLog}\left(3,ice^{2ia+2ibx}\right)}{4b^2} - \frac{\operatorname{PolyLog}\left(4,ice^{2ia+2ibx}\right)}{8b^3}$$

output
```
1/12*b*x^4-1/3*x^3*arctan(-c-(1-I*c)*cot(b*x+a))+1/6*I*x^3*ln(1-I*c*exp(2*
I*a+2*I*b*x))+1/4*x^2*polylog(2,I*c*exp(2*I*a+2*I*b*x))/b+1/4*I*x*polylog(
3,I*c*exp(2*I*a+2*I*b*x))/b^2-1/8*polylog(4,I*c*exp(2*I*a+2*I*b*x))/b^3
```

3.65.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.88

$$\int x^2 \arctan(c+(1-ic)\cot(a+bx))\,dx = \frac{1}{24}\left(8x^3\arctan(c+(1-ic)\cot(a+bx)) + 4ix^3\log\left(1+\frac{ie^{-2i(a+bx)}}{c}\right) - \frac{6x^2\,\mathrm{PolyLog}\left(2,-\frac{ie^{-2i(a+bx)}}{c}\right)}{b} + \frac{6ix\,\mathrm{PolyLog}\left(3,-\frac{ie^{-2i(a+bx)}}{c}\right)}{b^2} + \frac{3\,\mathrm{PolyLog}\left(4,-\frac{ie^{-2i(a+bx)}}{c}\right)}{b^3}\right)$$

input
```
Integrate[x^2*ArcTan[c + (1 - I*c)*Cot[a + b*x]],x]
```

output
```
(8*x^3*ArcTan[c + (1 - I*c)*Cot[a + b*x]] + (4*I)*x^3*Log[1 + I/(c*E^((2*I
)*(a + b*x)))] - (6*x^2*PolyLog[2, (-I)/(c*E^((2*I)*(a + b*x)))])/b + ((6*
I)*x*PolyLog[3, (-I)/(c*E^((2*I)*(a + b*x)))])/b^2 + (3*PolyLog[4, (-I)/(c
*E^((2*I)*(a + b*x)))])/b^3)/24
```

3.65.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.29, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {5696, 25, 2615, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(c+(1-ic)\cot(a+bx))\,dx$$

$$\Big\downarrow \ 5696$$

$$\frac{1}{3}x^3\arctan(c+(1-ic)\cot(a+bx)) - \frac{1}{3}ib\int -\frac{x^3}{e^{2ia+2ibx}c+i}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}ib\int \frac{x^3}{e^{2ia+2ibx}c+i}dx+\frac{1}{3}x^3\arctan(c+(1-ic)\cot(a+bx))$$

$$\Big\downarrow \text{2615}$$

$$\frac{1}{3}ib\left(ic\int \frac{e^{2ia+2ibx}x^3}{e^{2ia+2ibx}c+i}dx-\frac{ix^4}{4}\right)+\frac{1}{3}x^3\arctan(c+(1-ic)\cot(a+bx))$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{3}ib\left(ic\left(\frac{3i\int x^2\log\left(1-ice^{2ia+2ibx}\right)dx}{2bc}-\frac{ix^3\log\left(1-ice^{2ia+2ibx}\right)}{2bc}\right)-\frac{ix^4}{4}\right)+\frac{1}{3}x^3\arctan(c+(1-ic)\cot(a+bx))$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{3}ib\left(ic\left(\frac{3i\left(\frac{ix^2\operatorname{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{2b}-\frac{i\int x\operatorname{PolyLog}\left(2,ice^{2ia+2ibx}\right)dx}{b}\right)}{2bc}-\frac{ix^3\log\left(1-ice^{2ia+2ibx}\right)}{2bc}\right)-\frac{ix^4}{4}\right)+\frac{1}{3}x^3\arctan(c+(1-ic)\cot(a+bx))$$

$$\Big\downarrow \text{7163}$$

$$\frac{1}{3}ib\left(ic\left(\frac{3i\left(\frac{ix^2\operatorname{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{2b}-\frac{i\left(\frac{i\int \operatorname{PolyLog}\left(3,ice^{2ia+2ibx}\right)dx}{2b}-\frac{ix\operatorname{PolyLog}\left(3,ice^{2ia+2ibx}\right)}{2b}\right)}{b}\right)}{2bc}-\frac{ix^3\log\left(1-ice^{2ia+2ibx}\right)}{2bc}\right.\right.$$

$$\frac{1}{3}x^3\arctan(c+(1-ic)\cot(a+bx))$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{3}ib\left(ic\left(\frac{3i\left(\frac{ix^2\operatorname{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{2b}-\frac{i\left(\frac{\int e^{-2ia-2ibx}\operatorname{PolyLog}\left(3,ice^{2ia+2ibx}\right)de^{2ia+2ibx}}{4b^2}-\frac{ix\operatorname{PolyLog}\left(3,ice^{2ia+2ibx}\right)}{2b}\right)}{b}\right)}{2bc}-\frac{ix^3\log\left(1\right.}{}\right.\right.$$

$$\frac{1}{3}x^3\arctan(c+(1-ic)\cot(a+bx))$$

3.65. $\int x^2\arctan(c+(1-ic)\cot(a+bx))\,dx$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{3}x^3\arctan(c+(1-ic)\cot(a+bx))+\frac{1}{3}ib\left(ic\left(\frac{3i\left(\frac{ix^2\operatorname{PolyLog}(2,ice^{2ia+2ibx})}{2b}-\frac{i\left(\frac{\operatorname{PolyLog}(4,ice^{2ia+2ibx})}{4b^2}-\frac{ix\operatorname{PolyLog}(3,ice^{2ia+2ibx})}{2b}\right)}{b}\right)}{2bc}-\frac{ix^3\log\left(1-ice^{2ia+2ibx}\right)}{2bc}\right)\right.$$

input
```
Int[x^2*ArcTan[c + (1 - I*c)*Cot[a + b*x]],x]
```

output
```
(x^3*ArcTan[c + (1 - I*c)*Cot[a + b*x]])/3 + (I/3)*b*((-1/4*I)*x^4 + I*c*(
((-1/2*I)*x^3*Log[1 - I*c*E^((2*I)*a + (2*I)*b*x)])/(b*c) + (((3*I)/2)*(((
I/2)*x^2*PolyLog[2, I*c*E^((2*I)*a + (2*I)*b*x)])/b - (I*(((-1/2*I)*x*Poly
Log[3, I*c*E^((2*I)*a + (2*I)*b*x)])/b + PolyLog[4, I*c*E^((2*I)*a + (2*I)
*b*x)]/(4*b^2)))/b))/(b*c)))
```

3.65.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5696
```
Int[ArcTan[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Cot[a + b*x]]/(f*(m +
1))), x] - Simp[I*(b/(f*(m + 1)))   Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(2
*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && E
qQ[(c - I*d)^2, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.65.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.42 (sec) , antiderivative size = 1487, normalized size of antiderivative = 9.66

method	result	size
risch	Expression too large to display	1487

input
```
int(-x^2*arctan(-c-(1-I*c)*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/12*I*(2*I*Pi-2*ln(I+c)+I*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(I+c))*
csgn(I*(I+c)/(exp(2*I*(b*x+a))-1))+I*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*(I
+c)/(exp(2*I*(b*x+a))-1))*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-
1))+I*Pi*csgn(I*(exp(2*I*(b*x+a))*c+I))*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp
(2*I*(b*x+a))-1))^2+I*Pi*csgn(I*exp(2*I*(b*x+a)))^3-I*Pi*csgn(I*(exp(2*I*(
b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b
*x+a))-1))-I*Pi*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))*csgn(e
xp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))^2+I*Pi*csgn(I/(exp(2*I*(b*x+a)
)-1))*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))^2+I*Pi*csgn((exp
(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))^3-I*Pi*csgn((exp(2*I*(b*x+a))*c+I
)/(exp(2*I*(b*x+a))-1))^2-I*Pi*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x
+a))-1))^3+I*Pi*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))*csgn(e
xp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))-I*Pi*csgn(I*(I+c))*csgn(I*(I+c
)/(exp(2*I*(b*x+a))-1))^2-I*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*
x+a))*(I+c)/(exp(2*I*(b*x+a))-1))^2-I*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn
(I*(I+c)/(exp(2*I*(b*x+a))-1))^2-I*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*
(exp(2*I*(b*x+a))*c+I))*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1)
)-I*Pi*csgn(I*(I+c)/(exp(2*I*(b*x+a))-1))*csgn(I*exp(2*I*(b*x+a))*(I+c)/(e
xp(2*I*(b*x+a))-1))^2+I*Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+a)
))+I*Pi*csgn(I*(I+c)/(exp(2*I*(b*x+a))-1))^3+I*Pi*csgn(I*exp(2*I*(b*x+a...
```

3.65.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.08

$$\int x^2 \arctan(c + (1 - ic)\cot(a + bx))\,dx$$
$$= \frac{2\,b^4x^4 + 4i\,b^3x^3\log\left(-\frac{(c+i)e^{(2i\,bx+2i\,a)}}{ce^{(2i\,bx+2i\,a)}+i}\right) + 6\,b^2x^2\mathrm{Li}_2\left(i\,ce^{(2i\,bx+2i\,a)}\right) - 2\,a^4 - 4i\,a^3\log\left(\frac{ce^{(2i\,bx+2i\,a)}+i}{c}\right) + 6i\,bxp}{24\,b^3}$$

input
```
integrate(-x^2*arctan(-c-(1-I*c)*cot(b*x+a)),x, algorithm="fricas")
```

output
```
1/24*(2*b^4*x^4 + 4*I*b^3*x^3*log(-(c + I)*e^(2*I*b*x + 2*I*a)/(c*e^(2*I*b
*x + 2*I*a) + I)) + 6*b^2*x^2*dilog(I*c*e^(2*I*b*x + 2*I*a)) - 2*a^4 - 4*I
*a^3*log((c*e^(2*I*b*x + 2*I*a) + I)/c) + 6*I*b*x*polylog(3, I*c*e^(2*I*b*
x + 2*I*a)) - 4*(-I*b^3*x^3 - I*a^3)*log(-I*c*e^(2*I*b*x + 2*I*a) + 1) - 3
*polylog(4, I*c*e^(2*I*b*x + 2*I*a)))/b^3
```

3.65.6 Sympy [F(-2)]

Exception generated.

$$\int x^2 \arctan(c + (1 - ic)\cot(a + bx))\,dx = \text{Exception raised: CoercionFailed}$$

input
```
integrate(-x**2*atan(-c-(1-I*c)*cot(b*x+a)),x)
```

output
```
Exception raised: CoercionFailed >> Cannot convert _t0**2 - exp(2*I*a) of
type <class 'sympy.core.add.Add'> to QQ_I[x,b,_t0,exp(I*a)]
```

3.65.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 309 vs. 2(110) = 220.

Time = 0.21 (sec) , antiderivative size = 309, normalized size of antiderivative = 2.01

$$\int x^2 \arctan(c + (1 - ic)\cot(a + bx))\,dx$$
$$= \frac{\frac{4\left((bx+a)^3 - 3(bx+a)^2 a + 3(bx+a)a^2\right)\arctan((-ic+1)\cot(bx+a)+c)}{b^2} + \frac{\left(-3i(bx+a)^4 + 12i(bx+a)^3 a - 18i(bx+a)^2 a^2 - 2\left(4i(bx+a)^3 - 9i(bx+\right.\right.}{}}{}$$

input
```
integrate(-x^2*arctan(-c-(1-I*c)*cot(b*x+a)),x, algorithm="maxima")
```

output
```
1/12*(4*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arctan((-I*c + 1
)*cot(b*x + a) + c)/b^2 + (-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b
*x + a)^2*a^2 - 2*(4*I*(b*x + a)^3 - 9*I*(b*x + a)^2*a + 9*I*(b*x + a)*a^2
)*arctan2(c*cos(2*b*x + 2*a), c*sin(2*b*x + 2*a) + 1) - 3*(4*I*(b*x + a)^2
 - 6*I*(b*x + a)*a + 3*I*a^2)*dilog(I*c*e^(2*I*b*x + 2*I*a)) + (4*(b*x + a
)^3 - 9*(b*x + a)^2*a + 9*(b*x + a)*a^2)*log(c^2*cos(2*b*x + 2*a)^2 + c^2*
sin(2*b*x + 2*a)^2 + 2*c*sin(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polylog(3,
I*c*e^(2*I*b*x + 2*I*a)) + 6*I*polylog(4, I*c*e^(2*I*b*x + 2*I*a)))*(I*c -
 1)/(b^2*(c + I)))/b
```

3.65.8 Giac [F]

$$\int x^2 \arctan(c+(1-ic)\cot(a+bx))\,dx = \int -x^2 \arctan\left(-(-i\,c+1)\cot\left(bx+a\right)-c\right)\,dx$$

input
```
integrate(-x^2*arctan(-c-(1-I*c)*cot(b*x+a)),x, algorithm="giac")
```

output
```
integrate(-x^2*arctan(-(-I*c + 1)*cot(b*x + a) - c), x)
```

3.65.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \arctan(c+(1-ic)\cot(a+bx))\,dx = \int x^2\,\mathrm{atan}(c-\cot(a+b\,x)\,\left(-1+c\,1\mathrm{i}\right))\,dx$$

input
```
int(x^2*atan(c - cot(a + b*x)*(c*1i - 1)),x)
```

output
```
int(x^2*atan(c - cot(a + b*x)*(c*1i - 1)), x)
```

3.66 $\int x \arctan(c+(1-ic)\cot(a+bx))\,dx$

3.66.1 Optimal result

Integrand size = 19, antiderivative size = 123

$$\int x \arctan(c+(1-ic)\cot(a+bx))\,dx = \frac{bx^3}{6}+\frac{1}{2}x^2\arctan(c+(1-ic)\cot(a+bx)) + \frac{1}{4}ix^2\log\left(1-ice^{2ia+2ibx}\right) + \frac{x\,\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b} + \frac{i\,\text{PolyLog}\left(3,ice^{2ia+2ibx}\right)}{8b^2}$$

output
```
1/6*b*x^3-1/2*x^2*arctan(-c-(1-I*c)*cot(b*x+a))+1/4*I*x^2*ln(1-I*c*exp(2*I
*a+2*I*b*x))+1/4*x*polylog(2,I*c*exp(2*I*a+2*I*b*x))/b+1/8*I*polylog(3,I*c
*exp(2*I*a+2*I*b*x))/b^2
```

3.66.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.89

$$\int x \arctan(c+(1-ic)\cot(a+bx))\,dx = \frac{1}{2}x^2\arctan(c+(1-ic)\cot(a+bx)) + \frac{i\left(2b^2x^2\log\left(1+\frac{ie^{-2i(a+bx)}}{c}\right)+2ibx\,\text{PolyLog}\left(2,-\frac{ie^{-2i(a+bx)}}{c}\right)+\text{PolyLog}\left(3,-\frac{ie^{-2i(a+bx)}}{c}\right)\right)}{8b^2}$$

input
```
Integrate[x*ArcTan[c + (1 - I*c)*Cot[a + b*x]],x]
```

output
```
(x^2*ArcTan[c + (1 - I*c)*Cot[a + b*x]])/2 + ((I/8)*(2*b^2*x^2*Log[1 + I/(
c*E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, (-I)/(c*E^((2*I)*(a + b*x))
)] + PolyLog[3, (-I)/(c*E^((2*I)*(a + b*x)))]))/b^2
```

3.66.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.26, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {5696, 25, 2615, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan(c+(1-ic)\cot(a+bx))\,dx$$

$$\Big\downarrow \text{5696}$$

$$\frac{1}{2}x^2\arctan(c+(1-ic)\cot(a+bx))-\frac{1}{2}ib\int -\frac{x^2}{e^{2ia+2ibx}c+i}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}ib\int \frac{x^2}{e^{2ia+2ibx}c+i}dx+\frac{1}{2}x^2\arctan(c+(1-ic)\cot(a+bx))$$

$$\Big\downarrow \text{2615}$$

$$\frac{1}{2}ib\left(ic\int \frac{e^{2ia+2ibx}x^2}{e^{2ia+2ibx}c+i}dx-\frac{ix^3}{3}\right)+\frac{1}{2}x^2\arctan(c+(1-ic)\cot(a+bx))$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{2}ib\left(ic\left(\frac{i\int x\log\left(1-ice^{2ia+2ibx}\right)dx}{bc}-\frac{ix^2\log\left(1-ice^{2ia+2ibx}\right)}{2bc}\right)-\frac{ix^3}{3}\right)+\frac{1}{2}x^2\arctan(c+(1-ic)\cot(a+bx))$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{2}ib\left(ic\left(\frac{i\left(\frac{ix\,\text{PolyLog}(2,ice^{2ia+2ibx})}{2b}-\frac{i\int \text{PolyLog}(2,ice^{2ia+2ibx})dx}{2b}\right)}{bc}-\frac{ix^2\log\left(1-ice^{2ia+2ibx}\right)}{2bc}\right)-\frac{ix^3}{3}\right)+\frac{1}{2}x^2\arctan(c+(1-ic)\cot(a+bx))$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{2}ib\left(ic\left(\frac{i\left(\frac{ix\,\text{PolyLog}(2,ice^{2ia+2ibx})}{2b}-\frac{\int e^{-2ia-2ibx}\,\text{PolyLog}(2,ice^{2ia+2ibx})de^{2ia+2ibx}}{4b^2}\right)}{bc}-\frac{ix^2\log\left(1-ice^{2ia+2ibx}\right)}{2bc}\right)-\frac{ix^3}{3}\right)+\frac{1}{2}x^2\arctan(c+(1-ic)\cot(a+bx))$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{2}x^2\arctan(c+(1-ic)\cot(a+bx))+\frac{1}{2}ib\left(ic\left(\frac{i\left(\frac{ix\,\text{PolyLog}(2,ice^{2ia+2ibx})}{2b}-\frac{\text{PolyLog}(3,ice^{2ia+2ibx})}{4b^2}\right)}{bc}-\frac{ix^2\log\left(1-ice^{2ia+2ibx}\right)}{2bc}\right)-\frac{ix^3}{3}\right)$$

input
```
Int[x*ArcTan[c + (1 - I*c)*Cot[a + b*x]],x]
```

output
```
(x^2*ArcTan[c + (1 - I*c)*Cot[a + b*x]])/2 + (I/2)*b*((-1/3*I)*x^3 + I*c*(
((-1/2*I)*x^2*Log[1 - I*c*E^((2*I)*a + (2*I)*b*x)])/(b*c) + (I*(((I/2)*x*P
olyLog[2, I*c*E^((2*I)*a + (2*I)*b*x)])/b - PolyLog[3, I*c*E^((2*I)*a + (2
*I)*b*x)]/(4*b^2)))/(b*c)))
```

3.66.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5696
```
Int[ArcTan[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Cot[a + b*x]]/(f*(m +
1))), x] - Simp[I*(b/(f*(m + 1)))   Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(2
*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && E
qQ[(c - I*d)^2, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.66.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.10 (sec) , antiderivative size = 1452, normalized size of antiderivative = 11.80

method	result	size
risch	Expression too large to display	1452

input
```
int(-x*arctan(-c-(1-I*c)*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I/b*a*ln(1+I*exp(I*(b*x+a))*(-I*c)^(1/2))*x+1/6*b*x^3+1/2*I/b*ln(1-I*
exp(2*I*(b*x+a))*c)*a*x+1/2*I*x^2*ln(exp(I*(b*x+a)))-1/4*I*ln(exp(2*I*(b*x
+a))*c+I)*x^2+1/8*I/b^2*polylog(3,I*exp(2*I*(b*x+a))*c)-1/2*I/b*a*ln(1-I*e
xp(I*(b*x+a))*(-I*c)^(1/2))*x+1/4/b*polylog(2,I*exp(2*I*(b*x+a))*c)*x+1/4/
b^2*polylog(2,I*exp(2*I*(b*x+a))*c)*a-1/8*I*(2*I*Pi-2*ln(I+c)+I*Pi*csgn(I/
(exp(2*I*(b*x+a))-1))*csgn(I*(I+c))*csgn(I*(I+c)/(exp(2*I*(b*x+a))-1))+I*P
i*csgn(I*exp(2*I*(b*x+a)))*csgn(I*(I+c)/(exp(2*I*(b*x+a))-1))*csgn(I*exp(2
*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))+I*Pi*csgn(I*(exp(2*I*(b*x+a))*c+I)
)*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))^2+I*Pi*csgn(I*exp(2*
I*(b*x+a)))^3-I*Pi*csgn(I*(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))*csg
n((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))-I*Pi*csgn(I*exp(2*I*(b*x+a)
)*(I+c)/(exp(2*I*(b*x+a))-1))*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a)
)-1))^2+I*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(exp(2*I*(b*x+a))*c+I)/(e
xp(2*I*(b*x+a))-1))^2+I*Pi*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1
))^3-I*Pi*csgn((exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))^2-I*Pi*csgn(I*
(exp(2*I*(b*x+a))*c+I)/(exp(2*I*(b*x+a))-1))^3+I*Pi*csgn(I*exp(2*I*(b*x+a)
)*(I+c)/(exp(2*I*(b*x+a))-1))*csgn(exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a)
)-1))-I*Pi*csgn(I*(I+c))*csgn(I*(I+c)/(exp(2*I*(b*x+a))-1))^2-I*Pi*csgn(I*
exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a))*(I+c)/(exp(2*I*(b*x+a))-1))^2-I*
Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(I+c)/(exp(2*I*(b*x+a))-1))^2-I*...
```

3.66.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.17

$$\int x\arctan(c+(1-ic)\cot(a+bx))\,dx$$
$$=\frac{4\,b^3x^3+6i\,b^2x^2\log\left(-\frac{(c+i)e^{(2i\,bx+2i\,a)}}{ce^{(2i\,bx+2i\,a)}+i}\right)+4\,a^3+6\,bx\mathrm{Li}_2\left(i\,ce^{(2i\,bx+2i\,a)}\right)+6i\,a^2\log\left(\frac{ce^{(2i\,bx+2i\,a)}+i}{c}\right)-6\,(-i\,b^2x}{24\,b^2}$$

input
```
integrate(-x*arctan(-c-(1-I*c)*cot(b*x+a)),x, algorithm="fricas")
```

output
```
1/24*(4*b^3*x^3 + 6*I*b^2*x^2*log(-(c + I)*e^(2*I*b*x + 2*I*a)/(c*e^(2*I*b
*x + 2*I*a) + I)) + 4*a^3 + 6*b*x*dilog(I*c*e^(2*I*b*x + 2*I*a)) + 6*I*a^2
*log((c*e^(2*I*b*x + 2*I*a) + I)/c) - 6*(-I*b^2*x^2 + I*a^2)*log(-I*c*e^(2
*I*b*x + 2*I*a) + 1) + 3*I*polylog(3, I*c*e^(2*I*b*x + 2*I*a)))/b^2
```

3.66.6 Sympy [F(-2)]

Exception generated.

$$\int x \arctan(c + (1 - ic)\cot(a + bx))\,dx = \text{Exception raised: CoercionFailed}$$

input
```
integrate(-x*atan(-c-(1-I*c)*cot(b*x+a)),x)
```

output
```
Exception raised: CoercionFailed >> Cannot convert _t0**2 - exp(2*I*a) of
type <class 'sympy.core.add.Add'> to QQ_I[x,b,_t0,exp(I*a)]
```

3.66.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 217 vs. 2(88) = 176.

Time = 0.19 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.76

$$\int x \arctan(c + (1 - ic)\cot(a + bx))\,dx$$
$$= \frac{\frac{6\left((bx+a)^2-2\,(bx+a)a\right)\arctan((-i\,c+1)\cot(bx+a)+c)}{b} + \frac{\left(-4i\,(bx+a)^3+12i\,(bx+a)^2a-6i\,bx\mathrm{Li}_2\left(i\,ce^{(2i\,bx+2i\,a)}\right)-6\left(i\,(bx+a)^2-2i\,(bx+a)a\right)}{}}{}$$

input
```
integrate(-x*arctan(-c-(1-I*c)*cot(b*x+a)),x, algorithm="maxima")
```

output
```
1/12*(6*((b*x + a)^2 - 2*(b*x + a)*a)*arctan((-I*c + 1)*cot(b*x + a) + c)/
b + (-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*dilog(I*c*e^(2*I*b*x
+ 2*I*a)) - 6*(I*(b*x + a)^2 - 2*I*(b*x + a)*a)*arctan2(c*cos(2*b*x + 2*a)
, c*sin(2*b*x + 2*a) + 1) + 3*((b*x + a)^2 - 2*(b*x + a)*a)*log(c^2*cos(2*
b*x + 2*a)^2 + c^2*sin(2*b*x + 2*a)^2 + 2*c*sin(2*b*x + 2*a) + 1) + 3*poly
log(3, I*c*e^(2*I*b*x + 2*I*a)))*(I*c - 1)/(b*(c + I)))/b
```

3.66.8 Giac [F]

$$\int x\arctan(c+(1-ic)\cot(a+bx))\,dx=\int -x\arctan\left(-(-i\,c+1)\cot\left(bx+a\right)-c\right)\,dx$$

input
```
integrate(-x*arctan(-c-(1-I*c)*cot(b*x+a)),x, algorithm="giac")
```

output
```
integrate(-x*arctan(-(-I*c + 1)*cot(b*x + a) - c), x)
```

3.66.9 Mupad [F(-1)]

Timed out.

$$\int x\arctan(c+(1-ic)\cot(a+bx))\,dx=\int x\,\mathrm{atan}(c-\cot(a+b\,x)\,\left(-1+c\,1\mathrm{i}\right))\,dx$$

input
```
int(x*atan(c - cot(a + b*x)*(c*1i - 1)),x)
```

output
```
int(x*atan(c - cot(a + b*x)*(c*1i - 1)), x)
```

3.67 $\int \arctan(c+(1-ic)\cot(a+bx))\,dx$

3.67.1 Optimal result

Integrand size = 17, antiderivative size = 85

$$\int \arctan(c+(1-ic)\cot(a+bx))\,dx=\frac{bx^2}{2}+x\arctan(c+(1-ic)\cot(a+bx))+\frac{1}{2}ix\log\left(1-ice^{2ia+2ibx}\right)+\frac{\operatorname{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b}$$

output
```
1/2*b*x^2-x*arctan(-c-(1-I*c)*cot(b*x+a))+1/2*I*x*ln(1-I*c*exp(2*I*a+2*I*b
*x))+1/4*polylog(2,I*c*exp(2*I*a+2*I*b*x))/b
```

3.67.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 929 vs. 2(85) = 170.

Time = 11.67 (sec) , antiderivative size = 929, normalized size of antiderivative = 10.93

$$\int \arctan(c+(1-ic)\cot(a+bx))\,dx=x\arctan(c+(1-ic)\cot(a+bx))$$

$$-\frac{}{(i+\cot(a+bx))(1+ic+(i+c)\cot(a+bx))\left(2ibx+\log\left(1-\frac{\sec(bx)((-i+c)\cos(a)+i(i+c)\sin(a))(\cos(a+bx)-i\,s}{2c}\right.\right.}$$

input
```
Integrate[ArcTan[c + (1 - I*c)*Cot[a + b*x]],x]
```

output
```
x*ArcTan[c + (1 - I*c)*Cot[a + b*x]] - (I*x*Csc[a + b*x]^2*(2*b*x*Log[2*Co
s[b*x]*(Cos[b*x] - I*Sin[b*x])] + I*Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((I
+ c)*Cos[a + b*x] + (1 + I*c)*Sin[a + b*x]))/(2*c)]*Log[1 - I*Tan[b*x]] -
I*Log[(Sec[b*x]*((1 - I*c)*Cos[a + b*x] + (-I + c)*Sin[a + b*x]))/(2*Cos[a
] - (2*I)*Sin[a])]*Log[1 + I*Tan[b*x]] + I*PolyLog[2, -Cos[2*b*x] + I*Sin[
2*b*x]] + I*PolyLog[2, (Sec[b*x]*((-I + c)*Cos[a] + I*(I + c)*Sin[a])*(Cos
[a + b*x] - I*Sin[a + b*x]))/(2*c)] - I*PolyLog[2, (Sec[b*x]*((1 + I*c)*Co
s[a] - (I + c)*Sin[a])*(Cos[a + b*x] + I*Sin[a + b*x]))/2])*(Cos[b*x] - I*
Sin[b*x])*(Cos[b*x] + I*Sin[b*x]))/((I + Cot[a + b*x])*(1 + I*c + (I + c)*
Cot[a + b*x])*((2*I)*b*x + Log[1 - (Sec[b*x]*((-I + c)*Cos[a] + I*(I + c)*
Sin[a])*(Cos[a + b*x] - I*Sin[a + b*x]))/(2*c)] + Log[1 + (Sec[b*x]*((-1 -
 I*c)*Cos[a] + (I + c)*Sin[a])*(Cos[a + b*x] + I*Sin[a + b*x]))/2] + ((-I
+ c)*Cos[a + b*x]*(Log[1 - I*Tan[b*x]] - Log[1 + I*Tan[b*x]]))/((I + c)*Co
s[a + b*x] + (1 + I*c)*Sin[a + b*x]) + ((I + c)*(Log[1 - I*Tan[b*x]] - Log
[1 + I*Tan[b*x]])*Sin[a + b*x])/((1 - I*c)*Cos[a + b*x] + (-I + c)*Sin[a +
 b*x]) + 2*b*x*Tan[b*x] + I*Log[1 - (Sec[b*x]*((-I + c)*Cos[a] + I*(I + c)
*Sin[a])*(Cos[a + b*x] - I*Sin[a + b*x]))/(2*c)]*Tan[b*x] - I*Log[1 + (Sec
[b*x]*((-1 - I*c)*Cos[a] + (I + c)*Sin[a])*(Cos[a + b*x] + I*Sin[a + b*x])
)/2]*Tan[b*x] - I*Log[1 - I*Tan[b*x]]*Tan[b*x] + I*Cos[a]^2*Log[1 + I*Tan[
b*x]]*Tan[b*x] + I*Log[1 + I*Tan[b*x]]*Sin[a]^2*Tan[b*x] + (I*Log[(Sec[...
```

3.67.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.26, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {5688, 25, 2615, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(c+(1-ic)\cot(a+bx))\,dx$$

$$\Big\downarrow \text{ 5688}$$

$$x\arctan(c+(1-ic)\cot(a+bx))-ib\int -\frac{x}{e^{2ia+2ibx}c+i}dx$$

$$\Big\downarrow \text{ 25}$$

$$ib\int \frac{x}{e^{2ia+2ibx}c+i}dx+x\arctan(c+(1-ic)\cot(a+bx))$$

$$\Big\downarrow \text{ 2615}$$

$$ib\left(ic\int \frac{e^{2ia+2ibx}x}{e^{2ia+2ibx}c+i}dx - \frac{ix^2}{2}\right) + x\arctan(c+(1-ic)\cot(a+bx))$$

$\Big\downarrow$ 2620

$$ib\left(ic\left(\frac{i\int \log\left(1-ice^{2ia+2ibx}\right)dx}{2bc} - \frac{ix\log\left(1-ice^{2ia+2ibx}\right)}{2bc}\right) - \frac{ix^2}{2}\right) + x\arctan(c+(1-ic)\cot(a+bx))$$

$\Big\downarrow$ 2715

$$ib\left(ic\left(\frac{\int e^{-2ia-2ibx}\log\left(1-ice^{2ia+2ibx}\right)de^{2ia+2ibx}}{4b^2c} - \frac{ix\log\left(1-ice^{2ia+2ibx}\right)}{2bc}\right) - \frac{ix^2}{2}\right) + x\arctan(c+(1-ic)\cot(a+bx))$$

$\Big\downarrow$ 2838

$$x\arctan(c+(1-ic)\cot(a+bx)) + ib\left(ic\left(-\frac{\text{PolyLog}\left(2,ice^{2ia+2ibx}\right)}{4b^2c} - \frac{ix\log\left(1-ice^{2ia+2ibx}\right)}{2bc}\right) - \frac{ix^2}{2}\right)$$

input
```
Int[ArcTan[c + (1 - I*c)*Cot[a + b*x]],x]
```

output
```
x*ArcTan[c + (1 - I*c)*Cot[a + b*x]] + I*b*((-1/2*I)*x^2 + I*c*(((-1/2*I)*
x*Log[1 - I*c*E^((2*I)*a + (2*I)*b*x)])/(b*c) - PolyLog[2, I*c*E^((2*I)*a
+ (2*I)*b*x)]/(4*b^2*c)))
```

3.67.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5688
```
Int[ArcTan[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*ArcT
an[c + d*Cot[a + b*x]], x] - Simp[I*b   Int[x/(c - I*d - c*E^(2*I*a + 2*I*b
*x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - I*d)^2, -1]
```

3.67.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 583 vs. $2(73) = 146$.

Time = 1.07 (sec) , antiderivative size = 584, normalized size of antiderivative = 6.87

method	result
derivativedivides	$-\frac{\frac{\arctan(-c+\cot(bx+a)(ic-1))\ln(-i+\cot(bx+a)(ic-1)-c)c^2}{2i+2c}+\frac{2i\arctan(-c+\cot(bx+a)(ic-1))\ln(-i+\cot(bx+a)(ic-1)-c)c}{2i+2c}-\frac{\text{arct}}{}}{}$
default	$-\frac{\frac{\arctan(-c+\cot(bx+a)(ic-1))\ln(-i+\cot(bx+a)(ic-1)-c)c^2}{2i+2c}+\frac{2i\arctan(-c+\cot(bx+a)(ic-1))\ln(-i+\cot(bx+a)(ic-1)-c)c}{2i+2c}-\frac{\text{arct}}{}}{}$
risch	Expression too large to display

input
```
int(-arctan(-c-(1-I*c)*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/b/(-1+I*c)*(arctan(-c+cot(b*x+a)*(-1+I*c))/(2*I+2*c)*ln(-I+cot(b*x+a)*(
-1+I*c)-c)*c^2+2*I*arctan(-c+cot(b*x+a)*(-1+I*c))/(2*I+2*c)*ln(-I+cot(b*x+
a)*(-1+I*c)-c)*c-arctan(-c+cot(b*x+a)*(-1+I*c))/(2*I+2*c)*ln(-I+cot(b*x+a)
*(-1+I*c)-c)-arctan(-c+cot(b*x+a)*(-1+I*c))/(2*I+2*c)*ln(cot(b*x+a)*(-1+I*
c)+c+I)*c^2-2*I*arctan(-c+cot(b*x+a)*(-1+I*c))/(2*I+2*c)*ln(cot(b*x+a)*(-1
+I*c)+c+I)*c+arctan(-c+cot(b*x+a)*(-1+I*c))/(2*I+2*c)*ln(cot(b*x+a)*(-1+I*
c)+c+I)-(-1+I*c)^2*(-1/2/(I+c)*(1/2*I*(dilog(-1/2*I*(cot(b*x+a)*(-1+I*c)-c
+I))+ln(-I+cot(b*x+a)*(-1+I*c)-c)*ln(-1/2*I*(cot(b*x+a)*(-1+I*c)-c+I)))-1/
4*I*ln(-I+cot(b*x+a)*(-1+I*c)-c)^2)+1/2/(I+c)*(1/2*I*(dilog(-1/2*(cot(b*x+
a)*(-1+I*c)-c+I)/c)+ln(cot(b*x+a)*(-1+I*c)+c+I)*ln(-1/2*(cot(b*x+a)*(-1+I*
c)-c+I)/c))-1/2*I*(dilog((-I+cot(b*x+a)*(-1+I*c)-c)/(-2*I-2*c))+ln(cot(b*x
+a)*(-1+I*c)+c+I)*ln((-I+cot(b*x+a)*(-1+I*c)-c)/(-2*I-2*c))))))
```

3.67.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.32

$$\int \arctan(c+(1-ic)\cot(a+bx))\,dx = \frac{2\,b^2x^2 + 2i\,bx\log\left(-\frac{(c+i)e^{(2i\,bx+2i\,a)}}{ce^{(2i\,bx+2i\,a)}+i}\right) - 2\,a^2 - 2\left(-i\,bx - i\,a\right)\log\left(-i\,ce^{(2i\,bx+2i\,a)}+1\right) - 2i\,a\log\left(\frac{ce^{(2i\,bx+2i\,a)} \cdots}{c}\right. \cdots}{4\,b}$$

input
```
integrate(-arctan(-c-(1-I*c)*cot(b*x+a)),x, algorithm="fricas")
```

output
```
1/4*(2*b^2*x^2 + 2*I*b*x*log(-(c + I)*e^(2*I*b*x + 2*I*a)/(c*e^(2*I*b*x +
2*I*a) + I)) - 2*a^2 - 2*(-I*b*x - I*a)*log(-I*c*e^(2*I*b*x + 2*I*a) + 1)
- 2*I*a*log((c*e^(2*I*b*x + 2*I*a) + I)/c) + dilog(I*c*e^(2*I*b*x + 2*I*a)
))/b
```

3.67.6 Sympy [F(-2)]

Exception generated.

$$\int \arctan(c+(1-ic)\cot(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

input
```
integrate(-atan(-c-(1-I*c)*cot(b*x+a)),x)
```

output
```
Exception raised: CoercionFailed >> Cannot convert _t0**2 - exp(2*I*a) of
type <class 'sympy.core.add.Add'> to QQ_I[b,_t0,exp(I*a)]
```

3.67.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(64) = 128$.

Time = 0.36 (sec) , antiderivative size = 458, normalized size of antiderivative = 5.39

$$\int \arctan(c + (1 - ic)\cot(a + bx))\,dx =$$

$$-\frac{(ic-1)\left(\frac{4i\,(bx+a)\log\left(-\frac{2\left(i\,c^2-\left(c^2+1\right)\tan(bx+a)-2\,c-i\right)}{-2i\,c^2+2\left(c^2+1\right)\tan(bx+a)-2i}\right)}{i\,c-1} - \frac{i\left(4\,(bx+a)\left(\log\left(-i\,c^2+\left(c^2+1\right)\tan(bx+a)+2\,c+i\right)-\log\left(-i\,c^2+\left(c^2+1\right)\mathrm{t}\right.\right.}{}\right.}{}$$

input
```
integrate(-arctan(-c-(1-I*c)*cot(b*x+a)),x, algorithm="maxima")
```

output
```
-1/8*((I*c - 1)*(4*I*(b*x + a)*log(-2*(I*c^2 - (c^2 + 1)*tan(b*x + a) - 2*
c - I)/(-2*I*c^2 + 2*(c^2 + 1)*tan(b*x + a) - 2*I))/(I*c - 1) - I*(4*(b*x
+ a)*(log(-I*c^2 + (c^2 + 1)*tan(b*x + a) + 2*c + I) - log(-I*c^2 + (c^2 +
 1)*tan(b*x + a) - I)) - 2*I*log(-I*c^2 + (c^2 + 1)*tan(b*x + a) + 2*c + I
)*log(-1/2*((I*c + 1)*tan(b*x + a) + c + I)/c + 1) + 2*I*log(-I*c^2 + (c^2
 + 1)*tan(b*x + a) + 2*c + I)*log(tan(b*x + a) - I) - 2*I*log(1/2*(c - I)*
tan(b*x + a) - 1/2*I*c + 1/2)*log(tan(b*x + a) - I) - I*log(tan(b*x + a) -
 I)^2 - 2*I*log(c^2 + 1)*log(I*tan(b*x + a) + 1) + 2*I*log(tan(b*x + a) -
I)*log(-1/2*I*tan(b*x + a) + 1/2) + 2*I*log(c^2 + 1)*log(-I*tan(b*x + a) +
 1) - 2*I*dilog(-1/2*(c - I)*tan(b*x + a) + 1/2*I*c + 1/2) - 2*I*dilog(1/2
*((I*c + 1)*tan(b*x + a) + c + I)/c) + 2*I*dilog(1/2*I*tan(b*x + a) + 1/2)
)/(I*c - 1)) - 8*(b*x + a)*arctan(c + (-I*c + 1)/tan(b*x + a)) + 4*(-I*b*x
 - I*a)*log(-2*(I*c^2 - (c^2 + 1)*tan(b*x + a) - 2*c - I)/(-2*I*c^2 + 2*(c
^2 + 1)*tan(b*x + a) - 2*I)))/b
```

3.67.8 Giac [F]

$$\int \arctan(c+(1-ic)\cot(a+bx))\,dx=\int -\arctan\left(-(-i\,c+1)\cot\left(bx+a\right)-c\right)\,dx$$

input
```
integrate(-arctan(-c-(1-I*c)*cot(b*x+a)),x, algorithm="giac")
```

output
```
integrate(-arctan(-(-I*c + 1)*cot(b*x + a) - c), x)
```

3.67.9 Mupad [F(-1)]

Timed out.

$$\int \arctan(c+(1-ic)\cot(a+bx))\,dx=\int \mathrm{atan}(c-\cot(a+b\,x)\,\left(-1+c\,1\mathrm{i}\right))\,dx$$

input
```
int(atan(c - cot(a + b*x)*(c*1i - 1)),x)
```

output
```
int(atan(c - cot(a + b*x)*(c*1i - 1)), x)
```

3.68 $\int \frac{\arctan(c+(1-ic)\cot(a+bx))}{x}\,dx$

3.68.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{\arctan(c+(1-ic)\cot(a+bx))}{x}\,dx = \operatorname{Int}\left(\frac{\arctan(c+(1-ic)\cot(a+bx))}{x},x\right)$$

output
```
CannotIntegrate(-arctan(-c-(1-I*c)*cot(b*x+a))/x,x)
```

3.68.2 Mathematica [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{\arctan(c+(1-ic)\cot(a+bx))}{x}\,dx = \int \frac{\arctan(c+(1-ic)\cot(a+bx))}{x}\,dx$$

input
```
Integrate[ArcTan[c + (1 - I*c)*Cot[a + b*x]]/x,x]
```

output
```
Integrate[ArcTan[c + (1 - I*c)*Cot[a + b*x]]/x, x]
```

3.68.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(c+(1-ic)\cot(a+bx))}{x} \, dx$$
$$\Big\downarrow \; 7299$$
$$\int \frac{\arctan(c+(1-ic)\cot(a+bx))}{x}dx$$

input
```
Int[ArcTan[c + (1 - I*c)*Cot[a + b*x]]/x,x]
```

output
```
$Aborted
```

3.68.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.68.4 Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int -\frac{\arctan\left(-c-\left(-ic+1\right)\cot\left(bx+a\right)\right)}{x}dx$$

input
```
int(-arctan(-c-(1-I*c)*cot(b*x+a))/x,x)
```

output
```
int(-arctan(-c-(1-I*c)*cot(b*x+a))/x,x)
```

3.68.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.76

$$\int \frac{\arctan(c + (1 - ic)\cot(a + bx))}{x} \, dx = \int -\frac{\arctan\left(-(-i\,c + 1)\cot\left(bx + a\right) - c\right)}{x} \, dx$$

input
```
integrate(-arctan(-c-(1-I*c)*cot(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(1/2*I*log(-(c + I)*e^(2*I*b*x + 2*I*a)/(c*e^(2*I*b*x + 2*I*a) + I
))/x, x)
```

3.68.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan(c + (1 - ic)\cot(a + bx))}{x} \, dx = \text{Timed out}$$

input
```
integrate(-atan(-c-(1-I*c)*cot(b*x+a))/x,x)
```

output
```
Timed out
```

3.68.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(c + (1 - ic)\cot(a + bx))}{x} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(-arctan(-c-(1-I*c)*cot(b*x+a))/x,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c-1>0)', see `assume?` for more
details)Is
```

3.68.8 Giac [N/A]

Not integrable

Time = 1.80 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{\arctan(c+(1-ic)\cot(a+bx))}{x}\,dx=\int -\frac{\arctan\left(-(-i\,c+1)\cot\left(bx+a\right)-c\right)}{x}\,dx$$

input
```
integrate(-arctan(-c-(1-I*c)*cot(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(-arctan(-(-I*c + 1)*cot(b*x + a) - c)/x, x)
```

3.68.9 Mupad [N/A]

Not integrable

Time = 1.06 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{\arctan(c+(1-ic)\cot(a+bx))}{x}\,dx=\int \frac{\mathrm{atan}(c-\cot(a+b\,x)\,\left(-1+c\,1\mathrm{i}\right))}{x}\,dx$$

input
```
int(atan(c - cot(a + b*x)*(c*1i - 1))/x,x)
```

output
```
int(atan(c - cot(a + b*x)*(c*1i - 1))/x, x)
```

3.69 $\int x^2 \arctan(c+(-1-ic)\cot(a+bx))\,dx$

3.69.1 Optimal result

Integrand size = 21, antiderivative size = 155

$$\begin{aligned}\int x^2\arctan(c+(-1-ic)\cot(a+bx))\,dx &= -\frac{bx^4}{12}+\frac{1}{3}x^3\arctan(c-(1+ic)\cot(a+bx))\\ &\quad-\frac{1}{6}ix^3\log\left(1+ice^{2ia+2ibx}\right)\\ &\quad-\frac{x^2\operatorname{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}\\ &\quad-\frac{ix\operatorname{PolyLog}\left(3,-ice^{2ia+2ibx}\right)}{4b^2}\\ &\quad+\frac{\operatorname{PolyLog}\left(4,-ice^{2ia+2ibx}\right)}{8b^3}\end{aligned}$$

output
```
-1/12*b*x^4-1/3*x^3*arctan(-c+(1+I*c)*cot(b*x+a))-1/6*I*x^3*ln(1+I*c*exp(2
*I*a+2*I*b*x))-1/4*x^2*polylog(2,-I*c*exp(2*I*a+2*I*b*x))/b-1/4*I*x*polylo
g(3,-I*c*exp(2*I*a+2*I*b*x))/b^2+1/8*polylog(4,-I*c*exp(2*I*a+2*I*b*x))/b^
3
```

3.69.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.90

$$\int x^2 \arctan(c+(-1-ic)\cot(a+bx))\,dx = \frac{1}{3}x^3 \arctan(c+(-1-ic)\cot(a+bx)) - \frac{4ib^3x^3\log\left(1-\frac{ie^{-2i(a+bx)}}{c}\right) - 6b^2x^2\operatorname{PolyLog}\left(2,\frac{ie^{-2i(a+bx)}}{c}\right) + 6ibx\operatorname{PolyLog}\left(3,\frac{ie^{-2i(a+bx)}}{c}\right) + 3\operatorname{PolyLog}\left(\right.}{24b^3}$$

input
```
Integrate[x^2*ArcTan[c + (-1 - I*c)*Cot[a + b*x]],x]
```

output
```
(x^3*ArcTan[c + (-1 - I*c)*Cot[a + b*x]])/3 - ((4*I)*b^3*x^3*Log[1 - I/(c*
E^((2*I)*(a + b*x)))] - 6*b^2*x^2*PolyLog[2, I/(c*E^((2*I)*(a + b*x)))] +
(6*I)*b*x*PolyLog[3, I/(c*E^((2*I)*(a + b*x)))] + 3*PolyLog[4, I/(c*E^((2*
I)*(a + b*x)))])/(24*b^3)
```

3.69.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.28, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5696, 2615, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(c+(-1-ic)\cot(a+bx))\,dx$$

$$\Big\downarrow \text{ 5696}$$

$$\frac{1}{3}x^3\arctan(c-(1+ic)\cot(a+bx)) - \frac{1}{3}ib\int \frac{x^3}{i-ce^{2ia+2ibx}}dx$$

$$\Big\downarrow \text{ 2615}$$

$$\frac{1}{3}x^3\arctan(c-(1+ic)\cot(a+bx)) - \frac{1}{3}ib\left(-ic\int \frac{e^{2ia+2ibx}x^3}{i-ce^{2ia+2ibx}}dx - \frac{ix^4}{4}\right)$$

$$\Big\downarrow \text{ 2620}$$

$$\frac{1}{3}x^3\arctan(c-(1+ic)\cot(a+bx)) - \frac{1}{3}ib\left(-ic\left(\frac{ix^3\log\left(1+ice^{2ia+2ibx}\right)}{2bc} - \frac{3i\int x^2\log\left(ie^{2ia+2ibx}c+1\right)dx}{2bc}\right) - \frac{ix^4}{4}\right)$$

$$\Big\downarrow \; 3011$$

$$\frac{1}{3}x^3\arctan(c-(1+ic)\cot(a+bx))-\frac{1}{3}ib\left(-ic\left(\frac{ix^3\log\left(1+ice^{2ia+2ibx}\right)}{2bc}-\frac{3i\left(\frac{ix^2\operatorname{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{2b}-\frac{i\int x\operatorname{PolyLog}\left(2,-ice^{2ia+2ibx}\right)dx}{b}\right)}{2bc}\right)-\frac{ix^4}{4}\right)$$

$$\Big\downarrow \; 7163$$

$$\frac{1}{3}x^3\arctan(c-(1+ic)\cot(a+bx))-\frac{1}{3}ib\left(-ic\left(\frac{ix^3\log\left(1+ice^{2ia+2ibx}\right)}{2bc}-\frac{3i\left(\frac{ix^2\operatorname{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{2b}-\frac{i\left(\frac{i\int\operatorname{PolyLog}\left(3,-ice^{2ia+2ibx}\right)dx}{2b}-\frac{ix\operatorname{PolyLog}\left(3,-ice^{2ia+2ib\ldots}\right)}{2b}\right.}{b}\right.}{2bc}\right.\right.$$

$$\Big\downarrow \; 2720$$

$$\frac{1}{3}x^3\arctan(c-(1+ic)\cot(a+bx))-\frac{1}{3}ib\left(-ic\left(\frac{ix^3\log\left(1+ice^{2ia+2ibx}\right)}{2bc}-\frac{3i\left(\frac{ix^2\operatorname{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{2b}-\frac{i\left(\frac{\int e^{-2ia-2ibx}\operatorname{PolyLog}\left(3,-ice^{2ia+2ibx}\right)de^{2ia+2ibx}}{4b^2}-\frac{ix\operatorname{Pol}\ldots}{}\right.}{b}\right.}{2bc}\right.\right.$$

$$\Big\downarrow \; 7143$$

$$\frac{1}{3}x^3\arctan(c-(1+ic)\cot(a+bx))-\frac{1}{3}ib\left(-ic\left(\frac{ix^3\log\left(1+ice^{2ia+2ibx}\right)}{2bc}-\frac{3i\left(\frac{ix^2\operatorname{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{2b}-\frac{i\left(\frac{\operatorname{PolyLog}\left(4,-ice^{2ia+2ibx}\right)}{4b^2}-\frac{ix\operatorname{PolyLog}\left(3,-ice^{2ia+2ibx}\right)}{2b}\right)}{b}\right.}{2bc}\right.\right.$$

input
```
Int[x^2*ArcTan[c + (-1 - I*c)*Cot[a + b*x]],x]
```

3.69. $\int x^2\arctan(c+(-1-ic)\cot(a+bx))\,dx$

output
```
(x^3*ArcTan[c - (1 + I*c)*Cot[a + b*x]])/3 - (I/3)*b*((-1/4*I)*x^4 - I*c*(
((I/2)*x^3*Log[1 + I*c*E^((2*I)*a + (2*I)*b*x)])/(b*c) - (((3*I)/2)*(((I/2
)*x^2*PolyLog[2, (-I)*c*E^((2*I)*a + (2*I)*b*x)])/b - (I*(((-1/2*I)*x*Poly
Log[3, (-I)*c*E^((2*I)*a + (2*I)*b*x)])/b + PolyLog[4, (-I)*c*E^((2*I)*a +
 (2*I)*b*x)]/(4*b^2)))/b))/(b*c)))
```

3.69.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5696
```
Int[ArcTan[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Cot[a + b*x]]/(f*(m +
1))), x] - Simp[I*(b/(f*(m + 1)))   Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(2
*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && E
qQ[(c - I*d)^2, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.69.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.50 (sec) , antiderivative size = 1488, normalized size of antiderivative = 9.60

method	result	size
risch	Expression too large to display	1488

input
```
int(-x^2*arctan(-c-(-1-I*c)*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I/b^2*a^2*ln(1+I*exp(I*(b*x+a))*(I*c)^(1/2))*x-1/12*I*(-I*Pi*csgn(I*e
xp(2*I*(b*x+a)))^3+2*I*Pi-I*Pi*csgn((exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a
))-1))^2-I*Pi*csgn(I/(exp(2*I*(b*x+a))-1)*(c-I))^3-I*Pi*csgn(I*exp(2*I*(b*
x+a))*(c-I)/(exp(2*I*(b*x+a))-1))^3+I*Pi*csgn(exp(2*I*(b*x+a))*(c-I)/(exp(
2*I*(b*x+a))-1))^3-I*Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+a)))+
I*Pi*csgn(I/(exp(2*I*(b*x+a))-1)*(c-I))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp
(2*I*(b*x+a))-1))^2+2*ln(c-I)-I*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(c-
I))*csgn(I/(exp(2*I*(b*x+a))-1)*(c-I))-I*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(
I/(exp(2*I*(b*x+a))-1)*(c-I))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+
a))-1))-I*Pi*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))*csgn((exp
(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))^2+I*Pi*csgn(I/(exp(2*I*(b*x+a))-1
))*csgn(I/(exp(2*I*(b*x+a))-1)*(c-I))^2+I*Pi*csgn(I*(c-I))*csgn(I/(exp(2*I
*(b*x+a))-1)*(c-I))^2+I*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+a)
)*(c-I)/(exp(2*I*(b*x+a))-1))^2+I*Pi*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*
I*(b*x+a))-1))^3+I*Pi*csgn((exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))^3-
I*Pi*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))*csgn(exp(2*I*(b*x
+a))*(c-I)/(exp(2*I*(b*x+a))-1))-I*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*
(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))^2-I*Pi*csgn(I*(exp(2*I*(b*x+a
))*c-I))*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))^2+I*Pi*csgn(I
*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))*csgn((exp(2*I*(b*x+a))*c-...
```

3.69.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.07

$$\int x^2 \arctan(c+(-1-ic)\cot(a+bx))\,dx =$$
$$-\frac{2\,b^4x^4-4i\,b^3x^3\log\left(-\frac{(ce^{(2i\,bx+2i\,a)}-i)e^{(-2i\,bx-2i\,a)}}{c-i}\right)+6\,b^2x^2\mathrm{Li}_2\left(-i\,ce^{(2i\,bx+2i\,a)}\right)-2\,a^4-4i\,a^3\log\left(\frac{ce^{(2i\,bx+2i}\cdots}{c}\right.\cdots}{\cdots}$$

input
```
integrate(-x^2*arctan(-c-(-1-I*c)*cot(b*x+a)),x, algorithm="fricas")
```

output
```
-1/24*(2*b^4*x^4 - 4*I*b^3*x^3*log(-(c*e^(2*I*b*x + 2*I*a) - I)*e^(-2*I*b*
x - 2*I*a)/(c - I)) + 6*b^2*x^2*dilog(-I*c*e^(2*I*b*x + 2*I*a)) - 2*a^4 -
4*I*a^3*log((c*e^(2*I*b*x + 2*I*a) - I)/c) + 6*I*b*x*polylog(3, -I*c*e^(2*
I*b*x + 2*I*a)) + 4*(I*b^3*x^3 + I*a^3)*log(I*c*e^(2*I*b*x + 2*I*a) + 1) -
 3*polylog(4, -I*c*e^(2*I*b*x + 2*I*a)))/b^3
```

3.69.6 Sympy [F(-2)]

Exception generated.

$$\int x^2 \arctan(c + (-1 - ic)\cot(a + bx))\,dx = \text{Exception raised: CoercionFailed}$$

input
```
integrate(-x**2*atan(-c-(-1-I*c)*cot(b*x+a)),x)
```

output
```
Exception raised: CoercionFailed >> Cannot convert _t0**2 - exp(2*I*a) of
type <class 'sympy.core.add.Add'> to QQ_I[x,b,_t0,exp(I*a)]
```

3.69.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(109) = 218$.

Time = 0.22 (sec) , antiderivative size = 311, normalized size of antiderivative = 2.01

$$\int x^2 \arctan(c + (-1 - ic)\cot(a + bx))\,dx$$
$$= \frac{\frac{4\left((bx+a)^3-3\,(bx+a)^2a+3\,(bx+a)a^2\right)\arctan((-i\,c-1)\cot(bx+a)+c)}{b^2} - \frac{\left(-3i\,(bx+a)^4+12i\,(bx+a)^3a-18i\,(bx+a)^2a^2-2\left(-4i\,(bx+a)^3+9i\,(ba\ldots\right.\right.}{\ldots}}{\ldots}$$

input
```
integrate(-x^2*arctan(-c-(-1-I*c)*cot(b*x+a)),x, algorithm="maxima")
```

output
```
1/12*(4*((b*x + a)^3 - 3*(b*x + a)^2*a + 3*(b*x + a)*a^2)*arctan((-I*c - 1
)*cot(b*x + a) + c)/b^2 - (-3*I*(b*x + a)^4 + 12*I*(b*x + a)^3*a - 18*I*(b
*x + a)^2*a^2 - 2*(-4*I*(b*x + a)^3 + 9*I*(b*x + a)^2*a - 9*I*(b*x + a)*a^
2)*arctan2(c*cos(2*b*x + 2*a), -c*sin(2*b*x + 2*a) + 1) - 3*(4*I*(b*x + a)
^2 - 6*I*(b*x + a)*a + 3*I*a^2)*dilog(-I*c*e^(2*I*b*x + 2*I*a)) + (4*(b*x
+ a)^3 - 9*(b*x + a)^2*a + 9*(b*x + a)*a^2)*log(c^2*cos(2*b*x + 2*a)^2 + c
^2*sin(2*b*x + 2*a)^2 - 2*c*sin(2*b*x + 2*a) + 1) + 3*(4*b*x + a)*polylog(
3, -I*c*e^(2*I*b*x + 2*I*a)) + 6*I*polylog(4, -I*c*e^(2*I*b*x + 2*I*a)))*(
I*c + 1)/(b^2*(c - I)))/b
```

3.69.8 Giac [F]

$$\int x^2 \arctan(c+(-1-ic)\cot(a+bx))\,dx = \int -x^2 \arctan\left(-(-i\,c-1)\cot\left(bx+a\right)-c\right)\,dx$$

input
```
integrate(-x^2*arctan(-c-(-1-I*c)*cot(b*x+a)),x, algorithm="giac")
```

output
```
integrate(-x^2*arctan(-(-I*c - 1)*cot(b*x + a) - c), x)
```

3.69.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \arctan(c+(-1-ic)\cot(a+bx))\,dx = \int x^2\,\mathrm{atan}(c-\cot(a+b\,x)\,\left(1+c\,1\mathrm{i}\right))\,dx$$

input
```
int(x^2*atan(c - cot(a + b*x)*(c*1i + 1)),x)
```

output
```
int(x^2*atan(c - cot(a + b*x)*(c*1i + 1)), x)
```

3.70 $\int x \arctan(c + (-1 - ic)\cot(a + bx))\,dx$

3.70.1 Optimal result

Integrand size = 19, antiderivative size = 124

$$\int x \arctan(c + (-1 - ic)\cot(a + bx))\,dx = -\frac{bx^3}{6} + \frac{1}{2}x^2 \arctan(c - (1 + ic)\cot(a + bx)) - \frac{1}{4}ix^2 \log\left(1 + ice^{2ia+2ibx}\right) - \frac{x\,\text{PolyLog}\left(2, -ice^{2ia+2ibx}\right)}{4b} - \frac{i\,\text{PolyLog}\left(3, -ice^{2ia+2ibx}\right)}{8b^2}$$

output
```
-1/6*b*x^3-1/2*x^2*arctan(-c+(1+I*c)*cot(b*x+a))-1/4*I*x^2*ln(1+I*c*exp(2*
I*a+2*I*b*x))-1/4*x*polylog(2,-I*c*exp(2*I*a+2*I*b*x))/b-1/8*I*polylog(3,-
I*c*exp(2*I*a+2*I*b*x))/b^2
```

3.70.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.89

$$\int x \arctan(c + (-1 - ic)\cot(a + bx))\,dx = \frac{1}{2}x^2 \arctan(c + (-1 - ic)\cot(a + bx)) - \frac{i\left(2b^2x^2 \log\left(1 - \frac{ie^{-2i(a+bx)}}{c}\right) + 2ibx\,\text{PolyLog}\left(2, \frac{ie^{-2i(a+bx)}}{c}\right) + \text{PolyLog}\left(3, \frac{ie^{-2i(a+bx)}}{c}\right)\right)}{8b^2}$$

input
```
Integrate[x*ArcTan[c + (-1 - I*c)*Cot[a + b*x]],x]
```

output
```
(x^2*ArcTan[c + (-1 - I*c)*Cot[a + b*x]])/2 - ((I/8)*(2*b^2*x^2*Log[1 - I/
(c*E^((2*I)*(a + b*x)))] + (2*I)*b*x*PolyLog[2, I/(c*E^((2*I)*(a + b*x)))]
 + PolyLog[3, I/(c*E^((2*I)*(a + b*x)))]))/b^2
```

3.70.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.26, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5696, 2615, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan(c+(-1-ic)\cot(a+bx))\,dx$$

$$\Big\downarrow \text{5696}$$

$$\frac{1}{2}x^2\arctan(c-(1+ic)\cot(a+bx))-\frac{1}{2}ib\int\frac{x^2}{i-ce^{2ia+2ibx}}dx$$

$$\Big\downarrow \text{2615}$$

$$\frac{1}{2}x^2\arctan(c-(1+ic)\cot(a+bx))-\frac{1}{2}ib\left(-ic\int\frac{e^{2ia+2ibx}x^2}{i-ce^{2ia+2ibx}}dx-\frac{ix^3}{3}\right)$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{2}x^2\arctan(c-(1+ic)\cot(a+bx))-\frac{1}{2}ib\left(-ic\left(\frac{ix^2\log\left(1+ice^{2ia+2ibx}\right)}{2bc}-\frac{i\int x\log\left(ie^{2ia+2ibx}c+1\right)dx}{bc}\right)-\frac{ix^3}{3}\right)$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{2}x^2\arctan(c-(1+ic)\cot(a+bx))-\frac{1}{2}ib\left(-ic\left(\frac{ix^2\log\left(1+ice^{2ia+2ibx}\right)}{2bc}-\frac{i\left(\frac{ix\,\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{2b}-\frac{i\int\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)dx}{2b}\right)}{bc}\right)-\frac{ix^3}{3}\right)$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{2}x^2\arctan(c-(1+ic)\cot(a+bx))-$$
$$\frac{1}{2}ib\left(-ic\left(\frac{ix^2\log\left(1+ice^{2ia+2ibx}\right)}{2bc}-\frac{i\left(\frac{ix\,\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{2b}-\frac{\int e^{-2ia-2ibx}\,\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)de^{2ia+2ibx}}{4b^2}\right)}{bc}\right)-\frac{ix}{3}\right.$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{2}x^2\arctan(c-(1+ic)\cot(a+bx))-$$
$$\frac{1}{2}ib\left(-ic\left(\frac{ix^2\log\left(1+ice^{2ia+2ibx}\right)}{2bc}-\frac{i\left(\frac{ix\,\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{2b}-\frac{\text{PolyLog}\left(3,-ice^{2ia+2ibx}\right)}{4b^2}\right)}{bc}\right)-\frac{ix^3}{3}\right)$$

input
```
Int[x*ArcTan[c + (-1 - I*c)*Cot[a + b*x]],x]
```

output
```
(x^2*ArcTan[c - (1 + I*c)*Cot[a + b*x]])/2 - (I/2)*b*((-1/3*I)*x^3 - I*c*(
((I/2)*x^2*Log[1 + I*c*E^((2*I)*a + (2*I)*b*x)])/(b*c) - (I*(((I/2)*x*Poly
Log[2, (-I)*c*E^((2*I)*a + (2*I)*b*x)])/b - PolyLog[3, (-I)*c*E^((2*I)*a +
 (2*I)*b*x)]/(4*b^2)))/(b*c)))
```

3.70.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5696
```
Int[ArcTan[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_.
), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Cot[a + b*x]]/(f*(m +
1))), x] - Simp[I*(b/(f*(m + 1)))   Int[(e + f*x)^(m + 1)/(c - I*d - c*E^(2
*I*a + 2*I*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && E
qQ[(c - I*d)^2, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.70.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.10 (sec) , antiderivative size = 1453, normalized size of antiderivative = 11.72

method	result	size
risch	Expression too large to display	1453

input
```
int(-x*arctan(-c-(-1-I*c)*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/2*I/b^2*a^2*ln(1+I*exp(I*(b*x+a))*(I*c)^(1/2))-1/4*I*ln(I*exp(2*I*(b*x+a
))*c+1)*x^2-1/4*I/b^2*ln(I*exp(2*I*(b*x+a))*c+1)*a^2+1/2*I/b^2*a^2*ln(1-I*
exp(I*(b*x+a))*(I*c)^(1/2))+1/2*I/b*a*ln(1+I*exp(I*(b*x+a))*(I*c)^(1/2))*x
-1/4/b*polylog(2,-I*exp(2*I*(b*x+a))*c)*x-1/4/b^2*polylog(2,-I*exp(2*I*(b*
x+a))*c)*a-1/8*I/b^2*polylog(3,-I*exp(2*I*(b*x+a))*c)-1/4*I/b^2*a^2*ln(-ex
p(2*I*(b*x+a))*c+I)+1/4*I*x^2*ln(exp(2*I*(b*x+a))*c-I)-1/8*I*(-I*Pi*csgn(I
*exp(2*I*(b*x+a)))^3+2*I*Pi-I*Pi*csgn((exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x
+a))-1))^2-I*Pi*csgn(I/(exp(2*I*(b*x+a))-1)*(c-I))^3-I*Pi*csgn(I*exp(2*I*(
b*x+a))*(c-I)/(exp(2*I*(b*x+a))-1))^3+I*Pi*csgn(exp(2*I*(b*x+a))*(c-I)/(ex
p(2*I*(b*x+a))-1))^3-I*Pi*csgn(I*exp(I*(b*x+a)))^2*csgn(I*exp(2*I*(b*x+a))
)+I*Pi*csgn(I/(exp(2*I*(b*x+a))-1)*(c-I))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(e
xp(2*I*(b*x+a))-1))^2+2*ln(c-I)-I*Pi*csgn(I/(exp(2*I*(b*x+a))-1))*csgn(I*(
c-I))*csgn(I/(exp(2*I*(b*x+a))-1)*(c-I))-I*Pi*csgn(I*exp(2*I*(b*x+a)))*csg
n(I/(exp(2*I*(b*x+a))-1)*(c-I))*csgn(I*exp(2*I*(b*x+a))*(c-I)/(exp(2*I*(b*
x+a))-1))-I*Pi*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))*csgn((e
xp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1))^2+I*Pi*csgn(I/(exp(2*I*(b*x+a))
-1))*csgn(I/(exp(2*I*(b*x+a))-1)*(c-I))^2+I*Pi*csgn(I*(c-I))*csgn(I/(exp(2
*I*(b*x+a))-1)*(c-I))^2+I*Pi*csgn(I*exp(2*I*(b*x+a)))*csgn(I*exp(2*I*(b*x+
a))*(c-I)/(exp(2*I*(b*x+a))-1))^2+I*Pi*csgn(I*(exp(2*I*(b*x+a))*c-I)/(exp(
2*I*(b*x+a))-1))^3+I*Pi*csgn((exp(2*I*(b*x+a))*c-I)/(exp(2*I*(b*x+a))-1...
```

3.70.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.16

$$\int x\arctan(c+(-1-ic)\cot(a+bx))\,dx =$$
$$-\frac{4\,b^3x^3-6i\,b^2x^2\log\left(-\frac{(ce^{(2i\,bx+2i\,a)}-i)e^{(-2i\,bx-2i\,a)}}{c-i}\right)+4\,a^3+6\,bx\mathrm{Li}_2\left(-i\,ce^{(2i\,bx+2i\,a)}\right)+6i\,a^2\log\left(\frac{ce^{(2i\,bx+2i\,a)}\ldots}{c}\right.}{24\,b^2}$$

input
```
integrate(-x*arctan(-c-(-1-I*c)*cot(b*x+a)),x, algorithm="fricas")
```

output
```
-1/24*(4*b^3*x^3 - 6*I*b^2*x^2*log(-(c*e^(2*I*b*x + 2*I*a) - I)*e^(-2*I*b*
x - 2*I*a)/(c - I)) + 4*a^3 + 6*b*x*dilog(-I*c*e^(2*I*b*x + 2*I*a)) + 6*I*
a^2*log((c*e^(2*I*b*x + 2*I*a) - I)/c) + 6*(I*b^2*x^2 - I*a^2)*log(I*c*e^(
2*I*b*x + 2*I*a) + 1) + 3*I*polylog(3, -I*c*e^(2*I*b*x + 2*I*a)))/b^2
```

3.70.6 Sympy [F(-2)]

Exception generated.

$$\int x\arctan(c+(-1-ic)\cot(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

input
```
integrate(-x*atan(-c-(-1-I*c)*cot(b*x+a)),x)
```

output
```
Exception raised: CoercionFailed >> Cannot convert _t0**2 - exp(2*I*a) of
type <class 'sympy.core.add.Add'> to QQ_I[x,b,_t0,exp(I*a)]
```

3.70.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(87) = 174.

Time = 0.20 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.77

$$\int x\arctan(c+(-1-ic)\cot(a+bx))\,dx = \frac{\frac{6\left((bx+a)^2-2(bx+a)a\right)\arctan((-ic-1)\cot(bx+a)+c)}{b} - \frac{\left(-4i(bx+a)^3+12i(bx+a)^2a-6ibx\mathrm{Li}_2\left(-ice^{(2ibx+2ia)}\right)-6\left(-i(bx+a)^2+2i(bx+a\right.\right.}{}}{}$$

input
```
integrate(-x*arctan(-c-(-1-I*c)*cot(b*x+a)),x, algorithm="maxima")
```

output
```
1/12*(6*((b*x + a)^2 - 2*(b*x + a)*a)*arctan((-I*c - 1)*cot(b*x + a) + c)/
b - (-4*I*(b*x + a)^3 + 12*I*(b*x + a)^2*a - 6*I*b*x*dilog(-I*c*e^(2*I*b*x
 + 2*I*a)) - 6*(-I*(b*x + a)^2 + 2*I*(b*x + a)*a)*arctan2(c*cos(2*b*x + 2*
a), -c*sin(2*b*x + 2*a) + 1) + 3*((b*x + a)^2 - 2*(b*x + a)*a)*log(c^2*cos
(2*b*x + 2*a)^2 + c^2*sin(2*b*x + 2*a)^2 - 2*c*sin(2*b*x + 2*a) + 1) + 3*p
olylog(3, -I*c*e^(2*I*b*x + 2*I*a)))*(I*c + 1)/(b*(c - I)))/b
```

3.70.8 Giac [F]

$$\int x\arctan(c+(-1-ic)\cot(a+bx))\,dx=\int -x\arctan\left(-(-i\,c-1)\cot\left(bx+a\right)-c\right)\,dx$$

input
```
integrate(-x*arctan(-c-(-1-I*c)*cot(b*x+a)),x, algorithm="giac")
```

output
```
integrate(-x*arctan(-(-I*c - 1)*cot(b*x + a) - c), x)
```

3.70.9 Mupad [F(-1)]

Timed out.

$$\int x\arctan(c+(-1-ic)\cot(a+bx))\,dx=\int x\,\mathrm{atan}(c-\cot(a+b\,x)\,\left(1+c\,1\mathrm{i}\right))\,dx$$

input
```
int(x*atan(c - cot(a + b*x)*(c*1i + 1)),x)
```

output
```
int(x*atan(c - cot(a + b*x)*(c*1i + 1)), x)
```

3.71 $\int \arctan(c+(-1-ic)\cot(a+bx))\,dx$

3.71.1 Optimal result

Integrand size = 17, antiderivative size = 86

$$\int \arctan(c+(-1-ic)\cot(a+bx))\,dx = -\frac{bx^2}{2}+x\arctan(c-(1+ic)\cot(a+bx)) - \frac{1}{2}ix\log\left(1+ice^{2ia+2ibx}\right) - \frac{\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b}$$

output
```
-1/2*b*x^2-x*arctan(-c+(1+I*c)*cot(b*x+a))-1/2*I*x*ln(1+I*c*exp(2*I*a+2*I*
b*x))-1/4*polylog(2,-I*c*exp(2*I*a+2*I*b*x))/b
```

3.71.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 872 vs. 2(86) = 172.

Time = 6.71 (sec) , antiderivative size = 872, normalized size of antiderivative = 10.14

$$\int \arctan(c+(-1-ic)\cot(a+bx))\,dx = x\arctan(c+(-1-ic)\cot(a+bx)) + \frac{ix\csc(a+bx)\left(2bx\log(2\cos(bx)(\cos(bx)-i\sin(bx)))+i\log\left(\frac{\text{se}}{}\right.\right.}{(i+\cot(a+bx))((-i+c)\cos(a+bx)+i(i+c)\sin(a+bx))\left(-2ibx-\log\left(1-\frac{\sec(bx)((i+c)\cos(a)+(1+ic)}{}\right.\right.}$$

input
```
Integrate[ArcTan[c + (-1 - I*c)*Cot[a + b*x]],x]
```

output
```
x*ArcTan[c + (-1 - I*c)*Cot[a + b*x]] + (I*x*Csc[a + b*x]*(2*b*x*Log[2*Cos
[b*x]*(Cos[b*x] - I*Sin[b*x])] + I*Log[(Sec[b*x]*(Cos[a] - I*Sin[a])*((-I
+ c)*Cos[a + b*x] + I*(I + c)*Sin[a + b*x]))/(2*c)]*Log[1 - I*Tan[b*x]] -
I*Log[(Sec[b*x]*(Cos[a] + I*Sin[a])*((1 + I*c)*Cos[a + b*x] - (I + c)*Sin[
a + b*x]))/2]*Log[1 + I*Tan[b*x]] + I*PolyLog[2, -Cos[2*b*x] + I*Sin[2*b*x
]] + I*PolyLog[2, (Sec[b*x]*((I + c)*Cos[a] + (1 + I*c)*Sin[a])*(Cos[a + b
*x] - I*Sin[a + b*x]))/(2*c)] - I*PolyLog[2, ((Cos[a] + I*Sin[a])*((I + c)
*Cos[a] + (1 + I*c)*Sin[a])*(-I + Tan[b*x]))/2])*(Cos[b*x] - I*Sin[b*x])*(
Cos[b*x] + I*Sin[b*x]))/((I + Cot[a + b*x])*((-I + c)*Cos[a + b*x] + I*(I
+ c)*Sin[a + b*x])*((-2*I)*b*x - Log[1 - (Sec[b*x]*((I + c)*Cos[a] + (1 +
I*c)*Sin[a])*(Cos[a + b*x] - I*Sin[a + b*x]))/(2*c)] - (Log[1 - I*Tan[b*x]
]*((I + c)*Cos[a + b*x] + (1 + I*c)*Sin[a + b*x]))/((-I + c)*Cos[a + b*x]
+ I*(I + c)*Sin[a + b*x]) + (Log[1 + I*Tan[b*x]]*((I + c)*Cos[a + b*x] + (
1 + I*c)*Sin[a + b*x]))/((-I + c)*Cos[a + b*x] + I*(I + c)*Sin[a + b*x]) +
 (Log[(Sec[b*x]*(Cos[a] + I*Sin[a])*((1 + I*c)*Cos[a + b*x] - (I + c)*Sin[
a + b*x]))/2]*Sec[b*x]^2)/(1 + I*Tan[b*x]) - 2*b*x*Tan[b*x] - I*Log[1 - (S
ec[b*x]*((I + c)*Cos[a] + (1 + I*c)*Sin[a])*(Cos[a + b*x] - I*Sin[a + b*x]
))/(2*c)]*Tan[b*x] + I*Log[1 - I*Tan[b*x]]*Tan[b*x] - I*Log[1 + I*Tan[b*x]
]*Tan[b*x] + (I*Log[1 - ((Cos[a] + I*Sin[a])*((I + c)*Cos[a] + (1 + I*c)*S
in[a])*(-I + Tan[b*x]))/2]*Sec[b*x]^2)/(-I + Tan[b*x]) + (I*Log[(Sec[b*...
```

3.71.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.26, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {5688, 2615, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(c + (-1 - ic)\cot(a + bx))\,dx$$

$$\Big\downarrow \; 5688$$

$$x\arctan(c - (1 + ic)\cot(a + bx)) - ib\int \frac{x}{i - ce^{2ia+2ibx}}dx$$

$$\Big\downarrow \; 2615$$

$$x\arctan(c - (1 + ic)\cot(a + bx)) - ib\left(-ic\int \frac{e^{2ia+2ibx}x}{i - ce^{2ia+2ibx}}dx - \frac{ix^2}{2}\right)$$

$$\Big\downarrow \text{2620}$$

$$x\arctan(c-(1+ic)\cot(a+bx))-ib\left(-ic\left(\frac{ix\log\left(1+ice^{2ia+2ibx}\right)}{2bc}-\frac{i\int\log\left(ie^{2ia+2ibx}c+1\right)dx}{2bc}\right)-\frac{ix^2}{2}\right)$$

$$\Big\downarrow \text{2715}$$

$$x\arctan(c-(1+ic)\cot(a+bx))-ib\left(-ic\left(\frac{ix\log\left(1+ice^{2ia+2ibx}\right)}{2bc}-\frac{\int e^{-2ia-2ibx}\log\left(ie^{2ia+2ibx}c+1\right)de^{2ia+2ibx}}{4b^2c}\right)-\frac{ix^2}{2}\right)$$

$$\Big\downarrow \text{2838}$$

$$x\arctan(c-(1+ic)\cot(a+bx))-ib\left(-ic\left(\frac{\text{PolyLog}\left(2,-ice^{2ia+2ibx}\right)}{4b^2c}+\frac{ix\log\left(1+ice^{2ia+2ibx}\right)}{2bc}\right)-\frac{ix^2}{2}\right)$$

input
```
Int[ArcTan[c + (-1 - I*c)*Cot[a + b*x]],x]
```

output
```
x*ArcTan[c - (1 + I*c)*Cot[a + b*x]] - I*b*((-1/2*I)*x^2 - I*c*(((I/2)*x*L
og[1 + I*c*E^((2*I)*a + (2*I)*b*x)])/(b*c) + PolyLog[2, (-I)*c*E^((2*I)*a
+ (2*I)*b*x)]/(4*b^2*c)))
```

3.71.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5688
```
Int[ArcTan[(c_.) + Cot[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*ArcT
an[c + d*Cot[a + b*x]], x] - Simp[I*b   Int[x/(c - I*d - c*E^(2*I*a + 2*I*b
*x)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - I*d)^2, -1]
```

3.71.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 624 vs. $2(72) = 144$.

Time = 1.19 (sec) , antiderivative size = 625, normalized size of antiderivative = 7.27

method	result
derivativedivides	$-\frac{-\frac{\arctan(-c+(ic+1)\cot(bx+a))\ln(i+(ic+1)\cot(bx+a)-c)c^2}{2i-2c}+\frac{2i\arctan(-c+(ic+1)\cot(bx+a))\ln(i+(ic+1)\cot(bx+a)-c)c}{2i-2c}+\text{arct}}{}$
default	$-\frac{-\frac{\arctan(-c+(ic+1)\cot(bx+a))\ln(i+(ic+1)\cot(bx+a)-c)c^2}{2i-2c}+\frac{2i\arctan(-c+(ic+1)\cot(bx+a))\ln(i+(ic+1)\cot(bx+a)-c)c}{2i-2c}+\text{arct}}{}$
risch	Expression too large to display

input
```
int(-arctan(-c-(-1-I*c)*cot(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/b/(I*c+1)*(-arctan(-c+(I*c+1)*cot(b*x+a))/(2*I-2*c)*ln(I+(I*c+1)*cot(b*
x+a)-c)*c^2+2*I*arctan(-c+(I*c+1)*cot(b*x+a))/(2*I-2*c)*ln(I+(I*c+1)*cot(b
*x+a)-c)*c+arctan(-c+(I*c+1)*cot(b*x+a))/(2*I-2*c)*ln(I+(I*c+1)*cot(b*x+a)
-c)+arctan(-c+(I*c+1)*cot(b*x+a))/(2*I-2*c)*ln(-(I*c+1)*cot(b*x+a)-c+I)*c^
2-2*I*arctan(-c+(I*c+1)*cot(b*x+a))/(2*I-2*c)*ln(-(I*c+1)*cot(b*x+a)-c+I)*
c-arctan(-c+(I*c+1)*cot(b*x+a))/(2*I-2*c)*ln(-(I*c+1)*cot(b*x+a)-c+I)+(I*c
+1)^2*(-1/2/(I-c)*(-1/2*I*((ln(I+(I*c+1)*cot(b*x+a)-c)-ln(-1/2*I*(I+(I*c+1
)*cot(b*x+a)-c)))*ln(-1/2*I*(I-(I*c+1)*cot(b*x+a)+c))-dilog(-1/2*I*(I+(I*c
+1)*cot(b*x+a)-c)))+1/4*I*ln(I+(I*c+1)*cot(b*x+a)-c)^2)+1/2/(I-c)*(1/2*I*(
dilog((-I-(I*c+1)*cot(b*x+a)+c)/(-2*I+2*c))+ln(-(I*c+1)*cot(b*x+a)-c+I)*ln
((-I-(I*c+1)*cot(b*x+a)+c)/(-2*I+2*c)))-1/2*I*(dilog(1/2*(I-(I*c+1)*cot(b*
x+a)+c)/c)+ln(-(I*c+1)*cot(b*x+a)-c+I)*ln(1/2*(I-(I*c+1)*cot(b*x+a)+c)/c))
)))
```

3.71.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.30

$$\int \arctan(c+(-1-ic)\cot(a+bx))\,dx = -\frac{2b^2x^2 - 2ibx\log\left(-\frac{(ce^{(2i\,bx+2i\,a)}-i)e^{(-2i\,bx-2i\,a)}}{c-i}\right) - 2a^2 + 2(i\,bx+i\,a)\log\left(i\,ce^{(2i\,bx+2i\,a)}+1\right) - 2i\,a\log\left(\frac{ce}{}\right.}{4b}$$

input
```
integrate(-arctan(-c-(-1-I*c)*cot(b*x+a)),x, algorithm="fricas")
```

output
```
-1/4*(2*b^2*x^2 - 2*I*b*x*log(-(c*e^(2*I*b*x + 2*I*a) - I)*e^(-2*I*b*x - 2
*I*a)/(c - I)) - 2*a^2 + 2*(I*b*x + I*a)*log(I*c*e^(2*I*b*x + 2*I*a) + 1)
- 2*I*a*log((c*e^(2*I*b*x + 2*I*a) - I)/c) + dilog(-I*c*e^(2*I*b*x + 2*I*a
)))/b
```

3.71.6 Sympy [F(-2)]

Exception generated.

$$\int \arctan(c+(-1-ic)\cot(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

input
```
integrate(-atan(-c-(-1-I*c)*cot(b*x+a)),x)
```

output
```
Exception raised: CoercionFailed >> Cannot convert _t0**2 - exp(2*I*a) of
type <class 'sympy.core.add.Add'> to QQ_I[b,_t0,exp(I*a)]
```

3.71.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(63) = 126$.

Time = 0.30 (sec) , antiderivative size = 458, normalized size of antiderivative = 5.33

$$\int \arctan(c + (-1 - ic)\cot(a + bx))\,dx =$$

$$-\frac{(i\,c+1)\left(\frac{4i\,(bx+a)\log\left(-\frac{2\,\left(i\,c^2-\left(c^2+1\right)\tan(bx+a)+i\right)}{-2i\,c^2+2\,\left(c^2+1\right)\tan(bx+a)-4\,c+2i}\right)}{i\,c+1}+\frac{i\,\left(4\,(bx+a)\,(\log(-i\,c^2+(c^2+1)\tan(bx+a)-2\,c+i)-\log(-i\,c^2+(c^2+1)\cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
integrate(-arctan(-c-(-1-I*c)*cot(b*x+a)),x, algorithm="maxima")
```

output
```
-1/8*((I*c + 1)*(4*I*(b*x + a)*log(-2*(I*c^2 - (c^2 + 1)*tan(b*x + a) + I)
/(-2*I*c^2 + 2*(c^2 + 1)*tan(b*x + a) - 4*c + 2*I))/(I*c + 1) + I*(4*(b*x
+ a)*(log(-I*c^2 + (c^2 + 1)*tan(b*x + a) - 2*c + I) - log(-I*c^2 + (c^2 +
 1)*tan(b*x + a) - I)) - 2*I*log(-I*c^2 + (c^2 + 1)*tan(b*x + a) - 2*c + I
)*log(-1/2*((I*c - 1)*tan(b*x + a) + c - I)/c + 1) + 2*I*log(-I*c^2 + (c^2
 + 1)*tan(b*x + a) - 2*c + I)*log(tan(b*x + a) - I) - 2*I*log(-1/2*(c + I)
*tan(b*x + a) + 1/2*I*c + 1/2)*log(tan(b*x + a) - I) - I*log(tan(b*x + a)
- I)^2 - 2*I*log(c^2 + 1)*log(I*tan(b*x + a) + 1) + 2*I*log(tan(b*x + a) -
 I)*log(-1/2*I*tan(b*x + a) + 1/2) + 2*I*log(c^2 + 1)*log(-I*tan(b*x + a)
+ 1) - 2*I*dilog(1/2*(c + I)*tan(b*x + a) - 1/2*I*c + 1/2) - 2*I*dilog(1/2
*((I*c - 1)*tan(b*x + a) + c - I)/c) + 2*I*dilog(1/2*I*tan(b*x + a) + 1/2)
)/(I*c + 1)) - 8*(b*x + a)*arctan(c + (-I*c - 1)/tan(b*x + a)) + 4*(-I*b*x
 - I*a)*log(-2*(I*c^2 - (c^2 + 1)*tan(b*x + a) + I)/(-2*I*c^2 + 2*(c^2 + 1
)*tan(b*x + a) - 4*c + 2*I)))/b
```

3.71.8 Giac [F]

$$\int \arctan(c + (-1 - ic)\cot(a + bx))\,dx = \int -\arctan\left(-(-i\,c - 1)\cot\left(bx + a\right) - c\right)\,dx$$

input
```
integrate(-arctan(-c-(-1-I*c)*cot(b*x+a)),x, algorithm="giac")
```

output
```
integrate(-arctan(-(-I*c - 1)*cot(b*x + a) - c), x)
```

3.71.9 Mupad [F(-1)]

Timed out.

$$\int \arctan(c + (-1 - ic)\cot(a + bx))\,dx = \int \mathrm{atan}(c - \cot(a + b\,x)\,\left(1 + c\,1\mathrm{i}\right))\,dx$$

input
```
int(atan(c - cot(a + b*x)*(c*1i + 1)),x)
```

output
```
int(atan(c - cot(a + b*x)*(c*1i + 1)), x)
```

3.72 $\int \frac{\arctan(c+(-1-ic)\cot(a+bx))}{x}\,dx$

3.72.1 Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{\arctan(c+(-1-ic)\cot(a+bx))}{x}\,dx = \text{Int}\left(\frac{\arctan(c+(-1-ic)\cot(a+bx))}{x},x\right)$$

output
```
CannotIntegrate(-arctan(-c-(-1-I*c)*cot(b*x+a))/x,x)
```

3.72.2 Mathematica [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{\arctan(c+(-1-ic)\cot(a+bx))}{x}\,dx = \int \frac{\arctan(c+(-1-ic)\cot(a+bx))}{x}\,dx$$

input
```
Integrate[ArcTan[c + (-1 - I*c)*Cot[a + b*x]]/x,x]
```

output
```
Integrate[ArcTan[c + (-1 - I*c)*Cot[a + b*x]]/x, x]
```

3.72.3 Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(c+(-1-ic)\cot(a+bx))}{x} \, dx$$

$$\Big\downarrow \; 7299$$

$$\int \frac{\arctan(c+(-1-ic)\cot(a+bx))}{x} dx$$

input
```
Int[ArcTan[c + (-1 - I*c)*Cot[a + b*x]]/x,x]
```

output
```
$Aborted
```

3.72.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.72.4 Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int -\frac{\arctan\left(-c-(-ic-1)\cot\left(bx+a\right)\right)}{x} dx$$

input
```
int(-arctan(-c-(-1-I*c)*cot(b*x+a))/x,x)
```

output
```
int(-arctan(-c-(-1-I*c)*cot(b*x+a))/x,x)
```

3.72.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.76

$$\int \frac{\arctan(c + (-1 - ic)\cot(a + bx))}{x} \, dx = \int -\frac{\arctan\left(-(-i\,c - 1)\cot\left(bx + a\right) - c\right)}{x} \, dx$$

input
```
integrate(-arctan(-c-(-1-I*c)*cot(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(1/2*I*log(-(c*e^(2*I*b*x + 2*I*a) - I)*e^(-2*I*b*x - 2*I*a)/(c -
I))/x, x)
```

3.72.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan(c + (-1 - ic)\cot(a + bx))}{x} \, dx = \text{Timed out}$$

input
```
integrate(-atan(-c-(-1-I*c)*cot(b*x+a))/x,x)
```

output
```
Timed out
```

3.72.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(c + (-1 - ic)\cot(a + bx))}{x} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(-arctan(-c-(-1-I*c)*cot(b*x+a))/x,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(c-1>0)', see `assume?` for more
details)Is
```

3.72.8 Giac [N/A]

Not integrable

Time = 2.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.19

$$\int \frac{\arctan(c+(-1-ic)\cot(a+bx))}{x}\,dx = \int -\frac{\arctan\left(-(-i\,c-1)\cot\left(bx+a\right)-c\right)}{x}\,dx$$

input
```
integrate(-arctan(-c-(-1-I*c)*cot(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(-arctan(-(-I*c - 1)*cot(b*x + a) - c)/x, x)
```

3.72.9 Mupad [N/A]

Not integrable

Time = 1.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{\arctan(c+(-1-ic)\cot(a+bx))}{x}\,dx = \int \frac{\mathrm{atan}(c-\cot(a+b\,x)\,\left(1+c\,1\mathrm{i}\right))}{x}\,dx$$

input
```
int(atan(c - cot(a + b*x)*(c*1i + 1))/x,x)
```

output
```
int(atan(c - cot(a + b*x)*(c*1i + 1))/x, x)
```

3.73 $\int \arctan(\sinh(x))\,dx$

3.73.1 Optimal result

Integrand size = 3, antiderivative size = 39

$$\int \arctan(\sinh(x))\,dx = -2x\arctan\left(e^x\right)+x\arctan(\sinh(x)) + i\,\text{PolyLog}\left(2,-ie^x\right)-i\,\text{PolyLog}\left(2,ie^x\right)$$

output
```
-2*x*arctan(exp(x))+x*arctan(sinh(x))+I*polylog(2,-I*exp(x))-I*polylog(2,I
*exp(x))
```

3.73.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.44

$$\int \arctan(\sinh(x))\,dx = x\arctan(\sinh(x))-i(x(\log\left(1-ie^x\right)-\log\left(1+ie^x\right)) - \text{PolyLog}\left(2,-ie^x\right)+\text{PolyLog}\left(2,ie^x\right))$$

input
```
Integrate[ArcTan[Sinh[x]],x]
```

output
```
x*ArcTan[Sinh[x]] - I*(x*(Log[1 - I*E^x] - Log[1 + I*E^x]) - PolyLog[2, (-
I)*E^x] + PolyLog[2, I*E^x])
```

3.73.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 1.667$, Rules used = {5726, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(\sinh(x))\,dx$$

$$\Big\downarrow \text{5726}$$

$$x\arctan(\sinh(x)) - \int x\text{sech}(x)dx$$

$$\Big\downarrow \text{3042}$$

$$x\arctan(\sinh(x)) - \int x\csc\left(ix+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{4668}$$

$$i\int \log\left(1-ie^x\right)dx - i\int \log\left(1+ie^x\right)dx - 2x\arctan\left(e^x\right) + x\arctan(\sinh(x))$$

$$\Big\downarrow \text{2715}$$

$$i\int e^{-x}\log\left(1-ie^x\right)de^x - i\int e^{-x}\log\left(1+ie^x\right)de^x - 2x\arctan\left(e^x\right) + x\arctan(\sinh(x))$$

$$\Big\downarrow \text{2838}$$

$$-2x\arctan\left(e^x\right) + x\arctan(\sinh(x)) + i\,\text{PolyLog}\left(2,-ie^x\right) - i\,\text{PolyLog}\left(2,ie^x\right)$$

input
```
Int[ArcTan[Sinh[x]],x]
```

output
```
-2*x*ArcTan[E^x] + x*ArcTan[Sinh[x]] + I*PolyLog[2, (-I)*E^x] - I*PolyLog[
2, I*E^x]
```

3.73.3.1 Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5726
```
Int[ArcTan[u_], x_Symbol] :> Simp[x*ArcTan[u], x] - Int[SimplifyIntegrand[x
*(D[u, x]/(1 + u^2)), x], x] /; InverseFunctionFreeQ[u, x]
```

3.73.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.33

method	result	size
default	$x\arctan\left(\sinh\left(x\right)\right)-ix\left(\ln\left(1-i\mathrm{e}^{x}\right)-\ln\left(1+i\mathrm{e}^{x}\right)\right)+i\,\mathrm{dilog}\left(1+i\mathrm{e}^{x}\right)-i\,\mathrm{dilog}\left(1-i\mathrm{e}^{x}\right)$	52
parts	$x\arctan\left(\sinh\left(x\right)\right)-ix\left(\ln\left(1-i\mathrm{e}^{x}\right)-\ln\left(1+i\mathrm{e}^{x}\right)\right)+i\,\mathrm{dilog}\left(1+i\mathrm{e}^{x}\right)-i\,\mathrm{dilog}\left(1-i\mathrm{e}^{x}\right)$	52
risch	Expression too large to display	651

input
```
int(arctan(sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
x*arctan(sinh(x))-I*x*(ln(1-I*exp(x))-ln(1+I*exp(x)))+I*dilog(1+I*exp(x))-
I*dilog(1-I*exp(x))
```

3.73.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(26) = 52$.

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.49

$$\int \arctan(\sinh(x))\,dx = x\arctan\left(\sinh\left(x\right)\right) + i\,x\log\left(i\,\cosh\left(x\right) + i\,\sinh\left(x\right) + 1\right) - i\,x\log\left(-i\,\cosh\left(x\right) - i\,\sinh\left(x\right) + 1\right) - i\,\mathrm{Li}_2(i\,\cosh\left(x\right) + i\,\sinh\left(x\right)) + i\,\mathrm{Li}_2(-i\,\cosh\left(x\right) - i\,\sinh\left(x\right))$$

input
```
integrate(arctan(sinh(x)),x, algorithm="fricas")
```

output
```
x*arctan(sinh(x)) + I*x*log(I*cosh(x) + I*sinh(x) + 1) - I*x*log(-I*cosh(x
) - I*sinh(x) + 1) - I*dilog(I*cosh(x) + I*sinh(x)) + I*dilog(-I*cosh(x) -
 I*sinh(x))
```

3.73.6 Sympy [F]

$$\int \arctan(\sinh(x))\,dx = \int \operatorname{atan}\left(\sinh\left(x\right)\right)\,dx$$

input
```
integrate(atan(sinh(x)),x)
```

output
```
Integral(atan(sinh(x)), x)
```

3.73.7 Maxima [F]

$$\int \arctan(\sinh(x))\,dx = \int \arctan\left(\sinh\left(x\right)\right)\,dx$$

input
```
integrate(arctan(sinh(x)),x, algorithm="maxima")
```

output
```
x*arctan(1/2*(e^(2*x) - 1)*e^(-x)) - 2*integrate(x*e^x/(e^(2*x) + 1), x)
```

3.73.8 Giac [F]

$$\int \arctan(\sinh(x))\,dx = \int \arctan\left(\sinh\left(x\right)\right)\,dx$$

input
```
integrate(arctan(sinh(x)),x, algorithm="giac")
```

output
```
integrate(arctan(sinh(x)), x)
```

3.73.9 Mupad [F(-1)]

Timed out.

$$\int \arctan(\sinh(x))\,dx = \int \mathrm{atan}(\sinh(x))\,dx$$

input
```
int(atan(sinh(x)),x)
```

output
```
int(atan(sinh(x)), x)
```

3.74 $\int x \arctan(\sinh(x))\,dx$

3.74.1 Optimal result

Integrand size = 5, antiderivative size = 74

$$\int x \arctan(\sinh(x))\,dx = -x^2 \arctan(e^x) + \frac{1}{2}x^2 \arctan(\sinh(x)) + ix \operatorname{PolyLog}(2,-ie^x) - ix \operatorname{PolyLog}(2,ie^x) - i \operatorname{PolyLog}(3,-ie^x) + i \operatorname{PolyLog}(3,ie^x)$$

output
```
-x^2*arctan(exp(x))+1/2*x^2*arctan(sinh(x))+I*x*polylog(2,-I*exp(x))-I*x*p
olylog(2,I*exp(x))-I*polylog(3,-I*exp(x))+I*polylog(3,I*exp(x))
```

3.74.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.26

$$\int x \arctan(\sinh(x))\,dx = \frac{1}{2}x^2 \arctan(\sinh(x)) - \frac{1}{2}i\left(x^2 \log(1-ie^x) - x^2 \log(1+ie^x) - 2x \operatorname{PolyLog}(2,-ie^x) + 2x \operatorname{PolyLog}(2,ie^x) + 2 \operatorname{PolyLog}(3,-ie^x) - 2 \operatorname{PolyLog}(3,ie^x)\right)$$

input
```
Integrate[x*ArcTan[Sinh[x]],x]
```

output
```
(x^2*ArcTan[Sinh[x]])/2 - (I/2)*(x^2*Log[1 - I*E^x] - x^2*Log[1 + I*E^x] -
 2*x*PolyLog[2, (-I)*E^x] + 2*x*PolyLog[2, I*E^x] + 2*PolyLog[3, (-I)*E^x]
 - 2*PolyLog[3, I*E^x])
```

3.74.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.200, Rules used = {5728, 3042, 4668, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan(\sinh(x))\,dx$$

$$\Big\downarrow \text{ 5728}$$

$$\frac{1}{2}x^2 \arctan(\sinh(x)) - \frac{1}{2}\int x^2\text{sech}(x)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}x^2 \arctan(\sinh(x)) - \frac{1}{2}\int x^2 \csc\left(ix+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{ 4668}$$

$$\frac{1}{2}x^2 \arctan(\sinh(x)) + \frac{1}{2}\left(2i\int x\log\left(1-ie^x\right)dx - 2i\int x\log\left(1+ie^x\right)dx - 2x^2\arctan\left(e^x\right)\right)$$

$$\Big\downarrow \text{ 3011}$$

$$\frac{1}{2}x^2 \arctan(\sinh(x)) +$$
$$\frac{1}{2}\left(-2i\left(\int \text{PolyLog}\left(2,-ie^x\right)dx - x\,\text{PolyLog}\left(2,-ie^x\right)\right) + 2i\left(\int \text{PolyLog}\left(2,ie^x\right)dx - x\,\text{PolyLog}\left(2,ie^x\right)\right) - 2x^2$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{1}{2}x^2 \arctan(\sinh(x)) +$$
$$\frac{1}{2}\left(-2i\left(\int e^{-x}\,\text{PolyLog}\left(2,-ie^x\right)de^x - x\,\text{PolyLog}\left(2,-ie^x\right)\right) + 2i\left(\int e^{-x}\,\text{PolyLog}\left(2,ie^x\right)de^x - x\,\text{PolyLog}\left(2,ie\right.\right.\right.$$

$$\Big\downarrow \text{ 7143}$$

$$\frac{1}{2}x^2 \arctan(\sinh(x)) +$$
$$\frac{1}{2}\left(-2x^2\arctan\left(e^x\right) - 2i(\text{PolyLog}\left(3,-ie^x\right) - x\,\text{PolyLog}\left(2,-ie^x\right)) + 2i(\text{PolyLog}\left(3,ie^x\right) - x\,\text{PolyLog}\left(2,ie^x\right))\right)$$

input
```
Int[x*ArcTan[Sinh[x]],x]
```

output
```
(x^2*ArcTan[Sinh[x]])/2 + (-2*x^2*ArcTan[E^x] - (2*I)*(-(x*PolyLog[2, (-I)
*E^x]) + PolyLog[3, (-I)*E^x]) + (2*I)*(-(x*PolyLog[2, I*E^x]) + PolyLog[3
, I*E^x]))/2
```

3.74.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5728
```
Int[((a_.) + ArcTan[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(c + d*x)^(m + 1)*((a + b*ArcTan[u])/(d*(m + 1))), x] - Simp[b/(d*(m + 1)
)   Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(1 + u^2)), x], x], x]
 /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &
&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m +
 1, x]]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.74.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.56 (sec) , antiderivative size = 632, normalized size of antiderivative = 8.54

method	result	size
risch	Expression too large to display	632

input
```
int(x*arctan(sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*I*x^2*ln(exp(x)+I)-1/2*I*x^2*ln(exp(x)-I)+1/2*I*x^2*ln(1+I*exp(x))+I*x
*polylog(2,-I*exp(x))-I*polylog(3,-I*exp(x))-1/8*Pi*(csgn(I*(exp(x)-I))^2*
csgn(I*(exp(x)-I)^2)-2*csgn(I*(exp(x)-I))*csgn(I*(exp(x)-I)^2)^2+csgn(I*(e
xp(x)-I)^2)^3+csgn(I*(exp(x)-I)^2)*csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(x)-
I)^2)-csgn(I*(exp(x)-I)^2)*csgn(I*exp(-x)*(exp(x)-I)^2)^2-csgn(I*(exp(x)+I
))^2*csgn(I*(exp(x)+I)^2)+2*csgn(I*(exp(x)+I))*csgn(I*(exp(x)+I)^2)^2-csgn
(I*(exp(x)+I)^2)^3-csgn(I*(exp(x)+I)^2)*csgn(I*exp(-x))*csgn(I*exp(-x)*(ex
p(x)+I)^2)+csgn(I*(exp(x)+I)^2)*csgn(I*exp(-x)*(exp(x)+I)^2)^2-csgn(I*exp(
-x))*csgn(I*exp(-x)*(exp(x)-I)^2)^2+csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(x)
+I)^2)^2-csgn(I*exp(-x)*(exp(x)+I)^2)*csgn(exp(-x)*(exp(x)+I)^2)+csgn(exp(
-x)*(exp(x)+I)^2)^2+csgn(I*exp(-x)*(exp(x)-I)^2)*csgn(exp(-x)*(exp(x)-I)^2
)+csgn(exp(-x)*(exp(x)-I)^2)^2+csgn(I*exp(-x)*(exp(x)-I)^2)^3-csgn(I*exp(-
x)*(exp(x)-I)^2)*csgn(exp(-x)*(exp(x)-I)^2)^2-csgn(I*exp(-x)*(exp(x)+I)^2)
^3+csgn(I*exp(-x)*(exp(x)+I)^2)*csgn(exp(-x)*(exp(x)+I)^2)^2-csgn(exp(-x)*
(exp(x)+I)^2)^3-csgn(exp(-x)*(exp(x)-I)^2)^3-2)*x^2-1/2*I*x^2*ln(1-I*exp(x
))-I*x*polylog(2,I*exp(x))+I*polylog(3,I*exp(x))
```

3.74.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.26

$$\int x\arctan(\sinh(x))\,dx = \frac{1}{2}x^2\arctan(\sinh(x)) + \frac{1}{2}ix^2\log(i\cosh(x) + i\sinh(x) + 1) - \frac{1}{2}ix^2\log(-i\cosh(x) - i\sinh(x) + 1) - ix\mathrm{Li}_2(i\cosh(x) + i\sinh(x)) + ix\mathrm{Li}_2(-i\cosh(x) - i\sinh(x)) + i\,\mathrm{polylog}(3, i\cosh(x) + i\sinh(x)) - i\,\mathrm{polylog}(3, -i\cosh(x) - i\sinh(x))$$

input
```
integrate(x*arctan(sinh(x)),x, algorithm="fricas")
```

output
```
1/2*x^2*arctan(sinh(x)) + 1/2*I*x^2*log(I*cosh(x) + I*sinh(x) + 1) - 1/2*I
*x^2*log(-I*cosh(x) - I*sinh(x) + 1) - I*x*dilog(I*cosh(x) + I*sinh(x)) +
I*x*dilog(-I*cosh(x) - I*sinh(x)) + I*polylog(3, I*cosh(x) + I*sinh(x)) -
I*polylog(3, -I*cosh(x) - I*sinh(x))
```

3.74.6 Sympy [F]

$$\int x\arctan(\sinh(x))\,dx = \int x\,\mathrm{atan}(\sinh(x))\,dx$$

input
```
integrate(x*atan(sinh(x)),x)
```

output
```
Integral(x*atan(sinh(x)), x)
```

3.74.7 Maxima [F]

$$\int x\arctan(\sinh(x))\,dx = \int x\arctan(\sinh(x))\,dx$$

input
```
integrate(x*arctan(sinh(x)),x, algorithm="maxima")
```

output
```
1/2*x^2*arctan(1/2*(e^(2*x) - 1)*e^(-x)) - integrate(x^2*e^x/(e^(2*x) + 1)
, x)
```

3.74.8 Giac [F]

$$\int x \arctan(\sinh(x))\,dx = \int x \arctan(\sinh(x))\,dx$$

input
```
integrate(x*arctan(sinh(x)),x, algorithm="giac")
```

output
```
integrate(x*arctan(sinh(x)), x)
```

3.74.9 Mupad [F(-1)]

Timed out.

$$\int x \arctan(\sinh(x))\,dx = \int x \operatorname{atan}(\sinh(x))\,dx$$

input
```
int(x*atan(sinh(x)),x)
```

output
```
int(x*atan(sinh(x)), x)
```

3.75 $\int x^2 \arctan(\sinh(x))\,dx$

3.75.1 Optimal result

Integrand size = 7, antiderivative size = 108

$$\int x^2 \arctan(\sinh(x))\,dx = -\frac{2}{3}x^3 \arctan(e^x) + \frac{1}{3}x^3 \arctan(\sinh(x)) + ix^2 \operatorname{PolyLog}(2, -ie^x) - ix^2 \operatorname{PolyLog}(2, ie^x) - 2ix \operatorname{PolyLog}(3, -ie^x) + 2ix \operatorname{PolyLog}(3, ie^x) + 2i \operatorname{PolyLog}(4, -ie^x) - 2i \operatorname{PolyLog}(4, ie^x)$$

output
```
-2/3*x^3*arctan(exp(x))+1/3*x^3*arctan(sinh(x))+I*x^2*polylog(2,-I*exp(x))
-I*x^2*polylog(2,I*exp(x))-2*I*x*polylog(3,-I*exp(x))+2*I*x*polylog(3,I*ex
p(x))+2*I*polylog(4,-I*exp(x))-2*I*polylog(4,I*exp(x))
```

3.75.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.12

$$\int x^2 \arctan(\sinh(x))\,dx = \frac{1}{3}x^3 \arctan(\sinh(x)) - \frac{1}{3}i\left(x^3 \log(1 - ie^x) - x^3 \log(1 + ie^x) - 3x^2 \operatorname{PolyLog}(2, -ie^x) + 3x^2 \operatorname{PolyLog}(2, ie^x) + 6x \operatorname{PolyLog}(3, -ie^x) - 6x \operatorname{PolyLog}(3, ie^x) - 6 \operatorname{PolyLog}(4, -ie^x) + 6 \operatorname{PolyLog}(4, ie^x)\right)$$

input
```
Integrate[x^2*ArcTan[Sinh[x]],x]
```

output
```
(x^3*ArcTan[Sinh[x]])/3 - (I/3)*(x^3*Log[1 - I*E^x] - x^3*Log[1 + I*E^x] -
 3*x^2*PolyLog[2, (-I)*E^x] + 3*x^2*PolyLog[2, I*E^x] + 6*x*PolyLog[3, (-I
)*E^x] - 6*x*PolyLog[3, I*E^x] - 6*PolyLog[4, (-I)*E^x] + 6*PolyLog[4, I*E
^x])
```

3.75.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5728, 3042, 4668, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(\sinh(x))\,dx$$

$$\Big\downarrow \text{5728}$$

$$\frac{1}{3}x^3\arctan(\sinh(x))-\frac{1}{3}\int x^3\text{sech}(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}x^3\arctan(\sinh(x))-\frac{1}{3}\int x^3\csc\left(ix+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{4668}$$

$$\frac{1}{3}x^3\arctan(\sinh(x))+\frac{1}{3}\left(3i\int x^2\log\left(1-ie^x\right)dx-3i\int x^2\log\left(1+ie^x\right)dx-2x^3\arctan\left(e^x\right)\right)$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{3}x^3\arctan(\sinh(x))+$$
$$\frac{1}{3}\left(-3i\left(2\int x\,\text{PolyLog}\left(2,-ie^x\right)dx-x^2\,\text{PolyLog}\left(2,-ie^x\right)\right)+3i\left(2\int x\,\text{PolyLog}\left(2,ie^x\right)dx-x^2\,\text{PolyLog}\left(2,ie^x\right)\right.\right.$$

$$\Big\downarrow \text{7163}$$

$$\frac{1}{3}x^3\arctan(\sinh(x))+$$
$$\frac{1}{3}\left(-3i\left(2\left(x\,\text{PolyLog}\left(3,-ie^x\right)-\int\text{PolyLog}\left(3,-ie^x\right)dx\right)-x^2\,\text{PolyLog}\left(2,-ie^x\right)\right)+3i\left(2\left(x\,\text{PolyLog}\left(3,ie^x\right)-\right.\right.\right.$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{3}x^3\arctan(\sinh(x))+$$
$$\frac{1}{3}\left(-3i\left(2\left(x\operatorname{PolyLog}(3,-ie^x)-\int e^{-x}\operatorname{PolyLog}(3,-ie^x)\,de^x\right)-x^2\operatorname{PolyLog}(2,-ie^x)\right)+3i\left(2\left(x\operatorname{PolyLog}(3,i\right.\right.\right.$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{3}x^3\arctan(\sinh(x))+$$
$$\frac{1}{3}(-2x^3\arctan(e^x)-3i(2(x\operatorname{PolyLog}(3,-ie^x)-\operatorname{PolyLog}(4,-ie^x))-x^2\operatorname{PolyLog}(2,-ie^x))+3i(2(x\operatorname{PolyLog}(3$$

input
```
Int[x^2*ArcTan[Sinh[x]],x]
```

output
```
(x^3*ArcTan[Sinh[x]])/3 + (-2*x^3*ArcTan[E^x] - (3*I)*(-(x^2*PolyLog[2, (-
I)*E^x]) + 2*(x*PolyLog[3, (-I)*E^x] - PolyLog[4, (-I)*E^x])) + (3*I)*(-(x
^2*PolyLog[2, I*E^x]) + 2*(x*PolyLog[3, I*E^x] - PolyLog[4, I*E^x])))/3
```

3.75.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5728
```
Int[((a_.) + ArcTan[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[(c + d*x)^(m + 1)*((a + b*ArcTan[u])/(d*(m + 1))), x] - Simp[b/(d*(m + 1)
)   Int[SimplifyIntegrand[(c + d*x)^(m + 1)*(D[u, x]/(1 + u^2)), x], x], x]
 /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &
&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m +
 1, x]]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.75.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.74 (sec) , antiderivative size = 658, normalized size of antiderivative = 6.09

method	result	size
risch	Expression too large to display	658

input
```
int(x^2*arctan(sinh(x)),x,method=_RETURNVERBOSE)
```

output
```
1/3*I*x^3*ln(exp(x)+I)-1/3*I*x^3*ln(exp(x)-I)+1/3*I*x^3*ln(1+I*exp(x))+I*x
^2*polylog(2,-I*exp(x))-2*I*x*polylog(3,-I*exp(x))+2*I*polylog(4,-I*exp(x)
)-1/12*Pi*(csgn(I*(exp(x)-I))^2*csgn(I*(exp(x)-I)^2)-2*csgn(I*(exp(x)-I))*
csgn(I*(exp(x)-I)^2)^2+csgn(I*(exp(x)-I)^2)^3+csgn(I*(exp(x)-I)^2)*csgn(I*
exp(-x))*csgn(I*exp(-x)*(exp(x)-I)^2)-csgn(I*(exp(x)-I)^2)*csgn(I*exp(-x)*
(exp(x)-I)^2)^2-csgn(I*(exp(x)+I))^2*csgn(I*(exp(x)+I)^2)+2*csgn(I*(exp(x)
+I))*csgn(I*(exp(x)+I)^2)^2-csgn(I*(exp(x)+I)^2)^3-csgn(I*(exp(x)+I)^2)*cs
gn(I*exp(-x))*csgn(I*exp(-x)*(exp(x)+I)^2)+csgn(I*(exp(x)+I)^2)*csgn(I*exp
(-x)*(exp(x)+I)^2)^2-csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(x)-I)^2)^2+csgn(I
*exp(-x))*csgn(I*exp(-x)*(exp(x)+I)^2)^2-csgn(I*exp(-x)*(exp(x)+I)^2)*csgn
(exp(-x)*(exp(x)+I)^2)+csgn(exp(-x)*(exp(x)+I)^2)^2+csgn(I*exp(-x)*(exp(x)
-I)^2)*csgn(exp(-x)*(exp(x)-I)^2)+csgn(exp(-x)*(exp(x)-I)^2)^2+csgn(I*exp(
-x)*(exp(x)-I)^2)^3-csgn(I*exp(-x)*(exp(x)-I)^2)*csgn(exp(-x)*(exp(x)-I)^2
)^2-csgn(I*exp(-x)*(exp(x)+I)^2)^3+csgn(I*exp(-x)*(exp(x)+I)^2)*csgn(exp(-
x)*(exp(x)+I)^2)^2-csgn(exp(-x)*(exp(x)+I)^2)^3-csgn(exp(-x)*(exp(x)-I)^2)
^3-2)*x^3-1/3*I*x^3*ln(1-I*exp(x))-I*x^2*polylog(2,I*exp(x))+2*I*x*polylog
(3,I*exp(x))-2*I*polylog(4,I*exp(x))
```

3.75.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.16

$$\begin{aligned}\int x^2 \arctan(\sinh(x))\,dx =& \frac{1}{3}\,x^3 \arctan\left(\sinh\left(x\right)\right) + \frac{1}{3} i\,x^3 \log\left(i\,\cosh\left(x\right) + i\,\sinh\left(x\right) + 1\right) \\ &- \frac{1}{3} i\,x^3 \log\left(-i\,\cosh\left(x\right) - i\,\sinh\left(x\right) + 1\right) \\ &- i\,x^2 \mathrm{Li}_2(i\,\cosh\left(x\right) + i\,\sinh\left(x\right)) + i\,x^2 \mathrm{Li}_2(-i\,\cosh\left(x\right) - i\,\sinh\left(x\right)) \\ &+ 2i\,x \mathrm{polylog}(3, i\,\cosh\left(x\right) + i\,\sinh\left(x\right)) \\ &- 2i\,x \mathrm{polylog}(3, -i\,\cosh\left(x\right) - i\,\sinh\left(x\right)) \\ &- 2i\,\mathrm{polylog}(4, i\,\cosh\left(x\right) + i\,\sinh\left(x\right)) \\ &+ 2i\,\mathrm{polylog}(4, -i\,\cosh\left(x\right) - i\,\sinh\left(x\right))\end{aligned}$$

input
```
integrate(x^2*arctan(sinh(x)),x, algorithm="fricas")
```

output
```
1/3*x^3*arctan(sinh(x)) + 1/3*I*x^3*log(I*cosh(x) + I*sinh(x) + 1) - 1/3*I
*x^3*log(-I*cosh(x) - I*sinh(x) + 1) - I*x^2*dilog(I*cosh(x) + I*sinh(x))
+ I*x^2*dilog(-I*cosh(x) - I*sinh(x)) + 2*I*x*polylog(3, I*cosh(x) + I*sin
h(x)) - 2*I*x*polylog(3, -I*cosh(x) - I*sinh(x)) - 2*I*polylog(4, I*cosh(x
) + I*sinh(x)) + 2*I*polylog(4, -I*cosh(x) - I*sinh(x))
```

3.75.6 Sympy [F]

$$\int x^2 \arctan(\sinh(x))\,dx = \int x^2 \operatorname{atan}\left(\sinh\left(x\right)\right)dx$$

input
```
integrate(x**2*atan(sinh(x)),x)
```

output
```
Integral(x**2*atan(sinh(x)), x)
```

3.75.7 Maxima [F]

$$\int x^2 \arctan(\sinh(x))\,dx = \int x^2 \arctan\left(\sinh\left(x\right)\right)\,dx$$

input
```
integrate(x^2*arctan(sinh(x)),x, algorithm="maxima")
```

output
```
1/3*x^3*arctan(1/2*(e^(2*x) - 1)*e^(-x)) - 2*integrate(1/3*x^3*e^x/(e^(2*x
) + 1), x)
```

3.75.8 Giac [F]

$$\int x^2 \arctan(\sinh(x))\,dx = \int x^2 \arctan\left(\sinh\left(x\right)\right)\,dx$$

input
```
integrate(x^2*arctan(sinh(x)),x, algorithm="giac")
```

output
```
integrate(x^2*arctan(sinh(x)), x)
```

3.75.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \arctan(\sinh(x))\,dx = \int x^2\,\mathrm{atan}(\sinh(x))\;dx$$

input
```
int(x^2*atan(sinh(x)),x)
```

output
```
int(x^2*atan(sinh(x)), x)
```

3.76 $\int (e+fx)^3 \arctan(\tanh(a+bx))\,dx$

3.76.1 Optimal result

Integrand size = 15, antiderivative size = 299

$$\begin{aligned}\int (e+fx)^3 \arctan(\tanh(a+bx))\,dx = &-\frac{(e+fx)^4 \arctan\left(e^{2a+2bx}\right)}{4f}\\ &+\frac{(e+fx)^4 \arctan(\tanh(a+bx))}{4f}\\ &+\frac{i(e+fx)^3 \operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b}\\ &-\frac{i(e+fx)^3 \operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b}\\ &-\frac{3if(e+fx)^2 \operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{8b^2}\\ &+\frac{3if(e+fx)^2 \operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)}{8b^2}\\ &+\frac{3if^2(e+fx) \operatorname{PolyLog}\left(4,-ie^{2a+2bx}\right)}{8b^3}\\ &-\frac{3if^2(e+fx) \operatorname{PolyLog}\left(4,ie^{2a+2bx}\right)}{8b^3}\\ &-\frac{3if^3 \operatorname{PolyLog}\left(5,-ie^{2a+2bx}\right)}{16b^4}\\ &+\frac{3if^3 \operatorname{PolyLog}\left(5,ie^{2a+2bx}\right)}{16b^4}\end{aligned}$$

output
```
-1/4*(f*x+e)^4*arctan(exp(2*b*x+2*a))/f+1/4*(f*x+e)^4*arctan(tanh(b*x+a))/
f+1/4*I*(f*x+e)^3*polylog(2,-I*exp(2*b*x+2*a))/b-1/4*I*(f*x+e)^3*polylog(2
,I*exp(2*b*x+2*a))/b-3/8*I*f*(f*x+e)^2*polylog(3,-I*exp(2*b*x+2*a))/b^2+3/
8*I*f*(f*x+e)^2*polylog(3,I*exp(2*b*x+2*a))/b^2+3/8*I*f^2*(f*x+e)*polylog(
4,-I*exp(2*b*x+2*a))/b^3-3/8*I*f^2*(f*x+e)*polylog(4,I*exp(2*b*x+2*a))/b^3
-3/16*I*f^3*polylog(5,-I*exp(2*b*x+2*a))/b^4+3/16*I*f^3*polylog(5,I*exp(2*
b*x+2*a))/b^4
```

3.76.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 600 vs. 2(299) = 598.

Time = 0.79 (sec) , antiderivative size = 600, normalized size of antiderivative = 2.01

$$\int (e+fx)^3 \arctan(\tanh(a+bx))\,dx = \frac{1}{4}x\left(4e^3+6e^2fx+4ef^2x^2+f^3x^3\right)\arctan(\tanh(a+bx)) - \frac{i\left(8b^4e^3x\log\left(1-ie^{2(a+bx)}\right)+12b^4e^2fx^2\log\left(1-ie^{2(a+bx)}\right)+8b^4ef^2x^3\log\left(1-ie^{2(a+bx)}\right)+2b^4f^3x^4\log\right.}{}$$

input
```
Integrate[(e + f*x)^3*ArcTan[Tanh[a + b*x]],x]
```

output
```
(x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3)*ArcTan[Tanh[a + b*x]])/4 -
((I/16)*(8*b^4*e^3*x*Log[1 - I*E^(2*(a + b*x))] + 12*b^4*e^2*f*x^2*Log[1 -
 I*E^(2*(a + b*x))] + 8*b^4*e*f^2*x^3*Log[1 - I*E^(2*(a + b*x))] + 2*b^4*f
^3*x^4*Log[1 - I*E^(2*(a + b*x))] - 8*b^4*e^3*x*Log[1 + I*E^(2*(a + b*x))]
 - 12*b^4*e^2*f*x^2*Log[1 + I*E^(2*(a + b*x))] - 8*b^4*e*f^2*x^3*Log[1 + I
*E^(2*(a + b*x))] - 2*b^4*f^3*x^4*Log[1 + I*E^(2*(a + b*x))] - 4*b^3*(e +
f*x)^3*PolyLog[2, (-I)*E^(2*(a + b*x))] + 4*b^3*(e + f*x)^3*PolyLog[2, I*E
^(2*(a + b*x))] + 6*b^2*e^2*f*PolyLog[3, (-I)*E^(2*(a + b*x))] + 12*b^2*e*
f^2*x*PolyLog[3, (-I)*E^(2*(a + b*x))] + 6*b^2*f^3*x^2*PolyLog[3, (-I)*E^(
2*(a + b*x))] - 6*b^2*e^2*f*PolyLog[3, I*E^(2*(a + b*x))] - 12*b^2*e*f^2*x
*PolyLog[3, I*E^(2*(a + b*x))] - 6*b^2*f^3*x^2*PolyLog[3, I*E^(2*(a + b*x)
)] - 6*b*e*f^2*PolyLog[4, (-I)*E^(2*(a + b*x))] - 6*b*f^3*x*PolyLog[4, (-I
)*E^(2*(a + b*x))] + 6*b*e*f^2*PolyLog[4, I*E^(2*(a + b*x))] + 6*b*f^3*x*P
olyLog[4, I*E^(2*(a + b*x))] + 3*f^3*PolyLog[5, (-I)*E^(2*(a + b*x))] - 3*
f^3*PolyLog[5, I*E^(2*(a + b*x))]))/b^4
```

3.76.3 Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {5706, 3042, 4668, 3011, 7163, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^3 \arctan(\tanh(a+bx))\,dx$$

$$\Big\downarrow \text{5706}$$

$$\frac{(e+fx)^4 \arctan(\tanh(a+bx))}{4f}-\frac{b\int (e+fx)^4\text{sech}(2a+2bx)dx}{4f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(e+fx)^4 \arctan(\tanh(a+bx))}{4f}-\frac{b\int (e+fx)^4\csc\left(2ia+2ibx+\frac{\pi}{2}\right)dx}{4f}$$

$$\Big\downarrow \text{4668}$$

$$\frac{(e+fx)^4 \arctan(\tanh(a+bx))}{4f}-\frac{b\left(-\frac{2if\int (e+fx)^3\log\left(1-ie^{2a+2bx}\right)dx}{b}+\frac{2if\int (e+fx)^3\log\left(1+ie^{2a+2bx}\right)dx}{b}+\frac{(e+fx)^4\arctan\left(e^{2a+2bx}\right)}{b}\right)}{4f}$$

$$\Big\downarrow \text{3011}$$

$$\frac{(e+fx)^4 \arctan(\tanh(a+bx))}{4f}-\frac{b\left(\frac{2if\left(\frac{3f\int (e+fx)^2\text{PolyLog}\left(2,-ie^{2a+2bx}\right)dx}{2b}-\frac{(e+fx)^3\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{2b}\right)}{b}-\frac{2if\left(\frac{3f\int (e+fx)^2\text{PolyLog}\left(2,ie^{2a+2bx}\right)dx}{2b}-\frac{(e+fx)^3\text{PolyLog}\left(2,ie^{2\cdots}\right)}{2b}\cdots\right)}{b}\cdots\right)}{4f}$$

$$\Big\downarrow \text{7163}$$

$$\frac{(e+fx)^4 \arctan(\tanh(a+bx))}{4f}-\frac{b\left(\frac{2if\left(\frac{3f\left(\frac{(e+fx)^2\text{PolyLog}\left(3,-ie^{2a+2bx}\right)}{2b}-\frac{f\int (e+fx)\text{PolyLog}\left(3,-ie^{2a+2bx}\right)dx}{b}\right)}{2b}-\frac{(e+fx)^3\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{2b}\right)}{b}-\frac{2if\left(\frac{3f\left(\frac{(e+fx)^2\text{PolyLog}\left(\cdots\right)}{2b}\cdots\right)}{\cdots}\cdots\right)}{\cdots}\cdots\right)}{4f}$$

$\downarrow$ 7163

$$\frac{(e+fx)^4\arctan(\tanh(a+bx))}{4f}-$$

$$b\left(\frac{2if\left(\frac{3f\left(\frac{(e+fx)^2\text{PolyLog}\left(3,-ie^{2a+2bx}\right)}{2b}-\frac{f\left(\frac{(e+fx)\text{PolyLog}\left(4,-ie^{2a+2bx}\right)}{2b}-\frac{f\int\text{PolyLog}\left(4,-ie^{2a+2bx}\right)dx}{2b}\right)}{b}\right)}{2b}-\frac{(e+fx)^3\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{2b}\right)}{b}\right.$$

$\downarrow$ 2720

$$\frac{(e+fx)^4\arctan(\tanh(a+bx))}{4f}-$$

$$b\left(\frac{2if\left(\frac{3f\left(\frac{(e+fx)^2\text{PolyLog}\left(3,-ie^{2a+2bx}\right)}{2b}-\frac{f\left(\frac{(e+fx)\text{PolyLog}\left(4,-ie^{2a+2bx}\right)}{2b}-\frac{f\int e^{-2a-2bx}\text{PolyLog}\left(4,-ie^{2a+2bx}\right)de^{2a+2bx}}{4b^2}\right)}{b}\right)}{2b}-\frac{(e+fx)^3\text{PolyLog}\left(\right.}{2b}\right.}{b}\right.$$

$\downarrow$ 7143

3.76. $\int(e+fx)^3\arctan(\tanh(a+bx))\,dx$

$$\frac{(e+fx)^4\arctan(\tanh(a+bx))}{4f} - \frac{b\left(\frac{(e+fx)^4\arctan(e^{2a+2bx})}{b} + \frac{2if\left(\frac{3f\left(\frac{(e+fx)^2\operatorname{PolyLog}(3,-ie^{2a+2bx})}{2b} - \frac{f\left(\frac{(e+fx)\operatorname{PolyLog}(4,-ie^{2a+2bx})}{2b} - \frac{f\operatorname{PolyLog}(5,-ie^{2a+2bx})}{4b^2}\right)}{b}\right)}{2b} - \frac{(e+fx)^3}{\cdots}\right)}{b}\cdots\right)}{\cdots}$$

input
```
Int[(e + f*x)^3*ArcTan[Tanh[a + b*x]],x]
```

output
```
((e + f*x)^4*ArcTan[Tanh[a + b*x]])/(4*f) - (b*(((e + f*x)^4*ArcTan[E^(2*a
 + 2*b*x)])/b + ((2*I)*f*(-1/2*((e + f*x)^3*PolyLog[2, (-I)*E^(2*a + 2*b*x
)])/b + (3*f*(((e + f*x)^2*PolyLog[3, (-I)*E^(2*a + 2*b*x)])/(2*b) - (f*((
(e + f*x)*PolyLog[4, (-I)*E^(2*a + 2*b*x)])/(2*b) - (f*PolyLog[5, (-I)*E^(
2*a + 2*b*x)])/(4*b^2)))/b))/(2*b)))/b - ((2*I)*f*(-1/2*((e + f*x)^3*PolyL
og[2, I*E^(2*a + 2*b*x)])/b + (3*f*(((e + f*x)^2*PolyLog[3, I*E^(2*a + 2*b
*x)])/(2*b) - (f*(((e + f*x)*PolyLog[4, I*E^(2*a + 2*b*x)])/(2*b) - (f*Pol
yLog[5, I*E^(2*a + 2*b*x)])/(4*b^2)))/b))/(2*b)))/b))/(4*f)
```

3.76.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5706
```
Int[ArcTan[Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcTan[Tanh[a + b*x]]/(f*(m + 1))), x] - Simp[b/
(f*(m + 1))   Int[(e + f*x)^(m + 1)*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a,
b, e, f}, x] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.76. $\int (e+fx)^3 \arctan(\tanh(a+bx))\, dx$

3.76.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 15.92 (sec) , antiderivative size = 3570, normalized size of antiderivative = 11.94

method	result	size
risch	Expression too large to display	3570

input
```
int((f*x+e)^3*arctan(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/8*I/f*ln(exp(2*b*x+2*a)-I)*e^4+1/8*I*(f*x+e)^4/f*ln(exp(2*b*x+2*a)+I)-1
/8*I*f^3*ln(exp(2*b*x+2*a)-I)*x^4-1/8*I*f^3*ln(1-I*exp(2*b*x+2*a))*x^4-1/8
*I/f*e^4*ln(exp(2*b*x+2*a)+I)-1/2*I/b*e^3*dilog(((-I)^(1/2)-exp(b*x+a))/(-
I)^(1/2))-1/2*I/b*e^3*dilog(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))-1/2*I*e^3*
ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))*x-1/2*I*e^3*ln(((-I)^(1/2)+exp(b*x+
a))/(-I)^(1/2))*x+1/8*I*f^3*ln(1+I*exp(2*b*x+2*a))*x^4+1/8*I/f*e^4*ln(-exp
(2*b*x+2*a)+I)+1/2*I/b*e^3*dilog(1+exp(b*x+a)*(-1)^(3/4))+1/2*I/b*e^3*dilo
g(1-exp(b*x+a)*(-1)^(3/4))+1/2*I*e^3*ln(1+exp(b*x+a)*(-1)^(3/4))*x+1/2*I*e
^3*ln(1-exp(b*x+a)*(-1)^(3/4))*x-1/16*Pi*(csgn(I/(exp(2*b*x+2*a)+1))*csgn(
I*(exp(2*b*x+2*a)-I))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))-csgn(I
/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)+I))*csgn(I*(exp(2*b*x+2*a)+I)/
(exp(2*b*x+2*a)+1))-csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)-I)/(
exp(2*b*x+2*a)+1))^2+csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)+I)/
(exp(2*b*x+2*a)+1))^2-csgn(I*(exp(2*b*x+2*a)-I))*csgn(I*(exp(2*b*x+2*a)-I)
/(exp(2*b*x+2*a)+1))^2+csgn(I*(exp(2*b*x+2*a)+I))*csgn(I*(exp(2*b*x+2*a)+I
)/(exp(2*b*x+2*a)+1))^2-csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))*csgn
((1+I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))+csgn((1+I)*(exp(2*b*x+2*a)+I
)/(exp(2*b*x+2*a)+1))^2+csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))*csgn
((1-I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))+csgn((1-I)*(exp(2*b*x+2*a)-I
)/(exp(2*b*x+2*a)+1))^2+csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^3...
```

3.76.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1460 vs. 2(236) = 472.

Time = 0.35 (sec) , antiderivative size = 1460, normalized size of antiderivative = 4.88

$$\int (e+fx)^3 \arctan(\tanh(a+bx))\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*arctan(tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/8*(24*I*f^3*polylog(5, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) +
24*I*f^3*polylog(5, -1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 24*I
*f^3*polylog(5, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 24*I*f^3
*polylog(5, -1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 2*(b^4*f^3*
x^4 + 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x)*arctan(sinh(b*x + a
)/cosh(b*x + a)) - 4*(I*b^3*f^3*x^3 + 3*I*b^3*e*f^2*x^2 + 3*I*b^3*e^2*f*x
+ I*b^3*e^3)*dilog(1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 4*(I*b
^3*f^3*x^3 + 3*I*b^3*e*f^2*x^2 + 3*I*b^3*e^2*f*x + I*b^3*e^3)*dilog(-1/2*s
qrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 4*(-I*b^3*f^3*x^3 - 3*I*b^3*e*
f^2*x^2 - 3*I*b^3*e^2*f*x - I*b^3*e^3)*dilog(1/2*sqrt(-4*I)*(cosh(b*x + a)
 + sinh(b*x + a))) - 4*(-I*b^3*f^3*x^3 - 3*I*b^3*e*f^2*x^2 - 3*I*b^3*e^2*f
*x - I*b^3*e^3)*dilog(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + (
-I*b^4*f^3*x^4 - 4*I*b^4*e*f^2*x^3 - 6*I*b^4*e^2*f*x^2 - 4*I*b^4*e^3*x - 4
*I*a*b^3*e^3 + 6*I*a^2*b^2*e^2*f - 4*I*a^3*b*e*f^2 + I*a^4*f^3)*log(1/2*sq
rt(4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^4*f^3*x^4 - 4*I*b^4*e
*f^2*x^3 - 6*I*b^4*e^2*f*x^2 - 4*I*b^4*e^3*x - 4*I*a*b^3*e^3 + 6*I*a^2*b^2
*e^2*f - 4*I*a^3*b*e*f^2 + I*a^4*f^3)*log(-1/2*sqrt(4*I)*(cosh(b*x + a) +
sinh(b*x + a)) + 1) + (I*b^4*f^3*x^4 + 4*I*b^4*e*f^2*x^3 + 6*I*b^4*e^2*f*x
^2 + 4*I*b^4*e^3*x + 4*I*a*b^3*e^3 - 6*I*a^2*b^2*e^2*f + 4*I*a^3*b*e*f^2 -
 I*a^4*f^3)*log(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (...
```

3.76.6 Sympy [F]

$$\int (e+fx)^3 \arctan(\tanh(a+bx))\,dx = \int (e+fx)^3 \operatorname{atan}(\tanh(a+bx))\,dx$$

input
```
integrate((f*x+e)**3*atan(tanh(b*x+a)),x)
```

output
```
Integral((e + f*x)**3*atan(tanh(a + b*x)), x)
```

3.76.7 Maxima [F]

$$\int (e+fx)^3 \arctan(\tanh(a+bx))\,dx = \int (fx+e)^3 \arctan(\tanh(bx+a))\,dx$$

input
```
integrate((f*x+e)^3*arctan(tanh(b*x+a)),x, algorithm="maxima")
```

output
```
1/4*(f^3*x^4 + 4*e*f^2*x^3 + 6*e^2*f*x^2 + 4*e^3*x)*arctan((e^(2*b*x + 2*a
) - 1)/(e^(2*b*x + 2*a) + 1)) - integrate(1/2*(b*f^3*x^4*e^(2*a) + 4*b*e*f
^2*x^3*e^(2*a) + 6*b*e^2*f*x^2*e^(2*a) + 4*b*e^3*x*e^(2*a))*e^(2*b*x)/(e^(
4*b*x + 4*a) + 1), x)
```

3.76.8 Giac [F(-1)]

Timed out.

$$\int (e+fx)^3 \arctan(\tanh(a+bx))\,dx = \text{Timed out}$$

input
```
integrate((f*x+e)^3*arctan(tanh(b*x+a)),x, algorithm="giac")
```

output
```
Timed out
```

3.76.9 Mupad [F(-1)]

Timed out.

$$\int (e+fx)^3 \arctan(\tanh(a+bx))\,dx = \int \operatorname{atan}(\tanh(a+b\,x))\,(e+f\,x)^3\,dx$$

input
```
int(atan(tanh(a + b*x))*(e + f*x)^3,x)
```

output
```
int(atan(tanh(a + b*x))*(e + f*x)^3, x)
```

3.77 $\int (e+fx)^2 \arctan(\tanh(a+bx))\,dx$

3.77.1 Optimal result

Integrand size = 15, antiderivative size = 229

$$\begin{aligned}\int (e+fx)^2 \arctan(\tanh(a+bx))\,dx = &-\frac{(e+fx)^3 \arctan\left(e^{2a+2bx}\right)}{3f} \\ &+\frac{(e+fx)^3 \arctan(\tanh(a+bx))}{3f} \\ &+\frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} \\ &-\frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} \\ &-\frac{if(e+fx) \operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{4b^2} \\ &+\frac{if(e+fx) \operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)}{4b^2} \\ &+\frac{if^2 \operatorname{PolyLog}\left(4,-ie^{2a+2bx}\right)}{8b^3} \\ &-\frac{if^2 \operatorname{PolyLog}\left(4,ie^{2a+2bx}\right)}{8b^3}\end{aligned}$$

output
```
-1/3*(f*x+e)^3*arctan(exp(2*b*x+2*a))/f+1/3*(f*x+e)^3*arctan(tanh(b*x+a))/
f+1/4*I*(f*x+e)^2*polylog(2,-I*exp(2*b*x+2*a))/b-1/4*I*(f*x+e)^2*polylog(2
,I*exp(2*b*x+2*a))/b-1/4*I*f*(f*x+e)*polylog(3,-I*exp(2*b*x+2*a))/b^2+1/4*
I*f*(f*x+e)*polylog(3,I*exp(2*b*x+2*a))/b^2+1/8*I*f^2*polylog(4,-I*exp(2*b
*x+2*a))/b^3-1/8*I*f^2*polylog(4,I*exp(2*b*x+2*a))/b^3
```

3.77.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.64

$$\int (e+fx)^2 \arctan(\tanh(a+bx))\,dx = \frac{1}{3}x\left(3e^2+3efx+f^2x^2\right)\arctan(\tanh(a+bx))$$
$$-\frac{i\left(12b^3e^2x\log\left(1-ie^{2(a+bx)}\right)+12b^3efx^2\log\left(1-ie^{2(a+bx)}\right)+4b^3f^2x^3\log\left(1-ie^{2(a+bx)}\right)-12b^3e^2x\log\left(\right.\right.}{}$$

input
```
Integrate[(e + f*x)^2*ArcTan[Tanh[a + b*x]],x]
```

output
```
(x*(3*e^2 + 3*e*f*x + f^2*x^2)*ArcTan[Tanh[a + b*x]])/3 - ((I/24)*(12*b^3*
e^2*x*Log[1 - I*E^(2*(a + b*x))] + 12*b^3*e*f*x^2*Log[1 - I*E^(2*(a + b*x)
)] + 4*b^3*f^2*x^3*Log[1 - I*E^(2*(a + b*x))] - 12*b^3*e^2*x*Log[1 + I*E^(
2*(a + b*x))] - 12*b^3*e*f*x^2*Log[1 + I*E^(2*(a + b*x))] - 4*b^3*f^2*x^3*
Log[1 + I*E^(2*(a + b*x))] - 6*b^2*(e + f*x)^2*PolyLog[2, (-I)*E^(2*(a + b
*x))] + 6*b^2*(e + f*x)^2*PolyLog[2, I*E^(2*(a + b*x))] + 6*b*e*f*PolyLog[
3, (-I)*E^(2*(a + b*x))] + 6*b*f^2*x*PolyLog[3, (-I)*E^(2*(a + b*x))] - 6*
b*e*f*PolyLog[3, I*E^(2*(a + b*x))] - 6*b*f^2*x*PolyLog[3, I*E^(2*(a + b*x
))] - 3*f^2*PolyLog[4, (-I)*E^(2*(a + b*x))] + 3*f^2*PolyLog[4, I*E^(2*(a
+ b*x))]))/b^3
```

3.77.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {5706, 3042, 4668, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^2 \arctan(\tanh(a+bx))\,dx$$

$$\Big\downarrow \text{ 5706}$$

$$\frac{(e+fx)^3\arctan(\tanh(a+bx))}{3f}-\frac{b\int (e+fx)^3\text{sech}(2a+2bx)dx}{3f}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(e+fx)^3\arctan(\tanh(a+bx))}{3f}-\frac{b\int (e+fx)^3\csc\left(2ia+2ibx+\frac{\pi}{2}\right)dx}{3f}$$

$\Big\downarrow$ 4668

$$\frac{(e+fx)^3\arctan(\tanh(a+bx))}{3f}-\frac{b\left(-\frac{3if\int(e+fx)^2\log\left(1-ie^{2a+2bx}\right)dx}{2b}+\frac{3if\int(e+fx)^2\log\left(1+ie^{2a+2bx}\right)dx}{2b}+\frac{(e+fx)^3\arctan\left(e^{2a+2bx}\right)}{b}\right)}{3f}$$

$\Big\downarrow$ 3011

$$\frac{(e+fx)^3\arctan(\tanh(a+bx))}{3f}-$$
$$\frac{b\left(\frac{3if\left(\frac{f\int(e+fx)\,\text{PolyLog}\left(2,-ie^{2a+2bx}\right)dx}{b}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{2b}\right)}{2b}-\frac{3if\left(\frac{f\int(e+fx)\,\text{PolyLog}\left(2,ie^{2a+2bx}\right)dx}{b}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,ie^{2a+2}\right.}{2b}\right.}{2b}\right.}{3f}$$

$\Big\downarrow$ 7163

$$\frac{(e+fx)^3\arctan(\tanh(a+bx))}{3f}-$$
$$\frac{b\left(\frac{3if\left(\frac{f\left(\frac{(e+fx)\,\text{PolyLog}\left(3,-ie^{2a+2bx}\right)}{2b}-\frac{f\int\text{PolyLog}\left(3,-ie^{2a+2bx}\right)dx}{2b}\right)}{b}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{2b}\right)}{2b}-\frac{3if\left(\frac{f\left(\frac{(e+fx)\,\text{PolyLog}\left(3,ie^{2a+2bx}\right)}{2b}\right.}{}\right.}{}\right.}{3f}$$

$\Big\downarrow$ 2720

$$\frac{(e+fx)^3\arctan(\tanh(a+bx))}{3f}-$$
$$\frac{b\left(\frac{3if\left(\frac{f\left(\frac{(e+fx)\,\text{PolyLog}\left(3,-ie^{2a+2bx}\right)}{2b}-\frac{f\int e^{-2a-2bx}\,\text{PolyLog}\left(3,-ie^{2a+2bx}\right)de^{2a+2bx}}{4b^2}\right)}{b}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{2b}\right)}{2b}-\frac{3if\left(\frac{f\left(\frac{(e+fx)\,\text{Po}}{}\right.}{}\right.}{}\right.}{3f}$$

$\Big\downarrow$ 7143

3.77. $\int(e+fx)^2\arctan(\tanh(a+bx))\,dx$

$$\frac{(e+fx)^3 \arctan(\tanh(a+bx))}{3f} - \frac{b\left(\frac{(e+fx)^3 \arctan(e^{2a+2bx})}{b} + \frac{3if\left(\frac{f\left(\frac{(e+fx)\,\mathrm{PolyLog}(3,-ie^{2a+2bx})}{2b}-\frac{f\,\mathrm{PolyLog}(4,-ie^{2a+2bx})}{4b^2}\right)}{b}-\frac{(e+fx)^2\,\mathrm{PolyLog}(2,-ie^{2a+2bx})}{2b}\right)}{2b} - \frac{3if\left(\frac{f\left(\cdots\right.}{}\right.}{}\right.}{3f}$$

input
```
Int[(e + f*x)^2*ArcTan[Tanh[a + b*x]],x]
```

output
```
((e + f*x)^3*ArcTan[Tanh[a + b*x]])/(3*f) - (b*(((e + f*x)^3*ArcTan[E^(2*a
 + 2*b*x)])/b + (((3*I)/2)*f*(-1/2*((e + f*x)^2*PolyLog[2, (-I)*E^(2*a + 2
*b*x)])/b + (f*(((e + f*x)*PolyLog[3, (-I)*E^(2*a + 2*b*x)])/(2*b) - (f*Po
lyLog[4, (-I)*E^(2*a + 2*b*x)])/(4*b^2)))/b))/b - (((3*I)/2)*f*(-1/2*((e +
 f*x)^2*PolyLog[2, I*E^(2*a + 2*b*x)])/b + (f*(((e + f*x)*PolyLog[3, I*E^(
2*a + 2*b*x)])/(2*b) - (f*PolyLog[4, I*E^(2*a + 2*b*x)])/(4*b^2)))/b))/b))
/(3*f)
```

3.77.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5706
```
Int[ArcTan[Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcTan[Tanh[a + b*x]]/(f*(m + 1))), x] - Simp[b/
(f*(m + 1))   Int[(e + f*x)^(m + 1)*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a,
b, e, f}, x] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.77.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 10.45 (sec) , antiderivative size = 2668, normalized size of antiderivative = 11.65

method	result	size
risch	Expression too large to display	2668

input
```
int((f*x+e)^2*arctan(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/8*I*f^2*polylog(4,-I*exp(2*b*x+2*a))/b^3-1/8*I*f^2*polylog(4,I*exp(2*b*x
+2*a))/b^3-1/12*Pi*(csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)-I))*
csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))-csgn(I/(exp(2*b*x+2*a)+1))*c
sgn(I*(exp(2*b*x+2*a)+I))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))-cs
gn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^2+c
sgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^2-
csgn(I*(exp(2*b*x+2*a)-I))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^2
+csgn(I*(exp(2*b*x+2*a)+I))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^
2-csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))*csgn((1+I)*(exp(2*b*x+2*a)
+I)/(exp(2*b*x+2*a)+1))+csgn((1+I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^
2+csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))*csgn((1-I)*(exp(2*b*x+2*a)
-I)/(exp(2*b*x+2*a)+1))+csgn((1-I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^
2+csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^3-csgn(I*(exp(2*b*x+2*a)-I
)/(exp(2*b*x+2*a)+1))*csgn((1-I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^2-
csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^3+csgn(I*(exp(2*b*x+2*a)+I)/
(exp(2*b*x+2*a)+1))*csgn((1+I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^2-cs
gn((1+I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^3-csgn((1-I)*(exp(2*b*x+2*
a)-I)/(exp(2*b*x+2*a)+1))^3-1)*(f*x+e)^3/f+I*f/b^2*a^2*e*ln(((-I)^(1/2)-ex
p(b*x+a))/(-I)^(1/2))+I*f/b^2*a^2*e*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))
+I*f/b^2*a*e*dilog(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))+I*f/b^2*a*e*dilo...
```

3.77.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1002 vs. $2(180) = 360$.

Time = 0.33 (sec) , antiderivative size = 1002, normalized size of antiderivative = 4.38

$$\int (e+fx)^2 \arctan(\tanh(a+bx))\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*arctan(tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/6*(-6*I*f^2*polylog(4, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) -
6*I*f^2*polylog(4, -1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 6*I*f
^2*polylog(4, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 6*I*f^2*po
lylog(4, -1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 2*(b^3*f^2*x^3
 + 3*b^3*e*f*x^2 + 3*b^3*e^2*x)*arctan(sinh(b*x + a)/cosh(b*x + a)) - 3*(I
*b^2*f^2*x^2 + 2*I*b^2*e*f*x + I*b^2*e^2)*dilog(1/2*sqrt(4*I)*(cosh(b*x +
a) + sinh(b*x + a))) - 3*(I*b^2*f^2*x^2 + 2*I*b^2*e*f*x + I*b^2*e^2)*dilog
(-1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 3*(-I*b^2*f^2*x^2 - 2*I
*b^2*e*f*x - I*b^2*e^2)*dilog(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a
))) - 3*(-I*b^2*f^2*x^2 - 2*I*b^2*e*f*x - I*b^2*e^2)*dilog(-1/2*sqrt(-4*I)
*(cosh(b*x + a) + sinh(b*x + a))) + (-I*b^3*f^2*x^3 - 3*I*b^3*e*f*x^2 - 3*
I*b^3*e^2*x - 3*I*a*b^2*e^2 + 3*I*a^2*b*e*f - I*a^3*f^2)*log(1/2*sqrt(4*I)
*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^3*f^2*x^3 - 3*I*b^3*e*f*x^2
- 3*I*b^3*e^2*x - 3*I*a*b^2*e^2 + 3*I*a^2*b*e*f - I*a^3*f^2)*log(-1/2*sqrt
(4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b^3*f^2*x^3 + 3*I*b^3*e*f*
x^2 + 3*I*b^3*e^2*x + 3*I*a*b^2*e^2 - 3*I*a^2*b*e*f + I*a^3*f^2)*log(1/2*s
qrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b^3*f^2*x^3 + 3*I*b^3*
e*f*x^2 + 3*I*b^3*e^2*x + 3*I*a*b^2*e^2 - 3*I*a^2*b*e*f + I*a^3*f^2)*log(-
1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (3*I*a*b^2*e^2 - 3*I
*a^2*b*e*f + I*a^3*f^2)*log(I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x ...
```

3.77.6 Sympy [F]

$$\int (e+fx)^2 \arctan(\tanh(a+bx))\,dx = \int (e+fx)^2 \operatorname{atan}(\tanh(a+bx))\,dx$$

input
```
integrate((f*x+e)**2*atan(tanh(b*x+a)),x)
```

output
```
Integral((e + f*x)**2*atan(tanh(a + b*x)), x)
```

3.77.7 Maxima [F]

$$\int (e+fx)^2 \arctan(\tanh(a+bx))\,dx = \int (fx+e)^2 \arctan\left(\tanh\left(bx+a\right)\right)\,dx$$

input
```
integrate((f*x+e)^2*arctan(tanh(b*x+a)),x, algorithm="maxima")
```

output
```
1/3*(f^2*x^3 + 3*e*f*x^2 + 3*e^2*x)*arctan((e^(2*b*x + 2*a) - 1)/(e^(2*b*x
 + 2*a) + 1)) - integrate(2/3*(b*f^2*x^3*e^(2*a) + 3*b*e*f*x^2*e^(2*a) + 3
*b*e^2*x*e^(2*a))*e^(2*b*x)/(e^(4*b*x + 4*a) + 1), x)
```

3.77.8 Giac [F]

$$\int (e+fx)^2 \arctan(\tanh(a+bx))\,dx = \int (fx+e)^2 \arctan\left(\tanh\left(bx+a\right)\right)\,dx$$

input
```
integrate((f*x+e)^2*arctan(tanh(b*x+a)),x, algorithm="giac")
```

output
```
sage0*x
```

3.77.9 Mupad [F(-1)]

Timed out.

$$\int (e+fx)^2 \arctan(\tanh(a+bx))\,dx = \int \mathrm{atan}(\tanh(a+b\,x))\,\left(e+f\,x\right)^2\,dx$$

input
```
int(atan(tanh(a + b*x))*(e + f*x)^2,x)
```

output
```
int(atan(tanh(a + b*x))*(e + f*x)^2, x)
```

3.78 $\int (e+fx)\arctan(\tanh(a+bx))\,dx$

3.78.1 Optimal result

Integrand size = 13, antiderivative size = 159

$$\int (e+fx)\arctan(\tanh(a+bx))\,dx = -\frac{(e+fx)^2\arctan\left(e^{2a+2bx}\right)}{2f} + \frac{(e+fx)^2\arctan(\tanh(a+bx))}{2f} + \frac{i(e+fx)\operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} - \frac{i(e+fx)\operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} - \frac{if\operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{8b^2} + \frac{if\operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)}{8b^2}$$

output
```
-1/2*(f*x+e)^2*arctan(exp(2*b*x+2*a))/f+1/2*(f*x+e)^2*arctan(tanh(b*x+a))/
f+1/4*I*(f*x+e)*polylog(2,-I*exp(2*b*x+2*a))/b-1/4*I*(f*x+e)*polylog(2,I*e
xp(2*b*x+2*a))/b-1/8*I*f*polylog(3,-I*exp(2*b*x+2*a))/b^2+1/8*I*f*polylog(
3,I*exp(2*b*x+2*a))/b^2
```

3.78.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.49

$$\int (e+fx)\arctan(\tanh(a+bx))\,dx = ex\arctan(\tanh(a+bx))+\frac{1}{2}fx^2\arctan(\tanh(a+bx))$$
$$-\frac{ie\left(2bx\left(\log\left(1-ie^{2(a+bx)}\right)-\log\left(1+ie^{2(a+bx)}\right)\right)-\text{PolyLog}\left(2,-ie^{2(a+bx)}\right)+\text{PolyLog}\left(2,ie^{2(a+bx)}\right)\right)}{4b}$$
$$-\frac{if\left(2b^2x^2\log\left(1-ie^{2(a+bx)}\right)-2b^2x^2\log\left(1+ie^{2(a+bx)}\right)-2bx\,\text{PolyLog}\left(2,-ie^{2(a+bx)}\right)+2bx\,\text{PolyLog}\left(2,\right.\right.}{8b^2}$$

input
```
Integrate[(e + f*x)*ArcTan[Tanh[a + b*x]],x]
```

output
```
e*x*ArcTan[Tanh[a + b*x]] + (f*x^2*ArcTan[Tanh[a + b*x]])/2 - ((I/4)*e*(2*
b*x*(Log[1 - I*E^(2*(a + b*x))] - Log[1 + I*E^(2*(a + b*x))]) - PolyLog[2,
 (-I)*E^(2*(a + b*x))] + PolyLog[2, I*E^(2*(a + b*x))]))/b - ((I/8)*f*(2*b
^2*x^2*Log[1 - I*E^(2*(a + b*x))] - 2*b^2*x^2*Log[1 + I*E^(2*(a + b*x))] -
 2*b*x*PolyLog[2, (-I)*E^(2*(a + b*x))] + 2*b*x*PolyLog[2, I*E^(2*(a + b*x
))] + PolyLog[3, (-I)*E^(2*(a + b*x))] - PolyLog[3, I*E^(2*(a + b*x))]))/b
^2
```

3.78.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {5706, 3042, 4668, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)\arctan(\tanh(a+bx))\,dx$$

$$\Big\downarrow \text{5706}$$

$$\frac{(e+fx)^2\arctan(\tanh(a+bx))}{2f}-\frac{b\int (e+fx)^2\text{sech}(2a+2bx)dx}{2f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(e+fx)^2\arctan(\tanh(a+bx))}{2f}-\frac{b\int (e+fx)^2\csc\left(2ia+2ibx+\frac{\pi}{2}\right)dx}{2f}$$

$$\Big\downarrow \text{4668}$$

$$\frac{(e+fx)^2\arctan(\tanh(a+bx))}{2f}-\frac{b\left(-\frac{if\int(e+fx)\log\left(1-ie^{2a+2bx}\right)dx}{b}+\frac{if\int(e+fx)\log\left(1+ie^{2a+2bx}\right)dx}{b}+\frac{(e+fx)^2\arctan\left(e^{2a+2bx}\right)}{b}\right)}{2f}$$

$\downarrow$ 3011

$$\frac{(e+fx)^2\arctan(\tanh(a+bx))}{2f}-\frac{b\left(\frac{if\left(\frac{f\int \text{PolyLog}\left(2,-ie^{2a+2bx}\right)dx}{2b}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{2b}\right)}{b}-\frac{if\left(\frac{f\int \text{PolyLog}\left(2,ie^{2a+2bx}\right)dx}{2b}-\frac{(e+fx)\text{PolyLog}\left(2,ie^{2a+2bx}\right)}{2b}\right)}{b}+\frac{(e+fx)^2 \ldots}{\ldots}\right.}{2f}$$

$\downarrow$ 2720

$$\frac{(e+fx)^2\arctan(\tanh(a+bx))}{2f}-\frac{b\left(\frac{if\left(\frac{f\int e^{-2a-2bx}\text{PolyLog}\left(2,-ie^{2a+2bx}\right)de^{2a+2bx}}{4b^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{2b}\right)}{b}-\frac{if\left(\frac{f\int e^{-2a-2bx}\text{PolyLog}\left(2,ie^{2a+2bx}\right)de^{2a+2bx}}{4b^2}-\frac{(e+fx)\text{P}\ldots}{\ldots}\right.}{b}\right.}{2f}$$

$\downarrow$ 7143

$$\frac{(e+fx)^2\arctan(\tanh(a+bx))}{2f}-\frac{b\left(\frac{(e+fx)^2\arctan\left(e^{2a+2bx}\right)}{b}+\frac{if\left(\frac{f\text{PolyLog}\left(3,-ie^{2a+2bx}\right)}{4b^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{2b}\right)}{b}-\frac{if\left(\frac{f\text{PolyLog}\left(3,ie^{2a+2bx}\right)}{4b^2}-\frac{(e+fx)\text{PolyLog}\left(2,ie^{\ldots}\right.}{2b}\right.}{b}\right.}{2f}$$

input
```
Int[(e + f*x)*ArcTan[Tanh[a + b*x]],x]
```

output
```
((e + f*x)^2*ArcTan[Tanh[a + b*x]])/(2*f) - (b*(((e + f*x)^2*ArcTan[E^(2*a
 + 2*b*x)])/b + (I*f*(-1/2*((e + f*x)*PolyLog[2, (-I)*E^(2*a + 2*b*x)])/b
+ (f*PolyLog[3, (-I)*E^(2*a + 2*b*x)])/(4*b^2)))/b - (I*f*(-1/2*((e + f*x)
*PolyLog[2, I*E^(2*a + 2*b*x)])/b + (f*PolyLog[3, I*E^(2*a + 2*b*x)])/(4*b
^2)))/b))/(2*f)
```

3.78. $\int (e+fx)\arctan(\tanh(a+bx))\,dx$

3.78.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5706
```
Int[ArcTan[Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcTan[Tanh[a + b*x]]/(f*(m + 1))), x] - Simp[b/
(f*(m + 1))   Int[(e + f*x)^(m + 1)*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a,
b, e, f}, x] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.78.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.24 (sec) , antiderivative size = 1776, normalized size of antiderivative = 11.17

method	result	size
risch	Expression too large to display	1776

input
```
int((f*x+e)*arctan(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/4*Pi*(csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)-I))*csgn(I*(exp
(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))-csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(exp(
2*b*x+2*a)+I))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))-csgn(I/(exp(2
*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^2+csgn(I/(exp(
2*b*x+2*a)+1))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^2-csgn(I*(exp
(2*b*x+2*a)-I))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^2+csgn(I*(ex
p(2*b*x+2*a)+I))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^2-csgn(I*(e
xp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))*csgn((1+I)*(exp(2*b*x+2*a)+I)/(exp(2*
b*x+2*a)+1))+csgn((1+I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^2+csgn(I*(e
xp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))*csgn((1-I)*(exp(2*b*x+2*a)-I)/(exp(2*
b*x+2*a)+1))+csgn((1-I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^2+csgn(I*(e
xp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^3-csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*
x+2*a)+1))*csgn((1-I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)+1))^2-csgn(I*(exp
(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^3+csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+
2*a)+1))*csgn((1+I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^2-csgn((1+I)*(e
xp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)+1))^3-csgn((1-I)*(exp(2*b*x+2*a)-I)/(exp(
2*b*x+2*a)+1))^3-1)*(1/2*f*x^2+e*x)-1/4*I/b^2*f*a^2*ln(exp(2*b*x+2*a)+I)+1
/2*I/b*e*a*ln(exp(2*b*x+2*a)+I)+1/2*I*(1/2*f*x^2+e*x)*ln(exp(2*b*x+2*a)+I)
-1/2*I*e*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))*x-1/2*I*e*ln(((-I)^(1/2)+e
xp(b*x+a))/(-I)^(1/2))*x-1/2*I*e/b*dilog(((-I)^(1/2)-exp(b*x+a))/(-I)^(...
```

3.78.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 600 vs. 2(130) = 260.

Time = 0.33 (sec) , antiderivative size = 600, normalized size of antiderivative = 3.77

$$\int (e+fx)\arctan(\tanh(a+bx))\,dx$$

$$= \frac{2\left(b^2fx^2+2\,b^2ex\right)\arctan\left(\frac{\sinh(bx+a)}{\cosh(bx+a)}\right)-2\left(i\,bfx+i\,be\right)\mathrm{Li}_2\left(\frac{1}{2}\sqrt{4i}(\cosh\left(bx+a\right)+\sinh\left(bx+a\right))\right)-2\left(i\,b\right.}{}$$

input
```
integrate((f*x+e)*arctan(tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/4*(2*(b^2*f*x^2 + 2*b^2*e*x)*arctan(sinh(b*x + a)/cosh(b*x + a)) - 2*(I*
b*f*x + I*b*e)*dilog(1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 2*(I
*b*f*x + I*b*e)*dilog(-1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 2*
(-I*b*f*x - I*b*e)*dilog(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) -
 2*(-I*b*f*x - I*b*e)*dilog(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)
)) + (-I*b^2*f*x^2 - 2*I*b^2*e*x - 2*I*a*b*e + I*a^2*f)*log(1/2*sqrt(4*I)*
(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^2*f*x^2 - 2*I*b^2*e*x - 2*I*a
*b*e + I*a^2*f)*log(-1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) +
(I*b^2*f*x^2 + 2*I*b^2*e*x + 2*I*a*b*e - I*a^2*f)*log(1/2*sqrt(-4*I)*(cosh
(b*x + a) + sinh(b*x + a)) + 1) + (I*b^2*f*x^2 + 2*I*b^2*e*x + 2*I*a*b*e -
 I*a^2*f)*log(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (2*I*
a*b*e - I*a^2*f)*log(I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (2
*I*a*b*e - I*a^2*f)*log(-I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a))
+ (-2*I*a*b*e + I*a^2*f)*log(I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x +
 a)) + (-2*I*a*b*e + I*a^2*f)*log(-I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh
(b*x + a)) + 2*I*f*polylog(3, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a)
)) + 2*I*f*polylog(3, -1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 2*
I*f*polylog(3, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 2*I*f*pol
ylog(3, -1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))))/b^2
```

3.78.6 Sympy [F]

$$\int (e+fx)\arctan(\tanh(a+bx))\,dx=\int (e+fx)\operatorname{atan}\left(\tanh\left(a+bx\right)\right)dx$$

input
```
integrate((f*x+e)*atan(tanh(b*x+a)),x)
```

output
```
Integral((e + f*x)*atan(tanh(a + b*x)), x)
```

3.78.7 Maxima [F]

$$\int (e+fx)\arctan(\tanh(a+bx))\,dx = \int (fx+e)\arctan\left(\tanh\left(bx+a\right)\right)\,dx$$

input
```
integrate((f*x+e)*arctan(tanh(b*x+a)),x, algorithm="maxima")
```

output
```
1/2*(f*x^2 + 2*e*x)*arctan((e^(2*b*x + 2*a) - 1)/(e^(2*b*x + 2*a) + 1)) -
integrate((b*f*x^2*e^(2*a) + 2*b*e*x*e^(2*a))*e^(2*b*x)/(e^(4*b*x + 4*a) +
 1), x)
```

3.78.8 Giac [F]

$$\int (e+fx)\arctan(\tanh(a+bx))\,dx = \int (fx+e)\arctan\left(\tanh\left(bx+a\right)\right)\,dx$$

input
```
integrate((f*x+e)*arctan(tanh(b*x+a)),x, algorithm="giac")
```

output
```
sage0*x
```

3.78.9 Mupad [F(-1)]

Timed out.

$$\int (e+fx)\arctan(\tanh(a+bx))\,dx = \int \mathrm{atan}(\tanh(a+b\,x))\,\left(e+f\,x\right)\,dx$$

input
```
int(atan(tanh(a + b*x))*(e + f*x),x)
```

output
```
int(atan(tanh(a + b*x))*(e + f*x), x)
```

3.79 $\int \arctan(\tanh(a+bx))\,dx$

3.79.1 Optimal result

Integrand size = 7, antiderivative size = 74

$$\int \arctan(\tanh(a+bx))\,dx = -x\arctan\left(e^{2a+2bx}\right)+x\arctan(\tanh(a+bx)) + \frac{i\,\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} - \frac{i\,\text{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b}$$

output
```
-x*arctan(exp(2*b*x+2*a))+x*arctan(tanh(b*x+a))+1/4*I*polylog(2,-I*exp(2*b
*x+2*a))/b-1/4*I*polylog(2,I*exp(2*b*x+2*a))/b
```

3.79.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.23

$$\int \arctan(\tanh(a+bx))\,dx = x\arctan(\tanh(a+bx)) - \frac{i\left(2bx\left(\log\left(1-ie^{2(a+bx)}\right)-\log\left(1+ie^{2(a+bx)}\right)\right)-\text{PolyLog}\left(2,-ie^{2(a+bx)}\right)+\text{PolyLog}\left(2,ie^{2(a+bx)}\right)\right)}{4b}$$

input
```
Integrate[ArcTan[Tanh[a + b*x]],x]
```

output
```
x*ArcTan[Tanh[a + b*x]] - ((I/4)*(2*b*x*(Log[1 - I*E^(2*(a + b*x))] - Log[
1 + I*E^(2*(a + b*x))]) - PolyLog[2, (-I)*E^(2*(a + b*x))] + PolyLog[2, I*
E^(2*(a + b*x))]))/b
```

3.79.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {5702, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(\tanh(a+bx))\,dx$$

$$\Big\downarrow \; 5702$$

$$x\arctan(\tanh(a+bx))-b\int x\text{sech}(2a+2bx)dx$$

$$\Big\downarrow \; 3042$$

$$x\arctan(\tanh(a+bx))-b\int x\csc\left(2ia+2ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \; 4668$$

$$x\arctan(\tanh(a+bx))-b\left(-\frac{i\int\log\left(1-ie^{2a+2bx}\right)dx}{2b}+\frac{i\int\log\left(1+ie^{2a+2bx}\right)dx}{2b}+\frac{x\arctan\left(e^{2a+2bx}\right)}{b}\right)$$

$$\Big\downarrow \; 2715$$

$$x\arctan(\tanh(a+bx))-b\left(-\frac{i\int e^{-2a-2bx}\log\left(1-ie^{2a+2bx}\right)de^{2a+2bx}}{4b^2}+\frac{i\int e^{-2a-2bx}\log\left(1+ie^{2a+2bx}\right)de^{2a+2bx}}{4b^2}+\frac{x\arctan\left(e^{2a+2bx}\right)}{b}\right)$$

$$\Big\downarrow \; 2838$$

$$x\arctan(\tanh(a+bx))-b\left(\frac{x\arctan\left(e^{2a+2bx}\right)}{b}-\frac{i\,\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b^2}+\frac{i\,\text{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b^2}\right)$$

input
```
Int[ArcTan[Tanh[a + b*x]],x]
```

output
```
x*ArcTan[Tanh[a + b*x]] - b*((x*ArcTan[E^(2*a + 2*b*x)])/b - ((I/4)*PolyLo
g[2, (-I)*E^(2*a + 2*b*x)])/b^2 + ((I/4)*PolyLog[2, I*E^(2*a + 2*b*x)])/b^
2)
```

3.79.3.1 Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5702
```
Int[ArcTan[Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcTan[Tanh[a + b
*x]], x] - Simp[b   Int[x*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a, b}, x]
```

3.79.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 159 vs. 2(63) = 126.

Time = 0.82 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.16

method	result
derivativedivides	$\frac{\operatorname{arctanh}(\tanh(bx+a))\arctan(\tanh(bx+a))-\frac{i\operatorname{arctanh}(\tanh(bx+a))\left(\ln\left(1-\frac{i(\tanh(bx+a)+1)^2}{1-\tanh(bx+a)^2}\right)-\ln\left(1+\frac{i(\tanh(bx+a)+1)^2}{1-\tanh(bx+a)^2}\right)\right)}{2}}{b}$
default	$\frac{\operatorname{arctanh}(\tanh(bx+a))\arctan(\tanh(bx+a))-\frac{i\operatorname{arctanh}(\tanh(bx+a))\left(\ln\left(1-\frac{i(\tanh(bx+a)+1)^2}{1-\tanh(bx+a)^2}\right)-\ln\left(1+\frac{i(\tanh(bx+a)+1)^2}{1-\tanh(bx+a)^2}\right)\right)}{2}}{b}$
risch	Expression too large to display

input
```
int(arctan(tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b*(arctanh(tanh(b*x+a))*arctan(tanh(b*x+a))-1/2*I*arctanh(tanh(b*x+a))*(
ln(1-I*(tanh(b*x+a)+1)^2/(1-tanh(b*x+a)^2))-ln(1+I*(tanh(b*x+a)+1)^2/(1-ta
nh(b*x+a)^2)))+1/4*I*dilog(1+I*(tanh(b*x+a)+1)^2/(1-tanh(b*x+a)^2))-1/4*I*
dilog(1-I*(tanh(b*x+a)+1)^2/(1-tanh(b*x+a)^2)))
```

3.79.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(57) = 114$.

Time = 0.31 (sec) , antiderivative size = 334, normalized size of antiderivative = 4.51

$$\int \arctan(\tanh(a+bx))\,dx = \frac{2\,bx\arctan\left(\frac{\sinh(bx+a)}{\cosh(bx+a)}\right) + (-i\,bx - i\,a)\log\left(\frac{1}{2}\sqrt{4i}(\cosh(bx+a) + \sinh(bx+a)) + 1\right) + (-i\,bx - i\,a)\log}{}$$

input
```
integrate(arctan(tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*(2*b*x*arctan(sinh(b*x + a)/cosh(b*x + a)) + (-I*b*x - I*a)*log(1/2*sq
rt(4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b*x - I*a)*log(-1/2*sqr
t(4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b*x + I*a)*log(1/2*sqrt(-
4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b*x + I*a)*log(-1/2*sqrt(-4
*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + I*a*log(I*sqrt(4*I) + 2*cosh(b*
x + a) + 2*sinh(b*x + a)) + I*a*log(-I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sin
h(b*x + a)) - I*a*log(I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) -
I*a*log(-I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) - I*dilog(1/2*s
qrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - I*dilog(-1/2*sqrt(4*I)*(cosh(b
*x + a) + sinh(b*x + a))) + I*dilog(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b
*x + a))) + I*dilog(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))))/b
```

3.79.6 Sympy [F]

$$\int \arctan(\tanh(a+bx))\,dx = \int \operatorname{atan}\left(\tanh\left(a+bx\right)\right)dx$$

input
```
integrate(atan(tanh(b*x+a)),x)
```

output
```
Integral(atan(tanh(a + b*x)), x)
```

3.79.7 Maxima [F]

$$\int \arctan(\tanh(a+bx))\,dx = \int \arctan\left(\tanh\left(bx+a\right)\right)\,dx$$

input
```
integrate(arctan(tanh(b*x+a)),x, algorithm="maxima")
```

output
```
x*arctan((e^(2*b*x + 2*a) - 1)/(e^(2*b*x + 2*a) + 1)) - 2*b*integrate(x*e^
(2*b*x + 2*a)/(e^(4*b*x + 4*a) + 1), x)
```

3.79.8 Giac [F]

$$\int \arctan(\tanh(a+bx))\,dx = \int \arctan\left(\tanh\left(bx+a\right)\right)\,dx$$

input
```
integrate(arctan(tanh(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arctan(tanh(b*x + a)), x)
```

3.79.9 Mupad [F(-1)]

Timed out.

$$\int \arctan(\tanh(a+bx))\,dx = \int \operatorname{atan}(\tanh(a+b\,x))\;dx$$

input
```
int(atan(tanh(a + b*x)),x)
```

output
```
int(atan(tanh(a + b*x)), x)
```

3.80 $\int \frac{\arctan(\tanh(a+bx))}{e+fx}\,dx$

3.80.1 Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{\arctan(\tanh(a+bx))}{e+fx}\,dx = \text{Int}\left(\frac{\arctan(\tanh(a+bx))}{e+fx},x\right)$$

output
```
CannotIntegrate(arctan(tanh(b*x+a))/(f*x+e),x)
```

3.80.2 Mathematica [N/A]

Not integrable

Time = 11.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\arctan(\tanh(a+bx))}{e+fx}\,dx = \int \frac{\arctan(\tanh(a+bx))}{e+fx}\,dx$$

input
```
Integrate[ArcTan[Tanh[a + b*x]]/(e + f*x),x]
```

output
```
Integrate[ArcTan[Tanh[a + b*x]]/(e + f*x), x]
```

3.80.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(\tanh(a+bx))}{e+fx}\,dx$$

$$\Big\downarrow \; 7299$$

$$\int \frac{\arctan(\tanh(a+bx))}{e+fx}dx$$

input
```
Int[ArcTan[Tanh[a + b*x]]/(e + f*x),x]
```

output
```
$Aborted
```

3.80.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.80.4 Maple [N/A] (verified)

Not integrable

Time = 0.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\arctan\left(\tanh\left(bx+a\right)\right)}{fx+e}dx$$

input
```
int(arctan(tanh(b*x+a))/(f*x+e),x)
```

output
```
int(arctan(tanh(b*x+a))/(f*x+e),x)
```

3.80.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\arctan(\tanh(a+bx))}{e+fx}\,dx = \int \frac{\arctan(\tanh(bx+a))}{fx+e}\,dx$$

input
```
integrate(arctan(tanh(b*x+a))/(f*x+e),x, algorithm="fricas")
```

output
```
integral(arctan(tanh(b*x + a))/(f*x + e), x)
```

3.80.6 Sympy [N/A]

Not integrable

Time = 2.95 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{\arctan(\tanh(a+bx))}{e+fx}\,dx = \int \frac{\operatorname{atan}(\tanh(a+bx))}{e+fx}\,dx$$

input
```
integrate(atan(tanh(b*x+a))/(f*x+e),x)
```

output
```
Integral(atan(tanh(a + b*x))/(e + f*x), x)
```

3.80.7 Maxima [N/A]

Not integrable

Time = 1.56 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\arctan(\tanh(a+bx))}{e+fx}\,dx = \int \frac{\arctan(\tanh(bx+a))}{fx+e}\,dx$$

input
```
integrate(arctan(tanh(b*x+a))/(f*x+e),x, algorithm="maxima")
```

output
```
integrate(arctan(tanh(b*x + a))/(f*x + e), x)
```

3.80.8 Giac [N/A]

Not integrable

Time = 105.12 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.20

$$\int \frac{\arctan(\tanh(a+bx))}{e+fx}\,dx = \int \frac{\arctan\left(\tanh\left(bx+a\right)\right)}{fx+e}\,dx$$

input
```
integrate(arctan(tanh(b*x+a))/(f*x+e),x, algorithm="giac")
```

output
```
sage0*x
```

3.80.9 Mupad [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\arctan(\tanh(a+bx))}{e+fx}\,dx = \int \frac{\mathrm{atan}(\tanh(a+b\,x))}{e+f\,x}\,dx$$

input
```
int(atan(tanh(a + b*x))/(e + f*x),x)
```

output
```
int(atan(tanh(a + b*x))/(e + f*x), x)
```

3.81 $\int x^2 \arctan(c + d\tanh(a + bx))\,dx$

3.81.1 Optimal result

Integrand size = 15, antiderivative size = 355

$$\begin{aligned}\int x^2 \arctan(c + d\tanh(a + bx))\,dx &= \frac{1}{3}x^3 \arctan(c + d\tanh(a + bx)) \\ &+ \frac{1}{6}ix^3 \log\left(1 + \frac{(i - c - d)e^{2a+2bx}}{i - c + d}\right) \\ &- \frac{1}{6}ix^3 \log\left(1 + \frac{(i + c + d)e^{2a+2bx}}{i + c - d}\right) \\ &+ \frac{ix^2 \operatorname{PolyLog}\left(2, -\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b} \\ &- \frac{ix^2 \operatorname{PolyLog}\left(2, -\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b} \\ &- \frac{ix \operatorname{PolyLog}\left(3, -\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b^2} \\ &+ \frac{ix \operatorname{PolyLog}\left(3, -\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b^2} \\ &+ \frac{i \operatorname{PolyLog}\left(4, -\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{8b^3} \\ &- \frac{i \operatorname{PolyLog}\left(4, -\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{8b^3}\end{aligned}$$

output
```
1/3*x^3*arctan(c+d*tanh(b*x+a))+1/6*I*x^3*ln(1+(I-c-d)*exp(2*b*x+2*a)/(I-c
+d))-1/6*I*x^3*ln(1+(I+c+d)*exp(2*b*x+2*a)/(I+c-d))+1/4*I*x^2*polylog(2,-(
I-c-d)*exp(2*b*x+2*a)/(I-c+d))/b-1/4*I*x^2*polylog(2,-(I+c+d)*exp(2*b*x+2*
a)/(I+c-d))/b-1/4*I*x*polylog(3,-(I-c-d)*exp(2*b*x+2*a)/(I-c+d))/b^2+1/4*I
*x*polylog(3,-(I+c+d)*exp(2*b*x+2*a)/(I+c-d))/b^2+1/8*I*polylog(4,-(I-c-d)
*exp(2*b*x+2*a)/(I-c+d))/b^3-1/8*I*polylog(4,-(I+c+d)*exp(2*b*x+2*a)/(I+c-
d))/b^3
```

3.81.2 Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.23

$$\int x^2 \arctan(c+d\tanh(a+bx))\,dx = \frac{1}{3}x^3\arctan(c+d\tanh(a+bx)) - \frac{d\left(4b^3x^3\log\left(1+\frac{2(1+(c+d)^2)e^{2(a+bx)}}{2+2c^2-2d^2-4\sqrt{-d^2}}\right)-4b^3x^3\log\left(1+\frac{(1+(c+d)^2)e^{2(a+bx)}}{1+c^2-d^2+2\sqrt{-d^2}}\right)+6b^2x^2\,\mathrm{PolyLog}\left(2,\frac{(1+c^2+2cd+d^2}{-1-c^2+d^2+}\right.\right.}{}$$

input
```
Integrate[x^2*ArcTan[c + d*Tanh[a + b*x]],x]
```

output
```
(x^3*ArcTan[c + d*Tanh[a + b*x]])/3 - (d*(4*b^3*x^3*Log[1 + (2*(1 + (c + d
)^2)*E^(2*(a + b*x)))/(2 + 2*c^2 - 2*d^2 - 4*Sqrt[-d^2])] - 4*b^3*x^3*Log[
1 + ((1 + (c + d)^2)*E^(2*(a + b*x)))/(1 + c^2 - d^2 + 2*Sqrt[-d^2])] + 6*
b^2*x^2*PolyLog[2, ((1 + c^2 + 2*c*d + d^2)*E^(2*(a + b*x)))/(-1 - c^2 + d
^2 + 2*Sqrt[-d^2])] - 6*b^2*x^2*PolyLog[2, -(((1 + (c + d)^2)*E^(2*(a + b*
x)))/(1 + c^2 - d^2 + 2*Sqrt[-d^2]))] - 6*b*x*PolyLog[3, ((1 + c^2 + 2*c*d
 + d^2)*E^(2*(a + b*x)))/(-1 - c^2 + d^2 + 2*Sqrt[-d^2])] + 6*b*x*PolyLog[
3, -(((1 + (c + d)^2)*E^(2*(a + b*x)))/(1 + c^2 - d^2 + 2*Sqrt[-d^2]))] +
3*PolyLog[4, (-2*(1 + (c + d)^2)*E^(2*(a + b*x)))/(2 + 2*c^2 - 2*d^2 - 4*S
qrt[-d^2])] - 3*PolyLog[4, -(((1 + (c + d)^2)*E^(2*(a + b*x)))/(1 + c^2 -
d^2 + 2*Sqrt[-d^2]))]))/(24*b^3*Sqrt[-d^2])
```

3.81.3 Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.24, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5722, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(d\tanh(a+bx)+c)\,dx$$

$$\Big\downarrow \text{5722}$$

$$-\frac{1}{3}b(1+i(c+d))\int \frac{e^{2a+2bx}x^3}{-c+(-c-d+i)e^{2a+2bx}+d+i}dx+\frac{1}{3}b(1-i(c+d))\int \frac{e^{2a+2bx}x^3}{c+(c+d+i)e^{2a+2bx}-d+i}dx+\frac{1}{3}x^3\arctan(d\tanh(a+bx)+c)$$

$$\Big\downarrow \text{2620}$$

$$-\frac{1}{3}b(1+i(c+d))\left(\frac{x^3\log\left(1+\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b(-c-d+i)}-\frac{3\int x^2\log\left(\frac{e^{2a+2bx}(-c-d+i)}{-c+d+i}+1\right)dx}{2b(-c-d+i)}\right)+\frac{1}{3}b(1-i(c+d))\left(\frac{x^3\log\left(1+\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b(c+d+i)}-\frac{3\int x^2\log\left(\frac{e^{2a+2bx}(c+d+i)}{c-d+i}+1\right)dx}{2b(c+d+i)}\right)+\frac{1}{3}x^3\arctan(d\tanh(a+bx)+c)$$

$$\Big\downarrow \text{3011}$$

$$-\frac{1}{3}b(1+i(c+d))\left(\frac{x^3\log\left(1+\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b(-c-d+i)}-\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b}\right)}{2b(-c-d+i)}\right)+\frac{1}{3}b(1-i(c+d))\left(\frac{x^3\log\left(1+\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b(c+d+i)}-\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b}\right)}{2b(c+d+i)}\right)+\frac{1}{3}x^3\arctan(d\tanh(a+bx)+c)$$

$\downarrow$ 7163

$$d))\left(\frac{x^3\log\left(1+\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b(-c-d+i)}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b}-\frac{\int \text{PolyLog}\left(3,-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)dx}{2b}}{b}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{(-}{}\right.}{2b}\right.}{2b(-c-d+i)}\right.$$

$$-\frac{1}{3}b(1+i(c+$$

$$d))\left(\frac{x^3\log\left(1+\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b(c+d+i)}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b}-\frac{\int \text{PolyLog}\left(3,-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)dx}{2b}}{b}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{(c+d+i)}{c-}\right.}{2b}\right.}{2b(c+d+i)}\right.$$

$$\frac{1}{3}b(1-i(c+$$

$$\frac{1}{3}x^3\arctan(d\tanh(a+bx)+c)$$

$\downarrow$ 2720

$$-\frac{1}{3}b(1+i(c+$$

$$d))\left(\frac{x^3\log\left(1+\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b(-c-d+i)}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b}-\frac{\int e^{-2a-2bx}\,\text{PolyLog}\left(3,-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)de^{2a+2bx}}{4b^2}}{b}-\frac{x^2}{}\right.}{2b(-c-d+i)}\right.$$

$$\frac{1}{3}b(1-i(c+$$

$$d))\left(\frac{x^3\log\left(1+\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b(c+d+i)}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b}-\frac{\int e^{-2a-2bx}\,\text{PolyLog}\left(3,-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)de^{2a+2bx}}{4b^2}}{b}-\frac{x^2\,\text{Poly}}{}\right.}{2b(c+d+i)}\right.$$

$$\frac{1}{3}x^3\arctan(d\tanh(a+bx)+c)$$

$\downarrow$ 7143

3.81. $\int x^2\arctan(c+d\tanh(a+bx))\,dx$

$$\frac{1}{3}x^3\arctan(d\tanh(a+bx)+c)-\frac{1}{3}b(1+i(c+d))\left(\frac{x^3\log\left(1+\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b(-c-d+i)}-\frac{3\left(\frac{\frac{x\operatorname{PolyLog}\left(3,-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b}-\frac{\operatorname{PolyLog}\left(4,-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b^2}}{b}-\frac{x^2\operatorname{PolyLog}\left(2,-\frac{(-c-d}{}\right.}{2b}\right.}{2b(-c-d+i)}\right.$$

$$\frac{1}{3}b(1-i(c+d))\left(\frac{x^3\log\left(1+\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b(c+d+i)}-\frac{3\left(\frac{\frac{x\operatorname{PolyLog}\left(3,-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b}-\frac{\operatorname{PolyLog}\left(4,-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b^2}}{b}-\frac{x^2\operatorname{PolyLog}\left(2,-\frac{(c+d+i)e^{2a}}{c-d+i}\right.}{2b}\right.}{2b(c+d+i)}\right.$$

input
```
Int[x^2*ArcTan[c + d*Tanh[a + b*x]],x]
```

output
```
(x^3*ArcTan[c + d*Tanh[a + b*x]])/3 - (b*(1 + I*(c + d))*((x^3*Log[1 + ((I
 - c - d)*E^(2*a + 2*b*x))/(I - c + d)])/(2*b*(I - c - d)) - (3*(-1/2*(x^2
*PolyLog[2, -(((I - c - d)*E^(2*a + 2*b*x))/(I - c + d))])/b + ((x*PolyLog
[3, -(((I - c - d)*E^(2*a + 2*b*x))/(I - c + d))])/(2*b) - PolyLog[4, -(((
I - c - d)*E^(2*a + 2*b*x))/(I - c + d))]/(4*b^2))/b))/(2*b*(I - c - d))))
/3 + (b*(1 - I*(c + d))*((x^3*Log[1 + ((I + c + d)*E^(2*a + 2*b*x))/(I + c
 - d)])/(2*b*(I + c + d)) - (3*(-1/2*(x^2*PolyLog[2, -(((I + c + d)*E^(2*a
 + 2*b*x))/(I + c - d))])/b + ((x*PolyLog[3, -(((I + c + d)*E^(2*a + 2*b*x
))/(I + c - d))])/(2*b) - PolyLog[4, -(((I + c + d)*E^(2*a + 2*b*x))/(I +
c - d))]/(4*b^2))/b))/(2*b*(I + c + d))))/3
```

3.81.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5722
```
Int[ArcTan[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Tanh[a + b*x]]/(f*(m
+ 1))), x] + (Simp[I*b*((I - c - d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(E
^(2*a + 2*b*x)/(I - c + d + (I - c - d)*E^(2*a + 2*b*x))), x], x] - Simp[I*
b*((I + c + d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(E^(2*a + 2*b*x)/(I + c
 - d + (I + c + d)*E^(2*a + 2*b*x))), x], x]) /; FreeQ[{a, b, c, d, e, f},
x] && IGtQ[m, 0] && NeQ[(c - d)^2, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.81. $\int x^2 \arctan(c + d \tanh(a + bx)) \, dx$

3.81.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 26.22 (sec) , antiderivative size = 6917, normalized size of antiderivative = 19.48

method	result	size
risch	Expression too large to display	6917

input
```
int(x^2*arctan(c+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.81.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1289 vs. 2(263) = 526.

Time = 0.37 (sec) , antiderivative size = 1289, normalized size of antiderivative = 3.63

$$\int x^2 \arctan(c + d \tanh(a + bx)) \, dx = \text{Too large to display}$$

input
```
integrate(x^2*arctan(c+d*tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/6*(2*b^3*x^3*arctan((c*cosh(b*x + a) + d*sinh(b*x + a))/cosh(b*x + a)) +
 3*I*b^2*x^2*dilog(sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*
(cosh(b*x + a) + sinh(b*x + a))) + 3*I*b^2*x^2*dilog(-sqrt(-(c^2 - d^2 + 2
*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 3*I*
b^2*x^2*dilog(sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh
(b*x + a) + sinh(b*x + a))) - 3*I*b^2*x^2*dilog(-sqrt(-(c^2 - d^2 - 2*I*d
+ 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - I*a^3*log
(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(
b*x + a) + 2*(c^2 - d^2 - 2*I*d + 1)*sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 -
2*c*d + d^2 + 1))) - I*a^3*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2
*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) - 2*(c^2 - d^2 - 2*I*d + 1)*sqrt(-(
c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) + I*a^3*log(2*(c^2 + 2*c*
d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) + 2*(
c^2 - d^2 + 2*I*d + 1)*sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 +
1))) + I*a^3*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d
+ d^2 + 1)*sinh(b*x + a) - 2*(c^2 - d^2 + 2*I*d + 1)*sqrt(-(c^2 - d^2 - 2*
I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) - 6*I*b*x*polylog(3, sqrt(-(c^2 - d^2 +
 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 6*
I*b*x*polylog(3, -sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(
cosh(b*x + a) + sinh(b*x + a))) + 6*I*b*x*polylog(3, sqrt(-(c^2 - d^2 -...
```

3.81.6 Sympy [F(-1)]

Timed out.

$$\int x^2 \arctan(c + d \tanh(a + bx)) \, dx = \text{Timed out}$$

input
```
integrate(x**2*atan(c+d*tanh(b*x+a)),x)
```

output
```
Timed out
```

3.81.7 Maxima [F]

$$\int x^2 \arctan(c + d\tanh(a + bx))\, dx = \int x^2 \arctan\left(d\tanh\left(bx + a\right) + c\right)\, dx$$

input
```
integrate(x^2*arctan(c+d*tanh(b*x+a)),x, algorithm="maxima")
```

output
```
1/3*x^3*arctan(((c*e^(2*a) + d*e^(2*a))*e^(2*b*x) + c - d)/(e^(2*b*x) + 2*a
) + 1)) - 4*b*d*integrate(1/3*x^3*e^(2*b*x + 2*a)/(c^2 - 2*c*d + d^2 + (c^
2*e^(4*a) + 2*c*d*e^(4*a) + d^2*e^(4*a) + e^(4*a))*e^(4*b*x) + 2*(c^2*e^(2
*a) - d^2*e^(2*a) + e^(2*a))*e^(2*b*x) + 1), x)
```

3.81.8 Giac [F]

$$\int x^2 \arctan(c + d\tanh(a + bx))\, dx = \int x^2 \arctan\left(d\tanh\left(bx + a\right) + c\right)\, dx$$

input
```
integrate(x^2*arctan(c+d*tanh(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*arctan(d*tanh(b*x + a) + c), x)
```

3.81.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \arctan(c + d\tanh(a + bx))\, dx = \int x^2 \,\mathrm{atan}(c + d\tanh(a + b\,x))\, dx$$

input
```
int(x^2*atan(c + d*tanh(a + b*x)),x)
```

output
```
int(x^2*atan(c + d*tanh(a + b*x)), x)
```

3.82 $\int x \arctan(c + d \tanh(a + bx)) \, dx$

3.82.1 Optimal result

Integrand size = 13, antiderivative size = 267

$$\begin{aligned}\int x \arctan(c + d \tanh(a + bx)) \, dx = & \frac{1}{2}x^2 \arctan(c + d \tanh(a + bx)) \\ & + \frac{1}{4}ix^2 \log\left(1 + \frac{(i - c - d)e^{2a+2bx}}{i - c + d}\right) \\ & - \frac{1}{4}ix^2 \log\left(1 + \frac{(i + c + d)e^{2a+2bx}}{i + c - d}\right) \\ & + \frac{ix \operatorname{PolyLog}\left(2, -\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b} \\ & - \frac{ix \operatorname{PolyLog}\left(2, -\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b} \\ & - \frac{i \operatorname{PolyLog}\left(3, -\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{8b^2} \\ & + \frac{i \operatorname{PolyLog}\left(3, -\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{8b^2}\end{aligned}$$

output
```
1/2*x^2*arctan(c+d*tanh(b*x+a))+1/4*I*x^2*ln(1+(I-c-d)*exp(2*b*x+2*a)/(I-c
+d))-1/4*I*x^2*ln(1+(I+c+d)*exp(2*b*x+2*a)/(I+c-d))+1/4*I*x*polylog(2,-(I-
c-d)*exp(2*b*x+2*a)/(I-c+d))/b-1/4*I*x*polylog(2,-(I+c+d)*exp(2*b*x+2*a)/(
I+c-d))/b-1/8*I*polylog(3,-(I-c-d)*exp(2*b*x+2*a)/(I-c+d))/b^2+1/8*I*polyl
og(3,-(I+c+d)*exp(2*b*x+2*a)/(I+c-d))/b^2
```

3.82.2 Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.24

$$\int x\arctan(c+d\tanh(a+bx))\,dx=\frac{1}{2}x^2\arctan(c+d\tanh(a+bx))$$
$$-\frac{d\left(2b^2x^2\log\left(1+\frac{2(1+(c+d)^2)e^{2(a+bx)}}{2+2c^2-2d^2-4\sqrt{-d^2}}\right)-2b^2x^2\log\left(1+\frac{(1+(c+d)^2)e^{2(a+bx)}}{1+c^2-d^2+2\sqrt{-d^2}}\right)+2bx\operatorname{PolyLog}\left(2,\frac{(1+c^2+2cd+d^2)e}{-1-c^2+d^2+2\sqrt{}}\right.\right.}{}$$

input
```
Integrate[x*ArcTan[c + d*Tanh[a + b*x]],x]
```

output
```
(x^2*ArcTan[c + d*Tanh[a + b*x]])/2 - (d*(2*b^2*x^2*Log[1 + (2*(1 + (c + d
)^2)*E^(2*(a + b*x)))/(2 + 2*c^2 - 2*d^2 - 4*Sqrt[-d^2])] - 2*b^2*x^2*Log[
1 + ((1 + (c + d)^2)*E^(2*(a + b*x)))/(1 + c^2 - d^2 + 2*Sqrt[-d^2])] + 2*
b*x*PolyLog[2, ((1 + c^2 + 2*c*d + d^2)*E^(2*(a + b*x)))/(-1 - c^2 + d^2 +
 2*Sqrt[-d^2])] - 2*b*x*PolyLog[2, -(((1 + (c + d)^2)*E^(2*(a + b*x)))/(1
+ c^2 - d^2 + 2*Sqrt[-d^2]))] - PolyLog[3, (-2*(1 + (c + d)^2)*E^(2*(a + b
*x)))/(2 + 2*c^2 - 2*d^2 - 4*Sqrt[-d^2])] + PolyLog[3, -(((1 + (c + d)^2)*
E^(2*(a + b*x)))/(1 + c^2 - d^2 + 2*Sqrt[-d^2]))]))/(8*b^2*Sqrt[-d^2])
```

3.82.3 Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.28, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5722, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\arctan(d\tanh(a+bx)+c)\,dx$$

$$\Big\downarrow \text{5722}$$

$$-\frac{1}{2}b(1+i(c+d))\int\frac{e^{2a+2bx}x^2}{-c+(-c-d+i)e^{2a+2bx}+d+i}dx+\frac{1}{2}b(1-i(c+d))\int\frac{e^{2a+2bx}x^2}{c+(c+d+i)e^{2a+2bx}-d+i}dx+\frac{1}{2}x^2\arctan(d\tanh(a+bx)+c)$$

$$\Big\downarrow \text{2620}$$

$$-\frac{1}{2}b(1+i(c+d))\left(\frac{x^2\log\left(1+\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b(-c-d+i)}-\frac{\int x\log\left(\frac{e^{2a+2bx}(-c-d+i)}{-c+d+i}+1\right)dx}{b(-c-d+i)}\right)+\frac{1}{2}b(1-i(c+d))\left(\frac{x^2\log\left(1+\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b(c+d+i)}-\frac{\int x\log\left(\frac{e^{2a+2bx}(c+d+i)}{c-d+i}+1\right)dx}{b(c+d+i)}\right)+\frac{1}{2}x^2\arctan(d\tanh(a+bx)+c)$$

$\Big\downarrow$ 3011

$$-\frac{1}{2}b(1+i(c+d))\left(\frac{x^2\log\left(1+\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b(-c-d+i)}-\frac{\frac{\int \text{PolyLog}\left(2,-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)dx}{2b}-\frac{x\,\text{PolyLog}\left(2,-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b}}{b(-c-d+i)}\right)+\frac{1}{2}b(1-i(c+d))\left(\frac{x^2\log\left(1+\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b(c+d+i)}-\frac{\frac{\int \text{PolyLog}\left(2,-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)dx}{2b}-\frac{x\,\text{PolyLog}\left(2,-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b}}{b(c+d+i)}\right)+\frac{1}{2}x^2\arctan(d\tanh(a+bx)+c)$$

$\Big\downarrow$ 2720

$$-\frac{1}{2}b(1+i(c+d))\left(\frac{x^2\log\left(1+\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b(-c-d+i)}-\frac{\frac{\int e^{-2a-2bx}\text{PolyLog}\left(2,-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)de^{2a+2bx}}{4b^2}-\frac{x\,\text{PolyLog}\left(2,-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b}}{b(-c-d+i)}\right)+\frac{1}{2}b(1-i(c+d))\left(\frac{x^2\log\left(1+\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b(c+d+i)}-\frac{\frac{\int e^{-2a-2bx}\text{PolyLog}\left(2,-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)de^{2a+2bx}}{4b^2}-\frac{x\,\text{PolyLog}\left(2,-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b}}{b(c+d+i)}\right)+\frac{1}{2}x^2\arctan(d\tanh(a+bx)+c)$$

$\Big\downarrow$ 7143

3.82. $\int x\arctan(c+d\tanh(a+bx))\,dx$

$$\frac{1}{2}x^2\arctan(d\tanh(a+bx)+c)-\frac{1}{2}b(1+i(c+d))\left(\frac{x^2\log\left(1+\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b(-c-d+i)}-\frac{\frac{\operatorname{PolyLog}\left(3,-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b^2}-\frac{x\operatorname{PolyLog}\left(2,-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b}}{b(-c-d+i)}\right)+\frac{1}{2}b(1-i(c+d))\left(\frac{x^2\log\left(1+\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b(c+d+i)}-\frac{\frac{\operatorname{PolyLog}\left(3,-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b^2}-\frac{x\operatorname{PolyLog}\left(2,-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b}}{b(c+d+i)}\right)$$

input
```
Int[x*ArcTan[c + d*Tanh[a + b*x]],x]
```

output
```
(x^2*ArcTan[c + d*Tanh[a + b*x]])/2 - (b*(1 + I*(c + d))*((x^2*Log[1 + ((I
 - c - d)*E^(2*a + 2*b*x))/(I - c + d)])/(2*b*(I - c - d)) - (-1/2*(x*Poly
Log[2, -(((I - c - d)*E^(2*a + 2*b*x))/(I - c + d))])/b + PolyLog[3, -(((I
 - c - d)*E^(2*a + 2*b*x))/(I - c + d))]/(4*b^2))/(b*(I - c - d))))/2 + (b
*(1 - I*(c + d))*((x^2*Log[1 + ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)])
/(2*b*(I + c + d)) - (-1/2*(x*PolyLog[2, -(((I + c + d)*E^(2*a + 2*b*x))/(
I + c - d))])/b + PolyLog[3, -(((I + c + d)*E^(2*a + 2*b*x))/(I + c - d))]
/(4*b^2))/(b*(I + c + d))))/2
```

3.82.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5722
```
Int[ArcTan[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Tanh[a + b*x]]/(f*(m
+ 1))), x] + (Simp[I*b*((I - c - d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(E
^(2*a + 2*b*x)/(I - c + d + (I - c - d)*E^(2*a + 2*b*x))), x], x] - Simp[I*
b*((I + c + d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(E^(2*a + 2*b*x)/(I + c
 - d + (I + c + d)*E^(2*a + 2*b*x))), x], x]) /; FreeQ[{a, b, c, d, e, f},
x] && IGtQ[m, 0] && NeQ[(c - d)^2, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.82.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.00 (sec) , antiderivative size = 6567, normalized size of antiderivative = 24.60

method	result	size
risch	Expression too large to display	6567

input
```
int(x*arctan(c+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.82.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1067 vs. $2(197) = 394$.

Time = 0.36 (sec) , antiderivative size = 1067, normalized size of antiderivative = 4.00

$$\int x \arctan(c + d \tanh(a + bx)) \, dx = \text{Too large to display}$$

input
```
integrate(x*arctan(c+d*tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/4*(2*b^2*x^2*arctan((c*cosh(b*x + a) + d*sinh(b*x + a))/cosh(b*x + a)) +
 2*I*b*x*dilog(sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cos
h(b*x + a) + sinh(b*x + a))) + 2*I*b*x*dilog(-sqrt(-(c^2 - d^2 + 2*I*d + 1
)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 2*I*b*x*dilo
g(sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) +
sinh(b*x + a))) - 2*I*b*x*dilog(-sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*
d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + I*a^2*log(2*(c^2 + 2*c*d
+ d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) + 2*(c^
2 - d^2 - 2*I*d + 1)*sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1)
)) + I*a^2*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d +
d^2 + 1)*sinh(b*x + a) - 2*(c^2 - d^2 - 2*I*d + 1)*sqrt(-(c^2 - d^2 + 2*I*
d + 1)/(c^2 - 2*c*d + d^2 + 1))) - I*a^2*log(2*(c^2 + 2*c*d + d^2 + 1)*cos
h(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) + 2*(c^2 - d^2 + 2*I*
d + 1)*sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) - I*a^2*log
(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(
b*x + a) - 2*(c^2 - d^2 + 2*I*d + 1)*sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 -
2*c*d + d^2 + 1))) + (I*b^2*x^2 - I*a^2)*log(sqrt(-(c^2 - d^2 + 2*I*d + 1)
/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b^2*x^
2 - I*a^2)*log(-sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(co
sh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^2*x^2 + I*a^2)*log(sqrt(-(c^2...
```

3.82.6 Sympy [F]

$$\int x \arctan(c + d \tanh(a + bx)) \, dx = \int x \operatorname{atan}\left(c + d \tanh\left(a + bx\right)\right) \, dx$$

input
```
integrate(x*atan(c+d*tanh(b*x+a)),x)
```

output
```
Integral(x*atan(c + d*tanh(a + b*x)), x)
```

3.82.7 Maxima [F]

$$\int x \arctan(c + d \tanh(a + bx)) \, dx = \int x \arctan(d \tanh(bx + a) + c) \, dx$$

input
```
integrate(x*arctan(c+d*tanh(b*x+a)),x, algorithm="maxima")
```

output
```
1/2*x^2*arctan(((c*e^(2*a) + d*e^(2*a))*e^(2*b*x) + c - d)/(e^(2*b*x) + 2*a
) + 1)) - 2*b*d*integrate(x^2*e^(2*b*x + 2*a)/(c^2 - 2*c*d + d^2 + (c^2*e^
(4*a) + 2*c*d*e^(4*a) + d^2*e^(4*a) + e^(4*a))*e^(4*b*x) + 2*(c^2*e^(2*a)
- d^2*e^(2*a) + e^(2*a))*e^(2*b*x) + 1), x)
```

3.82.8 Giac [F]

$$\int x \arctan(c + d \tanh(a + bx)) \, dx = \int x \arctan(d \tanh(bx + a) + c) \, dx$$

input
```
integrate(x*arctan(c+d*tanh(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*arctan(d*tanh(b*x + a) + c), x)
```

3.82.9 Mupad [F(-1)]

Timed out.

$$\int x \arctan(c + d \tanh(a + bx)) \, dx = \int x \operatorname{atan}(c + d \tanh(a + b x)) \, dx$$

input
```
int(x*atan(c + d*tanh(a + b*x)),x)
```

output
```
int(x*atan(c + d*tanh(a + b*x)), x)
```

3.83 $\int \arctan(c + d \tanh(a + bx)) \, dx$

3.83.1 Optimal result

Integrand size = 11, antiderivative size = 174

$$\begin{aligned}\int \arctan(c+d\tanh(a+bx))\,dx = {} & x\arctan(c+d\tanh(a+bx)) \\ & +\frac{1}{2}ix\log\left(1+\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) \\ & -\frac{1}{2}ix\log\left(1+\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right) \\ & +\frac{i\,\mathrm{PolyLog}\left(2,-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b} \\ & -\frac{i\,\mathrm{PolyLog}\left(2,-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b}\end{aligned}$$

output
```
x*arctan(c+d*tanh(b*x+a))+1/2*I*x*ln(1+(I-c-d)*exp(2*b*x+2*a)/(I-c+d))-1/2
*I*x*ln(1+(I+c+d)*exp(2*b*x+2*a)/(I+c-d))+1/4*I*polylog(2,-(I-c-d)*exp(2*b
*x+2*a)/(I-c+d))/b-1/4*I*polylog(2,-(I+c+d)*exp(2*b*x+2*a)/(I+c-d))/b
```

3.83.2 Mathematica [A] (verified)

Time = 2.55 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.66

$$\int \arctan(c+d\tanh(a+bx))\,dx = x\arctan(c+d\tanh(a+bx))$$
$$+\frac{4a\sqrt{-d^2}\arctan\left(\frac{1+c^2-d^2+(1+c^2+2cd+d^2)e^{2(a+bx)}}{2d}\right)-2d(a+bx)\log\left(1+\frac{2(1+(c+d)^2)e^{2(a+bx)}}{2+2c^2-2d^2-4\sqrt{-d^2}}\right)+2d(a+bx)\log}{4b\sqrt{-}}$$

input
```
Integrate[ArcTan[c + d*Tanh[a + b*x]],x]
```

output
```
x*ArcTan[c + d*Tanh[a + b*x]] + (4*a*Sqrt[-d^2]*ArcTan[(1 + c^2 - d^2 + (1
 + c^2 + 2*c*d + d^2)*E^(2*(a + b*x)))/(2*d)] - 2*d*(a + b*x)*Log[1 + (2*(
1 + (c + d)^2)*E^(2*(a + b*x)))/(2 + 2*c^2 - 2*d^2 - 4*Sqrt[-d^2])] + 2*d*
(a + b*x)*Log[1 + ((1 + (c + d)^2)*E^(2*(a + b*x)))/(1 + c^2 - d^2 + 2*Sqr
t[-d^2])] + d*PolyLog[2, -(((1 + c^2 + 2*c*d + d^2)*E^(2*(a + b*x)))/(1 +
c^2 - d^2 + 2*Sqrt[-d^2]))] - d*PolyLog[2, ((1 + c^2 + 2*c*d + d^2)*E^(2*(
a + b*x)))/(-1 - c^2 + d^2 + 2*Sqrt[-d^2])])/(4*b*Sqrt[-d^2])
```

3.83.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.36, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {5714, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(d\tanh(a+bx)+c)\,dx$$

$$\Big\downarrow \text{5714}$$

$$-b(1+i(c+d))\int \frac{e^{2a+2bx}x}{-c+(-c-d+i)e^{2a+2bx}+d+i}dx+b(1-i(c+d))\int \frac{e^{2a+2bx}x}{c+(c+d+i)e^{2a+2bx}-d+i}dx+x\arctan(d\tanh(a+bx)+c)$$

$$\Big\downarrow \text{2620}$$

$$-b(1+i(c+d))\left(\frac{x\log\left(1+\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b(-c-d+i)}-\frac{\int\log\left(\frac{e^{2a+2bx}(-c-d+i)}{-c+d+i}+1\right)dx}{2b(-c-d+i)}\right)+b(1-i(c+d))\left(\frac{x\log\left(1+\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b(c+d+i)}-\frac{\int\log\left(\frac{e^{2a+2bx}(c+d+i)}{c-d+i}+1\right)dx}{2b(c+d+i)}\right)+x\arctan(d\tanh(a+bx)+c)$$

$$\Big\downarrow \text{2715}$$

$$-b(1+i(c+d))\left(\frac{x\log\left(1+\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b(-c-d+i)}-\frac{\int e^{-2a-2bx}\log\left(\frac{e^{2a+2bx}(-c-d+i)}{-c+d+i}+1\right)de^{2a+2bx}}{4b^2(-c-d+i)}\right)+b(1-i(c+d))\left(\frac{x\log\left(1+\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b(c+d+i)}-\frac{\int e^{-2a-2bx}\log\left(\frac{e^{2a+2bx}(c+d+i)}{c-d+i}+1\right)de^{2a+2bx}}{4b^2(c+d+i)}\right)+x\arctan(d\tanh(a+bx)+c)$$

$$\Big\downarrow \text{2838}$$

$$x\arctan(d\tanh(a+bx)+c)-b(1+i(c+d))\left(\frac{\text{PolyLog}\left(2,-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b^2(-c-d+i)}+\frac{x\log\left(1+\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b(-c-d+i)}\right)+b(1-i(c+d))\left(\frac{\text{PolyLog}\left(2,-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b^2(c+d+i)}+\frac{x\log\left(1+\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b(c+d+i)}\right)$$

input
```
Int[ArcTan[c + d*Tanh[a + b*x]],x]
```

output
```
x*ArcTan[c + d*Tanh[a + b*x]] - b*(1 + I*(c + d))*((x*Log[1 + ((I - c - d)
*E^(2*a + 2*b*x))/(I - c + d)])/(2*b*(I - c - d)) + PolyLog[2, -(((I - c -
 d)*E^(2*a + 2*b*x))/(I - c + d))]/(4*b^2*(I - c - d))) + b*(1 - I*(c + d)
)*((x*Log[1 + ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)])/(2*b*(I + c + d)
) + PolyLog[2, -(((I + c + d)*E^(2*a + 2*b*x))/(I + c - d))]/(4*b^2*(I + c
 + d)))
```

3.83. $\int \arctan(c+d\tanh(a+bx))\,dx$

3.83.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5714
```
Int[ArcTan[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Arc
Tan[c + d*Tanh[a + b*x]], x] + (Simp[I*b*(I - c - d)   Int[x*(E^(2*a + 2*b*
x)/(I - c + d + (I - c - d)*E^(2*a + 2*b*x))), x], x] - Simp[I*b*(I + c + d
)   Int[x*(E^(2*a + 2*b*x)/(I + c - d + (I + c + d)*E^(2*a + 2*b*x))), x],
x]) /; FreeQ[{a, b, c, d}, x] && NeQ[(c - d)^2, -1]
```

3.83.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 351 vs. 2(150) = 300.

Time = 2.18 (sec) , antiderivative size = 352, normalized size of antiderivative = 2.02

method	result
derivativedivides	$\frac{\arctan(c+d\tanh(bx+a))d\ln(-d\tanh(bx+a)-d)}{2}-\frac{\arctan(c+d\tanh(bx+a))d\ln(-d\tanh(bx+a)+d)}{2}+\frac{d^2\left(\frac{i\ln(-d\tanh(bx+a)+d)\ln\left(\ldots\right.}{2}\ldots\right.}{\ldots}$
default	$\frac{\arctan(c+d\tanh(bx+a))d\ln(-d\tanh(bx+a)-d)}{2}-\frac{\arctan(c+d\tanh(bx+a))d\ln(-d\tanh(bx+a)+d)}{2}+\frac{d^2\left(\frac{i\ln(-d\tanh(bx+a)+d)\ln\left(\ldots\right.}{2}\ldots\right.}{\ldots}$
risch	Expression too large to display

input
```
int(arctan(c+d*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b/d*(1/2*arctan(c+d*tanh(b*x+a))*d*ln(-d*tanh(b*x+a)-d)-1/2*arctan(c+d*t
anh(b*x+a))*d*ln(-d*tanh(b*x+a)+d)+1/2*d^2*(1/d*(1/2*I*ln(-d*tanh(b*x+a)+d
)*ln((I+d*tanh(b*x+a)+c)/(I+c+d))-1/2*I*ln(-d*tanh(b*x+a)+d)*ln((I-d*tanh(
b*x+a)-c)/(I-c-d))+1/2*I*dilog((I+d*tanh(b*x+a)+c)/(I+c+d))-1/2*I*dilog((I
-d*tanh(b*x+a)-c)/(I-c-d)))-1/d*(1/2*I*ln(-d*tanh(b*x+a)-d)*ln((I+d*tanh(b
*x+a)+c)/(I+c-d))-1/2*I*ln(-d*tanh(b*x+a)-d)*ln((I-d*tanh(b*x+a)-c)/(I-c+d
))+1/2*I*dilog((I+d*tanh(b*x+a)+c)/(I+c-d))-1/2*I*dilog((I-d*tanh(b*x+a)-c
)/(I-c+d))))
```

3.83.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 825 vs. $2(128) = 256$.

Time = 0.39 (sec) , antiderivative size = 825, normalized size of antiderivative = 4.74

$$\int \arctan(c + d\tanh(a + bx))\,dx$$
$$= \frac{2\,bx\arctan\left(\frac{c\cosh(bx+a)+d\sinh(bx+a)}{\cosh(bx+a)}\right) - i\,a\log\left(2\left(c^2+2\,cd+d^2+1\right)\cosh\left(bx+a\right)+2\left(c^2+2\,cd+d^2+1\right)\right.}{}$$

input
```
integrate(arctan(c+d*tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*(2*b*x*arctan((c*cosh(b*x + a) + d*sinh(b*x + a))/cosh(b*x + a)) - I*a
*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*s
inh(b*x + a) + 2*(c^2 - d^2 - 2*I*d + 1)*sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^
2 - 2*c*d + d^2 + 1))) - I*a*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) +
 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) - 2*(c^2 - d^2 - 2*I*d + 1)*sqrt(
-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) + I*a*log(2*(c^2 + 2*c*
d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) + 2*(
c^2 - d^2 + 2*I*d + 1)*sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 +
1))) + I*a*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d +
d^2 + 1)*sinh(b*x + a) - 2*(c^2 - d^2 + 2*I*d + 1)*sqrt(-(c^2 - d^2 - 2*I*
d + 1)/(c^2 - 2*c*d + d^2 + 1))) + (I*b*x + I*a)*log(sqrt(-(c^2 - d^2 + 2*
I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (
I*b*x + I*a)*log(-sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(
cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b*x - I*a)*log(sqrt(-(c^2 - d^2
- 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1)
 + (-I*b*x - I*a)*log(-sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 +
1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + I*dilog(sqrt(-(c^2 - d^2 + 2*I*
d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + I*dilog
(-sqrt(-(c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) +
sinh(b*x + a))) - I*dilog(sqrt(-(c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + ...
```

3.83.6 Sympy [F]

$$\int \arctan(c+d\tanh(a+bx))\,dx=\int \operatorname{atan}\left(c+d\tanh\left(a+bx\right)\right)dx$$

input
```
integrate(atan(c+d*tanh(b*x+a)),x)
```

output
```
Integral(atan(c + d*tanh(a + b*x)), x)
```

3.83.7 Maxima [F]

$$\int \arctan(c+d\tanh(a+bx))\,dx = \int \arctan\left(d\tanh\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(arctan(c+d*tanh(b*x+a)),x, algorithm="maxima")
```

output
```
-4*b*d*integrate(x*e^(2*b*x + 2*a)/(c^2 - 2*c*d + d^2 + (c^2*e^(4*a) + 2*c
*d*e^(4*a) + d^2*e^(4*a) + e^(4*a))*e^(4*b*x) + 2*(c^2*e^(2*a) - d^2*e^(2*
a) + e^(2*a))*e^(2*b*x) + 1), x) + x*arctan(((c*e^(2*a) + d*e^(2*a))*e^(2*
b*x) + c - d)/(e^(2*b*x + 2*a) + 1))
```

3.83.8 Giac [F]

$$\int \arctan(c+d\tanh(a+bx))\,dx = \int \arctan\left(d\tanh\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(arctan(c+d*tanh(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arctan(d*tanh(b*x + a) + c), x)
```

3.83.9 Mupad [F(-1)]

Timed out.

$$\int \arctan(c+d\tanh(a+bx))\,dx = \int \operatorname{atan}(c+d\tanh(a+b\,x))\,dx$$

input
```
int(atan(c + d*tanh(a + b*x)),x)
```

output
```
int(atan(c + d*tanh(a + b*x)), x)
```

3.84 $\int \frac{\arctan(c+d\tanh(a+bx))}{x}\,dx$

3.84.1 Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{\arctan(c+d\tanh(a+bx))}{x}\,dx = \text{Int}\left(\frac{\arctan(c+d\tanh(a+bx))}{x},x\right)$$

output
```
CannotIntegrate(arctan(c+d*tanh(b*x+a))/x,x)
```

3.84.2 Mathematica [N/A]

Not integrable

Time = 6.49 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\arctan(c+d\tanh(a+bx))}{x}\,dx = \int \frac{\arctan(c+d\tanh(a+bx))}{x}\,dx$$

input
```
Integrate[ArcTan[c + d*Tanh[a + b*x]]/x,x]
```

output
```
Integrate[ArcTan[c + d*Tanh[a + b*x]]/x, x]
```

3.84.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(d\tanh(a+bx)+c)}{x} \, dx$$

$$\Big\downarrow \; 7299$$

$$\int \frac{\arctan(d\tanh(a+bx)+c)}{x}dx$$

input
```
Int[ArcTan[c + d*Tanh[a + b*x]]/x,x]
```

output
```
$Aborted
```

3.84.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.84.4 Maple [N/A] (verified)

Not integrable

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\arctan\left(c+d\tanh\left(bx+a\right)\right)}{x}dx$$

input
```
int(arctan(c+d*tanh(b*x+a))/x,x)
```

output
```
int(arctan(c+d*tanh(b*x+a))/x,x)
```

3.84.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\arctan(c+d\tanh(a+bx))}{x}\,dx=\int \frac{\arctan\left(d\tanh\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arctan(c+d*tanh(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(arctan(d*tanh(b*x + a) + c)/x, x)
```

3.84.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan(c+d\tanh(a+bx))}{x}\,dx=\text{Timed out}$$

input
```
integrate(atan(c+d*tanh(b*x+a))/x,x)
```

output
```
Timed out
```

3.84.7 Maxima [N/A]

Not integrable

Time = 1.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\arctan(c+d\tanh(a+bx))}{x}\,dx=\int \frac{\arctan\left(d\tanh\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arctan(c+d*tanh(b*x+a))/x,x, algorithm="maxima")
```

output
```
integrate(arctan(d*tanh(b*x + a) + c)/x, x)
```

3.84.8 Giac [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\arctan(c+d\tanh(a+bx))}{x}\,dx = \int \frac{\arctan\left(d\tanh\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arctan(c+d*tanh(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(arctan(d*tanh(b*x + a) + c)/x, x)
```

3.84.9 Mupad [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\arctan(c+d\tanh(a+bx))}{x}\,dx = \int \frac{\mathrm{atan}(c+d\tanh(a+b\,x))}{x}\,dx$$

input
```
int(atan(c + d*tanh(a + b*x))/x,x)
```

output
```
int(atan(c + d*tanh(a + b*x))/x, x)
```

3.85 $\int x^2 \arctan(c+(i+c)\tanh(a+bx))\,dx$

3.85.1 Optimal result

Integrand size = 19, antiderivative size = 142

$$\int x^2 \arctan(c+(i+c)\tanh(a+bx))\,dx = -\frac{1}{12}ibx^4+\frac{1}{3}x^3\arctan(c+(i+c)\tanh(a+bx))+\frac{1}{6}ix^3\log\left(1+ice^{2a+2bx}\right) + \frac{ix^2\operatorname{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}-\frac{ix\operatorname{PolyLog}\left(3,-ice^{2a+2bx}\right)}{4b^2}+\frac{i\operatorname{PolyLog}\left(4,-ice^{2a+2bx}\right)}{8b^3}$$

output
```
-1/12*I*b*x^4+1/3*x^3*arctan(c+(I+c)*tanh(b*x+a))+1/6*I*x^3*ln(1+I*c*exp(2
*b*x+2*a))+1/4*I*x^2*polylog(2,-I*c*exp(2*b*x+2*a))/b-1/4*I*x*polylog(3,-I
*c*exp(2*b*x+2*a))/b^2+1/8*I*polylog(4,-I*c*exp(2*b*x+2*a))/b^3
```

3.85.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.94

$$\int x^2 \arctan(c+(i+c)\tanh(a+bx))\,dx = \frac{8b^3x^3\arctan(c+(i+c)\tanh(a+bx))+4ib^3x^3\log\left(1-\frac{ie^{-2(a+bx)}}{c}\right)-6ib^2x^2\operatorname{PolyLog}\left(2,\frac{ie^{-2(a+bx)}}{c}\right)-6ib}{24b^3}$$

input
```
Integrate[x^2*ArcTan[c + (I + c)*Tanh[a + b*x]],x]
```

output
```
(8*b^3*x^3*ArcTan[c + (I + c)*Tanh[a + b*x]] + (4*I)*b^3*x^3*Log[1 - I/(c*
E^(2*(a + b*x)))] - (6*I)*b^2*x^2*PolyLog[2, I/(c*E^(2*(a + b*x)))] - (6*I
)*b*x*PolyLog[3, I/(c*E^(2*(a + b*x)))] - (3*I)*PolyLog[4, I/(c*E^(2*(a +
b*x)))])/(24*b^3)
```

3.85.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.18, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {5718, 25, 2615, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(c+(c+i)\tanh(a+bx))\,dx$$

$$\Big\downarrow \text{5718}$$

$$\frac{1}{3}x^3\arctan(c+(c+i)\tanh(a+bx))-\frac{1}{3}b\int -\frac{x^3}{i-ce^{2a+2bx}}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}b\int \frac{x^3}{i-ce^{2a+2bx}}dx+\frac{1}{3}x^3\arctan(c+(c+i)\tanh(a+bx))$$

$$\Big\downarrow \text{2615}$$

$$\frac{1}{3}b\left(-ic\int \frac{e^{2a+2bx}x^3}{i-ce^{2a+2bx}}dx-\frac{ix^4}{4}\right)+\frac{1}{3}x^3\arctan(c+(c+i)\tanh(a+bx))$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{3}b\left(-ic\left(\frac{3\int x^2\log\left(ie^{2a+2bx}c+1\right)dx}{2bc}-\frac{x^3\log\left(1+ice^{2a+2bx}\right)}{2bc}\right)-\frac{ix^4}{4}\right)+\frac{1}{3}x^3\arctan(c+(c+i)\tanh(a+bx))$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{3}b\left(-ic\left(\frac{3\left(\frac{\int x\,\text{PolyLog}(2,-ice^{2a+2bx})dx}{b}-\frac{x^2\,\text{PolyLog}(2,-ice^{2a+2bx})}{2b}\right)}{2bc}-\frac{x^3\log\left(1+ice^{2a+2bx}\right)}{2bc}\right)-\frac{ix^4}{4}\right)+\frac{1}{3}x^3\arctan(c+(c+i)\tanh(a+bx))$$

$$\Big\downarrow \text{7163}$$

$$\frac{1}{3}b\left(-ic\left(\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-ice^{2a+2bx}\right)}{2b}-\frac{\int \text{PolyLog}\left(3,-ice^{2a+2bx}\right)dx}{2b}}{b}-\frac{x^2\,\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{2b}\right)}{2bc}-\frac{x^3\log\left(1+ice^{2a+2bx}\right)}{2bc}\right)-\frac{ix}{4}$$

$$\frac{1}{3}x^3\arctan(c+(c+i)\tanh(a+bx))$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{3}b\left(-ic\left(\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-ice^{2a+2bx}\right)}{2b}-\frac{\int e^{-2a-2bx}\,\text{PolyLog}\left(3,-ice^{2a+2bx}\right)de^{2a+2bx}}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{2b}\right)}{2bc}-\frac{x^3\log\left(1+ice^{2c}\right.}{2bc}\right.\right.$$

$$\frac{1}{3}x^3\arctan(c+(c+i)\tanh(a+bx))$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{3}x^3\arctan(c+(c+i)\tanh(a+bx))+$$

$$\frac{1}{3}b\left(-ic\left(\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-ice^{2a+2bx}\right)}{2b}-\frac{\text{PolyLog}\left(4,-ice^{2a+2bx}\right)}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{2b}\right)}{2bc}-\frac{x^3\log\left(1+ice^{2a+2bx}\right)}{2bc}\right)-\frac{ix^4}{4}\right)$$

input
```
Int[x^2*ArcTan[c + (I + c)*Tanh[a + b*x]],x]
```

output
```
(x^3*ArcTan[c + (I + c)*Tanh[a + b*x]])/3 + (b*((-1/4*I)*x^4 - I*c*(-1/2*(
x^3*Log[1 + I*c*E^(2*a + 2*b*x)])/(b*c) + (3*(-1/2*(x^2*PolyLog[2, (-I)*c*
E^(2*a + 2*b*x)])/b + ((x*PolyLog[3, (-I)*c*E^(2*a + 2*b*x)])/(2*b) - Poly
Log[4, (-I)*c*E^(2*a + 2*b*x)]/(4*b^2))/b))/(2*b*c))))/3
```

3.85. $\int x^2 \arctan(c+(i+c)\tanh(a+bx))\,dx$

3.85.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*(F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5718
```
Int[ArcTan[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Tanh[a + b*x]]/(f*(m
+ 1))), x] - Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a +
 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
d)^2, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.85.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.60 (sec) , antiderivative size = 1406, normalized size of antiderivative = 9.90

method	result	size
risch	Expression too large to display	1406

input
```
int(x^2*arctan(c+(I+c)*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/6*I*x^3*ln(1+I*c*exp(2*b*x+2*a))-1/2*I/b^2*ln(1+I*c*exp(2*b*x+2*a))*a^2*
x-1/6*I*x^3*ln(2*exp(2*b*x+2*a)*c-2*I)+1/2*I/b^2*a^2*ln(1+I*exp(b*x+a)*(I*
c)^(1/2))*x+1/2*I/b^2*a^2*ln(1-I*exp(b*x+a)*(I*c)^(1/2))*x-1/3*I/b^3*ln(1+
I*c*exp(2*b*x+2*a))*a^3-1/12*I*b*c/(I+c)*x^4+1/12*I/b^3*c/(I+c)*a^4-1/4*I*
x*polylog(3,-I*c*exp(2*b*x+2*a))/b^2+1/8*I*polylog(4,-I*c*exp(2*b*x+2*a))/
b^3+1/2*I/b^3*a^3*ln(1+I*exp(b*x+a)*(I*c)^(1/2))+1/2*I/b^3*a^2*dilog(1+I*e
xp(b*x+a)*(I*c)^(1/2))+1/2*I/b^3*a^2*dilog(1-I*exp(b*x+a)*(I*c)^(1/2))+1/4
*I*x^2*polylog(2,-I*c*exp(2*b*x+2*a))/b-1/4*I/b^3*polylog(2,-I*c*exp(2*b*x
+2*a))*a^2-1/6*I/b^3*a^3*ln(-exp(2*b*x+2*a)*c+I)-1/12*Pi*(csgn(I/(exp(2*b*
x+2*a)+1))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I
)/(exp(2*b*x+2*a)+1))-csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(2*I*exp(2*b*x+2*a
)+2*exp(2*b*x+2*a)*c))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp
(2*b*x+2*a)+1))-csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)
/(exp(2*b*x+2*a)+1))^2+csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(2*I*exp(2*b*x+2*
a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2-csgn(I*(2*exp(2*b*x+2*a)*c-2*
I))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))^2+csgn(I*(2*I*exp(
2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a
)*c)/(exp(2*b*x+2*a)+1))^2+csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)
+1))^3-csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))*csgn((2*exp(2*b
*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))^2+csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(e...
```

3.85.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(105) = 210$.

Time = 0.27 (sec) , antiderivative size = 293, normalized size of antiderivative = 2.06

$$\int x^2 \arctan(c + (i + c)\tanh(a + bx))\,dx$$
$$= \frac{-i\,b^4x^4 + 2i\,b^3x^3\log\left(-\frac{(c+i)e^{(2\,bx+2\,a)}}{ce^{(2\,bx+2\,a)}-i}\right) + 6i\,b^2x^2\mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4i\,c}e^{(bx+a)}\right) + 6i\,b^2x^2\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{-4i\,c}e^{(bx+a)}\right) + i\,a}{}$$

input
```
integrate(x^2*arctan(c+(I+c)*tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/12*(-I*b^4*x^4 + 2*I*b^3*x^3*log(-(c + I)*e^(2*b*x + 2*a)/(c*e^(2*b*x +
2*a) - I)) + 6*I*b^2*x^2*dilog(1/2*sqrt(-4*I*c)*e^(b*x + a)) + 6*I*b^2*x^2
*dilog(-1/2*sqrt(-4*I*c)*e^(b*x + a)) + I*a^4 - 2*I*a^3*log(1/2*(2*c*e^(b*
x + a) + I*sqrt(-4*I*c))/c) - 2*I*a^3*log(1/2*(2*c*e^(b*x + a) - I*sqrt(-4
*I*c))/c) - 12*I*b*x*polylog(3, 1/2*sqrt(-4*I*c)*e^(b*x + a)) - 12*I*b*x*p
olylog(3, -1/2*sqrt(-4*I*c)*e^(b*x + a)) - 2*(-I*b^3*x^3 - I*a^3)*log(1/2*
sqrt(-4*I*c)*e^(b*x + a) + 1) - 2*(-I*b^3*x^3 - I*a^3)*log(-1/2*sqrt(-4*I*
c)*e^(b*x + a) + 1) + 12*I*polylog(4, 1/2*sqrt(-4*I*c)*e^(b*x + a)) + 12*I
*polylog(4, -1/2*sqrt(-4*I*c)*e^(b*x + a)))/b^3
```

3.85.6 Sympy [F(-2)]

Exception generated.

$$\int x^2 \arctan(c + (i + c)\tanh(a + bx))\,dx = \text{Exception raised: CoercionFailed}$$

input
```
integrate(x**2*atan(c+(I+c)*tanh(b*x+a)),x)
```

output
```
Exception raised: CoercionFailed >> Cannot convert _t0**2*exp(2*a) + 1 of
type <class 'sympy.core.add.Add'> to QQ_I[x,b,_t0,exp(a)]
```

3.85.7 Maxima [A] (verification not implemented)

Time = 1.32 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.91

$$\int x^2 \arctan(c+(i+c)\tanh(a+bx))\,dx = \frac{1}{3}x^3\arctan\left((c+i)\tanh\left(bx+a\right)+c\right) + \frac{4}{9}\left(\frac{3x^4}{4ic-4} - \frac{4b^3x^3\log\left(ice^{(2bx+2a)}+1\right)+6b^2x^2\mathrm{Li}_2\left(-ice^{(2bx+2a)}\right)-6bx\mathrm{Li}_3(-ice^{(2bx+2a)})+3\mathrm{Li}_4(-\right.}{-2b^4(-ic+1)}$$

input
```
integrate(x^2*arctan(c+(I+c)*tanh(b*x+a)),x, algorithm="maxima")
```

output
```
1/3*x^3*arctan((c + I)*tanh(b*x + a) + c) + 4/9*(3*x^4/(4*I*c - 4) - (4*b^
3*x^3*log(I*c*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(-I*c*e^(2*b*x + 2*a))
 - 6*b*x*polylog(3, -I*c*e^(2*b*x + 2*a)) + 3*polylog(4, -I*c*e^(2*b*x + 2
*a)))/(b^4*(2*I*c - 2)))*b*(c + I)
```

3.85.8 Giac [F]

$$\int x^2 \arctan(c+(i+c)\tanh(a+bx))\,dx = \int x^2\arctan\left((c+i)\tanh\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(x^2*arctan(c+(I+c)*tanh(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*arctan((c + I)*tanh(b*x + a) + c), x)
```

3.85.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \arctan(c+(i+c)\tanh(a+bx))\,dx = \int x^2\,\mathrm{atan}(c+\tanh(a+b\,x)\,(c+1\mathrm{i}))\,dx$$

input
```
int(x^2*atan(c + tanh(a + b*x)*(c + 1i)),x)
```

output
```
int(x^2*atan(c + tanh(a + b*x)*(c + 1i)), x)
```

3.86 $\int x \arctan(c+(i+c)\tanh(a+bx))\,dx$

3.86.1 Optimal result

Integrand size = 17, antiderivative size = 113

$$\int x \arctan(c+(i+c)\tanh(a+bx))\,dx = -\frac{1}{6}ibx^3+\frac{1}{2}x^2\arctan(c+(i+c)\tanh(a+bx)) + \frac{1}{4}ix^2\log\left(1+ice^{2a+2bx}\right) + \frac{ix\,\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b} - \frac{i\,\text{PolyLog}\left(3,-ice^{2a+2bx}\right)}{8b^2}$$

output
```
-1/6*I*b*x^3+1/2*x^2*arctan(c+(I+c)*tanh(b*x+a))+1/4*I*x^2*ln(1+I*c*exp(2*
b*x+2*a))+1/4*I*x*polylog(2,-I*c*exp(2*b*x+2*a))/b-1/8*I*polylog(3,-I*c*ex
p(2*b*x+2*a))/b^2
```

3.86.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.91

$$\int x \arctan(c+(i+c)\tanh(a+bx))\,dx = \frac{2b^2x^2\left(2\arctan(c+(i+c)\tanh(a+bx))+i\log\left(1-\frac{ie^{-2(a+bx)}}{c}\right)\right)-2ibx\,\text{PolyLog}\left(2,\frac{ie^{-2(a+bx)}}{c}\right)-i\,\text{PolyL}}{8b^2}$$

input
```
Integrate[x*ArcTan[c + (I + c)*Tanh[a + b*x]],x]
```

output
```
(2*b^2*x^2*(2*ArcTan[c + (I + c)*Tanh[a + b*x]] + I*Log[1 - I/(c*E^(2*(a +
 b*x)))]) - (2*I)*b*x*PolyLog[2, I/(c*E^(2*(a + b*x)))] - I*PolyLog[3, I/(
c*E^(2*(a + b*x)))])/(8*b^2)
```

3.86.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.17, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {5718, 25, 2615, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\arctan(c+(c+i)\tanh(a+bx))\,dx$$

$$\Big\downarrow \text{5718}$$

$$\frac{1}{2}x^2\arctan(c+(c+i)\tanh(a+bx))-\frac{1}{2}b\int -\frac{x^2}{i-ce^{2a+2bx}}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}b\int \frac{x^2}{i-ce^{2a+2bx}}dx+\frac{1}{2}x^2\arctan(c+(c+i)\tanh(a+bx))$$

$$\Big\downarrow \text{2615}$$

$$\frac{1}{2}b\left(-ic\int \frac{e^{2a+2bx}x^2}{i-ce^{2a+2bx}}dx-\frac{ix^3}{3}\right)+\frac{1}{2}x^2\arctan(c+(c+i)\tanh(a+bx))$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{2}b\left(-ic\left(\frac{\int x\log\left(ie^{2a+2bx}c+1\right)dx}{bc}-\frac{x^2\log\left(1+ice^{2a+2bx}\right)}{2bc}\right)-\frac{ix^3}{3}\right)+\frac{1}{2}x^2\arctan(c+(c+i)\tanh(a+bx))$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{2}b\left(-ic\left(\frac{\frac{\int \text{PolyLog}(2,-ice^{2a+2bx})dx}{2b}-\frac{x\,\text{PolyLog}(2,-ice^{2a+2bx})}{2b}}{bc}-\frac{x^2\log\left(1+ice^{2a+2bx}\right)}{2bc}\right)-\frac{ix^3}{3}\right)+\frac{1}{2}x^2\arctan(c+(c+i)\tanh(a+bx))$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{2}b\left(-ic\left(\frac{\frac{\int e^{-2a-2bx}\,\mathrm{PolyLog}\left(2,-ice^{2a+2bx}\right)de^{2a+2bx}}{4b^2}-\frac{x\,\mathrm{PolyLog}\left(2,-ice^{2a+2bx}\right)}{2b}}{bc}-\frac{x^2\log\left(1+ice^{2a+2bx}\right)}{2bc}\right)-\frac{ix^3}{3}\right)+\frac{1}{2}x^2\arctan(c+(c+i)\tanh(a+bx))$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{2}x^2\arctan(c+(c+i)\tanh(a+bx))+\frac{1}{2}b\left(-ic\left(\frac{\frac{\mathrm{PolyLog}\left(3,-ice^{2a+2bx}\right)}{4b^2}-\frac{x\,\mathrm{PolyLog}\left(2,-ice^{2a+2bx}\right)}{2b}}{bc}-\frac{x^2\log\left(1+ice^{2a+2bx}\right)}{2bc}\right)-\frac{ix^3}{3}\right)$$

input
```
Int[x*ArcTan[c + (I + c)*Tanh[a + b*x]],x]
```

output
```
(x^2*ArcTan[c + (I + c)*Tanh[a + b*x]])/2 + (b*((-1/3*I)*x^3 - I*c*(-1/2*(
x^2*Log[1 + I*c*E^(2*a + 2*b*x)])/(b*c) + (-1/2*(x*PolyLog[2, (-I)*c*E^(2*
a + 2*b*x)])/b + PolyLog[3, (-I)*c*E^(2*a + 2*b*x)]/(4*b^2))/(b*c))))/2
```

3.86.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5718
```
Int[ArcTan[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Tanh[a + b*x]]/(f*(m
+ 1))), x] - Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a +
 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
d)^2, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.86.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.91 (sec) , antiderivative size = 1370, normalized size of antiderivative = 12.12

method	result	size
risch	Expression too large to display	1370

input
```
int(x*arctan(c+(I+c)*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I/b^2*a*dilog(1+I*exp(b*x+a)*(I*c)^(1/2))-1/2*I/b^2*a*dilog(1-I*exp(b
*x+a)*(I*c)^(1/2))-1/4*I*x^2*ln(2*exp(2*b*x+2*a)*c-2*I)+1/4*I/b^2*polylog(
2,-I*c*exp(2*b*x+2*a))*a+1/4*I/b^2*a^2*ln(-exp(2*b*x+2*a)*c+I)-1/6*I*b*c/(
I+c)*x^3-1/2*I/b^2*ln(1+I*exp(b*x+a)*(I*c)^(1/2))*a^2+1/4*I*x^2*ln(1+I*c*e
xp(2*b*x+2*a))-1/2*I/b^2*ln(1-I*exp(b*x+a)*(I*c)^(1/2))*a^2+1/4*I/b^2*ln(1
+I*c*exp(2*b*x+2*a))*a^2-1/8*I*polylog(3,-I*c*exp(2*b*x+2*a))/b^2+1/2*I/b*
ln(1+I*c*exp(2*b*x+2*a))*a*x-1/6*I/b^2*c/(I+c)*a^3+1/4*I*x*polylog(2,-I*c*
exp(2*b*x+2*a))/b+1/4*I*x^2*ln(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)-1/8*
Pi*(csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I))*csgn(I*(2*
exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))-csgn(I/(exp(2*b*x+2*a)+1))*csgn(
I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(2*I*exp(2*b*x+2*a)+2*ex
p(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))-csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(2*e
xp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))^2+csgn(I/(exp(2*b*x+2*a)+1))*csgn
(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2-csgn(I*(2
*exp(2*b*x+2*a)*c-2*I))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1)
)^2+csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(2*I*exp(2*b*x+
2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2+csgn(I*(2*exp(2*b*x+2*a)*c-
2*I)/(exp(2*b*x+2*a)+1))^3-csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)
+1))*csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))^2+csgn(I*(2*exp(2*b
*x+2*a)*c-2*I)/(exp(2*b*x+2*a)+1))*csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2...
```

3.86.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 247 vs. 2(83) = 166.

Time = 0.27 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.19

$$\int x\arctan(c+(i+c)\tanh(a+bx))\,dx$$
$$= \frac{-2i\,b^3x^3+3i\,b^2x^2\log\left(-\frac{(c+i)e^{(2\,bx+2\,a)}}{ce^{(2\,bx+2\,a)}-i}\right)-2i\,a^3+6i\,bx\mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4i\,c}e^{(bx+a)}\right)+6i\,bx\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{-4i\,c}e^{(bx+a)}\right)}{}$$

input
```
integrate(x*arctan(c+(I+c)*tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/12*(-2*I*b^3*x^3 + 3*I*b^2*x^2*log(-(c + I)*e^(2*b*x + 2*a)/(c*e^(2*b*x
+ 2*a) - I)) - 2*I*a^3 + 6*I*b*x*dilog(1/2*sqrt(-4*I*c)*e^(b*x + a)) + 6*I
*b*x*dilog(-1/2*sqrt(-4*I*c)*e^(b*x + a)) + 3*I*a^2*log(1/2*(2*c*e^(b*x +
a) + I*sqrt(-4*I*c))/c) + 3*I*a^2*log(1/2*(2*c*e^(b*x + a) - I*sqrt(-4*I*c
))/c) - 3*(-I*b^2*x^2 + I*a^2)*log(1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) - 3*(
-I*b^2*x^2 + I*a^2)*log(-1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) - 6*I*polylog(3
, 1/2*sqrt(-4*I*c)*e^(b*x + a)) - 6*I*polylog(3, -1/2*sqrt(-4*I*c)*e^(b*x
+ a)))/b^2
```

3.86.6 Sympy [F(-2)]

Exception generated.

$$\int x \arctan(c+(i+c)\tanh(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

input
```
integrate(x*atan(c+(I+c)*tanh(b*x+a)),x)
```

output
```
Exception raised: CoercionFailed >> Cannot convert _t0**2*exp(2*a) + 1 of
type <class 'sympy.core.add.Add'> to QQ_I[x,b,_t0,exp(a)]
```

3.86.7 Maxima [A] (verification not implemented)

Time = 1.26 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.94

$$\begin{aligned}&\int x \arctan(c+(i+c)\tanh(a+bx))\,dx\\&=\left(\frac{2\,x^3}{3i\,c-3}-\frac{2\,b^2x^2\log\left(i\,ce^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_2\left(-i\,ce^{(2\,bx+2\,a)}\right)-\mathrm{Li}_3\left(-i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^3(-i\,c+1)}\right)b(c+i)\\&\quad+\frac{1}{2}\,x^2\arctan\left((c+i)\tanh\left(bx+a\right)+c\right)\end{aligned}$$

input
```
integrate(x*arctan(c+(I+c)*tanh(b*x+a)),x, algorithm="maxima")
```

output
```
(2*x^3/(3*I*c - 3) - (2*b^2*x^2*log(I*c*e^(2*b*x + 2*a) + 1) + 2*b*x*dilog
(-I*c*e^(2*b*x + 2*a)) - polylog(3, -I*c*e^(2*b*x + 2*a)))/(b^3*(2*I*c - 2
)))*b*(c + I) + 1/2*x^2*arctan((c + I)*tanh(b*x + a) + c)
```

3.86.8 Giac [F]

$$\int x\arctan(c+(i+c)\tanh(a+bx))\,dx = \int x\arctan\left((c+i)\tanh\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(x*arctan(c+(I+c)*tanh(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*arctan((c + I)*tanh(b*x + a) + c), x)
```

3.86.9 Mupad [F(-1)]

Timed out.

$$\int x\arctan(c+(i+c)\tanh(a+bx))\,dx = \int x\,\mathrm{atan}(c+\tanh(a+b\,x)\,\,(c+1\mathrm{i}))\,dx$$

input
```
int(x*atan(c + tanh(a + b*x)*(c + 1i)),x)
```

output
```
int(x*atan(c + tanh(a + b*x)*(c + 1i)), x)
```

3.87 $\int \arctan(c+(i+c)\tanh(a+bx))\,dx$

3.87.1 Optimal result

Integrand size = 15, antiderivative size = 79

$$\int \arctan(c+(i+c)\tanh(a+bx))\,dx = -\frac{1}{2}ibx^2 + x\arctan(c+(i+c)\tanh(a+bx)) + \frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right) + \frac{i\,\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}$$

output
```
-1/2*I*b*x^2+x*arctan(c+(I+c)*tanh(b*x+a))+1/2*I*x*ln(1+I*c*exp(2*b*x+2*a)
)+1/4*I*polylog(2,-I*c*exp(2*b*x+2*a))/b
```

3.87.2 Mathematica [A] (verified)

Time = 1.66 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.90

$$\int \arctan(c+(i+c)\tanh(a+bx))\,dx = x\arctan(c+(i+c)\tanh(a+bx)) + \frac{i\left(2bx\log\left(1-\frac{ie^{-2(a+bx)}}{c}\right) - \text{PolyLog}\left(2,\frac{ie^{-2(a+bx)}}{c}\right)\right)}{4b}$$

input
```
Integrate[ArcTan[c + (I + c)*Tanh[a + b*x]],x]
```

output
```
x*ArcTan[c + (I + c)*Tanh[a + b*x]] + ((I/4)*(2*b*x*Log[1 - I/(c*E^(2*(a +
 b*x)))] - PolyLog[2, I/(c*E^(2*(a + b*x)))]))/b
```

3.87.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5710, 25, 2615, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(c+(c+i)\tanh(a+bx))\,dx$$

$$\Big\downarrow \text{5710}$$

$$x\arctan(c+(c+i)\tanh(a+bx))-b\int -\frac{x}{i-ce^{2a+2bx}}dx$$

$$\Big\downarrow \text{25}$$

$$b\int \frac{x}{i-ce^{2a+2bx}}dx+x\arctan(c+(c+i)\tanh(a+bx))$$

$$\Big\downarrow \text{2615}$$

$$b\left(-ic\int \frac{e^{2a+2bx}x}{i-ce^{2a+2bx}}dx-\frac{ix^2}{2}\right)+x\arctan(c+(c+i)\tanh(a+bx))$$

$$\Big\downarrow \text{2620}$$

$$b\left(-ic\left(\frac{\int \log\left(ie^{2a+2bx}c+1\right)dx}{2bc}-\frac{x\log\left(1+ice^{2a+2bx}\right)}{2bc}\right)-\frac{ix^2}{2}\right)+x\arctan(c+(c+i)\tanh(a+bx))$$

$$\Big\downarrow \text{2715}$$

$$b\left(-ic\left(\frac{\int e^{-2a-2bx}\log\left(ie^{2a+2bx}c+1\right)de^{2a+2bx}}{4b^2c}-\frac{x\log\left(1+ice^{2a+2bx}\right)}{2bc}\right)-\frac{ix^2}{2}\right)+x\arctan(c+(c+i)\tanh(a+bx))$$

$$\Big\downarrow \text{2838}$$

$$x\arctan(c+(c+i)\tanh(a+bx))+b\left(-ic\left(-\frac{\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b^2c}-\frac{x\log\left(1+ice^{2a+2bx}\right)}{2bc}\right)-\frac{ix^2}{2}\right)$$

input
```
Int[ArcTan[c + (I + c)*Tanh[a + b*x]],x]
```

output
```
x*ArcTan[c + (I + c)*Tanh[a + b*x]] + b*((-1/2*I)*x^2 - I*c*(-1/2*(x*Log[1
 + I*c*E^(2*a + 2*b*x)])/(b*c) - PolyLog[2, (-I)*c*E^(2*a + 2*b*x)]/(4*b^2
*c)))
```

3.87.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5710
```
Int[ArcTan[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Arc
Tan[c + d*Tanh[a + b*x]], x] - Simp[b   Int[x/(c - d + c*E^(2*a + 2*b*x)),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, -1]
```

3.87.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 544 vs. $2(65) = 130$.

Time = 1.03 (sec) , antiderivative size = 545, normalized size of antiderivative = 6.90

method	result
derivativedivides	$-\frac{\arctan(c+(i+c)\tanh(bx+a))\ln(i+c+(i+c)\tanh(bx+a))}{2i+2c}+\frac{2i\arctan(c+(i+c)\tanh(bx+a))\ln(i+c+(i+c)\tanh(bx+a))c}{2i+2c}+\arctan(c+($
default	$-\frac{\arctan(c+(i+c)\tanh(bx+a))\ln(i+c+(i+c)\tanh(bx+a))}{2i+2c}+\frac{2i\arctan(c+(i+c)\tanh(bx+a))\ln(i+c+(i+c)\tanh(bx+a))c}{2i+2c}+\arctan(c+($
risch	Expression too large to display

input
```
int(arctan(c+(I+c)*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b/(I+c)*(-arctan(c+(I+c)*tanh(b*x+a))/(2*I+2*c)*ln(I+c+(I+c)*tanh(b*x+a)
)+2*I*arctan(c+(I+c)*tanh(b*x+a))/(2*I+2*c)*ln(I+c+(I+c)*tanh(b*x+a))*c+ar
ctan(c+(I+c)*tanh(b*x+a))/(2*I+2*c)*ln(I+c+(I+c)*tanh(b*x+a))*c^2+arctan(c
+(I+c)*tanh(b*x+a))/(2*I+2*c)*ln(c-(I+c)*tanh(b*x+a)+I)-2*I*arctan(c+(I+c)
*tanh(b*x+a))/(2*I+2*c)*ln(c-(I+c)*tanh(b*x+a)+I)*c-arctan(c+(I+c)*tanh(b*
x+a))/(2*I+2*c)*ln(c-(I+c)*tanh(b*x+a)+I)*c^2-(I+c)^2*(1/2/(I+c)*(-1/2*I*(
(ln(I+c+(I+c)*tanh(b*x+a))-ln(-1/2*I*(I+c+(I+c)*tanh(b*x+a))))*ln(-1/2*I*(
I-c-(I+c)*tanh(b*x+a)))-dilog(-1/2*I*(I+c+(I+c)*tanh(b*x+a))))+1/4*I*ln(I+
c+(I+c)*tanh(b*x+a))^2)-1/2/(I+c)*(-1/2*I*(dilog(-1/2*(I-c-(I+c)*tanh(b*x+
a))/c)+ln(c-(I+c)*tanh(b*x+a)+I)*ln(-1/2*(I-c-(I+c)*tanh(b*x+a))/c))+1/2*I
*(dilog((-I-c-(I+c)*tanh(b*x+a))/(-2*I-2*c))+ln(c-(I+c)*tanh(b*x+a)+I)*ln(
(-I-c-(I+c)*tanh(b*x+a))/(-2*I-2*c))))))
```

3.87.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(58) = 116.

Time = 0.30 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.37

$$\int \arctan(c+(i+c)\tanh(a+bx))\,dx$$
$$= \frac{-i\,b^2x^2 + i\,bx\log\left(-\frac{(c+i)e^{(2\,bx+2\,a)}}{ce^{(2\,bx+2\,a)}-i}\right) + i\,a^2 + (i\,bx+i\,a)\log\left(\frac{1}{2}\sqrt{-4i\,c}e^{(bx+a)}+1\right) + (i\,bx+i\,a)\log\left(-\frac{1}{2}\sqrt{-}\right.}{}$$

input
```
integrate(arctan(c+(I+c)*tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*(-I*b^2*x^2 + I*b*x*log(-(c + I)*e^(2*b*x + 2*a)/(c*e^(2*b*x + 2*a) -
I)) + I*a^2 + (I*b*x + I*a)*log(1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) + (I*b*x
 + I*a)*log(-1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) - I*a*log(1/2*(2*c*e^(b*x +
 a) + I*sqrt(-4*I*c))/c) - I*a*log(1/2*(2*c*e^(b*x + a) - I*sqrt(-4*I*c))/
c) + I*dilog(1/2*sqrt(-4*I*c)*e^(b*x + a)) + I*dilog(-1/2*sqrt(-4*I*c)*e^(
b*x + a)))/b
```

3.87.6 Sympy [F(-2)]

Exception generated.

$$\int \arctan(c+(i+c)\tanh(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

input
```
integrate(atan(c+(I+c)*tanh(b*x+a)),x)
```

output
```
Exception raised: CoercionFailed >> Cannot convert _t0**2*exp(2*a) + 1 of
type <class 'sympy.core.add.Add'> to QQ_I[b,_t0,exp(a)]
```

3.87.7 Maxima [A] (verification not implemented)

Time = 1.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.01

$$\int \arctan(c+(i+c)\tanh(a+bx))\,dx$$
$$= 2\,b(c+i)\left(\frac{2\,x^2}{2i\,c-2} - \frac{2\,bx\log\left(i\,ce^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left(-i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^2(-i\,c+1)}\right)$$
$$+ x\arctan\left((c+i)\tanh\left(bx+a\right)+c\right)$$

input
```
integrate(arctan(c+(I+c)*tanh(b*x+a)),x, algorithm="maxima")
```

output
```
2*b*(c + I)*(2*x^2/(2*I*c - 2) - (2*b*x*log(I*c*e^(2*b*x + 2*a) + 1) + dil
og(-I*c*e^(2*b*x + 2*a)))/(b^2*(2*I*c - 2))) + x*arctan((c + I)*tanh(b*x +
 a) + c)
```

3.87.8 Giac [F]

$$\int \arctan(c+(i+c)\tanh(a+bx))\,dx = \int \arctan\left((c+i)\tanh\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(arctan(c+(I+c)*tanh(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arctan((c + I)*tanh(b*x + a) + c), x)
```

3.87.9 Mupad [F(-1)]

Timed out.

$$\int \arctan(c+(i+c)\tanh(a+bx))\,dx = \int \mathrm{atan}(c+\tanh(a+b\,x)\,\,(c+1\mathrm{i}))\,dx$$

input
```
int(atan(c + tanh(a + b*x)*(c + 1i)),x)
```

output
```
int(atan(c + tanh(a + b*x)*(c + 1i)), x)
```

3.88 $\int \frac{\arctan(c+(i+c)\tanh(a+bx))}{x}\,dx$

3.88.1 Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{\arctan(c+(i+c)\tanh(a+bx))}{x}\,dx = \text{Int}\left(\frac{\arctan(c+(i+c)\tanh(a+bx))}{x},x\right)$$

output
```
CannotIntegrate(arctan(c+(I+c)*tanh(b*x+a))/x,x)
```

3.88.2 Mathematica [N/A]

Not integrable

Time = 3.37 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{\arctan(c+(i+c)\tanh(a+bx))}{x}\,dx = \int \frac{\arctan(c+(i+c)\tanh(a+bx))}{x}\,dx$$

input
```
Integrate[ArcTan[c + (I + c)*Tanh[a + b*x]]/x,x]
```

output
```
Integrate[ArcTan[c + (I + c)*Tanh[a + b*x]]/x, x]
```

3.88.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(c+(c+i)\tanh(a+bx))}{x} \, dx$$
$$\Big\downarrow \; 7299$$
$$\int \frac{\arctan(c+(c+i)\tanh(a+bx))}{x}dx$$

input
```
Int[ArcTan[c + (I + c)*Tanh[a + b*x]]/x,x]
```

output
```
$Aborted
```

3.88.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.88.4 Maple [N/A] (verified)

Not integrable

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{\arctan \left(c+\left(i+c\right) \tanh \left(b x +a\right)\right)}{x}dx$$

input
```
int(arctan(c+(I+c)*tanh(b*x+a))/x,x)
```

output
```
int(arctan(c+(I+c)*tanh(b*x+a))/x,x)
```

3.88.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.95

$$\int \frac{\arctan(c + (i + c)\tanh(a + bx))}{x}\,dx = \int \frac{\arctan\left((c + i)\tanh\left(bx + a\right) + c\right)}{x}\,dx$$

input
```
integrate(arctan(c+(I+c)*tanh(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(1/2*I*log(-(c + I)*e^(2*b*x + 2*a)/(c*e^(2*b*x + 2*a) - I))/x, x)
```

3.88.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan(c + (i + c)\tanh(a + bx))}{x}\,dx = \text{Timed out}$$

input
```
integrate(atan(c+(I+c)*tanh(b*x+a))/x,x)
```

output
```
Timed out
```

3.88.7 Maxima [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 72, normalized size of antiderivative = 3.79

$$\int \frac{\arctan(c + (i + c)\tanh(a + bx))}{x}\,dx = \int \frac{\arctan\left((c + i)\tanh\left(bx + a\right) + c\right)}{x}\,dx$$

input
```
integrate(arctan(c+(I+c)*tanh(b*x+a))/x,x, algorithm="maxima")
```

output
```
I*b*x - 1/4*(4*pi - 4*I*a - 2*arctan(c) - I*log(c^2 + 1))*log(x) + 1/2*int
egrate(arctan(c*e^(2*b*x + 2*a))/x, x) - 1/4*I*integrate(log(c^2*e^(4*b*x
+ 4*a) + 1)/x, x)
```

3.88.8 Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(c+(i+c)\tanh(a+bx))}{x}\,dx = \int \frac{\arctan\left((c+i)\tanh\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arctan(c+(I+c)*tanh(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(arctan((c + I)*tanh(b*x + a) + c)/x, x)
```

3.88.9 Mupad [N/A]

Not integrable

Time = 1.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{\arctan(c+(i+c)\tanh(a+bx))}{x}\,dx = \int \frac{\mathrm{atan}(c+\tanh(a+b\,x)\,\,(c+1\mathrm{i}))}{x}\,dx$$

input
```
int(atan(c + tanh(a + b*x)*(c + 1i))/x,x)
```

output
```
int(atan(c + tanh(a + b*x)*(c + 1i))/x, x)
```

3.89 $\int x^2 \arctan(c-(i-c)\tanh(a+bx))\,dx$

3.89.1 Optimal result

Integrand size = 22, antiderivative size = 145

$$\int x^2 \arctan(c-(i-c)\tanh(a+bx))\,dx = \frac{1}{12}ibx^4+\frac{1}{3}x^3\arctan(c-(i-c)\tanh(a+bx))-\frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right) - \frac{ix^2\operatorname{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}+\frac{ix\operatorname{PolyLog}\left(3,ice^{2a+2bx}\right)}{4b^2}-\frac{i\operatorname{PolyLog}\left(4,ice^{2a+2bx}\right)}{8b^3}$$

output
```
1/12*I*b*x^4+1/3*x^3*arctan(c-(I-c)*tanh(b*x+a))-1/6*I*x^3*ln(1-I*c*exp(2*
b*x+2*a))-1/4*I*x^2*polylog(2,I*c*exp(2*b*x+2*a))/b+1/4*I*x*polylog(3,I*c*
exp(2*b*x+2*a))/b^2-1/8*I*polylog(4,I*c*exp(2*b*x+2*a))/b^3
```

3.89.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.92

$$\int x^2 \arctan(c-(i-c)\tanh(a+bx))\,dx = \frac{8b^3x^3\arctan(c+(-i+c)\tanh(a+bx))-4ib^3x^3\log\left(1+\frac{ie^{-2(a+bx)}}{c}\right)+6ib^2x^2\operatorname{PolyLog}\left(2,-\frac{ie^{-2(a+bx)}}{c}\right)+}{24b^3}$$

input
```
Integrate[x^2*ArcTan[c - (I - c)*Tanh[a + b*x]],x]
```

output
```
(8*b^3*x^3*ArcTan[c + (-I + c)*Tanh[a + b*x]] - (4*I)*b^3*x^3*Log[1 + I/(c
*E^(2*(a + b*x)))] + (6*I)*b^2*x^2*PolyLog[2, (-I)/(c*E^(2*(a + b*x)))] +
(6*I)*b*x*PolyLog[3, (-I)/(c*E^(2*(a + b*x)))] + (3*I)*PolyLog[4, (-I)/(c*
E^(2*(a + b*x)))])/(24*b^3)
```

3.89.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.17, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {5718, 2615, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(c-(-c+i)\tanh(a+bx))\,dx$$

$$\Big\downarrow \text{5718}$$

$$\frac{1}{3}x^3\arctan(c-(-c+i)\tanh(a+bx))-\frac{1}{3}b\int \frac{x^3}{e^{2a+2bx}c+i}dx$$

$$\Big\downarrow \text{2615}$$

$$\frac{1}{3}x^3\arctan(c-(-c+i)\tanh(a+bx))-\frac{1}{3}b\left(ic\int \frac{e^{2a+2bx}x^3}{e^{2a+2bx}c+i}dx-\frac{ix^4}{4}\right)$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{3}x^3\arctan(c-(-c+i)\tanh(a+bx))-$$
$$\frac{1}{3}b\left(ic\left(\frac{x^3\log\left(1-ice^{2a+2bx}\right)}{2bc}-\frac{3\int x^2\log\left(1-ice^{2a+2bx}\right)dx}{2bc}\right)-\frac{ix^4}{4}\right)$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{3}x^3\arctan(c-(-c+i)\tanh(a+bx))-$$
$$\frac{1}{3}b\left(ic\left(\frac{x^3\log\left(1-ice^{2a+2bx}\right)}{2bc}-\frac{3\left(\frac{\int x\,\text{PolyLog}(2,ice^{2a+2bx})dx}{b}-\frac{x^2\,\text{PolyLog}(2,ice^{2a+2bx})}{2b}\right)}{2bc}\right)-\frac{ix^4}{4}\right)$$

$$\Big\downarrow \text{7163}$$

$$\frac{1}{3}x^3\arctan(c-(-c+i)\tanh(a+bx))-\frac{1}{3}b\left(ic\left(\frac{x^3\log\left(1-ice^{2a+2bx}\right)}{2bc}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,ice^{2a+2bx}\right)}{2b}-\frac{\int \text{PolyLog}\left(3,ice^{2a+2bx}\right)dx}{2b}}{b}-\frac{x^2\,\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{2b}\right)}{2bc}\right)-\frac{ix^4}{4}\right)$$

$\downarrow$ 2720

$$\frac{1}{3}x^3\arctan(c-(-c+i)\tanh(a+bx))-\frac{1}{3}b\left(ic\left(\frac{x^3\log\left(1-ice^{2a+2bx}\right)}{2bc}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,ice^{2a+2bx}\right)}{2b}-\frac{\int e^{-2a-2bx}\,\text{PolyLog}\left(3,ice^{2a+2bx}\right)de^{2a+2bx}}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{2b}\right)}{2bc}\right.\right.$$

$\downarrow$ 7143

$$\frac{1}{3}x^3\arctan(c-(-c+i)\tanh(a+bx))-\frac{1}{3}b\left(ic\left(\frac{x^3\log\left(1-ice^{2a+2bx}\right)}{2bc}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,ice^{2a+2bx}\right)}{2b}-\frac{\text{PolyLog}\left(4,ice^{2a+2bx}\right)}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{2b}\right)}{2bc}\right)-\frac{ix^4}{4}\right)$$

input
```
Int[x^2*ArcTan[c - (I - c)*Tanh[a + b*x]],x]
```

output
```
(x^3*ArcTan[c - (I - c)*Tanh[a + b*x]])/3 - (b*((-1/4*I)*x^4 + I*c*((x^3*L
og[1 - I*c*E^(2*a + 2*b*x)])/(2*b*c) - (3*(-1/2*(x^2*PolyLog[2, I*c*E^(2*a
 + 2*b*x)])/b + ((x*PolyLog[3, I*c*E^(2*a + 2*b*x)])/(2*b) - PolyLog[4, I*
c*E^(2*a + 2*b*x)]/(4*b^2))/b))/(2*b*c))))/3
```

3.89.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5718
```
Int[ArcTan[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Tanh[a + b*x]]/(f*(m
+ 1))), x] - Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a +
 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
d)^2, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.89. $\int x^2 \arctan(c-(i-c)\tanh(a+bx))\,dx$

3.89.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.62 (sec) , antiderivative size = 1409, normalized size of antiderivative = 9.72

method	result	size
risch	Expression too large to display	1409

input
```
int(x^2*arctan(c-(I-c)*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I/b^3*a^3*ln(1-I*exp(b*x+a)*(-I*c)^(1/2))+1/4*I*x*polylog(3,I*c*exp(2
*b*x+2*a))/b^2-1/12*I*b*c/(I-c)*x^4-1/2*I/b^3*a^2*dilog(1-I*exp(b*x+a)*(-I
*c)^(1/2))+1/12*I/b^3*c*a^4/(I-c)-1/4*I*x^2*polylog(2,I*c*exp(2*b*x+2*a))/
b+1/3*I/b^3*ln(1-I*c*exp(2*b*x+2*a))*a^3-1/2*I/b^2*a^2*ln(1-I*exp(b*x+a)*(
-I*c)^(1/2))*x+1/6*I*x^3*ln(-2*exp(2*b*x+2*a)*c-2*I)+1/6*I/b^3*a^3*ln(exp(
2*b*x+2*a)*c+I)-1/6*I*x^3*ln(2*I*exp(2*b*x+2*a)-2*exp(2*b*x+2*a)*c)+1/4*I/
b^3*polylog(2,I*c*exp(2*b*x+2*a))*a^2-1/8*I*polylog(4,I*c*exp(2*b*x+2*a))/
b^3-1/2*I/b^3*a^2*dilog(1+I*exp(b*x+a)*(-I*c)^(1/2))-1/2*I/b^3*a^3*ln(1+I*
exp(b*x+a)*(-I*c)^(1/2))+1/2*I/b^2*ln(1-I*c*exp(2*b*x+2*a))*a^2*x+1/12*Pi*
(csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I))*csgn(I*(2*exp
(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))-csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(
-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp
(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))-csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(2*ex
p(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))^2+csgn(I/(exp(2*b*x+2*a)+1))*csgn(
I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2+csgn(I*(2
*exp(2*b*x+2*a)*c+2*I))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1)
)^2-csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(-2*I*exp(2*b*
x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2-csgn(I*(2*exp(2*b*x+2*a)*
c+2*I)/(exp(2*b*x+2*a)+1))^3+csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*
a)+1))*csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))^2+csgn(I*(2*ex...
```

3.89.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(105) = 210$.

Time = 0.27 (sec) , antiderivative size = 293, normalized size of antiderivative = 2.02

$$\int x^2 \arctan(c-(i-c)\tanh(a+bx))\,dx$$
$$= \frac{i\,b^4x^4 + 2i\,b^3x^3\log\left(-\frac{(ce^{(2\,bx+2\,a)}+i)e^{(-2\,bx-2\,a)}}{c-i}\right) - 6i\,b^2x^2\mathrm{Li}_2\left(\frac{1}{2}\sqrt{4i\,c}e^{(bx+a)}\right) - 6i\,b^2x^2\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{4i\,c}e^{(bx+a)}\right)}{}$$

input
```
integrate(x^2*arctan(c-(I-c)*tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/12*(I*b^4*x^4 + 2*I*b^3*x^3*log(-(c*e^(2*b*x + 2*a) + I)*e^(-2*b*x - 2*a
)/(c - I)) - 6*I*b^2*x^2*dilog(1/2*sqrt(4*I*c)*e^(b*x + a)) - 6*I*b^2*x^2*
dilog(-1/2*sqrt(4*I*c)*e^(b*x + a)) - I*a^4 + 2*I*a^3*log(1/2*(2*c*e^(b*x
+ a) + I*sqrt(4*I*c))/c) + 2*I*a^3*log(1/2*(2*c*e^(b*x + a) - I*sqrt(4*I*c
))/c) + 12*I*b*x*polylog(3, 1/2*sqrt(4*I*c)*e^(b*x + a)) + 12*I*b*x*polylo
g(3, -1/2*sqrt(4*I*c)*e^(b*x + a)) - 2*(I*b^3*x^3 + I*a^3)*log(1/2*sqrt(4*
I*c)*e^(b*x + a) + 1) - 2*(I*b^3*x^3 + I*a^3)*log(-1/2*sqrt(4*I*c)*e^(b*x
+ a) + 1) - 12*I*polylog(4, 1/2*sqrt(4*I*c)*e^(b*x + a)) - 12*I*polylog(4,
 -1/2*sqrt(4*I*c)*e^(b*x + a)))/b^3
```

3.89.6 Sympy [F(-2)]

Exception generated.

$$\int x^2 \arctan(c-(i-c)\tanh(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

input
```
integrate(x**2*atan(c-(I-c)*tanh(b*x+a)),x)
```

output
```
Exception raised: CoercionFailed >> Cannot convert _t0**2*exp(2*a) + 1 of
type <class 'sympy.core.add.Add'> to QQ_I[x,b,_t0,exp(a)]
```

3.89.7 Maxima [A] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.89

$$\int x^2 \arctan(c-(i-c)\tanh(a+bx))\,dx = \frac{1}{3}\,x^3 \arctan\left((c-i)\tanh\left(bx+a\right)+c\right) - \frac{4}{9}\left(\frac{3\,x^4}{4i\,c+4} - \frac{4\,b^3x^3\log\left(-i\,ce^{(2\,bx+2\,a)}+1\right)+6\,b^2x^2\mathrm{Li}_2\left(i\,ce^{(2\,bx+2\,a)}\right)-6\,bx\mathrm{Li}_3\left(i\,ce^{(2\,bx+2\,a)}\right)+3\,\mathrm{Li}_4\left(i\,ce\right.}{-2\,b^4(-i\,c-1)}\right.$$

input
```
integrate(x^2*arctan(c-(I-c)*tanh(b*x+a)),x, algorithm="maxima")
```

output
```
1/3*x^3*arctan((c - I)*tanh(b*x + a) + c) - 4/9*(3*x^4/(4*I*c + 4) - (4*b^
3*x^3*log(-I*c*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(I*c*e^(2*b*x + 2*a))
 - 6*b*x*polylog(3, I*c*e^(2*b*x + 2*a)) + 3*polylog(4, I*c*e^(2*b*x + 2*a
)))/(b^4*(2*I*c + 2)))*b*(c - I)
```

3.89.8 Giac [F]

$$\int x^2 \arctan(c-(i-c)\tanh(a+bx))\,dx=\int x^2 \arctan\left((c-i)\tanh\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(x^2*arctan(c-(I-c)*tanh(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*arctan((c - I)*tanh(b*x + a) + c), x)
```

3.89.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \arctan(c-(i-c)\tanh(a+bx))\,dx=\int x^2\,\mathrm{atan}(c+\tanh(a+b\,x)\,\,(c-\mathrm{i}))\,dx$$

input
```
int(x^2*atan(c + tanh(a + b*x)*(c - 1i)),x)
```

output
```
int(x^2*atan(c + tanh(a + b*x)*(c - 1i)), x)
```

3.90 $\int x\arctan(c-(i-c)\tanh(a+bx))\,dx$

3.90.1 Optimal result

Integrand size = 20, antiderivative size = 116

$$\int x\arctan(c-(i-c)\tanh(a+bx))\,dx=\frac{1}{6}ibx^3+\frac{1}{2}x^2\arctan(c-(i-c)\tanh(a+bx))-\frac{1}{4}ix^2\log\left(1-ice^{2a+2bx}\right)-\frac{ix\,\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}+\frac{i\,\text{PolyLog}\left(3,ice^{2a+2bx}\right)}{8b^2}$$

output
```
1/6*I*b*x^3+1/2*x^2*arctan(c-(I-c)*tanh(b*x+a))-1/4*I*x^2*ln(1-I*c*exp(2*b
*x+2*a))-1/4*I*x*polylog(2,I*c*exp(2*b*x+2*a))/b+1/8*I*polylog(3,I*c*exp(2
*b*x+2*a))/b^2
```

3.90.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.89

$$\int x\arctan(c-(i-c)\tanh(a+bx))\,dx$$
$$=\frac{2b^2x^2\left(2\arctan(c+(-i+c)\tanh(a+bx))-i\log\left(1+\frac{ie^{-2(a+bx)}}{c}\right)\right)+2ibx\,\text{PolyLog}\left(2,-\frac{ie^{-2(a+bx)}}{c}\right)+i\,\text{P}}{8b^2}$$

input
```
Integrate[x*ArcTan[c - (I - c)*Tanh[a + b*x]],x]
```

output
```
(2*b^2*x^2*(2*ArcTan[c + (-I + c)*Tanh[a + b*x]] - I*Log[1 + I/(c*E^(2*(a
+ b*x)))]) + (2*I)*b*x*PolyLog[2, (-I)/(c*E^(2*(a + b*x)))] + I*PolyLog[3,
 (-I)/(c*E^(2*(a + b*x)))])/(8*b^2)
```

3.90.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5718, 2615, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan(c-(-c+i)\tanh(a+bx))\,dx$$

$$\Big\downarrow \text{5718}$$

$$\frac{1}{2}x^2\arctan(c-(-c+i)\tanh(a+bx))-\frac{1}{2}b\int \frac{x^2}{e^{2a+2bx}c+i}dx$$

$$\Big\downarrow \text{2615}$$

$$\frac{1}{2}x^2\arctan(c-(-c+i)\tanh(a+bx))-\frac{1}{2}b\left(ic\int \frac{e^{2a+2bx}x^2}{e^{2a+2bx}c+i}dx-\frac{ix^3}{3}\right)$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{2}x^2\arctan(c-(-c+i)\tanh(a+bx))-\frac{1}{2}b\left(ic\left(\frac{x^2\log\left(1-ice^{2a+2bx}\right)}{2bc}-\frac{\int x\log\left(1-ice^{2a+2bx}\right)dx}{bc}\right)-\frac{ix^3}{3}\right)$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{2}x^2\arctan(c-(-c+i)\tanh(a+bx))-\frac{1}{2}b\left(ic\left(\frac{x^2\log\left(1-ice^{2a+2bx}\right)}{2bc}-\frac{\frac{\int \text{PolyLog}(2,ice^{2a+2bx})dx}{2b}-\frac{x\,\text{PolyLog}(2,ice^{2a+2bx})}{2b}}{bc}\right)-\frac{ix^3}{3}\right)$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{2}x^2\arctan(c-(-c+i)\tanh(a+bx))-\frac{1}{2}b\left(ic\left(\frac{x^2\log\left(1-ice^{2a+2bx}\right)}{2bc}-\frac{\frac{\int e^{-2a-2bx}\operatorname{PolyLog}(2,ice^{2a+2bx})de^{2a+2bx}}{4b^2}-\frac{x\operatorname{PolyLog}(2,ice^{2a+2bx})}{2b}}{bc}\right)-\frac{ix^3}{3}\right)$$

$$\Big\downarrow \ 7143$$

$$\frac{1}{2}x^2\arctan(c-(-c+i)\tanh(a+bx))-\frac{1}{2}b\left(ic\left(\frac{x^2\log\left(1-ice^{2a+2bx}\right)}{2bc}-\frac{\frac{\operatorname{PolyLog}(3,ice^{2a+2bx})}{4b^2}-\frac{x\operatorname{PolyLog}(2,ice^{2a+2bx})}{2b}}{bc}\right)-\frac{ix^3}{3}\right)$$

input
```
Int[x*ArcTan[c - (I - c)*Tanh[a + b*x]],x]
```

output
```
(x^2*ArcTan[c - (I - c)*Tanh[a + b*x]])/2 - (b*((-1/3*I)*x^3 + I*c*((x^2*L
og[1 - I*c*E^(2*a + 2*b*x)])/(2*b*c) - (-1/2*(x*PolyLog[2, I*c*E^(2*a + 2*
b*x)])/b + PolyLog[3, I*c*E^(2*a + 2*b*x)]/(4*b^2))/(b*c))))/2
```

3.90.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5718
```
Int[ArcTan[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Tanh[a + b*x]]/(f*(m
+ 1))), x] - Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)/(c - d + c*E^(2*a +
 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
d)^2, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.90.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.97 (sec) , antiderivative size = 1373, normalized size of antiderivative = 11.84

method	result	size
risch	Expression too large to display	1373

input
```
int(x*arctan(c-(I-c)*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/2*I/b*a*ln(1+I*exp(b*x+a)*(-I*c)^(1/2))*x+1/4*I*x^2*ln(-2*exp(2*b*x+2*a)
*c-2*I)+1/2*I/b^2*ln(1-I*exp(b*x+a)*(-I*c)^(1/2))*a^2-1/4*I/b^2*a^2*ln(exp
(2*b*x+2*a)*c+I)+1/2*I/b*a*ln(1-I*exp(b*x+a)*(-I*c)^(1/2))*x+1/2*I/b^2*a*d
ilog(1-I*exp(b*x+a)*(-I*c)^(1/2))-1/6*I*b*c/(I-c)*x^3+1/2*I/b^2*ln(1+I*exp
(b*x+a)*(-I*c)^(1/2))*a^2-1/6*I/b^2*c/(I-c)*a^3-1/4*I/b^2*ln(1-I*c*exp(2*b
*x+2*a))*a^2-1/4*I/b^2*polylog(2,I*c*exp(2*b*x+2*a))*a-1/4*I*x^2*ln(2*I*ex
p(2*b*x+2*a)-2*exp(2*b*x+2*a)*c)-1/4*I*x*polylog(2,I*c*exp(2*b*x+2*a))/b+1
/8*I*polylog(3,I*c*exp(2*b*x+2*a))/b^2+1/2*I/b^2*a*dilog(1+I*exp(b*x+a)*(-
I*c)^(1/2))+1/8*Pi*(csgn(I/(exp(2*b*x+2*a)+1))*csgn(I*(2*exp(2*b*x+2*a)*c+
2*I))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))-csgn(I/(exp(2*b*
x+2*a)+1))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(-2*I*e
xp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))-csgn(I/(exp(2*b*x+2*
a)+1))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))^2+csgn(I/(exp(2
*b*x+2*a)+1))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2
*a)+1))^2+csgn(I*(2*exp(2*b*x+2*a)*c+2*I))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)
/(exp(2*b*x+2*a)+1))^2-csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*cs
gn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)+1))^2-csgn(I
*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))^3+csgn(I*(2*exp(2*b*x+2*a)*c
+2*I)/(exp(2*b*x+2*a)+1))*csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1)
)^2+csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)+1))*csgn((2*exp(2*b...
```

3.90.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(83) = 166$.

Time = 0.28 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.13

$$\int x \arctan(c - (i - c)\tanh(a + bx))\, dx$$
$$= \frac{2i\, b^3x^3 + 3i\, b^2x^2 \log\left(-\frac{(ce^{(2\,bx+2\,a)}+i)e^{(-2\,bx-2\,a)}}{c-i}\right) + 2i\, a^3 - 6i\, bx\mathrm{Li}_2\left(\frac{1}{2}\sqrt{4i\, c}e^{(bx+a)}\right) - 6i\, bx\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{4i\, c}e^{(bx}\right.}{}$$

input
```
integrate(x*arctan(c-(I-c)*tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/12*(2*I*b^3*x^3 + 3*I*b^2*x^2*log(-(c*e^(2*b*x + 2*a) + I)*e^(-2*b*x - 2
*a)/(c - I)) + 2*I*a^3 - 6*I*b*x*dilog(1/2*sqrt(4*I*c)*e^(b*x + a)) - 6*I*
b*x*dilog(-1/2*sqrt(4*I*c)*e^(b*x + a)) - 3*I*a^2*log(1/2*(2*c*e^(b*x + a)
 + I*sqrt(4*I*c))/c) - 3*I*a^2*log(1/2*(2*c*e^(b*x + a) - I*sqrt(4*I*c))/c
) - 3*(I*b^2*x^2 - I*a^2)*log(1/2*sqrt(4*I*c)*e^(b*x + a) + 1) - 3*(I*b^2*
x^2 - I*a^2)*log(-1/2*sqrt(4*I*c)*e^(b*x + a) + 1) + 6*I*polylog(3, 1/2*sq
rt(4*I*c)*e^(b*x + a)) + 6*I*polylog(3, -1/2*sqrt(4*I*c)*e^(b*x + a)))/b^2
```

3.90.6 Sympy [F(-2)]

Exception generated.

$$\int x \arctan(c-(i-c)\tanh(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

input
```
integrate(x*atan(c-(I-c)*tanh(b*x+a)),x)
```

output
```
Exception raised: CoercionFailed >> Cannot convert _t0**2*exp(2*a) + 1 of
type <class 'sympy.core.add.Add'> to QQ_I[x,b,_t0,exp(a)]
```

3.90.7 Maxima [A] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.92

$$\int x \arctan(c-(i-c)\tanh(a+bx))\,dx = -\left(\frac{2\,x^3}{3i\,c+3} - \frac{2\,b^2x^2\log\left(-i\,ce^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_2\left(i\,ce^{(2\,bx+2\,a)}\right)-\mathrm{Li}_3\left(i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^3(-i\,c-1)}\right)b(c-i) + \frac{1}{2}\,x^2\arctan\left((c-i)\tanh\left(bx+a\right)+c\right)$$

input
```
integrate(x*arctan(c-(I-c)*tanh(b*x+a)),x, algorithm="maxima")
```

output
```
-(2*x^3/(3*I*c + 3) - (2*b^2*x^2*log(-I*c*e^(2*b*x + 2*a) + 1) + 2*b*x*dil
og(I*c*e^(2*b*x + 2*a)) - polylog(3, I*c*e^(2*b*x + 2*a)))/(b^3*(2*I*c + 2
)))*b*(c - I) + 1/2*x^2*arctan((c - I)*tanh(b*x + a) + c)
```

3.90.8 Giac [F]

$$\int x\arctan(c-(i-c)\tanh(a+bx))\,dx=\int x\arctan\left((c-i)\tanh\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(x*arctan(c-(I-c)*tanh(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*arctan((c - I)*tanh(b*x + a) + c), x)
```

3.90.9 Mupad [F(-1)]

Timed out.

$$\int x\arctan(c-(i-c)\tanh(a+bx))\,dx=\int x\,\mathrm{atan}(c+\tanh(a+b\,x)\,\left(c-\mathrm{i}\right))\,dx$$

input
```
int(x*atan(c + tanh(a + b*x)*(c - 1i)),x)
```

output
```
int(x*atan(c + tanh(a + b*x)*(c - 1i)), x)
```

3.91 $\int \arctan(c-(i-c)\tanh(a+bx))\,dx$

3.91.1 Optimal result

Integrand size = 18, antiderivative size = 82

$$\int \arctan(c-(i-c)\tanh(a+bx))\,dx=\frac{1}{2}ibx^2+x\arctan(c-(i-c)\tanh(a+bx))-\frac{1}{2}ix\log\left(1-ice^{2a+2bx}\right)-\frac{i\operatorname{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}$$

output
```
1/2*I*b*x^2+x*arctan(c-(I-c)*tanh(b*x+a))-1/2*I*x*ln(1-I*c*exp(2*b*x+2*a))
-1/4*I*polylog(2,I*c*exp(2*b*x+2*a))/b
```

3.91.2 Mathematica [A] (verified)

Time = 1.68 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.87

$$\int \arctan(c-(i-c)\tanh(a+bx))\,dx$$
$$=x\arctan(c+(-i+c)\tanh(a+bx))-\frac{i\left(2bx\log\left(1+\frac{ie^{-2(a+bx)}}{c}\right)-\operatorname{PolyLog}\left(2,-\frac{ie^{-2(a+bx)}}{c}\right)\right)}{4b}$$

input
```
Integrate[ArcTan[c - (I - c)*Tanh[a + b*x]],x]
```

output
```
x*ArcTan[c + (-I + c)*Tanh[a + b*x]] - ((I/4)*(2*b*x*Log[1 + I/(c*E^(2*(a
+ b*x)))] - PolyLog[2, (-I)/(c*E^(2*(a + b*x)))]))/b
```

3.91.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5710, 2615, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(c-(-c+i)\tanh(a+bx))\,dx$$

$$\Big\downarrow \; 5710$$

$$x\arctan(c-(-c+i)\tanh(a+bx))-b\int \frac{x}{e^{2a+2bx}c+i}dx$$

$$\Big\downarrow \; 2615$$

$$x\arctan(c-(-c+i)\tanh(a+bx))-b\left(ic\int \frac{e^{2a+2bx}x}{e^{2a+2bx}c+i}dx-\frac{ix^2}{2}\right)$$

$$\Big\downarrow \; 2620$$

$$x\arctan(c-(-c+i)\tanh(a+bx))-b\left(ic\left(\frac{x\log\left(1-ice^{2a+2bx}\right)}{2bc}-\frac{\int \log\left(1-ice^{2a+2bx}\right)dx}{2bc}\right)-\frac{ix^2}{2}\right)$$

$$\Big\downarrow \; 2715$$

$$x\arctan(c-(-c+i)\tanh(a+bx))-b\left(ic\left(\frac{x\log\left(1-ice^{2a+2bx}\right)}{2bc}-\frac{\int e^{-2a-2bx}\log\left(1-ice^{2a+2bx}\right)de^{2a+2bx}}{4b^2c}\right)-\frac{ix^2}{2}\right)$$

$$\Big\downarrow \; 2838$$

$$x\arctan(c-(-c+i)\tanh(a+bx))-b\left(ic\left(\frac{\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b^2c}+\frac{x\log\left(1-ice^{2a+2bx}\right)}{2bc}\right)-\frac{ix^2}{2}\right)$$

input
```
Int[ArcTan[c - (I - c)*Tanh[a + b*x]],x]
```

output
```
x*ArcTan[c - (I - c)*Tanh[a + b*x]] - b*((-1/2*I)*x^2 + I*c*((x*Log[1 - I*
c*E^(2*a + 2*b*x)])/(2*b*c) + PolyLog[2, I*c*E^(2*a + 2*b*x)]/(4*b^2*c)))
```

3.91.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5710
```
Int[ArcTan[(c_.) + (d_.)*Tanh[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*Arc
Tan[c + d*Tanh[a + b*x]], x] - Simp[b   Int[x/(c - d + c*E^(2*a + 2*b*x)),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, -1]
```

3.91.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 515 vs. $2(68) = 136$.

Time = 1.02 (sec) , antiderivative size = 516, normalized size of antiderivative = 6.29

method	result
derivativedivides	$\frac{\frac{\arctan(c+\tanh(bx+a)(c-i))\ln(-i+\tanh(bx+a)(c-i)+c)}{2i-2c}+\frac{2i\arctan(c+\tanh(bx+a)(c-i))\ln(-i+\tanh(bx+a)(c-i)+c)c}{2i-2c}-\frac{\arctan(c+t}{}}{}$
default	$\frac{\frac{\arctan(c+\tanh(bx+a)(c-i))\ln(-i+\tanh(bx+a)(c-i)+c)}{2i-2c}+\frac{2i\arctan(c+\tanh(bx+a)(c-i))\ln(-i+\tanh(bx+a)(c-i)+c)c}{2i-2c}-\frac{\arctan(c+t}{}}{}$
risch	Expression too large to display

input
```
int(arctan(c-(I-c)*tanh(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b/(c-I)*(arctan(c+tanh(b*x+a)*(c-I))/(2*I-2*c)*ln(-I+tanh(b*x+a)*(c-I)+c
)+2*I*arctan(c+tanh(b*x+a)*(c-I))/(2*I-2*c)*ln(-I+tanh(b*x+a)*(c-I)+c)*c-a
rctan(c+tanh(b*x+a)*(c-I))/(2*I-2*c)*ln(-I+tanh(b*x+a)*(c-I)+c)*c^2-arctan
(c+tanh(b*x+a)*(c-I))/(2*I-2*c)*ln(tanh(b*x+a)*(c-I)-c+I)-2*I*arctan(c+tan
h(b*x+a)*(c-I))/(2*I-2*c)*ln(tanh(b*x+a)*(c-I)-c+I)*c+arctan(c+tanh(b*x+a)
*(c-I))/(2*I-2*c)*ln(tanh(b*x+a)*(c-I)-c+I)*c^2+(I-c)^2*(1/2/(I-c)*(-1/4*I
*ln(-I+tanh(b*x+a)*(c-I)+c)^2+1/2*I*(dilog(-1/2*I*(tanh(b*x+a)*(c-I)+c+I))
+ln(-I+tanh(b*x+a)*(c-I)+c)*ln(-1/2*I*(tanh(b*x+a)*(c-I)+c+I))))-1/2/(I-c)
*(-1/2*I*(dilog((-I+tanh(b*x+a)*(c-I)+c)/(-2*I+2*c))+ln(tanh(b*x+a)*(c-I)-
c+I)*ln((-I+tanh(b*x+a)*(c-I)+c)/(-2*I+2*c)))+1/2*I*(dilog(1/2*(tanh(b*x+a
)*(c-I)+c+I)/c)+ln(tanh(b*x+a)*(c-I)-c+I)*ln(1/2*(tanh(b*x+a)*(c-I)+c+I)/c
)))))
```

3.91.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(58) = 116.

Time = 0.27 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.28

$$\int \arctan(c-(i-c)\tanh(a+bx))\,dx = \frac{i\,b^2x^2+i\,bx\log\left(-\frac{(ce^{(2\,bx+2\,a)}+i)e^{(-2\,bx-2\,a)}}{c-i}\right)-i\,a^2+(-i\,bx-i\,a)\log\left(\frac{1}{2}\sqrt{4i\,c}e^{(bx+a)}+1\right)+(-i\,bx-i\,a)\,\mathrm{lo}}{}$$

input
```
integrate(arctan(c-(I-c)*tanh(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*(I*b^2*x^2 + I*b*x*log(-(c*e^(2*b*x + 2*a) + I)*e^(-2*b*x - 2*a)/(c -
I)) - I*a^2 + (-I*b*x - I*a)*log(1/2*sqrt(4*I*c)*e^(b*x + a) + 1) + (-I*b*
x - I*a)*log(-1/2*sqrt(4*I*c)*e^(b*x + a) + 1) + I*a*log(1/2*(2*c*e^(b*x +
 a) + I*sqrt(4*I*c))/c) + I*a*log(1/2*(2*c*e^(b*x + a) - I*sqrt(4*I*c))/c)
 - I*dilog(1/2*sqrt(4*I*c)*e^(b*x + a)) - I*dilog(-1/2*sqrt(4*I*c)*e^(b*x
+ a)))/b
```

3.91.6 Sympy [F(-2)]

Exception generated.

$$\int \arctan(c-(i-c)\tanh(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

input
```
integrate(atan(c-(I-c)*tanh(b*x+a)),x)
```

output
```
Exception raised: CoercionFailed >> Cannot convert _t0**2*exp(2*a) + 1 of
type <class 'sympy.core.add.Add'> to QQ_I[b,_t0,exp(a)]
```

3.91.7 Maxima [A] (verification not implemented)

Time = 1.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

$$\begin{aligned}
&\int \arctan(c-(i-c)\tanh(a+bx))\,dx \\
&= -2\,b(c-i)\left(\frac{2\,x^2}{2i\,c+2}-\frac{2\,bx\log\left(-i\,ce^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left(i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^2(-i\,c-1)}\right) \\
&\quad + x\arctan\left((c-i)\tanh\left(bx+a\right)+c\right)
\end{aligned}$$

input
```
integrate(arctan(c-(I-c)*tanh(b*x+a)),x, algorithm="maxima")
```

output
```
-2*b*(c - I)*(2*x^2/(2*I*c + 2) - (2*b*x*log(-I*c*e^(2*b*x + 2*a) + 1) + d
ilog(I*c*e^(2*b*x + 2*a)))/(b^2*(2*I*c + 2))) + x*arctan((c - I)*tanh(b*x
+ a) + c)
```

3.91.8 Giac [F]

$$\int \arctan(c-(i-c)\tanh(a+bx))\,dx=\int \arctan\left((c-i)\tanh\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(arctan(c-(I-c)*tanh(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arctan((c - I)*tanh(b*x + a) + c), x)
```

3.91.9 Mupad [F(-1)]

Timed out.

$$\int \arctan(c-(i-c)\tanh(a+bx))\,dx=\int \mathrm{atan}(c+\tanh(a+b\,x)\,\left(c-\mathrm{i}\right))\,dx$$

input
```
int(atan(c + tanh(a + b*x)*(c - 1i)),x)
```

output
```
int(atan(c + tanh(a + b*x)*(c - 1i)), x)
```

3.92 $\int \frac{\arctan(c-(i-c)\tanh(a+bx))}{x}\,dx$

3.92.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\arctan(c-(i-c)\tanh(a+bx))}{x}\,dx = \text{Int}\left(\frac{\arctan(c-(i-c)\tanh(a+bx))}{x},x\right)$$

output
```
CannotIntegrate(arctan(c-(I-c)*tanh(b*x+a))/x,x)
```

3.92.2 Mathematica [N/A]

Not integrable

Time = 3.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\arctan(c-(i-c)\tanh(a+bx))}{x}\,dx = \int \frac{\arctan(c-(i-c)\tanh(a+bx))}{x}\,dx$$

input
```
Integrate[ArcTan[c - (I - c)*Tanh[a + b*x]]/x,x]
```

output
```
Integrate[ArcTan[c - (I - c)*Tanh[a + b*x]]/x, x]
```

3.92.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(c-(-c+i)\tanh(a+bx))}{x} \, dx$$
$$\Big\downarrow \; 7299$$
$$\int \frac{\arctan(c-(-c+i)\tanh(a+bx))}{x}dx$$

input
```
Int[ArcTan[c - (I - c)*Tanh[a + b*x]]/x,x]
```

output
```
$Aborted
```

3.92.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.92.4 Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\arctan\left(c-(i-c)\tanh\left(bx+a\right)\right)}{x}dx$$

input
```
int(arctan(c-(I-c)*tanh(b*x+a))/x,x)
```

output
```
int(arctan(c-(I-c)*tanh(b*x+a))/x,x)
```

3.92.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int \frac{\arctan(c-(i-c)\tanh(a+bx))}{x}\,dx = \int \frac{\arctan\left((c-i)\tanh\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arctan(c-(I-c)*tanh(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(1/2*I*log(-(c*e^(2*b*x + 2*a) + I)*e^(-2*b*x - 2*a)/(c - I))/x, x
)
```

3.92.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan(c-(i-c)\tanh(a+bx))}{x}\,dx = \text{Timed out}$$

input
```
integrate(atan(c-(I-c)*tanh(b*x+a))/x,x)
```

output
```
Timed out
```

3.92.7 Maxima [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 77, normalized size of antiderivative = 3.50

$$\int \frac{\arctan(c-(i-c)\tanh(a+bx))}{x}\,dx = \int \frac{\arctan\left((c-i)\tanh\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arctan(c-(I-c)*tanh(b*x+a))/x,x, algorithm="maxima")
```

output
```
-I*b*x - 1/2*pi*log(x) - 1/4*(2*pi + 4*I*a - 2*arctan(c) + I*log(c^2 + 1))
*log(x) + 1/2*integrate(arctan(c*e^(2*b*x + 2*a))/x, x) + 1/4*I*integrate(
log(c^2*e^(4*b*x + 4*a) + 1)/x, x)
```

3.92.8 Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{\arctan(c-(i-c)\tanh(a+bx))}{x}\,dx = \int \frac{\arctan\left((c-i)\tanh\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arctan(c-(I-c)*tanh(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(arctan((c - I)*tanh(b*x + a) + c)/x, x)
```

3.92.9 Mupad [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\arctan(c-(i-c)\tanh(a+bx))}{x}\,dx = \int \frac{\mathrm{atan}(c+\tanh(a+b\,x)\,\,(c-\mathrm{i}))}{x}\,dx$$

input
```
int(atan(c + tanh(a + b*x)*(c - 1i))/x,x)
```

output
```
int(atan(c + tanh(a + b*x)*(c - 1i))/x, x)
```

3.93 $\int (e+fx)^3 \arctan(\coth(a+bx))\,dx$

3.93.1 Optimal result

Integrand size = 15, antiderivative size = 299

$$\begin{aligned}\int (e+fx)^3 \arctan(\coth(a+bx))\,dx =& \frac{(e+fx)^4 \arctan\left(e^{2a+2bx}\right)}{4f} \\ &+ \frac{(e+fx)^4 \arctan(\coth(a+bx))}{4f} \\ &- \frac{i(e+fx)^3 \operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} \\ &+ \frac{i(e+fx)^3 \operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} \\ &+ \frac{3if(e+fx)^2 \operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{8b^2} \\ &- \frac{3if(e+fx)^2 \operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)}{8b^2} \\ &- \frac{3if^2(e+fx) \operatorname{PolyLog}\left(4,-ie^{2a+2bx}\right)}{8b^3} \\ &+ \frac{3if^2(e+fx) \operatorname{PolyLog}\left(4,ie^{2a+2bx}\right)}{8b^3} \\ &+ \frac{3if^3 \operatorname{PolyLog}\left(5,-ie^{2a+2bx}\right)}{16b^4} \\ &- \frac{3if^3 \operatorname{PolyLog}\left(5,ie^{2a+2bx}\right)}{16b^4}\end{aligned}$$

output
```
1/4*(f*x+e)^4*arctan(exp(2*b*x+2*a))/f+1/4*(f*x+e)^4*arctan(coth(b*x+a))/f
-1/4*I*(f*x+e)^3*polylog(2,-I*exp(2*b*x+2*a))/b+1/4*I*(f*x+e)^3*polylog(2,
I*exp(2*b*x+2*a))/b+3/8*I*f*(f*x+e)^2*polylog(3,-I*exp(2*b*x+2*a))/b^2-3/8
*I*f*(f*x+e)^2*polylog(3,I*exp(2*b*x+2*a))/b^2-3/8*I*f^2*(f*x+e)*polylog(4
,-I*exp(2*b*x+2*a))/b^3+3/8*I*f^2*(f*x+e)*polylog(4,I*exp(2*b*x+2*a))/b^3+
3/16*I*f^3*polylog(5,-I*exp(2*b*x+2*a))/b^4-3/16*I*f^3*polylog(5,I*exp(2*b
*x+2*a))/b^4
```

3.93.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 600 vs. 2(299) = 598.

Time = 0.30 (sec) , antiderivative size = 600, normalized size of antiderivative = 2.01

$$\int (e+fx)^3 \arctan(\coth(a+bx))\,dx = \frac{1}{4}x\left(4e^3+6e^2fx+4ef^2x^2+f^3x^3\right)\arctan(\coth(a+bx)) + \frac{i\left(8b^4e^3x\log\left(1-ie^{2(a+bx)}\right)+12b^4e^2fx^2\log\left(1-ie^{2(a+bx)}\right)+8b^4ef^2x^3\log\left(1-ie^{2(a+bx)}\right)+2b^4f^3x^4\log\right.}{}$$

input
```
Integrate[(e + f*x)^3*ArcTan[Coth[a + b*x]],x]
```

output
```
(x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3)*ArcTan[Coth[a + b*x]])/4 +
((I/16)*(8*b^4*e^3*x*Log[1 - I*E^(2*(a + b*x))] + 12*b^4*e^2*f*x^2*Log[1 -
 I*E^(2*(a + b*x))] + 8*b^4*e*f^2*x^3*Log[1 - I*E^(2*(a + b*x))] + 2*b^4*f
^3*x^4*Log[1 - I*E^(2*(a + b*x))] - 8*b^4*e^3*x*Log[1 + I*E^(2*(a + b*x))]
 - 12*b^4*e^2*f*x^2*Log[1 + I*E^(2*(a + b*x))] - 8*b^4*e*f^2*x^3*Log[1 + I
*E^(2*(a + b*x))] - 2*b^4*f^3*x^4*Log[1 + I*E^(2*(a + b*x))] - 4*b^3*(e +
f*x)^3*PolyLog[2, (-I)*E^(2*(a + b*x))] + 4*b^3*(e + f*x)^3*PolyLog[2, I*E
^(2*(a + b*x))] + 6*b^2*e^2*f*PolyLog[3, (-I)*E^(2*(a + b*x))] + 12*b^2*e*
f^2*x*PolyLog[3, (-I)*E^(2*(a + b*x))] + 6*b^2*f^3*x^2*PolyLog[3, (-I)*E^(
2*(a + b*x))] - 6*b^2*e^2*f*PolyLog[3, I*E^(2*(a + b*x))] - 12*b^2*e*f^2*x
*PolyLog[3, I*E^(2*(a + b*x))] - 6*b^2*f^3*x^2*PolyLog[3, I*E^(2*(a + b*x)
)] - 6*b*e*f^2*PolyLog[4, (-I)*E^(2*(a + b*x))] - 6*b*f^3*x*PolyLog[4, (-I
)*E^(2*(a + b*x))] + 6*b*e*f^2*PolyLog[4, I*E^(2*(a + b*x))] + 6*b*f^3*x*P
olyLog[4, I*E^(2*(a + b*x))] + 3*f^3*PolyLog[5, (-I)*E^(2*(a + b*x))] - 3*
f^3*PolyLog[5, I*E^(2*(a + b*x))]))/b^4
```

3.93.3 Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {5708, 3042, 4668, 3011, 7163, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^3 \arctan(\coth(a+bx))\,dx$$

$$\Big\downarrow \; 5708$$

$$\frac{b\int (e+fx)^4\text{sech}(2a+2bx)dx}{4f}+\frac{(e+fx)^4\arctan(\coth(a+bx))}{4f}$$

$$\Big\downarrow \; 3042$$

$$\frac{(e+fx)^4\arctan(\coth(a+bx))}{4f}+\frac{b\int (e+fx)^4\csc\left(2ia+2ibx+\frac{\pi}{2}\right)dx}{4f}$$

$$\Big\downarrow \; 4668$$

$$\frac{(e+fx)^4\arctan(\coth(a+bx))}{4f}+\frac{b\left(-\frac{2if\int (e+fx)^3\log\left(1-ie^{2a+2bx}\right)dx}{b}+\frac{2if\int (e+fx)^3\log\left(1+ie^{2a+2bx}\right)dx}{b}+\frac{(e+fx)^4\arctan\left(e^{2a+2bx}\right)}{b}\right)}{4f}$$

$$\Big\downarrow \; 3011$$

$$\frac{(e+fx)^4\arctan(\coth(a+bx))}{4f}+\frac{b\left(\frac{2if\left(\frac{3f\int (e+fx)^2\text{PolyLog}\left(2,-ie^{2a+2bx}\right)dx}{2b}-\frac{(e+fx)^3\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{2b}\right)}{b}-\frac{2if\left(\frac{3f\int (e+fx)^2\text{PolyLog}\left(2,ie^{2a+2bx}\right)dx}{2b}-\frac{(e+fx)^3\text{PolyLog}\left(2,ie^{2\cdots}\right)}{2b}\cdots\right.}{b}\cdots\right.}{4f}$$

$$\Big\downarrow \; 7163$$

$$\frac{(e+fx)^4\arctan(\coth(a+bx))}{4f}+\frac{b\left(\frac{2if\left(\frac{3f\left(\frac{(e+fx)^2\text{PolyLog}\left(3,-ie^{2a+2bx}\right)}{2b}-\frac{f\int (e+fx)\text{PolyLog}\left(3,-ie^{2a+2bx}\right)dx}{b}\right)}{2b}-\frac{(e+fx)^3\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{2b}\right)}{b}-\frac{2if\left(\frac{3f\left(\frac{(e+fx)^2\text{PolyLog}(\cdots}{2b}\cdots\right.}{\cdots}\cdots\right.}{\cdots}\cdots\right.}{4f}$$

$$\downarrow \; 7163$$

$$\frac{(e+fx)^4 \arctan(\coth(a+bx))}{4f} + b\left(\frac{2if\left(\frac{3f\left(\frac{(e+fx)^2 \operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{2b}-\frac{f\left(\frac{(e+fx)\operatorname{PolyLog}\left(4,-ie^{2a+2bx}\right)}{2b}-\frac{f\int \operatorname{PolyLog}\left(4,-ie^{2a+2bx}\right)dx}{2b}\right)}{b}\right)}{2b}-\frac{(e+fx)^3 \operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{2b}\right)}{b}\right.$$

$$\downarrow \; 2720$$

$$\frac{(e+fx)^4 \arctan(\coth(a+bx))}{4f} + b\left(\frac{2if\left(\frac{3f\left(\frac{(e+fx)^2 \operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{2b}-\frac{f\left(\frac{(e+fx)\operatorname{PolyLog}\left(4,-ie^{2a+2bx}\right)}{2b}-\frac{f\int e^{-2a-2bx}\operatorname{PolyLog}\left(4,-ie^{2a+2bx}\right)de^{2a+2bx}}{4b^2}\right)}{b}\right)}{2b}-\frac{(e+fx)^3 \operatorname{PolyLog}\left(\right.}{2b}\right.}{b}\right.$$

$$\downarrow \; 7143$$

3.93. $\int (e+fx)^3 \arctan(\coth(a+bx))\,dx$

$$\frac{(e+fx)^4\arctan(\coth(a+bx))}{4f}+$$

$$b\left(\frac{(e+fx)^4\arctan\left(e^{2a+2bx}\right)}{b}+\frac{2if\left(\frac{3f\left(\frac{(e+fx)^2\operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{2b}-\frac{f\left(\frac{(e+fx)\operatorname{PolyLog}\left(4,-ie^{2a+2bx}\right)}{2b}-\frac{f\operatorname{PolyLog}\left(5,-ie^{2a+2bx}\right)}{4b^2}\right)}{b}\right)}{2b}-\frac{(e+fx)^3}{}\right.}{b}\right.$$

input
```
Int[(e + f*x)^3*ArcTan[Coth[a + b*x]],x]
```

output
```
((e + f*x)^4*ArcTan[Coth[a + b*x]])/(4*f) + (b*(((e + f*x)^4*ArcTan[E^(2*a
 + 2*b*x)])/b + ((2*I)*f*(-1/2*((e + f*x)^3*PolyLog[2, (-I)*E^(2*a + 2*b*x
)])/b + (3*f*(((e + f*x)^2*PolyLog[3, (-I)*E^(2*a + 2*b*x)])/(2*b) - (f*((
(e + f*x)*PolyLog[4, (-I)*E^(2*a + 2*b*x)])/(2*b) - (f*PolyLog[5, (-I)*E^(
2*a + 2*b*x)])/(4*b^2)))/b))/(2*b)))/b - ((2*I)*f*(-1/2*((e + f*x)^3*PolyL
og[2, I*E^(2*a + 2*b*x)])/b + (3*f*(((e + f*x)^2*PolyLog[3, I*E^(2*a + 2*b
*x)])/(2*b) - (f*(((e + f*x)*PolyLog[4, I*E^(2*a + 2*b*x)])/(2*b) - (f*Pol
yLog[5, I*E^(2*a + 2*b*x)])/(4*b^2)))/b))/(2*b)))/b))/(4*f)
```

3.93.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5708
```
Int[ArcTan[Coth[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcTan[Coth[a + b*x]]/(f*(m + 1))), x] + Simp[b/
(f*(m + 1))   Int[(e + f*x)^(m + 1)*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a,
b, e, f}, x] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.93. $\int (e+fx)^3 \arctan(\coth(a+bx))\,dx$

3.93.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 15.86 (sec) , antiderivative size = 3570, normalized size of antiderivative = 11.94

method	result	size
risch	Expression too large to display	3570

input
```
int((f*x+e)^3*arctan(coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/8*I/f*e^4*ln(exp(2*b*x+2*a)+I)+1/8*I*f^3*ln(1-I*exp(2*b*x+2*a))*x^4+1/2*
I/b*e^3*dilog(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))+1/2*I/b*e^3*dilog(((-I)^
(1/2)+exp(b*x+a))/(-I)^(1/2))+1/2*I*e^3*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1
/2))*x+1/2*I*e^3*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))*x-1/8*I*f^3*ln(1+I
*exp(2*b*x+2*a))*x^4-1/8*I/f*e^4*ln(-exp(2*b*x+2*a)+I)-1/2*I*e^3*ln(1+exp(
b*x+a)*(-1)^(3/4))*x-1/2*I*e^3*ln(1-exp(b*x+a)*(-1)^(3/4))*x-1/2*I/b*e^3*d
ilog(1+exp(b*x+a)*(-1)^(3/4))-1/2*I/b*e^3*dilog(1-exp(b*x+a)*(-1)^(3/4))+1
/8*I*f^3*ln(exp(2*b*x+2*a)-I)*x^4+1/2*I*ln(exp(2*b*x+2*a)-I)*x*e^3+3/16*I*
f^3*polylog(5,-I*exp(2*b*x+2*a))/b^4+1/8*I/f*ln(exp(2*b*x+2*a)-I)*e^4-3/16
*I*f^3*polylog(5,I*exp(2*b*x+2*a))/b^4+1/16*Pi*(csgn(I*(exp(2*b*x+2*a)-I))
*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))-
csgn(I*(exp(2*b*x+2*a)-I))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^2
-csgn(I*(exp(2*b*x+2*a)+I))*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(exp(2*b*x+2
*a)+I)/(exp(2*b*x+2*a)-1))+csgn(I*(exp(2*b*x+2*a)+I))*csgn(I*(exp(2*b*x+2*
a)+I)/(exp(2*b*x+2*a)-1))^2+csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))*
csgn((1+I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))-csgn((1+I)*(exp(2*b*x+2*
a)-I)/(exp(2*b*x+2*a)-1))^2-csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))*
csgn((1-I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))-csgn((1-I)*(exp(2*b*x+2*
a)+I)/(exp(2*b*x+2*a)-1))^2-csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(exp(2*b*x+2
*a)-I)/(exp(2*b*x+2*a)-1))^2+csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(exp(2*b...
```

3.93.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1460 vs. 2(236) = 472.

Time = 0.36 (sec) , antiderivative size = 1460, normalized size of antiderivative = 4.88

$$\int (e+fx)^3 \arctan(\coth(a+bx))\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^3*arctan(coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/8*(-24*I*f^3*polylog(5, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) -
 24*I*f^3*polylog(5, -1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 24*
I*f^3*polylog(5, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 24*I*f^
3*polylog(5, -1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 2*(b^4*f^3
*x^4 + 4*b^4*e*f^2*x^3 + 6*b^4*e^2*f*x^2 + 4*b^4*e^3*x)*arctan(cosh(b*x +
a)/sinh(b*x + a)) - 4*(-I*b^3*f^3*x^3 - 3*I*b^3*e*f^2*x^2 - 3*I*b^3*e^2*f*
x - I*b^3*e^3)*dilog(1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 4*(-
I*b^3*f^3*x^3 - 3*I*b^3*e*f^2*x^2 - 3*I*b^3*e^2*f*x - I*b^3*e^3)*dilog(-1/
2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 4*(I*b^3*f^3*x^3 + 3*I*b^3*
e*f^2*x^2 + 3*I*b^3*e^2*f*x + I*b^3*e^3)*dilog(1/2*sqrt(-4*I)*(cosh(b*x +
a) + sinh(b*x + a))) - 4*(I*b^3*f^3*x^3 + 3*I*b^3*e*f^2*x^2 + 3*I*b^3*e^2*
f*x + I*b^3*e^3)*dilog(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) +
(I*b^4*f^3*x^4 + 4*I*b^4*e*f^2*x^3 + 6*I*b^4*e^2*f*x^2 + 4*I*b^4*e^3*x + 4
*I*a*b^3*e^3 - 6*I*a^2*b^2*e^2*f + 4*I*a^3*b*e*f^2 - I*a^4*f^3)*log(1/2*sq
rt(4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b^4*f^3*x^4 + 4*I*b^4*e*
f^2*x^3 + 6*I*b^4*e^2*f*x^2 + 4*I*b^4*e^3*x + 4*I*a*b^3*e^3 - 6*I*a^2*b^2*
e^2*f + 4*I*a^3*b*e*f^2 - I*a^4*f^3)*log(-1/2*sqrt(4*I)*(cosh(b*x + a) + s
inh(b*x + a)) + 1) + (-I*b^4*f^3*x^4 - 4*I*b^4*e*f^2*x^3 - 6*I*b^4*e^2*f*x
^2 - 4*I*b^4*e^3*x - 4*I*a*b^3*e^3 + 6*I*a^2*b^2*e^2*f - 4*I*a^3*b*e*f^2 +
 I*a^4*f^3)*log(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (...
```

3.93.6 Sympy [F]

$$\int (e+fx)^3 \arctan(\coth(a+bx))\,dx = \int (e+fx)^3 \operatorname{atan}(\coth(a+bx))\,dx$$

input
```
integrate((f*x+e)**3*atan(coth(b*x+a)),x)
```

output
```
Integral((e + f*x)**3*atan(coth(a + b*x)), x)
```

3.93.7 Maxima [F]

$$\int (e+fx)^3 \arctan(\coth(a+bx))\,dx = \int (fx+e)^3 \arctan\left(\coth\left(bx+a\right)\right)\,dx$$

input
```
integrate((f*x+e)^3*arctan(coth(b*x+a)),x, algorithm="maxima")
```

output
```
1/4*(f^3*x^4 + 4*e*f^2*x^3 + 6*e^2*f*x^2 + 4*e^3*x)*arctan2(e^(2*b*x + 2*a
) + 1, e^(2*b*x + 2*a) - 1) + integrate(1/2*(b*f^3*x^4*e^(2*a) + 4*b*e*f^2
*x^3*e^(2*a) + 6*b*e^2*f*x^2*e^(2*a) + 4*b*e^3*x*e^(2*a))*e^(2*b*x)/(e^(4*
b*x + 4*a) + 1), x)
```

3.93.8 Giac [F]

$$\int (e+fx)^3 \arctan(\coth(a+bx))\,dx = \int (fx+e)^3 \arctan\left(\coth\left(bx+a\right)\right)\,dx$$

input
```
integrate((f*x+e)^3*arctan(coth(b*x+a)),x, algorithm="giac")
```

output
```
sage0*x
```

3.93.9 Mupad [F(-1)]

Timed out.

$$\int (e+fx)^3 \arctan(\coth(a+bx))\,dx = \int \operatorname{atan}(\coth(a+b\,x))\,\left(e+f\,x\right)^3\,dx$$

input
```
int(atan(coth(a + b*x))*(e + f*x)^3,x)
```

output
```
int(atan(coth(a + b*x))*(e + f*x)^3, x)
```

3.94 $\int (e+fx)^2 \arctan(\coth(a+bx))\,dx$

3.94.1 Optimal result

Integrand size = 15, antiderivative size = 229

$$\int (e+fx)^2 \arctan(\coth(a+bx))\,dx = \frac{(e+fx)^3 \arctan\left(e^{2a+2bx}\right)}{3f} + \frac{(e+fx)^3 \arctan(\coth(a+bx))}{3f} - \frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} + \frac{i(e+fx)^2 \operatorname{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} + \frac{if(e+fx) \operatorname{PolyLog}\left(3,-ie^{2a+2bx}\right)}{4b^2} - \frac{if(e+fx) \operatorname{PolyLog}\left(3,ie^{2a+2bx}\right)}{4b^2} - \frac{if^2 \operatorname{PolyLog}\left(4,-ie^{2a+2bx}\right)}{8b^3} + \frac{if^2 \operatorname{PolyLog}\left(4,ie^{2a+2bx}\right)}{8b^3}$$

output
```
1/3*(f*x+e)^3*arctan(exp(2*b*x+2*a))/f+1/3*(f*x+e)^3*arctan(coth(b*x+a))/f
-1/4*I*(f*x+e)^2*polylog(2,-I*exp(2*b*x+2*a))/b+1/4*I*(f*x+e)^2*polylog(2,
I*exp(2*b*x+2*a))/b+1/4*I*f*(f*x+e)*polylog(3,-I*exp(2*b*x+2*a))/b^2-1/4*I
*f*(f*x+e)*polylog(3,I*exp(2*b*x+2*a))/b^2-1/8*I*f^2*polylog(4,-I*exp(2*b*
x+2*a))/b^3+1/8*I*f^2*polylog(4,I*exp(2*b*x+2*a))/b^3
```

3.94.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.64

$$\int (e+fx)^2 \arctan(\coth(a+bx))\,dx = \frac{1}{3}x\left(3e^2+3efx+f^2x^2\right)\arctan(\coth(a+bx)) + \frac{i\left(12b^3e^2x\log\left(1-ie^{2(a+bx)}\right)+12b^3efx^2\log\left(1-ie^{2(a+bx)}\right)+4b^3f^2x^3\log\left(1-ie^{2(a+bx)}\right)-12b^3e^2x\log\left(\right.\right.}{}$$

input
```
Integrate[(e + f*x)^2*ArcTan[Coth[a + b*x]],x]
```

output
```
(x*(3*e^2 + 3*e*f*x + f^2*x^2)*ArcTan[Coth[a + b*x]])/3 + ((I/24)*(12*b^3*
e^2*x*Log[1 - I*E^(2*(a + b*x))] + 12*b^3*e*f*x^2*Log[1 - I*E^(2*(a + b*x)
)] + 4*b^3*f^2*x^3*Log[1 - I*E^(2*(a + b*x))] - 12*b^3*e^2*x*Log[1 + I*E^(
2*(a + b*x))] - 12*b^3*e*f*x^2*Log[1 + I*E^(2*(a + b*x))] - 4*b^3*f^2*x^3*
Log[1 + I*E^(2*(a + b*x))] - 6*b^2*(e + f*x)^2*PolyLog[2, (-I)*E^(2*(a + b
*x))] + 6*b^2*(e + f*x)^2*PolyLog[2, I*E^(2*(a + b*x))] + 6*b*e*f*PolyLog[
3, (-I)*E^(2*(a + b*x))] + 6*b*f^2*x*PolyLog[3, (-I)*E^(2*(a + b*x))] - 6*
b*e*f*PolyLog[3, I*E^(2*(a + b*x))] - 6*b*f^2*x*PolyLog[3, I*E^(2*(a + b*x
))] - 3*f^2*PolyLog[4, (-I)*E^(2*(a + b*x))] + 3*f^2*PolyLog[4, I*E^(2*(a
+ b*x))]))/b^3
```

3.94.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {5708, 3042, 4668, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)^2 \arctan(\coth(a+bx))\,dx$$

$$\Big\downarrow \text{5708}$$

$$\frac{b\int (e+fx)^3\text{sech}(2a+2bx)dx}{3f}+\frac{(e+fx)^3\arctan(\coth(a+bx))}{3f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(e+fx)^3\arctan(\coth(a+bx))}{3f}+\frac{b\int (e+fx)^3\csc\left(2ia+2ibx+\frac{\pi}{2}\right)dx}{3f}$$

$\downarrow$ 4668

$$\frac{(e+fx)^3\arctan(\coth(a+bx))}{3f}+\frac{b\left(-\frac{3if\int(e+fx)^2\log\left(1-ie^{2a+2bx}\right)dx}{2b}+\frac{3if\int(e+fx)^2\log\left(1+ie^{2a+2bx}\right)dx}{2b}+\frac{(e+fx)^3\arctan\left(e^{2a+2bx}\right)}{b}\right)}{3f}$$

$\downarrow$ 3011

$$\frac{(e+fx)^3\arctan(\coth(a+bx))}{3f}+\frac{b\left(\frac{3if\left(\frac{f\int(e+fx)\,\text{PolyLog}\left(2,-ie^{2a+2bx}\right)dx}{b}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{2b}\right)}{2b}-\frac{3if\left(\frac{f\int(e+fx)\,\text{PolyLog}\left(2,ie^{2a+2bx}\right)dx}{b}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,ie^{2a+2}\right.}{2b}\right.}{2b}\right.}{3f}$$

$\downarrow$ 7163

$$\frac{(e+fx)^3\arctan(\coth(a+bx))}{3f}+\frac{b\left(\frac{3if\left(\frac{f\left(\frac{(e+fx)\,\text{PolyLog}\left(3,-ie^{2a+2bx}\right)}{2b}-\frac{f\int\text{PolyLog}\left(3,-ie^{2a+2bx}\right)dx}{2b}\right)}{b}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{2b}\right)}{2b}-\frac{3if\left(\frac{f\left(\frac{(e+fx)\,\text{PolyLog}\left(3,ie^{2a+2bx}\right)}{2b}\right.}{}\right.}{}\right.}{3f}$$

$\downarrow$ 2720

$$\frac{(e+fx)^3\arctan(\coth(a+bx))}{3f}+\frac{b\left(\frac{3if\left(\frac{f\left(\frac{(e+fx)\,\text{PolyLog}\left(3,-ie^{2a+2bx}\right)}{2b}-\frac{f\int e^{-2a-2bx}\,\text{PolyLog}\left(3,-ie^{2a+2bx}\right)de^{2a+2bx}}{4b^2}\right)}{b}-\frac{(e+fx)^2\,\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{2b}\right)}{2b}-\frac{3if\left(\frac{f\left(\frac{(e+fx)\,\text{Po}}{}\right.}{}\right.}{}\right.}{3f}$$

$\downarrow$ 7143

3.94. $\int(e+fx)^2\arctan(\coth(a+bx))\,dx$

$$\frac{(e+fx)^3\arctan(\coth(a+bx))}{3f}+\frac{b\left(\frac{(e+fx)^3\arctan(e^{2a+2bx})}{b}+\frac{3if\left(\frac{f\left(\frac{(e+fx)\operatorname{PolyLog}(3,-ie^{2a+2bx})}{2b}-\frac{f\operatorname{PolyLog}(4,-ie^{2a+2bx})}{4b^2}\right)}{b}-\frac{(e+fx)^2\operatorname{PolyLog}(2,-ie^{2a+2bx})}{2b}\right)}{2b}-\frac{3if\left(f\left(\right.\right.}{}\right.}{3f}$$

input
```
Int[(e + f*x)^2*ArcTan[Coth[a + b*x]],x]
```

output
```
((e + f*x)^3*ArcTan[Coth[a + b*x]])/(3*f) + (b*(((e + f*x)^3*ArcTan[E^(2*a
 + 2*b*x)])/b + (((3*I)/2)*f*(-1/2*((e + f*x)^2*PolyLog[2, (-I)*E^(2*a + 2
*b*x)])/b + (f*(((e + f*x)*PolyLog[3, (-I)*E^(2*a + 2*b*x)])/(2*b) - (f*Po
lyLog[4, (-I)*E^(2*a + 2*b*x)])/(4*b^2)))/b))/b - (((3*I)/2)*f*(-1/2*((e +
 f*x)^2*PolyLog[2, I*E^(2*a + 2*b*x)])/b + (f*(((e + f*x)*PolyLog[3, I*E^(
2*a + 2*b*x)])/(2*b) - (f*PolyLog[4, I*E^(2*a + 2*b*x)])/(4*b^2)))/b))/b))
/(3*f)
```

3.94.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5708
```
Int[ArcTan[Coth[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcTan[Coth[a + b*x]]/(f*(m + 1))), x] + Simp[b/
(f*(m + 1))   Int[(e + f*x)^(m + 1)*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a,
b, e, f}, x] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.94.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 10.37 (sec) , antiderivative size = 2668, normalized size of antiderivative = 11.65

method	result	size
risch	Expression too large to display	2668

input
```
int((f*x+e)^2*arctan(coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/8*I*f^2*polylog(4,I*exp(2*b*x+2*a))/b^3+1/2*I*e^2*ln(((-I)^(1/2)+exp(b*x
+a))/(-I)^(1/2))*x+1/6*I/f*e^3*ln(exp(2*b*x+2*a)+I)+1/6*I*f^2*ln(1-I*exp(2
*b*x+2*a))*x^3+1/2*I/b*e^2*dilog(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))+1/2*I
/b*e^2*dilog(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))+1/2*I*e^2*ln(((-I)^(1/2)-
exp(b*x+a))/(-I)^(1/2))*x+I*f/b*a*e*ln(1+exp(b*x+a)*(-1)^(3/4))*x+I*f/b*a*
e*ln(1-exp(b*x+a)*(-1)^(3/4))*x+1/3*I*f^2/b^3*ln(1+I*exp(2*b*x+2*a))*a^3-1
/4*I*f^2/b*polylog(2,-I*exp(2*b*x+2*a))*x^2+1/4*I*f^2/b^3*polylog(2,-I*exp
(2*b*x+2*a))*a^2+1/4*I*f^2/b^2*polylog(3,-I*exp(2*b*x+2*a))*x-1/2*I*f^2/b^
3*a^3*ln(1+exp(b*x+a)*(-1)^(3/4))-1/2*I*f^2/b^3*a^3*ln(1-exp(b*x+a)*(-1)^(
3/4))-1/2*I/b*a*e^2*ln(exp(2*b*x+2*a)+I)+1/2*I/b*e^2*ln(((-I)^(1/2)-exp(b*
x+a))/(-I)^(1/2))*a+1/2*I/b*e^2*ln(((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))*a-1
/6*I*f^2/b^3*a^3*ln(exp(2*b*x+2*a)+I)-1/4*I*f/b^2*e*polylog(3,I*exp(2*b*x+
2*a))+1/2*I*f*e*ln(1-I*exp(2*b*x+2*a))*x^2-1/3*I*f^2/b^3*ln(1-I*exp(2*b*x+
2*a))*a^3+1/4*I*f^2/b*polylog(2,I*exp(2*b*x+2*a))*x^2-1/4*I*f^2/b^3*polylo
g(2,I*exp(2*b*x+2*a))*a^2-1/4*I*f^2/b^2*polylog(3,I*exp(2*b*x+2*a))*x+1/2*
I*f^2/b^3*a^3*ln(((-I)^(1/2)-exp(b*x+a))/(-I)^(1/2))+1/2*I*f^2/b^3*a^3*ln(
((-I)^(1/2)+exp(b*x+a))/(-I)^(1/2))+1/2*I*f^2/b^3*a^2*dilog(((-I)^(1/2)-ex
p(b*x+a))/(-I)^(1/2))+1/2*I*f^2/b^3*a^2*dilog(((-I)^(1/2)+exp(b*x+a))/(-I)
^(1/2))-1/6*I*(f*x+e)^3/f*ln(exp(2*b*x+2*a)+I)+1/6*I*f^2*ln(exp(2*b*x+2*a)
-I)*x^3+1/2*I*ln(exp(2*b*x+2*a)-I)*x*e^2+1/6*I/f*ln(exp(2*b*x+2*a)-I)*e...
```

3.94.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1002 vs. $2(180) = 360$.

Time = 0.34 (sec) , antiderivative size = 1002, normalized size of antiderivative = 4.38

$$\int (e+fx)^2 \arctan(\coth(a+bx))\,dx = \text{Too large to display}$$

input
```
integrate((f*x+e)^2*arctan(coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/6*(6*I*f^2*polylog(4, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 6
*I*f^2*polylog(4, -1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 6*I*f^
2*polylog(4, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 6*I*f^2*pol
ylog(4, -1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 2*(b^3*f^2*x^3
+ 3*b^3*e*f*x^2 + 3*b^3*e^2*x)*arctan(cosh(b*x + a)/sinh(b*x + a)) - 3*(-I
*b^2*f^2*x^2 - 2*I*b^2*e*f*x - I*b^2*e^2)*dilog(1/2*sqrt(4*I)*(cosh(b*x +
a) + sinh(b*x + a))) - 3*(-I*b^2*f^2*x^2 - 2*I*b^2*e*f*x - I*b^2*e^2)*dilo
g(-1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 3*(I*b^2*f^2*x^2 + 2*I
*b^2*e*f*x + I*b^2*e^2)*dilog(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a
))) - 3*(I*b^2*f^2*x^2 + 2*I*b^2*e*f*x + I*b^2*e^2)*dilog(-1/2*sqrt(-4*I)*
(cosh(b*x + a) + sinh(b*x + a))) + (I*b^3*f^2*x^3 + 3*I*b^3*e*f*x^2 + 3*I*
b^3*e^2*x + 3*I*a*b^2*e^2 - 3*I*a^2*b*e*f + I*a^3*f^2)*log(1/2*sqrt(4*I)*(
cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b^3*f^2*x^3 + 3*I*b^3*e*f*x^2 + 3
*I*b^3*e^2*x + 3*I*a*b^2*e^2 - 3*I*a^2*b*e*f + I*a^3*f^2)*log(-1/2*sqrt(4*
I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^3*f^2*x^3 - 3*I*b^3*e*f*x^
2 - 3*I*b^3*e^2*x - 3*I*a*b^2*e^2 + 3*I*a^2*b*e*f - I*a^3*f^2)*log(1/2*sqr
t(-4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b^3*f^2*x^3 - 3*I*b^3*e
*f*x^2 - 3*I*b^3*e^2*x - 3*I*a*b^2*e^2 + 3*I*a^2*b*e*f - I*a^3*f^2)*log(-1
/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-3*I*a*b^2*e^2 + 3*I
*a^2*b*e*f - I*a^3*f^2)*log(I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x ...
```

3.94.6 Sympy [F]

$$\int (e+fx)^2 \arctan(\coth(a+bx))\,dx = \int (e+fx)^2 \operatorname{atan}(\coth(a+bx))\,dx$$

input
```
integrate((f*x+e)**2*atan(coth(b*x+a)),x)
```

output
```
Integral((e + f*x)**2*atan(coth(a + b*x)), x)
```

3.94.7 Maxima [F]

$$\int (e+fx)^2 \arctan(\coth(a+bx))\,dx = \int (fx+e)^2 \arctan(\coth(bx+a))\,dx$$

input
```
integrate((f*x+e)^2*arctan(coth(b*x+a)),x, algorithm="maxima")
```

output
```
1/3*(f^2*x^3 + 3*e*f*x^2 + 3*e^2*x)*arctan2(e^(2*b*x + 2*a) + 1, e^(2*b*x
+ 2*a) - 1) + integrate(2/3*(b*f^2*x^3*e^(2*a) + 3*b*e*f*x^2*e^(2*a) + 3*b
*e^2*x*e^(2*a))*e^(2*b*x)/(e^(4*b*x + 4*a) + 1), x)
```

3.94.8 Giac [F]

$$\int (e+fx)^2 \arctan(\coth(a+bx))\,dx = \int (fx+e)^2 \arctan(\coth(bx+a))\,dx$$

input
```
integrate((f*x+e)^2*arctan(coth(b*x+a)),x, algorithm="giac")
```

output
```
sage0*x
```

3.94.9 Mupad [F(-1)]

Timed out.

$$\int (e+fx)^2 \arctan(\coth(a+bx))\,dx = \int \operatorname{atan}(\coth(a+b\,x))\,(e+f\,x)^2\,dx$$

input
```
int(atan(coth(a + b*x))*(e + f*x)^2,x)
```

output
```
int(atan(coth(a + b*x))*(e + f*x)^2, x)
```

3.95 $\int (e+fx)\arctan(\coth(a+bx))\,dx$

3.95.1 Optimal result

Integrand size = 13, antiderivative size = 159

$$\int (e+fx)\arctan(\coth(a+bx))\,dx = \frac{(e+fx)^2\arctan\left(e^{2a+2bx}\right)}{2f} + \frac{(e+fx)^2\arctan(\coth(a+bx))}{2f} - \frac{i(e+fx)\,\mathrm{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b} + \frac{i(e+fx)\,\mathrm{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b} + \frac{if\,\mathrm{PolyLog}\left(3,-ie^{2a+2bx}\right)}{8b^2} - \frac{if\,\mathrm{PolyLog}\left(3,ie^{2a+2bx}\right)}{8b^2}$$

output
```
1/2*(f*x+e)^2*arctan(exp(2*b*x+2*a))/f+1/2*(f*x+e)^2*arctan(coth(b*x+a))/f
-1/4*I*(f*x+e)*polylog(2,-I*exp(2*b*x+2*a))/b+1/4*I*(f*x+e)*polylog(2,I*ex
p(2*b*x+2*a))/b+1/8*I*f*polylog(3,-I*exp(2*b*x+2*a))/b^2-1/8*I*f*polylog(3
,I*exp(2*b*x+2*a))/b^2
```

3.95.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.49

$$\int (e+fx)\arctan(\coth(a+bx))\,dx = ex\arctan(\coth(a+bx))+\frac{1}{2}fx^2\arctan(\coth(a+bx))+\frac{ie\left(2bx\left(\log\left(1-i\right.\right.\right.}{}$$
$$+\frac{if\left(2b^2x^2\log\left(1-ie^{2(a+bx)}\right)-2b^2x^2\log\left(1+ie^{2(a+bx)}\right)-2bx\,\text{PolyLog}\left(2,-ie^{2(a+bx)}\right)+2bx\,\text{PolyLog}\left(2,\right.\right.}{8b^2}$$

input
```
Integrate[(e + f*x)*ArcTan[Coth[a + b*x]],x]
```

output
```
e*x*ArcTan[Coth[a + b*x]] + (f*x^2*ArcTan[Coth[a + b*x]])/2 + ((I/4)*e*(2*
b*x*(Log[1 - I*E^(2*(a + b*x))] - Log[1 + I*E^(2*(a + b*x))]) - PolyLog[2,
 (-I)*E^(2*(a + b*x))] + PolyLog[2, I*E^(2*(a + b*x))]))/b + ((I/8)*f*(2*b
^2*x^2*Log[1 - I*E^(2*(a + b*x))] - 2*b^2*x^2*Log[1 + I*E^(2*(a + b*x))] -
 2*b*x*PolyLog[2, (-I)*E^(2*(a + b*x))] + 2*b*x*PolyLog[2, I*E^(2*(a + b*x
))] + PolyLog[3, (-I)*E^(2*(a + b*x))] - PolyLog[3, I*E^(2*(a + b*x))]))/b
^2
```

3.95.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {5708, 3042, 4668, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (e+fx)\arctan(\coth(a+bx))\,dx$$

$$\Big\downarrow \text{5708}$$

$$\frac{b\int (e+fx)^2\text{sech}(2a+2bx)dx}{2f}+\frac{(e+fx)^2\arctan(\coth(a+bx))}{2f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(e+fx)^2\arctan(\coth(a+bx))}{2f}+\frac{b\int (e+fx)^2\csc\left(2ia+2ibx+\frac{\pi}{2}\right)dx}{2f}$$

$$\Big\downarrow \text{4668}$$

$$\frac{(e+fx)^2\arctan(\coth(a+bx))}{2f}+\frac{b\left(-\frac{if\int(e+fx)\log\left(1-ie^{2a+2bx}\right)dx}{b}+\frac{if\int(e+fx)\log\left(1+ie^{2a+2bx}\right)dx}{b}+\frac{(e+fx)^2\arctan\left(e^{2a+2bx}\right)}{b}\right)}{2f}$$

$\downarrow$ 3011

$$\frac{(e+fx)^2\arctan(\coth(a+bx))}{2f}+\frac{b\left(\frac{if\left(\frac{f\int\text{PolyLog}\left(2,-ie^{2a+2bx}\right)dx}{2b}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{2b}\right)}{b}-\frac{if\left(\frac{f\int\text{PolyLog}\left(2,ie^{2a+2bx}\right)dx}{2b}-\frac{(e+fx)\text{PolyLog}\left(2,ie^{2a+2bx}\right)}{2b}\right)}{b}+\frac{(e+fx)^2 \cdots}{}\right.}{2f}$$

$\downarrow$ 2720

$$\frac{(e+fx)^2\arctan(\coth(a+bx))}{2f}+\frac{b\left(\frac{if\left(\frac{f\int e^{-2a-2bx}\text{PolyLog}\left(2,-ie^{2a+2bx}\right)de^{2a+2bx}}{4b^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{2b}\right)}{b}-\frac{if\left(\frac{f\int e^{-2a-2bx}\text{PolyLog}\left(2,ie^{2a+2bx}\right)de^{2a+2bx}}{4b^2}-\frac{(e+fx)\text{P}\cdots}{}\right.}{b}\right.}{2f}$$

$\downarrow$ 7143

$$\frac{(e+fx)^2\arctan(\coth(a+bx))}{2f}+\frac{b\left(\frac{(e+fx)^2\arctan\left(e^{2a+2bx}\right)}{b}+\frac{if\left(\frac{f\text{PolyLog}\left(3,-ie^{2a+2bx}\right)}{4b^2}-\frac{(e+fx)\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{2b}\right)}{b}-\frac{if\left(\frac{f\text{PolyLog}\left(3,ie^{2a+2bx}\right)}{4b^2}-\frac{(e+fx)\text{PolyLog}\left(2,ie^{\cdots}\right)}{2b}\right.}{b}\right.}{2f}$$

input
```
Int[(e + f*x)*ArcTan[Coth[a + b*x]],x]
```

output
```
((e + f*x)^2*ArcTan[Coth[a + b*x]])/(2*f) + (b*(((e + f*x)^2*ArcTan[E^(2*a
 + 2*b*x)])/b + (I*f*(-1/2*((e + f*x)*PolyLog[2, (-I)*E^(2*a + 2*b*x)])/b
+ (f*PolyLog[3, (-I)*E^(2*a + 2*b*x)])/(4*b^2)))/b - (I*f*(-1/2*((e + f*x)
*PolyLog[2, I*E^(2*a + 2*b*x)])/b + (f*PolyLog[3, I*E^(2*a + 2*b*x)])/(4*b
^2)))/b))/(2*f)
```

3.95. $\int (e+fx)\arctan(\coth(a+bx))\,dx$

3.95.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5708
```
Int[ArcTan[Coth[(a_.) + (b_.)*(x_)]]*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:> Simp[(e + f*x)^(m + 1)*(ArcTan[Coth[a + b*x]]/(f*(m + 1))), x] + Simp[b/
(f*(m + 1))   Int[(e + f*x)^(m + 1)*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a,
b, e, f}, x] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.95.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.34 (sec) , antiderivative size = 1777, normalized size of antiderivative = 11.18

method	result	size
risch	Expression too large to display	1777

input
```
int((f*x+e)*arctan(coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/4*I*f*ln(1+I*exp(2*b*x+2*a))*x^2+1/4*I*ln(exp(2*b*x+2*a)-I)*x^2*f+1/2*I
*ln(exp(2*b*x+2*a)-I)*e*x+1/2*I*(-1/2*f*x^2-e*x)*ln(exp(2*b*x+2*a)+I)+1/4*
Pi*(csgn(I*(exp(2*b*x+2*a)-I))*csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(exp(2*b*
x+2*a)-I)/(exp(2*b*x+2*a)-1))-csgn(I*(exp(2*b*x+2*a)-I))*csgn(I*(exp(2*b*x
+2*a)-I)/(exp(2*b*x+2*a)-1))^2-csgn(I*(exp(2*b*x+2*a)+I))*csgn(I/(exp(2*b*
x+2*a)-1))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))+csgn(I*(exp(2*b*x
+2*a)+I))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^2+csgn(I*(exp(2*b*
x+2*a)-I)/(exp(2*b*x+2*a)-1))*csgn((1+I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a
)-1))-csgn((1+I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^2-csgn(I*(exp(2*b*
x+2*a)+I)/(exp(2*b*x+2*a)-1))*csgn((1-I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a
)-1))-csgn((1-I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^2-csgn(I/(exp(2*b*
x+2*a)-1))*csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^2+csgn(I/(exp(2*b
*x+2*a)-1))*csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^2+csgn(I*(exp(2*
b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^3-csgn(I*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a
)-1))*csgn((1+I)*(exp(2*b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^2-csgn(I*(exp(2*b*
x+2*a)+I)/(exp(2*b*x+2*a)-1))^3+csgn(I*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-
1))*csgn((1-I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x+2*a)-1))^2+csgn((1+I)*(exp(2*
b*x+2*a)-I)/(exp(2*b*x+2*a)-1))^3+csgn((1-I)*(exp(2*b*x+2*a)+I)/(exp(2*b*x
+2*a)-1))^3+1)*(1/2*f*x^2+e*x)-1/4*I*f/b^2*a^2*ln(-exp(2*b*x+2*a)+I)+1/2*I
*e/b*a*ln(-exp(2*b*x+2*a)+I)+1/4*I*f*ln(1-I*exp(2*b*x+2*a))*x^2+1/2*I*e...
```

3.95.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 600 vs. 2(130) = 260.

Time = 0.34 (sec) , antiderivative size = 600, normalized size of antiderivative = 3.77

$$\int (e + fx)\arctan(\coth(a + bx))\,dx$$

$$= \frac{2\left(b^2fx^2 + 2\,b^2ex\right)\arctan\left(\frac{\cosh(bx+a)}{\sinh(bx+a)}\right) - 2\left(-i\,bfx - i\,be\right)\mathrm{Li}_2\left(\frac{1}{2}\sqrt{4i}(\cosh(bx+a) + \sinh(bx+a))\right) - 2\,(\cdots}{}$$

input
```
integrate((f*x+e)*arctan(coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/4*(2*(b^2*f*x^2 + 2*b^2*e*x)*arctan(cosh(b*x + a)/sinh(b*x + a)) - 2*(-I
*b*f*x - I*b*e)*dilog(1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) - 2*(
-I*b*f*x - I*b*e)*dilog(-1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) -
2*(I*b*f*x + I*b*e)*dilog(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)))
- 2*(I*b*f*x + I*b*e)*dilog(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)
)) + (I*b^2*f*x^2 + 2*I*b^2*e*x + 2*I*a*b*e - I*a^2*f)*log(1/2*sqrt(4*I)*(
cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b^2*f*x^2 + 2*I*b^2*e*x + 2*I*a*b
*e - I*a^2*f)*log(-1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-
I*b^2*f*x^2 - 2*I*b^2*e*x - 2*I*a*b*e + I*a^2*f)*log(1/2*sqrt(-4*I)*(cosh(
b*x + a) + sinh(b*x + a)) + 1) + (-I*b^2*f*x^2 - 2*I*b^2*e*x - 2*I*a*b*e +
 I*a^2*f)*log(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-2*I
*a*b*e + I*a^2*f)*log(I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + (
-2*I*a*b*e + I*a^2*f)*log(-I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)
) + (2*I*a*b*e - I*a^2*f)*log(I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x
+ a)) + (2*I*a*b*e - I*a^2*f)*log(-I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh
(b*x + a)) - 2*I*f*polylog(3, 1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a)
)) - 2*I*f*polylog(3, -1/2*sqrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 2*
I*f*polylog(3, 1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))) + 2*I*f*pol
ylog(3, -1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))))/b^2
```

3.95.6 Sympy [F]

$$\int (e+fx)\arctan(\coth(a+bx))\,dx=\int (e+fx)\operatorname{atan}(\coth(a+bx))\,dx$$

input
```
integrate((f*x+e)*atan(coth(b*x+a)),x)
```

output
```
Integral((e + f*x)*atan(coth(a + b*x)), x)
```

3.95.7 Maxima [F]

$$\int (e+fx)\arctan(\coth(a+bx))\,dx = \int (fx+e)\arctan\left(\coth\left(bx+a\right)\right)\,dx$$

input
```
integrate((f*x+e)*arctan(coth(b*x+a)),x, algorithm="maxima")
```

output
```
1/2*(f*x^2 + 2*e*x)*arctan2(e^(2*b*x + 2*a) + 1, e^(2*b*x + 2*a) - 1) + in
tegrate((b*f*x^2*e^(2*a) + 2*b*e*x*e^(2*a))*e^(2*b*x)/(e^(4*b*x + 4*a) + 1
), x)
```

3.95.8 Giac [F]

$$\int (e+fx)\arctan(\coth(a+bx))\,dx = \int (fx+e)\arctan\left(\coth\left(bx+a\right)\right)\,dx$$

input
```
integrate((f*x+e)*arctan(coth(b*x+a)),x, algorithm="giac")
```

output
```
sage0*x
```

3.95.9 Mupad [F(-1)]

Timed out.

$$\int (e+fx)\arctan(\coth(a+bx))\,dx = \int \mathrm{atan}(\coth(a+b\,x))\,\left(e+f\,x\right)\,dx$$

input
```
int(atan(coth(a + b*x))*(e + f*x),x)
```

output
```
int(atan(coth(a + b*x))*(e + f*x), x)
```

3.96 $\int \arctan(\coth(a+bx))\,dx$

3.96.1 Optimal result

Integrand size = 7, antiderivative size = 73

$$\int \arctan(\coth(a+bx))\,dx = x\arctan\left(e^{2a+2bx}\right)+x\arctan(\coth(a+bx)) - \frac{i\,\mathrm{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b}+\frac{i\,\mathrm{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b}$$

output
```
x*arctan(exp(2*b*x+2*a))+x*arctan(coth(b*x+a))-1/4*I*polylog(2,-I*exp(2*b*
x+2*a))/b+1/4*I*polylog(2,I*exp(2*b*x+2*a))/b
```

3.96.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.25

$$\int \arctan(\coth(a+bx))\,dx = x\arctan(\coth(a+bx)) + \frac{i\left(2bx\left(\log\left(1-ie^{2(a+bx)}\right)-\log\left(1+ie^{2(a+bx)}\right)\right)-\mathrm{PolyLog}\left(2,-ie^{2(a+bx)}\right)+\mathrm{PolyLog}\left(2,ie^{2(a+bx)}\right)\right)}{4b}$$

input
```
Integrate[ArcTan[Coth[a + b*x]],x]
```

output
```
x*ArcTan[Coth[a + b*x]] + ((I/4)*(2*b*x*(Log[1 - I*E^(2*(a + b*x))] - Log[
1 + I*E^(2*(a + b*x))]) - PolyLog[2, (-I)*E^(2*(a + b*x))] + PolyLog[2, I*
E^(2*(a + b*x))]))/b
```

3.96.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {5704, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(\coth(a+bx))\,dx$$

$$\Big\downarrow \text{5704}$$

$$b\int x\text{sech}(2a+2bx)dx + x\arctan(\coth(a+bx))$$

$$\Big\downarrow \text{3042}$$

$$x\arctan(\coth(a+bx)) + b\int x\csc\left(2ia+2ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{4668}$$

$$x\arctan(\coth(a+bx)) + b\left(-\frac{i\int \log\left(1-ie^{2a+2bx}\right)dx}{2b} + \frac{i\int \log\left(1+ie^{2a+2bx}\right)dx}{2b} + \frac{x\arctan\left(e^{2a+2bx}\right)}{b}\right)$$

$$\Big\downarrow \text{2715}$$

$$x\arctan(\coth(a+bx)) + b\left(-\frac{i\int e^{-2a-2bx}\log\left(1-ie^{2a+2bx}\right)de^{2a+2bx}}{4b^2} + \frac{i\int e^{-2a-2bx}\log\left(1+ie^{2a+2bx}\right)de^{2a+2bx}}{4b^2} + \frac{x\arctan\left(e^{2a+2bx}\right)}{b}\right)$$

$$\Big\downarrow \text{2838}$$

$$x\arctan(\coth(a+bx)) + b\left(\frac{x\arctan\left(e^{2a+2bx}\right)}{b} - \frac{i\,\text{PolyLog}\left(2,-ie^{2a+2bx}\right)}{4b^2} + \frac{i\,\text{PolyLog}\left(2,ie^{2a+2bx}\right)}{4b^2}\right)$$

input
```
Int[ArcTan[Coth[a + b*x]],x]
```

output
```
x*ArcTan[Coth[a + b*x]] + b*((x*ArcTan[E^(2*a + 2*b*x)])/b - ((I/4)*PolyLo
g[2, (-I)*E^(2*a + 2*b*x)])/b^2 + ((I/4)*PolyLog[2, I*E^(2*a + 2*b*x)])/b^
2)
```

3.96.3.1 Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5704
```
Int[ArcTan[Coth[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcTan[Coth[a + b
*x]], x] + Simp[b   Int[x*Sech[2*a + 2*b*x], x], x] /; FreeQ[{a, b}, x]
```

3.96.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 159 vs. 2(62) = 124.

Time = 0.84 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.19

method	result
derivativedivides	$\frac{\operatorname{arctanh}(\coth(bx+a))\arctan(\coth(bx+a))-\frac{i\operatorname{arctanh}(\coth(bx+a))\left(\ln\left(1-\frac{i(\coth(bx+a)+1)^2}{1-\coth(bx+a)^2}\right)-\ln\left(1+\frac{i(\coth(bx+a)+1)^2}{1-\coth(bx+a)^2}\right)\right)}{2}+}{b}$
default	$\frac{\operatorname{arctanh}(\coth(bx+a))\arctan(\coth(bx+a))-\frac{i\operatorname{arctanh}(\coth(bx+a))\left(\ln\left(1-\frac{i(\coth(bx+a)+1)^2}{1-\coth(bx+a)^2}\right)-\ln\left(1+\frac{i(\coth(bx+a)+1)^2}{1-\coth(bx+a)^2}\right)\right)}{2}+}{b}$
risch	Expression too large to display

input
```
int(arctan(coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b*(arctanh(coth(b*x+a))*arctan(coth(b*x+a))-1/2*I*arctanh(coth(b*x+a))*(
ln(1-I*(coth(b*x+a)+1)^2/(1-coth(b*x+a)^2))-ln(1+I*(coth(b*x+a)+1)^2/(1-co
th(b*x+a)^2)))+1/4*I*dilog(1+I*(coth(b*x+a)+1)^2/(1-coth(b*x+a)^2))-1/4*I*
dilog(1-I*(coth(b*x+a)+1)^2/(1-coth(b*x+a)^2)))
```

3.96.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(56) = 112$.

Time = 0.29 (sec) , antiderivative size = 334, normalized size of antiderivative = 4.58

$$\int \arctan(\coth(a+bx))\,dx$$
$$= \frac{2\,bx\arctan\left(\frac{\cosh(bx+a)}{\sinh(bx+a)}\right) + (i\,bx+i\,a)\log\left(\frac{1}{2}\sqrt{4i}(\cosh(bx+a)+\sinh(bx+a))+1\right) + (i\,bx+i\,a)\log\left(-\right.}{}$$

input
```
integrate(arctan(coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*(2*b*x*arctan(cosh(b*x + a)/sinh(b*x + a)) + (I*b*x + I*a)*log(1/2*sqr
t(4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b*x + I*a)*log(-1/2*sqrt(
4*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b*x - I*a)*log(1/2*sqrt(-4
*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*b*x - I*a)*log(-1/2*sqrt(-4
*I)*(cosh(b*x + a) + sinh(b*x + a)) + 1) - I*a*log(I*sqrt(4*I) + 2*cosh(b*
x + a) + 2*sinh(b*x + a)) - I*a*log(-I*sqrt(4*I) + 2*cosh(b*x + a) + 2*sin
h(b*x + a)) + I*a*log(I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) +
I*a*log(-I*sqrt(-4*I) + 2*cosh(b*x + a) + 2*sinh(b*x + a)) + I*dilog(1/2*s
qrt(4*I)*(cosh(b*x + a) + sinh(b*x + a))) + I*dilog(-1/2*sqrt(4*I)*(cosh(b
*x + a) + sinh(b*x + a))) - I*dilog(1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b
*x + a))) - I*dilog(-1/2*sqrt(-4*I)*(cosh(b*x + a) + sinh(b*x + a))))/b
```

3.96.6 Sympy [F]

$$\int \arctan(\coth(a+bx))\,dx = \int \operatorname{atan}(\coth(a+bx))\,dx$$

input
```
integrate(atan(coth(b*x+a)),x)
```

output
```
Integral(atan(coth(a + b*x)), x)
```

3.96.7 Maxima [F]

$$\int \arctan(\coth(a+bx))\,dx = \int \arctan(\coth(bx+a))\,dx$$

input
```
integrate(arctan(coth(b*x+a)),x, algorithm="maxima")
```

output
```
x*arctan2(e^(2*b*x + 2*a) + 1, e^(2*b*x + 2*a) - 1) + 2*b*integrate(x*e^(2
*b*x + 2*a)/(e^(4*b*x + 4*a) + 1), x)
```

3.96.8 Giac [F]

$$\int \arctan(\coth(a+bx))\,dx = \int \arctan(\coth(bx+a))\,dx$$

input
```
integrate(arctan(coth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arctan(coth(b*x + a)), x)
```

3.96.9 Mupad [F(-1)]

Timed out.

$$\int \arctan(\coth(a+bx))\,dx = \int \operatorname{atan}(\coth(a+b\,x))\,\,dx$$

input
```
int(atan(coth(a + b*x)),x)
```

output
```
int(atan(coth(a + b*x)), x)
```

3.97 $\int \frac{\arctan(\coth(a+bx))}{e+fx} \, dx$

3.97.1 Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{\arctan(\coth(a+bx))}{e+fx} \, dx = \text{Int}\left(\frac{\arctan(\coth(a+bx))}{e+fx}, x\right)$$

output
```
CannotIntegrate(arctan(coth(b*x+a))/(f*x+e),x)
```

3.97.2 Mathematica [N/A]

Not integrable

Time = 3.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\arctan(\coth(a+bx))}{e+fx} \, dx = \int \frac{\arctan(\coth(a+bx))}{e+fx} \, dx$$

input
```
Integrate[ArcTan[Coth[a + b*x]]/(e + f*x),x]
```

output
```
Integrate[ArcTan[Coth[a + b*x]]/(e + f*x), x]
```

3.97.3 Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(\coth(a+bx))}{e+fx} \, dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \frac{\arctan(\coth(a+bx))}{e+fx}dx$$

input
```
Int[ArcTan[Coth[a + b*x]]/(e + f*x),x]
```

output
```
$Aborted
```

3.97.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.97.4 Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\arctan \left(\coth \left(bx+a\right)\right)}{fx+e}dx$$

input
```
int(arctan(coth(b*x+a))/(f*x+e),x)
```

output
```
int(arctan(coth(b*x+a))/(f*x+e),x)
```

3.97.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\arctan(\coth(a+bx))}{e+fx}\,dx = \int \frac{\arctan\left(\coth\left(bx+a\right)\right)}{fx+e}\,dx$$

input
```
integrate(arctan(coth(b*x+a))/(f*x+e),x, algorithm="fricas")
```

output
```
integral(arctan(coth(b*x + a))/(f*x + e), x)
```

3.97.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan(\coth(a+bx))}{e+fx}\,dx = \text{Timed out}$$

input
```
integrate(atan(coth(b*x+a))/(f*x+e),x)
```

output
```
Timed out
```

3.97.7 Maxima [N/A]

Not integrable

Time = 1.55 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\arctan(\coth(a+bx))}{e+fx}\,dx = \int \frac{\arctan\left(\coth\left(bx+a\right)\right)}{fx+e}\,dx$$

input
```
integrate(arctan(coth(b*x+a))/(f*x+e),x, algorithm="maxima")
```

output
```
integrate(arctan(coth(b*x + a))/(f*x + e), x)
```

3.97.8 Giac [N/A]

Not integrable

Time = 90.48 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.20

$$\int \frac{\arctan(\coth(a+bx))}{e+fx}\,dx = \int \frac{\arctan\left(\coth\left(bx+a\right)\right)}{fx+e}\,dx$$

input
```
integrate(arctan(coth(b*x+a))/(f*x+e),x, algorithm="giac")
```

output
```
sage0*x
```

3.97.9 Mupad [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\arctan(\coth(a+bx))}{e+fx}\,dx = \int \frac{\mathrm{atan}(\coth(a+b\,x))}{e+f\,x}\,dx$$

input
```
int(atan(coth(a + b*x))/(e + f*x),x)
```

output
```
int(atan(coth(a + b*x))/(e + f*x), x)
```

3.98 $\int x^2 \arctan(c + d\coth(a + bx))\,dx$

3.98.1 Optimal result

Integrand size = 15, antiderivative size = 351

$$\begin{aligned}\int x^2 \arctan(c+d\coth(a+bx))\,dx &= \frac{1}{3}x^3\arctan(c+d\coth(a+bx))\\
&+\frac{1}{6}ix^3\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)\\
&-\frac{1}{6}ix^3\log\left(1-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)\\
&+\frac{ix^2\operatorname{PolyLog}\left(2,\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b}\\
&-\frac{ix^2\operatorname{PolyLog}\left(2,\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b}\\
&-\frac{ix\operatorname{PolyLog}\left(3,\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b^2}\\
&+\frac{ix\operatorname{PolyLog}\left(3,\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b^2}\\
&+\frac{i\operatorname{PolyLog}\left(4,\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{8b^3}\\
&-\frac{i\operatorname{PolyLog}\left(4,\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{8b^3}\end{aligned}$$

output
```
1/3*x^3*arctan(c+d*coth(b*x+a))+1/6*I*x^3*ln(1-(I-c-d)*exp(2*b*x+2*a)/(I-c
+d))-1/6*I*x^3*ln(1-(I+c+d)*exp(2*b*x+2*a)/(I+c-d))+1/4*I*x^2*polylog(2,(I
-c-d)*exp(2*b*x+2*a)/(I-c+d))/b-1/4*I*x^2*polylog(2,(I+c+d)*exp(2*b*x+2*a)
/(I+c-d))/b-1/4*I*x*polylog(3,(I-c-d)*exp(2*b*x+2*a)/(I-c+d))/b^2+1/4*I*x*
polylog(3,(I+c+d)*exp(2*b*x+2*a)/(I+c-d))/b^2+1/8*I*polylog(4,(I-c-d)*exp(
2*b*x+2*a)/(I-c+d))/b^3-1/8*I*polylog(4,(I+c+d)*exp(2*b*x+2*a)/(I+c-d))/b^
3
```

3.98.2 Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.26

$$\int x^2 \arctan(c+d\coth(a+bx))\,dx = \frac{1}{3}x^3\arctan(c+d\coth(a+bx)) + \frac{d\left(4b^3x^3\log\left(1-\frac{(1+(c+d)^2)e^{2(a+bx)}}{1+c^2-d^2+2\sqrt{-d^2}}\right) - 4b^3x^3\log\left(1+\frac{(1+(c+d)^2)e^{2(a+bx)}}{-1-c^2+d^2+2\sqrt{-d^2}}\right) + 6b^2x^2\,\mathrm{PolyLog}\left(2,\frac{(1+c^2+2cd+d^2)}{1+c^2-d^2+2}\right.\right.}{}$$

input
```
Integrate[x^2*ArcTan[c + d*Coth[a + b*x]],x]
```

output
```
(x^3*ArcTan[c + d*Coth[a + b*x]])/3 + (d*(4*b^3*x^3*Log[1 - ((1 + (c + d)^
2)*E^(2*(a + b*x)))/(1 + c^2 - d^2 + 2*Sqrt[-d^2])] - 4*b^3*x^3*Log[1 + ((
1 + (c + d)^2)*E^(2*(a + b*x)))/(-1 - c^2 + d^2 + 2*Sqrt[-d^2])] + 6*b^2*x
^2*PolyLog[2, ((1 + c^2 + 2*c*d + d^2)*E^(2*(a + b*x)))/(1 + c^2 - d^2 + 2
*Sqrt[-d^2])] - 6*b^2*x^2*PolyLog[2, -(((1 + (c + d)^2)*E^(2*(a + b*x)))/(
-1 - c^2 + d^2 + 2*Sqrt[-d^2]))] - 6*b*x*PolyLog[3, ((1 + c^2 + 2*c*d + d^
2)*E^(2*(a + b*x)))/(1 + c^2 - d^2 + 2*Sqrt[-d^2])] + 6*b*x*PolyLog[3, -((
(1 + (c + d)^2)*E^(2*(a + b*x)))/(-1 - c^2 + d^2 + 2*Sqrt[-d^2]))] - 3*Pol
yLog[4, ((1 + c^2 + 2*c*d + d^2)*E^(2*(a + b*x)))/(1 + c^2 - d^2 - 2*Sqrt[
-d^2])] + 3*PolyLog[4, ((1 + c^2 + 2*c*d + d^2)*E^(2*(a + b*x)))/(1 + c^2
- d^2 + 2*Sqrt[-d^2])]))/(24*b^3*Sqrt[-d^2])
```

3.98.3 Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.25, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5724, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(d\coth(a+bx)+c)\,dx$$

$$\Big\downarrow \text{5724}$$

$$\frac{1}{3}b(1+i(c+d))\int \frac{e^{2a+2bx}x^3}{-c-(-c-d+i)e^{2a+2bx}+d+i}dx-\frac{1}{3}b(1-i(c+d))\int \frac{e^{2a+2bx}x^3}{c-(c+d+i)e^{2a+2bx}-d+i}dx+\frac{1}{3}x^3\arctan(d\coth(a+bx)+c)$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{3}b(1+i(c+d))\left(\frac{3\int x^2\log\left(1-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)dx}{2b(-c-d+i)}-\frac{x^3\log\left(1-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b(-c-d+i)}\right)-\frac{1}{3}b(1-i(c+d))\left(\frac{3\int x^2\log\left(1-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)dx}{2b(c+d+i)}-\frac{x^3\log\left(1-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b(c+d+i)}\right)+\frac{1}{3}x^3\arctan(d\coth(a+bx)+c)$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{3}b(1+i(c+d))\left(\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b}\right)}{2b(-c-d+i)}-\frac{x^3\log\left(1-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b(-c-d+i)}\right)-\frac{1}{3}b(1-i(c+d))\left(\frac{3\left(\frac{\int x\,\text{PolyLog}\left(2,\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b}\right)}{2b(c+d+i)}-\frac{x^3\log\left(1-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b(c+d+i)}\right)+\frac{1}{3}x^3\arctan(d\coth(a+bx)+c)$$

$\Big\downarrow$ 7163

$$d))\left(\frac{\frac{1}{3}b(1+i(c+ 3\left(\frac{\frac{x\,\text{PolyLog}\left(3,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b}-\frac{\int \text{PolyLog}\left(3,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)dx}{2b}}{b}-\frac{x^2\,\text{PolyLog}\left(2,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b}\right)}{2b(-c-d+i)}-\frac{x^3\log\left(1-\frac{(-c-}{}\right.}{2b(-c-d}\right.$$

$$d))\left(\frac{\frac{1}{3}b(1-i(c+ 3\left(\frac{\frac{x\,\text{PolyLog}\left(3,\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b}-\frac{\int \text{PolyLog}\left(3,\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)dx}{2b}}{b}-\frac{x^2\,\text{PolyLog}\left(2,\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b}\right)}{2b(c+d+i)}-\frac{x^3\log\left(1-\frac{(c+d+i)e^2}{c-d+}\right.}{2b(c+d+i)}\right.$$

$$\frac{1}{3}x^3\arctan(d\coth(a+bx)+c)$$

$\Big\downarrow$ 2720

$$d))\left(\frac{\frac{1}{3}b(1+i(c+ 3\left(\frac{\frac{x\,\text{PolyLog}\left(3,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b}-\frac{\int e^{-2a-2bx}\,\text{PolyLog}\left(3,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)de^{2a+2bx}}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}\left(2,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b}\right)}{2b(-c-d+i)}-\frac{x^3\,\text{l}}{}\right.$$

$$d))\left(\frac{\frac{1}{3}b(1-i(c+ 3\left(\frac{\frac{x\,\text{PolyLog}\left(3,\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b}-\frac{\int e^{-2a-2bx}\,\text{PolyLog}\left(3,\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)de^{2a+2bx}}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}\left(2,\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b}\right)}{2b(c+d+i)}-\frac{x^3\log\left(\right.}{2}\right.$$

$$\frac{1}{3}x^3\arctan(d\coth(a+bx)+c)$$

$\Big\downarrow$ 7143

3.98. $\int x^2\arctan(c+d\coth(a+bx))\,dx$

$$\frac{1}{3}x^3\arctan(d\coth(a+bx)+c)+\frac{1}{3}b(1+i(c+d))\left(\frac{3\left(\frac{\frac{x\operatorname{PolyLog}\left(3,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b}-\frac{\operatorname{PolyLog}\left(4,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b^2}}{b}-\frac{x^2\operatorname{PolyLog}\left(2,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b}\right)}{2b(-c-d+i)}-\frac{x^3\log\left(1-\frac{(-c-d+i}{-c+}\right.}{2b(-c-d+}\right.$$

$$\frac{1}{3}b(1-i(c+d))\left(\frac{3\left(\frac{\frac{x\operatorname{PolyLog}\left(3,\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b}-\frac{\operatorname{PolyLog}\left(4,\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b^2}}{b}-\frac{x^2\operatorname{PolyLog}\left(2,\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b}\right)}{2b(c+d+i)}-\frac{x^3\log\left(1-\frac{(c+d+i)e^{2a+2}}{c-d+i}\right.}{2b(c+d+i)}\right.$$

input
```
Int[x^2*ArcTan[c + d*Coth[a + b*x]],x]
```

output
```
(x^3*ArcTan[c + d*Coth[a + b*x]])/3 + (b*(1 + I*(c + d))*(-1/2*(x^3*Log[1
- ((I - c - d)*E^(2*a + 2*b*x))/(I - c + d)])/(b*(I - c - d)) + (3*(-1/2*(
x^2*PolyLog[2, ((I - c - d)*E^(2*a + 2*b*x))/(I - c + d)])/b + ((x*PolyLog
[3, ((I - c - d)*E^(2*a + 2*b*x))/(I - c + d)])/(2*b) - PolyLog[4, ((I - c
 - d)*E^(2*a + 2*b*x))/(I - c + d)]/(4*b^2))/b))/(2*b*(I - c - d))))/3 - (
b*(1 - I*(c + d))*(-1/2*(x^3*Log[1 - ((I + c + d)*E^(2*a + 2*b*x))/(I + c
- d)])/(b*(I + c + d)) + (3*(-1/2*(x^2*PolyLog[2, ((I + c + d)*E^(2*a + 2*
b*x))/(I + c - d)])/b + ((x*PolyLog[3, ((I + c + d)*E^(2*a + 2*b*x))/(I +
c - d)])/(2*b) - PolyLog[4, ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)]/(4*
b^2))/b))/(2*b*(I + c + d))))/3
```

3.98.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5724
```
Int[ArcTan[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Coth[a + b*x]]/(f*(m
+ 1))), x] + (-Simp[I*b*((I - c - d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(
E^(2*a + 2*b*x)/(I - c + d - (I - c - d)*E^(2*a + 2*b*x))), x], x] + Simp[I
*b*((I + c + d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(E^(2*a + 2*b*x)/(I +
c - d - (I + c + d)*E^(2*a + 2*b*x))), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[m, 0] && NeQ[(c - d)^2, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.98. $\int x^2 \arctan(c + d\coth(a + bx))\, dx$

3.98.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 24.98 (sec) , antiderivative size = 6845, normalized size of antiderivative = 19.50

method	result	size
risch	Expression too large to display	6845

input
```
int(x^2*arctan(c+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.98.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1269 vs. $2(259) = 518$.

Time = 0.36 (sec) , antiderivative size = 1269, normalized size of antiderivative = 3.62

$$\int x^2 \arctan(c + d \coth(a + bx)) \, dx = \text{Too large to display}$$

input
```
integrate(x^2*arctan(c+d*coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/6*(2*b^3*x^3*arctan((d*cosh(b*x + a) + c*sinh(b*x + a))/sinh(b*x + a)) +
 3*I*b^2*x^2*dilog(sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(
cosh(b*x + a) + sinh(b*x + a))) + 3*I*b^2*x^2*dilog(-sqrt((c^2 - d^2 + 2*I
*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 3*I*b^
2*x^2*dilog(sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*
x + a) + sinh(b*x + a))) - 3*I*b^2*x^2*dilog(-sqrt((c^2 - d^2 - 2*I*d + 1)
/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - I*a^3*log(2*(
c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x
+ a) + 2*(c^2 - d^2 - 2*I*d + 1)*sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d
 + d^2 + 1))) - I*a^3*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2
 + 2*c*d + d^2 + 1)*sinh(b*x + a) - 2*(c^2 - d^2 - 2*I*d + 1)*sqrt((c^2 -
d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) + I*a^3*log(2*(c^2 + 2*c*d + d^
2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) + 2*(c^2 -
d^2 + 2*I*d + 1)*sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) +
I*a^3*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 +
 1)*sinh(b*x + a) - 2*(c^2 - d^2 + 2*I*d + 1)*sqrt((c^2 - d^2 - 2*I*d + 1)
/(c^2 - 2*c*d + d^2 + 1))) - 6*I*b*x*polylog(3, sqrt((c^2 - d^2 + 2*I*d +
1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 6*I*b*x*pol
ylog(3, -sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x +
 a) + sinh(b*x + a))) + 6*I*b*x*polylog(3, sqrt((c^2 - d^2 - 2*I*d + 1)...
```

3.98.6 Sympy [F(-1)]

Timed out.

$$\int x^2 \arctan(c + d\coth(a + bx))\, dx = \text{Timed out}$$

input
```
integrate(x**2*atan(c+d*coth(b*x+a)),x)
```

output
```
Timed out
```

3.98.7 Maxima [F]

$$\int x^2 \arctan(c + d\coth(a + bx))\,dx = \int x^2 \arctan\left(d\coth\left(bx + a\right) + c\right)\,dx$$

input
```
integrate(x^2*arctan(c+d*coth(b*x+a)),x, algorithm="maxima")
```

output
```
1/3*x^3*arctan2((c*e^(2*a) + d*e^(2*a))*e^(2*b*x) - c + d, e^(2*b*x + 2*a)
 - 1) + 4*b*d*integrate(1/3*x^3*e^(2*b*x + 2*a)/(c^2 - 2*c*d + d^2 + (c^2*
e^(4*a) + 2*c*d*e^(4*a) + d^2*e^(4*a) + e^(4*a))*e^(4*b*x) - 2*(c^2*e^(2*a
) - d^2*e^(2*a) + e^(2*a))*e^(2*b*x) + 1), x)
```

3.98.8 Giac [F]

$$\int x^2 \arctan(c + d\coth(a + bx))\,dx = \int x^2 \arctan\left(d\coth\left(bx + a\right) + c\right)\,dx$$

input
```
integrate(x^2*arctan(c+d*coth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*arctan(d*coth(b*x + a) + c), x)
```

3.98.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \arctan(c + d\coth(a + bx))\,dx = \int x^2 \,\mathrm{atan}(c + d\coth(a + b\,x))\,dx$$

input
```
int(x^2*atan(c + d*coth(a + b*x)),x)
```

output
```
int(x^2*atan(c + d*coth(a + b*x)), x)
```

3.99 $\int x \arctan(c + d\coth(a + bx))\,dx$

3.99.1 Optimal result

Integrand size = 13, antiderivative size = 265

$$
\begin{aligned}
\int x\arctan(c+d\coth(a+bx))\,dx =& \frac{1}{2}x^2\arctan(c+d\coth(a+bx)) \\
&+\frac{1}{4}ix^2\log\left(1-\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right) \\
&-\frac{1}{4}ix^2\log\left(1-\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right) \\
&+\frac{ix\,\mathrm{PolyLog}\left(2,\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b} \\
&-\frac{ix\,\mathrm{PolyLog}\left(2,\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b} \\
&-\frac{i\,\mathrm{PolyLog}\left(3,\frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{8b^2} \\
&+\frac{i\,\mathrm{PolyLog}\left(3,\frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{8b^2}
\end{aligned}
$$

output
```
1/2*x^2*arctan(c+d*coth(b*x+a))+1/4*I*x^2*ln(1-(I-c-d)*exp(2*b*x+2*a)/(I-c
+d))-1/4*I*x^2*ln(1-(I+c+d)*exp(2*b*x+2*a)/(I+c-d))+1/4*I*x*polylog(2,(I-c
-d)*exp(2*b*x+2*a)/(I-c+d))/b-1/4*I*x*polylog(2,(I+c+d)*exp(2*b*x+2*a)/(I+
c-d))/b-1/8*I*polylog(3,(I-c-d)*exp(2*b*x+2*a)/(I-c+d))/b^2+1/8*I*polylog(
3,(I+c+d)*exp(2*b*x+2*a)/(I+c-d))/b^2
```

3.99.2 Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.26

$$\int x\arctan(c+d\coth(a+bx))\,dx=\frac{1}{2}x^2\arctan(c+d\coth(a+bx))$$
$$+\frac{d\left(2b^2x^2\log\left(1-\frac{(1+(c+d)^2)e^{2(a+bx)}}{1+c^2-d^2+2\sqrt{-d^2}}\right)-2b^2x^2\log\left(1+\frac{(1+(c+d)^2)e^{2(a+bx)}}{-1-c^2+d^2+2\sqrt{-d^2}}\right)+2bx\,\mathrm{PolyLog}\left(2,\frac{(1+c^2+2cd+d^2)e^{2}}{1+c^2-d^2+2\sqrt{-}}\right.\right.}{}$$

input
```
Integrate[x*ArcTan[c + d*Coth[a + b*x]],x]
```

output
```
(x^2*ArcTan[c + d*Coth[a + b*x]])/2 + (d*(2*b^2*x^2*Log[1 - ((1 + (c + d)^
2)*E^(2*(a + b*x)))/(1 + c^2 - d^2 + 2*Sqrt[-d^2])] - 2*b^2*x^2*Log[1 + ((
1 + (c + d)^2)*E^(2*(a + b*x)))/(-1 - c^2 + d^2 + 2*Sqrt[-d^2])] + 2*b*x*P
olyLog[2, ((1 + c^2 + 2*c*d + d^2)*E^(2*(a + b*x)))/(1 + c^2 - d^2 + 2*Sqr
t[-d^2])] - 2*b*x*PolyLog[2, -(((1 + (c + d)^2)*E^(2*(a + b*x)))/(-1 - c^2
 + d^2 + 2*Sqrt[-d^2]))] + PolyLog[3, ((1 + c^2 + 2*c*d + d^2)*E^(2*(a + b
*x)))/(1 + c^2 - d^2 - 2*Sqrt[-d^2])] - PolyLog[3, ((1 + c^2 + 2*c*d + d^2
)*E^(2*(a + b*x)))/(1 + c^2 - d^2 + 2*Sqrt[-d^2])]))/(8*b^2*Sqrt[-d^2])
```

3.99.3 Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.28, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5724, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\arctan(d\coth(a+bx)+c)\,dx$$

$$\Big\downarrow \text{5724}$$

$$\frac{1}{2}b(1+i(c+d))\int\frac{e^{2a+2bx}x^2}{-c-(-c-d+i)e^{2a+2bx}+d+i}dx-\frac{1}{2}b(1-i(c+d))\int\frac{e^{2a+2bx}x^2}{c-(c+d+i)e^{2a+2bx}-d+i}dx+\frac{1}{2}x^2\arctan(d\coth(a+bx)+c)$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{2}b(1+i(c+d))\left(\frac{\int x\log\left(1-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)dx}{b(-c-d+i)}-\frac{x^2\log\left(1-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b(-c-d+i)}\right)-\frac{1}{2}b(1-i(c+d))\left(\frac{\int x\log\left(1-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)dx}{b(c+d+i)}-\frac{x^2\log\left(1-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b(c+d+i)}\right)+\frac{1}{2}x^2\arctan(d\coth(a+bx)+c)$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{2}b(1+i(c+d))\left(\frac{\frac{\int \text{PolyLog}\left(2,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)dx}{2b}-\frac{x\,\text{PolyLog}\left(2,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b}}{b(-c-d+i)}-\frac{x^2\log\left(1-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b(-c-d+i)}\right)-\frac{1}{2}b(1-i(c+d))\left(\frac{\frac{\int \text{PolyLog}\left(2,\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)dx}{2b}-\frac{x\,\text{PolyLog}\left(2,\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b}}{b(c+d+i)}-\frac{x^2\log\left(1-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b(c+d+i)}\right)+\frac{1}{2}x^2\arctan(d\coth(a+bx)+c)$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{2}b(1+i(c+d))\left(\frac{\frac{\int e^{-2a-2bx}\text{PolyLog}\left(2,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)de^{2a+2bx}}{4b^2}-\frac{x\,\text{PolyLog}\left(2,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b}}{b(-c-d+i)}-\frac{x^2\log\left(1-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b(-c-d+i)}\right)-\frac{1}{2}b(1-i(c+d))\left(\frac{\frac{\int e^{-2a-2bx}\text{PolyLog}\left(2,\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)de^{2a+2bx}}{4b^2}-\frac{x\,\text{PolyLog}\left(2,\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b}}{b(c+d+i)}-\frac{x^2\log\left(1-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b(c+d+i)}\right)+\frac{1}{2}x^2\arctan(d\coth(a+bx)+c)$$

$$\Big\downarrow \text{7143}$$

3.99. $\int x\arctan(c+d\coth(a+bx))\,dx$

$$\frac{1}{2}x^2\arctan(d\coth(a+bx)+c)+\frac{1}{2}b(1+i(c+d))\left(\frac{\frac{\operatorname{PolyLog}\left(3,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b^2}-\frac{x\operatorname{PolyLog}\left(2,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b}}{b(-c-d+i)}-\frac{x^2\log\left(1-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b(-c-d+i)}\right)-\frac{1}{2}b(1-i(c+d))\left(\frac{\frac{\operatorname{PolyLog}\left(3,\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b^2}-\frac{x\operatorname{PolyLog}\left(2,\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b}}{b(c+d+i)}-\frac{x^2\log\left(1-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b(c+d+i)}\right)$$

input
```
Int[x*ArcTan[c + d*Coth[a + b*x]],x]
```

output
```
(x^2*ArcTan[c + d*Coth[a + b*x]])/2 + (b*(1 + I*(c + d))*(-1/2*(x^2*Log[1
- ((I - c - d)*E^(2*a + 2*b*x))/(I - c + d)])/(b*(I - c - d)) + (-1/2*(x*P
olyLog[2, ((I - c - d)*E^(2*a + 2*b*x))/(I - c + d)])/b + PolyLog[3, ((I -
 c - d)*E^(2*a + 2*b*x))/(I - c + d)]/(4*b^2))/(b*(I - c - d))))/2 - (b*(1
 - I*(c + d))*(-1/2*(x^2*Log[1 - ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)
])/(b*(I + c + d)) + (-1/2*(x*PolyLog[2, ((I + c + d)*E^(2*a + 2*b*x))/(I
+ c - d)])/b + PolyLog[3, ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)]/(4*b^
2))/(b*(I + c + d))))/2
```

3.99.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5724
```
Int[ArcTan[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Coth[a + b*x]]/(f*(m
+ 1))), x] + (-Simp[I*b*((I - c - d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(
E^(2*a + 2*b*x)/(I - c + d - (I - c - d)*E^(2*a + 2*b*x))), x], x] + Simp[I
*b*((I + c + d)/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*(E^(2*a + 2*b*x)/(I +
c - d - (I + c + d)*E^(2*a + 2*b*x))), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[m, 0] && NeQ[(c - d)^2, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.99.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.42 (sec) , antiderivative size = 6495, normalized size of antiderivative = 24.51

method	result	size
risch	Expression too large to display	6495

input
```
int(x*arctan(c+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.99.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1051 vs. $2(195) = 390$.

Time = 0.37 (sec) , antiderivative size = 1051, normalized size of antiderivative = 3.97

$$\int x \arctan(c + d \coth(a + bx)) \, dx = \text{Too large to display}$$

input
```
integrate(x*arctan(c+d*coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/4*(2*b^2*x^2*arctan((d*cosh(b*x + a) + c*sinh(b*x + a))/sinh(b*x + a)) +
 2*I*b*x*dilog(sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh
(b*x + a) + sinh(b*x + a))) + 2*I*b*x*dilog(-sqrt((c^2 - d^2 + 2*I*d + 1)/
(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) - 2*I*b*x*dilog(
sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sin
h(b*x + a))) - 2*I*b*x*dilog(-sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d +
d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + I*a^2*log(2*(c^2 + 2*c*d + d^
2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) + 2*(c^2 -
d^2 - 2*I*d + 1)*sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) +
I*a^2*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 +
 1)*sinh(b*x + a) - 2*(c^2 - d^2 - 2*I*d + 1)*sqrt((c^2 - d^2 + 2*I*d + 1)
/(c^2 - 2*c*d + d^2 + 1))) - I*a^2*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x
+ a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) + 2*(c^2 - d^2 + 2*I*d + 1)
*sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) - I*a^2*log(2*(c^2
 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a
) - 2*(c^2 - d^2 + 2*I*d + 1)*sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d +
d^2 + 1))) + (I*b^2*x^2 - I*a^2)*log(sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2
*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b^2*x^2 - I*a^2
)*log(-sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a
) + sinh(b*x + a)) + 1) + (-I*b^2*x^2 + I*a^2)*log(sqrt((c^2 - d^2 - 2*...
```

3.99.6 Sympy [F(-1)]

Timed out.

$$\int x \arctan(c + d \coth(a + bx)) \, dx = \text{Timed out}$$

input
```
integrate(x*atan(c+d*coth(b*x+a)),x)
```

output
```
Timed out
```

3.99.7 Maxima [F]

$$\int x \arctan(c+d\coth(a+bx))\,dx=\int x\arctan\left(d\coth\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(x*arctan(c+d*coth(b*x+a)),x, algorithm="maxima")
```

output
```
1/2*x^2*arctan2((c*e^(2*a) + d*e^(2*a))*e^(2*b*x) - c + d, e^(2*b*x + 2*a)
 - 1) + 2*b*d*integrate(x^2*e^(2*b*x + 2*a)/(c^2 - 2*c*d + d^2 + (c^2*e^(4
*a) + 2*c*d*e^(4*a) + d^2*e^(4*a) + e^(4*a))*e^(4*b*x) - 2*(c^2*e^(2*a) -
d^2*e^(2*a) + e^(2*a))*e^(2*b*x) + 1), x)
```

3.99.8 Giac [F]

$$\int x \arctan(c+d\coth(a+bx))\,dx=\int x\arctan\left(d\coth\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(x*arctan(c+d*coth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*arctan(d*coth(b*x + a) + c), x)
```

3.99.9 Mupad [F(-1)]

Timed out.

$$\int x \arctan(c+d\coth(a+bx))\,dx=\int x\,\mathrm{atan}(c+d\coth(a+b\,x))\,dx$$

input
```
int(x*atan(c + d*coth(a + b*x)),x)
```

output
```
int(x*atan(c + d*coth(a + b*x)), x)
```

3.100 $\int \arctan(c + d\coth(a + bx))\,dx$

3.100.1 Optimal result

Integrand size = 11, antiderivative size = 174

$$\int \arctan(c + d\coth(a + bx))\,dx = x\arctan(c + d\coth(a + bx)) + \frac{1}{2}ix\log\left(1 - \frac{(i - c - d)e^{2a+2bx}}{i - c + d}\right) - \frac{1}{2}ix\log\left(1 - \frac{(i + c + d)e^{2a+2bx}}{i + c - d}\right) + \frac{i\,\mathrm{PolyLog}\left(2, \frac{(i-c-d)e^{2a+2bx}}{i-c+d}\right)}{4b} - \frac{i\,\mathrm{PolyLog}\left(2, \frac{(i+c+d)e^{2a+2bx}}{i+c-d}\right)}{4b}$$

output

```
x*arctan(c+d*coth(b*x+a))+1/2*I*x*ln(1-(I-c-d)*exp(2*b*x+2*a)/(I-c+d))-1/2
*I*x*ln(1-(I+c+d)*exp(2*b*x+2*a)/(I+c-d))+1/4*I*polylog(2,(I-c-d)*exp(2*b*
x+2*a)/(I-c+d))/b-1/4*I*polylog(2,(I+c+d)*exp(2*b*x+2*a)/(I+c-d))/b
```

3.100.2 Mathematica [A] (verified)

Time = 2.20 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.65

$$\int \arctan(c+d\coth(a+bx))\,dx = x\arctan(c+d\coth(a+bx)) + \frac{4a\sqrt{-d^2}\arctan\left(\frac{1+c^2-d^2-(1+c^2+2cd+d^2)e^{2(a+bx)}}{2d}\right)+2d(a+bx)\log\left(1-\frac{(1+(c+d)^2)e^{2(a+bx)}}{1+c^2-d^2+2\sqrt{-d^2}}\right)-2d(a+bx)\log}{4b\sqrt{-}}$$

input
```
Integrate[ArcTan[c + d*Coth[a + b*x]],x]
```

output
```
x*ArcTan[c + d*Coth[a + b*x]] + (4*a*Sqrt[-d^2]*ArcTan[(1 + c^2 - d^2 - (1
 + c^2 + 2*c*d + d^2)*E^(2*(a + b*x)))/(2*d)] + 2*d*(a + b*x)*Log[1 - ((1
+ (c + d)^2)*E^(2*(a + b*x)))/(1 + c^2 - d^2 + 2*Sqrt[-d^2])] - 2*d*(a + b
*x)*Log[1 + ((1 + (c + d)^2)*E^(2*(a + b*x)))/(-1 - c^2 + d^2 + 2*Sqrt[-d^
2])] + d*PolyLog[2, ((1 + c^2 + 2*c*d + d^2)*E^(2*(a + b*x)))/(1 + c^2 - d
^2 + 2*Sqrt[-d^2])] - d*PolyLog[2, -(((1 + c^2 + 2*c*d + d^2)*E^(2*(a + b*
x)))/(-1 - c^2 + d^2 + 2*Sqrt[-d^2]))])/(4*b*Sqrt[-d^2])
```

3.100.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.36, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {5716, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(d\coth(a+bx)+c)\,dx$$

$$\Big\downarrow \text{5716}$$

$$b(1+i(c+d))\int \frac{e^{2a+2bx}x}{-c-(-c-d+i)e^{2a+2bx}+d+i}dx - b(1-i(c+d))\int \frac{e^{2a+2bx}x}{c-(c+d+i)e^{2a+2bx}-d+i}dx + x\arctan(d\coth(a+bx)+c)$$

$$\Big\downarrow \text{2620}$$

$$b(1+i(c+d))\left(\frac{\int \log\left(1-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)dx}{2b(-c-d+i)}-\frac{x\log\left(1-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b(-c-d+i)}\right)-b(1-i(c+d))\left(\frac{\int \log\left(1-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)dx}{2b(c+d+i)}-\frac{x\log\left(1-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b(c+d+i)}\right)+x\arctan(d\coth(a+bx)+c)$$

$$\Big\downarrow \text{2715}$$

$$b(1+i(c+d))\left(\frac{\int e^{-2a-2bx}\log\left(1-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)de^{2a+2bx}}{4b^2(-c-d+i)}-\frac{x\log\left(1-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b(-c-d+i)}\right)-b(1-i(c+d))\left(\frac{\int e^{-2a-2bx}\log\left(1-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)de^{2a+2bx}}{4b^2(c+d+i)}-\frac{x\log\left(1-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b(c+d+i)}\right)+x\arctan(d\coth(a+bx)+c)$$

$$\Big\downarrow \text{2838}$$

$$x\arctan(d\coth(a+bx)+c)+b(1+i(c+d))\left(-\frac{\text{PolyLog}\left(2,\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{4b^2(-c-d+i)}-\frac{x\log\left(1-\frac{(-c-d+i)e^{2a+2bx}}{-c+d+i}\right)}{2b(-c-d+i)}\right)-b(1-i(c+d))\left(-\frac{\text{PolyLog}\left(2,\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{4b^2(c+d+i)}-\frac{x\log\left(1-\frac{(c+d+i)e^{2a+2bx}}{c-d+i}\right)}{2b(c+d+i)}\right)$$

input
```
Int[ArcTan[c + d*Coth[a + b*x]],x]
```

output
```
x*ArcTan[c + d*Coth[a + b*x]] + b*(1 + I*(c + d))*(-1/2*(x*Log[1 - ((I - c
 - d)*E^(2*a + 2*b*x))/(I - c + d)])/(b*(I - c - d)) - PolyLog[2, ((I - c
- d)*E^(2*a + 2*b*x))/(I - c + d)]/(4*b^2*(I - c - d))) - b*(1 - I*(c + d)
)*(-1/2*(x*Log[1 - ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)])/(b*(I + c +
 d)) - PolyLog[2, ((I + c + d)*E^(2*a + 2*b*x))/(I + c - d)]/(4*b^2*(I + c
 + d)))
```

3.100. $\int \arctan(c+d\coth(a+bx))\,dx$

3.100.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5716
```
Int[ArcTan[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Arc
Tan[c + d*Coth[a + b*x]], x] + (-Simp[I*b*(I - c - d)   Int[x*(E^(2*a + 2*b
*x)/(I - c + d - (I - c - d)*E^(2*a + 2*b*x))), x], x] + Simp[I*b*(I + c +
d)   Int[x*(E^(2*a + 2*b*x)/(I + c - d - (I + c + d)*E^(2*a + 2*b*x))), x],
 x]) /; FreeQ[{a, b, c, d}, x] && NeQ[(c - d)^2, -1]
```

3.100.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(150) = 300$.

Time = 2.57 (sec) , antiderivative size = 352, normalized size of antiderivative = 2.02

method	result
derivativedivides	$\frac{\arctan(c+d\coth(bx+a))d\ln(-d\coth(bx+a)-d)}{2}-\frac{\arctan(c+d\coth(bx+a))d\ln(-d\coth(bx+a)+d)}{2}+\frac{d^2\left(\frac{i\ln(-d\coth(bx+a)+d)\ln\left(\frac{i\cdots}{\cdots}\right.}{2}\right.}{\cdots}$
default	$\frac{\arctan(c+d\coth(bx+a))d\ln(-d\coth(bx+a)-d)}{2}-\frac{\arctan(c+d\coth(bx+a))d\ln(-d\coth(bx+a)+d)}{2}+\frac{d^2\left(\frac{i\ln(-d\coth(bx+a)+d)\ln\left(\frac{i\cdots}{\cdots}\right.}{2}\right.}{\cdots}$
risch	Expression too large to display

input
```
int(arctan(c+d*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b/d*(1/2*arctan(c+d*coth(b*x+a))*d*ln(-d*coth(b*x+a)-d)-1/2*arctan(c+d*c
oth(b*x+a))*d*ln(-d*coth(b*x+a)+d)+1/2*d^2*(1/d*(1/2*I*ln(-d*coth(b*x+a)+d
)*ln((I+d*coth(b*x+a)+c)/(I+c+d))-1/2*I*ln(-d*coth(b*x+a)+d)*ln((I-d*coth(
b*x+a)-c)/(I-c-d))+1/2*I*dilog((I+d*coth(b*x+a)+c)/(I+c+d))-1/2*I*dilog((I
-d*coth(b*x+a)-c)/(I-c-d)))-1/d*(1/2*I*ln(-d*coth(b*x+a)-d)*ln((I+d*coth(b
*x+a)+c)/(I+c-d))-1/2*I*ln(-d*coth(b*x+a)-d)*ln((I-d*coth(b*x+a)-c)/(I-c+d
))+1/2*I*dilog((I+d*coth(b*x+a)+c)/(I+c-d))-1/2*I*dilog((I-d*coth(b*x+a)-c
)/(I-c+d))))
```

3.100.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 813 vs. $2(128) = 256$.

Time = 0.39 (sec) , antiderivative size = 813, normalized size of antiderivative = 4.67

$$\int \arctan(c + d\coth(a + bx))\,dx$$
$$= \frac{2\,bx\arctan\left(\frac{d\cosh(bx+a)+c\sinh(bx+a)}{\sinh(bx+a)}\right) - i\,a\log\left(2\left(c^2+2\,cd+d^2+1\right)\cosh\left(bx+a\right)+2\left(c^2+2\,cd+d^2+1\right)\right.}{}$$

input
```
integrate(arctan(c+d*coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*(2*b*x*arctan((d*cosh(b*x + a) + c*sinh(b*x + a))/sinh(b*x + a)) - I*a
*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*s
inh(b*x + a) + 2*(c^2 - d^2 - 2*I*d + 1)*sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2
 - 2*c*d + d^2 + 1))) - I*a*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) +
2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) - 2*(c^2 - d^2 - 2*I*d + 1)*sqrt((
c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))) + I*a*log(2*(c^2 + 2*c*d
+ d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2 + 1)*sinh(b*x + a) + 2*(c^
2 - d^2 + 2*I*d + 1)*sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))
) + I*a*log(2*(c^2 + 2*c*d + d^2 + 1)*cosh(b*x + a) + 2*(c^2 + 2*c*d + d^2
 + 1)*sinh(b*x + a) - 2*(c^2 - d^2 + 2*I*d + 1)*sqrt((c^2 - d^2 - 2*I*d +
1)/(c^2 - 2*c*d + d^2 + 1))) + (I*b*x + I*a)*log(sqrt((c^2 - d^2 + 2*I*d +
 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (I*b*x
 + I*a)*log(-sqrt((c^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b
*x + a) + sinh(b*x + a)) + 1) + (-I*b*x - I*a)*log(sqrt((c^2 - d^2 - 2*I*d
 + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a)) + 1) + (-I*
b*x - I*a)*log(-sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cos
h(b*x + a) + sinh(b*x + a)) + 1) + I*dilog(sqrt((c^2 - d^2 + 2*I*d + 1)/(c
^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x + a))) + I*dilog(-sqrt((c
^2 - d^2 + 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(cosh(b*x + a) + sinh(b*x +
 a))) - I*dilog(sqrt((c^2 - d^2 - 2*I*d + 1)/(c^2 - 2*c*d + d^2 + 1))*(...
```

3.100.6 Sympy [F(-1)]

Timed out.

$$\int \arctan(c + d\coth(a + bx))\,dx = \text{Timed out}$$

input
```
integrate(atan(c+d*coth(b*x+a)),x)
```

output
```
Timed out
```

3.100.7 Maxima [F]

$$\int \arctan(c+d\coth(a+bx))\,dx=\int \arctan\left(d\coth\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(arctan(c+d*coth(b*x+a)),x, algorithm="maxima")
```

output
```
4*b*d*integrate(x*e^(2*b*x + 2*a)/(c^2 - 2*c*d + d^2 + (c^2*e^(4*a) + 2*c*
d*e^(4*a) + d^2*e^(4*a) + e^(4*a))*e^(4*b*x) - 2*(c^2*e^(2*a) - d^2*e^(2*a
) + e^(2*a))*e^(2*b*x) + 1), x) + x*arctan2((c*e^(2*a) + d*e^(2*a))*e^(2*b
*x) - c + d, e^(2*b*x + 2*a) - 1)
```

3.100.8 Giac [F]

$$\int \arctan(c+d\coth(a+bx))\,dx=\int \arctan\left(d\coth\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(arctan(c+d*coth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arctan(d*coth(b*x + a) + c), x)
```

3.100.9 Mupad [F(-1)]

Timed out.

$$\int \arctan(c+d\coth(a+bx))\,dx=\int \mathrm{atan}(c+d\coth(a+b\,x))\,dx$$

input
```
int(atan(c + d*coth(a + b*x)),x)
```

output
```
int(atan(c + d*coth(a + b*x)), x)
```

3.101 $\int \frac{\arctan(c+d\coth(a+bx))}{x}\,dx$

3.101.1 Optimal result

Integrand size = 15, antiderivative size = 15

$$\int \frac{\arctan(c+d\coth(a+bx))}{x}\,dx = \text{Int}\left(\frac{\arctan(c+d\coth(a+bx))}{x},x\right)$$

output
```
CannotIntegrate(arctan(c+d*coth(b*x+a))/x,x)
```

3.101.2 Mathematica [N/A]

Not integrable

Time = 6.78 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\arctan(c+d\coth(a+bx))}{x}\,dx = \int \frac{\arctan(c+d\coth(a+bx))}{x}\,dx$$

input
```
Integrate[ArcTan[c + d*Coth[a + b*x]]/x,x]
```

output
```
Integrate[ArcTan[c + d*Coth[a + b*x]]/x, x]
```

3.101.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(d\coth(a+bx)+c)}{x} \, dx$$

$$\Big\downarrow \; 7299$$

$$\int \frac{\arctan(d\coth(a+bx)+c)}{x} dx$$

input
```
Int[ArcTan[c + d*Coth[a + b*x]]/x,x]
```

output
```
$Aborted
```

3.101.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.101.4 Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\arctan\left(c+d\coth\left(bx+a\right)\right)}{x}dx$$

input
```
int(arctan(c+d*coth(b*x+a))/x,x)
```

output
```
int(arctan(c+d*coth(b*x+a))/x,x)
```

3.101.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\arctan(c + d\coth(a + bx))}{x}\,dx = \int \frac{\arctan\left(d\coth\left(bx + a\right) + c\right)}{x}\,dx$$

input
```
integrate(arctan(c+d*coth(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(arctan(d*coth(b*x + a) + c)/x, x)
```

3.101.6 Sympy [N/A]

Not integrable

Time = 161.55 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

$$\int \frac{\arctan(c + d\coth(a + bx))}{x}\,dx = \int \frac{\operatorname{atan}\left(c + d\coth\left(a + bx\right)\right)}{x}\,dx$$

input
```
integrate(atan(c+d*coth(b*x+a))/x,x)
```

output
```
Integral(atan(c + d*coth(a + b*x))/x, x)
```

3.101.7 Maxima [N/A]

Not integrable

Time = 1.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\arctan(c + d\coth(a + bx))}{x}\,dx = \int \frac{\arctan\left(d\coth\left(bx + a\right) + c\right)}{x}\,dx$$

input
```
integrate(arctan(c+d*coth(b*x+a))/x,x, algorithm="maxima")
```

output
```
integrate(arctan(d*coth(b*x + a) + c)/x, x)
```

3.101.8 Giac [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\arctan(c + d\coth(a + bx))}{x}\,dx = \int \frac{\arctan\left(d\coth\left(bx + a\right) + c\right)}{x}\,dx$$

input
```
integrate(arctan(c+d*coth(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(arctan(d*coth(b*x + a) + c)/x, x)
```

3.101.9 Mupad [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\arctan(c + d\coth(a + bx))}{x}\,dx = \int \frac{\operatorname{atan}(c + d\coth(a + b\,x))}{x}\,dx$$

input
```
int(atan(c + d*coth(a + b*x))/x,x)
```

output
```
int(atan(c + d*coth(a + b*x))/x, x)
```

3.102 $\int x^2 \arctan(c+(i+c)\coth(a+bx))\,dx$

3.102.1 Optimal result

Integrand size = 19, antiderivative size = 142

$$\int x^2 \arctan(c+(i+c)\coth(a+bx))\,dx = -\frac{1}{12}ibx^4+\frac{1}{3}x^3\arctan(c+(i+c)\coth(a+bx))+\frac{1}{6}ix^3\log\left(1-ice^{2a+2bx}\right) + \frac{ix^2\operatorname{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}-\frac{ix\operatorname{PolyLog}\left(3,ice^{2a+2bx}\right)}{4b^2}+\frac{i\operatorname{PolyLog}\left(4,ice^{2a+2bx}\right)}{8b^3}$$

output
```
-1/12*I*b*x^4+1/3*x^3*arctan(c+(I+c)*coth(b*x+a))+1/6*I*x^3*ln(1-I*c*exp(2
*b*x+2*a))+1/4*I*x^2*polylog(2,I*c*exp(2*b*x+2*a))/b-1/4*I*x*polylog(3,I*c
*exp(2*b*x+2*a))/b^2+1/8*I*polylog(4,I*c*exp(2*b*x+2*a))/b^3
```

3.102.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.94

$$\int x^2 \arctan(c+(i+c)\coth(a+bx))\,dx = \frac{8b^3x^3\arctan(c+(i+c)\coth(a+bx))+4ib^3x^3\log\left(1+\frac{ie^{-2(a+bx)}}{c}\right)-6ib^2x^2\operatorname{PolyLog}\left(2,-\frac{ie^{-2(a+bx)}}{c}\right)-6i}{24b^3}$$

input
```
Integrate[x^2*ArcTan[c + (I + c)*Coth[a + b*x]],x]
```

output
```
(8*b^3*x^3*ArcTan[c + (I + c)*Coth[a + b*x]] + (4*I)*b^3*x^3*Log[1 + I/(c*
E^(2*(a + b*x)))] - (6*I)*b^2*x^2*PolyLog[2, (-I)/(c*E^(2*(a + b*x)))] - (
6*I)*b*x*PolyLog[3, (-I)/(c*E^(2*(a + b*x)))] - (3*I)*PolyLog[4, (-I)/(c*E
^(2*(a + b*x)))])/(24*b^3)
```

3.102.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.18, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {5720, 25, 2615, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(c+(c+i)\coth(a+bx))\,dx$$

$$\Big\downarrow \text{ 5720}$$

$$\frac{1}{3}x^3\arctan(c+(c+i)\coth(a+bx))-\frac{1}{3}b\int -\frac{x^3}{e^{2a+2bx}c+i}dx$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{3}b\int \frac{x^3}{e^{2a+2bx}c+i}dx+\frac{1}{3}x^3\arctan(c+(c+i)\coth(a+bx))$$

$$\Big\downarrow \text{ 2615}$$

$$\frac{1}{3}b\left(ic\int \frac{e^{2a+2bx}x^3}{e^{2a+2bx}c+i}dx-\frac{ix^4}{4}\right)+\frac{1}{3}x^3\arctan(c+(c+i)\coth(a+bx))$$

$$\Big\downarrow \text{ 2620}$$

$$\frac{1}{3}b\left(ic\left(\frac{x^3\log\left(1-ice^{2a+2bx}\right)}{2bc}-\frac{3\int x^2\log\left(1-ice^{2a+2bx}\right)dx}{2bc}\right)-\frac{ix^4}{4}\right)+\frac{1}{3}x^3\arctan(c+(c+i)\coth(a+bx))$$

$$\Big\downarrow \text{ 3011}$$

$$\frac{1}{3}b\left(ic\left(\frac{x^3\log\left(1-ice^{2a+2bx}\right)}{2bc}-\frac{3\left(\frac{\int x\,\text{PolyLog}(2,ice^{2a+2bx})dx}{b}-\frac{x^2\,\text{PolyLog}(2,ice^{2a+2bx})}{2b}\right)}{2bc}\right)-\frac{ix^4}{4}\right)+\frac{1}{3}x^3\arctan(c+(c+i)\coth(a+bx))$$

$$\Big\downarrow \text{ 7163}$$

$$\frac{1}{3}b\left(ic\left(\frac{x^3\log\left(1-ice^{2a+2bx}\right)}{2bc}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,ice^{2a+2bx}\right)}{2b}-\frac{\int\text{PolyLog}\left(3,ice^{2a+2bx}\right)dx}{2b}}{b}-\frac{x^2\,\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{2b}\right)}{2bc}\right)-\frac{ix^4}{4}\right)+$$

$$\frac{1}{3}x^3\arctan(c+(c+i)\coth(a+bx))$$

$$\Big\downarrow \; 2720$$

$$\frac{1}{3}b\left(ic\left(\frac{x^3\log\left(1-ice^{2a+2bx}\right)}{2bc}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,ice^{2a+2bx}\right)}{2b}-\frac{\int e^{-2a-2bx}\,\text{PolyLog}\left(3,ice^{2a+2bx}\right)de^{2a+2bx}}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{2b}\right)}{2bc}\right.\right.$$

$$\frac{1}{3}x^3\arctan(c+(c+i)\coth(a+bx))$$

$$\Big\downarrow \; 7143$$

$$\frac{1}{3}x^3\arctan(c+(c+i)\coth(a+bx))+$$

$$\frac{1}{3}b\left(ic\left(\frac{x^3\log\left(1-ice^{2a+2bx}\right)}{2bc}-\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,ice^{2a+2bx}\right)}{2b}-\frac{\text{PolyLog}\left(4,ice^{2a+2bx}\right)}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{2b}\right)}{2bc}\right)-\frac{ix^4}{4}\right)$$

input
```
Int[x^2*ArcTan[c + (I + c)*Coth[a + b*x]],x]
```

output
```
(x^3*ArcTan[c + (I + c)*Coth[a + b*x]])/3 + (b*((-1/4*I)*x^4 + I*c*((x^3*L
og[1 - I*c*E^(2*a + 2*b*x)])/(2*b*c) - (3*(-1/2*(x^2*PolyLog[2, I*c*E^(2*a
 + 2*b*x)])/b + ((x*PolyLog[3, I*c*E^(2*a + 2*b*x)])/(2*b) - PolyLog[4, I*
c*E^(2*a + 2*b*x)]/(4*b^2))/b))/(2*b*c))))/3
```

3.102. $\int x^2\arctan(c+(i+c)\coth(a+bx))\,dx$

3.102.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5720
```
Int[ArcTan[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Coth[a + b*x]]/(f*(m
+ 1))), x] - Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a +
 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
d)^2, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.102.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.72 (sec) , antiderivative size = 1405, normalized size of antiderivative = 9.89

method	result	size
risch	Expression too large to display	1405

input
```
int(x^2*arctan(c+(I+c)*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/3*I/b^3*ln(1-I*c*exp(2*b*x+2*a))*a^3+1/2*I/b^3*a^3*ln(1-I*exp(b*x+a)*(-
I*c)^(1/2))+1/12*I/b^3*c/(I+c)*a^4-1/4*I*x*polylog(3,I*c*exp(2*b*x+2*a))/b
^2+1/8*I*polylog(4,I*c*exp(2*b*x+2*a))/b^3+1/2*I/b^3*a^2*dilog(1-I*exp(b*x
+a)*(-I*c)^(1/2))-1/6*I*x^3*ln(2*exp(2*b*x+2*a)*c+2*I)+1/2*I/b^3*a^2*dilog
(1+I*exp(b*x+a)*(-I*c)^(1/2))-1/12*I*b*c/(I+c)*x^4+1/2*I/b^2*a^2*ln(1-I*ex
p(b*x+a)*(-I*c)^(1/2))*x+1/2*I/b^3*a^3*ln(1+I*exp(b*x+a)*(-I*c)^(1/2))-1/4
*I/b^3*polylog(2,I*c*exp(2*b*x+2*a))*a^2+1/6*I*x^3*ln(1-I*c*exp(2*b*x+2*a)
)+1/4*I*x^2*polylog(2,I*c*exp(2*b*x+2*a))/b+1/6*I*x^3*ln(2*I*exp(2*b*x+2*a
)+2*exp(2*b*x+2*a)*c)-1/6*I/b^3*a^3*ln(exp(2*b*x+2*a)*c+I)-1/12*Pi*(csgn(I
/(exp(2*b*x+2*a)-1))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I))*csgn(I*(2*exp(2*b*x+
2*a)*c+2*I)/(exp(2*b*x+2*a)-1))-csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*I*exp
(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*
a)*c)/(exp(2*b*x+2*a)-1))-csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*exp(2*b*x+2
*a)*c+2*I)/(exp(2*b*x+2*a)-1))^2+csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*I*ex
p(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2-csgn(I*(2*exp(2*b*x
+2*a)*c+2*I))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))^2+csgn(I
*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(2*I*exp(2*b*x+2*a)+2*exp
(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2+csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(
2*b*x+2*a)-1))^3-csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))*csgn(
(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))^2+csgn(I*(2*exp(2*b*x+2*a)...
```

3.102.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 293 vs. 2(105) = 210.

Time = 0.27 (sec) , antiderivative size = 293, normalized size of antiderivative = 2.06

$$\int x^2 \arctan(c + (i + c) \coth(a + bx)) \, dx = \frac{-i b^4 x^4 + 2i b^3 x^3 \log\left(-\frac{(c+i)e^{(2bx+2a)}}{ce^{(2bx+2a)}+i}\right) + 6i b^2 x^2 \mathrm{Li}_2\left(\frac{1}{2}\sqrt{4i c}e^{(bx+a)}\right) + 6i b^2 x^2 \mathrm{Li}_2\left(-\frac{1}{2}\sqrt{4i c}e^{(bx+a)}\right) + i a^4 -$$

input
```
integrate(x^2*arctan(c+(I+c)*coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/12*(-I*b^4*x^4 + 2*I*b^3*x^3*log(-(c + I)*e^(2*b*x + 2*a)/(c*e^(2*b*x +
2*a) + I)) + 6*I*b^2*x^2*dilog(1/2*sqrt(4*I*c)*e^(b*x + a)) + 6*I*b^2*x^2*
dilog(-1/2*sqrt(4*I*c)*e^(b*x + a)) + I*a^4 - 2*I*a^3*log(1/2*(2*c*e^(b*x
+ a) + I*sqrt(4*I*c))/c) - 2*I*a^3*log(1/2*(2*c*e^(b*x + a) - I*sqrt(4*I*c
))/c) - 12*I*b*x*polylog(3, 1/2*sqrt(4*I*c)*e^(b*x + a)) - 12*I*b*x*polylo
g(3, -1/2*sqrt(4*I*c)*e^(b*x + a)) - 2*(-I*b^3*x^3 - I*a^3)*log(1/2*sqrt(4
*I*c)*e^(b*x + a) + 1) - 2*(-I*b^3*x^3 - I*a^3)*log(-1/2*sqrt(4*I*c)*e^(b*
x + a) + 1) + 12*I*polylog(4, 1/2*sqrt(4*I*c)*e^(b*x + a)) + 12*I*polylog(
4, -1/2*sqrt(4*I*c)*e^(b*x + a)))/b^3
```

3.102.6 Sympy [F(-2)]

Exception generated.

$$\int x^2 \arctan(c + (i + c) \coth(a + bx)) \, dx = \text{Exception raised: CoercionFailed}$$

input
```
integrate(x**2*atan(c+(I+c)*coth(b*x+a)),x)
```

output
```
Exception raised: CoercionFailed >> Cannot convert _t0**2*exp(2*a) - 1 of
type <class 'sympy.core.add.Add'> to QQ_I[x,b,_t0,exp(a)]
```

3.102.7 Maxima [A] (verification not implemented)

Time = 1.24 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.91

$$\int x^2 \arctan(c+(i+c)\coth(a+bx))\,dx = \frac{1}{3}x^3 \arctan\left((c+i)\coth\left(bx+a\right)+c\right) + \frac{4}{9}\left(\frac{3x^4}{4ic-4} - \frac{4b^3x^3\log\left(-ice^{(2bx+2a)}+1\right)+6b^2x^2\mathrm{Li}_2\left(ice^{(2bx+2a)}\right)-6bx\mathrm{Li}_3(ice^{(2bx+2a)})+3\,\mathrm{Li}_4(ice^{(2bx+2a)})}{-2b^4(-ic+1)}\right.$$

input
```
integrate(x^2*arctan(c+(I+c)*coth(b*x+a)),x, algorithm="maxima")
```

output
```
1/3*x^3*arctan((c + I)*coth(b*x + a) + c) + 4/9*(3*x^4/(4*I*c - 4) - (4*b^
3*x^3*log(-I*c*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(I*c*e^(2*b*x + 2*a))
 - 6*b*x*polylog(3, I*c*e^(2*b*x + 2*a)) + 3*polylog(4, I*c*e^(2*b*x + 2*a
)))/(b^4*(2*I*c - 2)))*b*(c + I)
```

3.102.8 Giac [F]

$$\int x^2 \arctan(c+(i+c)\coth(a+bx))\,dx = \int x^2 \arctan\left((c+i)\coth\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(x^2*arctan(c+(I+c)*coth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*arctan((c + I)*coth(b*x + a) + c), x)
```

3.102.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \arctan(c+(i+c)\coth(a+bx))\,dx = \int x^2 \operatorname{atan}(c+\coth(a+b\,x)\,(c+1\mathrm{i}))\,dx$$

input
```
int(x^2*atan(c + coth(a + b*x)*(c + 1i)),x)
```

output
```
int(x^2*atan(c + coth(a + b*x)*(c + 1i)), x)
```

3.103 $\int x \arctan(c+(i+c)\coth(a+bx))\,dx$

3.103.1 Optimal result

Integrand size = 17, antiderivative size = 113

$$\int x\arctan(c+(i+c)\coth(a+bx))\,dx=-\frac{1}{6}ibx^3+\frac{1}{2}x^2\arctan(c+(i+c)\coth(a+bx))+\frac{1}{4}ix^2\log\left(1-ice^{2a+2bx}\right)+\frac{ix\operatorname{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}-\frac{i\operatorname{PolyLog}\left(3,ice^{2a+2bx}\right)}{8b^2}$$

output
```
-1/6*I*b*x^3+1/2*x^2*arctan(c+(I+c)*coth(b*x+a))+1/4*I*x^2*ln(1-I*c*exp(2*
b*x+2*a))+1/4*I*x*polylog(2,I*c*exp(2*b*x+2*a))/b-1/8*I*polylog(3,I*c*exp(
2*b*x+2*a))/b^2
```

3.103.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.91

$$\int x\arctan(c+(i+c)\coth(a+bx))\,dx=\frac{2b^2x^2\left(2\arctan(c+(i+c)\coth(a+bx))+i\log\left(1+\frac{ie^{-2(a+bx)}}{c}\right)\right)-2ibx\operatorname{PolyLog}\left(2,-\frac{ie^{-2(a+bx)}}{c}\right)-i\operatorname{Poly}}{8b^2}$$

input
```
Integrate[x*ArcTan[c + (I + c)*Coth[a + b*x]],x]
```

output
```
(2*b^2*x^2*(2*ArcTan[c + (I + c)*Coth[a + b*x]] + I*Log[1 + I/(c*E^(2*(a +
 b*x)))]) - (2*I)*b*x*PolyLog[2, (-I)/(c*E^(2*(a + b*x)))] - I*PolyLog[3,
(-I)/(c*E^(2*(a + b*x)))])/(8*b^2)
```

3.103.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {5720, 25, 2615, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan(c+(c+i)\coth(a+bx))\,dx$$

$$\Big\downarrow \text{5720}$$

$$\frac{1}{2}x^2\arctan(c+(c+i)\coth(a+bx))-\frac{1}{2}b\int -\frac{x^2}{e^{2a+2bx}c+i}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}b\int \frac{x^2}{e^{2a+2bx}c+i}dx+\frac{1}{2}x^2\arctan(c+(c+i)\coth(a+bx))$$

$$\Big\downarrow \text{2615}$$

$$\frac{1}{2}b\left(ic\int \frac{e^{2a+2bx}x^2}{e^{2a+2bx}c+i}dx-\frac{ix^3}{3}\right)+\frac{1}{2}x^2\arctan(c+(c+i)\coth(a+bx))$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{2}b\left(ic\left(\frac{x^2\log\left(1-ice^{2a+2bx}\right)}{2bc}-\frac{\int x\log\left(1-ice^{2a+2bx}\right)dx}{bc}\right)-\frac{ix^3}{3}\right)+\frac{1}{2}x^2\arctan(c+(c+i)\coth(a+bx))$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{2}b\left(ic\left(\frac{x^2\log\left(1-ice^{2a+2bx}\right)}{2bc}-\frac{\frac{\int \text{PolyLog}(2,ice^{2a+2bx})dx}{2b}-\frac{x\,\text{PolyLog}(2,ice^{2a+2bx})}{2b}}{bc}\right)-\frac{ix^3}{3}\right)+\frac{1}{2}x^2\arctan(c+(c+i)\coth(a+bx))$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{2}b\left(ic\left(\frac{x^2\log\left(1-ice^{2a+2bx}\right)}{2bc}-\frac{\frac{\int e^{-2a-2bx}\,\text{PolyLog}\left(2,ice^{2a+2bx}\right)de^{2a+2bx}}{4b^2}-\frac{x\,\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{2b}}{bc}\right)-\frac{ix^3}{3}\right)+\frac{1}{2}x^2\arctan(c+(c+i)\coth(a+bx))$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{2}x^2\arctan(c+(c+i)\coth(a+bx))+\frac{1}{2}b\left(ic\left(\frac{x^2\log\left(1-ice^{2a+2bx}\right)}{2bc}-\frac{\frac{\text{PolyLog}\left(3,ice^{2a+2bx}\right)}{4b^2}-\frac{x\,\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{2b}}{bc}\right)-\frac{ix^3}{3}\right)$$

input
```
Int[x*ArcTan[c + (I + c)*Coth[a + b*x]],x]
```

output
```
(x^2*ArcTan[c + (I + c)*Coth[a + b*x]])/2 + (b*((-1/3*I)*x^3 + I*c*((x^2*L
og[1 - I*c*E^(2*a + 2*b*x)])/(2*b*c) - (-1/2*(x*PolyLog[2, I*c*E^(2*a + 2*
b*x)])/b + PolyLog[3, I*c*E^(2*a + 2*b*x)]/(4*b^2))/(b*c))))/2
```

3.103.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5720
```
Int[ArcTan[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Coth[a + b*x]]/(f*(m
+ 1))), x] - Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a +
 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
d)^2, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.103.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.01 (sec) , antiderivative size = 1369, normalized size of antiderivative = 12.12

method	result	size
risch	Expression too large to display	1369

input
```
int(x*arctan(c+(I+c)*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/4*I*x^2*ln(1-I*c*exp(2*b*x+2*a))-1/6*I/b^2*c/(I+c)*a^3+1/2*I/b*ln(1-I*c*
exp(2*b*x+2*a))*a*x-1/6*I*b*c/(I+c)*x^3-1/2*I/b*a*ln(1-I*exp(b*x+a)*(-I*c)
^(1/2))*x+1/4*I/b^2*ln(1-I*c*exp(2*b*x+2*a))*a^2+1/4*I/b^2*polylog(2,I*c*e
xp(2*b*x+2*a))*a+1/4*I*x^2*ln(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)-1/2*I
/b^2*a*dilog(1+I*exp(b*x+a)*(-I*c)^(1/2))-1/4*I*x^2*ln(2*exp(2*b*x+2*a)*c+
2*I)+1/4*I/b^2*a^2*ln(exp(2*b*x+2*a)*c+I)-1/2*I/b^2*ln(1-I*exp(b*x+a)*(-I*
c)^(1/2))*a^2-1/8*I*polylog(3,I*c*exp(2*b*x+2*a))/b^2-1/2*I/b^2*ln(1+I*exp
(b*x+a)*(-I*c)^(1/2))*a^2-1/2*I/b^2*a*dilog(1-I*exp(b*x+a)*(-I*c)^(1/2))-1
/8*Pi*(csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I))*csgn(I*
(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))-csgn(I/(exp(2*b*x+2*a)-1))*cs
gn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(2*I*exp(2*b*x+2*a)+2
*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))-csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(
2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))^2+csgn(I/(exp(2*b*x+2*a)-1))*c
sgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2-csgn(I
*(2*exp(2*b*x+2*a)*c+2*I))*csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)
-1))^2+csgn(I*(2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(2*I*exp(2*b
*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2+csgn(I*(2*exp(2*b*x+2*a)
*c+2*I)/(exp(2*b*x+2*a)-1))^3-csgn(I*(2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2
*a)-1))*csgn((2*exp(2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))^2+csgn(I*(2*exp(
2*b*x+2*a)*c+2*I)/(exp(2*b*x+2*a)-1))*csgn((2*exp(2*b*x+2*a)*c+2*I)/(ex...
```

3.103.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(83) = 166$.

Time = 0.28 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.19

$$\int x\arctan(c+(i+c)\coth(a+bx))\,dx = \frac{-2i\,b^3x^3 + 3i\,b^2x^2\log\left(-\frac{(c+i)e^{(2\,bx+2\,a)}}{ce^{(2\,bx+2\,a)}+i}\right) - 2i\,a^3 + 6i\,bx\mathrm{Li}_2\left(\frac{1}{2}\sqrt{4i\,c}e^{(bx+a)}\right) + 6i\,bx\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{4i\,c}e^{(bx+a)}\right) +}{}$$

input
```
integrate(x*arctan(c+(I+c)*coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/12*(-2*I*b^3*x^3 + 3*I*b^2*x^2*log(-(c + I)*e^(2*b*x + 2*a)/(c*e^(2*b*x
+ 2*a) + I)) - 2*I*a^3 + 6*I*b*x*dilog(1/2*sqrt(4*I*c)*e^(b*x + a)) + 6*I*
b*x*dilog(-1/2*sqrt(4*I*c)*e^(b*x + a)) + 3*I*a^2*log(1/2*(2*c*e^(b*x + a)
 + I*sqrt(4*I*c))/c) + 3*I*a^2*log(1/2*(2*c*e^(b*x + a) - I*sqrt(4*I*c))/c
) - 3*(-I*b^2*x^2 + I*a^2)*log(1/2*sqrt(4*I*c)*e^(b*x + a) + 1) - 3*(-I*b^
2*x^2 + I*a^2)*log(-1/2*sqrt(4*I*c)*e^(b*x + a) + 1) - 6*I*polylog(3, 1/2*
sqrt(4*I*c)*e^(b*x + a)) - 6*I*polylog(3, -1/2*sqrt(4*I*c)*e^(b*x + a)))/b
^2
```

3.103.6 Sympy [F(-2)]

Exception generated.

$$\int x\arctan(c+(i+c)\coth(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

input
```
integrate(x*atan(c+(I+c)*coth(b*x+a)),x)
```

output
```
Exception raised: CoercionFailed >> Cannot convert _t0**2*exp(2*a) - 1 of
type <class 'sympy.core.add.Add'> to QQ_I[x,b,_t0,exp(a)]
```

3.103.7 Maxima [A] (verification not implemented)

Time = 1.26 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.94

$$\begin{aligned}&\int x\arctan(c+(i+c)\coth(a+bx))\,dx\\&=\left(\frac{2\,x^3}{3i\,c-3}-\frac{2\,b^2x^2\log\left(-i\,ce^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_2\left(i\,ce^{(2\,bx+2\,a)}\right)-\mathrm{Li}_3\left(i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^3(-i\,c+1)}\right)b(c+i)\\&\quad+\frac{1}{2}\,x^2\arctan\left((c+i)\coth\left(bx+a\right)+c\right)\end{aligned}$$

input
```
integrate(x*arctan(c+(I+c)*coth(b*x+a)),x, algorithm="maxima")
```

output
```
(2*x^3/(3*I*c - 3) - (2*b^2*x^2*log(-I*c*e^(2*b*x + 2*a) + 1) + 2*b*x*dilo
g(I*c*e^(2*b*x + 2*a)) - polylog(3, I*c*e^(2*b*x + 2*a)))/(b^3*(2*I*c - 2)
))*b*(c + I) + 1/2*x^2*arctan((c + I)*coth(b*x + a) + c)
```

3.103.8 Giac [F]

$$\int x\arctan(c+(i+c)\coth(a+bx))\,dx=\int x\arctan\left((c+i)\coth\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(x*arctan(c+(I+c)*coth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*arctan((c + I)*coth(b*x + a) + c), x)
```

3.103.9 Mupad [F(-1)]

Timed out.

$$\int x\arctan(c+(i+c)\coth(a+bx))\,dx=\int x\,\mathrm{atan}\left(c+\coth(a+b\,x)\,\left(c+1\mathrm{i}\right)\right)\,dx$$

input
```
int(x*atan(c + coth(a + b*x)*(c + 1i)),x)
```

output
```
int(x*atan(c + coth(a + b*x)*(c + 1i)), x)
```

3.104 $\int \arctan(c+(i+c)\coth(a+bx))\,dx$

3.104.1 Optimal result

Integrand size = 15, antiderivative size = 79

$$\int \arctan(c+(i+c)\coth(a+bx))\,dx = -\frac{1}{2}ibx^2 + x\arctan(c+(i+c)\coth(a+bx)) + \frac{1}{2}ix\log\left(1-ice^{2a+2bx}\right) + \frac{i\operatorname{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b}$$

output
```
-1/2*I*b*x^2+x*arctan(c+(I+c)*coth(b*x+a))+1/2*I*x*ln(1-I*c*exp(2*b*x+2*a)
)+1/4*I*polylog(2,I*c*exp(2*b*x+2*a))/b
```

3.104.2 Mathematica [A] (verified)

Time = 1.48 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.90

$$\int \arctan(c+(i+c)\coth(a+bx))\,dx = x\arctan(c+(i+c)\coth(a+bx)) + \frac{i\left(2bx\log\left(1+\frac{ie^{-2(a+bx)}}{c}\right) - \operatorname{PolyLog}\left(2,-\frac{ie^{-2(a+bx)}}{c}\right)\right)}{4b}$$

input
```
Integrate[ArcTan[c + (I + c)*Coth[a + b*x]],x]
```

output
```
x*ArcTan[c + (I + c)*Coth[a + b*x]] + ((I/4)*(2*b*x*Log[1 + I/(c*E^(2*(a +
 b*x)))] - PolyLog[2, (-I)/(c*E^(2*(a + b*x)))]))/b
```

3.104.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5712, 25, 2615, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(c+(c+i)\coth(a+bx))\,dx$$

$$\Big\downarrow \text{5712}$$

$$x\arctan(c+(c+i)\coth(a+bx))-b\int -\frac{x}{e^{2a+2bx}c+i}dx$$

$$\Big\downarrow \text{25}$$

$$b\int \frac{x}{e^{2a+2bx}c+i}dx+x\arctan(c+(c+i)\coth(a+bx))$$

$$\Big\downarrow \text{2615}$$

$$b\left(ic\int \frac{e^{2a+2bx}x}{e^{2a+2bx}c+i}dx-\frac{ix^2}{2}\right)+x\arctan(c+(c+i)\coth(a+bx))$$

$$\Big\downarrow \text{2620}$$

$$b\left(ic\left(\frac{x\log\left(1-ice^{2a+2bx}\right)}{2bc}-\frac{\int \log\left(1-ice^{2a+2bx}\right)dx}{2bc}\right)-\frac{ix^2}{2}\right)+x\arctan(c+(c+i)\coth(a+bx))$$

$$\Big\downarrow \text{2715}$$

$$b\left(ic\left(\frac{x\log\left(1-ice^{2a+2bx}\right)}{2bc}-\frac{\int e^{-2a-2bx}\log\left(1-ice^{2a+2bx}\right)de^{2a+2bx}}{4b^2c}\right)-\frac{ix^2}{2}\right)+x\arctan(c+(c+i)\coth(a+bx))$$

$$\Big\downarrow \text{2838}$$

$$x\arctan(c+(c+i)\coth(a+bx))+b\left(ic\left(\frac{\text{PolyLog}\left(2,ice^{2a+2bx}\right)}{4b^2c}+\frac{x\log\left(1-ice^{2a+2bx}\right)}{2bc}\right)-\frac{ix^2}{2}\right)$$

input
```
Int[ArcTan[c + (I + c)*Coth[a + b*x]],x]
```

output
```
x*ArcTan[c + (I + c)*Coth[a + b*x]] + b*((-1/2*I)*x^2 + I*c*((x*Log[1 - I*
c*E^(2*a + 2*b*x)])/(2*b*c) + PolyLog[2, I*c*E^(2*a + 2*b*x)]/(4*b^2*c)))
```

3.104.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5712
```
Int[ArcTan[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Arc
Tan[c + d*Coth[a + b*x]], x] - Simp[b   Int[x/(c - d - c*E^(2*a + 2*b*x)),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, -1]
```

3.104.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 544 vs. $2(65) = 130$.

Time = 1.15 (sec) , antiderivative size = 545, normalized size of antiderivative = 6.90

method	result
derivativedivides	$\frac{\frac{\arctan(c+(i+c)\coth(bx+a))\ln(c-(i+c)\coth(bx+a)+i)}{2i+2c}-\frac{2i\arctan(c+(i+c)\coth(bx+a))\ln(c-(i+c)\coth(bx+a)+i)c}{2i+2c}-\frac{\arctan(c+(i+c}{}}{}$
default	$\frac{\frac{\arctan(c+(i+c)\coth(bx+a))\ln(c-(i+c)\coth(bx+a)+i)}{2i+2c}-\frac{2i\arctan(c+(i+c)\coth(bx+a))\ln(c-(i+c)\coth(bx+a)+i)c}{2i+2c}-\frac{\arctan(c+(i+c}{}}{}$
risch	Expression too large to display

input
```
int(arctan(c+(I+c)*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b/(I+c)*(arctan(c+(I+c)*coth(b*x+a))/(2*I+2*c)*ln(c-(I+c)*coth(b*x+a)+I)
-2*I*arctan(c+(I+c)*coth(b*x+a))/(2*I+2*c)*ln(c-(I+c)*coth(b*x+a)+I)*c-arc
tan(c+(I+c)*coth(b*x+a))/(2*I+2*c)*ln(c-(I+c)*coth(b*x+a)+I)*c^2-arctan(c+
(I+c)*coth(b*x+a))/(2*I+2*c)*ln(I+c+(I+c)*coth(b*x+a))+2*I*arctan(c+(I+c)*
coth(b*x+a))/(2*I+2*c)*ln(I+c+(I+c)*coth(b*x+a))*c+arctan(c+(I+c)*coth(b*x
+a))/(2*I+2*c)*ln(I+c+(I+c)*coth(b*x+a))*c^2-(I+c)^2*(1/2/(I+c)*(-1/2*I*((
ln(I+c+(I+c)*coth(b*x+a))-ln(-1/2*I*(I+c+(I+c)*coth(b*x+a))))*ln(-1/2*I*(I
-c-(I+c)*coth(b*x+a)))-dilog(-1/2*I*(I+c+(I+c)*coth(b*x+a))))+1/4*I*ln(I+c
+(I+c)*coth(b*x+a))^2)-1/2/(I+c)*(-1/2*I*(dilog(-1/2*(I-c-(I+c)*coth(b*x+a
))/c)+ln(c-(I+c)*coth(b*x+a)+I)*ln(-1/2*(I-c-(I+c)*coth(b*x+a))/c))+1/2*I*
(dilog((-I-c-(I+c)*coth(b*x+a))/(-2*I-2*c))+ln(c-(I+c)*coth(b*x+a)+I)*ln((
-I-c-(I+c)*coth(b*x+a))/(-2*I-2*c))))))
```

3.104.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(58) = 116$.

Time = 0.29 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.37

$$\int \arctan(c+(i+c)\coth(a+bx))\,dx$$
$$= \frac{-i\,b^2x^2 + i\,bx\log\left(-\frac{(c+i)e^{(2\,bx+2\,a)}}{ce^{(2\,bx+2\,a)}+i}\right) + i\,a^2 + (i\,bx+i\,a)\log\left(\frac{1}{2}\sqrt{4i\,c}e^{(bx+a)}+1\right) + (i\,bx+i\,a)\log\left(-\frac{1}{2}\sqrt{4i}\right.}{}$$

input
```
integrate(arctan(c+(I+c)*coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*(-I*b^2*x^2 + I*b*x*log(-(c + I)*e^(2*b*x + 2*a)/(c*e^(2*b*x + 2*a) +
I)) + I*a^2 + (I*b*x + I*a)*log(1/2*sqrt(4*I*c)*e^(b*x + a) + 1) + (I*b*x
+ I*a)*log(-1/2*sqrt(4*I*c)*e^(b*x + a) + 1) - I*a*log(1/2*(2*c*e^(b*x + a
) + I*sqrt(4*I*c))/c) - I*a*log(1/2*(2*c*e^(b*x + a) - I*sqrt(4*I*c))/c) +
 I*dilog(1/2*sqrt(4*I*c)*e^(b*x + a)) + I*dilog(-1/2*sqrt(4*I*c)*e^(b*x +
a)))/b
```

3.104.6 Sympy [F(-2)]

Exception generated.

$$\int \arctan(c + (i + c)\coth(a + bx))\,dx = \text{Exception raised: CoercionFailed}$$

input
```
integrate(atan(c+(I+c)*coth(b*x+a)),x)
```

output
```
Exception raised: CoercionFailed >> Cannot convert _t0**2*exp(2*a) - 1 of
type <class 'sympy.core.add.Add'> to QQ_I[b,_t0,exp(a)]
```

3.104.7 Maxima [A] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.01

$$\begin{aligned}
&\int \arctan(c + (i + c)\coth(a + bx))\,dx \\
&= 2\,b(c+i)\left(\frac{2\,x^2}{2i\,c-2} - \frac{2\,bx\log\left(-i\,ce^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left(i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^2(-i\,c+1)}\right) \\
&\quad + x\arctan\left((c+i)\coth\left(bx+a\right)+c\right)
\end{aligned}$$

input
```
integrate(arctan(c+(I+c)*coth(b*x+a)),x, algorithm="maxima")
```

output
```
2*b*(c + I)*(2*x^2/(2*I*c - 2) - (2*b*x*log(-I*c*e^(2*b*x + 2*a) + 1) + di
log(I*c*e^(2*b*x + 2*a)))/(b^2*(2*I*c - 2))) + x*arctan((c + I)*coth(b*x +
 a) + c)
```

3.104.8 Giac [F]

$$\int \arctan(c+(i+c)\coth(a+bx))\,dx = \int \arctan\left((c+i)\coth\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(arctan(c+(I+c)*coth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arctan((c + I)*coth(b*x + a) + c), x)
```

3.104.9 Mupad [F(-1)]

Timed out.

$$\int \arctan(c+(i+c)\coth(a+bx))\,dx = \int \mathrm{atan}\left(c+\coth(a+b\,x)\,\left(c+1\mathrm{i}\right)\right)\,dx$$

input
```
int(atan(c + coth(a + b*x)*(c + 1i)),x)
```

output
```
int(atan(c + coth(a + b*x)*(c + 1i)), x)
```

3.105 $\int \frac{\arctan(c+(i+c)\coth(a+bx))}{x}\,dx$

3.105.1 Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{\arctan(c+(i+c)\coth(a+bx))}{x}\,dx = \text{Int}\left(\frac{\arctan(c+(i+c)\coth(a+bx))}{x},x\right)$$

output
```
CannotIntegrate(arctan(c+(I+c)*coth(b*x+a))/x,x)
```

3.105.2 Mathematica [N/A]

Not integrable

Time = 3.43 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{\arctan(c+(i+c)\coth(a+bx))}{x}\,dx = \int \frac{\arctan(c+(i+c)\coth(a+bx))}{x}\,dx$$

input
```
Integrate[ArcTan[c + (I + c)*Coth[a + b*x]]/x,x]
```

output
```
Integrate[ArcTan[c + (I + c)*Coth[a + b*x]]/x, x]
```

3.105.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(c+(c+i)\coth(a+bx))}{x}dx$$
$$\Big\downarrow \text{7299}$$
$$\int \frac{\arctan(c+(c+i)\coth(a+bx))}{x}dx$$

input
```
Int[ArcTan[c + (I + c)*Coth[a + b*x]]/x,x]
```

output
```
$Aborted
```

3.105.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.105.4 Maple [N/A] (verified)

Not integrable

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{\arctan\left(c+(i+c)\coth\left(bx+a\right)\right)}{x}dx$$

input
```
int(arctan(c+(I+c)*coth(b*x+a))/x,x)
```

output
```
int(arctan(c+(I+c)*coth(b*x+a))/x,x)
```

3.105.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.95

$$\int \frac{\arctan(c+(i+c)\coth(a+bx))}{x}\,dx=\int \frac{\arctan\left((c+i)\coth\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arctan(c+(I+c)*coth(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(1/2*I*log(-(c + I)*e^(2*b*x + 2*a)/(c*e^(2*b*x + 2*a) + I))/x, x)
```

3.105.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan(c+(i+c)\coth(a+bx))}{x}\,dx=\text{Timed out}$$

input
```
integrate(atan(c+(I+c)*coth(b*x+a))/x,x)
```

output
```
Timed out
```

3.105.7 Maxima [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 77, normalized size of antiderivative = 4.05

$$\int \frac{\arctan(c+(i+c)\coth(a+bx))}{x}\,dx=\int \frac{\arctan\left((c+i)\coth\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arctan(c+(I+c)*coth(b*x+a))/x,x, algorithm="maxima")
```

output
```
I*b*x + 1/2*pi*log(x) - 1/4*(4*pi - 4*I*a - 2*arctan(c) - I*log(c^2 + 1))*
log(x) - 1/2*integrate(arctan(c*e^(2*b*x + 2*a))/x, x) - 1/4*I*integrate(l
og(c^2*e^(4*b*x + 4*a) + 1)/x, x)
```

3.105.8 Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(c+(i+c)\coth(a+bx))}{x}\,dx=\int \frac{\arctan\left((c+i)\coth\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arctan(c+(I+c)*coth(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(arctan((c + I)*coth(b*x + a) + c)/x, x)
```

3.105.9 Mupad [N/A]

Not integrable

Time = 1.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \frac{\arctan(c+(i+c)\coth(a+bx))}{x}\,dx=\int \frac{\mathrm{atan}\left(c+\coth(a+b\,x)\,\left(c+1\mathrm{i}\right)\right)}{x}\,dx$$

input
```
int(atan(c + coth(a + b*x)*(c + 1i))/x,x)
```

output
```
int(atan(c + coth(a + b*x)*(c + 1i))/x, x)
```

3.106 $\int x^2 \arctan(c-(i-c)\coth(a+bx))\,dx$

3.106.1 Optimal result

Integrand size = 22, antiderivative size = 145

$$\int x^2 \arctan(c-(i-c)\coth(a+bx))\,dx = \frac{1}{12}ibx^4+\frac{1}{3}x^3\arctan(c-(i-c)\coth(a+bx))-\frac{1}{6}ix^3\log\left(1+ice^{2a+2bx}\right) - \frac{ix^2\operatorname{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}+\frac{ix\operatorname{PolyLog}\left(3,-ice^{2a+2bx}\right)}{4b^2}-\frac{i\operatorname{PolyLog}\left(4,-ice^{2a+2bx}\right)}{8b^3}$$

output
```
1/12*I*b*x^4+1/3*x^3*arctan(c-(I-c)*coth(b*x+a))-1/6*I*x^3*ln(1+I*c*exp(2*
b*x+2*a))-1/4*I*x^2*polylog(2,-I*c*exp(2*b*x+2*a))/b+1/4*I*x*polylog(3,-I*
c*exp(2*b*x+2*a))/b^2-1/8*I*polylog(4,-I*c*exp(2*b*x+2*a))/b^3
```

3.106.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.92

$$\int x^2 \arctan(c-(i-c)\coth(a+bx))\,dx = \frac{8b^3x^3\arctan(c+(-i+c)\coth(a+bx))-4ib^3x^3\log\left(1-\frac{ie^{-2(a+bx)}}{c}\right)+6ib^2x^2\operatorname{PolyLog}\left(2,\frac{ie^{-2(a+bx)}}{c}\right)+6i}{24b^3}$$

input
```
Integrate[x^2*ArcTan[c - (I - c)*Coth[a + b*x]],x]
```

output
```
(8*b^3*x^3*ArcTan[c + (-I + c)*Coth[a + b*x]] - (4*I)*b^3*x^3*Log[1 - I/(c
*E^(2*(a + b*x)))] + (6*I)*b^2*x^2*PolyLog[2, I/(c*E^(2*(a + b*x)))] + (6*
I)*b*x*PolyLog[3, I/(c*E^(2*(a + b*x)))] + (3*I)*PolyLog[4, I/(c*E^(2*(a +
 b*x)))])/(24*b^3)
```

3.106.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.17, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {5720, 2615, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(c-(-c+i)\coth(a+bx))\,dx$$

$$\Big\downarrow \text{5720}$$

$$\frac{1}{3}x^3\arctan(c-(-c+i)\coth(a+bx))-\frac{1}{3}b\int \frac{x^3}{i-ce^{2a+2bx}}dx$$

$$\Big\downarrow \text{2615}$$

$$\frac{1}{3}x^3\arctan(c-(-c+i)\coth(a+bx))-\frac{1}{3}b\left(-ic\int \frac{e^{2a+2bx}x^3}{i-ce^{2a+2bx}}dx-\frac{ix^4}{4}\right)$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{3}x^3\arctan(c-(-c+i)\coth(a+bx))-\\ \frac{1}{3}b\left(-ic\left(\frac{3\int x^2\log\left(ie^{2a+2bx}c+1\right)dx}{2bc}-\frac{x^3\log\left(1+ice^{2a+2bx}\right)}{2bc}\right)-\frac{ix^4}{4}\right)$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{3}x^3\arctan(c-(-c+i)\coth(a+bx))-\\ \frac{1}{3}b\left(-ic\left(\frac{3\left(\frac{\int x\,\text{PolyLog}(2,-ice^{2a+2bx})dx}{b}-\frac{x^2\,\text{PolyLog}(2,-ice^{2a+2bx})}{2b}\right)}{2bc}-\frac{x^3\log\left(1+ice^{2a+2bx}\right)}{2bc}\right)-\frac{ix^4}{4}\right)$$

$$\Big\downarrow \text{7163}$$

$$\frac{1}{3}x^3\arctan(c-(-c+i)\coth(a+bx))-$$
$$\frac{1}{3}b\left(-ic\left(\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-ice^{2a+2bx}\right)}{2b}-\frac{\int \text{PolyLog}\left(3,-ice^{2a+2bx}\right)dx}{2b}}{b}-\frac{x^2\,\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{2b}\right)}{2bc}-\frac{x^3\log\left(1+ice^{2a+2bx}\right)}{2bc}\right)-\frac{ix}{4}\right.$$

$\big\downarrow$ 2720

$$\frac{1}{3}x^3\arctan(c-(-c+i)\coth(a+bx))-$$
$$\frac{1}{3}b\left(-ic\left(\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-ice^{2a+2bx}\right)}{2b}-\frac{\int e^{-2a-2bx}\,\text{PolyLog}\left(3,-ice^{2a+2bx}\right)de^{2a+2bx}}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{2b}\right)}{2bc}-\frac{x^3\log\left(1+ice^{2a}\right.}{2bc}\right.\right.$$

$\big\downarrow$ 7143

$$\frac{1}{3}x^3\arctan(c-(-c+i)\coth(a+bx))-$$
$$\frac{1}{3}b\left(-ic\left(\frac{3\left(\frac{\frac{x\,\text{PolyLog}\left(3,-ice^{2a+2bx}\right)}{2b}-\frac{\text{PolyLog}\left(4,-ice^{2a+2bx}\right)}{4b^2}}{b}-\frac{x^2\,\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{2b}\right)}{2bc}-\frac{x^3\log\left(1+ice^{2a+2bx}\right)}{2bc}\right)-\frac{ix^4}{4}\right)$$

input
```
Int[x^2*ArcTan[c - (I - c)*Coth[a + b*x]],x]
```

output
```
(x^3*ArcTan[c - (I - c)*Coth[a + b*x]])/3 - (b*((-1/4*I)*x^4 - I*c*(-1/2*(
x^3*Log[1 + I*c*E^(2*a + 2*b*x)])/(b*c) + (3*(-1/2*(x^2*PolyLog[2, (-I)*c*
E^(2*a + 2*b*x)])/b + ((x*PolyLog[3, (-I)*c*E^(2*a + 2*b*x)])/(2*b) - Poly
Log[4, (-I)*c*E^(2*a + 2*b*x)]/(4*b^2))/b))/(2*b*c))))/3
```

3.106.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5720
```
Int[ArcTan[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Coth[a + b*x]]/(f*(m
+ 1))), x] - Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a +
 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
d)^2, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.106. $\int x^2 \arctan(c-(i-c)\coth(a+bx))\,dx$

3.106.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.83 (sec) , antiderivative size = 1410, normalized size of antiderivative = 9.72

method	result	size
risch	Expression too large to display	1410

input
```
int(x^2*arctan(c-(I-c)*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/2*I/b^2*ln(1+I*c*exp(2*b*x+2*a))*a^2*x-1/2*I/b^2*a^2*ln(1+I*exp(b*x+a)*(
I*c)^(1/2))*x-1/2*I/b^3*a^3*ln(1+I*exp(b*x+a)*(I*c)^(1/2))-1/2*I/b^2*a^2*l
n(1-I*exp(b*x+a)*(I*c)^(1/2))*x-1/2*I/b^3*a^2*dilog(1+I*exp(b*x+a)*(I*c)^(
1/2))+1/3*I/b^3*ln(1+I*c*exp(2*b*x+2*a))*a^3-1/8*I*polylog(4,-I*c*exp(2*b*
x+2*a))/b^3+1/4*I*x*polylog(3,-I*c*exp(2*b*x+2*a))/b^2+1/12*I/b^3*c*a^4/(I
-c)-1/6*I*x^3*ln(2*I*exp(2*b*x+2*a)-2*exp(2*b*x+2*a)*c)-1/6*I*x^3*ln(1+I*c
*exp(2*b*x+2*a))+1/6*I/b^3*a^3*ln(-exp(2*b*x+2*a)*c+I)+1/6*I*x^3*ln(-2*exp
(2*b*x+2*a)*c+2*I)-1/2*I/b^3*a^3*ln(1-I*exp(b*x+a)*(I*c)^(1/2))-1/4*I*x^2*
polylog(2,-I*c*exp(2*b*x+2*a))/b+1/4*I/b^3*polylog(2,-I*c*exp(2*b*x+2*a))*
a^2+1/12*Pi*(csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I))*c
sgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))-csgn(I/(exp(2*b*x+2*a)-
1))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(-2*I*exp(2*b*
x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))-csgn(I/(exp(2*b*x+2*a)-1))*
csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))^2+csgn(I/(exp(2*b*x+2*
a)-1))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))
^2+csgn(I*(2*exp(2*b*x+2*a)*c-2*I))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2
*b*x+2*a)-1))^2-csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(-
2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2-csgn(I*(2*exp
(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))^3+csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(
exp(2*b*x+2*a)-1))*csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))^2+...
```

3.106.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 293 vs. 2(105) = 210.

Time = 0.29 (sec) , antiderivative size = 293, normalized size of antiderivative = 2.02

$$\int x^2 \arctan(c-(i-c)\coth(a+bx))\,dx$$
$$= \frac{i\,b^4x^4 + 2i\,b^3x^3\log\left(-\frac{(ce^{(2\,bx+2\,a)}-i)e^{(-2\,bx-2\,a)}}{c-i}\right) - 6i\,b^2x^2\mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4i\,c}e^{(bx+a)}\right) - 6i\,b^2x^2\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{-4i\,c}e^{(bx+a}\right.}{}$$

input
```
integrate(x^2*arctan(c-(I-c)*coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/12*(I*b^4*x^4 + 2*I*b^3*x^3*log(-(c*e^(2*b*x + 2*a) - I)*e^(-2*b*x - 2*a
)/(c - I)) - 6*I*b^2*x^2*dilog(1/2*sqrt(-4*I*c)*e^(b*x + a)) - 6*I*b^2*x^2
*dilog(-1/2*sqrt(-4*I*c)*e^(b*x + a)) - I*a^4 + 2*I*a^3*log(1/2*(2*c*e^(b*
x + a) + I*sqrt(-4*I*c))/c) + 2*I*a^3*log(1/2*(2*c*e^(b*x + a) - I*sqrt(-4
*I*c))/c) + 12*I*b*x*polylog(3, 1/2*sqrt(-4*I*c)*e^(b*x + a)) + 12*I*b*x*p
olylog(3, -1/2*sqrt(-4*I*c)*e^(b*x + a)) - 2*(I*b^3*x^3 + I*a^3)*log(1/2*s
qrt(-4*I*c)*e^(b*x + a) + 1) - 2*(I*b^3*x^3 + I*a^3)*log(-1/2*sqrt(-4*I*c)
*e^(b*x + a) + 1) - 12*I*polylog(4, 1/2*sqrt(-4*I*c)*e^(b*x + a)) - 12*I*p
olylog(4, -1/2*sqrt(-4*I*c)*e^(b*x + a)))/b^3
```

3.106.6 Sympy [F(-2)]

Exception generated.

$$\int x^2 \arctan(c-(i-c)\coth(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

input
```
integrate(x**2*atan(c-(I-c)*coth(b*x+a)),x)
```

output
```
Exception raised: CoercionFailed >> Cannot convert _t0**2*exp(2*a) - 1 of
type <class 'sympy.core.add.Add'> to QQ_I[x,b,_t0,exp(a)]
```

3.106.7 Maxima [A] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.89

$$\int x^2 \arctan(c-(i-c)\coth(a+bx))\,dx = \frac{1}{3}x^3\arctan\left((c-i)\coth\left(bx+a\right)+c\right) - \frac{4}{9}\left(\frac{3x^4}{4ic+4} - \frac{4b^3x^3\log\left(ice^{(2bx+2a)}+1\right)+6b^2x^2\mathrm{Li}_2\left(-ice^{(2bx+2a)}\right)-6bx\mathrm{Li}_3\left(-ice^{(2bx+2a)}\right)+3\,\mathrm{Li}_4(-}{-2b^4(-ic-1)}\right.$$

input
```
integrate(x^2*arctan(c-(I-c)*coth(b*x+a)),x, algorithm="maxima")
```

output
```
1/3*x^3*arctan((c - I)*coth(b*x + a) + c) - 4/9*(3*x^4/(4*I*c + 4) - (4*b^
3*x^3*log(I*c*e^(2*b*x + 2*a) + 1) + 6*b^2*x^2*dilog(-I*c*e^(2*b*x + 2*a))
 - 6*b*x*polylog(3, -I*c*e^(2*b*x + 2*a)) + 3*polylog(4, -I*c*e^(2*b*x + 2
*a)))/(b^4*(2*I*c + 2)))*b*(c - I)
```

3.106.8 Giac [F]

$$\int x^2 \arctan(c-(i-c)\coth(a+bx))\,dx = \int x^2 \arctan\left((c-i)\coth\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(x^2*arctan(c-(I-c)*coth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*arctan((c - I)*coth(b*x + a) + c), x)
```

3.106.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \arctan(c-(i-c)\coth(a+bx))\,dx = \int x^2 \operatorname{atan}(c+\coth(a+b\,x)\,\left(c-\mathrm{i}\right))\,dx$$

input
```
int(x^2*atan(c + coth(a + b*x)*(c - 1i)),x)
```

output
```
int(x^2*atan(c + coth(a + b*x)*(c - 1i)), x)
```

3.107 $\int x \arctan(c-(i-c)\coth(a+bx))\,dx$

3.107.1 Optimal result

Integrand size = 20, antiderivative size = 116

$$\int x\arctan(c-(i-c)\coth(a+bx))\,dx=\frac{1}{6}ibx^3+\frac{1}{2}x^2\arctan(c-(i-c)\coth(a+bx))-\frac{1}{4}ix^2\log\left(1+ice^{2a+2bx}\right)-\frac{ix\,\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}+\frac{i\,\text{PolyLog}\left(3,-ice^{2a+2bx}\right)}{8b^2}$$

output
```
1/6*I*b*x^3+1/2*x^2*arctan(c-(I-c)*coth(b*x+a))-1/4*I*x^2*ln(1+I*c*exp(2*b
*x+2*a))-1/4*I*x*polylog(2,-I*c*exp(2*b*x+2*a))/b+1/8*I*polylog(3,-I*c*exp
(2*b*x+2*a))/b^2
```

3.107.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.89

$$\int x\arctan(c-(i-c)\coth(a+bx))\,dx=\frac{2b^2x^2\left(2\arctan(c+(-i+c)\coth(a+bx))-i\log\left(1-\frac{ie^{-2(a+bx)}}{c}\right)\right)+2ibx\,\text{PolyLog}\left(2,\frac{ie^{-2(a+bx)}}{c}\right)+i\,\text{Poly}}{8b^2}$$

input
```
Integrate[x*ArcTan[c - (I - c)*Coth[a + b*x]],x]
```

output
```
(2*b^2*x^2*(2*ArcTan[c + (-I + c)*Coth[a + b*x]] - I*Log[1 - I/(c*E^(2*(a
+ b*x)))]) + (2*I)*b*x*PolyLog[2, I/(c*E^(2*(a + b*x)))] + I*PolyLog[3, I/
(c*E^(2*(a + b*x)))])/(8*b^2)
```

3.107.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5720, 2615, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan(c-(-c+i)\coth(a+bx))\,dx$$

$$\Big\downarrow \text{5720}$$

$$\frac{1}{2}x^2\arctan(c-(-c+i)\coth(a+bx))-\frac{1}{2}b\int\frac{x^2}{i-ce^{2a+2bx}}dx$$

$$\Big\downarrow \text{2615}$$

$$\frac{1}{2}x^2\arctan(c-(-c+i)\coth(a+bx))-\frac{1}{2}b\left(-ic\int\frac{e^{2a+2bx}x^2}{i-ce^{2a+2bx}}dx-\frac{ix^3}{3}\right)$$

$$\Big\downarrow \text{2620}$$

$$\frac{1}{2}x^2\arctan(c-(-c+i)\coth(a+bx))-\frac{1}{2}b\left(-ic\left(\frac{\int x\log\left(ie^{2a+2bx}c+1\right)dx}{bc}-\frac{x^2\log\left(1+ice^{2a+2bx}\right)}{2bc}\right)-\frac{ix^3}{3}\right)$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{2}x^2\arctan(c-(-c+i)\coth(a+bx))-\frac{1}{2}b\left(-ic\left(\frac{\frac{\int \text{PolyLog}(2,-ice^{2a+2bx})dx}{2b}-\frac{x\,\text{PolyLog}(2,-ice^{2a+2bx})}{2b}}{bc}-\frac{x^2\log\left(1+ice^{2a+2bx}\right)}{2bc}\right)-\frac{ix^3}{3}\right)$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{2}x^2\arctan(c-(-c+i)\coth(a+bx))-$$
$$\frac{1}{2}b\left(-ic\left(\frac{\frac{\int e^{-2a-2bx}\,\text{PolyLog}(2,-ice^{2a+2bx})de^{2a+2bx}}{4b^2}-\frac{x\,\text{PolyLog}(2,-ice^{2a+2bx})}{2b}}{bc}-\frac{x^2\log(1+ice^{2a+2bx})}{2bc}\right)-\frac{ix^3}{3}\right)$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{2}x^2\arctan(c-(-c+i)\coth(a+bx))-$$
$$\frac{1}{2}b\left(-ic\left(\frac{\frac{\text{PolyLog}(3,-ice^{2a+2bx})}{4b^2}-\frac{x\,\text{PolyLog}(2,-ice^{2a+2bx})}{2b}}{bc}-\frac{x^2\log(1+ice^{2a+2bx})}{2bc}\right)-\frac{ix^3}{3}\right)$$

input
```
Int[x*ArcTan[c - (I - c)*Coth[a + b*x]],x]
```

output
```
(x^2*ArcTan[c - (I - c)*Coth[a + b*x]])/2 - (b*((-1/3*I)*x^3 - I*c*(-1/2*(
x^2*Log[1 + I*c*E^(2*a + 2*b*x)])/(b*c) + (-1/2*(x*PolyLog[2, (-I)*c*E^(2*
a + 2*b*x)])/b + PolyLog[3, (-I)*c*E^(2*a + 2*b*x)]/(4*b^2))/(b*c))))/2
```

3.107.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5720
```
Int[ArcTan[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)]*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[(e + f*x)^(m + 1)*(ArcTan[c + d*Coth[a + b*x]]/(f*(m
+ 1))), x] - Simp[b/(f*(m + 1))   Int[(e + f*x)^(m + 1)/(c - d - c*E^(2*a +
 2*b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[(c -
d)^2, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.107.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.04 (sec) , antiderivative size = 1374, normalized size of antiderivative = 11.84

method	result	size
risch	Expression too large to display	1374

input
```
int(x*arctan(c-(I-c)*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/6*I*b*c/(I-c)*x^3-1/4*I*x^2*ln(1+I*c*exp(2*b*x+2*a))+1/2*I/b*a*ln(1+I*e
xp(b*x+a)*(I*c)^(1/2))*x-1/6*I/b^2*c/(I-c)*a^3-1/4*I/b^2*ln(1+I*c*exp(2*b*
x+2*a))*a^2-1/4*I*x*polylog(2,-I*c*exp(2*b*x+2*a))/b-1/4*I/b^2*polylog(2,-
I*c*exp(2*b*x+2*a))*a-1/4*I/b^2*a^2*ln(-exp(2*b*x+2*a)*c+I)+1/2*I/b^2*ln(1
+I*exp(b*x+a)*(I*c)^(1/2))*a^2+1/2*I/b^2*ln(1-I*exp(b*x+a)*(I*c)^(1/2))*a^
2+1/2*I/b^2*a*dilog(1-I*exp(b*x+a)*(I*c)^(1/2))+1/8*I*polylog(3,-I*c*exp(2
*b*x+2*a))/b^2-1/4*I*x^2*ln(2*I*exp(2*b*x+2*a)-2*exp(2*b*x+2*a)*c)+1/4*I*x
^2*ln(-2*exp(2*b*x+2*a)*c+2*I)-1/2*I/b*ln(1+I*c*exp(2*b*x+2*a))*a*x+1/8*Pi
*(csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I))*csgn(I*(2*ex
p(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))-csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*
(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(-2*I*exp(2*b*x+2*a)+2*ex
p(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))-csgn(I/(exp(2*b*x+2*a)-1))*csgn(I*(2*e
xp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))^2+csgn(I/(exp(2*b*x+2*a)-1))*csgn
(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2+csgn(I*(
2*exp(2*b*x+2*a)*c-2*I))*csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1
))^2-csgn(I*(-2*I*exp(2*b*x+2*a)+2*exp(2*b*x+2*a)*c))*csgn(I*(-2*I*exp(2*b
*x+2*a)+2*exp(2*b*x+2*a)*c)/(exp(2*b*x+2*a)-1))^2-csgn(I*(2*exp(2*b*x+2*a)
*c-2*I)/(exp(2*b*x+2*a)-1))^3+csgn(I*(2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2
*a)-1))*csgn((2*exp(2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))^2+csgn(I*(2*exp(
2*b*x+2*a)*c-2*I)/(exp(2*b*x+2*a)-1))*csgn((2*exp(2*b*x+2*a)*c-2*I)/(ex...
```

3.107.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(83) = 166$.

Time = 0.28 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.13

$$\int x \arctan(c-(i-c)\coth(a+bx))\,dx$$
$$= \frac{2i\,b^3x^3 + 3i\,b^2x^2\log\left(-\frac{(ce^{(2\,bx+2\,a)}-i)e^{(-2\,bx-2\,a)}}{c-i}\right) + 2i\,a^3 - 6i\,bx\mathrm{Li}_2\left(\frac{1}{2}\sqrt{-4i\,c}e^{(bx+a)}\right) - 6i\,bx\mathrm{Li}_2\left(-\frac{1}{2}\sqrt{-4i\,ce}\right.}{}$$

input
```
integrate(x*arctan(c-(I-c)*coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/12*(2*I*b^3*x^3 + 3*I*b^2*x^2*log(-(c*e^(2*b*x + 2*a) - I)*e^(-2*b*x - 2
*a)/(c - I)) + 2*I*a^3 - 6*I*b*x*dilog(1/2*sqrt(-4*I*c)*e^(b*x + a)) - 6*I
*b*x*dilog(-1/2*sqrt(-4*I*c)*e^(b*x + a)) - 3*I*a^2*log(1/2*(2*c*e^(b*x +
a) + I*sqrt(-4*I*c))/c) - 3*I*a^2*log(1/2*(2*c*e^(b*x + a) - I*sqrt(-4*I*c
))/c) - 3*(I*b^2*x^2 - I*a^2)*log(1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) - 3*(I
*b^2*x^2 - I*a^2)*log(-1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) + 6*I*polylog(3,
1/2*sqrt(-4*I*c)*e^(b*x + a)) + 6*I*polylog(3, -1/2*sqrt(-4*I*c)*e^(b*x +
a)))/b^2
```

3.107.6 Sympy [F(-2)]

Exception generated.

$$\int x\arctan(c-(i-c)\coth(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

input
```
integrate(x*atan(c-(I-c)*coth(b*x+a)),x)
```

output
```
Exception raised: CoercionFailed >> Cannot convert _t0**2*exp(2*a) - 1 of
type <class 'sympy.core.add.Add'> to QQ_I[x,b,_t0,exp(a)]
```

3.107.7 Maxima [A] (verification not implemented)

Time = 1.24 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.92

$$\int x\arctan(c-(i-c)\coth(a+bx))\,dx = -\left(\frac{2\,x^3}{3i\,c+3} - \frac{2\,b^2x^2\log\left(i\,ce^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_2\left(-i\,ce^{(2\,bx+2\,a)}\right)-\mathrm{Li}_3(-i\,ce^{(2\,bx+2\,a)})}{-2\,b^3(-i\,c-1)}\right)b(c-i) + \frac{1}{2}\,x^2\arctan\left((c-i)\coth\left(bx+a\right)+c\right)$$

input
```
integrate(x*arctan(c-(I-c)*coth(b*x+a)),x, algorithm="maxima")
```

output
```
-(2*x^3/(3*I*c + 3) - (2*b^2*x^2*log(I*c*e^(2*b*x + 2*a) + 1) + 2*b*x*dilo
g(-I*c*e^(2*b*x + 2*a)) - polylog(3, -I*c*e^(2*b*x + 2*a)))/(b^3*(2*I*c +
2)))*b*(c - I) + 1/2*x^2*arctan((c - I)*coth(b*x + a) + c)
```

3.107.8 Giac [F]

$$\int x\arctan(c-(i-c)\coth(a+bx))\,dx=\int x\arctan\left((c-i)\coth\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(x*arctan(c-(I-c)*coth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*arctan((c - I)*coth(b*x + a) + c), x)
```

3.107.9 Mupad [F(-1)]

Timed out.

$$\int x\arctan(c-(i-c)\coth(a+bx))\,dx=\int x\,\mathrm{atan}(c+\coth(a+b\,x)\,\left(c-\mathrm{i}\right))\,dx$$

input
```
int(x*atan(c + coth(a + b*x)*(c - 1i)),x)
```

output
```
int(x*atan(c + coth(a + b*x)*(c - 1i)), x)
```

3.108 $\int \arctan(c-(i-c)\coth(a+bx))\,dx$

3.108.1 Optimal result

Integrand size = 18, antiderivative size = 82

$$\int \arctan(c-(i-c)\coth(a+bx))\,dx = \frac{1}{2}ibx^2 + x\arctan(c-(i-c)\coth(a+bx)) - \frac{1}{2}ix\log\left(1+ice^{2a+2bx}\right) - \frac{i\,\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b}$$

output
```
1/2*I*b*x^2+x*arctan(c-(I-c)*coth(b*x+a))-1/2*I*x*ln(1+I*c*exp(2*b*x+2*a))
-1/4*I*polylog(2,-I*c*exp(2*b*x+2*a))/b
```

3.108.2 Mathematica [A] (verified)

Time = 1.43 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.87

$$\int \arctan(c-(i-c)\coth(a+bx))\,dx = x\arctan(c+(-i+c)\coth(a+bx)) - \frac{i\left(2bx\log\left(1-\frac{ie^{-2(a+bx)}}{c}\right)-\text{PolyLog}\left(2,\frac{ie^{-2(a+bx)}}{c}\right)\right)}{4b}$$

input
```
Integrate[ArcTan[c - (I - c)*Coth[a + b*x]],x]
```

output
```
x*ArcTan[c + (-I + c)*Coth[a + b*x]] - ((I/4)*(2*b*x*Log[1 - I/(c*E^(2*(a
+ b*x)))] - PolyLog[2, I/(c*E^(2*(a + b*x)))]))/b
```

3.108.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {5712, 2615, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(c-(-c+i)\coth(a+bx))\,dx$$

$$\Big\downarrow \ 5712$$

$$x\arctan(c-(-c+i)\coth(a+bx))-b\int \frac{x}{i-ce^{2a+2bx}}dx$$

$$\Big\downarrow \ 2615$$

$$x\arctan(c-(-c+i)\coth(a+bx))-b\left(-ic\int \frac{e^{2a+2bx}x}{i-ce^{2a+2bx}}dx-\frac{ix^2}{2}\right)$$

$$\Big\downarrow \ 2620$$

$$x\arctan(c-(-c+i)\coth(a+bx))-b\left(-ic\left(\frac{\int \log\left(ie^{2a+2bx}c+1\right)dx}{2bc}-\frac{x\log\left(1+ice^{2a+2bx}\right)}{2bc}\right)-\frac{ix^2}{2}\right)$$

$$\Big\downarrow \ 2715$$

$$x\arctan(c-(-c+i)\coth(a+bx))-b\left(-ic\left(\frac{\int e^{-2a-2bx}\log\left(ie^{2a+2bx}c+1\right)de^{2a+2bx}}{4b^2c}-\frac{x\log\left(1+ice^{2a+2bx}\right)}{2bc}\right)-\frac{ix^2}{2}\right)$$

$$\Big\downarrow \ 2838$$

$$x\arctan(c-(-c+i)\coth(a+bx))-b\left(-ic\left(-\frac{\text{PolyLog}\left(2,-ice^{2a+2bx}\right)}{4b^2c}-\frac{x\log\left(1+ice^{2a+2bx}\right)}{2bc}\right)-\frac{ix^2}{2}\right)$$

input
```
Int[ArcTan[c - (I - c)*Coth[a + b*x]],x]
```

output
```
x*ArcTan[c - (I - c)*Coth[a + b*x]] - b*((-1/2*I)*x^2 - I*c*(-1/2*(x*Log[1
 + I*c*E^(2*a + 2*b*x)])/(b*c) - PolyLog[2, (-I)*c*E^(2*a + 2*b*x)]/(4*b^2
*c)))
```

3.108.3.1 Defintions of rubi rules used

rule 2615
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x
_))))^(n_.)), x_Symbol] :> Simp[(c + d*x)^(m + 1)/(a*d*(m + 1)), x] - Simp[
b/a   Int[(c + d*x)^m*((F^(g*(e + f*x)))^n/(a + b*(F^(g*(e + f*x)))^n)), x]
, x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5712
```
Int[ArcTan[(c_.) + Coth[(a_.) + (b_.)*(x_)]*(d_.)], x_Symbol] :> Simp[x*Arc
Tan[c + d*Coth[a + b*x]], x] - Simp[b   Int[x/(c - d - c*E^(2*a + 2*b*x)),
x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[(c - d)^2, -1]
```

3.108.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 515 vs. $2(68) = 136$.

Time = 1.14 (sec) , antiderivative size = 516, normalized size of antiderivative = 6.29

method	result
derivativedivides	$\frac{\frac{\arctan(c+\coth(bx+a)(c-i))\ln(-i+\coth(bx+a)(c-i)+c)}{2i-2c}+\frac{2i\arctan(c+\coth(bx+a)(c-i))\ln(-i+\coth(bx+a)(c-i)+c)c}{2i-2c}-\frac{\arctan(c+\mathrm{co}}{}}{}$
default	$\frac{\frac{\arctan(c+\coth(bx+a)(c-i))\ln(-i+\coth(bx+a)(c-i)+c)}{2i-2c}+\frac{2i\arctan(c+\coth(bx+a)(c-i))\ln(-i+\coth(bx+a)(c-i)+c)c}{2i-2c}-\frac{\arctan(c+\mathrm{co}}{}}{}$
risch	Expression too large to display

input
```
int(arctan(c-(I-c)*coth(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b/(c-I)*(arctan(c+coth(b*x+a)*(c-I))/(2*I-2*c)*ln(-I+coth(b*x+a)*(c-I)+c
)+2*I*arctan(c+coth(b*x+a)*(c-I))/(2*I-2*c)*ln(-I+coth(b*x+a)*(c-I)+c)*c-a
rctan(c+coth(b*x+a)*(c-I))/(2*I-2*c)*ln(-I+coth(b*x+a)*(c-I)+c)*c^2-arctan
(c+coth(b*x+a)*(c-I))/(2*I-2*c)*ln(coth(b*x+a)*(c-I)-c+I)-2*I*arctan(c+cot
h(b*x+a)*(c-I))/(2*I-2*c)*ln(coth(b*x+a)*(c-I)-c+I)*c+arctan(c+coth(b*x+a)
*(c-I))/(2*I-2*c)*ln(coth(b*x+a)*(c-I)-c+I)*c^2+(I-c)^2*(1/2/(I-c)*(1/2*I*
(dilog(-1/2*I*(coth(b*x+a)*(c-I)+c+I))+ln(-I+coth(b*x+a)*(c-I)+c)*ln(-1/2*
I*(coth(b*x+a)*(c-I)+c+I)))-1/4*I*ln(-I+coth(b*x+a)*(c-I)+c)^2)-1/2/(I-c)*
(-1/2*I*(dilog((-I+coth(b*x+a)*(c-I)+c)/(-2*I+2*c))+ln(coth(b*x+a)*(c-I)-c
+I)*ln((-I+coth(b*x+a)*(c-I)+c)/(-2*I+2*c)))+1/2*I*(dilog(1/2*(coth(b*x+a)
*(c-I)+c+I)/c)+ln(coth(b*x+a)*(c-I)-c+I)*ln(1/2*(coth(b*x+a)*(c-I)+c+I)/c)
))))
```

3.108.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(58) = 116.

Time = 0.29 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.28

$$\int \arctan(c-(i-c)\coth(a+bx))\,dx = \frac{i\,b^2x^2+i\,bx\log\left(-\frac{\left(ce^{(2\,bx+2\,a)}-i\right)e^{(-2\,bx-2\,a)}}{c-i}\right)-i\,a^2+(-i\,bx-i\,a)\log\left(\frac{1}{2}\sqrt{-4i\,c}e^{(bx+a)}+1\right)+(-i\,bx-i\,a)}{}$$

input
```
integrate(arctan(c-(I-c)*coth(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*(I*b^2*x^2 + I*b*x*log(-(c*e^(2*b*x + 2*a) - I)*e^(-2*b*x - 2*a)/(c -
I)) - I*a^2 + (-I*b*x - I*a)*log(1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) + (-I*b
*x - I*a)*log(-1/2*sqrt(-4*I*c)*e^(b*x + a) + 1) + I*a*log(1/2*(2*c*e^(b*x
 + a) + I*sqrt(-4*I*c))/c) + I*a*log(1/2*(2*c*e^(b*x + a) - I*sqrt(-4*I*c)
)/c) - I*dilog(1/2*sqrt(-4*I*c)*e^(b*x + a)) - I*dilog(-1/2*sqrt(-4*I*c)*e
^(b*x + a)))/b
```

3.108.6 Sympy [F(-2)]

Exception generated.

$$\int \arctan(c-(i-c)\coth(a+bx))\,dx = \text{Exception raised: CoercionFailed}$$

input
```
integrate(atan(c-(I-c)*coth(b*x+a)),x)
```

output
```
Exception raised: CoercionFailed >> Cannot convert _t0**2*exp(2*a) - 1 of
type <class 'sympy.core.add.Add'> to QQ_I[b,_t0,exp(a)]
```

3.108.7 Maxima [A] (verification not implemented)

Time = 1.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

$$\int \arctan(c-(i-c)\coth(a+bx))\,dx = -2\,b(c-i)\left(\frac{2\,x^2}{2i\,c+2}-\frac{2\,bx\log\left(i\,ce^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left(-i\,ce^{(2\,bx+2\,a)}\right)}{-2\,b^2(-i\,c-1)}\right) + x\arctan\left((c-i)\coth\left(bx+a\right)+c\right)$$

input
```
integrate(arctan(c-(I-c)*coth(b*x+a)),x, algorithm="maxima")
```

output
```
-2*b*(c - I)*(2*x^2/(2*I*c + 2) - (2*b*x*log(I*c*e^(2*b*x + 2*a) + 1) + di
log(-I*c*e^(2*b*x + 2*a)))/(b^2*(2*I*c + 2))) + x*arctan((c - I)*coth(b*x
+ a) + c)
```

3.108.8 Giac [F]

$$\int \arctan(c-(i-c)\coth(a+bx))\,dx=\int \arctan\left((c-i)\coth\left(bx+a\right)+c\right)\,dx$$

input
```
integrate(arctan(c-(I-c)*coth(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arctan((c - I)*coth(b*x + a) + c), x)
```

3.108.9 Mupad [F(-1)]

Timed out.

$$\int \arctan(c-(i-c)\coth(a+bx))\,dx=\int \mathrm{atan}\left(c+\coth(a+b\,x)\,\left(c-\mathrm{i}\right)\right)\,dx$$

input
```
int(atan(c + coth(a + b*x)*(c - 1i)),x)
```

output
```
int(atan(c + coth(a + b*x)*(c - 1i)), x)
```

3.109 $\int \frac{\arctan(c-(i-c)\coth(a+bx))}{x}\,dx$

3.109.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\arctan(c-(i-c)\coth(a+bx))}{x}\,dx=\text{Int}\left(\frac{\arctan(c-(i-c)\coth(a+bx))}{x},x\right)$$

output
```
CannotIntegrate(arctan(c-(I-c)*coth(b*x+a))/x,x)
```

3.109.2 Mathematica [N/A]

Not integrable

Time = 3.45 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\arctan(c-(i-c)\coth(a+bx))}{x}\,dx=\int \frac{\arctan(c-(i-c)\coth(a+bx))}{x}\,dx$$

input
```
Integrate[ArcTan[c - (I - c)*Coth[a + b*x]]/x,x]
```

output
```
Integrate[ArcTan[c - (I - c)*Coth[a + b*x]]/x, x]
```

3.109.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(c-(-c+i)\coth(a+bx))}{x}\,dx$$
$$\Big\downarrow \; 7299$$
$$\int \frac{\arctan(c-(-c+i)\coth(a+bx))}{x}dx$$

input
```
Int[ArcTan[c - (I - c)*Coth[a + b*x]]/x,x]
```

output
```
$Aborted
```

3.109.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

3.109.4 Maple [N/A] (verified)

Not integrable

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\arctan\left(c-(i-c)\coth\left(bx+a\right)\right)}{x}dx$$

input
```
int(arctan(c-(I-c)*coth(b*x+a))/x,x)
```

output
```
int(arctan(c-(I-c)*coth(b*x+a))/x,x)
```

3.109.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int \frac{\arctan(c-(i-c)\coth(a+bx))}{x}\,dx = \int \frac{\arctan\left((c-i)\coth\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arctan(c-(I-c)*coth(b*x+a))/x,x, algorithm="fricas")
```

output
```
integral(1/2*I*log(-(c*e^(2*b*x + 2*a) - I)*e^(-2*b*x - 2*a)/(c - I))/x, x
)
```

3.109.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan(c-(i-c)\coth(a+bx))}{x}\,dx = \text{Timed out}$$

input
```
integrate(atan(c-(I-c)*coth(b*x+a))/x,x)
```

output
```
Timed out
```

3.109.7 Maxima [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 72, normalized size of antiderivative = 3.27

$$\int \frac{\arctan(c-(i-c)\coth(a+bx))}{x}\,dx = \int \frac{\arctan\left((c-i)\coth\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arctan(c-(I-c)*coth(b*x+a))/x,x, algorithm="maxima")
```

output
```
-I*b*x - 1/4*(2*pi + 4*I*a - 2*arctan(c) + I*log(c^2 + 1))*log(x) - 1/2*in
tegrate(arctan(c*e^(2*b*x + 2*a))/x, x) + 1/4*I*integrate(log(c^2*e^(4*b*x
 + 4*a) + 1)/x, x)
```

3.109.8 Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{\arctan(c-(i-c)\coth(a+bx))}{x}\,dx=\int \frac{\arctan\left((c-i)\coth\left(bx+a\right)+c\right)}{x}\,dx$$

input
```
integrate(arctan(c-(I-c)*coth(b*x+a))/x,x, algorithm="giac")
```

output
```
integrate(arctan((c - I)*coth(b*x + a) + c)/x, x)
```

3.109.9 Mupad [N/A]

Not integrable

Time = 1.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\arctan(c-(i-c)\coth(a+bx))}{x}\,dx=\int \frac{\mathrm{atan}(c+\coth(a+b\,x)\ (c-\mathrm{i}))}{x}\,dx$$

input
```
int(atan(c + coth(a + b*x)*(c - 1i))/x,x)
```

output
```
int(atan(c + coth(a + b*x)*(c - 1i))/x, x)
```

3.110 $\int \arctan(e^x)\,dx$

3.110.1 Optimal result

Integrand size = 4, antiderivative size = 31

$$\int \arctan(e^x)\,dx = \frac{1}{2}i\,\text{PolyLog}(2,-ie^x) - \frac{1}{2}i\,\text{PolyLog}(2,ie^x)$$

output
```
1/2*I*polylog(2,-I*exp(x))-1/2*I*polylog(2,I*exp(x))
```

3.110.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.90

$$\int \arctan(e^x)\,dx = x\arctan(e^x) - \frac{1}{2}i(x(\log(1-ie^x) - \log(1+ie^x)) - \text{PolyLog}(2,-ie^x) + \text{PolyLog}(2,ie^x))$$

input
```
Integrate[ArcTan[E^x],x]
```

output
```
x*ArcTan[E^x] - (I/2)*(x*(Log[1 - I*E^x] - Log[1 + I*E^x]) - PolyLog[2, (-
I)*E^x] + PolyLog[2, I*E^x])
```

3.110.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {2720, 5355, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan\left(e^x\right)\,dx$$

$$\Big\downarrow \text{ 2720}$$

$$\int e^{-x}\arctan\left(e^x\right)de^x$$

$$\Big\downarrow \text{ 5355}$$

$$\frac{1}{2}i\int e^{-x}\log\left(1-ie^x\right)de^x-\frac{1}{2}i\int e^{-x}\log\left(1+ie^x\right)de^x$$

$$\Big\downarrow \text{ 2838}$$

$$\frac{1}{2}i\,\text{PolyLog}\left(2,-ie^x\right)-\frac{1}{2}i\,\text{PolyLog}\left(2,ie^x\right)$$

input
```
Int[ArcTan[E^x],x]
```

output
```
(I/2)*PolyLog[2, (-I)*E^x] - (I/2)*PolyLog[2, I*E^x]
```

3.110.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5355
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Simp[I*(b/2)   Int[Log[1 - I*c*x]/x, x], x] - Simp[I*(b/2)   Int[Log[1
+ I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

3.110.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(21) = 42$.

Time = 0.31 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.71

method	result	size
parts	$x \arctan(e^x) + \frac{ix\ln(1+ie^x)}{2} - \frac{ix\ln(1-ie^x)}{2} + \frac{i\,\mathrm{dilog}(1+ie^x)}{2} - \frac{i\,\mathrm{dilog}(1-ie^x)}{2}$	53
derivativedivides	$\ln(e^x)\arctan(e^x) + \frac{i\ln(e^x)\ln(1+ie^x)}{2} - \frac{i\ln(e^x)\ln(1-ie^x)}{2} + \frac{i\,\mathrm{dilog}(1+ie^x)}{2} - \frac{i\,\mathrm{dilog}(1-ie^x)}{2}$	59
default	$\ln(e^x)\arctan(e^x) + \frac{i\ln(e^x)\ln(1+ie^x)}{2} - \frac{i\ln(e^x)\ln(1-ie^x)}{2} + \frac{i\,\mathrm{dilog}(1+ie^x)}{2} - \frac{i\,\mathrm{dilog}(1-ie^x)}{2}$	59
risch	$-\frac{ix\ln(1+ie^x)}{2} - \frac{i\ln(-i(-e^x+i))\ln(-ie^x)}{2} + \frac{i\ln(-i(-e^x+i))x}{2} - \frac{i\,\mathrm{dilog}(-ie^x)}{2} - \frac{i\,\mathrm{dilog}(1-ie^x)}{2}$	69

input
```
int(arctan(exp(x)),x,method=_RETURNVERBOSE)
```

output
```
x*arctan(exp(x))+1/2*I*x*ln(1+I*exp(x))-1/2*I*x*ln(1-I*exp(x))+1/2*I*dilog
(1+I*exp(x))-1/2*I*dilog(1-I*exp(x))
```

3.110.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 40 vs. $2(15) = 30$.

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.29

$$\int \arctan(e^x)\,dx = x\arctan(e^x) + \frac{1}{2}i\,x\log(i\,e^x+1) - \frac{1}{2}i\,x\log(-i\,e^x+1) - \frac{1}{2}i\,\mathrm{Li}_2(i\,e^x) + \frac{1}{2}i\,\mathrm{Li}_2(-i\,e^x)$$

input
```
integrate(arctan(exp(x)),x, algorithm="fricas")
```

output
```
x*arctan(e^x) + 1/2*I*x*log(I*e^x + 1) - 1/2*I*x*log(-I*e^x + 1) - 1/2*I*d
ilog(I*e^x) + 1/2*I*dilog(-I*e^x)
```

3.110.6 Sympy [F]

$$\int \arctan(e^x)\ dx = \int \operatorname{atan}(e^x)\,dx$$

input
```
integrate(atan(exp(x)),x)
```

output
```
Integral(atan(exp(x)), x)
```

3.110.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(15) = 30$.

Time = 0.30 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.10

$$\int \arctan(e^x)\ dx = x\arctan(e^x) - \frac{1}{4}\pi\log\left(e^{(2x)}+1\right) - \frac{1}{2}i\operatorname{Li}_2(i\,e^x+1) + \frac{1}{2}i\operatorname{Li}_2(-i\,e^x+1)$$

input
```
integrate(arctan(exp(x)),x, algorithm="maxima")
```

output
```
x*arctan(e^x) - 1/4*pi*log(e^(2*x) + 1) - 1/2*I*dilog(I*e^x + 1) + 1/2*I*d
ilog(-I*e^x + 1)
```

3.110.8 Giac [F]

$$\int \arctan(e^x)\ dx = \int \arctan(e^x)\ dx$$

input
```
integrate(arctan(exp(x)),x, algorithm="giac")
```

output
```
integrate(arctan(e^x), x)
```

3.110.9 Mupad [B] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.68

$$\int \arctan(e^x)\,dx = \frac{\mathrm{polylog}(2, -e^x\,1i)\,1i}{2} - \frac{\mathrm{polylog}(2, e^x\,1i)\,1i}{2}$$

input
```
int(atan(exp(x)),x)
```

output
```
(polylog(2, -exp(x)*1i)*1i)/2 - (polylog(2, exp(x)*1i)*1i)/2
```

3.111 $\int x \arctan(e^x)\, dx$

3.111.1 Optimal result

Integrand size = 6, antiderivative size = 63

$$\int x \arctan(e^x)\, dx = \frac{1}{2} i x \operatorname{PolyLog}(2, -ie^x) - \frac{1}{2} i x \operatorname{PolyLog}(2, ie^x) - \frac{1}{2} i \operatorname{PolyLog}(3, -ie^x) + \frac{1}{2} i \operatorname{PolyLog}(3, ie^x)$$

output
```
1/2*I*x*polylog(2,-I*exp(x))-1/2*I*x*polylog(2,I*exp(x))-1/2*I*polylog(3,-
I*exp(x))+1/2*I*polylog(3,I*exp(x))
```

3.111.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.79

$$\int x \arctan(e^x)\, dx = \frac{1}{2} i (x \operatorname{PolyLog}(2, -ie^x) - x \operatorname{PolyLog}(2, ie^x) - \operatorname{PolyLog}(3, -ie^x) + \operatorname{PolyLog}(3, ie^x))$$

input
```
Integrate[x*ArcTan[E^x],x]
```

output
```
(I/2)*(x*PolyLog[2, (-I)*E^x] - x*PolyLog[2, I*E^x] - PolyLog[3, (-I)*E^x]
 + PolyLog[3, I*E^x])
```

3.111.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5666, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\arctan(e^x)\,dx$$

$$\Big\downarrow \text{5666}$$

$$\frac{1}{2}i\int x\log(1-ie^x)\,dx-\frac{1}{2}i\int x\log(1+ie^x)\,dx$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{2}i\left(\int \text{PolyLog}(2,ie^x)\,dx-x\,\text{PolyLog}(2,ie^x)\right)-\frac{1}{2}i\left(\int \text{PolyLog}(2,-ie^x)\,dx-x\,\text{PolyLog}(2,-ie^x)\right)$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{2}i\left(\int e^{-x}\,\text{PolyLog}(2,ie^x)\,de^x-x\,\text{PolyLog}(2,ie^x)\right)-\frac{1}{2}i\left(\int e^{-x}\,\text{PolyLog}(2,-ie^x)\,de^x-x\,\text{PolyLog}(2,-ie^x)\right)$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{2}i(\text{PolyLog}(3,ie^x)-x\,\text{PolyLog}(2,ie^x))-\frac{1}{2}i(\text{PolyLog}(3,-ie^x)-x\,\text{PolyLog}(2,-ie^x))$$

input
```
Int[x*ArcTan[E^x],x]
```

output
```
(-1/2*I)*(-(x*PolyLog[2, (-I)*E^x]) + PolyLog[3, (-I)*E^x]) + (I/2)*(-(x*P
olyLog[2, I*E^x]) + PolyLog[3, I*E^x])
```

3.111.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5666
```
Int[ArcTan[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol] :
> Simp[I/2   Int[x^m*Log[1 - I*a - I*b*f^(c + d*x)], x], x] - Simp[I/2   In
t[x^m*Log[1 + I*a + I*b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] &
& IntegerQ[m] && m > 0
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.111.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.70

method	result	size
risch	$\frac{ix\operatorname{polylog}(2,-ie^{x})}{2}-\frac{ix\operatorname{polylog}(2,ie^{x})}{2}-\frac{i\operatorname{polylog}(3,-ie^{x})}{2}+\frac{i\operatorname{polylog}(3,ie^{x})}{2}$	44

input
```
int(x*arctan(exp(x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*I*x*polylog(2,-I*exp(x))-1/2*I*x*polylog(2,I*exp(x))-1/2*I*polylog(3,-
I*exp(x))+1/2*I*polylog(3,I*exp(x))
```

3.111.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03

$$\int x\arctan\left(e^x\right)\,dx = \frac{1}{2}x^2\arctan\left(e^x\right)+\frac{1}{4}i\,x^2\log\left(i\,e^x+1\right) - \frac{1}{4}i\,x^2\log\left(-i\,e^x+1\right)-\frac{1}{2}i\,x\mathrm{Li}_2(i\,e^x)+\frac{1}{2}i\,x\mathrm{Li}_2(-i\,e^x) + \frac{1}{2}i\,\mathrm{polylog}(3,i\,e^x)-\frac{1}{2}i\,\mathrm{polylog}(3,-i\,e^x)$$

input
```
integrate(x*arctan(exp(x)),x, algorithm="fricas")
```

output
```
1/2*x^2*arctan(e^x) + 1/4*I*x^2*log(I*e^x + 1) - 1/4*I*x^2*log(-I*e^x + 1)
 - 1/2*I*x*dilog(I*e^x) + 1/2*I*x*dilog(-I*e^x) + 1/2*I*polylog(3, I*e^x)
- 1/2*I*polylog(3, -I*e^x)
```

3.111.6 Sympy [F]

$$\int x\arctan\left(e^x\right)\,dx = \int x\,\mathrm{atan}\left(e^x\right)dx$$

input
```
integrate(x*atan(exp(x)),x)
```

output
```
Integral(x*atan(exp(x)), x)
```

3.111.7 Maxima [F]

$$\int x\arctan\left(e^x\right)\,dx = \int x\arctan\left(e^x\right)\,dx$$

input
```
integrate(x*arctan(exp(x)),x, algorithm="maxima")
```

output
```
1/2*x^2*arctan(e^x) - integrate(1/2*x^2*e^x/(e^(2*x) + 1), x)
```

3.111.8 Giac [F]

$$\int x \arctan(e^x)\,dx = \int x \arctan(e^x)\,dx$$

input
```
integrate(x*arctan(exp(x)),x, algorithm="giac")
```

output
```
integrate(x*arctan(e^x), x)
```

3.111.9 Mupad [F(-1)]

Timed out.

$$\int x \arctan(e^x)\,dx = \int x\,\mathrm{atan}(\mathrm{e}^x)\,dx$$

input
```
int(x*atan(exp(x)),x)
```

output
```
int(x*atan(exp(x)), x)
```

3.112 $\int x^2 \arctan(e^x) \, dx$

3.112.1 Optimal result

Integrand size = 8, antiderivative size = 91

$$\int x^2 \arctan(e^x) \, dx = \frac{1}{2} i x^2 \operatorname{PolyLog}(2, -ie^x) - \frac{1}{2} i x^2 \operatorname{PolyLog}(2, ie^x) - ix \operatorname{PolyLog}(3, -ie^x) + ix \operatorname{PolyLog}(3, ie^x) + i \operatorname{PolyLog}(4, -ie^x) - i \operatorname{PolyLog}(4, ie^x)$$

output
```
1/2*I*x^2*polylog(2,-I*exp(x))-1/2*I*x^2*polylog(2,I*exp(x))-I*x*polylog(3
,-I*exp(x))+I*x*polylog(3,I*exp(x))+I*polylog(4,-I*exp(x))-I*polylog(4,I*e
xp(x))
```

3.112.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.88

$$\int x^2 \arctan(e^x) \, dx = \frac{1}{2} i \left(x^2 \operatorname{PolyLog}(2, -ie^x) - x^2 \operatorname{PolyLog}(2, ie^x) + 2(-x \operatorname{PolyLog}(3, -ie^x) + x \operatorname{PolyLog}(3, ie^x) + \operatorname{PolyLog}(4, -ie^x) - \operatorname{PolyLog}(4, ie^x))\right)$$

input
```
Integrate[x^2*ArcTan[E^x],x]
```

output
```
(I/2)*(x^2*PolyLog[2, (-I)*E^x] - x^2*PolyLog[2, I*E^x] + 2*(-(x*PolyLog[3
, (-I)*E^x]) + x*PolyLog[3, I*E^x] + PolyLog[4, (-I)*E^x] - PolyLog[4, I*E
^x]))
```

3.112.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5666, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(e^x)\, dx$$

$$\Big\downarrow \text{5666}$$

$$\frac{1}{2} i \int x^2 \log(1-ie^x)\, dx-\frac{1}{2} i \int x^2 \log(1+ie^x)\, dx$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{2} i \left(2 \int x \operatorname{PolyLog}(2,ie^x)\, dx-x^2 \operatorname{PolyLog}(2,ie^x)\right)-\frac{1}{2} i \left(2 \int x \operatorname{PolyLog}(2,-ie^x)\, dx-x^2 \operatorname{PolyLog}(2,-ie^x)\right)$$

$$\Big\downarrow \text{7163}$$

$$\frac{1}{2} i \left(2 \left(x \operatorname{PolyLog}(3,ie^x)-\int \operatorname{PolyLog}(3,ie^x)\, dx\right)-x^2 \operatorname{PolyLog}(2,ie^x)\right)-\frac{1}{2} i \left(2 \left(x \operatorname{PolyLog}(3,-ie^x)-\int \operatorname{PolyLog}(3,-ie^x)\, dx\right)-x^2 \operatorname{PolyLog}(2,-ie^x)\right)$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{2} i \left(2 \left(x \operatorname{PolyLog}(3,ie^x)-\int e^{-x} \operatorname{PolyLog}(3,ie^x)\, de^x\right)-x^2 \operatorname{PolyLog}(2,ie^x)\right)-\frac{1}{2} i \left(2 \left(x \operatorname{PolyLog}(3,-ie^x)-\int e^{-x} \operatorname{PolyLog}(3,-ie^x)\, de^x\right)-x^2 \operatorname{PolyLog}(2,-ie^x)\right)$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{2} i (2(x \operatorname{PolyLog}(3,ie^x)-\operatorname{PolyLog}(4,ie^x))-x^2 \operatorname{PolyLog}(2,ie^x))-\frac{1}{2} i (2(x \operatorname{PolyLog}(3,-ie^x)-\operatorname{PolyLog}(4,-ie^x))-x^2 \operatorname{PolyLog}(2,-ie^x))$$

input
```
Int[x^2*ArcTan[E^x],x]
```

output
```
(-1/2*I)*(-(x^2*PolyLog[2, (-I)*E^x]) + 2*(x*PolyLog[3, (-I)*E^x] - PolyLo
g[4, (-I)*E^x])) + (I/2)*(-(x^2*PolyLog[2, I*E^x]) + 2*(x*PolyLog[3, I*E^x
] - PolyLog[4, I*E^x]))
```

3.112.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5666
```
Int[ArcTan[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol] :
> Simp[I/2   Int[x^m*Log[1 - I*a - I*b*f^(c + d*x)], x], x] - Simp[I/2   In
t[x^m*Log[1 + I*a + I*b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] &
& IntegerQ[m] && m > 0
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.112.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.77

method	result
risch	$\frac{ix^2 \operatorname{polylog}(2,-ie^x)}{2} - \frac{ix^2 \operatorname{polylog}(2,ie^x)}{2} - ix \operatorname{polylog}(3,-ie^x) + ix \operatorname{polylog}(3,ie^x) + i \operatorname{polylog}(4,-ie^x)$ -

input
```
int(x^2*arctan(exp(x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*I*x^2*polylog(2,-I*exp(x))-1/2*I*x^2*polylog(2,I*exp(x))-I*x*polylog(3
,-I*exp(x))+I*x*polylog(3,I*exp(x))+I*polylog(4,-I*exp(x))-I*polylog(4,I*e
xp(x))
```

3.112.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.96

$$\int x^2 \arctan(e^x)\, dx = \frac{1}{3}x^3 \arctan(e^x) + \frac{1}{6}i\,x^3 \log(i\,e^x + 1) - \frac{1}{6}i\,x^3 \log(-i\,e^x + 1) - \frac{1}{2}i\,x^2 \mathrm{Li}_2(i\,e^x) + \frac{1}{2}i\,x^2 \mathrm{Li}_2(-i\,e^x) + i\,x \operatorname{polylog}(3, i\,e^x) - i\,x \operatorname{polylog}(3, -i\,e^x) - i \operatorname{polylog}(4, i\,e^x) + i \operatorname{polylog}(4, -i\,e^x)$$

input
```
integrate(x^2*arctan(exp(x)),x, algorithm="fricas")
```

output
```
1/3*x^3*arctan(e^x) + 1/6*I*x^3*log(I*e^x + 1) - 1/6*I*x^3*log(-I*e^x + 1)
 - 1/2*I*x^2*dilog(I*e^x) + 1/2*I*x^2*dilog(-I*e^x) + I*x*polylog(3, I*e^x
) - I*x*polylog(3, -I*e^x) - I*polylog(4, I*e^x) + I*polylog(4, -I*e^x)
```

3.112.6 Sympy [F]

$$\int x^2 \arctan(e^x)\, dx = \int x^2 \operatorname{atan}(e^x)\, dx$$

input
```
integrate(x**2*atan(exp(x)),x)
```

output
```
Integral(x**2*atan(exp(x)), x)
```

3.112.7 Maxima [F]

$$\int x^2 \arctan(e^x) \, dx = \int x^2 \arctan(e^x) \, dx$$

input
```
integrate(x^2*arctan(exp(x)),x, algorithm="maxima")
```

output
```
1/3*x^3*arctan(e^x) - integrate(1/3*x^3*e^x/(e^(2*x) + 1), x)
```

3.112.8 Giac [F]

$$\int x^2 \arctan(e^x) \, dx = \int x^2 \arctan(e^x) \, dx$$

input
```
integrate(x^2*arctan(exp(x)),x, algorithm="giac")
```

output
```
integrate(x^2*arctan(e^x), x)
```

3.112.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \arctan(e^x) \, dx = \int x^2 \operatorname{atan}(e^x) \, dx$$

input
```
int(x^2*atan(exp(x)),x)
```

output
```
int(x^2*atan(exp(x)), x)
```

3.113 $\int \arctan\left(e^{a+bx}\right) dx$

3.113.1 Optimal result

Integrand size = 8, antiderivative size = 45

$$\int \arctan\left(e^{a+bx}\right) dx = \frac{i\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{2b} - \frac{i\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{2b}$$

output
```
1/2*I*polylog(2,-I*exp(b*x+a))/b-1/2*I*polylog(2,I*exp(b*x+a))/b
```

3.113.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.84

$$\int \arctan\left(e^{a+bx}\right) dx = x\arctan\left(e^{a+bx}\right) - \frac{i\left(bx\left(\log\left(1-ie^{a+bx}\right)-\log\left(1+ie^{a+bx}\right)\right)-\text{PolyLog}\left(2,-ie^{a+bx}\right)+\text{PolyLog}\left(2,ie^{a+bx}\right)\right)}{2b}$$

input
```
Integrate[ArcTan[E^(a + b*x)],x]
```

output
```
x*ArcTan[E^(a + b*x)] - ((I/2)*(b*x*(Log[1 - I*E^(a + b*x)] - Log[1 + I*E^
(a + b*x)]) - PolyLog[2, (-I)*E^(a + b*x)] + PolyLog[2, I*E^(a + b*x)]))/b
```

3.113.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2720, 5355, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan\left(e^{a+bx}\right) dx$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{\int e^{-a-bx}\arctan\left(e^{a+bx}\right)de^{a+bx}}{b}$$

$$\Big\downarrow \text{ 5355}$$

$$\frac{\frac{1}{2}i\int e^{-a-bx}\log\left(1-ie^{a+bx}\right)de^{a+bx}-\frac{1}{2}i\int e^{-a-bx}\log\left(1+ie^{a+bx}\right)de^{a+bx}}{b}$$

$$\Big\downarrow \text{ 2838}$$

$$\frac{\frac{1}{2}i\,\text{PolyLog}\left(2,-ie^{a+bx}\right)-\frac{1}{2}i\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b}$$

input
```
Int[ArcTan[E^(a + b*x)],x]
```

output
```
((I/2)*PolyLog[2, (-I)*E^(a + b*x)] - (I/2)*PolyLog[2, I*E^(a + b*x)])/b
```

3.113.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5355
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Simp[I*(b/2)   Int[Log[1 - I*c*x]/x, x], x] - Simp[I*(b/2)   Int[Log[1
+ I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

3.113.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(35) = 70$.

Time = 0.24 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.11

method	result
derivativedivides	$\frac{\ln(e^{bx+a})\arctan(e^{bx+a})+\frac{i\ln(e^{bx+a})\ln(1+ie^{bx+a})}{2}-\frac{i\ln(e^{bx+a})\ln(1-ie^{bx+a})}{2}+\frac{i\,\mathrm{dilog}(1+ie^{bx+a})}{2}-\frac{i\,\mathrm{dilog}(1-ie^{bx+a})}{2}}{b}$
default	$\frac{\ln(e^{bx+a})\arctan(e^{bx+a})+\frac{i\ln(e^{bx+a})\ln(1+ie^{bx+a})}{2}-\frac{i\ln(e^{bx+a})\ln(1-ie^{bx+a})}{2}+\frac{i\,\mathrm{dilog}(1+ie^{bx+a})}{2}-\frac{i\,\mathrm{dilog}(1-ie^{bx+a})}{2}}{b}$
parts	$x\arctan\left(e^{bx+a}\right)-\frac{-\frac{i(bx+a)\ln(1+ie^{bx+a})}{2}+\frac{i(bx+a)\ln(1-ie^{bx+a})}{2}-\frac{i\,\mathrm{dilog}(1+ie^{bx+a})}{2}+\frac{i\,\mathrm{dilog}(1-ie^{bx+a})}{2}-a\,\mathrm{arcta}}{b}$
risch	$-\frac{ix\ln(1+ie^{bx+a})}{2}+\frac{i\ln(-i(-e^{bx+a}+i))x}{2}-\frac{ia\ln(1+ie^{bx+a})}{2b}-\frac{i\ln(-i(-e^{bx+a}+i))\ln(-ie^{bx+a})}{2b}+\frac{i\ln(-i(-}{}$

input
```
int(arctan(exp(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
1/b*(ln(exp(b*x+a))*arctan(exp(b*x+a))+1/2*I*ln(exp(b*x+a))*ln(1+I*exp(b*x
+a))-1/2*I*ln(exp(b*x+a))*ln(1-I*exp(b*x+a))+1/2*I*dilog(1+I*exp(b*x+a))-1
/2*I*dilog(1-I*exp(b*x+a)))
```

3.113.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(29) = 58$.

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.29

$$\int \arctan\left(e^{a+bx}\right)\,dx$$
$$=\frac{2\,bx\arctan\left(e^{(bx+a)}\right)+i\,a\log\left(e^{(bx+a)}+i\right)-i\,a\log\left(e^{(bx+a)}-i\right)+(i\,bx+i\,a)\log\left(i\,e^{(bx+a)}+1\right)+(-i\,bx}{2\,b}$$

input
```
integrate(arctan(exp(b*x+a)),x, algorithm="fricas")
```

output
```
1/2*(2*b*x*arctan(e^(b*x + a)) + I*a*log(e^(b*x + a) + I) - I*a*log(e^(b*x
 + a) - I) + (I*b*x + I*a)*log(I*e^(b*x + a) + 1) + (-I*b*x - I*a)*log(-I*
e^(b*x + a) + 1) - I*dilog(I*e^(b*x + a)) + I*dilog(-I*e^(b*x + a)))/b
```

3.113.6 Sympy [F]

$$\int \arctan\left(e^{a+bx}\right)\,dx = \int \operatorname{atan}\left(e^{a+bx}\right)dx$$

input
```
integrate(atan(exp(b*x+a)),x)
```

output
```
Integral(atan(exp(a + b*x)), x)
```

3.113.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(29) = 58$.

Time = 0.29 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.40

$$\int \arctan\left(e^{a+bx}\right)\,dx = \frac{(bx+a)\arctan\left(e^{(bx+a)}\right)}{b} - \frac{\pi\log\left(e^{(2\,bx+2\,a)}+1\right) + 2i\,\mathrm{Li}_2\left(i\,e^{(bx+a)}+1\right) - 2i\,\mathrm{Li}_2\left(-i\,e^{(bx+a)}+1\right)}{4\,b}$$

input
```
integrate(arctan(exp(b*x+a)),x, algorithm="maxima")
```

output
```
(b*x + a)*arctan(e^(b*x + a))/b - 1/4*(pi*log(e^(2*b*x + 2*a) + 1) + 2*I*d
ilog(I*e^(b*x + a) + 1) - 2*I*dilog(-I*e^(b*x + a) + 1))/b
```

3.113.8 Giac [F]

$$\int \arctan\left(e^{a+bx}\right)\,dx = \int \arctan\left(e^{(bx+a)}\right)\,dx$$

input
```
integrate(arctan(exp(b*x+a)),x, algorithm="giac")
```

output
```
integrate(arctan(e^(b*x + a)), x)
```

3.113.9 Mupad [B] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int \arctan\left(e^{a+bx}\right)\,dx = -\frac{\mathrm{Li}_2\left(1-\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,1\mathrm{i}\right)\,1\mathrm{i}}{2\,b} + \frac{\mathrm{Li}_2\left(1+\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,1\mathrm{i}\right)\,1\mathrm{i}}{2\,b}$$

input
```
int(atan(exp(a + b*x)),x)
```

output
```
(dilog(exp(b*x)*exp(a)*1i + 1)*1i)/(2*b) - (dilog(1 - exp(b*x)*exp(a)*1i)*
1i)/(2*b)
```

3.114 $\int x \arctan \left(e^{a+bx}\right) dx$

3.114.1 Optimal result

Integrand size = 10, antiderivative size = 91

$$\int x \arctan \left(e^{a+bx}\right) dx = \frac{ix \operatorname{PolyLog}\left(2,-ie^{a+bx}\right)}{2b} - \frac{ix \operatorname{PolyLog}\left(2,ie^{a+bx}\right)}{2b} - \frac{i \operatorname{PolyLog}\left(3,-ie^{a+bx}\right)}{2b^2} + \frac{i \operatorname{PolyLog}\left(3,ie^{a+bx}\right)}{2b^2}$$

output
```
1/2*I*x*polylog(2,-I*exp(b*x+a))/b-1/2*I*x*polylog(2,I*exp(b*x+a))/b-1/2*I
*polylog(3,-I*exp(b*x+a))/b^2+1/2*I*polylog(3,I*exp(b*x+a))/b^2
```

3.114.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.78

$$\int x \arctan \left(e^{a+bx}\right) dx = \frac{i\left(bx \operatorname{PolyLog}\left(2,-ie^{a+bx}\right) - bx \operatorname{PolyLog}\left(2,ie^{a+bx}\right) - \operatorname{PolyLog}\left(3,-ie^{a+bx}\right) + \operatorname{PolyLog}\left(3,ie^{a+bx}\right)\right)}{2b^2}$$

input
```
Integrate[x*ArcTan[E^(a + b*x)],x]
```

output
```
((I/2)*(b*x*PolyLog[2, (-I)*E^(a + b*x)] - b*x*PolyLog[2, I*E^(a + b*x)] -
 PolyLog[3, (-I)*E^(a + b*x)] + PolyLog[3, I*E^(a + b*x)]))/b^2
```

3.114.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5666, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan\left(e^{a+bx}\right) dx$$

$$\Big\downarrow \text{5666}$$

$$\frac{1}{2}i\int x\log\left(1-ie^{a+bx}\right)dx-\frac{1}{2}i\int x\log\left(1+ie^{a+bx}\right)dx$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{2}i\left(\frac{\int \text{PolyLog}\left(2,ie^{a+bx}\right)dx}{b}-\frac{x\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b}\right)-\frac{1}{2}i\left(\frac{\int \text{PolyLog}\left(2,-ie^{a+bx}\right)dx}{b}-\frac{x\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b}\right)$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{2}i\left(\frac{\int e^{-a-bx}\,\text{PolyLog}\left(2,ie^{a+bx}\right)de^{a+bx}}{b^2}-\frac{x\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b}\right)-\frac{1}{2}i\left(\frac{\int e^{-a-bx}\,\text{PolyLog}\left(2,-ie^{a+bx}\right)de^{a+bx}}{b^2}-\frac{x\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b}\right)$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{2}i\left(\frac{\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^2}-\frac{x\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b}\right)-\frac{1}{2}i\left(\frac{\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^2}-\frac{x\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b}\right)$$

input
```
Int[x*ArcTan[E^(a + b*x)],x]
```

output
```
(-1/2*I)*(-((x*PolyLog[2, (-I)*E^(a + b*x)])/b) + PolyLog[3, (-I)*E^(a + b
*x)]/b^2) + (I/2)*(-((x*PolyLog[2, I*E^(a + b*x)])/b) + PolyLog[3, I*E^(a
+ b*x)]/b^2)
```

3.114.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5666
```
Int[ArcTan[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol] :
> Simp[I/2   Int[x^m*Log[1 - I*a - I*b*f^(c + d*x)], x], x] - Simp[I/2   In
t[x^m*Log[1 + I*a + I*b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] &
& IntegerQ[m] && m > 0
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.114.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 348 vs. 2(71) = 142.

Time = 0.38 (sec) , antiderivative size = 349, normalized size of antiderivative = 3.84

method	result
risch	$-\frac{i a^{2} \ln\left(1-i \mathrm{e}^{b x +a}\right)}{2 b^{2}}+\frac{i x \,\mathrm{polylog}\left(2,-i \mathrm{e}^{b x +a}\right)}{2 b}+\frac{i \,\mathrm{dilog}\left(-i \left(\mathrm{e}^{b x +a}+i\right)\right) a}{2 b^{2}}+\frac{i \,\mathrm{dilog}\left(-i \mathrm{e}^{b x +a}\right) a}{2 b^{2}}-\frac{i x \,\mathrm{polylog}\left(2, i \mathrm{e}^{b x +a}\right)}{2 b}-$ i po

input
```
int(x*arctan(exp(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I/b^2*a^2*ln(1-I*exp(b*x+a))+1/2*I*x*polylog(2,-I*exp(b*x+a))/b+1/2*I
/b^2*dilog(-I*(exp(b*x+a)+I))*a+1/2*I/b^2*dilog(-I*exp(b*x+a))*a-1/2*I*x*p
olylog(2,I*exp(b*x+a))/b-1/2*I*polylog(3,-I*exp(b*x+a))/b^2-1/2*I/b*ln(-I*
(-exp(b*x+a)+I))*a*x+1/2*I/b*ln(-I*(exp(b*x+a)+I))*a*x+1/2*I/b^2*ln(-I*exp
(b*x+a))*ln(-I*(-exp(b*x+a)+I))*a-1/2*I/b*ln(1-I*exp(b*x+a))*a*x+1/2*I*pol
ylog(3,I*exp(b*x+a))/b^2+1/2*I/b^2*polylog(2,-I*exp(b*x+a))*a+1/2*I/b*ln(1
+I*exp(b*x+a))*a*x-1/2*I/b^2*polylog(2,I*exp(b*x+a))*a-1/2*I/b^2*ln(-I*(-e
xp(b*x+a)+I))*a^2+1/2*I/b^2*ln(-I*(exp(b*x+a)+I))*a^2+1/2*I/b^2*a^2*ln(1+I
*exp(b*x+a))
```

3.114.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(61) = 122$.

Time = 0.28 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.66

$$\int x \arctan\left(e^{a+bx}\right) dx = \frac{2\,b^2x^2\arctan\left(e^{(bx+a)}\right) - 2i\,bx\mathrm{Li}_2\left(i\,e^{(bx+a)}\right) + 2i\,bx\mathrm{Li}_2\left(-i\,e^{(bx+a)}\right) - i\,a^2\log\left(e^{(bx+a)} + i\right) + i\,a^2\log\left(e^{(bx+a)}\right.}{}$$

input
```
integrate(x*arctan(exp(b*x+a)),x, algorithm="fricas")
```

output
```
1/4*(2*b^2*x^2*arctan(e^(b*x + a)) - 2*I*b*x*dilog(I*e^(b*x + a)) + 2*I*b*
x*dilog(-I*e^(b*x + a)) - I*a^2*log(e^(b*x + a) + I) + I*a^2*log(e^(b*x +
a) - I) + (I*b^2*x^2 - I*a^2)*log(I*e^(b*x + a) + 1) + (-I*b^2*x^2 + I*a^2
)*log(-I*e^(b*x + a) + 1) + 2*I*polylog(3, I*e^(b*x + a)) - 2*I*polylog(3,
 -I*e^(b*x + a)))/b^2
```

3.114.6 Sympy [F]

$$\int x \arctan\left(e^{a+bx}\right) dx = \int x \operatorname{atan}\left(e^a e^{bx}\right) dx$$

input
```
integrate(x*atan(exp(b*x+a)),x)
```

output
```
Integral(x*atan(exp(a)*exp(b*x)), x)
```

3.114.7 Maxima [F]

$$\int x \arctan\left(e^{a+bx}\right) dx = \int x \arctan\left(e^{(bx+a)}\right) dx$$

input
```
integrate(x*arctan(exp(b*x+a)),x, algorithm="maxima")
```

output
```
1/2*x^2*arctan(e^(b*x + a)) - b*integrate(1/2*x^2*e^(b*x + a)/(e^(2*b*x +
2*a) + 1), x)
```

3.114.8 Giac [F]

$$\int x \arctan\left(e^{a+bx}\right) dx = \int x \arctan\left(e^{(bx+a)}\right) dx$$

input
```
integrate(x*arctan(exp(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x*arctan(e^(b*x + a)), x)
```

3.114.9 Mupad [F(-1)]

Timed out.

$$\int x \arctan\left(e^{a+bx}\right) dx = \int x \operatorname{atan}\left(e^{a+bx}\right) dx$$

input
```
int(x*atan(exp(a + b*x)),x)
```

output
```
int(x*atan(exp(a + b*x)), x)
```

3.115 $\int x^2 \arctan \left(e^{a+bx}\right) \, dx$

3.115.1 Optimal result

Integrand size = 12, antiderivative size = 133

$$\int x^2 \arctan \left(e^{a+bx}\right) \, dx = \frac{ix^2 \operatorname{PolyLog}\left(2,-ie^{a+bx}\right)}{2b} - \frac{ix^2 \operatorname{PolyLog}\left(2,ie^{a+bx}\right)}{2b} - \frac{ix \operatorname{PolyLog}\left(3,-ie^{a+bx}\right)}{b^2} + \frac{ix \operatorname{PolyLog}\left(3,ie^{a+bx}\right)}{b^2} + \frac{i \operatorname{PolyLog}\left(4,-ie^{a+bx}\right)}{b^3} - \frac{i \operatorname{PolyLog}\left(4,ie^{a+bx}\right)}{b^3}$$

output
```
1/2*I*x^2*polylog(2,-I*exp(b*x+a))/b-1/2*I*x^2*polylog(2,I*exp(b*x+a))/b-I
*x*polylog(3,-I*exp(b*x+a))/b^2+I*x*polylog(3,I*exp(b*x+a))/b^2+I*polylog(
4,-I*exp(b*x+a))/b^3-I*polylog(4,I*exp(b*x+a))/b^3
```

3.115.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.86

$$\int x^2 \arctan \left(e^{a+bx}\right) \, dx = \frac{i\left(b^2x^2 \operatorname{PolyLog}\left(2,-ie^{a+bx}\right) - b^2x^2 \operatorname{PolyLog}\left(2,ie^{a+bx}\right) + 2\left(-bx \operatorname{PolyLog}\left(3,-ie^{a+bx}\right) + bx \operatorname{PolyLog}\left(3,ie\right.\right.\right.}{2b^3}$$

input
```
Integrate[x^2*ArcTan[E^(a + b*x)],x]
```

output
```
((I/2)*(b^2*x^2*PolyLog[2, (-I)*E^(a + b*x)] - b^2*x^2*PolyLog[2, I*E^(a +
 b*x)] + 2*(-(b*x*PolyLog[3, (-I)*E^(a + b*x)]) + b*x*PolyLog[3, I*E^(a +
b*x)] + PolyLog[4, (-I)*E^(a + b*x)] - PolyLog[4, I*E^(a + b*x)])))/b^3
```

3.115.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5666, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan\left(e^{a+bx}\right) dx$$

$$\Big\downarrow \text{5666}$$

$$\frac{1}{2}i\int x^2\log\left(1-ie^{a+bx}\right)dx-\frac{1}{2}i\int x^2\log\left(1+ie^{a+bx}\right)dx$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{2}i\left(\frac{2\int x\,\text{PolyLog}\left(2,ie^{a+bx}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b}\right)-\frac{1}{2}i\left(\frac{2\int x\,\text{PolyLog}\left(2,-ie^{a+bx}\right)dx}{b}-\frac{x^2\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b}\right)$$

$$\Big\downarrow \text{7163}$$

$$\frac{1}{2}i\left(\frac{2\left(\frac{x\,\text{PolyLog}(3,ie^{a+bx})}{b}-\frac{\int \text{PolyLog}(3,ie^{a+bx})dx}{b}\right)}{b}-\frac{x^2\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b}\right)-\frac{1}{2}i\left(\frac{2\left(\frac{x\,\text{PolyLog}(3,-ie^{a+bx})}{b}-\frac{\int \text{PolyLog}(3,-ie^{a+bx})dx}{b}\right)}{b}-\frac{x^2\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b}\right)$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{2}i\left(\frac{2\left(\frac{x\,\text{PolyLog}(3,ie^{a+bx})}{b}-\frac{\int e^{-a-bx}\,\text{PolyLog}(3,ie^{a+bx})de^{a+bx}}{b^2}\right)}{b}-\frac{x^2\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b}\right)-\frac{1}{2}i\left(\frac{2\left(\frac{x\,\text{PolyLog}(3,-ie^{a+bx})}{b}-\frac{\int e^{-a-bx}\,\text{PolyLog}(3,-ie^{a+bx})de^{a+bx}}{b^2}\right)}{b}-\frac{x^2\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b}\right)$$

$$\Big\downarrow 7143$$

$$\frac{1}{2}i\left(\frac{2\left(\frac{x\,\text{PolyLog}(3,ie^{a+bx})}{b}-\frac{\text{PolyLog}(4,ie^{a+bx})}{b^2}\right)}{b}-\frac{x^2\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b}\right)-$$
$$\frac{1}{2}i\left(\frac{2\left(\frac{x\,\text{PolyLog}(3,-ie^{a+bx})}{b}-\frac{\text{PolyLog}(4,-ie^{a+bx})}{b^2}\right)}{b}-\frac{x^2\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b}\right)$$

input
```
Int[x^2*ArcTan[E^(a + b*x)],x]
```

output
```
(-1/2*I)*(-((x^2*PolyLog[2, (-I)*E^(a + b*x)])/b) + (2*((x*PolyLog[3, (-I)
*E^(a + b*x)])/b - PolyLog[4, (-I)*E^(a + b*x)]/b^2))/b) + (I/2)*(-((x^2*P
olyLog[2, I*E^(a + b*x)])/b) + (2*((x*PolyLog[3, I*E^(a + b*x)])/b - PolyL
og[4, I*E^(a + b*x)]/b^2))/b)
```

3.115.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 5666
```
Int[ArcTan[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol] :
> Simp[I/2   Int[x^m*Log[1 - I*a - I*b*f^(c + d*x)], x], x] - Simp[I/2   In
t[x^m*Log[1 + I*a + I*b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] &
& IntegerQ[m] && m > 0
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.115.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 406 vs. $2(111) = 222$.

Time = 0.42 (sec) , antiderivative size = 407, normalized size of antiderivative = 3.06

method	result
risch	$-\frac{i\,\mathrm{dilog}\left(-i\,\mathrm{e}^{bx+a}\right)a^{2}}{2b^{3}}+\frac{i\ln\left(1-i\,\mathrm{e}^{bx+a}\right)a^{3}}{2b^{3}}-\frac{i\,\mathrm{dilog}\left(-i\left(\mathrm{e}^{bx+a}+i\right)\right)a^{2}}{2b^{3}}+\frac{i x^{2}\,\mathrm{polylog}\left(2,-i\,\mathrm{e}^{bx+a}\right)}{2b}+\frac{i x\,\mathrm{polylog}\left(3,i\,\mathrm{e}^{bx+a}\right)}{b^{2}}+$

input
```
int(x^2*arctan(exp(b*x+a)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I/b^3*dilog(-I*exp(b*x+a))*a^2+1/2*I/b^3*ln(1-I*exp(b*x+a))*a^3-1/2*I
/b^3*dilog(-I*(exp(b*x+a)+I))*a^2+1/2*I*x^2*polylog(2,-I*exp(b*x+a))/b+I*x
*polylog(3,I*exp(b*x+a))/b^2+1/2*I/b^2*ln(1-I*exp(b*x+a))*x*a^2-1/2*I/b^3*
a^3*ln(1+I*exp(b*x+a))-1/2*I/b^3*ln(-I*exp(b*x+a))*ln(-I*(-exp(b*x+a)+I))*
a^2+1/2*I/b^2*ln(-I*(-exp(b*x+a)+I))*a^2*x-I*polylog(4,I*exp(b*x+a))/b^3-1
/2*I/b^2*ln(1+I*exp(b*x+a))*a^2*x+1/2*I/b^3*polylog(2,I*exp(b*x+a))*a^2+1/
2*I/b^3*ln(-I*(-exp(b*x+a)+I))*a^3-1/2*I/b^3*ln(-I*(exp(b*x+a)+I))*a^3-1/2
*I/b^2*ln(-I*(exp(b*x+a)+I))*x*a^2+I*polylog(4,-I*exp(b*x+a))/b^3-1/2*I/b^
3*polylog(2,-I*exp(b*x+a))*a^2-I*x*polylog(3,-I*exp(b*x+a))/b^2-1/2*I*x^2*
polylog(2,I*exp(b*x+a))/b
```

3.115.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.41

$$\int x^2 \arctan\left(e^{a+bx}\right) dx$$
$$= \frac{2 b^3 x^3 \arctan\left(e^{(bx+a)}\right) - 3i b^2 x^2 \text{Li}_2\left(i e^{(bx+a)}\right) + 3i b^2 x^2 \text{Li}_2\left(-i e^{(bx+a)}\right) + i a^3 \log\left(e^{(bx+a)} + i\right) - i a^3 \log\left(e^{(b\right.}\right.}{}$$

input
```
integrate(x^2*arctan(exp(b*x+a)),x, algorithm="fricas")
```

output
```
1/6*(2*b^3*x^3*arctan(e^(b*x + a)) - 3*I*b^2*x^2*dilog(I*e^(b*x + a)) + 3*
I*b^2*x^2*dilog(-I*e^(b*x + a)) + I*a^3*log(e^(b*x + a) + I) - I*a^3*log(e
^(b*x + a) - I) + 6*I*b*x*polylog(3, I*e^(b*x + a)) - 6*I*b*x*polylog(3, -
I*e^(b*x + a)) + (I*b^3*x^3 + I*a^3)*log(I*e^(b*x + a) + 1) + (-I*b^3*x^3
- I*a^3)*log(-I*e^(b*x + a) + 1) - 6*I*polylog(4, I*e^(b*x + a)) + 6*I*pol
ylog(4, -I*e^(b*x + a)))/b^3
```

3.115.6 Sympy [F]

$$\int x^2 \arctan\left(e^{a+bx}\right) dx = \int x^2 \operatorname{atan}\left(e^a e^{bx}\right) dx$$

input
```
integrate(x**2*atan(exp(b*x+a)),x)
```

output
```
Integral(x**2*atan(exp(a)*exp(b*x)), x)
```

3.115.7 Maxima [F]

$$\int x^2 \arctan\left(e^{a+bx}\right) dx = \int x^2 \arctan\left(e^{(bx+a)}\right) dx$$

input
```
integrate(x^2*arctan(exp(b*x+a)),x, algorithm="maxima")
```

output
```
1/3*x^3*arctan(e^(b*x + a)) - b*integrate(1/3*x^3*e^(b*x + a)/(e^(2*b*x +
2*a) + 1), x)
```

3.115.8 Giac [F]

$$\int x^2 \arctan\left(e^{a+bx}\right)\,dx = \int x^2 \arctan\left(e^{(bx+a)}\right)\,dx$$

input
```
integrate(x^2*arctan(exp(b*x+a)),x, algorithm="giac")
```

output
```
integrate(x^2*arctan(e^(b*x + a)), x)
```

3.115.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \arctan\left(e^{a+bx}\right)\,dx = \int x^2 \operatorname{atan}\left(\mathrm{e}^{a+b\,x}\right)\,dx$$

input
```
int(x^2*atan(exp(a + b*x)),x)
```

output
```
int(x^2*atan(exp(a + b*x)), x)
```

3.116 $\int \arctan\left(a+bf^{c+dx}\right)\,dx$

3.116.1 Optimal result

Integrand size = 12, antiderivative size = 196

$$\int \arctan\left(a+bf^{c+dx}\right)\,dx = -\frac{\arctan\left(a+bf^{c+dx}\right)\log\left(\frac{2}{1-i(a+bf^{c+dx})}\right)}{d\log(f)} + \frac{\arctan\left(a+bf^{c+dx}\right)\log\left(\frac{2bf^{c+dx}}{(i-a)(1-i(a+bf^{c+dx}))}\right)}{d\log(f)} + \frac{i\,\text{PolyLog}\left(2,1-\frac{2}{1-i(a+bf^{c+dx})}\right)}{2d\log(f)} - \frac{i\,\text{PolyLog}\left(2,1-\frac{2bf^{c+dx}}{(i-a)(1-i(a+bf^{c+dx}))}\right)}{2d\log(f)}$$

output
```
-arctan(a+b*f^(d*x+c))*ln(2/(1-I*(a+b*f^(d*x+c))))/d/ln(f)+arctan(a+b*f^(d
*x+c))*ln(2*b*f^(d*x+c)/(I-a)/(1-I*(a+b*f^(d*x+c))))/d/ln(f)+1/2*I*polylog
(2,1-2/(1-I*(a+b*f^(d*x+c))))/d/ln(f)-1/2*I*polylog(2,1-2*b*f^(d*x+c)/(I-a
)/(1-I*(a+b*f^(d*x+c))))/d/ln(f)
```

3.116.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.85

$$\int \arctan \left(a+bf^{c+dx}\right) \, dx = x \arctan \left(a+bf^{c+dx}\right) - \frac{b\left(dx\log(f)\left(\log\left(1+\frac{b^2f^{c+dx}}{ab-\sqrt{-b^2}}\right)-\log\left(1+\frac{b^2f^{c+dx}}{ab+\sqrt{-b^2}}\right)\right)+\text{PolyLog}\left(2,-\frac{b^2f^{c+dx}}{ab-\sqrt{-b^2}}\right)-\text{PolyLog}\left(2,-\frac{b^2f^{c+dx}}{ab+\sqrt{-b^2}}\right)\right)}{2\sqrt{-b^2}d\log(f)}$$

input
```
Integrate[ArcTan[a + b*f^(c + d*x)],x]
```

output
```
x*ArcTan[a + b*f^(c + d*x)] - (b*(d*x*Log[f]*(Log[1 + (b^2*f^(c + d*x))/(a
*b - Sqrt[-b^2])] - Log[1 + (b^2*f^(c + d*x))/(a*b + Sqrt[-b^2])]) + PolyL
og[2, -((b^2*f^(c + d*x))/(a*b - Sqrt[-b^2]))] - PolyLog[2, -((b^2*f^(c +
d*x))/(a*b + Sqrt[-b^2]))]))/(2*Sqrt[-b^2]*d*Log[f])
```

3.116.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.90, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2720, 5570, 25, 27, 5381, 2849, 2752, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan \left(a+bf^{c+dx}\right) \, dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int f^{-c-dx} \arctan \left(bf^{c+dx}+a\right) df^{c+dx}}{d\log(f)}$$

$$\Big\downarrow \text{5570}$$

$$\frac{\int f^{-c-dx} \arctan \left(bf^{c+dx}+a\right) d\left(bf^{c+dx}+a\right)}{bd\log(f)}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int -f^{-c-dx} \arctan \left(bf^{c+dx}+a\right) d\left(bf^{c+dx}+a\right)}{bd\log(f)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int -\frac{f^{-c-dx}\arctan(bf^{c+dx}+a)}{b}d(bf^{c+dx}+a)}{d\log(f)}$$

$\downarrow$ 5381

$$-\frac{-\int \frac{\log\left(\frac{2}{1-i\left(bf^{c+dx}+a\right)}\right)}{f^{2c+2dx}+1}d(bf^{c+dx}+a)+\int \frac{\log\left(\frac{2bf^{c+dx}}{(i-a)\left(1-i\left(bf^{c+dx}+a\right)\right)}\right)}{f^{2c+2dx}+1}d(bf^{c+dx}+a)+\arctan\left(a+bf^{c+dx}\right)\log\left(\frac{}{1-i(}\right.}{d\log(f)}$$

$\downarrow$ 2849

$$-\frac{-i\int \frac{\log\left(\frac{2}{1-i\left(bf^{c+dx}+a\right)}\right)}{1-\frac{2}{1-i\left(bf^{c+dx}+a\right)}}d\frac{1}{1-i(bf^{c+dx}+a)}+\int \frac{\log\left(\frac{2bf^{c+dx}}{(i-a)\left(1-i\left(bf^{c+dx}+a\right)\right)}\right)}{f^{2c+2dx}+1}d(bf^{c+dx}+a)+\arctan\left(a+bf^{c+dx}\right)\log\left(\frac{}{1-i}\right.}{d\log(f)}$$

$\downarrow$ 2752

$$-\frac{\int \frac{\log\left(\frac{2bf^{c+dx}}{(i-a)\left(1-i\left(bf^{c+dx}+a\right)\right)}\right)}{f^{2c+2dx}+1}d(bf^{c+dx}+a)+\arctan\left(a+bf^{c+dx}\right)\log\left(\frac{2}{1-i(a+bf^{c+dx})}\right)-\arctan\left(a+bf^{c+dx}\right)\log\left(\frac{}{(}\right.}{d\log(f)}$$

$\downarrow$ 2897

$$-\frac{\arctan\left(a+bf^{c+dx}\right)\log\left(\frac{2}{1-i(a+bf^{c+dx})}\right)-\arctan\left(a+bf^{c+dx}\right)\log\left(\frac{2bf^{c+dx}}{(-a+i)(1-i(a+bf^{c+dx}))}\right)-\frac{1}{2}i\,\mathrm{PolyLog}\left(2,1-\right.}{d\log(f)}$$

input
```
Int[ArcTan[a + b*f^(c + d*x)],x]
```

output
```
-((ArcTan[a + b*f^(c + d*x)]*Log[2/(1 - I*(a + b*f^(c + d*x)))] - ArcTan[a
 + b*f^(c + d*x)]*Log[(2*b*f^(c + d*x))/((I - a)*(1 - I*(a + b*f^(c + d*x)
)))] - (I/2)*PolyLog[2, 1 - 2/(1 - I*(a + b*f^(c + d*x)))] + (I/2)*PolyLog
[2, 1 - (2*b*f^(c + d*x))/((I - a)*(1 - I*(a + b*f^(c + d*x))))])/(d*Log[f
]))
```

3.116. $\int \arctan\left(a+bf^{c+dx}\right)\,dx$

3.116.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5381
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] + Simp[b*(c/e)
   Int[Log[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Simp[b*(c/e)   Int[Log[2*
c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x]) /; FreeQ[{a
, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

rule 5570
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Simp[1/d   Subst[Int[(((d*e - c*f)/d + f*(x/d))^m*(a + b*A
rcTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && I
GtQ[p, 0]
```

3.116.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{\ln(-b f^{dx+c}) \arctan(a+b f^{dx+c}) - \frac{i \ln(-b f^{dx+c}) \ln\left(\frac{i+b f^{dx+c}+a}{i+a}\right)}{2} + \frac{i \ln(-b f^{dx+c}) \ln\left(\frac{i-b f^{dx+c}-a}{i-a}\right)}{2} - \frac{i \operatorname{dilog}\left(\frac{i+b f^{dx}}{i+}\right)}{2}}{d \ln(f)}$
default	$\frac{\ln(-b f^{dx+c}) \arctan(a+b f^{dx+c}) - \frac{i \ln(-b f^{dx+c}) \ln\left(\frac{i+b f^{dx+c}+a}{i+a}\right)}{2} + \frac{i \ln(-b f^{dx+c}) \ln\left(\frac{i-b f^{dx+c}-a}{i-a}\right)}{2} - \frac{i \operatorname{dilog}\left(\frac{i+b f^{dx}}{i+}\right)}{2}}{d \ln(f)}$
risch	$-\frac{ix \ln(1+i(a+b f^{dx+c}))}{2} + \frac{i \operatorname{dilog}\left(\frac{b f^{dx} f^{c}+a-i}{a-i}\right)}{2 \ln(f) d} + \frac{i \ln\left(\frac{b f^{dx} f^{c}+a-i}{a-i}\right) x}{2} + \frac{i \ln\left(\frac{b f^{dx} f^{c}+a-i}{a-i}\right) c}{2d} - \frac{ic \ln(i f^{dx} f}{2}$

input
```
int(arctan(a+b*f^(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d/ln(f)*(ln(-b*f^(d*x+c))*arctan(a+b*f^(d*x+c))-1/2*I*ln(-b*f^(d*x+c))*l
n((I+b*f^(d*x+c)+a)/(I+a))+1/2*I*ln(-b*f^(d*x+c))*ln((I-b*f^(d*x+c)-a)/(I-
a))-1/2*I*dilog((I+b*f^(d*x+c)+a)/(I+a))+1/2*I*dilog((I-b*f^(d*x+c)-a)/(I-
a)))
```

3.116.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.08

$$\int \arctan\left(a+bf^{c+dx}\right)\,dx$$
$$= \frac{2\,dx\arctan\left(bf^{dx+c}+a\right)\log\left(f\right)+i\,c\log\left(bf^{dx+c}+a+i\right)\log\left(f\right)-i\,c\log\left(bf^{dx+c}+a-i\right)\log\left(f\right)+\left(i\,dx\right.}{}$$

input
```
integrate(arctan(a+b*f^(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(2*d*x*arctan(b*f^(d*x + c) + a)*log(f) + I*c*log(b*f^(d*x + c) + a +
I)*log(f) - I*c*log(b*f^(d*x + c) + a - I)*log(f) + (I*d*x + I*c)*log(f)*l
og((a^2 + (a*b + I*b)*f^(d*x + c) + 1)/(a^2 + 1)) + (-I*d*x - I*c)*log(f)*
log((a^2 + (a*b - I*b)*f^(d*x + c) + 1)/(a^2 + 1)) + I*dilog(-(a^2 + (a*b
+ I*b)*f^(d*x + c) + 1)/(a^2 + 1) + 1) - I*dilog(-(a^2 + (a*b - I*b)*f^(d*
x + c) + 1)/(a^2 + 1) + 1))/(d*log(f))
```

3.116.6 Sympy [F]

$$\int \arctan \left(a+bf^{c+dx}\right) \, dx=\int \operatorname{atan}\left(a+bf^{c+dx}\right)\, dx$$

input
```
integrate(atan(a+b*f**(d*x+c)),x)
```

output
```
Integral(atan(a + b*f**(c + d*x)), x)
```

3.116.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.96

$$\int \arctan \left(a+bf^{c+dx}\right) \, dx=\frac{(dx+c)\arctan\left(bf^{dx+c}+a\right)}{d} - \frac{2\,(dx+c)\arctan\left(\frac{b^2f^{dx+c}+ab}{b}\right)\log(f)+\left(\pi-\arctan\left(\frac{1}{a}\right)\right)\log\left(b^2f^{2\,dx+2\,c}+2\,abf^{dx+c}+a^2+1\right)-\text{arctar}}{2\,d\log(f)}$$

input
```
integrate(arctan(a+b*f^(d*x+c)),x, algorithm="maxima")
```

output
```
(d*x + c)*arctan(b*f^(d*x + c) + a)/d - 1/2*(2*(d*x + c)*arctan((b^2*f^(d*
x + c) + a*b)/b)*log(f) + (pi - arctan(1/a))*log(b^2*f^(2*d*x + 2*c) + 2*a
*b*f^(d*x + c) + a^2 + 1) - arctan(b*f^(d*x + c) + a)*log(b^2*f^(2*d*x + 2
*c)/(a^2 + 1)) + I*dilog((I*b*f^(d*x + c) + I*a + 1)/(I*a + 1)) - I*dilog(
(I*b*f^(d*x + c) + I*a - 1)/(I*a - 1)))/(d*log(f))
```

3.116.8 Giac [F]

$$\int \arctan \left(a+bf^{c+dx}\right) \, dx=\int \arctan\left(bf^{dx+c}+a\right)\, dx$$

input
```
integrate(arctan(a+b*f^(d*x+c)),x, algorithm="giac")
```

output
```
integrate(arctan(b*f^(d*x + c) + a), x)
```

3.116.9 Mupad [F(-1)]

Timed out.

$$\int \arctan \left(a + bf^{c+dx}\right)\, dx = \int \operatorname{atan}\left(a + b\, f^{c+d\,x}\right)\, dx$$

input
```
int(atan(a + b*f^(c + d*x)),x)
```

output
```
int(atan(a + b*f^(c + d*x)), x)
```

3.117 $\int x \arctan\left(a+bf^{c+dx}\right)\,dx$

3.117.1 Optimal result

Integrand size = 14, antiderivative size = 232

$$\begin{aligned}\int x \arctan\left(a+bf^{c+dx}\right)\,dx &= \frac{1}{2}x^2\arctan\left(a+bf^{c+dx}\right)-\frac{1}{4}ix^2\log\left(1-\frac{ibf^{c+dx}}{1-ia}\right)\\ &+\frac{1}{4}ix^2\log\left(1+\frac{ibf^{c+dx}}{1+ia}\right)-\frac{ix\,\mathrm{PolyLog}\left(2,\frac{ibf^{c+dx}}{1-ia}\right)}{2d\log(f)}\\ &+\frac{ix\,\mathrm{PolyLog}\left(2,-\frac{ibf^{c+dx}}{1+ia}\right)}{2d\log(f)}\\ &+\frac{i\,\mathrm{PolyLog}\left(3,\frac{ibf^{c+dx}}{1-ia}\right)}{2d^2\log^2(f)}-\frac{i\,\mathrm{PolyLog}\left(3,-\frac{ibf^{c+dx}}{1+ia}\right)}{2d^2\log^2(f)}\end{aligned}$$

output
```
1/2*x^2*arctan(a+b*f^(d*x+c))-1/4*I*x^2*ln(1-I*b*f^(d*x+c)/(1-I*a))+1/4*I*
x^2*ln(1+I*b*f^(d*x+c)/(1+I*a))-1/2*I*x*polylog(2,I*b*f^(d*x+c)/(1-I*a))/d
/ln(f)+1/2*I*x*polylog(2,-I*b*f^(d*x+c)/(1+I*a))/d/ln(f)+1/2*I*polylog(3,I
*b*f^(d*x+c)/(1-I*a))/d^2/ln(f)^2-1/2*I*polylog(3,-I*b*f^(d*x+c)/(1+I*a))/
d^2/ln(f)^2
```

3.117.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.02

$$\int x \arctan\left(a+bf^{c+dx}\right)\,dx$$
$$=\frac{i\left(d^2x^2\log^2(f)\log\left(1-ia-ibf^{c+dx}\right)-d^2x^2\log^2(f)\log\left(1+ia+ibf^{c+dx}\right)-d^2x^2\log^2(f)\log\left(\frac{i+a+bf^{c+dx}}{i+a}\right)\right.}{}$$

input
```
Integrate[x*ArcTan[a + b*f^(c + d*x)],x]
```

output
```
((I/4)*(d^2*x^2*Log[f]^2*Log[1 - I*a - I*b*f^(c + d*x)] - d^2*x^2*Log[f]^2
*Log[1 + I*a + I*b*f^(c + d*x)] - d^2*x^2*Log[f]^2*Log[(I + a + b*f^(c + d
*x))/(I + a)] + d^2*x^2*Log[f]^2*Log[1 + (b*f^(c + d*x))/(-I + a)] + 2*d*x
*Log[f]*PolyLog[2, (b*f^(c + d*x))/(I - a)] - 2*d*x*Log[f]*PolyLog[2, -((b
*f^(c + d*x))/(I + a))] - 2*PolyLog[3, (b*f^(c + d*x))/(I - a)] + 2*PolyLo
g[3, -((b*f^(c + d*x))/(I + a))]))/(d^2*Log[f]^2)
```

3.117.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {5666, 3012, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan\left(a+bf^{c+dx}\right)\,dx$$
$$\Big\downarrow \; 5666$$
$$\frac{1}{2}i\int x\log\left(-ibf^{c+dx}-ia+1\right)dx-\frac{1}{2}i\int x\log\left(ibf^{c+dx}+ia+1\right)dx$$
$$\Big\downarrow \; 3012$$
$$\frac{1}{2}i\left(\int x\log\left(1-\frac{ibf^{c+dx}}{1-ia}\right)dx+\frac{1}{2}x^2\log\left(-ia-ibf^{c+dx}+1\right)-\frac{1}{2}x^2\log\left(1+\frac{bf^{c+dx}}{a+i}\right)\right)-\frac{1}{2}i\left(\int x\log\left(\frac{ibf^{c+dx}}{ia+1}+1\right)dx+\frac{1}{2}x^2\log\left(ia+ibf^{c+dx}+1\right)-\frac{1}{2}x^2\log\left(1-\frac{bf^{c+dx}}{-a+i}\right)\right)$$
$$\Big\downarrow \; 3011$$

$$\frac{1}{2}i\left(\frac{\int \operatorname{PolyLog}\left(2,-\frac{bf^{c+dx}}{a+i}\right)dx}{d\log(f)}-\frac{x\operatorname{PolyLog}\left(2,-\frac{bf^{c+dx}}{a+i}\right)}{d\log(f)}+\frac{1}{2}x^2\log\left(-ia-ibf^{c+dx}+1\right)-\frac{1}{2}x^2\log\left(1+\frac{bf^{c+dx}}{a+i}\right.\right.$$
$$\frac{1}{2}i\left(\frac{\int \operatorname{PolyLog}\left(2,\frac{bf^{c+dx}}{i-a}\right)dx}{d\log(f)}-\frac{x\operatorname{PolyLog}\left(2,\frac{bf^{c+dx}}{i-a}\right)}{d\log(f)}+\frac{1}{2}x^2\log\left(ia+ibf^{c+dx}+1\right)-\frac{1}{2}x^2\log\left(1-\frac{bf^{c+dx}}{-a+i}\right)\right)$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{2}i\left(\frac{\int f^{-c-dx}\operatorname{PolyLog}\left(2,-\frac{bf^{c+dx}}{a+i}\right)df^{c+dx}}{d^2\log^2(f)}-\frac{x\operatorname{PolyLog}\left(2,-\frac{bf^{c+dx}}{a+i}\right)}{d\log(f)}+\frac{1}{2}x^2\log\left(-ia-ibf^{c+dx}+1\right)-\frac{1}{2}x^2\log\left(\right.\right.$$
$$\frac{1}{2}i\left(\frac{\int f^{-c-dx}\operatorname{PolyLog}\left(2,\frac{bf^{c+dx}}{i-a}\right)df^{c+dx}}{d^2\log^2(f)}-\frac{x\operatorname{PolyLog}\left(2,\frac{bf^{c+dx}}{i-a}\right)}{d\log(f)}+\frac{1}{2}x^2\log\left(ia+ibf^{c+dx}+1\right)-\frac{1}{2}x^2\log\left(1-\right.\right.$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{2}i\left(\frac{\operatorname{PolyLog}\left(3,-\frac{bf^{c+dx}}{a+i}\right)}{d^2\log^2(f)}-\frac{x\operatorname{PolyLog}\left(2,-\frac{bf^{c+dx}}{a+i}\right)}{d\log(f)}+\frac{1}{2}x^2\log\left(-ia-ibf^{c+dx}+1\right)-\frac{1}{2}x^2\log\left(1+\frac{bf^{c+dx}}{a+i}\right)\right)$$
$$\frac{1}{2}i\left(\frac{\operatorname{PolyLog}\left(3,\frac{bf^{c+dx}}{i-a}\right)}{d^2\log^2(f)}-\frac{x\operatorname{PolyLog}\left(2,\frac{bf^{c+dx}}{i-a}\right)}{d\log(f)}+\frac{1}{2}x^2\log\left(ia+ibf^{c+dx}+1\right)-\frac{1}{2}x^2\log\left(1-\frac{bf^{c+dx}}{-a+i}\right)\right)$$

input
```
Int[x*ArcTan[a + b*f^(c + d*x)],x]
```

output
```
(-1/2*I)*((x^2*Log[1 + I*a + I*b*f^(c + d*x)])/2 - (x^2*Log[1 - (b*f^(c +
d*x))/(I - a)])/2 - (x*PolyLog[2, (b*f^(c + d*x))/(I - a)])/(d*Log[f]) + P
olyLog[3, (b*f^(c + d*x))/(I - a)]/(d^2*Log[f]^2)) + (I/2)*((x^2*Log[1 - I
*a - I*b*f^(c + d*x)])/2 - (x^2*Log[1 + (b*f^(c + d*x))/(I + a)])/2 - (x*P
olyLog[2, -((b*f^(c + d*x))/(I + a))])/(d*Log[f]) + PolyLog[3, -((b*f^(c +
 d*x))/(I + a))]/(d^2*Log[f]^2))
```

3.117. $\int x\arctan\left(a+bf^{c+dx}\right)\,dx$

3.117.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3012
```
Int[Log[(d_) + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g
_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(Log[d + e*(F^(c*(a +
b*x)))^n]/(g*(m + 1))), x] + (Int[(f + g*x)^m*Log[1 + (e/d)*(F^(c*(a + b*x)
))^n], x] - Simp[(f + g*x)^(m + 1)*(Log[1 + (e/d)*(F^(c*(a + b*x)))^n]/(g*(
m + 1))), x]) /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && NeQ[
d, 1]
```

rule 5666
```
Int[ArcTan[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol] :
> Simp[I/2   Int[x^m*Log[1 - I*a - I*b*f^(c + d*x)], x], x] - Simp[I/2   In
t[x^m*Log[1 + I*a + I*b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] &
& IntegerQ[m] && m > 0
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.117.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 671 vs. $2(200) = 400$.

Time = 0.88 (sec) , antiderivative size = 672, normalized size of antiderivative = 2.90

method	result
risch	$\frac{i\ln\left(1-\frac{ib\,f^{dx}f^{c}}{-ia-1}\right)x^2}{4}-\frac{i\ln\left(1-\frac{ib\,f^{dx}f^{c}}{-ia+1}\right)x^2}{4}+\frac{ic\ln\left(\frac{b\,f^{dx}f^{c}+a+i}{i+a}\right)x}{2d}+\frac{i\ln\left(1-\frac{ib\,f^{dx}f^{c}}{-ia-1}\right)c^2}{4d^2}-\frac{i\ln\left(1-\frac{ib\,f^{dx}f^{c}}{-ia+1}\right)c^2}{4d^2}+\frac{ic\,\mathrm{dil}}{}$

input
```
int(x*arctan(a+b*f^(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/4*I*ln(1-I*b/(-I*a-1)*f^(d*x)*f^c)*x^2-1/4*I*ln(1-I*b/(1-I*a)*f^(d*x)*f^
c)*x^2+1/2*I/d*c*ln((b*f^(d*x)*f^c+a+I)/(I+a))*x+1/4*I/d^2*ln(1-I*b/(-I*a-
1)*f^(d*x)*f^c)*c^2-1/4*I/d^2*ln(1-I*b/(1-I*a)*f^(d*x)*f^c)*c^2+1/2*I/ln(f
)/d^2*c*dilog((b*f^(d*x)*f^c+a+I)/(I+a))+1/4*I*x^2*ln(1-I*(a+b*f^(d*x+c)))
-1/2*I/ln(f)/d^2*polylog(2,I*b/(1-I*a)*f^(d*x)*f^c)*c-1/2*I/d^2*c^2*ln((b*
f^(d*x)*f^c+a-I)/(a-I))+1/4*I/d^2*c^2*ln(I*f^(d*x)*f^c*b+I*a+1)-1/2*I/ln(f
)^2/d^2*polylog(3,I*b/(-I*a-1)*f^(d*x)*f^c)+1/2*I/ln(f)^2/d^2*polylog(3,I*
b/(1-I*a)*f^(d*x)*f^c)-1/2*I/d*c*ln((b*f^(d*x)*f^c+a-I)/(a-I))*x-1/2*I/ln(
f)/d*polylog(2,I*b/(1-I*a)*f^(d*x)*f^c)*x+1/2*I/d^2*c^2*ln((b*f^(d*x)*f^c+
a+I)/(I+a))-1/4*I/d^2*c^2*ln(1-I*a-I*f^(d*x)*f^c*b)-1/2*I/ln(f)/d^2*c*dilo
g((b*f^(d*x)*f^c+a-I)/(a-I))+1/2*I/ln(f)/d^2*polylog(2,I*b/(-I*a-1)*f^(d*x
)*f^c)*c-1/4*I*x^2*ln(1+I*(a+b*f^(d*x+c)))+1/2*I/ln(f)/d*polylog(2,I*b/(-I
*a-1)*f^(d*x)*f^c)*x-1/2*I/d*ln(1-I*b/(1-I*a)*f^(d*x)*f^c)*c*x+1/2*I/d*ln(
1-I*b/(-I*a-1)*f^(d*x)*f^c)*c*x
```

3.117.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.31

$$\int x\arctan\left(a+bf^{c+dx}\right)\,dx$$
$$=\frac{2\,d^2x^2\arctan\left(bf^{dx+c}+a\right)\log\left(f\right)^2-i\,c^2\log\left(bf^{dx+c}+a+i\right)\log\left(f\right)^2+i\,c^2\log\left(bf^{dx+c}+a-i\right)\log\left(f\right)^2+}{}$$

input
```
integrate(x*arctan(a+b*f^(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*(2*d^2*x^2*arctan(b*f^(d*x + c) + a)*log(f)^2 - I*c^2*log(b*f^(d*x + c
) + a + I)*log(f)^2 + I*c^2*log(b*f^(d*x + c) + a - I)*log(f)^2 + 2*I*d*x*
dilog(-(a^2 + (a*b + I*b)*f^(d*x + c) + 1)/(a^2 + 1) + 1)*log(f) - 2*I*d*x
*dilog(-(a^2 + (a*b - I*b)*f^(d*x + c) + 1)/(a^2 + 1) + 1)*log(f) + (I*d^2
*x^2 - I*c^2)*log(f)^2*log((a^2 + (a*b + I*b)*f^(d*x + c) + 1)/(a^2 + 1))
+ (-I*d^2*x^2 + I*c^2)*log(f)^2*log((a^2 + (a*b - I*b)*f^(d*x + c) + 1)/(a
^2 + 1)) - 2*I*polylog(3, -(a*b + I*b)*f^(d*x + c)/(a^2 + 1)) + 2*I*polylo
g(3, -(a*b - I*b)*f^(d*x + c)/(a^2 + 1)))/(d^2*log(f)^2)
```

3.117.6 Sympy [F]

$$\int x \arctan \left(a+b f^{c+d x}\right) \, dx=\int x \operatorname{atan}\left(a+b f^{c+d x}\right) dx$$

input
```
integrate(x*atan(a+b*f**(d*x+c)),x)
```

output
```
Integral(x*atan(a + b*f**(c + d*x)), x)
```

3.117.7 Maxima [F]

$$\int x \arctan \left(a+b f^{c+d x}\right) \, dx=\int x \arctan \left(b f^{d x+c}+a\right) \, dx$$

input
```
integrate(x*arctan(a+b*f^(d*x+c)),x, algorithm="maxima")
```

output
```
-b*d*f^c*integrate(1/2*f^(d*x)*x^2/(b^2*f^(2*d*x)*f^(2*c) + 2*a*b*f^(d*x)*
f^c + a^2 + 1), x)*log(f) + 1/2*x^2*arctan(b*f^(d*x)*f^c + a)
```

3.117.8 Giac [F]

$$\int x \arctan \left(a+b f^{c+d x}\right) d x=\int x \arctan \left(b f^{d x+c}+a\right) d x$$

input
```
integrate(x*arctan(a+b*f^(d*x+c)),x, algorithm="giac")
```

output
```
integrate(x*arctan(b*f^(d*x + c) + a), x)
```

3.117.9 Mupad [F(-1)]

Timed out.

$$\int x \arctan \left(a+b f^{c+d x}\right) d x=\int x \operatorname{atan}\left(a+b f^{c+d x}\right) d x$$

input
```
int(x*atan(a + b*f^(c + d*x)),x)
```

output
```
int(x*atan(a + b*f^(c + d*x)), x)
```

3.118 $\int x^2 \arctan \left(a+bf^{c+dx}\right) \, dx$

3.118.1 Optimal result

Integrand size = 16, antiderivative size = 302

$$\begin{aligned}\int x^2 \arctan \left(a+bf^{c+dx}\right) \, dx &= \frac{1}{3}x^3 \arctan \left(a+bf^{c+dx}\right)-\frac{1}{6}ix^3 \log \left(1-\frac{ibf^{c+dx}}{1-ia}\right) \\ &\quad +\frac{1}{6}ix^3 \log \left(1+\frac{ibf^{c+dx}}{1+ia}\right)-\frac{ix^2 \text{PolyLog}\left(2,\frac{ibf^{c+dx}}{1-ia}\right)}{2d \log(f)} \\ &\quad +\frac{ix^2 \text{PolyLog}\left(2,-\frac{ibf^{c+dx}}{1+ia}\right)}{2d \log(f)} \\ &\quad +\frac{ix \text{PolyLog}\left(3,\frac{ibf^{c+dx}}{1-ia}\right)}{d^2 \log^2(f)}-\frac{ix \text{PolyLog}\left(3,-\frac{ibf^{c+dx}}{1+ia}\right)}{d^2 \log^2(f)} \\ &\quad -\frac{i \text{PolyLog}\left(4,\frac{ibf^{c+dx}}{1-ia}\right)}{d^3 \log^3(f)}+\frac{i \text{PolyLog}\left(4,-\frac{ibf^{c+dx}}{1+ia}\right)}{d^3 \log^3(f)}\end{aligned}$$

output
```
1/3*x^3*arctan(a+b*f^(d*x+c))-1/6*I*x^3*ln(1-I*b*f^(d*x+c)/(1-I*a))+1/6*I*
x^3*ln(1+I*b*f^(d*x+c)/(1+I*a))-1/2*I*x^2*polylog(2,I*b*f^(d*x+c)/(1-I*a))
/d/ln(f)+1/2*I*x^2*polylog(2,-I*b*f^(d*x+c)/(1+I*a))/d/ln(f)+I*x*polylog(3
,I*b*f^(d*x+c)/(1-I*a))/d^2/ln(f)^2-I*x*polylog(3,-I*b*f^(d*x+c)/(1+I*a))/
d^2/ln(f)^2-I*polylog(4,I*b*f^(d*x+c)/(1-I*a))/d^3/ln(f)^3+I*polylog(4,-I*
b*f^(d*x+c)/(1+I*a))/d^3/ln(f)^3
```

3.118.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.99

$$\int x^2 \arctan\left(a + bf^{c+dx}\right) dx = \frac{i\left(d^3x^3\log^3(f)\log\left(1 - ia - ibf^{c+dx}\right) - d^3x^3\log^3(f)\log\left(1 + ia + ibf^{c+dx}\right) - d^3x^3\log^3(f)\log\left(\frac{i+a+bf^{c+dx}}{i+a}\right)\right.}{}$$

input
```
Integrate[x^2*ArcTan[a + b*f^(c + d*x)],x]
```

output
```
((I/6)*(d^3*x^3*Log[f]^3*Log[1 - I*a - I*b*f^(c + d*x)] - d^3*x^3*Log[f]^3
*Log[1 + I*a + I*b*f^(c + d*x)] - d^3*x^3*Log[f]^3*Log[(I + a + b*f^(c + d
*x))/(I + a)] + d^3*x^3*Log[f]^3*Log[1 + (b*f^(c + d*x))/(-I + a)] + 3*d^2
*x^2*Log[f]^2*PolyLog[2, (b*f^(c + d*x))/(I - a)] - 3*d^2*x^2*Log[f]^2*Pol
yLog[2, -((b*f^(c + d*x))/(I + a))] - 6*d*x*Log[f]*PolyLog[3, (b*f^(c + d*
x))/(I - a)] + 6*d*x*Log[f]*PolyLog[3, -((b*f^(c + d*x))/(I + a))] + 6*Pol
yLog[4, (b*f^(c + d*x))/(I - a)] - 6*PolyLog[4, -((b*f^(c + d*x))/(I + a))
]))/(d^3*Log[f]^3)
```

3.118.3 Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5666, 3012, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan\left(a + bf^{c+dx}\right) dx$$

$$\Big\downarrow \text{5666}$$

$$\frac{1}{2}i\int x^2\log\left(-ibf^{c+dx} - ia + 1\right)dx - \frac{1}{2}i\int x^2\log\left(ibf^{c+dx} + ia + 1\right)dx$$

$$\Big\downarrow \text{3012}$$

$$\frac{1}{2}i\left(\int x^2\log\left(1 - \frac{ibf^{c+dx}}{1-ia}\right)dx + \frac{1}{3}x^3\log\left(-ia - ibf^{c+dx} + 1\right) - \frac{1}{3}x^3\log\left(1 + \frac{bf^{c+dx}}{a+i}\right)\right) - \frac{1}{2}i\left(\int x^2\log\left(\frac{ibf^{c+dx}}{ia+1} + 1\right)dx + \frac{1}{3}x^3\log\left(ia + ibf^{c+dx} + 1\right) - \frac{1}{3}x^3\log\left(1 - \frac{bf^{c+dx}}{-a+i}\right)\right)$$

$$\Big\downarrow \text{3011}$$

$$\frac{1}{2}i\left(\frac{2\int x\,\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{a+i}\right)dx}{d\log(f)}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{a+i}\right)}{d\log(f)}+\frac{1}{3}x^3\log\left(-ia-ibf^{c+dx}+1\right)-\frac{1}{3}x^3\log\left(1+\frac{bf}{a}\right.\right.$$
$$\frac{1}{2}i\left(\frac{2\int x\,\text{PolyLog}\left(2,\frac{bf^{c+dx}}{i-a}\right)dx}{d\log(f)}-\frac{x^2\,\text{PolyLog}\left(2,\frac{bf^{c+dx}}{i-a}\right)}{d\log(f)}+\frac{1}{3}x^3\log\left(ia+ibf^{c+dx}+1\right)-\frac{1}{3}x^3\log\left(1-\frac{bf^{c+dx}}{-a+i}\right.\right.$$

$$\Big\downarrow \text{7163}$$

$$\frac{1}{2}i\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,-\frac{bf^{c+dx}}{a+i}\right)}{d\log(f)}-\frac{\int \text{PolyLog}\left(3,-\frac{bf^{c+dx}}{a+i}\right)dx}{d\log(f)}\right)}{d\log(f)}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{a+i}\right)}{d\log(f)}+\frac{1}{3}x^3\log\left(-ia-ibf^{c+dx}+1\right.\right.$$
$$\frac{1}{2}i\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,\frac{bf^{c+dx}}{i-a}\right)}{d\log(f)}-\frac{\int \text{PolyLog}\left(3,\frac{bf^{c+dx}}{i-a}\right)dx}{d\log(f)}\right)}{d\log(f)}-\frac{x^2\,\text{PolyLog}\left(2,\frac{bf^{c+dx}}{i-a}\right)}{d\log(f)}+\frac{1}{3}x^3\log\left(ia+ibf^{c+dx}+1\right)-\frac{1}{3}\right.$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{2}i\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,-\frac{bf^{c+dx}}{a+i}\right)}{d\log(f)}-\frac{\int f^{-c-dx}\,\text{PolyLog}\left(3,-\frac{bf^{c+dx}}{a+i}\right)df^{c+dx}}{d^2\log^2(f)}\right)}{d\log(f)}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{a+i}\right)}{d\log(f)}+\frac{1}{3}x^3\log\left(-ia-ib\right.\right.$$
$$\frac{1}{2}i\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,\frac{bf^{c+dx}}{i-a}\right)}{d\log(f)}-\frac{\int f^{-c-dx}\,\text{PolyLog}\left(3,\frac{bf^{c+dx}}{i-a}\right)df^{c+dx}}{d^2\log^2(f)}\right)}{d\log(f)}-\frac{x^2\,\text{PolyLog}\left(2,\frac{bf^{c+dx}}{i-a}\right)}{d\log(f)}+\frac{1}{3}x^3\log\left(ia+ibf^{c+dx}\right.\right.$$

$$\Big\downarrow \text{7143}$$

3.118. $\int x^2\arctan\left(a+bf^{c+dx}\right)\,dx$

$$\frac{1}{2}i\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,-\frac{bf^{c+dx}}{a+i}\right)}{d\log(f)}-\frac{\text{PolyLog}\left(4,-\frac{bf^{c+dx}}{a+i}\right)}{d^2\log^2(f)}\right)}{d\log(f)}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{bf^{c+dx}}{a+i}\right)}{d\log(f)}+\frac{1}{3}x^3\log\left(-ia-ibf^{c+dx}+1\right)-\right.$$

$$\frac{1}{2}i\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,\frac{bf^{c+dx}}{i-a}\right)}{d\log(f)}-\frac{\text{PolyLog}\left(4,\frac{bf^{c+dx}}{i-a}\right)}{d^2\log^2(f)}\right)}{d\log(f)}-\frac{x^2\,\text{PolyLog}\left(2,\frac{bf^{c+dx}}{i-a}\right)}{d\log(f)}+\frac{1}{3}x^3\log\left(ia+ibf^{c+dx}+1\right)-\frac{1}{3}x^3\,\text{lo}\right.$$

input
```
Int[x^2*ArcTan[a + b*f^(c + d*x)],x]
```

output
```
(-1/2*I)*((x^3*Log[1 + I*a + I*b*f^(c + d*x)])/3 - (x^3*Log[1 - (b*f^(c +
d*x))/(I - a)])/3 - (x^2*PolyLog[2, (b*f^(c + d*x))/(I - a)])/(d*Log[f]) +
 (2*((x*PolyLog[3, (b*f^(c + d*x))/(I - a)])/(d*Log[f]) - PolyLog[4, (b*f^
(c + d*x))/(I - a)]/(d^2*Log[f]^2)))/(d*Log[f])) + (I/2)*((x^3*Log[1 - I*a
 - I*b*f^(c + d*x)])/3 - (x^3*Log[1 + (b*f^(c + d*x))/(I + a)])/3 - (x^2*P
olyLog[2, -((b*f^(c + d*x))/(I + a))])/(d*Log[f]) + (2*((x*PolyLog[3, -((b
*f^(c + d*x))/(I + a))])/(d*Log[f]) - PolyLog[4, -((b*f^(c + d*x))/(I + a)
)]/(d^2*Log[f]^2)))/(d*Log[f]))
```

3.118.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3012
```
Int[Log[(d_) + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g
_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(Log[d + e*(F^(c*(a +
b*x)))^n]/(g*(m + 1))), x] + (Int[(f + g*x)^m*Log[1 + (e/d)*(F^(c*(a + b*x)
))^n], x] - Simp[(f + g*x)^(m + 1)*(Log[1 + (e/d)*(F^(c*(a + b*x)))^n]/(g*(
m + 1))), x]) /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && NeQ[
d, 1]
```

rule 5666
```
Int[ArcTan[(a_.) + (b_.)*(f_)^((c_.) + (d_.)*(x_))]*(x_)^(m_.), x_Symbol] :
> Simp[I/2   Int[x^m*Log[1 - I*a - I*b*f^(c + d*x)], x], x] - Simp[I/2   In
t[x^m*Log[1 + I*a + I*b*f^(c + d*x)], x], x] /; FreeQ[{a, b, c, d, f}, x] &
& IntegerQ[m] && m > 0
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

3.118.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 757 vs. 2(268) = 536.

Time = 1.25 (sec) , antiderivative size = 758, normalized size of antiderivative = 2.51

method	result
risch	$-\frac{i \ln\left(1-\frac{i b f^{dx} f^{c}}{-ia-1}\right) c^{3}}{3 d^{3}}+\frac{i \ln\left(1-\frac{i b f^{dx} f^{c}}{-ia-1}\right) x^{3}}{6}-\frac{i \ln\left(1-\frac{i b f^{dx} f^{c}}{-ia+1}\right) x^{3}}{6}-\frac{i c^{3} \ln\left(i f^{dx} f^{c} b+ia+1\right)}{6 d^{3}}+\frac{i \ln\left(1-\frac{i b f^{dx} f^{c}}{-ia+1}\right) x c^{2}}{2 d^{2}}-$

input
```
int(x^2*arctan(a+b*f^(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/3*I/d^3*ln(1-I*b/(-I*a-1)*f^(d*x)*f^c)*c^3+1/6*I*ln(1-I*b/(-I*a-1)*f^(d
*x)*f^c)*x^3-1/6*I*ln(1-I*b/(1-I*a)*f^(d*x)*f^c)*x^3-1/6*I/d^3*c^3*ln(I*f^
(d*x)*f^c*b+I*a+1)+1/2*I/d^2*ln(1-I*b/(1-I*a)*f^(d*x)*f^c)*x*c^2-1/6*I*x^3
*ln(1+I*(a+b*f^(d*x+c)))-1/2*I/d^3*c^3*ln((b*f^(d*x)*f^c+a+I)/(I+a))+1/2*I
/d^3*c^3*ln((b*f^(d*x)*f^c+a-I)/(a-I))+I/d^2/ln(f)^2*polylog(3,I*b/(1-I*a)
*f^(d*x)*f^c)*x-I/d^3/ln(f)^3*polylog(4,I*b/(1-I*a)*f^(d*x)*f^c)+1/2*I/d/l
n(f)*polylog(2,I*b/(-I*a-1)*f^(d*x)*f^c)*x^2-I/d^2/ln(f)^2*polylog(3,I*b/(
-I*a-1)*f^(d*x)*f^c)*x+1/2*I/d^2*c^2*ln((b*f^(d*x)*f^c+a-I)/(a-I))*x+1/2*I
/d^3/ln(f)*polylog(2,I*b/(1-I*a)*f^(d*x)*f^c)*c^2+I/d^3/ln(f)^3*polylog(4,
I*b/(-I*a-1)*f^(d*x)*f^c)+1/2*I/d^3/ln(f)*c^2*dilog((b*f^(d*x)*f^c+a-I)/(a
-I))-1/2*I/d^2*c^2*ln((b*f^(d*x)*f^c+a+I)/(I+a))*x-1/2*I/d^3/ln(f)*c^2*dil
og((b*f^(d*x)*f^c+a+I)/(I+a))+1/3*I/d^3*ln(1-I*b/(1-I*a)*f^(d*x)*f^c)*c^3-
1/2*I/d^3/ln(f)*polylog(2,I*b/(-I*a-1)*f^(d*x)*f^c)*c^2-1/2*I/d/ln(f)*poly
log(2,I*b/(1-I*a)*f^(d*x)*f^c)*x^2+1/6*I/d^3*c^3*ln(1-I*a-I*f^(d*x)*f^c*b)
+1/6*I*x^3*ln(1-I*(a+b*f^(d*x+c)))-1/2*I/d^2*ln(1-I*b/(-I*a-1)*f^(d*x)*f^c
)*x*c^2
```

3.118.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.25

$$\int x^2 \arctan\left(a+bf^{c+dx}\right)\,dx$$
$$= \frac{2\,d^3x^3\arctan\left(bf^{dx+c}+a\right)\log\left(f\right)^3+3i\,d^2x^2\mathrm{Li}_2\left(-\frac{a^2+(ab+i\,b)f^{dx+c}+1}{a^2+1}+1\right)\log\left(f\right)^2-3i\,d^2x^2\mathrm{Li}_2\left(-\frac{a^2+(ab-i\,b}{a^2+}\right.}{}$$

input
```
integrate(x^2*arctan(a+b*f^(d*x+c)),x, algorithm="fricas")
```

output
```
1/6*(2*d^3*x^3*arctan(b*f^(d*x + c) + a)*log(f)^3 + 3*I*d^2*x^2*dilog(-(a^
2 + (a*b + I*b)*f^(d*x + c) + 1)/(a^2 + 1) + 1)*log(f)^2 - 3*I*d^2*x^2*dil
og(-(a^2 + (a*b - I*b)*f^(d*x + c) + 1)/(a^2 + 1) + 1)*log(f)^2 + I*c^3*lo
g(b*f^(d*x + c) + a + I)*log(f)^3 - I*c^3*log(b*f^(d*x + c) + a - I)*log(f
)^3 + (I*d^3*x^3 + I*c^3)*log(f)^3*log((a^2 + (a*b + I*b)*f^(d*x + c) + 1)
/(a^2 + 1)) + (-I*d^3*x^3 - I*c^3)*log(f)^3*log((a^2 + (a*b - I*b)*f^(d*x
+ c) + 1)/(a^2 + 1)) - 6*I*d*x*log(f)*polylog(3, -(a*b + I*b)*f^(d*x + c)/
(a^2 + 1)) + 6*I*d*x*log(f)*polylog(3, -(a*b - I*b)*f^(d*x + c)/(a^2 + 1))
 + 6*I*polylog(4, -(a*b + I*b)*f^(d*x + c)/(a^2 + 1)) - 6*I*polylog(4, -(a
*b - I*b)*f^(d*x + c)/(a^2 + 1)))/(d^3*log(f)^3)
```

3.118.6 Sympy [F]

$$\int x^2 \arctan\left(a + bf^{c+dx}\right) \, dx = \int x^2 \operatorname{atan}\left(a + bf^{c+dx}\right) dx$$

input
```
integrate(x**2*atan(a+b*f**(d*x+c)),x)
```

output
```
Integral(x**2*atan(a + b*f**(c + d*x)), x)
```

3.118.7 Maxima [F]

$$\int x^2 \arctan\left(a + bf^{c+dx}\right) \, dx = \int x^2 \arctan\left(bf^{dx+c} + a\right) \, dx$$

input
```
integrate(x^2*arctan(a+b*f^(d*x+c)),x, algorithm="maxima")
```

output
```
-b*d*f^c*integrate(1/3*f^(d*x)*x^3/(b^2*f^(2*d*x)*f^(2*c) + 2*a*b*f^(d*x)*
f^c + a^2 + 1), x)*log(f) + 1/3*x^3*arctan(b*f^(d*x)*f^c + a)
```

3.118.8 Giac [F]

$$\int x^2 \arctan\left(a + bf^{c+dx}\right) \, dx = \int x^2 \arctan\left(bf^{dx+c} + a\right) \, dx$$

input
```
integrate(x^2*arctan(a+b*f^(d*x+c)),x, algorithm="giac")
```

output
```
integrate(x^2*arctan(b*f^(d*x + c) + a), x)
```

3.118.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \arctan\left(a + bf^{c+dx}\right) \, dx = \int x^2 \operatorname{atan}\left(a + b\, f^{c+d\,x}\right) \, dx$$

input
```
int(x^2*atan(a + b*f^(c + d*x)),x)
```

output
```
int(x^2*atan(a + b*f^(c + d*x)), x)
```

3.119 $\int e^{-x} \arctan(e^x) \, dx$

3.119.1 Optimal result

Integrand size = 10, antiderivative size = 25

$$\int e^{-x} \arctan(e^x) \, dx = x - e^{-x} \arctan(e^x) - \frac{1}{2} \log(1 + e^{2x})$$

output
```
x-arctan(exp(x))/exp(x)-1/2*ln(1+exp(2*x))
```

3.119.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int e^{-x} \arctan(e^x) \, dx = x - e^{-x} \arctan(e^x) - \frac{1}{2} \log(1 + e^{2x})$$

input
```
Integrate[ArcTan[E^x]/E^x,x]
```

output
```
x - ArcTan[E^x]/E^x - Log[1 + E^(2*x)]/2
```

3.119.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5730, 25, 2720, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{-x}\arctan\left(e^x\right)dx$$

$$\Big\downarrow \text{5730}$$

$$-\int -\frac{1}{1+e^{2x}}dx-e^{-x}\arctan\left(e^x\right)$$

$$\Big\downarrow \text{25}$$

$$\int \frac{1}{1+e^{2x}}dx-e^{-x}\arctan\left(e^x\right)$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{2}\int \frac{e^{-2x}}{1+e^{2x}}de^{2x}-e^{-x}\arctan\left(e^x\right)$$

$$\Big\downarrow \text{47}$$

$$\frac{1}{2}\left(\int e^{-2x}de^{2x}-\int \frac{1}{1+e^{2x}}de^{2x}\right)-e^{-x}\arctan\left(e^x\right)$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{2}\left(\log\left(e^{2x}\right)-\int \frac{1}{1+e^{2x}}de^{2x}\right)-e^{-x}\arctan\left(e^x\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{2}\left(\log\left(e^{2x}\right)-\log\left(e^{2x}+1\right)\right)-e^{-x}\arctan\left(e^x\right)$$

input
```
Int[ArcTan[E^x]/E^x,x]
```

output
```
-(ArcTan[E^x]/E^x) + (Log[E^(2*x)] - Log[1 + E^(2*x)])/2
```

3.119.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 5730
```
Int[((a_.) + ArcTan[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Simp[(a + b*ArcTan[u])   w, x] - Simp[b   Int[SimplifyIntegrand[w*(D[u, x]
/(1 + u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x]
&& InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; Fre
eQ[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTan[u]), x]]
```

3.119.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$-\arctan\left(\mathrm{e}^{x}\right) \mathrm{e}^{-x}+\ln\left(\mathrm{e}^{x}\right)-\frac{\ln\left(1+\mathrm{e}^{2 x}\right)}{2}$	23
default	$-\arctan\left(\mathrm{e}^{x}\right) \mathrm{e}^{-x}+\ln\left(\mathrm{e}^{x}\right)-\frac{\ln\left(1+\mathrm{e}^{2 x}\right)}{2}$	23
parallelrisch	$\frac{\left(-\ln\left(1+\mathrm{e}^{2 x}\right) \mathrm{e}^{x}+2 x \,\mathrm{e}^{x}-2 \arctan\left(\mathrm{e}^{x}\right)\right) \mathrm{e}^{-x}}{2}$	29
risch	$\frac{i \mathrm{e}^{-x} \ln\left(1+i \mathrm{e}^{x}\right)}{2}-\frac{\ln\left(1+\mathrm{e}^{2 x}\right)}{2}+x-\frac{i \ln\left(1-i \mathrm{e}^{x}\right) \mathrm{e}^{-x}}{2}$	42

input
```
int(arctan(exp(x))/exp(x),x,method=_RETURNVERBOSE)
```

output
```
-arctan(exp(x))/exp(x)+ln(exp(x))-1/2*ln(exp(x)^2+1)
```

3.119.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int e^{-x}\arctan\left(e^{x}\right)\,dx=\frac{1}{2}\left(2\,xe^{x}-e^{x}\log\left(e^{(2\,x)}+1\right)-2\,\arctan\left(e^{x}\right)\right)e^{(-x)}$$

input
```
integrate(arctan(exp(x))/exp(x),x, algorithm="fricas")
```

output
```
1/2*(2*x*e^x - e^x*log(e^(2*x) + 1) - 2*arctan(e^x))*e^(-x)
```

3.119.6 Sympy [A] (verification not implemented)

Time = 1.42 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int e^{-x}\arctan\left(e^{x}\right)\,dx=x-\frac{\log\left(e^{2x}+1\right)}{2}-e^{-x}\,\mathrm{atan}\left(e^{x}\right)$$

input
```
integrate(atan(exp(x))/exp(x),x)
```

output
```
x - log(exp(2*x) + 1)/2 - exp(-x)*atan(exp(x))
```

3.119.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.76

$$\int e^{-x}\arctan\left(e^{x}\right)\,dx=-\arctan\left(e^{x}\right)e^{(-x)}-\frac{1}{2}\log\left(e^{(-2\,x)}+1\right)$$

input
```
integrate(arctan(exp(x))/exp(x),x, algorithm="maxima")
```

output
```
-arctan(e^x)*e^(-x) - 1/2*log(e^(-2*x) + 1)
```

3.119.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int e^{-x} \arctan \left(e^{x}\right) \, dx = -\arctan \left(e^{x}\right) e^{(-x)} + x - \frac{1}{2} \log \left(e^{(2\,x)} + 1\right)$$

input
```
integrate(arctan(exp(x))/exp(x),x, algorithm="giac")
```

output
```
-arctan(e^x)*e^(-x) + x - 1/2*log(e^(2*x) + 1)
```

3.119.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int e^{-x} \arctan \left(e^{x}\right) \, dx = x - \frac{\ln \left(\mathrm{e}^{2\,x} + 1\right)}{2} - \mathrm{atan}(\mathrm{e}^{x}) \, \mathrm{e}^{-x}$$

input
```
int(atan(exp(x))*exp(-x),x)
```

output
```
x - log(exp(2*x) + 1)/2 - atan(exp(x))*exp(-x)
```

3.120 $\int \frac{\arctan(x)}{(-1+x)^3} dx$

3.120.1 Optimal result

Integrand size = 8, antiderivative size = 45

$$\int \frac{\arctan(x)}{(-1+x)^3} dx = \frac{1}{4(1-x)} - \frac{\arctan(x)}{2(1-x)^2} - \frac{1}{4}\log(1-x) + \frac{1}{8}\log\left(1+x^2\right)$$

output
```
1/4/(1-x)-1/2*arctan(x)/(1-x)^2-1/4*ln(1-x)+1/8*ln(x^2+1)
```

3.120.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.78

$$\int \frac{\arctan(x)}{(-1+x)^3} dx = \frac{1}{8}\left(-\frac{2}{-1+x} - \frac{4\arctan(x)}{(-1+x)^2} - 2\log(1-x) + \log\left(1+x^2\right)\right)$$

input
```
Integrate[ArcTan[x]/(-1 + x)^3,x]
```

output
```
(-2/(-1 + x) - (4*ArcTan[x])/(-1 + x)^2 - 2*Log[1 - x] + Log[1 + x^2])/8
```

3.120.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5387, 480, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(x)}{(x-1)^3} \, dx$$

$$\Big\downarrow \text{5387}$$

$$\frac{1}{2}\int \frac{1}{(1-x)^2 (x^2+1)}dx-\frac{\arctan(x)}{2(1-x)^2}$$

$$\Big\downarrow \text{480}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{x+1}{(1-x) (x^2+1)}dx+\frac{1}{2(1-x)}\right)-\frac{\arctan(x)}{2(1-x)^2}$$

$$\Big\downarrow \text{657}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \left(\frac{x}{x^2+1}+\frac{1}{1-x}\right)dx+\frac{1}{2(1-x)}\right)-\frac{\arctan(x)}{2(1-x)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{1}{2}\log\left(x^2+1\right)-\log(1-x)\right)+\frac{1}{2(1-x)}\right)-\frac{\arctan(x)}{2(1-x)^2}$$

input
```
Int[ArcTan[x]/(-1 + x)^3,x]
```

output
```
-1/2*ArcTan[x]/(1 - x)^2 + (1/(2*(1 - x)) + (-Log[1 - x] + Log[1 + x^2]/2)
/2)/2
```

3.120.3.1 Defintions of rubi rules used

rule 480
```
Int[((c_) + (d_.)*(x_))^(n_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[d*((c
 + d*x)^(n + 1)/((n + 1)*(b*c^2 + a*d^2))), x] + Simp[b/(b*c^2 + a*d^2)   I
nt[(c + d*x)^(n + 1)*((c - d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d},
 x] && ILtQ[n, -1]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5387
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Simp[b*(
c/(e*(q + 1)))   Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b
, c, d, e, q}, x] && NeQ[q, -1]
```

3.120.4 Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.71

method	result
default	$-\frac{\arctan(x)}{2(x-1)^2}+\frac{\ln(x^2+1)}{8}-\frac{1}{4(x-1)}-\frac{\ln(x-1)}{4}$
parts	$-\frac{\arctan(x)}{2(x-1)^2}+\frac{\ln(x^2+1)}{8}-\frac{1}{4(x-1)}-\frac{\ln(x-1)}{4}$
parallelrisch	$-\frac{2\ln(x-1)x^2-\ln(x^2+1)x^2-2-4\ln(x-1)x+2\ln(x^2+1)x+2\ln(x-1)-\ln(x^2+1)+2x+4\arctan(x)}{8(x-1)^2}$
risch	$\frac{i\ln(ix+1)}{4(x-1)^2}-\frac{i(-2i\ln(x-1)x^2+i\ln(x^2+1)x^2+4i\ln(x-1)x-2i\ln(x^2+1)x-2i\ln(x-1)+i\ln(x^2+1)-2ix+2i+2\ln(-ix+1))}{8(x-1)^2}$

input
```
int(arctan(x)/(x-1)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/(x-1)^2*arctan(x)+1/8*ln(x^2+1)-1/4/(x-1)-1/4*ln(x-1)
```

3.120.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11

$$\int \frac{\arctan(x)}{(-1+x)^3}\,dx = \frac{(x^2-2\,x+1)\log\left(x^2+1\right)-2\,(x^2-2\,x+1)\log\left(x-1\right)-2\,x-4\,\arctan\left(x\right)+2}{8\,(x^2-2\,x+1)}$$

input
```
integrate(arctan(x)/(-1+x)^3,x, algorithm="fricas")
```

output
```
1/8*((x^2 - 2*x + 1)*log(x^2 + 1) - 2*(x^2 - 2*x + 1)*log(x - 1) - 2*x - 4
*arctan(x) + 2)/(x^2 - 2*x + 1)
```

3.120.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(31) = 62$.

Time = 0.22 (sec) , antiderivative size = 153, normalized size of antiderivative = 3.40

$$\int \frac{\arctan(x)}{(-1+x)^3}\,dx = -\frac{2x^2\log(x-1)}{8x^2-16x+8} + \frac{x^2\log(x^2+1)}{8x^2-16x+8} + \frac{4x\log(x-1)}{8x^2-16x+8} - \frac{2x\log(x^2+1)}{8x^2-16x+8} - \frac{2x}{8x^2-16x+8} - \frac{2\log(x-1)}{8x^2-16x+8} + \frac{\log(x^2+1)}{8x^2-16x+8} - \frac{4\operatorname{atan}(x)}{8x^2-16x+8} + \frac{2}{8x^2-16x+8}$$

input
```
integrate(atan(x)/(-1+x)**3,x)
```

output
```
-2*x**2*log(x - 1)/(8*x**2 - 16*x + 8) + x**2*log(x**2 + 1)/(8*x**2 - 16*x
 + 8) + 4*x*log(x - 1)/(8*x**2 - 16*x + 8) - 2*x*log(x**2 + 1)/(8*x**2 - 1
6*x + 8) - 2*x/(8*x**2 - 16*x + 8) - 2*log(x - 1)/(8*x**2 - 16*x + 8) + lo
g(x**2 + 1)/(8*x**2 - 16*x + 8) - 4*atan(x)/(8*x**2 - 16*x + 8) + 2/(8*x**
2 - 16*x + 8)
```

3.120.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.69

$$\int \frac{\arctan(x)}{(-1+x)^3}\,dx = -\frac{1}{4(x-1)} - \frac{\arctan(x)}{2(x-1)^2} + \frac{1}{8}\log(x^2+1) - \frac{1}{4}\log(x-1)$$

input
```
integrate(arctan(x)/(-1+x)^3,x, algorithm="maxima")
```

output
```
-1/4/(x - 1) - 1/2*arctan(x)/(x - 1)^2 + 1/8*log(x^2 + 1) - 1/4*log(x - 1)
```

3.120.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.71

$$\int \frac{\arctan(x)}{(-1+x)^3} \, dx = -\frac{1}{4(x-1)} - \frac{\arctan(x)}{2(x-1)^2} + \frac{1}{8} \log(x^2+1) - \frac{1}{4} \log(|x-1|)$$

input
```
integrate(arctan(x)/(-1+x)^3,x, algorithm="giac")
```

output
```
-1/4/(x - 1) - 1/2*arctan(x)/(x - 1)^2 + 1/8*log(x^2 + 1) - 1/4*log(abs(x
- 1))
```

3.120.9 Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.69

$$\int \frac{\arctan(x)}{(-1+x)^3} \, dx = \frac{\ln(x^2+1)}{8} - \frac{\ln(x-1)}{4} - \frac{\frac{x}{4} + \frac{\operatorname{atan}(x)}{2} - \frac{1}{4}}{(x-1)^2}$$

input
```
int(atan(x)/(x - 1)^3,x)
```

output
```
log(x^2 + 1)/8 - log(x - 1)/4 - (x/4 + atan(x)/2 - 1/4)/(x - 1)^2
```

3.121 $\int \frac{\arctan(1+2x)}{(4+3x)^3} \, dx$

3.121.1 Optimal result

Integrand size = 14, antiderivative size = 64

$$\int \frac{\arctan(1+2x)}{(4+3x)^3} \, dx = -\frac{1}{34(4+3x)} + \frac{8}{867}\arctan(1+2x) - \frac{\arctan(1+2x)}{6(4+3x)^2} + \frac{5}{289}\log(4+3x) - \frac{5}{578}\log\left(1+2x+2x^2\right)$$

output
```
-1/34/(4+3*x)+8/867*arctan(1+2*x)-1/6*arctan(1+2*x)/(4+3*x)^2+5/289*ln(4+3
*x)-5/578*ln(2*x^2+2*x+1)
```

3.121.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.27

$$\int \frac{\arctan(1+2x)}{(4+3x)^3} \, dx = \frac{-289\arctan(1+2x) + (4+3x)(-51 - (15-8i)(4+3x)\log(i+(1+i)x) - (15+8i)(4+3x)\log(1+(}{1734(4+3x)^2}$$

input
```
Integrate[ArcTan[1 + 2*x]/(4 + 3*x)^3,x]
```

output
```
(-289*ArcTan[1 + 2*x] + (4 + 3*x)*(-51 - (15 - 8*I)*(4 + 3*x)*Log[I + (1 +
 I)*x] - (15 + 8*I)*(4 + 3*x)*Log[1 + (1 + I)*x] + 120*Log[4 + 3*x] + 90*x
*Log[4 + 3*x]))/(1734*(4 + 3*x)^2)
```

3.121.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5568, 2081, 1145, 27, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(2x+1)}{(3x+4)^3} \, dx$$

$$\Big\downarrow \text{5568}$$

$$\frac{1}{3}\int \frac{1}{(3x+4)^2\left((2x+1)^2+1\right)}dx-\frac{\arctan(2x+1)}{6(3x+4)^2}$$

$$\Big\downarrow \text{2081}$$

$$\frac{1}{3}\int \frac{1}{(3x+4)^2\left(4x^2+4x+2\right)}dx-\frac{\arctan(2x+1)}{6(3x+4)^2}$$

$$\Big\downarrow \text{1145}$$

$$\frac{1}{3}\left(\frac{1}{34}\int \frac{2(1-3x)}{(3x+4)\left(2x^2+2x+1\right)}dx-\frac{3}{34(3x+4)}\right)-\frac{\arctan(2x+1)}{6(3x+4)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\left(\frac{1}{17}\int \frac{1-3x}{(3x+4)\left(2x^2+2x+1\right)}dx-\frac{3}{34(3x+4)}\right)-\frac{\arctan(2x+1)}{6(3x+4)^2}$$

$$\Big\downarrow \text{1200}$$

$$\frac{1}{3}\left(\frac{1}{17}\int \left(\frac{-30x-7}{17\left(2x^2+2x+1\right)}+\frac{45}{17(3x+4)}\right)dx-\frac{3}{34(3x+4)}\right)-\frac{\arctan(2x+1)}{6(3x+4)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\left(\frac{1}{17}\left(\frac{8}{17}\arctan(2x+1)-\frac{15}{34}\log\left(2x^2+2x+1\right)+\frac{15}{17}\log(3x+4)\right)-\frac{3}{34(3x+4)}\right)-\frac{\arctan(2x+1)}{6(3x+4)^2}$$

input
```
Int[ArcTan[1 + 2*x]/(4 + 3*x)^3,x]
```

output
```
-1/6*ArcTan[1 + 2*x]/(4 + 3*x)^2 + (-3/(34*(4 + 3*x)) + ((8*ArcTan[1 + 2*x
])/17 + (15*Log[4 + 3*x])/17 - (15*Log[1 + 2*x + 2*x^2])/34)/17)/3
```

3.121.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1145
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp
[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[m, -1]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2081
```
Int[(u_)^(m_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^m*ExpandToSum
[v, x]^p, x] /; FreeQ[{m, p}, x] && LinearQ[u, x] && QuadraticQ[v, x] &&  !
(LinearMatchQ[u, x] && QuadraticMatchQ[v, x])
```

rule 5568
```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_), x_Symbol] :> Simp[(e + f*x)^(m + 1)*((a + b*ArcTan[c + d*x])^p/(f*(m +
1))), x] - Simp[b*d*(p/(f*(m + 1)))   Int[(e + f*x)^(m + 1)*((a + b*ArcTan[
c + d*x])^(p - 1)/(1 + (c + d*x)^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && IGtQ[p, 0] && ILtQ[m, -1]
```

3.121.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.84

method	result
derivativedivides	$-\frac{2\arctan(1+2x)}{3(8+6x)^{2}}-\frac{1}{17(8+6x)}+\frac{5\ln(8+6x)}{289}-\frac{5\ln\left((1+2x)^{2}+1\right)}{578}+\frac{8\arctan(1+2x)}{867}$
default	$-\frac{2\arctan(1+2x)}{3(8+6x)^{2}}-\frac{1}{17(8+6x)}+\frac{5\ln(8+6x)}{289}-\frac{5\ln\left((1+2x)^{2}+1\right)}{578}+\frac{8\arctan(1+2x)}{867}$
parts	$-\frac{1}{34(4+3x)}+\frac{8\arctan(1+2x)}{867}-\frac{\arctan(1+2x)}{6(4+3x)^{2}}+\frac{5\ln(4+3x)}{289}-\frac{5\ln(2x^{2}+2x+1)}{578}$
parallelrisch	$\frac{810\ln(\frac{4}{3}+x)x^{2}-405\ln(x^{2}+x+\frac{1}{2})x^{2}+432\arctan(1+2x)x^{2}-612+2160\ln(\frac{4}{3}+x)x-1080\ln(x^{2}+x+\frac{1}{2})x+1152\arctan(1}{5202(4+3x)^{2}}$
risch	$\frac{i\ln(1+i(1+2x))}{12(4+3x)^{2}}-\frac{i(-270i\ln(2x+1-i)x^{2}-720i\ln(2x+1-i)x-306ix+960i\ln(4+3x)+1440i\ln(4+3x)x-720i\ln(2x+}{}$

input
```
int(arctan(1+2*x)/(4+3*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-2/3/(8+6*x)^2*arctan(1+2*x)-1/17/(8+6*x)+5/289*ln(8+6*x)-5/578*ln((1+2*x)
^2+1)+8/867*arctan(1+2*x)
```

3.121.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.20

$$\int \frac{\arctan(1+2x)}{(4+3x)^{3}}\,dx$$
$$=\frac{(48\,x^{2}+128\,x-11)\arctan(2\,x+1)-5\,(9\,x^{2}+24\,x+16)\log(2\,x^{2}+2\,x+1)+10\,(9\,x^{2}+24\,x+16)\log}{578\,(9\,x^{2}+24\,x+16)}$$

input
```
integrate(arctan(1+2*x)/(4+3*x)^3,x, algorithm="fricas")
```

output
```
1/578*((48*x^2 + 128*x - 11)*arctan(2*x + 1) - 5*(9*x^2 + 24*x + 16)*log(2
*x^2 + 2*x + 1) + 10*(9*x^2 + 24*x + 16)*log(3*x + 4) - 51*x - 68)/(9*x^2
+ 24*x + 16)
```

3.121.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(56) = 112$.

Time = 0.25 (sec) , antiderivative size = 223, normalized size of antiderivative = 3.48

$$\int \frac{\arctan(1+2x)}{(4+3x)^3}\,dx = \frac{90x^2\log(3x+4)}{5202x^2+13872x+9248} - \frac{45x^2\log(2x^2+2x+1)}{5202x^2+13872x+9248} + \frac{48x^2\,\mathrm{atan}(2x+1)}{5202x^2+13872x+9248} + \frac{240x\log(3x+4)}{5202x^2+13872x+9248} - \frac{120x\log(2x^2+2x+1)}{5202x^2+13872x+9248} + \frac{128x\,\mathrm{atan}(2x+1)}{5202x^2+13872x+9248} - \frac{51x}{5202x^2+13872x+9248} + \frac{160\log(3x+4)}{5202x^2+13872x+9248} - \frac{80\log(2x^2+2x+1)}{5202x^2+13872x+9248} - \frac{11\,\mathrm{atan}(2x+1)}{5202x^2+13872x+9248} - \frac{68}{5202x^2+13872x+9248}$$

input
```
integrate(atan(1+2*x)/(4+3*x)**3,x)
```

output
```
90*x**2*log(3*x + 4)/(5202*x**2 + 13872*x + 9248) - 45*x**2*log(2*x**2 + 2
*x + 1)/(5202*x**2 + 13872*x + 9248) + 48*x**2*atan(2*x + 1)/(5202*x**2 +
13872*x + 9248) + 240*x*log(3*x + 4)/(5202*x**2 + 13872*x + 9248) - 120*x*
log(2*x**2 + 2*x + 1)/(5202*x**2 + 13872*x + 9248) + 128*x*atan(2*x + 1)/(
5202*x**2 + 13872*x + 9248) - 51*x/(5202*x**2 + 13872*x + 9248) + 160*log(
3*x + 4)/(5202*x**2 + 13872*x + 9248) - 80*log(2*x**2 + 2*x + 1)/(5202*x**
2 + 13872*x + 9248) - 11*atan(2*x + 1)/(5202*x**2 + 13872*x + 9248) - 68/(
5202*x**2 + 13872*x + 9248)
```

3.121.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.84

$$\int \frac{\arctan(1+2x)}{(4+3x)^3}\,dx = -\frac{1}{34\,(3\,x+4)} - \frac{\arctan(2\,x+1)}{6\,(3\,x+4)^2} + \frac{8}{867}\arctan(2\,x+1) - \frac{5}{578}\log(2\,x^2+2\,x+1) + \frac{5}{289}\log(3\,x+4)$$

input
```
integrate(arctan(1+2*x)/(4+3*x)^3,x, algorithm="maxima")
```

output
```
-1/34/(3*x + 4) - 1/6*arctan(2*x + 1)/(3*x + 4)^2 + 8/867*arctan(2*x + 1)
- 5/578*log(2*x^2 + 2*x + 1) + 5/289*log(3*x + 4)
```

3.121.8 Giac [F]

$$\int \frac{\arctan(1+2x)}{(4+3x)^3}\,dx = \int \frac{\arctan\left(2\,x+1\right)}{\left(3\,x+4\right)^3}\,dx$$

input
```
integrate(arctan(1+2*x)/(4+3*x)^3,x, algorithm="giac")
```

output
```
sage0*x
```

3.121.9 Mupad [B] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.72

$$\int \frac{\arctan(1+2x)}{(4+3x)^3}\,dx = \frac{5\,\ln\left(x+\frac{4}{3}\right)}{289} - \frac{5\,\ln\left(x^2+x+\frac{1}{2}\right)}{578} + \frac{8\,\mathrm{atan}(2\,x+1)}{867} - \frac{\frac{3\,x}{34}+\frac{\mathrm{atan}(2\,x+1)}{6}+\frac{2}{17}}{\left(3\,x+4\right)^2}$$

input
```
int(atan(2*x + 1)/(3*x + 4)^3,x)
```

output
```
(5*log(x + 4/3))/289 - (5*log(x + x^2 + 1/2))/578 + (8*atan(2*x + 1))/867
- ((3*x)/34 + atan(2*x + 1)/6 + 2/17)/(3*x + 4)^2
```

3.122 $\int \arctan\left(\sqrt{1+x}\right)\,dx$

3.122.1 Optimal result

Integrand size = 8, antiderivative size = 30

$$\int \arctan\left(\sqrt{1+x}\right)\,dx=-\sqrt{1+x}+2\arctan\left(\sqrt{1+x}\right)+x\arctan\left(\sqrt{1+x}\right)$$

output
```
2*arctan((1+x)^(1/2))+x*arctan((1+x)^(1/2))-(1+x)^(1/2)
```

3.122.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \arctan\left(\sqrt{1+x}\right)\,dx=-\sqrt{1+x}+(2+x)\arctan\left(\sqrt{1+x}\right)$$

input
```
Integrate[ArcTan[Sqrt[1 + x]],x]
```

output
```
-Sqrt[1 + x] + (2 + x)*ArcTan[Sqrt[1 + x]]
```

3.122.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5726, 90, 27, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan\left(\sqrt{x+1}\right)dx$$

$$\Big\downarrow \ 5726$$

$$x\arctan\left(\sqrt{x+1}\right)-\int \frac{x}{\sqrt{x+1}(2x+4)}dx$$

$$\Big\downarrow \ 90$$

$$2\int \frac{1}{2\sqrt{x+1}(x+2)}dx+x\arctan\left(\sqrt{x+1}\right)-\sqrt{x+1}$$

$$\Big\downarrow \ 27$$

$$\int \frac{1}{\sqrt{x+1}(x+2)}dx+x\arctan\left(\sqrt{x+1}\right)-\sqrt{x+1}$$

$$\Big\downarrow \ 73$$

$$2\int \frac{1}{x+2}d\sqrt{x+1}+x\arctan\left(\sqrt{x+1}\right)-\sqrt{x+1}$$

$$\Big\downarrow \ 216$$

$$x\arctan\left(\sqrt{x+1}\right)+2\arctan\left(\sqrt{x+1}\right)-\sqrt{x+1}$$

input
```
Int[ArcTan[Sqrt[1 + x]],x]
```

output
```
-Sqrt[1 + x] + 2*ArcTan[Sqrt[1 + x]] + x*ArcTan[Sqrt[1 + x]]
```

3.122.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 5726
```
Int[ArcTan[u_], x_Symbol] :> Simp[x*ArcTan[u], x] - Int[SimplifyIntegrand[x
*(D[u, x]/(1 + u^2)), x], x] /; InverseFunctionFreeQ[u, x]
```

3.122.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$(1+x)\arctan\left(\sqrt{1+x}\right)-\sqrt{1+x}+\arctan\left(\sqrt{1+x}\right)$	25
default	$(1+x)\arctan\left(\sqrt{1+x}\right)-\sqrt{1+x}+\arctan\left(\sqrt{1+x}\right)$	25
parts	$2\arctan\left(\sqrt{1+x}\right)+x\arctan\left(\sqrt{1+x}\right)-\sqrt{1+x}$	25

input
```
int(arctan((1+x)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
(1+x)*arctan((1+x)^(1/2))-(1+x)^(1/2)+arctan((1+x)^(1/2))
```

3.122.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.60

$$\int \arctan\left(\sqrt{1+x}\right)\,dx = (x+2)\arctan\left(\sqrt{x+1}\right) - \sqrt{x+1}$$

input
```
integrate(arctan((1+x)^(1/2)),x, algorithm="fricas")
```

output
```
(x + 2)*arctan(sqrt(x + 1)) - sqrt(x + 1)
```

3.122.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int \arctan\left(\sqrt{1+x}\right)\,dx = x\,\mathrm{atan}\left(\sqrt{x+1}\right) - \sqrt{x+1} + 2\,\mathrm{atan}\left(\sqrt{x+1}\right)$$

input
```
integrate(atan((1+x)**(1/2)),x)
```

output
```
x*atan(sqrt(x + 1)) - sqrt(x + 1) + 2*atan(sqrt(x + 1))
```

3.122.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \arctan\left(\sqrt{1+x}\right)\,dx = (x+1)\arctan\left(\sqrt{x+1}\right) - \sqrt{x+1} + \arctan\left(\sqrt{x+1}\right)$$

input
```
integrate(arctan((1+x)^(1/2)),x, algorithm="maxima")
```

output
```
(x + 1)*arctan(sqrt(x + 1)) - sqrt(x + 1) + arctan(sqrt(x + 1))
```

3.122.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \arctan\left(\sqrt{1+x}\right)\,dx = (x+1)\arctan\left(\sqrt{x+1}\right) - \sqrt{x+1} + \arctan\left(\sqrt{x+1}\right)$$

input
```
integrate(arctan((1+x)^(1/2)),x, algorithm="giac")
```

output
```
(x + 1)*arctan(sqrt(x + 1)) - sqrt(x + 1) + arctan(sqrt(x + 1))
```

3.122.9 Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \arctan\left(\sqrt{1+x}\right)\,dx = \mathrm{atan}\left(\sqrt{x+1}\right) - \sqrt{x+1} + \mathrm{atan}\left(\sqrt{x+1}\right)\,(x+1)$$

input
```
int(atan((x + 1)^(1/2)),x)
```

output
```
atan((x + 1)^(1/2)) - (x + 1)^(1/2) + atan((x + 1)^(1/2))*(x + 1)
```

3.123 $\int \frac{1}{(1+x^2)(2+\arctan(x))}\,dx$

3.123.1 Optimal result

Integrand size = 14, antiderivative size = 5

$$\int \frac{1}{(1+x^2)(2+\arctan(x))}\,dx = \log(2+\arctan(x))$$

output
```
ln(2+arctan(x))
```

3.123.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+x^2)(2+\arctan(x))}\,dx = \log(2+\arctan(x))$$

input
```
Integrate[1/((1 + x^2)*(2 + ArcTan[x])),x]
```

output
```
Log[2 + ArcTan[x]]
```

3.123.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {5417}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x^2+1)(\arctan(x)+2)}dx$$

$$\Big\downarrow \; 5417$$

$$\log(\arctan(x)+2)$$

input
```
Int[1/((1 + x^2)*(2 + ArcTan[x])),x]
```

output
```
Log[2 + ArcTan[x]]
```

3.123.3.1 Defintions of rubi rules used

rule 5417
```
Int[1/(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)), x_Symbol]
 :> Simp[Log[RemoveContent[a + b*ArcTan[c*x], x]]/(b*c*d), x] /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d]
```

3.123.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.20

method	result	size
derivativedivides	$\ln(2+\arctan(x))$	6
default	$\ln(2+\arctan(x))$	6
parallelrisch	$\ln(2+\arctan(x))$	6
risch	$\ln(-\ln(-ix+1)+\ln(ix+1)+4i)$	21

input
```
int(1/(x^2+1)/(2+arctan(x)),x,method=_RETURNVERBOSE)
```

output
```
ln(2+arctan(x))
```

3.123.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+x^2)\,(2+\arctan(x))}\,dx = \log\left(\arctan\left(x\right)+2\right)$$

input
```
integrate(1/(x^2+1)/(2+arctan(x)),x, algorithm="fricas")
```

output
```
log(arctan(x) + 2)
```

3.123.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+x^2)\,(2+\arctan(x))}\,dx = \log\left(\operatorname{atan}\left(x\right)+2\right)$$

input
```
integrate(1/(x**2+1)/(2+atan(x)),x)
```

output
```
log(atan(x) + 2)
```

3.123.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+x^2)\,(2+\arctan(x))}\,dx = \log\left(\arctan\left(x\right)+2\right)$$

input
```
integrate(1/(x^2+1)/(2+arctan(x)),x, algorithm="maxima")
```

output
```
log(arctan(x) + 2)
```

3.123.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+x^2)(2+\arctan(x))}\,dx = \log\left(\arctan\left(x\right)+2\right)$$

input
```
integrate(1/(x^2+1)/(2+arctan(x)),x, algorithm="giac")
```

output
```
log(arctan(x) + 2)
```

3.123.9 Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1+x^2)(2+\arctan(x))}\,dx = \ln\left(\mathrm{atan}(x)+2\right)$$

input
```
int(1/((x^2 + 1)*(atan(x) + 2)),x)
```

output
```
log(atan(x) + 2)
```

3.124 $\int \frac{1}{(a+ax^2)(b-2b\arctan(x))}\,dx$

3.124.1 Optimal result

Integrand size = 19, antiderivative size = 17

$$\int \frac{1}{(a+ax^2)\,(b-2b\arctan(x))}\,dx = -\frac{\log(1-2\arctan(x))}{2ab}$$

output
```
-1/2*ln(1-2*arctan(x))/a/b
```

3.124.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+ax^2)\,(b-2b\arctan(x))}\,dx = -\frac{\log(-1+2\arctan(x))}{2ab}$$

input
```
Integrate[1/((a + a*x^2)*(b - 2*b*ArcTan[x])),x]
```

output
```
-1/2*Log[-1 + 2*ArcTan[x]]/(a*b)
```

3.124.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {5417}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax^2+a)(b-2b\arctan(x))}dx$$

$$\Big\downarrow \text{5417}$$

$$-\frac{\log(1-2\arctan(x))}{2ab}$$

input
```
Int[1/((a + a*x^2)*(b - 2*b*ArcTan[x])),x]
```

output
```
-1/2*Log[1 - 2*ArcTan[x]]/(a*b)
```

3.124.3.1 Defintions of rubi rules used

rule 5417
```
Int[1/(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)), x_Symbol]
 :> Simp[Log[RemoveContent[a + b*ArcTan[c*x], x]]/(b*c*d), x] /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d]
```

3.124.4 Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
parallelrisch	$-\frac{\ln\left(\arctan(x)-\frac{1}{2}\right)}{2ab}$	14
default	$-\frac{\ln(2b\arctan(x)-b)}{2ab}$	19
risch	$-\frac{\ln(-i-\ln(-ix+1)+\ln(ix+1))}{2ab}$	29

input
```
int(1/(a*x^2+a)/(b-2*b*arctan(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*ln(arctan(x)-1/2)/a/b
```

3.124.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a+ax^2)(b-2b\arctan(x))}\,dx = -\frac{\log(2\arctan(x)-1)}{2\,ab}$$

input
```
integrate(1/(a*x^2+a)/(b-2*b*arctan(x)),x, algorithm="fricas")
```

output
```
-1/2*log(2*arctan(x) - 1)/(a*b)
```

3.124.6 Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{1}{(a+ax^2)(b-2b\arctan(x))}\,dx = -\frac{\log\left(\operatorname{atan}(x)-\frac{1}{2}\right)}{2ab}$$

input
```
integrate(1/(a*x**2+a)/(b-2*b*atan(x)),x)
```

output
```
-log(atan(x) - 1/2)/(2*a*b)
```

3.124.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{1}{(a+ax^2)(b-2b\arctan(x))}\,dx = -\frac{\log(|2\arctan(x)-1|)}{2\,ab}$$

input
```
integrate(1/(a*x^2+a)/(b-2*b*arctan(x)),x, algorithm="maxima")
```

output
```
-1/2*log(abs(2*arctan(x) - 1))/(a*b)
```

3.124.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int \frac{1}{(a+ax^2)(b-2b\arctan(x))}\,dx = -\frac{\log(|2\arctan(x)-1|)}{2\,ab}$$

input
```
integrate(1/(a*x^2+a)/(b-2*b*arctan(x)),x, algorithm="giac")
```

output
```
-1/2*log(abs(2*arctan(x) - 1))/(a*b)
```

3.124.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a+ax^2)(b-2b\arctan(x))}\,dx = -\frac{\ln(2\,\mathrm{atan}(x)-1)}{2\,a\,b}$$

input
```
int(1/((a + a*x^2)*(b - 2*b*atan(x))),x)
```

output
```
-log(2*atan(x) - 1)/(2*a*b)
```

3.125 $\int \frac{x+x^3+(1+x)^2 \arctan(x)}{(1+x)^2(1+x^2)} dx$

3.125.1 Optimal result

Integrand size = 26, antiderivative size = 18

$$\int \frac{x+x^3+(1+x)^2 \arctan(x)}{(1+x)^2 (1+x^2)} dx = \frac{1}{1+x} + \frac{\arctan(x)^2}{2} + \log(1+x)$$

output
```
1/(1+x)+1/2*arctan(x)^2+ln(1+x)
```

3.125.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x+x^3+(1+x)^2 \arctan(x)}{(1+x)^2 (1+x^2)} dx = \frac{1}{1+x} + \frac{\arctan(x)^2}{2} + \log(1+x)$$

input
```
Integrate[(x + x^3 + (1 + x)^2*ArcTan[x])/((1 + x)^2*(1 + x^2)),x]
```

output
```
(1 + x)^(-1) + ArcTan[x]^2/2 + Log[1 + x]
```

3.125.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x+1)^2 \arctan(x) + x^3 + x}{(x+1)^2 (x^2+1)} \, dx$$

$$\Big\downarrow \text{ 7276}$$

$$\int \left(\frac{\arctan(x)}{x^2+1} + \frac{x}{(x+1)^2}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\arctan(x)^2}{2} + \frac{1}{x+1} + \log(x+1)$$

input
```
Int[(x + x^3 + (1 + x)^2*ArcTan[x])/((1 + x)^2*(1 + x^2)),x]
```

output
```
(1 + x)^(-1) + ArcTan[x]^2/2 + Log[1 + x]
```

3.125.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.125.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{1}{1+x}+\frac{\arctan(x)^2}{2}+\ln(1+x)$	17
parts	$\frac{1}{1+x}+\frac{\arctan(x)^2}{2}+\ln(1+x)$	17
parallelrisch	$\frac{\arctan(x)^2x+2\ln(1+x)x+2+\arctan(x)^2+2\ln(1+x)}{2+2x}$	33
risch	$-\frac{\ln(ix+1)^2}{8}+\frac{\ln(-ix+1)\ln(ix+1)}{4}+\frac{-\ln(-ix+1)^2x+8\ln(1+x)x-\ln(-ix+1)^2+8\ln(1+x)+8}{8+8x}$	74

input
```
int((x+x^3+(1+x)^2*arctan(x))/(1+x)^2/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/(1+x)+1/2*arctan(x)^2+ln(1+x)
```

3.125.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.44

$$\int \frac{x+x^3+(1+x)^2\arctan(x)}{(1+x)^2(1+x^2)}\,dx=\frac{(x+1)\arctan(x)^2+2(x+1)\log(x+1)+2}{2(x+1)}$$

input
```
integrate((x+x^3+(1+x)^2*arctan(x))/(1+x)^2/(x^2+1),x, algorithm="fricas")
```

output
```
1/2*((x + 1)*arctan(x)^2 + 2*(x + 1)*log(x + 1) + 2)/(x + 1)
```

3.125.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(15) = 30$.

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.94

$$\int \frac{x+x^3+(1+x)^2\arctan(x)}{(1+x)^2(1+x^2)}\,dx=\frac{2x\log(x+1)}{2x+2}+\frac{x\,\mathrm{atan}^2(x)}{2x+2}+\frac{2\log(x+1)}{2x+2}+\frac{\mathrm{atan}^2(x)}{2x+2}+\frac{2}{2x+2}$$

input
```
integrate((x+x**3+(1+x)**2*atan(x))/(1+x)**2/(x**2+1),x)
```

output
```
2*x*log(x + 1)/(2*x + 2) + x*atan(x)**2/(2*x + 2) + 2*log(x + 1)/(2*x + 2)
 + atan(x)**2/(2*x + 2) + 2/(2*x + 2)
```

3.125.7 Maxima [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{x+x^3+(1+x)^2\arctan(x)}{(1+x)^2\,(1+x^2)}\,dx = \frac{1}{2}\arctan{(x)}^2 + \frac{1}{x+1} + \log{(x+1)}$$

input
```
integrate((x+x^3+(1+x)^2*arctan(x))/(1+x)^2/(x^2+1),x, algorithm="maxima")
```

output
```
1/2*arctan(x)^2 + 1/(x + 1) + log(x + 1)
```

3.125.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(16) = 32$.

Time = 0.29 (sec) , antiderivative size = 104, normalized size of antiderivative = 5.78

$$\int \frac{x+x^3+(1+x)^2\arctan(x)}{(1+x)^2\,(1+x^2)}\,dx$$
$$= \frac{(x+1)\left(\frac{1}{x+1}-1\right)\arctan\left((x+1)\left(\frac{1}{x+1}-1\right)\right)^2 + 2\,(x+1)\left(\frac{1}{x+1}-1\right)\log\left(-(x+1)\left(\frac{1}{x+1}-1\right)+1\right) - \arctan}{2\left((x+1)\left(\frac{1}{x+1}-1\right)-1\right)}$$

input
```
integrate((x+x^3+(1+x)^2*arctan(x))/(1+x)^2/(x^2+1),x, algorithm="giac")
```

output
```
1/2*((x + 1)*(1/(x + 1) - 1)*arctan((x + 1)*(1/(x + 1) - 1))^2 + 2*(x + 1)
*(1/(x + 1) - 1)*log(-(x + 1)*(1/(x + 1) - 1) + 1) - arctan((x + 1)*(1/(x
+ 1) - 1))^2 - 2*log(-(x + 1)*(1/(x + 1) - 1) + 1) - 2)/((x + 1)*(1/(x + 1
) - 1) - 1)
```

3.125.9 Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{x+x^3+(1+x)^2\arctan(x)}{(1+x)^2\left(1+x^2\right)}\,dx=\ln\left(x+1\right)+\frac{1}{x+1}+\frac{\mathrm{atan}(x)^2}{2}$$

input
```
int((x + atan(x)*(x + 1)^2 + x^3)/((x^2 + 1)*(x + 1)^2),x)
```

output
```
log(x + 1) + 1/(x + 1) + atan(x)^2/2
```

3.126 $\int -x^3 \arctan\left(\sqrt{x}-\sqrt{1+x}\right)\,dx$

3.126.1 Optimal result

Integrand size = 21, antiderivative size = 68

$$\int -x^3 \arctan\left(\sqrt{x}-\sqrt{1+x}\right)\,dx = -\frac{\sqrt{x}}{8}+\frac{x^{3/2}}{24}-\frac{x^{5/2}}{40}+\frac{x^{7/2}}{56}+\frac{\pi x^4}{16}+\frac{\arctan\left(\sqrt{x}\right)}{8}-\frac{1}{8}x^4\arctan\left(\sqrt{x}\right)$$

output
```
1/24*x^(3/2)-1/40*x^(5/2)+1/56*x^(7/2)+1/16*Pi*x^4+1/8*arctan(x^(1/2))-1/8
*x^4*arctan(x^(1/2))-1/8*x^(1/2)
```

3.126.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.85

$$\int -x^3 \arctan\left(\sqrt{x}-\sqrt{1+x}\right)\,dx = \frac{\arctan\left(\sqrt{x}\right)}{8}-\frac{1}{840}\sqrt{x}\left(105-35x+21x^2-15x^3+210x^{7/2}\arctan\left(\sqrt{x}-\sqrt{1+x}\right)\right)$$

input
```
Integrate[-(x^3*ArcTan[Sqrt[x] - Sqrt[1 + x]]),x]
```

output
```
ArcTan[Sqrt[x]]/8 - (Sqrt[x]*(105 - 35*x + 21*x^2 - 15*x^3 + 210*x^(7/2)*A
rcTan[Sqrt[x] - Sqrt[1 + x]]))/840
```

3.126.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {25, 5682, 15, 5361, 60, 60, 60, 60, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int -x^3 \arctan\left(\sqrt{x}-\sqrt{x+1}\right) dx$$

$$\Big\downarrow \; 25$$

$$-\int x^3 \arctan\left(\sqrt{x}-\sqrt{x+1}\right) dx$$

$$\Big\downarrow \; 5682$$

$$\frac{\pi \int x^3dx}{4}-\frac{1}{2}\int x^3 \arctan\left(\sqrt{x}\right) dx$$

$$\Big\downarrow \; 15$$

$$\frac{\pi x^4}{16}-\frac{1}{2}\int x^3 \arctan\left(\sqrt{x}\right) dx$$

$$\Big\downarrow \; 5361$$

$$\frac{1}{2}\left(\frac{1}{8}\int \frac{x^{7/2}}{x+1}dx-\frac{1}{4}x^4 \arctan\left(\sqrt{x}\right)\right)+\frac{\pi x^4}{16}$$

$$\Big\downarrow \; 60$$

$$\frac{1}{2}\left(\frac{1}{8}\left(\frac{2x^{7/2}}{7}-\int \frac{x^{5/2}}{x+1}dx\right)-\frac{1}{4}x^4 \arctan\left(\sqrt{x}\right)\right)+\frac{\pi x^4}{16}$$

$$\Big\downarrow \; 60$$

$$\frac{1}{2}\left(\frac{1}{8}\left(\int \frac{x^{3/2}}{x+1}dx+\frac{2x^{7/2}}{7}-\frac{2x^{5/2}}{5}\right)-\frac{1}{4}x^4 \arctan\left(\sqrt{x}\right)\right)+\frac{\pi x^4}{16}$$

$$\Big\downarrow \; 60$$

$$\frac{1}{2}\left(\frac{1}{8}\left(-\int \frac{\sqrt{x}}{x+1}dx+\frac{2x^{7/2}}{7}-\frac{2x^{5/2}}{5}+\frac{2x^{3/2}}{3}\right)-\frac{1}{4}x^4 \arctan\left(\sqrt{x}\right)\right)+\frac{\pi x^4}{16}$$

$$\Big\downarrow \; 60$$

$$\frac{1}{2}\left(\frac{1}{8}\left(\int \frac{1}{\sqrt{x}(x+1)}dx+\frac{2x^{7/2}}{7}-\frac{2x^{5/2}}{5}+\frac{2x^{3/2}}{3}-2\sqrt{x}\right)-\frac{1}{4}x^4\arctan\left(\sqrt{x}\right)\right)+\frac{\pi x^4}{16}$$

$$\Big\downarrow \; 73$$

$$\frac{1}{2}\left(\frac{1}{8}\left(2\int \frac{1}{x+1}d\sqrt{x}+\frac{2x^{7/2}}{7}-\frac{2x^{5/2}}{5}+\frac{2x^{3/2}}{3}-2\sqrt{x}\right)-\frac{1}{4}x^4\arctan\left(\sqrt{x}\right)\right)+\frac{\pi x^4}{16}$$

$$\Big\downarrow \; 216$$

$$\frac{1}{2}\left(\frac{1}{8}\left(2\arctan\left(\sqrt{x}\right)+\frac{2x^{7/2}}{7}-\frac{2x^{5/2}}{5}+\frac{2x^{3/2}}{3}-2\sqrt{x}\right)-\frac{1}{4}x^4\arctan\left(\sqrt{x}\right)\right)+\frac{\pi x^4}{16}$$

input
```
Int[-(x^3*ArcTan[Sqrt[x] - Sqrt[1 + x]]),x]
```

output
```
(Pi*x^4)/16 + (-1/4*(x^4*ArcTan[Sqrt[x]]) + (-2*Sqrt[x] + (2*x^(3/2))/3 -
(2*x^(5/2))/5 + (2*x^(7/2))/7 + 2*ArcTan[Sqrt[x]])/8)/2
```

3.126.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5682
```
Int[ArcTan[(v_) + (s_.)*Sqrt[w_]]*(u_.), x_Symbol] :> Simp[Pi*(s/4)   Int[u
, x], x] + Simp[1/2   Int[u*ArcTan[v], x], x] /; EqQ[s^2, 1] && EqQ[w, v^2
+ 1]
```

3.126.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.66

method	result	size
default	$-\frac{x^{4} \arctan \left(\sqrt{x}-\sqrt{1+x}\right)}{4}+\frac{x^{\frac{7}{2}}}{56}-\frac{x^{\frac{5}{2}}}{40}+\frac{x^{\frac{3}{2}}}{24}-\frac{\sqrt{x}}{8}+\frac{\arctan \left(\sqrt{x}\right)}{8}$	45
parts	$-\frac{x^{4} \arctan \left(\sqrt{x}-\sqrt{1+x}\right)}{4}+\frac{x^{\frac{7}{2}}}{56}-\frac{x^{\frac{5}{2}}}{40}+\frac{x^{\frac{3}{2}}}{24}-\frac{\sqrt{x}}{8}+\frac{\arctan \left(\sqrt{x}\right)}{8}$	45

input
```
int(-x^3*arctan(x^(1/2)-(1+x)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-1/4*x^4*arctan(x^(1/2)-(1+x)^(1/2))+1/56*x^(7/2)-1/40*x^(5/2)+1/24*x^(3/2
)-1/8*x^(1/2)+1/8*arctan(x^(1/2))
```

3.126.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.59

$$\int -x^3 \arctan \left(\sqrt{x}-\sqrt{1+x}\right) \, dx = \frac{1}{4} \left(x^4-1\right) \arctan \left(\sqrt{x+1}-\sqrt{x}\right) + \frac{1}{840} \left(15 \, x^3-21 \, x^2+35 \, x-105\right) \sqrt{x}$$

input
```
integrate(-x^3*arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="fricas")
```

output
```
1/4*(x^4 - 1)*arctan(sqrt(x + 1) - sqrt(x)) + 1/840*(15*x^3 - 21*x^2 + 35*
x - 105)*sqrt(x)
```

3.126.6 Sympy [F(-1)]

Timed out.

$$\int -x^3 \arctan \left(\sqrt{x} - \sqrt{1+x}\right) \, dx = \text{Timed out}$$

input
```
integrate(-x**3*atan(x**(1/2)-(1+x)**(1/2)),x)
```

output
```
Timed out
```

3.126.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.65

$$\int -x^3 \arctan \left(\sqrt{x} - \sqrt{1+x}\right) \, dx = \frac{1}{4} x^4 \arctan \left(\sqrt{x+1} - \sqrt{x}\right) + \frac{1}{56} x^{\frac{7}{2}} - \frac{1}{40} x^{\frac{5}{2}} + \frac{1}{24} x^{\frac{3}{2}} - \frac{1}{8} \sqrt{x} + \frac{1}{8} \arctan \left(\sqrt{x}\right)$$

input
```
integrate(-x^3*arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="maxima")
```

output
```
1/4*x^4*arctan(sqrt(x + 1) - sqrt(x)) + 1/56*x^(7/2) - 1/40*x^(5/2) + 1/24
*x^(3/2) - 1/8*sqrt(x) + 1/8*arctan(sqrt(x))
```

3.126.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.65

$$\int -x^3 \arctan\left(\sqrt{x}-\sqrt{1+x}\right)\,dx = -\frac{1}{4}\,x^4 \arctan\left(-\sqrt{x+1}+\sqrt{x}\right)+\frac{1}{56}\,x^{\frac{7}{2}} - \frac{1}{40}\,x^{\frac{5}{2}}+\frac{1}{24}\,x^{\frac{3}{2}}-\frac{1}{8}\,\sqrt{x}+\frac{1}{8}\,\arctan\left(\sqrt{x}\right)$$

input
```
integrate(-x^3*arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="giac")
```

output
```
-1/4*x^4*arctan(-sqrt(x + 1) + sqrt(x)) + 1/56*x^(7/2) - 1/40*x^(5/2) + 1/
24*x^(3/2) - 1/8*sqrt(x) + 1/8*arctan(sqrt(x))
```

3.126.9 Mupad [B] (verification not implemented)

Time = 1.92 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.06

$$\int -x^3 \arctan\left(\sqrt{x}-\sqrt{1+x}\right)\,dx = \frac{x^{3/2}}{24}-\frac{\sqrt{x}}{8}-\frac{x^{5/2}}{40}+\frac{x^{7/2}}{56} + \frac{\mathrm{atan}\left(\sqrt{x+1}-\sqrt{x}\right)\,\left(\frac{x^5}{2}+\frac{x^4}{2}\right)}{2\,x+2}+\frac{\ln\left(\frac{{\left(-1+\sqrt{x}\,1{}\mathrm{i}\right)}^2}{x+1}\right)\,1{}\mathrm{i}}{16}$$

input
```
int(x^3*atan((x + 1)^(1/2) - x^(1/2)),x)
```

output
```
(log((x^(1/2)*1i - 1)^2/(x + 1))*1i)/16 - x^(1/2)/8 + x^(3/2)/24 - x^(5/2)
/40 + x^(7/2)/56 + (atan((x + 1)^(1/2) - x^(1/2))*(x^4/2 + x^5/2))/(2*x +
2)
```

3.127 $\int -x^2 \arctan \left(\sqrt{x}-\sqrt{1+x}\right) \, dx$

3.127.1 Optimal result

Integrand size = 21, antiderivative size = 59

$$\int -x^2 \arctan \left(\sqrt{x}-\sqrt{1+x}\right) \, dx=\frac{\sqrt{x}}{6}-\frac{x^{3/2}}{18}+\frac{x^{5/2}}{30}+\frac{\pi x^3}{12}-\frac{\arctan \left(\sqrt{x}\right)}{6}-\frac{1}{6}x^3 \arctan \left(\sqrt{x}\right)$$

output
```
-1/18*x^(3/2)+1/30*x^(5/2)+1/12*Pi*x^3-1/6*arctan(x^(1/2))-1/6*x^3*arctan(
x^(1/2))+1/6*x^(1/2)
```

3.127.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int -x^2 \arctan \left(\sqrt{x}-\sqrt{1+x}\right) \, dx = \frac{1}{90}\left(-15 \arctan \left(\sqrt{x}\right)-\sqrt{x}\left(-15+5 x-3 x^2 +30 x^{5/2} \arctan \left(\sqrt{x}-\sqrt{1+x}\right)\right)\right)$$

input
```
Integrate[-(x^2*ArcTan[Sqrt[x] - Sqrt[1 + x]]),x]
```

output
```
(-15*ArcTan[Sqrt[x]] - Sqrt[x]*(-15 + 5*x - 3*x^2 + 30*x^(5/2)*ArcTan[Sqrt
[x] - Sqrt[1 + x]]))/90
```

3.127.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {25, 5682, 15, 5361, 60, 60, 60, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int -x^2 \arctan\left(\sqrt{x}-\sqrt{x+1}\right) \, dx$$

$$\Big\downarrow \; 25$$

$$-\int x^2 \arctan\left(\sqrt{x}-\sqrt{x+1}\right) \, dx$$

$$\Big\downarrow \; 5682$$

$$\frac{\pi \int x^2 dx}{4}-\frac{1}{2}\int x^2 \arctan\left(\sqrt{x}\right) dx$$

$$\Big\downarrow \; 15$$

$$\frac{\pi x^3}{12}-\frac{1}{2}\int x^2 \arctan\left(\sqrt{x}\right) dx$$

$$\Big\downarrow \; 5361$$

$$\frac{1}{2}\left(\frac{1}{6}\int \frac{x^{5/2}}{x+1}dx-\frac{1}{3}x^3 \arctan\left(\sqrt{x}\right)\right)+\frac{\pi x^3}{12}$$

$$\Big\downarrow \; 60$$

$$\frac{1}{2}\left(\frac{1}{6}\left(\frac{2x^{5/2}}{5}-\int \frac{x^{3/2}}{x+1}dx\right)-\frac{1}{3}x^3 \arctan\left(\sqrt{x}\right)\right)+\frac{\pi x^3}{12}$$

$$\Big\downarrow \; 60$$

$$\frac{1}{2}\left(\frac{1}{6}\left(\int \frac{\sqrt{x}}{x+1}dx+\frac{2x^{5/2}}{5}-\frac{2x^{3/2}}{3}\right)-\frac{1}{3}x^3 \arctan\left(\sqrt{x}\right)\right)+\frac{\pi x^3}{12}$$

$$\Big\downarrow \; 60$$

$$\frac{1}{2}\left(\frac{1}{6}\left(-\int \frac{1}{\sqrt{x}(x+1)}dx+\frac{2x^{5/2}}{5}-\frac{2x^{3/2}}{3}+2\sqrt{x}\right)-\frac{1}{3}x^3 \arctan\left(\sqrt{x}\right)\right)+\frac{\pi x^3}{12}$$

$$\Big\downarrow \; 73$$

$$\frac{1}{2}\left(\frac{1}{6}\left(-2\int \frac{1}{x+1}d\sqrt{x}+\frac{2x^{5/2}}{5}-\frac{2x^{3/2}}{3}+2\sqrt{x}\right)-\frac{1}{3}x^3\arctan\left(\sqrt{x}\right)\right)+\frac{\pi x^3}{12}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(\frac{1}{6}\left(-2\arctan\left(\sqrt{x}\right)+\frac{2x^{5/2}}{5}-\frac{2x^{3/2}}{3}+2\sqrt{x}\right)-\frac{1}{3}x^3\arctan\left(\sqrt{x}\right)\right)+\frac{\pi x^3}{12}$$

input
```
Int[-(x^2*ArcTan[Sqrt[x] - Sqrt[1 + x]]),x]
```

output
```
(Pi*x^3)/12 + ((2*Sqrt[x] - (2*x^(3/2))/3 + (2*x^(5/2))/5 - 2*ArcTan[Sqrt[
x]])/6 - (x^3*ArcTan[Sqrt[x]])/3)/2
```

3.127.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5682
```
Int[ArcTan[(v_) + (s_.)*Sqrt[w_]]*(u_.), x_Symbol] :> Simp[Pi*(s/4)   Int[u
, x], x] + Simp[1/2   Int[u*ArcTan[v], x], x] /; EqQ[s^2, 1] && EqQ[w, v^2
+ 1]
```

3.127.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.68

method	result	size
default	$-\frac{x^{3} \arctan \left(\sqrt{x}-\sqrt{1+x}\right)}{3}+\frac{x^{\frac{5}{2}}}{30}-\frac{x^{\frac{3}{2}}}{18}+\frac{\sqrt{x}}{6}-\frac{\arctan \left(\sqrt{x}\right)}{6}$	40
parts	$-\frac{x^{3} \arctan \left(\sqrt{x}-\sqrt{1+x}\right)}{3}+\frac{x^{\frac{5}{2}}}{30}-\frac{x^{\frac{3}{2}}}{18}+\frac{\sqrt{x}}{6}-\frac{\arctan \left(\sqrt{x}\right)}{6}$	40

input
```
int(-x^2*arctan(x^(1/2)-(1+x)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-1/3*x^3*arctan(x^(1/2)-(1+x)^(1/2))+1/30*x^(5/2)-1/18*x^(3/2)+1/6*x^(1/2)
-1/6*arctan(x^(1/2))
```

3.127.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.59

$$\int -x^2 \arctan \left(\sqrt{x}-\sqrt{1+x}\right) \, dx = \frac{1}{3} \left(x^{3} + 1\right) \arctan \left(\sqrt{x + 1} - \sqrt{x}\right) + \frac{1}{90} \left(3 \, x^{2} - 5 \, x + 15\right) \sqrt{x}$$

input
```
integrate(-x^2*arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="fricas")
```

output
```
1/3*(x^3 + 1)*arctan(sqrt(x + 1) - sqrt(x)) + 1/90*(3*x^2 - 5*x + 15)*sqrt
(x)
```

3.127.6 Sympy [A] (verification not implemented)

Time = 175.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.78

$$\int -x^2 \arctan\left(\sqrt{x}-\sqrt{1+x}\right)\,dx = \frac{x^{\frac{5}{2}}}{30} - \frac{x^{\frac{3}{2}}}{18} + \frac{\sqrt{x}}{6} - \frac{x^3 \operatorname{atan}\left(\sqrt{x}-\sqrt{x+1}\right)}{3} - \frac{\operatorname{atan}\left(\sqrt{x}\right)}{6}$$

input
```
integrate(-x**2*atan(x**(1/2)-(1+x)**(1/2)),x)
```

output
```
x**(5/2)/30 - x**(3/2)/18 + sqrt(x)/6 - x**3*atan(sqrt(x) - sqrt(x + 1))/3
 - atan(sqrt(x))/6
```

3.127.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.66

$$\int -x^2 \arctan\left(\sqrt{x}-\sqrt{1+x}\right)\,dx = \frac{1}{3}\,x^3 \arctan\left(\sqrt{x+1}-\sqrt{x}\right) + \frac{1}{30}\,x^{\frac{5}{2}} - \frac{1}{18}\,x^{\frac{3}{2}} + \frac{1}{6}\,\sqrt{x} - \frac{1}{6}\,\arctan\left(\sqrt{x}\right)$$

input
```
integrate(-x^2*arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="maxima")
```

output
```
1/3*x^3*arctan(sqrt(x + 1) - sqrt(x)) + 1/30*x^(5/2) - 1/18*x^(3/2) + 1/6*
sqrt(x) - 1/6*arctan(sqrt(x))
```

3.127.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.66

$$\int -x^2 \arctan\left(\sqrt{x}-\sqrt{1+x}\right)\,dx = -\frac{1}{3}\,x^3 \arctan\left(-\sqrt{x+1}+\sqrt{x}\right) + \frac{1}{30}\,x^{\frac{5}{2}} - \frac{1}{18}\,x^{\frac{3}{2}} + \frac{1}{6}\,\sqrt{x} - \frac{1}{6}\,\arctan\left(\sqrt{x}\right)$$

input
```
integrate(-x^2*arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="giac")
```

output
```
-1/3*x^3*arctan(-sqrt(x + 1) + sqrt(x)) + 1/30*x^(5/2) - 1/18*x^(3/2) + 1/
6*sqrt(x) - 1/6*arctan(sqrt(x))
```

3.127.9 Mupad [B] (verification not implemented)

Time = 1.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.10

$$\int -x^2 \arctan \left(\sqrt{x}-\sqrt{1+x}\right) \, dx = \frac{\sqrt{x}}{6} - \frac{x^{3/2}}{18} + \frac{x^{5/2}}{30} + \frac{\mathrm{atan}\left(\sqrt{x+1}-\sqrt{x}\right)\,\left(\frac{2\,x^4}{3}+\frac{2\,x^3}{3}\right)}{2\,x+2} + \frac{\ln\left(\frac{{\left(\sqrt{x}-\mathrm{i}\right)}^2}{x+1}\right)\,1\mathrm{i}}{12}$$

input
```
int(x^2*atan((x + 1)^(1/2) - x^(1/2)),x)
```

output
```
(log((x^(1/2) - 1i)^2/(x + 1))*1i)/12 + x^(1/2)/6 - x^(3/2)/18 + x^(5/2)/3
0 + (atan((x + 1)^(1/2) - x^(1/2))*((2*x^3)/3 + (2*x^4)/3))/(2*x + 2)
```

3.128 $\int -x \arctan \left(\sqrt{x}-\sqrt{1+x}\right) dx$

3.128.1 Optimal result

Integrand size = 19, antiderivative size = 50

$$\int -x \arctan \left(\sqrt{x}-\sqrt{1+x}\right) dx = -\frac{\sqrt{x}}{4}+\frac{x^{3/2}}{12}+\frac{\pi x^2}{8}+\frac{\arctan \left(\sqrt{x}\right)}{4}-\frac{1}{4}x^2 \arctan \left(\sqrt{x}\right)$$

output
```
1/12*x^(3/2)+1/8*Pi*x^2+1/4*arctan(x^(1/2))-1/4*x^2*arctan(x^(1/2))-1/4*x^
(1/2)
```

3.128.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int -x \arctan \left(\sqrt{x}-\sqrt{1+x}\right) dx = \frac{1}{12}\left(3 \arctan \left(\sqrt{x}\right) - \sqrt{x}\left(3-x+6x^{3/2} \arctan \left(\sqrt{x}-\sqrt{1+x}\right)\right)\right)$$

input
```
Integrate[-(x*ArcTan[Sqrt[x] - Sqrt[1 + x]]),x]
```

output
```
(3*ArcTan[Sqrt[x]] - Sqrt[x]*(3 - x + 6*x^(3/2)*ArcTan[Sqrt[x] - Sqrt[1 +
x]]))/12
```

3.128.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {25, 5682, 15, 5361, 60, 60, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int -x\arctan\left(\sqrt{x}-\sqrt{x+1}\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int x\arctan\left(\sqrt{x}-\sqrt{x+1}\right)dx$$

$$\Big\downarrow \text{5682}$$

$$\frac{\pi\int xdx}{4}-\frac{1}{2}\int x\arctan\left(\sqrt{x}\right)dx$$

$$\Big\downarrow \text{15}$$

$$\frac{\pi x^2}{8}-\frac{1}{2}\int x\arctan\left(\sqrt{x}\right)dx$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{2}\left(\frac{1}{4}\int \frac{x^{3/2}}{x+1}dx-\frac{1}{2}x^2\arctan\left(\sqrt{x}\right)\right)+\frac{\pi x^2}{8}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{2x^{3/2}}{3}-\int \frac{\sqrt{x}}{x+1}dx\right)-\frac{1}{2}x^2\arctan\left(\sqrt{x}\right)\right)+\frac{\pi x^2}{8}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\int \frac{1}{\sqrt{x}(x+1)}dx+\frac{2x^{3/2}}{3}-2\sqrt{x}\right)-\frac{1}{2}x^2\arctan\left(\sqrt{x}\right)\right)+\frac{\pi x^2}{8}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(2\int \frac{1}{x+1}d\sqrt{x}+\frac{2x^{3/2}}{3}-2\sqrt{x}\right)-\frac{1}{2}x^2\arctan\left(\sqrt{x}\right)\right)+\frac{\pi x^2}{8}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(2\arctan\left(\sqrt{x}\right)+\frac{2x^{3/2}}{3}-2\sqrt{x}\right)-\frac{1}{2}x^2\arctan\left(\sqrt{x}\right)\right)+\frac{\pi x^2}{8}$$

input
```
Int[-(x*ArcTan[Sqrt[x] - Sqrt[1 + x]]),x]
```

output
```
(Pi*x^2)/8 + (-1/2*(x^2*ArcTan[Sqrt[x]]) + (-2*Sqrt[x] + (2*x^(3/2))/3 + 2
*ArcTan[Sqrt[x]])/4)/2
```

3.128.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5682
```
Int[ArcTan[(v_) + (s_.)*Sqrt[w_]]*(u_.), x_Symbol] :> Simp[Pi*(s/4)   Int[u
, x], x] + Simp[1/2   Int[u*ArcTan[v], x], x] /; EqQ[s^2, 1] && EqQ[w, v^2
+ 1]
```

3.128.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.70

method	result	size
default	$-\frac{x^{2} \arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{2}+\frac{x^{\frac{3}{2}}}{12}-\frac{\sqrt{x}}{4}+\frac{\arctan\left(\sqrt{x}\right)}{4}$	35
parts	$-\frac{x^{2} \arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{2}+\frac{x^{\frac{3}{2}}}{12}-\frac{\sqrt{x}}{4}+\frac{\arctan\left(\sqrt{x}\right)}{4}$	35

input
```
int(-x*arctan(x^(1/2)-(1+x)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x^2*arctan(x^(1/2)-(1+x)^(1/2))+1/12*x^(3/2)-1/4*x^(1/2)+1/4*arctan(x
^(1/2))
```

3.128.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.56

$$\int -x \arctan \left(\sqrt{x}-\sqrt{1+x}\right)\,dx=\frac{1}{2}\left(x^{2}-1\right)\arctan \left(\sqrt{x+1}-\sqrt{x}\right)+\frac{1}{12}\left(x-3\right)\sqrt{x}$$

input
```
integrate(-x*arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="fricas")
```

output
```
1/2*(x^2 - 1)*arctan(sqrt(x + 1) - sqrt(x)) + 1/12*(x - 3)*sqrt(x)
```

3.128.6 Sympy [A] (verification not implemented)

Time = 34.99 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78

$$\int -x\arctan\left(\sqrt{x}-\sqrt{1+x}\right)\,dx=\frac{x^{\frac{3}{2}}}{12}-\frac{\sqrt{x}}{4}-\frac{x^2\operatorname{atan}\left(\sqrt{x}-\sqrt{x+1}\right)}{2}+\frac{\operatorname{atan}\left(\sqrt{x}\right)}{4}$$

input
```
integrate(-x*atan(x**(1/2)-(1+x)**(1/2)),x)
```

output
```
x**(3/2)/12 - sqrt(x)/4 - x**2*atan(sqrt(x) - sqrt(x + 1))/2 + atan(sqrt(x
))/4
```

3.128.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.68

$$\int -x\arctan\left(\sqrt{x}-\sqrt{1+x}\right)\,dx=\frac{1}{2}x^2\arctan\left(\sqrt{x+1}-\sqrt{x}\right)+\frac{1}{12}x^{\frac{3}{2}}-\frac{1}{4}\sqrt{x}+\frac{1}{4}\arctan\left(\sqrt{x}\right)$$

input
```
integrate(-x*arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="maxima")
```

output
```
1/2*x^2*arctan(sqrt(x + 1) - sqrt(x)) + 1/12*x^(3/2) - 1/4*sqrt(x) + 1/4*a
rctan(sqrt(x))
```

3.128.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.68

$$\int -x\arctan\left(\sqrt{x}-\sqrt{1+x}\right)\,dx=-\frac{1}{2}x^2\arctan\left(-\sqrt{x+1}+\sqrt{x}\right)+\frac{1}{12}x^{\frac{3}{2}}-\frac{1}{4}\sqrt{x}+\frac{1}{4}\arctan\left(\sqrt{x}\right)$$

input
```
integrate(-x*arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="giac")
```

output
```
-1/2*x^2*arctan(-sqrt(x + 1) + sqrt(x)) + 1/12*x^(3/2) - 1/4*sqrt(x) + 1/4
*arctan(sqrt(x))
```

3.128.9 Mupad [B] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.16

$$\int -x\arctan\left(\sqrt{x}-\sqrt{1+x}\right)\,dx=\frac{x^{3/2}}{12}-\frac{\sqrt{x}}{4}+\frac{\mathrm{atan}\left(\sqrt{x+1}-\sqrt{x}\right)\,\left(x^3+x^2\right)}{2\,x+2}+\frac{\ln\left(\frac{\left(-1+\sqrt{x}\,1\mathrm{i}\right)^2}{x+1}\right)\,1\mathrm{i}}{8}$$

input
```
int(x*atan((x + 1)^(1/2) - x^(1/2)),x)
```

output
```
(log((x^(1/2)*1i - 1)^2/(x + 1))*1i)/8 - x^(1/2)/4 + x^(3/2)/12 + (atan((x
 + 1)^(1/2) - x^(1/2))*(x^2 + x^3))/(2*x + 2)
```

3.129 $\int -\arctan\left(\sqrt{x}-\sqrt{1+x}\right)\,dx$

3.129.1 Optimal result

Integrand size = 18, antiderivative size = 37

$$\int -\arctan\left(\sqrt{x}-\sqrt{1+x}\right)\,dx=\frac{\sqrt{x}}{2}+\frac{\pi x}{4}-\frac{\arctan\left(\sqrt{x}\right)}{2}-\frac{1}{2}x\arctan\left(\sqrt{x}\right)$$

output
```
1/4*Pi*x-1/2*arctan(x^(1/2))-1/2*x*arctan(x^(1/2))+1/2*x^(1/2)
```

3.129.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int -\arctan\left(\sqrt{x}-\sqrt{1+x}\right)\,dx=\frac{\sqrt{x}}{2}-(1+x)\arctan\left(\sqrt{x}-\sqrt{1+x}\right)$$

input
```
Integrate[-ArcTan[Sqrt[x] - Sqrt[1 + x]],x]
```

output
```
Sqrt[x]/2 - (1 + x)*ArcTan[Sqrt[x] - Sqrt[1 + x]]
```

3.129.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {25, 5682, 24, 5345, 60, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int -\arctan\left(\sqrt{x}-\sqrt{x+1}\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \arctan\left(\sqrt{x}-\sqrt{x+1}\right)dx$$

$$\Big\downarrow \text{5682}$$

$$\frac{\pi\int 1dx}{4}-\frac{1}{2}\int \arctan\left(\sqrt{x}\right)dx$$

$$\Big\downarrow \text{24}$$

$$\frac{\pi x}{4}-\frac{1}{2}\int \arctan\left(\sqrt{x}\right)dx$$

$$\Big\downarrow \text{5345}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{\sqrt{x}}{x+1}dx-x\arctan\left(\sqrt{x}\right)\right)+\frac{\pi x}{4}$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(2\sqrt{x}-\int \frac{1}{\sqrt{x}(x+1)}dx\right)-x\arctan\left(\sqrt{x}\right)\right)+\frac{\pi x}{4}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(2\sqrt{x}-2\int \frac{1}{x+1}d\sqrt{x}\right)-x\arctan\left(\sqrt{x}\right)\right)+\frac{\pi x}{4}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(2\sqrt{x}-2\arctan\left(\sqrt{x}\right)\right)-x\arctan\left(\sqrt{x}\right)\right)+\frac{\pi x}{4}$$

input
```
Int[-ArcTan[Sqrt[x] - Sqrt[1 + x]],x]
```

output
```
(Pi*x)/4 + ((2*Sqrt[x] - 2*ArcTan[Sqrt[x]])/2 - x*ArcTan[Sqrt[x]])/2
```

3.129.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5682
```
Int[ArcTan[(v_) + (s_.)*Sqrt[w_]]*(u_.), x_Symbol] :> Simp[Pi*(s/4)   Int[u
, x], x] + Simp[1/2   Int[u*ArcTan[v], x], x] /; EqQ[s^2, 1] && EqQ[w, v^2
+ 1]
```

3.129.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

method	result	size
default	$-x \arctan\left(\sqrt{x}-\sqrt{1+x}\right)+\frac{\sqrt{x}}{2}-\frac{\arctan\left(\sqrt{x}\right)}{2}$	28
parts	$-x \arctan\left(\sqrt{x}-\sqrt{1+x}\right)+\frac{\sqrt{x}}{2}-\frac{\arctan\left(\sqrt{x}\right)}{2}$	28

input
```
int(-arctan(x^(1/2)-(1+x)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-x*arctan(x^(1/2)-(1+x)^(1/2))+1/2*x^(1/2)-1/2*arctan(x^(1/2))
```

3.129.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.59

$$\int -\arctan\left(\sqrt{x}-\sqrt{1+x}\right)\,dx=(x+1)\arctan\left(\sqrt{x+1}-\sqrt{x}\right)+\frac{1}{2}\sqrt{x}$$

input
```
integrate(-arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="fricas")
```

output
```
(x + 1)*arctan(sqrt(x + 1) - sqrt(x)) + 1/2*sqrt(x)
```

3.129.6 Sympy [A] (verification not implemented)

Time = 8.41 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int -\arctan\left(\sqrt{x}-\sqrt{1+x}\right)\,dx=\frac{\sqrt{x}}{2}-x\,\mathrm{atan}\left(\sqrt{x}-\sqrt{x+1}\right)-\frac{\mathrm{atan}\left(\sqrt{x}\right)}{2}$$

input
```
integrate(-atan(x**(1/2)-(1+x)**(1/2)),x)
```

output
```
sqrt(x)/2 - x*atan(sqrt(x) - sqrt(x + 1)) - atan(sqrt(x))/2
```

3.129.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70

$$\int -\arctan\left(\sqrt{x}-\sqrt{1+x}\right)\,dx = x\arctan\left(\sqrt{x+1}-\sqrt{x}\right)+\frac{1}{2}\,\sqrt{x}-\frac{1}{2}\,\arctan\left(\sqrt{x}\right)$$

input
```
integrate(-arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="maxima")
```

output
```
x*arctan(sqrt(x + 1) - sqrt(x)) + 1/2*sqrt(x) - 1/2*arctan(sqrt(x))
```

3.129.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int -\arctan\left(\sqrt{x}-\sqrt{1+x}\right)\,dx = -x\arctan\left(-\sqrt{x+1}+\sqrt{x}\right)+\frac{1}{2}\,\sqrt{x}-\frac{1}{2}\,\arctan\left(\sqrt{x}\right)$$

input
```
integrate(-arctan(x^(1/2)-(1+x)^(1/2)),x, algorithm="giac")
```

output
```
-x*arctan(-sqrt(x + 1) + sqrt(x)) + 1/2*sqrt(x) - 1/2*arctan(sqrt(x))
```

3.129.9 Mupad [B] (verification not implemented)

Time = 1.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.08

$$\int -\arctan\left(\sqrt{x}-\sqrt{1+x}\right)\,dx = x\,\mathrm{atan}\left(\sqrt{x+1}-\sqrt{x}\right)+\frac{\sqrt{x}}{2}-\frac{\ln\left(\frac{\left(-1+\sqrt{x}\,1\mathrm{i}\right)^2}{x+1}\right)\,1\mathrm{i}}{4}$$

input
```
int(atan((x + 1)^(1/2) - x^(1/2)),x)
```

output
```
x*atan((x + 1)^(1/2) - x^(1/2)) - (log((x^(1/2)*1i - 1)^2/(x + 1))*1i)/4 +
 x^(1/2)/2
```

3.130 $\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x}\,dx$

3.130.1 Optimal result

Integrand size = 21, antiderivative size = 42

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x}\,dx = \frac{1}{4}\pi\log(x)-\frac{1}{2}i\,\text{PolyLog}\left(2,-i\sqrt{x}\right)+\frac{1}{2}i\,\text{PolyLog}\left(2,i\sqrt{x}\right)$$

output
```
1/4*Pi*ln(x)-1/2*I*polylog(2,-I*x^(1/2))+1/2*I*polylog(2,I*x^(1/2))
```

3.130.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.00

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x}\,dx = -\arctan\left(\sqrt{x}-\sqrt{1+x}\right)\log(x) + \frac{1}{4}i\left(\left(\log\left(1-i\sqrt{x}\right)-\log\left(1+i\sqrt{x}\right)\right)\log(x) - 2\,\text{PolyLog}\left(2,-i\sqrt{x}\right)+2\,\text{PolyLog}\left(2,i\sqrt{x}\right)\right)$$

input
```
Integrate[-(ArcTan[Sqrt[x] - Sqrt[1 + x]]/x),x]
```

output
```
-(ArcTan[Sqrt[x] - Sqrt[1 + x]]*Log[x]) + (I/4)*((Log[1 - I*Sqrt[x]] - Log
[1 + I*Sqrt[x]])*Log[x] - 2*PolyLog[2, (-I)*Sqrt[x]] + 2*PolyLog[2, I*Sqrt
[x]])
```

3.130.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {25, 5682, 14, 5359, 5355, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{x+1}\right)}{x} \, dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\arctan\left(\sqrt{x}-\sqrt{x+1}\right)}{x}dx$$

$$\Big\downarrow \text{5682}$$

$$\frac{1}{4}\pi\int\frac{1}{x}dx-\frac{1}{2}\int\frac{\arctan\left(\sqrt{x}\right)}{x}dx$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{4}\pi\log(x)-\frac{1}{2}\int\frac{\arctan\left(\sqrt{x}\right)}{x}dx$$

$$\Big\downarrow \text{5359}$$

$$\frac{1}{4}\pi\log(x)-\int\frac{\arctan\left(\sqrt{x}\right)}{\sqrt{x}}d\sqrt{x}$$

$$\Big\downarrow \text{5355}$$

$$-\frac{1}{2}i\int\frac{\log\left(1-i\sqrt{x}\right)}{\sqrt{x}}d\sqrt{x}+\frac{1}{2}i\int\frac{\log\left(i\sqrt{x}+1\right)}{\sqrt{x}}d\sqrt{x}+\frac{1}{4}\pi\log(x)$$

$$\Big\downarrow \text{2838}$$

$$-\frac{1}{2}i\,\text{PolyLog}\left(2,-i\sqrt{x}\right)+\frac{1}{2}i\,\text{PolyLog}\left(2,i\sqrt{x}\right)+\frac{1}{4}\pi\log(x)$$

input
```
Int[-(ArcTan[Sqrt[x] - Sqrt[1 + x]]/x),x]
```

output
```
(Pi*Log[x])/4 - (I/2)*PolyLog[2, (-I)*Sqrt[x]] + (I/2)*PolyLog[2, I*Sqrt[x
]]
```

3.130.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5355
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Simp[I*(b/2)   Int[Log[1 - I*c*x]/x, x], x] - Simp[I*(b/2)   Int[Log[1
+ I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

rule 5359
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1
/n   Subst[Int[(a + b*ArcTan[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c,
n}, x] && IGtQ[p, 0]
```

rule 5682
```
Int[ArcTan[(v_) + (s_.)*Sqrt[w_]]*(u_.), x_Symbol] :> Simp[Pi*(s/4)   Int[u
, x], x] + Simp[1/2   Int[u*ArcTan[v], x], x] /; EqQ[s^2, 1] && EqQ[w, v^2
+ 1]
```

3.130.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(28) = 56$.

Time = 0.76 (sec) , antiderivative size = 374, normalized size of antiderivative = 8.90

method	result
default	$-2\arctan\left(\sqrt{x}-\sqrt{1+x}\right)\ln\left(1+\frac{\left(1+i\left(\sqrt{x}-\sqrt{1+x}\right)\right)^{4}}{\left(\left(\sqrt{x}-\sqrt{1+x}\right)^{2}+1\right)^{2}}\right)+\frac{i\,\mathrm{polylog}\left(2,-\frac{\left(1+i\left(\sqrt{x}-\sqrt{1+x}\right)\right)^{4}}{\left(\left(\sqrt{x}-\sqrt{1+x}\right)^{2}+1\right)^{2}}\right)}{2}+2\arctan\left(\sqrt{\right.$

input
```
int(-arctan(x^(1/2)-(1+x)^(1/2))/x,x,method=_RETURNVERBOSE)
```

output
```
-2*arctan(x^(1/2)-(1+x)^(1/2))*ln(1+(1+I*(x^(1/2)-(1+x)^(1/2)))^4/((x^(1/2
)-(1+x)^(1/2))^2+1)^2)+1/2*I*polylog(2,-(1+I*(x^(1/2)-(1+x)^(1/2)))^4/((x^
(1/2)-(1+x)^(1/2))^2+1)^2)+2*arctan(x^(1/2)-(1+x)^(1/2))*ln(1-(1+I*(x^(1/2
)-(1+x)^(1/2)))/((x^(1/2)-(1+x)^(1/2))^2+1)^(1/2))-2*I*polylog(2,(1+I*(x^(
1/2)-(1+x)^(1/2)))/((x^(1/2)-(1+x)^(1/2))^2+1)^(1/2))+2*arctan(x^(1/2)-(1+
x)^(1/2))*ln(1+(1+I*(x^(1/2)-(1+x)^(1/2)))^2/((x^(1/2)-(1+x)^(1/2))^2+1))-
I*polylog(2,-(1+I*(x^(1/2)-(1+x)^(1/2)))^2/((x^(1/2)-(1+x)^(1/2))^2+1))+2*
arctan(x^(1/2)-(1+x)^(1/2))*ln(1+(1+I*(x^(1/2)-(1+x)^(1/2)))/((x^(1/2)-(1+
x)^(1/2))^2+1)^(1/2))-2*I*polylog(2,-(1+I*(x^(1/2)-(1+x)^(1/2)))/((x^(1/2)
-(1+x)^(1/2))^2+1)^(1/2))
```

3.130.5 Fricas [F]

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x}\,dx=\int -\frac{\arctan\left(-\sqrt{x+1}+\sqrt{x}\right)}{x}\,dx$$

input
```
integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x,x, algorithm="fricas")
```

output
```
integral(arctan(sqrt(x + 1) - sqrt(x))/x, x)
```

3.130.6 Sympy [F]

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x}\,dx=-\int \frac{\operatorname{atan}\left(\sqrt{x}-\sqrt{x+1}\right)}{x}\,dx$$

input
```
integrate(-atan(x**(1/2)-(1+x)**(1/2))/x,x)
```

output
```
-Integral(atan(sqrt(x) - sqrt(x + 1))/x, x)
```

3.130.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x}\,dx = \frac{1}{4}\pi\log\left(x+1\right)+\arctan\left(\sqrt{x+1}-\sqrt{x}\right)\log\left(x\right) + \frac{1}{2}i\,\mathrm{Li}_2\left(i\sqrt{x}+1\right)-\frac{1}{2}i\,\mathrm{Li}_2\left(-i\sqrt{x}+1\right)$$

input
```
integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x,x, algorithm="maxima")
```

output
```
1/4*pi*log(x + 1) + arctan(sqrt(x + 1) - sqrt(x))*log(x) + 1/2*I*dilog(I*s
qrt(x) + 1) - 1/2*I*dilog(-I*sqrt(x) + 1)
```

3.130.8 Giac [F]

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x}\,dx = \int -\frac{\arctan\left(-\sqrt{x+1}+\sqrt{x}\right)}{x}\,dx$$

input
```
integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x,x, algorithm="giac")
```

output
```
integrate(-arctan(-sqrt(x + 1) + sqrt(x))/x, x)
```

3.130.9 Mupad [F(-1)]

Timed out.

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x}\,dx = \int \frac{\mathrm{atan}\left(\sqrt{x+1}-\sqrt{x}\right)}{x}\,dx$$

input
```
int(atan((x + 1)^(1/2) - x^(1/2))/x,x)
```

output
```
int(atan((x + 1)^(1/2) - x^(1/2))/x, x)
```

3.131 $\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x^2}\,dx$

3.131.1 Optimal result

Integrand size = 21, antiderivative size = 41

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x^2}\,dx = -\frac{\pi}{4x}+\frac{1}{2\sqrt{x}}+\frac{\arctan\left(\sqrt{x}\right)}{2}+\frac{\arctan\left(\sqrt{x}\right)}{2x}$$

output
```
-1/4*Pi/x+1/2*arctan(x^(1/2))+1/2*arctan(x^(1/2))/x+1/2/x^(1/2)
```

3.131.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x^2}\,dx = \frac{1}{2\sqrt{x}}+\frac{\arctan\left(\sqrt{x}\right)}{2}+\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x}$$

input
```
Integrate[-(ArcTan[Sqrt[x] - Sqrt[1 + x]]/x^2),x]
```

output
```
1/(2*Sqrt[x]) + ArcTan[Sqrt[x]]/2 + ArcTan[Sqrt[x] - Sqrt[1 + x]]/x
```

3.131.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {25, 5682, 15, 5361, 61, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{x+1}\right)}{x^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\arctan\left(\sqrt{x}-\sqrt{x+1}\right)}{x^2}dx$$

$$\Big\downarrow \text{5682}$$

$$\frac{1}{4}\pi\int \frac{1}{x^2}dx-\frac{1}{2}\int \frac{\arctan\left(\sqrt{x}\right)}{x^2}dx$$

$$\Big\downarrow \text{15}$$

$$-\frac{1}{2}\int \frac{\arctan\left(\sqrt{x}\right)}{x^2}dx-\frac{\pi}{4x}$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\sqrt{x}\right)}{x}-\frac{1}{2}\int \frac{1}{x^{3/2}(x+1)}dx\right)-\frac{\pi}{4x}$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\int \frac{1}{\sqrt{x}(x+1)}dx+\frac{2}{\sqrt{x}}\right)+\frac{\arctan\left(\sqrt{x}\right)}{x}\right)-\frac{\pi}{4x}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(2\int \frac{1}{x+1}d\sqrt{x}+\frac{2}{\sqrt{x}}\right)+\frac{\arctan\left(\sqrt{x}\right)}{x}\right)-\frac{\pi}{4x}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\sqrt{x}\right)}{x}+\frac{1}{2}\left(2\arctan\left(\sqrt{x}\right)+\frac{2}{\sqrt{x}}\right)\right)-\frac{\pi}{4x}$$

input
```
Int[-(ArcTan[Sqrt[x] - Sqrt[1 + x]]/x^2),x]
```

output
```
-1/4*Pi/x + (ArcTan[Sqrt[x]]/x + (2/Sqrt[x] + 2*ArcTan[Sqrt[x]])/2)/2
```

3.131.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5682
```
Int[ArcTan[(v_) + (s_.)*Sqrt[w_]]*(u_.), x_Symbol] :> Simp[Pi*(s/4)   Int[u
, x], x] + Simp[1/2   Int[u*ArcTan[v], x], x] /; EqQ[s^2, 1] && EqQ[w, v^2
+ 1]
```

3.131. $\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x^2}\,dx$

3.131.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(27) = 54.

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.39

method	result	size
default	$\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x}+\frac{1}{2\sqrt{x}}+\frac{\operatorname{arctanh}\left(\sqrt{1+x}\right)}{2}+\frac{\arctan\left(\sqrt{x}\right)}{2}-\frac{\ln\left(1+\sqrt{1+x}\right)}{4}+\frac{\ln\left(\sqrt{1+x}-1\right)}{4}$	57
parts	$\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x}+\frac{1}{2\sqrt{x}}+\frac{\operatorname{arctanh}\left(\sqrt{1+x}\right)}{2}+\frac{\arctan\left(\sqrt{x}\right)}{2}-\frac{\ln\left(1+\sqrt{1+x}\right)}{4}+\frac{\ln\left(\sqrt{1+x}-1\right)}{4}$	57

input
```
int(-arctan(x^(1/2)-(1+x)^(1/2))/x^2,x,method=_RETURNVERBOSE)
```

output
```
arctan(x^(1/2)-(1+x)^(1/2))/x+1/2/x^(1/2)+1/2*arctanh((1+x)^(1/2))+1/2*arc
tan(x^(1/2))-1/4*ln(1+(1+x)^(1/2))+1/4*ln((1+x)^(1/2)-1)
```

3.131.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x^2}\,dx = -\frac{2\,(x+1)\arctan\left(\sqrt{x+1}-\sqrt{x}\right)-\sqrt{x}}{2\,x}$$

input
```
integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x^2,x, algorithm="fricas")
```

output
```
-1/2*(2*(x + 1)*arctan(sqrt(x + 1) - sqrt(x)) - sqrt(x))/x
```

3.131.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 537 vs. $2(31) = 62$.

Time = 88.10 (sec) , antiderivative size = 537, normalized size of antiderivative = 13.10

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x^2}\,dx = -\frac{2x^{\frac{5}{2}}\sqrt{x+1}\operatorname{atan}\left(\sqrt{x}-\sqrt{x+1}\right)}{-2x^{\frac{5}{2}}\sqrt{x+1}-2x^{\frac{3}{2}}\sqrt{x+1}+2x^3+2x^2} + \frac{x^{\frac{5}{2}}}{-2x^{\frac{5}{2}}\sqrt{x+1}-2x^{\frac{3}{2}}\sqrt{x+1}+2x^3+2x^2} - \frac{4x^{\frac{3}{2}}\sqrt{x+1}\operatorname{atan}\left(\sqrt{x}-\sqrt{x+1}\right)}{-2x^{\frac{5}{2}}\sqrt{x+1}-2x^{\frac{3}{2}}\sqrt{x+1}+2x^3+2x^2} + \frac{x^{\frac{3}{2}}}{-2x^{\frac{5}{2}}\sqrt{x+1}-2x^{\frac{3}{2}}\sqrt{x+1}+2x^3+2x^2} - \frac{2\sqrt{x}\sqrt{x+1}\operatorname{atan}\left(\sqrt{x}-\sqrt{x+1}\right)}{-2x^{\frac{5}{2}}\sqrt{x+1}-2x^{\frac{3}{2}}\sqrt{x+1}+2x^3+2x^2} + \frac{2x^3\operatorname{atan}\left(\sqrt{x}-\sqrt{x+1}\right)}{-2x^{\frac{5}{2}}\sqrt{x+1}-2x^{\frac{3}{2}}\sqrt{x+1}+2x^3+2x^2} - \frac{x^2\sqrt{x+1}}{-2x^{\frac{5}{2}}\sqrt{x+1}-2x^{\frac{3}{2}}\sqrt{x+1}+2x^3+2x^2} + \frac{4x^2\operatorname{atan}\left(\sqrt{x}-\sqrt{x+1}\right)}{-2x^{\frac{5}{2}}\sqrt{x+1}-2x^{\frac{3}{2}}\sqrt{x+1}+2x^3+2x^2} - \frac{x\sqrt{x+1}}{-2x^{\frac{5}{2}}\sqrt{x+1}-2x^{\frac{3}{2}}\sqrt{x+1}+2x^3+2x^2} + \frac{2x\operatorname{atan}\left(\sqrt{x}-\sqrt{x+1}\right)}{-2x^{\frac{5}{2}}\sqrt{x+1}-2x^{\frac{3}{2}}\sqrt{x+1}+2x^3+2x^2}$$

input
```
integrate(-atan(x**(1/2)-(1+x)**(1/2))/x**2,x)
```

output
```
-2*x**(5/2)*sqrt(x + 1)*atan(sqrt(x) - sqrt(x + 1))/(-2*x**(5/2)*sqrt(x +
1) - 2*x**(3/2)*sqrt(x + 1) + 2*x**3 + 2*x**2) + x**(5/2)/(-2*x**(5/2)*sqr
t(x + 1) - 2*x**(3/2)*sqrt(x + 1) + 2*x**3 + 2*x**2) - 4*x**(3/2)*sqrt(x +
 1)*atan(sqrt(x) - sqrt(x + 1))/(-2*x**(5/2)*sqrt(x + 1) - 2*x**(3/2)*sqrt
(x + 1) + 2*x**3 + 2*x**2) + x**(3/2)/(-2*x**(5/2)*sqrt(x + 1) - 2*x**(3/2
)*sqrt(x + 1) + 2*x**3 + 2*x**2) - 2*sqrt(x)*sqrt(x + 1)*atan(sqrt(x) - sq
rt(x + 1))/(-2*x**(5/2)*sqrt(x + 1) - 2*x**(3/2)*sqrt(x + 1) + 2*x**3 + 2*
x**2) + 2*x**3*atan(sqrt(x) - sqrt(x + 1))/(-2*x**(5/2)*sqrt(x + 1) - 2*x*
*(3/2)*sqrt(x + 1) + 2*x**3 + 2*x**2) - x**2*sqrt(x + 1)/(-2*x**(5/2)*sqrt
(x + 1) - 2*x**(3/2)*sqrt(x + 1) + 2*x**3 + 2*x**2) + 4*x**2*atan(sqrt(x)
- sqrt(x + 1))/(-2*x**(5/2)*sqrt(x + 1) - 2*x**(3/2)*sqrt(x + 1) + 2*x**3
+ 2*x**2) - x*sqrt(x + 1)/(-2*x**(5/2)*sqrt(x + 1) - 2*x**(3/2)*sqrt(x + 1
) + 2*x**3 + 2*x**2) + 2*x*atan(sqrt(x) - sqrt(x + 1))/(-2*x**(5/2)*sqrt(x
 + 1) - 2*x**(3/2)*sqrt(x + 1) + 2*x**3 + 2*x**2)
```

3.131.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.71

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x^2}\,dx = -\frac{\arctan\left(\sqrt{x+1}-\sqrt{x}\right)}{x}+\frac{1}{2\sqrt{x}}+\frac{1}{2}\arctan\left(\sqrt{x}\right)$$

input
```
integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x^2,x, algorithm="maxima")
```

output
```
-arctan(sqrt(x + 1) - sqrt(x))/x + 1/2/sqrt(x) + 1/2*arctan(sqrt(x))
```

3.131.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x^2}\,dx = \frac{\arctan\left(-\sqrt{x+1}+\sqrt{x}\right)}{x}+\frac{1}{2\sqrt{x}}+\frac{1}{2}\arctan\left(\sqrt{x}\right)$$

input
```
integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x^2,x, algorithm="giac")
```

output
```
arctan(-sqrt(x + 1) + sqrt(x))/x + 1/2/sqrt(x) + 1/2*arctan(sqrt(x))
```

3.131.9 Mupad [B] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x^2}\,dx = -\frac{\mathrm{atan}\left(\sqrt{x+1}-\sqrt{x}\right)-\frac{\sqrt{x}}{2}}{x}+\frac{\ln\left(\frac{\left(-1+\sqrt{x}\,1\mathrm{i}\right)^2}{x+1}\right)\,1\mathrm{i}}{4}$$

input
```
int(atan((x + 1)^(1/2) - x^(1/2))/x^2,x)
```

output
```
(log((x^(1/2)*1i - 1)^2/(x + 1))*1i)/4 - (atan((x + 1)^(1/2) - x^(1/2)) -
x^(1/2)/2)/x
```

3.132 $\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x^3}\,dx$

3.132.1 Optimal result

Integrand size = 21, antiderivative size = 50

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x^3}\,dx = -\frac{\pi}{8x^2}+\frac{1}{12x^{3/2}}-\frac{1}{4\sqrt{x}}-\frac{\arctan\left(\sqrt{x}\right)}{4}+\frac{\arctan\left(\sqrt{x}\right)}{4x^2}$$

output
```
-1/8*Pi/x^2+1/12/x^(3/2)-1/4*arctan(x^(1/2))+1/4*arctan(x^(1/2))/x^2-1/4/x
^(1/2)
```

3.132.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x^3}\,dx = -\frac{\sqrt{x}(-1+3x)+3x^2\arctan\left(\sqrt{x}\right)-6\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{12x^2}$$

input
```
Integrate[-(ArcTan[Sqrt[x] - Sqrt[1 + x]]/x^3),x]
```

output
```
-1/12*(Sqrt[x]*(-1 + 3*x) + 3*x^2*ArcTan[Sqrt[x]] - 6*ArcTan[Sqrt[x] - Sqr
t[1 + x]])/x^2
```

3.132.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {25, 5682, 15, 5361, 61, 61, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{x+1}\right)}{x^3}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\arctan\left(\sqrt{x}-\sqrt{x+1}\right)}{x^3}dx$$

$$\Big\downarrow \text{5682}$$

$$\frac{1}{4}\pi\int \frac{1}{x^3}dx-\frac{1}{2}\int \frac{\arctan\left(\sqrt{x}\right)}{x^3}dx$$

$$\Big\downarrow \text{15}$$

$$-\frac{1}{2}\int \frac{\arctan\left(\sqrt{x}\right)}{x^3}dx-\frac{\pi}{8x^2}$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\sqrt{x}\right)}{2x^2}-\frac{1}{4}\int \frac{1}{x^{5/2}(x+1)}dx\right)-\frac{\pi}{8x^2}$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\int \frac{1}{x^{3/2}(x+1)}dx+\frac{2}{3x^{3/2}}\right)+\frac{\arctan\left(\sqrt{x}\right)}{2x^2}\right)-\frac{\pi}{8x^2}$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(-\int \frac{1}{\sqrt{x}(x+1)}dx+\frac{2}{3x^{3/2}}-\frac{2}{\sqrt{x}}\right)+\frac{\arctan\left(\sqrt{x}\right)}{2x^2}\right)-\frac{\pi}{8x^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(-2\int \frac{1}{x+1}d\sqrt{x}+\frac{2}{3x^{3/2}}-\frac{2}{\sqrt{x}}\right)+\frac{\arctan\left(\sqrt{x}\right)}{2x^2}\right)-\frac{\pi}{8x^2}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(\frac{1}{4}\left(-2\arctan\left(\sqrt{x}\right)+\frac{2}{3x^{3/2}}-\frac{2}{\sqrt{x}}\right)+\frac{\arctan\left(\sqrt{x}\right)}{2x^2}\right)-\frac{\pi}{8x^2}$$

input
```
Int[-(ArcTan[Sqrt[x] - Sqrt[1 + x]]/x^3),x]
```

output
```
-1/8*Pi/x^2 + ((2/(3*x^(3/2)) - 2/Sqrt[x] - 2*ArcTan[Sqrt[x]])/4 + ArcTan[
Sqrt[x]]/(2*x^2))/2
```

3.132.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

3.132. $\int -\frac{\arctan(\sqrt{x}-\sqrt{1+x})}{x^3}\,dx$

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5682
```
Int[ArcTan[(v_) + (s_.)*Sqrt[w_]]*(u_.), x_Symbol] :> Simp[Pi*(s/4)   Int[u
, x], x] + Simp[1/2   Int[u*ArcTan[v], x], x] /; EqQ[s^2, 1] && EqQ[w, v^2
+ 1]
```

3.132.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.70

method	result	size
default	$\frac{\arctan(\sqrt{x}-\sqrt{1+x})}{2x^2}+\frac{1}{12x^{\frac{3}{2}}}-\frac{1}{4\sqrt{x}}-\frac{\arctan(\sqrt{x})}{4}$	35
parts	$\frac{\arctan(\sqrt{x}-\sqrt{1+x})}{2x^2}+\frac{1}{12x^{\frac{3}{2}}}-\frac{1}{4\sqrt{x}}-\frac{\arctan(\sqrt{x})}{4}$	35

input
```
int(-arctan(x^(1/2)-(1+x)^(1/2))/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*arctan(x^(1/2)-(1+x)^(1/2))/x^2+1/12/x^(3/2)-1/4/x^(1/2)-1/4*arctan(x^
(1/2))
```

3.132.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.70

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x^3}\,dx=\frac{6\,(x^2-1)\arctan\left(\sqrt{x+1}-\sqrt{x}\right)-(3\,x-1)\sqrt{x}}{12\,x^2}$$

input
```
integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x^3,x, algorithm="fricas")
```

output
```
1/12*(6*(x^2 - 1)*arctan(sqrt(x + 1) - sqrt(x)) - (3*x - 1)*sqrt(x))/x^2
```

3.132.6 Sympy [F(-1)]

Timed out.

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x^3}\,dx = \text{Timed out}$$

input
```
integrate(-atan(x**(1/2)-(1+x)**(1/2))/x**3,x)
```

output
```
Timed out
```

3.132.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.68

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x^3}\,dx = -\frac{1}{4\sqrt{x}} - \frac{\arctan\left(\sqrt{x+1}-\sqrt{x}\right)}{2x^2} + \frac{1}{12x^{\frac{3}{2}}} - \frac{1}{4}\arctan\left(\sqrt{x}\right)$$

input
```
integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x^3,x, algorithm="maxima")
```

output
```
-1/4/sqrt(x) - 1/2*arctan(sqrt(x + 1) - sqrt(x))/x^2 + 1/12/x^(3/2) - 1/4*
arctan(sqrt(x))
```

3.132.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.68

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x^3}\,dx = -\frac{3x-1}{12x^{\frac{3}{2}}} + \frac{\arctan\left(-\sqrt{x+1}+\sqrt{x}\right)}{2x^2} - \frac{1}{4}\arctan\left(\sqrt{x}\right)$$

input
```
integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x^3,x, algorithm="giac")
```

output
```
-1/12*(3*x - 1)/x^(3/2) + 1/2*arctan(-sqrt(x + 1) + sqrt(x))/x^2 - 1/4*arc
tan(sqrt(x))
```

3.132.9 Mupad [B] (verification not implemented)

Time = 1.44 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x^3}\,dx = -\frac{\frac{\mathrm{atan}\left(\sqrt{x+1}-\sqrt{x}\right)}{2}-\frac{\sqrt{x}}{12}+\frac{x^{3/2}}{4}}{x^2}+\frac{\ln\left(\frac{\left(\sqrt{x}-\mathrm{i}\right)^2}{x+1}\right)\,1\mathrm{i}}{8}$$

input
```
int(atan((x + 1)^(1/2) - x^(1/2))/x^3,x)
```

output
```
(log((x^(1/2) - 1i)^2/(x + 1))*1i)/8 - (atan((x + 1)^(1/2) - x^(1/2))/2 -
x^(1/2)/12 + x^(3/2)/4)/x^2
```

3.133 $\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x^4}\,dx$

3.133.1 Optimal result

Integrand size = 21, antiderivative size = 59

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x^4}\,dx = -\frac{\pi}{12x^3}+\frac{1}{30x^{5/2}}-\frac{1}{18x^{3/2}}+\frac{1}{6\sqrt{x}}+\frac{\arctan\left(\sqrt{x}\right)}{6}+\frac{\arctan\left(\sqrt{x}\right)}{6x^3}$$

output
```
-1/12*Pi/x^3+1/30/x^(5/2)-1/18/x^(3/2)+1/6*arctan(x^(1/2))+1/6*arctan(x^(1
/2))/x^3+1/6/x^(1/2)
```

3.133.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x^4}\,dx = \frac{1}{90}\left(-\frac{-3+5x-15x^2}{x^{5/2}}+15\arctan\left(\sqrt{x}\right)+\frac{30\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x^3}\right)$$

input
```
Integrate[-(ArcTan[Sqrt[x] - Sqrt[1 + x]]/x^4),x]
```

output
```
(-((-3 + 5*x - 15*x^2)/x^(5/2)) + 15*ArcTan[Sqrt[x]] + (30*ArcTan[Sqrt[x]
- Sqrt[1 + x]])/x^3)/90
```

3.133.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {25, 5682, 15, 5361, 61, 61, 61, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{x+1}\right)}{x^4}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\arctan\left(\sqrt{x}-\sqrt{x+1}\right)}{x^4}dx$$

$$\Big\downarrow \text{5682}$$

$$\frac{1}{4}\pi\int \frac{1}{x^4}dx-\frac{1}{2}\int \frac{\arctan\left(\sqrt{x}\right)}{x^4}dx$$

$$\Big\downarrow \text{15}$$

$$-\frac{1}{2}\int \frac{\arctan\left(\sqrt{x}\right)}{x^4}dx-\frac{\pi}{12x^3}$$

$$\Big\downarrow \text{5361}$$

$$\frac{1}{2}\left(\frac{\arctan\left(\sqrt{x}\right)}{3x^3}-\frac{1}{6}\int \frac{1}{x^{7/2}(x+1)}dx\right)-\frac{\pi}{12x^3}$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}\left(\frac{1}{6}\left(\int \frac{1}{x^{5/2}(x+1)}dx+\frac{2}{5x^{5/2}}\right)+\frac{\arctan\left(\sqrt{x}\right)}{3x^3}\right)-\frac{\pi}{12x^3}$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}\left(\frac{1}{6}\left(-\int \frac{1}{x^{3/2}(x+1)}dx-\frac{2}{3x^{3/2}}+\frac{2}{5x^{5/2}}\right)+\frac{\arctan\left(\sqrt{x}\right)}{3x^3}\right)-\frac{\pi}{12x^3}$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}\left(\frac{1}{6}\left(\int \frac{1}{\sqrt{x}(x+1)}dx-\frac{2}{3x^{3/2}}+\frac{2}{5x^{5/2}}+\frac{2}{\sqrt{x}}\right)+\frac{\arctan\left(\sqrt{x}\right)}{3x^3}\right)-\frac{\pi}{12x^3}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{1}{6}\left(2\int\frac{1}{x+1}d\sqrt{x}-\frac{2}{3x^{3/2}}+\frac{2}{5x^{5/2}}+\frac{2}{\sqrt{x}}\right)+\frac{\arctan\left(\sqrt{x}\right)}{3x^3}\right)-\frac{\pi}{12x^3}$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(\frac{1}{6}\left(2\arctan\left(\sqrt{x}\right)-\frac{2}{3x^{3/2}}+\frac{2}{5x^{5/2}}+\frac{2}{\sqrt{x}}\right)+\frac{\arctan\left(\sqrt{x}\right)}{3x^3}\right)-\frac{\pi}{12x^3}$$

input
```
Int[-(ArcTan[Sqrt[x] - Sqrt[1 + x]]/x^4),x]
```

output
```
-1/12*Pi/x^3 + (ArcTan[Sqrt[x]]/(3*x^3) + (2/(5*x^(5/2)) - 2/(3*x^(3/2)) +
 2/Sqrt[x] + 2*ArcTan[Sqrt[x]])/6)/2
```

3.133.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

3.133. $\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x^4}\,dx$

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5682
```
Int[ArcTan[(v_) + (s_.)*Sqrt[w_]]*(u_.), x_Symbol] :> Simp[Pi*(s/4)   Int[u
, x], x] + Simp[1/2   Int[u*ArcTan[v], x], x] /; EqQ[s^2, 1] && EqQ[w, v^2
+ 1]
```

3.133.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.68

method	result	size
default	$\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{3x^3}+\frac{1}{30x^{\frac{5}{2}}}-\frac{1}{18x^{\frac{3}{2}}}+\frac{1}{6\sqrt{x}}+\frac{\arctan\left(\sqrt{x}\right)}{6}$	40
parts	$\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{3x^3}+\frac{1}{30x^{\frac{5}{2}}}-\frac{1}{18x^{\frac{3}{2}}}+\frac{1}{6\sqrt{x}}+\frac{\arctan\left(\sqrt{x}\right)}{6}$	40

input
```
int(-arctan(x^(1/2)-(1+x)^(1/2))/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/3*arctan(x^(1/2)-(1+x)^(1/2))/x^3+1/30/x^(5/2)-1/18/x^(3/2)+1/6/x^(1/2)+
1/6*arctan(x^(1/2))
```

3.133.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.68

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x^4}\,dx=-\frac{30\,(x^3+1)\arctan\left(\sqrt{x+1}-\sqrt{x}\right)-(15\,x^2-5\,x+3)\sqrt{x}}{90\,x^3}$$

input
```
integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x^4,x, algorithm="fricas")
```

output
```
-1/90*(30*(x^3 + 1)*arctan(sqrt(x + 1) - sqrt(x)) - (15*x^2 - 5*x + 3)*sqr
t(x))/x^3
```

3.133.6 Sympy [F(-1)]

Timed out.

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x^4}\,dx = \text{Timed out}$$

input
```
integrate(-atan(x**(1/2)-(1+x)**(1/2))/x**4,x)
```

output
```
Timed out
```

3.133.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.66

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x^4}\,dx = \frac{1}{6\sqrt{x}} - \frac{1}{18\,x^{\frac{3}{2}}} - \frac{\arctan\left(\sqrt{x+1}-\sqrt{x}\right)}{3\,x^3} + \frac{1}{30\,x^{\frac{5}{2}}} + \frac{1}{6}\arctan\left(\sqrt{x}\right)$$

input
```
integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x^4,x, algorithm="maxima")
```

output
```
1/6/sqrt(x) - 1/18/x^(3/2) - 1/3*arctan(sqrt(x + 1) - sqrt(x))/x^3 + 1/30/
x^(5/2) + 1/6*arctan(sqrt(x))
```

3.133.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.66

$$\int -\frac{\arctan\left(\sqrt{x}-\sqrt{1+x}\right)}{x^4}\,dx = \frac{15\,x^2-5\,x+3}{90\,x^{\frac{5}{2}}} + \frac{\arctan\left(-\sqrt{x+1}+\sqrt{x}\right)}{3\,x^3} + \frac{1}{6}\arctan\left(\sqrt{x}\right)$$

input
```
integrate(-arctan(x^(1/2)-(1+x)^(1/2))/x^4,x, algorithm="giac")
```

output
```
1/90*(15*x^2 - 5*x + 3)/x^(5/2) + 1/3*arctan(-sqrt(x + 1) + sqrt(x))/x^3 +
 1/6*arctan(sqrt(x))
```

3.133.9 Mupad [B] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.95

$$\int -\frac{\arctan \left(\sqrt{x}-\sqrt{1+x}\right)}{x^4}\,dx = -\frac{\frac{\mathrm{atan}\left(\sqrt{x+1}-\sqrt{x}\right)}{3}-\frac{\sqrt{x}}{30}+\frac{x^{3/2}}{18}-\frac{x^{5/2}}{6}}{x^3}+\frac{\ln\left(\frac{\left(-1+\sqrt{x}\,1\mathrm{i}\right)^2}{x+1}\right)\,1\mathrm{i}}{12}$$

input
```
int(atan((x + 1)^(1/2) - x^(1/2))/x^4,x)
```

output
```
(log((x^(1/2)*1i - 1)^2/(x + 1))*1i)/12 - (atan((x + 1)^(1/2) - x^(1/2))/3
 - x^(1/2)/30 + x^(3/2)/18 - x^(5/2)/6)/x^3
```

3.134 $\int \frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^m}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx$

3.134.1 Optimal result

Integrand size = 39, antiderivative size = 63

$$\int \frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^m}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx=\frac{\sqrt{a-c^2x^2}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^{1+m}}{c(1+m)\sqrt{d-\frac{c^2dx^2}{a}}}$$

output
```
arctan(c*x/(-c^2*x^2+a)^(1/2))^(1+m)*(-c^2*x^2+a)^(1/2)/c/(1+m)/(d-c^2*d*x
^2/a)^(1/2)
```

3.134.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^m}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx=\frac{\sqrt{a-c^2x^2}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^{1+m}}{c(1+m)\sqrt{d-\frac{c^2dx^2}{a}}}$$

input
```
Integrate[ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^m/Sqrt[d - (c^2*d*x^2)/a],x]
```

output
```
(Sqrt[a - c^2*x^2]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^(1 + m))/(c*(1 + m)*Sqr
t[d - (c^2*d*x^2)/a])
```

3.134.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {5680, 5678}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^m}{\sqrt{d-\frac{c^2dx^2}{a}}}dx$$

$$\Big\downarrow \text{5680}$$

$$\frac{\sqrt{a-c^2x^2}\int \frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^m}{\sqrt{a-c^2x^2}}dx}{\sqrt{d-\frac{c^2dx^2}{a}}}$$

$$\Big\downarrow \text{5678}$$

$$\frac{\sqrt{a-c^2x^2}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^{m+1}}{c(m+1)\sqrt{d-\frac{c^2dx^2}{a}}}$$

input
```
Int[ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^m/Sqrt[d - (c^2*d*x^2)/a],x]
```

output
```
(Sqrt[a - c^2*x^2]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^(1 + m))/(c*(1 + m)*Sqr
t[d - (c^2*d*x^2)/a])
```

3.134.3.1 Defintions of rubi rules used

rule 5678
```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(a_.) + (b_.
)*(x_)^2], x_Symbol] :> Simp[ArcTan[c*(x/Sqrt[a + b*x^2])]^(m + 1)/(c*(m +
1)), x] /; FreeQ[{a, b, c, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

rule 5680
```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Simp[Sqrt[a + b*x^2]/Sqrt[d + e*x^2]   Int[ArcTan[c
*(x/Sqrt[a + b*x^2])]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}
, x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

3.134.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{\arctan\left(\frac{cx}{\sqrt{-c^2x^2+a}}\right)^{1+m}\sqrt{-c^2x^2+a}}{c(1+m)\sqrt{\frac{d(-c^2x^2+a)}{a}}}$	59

input
```
int(arctan(c*x/(-c^2*x^2+a)^(1/2))^m/(d-c^2*d*x^2/a)^(1/2),x,method=_RETUR
NVERBOSE)
```

output
```
arctan(c*x/(-c^2*x^2+a)^(1/2))^(1+m)/c/(1+m)*(-c^2*x^2+a)^(1/2)/(d*(-c^2*x
^2+a)/a)^(1/2)
```

3.134.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(57) = 114.

Time = 0.30 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.00

$$\int \frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^m}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx = -\frac{\sqrt{-c^2x^2+a}\,a\left(-\arctan\left(\frac{\sqrt{-c^2x^2+a}\,cx}{c^2x^2-a}\right)\right)^m\sqrt{-\frac{c^2dx^2-ad}{a}}\arctan\left(\frac{\sqrt{-c^2x^2+a}\,cx}{c^2x^2-a}\right)}{acdm+acd-(c^3dm+c^3d)x^2}$$

input
```
integrate(arctan(c*x/(-c^2*x^2+a)^(1/2))^m/(d-c^2*d*x^2/a)^(1/2),x, algori
thm="fricas")
```

output
```
-sqrt(-c^2*x^2 + a)*a*(-arctan(sqrt(-c^2*x^2 + a)*c*x/(c^2*x^2 - a)))^m*sq
rt(-(c^2*d*x^2 - a*d)/a)*arctan(sqrt(-c^2*x^2 + a)*c*x/(c^2*x^2 - a))/(a*c
*d*m + a*c*d - (c^3*d*m + c^3*d)*x^2)
```

3.134.6 Sympy [F]

$$\int \frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^m}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx=\int \frac{\operatorname{atan}^m\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}{\sqrt{-d\left(-1+\frac{c^2x^2}{a}\right)}}\,dx$$

input
```
integrate(atan(c*x/(-c**2*x**2+a)**(1/2))**m/(d-c**2*d*x**2/a)**(1/2),x)
```

output
```
Integral(atan(c*x/sqrt(a - c**2*x**2))**m/sqrt(-d*(-1 + c**2*x**2/a)), x)
```

3.134.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^m}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(c*x/(-c^2*x^2+a)^(1/2))^m/(d-c^2*d*x^2/a)^(1/2),x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.134.8 Giac [F]

$$\int \frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^m}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx=\int \frac{\arctan\left(\frac{cx}{\sqrt{-c^2x^2+a}}\right)^m}{\sqrt{-\frac{c^2dx^2}{a}+d}}\,dx$$

input
```
integrate(arctan(c*x/(-c^2*x^2+a)^(1/2))^m/(d-c^2*d*x^2/a)^(1/2),x, algori
thm="giac")
```

output
```
integrate(arctan(c*x/sqrt(-c^2*x^2 + a))^m/sqrt(-c^2*d*x^2/a + d), x)
```

3.134.9 Mupad [B] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^m}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx = \frac{\operatorname{atan}\left(\frac{c\,x}{\sqrt{a-c^2\,x^2}}\right)^{m+1}\sqrt{a-c^2\,x^2}}{c\,(m+1)\,\sqrt{d-\frac{c^2\,d\,x^2}{a}}}$$

input
```
int(atan((c*x)/(a - c^2*x^2)^(1/2))^m/(d - (c^2*d*x^2)/a)^(1/2),x)
```

output
```
(atan((c*x)/(a - c^2*x^2)^(1/2))^(m + 1)*(a - c^2*x^2)^(1/2))/(c*(m + 1)*(
d - (c^2*d*x^2)/a)^(1/2))
```

3.135 $\int \frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx$

3.135.1 Optimal result

Integrand size = 39, antiderivative size = 59

$$\int \frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx=\frac{\sqrt{a-c^2x^2}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^3}{3c\sqrt{d-\frac{c^2dx^2}{a}}}$$

output
```
1/3*arctan(c*x/(-c^2*x^2+a)^(1/2))^3*(-c^2*x^2+a)^(1/2)/c/(d-c^2*d*x^2/a)^
(1/2)
```

3.135.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

$$\int \frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx=\frac{\sqrt{a-c^2x^2}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^3}{3c\sqrt{d-\frac{c^2dx^2}{a}}}$$

input
```
Integrate[ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^2/Sqrt[d - (c^2*d*x^2)/a],x]
```

output
```
(Sqrt[a - c^2*x^2]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^3)/(3*c*Sqrt[d - (c^2*d
*x^2)/a])
```

3.135.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {5680, 5678}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}{\sqrt{d-\frac{c^2dx^2}{a}}}dx$$

$$\Big\downarrow \text{5680}$$

$$\frac{\sqrt{a-c^2x^2}\int \frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}{\sqrt{a-c^2x^2}}dx}{\sqrt{d-\frac{c^2dx^2}{a}}}$$

$$\Big\downarrow \text{5678}$$

$$\frac{\sqrt{a-c^2x^2}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^3}{3c\sqrt{d-\frac{c^2dx^2}{a}}}$$

input
```
Int[ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^2/Sqrt[d - (c^2*d*x^2)/a],x]
```

output
```
(Sqrt[a - c^2*x^2]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^3)/(3*c*Sqrt[d - (c^2*d
*x^2)/a])
```

3.135.3.1 Defintions of rubi rules used

rule 5678
```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(a_.) + (b_.
)*(x_)^2], x_Symbol] :> Simp[ArcTan[c*(x/Sqrt[a + b*x^2])]^(m + 1)/(c*(m +
1)), x] /; FreeQ[{a, b, c, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

rule 5680
```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Simp[Sqrt[a + b*x^2]/Sqrt[d + e*x^2]   Int[ArcTan[c
*(x/Sqrt[a + b*x^2])]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}
, x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

3.135.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{\sqrt{\frac{d\left(-c^{2} x^{2}+a\right)}{a}}\, \arctan\left(\frac{c x}{\sqrt{-c^{2} x^{2}+a}}\right)^{3} a}{3 \sqrt{-c^{2} x^{2}+a}\, d c}$	57

input
```
int(arctan(c*x/(-c^2*x^2+a)^(1/2))^2/(d-c^2*d*x^2/a)^(1/2),x,method=_RETUR
NVERBOSE)
```

output
```
1/3/(-c^2*x^2+a)^(1/2)*(d*(-c^2*x^2+a)/a)^(1/2)/d/c*arctan(c*x/(-c^2*x^2+a
)^(1/2))^3*a
```

3.135.5 Fricas [F]

$$\int \frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx=\int \frac{\arctan\left(\frac{cx}{\sqrt{-c^2x^2+a}}\right)^2}{\sqrt{-\frac{c^2dx^2}{a}+d}}\,dx$$

input
```
integrate(arctan(c*x/(-c^2*x^2+a)^(1/2))^2/(d-c^2*d*x^2/a)^(1/2),x, algori
thm="fricas")
```

output
```
integral(-a*sqrt(-(c^2*d*x^2 - a*d)/a)*arctan(sqrt(-c^2*x^2 + a)*c*x/(c^2*
x^2 - a))^2/(c^2*d*x^2 - a*d), x)
```

3.135.6 Sympy [F]

$$\int \frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx=\int \frac{\operatorname{atan}^2\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}{\sqrt{-d\left(-1+\frac{c^2x^2}{a}\right)}}\,dx$$

input
```
integrate(atan(c*x/(-c**2*x**2+a)**(1/2))**2/(d-c**2*d*x**2/a)**(1/2),x)
```

output
```
Integral(atan(c*x/sqrt(a - c**2*x**2))**2/sqrt(-d*(-1 + c**2*x**2/a)), x)
```

3.135.7 Maxima [F]

$$\int \frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx = \int \frac{\arctan\left(\frac{cx}{\sqrt{-c^2x^2+a}}\right)^2}{\sqrt{-\frac{c^2dx^2}{a}+d}}\,dx$$

input
```
integrate(arctan(c*x/(-c^2*x^2+a)^(1/2))^2/(d-c^2*d*x^2/a)^(1/2),x, algori
thm="maxima")
```

output
```
integrate(arctan(c*x/sqrt(-c^2*x^2 + a))^2/sqrt(-c^2*d*x^2/a + d), x)
```

3.135.8 Giac [F]

$$\int \frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx = \int \frac{\arctan\left(\frac{cx}{\sqrt{-c^2x^2+a}}\right)^2}{\sqrt{-\frac{c^2dx^2}{a}+d}}\,dx$$

input
```
integrate(arctan(c*x/(-c^2*x^2+a)^(1/2))^2/(d-c^2*d*x^2/a)^(1/2),x, algori
thm="giac")
```

output
```
integrate(arctan(c*x/sqrt(-c^2*x^2 + a))^2/sqrt(-c^2*d*x^2/a + d), x)
```

3.135.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx = \int \frac{\mathrm{atan}\left(\frac{c\,x}{\sqrt{a-c^2\,x^2}}\right)^2}{\sqrt{d-\frac{c^2\,d\,x^2}{a}}}\,dx$$

input
```
int(atan((c*x)/(a - c^2*x^2)^(1/2))^2/(d - (c^2*d*x^2)/a)^(1/2),x)
```

output
```
int(atan((c*x)/(a - c^2*x^2)^(1/2))^2/(d - (c^2*d*x^2)/a)^(1/2), x)
```

3.136 $\int \frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx$

3.136.1 Optimal result

Integrand size = 37, antiderivative size = 59

$$\int \frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx=\frac{\sqrt{a-c^2x^2}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}{2c\sqrt{d-\frac{c^2dx^2}{a}}}$$

output
```
1/2*arctan(c*x/(-c^2*x^2+a)^(1/2))^2*(-c^2*x^2+a)^(1/2)/c/(d-c^2*d*x^2/a)^
(1/2)
```

3.136.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

$$\int \frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx=\frac{\sqrt{a-c^2x^2}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}{2c\sqrt{d-\frac{c^2dx^2}{a}}}$$

input
```
Integrate[ArcTan[(c*x)/Sqrt[a - c^2*x^2]]/Sqrt[d - (c^2*d*x^2)/a],x]
```

output
```
(Sqrt[a - c^2*x^2]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^2)/(2*c*Sqrt[d - (c^2*d
*x^2)/a])
```

3.136.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.054, Rules used = {5680, 5678}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}{\sqrt{d-\frac{c^2dx^2}{a}}}dx$$

$$\Big\downarrow \text{ 5680}$$

$$\frac{\sqrt{a-c^2x^2}\int \frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}{\sqrt{a-c^2x^2}}dx}{\sqrt{d-\frac{c^2dx^2}{a}}}$$

$$\Big\downarrow \text{ 5678}$$

$$\frac{\sqrt{a-c^2x^2}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}{2c\sqrt{d-\frac{c^2dx^2}{a}}}$$

input
```
Int[ArcTan[(c*x)/Sqrt[a - c^2*x^2]]/Sqrt[d - (c^2*d*x^2)/a],x]
```

output
```
(Sqrt[a - c^2*x^2]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^2)/(2*c*Sqrt[d - (c^2*d
*x^2)/a])
```

3.136.3.1 Defintions of rubi rules used

rule 5678
```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(a_.) + (b_.
)*(x_)^2], x_Symbol] :> Simp[ArcTan[c*(x/Sqrt[a + b*x^2])]^(m + 1)/(c*(m +
1)), x] /; FreeQ[{a, b, c, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

rule 5680
```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Simp[Sqrt[a + b*x^2]/Sqrt[d + e*x^2]   Int[ArcTan[c
*(x/Sqrt[a + b*x^2])]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}
, x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

3.136.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{\sqrt{\frac{d\left(-c^{2}x^{2}+a\right)}{a}}\,\arctan\left(\frac{cx}{\sqrt{-c^{2}x^{2}+a}}\right)^{2}a}{2\sqrt{-c^{2}x^{2}+a}\,dc}$	57

input
```
int(arctan(c*x/(-c^2*x^2+a)^(1/2))/(d-c^2*d*x^2/a)^(1/2),x,method=_RETURNV
ERBOSE)
```

output
```
1/2/(-c^2*x^2+a)^(1/2)*(d*(-c^2*x^2+a)/a)^(1/2)/d/c*arctan(c*x/(-c^2*x^2+a
)^(1/2))^2*a
```

3.136.5 Fricas [F]

$$\int \frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx = \int \frac{\arctan\left(\frac{cx}{\sqrt{-c^2x^2+a}}\right)}{\sqrt{-\frac{c^2dx^2}{a}+d}}\,dx$$

input
```
integrate(arctan(c*x/(-c^2*x^2+a)^(1/2))/(d-c^2*d*x^2/a)^(1/2),x, algorith
m="fricas")
```

output
```
integral(a*sqrt(-(c^2*d*x^2 - a*d)/a)*arctan(sqrt(-c^2*x^2 + a)*c*x/(c^2*x
^2 - a))/(c^2*d*x^2 - a*d), x)
```

3.136.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx = \text{Timed out}$$

input
```
integrate(atan(c*x/(-c**2*x**2+a)**(1/2))/(d-c**2*d*x**2/a)**(1/2),x)
```

output
```
Timed out
```

3.136.7 Maxima [F]

$$\int \frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx = \int \frac{\arctan\left(\frac{cx}{\sqrt{-c^2x^2+a}}\right)}{\sqrt{-\frac{c^2dx^2}{a}+d}}\,dx$$

input
```
integrate(arctan(c*x/(-c^2*x^2+a)^(1/2))/(d-c^2*d*x^2/a)^(1/2),x, algorith
m="maxima")
```

output
```
integrate(arctan(c*x/sqrt(-c^2*x^2 + a))/sqrt(-c^2*d*x^2/a + d), x)
```

3.136.8 Giac [F]

$$\int \frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx = \int \frac{\arctan\left(\frac{cx}{\sqrt{-c^2x^2+a}}\right)}{\sqrt{-\frac{c^2dx^2}{a}+d}}\,dx$$

input
```
integrate(arctan(c*x/(-c^2*x^2+a)^(1/2))/(d-c^2*d*x^2/a)^(1/2),x, algorith
m="giac")
```

output
```
integrate(arctan(c*x/sqrt(-c^2*x^2 + a))/sqrt(-c^2*d*x^2/a + d), x)
```

3.136.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}{\sqrt{d-\frac{c^2dx^2}{a}}}\,dx = \int \frac{\mathrm{atan}\left(\frac{c\,x}{\sqrt{a-c^2\,x^2}}\right)}{\sqrt{d-\frac{c^2\,d\,x^2}{a}}}\,dx$$

input
```
int(atan((c*x)/(a - c^2*x^2)^(1/2))/(d - (c^2*d*x^2)/a)^(1/2),x)
```

output
```
int(atan((c*x)/(a - c^2*x^2)^(1/2))/(d - (c^2*d*x^2)/a)^(1/2), x)
```

3.137 $\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}\,dx$

3.137.1 Optimal result

Integrand size = 39, antiderivative size = 55

$$\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}\,dx=\frac{\sqrt{a-c^2x^2}\log\left(\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)\right)}{c\sqrt{d-\frac{c^2dx^2}{a}}}$$

output
```
ln(arctan(c*x/(-c^2*x^2+a)^(1/2)))*(-c^2*x^2+a)^(1/2)/c/(d-c^2*d*x^2/a)^(1
/2)
```

3.137.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}\,dx=\frac{\sqrt{a-c^2x^2}\log\left(\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)\right)}{c\sqrt{d-\frac{c^2dx^2}{a}}}$$

input
```
Integrate[1/(Sqrt[d - (c^2*d*x^2)/a]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]),x]
```

output
```
(Sqrt[a - c^2*x^2]*Log[ArcTan[(c*x)/Sqrt[a - c^2*x^2]]])/(c*Sqrt[d - (c^2*
d*x^2)/a])
```

3.137.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {5680, 5676}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)\sqrt{d-\frac{c^2dx^2}{a}}}dx$$

$$\Big\downarrow \text{5680}$$

$$\frac{\sqrt{a-c^2x^2}\int \frac{1}{\sqrt{a-c^2x^2}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}dx}{\sqrt{d-\frac{c^2dx^2}{a}}}$$

$$\Big\downarrow \text{5676}$$

$$\frac{\sqrt{a-c^2x^2}\log\left(\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)\right)}{c\sqrt{d-\frac{c^2dx^2}{a}}}$$

input
```
Int[1/(Sqrt[d - (c^2*d*x^2)/a]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]),x]
```

output
```
(Sqrt[a - c^2*x^2]*Log[ArcTan[(c*x)/Sqrt[a - c^2*x^2]]])/(c*Sqrt[d - (c^2*
d*x^2)/a])
```

3.137.3.1 Defintions of rubi rules used

rule 5676
```
Int[1/(ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*Sqrt[(a_.) + (b_.)*(
x_)^2]), x_Symbol] :> Simp[(1/c)*Log[ArcTan[c*(x/Sqrt[a + b*x^2])]], x] /;
FreeQ[{a, b, c}, x] && EqQ[b + c^2, 0]
```

rule 5680
```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Simp[Sqrt[a + b*x^2]/Sqrt[d + e*x^2]   Int[ArcTan[c
*(x/Sqrt[a + b*x^2])]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}
, x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

3.137.4 Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{\sqrt{\frac{d\left(-c^{2} x^{2}+a\right)}{a}}\, \ln\left(\arctan\left(\frac{c x}{\sqrt{-c^{2} x^{2}+a}}\right)\right) a}{\sqrt{-c^{2} x^{2}+a}\, d c}$	55

input
```
int(1/arctan(c*x/(-c^2*x^2+a)^(1/2))/(d-c^2*d*x^2/a)^(1/2),x,method=_RETUR
NVERBOSE)
```

output
```
1/(-c^2*x^2+a)^(1/2)*(d*(-c^2*x^2+a)/a)^(1/2)/d/c*ln(arctan(c*x/(-c^2*x^2+
a)^(1/2)))*a
```

3.137.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.51

$$\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}\,dx = -\frac{\sqrt{-c^2x^2+a}\,a\sqrt{-\frac{c^2dx^2-ad}{a}}\log\left(2\arctan\left(\frac{\sqrt{-c^2x^2+a}\,cx}{c^2x^2-a}\right)\right)}{c^3dx^2-acd}$$

input
```
integrate(1/arctan(c*x/(-c^2*x^2+a)^(1/2))/(d-c^2*d*x^2/a)^(1/2),x, algori
thm="fricas")
```

output
```
-sqrt(-c^2*x^2 + a)*a*sqrt(-(c^2*d*x^2 - a*d)/a)*log(2*arctan(sqrt(-c^2*x^
2 + a)*c*x/(c^2*x^2 - a)))/(c^3*d*x^2 - a*c*d)
```

3.137.6 Sympy [F]

$$\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}\,dx=\int \frac{1}{\sqrt{-d\left(-1+\frac{c^2x^2}{a}\right)}\operatorname{atan}\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}\,dx$$

input
```
integrate(1/atan(c*x/(-c**2*x**2+a)**(1/2))/(d-c**2*d*x**2/a)**(1/2),x)
```

output
```
Integral(1/(sqrt(-d*(-1 + c**2*x**2/a))*atan(c*x/sqrt(a - c**2*x**2))), x)
```

3.137.7 Maxima [F]

$$\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}\,dx=\int \frac{1}{\sqrt{-\frac{c^2dx^2}{a}+d}\arctan\left(\frac{cx}{\sqrt{-c^2x^2+a}}\right)}\,dx$$

input
```
integrate(1/arctan(c*x/(-c^2*x^2+a)^(1/2))/(d-c^2*d*x^2/a)^(1/2),x, algori
thm="maxima")
```

output
```
integrate(1/(sqrt(-c^2*d*x^2/a + d)*arctan(c*x/sqrt(-c^2*x^2 + a))), x)
```

3.137.8 Giac [F]

$$\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}\,dx=\int \frac{1}{\sqrt{-\frac{c^2dx^2}{a}+d}\arctan\left(\frac{cx}{\sqrt{-c^2x^2+a}}\right)}\,dx$$

input
```
integrate(1/arctan(c*x/(-c^2*x^2+a)^(1/2))/(d-c^2*d*x^2/a)^(1/2),x, algori
thm="giac")
```

output
```
integrate(1/(sqrt(-c^2*d*x^2/a + d)*arctan(c*x/sqrt(-c^2*x^2 + a))), x)
```

3.137.9 Mupad [B] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}\,dx=\frac{\ln\left(\mathrm{atan}\left(\frac{c\,x}{\sqrt{a-c^2\,x^2}}\right)\right)\,\sqrt{a-c^2\,x^2}}{c\,\sqrt{d-\frac{c^2\,d\,x^2}{a}}}$$

input
```
int(1/(atan((c*x)/(a - c^2*x^2)^(1/2))*(d - (c^2*d*x^2)/a)^(1/2)),x)
```

output
```
(log(atan((c*x)/(a - c^2*x^2)^(1/2)))*(a - c^2*x^2)^(1/2))/(c*(d - (c^2*d*
x^2)/a)^(1/2))
```

3.138 $\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}\,dx$

3.138.1 Optimal result

Integrand size = 39, antiderivative size = 57

$$\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}\,dx=-\frac{\sqrt{a-c^2x^2}}{c\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}$$

output
```
-(-c^2*x^2+a)^(1/2)/c/arctan(c*x/(-c^2*x^2+a)^(1/2))/(d-c^2*d*x^2/a)^(1/2)
```

3.138.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}\,dx=-\frac{\sqrt{a-c^2x^2}}{c\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}$$

input
```
Integrate[1/(Sqrt[d - (c^2*d*x^2)/a]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^2),x]
```

output
```
-(Sqrt[a - c^2*x^2]/(c*Sqrt[d - (c^2*d*x^2)/a]*ArcTan[(c*x)/Sqrt[a - c^2*x
^2]]))
```

3.138.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {5680, 5678}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2\sqrt{d-\frac{c^2dx^2}{a}}}dx$$
$$\Big\downarrow \ 5680$$
$$\frac{\sqrt{a-c^2x^2}\int \frac{1}{\sqrt{a-c^2x^2}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}dx}{\sqrt{d-\frac{c^2dx^2}{a}}}$$
$$\Big\downarrow \ 5678$$
$$-\frac{\sqrt{a-c^2x^2}}{c\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)\sqrt{d-\frac{c^2dx^2}{a}}}$$

input
```
Int[1/(Sqrt[d - (c^2*d*x^2)/a]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^2),x]
```

output
```
-(Sqrt[a - c^2*x^2]/(c*Sqrt[d - (c^2*d*x^2)/a]*ArcTan[(c*x)/Sqrt[a - c^2*x
^2]]))
```

3.138.3.1 Defintions of rubi rules used

rule 5678
```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(a_.) + (b_.
)*(x_)^2], x_Symbol] :> Simp[ArcTan[c*(x/Sqrt[a + b*x^2])]^(m + 1)/(c*(m +
1)), x] /; FreeQ[{a, b, c, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

rule 5680
```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Simp[Sqrt[a + b*x^2]/Sqrt[d + e*x^2]   Int[ArcTan[c
*(x/Sqrt[a + b*x^2])]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}
, x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

3.138.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

method	result	size
default	$-\frac{\sqrt{\frac{d\left(-c^{2} x^{2}+a\right)}{a}}\, a}{\sqrt{-c^{2} x^{2}+a}\, d c \arctan\left(\frac{c x}{\sqrt{-c^{2} x^{2}+a}}\right)}$	57

input
```
int(1/arctan(c*x/(-c^2*x^2+a)^(1/2))^2/(d-c^2*d*x^2/a)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/(-c^2*x^2+a)^(1/2)*(d*(-c^2*x^2+a)/a)^(1/2)/d/c*a/arctan(c*x/(-c^2*x^2+
a)^(1/2))
```

3.138.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.44

$$\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}\,dx = -\frac{\sqrt{-c^2x^2+a}a\sqrt{-\frac{c^2dx^2-ad}{a}}}{(c^3dx^2-acd)\arctan\left(\frac{\sqrt{-c^2x^2+a}cx}{c^2x^2-a}\right)}$$

input
```
integrate(1/arctan(c*x/(-c^2*x^2+a)^(1/2))^2/(d-c^2*d*x^2/a)^(1/2),x, algo
rithm="fricas")
```

output
```
-sqrt(-c^2*x^2 + a)*a*sqrt(-(c^2*d*x^2 - a*d)/a)/((c^3*d*x^2 - a*c*d)*arct
an(sqrt(-c^2*x^2 + a)*c*x/(c^2*x^2 - a)))
```

3.138.6 Sympy [F]

$$\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}\,dx = \int \frac{1}{\sqrt{-d\left(-1+\frac{c^2x^2}{a}\right)}\operatorname{atan}^2\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}\,dx$$

input
```
integrate(1/atan(c*x/(-c**2*x**2+a)**(1/2))**2/(d-c**2*d*x**2/a)**(1/2),x)
```

output
```
Integral(1/(sqrt(-d*(-1 + c**2*x**2/a))*atan(c*x/sqrt(a - c**2*x**2))**2),
 x)
```

3.138.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.51

$$\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}\,dx=-\frac{\sqrt{a}}{c\sqrt{d}\arctan\left(cx,\sqrt{-c^2x^2+a}\right)}$$

input
```
integrate(1/arctan(c*x/(-c^2*x^2+a)^(1/2))^2/(d-c^2*d*x^2/a)^(1/2),x, algo
rithm="maxima")
```

output
```
-sqrt(a)/(c*sqrt(d)*arctan2(c*x, sqrt(-c^2*x^2 + a)))
```

3.138.8 Giac [F]

$$\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}\,dx=\int \frac{1}{\sqrt{-\frac{c^2dx^2}{a}+d}\arctan\left(\frac{cx}{\sqrt{-c^2x^2+a}}\right)^2}\,dx$$

input
```
integrate(1/arctan(c*x/(-c^2*x^2+a)^(1/2))^2/(d-c^2*d*x^2/a)^(1/2),x, algo
rithm="giac")
```

output
```
integrate(1/(sqrt(-c^2*d*x^2/a + d)*arctan(c*x/sqrt(-c^2*x^2 + a))^2), x)
```

3.138.9 Mupad [B] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}\,dx=-\frac{\sqrt{a-c^2\,x^2}}{c\,\mathrm{atan}\left(\frac{c\,x}{\sqrt{a-c^2\,x^2}}\right)\,\sqrt{d-\frac{c^2\,d\,x^2}{a}}}$$

input
```
int(1/(atan((c*x)/(a - c^2*x^2)^(1/2))^2*(d - (c^2*d*x^2)/a)^(1/2)),x)
```

output
```
-(a - c^2*x^2)^(1/2)/(c*atan((c*x)/(a - c^2*x^2)^(1/2))*(d - (c^2*d*x^2)/a
)^(1/2))
```

3.138. $\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}\,dx$

3.139 $\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^3}\,dx$

3.139.1 Optimal result

Integrand size = 39, antiderivative size = 59

$$\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^3}\,dx=-\frac{\sqrt{a-c^2x^2}}{2c\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}$$

output
```
-1/2*(-c^2*x^2+a)^(1/2)/c/arctan(c*x/(-c^2*x^2+a)^(1/2))^2/(d-c^2*d*x^2/a)
^(1/2)
```

3.139.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^3}\,dx=-\frac{\sqrt{a-c^2x^2}}{2c\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2}$$

input
```
Integrate[1/(Sqrt[d - (c^2*d*x^2)/a]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^3),x]
```

output
```
-1/2*Sqrt[a - c^2*x^2]/(c*Sqrt[d - (c^2*d*x^2)/a]*ArcTan[(c*x)/Sqrt[a - c^
2*x^2]]^2)
```

3.139.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.051, Rules used = {5680, 5678}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^3\sqrt{d-\frac{c^2dx^2}{a}}}dx$$
$$\Big\downarrow \ 5680$$
$$\frac{\sqrt{a-c^2x^2}\int \frac{1}{\sqrt{a-c^2x^2}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^3}dx}{\sqrt{d-\frac{c^2dx^2}{a}}}$$
$$\Big\downarrow \ 5678$$
$$-\frac{\sqrt{a-c^2x^2}}{2c\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^2\sqrt{d-\frac{c^2dx^2}{a}}}$$

input
```
Int[1/(Sqrt[d - (c^2*d*x^2)/a]*ArcTan[(c*x)/Sqrt[a - c^2*x^2]]^3),x]
```

output
```
-1/2*Sqrt[a - c^2*x^2]/(c*Sqrt[d - (c^2*d*x^2)/a]*ArcTan[(c*x)/Sqrt[a - c^
2*x^2]]^2)
```

3.139.3.1 Defintions of rubi rules used

rule 5678
```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(a_.) + (b_.
)*(x_)^2], x_Symbol] :> Simp[ArcTan[c*(x/Sqrt[a + b*x^2])]^(m + 1)/(c*(m +
1)), x] /; FreeQ[{a, b, c, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

rule 5680
```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Simp[Sqrt[a + b*x^2]/Sqrt[d + e*x^2]   Int[ArcTan[c
*(x/Sqrt[a + b*x^2])]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}
, x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

3.139.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97

method	result	size
default	$-\frac{\sqrt{\frac{d\left(-c^{2}x^{2}+a\right)}{a}}\, a}{2\sqrt{-c^{2}x^{2}+a}\, d c \arctan\left(\frac{c x}{\sqrt{-c^{2}x^{2}+a}}\right)^{2}}$	57

input
```
int(1/arctan(c*x/(-c^2*x^2+a)^(1/2))^3/(d-c^2*d*x^2/a)^(1/2),x,method=_RET
URNVERBOSE)
```

output
```
-1/2/(-c^2*x^2+a)^(1/2)*(d*(-c^2*x^2+a)/a)^(1/2)/d/c*a/arctan(c*x/(-c^2*x^
2+a)^(1/2))^2
```

3.139.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.39

$$\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^3}\,dx = \frac{\sqrt{-c^2x^2+a}\,a\sqrt{-\frac{c^2dx^2-ad}{a}}}{2\left(c^3dx^2-acd\right)\arctan\left(\frac{\sqrt{-c^2x^2+a}\,cx}{c^2x^2-a}\right)^2}$$

input
```
integrate(1/arctan(c*x/(-c^2*x^2+a)^(1/2))^3/(d-c^2*d*x^2/a)^(1/2),x, algo
rithm="fricas")
```

output
```
1/2*sqrt(-c^2*x^2 + a)*a*sqrt(-(c^2*d*x^2 - a*d)/a)/((c^3*d*x^2 - a*c*d)*a
rctan(sqrt(-c^2*x^2 + a)*c*x/(c^2*x^2 - a))^2)
```

3.139.6 Sympy [F]

$$\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^3}\,dx = \int \frac{1}{\sqrt{-d\left(-1+\frac{c^2x^2}{a}\right)}\operatorname{atan}^3\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)}\,dx$$

input
```
integrate(1/atan(c*x/(-c**2*x**2+a)**(1/2))**3/(d-c**2*d*x**2/a)**(1/2),x)
```

output
```
Integral(1/(sqrt(-d*(-1 + c**2*x**2/a))*atan(c*x/sqrt(a - c**2*x**2))**3),
 x)
```

3.139.7 Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.49

$$\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^3}\,dx = -\frac{\sqrt{a}}{2\,c\sqrt{d}\arctan\left(cx,\sqrt{-c^2x^2+a}\right)^2}$$

input
```
integrate(1/arctan(c*x/(-c^2*x^2+a)^(1/2))^3/(d-c^2*d*x^2/a)^(1/2),x, algo
rithm="maxima")
```

output
```
-1/2*sqrt(a)/(c*sqrt(d)*arctan2(c*x, sqrt(-c^2*x^2 + a))^2)
```

3.139.8 Giac [F]

$$\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^3}\,dx = \int \frac{1}{\sqrt{-\frac{c^2dx^2}{a}+d}\arctan\left(\frac{cx}{\sqrt{-c^2x^2+a}}\right)^3}\,dx$$

input
```
integrate(1/arctan(c*x/(-c^2*x^2+a)^(1/2))^3/(d-c^2*d*x^2/a)^(1/2),x, algo
rithm="giac")
```

output
```
integrate(1/(sqrt(-c^2*d*x^2/a + d)*arctan(c*x/sqrt(-c^2*x^2 + a))^3), x)
```

3.139.9 Mupad [B] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^3}\,dx = -\frac{\sqrt{a-c^2\,x^2}}{2\,c\,\mathrm{atan}\left(\frac{c\,x}{\sqrt{a-c^2\,x^2}}\right)^2\sqrt{d-\frac{c^2\,d\,x^2}{a}}}$$

input
```
int(1/(atan((c*x)/(a - c^2*x^2)^(1/2))^3*(d - (c^2*d*x^2)/a)^(1/2)),x)
```

output
```
-(a - c^2*x^2)^(1/2)/(2*c*atan((c*x)/(a - c^2*x^2)^(1/2))^2*(d - (c^2*d*x^
2)/a)^(1/2))
```

3.139. $\int \frac{1}{\sqrt{d-\frac{c^2dx^2}{a}}\arctan\left(\frac{cx}{\sqrt{a-c^2x^2}}\right)^3}\,dx$

3.140 $\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^m}{\sqrt{a+bx^2}}\,dx$

3.140.1 Optimal result

Integrand size = 40, antiderivative size = 72

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^m}{\sqrt{a+bx^2}}\,dx=\frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^{1+m}}{e(1+m)\sqrt{a+bx^2}}$$

output
```
arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^(1+m)*(-a*e^2/b-e^2*x^2)^(1/2)/e/(1+m
)/(b*x^2+a)^(1/2)
```

3.140.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.92

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^m}{\sqrt{a+bx^2}}\,dx=\frac{\sqrt{-\frac{e^2(a+bx^2)}{b}}\arctan\left(\frac{ex}{\sqrt{-\frac{e^2(a+bx^2)}{b}}}\right)^{1+m}}{e(1+m)\sqrt{a+bx^2}}$$

input
```
Integrate[ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]^m/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[-((e^2*(a + b*x^2))/b)]*ArcTan[(e*x)/Sqrt[-((e^2*(a + b*x^2))/b)]]^(
1 + m))/(e*(1 + m)*Sqrt[a + b*x^2])
```

3.140.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {5680, 5678}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{e^2(-x^2)-\frac{ae^2}{b}}}\right)^m}{\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{ 5680}$$

$$\frac{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}\int \frac{\arctan\left(\frac{ex}{\sqrt{-x^2e^2-\frac{ae^2}{b}}}\right)^m}{\sqrt{-x^2e^2-\frac{ae^2}{b}}}dx}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 5678}$$

$$\frac{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}\arctan\left(\frac{ex}{\sqrt{e^2(-x^2)-\frac{ae^2}{b}}}\right)^{m+1}}{e(m+1)\sqrt{a+bx^2}}$$

input
```
Int[ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]^m/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[-((a*e^2)/b) - e^2*x^2]*ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]^(
1 + m))/(e*(1 + m)*Sqrt[a + b*x^2])
```

3.140.3.1 Defintions of rubi rules used

rule 5678
```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(a_.) + (b_.
)*(x_)^2], x_Symbol] :> Simp[ArcTan[c*(x/Sqrt[a + b*x^2])]^(m + 1)/(c*(m +
1)), x] /; FreeQ[{a, b, c, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

rule 5680
```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Simp[Sqrt[a + b*x^2]/Sqrt[d + e*x^2]   Int[ArcTan[c
*(x/Sqrt[a + b*x^2])]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}
, x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

3.140.4 Maple [F]

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{a\,e^2}{b}-e^2x^2}}\right)^m}{\sqrt{x^2b+a}}dx$$

input
```
int(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^m/(b*x^2+a)^(1/2),x)
```

output
```
int(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^m/(b*x^2+a)^(1/2),x)
```

3.140.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.83

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^m}{\sqrt{a+bx^2}}\,dx = -\frac{\sqrt{bx^2+a}\left(-\arctan\left(\frac{bx\sqrt{-\frac{be^2x^2+ae^2}{b}}}{bex^2+ae}\right)\right)^m\sqrt{-\frac{be^2x^2+ae^2}{b}}\arctan\left(\frac{bx\sqrt{-\frac{be^2x^2+ae^2}{b}}}{bex^2+ae}\right)}{aem+(bem+be)x^2+ae}$$

input
```
integrate(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^m/(b*x^2+a)^(1/2),x, algori
thm="fricas")
```

output
```
-sqrt(b*x^2 + a)*(-arctan(b*x*sqrt(-(b*e^2*x^2 + a*e^2)/b)/(b*e*x^2 + a*e)
))^m*sqrt(-(b*e^2*x^2 + a*e^2)/b)*arctan(b*x*sqrt(-(b*e^2*x^2 + a*e^2)/b)/
(b*e*x^2 + a*e))/(a*e*m + (b*e*m + b*e)*x^2 + a*e)
```

3.140.6 Sympy [F]

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^m}{\sqrt{a+bx^2}}\,dx=\int \frac{\operatorname{atan}^m\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}{\sqrt{a+bx^2}}\,dx$$

input
```
integrate(atan(e*x/(-a*e**2/b-e**2*x**2)**(1/2))**m/(b*x**2+a)**(1/2),x)
```

output
```
Integral(atan(e*x/sqrt(-a*e**2/b - e**2*x**2))**m/sqrt(a + b*x**2), x)
```

3.140.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^m}{\sqrt{a+bx^2}}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^m/(b*x^2+a)^(1/2),x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: sign: argument cannot be imagi
nary; found  sqrt((-_SAGE_VAR_b*_SAGE_VAR_x^2)-_SAGE_VAR_a)
```

3.140.8 Giac [F]

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^m}{\sqrt{a+bx^2}}\,dx=\int \frac{\arctan\left(\frac{ex}{\sqrt{-e^2x^2-\frac{ae^2}{b}}}\right)^m}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^m/(b*x^2+a)^(1/2),x, algori
thm="giac")
```

output
```
integrate(arctan(e*x/sqrt(-e^2*x^2 - a*e^2/b))^m/sqrt(b*x^2 + a), x)
```

3.140.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^m}{\sqrt{a+bx^2}}\,dx = \int \frac{\operatorname{atan}\left(\frac{e\,x}{\sqrt{-e^2\,x^2-\frac{a\,e^2}{b}}}\right)^m}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int(atan((e*x)/(- e^2*x^2 - (a*e^2)/b)^(1/2))^m/(a + b*x^2)^(1/2),x)
```

output
```
int(atan((e*x)/(- e^2*x^2 - (a*e^2)/b)^(1/2))^m/(a + b*x^2)^(1/2), x)
```

3.141 $\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}{\sqrt{a+bx^2}}\,dx$

3.141.1 Optimal result

Integrand size = 40, antiderivative size = 68

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}{\sqrt{a+bx^2}}\,dx=\frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^3}{3e\sqrt{a+bx^2}}$$

output
```
1/3*arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^3*(-a*e^2/b-e^2*x^2)^(1/2)/e/(b*x
^2+a)^(1/2)
```

3.141.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}{\sqrt{a+bx^2}}\,dx=\frac{\sqrt{-\frac{e^2(a+bx^2)}{b}}\arctan\left(\frac{ex}{\sqrt{-\frac{e^2\left(a+bx^2\right)}{b}}}\right)^3}{3e\sqrt{a+bx^2}}$$

input
```
Integrate[ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]^2/Sqrt[a + b*x^2],x]
```

3.141. $\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}{\sqrt{a+bx^2}}\,dx$

output
```
(Sqrt[-((e^2*(a + b*x^2))/b)]*ArcTan[(e*x)/Sqrt[-((e^2*(a + b*x^2))/b)]]^3
)/(3*e*Sqrt[a + b*x^2])
```

3.141.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {5680, 5678}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{e^2(-x^2)-\frac{ae^2}{b}}}\right)^2}{\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{5680}$$

$$\frac{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}\int \frac{\arctan\left(\frac{ex}{\sqrt{-x^2e^2-\frac{ae^2}{b}}}\right)^2}{\sqrt{-x^2e^2-\frac{ae^2}{b}}}dx}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{5678}$$

$$\frac{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}\arctan\left(\frac{ex}{\sqrt{e^2(-x^2)-\frac{ae^2}{b}}}\right)^3}{3e\sqrt{a+bx^2}}$$

input
```
Int[ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]^2/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[-((a*e^2)/b) - e^2*x^2]*ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]^3
)/(3*e*Sqrt[a + b*x^2])
```

3.141.3.1 Defintions of rubi rules used

rule 5678
```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(a_.) + (b_.
)*(x_)^2], x_Symbol] :> Simp[ArcTan[c*(x/Sqrt[a + b*x^2])]^(m + 1)/(c*(m +
1)), x] /; FreeQ[{a, b, c, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

rule 5680
```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Simp[Sqrt[a + b*x^2]/Sqrt[d + e*x^2]   Int[ArcTan[c
*(x/Sqrt[a + b*x^2])]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}
, x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

3.141.4 Maple [F]

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{a e^2}{b}-e^2x^2}}\right)^2}{\sqrt{x^2b+a}}dx$$

input
```
int(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^2/(b*x^2+a)^(1/2),x)
```

output
```
int(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^2/(b*x^2+a)^(1/2),x)
```

3.141.5 Fricas [F]

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}{\sqrt{a+bx^2}}\,dx = \int \frac{\arctan\left(\frac{ex}{\sqrt{-e^2x^2-\frac{ae^2}{b}}}\right)^2}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^2/(b*x^2+a)^(1/2),x, algori
thm="fricas")
```

output
```
integral(arctan(b*x*sqrt(-(b*e^2*x^2 + a*e^2)/b)/(b*e*x^2 + a*e))^2/sqrt(b
*x^2 + a), x)
```

3.141.6 Sympy [F]

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}{\sqrt{a+bx^2}}\,dx = \int \frac{\operatorname{atan}^2\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}{\sqrt{a+bx^2}}\,dx$$

input
```
integrate(atan(e*x/(-a*e**2/b-e**2*x**2)**(1/2))**2/(b*x**2+a)**(1/2),x)
```

output
```
Integral(atan(e*x/sqrt(-a*e**2/b - e**2*x**2))**2/sqrt(a + b*x**2), x)
```

3.141.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}{\sqrt{a+bx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^2/(b*x^2+a)^(1/2),x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: sign: argument cannot be imagi
nary; found  sqrt((-_SAGE_VAR_b*_SAGE_VAR_x^2)-_SAGE_VAR_a)
```

3.141.8 Giac [F]

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}{\sqrt{a+bx^2}}\,dx = \int \frac{\arctan\left(\frac{ex}{\sqrt{-e^2x^2-\frac{ae^2}{b}}}\right)^2}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^2/(b*x^2+a)^(1/2),x, algori
thm="giac")
```

output
```
integrate(arctan(e*x/sqrt(-e^2*x^2 - a*e^2/b))^2/sqrt(b*x^2 + a), x)
```

3.141.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}{\sqrt{a+bx^2}}\,dx = \int \frac{\mathrm{atan}\left(\frac{e\,x}{\sqrt{-e^2\,x^2-\frac{a\,e^2}{b}}}\right)^2}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int(atan((e*x)/(- e^2*x^2 - (a*e^2)/b)^(1/2))^2/(a + b*x^2)^(1/2),x)
```

output
```
int(atan((e*x)/(- e^2*x^2 - (a*e^2)/b)^(1/2))^2/(a + b*x^2)^(1/2), x)
```

3.142 $\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}{\sqrt{a+bx^2}}\,dx$

3.142.1 Optimal result

Integrand size = 38, antiderivative size = 68

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}{2e\sqrt{a+bx^2}}$$

output
```
1/2*arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^2*(-a*e^2/b-e^2*x^2)^(1/2)/e/(b*x
^2+a)^(1/2)
```

3.142.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{-\frac{e^2(a+bx^2)}{b}}\arctan\left(\frac{ex}{\sqrt{-\frac{e^2(a+bx^2)}{b}}}\right)^2}{2e\sqrt{a+bx^2}}$$

input
```
Integrate[ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]/Sqrt[a + b*x^2],x]
```

---

output
```
(Sqrt[-((e^2*(a + b*x^2))/b)]*ArcTan[(e*x)/Sqrt[-((e^2*(a + b*x^2))/b)]]^2
)/(2*e*Sqrt[a + b*x^2])
```

3.142.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {5680, 5678}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{e^2(-x^2)-\frac{ae^2}{b}}}\right)}{\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{5680}$$

$$\frac{\sqrt{e^2(-x^2)-\frac{ae^2}{b}}\int \frac{\arctan\left(\frac{ex}{\sqrt{-x^2e^2-\frac{ae^2}{b}}}\right)}{\sqrt{-x^2e^2-\frac{ae^2}{b}}}dx}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{5678}$$

$$\frac{\sqrt{e^2(-x^2)-\frac{ae^2}{b}}\arctan\left(\frac{ex}{\sqrt{e^2(-x^2)-\frac{ae^2}{b}}}\right)^2}{2e\sqrt{a+bx^2}}$$

input
```
Int[ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[-((a*e^2)/b) - e^2*x^2]*ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]^2
)/(2*e*Sqrt[a + b*x^2])
```

3.142.3.1 Defintions of rubi rules used

rule 5678
```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(a_.) + (b_.
)*(x_)^2], x_Symbol] :> Simp[ArcTan[c*(x/Sqrt[a + b*x^2])]^(m + 1)/(c*(m +
1)), x] /; FreeQ[{a, b, c, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

rule 5680
```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Simp[Sqrt[a + b*x^2]/Sqrt[d + e*x^2]   Int[ArcTan[c
*(x/Sqrt[a + b*x^2])]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}
, x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

3.142.4 Maple [F]

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{a\,e^2}{b}-e^2x^2}}\right)}{\sqrt{x^2b+a}}dx$$

input
```
int(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))/(b*x^2+a)^(1/2),x)
```

output
```
int(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))/(b*x^2+a)^(1/2),x)
```

3.142.5 Fricas [F]

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}{\sqrt{a+bx^2}}\,dx = \int \frac{\arctan\left(\frac{ex}{\sqrt{-e^2x^2-\frac{ae^2}{b}}}\right)}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))/(b*x^2+a)^(1/2),x, algorith
m="fricas")
```

output
```
integral(-arctan(b*x*sqrt(-(b*e^2*x^2 + a*e^2)/b)/(b*e*x^2 + a*e))/sqrt(b*
x^2 + a), x)
```

3.142.6 Sympy [F]

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}{\sqrt{a+bx^2}}\,dx = \int \frac{\operatorname{atan}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}{\sqrt{a+bx^2}}\,dx$$

input
```
integrate(atan(e*x/(-a*e**2/b-e**2*x**2)**(1/2))/(b*x**2+a)**(1/2),x)
```

output
```
Integral(atan(e*x/sqrt(-a*e**2/b - e**2*x**2))/sqrt(a + b*x**2), x)
```

3.142.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}{\sqrt{a+bx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))/(b*x^2+a)^(1/2),x, algorith
m="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: sign: argument cannot be imagi
nary; found  sqrt((-_SAGE_VAR_b*_SAGE_VAR_x^2)-_SAGE_VAR_a)
```

3.142.8 Giac [F]

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}{\sqrt{a+bx^2}}\,dx = \int \frac{\arctan\left(\frac{ex}{\sqrt{-e^2x^2-\frac{ae^2}{b}}}\right)}{\sqrt{bx^2+a}}\,dx$$

input
```
integrate(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))/(b*x^2+a)^(1/2),x, algorith
m="giac")
```

output
```
integrate(arctan(e*x/sqrt(-e^2*x^2 - a*e^2/b))/sqrt(b*x^2 + a), x)
```

3.142.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}{\sqrt{a+bx^2}}\,dx = \int \frac{\operatorname{atan}\left(\frac{e\,x}{\sqrt{-e^2\,x^2-\frac{a\,e^2}{b}}}\right)}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int(atan((e*x)/(- e^2*x^2 - (a*e^2)/b)^(1/2))/(a + b*x^2)^(1/2),x)
```

output
```
int(atan((e*x)/(- e^2*x^2 - (a*e^2)/b)^(1/2))/(a + b*x^2)^(1/2), x)
```

3.143 $\int \frac{1}{\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}\,dx$

3.143.1 Optimal result

Integrand size = 40, antiderivative size = 64

$$\int \frac{1}{\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}\,dx = \frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}\log\left(\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)\right)}{e\sqrt{a+bx^2}}$$

output
```
ln(arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2)))*(-a*e^2/b-e^2*x^2)^(1/2)/e/(b*x^2
+a)^(1/2)
```

3.143.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}\,dx = \frac{\sqrt{-\frac{e^2(a+bx^2)}{b}}\log\left(\arctan\left(\frac{ex}{\sqrt{-\frac{e^2(a+bx^2)}{b}}}\right)\right)}{e\sqrt{a+bx^2}}$$

input
```
Integrate[1/(Sqrt[a + b*x^2]*ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]),x
]
```

output
```
(Sqrt[-((e^2*(a + b*x^2))/b)]*Log[ArcTan[(e*x)/Sqrt[-((e^2*(a + b*x^2))/b)
]]])/(e*Sqrt[a + b*x^2])
```

3.143.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {5680, 5676}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{e^2(-x^2)-\frac{ae^2}{b}}}\right)}dx$$

$$\Big\downarrow \text{5680}$$

$$\frac{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}\int \frac{1}{\sqrt{-x^2e^2-\frac{ae^2}{b}}\arctan\left(\frac{ex}{\sqrt{-x^2e^2-\frac{ae^2}{b}}}\right)}dx}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{5676}$$

$$\frac{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}\log\left(\arctan\left(\frac{ex}{\sqrt{e^2(-x^2)-\frac{ae^2}{b}}}\right)\right)}{e\sqrt{a+bx^2}}$$

input
```
Int[1/(Sqrt[a + b*x^2]*ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]),x]
```

output
```
(Sqrt[-((a*e^2)/b) - e^2*x^2]*Log[ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2
]]])/(e*Sqrt[a + b*x^2])
```

3.143.3.1 Defintions of rubi rules used

rule 5676
```
Int[1/(ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]*Sqrt[(a_.) + (b_.)*(
x_)^2]), x_Symbol] :> Simp[(1/c)*Log[ArcTan[c*(x/Sqrt[a + b*x^2])]], x] /;
FreeQ[{a, b, c}, x] && EqQ[b + c^2, 0]
```

rule 5680
```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Simp[Sqrt[a + b*x^2]/Sqrt[d + e*x^2]   Int[ArcTan[c
*(x/Sqrt[a + b*x^2])]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}
, x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

3.143.4 Maple [F]

$$\int \frac{1}{\arctan\left(\frac{ex}{\sqrt{-\frac{a\,e^2}{b}-e^2x^2}}\right)\sqrt{x^2b+a}}dx$$

input
```
int(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))/(b*x^2+a)^(1/2),x)
```

output
```
int(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))/(b*x^2+a)^(1/2),x)
```

3.143.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.30

$$\int \frac{1}{\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}dx = \frac{\sqrt{bx^2+a}\sqrt{-\frac{be^2x^2+ae^2}{b}}\log\left(2\arctan\left(\frac{bx\sqrt{-\frac{be^2x^2+ae^2}{b}}}{bex^2+ae}\right)\right)}{bex^2+ae}$$

input
```
integrate(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))/(b*x^2+a)^(1/2),x, algori
thm="fricas")
```

output
```
sqrt(b*x^2 + a)*sqrt(-(b*e^2*x^2 + a*e^2)/b)*log(2*arctan(b*x*sqrt(-(b*e^2
*x^2 + a*e^2)/b)/(b*e*x^2 + a*e)))/(b*e*x^2 + a*e)
```

3.143.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}\,dx = \int \frac{1}{\sqrt{a+bx^2}\,\mathrm{atan}\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}\,dx$$

input
```
integrate(1/atan(e*x/(-a*e**2/b-e**2*x**2)**(1/2))/(b*x**2+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*x**2)*atan(e*x/sqrt(-a*e**2/b - e**2*x**2))), x)
```

3.143.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))/(b*x^2+a)^(1/2),x, algori
thm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: sign: argument cannot be imagi
nary; found  sqrt((-_SAGE_VAR_b*_SAGE_VAR_x^2)-_SAGE_VAR_a)
```

3.143.8 Giac [F]

$$\int \frac{1}{\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}\,dx = \int \frac{1}{\sqrt{bx^2+a}\arctan\left(\frac{ex}{\sqrt{-e^2x^2-\frac{ae^2}{b}}}\right)}\,dx$$

input
```
integrate(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))/(b*x^2+a)^(1/2),x, algori
thm="giac")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*arctan(e*x/sqrt(-e^2*x^2 - a*e^2/b))), x)
```

3.143.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}\,dx = \int \frac{1}{\mathrm{atan}\left(\frac{e\,x}{\sqrt{-e^2\,x^2-\frac{a\,e^2}{b}}}\right)\sqrt{b\,x^2+a}}\,dx$$

input
```
int(1/(atan((e*x)/(- e^2*x^2 - (a*e^2)/b)^(1/2))*(a + b*x^2)^(1/2)),x)
```

output
```
int(1/(atan((e*x)/(- e^2*x^2 - (a*e^2)/b)^(1/2))*(a + b*x^2)^(1/2)), x)
```

3.144 $\int \frac{1}{\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}\,dx$

3.144.1 Optimal result

Integrand size = 40, antiderivative size = 66

$$\int \frac{1}{\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}\,dx=-\frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}}{e\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}$$

output
```
-(-a*e^2/b-e^2*x^2)^(1/2)/e/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))/(b*x^2+a)
^(1/2)
```

3.144.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}\,dx=\frac{e\sqrt{a+bx^2}}{b\sqrt{-\frac{e^2(a+bx^2)}{b}}\arctan\left(\frac{ex}{\sqrt{-\frac{e^2(a+bx^2)}{b}}}\right)}$$

input
```
Integrate[1/(Sqrt[a + b*x^2]*ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]^2)
,x]
```

output
```
(e*Sqrt[a + b*x^2])/(b*Sqrt[-((e^2*(a + b*x^2))/b)]*ArcTan[(e*x)/Sqrt[-((e
^2*(a + b*x^2))/b)]])
```

3.144.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {5680, 5678}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{e^2(-x^2)-\frac{ae^2}{b}}}\right)^2}dx$$

$$\Big\downarrow \text{ 5680}$$

$$\frac{\sqrt{e^2(-x^2)-\frac{ae^2}{b}}\int \frac{1}{\sqrt{-x^2e^2-\frac{ae^2}{b}}\arctan\left(\frac{ex}{\sqrt{-x^2e^2-\frac{ae^2}{b}}}\right)^2}dx}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 5678}$$

$$-\frac{\sqrt{e^2(-x^2)-\frac{ae^2}{b}}}{e\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{e^2(-x^2)-\frac{ae^2}{b}}}\right)}$$

input
```
Int[1/(Sqrt[a + b*x^2]*ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]^2),x]
```

output
```
-(Sqrt[-((a*e^2)/b) - e^2*x^2]/(e*Sqrt[a + b*x^2]*ArcTan[(e*x)/Sqrt[-((a*e
^2)/b) - e^2*x^2]]))
```

3.144.3.1 Defintions of rubi rules used

rule 5678
```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(a_.) + (b_.
)*(x_)^2], x_Symbol] :> Simp[ArcTan[c*(x/Sqrt[a + b*x^2])]^(m + 1)/(c*(m +
1)), x] /; FreeQ[{a, b, c, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

rule 5680
```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Simp[Sqrt[a + b*x^2]/Sqrt[d + e*x^2]   Int[ArcTan[c
*(x/Sqrt[a + b*x^2])]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}
, x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

3.144.4 Maple [F]

$$\int \frac{1}{\arctan\left(\frac{ex}{\sqrt{-\frac{a\,e^2}{b}-e^2x^2}}\right)^2\sqrt{x^2b+a}}\,dx$$

input
```
int(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^2/(b*x^2+a)^(1/2),x)
```

output
```
int(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^2/(b*x^2+a)^(1/2),x)
```

3.144.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.24

$$\int \frac{1}{\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}\,dx = \frac{\sqrt{bx^2+a}\sqrt{-\frac{be^2x^2+ae^2}{b}}}{(bex^2+ae)\arctan\left(\frac{bx\sqrt{-\frac{be^2x^2+ae^2}{b}}}{bex^2+ae}\right)}$$

input
```
integrate(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^2/(b*x^2+a)^(1/2),x, algo
rithm="fricas")
```

output
```
sqrt(b*x^2 + a)*sqrt(-(b*e^2*x^2 + a*e^2)/b)/((b*e*x^2 + a*e)*arctan(b*x*s
qrt(-(b*e^2*x^2 + a*e^2)/b)/(b*e*x^2 + a*e)))
```

3.144.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}\,dx = \int \frac{1}{\sqrt{a+bx^2}\operatorname{atan}^2\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}\,dx$$

input
```
integrate(1/atan(e*x/(-a*e**2/b-e**2*x**2)**(1/2))**2/(b*x**2+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*x**2)*atan(e*x/sqrt(-a*e**2/b - e**2*x**2))**2), x)
```

3.144.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^2/(b*x^2+a)^(1/2),x, algo
rithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: sign: argument cannot be imagi
nary; found  sqrt((-_SAGE_VAR_b*_SAGE_VAR_x^2)-_SAGE_VAR_a)
```

3.144.8 Giac [F]

$$\int \frac{1}{\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}\,dx = \int \frac{1}{\sqrt{bx^2+a}\arctan\left(\frac{ex}{\sqrt{-e^2x^2-\frac{ae^2}{b}}}\right)^2}\,dx$$

input
```
integrate(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^2/(b*x^2+a)^(1/2),x, algo
rithm="giac")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*arctan(e*x/sqrt(-e^2*x^2 - a*e^2/b))^2), x)
```

3.144.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}\,dx = \int \frac{1}{\mathrm{atan}\left(\frac{e\,x}{\sqrt{-e^2\,x^2-\frac{a\,e^2}{b}}}\right)^2\sqrt{b\,x^2+a}}\,dx$$

input
```
int(1/(atan((e*x)/(- e^2*x^2 - (a*e^2)/b)^(1/2))^2*(a + b*x^2)^(1/2)),x)
```

output
```
int(1/(atan((e*x)/(- e^2*x^2 - (a*e^2)/b)^(1/2))^2*(a + b*x^2)^(1/2)), x)
```

3.145 $\int \frac{1}{\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^3}\,dx$

3.145.1 Optimal result

Integrand size = 40, antiderivative size = 68

$$\int \frac{1}{\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^3}\,dx = -\frac{\sqrt{-\frac{ae^2}{b}-e^2x^2}}{2e\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^2}$$

output
```
-1/2*(-a*e^2/b-e^2*x^2)^(1/2)/e/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^2/(b*
x^2+a)^(1/2)
```

3.145.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^3}\,dx = -\frac{\sqrt{-\frac{e^2(a+bx^2)}{b}}}{2e\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{-\frac{e^2(a+bx^2)}{b}}}\right)^2}$$

input
```
Integrate[1/(Sqrt[a + b*x^2]*ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]^3)
,x]
```

output
```
-1/2*Sqrt[-((e^2*(a + b*x^2))/b)]/(e*Sqrt[a + b*x^2]*ArcTan[(e*x)/Sqrt[-((
e^2*(a + b*x^2))/b)]]^2)
```

3.145.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {5680, 5678}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{e^2(-x^2)-\frac{ae^2}{b}}}\right)^3}dx$$

$$\Big\downarrow \text{ 5680}$$

$$\frac{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}\int \frac{1}{\sqrt{-x^2e^2-\frac{ae^2}{b}}\arctan\left(\frac{ex}{\sqrt{-x^2e^2-\frac{ae^2}{b}}}\right)^3}dx}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 5678}$$

$$-\frac{\sqrt{e^2\left(-x^2\right)-\frac{ae^2}{b}}}{2e\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{e^2(-x^2)-\frac{ae^2}{b}}}\right)^2}$$

input
```
Int[1/(Sqrt[a + b*x^2]*ArcTan[(e*x)/Sqrt[-((a*e^2)/b) - e^2*x^2]]^3),x]
```

output
```
-1/2*Sqrt[-((a*e^2)/b) - e^2*x^2]/(e*Sqrt[a + b*x^2]*ArcTan[(e*x)/Sqrt[-((
a*e^2)/b) - e^2*x^2]]^2)
```

3.145.3.1 Defintions of rubi rules used

rule 5678
```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(a_.) + (b_.
)*(x_)^2], x_Symbol] :> Simp[ArcTan[c*(x/Sqrt[a + b*x^2])]^(m + 1)/(c*(m +
1)), x] /; FreeQ[{a, b, c, m}, x] && EqQ[b + c^2, 0] && NeQ[m, -1]
```

rule 5680
```
Int[ArcTan[((c_.)*(x_))/Sqrt[(a_.) + (b_.)*(x_)^2]]^(m_.)/Sqrt[(d_.) + (e_.
)*(x_)^2], x_Symbol] :> Simp[Sqrt[a + b*x^2]/Sqrt[d + e*x^2]   Int[ArcTan[c
*(x/Sqrt[a + b*x^2])]^m/Sqrt[a + b*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}
, x] && EqQ[b + c^2, 0] && EqQ[b*d - a*e, 0]
```

3.145.4 Maple [F]

$$\int \frac{1}{\arctan\left(\frac{ex}{\sqrt{-\frac{a\,e^2}{b}-e^2x^2}}\right)^3\sqrt{x^2b+a}}dx$$

input
```
int(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^3/(b*x^2+a)^(1/2),x)
```

output
```
int(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^3/(b*x^2+a)^(1/2),x)
```

3.145.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.22

$$\int \frac{1}{\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^3}\,dx = -\frac{\sqrt{bx^2+a}\sqrt{-\frac{be^2x^2+ae^2}{b}}}{2\,(bex^2+ae)\arctan\left(\frac{bx\sqrt{-\frac{be^2x^2+ae^2}{b}}}{bex^2+ae}\right)^2}$$

input
```
integrate(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^3/(b*x^2+a)^(1/2),x, algo
rithm="fricas")
```

output
```
-1/2*sqrt(b*x^2 + a)*sqrt(-(b*e^2*x^2 + a*e^2)/b)/((b*e*x^2 + a*e)*arctan(
b*x*sqrt(-(b*e^2*x^2 + a*e^2)/b)/(b*e*x^2 + a*e))^2)
```

3.145.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^3}\,dx = \int \frac{1}{\sqrt{a+bx^2}\,\mathrm{atan}^3\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)}\,dx$$

input
```
integrate(1/atan(e*x/(-a*e**2/b-e**2*x**2)**(1/2))**3/(b*x**2+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(a + b*x**2)*atan(e*x/sqrt(-a*e**2/b - e**2*x**2))**3), x)
```

3.145.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^3/(b*x^2+a)^(1/2),x, algo
rithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: sign: argument cannot be imagi
nary; found  sqrt((-_SAGE_VAR_b*_SAGE_VAR_x^2)-_SAGE_VAR_a)
```

3.145.8 Giac [F]

$$\int \frac{1}{\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^3}\,dx = \int \frac{1}{\sqrt{bx^2+a}\arctan\left(\frac{ex}{\sqrt{-e^2x^2-\frac{ae^2}{b}}}\right)^3}\,dx$$

input
```
integrate(1/arctan(e*x/(-a*e^2/b-e^2*x^2)^(1/2))^3/(b*x^2+a)^(1/2),x, algo
rithm="giac")
```

output
```
integrate(1/(sqrt(b*x^2 + a)*arctan(e*x/sqrt(-e^2*x^2 - a*e^2/b))^3), x)
```

3.145.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\arctan\left(\frac{ex}{\sqrt{-\frac{ae^2}{b}-e^2x^2}}\right)^3}\,dx = \int \frac{1}{\mathrm{atan}\left(\frac{e\,x}{\sqrt{-e^2\,x^2-\frac{a\,e^2}{b}}}\right)^3\sqrt{b\,x^2+a}}\,dx$$

input
```
int(1/(atan((e*x)/(- e^2*x^2 - (a*e^2)/b)^(1/2))^3*(a + b*x^2)^(1/2)),x)
```

output
```
int(1/(atan((e*x)/(- e^2*x^2 - (a*e^2)/b)^(1/2))^3*(a + b*x^2)^(1/2)), x)
```

3.146 $\int \frac{\arctan(c(a+bx))\log(d(a+bx))}{a+bx}\,dx$

3.146.1 Optimal result

Integrand size = 24, antiderivative size = 101

$$\int \frac{\arctan(c(a+bx))\log(d(a+bx))}{a+bx}\,dx = \frac{i\log(d(a+bx))\operatorname{PolyLog}(2,-ic(a+bx))}{2b} - \frac{i\log(d(a+bx))\operatorname{PolyLog}(2,ic(a+bx))}{2b} - \frac{i\operatorname{PolyLog}(3,-ic(a+bx))}{2b} + \frac{i\operatorname{PolyLog}(3,ic(a+bx))}{2b}$$

output
```
1/2*I*ln(d*(b*x+a))*polylog(2,-I*c*(b*x+a))/b-1/2*I*ln(d*(b*x+a))*polylog(
2,I*c*(b*x+a))/b-1/2*I*polylog(3,-I*c*(b*x+a))/b+1/2*I*polylog(3,I*c*(b*x+
a))/b
```

3.146.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.78

$$\int \frac{\arctan(c(a+bx))\log(d(a+bx))}{a+bx}\,dx = \frac{i(\log(d(a+bx))\operatorname{PolyLog}(2,-ic(a+bx)) - \log(d(a+bx))\operatorname{PolyLog}(2,ic(a+bx)) - \operatorname{PolyLog}(3,-ic(a+}{2b}$$

input
```
Integrate[(ArcTan[c*(a + b*x)]*Log[d*(a + b*x)])/(a + b*x),x]
```

output
```
((I/2)*(Log[d*(a + b*x)]*PolyLog[2, (-I)*c*(a + b*x)] - Log[d*(a + b*x)]*P
olyLog[2, I*c*(a + b*x)] - PolyLog[3, (-I)*c*(a + b*x)] + PolyLog[3, I*c*(
a + b*x)]))/b
```

3.146.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {5732, 2894, 2881, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan (c(a+bx)) \log (d(a+bx))}{a+bx} \, dx$$

$$\Big\downarrow \text{5732}$$

$$\frac{1}{2} i \int \frac{\log (d(a+bx)) \log (1-ic(a+bx))}{a+bx}dx-\frac{1}{2} i \int \frac{\log (d(a+bx)) \log (ic(a+bx)+1)}{a+bx}dx$$

$$\Big\downarrow \text{2894}$$

$$\frac{1}{2} i \int \frac{\log (d(a+bx)) \log (-iac-ibxc+1)}{a+bx}dx-\frac{1}{2} i \int \frac{\log (d(a+bx)) \log (iac+ibxc+1)}{a+bx}dx$$

$$\Big\downarrow \text{2881}$$

$$\frac{i \int \frac{\log (d(a+bx)) \log (1-ic(a+bx))}{a+bx}d(a+bx)}{2 b}-\frac{i \int \frac{\log (d(a+bx)) \log (ic(a+bx)+1)}{a+bx}d(a+bx)}{2 b}$$

$$\Big\downarrow \text{2821}$$

$$\frac{i \left(\int \frac{\text{PolyLog}(2,ic(a+bx))}{a+bx}d(a+bx)-\text{PolyLog}(2,ic(a+bx)) \log (d(a+bx))\right)}{2 b}-\frac{i \left(\int \frac{\text{PolyLog}(2,-ic(a+bx))}{a+bx}d(a+bx)-\text{PolyLog}(2,-ic(a+bx)) \log (d(a+bx))\right)}{2 b}$$

$$\Big\downarrow \text{7143}$$

$$\frac{i (\text{PolyLog}(3,ic(a+bx))-\text{PolyLog}(2,ic(a+bx)) \log (d(a+bx)))}{2 b}-\frac{i (\text{PolyLog}(3,-ic(a+bx))-\text{PolyLog}(2,-ic(a+bx)) \log (d(a+bx)))}{2 b}$$

input
```
Int[(ArcTan[c*(a + b*x)]*Log[d*(a + b*x)])/(a + b*x),x]
```

output
```
((-1/2*I)*(-(Log[d*(a + b*x)]*PolyLog[2, (-I)*c*(a + b*x)]) + PolyLog[3, (
-I)*c*(a + b*x)]))/b + ((I/2)*(-(Log[d*(a + b*x)]*PolyLog[2, I*c*(a + b*x)
]) + PolyLog[3, I*c*(a + b*x)]))/b
```

3.146.3.1 Defintions of rubi rules used

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2881
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Simp[1/e   Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*
((e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

rule 2894
```
Int[((a_.) + Log[(c_.)*(v_)^(n_.)]*(b_.))^(p_.)*(u_.), x_Symbol] :> Int[u*(
a + b*Log[c*ExpandToSum[v, x]^n])^p, x] /; FreeQ[{a, b, c, n, p}, x] && Lin
earQ[v, x] &&  !LinearMatchQ[v, x] &&  !(EqQ[n, 1] && MatchQ[c*v, (e_.)*((f
_) + (g_.)*x) /; FreeQ[{e, f, g}, x]])
```

rule 5732
```
Int[(ArcTan[v_]*Log[w_])/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[I/2   Int[
Log[1 - I*v]*(Log[w]/(a + b*x)), x], x] - Simp[I/2   Int[Log[1 + I*v]*(Log[
w]/(a + b*x)), x], x] /; FreeQ[{a, b}, x] && LinearQ[v, x] && LinearQ[w, x]
 && EqQ[Simplify[D[v/(a + b*x), x]], 0] && EqQ[Simplify[D[w/(a + b*x), x]],
 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

3.146. $\int \frac{\arctan(c(a+bx))\log(d(a+bx))}{a+bx}\,dx$

3.146.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.95 (sec) , antiderivative size = 1087, normalized size of antiderivative = 10.76

method	result	size
risch	Expression too large to display	1087

input
```
int(arctan(c*(b*x+a))*ln(d*(b*x+a))/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/4/b*Pi*csgn(I*d)*csgn(I*d*(b*x+a))^2*dilog(-I*(c*(b*x+a)+I))+1/4/b*Pi*cs
gn(I*(b*x+a))*csgn(I*d*(b*x+a))^2*dilog(-I*(c*(b*x+a)+I))-1/4/b*Pi*csgn(I*
d)*csgn(I*(b*x+a))*csgn(I*d*(b*x+a))*dilog(-I*(c*(b*x+a)+I))-1/4/b*ln(-I*(
-c*(b*x+a)+I))*ln(b*x+a)*Pi*csgn(I*d)*csgn(I*d*(b*x+a))^2-1/4/b*ln(-I*(-c*
(b*x+a)+I))*ln(b*x+a)*Pi*csgn(I*(b*x+a))*csgn(I*d*(b*x+a))^2+1/4/b*ln(-I*(
-c*(b*x+a)+I))*ln(-I*c*(b*x+a))*Pi*csgn(I*d)*csgn(I*d*(b*x+a))^2+1/4/b*ln(
-I*(-c*(b*x+a)+I))*ln(-I*c*(b*x+a))*Pi*csgn(I*(b*x+a))*csgn(I*d*(b*x+a))^2
-1/4/b*dilog(-I*c*(b*x+a))*Pi*csgn(I*d)*csgn(I*(b*x+a))*csgn(I*d*(b*x+a))-
1/4*I*(-I*Pi*ln(b*x+a)*csgn(I*d)*csgn(I*(b*x+a))*csgn(I*d*(b*x+a))+I*Pi*ln
(b*x+a)*csgn(I*d)*csgn(I*d*(b*x+a))^2+I*Pi*ln(b*x+a)*csgn(I*(b*x+a))*csgn(
I*d*(b*x+a))^2-I*Pi*ln(b*x+a)*csgn(I*d*(b*x+a))^3+2*ln(d)*ln(b*x+a)+ln(b*x
+a)^2)/b*ln(1+I*c*(b*x+a))-1/4/b*Pi*csgn(I*d*(b*x+a))^3*dilog(-I*(c*(b*x+a
)+I))+1/2*I*polylog(3,I*c*(b*x+a))/b-1/4/b*dilog(-I*c*(b*x+a))*Pi*csgn(I*d
*(b*x+a))^3+1/4*I/b*ln(b*x+a)^2*ln(1+I*c*(b*x+a))+1/2*I/b*ln(b*x+a)*polylo
g(2,-I*c*(b*x+a))-1/2*I/b*dilog(-I*c*(b*x+a))*ln(d)+1/4/b*ln(-I*(-c*(b*x+a
)+I))*ln(b*x+a)*Pi*csgn(I*d*(b*x+a))^3-1/4/b*ln(-I*(-c*(b*x+a)+I))*ln(-I*c
*(b*x+a))*Pi*csgn(I*d*(b*x+a))^3+1/4/b*dilog(-I*c*(b*x+a))*Pi*csgn(I*d)*cs
gn(I*d*(b*x+a))^2+1/4/b*dilog(-I*c*(b*x+a))*Pi*csgn(I*(b*x+a))*csgn(I*d*(b
*x+a))^2+1/2*I/b*ln(-I*(-c*(b*x+a)+I))*ln(b*x+a)*ln(d)-1/2*I/b*ln(-I*(-c*(
b*x+a)+I))*ln(-I*c*(b*x+a))*ln(d)-1/4*I/b*ln(b*x+a)^2*ln(1-I*c*(b*x+a))...
```

3.146.5 Fricas [F]

$$\int \frac{\arctan(c(a+bx))\log(d(a+bx))}{a+bx}\,dx=\int \frac{\arctan\left((bx+a)c\right)\log\left((bx+a)d\right)}{bx+a}\,dx$$

input
```
integrate(arctan(c*(b*x+a))*log(d*(b*x+a))/(b*x+a),x, algorithm="fricas")
```

output
```
integral(arctan(b*c*x + a*c)*log(b*d*x + a*d)/(b*x + a), x)
```

3.146.6 Sympy [F]

$$\int \frac{\arctan(c(a+bx))\log(d(a+bx))}{a+bx}\,dx = \int \frac{\log(ad+bdx)\,\mathrm{atan}\,(ac+bcx)}{a+bx}\,dx$$

input
```
integrate(atan(c*(b*x+a))*ln(d*(b*x+a))/(b*x+a),x)
```

output
```
Integral(log(a*d + b*d*x)*atan(a*c + b*c*x)/(a + b*x), x)
```

3.146.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(c(a+bx))\log(d(a+bx))}{a+bx}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(c*(b*x+a))*log(d*(b*x+a))/(b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

3.146.8 Giac [F]

$$\int \frac{\arctan(c(a+bx))\log(d(a+bx))}{a+bx}\,dx = \int \frac{\arctan\left((bx+a)c\right)\log\left((bx+a)d\right)}{bx+a}\,dx$$

input
```
integrate(arctan(c*(b*x+a))*log(d*(b*x+a))/(b*x+a),x, algorithm="giac")
```

output
```
integrate(arctan((b*x + a)*c)*log((b*x + a)*d)/(b*x + a), x)
```

3.146.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(c(a+bx))\log(d(a+bx))}{a+bx}\,dx = \int \frac{\mathrm{atan}(c\,(a+b\,x))\,\ln\left(d\,(a+b\,x)\right)}{a+b\,x}\,dx$$

input
```
int((atan(c*(a + b*x))*log(d*(a + b*x)))/(a + b*x),x)
```

output
```
int((atan(c*(a + b*x))*log(d*(a + b*x)))/(a + b*x), x)
```

3.147 $\int e^{c(a+bx)} \arctan(\sinh(ac+bcx))\,dx$

3.147.1 Optimal result

Integrand size = 20, antiderivative size = 48

$$\int e^{c(a+bx)} \arctan(\sinh(ac+bcx))\,dx = \frac{e^{ac+bcx}\arctan(\sinh(c(a+bx)))}{bc} - \frac{\log\left(1+e^{2c(a+bx)}\right)}{bc}$$

output
```
exp(b*c*x+a*c)*arctan(sinh(c*(b*x+a)))/b/c-ln(1+exp(2*c*(b*x+a)))/b/c
```

3.147.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.27

$$\int e^{c(a+bx)} \arctan(\sinh(ac+bcx))\,dx = -\frac{e^{c(a+bx)}\arctan\left(\frac{1}{2}e^{-c(a+bx)}-\frac{1}{2}e^{c(a+bx)}\right)+\log\left(1+e^{2c(a+bx)}\right)}{bc}$$

input
```
Integrate[E^(c*(a + b*x))*ArcTan[Sinh[a*c + b*c*x]],x]
```

output
```
-((E^(c*(a + b*x))*ArcTan[1/(2*E^(c*(a + b*x))) - E^(c*(a + b*x))/2] + Log
[1 + E^(2*c*(a + b*x))])/(b*c))
```

3.147.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {7281, 5730, 2720, 27, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c(a+bx)} \arctan(\sinh(ac+bcx)) \, dx$$

$$\Big\downarrow \text{ 7281}$$

$$\frac{\int e^{ac+bxc} \arctan(\sinh(ac+bxc))d(ac+bxc)}{bc}$$

$$\Big\downarrow \text{ 5730}$$

$$\frac{e^{ac+bcx} \arctan(\sinh(ac+bcx)) - \int e^{ac+bxc}\text{sech}(ac+bxc)d(ac+bxc)}{bc}$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{e^{ac+bcx} \arctan(\sinh(ac+bcx)) - \int \frac{2e^{ac+bxc}}{1+e^{2ac+2bxc}}de^{ac+bxc}}{bc}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{e^{ac+bcx} \arctan(\sinh(ac+bcx)) - 2\int \frac{e^{ac+bxc}}{1+e^{2ac+2bxc}}de^{ac+bxc}}{bc}$$

$$\Big\downarrow \text{ 240}$$

$$\frac{e^{ac+bcx} \arctan(\sinh(ac+bcx)) - \log\left(e^{2ac+2bcx}+1\right)}{bc}$$

input
```
Int[E^(c*(a + b*x))*ArcTan[Sinh[a*c + b*c*x]],x]
```

output
```
(E^(a*c + b*c*x)*ArcTan[Sinh[a*c + b*c*x]] - Log[1 + E^(2*a*c + 2*b*c*x)])
/(b*c)
```

3.147.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 5730
```
Int[((a_.) + ArcTan[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Simp[(a + b*ArcTan[u])   w, x] - Simp[b   Int[SimplifyIntegrand[w*(D[u, x]
/(1 + u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x]
&& InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; Fre
eQ[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTan[u]), x]]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

3.147.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.11 (sec) , antiderivative size = 1299, normalized size of antiderivative = 27.06

method	result	size
risch	Expression too large to display	1299

input
```
int(exp(c*(b*x+a))*arctan(sinh(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

output
```
1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))-I)^2)*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x
+a))-I)^2)^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a)))*csgn(I*exp(
-c*(b*x+a))*(exp(c*(b*x+a))-I)^2)^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*exp(-
c*(b*x+a))*(exp(c*(b*x+a))-I)^2)*csgn(exp(-c*(b*x+a))*(exp(c*(b*x+a))-I)^2
)^2*exp(c*(b*x+a))+1/2/b/c*Pi*csgn(I*(exp(c*(b*x+a))-I))*csgn(I*(exp(c*(b*
x+a))-I)^2)^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))-I))^2*csgn(
I*(exp(c*(b*x+a))-I)^2)*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a))*(
exp(c*(b*x+a))+I)^2)*csgn(exp(-c*(b*x+a))*(exp(c*(b*x+a))+I)^2)^2*exp(c*(b
*x+a))-1/2/b/c*Pi*csgn(I*(exp(c*(b*x+a))+I))*csgn(I*(exp(c*(b*x+a))+I)^2)^
2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))+I)^2)*csgn(I*exp(-c*(b*
x+a))*(exp(c*(b*x+a))+I)^2)^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(exp(c*(b*x
+a))+I))^2*csgn(I*(exp(c*(b*x+a))+I)^2)*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*e
xp(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))+I)^2)^2*exp(c*(b*x+
a))-1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))-I)^2)*csgn(exp(-c*(b
*x+a))*(exp(c*(b*x+a))-I)^2)*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*exp(-c*(b*x+
a))*(exp(c*(b*x+a))+I)^2)*csgn(exp(-c*(b*x+a))*(exp(c*(b*x+a))+I)^2)*exp(c
*(b*x+a))-1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a))*(exp(c*(b*x+a))-I)^2)^3*exp(c*
(b*x+a))+1/4/b/c*Pi*csgn(exp(-c*(b*x+a))*(exp(c*(b*x+a))-I)^2)^3*exp(c*(b*
x+a))-1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))-I)^2)^3*exp(c*(b*x+a))+1/4/b/c*Pi*
csgn(exp(-c*(b*x+a))*(exp(c*(b*x+a))+I)^2)^3*exp(c*(b*x+a))+1/4/b/c*Pi*...
```

3.147.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.56

$$\int e^{c(a+bx)} \arctan(\sinh(ac+bcx))\,dx = \frac{(\cosh(bcx+ac)+\sinh(bcx+ac))\arctan(\sinh(bcx+ac)) - \log\left(\frac{2\cosh(bcx+ac)}{\cosh(bcx+ac)-\sinh(bcx+ac)}\right)}{bc}$$

input
```
integrate(exp(c*(b*x+a))*arctan(sinh(b*c*x+a*c)),x, algorithm="fricas")
```

output
```
((cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*arctan(sinh(b*c*x + a*c)) - log(2
*cosh(b*c*x + a*c)/(cosh(b*c*x + a*c) - sinh(b*c*x + a*c))))/(b*c)
```

3.147.6 Sympy [F]

$$\int e^{c(a+bx)}\arctan(\sinh(ac+bcx))\,dx = e^{ac}\int e^{bcx}\operatorname{atan}\left(\sinh\left(ac+bcx\right)\right)dx$$

input
```
integrate(exp(c*(b*x+a))*atan(sinh(b*c*x+a*c)),x)
```

output
```
exp(a*c)*Integral(exp(b*c*x)*atan(sinh(a*c + b*c*x)), x)
```

3.147.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int e^{c(a+bx)}\arctan(\sinh(ac+bcx))\,dx = \frac{\arctan\left(\sinh\left(bcx+ac\right)\right)e^{\left((bx+a)c\right)}}{bc} - \frac{\log\left(e^{\left(2\,bcx+2\,ac\right)}+1\right)}{bc}$$

input
```
integrate(exp(c*(b*x+a))*arctan(sinh(b*c*x+a*c)),x, algorithm="maxima")
```

output
```
arctan(sinh(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) - log(e^(2*b*c*x + 2*a*c)
+ 1)/(b*c)
```

3.147.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.35

$$\int e^{c(a+bx)}\arctan(\sinh(ac+bcx))\,dx = \frac{\left(\arctan\left(\frac{1}{2}\,e^{\left(bcx+ac\right)}-\frac{1}{2}\,e^{\left(-bcx-ac\right)}\right)e^{\left(bcx\right)}-e^{\left(-ac\right)}\log\left(e^{\left(2\,bcx+2\,ac\right)}+1\right)\right)e^{\left(ac\right)}}{bc}$$

input
```
integrate(exp(c*(b*x+a))*arctan(sinh(b*c*x+a*c)),x, algorithm="giac")
```

output
```
(arctan(1/2*e^(b*c*x + a*c) - 1/2*e^(-b*c*x - a*c))*e^(b*c*x) - e^(-a*c)*l
og(e^(2*b*c*x + 2*a*c) + 1))*e^(a*c)/(b*c)
```

3.147.9 Mupad [B] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.38

$$\int e^{c(a+bx)} \arctan(\sinh(ac+bcx))\,dx$$
$$= \frac{e^{b\,c\,x}\,e^{a\,c}\,\mathrm{atan}\left(\frac{e^{b\,c\,x}\,e^{a\,c}}{2} - \frac{e^{-b\,c\,x}\,e^{-a\,c}}{2}\right)}{b\,c} - \frac{\ln\left(e^{2\,b\,c\,x}\,e^{2\,a\,c} + 1\right)}{b\,c}$$

input
```
int(exp(c*(a + b*x))*atan(sinh(a*c + b*c*x)),x)
```

output
```
(exp(b*c*x)*exp(a*c)*atan((exp(b*c*x)*exp(a*c))/2 - (exp(-b*c*x)*exp(-a*c)
)/2))/(b*c) - log(exp(2*b*c*x)*exp(2*a*c) + 1)/(b*c)
```

3.148 $\int e^{c(a+bx)} \arctan(\cosh(ac+bcx))\,dx$

3.148.1 Optimal result

Integrand size = 20, antiderivative size = 103

$$\int e^{c(a+bx)} \arctan(\cosh(ac+bcx))\,dx = \frac{e^{ac+bcx}\arctan(\cosh(c(a+bx)))}{bc} - \frac{(1-\sqrt{2})\log\left(3-2\sqrt{2}+e^{2c(a+bx)}\right)}{2bc} - \frac{(1+\sqrt{2})\log\left(3+2\sqrt{2}+e^{2c(a+bx)}\right)}{2bc}$$

output
```
exp(b*c*x+a*c)*arctan(cosh(c*(b*x+a)))/b/c-1/2*ln(3+exp(2*c*(b*x+a))-2*2^(
1/2))*(1-2^(1/2))/b/c-1/2*ln(3+exp(2*c*(b*x+a))+2*2^(1/2))*(1+2^(1/2))/b/c
```

3.148.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.42

$$\int e^{c(a+bx)} \arctan(\cosh(ac+bcx))\,dx = \frac{-4c(a+bx)+2e^{c(a+bx)}\arctan\left(\frac{1}{2}e^{-c(a+bx)}\left(1+e^{2c(a+bx)}\right)\right)+\text{RootSum}\left[1+6\#1^2+\#1^4\&,\frac{ac+bcx-\log\left(e^{c(a+\cdots}\right.}{\cdots}\right.}{2bc}$$

input
```
Integrate[E^(c*(a + b*x))*ArcTan[Cosh[a*c + b*c*x]],x]
```

output
```
(-4*c*(a + b*x) + 2*E^(c*(a + b*x))*ArcTan[(1 + E^(2*c*(a + b*x)))/(2*E^(c
*(a + b*x)))] + RootSum[1 + 6*#1^2 + #1^4 & , (a*c + b*c*x - Log[E^(c*(a +
 b*x)) - #1] + 7*a*c*#1^2 + 7*b*c*x*#1^2 - 7*Log[E^(c*(a + b*x)) - #1]*#1^
2)/(1 + 3*#1^2) & ])/(2*b*c)
```

3.148.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {7281, 5730, 2720, 27, 1576, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c(a+bx)} \arctan(\cosh(ac+bcx))\,dx$$

$$\Big\downarrow \; 7281$$

$$\frac{\int e^{ac+bxc} \arctan(\cosh(ac+bxc))d(ac+bxc)}{bc}$$

$$\Big\downarrow \; 5730$$

$$\frac{e^{ac+bcx} \arctan(\cosh(ac+bcx)) - \int \frac{e^{ac+bxc}\sinh(ac+bxc)}{\cosh^2(ac+bxc)+1}d(ac+bxc)}{bc}$$

$$\Big\downarrow \; 2720$$

$$\frac{e^{ac+bcx} \arctan(\cosh(ac+bcx)) - \int -\frac{2e^{ac+bxc}\left(1-e^{2ac+2bxc}\right)}{1+6e^{2ac+2bxc}+e^{4ac+4bxc}}de^{ac+bxc}}{bc}$$

$$\Big\downarrow \; 27$$

$$\frac{2\int \frac{e^{ac+bxc}\left(1-e^{2ac+2bxc}\right)}{1+6e^{2ac+2bxc}+e^{4ac+4bxc}}de^{ac+bxc} + e^{ac+bcx} \arctan(\cosh(ac+bcx))}{bc}$$

$$\Big\downarrow \; 1576$$

$$\frac{\int \frac{-ac-bxc+1}{1+7e^{2ac+2bxc}}de^{2ac+2bxc} + e^{ac+bcx} \arctan(\cosh(ac+bcx))}{bc}$$

$$\Big\downarrow \; 1141$$

$$\frac{\int \left(-\frac{2-\sqrt{2}}{2\left(4-3\sqrt{2}-\sqrt{2}e^{2ac+2bxc}\right)}-\frac{1+\sqrt{2}}{2\left(3+2\sqrt{2}+e^{2ac+2bxc}\right)}\right)de^{2ac+2bxc}+e^{ac+bcx}\arctan(\cosh(ac+bcx))}{bc}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{e^{ac+bcx}\arctan(\cosh(ac+bcx))-\frac{1}{2}(1+\sqrt{2})\log\left(e^{2ac+2bcx}+3+2\sqrt{2}\right)-\frac{1}{2}(1-\sqrt{2})\log\left(-\sqrt{2}e^{2ac+2bcx}+4-3\sqrt{2}\right.}{bc}$$

input
```
Int[E^(c*(a + b*x))*ArcTan[Cosh[a*c + b*c*x]],x]
```

output
```
(E^(a*c + b*c*x)*ArcTan[Cosh[a*c + b*c*x]] - ((1 + Sqrt[2])*Log[3 + 2*Sqrt
[2] + E^(2*a*c + 2*b*c*x)])/2 - ((1 - Sqrt[2])*Log[4 - 3*Sqrt[2] - Sqrt[2]
*E^(2*a*c + 2*b*c*x)])/2)/(b*c)
```

3.148.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 5730
```
Int[((a_.) + ArcTan[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Simp[(a + b*ArcTan[u])   w, x] - Simp[b   Int[SimplifyIntegrand[w*(D[u, x]
/(1 + u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x]
&& InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; Fre
eQ[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTan[u]), x]]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

3.148.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 5.59 (sec) , antiderivative size = 1371, normalized size of antiderivative = 13.31

method	result	size
risch	Expression too large to display	1371

input
```
int(exp(c*(b*x+a))*arctan(cosh(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I/b/c*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))+1-2*I*exp(c*(b*x+a)))+1/4/b/
c*Pi*csgn(-I*exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-I)*csgn(I*exp(-c*(b*x+a))*(
exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I
*exp(-c*(b*x+a)))*csgn(-I*exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-I)*csgn(I*exp(
-c*(b*x+a))*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))*exp(c*(b*x+a))+1/4/b/
c*Pi*csgn(I*exp(-c*(b*x+a))*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))^3*exp
(c*(b*x+a))-1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(exp
(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*ex
p(-c*(b*x+a))*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))*csgn(exp(-c*(b*x+a)
)*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))^2*exp(c*(b*x+a))+1/4/b/c*Pi*csg
n(I*exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a))))^3*exp(c*(b*x
+a))+1/4/b/c*Pi*csgn(I*exp(2*c*(b*x+a))+I+2*exp(c*(b*x+a)))*csgn(I*exp(-c*
(b*x+a))*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a))))^2*exp(c*(b*x+a))+1/4/b/
c*Pi*csgn(I*exp(-c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(-exp(2*c*(b*x+a))-1+2
*I*exp(c*(b*x+a))))^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*exp(-c*(b*x+a)))*cs
gn(I*exp(2*c*(b*x+a))+I+2*exp(c*(b*x+a)))*csgn(I*exp(-c*(b*x+a))*(-exp(2*c
*(b*x+a))-1+2*I*exp(c*(b*x+a))))*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*exp(-c*(
b*x+a))*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a))))*csgn(exp(-c*(b*x+a))*(-e
xp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a))))^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(ex
p(-c*(b*x+a))*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a))))^3*exp(c*(b*x+a))...
```

3.148.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(86) = 172$.

Time = 0.27 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.15

$$\int e^{c(a+bx)}\arctan(\cosh(ac+bcx))\,dx$$

$$= \frac{2\left(\cosh\left(bcx+ac\right)+\sinh\left(bcx+ac\right)\right)\arctan\left(\cosh\left(bcx+ac\right)\right)+\sqrt{2}\log\left(-\frac{3\left(2\sqrt{2}-3\right)\cosh(bcx+ac)^2-4\left(3\sqrt{2}-4\right)\ldots}{\cosh\ldots}\right.}{2\ldots}$$

input
```
integrate(exp(c*(b*x+a))*arctan(cosh(b*c*x+a*c)),x, algorithm="fricas")
```

output
```
1/2*(2*(cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*arctan(cosh(b*c*x + a*c)) +
 sqrt(2)*log(-(3*(2*sqrt(2) - 3)*cosh(b*c*x + a*c)^2 - 4*(3*sqrt(2) - 4)*c
osh(b*c*x + a*c)*sinh(b*c*x + a*c) + 3*(2*sqrt(2) - 3)*sinh(b*c*x + a*c)^2
 + 2*sqrt(2) - 3)/(cosh(b*c*x + a*c)^2 + sinh(b*c*x + a*c)^2 + 3)) - log(2
*(cosh(b*c*x + a*c)^2 + sinh(b*c*x + a*c)^2 + 3)/(cosh(b*c*x + a*c)^2 - 2*
cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + sinh(b*c*x + a*c)^2)))/(b*c)
```

3.148.6 Sympy [F]

$$\int e^{c(a+bx)}\arctan(\cosh(ac+bcx))\,dx = e^{ac}\int e^{bcx}\operatorname{atan}(\cosh(ac+bcx))\,dx$$

input
```
integrate(exp(c*(b*x+a))*atan(cosh(b*c*x+a*c)),x)
```

output
```
exp(a*c)*Integral(exp(b*c*x)*atan(cosh(a*c + b*c*x)), x)
```

3.148.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.27

$$\int e^{c(a+bx)}\arctan(\cosh(ac+bcx))\,dx = \frac{\arctan(\cosh(bcx+ac))\,e^{((bx+a)c)}}{bc} - \frac{\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(-2\,bcx-2\,ac)}-3}{2\sqrt{2}+e^{(-2\,bcx-2\,ac)}+3}\right)}{2\,bc} - \frac{2\,(bcx+ac)}{bc} - \frac{\log\left(6\,e^{(-2\,bcx-2\,ac)}+e^{(-4\,bcx-4\,ac)}+1\right)}{2\,bc}$$

input
```
integrate(exp(c*(b*x+a))*arctan(cosh(b*c*x+a*c)),x, algorithm="maxima")
```

output
```
arctan(cosh(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) - 1/2*sqrt(2)*log(-(2*sqrt
(2) - e^(-2*b*c*x - 2*a*c) - 3)/(2*sqrt(2) + e^(-2*b*c*x - 2*a*c) + 3))/(b
*c) - 2*(b*c*x + a*c)/(b*c) - 1/2*log(6*e^(-2*b*c*x - 2*a*c) + e^(-4*b*c*x
 - 4*a*c) + 1)/(b*c)
```

3.148.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.50

$$\int e^{c(a+bx)}\arctan(\cosh(ac+bcx))\,dx = \frac{\left(\sqrt{2}e^{(-ac)}\log\left(-\frac{2\sqrt{2}e^{(2\,ac)}-e^{(2\,bcx+4\,ac)}-3\,e^{(2\,ac)}}{2\sqrt{2}e^{(2\,ac)}+e^{(2\,bcx+4\,ac)}+3\,e^{(2\,ac)}}\right)+2\,\arctan\left(\frac{1}{2}\,e^{(bcx+ac)}+\frac{1}{2}\,e^{(-bcx-ac)}\right)e^{(bcx)}-e^{(-ac)}\log\left(e^{(4\,bc}\right.\right.}{2\,bc}$$

input
```
integrate(exp(c*(b*x+a))*arctan(cosh(b*c*x+a*c)),x, algorithm="giac")
```

output
```
1/2*(sqrt(2)*e^(-a*c)*log(-(2*sqrt(2)*e^(2*a*c) - e^(2*b*c*x + 4*a*c) - 3*
e^(2*a*c))/(2*sqrt(2)*e^(2*a*c) + e^(2*b*c*x + 4*a*c) + 3*e^(2*a*c))) + 2*
arctan(1/2*e^(b*c*x + a*c) + 1/2*e^(-b*c*x - a*c))*e^(b*c*x) - e^(-a*c)*lo
g(e^(4*b*c*x + 4*a*c) + 6*e^(2*b*c*x + 2*a*c) + 1))*e^(a*c)/(b*c)
```

3.148.9 Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.29

$$\int e^{c(a+bx)}\arctan(\cosh(ac+bcx))\,dx=\frac{\ln\left(-8\,{\mathrm{e}}^{2\,c\,(a+b\,x)}-2\,\sqrt{2}-6\,\sqrt{2}\,{\mathrm{e}}^{2\,c\,(a+b\,x)}\right)\,\left(\sqrt{2}-1\right)}{2\,b\,c}-\frac{\ln\left(2\,\sqrt{2}-8\,{\mathrm{e}}^{2\,c\,(a+b\,x)}+6\,\sqrt{2}\,{\mathrm{e}}^{2\,c\,(a+b\,x)}\right)\,\left(\sqrt{2}+1\right)}{2\,b\,c}+\frac{{\mathrm{e}}^{a\,c+b\,c\,x}\,\mathrm{atan}\left(\frac{{\mathrm{e}}^{b\,c\,x}\,{\mathrm{e}}^{a\,c}}{2}+\frac{{\mathrm{e}}^{-b\,c\,x}\,{\mathrm{e}}^{-a\,c}}{2}\right)}{b\,c}$$

input
```
int(exp(c*(a + b*x))*atan(cosh(a*c + b*c*x)),x)
```

output
```
(log(- 8*exp(2*c*(a + b*x)) - 2*2^(1/2) - 6*2^(1/2)*exp(2*c*(a + b*x)))*(2
^(1/2) - 1))/(2*b*c) - (log(2*2^(1/2) - 8*exp(2*c*(a + b*x)) + 6*2^(1/2)*e
xp(2*c*(a + b*x)))*(2^(1/2) + 1))/(2*b*c) + (exp(a*c + b*c*x)*atan((exp(b*
c*x)*exp(a*c))/2 + (exp(-b*c*x)*exp(-a*c))/2))/(b*c)
```

3.149 $\int e^{c(a+bx)} \arctan(\tanh(ac+bcx))\,dx$

3.149.1 Optimal result

Integrand size = 20, antiderivative size = 180

$$\int e^{c(a+bx)} \arctan(\tanh(ac+bcx))\,dx = \frac{\arctan\left(1-\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}bc} - \frac{\arctan\left(1+\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}bc} + \frac{e^{ac+bcx}\arctan(\tanh(c(a+bx)))}{bc} - \frac{\log\left(1+e^{2c(a+bx)}-\sqrt{2}e^{ac+bcx}\right)}{2\sqrt{2}bc} + \frac{\log\left(1+e^{2c(a+bx)}+\sqrt{2}e^{ac+bcx}\right)}{2\sqrt{2}bc}$$

output
```
exp(b*c*x+a*c)*arctan(tanh(c*(b*x+a)))/b/c-1/2*arctan(-1+exp(b*c*x+a*c)*2^
(1/2))/b/c*2^(1/2)-1/2*arctan(1+exp(b*c*x+a*c)*2^(1/2))/b/c*2^(1/2)-1/4*ln
(1+exp(2*c*(b*x+a))-exp(b*c*x+a*c)*2^(1/2))/b/c*2^(1/2)+1/4*ln(1+exp(2*c*(
b*x+a))+exp(b*c*x+a*c)*2^(1/2))/b/c*2^(1/2)
```

3.149.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.49

$$\int e^{c(a+bx)}\arctan(\tanh(ac+bcx))\,dx$$
$$=\frac{2e^{c(a+bx)}\arctan\left(\frac{-1+e^{2c(a+bx)}}{1+e^{2c(a+bx)}}\right)+\text{RootSum}\left[1+\#1^4\&,\frac{ac+bcx-\log\left(e^{c(a+bx)}-\#1\right)}{\#1}\&\right]}{2bc}$$

input
```
Integrate[E^(c*(a + b*x))*ArcTan[Tanh[a*c + b*c*x]],x]
```

output
```
(2*E^(c*(a + b*x))*ArcTan[(-1 + E^(2*c*(a + b*x)))/(1 + E^(2*c*(a + b*x)))
] + RootSum[1 + #1^4 & , (a*c + b*c*x - Log[E^(c*(a + b*x)) - #1])/#1 & ])
/(2*b*c)
```

3.149.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.97, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {7281, 5730, 27, 2679, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c(a+bx)}\arctan(\tanh(ac+bcx))\,dx$$
$$\Big\downarrow \text{ 7281}$$
$$\frac{\int e^{ac+bxc}\arctan(\tanh(ac+bxc))d(ac+bxc)}{bc}$$
$$\Big\downarrow \text{ 5730}$$
$$\frac{e^{ac+bcx}\arctan(\tanh(ac+bcx))-\int\frac{2e^{3(ac+bxc)}}{1+e^{4(ac+bxc)}}d(ac+bxc)}{bc}$$
$$\Big\downarrow \text{ 27}$$
$$\frac{e^{ac+bcx}\arctan(\tanh(ac+bcx))-2\int\frac{e^{3(ac+bxc)}}{1+e^{4(ac+bxc)}}d(ac+bxc)}{bc}$$

$$\Big\downarrow \text{2679}$$

$$\frac{e^{ac+bcx}\arctan(\tanh(ac+bcx))-2\int \frac{e^{2ac+2bxc}}{1+e^{4ac+4bxc}}de^{ac+bxc}}{bc}$$

$$\Big\downarrow \text{826}$$

$$\frac{e^{ac+bcx}\arctan(\tanh(ac+bcx))-2\left(\frac{1}{2}\int \frac{1+e^{2ac+2bxc}}{1+e^{4ac+4bxc}}de^{ac+bxc}-\frac{1}{2}\int \frac{1-e^{2ac+2bxc}}{1+e^{4ac+4bxc}}de^{ac+bxc}\right)}{bc}$$

$$\Big\downarrow \text{1476}$$

$$\frac{e^{ac+bcx}\arctan(\tanh(ac+bcx))-2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{1-\sqrt{2}e^{ac+bxc}+e^{2ac+2bxc}}de^{ac+bxc}+\frac{1}{2}\int \frac{1}{1+\sqrt{2}e^{ac+bxc}+e^{2ac+2bxc}}de^{ac+bxc}\right)-\frac{1}{2}\ \cdots\right.}{bc}$$

$$\Big\downarrow \text{1082}$$

$$\frac{e^{ac+bcx}\arctan(\tanh(ac+bcx))-2\left(\frac{1}{2}\left(\frac{\int \frac{1}{-1-e^{2ac+2bxc}}d\left(1-\sqrt{2}e^{ac+bxc}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-1-e^{2ac+2bxc}}d\left(1+\sqrt{2}e^{ac+bxc}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-e^{2ac+\cdots}}{1+e^{4ac+\cdots}}\cdots\right.}{bc}$$

$$\Big\downarrow \text{217}$$

$$\frac{e^{ac+bcx}\arctan(\tanh(ac+bcx))-2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}e^{ac+bcx}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-e^{2ac+2bxc}}{1+e^{4ac+4bxc}}de^{ac+bxc}\right)}{bc}$$

$$\Big\downarrow \text{1479}$$

$$\frac{e^{ac+bcx}\arctan(\tanh(ac+bcx))-2\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-2e^{ac+bxc}}{1-\sqrt{2}e^{ac+bxc}+e^{2ac+2bxc}}de^{ac+bxc}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(1+\sqrt{2}e^{ac+bxc}\right)}{1+\sqrt{2}e^{ac+bxc}+e^{2ac+2bxc}}de^{ac+bxc}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\text{arc}\cdots}{\cdots}\right.\right.}{bc}$$

$$\Big\downarrow \text{25}$$

$$\frac{e^{ac+bcx}\arctan(\tanh(ac+bcx))-2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2e^{ac+bxc}}{1-\sqrt{2}e^{ac+bxc}+e^{2ac+2bxc}}de^{ac+bxc}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(1+\sqrt{2}e^{ac+bxc}\right)}{1+\sqrt{2}e^{ac+bxc}+e^{2ac+2bxc}}de^{ac+bxc}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\text{arct}\cdots}{\cdots}\right.\right.}{bc}$$

$$\Big\downarrow \text{27}$$

3.149. $\int e^{c(a+bx)}\arctan(\tanh(ac+bcx))\,dx$

$$\frac{e^{ac+bcx}\arctan(\tanh(ac+bcx))-2\left(\frac{1}{2}\left(-\frac{\int\frac{\sqrt{2}-2e^{ac+bxc}}{1-\sqrt{2}e^{ac+bxc}+e^{2ac+2bxc}}de^{ac+bxc}}{2\sqrt{2}}-\frac{1}{2}\int\frac{1+\sqrt{2}e^{ac+bxc}}{1+\sqrt{2}e^{ac+bxc}+e^{2ac+2bxc}}de^{ac+bxc}\right)+\frac{1}{2}\left(\right.}{bc}$$

$$\Big\downarrow \text{1103}$$

$$\frac{e^{ac+bcx}\arctan(\tanh(ac+bcx))-2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}e^{ac+bcx}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}e^{ac+bcx}+e^{2ac+2bcx}+1\right.}{2\sqrt{2}}\right.\right.}{bc}$$

input
```
Int[E^(c*(a + b*x))*ArcTan[Tanh[a*c + b*c*x]],x]
```

output
```
(E^(a*c + b*c*x)*ArcTan[Tanh[a*c + b*c*x]] - 2*((-(ArcTan[1 - Sqrt[2]*E^(a
*c + b*c*x)]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*E^(a*c + b*c*x)]/Sqrt[2])/2 + (
Log[1 - Sqrt[2]*E^(a*c + b*c*x) + E^(2*a*c + 2*b*c*x)]/(2*Sqrt[2]) - Log[1
 + Sqrt[2]*E^(a*c + b*c*x) + E^(2*a*c + 2*b*c*x)]/(2*Sqrt[2]))/2))/(b*c)
```

3.149.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

rule 5730
```
Int[((a_.) + ArcTan[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Simp[(a + b*ArcTan[u])   w, x] - Simp[b   Int[SimplifyIntegrand[w*(D[u, x]
/(1 + u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x]
&& InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; Fre
eQ[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTan[u]), x]]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

3.149. $\int e^{c(a+bx)} \arctan(\tanh(ac+bcx))\,dx$

3.149.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.38 (sec) , antiderivative size = 1355, normalized size of antiderivative = 7.53

method	result	size
risch	Expression too large to display	1355

input
```
int(exp(c*(b*x+a))*arctan(tanh(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

output
```
1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))-I))*csgn(I*(exp(2*c*(b*x+a))-I)/(1+exp
(2*c*(b*x+a))))^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I/(1+exp(2*c*(b*x+a))))*c
sgn(I*(exp(2*c*(b*x+a))-I)/(1+exp(2*c*(b*x+a))))^2*exp(c*(b*x+a))+1/4/b/c*
Pi*csgn(I*(exp(2*c*(b*x+a))-I)/(1+exp(2*c*(b*x+a))))*csgn((1-I)*(exp(2*c*(
b*x+a))-I)/(1+exp(2*c*(b*x+a))))^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I/(1+exp
(2*c*(b*x+a))))*csgn(I*(exp(2*c*(b*x+a))+I)/(1+exp(2*c*(b*x+a))))^2*exp(c*
(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+I))*csgn(I*(exp(2*c*(b*x+a))+
I)/(1+exp(2*c*(b*x+a))))^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+
a))+I)/(1+exp(2*c*(b*x+a))))*csgn((1+I)*(exp(2*c*(b*x+a))+I)/(1+exp(2*c*(b
*x+a))))^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))-I)/(1+exp(2*
c*(b*x+a))))*csgn((1-I)*(exp(2*c*(b*x+a))-I)/(1+exp(2*c*(b*x+a))))*exp(c*(
b*x+a))+1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+I)/(1+exp(2*c*(b*x+a))))*csgn(
(1+I)*(exp(2*c*(b*x+a))+I)/(1+exp(2*c*(b*x+a))))*exp(c*(b*x+a))+1/4/b/c*Pi
*csgn(I*(exp(2*c*(b*x+a))+I)/(1+exp(2*c*(b*x+a))))^3*exp(c*(b*x+a))+1/4/b/
c*Pi*csgn((1+I)*(exp(2*c*(b*x+a))+I)/(1+exp(2*c*(b*x+a))))^3*exp(c*(b*x+a)
)-1/4/b/c*Pi*csgn((1-I)*(exp(2*c*(b*x+a))-I)/(1+exp(2*c*(b*x+a))))^2*exp(c
*(b*x+a))-1/4/b/c*Pi*csgn((1+I)*(exp(2*c*(b*x+a))+I)/(1+exp(2*c*(b*x+a))))
^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))-I)/(1+exp(2*c*(b*x+a
))))^3*exp(c*(b*x+a))+1/4/b/c*Pi*csgn((1-I)*(exp(2*c*(b*x+a))-I)/(1+exp(2*
c*(b*x+a))))^3*exp(c*(b*x+a))-1/2*I/b/c*exp(c*(b*x+a))*ln(exp(2*c*(b*x+...
```

3.149.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.43

$$\int e^{c(a+bx)} \arctan(\tanh(ac+bcx))\, dx =$$

$$-\frac{bc\left(-\frac{1}{b^4c^4}\right)^{\frac{1}{4}} \log\left(b^3c^3\left(-\frac{1}{b^4c^4}\right)^{\frac{3}{4}} + \cosh(bcx+ac) + \sinh(bcx+ac)\right) - i\,bc\left(-\frac{1}{b^4c^4}\right)^{\frac{1}{4}} \log\left(i\,b^3c^3\left(-\frac{1}{b^4c^4}\right)^{\frac{3}{4}} \cdots\right.}{\cdots}$$

input
```
integrate(exp(c*(b*x+a))*arctan(tanh(b*c*x+a*c)),x, algorithm="fricas")
```

output
```
-1/2*(b*c*(-1/(b^4*c^4))^(1/4)*log(b^3*c^3*(-1/(b^4*c^4))^(3/4) + cosh(b*c
*x + a*c) + sinh(b*c*x + a*c)) - I*b*c*(-1/(b^4*c^4))^(1/4)*log(I*b^3*c^3*
(-1/(b^4*c^4))^(3/4) + cosh(b*c*x + a*c) + sinh(b*c*x + a*c)) + I*b*c*(-1/
(b^4*c^4))^(1/4)*log(-I*b^3*c^3*(-1/(b^4*c^4))^(3/4) + cosh(b*c*x + a*c) +
 sinh(b*c*x + a*c)) - b*c*(-1/(b^4*c^4))^(1/4)*log(-b^3*c^3*(-1/(b^4*c^4))
^(3/4) + cosh(b*c*x + a*c) + sinh(b*c*x + a*c)) - 2*(cosh(b*c*x + a*c) + s
inh(b*c*x + a*c))*arctan(sinh(b*c*x + a*c)/cosh(b*c*x + a*c)))/(b*c)
```

3.149.6 Sympy [F]

$$\int e^{c(a+bx)}\arctan(\tanh(ac+bcx))\,dx = e^{ac}\int e^{bcx}\,\mathrm{atan}\left(\tanh\left(ac+bcx\right)\right)dx$$

input
```
integrate(exp(c*(b*x+a))*atan(tanh(b*c*x+a*c)),x)
```

output
```
exp(a*c)*Integral(exp(b*c*x)*atan(tanh(a*c + b*c*x)), x)
```

3.149.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.93

$$\begin{aligned}\int e^{c(a+bx)}\arctan(\tanh(ac+bcx))\,dx =& \frac{\arctan\left(\tanh\left(bcx+ac\right)\right)e^{((bx+a)c)}}{bc} \\ &- \frac{\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\,e^{(bcx+ac)}\right)\right)}{2\,bc} \\ &- \frac{\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\,e^{(bcx+ac)}\right)\right)}{2\,bc} \\ &+ \frac{\sqrt{2}\log\left(\sqrt{2}e^{(bcx+ac)}+e^{(2\,bcx+2\,ac)}+1\right)}{4\,bc} \\ &- \frac{\sqrt{2}\log\left(-\sqrt{2}e^{(bcx+ac)}+e^{(2\,bcx+2\,ac)}+1\right)}{4\,bc}\end{aligned}$$

input
```
integrate(exp(c*(b*x+a))*arctan(tanh(b*c*x+a*c)),x, algorithm="maxima")
```

output
```
arctan(tanh(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) - 1/2*sqrt(2)*arctan(1/2*s
qrt(2)*(sqrt(2) + 2*e^(b*c*x + a*c)))/(b*c) - 1/2*sqrt(2)*arctan(-1/2*sqrt
(2)*(sqrt(2) - 2*e^(b*c*x + a*c)))/(b*c) + 1/4*sqrt(2)*log(sqrt(2)*e^(b*c*
x + a*c) + e^(2*b*c*x + 2*a*c) + 1)/(b*c) - 1/4*sqrt(2)*log(-sqrt(2)*e^(b*
c*x + a*c) + e^(2*b*c*x + 2*a*c) + 1)/(b*c)
```

3.149.8 Giac [F]

$$\int e^{c(a+bx)}\arctan(\tanh(ac+bcx))\,dx=\int \arctan\left(\tanh\left(bcx+ac\right)\right)e^{((bx+a)c)}\,dx$$

input
```
integrate(exp(c*(b*x+a))*arctan(tanh(b*c*x+a*c)),x, algorithm="giac")
```

output
```
sage0*x
```

3.149.9 Mupad [B] (verification not implemented)

Time = 1.59 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.91

$$\int e^{c(a+bx)}\arctan(\tanh(ac+bcx))\,dx$$
$$=\frac{4\,e^{a\,c+b\,c\,x}\,\mathrm{atan}\left(\frac{e^{2\,b\,c\,x}\,e^{2\,a\,c}-1}{e^{2\,b\,c\,x}\,e^{2\,a\,c}+1}\right)+\sqrt{2}\,\ln\left(\sqrt{2}\,(-4-4\mathrm{i})+e^{b\,c\,x}\,e^{a\,c}\,8\mathrm{i}\right)\,(-1-\mathrm{i})+\sqrt{2}\,\ln\left(\sqrt{2}\,(-4+4\mathrm{i})-e^{b\,c\,x}\cdots\right.}{4\,b\cdots}$$

input
```
int(exp(c*(a + b*x))*atan(tanh(a*c + b*c*x)),x)
```

output
```
(2^(1/2)*log(2^(1/2)*(4 - 4i) - exp(b*c*x)*exp(a*c)*8i)*(1 - 1i) - 2^(1/2)
*log(- 2^(1/2)*(4 - 4i) - exp(b*c*x)*exp(a*c)*8i)*(1 - 1i) - 2^(1/2)*log(e
xp(b*c*x)*exp(a*c)*8i - 2^(1/2)*(4 + 4i))*(1 + 1i) + 2^(1/2)*log(2^(1/2)*(
4 + 4i) + exp(b*c*x)*exp(a*c)*8i)*(1 + 1i) + 4*exp(a*c + b*c*x)*atan((exp(
2*b*c*x)*exp(2*a*c) - 1)/(exp(2*b*c*x)*exp(2*a*c) + 1)))/(4*b*c)
```

3.150 $\int e^{c(a+bx)} \arctan(\coth(ac+bcx))\,dx$

3.150.1 Optimal result

Integrand size = 20, antiderivative size = 180

$$\int e^{c(a+bx)} \arctan(\coth(ac+bcx))\,dx = -\frac{\arctan\left(1-\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}bc}+\frac{\arctan\left(1+\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}bc}+\frac{e^{ac+bcx}\arctan(\coth(c(a+bx)))}{bc}+\frac{\log\left(1+e^{2c(a+bx)}-\sqrt{2}e^{ac+bcx}\right)}{2\sqrt{2}bc}-\frac{\log\left(1+e^{2c(a+bx)}+\sqrt{2}e^{ac+bcx}\right)}{2\sqrt{2}bc}$$

output
```
exp(b*c*x+a*c)*arctan(coth(c*(b*x+a)))/b/c+1/2*arctan(-1+exp(b*c*x+a*c)*2^
(1/2))/b/c*2^(1/2)+1/2*arctan(1+exp(b*c*x+a*c)*2^(1/2))/b/c*2^(1/2)+1/4*ln
(1+exp(2*c*(b*x+a))-exp(b*c*x+a*c)*2^(1/2))/b/c*2^(1/2)-1/4*ln(1+exp(2*c*(
b*x+a))+exp(b*c*x+a*c)*2^(1/2))/b/c*2^(1/2)
```

3.150.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.49

$$\int e^{c(a+bx)} \arctan(\coth(ac+bcx)) \, dx = \frac{2e^{c(a+bx)} \arctan\left(\frac{1+e^{2c(a+bx)}}{-1+e^{2c(a+bx)}}\right) + \text{RootSum}\left[1+\#1^4\&, \frac{-ac-bcx+\log\left(e^{c(a+bx)}-\#1\right)}{\#1}\&\right]}{2bc}$$

input
```
Integrate[E^(c*(a + b*x))*ArcTan[Coth[a*c + b*c*x]],x]
```

output
```
(2*E^(c*(a + b*x))*ArcTan[(1 + E^(2*c*(a + b*x)))/(-1 + E^(2*c*(a + b*x)))
] + RootSum[1 + #1^4 & , (-(a*c) - b*c*x + Log[E^(c*(a + b*x)) - #1])/#1 &
 ])/(2*b*c)
```

3.150.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.97, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {7281, 5730, 27, 2679, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c(a+bx)} \arctan(\coth(ac+bcx)) \, dx$$

$$\Big\downarrow \text{7281}$$

$$\frac{\int e^{ac+bxc} \arctan(\coth(ac+bxc)) d(ac+bxc)}{bc}$$

$$\Big\downarrow \text{5730}$$

$$\frac{e^{ac+bcx} \arctan(\coth(ac+bcx)) - \int -\frac{2e^{3(ac+bxc)}}{1+e^{4(ac+bxc)}} d(ac+bxc)}{bc}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{e^{3(ac+bxc)}}{1+e^{4(ac+bxc)}} d(ac+bxc) + e^{ac+bcx} \arctan(\coth(ac+bcx))}{bc}$$

$$\Big\downarrow \ 2679$$

$$\frac{2\int \frac{e^{2ac+2bxc}}{1+e^{4ac+4bxc}}de^{ac+bxc}+e^{ac+bcx}\arctan(\coth(ac+bcx))}{bc}$$

$$\Big\downarrow \ 826$$

$$\frac{2\left(\frac{1}{2}\int \frac{1+e^{2ac+2bxc}}{1+e^{4ac+4bxc}}de^{ac+bxc}-\frac{1}{2}\int \frac{1-e^{2ac+2bxc}}{1+e^{4ac+4bxc}}de^{ac+bxc}\right)+e^{ac+bcx}\arctan(\coth(ac+bcx))}{bc}$$

$$\Big\downarrow \ 1476$$

$$\frac{2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{1-\sqrt{2}e^{ac+bxc}+e^{2ac+2bxc}}de^{ac+bxc}+\frac{1}{2}\int \frac{1}{1+\sqrt{2}e^{ac+bxc}+e^{2ac+2bxc}}de^{ac+bxc}\right)-\frac{1}{2}\int \frac{1-e^{2ac+2bxc}}{1+e^{4ac+4bxc}}de^{ac+bxc}\right)+e^{ac+bcx}\text{ar}}{bc}$$

$$\Big\downarrow \ 1082$$

$$\frac{2\left(\frac{1}{2}\left(\frac{\int \frac{1}{-1-e^{2ac+2bxc}}d\left(1-\sqrt{2}e^{ac+bxc}\right)}{\sqrt{2}}-\frac{\int \frac{1}{-1-e^{2ac+2bxc}}d\left(1+\sqrt{2}e^{ac+bxc}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-e^{2ac+2bxc}}{1+e^{4ac+4bxc}}de^{ac+bxc}\right)+e^{ac+bcx}\arctan(\text{cot}}{bc}$$

$$\Big\downarrow \ 217$$

$$\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}e^{ac+bcx}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-e^{2ac+2bxc}}{1+e^{4ac+4bxc}}de^{ac+bxc}\right)+e^{ac+bcx}\arctan(\coth(ac+bcx))}{bc}$$

$$\Big\downarrow \ 1479$$

$$\frac{2\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-2e^{ac+bxc}}{1-\sqrt{2}e^{ac+bxc}+e^{2ac+2bxc}}de^{ac+bxc}}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(1+\sqrt{2}e^{ac+bxc}\right)}{1+\sqrt{2}e^{ac+bxc}+e^{2ac+2bxc}}de^{ac+bxc}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}e^{ac+bcx}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}e^{ac+}\right.}{\sqrt{2}}\right.}{bc}$$

$$\Big\downarrow \ 25$$

$$\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2e^{ac+bxc}}{1-\sqrt{2}e^{ac+bxc}+e^{2ac+2bxc}}de^{ac+bxc}}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(1+\sqrt{2}e^{ac+bxc}\right)}{1+\sqrt{2}e^{ac+bxc}+e^{2ac+2bxc}}de^{ac+bxc}}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}e^{ac+bcx}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}e^{ac+b}\right.}{\sqrt{2}}\right.}{bc}$$

$$\Big\downarrow \ 27$$

3.150. $\int e^{c(a+bx)}\arctan(\coth(ac+bcx))\,dx$

$$\frac{2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2e^{ac+bxc}}{1-\sqrt{2}e^{ac+bxc}+e^{2ac+2bxc}}de^{ac+bxc}}{2\sqrt{2}}-\frac{1}{2}\int \frac{1+\sqrt{2}e^{ac+bxc}}{1+\sqrt{2}e^{ac+bxc}+e^{2ac+2bxc}}de^{ac+bxc}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}e^{ac+bcx}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}e\right.}{\sqrt{2}}\right.\right.}{bc}$$

$$\Big\downarrow \text{1103}$$

$$\frac{2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}e^{ac+bcx}+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}e^{ac+bcx}\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}e^{ac+bcx}+e^{2ac+2bcx}+1\right)}{2\sqrt{2}}-\frac{\log\left(\sqrt{2}e^{ac+bcx}+e^{2ac+2bcx}+1\right)}{2\sqrt{2}}\right)\right)}{bc}$$

input
```
Int[E^(c*(a + b*x))*ArcTan[Coth[a*c + b*c*x]],x]
```

output
```
(E^(a*c + b*c*x)*ArcTan[Coth[a*c + b*c*x]] + 2*((-(ArcTan[1 - Sqrt[2]*E^(a
*c + b*c*x)]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*E^(a*c + b*c*x)]/Sqrt[2])/2 + (
Log[1 - Sqrt[2]*E^(a*c + b*c*x) + E^(2*a*c + 2*b*c*x)]/(2*Sqrt[2]) - Log[1
 + Sqrt[2]*E^(a*c + b*c*x) + E^(2*a*c + 2*b*c*x)]/(2*Sqrt[2]))/2))/(b*c)
```

3.150.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2679
```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[d*e*(Log[F]/(g*h*Log
[G]))]}, Simp[Denominator[m]/(g*h*Log[G])   Subst[Int[x^(Denominator[m] - 1
)*(a + b*F^(c*e - d*e*(f/g))*x^Numerator[m])^p, x], x, G^(h*((f + g*x)/Deno
minator[m]))], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e
, f, g, h, p}, x]
```

rule 5730
```
Int[((a_.) + ArcTan[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Simp[(a + b*ArcTan[u])   w, x] - Simp[b   Int[SimplifyIntegrand[w*(D[u, x]
/(1 + u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x]
&& InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; Fre
eQ[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTan[u]), x]]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

3.150. $\int e^{c(a+bx)} \arctan(\coth(ac+bcx))\,dx$

3.150.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.23 (sec) , antiderivative size = 1355, normalized size of antiderivative = 7.53

method	result	size
risch	Expression too large to display	1355

input
```
int(exp(c*(b*x+a))*arctan(coth(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

output
```
1/4/b/c*Pi*csgn(I/(exp(2*c*(b*x+a))-1))*csgn(I*(exp(2*c*(b*x+a))+I)/(exp(2
*c*(b*x+a))-1))^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I/(exp(2*c*(b*x+a))-1))*c
sgn(I*(exp(2*c*(b*x+a))-I)/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a))+1/4/b/c*
Pi*csgn(I*(exp(2*c*(b*x+a))+I))*csgn(I*(exp(2*c*(b*x+a))+I)/(exp(2*c*(b*x+
a))-1))^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+I)/(exp(2*c*(
b*x+a))-1))*csgn((1-I)*(exp(2*c*(b*x+a))+I)/(exp(2*c*(b*x+a))-1))^2*exp(c*
(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))-I))*csgn(I*(exp(2*c*(b*x+a))-
I)/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+
a))-I)/(exp(2*c*(b*x+a))-1))*csgn((1+I)*(exp(2*c*(b*x+a))-I)/(exp(2*c*(b*x
+a))-1))^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+I)/(exp(2*c*
(b*x+a))-1))*csgn((1-I)*(exp(2*c*(b*x+a))+I)/(exp(2*c*(b*x+a))-1))*exp(c*(
b*x+a))+1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))-I)/(exp(2*c*(b*x+a))-1))*csgn(
(1+I)*(exp(2*c*(b*x+a))-I)/(exp(2*c*(b*x+a))-1))*exp(c*(b*x+a))-1/2*I/b/c*
exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))+I)-1/4/b/c*ln(exp(c*(b*x+a))+1/2*2^(1/2
)+1/2*I*2^(1/2))*2^(1/2)-1/4/b/c*ln(exp(c*(b*x+a))+1/2*2^(1/2)-1/2*I*2^(1/
2))*2^(1/2)+1/4/b/c*ln(exp(c*(b*x+a))-1/2*I*2^(1/2)-1/2*2^(1/2))*2^(1/2)+1
/4/b/c*ln(exp(c*(b*x+a))+1/2*I*2^(1/2)-1/2*2^(1/2))*2^(1/2)+1/4/b/c*exp(c*
(b*x+a))*Pi-1/4/b/c*Pi*csgn((1-I)*(exp(2*c*(b*x+a))+I)/(exp(2*c*(b*x+a))-1
))^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn((1+I)*(exp(2*c*(b*x+a))-I)/(exp(2*c*(b
*x+a))-1))^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+I)/(exp...
```

3.150.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.43

$$\int e^{c(a+bx)}\arctan(\coth(ac+bcx))\,dx$$
$$= \frac{bc\left(-\frac{1}{b^4c^4}\right)^{\frac14}\log\left(b^3c^3\left(-\frac{1}{b^4c^4}\right)^{\frac34}+\cosh(bcx+ac)+\sinh(bcx+ac)\right)-i\,bc\left(-\frac{1}{b^4c^4}\right)^{\frac14}\log\left(i\,b^3c^3\left(-\frac{1}{b^4c^4}\right)^{\frac34}+c\right.}{}$$

input
```
integrate(exp(c*(b*x+a))*arctan(coth(b*c*x+a*c)),x, algorithm="fricas")
```

output
```
1/2*(b*c*(-1/(b^4*c^4))^(1/4)*log(b^3*c^3*(-1/(b^4*c^4))^(3/4) + cosh(b*c*
x + a*c) + sinh(b*c*x + a*c)) - I*b*c*(-1/(b^4*c^4))^(1/4)*log(I*b^3*c^3*(
-1/(b^4*c^4))^(3/4) + cosh(b*c*x + a*c) + sinh(b*c*x + a*c)) + I*b*c*(-1/(
b^4*c^4))^(1/4)*log(-I*b^3*c^3*(-1/(b^4*c^4))^(3/4) + cosh(b*c*x + a*c) +
sinh(b*c*x + a*c)) - b*c*(-1/(b^4*c^4))^(1/4)*log(-b^3*c^3*(-1/(b^4*c^4))^
(3/4) + cosh(b*c*x + a*c) + sinh(b*c*x + a*c)) + 2*(cosh(b*c*x + a*c) + si
nh(b*c*x + a*c))*arctan(cosh(b*c*x + a*c)/sinh(b*c*x + a*c)))/(b*c)
```

3.150.6 Sympy [F]

$$\int e^{c(a+bx)}\arctan(\coth(ac+bcx))\,dx = e^{ac}\int e^{bcx}\operatorname{atan}\left(\coth\left(ac+bcx\right)\right)dx$$

input
```
integrate(exp(c*(b*x+a))*atan(coth(b*c*x+a*c)),x)
```

output
```
exp(a*c)*Integral(exp(b*c*x)*atan(coth(a*c + b*c*x)), x)
```

3.150.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.93

$$\begin{aligned}\int e^{c(a+bx)}\arctan(\coth(ac+bcx))\,dx = {} & \frac{\arctan\left(\coth\left(bcx+ac\right)\right)e^{\left((bx+a)c\right)}}{bc} \\ & + \frac{\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\,e^{(bcx+ac)}\right)\right)}{2\,bc} \\ & + \frac{\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\,e^{(bcx+ac)}\right)\right)}{2\,bc} \\ & - \frac{\sqrt{2}\log\left(\sqrt{2}e^{(bcx+ac)}+e^{(2\,bcx+2\,ac)}+1\right)}{4\,bc} \\ & + \frac{\sqrt{2}\log\left(-\sqrt{2}e^{(bcx+ac)}+e^{(2\,bcx+2\,ac)}+1\right)}{4\,bc}\end{aligned}$$

input
```
integrate(exp(c*(b*x+a))*arctan(coth(b*c*x+a*c)),x, algorithm="maxima")
```

output
```
arctan(coth(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) + 1/2*sqrt(2)*arctan(1/2*s
qrt(2)*(sqrt(2) + 2*e^(b*c*x + a*c)))/(b*c) + 1/2*sqrt(2)*arctan(-1/2*sqrt
(2)*(sqrt(2) - 2*e^(b*c*x + a*c)))/(b*c) - 1/4*sqrt(2)*log(sqrt(2)*e^(b*c*
x + a*c) + e^(2*b*c*x + 2*a*c) + 1)/(b*c) + 1/4*sqrt(2)*log(-sqrt(2)*e^(b*
c*x + a*c) + e^(2*b*c*x + 2*a*c) + 1)/(b*c)
```

3.150.8 Giac [F]

$$\int e^{c(a+bx)} \arctan(\coth(ac+bcx))\, dx = \int \arctan\left(\coth\left(bcx+ac\right)\right) e^{\left((bx+a)c\right)}\, dx$$

input
```
integrate(exp(c*(b*x+a))*arctan(coth(b*c*x+a*c)),x, algorithm="giac")
```

output
```
sage0*x
```

3.150.9 Mupad [B] (verification not implemented)

Time = 1.80 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.91

$$\int e^{c(a+bx)} \arctan(\coth(ac+bcx))\, dx$$
$$= \frac{4\,\mathrm{e}^{a\,c+b\,c\,x}\,\mathrm{atan}\left(\frac{\mathrm{e}^{2\,b\,c\,x}\,\mathrm{e}^{2\,a\,c}+1}{\mathrm{e}^{2\,b\,c\,x}\,\mathrm{e}^{2\,a\,c}-1}\right)+\sqrt{2}\,\ln\left(\sqrt{2}\,(-4-4\mathrm{i})-\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}\,8\mathrm{i}\right)\,(-1-\mathrm{i})+\sqrt{2}\,\ln\left(\sqrt{2}\,(-4+4\mathrm{i})+\mathrm{e}^{b\,c\,x}\right.}{4\,b}$$

input
```
int(exp(c*(a + b*x))*atan(coth(a*c + b*c*x)),x)
```

output
```
(2^(1/2)*log(2^(1/2)*(4 - 4i) + exp(b*c*x)*exp(a*c)*8i)*(1 - 1i) - 2^(1/2)
*log(exp(b*c*x)*exp(a*c)*8i - 2^(1/2)*(4 - 4i))*(1 - 1i) - 2^(1/2)*log(- 2
^(1/2)*(4 + 4i) - exp(b*c*x)*exp(a*c)*8i)*(1 + 1i) + 2^(1/2)*log(2^(1/2)*(
4 + 4i) - exp(b*c*x)*exp(a*c)*8i)*(1 + 1i) + 4*exp(a*c + b*c*x)*atan((exp(
2*b*c*x)*exp(2*a*c) + 1)/(exp(2*b*c*x)*exp(2*a*c) - 1)))/(4*b*c)
```

3.151 $\int e^{c(a+bx)} \arctan(\mathbf{sech}(ac+bcx))\,dx$

3.151.1 Optimal result

Integrand size = 20, antiderivative size = 103

$$\int e^{c(a+bx)} \arctan(\operatorname{sech}(ac+bcx))\,dx = \frac{e^{ac+bcx}\arctan(\operatorname{sech}(c(a+bx)))}{bc} + \frac{(1-\sqrt{2})\log\left(3-2\sqrt{2}+e^{2c(a+bx)}\right)}{2bc} + \frac{(1+\sqrt{2})\log\left(3+2\sqrt{2}+e^{2c(a+bx)}\right)}{2bc}$$

output
```
exp(b*c*x+a*c)*arctan(sech(c*(b*x+a)))/b/c+1/2*ln(3+exp(2*c*(b*x+a))-2*2^(
1/2))*(1-2^(1/2))/b/c+1/2*ln(3+exp(2*c*(b*x+a))+2*2^(1/2))*(1+2^(1/2))/b/c
```

3.151.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.41

$$\int e^{c(a+bx)} \arctan(\operatorname{sech}(ac+bcx))\,dx = \frac{4c(a+bx)+2e^{c(a+bx)}\arctan\left(\frac{2e^{c(a+bx)}}{1+e^{2c(a+bx)}}\right)+\operatorname{RootSum}\left[1+6\#1^2+\#1^4\&,\frac{-ac-bcx+\log\left(e^{c(a+bx)}-\#1\right)-7ac\#1}{1+3}\right.}{2bc}$$

input
```
Integrate[E^(c*(a + b*x))*ArcTan[Sech[a*c + b*c*x]],x]
```

output
```
(4*c*(a + b*x) + 2*E^(c*(a + b*x))*ArcTan[(2*E^(c*(a + b*x)))/(1 + E^(2*c*
(a + b*x)))] + RootSum[1 + 6*#1^2 + #1^4 & , (-(a*c) - b*c*x + Log[E^(c*(a
 + b*x)) - #1] - 7*a*c*#1^2 - 7*b*c*x*#1^2 + 7*Log[E^(c*(a + b*x)) - #1]*#
1^2)/(1 + 3*#1^2) & ])/(2*b*c)
```

3.151.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {7281, 5730, 25, 2720, 27, 1576, 1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c(a+bx)} \arctan(\text{sech}(ac+bcx)) \, dx$$

$$\Big\downarrow \text{7281}$$

$$\frac{\int e^{ac+bxc} \arctan(\text{sech}(ac+bxc))d(ac+bxc)}{bc}$$

$$\Big\downarrow \text{5730}$$

$$\frac{e^{ac+bcx} \arctan(\text{sech}(ac+bcx)) - \int -\frac{e^{ac+bxc}\text{sech}(ac+bxc)\tanh(ac+bxc)}{\text{sech}^2(ac+bxc)+1}d(ac+bxc)}{bc}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{e^{ac+bxc}\text{sech}(ac+bxc)\tanh(ac+bxc)}{\text{sech}^2(ac+bxc)+1}d(ac+bxc) + e^{ac+bcx} \arctan(\text{sech}(ac+bcx))}{bc}$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int -\frac{2e^{ac+bxc}\left(1-e^{2ac+2bxc}\right)}{1+6e^{2ac+2bxc}+e^{4ac+4bxc}}de^{ac+bxc} + e^{ac+bcx} \arctan(\text{sech}(ac+bcx))}{bc}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^{ac+bcx} \arctan(\text{sech}(ac+bcx)) - 2\int \frac{e^{ac+bxc}\left(1-e^{2ac+2bxc}\right)}{1+6e^{2ac+2bxc}+e^{4ac+4bxc}}de^{ac+bxc}}{bc}$$

$$\Big\downarrow \text{1576}$$

$$\frac{e^{ac+bcx}\arctan(\operatorname{sech}(ac+bcx))-\int\frac{-ac-bxc+1}{1+7e^{2ac+2bxc}}de^{2ac+2bxc}}{bc}$$

$\downarrow$ 1141

$$\frac{e^{ac+bcx}\arctan(\operatorname{sech}(ac+bcx))-\int\left(-\frac{2-\sqrt{2}}{2\left(4-3\sqrt{2}-\sqrt{2}e^{2ac+2bxc}\right)}-\frac{1+\sqrt{2}}{2\left(3+2\sqrt{2}+e^{2ac+2bxc}\right)}\right)de^{2ac+2bxc}}{bc}$$

$\downarrow$ 2009

$$\frac{e^{ac+bcx}\arctan(\operatorname{sech}(ac+bcx))+\frac{1}{2}(1+\sqrt{2})\log\left(e^{2ac+2bcx}+3+2\sqrt{2}\right)+\frac{1}{2}(1-\sqrt{2})\log\left(-\sqrt{2}e^{2ac+2bcx}+4-3\sqrt{2}\right.}{bc}$$

input
```
Int[E^(c*(a + b*x))*ArcTan[Sech[a*c + b*c*x]],x]
```

output
```
(E^(a*c + b*c*x)*ArcTan[Sech[a*c + b*c*x]] + ((1 + Sqrt[2])*Log[3 + 2*Sqrt
[2] + E^(2*a*c + 2*b*c*x)])/2 + ((1 - Sqrt[2])*Log[4 - 3*Sqrt[2] - Sqrt[2]
*E^(2*a*c + 2*b*c*x)])/2)/(b*c)
```

3.151.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 5730
```
Int[((a_.) + ArcTan[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Simp[(a + b*ArcTan[u])   w, x] - Simp[b   Int[SimplifyIntegrand[w*(D[u, x]
/(1 + u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x]
&& InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; Fre
eQ[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTan[u]), x]]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

3.151.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.97 (sec) , antiderivative size = 838, normalized size of antiderivative = 8.14

method	result	size
risch	Expression too large to display	838

input
```
int(exp(c*(b*x+a))*arctan(sech(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I/b/c*exp(c*(b*x+a))*ln(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a)))-1/4/b/
c*Pi*csgn(I*exp(2*c*(b*x+a))+I+2*exp(c*(b*x+a)))*csgn(I/(1+exp(2*c*(b*x+a)
)))*csgn(I*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a)))/(1+exp(2*c*(b*x+a))))*
exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*exp(2*c*(b*x+a))+I+2*exp(c*(b*x+a)))*csgn
(I*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a)))/(1+exp(2*c*(b*x+a))))^2*exp(c*
(b*x+a))-1/4/b/c*Pi*csgn(I/(1+exp(2*c*(b*x+a))))*csgn(I*(-exp(2*c*(b*x+a))
-1+2*I*exp(c*(b*x+a)))/(1+exp(2*c*(b*x+a))))^2*exp(c*(b*x+a))+1/4/b/c*Pi*c
sgn(-I*exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-I)*csgn(I/(1+exp(2*c*(b*x+a))))*c
sgn(I*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a)))/(1+exp(2*c*(b*x+a))))*exp(c*
(b*x+a))+1/4/b/c*Pi*csgn(I/(1+exp(2*c*(b*x+a))))*csgn(I*(exp(2*c*(b*x+a))+
1+2*I*exp(c*(b*x+a)))/(1+exp(2*c*(b*x+a))))^2*exp(c*(b*x+a))-1/4/b/c*Pi*cs
gn(I*(-exp(2*c*(b*x+a))-1+2*I*exp(c*(b*x+a)))/(1+exp(2*c*(b*x+a))))^3*exp(
c*(b*x+a))-1/4/b/c*Pi*csgn(-I*exp(2*c*(b*x+a))+2*exp(c*(b*x+a))-I)*csgn(I*
(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a)))/(1+exp(2*c*(b*x+a))))^2*exp(c*(b*x
+a))-1/4/b/c*Pi*csgn(I*(exp(2*c*(b*x+a))+1+2*I*exp(c*(b*x+a)))/(1+exp(2*c*
(b*x+a))))^3*exp(c*(b*x+a))-1/2/b/c*2^(1/2)*ln(exp(2*c*(b*x+a))+(2^(1/2)-1
)^2)+1/2/b/c*2^(1/2)*ln(exp(2*c*(b*x+a))+(1+2^(1/2))^2)-2*a/b+1/2*I/b/c*ex
p(c*(b*x+a))*ln(exp(2*c*(b*x+a))+1-2*I*exp(c*(b*x+a)))+1/2/b/c*ln(exp(2*c*
(b*x+a))+(2^(1/2)-1)^2)+1/2/b/c*ln(exp(2*c*(b*x+a))+(1+2^(1/2))^2)
```

3.151.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(86) = 172$.

Time = 0.29 (sec) , antiderivative size = 276, normalized size of antiderivative = 2.68

$$\int e^{c(a+bx)}\arctan(\operatorname{sech}(ac+bcx))\,dx$$

$$= \frac{2\left(\cosh\left(bcx+ac\right)+\sinh\left(bcx+ac\right)\right)\arctan\left(\frac{2\left(\cosh(bcx+ac)+\sinh(bcx+ac)\right)}{\cosh(bcx+ac)^2+2\cosh(bcx+ac)\sinh(bcx+ac)+\sinh(bcx+ac)^2+1}\right)+\sqrt{2}\log}{}$$

input
```
integrate(exp(c*(b*x+a))*arctan(sech(b*c*x+a*c)),x, algorithm="fricas")
```

output
```
1/2*(2*(cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*arctan(2*(cosh(b*c*x + a*c)
 + sinh(b*c*x + a*c))/(cosh(b*c*x + a*c)^2 + 2*cosh(b*c*x + a*c)*sinh(b*c*
x + a*c) + sinh(b*c*x + a*c)^2 + 1)) + sqrt(2)*log((3*(2*sqrt(2) + 3)*cosh
(b*c*x + a*c)^2 - 4*(3*sqrt(2) + 4)*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) +
3*(2*sqrt(2) + 3)*sinh(b*c*x + a*c)^2 + 2*sqrt(2) + 3)/(cosh(b*c*x + a*c)^
2 + sinh(b*c*x + a*c)^2 + 3)) + log(2*(cosh(b*c*x + a*c)^2 + sinh(b*c*x +
a*c)^2 + 3)/(cosh(b*c*x + a*c)^2 - 2*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) +
 sinh(b*c*x + a*c)^2)))/(b*c)
```

3.151.6 Sympy [F]

$$\int e^{c(a+bx)} \arctan(\operatorname{sech}(ac+bcx))\,dx = e^{ac} \int e^{bcx} \operatorname{atan}(\operatorname{sech}(ac+bcx))\,dx$$

input
```
integrate(exp(c*(b*x+a))*atan(sech(b*c*x+a*c)),x)
```

output
```
exp(a*c)*Integral(exp(b*c*x)*atan(sech(a*c + b*c*x)), x)
```

3.151.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.64

$$\int e^{c(a+bx)} \arctan(\operatorname{sech}(ac+bcx))\,dx = \frac{\arctan(\operatorname{sech}(bcx+ac))\,e^{((bx+a)c)}}{bc} - \frac{3\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(2\,bcx+2\,ac)}-3}{2\sqrt{2}+e^{(2\,bcx+2\,ac)}+3}\right)}{8\,bc} + \frac{\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(-2\,bcx-2\,ac)}-3}{2\sqrt{2}+e^{(-2\,bcx-2\,ac)}+3}\right)}{8\,bc} + \frac{\log\left(e^{(4\,bcx+4\,ac)}+6\,e^{(2\,bcx+2\,ac)}+1\right)}{2\,bc}$$

input
```
integrate(exp(c*(b*x+a))*arctan(sech(b*c*x+a*c)),x, algorithm="maxima")
```

output
```
arctan(sech(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) - 3/8*sqrt(2)*log(-(2*sqrt
(2) - e^(2*b*c*x + 2*a*c) - 3)/(2*sqrt(2) + e^(2*b*c*x + 2*a*c) + 3))/(b*c
) + 1/8*sqrt(2)*log(-(2*sqrt(2) - e^(-2*b*c*x - 2*a*c) - 3)/(2*sqrt(2) + e
^(-2*b*c*x - 2*a*c) + 3))/(b*c) + 1/2*log(e^(4*b*c*x + 4*a*c) + 6*e^(2*b*c
*x + 2*a*c) + 1)/(b*c)
```

3.151.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.50

$$\int e^{c(a+bx)}\arctan(\operatorname{sech}(ac+bcx))\,dx = -\frac{\left(\sqrt{2}e^{(-ac)}\log\left(-\frac{2\sqrt{2}e^{(2\,ac)}-e^{(2\,bcx+4\,ac)}-3\,e^{(2\,ac)}}{2\sqrt{2}e^{(2\,ac)}+e^{(2\,bcx+4\,ac)}+3\,e^{(2\,ac)}}\right)-2\,\arctan\left(\frac{2}{e^{(bcx+ac)}+e^{(-bcx-ac)}}\right)e^{(bcx)}-e^{(-ac)}\log\left(e^{(4\,bcx+4\,ac)}\right.\right.}{2\,bc}$$

input
```
integrate(exp(c*(b*x+a))*arctan(sech(b*c*x+a*c)),x, algorithm="giac")
```

output
```
-1/2*(sqrt(2)*e^(-a*c)*log(-(2*sqrt(2)*e^(2*a*c) - e^(2*b*c*x + 4*a*c) - 3
*e^(2*a*c))/(2*sqrt(2)*e^(2*a*c) + e^(2*b*c*x + 4*a*c) + 3*e^(2*a*c))) - 2
*arctan(2/(e^(b*c*x + a*c) + e^(-b*c*x - a*c)))*e^(b*c*x) - e^(-a*c)*log(e
^(4*b*c*x + 4*a*c) + 6*e^(2*b*c*x + 2*a*c) + 1))*e^(a*c)/(b*c)
```

3.151.9 Mupad [B] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.31

$$\int e^{c(a+bx)}\arctan(\operatorname{sech}(ac+bcx))\,dx = \frac{\mathrm{e}^{a\,c+b\,c\,x}\,\mathrm{atan}\left(\frac{1}{\frac{\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}}{2}+\frac{\mathrm{e}^{-b\,c\,x}\,\mathrm{e}^{-a\,c}}{2}}\right)}{b\,c} + \frac{\ln\left(8\,\mathrm{e}^{2\,c\,(a+b\,x)}-2\,\sqrt{2}-6\,\sqrt{2}\,\mathrm{e}^{2\,c\,(a+b\,x)}\right)\,\left(\sqrt{2}+1\right)}{2\,b\,c} - \frac{\ln\left(8\,\mathrm{e}^{2\,c\,(a+b\,x)}+2\,\sqrt{2}+6\,\sqrt{2}\,\mathrm{e}^{2\,c\,(a+b\,x)}\right)\,\left(\sqrt{2}-1\right)}{2\,b\,c}$$

input
```
int(atan(1/cosh(a*c + b*c*x))*exp(c*(a + b*x)),x)
```

output
```
(exp(a*c + b*c*x)*atan(1/((exp(b*c*x)*exp(a*c))/2 + (exp(-b*c*x)*exp(-a*c)
)/2)))/(b*c) + (log(8*exp(2*c*(a + b*x)) - 2*2^(1/2) - 6*2^(1/2)*exp(2*c*(
a + b*x)))*(2^(1/2) + 1))/(2*b*c) - (log(8*exp(2*c*(a + b*x)) + 2*2^(1/2)
+ 6*2^(1/2)*exp(2*c*(a + b*x)))*(2^(1/2) - 1))/(2*b*c)
```

3.151. $\int e^{c(a+bx)} \arctan(\operatorname{sech}(ac+bcx))\,dx$

3.152 $\int e^{c(a+bx)} \arctan(\mathbf{csch}(ac+bcx))\,dx$

3.152.1 Optimal result

Integrand size = 20, antiderivative size = 47

$$\int e^{c(a+bx)} \arctan(\operatorname{csch}(ac+bcx))\,dx = \frac{e^{ac+bcx}\arctan(\operatorname{csch}(c(a+bx)))}{bc} + \frac{\log\left(1+e^{2c(a+bx)}\right)}{bc}$$

output
```
exp(b*c*x+a*c)*arctan(csch(c*(b*x+a)))/b/c+ln(1+exp(2*c*(b*x+a)))/b/c
```

3.152.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.21

$$\int e^{c(a+bx)} \arctan(\operatorname{csch}(ac+bcx))\,dx = \frac{e^{c(a+bx)}\arctan\left(\frac{2e^{c(a+bx)}}{-1+e^{2c(a+bx)}}\right) + \log\left(1+e^{2c(a+bx)}\right)}{bc}$$

input
```
Integrate[E^(c*(a + b*x))*ArcTan[Csch[a*c + b*c*x]],x]
```

output
```
(E^(c*(a + b*x))*ArcTan[(2*E^(c*(a + b*x)))/(-1 + E^(2*c*(a + b*x)))] + Lo
g[1 + E^(2*c*(a + b*x))])/(b*c)
```

3.152.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {7281, 5730, 25, 2720, 27, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c(a+bx)} \arctan(\operatorname{csch}(ac+bcx))\,dx$$

$$\Big\downarrow \text{7281}$$

$$\frac{\int e^{ac+bxc}\arctan(\operatorname{csch}(ac+bxc))d(ac+bxc)}{bc}$$

$$\Big\downarrow \text{5730}$$

$$\frac{e^{ac+bcx}\arctan(\operatorname{csch}(ac+bcx))-\int -e^{ac+bxc}\operatorname{sech}(ac+bxc)d(ac+bxc)}{bc}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int e^{ac+bxc}\operatorname{sech}(ac+bxc)d(ac+bxc)+e^{ac+bcx}\arctan(\operatorname{csch}(ac+bcx))}{bc}$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{2e^{ac+bxc}}{1+e^{2ac+2bxc}}de^{ac+bxc}+e^{ac+bcx}\arctan(\operatorname{csch}(ac+bcx))}{bc}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{e^{ac+bxc}}{1+e^{2ac+2bxc}}de^{ac+bxc}+e^{ac+bcx}\arctan(\operatorname{csch}(ac+bcx))}{bc}$$

$$\Big\downarrow \text{240}$$

$$\frac{e^{ac+bcx}\arctan(\operatorname{csch}(ac+bcx))+\log\left(e^{2ac+2bcx}+1\right)}{bc}$$

input
```
Int[E^(c*(a + b*x))*ArcTan[Csch[a*c + b*c*x]],x]
```

output
```
(E^(a*c + b*c*x)*ArcTan[Csch[a*c + b*c*x]] + Log[1 + E^(2*a*c + 2*b*c*x)])
/(b*c)
```

3.152.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 5730
```
Int[((a_.) + ArcTan[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Simp[(a + b*ArcTan[u])   w, x] - Simp[b   Int[SimplifyIntegrand[w*(D[u, x]
/(1 + u^2)), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x]
&& InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; Fre
eQ[{c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTan[u]), x]]
```

rule 7281
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLinear[u, x]}, Simp[1/lst[[3]]
 Subst[Int[lst[[1]], x], x, lst[[2]] + lst[[3]]*x], x] /;  !FalseQ[lst]]
```

3.152.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.02 (sec) , antiderivative size = 885, normalized size of antiderivative = 18.83

method	result	size
risch	Expression too large to display	885

input
```
int(exp(c*(b*x+a))*arctan(csch(b*c*x+a*c)),x,method=_RETURNVERBOSE)
```

output
```
-I/b/c*exp(c*(b*x+a))*ln(exp(c*(b*x+a))+I)-1/4/b/c*Pi*csgn(I/(exp(2*c*(b*x
+a))-1))*csgn(I*(exp(c*(b*x+a))-I)^2/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a)
)+1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))-I)^2)*csgn(I/(exp(2*c*(b*x+a))-1))*csg
n(I*(exp(c*(b*x+a))-I)^2/(exp(2*c*(b*x+a))-1))*exp(c*(b*x+a))+1/4/b/c*Pi*c
sgn(I/(exp(2*c*(b*x+a))-1))*csgn(I*(exp(c*(b*x+a))+I)^2/(exp(2*c*(b*x+a))-
1))^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))+I)^2)*csgn(I/(exp(2
*c*(b*x+a))-1))*csgn(I*(exp(c*(b*x+a))+I)^2/(exp(2*c*(b*x+a))-1))*exp(c*(b
*x+a))+1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))-I)^2/(exp(2*c*(b*x+a))-1))^3*exp(
c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))-I)^2)*csgn(I*(exp(c*(b*x+a))-
I)^2/(exp(2*c*(b*x+a))-1))^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(exp(c*(b*x+
a))-I)^2)^3*exp(c*(b*x+a))-1/2/b/c*Pi*csgn(I*(exp(c*(b*x+a))-I))*csgn(I*(e
xp(c*(b*x+a))-I)^2)^2*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))-I))
^2*csgn(I*(exp(c*(b*x+a))-I)^2)*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(c*(b
*x+a))+I)^2/(exp(2*c*(b*x+a))-1))^3*exp(c*(b*x+a))+1/4/b/c*Pi*csgn(I*(exp(
c*(b*x+a))+I)^2)*csgn(I*(exp(c*(b*x+a))+I)^2/(exp(2*c*(b*x+a))-1))^2*exp(c
*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))+I))^2*csgn(I*(exp(c*(b*x+a))+I
)^2)*exp(c*(b*x+a))+1/2/b/c*Pi*csgn(I*(exp(c*(b*x+a))+I))*csgn(I*(exp(c*(b
*x+a))+I)^2)^2*exp(c*(b*x+a))-1/4/b/c*Pi*csgn(I*(exp(c*(b*x+a))+I)^2)^3*ex
p(c*(b*x+a))-2*a/b+I/b/c*exp(c*(b*x+a))*ln(exp(c*(b*x+a))-I)+ln(1+exp(2*c*
(b*x+a)))/b/c
```

3.152.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(45) = 90$.

Time = 0.27 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.79

$$\int e^{c(a+bx)} \arctan(\text{csch}(ac+bcx))\,dx$$
$$= \frac{(\cosh(bcx+ac)+\sinh(bcx+ac))\arctan\left(\frac{2(\cosh(bcx+ac)+\sinh(bcx+ac))}{\cosh(bcx+ac)^2+2\cosh(bcx+ac)\sinh(bcx+ac)+\sinh(bcx+ac)^2-1}\right)+\log\left(\frac{}{\cosh(}\right.}{bc}$$

input
```
integrate(exp(c*(b*x+a))*arctan(csch(b*c*x+a*c)),x, algorithm="fricas")
```

output
```
((cosh(b*c*x + a*c) + sinh(b*c*x + a*c))*arctan(2*(cosh(b*c*x + a*c) + sin
h(b*c*x + a*c))/(cosh(b*c*x + a*c)^2 + 2*cosh(b*c*x + a*c)*sinh(b*c*x + a*
c) + sinh(b*c*x + a*c)^2 - 1)) + log(2*cosh(b*c*x + a*c)/(cosh(b*c*x + a*c
) - sinh(b*c*x + a*c))))/(b*c)
```

3.152.6 Sympy [F]

$$\int e^{c(a+bx)}\arctan(\operatorname{csch}(ac+bcx))\,dx = e^{ac}\int e^{bcx}\operatorname{atan}\left(\operatorname{csch}\left(ac+bcx\right)\right)dx$$

input
```
integrate(exp(c*(b*x+a))*atan(csch(b*c*x+a*c)),x)
```

output
```
exp(a*c)*Integral(exp(b*c*x)*atan(csch(a*c + b*c*x)), x)
```

3.152.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int e^{c(a+bx)}\arctan(\operatorname{csch}(ac+bcx))\,dx = \frac{\arctan\left(\operatorname{csch}\left(bcx+ac\right)\right)e^{\left((bx+a)c\right)}}{bc} + \frac{\log\left(e^{\left(2\,bcx+2\,ac\right)}+1\right)}{bc}$$

input
```
integrate(exp(c*(b*x+a))*arctan(csch(b*c*x+a*c)),x, algorithm="maxima")
```

output
```
arctan(csch(b*c*x + a*c))*e^((b*x + a)*c)/(b*c) + log(e^(2*b*c*x + 2*a*c)
+ 1)/(b*c)
```

3.152.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.40

$$\int e^{c(a+bx)}\arctan(\operatorname{csch}(ac+bcx))\,dx = \frac{\left(\arctan\left(\frac{2}{e^{(bcx+ac)}-e^{(-bcx-ac)}}\right)e^{(bcx)}+e^{(-ac)}\log\left(e^{(2\,bcx+2\,ac)}+1\right)\right)e^{(ac)}}{bc}$$

input
```
integrate(exp(c*(b*x+a))*arctan(csch(b*c*x+a*c)),x, algorithm="giac")
```

output
```
(arctan(2/(e^(b*c*x + a*c) - e^(-b*c*x - a*c)))*e^(b*c*x) + e^(-a*c)*log(e
^(2*b*c*x + 2*a*c) + 1))*e^(a*c)/(b*c)
```

3.152.9 Mupad [B] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.43

$$\int e^{c(a+bx)} \arctan(\operatorname{csch}(ac+bcx))\,dx = \frac{\ln\left(\mathrm{e}^{2\,b\,c\,x}\,\mathrm{e}^{2\,a\,c}+1\right)}{b\,c} + \frac{\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}\,\mathrm{atan}\left(\frac{1}{\frac{\mathrm{e}^{b\,c\,x}\,\mathrm{e}^{a\,c}}{2}-\frac{\mathrm{e}^{-b\,c\,x}\,\mathrm{e}^{-a\,c}}{2}}\right)}{b\,c}$$

input
```
int(atan(1/sinh(a*c + b*c*x))*exp(c*(a + b*x)),x)
```

output
```
log(exp(2*b*c*x)*exp(2*a*c) + 1)/(b*c) + (exp(b*c*x)*exp(a*c)*atan(1/((exp
(b*c*x)*exp(a*c))/2 - (exp(-b*c*x)*exp(-a*c))/2)))/(b*c)
```

3.153 $\int \frac{(a+b\arctan(cx^n))(d+e\log(fx^m))}{x}\,dx$

3.153.1 Optimal result

Integrand size = 24, antiderivative size = 163

$$\int \frac{(a+b\arctan(cx^n))\,(d+e\log(fx^m))}{x}\,dx = ad\log(x)+\frac{ae\log^2(fx^m)}{2m}+\frac{ibd\,\text{PolyLog}(2,-icx^n)}{2n}+\frac{ibe\log(fx^m)\,\text{PolyLog}(2,-icx^n)}{2n} - \frac{ibd\,\text{PolyLog}(2,icx^n)}{2n}-\frac{ibe\log(fx^m)\,\text{PolyLog}(2,icx^n)}{2n} - \frac{ibem\,\text{PolyLog}(3,-icx^n)}{2n^2}+\frac{ibem\,\text{PolyLog}(3,icx^n)}{2n^2}$$

output
```
a*d*ln(x)+1/2*a*e*ln(f*x^m)^2/m+1/2*I*b*d*polylog(2,-I*c*x^n)/n+1/2*I*b*e*
ln(f*x^m)*polylog(2,-I*c*x^n)/n-1/2*I*b*d*polylog(2,I*c*x^n)/n-1/2*I*b*e*l
n(f*x^m)*polylog(2,I*c*x^n)/n-1/2*I*b*e*m*polylog(3,-I*c*x^n)/n^2+1/2*I*b*
e*m*polylog(3,I*c*x^n)/n^2
```

3.153.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.20 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.71

$$\int \frac{(a+b\arctan(cx^n))(d+e\log(fx^m))}{x}\,dx$$
$$= -\frac{bcemx^n\,{}_4F_3\left(\frac{1}{2},\frac{1}{2},\frac{1}{2},1;\frac{3}{2},\frac{3}{2},\frac{3}{2};-c^2x^{2n}\right)}{n^2} + \frac{bcx^n\,{}_3F_2\left(\frac{1}{2},\frac{1}{2},1;\frac{3}{2},\frac{3}{2};-c^2x^{2n}\right)(d+e\log(fx^m))}{n} + \frac{1}{2}a\log(x)\left(2d-em\log(x)+2e\log(fx^m)\right)$$

input
```
Integrate[((a + b*ArcTan[c*x^n])*(d + e*Log[f*x^m]))/x,x]
```

output
```
-((b*c*e*m*x^n*HypergeometricPFQ[{1/2, 1/2, 1/2, 1}, {3/2, 3/2, 3/2}, -(c^
2*x^(2*n))])/n^2) + (b*c*x^n*HypergeometricPFQ[{1/2, 1/2, 1}, {3/2, 3/2},
-(c^2*x^(2*n))]*(d + e*Log[f*x^m]))/n + (a*Log[x]*(2*d - e*m*Log[x] + 2*e*
Log[f*x^m]))/2
```

3.153.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx^n))(d+e\log(fx^m))}{x}\,dx$$
$$\Big\downarrow \; 7293$$
$$\int \left(\frac{d(a+b\arctan(cx^n))}{x}+\frac{e\log(fx^m)(a+b\arctan(cx^n))}{x}\right)dx$$
$$\Big\downarrow \; 2009$$
$$ad\log(x)+\frac{ae\log^2(fx^m)}{2m}+\frac{ibd\,\text{PolyLog}(2,-icx^n)}{2n}-\frac{ibd\,\text{PolyLog}(2,icx^n)}{2n}+\frac{ibe\,\text{PolyLog}(2,-icx^n)\log(fx^m)}{2n}-\frac{ibe\,\text{PolyLog}(2,icx^n)\log(fx^m)}{2n}-\frac{ibem\,\text{PolyLog}(3,-icx^n)}{2n^2}+\frac{ibem\,\text{PolyLog}(3,icx^n)}{2n^2}$$

input
```
Int[((a + b*ArcTan[c*x^n])*(d + e*Log[f*x^m]))/x,x]
```

output
```
a*d*Log[x] + (a*e*Log[f*x^m]^2)/(2*m) + ((I/2)*b*d*PolyLog[2, (-I)*c*x^n])
/n + ((I/2)*b*e*Log[f*x^m]*PolyLog[2, (-I)*c*x^n])/n - ((I/2)*b*d*PolyLog[
2, I*c*x^n])/n - ((I/2)*b*e*Log[f*x^m]*PolyLog[2, I*c*x^n])/n - ((I/2)*b*e
*m*PolyLog[3, (-I)*c*x^n])/n^2 + ((I/2)*b*e*m*PolyLog[3, I*c*x^n])/n^2
```

3.153.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

3.153.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 212.62 (sec) , antiderivative size = 547, normalized size of antiderivative = 3.36

method	result
risch	$\frac{\left(-\frac{ie\pi\ \mathrm{csgn}(if)\,\mathrm{csgn}(ix^{m})\,\mathrm{csgn}(if\,x^{m})}{4}+\frac{ie\pi\ \mathrm{csgn}(if)\,\mathrm{csgn}(if\,x^{m})^{2}}{4}+\frac{ie\pi\ \mathrm{csgn}(ix^{m})\,\mathrm{csgn}(if\,x^{m})^{2}}{4}-\frac{ie\pi\,\mathrm{csgn}(if\,x^{m})^{3}}{4}+\frac{e\ln(f)}{2}+\frac{d}{2}\right)(-ib\,\mathrm{dilog}}{n}$

input
```
int((a+b*arctan(c*x^n))*(d+e*ln(f*x^m))/x,x,method=_RETURNVERBOSE)
```

output
```
(-1/4*I*e*Pi*csgn(I*f)*csgn(I*x^m)*csgn(I*f*x^m)+1/4*I*e*Pi*csgn(I*f)*csgn
(I*f*x^m)^2+1/4*I*e*Pi*csgn(I*x^m)*csgn(I*f*x^m)^2-1/4*I*e*Pi*csgn(I*f*x^m
)^3+1/2*e*ln(f)+1/2*d)/n*(-I*b*dilog(1-I*c*x^n)+2*ln(x^n)*a+I*b*dilog(1+I*
c*x^n))+1/2*I*e*b*ln(x)^2*ln(-I*(c*x^n+I))*m+1/2*I*b*e*m*polylog(3,I*c*x^n
)/n^2-1/2*I*e*b/n*dilog(-I*(c*x^n+I))*ln(x^m)-1/2*I*e*b*ln(x)^2*ln(-I*(-c*
x^n+I))*m+1/2*I*e*b*ln(x)*ln(1-I*c*x^n)*ln(x^m)+1/2*I*e*b*m/n*ln(x)*polylo
g(2,-I*c*x^n)-1/2*I*e*b/n*dilog(-I*c*x^n)*ln(x^m)-1/2*I*e*b/n*ln(-I*(-c*x^
n+I))*ln(-I*c*x^n)*ln(x^m)+1/2*e*a/m*ln(x^m)^2-1/2*I*e*b*ln(x)*ln(x^m)*ln(
1+I*c*x^n)-1/2*I*e*b*ln(x)^2*ln(1-I*c*x^n)*m+1/2*I*e*b*ln(x)^2*ln(1+I*c*x^
n)*m+1/2*I*e*b/n*ln(-I*(-c*x^n+I))*ln(-I*c*x^n)*m*ln(x)-1/2*I*e*b*ln(x)*ln
(-I*(c*x^n+I))*ln(x^m)+1/2*I*e*b/n*dilog(-I*(c*x^n+I))*m*ln(x)-1/2*I*b*e*m
*polylog(3,-I*c*x^n)/n^2-1/2*I*e*b*m/n*ln(x)*polylog(2,I*c*x^n)+1/2*I*e*b/
n*dilog(-I*c*x^n)*m*ln(x)+1/2*I*e*b*ln(x)*ln(x^m)*ln(-I*(-c*x^n+I))
```

3.153.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 250 vs. 2(121) = 242.

Time = 0.29 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.53

$$\int \frac{(a + b\arctan(cx^n))(d + e\log(fx^m))}{x}\,dx$$
$$= \frac{2\,aemn^2\log(x)^2 + 2i\,bem\text{polylog}(3, i\,cx^n) - 2i\,bem\text{polylog}(3, -i\,cx^n) + 2\left(bemn^2\log(x)^2 + 2\left(ben^2\log(\right.\right.}{}$$

input
```
integrate((a+b*arctan(c*x^n))*(d+e*log(f*x^m))/x,x, algorithm="fricas")
```

output
```
1/4*(2*a*e*m*n^2*log(x)^2 + 2*I*b*e*m*polylog(3, I*c*x^n) - 2*I*b*e*m*poly
log(3, -I*c*x^n) + 2*(b*e*m*n^2*log(x)^2 + 2*(b*e*n^2*log(f) + b*d*n^2)*lo
g(x))*arctan(c*x^n) - 2*(I*b*e*m*n*log(x) + I*b*e*n*log(f) + I*b*d*n)*dilo
g(I*c*x^n) - 2*(-I*b*e*m*n*log(x) - I*b*e*n*log(f) - I*b*d*n)*dilog(-I*c*x
^n) + (I*b*e*m*n^2*log(x)^2 - 2*(-I*b*e*n^2*log(f) - I*b*d*n^2)*log(x))*lo
g(I*c*x^n + 1) + (-I*b*e*m*n^2*log(x)^2 - 2*(I*b*e*n^2*log(f) + I*b*d*n^2)
*log(x))*log(-I*c*x^n + 1) + 4*(a*e*n^2*log(f) + a*d*n^2)*log(x))/n^2
```

3.153.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx^n))\,(d+e\log(fx^m))}{x}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x**n))*(d+e*ln(f*x**m))/x,x)
```

output
```
Timed out
```

3.153.7 Maxima [F]

$$\int \frac{(a+b\arctan(cx^n))\,(d+e\log(fx^m))}{x}\,dx = \int \frac{(b\arctan(cx^n)+a)(e\log(fx^m)+d)}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x^n))*(d+e*log(f*x^m))/x,x, algorithm="maxima")
```

output
```
1/2*a*e*log(f*x^m)^2/m + a*d*log(x) - 1/2*(b*e*m*log(x)^2 - 2*b*e*log(x)*l
og(x^m) - 2*(b*e*log(f) + b*d)*log(x))*arctan(c*x^n) - integrate(-1/2*(b*c
*e*m*n*x^n*log(x)^2 - 2*b*c*e*n*x^n*log(x)*log(x^m) - 2*(b*c*e*log(f) + b*
c*d)*n*x^n*log(x))/(c^2*x*x^(2*n) + x), x)
```

3.153.8 Giac [F]

$$\int \frac{(a+b\arctan(cx^n))\,(d+e\log(fx^m))}{x}\,dx = \int \frac{(b\arctan(cx^n)+a)(e\log(fx^m)+d)}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x^n))*(d+e*log(f*x^m))/x,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x^n) + a)*(e*log(f*x^m) + d)/x, x)
```

3.153.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx^n))(d+e\log(fx^m))}{x}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x^n))\,(d+e\,\ln(f\,x^m))}{x}\,dx$$

input
```
int(((a + b*atan(c*x^n))*(d + e*log(f*x^m)))/x,x)
```

output
```
int(((a + b*atan(c*x^n))*(d + e*log(f*x^m)))/x, x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                               convert(leaf_count_result,string)," vs. $2 (",
                               convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```python
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
        return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
return grade, grade_annotation
```